Amplify Math

Algebra 1

Volume 1: Units 1–3

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Hello, curious mind!

Welcome to Algebra 1. The word "algebra" originates from the Arabic word "al-jabr," which means "the reunion of broken parts." Don't worry, you'll stay in one piece. But this is a year to break up the mathematics, determine what you know, and develop strategies to solve all sorts of problems.

You see, this year, you will break the constraints (and conventions) of homecoming, plan for life after high school, overcome "choice overload," and strategize your fundraising efforts to support the environment. And that's just Unit 1!

We promise you, it's all possible. This year, you'll see not only that math makes sense, but it can be fun, too.

Before you dig in, we want you to know two things:

This book is special! It's where you'll record your thoughts, strategies, explanations, and, occasionally, any award-winning doodles that might come to mind.

When you go online, you won't be mindlessly plugging numbers into your device . . . You'll be pushing, pulling, crawling, teleporting, melting . . . , well, let's just say you'll be doing a lot of things, and you'll be teaming up with your classmates as you do.

Let's dig in!

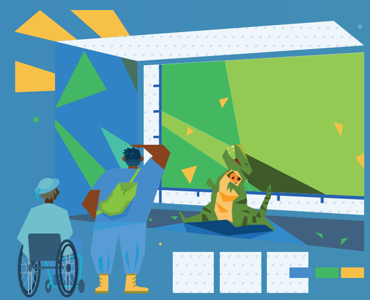
Sincerely,
The Amplify Math Team

Volume 1

Unit 1 Linear Equations, Inequalities, and Systems

In this unit, you will encounter a range of situations, from high school to adulthood. Along the way, you will discover how constraints connect to equations and inequalities, and how they can be used to help with decision-making.

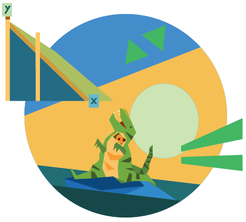
Unit Narrative:
Adulting (Making Life Decisions)



How did a tragic accident end a three-month strike?
Revisit how equations and inequalities can be used to model real-world situations, and how they can help you make decisions.

How do first-gen Americans vault the hurdles of college?
"Solving" an equation doesn't always mean finding an unknown value — sometimes it can mean changing the equation's very structure.

What's after high school?
Whether you work, intern, attend college, or do something else, inequalities can be used to model your time and money, helping you manage both of them.



Are you a 'Boomerang-er'?
For better or for worse, life is full of constraints. Discover new strategies for solving problems with multiple constraints, which you will see time and again.

Is there such a thing as too much choice?
What happens when the decisions become more complicated? Look at the big picture and then fine-tune where the decision overlaps.

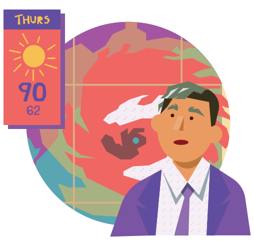
Unit 2 Data Analysis and Statistics

Unit Narrative:
Analyzing Climate Change

In this unit, you will explore data sets with one or two variables, often related to one of the most pressing threats we face as humanity: climate change. Along the way, you will encounter new statistical measures of center, spread, and association.

How can we protect ourselves from a zombie virus?

Remember dot plots, histograms, and box plots? Revisit them through temperature data, while describing the data's center and variability.



Is Sandy the new normal?

Meet the most commonly used measure of variability: standard deviation.

What is "Day Zero"?

You've seen linear models before, but now you will (finally!) see how to identify the "best" model, using what are called residuals.

What makes storms worse and has nothing to do with the weather?

Use two-way tables to see how the changing climate has affected marginalized people around the world.

Who is the "water warrior"?

Finally, you can say just how strong or weak an association is using a new statistical measure: the correlation coefficient.

Unit 3 Functions and Their Graphs

Unit Narrative:
Artscapes

You will expand your understanding of functions, their representations and graphs. Along the way, you will write, graph, and interpret a variety of functions and their inverses.

How did the blues find a home in Memphis?
Remember representing functions with description, tables, graphs, and equations? Take another look as you visit Memphis, and meet another common tool to represent and interpret functions: *function notation*.

What's the function of a jazz solo?
The way you describe a graph helps you gain insight on the relationship it represents. *Average rate of change*, *domain*, and range help to construct and interpret graphs more precisely.

Where did the world meet soul?
Piecewise functions allow you to represent a relationship between two quantities as a set of rules. The absolute value function is a specific piecewise function that represents the distance from zero.

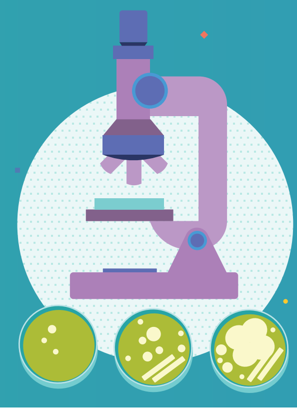
How do you get Sunday shoppers to hear your song?
What happens if you reverse the process used in a relationship between two quantities? What changes? What stays the same? You can explore these questions using inverses of functions.

Volume 2

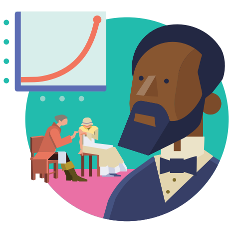
Unit 4 Introducing Exponential Functions

This is a unit of mathematical discovery, where relationships between quantities are unlike any function you have seen up to this point. You will encounter the explosiveness of exponential growth, as well as the lingering of exponential decay, through the lenses of infectious disease, vaccination, and prescription drug costs.

Unit Narrative: Infectious Diseases, Vaccines, and Costs



Where do baby bacteria come from?
Examine nonlinear functions using tables and graphs, before defining an exponential relationship. You'll represent exponential growth using an equation and explore it in context.

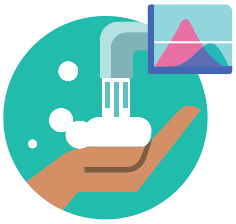


How did an enslaved person save the city of Boston?
Examine growth factors between 0 and 1, as you develop an understanding of exponential decay.

What does growing or shrinking look like on a graph?
Identify exponential relationships as exponential functions, and whether a graph is discrete.

Want to be CEO for a day?

Make sense of repeated percent increase and see how it relates to compound interest.



How does distance make the curve grow flatter?

Compare the growth of different kinds of functions and finish with an exploration of how social distancing can combat the dangers of an epidemic.

Unit 5 Introducing Quadratic Functions

Unit Narrative:
Squares in Motion

What goes up must come down. In this unit, you will study quadratic functions. By analyzing and comparing patterns, tables, graphs, and equations, you will gain an appreciation for the special features of quadratic functions and the situations they represent.

Unit 6 Quadratic Equations

In this unit, you will explore how people have learned to solve quadratic equations throughout history. You will write, solve, and explore strategies for solving quadratic equations.

Unit Narrative:
The Evolution of Solving Quadratic Equations

How did the Nile River spur on Egyptian mathematics?
Revisit projectile motion and maximizing revenue, as you discover new meanings for the zeros of a quadratic function.

When is zero more than nothing?
Understand the importance of zero when solving quadratic equations. Then, efficiently factor quadratics using a variety of strategies.

How many ways can you crack an egg?
Discover the ancient art of taking a quadratic expression and completing the square. It's all about that missing piece.

Where does a number call its home?
Subtraction and division took you from whole numbers to rationals. Now you must look beyond them as you operate with irrational numbers.

What was the House of Wisdom?

Discover strategies for solving any quadratic equation. You will also determine which strategies are more efficient.

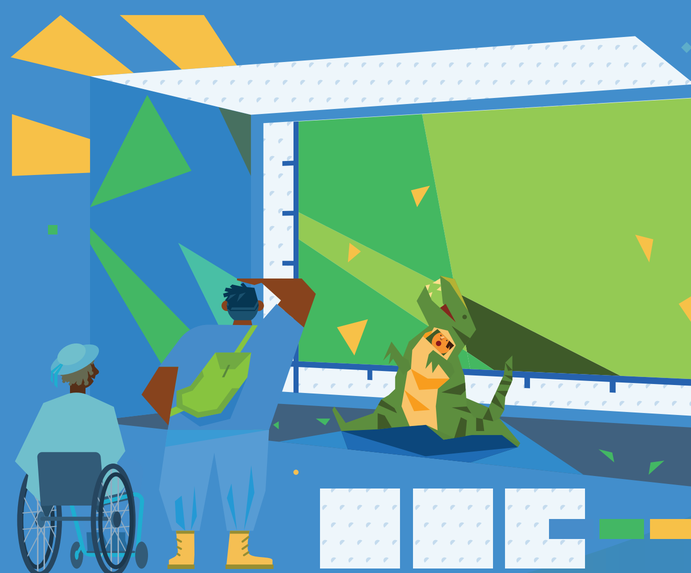
UNIT 1

Linear Equations, Inequalities, and Systems

In this unit, you will encounter a range of situations, taken from high school to adulthood. Along the way, you will discover how *constraints* connect to equations and inequalities, and how they can be used to help with decision-making.

Essential Questions

- How can equations and inequalities help you solve problems?
- Why is it useful to have different forms of linear equations?
- How can you use systems of equations or inequalities to model situations and solve problems?
- *(By the way, how can you make a decision if there are infinitely many possibilities?)*



SUB-UNIT

1 Writing and Modeling With Equations and Inequalities

Narrative: Making life decisions involves understanding unknowns and limitations — algebra can help!

You'll learn . . .

- how equations and inequalities can model problems involving constraints.

SUB-UNIT

2 Manipulating Equations and Understanding Their Structure

Narrative: Understanding equations can help you navigate decisions beyond high school.

You'll learn . . .

- how to rearrange equations and maintain equivalence.

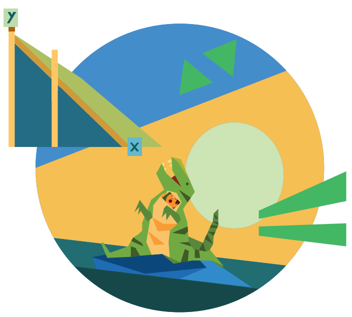
SUB-UNIT

3 Solving Inequalities and Graphing Their Solutions

Narrative: Your old friend — the inequality — is here to help you decide what's next after senior year.

You'll learn . . .

- how inequalities can be used to manage time and money.



SUB-UNIT

4 Systems of Linear Equations in Two Variables

Narrative: Where to live after high school — systems of linear equations to the rescue!

You'll learn . . .

- how to solve problems that involve multiple constraints.

SUB-UNIT

5 Systems of Linear Inequalities in Two Variables

Narrative: Discover how algebra can help you avoid choice overload.

You'll learn . . .

- how systems of linear inequalities can model real-world problems.

Unit 1 | Lesson 1 – Launch

Homecoming in Style

Let's see how constraints can affect preparation for homecoming.

Warm-up The School Dance

1. What school event are you most looking forward to this year? Why?

2. Do you want to go to a school dance? If so, what specifically are you looking forward to? If not, what would you like to do instead?

3. In many schools in the U.S., the homecoming dance is a major event. If you were to attend, what would you need in order to prepare for it?

4. What are some things to consider while preparing for homecoming?

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 Planning for Homecoming

Jada is preparing for a homecoming dance and has a budget of $400. She must spend her money on five total items, each from a different category: transportation, clothing, shoes, accessories, and hair. Each item also comes with "style points." Jada would like to maximize her style points, while staying within her $400 budget. Jada's need to have the best style and experience within a certain budget is similar to a famous problem in mathematics, known as the "knapsack" problem, which was recently studied by Ce Jin, a former student of Professor Jelani Nelson in Berkeley, California.

You will be given cards with items to choose from for Jada. To help Jada, her mom has created the following table with five items, their cost, and their style points.

Category	Item/experience	Cost ($)	Style points
Transportation	Ride-sharing	50	50
Clothing	New	150	100
Shoes	New heels	150	100
Accessories	Flowers	50	25
Hair	Do-it-yourself	0	50
Total		**400**	**325**

1. Plan a homecoming experience for Jada that stays within the $400 budget but has *more* style points than Jada's mom's choices.

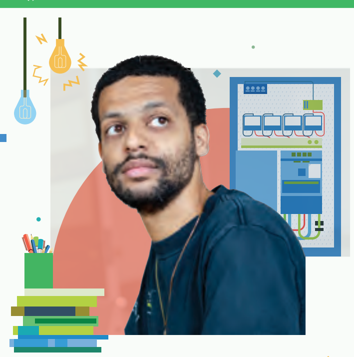
Category	Item/experience	Cost ($)	Style points
Transportation			
Clothing			
Shoes			
Accessories			
Hair			
Total			

Activity 1 Planning for Homecoming (continued)

2. How did you decide which options to choose for Jada in each category?

3. If Jada had an unlimited budget, which options should she choose to maximize her style points?

Featured Mathematician



Jelani Nelson

Born on the island of St. Thomas in the Caribbean, Jelani Nelson is a Professor of Electrical Engineering and Computer Science at Berkeley University in California. He develops and analyzes fast algorithms, efficient methods for working with large data sets. He also teaches programming courses in Addis Ababa, Ethiopia.

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Name: ________________ Date: ________ Period: ________

Activity 2 A Homecoming Couple

Shawn and Noah are going to their homecoming dance together. They decide to put their money together to create an $800 budget, which they will spend together. They will ride to and from the dance together, so they must choose the same form of transportation.

You will be given cards of each person's options, including how much they cost and their number of style points.

1. Determine a combination of options that results in at least 700 total style points, while making sure they each have at least 300 style points (so that they both have a good time at the dance).

	Shawn	Noah		
Category	Item/ experience	Item/ experience	Combined cost ($)	Combined style points
Transportation				
Clothing				
Shoes				
Accessories				
Hair				

2. Describe your method for choosing options for Shawn and Noah.

3. Was it more challenging to choose the options for Shawn and Noah, or for Jada in Activity 1? Explain your thinking.

STOP

Narrative Connections

Unit 1 Linear Equations, Inequalities, and Systems

Adulting (Making Life Decisions)

Pep rallies, bouquets, the crowning of a student court — homecoming can be a watershed moment in many teenagers' lives. It celebrates a school football team's first home game of a season. Students past and present "come home" to the school to celebrate, with the week's festivities culminating in a homecoming dance. Whether you choose to participate in homecoming or other activities, high school can be an exciting time. For many, it marks a step into the world of adulthood.

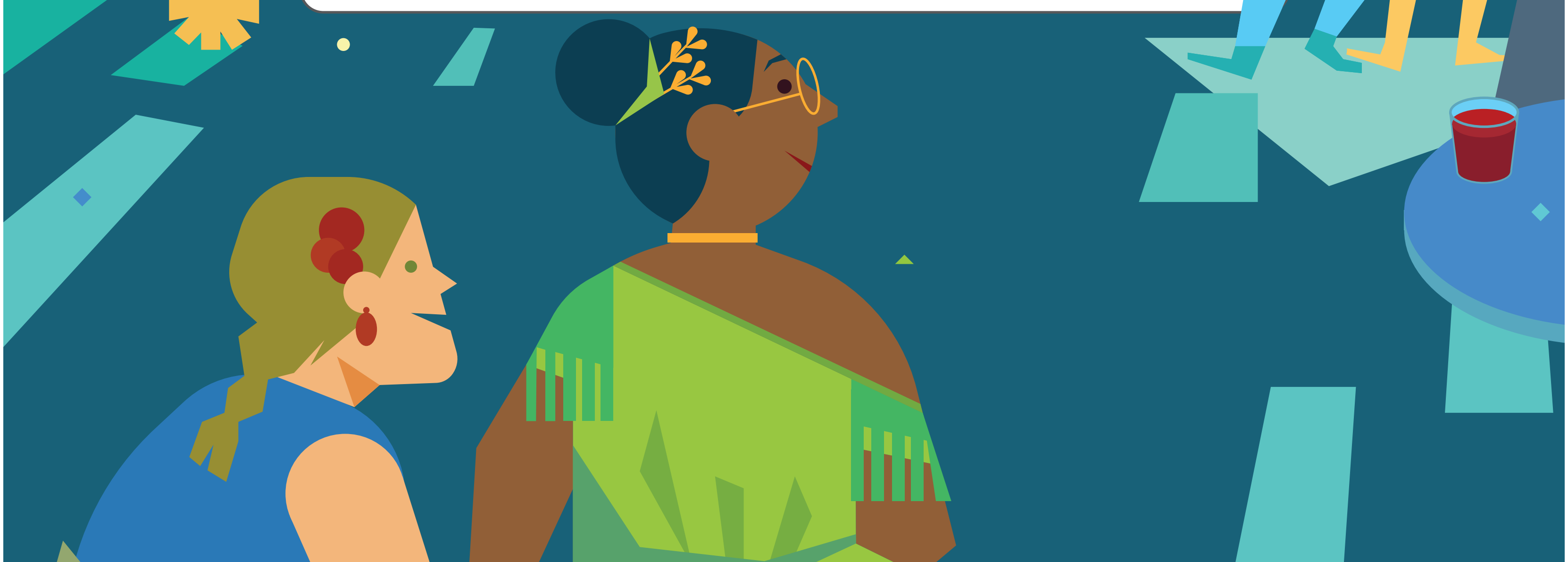
But becoming an adult can be a mixed bag of increased responsibility and freedom. Part of that process includes learning how to make choices.

In the years to come, you'll face all kinds of decisions — some as small as deciding what to have for lunch; and some big enough to impact the rest of your life (and even the lives of others). You'll have to make decisions about where you will live; what kind of work you will do; and what issues you will stand up for. And no matter what those choices are, there will be trade-offs to consider. It won't always be clear which option is best.

Constraints, like the budgets you saw in this lesson, can help frame those options. They can guide you in determining what your priorities are and establish what you *are* and *aren't* willing to compromise on.

In these next lessons, you will explore ways constraints can interact, as you build systems of linear equations and inequalities. These equations and systems will be powerful tools, allowing you to make choices that are thoughtful and consistent with the values that matter to you.

Welcome to Algebra 1.



Name: ________________ Date: ________ Period: ________

Practice

1. Tyler brought his car to the mechanic with a budget of $550 for repairs. The mechanic informed Tyler of the things that should be replaced or repaired. The mechanic gave Tyler the table provided, which shows the cost and the importance rating of each repair.

	Cost ($)	Importance rating
Spark plug replacement	100	80
Muffler replacement	200	20
New tires	400	100
New brake pads	225	70

What repairs should Tyler choose so that he stays within his budget and maximizes the combined importance rating?

2. Solve each equation.

a. $5x = 20$

b. $\frac{1}{4}x = 80$

c. $6x = \frac{3}{5}$

3. The table gives values from a linear relationship.

a. Complete the table.

b. Write an equation that represents the relationship shown in the table.

x	y
12	9
20	15
	24
48	

Practice

Name: ________ Date: ________ Period: ________

4. Han solved the equation below incorrectly. Identify Han's error and then solve the equation correctly.

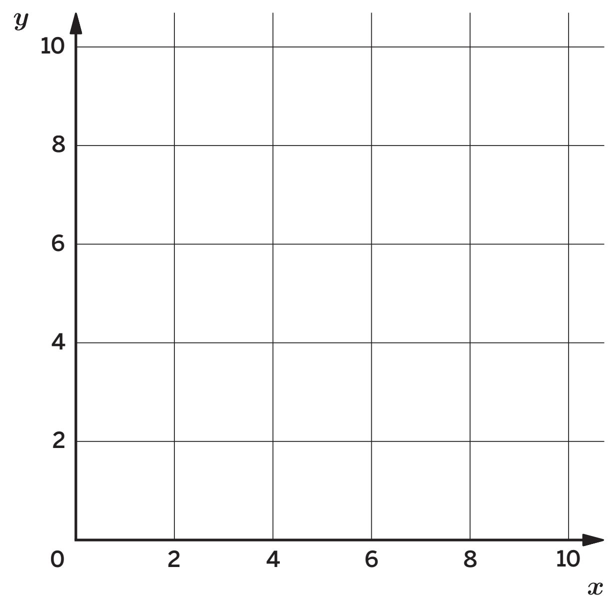
Han's work:

$$\frac{3}{4}(x-4)=\frac{1}{2}x$$
$$\frac{3}{4}x-3=\frac{1}{2}x$$
$$\frac{2}{2}x-3=0$$
$$x-3=0$$
$$x=3$$

5. Graph the following two equations:

Equation A: $y=\frac{1}{2}x$

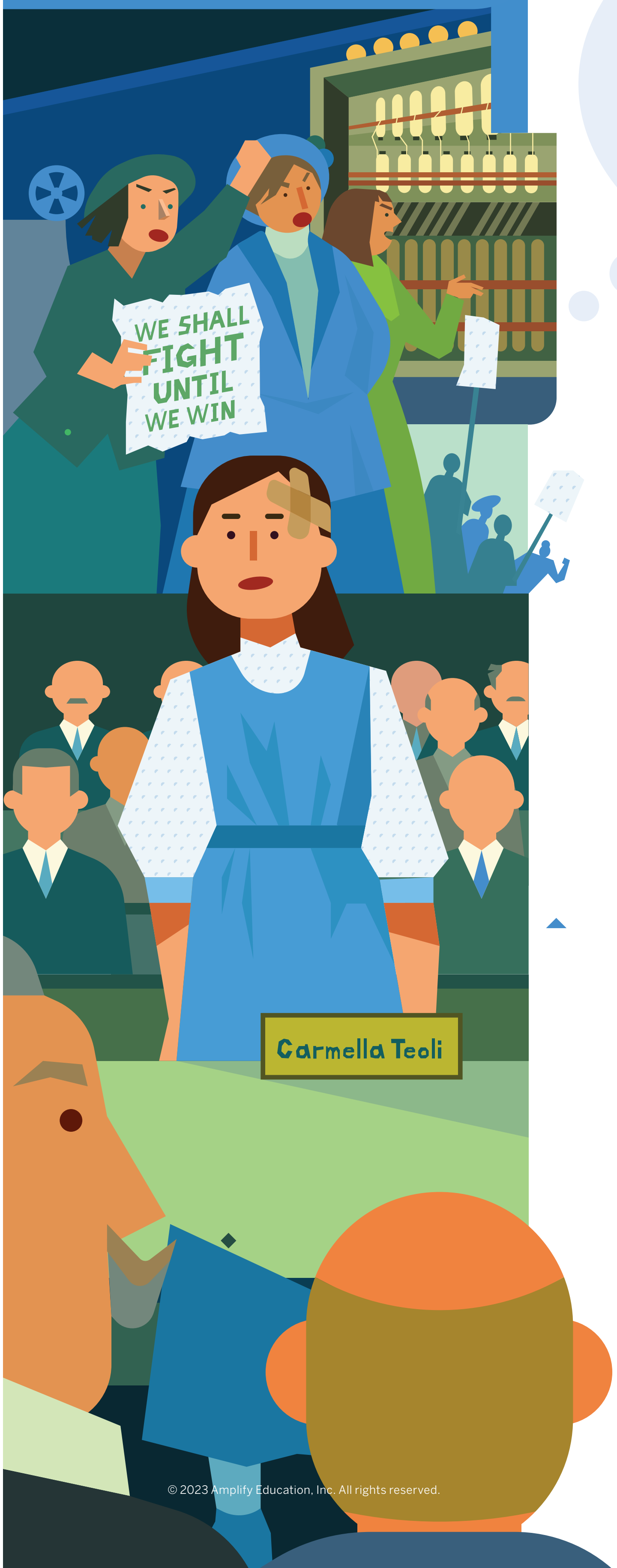
Equation B: $y=2x+4$



6. A cellphone plan costs $30 a month and has a one-time fee of $50. Write an equation that represents the cost y of the cellphone plan, in dollars, after x months.

SUB-UNIT

1 Writing and Modeling With Equations and Inequalities



How did a tragic accident end a three-month strike?

In the early 20th century, American factories employed children and young teenagers, expecting them to work long hours on dangerous machinery. Carmella Teoli was one such teenager. At 13, her hair got caught in the machinery at the mill where she worked. The incident left her hospitalized with a six-inch scar on her head.

At the same time, discontent was growing among the mill workers of New England — the majority of whom were immigrant women. Their hours were being cut and their wages lowered. By the time Teoli was released from the hospital, the Lawrence Textile Strike of 1912 was underway.

Teoli joined the striking workers. And when a Congressional hearing was called, she testified. Her account was so moving that President Taft launched a national investigation into factory working conditions. Three months after the strike began, the mill owners bowed to the strikers' demands.

As Teoli's story shows, working teenagers didn't always have options when it came to how they were treated. But thanks to the reforms spurred on by Teoli and the other strikers, teenagers working today have better choices when it comes to where and how they work. But the best choices aren't always obvious.

Will a summer job let you save enough money for a new car? Should you work somewhere closer to home for less money, or take a job farther away that you would have to commute to?

Making decisions like these can involve understanding what your unknowns are, taking into account the relevant facts, and being able to express your situation mathematically.

Unit 1 | Lesson 2

Writing Equations to Model Relationships

Let's write equations or inequalities that help us to model quantities and constraints.

Warm-up Math Talk

What strategy would you use to evaluate each expression? Use your strategy to find the value. Explain your thinking.

1. 25% of 200

   Strategy:

   Solution:

2. 8% of 200

   Strategy:

   Solution:

3. 12% of 200

   Strategy:

   Solution:

4. p% of 200

   Strategy:

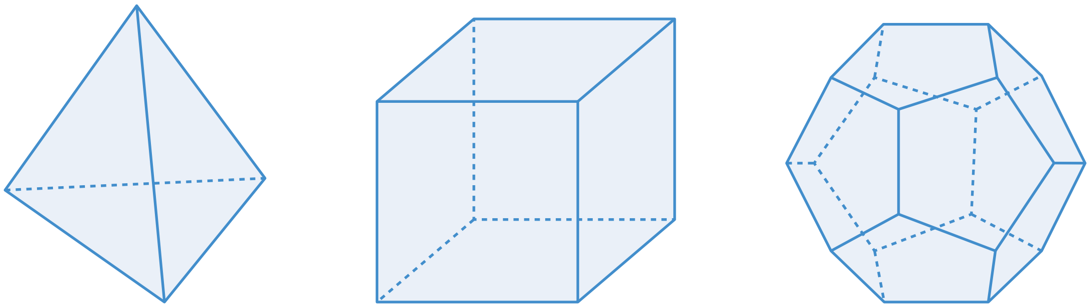
   Solution:

Log in to Amplify Math to complete this lesson online.

Name: ______________ Date: ________ Period: ________

Activity 1 A Platonic Relationship

These three figures are called Platonic solids. The faces of a Platonic solid are all congruent regular polygons, meaning they all have congruent angles and sides.



The table shows the number of faces, vertices, and edges for a tetrahedron and a dodecahedron.

	Faces (F)	Vertices (V)	Edges (E)
Tetrahedron	4	4	6
Cube			
Dodecahedron	12	20	30

1. Complete the missing values for the cube. What observations can you make about the number of faces, edges, and vertices in a Platonic solid?

Activity 1 A Platonic Relationship (continued)

2. There are some interesting relationships between the number of faces (F), edges (E), and vertices (V) in all Platonic solids. Use the following information to complete the table. The first two algebraic representations have been completed for you.

   a. *The number of edges of a Platonic solid is always greater than the number of its faces*. Write the inequalities to verify that this is true for each Platonic solid in the first row of the table.

   b. *The number of edges of a Platonic solid is always less than the sum of the number of its faces and the number of its vertices*. Write an inequality to verify that this is true for each Platonic solid in the second row of the table.

   c. The relationship between F, V, and E can be expressed as an equation, which was studied by mathematician Leonhard Euler. Can you determine this equation? Write the equation in the last row of the table's first column. Then, verify that the equation is true for each Platonic solid in the last row of the table. **Hint:** Examine the values in the first table to help you.

Algebraic representation	Tetrahedron	Cube	Dodecahedron
$E > F$			
$E < F + V$			

Featured Mathematician

Leonhard Euler

Leonhard Euler was a prolific Swiss mathematician who made significant contributions to mathematics, the physical sciences, and astronomy. He proved theorems about prime numbers (including the largest known prime at the time), he has two constants named after him (which you may learn about in a later algebra course), and he developed much of today's mathematical notation. He also discovered a relationship between the number of vertices, edges, and faces of any convex polyhedron, or three-dimensional solid.

NoPainNoGain/Shutterstock.com

Name: .... Date: .... Period: ....

Activity 2 Bus Fares and Summer Earnings

Diego, Clare, and Lin each have a summer job to earn extra money. They use public transportation to get there.

1. Write an equation to represent each scenario.

   a. It costs $2 for a one-way fare on the local bus. Diego buys b bus fares and pays $20 to get to work during the week.

   b. It costs $2 for a one-way fare on the local bus. Clare buys t bus fares and pays c dollars to get to work during the week.

   c. It costs d dollars for a one-way fare on the local bus. Lin buys q bus fares and pays t dollars to get to work during the week.

   d. Diego earned n dollars over the summer. Lin earned $975, which is $45 more than Diego.

   e. Diego earned v dollars over the summer. Lin earned m dollars, which is $45 more than Diego.

   f. Diego earned w dollars over the summer. Lin earned x dollars, which is y dollars more than Diego.

2. How are the equations you wrote to represent the bus fare scenarios in parts a–c similar to the equations you wrote to represent the summer earnings scenarios in parts d–f? How are they different?

Similarities	Differences

3. Lin worked a total of h hours this summer. Clare worked m hours more than Lin did. Together, they worked a total of s hours. Write an equation that relates the number of hours Lin and Clare worked this summer.

Activity 3 Car Prices

Clare's parents are planning to buy her a car so she will no longer have to take the bus. The sales tax on a car in her state is 6%. At the local dealership, a car purchase also requires $120 for the title and registration fees.

1. There are several quantities in this scenario: the original car price, sales tax, title and registration fees, and the total price. For each of the following statements, write an equation that relates all the quantities. If you use a variable, specify what it represents.

   a. The original price is $9,500.

   b. The original price is $14,699.

   c. The total price is $22,480.

   d. The original price is p.

2. How would each of your equations in Problem 1 change if the tax were r% and the title and registration fees were m dollars?

   a. The original price is $9,500.

   b. The original price is $14,699.

   c. The total price is $22,480.

   d. The original price is p.

3. Besides the sales tax and title and registration fees, what other costs are associated with owning a car? What are the benefits or drawbacks of owning a car?

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You studied several scenarios in which quantities were unknown, varied, or remained constant (fixed). You used letters — *variables* — to model the quantities and constraints in each scenario.

Variables are helpful for representing quantities that have some unknown fixed value, or that are unknown but have a value that may vary. Variables are also helpful when you want to understand the relationship between quantities better, or how one quantity depends on another (rather than just using a few specific values).

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. At a certain pizzeria, a large cheese pizza is \$5 and a large one-topping pizza is \$6. Write an equation that represents the total cost T of c large cheese pizzas and d large one-topping pizzas.

2. Mai plans to serve milk and scones at her book club meeting. She is preparing 12 oz of milk per person and 4 scones per person. Including herself, there are 15 people in the club.

   - A package of scones contains 24 scones and costs \$4.50.
   - A 1-gallon jug of milk contains 128 oz and costs \$3.

   Let n represent the number of people in the club, m represent the total number of ounces of milk, s represent the total number of scones, and b represent Mai's budget, in dollars. Select *all* of the equations that could represent the quantities and constraints in this scenario.

   A. $s = 4n$

   B. $s = 4(4.50)$

   C. $m = 12(15)$

   D. $b = 3m + 4.5s$

   E. $b = 2(3) + 3(4.50)$

3. A student on the track team runs 45 minutes each day as part of his training. He begins his workout by running at a constant rate of 8 mph for a minutes, then slows to a constant rate of 7.5 mph for b minutes. Which equation describes the relationship between the distance he runs in miles D and his running speed, in miles per hour?

   A. $a + b = 45$

   B. $8a + 7.5b = D$

   C. $8\left(\frac{a}{60}\right) + 7.5\left(\frac{b}{60}\right) = D$

   D. $8(45 - b) + 7.5b = D$

Name: ______ Date: ______ Period: ______

Practice

4. Solve this equation. Explain or show your thinking.

$\frac{1}{4}x - 5 = \frac{1}{3}(x - 12)$

5. Solve each equation.

a. $3x + 10 = 20$

b. $4x - 5 = 12 - 2x$

6. Complete each table so that each pair of numbers makes the equation true.

a. $y = 5x$

x	y
3	
	90
$\frac{4}{5}$	

b. $y = 3x - 1$

x	y
2	
	11
	-28

c. $y = \frac{x+1}{2}$

x	y
-2	
-1	
	4

d. $y = \frac{24}{x}$

x	y
-4	
	-8
9	

Unit 1 | Lesson 3

Strategies for Determining Relationships

Let's use patterns to help us write equations.

Warm-up Math Talk

Here is a table of values. The two quantities, x and y, are related.

x	y
1	0
3	8
5	24
7	48

What strategies could you use to find a relationship between x and y? Talk about it.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Something About 400

Part 1 You and a partner will take turns describing in words how the two quantities in each table are related. As one partner explains, the other partner's role is to listen carefully, agree or disagree, and then explain why. If you and your partner disagree, work together to reach an agreement.

1.

Number of laps, x	0	1	2.5	6	9
Meters run, y	0	400	1,000	2,400	3,600

Partner ______:

Partner ______:

2.

Distance from home (m), x	0	75	128	319	396
Distance from school (m), y	400	325	272	81	4

Partner ______:

Partner ______:

3.

Number of transfer students, x	85	124	309	816
High school population, y	485	524	709	1,216

Partner ______:

Partner ______:

4.

Monthly earnings ($), x	872	998	1,015	2,110
Amount deposited ($), y	472	598	615	1,710

Partner ______:

Partner ______:

Activity 1 Something About 400 (continued)

Part 2

5. Match each table with the equation that represents the relationship.

a

Number of laps, x	0	1	2.5	6	9
Meters run, y	0	400	1,000	2,400	3,600

b

Distance from home (m), x	0	75	128	319	396
Distance from school (m), y	400	325	272	81	4

c

Number of transfer students, x	85	124	309	816
High school population, y	485	524	709	1,216

d

Monthly earnings ($), x	872	998	1,015	2,110
Amount deposited ($), y	472	598	615	1,710

.... $400 + x = y$

.... $x - 400 = y$

.... $x + y = 400$

.... $400 \bullet x = y$

Are you ready for more?

On a separate sheet of paper, express every number between 1 and 20 at least one way using exactly four 4's and any operation or mathematical symbol. For example, 1 could be written as $\frac{4}{4} + 4 - 4$.

Name: .... Date: .... Period: ....

Activity 2 What Are the Relationships?

As you describe the relationship between the quantities in each scenario, use words, expressions, or equations to help explain your thinking.

1. A geometry teacher draws several parallelograms. The table represents the relationship between the base length and the height of some of the parallelograms. What is the relationship between the base length and the height of these parallelograms? Explain your thinking.

Base length (in.)	1	2	3	4	6
Height (in.)	48	24	16	12	8

2. At a high school pep rally, students are challenged to guess the number of marbles in a jar. The student who guesses the correct number wins \$300. If multiple students guess correctly, the prize will be divided evenly among them. What is the relationship between the number of students who guess correctly and the amount of money each student will receive? Explain your thinking.

3. A cafeteria worker is preparing cups of milk for students. A half-gallon jug of milk can fill 8 cups, while 32 fl oz of milk can fill 4 cups. What is the relationship between the number of gallons and ounces? Explain your thinking.

STOP

Summary

In today's lesson . . .

You used a variety of strategies to reason about the relationship between quantities and write equations to represent those relationships.

Some strategies that are helpful include:

- Creating a table to see how a quantity changes or determine how two quantities are related.
- Looking for patterns within a table.
- Testing different values for one variable and observing its effects on the other variable.

Sometimes the relationship between two quantities is apparent. But other times, the relationship is not apparent and requires you to perform some calculations.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Members of a band sold t-shirts and hats at a college football game to raise money for an upcoming trip. The band raised $\$2{,}000$, which will be divided equally among the m members of the band. Which equation represents the amount A that each member receives?

A. $A = \frac{m}{2000}$

B. $A = \frac{2000}{m}$

C. $A = 2000m$

D. $A = 2000 - m$

2. Tyler is completing a table for his consumer science class. Each row shows an equivalent amount. He knows that 1 tablespoon contains 3 teaspoons and that 1 cup contains 16 tablespoons.

Teaspoons	Tablespoons	Cups
		2
36	12	0.75
	48	3

a. Complete the missing values in the table.

b. Write an equation that represents the number of teaspoons t contained in C cups.

3. The volume of dry goods, like apples or peaches, can be measured using bushels, pecks, and quarts. A bushel is equivalent to 4 pecks, and a peck is equivalent to 8 quarts. For a given container, what is the relationship between its volume in bushels b and its volume in quarts q? Create a table to show your thinking.

Practice

Name: ______ Date: ______ Period: ______

4. At a hamburger stand, a burger is $\$6$, a drink is $\$3$, and there is currently a special offer for a $\$1$ discount on every order. Write an equation that represents the total cost C of b burgers and d drinks.

5. Jada is driving to a different city to visit her friend. She breaks the trip up into two days. On the first day, she drives for x minutes at an average speed of 55 mph. The second day she drives for z minutes at an average speed of 65 mph. Write an equation that represents the distance in miles D that Jada travels.

6. Clare filled her car with gas. She paid $\$2$ per gallon. For being part of the gas station's rewards program, she then received a discount of $\$5$ off her final bill. In total, she spent $\$35$.

a Write an equation that models the following scenario.

b Did Claire buy 17 gallons of gas? Show or explain your thinking.

Name: ______ Date: ______ Period: ______

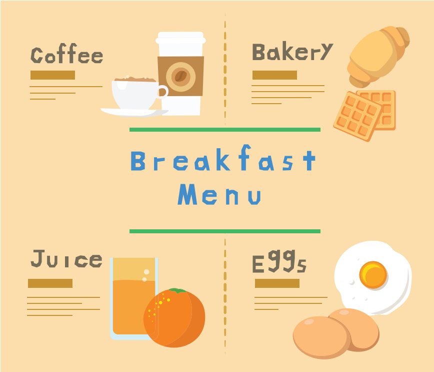
Unit 1 | Lesson 4

Equations and Their Solutions

Let's recall what we know about solutions to equations.

Warm-up What is a Solution?

Bard orders veggie toast v for the table while dining with friends. Veggie toast is $2.49. The total of the order is $19.33. The equation $2.49v + 4.39 = 19.33$ represents the relationship between the quantities.



1. What could the 4.39 represent in the equation?

2. If Bard orders 7 orders of veggie toast, is the total $19.33? Explain or show your thinking.

3. If Bard orders 2 orders of veggie toast, is the total $19.33? Explain or show your thinking.

4. How many orders of veggie toast does Bard order?

Log in to Amplify Math to complete this lesson online.

Activity 1 Weekend Earnings

Lin works at Market Fresh on the weekend and earns $\$12.20$ each hour. Each day, she spends $\$7.15$ on bus fare to commute to work.

1. Write an expression that represents Lin's take-home earnings in dollars if she works at Market Fresh for h hours in one day.

2. One day, Lin's take-home earnings are $\$90.45$ after working h hours and paying the bus fare. Write an equation to represent her take-home earnings on this day.

3. Is either 4 or 7 a solution to the equation you wrote in Problem 2?
   - If so, explain how you know one or both values are solutions.
   - If not, explain why one or both values are not solutions. Then determine the solution.

4. For Problem 2, what does the solution to the equation represent?

Are you ready for more?

Lin has another opportunity to earn money. She can help her neighbors with their errands for $\$11$ an hour. Lin considers her schedule and determines she has about 9 hours available to work one day of the weekend.

Should Lin keep her job at Market Fresh or help her neighbors? Explain your thinking.

Name: ______ Date: ______ Period: ______

Activity 2 Customer Receipts

At Market Fresh, a salmon burger costs $3 and a side salad costs $2. Lin's next five customers all order salmon burgers and side salads. Match each customer receipt with the correct customer order.

Receipt:

Receipt for Customer ______
Amount Paid: $21

Receipt for Customer ______
Amount Paid: $40

Receipt for Customer ______
Amount Paid: $32

Receipt for Customer ______
Amount Paid: $12

Receipt for Customer ______
Amount Paid: $43

Customer order:

Customer 120:
2 salmon burgers + 3 side salads

Customer 121:
3 salmon burgers + 6 side salads

Customer 122:
10 salmon burgers + 5 side salads

Customer 123:
7 salmon burgers + 11 side salads

Customer 124:
8 salmon burgers + 4 side salads

Activity 3 What Did They Order?

Priya also works at Market Fresh. A customer pays $24 for b salmon burgers and s side salads. The equation $3b + 2s = 24$ represents the relationship between these quantities.

1. Determine if each of the following orders could be the number of salmon burgers and side salads that Priya's customer ordered. Explain or show your thinking.

   a 5 salmon burgers and 4 side salads.

   b 2 salmon burgers and 9 side salads.

   c 8 salmon burgers and a side salad.

2. If Priya's customer ordered 6 salmon burgers, how many side salads did they order? Explain or show your thinking.

3. If Priya's customer did not order side salads, how many salmon burgers did they order?

4. What does a solution to the equation $3b + 2s = 24$ represent?

5. Could Priya's customer have ordered 3 salmon burgers? Why or why not?

Reflect: How well did you justify your responses to Problems 1–5? Ask your partner for feedback.

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You reviewed the meaning of a solution to an equation in a context.

An equation with only one unknown quantity is called an equation in one variable. To solve this kind of equation means to find a value that makes the equation true.

An equation with two unknown quantities is called an equation in two variables. When you solve these types of equations, you are looking for a pair of values that make the equation true. Equations in two variables often have multiple solutions.

For equations in one variable and equations in two variables, you can determine whether a value, or a pair of values, is a solution by substituting them into the equation and evaluating if the resulting statement is true or false.

Reflect:

Practice

Name: ____________ Date: ____________ Period: ____________

1. An artist sells bracelets and necklaces. Bracelets cost $\$1.50$ each and necklaces cost $\$2.25$ each. Select *all* combinations of bracelets and necklaces that the artist could sell for exactly $\$12$.

   A. 0 bracelets and 6 necklaces

   B. 1 bracelet and 5 necklaces

   C. 2 bracelets and 4 necklaces

   D. 5 bracelets and 3 necklaces

   E. 5 bracelets and 2 necklaces

   F. 8 bracelets and 0 necklaces

2. Volunteer drivers are needed to transport 80 students to the championship baseball game. Some drivers have cars, which can seat 4 students, and other drivers have vans, which can seat 6 students. The equation $4c + 6v = 80$ models the relationship between the number of cars c and number of vans v that can transport exactly 80 students. Select *all* statements that are true about this scenario.

   A. If 20 volunteers have cars, then no vans are needed.

   B. If 12 volunteers have cars, then 2 vans are needed.

   C. $c = 14$ and $v = 4$ are a pair of solutions to the equation.

   D. 10 cars and 8 vans are not enough to transport all the students.

   E. If 6 volunteers have cars and 11 have vans, there will be extra space.

   F. 8 vans and 8 cars are values that meet the constraints in this scenario.

3. The drama club purchases t-shirts for its members. The t-shirt company charges a certain amount for each shirt plus a printing fee of $\$40$. There are 21 members in the drama club.

   a The equation $187 = 40 + 21p$ represents the total cost of the t-shirt order from this company. What is the price p of each t-shirt? Explain or show your thinking.

   b The equation $201.50 = f + 6.50(21)$ represents the cost of purchasing t-shirts at a different t-shirt company. Determine the solution of the equation and explain what it represents in this scenario.

Name: ______ Date: ______ Period: ______

Practice

4. A high school Environmental Awareness Club held a fundraiser to raise money to donate to a variety of environment focused non-profit organizations. The club raised $\$5{,}500$, which will be divided equally among the p organizations that they will donate to. Write an equation that represents the amount F that each organization will receive.

5. Bard is buying books and gift certificates for each friend in a group of friends who has an upcoming birthday. Bard is planning to buy 2 books per person, and 3 gift certificate for each friend. Bard has 3 friends with upcoming birthdays.

- A book costs $\$10$.
- Bard is buying $\$5$ gift certificates.

Let f represent the number of friends with upcoming birthdays, b represent the number of books Bard buys, g represent the number of gift certificates Bard buys, and t represents the amount Bard spends, in dollars. Select *all* of the equations that could represent the quantities and constraints in this scenario.

A. $g = 3f$

B. $b = 2(3)$

C. $t = f + b$

D. $t = 2(10)(3) + 3(5)(3)$

6. Write an inequality that can be used to model each statement. If you use a variable, specify what it represents.

a. Clare has more than $\$200$ in her savings account.

b. Jada is younger than Tyler.

c. Mai's bowling score is more than Clare's and Han's combined.

Unit 1 | Lesson 5

Writing Inequalities to Model Relationships

Let's use inequalities to represent constraints in a context.

Warm-up What Do Those Symbols Mean Again?

Match each inequality with the meaning of its symbol(s). Then respond to the questions that follow.

Inequality:	Meaning of symbol(s):
a $h > 50$	.... less than or equal to
b $h \leq 20$	.... between
c $30 \geq h$	.... greater than or equal to
d $20 < h < 30$	.... greater than

1. Does 25 satisfy any of the inequalities? If so, which one(s)?

2. Does 40 satisfy any of the inequalities? If so, which one(s)?

3. Does 30 satisfy any of the inequalities? If so, which one(s)?

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Planning the Freshman Mixer

A senior class tradition at a high school is to plan a mixer for incoming freshmen. The seniors on the student council are creating a budget for the event and have gathered the following information:

- **Last year, 120 students attended. This year, the Freshmen Mixer is expected to draw as many as 200 students.**
- **For every 20 students, there should be at least 1 adult chaperone.**
- **The ticket price cannot exceed $20 per student.**
- **The revenue from ticket sales must cover the cost of meals and entertainment, and make a profit of at least $200.**

Three Reads: To make sense of this information, you will read this text three times. Your teacher will instruct you on what to focus for each read.

The senior class uses the following inequalities to model some of the information gathered. Each variable represents a quantity in the inequality. Use the given information to determine what each inequality represents.

1. $t \leq 20$

2. $120 \leq p \leq 200$

3. $a \geq \frac{p}{20}$

4. $pt - m \geq 200$

Are you ready for more?

Kiran says the senior class should add the constraint $t \geq 0$ to their information.

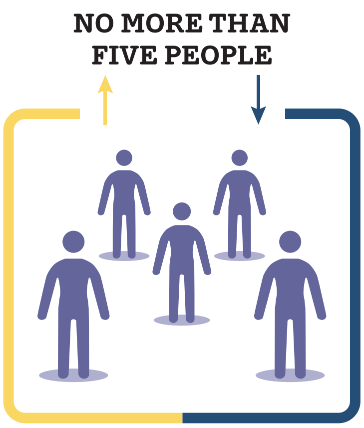
1. Why should this constraint be added?

2. Are there other similar constraints that should be added? Explain your thinking.

Activity 2 Mix and Mingle

The Freshmen Mixer is held on the 20th floor of a building at a local park, which has the best views of the city skyline. Adult chaperones are using an elevator to carry party supplies up to the 20th floor. The signs shown are posted in the building's elevator.

- **Suppose an average adult weighs 77 kg.**
- **Some of the adults each carry an additional 12 kg of party supplies.**
- **Each person carries 4 kg of personal belongings on the elevator with them.**



1. Write as many equations and inequalities as you can to represent these constraints. Be sure to specify the meaning of any variables that you use. (Avoid using the letters z, x, or d, which you will use later in this activity.)

Name: ______ Date: ______ Period: ______

Activity 2 Mix and Mingle (continued)

2. Now you will use the *Mix and Mingle* routine.

   Round 1: Trade your work with a partner and read each other's equations and inequalities. Take brief notes on what you observe or any questions you have for your partner.

   Round 2: Explain to your partner what you think their equations and inequalities represent, and listen to their explanation of yours. If needed, make adjustments to your equations and inequalities based on your partner's feedback, so that they are communicated more clearly.

3. Rewrite your equations and inequalities so that they would work for a different building in which:
   - An elevator car can hold at most z people.
   - Each elevator car can carry a maximum of x lb.
   - Each adult carries d lb of decorations.

STOP

Summary

In today's lesson . . .

You revisited previously learned concepts about inequalities from Grade 7. You recalled that some relationships and constraints cannot be modeled with symbols of equality.

In some situations, one quantity is, or needs to be greater than or less than another. The symbols, $>$, $<$, $\geq$, or $\leq$ are used to represent these situations. Some keywords used to help cue the use of inequalities, include but are not limited to:

$>$	Greater than, more than, above, exceeds
$\geq$	Greater than or equal to, at least, minimum, not below, no less than
$<$	Less than, smaller than, below
$\leq$	Less than or equal to, no more than, maximum

Understanding these terms and symbols enables you to interpret and write inequalities to model the constraints in various situations. Similar to equations, inequalities provide you with ways to represent relationships, but between quantities that are not equal.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Tyler has a budget of $125 to spend at the store. Which inequality represents x, the amount in dollars Tyler can spend at the store?

   A. $x > 125$
   B. $x < 125$
   C. $x \leq 125$
   D. $x \geq 125$

2. Jada wants to make lemonade for a study session with her friends. She expects to host a total of 5 to 8 people (including herself) and plans to serve 2 cups of lemonade per person. The lemonade recipe requires 4 scoops of lemonade mix for each quart of water. Each quart is equivalent to 4 cups. Let n represent the number of people at the study session, c the number of cups of water, and l the number of scoops of lemonade mix. Select *all* the mathematical statements that represent the quantities and constraints in the scenario.

   A. $l = c$
   B. $c = 2n$
   C. $5 < n < 8$
   D. $5 \leq n \leq 8$
   E. $10 < c < 16$
   F. $10 \leq l \leq 16$

3. A doctor sees between 7 and 12 patients each day. On Mondays and Tuesdays, the appointment times are 15 minutes. On Wednesdays and Thursdays, appointment times are 30 minutes. On Fridays, appointment times are one hour long. The doctor works no more than 8 hours a day. Here are some inequalities that represent the situation:

   $7 \leq x \leq 12$ $\quad 0.25 \leq y \leq 1$ $\quad xy \leq 8$

   a. What does each variable represent?

   b. What does the expression xy in the last inequality represent in this context?

4. A landscaping company is delivering crushed stone to a construction site. The table shows W, the total weight in pounds of n loads of crushed stone. Which equation could represent the total weight, in pounds, for n loads of crushed stone?

Number of loads, n	0	1	2	3
Total weight (lb), W	0	2,000	4,000	6,000

   A. $W = \frac{6000}{n}$
   B. $W = 2000n$
   C. $W = n + 2000$
   D. $W = 6000 - 2000n$

Practice

Name: ______________________ Date: __________ Period: __________

5. Han is a baker and needs to convert some measurements for his recipes. He knows that 1 gallon contains 8 pints, and 1 pint contains 2 cups.

a Complete the missing values in the table.

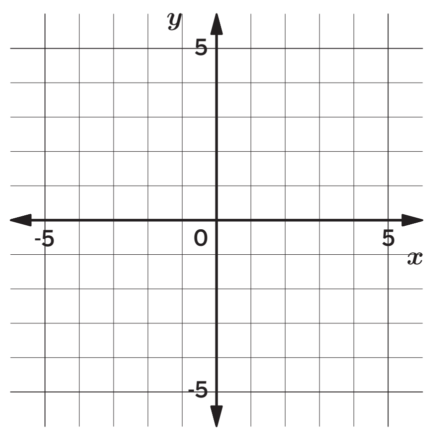
b Write an equation that represents the number of cups c contained in g gallons.

Gallons	Pints	Cups
2.5	20	40
	256	
	9	18

6. Sketch the graph of $y = x + 3$ on the graph shown.

a Does the point $(4, 1)$ lie on the graph? What about $(1, 4)$?

b How can you use the equation to show whether these points lie on the graph? Show or explain your thinking.



Name: ________ Date: ________ Period: ________

Unit 1 | Lesson 6

Equations and Their Graphs

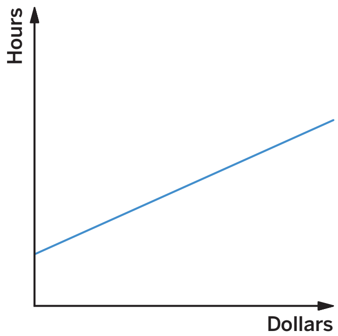
Let's graph equations in two variables.

Warm-up Which One Doesn't Belong?

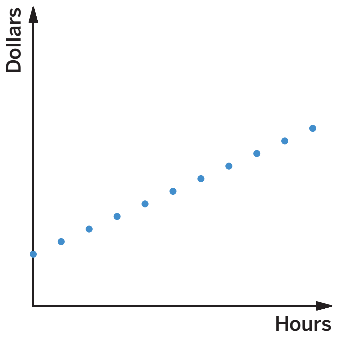
Which of the following graphs doesn't belong? Explain your thinking.

A.

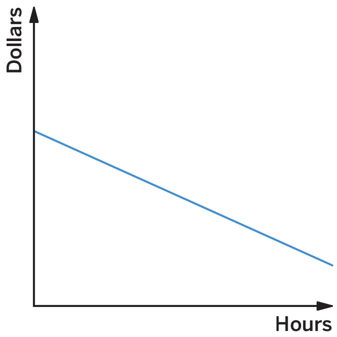
C.



B.



D.



Log in to Amplify Math to complete this lesson online.

Activity 1 Movie Night Snacks

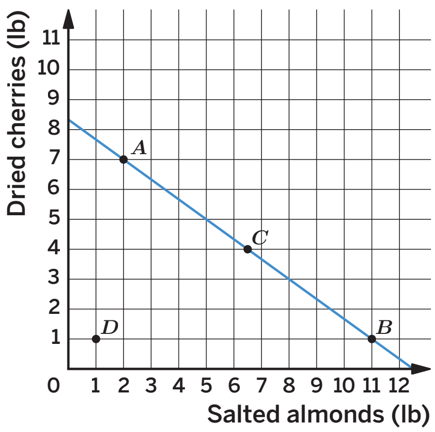
Clare invited some friends over to watch movies where she plans to serve some healthy snacks. Clare visits a wholesale store, where she can buy any quantity of reasonably priced products by the pound. She purchases some salted almonds at $6 per pound and some dried cherries at $9 per pound. She spends $75 before tax.

1. If Clare buys 2 lb of salted almonds, how many pounds of dried cherries does she buy?

2. If Clare buys 1 lb of dried cherries, how many pounds of salted almonds does she buy?

3. Write an equation that describes the relationship between the pounds of dried cherries and pounds of salted almonds Clare buys, and the total dollar amount she spends. Be sure to specify what each variable represents.

4. The graph represents the relationship between the pounds of salted almonds and dried cherries.



   a Choose a point on the line, and record its coordinates. Explain what the point represents in this context.

   b Choose a point that is *not* on the line, and record its coordinates. Explain what the point represents in this context.

Name: ______ Date: ______ Period: ______

Activity 2 Graph It!

Let's explore the graph of the equation $y = -\frac{2}{3}x + \frac{25}{3}$. You will use graphing technology in this activity.

1. Enter the equation, $y = -\frac{2}{3}x + \frac{25}{3}$.

2. Adjust your axes to view the first quadrant of the graph. Record the scales you used:

   x min: **y min:**

   x max: **y max:**

3. Use graphing technology to determine the y-intercept of the equation.

   y-intercept:

4. Use graphing technology to determine the x-intercept of the equation.

   x-intercept:

5. Navigate to the table of values that corresponds to the graph. Use the table to determine the y-coordinate that corresponds to $x = 50$.

6. Graph the equation, $y = -\frac{3}{2}x - 7$. Use graphing technology to determine the x- and y-intercept of the equation.

   x-intercept:

   y-intercept:

Activity 3 Saving and Gaming

1. Andre has $475 in a bank account (without interest). He deposits $125 of his paycheck into the account every week. Andre's goal is to save $7,000 for college.

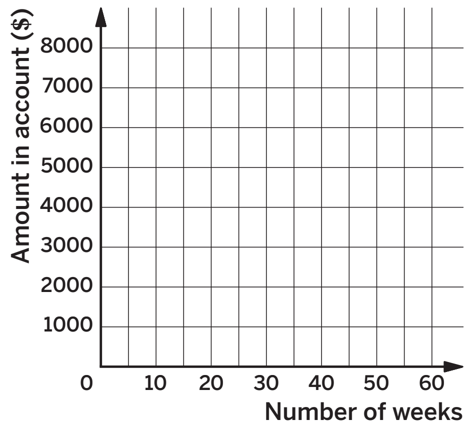
a How much will be in the account after 3 weeks?

b How long will it take until Andre has $1,350?

c Write an equation that represents the relationship between the dollar amount in Andre's account and the number of weeks since he started making deposits. Be sure to specify what each variable represents.

d Use graphing technology to graph your equation. Sketch the graph and label points on the graph that represent the amount after 3 weeks, and when he has $1,350.

e Use graphing technology to determine how long it will take Andre to reach his goal.



2. A gamer has a monthly plan that allocates 4,500 megabytes (MB) of data for gaming. She averages 200 MB of data per hour during gameplay.

a How many MB will she have left after 7 hours?

b How long will it take until she has 2,000 MB of data left?

c Write an equation that represents the relationship between the amount of data the gamer has left and the number of hours she spends gaming. Be sure to specify what each variable represents.

d Use graphing technology to graph your equation. In the space provided here, sketch the graph and label the points that represent the data left after 7 hours of gameplay, and when 2,000 MB of data are left.

e Use graphing technology to determine how long it will take before her data runs out.

STOP

Name: ______________ Date: ______________ Period: ______________

Summary

In today's lesson . . .

You graphed linear equations that modeled the constraints and the relationship between two quantities in a given scenario.

You also made connections between a given scenario, its graph, and its equation. The x- and y-coordinates of the points on the line are solutions to the corresponding linear equation, and these are the values that satisfy the constraints in the scenario.

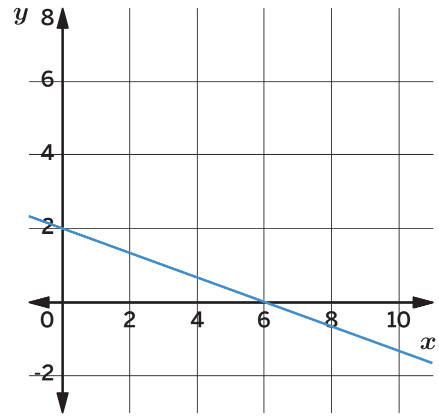
On the other hand, points that are *not* on the line are not solutions to the equation of the graph, and they represent values that do *not* satisfy those same constraints.

Reflect:

Practice

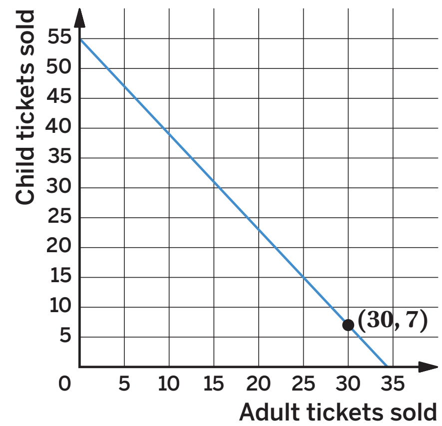
Name: .... Date: .... Period: ....

1. Refer to the graph of the equation $x + 3y = 6$. Select *all* ordered pairs that represent a solution to the equation.



A. $(0, 2)$
B. $(0, 6)$
C. $(2, 6)$
D. $(3, 1)$
E. $(4, 1)$
F. $(6, 2)$

2. A theater is selling tickets to a play. An adult ticket costs $\$8$ and a child ticket costs $\$5$. The theater collects $\$275$ after selling x adult tickets and y child tickets. What does the point $(30, 7)$ represent in this situation?



3. *Technology required*. Priya starts with $\$50$ in her bank account. She then deposits $\$20$ each week for 12 weeks (without interest).

a. Write an equation that represents the relationship between the dollar amount in her bank account and the number of weeks of depositing money. Be sure to specify what each variable represents.

b. Graph your equation using graphing technology. In the space provided here, sketch the graph and label the point on the graph that represents the amount in the bank account after 3 weeks. How much is in her account?

c. Use the graph to determine the number of weeks it takes Priya to have $\$250$ in her bank account. Label the point on the graph and write its coordinates.

Name: .... Date: .... Period: ....

Practice

4. Mai has an internship for the summer. Her internship requires her to work at least 17 hours a week, and at most 40 hours a week. Write an inequality that represents h, the hours per week Mai works as her internship.

5. A student on the cross-country team runs 30 minutes a day as part of her training. Write an equation to describe the relationship between the distance she runs in miles D and her running speed, in miles per hour (mph), when she runs:

   a. 4 mph for the entire 30 minutes.

   b. 5 mph for the first 20 minutes, and then 4 mph the last 10 minutes.

   c. 6 mph the first 15 minutes, and then 5.5 mph for the remaining 15 minutes.

   d. a mph the first 6 minutes, and then 6.5 mph for the remaining 24 minutes.

6. Select *all* of the following equations that have a solution of $x = -4$.

   A. $\frac{1}{3x} = -12$

   B. $-3x = 12$

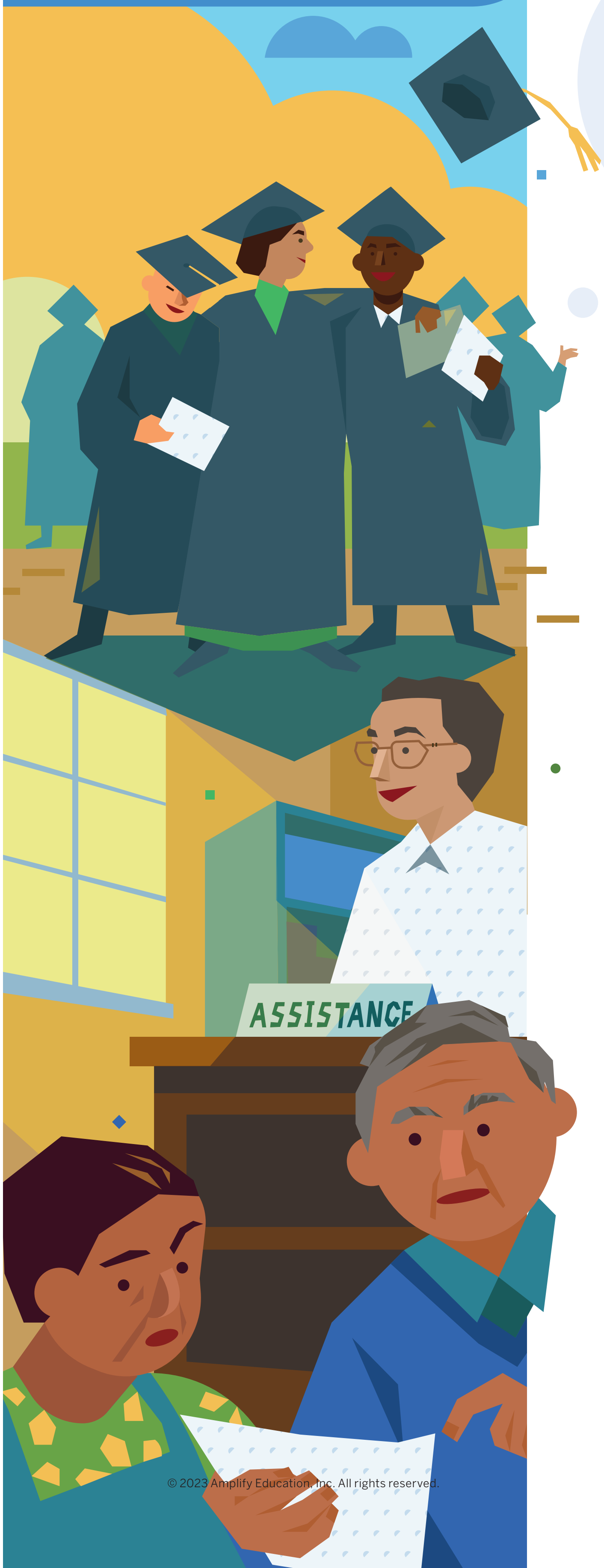
   C. $7 - x = 3$

   D. $12 - x = 16$

My Notes:

SUB-UNIT

2 Manipulating Equations and Understanding Their Structure



Narrative Connections

How do first-gen Americans vault the hurdles of college?

College can be a time for finding yourself, making new friends, trying new experiences, and developing new intellectual passions. But for the children of immigrants, applying to college can come with its own unique challenges. For many immigrant homes, these students are the first in their family to attend college. Many parents and guardians are new to things like financial aid, scholarships, tuition fees, and student housing.

But these challenges are far from the whole story. First-generation students who thrive in college often have support in their schools and communities. Guidance counselors can help navigate the application process and steer students toward financial aid and scholarships. Many colleges also have departments whose goal is to support first-generation students. These departments offer mentoring services, textbook exchange programs, and even emergency funds when students encounter sudden, unexpected costs. There are even government funded programs — such as TRIO and the College Assistance Migrant Program (CAMP) — designed to provide services, counseling, and aid to first-generation students.

Above all, students have reported that simply having someone to talk to who looked like them helps with the challenges of college life.

All college-bound students will face decisions and trade-offs around budget, access to resources, and proximity to home. Equations help you model and weigh these sorts of decisions. But to efficiently solve equations, you have to be able to work with them: manipulating them, rearranging them, and graphing them. And that's exactly what you'll be doing over the next few lessons.

Unit 1 | Lesson 7

Equivalent Equations

Let's investigate what makes equations equivalent.

Warm-up Take Two

You will be assigned Expression A or Expression B.

Expression A	Expression B
$\frac{n^2-9}{2(-1)^2}$	$(n+3)\cdot\frac{n-3}{\sqrt{25}-\sqrt{9}}$

Evaluate your expression for each of the following values.

1. $n = 5$

2. $n = 7$

3. $n = 13$

4. $n = -1$

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Activity 1 Mix and Mingle

1. The table gives information about college tuition in California and Virginia.

California	Virginia
In 2018, the least expensive in-state college tuition was $\$48,000$ less than the most expensive tuition, which was $\$55,000$.	In 2018, the average in-state college tuition was $\$20,000$ more than the least expensive tuition.

Use the table to write as many equations as possible that could represent the information about college tuition in California and Virginia. If you use a variable, specify what it represents.

a **California:**

b **Virginia:**

2. Tyler uses the information about college tuition in Virginia and writes the equation $2a - 2\ell = 40000$, where a is the average cost of tuition and ℓ is the least expensive cost of tuition. Explain why Tyler's equation is correct.

Are you ready for more?

In 2010, a parent was 1 more than 10 times the age of their child. In 2015, the parent was 16 more than 4 times the age of their child. Let x represent the age of the child in 2010, and $x + 5$ represent the age of their child in 2015. How old was the child in 2010? Show your thinking.

Activity 2 Examining Equivalent Equations

1. Which of the following equations are equivalent? Explain or show your thinking.

   Equation A: $2p + 4 = 9.60$

   Equation B: $2(p + 0.25) = 6.10$

   Equation C: $\frac{1}{2}(2p + 0.50) = 3.05$

   Equation D: $6p + 1.50 = 18.30$

2. Which of the following equations are equivalent? Explain or show your thinking.

   Equation A: $y = 2x + 3$

   Equation B: $-4x + 2y = 6$

   Equation C: $x = \frac{y}{2} + 3$

3. Consider the following equations.

$m + m = N$	$N + N = p$	$m + p = Q$	$p + Q = ?$

Using these equations, determine which of the following expressions are equivalent to the expression $p + Q$. Explain or show your thinking.

A. $2p + m$

B. $4m + N$

C. $3N$

D. $9m$

Reflect: How did following the rules of the activity help make your experience successful?

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Activity 3 Jigsaw: Buying College Textbooks

Noah needs to purchase a textbook for his biology class. The bookstore sells his textbook for $275, but Noah thinks he can get a better deal online. An online textbook store sells the book for just $56.70, a price that includes $2.70 in sales tax and a coupon for 10% off.

Noah's purchase is modeled by the equation $x - 0.1x + 2.70 = 56.70$.

1. What does the solution to the equation represent in this scenario?

2. Explain why 70 is not a solution to the equation and 60 is the solution.

3. Consider different equations in the Jigsaw. Your group will be assigned one card. For each equation:

- Determine either the operation(s) performed or how the equation could be interpreted in terms of the original scenario.
- Determine if the equation has the same solution as Noah's original equation.

4. You will be assigned to a new group, where each group member has a different card. For each equation:

- Discuss the operation(s) performed and how the equation could be interpreted in terms of the original scenario.
- Discuss whether the new equations are equivalent to Noah's original equation.
- Explain or show your thinking.

STOP

Summary

In today's lesson . . .

You wrote, interpreted, and analyzed ***equivalent equations***, which are equations that have the same solutions. You saw that *equivalent equations* can describe the same scenario in different ways.

There are certain steps that can be taken to rewrite equations as *equivalent equations.* These steps include:

- Applying the *Distributive Property.*
- *Addition Property of Equality:* Adding the same value to both sides of the equation.
- *Subtraction Property of Equality:* Subtracting the same value to both sides of the equation.
- *Multiplication Property of Equality:* Multiplying the same value to both sides of the equation.
- *Division Property of Equality:* Dividing the same value to both sides of the equation.

You can determine if equations are equivalent by checking if they have the same solution, or if they are the same after performing any mathematically correct steps.

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. Which equation is equivalent to $6x + 9 = 12$?

A. $x + 9 = 6$

B. $2x + 3 = 4$

C. $3x + 9 = 6$

D. $6x + 12 = 9$

2. Select *all* equations that have the same solution as $3x - 12 = 24$.

A. $15x - 60 = 120$

B. $3x = 12$

C. $3x = 36$

D. $x - 4 = 8$

E. $12x - 12 = 24$

3. Which equation is equivalent to $0.05n + 0.1d = 3.65$?

A. $5n + d = 365$

B. $0.5n + d = 365$

C. $5n + 10d = 365$

D. $0.05d + 0.1n = 365$

4. Kiran collects dimes and quarters in his coin jar. He has collected \$10 so far. The relationship between the number of dimes d and quarters q, and the amount of money in dollars, is represented by the equation $0.1d + 0.25q = 10$. Select *all* the values of (d, q) that could be solutions to the equation.

A. $(100, 0)$

B. $(20, 50)$

C. $(50, 20)$

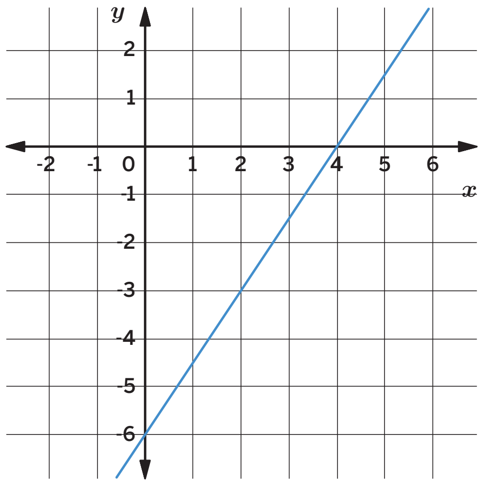
D. $(0, 100)$

E. $(10, 36)$

Practice

Name: ______ Date: ______ Period: ______

5. Consider the graph of the equation $3x - 2y = 12$. Select *all* the coordinates of points that represent a solution to the equation.



A. $(2, 3)$

B. $(4, 0)$

C. $(5, -1)$

D. $(0, -6)$

E. $(2, -3)$

6. Select *all* of the following expressions that are equivalent to $4(x-3) + 2x$.

A. $4x - 12 + 2x$

B. $6x - 3$

C. x

D. $6x - 12$

Name: .... Date: .... Period: ....

Unit 1 | Lesson 8

Explaining Steps for Rewriting Equations

Let's investigate why some steps for rewriting equations work but other steps do not.

Warm-up Matching Properties

Match each equation with the property it demonstrates. Note that not all the properties will be used.

Equations

a. $(2 \cdot 4x) \cdot 6 = 2 \cdot (4x \cdot 6)$

b. $3(x + 2) = 3x + 6$

c. $4 - x = -x + 4$

d. $(8x + 4x) + 6x = 8x + (4x + 6x)$

Properties

.... Distributive Property

.... Commutative Property of Addition

.... Commutative Property of Multiplication

.... Associative Property of Addition

.... Associative Property of Multiplication

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Activity 1 Could It Be Zero?

For each equation, determine whether this statement is *true* or *false*: The solution to the equation is 0.

- **If true, explain the strategy you used to determine that the solution was 0.**
- **If false, explain the strategy you used to determine that the solution was *not* 0. Then determine the solution to the equation.**

1. $12 - 8x = 3(x + 4)$

2. $4(x + 2) = 10$

3. $5x = \frac{1}{2}x$

4. $\frac{6}{x} + 1 = 8$

Are you ready for more?

1. 100 cannot be divided by 0 because dividing by 0 is undefined.

a Divide 100 by 10, then by 1, then 0.1, then 0.01. Describe what happens when you divide by smaller numbers.

b Divide -100 by 10, by 1, by 0.1, and by 0.01. Compare your solutions to part a.

2. The tape diagram illustrates $6 \div 2 = 3$.

2	2	2

a Draw a tape diagram that illustrates $6 \div \frac{1}{2} = 12$.

b Is it possible to draw a tape diagram that illustrates $6 \div 0$? Explain your thinking.

Name: ______ Date: ______ Period: ______

Activity 2 Take Turns

You will be given a set of shaded cards and a set of unshaded cards. Each card contains a pair of equations. You will take turns as either the speaker or the listener.

When you are the speaker:	When you are the listener:
1. Choose a card from the shaded deck. Without solving each equation, explain to your partner why the equations are equivalent. **2.** Choose a card from the unshaded deck. Without solving each equation, explain to your partner why the equations are *not* equivalent.	**1.** Actively listen to your partner. **2.** After the speaker shares, use these sentence stems to help clarify your partner's thinking: • Can you restate . . .? • Why do you think . . .? • How do you know . . .? • How did you get . . .? **3.** Discuss your thinking. If you disagree, ask your partner to support their case and listen. Continue to discuss until you reach an agreement.

After you have discussed one shaded card and one unshaded card, switch roles.

Activity 3 It Doesn't Work!

Bard is having trouble solving the two following equations. In each case, Bard thought certain steps were acceptable and took those steps, but ended up with false statements.

Analyze Bard's work and the operations performed.

Equation 1:

a Did Bard perform acceptable operations?

b Why do you think the last statement is a false equation?

Equation 1	
$x + 6 = 4x + 1 - 3x$	Original equation
$x + 6 = 4x - 3x + 1$	Apply the commutative property.
$x + 6 = x + 1$	Combine like terms.
$6 = 1$	Subtract x from each side.

Equation 2:

a Did Bard perform acceptable operations?

b Why do you think the last statement is a false equation?

Equation 2	
$2(5 + x) - 1 = 3x + 9$	Original equation
$10 + 2x - 1 = 3x + 9$	Apply the Distributive Property.
$2x - 1 = 3x - 1$	Subtract 10 from each side.
$2x = 3x$	Add 1 to each side.
$2 = 3$	Divide each side by x.

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You examined acceptable steps for rewriting *equivalent equations*. These steps include:

- Adding or subtracting the same value to both sides of the equation.
- Multiplying the same value — but not 0 — on both sides of the equation.
- Dividing by the same value — but not 0 — on both sides of the equation.
- Applying the Distributive Property.
- Rewriting expressions using the commutative or associative properties.

You also reviewed the properties of operations and recalled that dividing by zero is undefined. You saw that dividing by a variable can lead to a false statement and is not valid when that variable equals zero. You also investigated an equation with no solution, meaning no value of x could make the equation true.

Reflect:

Practice

Name: ____________ Date: ____________ Period: ____________

1. Match each of the three equations in Set A with an equivalent equation in Set B. Note that not all of the answer choices in Set B will be used.

Set A

a. $3x + 6 = 4x + 7$

b. $3(x + 6) = 4x + 7$

c. $4x + 3x = 7 - 6$

Set B

_______ $9x = 4x + 7$

_______ $3x + 18 = 4x + 7$

_______ $3x = 4x + 7$

_______ $3x - 1 = 4x$

_______ $7x = 1$

2. Equations A and B have the same solution. Select the statement that explains why this is true.

Equation A: $-3(x + 7) = 24$

Equation B: $x + 7 = -8$

A. Adding 3 to both sides of Equation A results in $x + 7 = -8$.

B. Dividing both sides of Equation A by -3 results in $x + 7 = -8$.

C. Subtracting 3 from both sides of Equation A results in $x + 7 = -8$.

D. Applying the Distributive Property to Equation A results in $x + 7 = -8$.

3. Is 0 a solution to the equation $2x + 10 = 4x + 10$? Explain or show your thinking.

4. The on-campus entrepreneurship club wants to order potted plants for all 36 of its sponsors. The first store it called charges $\$8.50$ for each plant plus a delivery fee of $\$20$. The equation $320 = x + 7.50(36)$ represents the cost of ordering potted plants from the second store it called. What does the x likely represent in this scenario?

A. The delivery fee at the second store.

B. The cost for each potted plant at the second store.

C. The number of sponsors of the entrepreneurship club.

D. The total cost of ordering potted plants at the second store.

5. The on-campus environmental science club is printing t-shirts for its 15 members. The printing company charges a certain amount for each shirt plus a setup fee of $\$12$. If the t-shirt order costs a total of $\$154.50$, how much does the company charge for each t-shirt?

6. Select *all* the expressions that are equivalent to the expression $\frac{-8x-6}{2}$.

A. $4x + 3$

B. $-4x - 3$

C. $-\frac{8x+6}{2}$

D. $\frac{1}{2}(-4x - 3)$

E. $-\frac{1}{2}(8x + 6)$

Unit 1 | Lesson 9

Rearranging Equations (Part 1)

Let's rearrange equations to determine certain quantities.

Warm-up Which Equation?

At many colleges, students who live on campus are required to purchase a meal plan. Meal plans require students to prepay for their meals at the beginning of the semester, at a discounted rate. Students who live off campus can purchase meal plans on a weekly or monthly basis.

The Collegiate University meal plan has a fee for a set number of meals each week. There are also "half-meals," which include snacks between dining hours and after-hour meals.

1. The table shows the relationship between the number of meals m allowed each week and the cost c, in dollars. Complete the table.

Number of meals, m	Cost (\$), c
1	10
5	50
7	70
$\frac{21}{2}$	
15.5	
	210

2. Which equations could represent the relationship between m and c? Be prepared to explain your thinking.

   A. $m = 10c$ **B.** $m = \frac{c}{10}$ **C.** $c = \frac{m}{10}$ **D.** $c = 10m$ **E.** $\frac{m}{c} = 10$

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 College Housing Decisions

If you attend college, your living arrangements may be one of the first "adult" decisions you will make. Some options include:

- **Living at home and commuting.**
- **Renting an off-campus apartment or house alone.**
- **Living on-campus.**
- **Renting an off-campus apartment or house with roommates.**

Each option has its own advantages and disadvantages. Priya decides to live off campus and rent a house. The rent is $1,700 per month, with utilities included. Priya cannot afford to rent this house alone, but is unsure how many roommates she may need to afford the rent.

1. For each number of people, determine how much Priya would pay for rent each month. Explain or show your thinking.

   a. 2 total people

   b. 3 total people

   c. 7 total people

2. Write an equation to determine the cost per person c, if a total of p people live in the house.

3. Determine the number of people living in the house if each person pays the following monthly amount. Explain or show your thinking.

   a. $340

   b. $212.50

   c. $154.54

4. Write an equation to determine p, the total number of people living in the house that each pay a monthly amount c.

5. If Priya wants to pay at most $500 each month, how many roommates will she need? Explain your thinking.

Activity 2 Out of Gas

Priya and her roommates take a day trip to the beach using two cars. The group in Car A notice their gas tank has 2 gallons of gas remaining, so they stop at a gas station. The gas pump fills the tank at a constant rate of 8 gallons per minute.

1. How many gallons of gas remain in the tank after each number of minutes?

   a. 0.5 minutes
   b. 1.5 minutes
   c. 2 minutes

2. Write an equation to determine g, the number of gallons of gas in the tank, after m minutes.

3. Given the number of gallons of gas in Car A's tank, determine the number of minutes that have passed while waiting at the pump.

   a. 10 gallons
   b. 12 gallons
   c. 14 gallons

4. Write an equation to determine the number of minutes m that have passed while waiting at the pump if Car A's gas tank contained g gallons of gas.

The group in Car B fill its 18-gallon gas tank along the highway. The car then uses gas at a rate of 0.05 gallons per minute.

5. How many gallons of gas remain in the tank after each number of minutes?

   a. 1 minute
   b. 10 minutes
   c. 100 minutes

6. Write an equation to determine v, the number of gallons of gas in the tank after t minutes.

7. Given the number of gallons of gas remaining in Car B's tank, determine the number of minutes t that have passed since filling the tank along the highway.

   a. 16 gallons
   b. 9 gallons
   c. 4.5 gallons

8. Write an equation to determine t, the number of minutes that have passed since filling the tank, if there are v gallons of gas remaining in the tank.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

Rearranging an equation to isolate one variable is called solving for a variable. You wrote and rearranged equations to solve for different variables, depending on which quantity you wanted to determine.

You also saw that relationships between quantities can be described in more than one way. Some ways are more helpful than others, depending on what you want to determine.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. A basketball coach purchases bananas for $\$0.59$ each and a 5-gallon cooler of Electro-ade for $\$12.99$ for her team.

a Write an equation that would efficiently determine the total cost C in dollars, without tax, for the purchase of b bananas and Electro-ade.

b If the coach spent $\$27.74$, without tax, how many bananas did she purchase?

c Write an equation that would efficiently determine b, the number of bananas she purchased, if the total cost of the bananas and Electro-ade, without tax, is C.

2. A chef purchased $\$17.01$ worth of ribs and chicken. Ribs cost $\$1.89$ per pound and chicken costs $\$0.90$ per pound. The equation $0.9c + 1.89r = 17.01$ represents the relationship between the quantities in this scenario.

a Show that the equation $c = 18.9 - 2.1r$ is equivalent to $0.9c + 1.89r = 17.01$. Then explain when it might be helpful to write the equation in this form.

b Show that the equation $r = -\frac{10}{21}c + 9$ is equivalent to $0.9c + 1.89r = 17.01$. Then explain when it might be helpful to write the equation in this form.

Practice

Name: ______ Date: ______ Period: ______

3. Consider the linear equation $2x + 4y - 31 = 123$.

a Solve the equation for x. Show your thinking.

b Solve the equation for y. Show your thinking.

4. Bananas cost $\$0.50$ each and apples cost $\$1.00$ each. Select *all* combinations of bananas and apples that Elena could buy for exactly $\$3.50$.

A. 1 banana and 2 apples

B. 5 bananas and 1 apple

C. 1 banana and 3 apples

D. 2 bananas and 2 apples

E. 3 bananas and 2 apples

F. 5 bananas and 2 apples

5. Write an inequality to represent each statement. Be sure to specify what each variable represents.

a Lin has at least $\$100$ in her savings account.

b Mai has more than 5 pencils in her backpack.

c Andre can work at most 20 hours at his new job.

6. The perimeter of Priya's bedroom is 42 ft. The relationship between the length ℓ, the width w, and the perimeter of the rectangle can be described by the equation $2\ell + 2w = 42$.

a Determine the length of her room if the width is 10 ft.

b Determine the length of her room if the width is 8.4 ft.

c Write an equation to determine the length ℓ of her room if the width is w ft.

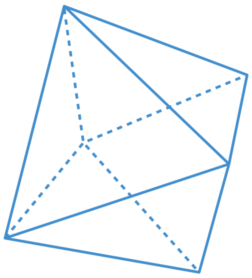
Rearranging Equations (Part 2)

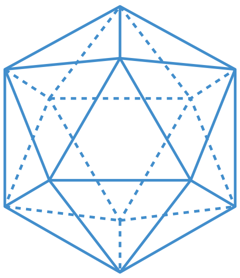
Let's rearrange equations to solve for one of its variables.

Warm-up Faces, Vertices, and Edges

In Lesson 2, you saw Euler's polyhedral equation, $V + F - 2 = E$, which relates the number of vertices, faces, and edges in Platonic solids.

1. Write an equation that would efficiently determine the number of vertices in each of the Platonic solids.

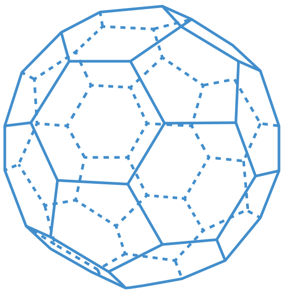




a. An octahedron, which has 8 faces.

b. An icosahedron, which has 30 edges.

2. A Buckminsterfullerene (also called a "Buckyball") is a polyhedron with 60 vertices. It is not a Platonic solid, but the equation $V + F - 2 = E$ still applies. Write an equation that would efficiently determine the number of faces a Buckyball has if the number of edges is known.



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Activity 1 Budgeting Goals

Kiran decides to opt out of his college's meal plan option and budget his own money for meals. He has a total of $350 to spend on restaurants and groceries each month. On average, Kiran spends $55 each time he visits a grocery store and $12 each time he eats at a restaurant.

1. Write an equation to represent how much money, in dollars, Kiran spends on food each month if he visits the grocery store g times and eats at r restaurants.

Kiran often runs out of money before the end of each month, so he decides to plan ahead.

2. For the next month, determine how many times Kiran can eat at a restaurant if he visits the grocery store the following numbers of times.

   a. 3 grocery visits b. 5 grocery visits c. g grocery visits

3. Kiran is deciding how many times he would like to visit a restaurant next month. Each month has approximately 4 weeks.

   a. Write an equation that Kiran could use to efficiently determine the number of grocery visits he can make next month if the number of restaurant visits is known.

   b. Determine the number of grocery visits Kiran can make in 4 weeks, if he eats at a restaurant 5 times each week. Explain or show your thinking.

   c. Determine the number of grocery visits Kiran can make in 4 weeks, if he eats at a restaurant for lunch and dinner 3 times each week. Explain or show your thinking.

4. What would you do in Kiran's situation? Do you care more about eating at restaurants or using your money for something else?

Activity 2 Budgeting Woes

Despite Kiran's best budgeting efforts, an emergency changed his spending plan. He will receive his next paycheck in three days, and currently has $18.25 left for food. Kiran decides to spend this money on boxes of macaroni and cheese for $1.15 each, including tax, and frozen veggie pizzas for $2.75 each, including tax.

1. Let m represent the number of boxes of macaroni and cheese and p represent the number of the frozen veggie pizzas.
   a. Write an equation that represents the relationship between the number of boxes of macaroni and cheese and the number of frozen veggie pizzas Kiran can purchase if he spends the rest of his money.
   b. Solve your equation for p. Explain what the solution represents in this scenario.
   c. Solve your equation for m. Explain what the solution represents in this scenario.

2. Kiran wants to use his entire budget. He first buys 9 boxes of macaroni and cheese.
   a. Explain which equation would be most efficient to determine how many frozen veggie pizzas he could purchase.
   b. Determine how many frozen veggie pizzas Kiran could purchase if he buys 9 boxes of macaroni and cheese.

Are you ready for more?

Kiran decides he also wants to purchase cartons of eggs for $2.10 per carton, including tax.

1. Write an equation to determine the number of cartons of eggs c that can be purchased if the number of boxes of macaroni and cheese m and frozen veggie pizzas p is known.
2. Determine the number of cartons of eggs Kiran can purchase if he already bought 5 boxes of macaroni and cheese and 3 frozen veggie pizzas.

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Activity 3 Spreadsheets, Streets, and Staffing

Collegiate University's Department of Campus Planning and Facilities has a budget of $\$1{,}962{,}800$ for resurfacing roads and hiring additional workers this year.

It costs approximately $\$84{,}000$ to resurface each mile of a two-lane road. The average starting salary of a worker in the department is $\$36{,}000$ per year.

1. Write an equation that represents the relationship between the miles m of two-lane roads the department could resurface and the number of new workers w it could hire, if the department spends the entire budget.

2. The department wants to determine the number of new workers it could hire if the number of miles of resurfaced roads is known and the entire budget is used.

a Write an equation to determine the number of workers that could be hired.

b Use spreadsheet technology to determine the number of new workers that could be hired if 10 miles are resurfaced.

- In a blank spreadsheet, label the cells **A1** and **B1** with "miles" and "workers."
- In cell **A2**, enter the value for the number of miles, 10.
- In cell **B2**, enter your equation, starting with "=" and replacing the variable m with **A2**. Remember to use parentheses around the entire numerator.

How many workers could be hired if 10 miles are resurfaced?

c Is it possible for the department to resurface 20 miles and hire 8 workers? Explain your thinking.

Activity 3 Spreadsheets, Streets, and Staffing (continued)

3. The department wants to determine the number of miles that could be resurfaced if the number of new workers is known and the entire budget is used.

   a. Write an equation to determine how many miles could be resurfaced.

   b. Use spreadsheet technology, to determine the number of miles that could be resurfaced if 15 new workers were hired.

      - Label the cells **D1** and **E1** with "workers" and "miles."
      - In cell **D2**, enter the value for the number of new workers, 15.
      - In cell **E2**, enter your equation, starting with "=" and replacing the variable w with **D2**.

      How many miles could be resurfaced if 15 new workers are hired?

   c. Is it possible for the department to hire 10 workers and resurface 20 miles? Explain your thinking.

4. The department wants to resurface as many miles as possible, but also wants to hire at least 12 workers.

   a. Which equation would you use to make sense of this situation? Explain your thinking.

   b. How many new workers do you suggest the department hire? How many miles do you suggest it resurface? Explain your thinking.

STOP

Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that solving for a variable is an efficient way to determine the values that meet the constraints in a scenario. Solving for a variable — before substituting any known values — makes it more efficient to test different values of one variable, seeing how they affect the other variable. It can save you the trouble of doing the same calculation over and over!

As an example, here is how you could solve the equation $60x + 150y = 3000$ for both x and y:

Solve the equation for x.

$$60x + 150y = 3000$$
$$60x = 3000 - 150y$$
$$x = \frac{3000 - 150y}{60}$$
$$x = 50 - 2.5y$$

Solve the equation for y.

$$60x + 150y = 3000$$
$$150y = 3000 - 60x$$
$$y = \frac{3000 - 60x}{150}$$
$$y = 20 - 0.4x$$

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A car has a 16-gallon fuel tank. When driven on a highway, it has a gas mileage of 30 miles per gallon. The gas mileage is the number of miles the car can travel using 1 gallon of gasoline. After the gas tank has been filled, the car is driven on the highway for a while.

a. How many miles has the car traveled if it has the following amounts of gas *left* in the tank?

15 gallons   10 gallons   2.5 gallons

b. Write an equation that represents the relationship between the distance d the car has traveled in miles and the amount of gas left in the tank x, in gallons.

c. How many gallons are *left* in the tank when the car has traveled the following distances on the highway?

90 miles   246 miles

d. Write an equation that could be used to determine the amount of gallons of gas left in the tank x if you know the car has traveled d miles.

2. Diego helps collect the entry fees at his school's basketball game. Student entry costs \$2.75 and adult entry costs \$5.25. At the end of the game, Diego collected \$281.25. Select *all* equations that could represent the relationship between the number of students s, the number of adults a, and the amount of money received at the game.

A. $a = 53.57 - 0.52s$

B. $281.25 - 5.25a = 2.75s$

C. $281.25 - 5.25s = 2.75a$

D. $281.25 - 5.25\text{s} = a$

E. $281.25 + 2.75a = s$

F. $281.25 + 5.25s = a$

Practice

Name: ______ Date: ______ Period: ______

3. An equation to calculate the volume of a cylinder V is $V = \pi r^2 h$ where r represents the cylinder's radius and h represents its height. Which could be used to efficiently calculate the height of the cylinder?

A. $r^2 h = \frac{V}{\pi}$

B. $h = V - \pi r^2$

C. $h = \frac{V}{\pi r^2}$

D. $\pi h = V r^2$

4. A catering company is setting up for a wedding. It expects 150 people to attend. It can set up small tables that seat 6 people and large tables that seat 10 people.

a. Determine a combination of small and large tables that seat exactly 150 people.

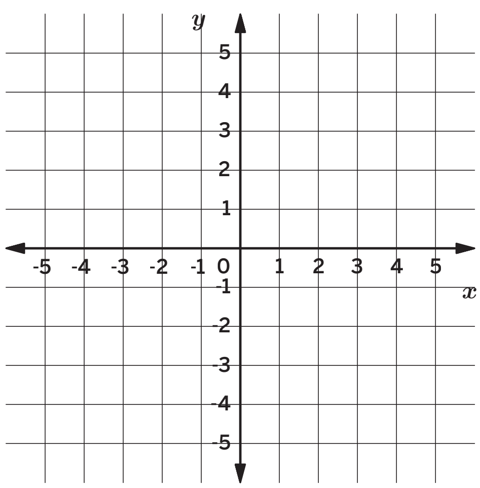
b. Let x represent the number of small tables and y represent the number of large tables. Write an equation to represent the relationship between x and y.

c. Explain what the coordinate point $(20, 5)$ represents in this scenario.

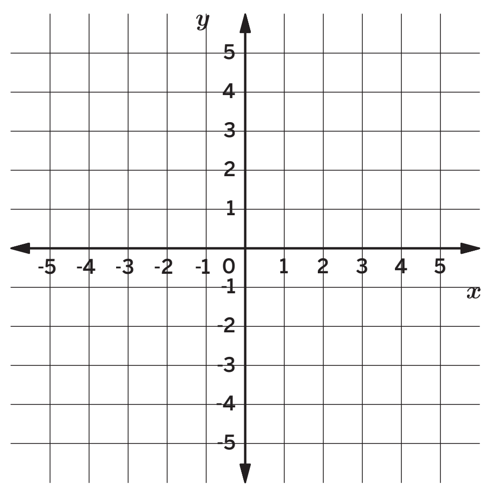
d. Is the coordinate point $(20, 5)$ a solution to the equation you wrote? Explain your thinking.

5. Graph each equation on the coordinate plane.

a. $y = 3x + 1$



b. $2x - 4y = 8$



Unit 1 | Lesson 11

Connecting Equations in Standard Form to Their Graphs

Let's investigate what graphs can tell us about the equations and relationships they represent.

Warm-up Algebra Talk

Collegiate University hosts its annual Spring Carnival. Jada has \$20 to spend on games and bottles of water. Let x represent the number of games she plays and y represent the number of bottles of water she purchases.

Each of the following equations is presented in standard form, $ax + by = c$. How would you interpret each equation? Discuss your responses with a partner.

1. What does the equation $4x + 2y = 20$ represent in this scenario? Explain your thinking.

2. What does the equation $2x + y = 20$ represent in this scenario? Explain your thinking.

3. What does the equation $x + y = 20$ represent in this scenario? Explain your thinking.

Compare and Connect: Study the three equations. What connections do you see among the coefficients and constants and what they represent in this scenario?

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Jigsaw: Games and Bottles of Water

Each equation represents a relationship between the number of games x, the number of bottles of water y, and the dollar amount a student spends on each at their school's Orientation Carnival.

