RESEARCH PROJECT

Finding a Path for Equitable Mathematical Student Discourse: Targeted Universalism Meets Human-Centered Design

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Amplify.

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Profound things happen when one human being lets another human being know that their voice matters. For students, the more they feel their voice is heard, the greater their opportunity to learn. These sentiments were the driving force for the project team that created the prototype for the 5 Practices Pathfinder, a teacher-facing app designed to support the implementation of the 5 Practices for Orchestrating Productive Mathematics Discussions (Smith & Stein, 2011).

The 5 Practices encourage teachers to set a learning goal and choose a high-level mathematical task that aligns with the goal, *Anticipate* student strategies, *Monitor* as students work in groups, *Select* and *Sequence* strategies in a coherent manner, and facilitate a discussion to Connect the strategies and ideas in a way that helps students understand the mathematics.

As a pedagogical strategy for supporting student voice, the 5 Practices are well-regarded in the world of mathematics education as being beneficial for all students. Student discourse and increased accountable student talk have been shown to build conceptual understanding in mathematics (Resnick, 1999; Walshaw & Anthony, 2008) and the 5 Practices book is the National Council of Teachers of Mathematics' bestselling book (NCTM, 2020).

When it comes to implementation, however, teachers experience challenges using the 5 Practices in their classrooms (Smith & Sherin, 2019). These challenges can specifically affect Black and Latinx students, who are disproportionately more likely than white students to attend mathematics classrooms that have fewer opportunities for rich mathematical discourse and are more likely than white students to engage in learning that is rote-oriented and remedial (Delpit, 2012; Noguera, Darling-Hammond, & Friedlaender, 2015; TNTP, 2018). At the same time, research has shown that these priority students[1] benefit from discourse-focused mathematics instruction, but that these assets are currently under-leveraged in most classrooms (Archer, 1993; Darragh, 2016; Gray, Hope, & Matthews, 2018).

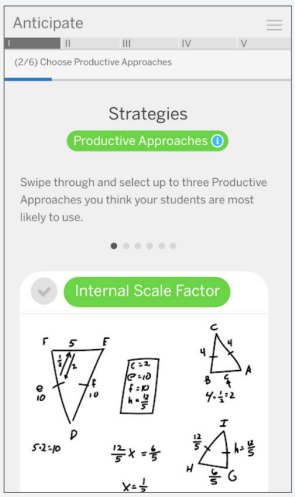
Through conversations with 5 Practices creator Margaret (Peg) Smith and colleagues, Amplify learned that there has been academic and professional interest in developing a digital solution for supporting and streamlining teachers' use of the 5 Practices in the hopes of overcoming these challenges. Through a partnership with the Bill & Melinda Gates Foundation, the project team set out to create a proof of concept prototype that a digital tool could meet the unique needs of teachers of both priority student and general student populations in using the 5 Practices. The findings and conclusions of the project are those of the authors and do not necessarily reflect positions or policies of the Gates Foundation.

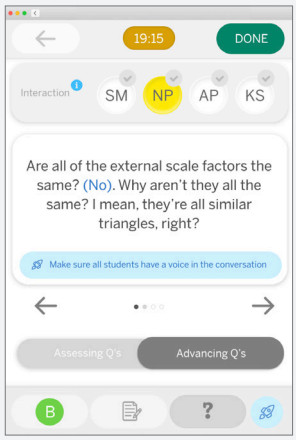
The result of this endeavor is the freely available prototype for the 5 Practices Pathfinder app, hereafter referred to as the *prototype*. This paper captures the structure and learnings of the project that created the prototype. First, the purpose of the project, including the problems and solution as the project team conceptualized them, are detailed. Second, the 5 Practices and the associated challenges teachers face while implementing them are discussed. Third, the theories and concepts driving the prototype's design are considered. Fourth, the design principles and process are explained. Fifth, the research methodology is described. Sixth, the main section of the paper discusses the key themes and features that emerged in the project and how the design of the prototype evolved over the course of the project. Last, implications and details for next steps in this work are shared. The project team hopes that product designers and managers, curriculum writers, and mathematics educators and supervisors find this information useful for their future endeavors.

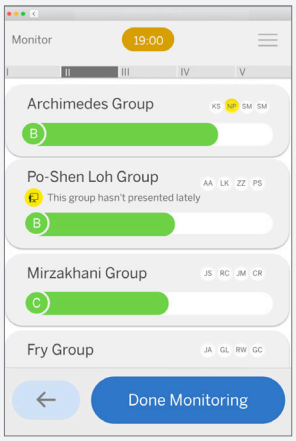
1 The project team chooses to call Black and Latinx students "priority students" over other wording commonly used such as "minority students," "underrepresented students," or "underachieving students," as such labels systemically denote a hierarchical structure in which this population of students find themselves on the bottom.

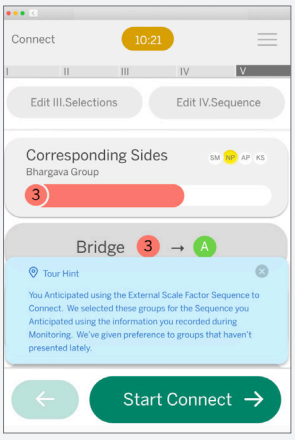
How to explore the 5 Practices Pathfinder prototype

Visit **https://five-practices.prod.learning.amplify.com**









- Take on the mindset of a teacher planning and teaching with the 5 Practices for Orchestrating Productive Mathematics Discussions.
- The prototype is designed for tablets.
- If exploring on a phone or computer, note:
  - The prototype may not display properly screens smaller or larger than tablet size.
  - When viewing on a computer without a touchscreen, you will not be able to swipe left-right or up-down; at those moments, you can click and drag to explore those features.
- Since it is a prototype, not all buttons will work.
- You will see the names of sample students with which to explore the app.
- You have been jumped ahead in time so that you can see some information about the students that you can imagine you have collected during prior lessons.
  - For example, keep an eye out for student initials highlighted in yellow—these are students you have not noted as recently interacting with classmates, teacher, and or the math.

Purpose

The purpose of this project was to create a digital tool to support teachers in operationalizing the pedagogical model of the 5 Practices. Teachers have created and used a variety of paper-based *Monitoring* tools for *Anticipating* strategies, tracking which students are trying which strategy, and identifying the order for students to share their strategies. While we began the project with the idea that technology could overcome 5 Practices challenges in a way that paper-based tools could not, creating a digital tool for the 5 Practices that increases equitable student discourse was not as simple as taking a useful paper-and-pencil instructional material and adding some technology to it.

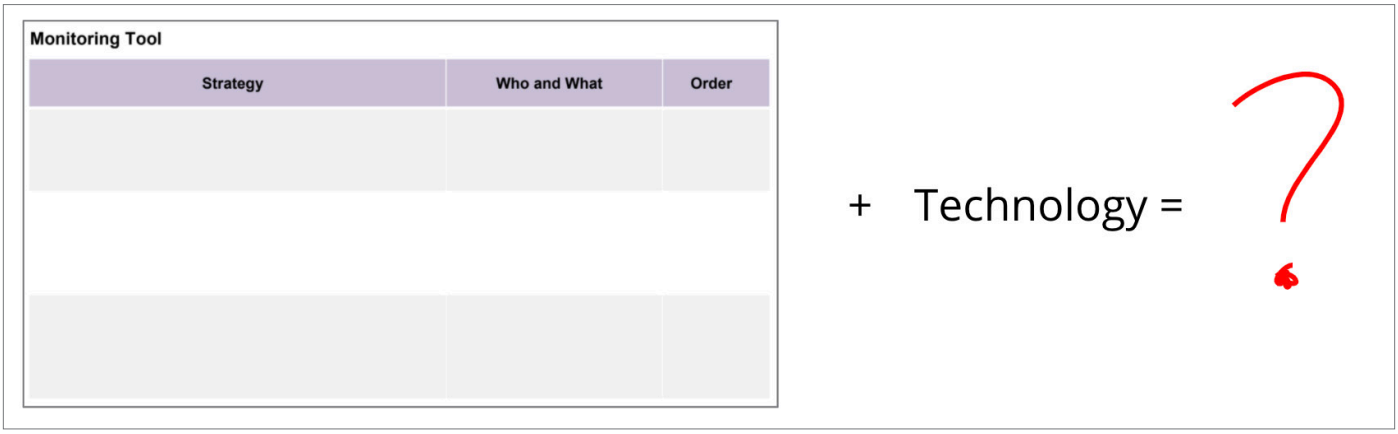


Figure 1: *Monitoring* tool courtesy of Peg Smith

The project team hypothesized that *creating a useful digital teacher-facing tool for the 5 Practices would increase teachers' self-efficacy with the 5 Practices. With this digital teacher-facing tool, teachers of Black and Latinx students would perceive the 5 Practices as being less challenging, and this work would benefit teachers of all students*. The project team studied this hypothesis through the theory of Targeted Universalism tested with Human-Centered Design and design thinking protocols, centering the design of the prototype on ongoing mixed methods research with teachers of priority students. The key idea of Targeted Universalism is that better understanding the unmet needs for priority users may provide insights that improve usability and value-add for all, while Human-Centered Design calls for starting product design with understanding the needs of the users you are creating for and ensuring that the ongoing design process is continually based on users' perspectives.

The six-month project for creating a proof-of-concept prototype ran from April to October 2020. Ongoing mixed methods research with a six-member Teacher Advisory Board (TAB) and advisors informed the creation of the digital solution. The high-quality instructional material (HQIM) and open education resource Illustrative Mathematics (IM) curriculum was used as a basis for the instructional materials. The goals of the project were to (1) create a proof-of-concept prototype of a digital tool that supports the implementation of the 5 Practices with the IM curriculum, (2) solve the top challenges teachers of priority students face while implementing the 5 Practices, and (3) incorporate Targeted Universalism with Human-Centered Design to include equity in the design process.

The Problem: A Lack of Equitable Discourse

Remember your middle school mathematics class? Unless you were one of the lucky ones, most of us sat in tidy rows listening to the teacher talking, possibly thinking, "Wouldn't this be easier if I could somehow take all of the teacher's knowledge and just put it in my brain?" It was hard to realize then, but looking back, this thought would have been a manifestation of a mathematics classroom where it's the job of teachers to lecture and students to take notes—where students are seen as empty vessels into which teachers pour knowledge. However, research states this is not predominantly how people learn (Adams, 2006; Powell & Kalina, 2009).

Mathematical discourse has been identified as critical to transformational student-centered pedagogy, and it was therefore at the heart of the purpose of this project (NCTM, 2014; Smith & Stein, 2011). Generally, discourse in the classroom refers to the socio-cultural language structure and interactions that teachers and students use to communicate thoughts via words, symbols, or verbalizations while interacting with each other, and is the basis of the meaning teachers and students make of social interactions, and is a central contributor to individual and group identity (Cazden, 2001). Mathematical discourse is a subset of classroom discourse that specifically engages students and teacher in talking about mathematics to build understanding of procedures and fluency, conceptual understanding, and application (Gresham & Shannon, 2017). Research suggests that equitably using routines for mathematical discourse supports a collaborative environment where student thought, action, and language are used as a bridge to formalized mathematics (Zwiers et al., 2017).

Although this existing research has conceptualized how mathematics classrooms should look and feel in terms of equitable student discourse, accomplishing this can be difficult when it comes to teacher implementation. Transitioning to more student-centered models of discourse is at best, difficult, and at worst, fought against, in order to maintain the status quo of the traditional classroom (Nank, 2011b). Teachers do the best they can with what they have and what they need most is support in enacting a better way to encourage open discourse in mathematics classrooms—to help them wonder what students think, what they would say, and how they make sense of the beauty of mathematics that is intricately woven into the very fabric of our existence.

Implicit Bias

The barriers to equitable student voice via productive mathematical discourse are further exacerbated when implicit biases manifest in mathematics classrooms (Archer, 1993; Darling-Hammond, 2007; Gray, Hope, & Matthews, 2018; Khisty & Chval, 2002). Implicit bias is, well, implicit. By definition, it is difficult to track with 100 percent certainty that every student experiences equality of voice and interactions while uncovering and addressing implicit patterns of conversations in the classroom. Project advisor Michael D. Steele provides this example of implicit bias: "A teacher who feels a gravitational pull towards privileging voices that already have mathematical standing and authority." Although the teacher may not mean to invoke a hierarchical structure of value and equity in the classroom, they may value certain peoples' voices over others.' Even if a teacher wants to improve equity, access, social justice, and interactions in classrooms, uncovering implicit bias is like asking someone to reflect on the details of the air they've been breathing the last few moments. People need help seeing what they do not see and acknowledging what has always felt like it naturally belongs. Without some external factor or support, seeing one's own implicit bias can be near impossible.

From the beginning of this project, the project team acknowledged the importance of priority students especially in regard to teachers' implicit bias. The team defined *implicit bias* as the unconscious ways that attitudes, beliefs, stereotypes, and past experiences influence, in covert and overt ways, teachers' actions, intentions, and decisions in the classroom. The danger of implicit bias lies in the cultivation of damaging behaviors in the classroom that cumulatively and collectively occur without awareness, endorsement, or explicit intentionality (Holroyd, Scaife, & Stafford, 2017). The project team supported a revisionist perspective of moral responsibility. Revisionism assumes people are morally responsible when their implicit biases influence their actions and interactions (Glasgow, 2016). The responsibility to address implicit bias is exacerbated by the realization that actions that materialize from implicit biases harm more than just the perpetrator, having a spectrum of harmful outcomes for students exposed to the manifestations of implicit bias (Faucher, 2016).

High-Quality Instructional Materials

High-quality instructional materials (HQIM) were defined in this project as materials that aid teachers in building content knowledge while supporting teachers in lesson planning and suggested pedagogical routines that foster collaboration among teachers outside the classroom and among students inside the classroom. Unfortunately, HQIM are necessary but not sufficient conditions for stronger student learning as teachers must also be prepared to implement appropriate pedagogical strategies (NCTM, 2014). Therefore, a teacher using HQIM is not necessarily going to automatically increase the amount of equitable student discourse in their classroom.

At the same time, HQIMs can provide the basis for high-quality instruction. Since the 5 Practices are specifically designed as a pedagogical strategy aligned to rich mathematical tasks, the project team decided to use an existing HQIM for the tasks in the prototype, creating space in the project to focus on supporting teachers in using this HQIM to achieve equitable mathematical discourse. The basis for this decision lies in Stein and Lane's (1996) assertion that high-quality curricula, no matter the quality of implementation, results in gains in student performance and gains in student learning outcomes. The way to achieve the highest results in terms of equitable student discourse is to use high-quality tasks and to implement them at a high level; according to project advisor Peg Smith, "Students learn most when they have ongoing opportunities to engage with high-level tasks at high levels." If one starts with a low-level task, the bar has already been set low, thus ensuring low results.

Priority Students

Black, Latinx, and other priority students have yet to be supported in educational settings with true and meaningful equality of voice, perspective, discourse, and views on a systemic level (TNTP, 2018). Incorporating structures to provide a sense of belonging for priority students while providing equal opportunities for voice and access is desperately needed in classrooms and schools (Gray, Hope, & Matthews, 2018). High-quality discourse, if it occurs, is generally relegated to students of privilege, wealth, and of European descent as Black and Latinx students are more likely than their white counterparts to spend most of their time in school without access to grade-appropriate work, strong instruction, deep engagement, and teachers who hold high expectations (TNTP, 2018). A shift needs to occur so that Black and Latinx students have access to both high quality instructional materials that support discourse and pedagogical strategies that aid in such discourse (TNTP, 2018).

When we embrace voice and choice in classrooms, we create an environment where all students can thrive. Providing a voice to all students can increase achievement, foster interactions around mathematics, and uncover the richness and beauty inherent in mathematics that could not be seen without diverse perspectives and thoughts (Gresham & Shannon, 2017). As such, it's important to consider the strengths priority students bring to this area, in addition to needs, in order to combat deficit thinking. Deficit thinking is harmful to any student but can perpetuate inequity when teachers endorse and engage in deficit thinking and language more dominantly with priority students (Battey & Franke, 2015). Being culturally responsive to the strengths in the ways priority students may interact with mathematics enables teachers to create mathematics classrooms in which mathematics is experienced as problem-solving and ways to critique and understand the world, which helps priority students identify themselves as doers of mathematics (Thomas & Berry, 2019). When this is not done, it is a missed opportunity—both for the students being left out of discourse and because others do not benefit from their perspective.

A Digital Tool as a Solution

In light of these problems, the project team committed to the development of a digital prototype that builds on a HQIM and the structure provided by the 5 Practices to facilitate student-centered conversations around student thought, vocabulary, interpretations, and meaning (Smith & Stein, 2011). The culmination of the research into these problems resulted in a teacher-facing tool with features designed within four main themes: **support equity, suppor pedagogy and curricula, save teachers time,** and **enable teacher superpowers.** These features provide support for teachers to continually do and be better with the practices and challenges, thus supporting all students in actionable and meaningful ways.

For the purposes of testing proof of concept, the project team investigated the most appropriate form factor for the prototype with teachers of priority students. Of utmost importance was being able to access the prototype offline, given that online connectivity was reported to be sporadic at best in many of the schools. Therefore, offline caching was developed for the prototype so that teachers could access the features regardless of internet connection. In order to at minimum substitute in for the paper *Monitoring* tools teachers carried around on clipboards, the project team also knew that the prototype needed to be usable on a mobile device (Puentedura, 2013). Teachers of priority students expressed concerns about using personal mobile phones to access the prototype, due to both the size of the screen and school and district rules and norms regarding the use of mobile phones in classrooms. Therefore, the prototype was built for use on an iPad or tablet.

The Practices and Challenges

The 5 Practices

In the 1990's, with the exception of pockets in the educational system that were committed to research-based practices and had resources for professional learning for teachers, student discourse in mathematics classrooms was elusive. During this era, Silver, Smith, and Nelson (1995) set out to improve participation in mathematics classrooms through the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project. The culmination of their theory for facilitating meaningful discourse was the creation of the 5 Practices (Smith & Stein, 2011). The 5 Practices originated with five distinct Practices for facilitating discourse, with a practice 0 that was added slightly later at the behest of a teacher.

The Practices are:

0. *Setting Goals and* Selecting *a Task*: Specifying what you want students to learn about mathematics as a result of engaging in a particular lesson and identifying a high-level task that aligns with your goals and provides all students with access.
1. *Anticipating:* Carefully considering the strategies students are likely to use to approach or solve a challenging mathematical task; how to respond to the work that students are likely to produce; and which student strategies are likely to be most useful in addressing the mathematics to be learned.
2. *Monitoring:* Listening in on what students are saying and observing what they are doing; asking questions to assess what students understand and to advance them towards the goals of the lesson; and keeping track of the approaches that they are using.
3. *Selecting:* Determining what strategies—and what mathematics—will be the focus of the whole class discussion; choosing particular students to present because of the mathematics available in their responses; and making sure that all students have the opportunity to be seen as authors of mathematical ideas over time.
4. *Sequencing:* Purposefully ordering the solutions that will be presented; making the mathematics accessible to all students; and building a mathematically coherent storyline.
5. *Connecting:* Asking questions that link different solution strategies; asking questions that make the key ideas that are target in the lesson public; and making sure all students are making sense of the ideas (Smith & Stein, 2018).

The 5 Practices are ideal for facilitating high quality student discourse because it is a deliberate pedagogical structure and set of strategies for enabling teachers to move from models of teaching where the teacher is center stage, such as the show-and-tell model (Stein, Engle, Smith, & Hughes, 2013) or the see-this-do-that model (Nank, 2011b) towards a model of teaching that centers student discourse, thought, and mathematical reasoning. Building mathematical discourse in students increases student achievement and fosters greater conceptual understanding (Gresham & Shannon, 2017). If every lesson, regardless of the instructional approach taken, incorporated at the very least *Anticipating* student thinking and *Monitoring* for such thinking, classrooms would be more student-centered, elevating the value of every students' voice.

The 19 Challenges

Setting Goals & Selecting *a Task*, *Anticipating*, *Monitoring, Selecting, Sequencing*, and *Connecting* can be intimidating for teachers. *What if I Anticipate a strategy students do not use? What if I misinterpret a strategy while Monitoring? What if I Select the "wrong" group? What if my Sequence does not exactly capture the increasing sophistication of the ways students approached the task? What if I do not Connect all of the students' ideas in a coherent manner, thus leaving the students confused and frustrated?* These are only a small selection of the worries a teacher may face when using the 5 Practices to support student discourse.

The complexity of supporting student discourse is explained in the theory of *accountable talk*. Resnick (1999) states that accountable talk in classrooms embodies the belief that to sustain learning, discourse in classrooms should embrace evidence and vocabulary that are relevant to the subject matter while embracing socially negotiated and established norms of reasoning and communication. When this occurs, socializing intelligence reinforces expectations for the students and the teacher to expect all to take responsibility for their roles in teaching and learning.

Seeing that the complexity of a rich pedagogical undertaking such as mathematics discourse can be daunting (NCTM, 2014), Smith and Sherin (2019), with collaboration from Victoria Bill and Michael D. Steele and based on the group's collective experiences in working with teachers and the 5 Practices for the past decade, identified 19 Challenges teachers may encounter while implementing the 5 Practices in a classroom setting. These 19 Challenges impact every student's ability to learn mathematics with understanding. The 19 Challenges are detailed in Table 1.

Table 1: The 19 Challenges

0.	*Setting Goals and Selecting a Task*	
1	Identifying learning goals	Goal needs to focus on what students will learn as a result of engaging in the task, not on what students will do. Clarity on goals sets the stage for everything else (p. 21).
2	Identifying a doing-mathematics task	While *doing-mathematics* tasks provide the greatest opportunities for student learning, they are not readily available in some textbooks. Teachers may need to adapt an existing task, find a task in another resource, or create a task (p. 21).
3	Ensuring alignment between task and goals	Even with learning goals specified, teachers may select a task that does not allow students to make progress on those particular goals (p. 21).
4	Launching a task to ensure student access	Teachers need to provide access to the context and the mathematics in the launch but not so much that the mathematical demands are reduced and key ideas are *given away* (p. 21).
1.	*Anticipating Student Responses*	
5	Moving beyond the way *you* solve a problem	Teachers often feel limited by their own experiences. They know how to solve a task but may not have access to the array of strategies that students are likely to use (p. 48).
6	Being prepared to help students who cannot get started on a task	Teachers need to be prepared to provide support to students who do not know how to begin work on the task so that they can make progress without being told exactly what to do and how (p. 48).
7	Creating questions that move students toward the mathematical goals	The questions teachers ask need to be driven by the mathematical goals of the lesson. The focus needs to be on ensuring that students *understand* the key mathematical ideas, not just on producing a solution to the task (p. 48).

2. *Monitoring*		
8	Trying to understand what students are thinking	Students do not always articulate their thinking clearly. It can be quite demanding for teachers, in the moment, to figure out what a student means or is trying to say. This requires teachers to listen carefully to what students are saying and to ask questions that help them better explain what they are thinking (p. 79).
9	Keeping track of group progress—which groups you visited and what you left them to work on	As teachers are running from group to group, providing support, they need to be able to keep track of what each group is doing and what they left students to work on. Also, it is important for a teacher to return to a group to determine whether the advancing question given to them helped them make progress (p. 79).
10	Involving all members of a group	All individuals in the group need to be challenged to answer assessing and advancing questions. For individuals to benefit from the thinking of their peers, they need to be held accountable for listening to and adding on, repeating and summarizing what others are saying (p. 79).
3,4. *Selecting and Sequencing*		
11	*Selecting* only solutions that are most relevant to the learning goals	Teachers need to *Select* a limited number of solutions that will help achieve the mathematical goals of the lesson. Sharing solutions that are not directly relevant can take a discussion off track, and sharing too many solutions (even if they are relevant) can lead to student disengagement (p.111).
12	Expanding beyond the usual student presenters	Teachers often select students who are articulate and on whom they can count for a coherent explanation. Teachers need to look for opportunities to position each and every student as a presenter and help students develop their ability to explain their thinking (p.111).
13	Deciding what work to share when the majority of students were not able to solve the task and your initial goal no longer seems obtainable	Teachers may on occasion find that the task was too challenging for most students and that they were not able to engage as intended. This situation requires the teacher to modify her initial plan and determine how to focus the discussion so students can make progress (p.111).
14	Moving forward when a key strategy is not produced by students	In planning the lesson, a teacher may determine that a particular strategy is critical to accomplishing the lesson goals. If the success of a lesson hinges on the availability of a particular strategy, then the teacher needs to be prepared to introduce the strategy through some means (p.111).
15	Determining how to *Sequence* errors, misconceptions, and/or incomplete solutions	Teachers often choose not to share work that is not complete and correct for fear that students will remember incorrect methods. Sharing solutions that highlight key misconceptions in a domain can provide all students with an opportunity to analyze why a particular approach does not work. Sharing incomplete or partial solutions can provide all students with the opportunity to consider how such work can be *connected* to more robust solutions (p. 112).

5.	*Connecting*	
16	Keeping the entire class engaged and accountable during individual presentations	Often, the sharing of solutions turns into a *show and tell* or a dialogue between the teacher and the presenter. The rest of the class needs to be held accountable for understanding and making sense of the solutions that are presented (p. 144).
17	Ensuring key mathematical ideas are made public and remain the focus	It is possible to have students share and discuss a lot of interesting solutions and never get to the point of the lesson. It is critical that the key mathematical ideas that are being targeted in the lesson are explicitly discussed (p. 144).
18	Making sure that you do not take over the discussion and do the explaining	As students are presenting their solutions, the teacher needs to ask questions that engage the presenters and the rest of the class in explaining and making sense of the solutions. There is a temptation for the teacher to take over and tell the students what they need to know. When this happens, opportunities for learning are diminished. Remember whoever is doing the talking is doing the thinking (p. 144).
19	Running out of time	Teachers may not have enough time to conduct the whole class discussion the way they had planned it. In such cases it is important to come up with a *Plan B* that provides some closure to the lesson but does not turn into telling (p. 144).

(Smith & Sherin, 2019)

The 19 Challenges demonstrate that we are all complex beings with histories, meanings, and perspectives that are all pertinent to discourse, understanding, and learning. If done in a meaningful way, teaching is hard. Time, support, and a growth mindset (Boaler, 2016; Dweck, 2006) are needed from all members of the educational community as we realize a history of student silence in mathematics classrooms cannot be undone in a moment. The 19 Challenges provide specific sustained pedagogical strategies for promoting discourse in all classrooms for all students; therefore the project team decided to base our work with priority students on the challenges. First, the team needed to concretely understand the ways the challenges manifested with teachers of priority students through embracing the tenets of Targeted Universalism and Human-Centered Design.

Theoretical and Conceptual Frameworks

Targeted Universalism

Targeted Universalism, the theoretical framework of the project, posits that better understanding the unmet needs of targeted users can provide insights that improve the needs of all users. As such, it is an approach to equity both through the process of Targeted Universalism and in the outcomes of that process (Gates, IDEO, & Bellwether, 2020). It involves the setting of a universal goal, assessing the goal, identifying groups that are performing differently in regards to the universal goal, understanding why that difference is occurring, and designing targeted support strategies that should be highly effective for a target population and can also be used by all who have a vested interest in meeting the universal goal, creating a targeted approach universally (Gates, IDEO, & Bellwether, 2020; Wright, Price, & Anidi, 2018).

Using Targeted Universalism, the project team set the universal goal as facilitating equality of voice and opportunity to engage in high quality mathematical discourse. Black and Latinx students were the group of priority students identified through background research as having particular barriers and lack of opportunity to achieve that universal goal, although all students do face challenges and barriers to engaging in equitable discourse, as cited through research above (Gray, Hope, & Matthews, 2018; TNTP, 2018).

Targeted Universalism ensured that students who traditionally are not afforded equality of voice and considerations when designing curricula and educational technology products had voice and representation in this project. In this sense, all perspectives are represented in the goals and outcomes (Powell, Mendendian, & Ake, 2019). Giving marginalized stakeholders—in our case, the teachers of priority students who formed our Teacher Advisory Board (TAB)—a seat at the table during the design process should yield a better solution with greater buy-in (Gates, IDEO, & Bellwether, 2020). Creating the prototype with a Targeted Universalist approach increased the opportunity to build buy-in from all educators and students who had the potential to be supported with the prototype, with a special focus on priority students.

Human-Centered Design

Human-Centered Design is a conceptual construct that designers embrace for problem solving and providing solutions while iteratively involving humanistic perspectives from the people for whom one is solving the problem. Human-Centered Design is a means to focus on the user, keeping their perspectives and the meaning they make of the solution central to the design process (IDEO, 2016).

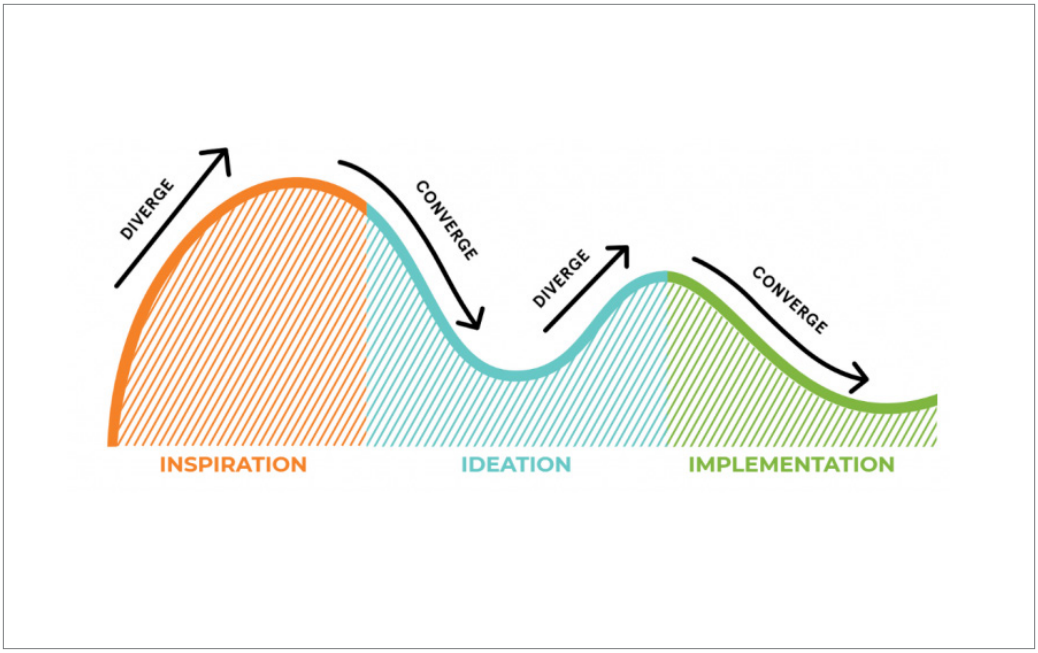


Figure 2: Human-Centered Design Construct, Hoover (2018)

Design thinking is a process that designers use to enact Human-Centered Design that embraces an iterative process to understand users' needs. The five steps of design thinking are 1) empathize with the users, 2) define the needs, 3) ideate possible solutions, 4) prototype from one or more of the ideas, and 5) test the ideas with users and learn from their feedback (Dam & Siang, 2018). Design thinking provides a means to prototype and test iterations of an idea using rational problem-solving techniques (Foster, 2019) and incorporates the process made popular by Stanford's d.school (2020).

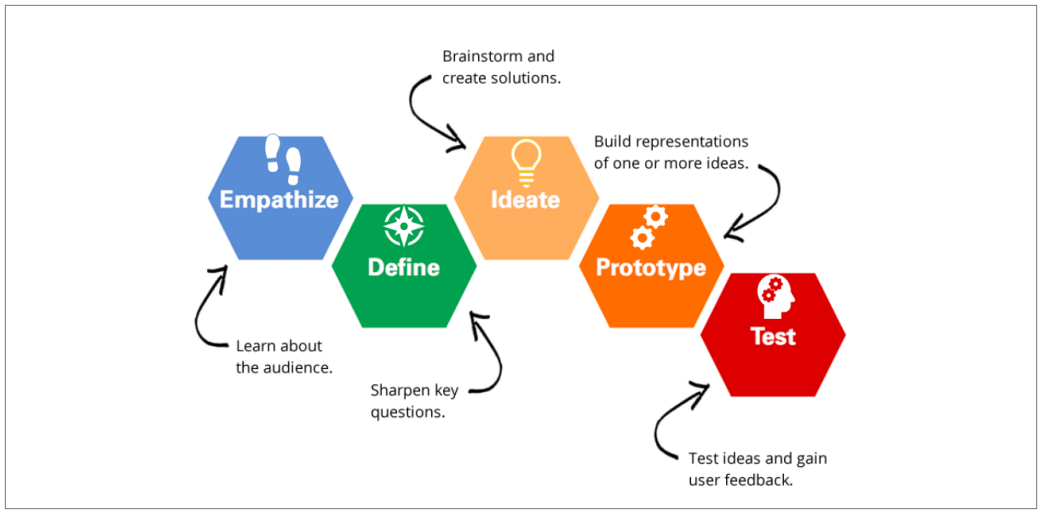


Figure 3: Design Thinking Process, Hoover (2018)

Design thinking adds an additional layer to Human-Centered Design by continually returning to the user's perspective so the solution is relevant and beneficial in the long while and embracing the importance of serving the needs of the people using the solution (Hoover, 2018).

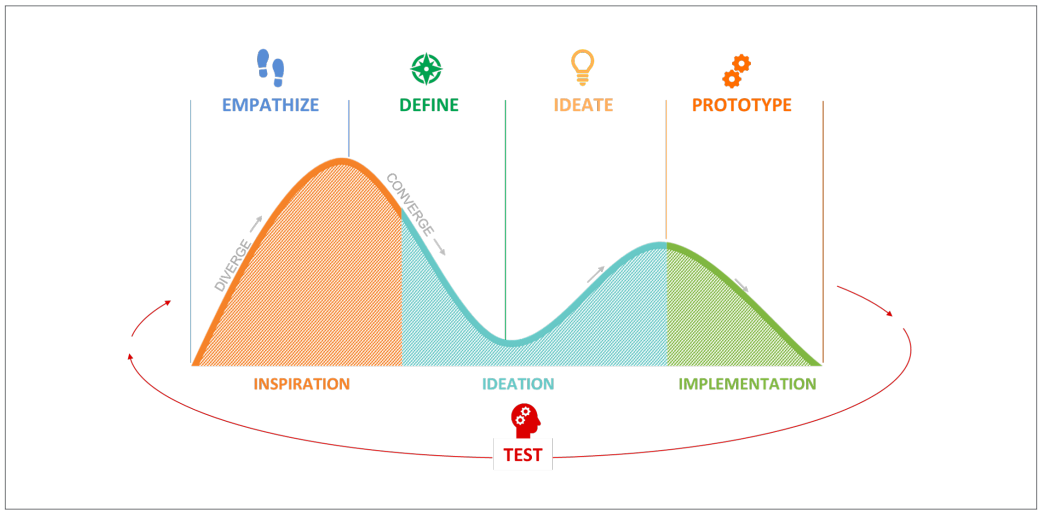


Figure 4: The design thinking process overlaid on the Human-Centered Design construct, Hoover (2018)

The project team integrated the theory of Targeted Universalism and the construct of Human-Centered Design, using the iterative process of design thinking to continuously empathize, define, ideate, prototype, and test to understand the barriers and lack of opportunities teachers of priority students face and to develop targeted strategies that would theoretically support both priority students and all students in overcoming those barriers. The model for using Human-Centered Design to support Targeted Universalism is as such:

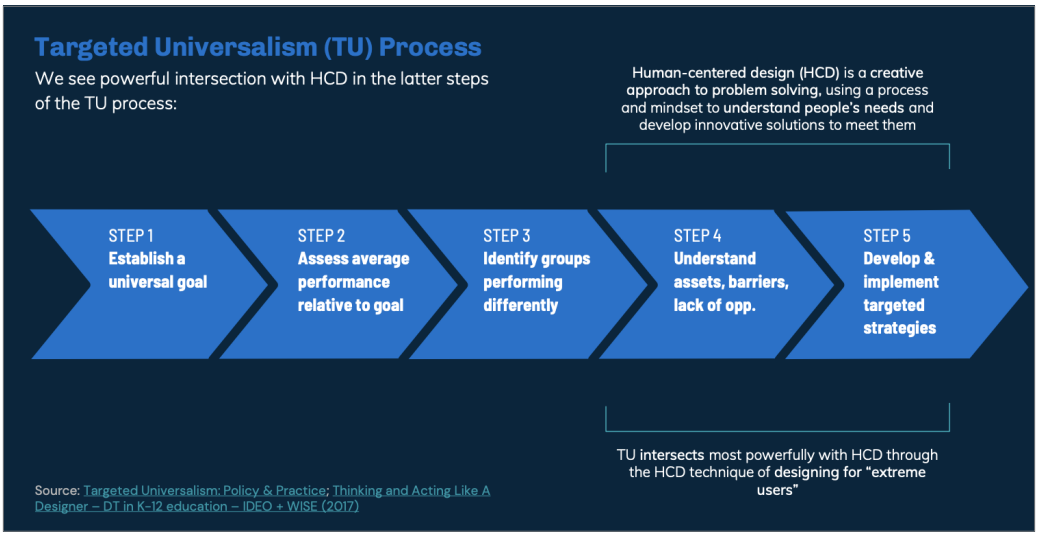


Figure 5: Targeted Universalism. Gates, IDEO, & Bellwether (2020)

In such a model, research focuses on the experiences and unmet needs of marginal and non-users (often called "extreme users")—both students and the teachers who serve them. The process we created to conduct this research is detailed in the next section.

Design Process

As the painter Gustave Flaubert said, "Be regular and orderly in your life, so that you may be violent and original in your work" (Winock, 2016). We're not so sure about the violence; however, borrowing Flaubert's sentiment, the project team found that it was important, with such a complicated project, to build a design process that allowed the team to put their energy towards thinking creatively about the task at hand rather than spending time thinking creatively about the process that was supposed to elicit creativity.

A Targeted Universalist hypothesis needing to be rapidly tested through the construct of Human-Centered Design called for a unique, yet regular and orderly, set of processes to be developed for this project. Complicating the research and learning-focused design thinking process was the project's primary deliverable: a working prototype. Therefore, engineering development needed to happen at nearly the same time as design. The project team used a combination of the Google Ventures Sprint protocols (Knapp, Zeratsky, & Kowitz, 2016) with Agile Scrum development framework protocols (Schwaber & Sutherland, 2018) to create an evolution of Human-Centered design: seven two-week iterative design sprints incorporating targeted user feedback, each followed by a two-week development sprint building and developing what had emerged from the design sprints as strongly meeting users' needs. In true educator fashion, the project team borrowed from existing structures and combined them to make something that would work well for this project. In the case of iterative design, we would augment Flaubert's quote to say, "Be regular and orderly in your sprints, so that you can be creative and original in your deliverables."

The Design Sprint Process

Each of the seven two-week design sprints, Sprint A through Sprint G, followed the same trajectory:

Monday	Tuesday	Wednesday	Thursday	Friday
Sprint Planning 1.5hrs	**Sketching** 1.5hrs **Deciding** 1.5hrs **Empathy Mapping** 1hr **Storyboarding** 1.5hrs	**Design Time**	**Design Time**	**Design Time**
Planning User Feedback Sessions 1.5hrs	**Internal Feedback Session** 1hr	**TAB Feedback Sessions** 6 x 1hr	**Advisor Feedback Sessions** 1hr **Feedback Synthesis** 2hrs	**Sprint Review and Retrospective** 1hr

Figure 6: Two-week sprint protocol

Sprint Planning

Each sprint began with a plan (Schwaber & Sutherland, 2018). In Sprint A, planning was based on initial and background research. In subsequent sprints, planning was based on the learnings from the previous sprints. During planning, the project team reviewed the learnings about priority students gathered thus far against the broad complex challenge of the 5 Practices as a whole to create a specific target to accomplish in the next two weeks. Note-taking on big ideas from previous learnings, theming, discussing, and voting comprised a focused collaborative planning session that provided structure for the work of the sprint and kept the team focused on the needs of our target users (Knapp, et al., 2016). In each sprint planning session, the project team asked us, what's the biggest moment of opportunity for us? What moment provides us with the biggest opportunity to learn and apply our learning in future sprints? (Knapp, et al., 2016).

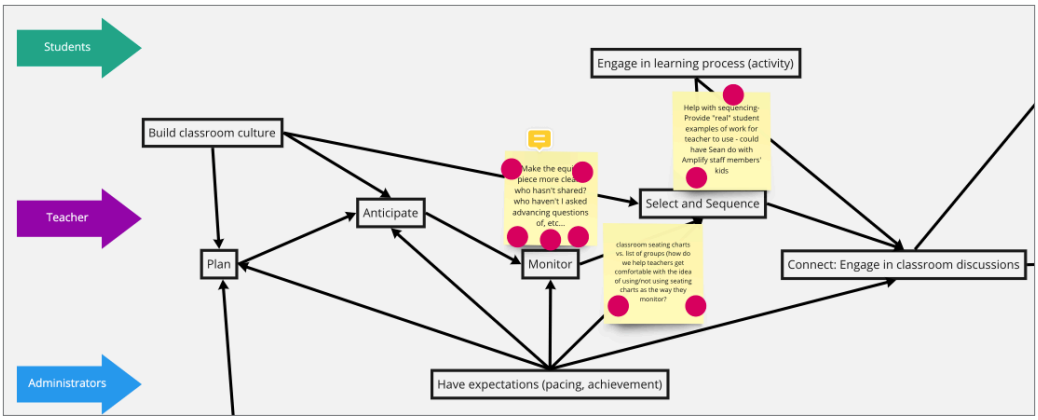


Figure 7: Target planning map for Sprint B

Empathy Mapping

Building empathy for target users enabled the project team to understand the experiences of the teachers on the TAB while taking action on past learnings. Empathy mapping provides a protocol to visualize and articulate users' point of view relative to the project at hand (Gibbons, 2018) and helps the team to guide the decision-making process with the users' needs in mind, prioritize the needs, and discover gaps in their understanding of the prototype and users' perspectives. The result is a thick description (Geertz, 1973) with rich details (Agar, 1996) that enables understanding of users' point of view (Erickson, 1986). Empathy mapping answers questions such as *Who are we empathizing with? What do they need to do? What do they see? What are their pains?* (Gray, 2017). The project team had to be especially deliberate about building empathy for the users because due to the COVID-19 pandemic they were unable to visit the TAB members to see them and their students in action in their classrooms. Meeting the teachers and spending an hour hearing about their experiences with the 5 Practices gave the project team initial context that enabled them to begin crafting empathy maps and user profiles of the teachers, which were worked on throughout the project as they met with each TAB member every other week for one hour. Empathy mapping facilitated a community of understanding within the project team about the teachers' experiences and fundamentally influenced both major and minor decisions in every sprint.

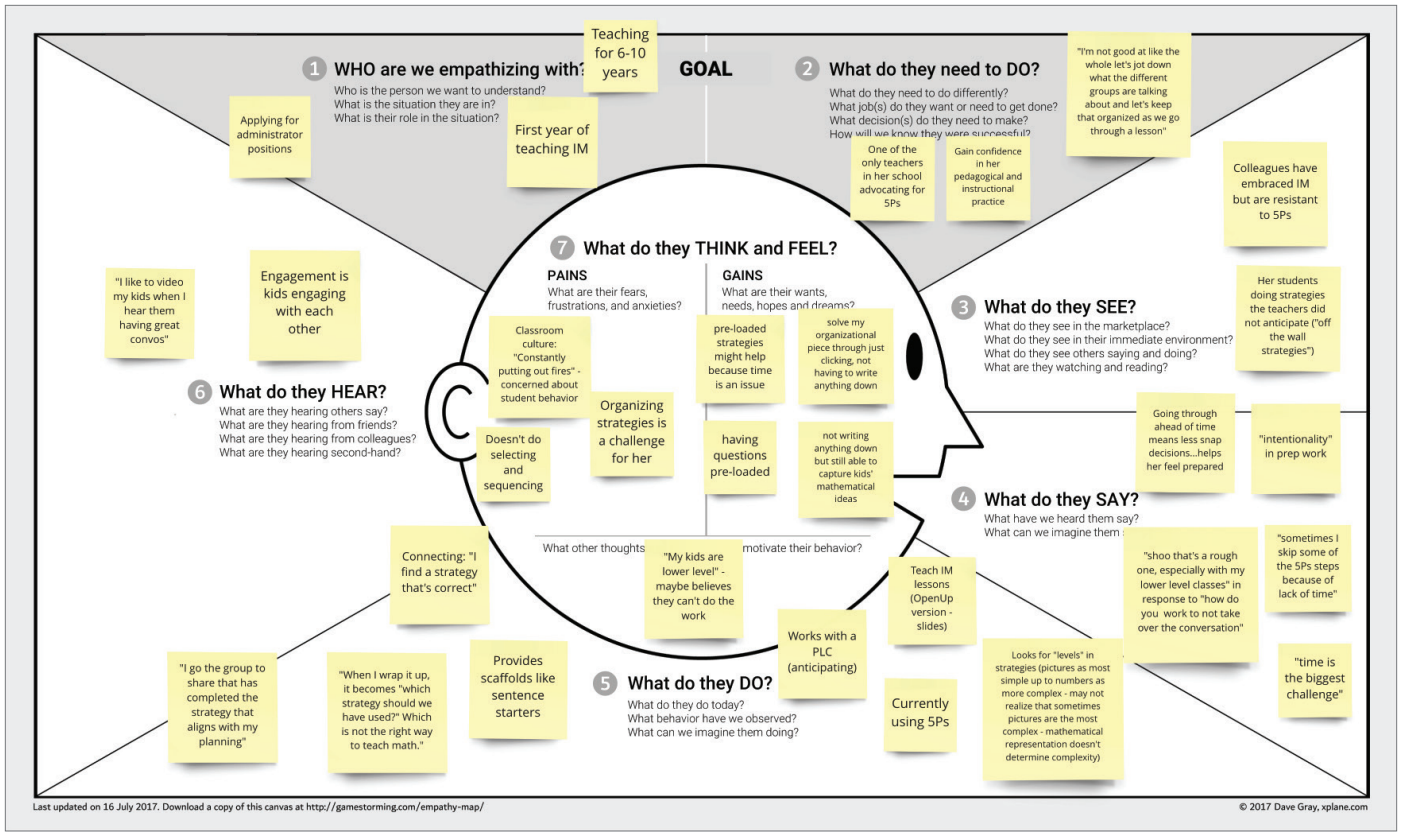


Figure 8: Empathy map for a Teacher Advisory Board member

Sketching

Once the target for the sprint was planned, the project team moved into sketching, which Knapp et al. (2016) considers "the easiest and fastest way to transform abstract ideas into concrete solutions" (p.107). Rather than try to all sketch together, a recipe for mediocrity and groupthink, the project team sketched independently and then shared ideas, as seen in Figure 9. Reviewing key information, doodling rough solutions, trying rapid variations, and finally putting together a solution sketch with details that would be shared valued every person's unique perspectives and ideas, providing equality of voice and perspectives to better shape the prototype (Knapp, et al., 2016).

Deciding

Deciding involved taking all of the different solutions the project team proposed, noting interesting parts, speed critiquing standout ideas, and voting on what should be prototyped in the current sprint (Knapp, et al., 2016). Voting was an equitable method for valuing every team member's voice and can be seen via the dots in Figure 9.

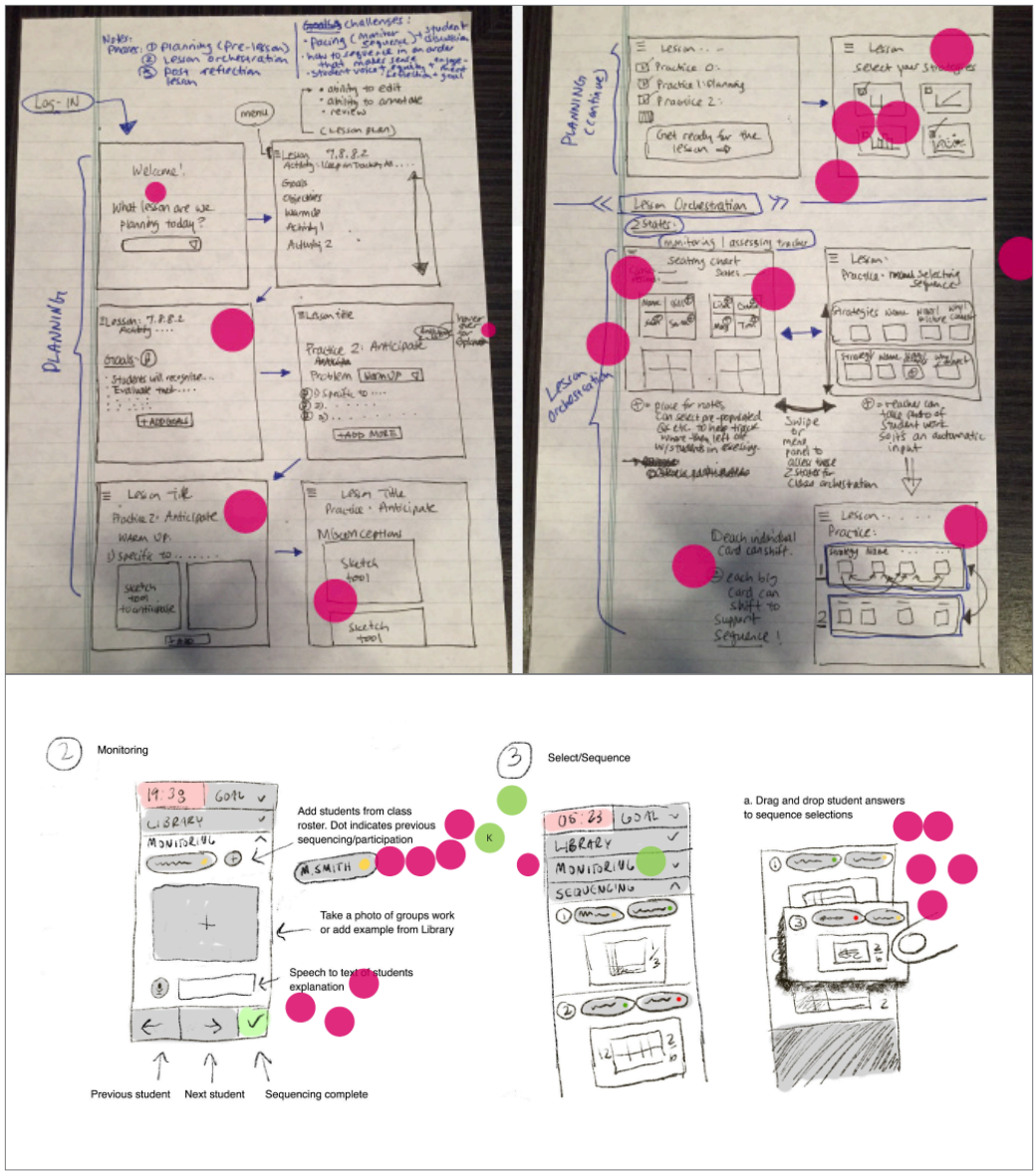


Figure 9. Two sketches with voting dots from Sprint A

Storyboarding

During storyboarding, the team worked together to imagine the finished prototype for the sprint that they would then obtain feedback on during the qualitative sprint interviews. With one team member holding the pen, the project team took the sketches with the most votes and wove them together into a cohesive story that the product designers could make into a clickable prototype during the design days (Knapp, et al., 2016).

