Amplify Math

Grade 6

Volume 1: Units 1–4

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify Math is based on the Illustrative Mathematics (IM) curriculum. IM 6–8 Math™ was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is © 2017–2019 Open Up Resources. Additional adaptations and updates to IM 6–8 Math™ are © 2019 Illustrative Mathematics. IM 9–12 Math™ is © 2019 Illustrative Mathematics. IM 6–8 Math™ and IM 9–12 Math™ are licensed under the Creative Commons Attribution 4.0 International license (CC BY 4.0). Additional modifications contained in Amplify Math are © 2020 Amplify Education, Inc. and its licensors. Amplify is not affiliated with the Illustrative Mathematics organization.

Desmos® is a registered trademark of Desmos, Inc.

English Learners Success Forum is a fiscally sponsored project of the New Venture Fund (NVF), a 501(c)(3) public charity.

Universal Design for Learning Guidelines and framework are developed by the Center for Applied Special Technology. © 2018 CAST.

The Effective Mathematics Teaching Practices are developed by NCTM in *Principles to Actions: Ensuring mathematical success for all.* © 2014 NCTM.

Notice and Wonder and I Notice/I Wonder are trademarks of NCTM and the Math Forum.

No part of this publication may be reproduced or distributed in its original form, or stored in a database or retrieval system, without the prior written consent of Amplify Education, Inc., except for the classroom use of the worksheets included for students in some lessons.

Cover illustration by Caroline Hadilaksono.

© 2023 by Amplify Education, Inc.
55 Washington Street, Suite 800,
Brooklyn, NY 11201
www.amplify.com

ISBN: 978-1-63643-000-3
Printed in [e.g., the United States of America]
[# of print run] [print vendor] [year of printing]

Hello, curious mind!

Welcome to Grade 6. For many students, it's a year of change and growth . . . and the same goes for what you'll be doing in math.

This year, you'll meet the Song dynasty of China and solve a few tangram puzzles they inspired, learn how a misplaced ruler led to the invention of the cardboard box, construct rhombicuboctahedrons (say that five times fast), and design a suspended tent for you and three friends to camp out in. And that's just Unit 1!

We promise you, it's all possible. This year, you'll see not only that math makes sense, but it can be fun, too.

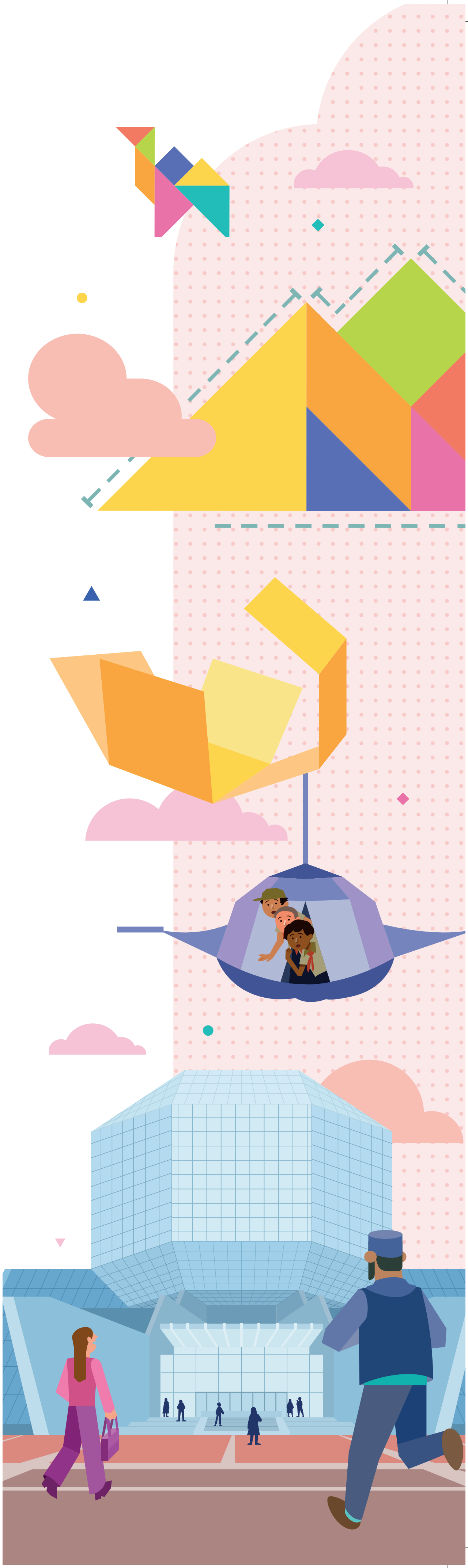
Before you dig in, we want you to know two things:

This book is special! It's where you'll record your thoughts, strategies, explanations, and, occasionally, any award-winning doodles that might come to mind.