1. Interpret each equation.

Equation 1: $x + y = 20$
Interpretation:

Equation 2: $2.50x + y = 15$
Interpretation:

Equation 3: $x + 4y = 28$
Interpretation:

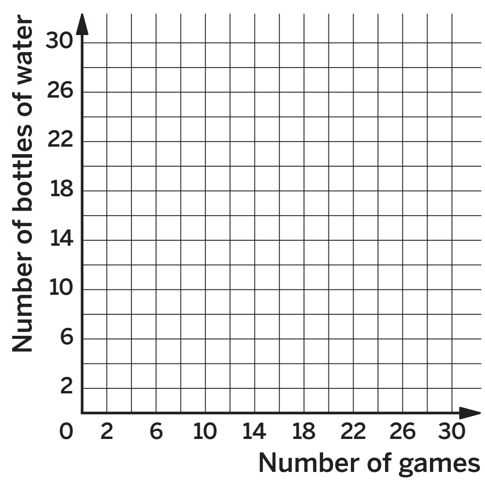
Your group will be assigned one of the three equations.

- Complete each problem using your assigned equation.
- Be prepared to explain your thinking and what you notice.

Then you will be assigned to a new group where each member has a different equation.

- Share both forms of your equation.
- Discuss the connections you noticed between your graph and your equations.

2. Determine the number of bottles of water the student can purchase if they do not play any games. Then on the coordinate plane, mark the point that represents this scenario and label its coordinates.



3. Determine the number of games the student can play if they do not purchase any bottles of water. Then on the coordinate plane, mark the point that represents this scenario and label its coordinates.

4. Draw a line to connect the two points.

Activity 1 Jigsaw: Games and Bottles of Water (continued)

5. Complete the statements.

a If the student played no games, they can purchase .... bottles of water.

b For every additional game that they play x, the possible number of bottles of water y, ... (increases or decreases) by ....

6. Study the graph.

a Determine the slope of the graph.

b Determine the coordinates of the y-intercept.

7. Solve the equation for y.

8. Consider the original equation, the equation solved for y in Problem 7, and the graph.

a What connections can you make between the equation solved for y and the graph?

b What connections can you make between the original equation and the graph?

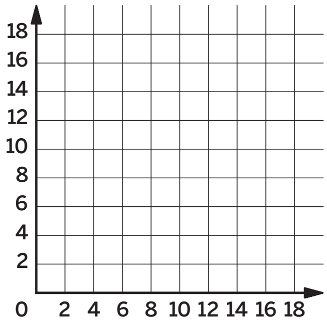
Name: .... Date: .... Period: ....

Activity 2 Nickels and Dimes

Collegiate University's Winter Festival offers discounted snacks for nickels and dimes. Andre has 85 cents in his coin jar, which contains only nickels and dimes.

1. Write an equation that relates the number of nickels n, the number of dimes d, and the amount of money, in cents, in Andre's coin jar.

2. Graph your equation on the coordinate plane. Label the axes.



3. Determine the number of nickels in the coin jar if there are no dimes. Explain your thinking.

4. Determine the number of dimes in the coin jar if there are no nickels. Explain your thinking.

Are you ready for more?

Determine all the different ways the coin jar could have 85 cents, if it also contains quarters.

STOP

Summary

In today's lesson . . .

You recalled that linear equations can be written in different forms. Each form allows you to see the relationship between quantities or to predict the graph of the equation.

When you consider equations written in standard form, $ax + by = c$, where x and y are variables and a, b, and c are constants, you can efficiently determine:

- The x-intercept — when the value of y is 0.
- The y-intercept — when the value of x is 0.

Another strategy to determine more information about the graph of the standard form equation $ax + by = c$ is to solve the equation for y. When you write the resulting equation in slope-intercept form, $y = mx + b$, where x and y are variables and m and b are constants, you can efficiently determine:

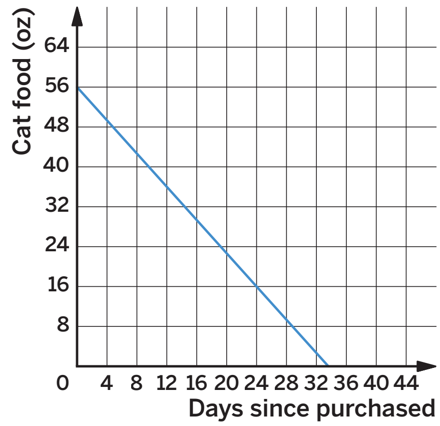
- The slope m of the graph.
- The y-intercept b, where the graph intersects the y-axis.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. Andre purchased a new bag of cat food. The next day, he opened it to feed his cat. The graph illustrates how many ounces were left in the bag on the days after it was purchased.



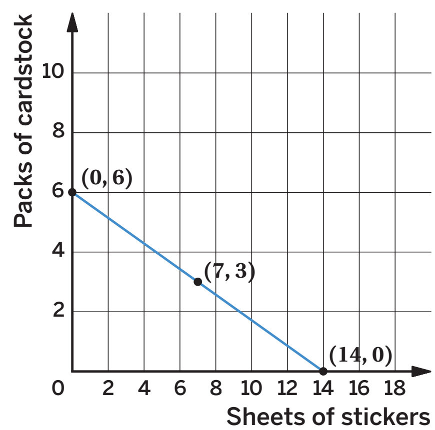
a Determine how many ounces of cat food were in the bag 12 days after Andre purchased it.

b After how many days did the bag contain 16 oz of cat food?

c Determine the weight of the bag before it was opened.

d Determine how many days it took for the bag to be emptied.

2. A kindergarten teacher bought $21 worth of stickers and cardstock for his class. The stickers cost $1.50 per sheet and the cardstock cost $3.50 per pack. The equation $1.5s + 3.5c = 21$ represents the relationship between sheets of stickers s, packs of cardstock c, and the dollar amount spent on supplies.



a Explain why the graph represents the equation $1.5s + 3.5c = 21$.

b Explain what the vertical and horizontal intercepts represent in this scenario.

Practice

Name: .... Date: .... Period: ....

3. A Little League Baseball team is ordering hats. The graph illustrates the relationship between the total cost, in dollars, and the number of hats ordered. What does the slope of the graph represent in this scenario?

A. The slope represents the fixed cost of approximately $\$35$ for ordering hats.

B. The slope represents the amount that the total cost increases for each additional hat ordered.

C. The slope represents that when 9 hats are ordered, the total cost is approximately $\$160$.

D. The slope represents that when the number of hats ordered increases by 10, the total cost increases by approximately $\$175$.

4. A soccer team needs to raise $\$460$ for uniforms and travel expenses. It decides to hold a car wash in a part of town with a lot of car and truck traffic. The team spends $\$90$ on sponges and soap. It plans to charge $\$10$ per car and $\$20$ per truck. Determine the number of cars volunteers have to wash if they washed the following number of trucks.

a. 4 trucks

b. 15 trucks

c. 27 trucks

d. t trucks

5. For each equation, identify the slope and the coordinates of the y-intercept of its graph.

a. $y = 2x - 7$

Slope:

y-intercept:

b. $y + 3 = 6x$

Slope:

y-intercept:

c. $y = \frac{x}{4} + 2$

Slope:

y-intercept:

Name: .... Date: .... Period: ....

Unit 1 | Lesson 12

Connecting Equations in Slope-Intercept Form to Their Graphs

Let's analyze different forms of linear equations and how the forms relate to their graphs.

Warm-up Which One Doesn't Belong?

Which of the following equations doesn't belong? Circle your choice and explain your thinking.

A. $-6x + 2y = 4$

B. $y = -2x + 1$

C. $y = \frac{1}{2}x + \frac{5}{2}$

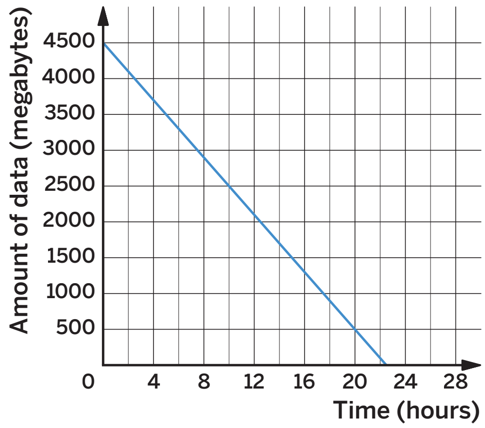
D. $y = 3x - 5$

Log in to Amplify Math to complete this lesson online.

Activity 1 Graphs of Two Equations

The two graphs represent scenarios you examined in Lesson 6.

1. The graph represents $a = 4500 - 200h$, the relationship between a gamer's amount of data used in megabytes a and hours spent gaming h.



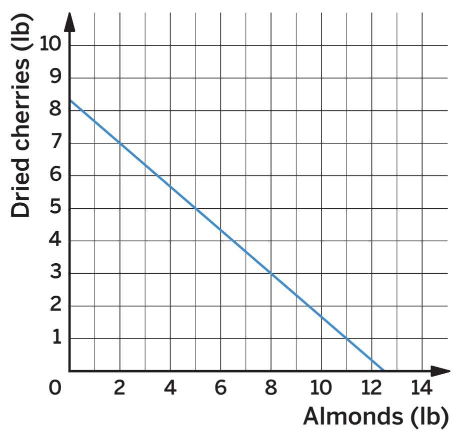
a Describe where the $4{,}500$ is visible on the graph.

b Explain what the $4{,}500$ represents in this scenario.

c Describe where the -200 is visible on the graph.

d Explain what the -200 represents in this scenario.

2. The graph represents $6x + 9y = 75$, the relationship between the pounds of almonds and dried cherries, and the dollar amount Clare spent in total for her movie night. Tyler states, "This graph does not represent the equation $6x + 9y = 75$ because the 6, 9, and 75 are not visible on the graph." Explain how you could show Tyler that the graph does, in fact, represent the equation.



Name: ______ Date: ______ Period: ______

Activity 2 Matching Equation Terms

Plan ahead: How can you use organization skills to help keep you focused on the activity?

1. Match each equation with the corresponding slope m and y-intercept of its graph. Not all options will be matched.

a. $-4x + 3y = 3$

b. $12x - 4y = 8$

c. $8x + 2y = 16$

d. $-x + \frac{1}{3}y = \frac{1}{3}$

e. $-4x + 3y = 24$

______ $m = 3$, y-intercept: $(0, 1)$

______ $m = \frac{4}{3}$, y-intercept: $(0, 1)$

______ $m = \frac{4}{3}$, y-intercept: $(0, 8)$

______ $m = -4$, y-intercept: $(0, 8)$

______ $m = -4$, y-intercept: $(0, -2)$

______ $m = 3$, y-intercept: $(0, -2)$

2. Use the unmatched slope and y-intercept to complete the following.

a. Write the equation of the line in slope-intercept form.

b. Write an equation of the line in standard form.

Are you ready for more?

Consider the equation $ax + by = c$.

1. What are the coordinates of the x-intercept in terms of a, b, and c?

2. What are the coordinates of the y-intercept in terms of a, b, and c?

Activity 3 Info Gap: Forms of Equations

You will be given either a problem card or a data card. Do not show or read your card to your partner.

If you are given the *data card*:	If you are given the *problem card*:
1. Silently read the information on your card.	**1.** Silently read your card and think about what information you need to solve the problem.
2. Ask your partner, "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card.	**2.** Ask your partner for the specific information that you need.
3. Before telling your partner the information, ask, "Why do you need to know (that piece of information)?"	**3.** Explain to your partner how you are using the information to solve the problem.
4. Read the problem card, and solve the problem independently.	**4.** Share the problem card with your partner, and solve the problem independently.
5. Share the data card, and discuss your thinking.	**5.** Read the data card, and discuss your thinking.

STOP

Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You took equations in standard form and rearranged them so they were in slope-intercept form. By solving for y, you can more efficiently determine the slope and y-intercept (vertical intercept) of their graphs.

You also observed patterns when manipulating the equation $ax + by = c$. For example, when a, b, and c are all positive, its graph slants downward from left to right.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. What is the slope of the graph of $5x - 2y = 20$?

A. $m = 5$

B. $m = \frac{5}{2}$

C. $m = -10$

D. $m = -\frac{2}{5}$

2. Determine the y-intercept of each of the following equations.

a. $y = 6x + 2$

b. $10x + 5y = 30$

c. $y - 6 = 2(3x - 4)$

3. Han incorrectly determines the x-intercept and y-intercept of the equation $10x + 4y = 20$. Consider his work shown. Han concludes the x-intercept is $\left(\frac{1}{2}, 0\right)$ and the y-intercept is $(0, 5)$.

Han's Work:

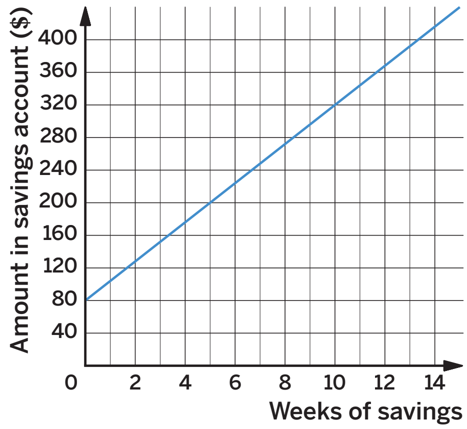
$$10x + 4y = 20$$
$$4y = 20 - 10x$$
$$y = 5 - 10x$$

a. Describe Han's error.

b. Determine the x-intercept and y-intercept of the equation. Explain or show your thinking.

Name: .... Date: .... Period: ....

4. The graph shows how much money Priya has in her savings account each week after she started saving on a regular basis.



a How much money did Priya have in her savings account when she started to save regularly?

b Determine the amount of money Priya has in her account after 10 weeks.

c Determine how long it took Priya to save a total of $200.

d Write an equation to represent the relationship between the dollar amount in her savings account and the number of weeks of saving. Specify what each variable represents.

5. A pizza costs $12.49 plus $1.50 for each vegetable topping. Noah orders a pizza with t vegetable toppings that costs a total of d dollars. Select *all* of the equations that represent the relationship between the total cost d of the pizza with t vegetable toppings.

A. $12.49 + t = d$

B. $12.49 + 1.50t = d$

C. $12.49 + 1.50d = t$

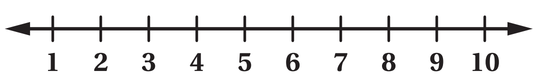
D. $12.49 = d + 1.50t$

E. $t = \frac{d - 12.49}{1.50}$

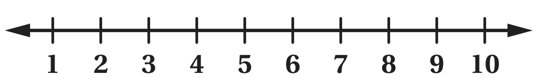
F. $t = d - \frac{12.49}{1.50}$

6. Graph each inequality on the following number lines.

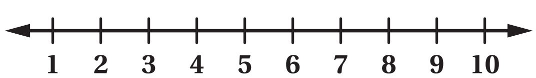
a $x > 3$



b $x \geq 5$



c $x < 2$



My Notes:

SUB-UNIT

3 Solving Inequalities and Graphing Their Solutions

What's after high school?

Imagine it's senior year. In a few short months, you'll finally be out of high school. Most of your classmates will likely go straight to college or enter the workforce. For many, those might be the right choices. But those aren't the *only* choices.

Some graduates take what's called a "gap year." These students take a year off from their studies or employment to travel, work on community projects, or develop a skill. That way, when they continue their education or start a new job, they're more experienced, mature, and have a stronger sense of purpose.

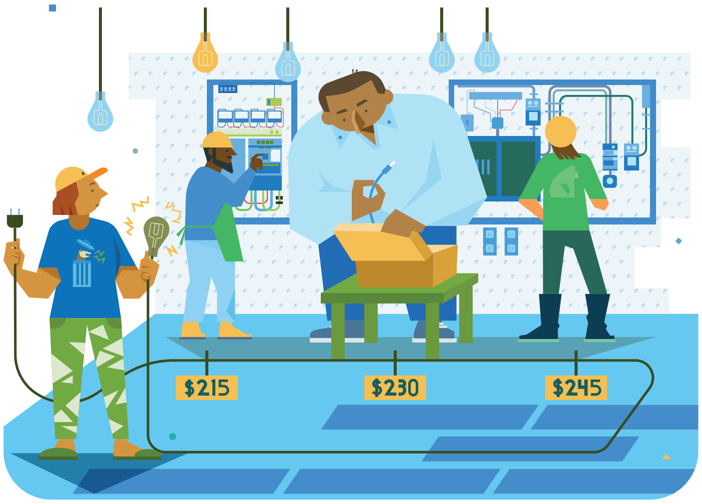
What life after high school looks like will come down to any constraints that might be in place, as well as the choices you make — whether it's taking a gap year, attending a college, or pursing a career. You will have to weigh your options carefully, scrutinizing what each path offers and comparing their pros and cons.

In this next set of lessons, You will learn how to make these kinds of comparisons mathematically. That way, you can see the impact of those options more clearly and choose the one that's best for you. But in order to do that, we must first pay a visit to our old friend: inequalities.

Unit 1 | Lesson 13

Inequalities and Their Solutions

Let's solve problems by writing and solving inequalities in one variable.



Warm-up How Many Solutions?

1. List four different solutions for the inequality $y \leq 9.2$.

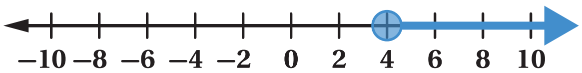
2. Write at least one solution for the inequality $7(3 - x) > 14$. Explain your thinking.

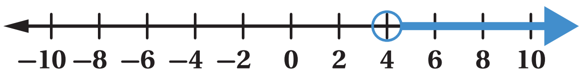
Log in to Amplify Math to complete this lesson online.

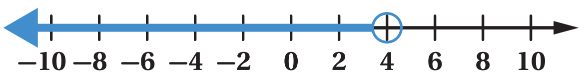
Name: ______ Date: ______ Period: ______

Activity 1 Matching Inequalities and Solutions

1. Which number line represents the inequality $x > 4$? Explain your thinking.

A. 

B. 

C. 

2. Match each inequality to the graph that represents its solutions.

a. $6x \leq 3x$ ______

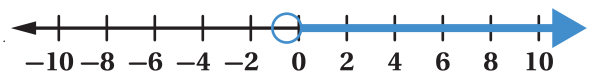
b. $\frac{1}{4}x > -\frac{1}{2}$ ______

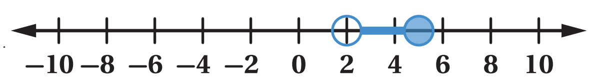
c. $16 < 4(x + 2) \leq 28$ ______

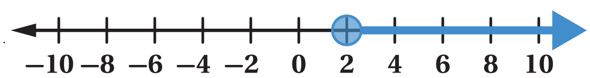
d. $8x - 2 < -4(x - 1)$ ______

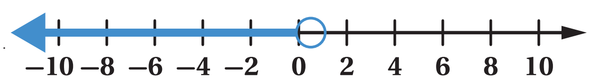
e. $\frac{4x - 1}{3} > -1$ ______

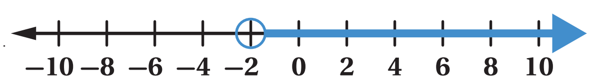
f. $\frac{12}{5} - \frac{x}{5} \leq x$ ______

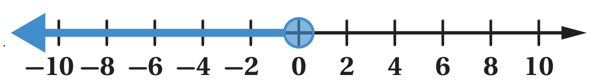












3. How did you solve the inequalities with variables on both sides of the inequality symbol?

Stronger and Clearer: After you complete Problem 3, your teacher will provide you some time to work with a partner to clarify and revise your thinking.

Activity 2 Choosing an Electrician

The lights in Kiran's condo are flickering. He needs an electrician to fix the wiring. Kiran contacts four different electricians to learn about their pricing.

Pixel-Shot/Shutterstock.com

Electrician 1: Charges an initial fee of \$50 and \$45 per hour.

Electrician 2: Charges an initial fee of \$45 and \$50 per hour.

Electrician 3: Charges an initial fee of \$45 and \$80 per hour.

Electrician 4: Charges an initial fee of \$80 and \$45 per hour.

1. Determine which electrician would be cheapest for each amount of hours it takes to fix Kiran's lights.

   a. 1 hour b. 2.5 hours c. 5 hours

2. Kiran can spend no more than \$200. Match each inequality with the electrician's pricing plan it represents.

   a. $45 + 50x \le 200$ .... Electrician 1

   b. $80x + 45 \le 200$ .... Electrician 2

   c. $45x + 50 \le 200$ .... Electrician 3

   d. $45x + 80 \le 200$ .... Electrician 4

3. What does the variable x represent in the inequalities?

4. What does the inequality symbol and the 200 represent in the inequalities?

Are you ready for more?

Using positive integers between 1 and 9 at most once each, determine values to create two inequalities, so that $x = 7$ is the only integer that satisfies each inequality.

$\square x + \square < \square x + \square$

$\square x + \square > \square x + \square$

Name: .... Date: .... Period: ....

Activity 3 Union Dues

Han is a member of the Construction Workers Labor Union. Union members pay annual dues. In return, union leaders help their members negotiate better working conditions and other benefits through collective bargaining (negotiating as a large group of employees, rather than as individuals).

Han's annual union dues are $\$378$ and he has already paid part of the amount. Han makes monthly payments of $\$42$ toward his dues until they are fully paid.

1. Write one or more inequalities to represent all the possible values of x, the number of payments it will take Han to fully pay his annual union dues.

2. What are all the possible values for x? Explain your thinking.

With the union, Han plans to volunteer at least 10 hours, but no more than 30 hours. He wants to volunteer the same amount of time each month.

3. Write an inequality (or inequalities) to represent all possible values of h, the number of hours per month Han volunteers with the Construction Workers Labor Union.

4. What are all the possible values for h? Explain your thinking.

STOP

Summary

In today's lesson . . .

You wrote and solved inequalities in one variable to help make sense of the constraints in a scenario. You used the same strategies for solving inequalities in one variable that are used when solving equations in one variable.

While an equation can result in one solution, an inequality results in a ***solution set***, which is a set of values that all satisfy the inequality. The solution set to an inequality in context may also be constrained by other factors of the scenario. For example, sometimes only positive values may be realistic.

Remember, solutions to inequalities in one variable can be represented using a number line. An open circle *does not* include the value in the solution, while a filled in circle *does* include the value. The direction of the ray you draw reflects the values in the solution set.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. Solve the inequality $2x < 10$. Explain how you determined the solution set. Graph the solution set on a number line.

2. Diego is solving the inequality $-15 + x < -14$. He knows the solution to the equation $-15 + x = -14$ is $x = 1$. How can Diego determine whether $x < 1$ or $x > 1$ is the solution set to the inequality?

3. A cellphone company offers two texting plans. People who use Plan 1 pay 10 cents for each text sent or received. People who use Plan 2 pay \$12 per month, and then pay an additional 2 cents for each text sent or received. Han determines that it is cheaper for him to use Plan 1 rather than Plan 2. Write an inequality to represent this situation. Use x to represent the number of texts he sends.

4. Tyler is a professional landscaper. He charges for the number of gallons of gasoline his equipment uses, as well as an hourly rate. The equation $5g + 15h = 35$ represents the amount of money he made using g gallons of gasoline and working h hours. Select *all* the values (g, h) that could be solutions to the equation.

   A. $(2, 3)$

   B. $(1, 2)$

   C. $(5, 2)$

   D. $(4, 1)$

Practice

Name: ______ Date: ______ Period: ______

5. Shawn's work for solving an equation is shown. Are the operations performed by Shawn correct? Why do you think Shawn ended up with a false equation?

Shawn's Work:

$$2(x-1)=4x-2$$
$$2x-2=4x-2$$
$$2x=4x$$
$$2=4$$

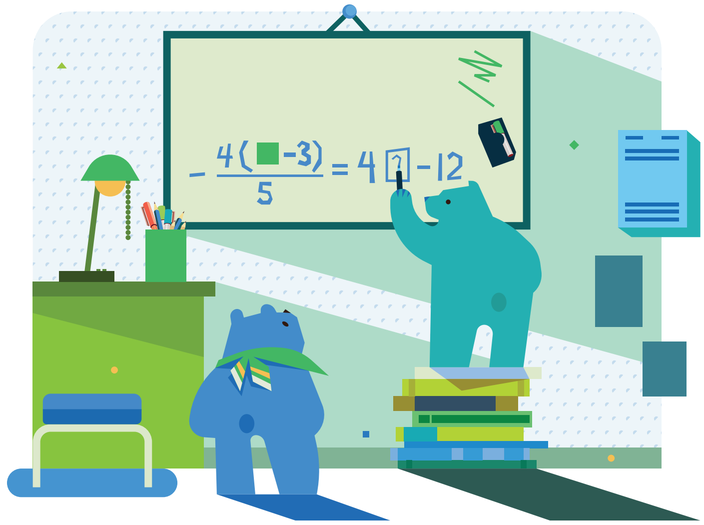
6. Kiran argues that $x=5$ is a solution to the inequality $-3x>9$ since it is possible to divide both sides by -3 and get $x>-3$. Do you agree with Kiran? Explain your thinking.

Name: ______________________ Date: __________ Period: __________

Unit 1 | Lesson 14

Solving Two-Variable Linear Inequalities

Let's practice writing and solving linear inequalities in two variables.



Warm-up Matching to Graphs

Match each equation or inequality with its graph. Not all of the graphs will have a matching equation or inequality.

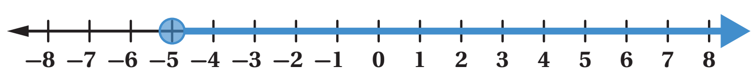
a. $3x - 5 = 10$ ....

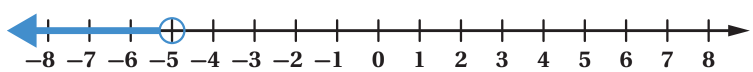
b. $3x - 5 \geq 10$ ....

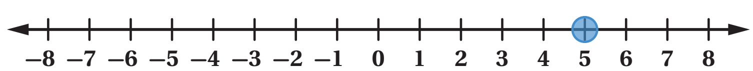
c. $-3x - 5 > 10$ ....

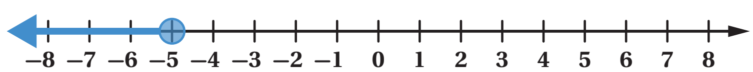
d. $3x - 5 < 10$ ....

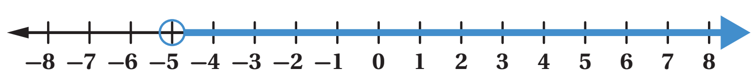
e. $-3x - 5 \leq 10$ ....











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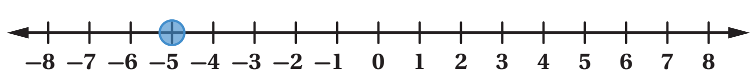

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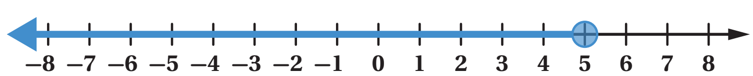

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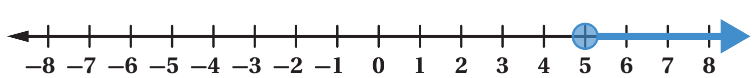
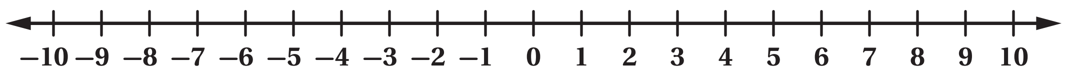


Log in to Amplify Math to complete this lesson online.

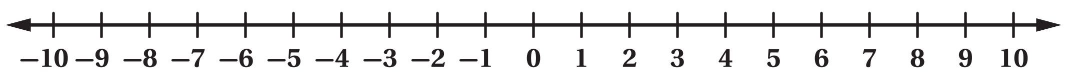
Activity 1 Multiplying by a Negative Value

1. Tyler is solving the equation $-x + 6 = 0$. He first multiplies both sides of the equation by -1, which gives $x - 6 = 0$. Then he adds 6 to both sides of the equation, determining that $x = 6$. Do you agree or disagree with Tyler's work? Explain your thinking.

2. Tyler is now solving the inequality $-x + 6 > 0$. Use the number line to graph the solutions for this inequality, and verify your answer by checking specific points on the number line.



3. Tyler wants to solve the inequality by multiplying both sides by -1. So, he writes $x - 6 > 0$. Use the number line to graph the inequality $x - 6 > 0$.



4. When you perform the same operation to both sides of an equation (or inequality), the solution (or solution set) should not change. When Tyler multiplied both sides of the inequality by -1, what else should he have done?

5. In your own words, explain why your method in Problem 4 works for inequalities.

Name: ______ Date: ______ Period: ______

Activity 2 Equality or Inequality

Part 1

You and your partner will each study one of these two strategies shown for solving inequalities.

Strategy 1:

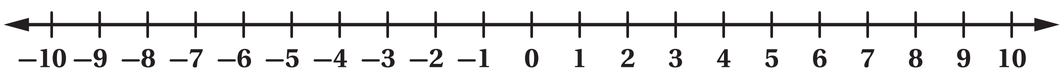
1. Use a separate sheet of paper to solve $-\frac{4(x+3)}{5} = 4x - 12$. Check your solution.

2. Consider the inequality $-\frac{4(x+3)}{5} \leq 4x - 12$.

a. Choose some values of x that are less than 2. Then choose some values of x that are greater than 2. Are any of the values you chose solutions to the inequality?

b. Choose 2 for the value of x. Is it a solution?

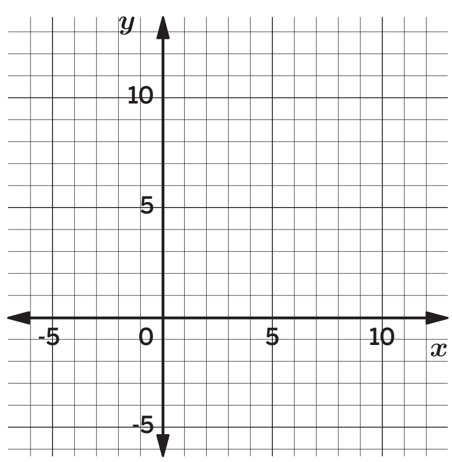
c. Graph the solution set of the inequality on the number line.



Strategy 2:

3. Use a separate sheet of paper to solve $-\frac{1}{2}x + 6 = 4x - 3$. Check your solution.

4. Graph each equation on the coordinate plane.
$y = -\frac{1}{2}x + 6$ and $y = 4x - 3$



5. Consider the inequality $-\frac{1}{2}x + 6 < 4x - 3$.

a. What value of x makes $-\frac{1}{2}x + 6$ and $4x - 3$ equal?

b. For what values of x is $-\frac{1}{2}x + 6$ less than $4x - 3$? Greater than $4x - 3$?

c. What is the solution to $-\frac{1}{2}x + 6 < 4x - 3$? Explain your thinking.

Activity 2 Equality or Inequality (continued)

Part 2

Two strategies for solving the inequality $2(2000m) + 1500 < 19500 - 2000m$ are shown. You and your partner will choose a different strategy to study. Explain the solution method for your chosen strategy.

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Strategy 3:

Solution method:

$$\begin{aligned} 2(2000m) + 1500 &= 19500 - 2000m \\ 4000m + 1500 &= 19500 - 2000m \\ 4000m - 18000 &= -2000m \\ -18000 &= -6000m \\ 3 &= m \end{aligned}$$

3 is the boundary value. Test a value greater than 3. Is $m = 4$ a solution?

$$\begin{aligned} 2(2000 \cdot 4) + 1500 &< 19500 - 2000 \cdot 4 \\ 17500 &< 11500 \end{aligned}$$

The inequality is false, so the solution must be less than 3, or $m < 3$.

</td>
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Explanation:

</td>
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Strategy 4:

Solution method:

$$\begin{aligned} 2(2000m) + 1500 &= 19500 - 2000m \\ 4000m + 1500 &= 19500 - 2000m \\ 6000m + 1500 &= 19500 \\ 6000m &= 18000 \\ m &= 3 \end{aligned}$$

Looking back at the second line, for $4000m + 1500 < 19500 - 2000m$ to be true, m must include negative numbers.

So, the solution to the inequality is $m < 3$.

</td>
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Explanation:

</td>
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Part 3

With your partner, choose one of the four inequalities shown to solve. Determine the boundary value and use any of the previous strategies. Show your thinking on a separate sheet of paper.

$3(x + 2) + 2x < 16$	$\dfrac{2(x-1)}{3} \geq 6$	$5x + 3 \leq 8x + 21$	$\dfrac{3(x-2)}{5} < x$

Name: ______ Date: ______ Period: ______

Activity 3 Inequality Bash

Solve each inequality for y. Check your work by finding an ordered pair (x, y) that is a solution to both the original inequality as well as the one you wrote.

1. $-3y - 1 > 4x + 5$

2. $-4\left(-\frac{1}{2}y - 4\right) \geq -3(x - 2)$

3. $2x - 10 \leq -\frac{y}{2}$

4. $-2y + 4 \leq 5\left(y - \frac{3}{5}\right)$

5. $-\frac{3}{2}\left(\frac{1}{6}y + 4\right) < -2x$

6. $4x + 6 + 2x \leq -2(3 + 3y)$

STOP

Summary

In today's lesson . . .

You revisited how multiplying or dividing an inequality by a negative value affects the inequality symbol. Specifically, the direction of an inequality symbol changes whenever you multiply or divide both sides by a negative value.

You solved multi-step inequalities in one and two variables by applying the properties of equality, and using a variety of strategies including:

- Testing different values.
- Relating the inequality to an equation.
- Graphing each side of an inequality separately.
- Reasoning about parts of an inequality or its structure.
- Rearranging an inequality to isolate y, creating an equivalent inequality in slope-intercept form.

You observed that the solutions of linear inequalities in two variables are ordered pairs, similar to solutions of equations in two variables.

Reflect:

Name: ________________ Date: ________ Period: ________

Practice

1. Consider the inequality $\frac{7x + 6}{2} \leq 3x + 2$. Select *all* of the values that are a solution to the inequality.

A. $x = -3$

B. $x = -2$

C. $x = -1$

D. $x = 0$

E. $x = 1$

F. $x = 2$

2. Solve the inequality $\frac{1}{2}(-8x - 6) > \frac{2x - 26}{2}$. Show your thinking.

3. Noah is solving the inequality $7x + 5 > 2x + 35$. He solves the equation $7x + 5 = 2x + 35$ and gets $x = 6$. How does the solution to the equation $7x + 5 = 2x + 35$ help Noah solve the inequality $7x + 5 > 2x + 35$? Explain your thinking.

4. Elena argues that $3x - 1 = 40$ has the same solution as $3(3x - 1) = 80$. Do you agree with her? Explain your thinking.

Practice

Name: .... Date: .... Period: ....

5. Bard creates a table to keep track of measurements for baking pies. Bard knows that 1 quart contains 4 cups and that 1 gallon contains 4 quarts.

a Complete the missing values in the table.

Cups	Quarts	Gallons
	3	0.75
15		0.94
	5	
28		

b Use the table to write an equation that represents the number of gallons g contained in one cup c.

6. For the expression $4x - 5(y - 1)$, which of the following ordered pairs makes the value of the expression greater than 20?

A. $(0, 5)$

B. $(8, 10)$

C. $(5, 0)$

D. $(10, 8)$

Name: ______ Date: ______ Period: ______

Unit 1 | Lesson 15

Graphing Two-Variable Linear Inequalities (Part 1)

Let's use graphs to represent solutions of two-variable linear inequalities.

Warm-up Algebra Talk

What strategies would you use to determine if the values in each ordered pair, (x, y), make $2x + 3y$ less than, greater than, or equal to 12? Use your strategy to determine the solution.

1. $(0, 5)$

Strategy:

Solution:

2. $(6, 0)$

Strategy:

Solution:

3. $(-1, -1)$

Strategy:

Solution:

4. $(-5, 10)$

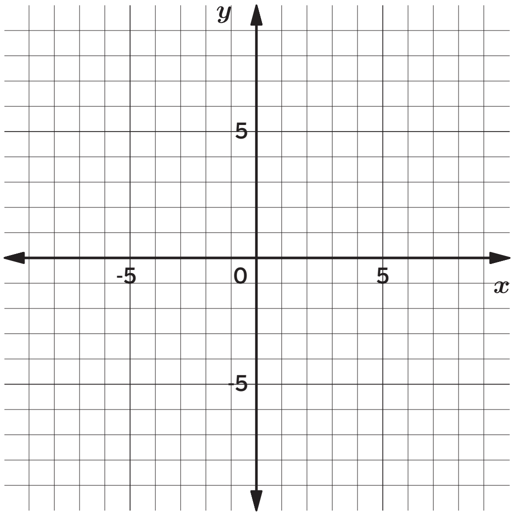
Strategy:

Solution:

Log in to Amplify Math to complete this lesson online.

Activity 1 Solutions and Non-Solutions

In the Warm-up, given different ordered pairs, you compared the value of the expression $2x + 3y$ to 12. Now consider the inequality $2x + 3y < 12$.



1. Choose as many ordered pairs that would make the inequality true and plot those ordered pairs on the graph with a dot. Then choose as many ordered pairs that make the inequality false and plot those ordered pairs on the graph with an "X."

2. What do you notice or wonder about the solutions of the inequality?

3. What do you notice or wonder about the non-solutions of the inequality?

4. Four inequalities are shown on the next page. Your group will be assigned one or more inequalities. For each inequality assigned to your group:
   - Choose three points from each quadrant and one point on each axis that you will test in your inequality.

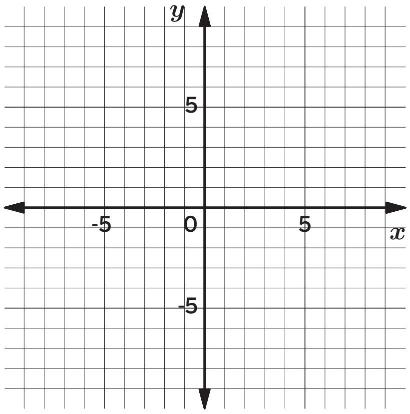
Quadrant I	Quadrant II	Quadrant III	Quadrant IV	x-axis	y-axis

   - Determine which ordered pairs represent solutions to the inequality and which ordered pairs do not.
   - Plot the points that are solutions with a dot. Plot points that are not solutions with an "X."
   - Continue plotting enough points until you start to see the region that contains solutions and the region that contains non-solutions.
   - Look for a pattern to help determine the region of solutions.

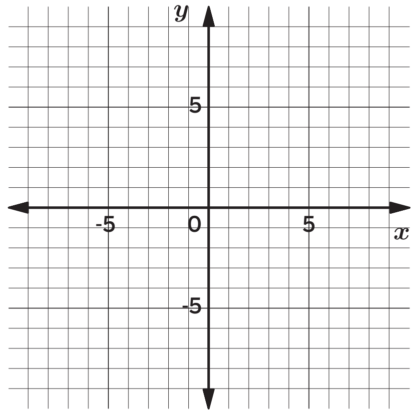
Name: ____________ Date: ________ Period: ________

Activity 1 Solutions and Non-Solutions (continued)

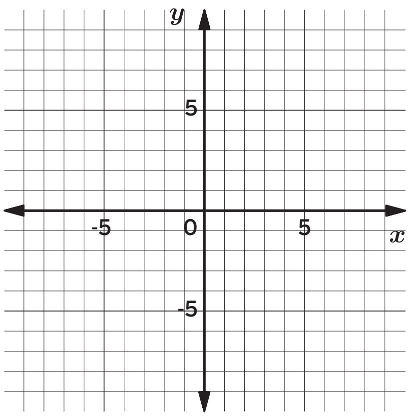
Inequality 1: $y \leq x$



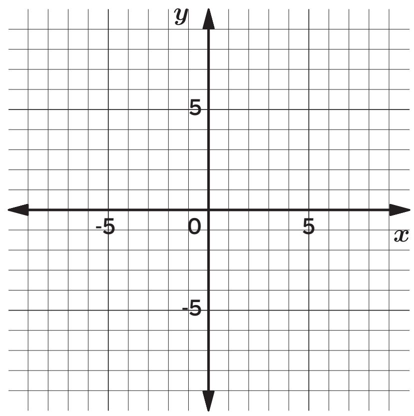
Inequality 2: $-2y \leq -4$



Inequality 3: $3x < 0$



Inequality 4: $x + y > 10$



5. Without giving the ordered pairs, what points are solutions to $y \leq x$, but not $y < x$? Explain your thinking.

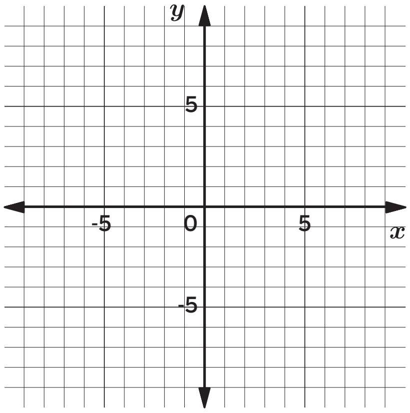
6. How could you show all the possible solutions of a linear inequality in two variables without plotting individual points?

7. How could you use the inequalities to determine the equation for the boundary line that separates the two regions of solutions and non-solutions?

8. Sketch the boundary line for your assigned inequality.

Activity 2 Sketching Solutions to Inequalities

1. Graph the equation $x - y = 5$.

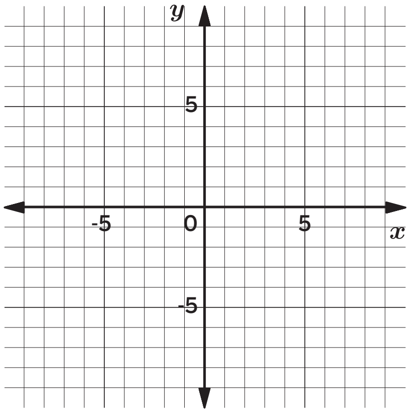


a What do the points on the line represent?

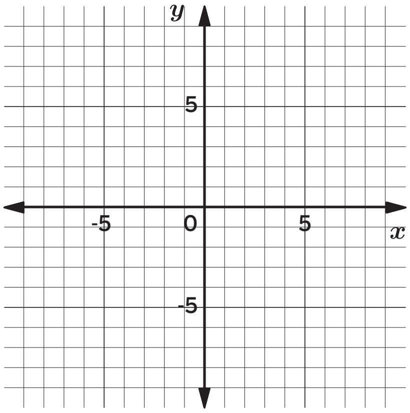
2. Sketch the following graphs representing the solutions to each of these inequalities.

- Make the boundary line solid if it is part of the solution, and dashed if it is not part of the solution.
- Shade the region containing the solutions.

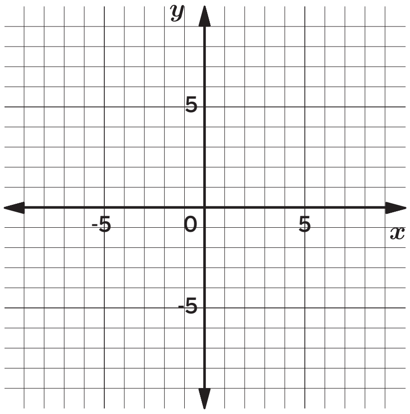
a $x - y < 5$



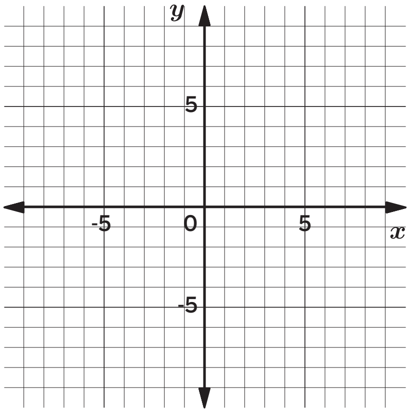
b $x - y \leq 5$



c $x - y > 5$



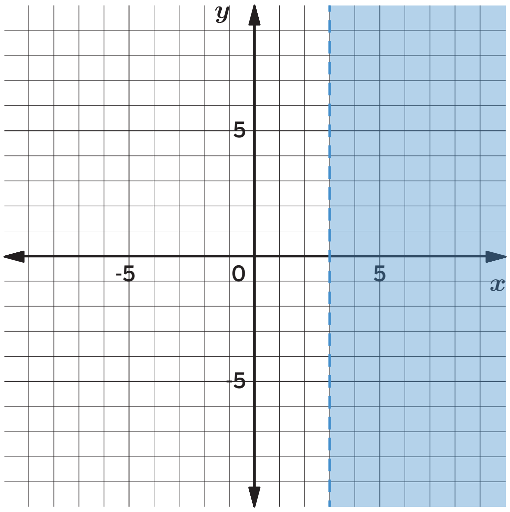
d $x - y \geq 5$



Activity 2 Sketching Solutions to Inequalities (continued)

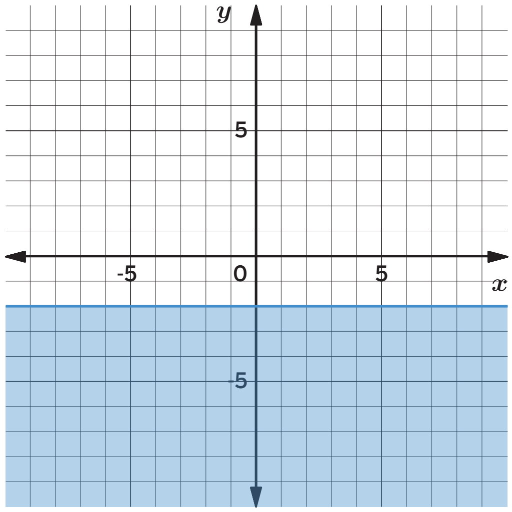
3. For each graph, write an inequality whose solutions are represented by the shaded part of the graph.

a



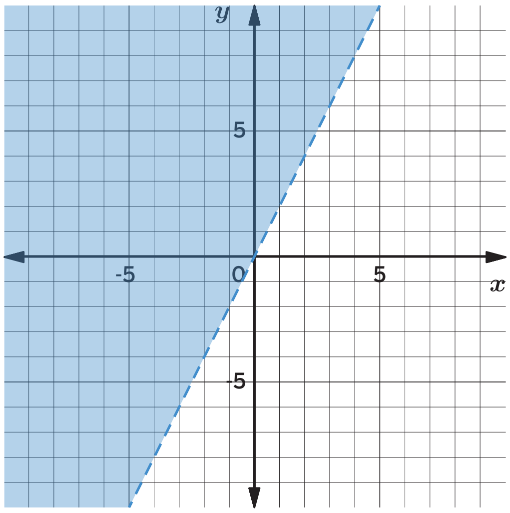
Inequality:

b



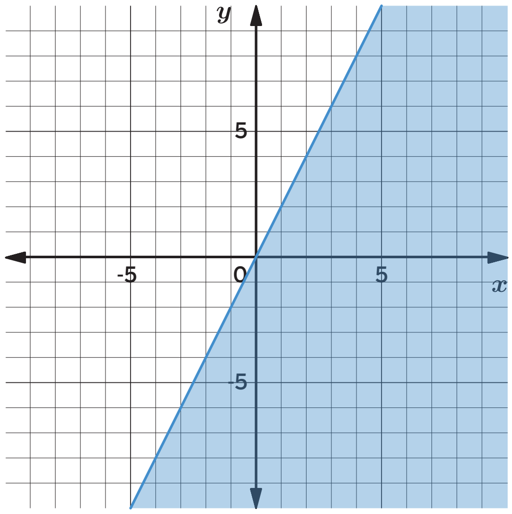
Inequality:

c



Inequality:

d

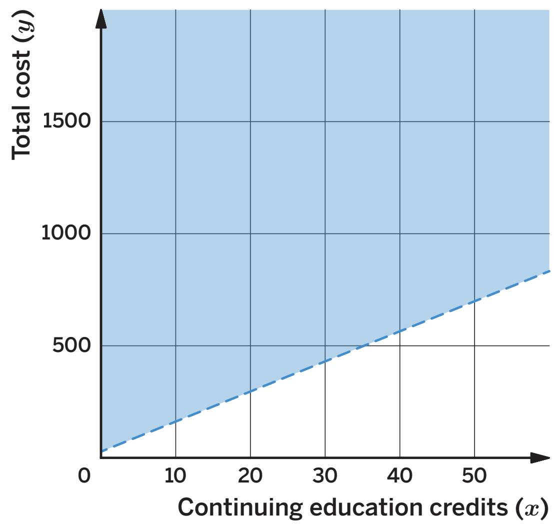


Inequality:

Activity 3 Continuing Education

Although people make money at their jobs, they also spend money for their jobs for technology, clothing, and transportation. Some professions, such as lawyers and teachers, also require continuing education, that costs money.

Tyler and Elena are both lawyers at the same law firm. The inequality shown in the graph, $y - 135x > 200$, represents how much money they each spent to maintain their jobs y, while taking x continuing education credits.



In their first year at the law firm, $(20, 4000)$ represents the number of continuing education credits Tyler takes and how much money he spends on his job in dollars, and $(20, 4500)$ represents the same for Elena.

1. What does the 135 represent in the inequality?

2. Compute $y - 135x$ for both these points.

3. Which point, $(20, 4000)$ or $(20, 4500)$, is closer to satisfying the equation $y - 135x = 200$? That is, for which point is $y - 135x$ closest to 200? Explain your thinking.

4. Which person's total cost is closer to the minimum cost of maintaining their jobs and taking 20 credits? Explain your thinking.

In his second year, Tyler wants to decrease how much he spends on his job by reducing the number of credits he takes as well as reducing additional expenses. $(12, 2700)$ represents the number of credits Tyler takes and the total cost, in dollars, for his second year.

5. Did Tyler lower his expenses outside of continuing education costs? Explain your thinking.

6. Tyler plans to keep his additional expenses the same next year. If he plans on taking 15 credits, what will his total cost be?

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You examined how to determine the solutions for a two-variable linear inequality, and how to graphically indicate all the points that are part of the solution.

1. **Graph the boundary line.**
   This ***boundary line*** represents the boundary between the region containing solutions and the region containing non-solutions. Each of these regions are considered ***half-planes***. You graphed this line by changing the inequality symbol to an equal sign and graphing the line represented by this equation.

2. **Determine if the line is dashed or solid.**
   If the points that lie on the line are solutions, the line should be solid ($\geq$ or $\leq$ inequalities). If the points along the line are *not* solutions, the line should be dashed ($>$ or $<$ inequalities).

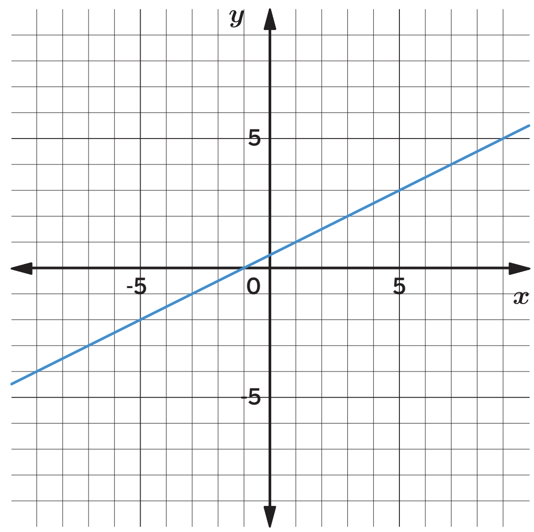
3. **Test points to determine the solution region and where to shade.**
   You can choose a point on either side of the line and substitute its coordinates into the inequality to see if it is a solution. This will help you determine which side of the line should be shaded.

Reflect:

Practice

Name: ______________________ Date: __________ Period: __________

1. Refer to the graph of the equation $2y - x = 1$.



a. Are the points $(0, 0.5)$ and $(-7, -3)$ solutions to the equation? Explain your thinking.

b. Select *all* points that are solutions to the inequality $2y - x > 1$.

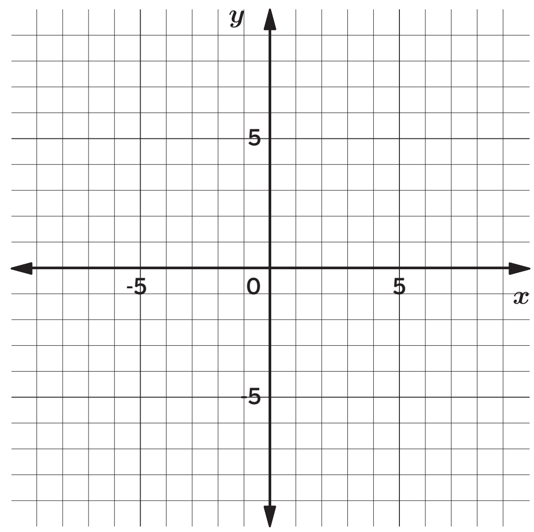
A. $(0, 2)$

B. $(8, 0.5)$

C. $(-6, 3)$

D. $(-7, -3)$

c. Revise the original graph, and shade the region that represents the solution set to the inequality $2y - x > 1$.



d. Are the points on the line included in the solution set? Explain your thinking.

2. Select *all* ordered pairs that are solutions to the inequality $5x + 9y < 45$.

A. $(0, 0)$

B. $(5, 0)$

C. $(9, 0)$

D. $(0, 5)$

E. $(0, 9)$

F. $(-5, -9)$

Name: .... Date: .... Period: ....

Practice

3. Consider the inequality $2y - 3x < 5$.

a Are $(-1, 1)$ and $(4, 1)$ solutions to the inequality? Explain your thinking.

b Explain how you can use your response to part a to graph the solution set to the inequality.

4. Kiran wants to buy dinner for his drama club on the evening of their final rehearsal. The budget for dinner is $\$60$. Kiran plans to buy some prepared dishes from a supermarket. The prepared dishes are sold by the pound, at $\$5.29$ a pound. He also plans to buy two large bottles of sparkling water at $\$2.49$ each. Let p represent the amount of prepared food, in pounds, Kiran could buy without going over budget. Represent the constraints in the situation mathematically.

5. Which equation is equivalent to $0.3x + 0.06y = 4.3$?

A. $3x + 6y = 43$

B. $30x + 60y = 43$

C. $3x + 0.6y = 430$

D. $30x + 6y = 430$

6. Shawn is shopping for back to school clothing at the end of the summer. New pants cost $\$50$ a pair and a new shirt costs $\$30$. Shawn wants to spend no more than $\$200$ on clothing in total. If p represents the number of pairs of pants purchased and s represents the number of shirts purchased, write an inequality to represent this scenario.

Unit 1 | Lesson 16

Graphing Two-Variable Linear Inequalities (Part 2)

Let's practice writing, interpreting, and graphing solutions to linear inequalities in two variables.

Warm-up Notice and Wonder

Clare solves three linear inequalities. Her work is shown. What do you notice? What do you wonder?

Inequality 1: $-y \leq x$

$$\frac{-y}{-1} \leq \frac{x}{-1}$$
$$y \geq -x$$

Inequality 2: $x - y < 6$

$$-y < 6 - x$$
$$\frac{-y}{-1} < \frac{6-x}{-1}$$
$$y > -6 + x$$

Inequality 3: $-4y < 8x + 16$

$$\frac{-4y}{-4} < \frac{8x + 16}{-4}$$
$$y > -2x - 4$$

1. I notice . . .

2. I wonder . . .

Co-craft Questions: Share your responses to Problem 2 with a partner. Work together to generate 1–2 questions that you would like to answer during today's lesson.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Gap Year Options

A gap year can be taken after high school or after post-secondary education in order to gain a better sense of oneself and the world, pursue a passion or unique experience, or work to save money for further education. Common experiences during a gap year include volunteering, working, completing an internship, and traveling.

You will be given a page for each experience. Choose one experience to work on. There are two questions about each experience. For each problem that you work on:

- Write an inequality to describe the constraints. Specify what each variable represents.
- Use graphing technology to graph the inequality.
- Sketch the graph and label the axes.
- Complete the problems that go with each scenario.

Reflect: What part of this activity played into your strengths? How did you overcome any limitations?

Activity 2 Card Sort: Gap Year Experiences

You will be given a set of cards. Take turns with your partner to match a set of four cards that contain:

- **A description of a scenario.**
- **An inequality that represents the scenario.**
- **A graph that represents the solution region.**
- **A solution written as an ordered pair.**

For each match that you determine, discuss your thinking with your partner.

For each match that your partner determines, listen carefully to their explanation. If you disagree, discuss your thinking and work together to reach an agreement.

Record your matches in the table.

	Set 1	Set 2	Set 3	Set 4
Scenario				
Inequality				
Graph				
Solution				

Name: .... Date: .... Period: ....

Activity 3 What Went Wrong?

Jada goes back to school to earn a graduate degree where she works in the science laboratory. Jada earns \$20 per hour and spends \$30 per week on school expenses. Jada must save at least \$18,000 to help pay for her living expenses during the academic year. To analyze her earnings and expenses, she creates the inequality $20h - 30w \geq 18{,}000$ where w represents weeks and h represents hours. Her work is shown:

First, Jada isolates w.

$20h - 30w \geq 18000$

$-30w \geq 18000 - 20h$

$\frac{-30w}{-30} \geq \frac{18000 - 20h}{-30}$

$w \geq -600 + \frac{2}{3}h$

Next, she tests a point.

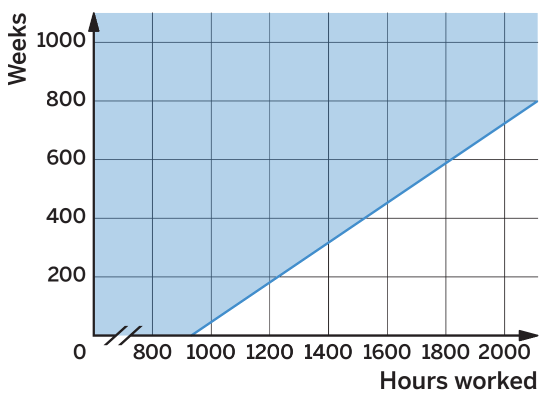
$(150, 200)$

$w \geq -600 + \frac{2}{3}h$

$200 \geq -600 + \frac{2}{3}(150)$

$200 \geq -500$

Finally, she graphs the inequality.

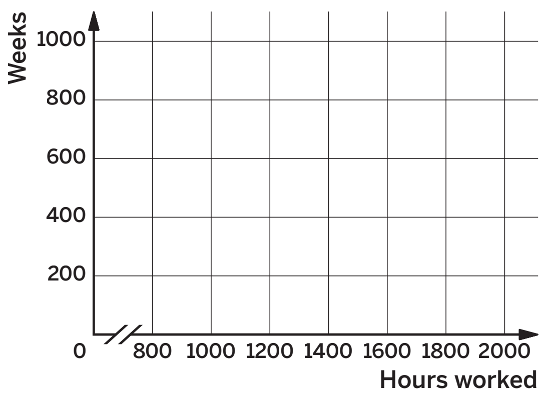


1. Analyze Jada's work to determine the mistake she made.

   a. Describe her mistake. Then fix her mistake and write the inequality that represents the situation.

   b. Graph the inequality that you wrote in part a.

   c. How does Jada's incorrect graph compare to your graph?



2. Name two possible combinations of number of hours and number of weeks that would allow Jada to meet her goal.

3. Many of the work opportunities on college campuses offer low pay. What issues do you think this could cause for students?

STOP

Summary

In today's lesson . . .

You connected different representations of constraints (graphs, inequalities, and descriptions) that were represented by two variables. You also identified and interpreted the meaning of solutions in context.

You used graphing technology to graph linear inequalities in two variables. Some tools, however, may require the inequalities to be in specific type of form before displaying the solution region. Be sure to learn how to use the graphing technology available in your classroom.

Although graphing using technology is efficient, you should still analyze any graph with care. For example, if the graphing window is too small, you may not be able to clearly see the solution region or the boundary line. You should also always think about the meaning of solution points in context.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. This year, students in the 9th grade are collecting dimes and quarters for a school fundraiser. They are trying to collect more money than the students who were in the 9th grade last year, who collected $\$143.88$. Using d to represent the number of dimes collected and q to represent the number of quarters, which statement best represents this situation?

A. $0.25d + 0.1q \geq 143.88$

B. $0.25q + 0.1d \geq 143.88$

C. $0.25d + 0.1q > 143.88$

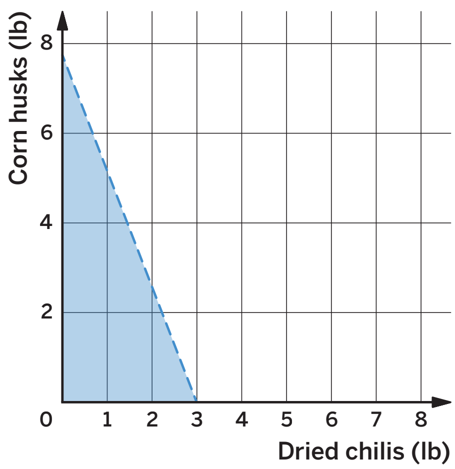
D. $0.25q + 0.1d > 143.88$

2. A farmer is creating a budget for planting soybeans and wheat. Planting soybeans costs $\$200$ per acre and planting wheat costs $\$500$ per acre. He wants to spend no more than $\$100{,}000$ planting soybeans and wheat.

a. Write an inequality to describe the constraints. Specify what each variable represents.

b. Write one solution to the inequality and explain what it represents in that situation.

3. Clare is ordering dried chili peppers and corn husks for her cooking class. Chili peppers cost $\$16.95$ per pound and corn husks cost $\$6.49$ per pound. Clare spends less than $\$50$ on d lb of dried chili peppers and h lb of corn husks. The graph shown represents this situation.



a. Write an inequality that represents this situation.

b. Can Clare purchase 2 lb of dried chili peppers and 4 lb of corn husks and spend less than $\$50$? Explain your thinking.

c. Can Clare purchase 1.5 lb of dried chili peppers and 3 lb of corn husks and spend less than $\$50$? Explain your thinking.

Practice

Name: ______________________ Date: __________ Period: __________

4. Jada has a sleeping bag that is rated for $30°F$. This means that if the temperature outside is at least $30°F$, Jada will be able to stay warm in her sleeping bag.

a Write an inequality that represents the outdoor temperature at which Jada will be able to stay warm in her sleeping bag.

b Write an inequality that represents the outdoor temperature at which a thicker or warmer sleeping bag would be needed to keep Jada warm.

5. Select *all* the equations that have the same solution as $3x + 5 = 20 - x$.

A. $4x = 15$

B. $2x = 25$

C. $-x + 20 = 5 + 3x$

D. $-4x + 20 = -5$

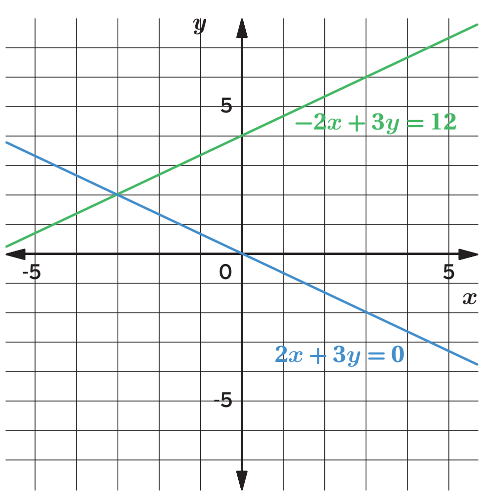
E. $4x - 15 = 0$

F. $x - 20 = 5 + 3x$

6. The graphs represent a system of equations:

$$\begin{cases} -2x + 3y = 12 \\ 2x + 3y = 0 \end{cases}$$

Solve the system of equations. Explain or show your thinking.



SUB-UNIT

4 Systems of Linear Equations in Two Variables

Narrative Connections

Are you a 'Boomerang-er'?

When we think about being an adult, we think of being independent and living on one's own. But recently, more and more young adults move back home after college.

According to the U.S. Census, over the last 20 years there has been an increase of roughly 2 million people between the ages of 25 and 34 who move in with their parents. Dubbed the "Boomerang Generation," these individuals often found it difficult to afford living on their own, given a scarcity of high-paying jobs and rising levels of student debt.

This trend might not seem so strange for many cultures outside the U.S., where multiple generations can be found living under the same roof. Moving back home is not only more affordable — it allows you to be in a system of mutual support and care with the rest of your family. And for city dwellers returning to the suburbs, access to a washer and dryer is a notable perk!

But there is much to consider when choosing the right living arrangement. Does the price of gas commuting from your family's home cost more than public transit over time? Does splitting the electric bill with several people cost less than paying only for yourself?

Earlier in this unit, you learned to solve problems with one variable. But the complexities of life can often involve multiple variables. This next set of lessons will show how to solve equations with more than one variable, so that you can make a confident decision, no matter where you hang your hat!

Unit 1 | Lesson 17

Writing and Graphing Systems of Linear Equations

Let's recall what it means to solve a system of linear equations and represent the solution graphically.

Warm-up Grocery Shopping With Roomies

Kiran, Lin, Mai, and Noah share an apartment. They usually go grocery shopping together to split their grocery expenses.

- Kiran and Mai purchased a total of 7 items from the supermarket.
- Together, Kiran and Lin purchased 5 grocery items.
- If Mai and Noah put all their grocery items together, they would have 12 in total.
- If Noah and Lin put all their grocery items in one cart, the cart would have 10 items.
- The 4 roommates purchased 17 items.

What is the possible number of items each roommate could have purchased?

Ljupco Smokovski/Shutterstock.com

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Working Two Jobs

Diego works as a customer service representative during the day and earns \$18 per hour. He has a second job as a night security guard where he earns \$24 per hour. This week, Diego's total earnings from both jobs are \$864.

1. Decide whether Diego could have worked the set of hours shown for each job this week. Be prepared to explain your thinking.

   a. 40 day hours and 6 night hours

   b. 35 day hours and 10 night hours

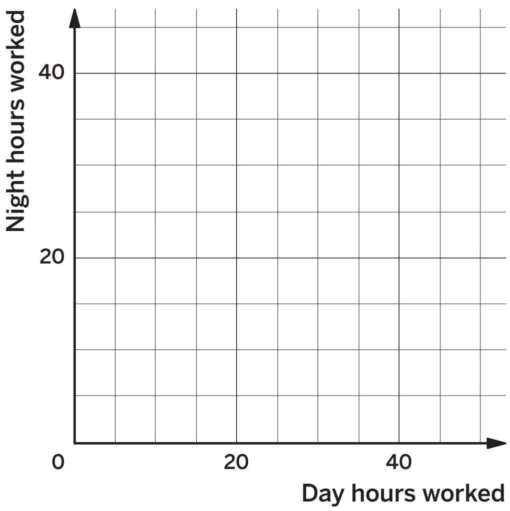
   c. 31 day hours and 13.5 night hours

   d. 25 day hours and 17.25 night hours

2. Read the scenario again. Complete each of the following problems.

   a. Write an equation to represent the number of hours Diego works at each job this week, given that his total earnings are \$864. Let x represent the number of day hours worked and y represent the number of night hours worked.

   b. Graph the equation.



   c. Complete the table with the number of hours Diego could work at one job, given the number of hours worked at the other.

Number of day hours worked	0	10			34	42
Number of night hours worked			24.75	20.25		

Activity 1 Working Two Jobs (continued)

3. Diego works a total of 44.5 hours this week.

   a. Write an equation to represent this new constraint. Let x be the number of day hours worked per week and y be the number of night hours worked per week.

   b. Graph your equation from Problem 3a on the same graph used in Problem 2b.

   c. Complete the table with the number of hours Diego could work at one job, given the number of hours worked at the other.

Number of day hours worked	0	10			34	42
Number of night hours worked			24.75	20.25		

4. This week Diego works 44.5 hours and earns $864. How many hours does he work at each job? Explain or show your thinking.

Name: .... Date: .... Period: ....

Activity 2 Meeting Both Constraints

Each problem has two related quantities and involves two constraints. For each problem, determine the pair of values that meet both constraints. Explain or show your thinking.

1. A restaurant has a total of 25 tables. Some tables are rectangular and can seat 8 people, while other tables are round and can seat 6 people. On a busy evening, all 190 seats at the tables are occupied. How many rectangular tables x, and round tables y, are there? Explain your thinking.

2. A family buys 16 tickets to a magic show. Adult tickets are $\$10.50$ each and child tickets are $\$7.50$ each. The family pays a total of $\$141$. How many adult tickets a, and child tickets c, did they buy? Explain your thinking.

3. Han pays $\$16.80$ for 2 large posters and 3 small posters of his favorite band. Kiran pays $\$14.15$ for 1 large poster and 4 small posters of his favorite movie stars. Posters of the same size have the same price. Determine the price of a large poster ℓ, and a small poster s.

Are you ready for more?

1. Write equations for two lines that intersect at the point $(4, 1)$.

2. Write equations for three lines whose intersection points form a triangle with vertices located at $(-4, 0)$, $(2, 9)$, and $(6, 5)$.

STOP

Summary

In today's lesson . . .

You revisited systems of linear equations, which are formed by two (or more) equations. A system of equations can be used to represent multiple constraints in a context.

A curly bracket is commonly used to indicate a system of equations, like this:

$$\begin{cases} x + y = 4 \\ 5x + 10y = 25 \end{cases}$$

A solution to a system is a pair of values that make all the equations in the system true.

You saw that graphing a system of equations is an efficient strategy for determining its solutions. On a coordinate plane, the solution to a system of equations is the intersection of the lines of the equations in the system.

The solution to a system of equations can be verified by substituting the x- and y-coordinates of the point of intersection into each equation in the system. If the pair is a solution, each equation will be true.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. The knitting club sold 40 scarves and hats at a winter festival and made $\$700$ from the sales. They charged $\$18$ for each scarf and $\$14$ for each hat. If s represents the number of scarves sold and h represents the number of hats sold, which system of equations represents the constraints in this situation?

A. $\begin{cases} 40s + h = 700 \\ 18s + 14h = 700 \end{cases}$

B. $\begin{cases} 18s + 14h = 40 \\ s + h = 700 \end{cases}$

C. $\begin{cases} s + h = 40 \\ 18s + 14h = 700 \end{cases}$

D. $\begin{cases} 40(s + h) = 700 \\ 18s = 14h \end{cases}$

2. Consider these two equations:

Equation 1: $6x + 4y = 34$ **Equation 2:** $5x - 2y = 15$

Determine whether each ordered pair is a solution to *one equation*, *both equations*, or *neither of the equations*. Then respond to part e.

a. $(3, 4)$

b. $(4, 2.5)$

c. $(5, 5)$

d. $(3, 2)$

e. Is it possible to have more than one ordered pair that is a solution to both equations? Explain or show your thinking.

3. Explain or show that the point $(5, -4)$ is a solution to this system of equations:

$\begin{cases} 3x - 2y = 23 \\ 2x + y = 6 \end{cases}$

Practice

Name: ______________ Date: ______________ Period: ______________

4. The table shows the volume of water in a tank after it has been filled to a certain height. Which equation could represent the volume of water in cubic inches V, when the height is h in.?

Height of water (in.)	Volume of water (in^3)
0	0
1	1.05
2	8.40
3	28.35

A. $h = V$

B. $h = \frac{V}{4}$

C. $V = h^2 + 0.05$

D. $V = 1.05h^3$

5. Andre does not understand why a solution to the equation $3 - x = 4$ must also be a solution to the equation $12 = 9 - 3x$. Write a convincing explanation as to why this is true.

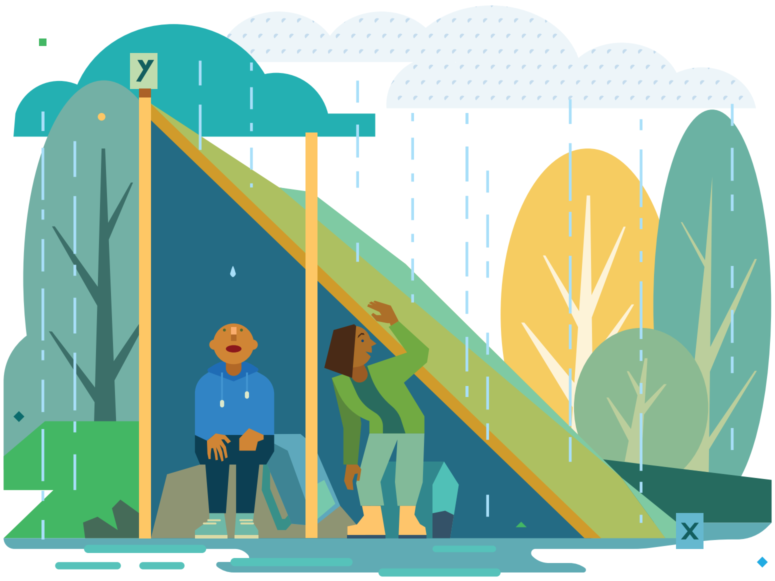
6. Solve the system of equations without graphing. Show your thinking.

$$\begin{cases} y + \frac{1}{3}x = 3 \\ y = 2x - 4 \end{cases}$$

Name: ______ Date: ______ Period: ______

Unit 1 | Lesson 18

Solving Systems by Substitution

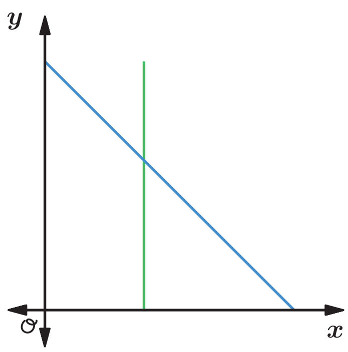


Let's use substitution to solve systems of linear equations.

Warm-up Math Talk

The graph of two equations in a system is shown.

What strategies would you use to determine if each of these systems could be represented by the graph? Use your strategy to determine whether the system could be represented by the graph. Circle *yes* or *no*.



a $\begin{cases} x + 2y = 8 \\ x = -5 \end{cases}$

Strategy:

Yes

No

b $\begin{cases} 3x = 8 \\ 3x + y = 15 \end{cases}$

Strategy:

Yes

No

c $\begin{cases} y = -7x + 13 \\ y = -1 \end{cases}$

Strategy:

Yes

No

d $\begin{cases} y = 2x - 7 \\ 4 + y = 12 \end{cases}$

Strategy:

Yes

No

Log in to Amplify Math to complete this lesson online.

Activity 1 Four Systems

The four systems of linear equations from the Warm-up are shown. Solve each system algebraically. Then check your solutions by substituting them into the original equations to see if the equations are true.

1. $\begin{cases} x + 2y = 8 \\ x = -5 \end{cases}$

2. $\begin{cases} y = -7x + 13 \\ y = -1 \end{cases}$

3. $\begin{cases} 3x = 8 \\ 3x + y = 15 \end{cases}$

4. $\begin{cases} y = 2x - 7 \\ 4 + y = 12 \end{cases}$

Name: ______ Date: ______ Period: ______

Activity 2 Which Strategy Is Most Efficient?

Study the systems of linear equations. Complete each problem on your own, and then discuss your thinking with your partner. Decide whether you agree or disagree with your partner, supporting your thinking with evidence. Move on to the next problem after you both reach a shared agreement.

System A	System B	System C
$\begin{cases} y = -2x + 5 \\ y = x + 3 \end{cases}$	$\begin{cases} 2x + y = 5 \\ y = x - 1 \end{cases}$	$\begin{cases} x + 5y = 1 \\ 2x + 3y = 9 \end{cases}$

1. Which system(s) is most efficiently solved by graphing? Explain or show your thinking.

2. Which system(s) is most efficiently solved by substitution? Explain or show your thinking.

Compare and Connect:
Look back at the three systems. What connections do you see between the structure of the equations and the method you thought was the most efficient?

Activity 3 What About Now?

Solve each system without graphing. Show your thinking.

1. $\begin{cases} 5x - 2y = 26 \\ y + 4 = x \end{cases}$

2. $\begin{cases} 2m - 2p = -6 \\ p = 2m + 10 \end{cases}$

3. $\begin{cases} 2d = 8f \\ 18 - 4f = 2d \end{cases}$

4. $\begin{cases} w + \frac{1}{7}z = 4 \\ z = 3w - 2 \end{cases}$

Are you ready for more?

Solve this system with four equations.

$$\begin{cases} 3x + 2y - z + 5w = 20 \\ y = 2z - 3w \\ z = w + 1 \\ 2w = 8 \end{cases}$$

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You reviewed solving a system of linear equations algebraically using substitution. You examined the structure of equations in different systems and reasoned about what makes substitution an efficient way to solve some systems.

Substitution is useful when one variable is already isolated or can be readily isolated so that the value (or expression) of that variable can be substituted into the other equation in the system without much manipulation.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. Identify a solution to this system of equations: $\begin{cases} -4x + 3y = 23 \\ x - y = -7 \end{cases}$

A. $(-5, 2)$

B. $(-2, 5)$

C. $(-3, 4)$

D. $(4, -3)$

2. Lin is solving this system of equations: $\begin{cases} 6x - 5y = 34 \\ 3x + 2y = 8 \end{cases}$

She starts by rearranging the second equation to isolate the y variable: $y = 4 - 1.5x$. Then she substitutes the expression $4 - 1.5x$ for y in the first equation, as shown:

$$\begin{aligned} 6x - 5(4 - 1.5x) &= 34 \\ 6x - 20 - 7.5x &= 34 \\ -1.5x &= 54 \\ x &= -36 \end{aligned} \qquad \begin{aligned} y &= 4 - 1.5x \\ y &= 4 - 1.5\,(-36) \\ y &= 58 \end{aligned}$$

a. Check to see if Lin's solution of $(-36, 58)$ makes both equations in the system true. Explain your thinking.

b. If your answer to the previous question was "no," find and explain her mistake. If your answer was "yes," graph the equations to verify the solution of the system.

3. Solve each system of equations. Show your thinking.

a. $\begin{cases} 2x - 4y = 20 \\ x = 4 \end{cases}$

b. $\begin{cases} y = 6x + 11 \\ 2x - 3y = 7 \end{cases}$

Practice

Name: ______________________ Date: __________ Period: __________

4. Kiran buys supplies for his school's greenhouse. He buys f bags of fertilizer and s bags of soil. He pays \$5 for each bag of fertilizer and \$2 for each bag of soil and spends a total of \$90. The equation $5f + 2s = 90$ describes this relationship. If Kiran solves the equation for s, which equation would result?

A. $2s = 90 - 5f$

B. $s = \dfrac{5f - 90}{2}$

C. $s = 45 - 2.5f$

D. $s = \dfrac{85f}{2}$

5. Use the given equations to answer each of the following. Graphing technology should not be used.

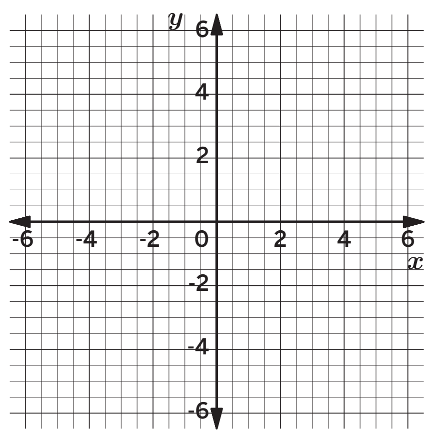
Equation 1: $y = 3x + 8$ **Equation 2:** $2x - y = -6$

a. Identify a point that is a solution to Equation 1, but not a solution to Equation 2.

b. Identify a point that is a solution to Equation 2, but not a solution to Equation 1.

c. Graph the two equations.

d. Identify a point that is a solution to both equations.



6. Rewrite each expression by combining like terms.

a. $5t + 3z - 2t$

b. $3(c - 5) + 2c$

c. $23s - (13t + 7t)$

d. $5x + 4y - (5x + 7y)$

e. $7t + 18r + (2r - 5t)$

f. $6x + 12y + 2(3x - 6y)$

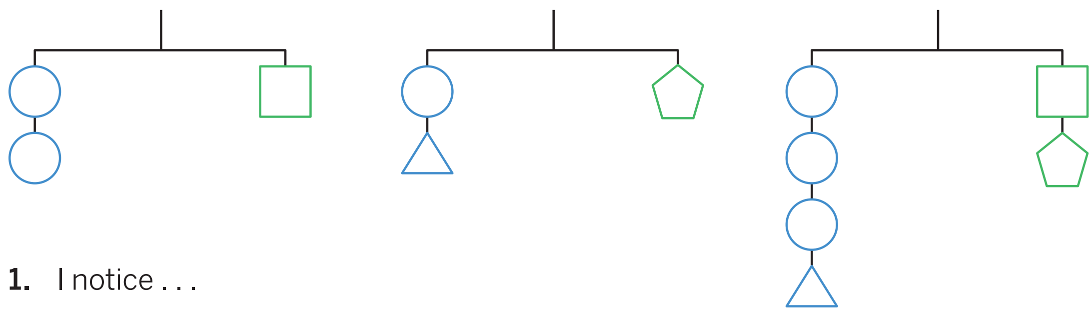
Unit 1 | Lesson 19

Solving Systems by Elimination: Adding and Subtracting (Part 1)

Let's investigate how adding or subtracting equations can help us solve systems of linear equations.

Warm-up Notice and Wonder

Study the three hanger diagrams.



1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Adding Equations

The step-by-step solution for the following system of equations is shown.

$$\begin{cases} \mathbf{4x + 3y = 10} \\ \mathbf{-4x + 5y = 6} \end{cases}$$

$$\begin{array}{rl} 4x + 3y &= 10 \\ (+)\ -4x + 5y &= 6 \\ \hline 0 + 8y &= 16 \\ y &= 2 \end{array} \qquad \begin{array}{rl} 4x + 3(2) &= 10 \\ 4x + 6 &= 10 \\ 4x &= 4 \\ x &= 1 \end{array}$$

1. Consider the work shown. Share your thinking with your partner.

a Describe how the solution for y is determined.

b The solution for y is substituted into the first equation of the system, giving the solution $x = 1$. If y is instead substituted into the second equation of the system, is $x = 1$ still the solution? Explain or show your thinking.

c Is the pair of values for x and y a solution to the system? Explain your thinking.

2. Do you think this strategy would work for the following two systems?

- If *yes*, use the strategy to determine the solution.
- If *no*, explain how you would solve the system. Then determine the solution.

a $\begin{cases} 2x + y = 4 \\ x - y = 11 \end{cases}$

b $\begin{cases} 8x + 11y = 37 \\ 8x + y = 7 \end{cases}$

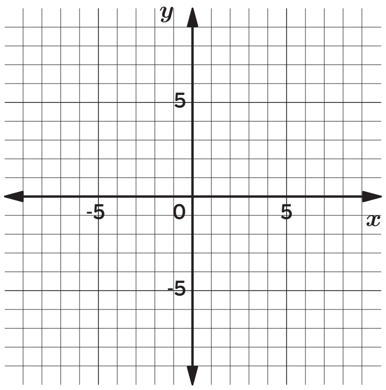
Activity 2 Adding and Subtracting Equations to Solve Systems

You will be assigned one of the three systems of linear equations from Activity 1.

My assigned system of linear equations is System ....

System A	System B	System C
$\begin{cases} 4x + 3y = 10 \\ -4x + 5y = 6 \end{cases}$	$\begin{cases} 2x + y = 4 \\ x - y = 11 \end{cases}$	$\begin{cases} 8x + 11y = 37 \\ 8x + y = 7 \end{cases}$

1. Graph the two equations from your assigned system. Identify the coordinates of the solution to the system.



2. Add or subtract the two equations in your system to obtain a new equation in which one of the variables, x or y, has been eliminated, meaning the variable does not appear in your new equation. Note which variable you eliminated.

3. Graph your equation from Problem 2 on the same graph used in Problem 1. What do you notice about this graph? Explain your thinking.

Name: ______ Date: ______ Period: ______

Activity 3 Which Strategy Is Most Efficient?

Study the systems of linear equations. Respond to each problem on your own. Then discuss your thinking with your partner. Decide whether you agree or disagree with your partner, supporting your thinking with evidence. Move on to the next problem after you reach a shared agreement.

System A	System B	System C
$\begin{cases} 3x + y = 71 \\ 2x - y = 30 \end{cases}$	$\begin{cases} 4x + y = 1 \\ y = -2x + 9 \end{cases}$	$\begin{cases} 5x + 4y = 15 \\ 5x + 11y = 22 \end{cases}$

1. Which system(s) is most efficiently solved using substitution? Explain or show your thinking.

2. Which system(s) is most efficiently solved using elimination by addition? Explain or show your thinking.

3. Which system(s) is most efficiently solved using elimination by subtraction? Explain or show your thinking.

Reflect: How will you show respect for your partner while discussing how you solved the systems, especially if you disagree?

STOP

Summary

In today's lesson . . .

You learned another strategy for solving systems of linear equations algebraically called ***elimination***. Just like in substitution, the goal is to eliminate one variable so you can solve for the other variable. Using elimination, one variable is eliminated by either adding or subtracting the equations in the system. This creates a new equation that can be used to solve for the other variable.

You graphed the third equation created from elimination and saw that the intersection of the three equations was the solution to the system.

You analyzed different systems of linear equations and determined which strategy was most efficient for solving them.

- Substitution is efficient when a system has an equation where a variable is already isolated.
- Elimination by adding is efficient when a system has one equation containing a term whose coefficient is the opposite of the coefficient in the other equation in the system.
- Elimination by subtracting is efficient when a system has two equations with exactly the same term.

Reflect:

Practice

Name: ______________________ Date: ____________ Period: __________

1. Which equation is the result of adding these two equations?

$$\begin{cases} -2x + 4y = 17 \\ 3x - 10y = -3 \end{cases}$$

A. $-5x - 6y = 14$ **B.** $-x - 6y = 14$ **C.** $x - 6y = 14$ **D.** $5x + 14y = 20$

2. Solve the following system of equations without graphing.

$$\begin{cases} 5x + 2y = 29 \\ 5x - 2y = 41 \end{cases}$$

3. Which strategy would be most efficient for solving this system of equations? Explain your thinking.

$$\begin{cases} 6x + 21y = 103 \\ -6x + 23y = 51 \end{cases}$$

4. Solve each system of equations. Show your thinking.

a $\begin{cases} 2x + 3y = 2 \\ 2x = 8y + 24 \end{cases}$

b $\begin{cases} 5x + 3y = 23 \\ 3y = 15x - 21 \end{cases}$

Practice

Name: .... Date: .... Period: ....

5. Kiran sells f full boxes and h half-boxes of fruit to raise money for a band trip. He earns $\$5$ for each full box and $\$3$ for each half-box of fruit he sells and earns a total of $\$100$ toward the cost of his band trip. The equation $5f + 3h = 100$ describes this relationship. Solve the equation for f.

6. Elena and Kiran are playing a board game. After one round, Elena says, "You earned so many more points than I did. If you had earned 5 more points, your score would be twice mine!" Kiran says, "Oh, I don't think I did that much better. I only scored 9 points more than you did."

a Write a system of equations to represent each student's comment. Be sure to specify what each variable represents.

b If both students were correct, how many points did each student score? Show or explain your thinking.

Name: ________ Date: ________ Period: ________

Unit 1 | Lesson 20

Solving Systems by Elimination: Adding and Subtracting (Part 2)

Let's think about why adding and subtracting equations works for solving systems of linear equations.

Warm-up True or False?

Study the equation: $50 + 1 = 51$.

1. Perform each of the following operations. What is the new equation? Is it still true?

   a. Add 12 to each side of the equation.

   b. Add $10 + 2$ to the left side of the equation and 12 to the right side.

   c. Add the sides of the equation $4 + 3 = 7$ to the corresponding sides of the equation $50 + 1 = 51$.

2. Determine an equation to add to $50 + 1 = 51$ that results in a sum that is true. Write the sum of the two equations here. Why is the sum true?

Log in to Amplify Math to complete this lesson online.

Activity 1 Apples and Oranges

Kiran plans a weekend camp for children at the community center. He budgets \$70 for healthy snacks. He first purchases 7 bags of apples and 5 bags of oranges. Kiran needs more snacks. He purchases 3 more bags of apples and 1 more bag of oranges.

Szasz-Fabian Ilka Erika/Shutterstock.com

design56/Shutterstock.com

The following system represents the constraints in this situation, where a and b represent the unit price of a bag of apples and oranges, respectively:

$$\begin{cases} 7a + 5b = 51.73 \\ 3a + b = 15.13 \end{cases}$$

1. Use the system to complete the following problems.

a. What do the solutions to the first equation represent?

b. What do the solutions to the second equation represent?

c. How many possible solutions are there for each equation? Explain your thinking.

d. What does the solution to this system of equations represent?

e. Determine the solution to the system. Explain or show your thinking.

Name: ______ Date: ______ Period: ______

Activity 1 Apples and Oranges (continued)

2. Kiran wants to be reimbursed for the cost of the snacks. He records, "Items purchased: 10 bags of apples and 6 bags of oranges. Amount: $\$66.86$" in a ledger.

a Write an equation to represent the relationship between the number of bags of apples and oranges purchased, the prices of each, and the total amount spent. Show your thinking.

b How is this equation related to the first two equations?

c In this situation, what do the solutions of this equation represent?

d How many possible solutions does this equation have? How many solutions make sense in this situation? Explain your thinking.

Are you ready for more?

This system has three equations.

$$\begin{cases} 3x + 2y - z = 7 \\ -3x + y + 2z = -14 \\ 3x + y - z = 10 \end{cases}$$

1. Add the first two equations to get a new equation.
2. Add the second two equations to get a new equation.
3. Solve the system of your two new equations.
4. What is the solution to the original system of equations?
5. If two variables describe a line, what do three variables describe? And, what do the solution values of x, y, and z describe?

Activity 2 A Bunch of Systems

Solve each system of linear equations without graphing. Check your solutions. Explain or show your thinking.

1. $\begin{cases} 2x + 3y = 7 \\ -2x + 4y = 14 \end{cases}$

2. $\begin{cases} 2x + 3y = 7 \\ 3x - 3y = 3 \end{cases}$

3. $\begin{cases} 2x + 3y = 5 \\ 2x + 4y = 9 \end{cases}$

4. $\begin{cases} 2x + 3y = 16 \\ 6x - 5y = 20 \end{cases}$

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You further developed your understanding of solving systems of linear equations by elimination using addition or subtraction. You examined the third equation formed by combining the original equations in a system to understand why that equation shares a solution with the system.

Why do these strategies work? Remember that an equation is a statement that says two things are equal. As long as you add or subtract an equal amount to both sides of a true equation, the two sides of the resulting equation will remain equal. You can use the same reasoning for adding or subtracting entire equations in a system. This is why adding or subtracting two equations in a system results in a new equation that is also true.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Solve this system of linear equations without graphing: $\begin{cases} 5x + 4y = 8 \\ 10x - 4y = 46 \end{cases}$
Show your thinking.

2. Select *all* the equations that share a solution with this system of equations. Explain your thinking.

$$\begin{cases} 5x + 4y = 24 \\ 2x - 7y = 26 \end{cases}$$

A. $7x + 3y = 50$ **B.** $7x - 3y = 50$ **C.** $3x - 11y = -2$ **D.** $3x + 11y = -2$

3. Students performed their one-act play as an online webcast on a Friday and a Saturday. For both performances, adults donated a dollars each and students donated s dollars each. On Friday, they received 125 adult donations and 65 student donations, totaling $\$1,200$. On Saturday, they received 140 adult donations and 50 student donations totaling $\$1,230$. This scenario is represented by this system of equations:

$$\begin{cases} 125a + 65s = 1200 \\ 140a + 50s = 1230 \end{cases}$$

a What could the equation $265a + 115s = 2430$ mean in this scenario?

b The solution to the original system is the pair $a = 7$ and $s = 5$. Explain why it makes sense that this pair of values is also the solution to the equation $265a + 115s = 2430$.

Name: ______________ Date: ______________ Period: ______________

Practice

4. Solve this system of equations: $\begin{cases} y + 3x = 14 \\ 3x - 5y = -4 \end{cases}$
Show your thinking.

5. Solve each system of equations. Show your thinking.

a $\begin{cases} 7x + 12y = 180 \\ 7x = 84 \end{cases}$

b $\begin{cases} -16y = 4x \\ 4x + 27y = 11 \end{cases}$

6. How could you solve this system of equations using elimination? What could you do first to one equation in order to eliminate a variable when adding the equations? Explain your thinking.

$\begin{cases} x + y = 12 \\ 3x - 5y = 4 \end{cases}$

Solving Systems by Elimination: Multiplying

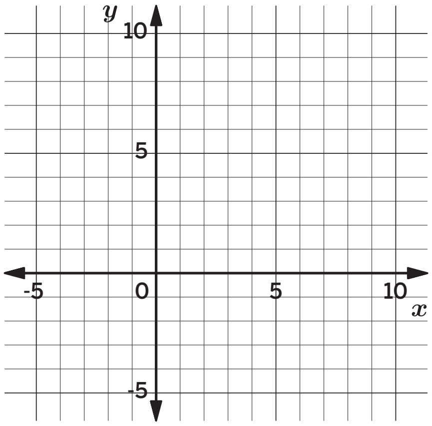
Let's investigate how multiplying equations by a factor can help us solve systems of linear equations.

Warm-up Equivalent Equations

Consider this system of equations:

$$\begin{cases} 4x + y = 1 \\ x + 2y = 9 \end{cases}$$

1. Use graphing technology to graph the system of equations. Sketch the graph. Then identify the coordinates of the solution.



2. For the equation $4x + y = 1$, multiply each side by the factor shown. Write the resultant equation.

   a. 2

   b. 5

   c. $\frac{1}{2}$

3. Graph your equations from Problem 2 on your graph from Problem 1. What do you notice about the graphs?

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Which Variable?

Here is the system you solved by graphing in the Warm-up: $\begin{cases} 4x + y = 1 \\ x + 2y = 9 \end{cases}$

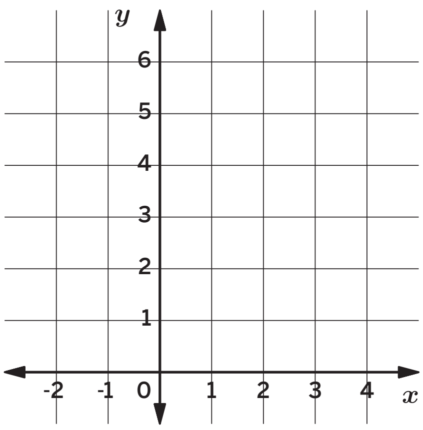
Partial work for two possible approaches to solving the system are shown.

Work Sample 1:	Work Sample 2:
$-2(4x + y = 1) \Rightarrow -8x - 2y = -2$	$4x + y = 1 \qquad 4x + y = 1$
$x + 2y = 9 \qquad x + 2y = 9$	$-4(x + 2y = 9) \Rightarrow -4x - 8y = -36$

1. Compare the work samples. What is happening in the first step of each work sample? Why are these steps possible? Explain your thinking.

2. Select one work sample to complete, and your partner will complete the other work sample. Compare your solutions. What do you notice? Explain your thinking.

3. Graph all the original systems of linear equations, and the two systems from the work samples. What do you notice about the graphs? Explain or show your thinking.



Activity 2 Building Equivalent Systems

Tyler was asked to solve this system of equations: $\begin{cases} 12a + 5b = -15 \\ 8a + b = 11 \end{cases}$

1. These were his first two steps:

Step 1:	**Step 2:**
$\begin{cases} 12a + 5b = -15 \\ -40a - 5b = -55 \end{cases}$	$\begin{cases} 12a + 5b = -15 \\ -28a \quad = -70 \end{cases}$

What operations did Tyler use to create each equivalent system of equations? Do the systems in Step 1 and Step 2 have the same solution as the original system? Explain your thinking.

2. Tyler's approach eliminates b. What operations would you use to eliminate a? Show or explain your thinking.

3. Use your equivalent system of equations from Problem 2 to solve the original system. Check your solution by substituting the pair of values into the original system.

Name: .... Date: .... Period: ....

Activity 3 Card Sort: What Comes Next?

You will be given cards with equivalent systems of linear equations written on them. Each system represents a step in solving the following system.

$$\begin{cases} \frac{4}{5}x + 6y = 15 \\ -x + 18y = 11 \end{cases}$$

1. Arrange the systems in the order that would lead to a solution, and describe how each system of equations was created from the system in the previous step. Record your responses on the cards.