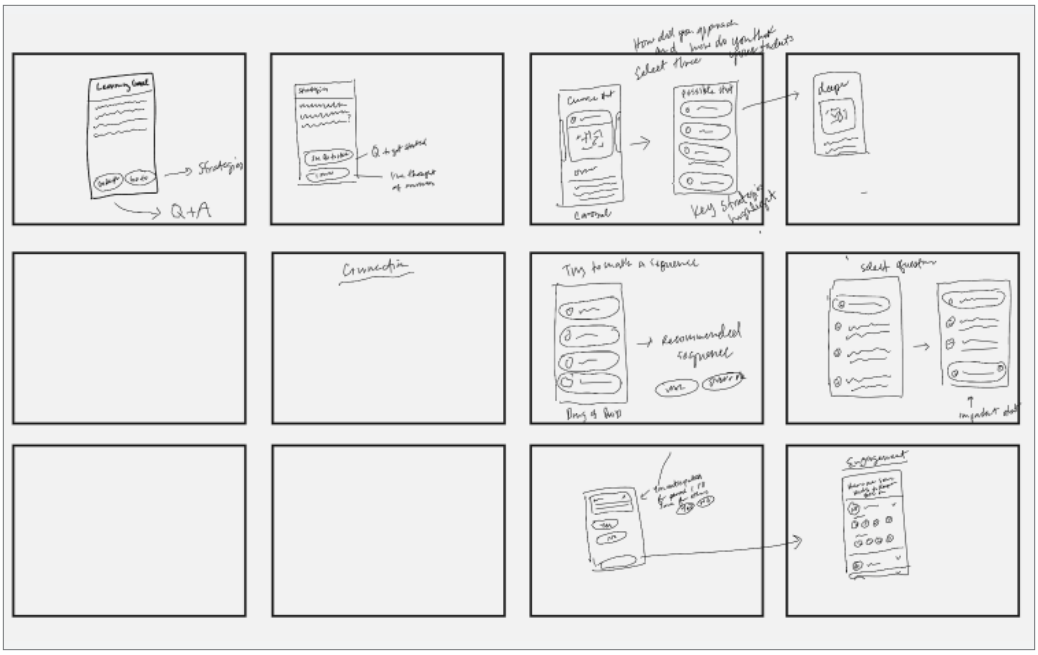


Figure 10: Storyboard from Sprint D

Planning User Feedback Sessions

Once the clickable prototype for the sprint was designed, the team reviewed the prototype and created an interview plan. As part of that process, we asked ourselves the following: What questions would we need to ask in order to determine if our solution was overcoming the barriers and challenges felt by the users? (Knapp, et al., 2016).

Internal Feedback Sessions

While the project team believed that internal staff should never be used as the sole source of user feedback as we are not our users (Budiu, 2017), the team took a balanced approach and conducted feedback sessions with Amplify curriculum developers who had teaching expertise (some with non-priority students) before leading feedback sessions with the TAB. These internal feedback sessions allowed the team to test designs and interview questions; the team typically made revisions and clarifications to questions before running the sessions with the TAB. The project team was also able to integrate this additional educator feedback into user synthesis sessions.

TAB Feedback Sessions

The feedback sessions with the TAB were the most important, tiring, and invigorating aspect of every sprint. Six one-hour feedback sessions over the course of one day provided an immediate deluge of information on whether the solution being tested in that sprint actually helped the target users overcome their barriers and challenges or not.

Advisor Feedback Sessions

In every sprint, the team also spent an hour with project advisors Peg Smith and Michael D. Steele, checking the ideas in the prototype against their view of the 5 Practices and leveraging their mathematical and instructional expertise. This provided specific and global insight into the usability of the prototype and mathematical considerations, allowing the project team to draw from the advisors' extensive experience helping people throughout the country enact the 5 Practices in a meaningful way. While the advisors were not the target audience of users for the prototype, their input was invaluable.

Feedback Synthesis

After collecting such rich information, the project team engaged in data analysis and synthesis of the feedback, examining what was said, what was not said, and what the users did with the prototype in terms of themes and trends. An important aspect of synthesis was listening and looking for implicit bias. Given the project team's focus on priority students, the team did not want to accidentally build implicit bias into the prototype by blindly following feedback that may have been given with the best of intentions but that was unconsciously biased against priority students. As a result of this regular examination the project team decided to also conduct a thorough literature review into equity in mathematics education, an important deviation from traditional Human-Centered Design that allowed the project team during synthesis to deliberately check what users were saying against established academic research. At this point, the ideas in the sprint met one of three fates:

- Stop: The project team decided to stop working on the idea either because it was not helping our users overcome their barriers or because it was a promising idea that the team would not have time to fully develop within the scope of the project.
- Keep working: The project team decided the idea was promising, fit within the scope of the project, and needed additional iterative thinking in a future sprint to be considered a working idea.
- Make: The project team decided that the idea worked to overcome barriers users faced and was ready to be refined for the development team to build.

Sprint Review and Retrospective

At the conclusion of each sprint, the team invited stakeholders to join the entire team for a review to see a demonstration of the ideas that had been tested in that sprint and to hear about the feedback received (Schwaber & Sutherland, 2018). Immediately following, the project team engaged in a retrospective session that provided a forum in which the project team reflected on how well the sprint worked and what changes could be made in the next sprint to make the prototype even better (Schwaber & Sutherland, 2018).

The project team did not think any designer, teacher, or engineer would ever say they have enough time to create everything, and 14 weeks to create a prototype certainly is not anyone's idea of enough time. That being said, a defined cyclical period of time helped the team set ambitious, yet doable, expectations for accomplishing such a lofty goal. Once the project team was familiar with the two-week sprint cycle, they could predict how much work they could accomplish in each sprint which made fielding feedback insights easier to judge in terms of what they could act on.

The Project Team

Successfully achieving the goals of this process required a collaborative cross-functional project team with expertise in mathematics education, product design, research, product management, and development. If the project team was to embrace Human-Centered Design, then the project team needed to embrace the value of each individual on the team as an authentic human being, which confirmed the importance of developing "traditions, behavioral standards, and unwritten rules" for the team that would enhance and build psychological safety and enable the team to become more collectively intelligent and greater than the sum of its parts (Duhigg, 2016, para. 19). *Psychological safety* refers to a group culture in which all members of the team believe that the team is safe for interpersonal risk-taking and in which the team climate is characterized by interpersonal trust and mutual respect (Duhigg, 2016).

Complicating this push for psychological safety was the timing of the project: in the middle of the COVID-19 pandemic, which meant that the work of the team needed to be 100 percent distributed and virtual. This was also a new team in which very few members had worked together previously. These factors presented a tangible risk to the project in that the creativity and innovation needed would be difficult to muster without the project team ever being together in person. The two main components of psychological safety—ensuring equality in time spent talking and taking the time to intuit how others in the group are feeling—led the team to use video calls in deliberate ways: keeping cameras on, staying attentive to when someone went off mute, asking open-ended questions and providing wait time, in addition to consistently using a virtual whiteboard space for the sprint process for real-time collaboration to substitute for gathering around a whiteboard together in person.

Another norm the team embraced that contributed to psychological safety was deliberately valuing the perspectives of those on the project team who are outside of the primary field of the project—in this case, mathematics education. These valued outside perspectives helped the project team to make something familiar that they all participated in many times, a mathematics classroom, feel strange (Spindler & Spindler, 2000) by looking at it with fresh eyes. This involved actions such as not assuming anything, making it okay to take risks and be wrong, asking many questions, and embracing the complexity of the issues we were grappling with. An example of embracing inquiry and making the familiar strange came early in the project when one of the product designers asked, "Why do teachers try to fit all this [the 5 Practices] into one lesson or class period?" It would have been easy for the mathematics education experts on the team to quickly answer the question based on the structural and historical reasons why mathematics classes are set up to teach one lesson in each class period and move on. But pausing to consider that question while providing space for the discussion opened up the team to possibilities for the prototype outside of the traditional and historical structure of the mathematics classroom and added authentic value to everyone's voice that was carried forward through the project.

Key to the project were the target users whom the project team engaged, learned from, and co-created and prototyped with. Asking these people to be an ongoing part of the team ensured that both the universal goal and priority student groups were being considered in every iteration of the prototype as these team members provided ongoing feedback. The project's TAB consisted of six actively-teaching eighth-grade public school teachers who taught student populations that were majority priority students. Other key regular contributors to the project were 5 Practices authors and project advisors Peg Smith and Michael D. Steele, Amplify mathematics curriculum developers (former teachers who did not teach only priority students), and stakeholders from the Gates Foundation and Amplify.

The project team found that transformational things can occur when you get high-quality people with good intentions together to collaborate over a project, bringing their individualized passion based on their particular expertise to bear on the conversations while genuinely getting to know each other as human beings. If we embrace Targeted Universalism with Human-Centered Design, then the more the team is humanized, the more we will humanize others and, hence, the product.

Research Methodology and Results

Research for the project was conducted using a transformative mixed-methods design (Mertler, 2016) to guide qualitative and quantitative data collection around what teachers would find useful in a digital tool, helping them overcome their barriers and challenges in implementing the 5 Practices. The overarching research questions were based on the existing Smith and colleagues work on the 19 Challenges: *What was it about those challenges that presented barriers and lack of opportunities for mathematical discourse with the 5 Practices for teachers of our priority students and for all students? How could we create a useful digital tool that would support strong implementation of the 5 Practices by teachers, specifically with the Illustrative Mathematics (IM) curriculum?*

Teachers across the United States were invited to apply for a position on the Teacher Advisory Board (TAB). A variety of methods were used to recruit interested teachers, such as posting in mathematics educator communities online, reaching out to extended networks and asking those extended networks to reach out to their networks, sending targeted emails, and tweeting and requesting prominent members of the mathematics educator twitter community to retweet. All TAB members were required to teach student populations that were majority priority students, to have a range of experience with the 5 Practices, and to have past experience teaching with the Illustrative Mathematics grade 6-8 curriculum, specifically the eighth grade curriculum. Applicants were also screened for geographic diversity. Interested teachers attended an informational session about the opportunity and selected teachers were asked to commit to joining for the entirety of the project.

Data collection and analysis occurred in three phases.

Phase One (Initial inspiration: empathizing and defining):

- Quantitative:
  - Review of a large-scale survey on the 19 Challenges recently administered by Peg Smith to understand what teachers of all students see as barriers and challenges to the 5 Practices
  - Pre-survey on the challenges administered to TAB members
- Qualitative:
  - Review of a range of 5 Practices paper-and-pencil tools teachers currently use
  - Semi-structured interviews with TAB members, curriculum developers, and project advisors to gather background and context on their perspectives of the challenges

Phase Two (Repeated ideation and implementation: prototyping and testing, leading to further inspiration):

- Qualitative:
  - Bi-weekly semi-structured feedback sessions with TAB members, project advisors, and curriculum developers
  - Ongoing literature review on equity and discourse in mathematics education

Phase Three (Implementation: final testing):

- Quantitative:
  - Post-surveys on the challenges and usefulness of the prototype administered to TAB members
  - Post-survey on the usefulness of the prototype administered to a larger pool of mathematics teachers unfamiliar with the prototype but familiar with the 5 Practices, not constrained to teachers of majority priority students
- Qualitative:
  - Semi-structured interviews with TAB members and advisors to gather additional context and information on their post-perceptions of challenges and usefulness

All data had a high level of interaction with sequential timing and mixing during data collection. All data were collected between May and October 2020. Because the TAB members' prior experience with the 5 Practices, according to the pre-survey, ranged from "I have heard of it but don't know what it is" to "I've used it so much I could teach others about it," each member received a copy of Smith and Sherin's (2019) 5 Practices in Practice book and attended a series of short introductory 5 Practices workshops with Peg Smith. The project team believed that this preparation approximated the level of professional development a typical teacher may engage in before attempting a new pedagogical structure in their classroom. It also allowed the project team to communicate with the TAB using a shared understanding of 5 Practices terminology and concepts.

Quantitative Data Collection and Analysis

Phase One

On April 8, 2020, during NCTM's 100 Days of Professional Learning webinar series, Peg Smith led a webinar for 202 teachers entitled "Orchestrating Productive Discussions: Overcoming the Challenges" targeted at grades 6-8. The attending teachers were asked to identify "the challenge that you struggle with most." Although the intent was to have teachers identify one challenge, since the challenges were split into two groups (1-10 and 11-19) all teachers selected at least one challenge from each group. While most teachers selected two challenges, some teachers appeared to select all that applied. The greatest number of challenges selected by one teacher was seven and the total number of responses was 604. This survey was useful as background information in building the project team's understanding of the challenges and what teachers of all students see as barriers to engaging in student discourse using the 5 Practices.

Table 2: Top 10 challenges from Smith's Overcoming the Challenges survey

Practice	Challenge	# Responses
2	C10. Involving all members of a group	106
5	C16. Keeping the entire class engaged and accountable during individual presentations	99
5	C19. Running out of time	52
1	C05. Moving beyond the way you solve a problem	43
5	C18. Making sure that you do not take over the discussion and do the explaining	38
3/4	C14. Moving forward when a key strategy is not produced by students	34
3/4	C15. Determining how to sequence errors, misconceptions, and/or incomplete solutions	32
1	C06. Being prepared to help students who cannot get started on a task	28
3/4	C13. Deciding what work to share when the majority of students were not able to solve the task and your initial goal no longer seems obtainable	24
1	C07. Creating questions that move students toward the mathematical goals	23

The internally-administered component of Phase One consisted of a pre-survey on the 19 Challenges for the members of the TAB, called the "5 Practices Survey." Each challenge was presented as a survey item with five response categories: 1) Not A Challenge, 2) A Minor Challenge, 3) A Moderate Challenge, 4) A Major Challenge, and 5) I Don't Know[2,3], Because some of the TAB members had little to no prior experience with using the 5 Practices, all members were instructed, at baseline, to rate each item based on the anticipated level of challenge were they to implement the 5 Practices in the new school year. This survey was developed specifically for user research. It was framed as a tool to help facilitate reflection and conversation about the experience around the 5 Practices and as such may not be applicable in other contexts. The survey development process is described and the survey instrument is linked to in Appendix A. It may be used under CC-BY license for purposes of reflection and conversation.

2 The response categories were adapted from a survey item developed by The University of Chicago Consortium on Chicago School Research (CCSR). The CCSR response categories were "Not a barrier," "A minimal barrier," "A moderate barrier," and "A major barrier." The survey was retrieved from https://www.uchicagoimpact.org/sites/default/files/2017%205E%20Teacher%20Survey%20FINAL.pdf

3 The 5 Practices survey also measured the TAB members' confidence in overcoming each of the challenges. Data on perceived confidence, however, were not found to be useful during the design process and therefore are not reported here.

Simple descriptive analysis of results from the pre-survey was conducted to identify which challenges the TAB members struggled with most at baseline. A challenge was flagged if it was rated as a moderate or major challenge by three or more TAB members. Table 3 lists these challenges and Table 4 shows these challenges alongside the challenges from Smith's Overcoming the Challenges Survey.

Table 3: Challenges survey items rated as a moderate or major challenge by three or more TAB members at baseline

Practice	Challenge
2	C9. Keeping track of group progress—which groups you visited and what you left them to work on
2	C10. Involving all members of a group
3/4	C13. Deciding what work to share when the majority of students were not able to solve the task and your initial goal no longer seems obtainable
3/4	C14. Moving forward when a key strategy is not produced by students
3/4	C15. Determining how to *Sequence* errors, misconceptions, and/or incomplete solutions
5	C16. Keeping the entire class engaged and accountable during individual presentations
5	C18. Making sure that you do not take over the discussion and do the explaining
5	C19. Running out of time

Table 4: Top challenges from Overcoming the Challenges survey vs. 5 Practices survey: Where did the two survey results converge?

Practice	Challenge	# NCTM	# TAB
2	C10. Involving all members of a group	106	4
5	C16. Keeping the entire class engaged and accountable during individual presentations	99	3
5	C19. Running out of time	52	5
5	C18. Making sure that you do not take over the discussion and do the explaining	38	4
3/4	C14. Moving forward when a key strategy is not produced by students	34	4
3/4	C15. Determining how to *Sequence* errors, misconceptions, and/or incomplete solutions	32	3
3/4	C13. Deciding what work to share when the majority of students were not able to solve the task and your initial goal no longer seems obtainable	24	4

These were the challenges the project team therefore focused on as these challenges were identified as being the largest barriers by teachers of priority students as well as by teachers of the general student population.

Phase Three

At the conclusion of the project the 5 Practices Survey was re-administered to the TAB to assess which of the initial challenges were ameliorated at follow-up as a result of the prototype. Due to TAB members not being present in their classrooms because of the COVID-19 pandemic, TAB members were instructed to rate the items based on the anticipated level of challenge if they were using the Pathfinder prototype in the context of post-pandemic, in-person classroom instruction.

At follow-up, the majority of challenges were no longer a moderate or major challenge for three or more TAB members; two challenges remained a moderate or major challenge for three or more TAB members: moving forward when a key strategy is not produced by students and running out of time. Figure 5 shows the number of moderate and major ratings at baseline and at follow-up for each of the target challenges.

Table 5: Number of moderate and major ratings at baseline and at follow-up

Practice	Challenge	Baseline	Followup
2	C9	4	0
2	C10	4	0
3/4	C13	4	2
3/4	C14	4	3
3/4	C15	3	0
5	C16	3	1
5	C18	4	1
5	C19	5	5

In addition to re-administering the 5 Practices survey at the close of the project, the team developed an additional post-survey based on the Technology Acceptance Model (TAM) (Davis, 1989) to measure the perceived usefulness of the prototype. Due to the COVID-19 pandemic, the opportunity for users to test the prototype in classrooms did not come to fruition. As a result, perception and intention were measured, instead of actual use. According to Davis, user acceptance of a technology or system is related to the perceived usefulness and perceived ease of use of the system (Davis, 1989; Davis, Bagozzi, & Warshaw, 1989). Davis developed a TAM scale that consisted of six items to measure perceived usefulness (Cronbach alpha = 0.98) and six items to measure perceived ease of use (Cronbach alpha = 0.94). This scale has been widely used and adapted for research on a variety of technological systems (Hess, McNab, & Basoflu, 2014).

The post-survey measured the perceived usefulness of different features of the prototype. Perceived ease of use items were not included because feedback obtained during survey pretesting with Amplify mathematics curriculum developers indicated that respondents had difficulty rating the ease of use of features due to COVID-19 restrictions disallowing use in the classroom. Excluding ease of use items was not deemed to be problematic because Davis (1989) found that actual usage of a technology or system was more strongly predicted by perceived usefulness ($p < .001$) than by ease of use.

Figure 11 lists the six perceived usefulness items adapted from the TAM scale. Each item was scored on a seven-point scale, with 1 = strongly disagree, 2 = moderately disagree, 3 = slightly disagree, 4 = neither disagree nor agree, 5 = slightly agree, 6 = moderately agree, and 7 = strongly agree. Because the six items were replicated for each feature, the final survey consisted of seven six-item scales on perceived usefulness. Open-ended items were also included.

To what extent do you disagree or agree with the following statements about the usefulness of the _____ feature (i.e., the degree to which this feature supports your teaching with the 5 Practices)?

1. The _____ feature will enable me to teach with the 5 Practices more quickly.
2. Using the _____ feature will improve my teaching with the 5 Practices.
3. Using the _____ feature will increase my productivity when teaching with the 5 Practices.
4. Using the _____ feature will enhance my effectiveness when teaching with the 5 Practices.
5. Using the _____ feature will make teaching with the 5 Practices easier.
6. Overall, I find the _____ feature useful for teaching with the 5 Practices.

Figure 11: Perceived usefulness items in the post-survey for each feature

The post-survey was administered to the six TAB members and to a larger sample of middle school mathematics teachers who are currently teaching or have taught within the last three years and who were not restricted based on the make-up of their student populations. Because the prototype was designed with Targeted Universalism in mind, the project team expected that data from the larger sample would provide insight into the universal benefits of the prototype.

Of the N = 41 teachers who expressed interest in providing feedback on the prototype, 24 completed the survey, for a response rate for the larger sample survey of 58%. Half of the teachers who completed the survey teach majority (i.e., more than 50%) White students (n = 12) while one-third teach majority priority students (n=8). The rest teach a diverse student population with no clear majority (n = 4).

Table 6: Summary statistics of scale scores for each feature from TAB members and from the larger sample of teachers

	TAB (N = 6)		**Larger Sample of Teachers (N = 24)**			
	Majority Priority Students		**Majority Priority Students (n = 8)**		**Not Majority Priority Students (n = 16)**	
Feature	Mean (SD)	[Min, Max]	Mean (SD)	[Min, Max]	Mean (SD)	[Min, Max]
Helpful Hints	5.22[6] (1.49)	[1,7]	6.48[2] (0.71)	[5,7]	5.61[6] (1.07)	[2,7]
Key Ideas	6.39[2] (1.05)	[3,7]	5.85[5] (1.22)	[2,7]	5.63[5] (1.17)	[2,7]
Interactions	5.72[5] (1.54)	[3,7]	5.52[6] (1.60)	[1,7]	4.90[7] (1.53)	[1,7]
Who's Recently Presented	6.19[3] (0.95)	[3,7]	5.19[7] (1.99)	[1,7]	5.65[4] (1.42)	[2,7]
Student Work Photo	5.17[7] (1.28)	[3,7]	6.13[3] (1.23)	[1,7]	5.97[2] (1.17)	[2,7]
Anticipating	6.44[1] (0.91)	[3,7]	6.81[1] (0.39)	[6,7]	6.27[1] (1.07)	[3,7]
Selecting and *Sequencing*	6.06[4] (1.17)	[4,7]	6.13[4] (0.82)	[5,7]	5.84[3] (1.46)	[2,7]

Note: Shading indicates the highest and lowest mean scale scores. Superscripted numbers indicate the order of mean scale scores, from 1 = highest to 7 = lowest.

For each feature, respondents' ratings were averaged across all six items to create an overall scale score. Higher scale scores indicate greater agreement with the statements on the usefulness of the feature, with 7.00 as the highest possible score. Table 6 reports the scale scores for each feature averaged across the TAB members (all of whom teach majority Priority students), teachers in the larger sample who teach majority Priority students, and teachers in the larger sample who do not teach majority Priority students (i.e., teachers with majority White students or no clear majority).

Data from the larger sample of teachers were also disaggregated by level of experience with the 5 Practices. On average, teachers with some experience with the 5 Practices (i.e., "I have used the 5 Practices in my classroom within the past 3 years" or "I have used the 5 Practices so much that I could teach others about it") rated *Anticipating* as the most useful feature. For teachers with no experience with the 5 Practices, the most useful feature was *Helpful Hints*.

Table 7: Summary statistics of scale scores for each feature from the larger sample of teachers by level of experience with the 5 Practices

	Larger Sample of Teachers (N = 24)					
	None (n = 3)		**Medium (n = 13)**		**High (n = 8)**	
Feature	Mean (SD)	[Min, Max]	Mean (SD)	[Min, Max]	Mean (SD)	[Min, Max]
Helpful Hints	6.56[1] (0.51)	[6,7]	5.69[5] (1.11)	[3,7]	6.00[4] (0.99)	[2,7]
Key Ideas	4.61[7] (1.54)	[2,7]	5.95[3] (1.02)	[3,7]	5.71[5] (1.09)	[2,7]
Interactions	5.28[5] (1.93)	[1,7]	4.91[7] (1.65)	[1,7]	5.35[7] (1.26)	[2,7]
Who's Recently Presented	4.94[6] (2.88)	[1,7]	5.63[6] (1.53)	[2,7]	5.48[6] (1.09)	[3,7]
Student Work Photo	6.06[2] (1.59)	[1,7]	5.97[2] (1.08)	[4,7]	6.08[3] (1.22)	[2,7]
Anticipating	6.06[3] (1.43)	[3,7]	6.55[1] (0.66)	[5,7]	6.44[1] (1.07)	[4,7]
Selecting and *Sequencing*	5.67[4] (1.28)	[4,7]	5.82[4] (1.47)	[2,7]	6.23[2] (0.88)	[5,7]

Note. None = "I have not used the 5 Practices in my classroom"; Medium = "I have used the 5 Practices in my classroom within the past 3 years"; High = "I have used the 5 Practices so much that I could teach others about it." Shading indicates the highest and lowest mean scale scores. Subscripted numbers indicate the order of mean scale scores, from 1 = highest to 7 = lowest.

Final quantitative analysis showed that all the features received favorable (above neutral=4) ratings, with mean scale scores of 5.17 to 6.44 from the TAB, 5.19 to 6.81 from teachers with majority Black and/or Latinx students, and 4.90 to 6.27 from teachers who do not teach majority Black and/or Latinx students.

Qualitative Data Collection and Analysis

The primary method of qualitative research was semi-structured interviews. Each interview was one hour in duration, was recorded, and took place over virtual web-based meetings. All interviews were conducted with individuals, with the exception of having our two advisors Smith and Steele interview together. From the project team, there were always at least two people on each interview, one as an interviewer and one as a notetaker. Whenever possible, others on the team would join interviews to listen. While interviewing, only the interviewer and interviewee had their cameras on and were unmuted.

The qualitative interview process also afforded the opportunity to listen for and inquire about the perceptions of the teachers about the effect the prototype could have on their ability to support their priority students. Whenever possible, the implicit bias expert of the team attended interviews to specifically monitor for biases when TAB members were answering questions and exploring the prototype.

Phase One

Initial qualitative data consisted primarily of pre-semi-structured interviews with TAB members and advisors. For the initial interviews, qualitative data were hand-coded with a two person interrater reliability protocol (Armstrong, Gosling, Weinman, & Marteau, 1997; Belotto, 2018). Two researchers individually transcribed and coded each interview and met to compare coding of data via spreadsheets, discuss similarities, address discrepancies, and adjust accordingly for the next round of coding.

Table 8: Examples of questions from the TAB qualitative pre-interviews

Category	Target	Question
Broad	Behavioral beliefs	Tell us briefly about your experiences with the 5Ps.
Broad	5 Practices	What do you see as the advantages to implementing the 5Ps in your classroom?
Broad	5 Practices	What are some challenges associated with implementing the 5P's in a classroom? Why is that a challenge?
Questions targeting individual 5P's	Practice 0 & *Anticipating*	When preparing to teach a lesson using the 5 P's structure, what do you do?
Questions targeting individual 5P's	*Monitoring*	How do you evaluate student comprehension of the mathematical concepts housed within the lesson?
Prototype		Are there any features that you think will be critical for the successful use of a 5Ps app within a classroom environment?

Incorporated into some of the questions was an eye toward identifying implicit biases. For example, the questions about advantages and challenges associated with implementing the 5 Practices were formulated partly to understand what the question explicitly asks; attention was also paid to the answers the TAB members provided with an access, equity, and social justice lens. The implicit bias expert of the project team looked over the qualitative data, especially with questions such as these, to identify if someone categorized the advantages or challenges as being specific to a gender or ethnicity. This was partly to better understand the TAB members and also to ensure suggestions that influenced the prototype were screened for implicit bias. For example, if someone said that something in the prototype would help priority students, white students, or tagged their response with a gender-specific caveat, the project team needed to know this in order to disallow potential implicit bias from filtering into the design.

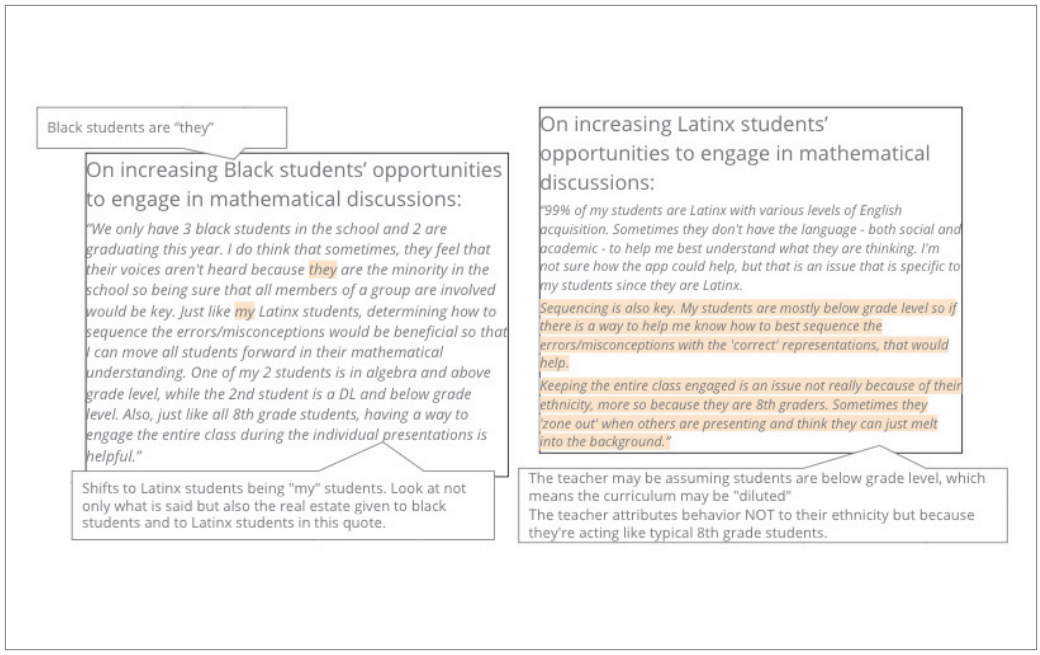


Figure 12: Example of analyzing TAB responses for implicit bias

The project team engaged in conversations based on the ongoing reviews for implicit bias, which led to revisions of the Empathy Maps, one example illustrated in Figure 8, as well as User Personas (two examples anonymized and shown in Figure 13). The User Personas were crucial to Human-Centered Design and Targeted Universalism because in order to understand the meaning the TAB members made of the 5 Practices and the prototype, the project team needed to understand who they are. The User Personas the project team used in the design were detailed and personal. This was necessary to empathize with the TAB members and understand the meaning they made. To protect the identity of the TAB members, identifiable aspects of the User Personas were removed from this paper.

Shannon

User profile

# of years teaching: 6-10

Currently Teaches: 8th grade + Algebra 1

Comfort Level with the 5 Practices: I have heard of it but don't know what it is

Goal:
- Develop student confidence so all groups can have mathematically rich discussions

Pain points:
- Differentiating for equality of voice for all levels of students
- Some students need to have step-by-step strategies to experience success
- Having the "higher level" kids in class with "lower level" students
- Students may know their ranking in class
- "Lower level" groups see "higher level" groups move through the problems faster

Needs:
- Every "level" of student participating
- Having time to Anticipate, especially anticipating strategies
- Support choosing multiple strategies - as opposed to choosing different stages of completion for the one "right" strategy

Supports Needed:
- How to build mathematical confidence for all "levels" of students

School Profile

Type: Public

Title I status: Yes

FRL Status: 45%

Student Demographics:
White: 23.4%
Black: 59.6%
Asian: 1.8%
Hispanic: 10.3%
Native Hawaiian/ Pacific Islander: .5%
American Indian: 0%
Two or more race: 4.4%

Student Device Access:
- 75% of students have reliable internet access at home
- Most students have access via smartphone

Iris

User profile

# of years teaching: 1-5

Currently Teaches: 8th grade

Comfort Level with the 5 Practices: I have used it in my classroom in the past, but stopped using it

Goals:
- Foster a classroom community "where students feel comfortable talking to each other about math even if they might be wrong"
- Collaborate with other educators around what's happening in his classroom in order to grow own practice - including understanding multiple representations

Pain points:
- Only 8th grade math teacher at school
- No time to plan and anticipate - has led to stop engaging in 5Ps
- Nervous about selecting, sequencing, and finishing the lesson

Needs:
- Collaboration with other educators, particularly for ideas for multiple representations and ways to select and sequence

Supports Needed:
- Ways to learn and improve in the areas they sees as areas of growth
- Questions for students who participate more orally than written
- Ways to monitor math and safety/on-task behavior
- Ways to provide "frequent check-ins" for some students
- Support for active working memory while students are working

School Profile

Type: Charter

Title I status: Yes

FRL Status: 61%

Student Demographic:
White: 4.02%
Black: 15%
Asian: 13.9%
Hispanic: 56.3%
Native Hawaiian/ Pacific Islander: 0%
American Indian: .28%
Two or more race: 10.5%

Student Device Access:
- Majority of students have access to internet through home wifi and cell phones
- Majority have consistent access to devices

Figure 13: Two examples of user personas

Qualitative data in this phase also included analysis of existing teacher-created 5 Practices *Monitoring* templates. Qualitative document collection occurred as teachers across the country were invited to share what they currently use in the classroom as a *Monitoring* template for the 5 Practices. 22 distinct tools were collected and used to inform the creation of a 5 Practices Tools Rubric to assess the features of the existing teacher-created *Monitoring* templates (see Appendix B). The team and advisors collaboratively used two of the teacher-created *Monitoring* templates to norm on the Rubric and revise it, and then engaged in rounds of individual and collaborative scoring based on the Rubric, including making qualitative notes. The *Monitoring* templates that scored the highest on the Rubric were then studied by the entire project team via Lightning Demos (Knapp, et al., 2016) and the stand-out features from the top *Monitoring* templates were used for inspiration in informing the iterations of the prototype as potential features of value, given that teachers were already using them in classrooms in paper-and-pencil form.

Phase Two

A literature review was conducted throughout the project, including during every sprint. The literature review consisted of a variety of articles related to the project and mainly focused on equity, access, priority students, and social justice in mathematics classrooms. The project team knew from the beginning that Targeted Universalism required a balance of the TAB members' views with current research about Black and Latinx students' experiences in the classroom, confirmed when an advisor noted of the top challenges, "One thing that comes to mind for me is a number of those challenges that rose to the top for teachers related to Black and Latinx students seemed to have a bit of a 'classroom management' flavor to them (participation, engagement). There's lots of discussion right now in the education community about management pedagogies that really are implicitly about controlling black and brown bodies. I think as we go, we need to be attentive to asking teachers to unpack how they think addressing these challenges relates to elevating black and brown students and their mathematical success in the classroom" (Sprint A Interview, 6/26/2020). The project team felt that a consistent focus on researched-based ideas would balance any implicit bias in feedback.

Seven interviews were conducted with each TAB member and advisor, one during each two-week sprint. Qualitative data were analyzed using the iterative process (Agar, 1996; Hammersley & Atkinson, 1995). The videos were transcribed and both the notes and transcriptions were used to refine the empathy maps and user personas, determine what to stop, keep working on, and make for the prototype, and what to concentrate on in the next sprint.

Table 9: Examples of questions from iterative feedback sessions

Sprint	Sample Questions
A	What are you looking at here?
	What would you want to see and do as you're engaging in *Anticipating*?
B	Is there anything you might have done during the *Anticipating* step that you would want to see again here?
	If you were looking at this on your device, what would be happening at this moment in your real classroom?
C	Prompt if necessary: Pretend that the group is working on strategy B, and it looks like they're about halfway through Strategy B. How would you record that? [May need to prompt them to try it out]
	Are there certain students with whom you have to do more to keep them engaged in the class discussion? How do you do this?
D	Wait, before you do anything, just pause, look around, what do you think you can do here?
	Prompt if necessary: What might you expect to see if you click on the "i" icon?
E	Looking at the "Reflections" section, what would you do with these questions at this moment when you're starting to engage in *Anticipating*?
	Is there anything else you would want to do during *Anticipating* that the app could help with?
F	If you had heeded the helpful hint and included Nicky in the conversation, what would you do?
	How would you use what we showed you today?
G	If needed: Was there anything you expected to see in the designs for this moment/feature that you didn't see?
	Thinking about Practice 2 - *Monitoring*, if we had constraints on what we could make and we had to get rid of every feature except ONE, prioritize for us. Which is the one feature you would tell us we HAD to keep? Why?

Notice Sprint C question number two. Although every piece of qualitative data were analyzed for implicit bias, there were certain questions such as this one built to identify implicit bias. Other questions encouraged the TAB members to tell a story about how they lesson plan or how they use the 5 Practices in the classroom. Particular attention occurred if TAB members referenced any subset of students or referred to any students as exclusionary.

Phase Three

The post-interview for the TAB members was structurally different from the pre-interview and all sprint interviews. The TAB members perused the prototype on their own time and completed the quantitative post-survey. Then the project team analyzed the results of the survey for individual TAB members and holistically. As a result, some questions were templated but targeted to add clarity to their survey responses while other questions were more open-ended in nature. Survey-dependent questions used questions and follow-up questions. Survey-independent questions used prompts and questions. Table 10 provides examples of both types.

Table 10: Examples of questions from post-interviews

Type of question	Question	Follow-up question
Survey-dependent	We didn't see a change from your pre- to your post-survey response on keeping track of group progress—that is, which groups you visited and what you left them to work on. On both surveys you indicated that this was a minor challenge. Why wasn't there a change?	What would you have liked to see in the app to help alleviate this challenge?
	On your pre-project survey you indicated that keeping track of group progress that is, which groups you visited and what you left them to work on—was a moderate challenge. On your post-project survey you indicated that this is NOT a challenge. What in the app was responsible for this change?	Was there anything else in this project that contributed to this change?
	When asked to rate whether Helpful Hints and Key Ideas would enable you to teach with the 5 Practices more quickly you responded with Slightly Disagree. Tell us more about why you slightly disagreed with this statement when rating these features.	How could the Helpful Hints feature be modified so that it enables you to teach with the 5Ps more quickly? How could the Key Ideas feature be modified so that it enables you to teach with the 5 Ps more quickly?

Table 10 (continued)

Type of question	Prompt	Question
Survey-independent	As we developed the app, four themes emerged: supporting pedagogy, supporting equity, saving teachers time, and enabling teacher superpowers. Let's start with supporting pedagogy. Supporting pedagogy is really about helping teachers use productive 5 Practices teaching practices.	Does the app support pedagogy? How?
	Next let's talk about how the app supports equity. Our goal was to raise awareness around equity by helping teachers engage with all students within the classroom.	Does the app support equity? How?
	As you know, this was a fast-paced project with the goal of developing a prototype. We accomplished a lot in a short period of time, but there is so much more than can be done. I'd like you to shift gears for a moment and share thoughts on additional features that are NOT currently included in the app, but that you think would be important additions.	(If time) Are there additional features that you think should be included to support [name of theme]?

The post-interviews served multiple purposes. The interviews helped to frame the project team's understanding of how well Targeted Universalism with Human-Centered Design worked as well as gauge, as best as possible, how teachers might have used the prototype had they been in classrooms if COVID-19 never occurred. On a practical level, it also helped to collect data and frame the next steps for the prototype. The post-interviews also afforded an opportunity to learn the TAB members' perspectives while understanding the meaning they made overarchingly of the prototype.

The intention of the bi-weekly and post-synthesis of qualitative data coupled with the quantitative data were to understand the meaning of the 5 Practices and iterations of the prototype from the participants' point of view (Erickson, 1986; Geertz, 1973) while quantitatively prioritizing themes and features and gauging the TAB members' view of the prototype from multiple lenses. The quantitative data were critically important in conceptualizing initial iterations and deciding through the identification of design themes and features what to prioritize. Human-Centered Design would not have been possible without the qualitative data. No one set of data were more or less useful, but instead both had equal status in understanding users' needs.

Design Themes and Features

As the project team embarked on the design process to create the prototype, they followed the needs the users expressed, and as such engaged in a non-linear creation process. In this sense, this paper cannot point to one day, one time, one moment that definitively elicited the major themes that emerged from the development of the prototype. Across the course of the project, individually and collectively held ideas for what the prototype may look like, what content could be housed in the prototype, how it could be presented to users, and what the most important features would be emerged, were tested and evolved. Four major themes emerged from the iterative feedback:

1. Support Equity: Help teachers to engage with all students in the classroom
2. Support Pedagogy and Curricula: Help teachers use productive 5 Practices approaches
3. Save Teachers Time: Maximize in-classroom interactions and practices
4. Enable Teacher Superpowers: Make it possible for teachers do things they could not do before

As the work started on the prototype, the project team utilized the Phase One research to inform the work of Sprint A. Once Phase Two started and the team began to receive feedback on iterative ideas, these themes began to take shape. The themes were named and defined at the end of Sprint B, creating a roadmap for the features that needed to be created to enable teachers to overcome their barriers with the 5 Practices. In each sprint, the team checked understanding of these four themes against the feedback being received to ensure the themes were still accurate. In addition, the quantitative data from Phase One were consistently used to help the team make decisions about what to focus on when multiple options presented themselves.

For the sake of ease of prototyping and testing, the project team used one high-level task throughout the entire design sprint process, which is seen in the figures throughout this section, creating content for the remaining nine lessons of the sprint at the conclusion of the final prototype for the prototype. See Appendix C for the content used for all 10 lessons. The task was an eighth-grade Illustrative Mathematics problem involving using scale factors to find the length of similar triangles, chosen since all TAB members currently taught grade eight mathematics and the team felt it would not face any content knowledge deficits on the part of the TAB members that could impede feedback. The problem states, "Triangles ABC, EFD and GHI are all similar. The side lengths of the triangles all have the same units. Find the unknown side lengths" (Illustrative Mathematics, 2020).

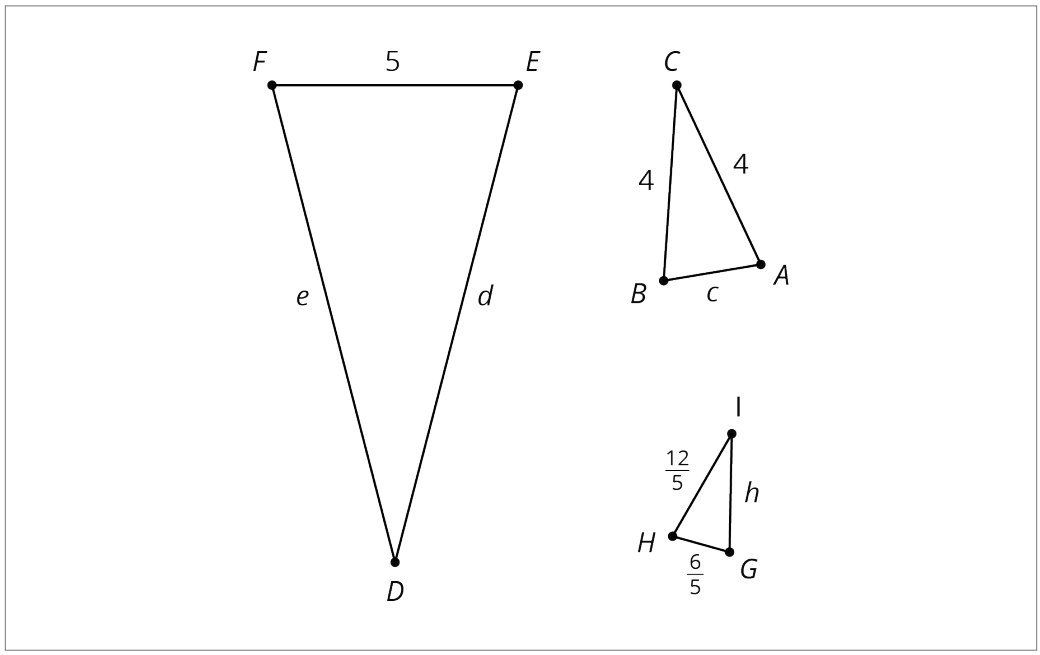


Figure 14: Image for high-level task used in interactive prototyping

Human-Centered Design was essential for the evolution of the themes and features. In terms of design, it is important to listen to where the data, the people, and the iterations take you. Covert and overt hints were strewn throughout the design process and it was in listening to these points the team allowed sprints to go where they needed to at each point in the process. This section will trace the iterative design process for several select features across the four main themes.

Theme 1: Support Equity: Help teachers to engage with all students in the classroom

Equity was a natural important theme to emerge for the project, due to the project's focus on priority students—the 5 Practices should facilitate not just discussions in mathematics classrooms but in particular equitable discussions. It was clear from the Phase One data that a large number of the challenges teachers of both priority students and all students faced were related to the struggle to facilitate equitable discussions. This theme was also distinct from the others and from traditional Human-Centered Design because in this theme the project team felt that they could not listen to only user feedback—due to the danger that the team could build inequities and bias into the app—but also needed to balance user feedback with research into best practices for equity in mathematics classrooms. The paper will highlight three features in particular that emerged from this combination of user feedback and academic research: (1) Interactions, (2) Who's Recently Presented, and (3) Student Work Photos.

(1) Interactions

Have you ever used a word that just does not fit? You cannot put your finger on it, there is nothing wrong per se with the word; it just does not exactly point in the right direction. The Interactions feature of the prototype suffered from this problem for quite some time in the design process when it initially began its life as "Engagement." Starting in Phase One, there were many quantitative and qualitative data points that told the project team that a feature that had something to do with student engagement and participation would solve challenges many teachers experienced. But engagement turned out to not quite embody the need to be solved.

During the initial TAB interviews, Nia[4] said, "I think with this, with the five practices, it might give me more leeway on *getting* those kids who are really, really struggling" (Pre-Interview, 6/3/2020). This comment brought two things front and center to the conversations about the prototype. The first is "those kids"—which was an input to the team in recognizing the danger of creating an implicit categorization of certain students who struggle. The second aspect is that it is hard to bring students who may not cognitively understand the mathematical activity immediately into the conversation.

4 All TAB member names are pseudonyms.

Another TAB member, Iris, said she had been wanting to try something she had heard at a conference where a speaker talked about tracking "the way conversations were occurring at a table ... was it a comment, was it a correct comment ... something that adds some kind of qualitative measure of what that contribution was to the conversation ..." (Pre-Interview, 6/2/2020). Shannon also suggested that, "Progress monitoring tracking for like every student ... or groups of students ... because that's been my professional growth plan for the last two years is working on progress monitoring and like monitoring every student" (Pre-Interview, 6/2/2020).

All six TAB members talked during the pre-interview specifically about how the prototype would help them if there was a component that would enable teachers to track student participation and would remind the teacher of who has spoken and who has not spoken. Quantitatively, four of the six TAB members denoted challenge #9, "Keeping track of group progress —which groups you visited and what you left them to work on" (Smith & Sherin, 2019) during the *Monitoring* practice as a challenge.

These data convinced the project team that this was an important need to focus on solving. During Sprint A, the team started designing with the *Monitoring* practice, initially assuming due to the number of challenges present in this practice that this is where the prototype would thrive. The assumption based on the initial data was that *Monitoring* afforded the greatest opportunity to help teachers implement the 5 Practices. During Sprint A, Interactions started with what the project team referred to originally as a "strength icon" that would show up if a teacher marked a student as not "participating." In figure 15, notice Sandra and Nicky have these icons.

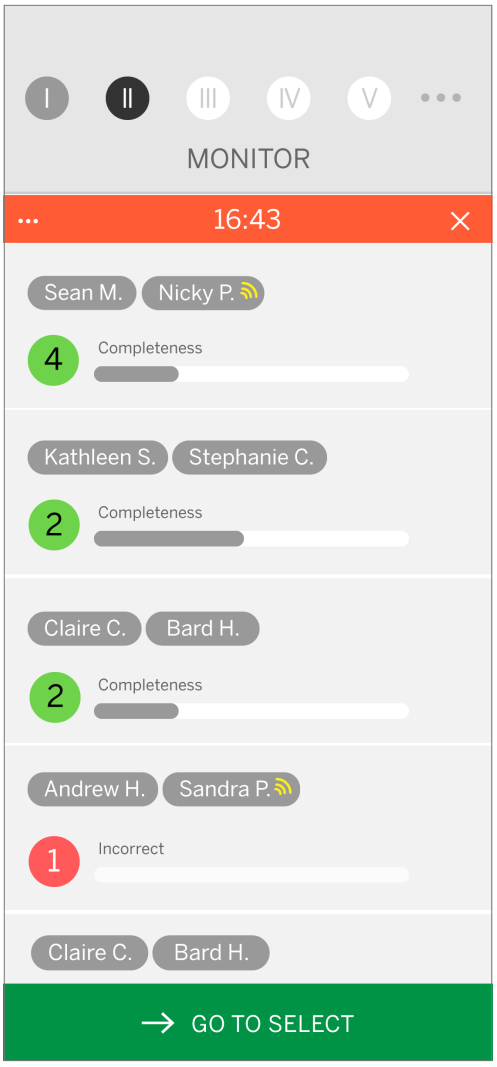


Figure 15: Initial concept for engagement from Sprint A

The project team hypothesized that if a teacher saw that icon by a student's name, they would elicit student participation or engagement individually, in a group setting, or when the class as a whole engaged in the *Connecting* practice. The advisors and TAB members universally appreciated the notion of a signal for student engagement. However, the signal was initially confusing. For example, a TAB member acknowledged the confusion of the symbol, stating, "I was kind of thinking that these would be the ones that you've already, like, they've already voiced their opinion" (Sprint A Interview, 6/24/2020). An advisor suggested that "color could be one way we designate that [engagement]" (Sprint A Interview, 6/25/2020). This led the team to keep the idea of an icon for engagement or participation but to replace it with a yellow circle around student initials.