When you go online, you won't be mindlessly plugging numbers into your device . . . You'll be pushing, pulling, crawling, teleporting, melting . . . , well, let's just say you'll be doing a lot of things, and you'll be teaming up with your classmates as you do.

Let's dig in!

Sincerely,
The Amplify Math Team

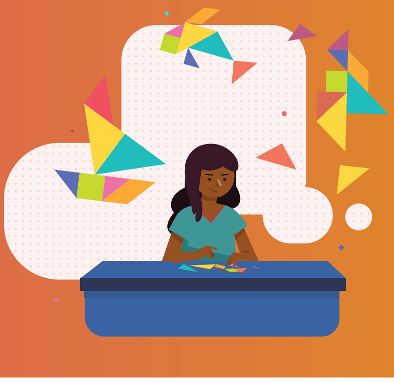


Volume 1

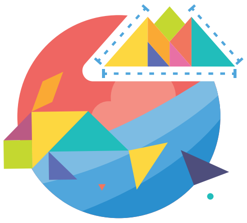
Unit 1 Area and Surface Area

Unit Narrative:
A Place for Space

Geometry is the mathematics of space and all the shapes and sizes within it, and even dimensions. You know the names of many special two-dimensional and three-dimensional figures, and have worked with the area of very basic shapes before. But, now, it is time to cover anything and everything, literally.



LAUNCH



Sub-Unit 1 Area of Special Polygons

Can a sum ever really be greater than its parts?

Polygons are shapes whose sides are all line segments, and they can be decomposed and rearranged without changing their area.

Sub-Unit 2 Nets and Surface Area

How did a misplaced ruler change the way you shop?

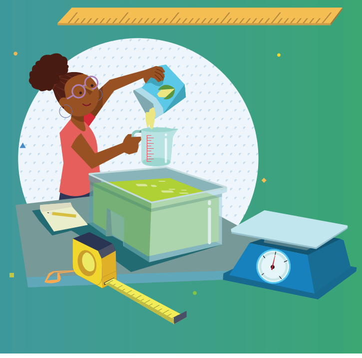
Polyhedra are three-dimensional figures composed of polygon faces. Their surfaces can be decomposed.

CAPSTONE

Unit 2 Introducing Ratios

A little bit of this and a little bit of that. Well, maybe a lot of that? Wait, I think a ratio can help with this dilemma! Ratios help us see the relationship between one number and another, so when we make guacamole, it doesn't taste awful.

Unit Narrative:
Sensing a Ratio



Sub-Unit 1 What Are Ratios? 141

How does an eggplant become a plum?

Ratios represent comparisons between quantities by multiplication or division. First, you must first learn the language of ratios and how quantities "communicate."

Sub-Unit 2 Equivalent Ratios 171

How do you put your music where your mouth is?

Equivalent ratios involve relationships between ratios themselves. They speak to each other through music and rhythm, beats and time.

Sub-Unit 3 Solving Ratio Problems 225

Who brought Italy to India and back again?

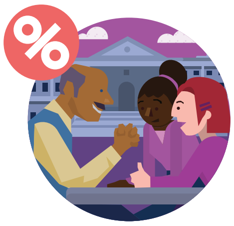
Now it is your turn to choose the information to represent and compare ratios.

Unit 3 Rates and Percentages

One black truffle costs how much?! A hummingbird flaps its wings how many times in one minute?! Unit rates — how much per one — are useful ratios. And sometimes how much per one hundred, a percentage, is useful too — if you want to know: Who should take the technical foul shot with no time on the clock? Do people really like dogs better than cats? Who won the election?

Unit Narrative:
Stand and Be Counted

LAUNCH



Sub-Unit 1 Rates ... 281

How did student governments come to be?