2. Using graphing technology, graph the systems for as many steps as you can. What do you notice about the systems of linear equations? Explain your thinking.

Are you ready for more?

The following system of equations has the solution (5, –2).

$$\begin{cases} Ax - By = 24 \\ Bx + Ay = 31 \end{cases}$$

Find the missing values, A and B.

STOP

Summary

In today's lesson . . .

You learned that sometimes solving a system of equations by elimination requires multiple steps before you can add or subtract to eliminate a variable.
The operations performed in each step form an ***equivalent system*** of equations.

$$\begin{cases}2x + 3y = 15 \\ 3x - 9y = 18\end{cases} \rightarrow \begin{cases}6x + 9y = 45 \\ 3x - 9y = 18\end{cases}$$

These steps include:

- Multiplying both sides of an equation by the same factor and then applying the distributive property.
- Adding or subtracting the equations in a system.

While the equations may change with every step, the system will always have the same point of intersection, or solution, when graphed.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Solve each system of equations. Show your thinking.

a $\begin{cases} 2x - 4y = 10 \\ x + 5y = 40 \end{cases}$

b $\begin{cases} 3x - 5y = 4 \\ -2x + 6y = 18 \end{cases}$

2. Consider these potential strategies for solving the following system.

$\begin{cases} 4p + 2q = 62 \\ 8p - q = 59 \end{cases}$

- Multiply $4p + 2q = 62$ by 2. Then subtract $8p - q = 59$ from the result.
- Multiply $8p - q = 59$ by 2. Then add the result to $4p + 2q = 62$.

Do both strategies work for solving the system? Explain or show your thinking.

3. Select all systems that are equivalent to this system.

$\begin{cases} 6d + 4.5e = 16.5 \\ 5d + 0.5e = 4 \end{cases}$

A. $\begin{cases} 6d + 4.5e = 16.5 \\ 45d + 4.5e = 4 \end{cases}$

B. $\begin{cases} 6d + 4.5e = 16.5 \\ 6d + 0.6e = 4.8 \end{cases}$

C. $\begin{cases} 30d + 22.5e = 82.5 \\ 30d + 3e = 24 \end{cases}$

D. $\begin{cases} 30d + 22.5e = 82.5 \\ 5d + 0.5e = 4 \end{cases}$

E. $\begin{cases} 12d + 9e = 33 \\ 10d + 0.5e = 8 \end{cases}$

F. $\begin{cases} 6d + 4.5e = 16.5 \\ 11d + 5e = 20.5 \end{cases}$

Practice

Name: ________ Date: ________ Period: ________

4. The cost to mail a package is $\$5$. Noah has postcard stamps that are worth $\$0.34$ each and First Class stamps that are worth $\$0.49$ each.

a Write an equation that relates the number of postcard stamps p, the number of First Class stamps f, and the cost of mailing the package.

b Solve the equation for f.

c Solve the equation for p.

d If Noah puts 7 First Class stamps on the package, how many postcard stamps will he need?

5. Priya buys 2.4 lb of bananas and 3.6 lb of grapes for $\$9.38$ at a grocery store. At the same grocery store, Andre buys 1.2 lb of bananas and 1.8 lb of grapes for $\$4.69$. This information can be represented by the following system of equations:

$$\begin{cases} 2.4b + 3.6g = 9.38 \\ 1.2b + 1.8g = 4.69 \end{cases}$$

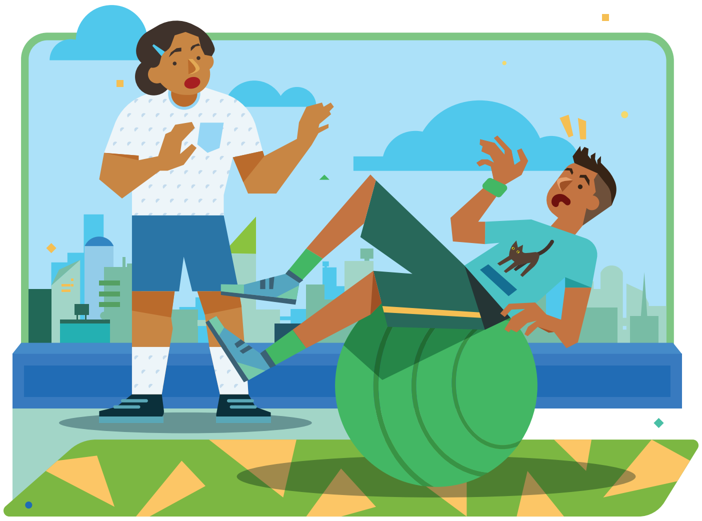
What happens when you try to solve the system of equations using elimination? Explain or show your thinking.

Name: ______________ Date: ______________ Period: ______________

Unit 1 | Lesson 22

Systems of Linear Equations and Their Solutions

Let's determine how many solutions there are to a system of linear equations.



Warm-up A Curious System

Consider the following system of equations.

$$\begin{cases} x + y = 3 \\ 4x = 12 - 4y \end{cases}$$

1. Choose any two numbers that add up to 3. Let the first number be the value for x and the second number be the value for y.

2. The pair of numbers you chose is a solution to $x + y = 3$. Determine if the pair is also a solution to the second equation in the system. Share your thinking with your group.

3. How many solutions does this system have? Explain or show your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Gym Membership and Personal Training

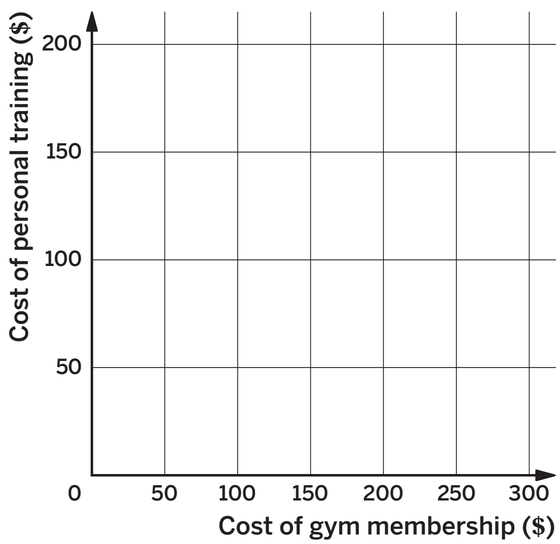
Clare wants to be financially responsible and looks for the best gym option. She settles on two gyms and compares their options.

- **Gym A offers 4 months and 2 personal training sessions for $342.**
- **Gym B offers 2 months and 1 personal training session for $190.**

1. Write a system of equations that represents the relationships between each gym membership, personal training sessions included, and the costs. Be sure to define what each variable represents.

2. Determine the cost of a gym membership and one personal training session by solving your system in Problem 1, algebraically. Explain or show your thinking.

3. Use graphing technology to graph the system. What do you notice about your graphs?



4. Study the system of equations you wrote in Problem 1. What do you notice about the structure of the equations?

Name: .... Date: .... Period: ....

Activity 2 Card Sort: Sorting Systems

You will be given a set of cards. Sort them into the following three groups: *one solution*, *infinitely many solutions*, and *no solution*. Be prepared to explain your thinking.

One solution	Infinitely many solutions	No solution

1. What do you notice about systems of linear equations that have:

   a. One solution?

   b. Infinitely many solutions?

   c. No solution?

Are you ready for more?

1. For each system with one solution, change a single constant term so that there are infinitely many solutions to the system.

2. For each system with infinitely many solutions, change a single constant term so that there are no solutions to the system.

3. Explain why it is impossible to change a single constant term so that there is exactly one solution to a system that originally has no solution or infinitely many solutions.

Activity 3 One, Zero, Infinitely Many

For each given equation, create a second equation that would make a system of equations with one solution, no solution, and infinitely many solutions. Use graphing, substitution, or elimination to verify your thinking.

1. $5x - 2y = 10$

   a. System with one solution:

   b. System with no solution:

   c. System with infinitely many solutions:

Name: ________ Date: ________ Period: ________

Activity 3 One, Zero, Infinitely Many (continued)

2. $y = \frac{1}{3}x - 4$

   a. System with one solution:

   b. System with no solution:

   c. System with infinitely many solutions:

STOP

Summary

In today's lesson . . .

You learned that some systems of linear equations do not always have one solution.

- Some systems have no solution and the equations in these systems have the same slope but different y-intercepts. When a system of linear equations has no solutions, the graph is represented by two parallel lines, or lines that never intersect.
- Other systems of linear equations have infinitely many solutions and each equation in these systems have the same slope and y-intercept and are equivalent equations. When a system of linear equations has infinitely many solutions, the graph is represented by two identical lines.

	Graphs: What are the characteristics of the graphs of the equations in the system?	**Equations:** What are the characteristics of the equations in the system?	**Solution:** What happens when you solve the system algebraically?
One solution	The graphs intersect in one point.	The slopes are different. The y-intercepts are different.	There is one ordered pair that is a solution to each equation.
No solution	The graphs are parallel and do not intersect.	The slopes are the same. The y-intercepts are different.	You arrive at a false statement, such as $2 = 3$.
Infinitely many solutions	The graphs intersect in infinitely many points. They are the same line.	The slopes are the same. The y-intercepts are the same.	You arrive at a statement that is always true, such as $5 = 5$.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Consider the following system of equations:

$$\begin{cases} y = \frac{4}{5}x - 3 \\ y = \frac{4}{5}x + 1 \end{cases}$$

a. Without graphing, determine how many solutions you would expect this system of equations to have. Explain your thinking.

b. Describe the result if you tried solving this system algebraically.

2. How many solutions does this system of equations have? Explain your thinking.

$$\begin{cases} 9x - 3y = -6 \\ 5y = 15x + 10 \end{cases}$$

3. Select *all* systems of linear equations that have no solutions.

A. $\begin{cases} y = 5 - 3x \\ y = -3x + 4 \end{cases}$

B. $\begin{cases} y = 4x - 14 \\ y = 16x - 4 \end{cases}$

C. $\begin{cases} 5x - 2y = 3 \\ 10x - 4y = 6 \end{cases}$

D. $\begin{cases} 3x + 7y = 42 \\ 6x + 14y = 50 \end{cases}$

E. $\begin{cases} y = 5 + 2x \\ y = 5x + 2 \end{cases}$

Practice

Name: ______ Date: ______ Period: ______

4. Solve each of the following systems of linear equations without graphing. Show your thinking.

a. $\begin{cases} 2v + 6w = -36 \\ 5v + 2w = 1 \end{cases}$

b. $\begin{cases} 6t - 9u = 10 \\ 2t + 3u = 4 \end{cases}$

5. Select *all* ordered pairs that are solutions to the inequality $-3x + 6y \geq 10$.

A. $(0, 1)$

B. $(1, 0)$

C. $(2, 4)$

D. $(4, 2)$

E. $(1, 5)$

F. $(-4, 4)$

6. Study the system linear inequalities. Compare this system to other systems you have studied so far. What do you notice or wonder about it? What might the graph of this system look like?

$$\begin{cases} y < 2x + 1 \\ y \geq -x - 3 \end{cases}$$

SUB-UNIT

5 Systems of Linear Inequalities in Two Variables

Is there such a thing as *too much* choice?

Next time you're at the supermarket, take a walk down the cereal aisle.

There, you will find dozens of boxes in a wide assortment of colors, flavors, and mascots. Some will have marshmallows, others whole-grain oats, puffed corn kernels, or almond clusters. There are cereals shaped like animals, hearts, O's, or squares. With the vast number of choices at your disposal, a shopper could be excused for feeling downright exhausted.

Some psychologists call this phenomenon "overchoice," or "choice overload." They believe that whenever a person is confronted with too many options, their mind can become overwhelmed by the risks and outcomes should they make the wrong choice.

As far as cereals go, the stakes aren't exactly high. But it's a different story altogether when it comes to things like choosing a career or picking a college major. These decisions can lead to an individual to agonize, procrastinate, or be unmotivated, to the point of not making any choice at all.

The key is being able to whittle down your choices, taking into account constraints. By putting boundaries around what might feel like a sea of options, you can reduce them to a size that is manageable. One way to do this is to express your constraints mathematically, such as with a system of inequalities. The more constraints you have, the more you can narrow your options and avoid the trap of overchoice.

Unit 1 | Lesson 23

Graphing Systems of Linear Inequalities

Let's solve problems by graphing systems of inequalities in two variables.

Warm-up A Riddle

Study these clues.

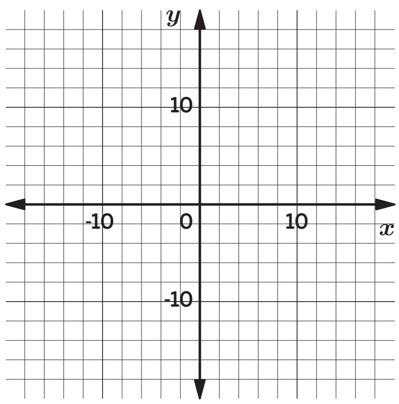
Clue 1: The sum of two numbers is at least 6.
Clue 2: The difference of two numbers is at most 10.

1. Name any pair of numbers whose sum is 6.

2. Name any pair of numbers whose difference is 10.

3. Write an inequality for each of the clues.

4. Graph your inequalities for Problem 3 on the same coordinate plane.



5. What are two possible coordinate points that satisfy the system? Use your graph to help explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

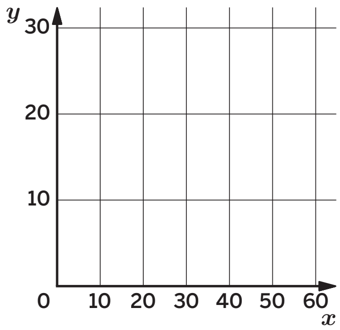
Activity 1 Scavenger Hunt

The high school math club hosts a scavenger hunt. The members hide three items in a rectangular park that measures 50 m by 20 m. Clues about the locations of the items are written as systems of inequalities. One of the clues has no solutions. Solving the systems by graphing will reveal where each item could be hidden.

1. Graph each system of inequalities.

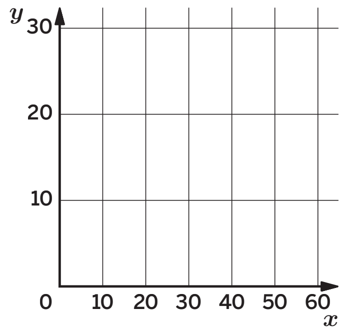
a **Clue 1:**

$$\begin{cases} y \geq 14 \\ x \leq 10 \end{cases}$$



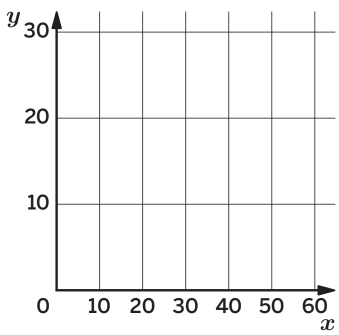
b **Clue 2:**

$$\begin{cases} x + y < 20 \\ x \geq 6 \end{cases}$$



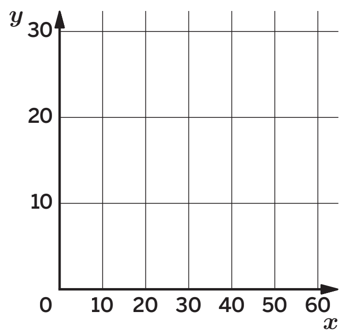
c **Clue 3:**

$$\begin{cases} y < -2x + 10 \\ y < -2x + 10 \end{cases}$$



d **Clue 4:**

$$\begin{cases} y \geq x + 10 \\ x > y \end{cases}$$



2. Using your graphs, where could each of the items be hidden? Explain your thinking.

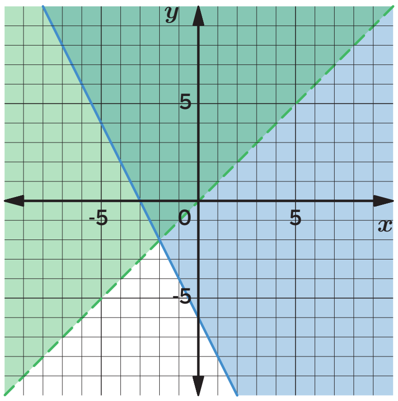
3. Which system has no solutions? Explain your thinking.

4. If possible, give one coordinate point for each system that could be a solution. Explain your thinking.

Activity 2 Focusing on the Details

Given the system of linear inequalities and the corresponding graph, decide whether each ordered pair is a solution to the system. Explain or show your thinking.

$$\begin{cases} x < y \\ y \geq -2x - 6 \end{cases}$$



1. $(3, -5)$

2. $(0, 5)$

3. $(-6, 6)$

4. $(3, 3)$

5. $(-2, -2)$

6. $(5, 20)$

Critique and Correct: Your teacher will present you with an incorrect statement. WIth your partner, determine why it is incorrect and then correct it.

Name: .... Date: .... Period: ....

Activity 3 Info Gap: Terms of a Team

You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given the *data card*:	If you are given the *problem card*:
1. Silently read the information on your card.	**1.** Silently read your card and think about what information you need to answer the problem.
2. Ask your partner, "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card.	**2.** Ask your partner for the specific information that you need.
3. Before telling your partner the information, ask, "Why do you need to know (that piece of information)?"	**3.** Explain to your partner how you are using the information to solve the problem.
4. Read the problem card, and solve the problem independently.	**4.** When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Share the data card, and discuss your thinking.	**5.** Read the data card, and discuss your thinking.

STOP

Summary

In today's lesson . . .

You determined that the solutions to a ***system of linear inequalities*** are the ordered pairs that satisfy both inequalities at the same time. Graphically, this is represented by the ***overlap of the graphs of the inequalities***, where the graphs of the individual inequalities in the system overlap.

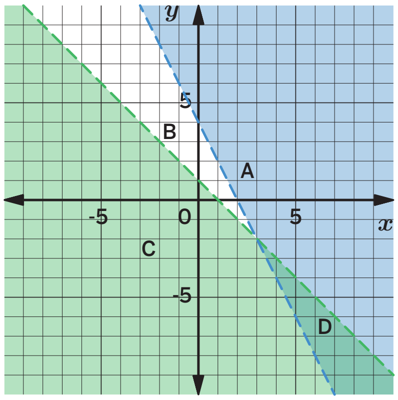
You also learned how to determine whether ordered pairs on the boundary lines of the solution region are included in the solution. If an ordered pair is on a solid line as part of the solution region, then the ordered pair is a solution. If an ordered pair is on a dashed line as part of the solution region, then the ordered pair is not a solution.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. The graph represents a system of linear inequalities. Which region represents the solution to the system? Explain your thinking.



2. Select all points that are solutions to the following system of inequalities.

$$\begin{cases} y \le -2x + 6 \\ x - y < 6 \end{cases}$$

A. $(0, 0)$

B. $(-5, -15)$

C. $(4, -2)$

D. $(3, 0)$

E. $(10, 0)$

F. $(10, 10)$

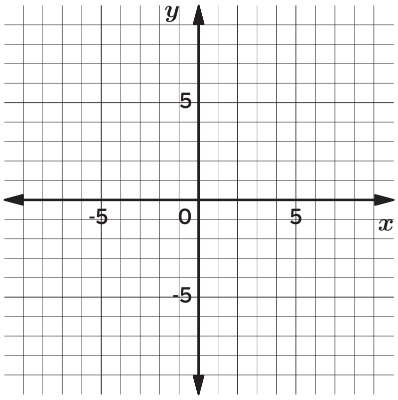
3. Consider the following system of inequalities.

$$\begin{cases} y < -3x + 9 \\ y < 3x - 9 \end{cases}$$

a. Graph the system of inequalities and shade the solution region.

b. Identify a point that is a solution to the system.

c. Are points on the boundary lines of the solution region also solutions? Explain your thinking.



Practice

Name: .... Date: .... Period: ....

4. Which ordered pair is a solution to the inequality $4x - 2y < 22$?

A. $(4, -3)$

B. $(4, 3)$

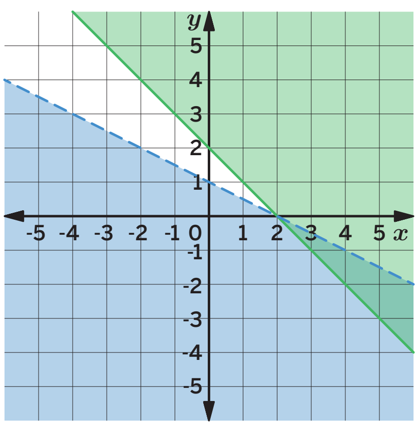
C. $(8, -3)$

D. $(8, 3)$

5. Solve the following system of equations. Show your thinking.

$$\begin{cases} y = 2x - 1 \\ -10x + 5y = -5 \end{cases}$$

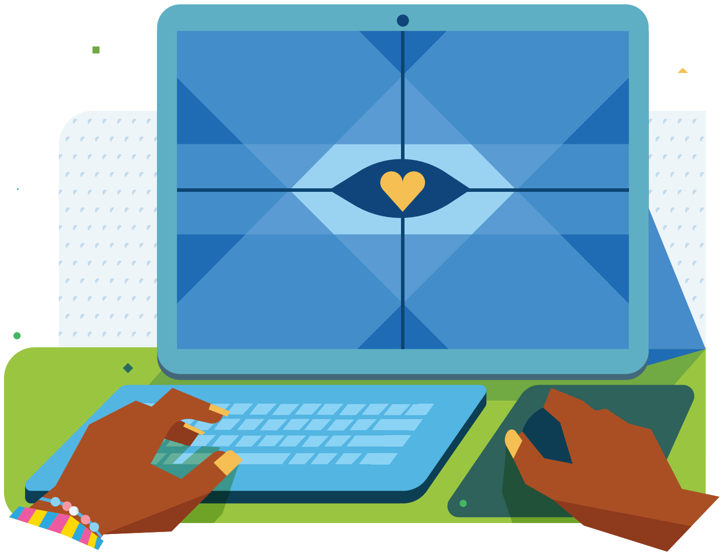
6. Write the system of inequalities represented by the graph.



Name: .... Date: .... Period: ....

Unit 1 | Lesson 24

Solving and Writing Systems of Linear Inequalities



Let's explore the use of systems of linear inequalities in design.

Warm-up Matching the Inequalities to Graphs

Consider the graph of the system of inequalities.

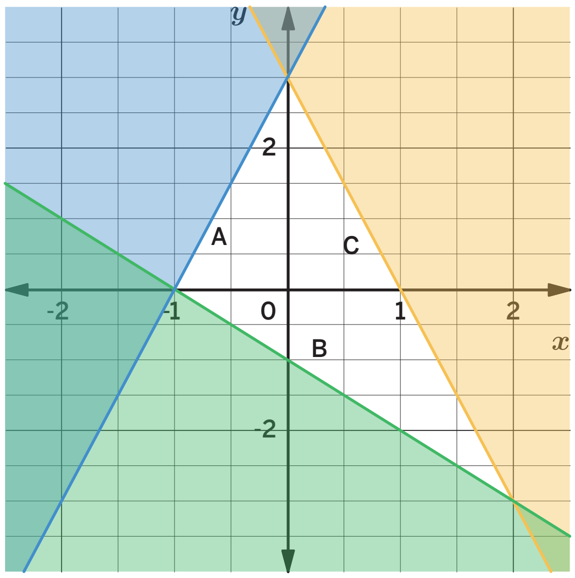
1. Match each graph with the linear inequality that defines it. Not all inequalities will have a matching graph.

   a. Graph A .... $y + 1 \leq -x$

   b. Graph B .... $-y + 3 \leq 3x$

   c. Graph C .... $y \geq -x + 3$

   .... $-3x \geq -y + 3$



2. How do you use the shading of the graph to determine the inequality symbol used to write an inequality?

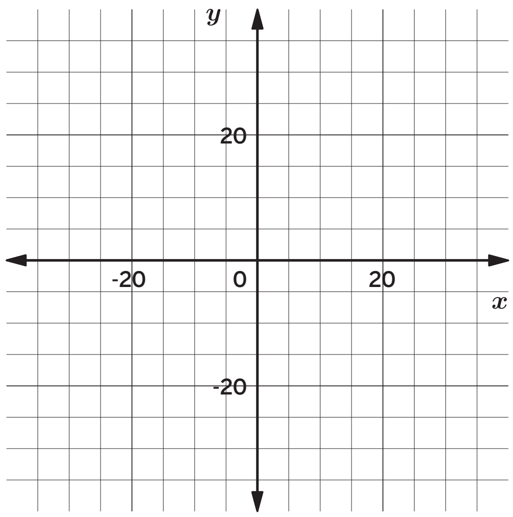
Log in to Amplify Math to complete this lesson online.

Activity 1 Designing a Company Logo

Mai is an entrepreneur, starting her own graphic design firm. She is creating a logo for a software company and uses a system of linear inequalities to design her first draft of the company's logo.

1. Graph the system of inequalities.

$$\begin{cases} y + x \geq 20 \\ y - x \geq 20 \\ -y \geq 20 + x \\ -3y \geq -3x + 60 \\ y \geq 8 \\ y \leq -8 \end{cases}$$



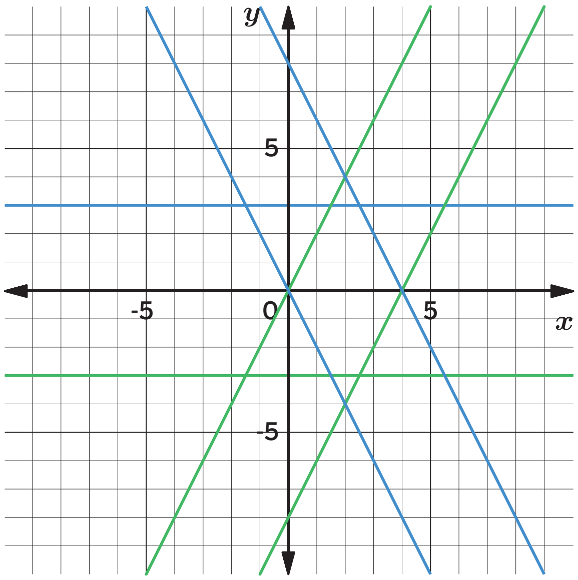
2. Mai realizes that the logo should instead be the unshaded region of the graph you made in Problem 1. How could Mai change her system of inequalities so that her solution is the unshaded region of the original graph?

3. Shade the area you think should represent the logo.

Name: ______ Date: ______ Period: ______

Activity 2 Help Design the Graphic

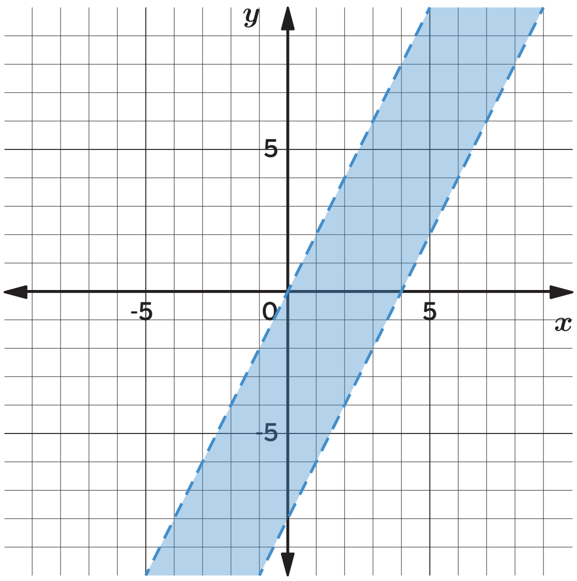
A technology company hires Mai's design firm to design their company website's main graphic. The technology company is not exactly sure what they want the graphic to look like, but, due to space constraints on their website, they provide Mai with this outline for the logo.



1. Write a system of six equations that represent the boundary lines drawn in the outline the company gave to Mai.

Mai starts her design by shading the given region.

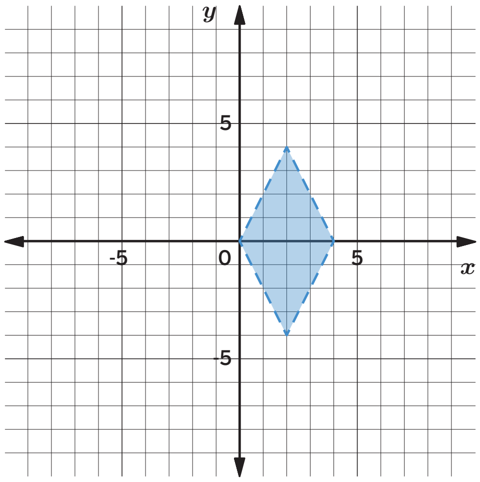
2. Use two of your boundary lines to create a system of inequalities that represents this region.



Activity 2 Help Design the Graphic (continued)

Mai finished her design, but needs help determining the other inequalities that represent her sketch.

3. Create a system of inequalities so that the solution is represented by Mai's shaded region.



Mai feels that the design could be more creative and hires you to use the boundary lines to make your own design.

4. Create your own design by shading the region(s) on the graph.

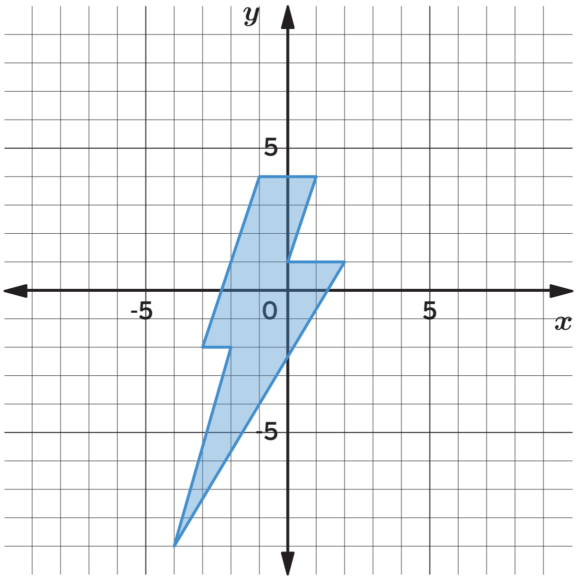
5. Use your equations for the boundary lines and your shaded region to write a system of inequalities that reflects your design.

Name: ________ Date: ________ Period: ________

Activity 3 The Design Plan

A smartphone app company hires Mai's design firm to help with the icon for their new app. The company gives Mai the design already drawn.

Mai wants to make sure her team has a system of inequalities to represent the drawing, in case it gets lost or needs to be quickly copied. She hires you to help.



Write a system of inequalities so that the solution is represented by the lightning bolt.

Use a ruler or a straightedge to extend each side of the lightning bolt to help you determine the boundary line for each inequality in your system.

STOP

Summary

In today's lesson . . .

You graphed systems of linear inequalities that had more than two inequalities. You used the same methods you previously used when there were only two inequalities.

- Graph the boundary line for each inequality.
- Determine whether each boundary line should be dashed or solid.
- Test a point on one side of the boundary line to determine where to shade for that inequality.

You sometimes isolated y in an inequality to identify the slope and y-intercept, which you used to graph the inequality.

You also wrote systems of linear inequalities that represent given graphs. You used a ruler or straightedge to extend boundary lines, which helped you identify their slope and intercept so you could more efficiently write them as inequalities. You then used the shaded region on the graph to determine which inequality symbol to use.

Reflect:

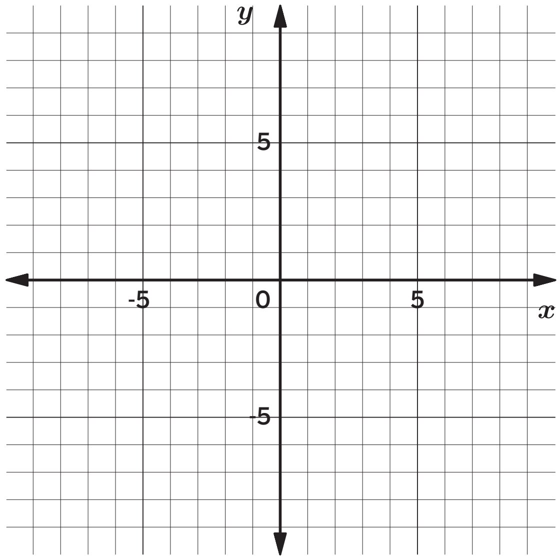
Practice

Name: ______________________ Date: ____________ Period: ____________

1. Consider the following system of inequalities.

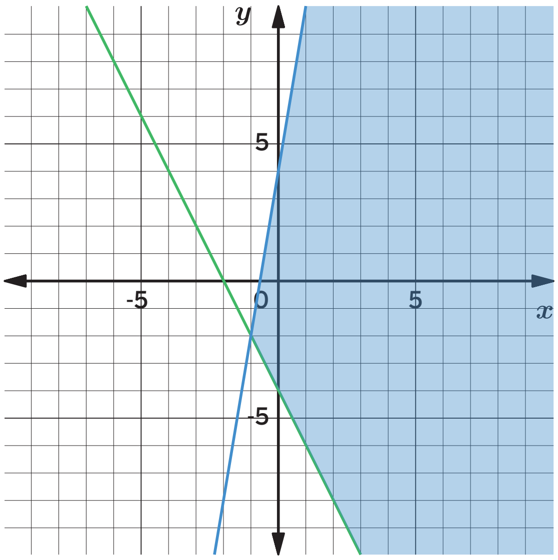
$$\begin{cases} 2x + 3y > 12 \\ -4y < 30 - 8x \end{cases}$$

a Graph the system of inequalities.



b Write two ordered pairs that are solutions to the system. Explain or show how you know the two ordered pairs are solutions.

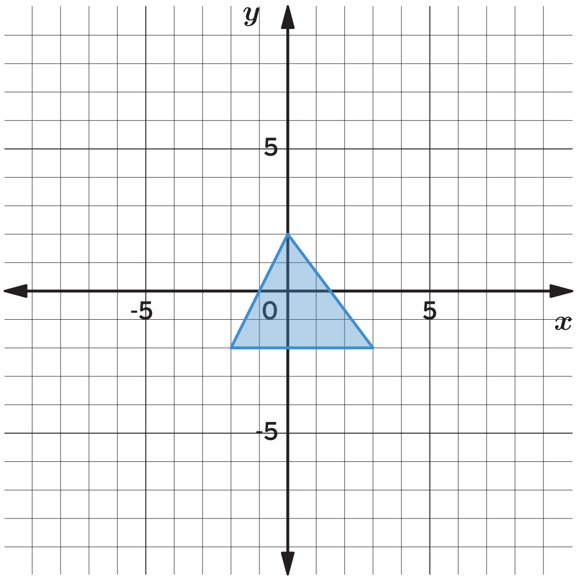
2. Write a system of inequalities whose solution is represented by the shaded region of the graph.



Practice

Name: .... Date: .... Period: ....

3. Write a system of inequalities whose solution is represented by the shaded region of the graph.



4. Elena is solving this system of equations.

$$\begin{cases} 10x - 6y = 16 \\ 5x - 3y = 8 \end{cases}$$

She multiplies the second equation by 2, then subtracts the resulting equation from the first. To her surprise, she gets the equation $0 = 0$. What is special about this system of equations? Why does she get this result, and what does it mean about the solutions?

5. Tyler is making a scarf. Red yarn costs $\$0.07$ per yard and yellow yarn costs $\$0.08$ per yard. He has a budget of $\$60$ but knows he needs at least 600 yd of yarn. Write a system of inequalities representing the number of yards x of red yarn and number of yards y of yellow yarn he could purchase.

Name: ______ Date: ______ Period: ______

Unit 1 | Lesson 25

Modeling With Systems of Linear Inequalities

Let's create mathematical models using systems of linear inequalities.

Warm-up A Solution to Which Inequality?

1. Choose the inequalities for which the ordered pair $(3.5, 1.5)$ is a solution. Be prepared to explain your thinking.

   A. $2x \geq 10 - 2y$

   B. $-4x - 4y < -20$

   C. $-4(x - 2) \geq -12 + 4y$

2. If these three inequalities represent a system, what is the solution(s)? Explain your thinking or show your thinking by graphing.

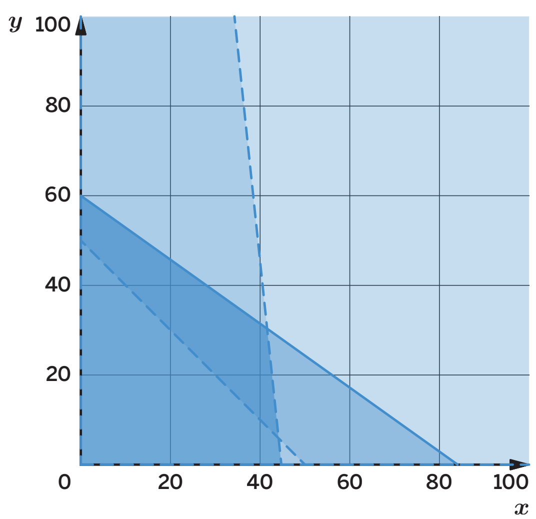
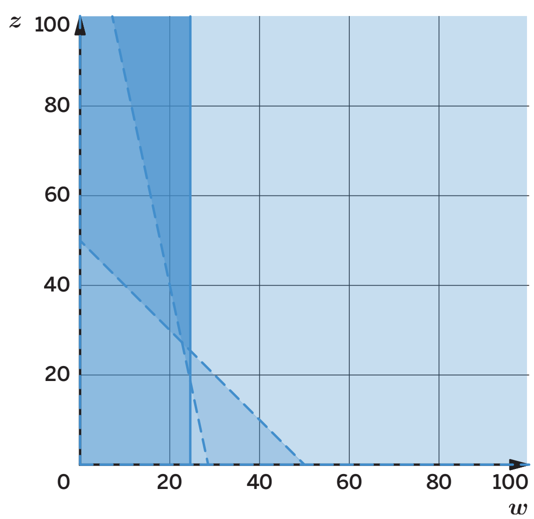
Log in to Amplify Math to complete this lesson online.

Activity 1 Custom Food Bars

Tyler is a food scientist and Elena is an entrepreneur. They have teamed up to found a company aimed at breaking into the health food market. They will each design a custom food bar with two main ingredients.

You will be given a table of the nutrition information of the ingredients from which they can choose.

Tyler and Elena wrote inequalities and created graphs to represent the constraints of their customized food bars.

Tyler	Elena
$\begin{cases} x + y > 50 \\ 4.76x + 6.67y \leq 400 \\ 0.67x + 0.07y < 30 \\ x > 0 \\ y > 0 \end{cases}$	$\begin{cases} w + z > 50 \\ 0.14w + 0.03z > 4 \\ 0.61w \leq 15 \\ w > 0 \\ z > 0 \end{cases}$
	

Name: ____________ Date: ________ Period: ________

Activity 1 Custom Food Bars (continued)

Use the inequalities and graphs from the previous page to respond to these problems about each food bar. Be prepared to explain your thinking.

1. Which two ingredients did they choose?

Tyler:

Elena:

2. What do their variables represent?

Tyler:

Elena:

3. What does each constraint mean?

Tyler:

Elena:

4. Which graph represents which constraint?

Tyler:

Elena:

5. Name one possible combination of ingredients for their food bar.

Tyler:

Elena:

Activity 2 Design Your Own

Plan ahead: What skills will you apply when preparing your summary to make sure you communicate clearly to others?

It's your turn to create your own food bar to compete with Tyler's and Elena's company.

1. Think about the constraints for your food bar.

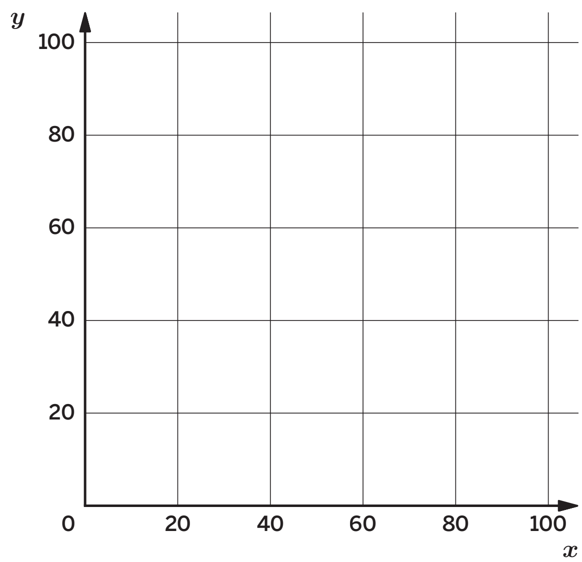
   a. Who is your target audience? *(Bodybuilders, runners, the everyday person, etc.)*

   b. What is the goal of your food bar? *(Gain muscle, lose weight, etc.)*

   c. What do you want to be true about the food bar's calories, protein, sugar, fat, and/or fiber?

2. Choose two main ingredients for your food bar.

3. Write inequalities to represent your constraints. Then graph the inequalities.



4. Is it possible to make a food bar that meets all your constraints using your ingredients? If not, make changes to your constraints.

5. Write a possible combination of ingredients for your food bar.

6. Write a 90-second pitch (a short summary) of your food bar that you could use to convince investors to invest in your company. Highlight the following:
   - The food bar name.
   - The intended customers.
   - The purpose or goal of the food bar.
   - A unique feature of your food bar.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You used a set of data, a system of linear inequalities, and a graph of this system to interpret the constraints used in a mathematical model of a situation. You also used data to create a system of linear inequalities to model your own chosen constraints for a situation.

You used the graph of your system of linear inequalities to determine the solution set that would meet your chosen constraints. A system of linear inequalities can be used to determine which choices or options of a situation meet several constraints.

Reflect:

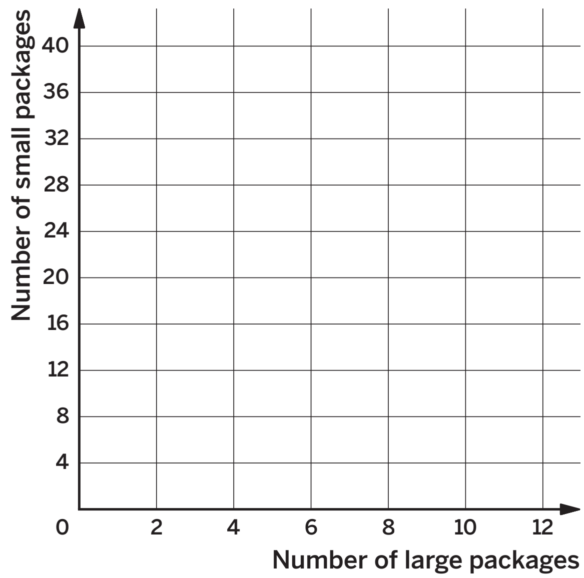
Practice

Name: ______________________ Date: ____________ Period: ____________

1. The organizers of a conference need to prepare at least 200 notepads for the event and have a budget of $\$160$. A store sells notepads in packages of 24 and packages of 6. The following system of inequalities represent these constraints.

$$\begin{cases} 24x + 6y \geq 200 \\ 16x + 5.40y \leq 160 \end{cases}$$

a What does the second inequality in the system tell you about the situation?

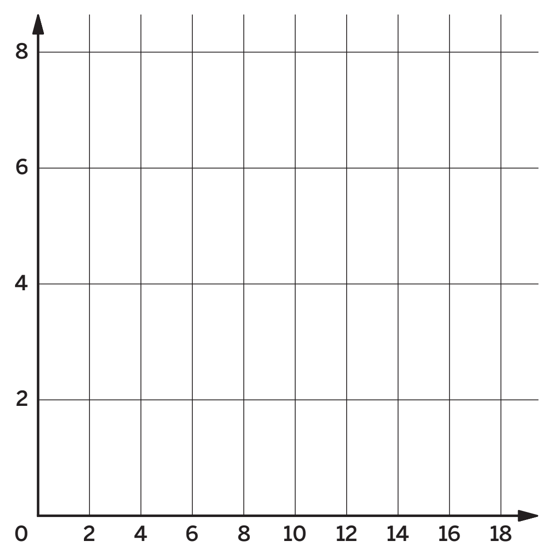


b Graph the solution set to the system of inequalities.

c Determine a possible combination of large and small packages of notepads the organizer could order.

2. A hair stylist charges $\$15$ for a haircut and $\$30$ for hair coloring. A haircut takes 30 minutes, while coloring takes 2 hours. The stylist works up to 8 hours in a day, and she needs to earn a minimum of $\$150$ a day.

a Create a system of inequalities that describe the constraints in this situation. Specify what each variable represents.



b Graph the inequalities and show the solution set.

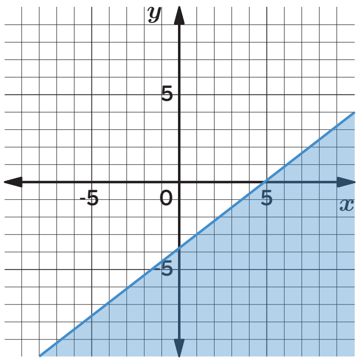
c Identify a point that meets the stylist's requirements.

Name: .... Date: .... Period: ....

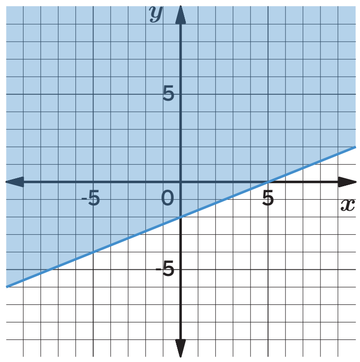
d Identify a point that is a solution to the system but is not possible or not likely in the situation. Explain your thinking.

3. Match each inequality to the graph of its solution.

a



b



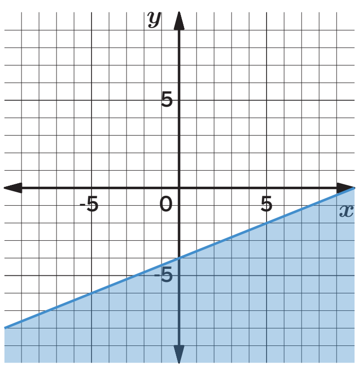
.... $2x - 5y \geq 20$

.... $5x + 2y \geq 20$

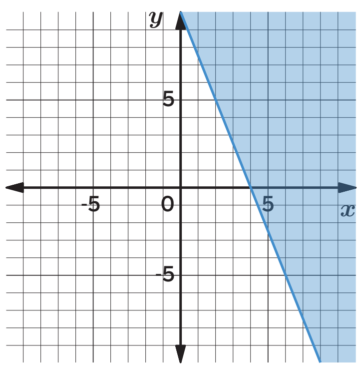
.... $4x - 10y \leq 20$

.... $4x - 5y \geq 20$

c



d



4. Of the order pairs $(-1, 1)$, $(2, 3)$, or $(4, -1)$, which results in the greatest value of the expression $2x - y$?

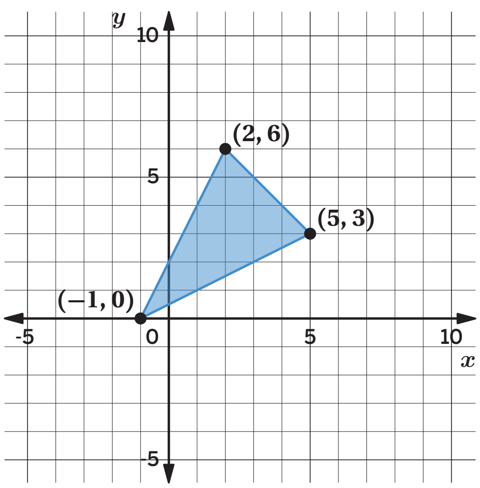
Unit 1 | Lesson 26 – Capstone

Linear Programming

Let's investigate how to maximize time and revenue within given constraints.

Warm-up Staying in Bounds

Consider the following shaded polygon.



Of these six inequalities, determine which three form a system whose solution is the triangle's interior.

A. $y \leq x + 2$	**C.** $y \geq -x + 8$	**E.** $y \geq \frac{1}{2}x + \frac{1}{2}$
B. $y \leq 2x + 2$	**D.** $y \leq -x + 8$	**F.** $y \geq \frac{1}{2}x - 1$

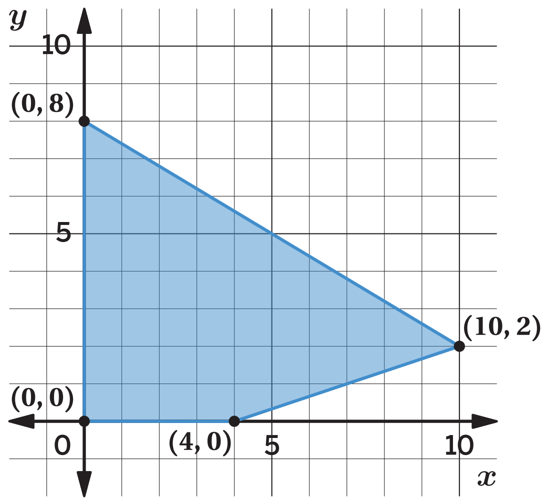
Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Optimal Solutions

Consider the shaded quadrilateral.

1. Determine the system of inequalities that describes the quadrilateral.



2. Your goal is to find the point (x, y) in the quadrilateral (including its edges) with the greatest value of $x + y$.

a Choose any three points in the quadrilateral that you think might yield the maximum value of $x + y$.

Point 1: **Point 2:** **Point 3:**

b Determine the value of $x + y$ for each point.

Point 1: **Point 2:** **Point 3:**

c Describe where each point is located in the quadrilateral.

Point 1: **Point 2:** **Point 3:**

d Which of your three points resulted in the greatest value of $x + y$? Where is the corresponding point located in the quadrilateral?

e Record your points and maximum values in the class table.

Activity 1 Optimal Solutions (continued)

3. Your next goal is to find the point (x, y) in the quadrilateral (including its edges) with the greatest value of $x + 5y$.

a Choose any three coordinates of points within the solution region that you think might yield the maximum value of $x + 5y$.

Point 1: **Point 2:** **Point 3:**

b Explain why you chose each of the three points.

Point 1: **Point 2:** **Point 3:**

c Determine the value of $x + 5y$ for each point.

Point 1: **Point 2:** **Point 3:**

d Which of your three points resulted in the greatest value of $x + 5y$? Where is the point located in the quadrilateral?

4. Make a conjecture about where the maximum value of some expression of x and y will occur for any polygon. Explain your thinking.

Name: ________________ Date: ________ Period: ________

Activity 2 Putting the "Fun" in Fundraising

Inspired by the work of Dr. Warren Washington, Shawn and Bard join a fundraiser at their company to build a garden, which will reduce their company's ecological footprint. Shawn and Bard are tasked with creating designs for two cutouts that people can sign and display if they donate to the fundraiser. Their designs must meet certain constraints.

Your group will be provided with each design, a heart and a star, in order to determine how much time it takes, on average, to trace and cut each design. One student will trace the designs, one student will cut the designs, and the third student will be the timer and recorder. Follow these instructions:

Gathering the data:

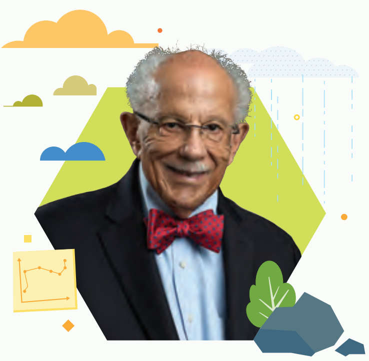
1. One student will trace the heart five times. Do not rush tracing, and be as accurate as possible.
2. A second student will cut out the five hearts. A third student will start the stopwatch when the first tracing starts and stop the stopwatch when the fifth heart is cut out. Record the total cutting time in the table.
3. Repeat these steps for the star.

Calculating the averages:

1. Calculate the average time to trace the heart by dividing the total tracing time by 5. Round to the nearest whole second. Repeat this process for the star. Record these values in the table.
2. Calculate the average time to cut the heart by dividing the total cutting time by 5. Round to the nearest whole second. Repeat this process for the star. Record the values in the table.

Design	Total time to trace and cut (seconds)	$\frac{\text{Total time}}{5}$	Average time (seconds)
Heart			
Star			

Featured Mathematician



Warren Washington

A Nobel Prize winner and presidential advisor, Warren Washington was among the first to develop atmospheric computer models to help scientists understand our climate and predict what could happen to the future of our planet.

Climate modeling combines mathematics and physics to make sense of weather phenomena. Washington's climate models provide the data for climate risk-related decisions being made by policymakers and businesses.

Oregon State University/Flickr. (CC BY-SA 2.0)

Activity 3 Optimizing Revenue

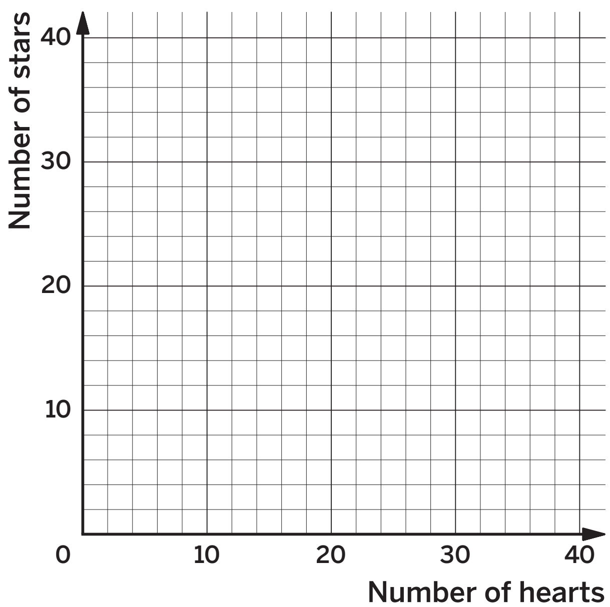
Using the average values for tracing and cutting each design you calculated from the previous activity, along with the following given information, write inequalities to represent the constraints Shawn and Bard must meet if x represents the number of hearts made and y represents the number of stars made:

- **Making fewer than zero of either type of design is not possible.**
- **The total time available to trace and cut both the heart and star cannot exceed 30 minutes. Use the average time for tracing and cutting the heart and star.**
- **The total number of each design that can be made cannot exceed 45.**
- **The revenue made from each heart is $0.75.**
- **The revenue made from each star is $1.50.**

1. Write an equation to represent the revenue made from each design. Be sure to specify what your variables represent.

2. Write a system of inequalities that represents the constraints.

3. Graph the inequalities on the same coordinate plane.



Name: ________ Date: ________ Period: ________

Activity 3 Optimizing Revenue (continued)

4. Determine the intersection coordinates of the boundary lines that represent the constraints. Use your expression from Problem 1 to determine the revenue from each coordinate point, which represents a possible combination of hearts and stars.

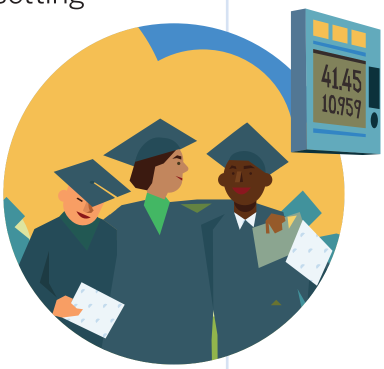
Coordinates	Revenue ($)

5. How many of each type of heart and star should be made to maximize revenue?

STOP

Unit Summary

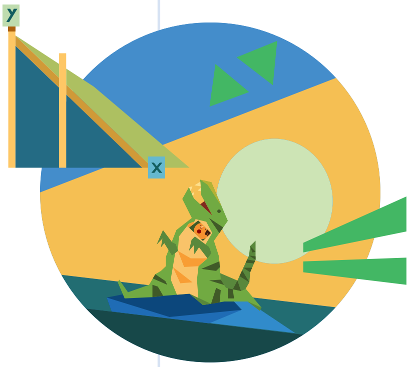
In the months and years ahead, you'll notice increasing amounts of freedom *and* responsibility. From how you spend your money, to choosing a college or setting on a career path, you'll face many decisions that will shape the adult you become.



At times, you will feel overwhelmed by all the choices in front of you:

What are your plans after graduation? Should you take a year off and find yourself? How do you plan on supporting yourself? How will you travel to and from work? Where will you live? Will it be with your parents, or with roommates?

It's easy to get lost in this jungle of possibilities. But remember: To make the best choice, you should be clear about what you want and what you would be willing to give up to get what you most want.



Linear equations and inequalities offer a way to help you wade through the enormity of some of these choices. By identifying your constraints and then breaking down your options into algebraic expressions, you can tease out what you stand to gain or lose in any scenario.

Equipped with that knowledge, there's no telling all the places you'll go . . .

See you in Unit 2.

Name: ______________________ Date: __________ Period: __________

1. Consider the system of linear inequalities.

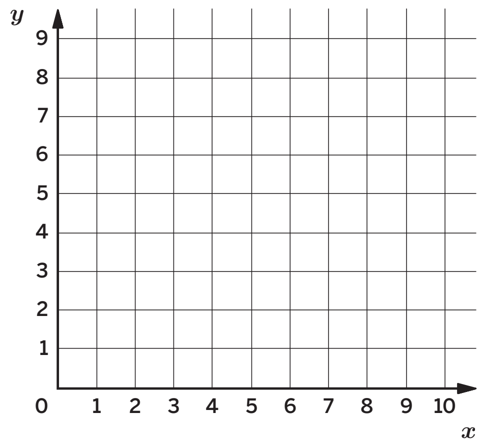
a Graph the solution bounded by the following inequalities.

$$y \geq -4x + 9$$

$$y \geq -\frac{1}{4}x + \frac{3}{2}$$

$$y \leq \frac{2}{3}x + \frac{13}{3}$$

$$y \leq -\frac{7}{2}x + 21$$



b Determine the maximum value of $-4x + 2y$. Show or explain your thinking.

2. Bard has nickels and quarters in their pocket. Bard has 27 coins altogether, worth a total of \$2.75.

a Write a system of equations to represent the relationships between the number of nickels, the number of quarters, and the dollar amount in this situation. Be sure to specify what your variables represent.

b How many nickels and quarters are in Bard's pocket? Show or explain your thinking.

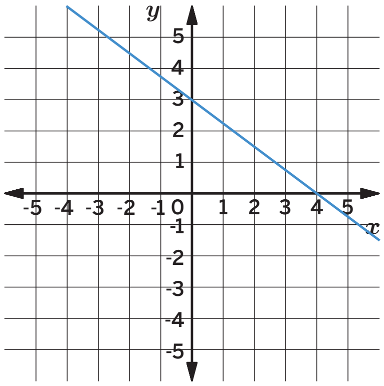
Name: .... Date: .... Period: ....

Practice

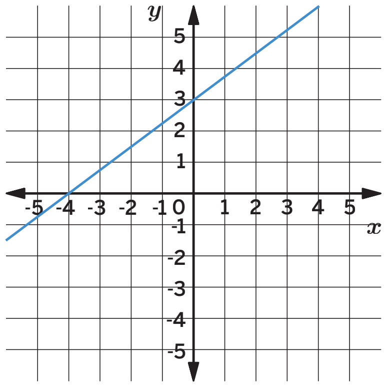
3. Students at the community college are allowed to work on campus no more than 20 hours per week. The jobs that are available pay different rates, starting from \$8.75 an hour. Students can earn a maximum of \$320 per week. Write at least two inequalities that could represent the constraints in this situation. Be sure to specify what your variables represent.

4. Which graph represents the equation $12 = 3x + 4y$? Explain or show your thinking.

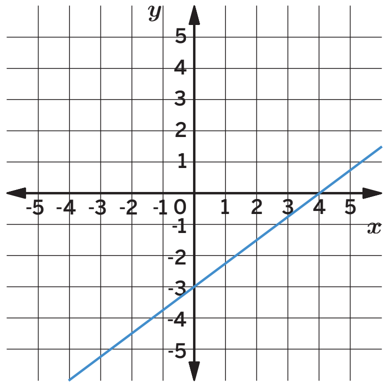
Graph A



Graph B



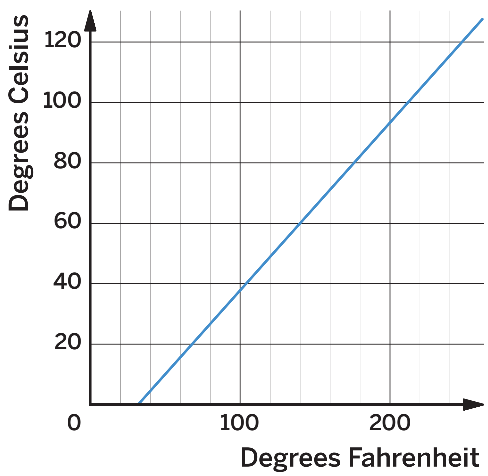
Graph C



5. The graph shows the relationship between temperature in degrees Celsius and temperature in degrees Fahrenheit.

a. Determine the temperature in Celsius when it is $60\ ^\circ\text{F}$.

b. Water boils at $100\ ^\circ\text{C}$. Explain how to use the graph to approximate the boiling temperature in Fahrenheit.



My Notes:

UNIT 2

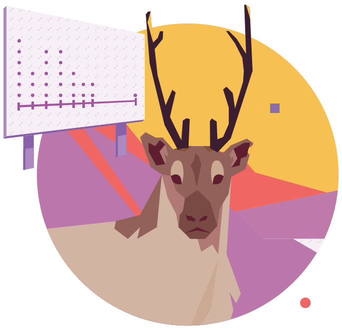
Data Analysis and Statistics

In this unit, you will explore data sets with one or two variables, often related to one of the most pressing threats we face as humanity: climate change. Along the way, you will encounter new statistical measures of center, spread, and association.

Essential Questions

- What is data and who uses it?
- How do people understand and communicate data?
- How can graphical displays be manipulated to present misleading information?
- *(By the way, when making decisions, do you think there is too much data or not enough data?)*

If you have five values, what is the greatest number of them that can be more than one standard deviation from their mean?

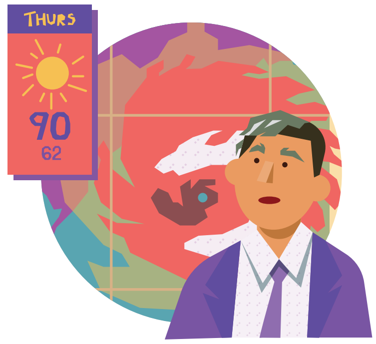


SUB-UNIT

1 Data Distributions

Narrative: Understanding statistics can help protect the world from a "zombie virus".

You'll learn . . .
- about the shapes of data distributions and what they tell you.



SUB-UNIT

2 Standard Deviation

Narrative: A *new measure* can help determine whether the current weather is the *new normal*.

You'll learn . . .
- how to visualize, calculate, and interpret the standard deviation of a data set.

SUB-UNIT

3 Bivariate Data

Narrative: Linear models can help predict when — or if — a shortage of natural resources may occur.

You'll learn . . .
- how residuals help you judge the fit of a linear model.

SUB-UNIT

4 Categorical Data

Narrative: Mathematical tables can help you understand the effects of climate change.

You'll learn . . .
- how two-way tables help you look for associations among bivariate categorical data.

SUB-UNIT

5 Correlation

Narrative: Analyze possible associations related to climate change and global sustainability.

You'll learn . . .
- how the correlation coefficient measures association.

Unit 2 | Lesson 1 – Launch

What Is a Statistical Question?

Let's investigate the information data representations can reveal and the statistical questions those representations can answer.

Warm-up Slow Reveal

You will be shown three different displays of the same data set. For each display, what do you notice? What do you wonder?

	I notice . . .	I wonder . . .
Display 1		
Display 2		
Display 3		

Log in to Amplify Math to complete this lesson online.

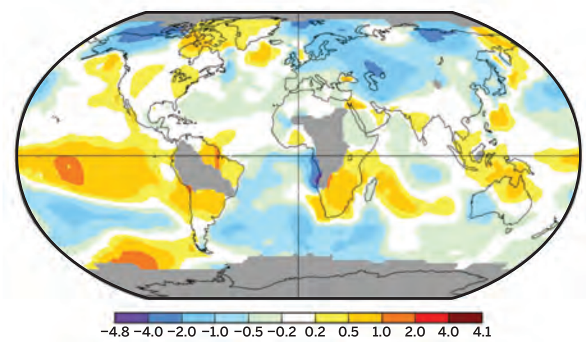
Name: ________________ Date: ________ Period: ________

Activity 1 Temperature on a Global Scale

Part 1

Researchers have been tracking global temperatures for more than a century. And throughout this time, scientists like John Tyndall have worked to explain the trends they observed. You will be shown data from five maps that display global temperature changes over time. The first map is shown here.

Global Temperatures: 1900 (°C) Anomaly vs 1900–2019



Note: Gray areas signify missing data.

GISTEMP Team, 2020: GISS Surface Temperature Analysis (GISTEMP), version 4. NASA Goddard Institute for Space Studies. Dataset accessed 2020-12-23 at https://data.giss.nasa.gov/gistemp/. Lenssen, N., G. Schmidt, J. Hansen, M. Menne, A. Persin, R. Ruedy, and D. Zyss, 2019

1. Record your observations.

2. After examining each display, which statement do you agree with? Use the information from the displays to explain your thinking.

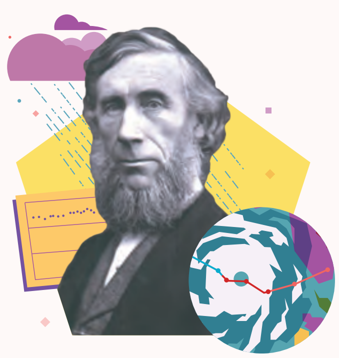
A. Global temperatures seem to be increasing at a steady rate throughout the last century.

B. Global temperatures seem to be increasing more rapidly over time.

C. Global temperatures seem to remain unchanged.

D. Global temperatures seem to be decreasing.

Featured Mathematician



John Tyndall

John Tyndall was a 19th century Irish physicist. In addition to studying magnetism, he developed early theories connecting carbon dioxide and heat in Earth's atmosphere. He also helped to popularize the sciences, especially physics, delivering hundreds of lectures in England and across America.

Portrait of John Tyndall. Woodburytype by Lock and Whitfield. Courtesy of the Smithsonian Libraries.

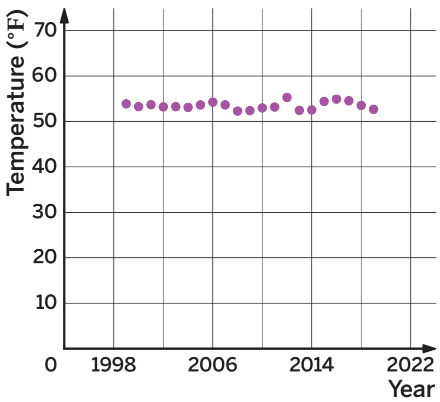
Activity 1 Temperature on a Global Scale (continued)

Part 2

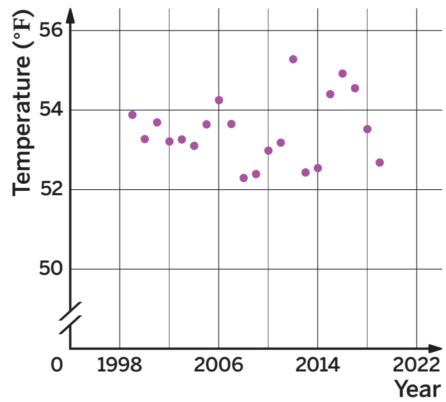
The following four graphs all show the annual average U.S. temperature, but with different scales on the axes.

Collect and Display: Your teacher will highlight the math language you use to discuss these graphs and add it to a class display. Refer to the display during future discussions.

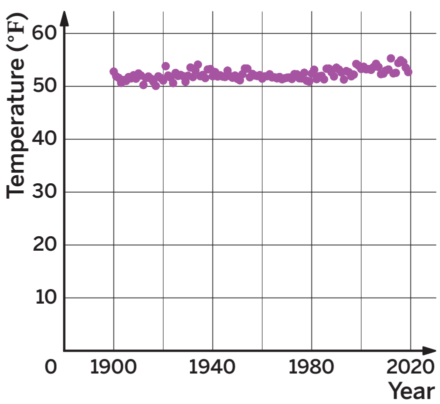
Graph A



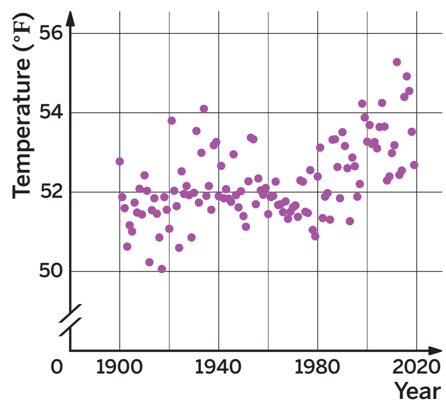
Graph B



Graph C



Graph D



3. While all four graphs show the same data, how do they appear different?

4. Which graph(s) support the statement, "The U.S. average temperature is rising"? Explain your thinking.

5. If you wanted to convince someone the average U.S. temperature was *not* rising, which graph would you show them? Explain your thinking.

Name: ________________ Date: ________ Period: ________

Activity 2 Weather vs. Climate

In recent years, the Greenland ice sheet has been melting due to rising temperatures. This melting has been the focus of much scientific research. Consider these three statements about Greenland's temperatures:

Statement 1: From 1991 to 2019, the winter temperatures of Greenland's coastal region increased by $7.9°F$.

Statement 2: From 1991 to 2019, the summer temperatures of Greenland's coastal region increased by $3.1°F$.

Statement 3: On January 2, 2020, Greenland recorded its lowest temperature ever, $-86°F$.

1. What are the differences and similarities among the statements?

2. Assume that each statement is an answer to a question that a researcher asked. Write a possible question the researcher may have asked for each statement.

A *statistical question* is a question that can only be answered using data that varies, meaning there are multiple data points that are not all the same.

3. Which of your questions in Problem 2 are statistical questions? Explain your thinking.

4. After reading Statement 3, a blogger says that Greenland's ice sheet cannot be getting warmer because Greenland recently recorded its coldest temperature. How would you respond to this blogger?

STOP

Narrative Connections

Unit 2 Data Analysis and Statistics

Analyzing Climate Change

Facts are facts — and that's all there is to say. Right?

Not quite.

While any search for truth begins with data, data can be misleading. It can be manipulated. It might not show us the whole picture, or lead us to make the wrong conclusions.

In the Indian subcontinent, there is a story of three men who couldn't see. One day they encounter something strange by a pond. Unable to see the elephant that is standing there, each man touches a different part of the animal. One man, touching the trunk, is convinced it is a snake; another, placing his hand on its ear, is convinced it is a fan; a third, feeling the rough skin of its leg, is sure it is a tree.

Each man used what they thought was the best data available, and yet missed the full picture.

One of the most hotly contested issues since the mid-20th century has been climate change. If you were to study global temperatures over the last few years, you might not think they were changing too much. But if you zoom further out, looking across multiple decades rather than a few years, a clear trend emerges. At the same time, scientists are observing many other changes, such as world-wide increases in hurricane activity and intensity. Without a doubt, something is happening to our climate. But what is it? And how do we measure and interpret it?

In these next lessons, you will take on the role of an investigator as you tackle these questions. You will comb through and analyze data, and — with patience and an open mind — see the seven-ton elephant standing in the room.

Welcome to Unit 2.

Name: ______ Date: ______ Period: ______

Practice

1. Select *all* questions that are statistical questions.

   A. What is the typical amount of rainfall for the month of June in the Galapagos Islands?

   B. How much did it rain yesterday at the Mexico City International Airport?

   C. Why do you like to listen to music?

   D. How many songs does the class usually listen to each day?

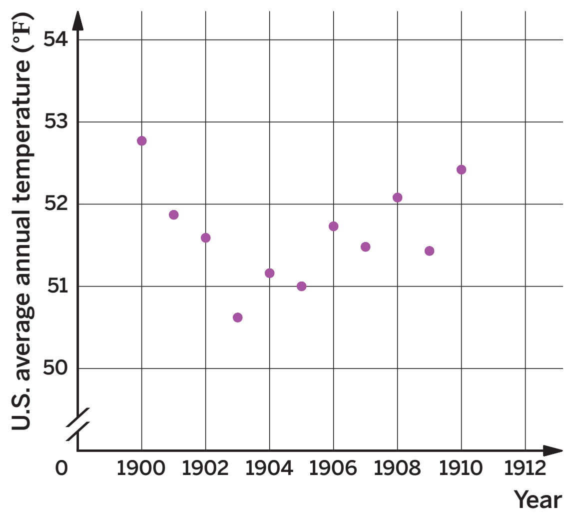
   E. How many songs did you listen to today?

   F. What is the capital of Canada?

   G. How long does it typically take for high schoolers to walk to our school?

2. Write a statistical question about the climate of the North American continent.

3. A climatologist (a scientist who studies the climate) wants to study the temperatures of the United States in the early 1900s. She decides to use the following graph in her study. Is the graph useful? Explain your thinking.



Practice

Name: ______ Date: ______ Period: ______

4. The data set represents the number of days each week it rained for the last 10 weeks. Create a dot plot of the data.

$0, 0, 1, 1, 2, 3, 4, 4, 5, 5$

5. Solve the following system of equations. Show your thinking.

$$\begin{cases} 4x - 2y = 10 \\ 2y - 2x = 8 \end{cases}$$

6. Create a histogram to display the given data set.

$2, 3, 3, 4, 8, 10, 12, 20, 22, 24$

SUB-UNIT

1 Data Distributions

Narrative Connections

How can we protect ourselves from a zombie virus?

When you read “zombie virus,” you might imagine a virus that turns humans into undead monsters. In the world of science, however, it refers to a past disease that has been revived. For example, consider the strange case of a reindeer carcass found in the Siberian Arctic permafrost. The reindeer, which had died from an anthrax outbreak in 1941, was buried and frozen in the Arctic’s permafrost. But in 2016, a record-setting heat wave caused that permafrost to thaw.

While the deer itself remained dead, the anthrax bacteria had other plans. It made its way from the carcass to the topsoil to the water. From there, it went into the bodies of 2,000 living reindeer and the people who lived alongside them. In total, 115 people were hospitalized and one 12-year-old boy died.

This might seem like an isolated case. But as Arctic temperatures rise, more and more viruses and bacteria once frozen could be resurrected.

Some scientists have downplayed the risk, noting that bacteria like anthrax are commonly found in warmer climates. But others are less quick to dismiss the threat, warning that an outbreak could be disastrous when coupled with higher rates of exposure.

In order to assess the risk these viruses might pose, we must first make sense of the temperature data. What is a typical temperature in the Arctic? What is an extreme temperature? And how much does it vary? To help us answer these questions, we turn to data distributions.

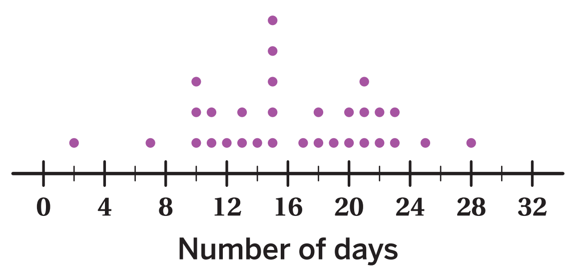
Unit 2 | Lesson 2

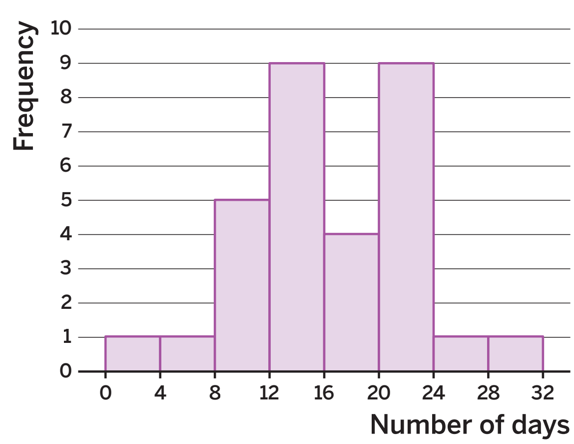
Data Representations

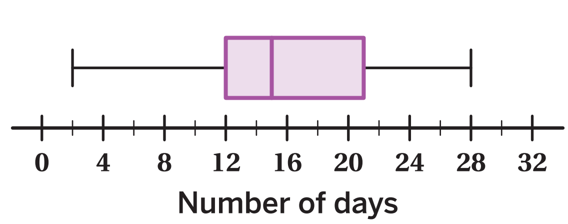
Let's make, compare, and interpret representations of data.

Warm-up Notice and Wonder

In June 1960, the average temperature in Anchorage, Alaska, was 55.1°F. The dot plot, histogram, and box plot show how many days in June were *warmer* than 55.1°F, for each year between 1990 and 2020. What do you notice? What do you wonder?







1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Revisiting Dot Plots and Histograms

In June of 1960, the highest temperature recorded in Anchorage was 70°F. You will be given a table that shows the number of days in June the high temperature was greater than 70°F, for each year between 1990 and 2020.

A dot plot and a histogram can be used to represent distributions of numerical data.

1. Create a dot plot in the space provided.

- **a** Choose a horizontal scale that contains all the values in the data set. Use the least and greatest values to help determine your scale.
- **b** For each value in the table, draw a dot above the value on your horizontal line, stacking dots with the same value.

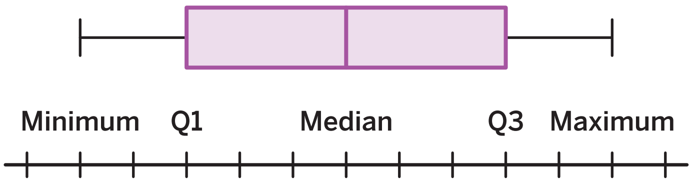
2. Create a histogram in the space provided.

- **a** Choose equal-sized intervals that contain all values in the data set. Complete the first column of the table provided here with these intervals.
- **b** Determine the frequency of the data within each interval to complete the second column.
- **c** Use your table to draw a histogram by counting the number of values from the data set within each interval. Then draw a rectangular bar over that interval whose height matches the count and whose width is the interval length.

Number of days in June warmer than 70°F	Frequency

Activity 2 Five-Number Summary and Box Plots

Box plots are constructed using five values calculated from a set of data. Together, these values are called the *five-number summary*. The box plot shown shows these five values.



Using the data set from Activity 1, complete each of the following to help determine the values in the five-number summary.

1. List the data values from least to greatest.

2. Determine each of the following values. Then explain what these values represent in this context.

a The minimum and maximum values.

b The median (the middle value) of the data.

c The median of the *lesser half* of the data. This value is called the first quartile (Q1).

d The median of the *greater half* of the data. This value is called the third quartile (Q3).

3. Using your responses to Problem 2, record the values of the five-number summary.

Minimum: **Q1:** **Median:** **Q3:** **Maximum:**

4. Use the five-number summary to create a box plot representing the number of days in June the temperature was greater than 70°F in Anchorage, from 1990 to 2020.

Name: ________________ Date: ________ Period: ________

Activity 3 Data Displays

Your group will be given a data set and a statistical question.

1. Create a dot plot, histogram, and box plot to represent the data on a display or in the space provided.

2. Respond to the statistical question.

3. Write at least two more statements that interpret the data on your display.

If you visit each display, write at least two sentences here, summarizing the information in the display.

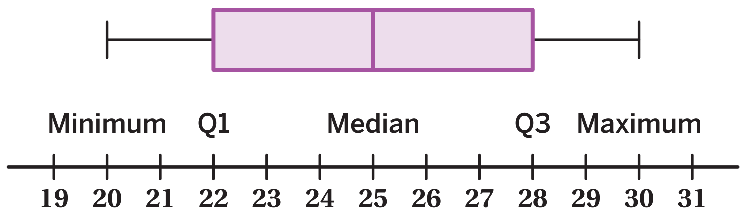
STOP

Summary

In today's lesson . . .

You represented a data set using dot plots, histograms, and box plots.

- A dot plot is created by marking a dot for each value above its position on a number line.
- A histogram is created by counting the number of values from the data set within certain intervals and drawing a bar over that interval whose height matches the count.
- To create a box plot, determine the ***five-number summary***: the minimum, first quartile (Q1), median, third quartile (Q3), and maximum values for the data set. Draw vertical marks for each number and then connect them as shown:



You also observed that . . .

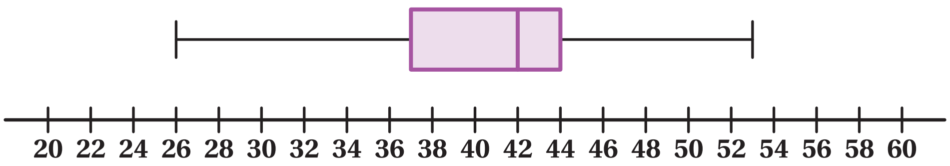
- Dot plots are most useful for observing the frequency, range, and number of points in a data set.
- Histograms are useful for observing the shape of a distribution.
- Box plots are useful for observing the minimum, maximum, and median values of a data set.

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. The box plot represents the distribution of average daily temperatures of a town during 20 days of winter. Determine the five-number summary that represents this data.



Minimum: **Q1:** **Median:** **Q3:** **Maximum:**

2. The table summarizes the daily minimum temperature of the first 16 days of 2020 in Atlanta, Georgia.

36	46	50	37	32	38	43	36
37	50	60	49	59	60	56	50

a. Determine the five-number summary that represents this data.

Minimum: **Q1:** **Median:** **Q3:** **Maximum:**

b. Create a box plot that represents the data.

Practice

Name: ________ Date: ________ Period: ________

3. The table summarizes the daily maximum temperatures, in degrees Fahrenheit, of San Antonio, Texas, in February 2020.

72	76	73	78	50	59	74	70	78	73
51	68	61	59	64	72	81	75	53	50
57	64	71	73	68	56	62	73	76	

Use the table to create a frequency table for the data. Then use the table to create a histogram.

Daily maximum temperature (°F)	Frequency

4. Solve the following system of equations. Show your thinking.

$$\begin{cases} y = 4x + 10 \\ -5x + y = 3 \end{cases}$$

5. Determine which statements about the box plot are true. Select *all* that apply.

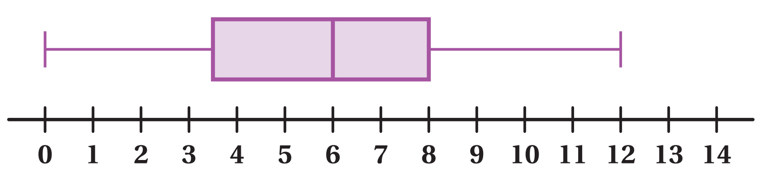
A. The range of the data is 12.

B. The first quartile is 4.

C. The IQR (interquartile range) is 4.5.

D. The mean is 6.

E. The median is 6.



Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 3

The Shape of Distributions

Let's describe data distributions.

Warm-up Describing Distributions

1. Describe each of the following terms in your own words.

   a. Skewed

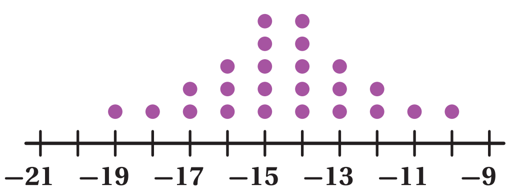
   b. Uniform

   c. Bell-shaped

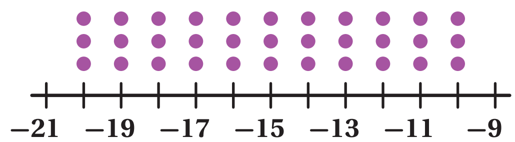
   d. Bimodal

2. Use one of the terms in Problem 1 to describe each distribution. Explain your thinking.

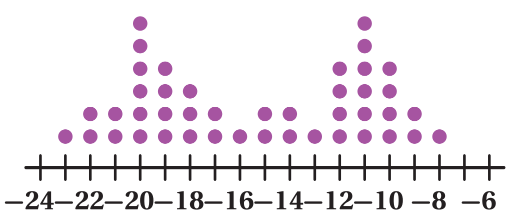
   a.



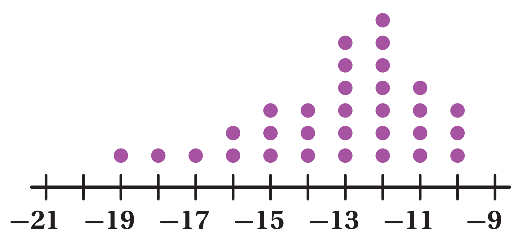
   b.



   c.



   d.



Log in to Amplify Math to complete this lesson online.

Activity 1 Card Sort: Matching Distributions

You will be given a set of cards summarizing U.S. temperature and snow coverage data in North America.

Take turns with your partner matching two different representations for the same set of data.

- For each set that you match, explain why it is a match.
- For each set that your partner matches, listen carefully to their explanation. If you disagree, discuss your thinking and work together to reach an agreement.
- Describe the shape of each distribution using one or more of the following terms: *skewed (left or right)*, *bell-shaped*, *uniform*, *bimodal*.

	Cards	Shape of distribution
Pair 1		
Pair 2		
Pair 3		
Pair 4		
Pair 5		

Name: ________ Date: ________ Period: ________

Activity 2 Examining the Pairs

Snow is often associated with cold weather, but snow influences temperature as well. Snow can reflect heat from the Sun back into space, cooling the planet. The presence or absence of snow contributes to patterns of warming and cooling.

1. A journalist uses the data cards from Activity 1 to investigate snowfall coverage in North America and U.S. temperatures from recent decades. Which pair of cards does not belong? Explain your thinking.

2. The journalist writes an article about snow coverage and temperature during summer in recent decades. Which pair(s) of cards do you think should be used for the article? Explain your thinking.

3. What statistical questions might the journalist have wanted answers to, based on the data used in the article? Be prepared to explain your thinking.

4. Examine the cards with a bimodal distribution.

   a. Why do you think this data set is bimodal?

   b. Which months of the year do you think the smaller peak of data values are from? The larger peak of data values? Explain your thinking.

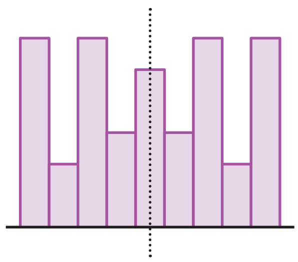
STOP

Summary

In today's lesson . . .

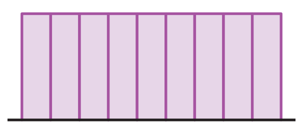
You described the shapes of different data distributions, such as the ones shown.

Symmetric



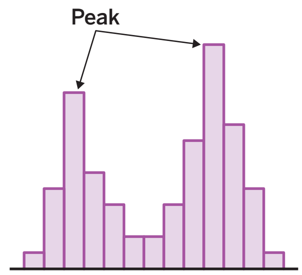
The distribution has a vertical line of symmetry. The mean is equal to the median.

Uniform



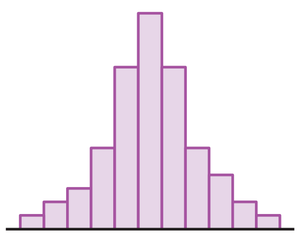
Data is evenly distributed throughout the range.

Bimodal



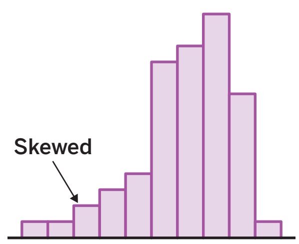
There are two distinct peaks in the distribution.

Bell-shaped



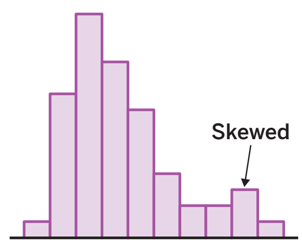
The distribution looks like a bell, with most of the data near the center and fewer points farther from the center.

Skewed left



A distribution with a long left tail, where data extends far away from the center.

Skewed right



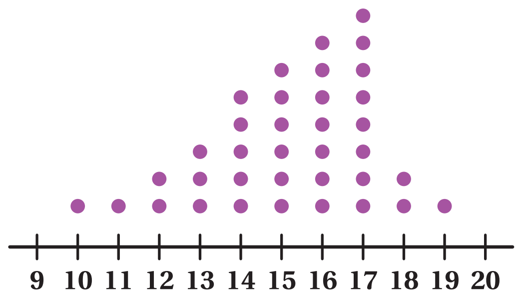
A distribution with a long right tail, where data extends far away from the center.

Reflect:

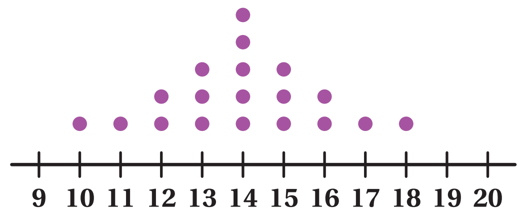
Name: ______________________ Date: __________ Period: __________

1. Select *all* dot plots that have a symmetric, or approximately symmetric, distribution.

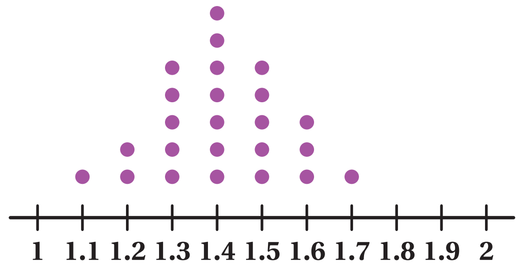
A.



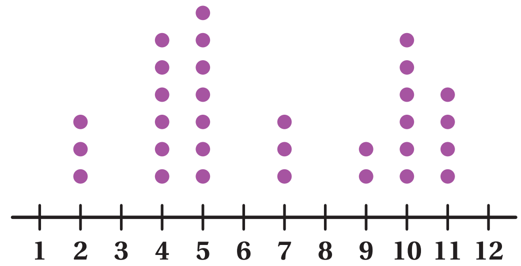
C.



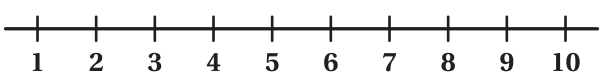
B.



D.

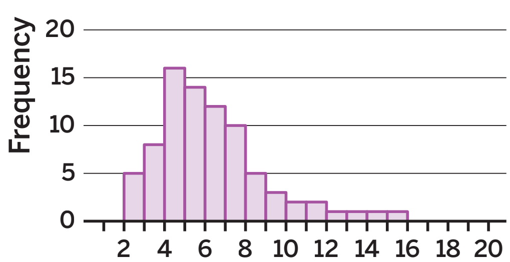


2. Use the number line to create a dot plot with a uniform distribution.

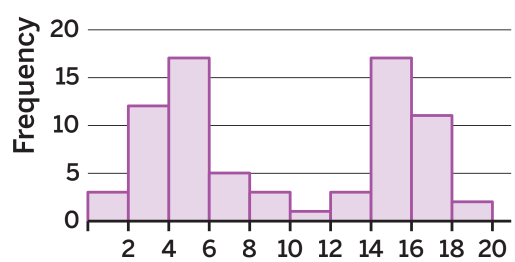


3. Describe the shape of each distribution shown.

a



b



Practice

Name: ______ Date: ______ Period: ______

4. Here are the maximum daily temperatures, in degrees Fahrenheit, for Denver, Colorado, during May 2020.

53, 60, 63, 65, 65, 65, 68, 68, 70, 70, 70, 71, 72, 73, 73, 74, 75, 76, 77, 77, 79, 80, 82, 83, 83, 85, 88, 88, 89, 90, 92

a Determine the five-number summary for the data.

b Create a box plot for the data.

5. Solve the following system of equations. Show your thinking.

$$\begin{cases} 2x - 6y = -4 \\ 3x + 2y = 5 \end{cases}$$

6. Calculate the mean and the mean absolute deviation (MAD) of the following data set. Do you think the data set has an outlier? Explain your thinking.

40, 43, 67, 185, 52, 32, 45, 49, 50, 53, 56, 58

Name: ______ Date: ______ Period: ______

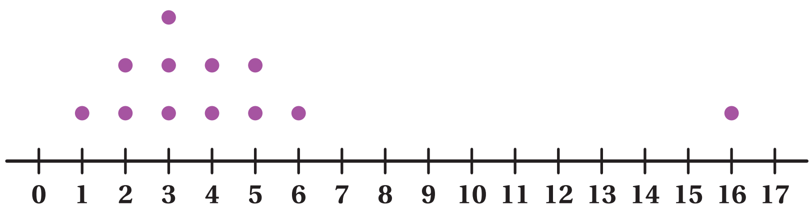
Unit 2 | Lesson 4

Deviation From the Center

Let's calculate measures of center and variability, and use them to describe distributions of data.

Warm-up True or False

Determine whether each statement about the dot plot shown is true or false. Explain your thinking.



Statement	True or False?
1. The mean is greater than the median.	
2. Moving the dot from 16 to 12 decreases the median.	
3. Removing the dot at 16 from the data set changes the median.	
4. Excluding the value 16 from the data set makes the median equal to the mean.	

Log in to Amplify Math to complete this lesson online.

Activity 1 Center and Spread on a Stick

Your group will be given a yardstick and 10 pennies. Use your pennies and the yardstick to create distributions that meet the given descriptions. Record the location (inch mark) of the pennies for each problem.

1. A distribution where the average of the distances from the mean is 0.

2. A distribution where the difference between Q1 and Q3 of the data set is 36.

3. A distribution where the median is greater than the mean.

4. Several pennies are placed along the inch marks on a yardstick and these locations are recorded. The mean is 10 in. and the average distance from each penny to the mean is 6 in. Are the pennies placed on values lower or higher than the 10 in. mark? Explain your thinking.

Reflect: In what ways did you show empathy and respect towards members of your group during the activity?

Are you ready for more?

Suppose there are 6 pennies on a yardstick so that the mean position is 21 in. and the average of the distances from the mean is 4 in.

1. Determine possible locations for the 6 pennies.

2. Determine a different set of possible locations for the 6 pennies.

Name: .... Date: .... Period: ....

Activity 2 Global Temperatures From the Early 1900s

The Global Land-Ocean Temperature Index measures the change in global surface temperature relative to the average temperature of the 20th century. For example, a value of -0.29 in the year 1880 means that the global surface temperature in 1880 was $0.29°F$ lower than the average temperature from 1901 to 2000.

Climate scientists, including Nicole Hernandez Hammer, use baseline temperatures such as the Global Land-Ocean Temperature Index to understand how the climate has changed and its effect on different communities.

The table shows the Global Land-Ocean Temperature Index from 1910 to 1919.

Year	Value (°F)	Year	Value (°F)	Year	Value (°F)
1910	-0.77	1914	-0.27	1918	-0.52
1911	-0.79	1915	-0.25	1919	-0.49
1912	-0.65	1916	-0.65		
1913	-0.63	1917	-0.83		

1. Create a box plot for the data set.

2. Determine the number of values in the data that are:

   a. Less than quartile 1 (Q1).

   b. Greater than quartile 3 (Q3).

   c. Between quartile 1 (Q1) and the median.

   d. Between the median and quartile 3 (Q3).

3. The *interquartile range* (IQR) is the difference between Q3 and Q1. Calculate the IQR. What does this statistic represent in this context?

Activity 2 Global Temperatures From the Early 1900s (continued)

4. The Global Land-Ocean Temperature Index is available for every year from 1880 to 2019 (118 years). The median value for this period is -0.13°F. What information does this reveal about the average global surface temperature between 1880 to 2019?

5. From 1980 to 2019 (40 years), the IQR is about 0.612°F. What information does this reveal about the average global land-ocean temperature between 1980 and 2019?

Featured Mathematician

Nicole Hernandez Hammer

Nicole Hernandez Hammer, born in Guatemala, is an American climate scientist and activist who studies the change in global temperatures and the accompanying changes in sea level. Her research focuses on how these changes have disproportionately affected communities of color and low-income communities. Through her public outreach, she makes climate change information more accessible, and wants to empower the Latino communities to talk to their government officials.