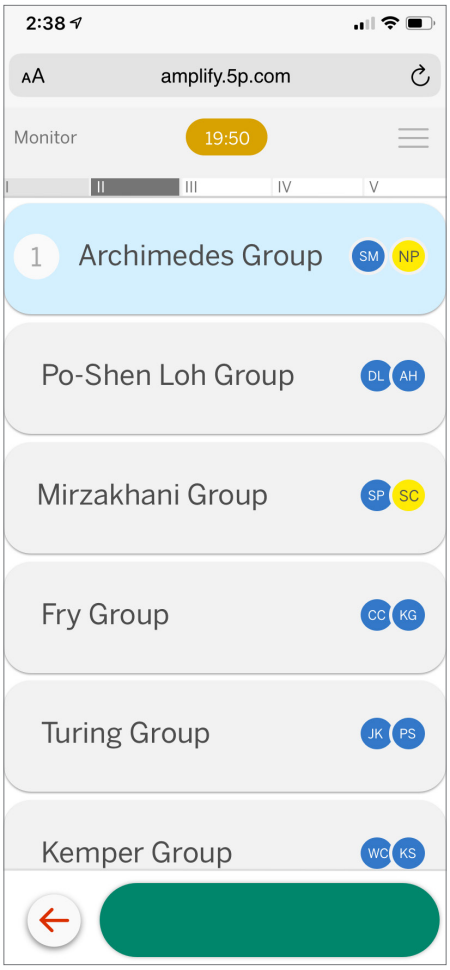


Figure 16: Next iteration of engagement, from sprint B

In this iteration, students with a blue circle had been marked as engaged while students in yellow had not—everywhere the student's initials appeared in the prototype. The project team's intention with the icon was to implicitly call out implicit bias. Seeing the students who had yellow circles would allow the prototype to, in a non-threatening way, signal to the teacher "Hey, go talk to this student, see what's up!" Shannon voiced this idea: "The yellow is now indicating I need to talk to them more" (Sprint B interview, 6/24/2020).

Further, the teacher could click to open an "Engagement" drawer. Here the teacher could click one, two, three, or four check marks indicating the degree to which each student was engaged, which would then tell the prototype whether to highlight the student's initials in yellow or blue.

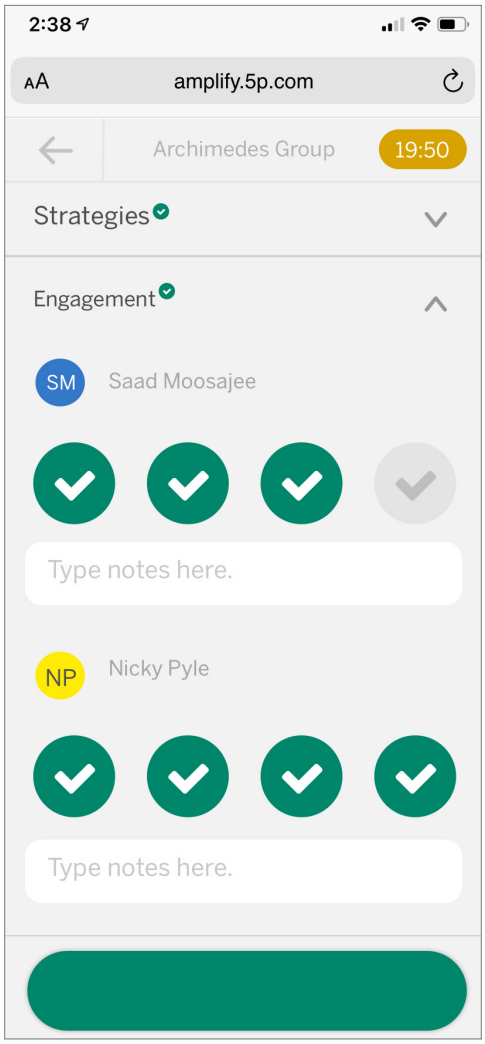


Figure 17: "Engagement Drawer" from Sprint B

The team, based on consistent positive feedback for this idea from teachers who felt like this could be flexible enough to be usable with the different ways they track participation, turned attention to other features for several sprints, keeping this idea consistent while other features around it changed. Meanwhile, the team was engaged in background research into equity in mathematics class—research that proved fruitful when Shannon asked, "So like, wait, what does engagement really mean?" (Sprint E Interview, 8/19/2020).

During Sprint E, the team then talked at length about the word "engagement." There's a Buddhist saying that goes, "the finger pointing at the moon is not the moon." The word *engagement* is just that, a word. But what meaning did this word point to? And did it point to the meaning we wished to embrace in the prototype? Or did it embody implicit biases, systemic -isms, and an identification that if a student is not learning, it is their fault? Reviewing feedback from previous sprints led the team to conclude that this was indeed happening. In Sprint B, Guillermo said, "I would use this to grade them on the effort they put forth ... it's more of like a behavior management strike" (Sprint B Interview, 7/8/2020). A careful analysis revealed that this interpretation of this feature was common across the entire TAB.

Moving away from the user feedback and into the academic research led us to the concept of "interactions." Joseph, Hailu, and Matthews (2019) suggest the way to address the devaluation of Black women throughout history and, hence, in classroom settings, is to realize mathematics is "particularly susceptible to inequities in the classroom because it is shrouded in a myth of objectivity" (p. 135). The key to addressing the inequities in the classroom lies in teachers explicitly encouraging interactions—between students, teacher, and the math, thus providing opportunities for empathy and others' perspectives. Archer (1993) and Jackson and Wilson (2012) also address the importance of interactions, particularly teacher-student interactions, in which the responsibility lies with the teacher to teach and reinforce interactions to promote equity. Interactions widen the scope of how a sense of belonging occurs. Interactions can be with anyone or with mathematics and interactions can denote a relational sense of being in the classroom where the teacher, not just the student, is responsible for embracing the discourse for all students (Archer, 1993; Jackson & Wilson, 2012).

With the realization of the importance of conceptualizing student learning and discourse in terms of *interaction* rather than engagement, pivotal aspects changed in terms of this feature of the prototype: streamlined interactions, more use of the interactions component, and explicit reminders to the teacher to include certain students. Sprint F therefore welcomed interactions into the prototype more prominently displayed toward the top of the screen while *Monitoring*. An advisor agreed with the assertion that the bar being on the top of the page for the prototype draws more importance to the feature, stating, "Yeah, it's, it's kind of that like this being at the top it signals to me, that's the first thing I should be paying attention to" (Sprint F Interview, 9/3/2020).

Rather than opening another screen to click between one and four check marks, the teacher could now click on the student from the main screen with a binary response to mark engagement. The prototype will also suggest to "make sure to include [name] in the conversation" when a student has not been marked as interacting in the past two weeks—a suggestion to combat implicit bias from a place of calm peace (Lindsey, Nuri-Robins, Terrell, & Lindsey, 2019). The prototype does not know if the student was not called on because she has not been present, is quiet by nature, is in a part of the room the teacher normally does not circulate toward, or if it is because the teacher defaults to calling on white boys over Latina girls. What the prototype does know is if the teacher has marked a student as interacting or not in the past couple weeks. In this way, the prototype is not passing judgment but instead is noticing patterns of interaction or lack thereof and nudging the teacher's behavior accordingly. With approximately 30 students in a classroom, it is impossible for teachers to keep accurate track of these patterns without technological assistance. The prototype allows teachers to see what they could not have seen and which may or may not point to implicit bias, and take action on it.

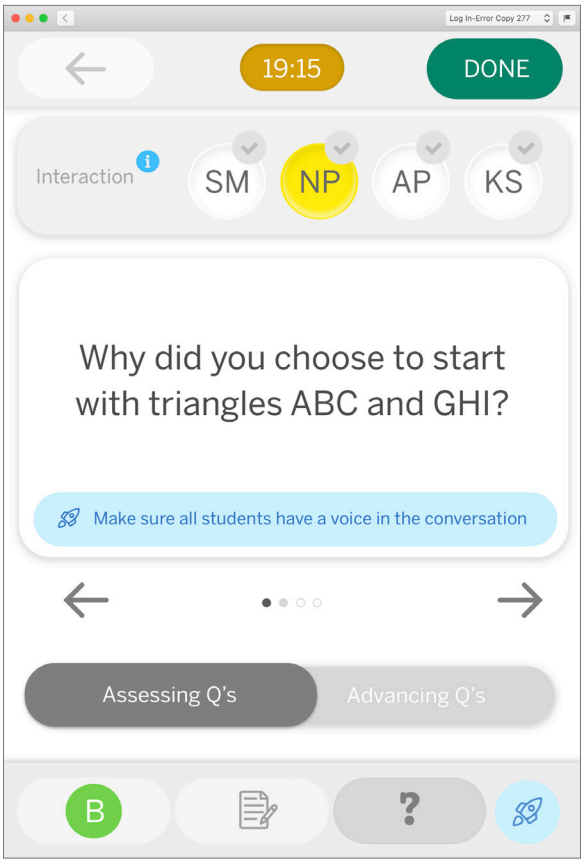


Figure 18: Final iteration of Interaction

Facts and concrete aspects of situations do not determine how someone acts and interacts. Instead, "the way someone acts in a situation is dependent on how they define a situation" (Nank, 2011b, p. 19). What this means for students in classrooms is that they act, react, and interact differently whether we are in an one-on-one, group, or whole class setting. Who is and is not in a student's group could affect interactions as could a thousand other variables the student uses to define the particular situation. In this sense, a student could never say a word in a whole class setting but be the model of interaction once they are in a specific small group. Therefore, it is important for teachers to understand overarching patterns of interactions as well as situational patterns of interactions.

Most of the TAB members pointed out, rightly, that this feature does not guarantee that the teacher will act on the information. However, it ensures the pattern is literally in front of the teacher at every stage. For this reason coupled with the potential desire to provide more details in relation to the nature of the student interaction, the prototype provides a place to make notes as well. TAB member Deja reinforced the project team's assumption that teachers must ultimately choose what to do with the patterns of interactions the prototype highlights when she said, "I do think there's still some teacher responsibility there and we still, you know, just have so much control as a teacher to either do it [foster student interaction] or not do it" (Post-Interview, 10/7/2020).

Guillermo also liked the interaction feature. The interviewer asked, "Are there any other features in the prototype that you think could support pedagogy?" Guillermo immediately responded, "I think the interactions one does. If you're looking at, you know, your level of teaching and your level of engaging your kids" (Post-Interview, 10/6/2020). Interactions assumes it is the student AND teacher's responsibility whereas engagement places the onus solely on the student. This sentiment was a theme across the TAB and reflects a swing away from teachers viewing student participation as a barrier to student discourse and towards a specific action they could take to elicit interactions to achieve more equitable discourse. Shannon pointed out that the interactions feature could help teachers remember who to interact with: "It's going to flag that you have not interacted with that student. So you need to interact with that student. So it's not even just like, that student didn't do his job, but like, maybe you didn't do yours. So like, go talk to them" (Post-Interview, 10/7/2020). In this quote, Shannon sums up the project team's intention of the Interactions feature and why, based on research, the feature is denoted as "interactions" rather than "engagement." The quest to find the best word to convey the meaning the project team had been seeking was successful. This sentiment was a theme across the TAB and reflects a swing away from teachers viewing student participation as a barrier to student discourse and towards understanding a specific step they could take to elicit interactions to achieve more equitable discourse.

With the interaction feature, the teacher could click on students' initials in the prototype, checking off who was interacting at different times while taking notes for the particular student. Teachers then would have data for interactions in group settings, while the group is presenting, and while the whole class is engaged in discourse. This embraces the assumption related to the theory of Definition of the Situation (Bellack, 1978; Lauer & Handel, 1977; Nank, 2011b; Waller, 1961) in that people act differently depending on the context. Students may interact more or less given the milieu of individual, group, whole class discussion, or while presenting a strategy to the class. One consideration the project team took in creating the interaction feature is that any teacher, or person in general, would likely not respond favorably if you were to point directly to their implicit bias. If the prototype informed the teacher in the middle of a classroom setting that they were neglecting the Black girls in the classroom, the possibility exists for the teacher to stop using the prototype. Instead of directly pointing out that the teacher is ignoring certain students or groups, the prototype tells the teacher that a student or group has not interacted in a while and encourages the teacher to be inclusive in regard to the student or group.

(2) Who Has Recently Presented

Individual student interactions are critically important in mathematics classrooms. So too are group interactions and opportunities. The Who Has Recently Presented feature aims to highlight that importance, as well as to support teachers' struggles with deciding what work to share during *Selecting*, *Sequencing*, and *Connecting*. The project team heard early on in the sprint process that this decision point was a make-or-break moment for the use of the 5 Practices in the classroom and that the teachers were interested in tracking students who have presented. This was even an exciting feature observed in several paper-and-pencil *Monitoring* tools, although the majority of the TAB expressed that this was too difficult to do with consistency. Deja shared, "I just can't carry around a stack of papers" (Pre-Interview, 6/3/2020).

The project team studied how teachers currently decide who to ask to share in order to fully understand why this is a barrier to equitable student discourse. TAB member Nia stated, "So more times I will choose somebody that you know, is on the right path. So something that's correct first" (Pre-Interview, 6/3/2020). Other TAB members concurred. Guillermo said, "I go to the group to share that has completed the strategy that aligns with my planning … when I wrap it up, it becomes, 'which strategy should we have used?' which is not the right way to teach math" (Pre-Interview, 6/3/2020). This told the project team that equality of opportunity may not be the indicator for the teacher for who to ask to present, but rather the ability to come to a correct solution. Hence, the project team thought it important to track with the prototype who has presented to monitor for equality of voice.

The project team also knew from the beginning that two TAB members were focusing on equality of student voice as an overarching goal. TAB member Mark said, "...my big thing that [at our school] we've been focusing on is equity of voice. So that's why I also have been using that [*Monitoring*] chart, because then I can see, oh I'm always calling on this person. I mean, in your head you kind of know, and then you're kind of thinking, well, I think I'm not calling on them as much, but you could totally be still calling on the same people" (Pre-Interview, 6/3/2020). What stood out most in this quote was the affirmation that sometimes teachers, and people in general, think they are doing one thing but are in fact not doing so. Mark didn't mean to enact implicit bias or not call on a particular student or ignore a group of students but sometimes these things happen.

During the Sprint D interviews, some TAB members questioned whether or not the prototype was showing them if students or groups recently presented. While looking at the student engagement part of the prototype, Mark stated, "So it shows me Nicky's [student] previous ... oops, shoot, sorry, group work? Or didn't he present or maybe he did present?" (8/5/2020). Mark was trying to figure out if a yellow icon denoted if students were engaged or if they had presented to the class. This helped the team realize that the prototype needed to treat presenting separately from interacting.

First, the project team thought about different markers for student engagement (at this point, the project team had not yet gone through the above considerations about this terminology). The project team incorporated multiple levels of interactions including students working in their group, students involved in whole class discussion, students who have presented to the class, and students who have engaged with mathematics.

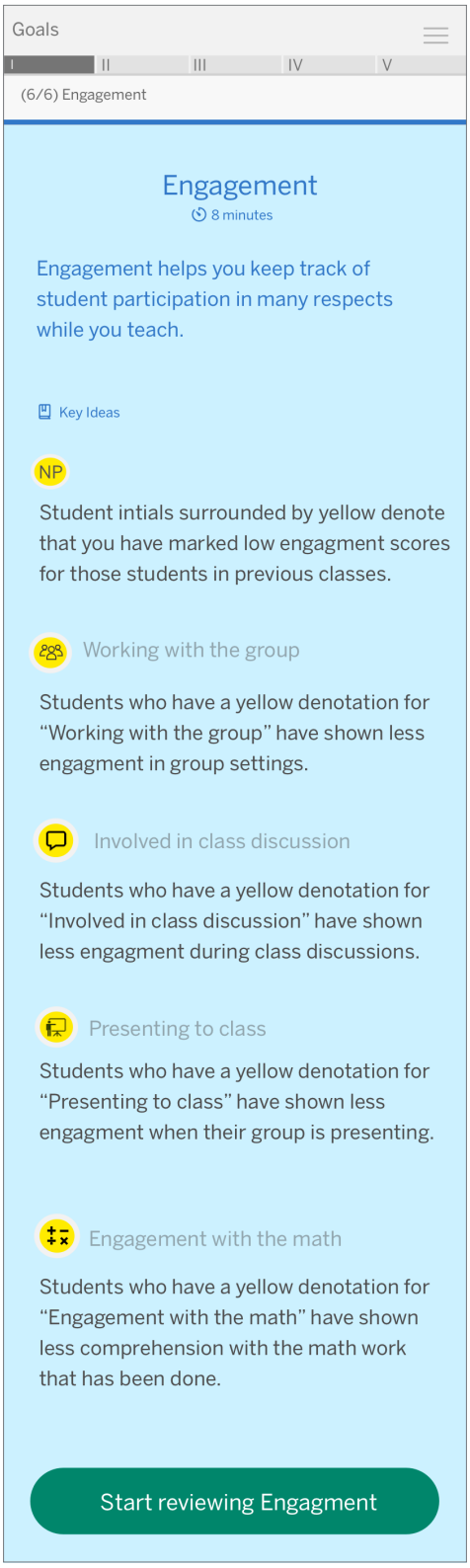


Figure 19: Adding the concept of "presenting"

Deja alluded to different facets of engagement when she stated, "Now this is interesting. So there are certain students who are showing less engagement in group settings. I'm going to assume that's a small group setting, not a whole group?" (Sprint E Interview, 8/19/2020). Guillermo said, "So you have your kids that have low engagement with the class, but then I like the differentiations of the engagements because not every kid is going to share out in the whole class discussion, but they might be engaged with their group or with the math" (Sprint E Interview, 8/19/2020). This spoke to the need for the prototype to possibly present different types of engagement, including individual, group, and whole class engagement. However, this level of detail proved too complicated for the TAB members to imagine themselves using in the middle of a busy classroom. Nia shared, "It's just too many different things to think about when I'm with the kids" (Sprint E Interview, 8/19/2020). So during Sprint G, the team settled on a simple way to integrate group interactions with a different yellow symbol to denote groups that had not presented in the past two weeks.

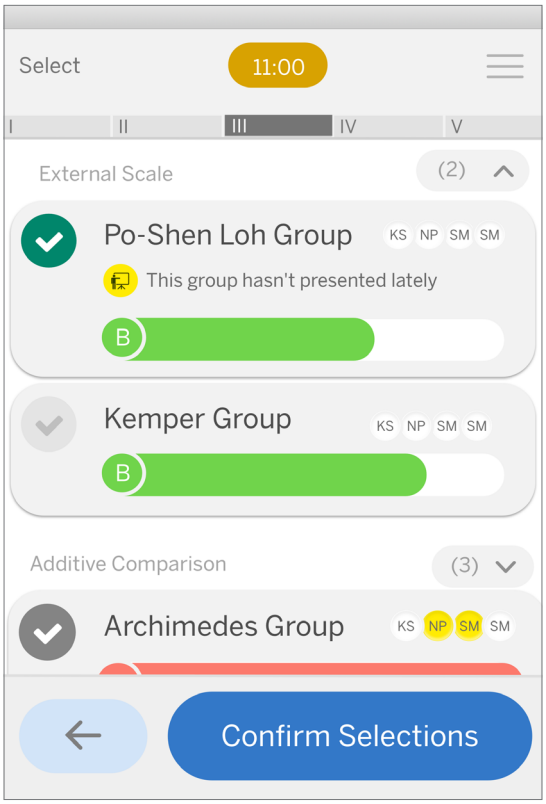


Figure 20: The final iteration, showing information about a group's history of presenting

TAB members responded well to this feature. While reflecting on the interaction component of the prototype, Nia stated, "On the prototype, it does have that feature of where you, again, keeping track of the kids ... it kind of helps me with kind of getting them in that mode [of interacting on multiple levels]" (Post-Interview, 10/7/2020). Immediately before saying this, Nia talked about how the Who's Recently Presented component of the prototype could facilitate more student confidence in mathematics. With more opportunities to share their ideas with the class, comes greater buy-in, a better classroom culture, and a growth mindset that helps students understand they are all mathematical learners and that their voice and perspectives matter.

(3) Student Work Photos

The idea of encouraging teachers to capture student work was constantly present in the prototype since the beginning. While engaging in the practice of *Anticipating*, teachers try to think of productive approaches and points of growth[5] students may use. However, no one teacher can *Anticipate* every strategy—and one of the top challenges for our teachers of priority students was moving beyond the way they as teachers solve a problem. The beauty of classrooms in which all students have a voice, feel valued, and are encouraged to explore mathematics is that students think of wonderfully diverse strategies that the teacher never *Anticipated*. Indeed, research has shown that priority students often engage in productive approaches, but in non-traditional ways that are not recognized by their teacher (Khisty & Chval, 2002). Many times, these strategies are enlightening and propel other students toward further procedural and conceptual understanding of the mathematical concept at hand and are therefore worthy of capturing.

5 The prototype describes productive approaches as, "Mathematically sound strategies students can use to make sense of and solve the math problem. These are strategies you should actively look for and Sequence for the 5Ps activity." Points of growth are described as, "These are common mistakes, misconceptions, and incomplete understandings students may use. Address these points of growth individually or with groups. Sequence these strategies if five or more students or two or more groups are using them. You may also want to sequence the strategy if you notice the Points of Growth being used in previous days/lessons. We advise calling only on students who understand this is a Point of Growth so the experience does not negatively influence their perception of themselves as learners of mathematics." The project team felt words were important as they shape our conceptions of mathematics, ourselves, and others. Particularly, the project team spent time finding a better phrase than "misconceptions" because every mistake, every thought, every "misconception" is an opportunity, a point in time to grow, hence the phrase "Points of Growth."

When this happens, the teacher needs a way to capture the unanticipated approach. TAB members Nia and Deja both brought this up during the initial interviews before Sprint A began. Nia stated, "Sometimes I come up with two or three strategies, and the kids kind of surprise me with something and I'm like oh, really, I didn't really think about that ... so while I'm teaching, I'm like okay, yeah, that actually works" (Pre-Interview, 6/3/2020). Deja said, "And I want to take snapshots and build ... this kind of picture of what students are thinking and then present it to [the class]" (Pre-Interview, 6/3/2020). TAB members continued to share in future interviews about how the ability to take a picture saves time in the classroom and also allows them a space to accurately remember what the student wrote while being able to share the strategy. During Sprint A, the project team designed such an ability.

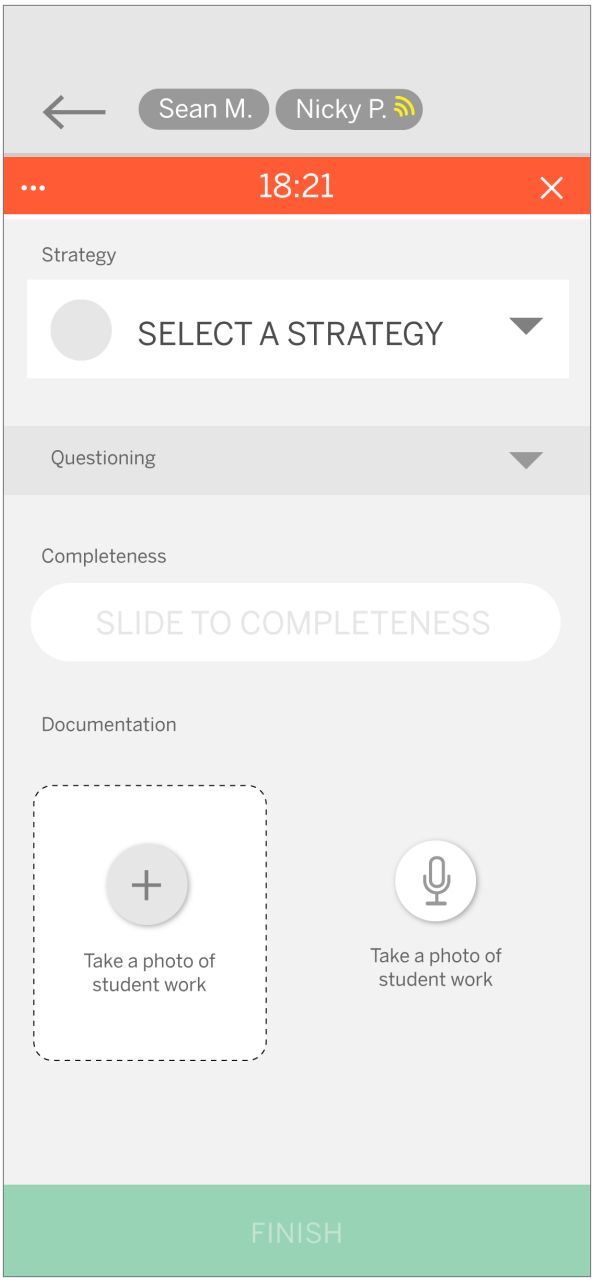


Figure 21: Taking pictures as it appeared in Sprint A

The intention from the team was for teachers to take a picture of student work to look at later and also to have at their disposal to sequence unanticipated productive approaches or points of growth. Feedback was mixed regarding the recording of student explanations so that portion of the prototype was abandoned. For example, Deja commented on the Photo feature saying, "I really like the idea of having, of being able to take snapshots on my phone" (Sprint A Interview, 6/24/2020). One advisor's interpretation of the Photo feature was to project student images; they asked, "Can pictures of student work then be projected from the prototype to a SMART Board?" (Sprint A Interview, 6/26/2020). Another advisor's interpretation of the Photo feature and its prominent placement was that the intention was for the teacher to take a picture of every students' and groups' strategy, which would take far too long to do. Due to this feedback, the project team moved this feature to a less prominent location so that teachers could use it if needed to value students who develop strategies teachers and curriculum writers did not anticipate.

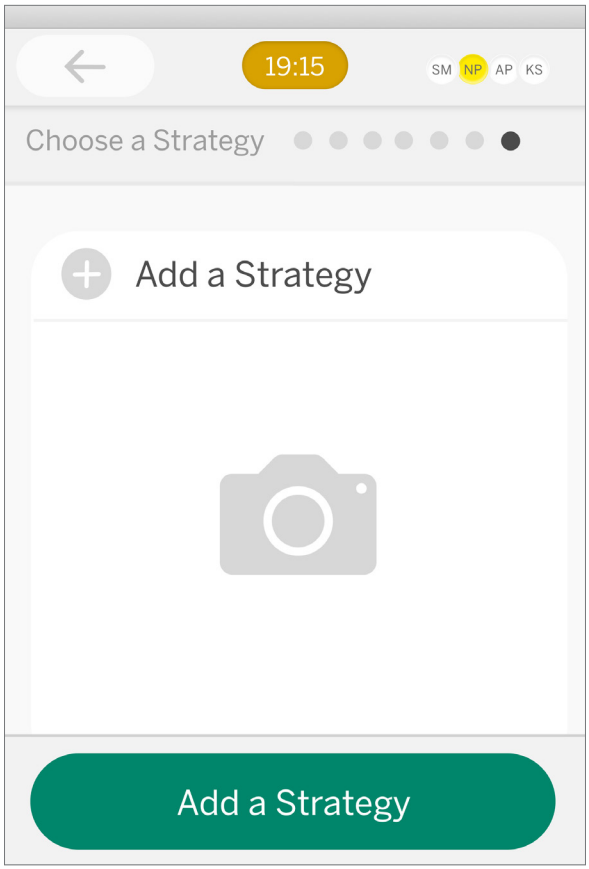


Figure 22: Final iteration of taking photos

Capturing unanticipated strategies from students also evolved into a desire to capture unanticipated strategies from the teachers. Guillermo stated, "This would be strategies that you *Anticipate* them using. So these will be ones that you've [teacher] drawn out and taken a picture of" (Sprint A Interview, 6/24/2020). As such, the Photo feature was also added for teachers to use with their own strategies during *Anticipating* in addition to *Monitoring*.

The TAB members appreciated the Photo feature. During the last interview, Guillermo stated, "I'm just better digitally than handwriting stuff out. And it's faster when you can, you know, check a strategy than trying to jot down on a notebook, what this good strategy was, or having to draw pictures of all your different groups ..." (Post-Interview, 10/6/2020). Shannon echoed this sentiment, "I could snap a shot of the student work, and then ideally be able to broadcast that on the screen in front of them ..." (Post-Interview, 10/7/2020). Nia also commented positively on the Photo feature, suggesting they could even use it during team meetings to take pictures of strategies their team came up with during meetings. TAB members never wavered from their initial feedback that providing space within the prototype to place unanticipated strategies was valuable for equality of student voice.

Iris also commented on the Photo feature. She alluded to the longevity of the data she could have gathered in her first five years of teaching as she said, "Yeah, I think the inclusion of the snapshots when I'm actually working with the groups, I think, is what I was missing and was hoping for. Because again, I'm thinking ... in the long run ... I could have had five years worth of students' strategies somewhere, but I don't. And I think having that, while I'm already using it with students I think could have been helpful in the long run" (Post-Interview, 10/7/2020). The photos could also provide a reflective avenue for teachers to see the influence of curricula and pedagogy over the years on different unanticipated strategies.

Theme 2: Support Pedagogy and Curricula: Help teachers use productive 5 Practices approaches

At the core of the second theme is the realization that there are many moments when a teacher asks themself, "Am I doing this well or am I blowing it?" The benefit in asking such a reflective question is that teachers can begin learning how to do and be better. The detriment is that the answer might be, "No, I am not doing this well, so I need to never do it again." There are many decision points teachers experience while using the 5 Practices—and every one of these points affords an opportunity for teachers to take the path of never doing it again.

One of the broadest trends in the pre-interviews in Phase One was a sense about the 5 Practices of, "it's wonderful but ..." followed by a reiteration of the different challenges and barriers. This sentiment was the basis for the theme of Supporting Pedagogy and Curricula as there is a learning curve and teachers need to be supported while embracing a growth mindset (Boaler, 2016; Dweck, 2006) as they learn how to facilitate equitable discourse through the 5 Practices. This section provides insights on two features: (1) Helpful Hints and (2) Key Ideas, which are designed to support teachers in overcoming barriers and work through these pivotal and transformative moments in the classroom.

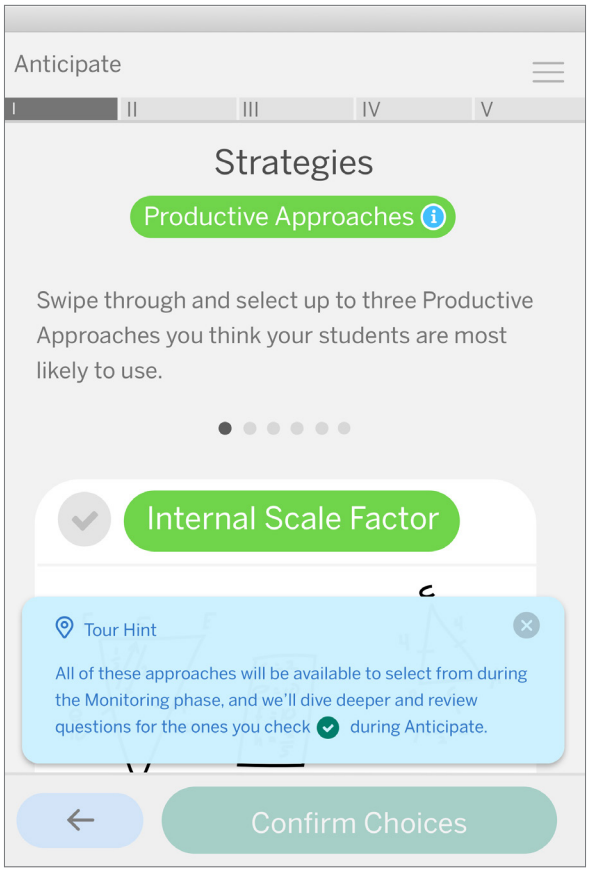


Figure 23: A helpful hint

(1) Helpful Hints

Helpful hints are meant to help teachers enact the 5 Practices with fast on-the-spot, in-the-moment assistance at key decision-making points. Their content helps teachers learn about the 5 Practices and how to effectively implement them, providing support at the main teacher decision points throughout the 5 Practices, including information to help teachers understand the mathematics of the task at hand.

Helpful hints evolved as a direct response to user feedback. Whenever there was a decision point where the teacher may choose to keep going or stop the use of the 5 Practices in their classroom, the team considered if a helpful hint might be useful. A comment an advisor and interviewer made during the Sprint E interview succinctly captures the intent of the helpful hints. The advisor was reflecting on how they coach teachers in classrooms on the use of the 5 Practices and said, "Definitely. Yeah, in an ideal world, that is exactly what's happening. It's taking the stress of those decisions and being there and saying [to the teacher], well, based on this, how about this? I think this would be best" (8/20/2020). The interviewer responded, partly with a joke, "To your point ... it's like having your [advisor] in the classroom with them [teacher] ... we're still working on the [advisors] caricatures that can pop up at key moments [to help the teacher]" (Sprint E Interview, 8/20/2020). While caricatures of Peg Smith and Michael D. Steele were a bit out of scope for the project, helpful hints could serve in their place.

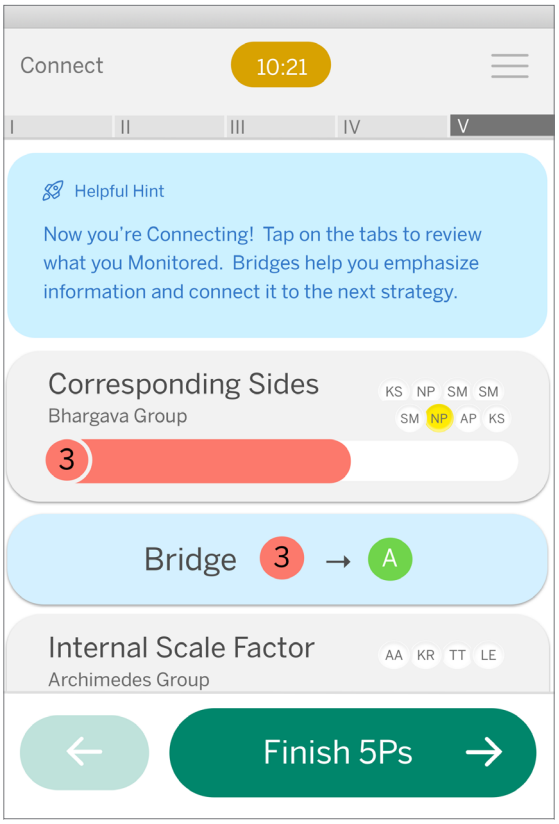


Figure 24: A helpful hint

Prior to Sprint C it was clear that different locations in the prototype would require a prompt for the teacher to interact and engage with the content of the prototype. For example, during Sprint A an advisor suggested there were screens in the prototype "that if there's no interactivity other than move on, then there might not be engagement with the content" (Sprint A Interview, 6/25/2020). Helpful hints first materialized in Sprint C. For example, in the *Connect* practice, a helpful hint clarified that the prototype chose groups and *Sequenced* them based on what the teacher marked during the *Monitor* practice.

Helpful hints were iterated further in Sprint D in thinking about how to provide collaboration opportunities for teachers via the prototype. Although this capability did not come to fruition due to time constraints, it is a possibility for future iterations of the prototype. This specific helpful hint anonymously tracks how many teachers choose specific strategies and helps teachers move beyond the way they solve a problem, one of the main challenges for our teachers of priority students.

TAB members consistently perceived the helpful hints as being useful. Iris said, "That hint is very powerful. I think about pods of students when *Monitoring*. When I am trying to be the best teacher-version of myself and I try to monitor for equity, that helps remind me of that" (Sprint G Interview, 9/16/2020). Shannon shared in the final interview "I don't want the hints to help me speed it up, I want it to help me implement with fidelity, the best instruction with the 5 Practices, not say, 'come on, let's go,' other parts in the app are for making it go quicker, and the hints are not that and I'm okay with that" (Post-Interview, 10/7/2020).

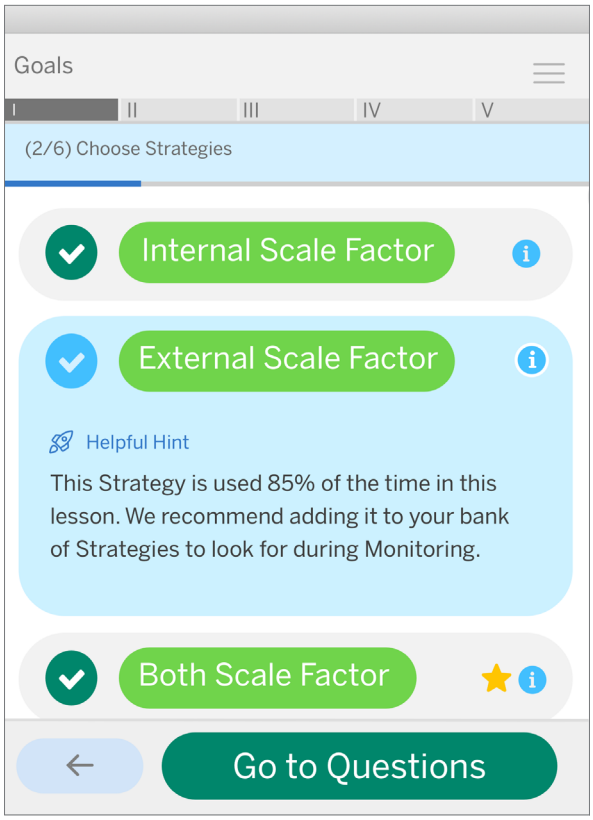


Figure 25:A helpful hint

(2) Key Ideas

Key ideas similarly support teachers' acquisition of pedagogical and content strategies for overcoming their challenges. Key ideas is another iteration within the prototype to help teachers choose to stick with the 5 Practices when they experience that critical decision point to embrace the complexity of student-centered high quality discourse in the classroom or default to less meaningful and less collaborative pedagogy. Unlike the helpful hints that appear during class time at point-of-use, key ideas are a bit longer and more in-depth and appear only in the *Anticipating* section as they are meant to be read and understood while the teacher is planning their lesson.

The key ideas section was first presented for feedback during Sprint E. The key ideas icon is not a link but instead operates as a signpost for the user to realize the text is important in order to understand the pedagogy and content woven throughout the 5 Practices and, hence, the prototype. In Sprint E, the key ideas feature was first used to denote the difference between different questioning strategies teachers can use to help involve all students, one of the top challenges faced by TAB members.

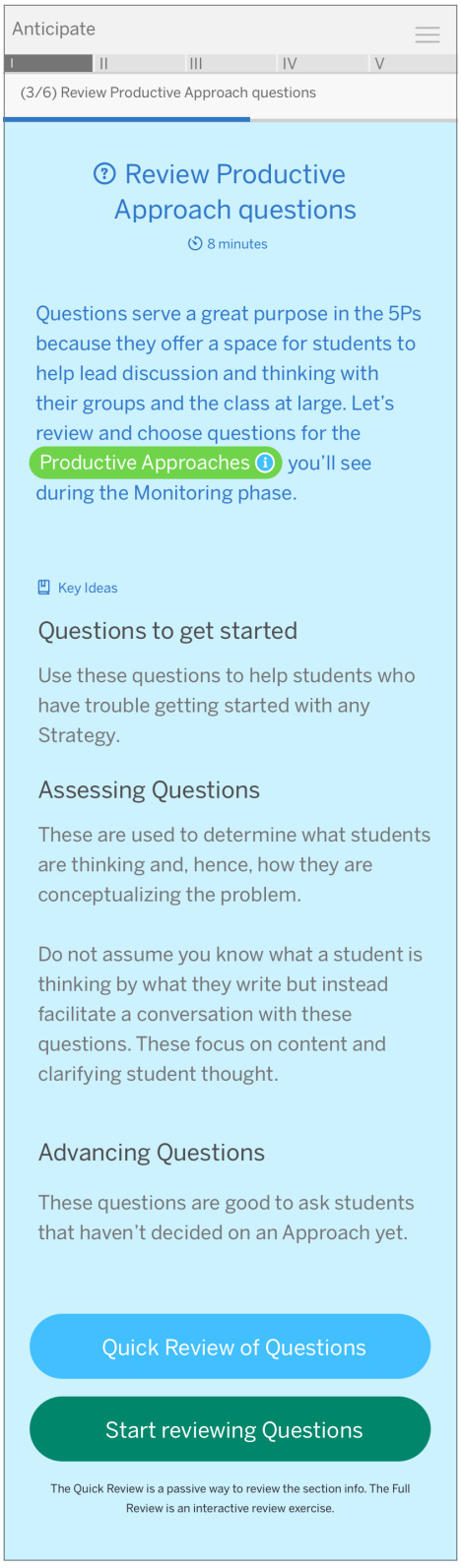


Figure 26: Key Ideas about Questions

Also in Sprint E, teachers got to experience using a key ideas section to understand the different types of strategies students may take while working on a high-level task: Productive Approaches and Points of Growth.

Talking about key ideas, Nia shared, "I like the fact that the information is all in one place. I can't stress that enough. I usually have five books in front of me and I can't remember where I put them. I like that it has everything in one spot. It's just toggling [to] what comes next" (Sprint E interview, 8/19/2020). Based on the solid feedback for key ideas, this feature stayed consistent in the prototype. The helpful hints and key ideas are meant to help the user make sense of the mathematical knowledge, curricular philosophy, and pedagogical moves baked into the prototype, helping teachers overcome their challenges to engaging in the 5 Practices.

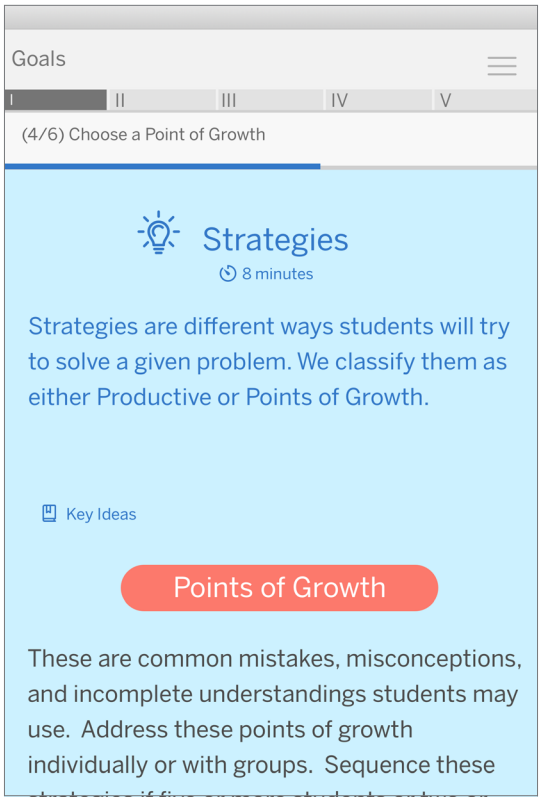


Figure 27: Key Ideas about Points of Growth

Theme 3: Save Teachers Time: Maximize in-classroom interactions and practices

Anticipating

"Teachers very rarely call aspects of *Anticipating* a challenge—but if they *Anticipated*, they wouldn't have all the challenges they have in the other practices" (Pre-Interview, 6/15/2020). One of our advisors told us this the very first time they met with the project team. At first, the team was not sure about this; users were indeed saying that their main challenges were during the in-classroom practices, and so the project team started there. But as the project team iterated ideas to support those practices, a problem kept coming up: teachers did not have time to process and use those ideas during class. The top challenge for the TAB teachers was already "Running out of time" before they saw any new ideas to try. For example, helping teachers with Challenge 15—Determining how to sequence errors, misconceptions, and/or incomplete solutions—seemed to be well-supported through providing an overview of how those points of growth connected to the learning goal of the task. TAB members appreciated that information. Guillermo said, "This helps me make sense of the math—why the strategies are what the strategies are is huge, is super helpful" (Sprint C feedback, 7/22/2020). At the same time, they struggled to imagine being able to make use of this information in the classroom. Deja said, "I just want a quick reminder … all of these things to me would be helpful for planning, but … it's a heavy cognitive load [during class]" (Sprint C feedback, 7/22/2020). The project team realized in order to save time during class, teachers needed help before class started, and so the prototype needed to support *Anticipating*.

When you picture *Anticipating's* place in the 5 Practices, think of a wedding reception. The clothes, music, food, tables, decorations, toasts, first dance, and a thousand other details are packed into a small amount of time. If all goes well the day of the wedding, it is generally due in large part to the planning that happened in advance. Applying this analogy to the 5 Practices, the *Monitoring*, *Selecting*, *Sequencing*, and *Connecting* practices are all invited to the party but it would not go well if the *Anticipating* practice was missing. All of the work before the teacher is standing in front of the classroom critically influences how the lesson goes. The better the *Anticipating* practice, the more likely that high quality mathematical discourse will occur. If a 5 Practices lesson goes wrong, the origin usually tracks back to *Anticipating* in some way, but sometimes teachers simply run out of time to *Anticipate*, or struggle to think mathematically about all the different aspects of the task, such as strategies, questions, and *Sequences*.

When the project team reviewed what the TAB said about the barriers to and important aspects of *Anticipating*, they had had a lot to say. TAB members Guillermo and Mark talked about the importance of *Anticipating* during their initial interviews. Guillermo stated that when he does *Anticipate*, "you're more prepared for those off-the-wall strategies that a kid might come up with because you've already thought through a lot of them. There still might be one or two strategies that come up that you're like, 'I never thought of doing it that way' but [at least] you don't have four or five of those" (Pre-Interview, 6/10/2020). Mark also talked about *Anticipating* when he said, "I'm always trying to *Anticipate* what the kids are gonna say. I'm not always good at that" (Pre-Interview, 6/3/2020). Mark also went on to state what other TAB members periodically echoed: *Anticipating* can be difficult and takes time. Further, some of the TAB members, such as Iris, were the only eighth grade teachers in their school and therefore when they engaged in *Anticipating* had to do it alone, without the perspectives and strategies other colleagues may have thought of.

Given the lack of top challenges in the *Anticipating* practice, the first time the project team included *Anticipating* was a very simple iteration: the teacher selects their task, peruses possible strategies built into the prototype, and continues immediately to *Monitoring*.

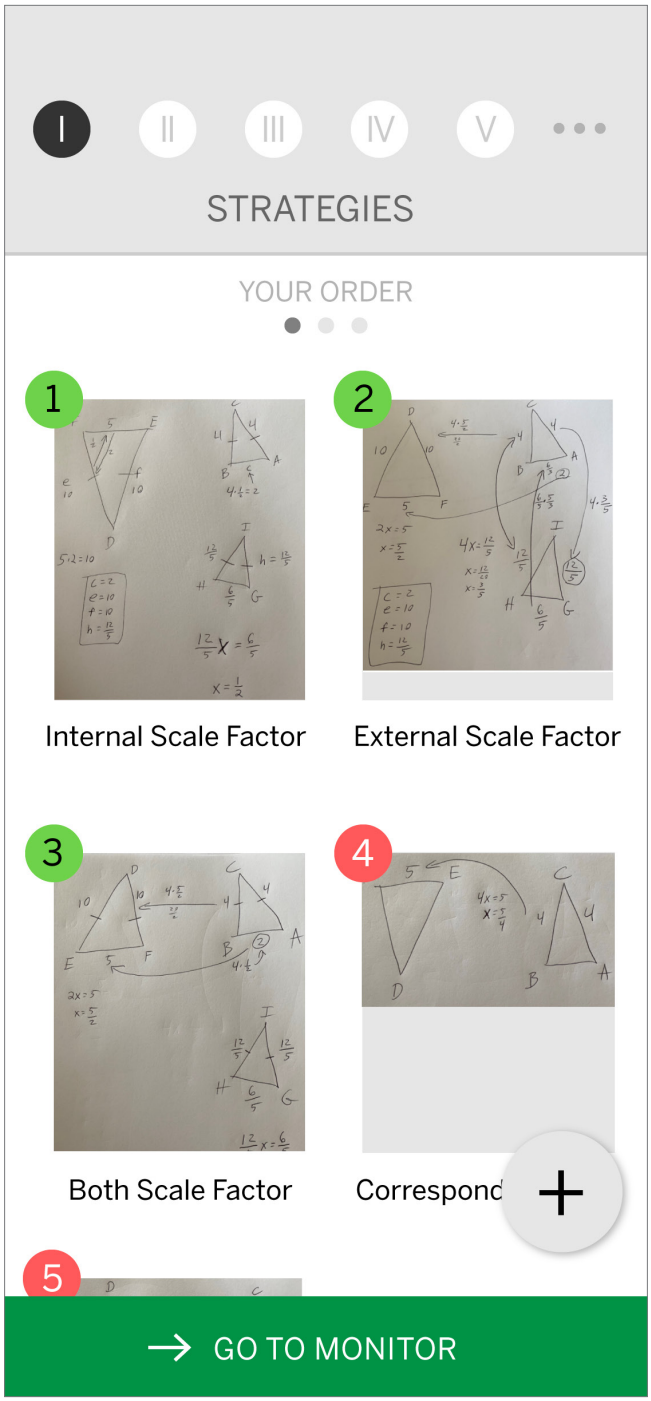


Figure 28: *Anticipating* in Sprint B

During the Sprint B interviews, the project team specifically asked the TAB members what they would like to see during the *Anticipating* practice. Deja wanted to be assured that whatever was in the *Anticipating* part of the prototype would be available in the *Monitoring* part of the prototype. Deja said, "maybe from the *Anticipating* phase ... with each strategy you would kind of hope to see it again or expect to see it again to help you make those decisions" (7/8/2020). Deja also noted that *Sequences* would be nice to have in the *Anticipating* part of the prototype stating, "And perhaps these [*Sequences*] might be *Sequences* that you've already seen in the *Anticipate* phase ... you know, things that the prototype had already suggested that you had seen in the *Anticipating* phase and then it's taking that and kind of grouping or *sequencing* your groups based on that" (7/8/2020).

Nia suggested, "Okay, what would I hope to see if I scroll down to 'no strategy selected,' what would I hope to find there in terms of information? Probably *Anticipated* strategies I can probably use ... So perhaps in your *Anticipation* prep, you've seen that .... I would like to *Sequence* the strategy in [*Anticipating*] somewhere" (Sprint B Interview, 7/8/2020).

Mark, Iris, and Guillermo all spoke of the importance of the *Anticipating* practice for informing and making easier the *Monitoring*, *Selecting*, and *Sequencing* practices, specifically pointing to the importance of *Anticipating* how strategies will be potentially *Sequenced* in the classroom. Because of this, *Anticipating* was iterated in Sprint D with the addition of more in-depth strategy overviews, relations between the strategies and the learning goal, suggested *Sequences*, and questioning strategies.