Rates describe relationships between quantities like price and speed. Unit rates reveal which is a better deal or who is faster.

Sub-Unit 2 Percentages ... 323

What can a corpse teach us about governing?

Percentages are rates per 100. They can compare relationships between parts and wholes, even when two quantities have different total amounts.

CAPSTONE

Unit 4 Dividing Fractions

Division can be used to solve equal-sized groups problems, including when the size of a group and even the number of groups are represented by fractions. See how you can apply what you already know about multiplication and division to follow the mysteries within Spöklik Furniture and fraction division.

Unit Narrative:
Crossing the Fractional Divide

Which item costs between 100 and 1,000 spök-bucks?

Multiplication and division are related, and the relationship between fractions and division can be used to estimate quotients.

How long is the bolt Samira needs?

To divide fractions, you can use multiplication, common denominators, or an algorithm. Apply these to determine the length of an oddly labeled bolt.

How can Maya fit Penny in the box?

When you know an area or volume, but not every side length, you will often divide fractions.

Volume 2

Unit 5 Arithmetic in Base Ten

Decimals embody the numerical language of precision. And, because we use a base ten number system, and the world is a messy place, decimals are everywhere. Being able to add, subtract, multiply, and divide any numbers with any number of decimal places can help you determine and make sense of some astonishing facts and human accomplishments that are world records.

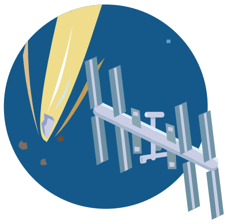
Unit Narrative:
Making Moves With Decimals

How did a decimal decide an Olympic race?

Determine the results of high stakes competitions and identify record-setting moments by adding and subtracting decimals, as precisely as you need.

What happens when you make a small change to a big bridge?

To reproduce something at large or small scales so it looks the same, you need decimals and multiplication.

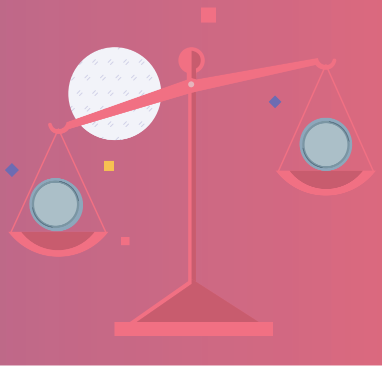


How do you dodge a piece of space junk?

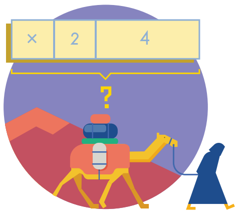
Dividing whole numbers and decimals with many digits is the final set of operations you need to complete your trophy case.

Unit 6 Expressions and Equations

Unit Narrative:
The Power of Balance



Up until now, an equal sign meant you were being asked to calculate an answer. In this unit, you'll learn about its other meaning — balance. And when things are in balance, it becomes possible to know the unknown.



What's a bag of chips worth in Timbuktu?

Learn about the 14th century African salt trade, as you explore expressions and equations with tape diagrams and hanger diagrams.

How did a Welshman equalize England's upper crust with its common folk?

Extend the concept of equality as you investigate equivalent expressions, the all-important Distributive Property, and exponents.

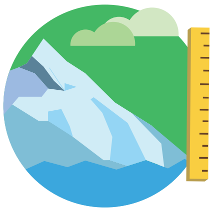
What's more dangerous: a pack of wolves or a gang of elk?

Balance is everywhere, especially in ecosystems. You'll look at systems that are in and out of balance.

Unit 7 Rational Numbers

Think back to when you first learned about whole numbers and used them to count. Later, you saw there were numbers between them: fractions and decimals. Up until now, every number you've encountered has always been greater than 0. But no more. There is an entire set of numbers (just as many, in fact), lurking on the *other* side of every number line.

Unit Narrative:
Getting Where We're Going



What's the tallest mountain in the world?

Consider the most extreme locations on Earth as you discover negative numbers, which lend new meaning to positive numbers and zero.

How do you keep a quantity from wandering off?

A variable represents an unknown quantity. And sometimes it represents many possible values, which can be expressed as an inequality.