Nicole Hernandez Hammer

Name: ______ Date: ______ Period: ______

Activity 3 Deviation in Global Temperature

The Global Land-Ocean Temperature Index for 2019 has been added to the data set from Activity 2.

Year	Data value (°F)	Deviation from the mean	Absolute deviation from the mean
1910	-0.77		
1911	-0.79		
1912	-0.65		
1913	-0.63		
1914	-0.27		
1915	-0.25		
1916	-0.65		
1917	-0.83		
1918	-0.52		
1919	-0.49		
2019	1.76		

1. Calculate the mean value of the data.

2. Complete the last two columns of the table. The *deviation from the mean* of a data set is the difference between each value and the mean. The *absolute deviation from the mean* is the absolute value of the deviation.

3. The *mean absolute deviation*, or MAD, is the average of the absolute deviations. Calculate the MAD for this data set.

Activity 3 Deviation in Global Temperature (continued)

4. Create a dot plot of the year and Global Land-Ocean Temperature Index data set, rounded to the nearest tenth.

5. Using your dot plot, and without performing additional calculations, determine if:

a The mean is less than, greater than, or equal to the mean of the original data set in Activity 2. Explain your thinking.

b The MAD is less than, greater than, or equal to the MAD of the original data set in Activity 2. Explain your thinking.

c The data set from Activity 2 or Activity 3 has a greater variability. Explain your thinking.

6. How does the MAD describe the variability of a data set?

7. For another Global Land-Ocean Temperature Index data set, all the values are $1°F$ above the mean or $1°F$ below the mean. Is this enough information to determine the MAD for this data set? If so, determine the MAD. If not, what other information is needed? Explain your thinking.

Stronger and Clearer: Share your responses to Problems 6 and 7 with another pair of students to receive feedback. Use this feedback to revise and improve your initial responses.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You reviewed two measures of center: mean and median. Measures of center are used to approximate the middle of a data set or to describe a typical value.

The interquartile range (IQR) and mean absolute deviation (MAD) are two measures of variability, which tell you how spread out the data is. The IQR is the range of the middle 50% of the data, while the MAD is the average distance between each data value and the mean.

Extremely small or large values in a data set tend to affect the MAD more than the IQR, because they are not in the middle 50% of the data.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. The following data set represents the number of errors different students made on a typing test.

$5, 6, 8, 8, 9, 9, 10, 10, 10, 12$

a. What is the median? What is the meaning of this value in context?

b. What is the IQR of the data set?

2. The data set represents the heights, in centimeters, of ten model bridges made for an engineering competition.

$13, 14, 14, 16, 16, 16, 16, 18, 18, 19$

a. What is the mean of the data set?

b. What is the MAD of the data set?

3. A pod of dolphins contains 800 dolphins of various lengths. The median length of dolphins in this pod is 5.8 ft. What information does this tell you about the length of dolphins in this pod?

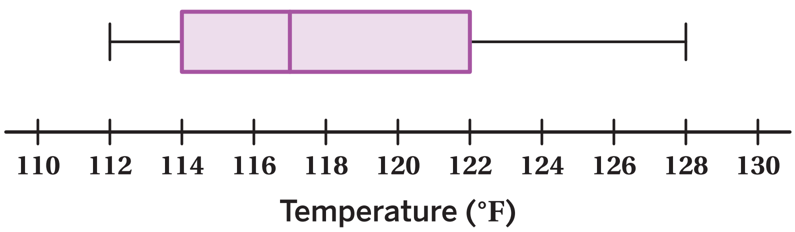
Name: ______________________ Date: ____________ Period: ____________

Practice

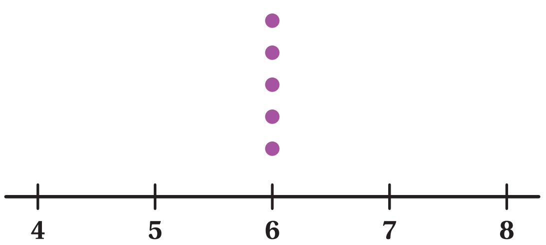
4. Solve the following system of equations. Show your thinking.

$$\begin{cases} 4y + 14x = -10 \\ -14x = 24 - 10y \end{cases}$$

5. The box plot displays the temperature of saunas in degrees Fahrenheit. Determine the five-number summary of the data set.



6. Refer to the dot plot. If the values of 4 and 8 are added to the data set, will the mean or median change? Explain your thinking.



Unit 2 | Lesson 5

Measuring Outliers

Let's look at a definition for outliers and apply it.

Warm-up Math Talk

1. Mentally determine the mean and median of each data set. Record the strategy you used and discuss it with your partner.

   a. 27, 30, 33

   Strategy:

   Solution:

   b. 0, 100, 100, 100, 100

   Strategy:

   Solution:

   c. 61, 71, 81, 91, 101

   Strategy:

   Solution:

   d. 0, 5, 6, 7, 12

   Strategy:

   Solution:

2. Which data set do you think has an outlier? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

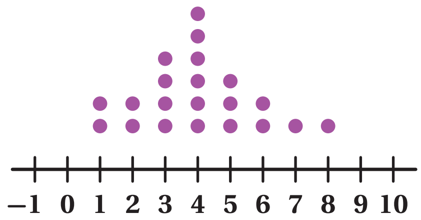
Name: ______ Date: ______ Period: ______

Activity 1 What Is an Outlier?

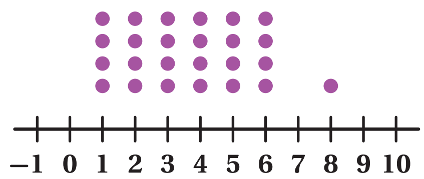
Refer to the four data sets shown.

1. Which data set(s) do you think contain an outlier(s)? Explain your thinking.

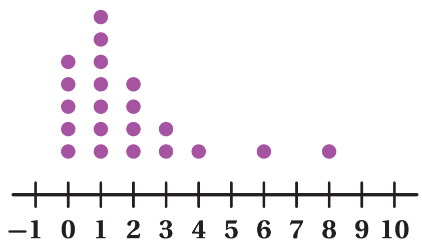
Data set A



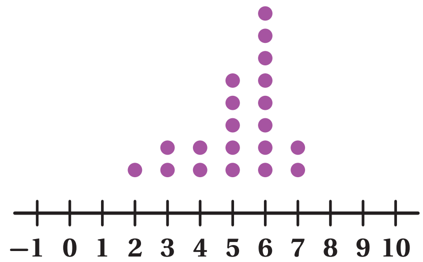
Data set B



Data set C



Data set D



2. Determine the IQR of each data set.

One way an outlier can be mathematically defined uses the IQR. An outlier is defined as a value that is at least 1.5 IQRs *less than Q1*, or at least 1.5 IQRs *greater than Q3*.

3. Use the definition to determine which data set(s) contain an outlier. Explain your thinking. The calculations and work for Data set B have been completed for you.

Data set B:

Q1 is 2 and Q3 is 5, so the IQR is 3.

$Q1 - 1.5 \cdot IQR = 2 - 1.5(3) = -2.5$

$Q3 + 1.5 \cdot IQR = 5 + 1.5(3) = 9.5$

Because there are no values that are less than −2.5 or greater than 9.5, Data set B does not contain any outliers.

Activity 2 Mean, Median, and Outliers

Consider this data set: 6, 7, 8, 8, 9, 9, 9, 10, 10, 10, 10, 11, 11, 11, 12, 12, 13, 14.

1. Create a dot plot of the data set and describe the shape of the distribution.

2. Determine the mean and median of the data set.

3. Suppose two outliers are added to the original data set.

   a. If both outliers are much greater than 14, will the mean change? Will the median change?

   b. If both outliers are much less than 6, will the mean change? Will the median?

4. How many values much greater than 14 must be added to the original data set in order to increase the median?

5. If you want to describe the center of a data set that is very skewed or has outliers, which measure of center would you use: mean or median? Explain your thinking.

Are you ready for more?

A government agency is setting its budget for fighting forest fires over the next 10 years. Each year, the distribution of the cost per fire is very skewed to the right. Which measure of center should the government agency use to determine its budget: mean or median?

Name: ______ Date: ______ Period: ______

Activity 3 Plots Matching Measures

Add five values to each of the following data sets so that they meet the given conditions. At least three of the values that you add should be different. Then create a dot plot of your new data set.

Use this data set for Problems 1 and 2: 5, 25, 25, 30, 30, 35, 35.

1. A distribution that has both a mean and median of 30.

   Data set: **Dot plot:**

2. A distribution that has both a mean and median of 20.

   Data set: **Dot plot:**

Use the following data set for Problems 3 and 4: 55, 55, 60, 60, 60, 65, 65.

3. A distribution that has a median of 57.5 and a mean greater than the median.

   Data set: **Dot plot:**

4. A distribution that has a median of 62.5 and a median greater than the mean.

   Data set: **Dot plot:**

Activity 3 Plots Matching Measures (continued)

5. Which of the data sets that you created include outliers, if any? Explain or show your thinking.

Are you ready for more?

A *stem and leaf plot* is a table where each data point is indicated by writing the first digit(s) on the left (the stem) and the last digit(s) on the right (the leaves). Each stem is written only once and shared by all data points with the same first digit(s). For example, the stem and leaf plot for the values 31, 32, and 45 are shown.

Stem	Leaf
3	1 2
4	5

Key: 3 | 1 = 31

The data set represents exam scores of a math class.
21, 86, 73, 85, 86, 72, 94, 88, 98, 87, 86, 85, 93, 75, 64, 82, 95, 99, 76, 84, 68

1. Create a stem and leaf plot for this data set.
2. How can you see the shape of the distribution from this plot?
3. What characteristics of the stem and leaf plot would suggest that the data set has an outlier?

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You were introduced to a mathematical method for determining whether a value in a data set is an outlier. A value is an outlier if it is less than Q1 – $1.5 \cdot$ IQR or greater than Q3 + $1.5 \cdot$ IQR. Outliers generally influence the mean more than they influence the median.

- When a distribution is skewed or includes outliers, the median is the preferred measure of center because these changes usually do not influence it.
- When a distribution is symmetric, the mean is the preferred measure of center because it gives equal importance to each value in the data set.
  - » For a distribution that is skewed right, the mean is typically greater than the median because the points far to the right do not affect the median.
  - » For a distribution that is skewed left, the mean is typically less than the median.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. For each distribution shape, determine if it is more appropriate to use the mean or median as a measure of center.

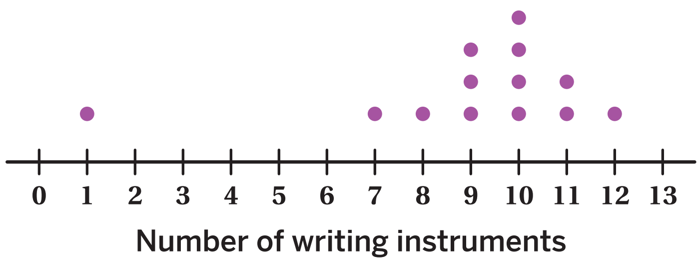
   a. Bell-shaped

   b. Symmetric

   c. Skewed

   d. Uniform

2. The number of writing instruments in each of several students' desks is displayed in the dot plot. Which is greater, the mean or the median? Explain your thinking using the shape of the distribution.



3. The data set represents the scores of Bard's assignments.
   $0, 40, 60, 70, 75, 80, 85, 95, 95, 100$

   a. Is 0 an outlier? Explain your thinking.

   b. The teacher is considering dropping the lowest score. What effect does eliminating the lowest value, 0, from the data set have on the mean and median?

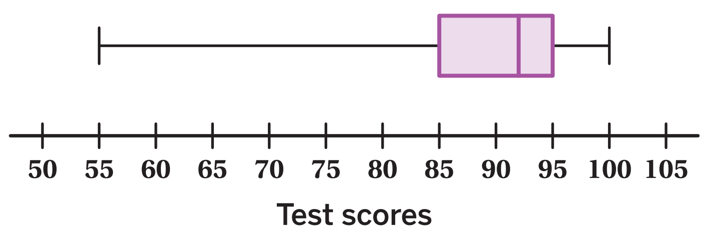
Name: .... Date: .... Period: ....

Practice

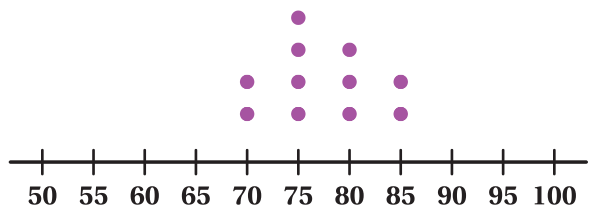
4. Solve the system of equations. Show your thinking.

$$\begin{cases} \frac{1}{2}x - y = 3 \\ 2x - 4y = 12 \end{cases}$$

5. Refer to the box plot summarizing the test scores of 20 students. Describe the shape of the distribution of data.



6. Jada wants to determine the mean of the dot plot shown. She added up all of the values and then divided their sum by 15. Did Jada calculate the mean correctly? Explain your thinking.



Unit 2 | Lesson 6

Data With Spreadsheets

Let's use technology to organize and visualize data.

Warm-up It Begins With Data

For two years, Jada recorded the number of days that the temperature was above the historical average in her hometown each month. The table shows her data.

7	1	18	3	5	11	3	3	4	20	4	4
3	6	6	7	0	8	9	10	3	14	1	4

Enter Jada's data in a spreadsheet.

1. Open a blank spreadsheet. In **A1**, enter the title "Number of days above average". Then enter Jada's data so that each value is in its own cell in Column A.

2. Use your spreadsheet to check the measures of center of Jada's data set. Record the value for each measure.

   a. Enter "Mean" in **D2**. Then, in cell **E2**, enter "**=AVERAGE(A2:A25)**".

   b. Enter "Median" in **D3**. Then, in cell **E3**, enter "**=MEDIAN(A2:A25)**".

   c. Enter "Maximum" in **D4**. Then, in cell **D4**, enter "**=MAX(A2:A25)**".

   d. Enter "Minimum" in **D5**. Then, in cell **D5**, enter "**=MIN(A2:A25)**".

3. What does **A2:A25** represent in each equation entered into the spreadsheet in Problem 2?

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 From Spreadsheets to Histograms and Dot Plots

Spreadsheets can be a helpful way to create data representations. Complete each step to create a histogram of Jada's data set from the Warm-up.

1. To create the histogram:

- **a** Highlight Column A by selecting the letter **A**.
- **b** Select the **Insert** dropdown from the menu bar at the top of the page and select **Chart**.
- **c** Select **Histogram**.

2. The axes of the histogram are automatically created. Select the histogram to reformat the axes. Using the menu options, you can change the chart title, axes titles, interval size, and the maximum/minimum of the axes. Change the:

- **a** Horizontal axis title to "Number of days above average."
- **b** Vertical axis title to "Frequency."
- **c** Interval size to 2.

3. Sketch the histogram from your spreadsheet.

4. Describe the shape of the histogram.

5. Do you think there are any outliers in the data set?

Activity 1 From Spreadsheets to Histograms and Dot Plots (continued)

To help study individual data, you can use spreadsheets to create dot plots. Complete each step to create a dot plot of Jada's data set from the Warm-up.

6. You first need to sort the data from least to greatest. Highlight all the data by selecting **A2**, and then dragging your cursor to **A25**.

7. Select the **Sort** option to sort the data from least to greatest.

8. In **B1**, enter "Count." In Column B, enter the number of times each value occurs in Column A. The count of 0 and 1 have already been completed for you.

	A	B
1	Number of days above average	Count
2	0	1
3	1	1
4	1	2

9. To create the dot plot:

- **a** Highlight both columns of values and their titles.
- **b** Select **Insert** from the menu bar and select **Chart**.
- **c** Select **Dot Plot**.

10. Sketch the dot plot from your spreadsheet.

11. Describe the shape of the dot plot.

12. Is the shape of the dot plot similar to the shape of the histogram?

Name: .... Date: .... Period: ....

Activity 2 Using Spreadsheets to Create Box Plots

You will be given a copy of a data set showing the change in global ocean temperatures relative to the average temperature in the 20th century. For example, a value of 0.3 for 2000 means that the average global ocean temperature in 2000 was $0.3°C$ higher than the 20th century average.

In addition to creating histograms and dot plots, you can also use spreadsheets to create box plots. Complete each step to create a box plot of the data from 1901 to 1910.

1. In cell **A1** enter "Year" and in cell **B1** enter "Value". Enter the years of the data set in Column A and the respective values in Column B.

2. Some spreadsheet technology will create a box plot for you. If yours can do so, highlight the data in Column B. Select **Insert** from the menu bar and select **Chart**. Select **Box Plot**. Then proceed to Problem 8.

3. If your spreadsheet technology cannot create a box plot, you can still compute the five-number summary. Enter the following in each given cell.

C2: "Global Ocean Temperature Anomalies for 1901–1910"

D1: "Minimum"

E1: "Q1"

F1: "Median"

G1: "Q3"

H1: "Maximum"

4. In cell **D2**, enter "**=MIN(B2:B11,1)**" to identify the minimum. Why are the cells **B2:B11** selected?

5. In cell **E2**, enter "**=QUARTILE(B2:B11,1).**" A comma then 1 is entered to indicate Quartile 1.

6. In **F2**, enter "**=QUARTILE(B2:B11,3).**" A comma then 3 is entered to indicate Quartile 3.

7. In **G2**, enter "**=MAX(B2:B11,1)**" to identify the maximum.

8. Sketch the box plot, either based on the box plot created by your spreadsheet technology or from your five-number summary.

Activity 3 Comparing Representations

Let's take a closer look at the data set from Activity 2.

1. Using a spreadsheet and the data set from Activity 2, create a box plot and histogram for the following two time periods: 1901–1960 and 1961–2019. Sketch the box plots and histograms from your spreadsheet.

2. Describe the distribution for 1901–1960.

3. What does the data reveal about the change in ocean temperatures over the last 120 years?

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You created histograms, dot plots, and box plots. Data representations can be very useful for quickly understanding a large amount of information. However, they can take a long time to construct using pencil and paper.

Spreadsheet technology can help create these representations more efficiently and also calculate useful statistics. For very large data sets, spreadsheet technology is essential for organizing and representing information so it can be better understood.

As always, be mindful that the act of choosing which data to portray (or to omit) can lead to misleading representations. Always pay close attention to the data set used, as well as the representation's title, axes labels, and intervals.

Reflect:

Practice

Name: ____________ Date: ____________ Period: ____________

1. The data set represents the average customer rating for several items sold online.
0.5, 3.2, 1.8, 1.3, 2.6, 2.6, 3.1, 3.3, 3.4, 4.5, 3.5, 3.6, 3.7, 4, 4.1, 4.1, 4.2, 4.2, 4.5, 4.7, 4.8, 3.7

a. Use spreadsheet technology to create a histogram for the data with an interval size of 0.5. Sketch the histogram here.

b. Describe the shape of the distribution.

c. Which interval has the highest frequency?

2. The data set represents the amount of corn, in bushels per acre, harvested from different locations.
133, 133, 134, 134, 134, 135, 135, 135, 135, 135, 135, 136, 136, 136, 137, 137, 138, 138, 139, 140

a. Use spreadsheet technology to create a dot plot and a box plot. Sketch both data representations here.

b. What is the shape of the distribution?

c. Compare the information displayed by the dot plot and box plot.

Name: .... Date: .... Period: ....

3. Tyler recorded the number of days it rained each month for two years. The table shows his data.

5	2	2	2	4	4	4	4
4	4	7	7	7	7	7	9
7	9	10	9	13	18	17	21

a Use spreadsheet technology to create a data representation to show where data is clustered. Sketch the data representation.

b Calculate the median and mean of the data set.

4. Calculate the MAD and IQR of this data set: $3, 3, 4, 5, 5, 6, 7, 10, 14, 16, 22$.

5. The dot plot represents the distribution of satisfaction ratings for a landscaping company on a scale of 1 to 10. 25 customers were surveyed.

Determine each measure of center or variability then determine what it means in context:

a Mean:

b Median:

c MAD:

d IQR:

My Notes:

SUB-UNIT

2 Standard Deviation

Narrative Connections

Is Sandy the new normal?

New York City is known as the city that never sleeps. But when Hurricane Sandy slammed into the east coast of the U.S. in 2012, it brought much of the city to a standstill. The hurricane created a massive storm surge — a 14-foot wall of water — that poured into streets and subways. It paralyzed the city and devastated the surrounding areas. By the time the storm had passed, it had killed over 200 people and caused $70 billion worth of damage to the Caribbean and the eastern U.S.

Usually, storms like Sandy are pushed away into the North Atlantic by air currents known as the jet stream. But an unusual bend in the jet stream caused by a mass of high-pressure cold air redirected Sandy toward the northeastern U.S. Combined with hurricane winds, this created a rare post-tropical cyclone called a “superstorm.” Through warming oceans, rising sea levels, and increased atmospheric moisture, researchers believe that superstorm events could become more frequent. Rather than occuring once every 400 years, they believe events like Sandy will occur about once every 23 years by the end of the century.

Researchers reached these conclusions by investigating historical data and analyzing climate models. By observing how distributions of storm data change over time, they can determine how likely extreme events (or outliers) like Sandy occur. You already encountered different ways of measuring how spread out a distribution is to identify outliers. However, there is one measure that is used more frequently than the others. In the next few lessons, you will learn about this measure. Then, you will be able to determine for yourself whether superstorms like Sandy are the new normal.

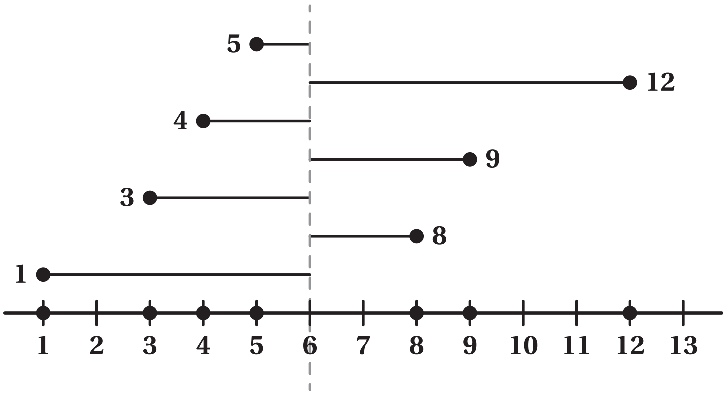
Unit 2 | Lesson 7

Standard Deviation

Let's explore another measure of variability.

Warm-up The Average Distance

Mai created the diagram shown to represent the following data: 1, 3, 4, 5, 8, 9, 12. The mean of the data is 6.



Co-craft Questions: Work with a partner to write 1–2 mathematical questions you have about this diagram before completing the Warm-up independently.

1. The line segments represent the distance between each data point and the mean. Label each segment on the diagram with its length.

2. What is the average length of the seven line segments?

3. What does your response to Problem 2 represent? Explain your thinking.

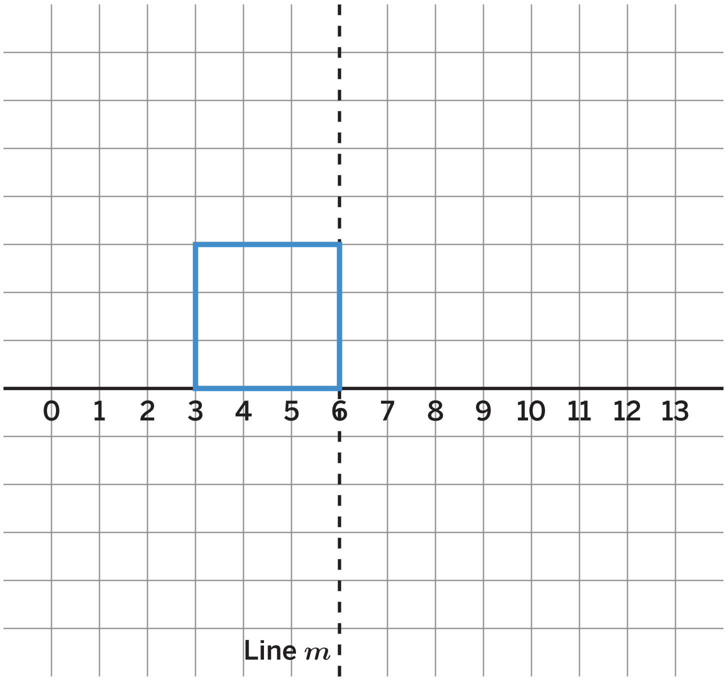
Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 Another Measure of Variability

In previous lessons, you explored two measures of variability: IQR and MAD. In this activity, you will encounter yet another measure of variability — the standard deviation.

1. Complete these steps to calculate the standard deviation of the data set: $3, 8, 5, 1, 12, 9, 4$. (The mean of this data set is 6.) A horizontal number line containing the seven data points and a vertical line at the mean are shown.



a For each data point, draw a square with a corner at the data point and another corner at $(6, 0)$. The first square is shown.

b Label each square with its area.

c Calculate the average area of the squares.

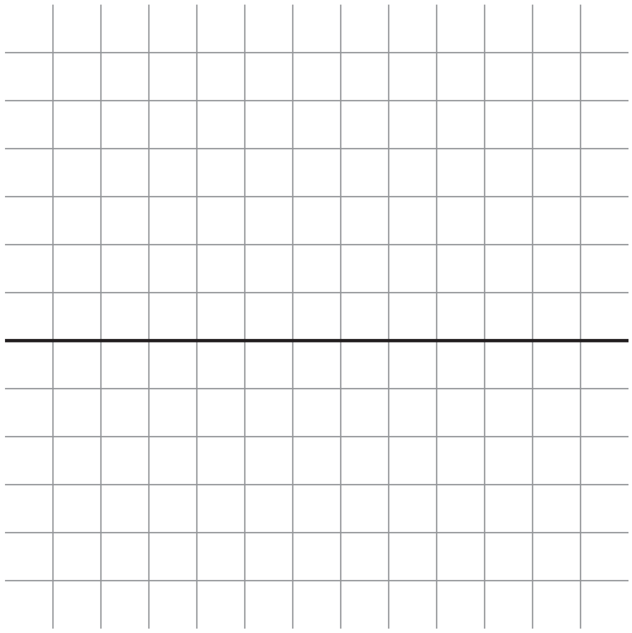
d Try to draw a square whose area is the average area you calculated in part c. This represents the "average square" between each data point and the mean. How long is each side of this "average square"?

The side length of the average square that you calculated in Problem 1 is the ***standard deviation*** of the data set. Standard deviation (SD) is a popular measure of variability, and is more commonly used than the MAD.

2. What similarities and differences do you notice between calculating the MAD and calculating the standard deviation?

Activity 1 Another Measure of Variability (continued)

3. Complete the steps to calculate the standard deviation for the following data set: $-4, 4, -3, 5, 2, -1, 4$.



a. Calculate the mean of the data set.

b. On the diagram, label the horizontal number line with the data points and draw a vertical line at the mean. Then draw squares connecting each data point to the vertical line, and label each square with its area.

c. Calculate the area of the "average square."

d. Calculate the standard deviation of the data set (the side length of the "average square").

Are you ready for more?

The Warm-up and Problem 1 of this activity used the same data set. In the Warm-up, you calculated the data set's MAD. In Problem 1, you calculated its standard deviation.

1. Which was greater, the MAD or the standard deviation?

2. Why do you think this particular measure of variability is greater? Will this always be the case?

Name: ______ Date: ______ Period: ______

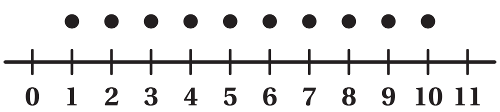
Activity 2 Mean and Standard Deviation

Plan ahead: What rules will you apply in order to stay safe when using technology?

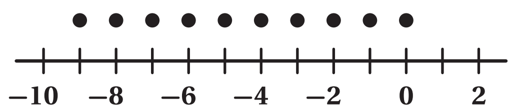
Part 1 You and your partner will use spreadsheet technology to determine the mean and standard deviation for each data set. To calculate the standard deviation, use the formula "=STDEV()" with the range of cells in the parentheses. Round each value to the nearest hundredth.

Partner 1	Partner 2

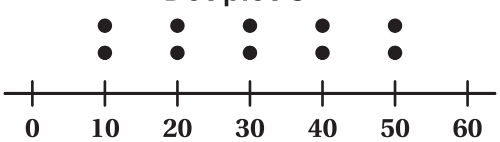
Dot plot A



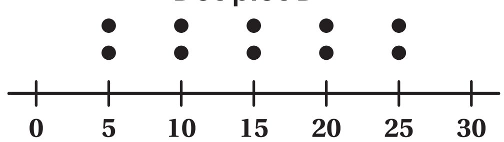
Dot plot B



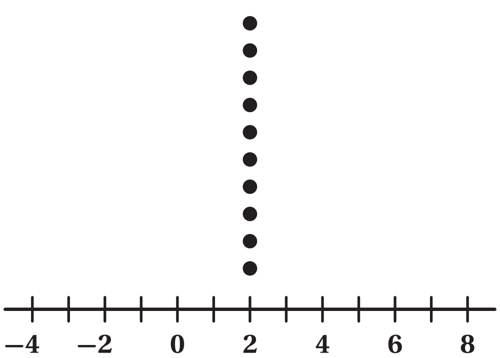
Dot plot C



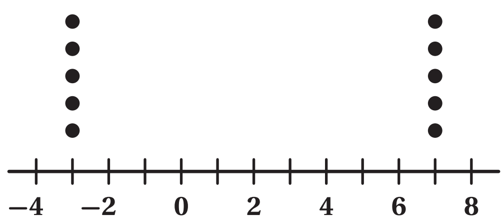
Dot plot D



Dot plot E



Dot plot F



Part 2 For each pair of dot plots, compare your statistics with your partner's. Come to a consensus as to how the statistics are similar or different.

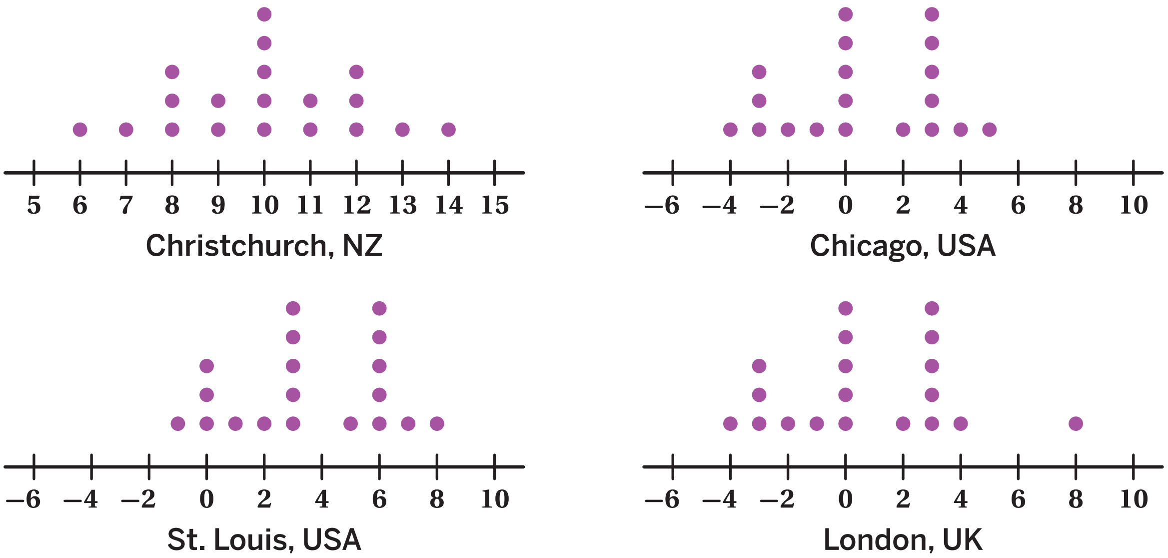
Dot plots A and B:

Dot plots C and D:

Dot plots E and F:

Activity 3 Comparing Statistics

The dot plots show the low temperatures, in degrees Celsius, for four different cities during the same set of days. Determine whether each statement is true or false. Explain your thinking.



1. The data set of temperatures for Christchurch is symmetric.

2. The IQR of London’s temperatures is greater than 12.

3. The median of London’s temperatures is greater than the median of Chicago’s temperatures, but their means are equal.

4. The mean of London’s temperatures is greater than the mean of Chicago’s temperatures, but their medians are equal.

5. The standard deviation of the temperatures in St. Louis is greater than the standard deviation of those in London.

6. The standard deviation of St. Louis’s temperatures is equal to the standard deviation of Chicago’s temperatures.

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You were introduced to ***standard deviation***, another measure of variability. You represented a data set on a number line, sketched the squares of their distances from the mean, and computed the average of the squares. Finally, you determined the side length of this "average square," which was the standard deviation.

Calculating the standard deviation is similar to calculating the mean absolute deviation (MAD). However, while the MAD is the average *distance* to the mean, the standard deviation involves the average of the *squares* of these distances — and then taking the square root at the end (giving a distance, rather than an area).

You also used a new spreadsheet function (=STDEV) to calculate the standard deviation of a data set.

Reflect:

Practice

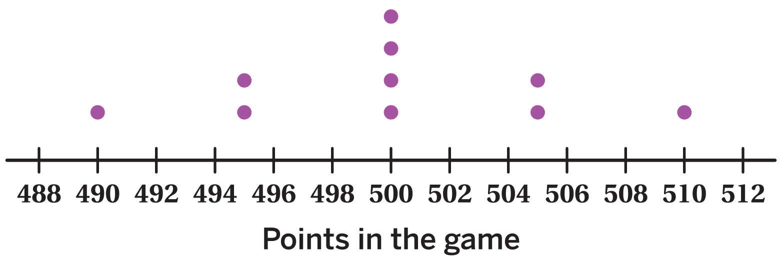
Name: .... Date: .... Period: ....

1. The data set represents the shoe size for all pairs of shoes in Clare's closet.
$7, 7, 7, 7, 7, 7, 7, 7, 7, 7$

   a. What is the mean?

   b. What is the standard deviation?

2. Refer to the dot plot. Determine which of these best estimates the standard deviation of the points in a card game.



   A. 5 points
   B. 20 points
   C. 50 points
   D. 500 points

3. The mean of Data set A is 43.5 and the standard deviation is 3.7.
The mean of Data set B is 12.8 and the standard deviation is 4.1.

   a. Which data set shows greater variability? Explain your thinking.

   b. What differences would you expect to see when comparing the dot plots of the two data sets?

4. Consider this data set: $1, 3, 3, 3, 4, 8, 9, 10, 10, 17$.

   a. What is the five-number summary that represents this data set?

   b. Suppose the maximum value is removed from the data set. What is the five-number summary that represents this new data set?

Name: .... Date: .... Period: ....

Practice

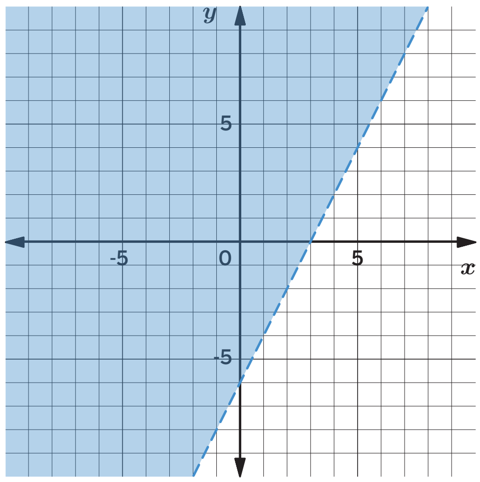
5. Which inequality is represented by the graph?

A. $4x - 2y > 12$

B. $4x - 2y < 12$

C. $4x + 2y > 12$

D. $4x + 2y < 12$



6. The tables show the players' heights, in feet and inches, on two NBA championship teams, Team A and Team B.

Team A

6'10"	6'5"	6'10"	6'4"	6'8"	6'1"
6'3"	6'5"	6'6"	6'10"	7'0"	6'6"
6'8"	6'1"	6'6"	6'9"	6'8"	

Team B

6'9"	6'6"	6'5"	6'6"	6'8"	6'8"
6'11"	6'3"	6'6"	7'0"	6'11"	6'3"
6'7"	6'8"	6'6"	6'2"	6'7"	

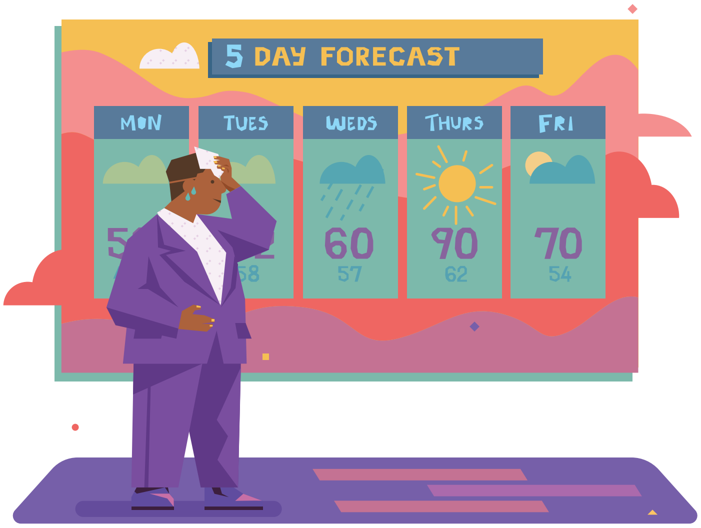
a. The median height of the players on Team A is 6'6". Explain what the value of the median represents in this situation.

b. The IQR of the players' heights on Team A is 4 in. and the IQR for the players' heights on Team B is 2 in. What does the IQR tell you about the data for each team?

Unit 2 | Lesson 8

Choosing Appropriate Measures (Part 1)

Let's investigate relationships between shapes of distributions and measures of center and variability.



Warm-up Data Talk

You will be given four dot plots. Study each display. Determine the mean and median of each distribution mentally. Estimate the standard deviation and IQR. Record your solutions and the strategies you used.

1. Dot plot A
Strategy:

Solutions:

2. Dot plot B
Strategy:

Solutions:

3. Dot plot C
Strategy:

Solutions:

4. Dot plot D
Strategy:

Solutions:

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Extreme Heat and Extreme Cold

The temperatures, in degrees Fahrenheit, of a city during a five-week period are shown in the table. Use spreadsheet technology to respond to the following problems.

52	59	60	61	65	64	66
54	58	61	62	64	65	67
56	59	60	63	65	64	66
57	60	61	62	64	66	68
58	61	60	64	65	67	70

1. Determine the mean and the median, then compare their values.

2. Determine the standard deviation and the IQR, then compare their values.

3. Construct a histogram of the data then describe its shape.

Activity 1 Extreme Heat and Extreme Cold (continued)

4. Suppose a record-breaking high temperature of $90°F$ occurs during the five-week period. Replace the maximum value of the original data set with 90.

a Using the new maximum value, determine the mean, median, standard deviation, and IQR.

b How does changing the maximum value affect the original statistics you determined in Problems 1 and 2? Explain your thinking.

5. Suppose a record-breaking low temperature of $30°F$ occurs during the five-week period. Replace the minimum value of the original data set with 30.

a Using the new minimum value, determine the mean, median, standard deviation, and IQR.

b How does changing the minimum value affect the original statistics you determined in Problems 1 and 2? Explain your thinking.

6. Which measure of center, the mean or median, do you think is more affected by extreme values? Explain your thinking.

7. Which measure of variability, the standard deviation or IQR, do you think is more affected by extreme values? Explain your thinking.

Name: ______ Date: ______ Period: ______

Activity 2 Three Distributions

You will be given three different data distributions. Complete the following problems for each data distribution.

1. Data Set A

a. Is this distribution skewed? If so, in which direction?

b. Which measure of center is greater: the mean or the median?

c. Which measure of center do you think is more appropriate for this distribution: the mean or the median? Explain your thinking.

d. Which measure of variability do you think is more appropriate for this distribution: the standard deviation or the IQR? Explain your thinking.

2. Data Set B

a. Is the distribution skewed? If so, in which direction?

b. Which measure of center is greater: the mean or the median?

c. Which measure of center do you think is more appropriate for this distribution: the mean or the median? Explain your thinking.

d. Which measure of variability do you think is more appropriate for this distribution: the standard deviation or the IQR? Explain your thinking.

Activity 2 Three Distributions (continued)

3. Data Set C

a Is the distribution skewed? If so, in which direction?

b Which measure of center is greater: the mean or the median?

c Which measure of center do you think is more appropriate for this distribution: the mean or the median? Explain your thinking.

d Which measure of variability do you think is more appropriate for this distribution: the standard deviation or the IQR? Explain your thinking.

4. If the mean is a more appropriate measure of center for a data set, which measure of variability do you think would be more appropriate: the standard deviation or the IQR? Explain your thinking.

5. If the median is a more appropriate measure of center for a data set, which measure of variability do you think would be more appropriate: the standard deviation or the IQR? Explain your thinking.

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You determined that the mean and the standard deviation are more appropriate measures for some distributions, while the median and the IQR are more appropriate measures for other distributions.

Outliers and skewness have a greater effect on the mean than on the median. The inclusion of extreme values also increases the variability of a data set. For distributions with skew or outliers, the median and IQR may be more appropriate measures.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. In science class, Clare and Lin each record their estimates of the masses of eight different objects. The actual weight of each object is 2,000 g. The mean, MAD, median, and IQR have been calculated using their respective estimates. Which student was more accurate when estimating the masses of the objects? Explain your thinking.

Clare:
Mean: 2,000 g
MAD: 275 g
Median: 2,000 g
IQR: 500 g

Lin:
Mean: 2,000 g
MAD: 225 g
Median: 1,950 g
IQR: 350 g

2. Consider this data set: 1, 2, 3, 3, 4, 4, 4, 4, 5, 5, 6, 7.

a. What happens to the mean and standard deviation of the data set if the 7 is changed to a 70?

b. If the 7 is changed to a 70, why would the median be a more appropriate measure of center than the mean?

3. The following data set represents the expected number of paintings an artist will produce each day for 10 days.
0, 0, 0, 1, 1, 1, 2, 2, 3, 5

a. Use graphing or spreadsheet technology to determine the mean and standard deviation of the data set.

b. The artist is not pleased with these statistics. If the 5 is increased to a larger value, how does this impact the mean, median, and standard deviation?

Name: ______ Date: ______ Period: ______

Practice

4. The following data set represents the number of cans collected by 12 different classes for a service project.
$12, 14, 22, 14, 18, 23, 42, 13, 9, 19, 22, 14$

a. Determine the mean.

b. Determine the median.

c. Eliminate the greatest value, 42, from the data set. Explain how the measures of center change.

5. Select the solution set to the inequality $2x - 3 > \frac{2x - 5}{2}$.

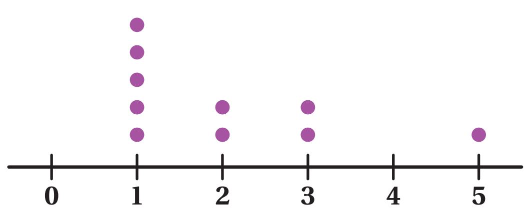
A. $x < \frac{1}{2}$

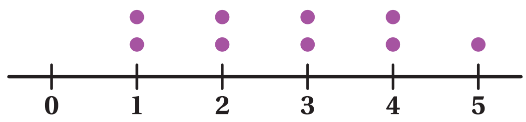
B. $x > \frac{1}{2}$

C. $x \leq \frac{1}{2}$

D. $x \geq \frac{1}{2}$

6. Determine which measure of center – the *mean* or the *median* – best describes each distribution and explain your thinking. Then calculate the most appropriate measure(s) of center.

a.


b.


Unit 2 | Lesson 9

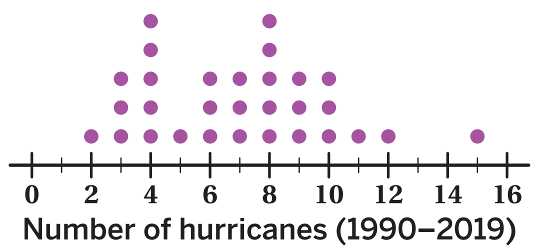
Choosing Appropriate Measures (Part 2)

Let's apply what you have learned about statistics to understand hurricanes.

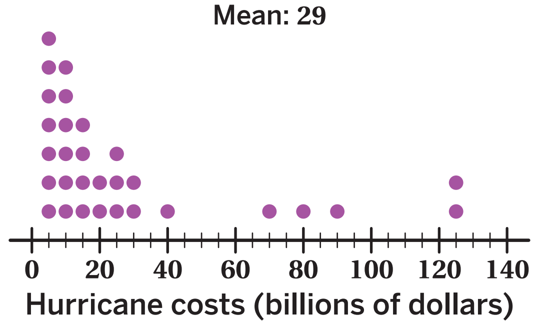
Warm-up Recent Hurricanes

The two dot plots summarize hurricane data from 1990 to 2019. Dot plot A shows the number of hurricanes each year, while Dot plot B shows the approximate damage (in billions of dollars) of the 30 costliest hurricanes. The approximate mean of each data set is also given.

Dot plot A
Mean: 7



Dot plot B
Mean: 29



1. Is the mean representative of a typical number of hurricanes in Dot plot A? Explain your thinking.

2. Is the mean representative of the typical cost of damage of a severe hurricane in Dot plot B? Explain your thinking.

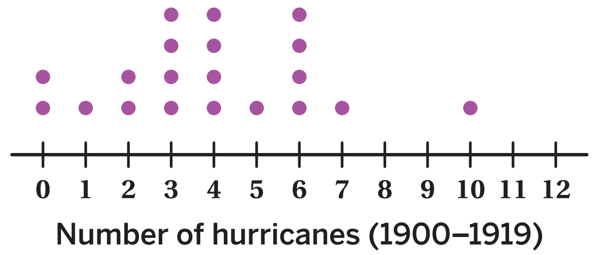
3. The Federal Emergency Management Agency (FEMA) provides hurricane relief throughout the country and is planning to use Dot plot B's mean to determine its 2020 budget for hurricane relief. Do you agree with their decision? Explain your thinking.

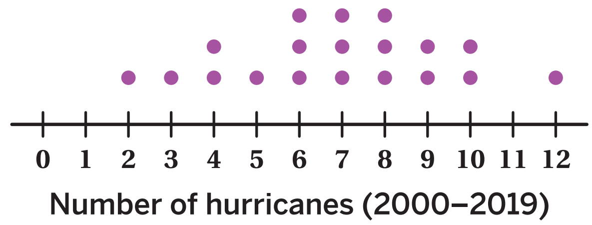
Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Hurricane Frequency

A tropical storm is classified as a hurricane when it produces sustained wind speeds of at least 74 mph. Refer to these dot plots shown.

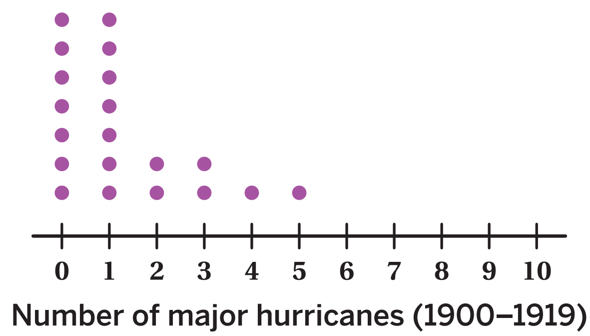


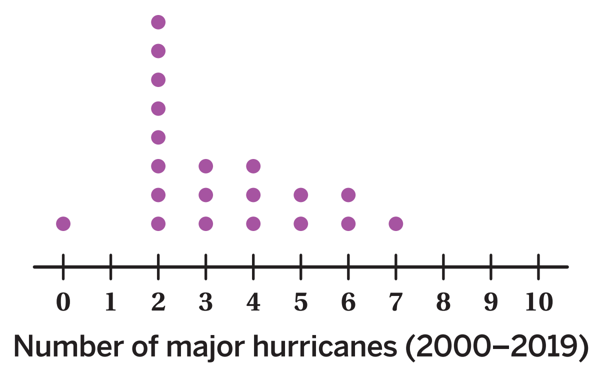


1. Which measure of center would you use to compare these data sets? Explain your thinking.

2. Which measure of variability would you use to compare these data sets? Explain your thinking.

A hurricane is considered a major hurricane when its wind speeds reach at least 111 mph. Refer to these dot plots shown.





3. What measure of center and measure of variability would you use to compare these distributions? Explain your thinking.

4. Is 0 an outlier for the data recorded during 2000–2019? Explain your thinking.

Activity 1 Hurricane Frequency (continued)

5. Use technology to determine the following statistics for each number of major hurricanes data set showing the number of major hurricanes.

	Mean	Median	IQR	SD
1900–1919				
2000–2019				

6. Use the measure of center and the measure of variability you chose in Problem 3 to compare the data for the number of major hurricanes for each time period.

7. Which statistic(s) might be of greater interest to a meteorologist? Explain your thinking.

8. Which statistic(s) might be of greater interest to a climatologist? Explain your thinking.

Are you ready for more?

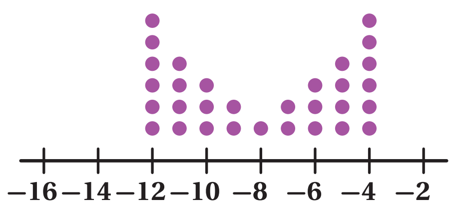
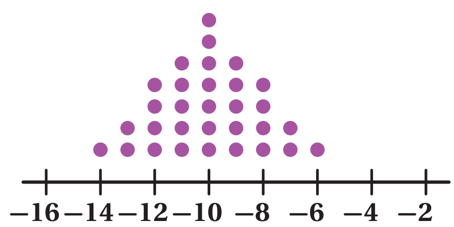
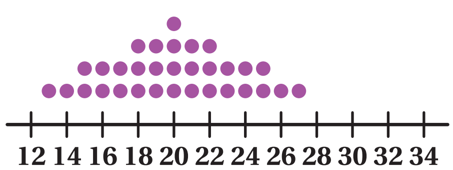
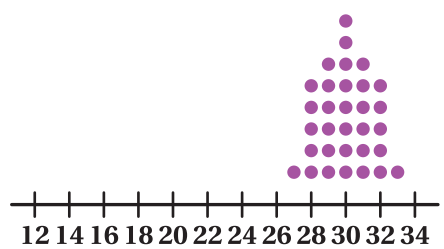
How might the frequency of major hurricanes from 2100–2119 compare to the distributions shown in the activity? Create a dot plot showing the frequency of hurricanes for this time period.

Name: ________ Date: ________ Period: ________

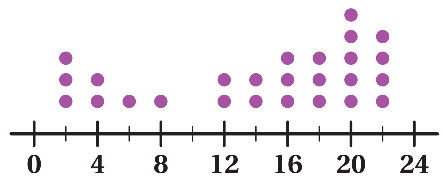
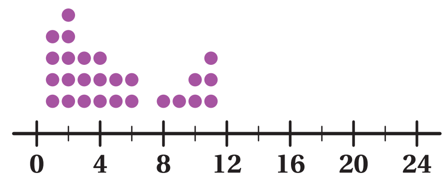
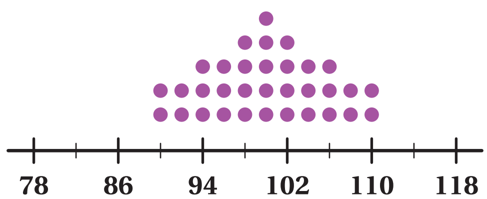
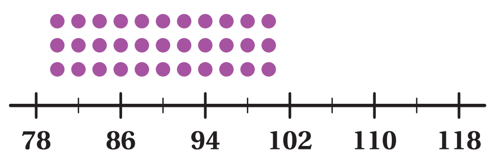
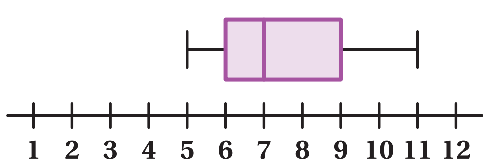
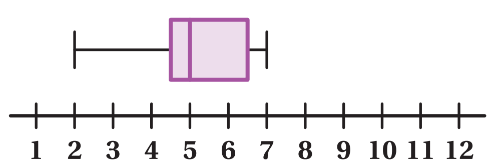
Activity 2 Comparing Measures

Follow these instructions to complete this activity.

- **Partner A will determine the most appropriate measure of center and measure of variability to use, based on the distributions in Set A.**
- **Partner B will determine the most appropriate measure of center and measure of variability to use, based on the distributions in Set B.**
- **After you and your partner have come to a consensus for each pair of data sets, determine which distribution has the greater measure of center and variability.**

	Set A	Set B
1.		
2.		

Activity 2 Comparing Measures (continued)

	Set A	Set B
3.		
4.		
5.		
6.	A political podcast has reviews that mostly either love the podcast or hate it.	A data science podcast has reviews that neither hate nor love the podcast.

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

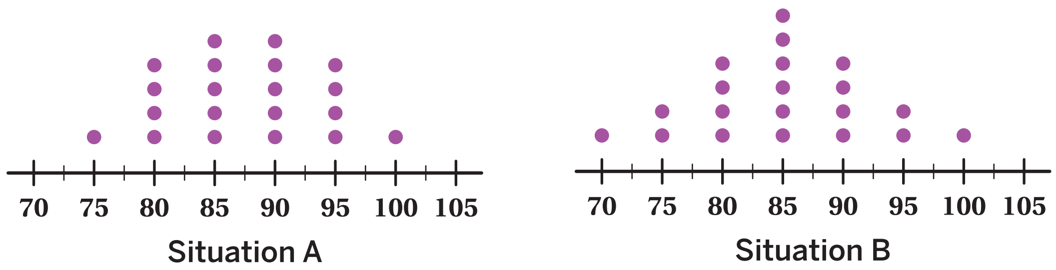
You applied your knowledge from previous lessons to compare data sets using measures of center and measures of variability. You used your experiences with symmetric and uniform distributions to summarize data sets using the mean and the standard deviation and summarized skewed distributions using the median and the IQR. You used hurricane data to make decisions and predictions, just as city planners and researchers would.

Reflect:

Practice

Name: ____________ Date: ____________ Period: ____________

1. 20 students participated in a psychology experiment in which their heart rates were measured in two different situations. The dot plots represent the frequency of their heart rates in each situation.



a. What are the appropriate measures of center and variability to use with each data set? Explain your thinking.

b. Which situation shows a greater typical heart rate? Explain your thinking.

c. Which situation shows a greater variability? Explain your thinking.

2. The mean exam score for the first group of 20 examinees applying for a security job is 35.3 with a standard deviation of 3.6. The mean exam score for the second group of 20 examinees is 34.1 with a standard deviation of 0.5. Both distributions are close to being symmetric in shape.

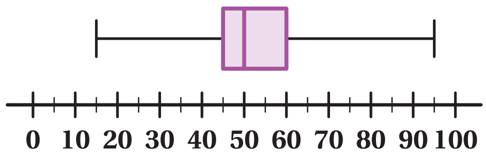
a. Use the mean and standard deviation to compare the scores of the two groups.

b. The minimum score required for an in-person interview is 33. Which group do you think has more people who qualify for an in-person interview? Explain your thinking.

Name: .... Date: .... Period: ....

Practice

3. Consider the box plot shown. Create another box plot that has a greater measure of variability, but the same minimum and maximum values.



4. The height, in inches, of a class of 24 students is measured. The median height is 64 in. and the IQR is 10. What information does this tell you about the height of the students in this class?

5. Tyler records the number of letters he receives in the mail over the past week.
$2, 3, 5, 5, 5, 15$

a. Which value appears to be an outlier?

b. How can you determine if the value you chose is an outlier?

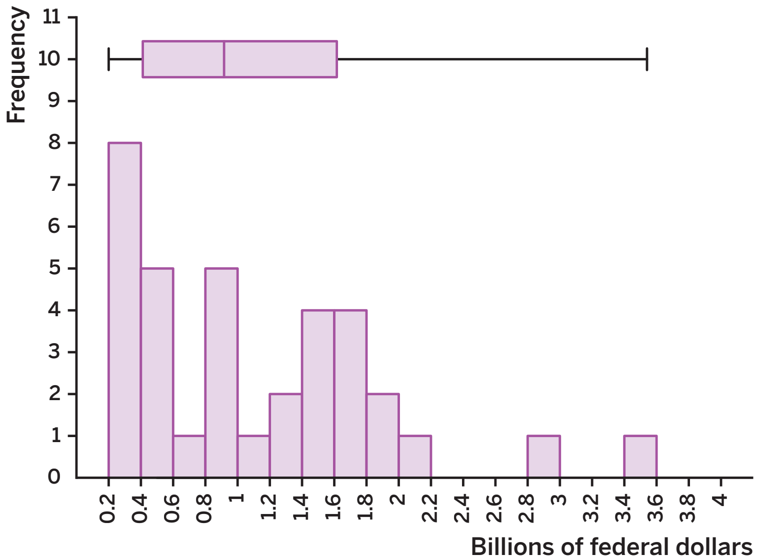
Unit 2 | Lesson 10

Outliers and Standard Deviation

Let's take a closer look at outliers, now that we know more about standard deviation.

Warm-up The Cost of Wildfires

Millions of acres of land are consumed by wildfires each year in the U.S. Warmer temperatures, shorter winters, and dryer forests increase the risk of wildfires, which has led to more federal spending to fight these fires. The displays show how many billions of dollars the U.S. government has spent fighting wildfires between 1985 and 2019.



1. Describe the distribution of this data set, including any unique features.

2. Does the data set have an outlier? Explain your thinking.

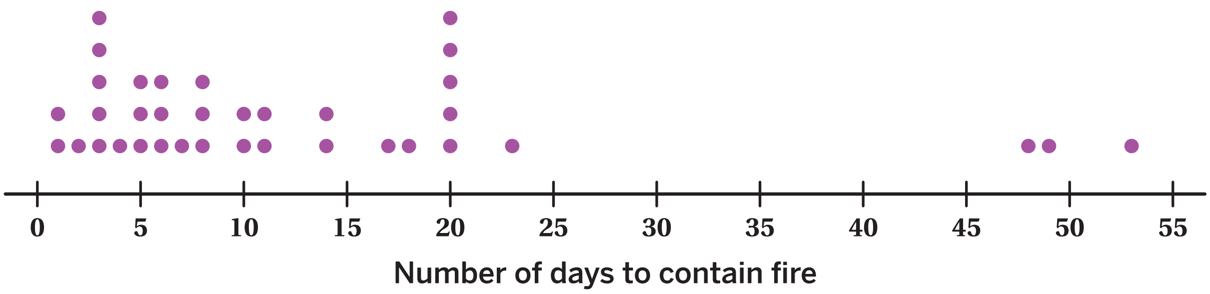
Log in to Amplify Math to complete this lesson online.

Name: ____________ Date: ________ Period: ________

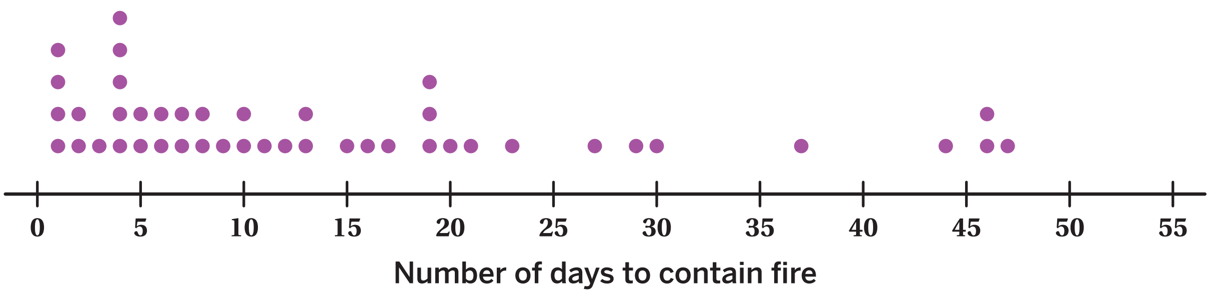
Activity 1 Investigating Outliers

Wildfires particularly affect parts of the western United States. The dot plots show the number of days in 2020 it took to contain different wildfires in Arizona and California. The mean and the standard deviation for each data set are also shown.

Arizona wildfires **mean: 12.92 days, standard deviation: 13.07 days**



California wildfires **mean: 14.45 days, standard deviation: 13.28 days**



1. Interpret the mean in terms of the situation.

2. Interpret the standard deviation in terms of the situation.

Activity 1 Investigating Outliers (continued)

3. Use the mean and standard deviation to compare the two data sets.

4. How might you use the standard deviation and the mean of each data set to determine if a value is an outlier? Write your own mathematical definition for an outlier that involves the standard deviation and mean.

5. Select one of the data sets (Arizona or California). Use your mathematical definition of an outlier from Problem 4 to determine if there are any outliers in this data set.

Name: .... Date: .... Period: ....

Activity 2 Where Do Outliers Come From?

1. The data set shows the number of wildfires caused by lightning in Southern California each year from 2001 to 2019. Some statistics for the data are also shown.

832	179	428	323	272	409	291	174	179	216
258	266	274	259	397	96	188	131	76	

Mean: 276.21 **Minimum:** 76 **SD:** 166.95 **Maximum:** 832
Q1: 179 **Median:** 259 **Q3:** 323

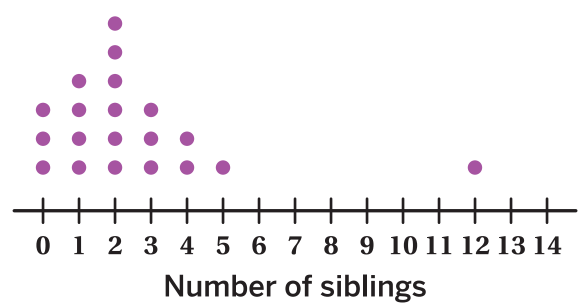
a Are any of the values outliers? Use the standard deviation to explain or show your thinking.

b If there are outliers, why do you think they exist? Do you think they should be included in the analysis of the data?

c The presence of an outlier may indicate some sort of problem, such as an error in measurement, data collection, or recording. Suppose any outlier you identified *is* an error. What might be done to handle it?

2. The following scenarios have an outlier. For each scenario, how would you determine whether it is appropriate to keep or remove the outlier when analyzing the data? Discuss your thinking with your partner.

a The dot plot represents the distribution of the number of siblings reported by a group of 20 people.



Activity 2 Where Do Outliers Come From? (continued)

b Tyler rolls a standard number cube 15 times and records his data.
1, 1, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 20

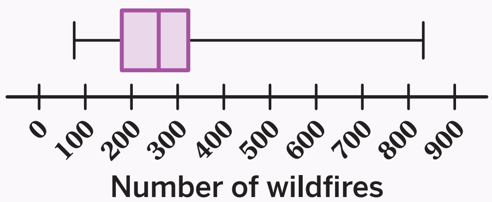
c In a science class, 12 groups of students are synthesizing biodiesel. At the end of the experiment, each group records the mass in grams of the biodiesel they synthesized.

0, 1.245, 1.292, 1.375, 1.383, 1.412, 1.435, 1.471, 1.482, 1.501, 1.532

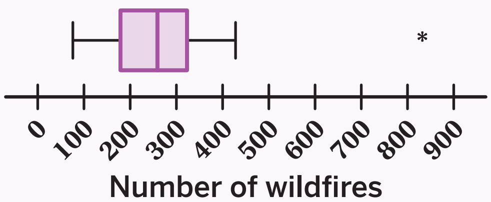
Are you ready for more?

Two different types of box plots are shown using the data from Problem 1.

A *standard* box plot uses the five-number summary of a data set where the endpoints of the whiskers are the minimum and maximum values of that set.



A *modified* box plot marks any outliers in the data set and the whiskers go only as far as the highest and lowest values that are *not* outliers.



1. What information does a modified box plot show that a standard box plot does not?

2. The following data set shows the number of lightning-caused wildfires in 20 eastern states in the U.S. from 2001 to 2019. Use the data to construct a modified box plot.

38	69	59	107	86	34	54	205	161	169
62	171	330	256	175	88	102	372	889	

Name: ______ Date: ______ Period: ______

Activity 3 Interpreting Outliers

The following data set was used to create the displays from the Warm-up.

0.240	0.398	0.477	1.411	0.819	0.810	2.131
0.203	0.206	0.701	0.953	1.704	1.375	1.976
0.335	0.377	0.284	1.674	1.620	1.902	2.918
0.579	0.240	0.417	1.327	1.586	1.741	3.543
0.500	0.918	0.516	1.007	0.921	1.522	1.590

1. Use spreadsheet technology to determine the mean, standard deviation, and five-number summary for how much money (in billions of dollars) the U.S. government spent suppressing wildfires. Round to the nearest thousandth.

2. The maximum value, which happened to come from the year 2018, is an outlier. Use the standard deviation to explain or show why this value is an outlier.

3. Oops! A discrepancy was discovered in the reporting of federal spending on wildfire suppression in the year 2018. Although outliers should not be removed without considering their cause, it is important to see how influential outliers can be for various statistics. Remove the outlier from the data set.

a Use technology to calculate the new mean, standard deviation, and five-number summary.

b After the outlier is removed, how do the mean and standard deviation of the data set compare to the same statistics of the original data set?

Reflect: How did you demonstrate that you were actively listening to your partner?

STOP

Summary

In today's lesson . . .

You were introduced to a new way of mathematically determining whether a value is an outlier — if it is at least 3 standard deviations above (or below) the mean. **Note:** This is a good rule of thumb. However, when working with very large data sets with thousands of data points, it is perfectly normal to encounter a few data points that are 3 or more standard deviations from the mean and would not be considered outliers.

It is important to identify an outlier's source, because outliers can significantly affect measures of center and variability. Outliers can reveal cases that are worth studying in greater detail, if they represent accurate values in the data set.

An outlier can also reveal errors in the data collection process. If an outlier is a result of an error, it can be removed. To avoid tampering with the data and to report accurate results, data values should not be deleted unless they are confirmed to be errors in the data collection or entry process.

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. The widths, in millimeters, of fabric produced at a ribbon factory are collected. The mean is approximately 23 mm and the standard deviation is approximately 0.06 mm. What information does the mean and standard deviation provide about the fabric? Explain or show your thinking.

2. Elena collects 112 specimens of beetle and records their lengths for an ecology research project. Elena incorrectly records one of the lengths of the beetles as 122 cm (about 4 ft). What should she do with the outlier, 122 cm, when she analyzes her data?

3. Mai surveys the students in her class to determine how many hours each student spends reading each week. Mai then calculates the following statistics using the data she gathered.

Mean: 8.5 hours **SD:** 5.3 hours
Q1: 5 hours **Median:** 7 hours **Q3:** 11 hours

a. Give an example of a number of hours greater than the median which would be an outlier. Explain your thinking.

b. Are there any outliers less than the median? Explain your thinking.

Name: ______ Date: ______ Period: ______

Practice

4. Select *all* questions that are statistical questions.

A. How cold is it normally in Florida in the winter?

B. How much snow did Chicago get on January 1, 2019?

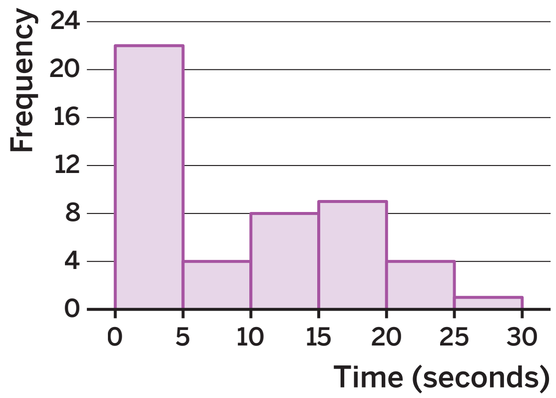
C. What is your favorite sport?

D. What is the average height of the students in your classroom?

E. How many steps did you take today?

F. What is the largest city in the United States?

5. The histogram represents the distribution of the number of seconds it took for 50 students to determine the answer to a trivia question. Which interval contains the median?



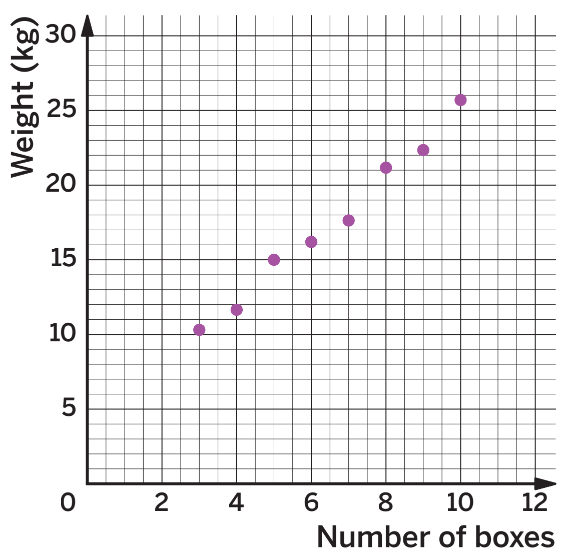
A. 0 to 5 seconds

B. 5 to 10 seconds

C. 10 to 15 seconds

D. 15 to 20 seconds

6. The scatter plot shows the weight of boxes of oranges at the grocery store.



a. Describe the association between the number of boxes of oranges and weight.

b. What is the weight of 6 boxes of oranges?

c. Predict the weight of 12 boxes of oranges.

SUB-UNIT

3

Bivariate Data

Narrative Connections

What is "Day Zero"?

In January 2018, the air was charged in the city of Cape Town, South Africa. Men and women stood in lines with jugs, barrels, and buckets to draw water from the city's springs and communal taps. Cape Town was facing the worst drought the region had seen in a century. And now its four million residents were bracing for the worst — what officials were calling "Day Zero."

On that day, city officials would shut off the public water supply and begin water rationing measures. Residents would have to gather their allotted 25 liters of clean water from the 149 police-protected collection points throughout the city.

The measures were drastic, but necessary. The drought had begun in 2015, when the city's water reserves fell from 72% down to 50%. By late 2017, that number had dipped as low as 15%. Officials were anticipating a crisis of massive proportions, leading to food shortages, social upheaval, and widespread death and disease.

But by June 2018, the winter rains had restored Cape Town's dams, eventually ending the three-year drought.

How did city officials know when Day Zero would occur? They analyzed the data of how stored water changed over time, observed a linear trend, and made a prediction. Linear models are powerful tools for noticing trends and making predictions. In the lessons that follow, you'll explore just how they work.

Unit 2 | Lesson 11

Representing Data With Two Variables

Let's model data with two variables.

Warm-up A Tree-mendous Resource

Trees are among the world's most critical resources. We breathe in the oxygen they release, while they in turn absorb the carbon dioxide we emit. Trees also serve as a source of food and fuel, and provide material for furniture, medicine, and paper products.

The following data set shows the area of the U.S. covered by forest, in millions of acres, for certain years between 1920 and 2012.
721, 738, 742, 753, 742, 733, 742, 752, 766

1. Choose and create a representation to display this data.

2. Which measures of center and variability would be most appropriate for this data set? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

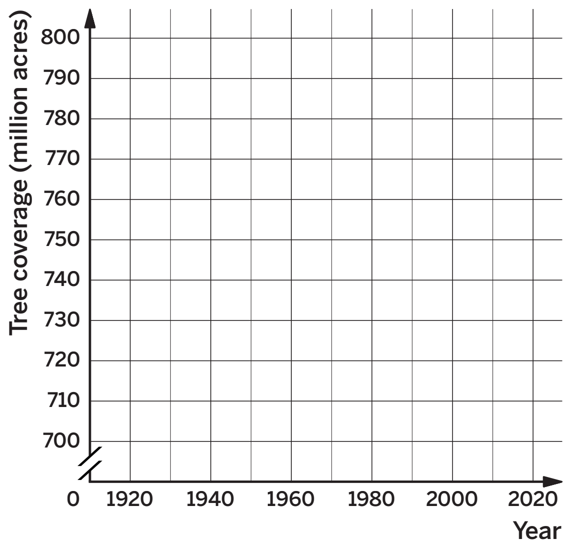
Name: ______ Date: ______ Period: ______

Activity 1 Plotting Points

The total area of the U.S. covered by forest for certain years between 1920 and 2012 is shown in the table.

Year	1920	1940	1953	1963	1977	1987	1997	2007	2012
Area (million acres)	721	738	742	753	742	733	742	752	766

1. Plot the points shown in the table on the graph.



2. According to your *scatter plot*, how did the area of tree coverage change from 1920 to 2012? Does there appear to be a trend?

3. What are some factors that might contribute to an increase in tree coverage? What might contribute to a decrease in tree coverage?

4. What would be a good estimate for the area of tree coverage in 1930? Explain your thinking.

5. Can you use your scatter plot to predict the area of tree coverage in 2030? Explain your thinking.

Critique and Correct: Critique the following statement to identify and correct any errors: 780 million acres of tree coverage is a reasonable prediction for the year 2030, based on the data.

Activity 2 Defoliating Insects

One factor that has contributed to decreases in tree coverage is "defoliating" insects, so named because they eat the leaves of their tree hosts. This makes the trees less likely to grow, and more likely to be attacked by other insects or diseases or even die.

Safwan Abd Rahman/Shutterstock.com

Part 1

You will be given a scatter plot showing forest area, in millions of acres, that has been defoliated by a certain species of insect over time. Use the scatter plot to respond to the following problems.

1. Describe the trend, if any, in the scatter plot. What does it tell you about the effect of the defoliating insect on the forest's area?

2. Is the trend linear or nonlinear? Explain your thinking.

3. Use the trend to make a prediction of the area of forest that will be defoliated by the insect in 2020.

Part 2

Share your scatter plot with your group members and describe its trend. After everyone has had a chance to share, compare your scatter plots. Come to a consensus about which defoliating insect species was most destructive and which was least destructive between 1980 and 2010. Explain your thinking.

Most destructive defoliating insect:

Least destructive defoliating insect:

Name: ______________________ Date: __________ Period: __________

Activity 3 Card Sort: Describing Data Patterns

You will be given a set of cards. Each card contains a different scatter plot.

1. Sort the cards into the following groups. Ensure that you and your partner agree before moving on to the next card.

   a. Linear and nonlinear

   b. Increasing and decreasing

2. For each card, describe the relationship between the independent and dependent variables.

Card	Relationship between independent and dependent variables
A	
B	
C	
D	
E	
F	
G	
H	

STOP

Summary

In today's lesson . . .

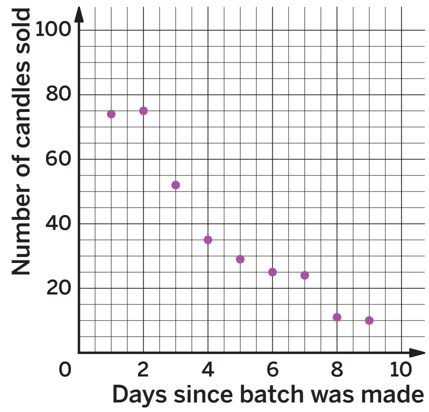
You created a scatter plot to help you analyze bivariate data, or data in two variables. You described the relationship between the independent and dependent variables of scatter plots with different trends, identifying whether the direction was increasing, decreasing, or neither.

You were also able to categorize scatter plots as having linear or nonlinear trends. When data has a linear trend, you can use a linear model to predict values that may not be represented in the scatter plot.

Reflect:

Name: .... Date: .... Period: ....

1. Bard and Mai made and packaged a batch of candles to sell on their online store. The scatter plot shows the number of candles sold each day for 9 days.

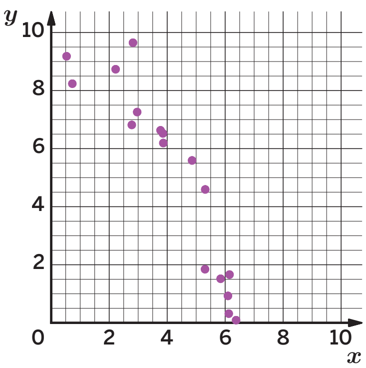


a Is the data linear or nonlinear?

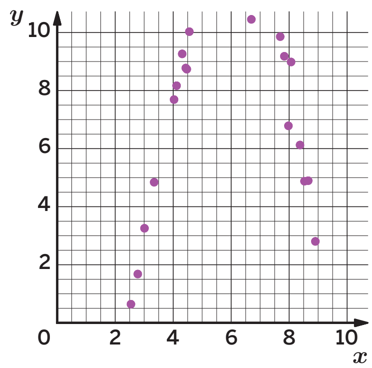
b The scatter plot includes the point $(7, 24)$. Describe its meaning in this situation.

2. Which of the following scatter plots contain data best fit by a linear model?

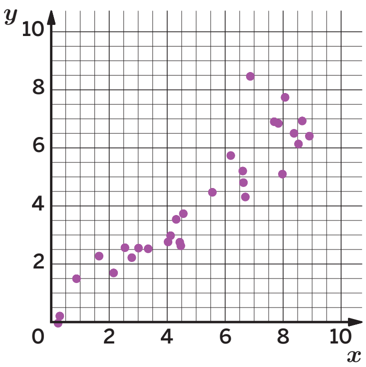
A.



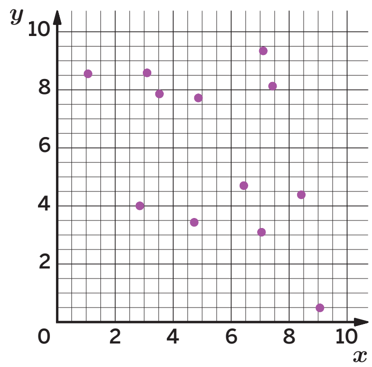
B.



C.



D.



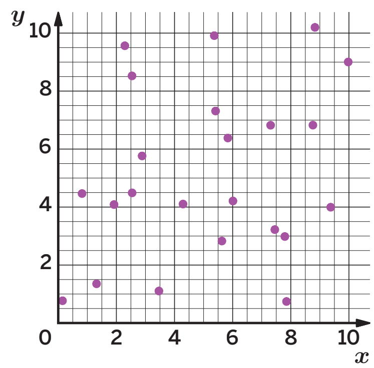
3. Which of these statements is true about the data in the scatter plot?

A. As x increases, y tends to increase.

B. As x increases, y tends to decrease.

C. As x increases, y tends to stay unchanged.

D. x and y appear to be unrelated.



Practice

Name: ______________________ Date: ____________ Period: ____________

4. Consider the following data set.
$11.5, 12.3, 13.5, 15.6, 16.7, 17.2, 18.4, 19, 21.5$

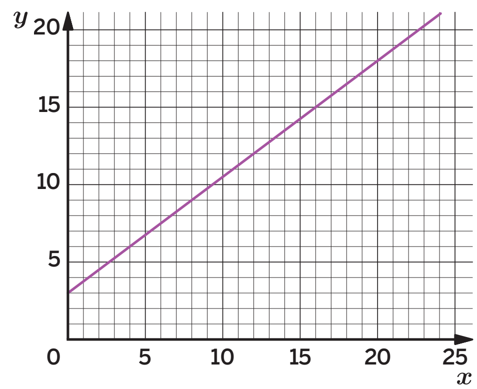
If 5 is added to each value in the data set, determine the impact on:

a. The shape of the distribution.

b. The measures of center.

c. The measures of variability.

5. Refer to the graph of the equation $y = 0.75x + 3$. Select all coordinate pairs that represent a solution to the equation.



A. $(8, 9)$

B. $(10, 10)$

C. $(12, 12)$

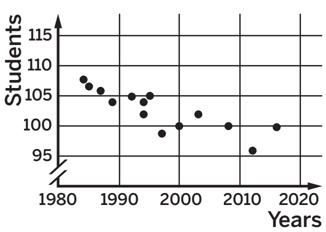
D. $(14, 13.75)$

E. $(16, 15.25)$

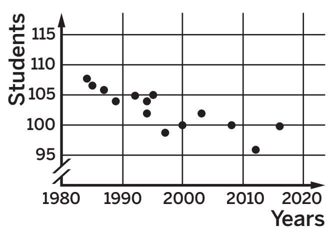
F. $(18, 16.5)$

6. Three lines of fit are given for the data set. Which line seems to be the *best* fit for the data? Explain your thinking.

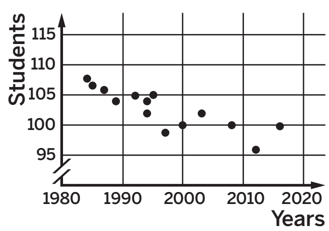
A.



B.



C.



Name: ________ Date: ________ Period: ________

Unit 2 | Lesson 12

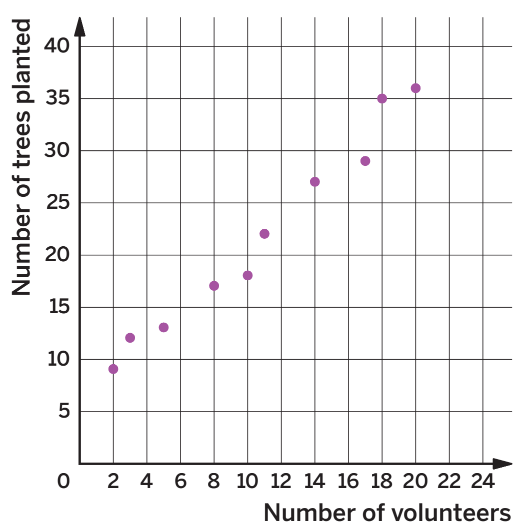
Linear Models

Let's explore the relationships between two numerical variables.

Warm-up Planting Trees

Wildfires can cause severe environmental damage. Burned forests can take decades to recover. Burning trees also release carbon dioxide, a gas that can trap heat in the Earth's atmosphere, warming the Earth's surface through the "greenhouse effect." Planting new trees can restore forests and counteract years of carbon emissions. For these reasons, several organizations have pledged to restore hundreds of millions of acres of deforested land around the world.

The scatter plot shows the number of trees one such organization plants on different days, along with how many volunteers it had each day.



1. What does the coordinate (10, 18) mean in this context?

2. Approximately how many trees would you expect 15 volunteers to plant?

3. What can you do to help you make predictions about points that are not on the graph?

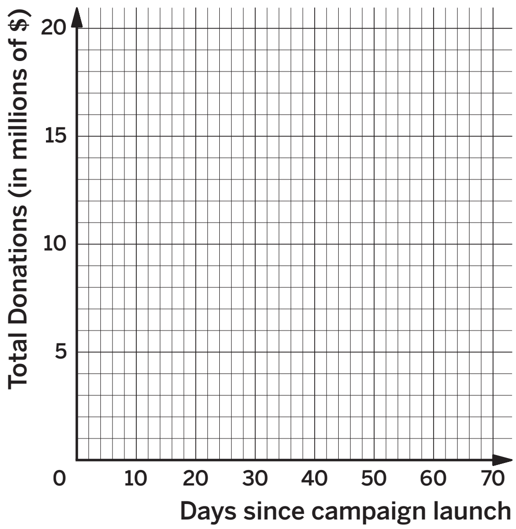
Log in to Amplify Math to complete this lesson online.

Activity 1 Team Trees

In May 2019, two social media influencers launched a campaign to plant 20 million trees by 2020. They partnered with the Arbor Day Foundation, which plants one tree for every dollar donated. The table shows the amount of money donated to plant a tree since the campaign launched.

Days since campaign launch	Total donations (millions of $)
7	11.8
11	12.9
13	14.3
18	15.1
31	16.2
41	17.2
44	17.5
49	18.4
54	19.5

1. Create a scatter plot of the data. Then draw a line that fits the data.



2. Estimate the slope of the line that you drew. What does the slope represent?

3. The campaign launched on October 25, 2019. If the goal was to plant 20 million trees by January 1, 2020, 68 days after the campaign launch, and Team Trees continued receiving donations at the same rate, predict whether it reached its goal. Explain your thinking.

4. Which point(s) fit your linear model well? Explain your thinking.

5. Which point(s) do not fit your linear model well? Explain your thinking.

Name: ______ Date: ______ Period: ______

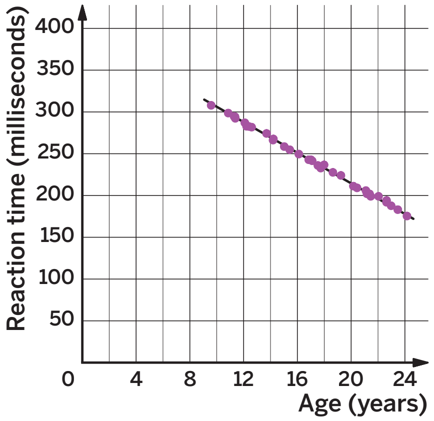
Activity 2 The Slope Is the Thing

Three scatter plots are shown, along with equations for lines that fit the data, where x represents the horizontal axis and y represents the vertical axis.

For each scatter plot:

- **Interpret the slope of the linear model.**
- **Interpret the y-intercept of each linear model.**

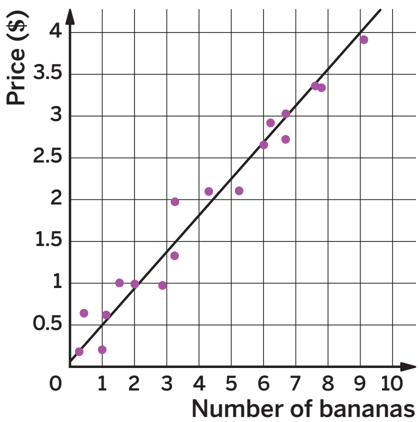
1. $y = -9.25x + 400$



Slope:

y-intercept:

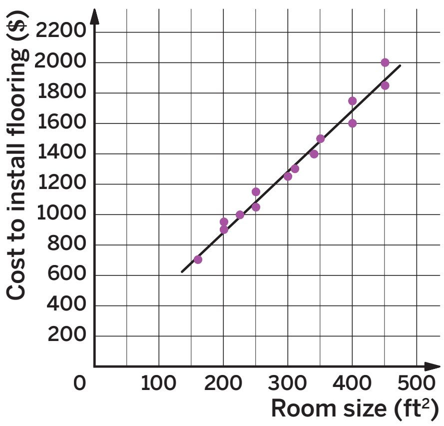
2. $y = 0.44x + 0.04$



Slope:

y-intercept:

3. $y = 4x + 87$



Slope:

y-intercept:

Activity 3 Planting Mangroves

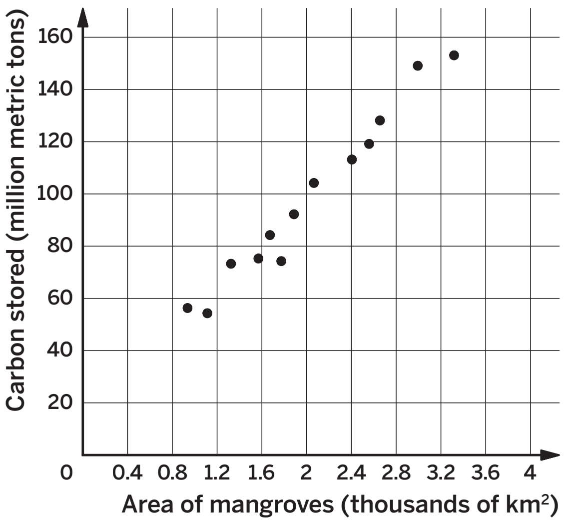
Efforts have been made in recent years to plant (and replant) mangroves along the shorelines of coastal cities. Mangroves are trees that can help to form a natural barrier against rising sea levels and storm surges. Mangrove forests also capture 10 times more carbon than forests on land.

The scatter plot shows the amount of carbon stored, in millions of metric tons, in the mangrove forests of 13 different countries based on their area, in thousands of square kilometers.

KONGKOON/Shutterstock.com

1. Draw a line that fits the data on the scatter plot.

2. What does the slope of your line represent in this context?



3. Plot the point (3, 20) on the scatter plot.
   - **a** What does this point represent in this context?
   - **b** Do you think this point is an outlier? Explain your thinking.

4. Suppose the point (3, 20) is added to the data set. Draw a line that you think fits the data now.

5. How does the new line of fit compare to the original line that you drew?

STOP

Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You recalled how to create a linear model for a scatter plot, and interpreted its slope and vertical intercept. Other data may have a nonlinear trend or no apparent trend at all.

The equation of a linear model is helpful for determining how y changes with respect to x and for estimating or making predictions about values not represented on the scatter plot.

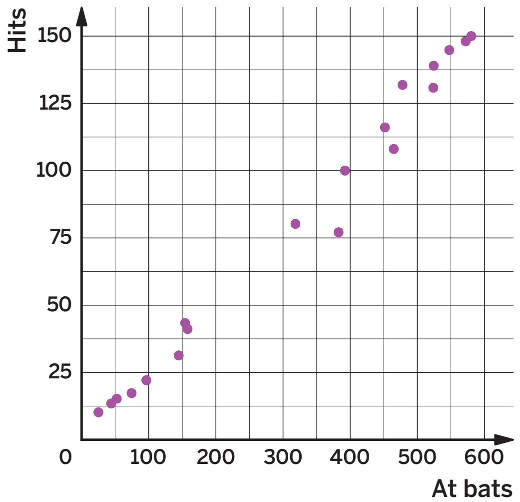
Outliers can sometimes have a strong effect on linear models. As with any outlier, you should closely examine it and determine its cause before removing it from your data set.

Reflect:

Practice

Name: .... Date: .... Period: ....

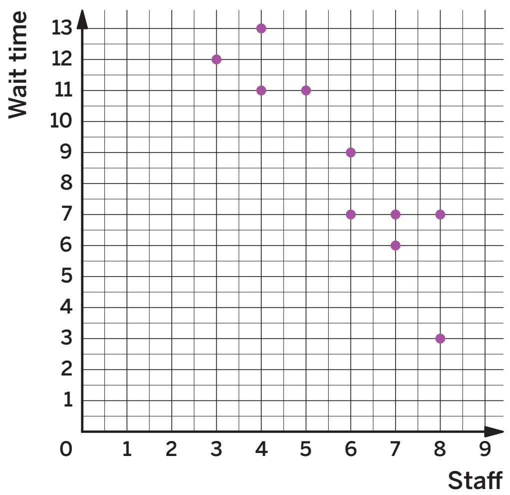
1. The scatter plot shows the number of times a player was at bat and the number of hits they had.



a. The scatter plot includes a point at $(318, 80)$. What does this point mean in context?

b. Suppose the point $(100, 95)$ is added to the data. Is it an outlier? Explain your thinking. What does this point mean in context?

2. The scatter plot shows the number of minutes customers had to wait for service at a restaurant and the number of staff working at the time. The equation for the line of fit is given by $y = -1.62x + 18$, where y represents the wait time, and x represents the number of staff working.



a. The slope of the line is -1.62. What does this mean in this context? Is it realistic?

b. The y-intercept is $(0, 18)$. What does this mean in this context? Is it realistic?

3. A taxi driver records the time required to complete various trips and the distance for each trip. The equation for the line of fit is given by $y = 0.467x + 0.417$, where y represents the distance in miles, and x represents the time for the trip in minutes.

a. Use the linear model to estimate the distance for a trip that takes 20 minutes. Show your thinking.

b. Use the linear model to estimate the time for a trip that is 6 miles long. Show your thinking.

4. Consider the following data set: 3, 9, 1, 10, 3, 7, 8, 2, 2, 11, 1, 35.
Are there any outliers in this data set? Show or explain your thinking.

5. *Technology required*. Use the table to respond to the following problems.

x	2.3	2.8	3.1	3	3.5	3.8
y	6.2	5.7	4.7	3.2	3	2.8

a What is the equation for a line of fit? Round to the nearest hundredth.

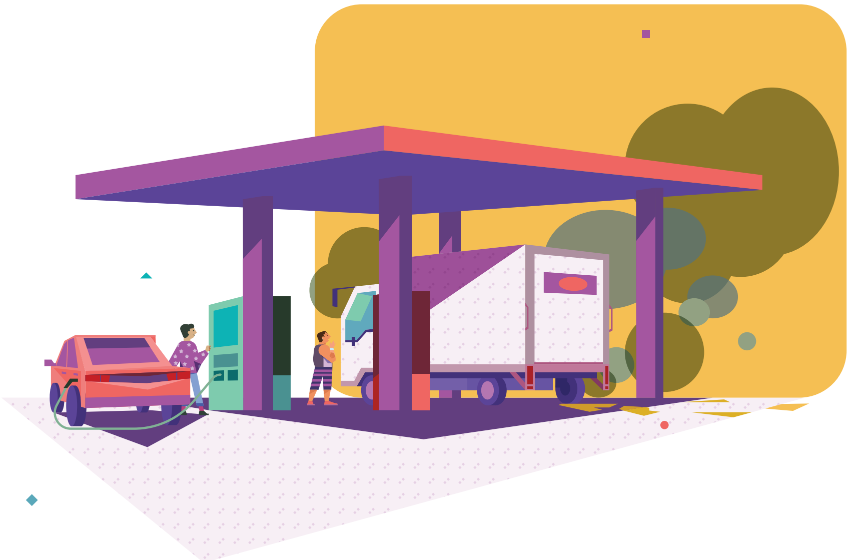
b Use your equation from part a to estimate y when x is 2.3. Round to the nearest thousandth.

c How does the estimated value compare to the actual value from the table when x is 2.3?

d How does the estimated value compare to the actual value from the table when x is 3?

Residuals

Let's examine how close linear models are to the data they represent.



Warm-up Differences in Expectations

Most carbon dioxide (CO_2) emissions in the U.S. come from burning fossil fuels, such as the gasoline for cars and trucks. A zero emissions vehicle (ZEV) does not run on fossil fuels and does not emit any CO_2.

The fuel economy of ZEVs is measured in MPGe, a unit that represents the number of miles traveled on the electric equivalent of one gallon of gas. The table shows the advertised fuel economies for five different ZEVs, compared to their average fuel economies reported by drivers.

	Average fuel economy (MPGe)	Advertised fuel economy (MPGe)	Difference
Suzuku Electron	147.4	130	
Zenith E Series	124.1	116	
Privvus Bolt	150	123	
Matsubishi Nil X	112.6	116	
Vitara Cipher	89.7	121	

1. Complete the table by calculating the difference between average fuel economy and advertised fuel economy.
2. Which vehicle's average fuel economy do you think most surprised its drivers? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Creating a Residual Plot

You will be given a scatter plot and a linear model. The table shows the coordinates of the data points. The equation of the linear model is $y = 2.2x + 3.2$.

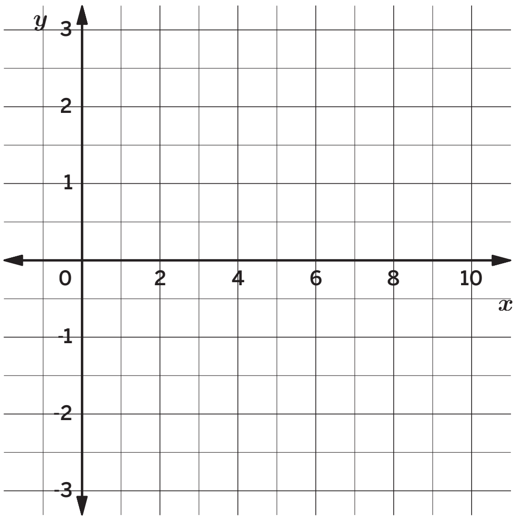
x	y	Predicted y-coordinate	Residual
1	4		
2	9		
3	10		
4	12		
5	15		
6	16		
7	18		
8	21		
9	22		
10	26		

1. Use the equation to determine the predicted y-coordinates for each value of x estimated by the linear model. Record the predicted values in the table.

2. For each data point, subtracting the predicted y-coordinate from the actual y-coordinate gives you that point's ***residual***. Calculate the residuals and record them in the table.