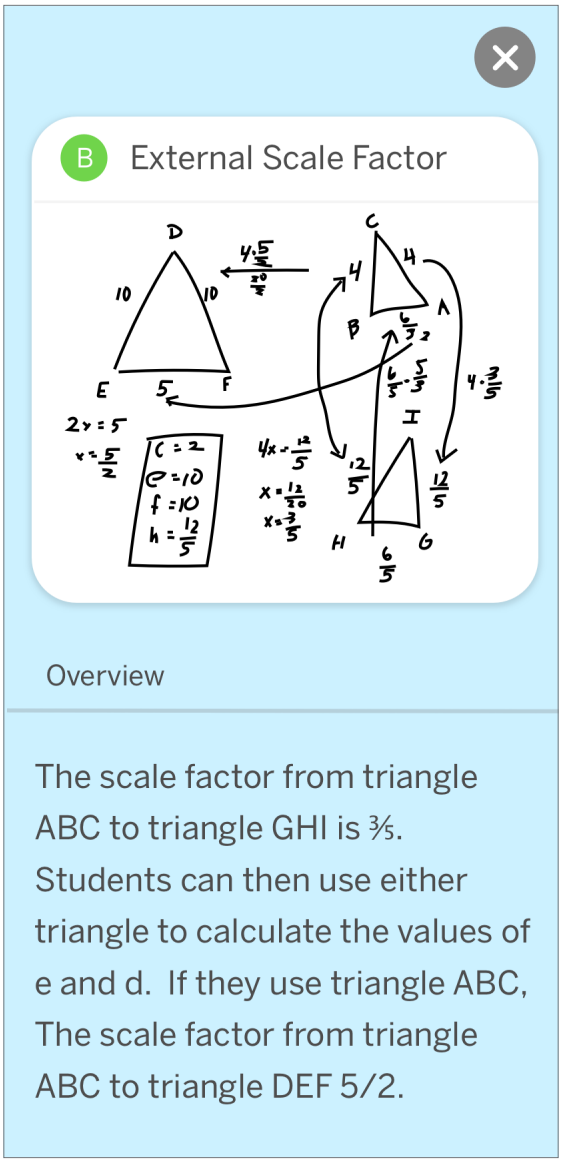


Figure 29: Information about a strategy provided during *Anticipating*

During the Sprint D interview, the TAB members were encouraged to describe how they currently use the *Anticipate* practice to ready themselves for the classroom without the app. Shannon described aspects of *Anticipating* as, "We [eighth-grade teacher team] weren't going through possible solutions, it was more of where we want them to get at the end instead of coming up with all the options" (Sprint D Interview, 8/5/2020). Guillermo liked the possibility of the prototype providing strategy options in the *Anticipating* portion of the prototype. Guillermo stated, "So if there's one [strategy] that maybe we didn't think of, see, but it's kind of already thought out for you. That's helpful because we don't always think the way our kids think even when we try to ..." (Sprint D Interview, 8/5/2020). Iris also liked the strategies being *Anticipated*. She reacted positively and stated, "It looks like I can click on strategies to see ways that have already been *Anticipated* for me ...Cool. So those are other strategies I'm noticing. I can select more than one strategy meaning maybe these are strategies that my students will be using, based on my work with them" (Sprint D Interview, 8/5/2020).

Shannon was open to the idea of using *Anticipating* to *Sequence* the strategies as she said, "it's [*Anticipating*, *Sequencing*] that's also going to help lead the class to *Connect* the strategy to the learning goal. So it kind of reinforces ... a learning target for the teacher in this stage." (Sprint D Interview, 8/5/2020). Shannon understood that *Anticipating* strategies and *Sequences* could help while *Monitoring* and *Connecting* in the classroom.

An advisor reflected further on how teachers think about *Anticipating*: "but they only think about one way to do it right. And they may not even engage in the tasks themselves. So they don't actually do the task. It's like, 'I know how to solve for missing value in a proportion, I don't need to work out this task'" (Sprint D Interview, 8/6/2020).

In Sprints E and F, the project team focused on what exactly to include in *Anticipating* that TAB teachers could stick with and would set them up for success during class. The end result was an *Anticipating* section of the prototype that encourages teachers to interact with Productive Approaches, Points of Growth, questions[6], sequences, and prompts for facilitating a *Connection* conversation.

6 Two types of questions were included. Assessing questions are meant to help teachers understand what students were thinking mathematically. Advancing questions are meant to help students attain the next level of procedural and/or conceptual mathematical understanding.

TAB members were particularly interested in anticipating questions and spoke at length about questioning strategies during the initial interviews. The TAB members' comments point to the importance of different types of questions, using questions to save teachers time, and having questions ready for students who cannot get started.

Shannon highlighted the difficulty in knowing how to help students who could not get started: "And then the other thing is when kids are like, 'I don't even know where to start. I don't have anything'" (Pre-Interview, 6/3/2020). Although Shannon did not specifically talk about questioning strategies to help students who cannot get started, the project team quickly realized leveraging specific and actionable questions could help the teacher support the students who for varied reasons cannot find an entry point to the mathematical problem or concept at hand for the activity without the teacher taking the struggle away from the students.

Nia spoke of overarching questioning strategies when she said, "Maybe some sample questions kind of ... like some generic questions that would kind of push kids towards higher levels of thinking" (Pre-Interview, 6/3/2020). Nia also touched on Shannon's point stating, "For the kids who have not done or started anything? You know, those are the groups that would be [I'm] trying to ask them questions or guide them as to where do you think you should start? Rather than tell them you know, this is where you should start ... like my old version of me telling them exactly what to do so questioning is really important" (Pre-Interview, 6/3/2020).

Mark stated, "I don't write out my questions for each of the *Anticipated* strategies; I do write out questions of how I want to move the class along like in general. I [try to] script myself, but I don't always ..." (Pre-Interview, 6/3/2020). From Mark's comment, the project team realized teachers may not have questions ready as they engage in the 5 Practices in the classroom. Building questioning strategies in the prototype may encourage teachers to increase questioning techniques and use them more effectively in the classroom.

During the Sprint D interview, an advisor talked about different types of questions, saying, 'I think that you should ask any group no matter what they've done 'can you tell me what you did?' because I think that's a lot of the information that's in these other [mathematical] questions ... I've always thought that question was a way of saying to the student, 'I care about how you're thinking and working on this'" (Sprint D Interview, 8/6/2020). With the TAB members, the mathematically-based questions resonated but the advisors tended to embrace the more overarching questions, generally inviting students' perspectives. Therefore, the project team decided to include both types of questions.

Reviewing questions during *Anticipating* can save teachers time during class time because it is difficult at best to write assessing and advancing questions, let alone create questions to ask in the moment. In the classroom, better questions facilitate stronger student discourse through better reflection, more student opportunities to talk, and better understanding of what students are thinking.

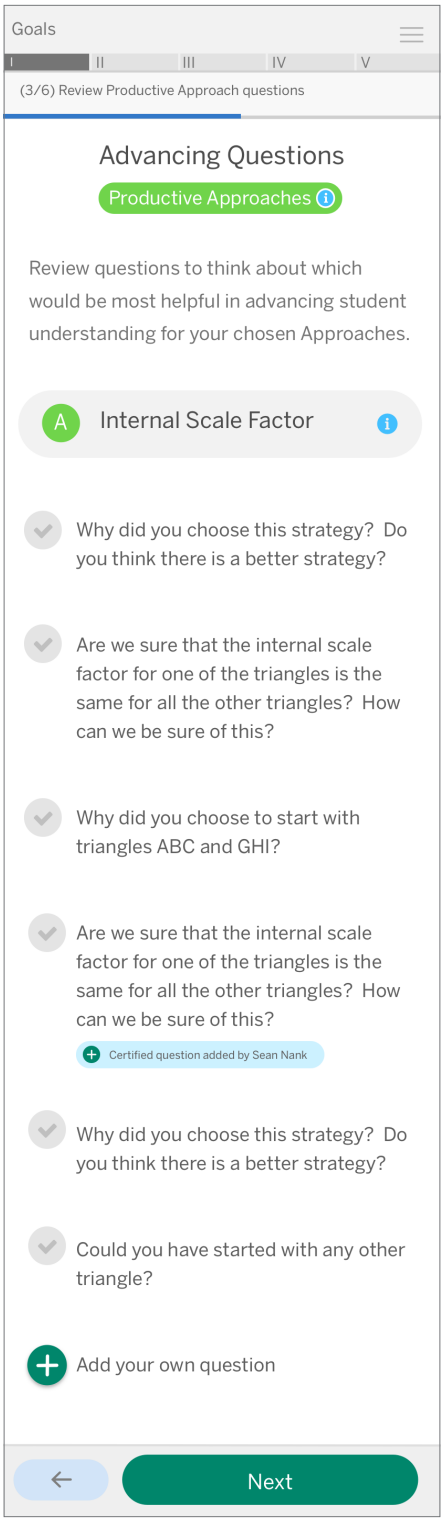


Figure 30: Questions to review during *Anticipating*

Overall, Nia alluded to the importance of the *Anticipating* practice being substantially recognized in the prototype, "Because it does have all the features, the *Anticipating*, and I think that's one of the major things that I think some teachers don't want to do ... So I think it [the prototype] helps in that way. I'm also going through like how to monitor kids while you're teaching, while they're working and questions to ask. And I think all of that helps to, you know, make a better teacher out of anyone" (Post-Interview, 10/7/2020).

Mark and Deja appreciated how the *Anticipating* part of the prototype helped them to plan as well as helping them in the context of the classroom. Deja liked that the *Anticipating* part of the prototype saved time as she stated, "The *Anticipating* part that shows different students' strategy, it saves time, I don't have to go sifting through old papers or looking online to see how it can be solved. That's all there for me ... that is very useful and saves time (Post-Interview, 10/7/2020). Mark stated, "So the fact that you have the *Anticipated* strategies, and then it has suggested *Sequences* to get to the main idea, and then the helpful hints that more flesh out the mathematics/main mathematical concepts, I think all of those are beneficial and help with the pedagogy because like, I don't always feel comfortable that I know the *Sequence* that would move me in the direction more smoothly than others. So I appreciate that" (Post-Interview, 10/6/2020).

Iris liked the time the *Anticipating* portion of the prototype could save. Iris stated, "I think the *Anticipation* stage is sped up with the help of the app. And having access to the different strategies, and maybe even strategies that you wouldn't have thought of yourself. I think the suggested *Sequences* also really help with that. And then the way that the app speaks to itself, so that when you're ready to *Connect*, it tells you what group you can call on. I think those features do help with the speed" (Post-Interview, 10/7/2020).

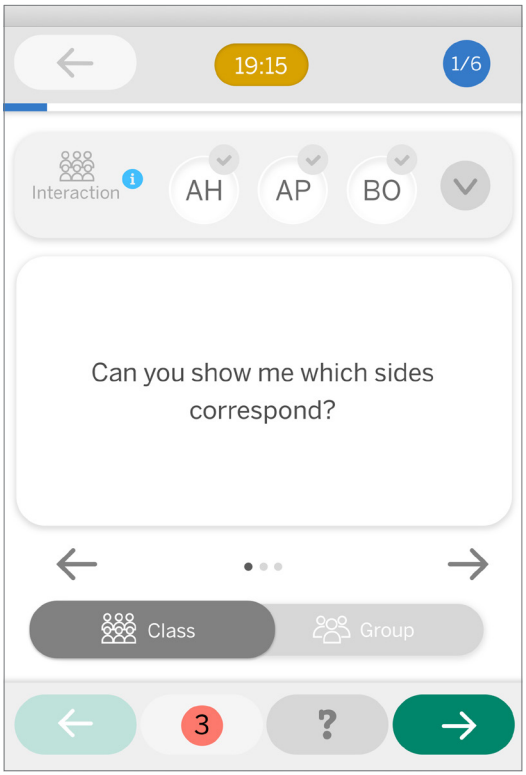


Figure 31: An *Anticipated* question appearing during *Connecting*

Theme 4: Enable Teacher Superpowers: Make it possible for teachers do things they could not do before

It is estimated that teachers engage in between 1200 and 1500 interactions and decisions in a single day (Cuban, 2011; Jackson, 1990). Now welcome the 5 Practices into a classroom and more decisions need to be made. While the *Anticipating* practice can help with some of these decisions, what about the decisions that happen during class that can't be *Anticipated*?

Selecting and Sequencing

Not only were several of the top challenges in the areas of *Selecting* and *Sequencing*, these practices also tended to be where the TAB expressed a great deal of anxiety. All six TAB members spoke about *Selecting* and *Sequencing* during the initial interviews. Nia said, "*Sequencing* for me has been the toughest part of it, really more so than the *Connection*. Because I don't know whether to go with the kids who are making the mistake first ... So I've been struggling with which one goes first, or what should I talk about first? Should I be [*Selecting*] the ones that are complete, the ones that are not complete, or the ones that are with the misconceptions? I don't want to do it wrong" (Pre-Interview, 6/3/2020). When asked about which practice is the hardest, Iris answered, "The one that comes to mind is definitely *Sequencing* because I'm constantly thinking about that as I'm *Monitoring* and also as I'm planning" (Pre-Interview, 6/2/2020).

Several TAB members confessed to giving up at the *Selecting* and *Sequencing* point. Shannon shared, "I pick the strategy that I want" (Pre-Interview, 6/3/2020). Shannon bypassed the stress and complexity of *Selecting* and sequencing by *Monitoring* for only one strategy and having that student or group present that one strategy. When talking about *Selecting*, *Sequencing*, and *Connecting*, Guillermo stated, "You know I skip some of those steps for lack of time where I think we sometimes fall apart is in the *Monitoring* and then the *Selecting*" (Pre-Interview, 6/10/2020). Deja confirmed this sentiment, saying that *Selecting* and *Sequencing* is "the hardest part to pull off everyday" (Pre-Interview, 6/3/2020).

During Sprint A, the prototype had the teachers *Select* and *Sequence* groups to present based on the rich data they collected during *Monitoring*.

However, the team soon realized that this moment in the prototype was just as overwhelming to teachers as *Selecting* and *Sequencing* already was, and it didn't help teachers overcome their anxiety about *Selecting* and *Sequencing*. TAB member Mark stated, "So you would order, like often I would order this to be able to *Sequence* it, right? So that's why you have the drag and drop. Is that helpful at this point? ... So I'm confused a little bit. So because right now I'm thinking of like, how would this help me?" (Sprint A Interview, 6/24/2020).

This and similar feedback led the project team to experiment with several different iterations of *Selecting* and *Sequencing*. During Sprint D, the prototype let the teachers create a *Sequence* and then showed the teachers several suggested *Sequences*. The TAB members did not respond favorably to this iteration. The TAB members liked the suggested *Sequence* but felt as if they were being tested and had failed. Mark stated, "Instead [of me *Sequencing* first], I suggest you make this suggestion. I would find that very useful because I think that *Sequencing* is one of the harder things" (Sprint D Interview, 8/5/2020). Deja also stated, "I think it'd be nice just to have some guidance around what the *Sequence* should be taking a little bit of that cognitive load off of the teacher" (Sprint D Interview, 8/4/2020).

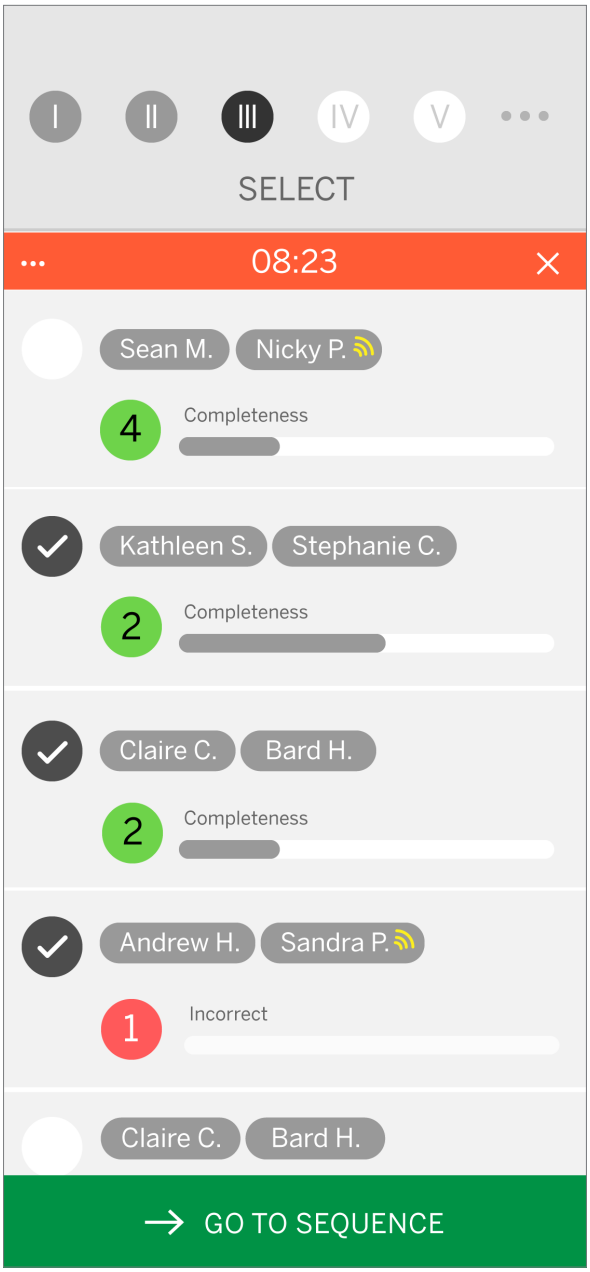


Figure 32: Sprint A *Selecting* and *Sequencing*

The team then suggested the opposite order instead: teachers could first peruse three suggested *Sequences*, such as finishing with the strategy most aligned to the learning goal or starting with the strategy the students were most likely to use the most and could then decide which *Sequence* made the most sense for their classroom. While teachers liked this better, they questioned the need to do this in the moment. Iris pointed out that, "I want to go into my lesson or task having an idea of the *Sequence* that I'm looking for" (Sprint D Interview, 8/4/2020).

Given the work the team did to build out support for *Anticipating*, they could build on that *Anticipated Sequence*, and a superpower was born: The prototype now uses the teacher's information from *Monitoring* to *Select* student groups into the *Sequence* the teacher *Anticipated*. Groups that have not presented lately are given preference. The teacher could change the suggestions for *Selecting* and *Sequencing*, but the prototype performing the *Selecting* and *Sequencing* action for the teachers could reduce the stress and cognitive load for the teachers.

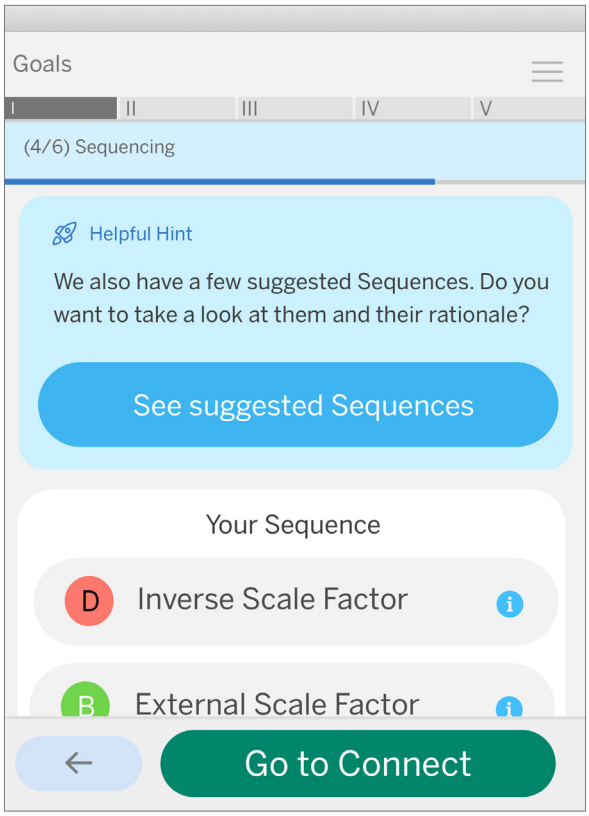


Figure 33: A helpful hint for *Sequencing*

The culmination of this idea came in Sprint G, where the "Has not presented recently" icon is presented and the prototype uses this information as background data to inform its suggestion of which groups the teacher should *Select* to present. Using this logic, a group who has the same strategy but has not completed as much of the solution as another group who is farther along in the process but has presented recently has preference of presenting. The teacher can call on the group who has not recently presented, have them share their work so far on the strategy, and then call on another group to complete the strategy. If two groups share the same strategy and have not presented recently, the group who is more complete in the strategy gains preference. When two groups have been marked by the teacher as using the same strategy, the prototype uses the following logic to suggest a group to *Select*:

1. The group in which any student is "no" for presenting in the past week and the group has a greater degree of completeness than other groups that use the same strategy.
2. The group in which any student is "no" for presenting in the past week and the group has a lesser degree of completeness than other groups that use the same strategy.
3. The group in which all students are "yes" for sharing their approach in the past week and the group has a greater degree of completeness than other groups that use the same strategy.
4. The group in which all students are "yes" for sharing their approach in the past week and the group has a lesser degree of completeness than other groups that use the same strategy.

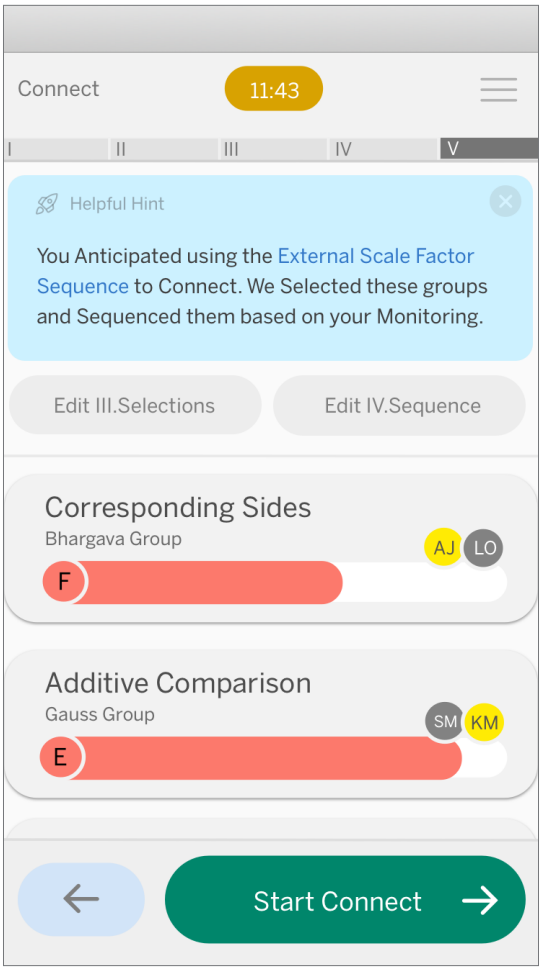


Figure 34: Adjusting *Selecting* and *Sequencing*

TAB members liked the suggestion of *Sequences* and also liked that they could change the *Sequence* in the *Anticipate*, *Sequence* or *Connect* practice. Iris commented on the benefit of having a predetermined *Sequence* in the prototype when she said, "having access to the different strategies, and maybe even strategies that you wouldn't have thought of yourself. I think the suggested *Sequences* also really helps with that. And then just like, the way that the app speaks to itself, so that when you're ready to *Connect*, it tells you like, what group you can like, call on. I think those features do help with the speed" (Post-Interview, 10/7/2020). Iris continued, saying, "I know that there's suggested *Sequences* and suggested groups. And at the end of the day, it's really up to the teacher to do something with that information" (Post-Interview, 10/7/2020). Giving teachers suggested *Sequences* with the ability to adjust the *Sequences* at multiple points provided the support needed from the prototype while still respecting the professional perspective of teachers.

Figure 35 provides examples of the feedback that made the project team confident that these were strong features to include to ameliorate the challenges faced by teachers with the 5 Practices. These features, as well as others not detailed in this paper, were designed based on iterative feedback to help teachers overcome the barriers they expressed facing when attempting to facilitate student discourse using the 5 Practices. As is the case with anything teachers bring into the classroom, they can choose to use any or all of the features they find most helpful.

Theme One: *Support Equity: Help teachers to engage with all students in the classroom* "When you're in the monitoring phase, you can select the specific kids who are engaging with each other or with you and your questions. And it [the app] highlights it [the student] yellow. [That's a] big boost to my ability to reach all the kids" (Guillermo Post-Interview, 10/6/2020).
Theme Two: *Support Pedagogy and Curricula: Help teachers use productive 5 Practices approaches* "I love the helpful hints…to help me implement, with fidelity, the best instruction with the 5 Practices" (Shannon Post-Interview, 10/7/2020).
Theme Three: *Save Teachers Time: Maximize in-classroom interactions and practices* "The app has really improved the ability to do all of these things that are time-intensive from a teacher's perspective. My overall comfort with the 5 Practices in general…I feel better because of the app" (Mark Post-Interview, 10/6/2020).
Theme Four: *Enable Teacher Superpowers: Make it possible for teachers do things they could not do before* "Before the app, I was like, I don't know where to start [with Selecting and Sequencing]. But the app gives you those bridging things to follow, gives you suggestions of how to link productive and non-productive strategies…to help students make sense of the problem…the app gives you suggestions that you can alter or change" (Deja Post-Interview, 10/7/2020).

Figure 35: The four main themes and an example of feedback that supported each theme

Implications and Next Steps

Implications

Imagine an educational system with an expanse of classrooms filled with students who unequivocally know that they—their voices, their perspectives, their thoughts—matter. Imagine a conversation centered around mathematical discourse where everyone in the conversation realizes that people do and become better whenever they listen to others who equally listen to them. Imagine a student who hesitates for a moment in a classroom because they are not certain of their thought but still feels included and valued enough to speak the words only they could in that moment. These visions, above all else, are the heartbeat of the 5 Practices Pathfinder prototype. One cannot support students without also supporting teachers. The 5 Practices Pathfinder prototype is meant to facilitate equality of discourse in mathematics classrooms through offering teachers insights, support, and encouragement to enact a difficult and rewarding form of pedagogy.

Every students' voice matters regardless of their status, gender, ethnicity, race, or any other identifiable characteristics that may precipitate the perpetuation of any "-ism" in the classroom. Valuing *some* students' mathematical discourse deprives many students from finding their voice and understanding of mathematics and also potentially deprives the world of beautifully rich and diverse mathematical thinking. The implications for learning if teachers are supported in using a high-quality pedagogical strategy like the 5 Practices coupled with consistent *Monitoring* for implicit bias run the spectrum from profound to life changing.

The iterative feedback, qualitiative semi-structured interviews, and quantitative pre- and post-surveys showed that teachers of both priority students and all students felt their challenges would be less of a barrier with the prototype and that the features in the prototype would be useful in their classrooms. Since COVID-19 quarantines and school shutdowns occurred as this project was starting, the project team focused on measuring how and if the choices made in the process of creating the prototype have the potential to impact what occurs in future in-person classrooms. In-classroom testing of the ideas are needed. In advance of such testing, there are several definitive implications from this project.

1. *Targeted Universalism tested through Human-Centered Design and design thinking processes can be used successfully in edtech product development to include equity in the design process by focusing research on priority students and checking for universal application*. Targeted Universalism embodies the idea that if one designs, iterates, and attempts to solve problems with priority students at the core of the process, then the end result can be something that will promote true equality for all and benefit the entire population. Further, in humanizing the users, we humanize ourselves, our teams, and everyone around us, which has the potential to permeate users' perspectives in subtly profound ways. The true beauty of this is that the project team and users may never fully understand the reach of such implications. For example, if a teacher, through the use of the prototype, realizes that they typically call on one group or one subset of students over others and purposefully changes that pattern in their classroom, that could be the moment where a student realizes that their teacher values their voice and perspective. Imagine that type of impact happening in other product projects on other topics as a result of taking a Targeted Universalist approach to a Human-Centered Design project. Using a mixed methods design approach to inform this incorporated multiple perspectives, lenses, and data to make sense of what the users thought and needed. Both quantitative and qualitative data are critically important for empathizing with users in an authentic way. Both allow the project team to understand the users' points of view. Quantitative and qualitative methods used in this manner breaks down a traditional false dichotomy, frees the methods of their imposed conflict, and brings to the research and design world an understanding of the symbiotic nature of the two views. Both perspectives are needed to truly understand and empathize with the users.

2. *Technology has the potential to be useful in the classroom for supporting mathematical discourse and making curriculum-based pedagogical practices for mathematical discourse, such as the 5 Practices, less challenging for teachers*. The 5 Practices provide a pedagogical template for teachers to value equality of student voice and discourse in the classroom. The 5 Practices, like all approaches to equitable student discourse, contain challenges for teachers. By digitizing a support structure for the 5 Practices, teachers may be enabled to more effectively use this pedagogical approach to foster equality of student-centered discourse in mathematics classrooms.

3. *The themes uncovered in this project—supporting pedagogy, supporting equity, saving teachers time, and enabling teacher superpowers—may be applicable to other ed tech product development projects*. The prototype enables teachers to concentrate more on what they as professionals should think about and less on what they should not think about. Teachers are extraordinarily busy people who thrive in the complexity of the classroom. Making the classroom a little less complex, helping teachers with decisions and moves they can make in the classroom, and enabling teachers to fully leverage every single student's perspective and voice creates multiple opportunities to meaningfully experience mathematics and each other in authentically diverse ways.

Next Steps

In addition to the widely applicable implications above, there are specific next steps that arose from user feedback that could be taken with the 5 Practices Pathfinder prototype to create a functioning app. The next steps fall within 4 areas: equity, collaboration, flexibility, and online/blended instruction.

Equity

Data experiences for the user could be expanded upon as data from the *Monitoring* practice for individual students, groups, and class as a whole could be analyzed and presented to the user as areas for reflection and action. Multiple TAB members expressed a desire for more ability to interact with the data the prototype was collecting, specifically the interactions data. Both Iris and Shannon told us, "show me the data!" (Post-Interviews, 10/7/2020). Making transparent trends from an equity perspective, reflective questions about the data, and possible instructional or pedagogical suggestions could enable the prototype to effectively present complex patterns that the teacher would not be able to easily take action on without the prototype.

Iterations for supporting English Language Learners and students with disabilities could be prototyped and tested. Features such as questions in languages other than English, built-in key ideas for working with students with disabilities, and mathematical language routine helpful hints could be incorporated into the prototype.

Collaboration

A collaboration network could be integrated into the prototype. Teachers loved the idea of seeing strategies and questions from fellow teachers, especially the teachers who were the only math teachers at their grade level at their school. The team generated many questions to study about this idea: *Whose ideas would teachers like to see? Would they want to add connections or follow specific colleagues? Would teachers want to share their strategies and questions with others? Would teachers want content to go through a curation or vetting process? Would teachers want spaces in which to collaborate and discuss tasks?*

Further, the team considered same-site collaboration. *Could teachers share work with their co-workers? If there was a PLC of four teachers, could they all share their work from the prototype during Anticipating? Could one teacher Monitor an unanticipated strategy from period two and send it to other teachers who teach the same course for the remainder of the periods in the day? Would teachers be able to see live updates about the class from teachers co-teaching and Monitoring in the same classroom?*

Some of the TAB members either taught in isolation, being the only eighth grade teacher at their site or being the only teacher at their site to break from traditional lecturing pedagogy. Building connections to other colleagues and communities could mean the difference between a 5 Practices user facing challenges continuing to transform their pedagogy or reverting back to the way things had always been.

Flexibility

The prototype presents linearly to the user in order to help teachers overcome each of the barriers and decision points they encounter as they proceed through the 5 Practices. A next step would be to revise the flow of the prototype, inviting non-linear presentation and use. There could be a computer-based, instead of mobile-based, version of the *Anticipation* practice, providing the screen space needed to think about strategies, questions, and *Sequences* at the same time instead of one after another. Teachers could then send that information to their mobile device for use during class time. Additional features such as the ability to track different strategies used by the same group of students and the ability to work on multiple tasks and have several smaller discussions within one class period could support rough draft thinking about mathematics (Jansen, 2020).

A "lighter" version of the prototype incorporating universal pedagogical aspects of the 5 Practices could be developed for teachers to use daily regardless of whether they were doing all 5 Practices, and thus be useful for any problem-based lesson. The ability to seamlessly add additional questions, bridges, and *Sequences* could provide further flexibility.

Online/Blended Instruction

Additional superpowers could be created to support online/blended instruction, such as the ability to present or display student work or to integrate with other technological resources such as google slides and video conferencing software. While the Pathfinder project focused on in-person instruction, online and blended learning is here to stay and work in supporting the facilitation of effective and equitable discourse in these settings will be needed.

Conclusion

The work of ensuring that High-Quality Instructional Materials are leveraged with high level pedagogical strategies to achieve true equality of all students' voices is never complete. In this regard, the project team views the current 5 Practices Pathfinder prototype as the first of many stages of iteration with profound possibilities. Helping teachers to *Anticipate*, *Monitor*, *Select*, *Sequence*, and *Connect* strategies in an inclusive classroom in a coherent manner while alleviating the roadblocks to successful implementation, especially for teachers of priority students, was the overarching goal of this project. Inherent in the prototype for the 5 Practices Pathfinder is a design which encourages equality of student voice, support at the most difficult points of implementation, and the ability to transform the 5 Practices so users can accomplish things they could not have done before. To this end, the work has only begun and will not be completed until all teachers are supported in the implementation of student-centered pedagogy and every students' voice matters.

There are many things that contribute to students losing their voice in mathematics classrooms. Some of these contributors may have occurred in classrooms and centered on mathematics. Other contributors live worlds away from the classroom and have nothing to do with mathematics but find their way into the classroom, flowing like water through the cracks, permeating the culture of the mathematics classroom. Even with all these possibilities to stifle or create inequities in students' voices, one solution remains constant: what adults in the educational system choose to do and, sometimes more powerfully, choose not to do , has the potential to permeate generations of students' perceptions of themselves as mathematical thinkers. It is the project team's goal to help teachers to help their students, to make it easier to incorporate a high quality pedagogical strategy, to identify and target their own personal implicit biases, and to see the inherent beauty in a classroom with a diversity of voices.

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5 Practices Teacher Advisory Board: Bethany, Casey, Ernesto, Lorri, Sharon, and Wendy

Project Advisors: Margaret (Peg) Smith and Michael D. Steele

Additional guidance and support from: Larry Berger, Jaclyn Claiborne, Jaymes Hanna, Lynn Harris, Chris Ignaciuk, Justine Jackson, Yuri Kim, Joshua Leferman, Suzanne Margaree, Jaclyn Murawska, Ian Nies, Gabriel Parra, Mariana Preciado, Todd Rawson, Brad Shank, Jason Siegel, Ben Simon, Zachary Slack, Zachary Wissner-Gross

Special thanks to all the teachers who reviewed the prototype and provided feedback.

This report is based on research funded by the Bill & Melinda Gates Foundation. The findings and conclusions contained within are those of the authors and do not necessarily reflect positions or policies of the Bill & Melinda Gates Foundation.

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Assessing Teachers' Understanding of the 5 Practices and their Confidence in Applying Them to their Classrooms

Introduction

The 5 Practices Survey is a self-report measure of teachers' perceived difficulty and confidence with the 5 Practices (5Ps). It was developed by Amplify and administered to the 5 Practices project Teacher Advisory Board (TAB) members at the start of the 5 Practices project to inform the initial design of the 5 Practices prototype to obtain baseline measures of the TAB's self-assessment on the 5Ps. The survey was administered again at the end of the project to assess changes in self-assessment with the prototype. The survey design process is described below.

Research

Before developing the survey, the project team conducted a review of the research literature to identify factors that have been shown to explain or predict behavior, since the purpose of the project is to develop a prototype that could directly or indirectly lead to increased use of the 5Ps among math teachers. One widely used framework for understanding behavior is the theory of planned behavior (TPB; Ajzen, 1991), shown in Figure 1. TPB posits that behavior is influenced by a person's intention to perform the behavior. Behavioral intention, in turn, is influenced by one's attitude toward the behavior, subjective norm, and perceived behavioral control. Attitude is "the individual's positive or negative evaluation of performing the particular behavior of interest", subjective norm is "the person's perception of social pressure to perform or not perform the behavior under consideration," and perceived behavioral control is "the sense of self-efficacy or ability to perform the behavior of interest" (Ajzen, 2005, p. 118). According to Ajzen (n.d.),

> Interventions designed to change behavior can be directed at one or more of its determinants: attitudes, subjective norms, or perceptions of behavioral control. Changes in these factors should produce changes in behavioral intentions and, given adequate control over the behavior, the new intentions should be carried out under appropriate circumstances. (p. 1)

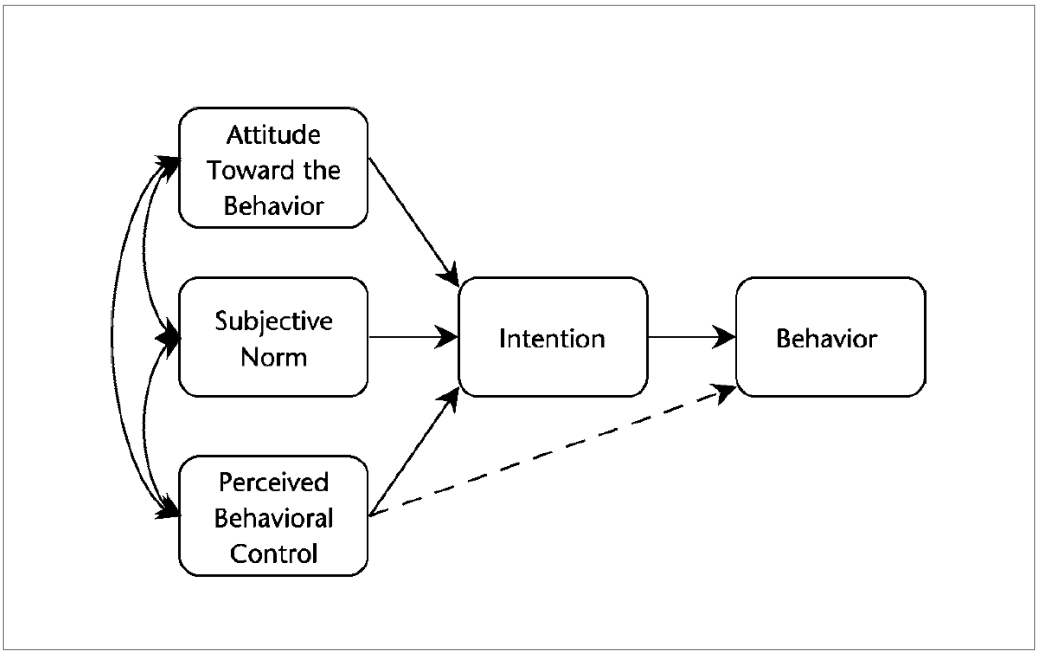


Figure A.1. The Theory of Planned Behavior
Source: Ajzen (2005)

Note. The TPB model presents perceived behavioral control as a single construct. However, research has shown that it should be conceptualized as multidimensional. Trafimow, Sheeran, Conner, and Finlay (2002), for example, demonstrated that perceived behavioral control can be decomposed into perceived difficulty (i.e., "the extent to which the behaviour is perceived to be easy or difficult for the person to perform") and perceived control (i.e., "the extent to which the behaviour is perceived to be under a person's voluntary control") (p. 103). Other researchers (e.g., Kraft, Rise, Sutton, & Roysamb, 2005; Rodgers, Conner, & Murray, 2008) have suggested the presence of a third dimension, perceived confidence, defined as "perceptions of having the ability, or being capable of performing the behaviour" (Rodgers et al., 2008, p. 619).

TPB has been used in educational research to investigate a range of behavioral intentions, such as teachers' intentions to teach environmental issues and to use educational technology (see Cooper, Barkatsas, & Strathdee, 2016).

In addition to reviewing the literature on TPB, the project team conducted a search on Google, Academic Search Premier, and the NCTM website for existing 5Ps survey items that could be adapted for the project. We were able to identify one survey that was administered by Peg Smith during a webinar for middle school teachers on the 5Ps, presented in April 2020 as part of NCTM's 100 Days of Professional Learning series. During the webinar, teachers were presented with a list of 19 "challenges" (e.g., "creating questions that move students toward the mathematical goals") and asked to "identify the challenge that you struggle with most." Each of the challenges was associated with a practice (or set of practices), as shown in Figure 2.

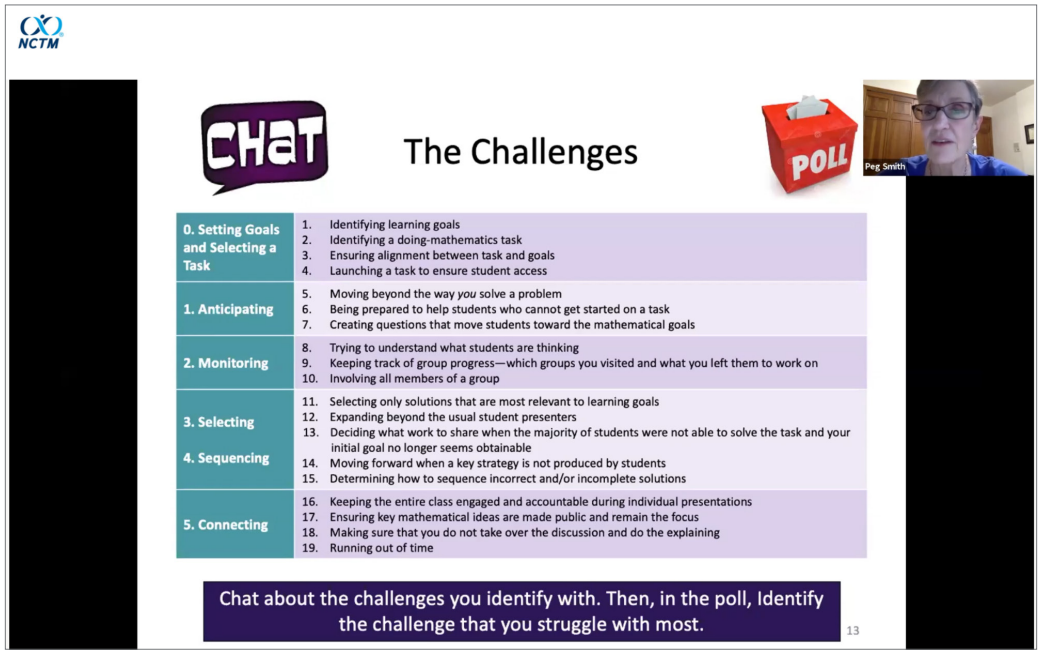


Figure A.2. Peg Smith's NCTM Survey
Source: https://nctm.wistia.com/medias/4o06rp2uld

Constructing the Survey Items

For this project, we chose to construct the 5Ps survey around the 19 challenges for three reasons. First, the challenges align with the perceived difficulty and perceived confidence constructs within TPB. Second, the challenges were developed by Smith and colleagues from decades of experience working with classroom teachers. Lastly, the TAB members were each provided with a copy of the book The 5 Practices in Practice: Successfully Orchestrating Mathematical Discussions in Your Middle School Classroom (Smith & Sherin, 2019), which details the 19 challenges. The descriptions of the challenges provided in the book would ensure a common understanding and interpretation of the survey content among the TAB members. These last two points are especially salient in light of Cooper et al.'s (2016) caution that

> Defining behaviour of interest carefully with consistency across constructs is a crucial element of research validity when using the TPB (Ajzen, 1991). Failure to define the behaviour of interest carefully and with consistency across elements is likely to substantially impact, or void, the validity of the results. (Ajzen, 2005, p. 147)

To construct survey items based on the TPB, Cooper et al. recommend defining behavior in terms of its target, action, context, and time (TACT). The TACT elements for the initial draft of the 5Ps survey are presented in Table 1.

Table A.1. TACT Elements for the Initial Draft of the 5Ps Survey

TACT element	Description
Target	Middle school math teachers
Actions	The 19 challenges associated with implementing the 5Ps (e.g., "creating questions that move students toward the mathematical goals")
Context	Teaching the Illustrative Mathematics curriculum
Time	The 2020-2021 school year

Note. The time element was set as the upcoming school year rather than the current or past school years because some of the TAB members had not used the 5Ps prior to their involvement with the project.

We selected perceived difficulty and perceived confidence as the constructs to measure, with the hypothesis that use of the prototype would affect a change in perceived difficulty and perceived confidence with the 5Ps, and that change would then lead to an increase in intention to use the 5Ps. A sample of the initial draft of a perceived difficulty item and the associated TACT elements are shown in Figure 3.

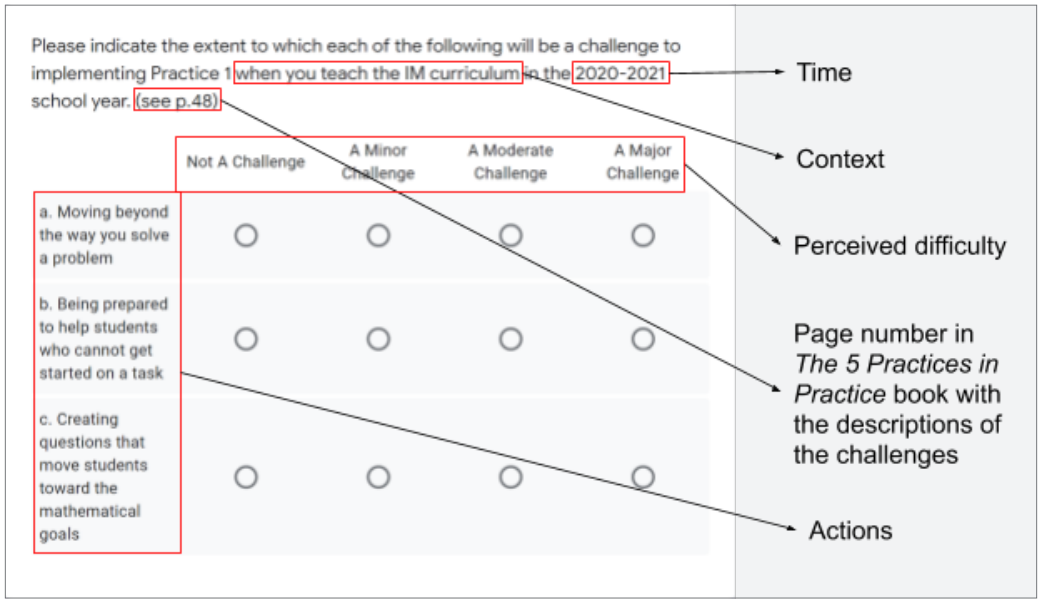


Figure A.3. Initial Draft of a Perceived Difficulty Item and the Actions, Time, and Context Elements

Pretesting the Survey Items

The initial draft of the survey was pretested using the cognitive interview approach to identify and correct any problems before it was administered to the TAB members. Three curriculum developers from Amplify's math team were recruited to pretest the survey. All three are involved in developing Amplify's Mathematics curriculum. In addition, they each have 9-10 years of prior experience teaching 7th-12th grade math.

The pretest respondents participated in a training session on cognitive interviewing prior to their individual cognitive interview sessions. The training included watching a YouTube video on cognitive interviewing produced by University of Southampton's National Centre for Research Methods, followed by practice in "thinking aloud" responses to questions such as "What is your favorite color?" and "How many windows are there in your home?"

For the individual cognitive interview sessions, we adapted Irwin and Stafford's (2016) sample cognitive interview protocol, which includes examples of probing questions to ask pretest respondents. The probing questions about the 5Ps survey are listed in Table 2.

Probes about the survey introduction:

- What, if anything, was confusing or unnecessary in the introduction?
- What, if anything, was missing from the introduction?
- What can we do to improve the introduction?

Probes about the survey items:

- Was there something about the item that was unclear to you?
- Was there a response option that you were looking for but was not included?
- Was it clear to you that you need to look in the book for more information about the challenges?
- What can we do to improve the item?

Probes about overall impressions of the survey:

- What did you think of the length of the survey?
- How many minutes do you think it would take to complete the survey?
- What did you think about the flow of the survey?
- Would you suggest any reordering of the items?

Table A.2. Probing Questions During the 5Ps Cognitive Interview Sessions

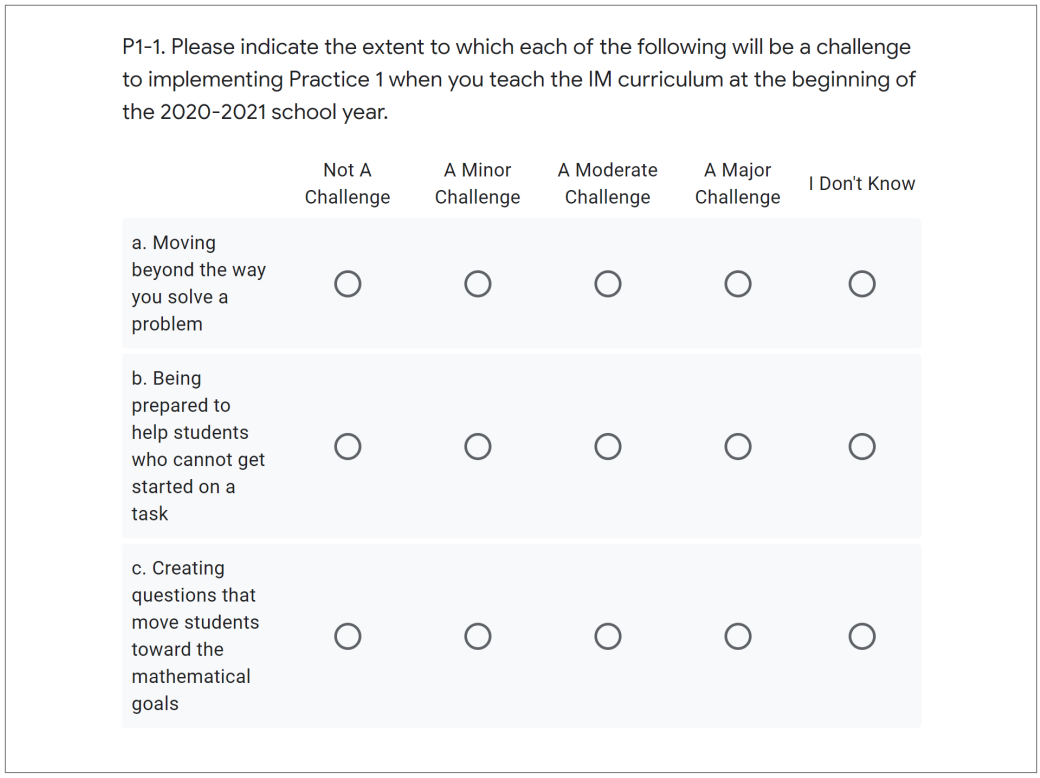
Feedback from the cognitive interview sessions led to the following survey revisions:

1. a shorter introduction
2. the addition of "I Don't Know" as a response category
3. a refinement of the time element from "the 2020–2021 school year" to "the beginning of the 2020–2021 school year"
4. a reframing of the perceived confidence items from "How confident are you in your ability to implement ... ?" to "How confident are you in your ability to overcome each of the challenges ... ?"