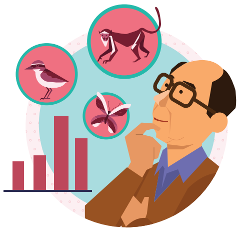
How did Greenland get so big?

Armed with the opposites of positive rational numbers, it's time you expanded your coordinate plane. Welcome to the four quadrants!

Unit 8 Data Sets and Distributions

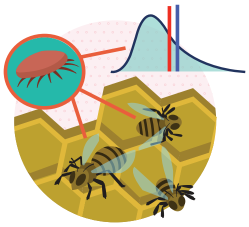
Statistics is the science of collecting and analyzing data. It is one of the most relevant aspects of mathematics in everyday life. And it is also used by researchers in many fields, such as zoologists identifying new species and studying populations of endangered species. In all cases, knowing what is typical is critical to understanding what is not.

Unit Narrative:
Walk on the Wild Side with Data



How do you keep track of a disappearing animal?

When questions have more than one answer, it is helpful to visualize and describe a typical answer. For numbers, you can also identify the center and describe the spread of the numbers.



What's the buzz on honey bees?

For numerical data, you can summarize an entire data set by a single value representing the center of the distribution. The mean and the median represent two ways you can do this.

Where have the giant sea cows gone?

For numerical data, you can summarize an entire data set by a single value representing the variability of the distribution. The MAD, range, and IQR represent three ways you can do this.

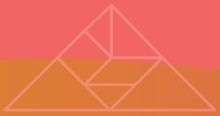
UNIT 1

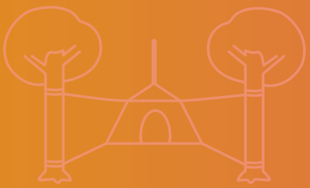
Area and Surface Area

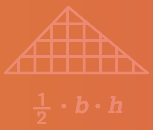
Geometry is the mathematics of space and all the objects in it, which come in various shapes and sizes, and even dimensions. You know the names of many special two-dimensional and three-dimensional figures, and have worked with the area of very basic shapes before. But now it is time to cover anything and everything, literally.

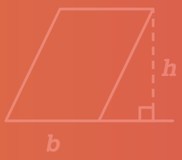
Essential Questions

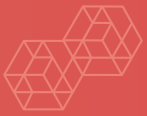
- What does it mean when you say two shapes have the same area?
- How is surface area different from volume?
- *(By the way, what do you get when you stitch together 12 pentagons and 20 hexagons?)*











SUB-UNIT

1 Area of Special Polygons

Narrative: Discover how something can never be greater than the sum of its parts.

You'll learn . . .

- about decomposing shapes.
- formulas for the areas of triangles and parallelograms.

SUB-UNIT

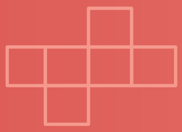
2 Nets and Surface Area

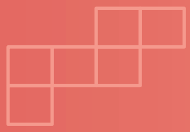
Narrative: From cardboard boxes to suspended tents, areas folded in three dimensions have got you covered!

You'll learn . . .

- how nets can be used to compute surface area.
- to represent repeated multiplication with exponents.

A cube has N unique nets, two of which are shown below. If you add up the digits of N and square the result, what number do you get?