3. If the actual y-coordinate is more than the predicted y-coordinate, will the residual be positive or negative? Does this indicate an overestimate or an underestimate?

4. A ***residual plot*** shows the residuals on the vertical axis, with the independent variable (x) on the horizontal axis. Create a residual plot on the axes shown.



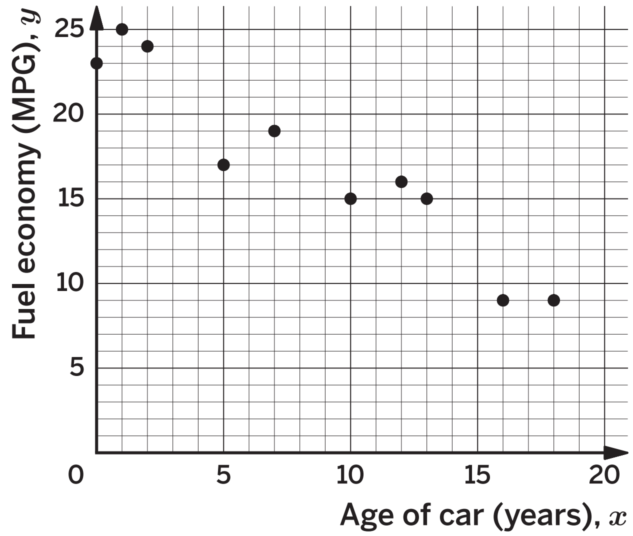
5. How does the residual plot compare to the scatter plot?

Activity 2 Which Line Fits Better?

Plan ahead: What will you do if you have an impulse to quit because your stress level is rising during the activity?

Clare has been tracking the fuel economy of her gas-powered car since she bought it from the dealership. Consider the scatter plot showing the relationship between the age of Clare's car, in years, and its fuel economy, in miles per gallon.

1. Draw a line to fit the data. Then write an equation for the line you drew.



2. The table shows the x- and y-coordinates of each data point. Calculate the predicted y-coordinate (according to your model) and the residual for each data point.

x	y	Predicted y-coordinate	Residual
0	23		
1	25		
2	24		
5	17		
7	19		
10	15		
12	16		
13	15		
16	9		
18	9		

Name: ______ Date: ______ Period: ______

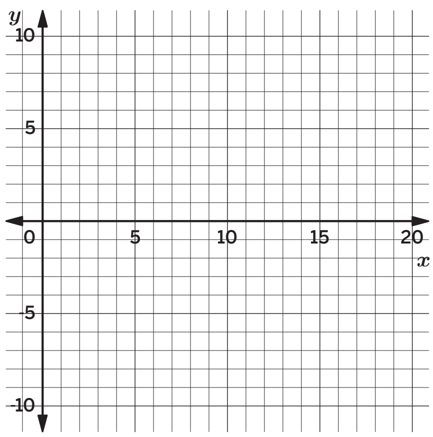
Activity 2 Which Line Fits Better? (continued)

Clare calculated the residuals for the data based on her own linear model. Her results are shown in the table.

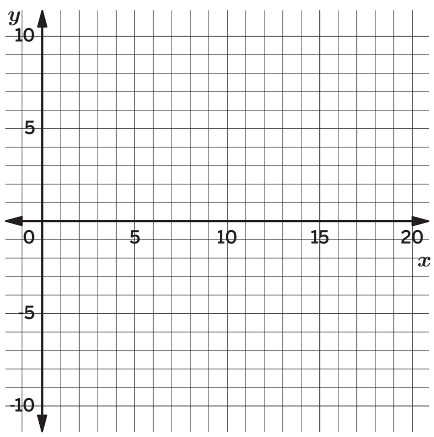
x	y	Predicted y-coordinate	Residual
0	23	28	−5
1	25	27.25	−2.25
2	24	26.5	−2.5
5	17	24.25	−7.25
7	19	22.75	−3.75
10	15	20.5	−5.5
12	16	19	−3
13	15	18.25	−3.25
16	9	16	−7
18	9	14.5	−5.5

3. Draw a residual plot for your linear model and for Clare's linear model on the graph provided.

Your Model:



Clare's Model:



4. Based on your residual plots, whose linear model do you think fits the data better: yours or Clare's? Explain your thinking.

Activity 3 Reducing Emissions

Many car companies manufacture alternative-fuel vehicles that use fuels with fewer CO_2 emissions, such as plug-in hybrid electric vehicles (PHEVs). The following table shows the fuel economy of eight PHEVs and their CO_2 emissions. The residuals have been calculated from a linear model with the equation $y = -1.10x + 156.50$.

Fuel Economy (mpg), x	CO_2 Emission (g/km), y	Residuals
88	62.01	2.31
79	69.08	−0.52
78	69.96	−0.74
76	71.80	−1.1
66	82.68	−1.22
61	89.46	0.06
59	92.49	0.89
58	94.09	1.39

1. In this context . . .

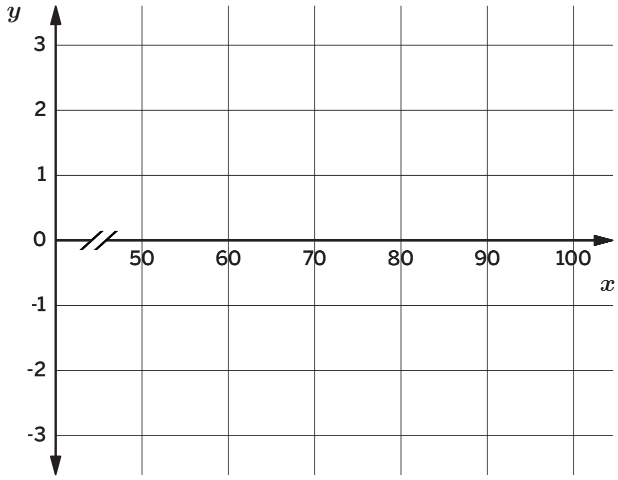
a. What does a positive residual represent? A negative residual?

b. Are positive or negative residuals more desirable? Explain your thinking.

Name: ______ Date: ______ Period: ______

Activity 3 Reducing Emissions (continued)

2. Create a residual plot for this data.



3. What is the shape of this residual plot? How does it compare to the residual plot you created in Activity 1?

4. Based on your residual plot, do you think a linear model is a good fit for this data? Explain your thinking.

5. Based on all your residual plots in this lesson, what do you think a residual plot should look like for a linear model that fits a data set well?

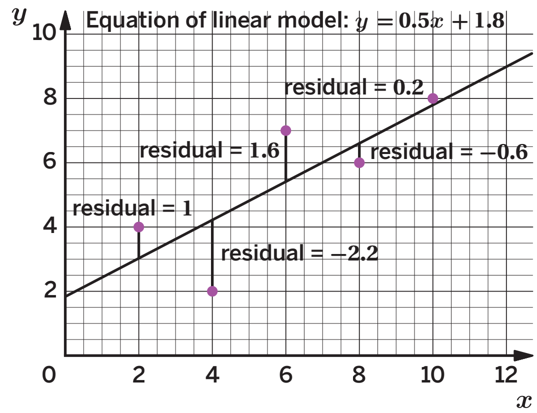
STOP

Summary

In today's lesson . . .

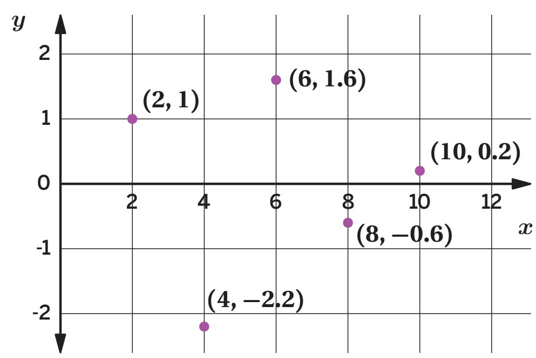
You learned how to calculate ***residuals*** for a linear model. A residual is the difference between the y-coordinate of the actual data and the y-coordinate predicted by the model for the same x-coordinate.

- If the actual value is greater than the predicted value, the residual is positive.
- If the actual value is less than the predicted value, the residual is negative.



You also constructed ***residual plots*** for data sets.

- When the residuals are close to zero and appear to be randomly distributed above and below the x-axis on a residual plot, it indicates that a linear model is a good fit for the data.
- However, if the residuals follow a pattern, it indicates that a linear model may not be a good fit.



Reflect:

Name: .... Date: .... Period: ....

Practice

1. Han creates a scatter plot that displays the relationship between the number of items sold x, and the total revenue y, in dollars. Han creates a line of fit and finds that the point $(13, 930)$ has a residual of -40. Interpret the meaning of -40 in the context of the problem.

2. The equation of a line that fits a data set is $y = 1.1x + 3.4$. Calculate the residual for each of the coordinate pairs, (x, y).

   a. $(5, 8.8)$

   b. $(2.5, 5.95)$

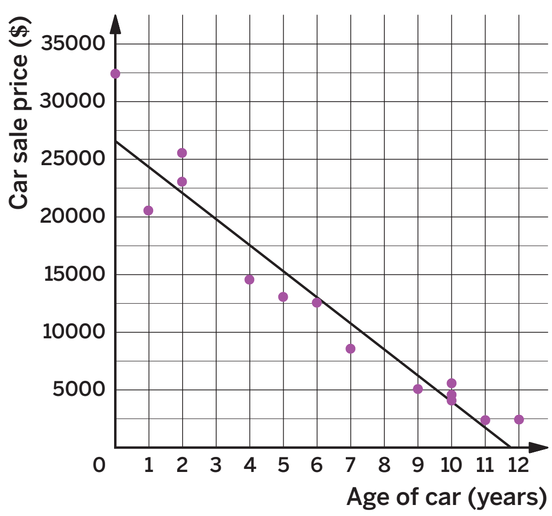
   c. $(0, 3.72)$

   d. $(1.5, 5.05)$

   e. $(-3, 0)$

   f. $(-5, -4.86)$

3. A local car salesperson created the scatter plot shown to display the relationship between a car's sale price in dollars y, and the age of the car in years x. A linear model that fits the data is shown on the graph.



   a. For a car that is 4 years old, does the salesperson sell above or below her average selling price? Explain your thinking.

   b. For a car that is 12 years old, does the salesperson sell above or below her average selling price? Explain your thinking.

Practice

Name: ______________________ Date: __________ Period: __________

4. Consider this data set: $-6, 3, 3, 3, 3, 5, 6, 6, 8, 10$. How does eliminating the least value from the data set affect the mean and median?

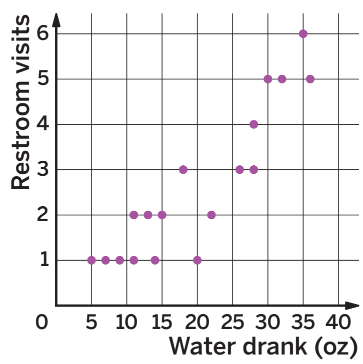
5. Consider these two data sets. Without performing any calculations, which data set will have the greater standard deviation? Explain your thinking.

Data Set 1: $10, 11, 15, 12, 13, 10$

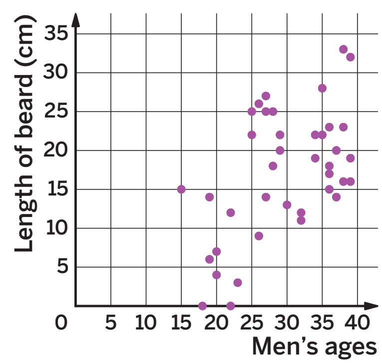
Data Set 2: $10, 11, 15, 12, 13, 55$

6. Order these scatter plots by how well a linear model would fit the data.

Scatter plot A

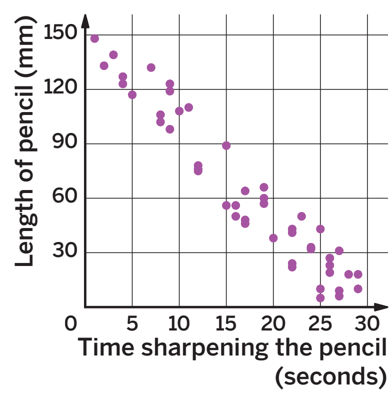


Scatter plot B



Scatter plot C

Scatter plot D



A linear model is not a good fit.

A linear model is an excellent fit.

Name: ______ Date: ______ Period: ______

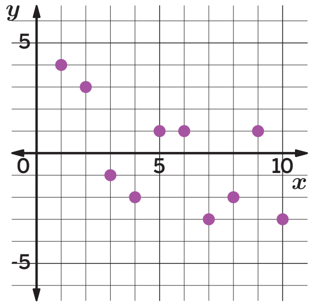
Unit 2 | Lesson 14

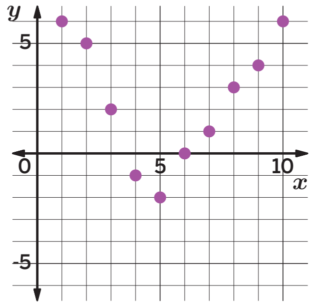
Line of Best Fit

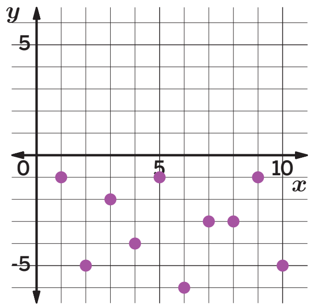
Let's figure out which linear model is the *best* linear model for a data set.

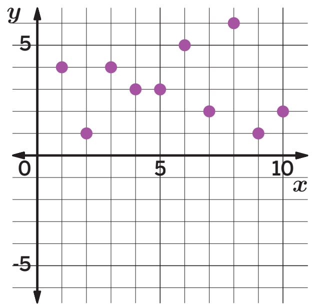
Warm-up Which One Doesn't Belong?

Which of the following residual plots does not belong with the others? Explain your thinking.

A. 

B. 

C. 

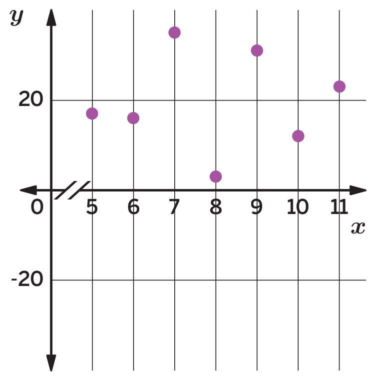
D. 

Log in to Amplify Math to complete this lesson online.

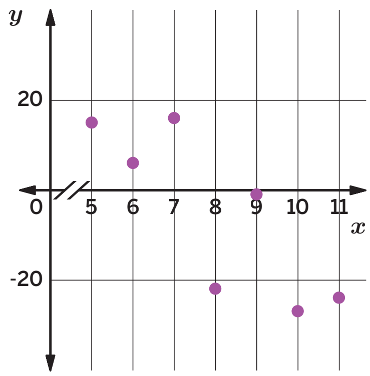
Activity 1 Which Line Is “Better”?

Up to this point, you have been fitting lines to data by eye. This meant you and your classmates often drew slightly different lines. But some lines fit the data better than others. Diego, Lin, and Kiran each started with the same data set and created their own linear models. They plotted the residuals for their models, as shown.

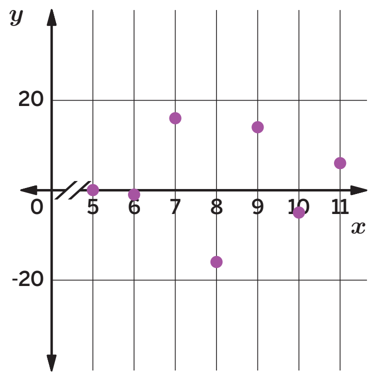
Diego



Lin



Kiran



1. Based on these residual plots, whose line best fit the data? Explain your thinking.

The table shows the values of the residuals for all three models.

2. Using the data in the table, develop *your own* measure for precisely how well each model fits the data. Describe your measure.

3. According to your measure, whose line best fits the data? Does this agree with your response to Problem 1?

x	Diego’s residuals	Lin’s residuals	Kiran’s residuals
5	17	15	0
6	16	6	–1
7	35	16	16
8	3	–22	–16
9	31	–1	14
10	12	–27	–5
11	23	–24	6

Name: ______ Date: ______ Period: ______

Activity 2 Summing the Squares

Residuals can show you which lines of fit are better than others. But is there one line that is the line of *best* fit? Mathematicians have agreed that adding up the squares of the residuals can help you determine the line of best fit. (Squares show up here, just as they did with standard deviations!)

Your group will be assigned to a linear model.

x	y	$y = -0.55x + 2.96$ Residuals	$y = -0.5x + 1$ Residuals	$y = -0.5x + 4$ Residuals	$y = -0.55x + 3.5$ Residuals
-1	4				
0	2				
1	3				
2	1				
3	2				
4	1.5				
5	-0.5				

1. Calculate the residuals for your assigned model and record them in the table.

2. Add up the squares of the residuals for your model and record the sum here. Round to the nearest hundredth.

3. Compare the sum of the squares from your model with those of the other models. Based on these results, which model do you think is the line of *best* fit among the four? Explain your thinking.

Activity 3 Hybrid Cars

Hybrid cars use less fuel than traditional gas-powered vehicles. They are powered by a gas engine part of the time and by an electric motor (which does not use gas) the rest of the time. The table shows the fuel economy for 10 hybrid cars, in miles per gallon, and their approximate mass, in thousands of kilograms.

Mass (thousands of kg), x	Fuel economy (MPG), y	Predicted value of fuel economy (MPG)	Residual
1.123	38		
1.277	39		
1.252	35		
1.368	36		
1.571	31		
1.663	27		
1.698	28		
1.720	26		
2.029	24		
2.065	22		

1. Use graphing technology to create a scatter plot for this data. With your group, come up with your own linear model, and record it here.

2. Complete the table. Use your linear model from Problem 1 to estimate the fuel economy of each hybrid car, based on its mass. Then calculate the residuals.

3. Calculate the sum of the squares of your model's residuals.

4. Your teacher will ask you to share your linear models and responses to Problem 3 with other groups. How will you know if your linear model was the "best" line of fit?

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You explored different ways to evaluate how well a linear model fits a set of data. You can evaluate the graph of the line of the linear model by eye, making sure it captures the trend of the data. You can also examine the residual plot. If a linear model is a good fit, its residual plot should be close to the x-axis and have no pattern.

Finally, you looked at a mathematically precise way to evaluate a line of fit. You did this by calculating the sum of the squares of the residuals. The ***line of best fit*** is the linear model that *minimizes* the sum of the squares of the residuals. Calculators and computers have exact ways of finding the line of best fit, which you can learn about in a later statistics course.

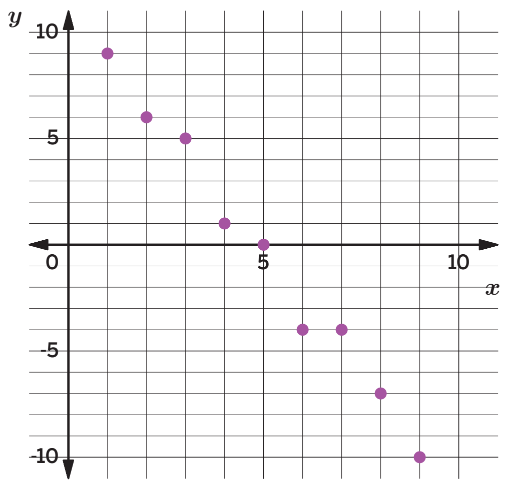
Reflect:

Name: .... Date: .... Period: ....

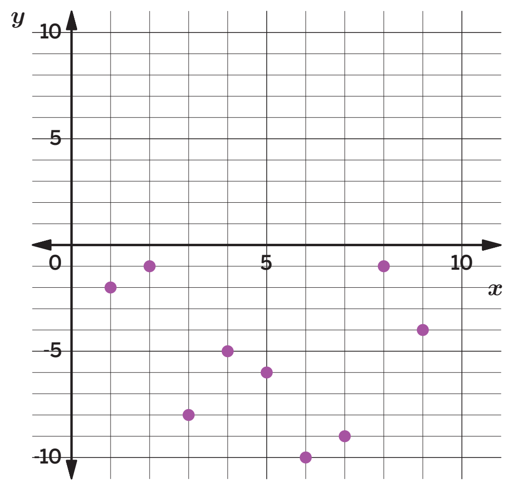
Practice

1. For the same set of data, four different residual plots are shown, representing four different models for the line of fit. Which of the following represents the line of best fit? Explain your thinking.

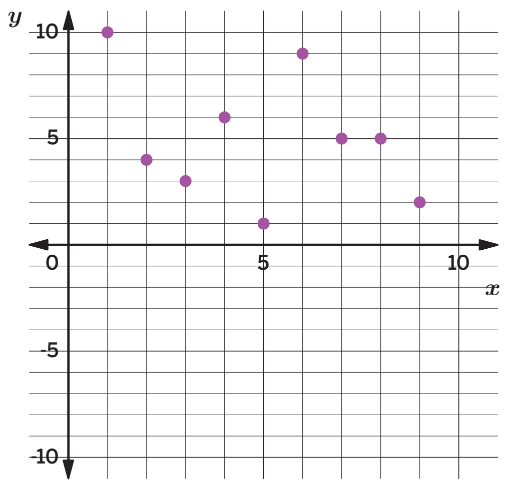
A.



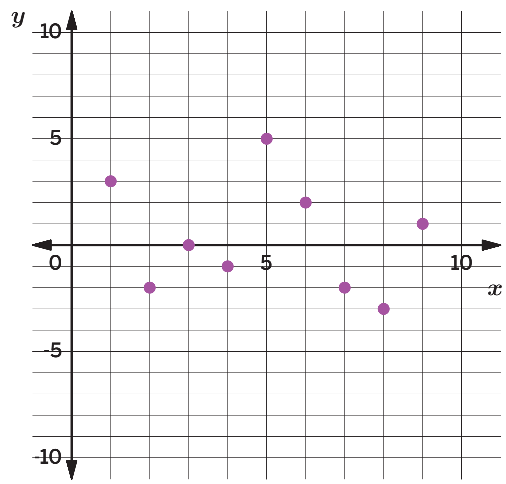
B.



C.



D.



Name: ________________ Date: ________ Period: ________

Practice

2. For a data set, Bard and Shawn each calculate a line of fit. For Bard's line of fit, the sum of squared residuals is 3.48. For Shawn's line of fit, the sum of squared residuals is 5.44.

   a. Whose line of fit is better? Explain your thinking.

   b. For the better line of fit, was this line also the line of best fit? Explain your thinking.

3. The line of best fit for a data set is $y = -4.3x + 1.2$. Find the residual value for each coordinate pair.

   a. $(0, 2)$

   b. $(1, -3)$

   c. $(-1, 4.9)$

4. Jada is measuring the growth of her corn stalk over time. Each day, she measures the height of the corn stalk, in inches, until it is fully grown. Jada finds that the line of best fit for her growing corn stalk is $y = 1.1x - 2.4$.

   a. Interpret the slope in this context.

   b. Interpret the y-intercept in this context. Does it make sense?

5. Jada is comparing two recipes for making pita bread.

   a. Which recipe calls for more flour?

   b. Which ingredients have the same measurements in both recipes?

	Yeast (tsp)	Honey (tsp)	Flour (cups)	Kosher salt (tsp)	Olive oil (Tbsp)
Recipe A	2	$\frac{1}{2}$	$2\frac{3}{4}$	1	2
Recipe B	2	$\frac{1}{2}$	3	$1\frac{1}{2}$	2

   c. Circle the value $1\frac{1}{2}$ in the table. What does it represent?

My Notes:

SUB-UNIT

4 Categorical Data

Narrative Connections

What makes storms worse and has nothing to do with the weather?

When Hurricane Harvey hit Louisiana and Texas in 2017, it arrived with shocking force. Making landfall with 130-mile-per-hour winds, the storm and flooding killed dozens, caused $125 billion worth of damage, and forced 32,000 people from their homes. It was the worst natural disaster the country had seen since Hurricane Katrina.

What you may not realize is that small differences in where people live within a town or city can make a big difference in how they are affected by a regional event. For example, when it came to Houston's embankments, some communities were better protected than others. It turned out that some of the city's low-income housing was located directly in a high-risk flood zone! Meanwhile, people who lived closer to chemical plants were at greater risk to sources of toxic spillage from the flooding.

While natural disasters affect everyone, they can inflict the worst damage on those who are least able to withstand it. Imagine if you were a city planner or in charge of infrastructure. You wouldn't want to leave anything, or anyone, to chance.

To do that, you have to break the numbers down. Mathematical tools such as two-way tables and relative frequency tables can help us understand data by seeing which categories might be associated. With these tools, we can parse the effects of environmental damage on different groups, and hopefully even fend off future disasters.

Unit 2 | Lesson 15

Two-Way Tables

Let's create and interpret categorical data using two-way tables.

Warm-up Census Data

A census is an official count or survey of a population, and the U.S. conducts one every 10 years. Here are some results surrounding poverty from various places across the U.S. in 2019:

- **There were approximately 188,935,435 people in the Midwest and South regions of the U.S.**
- **7,837,254 people were below the poverty line in the Midwest.**
- **122,409,414 people lived in the South.**
- **164,440,192 people lived above the poverty line in both regions.**

Using this information, explain or show your thinking for each of the following:

1. In 2019, how many people lived in the Midwest?

2. In 2019, how many people lived below the poverty line in the Midwest and South?

3. In 2019, how many people in the South lived below the poverty line?

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Social Impacts of Climate Change

Climate change affects everyone, but it especially affects people in socioeconomically disadvantaged communities. People in these communities are more likely to experience the effects of climate change because they have fewer resources to address its challenges.

For example, in August 2005, Hurricane Katrina devastated many communities across the Southeastern United States and the Bahamas. New Orleans, Louisiana was particularly affected because the city is close to the ocean, it is below sea-level, and its levees failed.

The *two-way table* organizes those most affected by Hurricane Katrina in Louisiana into categories, by poverty level and race.

	Above poverty line	Below poverty line	Total
Black residents	1,206	256	1,462
Non-Black residents	2,855	152	3,007
Total	4,061	408	4,469

1. What does the value 1,206 represent in the table?

2. What does the value 408 represent in the table?

3. What does the value 4,469 represent in the table?

4. How many Black residents were most affected by Hurricane Katrina in Louisiana?

5. How many non-Black residents who were most affected by Hurricane Katrina lived below the poverty line?

6. How many people who were most affected by Hurricane Katrina in Louisiana lived above the poverty line?

Activity 2 Info Gap: Droughts and Flooding

Droughts and flooding are documented results of a changing climate. Droughts and flooding cause disadvantaged and marginalized groups, who have greater difficulty recovering from these disasters, to migrate (or move) within the U.S.

You will be given either a problem card or a data card. Do not show or read your card to your partner.

If are given the *data card*:	If are given the *problem card*:
1. Silently read the information on your card.	**1.** Silently read your card and think about what information you need to answer the problem.
2. Ask your partner, "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card.	**2.** Ask your partner for the specific information that you need.
3. Before telling your partner the information, ask, "Why do you need to know (that piece of information)?"	**3.** Explain to your partner how you are using the information to solve the problem.
4. Read the problem card, and solve the problem independently.	**4.** When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Share the data card, and discuss your thinking.	**5.** Read the data card, and discuss your thinking.

STOP

Name: ______________________ Date: __________ Period: ________

Summary

In today's lesson . . .

You explored statistics and saw that a *variable* (in statistics) is a characteristic that can be measured or counted. A ***categorical variable*** is one that can be partitioned into groups or categories. Data from two categorical variables can be organized using a two-way table.

In a ***two-way table***, the categories for each variable should not overlap, so that each data value is recorded in exactly one of the cells in the table, rather than in multiple cells.

The total of each row and column is represented in the rightmost column and bottom row, with the total of all the cells in the bottom-right corner.

	Category 1	Category 2	Total
Category A			Row total
Category B			Row total
Total	Column total	Column total	Total of all cells

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Tyler surveys some of his classmates to determine whether or not they bring their lunch from home. He records their grade levels and responses to his question. Some of the results of his survey are shown in the table. Complete the two-way table.

	Brings lunch from home	Does not bring lunch from home	Total
10th Grade	29		
12th Grade			60
Total	39		105

2. Mai conducts a survey in her community asking if the homeowners think they have safe drinking water and if they think more efforts should be made to provide clean water. The results of her survey are shown.

	More can be done to provide clean water	More cannot be done to provide clean water	Total
Have safe drinking water	32	23	55
Do not have safe drinking water	61	11	72
Total	93	34	127

a. How many members of the community participated in Mai's survey?

b. How many members of the community think more cannot be done to provide clean water?

c. Of the members of the community who do not have safe drinking water, how many think more can be done to provide clean water?

Name: ________ Date: ________ Period: ________

Practice

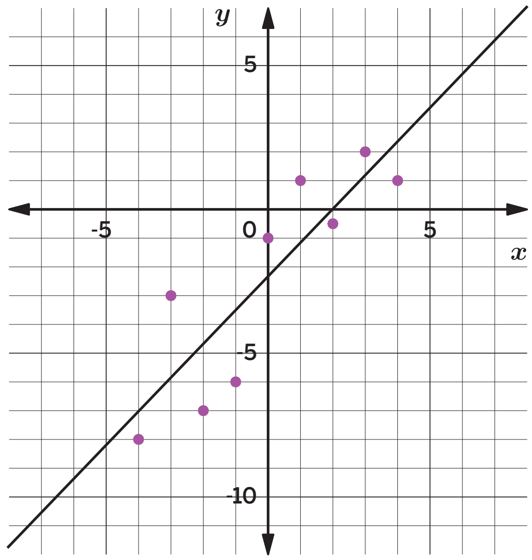
3. Which of the following could *not* be the equation for the line of best fit for the scatter plot shown? Explain your thinking.

A. $y = 1.18x - 2.39$

B. $y = 1.19x - 2.41$

C. $y = x - 2$

D. $y = 1.18x + 2.39$



4. The same data from Mai's survey is shown in the two way table.

	More can be done to provide clean water	More cannot be done to provide clean water	Total
Have safe drinking water	32	23	55
Do not have safe drinking water	61	11	72
Total	93	34	127

a. What percent of community members surveyed do not have safe drinking water?

b. What percent of community members think more can be done to provide clean water?

Unit 2 | Lesson 16

Relative Frequency Tables

Let's analyze two-way tables relative to their totals.

Warm-up Notice and Wonder

Millions of people in California and Arizona were affected by wildfires in 2020, meaning they were displaced from their homes, affected by smoke, or sought medical treatment. The table includes data from a population affected by these wildfires — specifically, whether their income was below or above the poverty line, and whether they had difficulty paying medical bills. What do you notice? What do you wonder?

	Income below poverty line	Income above poverty line
Difficulties paying medical bills	69%	33%
No difficulties paying medical bills	31%	67%

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Age and Wildfires

Wildfires do more than just destroy property. Inhalation of smoke can cause damage to lungs and respiratory health. Those who are 65 years of age or older can have an even harder time recovering from this lung damage. Study the following *relative frequency table*, which shows how the population of California was affected by wildfires in 2020, based on their age.

	Health or property affected by wildfires	Health or property not affected by wildfires	Total
Under age 65	40%	45%	85%
Over age 65	4%	11%	15%
Total	44%	56%	100%

1. What does the value 15% represent in this context? The value 44%?

2. What percent of people under the age of 65 were not affected by wildfires?

3. What percent of people affected by wildfires were over the age of 65?

Here is the same frequency table, but with the frequency counts, instead of the relative frequency.

	Health or property affected by wildfires	Health or property not affected by wildfires	Total
Under age 65	15,750,767	17,913,647	33,664,414
Over age 65	1,603,414	4,244,395	5,847,809
Total	17,354,181	22,158,042	39,512,223

4. How do you think the values in the relative frequency table were calculated using these values? Explain your thinking.

Activity 2 Flint, Michigan

Flint, Michigan has been the subject of an ongoing water crisis since 2014. It was discovered that lead from aging pipes was contaminating the water, exposing hundreds of thousands of residents to heavy metal contamination. The following two-way table shows how children were exposed according to the levels of lead (a heavy metal) in their blood.

	Elevated levels of lead in blood	Normal levels of lead in blood	Total
Black children, ages 1–5	338	5,697	6,035
Non-Black children, ages 1–5	161	7,532	7,693
Total	499	13,229	13,728

1. Calculate the percentage of total children in each cell in the table. Round to the nearest tenth. The first cell is completed for you.

	Elevated levels of lead in blood	Normal levels of lead in blood
Black children, ages 1–5	$\frac{338}{13728} = 2.5\%$	
Non-Black children, ages 1–5		

2. Among children with elevated levels of lead in their blood, what percentage are Black?

3. For each cell in the following table, calculate the percentage of children relative to the total in the column. Round to the nearest percent.

	Elevated levels of lead in blood	Normal levels of lead in blood
Black children, ages 1–5		
Non-Black children, ages 1–5		

Name: ________________ Date: ________ Period: ________

Activity 2 Flint, Michigan (continued)

The table you completed in Problem 3 shows *relative frequency by column*. You can use tables like this to respond to Problem 2 and other questions about two-way tables.

4. Use your table to answer the question: *Among children with elevated levels of lead in their blood, what percentage were not Black?*

5. Among Black children in Flint, what percentage had elevated levels of lead in their blood?

6. For each cell in the following table, calculate the percentage of children relative to the total in the row. Round to the nearest percent.

	Elevated levels of lead in blood	Normal levels of lead in blood
Black children, ages 1–5		
Non-Black children, ages 1–5		

The table you completed in Problem 6 shows *relative frequency by row*. You can use tables like this to respond to Problem 5 and other questions about two-way tables.

7. Use your table to answer the question: *Among Black children, what percentage had normal blood levels?*

8. The researchers who collected this data want to determine whether race was an indicator for blood lead levels. Which of your relative frequency tables best answers this question? Explain your thinking.

STOP

Summary

In today's lesson . . .

You saw that converting two-way tables to relative frequency tables can reveal patterns in paired categorical variables. A ***relative frequency*** table shows the proportion of each value — expressed as fractions, decimals, or percentages — compared to the total. This total could be:

- The total number of responses,
- The total number of responses for each column, or
- The total number of responses for each row.

Depending on the question being asked, some types of relative frequency tables are more useful than others. Here are the three types of tables, applied to the same data:

Total Relative Frequency
Calculate the proportion of the total.

	Headache	No headache
Medication	$\frac{40}{100}$	$\frac{20}{100}$
No medication	$\frac{5}{100}$	$\frac{35}{100}$

Column Relative Frequency
Calculate the proportion of each column total.

	Headache	No headache
Medication	$\frac{40}{45}$	$\frac{20}{55}$
No medication	$\frac{5}{45}$	$\frac{35}{55}$

Row Relative Frequency
Calculate the proportion of each row total.

	Headache	No headache
Medication	$\frac{40}{60}$	$\frac{20}{60}$
No medication	$\frac{5}{40}$	$\frac{35}{40}$

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. A researcher conducted a survey in a city to investigate the relationship between health insurance and employment status. The relative frequency table displays some of the data they collected.

	Health insurance	No health insurance
Employed	77%	46%
Unemployed	23%	54%

a) What does the value 77% represent?

b) What does the value 54% represent?

2. A small city that is susceptible to flooding also has a high poverty rate. The city has 3,400 residents, and only some can afford flood insurance. The table shows the data collected from the residents of the city. Complete the relative frequency table to show the percentages of flood insurance, based on poverty designation.

	Flood insurance	No flood insurance
Income below poverty line	200	700
Income above poverty line	2,000	500

Relative frequency table

	Flood insurance	No flood insurance
Income below poverty line		
Income above poverty line		

Practice

Name: .... Date: .... Period: ....

3. Solve the following system of linear equations using any method:

$$\begin{cases} 4x + 3y = 12 \\ -2x + y = 4 \end{cases}$$

4. Consider the fraction $\frac{3}{8}$.

a Which of the following represents $\frac{3}{8}$ as a decimal?

- **A.** 0.3
- **B.** 0.8
- **C.** 0.375
- **D.** 2.67

b Which of the following represents $\frac{3}{8}$ as a percentage?

- **A.** 0.375%
- **B.** 3.75%
- **C.** 37.5%
- **D.** 375%

Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 17

Associations in Categorical Data

Let's look for associations in categorical data.

Warm-up Typhoon Milenyo

The Philippines in the western Pacific Ocean is subject to many typhoons (hurricanes in the Pacific) and flooding. In East Laguna Village, after Typhoon Milenyo in 2006, data were collected from households consuming less rice, protein, or other foods to cope with the flooding. The two-way table summarizes some of the data collected.

	Households that owned land	Households that did not own land	Total
Only consumed less rice	3%	15%	18%
Only consumed less protein	6%	27%	33%
Consumed less of other foods	15%	34%	49%
Total	24%	76%	100%

1. What does the value 3% represent in the table?

2. What does the value 76% represent in the table?

Log in to Amplify Math to complete this lesson online.

Activity 1 Droughts in Kenya

In Kenya, a country in eastern Africa, the changing climate causes droughts that affect millions of people. In addition, some Kenyans already suffer from food insecurity, or the disruption of food intake due to lack of money or resources. The following two-way table shows how many of the approximate 1.6 million people who were affected by drought also suffered from malnourishment and food insecurity.

	Suffered from food insecurity	Did not suffer from food insecurity	Total
Suffered from malnourishment	910,000	110,000	1,020,000
Did not suffer from malnourishment	390,000	190,000	580,000
Total	1,300,000	300,000	1,600,000

1. Complete the two-way relative frequency table by columns.

	Suffered from food insecurity	Did not suffer from food insecurity
Suffered from malnourishment		
Did not suffer from malnourishment		

2. When there was food insecurity, which had a higher relative frequency: suffering from malnourishment or not suffering from malnourishment?

3. When there was food security, which had a higher relative frequency, suffering from malnourishment or not suffering from malnourishment?

4. Based on this data, is there an association between food insecurity and malnourishment? Explain your thinking.

Name: ____________ Date: ________ Period: ________

Activity 2 Card Sort: Looking for Associations

Looking for *associations* in data is a critical part of analyzing data. In statistics, an *association* exists when two variables are statistically related to each other (if one of the variables can be used to estimate the value of the other).

In Activity 1, you saw an example of a two-way table where an association existed between two variables. You will be provided with cards with two-way tables and will sort them into two categories: *likely* and *unlikely* associations. Record the card numbers in the table.

Likely association	Unlikely association

1. What do you notice about the two-way tables from cards that had a likely association?

2. What do you notice about the two-way tables from cards that had an unlikely association?

3. What were some strategies that you used to sort the cards?

STOP

Summary

In today's lesson . . .

An ***association*** between two variables means that the two variables are statistically related to each other. Noticing a pattern in the raw data can be difficult depending on the numbers, so converting into a row or column relative frequency table can be helpful when looking for an association.

Here are two examples showing likely and unlikely association.

Likely association:

	Likes school	Dislikes school
Part of a club	23 (71.875%)	5 (≈ 20.8%)
Not part of a club	9 (28.125%)	19 (≈ 79.2%)

Unlikely association:

	Left handed	Right handed
Composts food waste	10 (10%)	100 (10%)
Does not compost food waste	90 (90%)	900 (90%)

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Which value would best fit in the missing cell to suggest there is no evidence of an association?

	Digital watch	Analog watch
Displays the date	54	27
No date display	18	

A. 9 B. 18 C. 27 D. 54

2. The relative frequency table shows the percentage of each type of art (painting or sculpture) in a museum would be classified in the different styles (modern or classical). Based on these percentages, is there evidence to suggest an association between the variables? Explain your thinking.

	Modern	Classical
Painting	41%	59%
Sculpture	38%	62%

3. An automobile dealership keeps track of the number of cars and trucks they have for sale, as well as whether they are new or used. Based on the data, does there appear to be an association between the type of automobile and whether it is new or used? Explain your thinking.

	Car	Truck
New	812	233
Used	422	51

Practice

Name: ____________ Date: ____________ Period: ____________

4. A survey is given to $1,432$ people about whether they take daily supplemental vitamins and whether they eat breakfast on a regular basis. The results are shown in the table. Create a relative frequency table that shows the percentage of the entire group that is in each cell.

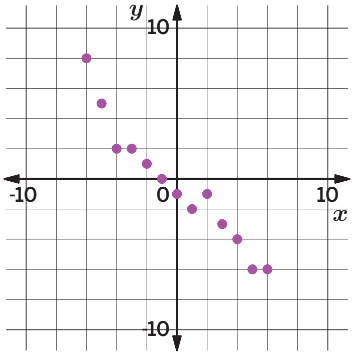
	Take daily vitamins	No daily vitamins
Eat breakfast	384	476
No breakfast	268	304

5. Several college students are surveyed about their college location and preferred locations for a spring break trip.

	College near coast	College away from the coast
Beach break	37	54
Ski break	24	36

What percentage of people who prefer to spend spring break at the beach go to a college away from the coast? What percentage of people who prefer to spend spring break skiing go to a college away from the coast?

6. The following plot shows data on a coordinate grid. What is the most accurate description of the strength of the association in the data?



A. No association

B. Weak association

C. Strong association

D. Cannot be determined

SUB-UNIT

5 Correlation

Narrative Connections

Who is the "water warrior"?

During the Assembly of First Nations' annual gathering in Quebec, a 12-year-old Autumn Peltier took the stage. As a member of Wiikwemkoong First Nation, she was there to confront Canada's prime minister Justin Trudeau.

For years, Canada's policies had endangered its Indigenous communities' ability to get clean water. Rising temperatures are a major threat to Canada's water system. Warmer temperatures increase the spread of water-borne pathogens in Canada's Great Lakes. The increased temperatures also lead to increased evaporation, which can disrupt seasonal rainfall patterns.

Along with Canada's expanding oil pipelines, climate change has endangered water access for Canada's Indigenous populations. These communities face boil-water advisories (which warn residents of sewage contamination in tap water), oil spills, and high lead content in their water. It was these threats that spurred Peltier to become an advocate for her people.

Dubbed the "water warrior," Peltier took the position of Water Commissioner of the Anishinabek Nation at the age of 14. In an address at the United Nations, she urged governments to protect the world's water through sustainability measures like banning plastic.

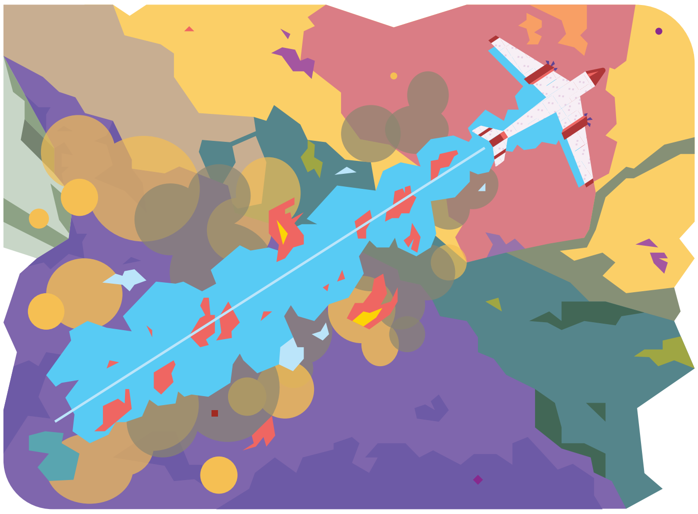
The ripple effects of climate change can be overwhelming, especially for vulnerable populations. For any sustainability measure to be successful, governments must understand the impact their policies may have.

Learning how to look for patterns of association in bivariate data can help you determine whether two quantities are merely *related*, or if changes in one quantity might be *causing* changes in the other quantity.

Unit 2 | Lesson 18

“Strength” of Association

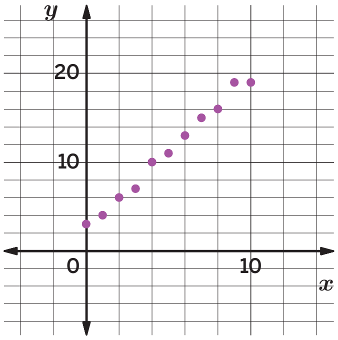
Let’s measure associations in data.



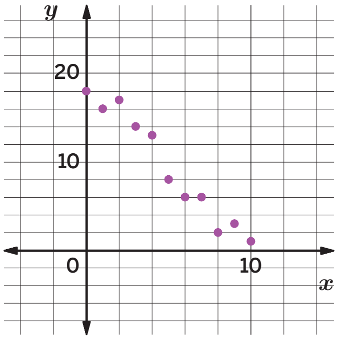
Warm-up Which One Doesn’t Belong?

Which of the following scatter plots doesn’t belong? Explain your thinking.

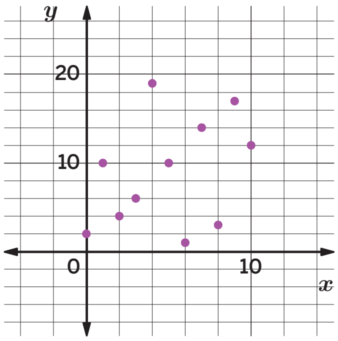
A.



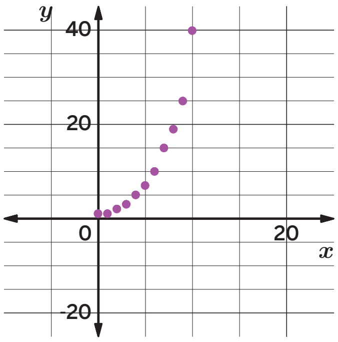
B.



C.



D.



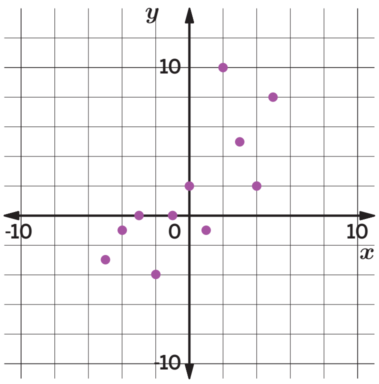
Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

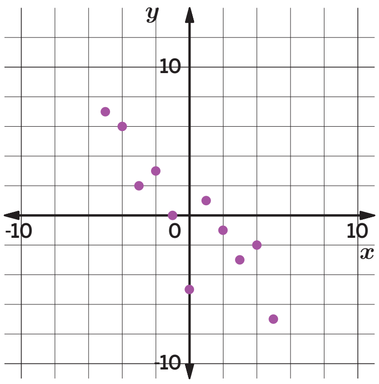
Activity 1 Ranking Associations

Associations can be strong or weak. When an association is strong, the data are closer to the trend and the association is clearer. Study these scatter plots.

Scatter plot A

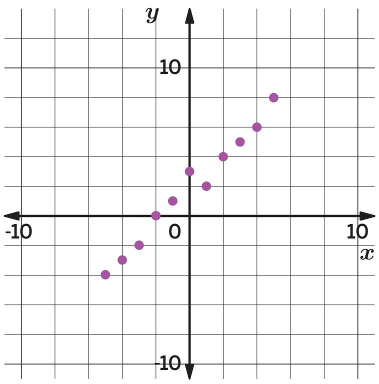


Scatter plot B

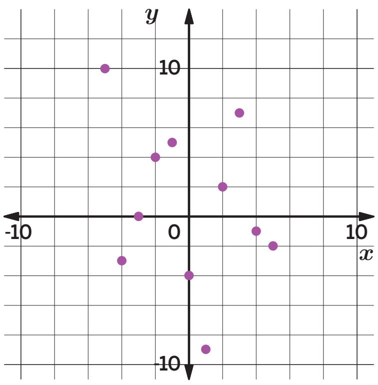


Compare and Connect: As you study these scatter plots, what similarities do you notice? What differences do you notice?

Scatter plot C



Scatter plot D



1. Order the scatter plots from the weakest association to the strongest association. Explain your thinking.

Weakest association **Strongest association**

2. What are some pros and cons of visually determining if a scatter plot has a strong or weak association?

Pros:

Cons:

Activity 2 Associations in Two-Way Tables

A survey was conducted on whether residents from California and New York have been affected by wildfires, either directly or indirectly. The results are shown in the two-way table.

	Affected by wildfires	Not affected by wildfires	Total
California resident	10	21	31
New York resident	1	20	21
Total	11	41	52

1. Complete the two-way relative frequency table. Round to the nearest percent.

	Affected by wildfires	Not affected by wildfires	Total
California resident	32%		100%
New York resident			

2. Do you think there is an association between someone living in California or New York and whether they have been affected by wildfires? Explain your thinking.

3. How does looking for association in two-way relative frequency tables compare to making observations about scatter plots having a strong or weak association? Explain your thinking.

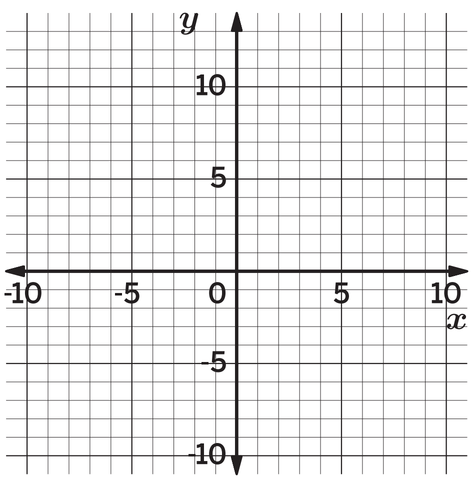
Name: ______ Date: ______ Period: ______

Activity 3 Measuring Association

If you can tell by eye whether an association is strong or weak, there must be some way to quantify, or measure, the strength of an association. Lin decides to come up with a method for quantifying the strength of an association, using the following steps. You will be given a data set. Follow the steps Lin takes to measure the strength of an association.

1. Determine the mean of x and y.

2. Create a scatter plot of your data and draw a vertical line to represent the mean of x and a horizontal line to represent the mean of y.



Activity 3 Measuring Association (continued)

3. Organize the data into the following two-way table, based on how the values compare to the means of x and y (the vertical and horizontal lines you drew).

	Greater than or equal to the mean of x	Less than the mean of x
Greater than or equal to the mean of y		
Less than the mean of y		

4. Organize your data into the following relative frequency table.

	Greater than or equal to the mean of x	Less than the mean of x
Greater than or equal to the mean of y		
Less than the mean of y		

5. Lin adds the percent of data from the top left cell to the data from the bottom right cell. She adds the data from the top right cell to the data from the bottom left cell. She subtracts the second value from the first. Compute Lin's results for your data set.

6. Compare these values to the scatter plots and calculations your classmates made. Use your results from Problem 5 to draw conclusions about the association of the four different data sets.

7. What are some reasons you might not want to measure associations using Lin's method?

8. What improvements might you make to Lin's method of measuring association?

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You learned that *association* is a measure of how strong the relationship is between two variables. Specifically, you examined linear associations to visually determine if there was a strong or weak association.

Association can be found in both categorical and quantitative data, so you used frequency tables to help quantify the strength of associations.

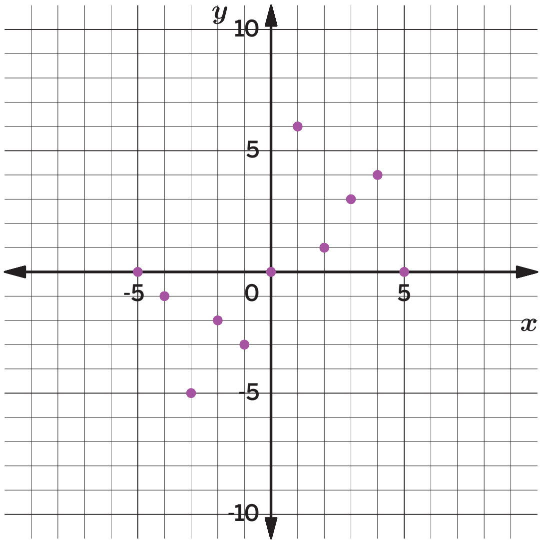
Visually determining association leaves room for interpretation and disagreement when comparing scatter plots. Today, you saw one possible way to quantify (that is, to measure using numbers) associations and you suggested how to improve this approach.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. Which of the following is the *best* description for the following scatter plot?



A. Strong association and increasing.
B. Strong association and decreasing.
C. Weak association and increasing.
D. Weak association and decreasing.

2. Shawn is interested in measuring the association of some data displayed on a scatter plot. Shawn decides to use the MAD as the statistic and determines the MAD of the y-values in the data set. Shawn determines that the value is small and concludes that because this measurement is small, there is little variation in the data set, so there is a strong association. What are some pros and cons of Shawn's method?

Name: .... Date: .... Period: ....

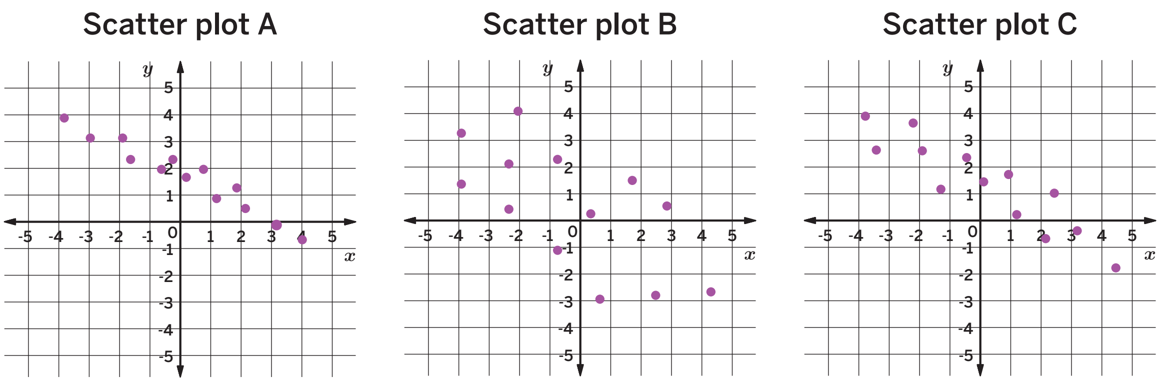
Practice

3. For a data set, Lin and Diego each calculate a line of fit. For Lin's line of fit, her sum of squared residuals is 0.55. For Diego's line of fit, his sum of squared residuals is 1.09.

a. Whose line of fit is better? Explain your thinking.

b. For the better line of fit, was this line also the line of best fit? Explain your thinking.

4. Order the three scatter plots from *weakest* association to *strongest* association.



....

Weakest association **Strongest association**

Unit 2 | Lesson 19

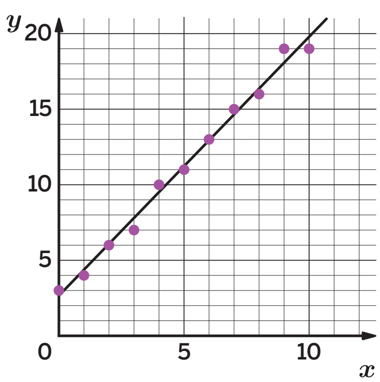
Correlation Coefficient (Part 1)

Let's put a number to the strength of association in data.

Warm-up Notice and Wonder

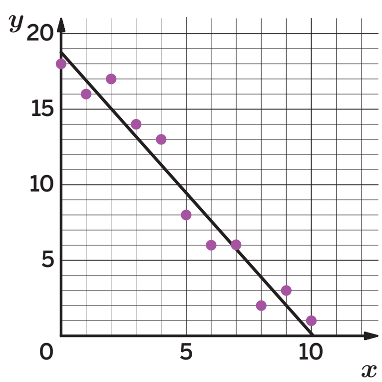
Three scatter plots are given with their line of best fit, along with a corresponding value r, which you will explore in the next activity. What do you notice? What do you wonder?

Scatter plot A



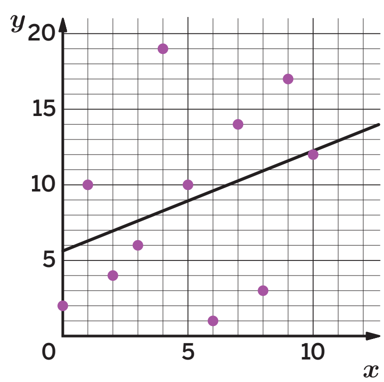
$y = 1.718x + 2.591$
$r = 0.9956$

Scatter plot B



$y = -1.863x + 18.773$
$r = -0.9764$

Scatter plot C



$y = 0.664x + 5.591$
$r = 0.3557$

1. I notice . . .

2. I wonder . . .

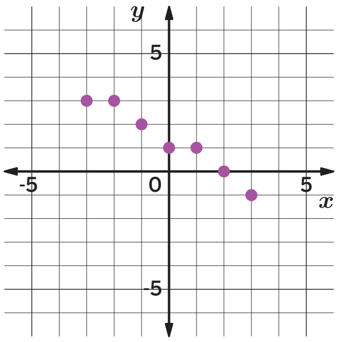
Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

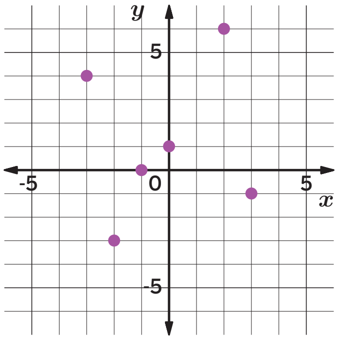
Activity 1 Analyzing Scatter Plots

1. Consider these scatter plots. Classify each as having a *strong* linear relationship, *weak* linear relationship, or *no* linear relationship. Explain your thinking.

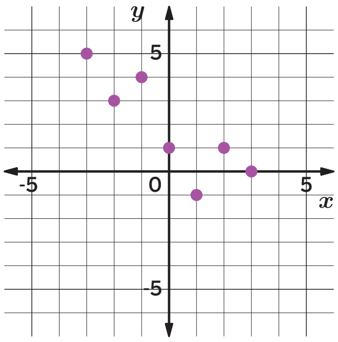
Scatter plot A



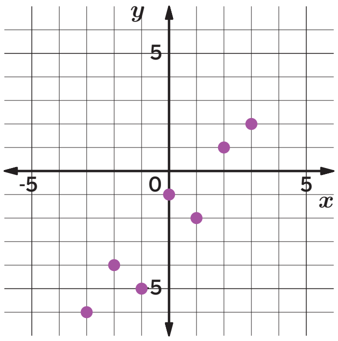
Scatter plot B



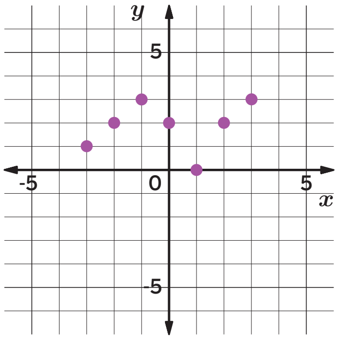
Scatter plot C



Scatter plot D



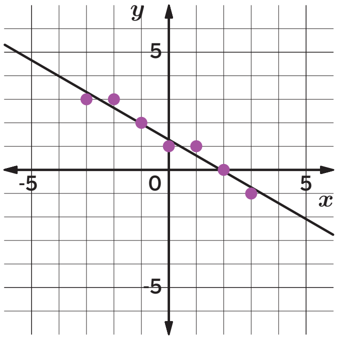
Scatter plot E



Activity 1 Analyzing Scatter Plots (continued)

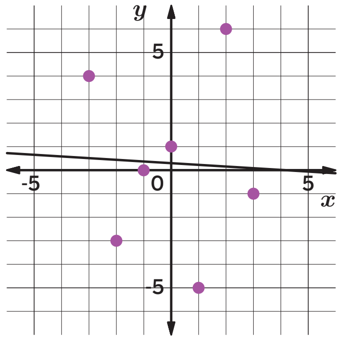
2. The same five scatter plots are given with their equations for the line of best fit and values of r.

Scatter plot A



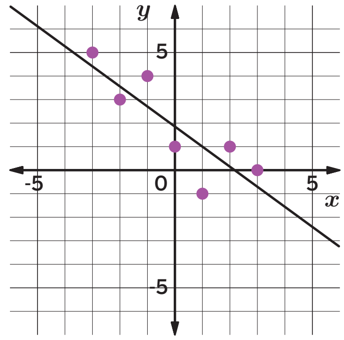
$y = -0.68x + 1.29$
$r = -0.98$

Scatter plot B



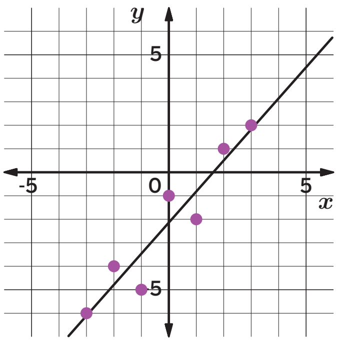
$y = -0.07x + 0.29$
$r = -0.04$

Scatter plot C



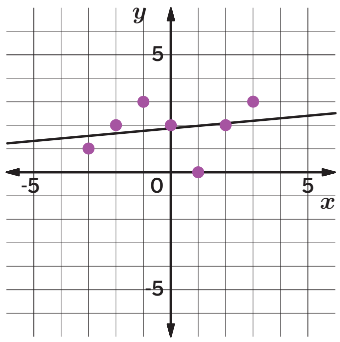
$y = -0.86x + 1.86$
$r = -0.84$

Scatter plot D



$y = 1.32x - 2.14$
$r = 0.94$

Scatter plot E



$y = 0.11x + 1.86$
$r = 0.22$

a Describe the data points in relation to the line of fit for each scatter plot. For example, are the points generally above the line, below the line, near or far?

b Consider the value of r for each of the five scatter plots. In your own words, explain what you think the value of r represents.

Name: ______ Date: ______ Period: ______

Activity 2 r We There Yet?

You have seen that r, known as the *correlation coefficient*, describes the strength and direction of a linear relationship between two variables. But how is r calculated? You will be given a scatter plot and data for which you will calculate the value of r.

1. First, make a prediction for the value of r for your data set.

On your scatter plot, you will also see a vertical line, a horizontal line, and rectangles. The vertical line is the mean of the x-coordinates, while the horizontal line is the mean of the y-coordinates. The rectangles show the distances from each point to the means of x and y.

2. Calculate and record the area of each rectangle by multiplying the difference from the mean of x by the difference from the mean in y. Round to the nearest hundredth. Even though area cannot be negative, retain the negative sign on any calculations in which the differences have opposite signs.

3. Determine the average area of all rectangles. Retain the negative sign on any calculations.

4. Calculate the area for a "typical" point by multiplying the standard deviation for x by the standard deviation for y. Round to the nearest hundredth.

5. Divide your response from Problem 3 by your response from Problem 4. Round to the nearest hundredth.

The value you just determined is your data set's correlation coefficient, r.

6. How close was your predicted value to the actual value?

7. What does this correlation coefficient, r, tell you about the data?

STOP

Summary

In today's lesson . . .

You recalled that linear models sometimes fit data well, while other times they do not. Visualizing and sketching a line of fit on a scatter plot is a good place to start when determining how well a linear model fits a data set, but you can be more precise.

You saw that r, the data's ***correlation coefficient***, measures linear association between two variables. The correlation coefficient is always between -1 and 1, and is interpreted as follows:

r close to -1	r close to 1	r close to 0
Indicates a strong negative (decreasing) linear association.	Indicates a strong positive (increasing) linear association.	Indicates no linear association.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Select *all* values for r that indicate a positive slope for the line of best fit.

A. $r = 1$

B. $r = -1$

C. $r = 0.5$

D. $r = -0.5$

E. $r = 0$

F. $r = 0.45$

2. Six different lines are fit to a set of data. Their correlation coefficients, r, are given. Which value indicates the *best* fit for the data?

A. $r = -0.94$

B. $r = -0.46$

C. $r = -0.11$

D. $r = -0.59$

E. $r = -0.77$

F. $r = -0.89$

3. Match each with the correlation coefficient that best represents it.

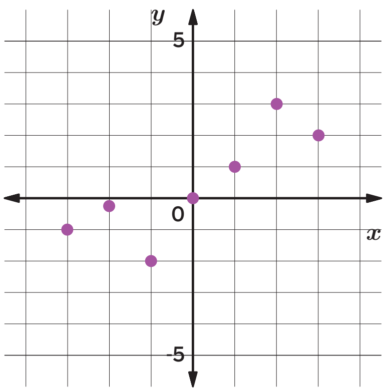
a. $r = -1$

b. $r = 0.8$

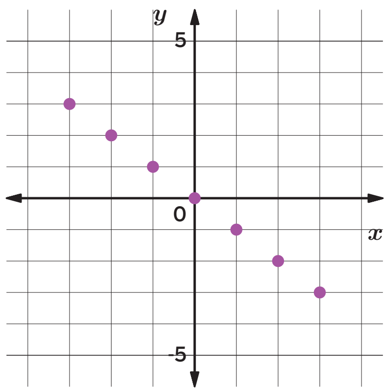
c. $r = -0.9$

d. $r = -0.19$

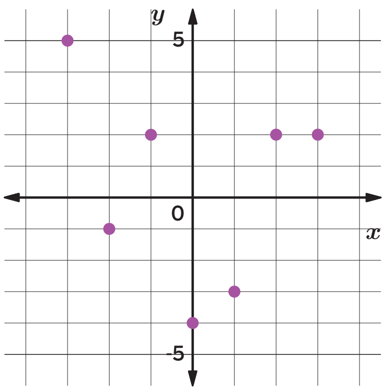
.... **Scatter plot 1**



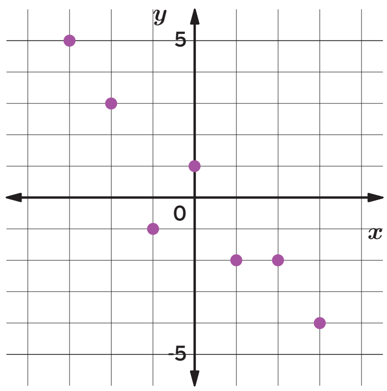
.... **Scatter plot 2**



.... **Scatter plot 3**



.... **Scatter plot 4**



Practice

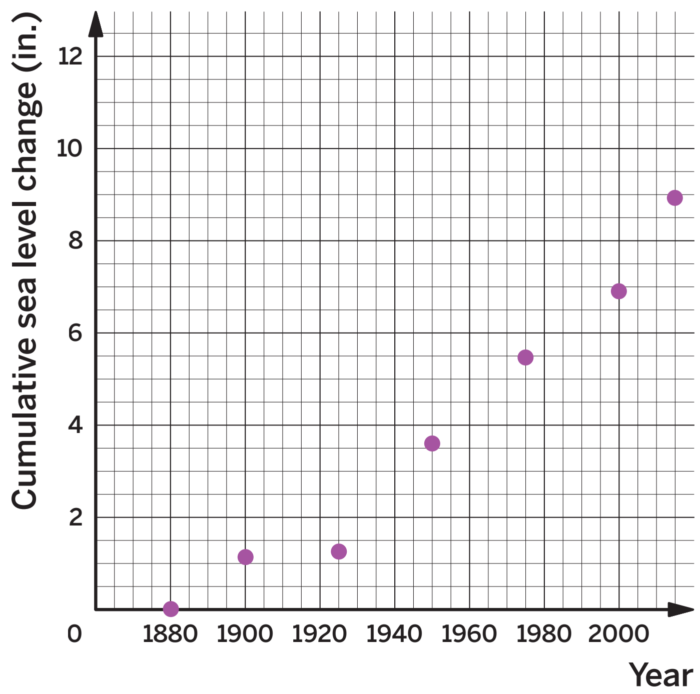
Name: ______________________ Date: ____________ Period: ____________

4. Consider the data set: $8, 3, 4, 10, 5, 5, 9, 7, 8, 11, 6, 6, 10, 12, 45$.

a Calculate the mean and median of the data set.

b Which measure of center is a more accurate description of the center of the data? Explain your thinking.

5. The following scatter plot shows how the cumulative sea level has changed from 1880 to 2015. Which correlation coefficient best represents the scatter plot?

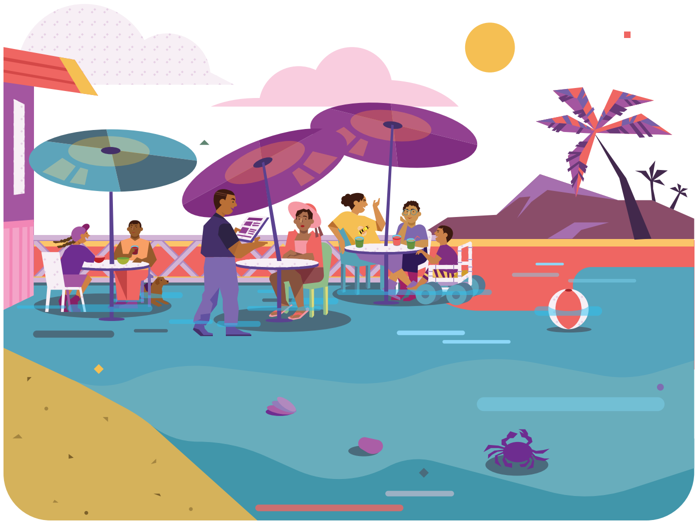


A. $r = -0.98$ **B.** $r = -0.5$ **C.** $r = 0.5$ **D.** $r = 0.98$

Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 20

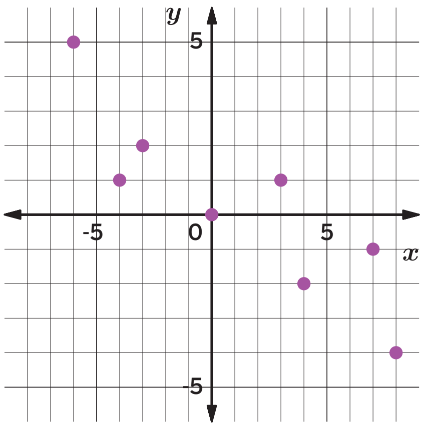
Correlation Coefficient (Part 2)



Let's calculate and use the correlation coefficient to describe linear models in real-world scenarios.

Warm-up True or False

Refer to the scatter plot shown. Determine whether each statement is true or false. Explain your thinking.



Statement	True or False
1. A nonlinear model would best fit the data.	
2. The line of best fit will have a negative slope.	
3. The correlation coefficient is approximately 0.15.	
4. The correlation coefficient is approximately -0.85.	

Log in to Amplify Math to complete this lesson online.

Activity 1 Sea Level Change

Changes in sea level can affect human activities in coastal areas. Rising sea levels can erode shorelines, cause coastal flooding, and make coastal infrastructure vulnerable.

You will be given data on global sea level, which shows how sea levels have changed, in inches, relative to 1880. For example, a value of 0.5 means the sea level rose 0.5 in. since 1880.

1. Enter the data into a spreadsheet. In cell **A1** enter the label "Year" and in cell **B1** enter the label "Cumulative sea level change (in.)". Enter each year into cells **A1** through **A16**. Enter the sea level change into cells **B2** through **B16**.

2. Create a scatter plot in the spreadsheet.

   a. Does the scatter plot show a linear or nonlinear association for the data?

   b. If there is a linear association, is it strong or weak? Explain your thinking.

   c. Does the data have a positive or negative trend? Predict the value of the correlation coefficient.

3. On the scatter plot in the spreadsheet, display the **Trendline**, the equation for the trendline, and R^2.

   a. What equation is shown? What does the slope represent about sea level change?

   b. What is the value of R^2?

   c. Calculate the correlation coefficient by taking the square root of R^2.

   d. What information does the correlation coefficient provide about the change in sea level?

4. A *trendline* is the line of best fit. Use the **Trendline** to predict the number of inches the sea level will have changed between the years 1880 and 2030.

Name: ______ Date: ______ Period: ______

Activity 2 An Incomplete Story

Han does not believe that the sea level is actually rising. He analyzed the global sea level data from 1981–1990 and determined that the sea level has not changed much during this time.

You will be given sea level data from specific timeframes. Use spreadsheet technology to complete each problem.

1. Create a scatter plot of the data including the **Trendline**. Sketch your scatter plot.

2. What is the correlation coefficient? Show your thinking.

Activity 2 An Incomplete Story (continued)

3. What information does the correlation coefficient provide about the changes in sea level for your given timeframe? Explain your thinking.

4. Use your **Trendline** to predict the number of inches the sea level changes 10 years from the end of the given timeframe.

5. Use your **Trendline** to predict the number of inches the sea level will change by 2025.

6. Do you agree or disagree with Han's conclusion that the sea level does not change much over time? Explain your thinking.

Name: ________________ Date: ________ Period: ________

Activity 3 Two Truths and a Lie

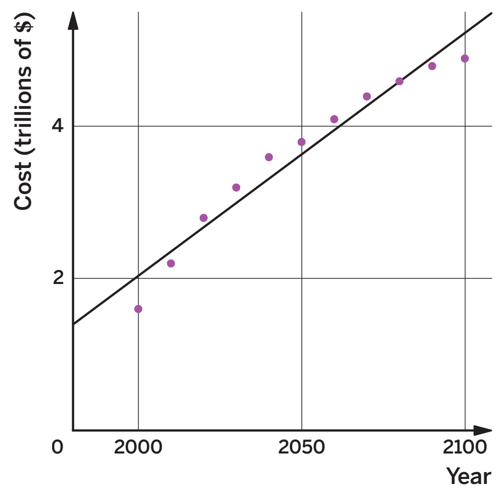
In this unit, you examined data addressing temperature, wildfires, and sea level. These sorts of changes in climate can also have a significant financial impact.

1. Rising sea levels pose risks and require action along coastlines, such as damage control and prevention measures to protect people and property. The scatter plot projects future costs if rising sea levels are not addressed. The equation for the line of best fit is $y = 0.032x - 61.964$. Which of the following statements is *false*? Explain your thinking.

A. There is a positive association between the years and cost.

B. For every one year increase, the cost associated with the rising sea level is projected to increase by $\$0.032$ trillion.

C. A correlation coefficient close to -1 is appropriate for the scatter plot and line of best fit.

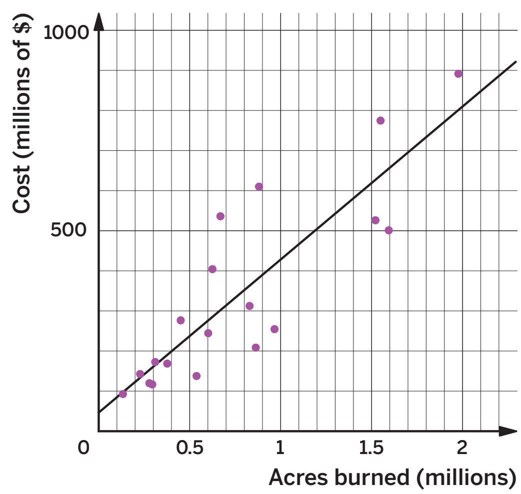


2. The scatter plot shows the data for the total area of California wildfires (in millions of acres) and their damage (in millions of dollars). The equation for the line of best fit is $y = 382.739x + 43.472$. Which of the following statements is *false*? Explain your thinking.

A. A correlation coefficient close to 0.8 is appropriate for the scatter plot and line of best fit.

B. For every additional 1 million acres burned, the additional damage is estimated to be about $\$43.47$ million.

C. If 2 million acres are burned, the damage is estimated to be about $\$809$ million.



Reflect: How did studying what others might believe create empathy and influence your perspective?

STOP

Summary

In today's lesson . . .

You used spreadsheet technology to:

- Create a scatter plot.
- Display the line of best fit on a scatter plot.
- Determine the equation of the line of best fit.
- Show the value of R^2.

You calculated the correlation coefficient by taking the square root of R^2, and you interpreted the equation of the line of best fit in terms of global sea level changes and used it to predict the financial impact of climate events.

You also observed that choosing data from limited intervals can tell a different story from that of the whole data set. This can lead to misrepresentations, and can wrongly influence decision making.

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. The following data shows how much hazardous waste (in millions of tons) has been disposed of on land in the U.S. every other year from 2001 to 2017.

Year	2001	2003	2005	2007	2009	2011	2013	2015	2017
Waste disposed (millions of tons)	19.5	16.1	23.7	24.3	21.6	24.0	25.4	25.0	25.4

a. Use graphing technology to determine the equation for the line of best fit.

b. Use graphing technology to calculate the correlation coefficient. What does the correlation coefficient tell you about the data? Explain your thinking.

2. Priya is analyzing the set of data given. Instead of finding the line of best fit and correlation coefficient for the entire data set, she decides the first five data points are enough to determine a pattern, and concludes her line of best fit and correlation coefficient are representative of her entire data set. What are some possible mistakes or errors she could be making in doing this?

x	1	2	3	4	5	6	7	8	9	10
y	1.9	1.8	1.5	1.4	1.1	1.1	1.2	1.4	1.7	1.6

Practice

Name: .... Date: .... Period: ....

3. The line of best fit $y = -1.83x - 67.4$ was calculated for a set of data. Which of the following are *not* possible values for the correlation coefficient?

A. $r = -0.57$

B. $r = 0.57$

C. $r = 0.91$

D. $r = -0.65$

E. $r = -1.1$

F. $r = -0.99$

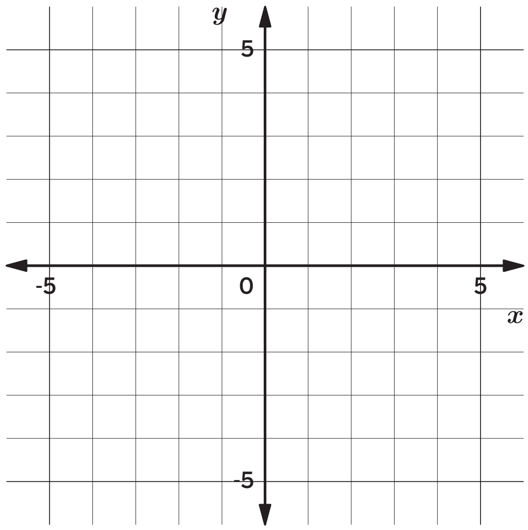
4. Consider the following system of inequalities.

$$\begin{cases} y < -2x + 4 \\ y < 2x + 5 \end{cases}$$

a Graph the system of inequalities and shade the solution region.

b Identify a point that is a solution to the system.

c Are points on the boundary lines of the solution region also solutions? Explain your thinking.



5. Jada collected data on the amount of rainfall each month and number of car accidents and calculated a correlation coefficient of $r = 0.8$. She concluded that high rainfall causes a higher rate of car accidents. Do you agree? Explain your thinking.

Name: .... Date: .... Period: ....

Unit 2 | Lesson 21

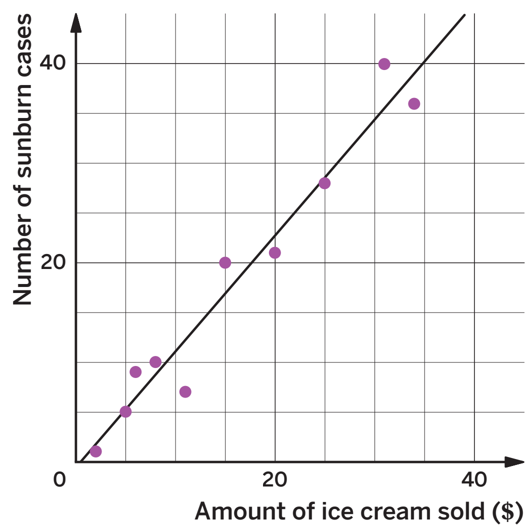
Correlation vs. Causation

Let's determine the difference between correlation and causation.

Warm-up Ice Cream and Sunburns

Diego created a scatter plot showing the relationship between ice cream sales, in dollars, and how many people were treated for sunburn in his hometown last year. Each data point represents one day of data. Diego then added the line of best fit and calculated the correlation coefficient.

Based on this information, Diego concludes: "When more ice cream is sold, there are more cases of sunburn. That must mean that the ice cream causes the sunburn." Do you agree or disagree with Diego's conclusion? Explain your thinking.



$y = 1.2x - 0.6$

$r = 0.98$

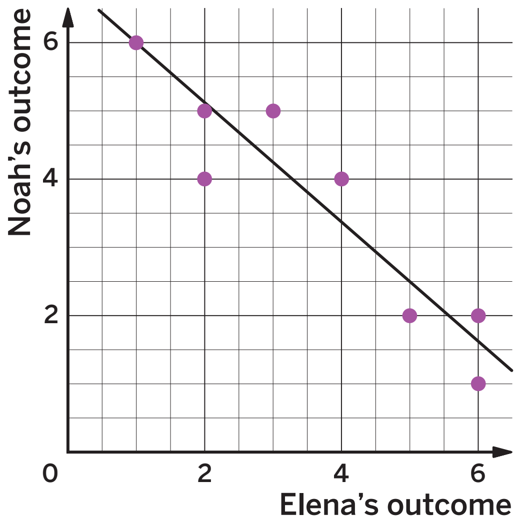
Log in to Amplify Math to complete this lesson online.

Activity 1 Rolling the Dice

Part 1

Elena and Noah decide to see if there is an association between their dice rolls. Elena rolls one die, Noah the other, and they write their outcomes as ordered pairs. They conduct 10 trials before creating a scatter plot with the line of best fit and computing a correlation coefficient.

Elena's outcome	Noah's outcome	Coordinate
1	6	(1, 6)
2	4	(2, 4)
1	6	(1, 6)
5	2	(5, 2)
6	2	(6, 2)
3	5	(3, 5)
4	4	(4, 4)
3	5	(3, 5)
2	5	(2, 5)
6	1	(6, 1)



$y = -0.87x + 6.88$
$r = -0.93$

1. Based on this information, Elena and Noah claim, "As Elena's outcome on her die increases, Noah's outcome decreases. So, Elena's die is causing Noah's die to result in smaller outcomes." What do you think of the claim they are making?

2. Why do you think there is an association between Elena and Noah's outcomes on their dice? Explain your thinking.

Name: ______________ Date: ______________ Period: ______________

Activity 1 Rolling the Dice (continued)

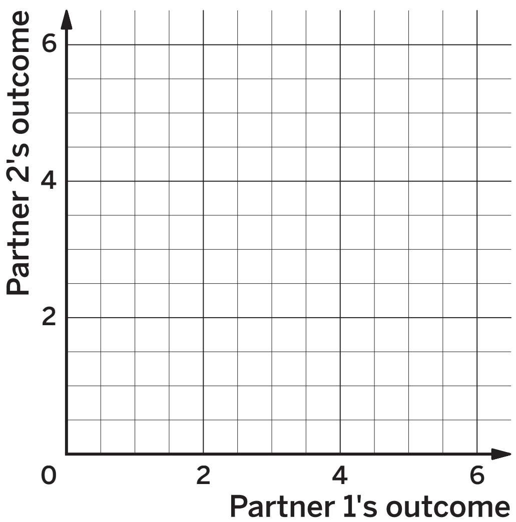
Part 2

Now, you and your partner will have the opportunity to try this out on your own. You will be provided with two dice. Decide who will be Partner 1 and Partner 2.

3. You and your partner will roll both dice, record the outcomes in the table, and write them as coordinate points.

Partner 1	Partner 2	Ordered pair

4. Create a scatter plot with the line of best fit using your data.



5. Use spreadsheet technology to calculate the correlation coefficient.

6. What does the correlation coefficient tell you about your data?

7. Do you think an association between two variables means that one variable causes another to change? Explain your thinking.

Activity 2 What Comes Next?

In many disciplines, *causation* is proven through carefully designed experiments in which researchers control for outside influences. In doing so, they can test whether changing one variable (such as temperature, nutrient level, medicine dosage, etc.) causes a change in another.

Noah and Bard are interested in how temperature affects the growth of crops in the United States. They design a statistical experiment to see if there is a causal relationship between temperature levels and the growth of certain crops.

You will receive cards that represent the steps for Noah and Bard's experiment.

Three Reads: Prepare yourself for this activity by reading the information multiple times.

1. Make sense of the experiment.
2. Identify possible places to each card could belong.
3. Brainstorm strategies to arrange the cards.

1. Arrange the cards in order. Record the card numbers in the table.

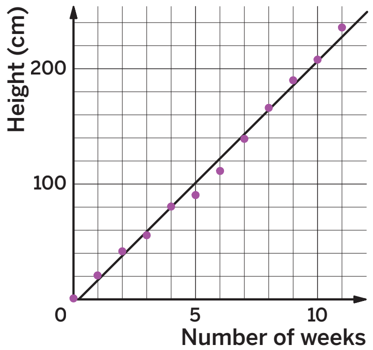
Step 1	Step 2	Step 3
Step 4	**Step 5**	**Step 6**
Step 7	**Step 8**	**Step 9**
Step 10	**Step 11**	**Step 12**

Name: ______ Date: ______ Period: ______

Activity 2 What Comes Next? (continued)

2. To analyze their data, Noah and Bard create the following three scatter plots along with the equation for the line of best fit and correlation coefficient.

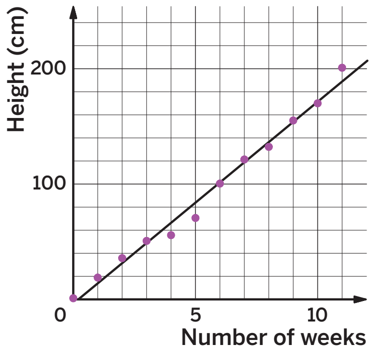
Room Temperature:



$y = 21.27x - 5.67$

$r = 0.997$

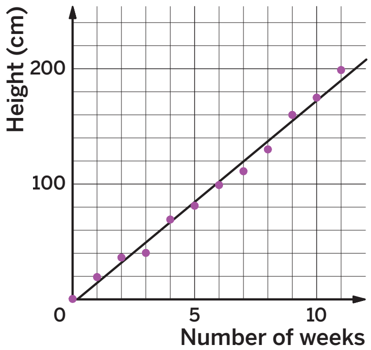
5 degrees warmer:



$y = 17.68x - 5$

$r = 0.994$

5 degrees cooler:



$y = 17.67x - 3.96$

$r = 0.996$

a What do you notice when you compare the correlation coefficients of all three scatter plots? Explain your thinking.

b What do you notice when you compare the slopes of the equations for all three lines of best fit? Explain your thinking.

3. Based on these findings, Noah and Bard concluded that the corn grows best at room temperature, and not when it is warmer or cooler than average. Are Noah and Bard correct in their conclusion? Explain your thinking.

Activity 3 Design Your Own Experiment

Throughout this unit, you have analyzed data involving:

- **City, national, and global extreme temperatures**
- **Hurricanes (frequency and strength)**
- **Wildfires (acres burned and cost)**
- **Ocean temperature**
- **Sea level change**
- **Snow coverage**

Now you and your group will have the opportunity to think about how you would design your own experiment to test the effect one variable has on another.

1. From the list provided, select two variables: one to represent x, the other to reprepresent y.

2. How would you set up the experiment so you can measure the changes in the two variables you selected?

3. How will data be measured throughout the course of the experiment?

4. When the experiment is over, how will data be analyzed? What types of statistics and data displays will be calculated?

5. Do you think this experiment you described is one that researchers and scientists would do themselves? Why or why not?

6. What are some of the obstacles and challenges that researchers and scientists might have to overcome when designing their own experiment to determine causal relationships in nature and the climate?

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You observed that just because two variables are correlated does not necessarily mean that one variable causes the change in another variable. *Correlation* and ***causation*** do not mean the same thing.

It is true that if a change in one variable causes a change in another, the variables will be correlated. However, correlation can also be caused by some third variable that causes the two observed variables. Correlation can even be caused by coincidence — if you study lots of variables, then it is very likely that at least two of them will be somewhat correlated.

To determine whether causation is present, researchers must perform careful experiments that must control for other variables and rule out coincidence.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Which of the following relationships is most likely to be related by causation?

   A. Number of shoes owned and sea level change.

   B. Social media use and acres burned by wildfires.

   C. Number of dogs owned and number of cars owned.

   D. Global temperature change and snow cover change.

2. Which of the following are parts of a statistical experiment? Select *all* that apply.

   A. Identifying the population of interest.

   B. Determining the treatments that different groups in the experiment will receive.

   C. Choosing values to be part of a sample based on convenience and values that fit a desired outcome.

   D. Representing data using data displays.

   E. Only summarizing data from specific portions of the entire sample.

3. A news website shows a scatter plot with a negative linear association between the amount of sugar eaten and happiness levels. The headline reads, "Eating sugar causes happiness to decrease!"

   a What is wrong with this claim?

   b What would need to be done to show causality?

Name: ________ Date: ________ Period: ________

Practice

4. A group of college students are surveyed about their note taking and study habits. The results are represented in the table.

	Prefers writing notes by hand	Prefers typing notes	Does not take notes	Total
Study for less than one hour	22	26	8	56
Study for one hour or more	38	28	3	69
Total	60	54	11	125

a What does the value 11 represent?

b How many college students prefer typing notes and study for one hour or more?

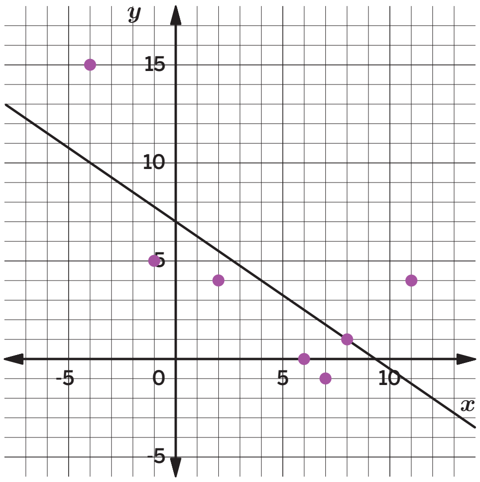
5. Which of the following is the *best* estimate for the correlation coefficient of the scatter plot and line of fit shown?

A. $r = 0.7$

B. $r = 0.95$

C. $r = -0.7$

D. $r = -0.95$



6. Diego analyzes two scatter plots. One shows a strong increasing association between global carbon dioxide levels and global temperatures. Another shows a strong increasing association between global carbon dioxide levels and social media usage. Which one would you choose to investigate to determine if there is a causal association?

Unit 2 | Lesson 22 – Capstone

Cutting Through Misleading Statistical Claims

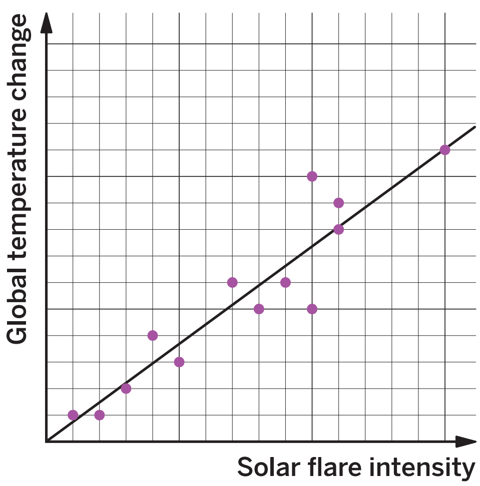
Numbers might not lie, but what about their interpretation? Let's explore this further.

Warm-up Newspaper Headlines

Two newspapers have catchy headlines and data displays to make claims about mankind's connection to climate change.

Newspaper A:

"Are Solar Flares the Real Cause of Climate Change?"



Newspaper B:

"Study Shows High CO_2 Levels Do Not Affect Crop Growth"

	Low crop growth	Normal crop growth
High CO_2	25	25
Normal CO_2	9	325

Identify at least one error that is made by each of these newspaper headlines.

Newspaper A:

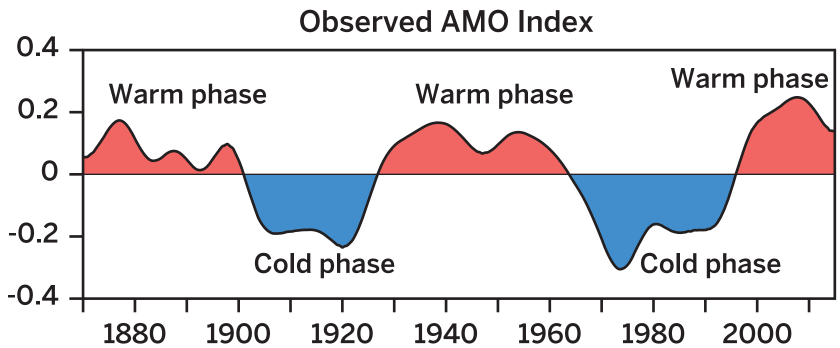
Newspaper B:

Log in to Amplify Math to complete this lesson online.

Name: Date: Period:

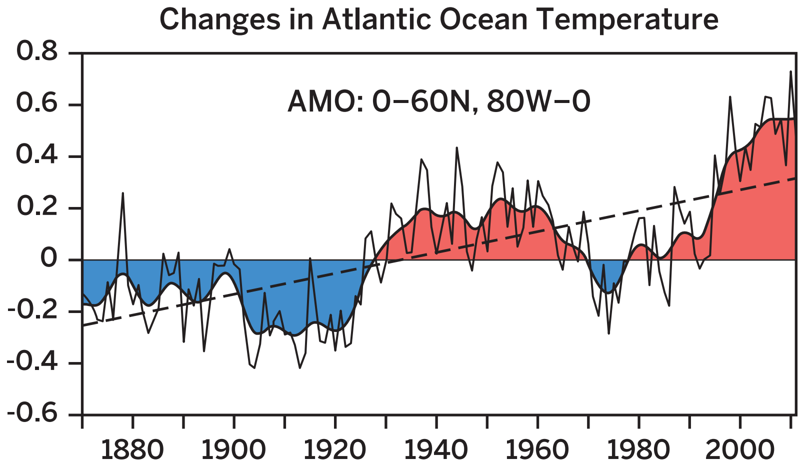
Activity 1 Increasing Number of Hurricanes

Researchers agree that there have been more hurricanes in recent decades. While most have attributed this increase to global warming, some people blame the Atlantic Multidecadal Oscillation (AMO), a natural cycle in Atlantic Ocean temperature. The AMO consists of warm phases with more hurricane activity and cold phases with less hurricane activity, as shown.



1. Based on the graph, make a prediction about the frequency of storms and hurricanes in the next few decades. Do you think this contradicts global warming as a cause of increased hurricanes?

The AMO is not a temperature — it is the *difference* between the Atlantic Ocean temperature and the global sea surface temperature. However, the global sea surface temperature has itself increased in recent decades. Rather than showing this difference, the following graph shows changes in the Atlantic Ocean temperature over time.



2. Based on this graph, would you change your prediction about the frequency of storms and hurricanes in the next few decades? Explain your thinking.

Activity 2 Global Average Temperature Change

In this activity, you and your group members will try to convince another group that climate change is not real. Your group will be given a large data set from which you will choose a time interval to analyze. You will construct your argument using spreadsheet technology.

Since 1880, data has been collected comparing the global temperature change relative to the average temperature from 1951 to 1980. A negative value means a certain year's temperature was below the average, while a positive value means it was above the average.

1. Create a scatter plot with the line of best fit for the time interval you selected.

2. Determine the following:

a. The equation for the line of best fit.

b. The correlation coefficient.

3. What does the slope of the line of best fit mean in the context of this situation?

4. What does the correlation coefficient tell you? Explain your thinking.

Name: ____________ Date: ____________ Period: ____________

Activity 2 Global Average Temperature Change (continued)

Now, you and your group will analyze the entire data set of values from 1880–2020.

5. Create a scatter plot with the line of best fit.

6. Determine the following:

a The equation for the line of best fit.

b The correlation coefficient.

7. What does the slope of the line of best fit mean in this context?

8. What does the correlation coefficient tell you? Explain your thinking.

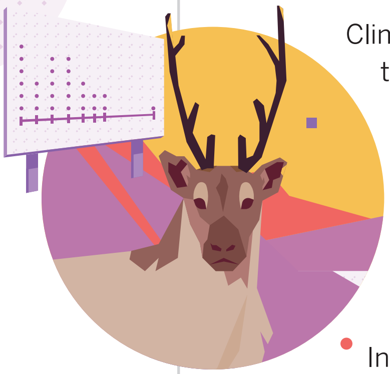
9. How does the equation and correlation coefficient you calculated for the entire data set compare to the ten year time interval you chose earlier?