Due to time constraints, we were unable to conduct a second round of cognitive interview sessions to pretest the revisions before administering the survey to the TAB.

Final 5Ps Survey

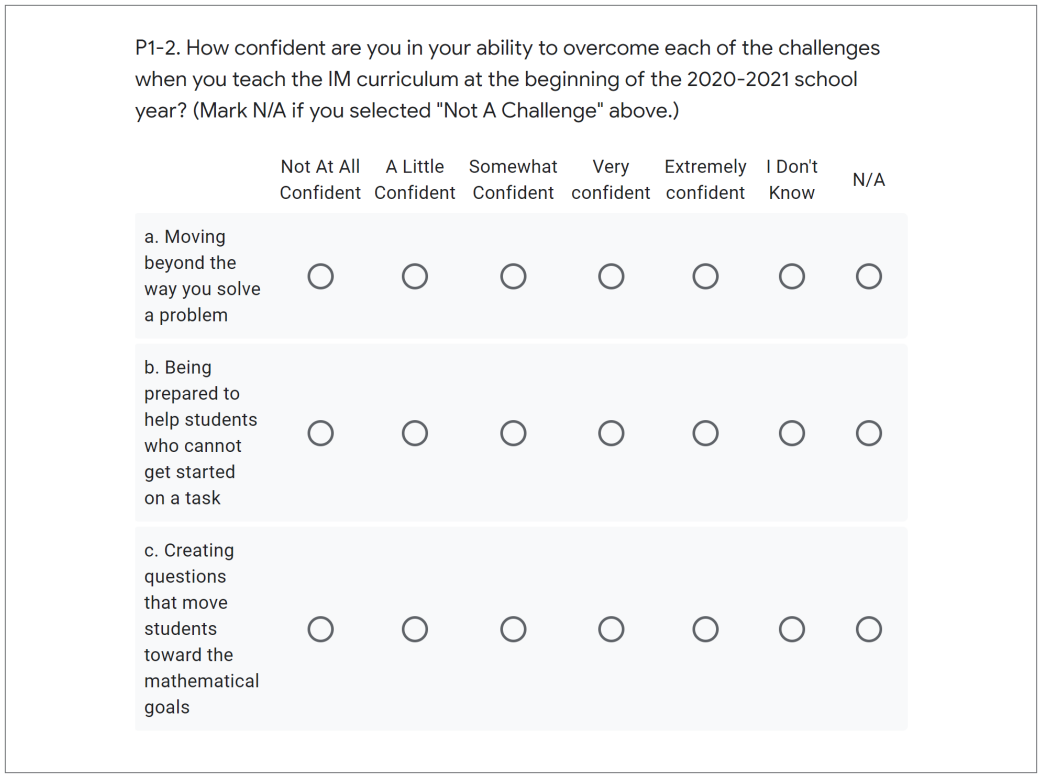
Samples of perceived difficulty and perceived confidence items that were administered to the TAB at the start of the project are shown in Figures 4 and 5. For the second administration at the end of the project, the context element (i.e., "when you teach the IM curriculum") was revised to include teaching with the prototype.



P1-1. Please indicate the extent to which each of the following will be a challenge to implementing Practice 1 when you teach the IM curriculum at the beginning of the 2020-2021 school year.

	Not A Challenge	A Minor Challenge	A Moderate Challenge	A Major Challenge	I Don't Know
a. Moving beyond the way you solve a problem	○	○	○	○	○
b. Being prepared to help students who cannot get started on a task	○	○	○	○	○
c. Creating questions that move students toward the mathematical goals	○	○	○	○	○

Figure A.4. A Perceived Difficulty Item Administered at the Start of the Project



P1-2. How confident are you in your ability to overcome each of the challenges when you teach the IM curriculum at the beginning of the 2020-2021 school year? (Mark N/A if you selected "Not A Challenge" above.)

	Not At All Confident	A Little Confident	Somewhat Confident	Very confident	Extremely confident	I Don't Know	N/A
a. Moving beyond the way you solve a problem	○	○	○	○	○	○	○
b. Being prepared to help students who cannot get started on a task	○	○	○	○	○	○	○
c. Creating questions that move students toward the mathematical goals	○	○	○	○	○	○	○

Figure A.5. A Perceived Confidence Item Administered at the Start of the Project

Because data from the 5Ps survey were collected from only six TAB members, the project team does not intend to make generalized inferences from the results. The project team also notes, as a final point, that the sample size is also too small to test the psychometric properties of the survey. Because the project team was aware of this limitation from the start, the team focused on developing a survey that is carefully grounded in both a research-based theoretical framework and the expertise of the 5Ps authors.

5 Practices Tools Rubric

Category	3	2	1	0
Goal(s)	The goal(s) of the lesson is prominent, focused on student learning, and tied to other areas of the tool (for example, evident in the *Monitoring* component)	The goal(s) of the lesson is prominent, focused on student learning but not tied to other areas of the tool.	The goal(s) of the lesson are used more procedurally and/or concentrate on answers over student thinking.	The goal(s) of the lesson are absent from the tool.
Anticipate	The tool provides room for visual and verbal representations of anticipated strategies, including incorrect/ incomplete work	The tool provides room for visual *and* verbal representations of anticipated strategies	The tool provides room for visual *or* verbal representations of anticipated strategies	The tool has no *Anticipate* component
Monitor	The tool provides space and assessing/ advancing questions for teachers to *Monitor* students, groups, *Anticipated* strategies, incorrect /incomplete work, and unanticipated strategies	The tool provides space and guidance for three of the five factors including integration of assessing/advancing questions.	The tool provides space for *Monitoring* but no guidance for *Monitoring*. The tool does not integrate space for assessing/ advancing questions.	The tool has no *Monitoring* component
Select	The tool integrates the *Selection* process within the *Monitoring* component	The tool correlates in a straightforward manner the *Selection* process with the *Monitoring* component	The tool provides a *Select* component void of *Connections* with the *Monitoring* component	The tool has no *Select* component

Category	3	2	1	0
Sequence	The tool provides a sequencing component integrated with the *Select* component	The tool provides a sequencing component referencing the *Select* component	The tool provides a sequencing component with no *Connection* to the *Select* component	The tool has no *Sequence* component
Connect	The tool provides an integrated way to *Connect* the *Sequenced* strategies that are *Monitored*	The tool provides a segregated way to *Connect* the *Sequenced* strategies that are *Monitored*	The tool provides a *Connect* component that is not tied to the other 5 Practices	The tool has no *Connect* component
Reflection	The tool provides space for in the moment reflection concerning the 5 Practices as it relates to groups, students, and ideas	The tool provides space for two of the three reflection components	The tool provides space for one of the three reflection components	The tool provides no reflection component
Usability	The layout of the tool is highly usable and adaptable for teacher in the moment	The layout of the tool is usable and adaptable for the teacher in the moment	The layout is absent of the usability or adaptability component for teachers in the moment	The layout is void of adaptability and usability for the teacher in the moment

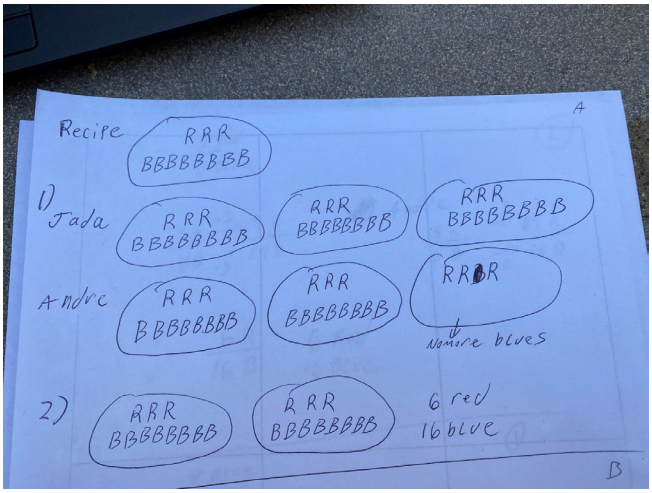
Ten Example 5 Practices Teacher Materials

Aligned to activities from the Illustrative Mathematics 6–8 curriculum

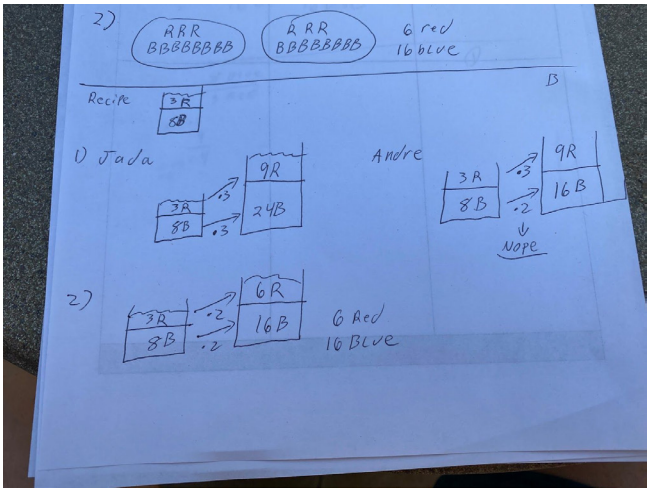
Grade Level	6
Unit Number	2
Lesson Number	4
Activity Number	3
Activity Name	Perfect Purple Water
Learning Goal	- Students will recognize if two ratios are equivalent and explain equivalence in the terms of each number in the ratio being multiplied by the same number to justify equivalence. - Students will use discrete diagrams with circled groups to represent ratios.
Image of Student-Facing Activity	The recipe for Perfect Purple Water says, “Mix 8 ml of blue water with 3 ml of red water.” Jada mixes 24 ml of blue water with 9 ml of red water. Andre mixed 16 ml of blue water with 9 ml of red water. 1. Which person will get a color mixture that is the same shade as Perfect Purple Water? Explain or show your reasoning. 2. Find another combination of blue water and red water that will also result in the same shade as Perfect Purple Water. Explain or show your reasoning.

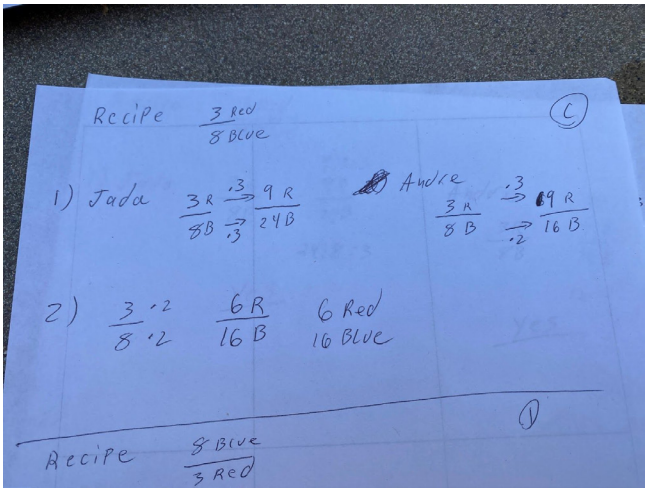
Productive Approaches

Name of Strategy	Grouping
Letter of Strategy	A
Color of Letter Circle	Green

Image of Strategy	
Helpful Hint	This provides a visual link to equivalent ratios.
Strategy Overview	- Draw the number ml of blue water and red water. - Then circle or box an equal number of blue and red water. - Use the Grouping to determine equivalence.
Relation to Learning Goal	- This Strategy allows students to visually see if the measurements are in equivalence. - Students can justify their reasoning based on the discrete diagraming of the Grouping.
Assessing Questions	Why did you decide to put 3 reds and 8 blues in each group?
	Why are there no blues in Andre's third circle?
Advancing Questions	Could we use this strategy for figuring out if it's the same shade of purple if we had really big numbers, like 192 ml of blue water and 66 ml of red water?
	What can we learn from the Grouping that we can use so we wouldn't have to make a ton of circles?

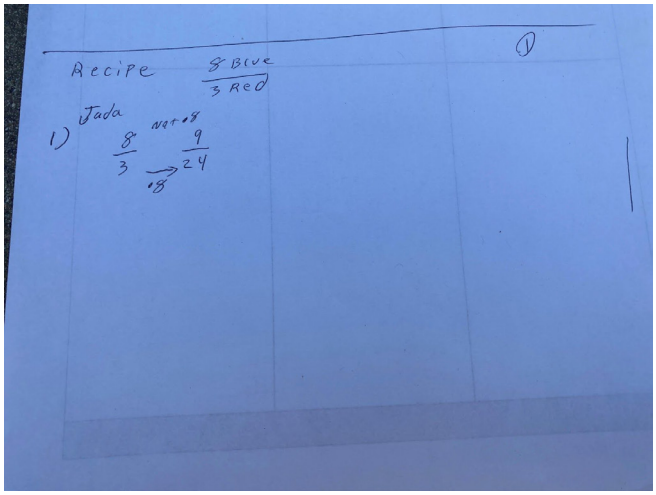
Name of Strategy	Drawing
Letter of Strategy	B
Color of Letter Circle	Green

Image of Strategy	
Helpful Hint	This strategy helps students visualize equivalence by pictorially pouring the blue and red water into a container.
Strategy Overview	- Make a “cup” with 8 ml of blue water and 3 ml of red water. - Use this mixture to compare with Jada and Andre’s mixture to determine equivalence.
Relation to Learning Goal	- This Strategy encourages students to determine if the measurement of the blue water and red water were increased by the same multiplier. - Students can justify their reasoning based on the multiplier being the same or different.
Assessing Questions	How did you get this number here (point to the 3)? How about this one (point to the 2)?
	Why did you decide to draw a container, how did that help?
Advancing Questions	What would I need to do in order to make Andre’s water the same shade of purple?
	Could you have multiplied by another number here (point to the number for their own mixture) and still have the same shade of purple? Why or why not?

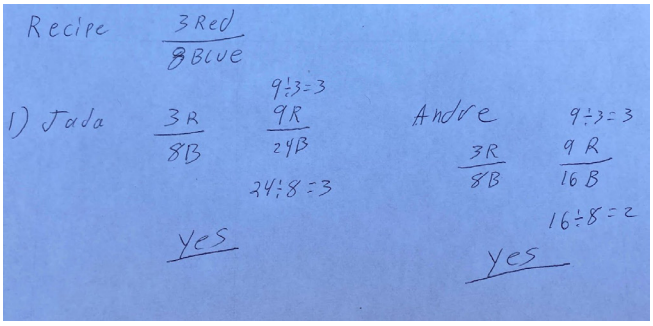
Name of Strategy	Fractions ⭐
Letter of Strategy	C
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	This is a critical Strategy because it can be referenced in future lessons to use fractions to visualize and denote ratios.
Strategy Overview	- Write a fraction with the blue measurement in the numerator and red in the denominator (or switch this but stay consistent). - Create fractions for Jada and Andre's mixtures and see if the numbers you use to multiply the numerator and denominator by relate the two fractions are equivalent.
Relation to Learning Goal	- This Strategy encourages students to determine if the measurements of the blue water and red water were increased by the same multiplier. - Students can justify their reasoning based on the multiplier being the same or different.
Assessing Questions	How did you get this number here (point to the 3)? How about this one (point to the 2)?
	Why did you decide to make a fraction? How did that help?

Advancing Questions	What would I need to do in order to make Andre's water the same shade of purple?
	Could you have multiplied by another number here (point to the number for their own mixture) and still have the same shade of purple? Why or why not?

Points of Growth

Name of Strategy	Reciprocal
Number of Strategy	1
Color of Letter Circle	Red
Image of Strategy	
Strategy Overview	- Students may use the reciprocal fraction and misalign the numbers denoting blue and red.
Relation to Learning Goal	- It is important that students realize how to align the numbers according to what they contextually denote before determining equivalence through multiplication. - If many students experience this Point of Growth, Sequence it before the Fractions Productive Approach to connect the two Strategies.
Assessing Questions	Why did you put this number in the numerator (8) and this one in the denominator (3)?

	Why did you put this number in the numerator (9) and this one in the denominator (24)?
Advancing Questions	Is it important to have the blues in one part of the fraction and the reds in the other? Why or why not?
	When we compare this number (8) with this one (9), what are we comparing?
	How are these values related and why is it important to align the same values?

Name of Strategy	Dividing
Number of Strategy	2
Color of Letter Circle	Red
Image of Strategy	
Strategy Overview	Students may think that because 8 is a factor of 16 and 3 is a factor of 9, that the two ratios are equivalent.

Questions to help get started

Helpful Hint	Remember students always have a place to start and sometimes we need to listen so we can remind students they understand mathematics on some level.
Questions	How can you arrange these numbers so you can compare them?

	Remember the cylinders in the warm-up? How about we try that strategy?
	Can you explore what we just talked about and I'll check back with you in a moment?

Sequence/Connections

Name of Sequence	Grouping First
Description of Sequence	- Use this sequence if at least one group uses Grouping. This will provide a visual reference for the rest of the Strategies. - Then use the Drawing Strategy to further emphasize the visualization of the equivalence. - Finish with the Fractions to reinforce the multiplicative relationship. - If a lot of students use Drawing, use suggested sequence #2.
A	Grouping
Color of Letter Circle	Green
Question(s) for the Group	How did you decide to group the reds and the blues?
	How did you use the groups to figure out if the mixture would be the same shade of purple?
Question(s) for the Whole Class	Can someone tell me if there's another way to group the reds and the blues?
	Can someone tell me if you'd use this Strategy for figuring out if it's the same shade of purple if we had really big numbers? Why or why not?
Bridge	- This is a nice visual for seeing how many reds and blues we need. - I wonder if there's another way to visualize this.
B	Drawing
Color of Letter Circle	Green
Question(s) for the Group	What gave you the idea to draw the cups?

	How did you come up with this number (3 & 2)?
Question(s) for the Whole Class	Can someone tell me why we are multiplying by the 3? Why not add a number instead?
	Can someone tell me if there are similarities or differences between this Strategy and the Grouping?
	When would we use the Drawing? When would we use the Grouping instead?
Bridge	- I definitely see a similar pattern here. - I love these visuals. I wonder if there's a way to do this with just the numbers.
C	Fractions ⭐
Color of Letter Circle	Green
Question(s) for the Group	Why did you put the 24 in the numerator and the 9 in the denominator?
	Why did you decide to multiply by this number (3 & 2)?
Question(s) for the Whole Class	Can someone tell me if we could have put the red in the numerator and the blue in the denominator instead? Why or why not?
	Can someone tell me why I need to multiply by the same number to say the shade of purple is the same?
	Can someone tell me if there's any similarities or differences between all three Strategies?
Activity Conclusion	- How did each Strategy help you decide if the shade of purple would be the same? - Which Strategy do you prefer? Why? - Do we have to multiply the reds and the blues by the same number to get the same shade of purple? Why or why not?

Name of Sequence	Drawing First
Description of Sequence	- Use this sequence if a lot of groups use the Drawing. This will provide a visual reference and

	allow for many students to talk about the first Strategy. - Then use the Grouping Strategy to further emphasize the visualization of the equivalence. - Finish with the Fractions to reinforce the multiplicative relationship. - If a lot of students use the Grouping Strategy, use suggested sequence #1.
B	Drawing
Color of Letter Circle	Green
Question(s) for the Group	What gave you the idea to draw the cups?
	How did you come up with this number (3 & 2)?
Question(s) for the Whole Class	Can someone tell me why we are multiplying by the 3? Why not add a number instead?
	Can someone tell me if you'd use this Strategy for figuring out if it's the same shade of purple if we had really big numbers? Why or why not?
Bridge	- This is a nice visual for seeing how many reds and blues we need. - I wonder if there's another way to visualize this.
A	Grouping
Color of Letter Circle	Green
Question(s) for the Group	How did you decide to group the reds and the blues?
	How did you use the Groups to figure out if the mixture would be the same shade of purple?
Question(s) for the Whole Class	Can someone tell me if there's another way to group the reds and the blues?
	Can someone tell me if there are similarities or differences between this Strategy and the Grouping?
	When would we use the Drawing? When would we use the Grouping instead?
Bridge	- I definitely see a similar pattern here. - I love these visuals. I wonder if there's a way to do this with just the numbers.

C	Fractions ☆
Color of Letter Circle	Green
Question(s) for the Group	Why did you put the 24 in the numerator and the 9 in the denominator?
	Why did you decide to multiply by this number (3 & 2)?
Question(s) for the Whole Class	Can someone tell me if we could have put the red in the numerator and the blue in the denominator instead? Why or why not?
	Can someone tell me why I need to multiply by the same number to say the shade of purple is the same?
	Can someone tell me if there's any similarities or differences between all three Strategies?
Activity Conclusion	- How did each Strategy help you decide if the shade of purple would be the same? - Which Strategy do you like the most? Why? - Do we have to multiply the reds and the blues by the same number to get the same shade of purple? Why or why not?

Name of Sequence	Using the Reciprocal
Description of Sequence	- Use this sequence if a lot of groups use the Reciprocal Point of Growth. - Sequence it right before the Fractions Productive Approach to connect the Strategies and emphasize how important it is to align the reds and the blues.
A	Grouping
Color of Letter Circle	Green
Question(s) for the Group	How did you decide to group the reds and the blues?
	How did you use the Groups to figure out if the mixture would be the same shade of purple?
Question(s) for the Whole Class	Can someone tell me if there's another way to group the reds and the blues?

	Can someone tell me if you'd use this Strategy for figuring out if it's the same shade of purple if we had really big numbers? Why or why not?
Bridge	- This is a nice visual for seeing how many reds and blues we need. - I wonder if there's another way to visualize this.
B	Drawing
Color of Letter Circle	Green
Question(s) for the Group	What gave you the idea to draw the cups?
	How did you come up with this number (3 & 2)?
Question(s) for the Whole Class	Can someone tell me why we are multiplying by the 3? Why not add a number instead?
	Can someone tell me if there are similarities or differences between this Strategy and the Grouping?
	When would we use the Drawing? When would we use the Grouping instead?
Bridge	- I definitely see a similar pattern here. - I love these visuals. I wonder if there's a way to do this with just the numbers.
1	Reciprocal
Color of Letter Circle	Red
Question(s) for the Group	Why did you put this number in the numerator (8) and this one in the denominator (3)?
	Why did you put this number in the numerator (9) and this one in the denominator (24)?
Question(s) for the Whole Class	Can someone tell me what's misaligned with these fractions?
	How would we use these fractions and ensure the blues match up with the blues and the reds match up with the reds?
Bridge	- So the fractions are okay, it's just the alignment. - I wonder how we can use these and see if the shades of purple are the same.

C	Fractions ⭐
Color of Letter Circle	Green
Question(s) for the Group	Why did you put the 24 in the numerator and the 9 in the denominator?
	Why did you decide to multiply by this number (3 & 2)?
Question(s) for the Whole Class	Can someone tell me if we could have put the red in the numerator and the blue in the denominator instead? Why or why not?
	Can someone tell me why I need to multiply by the same number to say the shade of purple is the same?
	Can someone tell me if there's any similarities or differences between all three Strategies?
Activity Conclusion	- How did each Strategy help you decide if the shade of purple would be the same? - Which Strategy do you like the most? Why? - Do we have to multiply the reds and the blues by the same number to get the same shade of purple? Why or why not?

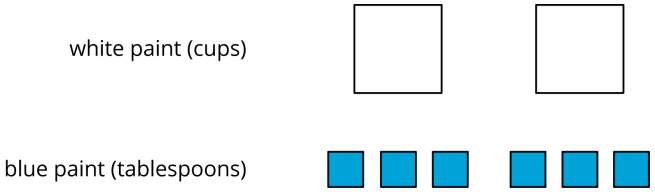
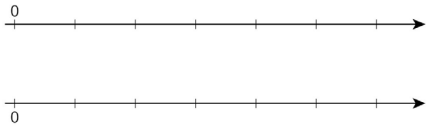
Reflection Questions

Reflection Question(s)	- How could I re-sequence the Strategies my students used to emphasize the importance of the multiplicative relationship? - What is the best way to show that all Strategies are checking to see if the proportion of blues and reds are preserved? - Is there a pattern I noticed when interacting with students or when I called on students/groups to present? Am I leaving anyone out? - Is there anything I did during Monitoring, Selecting, Sequencing, or Connecting that I can learn from for the next 5Ps activity?

Grade Level	6
Unit Number	2
Lesson Number	4
Activity Number	3
Activity Name	Perfect Purple Water
Learning Goal	- Students will recognize if two ratios are equivalent and explain equivalence in the terms of each number in the ratio being multiplied by the same number to justify equivalence. - Students will use discrete diagrams with circled groups to represent ratios.
Image of Student-Facing Activity	The recipe for Perfect Purple Water says, “Mix 8 ml of blue water with 3 ml of red water.” Jada mixes 24 ml of blue water with 9 ml of red water. Andre mixed 16 ml of blue water with 9 ml of red water. 1. Which person will get a color mixture that is the same shade as Perfect Purple Water? Explain or show your reasoning. 2. Find another combination of blue water and red water that will also result in the same shade as Perfect Purple Water. Explain or show your reasoning.

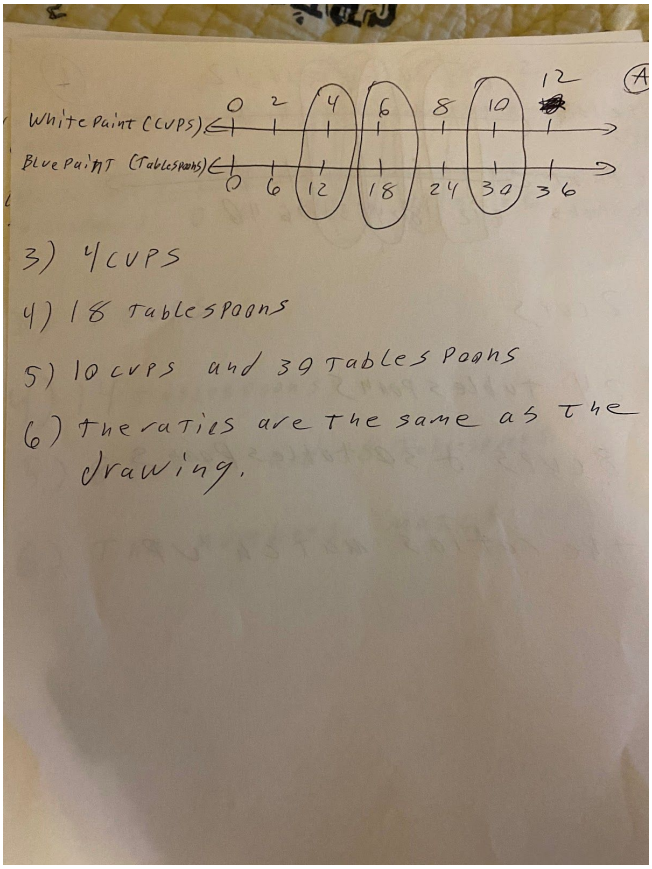
Productive Approaches

Name of Strategy	Grouping
Letter of Strategy	A
Color of Letter Circle	Green

Grade Level	6
Unit Number	2
Lesson Number	6
Activity Number	3
Activity Name	Blue Paint on a Double Number Line
Learning Goal	- Students will compare and contrast discrete values on double number line diagrams. - Students will recognize how to use a double number line diagram to find equivalent ratios, and label and interpret the data in context.
Image of Student-Facing Activity	Here is a diagram showing Elena's recipe for light blue paint. (Figure 1)  1. Complete the double number line diagram to show the amounts of white paint and blue paint in different-sized batches of light blue paint. (Figure 2)  2. Compare your double number line diagram with your partner. Discuss your thinking. If needed, revise your diagram. 3. How many cups of white paint should Elena mix with 12 tablespoons of blue paint? How many batches would this make? 4. How many tablespoons of blue paint should Elena mix with 6 cups of white paint? How many batches would this make?

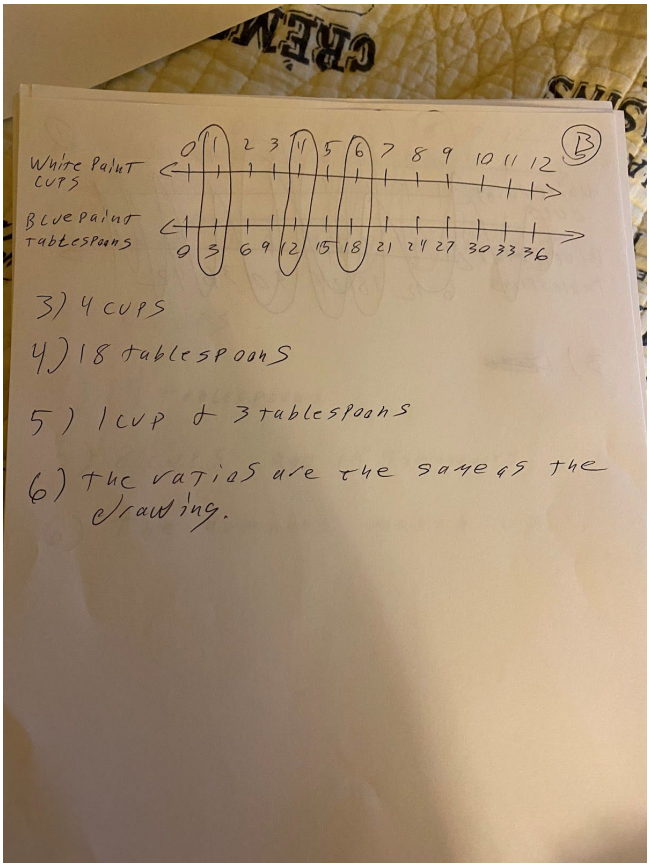
	5. Use your double number line diagram to find another amount of white paint and blue paint that would make the same shade of light blue paint. 6. How do you know that these mixtures would make the same shade of light blue paint?

Productive Approaches

Name of Strategy	2 on Top, 6 Below
Letter of Strategy	A
Color of Letter Circle	Green
Image of Strategy	

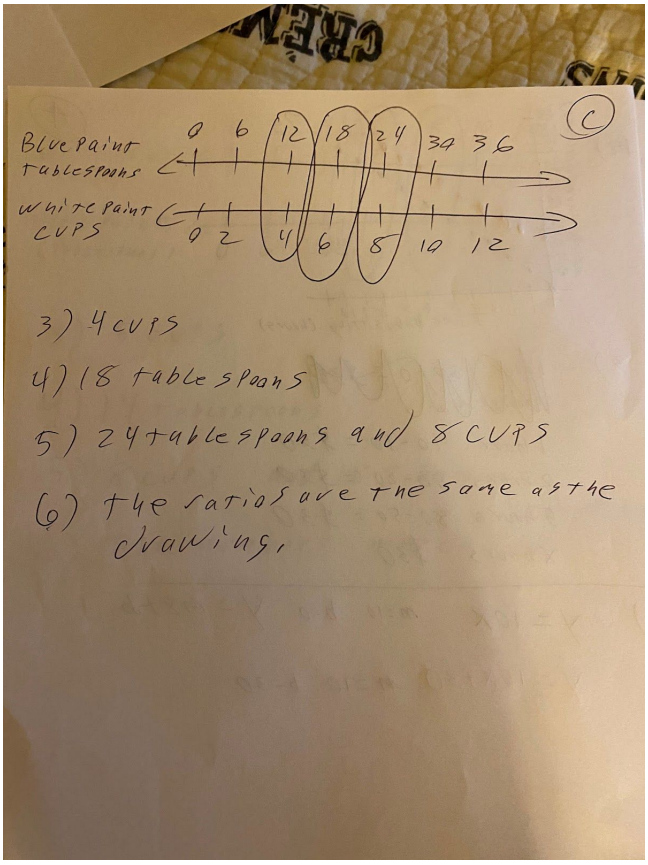
Helpful Hint	This Strategy uses the visualization provided in the problem.
Strategy Overview	- Draw a number line for the white paint. - Draw a number line below for the blue paint. - Expand both number lines to answer the questions.
Relation to Learning Goal	- This Strategy allows students to visually align the white and blue paint. - Students can justify their reasoning based on the double number line. - Students may visually expand the double number line to compare rates and predict outcomes.
Assessing Questions	You lined up the white and blue paint here like this (point to the numbers). Why did you choose those numbers to line up?
	How did you know when to stop expanding the double number line?
Advancing Questions	Does it matter which number line is on the top or the bottom? Why or why not?
	Are there any contexts or numbers you wouldn't want to use a double number line for? (Yes) What would that be?

Name of Strategy	1 on Top, 3 Below ☆
Letter of Strategy	B
Color of Letter Circle	Green

Image of Strategy	
Helpful Hint	This is a Critical Strategy because students are calculating what they will learn to be the unit rate.
Strategy Overview	- Draw a number line for the white paint. - Draw a number line below for the blue paint. - Expand both number lines to answer the questions, adjusting the scale to the unit rate.
Relation to Learning Goal	- This Strategy allows students to visually align the white and blue paint. - Students can justify their reasoning based on the double number line. - Students may visually expand the double number line to compare rates and predict outcomes.

Assessing Questions	You lined up the white and blue paint here like this (point to the numbers). Why did you choose those numbers to line up?
	How did you know when to stop expanding the double number line?
Advancing Questions	Does it matter which number line is on the top or the bottom? Why or why not?
	Are there any contexts or numbers you wouldn't want to use a double number line for? (Yes) What would that be?
	Why did you decide to count by 1s and 3s instead of 2s and 6s?

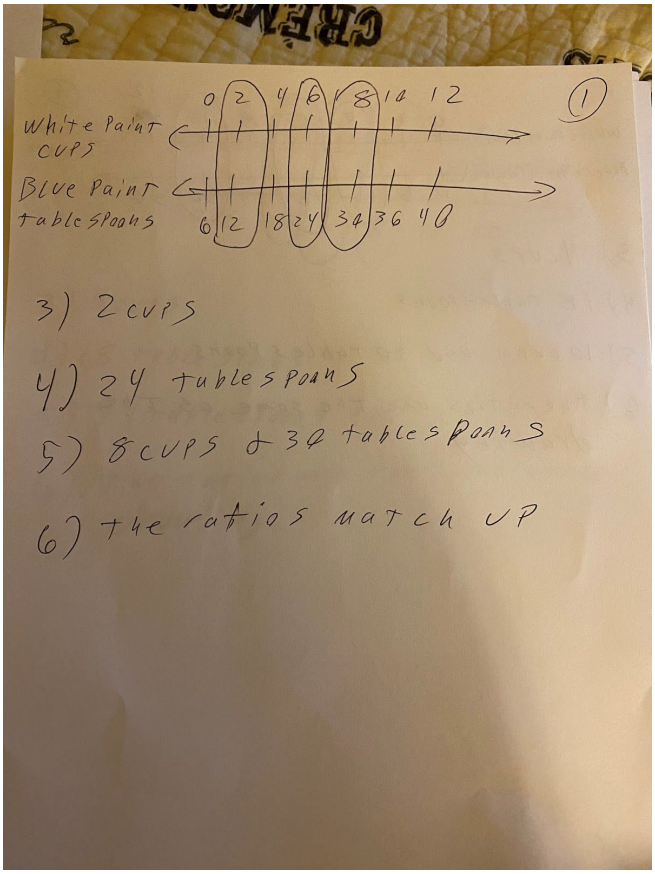
Name of Strategy	6 on Top, 2 Below
Letter of Strategy	C
Color of Letter Circle	Green

Image of Strategy	
Helpful Hint	This Strategy uses the visualization provided in the problem.
Strategy Overview	- Draw a number line for the blue paint. - Draw a number line below for the white paint. - Expand both number lines to answer the questions.
Relation to Learning Goal	- This Strategy allows students to visually align the white and blue paint. - Students can justify their reasoning based on the double number line. - Students may visually expand the double number line to compare rates and predict outcomes.

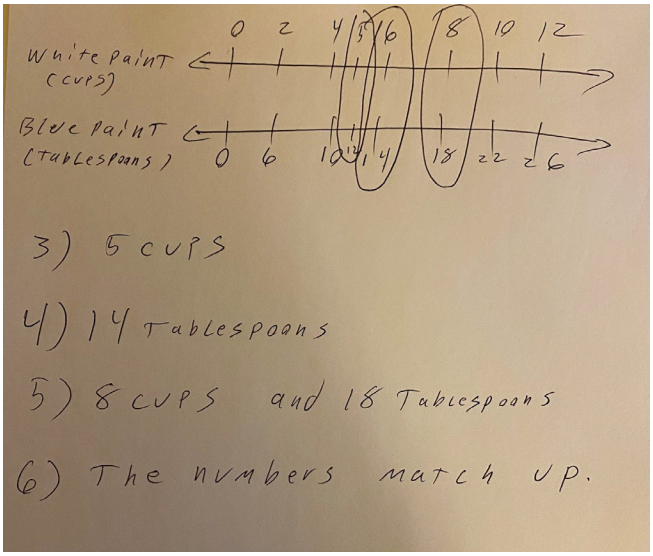
Assessing Questions	You lined up the white and blue paint here like this (point to the numbers). Why did you choose those numbers to line up?
	How did you know when to stop expanding the double number line?
Advancing Questions	Does it matter which number line is on the top or the bottom? Why or why not?
	Are there any contexts or numbers you wouldn't want to use a double number line for? (Yes) What would that be?

Points of Growth

Name of Strategy	Start at Zero
Number of Strategy	1
Color of Letter Circle	Red

Image of Strategy	
Strategy Overview	- Students may start in different places on the number lines, thus misaligning the relationship. - Students then use the misaligned number lines to derive their answers to the problem.
Relation to Learning Goal	- It is important that students take time to align the double number line appropriately. - If many students experience this Point of Growth, sequence this approach in the beginning to facilitate conversations about the importance of starting at zero for both number lines.
Assessing Questions	What number did you start with for the number line up top? Why did you start there?

	What number did you start with for the number line down below? Why did you start there?
Advancing Questions	What happens to the numbers when we don't start both number lines at zero?
	Why would we want to start both number lines at zero?
	What happens when we don't start both number lines aligned at zero?

Name of Strategy	Skip Count
Number of Strategy	2
Color of Letter Circle	Red
Image of Strategy	
Strategy Overview	Students may make a computational error while skip counting and align different numbers from what the ratio would produce.

Questions to help get started

Helpful Hint	Remember students always have a place to start and sometimes we need to listen so we can remind students they understand mathematics on some level.
Questions	How can I use these visuals with the paint to get started?
	They say to use two number lines, how can I use them both with the white and blue paint?
	Can you explore what we just talked about and I'll check back with you in a moment?

Sequence/Connections

Name of Sequence	Start with 2 and 6
Description of Sequence	- Use this sequence if a lot of groups place the white paint on the top and count by 2s while placing the blue paint on the bottom and counting by 6s. - Then show the double number lines counting by 1s and 3s. - Finish with the blue paint number line on the top and the white paint number line on the bottom. - If more students count by 1s and 3s, use suggested sequence #2.
A	2 on Top, 6 Below
Color of Letter Circle	Green
Question(s) for the Group	Why did you decide to put the white paint on top and the blue paint on the bottom?
	Why did you count by 2s and 6s?

Question(s) for the Whole Class	Can someone tell me if they had to go this far with the number lines? Why or why not?
	Can someone tell me how they might have made sure they lined up the numbers correctly?
Bridge	- This is a nice visual for seeing where the blue and white paint line up. - I wonder if there's another way to visualize these numbers.
B	1 on Top, 3 Below ☆
Color of Letter Circle	Green
Question(s) for the Group	Why did you decide to count by 1s and 3s?
	What numbers does this double number line have in common with the first Strategy? Why?
Question(s) for the Whole Class	Can someone tell me what the similarities or differences are between these two Strategies?
	Can someone tell me if there's a benefit to using one of these Strategies over the other?
	What would it take to get you to use the Strategy you didn't choose?
Bridge	- I like that I have two different double number lines to choose from. - I wonder if there's a third way to think about these number lines.
C	6 on Top, 2 Below
Color of Letter Circle	Green
Question(s) for the Group	Why did you decide to put the blue paint on top and the white paint on the bottom?
	Is this Strategy more like the first or second one we saw? Why?
Question(s) for the Whole Class	Can someone tell me if it matters which paint is on the top or bottom? Why or why not?
	Which of the three Strategies do we like the most? Why?

Activity Conclusion	- How did each Strategy help you to answer the questions? - Does it matter which Strategy I use to find the answers? Why or why not? - What would make you choose one Strategy over another?

Name of Sequence	Start with 1 and 3
Description of Sequence	- Use this sequence if a lot of groups place the white paint on the top and count by 1s while placing the blue paint on the bottom and counting by 3s. - Then show the double number lines counting by 2s and 6s. - Finish with the blue paint number line on the top and the white paint number line on the bottom. - If more students count by 2s and 6s, use suggested sequence #1.
B	1 on Top, 3 Below ⭐
Color of Letter Circle	Green
Question(s) for the Group	Why did you decide to count by 1s and 3s?
	Why did you decide to put the white paint on top and the blue paint on the bottom?
Question(s) for the Whole Class	Can someone tell me if they had to go this far with the number lines? Why or why not?
	Can someone tell me how they might have made sure they lined up the numbers correctly?
Bridge	- This is a nice visual for seeing where the blue and white paint line up. - I wonder if there's another way to visualize these numbers.
A	2 on Top, 6 Below
Color of Letter Circle	Green
Question(s) for the Group	Why did you count by 2s and 6s?

	What numbers does this double number line have in common with the first Strategy? Why?
Question(s) for the Whole Class	Can someone tell me what the similarities or differences are between these two Strategies?
	Can someone tell me if there's a benefit to using one of these Strategies over the other?
	What would it take to get you to use the Strategy you didn't choose?
Bridge	- I like that I have two different double number lines to choose from. - I wonder if there's a third way to think about these number lines.
C	6 on Top, 2 Below
Color of Letter Circle	Green
Question(s) for the Group	Why did you decide to put the blue paint on top and the white paint on the bottom?
	Is this Strategy more like the first or second one we saw? Why?
Question(s) for the Whole Class	Can someone tell me if it matters which paint is on the top or bottom? Why or why not?
	Which of the three Strategies do we like the most? Why?
Activity Conclusion	- How did each Strategy help you to answer the questions? - Does it matter which Strategy I use to find the answers? Why or why not? - What would make you choose one Strategy over another?

Name of Sequence	Consistently Start at Zero
Description of Sequence	- Use this sequence if a lot of groups use the Start at Zero Point of Growth.

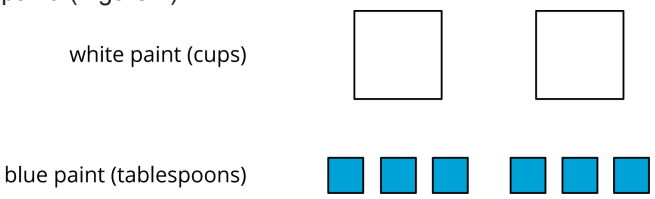
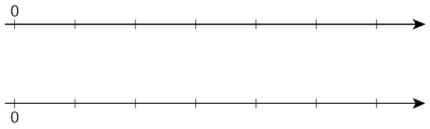
	- Sequence it at the beginning to facilitate a discussion about the importance of starting the alignment of the ratios accurately.
1	Start at Zero
Color of Letter Circle	Red
Question(s) for the Group	Why did you start at 0 here but you started at 6 here?
	How did you know that the numbers didn't line up like they should have?
Question(s) for the Whole Class	Can someone tell me what we need to do to align these number lines?
	Can someone tell me why we should start at 0 with both number lines?
Bridge	- It's important to start both number lines at zero. - Let's see what happens when we do that.
A	2 on Top, 6 Below
Color of Letter Circle	Green
Question(s) for the Group	Why did you decide to put the white paint on top and the blue paint on the bottom?
	Why did you count by 2s and 6s?
Question(s) for the Whole Class	Can someone tell me if they had to go this far with the number lines? Why or why not?
	Can someone tell me how they might have made sure they lined up the numbers correctly?
Bridge	- This is a nice visual for seeing where the blue and white paint line up. - I wonder if there's another way to visualize these numbers.
B	1 on Top, 3 Below ⭐
Color of Letter Circle	Green
Question(s) for the Group	Why did you decide to count by 1s and 3s?

	What numbers does this double number line have in common with the first Strategy? Why?
Question(s) for the Whole Class	Can someone tell me what the similarities or differences are between these two Strategies?
	Can someone tell me if there's a benefit to using one of these Strategies over the other?
	What would it take to get you to use the Strategy you didn't choose?
Bridge	- I like that I have two different double number lines to choose from. - I wonder if there's a third way to think about these number lines.
C	6 on Top, 2 Below
Color of Letter Circle	Green
Question(s) for the Group	Why did you decide to put the blue paint on top and the white paint on the bottom?
	Is this Strategy more like the first or second one we saw? Why?
Question(s) for the Whole Class	Can someone tell me if it matters which paint is on the top or bottom? Why or why not?
	Which of the three Strategies do we like the most? Why?
Activity Conclusion	- How did each Strategy help you to answer the questions? - Does it matter which Strategy I use to find the answers? Why or why not? - What would make you choose one Strategy over another?

Reflection Questions

Reflection Question(s)	- How could I re-sequence the Strategies so students better connect similarities between the three different ways to make a double number

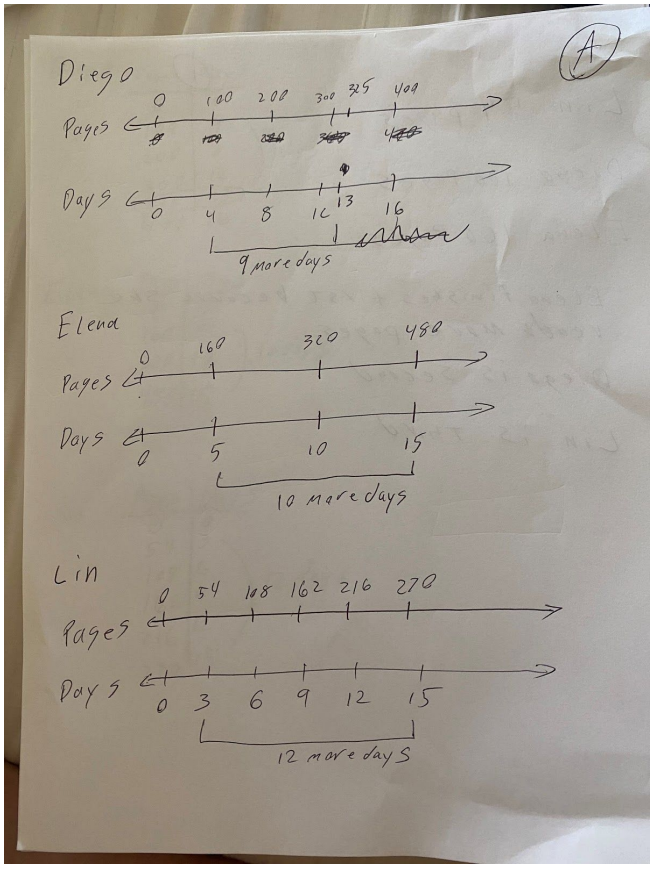
	line? - Based on what my students did, how could I informally continue to build the concept of unit rate? - Is there a pattern I noticed when interacting with students or when I called on students/groups to present? Am I leaving anyone out? - Is there anything I did during Monitoring, Selecting, Sequencing, or Connecting that I can learn from for the next 5Ps activity?

Grade Level	6
Unit Number	2
Lesson Number	6
Activity Number	3
Activity Name	Blue Paint on a Double Number Line
Learning Goal	- Students will compare and contrast discrete values on double number line diagrams. - Students will recognize how to use a double number line diagram to find equivalent ratios, and label and interpret the data in context.
Image of Student-Facing Activity	Here is a diagram showing Elena's recipe for light blue paint. (Figure 1)  1. Complete the double number line diagram to show the amounts of white paint and blue paint in different-sized batches of light blue paint. (Figure 2)  2. Compare your double number line diagram with your partner. Discuss your thinking. If needed, revise your diagram. 3. How many cups of white paint should Elena mix with 12 tablespoons of blue paint? How many batches would this make? 4. How many tablespoons of blue paint should Elena mix with 6 cups of white paint? How many batches would this make?

Grade Level	6
Unit Number	2
Lesson Number	14
Activity Number	3
Activity Name	Keeping Track of all Possible Outcomes
Learning Goal	- Students will recognize and determine what information is needed to solve a problem involving equivalent ratios. - Students will choose a visual or numerical strategy to compare rates and extend the comparison to predict outcomes.
Image of Student-Facing Activity	- Lin read the first 54 pages from a 270-page book in the last 3 days. - Diego read the first 100 pages from a 320-page book in the last 4 days. - Elena read the first 160 pages from a 480-page book in the last 5 days If they continue to read every day at these rates, who will finish first, second, and third? Explain or show your reasoning.