Unit 1 | **Lesson 1 – Launch**

The Tangram

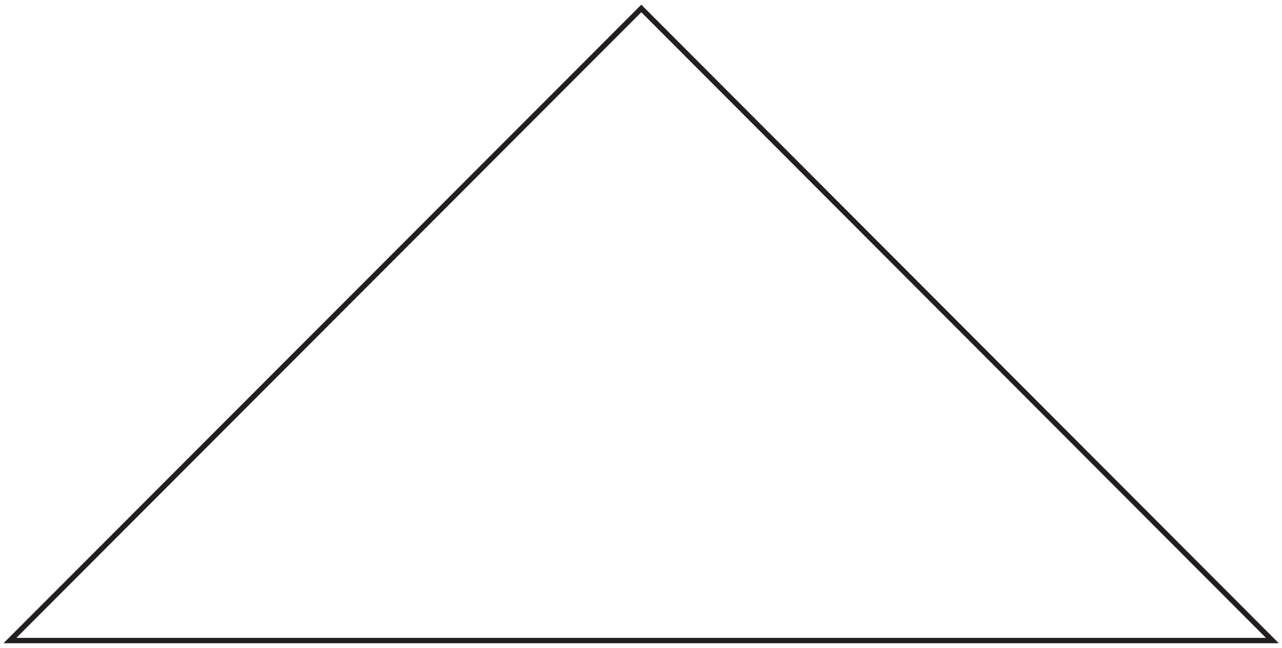
Let's discover the tangram.

Warm-up Working Together

Work with your partner to create the figure shown here using all seven of your tangram pieces.

As you work, pay attention to and be prepared to share your responses to the following with the class:

- What happened next when you and your partner "got stuck?"
- How were moments of disagreement resolved?
- How were you and your partner able to use your time productively?



Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

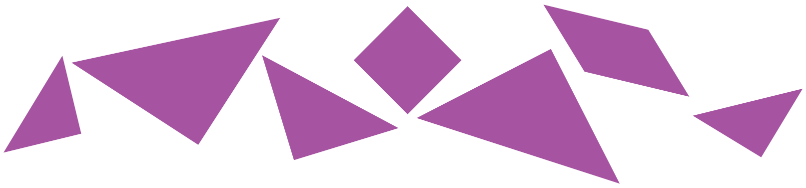
Activity 1 The Tangram Legend

Use your seven tangram pieces to recreate the images from *The Tangram Legend.*

Many centuries ago, a king ordered a window be created for his palace in the shape of a perfect square. A sage was sent out on the arduous journey to collect the glass from an artisan who lived on the opposite side of the kingdom.

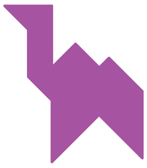
The long route involved navigating a vast array of landscapes — fields, forests, deserts, and rivers. Nearing her final destination, the sage climbed the rockiest peak of the final mountain range, and the palace came back into view.

Overjoyed by her imminent arrival and a sure-to-be pleased king awaiting, the sage took a hasty step and tripped. The tumble down the mountain broke the precious glass. But intriguingly, it was not shattered — rather, seven geometric shapes, each equally impressive, were formed.

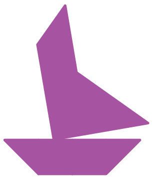


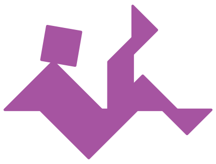
The heart-broken sage came before the king and recounted her treacherous journey. As she spoke, she skillfully moved the shapes around and formed many images to recreate the journey.

The *home* of the artisan. A *camel* used to cross the desert.

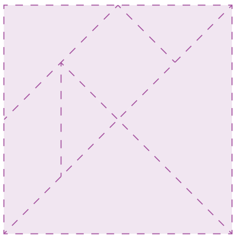


A *boat* for sailing across the river. The infamous *mountain range* and the apologetic *falling*.