10. Given a large data set, does it make sense to select a time interval and draw conclusions about trends and associations? Explain your thinking.

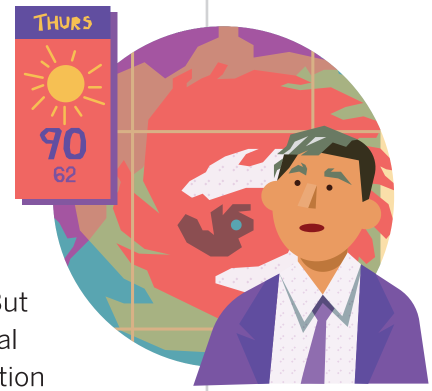
STOP

Narrative Connections

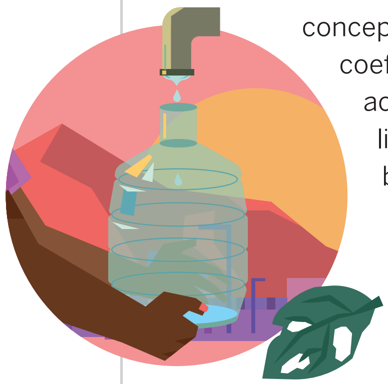
Unit Summary



Climate change is causing sea levels to rise, shorelines to erode, and hurricanes and wildfires to intensify and become more frequent. As a result, people all over the world are losing their homes and livelihoods, they're breathing unclean air, and wildlife are seeing their habitats depleted and destroyed.



In terms of geologic time, where change typically takes tens of thousands of years to occur, this climate change has occurred *very* quickly. But to human eyes, it can still seem gradual. Statistical concepts, such as standard deviation, correlation coefficient, and the line of best fit can give us a more accurate and complete picture, accounting for the limits of our own perception and helping us to better understand how the climate is changing.



But be careful! As useful as statistics are, they can also be manipulated, distorted, and misinterpreted. Efforts to deny climate change — as well as humanity's role in climate change — often rely on such manipulations. Just because a statistical argument *sounds* convincing, that doesn't mean it is. These days, being able to interpret and analyze statistics is more important than ever.

With the tools from this unit, you can reveal the true story being told by statistics and perceive more than what your eyes tell you. When we understand exactly what is happening to our Earth, and how, we can take the right steps to change it.

See you in Unit 3.

Name: .... Date: .... Period: ....

Practice

1. Which of the following would *not* be part of a statistical experiment?

   A. Determining the treatments that different groups in the experiment will receive.

   B. Collecting data over time.

   C. Changing a treatment part way through an experiment.

   D. Summarizing data using statistics and data displays.

2. A news website reports a scatter plot showing a positive linear association between the number of hours spent exercising and overall health. What are some other variables that could affect overall health besides time spent exercising?

3. The following describes an outline for the steps of an experiment that Priya is designing. Write the step number for each description in the order that makes logical sense.

Step: .... Priya decides on three treatment levels: one group of corn will have no pesticides, one will have a low level of pesticides, one will have a high level of pesticides.	**Step:** .... Priya summarizes her findings and draws conclusions about the effect the different levels of pesticides have on the growth of corn.	**Step:** .... Priya describes how her experiment could be improved or what changes to make for a later experiment.
Step: .... Priya applies the three different amounts of pesticides to the corn and allows the corn to grow over time.	**Step:** .... Priya decides to test the effects of various pesticides on the growth of corn.	**Step:** .... Priya takes her data and analyzes it using statistics and data displays.

Practice

Name: ________ Date: ________ Period: ________

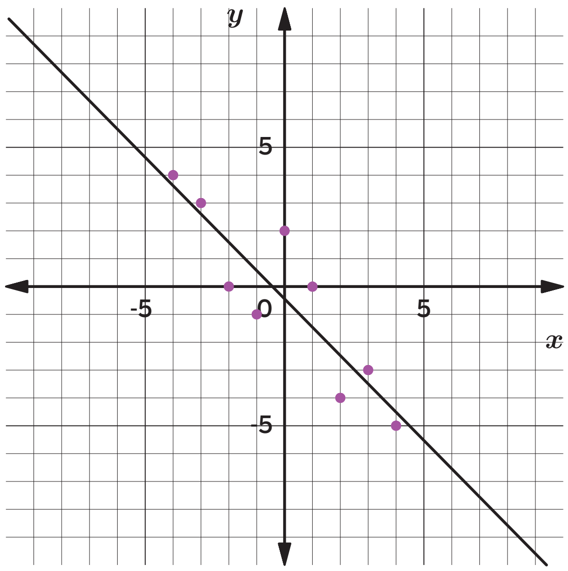
4. Use the scatter plot and line of best fit shown to select the best equation and correlation coefficient.

A. $y = 1.02x + 0.44$; $r = 0.89$

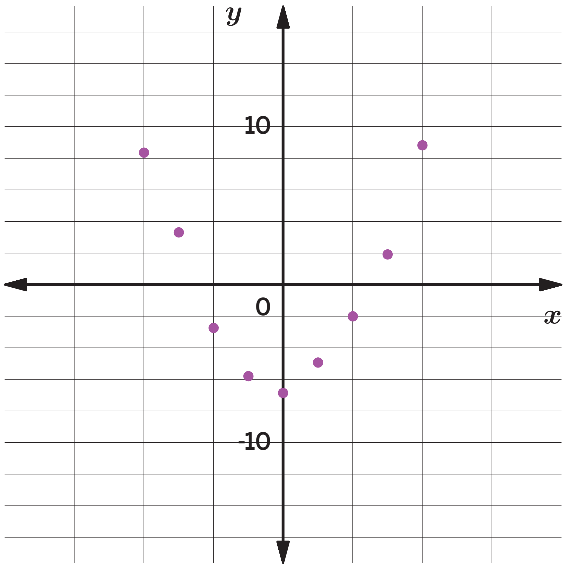
B. $y = -1.02x + 0.44$; $r = -0.89$

C. $y = 1.02x - 0.44$; $r = -0.89$

D. $y = -1.02x - 0.44$; $r = -0.89$



5. What does the following residual plot tell you about the line of fit for the data from which it was calculated?



My Notes:

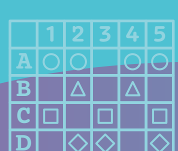
UNIT 3

Functions and Their Graphs

You will expand your understanding of functions, their representations, and their graphs through music and cityscapes. Along the way, you will write, graph, and interpret a variety of functions and their inverses.

Essential Questions

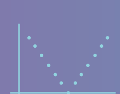
- How can you represent and describe functions?
- Why are relations and functions represented in multiple ways?
- Why should functions be analyzed graphically?
- *(By the way, what is the inverse of putting on your socks and shoes?)*

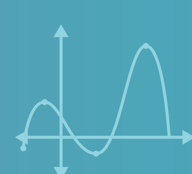


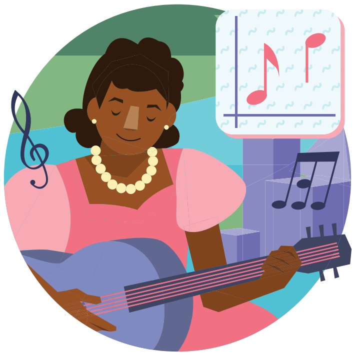
Find all the solutions to the following system of equations. Then add up the absolute values of all the x- and y-coordinates. What do you get?

$$y = x + |x| - 3$$

$$y = x - |x| + 3$$







SUB-UNIT

1 Functions and Their Representations

Narrative: Much like playing an instrument, you can "play" a mathematical function.

You'll learn . . .
- a new tool for communication about functions — *function notation*.

SUB-UNIT

2 Analyzing and Creating Graphs of Functions

Narrative: Reading a sheet of music is similar to interpreting the graph of a function.

You'll learn . . .
- about the domain, range, and average rate of change of functions.

SUB-UNIT

3 Piecewise Functions

Narrative: Explore how a function can model the *pieces* of sound.

You'll learn . . .
- about piecewise and absolute value functions.

SUB-UNIT

4 Inverses of Functions

Narrative: Functions and their inverses can help you go from acoustics to amplified sounds and back again.

You'll learn . . .
- what it means for a function to have an inverse.

Unit 3 | Lesson 1 – Launch

Music to Our Ears

Let's determine how graphs and functions can be used to tell a story.

Warm-up Slow Reveal

Your teacher will play a sound and then show you a graph. Then your teacher will play the sound and show you a graph at the same time. What do you notice? What do you wonder?

	I notice . . .	I wonder . . .
Sound		
Graph		
Both		

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Take Turns: Music Is All Around Us!

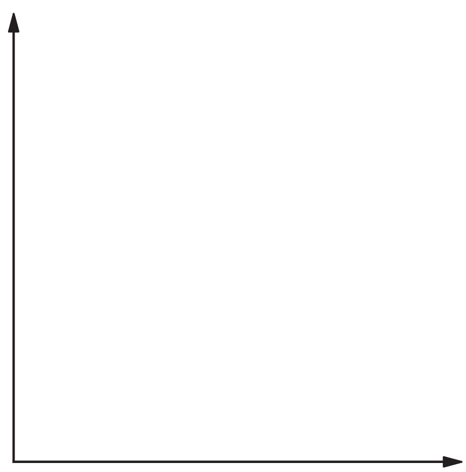
Music comes in many forms and unique styles. Music you create does not need to be complex or require instruments, just your own hands, feet, or voice! You and a partner will have the opportunity to create some of your own sounds, beats, or music.

You will be given a set of cards. Each card contains a sound you will make with your hands, feet, or voice. Take turns with your partner as either the sound maker or the listener.

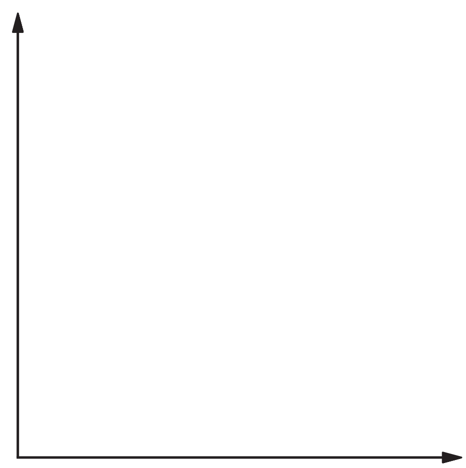
When you are the sound maker:	When you are the listener:
1. Select a card from the shaded deck. Create the sound described on the card using your hands or feet. **2.** Select a card from the unshaded deck. Create the sound described on the card using your voice.	**1.** Actively listen to your partner and the sound they are making. **2.** After your partner shares their sound, use these sentence stems to help clarify your partner's actions: "Can you recreate the sound of . . .?" "How did you create . . .?" **3.** Sketch a graph to interpret how the sound your partner created changes over time.

When you are the listener, use the space provided to sketch the graph that corresponds with each card.

Card 1:

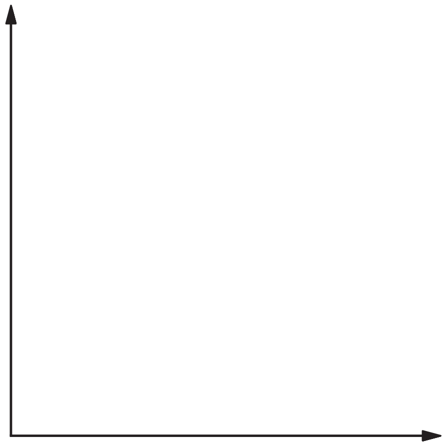


Card 2:

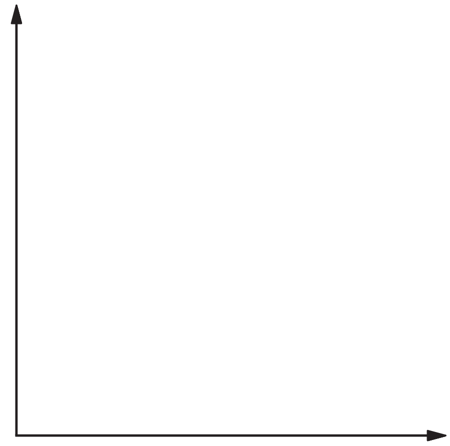


Activity 1 Take Turns: Music Is All Around Us! (continued)

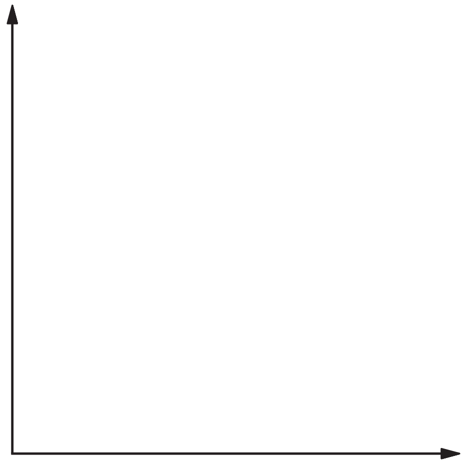
Card 3:



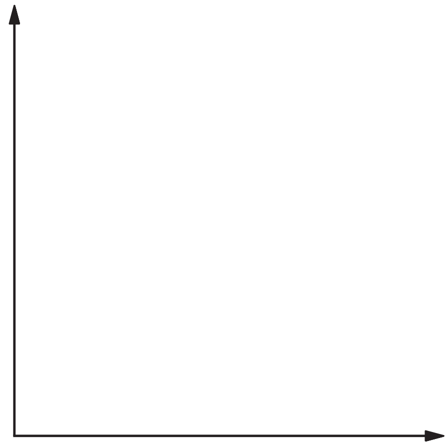
Card 4:



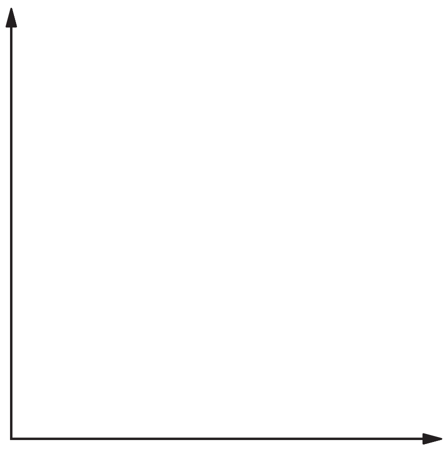
Card 5:



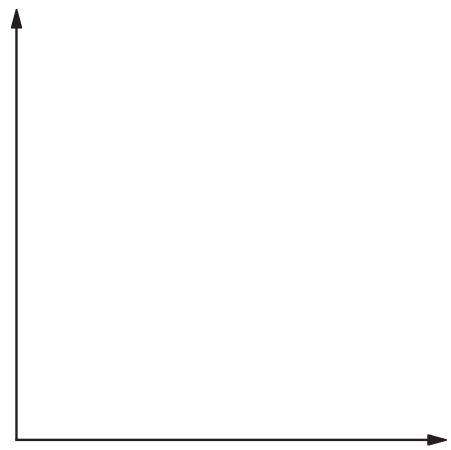
Card 6:



Card 7:



Card 8:



Name: ______ Date: ______ Period: ______

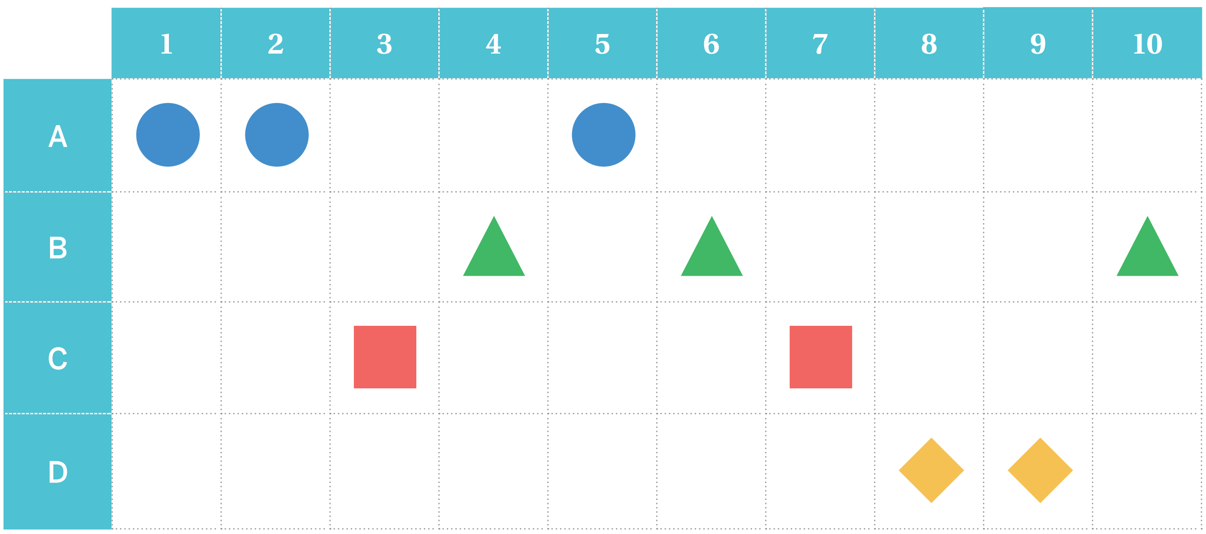
Activity 2 One-Person Band

A one-person band is a musician who plays multiple instruments by themselves. Consider a musician in a one-person band who must move from one instrument to the next in order to play it.

Africa Studio/Shutterstock.com

Sheet music, or musical notation, is printed music, with various notes and symbols written on different lines. Another common way to represent music visually is by the use of graphic notation. Graphic notation does not use all of the formal symbols of musical notation, but it is influenced by it. Do not worry! You do not need to be an expert to interpret the symbols you see, that is part of what music is all about; experimenting and expression!

1. Consider this example of a graphic score used by a one-person band.



a What do you think each number represents?

b What do you think each symbol represents?

Activity 2 One-Person Band (continued)

2. Consider this graphic score of a one-person band, similar to the one in Problem 1.

	1	2	3	4	5	6	7	8	9	10
A	●	●		●	●		●	●		●
B		▲		▲		▲		▲		▲
C	■		■		■		■		■	
D		◆	◆		◆	◆		◆	◆	

a What is different about this graphic score?

b What do you think this difference means? Explain your thinking.

This idea that some instruments can only play certain sounds at certain times relates to mathematical *functions*. A ***function*** takes each input from one set and assigns it to exactly one output from another set.

Reflect: How can music help you relax and lower your stress level?

3. Explain why the second graphic score from Problem 2 is *not* a function.

4. Using the blank table, create a graphic score that is a function.

	1	2	3	4	5	6	7	8	9	10
A										
B										
C										
D										

STOP

Unit 3 Functions and Their Graphs

Artscapes

Music moves: from note-to-note, beat-to-beat. It moves through us, into us, out of us. There is the movement of air in a trumpeter's lungs before the blast, or the stamping of feet against a dance hall floor.

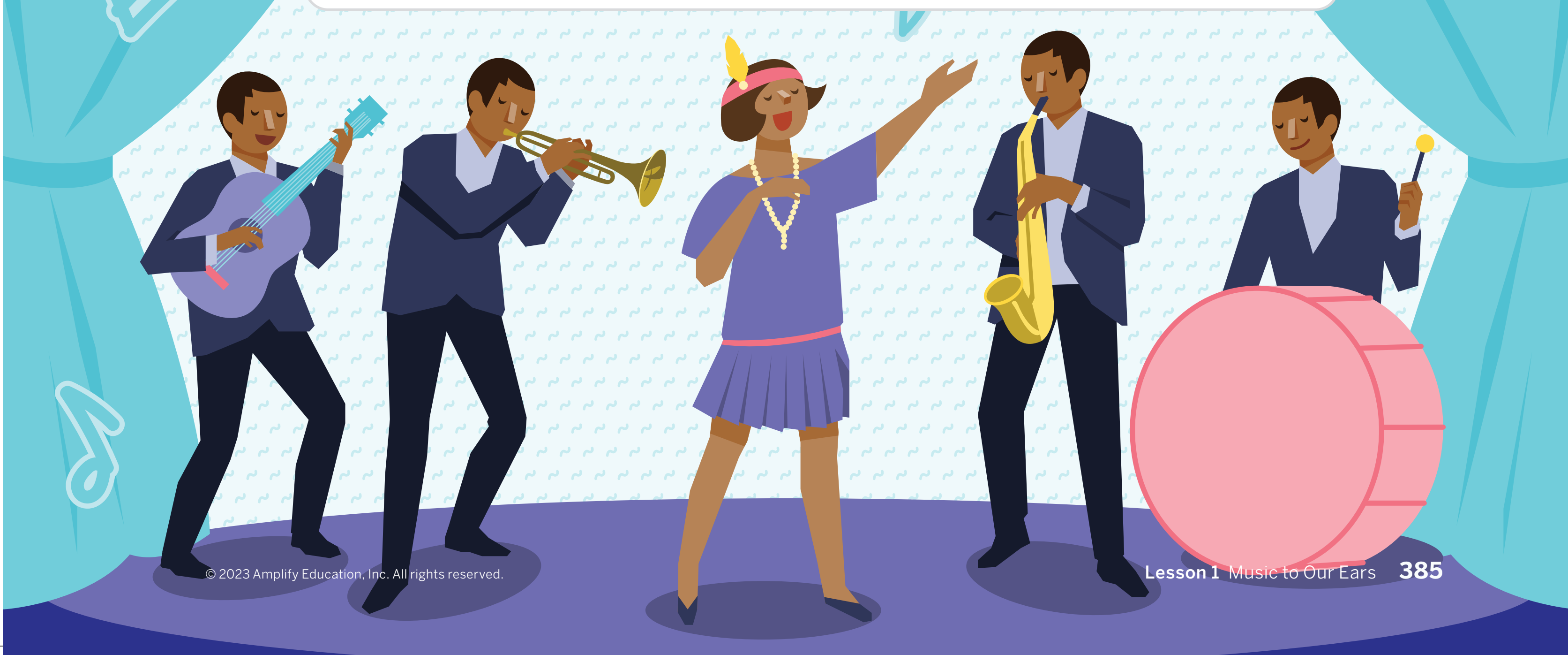
According to the philosopher Plato, "Music gives soul to the universe, wings to the mind, flight to the imagination, and life to everything." Music can lift us up when we are down, say the words we do not know how, and let us connect with a stadium full of people we have never met.

And yet, the music you know and love today did not just spring up fully formed. The jams on your playlist are the end result of hundreds of years and thousands upon thousands of miles of shifting cultural forces. To understand what gives our most beloved music its particular sound, we have to look at where that music came from.

There is no musical art form more quintessentially American than blues and jazz. Their legacy can still be found in rock, country, hip-hop, and R&B. But the story of blues music begins hundreds of years ago with the rhythms of the enslaved West African people, then to the Great Migration, where Black men, women, and children left the rural South to settle northward in America's growing cities. It was in these cities that their music gathered, mingled, and morphed.

In these next lessons, we will look at the way music such as blues and jazz were functions of the cities they have passed through, responding to the forces of place, time, people, and history.

Welcome to Unit 3.



Practice

Name: ____________ Date: ____________ Period: ____________

1. Diego claps his hands twice quickly, pauses, stomps his feet twice quickly, pauses, and repeats this process for a few seconds. Sketch a graph to represent the sounds Diego makes.

2. The table shows every month of the year and how many days are in each month. Let the input be represented by the month and the output be represented by the number of days in the month.

	J	F	M	A	M	J	J	A	S	O	N	D
28		✓										
29		✓										
30				✓		✓			✓		✓	
31	✓		✓		✓		✓	✓		✓		✓

Does this table represent a function? Explain your thinking.

Name: .... Date: .... Period: ....

3. The following data set represents the quiz scores of eleven students in a math class.
39, 77, 71, 85, 94, 63, 79, 80, 89, 75, 92

a What is the mean of the data set?

b What is the median of the data set?

c Is the mean or median a more appropriate measure of center for this data set? Explain your thinking.

4. Consider the input-output rule that assigns exactly one output to each allowable input. Use the rule to complete the table.

Input → **Add 2 and then multiply by 3.** → **Output**

Input	Output
-5	
$\frac{2}{3}$	
2.58	
10	

My Notes:

SUB-UNIT

1 Functions and Their Representations

Narrative Connections

How did the blues find a home in Memphis?

Follow the Mississippi River long enough and you'll end up in the city of Memphis, Tennessee. For more than a hundred years, this city has been a waystation for countless blues legends.

This story begins after the Civil War. The city's cotton sellers had established a trade association, which allowed them to control the prices of cotton being sold in Memphis. This kept the city's economy strong, drawing in more workers from throughout the South. As more jobs came, so too did traveling performers. These performers exposed their Memphis audiences to the music that originated in rural Black communities.

By the early 1900s, vaudeville acts overtook the older performance acts. Places like Beale Street and Church Park blossomed, becoming centers for Black business and culture. Musicians brought the sounds of the country to the city. Work songs, ragtime, and country blues bursted in every Memphis theater, dance hall, and juke joint.

In the years to come, the city became home to performers like Memphis Minnie, Furry Lewis, Sleepy John Estes, and the "Father of the Blues" himself: W.C. Handy. In 1912, Handy composed "The Memphis Blues," a 12-bar musical composition set down in sheet music. It was this form and structure that would inspire blues players for generations to come.

It's not just music that has its own notation. As you'll see in the next few lessons, the same goes for functions. Just as a musician can write and play from sheet music, a mathematician can concisely write a function and even "play" it, by analyzing its structure and studying its graph.

Unit 3 | Lesson 2

Describing and Graphing Situations

Let's explore different ways to represent a relationship between two quantities.

Warm-up Notice and Wonder

Study the table. What do you notice? What do you wonder?

Number of tickets	Total ticket price ($)
1	9
5	40
10	60
20	100

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

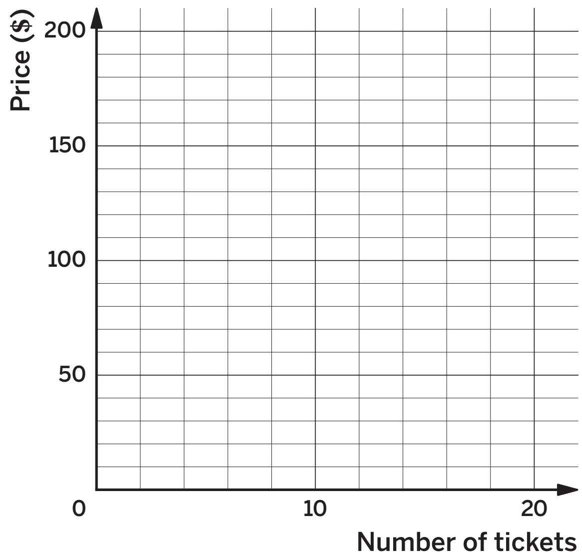
Name: .... Date: .... Period: ....

Activity 1 Going to the Museum

The Stax Museum of American Soul Music is located in the Soulsville neighborhood of Memphis, Tennessee, at the site of the original and iconic Stax Records studio. Since the 1950s, Stax Records has produced some of the earliest recordings by musical legends such as Isaac Hayes, Otis Redding, and Booker T. Jones.

1. A teacher organizes a field trip to the Stax museum and researches admission prices. She determines that the price for one ticket is $9. Complete the table with the price for each number of tickets purchased. Plot the corresponding points on the graph.

Number of tickets	Total ticket price ($)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
20	



Activity 1 Going to the Museum (continued)

Upon further research, the teacher determines that the museum offers a discounted rate on student tickets. The student tickets are sold in packages of 5, 10, and 20. The number of tickets in each package and their corresponding price is shown in the table.

Number of tickets	Total ticket price ($)
5	40
10	60
20	100

2. The teacher determines the price of the trip for 32 students will be $178. Jada, Priya, and Han disagree with their teacher's calculation.

- Jada says to her friends, "I think the total ticket price should be $288."
- Priya says, "I think the price of the trip should be $258."
- Han says, "No, I think the total should be $198."

Explain how the teacher, Jada, Priya, and Han could each be correct.

3. Graph the student discounted ticket package prices for 5, 10 and 20 tickets on the same coordinate plane used in Problem 1. Why does the relationship between the number of tickets and total ticket price not represent a function?

Collect and Display: Your teacher will start a class display of math terms and phrases related to functions. Refer to this display and help add to it as you progress through this unit.

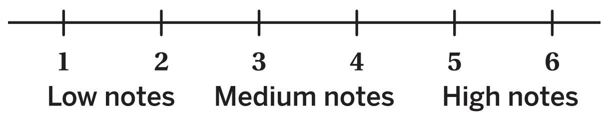
Name: ________ Date: ________ Period: ________

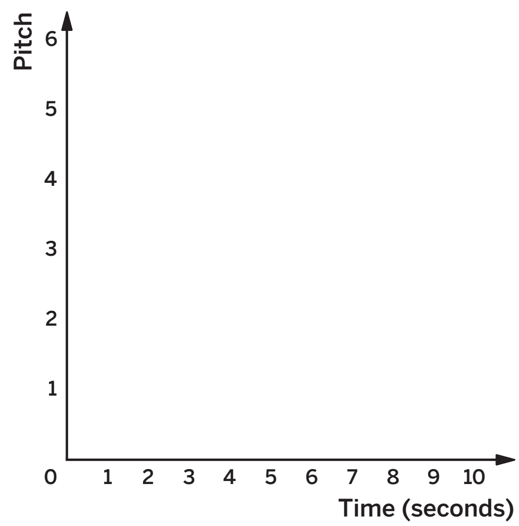
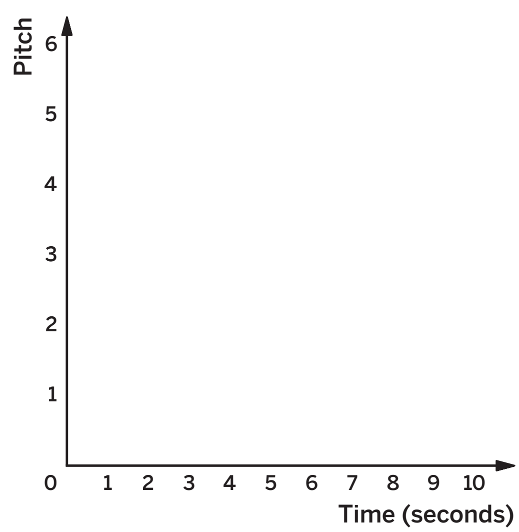
Activity 2 Making Music

Before the field trip to the Stax museum, the class explores how to make music using straws. They build straw whistles and experiment creating different pitches or notes. You will be shown a series of videos that demonstrate students playing the straw whistle in different ways. Three descriptions of the sounds created by these students are given.

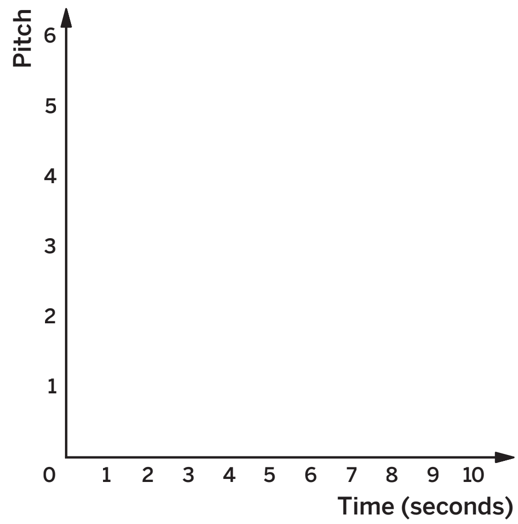
1. Sketch a graph that could represent the pitch of the notes being played at different times. **Hint:** Use this pitch scale shown here to help you sketch your graph.

Pitch scale



Description	Graph
The pitch remains constant as the student plays the same note on a 5-in. straw.	
The pitch begins low and increases as the student cuts the length of a 10-in. straw and plays different notes.	

Activity 2 Making Music (continued)

Description	Graph
The student plays straws of different lengths from the highest to the lowest pitch.	

2. What are the variables in this situation?

3. Which variable represents the input? Explain your thinking.

4. Which variable represents the output? Explain your thinking.

5. Is the relationship between pitch and time a function? Explain your thinking.

Are you ready for more?

Use a straw to build your own whistle. Experiment creating different sounds, then sketch a graph of the pitch of the sounds you create on a separate sheet of paper.

Name: ______ Date: ______ Period: ______

Activity 3 Partner Activity: Talk About a Function

In each of the following scenarios, the relationship between each quantity can be expressed as a function.

- **Scenario 1: An elevator's height, in feet, from the ground and the length of time, in seconds, after it starts moving.**
- **Scenario 2: The time, in hours, since a museum opened and the number of tickets sold each hour.**

You and your partner will each select one of these scenarios and analyze the relationship between the quantities. Which scenario did you choose? Scenario ______

1. Which quantity represents the independent variable? Explain your thinking.

2. Which quantity represents the dependent variable? Explain your thinking.

3. Use your responses from Problems 1 and 2 to complete the sentence for your scenario.

Scenario 1: "The ______ depends on the ______."

Scenario 2: "The ______ depends on the ______ since the museum opened."

4. Sketch a possible graph of the relationship between the quantities in your scenario. Be sure to label the axes. Be prepared to explain what each part of your graph represents.

STOP

Summary

In today's lesson . . .

You analyzed the relationship between two quantities in varying contexts and identified the *independent variable* (input) and *dependent variable* (output). The output is a *function* of the input if there is only one output for each possible input. When a function is represented with a graph, each point on the graph is an ordered pair of input and output values.

You also observed a variety of ways to represent functions:

- Verbal descriptions
- Tables
- Graphs

Reflect:

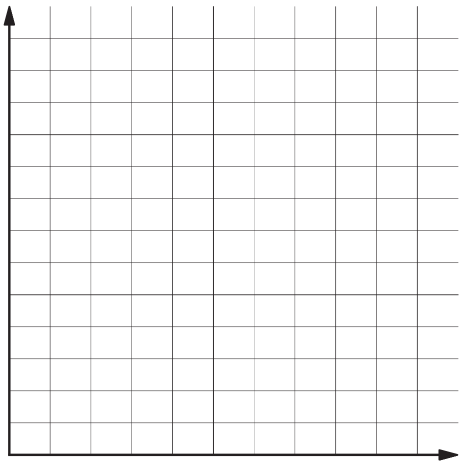
Name: ______ Date: ______ Period: ______

Practice

1. The relationship between the amount of time a car is parked and the cost of parking can be expressed as a function.

   a. Identify the independent variable and the dependent variable of this function.

   b. Suppose it costs $\$3$ per hour to park, with a maximum cost of $\$12$. Sketch a possible graph of the function. Be sure to label the axes.



2. Determine whether each of the following descriptions represents a function. Explain your thinking.

   a. The input is a year. The output is the population of the United States during that year.

   b. The input is the distance a person is from the ground. The output is the time related to that height as the person rides a Ferris wheel.

   c. The input is a person's name. The output is that person's phone number.

3. The distance a person walks d, in kilometers, is a function of the time t, in minutes, since they began walking. Select *all* the true statements about this function.

   A. The distance walked is the input.

   B. The time of day is the input.

   C. The input is measured in hours.

   D. The variable t represents the input.

   E. The variable d represents the input.

   F. The input is not measured in any particular unit.

   G. The time since the person began walking is the input.

   H. For each input, there are sometimes two outputs.

Practice

Name: .... Date: .... Period: ....

4. For the expression $4x - 5(y - 1)$, which of the following ordered pairs makes the value of the expressions greater than 20?

A. $(8, 10)$

B. $(5, 0)$

C. $(10, 8)$

D. $(0, 5)$

5. A randomly selected group of 100 employed adults were surveyed about whether they earned a high school diploma and whether their current annual income was greater than $\$30{,}000$. The following table shows some of the results. Complete the two-way table.

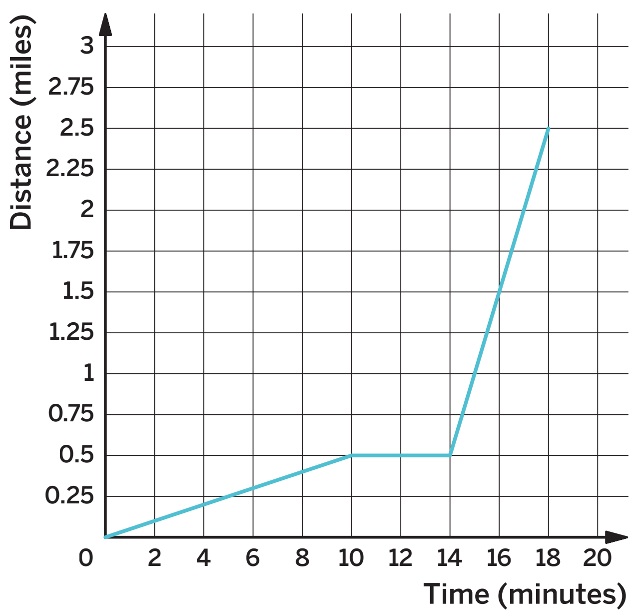
	$30,000 or less	Greater than $30,000	Total
High school diploma		68	89
No high school diploma	9		
Total		70	

6. Han walked to his bus stop and then took the bus for one stop to his school. The graph shows the relationship between his distance d, in miles, from home and time t, in minutes.

a What is Han's distance from home after 10 minutes?

b How long does it take Han to travel a distance of 1.5 miles?

c Based on the graph, how long did Han have to wait for the bus to arrive after walking to his bus stop? Explain your thinking.



Name: ______ Date: ______ Period: ______

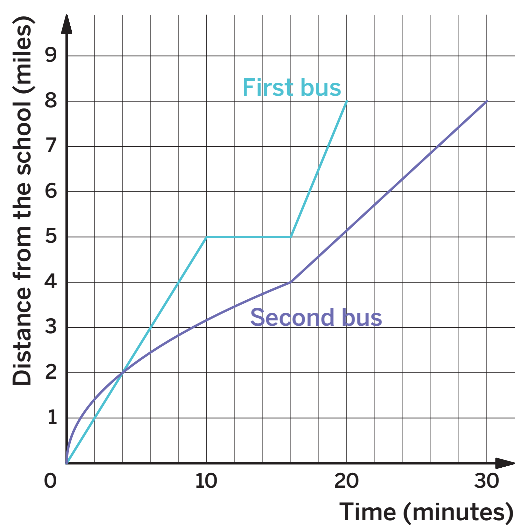
Unit 3 | Lesson 3

Function Notation

Let's explore a more efficient way to refer to and communicate about functions.

Warm-up Back to the Museum

Two school buses transport students on a field trip to a soul music museum. Each bus takes a different route to the museum. Consider the graphs representing each bus's distance from the school as a function of time.



1. How many miles from the school has each bus traveled after 16 minutes?

   a. First bus

   b. Second bus

2. How many minutes does it take each bus to travel 3 miles from the school?

   a. First bus

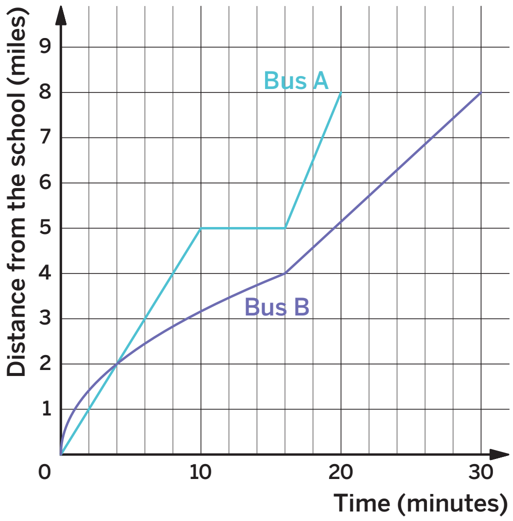
   b. Second bus

3. Consider the statement, "The bus was 8 miles away from the school after 20 minutes." Do you agree with this statement?

Log in to Amplify Math to complete this lesson online.

Activity 1 A Handy Notation

It is important to have a symbolic language that everyone agrees on in order to discuss mathematics. Imagine how confusing it would be if every person used a different name to refer to the same function! Not only that, but just saying First bus or Second bus does not tell you the independent or dependent variables that the function describes. *Function notation* is an efficient way to refer to a function and describe its input and output values.



Using the same two buses from the Warm-up, you will describe their movements using function notation. Let's start by choosing a name for each bus. The first bus will be named Bus A, and the second bus will be named Bus B. The input of each function is the time t, in minutes.

1. Complete each sentence using the graph.

a After 4 minutes, Bus A is .... miles away from the school.

b After 9 minutes, Bus B is .... miles away from the school.

With function notation, you can write these statements more efficiently and know which variables are the independent and dependent variables. For example, the statement in Problem 1a can be written as $A(4) = 2$ because 4 is the input and 2 is the output.

2. Use the definition of function notation to translate each verbal statement to function notation.

a The distance Bus A has traveled from the school after 4 minutes.

b The distance Bus A has traveled from the school after 9 minutes.

c The distance Bus B has traveled from the school after 4 minutes.

d The distance Bus B has traveled from the school after 9 minutes.

Name: ______ Date: ______ Period: ______

Activity 1 A Handy Notation (continued)

3. Translate each expression written in function notation to its verbal statement.

a. $A(16)$

b. $B(4)$

c. $A(t)$

4. The function notation statement $A(4) = 2$ means, "4 minutes after Bus A left the school, the bus was 2 miles away from the school." Describe what each function notation statement means in this situation.

a. $A(9) = 4.5$

b. $B(16) = 4$

c. $A(t) = 5$

d. $B(t) = d$

5. Refer to the function notations and descriptions in Problems 2–4. Describe one advantage of using a verbal description to describe a situation and one advantage to using function notation to describe a situation.

Critique and Correct: Your teacher will display an incorrect statement. Work with your partner to critique and correct the statement. Then discuss how you know your statement is correct.

6. Refer to the graph that represents Bus B, or function B. Use function notation to describe the output when the input is 30 minutes. Explain what the statement means in the context of the problem.

Activity 2 Name That Song

Memphis, Tennessee, is often called the birthplace of rock 'n' roll because of its rich musical history. The legendary king of rock 'n' roll, Elvis Presley, lived in Memphis and recorded his first records at the famous Sun Records studio. The table displays some of the hit songs by Elvis and the year that they were released.

Consider the following statements describing two possible relationships between the song titles and the years they were released.

- Relationship T takes the song title as its input and gives the release date as its output.
- Relationship S takes the release date as its input and gives the song title as its output.

Song title	Release date
"Can't Help Falling in Love"	1961
"Jailhouse Rock"	1973
"All Shook Up"	1957
"Blue Suede Shoes"	1956
"Always on My Mind"	1973
"Are You Lonesome Tonight?"	1960
"Love Me Tender"	1957

1. If each song title is used as the input for T, how many outputs are possible for each input? Explain your thinking.

2. If each release date is used as the input for S, how many outputs are possible for each input? Explain your thinking.

3. One of the relationships is a function while the other is not. Which relationship is a function? Explain your thinking.

4. Choose one set of input and output pairs from the table and write a statement describing the input-output relationship using function notation. Describe what your function notation statement means within this context.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored how ***function notation*** is an efficient way to communicate information about a function without having to write a verbal description. In general, function notation has the form:

Name of function   Input of function

$$f(x)$$

Output of function

The notation $f(x)$ is read as "f of x" and can be interpreted to mean $f(x)$ is the output of a function f, when x is the input. The statement $f(x) = y$ is read "f of x is equal to y" and tells you that the output $f(x)$ has the same value as y.

Function notation is a way of expressing the specific input and output values of a function that you have named. Remember that a function is a relationship between two quantities in which there is exactly one output value for each input value.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. The function P represents the height of water in a bathtub w, in inches, as a function of time t, in minutes. Match each verbal statement with its corresponding function notation.

   a. After 20 minutes, the bathtub is empty.

   b. At the start, the bathtub is empty.

   c. After 10 minutes, the height of the water is 4 in.

   d. The height of the water is 10 in. after 4 minutes.

   e. The height of the water is w in. after t minutes.

   .... $P(10) = 4$

   .... $P(t) = w$

   .... $P(20) = 0$

   .... $P(0) = 0$

   .... $P(4) = 10$

2. Suppose a function M takes time as its input and gives a student's Monday class as its output.

   a. Use function notation to represent the statement, "A student has English class at 10:00 a.m."

   b. Write a statement to describe the meaning of M(11:15) = Chemistry.

3. The function C gives the cost, in dollars, of buying n apples. What does each expression or equation represent in this context?

   a. $C(5) = 4.50$

   b. $C(2)$

4. It costs \$3 per hour to park in a parking lot, with a maximum cost of \$12. Explain why the amount of time a car is parked is not a function of the parking cost.

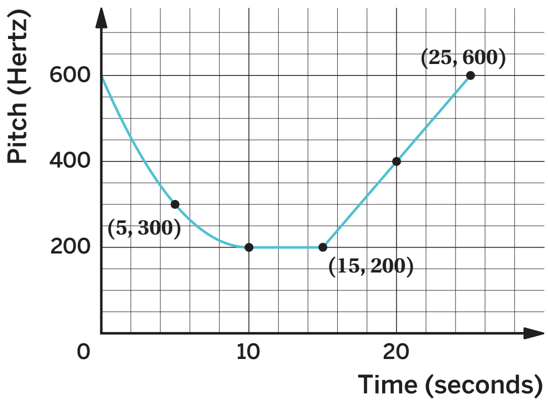
Name: .... Date: .... Period: ....

5. Here are two clues for a puzzle involving two numbers.

- Seven times the first number plus six times the second number equals 31.
- Three times the first number minus ten times the second number is 29.

What are the two numbers? Explain or show your thinking.

6. The graph represents the pitch of the notes played by a trumpet in the first 25 seconds of a song. A high pitch means high frequency, while a low pitch means low frequency. The pitch depends on the frequency of a sound wave and is measured in Hertz, or vibrations per second.



a. Complete the sentence: The pitch of the trumpet at .... is 300 Hertz.

b. Complete the sentence: The pitch of the trumpet at 25 seconds is ....

c. Use function notation to represent the statements in parts a and b.

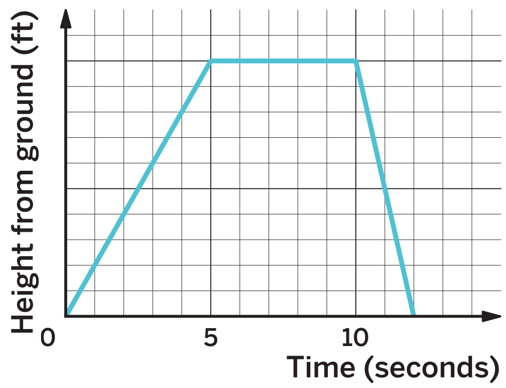
Unit 3 | Lesson 4

Interpreting and Using Function Notation

Let's explore how to use function notation to describe quantities in a graph.

Warm-up Graphing Story

Tennessee's Memphis Zoo is one of the few zoos in the nation that hosts a pair of giant pandas. Originally from China, giant pandas are an endangered species and, despite their large size, are very skilled at climbing trees. The graph shows the panda's height as it climbs a tree, in feet, as a function of the time, in seconds.



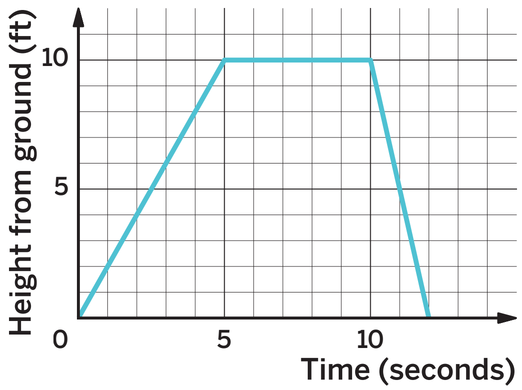
Is the panda at a greater height after 6 seconds or after 11 seconds? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 How High?

Consider the graph, which is the same graph that you saw in the Warm-up. The function f represents the panda's height from the ground, in feet, as it climbs a tree, t seconds after it leaves the ground.



1. Determine which value in each pair of values is greater. Explain your thinking.
   - a) $f(3)$ or $f(8)$
   - b) $f(5)$ or $f(10)$
   - c) $f(t)$ or $f(t+1)$

2. Explain each statement within the context of the problem.
   - a) $f(11) < f(4)$
   - b) $f(0) = f(12)$

Activity 2 How Heavy?

Plan ahead: How do you think function notation will help you organize and interpret the information in the problem?

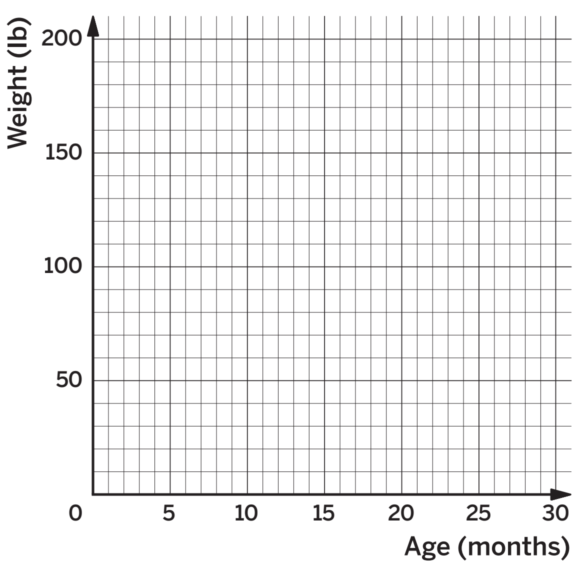
Adult pandas can weigh up to 300 lb. This weight is 900 times heavier than the weight of a typical baby panda, which is 3.5 oz at birth. The function P gives the weight of a baby panda, in pounds, x months after it is born.

1. Explain what each function notation statement means in this context.
   - a $P(10) = 44.1$
   - b $P(0) = 0.2$

2. Use function notation to represent each statement.
   - a At two years old, the baby panda weighs 143 lb.
   - b At the age of one year and five months old, the baby panda weighs 110.5 lb.

3. Clare is curious about the value of x in the function notation statement $P(x) = 100$.
   - a What would the value of x tell Clare about this context?
   - b Do you think 5 is a reasonable value of x to make the statement true? Explain your thinking.

4. Use the information from Problems 1 and 2 to sketch a graph of the function P.



Name: ________________ Date: ________ Period: ________

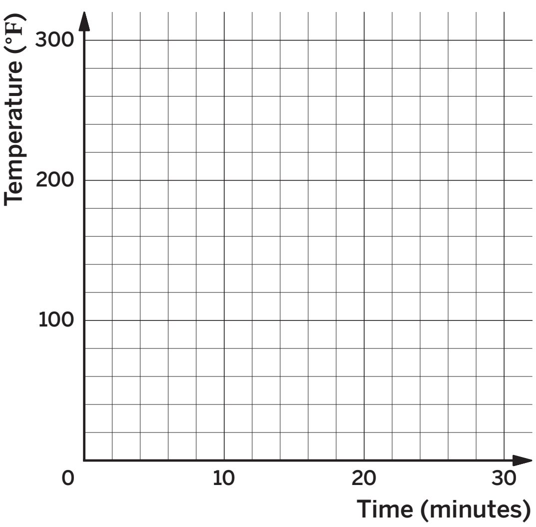
Activity 3 Partner Problems: Boiling Water

One partner will complete Column A and one will complete Column B. Complete the problems in your column, and then compare responses with your partner. If your responses are not the same, discuss and resolve any differences.

For each column, explain the meaning of each statement for the following scenario: The function W gives the temperature, in degrees Fahrenheit, of the water in a pot placed on a stove t minutes after the stove is turned on.

Column A	Column B
1. $W(0) = 72$	**1.** $W(10) = 212$
2. $W(5) > W(2)$	**2.** $W(15) > W(30)$
3. $W(12) = W(10)$	**3.** $W(0) < W(30)$

4. Use each statement from both columns in Problems 1–3 to describe the temperature at specific times. Sketch a possible graph of function W. Be prepared to explain how each statement is represented on your graph.



STOP

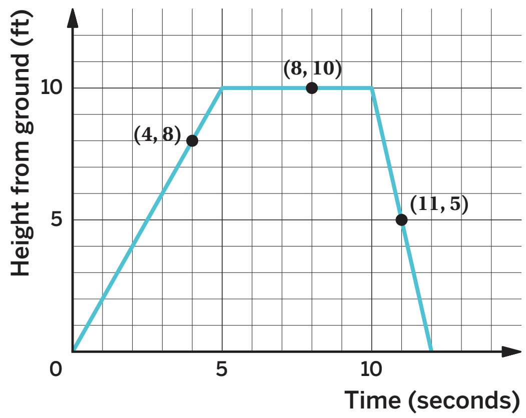
Summary

In today's lesson . . .

You described connections between function notation statements and how they are represented on the graph of a function. You also represented a relationship between real-world quantities by converting between verbal descriptions and function notation.

When given a statement written in function notation, you described the pairs of input and output values as ordered pairs with the coordinates $(x, f(x))$. You used these ordered pairs to sketch a graph of the function.

Consider the graph from the Warm-up and Activity 1, which represents a panda's height from the ground at different times as it climbs a tree.



The function f relates the height in feet to the time in seconds.

- $f(\text{time}) = \text{height}$
- $f(t) = h$ means that the panda is h feet from the ground after t seconds. For example, $f(11) = 5$ means the height of the panda is 5 ft from the ground after 11 seconds.
- Each pair of input and output values corresponds to a point on the graph with the coordinates $(t, f(t))$ and describes the height at a specific time. For example, the point $(11, 5)$ represents the function notation statement $f(11) = 5$.

Reflect:

Name: ____________ Date: ____________ Period: ____________

Practice

1. The function f gives the temperature, in degrees Celsius, t hours after midnight. Select the equation that represents the statement, "At 1:30 p.m., the temperature was $20^\circ C$."

A. $f(1.30) = 20$

B. $f(1.50) = 20$

C. $f(13.30) = 20$

D. $f(13.50) = 20$

2. Tyler filled up his bathtub, took a bath, and then drained the tub. The function B gives the depth of the water, in inches, t minutes after Tyler began to fill the bathtub. Explain the meaning of each statement in this scenario.

a. $B(0) = 0$

b. $B(1) < B(7)$

c. $B(9) = 11$

d. $B(10) = B(22)$

e. $B(20) > B(40)$

3. The function f gives the temperature in degrees Celsius, t hours after midnight. Use function notation to represent each statement.

a. The temperature at 12 p.m.

b. The temperature was the same at 9 a.m. and at 4 p.m.

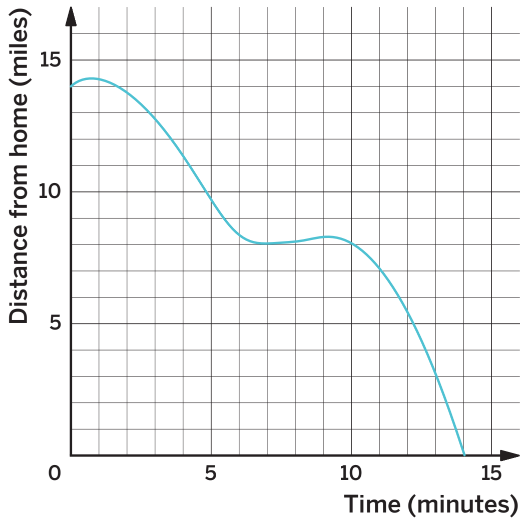
c. It was warmer at 9 a.m. than at 6 p.m.

d. Some time after midnight, the temperature was $24^\circ C$.

Practice

Name: ______ Date: ______ Period: ______

4. Consider the graph of the function h, which gives the distance, in miles, a student is from home as they ride their bicycle, as a function of time t, in minutes. Determine the values of $h(7)$ and $h(11)$.



5. A number of identical cups are stacked. The number of cups in a stack and the height of the stack in centimeters are related.

a Is the height of the stack a function of the number of cups in the stack? Explain your thinking.

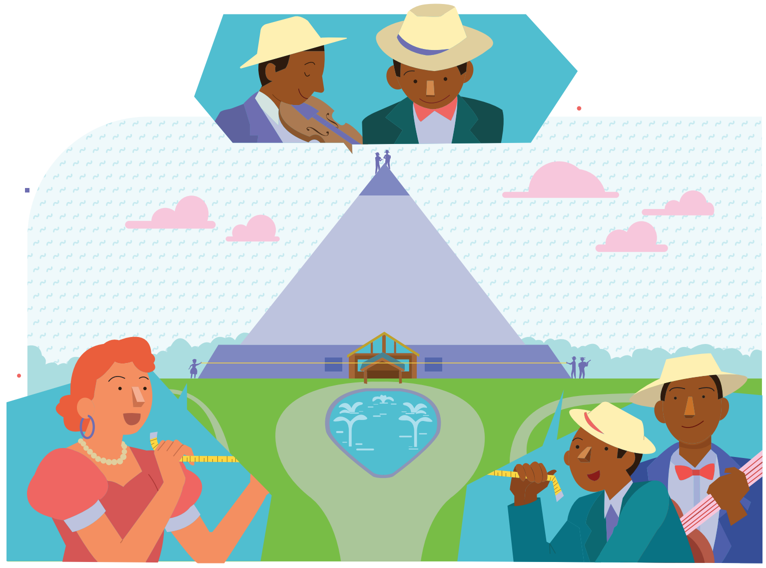
b Are the number of cups in a stack a function of the height of the stack? Explain your thinking.

6. Determine three points that are on the graph $y = -\frac{1}{2}x + 4$.

Name: ________ Date: ________ Period: ________

Unit 3 | Lesson 5

Using Function Notation to Describe Rules (Part 1)



Let's explore how to use function notation to write equations that represent function rules.

Warm-up Notice and Wonder

Study the tables. What do you notice? What do you wonder?

n	$f(n) = 1 + 3n$
0	1
1	4
2	7
3	10

n	$g(n) = 16 - 4n$
0	16
1	12
2	8
3	4

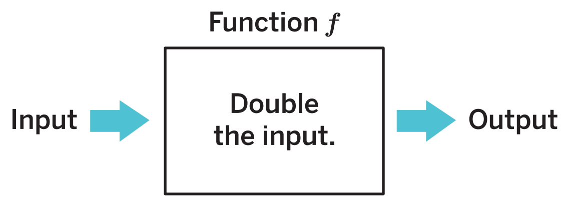
1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Jigsaw: Four Functions

A function machine is a diagram that takes an input value, applies a rule, such as a set of operations, and gives an output value. Consider the function machine that takes an input and doubles it to generate an output.



1. Use the function machine to complete the table. In the last column, use function notation to write the relationship between the input and output values.

Input, x	Process	Output, y	$f(x) = y$
0			
1			
3			
x			

2. You will be given a card that contains a function machine. On your card, complete the table and write the relationship between the input and output values using function notation.

3. Next, you will be assigned to a new group, where each group member has a different card. Use your table to respond to the following problems.

   a. For one of the four functions, when the input is 6, the output is -3. Which is that function: g, h, k or m? Explain your thinking.

   b. Which function, $g(x)$, $h(x)$, $k(x)$ or $m(x)$, has the greatest value when $x = 0$? When the input is 10.25?

Are you ready for more?

Mai claims that $g(x)$ is always greater than $h(x)$ for the same value of x. Is her claim true? Explain your thinking.

Name: ______ Date: ______ Period: ______

Activity 2 The Memphis Pyramid

The Memphis Pyramid in Tennessee is a 32-story square pyramid. It was built to honor the city's connection to the famous Pyramids of Giza in the ancient city of Memphis, Egypt.

Joseph Sohm/Shutterstock.com

1. The base of the square pyramid has a side length of 200 yd and an area of 40,000 yd^2. The relationship between the side length and the area of a square is a function.

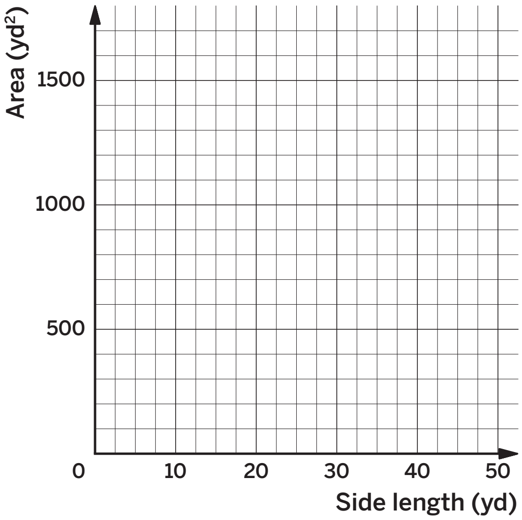
   a. Complete the table with the area for each given side length.

Side length (yd)	Area (yd^2)
10	
20	
30	
40	
s	

   b. Write a rule for the area function A, using function notation.

   c. What does $A(20)$ represent in this situation? What is its value?

   d. Sketch a graph of the area function A.



Activity 2 The Memphis Pyramid (continued)

2. The Memphis Pyramid is located in the historic downtown area which borders the Mississippi river and overlooks Mud Island River park. Suppose the city council plans to expand the park and wants to add a playground. The city budget and land restrictions require the width of the playground to be 20 yd.

a If a rectangular playground has a width of 20 yd and a length of 50 yd, what is the perimeter of the playground?

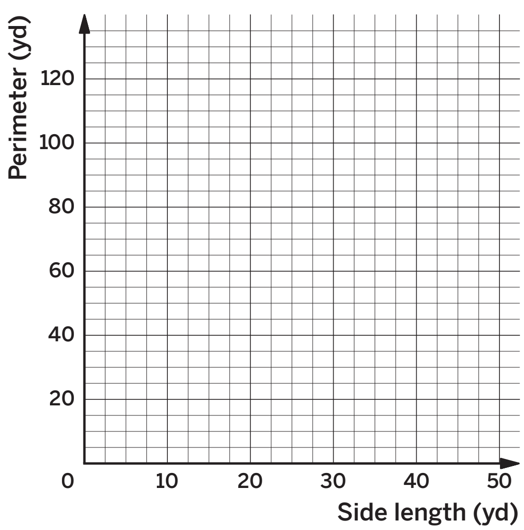
b Now you will vary the length of a playground that has a fixed width of 20 yd. Complete the following table by determining the perimeter for each given side length.

Side length (yd)	Perimeter (yd)
10	
20	
30	
45	
l	

c Write a rule for the perimeter function P using function notation.

d What does $P(25)$ represent in this situation? What is its value?

e Sketch a graph of the perimeter function P.



STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You described a function as a "machine" that uses a rule to compute an output value, given an input value. These function rules can be represented with verbal descriptions, tables, graphs, and equations. You also interpreted and wrote the equation of a function using function notation.

For example, consider the following function:

Verbal description:

The area of a square is a function of the length of its side.

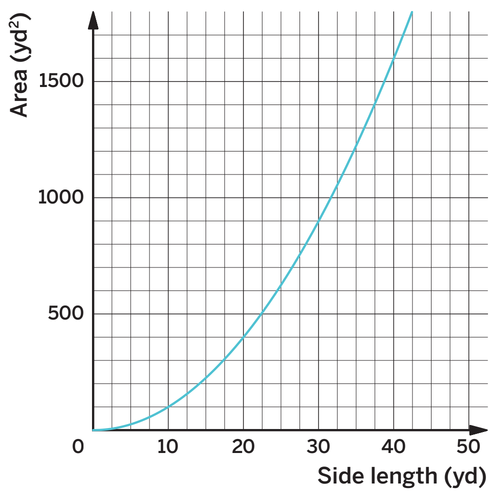
Equation:

$f(s) = s^2$

Table:

Side length (yd)	Area (yd^2)
10	100
20	400
30	900
40	1,600
s	s^2

Graph:



Reflect:

Practice

Name: .... Date: .... Period: ....

1. Match each statement with a description of the function it represents.

a $f(x) = 2x + 4$ .... To get the output value, add 4 to the input value, then multiply the result by 2.

b $g(x) = 2(x + 4)$ .... To get the output value, add 2 to the input value, then multiply the result by 4.

c $h(x) = 4x + 2$ .... To get the output value, multiply the input value by 2, then add 4 to the result.

d $k(x) = 4(x + 2)$ .... To get the output value, multiply the input value by 4, then add 2 to the result.

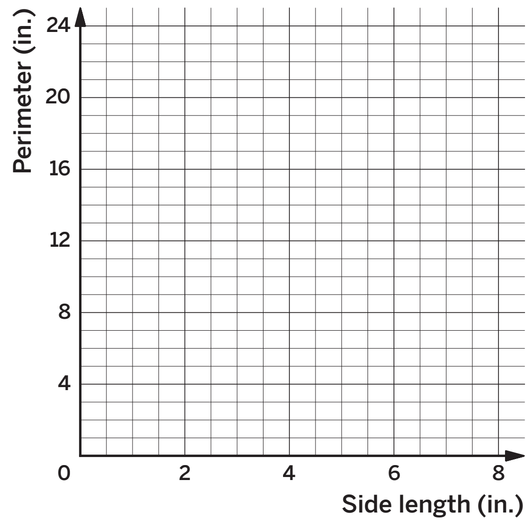
2. The function P represents the perimeter, in inches, of a square with side length x in.

a Complete the table.

x	0	1	2	3	4	5	6
$P(x)$							

b Write a function notation statement to represent the function P.

c Sketch a graph of function P.



3. Functions f and A are defined by these equations.

$f(x) = 80 - 15x$ $\quad$ $A(x) = 25 + 10x$

Which function has a greater output value when x is 2.5?

Name: ______________________ Date: ____________ Period: ____________

4. Tyler is using a garden hose to fill a child's pool. The pool has a capacity of 90 gallons. Think of two quantities that are related in this situation.

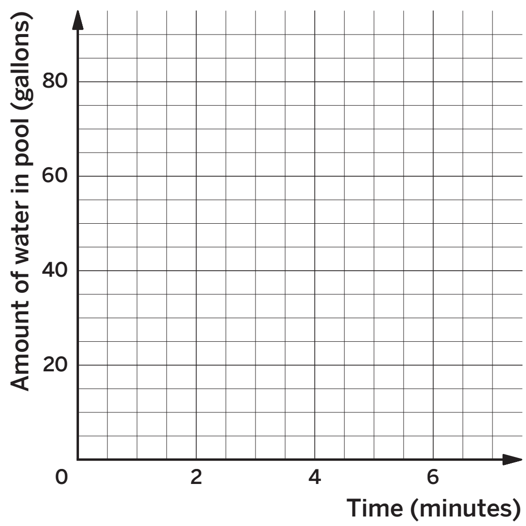
a Define the relationship between the quantities as a function and complete the sentence. Be sure to consider the units of measurement.

The ______________________________ is a function

of the ______________________________ .

b Sketch a possible graph of the function.

c Identify the coordinates of one point on the graph and explain its meaning.



5. Complete the table of input-output pairs that represent the area of a circle A with radius r. Then write an equation in function notation that represents the rule, using the variables A and r. Write your response in terms of π.

Radius, r	Area, A
2	
3	
5	
8	
10	

Unit 3 | Lesson 6

Using Function Notation to Describe Rules (Part 2)

Let's explore different ways to determine the input value of a function, given its output value, and vice versa.

Warm-up Make It True

Consider the equation $y = 4 + 0.8x$. Be prepared to explain your thinking.

1. Determine which value of y would make the equation true when:

   a. x is 7

   b. x is 100

2. Determine which value of x would make the equation true when:

   a. y is 12

   b. y is 60

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 A Steady Pace

To raise money for their track team, Andre and Elena sign up for The Great American River Run held in Memphis, Tennessee. One company sponsors Andre with a $40 pledge plus an additional $8.50 per mile. Another company promises to donate $125 to Elena no matter how far she runs. The amount of money raised by each student is a function of the number of miles they run and can be defined by the following equations.

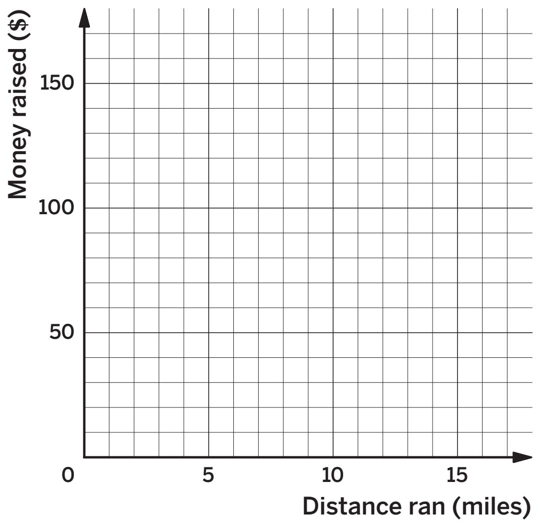
Andre: $A(x) = 8.5x + 40$ **Elena:** $E(x) = 125$

1. Andre and Elena want to compare the amount of money they can each raise by running different distances. Determine each value.

   a. $A(5)$ and $E(5)$

   b. $A(12)$ and $E(12)$

2. Graph each function on the coordinate plane.



3. Which student will raise more money? Explain your thinking.

4. For how many miles will both companies donate the same amount? Explain your thinking.

5. Which student do you think might be more motivated to run a greater distance? Explain your thinking.

Activity 2 Function Notation and Graphing Technology

In this activity, you will graph and view the function $A(x) = 8.5x + 40$ using graphing technology.

1. Enter the equation $y = 8.5x + 40$, which is given in slope-intercept form.

2. Adjust the axes scales to view the first quadrant of the graph. Record the scales you used:

   x min: y min:

   x max: y max:

3. Use the graphing technology tools to determine the value of each expression.

   a. $A(6)$ b. $A(9.6)$ c. $A(1.48)$

4. Use the graphing technology tools to determine what value of x makes each function notation statement true.

   a. $A(x) = 106.30$ b. $A(x) = 54.62$ c. $A(x) = 133.50$

Are you ready for more?

Use graphing technology to create a drawing of the outline of the Memphis Pyramid. Write equations of three lines that intersect to represent the sides of the triangle. Solve the system of equations and calculate the coordinates of each vertex point.

Marina Podre/Shutterstock.com

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You used rules of functions to determine the output when the input was given. You also solved equations to determine the input when the output was given.

In the context of a real-world situation, you worked with a variety of ways to represent functions:

- Verbal descriptions
- Tables
- Graphs
- Statements in function notation
- Equations, including those written in function notation

You specifically focused on ***linear functions***, which have a constant rate of change (or slope) and graphs that are lines. For example, $f(x) = 4x + 3$ defines a linear function. Any time x increases by 1, $f(x)$ increases by 4, so the slope of $f(x)$ is 4.

Reflect:

Practice

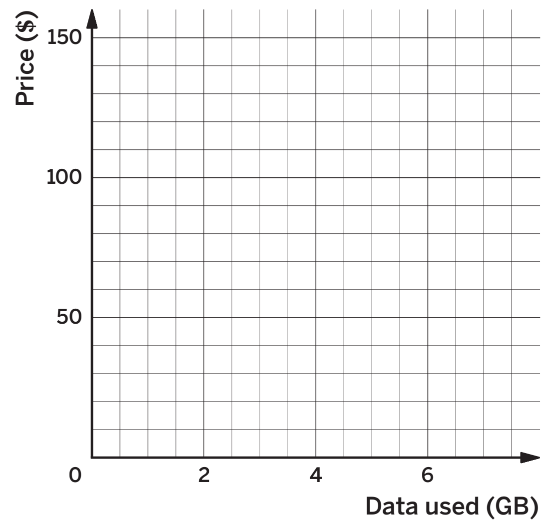
Name: ________________ Date: ________ Period: ________

1. Company C and D both offer cell phone plans as described in the table. The function representing each plan gives the monthly cost, in dollars, of using g gigabytes (GB) of data.

Company C:	Company D:
\$10 per month, plus \$15 per gigabyte used. $C(g) = 15g + 10$	\$80 per month, with unlimited data. $D(g) = 80$

a. Write a sentence describing the meaning of the statement $C(2) = 40$.

b. Draw and label the graph of each function.



c. Which is less, $C(4)$ or $D(4)$? What does this mean for the two phone plans?

d. Which is less, $C(5)$ or $D(5)$?

e. For what number g is $C(g) = 130$?

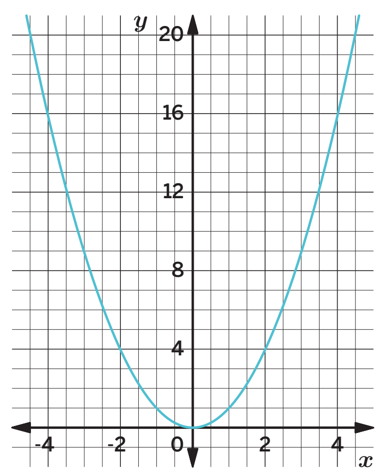
2. The function g is represented by the graph. For what input value or values is $g(x) = 4$?

A. 2

B. −2 and 2

C. 16

D. None



3. The function P gives the perimeter of an equilateral triangle of side length s. It is represented by the equation $P(s) = 3s$.

a. What does $P(s) = 60$ mean in this situation?

b. Determine a value of s that makes the equation $P(s) = 60$ true.

Practice

Name: .... Date: .... Period: ....

4. The function W gives the weight of a puppy, in pounds, as a function of its age t, in months. Describe the meaning of each statement.

   a. $W(2) = 5$

   b. $W(6) > W(4)$

   c. $W(12) = W(15)$

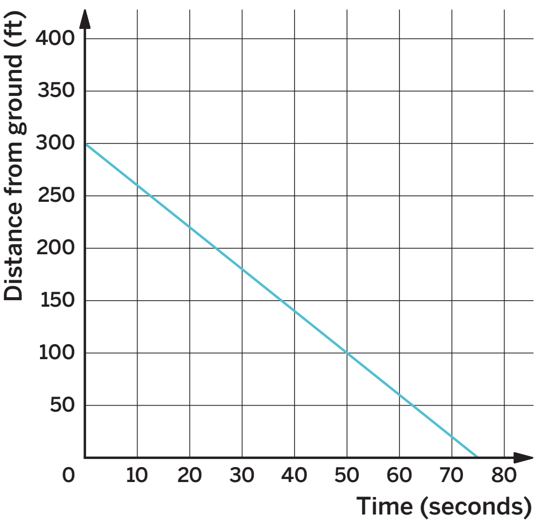
5. Diego is building a fence for a rectangular garden. It needs to be at least 10 ft wide and at least 8 ft long. The fencing costs \$3 per foot. His budget is \$120. He wrote the following to represent the constraints in this context.

$f = 2x + 2y$	$x \geq 10$	$y \geq 8$	$3f \leq 120$

   a. Explain what each equation or inequality represents in this context.

   b. Diego's mom says he should also include the inequality $f > 0$. Do you agree? Explain your thinking.

6. An elevator descends from the top floor of a building at a speed of 4 ft per second. The elevator's distance from the ground in feet is a function of the time in seconds and can be represented with the graph.



   a. Write an equation for function H, which gives the distance from the ground, in feet, t seconds after the elevator begins its descent.

   b. Use the graph to determine the value of the expression $H(0)$ and explain its meaning.

   c. Use the graph to estimate the solution to the equation $H(t) = 0$ and explain what the solution represents in context.

My Notes:

SUB-UNIT

2 Analyzing and Creating Graphs of Functions

Narrative Connections

What's the function of a jazz solo?

Jazz has its roots in the history and culture of New Orleans, Louisiana. Its distinctive sound draws from numerous threads of New Orleans culture.

In the 18th century, enslaved Africans held dance and music gatherings in what is now known as Congo Square. West African and Caribbean drumming and call-and-response chanting were combined with the instruments of European colonizers: guitars, trumpets, and pianos.

It is from this blend of cultures that jazz was born. One of jazz music's most defining characteristics is its improvisation. In a band, each musician might take up the main theme of a song and riff on it, weaving in and out of each other's lines. A soloist such as the New Orleans-born trumpeter Louis Armstrong can use a song's chords to invent a brand new melody on the spot.

In each case, the musician takes an input — a chord progression, a theme, or a melody — then applies knowledge, creativity, and skill to transform it into something new. And just as a solo can be transcribed into sheet music, a function can also be graphed in order to be analyzed and understood.

Unit 3 | Lesson 7

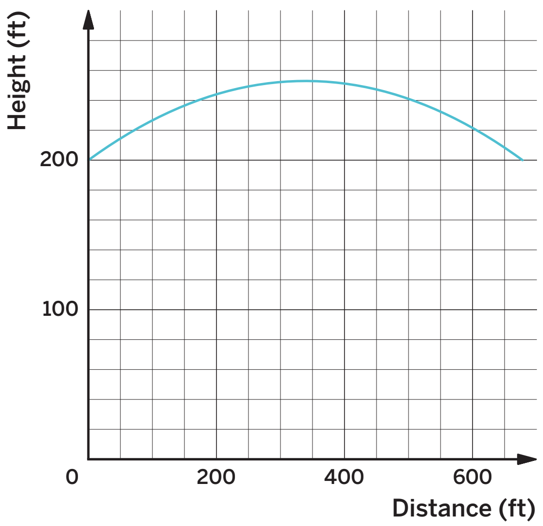
Features of Graphs

Let's determine important features of graphs of functions.

Warm-up Superdome

Built in 1975, the New Orleans' Superdome is the largest fixed dome structure in the world.

The graph shown is a representation of the Superdome's roof. The function h, gives the height of the roof, in terms of the distance d along the base.



Use the graph to determine or estimate each of the following.

1. $h(0)$

2. $h(340)$

3. The value of d that makes the equation $h(d) = 220$ true.

4. The value of d that makes the equation $h(d) = 0$ true.

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 We Have Liftoff!

A toy rocket and a drone are launched at the same time. You will be given a graph that represents either the toy rocket or the drone. Do not show your graph to your partner.

1. Analyze the graph. Use precise language to describe what is happening to the object.

2. Which parts or features of the graph show important information about the object's movement? List the features or mark them on your graph.

3. Take turns with your partner describing your graph by giving important features. While the graph is being described to you by your partner, use a separate sheet of paper to sketch a graph that represents your partner's description.

Having a common language to refer to important features of graphs is critical. The table lists some important features of graphs you will see in this unit.

Important features of graphs:

- The horizontal intercept
- The vertical intercept
- The maximum value
- The minimum value
- Where the graph is increasing
- Where the graph is decreasing

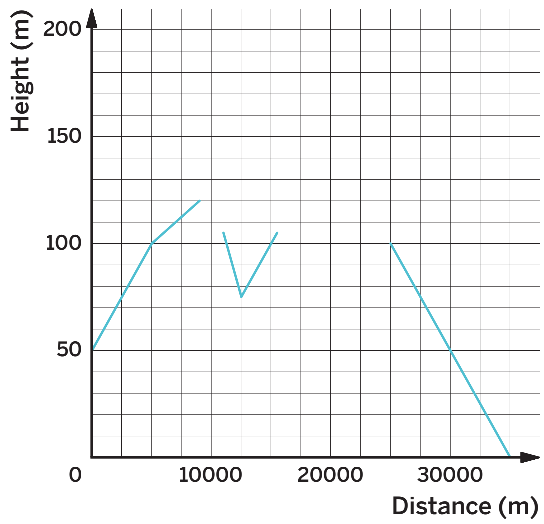
Activity 2 Mountain Range

Plan ahead: What will you do if you disagree with a partner about how the graph should be completed?

The United States Geological Survey (USGS) is working to map part of an incomplete mapping data about a mountain range. They send Diego and Priya to gather data on the positions of different peaks.

- **Diego reached a mountain peak that was 125 m high.**
- **Priya reached a mountain peak after hiking a distance of 20,000 m.**
- **After hiking a distance of 12,500 m, Priya hiked uphill at a constant increase in height until she reached a mountain peak at a distance of 20,000 m.**
- **One of these mountain peaks was the highest of the entire mountain range.**

This sketch describes the incomplete mountain range the USGS had prior to sending out Diego and Priya.



1. Using Diego's and Priya's information, who hiked to the highest mountain peak? Explain your thinking.

2. Approximate the height of the highest peak.

3. Both Diego and Priya claimed to have reached a maximum height. Can they both be correct? Explain your thinking.

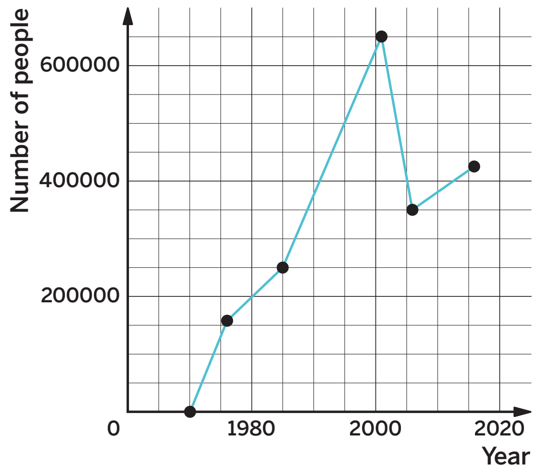
When the value of a function is greater or less than *all* other values of the function, this is called a ***global maximum*** or ***global minimum***, respectively.

When the value of a function is greater or less than the nearby or surrounding values, this is called a ***local maximum*** or ***local minimum***, respectively.

Name: ______ Date: ______ Period: ______

Activity 3 Jazz and Heritage Festival

Starting in 1970, The New Orleans Jazz and Heritage Festival has become one of the largest music festivals. Many musicians have performed at the festival, including New Orleans' own Neville Brothers. The graph shows how the Jazz and Heritage Festival attendance has changed over time. Note: In 1970, 350 people attended the festival. The function P gives the number of people in attendance as a function of time t in years.



1. For each description of the attendance, record its corresponding equation and feature on the graph in the table. Use the equations, interval, and features shown.

Equations or interval	Graph features
$P(t) = 350000$	Global maximum
$P(t) = 0$	Global minimum
$P(t) = 350$	Horizontal intercept
$P(t) = 650000$	Local minimum
$t = 2001$ to $t = 2006$	Decreasing

Description	Equation or interval	Graph feature
The greatest attendance.		
The least attendance.		
The years when attendance fell.		
The least attendance between 2001 and 2016.		

2. One equation and graph feature do not have a matching description. Record the equation and graph feature here and write a corresponding description.

Description	Equation or interval	Graph feature

STOP

Summary

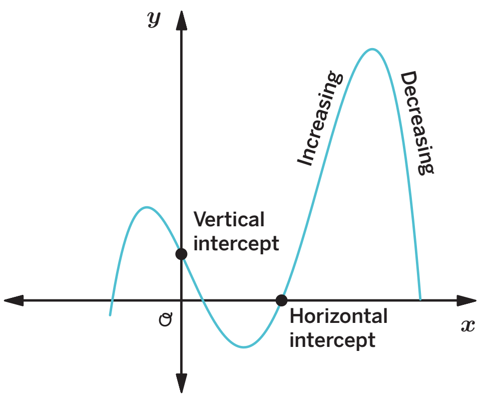
In today's lesson . . .

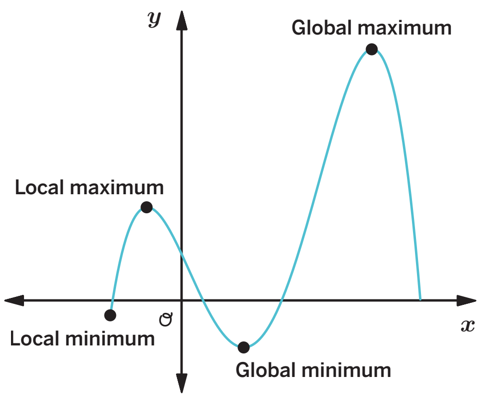
You observed that the graph of a function can reveal important information about the quantities in a real-world context. You identified how graphs change over different intervals and the locations of certain features.

Some important features of graphs that you can now identify are shown in the table and highlighted on the following graphs.

Important features of graphs:

- The *horizontal intercept*
- The *vertical intercept*
- Where the graph is *increasing*
- Where the graph is *decreasing*
- The ***local maximum***
- The ***local minimum***
- The ***global maximum***
- The ***global minimum***





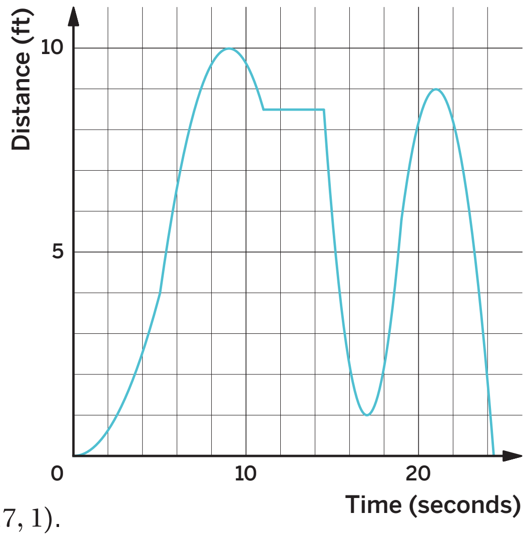
Not all functions or graphs will contain each of these features, so it is important to pay attention to both the real-world context and how the context is represented in the graph.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. The graph represents the distance from a fountain while Bard walks in the park. Determine whether the following statements are true or false.



a. The graph has multiple horizontal intercepts.

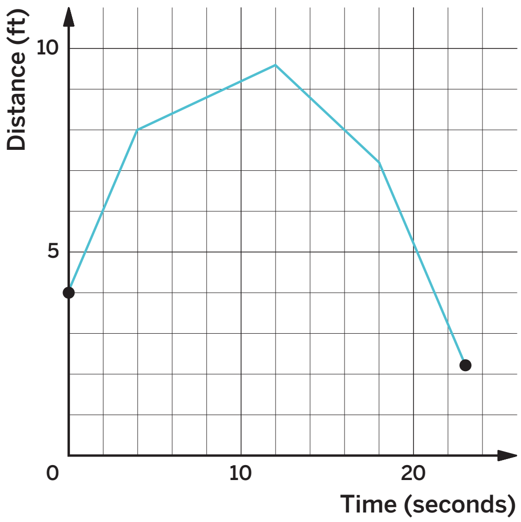
b. A horizontal intercept of the graph represents the time when Bard was right next to the fountain.

c. The global minimum of the graph is located at $(17, 1)$.

d. The graph has two global maximums.

e. About 9 seconds after Bard left the fountain at the beginning of the walk, Bard was the farthest away from it, about 10 ft.

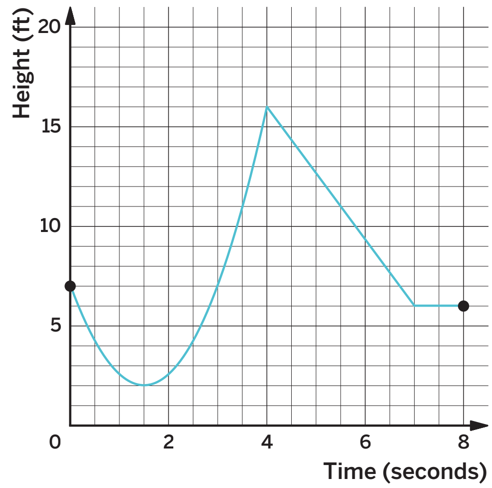
2. The graph represents the distance Lin is away from her bedroom as a function of time. Describe Lin's distance from her bedroom over time. Identify key features of the graph.



Practice

Name: .... Date: .... Period: ....

3. Consider the graph of the function $h(t)$. Match each feature of the graph with a corresponding statement in function notation.



a Maximum height .... $h(0) = 7$

b Minimum height .... $h(1.5)$

c Height staying the same .... $h(4)$

d Starting height .... $h(t) = 6$ between $t = 7$ and $t = 8$

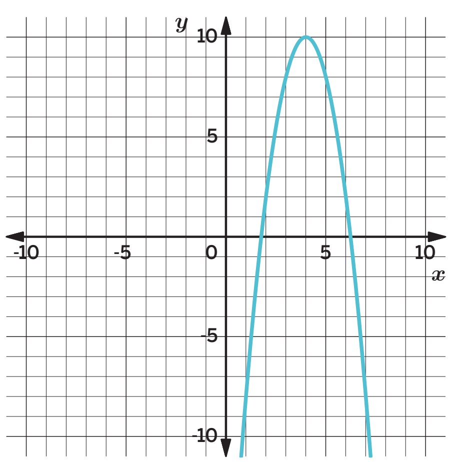
4. Consider the function $f(x) = x^2$.

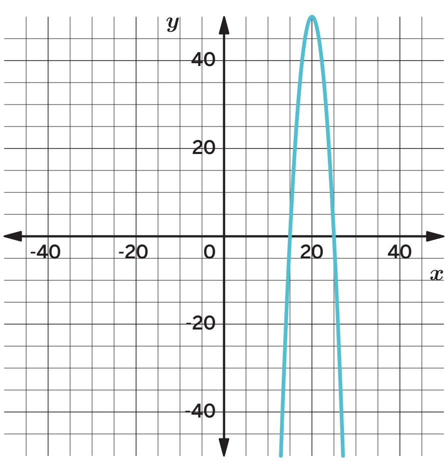
a What is $f(2)$?

b What is $f(3)$?

c Explain why $f(2) + f(3) \neq f(5)$.

5. What are the similarities and differences between the two graphs shown?

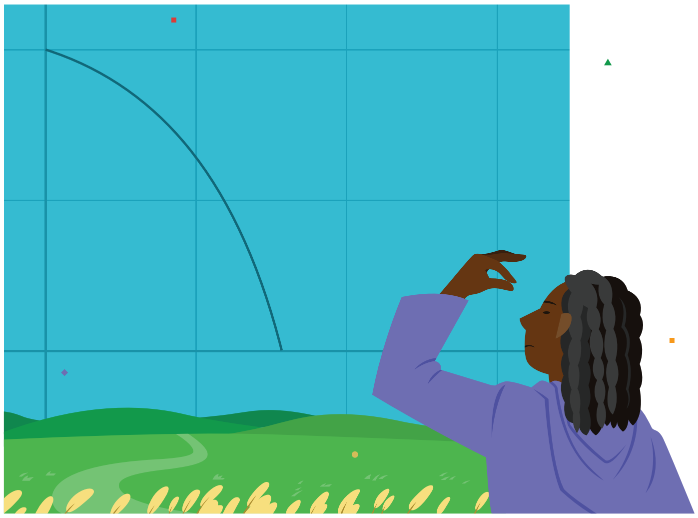




Name: ______ Date: ______ Period: ______

Unit 3 | Lesson 8

Understanding Scale

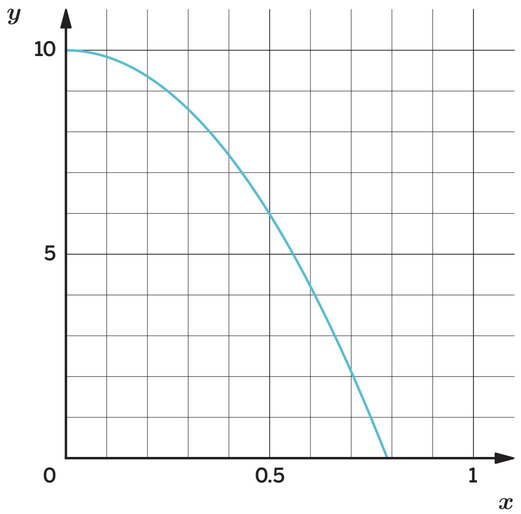


Let's interpret the scale of a graph and determine whether the points on a graph should be connected.

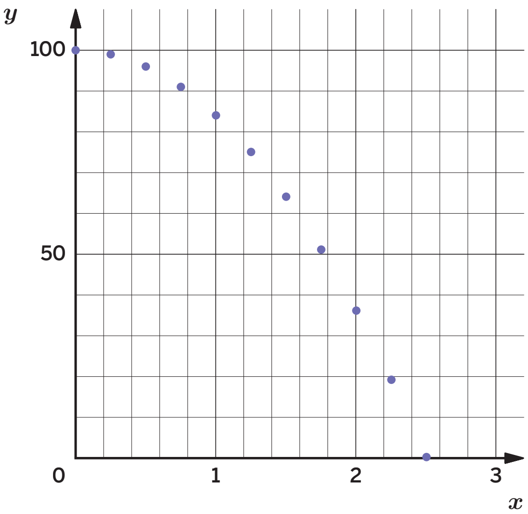
Warm-up Notice and Wonder

Compare the two graphs. What do you notice? What do you wonder?

Graph A



Graph B



1. I notice . . .

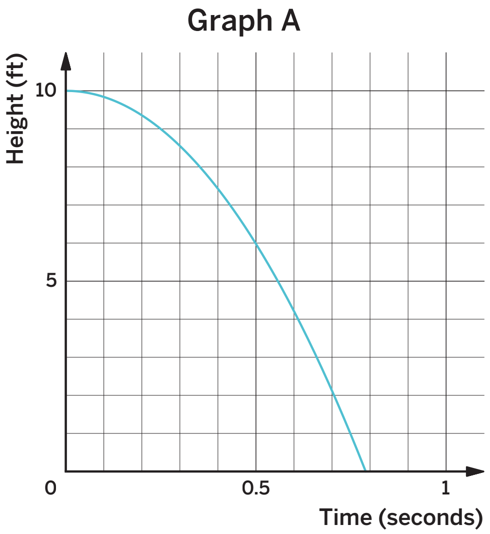
2. I wonder . . .

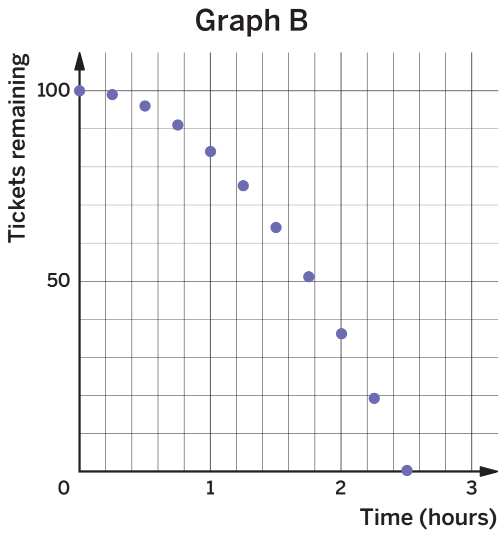
Log in to Amplify Math to complete this lesson online.

Activity 1 Interpreting Scale

When interpreting or creating a graph to represent the function, it is important to consider the scale of the axes. Each axis can be scaled by assigning each tick mark to represent different increments of values. In doing so, important features can be represented on a graph of a function.

The two graphs from the Warm-up are shown, now the axes labeled as to the contexts they represent.





1. What does the scale for the vertical and horizontal axes for Graph A show?

2. What does the scale for the vertical and horizontal axes for Graph B show?

3. Determine and interpret the vertical and horizontal intercepts for:

   a. Graph A

   b. Graph B

4. Why does Graph A show a connected curve but Graph B show disconnected points?

Name: ______________________ Date: ____________ Period: __________

Activity 2 Card Sort: Discrete or Not?

Sometimes, depending on the context of a scenario, only certain values make sense. For example, to measure the number of students in a class, only whole number values make sense. (It is not possible to have 3.14 students!) These types of data sets are *discrete*, because they can only include specific, individual values.

You will be given two sets of cards: one with verbal descriptions of scenarios and one with graphs. Match the cards from each set and determine whether each pair represents a *discrete* or *not discrete* scenario. Record your matches and responses in the table.

Description card	Graph card	Discrete or not discrete?

1. For the scenarios you identified as discrete, explain how you identified these cards.

2. For the scenarios you identified as not discrete, explain how you identified these cards.

3. Describe two scenarios, one that is discrete, and one that is not discrete. Explain your thinking.

   a. Discrete

   b. Not discrete

STOP

Summary

In today's lesson . . .