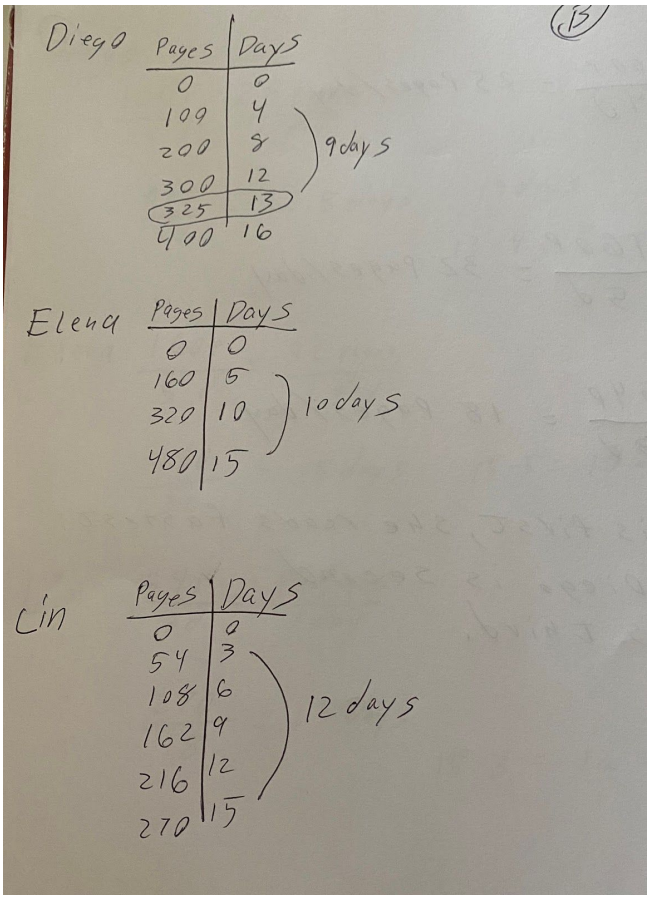
Productive Approaches

Name of Strategy	Double Number Line
Letter of Strategy	A
Color of Letter Circle	Green

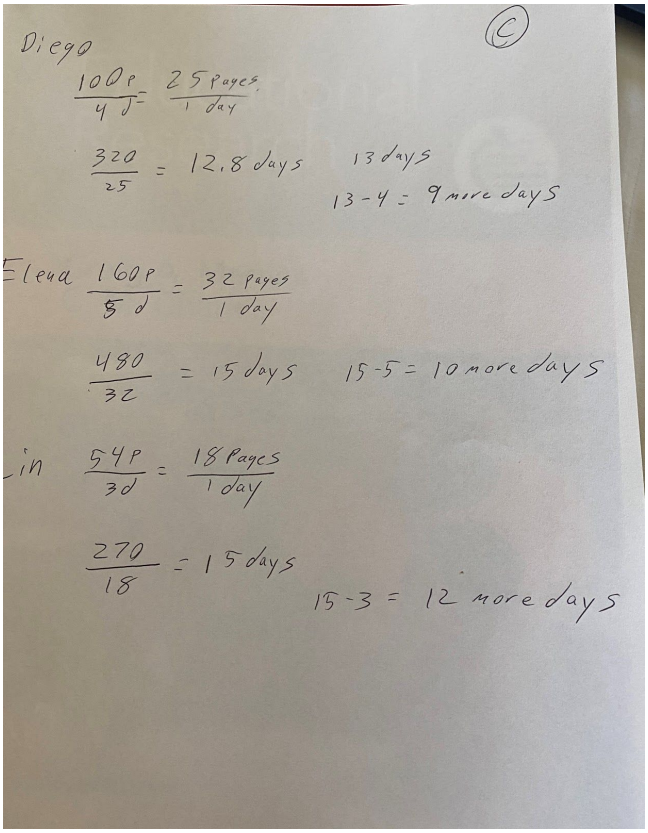
Image of Strategy	
Helpful Hint	This is a familiar strategy students have used in many contexts before.
Strategy Overview	- Draw a number line for the number of pages read. - Draw a number line below for the number of days. - Expand both number lines until the final page in the book is reached.
Relation to Learning Goal	- This Strategy allows students to visually align the number of pages with the number of days. - Students can justify their reasoning based on the Double Number Line. - Students may visually extend the Double Number Line to compare rates and predict outcomes.

Assessing Questions	You lined up the number of pages with the number of days here like this (point to the numbers). Why did you choose those numbers to line up?
	How did you know when to stop expanding the Double Number Lines?
Advancing Questions	Does it matter which number line is on the top or the bottom? Why or why not?
	Are there any contexts or numbers you wouldn't want to use a double number line for? (Yes) What would that be?

Name of Strategy	Table
Letter of Strategy	B
Color of Letter Circle	Green

Image of Strategy	
Helpful Hint	This strategy creates a way for students to keep track of the data as they continue to build out the numbers until the total number of pages is reached.
Strategy Overview	- Draw a table with the left column denoting the number of pages read. - The right column denotes the number of days. - Expand the table until the final page in the book is reached.
Relation to Learning Goal	- This Strategy allows students to numerically align the number of pages with the number of days. - Students can justify their reasoning based on the Table. - Students may extend the Table to compare rates and predict outcomes.

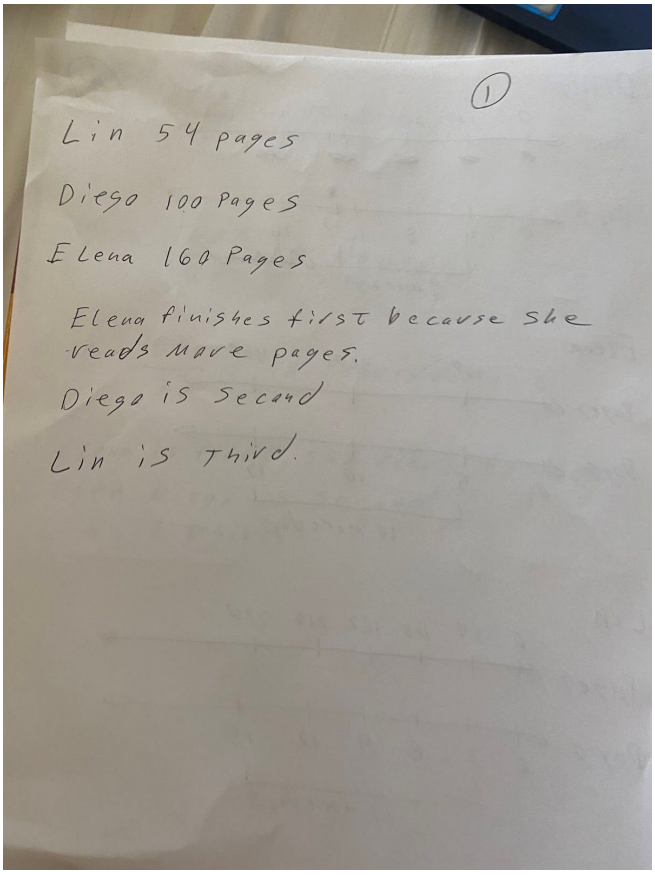
Assessing Questions	You lined up the number of pages with the number of days here like this (point to the numbers). Why did you choose those numbers to line up?
	How did you know when to stop expanding the Table?
Advancing Questions	Does it matter which number is on the left or right (top or bottom)? Why or why not?
	Are there any contexts or numbers you wouldn't want to use a Table for? (Yes) What would that be?

Name of Strategy	Per Day ⭑
Letter of Strategy	C
Color of Letter Circle	Green
Image of Strategy	

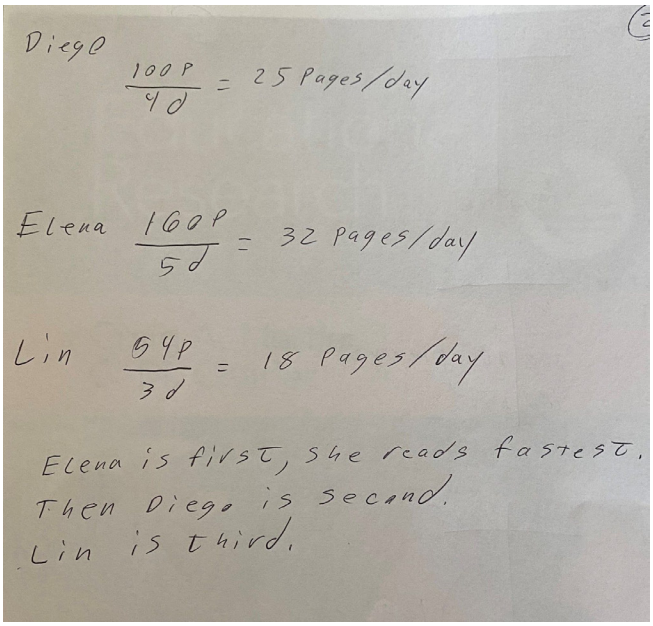
Helpful Hint	This is a critical Strategy because it uses numerical reasoning to calculate the number of pages each student reads per day. The strategy is important for future discussions of unit rate.
Strategy Overview	- Divide the number of pages read by the number of days it took to read them. This gives the number of pages read per day. - Divide the total number of pages in the book by the number of pages read per day. This gives the number of days it will take to read the entire book.
Relation to Learning Goal	- This Strategy allows students to numerically compare rates and identify the unit rate. - Students can then use the unit rate to calculate the outcome. - Sequence this strategy last so you have the other Productive Approaches as visuals to reference for the meaning of the calculations.
Assessing Questions	Why did you decide to divide these two numbers here first?
	Can you tell me what you're doing here when you take the number of pages and divide that by this number you got here?
Advancing Questions	When you got the numbers here (point to 18, 25, or 32), what do these numbers mean in terms of the problem?
	Why did you start working with the pages read and the days? Why not start with the total pages?

Points of Growth

Name of Strategy	Pages Read
Number of Strategy	1
Color of Letter Circle	Red

Image of Strategy	
[H2] Helpful Hint - NONE FOR THIS ONE	
Strategy Overview	- Students may look at the pages read and conclude that the larger the number, the faster someone reads, not taking into consideration the number of days it took to read the pages. - Students may also think a bigger book will take longer to read and not consider how fast someone is reading.
Relation to Learning Goal	- It is important that students realize the ratios and the total number of pages should be considered to ascertain who will be finished first. - If many students experience this Point of Growth, sequence this approach at the beginning to bridge the conversation to a comparison or calculation.

Assessing Questions	What numbers did you compare to find out who finishes first, second, and third?
	What did you do with these numbers (point to 3, 4, 5 days)?
Advancing Questions	If someone reads 100 pages, are they reading faster or slower if they did it in 1 day or in 4 days?
	Can I compare just the number of pages? (No) What if it took me 2 minutes to read a page and it took you 1, who is faster? Why?

Name of Strategy	Total Pages Not Used
Number of Strategy	2
Color of Letter Circle	Red
Image of Strategy	
Strategy Overview	Students may stop once they've calculated the rate at which students read per day and use that number to ascertain who will finish first, second, and third.

Questions to help get started

Helpful Hint	Remember students always have a place to start and sometimes we need to listen so we can remind students they understand mathematics on some level.
Questions	What do I need to figure out first?
	Think about strategies we've used before. Is there one we can use here?
	Can you explore what we just talked about and I'll check back with you in a moment?

Sequence/Connections

Name of Sequence	Double Number Line First
Description of Sequence	- Use this sequence if a lot of groups use a Double Number Line. This will provide multiple points of entry to the conversation for many students. - Then use a Table to show a similar but visually different way to organize the data. - Finish with the Per Day to emphasize that this strategy is useful when using large numbers and you don't want to build out the Table or Double Number Line. - If more students use the Table Strategy, use suggested sequence #2.
A	Double Number Line
Color of Letter Circle	Green
Question(s) for the Group	How did you align these numbers (point to the top number line) with these down here (point to the bottom number line)?
	How did you know when to stop building the Double Number Line out?

Question(s) for the Whole Class	Can someone tell me why a Double Number Line works well for this problem?
	Can someone tell me what would make you not want to use a Double Number Line?
Bridge	- This is a nice visual for seeing where the pages and days line up. - I wonder if there's another way to visualize these numbers.
B	Table
Color of Letter Circle	Green
Question(s) for the Group	How did you align these numbers (point to the left column) with these down here (point to the right column)?
	What made you decide to use a Table over a Double Number Line?
Question(s) for the Whole Class	Can someone tell me what the similarities or differences are between the Double Number Line and the Table?
	Can someone tell me if there's a benefit to using one of these Strategies over the other?
	What would it take to get you to use the Strategy you didn't choose to use?
Bridge	- I like that I have two different visual Strategies to choose from. - I wonder if there's a third way to think about this problem that involves calculations.
C	Per Day ☆
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to do these calculations over a Double Number Line or a Table?
	What made you decide to divide by the number of pages per day?
Question(s) for the Whole Class	I see the 25 pages per day, 32 pages per day, and 18 pages per day in the calculations. Can someone tell me

	where these numbers are in the Double Number Line and the Table?
	Which of the three Strategies do we like the most? Why?
Activity Conclusion	- How did each Strategy help you to know who would finish first, second, and third? - Elena and Lin both finished in 15 days total, so why was Lin listed as second and Elena as third? - There are two very visual Strategies and one calculation. What would make you choose one Strategy over another?

Name of Sequence	Table First
Description of Sequence	- Use this sequence if a lot of groups use a Table. This will provide multiple points of entry to the conversation for many students. - Then use a Double Number Line to show a similar but visually different way to organize the data. - Finish with the Per Day to emphasize that this strategy is useful when using large numbers and you don't want to build out the Table or Double Number Line. - If more students use the Double Number Line Strategy, use suggested sequence #1.
B	Table
Color of Letter Circle	Green
Question(s) for the Group	How did you align these numbers (point to the left column) with these down here (point to the right column)?
	What made you decide to use a Table over a Double Number Line?
Question(s) for the Whole Class	Can someone tell me what the similarities or differences are between the Double Number Line and the Table?

	Can someone tell me if there's a benefit to using one of these Strategies over the other?
	What would it take to get you to use the Strategy you didn't choose to use?
Bridge	- This is a nice visual for seeing where the pages and days line up. - I wonder if there's another way to visualize these numbers.
A	Double Number Line
Color of Letter Circle	Green
Question(s) for the Group	How did you align these numbers (point to the top number line) with these down here (point to the bottom number line)?
	How did you know when to stop building the Double Number Line out?
Question(s) for the Whole Class	Can someone tell me why a Double Number Line works well for this problem?
	Can someone tell me what would make you not want to use a Double Number Line?
Bridge	- I like that I have two different visual Strategies to choose from. - I wonder if there's a third way to think about this problem that involves calculations.
C	Per Day ⭑
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to do these calculations over a Double Number Line or a Table?
	What made you decide to divide by the number of pages per day?
Question(s) for the Whole Class	I see the 25 pages per day, 32 pages per day, and 18 pages per day in the calculations. Can someone tell me where these numbers are in the Double Number Line and the Table?

	Which of the three Strategies do we like the most? Why?
Activity Conclusion	- How did each Strategy help you to know who would finish first, second, and third? - Elena and Lin both finished in 15 days total, so why was Lin listed as second and Elena as third? - There are two very visual Strategies and one calculation. What would make you choose one Strategy over another?

Name of Sequence	Pages Read Assumption
Description of Sequence	- Use this sequence if a lot of groups use the Pages Read Point of Growth. - Sequence it right before the Per Day Productive Approach since the calculations for this Point of Growth are used in the Per Day Productive Approach.
A	Double Number Line
Color of Letter Circle	Green
Question(s) for the Group	How did you align these numbers (point to the top number line) with these down here (point to the bottom number line)?
	How did you know when to stop building the Double Number Line out?
Question(s) for the Whole Class	Can someone tell me why a Double Number Line works well for this problem?
	Can someone tell me what would make you not want to use a Double Number Line?
Bridge	- This is a nice visual for seeing where the pages and days line up. - I wonder if there's another way to visualize these numbers.
B	Table
Color of Letter Circle	Green

Question(s) for the Group	How did you align these numbers (point to the left column) with these down here (point to the right column)?
	What made you decide to use a Table over a Double Number Line?
Question(s) for the Whole Class	Can someone tell me what the similarities or differences are between the Double Number Line and the Table?
	Can someone tell me if there's a benefit to using one of these Strategies over the other?
	What would it take to get you to use the Strategy you didn't choose to use?
Bridge	- I like that I have two different visual Strategies to choose from. - I wonder if there's a third way to think about this problem that involves calculations.
1	Pages Read
Color of Letter Circle	Red
Question(s) for the Group	How did you use these numbers to answer the problem (point to the 25, 32, and 18)?
	Why did you decide not to use the total number of pages in the book?
Question(s) for the Whole Class	Can someone tell me if Elena reads faster but hasn't read for as many days as Diego or Lin, could Diego or Lin be ahead of Elena?
	Can someone tell me if or why the total number of pages in the book might matter?
Bridge	- So it looks like these numbers are usable, we just need to do more here. - Let's look at another approach to continue this great start.
C	Per Day ⭑
Color of Letter Circle	Green

Question(s) for the Group	Why did you choose to do these calculations over a Double Number Line or a Table?
	What made you decide to divide by the number of pages per day?
Question(s) for the Whole Class	I see the 25 pages per day, 32 pages per day, and 18 pages per day in the calculations. Can someone tell me where these numbers are in the Double Number Line and the Table?
	Which of the three Strategies do we like the most? Why?
Activity Conclusion	- How did each Strategy help you to know who would finish first, second, and third? - Elena and Lin both finished in 15 days total, so why was Lin listed as second and Elena as third? - There are two very visual Strategies and one calculation. What would make you choose one Strategy over another?

Reflection Questions

Reflection Question(s)	- How could I re-sequence the Strategies to help my students connect the visual representations to the calculations? - What is the best way to draw out the connection between the strategies to show how they all use 25, 18, and 32 pages per day? - Is there a pattern I noticed when interacting with students or when I called on students/groups to present? Am I leaving anyone out? - Is there anything I did during Monitoring, Selecting, Sequencing, or Connecting that I can learn from for the next 5Ps activity?

Grade Level	6
Unit Number	2
Lesson Number	14
Activity Number	3
Activity Name	Keeping Track of all Possible Outcomes
Learning Goal	- Students will recognize and determine what information is needed to solve a problem involving equivalent ratios. - Students will choose a visual or numerical strategy to compare rates and extend the comparison to predict outcomes.
Image of Student-Facing Activity	- Lin read the first 54 pages from a 270-page book in the last 3 days. - Diego read the first 100 pages from a 320-page book in the last 4 days. - Elena read the first 160 pages from a 480-page book in the last 5 days If they continue to read every day at these rates, who will finish first, second, and third? Explain or show your reasoning.

Productive Approaches

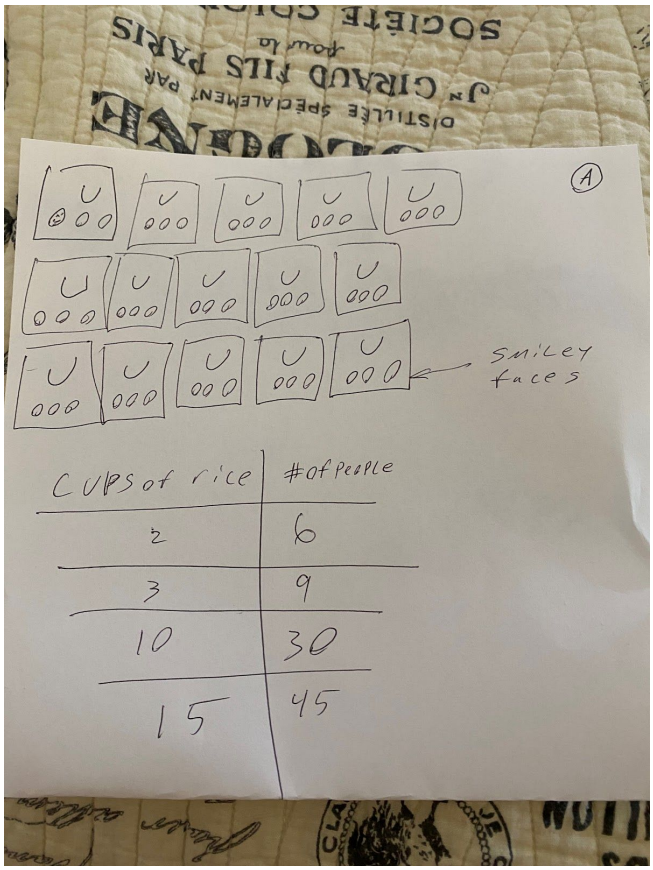
Name of Strategy	Double Number Line
Letter of Strategy	A
Color of Letter Circle	Green

<table>
<tr><td>Grade Level</td><td>7</td></tr>
<tr><td>Unit Number</td><td>2</td></tr>
<tr><td>Lesson Number</td><td>2</td></tr>
<tr><td>Activity Number</td><td>2</td></tr>
<tr><td>Activity Name</td><td>Feeding a Crowd</td></tr>
<tr><td>Learning Goal</td><td>- Students will comprehend that “proportional relationships” refers to when two quantities are related by multiplying by a “constant of proportionality.”
- Students will use tables to express proportional relationships and calculate missing values in a table.</td></tr>
<tr><td>Image of Student-Facing Activity</td><td>1. A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.
a. How many people will 10 cups of rice serve?
b. How many cups of rice are needed to serve 45 people?
<table><tr><th>cups of rice</th><th>number of people</th></tr><tr><td>2</td><td>6</td></tr><tr><td>3</td><td>9</td></tr><tr><td>10</td><td></td></tr><tr><td></td><td>45</td></tr></table>2. A recipe says that 6 spring rolls will serve 3 people. Complete the table.</td></tr>
</table>

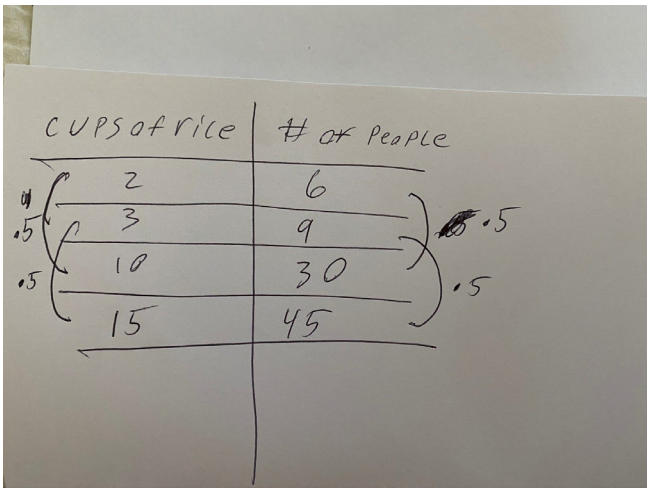
	<table><tr><th>number of spring rolls</th><th>number of people</th></tr><tr><td>6</td><td>3</td></tr><tr><td>30</td><td></td></tr><tr><td>40</td><td></td></tr><tr><td></td><td>28</td></tr></table>

Productive Approaches

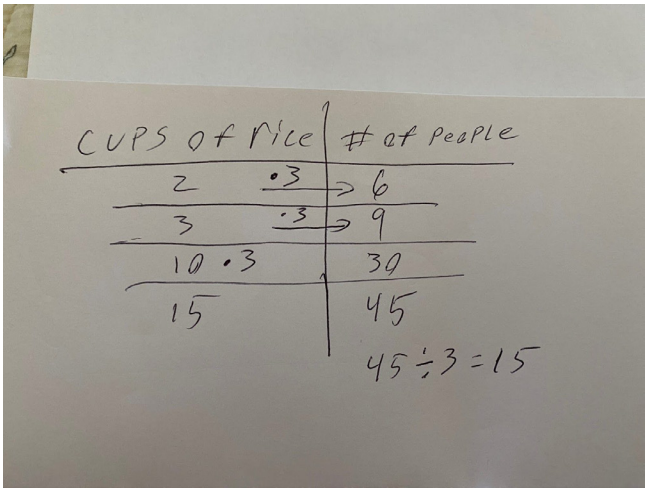
Name of Strategy	Drawing
Letter of Strategy	A
Color of Letter Circle	Green

Image of Strategy	
Helpful Hint	This provides a visual link to the unit rate.
Strategy Overview	- Draw the number of people and cups of rice. - Then circle or box an equal number of people with each cup of rice. - Use the Drawing to fill in the missing values.
Relation to Learning Goal	- This Strategy allows students to visually see how many people can be fed with each cup of rice, implicitly using the Unit Rate. - Students can justify their reasoning based on the Drawing. - Students may not draw out all the cups of rice but instead use the Unit Rate to finish the problem.
Assessing Questions	Which numbers did you use from the table to start the Drawing?

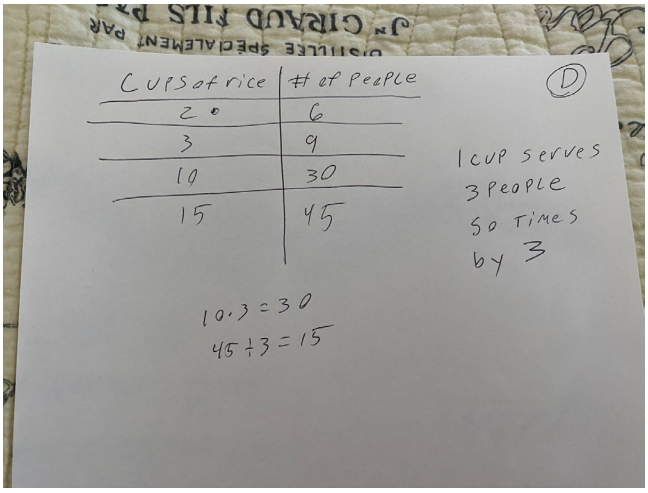
	How did you know how many people to group together with each cup of rice?
Advancing Questions	Do we have to keep drawing this out if we had 100 cups of rice? (That would take way too long)
	What can we learn from the Drawing that we can use so we wouldn't have to draw this out 100 times?

Name of Strategy	Moving Down a Table
Letter of Strategy	B
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	This strategy helps students see the recursive and vertical patterns in the sets of numbers.
Strategy Overview	- Identify the pattern to move down the left side of the column and use that same pattern to move down the right side of the column. - Although the Strategy denotes multiplication, addition can be used as well.
Relation to Learning Goal	- This Strategy allows students to proportional relationships to fill in the table vertically. - Students can justify their reasoning based on the table and multiplier.

Assessing Questions	How did you get this number here (point to the 5)?
	What did you do with this number in order to fill in these blanks in the table?
Advancing Questions	Do you think I'd multiply by this number (5) to get anywhere in the table? Why or why not?
	I notice there's gaps in the table (like between 3 and 10). Does this affect your Strategy at all?

Name of Strategy	Moving Across a Table
Letter of Strategy	C
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	This strategy helps students see the explicit and horizontal patterns in the sets of numbers.
Strategy Overview	- Identify the pattern to move from the left to the right side of the table. - Use the pattern (multiply by 3) to move from the left to the right side of the table, filling in the blank. - Use the inverse (divide by 3) to move from the right to the left side of the table.
Relation to Learning Goal	- This Strategy allows students to use proportional relationships to fill in the table horizontally.

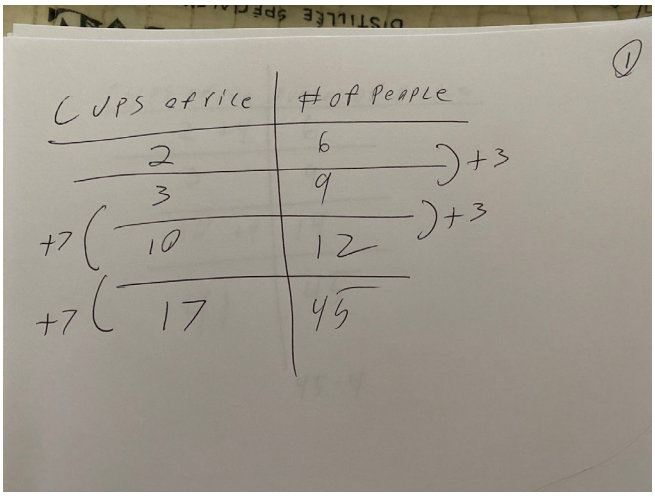
	- Students implicitly use the Unit Rate to calculate the missing numbers. - Students can justify their reasoning based on the table and multiplier.
Assessing Questions	How did you get this number here (point to the 3)?
	What did you do with this number in order to fill in these blanks in the table?
Advancing Questions	I notice there's gaps in the table (like between 3 and 10). Does this affect your Strategy at all?
	Why do you think we can use the same number (3) every time no matter where we are in the table?

Name of Strategy	Unit Rate ⭐
Letter of Strategy	D
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	This is a critical Strategy because it is used through all Strategies and will be the main Strategy in future lessons.
Strategy Overview	- Divide the output (number of people) by the input (cups of rice) to determine the Unit Rate.

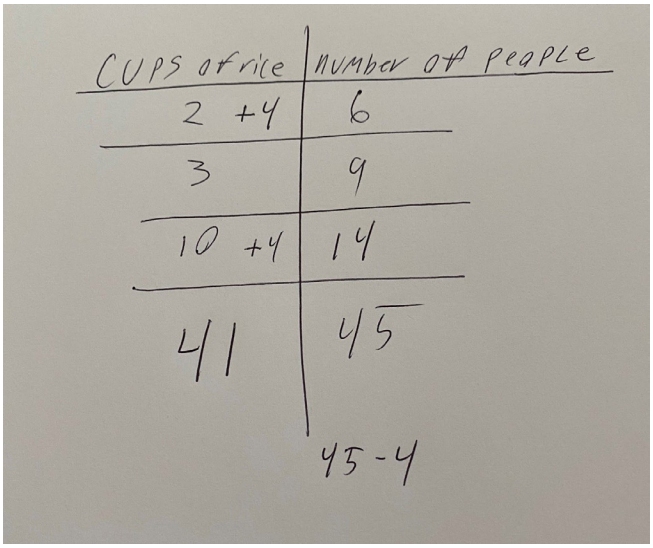
	- Multiply by the Unit Rate to move to the right side of the table and divide by the Unit Rate to move to the left side of the table..
Relation to Learning Goal	- This Strategy allows students to explicitly identify the Unit Rate. - Students can then use the Unit Rate to calculate the outcome. - Sequence this strategy last so you can connect the Unit Rate to the previously Sequenced Strategies.
Assessing Questions	How did you get this number here (point to the 3)?
	What did you do with this number in order to fill in these blanks in the table?
Advancing Questions	I notice there's gaps in the table (like between 3 and 10). Does this affect your Strategy at all?
	Why do you think we can use the same number (3) every time no matter where we are in the table?

Points of Growth

Name of Strategy	Adding Down
Number of Strategy	1
Color of Letter Circle	Red

Image of Strategy	
Strategy Overview	- Students may look at the pattern and use addition inappropriately to fill in the missing numbers. - Although addition can be used in this pattern, it is not consistent when gaps are presented, like between 3 and 10.
Relation to Learning Goal	- It is important that students realize how to use multiplication or addition depending on the consistency of the data presented. - If many students experience this Point of Growth, sequence this approach before the Moving Down the Table Productive Approach to highlight how to effectively use such patterns.
Assessing Questions	How did you get this number here (point to the 7 and 3)?
	What did you do with this number in order to fill in these blanks in the table?
Advancing Questions	Would this number (+7) work if we moved from 2 to 3? Why not?
	Would this number (+3) work if we moved from 12 to 45? Why not?

Name of Strategy	Adding Across
Number of Strategy	2

Color of Letter Circle	Red
Image of Strategy	
Static text under image - ONLY FOR THIS STRATEGY IN THIS LESSON	Certified Strategy added by [name and school from copy doc]
Helpful Hint- NONE FOR THIS ONE	
Strategy Overview	Students may think there is an additive, rather than multiplicative, relationship horizontally in the table. They are correct only if they add 2 more than the previously added number, like add 4, then 6, then 8.

Questions to help get started

Helpful Hint	Remember students always have a place to start and sometimes we need to listen so we can remind students they understand mathematics on some level.
Questions	What pattern do we see in these numbers?
	Remember the paper towels in the warm-up? How about we try that strategy?

	Can you explore what we just talked about and I'll check back with you in a moment?

Sequence/Connections

Name of Sequence	Use the Drawing
Description of Sequence	- Use this sequence if at least one group uses a Drawing. This will provide a visual reference for the rest of the Strategies. - Then use a Table to show the vertical and horizontal patterns. - Finish with the Unit Rate to emphasize that all the Strategies embrace and use the Unit Rate in some manner. - If no student or group uses a Drawing, use suggested sequence #2.
A	Drawing
Color of Letter Circle	Green
Question(s) for the Group	How did you decide to group the cups of rice and the number of people together?
	How did you use the Drawing to fill in the table?
Question(s) for the Whole Class	Can someone tell me if there's another way to group the cups of rice and the number of people while still keeping the groups the same?
	Can someone tell me if you'd use this Strategy for figuring out how many people 100 cups of rice would feed?
Bridge	- This is a nice visual for seeing how many cups of rice feed a certain number of people. - I wonder if there's another way to look at the table.
B	Moving Down a Table
Color of Letter Circle	Green

Question(s) for the Group	Can you tell me what pattern you noticed moving down this table?
	How did you come up with this number (5)?
Question(s) for the Whole Class	Can someone tell me if we could add a number instead of multiply by 5 to move down the table?
	Can someone tell me if there are similarities or differences between this Strategy and the Drawing?
	When would we use the Drawing? When would we use this Strategy instead?
Bridge	- I definitely see a pattern moving down the table. - I wonder if there's a pattern moving across the table.
C	Moving Across a Table
Color of Letter Circle	Green
Question(s) for the Group	How did you get this number here (3)?
	Why did you decide to multiply by this number (3)?
Question(s) for the Whole Class	Can someone tell me if I could add a number instead of multiplying to go from left to right?
	Can someone tell me why I had to divide by 3 to move from right to left?
	Can someone tell me if there's any similarities between moving down the table and moving across the table?
Bridge	- I see that I can move down the table or across the table to find the missing values. - I wonder if there's a consistent number I see in all these Strategies
D	Unit Rate ⭐
Color of Letter Circle	Green
Question(s) for the Group	Which Strategy above most helps you to explain why you're using the number 3?
	Why are you multiplying by 3? Why not add?

Question(s) for the Whole Class	Can someone tell me if the number 3 shows up in all these Strategies?
	Can someone show me where the number 3 is in all these Strategies?
	Are we using the number 3 in the same way throughout the Strategies? Why or why not?
Activity Conclusion	- How did each Strategy help you to fill in the missing parts of the table? - Which Strategy do you most readily see the 3 in? Why? - When we use a Strategy like these we say there is a proportional relationship. Has anyone heard this phrase before?

Name of Sequence	No Drawing
Description of Sequence	- Use this sequence if no one uses a Drawing. - Start with a Table to show the vertical and horizontal patterns. - Finish with the Unit Rate to emphasize that all the Strategies embrace and use the Unit Rate in some manner. - If any student or group uses a Drawing, use suggested sequence #1.
B	Moving Down a Table
Color of Letter Circle	Green
Question(s) for the Group	Can you tell me what pattern you noticed moving down this table?
	How did you come up with this number (5)?
Question(s) for the Whole Class	Can someone tell me if we could add a number instead of multiply by 5 to move down the table?
	Can someone tell me if there are similarities or differences between this Strategy and the Drawing?
	Is there a way we can visualize this pattern?
Bridge	- I definitely see a pattern moving down the table.

	- I wonder if there's a pattern moving across the table.
C	Moving Across a Table
Color of Letter Circle	Green
Question(s) for the Group	How did you get this number here (3)?
	Why did you decide to multiply by this number (3)?
Question(s) for the Whole Class	Can someone tell me if I could add a number instead of multiplying to go from left to right?
	Can someone tell me why I had to divide by 3 to move from right to left?
	Can someone tell me if there's any similarities between moving down the table and moving across the table?
Bridge	- I see that I can move down the table or across the table to find the missing values. - I wonder if there's a consistent number I see in both of these Strategies
D	Unit Rate ⭐
Color of Letter Circle	Green
Question(s) for the Group	Which Strategy above most helps you to explain why you're using the number 3?
	Why are you multiplying by 3? Why not add?
Question(s) for the Whole Class	Can someone tell me if the number 3 shows up in all these Strategies?
	Can someone show me where the number 3 is in all these Strategies?
	Are we using the number 3 in the same way throughout the Strategies? Why or why not?
Activity Conclusion	- How did each Strategy help you to fill in the missing parts of the table? - Which Strategy do you most readily see the 3 in? Why?

	- When we use a Strategy like these we say there is a proportional relationship. Has anyone heard this phrase before?

Name of Sequence	Adding Down the Table
Description of Sequence	- Use this sequence if a lot of groups use the Adding Down Point of Growth. - Sequence it right before the Moving Down a Table Productive Approach to connect when use addition and when to use multiplication to distinguish a pattern.
A	Drawing
Color of Letter Circle	Green
Question(s) for the Group	How did you decide to group the cups of rice and the number of people together?
	How did you use the Drawing to fill in the table?
Question(s) for the Whole Class	Can someone tell me if there's another way to group the cups of rice and the number of people while still keeping the groups the same?
	Can someone tell me if you'd use this Strategy for figuring out how many people 100 cups of rice would feed?
Bridge	- This is a nice visual for seeing how many cups of rice feed a certain number of people. - I wonder if there's another way to look at the table.
1	Adding Down
Color of Letter Circle	Red
Question(s) for the Group	How did you come up with the numbers 7 and 3?
	What made you decide to add these numbers?
Question(s) for the Whole Class	Can someone point out where this pattern might not work within the table?

	Would it make sense that 3 cups of rice serves 9 people and 10 cups of rice serves 12 people? Why or why not?
	Did anyone else try to add down the table? If so, did it work for you? Why or why not?
Bridge	- We can still find a pattern moving down the table. - Let's see how we can find a consistent pattern.
B	Moving Down a Table
Color of Letter Circle	Green
Question(s) for the Group	Can you tell me what pattern you noticed moving down this table?
	How did you come up with this number (5)?
Question(s) for the Whole Class	Can someone tell me if we could add a number instead of multiplying by 5 to move down the table?
	Can someone tell me if there are similarities or differences between this Strategy and the Drawing?
	When would we use the Drawing? When would we use this Strategy instead?
Bridge	- I definitely see a pattern moving down the table. - I wonder if there's a pattern moving across the table.
C	Moving Across a Table
Color of Letter Circle	Green
Question(s) for the Group	How did you get this number here (3)?
	Why did you decide to multiply by this number (3)?
Question(s) for the Whole Class	Can someone tell me if I could add a number instead of multiplying to go from left to right?
	Can someone tell me why I had to divide by 3 to move from right to left?
	Can someone tell me if there's any similarities between moving down the table and moving across the table?

Bridge	- I see that I can move down the table or across the table to find the missing values. - I wonder if there's a consistent number I see in all these Strategies
D	Unit Rate ⭐
Color of Letter Circle	Green
Question(s) for the Group	Which Strategy above most helps you to explain why you're using the number 3?
	Why are you multiplying by 3? Why not add?
Question(s) for the Whole Class	Can someone tell me if the number 3 shows up in all these Strategies?
	Can someone show me where the number 3 is in all these Strategies?
	Are we using the number 3 in the same way throughout the Strategies? Why or why not?
Activity Conclusion	- How did each Strategy help you to fill in the missing parts of the table? - Which Strategy do you most readily see the 3 in? Why? - When we use a Strategy like these we say there is a proportional relationship. Has anyone heard this phrase before?

Reflection Questions

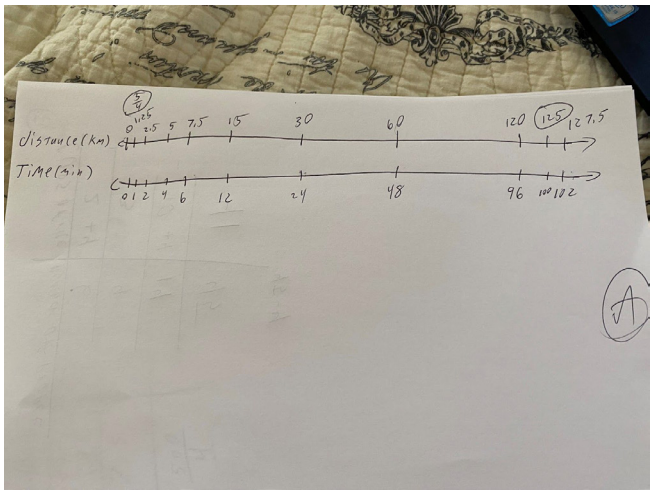
Reflection Question(s)	- How could I re-sequence the Strategies to lead to the formal definition of "proportional relationships?" - What is the best way to emphasize the connections between the Productive Approaches, especially with the Unit Rate? - Is there a pattern I noticed when interacting with students or when I called on students/groups to present? Am I leaving anyone out?

	- Is there anything I did during Monitoring, Selecting, Sequencing, or Connecting that I can learn from for the next 5Ps activity?

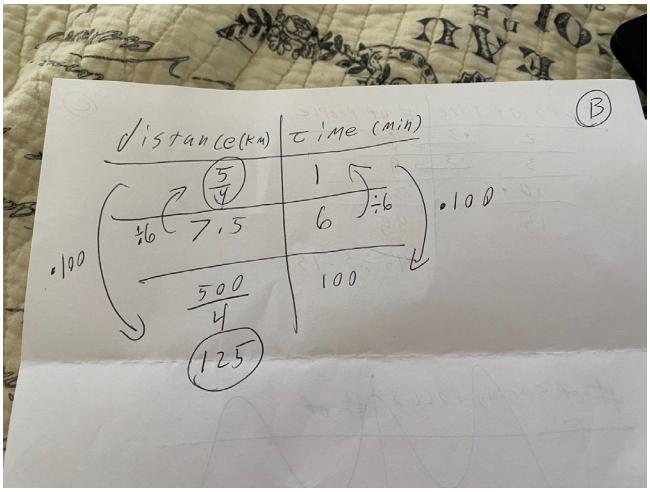
Grade Level	7
Unit Number	4
Lesson Number	2
Activity Number	2
Activity Name	A Train is Traveling at...
Learning Goal	- Students will compare and contrast different strategies for solving problems with equivalent ratios with fractional quantities. - Students will recognize how to find and use a unit rate to solve a problem involving fractional quantities.
Image of Student-Facing Activity	A train is traveling at a constant speed and goes 7.5 kilometers in 6 minutes. At that rate: 1. How far does the train go in 1 minute? 2. How far does the train go in 100 minutes? 

Productive Approaches

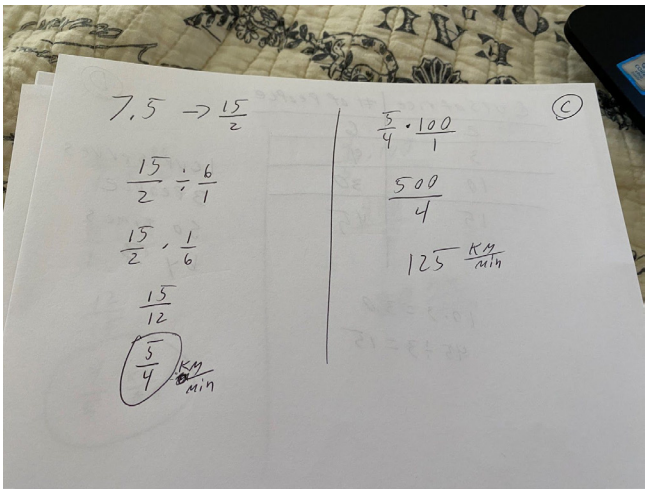
Name of Strategy	Double Number Line
Letter of Strategy	A
Color of Letter Circle	Green

Image of Strategy	
Helpful Hint	This is a familiar strategy students have used in many contexts before.
Strategy Overview	- Draw a number line for the distance. - Draw a number line below for the time. - Expand both number lines you reach 1 minute and 100 minutes.
Relation to Learning Goal	- This Strategy allows students to visually align the distance with the time. - Students can justify their reasoning based on the Double Number Line. - Students may visually expand the Double Number Line to compare rates and predict outcomes.
Assessing Questions	You lined up the distance with the time here like this (point to the numbers). Why did you choose those numbers to line up?
	How did you know when to stop expanding the Double Number Lines?
Advancing Questions	Does it matter which number line is on the top or the bottom? Why or why not?
	Are there any contexts or numbers you wouldn't want to use a double number line for? (Yes) What would that be?

Name of Strategy	Table

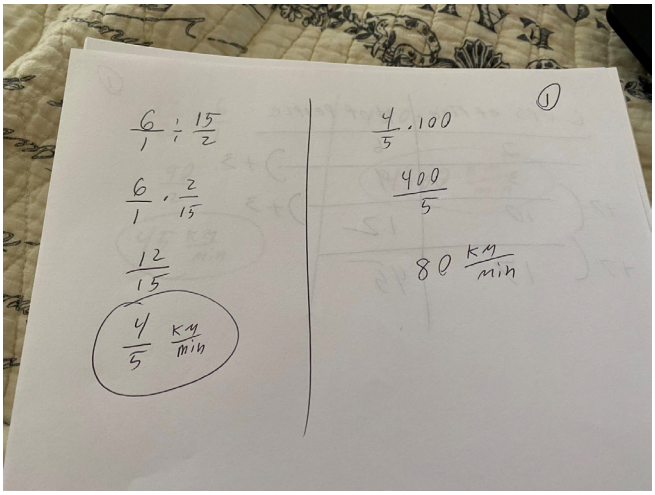
Letter of Strategy	B
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	This strategy creates a way for students to keep track of the data as they continue to build out the numbers until 1 minute and 100 minutes are reached.
Strategy Overview	- Draw a table with the left column denoting the distance. - The right column denotes the time. - Expand the table until 1 minute and 100 minutes is reached with the vertical or horizontal pattern in the table.
Relation to Learning Goal	- This Strategy allows students to numerically align the distance with the time. - Students can justify their reasoning based on the Table. - Students may extend the Table to compare numbers and find the distance at 1 and 100 miles.
Assessing Questions	You lined up the distance with the time here like this (point to the numbers). Why did you choose those numbers to line up?
	How did you know when to stop expanding the Table?
Advancing Questions	Does it matter which number is on the left or right (top or bottom)? Why or why not?

	Are there any contexts or numbers you wouldn't want to use a Table for? (Yes) What would that be?

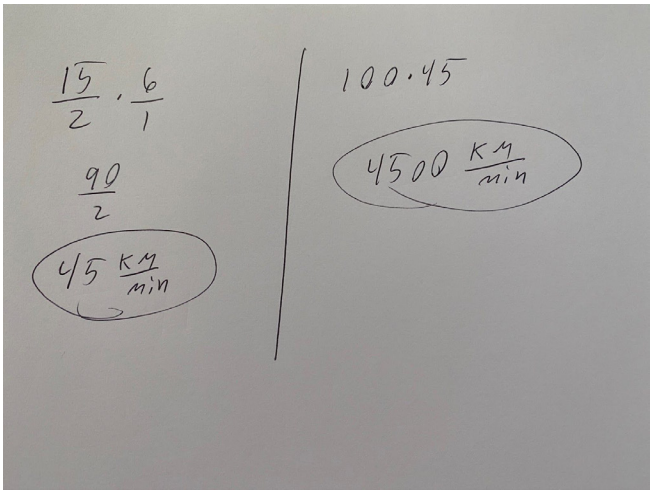
Name of Strategy	Unit Rate ⭐
Letter of Strategy	C
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	This is a critical Strategy because it uses numerical reasoning to calculate the number of pages each student reads per day. The strategy is important in further solidifying understanding of Unit Rate.
Strategy Overview	- Divide the distance by the time. This gives the Unit Rate. - Multiply the Unit Rate by 100 minutes. This gives the distance traveled at 100 minutes.
Relation to Learning Goal	- This Strategy allows students to numerically compare rates and identify the unit rate. - Students can then use the unit rate to calculate the distance. - Sequence this strategy last so you have the other Productive Approaches as visuals to reference for the meaning of the calculations.
Assessing Questions	Why did you decide to divide these two numbers here first?

	Can you tell me what you're doing here when you take the distance and divide by the time?
Advancing Questions	When you got the number here (5/4), what does this number mean in terms of the problem?
	Why did you multiply by the Unit Rate here? Is there a time I would want to divide by the Unit Rate?

Points of Growth

Name of Strategy	Switch Divisor and Dividend
Number of Strategy	1
Color of Letter Circle	Red
Image of Strategy	
Strategy Overview	- Students may switch the dividend and the divisor, thus using the reciprocal of the Unit Rate. - Students then use this reciprocal in the proper manner to calculate the distance after 100 minutes.
Relation to Learning Goal	- It is important that students realize which ratio is most effective and makes the most amount of sense given the context of a problem.

	- If many students experience this Point of Growth, sequence this approach before the Unit Rate Productive Approach to connect the Strategies.
Assessing Questions	What number did you divide by here?
	What does this mean in terms of the units here if I multiply 100 minutes by ⅙?
Advancing Questions	What Unit Rate does that give me if I divide like this? Km per minute or minute per Km?
	Which Unit Rate would be most useful given the context of this problem?

Name of Strategy	Did Not Divide
Number of Strategy	2
Color of Letter Circle	Red
Image of Strategy	
Strategy Overview	Students may remember how to set up a Strategy to calculate the Unit Rate but forget that we should divide, not multiply.

Questions to help get started

Helpful Hint	Remember students always have a place to start and sometimes we need to listen so we can remind students they understand mathematics on some level.
Questions	What do I need to figure out first?
	We're talking about distance and time. How do we usually want to measure that, like how fast we're going in a car?
	Can you explore what we just talked about and I'll check back with you in a moment?

Sequence/Connections

Name of Sequence	Double Number Line First
Description of Sequence	- Use this sequence if a lot of groups use a Double Number Line. This will provide multiple points of entry to the conversation for many students. - Then use a Table to show a similar but visually different way to organize the data. - Finish with the Unit Rate to emphasize that this strategy is useful when using large numbers and you don't want to build out the Table or Double Number Line. - If more students use the Table Strategy, use suggested sequence #2.
A	Double Number Line
Color of Letter Circle	Green
Question(s) for the Group	How did you align these numbers (point to the top number line) with these down here (point to the bottom number line)?
	How did you know when to stop building the Double Number Line out?
Question(s) for the Whole Class	Can someone tell me why a Double Number Line works well for this problem?