The king was so fascinated by the multitude of geometric images that could be created from the pieces that he had the shapes recreated out of wood. Thus, the *tangram square* was invented.



Activity 2 Tangram Paradoxes

A *paradox* is a statement that seems like it cannot be true at first, but after investigating it or thinking about it in a different way, it seems like it *could* be true.

Work with your partner to recreate both figures in each of these two paradoxes and explain what is happening that allows each to be possible.

1. Figures A and B are both squares that can be created using all seven tangram pieces, but one has a hole in the middle.

Figure A

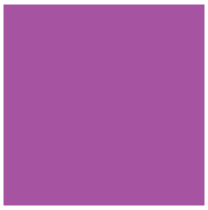
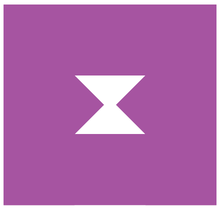


Figure B



2. Figures C and D are both side views of a person that can be created using all seven tangram pieces, but one has feet!

Figure C

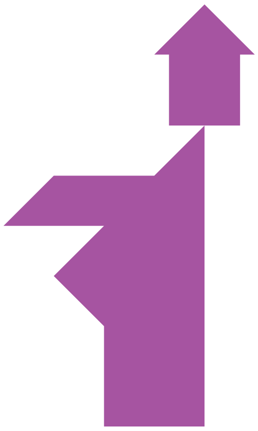
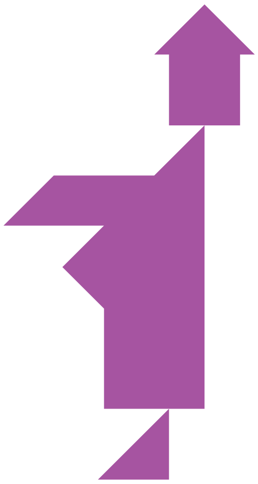


Figure D



STOP

Unit 1 Area and Surface Area

A Place for Space

At first glance, a tangram might look like a simple toy: a flat square, made up of seven smaller shapes, called “tans.” But using only these seven shapes, you transformed the square into a house, a mountain, a camel, and a boat. No one knows the true history of the tangram, but the legend of the sage who shattered a precious glass pane shows how entire stories can be woven together with just those seven shapes.

It was the artist Michelangelo who said, “Every block of stone has a statue inside it.” He could have just as easily been talking about tangrams — or even *math students*, for that matter.

As you start the year, think about the humble square. Think about all the possibilities tucked inside it. And just as there is a statue inside every block of stone, there is a mathematician within every student. It’s right there, just waiting to be brought to the surface.

With an open mind, some elbow grease, and a little imagination, we’ll get there together.

Let’s see how . . .

Practice

Name: ________________ Date: ________ Period: ________

1. Identify two of your strengths as a math student and two areas in which you would like to grow or improve as a math student.

2. What motivates you to do your best?

3. What does it mean to be accountable to yourself? To your peers?

Name: .... Date: .... Period: ....

4. How can you promote positive and effective communication with others?

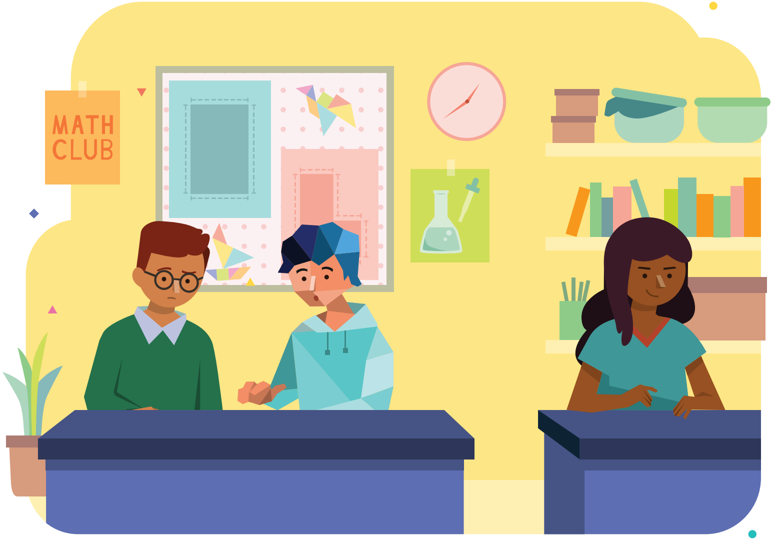
5. What are some qualities that you would like your peers to display when collaborating and working together?