You discovered that paying attention to the scale of graphs is important when interpreting features of graphs. Changing the scale of the vertical or horizontal axis can be useful for revealing these important features that otherwise might not be noticeable.

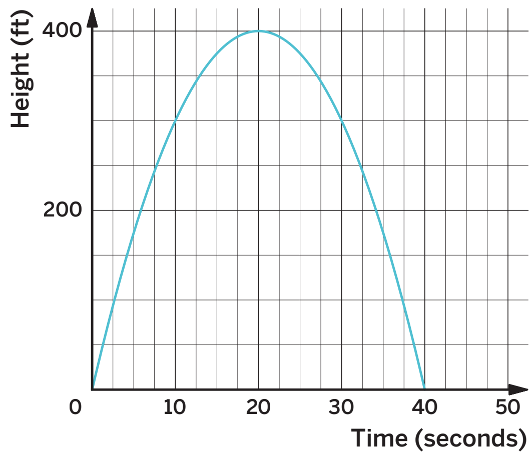
You also saw that sometimes only specific values make sense in different scenarios. These scenarios are ***discrete***. Discrete scenarios have graphs with distinct, disconnected points. If a situation is not discrete, and any value makes sense in context, the graphs are typically made up of connected curves and/or lines.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. The graph describes the launch of a toy rocket into the air. Which of the following statements are true about the graph? Select *all* that apply.



A. The rocket reaches a maximum height of 400 ft.

B. Each tick mark on the horizontal axis represents 1 second.

C. Each tick mark on the vertical axis represents 50 ft.

D. The rocket is in the air for 40 seconds.

E. The rocket reaches its maximum height at 40 seconds.

2. Which of the following scenarios would be best described as discrete? Select *all* that apply.

A. The number of albums sold every year by a musician.

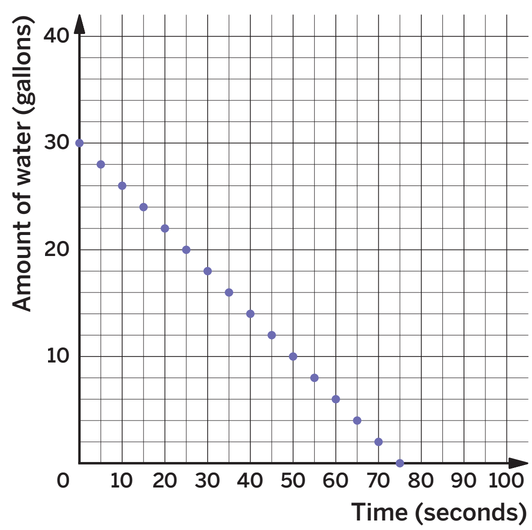
B. The total number of people at every game throughout a year.

C. The distance a person travels from home throughout the day.

D. The height of a toy rocket over time, after it is launched.

E. The number of users on a social media platform every hour.

3. Han fills his bathtub up with water until the tub contains 30 gallons. He then drains the tub and measures the amount of water left in the tub every 5 minutes. He creates the following graph using his data. Explain Han's mistake in creating his graph.



Practice

Name: .... Date: .... Period: ....

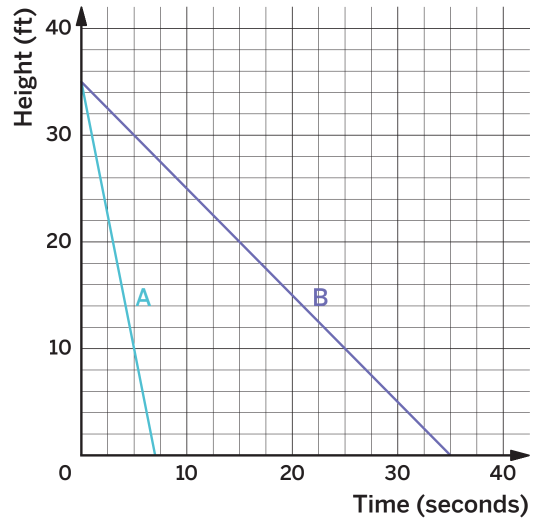
4. Consider the function $f(x) = -3(x - 5)$.

a Determine $f(-5)$.

b Determine the value of x such that the equation $f(x) = 90$ is true.

5. The graph shows the time it took Bard to go down two different slides, A and B.

a Without doing any calculations, was Bard's speed faster on Slide A or Slide B. Explain your thinking.



b Determine Bard's velocity on each slide to verify your response to part a. Show or explain your thinking.

Name: ______________ Date: ________ Period: ________

Unit 3 | Lesson 9

How Do Graphs Change?

Let's determine how to calculate and interpret the average rate of change.

Warm-up Temperature Drop

The table shows the recorded temperatures at three different times during one evening.

Time	4 p.m.	6 p.m.	10 p.m.
Temperature (°F)	25	17	8

Tyler and Mai make the following observations:

- **Tyler says the temperature dropped faster between 4 p.m. and 6 p.m.**
- **Mai says the temperature dropped faster between 6 p.m. and 10 p.m.**

Do you agree with Tyler or Mai? Explain your thinking.

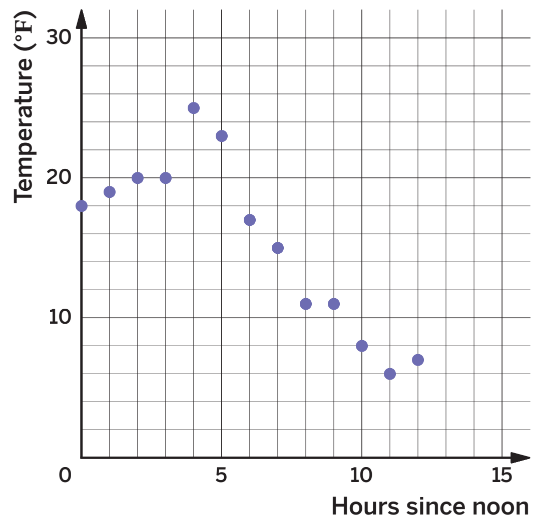
Log in to Amplify Math to complete this lesson online.

Activity 1 Temperature Change

You previously learned how the rate of change, or slope, of a linear function describes the ratio of the change in output to the change in input. Similarly, the *average rate of change* is the ratio of the change in output to the change in input for a particular interval.

The table and graph show how the temperature changes over the course of the day from the same scenario as the Warm-up. The function T represents the temperature in degrees Fahrenheit given the number of hours since noon h.

h	0	1	2	3	4	5	6	7	8	9	10	11	12
$T(h)$	18	19	20	20	25	23	17	15	11	11	8	6	7



1. Determine the average rate of change for each interval. Explain your thinking.

a Between noon and 1 p.m.

b Between noon and 4 p.m.

c Between noon and midnight.

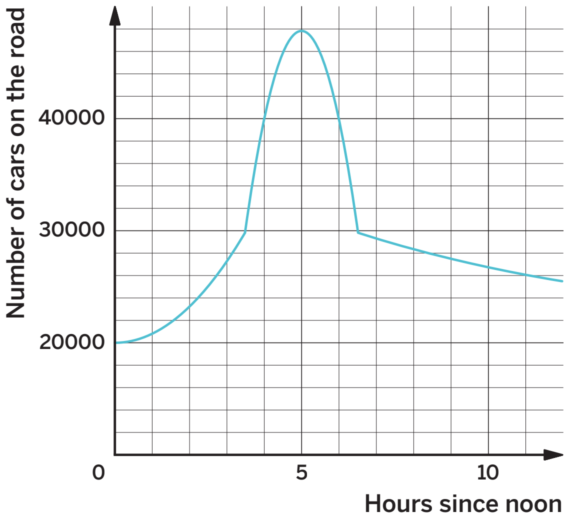
2. Refer back to the statements that Tyler and Mai made in the Warm-up. Now that you have more data, determine whether either or both of their statements were correct.

Name: ______ Date: ______ Period: ______

Activity 2 Traffic in New Orleans

New Orleans, Louisiana, is the largest city in Louisiana, and is known for its music, cuisine, culture, and festivals. Like any large city, people use many different forms of transportation to go about their daily lives, such as cars, buses, bicycles, ferries, and even streetcars (or trolleys).

Consider the graph of the function f, which shows how the traffic in New Orleans changes over the course of a typical day, starting at noon.



1. For each of the following intervals, determine if the average rate of change in the number of cars is positive or negative. Explain your thinking.
   a. Between noon and 5 p.m.
   b. Between 5 p.m. and 11 p.m.

2. Use the graph to estimate each value and interpret what it means in this context.
   a. $f(0)$
   b. $f(5)$
   c. $f(11)$

3. Estimate the average rate of change in the number of cars on the road between noon and 5 p.m.

4. Estimate the average rate of change in the number of cars on the road between 5 p.m. and 11 p.m.

5. What does each average rate of change mean in this context?

STOP

Summary

In today's lesson . . .

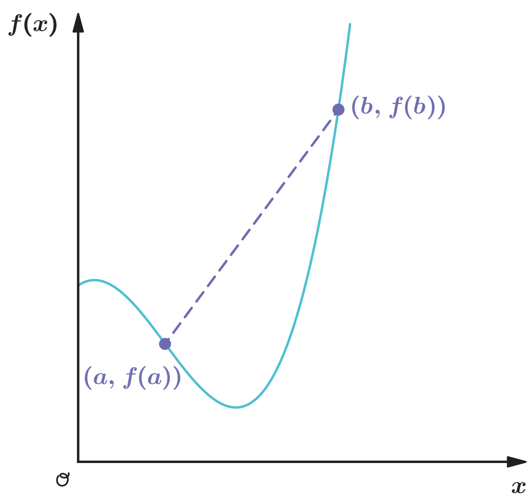
You observed that functions can have different rates of change over different intervals. In order to compare the rates of change over two different intervals, you calculated the ***average rate of change***. The average rate of change is similar to the slope, and is used to find how any function, not just linear functions, change over a given interval.

The average rate of change of a function $f(x)$ on an x-interval from a to b is given by the formula shown here.

Average rate of change:

$$\frac{f(b) - f(a)}{b - a}$$

In other words, it represents the slope of the line joining the points $(a, f(a))$ and $(b, f(b))$.

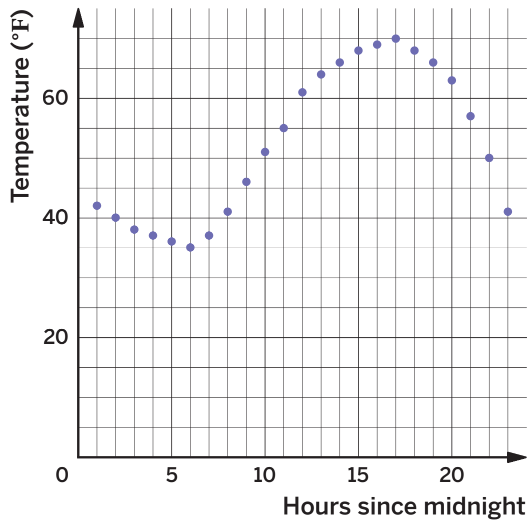


Reflect:

Name: .... Date: .... Period: ....

Practice

1. The temperature was recorded at several times during the day. The function T represents the temperature in degrees Fahrenheit, given the number of hours since midnight n. Use the graph to determine if the average rate of change for each interval is *positive*, *negative*, or *zero*.



a. $n = 1$ to $n = 5$

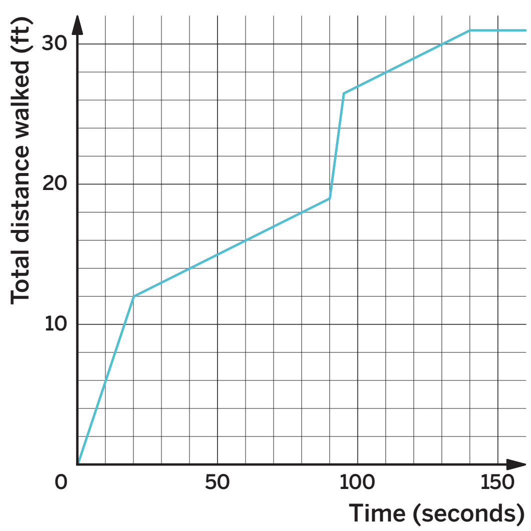
b. $n = 5$ to $n = 8$

c. $n = 10$ to $n = 20$

d. $n = 15$ to $n = 18$

e. $n = 20$ to $n = 23$

2. The graph shows the total distance, in feet, walked by Priya as a function of time, in seconds.



a. Was Priya walking faster between 20 and 40 seconds or between 80 and 100 seconds? Explain your thinking.

b. Was Priya walking faster between 0 and 40 seconds or between 40 and 100 seconds? Explain your thinking.

Practice

Name: .... Date: .... Period: ....

3. The percentage of voters between the ages of 18 and 29 that participated in each United States presidential election between the years 1988 and 2016 are shown in the table. The function P gives the percentage of voters between 18 and 29 years old that participated in the election during year t.

Year	1988	1992	1996	2000	2004	2008	2012	2016
Percentage of voters between ages 18–29	35.7	42.7	33.1	34.5	45.0	48.4	40.9	43.4

a. Determine the average rate of change for P between 1992 and 2000.

b. Select two different values of t so that the function has a negative average rate of change between the two values. Determine the average rate of change.

c. Select two values of t so that the function has a positive average rate of change between the two values. Determine the average rate of change.

4. Jada walks to school. The function D gives her distance from the school, in meters, t minutes since she left home. Which equation represents the statement, "Jada is located 600 m from the school after 5 minutes"?

A. $D(5) = 600$ **B.** $D(600) = 5$ **C.** $t(5) = 600$ **D.** $t(600) = 5$

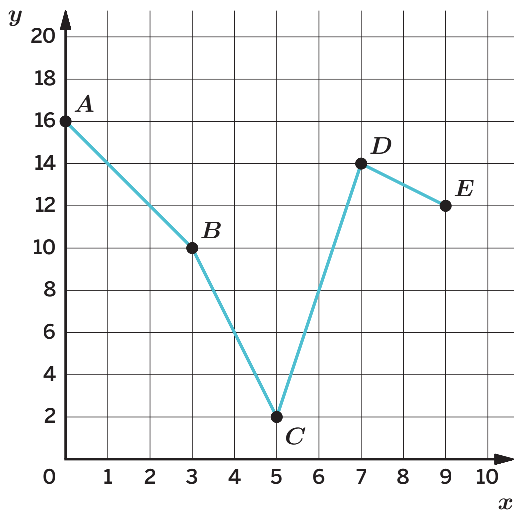
5. For each segment, identify whether the graph is increasing or decreasing.

a. Segment AB

b. Segment BC

c. Segment CD

d. Segment DE



Name: ________ Date: ________ Period: ________

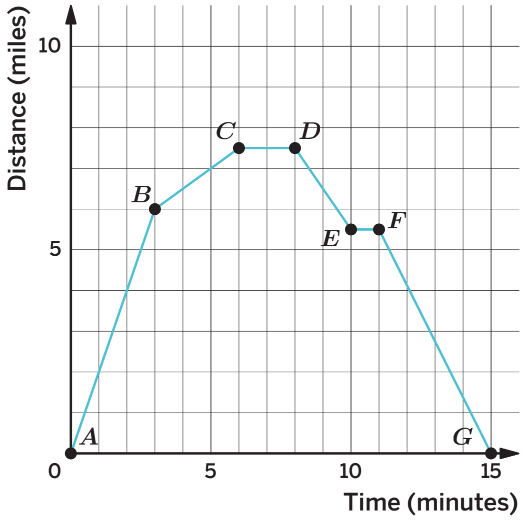
Unit 3 | Lesson 10

Where Are Functions Changing?

Let's determine reasonable input and output values of a function.

Warm-up Author Your Own Story

Consider the following graph representing a scenario.



1. Determine each of the following.

   a. Segments where the graph is increasing.

   b. Segments where the graph is decreasing.

   c. Segments where the graph is constant.

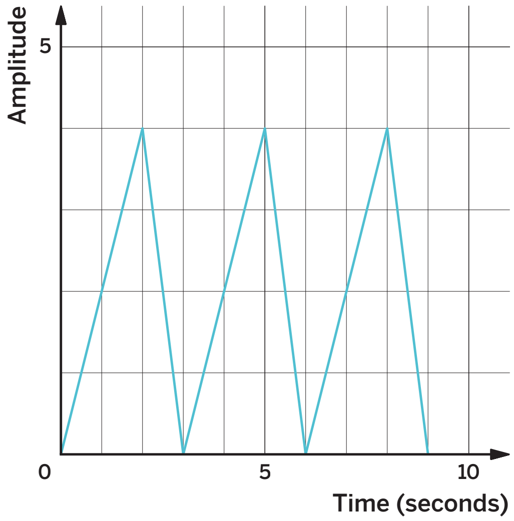
2. What scenario could this graph represent? Write a short description that matches the graph.

Log in to Amplify Math to complete this lesson online.

Activity 1 Reasonable Inputs

Not all input values make sense for every graph or scenario. You will explore this further in this activity.

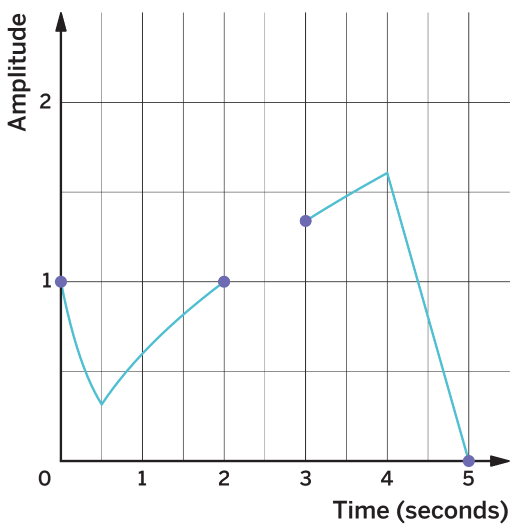
1. A synthesizer is used to digitally create some music and sound effects. The graph represents the sound waves that are created.



a What are some reasonable input values for the function represented by this graph? Explain your thinking.

b What are some unreasonable input values for the function represented by this graph? Explain your thinking.

2. Bard mixes a sound and creates the graph shown to represent the amplitude of the sound sampled over time.



a What are some reasonable input values for the function represented by this graph? Explain your thinking.

b What are some unreasonable input values for the function represented by this graph? Explain your thinking.

c What might be some reasons for the gap in the graph?

Name: ________________ Date: ________ Period: ________

Activity 2 Card Sort: Do the Input Values Make Sense?

You will receive a set of cards with different numbers on them. Decide whether each number is a reasonable input value for the functions described. Sort the cards into two categories of input values: reasonable and not reasonable.

1. Tyler records himself singing and uses computer software to lower the pitch of his voice. The frequency of the pitch is a function of time, in seconds, given by the function $F(t) = 200 - 10t$.

   a. Record the card numbers in each group.

Reasonable input value	Not a reasonable input value

   b. If there are input values that do not make sense, what makes them unreasonable? Explain your thinking.

2. Mai hosts a summer music camp and charges $\$40$ per camper each day. She needs at least 5 campers to sign up in order to open each day. Camp enrollment is limited to 16 campers per day. The amount of revenue, in dollars, the camp collects is a function of the number of campers enrolled. The function is defined by $R(n) = 40n$, where n is the number of campers enrolled.

   a. Record the card numbers in each group.

Reasonable input value	Not a reasonable input value

   b. If there are input values that do not make sense, what makes them unreasonable? Explain your thinking.

Activity 2 Card Sort: Do the Input Values Make Sense? (continued)

3. The area of a square, in square centimeters, is a function of its side length s, in centimeters, given by the function $A(s) = s^2$.

   a. Record the card numbers in each group.

Reasonable input value	Not a reasonable input value

   b. If there are input values that do not make sense, what makes them unreasonable? Explain your thinking.

4. Money is invested in a bank account and increases in value over time. The function P describes how much money accumulates in the bank account over time, where $P(t) = 1500(1.01075)^t$ and t represents the number of years.

   a. Record the card numbers in each group.

Reasonable input value	Not a reasonable input value

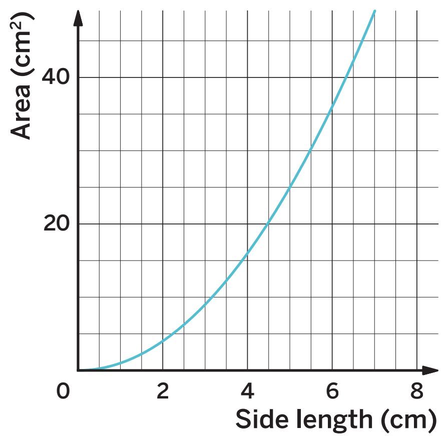
   b. If there are input values that do not make sense, what makes them unreasonable? Explain your thinking.

Name: ________ Date: ________ Period: ________

Activity 3 What About the Output Values?

In Activity 2, you determined reasonable input values for four different functions. What about the output values of those functions? Are they reasonable?

1. Consider the graph of the function A, where $A(s) = s^2$, which represents the area of a square as a function of its side lengths.



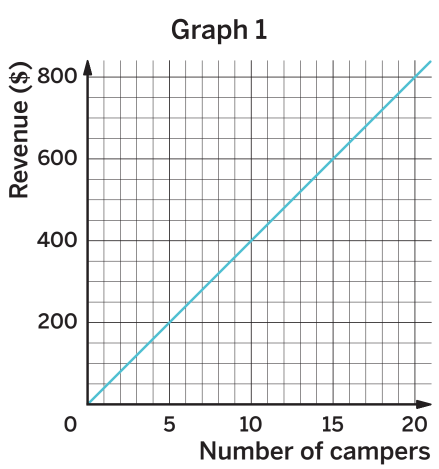
a Write three equations in function notation that represent reasonable side lengths and corresponding areas of several squares.

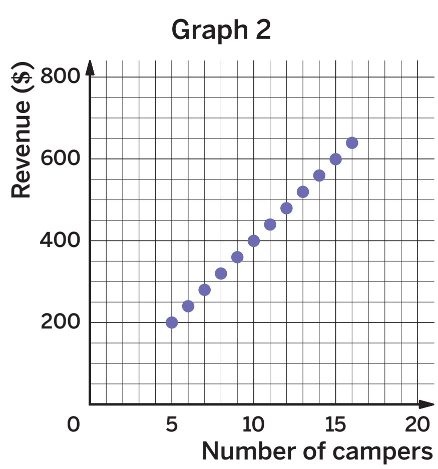
b How would you describe all reasonable output values of A?

2. The function $R(n) = 40n$ gives the revenue generated by a music camp depending on the number of campers enrolled. The camp needs at least 5 campers in order to open each day. Camp enrollment is limited to 16 campers per day.

a Is 100 a reasonable revenue amount? Explain your thinking.

b Which of these two graphs best represents the function R? Explain your thinking.





STOP

Summary

In today's lesson . . .

You were given graphs with varying levels of description, and you determined what could be some reasonable explanations for changes in the context of a scenario.

You also saw that depending on the function and context, some input and output values make sense and others do not make sense. Often, negative values do not make sense when measuring the length of an object or referring to time. Fractional values often do not make sense when referring to quantities that can only be measured in whole number values, such as the number of people.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. The cost for an upcoming field trip is \$30 per student. The function $C(s) = 30s$ gives the cost in dollars given the number of students s. Select *all* reasonable output values for this function.

   A. 20

   B. 30

   C. 50

   D. 90

   E. 100

2. A rectangle has an area of 24 cm^2. A function f gives the length of the rectangle, in centimeters, when the width is w cm. Determine whether each value, in centimeters, is a reasonable input of the function. For the values that are not reasonable inputs, explain your thinking.

   a. 3 b. 0.5 c. 48 d. -6 e. 0

3. Diego is recording a song and uses computer software to adjust the pitch of his voice. The frequency of the pitch is a function of time, in seconds, given by the function $F(t) = 180 + 12t$.

   a. What are the reasonable input values of this function?

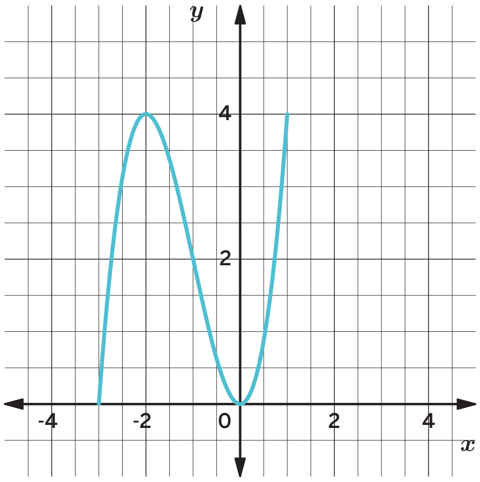
   b. What are the reasonable output values of this function?

4. The graph of the function f passes through the points $(0, 3)$ and $(4, 6)$. Use function notation to write the information about function f that each point gives.

Practice

Name: ____________ Date: ____________ Period: ____________

5. Use the graph to determine if the average rate of change is *positive*, *negative*, or *zero* for each of the given intervals.



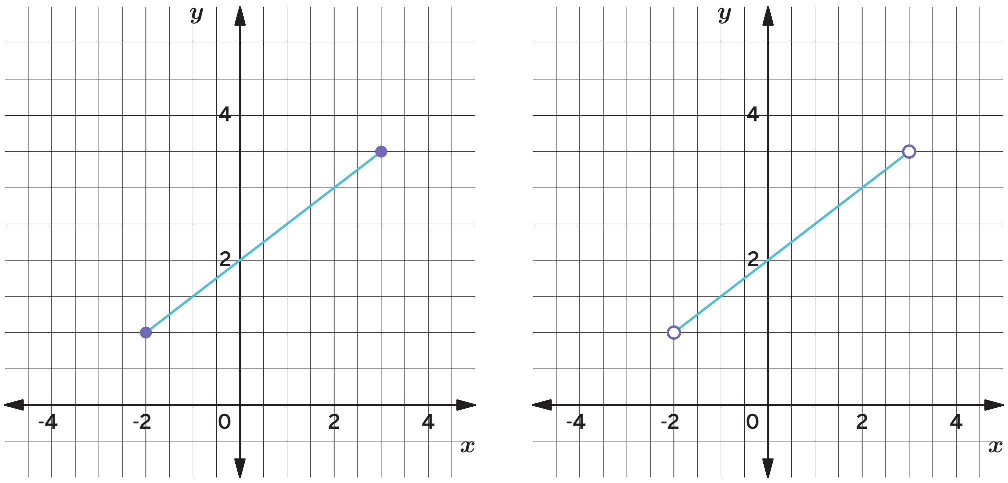
a $x = -3$ to $x = -2$

b $x = -2$ to $x = 0$

c $x = 0$ to $x = 1$

d $x = -3$ to $x = 0$

6. How are these two graphs different? What do you think the difference between the graphs represents?



Name: ______ Date: ______ Period: ______

Unit 3 | Lesson 11

Domain and Range

Let's represent the input and output values of a function using interval notation.

Warm-up True or False

The solutions to an inequality are represented on the number line. Determine whether each statement is true or false. Explain your thinking.

Statement	True or False
1. The solutions only include values greater than -2.	
2. The solutions can be represented by the inequality $x \geq -2$.	
3. The value 50 is a solution to the inequality.	
4. The solutions represented on the number line continue forever.	

Log in to Amplify Math to complete this lesson online.

Activity 1 Two Truths and a Lie: Using Inequalities

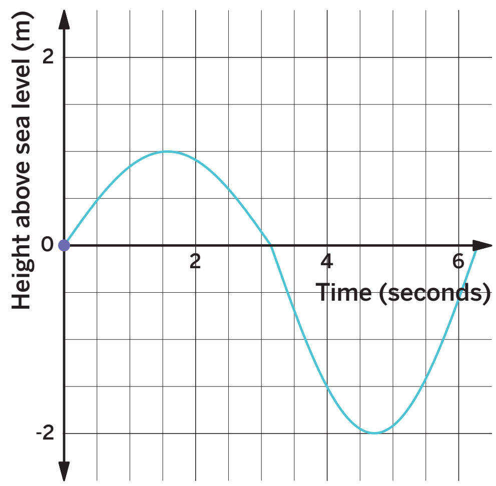
You have seen how different functions can have different possibilities for the input and output values depending on the context of the scenario. Mathematicians refer to the input and output values of a function as the function's *domain* and *range*, respectively. You can use inequalities or lists to represent the domain and range in an efficient way.

1. A dolphin jumps out of the water, into the air, and dives back under water before returning to the surface. The graph shows the path the dolphin takes. Which of the following statements is *false*? Explain your thinking.

A. The domain is approximately $0 \leq x \leq 6.2$.

B. The range is $\{-2, 0, 1\}$.

C. The range is $-2 \leq y \leq 1$.

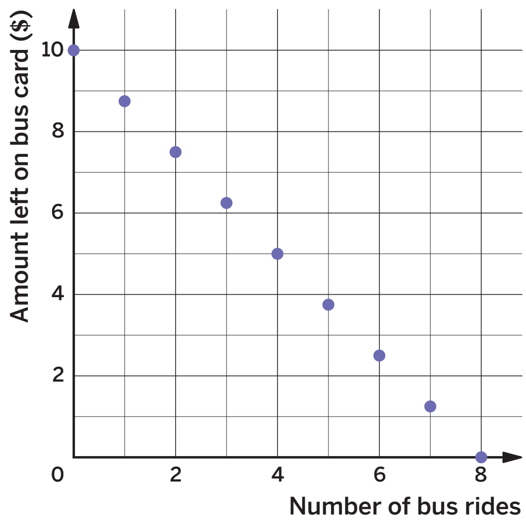


2. Jada has a prepaid bus card to use while she is in New Orleans. The bus card has $10 on it, and bus fare costs $1.25 for a single ride. The graph represents the amount of money left on Jada's bus card. Which of the following statements is *false*? Explain your thinking.

A. The domain is $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$.

B. The range is $\{0, 1.25, 2.5, 3.75, 5, 6.25, 7.5, 8.75, 10\}$.

C. It is possible for Jada to have $4 left on her bus card.

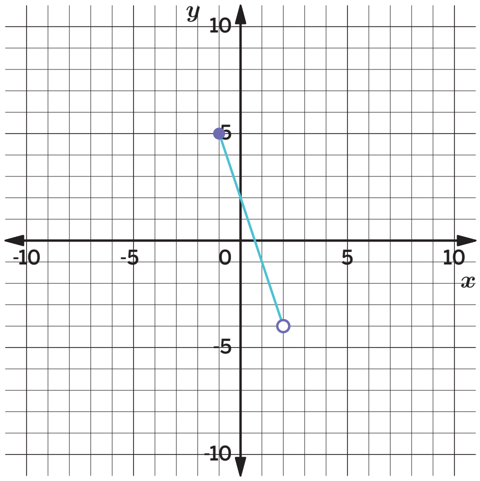


Name: .... Date: .... Period: ....

Activity 2 To Infinity, and Beyond!

Describe the domain and range for these two graphs

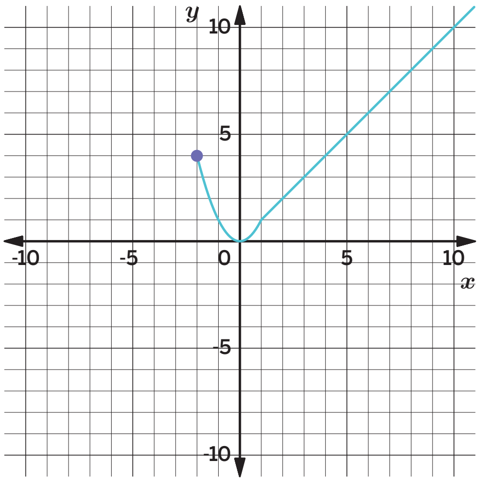
1.



Domain:

Range:

2.



Domain:

Range:

3. What are some differences between the domains of each graph? Between the ranges of each graph?

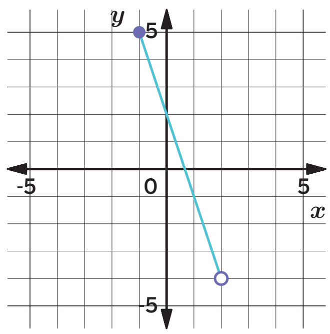
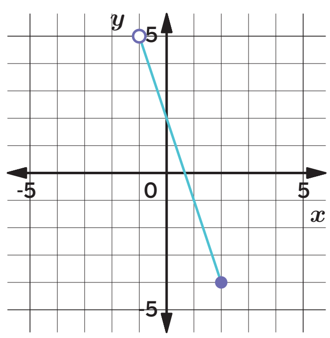
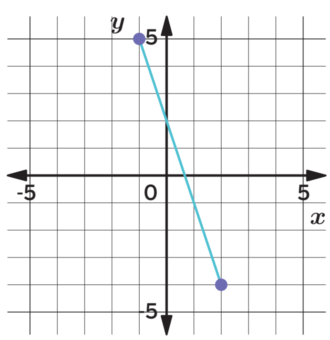
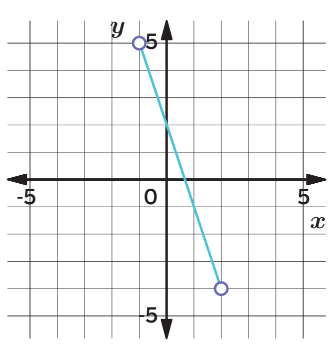
4. What is the largest number you can think of? The smallest number?

5. Is there any limit to how large or small a number can be? Explain your thinking.

6. If there is a limit, what do you think it is? If there is no limit, how could you represent that mathematically?

Activity 3 Interval Notation

Part A: You have described domain and range using words, lists, and inequalities. *Interval notation* is another way to describe the domain and range by representing intervals of values. For each graph, determine the domain and range using inequalities. Then select the corresponding representation in interval notation.

Graph	Inequality	Select the corresponding interval notation.
1. 	Domain: Range:	Domain: A. $[-1, 2)$ C. $(-1, 2)$ B. $(-1, 2]$ D. $[-1, 2]$ Range: A. $[-4, 5)$ C. $(-4, 5)$ B. $(-4, 5]$ D. $[-4, 5]$
2. 	Domain: Range:	Domain: A. $[-1, 2)$ C. $(-1, 2)$ B. $(-1, 2]$ D. $[-1, 2]$ Range: A. $[-4, 5)$ C. $(-4, 5)$ B. $(-4, 5]$ D. $[-4, 5]$
3. 	Domain: Range:	Domain: A. $[-1, 2)$ C. $(-1, 2)$ B. $(-1, 2]$ D. $[-1, 2]$ Range: A. $[-4, 5)$ C. $(-4, 5)$ B. $(-4, 5]$ D. $[-4, 5]$
4. 	Domain: Range:	Domain: A. $[-1, 2)$ C. $(-1, 2)$ B. $(-1, 2]$ D. $[-1, 2]$ Range: A. $[-4, 5)$ C. $(-4, 5)$ B. $(-4, 5]$ D. $[-4, 5]$

Name: ________________ Date: ________ Period: ________

Activity 3 Interval Notation (continued)

If a graph contains no endpoint(s), and continues on forever, use ∞ or $-\infty$ in interval notation. Note that ∞ or $-\infty$ cannot be included as endpoints.

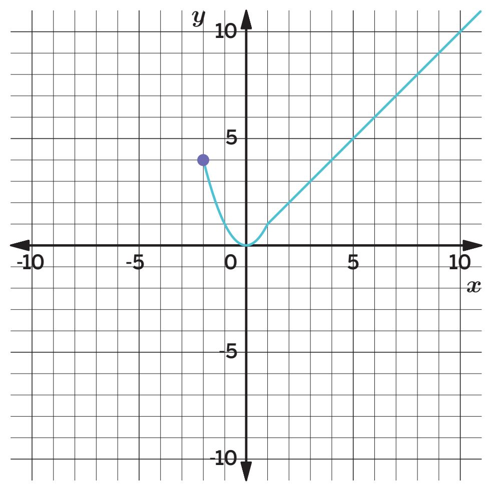
5. Determine the domain and range using both inequalities and interval notation for the graph shown.

Inequality:

Domain: Range:

Interval notation:

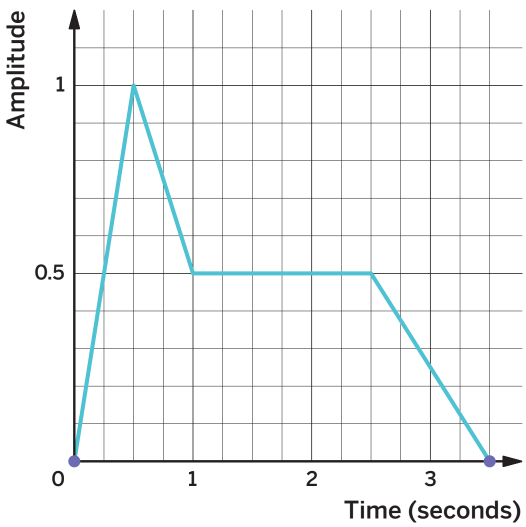
Domain: Range:



Part B: **Sound and music can change as time passes. One of these changes can be the amplitude, or volume, of the sound. The following graph shows how the amplitude of a sound changes as time passes.**

6. Determine the domain using interval notation.

7. Determine the range using interval notation.



8. Determine the interval(s) in which the function represented by the graph is:

a Increasing

b Decreasing

c Constant

STOP

Summary

In today's lesson . . .

You explored the common mathematical language that is used to communicate about input and output values; ***domain*** and ***range***. The domain of a function is the set of all possible input values. The range of a function is the set of all possible output values.

When values continue on forever without end, these values are ***infinite***.

- For values that continue on forever in the positive direction, use the infinity symbol, ∞.
- For values that continue on forever in the negative direction, use the negative infinity symbol, $-\infty$.

Inequalities can be used to represent domain and range, as they can help describe domain and range more efficiently than writing a description.

You also saw that in addition to inequalities, ***interval notation*** can be used to represent the domain and range of a function in another way. Interval notation has its own efficiencies for expressing intervals of values, and is used widely among mathematicians and scientists.

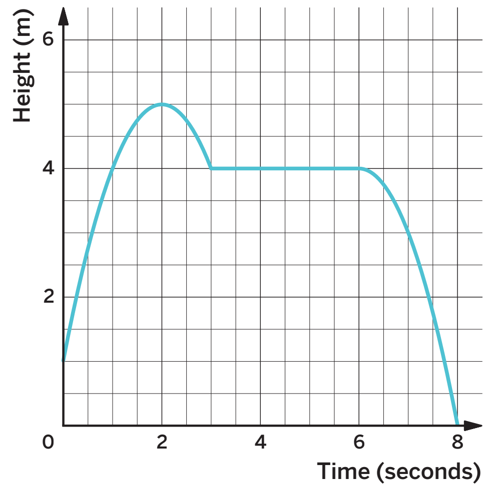
- Parentheses indicate the beginnings or endings of intervals in which the endpoint values are not included. For example, $(-\infty, 5)$ represents the interval from negative infinity all the way up to 5, but does not include the value 5.
- Brackets indicate the beginnings or endings of intervals in which the endpoint values are included. For example, $(-\infty, 5]$ represents the interval from negative infinity all the way up to 5, and includes the value 5.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Kiran tosses a ball up in the air. It gets stuck in a tree for a while before a light breeze pushes the ball out of the tree and it falls back down to the ground. A graph of the scenario is shown. Which of the following *best* represents the range of this function?



A. $[0, 8]$

B. $(0, 8)$

C. $[0, 5]$

D. $(0, 5)$

2. Mai has a gift card she can use on music lessons. The gift card has $\$120$ on it and every lesson costs $\$15$. The amount of money left on the gift card is a function of the number of lessons taken. Which of the following *best* represents the range?

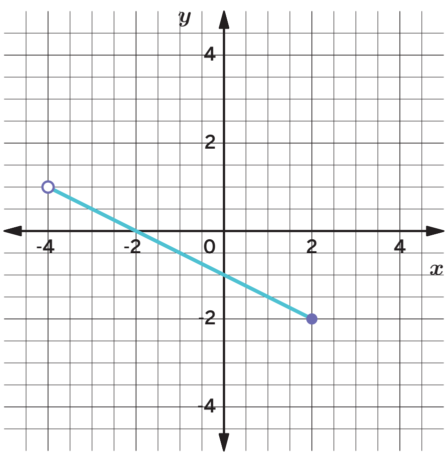
A. $\{0, 15, 30, 45, 60, 75, 90, 105, 120\}$

B. $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

C. $[0, 120]$

D. $[0, 8]$

3. Priya and Tyler disagree about the domain of the graph of the function shown. Priya thinks the domain is $(-4, 2]$ and Tyler thinks the domain is $[-2, 1)$. Who is correct? Why is the other person incorrect? Explain your thinking.



Name: .... Date: .... Period: ....

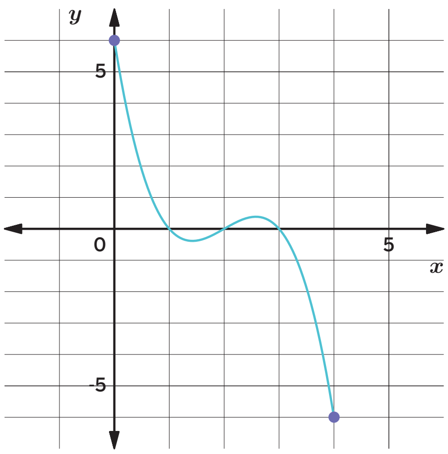
4. Elena is choosing between two cafeteria meal plans. Under Plan A, each meal costs $\$2.50$. Under Plan B, one month of unlimited meals costs $\$30$.

a Write an equation for function A, which gives the cost, in dollars, of buying n meals under Plan A for one month.

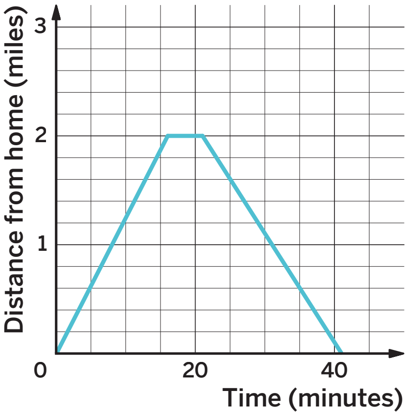
b Write an equation for function B, which gives the cost, in dollars, of buying n meals under Plan B for one month.

c Elena estimates that she will buy 15 meals per month. Which meal plan should she choose? Explain your thinking.

5. Use the graph to determine the average rate of change between $x = 0$ and $x = 4$.



6. Han starts at home and runs 2 miles. He stops to rest before running the two miles back home.



a Which section of the graph is increasing?

b What does the increasing section of the graph represent?

c Which section of the graph is constant?

d What does the constant section of the graph represent?

e Which section of the graph is decreasing?

f What does the decreasing section of the graph represent?

Name: ______________________ Date: ____________ Period: ____________

Unit 3 | Lesson 12

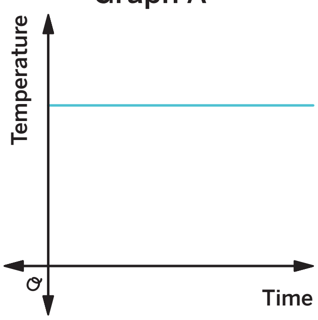
Interpreting Graphs

Let’s describe and interpret important features of graphs.

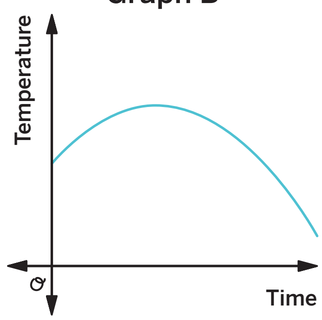
Warm-up Which One Doesn’t Belong?

Which of these graphs does not belong with the others? Explain your thinking.

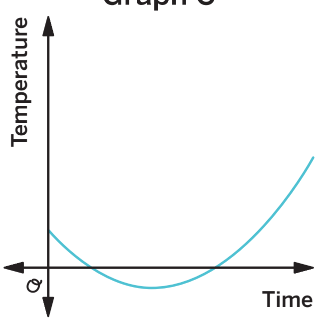
Graph A



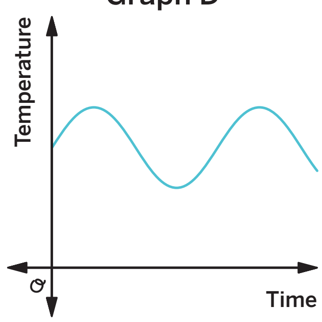
Graph B



Graph C



Graph D



Log in to Amplify Math to complete this lesson online.

Activity 1 The Flag of St. Louis

The St. Louis flag is among the most popular flags in the world. The height of the flag, as it is being raised, is a function of time. You will be given a set of graphs that represent possible scenarios of the flag's height, in feet, as it is being raised over time, in seconds.

Public Domain

1. For each graph, provide a description of what could be happening to the flag.

Graph A:

Graph B:

Graph C:

Graph D:

Graph E:

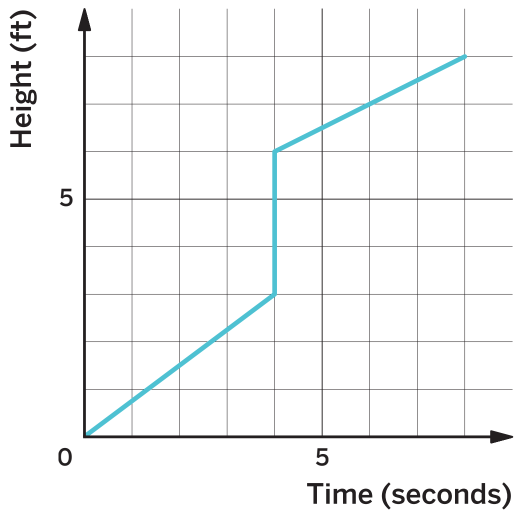
Graph F:

2. Which graph(s) appear to be the most realistic? Explain your thinking.

3. Refer to the graph shown.

a Could this graph represent the time and height of the flag? Explain your thinking.

b Is this a graph of a function? Explain your thinking.

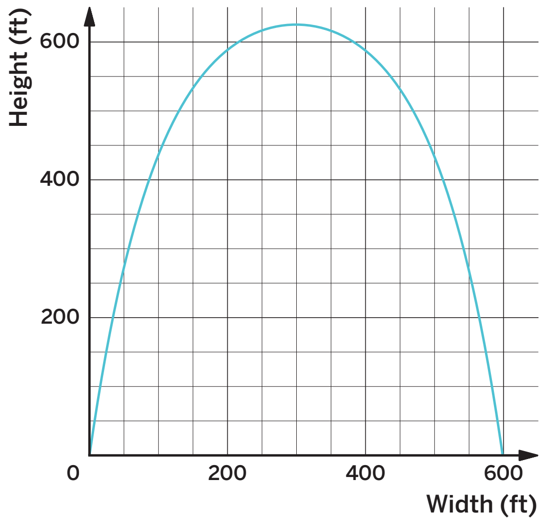


Name: ______ Date: ______ Period: ______

Activity 2 The Gateway Arch

The Gateway Arch in St. Louis, built in 1965, is the world's tallest arch, and is commonly called "The Gateway to the West." With the 65th anniversary of the arch approaching, it is set to undergo a $400 million renovation, requiring workers to ascend its "sides" to remove stains.

The graph shown is a mathematical model of The Gateway Arch.



1. Using the graph:

   a. Approximately, what is the distance between the starting and ending points of the arch? Explain your thinking.

   b. Which mathematical term best describes the distance between the starting and ending points of the arch?

   c. Write the distance between the starting and ending points of the arch using interval notation.

In order to clean the Gateway Arch, a team ascends one "side," and descends the other, and removes stains along the way.

2. Determine in which interval(s) on the graph the team would be:

   a. Ascending

   b. Descending

3. Estimate and interpret the average rate of change as the team moves along the arch where the the Gateway Arch is:

   a. Increasing

   b. Decreasing

Reflect: How did the graph relate information about The Gateway Arch that you did not know before?

STOP

Summary

In today's lesson . . .

You observed that graphs can help you visualize functions that represent different scenarios for a given context. Some of these graphs can realistically represent a situation, but other graphs may not make sense in context. It is important to pay attention to key features of the graphs when determining which graph best represents a situation.

The table shows some of the key features of graphs you have identified.

Key features of graphs:

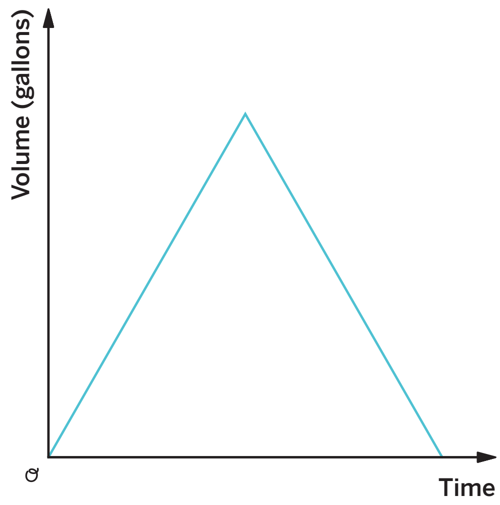
- Domain
- Range
- Increasing intervals
- Decreasing intervals
- Average rate of change

Reflect:

Name: ________ Date: ________ Period: ________

Practice

1. The graph represents the volume of water in a tank as a function of time. Which of the descriptions matches the graph?



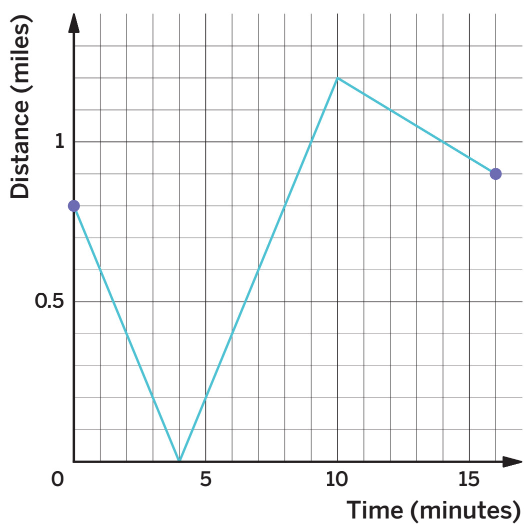
A. An empty 20-gallon water tank is filled at a constant rate for 3 minutes until it is half full. Then it is emptied at a constant rate for 3 minutes.

B. A full 10-gallon water tank is drained for 30 seconds until it is half full. Afterwards, it gets refilled.

C. A $2,000$-gallon water tank starts out empty. It is filled for 5 hours, slowly at first, and faster later.

D. An empty 100-gallon water tank is filled in 50 minutes. Then a dog jumps in and splashes around for 10 minutes, letting 7 gallons of water out. The tank is refilled afterwards.

2. Priya rode her bicycle around her town on Saturday. The graph represents the function D, which gives the distance between Priya and her home as a function of time t.



a. Determine and interpret the domain of the function.

b. Determine and interpret the range of the function.

c. Determine and interpret the average rate of change of the function for the interval $[0, 10]$.

Practice

Name: ______ Date: ______ Period: ______

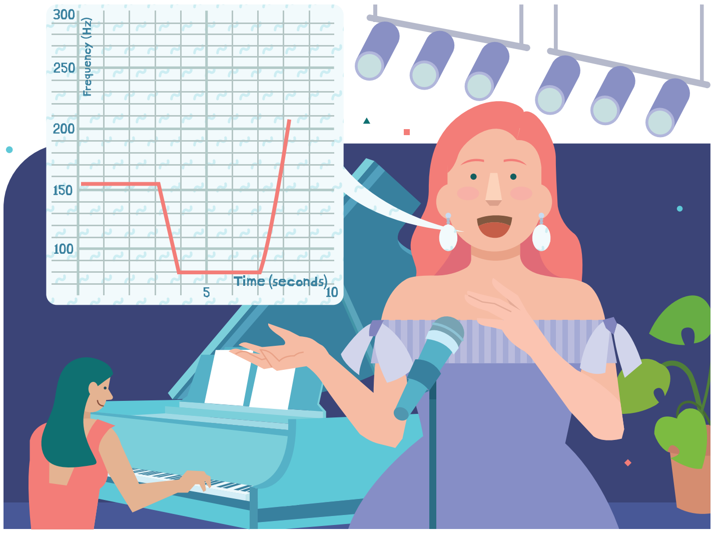
3. Consider the function $f(x) = 3x - 7$. For what value of x is the function notation statement $f(x) = 20$ true?

4. While scuba diving, Han swims underwater for a few minutes, stays about 60 ft below the surface for a few minutes, then swims back to the surface. After returning to the surface, he returns to land and travels uphill to go back home. Sketch a possible graph describing Han's height relative to sea level as a function of time. Be sure to label the axes.

Name: ________ Date: ________ Period: ________

Unit 3 | Lesson 13

Creating Graphs of Functions



Let's create graphs of functions to represent real-world contexts and highlight their important features.

Warm-up Ball Bounce

Suppose Lonzo Ball, a professional basketball player for the New Orleans Pelicans, holds a ball in his hand, raises his arm, and lets the ball fall to the ground. This can be represented by the function h, which gives the height of the ball, in feet, as a function of time t, in seconds.

1. Sketch two graphs of this scenario: a realistic representation and an unrealistic representation.

   Realistic representation:

   Unrealistic representation:

2. Explain why each graph is a realistic and unrealistic representation of this scenario.

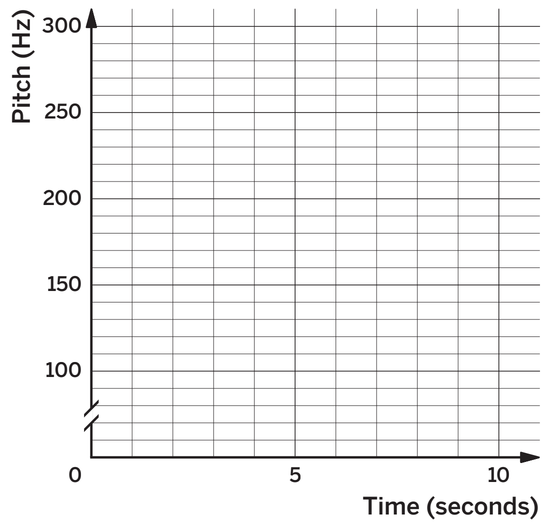
Log in to Amplify Math to complete this lesson online.

Activity 1 Changing Pitch

New Orleans is home to many famous musicians and singers, such as Billie Holiday and Louis Armstrong. Many singers use vocal exercises to warm up their voice to avoid damaging their vocal cords over time.

Jada, a jazz singer in New Orleans, warms up her voice before singing Louis Armstrong's "What a Wonderful World." The pitch of Jada's voice is measured in units of Hertz (Hz).

1. Jada holds a constant pitch of 255 Hz for 3 seconds before lowering her voice to 105 Hz for 1.5 seconds. She holds a constant pitch of 105 Hz for 2.5 seconds before rapidly raising her voice to 233 Hz for 1 second. Sketch a graph that represents how Jada's pitch changes over time.



2. What are the minimum and maximum pitches that Jada sings?

3. How long is Jada's vocal exercise? Explain or show your thinking.

4. What is a reasonable domain and range for this scenario?

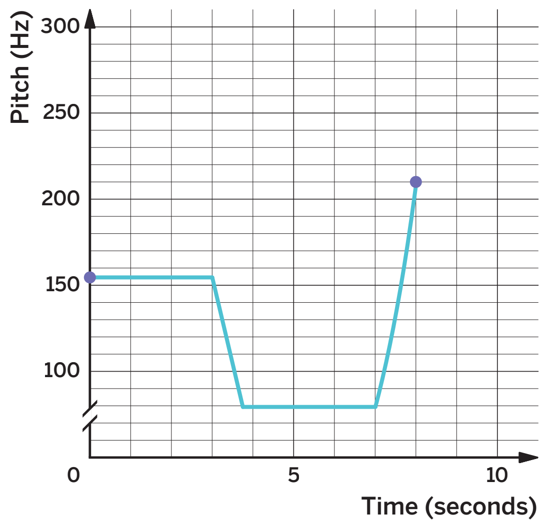
5. For which time interval(s) does Jada's pitch decrease? Increase? Remain constant?

6. For what time interval does Jada's pitch change the "fastest"? Explain or show your thinking.

Name: ________ Date: ________ Period: ________

Activity 2 Comparing Scenarios

Now Jada's friend, Tyler, decides to do vocal exercises to warm up his voice while Jada measures the change in his pitch over time. The graph shows how Tyler's pitch changes over time.



1. Determine whether each statement is *true* or *false*. Explain your thinking.

a Tyler's minimum pitch and maximum pitch are both lower than Jada's.

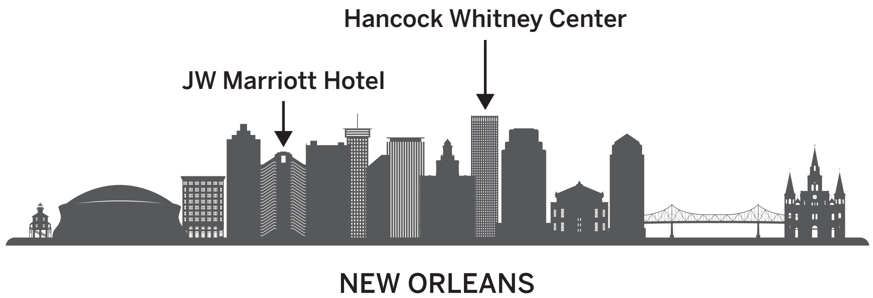
b Tyler's graph has the same domain and range when compared to Jada's.

c Both Tyler and Jada have an increase in pitch over the same time interval(s).

d Both Tyler and Jada have approximately the same average rate of change in pitch over the interval $[7, 8]$.

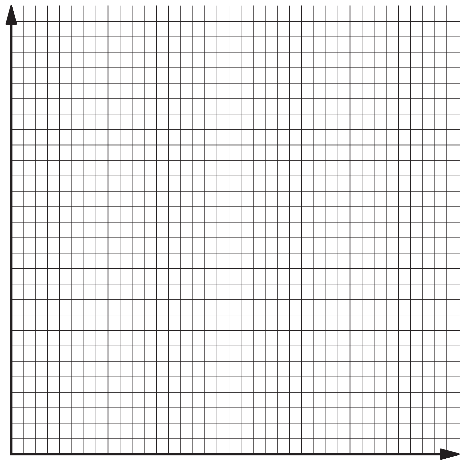
Activity 3 The New Orleans Skyline

The New Orleans skyline is home to many iconic buildings and structures. In the state of Louisiana, nine of the ten tallest buildings are located in New Orleans. The tallest building in New Orleans is the Hancock Whitney Center, and the thirteenth tallest building is the JW Marriott Hotel. The Hancock Whitney Center stands 697 ft tall, and is approximately 61 ft wide. The JW Marriott Hotel is 331 ft tall, and is approximately 70 ft wide at its widest part of the building. The roof begins to narrow approximately 100 ft from the top point.



Greens87/Shutterstock.com

1. Create a sketch of the outline of the two buildings on the same coordinate plane. (Be sure to consider your scale.)



2. How did you use the given information about both buildings to sketch their outlines?

Name: ________ Date: ________ Period: ________

Activity 3 The New Orleans Skyline (continued)

3. Use the graph you created in Problem 1 to determine the domain and range represented by each of the following:

   a. The rooftop of the Hancock Whitney Center.

   b. The rooftop of the JW Marriott Hotel.

4. Determine the maximum and minimum heights of each of the following:

   a. The Hancock Whitney Center.

   b. The JW Marriott Hotel.

Are you ready for more?

Write an equation that could represent each side of the roof of the JW Marriott Hotel.

STOP

Summary

In today's lesson . . .

You created graphs of functions given their descriptions. This included determining what a reasonable sketch of a graph could look like, and then turning that sketch into a graph.

It is important to take into account key features of graphs when sketching and comparing them, such as the ones shown in the table.

Key features of graphs:
• The scale of each axis. • The domain and range. • The intervals for which the function is increasing and decreasing. • The minimum and maximum values of the function. • The average rate of change over specified intervals.

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. Clare describes her visit to the art museum: "I entered the museum on the first floor and walked up the stairs to the third floor to view an exhibit. I spent an hour on the third floor. Then I walked down to the basement and spent an hour viewing several exhibits there. Lastly, I walked up to the first floor and sat outside to eat my lunch." Sketch a possible graph of Clare's height from the first floor as a function of time. Be sure to label your axes.

2. Mai fills up her bathtub slowly at first, then more quickly for 10 minutes until the water measures 15 in. deep. She then plugs the drain, takes a 15-minute bath, and lets the bath drain at a constant rate for 10 minutes.

   a. Sketch a graph to represent this scenario. Be sure to label your axes.

   b. What is the domain of this scenario and what does it represent?

   c. What is the average rate of change on the interval $[0, 10]$? What does this mean in context?

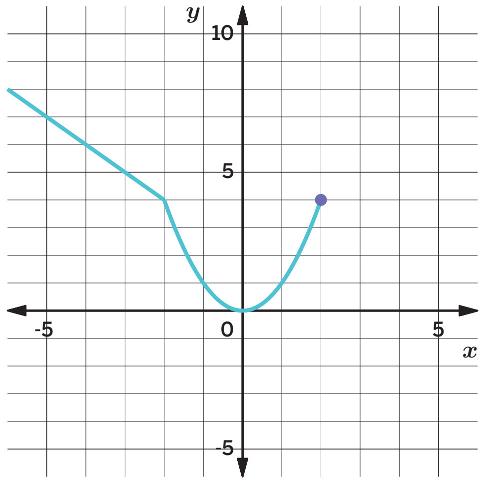
Practice

Name: ____________ Date: ____________ Period: ____________

3. Consider the graph of a function.

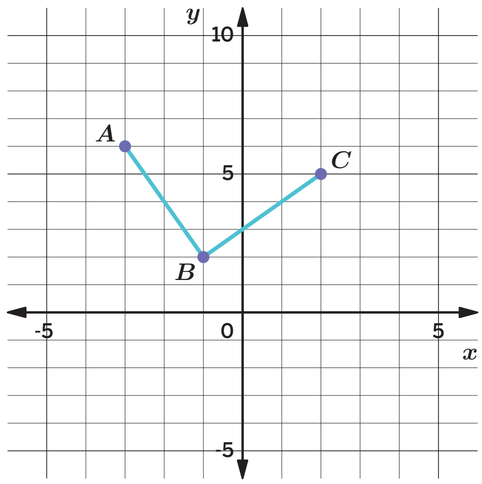
a Write the domain and range using inequalities.

b Write the domain and range using interval notation.



4. Answer each question based on the graph shown.

a Describe the key features of the graph.



b What is the slope of segment AB

c What is the slope of segment BC

SUB-UNIT

3 Piecewise Functions

Narrative Connections

Where did the world meet soul?

Ray Charles sat down with the executives of Atlantic Records at The Royal Peacock club. Charles had been on the road, opening for other artists. For the last three years, the 24-year-old musician had spent his career imitating the jazz style of singers like Nat “King” Cole — but now he wanted something different. Solemnly, Charles took the stage, and launched into a new song, “I Got a Woman.” It was a fiery, gospel-inflected blues/jazz fusion whose driving rhythms were something many executives had never heard before.

And so, on a November day in 1954, in a club in the Sweet Auburn district of Atlanta, Ray Charles introduced the world to soul music. The Sweet Auburn neighborhood of Atlanta is full of stories of Black cultural excellence. Miles Davis, B.B. King, Nina Simone, Sam Cooke, and Gladys Knight are just a handful of the artists that have played in Sweet Auburn.

The neighborhood was formed in the early 1900s, when many Black-owned businesses relocated from Atlanta’s downtown area to Auburn Avenue. The area became home to the Ebenezer Baptist Church, where Martin Luther King was pastor, as well as one of the earliest and most influential Black-owned newspapers, the Atlanta Daily World. Over time, the neighborhood, and the institutions within it, transformed Atlanta into a hub for culture and civil rights.

The next time you listen to “I Got a Woman,” pay special attention to the saxophone solo in the middle. If you graphed the solo, what would it look like? Whether you’re plotting the notes or the rhythm, your graph will probably have a lot in common with the piecewise functions you’ll encounter in the next few lessons.

Unit 3 | Lesson 14

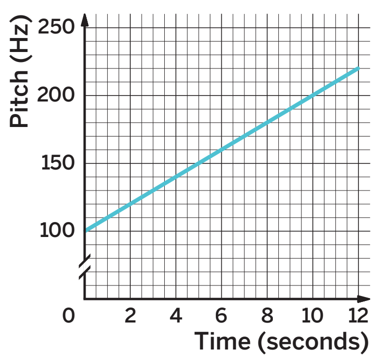
Piecewise Functions (Part 1)

Let's look at functions that are defined in pieces.

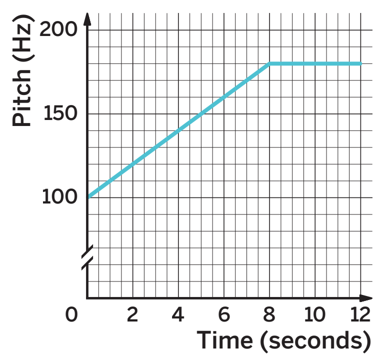
Warm-up A Sound's Pitch

A singer hums a short warm-up, beginning at 100 Hz and increasing her pitch by 10 Hz per second until 8 seconds, when her pitch remains the same for 4 more seconds. Select the graph that represents the singer's pitch as a function of time. Explain your thinking.

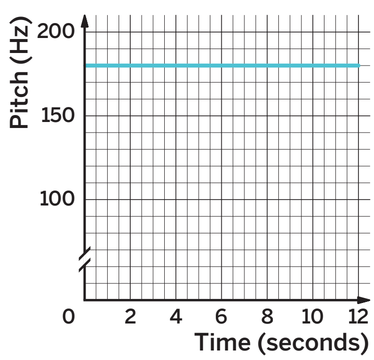
A.



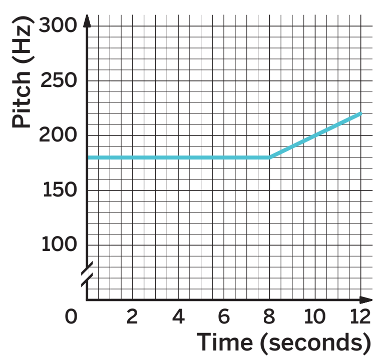
B.



C.



D.



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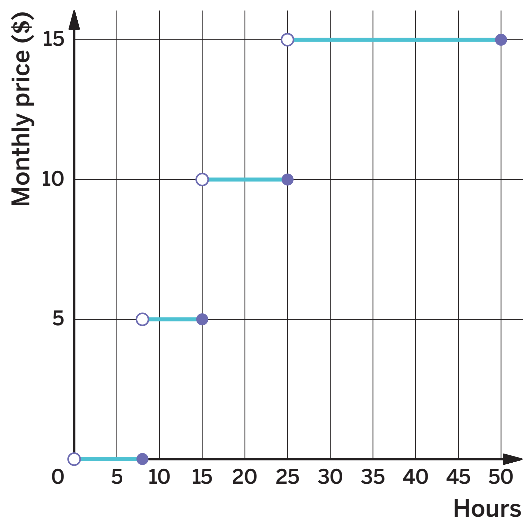
Name: .... Date: .... Period: ....

Activity 1 Digital Music Subscription

Sound recording was revolutionized by the invention of digital recording. Digital sound is created by using a microphone to turn recorded sound into an electrical signal which is then converted into digital software. Because digital recordings' quality lasts practically forever, and copies can be made at almost no cost, the widespread use of digital recording has forever changed the price of music.

A streaming music platform's monthly subscription price depends on the number of hours a month the subscriber listens to music. The relationship between the price and the number of hours of music can be defined by a *piecewise function*, because the overall function consists of different "pieces" of functions over different intervals. The graph shows the monthly subscription price.



1. Tyler and Mai were each studying the graph. Critique each person's statement and determine which statement is correct. Explain your thinking.

   Tyler: This graph is not a function because there are two output values for $x = 8$, $x = 15$, and $x = 25$.

   Mai: This graph is a function because each open circle means that the endpoint value is not included.

2. Determine the monthly subscription price for each of the following number of hours per month of music.

   a. 15 hours
   b. 15.1 hours
   c. 14.9 hours

3. Suppose the bill for one month of this subscription was $10. Describe the possible number of hours that could have been spent listening to music.

Activity 1 Digital Music Subscription (continued)

4. Tyler and Mai each wrote some rules to represent the monthly subscription as a function, but they each made some errors. The function T represents Tyler's work and the function M represents Mai's work.

$$T(h) = \begin{cases} 0, & 0 \leq h \leq 8 \\ 5, & 8 \leq h \leq 15 \\ 10, & 15 \leq h \leq 25 \\ 15, & 25 \leq h \leq 50 \end{cases} \qquad M(h) = \begin{cases} 0, & 0 < h < 8 \\ 5, & 8 < h < 15 \\ 10, & 15 < h < 25 \\ 15, & 25 < h < 50 \end{cases}$$

Identify the error in each person's work and write a corrected set of rules.

Are you ready for more?

The table shows how the company specifies the different monthly subscription prices. Notice that it uses the language "Time listening, not over (hours)" to describe the different rates. Explain or use a sketch to show how the graph would change if it used "under" instead of "not over."

Time listening, not over (hours)	Price($)
8	0
15	5
25	10
50	15

Name: ______ Date: ______ Period: ______

Activity 2 The Cost of Connectivity

Streaming music services are built on the infrastructure of satellites and towers. Mary Golda Ross, the first known Native American female engineer, worked on top secret satellite projects in the mid 1900s that set the foundation for modern computers and smartphones to access signals all over the world. Now, if you have access to a signal, you can practically listen to any music you want wherever you are.

Some cell phone companies charge a higher rate called "roaming" if a signal is accessed outside of their network.

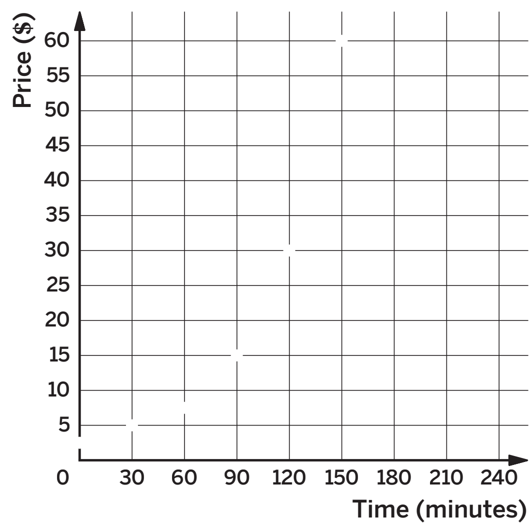
The function P represents the dollar price of a cell phone company's roaming service for t minutes. Here are the rules describing the function:

$$P(t) = \begin{cases} 2.50, & 0 < t \leq 30 \\ 5.00, & 30 < t \leq 60 \\ 7.50, & 60 < t \leq 90 \\ 15.00, & 90 < t \leq 120 \\ 30.00, & 120 < t \leq 150 \\ 60.00, & t > 150 \end{cases}$$

1. Complete the table with the price for each given roaming time.

t (minutes)	P ($)
0	
10	
25	
60	
75	
130	
180	

2. Sketch a graph of the function for all values of t that are greater than 0 minutes and at most 240 minutes.



Activity 2 The Cost of Connectivity (continued)

3. Use the function to translate the pricing rules for the company's roaming service to a verbal statement.

4. Determine the domain and range of this function.

Stronger and Clearer: After writing a draft response to Problem 3, meet with 2–3 partners to give and receive feedback. Use the feedback to refine your response.

Featured Mathematician

Mary Golda Ross

Mary Golda Ross is the first known Native American and Cherokee female engineer of the mid 1900s. One of 40 founding engineers for the top secret spy plane project at the aerospace company, Lockheed Martin, she originally only worked with a slide rule and Friden computer to help advance her theories into reality. She pioneered ballistic missile, satellite, and manned/unmanned flight technology, and studied the effects of ocean waves on submarines.

STOP

Walter P. Reuther Library, Archives of Labor and Urban Affairs, Wayne State University.

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored a different type of function called a piecewise function. A ***piecewise function*** has different descriptions or rules for different parts of its domain. The graph of a piecewise function is often composed of pieces or segments of functions. The pieces can be connected or disconnected. When disconnected, the graph appears to have breaks or steps. A piecewise function in which the pieces represent constant values is called a ***step function***, because its graph looks like a series of steps.

It is important to consider the value of the function at places where the graph is disconnected or where the graph "breaks." Examining the domain of each piece will help determine the value of the function at these points.

Reflect:

Practice

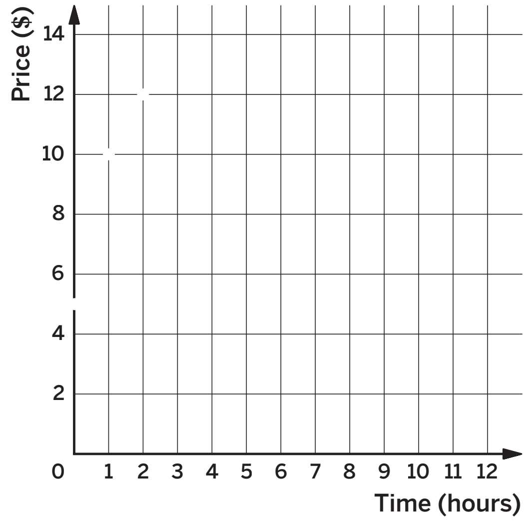
Name: .... Date: .... Period: ....

1. A parking garage charges \$5 for the first hour, \$10 for up to two hours, and \$12 for more than two hours. Let G represent the dollar price of parking for t hours.

a. Complete the table.

t (hours)	G (\$)
0	
0.5	
1	
1.75	
2	
5	

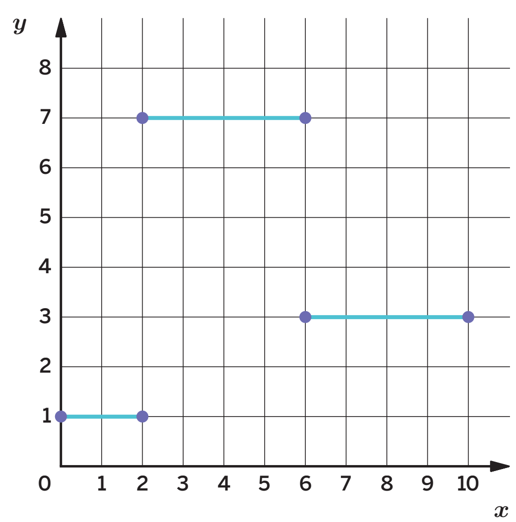
b. Graph the function G for $0 \le t \le 12$.



c. Is G a function of t? Explain your thinking.

d. Is t a function of G? Explain your thinking.

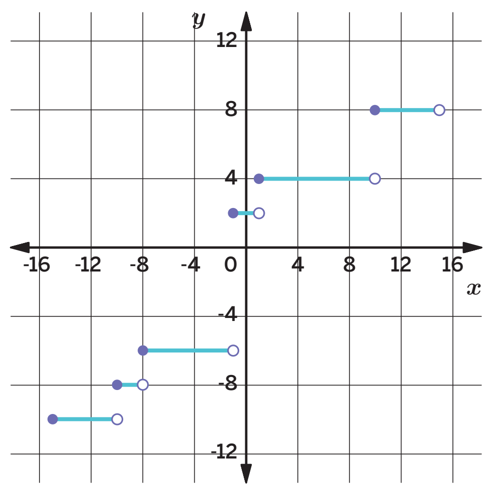
2. Consider the graph. Does the graph represent a function? Explain your thinking.



Name: .... Date: .... Period: ....

3. Use the graph of the function g to complete the following problems. Some of the values given by the function rule are missing.

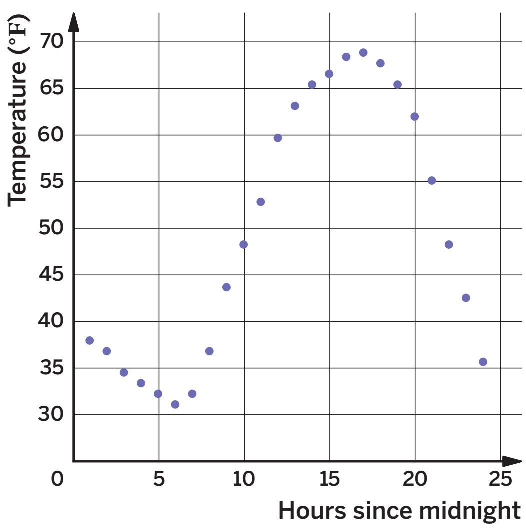
$$g(x) = \begin{cases} -10, & -15 \le x < -10 \\ \text{.........}, & -10 \le x < -8 \\ -6, & \text{.........} \le x < -1 \\ \text{.........}, & -1 \le x < 1 \\ 4, & \text{.........} \le x < \text{.........} \\ 8, & 10 \le x < 15 \end{cases}$$



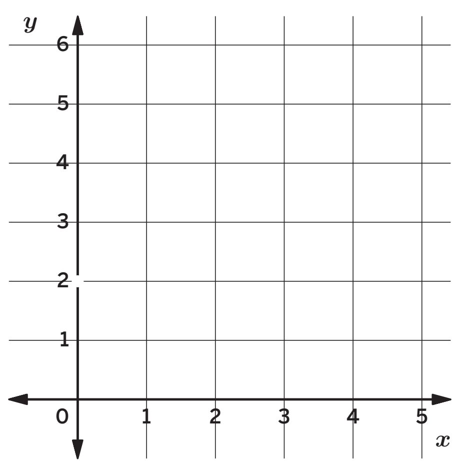
a What are the values of $g(1)$, $g(-12)$, and $g(15)$?

b Complete the rule for $g(x)$ so that the graph represents the function.

4. The function T gives the temperature in degrees Fahrenheit, n hours since midnight. The graph shows this function. Based on the graph, did the temperature change more quickly between 10:00 a.m. and noon, or between 8:00 p.m. and 10:00 p.m.? Explain your thinking.



5. Graph the function $f(x) = x + 2$ over the domain $0 < x \le 3$.



Unit 3 | Lesson 15

Piecewise Functions (Part 2)

Let's see how piecewise functions can represent design and sound.

Warm-up Atlanta's Famous Theatre

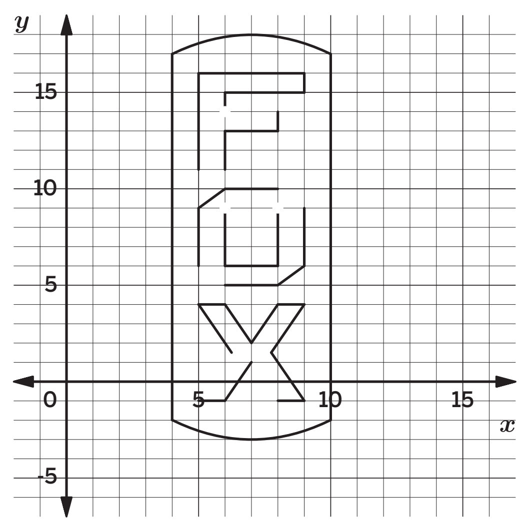
Atlanta, Georgia is home to the former Palace Theatre. This performance venue opened in 1929 and has been a cultural and artistic center for almost a century. A variety of famous artists have performed here over the years, including Elvis Presley, Ray Charles, and Bob Marley.

Part of this theater's sign is created for you. The missing parts of each letter are represented by the three piecewise functions shown. Graph these piecewise functions to complete the sign. While the name of the theater may be obvious, pay attention to how the rules relate to the missing pieces on the graph.

$$f(x) = \begin{cases} 11, & 5 \le x \le 6 \\ 14, & 6 < x \le 8 \end{cases}$$

$$g(x) = \begin{cases} -x + 11, & 5 \le x \le 6 \\ 9, & 6 < x < 8 \\ -x + 18, & 8 \le x \le 9 \end{cases}$$

$$h(x) = \begin{cases} 2x - 11, & 5 \le x \le 6.25 \\ -2x + 15, & 7 \le x \le 8 \end{cases}$$



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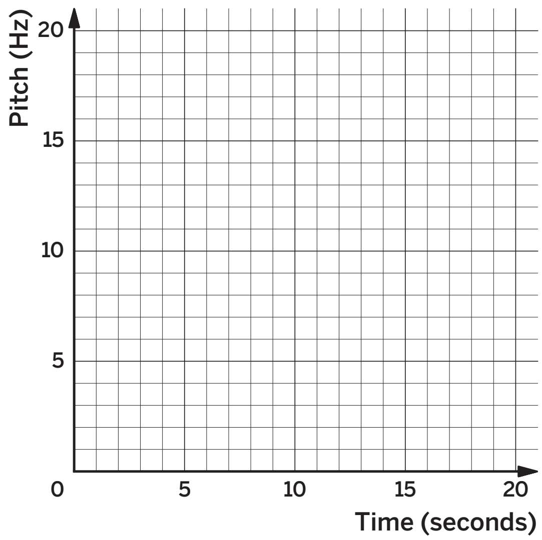
Name: .... Date: .... Period: ....

Activity 1 The Pieces of Sound

Sounds can be represented in a coordinate plane with linear and nonlinear functions. Some sounds are best represented by piecewise functions.

Your teacher will play a stream of sound. Recall that *pitch* is the aspect of a sound that makes it possible to judge sounds as "higher" or "lower" than other sounds. Use this term to help describe what you hear and see.

1. Draw a sketch of the pitch of the sound here.



Your teacher will play the sound again, while also displaying the graph of the piecewise function that represents the sound.

2. How is the sound represented by the piecewise function?

3. How does the changing pitch of the sound relate to the slope of each piece of the function?

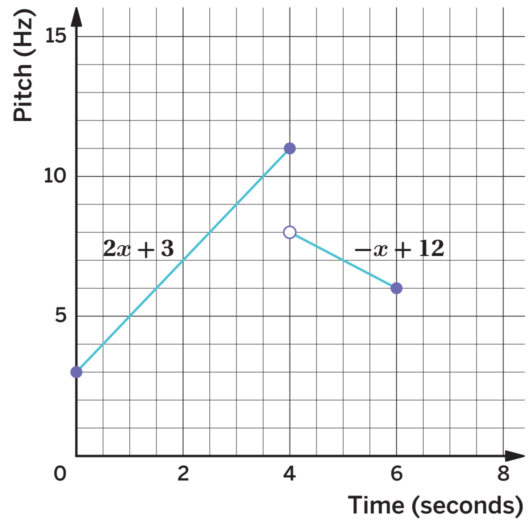
Activity 2 Creating a Melody

For some musicians, to create a pleasant-sounding melody, the sound should have no sudden breaks or jumps in pitch.

1. Consider the melody represented by the piecewise function.

$$f(x) = \begin{cases} 2x + 3, & 0 \le x \le 4 \\ -x + 12, & 4 < x \le 6 \end{cases}$$

Change the rule of the second piece so that the end of the first piece intersects with the beginning of the second piece, eliminating the jump in pitch. Write the new piecewise function here.



2. Substitute $x = 4$ into the expression for each piece of f. What do you notice? How does this confirm that the melody has no sudden breaks or jumps?

3. Here is another piecewise function representing a melody.

$$g(x) = \begin{cases} x + 4, & 0 \le x \le 10 \\ 2x - 6, & 11 < x \le 12 \end{cases}$$

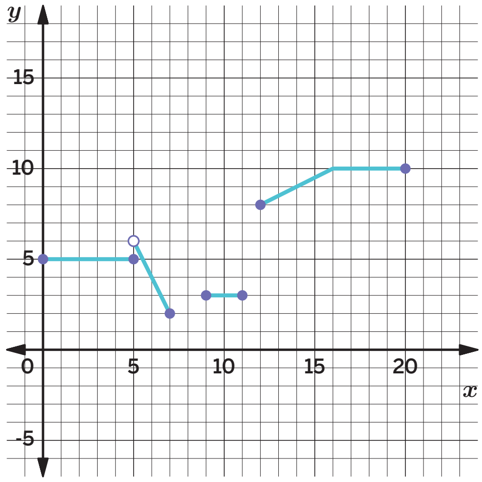
a Are there breaks in the sound? If so, where?

b Change the domain of the second piece. Write a new piecewise function so that it represents a melody with no breaks or jumps, and so that there is only one sound being played at any given time. Explain the changes you made.

Name: .... Date: .... Period: ....

Activity 2 Creating a Melody (continued)

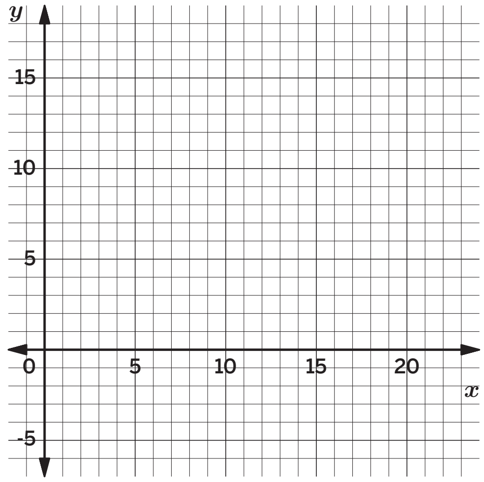
4. Suppose you are a sound engineer and want to adjust the pitches of a recording to eliminate all breaks and jumps, and so that only one melody plays at a time. Currently, there are several breaks in the recording. Consider the graph and the function of the original recording.



$$f(x) = \begin{cases} 5, & 0 \le x \le 5 \\ -2x + 16, & 5 < x \le 7 \\ 3, & 9 \le x \le 11 \\ \frac{1}{2}x + 2, & 12 \le x \le 16 \\ 10, & 16 < x \le 20 \end{cases}$$

a Write a new piecewise function so that all breaks and jumps in the recording are eliminated.

b Graph your piecewise function on the coordinate plane to confirm that there are no breaks or jumps in the graph.



STOP

Summary

In today's lesson . . .

You observed that a piecewise function can be represented by different rules for different intervals (pieces) of its domain. These pieces can be linear or nonlinear. A rule represents each piece of the function, where each rule describes the range for different domain intervals.

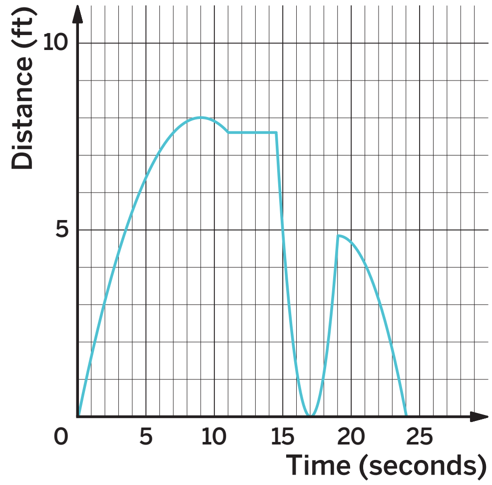
The pieces may connect, or there may be breaks in between each piece. When the pieces connect, it is important to consider which rule applies to the value of x when evaluating the function.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. This graph represents Andre's distance from his bicycle as he walks in a park.

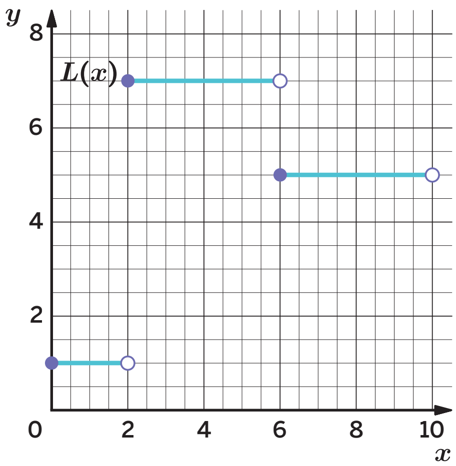


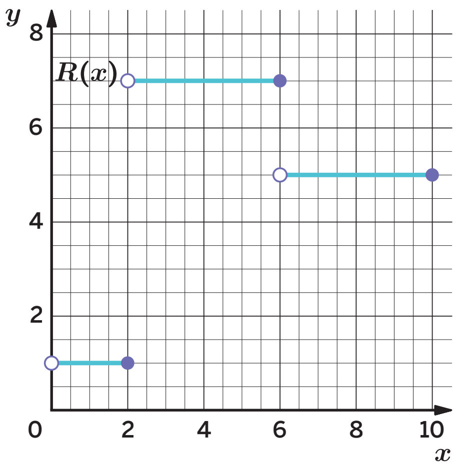
a Determine on which interval(s) on the graph Andre's distance from his bicycle is decreasing.

b On which interval(s) is Andre's distance from his bicycle increasing?

c Describe Andre's location during the time in which the value of the function is increasing.

2. Refer to the graphs of the functions $L(x)$ and $R(x)$.





a What are the values of $L(0)$ and $R(0)$?

b What are the values of $L(2)$ and $R(2)$?

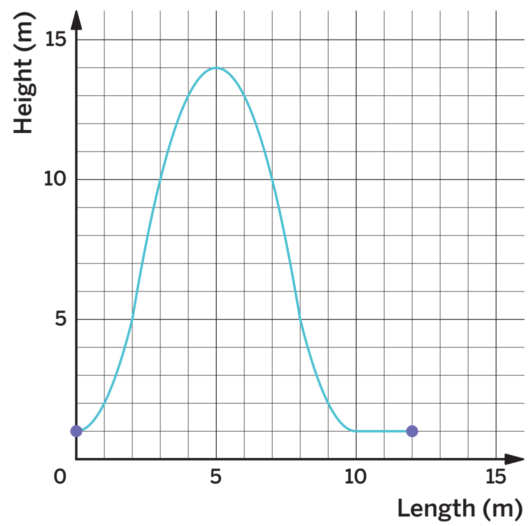
c For what values of x is the function notation statement $L(x) = 7$ true?

d For what values of x is the function notation statement $R(x) = 7$ true?

Practice

Name: ________ Date: ________ Period: ________

3. Jada rides a rollercoaster that consists of repeating the same loop. One loop of the rollercoaster ride is represented by the graph of the piecewise function.

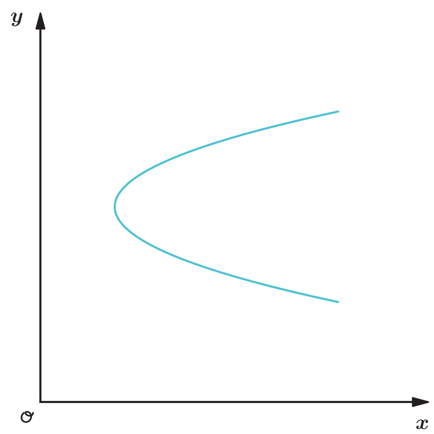


a. For which interval is the piecewise function nonlinear?

b. For which interval is the piecewise function linear?

c. What is different about the beginning and end of the loop?

4. Explain why this graph does not represent a function.



5. Diego went to a carnival and played a game in which he had to guess the number of marbles in a jar. Diego guessed 30. Mai, the carnival employee who runs the game, responded, "I am sorry, but you are incorrect. The number of marbles in the jar is 45, so you were −15 away from the actual number of marbles." Do you agree with Mai's statement? Explain your thinking.

Name: ______________ Date: ______________ Period: ______________

Unit 3 | Lesson 16

Another Function?

Let's make some guesses and see how close they are to actual values.

Warm-up How Close Were the Guesses?

You will guess the number of objects in a jar. The guesses of all students will be collected. Your teacher will share the data and reveal the actual number of objects in the jar. Record your guess and 11 of your classmates' guesses in the table.

Use the actual number of objects in the jar to calculate the absolute guessing error of each guess, or how far the guess is from the actual number. For example, suppose the actual number of objects is 100.

- If the guess is 75, then the absolute guessing error is 25.
- If the guess is 110, then the absolute guessing error is 10.

Record the absolute guessing error of the 12 guesses in the table.

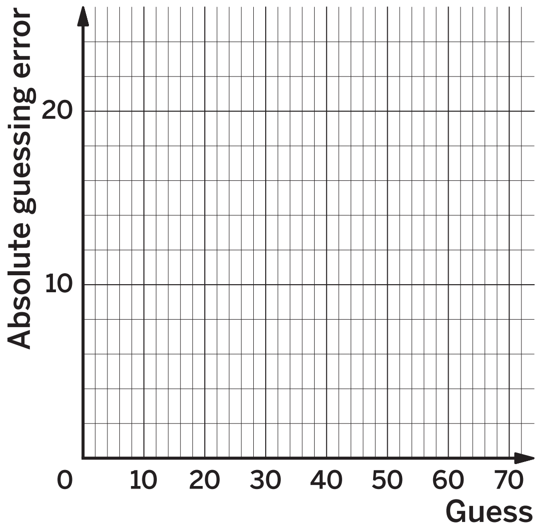
Guess	Absolute guessing error

Log in to Amplify Math to complete this lesson online.

Activity 1 Plotting the Guesses

Refer to the table you completed in the Warm-up.

1. Plot the values from your table on the coordinate plane.



2. Write three observations about your graph.

3. Is the absolute guessing error a function of the guess? Explain your thinking.

Are you ready for more?

Suppose there is another guessing contest to win a prize. Each class can submit one guess. It is up to the students to decide on the number to be submitted. Here are some ideas that have been proposed on how to decide on that number:

- **Option A: Ask a person who did really well in the previous guessing game to make a guess.**
- **Option B: Ask everyone to make a guess and have a discussion to narrow the list.**
- **Option C: Ask everyone to make a guess and determine the mean of all the guesses.**
- **Option D: Ask everyone to make a guess and determine the median value.**

Which approach do you think would give your class the best chance of winning? Explain your thinking.

Name: .... Date: .... Period: ....

Activity 2 Oops, Try Again!

Earlier, you guessed the number of objects in a jar and then your teacher told you the actual number. Suppose your teacher made a mistake about the number of objects in the jar and would like to correct it. The actual number of objects in the jar is ....

1. Determine the new absolute guessing errors based on this new information. Record the errors in the table.

Guess	Absolute guessing error

2. What do you notice about the new set of absolute guessing errors?

3. Predict how the scatter plot would remain the same and how it would change given the new actual number of objects.

4. Use graphing technology to plot the points to check if your prediction is true.

5. Write a rule to determine the output (absolute guessing error) given the input (a guess).

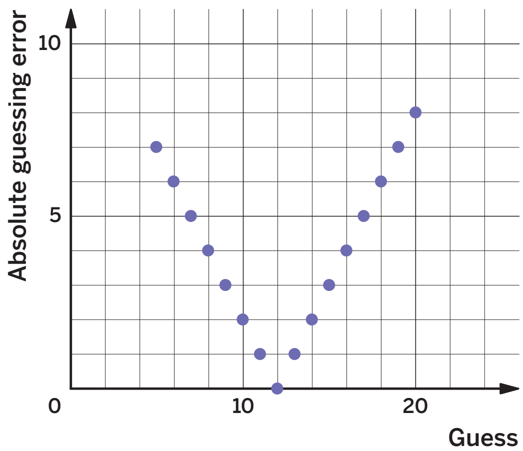
STOP

Summary

In today's lesson . . .

You calculated and graphed how far guesses are from a target number. It does not matter if the guess is above or below the target number. What matters is how far off the guess is from the target number, which is the absolute guessing error. The smaller the absolute guessing error, or the closer it is to 0, the better the guess.

If you plot the guesses and the absolute guessing errors on a coordinate plane, the points form a V shape. Notice that the V shape is on or above the horizontal axis, suggesting that all values are non-negative.