	Can someone tell me what would make you not want to use a Double Number Line?
Bridge	- This is a nice visual for seeing where the pages and days line up. - I wonder if there's another way to visualize these numbers.
B	Table
Color of Letter Circle	Green
Question(s) for the Group	How did you align these numbers (point to the left column) with these down here (point to the right column)?
	What made you decide to use a Table over a Double Number Line?
Question(s) for the Whole Class	Can someone tell me what the similarities or differences are between the Double Number Line and the Table?
	Can someone tell me if there's a benefit to using one of these Strategies over the other?
	What would it take to get you to use the Strategy you didn't choose to use?
Bridge	- I like that I have two different visual Strategies to choose from. - I wonder if there's a third way to think about this problem that involves calculations.
C	Unit Rate ⭐
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to do these calculations over a Double Number Line or a Table?
	What made you decide to divide by the time?
Question(s) for the Whole Class	I see the 5/4 for the Unit Rate. Can someone tell me where these numbers are in the Double Number Line and the Table?
	Which of the three Strategies do we like the most? Why?

Activity Conclusion	- How did each Strategy help you to find the Unit Rate? - Does it matter which Strategy I use to find the Unit Rate? Why or why not? - There are two very visual Strategies and one calculation. What would make you choose one Strategy over another?

Name of Sequence	Table First
Description of Sequence	- Use this sequence if a lot of groups use a Table. This will provide multiple points of entry to the conversation for many students. - Then use a Double Number Line to show similar but visually different ways to organize the data. - Finish with the Unit Rate to emphasize that this strategy is useful when using large numbers and you don't want to build out the Table or Double Number Line. - If more students use the Double Number Line Strategy, use suggested sequence #1.
B	Table
Color of Letter Circle	Green
Question(s) for the Group	How did you align these numbers (point to the left column) with these down here (point to the right column)?
	What made you decide to use a Table over a Double Number Line?
Question(s) for the Whole Class	Can someone tell me what the similarities or differences are between the Double Number Line and the Table?
	Can someone tell me if there's a benefit to using one of these Strategies over the other?
	What would it take to get you to use the Strategy you didn't choose to use?
Bridge	- This is a nice visual for seeing where the pages and days line up.

	- I wonder if there's another way to visualize these numbers.
A	Double Number Line
Color of Letter Circle	Green
Question(s) for the Group	How did you align these numbers (point to the top number line) with these down here (point to the bottom number line)?
	How did you know when to stop building the Double Number Line out?
Question(s) for the Whole Class	Can someone tell me why a Double Number Line works well for this problem?
	Can someone tell me what would make you not want to use a Double Number Line?
Bridge	- I like that I have two different visual Strategies to choose from. - I wonder if there's a third way to think about this problem that involves calculations.
C	Unit Rate ⭐
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to do these calculations over a Double Number Line or a Table?
	What made you decide to divide by the time?
Question(s) for the Whole Class	I see the 5/4 for the Unit Rate. Can someone tell me where these numbers are in the Double Number Line and the Table?
	Which of the three Strategies do we like the most? Why?
Activity Conclusion	- How did each Strategy help you to find the Unit Rate? - Does it matter which Strategy I use to find the Unit Rate? Why or why not? - There are two very visual Strategies and one calculation. What would make you choose one Strategy over another?

Name of Sequence	Students Switched the Divisor and Dividend
Description of Sequence	- Use this sequence if a lot of groups use the Switch Divisor and DIvidend Point of Growth. - Sequence it right before the Unit Rate Productive Approach since the calculations for this Point of Growth are used in the Unit Rate Productive Approach.
A	Double Number Line
Color of Letter Circle	Green
Question(s) for the Group	How did you align these numbers (point to the top number line) with these down here (point to the bottom number line)?
	How did you know when to stop building the Double Number Line out?
Question(s) for the Whole Class	Can someone tell me why a Double Number Line works well for this problem?
	Can someone tell me what would make you not want to use a Double Number Line?
Bridge	- This is a nice visual for seeing where the pages and days line up. - I wonder if there's another way to visualize these numbers.
B	Table
Color of Letter Circle	Green
Question(s) for the Group	How did you align these numbers (point to the left column) with these down here (point to the right column)?
	What made you decide to use a Table over a Double Number Line?
Question(s) for the Whole Class	Can someone tell me what the similarities or differences are between the Double Number Line and the Table?

	Can someone tell me if there's a benefit to using one of these Strategies over the other?
	What would it take to get you to use the Strategy you didn't choose to use?
Bridge	- I like that I have two different visual Strategies to choose from. - I wonder if there's a third way to think about this problem that involves calculations.
1	Switch Divisor and Dividend
Color of Letter Circle	Red
Question(s) for the Group	We're using the right numbers but what's happening here (point to the divisor or dividend)?
	Where in the Strategy did you notice something was off?
Question(s) for the Whole Class	Can someone tell me if it is okay to divide by the distance instead of the time? (Yes) Great, so why don't we want to do it for this particular problem?
	Can someone tell me if there's a way to use this Strategy but change it a bit so it's more applicable to the context of this problem?
Bridge	- So it looks like these numbers are usable, we just need to use them in a different way. - Let's look at a Strategy to continue this great start.
C	Unit Rate ⭐
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to do these calculations over a Double Number Line or a Table?
	What made you decide to divide by the time?
Question(s) for the Whole Class	I see the 5/4 for the Unit Rate. Can someone tell me where these numbers are in the Double Number Line and the Table?
	Which of the three Strategies do we like the most? Why?

Activity Conclusion	- How did each Strategy help you to find the Unit Rate? - Does it matter which Strategy I use to find the Unit Rate? Why or why not? - There are two very visual Strategies and one calculation. What would make you choose one Strategy over another?

Reflection Questions

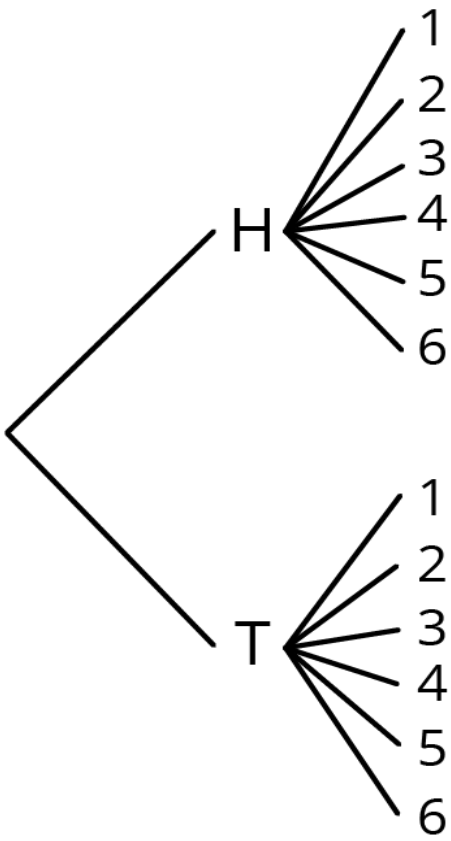
Reflection Question(s)	- How could I re-sequence the Strategies to help students connect the visual representations to the calculations? - What is the best way to draw out the connection between the Unit Rate in all of the Strategies? - Is there a pattern I noticed when interacting with students or when I called on students/groups to present? Am I leaving anyone out? - Is there anything I did during Monitoring, Selecting, Sequencing, or Connecting that I can learn from for the next 5Ps activity?

Grade Level	7
Unit Number	4
Lesson Number	2
Activity Number	2
Activity Name	A Train is Traveling at...
Learning Goal	- Students will compare and contrast different strategies for solving problems with equivalent ratios with fractional quantities. - Students will recognize how to find and use a unit rate to solve a problem involving fractional quantities.
Image of Student-Facing Activity	A train is traveling at a constant speed and goes 7.5 kilometers in 6 minutes. At that rate: 1. How far does the train go in 1 minute? 2. How far does the train go in 100 minutes? 

Productive Approaches

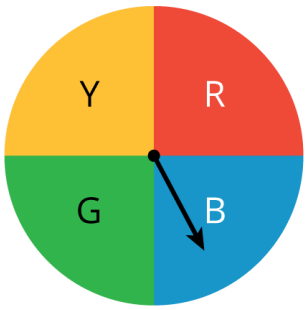
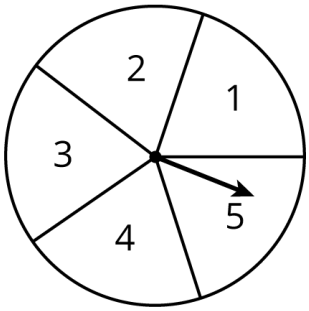
Name of Strategy	Double Number Line
Letter of Strategy	A
Color of Letter Circle	Green

<table>
<tr><td>Grade Level</td><td>7</td></tr>
<tr><td>Unit Number</td><td>8</td></tr>
<tr><td>Lesson Number</td><td>8</td></tr>
<tr><td>Activity Number</td><td>2</td></tr>
<tr><td>Activity Name</td><td>Keeping Track of all Possible Outcomes</td></tr>
<tr><td>Learning Goal</td><td>- Students will recognize the pros and cons of different methods to represent sample space with compound events and choose a method based on the context and structure of a problem
- Students will use the chosen strategy to determine the total possible outcomes for the event and justify their reasoning.</td></tr>
<tr><td>Image of Student-Facing Activity</td><td>Consider the experiment: Flip a coin, and then roll a number cube.
Elena, Kiran, and Priya each use a different method for finding the sample space of this experiment.
● Elena carefully writes a list of all the options: Heads 1, Heads 2, Heads 3, Heads 4, Heads 5, Heads 6, Tails 1, Tails 2, Tails 3, Tails 4, Tails 5, Tails 6.
● Kiran makes a table. (Figure 1)
<table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>H</td><td>H1</td><td>H2</td><td>H3</td><td>H4</td><td>H5</td><td>H6</td></tr><tr><td>T</td><td>T1</td><td>T2</td><td>T3</td><td>T4</td><td>T5</td><td>T6</td></tr></table>● Priya draws a tree with branches in which each pathway represents a different outcome. (Figure 2)</td></tr>
</table>

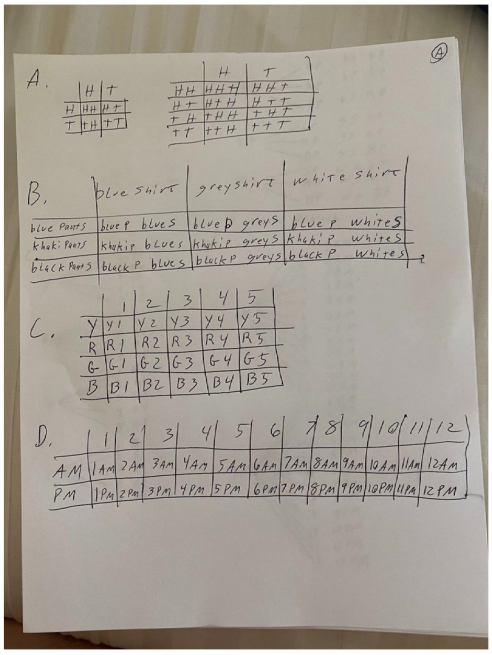


1. Compare the three methods. What is the same about each method? What is different? Be prepared to explain why each method produces all the different outcomes without repeating any.
2. Which method do you prefer for this situation? Pause here so your teacher can review your work.
3. Find the sample space for each of these experiments using any method. Make sure you list every possible outcome without repeating any.

a. Flip a dime, then flip a nickel, and then flip a penny. Record whether each lands heads or tails up.
b. Han's closet has: a blue shirt, a gray shirt, a white shirt, blue pants, khaki pants, and black pants. He must select one shirt and one pair of pants to wear for the day.
c. Spin a color, and then spin a number.

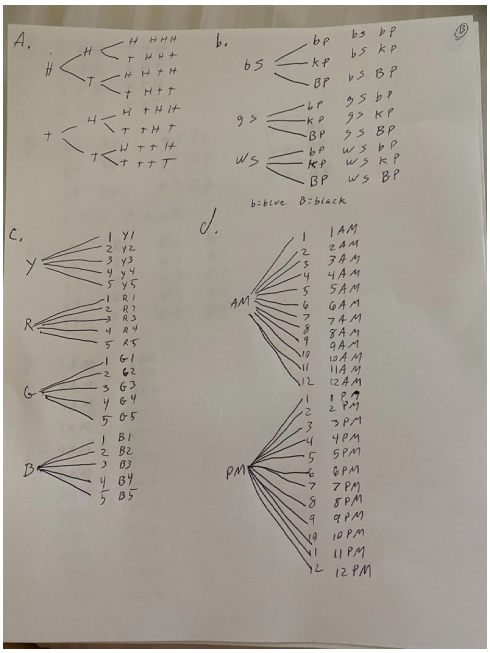
	  d. Spin the hour hand on an analog clock, and then choose a.m. or p.m.

Productive Approaches

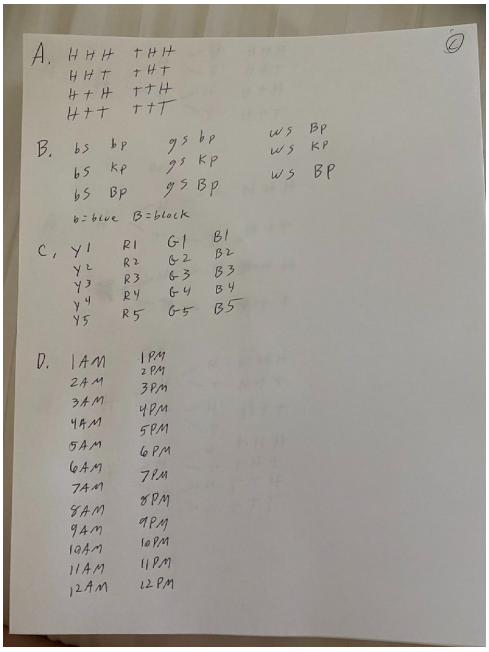
Name of Strategy	Table ☆
Letter of Strategy	A
Color of Letter Circle	Green
Image of Strategy	

Helpful Hint	This is a critical strategy because it provides an efficient, visual, and compact way to list all possible outcomes of an event.
Strategy Overview	- Draw a table, placing one set of outcomes vertically and the other set horizontally. - Use the table to fill in the possible outcomes based on the horizontal and vertical information.
Relation to Learning Goal	- This Strategy allows students to organize data ensuring there is no repetition or missing outcomes. - Students can justify their reasoning based on the visualization represented in the table. - Students may try to use a table for flipping the dime, nickel, and penny. Although this can be done, encourage students to use a list or tree diagram instead.
Assessing Questions	Why did you decide to put this event (point to outcome) up here in the table?
	How did you get all these outcomes here in the table?
Advancing Questions	Do you have to put this event (point to the outcome) on this side of the table? (No) How come?
	How are we sure with this Strategy that I listed every possible outcome?
	Would I get more, fewer, or the same number of outcomes if I used a different visualization for finding the outcomes? Why?

Name of Strategy	Tree Diagram
Letter of Strategy	B
Color of Letter Circle	Green

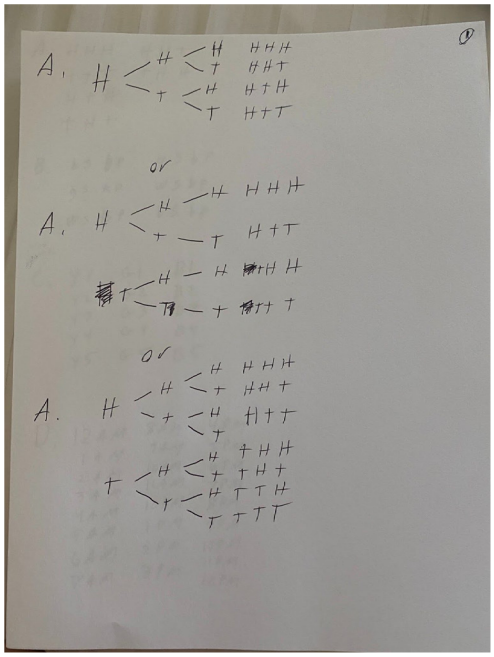
Image of Strategy	
Helpful Hint	This strategy may be easier for students to grasp if they've used "factor trees" a lot in the past.
Strategy Overview	- Draw a tree diagram for each of the events. - Then follow the branches to determine possible outcomes.
Relation to Learning Goal	- This strategy allows students to visualize systematically through a strategy used before (factor trees) the outcomes. - This Productive Approach may take more time than others but provides a touch point visual for making sense of the possible outcomes.
Assessing Questions	Why did you decide to make the first branch with this event?
	How did you get all these outcomes from the tree diagram?
Advancing Questions	Do you have to start the tree diagram with this outcome first? (No) How come?
	How are we sure with this Strategy that I listed every possible outcome?

	Would I get more, fewer, or the same number of outcomes if I used a different visualization for finding the outcomes? Why?

Name of Strategy	List
Letter of Strategy	C
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	Watch for students repeating or missing outcomes if they use this Strategy.
Strategy Overview	- Create a list of possible outcomes using one outcome consistently as the first entry and the other as the second entry. - Check the list to ensure no repeats and that all possible outcomes are written.
Relation to Learning Goal	- This Strategy provides a way to think through possible outcomes for the compound event. - The Strategy will help to see the strength of the other two strategies when compared.

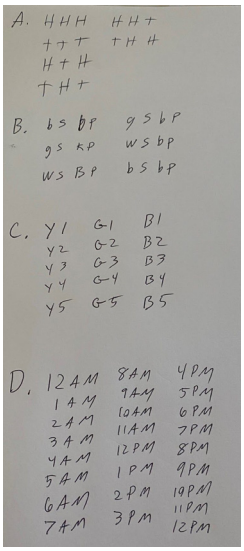
Assessing Questions	Why did you decide to list this event first?
	How many outcomes do we have here? (12) Can you use the work you showed to convince me there's exactly 12 outcomes?
Advancing Questions	Do you have to start your list with this event first? (No) How come?
	How are we sure with this Strategy that I listed every possible outcome?
	Would I get more, fewer, or the same number of outcomes if I used a different visualization for finding the outcomes? Why?

Points of Growth

Name of Strategy	Branches of the Tree Diagram
Number of Strategy	1
Color of Letter Circle	Red
Image of Strategy	

Strategy Overview	- Students may not be able to follow the tree branches to obtain all possible outcomes. - Students may also miss creating branches.
Relation to Learning Goal	- It is important that students realize the importance of listing all events in each branch while following every branch to the outcome. - If many students experience this Point of Growth, sequence this approach before the Tree Diagram Productive Approach because this Strategy can be completed with the Tree Diagram Strategy.
Assessing Questions	Can you walk me through all of the branches you drew?
	Can you show me which branches you followed to get each outcome?
Advancing Questions	If I have (state the number of options for the event), how many branches do I need here? What happens if I miss a branch?
	Is there another way to mark the branches I used to find each outcome? (Yes) Can you show me how?

Name of Strategy	Incomplete List
Number of Strategy	2
Color of Letter Circle	Red

Image of Strategy	
Strategy Overview	Students may repeat or miss outcomes based on a list that is not systematically thought out.

Questions to help get started

Helpful Hint	Remember students always have a place to start and sometimes we need to listen so we can remind students they understand mathematics on some level.
Questions	How many events do I have here?
	Think about the example we looked at with the coins and numbers. Which one do you want to try using here? Can you show me how we can start the problem using that Strategy?
	Can you explore what we just talked about and I'll check back with you in a moment?

Sequence/Connections

Name of Sequence	Tree Diagram First
Description of Sequence	- Use this sequence if a lot of groups use a Tree Diagram. This will provide multiple points of entry to the conversation for many students. - Then use a table to show another visualization of the outcomes. - Finish with a List to emphasize that the List is like the Table, only you do not have to build out the table. - If more students use the Table Strategy, use suggested sequence #2.
B	Tree Diagram
Color of Letter Circle	Green
Question(s) for the Group	Why did you put this event first?
	Do you think the outcomes would change if you put another event first? (the order would change but the outcomes wouldn't)
Question(s) for the Whole Class	If I put another event first then I notice the order would change, like from Y2 to 2Y. Can someone tell me if these are two different outcomes? Why or why not?
	How can we be sure that we covered all of the outcomes without repeating any using a Tree Diagram?
Bridge	- This is a nice visual but it takes up a lot of space and time. - I wonder if there's a more compact way to think about all the outcomes.
A	Table ☆
Color of Letter Circle	Green
Question(s) for the Group	I notice with the Tree Diagram Strategy, this event is here and in the Table Strategy, you put the same event here. Is there any relationship there?
	Are there any of these problems that lend themselves to using a Tree Diagram instead of a Table?

Question(s) for the Whole Class	Can someone tell me if I get the same answers with a Tree Diagram or a Table, when would I choose one over the other?
	Can someone tell me if you prefer one of these Strategies over the other?
	What would it take to get you to use the Strategy you don't prefer?
Bridge	- I like that I have two different strategies to choose from. - I wonder if there's a third way to think about the events and outcomes.
C	List
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to make a List over a Table or Tree Diagram?
	How did you keep track of all the outcomes with a List, especially since you don't have a Table or branches?
Question(s) for the Whole Class	Can someone tell me what's the same and what's different between the Table and the List?
	Which of the three Strategies do we like the most? Why?
Activity Conclusion	- How did each strategy help you to know you listed all possible outcomes without duplicating or skipping some? - Would each of the methods work for any of the scenarios? - Say I have 50 different shirts and 40 different pants and I don't want to list all those out. Do you notice a pattern here? Can I find out how many total outcomes there will be without listing them all out? - When would you choose to use a List, or a Table, or a Tree Diagram?

Name of Sequence	Table First

Description of Sequence	- Use this sequence if a lot of groups use a Table. This will provide multiple points of entry to the conversation for many students. - Then use a Tree Diagram to show another visualization of the outcomes. - Finish with a List to emphasize that the List is like the Table, only you do not have to build out the table. - If more students use the Tree Diagram Strategy, use suggested sequence #1.
A	Table ⭐
Color of Letter Circle	Green
Question(s) for the Group	I notice with the Tree Diagram Strategy, this event is here and in the Table Strategy, you put the same event here. Is there any relationship there?
	Are there any of these problems that lend themselves to using a Tree Diagram instead of a Table?
Question(s) for the Whole Class	Can someone tell me if I get the same answers with a Tree Diagram or a Table, when would I choose one over the other?
	Can someone tell me if you prefer one of these Strategies over the other?
	What would it take to get you to use the Strategy you don't prefer?
Bridge	- This is a nice visual for organizing the outcomes. - II wonder if there's another way of thinking about the outcomes.
B	Tree Diagram
Color of Letter Circle	Green
Question(s) for the Group	Why did you put this event first?
	Do you think the outcomes would change if you put another event first? (the order would change but the outcomes wouldn't)

Question(s) for the Whole Class	If I put another event first then I notice the order would change, like from Y2 to 2Y. Can someone tell me if these are two different outcomes? Why or why not?
	How can we be sure that we covered all of the outcomes without repeating any using a Tree Diagram?
Bridge	- This is a nice visual but it takes up a lot of space and time. - I wonder if there's a third way to think about the events and outcomes.
C	List
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to make a List over a Table or Tree Diagram?
	How did you keep track of all the outcomes with a List, especially since you don't have a Table or branches?
Question(s) for the Whole Class	Can someone tell me what's the same and what's different between the Table and the List?
	Which of the three Strategies do we like the most? Why?
Activity Conclusion	- How did each strategy help you to know you listed all possible outcomes without duplicating or skipping some? - Would each of the methods work for any of the scenarios? - Say I have 50 different shirts and 40 different pants and I don't want to list all those out. Do you notice a pattern here? Can I find out how many total outcomes there will be without listing them all out? - When would you choose to use a List, or a Table, or a Tree Diagram?

Name of Sequence	Branches Point of Growth
Description of Sequence	- Use this sequence if a lot of groups use the Branches of the Tree Diagram Point of Growth.

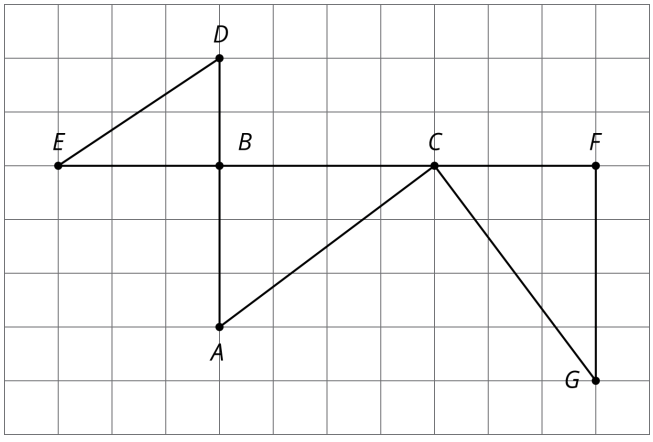
	Sequence it right before the Tree Diagram Productive Approach - Then use a table to show another visualization of the outcomes. - Finish with a List to emphasize that the List is like the Table, only you do not have to build out the table.
1	Branches of the Tree Diagram
Color of Letter Circle	Red
Question(s) for the Group	Can you tell me why you put the branches here?
	How can we tell that there's not enough outcomes (or branches)?
Question(s) for the Whole Class	Can someone tell me where I need more branches (or outcomes)?
	Can someone tell me how I can be sure I have all the outcomes with a Tree Diagram?
Bridge	- It's good to check that I have all possible outcomes with this and any Strategy. - Now that I know how to check, let's take another look at the Tree Diagram.
B	Tree Diagram
Color of Letter Circle	Green
Question(s) for the Group	Why did you put this event first?
	Do you think the outcomes would change if you put another event first? (the order would change but the outcomes wouldn't)
Question(s) for the Whole Class	If I put another event first then I notice the order would change, like from Y2 to 2Y. Can someone tell me if these are two different outcomes? Why or why not?
	How can we be sure that we covered all of the outcomes without repeating any using a Tree Diagram?
Bridge	- This is a nice visual but it takes up a lot of space and time. - I wonder if there's a more compact way to think about all the outcomes.

A	Table ☆
Color of Letter Circle	Green
Question(s) for the Group	I notice with the Tree Diagram Strategy, this event is here and in the Table Strategy, you put the same event here. Is there any relationship there?
	Are there any of these problems that lend themselves to using a Tree Diagram instead of a Table?
Question(s) for the Whole Class	Can someone tell me if I get the same answers with a Tree Diagram or a Table, when would I choose one over the other?
	Can someone tell me if you prefer one of these Strategies over the other?
	What would it take to get you to use the Strategy you don't prefer?
Bridge	- I like that I have two different strategies to choose from. - I wonder if there's a third way to think about the events and outcomes.
C	List
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to make a List over a Table or Tree Diagram?
	How did you keep track of all the outcomes with a List, especially since you don't have a Table or branches?
Question(s) for the Whole Class	Can someone tell me what's the same and what's different between the Table and the List?
	Which of the three Strategies do we like the most? Why?
Activity Conclusion	- How did each strategy help you to know you listed all possible outcomes without duplicating or skipping some? - Would each of the methods work for any of the scenarios? - Say I have 50 different shirts and 40 different pants and I don't want to list all those out. Do

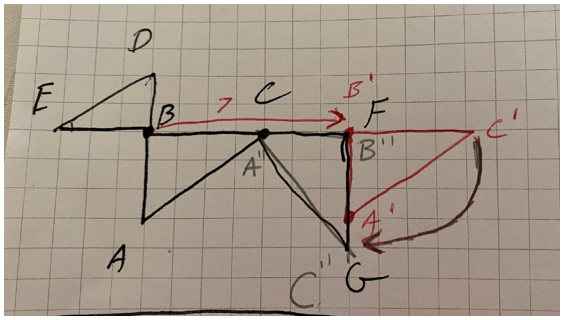
	you notice a pattern here? Can I find out how many total outcomes there will be without listing them all out? - When would you choose to use a List, or a Table, or a Tree Diagram?

Reflection Questions

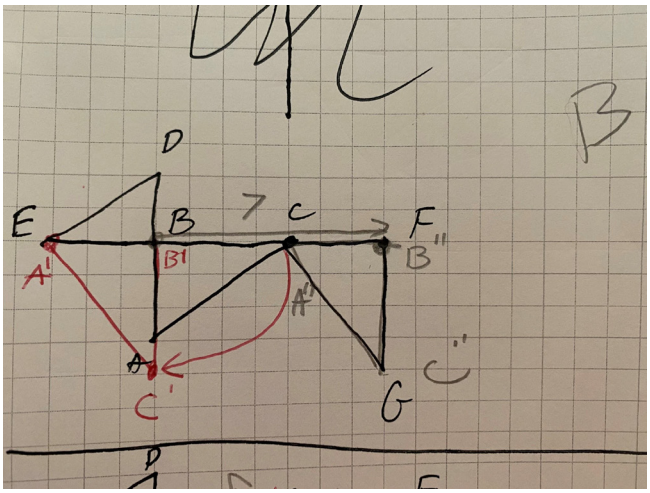
Reflection Question(s)	- How could I re-sequence the Strategies so students better understand the pros and cons of each strategy? - What is the best way to draw out of the connection that order for these types of events doesn't matter (like Y2 is the same as 2Y)? - Is there a pattern I noticed when interacting with students or when I called on students/groups to present? Am I leaving anyone out? - Is there anything I did during Monitoring, Selecting, Sequencing, or Connecting that I can learn from for the next 5Ps activity?

Grade Level	8
Unit Number	1
Lesson Number	7
Activity Number	3
Activity Name	Which One?
Learning Goal	- Students will recognize that the only transformation that changes the size of an object is a dilation. Rotations, reflections, and translations are “rigid transformations” in that they preserve the size of the shape. - Rigid transformations preserve side and angle measures whereas dilations only preserve angle measures. - Students will use side lengths and angle measures to align corresponding parts while describing in detail the sequence of rigid transformations using drawings, verbalization, and writing.
Image of Student-Facing Activity	Here is a grid showing triangle ABC and two other triangles. You can use a **rigid transformation** to take triangle ABC to *one* of the other triangles. 1) Which one? Explain how you know. 2) Describe a rigid transformation that takes ABC to the triangle you selected. 

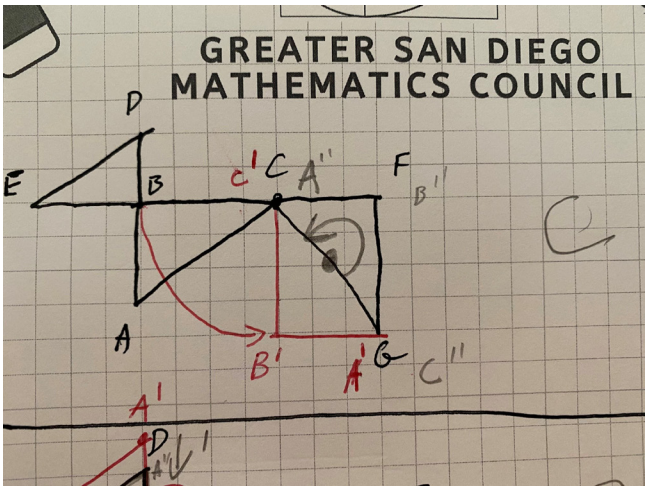
Productive Approaches

Name of Strategy	Translate Vertex B
Letter of Strategy	A
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	This strategy may be easier for students to grasp because the first step is based on a translation, affording more entry points for students who may have difficulties with reflections or rotations.
Strategy Overview	- Translate vertex B 7 units to the right. - Then rotate triangle ABC 90 degrees clockwise about vertex B.
Relation to Learning Goal	- This strategy allows students to translate and rotate triangle ABC to verify it aligns with triangle CFG. - Students remember that rotations and translations preserve the size and shape of an object.
Assessing Questions	How did you move triangle ABC to get vertex B to line up with vertex F?
	How many degrees did you rotate triangle ABC and about which point?
Advancing Questions	Why did you decide to translate the triangle first? Did you have to translate it first? (No) How come?
	Could you rotate the triangle the other way? (Yes) Why can we rotate it the other way? How many degrees would I have to rotate it the other way?

	Is there any other point I could rotate triangle ABC about to get it to line up with triangle CFG? Why (or why not)?

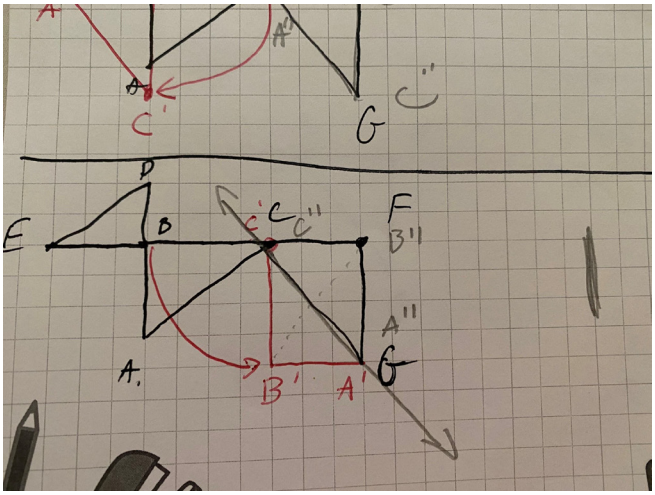
Name of Strategy	Rotate Vertex B⭐
Letter of Strategy	B
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	This is a critical strategy because it provides a fast visual for triangle ABC being oriented the same as triangle CFG, offering guidance for exactly how far to translate the object.
Strategy Overview	- Rotate triangle ABC 90 degrees about point B. - Then translate triangle ABC 7 units to the right.
Relation to Learning Goal	This strategy allows students to quickly visualize the alignment of corresponding parts of each triangle, helping them to move triangle ABC to triangle CFG.
Assessing Questions	How did you move triangle ABC to get vertex B to line up with vertex F?
	How many degrees did you rotate triangle ABC and about which point?
Advancing Questions	Why did you decide to rotate the triangle first? Did you have to rotate it first? (No) How come?

	Could you rotate the triangle the other way? (Yes) Why can we rotate it the other way? How many degrees would I have to rotate it the other way?
	Is there any other point I could rotate triangle ABC about to get it to line up with triangle CFG? Why (or why not)?

Name of Strategy	Rotate Twice
Letter of Strategy	C
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	This strategy reinforces that we can rotate an object around any point, not just vertices.
Strategy Overview	- Rotate triangle ABC 90 degrees counterclockwise about vertex C. - Then rotate triangle ABC 180 degrees around the midpoint of line segment AC.
Relation to Learning Goal	- This strategy allows students to rotate triangle ABC twice. - Students remember that rotations preserve the size and shape of an object.
Assessing Questions	How many degrees did you rotate triangle ABC and about which point?

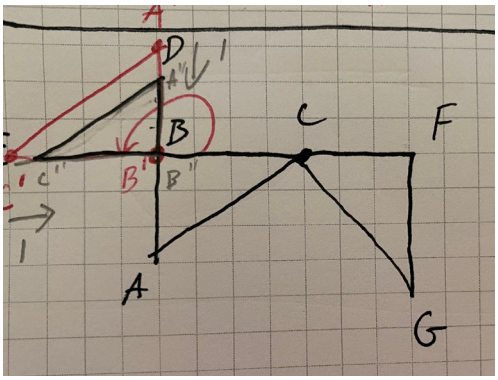
	Why did you choose to do two rotations?
	Which rotation did you do first? Can you show me how?
Advancing Questions	Could you rotate triangle ABC about that midpoint first? (Yes) If so, is there another transformation you could do right after to overlay it on triangle CFG? (Yes) Can you show me?
	Could you rotate the triangle about the midpoint first and then about vertex C? Why (or why not)?
	There's that nice line segment CG right here, why didn't you just reflect triangle ABC about that line segment?

Points of Growth

Name of Strategy	Only Isosceles Right Triangles
Number of Strategy	1
Color of Letter Circle	Red
Image of Strategy	
[H2] Helpful Hint - NONE FOR THIS ONE	

Strategy Overview	- Students may think this strategy will align vertex B with vertex F but this will only occur if the triangle is an isosceles right triangle. - Rotate triangle ABC 90 degrees counterclockwise about vertex C. Then reflect triangle ABC about line segment AC.
Relation to Learning Goal	- It is important that students realize the corresponding sides and angles of the congruent triangle should match up once the transformations are complete. - If many students experience this Point of Growth, sequence this approach before the Rotate Twice Productive Approach because this Strategy can be completed with the Rotate Twice Strategy.
Assessing Questions	Can you walk me through how you rotated triangle ABC here (point to vertex C)?
	Why did you decide to reflect triangle ABC about this line segment (point to segment CG)?
Advancing Questions	Let's fold our paper on line segment CG. Where does vertex B end up? Does it line up exactly with vertex F? (No) How come?
	Let's stick with this though, the rotation is okay. We know we cannot reflect it about line segment CG but is there another rigid transformation we can do from here?

Name of Strategy	Dilation
Number of Strategy	2
Color of Letter Circle	Red

Image of Strategy	
Strategy Overview	- Students may forget that dilations are NOT rigid transformations. - Rotate triangle ABC 180 degrees about vertex B. Then dilate triangle ABC reducing the length of AB and BC 1 unit.

Questions to help get started

Helpful Hint	Remember students always have a place to start and sometimes we need to listen so we can remind students they understand mathematics on some level.
Questions	What do you notice about the triangles?
	Think about the activity we just did with sides and angles. Why do we call these Rigid Transformations? (Because the size and shape stay the same) Great! How can we use that here?
	Can you explore what we just talked about and I'll check back with you in a moment?

Sequence/Connections

Name of Sequence	Rotate Vertex B First

Description of Sequence	- Use this sequence if a lot of groups use Rotate Vertex B. This will provide multiple points of entry to the conversation for many students. - Conclude the sequencing with Rotate Twice because rotating around a midpoint instead of a vertex might not be a “go to” Strategy for students.. - If more students use the Translate Vertex B Strategy, use suggested sequence #2.
B	Rotate Vertex B⭐
Color of Letter Circle	Green
Question(s) for the Group	Why did you decide to rotate and then translate the figure?
	How did you know when to stop rotating and translating?
Question(s) for the Whole Class	Can someone tell me if I could use triangle BDE in this problem? (No, you cannot) Why not?
	How can we be sure that we didn’t change any of the measurements for the triangle I was moving?
Bridge	- So we rotated and then translated. - I wonder what would change if I decided to translate first.
A	Translate Vertex B
Color of Letter Circle	Green
Question(s) for the Group	Why did you decide to translate and then rotate the figure?
	How did you know when to stop translating and rotating?
Question(s) for the Whole Class	Can someone tell me if I rotated first or if I translated first, did that change how many degrees I rotated or how far I translated the triangle? (No, it didn’t) Why not?
	Does this mean when we do a sequence of transformations, it does not matter what order we do it in?

Bridge	- It looks like we can rotate first or translate first for this problem. - I wonder if we can only do translations or only do rotations.
C	Rotate Twice
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to only do rotations?
	Did you set out to only do this or did you decide part way into the problem?
Question(s) for the Whole Class	Can someone tell me if we have to do the rotations in this order?
	Can someone tell me how the group might have thought to rotate the figure about the midpoint?
Activity Conclusion	- Why didn't we use triangle BDE in this problem? - Are there other ways to perform rigid transformations to move triangle ABC to triangle CFG? - There are many different ways to perform transformations. But rigid transformations are restricted to reflections, rotations, and translations. Although dilations preserve angle measure, they do not preserve side lengths.

Name of Sequence	Translate Vertex B First
Description of Sequence	- Use this sequence if a lot of groups use Translate Vertex B. This will provide multiple points of entry to the conversation for many students. - Conclude the sequencing with Rotate Twice because rotating around a midpoint instead of a vertex might not be a "go to" Strategy for students.. - If more students use the Rotate Vertex B Strategy, use suggested sequence #1.
A	Translate Vertex B
Color of Letter Circle	Green

Question(s) for the Group	Why did you decide to translate and then rotate the figure?
	How did you know when to stop translating and rotating?
Question(s) for the Whole Class	Can someone tell me if I could use triangle BDE in this problem? (No, you cannot) Why not?
	How can we be sure that we didn't change any of the measurements for the triangle I was moving?
Bridge	- So we translated and then rotated. - I wonder what would change if I decided to rotate first.
B	Rotate Vertex B⭐
Color of Letter Circle	Green
Question(s) for the Group	Why did you decide to rotate and then translate the figure?
	How did you know when to stop rotating and translating?
Question(s) for the Whole Class	Can someone tell me if I rotated first or if I translated first, did that change how many degrees I rotated or how far I translated the triangle? (No, it didn't) Why not?
	Does this mean when we do a sequence of transformations, it does not matter what order we do it in?
Bridge	- It looks like we can rotate first or translate first for this problem. - I wonder if we can only do translations or only do rotations.
C	Rotate Twice
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to only do rotations?
	Did you set out to only do this or did you decide part way into the problem?

Question(s) for the Whole Class	Can someone tell me if we have to do the rotations in this order?
	Can someone tell me how the group might have thought to rotate the figure about the midpoint?
Activity Conclusion	- Why didn't we use triangle BDE in this problem? - Are there other ways to perform rigid transformations to move triangle ABC to triangle CFG? - There are many different ways to perform transformations. But rigid transformations are restricted to reflections, rotations, and translations. Although dilations preserve angle measure, they do not preserve side lengths.

Name of Sequence	Only Isosceles Right Triangles Reflection
Description of Sequence	- Use this sequence if a lot of groups align line segment CG with line segment AC and try to reflect triangle ABC about the segment. - Conclude the sequencing with Rotate Twice because rotating around a midpoint instead of a vertex might not be a "go to" Strategy for students. - It is important to address this Point of Growth before the Rotate Twice Productive Approach because this Strategy can be completed with the Rotate Twice Strategy.
B	Rotate Vertex B⭐
Color of Letter Circle	Green
Question(s) for the Group	Why did you decide to rotate and then translate the figure?
	How did you know when to stop rotating and translating?
Question(s) for the Whole Class	Can someone tell me if I could use triangle BDE in this problem? (No, you cannot) Why not?
	How can we be sure that we didn't change any of the measurements for the triangle I was moving?

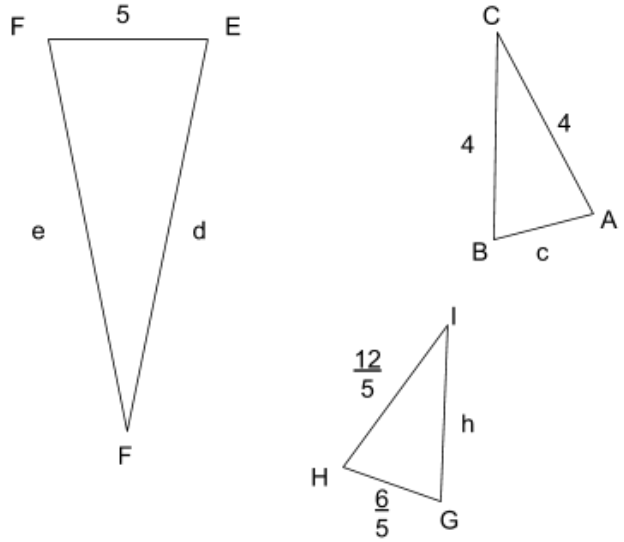
Bridge	- So we rotated and then translated. - I wonder what would change if I decided to translate first.
A	Translate Vertex B
Color of Letter Circle	Green
Question(s) for the Group	Why did you decide to translate and then rotate the figure?
	How did you know when to stop translating and rotating?
Question(s) for the Whole Class	Can someone tell me if I rotated first or if I translated first, did that change how many degrees I rotated or how far I translated the triangle? (No, it didn't) Why not?
	Does this mean when we do a sequence of transformations, it does not matter what order we do it in?
Bridge	- It looks like we can rotate first or translate first for this problem. - Let's look at a reflection now.
1	Only Isosceles Right Triangles
Color of Letter Circle	Red
Question(s) for the Group	Why did you decide to reflect triangle ABC about the line segment CG?
	Did vertex B align with vertex F when you reflected the triangle? (No) Why do you think those vertices might have aligned?
Question(s) for the Whole Class.	Can someone tell me why vertex B and F did not align?
	Can someone tell me what type of shape BCFG is now? (A rectangle) What type of shape would BCFG have to be for the reflection to work? (A square) How come?
	Do we want to keep going with this strategy? Is there a way we can do the first rotation and then something from here to align the figures?

Bridge	- Let's leave triangle ABC where it is with this Point of Growth after the rotation - I wonder if we can only do something from here to align triangle ABC with triangle CFG.
C	Rotate Twice
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to only do rotations?
	Did you set out to only do this or did you decide part way into the problem?
Question(s) for the Whole Class	Can someone tell me if we have to do the rotations in this order?
	Can someone tell me how the group might have thought to rotate the figure about the midpoint?
Activity Conclusion	- Why didn't we use triangle BDE in this problem? - Are there other ways to perform rigid transformations to move triangle ABC to triangle CFG? - There are many different ways to perform transformations. But rigid transformations are restricted to reflections, rotations, and translations. Although dilations preserve angle measure, they do not preserve side lengths.

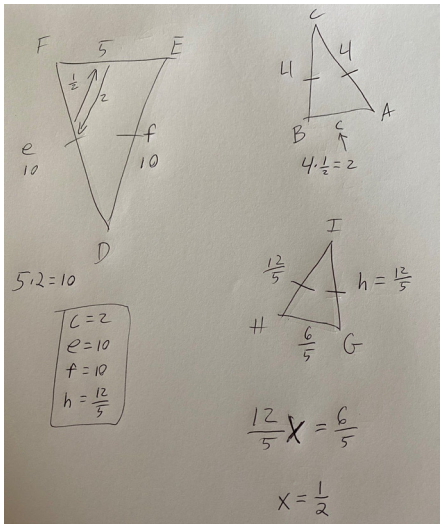
Reflection Questions

Reflection Question(s)	- How could I re-sequence the Strategies so students understand the difference between rigid and non-rigid transformations? - Based on the work my students did, what is the best way to show when the order of a sequence of transformations might matter and when it might not matter? - Is there a pattern I noticed when interacting with students or when I called on students/groups to present? Am I leaving anyone out?

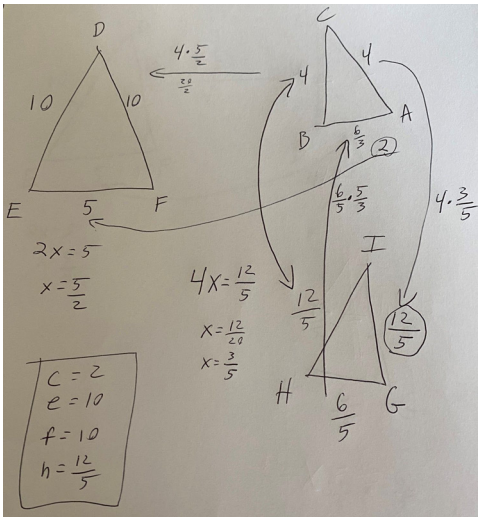
	- Is there anything I did during Monitoring, Selecting, Sequencing, or Connecting that I can learn from for the next 5Ps activity?

Grade Level	8
Unit Number	2
Lesson Number	9
Activity Number	3
Activity Name	Using Side Quotients to Find Side Lengths of Similar Triangles
Learning Goal	- Students will recognize that the corresponding parts of similar triangles have common scale factors within one triangle and between other triangles and that missing values in similar triangles can be found by determining these scale factors. - Students will generalize strategies using corresponding sides of similar triangles and scale factors to calculate unknown side lengths, fluidly using internal and external scale factors to find the length of missing sides.
Image of Student-Facing Activity	Triangles ABC, EFD and GHI are all similar. The side lengths of the triangles all have the same units. Find the unknown side lengths. 

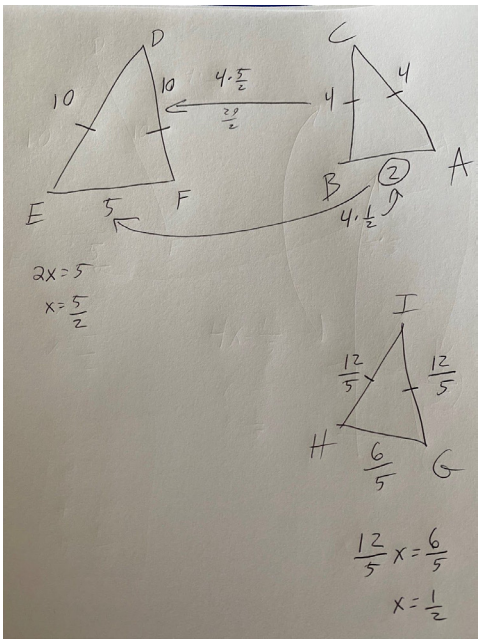
Productive Approaches

Name of Strategy	Internal Scale Factor
Letter of Strategy	A
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	This strategy may be easier for students to grasp because they're working within one triangle, hence the Point of Growth with corresponding sides is avoided.
Strategy Overview	All triangles are Isosceles and the internal scale factor is 2 or ½.
Relation to Learning Goal	- This is one of the two main strategies for finding missing values in similar triangles. - Students learn to work within a single triangle to find missing values.
Assessing Questions	How do we know these triangles are isosceles?
	How did you get this number here (point to Internal Scale Factor)?
Advancing Questions	Are we sure that the Internal Scale Factor for one of the triangles is the same for all of the triangles? (Yes) How can we be sure of this?

	Is there another relationship between the triangles?
	Can I use Internal Scale Factors in this manner for any type of triangle? Why (or why not)?

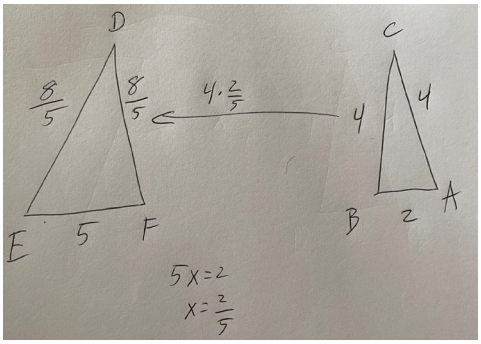
Name of Strategy	External Scale Factor
Letter of Strategy	B
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	While monitoring, if students are not using the external scale factor, reference the cool down from lesson eight or the first activity from this lesson.
Strategy Overview	- The scale factor from triangle ABC to triangle GHI is ⅗. Students can then use either triangle to calculate the values of e and d. - If students use triangle ABC, the scale factor from triangle ABC to triangle DEF is 5/2.
Relation to Learning Goal	This strategy reminds students to use proportional reasoning when determining side lengths between two similar triangles.
Assessing Questions	Why did you choose to start with these two triangles?