Reflect:

Practice

Name: ______ Date: ______ Period: ______

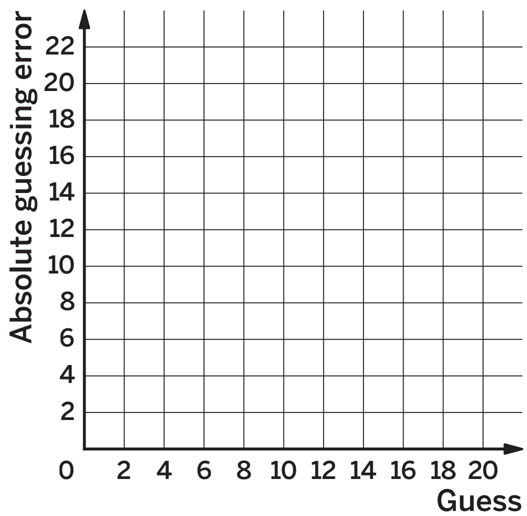
1. A group of 10 friends played a number guessing game. They were asked to select a number between 1 and 20. The person closest to the target number wins. The table shows the guesses made by each of the 10 friends.

Guess	2	15	10	8	12	19	20	5	7	9
Absolute guessing error										

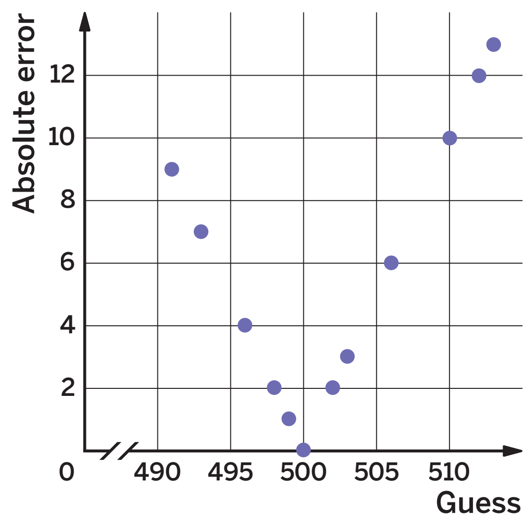
a The actual number was 14. Complete the table with the absolute guessing errors.

b Graph each guess and its corresponding absolute guessing error on the coordinate plane.

c Is the absolute guessing error a function of the guess? Explain your thinking.



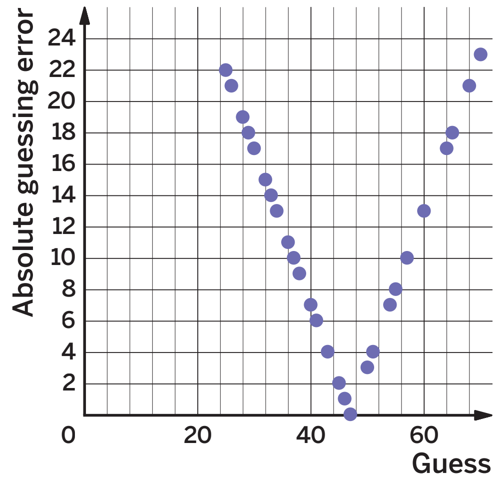
2. Bags of walnuts from a food producer are advertised to weigh 500 g each. In a certain batch of 20 bags, most bags have an absolute error that is less than 4 g. Could this scatter plot represent the 20 bags of walnuts in the batch and their absolute errors? Explain your thinking.



Practice

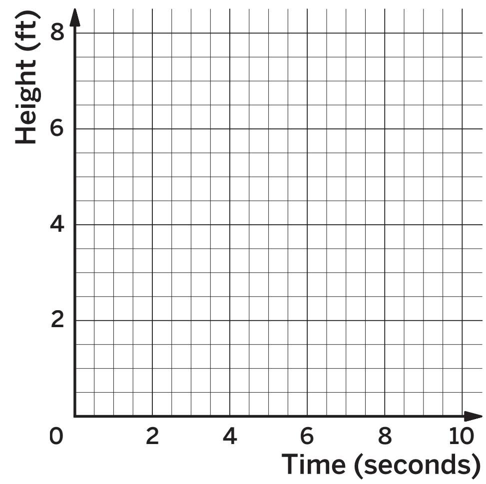
Name: .... Date: .... Period: ....

3. The class guessed how many objects were placed in a mason jar. The graph displays the class results, with an actual number of objects in the mason jar being 47. Suppose a mistake was made, and the actual number is 45. Explain how the graph would change, given the new actual number.



4. The function D gives the height of a drone in feet, t seconds after it lifts off. Sketch a possible graph for this function given that:

- $D(3) = 4$
- $D(10) = 0$
- $D(5) > D(3)$



5. The population of a city grew from $23{,}000$ in 2010 to $25{,}000$ in 2015.

a. What was the average rate of change during this time interval?

b. What does the average rate of change mean in this context?

6. Match each scenario with the absolute value expression that represents the absolute guessing error.

a. Students were shown a jar containing 50 marbles. One student guessed there were 45 marbles inside.	....	$\lvert 50 - 55 \rvert$
b. Students were shown a jar containing 50 marbles. One student guessed there were 55 marbles inside.	....	$\lvert 50 - 45 \rvert$
c. Students were shown a jar containing 60 marbles. One student guessed there were 50 marbles inside.	....	$\lvert 60 - 85 \rvert$
d. Students were shown a jar containing 60 marbles. One student guessed there were 85 marbles inside.	....	$\lvert 60 - 50 \rvert$

Name: ______ Date: ______ Period: ______

Unit 3 | Lesson 17

Absolute Value Functions

Let's investigate distance as a function.

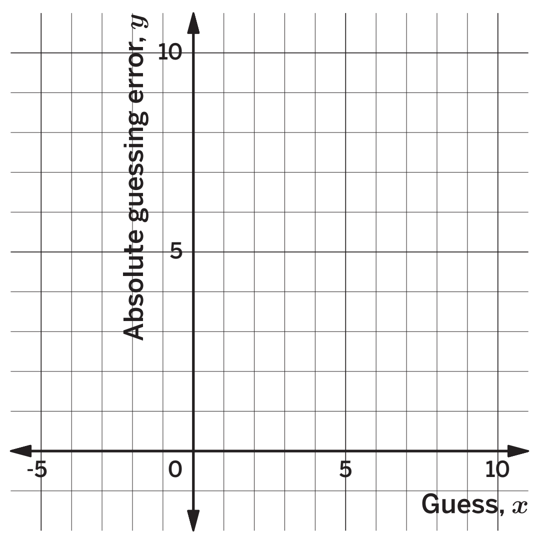
Warm-up Hip-Hop's Popularity

During the first two decades of the 2000s, Atlanta became one of the national centers for hip-hop music production, with top-selling and acclaimed artists, such as Outkast, Usher, and Childish Gambino originating from the area. In general, hip-hop had a steady rise in popularity in the U.S. during this time.

From 2017 to 2018, there was a change in the percentage of hip-hop songs consumed. Here are 12 guesses of the percent change.

5	2	–5	3	0	–1	1.5	4	–2.5	6	4.6	7

1. In 2017, hip-hop songs accounted for 20.9% of the songs consumed, with this number increasing to 24.7% in 2018. So, the actual change was 3.8%. Use this information to sketch a scatter plot representing the guesses x and the corresponding absolute guessing errors y.



2. What rule can you write to determine the output value given the input value?

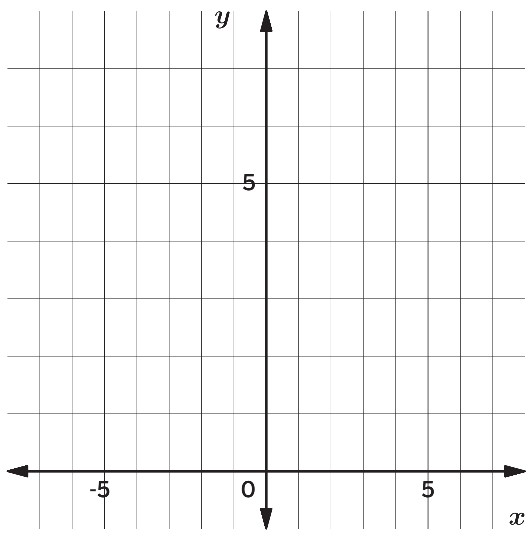
Activity 1 The Distance Function

Plan ahead: How can you apply what you know about the structure of linear functions to absolute value functions?

The function A gives the distance of x from 0 on the number line.

1. Complete the table and sketch a graph of the function A.

x	$A(x)$
8	
	5.6
3.14	
$\frac{1}{2}$	
	1
0	
$-\frac{1}{2}$	
-1	
-5.6	
	8



2. Andre and Elena write a rule for this function.

   Andre writes: $A(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

   Elena writes: $A(x) = |x|$

   Explain why both equations correctly represent the function A.

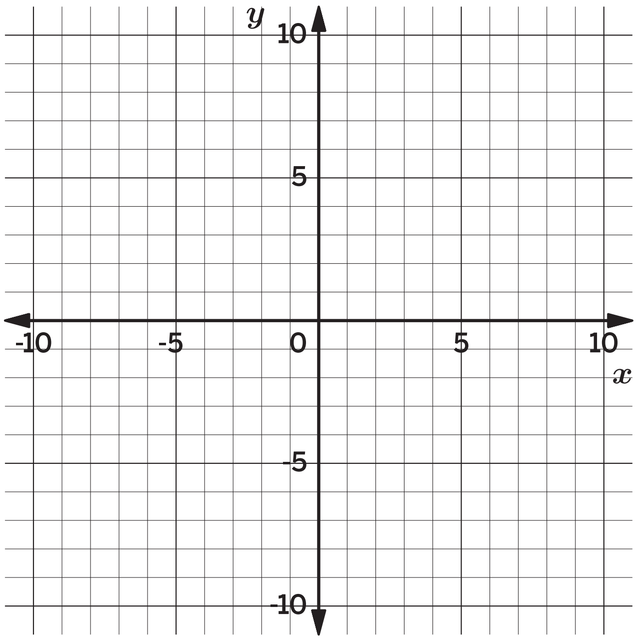
Name: ______________ Date: ______________ Period: ______________

Activity 2 Moving Graphs Around

$f(x) = |x|$ is an *absolute value function* because the output values represent the distance between each x-value and 0. The graph of f has a vertex at $(0, 0)$, which is where the graph changes from having a negative slope to a positive slope.

Absolute value functions can be transformed like any other function. The functions $p(x) = |x - h|$ and $g(x) = |x| + k$ are absolute value functions transformed by the constants h and k.

1. Graph the functions $p(x)$ and $g(x)$ using graphing technology. Experiment using different positive and negative values of h and k. Sketch at least four functions on the same coordinate plane and label each graph with its function.



2. How does changing the value of h affect the graph of an absolute value function and its vertex?

3. How does changing the value of k affect the graph of an absolute value function and its vertex?

Compare and Connect:
Your teacher may ask you to create a graphic organizer or display that summarizes how changing the values of h and k affects the graph.

Activity 2 Moving Graphs Around (continued)

4. Match each equation with the graph that it represents. One graph is not shown. Graph the remaining equation that could not be matched with the other graphs.

a $y = |x - 3|$ ....

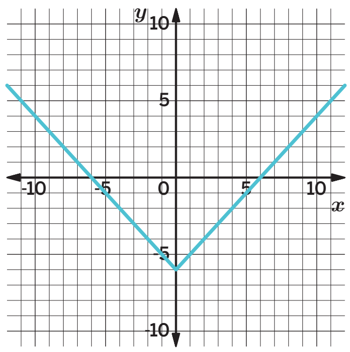
b $y = |x - 9| + 3$ ....

c $y = |x| - 6$ ....

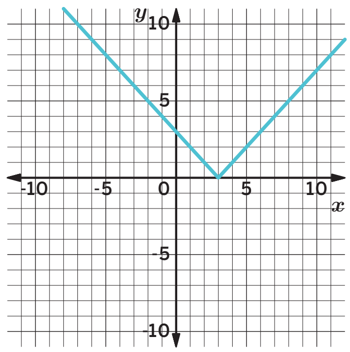
d $y = |x + 3|$ ....

e $y = |x + 3| - 6$ ....

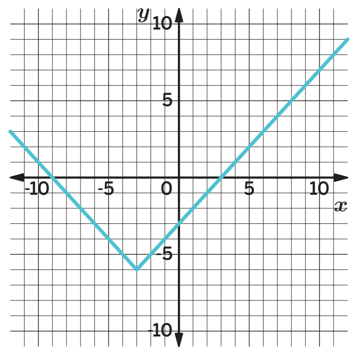
Graph 1



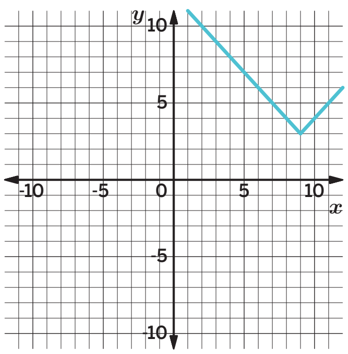
Graph 2

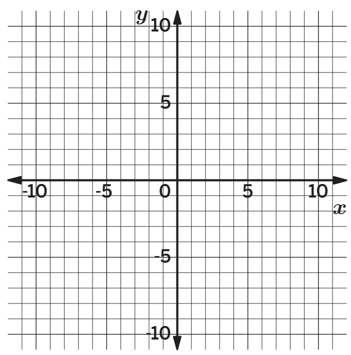


Graph 3



Graph 4





Are you ready for more?

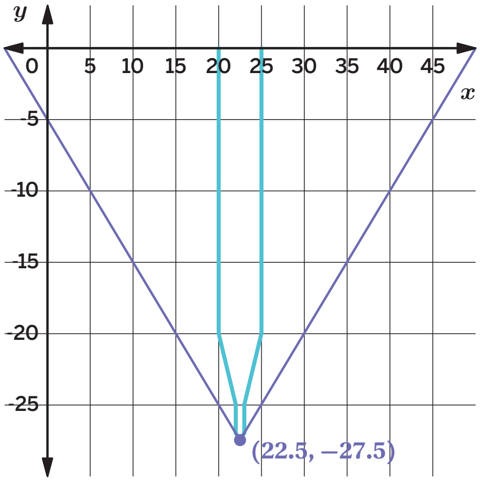
1. In Problem 1, look for the minimum value of each function. For the function p, what value of x gives the least output value? For the function g?

2. Another function is defined by $m(x) = |x + 11.5|$. What value of x produces the least output of function m? Describe the graph of m.

Name: .... Date: .... Period: ....

Activity 3 The Atlanta Skyline

Walking along Piedmont Park in Atlanta, you can see the reflection of the skyline in the waters of Lake Clara Meer. Consider the coordinate plane, which contains the graph of Bank of America Plaza's reflection.



Sean Pavone/Shutterstock.com

1. The tip of the building's reflection can be modeled by an absolute value function. Use the graph to write the function h and its domain to represent the tip of the reflection.

2. As you walk around the park, your viewpoint of the reflection changes. Write a new function to represent the tip of the building's reflection if the reflection moves:

- **a** 2 units to the right
- **b** 4 units up
- **c** 3 units left and 0.5 units down

3. Bard claims that the absolute value function from Problem 1 can model the actual tip of the building (and not its reflection) by changing the constant values of the function to represent the vertex of the building. Do you agree with Bard? Explain your thinking.

STOP

Summary

In today's lesson . . .

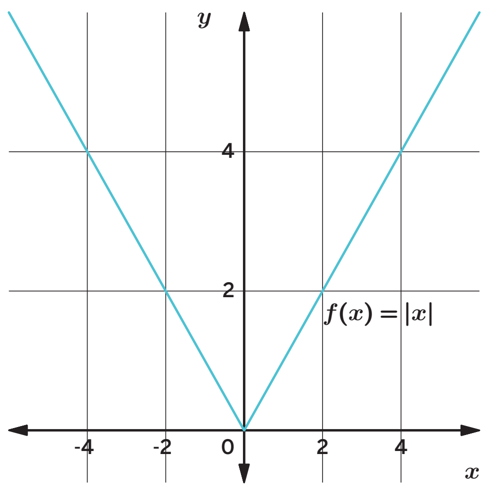
You observed that in a guessing game, each guess can be seen as an input value of a function and each absolute guessing error as an output value. Because absolute guessing error determines how far a guess is from a target number, the output values represent distances.

If the function f gives the distance of x from 0, it can be defined with the equation: $f(x) = |x - 0|$, or simply $f(x) = |x|$.

The function f is the ***absolute value function***. It gives the distance of x from 0 by determining the absolute value of x.

The graph of function f is a V shape with the two lines meeting at $(0, 0)$.

This point is called the ***vertex*** of the graph. It is the point where a graph changes direction, from decreasing to increasing when reading the graph from left to right.



Reflect:

Name: .... Date: .... Period: ....

Practice

1. The absolute value function can be defined using piecewise notation.

$$A(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Use this notation to determine each value.

a) $A(10)$

b) $A(0)$

c) $A(-3)$

d) $A(3.14159)$

e) $A(x) = 7$

f) $A(x) = -7$

2. Consider the four absolute value functions and three ordered pairs. Each ordered pair represents the vertex of the graph of an absolute value function. Match each function with the coordinates of its vertex. The vertex coordinates of the graph of one equation are not shown.

Function	Vertex coordinates
a) $p(x) = \lvert x - 9 \rvert$	.... $(-9, 0)$
b) $q(x) = \lvert x \rvert + 9$	.... $(9, 0)$
c) $r(x) = \lvert x + 9 \rvert$	.... $(0, -9)$
d) $t(x) = \lvert x \rvert - 9$	

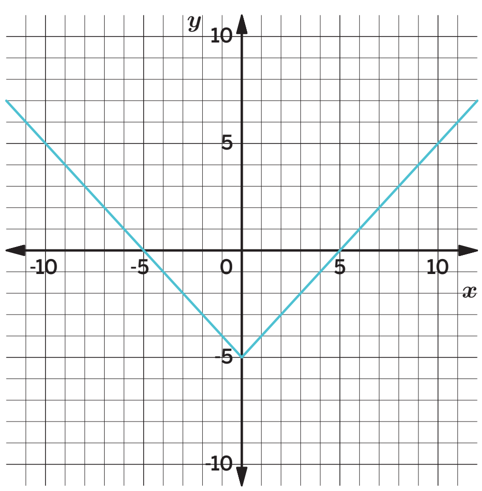
3. The graph of a function is shown. Which function represents the graph?

A. $f(x) = \lvert x \rvert - 5$

B. $f(x) = \lvert x \rvert + 5$

C. $f(x) = \lvert x - 5 \rvert$

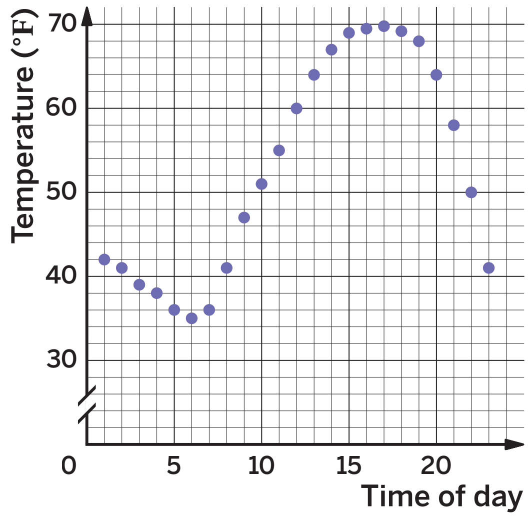
D. $f(x) = \lvert x + 5 \rvert$



Practice

Name: ______ Date: ______ Period: ______

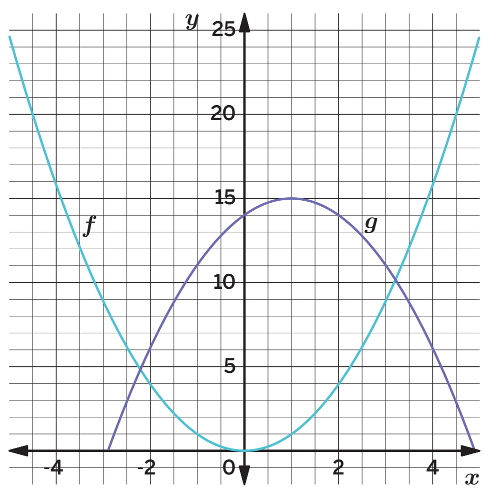
4. The temperature was recorded several times during the day. The function T gives the temperature in degrees Fahrenheit, n hours since midnight. The graph of this function is shown.



a Select two consecutive points and connect them with a line segment. Estimate the slope of that line. Explain what that estimated value means in this situation.

b Select two non-consecutive points and connect them with a line segment. Estimate the slope of that line. Explain what that estimated value means in this situation.

5. The graphs of functions f and g are shown. Identify at least two values of x at which the inequality $g(x) > f(x)$ is true.



6. The text "I LOVE ALGEBRA" is encoded using a cipher so that the text now reads "J MPWF BMHFCSB." How was the original message encoded?

SUB-UNIT

4 Inverses of Functions

Narrative Connections

How do you get Sunday shoppers to hear your song?

For much of the 20th century, Maxwell Street in Chicago was a lively outdoor marketplace. Immigrants from various countries in Europe sold goods from pushcarts and stalls. Anything you wanted to buy, from produce and pants, to pots and pans, could be found on Maxwell Street. There was also one unusual thing on offer: live blues music.

African-American blues musicians who had migrated to Chicago from the South came to Maxwell Street to play for the passing crowds in exchange for tips. Great musicians such as Muddy Waters and Howlin' Wolf could, at various times, be found busking for the crowd of a Sunday afternoon.

These artists faced an unusual challenge, playing not in a club or theater, but on a busy street in broad daylight, alongside vendors hawking and cars honking. How did they cut through the din and the chaos to make their music heard?

For many of them, the answer was to plug in their instruments and amplify them.

As a result, Chicago is known for its electric blues, and many credit the noise on Maxwell Street as one of the factors leading blues artists to switch from acoustic instruments to electrified, amplified ones. You can think of the acoustic sound as an input and the amplified sound as an output. But what if you heard the amplified sound and wanted to know what the acoustic version sounded like? For that, you'd need inverses of functions, which you'll encounter in these next few lessons.

Unit 3 | Lesson 18

Inverses of Functions

Let's explore what happens when the input and output values trade places.

Warm-up Center of Blues

A major city in the midwest is considered a center of blues music in America. In this city, one of the largest open-air markets became the birthplace for this unique form of blues. Most early blues musicians in this city started out as street performers entertaining local residents who frequented the market.

The name of the city and the market have been converted into a code.

BGHBZFN'R LZWVDKK RSQDDS

Can you determine the name of the city and market in English? How was the original message encoded?

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Encoder and Decoder

Encoded messages make for a fun puzzle, like the one you saw in the Warm-up. They have been used throughout our country's history for secret messages, such as in Chicago during the time of prohibition of alcohol. People used secret codes to sell alcohol, and these secret codes were cracked by the FBI.

The inventor Hedy Lamarr patented technology to prevent secret radio signals from being detected and jammed in the mid-1900s, and to this day, codebreakers and hackers are sought out by government agencies. Lets see how you do as an encoder and decoder!

1. It's your turn to write a secret code!

a Write a short and friendly message using 3–4 words.

b Select a number from 1 to 10. Encode your message by shifting each letter that many steps forward or backward in the alphabet, wrapping around from Z to A as needed. Consider using these tables to create a key for your encoded text.

Plain text	A	B	C	D	E	F	G	H	I	J	K	L	M
Encoded text													

Plain text	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Encoded text													

c Give your encoded message to a partner to decode. If requested, give the number you used.

d Decode the message from your partner. Ask for their number, if needed.

Activity 1 Encoder and Decoder (continued)

2. Suppose m and c each represent the position number of a letter in the alphabet, but m represents the letters in the original message and c represents the letters in your secret code.

   a. Complete the table.

Letter in message	F	I	S	H
m	6	9	19	8
c				
Letter in code				

   b. Use m and c to write an equation that can be used to encode an original message into your secret code.

   c. Use m and c to write an equation that can be used to decode your secret code into the original message.

Two relationships are ***inverses*** of each other if their input-output pairs are reversed, so that if one function takes a as an input and gives b as an output, then its inverse takes b as an input and gives a as an output.

3. Are the two equations you wrote inverses of each other? Explain your thinking.

Featured Mathematician

Hedy Lamarr

Many people knew Hedy Lamarr as a beautiful actress of the mid-1900s, but she was also an innovative inventor who created signal technology still used today. Lamarr realized that by transmitting radio signals along rapidly changing frequencies, these signals would be much less likely to be detected or "jammed." This technology was highly useful for the American military's radio-guided weapons and is a basis of cell-phone signal technology used today.

"Hedy Lamarr in 'The Heavenly Body'. Movie of MGM (1944)" by MGM Public Domain via Wikimedia Commons.

Name: ________ Date: ________ Period: ________

Activity 2 Exchange Rates

In the early 1960s, many rock and roll bands from Great Britain, such as the Rolling Stones, were heavily influenced by Chicago blues artists.

Suppose an American musician tours in Great Britain and exchanges U.S. dollars for British pounds. At the time of his travel, 1 U.S. dollar can be exchanged for 0.74 British pounds. At the same time, a British musician tours in the United States and she exchanges British pounds for U.S. dollars at the same exchange rate.

1. Determine the amount of money in British pounds that the American musician would receive if he exchanged:

a 100 U.S. dollars

b 500 U.S. dollars

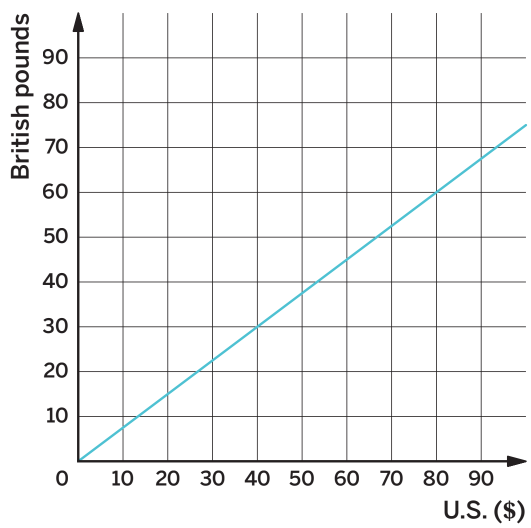
2. Write an equation that gives the amount of money in British pounds b as a function of the U.S. dollar amount d being exchanged.

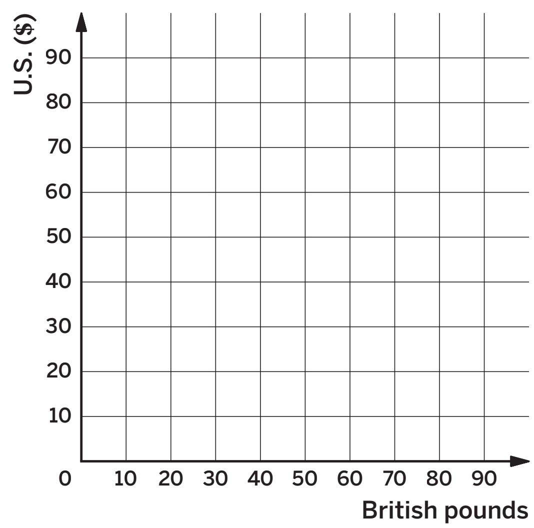
3. Determine the amount that the British musician would receive if she exchanged:

a $1{,}000$ British pounds

b $5{,}000$ British pounds

4. Consider the graph of the equation that gives the amount of money in British pounds b, as a function of the U.S. dollar amount d. Graph its inverse.





What do you notice?

5. Explain why it might be helpful to write the inverse of the function you wrote earlier. Then write an equation that defines the inverse.

STOP

Summary

In today's lesson . . .

You encoded a message, and then attempted to determine another student's method and decode their message. To encode a message, you took an original letter as your input, changed it according to your method chosen, and then had an encoded letter as your output. To decode a message, this process was reversed. The input became the encoded letter, and the output became the original letter. Decoding and encoding is an example of an inverse relationship.

The ***inverse of a function*** reverses the input and output values of a function so that the original output is now the input. In general, if a function takes x as its input and gives y as its output, its inverse function takes y as the input and gives x as the output.

Reflect:

Name: ____________ Date: ____________ Period: ____________

Practice

1. Noah's cousin is exactly 7 years younger than Noah. Let C represent Noah's cousin's age and N represent Noah's age. Ages are measured in years.

   a. Write a function that defines the cousin's age as a function of Noah's age. What quantities represent the input and output of this function?

   b. Write the inverse of the function you wrote in part a. What quantities represent the input and output of this inverse?

2. Tyler's brother is exactly 7 years younger than Tyler. Let B represent Tyler's brother's age in months and T represent Tyler's age in years.

   a. If Tyler is 15 years old, how old is his brother, in months?

   b. When Tyler's brother is 132 months old, how old is Tyler, in years?

   c. Write a function that gives the age of Tyler's brother in months, as a function of Tyler's age in years.

   d. Write the inverse of the function you wrote in part c. What quantities represent the input and the output of this inverse?

3. Each equation represents a function. For each, write the equation that represents the inverse of the function.

   a. $c = w + 3$

   b. $y = x - 2$

   c. $y = 5x$

   d. $w = \frac{d}{7}$

Practice

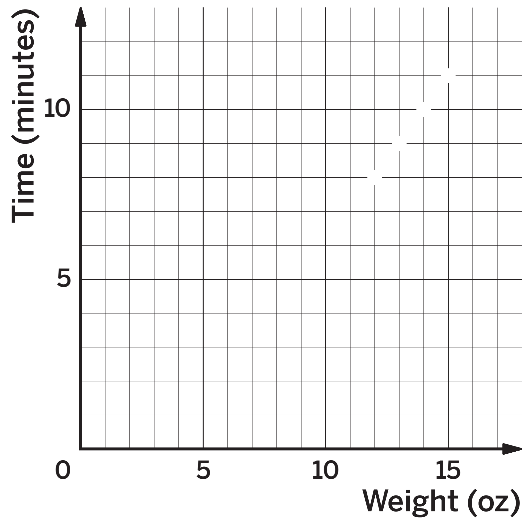
Name: ________________ Date: ________ Period: ________

4. The instructions for cooking a steak with a pressure cooker can be represented with this set of rules, where x represents the weight of a steak in ounces and $f(x)$ represents the cooking time in minutes.

$$f(x) = \begin{cases} 7, & 8 \leq x \leq 12 \\ 8, & 12 < x \leq 13 \\ 9, & 13 < x \leq 14 \\ 10, & 14 < x \leq 15 \\ 11, & 15 < x \leq 16 \end{cases}$$

a. Describe the instructions in words so that they can be followed by someone using the pressure cooker.

b. Graph the function f on the coordinate plane.



5. The absolute value function $Q(x) = |x|$ gives the distance from 0 of the point x on the number line.

Q can also be defined using piecewise notation: $Q(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Determine whether each point is on the graph of Q. For each point that you determine is not on the graph of Q, change the output coordinate so that the point is on the graph of Q.

a. $(-3, 3)$

b. $(-72, 72)$

c. $(0, 0)$

d. $\left(\frac{4}{5}, -\frac{4}{5}\right)$

e. $(-5, -5)$

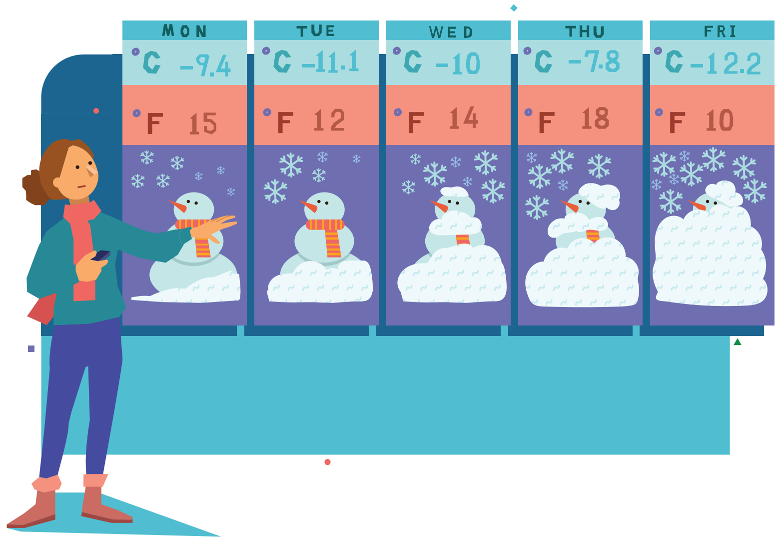
6. Solve the equation $y = \frac{2}{3}x$ for x. Show your thinking.

Name: ______________________ Date: __________ Period: __________

Unit 3 | Lesson 19

Finding and Interpreting Inverses of Functions

Let's determine the inverses of linear functions.



Warm-up Vinyl Record Sales

In 2020, vinyl record sales made more money than CD sales, continuing the trend of vinyl record's increase in popularity over the past couple of decades. Many listeners find that music on vinyl sounds better compared to music played through streaming services.

Nattapol Meechart/Shutterstock.com

Lin compares the price of vinyl records from different online stores.

- **Store A: $20 each and offers free shipping.**
- **Store B: $20 each and charges $5 for shipping.**
- **Store C: p dollars each and charges $5 for shipping.**
- **Store D: p dollars each and charges f dollars for shipping.**

1. Write an equation to represent the total price T in dollars as a function of n records bought at each store.

a Store A **b** Store B **c** Store C **d** Store D

2. Write an equation to determine the number of records n that Lin could buy if she spent T dollars at each store.

a Store A **b** Store B **c** Store C **d** Store D

Log in to Amplify Math to complete this lesson online.

Activity 1 Chicago's Cold Weather Blues

Chicago is known for its intensely cold winters. Some mid-1900s Chicago-based blues artists wrote songs to help make it through the dark and cold days of winter such as Muddy Water's "Cold Weather Blues" and Sonny Boy Williamson's "Nine Below Zero."

If you know the temperature of Chicago in degrees Celsius C, you can determine the temperature in degrees Fahrenheit F using the equation: $F = \frac{9}{5}C + 32$. The table shows the daily low temperatures in Chicago for the first week of January 2020.

C (°C)			−10			−22.2	
F (°F)	15	12		18	10		2

1. Complete the table with the temperatures in degrees Fahrenheit or degrees Celsius.
2. The equation $F = \frac{9}{5}C + 32$ represents a function. Write an equation to represent the inverse if degrees Fahrenheit is now the input and degrees Celsius is the output.
3. The equation $R = \frac{9}{5}(C + 273.15)$ defines the temperature in degrees Rankine as a function of the temperature in degrees Celsius. Show that the equation $C = (R - 491.67) \cdot \frac{5}{9}$ defines the inverse if degrees Rankine is the input and degrees Celsius is the output.

Are you ready for more?

The temperature was so cold in Chicago one day that the temperature was the same in degrees Fahrenheit and degrees Celsius. What was the temperature in degrees Fahrenheit? Explain or show your thinking.

Name: ______ Date: ______ Period: ______

Activity 2 Info Gap: Merchandise Sales

With the popularity of internet streaming and digital music services, musicians are making less money from music sales today than ever before. Many musicians now focus on live performances, advertising, and merchandise sales to increase their income.

You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.

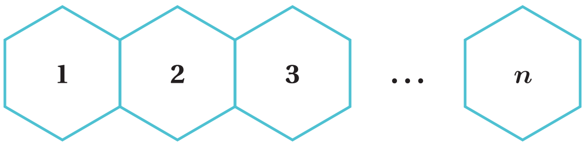
If are given the *data card*:	If are given the *problem card*:
1. Silently read the information on your card.	**1.** Silently read your card and think about what information you need to solve the problem.
2. Ask your partner, "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card.	**2.** Ask your partner for the specific information that you need.
3. Before telling your partner the information, ask, "Why do you need to know (that piece of information)?"	**3.** Explain to your partner how you are using the information to solve the problem.
4. Read the problem card, and solve the problem independently.	**4.** When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Share the data card, and discuss your thinking.	**5.** Read the data card, and discuss your thinking.

Discussion Support:
Ask your partner for more details using these prompts:

- "How do you know what the band can afford?"
- "How does the equation show that?"
- "Does my reasoning make sense?"

Activity 3 Tables and Seats

At a blues club, hexagonal tables are placed side by side to form one long line of tables, as shown here. One seat is placed at each side of the table.



1. Write an equation that represents the number of seats S as a function of the number of tables n.

2. What domain and range make sense for this function?

3. Write an equation to represent the inverse of the function you wrote in Problem 1. What is the input and output of the inverse equation?

4. How many tables are needed if the following number of people are attending a show? Explain your thinking.

a 94 people

b 95 people

5. What is the domain of the inverse that you found in Problem 3? Is it the same set of values as the range of the original function? Explain your thinking.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You wrote the equation that represents the inverse of a function. Just as a function tells you the output value when you know the input value, you can use the inverse of a function to determine the input value when you know the function's output value.

You determined the inverse of a function by isolating the value that represents the input. You determined the equation defining the inverse by reversing the process that defined the original function.

Consider the function $p = 15t + 300$. By solving the equation for t, you can determine that the inverse of the function is $t = \frac{p - 300}{15}$. In the original function, the input is t and the output is p. In the inverse, the input is p and the output is t. Recall that the inverse reverses the function's input and output.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Tickets to a family concert cost $\$10$ for adults and $\$3$ for children. The concert organizers collected a total of $\$900$ from ticket sales.

   a. In this situation, what is the meaning of each variable in the equation $10A + 3C = 900$?

   b. If 42 adults were at the concert, how many children attended? Show your thinking.

   c. If 140 children were at the concert, how many adults attended? Show your thinking.

   d. Write an equation to represent C in terms of A. What is the input and output?

   e. Write an equation to represent A in terms of C. What is the input and output?

2. A school club has $\$600$ to spend on t-shirts. The club is buying from a store that gives it a $\$5$ discount off the regular price per shirt.

   $n = \dfrac{600}{p-5}$ represents the number of shirts n that can be purchased at a regular price p.

   $p = \dfrac{600}{n} + 5$ represents the regular price p of a shirt when n shirts are purchased.

   a. What is n when p is 20?

   b. What is p when n is 40?

   c. Is one equation an inverse of the other? Explain your thinking.

3. Functions f and g are inverses, and $f(-2) = 3$. Is the point $(3, -2)$ on the graph of f, on the graph of g, or neither? Explain your thinking.

Name: .... Date: .... Period: ....

Practice

4. Elena plays the piano for 30 minutes each practice day. The total number of minutes p that Elena practiced last week is a function of n, the number of practice days. Determine the domain and range for this function.

5. Two children set up a lemonade stand in their front yard. They charge $\$1$ for every cup. They sell a total of 15 cups of lemonade. The amount of money the children earned, R dollars, is a function of the number of cups of lemonade they sold n.

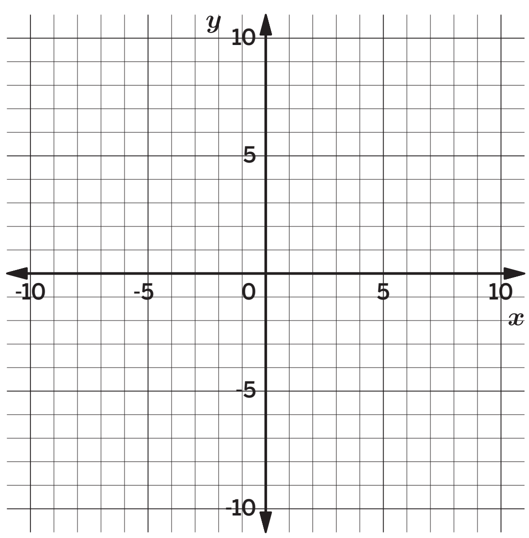
   a. Is 20 part of the domain of this function? Explain your thinking.

   b. What does the range of this function represent?

   c. Describe the set of values in the range of R.

   d. Is the graph of this function discrete or not discrete? Explain your thinking.

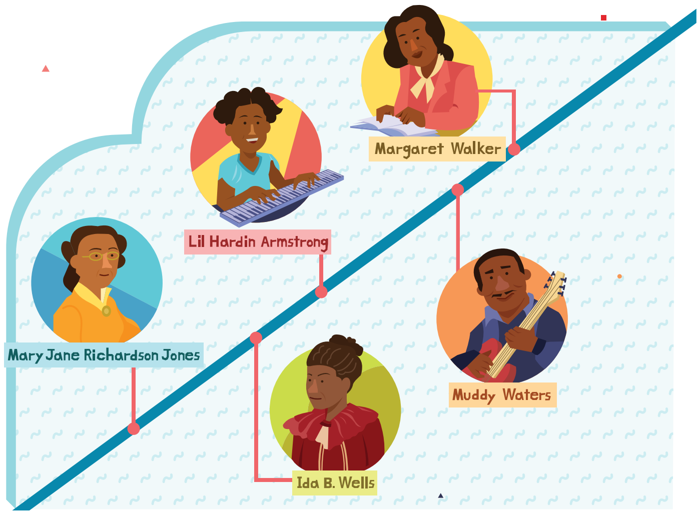
6. Graph $y = 2x + 6$ and $y = \frac{1}{2}x - 3$. What do you notice about the equations and the graphs?



Unit 3 | Lesson 20

Writing Inverses of Functions to Solve Problems

Let's use inverses of functions to solve real-world problems.



Warm-up The Great Migration

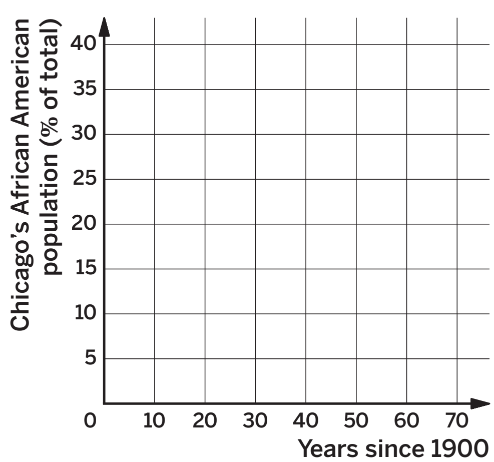
The Great Migration was the movement of more than 6 million African Americans from the rural South to more urban areas in the North, Midwest, and West from 1900 to 1980. It was caused by poor economic conditions and racial discrimination at the time. The Great Migration brought African-American musicians to Chicago, along with their traditional jazz and blues music, resulting in the development of the "Chicago blues."

The function $p(t) = 1.5 + 0.5t$ represents the percentage of Chicago's population that was African American, t years since 1900, during the Great Migration.

1. How did the African American population of Chicago change during this time period?

2. What does $p(t)$ represent? Is $p(t)$ the input or the output of this function?

3. Sketch a graph of the function.



Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Another Look at the Great Migration

The function from the Warm-up, $p(t) = 1.5 + 0.5t$, represents the percentage of Chicago's population that was African American, t years since 1900, during the Great Migration.

1. What does the value 1.5 represent in this function?

2. What does the value of $p(0)$ represent?

3. Determine the percentage of the Chicago population that was African American in each of the following years, based on this function.

   a. 1920

   b. 1980

4. In which year does the function predict that the percentage of African Americans in Chicago reached 20%?

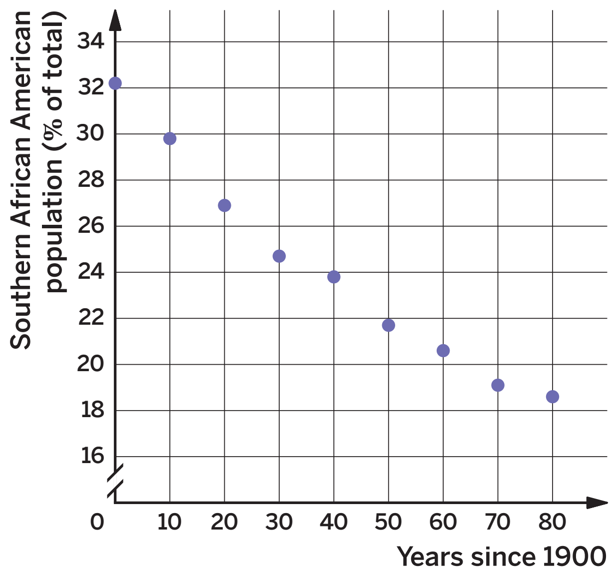
5. In this situation, what information can be determined using the inverse of function p?

6. Determine the inverse of function p, using t to represent the inverse of the function. Be prepared to explain your thinking.

Activity 2 The Great Migration From the South

As African Americans migrated to other areas of the U.S., the percentage of the Southern population that was African American decreased. The table and graph show how many African Americans lived in the South from 1900 to 1980, along with the corresponding percentage of the total Southern population.

Years since 1900	Percentage
0	32.2
10	29.8
20	26.9
30	24.7
40	23.8
50	21.7
60	20.6
70	19.1
80	18.6



1. Suppose a linear function P gives the number of African Americans in the South, as a percentage of the total Southern population, as a function of years t since 1900. Draw a line of fit to represent this function. Write the equation for your linear model.

2. Use your equation to determine the value of the expression $P(65)$. Explain what it means in this situation.

3. Use your equation to determine the value of t that makes the function notation statement $P(t) = 35$ true. What does this solution represent in context?

Name: .... Date: .... Period: ....

Activity 2 The Great Migration From the South (continued)

4. Suppose you want to know when the Southern population that was African American was a certain percentage, or when it might be predicted to be that percentage. What equation could you write to help determine when these percentages occurred (or will occur)? Explain your thinking.

Are you ready for more?

How well do you think your linear model would represent the percentage of the Southern population that was African American between 1980 and 2000? Explain your thinking.

Activity 3 The Electric Guitar

The Chicago blues expanded blues music with the inclusion of the electric guitar. Electric guitars have metal strings that run from the guitar's neck to its body. Sound is made by plucking the strings, causing them to vibrate.

The function $f(v) = \frac{v}{2L}$ represents the pitch of a sound, measured in Hertz (Hz), made by a guitar string that is L meters long, and on which vibrations move at a speed of v meters per second.

4 PM production/Shutterstock.com

1. Write a function f that relates the pitch v and the vibrating speed of a 0.65 m long guitar string.

2. What does the expression $f(80)$ represent?

3. Use your function from Problem 1 to determine the value of v that would make the function notation statement $f(v) = 110$ true. What does this solution represent?

The vibrating speed of a guitar string is affected by the tension in the string, which can be adjusted by turning the knobs on a guitar.

4. A musician adjusts the tension in a 0.65 m long guitar string. She plucks the string and measures the pitch to be 80 Hz.

   a. Which equation should she use to determine the vibrating speed of the string? f or the inverse of f? Explain your thinking.

   b. Use the equation you chose to determine the string's vibrating speed that produces a pitch of 80 Hz. Explain your thinking.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You determined the inverses of functions that were given in function notation. You wrote linear functions to model data, determined the inverses of these functions, and used both functions and their inverses to solve problems in context.

The inverse of a function can be useful when the function's output values are known, and you are determining the input values that produce these output values. Substituting an output value into the inverse of a function will efficiently determine its corresponding input value.

Consider the function $P(w) = 30 + 2w$, which represents the perimeter P of a rectangle with a fixed length of 15, given the width w. The input is w and the output is P. The inverse of this function can be written as the equation $w = \frac{P - 30}{2}$, where the input is P and the output is w.

You can use the inverse equation to efficiently determine the width of the rectangle, given the perimeter.

Reflect:

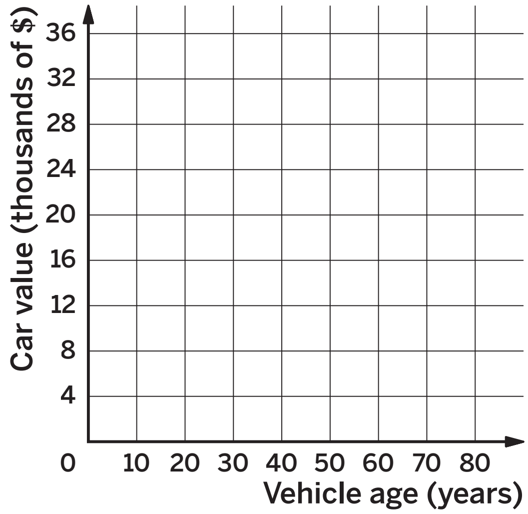
Practice

Name: .... Date: .... Period: ....

1. The table shows the value of a car, in thousands of dollars, each year after it was purchased.

a Plot the data values, and draw a line that fits the data.

Age (years)	Value (thousands of $)
0	30.0
1	22.5
2	19.0
3	16.0
4	13.5
5	11.4



b Use your line to write an equation for the linear function C that gives the value of the car, in thousands of dollars, when its age is t years.

c What is the value of $C(6)$? What does it mean in this situation?

d For what value of t is the function notation statement $C(t) = 2$ true? Interpret this within the context of the situation.

e Write an equation that would predict the age of the car when $C(t)$ is known.

f Use your equation from part e to predict the vehicle age when the value of the car will be $500.

Name: .... Date: .... Period: ....

Practice

2. The distance d, in kilometers, that a car travels at a speed of 80 km per hour, for t hours, is given by the equation $d = 80t$.

   a. If the car has traveled 120 km, how long has it been traveling?

   b. Rewrite the equation to represent time t as a function of distance d.

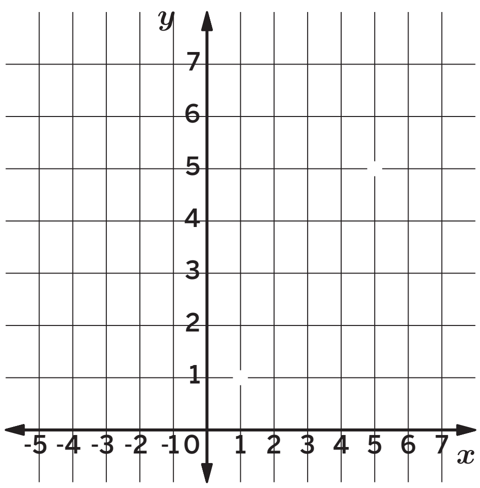
3. Match each function to its inverse.

Function	Inverse
a. $r = 2t - 3$	.... $t = \frac{r+2}{3}$
b. $r = 3t$	.... $t = \frac{r+3}{2}$
c. $r = 3t - 2$	.... $t = r - 2$
d. $r = t - 2$	.... $t = r + 2$
e. $r = t + 2$	.... $t = \frac{r}{3}$

4. Refer to these rules that define the function f.

$$f(x) = \begin{cases} 2, & -5 \le x \le 1 \\ x, & 1 < x < 5 \\ 7, & 5 \le x \le 7 \end{cases}$$

Draw the graph of f.



5. The input is k and the output is j for the following functions. Determine an equation to represent the inverse of each function.

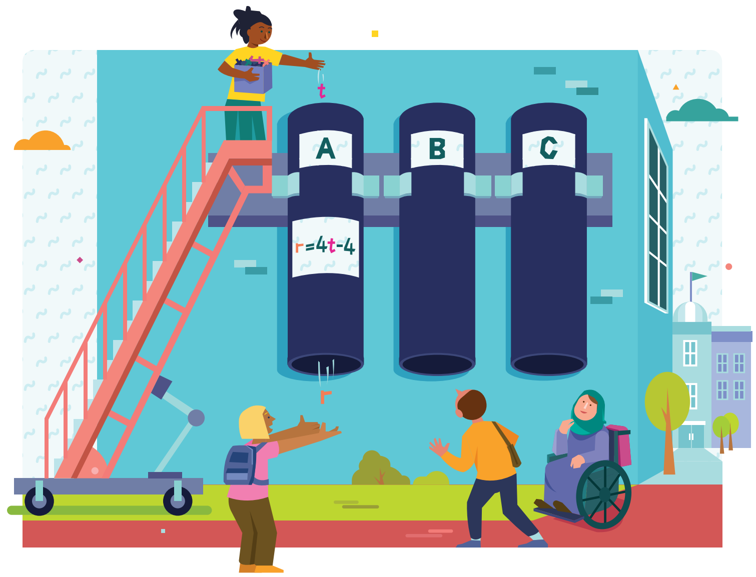
   a. $j = 4k$

   b. $j = 2k + 10$

Unit 3 | Lesson 21

Graphing Inverses of Functions

Let's examine the relationship between the graph of a function and its inverse.



Warm-up A Function and Its Inverse

Here are several functions with the variable that represents the input and output of each function.

Function A: $r = 4t - 4$
Input: t
Output: r

Function B: $p = -3f + 9$
Input: f
Output: p

Function C: $h = \frac{1}{2}k + 3$
Input: k
Output: h

1. Determine an equation to represent the inverse of each function. Identify the variable that represents the input and output values of the inverse.

a. Function A:

Input:
Output:

b. Function B:

Input:
Output:

c. Function C:

Input:
Output:

2. How are the input and output values related for each function and its inverse?

3. Here is a pair of input and output values for Function A. Complete the table for the inverse of this function.

Function

Input	Output
0	−4

Inverse

Input	Output

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Input and Output Values of an Inverse

Use the functions and their inverses you determined in the Warm-up to complete the following problems.

1. Complete the table of values of the function and its inverse. For Function C, you will select the input values to complete the table.

Function A

Input	Output
0	
1	
2	
3	
4	

Inverse of Function A

Input	Output
	0
	1
	2
	3
	4

Function B

Input	Output
–2	
0	
2	
4	
6	

Inverse of Function B

Input	Output

2. How did you complete the table of values for the inverse of each function?

Function C

Input	Output

Inverse of Function C

Input	Output

3. What do you notice and wonder about the table of a function and its inverse?

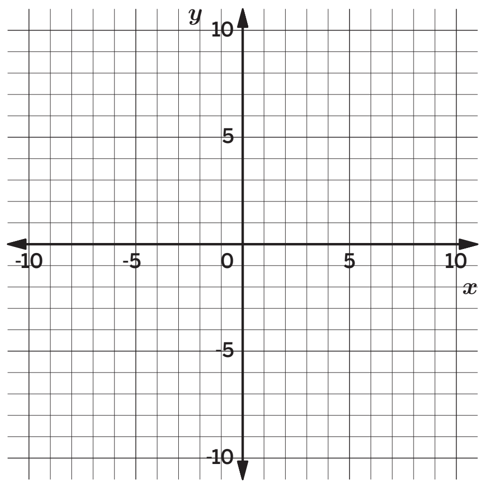
a I notice . . .

b I wonder . . .

Activity 2 The Graph of a Function and Its Inverse

You and your partner will be assigned one of the functions and its inverse from Activity 1. Use graphing technology and let x represent the input of the original function and y represent the output.

1. Use the table of values from Activity 1 to graph your function and its inverse on the same coordinate plane. Label each line.



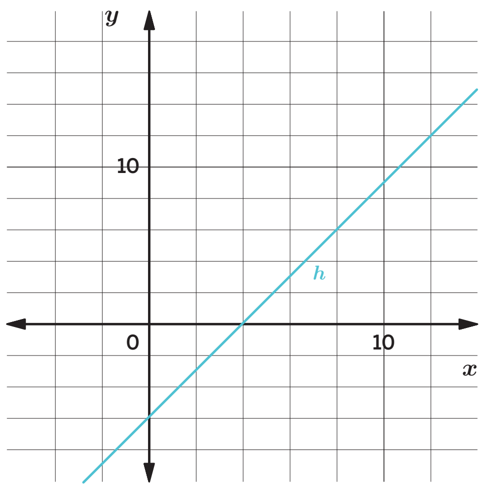
2. Examine your findings from Activity 1 and the graph of your function and its inverse. What do you notice? What do you wonder?

a I notice . . .

b I wonder . . .

3. If you are given just the graph of a function, how could you draw the graph of the inverse of the function?

4. Refer to the graph of h. Use your response to Problem 3 to sketch the graph of its inverse on the same coordinate plane. Explain the strategy you used to sketch the inverse.



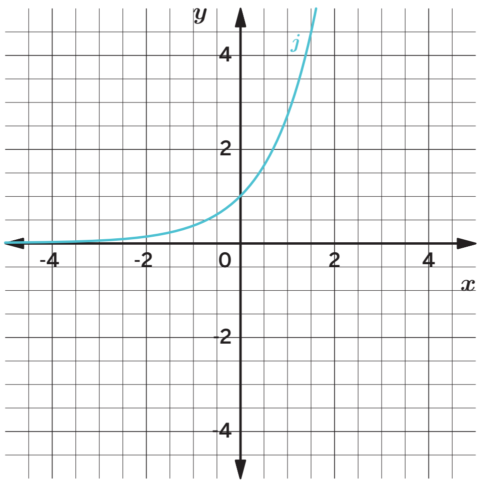
Reflect: How did the structure of the coordinate grid make graphing a function and its inverse less stressful?

Name: ______ Date: ______ Period: ______

Activity 3 The Inverses of Nonlinear Functions

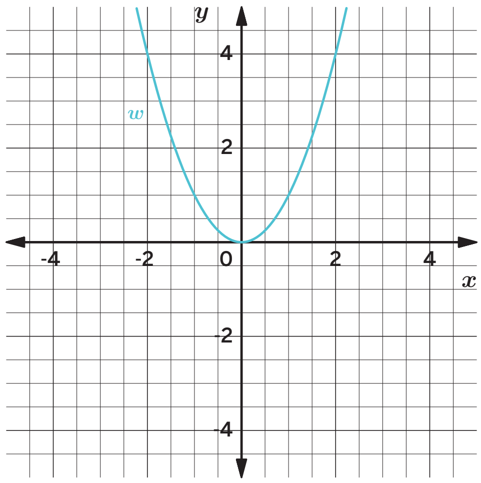
The same strategies used to graph the inverse of linear functions can be used to graph the inverse of nonlinear functions.

Refer to the graph of the function j.



1. Sketch the inverse of j on the same coordinate plane.
2. Is the inverse of j a function? Explain your thinking.

Refer to the graph of the function w.



3. Sketch the inverse of w on the same coordinate plane.
4. Is the inverse of w a function? Explain your thinking.

Are you ready for more?

Looking at just the graph of a function, how can you determine whether its inverse is also a function?

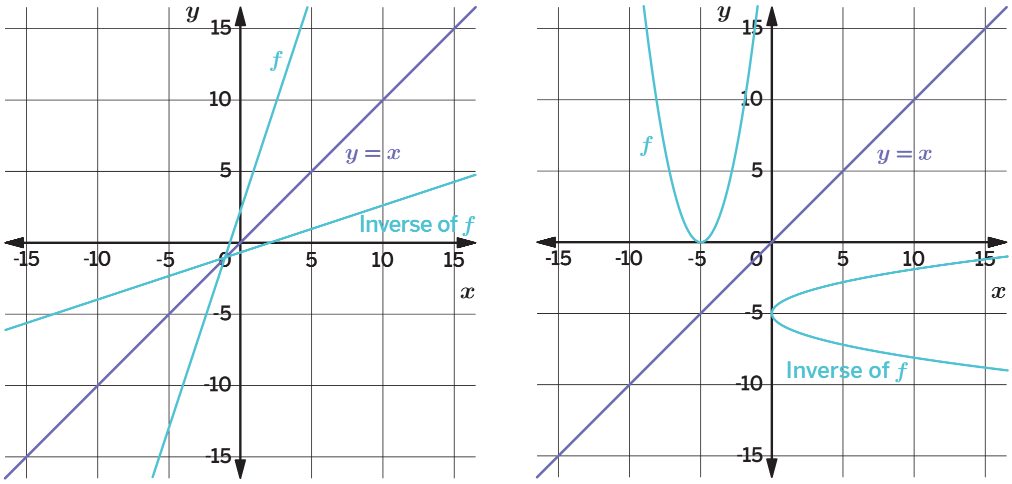
STOP

Summary

In today's lesson . . .

You graphed the inverse of a function by switching the input-output pairs of the function and plotting these ordered pairs. You noticed that switching the coordinates of the points is the same as reflecting the graph of a function across the line $y = x$ to produce the graph of its inverse.

If every input value of an inverse produced only one output value, then the inverse is also a function. This is not always the case, so sometimes the inverse of a function is itself *not* a function.



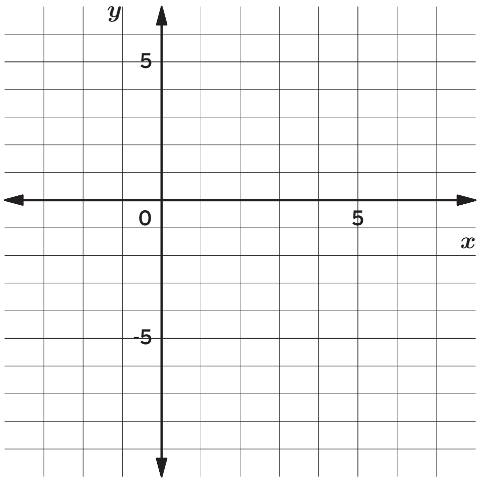
Reflect:

Name: .... Date: .... Period: ....

Practice

1. Clare attempts to determine the inverse of the function $f(x) = 6x - 12$. She graphs the inverse of f, and determines the graph of the inverse of f is a line with a slope of -6 and a vertical intercept of 12, because the inverse is found by reversing the operations of the original function. Do you agree with Clare? Explain your thinking.

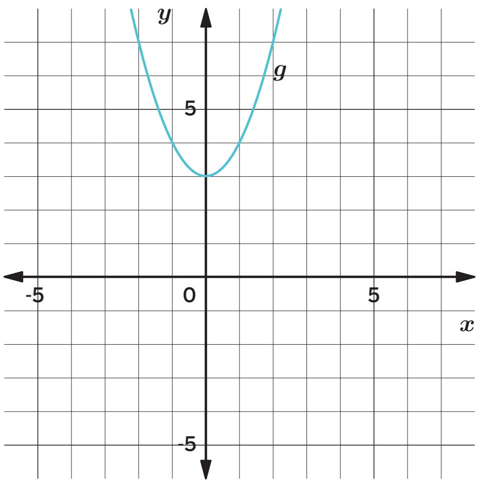
2. Graph the function $h(x) = 3x - 9$ and its inverse on the coordinate plane. Be sure to label each graph.



3. Consider the graph of the function g.

   a Graph the inverse of g.

   b Is the inverse of g a function? Explain your thinking.



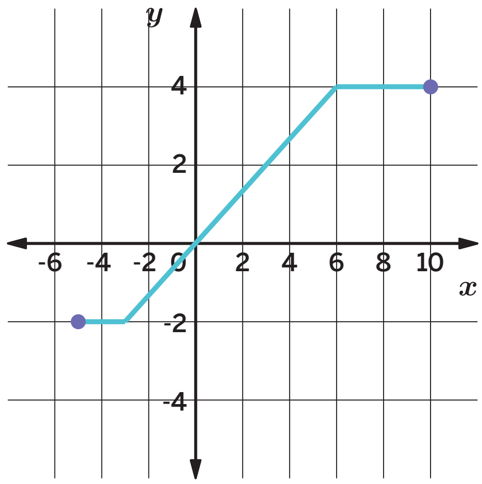
Name: .... Date: .... Period: ....

4. The functions h and j are inverses. When x is -10, the value of $h(x)$ is 7, or $h(-10) = 7$.

a What is the value of $j(7)$?

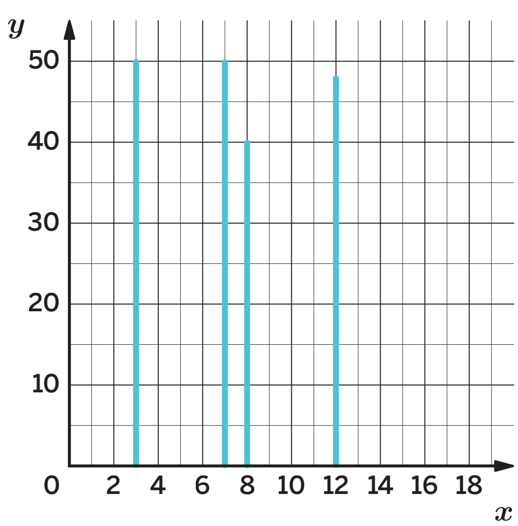
b Is $(-10, 7)$ on the graph of h, on the graph of j, or neither? Explain your thinking.

5. Refer to the graph of the function shown. Represent the domain and range of the function using interval notation.



6. The graph models the skyline of two buildings, but the graph is incomplete. The piecewise function shown models the missing sections of the graph. Graph the piecewise function to complete the skyline of the buildings.

$$f(x) = \begin{cases} |x - 5| + 48, & 3 < x < 7 \\ 2x + 24, & 8 < x < 12 \end{cases}$$



Name: ______ Date: ______ Period: ______

Unit 3 | Lesson 22 – Capstone

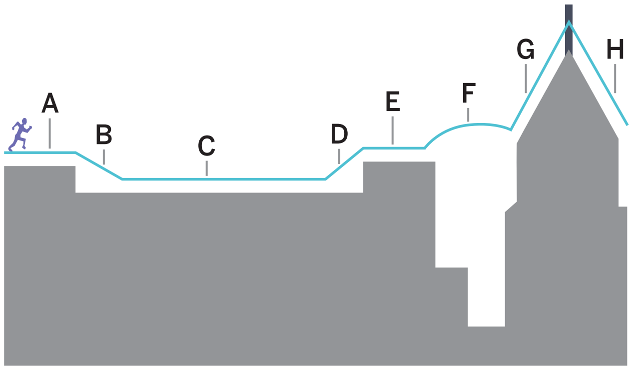
Freerunning Functions

Let's create piecewise functions using descriptions of their key features and parts of their graphs.

Warm-up Freerunning

Freerunning is a sport where athletes travel from one point to another as fast as possible by running, swinging, jumping, and climbing. Freerunners often run up and across buildings and rooftops, as they cautiously traverse cityscapes and skylines.

Consider the path of a freerunner crossing over part of the Memphis skyline. Each section of the freerunner's path is labeled.



1. Determine the section(s) of the freerunner's path that are:

   a. Increasing

   b. Decreasing

   c. Constant

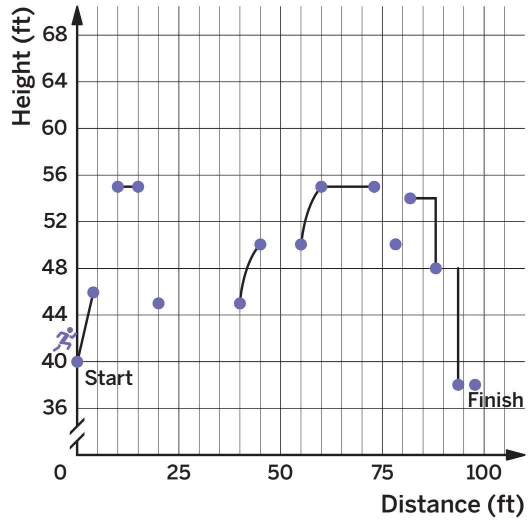
2. Between which two sections is the global maximum contained?

3. The freerunner starts running at a height of 100 ft. Then, after running for 50 ft, she jumps down along the line represented by Section B. Write an equation that represents Section A.

Log in to Amplify Math to complete this lesson online.

Activity 1 Completing a Freerunnner's Path

Elena is a freerunner who is designing a route through her city. There are tops of buildings she has to run over, breaks between buildings she has to jump, and other obstacles to ascend and descend. So far, she has completed the following design of her route.



1. Sketch the missing pieces of her route using the graphs of linear, constant, and absolute value functions. Label each piece you sketch, "Section A," "Section B," etc.

2. Use your sketch to write a piecewise function that models the missing pieces of Elena's design.

3. Using interval notation, determine the intervals of her entire completed route that are:

   a. Increasing

   b. Decreasing

   c. Constant

4. What are the global maximum and global minimum of Elena's route?

5. Of the sections that you sketched, which one has the greatest average rate of change? Explain your thinking.

Name: .... Date: .... Period: ....

Activity 2 Creating a Freerunner Course

Freerunners often use the buildings and structures within a city or town. There are also freerunning competitions, in which freerunners compete against each other on a predesigned course with walls, raised platforms, and ramps.

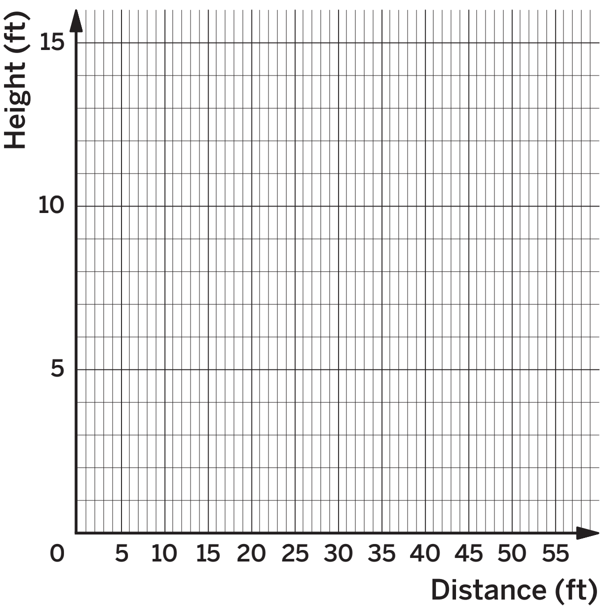
Krasovski Dmitri/Shutterstock.com

Suppose you are in charge of constructing a freerunning course for an upcoming freerunning competition. The route a successful freerunner takes on your course must meet the following requirements:

- **The freerunner starts at the origin and moves upward until they reach a constant piece.**
- **10 ft from the start, the freerunner falls vertically 2 ft.**
- **The range is $[0, 10]$.**
- **Two different absolute value functions model the course on the intervals $[14, 18]$ and $[41, 50]$.**
- **There is a local minimum of 1 ft at $x = 17$ ft and another of 5 ft at $x = 45$ ft.**
- **The course is constant over the intervals $[2, 6]$, $[10, 14]$, $[18, 22]$, $[36, 41]$, $[50, 52]$, $[52, 54]$, and $[54, 58]$.**

1. Sketch the route on the coordinate plane using a combination of linear, constant, and absolute value pieces.

2. After the route is complete, draw walls, raised platforms, ramps, and any other obstacles that you want to include in your course.



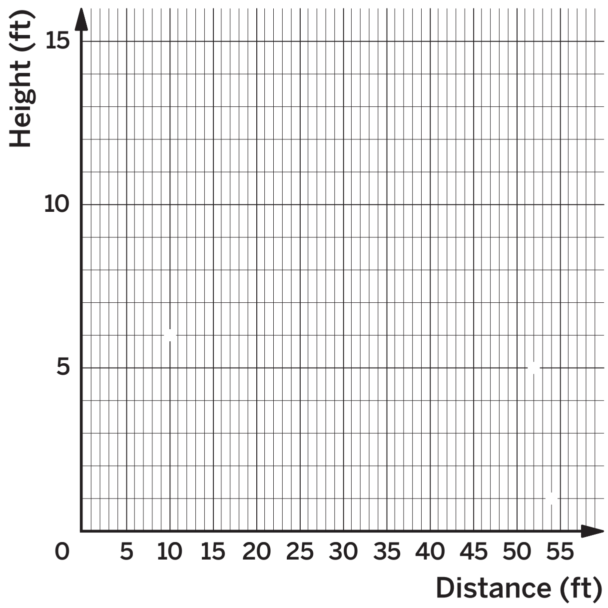
Activity 3 Checking the Course

You and your partner will now check the accuracy of each other's freerunner courses.

1. Using the graph that you created in Activity 2, write a piecewise function that represents the graph of your course (do not include any vertical pieces).

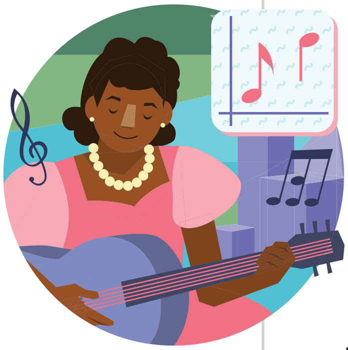
2. Trade books with your partner. You will now check your partner's course by graphing their piecewise function here, and checking to see that the graph meets all the criteria set in Activity 2.

3. Does the piecewise function meet all the criteria? If not, provide any feedback for your partner here.



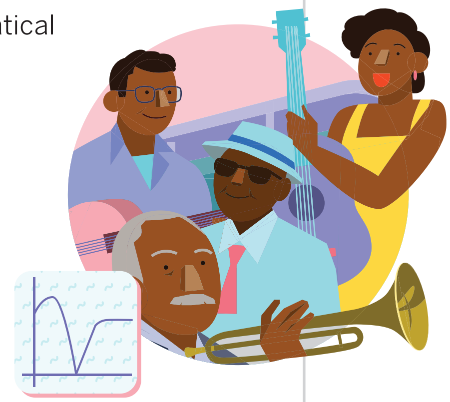
STOP

Unit Summary



People tend to think that math and science are one part of the human experience, while art and music are another. But that is not true! The two can weave and intermingle. Nowhere is that more plainly seen than in the world of functions.

Look at the breath-taking architecture of America's cities. Listen to the rhythms and grooves that play in its streets. These are things that can move our souls and give us a sense of place. And just as we find ourselves rooted in these places, their forms can be rooted in mathematical functions.



At its heart, functions help us mathematically describe relationships. And for different relationships there are different kinds of functions. Linear functions describe relationships with a constant rate of change, like a building's roof when it slants.

Meanwhile, piecewise and absolute value functions help model relationships that may have sudden, dramatic changes, like the staccato blasts of a brass band's right when the music starts.

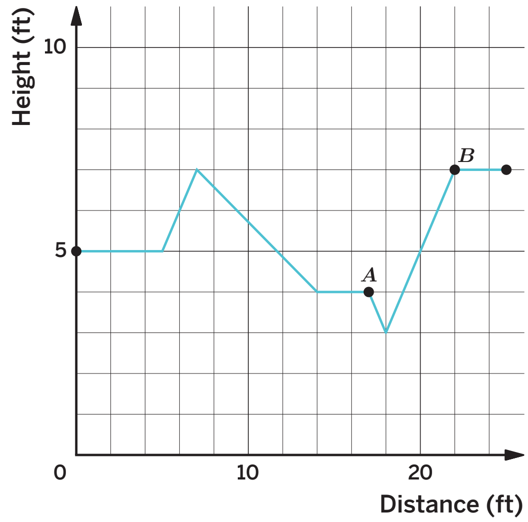
With functions, you can do more than hear the music. You can see it and give voice to precise changes and movements within a composition. Things like volume, pitch, rhythm, and tempo can all be described in terms of intervals, constants, domain, and range. And now that you've seen the math behind the music, maybe you can also hear the music behind the math!

See you in Unit 4.

Practice

Name: .... Date: .... Period: ....

1. A freerunner traverses a skyline. Her path is modeled in the graph.

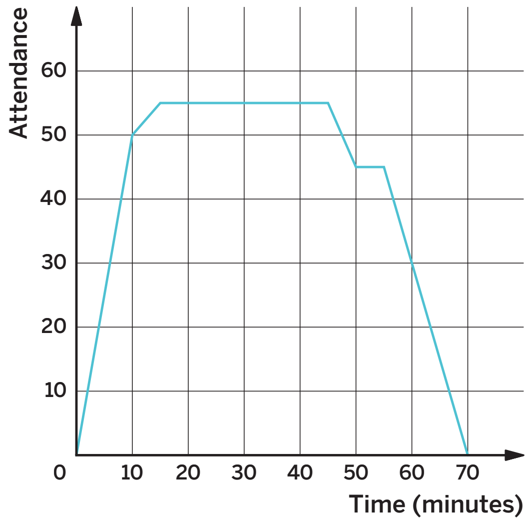


a. List the intervals in which her path is constant.

b. Write a function f with its domain, that represents the path between points A and B.

c. For the path between points A and B, what is the local maximum and local minimum?

2. The graph shows the attendance at a sports game as a function of time in minutes.



a. Describe the domain.

b. Describe the range.

c. Describe how the attendance changed over time.

3. Consider each function. Write an equation that represents the function's inverse. What is the input and output of the inverse?

a. $y(x) = 65 + 5x$

b. $P(n) = \frac{n}{3} - 1.2$

Name: ______________________ Date: ____________ Period: ____________

Practice

4. The number of chirps that crickets make is closely related to the temperature of their environment. When the temperature is between 12 and 38 degrees Celsius, it is possible to determine the temperature by counting the number of chirps! A formula that is commonly used to determine the temperature in degrees Celsius is to count the number of chirps in 25 seconds, divide by 3, and then add 4 to get the temperature.

Let m be the number of chirps that crickets make in 25 seconds and C be the temperature in degrees Celsius.

a What is the temperature when 84 chirps are heard in 25 seconds?

b Write an equation that defines C as a function of m.

c How many chirps would you expect to hear in 25 seconds when it is $14°\text{C}$?

d Write an equation that defines the inverse of the function you wrote. Explain what the inverse can determine about the situation.

5. A college student borrows $\$360$ from his cousin to repair his car. He agrees to pay $\$15$ per week until the loan is paid off.

a Function L represents the amount owed w weeks after the student borrows the money. Write an equation to represent this function using function notation.

b Write an equation to represent the inverse of function L. Explain what information it tells you about the situation.

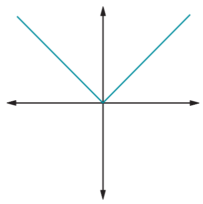
c How many weeks will it take the student to pay off the loan?

Glossary/Glosario

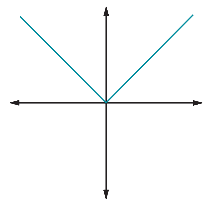
English | **Español**

A

absolute value function A function whose output value is the distance of its input value from 0. In other words, the absolute value function is a piecewise function that takes negative input values and makes them positive.



función de valor absoluto Función cuya salida es la distancia entre su entrada y 0. En otras palabras, la función de valor absoluto es una función definida a trozos que toma entradas negativas y las hace positivas.



association When a change in one variable suggests another may change as well, the variables have an *association* and are said to be *associated* with one another.

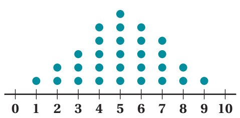
asociación Cuando un cambio en una variable sugiere que otra también podría cambiar, las variables tienen una *asociación* y están *asociadas* entre sí.

average rate of change The ratio of the change in the outputs to the change in the inputs, for a given interval of a function.

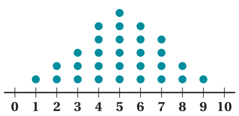
tasa de cambio promedio Razón entre el cambio de las salidas y el cambio de las entradas para un determinado intervalo de una función.

B

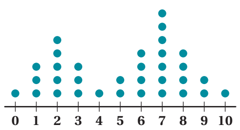
bell shaped A distribution that looks like a bell, with most of the data near the center and fewer points farther from the center, is called *bell shaped*.



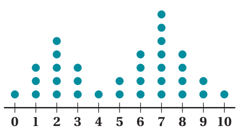
acampanada Una distribución que asemeja a una campana, con la mayoría de los datos cerca del centro y una menor cantidad de puntos más lejos del centro, es llamada *acampanada*.



bimodal A distribution with two distinct peaks is called *bimodal*.



bimodal Una distribución con dos picos distintivos es llamada *bimodal*.



boundary line The line that represents the boundary between the region containing solutions and the region containing non-solutions for an inequality.

línea límite Línea que representa el límite entre la región que contiene soluciones a una desigualdad y la región que contiene no-soluciones.

Glossary/Glosario

English	Español
C	**C**
categorical variable A variable that can be partitioned into groups or categories.	**variable categórica** Variable que puede partirse en grupos o categorías.
causation When a change in one variable is shown, through careful experimentation, to cause a change in another variable.	**causalidad** Cuando se muestra que un cambio en una variable causa un cambio en otra variable, a través de cuidadosa experimentación.
common difference The difference between two consecutive terms in a linear pattern.	**diferencia común** Diferencia entre dos términos consecutivos de un patrón lineal.
common factor The factor by which each term is multiplied to generate an exponential pattern.	**factor común** Factor por el cual multiplicamos cada término para generar un patrón exponencial.
commutative property Changing the order in which numbers are either added or multiplied does not change the value of the sum or product.	**propiedad conmutativa** Cambiar el orden en que los números se suman o multiplican no cambia el valor de la suma o el producto.
completing the square Completing the square in a quadratic expression means transforming it into the form $a(x-h)^2+k$.	**completar el cuadrado** Completar el cuadrado en una expresión cuadrática significa transformarla en la forma $a(x-h)^2+k$.
compounding (interest) When interest itself earns further interest, it is said to be compounded, or applied to itself multiple times.	**(interés) compuesto** Cuando el interés genera más interés, se dice que es compuesto, o que se aplica a sí mismo múltiples veces.
constraint A limitation on the possible values of variables, often expressed by equations or inequalities. For example, distance above the ground d might be constrained to be non-negative: $d \geq 0$.	**limitación** Restricción de los posibles valores de las variables, usualmente expresada por ecuaciones o desigualdades. Por ejemplo, la distancia desde el suelo d puede ser limitada a ser no negativa: $d \geq 0$.
correlation coefficient A value that describes the strength and direction of a linear association between two variables. Strong positive associations have correlation coefficients close to 1, strong negative associations have correlation coefficients close to -1, and weak associations have correlation coefficients close to 0.	**coeficiente de correlación** Valor que describe la fuerza y dirección de una asociación lineal entre dos variables. Asociaciones positivas fuertes tienen coeficientes de correlación cercanos a 1, mientras que asociaciones negativas fuertes tienen coeficientes de correlación cercanos a -1, y asociaciones débiles tienen coeficientes de correlación cercanos a 0.
D	**D**
decay factor A common factor in an exponential pattern that is between 0 and 1.	**factor de decaimiento** Factor común en un patrón exponencial que se encuentra entre 0 y 1.
difference of squares Two squared terms that are separated by a subtraction sign.	**diferencia de cuadrados** Dos términos al cuadrado que están separados por un signo de resta.
discrete Separate and distinct values or points.	**discreto** Valores o puntos separados y distintivos.
discriminant For a quadratic equation of the form $ax^2+bx+c=0$, the discriminant is b^2-4ac.	**discriminante** Para una ecuación cuadrática de la forma $ax^2+bx+c=0$, el discriminante es b^2-4ac.
domain The set of all of possible input values for a given function.	**dominio** Conjunto de todos los posibles valores de entrada para una determinada función.

English | **Español**

E

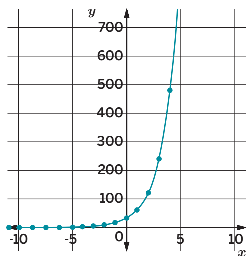
effective rate The actual interest amount earned over a year, taking into account the interest payment.

elimination The removal of a variable from a system of equations by adding or subtracting equations.

equivalent equations Equations that have the same solution or solutions.

equivalent systems Systems of equations that have the exact same solution or solutions.

exponential (growth) Describes a change characterized by the repeated multiplication of a common factor.



exponential function A one-to-one relationship in which a constant is raised to a variable power.

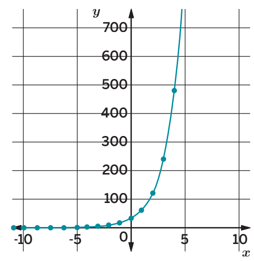
tasa efectiva Monto del interés real ganado en un año, después de tomar en cuenta el pago de intereses.

eliminación Anulación de una variable de un sistema de ecuaciones por medio de la suma o resta de ecuaciones.

ecuaciones equivalentes Ecuaciones que tienen la misma solución o soluciones.

sistemas equivalentes Sistemas de ecuaciones que tienen exactamente la misma solución o soluciones.

(crecimiento) exponencial Describe un cambio caracterizado por la multiplicación repetida de un factor común.



función exponencial Relación uno a uno en la cual una constante se eleva a una potencia variable.

F

factored form (of a quadratic expression) A quadratic expression that is written as the product of a constant and two linear factors is said to be in factored form.

first difference The difference between two consecutive dependent terms for a function.

function notation A way of writing the output of a named function. For example, if the function f has an input x, then $f(x)$ denotes the corresponding output.

forma factorizada (de una expresión cuadrática) Una expresión cuadrática escrita como el producto de una constante multiplicada por dos factores lineales se considera que está en forma factorizada.

primera diferencia Diferencia entre dos términos dependientes y consecutivos de una función.

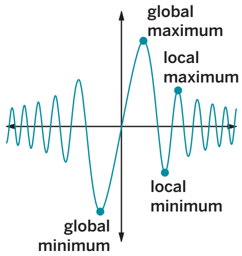
notación de función Forma de escribir la salida de una determinada función. Por ejemplo, si la función f tiene una entrada x, entonces $f(x)$ denota la salida correspondiente.

Glossary/Glosario

English | **Español**

G

global maximum The greatest value of a function over its entire domain.

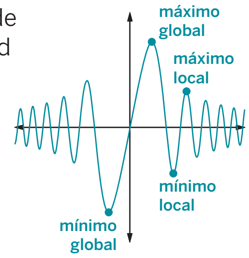


global minimum The least value of a function over its entire domain.

growth factor The common factor that is multiplied over equal intervals in an exponential pattern. In exponential functions of the form $f(x) = a \cdot (1 + r)^x$, the growth factor is $1 + r$.

growth rate The percent change of an exponential function. In exponential functions of the form $f(x) = a \cdot (1 + r)^x$, the growth rate is r.

máximo global El mayor valor de una función por sobre la totalidad de su dominio.



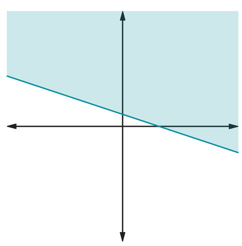
mínimo global El menor valor de una función por sobre la totalidad de su dominio.

factor de crecimiento Factor común que es multiplicado en intervalos iguales como parte de un patrón exponencial. En funciones exponenciales de la forma $f(x) = a \cdot (1 + r)^x$, el factor de crecimiento es $1 + r$.

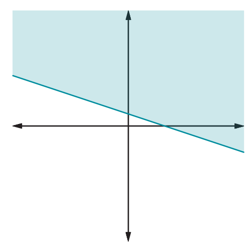
tasa de crecimiento El cambio porcentual de una función exponencial. En funciones exponenciales de la forma $f(x) = a \cdot (1 + r)^x$, la tasa de crecimiento es r.

H

half-plane The set of points in the coordinate plane on one side of a boundary line.



medio plano Conjunto de puntos en el plano de coordenadas que está a un solo lado de una línea límite.



I

index fund An investment fund constructed to track segments of a financial market.

infinity A boundless value, greater than that of any real number.

interest A percentage of the principal that is paid or owed over a specific amount of time.

interval notation A way to represent a set of numbers using parentheses and brackets. For example, the interval $(3, 5]$ represents all the values greater than 3 and less than or equal to 5.

inverse of a function The inverse of a function is created by reversing all of the function's input-output pairs. It can be determined by reversing the process that defined the original function.

fondo indexado Fondo de inversiones elaborado para seguir segmentos de un mercado financiero.

infinito Valor ilimitado, mayor que el valor de cualquier número real.

interés Un porcentaje del principal que se paga o debe durante un periodo de tiempo específico.

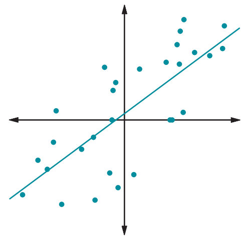
notación de intervalo Forma de representar un conjunto de números por medio de paréntesis y corchetes. Por ejemplo, el intervalo $(3, 5]$ representa todos los valores mayores que 3 y menores o iguales que 5.

inverso de una función El inverso de una función es creado al revertir todos los pares entrada-salida de la función. Se le puede determinar revirtiendo el proceso que definió a la función original.

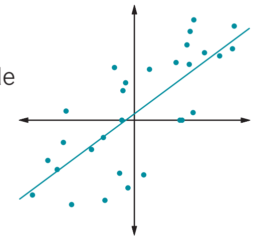
English | Español

L

line of best fit The linear model that has the smallest possible sum of the squares of the residuals.



línea de ajuste óptimo Modelo lineal que tiene la menor suma posible de los cuadrados de los residuos.



linear function A function with a constant rate of change.

función lineal Función con una tasa de cambio constante.

local maximum The value of a function that is greater than the nearby or surrounding values of the function.

máximo local Valor de una función que es mayor a los valores cercanos o circundantes de la función.

local minimum The value of a function that is less than the nearby or surrounding values of the function.

mínimo local Valor de una función que es menor a los valores cercanos o circundantes de la función.

M

monic quadratic An expression of the form $x^2 + bx + c$, where the coefficient of the x^2 term is 1.

ecuación cuadrática mónica Expresión de la forma $x^2 + bx + c$, en la cual el coeficiente del término x^2 es 1.

N

nominal rate The stated or published rate.

tasa nominal Tasa declarada o publicada.

non-monic quadratic An expression of the form $ax^2 + bx + c$, where a does not equal 1 or 0.

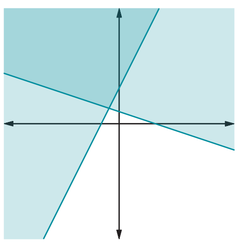
ecuación cuadrática no mónica Expresión de la forma $ax^2 + bx + c$, en la cual a no es igual a 1 o 0.

nonlinear relationship A relationship between two quantities in which there is no constant rate of change.

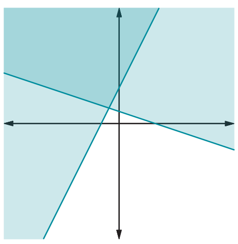
relación no lineal Una relación entre dos cantidades que no tiene una tasa de cambio constante.

O

overlap of graphs of inequalities The set of points that satisfy two or more inequalties.



superposición de gráficas de desigualdades Conjunto de puntos que satisfacen dos o más desigualdades.



Glossary/Glosario

English	Español

P

piecewise function A function defined using different expressions for different intervals in its domain.

función definida a trozos Función definida por el uso de diferentes expresiones para diferentes intervalos de su dominio.

plus-or-minus symbol A symbol used to represent both the positive and negative of a number ($\pm$).

símbolo de más menos Usado para representar tanto el positivo como el negativo de un número ($\pm$).

principal Initial amount of a loan, investment, or deposit.

principal Monto inicial de un préstamo, inversión o depósito.

Q

quadratic equation An equation in which the highest power of the variable is 2. Also called an equation of the second degree.

ecuación cuadrática Ecuación en la cual la potencia más alta de la variable es 2. También se llama ecuación de segundo grado.

Quadratic Formula The formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ that gives the solutions to the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$.

Fórmula cuadrática Fórmula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ que provee las soluciones de la ecuación cuadrática $ax^2 + bx + c = 0$, en la cual $a \neq 0$.

quadratic function A function in which the output is given by a quadratic expression.

función cuadrática Función en la cual la salida está dada por una expresión cuadrática.

R

range In algebra, a function's *range* is the set of all possible output values for the function. In statistics, the *range* of a data distribution is the difference between the maximum and minimum occurring values.

rango En algebra, el *rango* de una función es el conjunto de todos los posibles valores de salida de la función. En estadística, el *rango* de una distribución de datos es la diferencia entre los valores máximo y mínimo existentes.

relative frequency table A two-way table that shows the proportion of each value — expressed as fractions, decimals, or percentages — compared to the total in each row, column or in the entire table.

tabla de frecuencia relativa Tabla de doble entrada que muestra la proporción de cada valor (expresada como fracciones, decimales o porcentajes), en comparación con el total de cada fila, columna o con toda la tabla.

residual The difference between the actual y-coordinate and y-coordinate predicted by a model, given the x-coordinate.

residuo Diferencia entre la coordenada y real y la coordenada y pronosticada por un modelo, dada la coordenada x.

revenue The income generated from selling of a product or service.

ingreso Entrada de dinero generada por la venta de un producto o servicio.

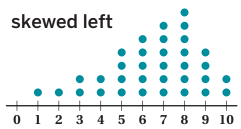
English | **Español**

S

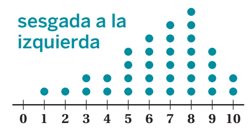
second difference The difference between two consecutive first differences.

segunda diferencia Diferencia entre dos primeras diferencias consecutivas.

skewed A distribution with a long tail, where data extends far away from the center, is called *skewed*.



sesgada Una distribución de cola larga, en la cual los datos se extienden en dirección opuesta al centro, se conoce como *sesgada*.



solution set The set of all values that satisfy an equation or inequality.

conjunto de soluciones Conjunto de todos los valores que satisfacen una ecuación o una desigualdad.

square expression An expression that represents the product of two identical expressions.

expresión cuadrada Expresión que representa el producto de dos expresiones idénticas.

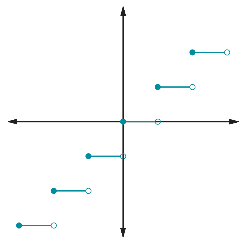
standard deviation A commonly used measure of variability. It is the square root of the average of the squares of the distances between data values and the mean.

desviación estándar Medida de variabilidad de uso común. Se trata de la raíz cuadrada del promedio de las distancias elevadas al cuadrado entre los valores de los datos y la media.

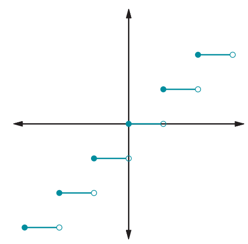
standard form (of a quadratic expression) The standard form of a quadratic expression in x is $Ax^2 + Bx + C$, where A, B, and C are constants, and $A \neq 0$.

forma estándar (de una expresión cuadrática) La forma estándar de una expresión cuadrática en x es $Ax^2 + Bx + C$, en la cual A, B y C son constantes, y $A \neq 0$.

step function A piecewise function whose pieces are all constant values.



función escalonada Función definida a trozos, cuyos trozos son todos valores constantes.



system of linear inequalities Two or more inequalities that represent the constraints in the same situation.

sistema de desigualdades lineales Dos o más desigualdades que representan las limitaciones en la misma situación.

T

two-way table A table that organizes categorical data into cells. The categories do not overlap, so that each data value is recorded in exactly one cell.

tabla de doble entrada Tabla que organiza datos categóricos en celdas. Las categorías no se superponen, de manera que el valor de cada dato es registrado exactamente en una sola celda.

U

uniform A distribution in which data is evenly distributed throughout the range is called *uniform*.

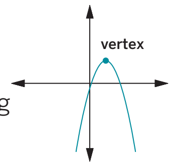
uniforme Distribución en la cual los datos son distribuidos de manera regular a través del rango.

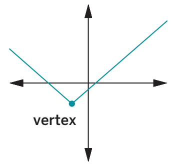
Glossary/Glosario

English

V

vertex (of a graph) The *vertex* of the graph of a quadratic function or of an absolute value function is the point where the graph changes from increasing to decreasing or vice versa. It is the highest or lowest point on the graph.





vertex form An equation of the form $y = a(x - h)^2 + k$ where (h, k) represents the coordinates of the vertex of a quadratic function.

Z

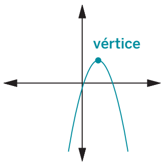
Zero Product Principle This principle states that $a \cdot b = 0$, if and only if $a = 0$ or $b = 0$.

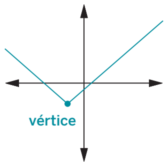
zeros (of a function) The values at which the function is zero.

Español

V

vértice (de una gráfica) El *vértice* de la gráfica de una función cuadrática o de una función de valor absoluto es el punto en que la tendencia de la gráfica cambia de aumentar a disminuir o viceversa. Es el punto más alto o más bajo de la gráfica.





forma de vértice Ecuación de la forma $y = a(x - h)^2 + k$, en la cual (h, k) representa las coordenadas del vértice de una función cuadrática.

Z

Principio de producto cero Este principio establece que $a \cdot b = 0$ si y solo si $a = 0$ o $b = 0$.

ceros (de una función) Valores para los cuales la función es cero.

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