	How did you get this number here (point to external scale factor)?
Advancing Questions	Are all of the external scale factors the same? (No) All the triangles are similar so how can all the scale factors be different?
	Is there a relationship within one triangle as well?

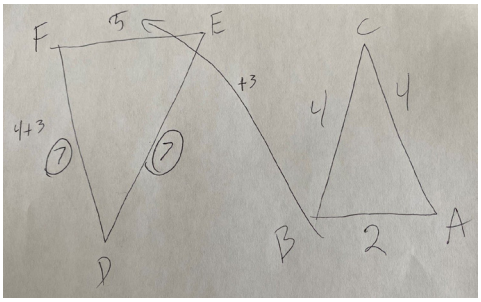
Name of Strategy	Both Scale Factors ⭐
Letter of Strategy	C
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	This is a critical strategy because it combines the use of both Internal and External Scale Factors and should be shown last as a bridge to integrated approaches.
Strategy Overview	Students use internal and external scale factors in a fluid manner, deciding which one makes more sense to use at different stages within the problem.

Relation to Learning Goal	- This is a critical strategy because students realize they can shift from one scale factor to the other at different stages of the problem. - While connecting, this strategy is best sequenced last.
Assessing Questions	Why did you choose to start with the internal (or external) scale factor?
Advancing Questions	When did you decide to switch from the internal/external scale factor to the external/internal scale factor and why?
	Do you think there are problems where I can only use Internal (or External) Scale Factor? Why (or why not)?

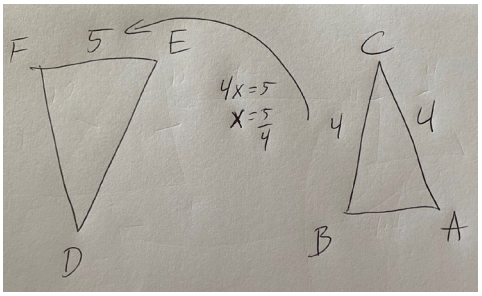
Points of Growth

Name of Strategy	Inverse Scale Factor
Number of Strategy	1
Color of Letter Circle	Red
Image of Strategy	
Strategy Overview	- Students use the reciprocal of the scale factor. - For example, students use ½ where they should use 2.
Relation to Learning Goal	- It is important that students realize multiplying by a number greater than 1 creates a larger triangle and multiplying by a number less than 1 creates a smaller triangle. - If many students experience this Point of Growth, sequence this approach at the beginning, before

	any Productive Approach so you can have a conversation about how multiplying by a number greater than 1 yields a bigger shape and smaller than 1 yields a smaller shape.
Assessing Questions	Can you tell me how you got this number here (point at the scale factor)?
Advancing Questions	Which shape is bigger?
	So if I multiply by this number will I get a bigger or smaller number?

Name of Strategy	Additive Comparison
Number of Strategy	2
Color of Letter Circle	Red
Image of Strategy	
Helpful Hint- NONE FOR THIS ONE	
Strategy Overview	Students use internal or external scale factors rooted in addition instead of multiplication.
Relation to Learning Goal	- Similar triangles are proportionally related which means they are related via multiplication, not addition. - If many students experience this Point of Growth, sequence it at the beginning, before any Productive Approach so you can have a conversation about similar shapes being proportionally related which means multiplication (not addition) links the shapes together.

	- Think of this Point of Growth as the students understanding what to do but using the improper operation to do so.
Assessing Questions	How did you find this number (point at the number they're adding to the sides)?
	For the warm-up, how did you determine if the triangles were similar? (I saw if I could multiply by the same number)
Advancing Questions	If the triangles are similar, should we be multiplying or adding with these numbers? Why?
	Why would we need to multiply instead of add here?
	Is there a time I would need to add scale factors? Why (or why not)?

Name of Strategy	Corresponding Sides
Number of Strategy	3
Color of Letter Circle	Red
Image of Strategy	
Strategy Overview	- Students have trouble locating and aligning the corresponding sides.

Questions to help get started

Helpful Hint	Remember students always have a place to start and sometimes we need to listen so we can remind students they understand mathematics on some level.

Questions	What do you notice about the triangles?
	What approach did you use in the warm up today? (Scale Factors) Can we use that here?
	Can you explore what we just talked about and I'll check back with you in a moment?

Sequence/Connections

Name of Sequence	Internal Scale Factor First
Description of Sequence	- Use this sequence if a lot of groups use Internal Scale Factor. This will provide multiple points of entry to the conversation for many students. - Conclude the sequencing with Both Scale Factors so in the Connect phase you can point out that both approaches can be used in one problem. - If more students use the External Scale Factor than the Internal Scale Factor, use suggested sequence #2.
A	Internal Scale Factor
Color of Letter Circle	Green
Question(s) for the Group	How did you notice these were isosceles triangles?
Question(s) for the Whole Class	If this one triangle is isosceles, is anybody convinced that all the triangles are isosceles? (Yes) Why?
	Why can we use the same internal scale factor for all these triangles even though they are all different sizes?
Bridge	- So the ½ or 2 as a scale factor can be used within each of the triangles. - With this next strategy, notice how we'll use scale factors in a different way.
B	External Scale Factor
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to use these two triangles first?

	So what's the External Scale Factor for triangles ABC and GHI? How about ABC and EFD? How about EFD and HGI?
Question(s) for the Whole Class	Can someone tell me what the relationship between all the different External Scale Factors is?
	Can someone tell me why the internal scale factors are the same but the external scale factors are different?
Bridge	- So the scale factor is determined based on which triangle I'm starting at and which one I'm going to. - I wonder if we can use this strategy (point to Internal Scale Factor) and this strategy (External Scale Factor) together.
C	Both Scale Factors
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to start with the internal (or external) scale factor?
Question(s) for the Whole Class	Can someone tell me where they switched from the internal/external scale factor to the external/internal scale factor?
	Do we agree with this? If so, why? If not, what would you have done differently?
Activity Conclusion	- Which method is more efficient? - How did you know which side corresponded to the other? - It's okay to use either scale factor. One method is not better than another, it mostly depends on what sides are missing. It's okay, actually encouraged, to use both Internal and External Scale Factors throughout any problem if it makes sense to do so.

Name of Sequence	External Scale Factor First
Description of Sequence	- Use this sequence if a lot of groups use External Scale Factor. This will provide multiple points of entry to the conversation for many students.

	- Conclude the sequencing with Both Scale Factors so in the Connect phase you can point out that both approaches can be used in one problem. - If more students use the Internal Scale Factor than the External Scale Factor, use suggested sequence #1. - If more students use the Internal Scale Factor than the External Scale Factor, use suggested sequence #1.
B	External Scale Factor
Color of Letter Circle	Green
Questions for the Group	Why did you choose to use these two triangles first?
	So what's the External Scale Factor for triangles ABC and GHI? How about ABC and EFD? How about EFD and HGI?
Question(s) for the Whole Class	Can someone tell me what the relationship between all the different External Scale Factors is?
	Can someone tell me why the internal scale factors are the same but the external scale factors are different?
Bridge	- So the scale factor is determined based on which triangle I'm starting at and which one I'm going to. - With this next strategy, notice how we'll use scale factors in a different way.
A	Internal Scale Factor
Color of Letter Circle	Green
Questions for the Group	How did you notice these were isosceles triangles?
Question(s) for the Whole Class	If this one triangle is isosceles, are we convinced that all the triangles are isosceles? (Yes) Why?
	Why can we use the same internal scale factor for all these triangles even though they are all different sizes?
Bridge	- So the ½ or 2 as a scale factor can be used within each of the triangles.

	- I wonder if we can use this strategy (point to Internal Scale Factor) and this strategy (External Scale Factor) together.
C	Both Scale Factors
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to start with the internal (or external) scale factor?
Question(s) for Whole Class	Can someone tell me where they switched from the internal/external scale factor to the external/internal scale factor?
	Do we agree with this? If so, why? If not, what would you have done differently?
Activity Conclusion	- Which method is more efficient? - How did you know which side corresponded to the other? - It's okay to use either scale factor. One method is not better than another, it mostly depends on what sides are missing. It's okay, actually encouraged, to use both Internal and External Scale Factors throughout any problem if it makes sense to do so.

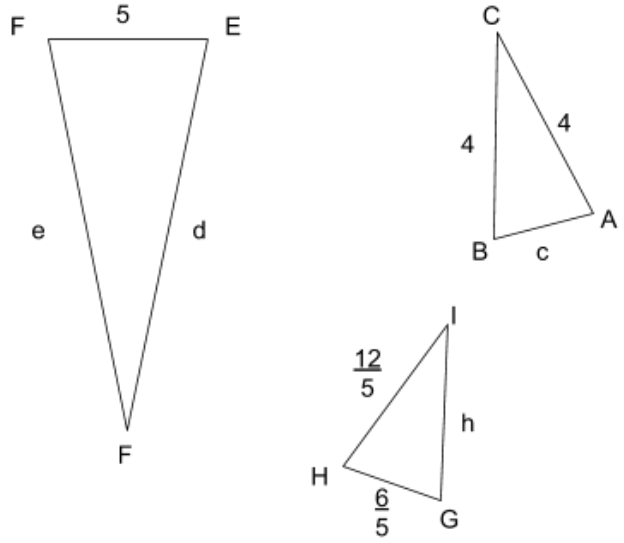
Name of Sequence	Corresponding Sides is Common
Description of Sequence	- Use this sequence if you notice a lot of groups or students use the Corresponding Sides Point of Growth. - Conclude the sequencing with Both Scale Factors so in the Connect phase you can point out that both approaches can be used in one problem. - It is important to address this Point of Growth before the External Scale Factor Productive Approach so students who did not align the sides properly can discuss the importance of doing so.
A	Internal Scale Factor
Color of Letter Circle	Green

Questions for the Presenting Group	How did you notice these were isosceles triangles?
Question(s) for the Whole Class	If this one triangle is isosceles, are we convinced that all the triangles are isosceles? (Yes) Why?
	Why can we use the same internal scale factor for all these triangles even though they are all different sizes?
Bridge	- So the ½ or 2 as a scale factor can be used within each of the triangles. - Before we dive into another Strategy, let's take a look at how we match up the sides of different triangles.
3	Corresponding Sides
Color of Letter Circle	Red
Questions for the Presenting Group	Did you try a transformation? (Yes) How did that help you?
	Can we line up any of the sides we want? (No) Why not?
Question(s) for the Whole Class	Can someone tell me why it is important to make sure we line up the sides in the correct manner?
Bridge	- Remember to always pause and make sure those sides are in fact corresponding. This is the key to doing the next strategy well. - With this next strategy, notice how we'll use scale factors in a different way.
B	External Scale Factor
Color of Letter Circle	Green
Questions for the Presenting Group	Why did you choose to use these two triangles first?
	So what's the External Scale Factor for triangles ABC and GHI? How about ABC and EFD? How about EFD and HGI?
Question(s) for the Whole Class	Can someone tell me what the relationship between all the different External Scale Factors is?
	Can someone tell me why the internal scale factors are the same but the external scale factors are different?

Bridge	- So the scale factor is determined based on which triangle I'm starting at and which one I'm going to. - I wonder if we can use this strategy (point to Internal Scale Factor) and this strategy (External Scale Factor) together.
C	Both Scale Factors
Color of Letter Circle	Green
Questions for the Presenting Group	Why did you choose to start with the internal (or external) scale factor?
Question(s) for the Whole Class	Can someone tell me where they switched from the internal/external scale factor to the external/internal scale factor?
	Do we agree with this? If so, why? If not, what would you have done differently?
Activity Conclusion	- Which method is more efficient? - How did you know which side corresponded to the other? - It's okay to use either scale factor. One method is not better than another, it mostly depends on what sides are missing. It's okay, actually encouraged, to use both Internal and External Scale Factors throughout any problem if it makes sense to do so.

Reflection Questions

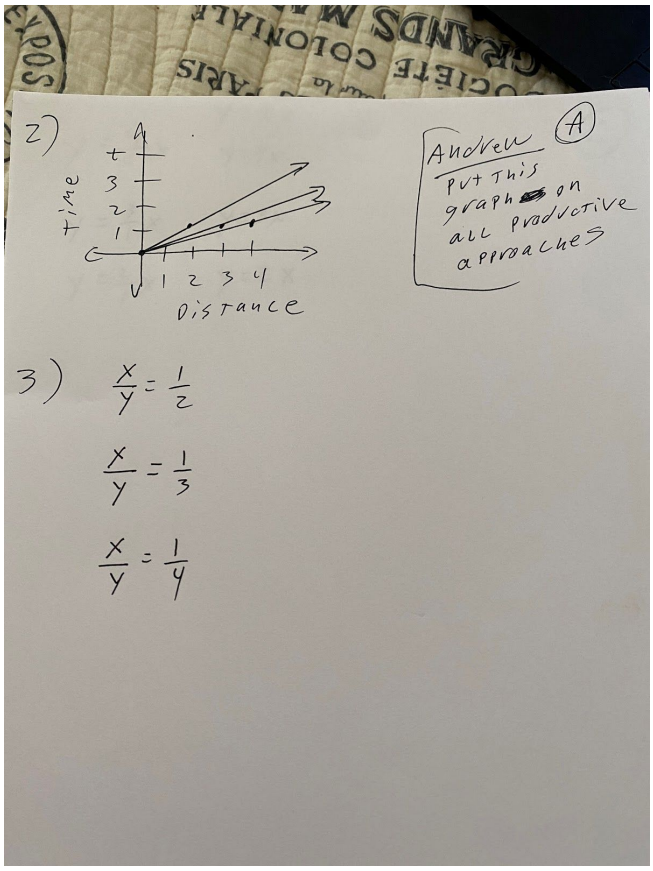
Reflection Question(s)	- How could I re-sequence the Strategies so students discover either Internal or External Scale Factor can be used at different times within one problem? - What is the best way to reinforce External Scale Factor given what students said today? - Is there a pattern I noticed when interacting with students or when I called on students/groups to present? Am I leaving anyone out? - Is there anything I did during Monitoring, Selecting, Sequencing, or Connecting that I can learn from for the next 5Ps activity?

Grade Level	8
Unit Number	2
Lesson Number	9
Activity Number	3
Activity Name	Using Side Quotients to Find Side Lengths of Similar Triangles
Learning Goal	- Students will recognize that the corresponding parts of similar triangles have common scale factors within one triangle and between other triangles and that missing values in similar triangles can be found by determining these scale factors. - Students will generalize strategies using corresponding sides of similar triangles and scale factors to calculate unknown side lengths, fluidly using internal and external scale factors to find the length of missing sides.
Image of Student-Facing Activity	Triangles ABC, EFD and GHI are all similar. The side lengths of the triangles all have the same units. Find the unknown side lengths. 

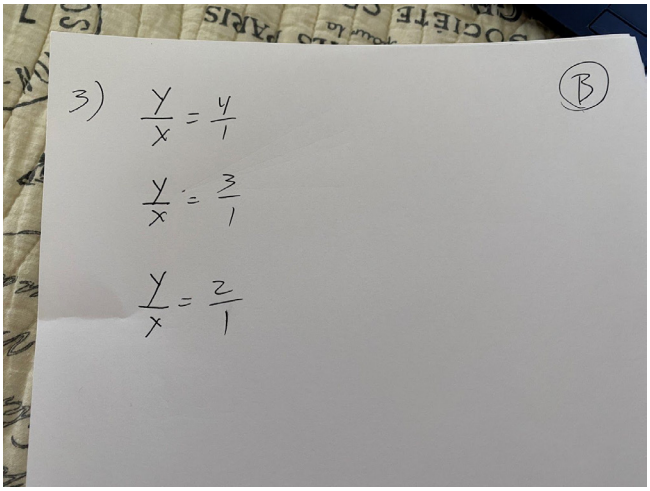
Grade Level	8
Unit Number	3
Lesson Number	1
Activity Number	3
Activity Name	Moving Twice as Fast
Learning Goal	- Students will recognize that an equation for a proportional relationship given by y=kx represents the constant of proportionality. - Students will create and interpret graphs and equations representing and proportional relationships in context.
Image of Student-Facing Activity	Refer to the tick-mark diagrams and graph in the earlier activity in this lesson when needed. 1. Imagine a bug that is moving twice as fast as the ladybug. On each tick-mark diagram, mark the position of this bug. 2. Plot this bug's positions on the coordinate axes with lines u and v, and connect them with a line. 3. Write an equation for each of the three lines.

Productive Approaches

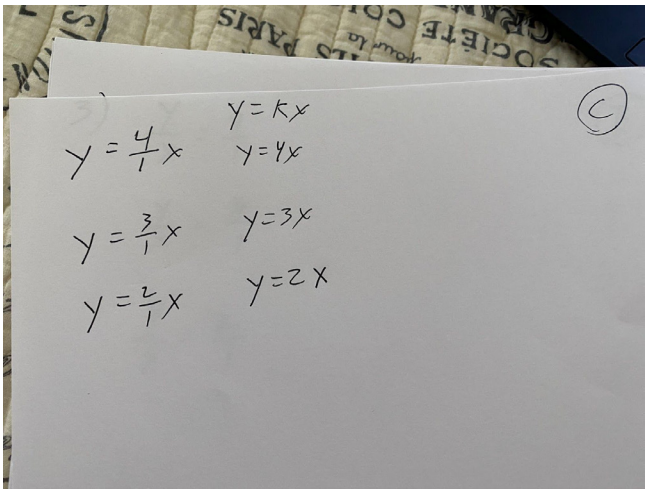
Name of Strategy	x on Top
Letter of Strategy	A
Color of Letter Circle	Green

Image of Strategy	
Helpful Hint	This Strategy uses proportional reasoning with the reciprocal of the slope.
Strategy Overview	- Draw the line for the pattern described. - Write a proportional relationship with x/y set equivalent to the run over rise of the line.
Relation to Learning Goal	- This Strategy allows students to visually find the proportional relationship of graphed lines.
Assessing Questions	Where did you get these numbers (point to the 1 & 2)?
	How did you know which one goes with x and which one goes with y?
Advancing Questions	Does it matter which number you put in the numerator or denominator? Why or why not?

	What do these numbers tell us in terms of the steepness of the line?
	If these numbers were bigger, say a rise of 50 and a run of 25, would it change these equations? Why or why not?

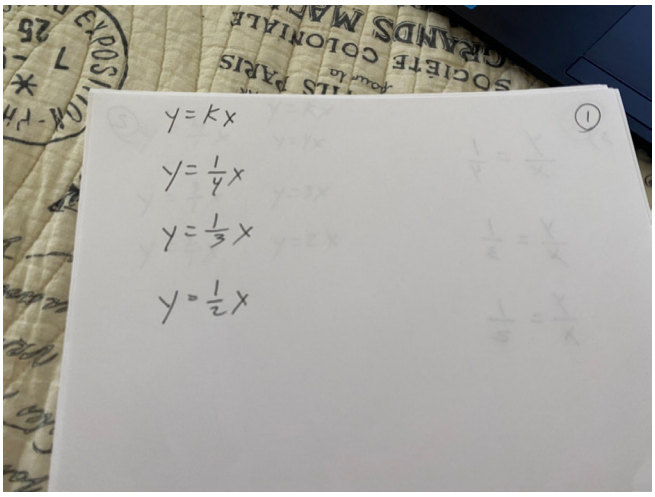
Name of Strategy	y on Top
Letter of Strategy	B
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	This Strategy uses proportional reasoning aligned with the slope.
Strategy Overview	- Draw the line for the pattern described. - Write a proportional relationship with y/x set equivalent to the rise over run of the line.
Relation to Learning Goal	- This Strategy allows students to visually find the proportional relationship of graphed lines.
Assessing Questions	Where did you get these numbers (point to the 1 & 2)?
	How did you know which one goes with x and which one goes with y?
Advancing Questions	Does it matter which number you put in the numerator or denominator? Why or why not?

	What do these numbers tell us in terms of the steepness of the line?
	If these numbers were bigger, say a rise of 50 and a run of 25, would it change these equations? Why or why not?

Name of Strategy	Equation ⭐
Letter of Strategy	C
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	This Strategy uses the formal equation for proportional relationships with a y-intercept of zero.
Strategy Overview	- Draw the line for the pattern described. - Write a proportional relationship with the equation y=kx where k is the slope of the line.
Relation to Learning Goal	- This Strategy allows students to visually find the proportional relationship of graphed lines using the constant of proportionality in the formalized equation
Assessing Questions	Where did you get these numbers (point to the 1 & 2)?
	How did you know that the number goes next to the x and not the y?

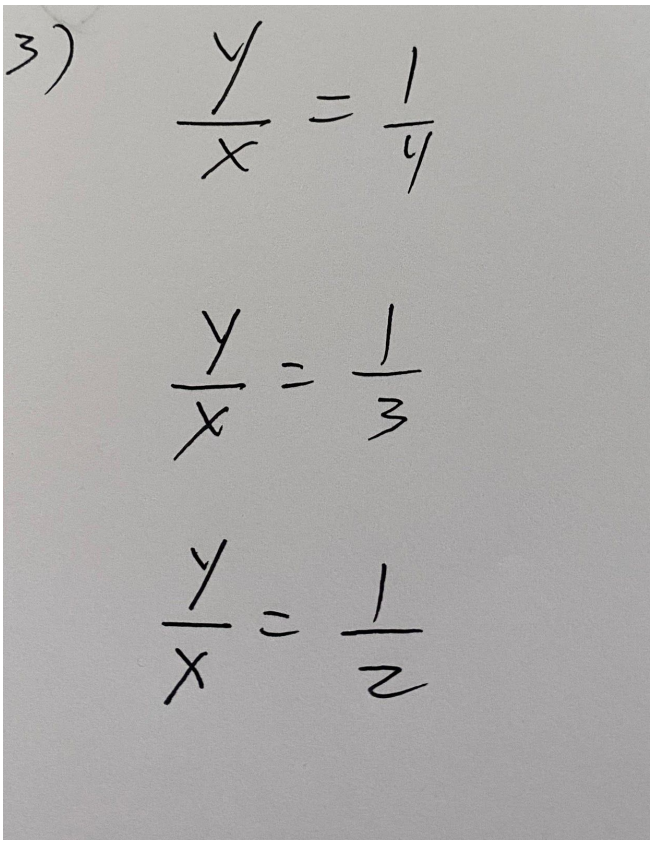
Advancing Questions	Does it matter which number you put in the numerator or denominator? Why or why not?
	What do these numbers tell us in terms of the steepness of the line?
	If these numbers were bigger, say a rise of 50 and a run of 25, would it change these equations? Why or why not?

Points of Growth

Name of Strategy	Reciprocal
Number of Strategy	1
Color of Letter Circle	Red
Image of Strategy	
Strategy Overview	- Students may enter the reciprocal of the constant of proportionality into the formal equation.
Relation to Learning Goal	- It is important that students understand for the line that the constant of proportionality, k, is y/x and not x/y. - If many students experience this Point of Growth, sequence this approach before the Equation Productive Approach to facilitate conversations about the connections between the Strategies.

Assessing Questions	Where did you get these numbers (point to the 1 & 2)?
	How did you know which one goes with x and which one goes with y?
Advancing Questions	Does it matter which number you put in the numerator or denominator? Why or why not?
	What do these numbers tell us in terms of the steepness of the line?
	Let's plug in a point, does it work with this equation? (No) Why not?

Name of Strategy	Misaligned
Number of Strategy	2
Color of Letter Circle	Red

Image of Strategy	
Strategy Overview	Students may misalign the proportional relationship, using the reciprocal of the constant of proportionality.

Questions to help get started

Helpful Hint	Remember students always have a place to start and sometimes we need to listen so we can remind students they understand mathematics on some level.
Questions	How can I use these visuals from the graph on the previous problem we just did to get started?
	How can I graph a line like they already did for me with line u and line v?

	Can you explore what we just talked about and I'll check back with you in a moment?

Sequence/Connections

Name of Sequence	Start with x on Top
Description of Sequence	- Use this sequence if a lot of groups set up the proportional relationship with x in the numerator and y in the denominator. - Next, show the y on Top Productive Approach to emphasize that x on Top is valid but try to transition to y/x for a connection to the slope in later lessons. - Finish with the proportional relationship Equations. - If more students use the y on Top Productive Approach, use suggested sequence #2.
A	x on Top
Color of Letter Circle	Green
Question(s) for the Group	Where did you get these numbers (point to the 1 & 2)?
	How did you know which one goes with x and which one goes with y?
Question(s) for the Whole Class	Can someone tell me if it's okay to write this as x/y? Why or why not?
	Can someone tell me how to check and see if this equation works?
Bridge	- We took the time to line up the x's and y's. - I wonder if there's another way to set this up.
B	y on Top
Color of Letter Circle	Green
Question(s) for the Group	Where did you get these numbers (point to the 1 & 2)?
	How did you know which one goes with x and which one goes with y?

Question(s) for the Whole Class	Can someone tell me if it's okay to write this as y/x? Why or why not?
	Can someone tell me what's similar and what's different about the two Strategies?
	What would it take to get you to use the Strategy you didn't choose?
Bridge	- I like that we have two different ways of thinking about this - I wonder if there's a third way to write the equations for these lines.
C	Equation ⭐
Color of Letter Circle	Green
Question(s) for the Group	Where did you get these numbers (point to the 1 & 2)?
	How did you know that the number goes next to the x and not the y?
Question(s) for the Whole Class	Can someone tell me if it matters if I write the coefficient as 4/1 or ¼? Why or why not?
	Which of the three Strategies do we like the most? Why?
Activity Conclusion	- How did each Strategy help you to see how steep the lines are? - Does it matter which Strategy I use to find the equations? Why or why not? - What would make you choose one Strategy over another?

Name of Sequence	Start with y on Top
Description of Sequence	- Use this sequence if a lot of groups set up the proportional relationship with y in the numerator and x in the denominator. - Next, show the x on Top Productive Approach to emphasize that both Strategies are valid as long as the x and y values line up. - Finish with the proportional relationship Equations.

	- If more students use the x on Top Productive Approach, use suggested sequence #1.
B	y on Top
Color of Letter Circle	Green
Question(s) for the Group	Where did you get these numbers (point to the 1 & 2)?
	How did you know which one goes with x and which one goes with y?
Question(s) for the Whole Class	Can someone tell me if it's okay to write this as y/x? Why or why not?
	Can someone tell me how to check and see if this equation works?
Bridge	- I like how we took the time to line up the x's and y's. - I wonder if there's another way to set this up.
A	x on Top
Color of Letter Circle	Green
Question(s) for the Group	Where did you get these numbers (point to the 1 & 2)?
	How did you know which one goes with x and which one goes with y?
Question(s) for the Whole Class	Can someone tell me if it's okay to write this as x/y? Why or why not?
	Can someone tell me what's similar and what's different about the two Strategies?
	What would it take to get you to use the Strategy you didn't choose?
Bridge	- We have two different ways of thinking about this - I wonder if there's a third way to write the equations for these lines.
C	Equation ⭑
Color of Letter Circle	Green
Question(s) for the Group	Where did you get these numbers (point to the 1 & 2)?

	How did you know that the number goes next to the x and not the y?
Question(s) for the Whole Class	Can someone tell me if it matters if I write the coefficient as 4/1 or ¼? Why or why not?
	Which of the three Strategies do we like the most? Why?
Activity Conclusion	- How did each Strategy help you to see how steep the lines are? - Does it matter which Strategy I use to find the equations? Why or why not? - What would make you choose one Strategy over another?

Name of Sequence	Using the Reciprocal
Description of Sequence	- Use this sequence if a lot of groups use the Reciprocal Point of Growth. - Sequence it at the beginning to facilitate a discussion about the importance of starting the alignment of the ratios accurately.
1	Reciprocal
Color of Letter Circle	Red
Question(s) for the Group	Where did you get these numbers (point to the 1 & 2)?
	How did you know which one goes with x and which one goes with y?
	Where did you start to see this wasn't going to work?
Question(s) for the Whole Class	Can someone tell me if it matters which number is in the numerator and denominator? Why or why not?
	What do these numbers tell us in terms of the steepness of the line?
Bridge	- I see we need to watch closely where we put these numbers in our fraction.

	- Let's look at another way to think about this.
A	x on Top
Color of Letter Circle	Green
Question(s) for the Group	Where did you get these numbers (point to the 1 & 2)?
	How did you know which one goes with x and which one goes with y?
Question(s) for the Whole Class	Can someone tell me if it's okay to write this as x/y? Why or why not?
	Can someone tell me how to check and see if this equation works?
Bridge	- We took the time to line up the x's and y's. - I wonder if there's another way to set this up.
B	y on Top
Color of Letter Circle	Green
Question(s) for the Group	Where did you get these numbers (point to the 1 & 2)?
	How did you know which one goes with x and which one goes with y?
Question(s) for the Whole Class	Can someone tell me if it's okay to write this as y/x? Why or why not?
	Can someone tell me what's similar and what's different about the two Strategies?
	What would it take to get you to use the Strategy you didn't choose?
Bridge	- We have two different ways of thinking about this - I wonder if there's a third way to write the equations for these lines.
C	Equation ⋆
Color of Letter Circle	Green
Question(s) for the Group	Where did you get these numbers (point to the 1 & 2)?

	How did you know that the number goes next to the x and not the y?
Question(s) for the Whole Class	Can someone tell me if it matters if I write the coefficient as 4/1 or ¼? Why or why not?
	Which of the three Strategies do we like the most? Why?
Activity Conclusion	- How did each Strategy help you to see how steep the lines are? - Does it matter which Strategy I use to find the equations? Why or why not? - What would make you choose one Strategy over another?

Reflection Questions

Reflection Question(s)	- How could I re-sequence the Strategies to help students connect the similarities between the Strategies? - Based on the work my students did, how could I continue to informally build the concept of slope? - Is there a pattern I noticed when interacting with students or when I called on students/groups to present? Am I leaving anyone out? - Is there anything I did during Monitoring, Selecting, Sequencing, or Connecting that I can learn from for the next 5Ps activity?

Grade Level	8
Unit Number	3
Lesson Number	1
Activity Number	3
Activity Name	Moving Twice as Fast
Learning Goal	- Students will recognize that an equation for a proportional relationship given by y=kx represents the constant of proportionality. - Students will create and interpret graphs and equations representing and proportional relationships in context.
Image of Student-Facing Activity	Refer to the tick-mark diagrams and graph in the earlier activity in this lesson when needed. 1. Imagine a bug that is moving twice as fast as the ladybug. On each tick-mark diagram, mark the position of this bug. 2. Plot this bug's positions on the coordinate axes with lines u and v, and connect them with a line. 3. Write an equation for each of the three lines.

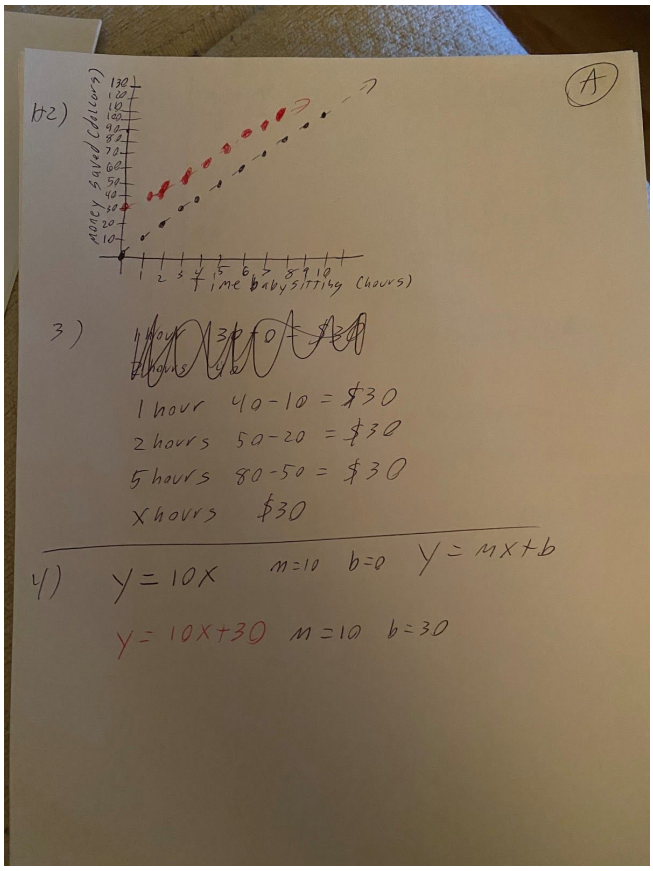
Productive Approaches

Name of Strategy	x on Top
Letter of Strategy	A
Color of Letter Circle	Green

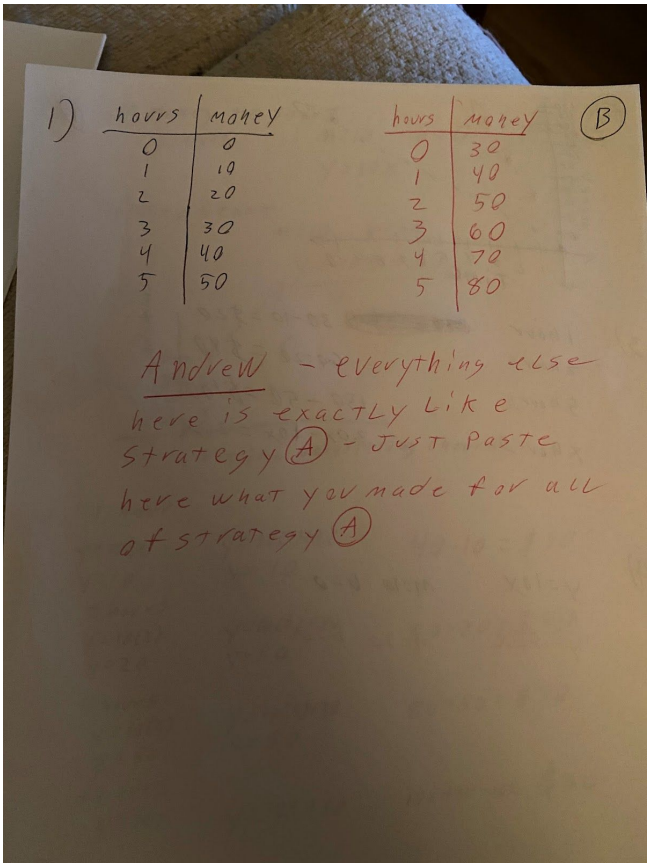
Grade Level	8
Unit Number	3
Lesson Number	8
Activity Number	2
Activity Name	Increased Savings
Learning Goal	- Students will recognize the features of the equation y=mx+b and see the relationship with the graph, including lines with a negative y-intercept. - Students will create and compare graphs that represent linear relationships with the same rate of change but different initial values.
Image of Student-Facing Activity	1. Diego earns $10 per hour babysitting. Assume that he has no money saved before he starts babysitting and plans to save all of his earnings. Graph how much money, y, he has after x hours of babysitting. 2. Now imagine that Diego started with $30 saved before he starts babysitting. On the same set of axes, graph how much money, y, he would have after x hours of babysitting. 3. Compare the second line with the first line. How much more money does Diego have after 1 hour of babysitting? 2 hours? 5 hours? x hours? 4. Write an equation for each line.

Productive Approaches

Name of Strategy	Graph
Letter of Strategy	A
Color of Letter Circle	Green

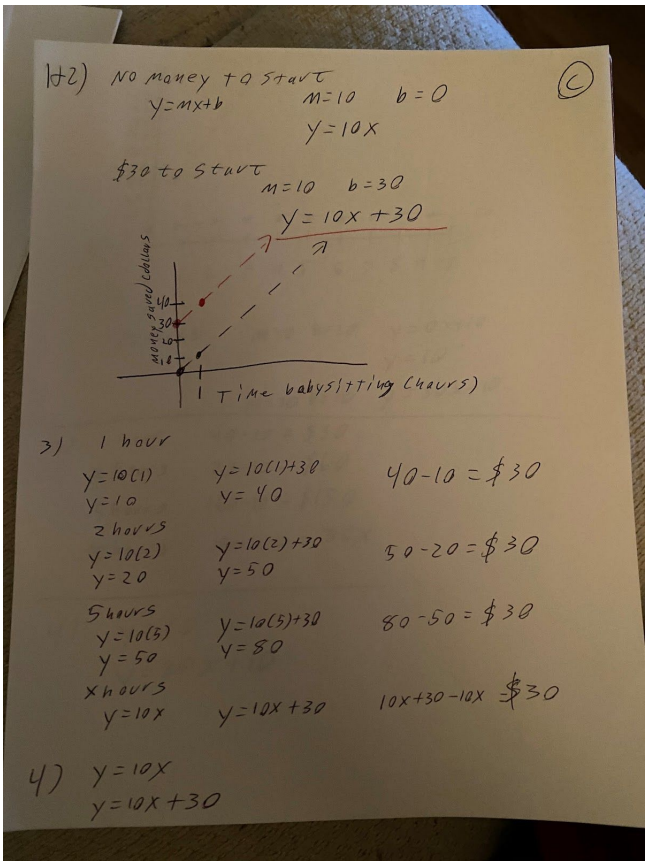
Image of Strategy	
Helpful Hint	This is a Strategy students might use if they are comfortable with graphs but uncomfortable with equations.
Strategy Overview	- Draw the Cartesian Plane and plot the points directly without an Equation or Table. - Use the information from the graph to answer the questions and write the equation.
Relation to Learning Goal	- This Strategy allows students to visually plot and make sense of the data. - Students can justify their reasoning based on the Graph.
Assessing Questions	Can you tell me about these points here (point to the y-intercepts)?

	How did you know where to plot the points?
Advancing Questions	What is the same about the graphs and why?
	What is different about the graphs and why?
	Will these lines ever cross? Why or why not?

Name of Strategy	Table
Letter of Strategy	B
Color of Letter Circle	Green
Image of Strategy	
Helpful Hint	This strategy creates a way for students to keep track of the data in a Table before they Graph the points or write the Equations.

Strategy Overview	- Draw a Table of values for both scenarios. - Use the data from the Table to Graph the points on the Cartesian Plane. - Either expand the Table or use the Graph to answer the questions and write the Equation.
Relation to Learning Goal	- This Strategy allows students to organize the data in order to identify the slope and the y-intercept. - Students can justify their reasoning based on the Table. - Students may extend the Table to answer the questions.
Assessing Questions	Can you tell me about these points here (point to the y-intercepts) and where they came from the table?
	How did you know where to plot the points?
Advancing Questions	What is the same about the graphs and why?
	What is different about the graphs and why?
	Will these lines ever cross? Why or why not?

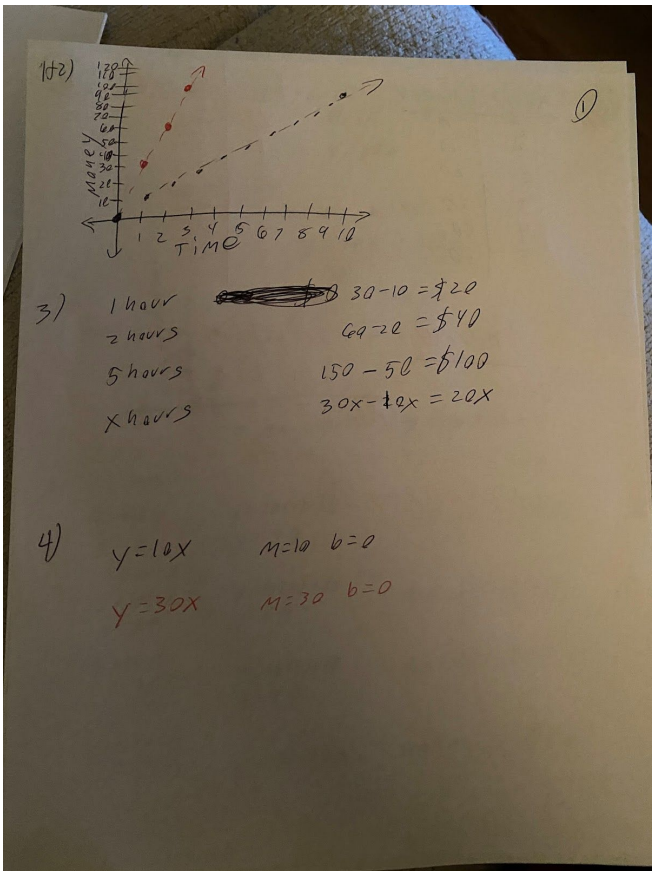
Name of Strategy	Equations ☆
Letter of Strategy	C
Color of Letter Circle	Green

Image of Strategy	
Helpful Hint	This is a critical Strategy because it allows students to identify the slope and y-intercept while using that information to directly write the equation of the line slope intercept form.
Strategy Overview	- Use the given information to identify the slope and y-intercept. - Use the equation y=mx+b to write both Equations for both scenarios. - Use the Equations to graph the lines and answer the questions
Relation to Learning Goal	- This Strategy allows students to identify the slope and y-intercept from the problem and use it to directly write the Equations. - Sequence this strategy last so you have the other Productive Approaches as visuals to reference for

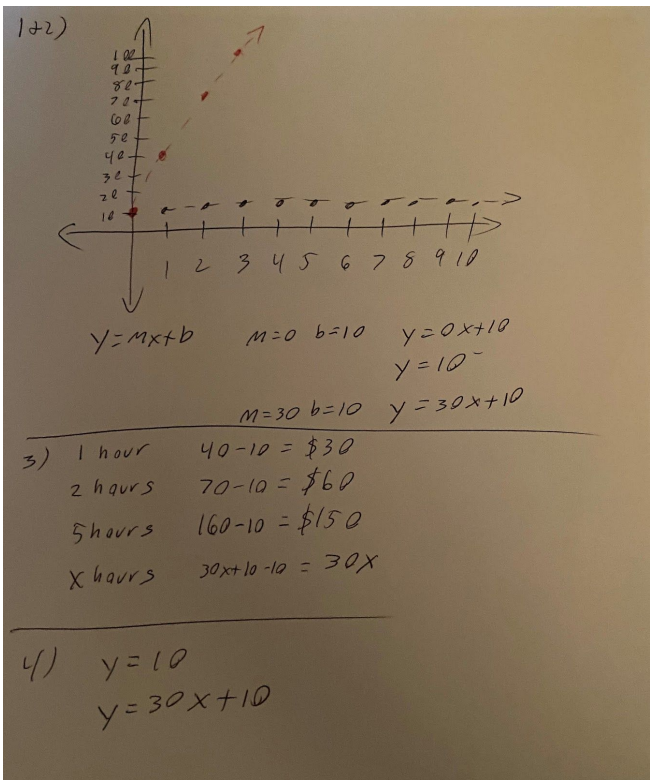
	similarities and differences when writing the Equation first or last.
Assessing Questions	Can you tell me how you wrote these two equations?
	How did you use the equations to graph the lines and answer the questions?
Advancing Questions	How did you know 10 was the slope?
	How did you know 0 and 30 were the y-intercepts?

Points of Growth

Name of Strategy	Same y-intercept
Number of Strategy	1
Color of Letter Circle	Red

Image of Strategy	
Strategy Overview	- Students may switch the slope and y-intercept for one or both of the Equations and Graphs. - Students then use this to find the answers.
Relation to Learning Goal	- It is important that students can contextually identify the slope and y-intercept. - If many students experience this Point of Growth, sequence this approach at the beginning to talk about identifying the slope and y-intercept.
Assessing Questions	What number did you use to place this point (y-intercept) here and why did you use it?
	How did you answer the questions and write the equations from the graph?

Advancing Questions	If he saved $10 per day no matter what, how would the graphs need to look?
	Did he start with the same amount of money? (No) How would that influence where we have the y-intercept or the amount of money he had before saving $10 per day?

Name of Strategy	Switch Slope and Intercept
Number of Strategy	2
Color of Letter Circle	Red
Image of Strategy	
Strategy Overview	Students may remember the slope intercept form for the equation of a line but reverse which number contextually represents the slope and which one represents the y-intercept.

Questions to help get started

Helpful Hint	Remember students always have a place to start and sometimes we need to listen so we can remind students they understand mathematics on some level.
Questions	Let's start with $10 and $30. What do these mean?
	Do you remember using the equation y=mx+b? Which of these numbers is the slope and which one is the y-intercept?
	Can you explore what we just talked about and I'll check back with you in a moment?

Sequence/Connections

Name of Sequence	Graph It First
Description of Sequence	- Use this sequence if a lot of groups use a Graph to start the problem. This will provide multiple points of entry to the conversation for many students. - Then use the Graph to answer the questions. - Finish with using the Graph to write the Equations. - As groups come up to show the Table and Equation Strategies, have them add to the Graph Strategy and reference the work already on the board. - If more students use the Table Strategy, use suggested sequence #2.
A	Graph
Color of Letter Circle	Green
Question(s) for the Group	How did you get these points here (point to the y-intercept and the first value at one hour)?
	How did you know where to plot the next set of points?

Question(s) for the Whole Class	Can someone tell me why starting with a Graph works well for this problem?
	Can someone tell me what would make you not want to use a Graph first?
Bridge	- This is a nice visual for identifying the values. - I wonder if there's another way to visualize these numbers.
B	Table
Color of Letter Circle	Green
Question(s) for the Group	How did you align these numbers (point to the left column) with these down here (point to the right column)?
	What made you decide to use a Table over a Graph to start?
Question(s) for the Whole Class	Can someone tell me where the points on the Graph came from the Table?
	Can someone tell me if there's a benefit to using one of these Strategies over the other?
	What would it take to get you to use the Strategy you didn't choose to use?
Bridge	- We're getting to the graph quickly. - I wonder if there's a way to go straight to the Equations.
C	Equations ⭐
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to go straight to the equation?
	How did you know from the problem which number is the slope and which number is the y-intercept?
Question(s) for the Whole Class	Can someone tell me the benefits (or detriments) to starting with the Equations first?
	Which of the three Strategies do we like the most? Why?

Activity Conclusion	- How did each Strategy help you to find the equation of the lines? - Does it matter which Strategy I use to find the equation? Why or why not? - What would make you choose one Strategy over another?

Name of Sequence	Table First
Description of Sequence	- Use this sequence if a lot of groups use a Table. This will provide multiple points of entry to the conversation for many students. - Then build the Graph, answer the questions, and write the equations using the Table. - Finish with the Equation Strategy. - Consider having the Table group provide the table and start the Graph, then have the Graph group finish the Graph and answer the questions, then have the Equations group write the equations and all groups explain the ways they came up with their answers. - If students do not use the Table Strategy, use suggested sequence #1.
B	Table
Color of Letter Circle	Green
Question(s) for the Group	How did you align these numbers (point to the left column) with these down here (point to the right column)?
	How did you build the Graph from the Tablet?
Question(s) for the Whole Class	Can someone tell me where the points on the Graph came from the Table?
	Can someone tell me why starting with a Table works well for this problem?
	Can someone tell me what would make you not want to use a Table first?
Bridge	- This is a nice way to organize the points before I graph it.

	- I wonder if anyone skipped the Table and went right to the Graph.
A	Graph
Color of Letter Circle	Green
Question(s) for the Group	How did you get these points here (point to the y-intercept and the first value at one hour?
	How did you know where to plot the next set of points?
Question(s) for the Whole Class	Can someone tell me why starting with a Graph works well for this problem?
	Can someone tell me what would make you not want to use a Graph first?
	What would it take to get you to use the Strategy you didn't choose to use?
Bridge	- We're getting to the graph quickly. - I wonder if there's a way to go straight to the Equations.
C	Equations ⭐
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to go straight to the equation?
	How did you know from the problem which number is the slope and which number is the y-intercept?
Question(s) for the Whole Class	Can someone tell me the benefits (or detriments) to starting with the Equations first?
	Which of the three Strategies do we like the most? Why?
Activity Conclusion	- How did each Strategy help you to find the equation of the lines? - Does it matter which Strategy I use to find the equation? Why or why not? - What would make you choose one Strategy over another?

Name of Sequence	y-intercept is the Same
Description of Sequence	- Use this sequence if a lot of groups use the Same y-Intercept Point of Growth. - Sequence it at the beginning and then move into the Graph Productive Approach to discuss the y-intercepts.
1	Same y-intercept
Color of Letter Circle	Red
Question(s) for the Group	What numbers from the problem did you use to get the y-intercepts?
	What numbers did you use to get the graph?
	Why is the initial money he has the y-intercept and what he saves is not?
Question(s) for the Whole Class	Can someone tell me why the y-intercepts should be different?
	Can someone tell me why the lines they graphed are dashed and not solid?
	Can we still use some of what is up here? Why or why not?
Bridge	- It looks like we can use one of these lines. - I wonder how we can graph the other one with a different y-intercept.
A	Graph
Color of Letter Circle	Green
Question(s) for the Group	How did you get these points here (point to the y-intercept and the first value at one hour?
	How did you know where to plot the next set of points?
Question(s) for the Whole Class	Can someone tell me why starting with a Graph works well for this problem?

	Can someone tell me what would make you not want to use a Graph first?
Bridge	- This is a nice visual for identifying the values. - I wonder if there's another way to visualize these numbers.
B	Table
Color of Letter Circle	Green
Question(s) for the Group	How did you align these numbers (point to the left column) with these down here (point to the right column)?
	What made you decide to use a Table over a Graph to start?
Question(s) for the Whole Class	Can someone tell me where the points on the Graph came from the Table?
	Can someone tell me if there's a benefit to using one of these Strategies over the other?
	What would it take to get you to use the Strategy you didn't choose to use?
Bridge	- We're getting to the graph quickly. - I wonder if there's a way to go straight to the Equations.
C	Equations ⭐
Color of Letter Circle	Green
Question(s) for the Group	Why did you choose to go straight to the equation?
	How did you know from the problem which number is the slope and which number is the y-intercept?
Question(s) for the Whole Class	Can someone tell me the benefits (or detriments) to starting with the Equations first?
	Which of the three Strategies do we like the most? Why?
Activity Conclusion	- How did each Strategy help you to find the equation of the lines?

	- Does it matter which Strategy I use to find the equation? Why or why not? - What would make you choose one Strategy over another?

Reflection Questions

Reflection Question(s)	- How could I re-sequence the Strategies to help students understand the connection between the Graph, Table, and Equations? - Based on what my students did, how could I draw out the connection between the context of the problem and which value is the y-intercept and which one is the slope? - Is there a pattern I noticed when interacting with students or when I called on students/groups to present? Am I leaving anyone out? - Is there anything I did during Monitoring, Selecting, Sequencing, or Connecting that I can learn from for the next 5Ps activity?

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