6. Using the tangrams from class, make a quadrilateral. Sketch your quadrilateral.

Unit 1 | Lesson 2 – Launch

Exploring the Tangram

Let's make patterns using tangram pieces.



Warm-up How Many Squares?

Using four or more of the pieces from a tangram set, how many different ways can you build a square of any size? Record the tangram pieces used for each square that your group builds, and then sketch the square.

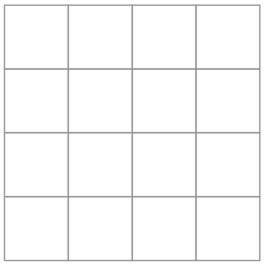
Compare and Connect:
After completing the Warm-up, share with a partner what is different and what is similar among the ways the squares have been composed.

Name: ________________ Date: ________ Period: ________

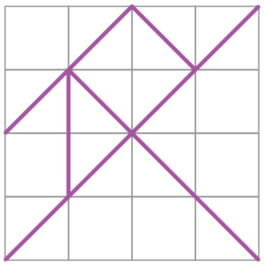
Activity 1 Making a Tangram Set

You will be given a square piece of paper. Follow the steps to create your own set of tangram pieces from that square.

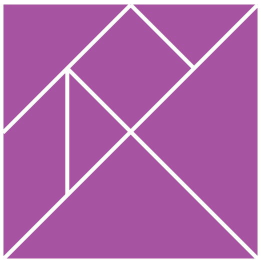
1. Fold the square piece of paper in half horizontally, and then fold it in half again vertically. Repeat these two types of folds one more time. When you unfold the paper, the folds create sixteen equal-sized squares.



2. Draw lines on your square as shown here, and then cut your paper along the lines.



3. You will now have a set of the seven standard tangram pieces:
- one small square
- two small triangles
- one medium triangle
- two large triangles
- one parallelogram



Are you ready for more?

Each of these figures represents a different paradox. They can all be solved using all seven tangram pieces. But they can also all be solved using only six tangram pieces. Try to solve one (or more) of these tangram puzzles, first using all seven pieces, and then again using only six pieces.

Figure A

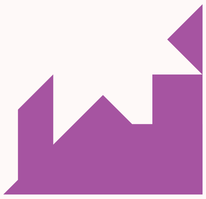


Figure B

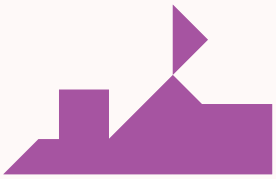


Figure C

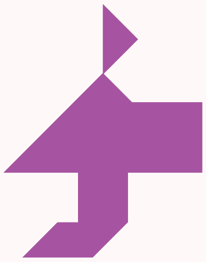
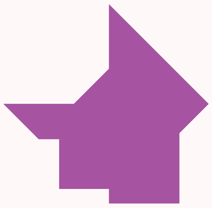


Figure D



Activity 2 Creating Your Own Tangram Puzzle

The classic rules of tangram puzzles are:

- All seven tangram pieces must be used in the puzzle.
- All pieces must lie flat.
- Each piece must touch at least one other piece.
- No pieces can overlap.

Create a puzzle using the tangram pieces you created in Activity 1. Draw an outline of your puzzle and its pieces here. Then glue the pieces on a separate sheet of paper and color them.

STOP

A Place for Space, continued

Originating in China during the Song dynasty, tangram puzzles spread to the U.S. and Europe in the early 1800s. They fascinated figures as wide-ranging as Napoleon Bonaparte, Edgar Allen Poe, John Quincy Adams, and Lewis Carroll.

The beauty of tangrams is that they take something simple—a square—and turn it into something deeply complex. And just as we rearrange tans to compose a shape, we also have to rearrange the way we think to arrive at our solutions.

In some ways, your math class is like a tangram. It's a whole unit on its own, but it's also made up of smaller individual pieces: you, your classmates, and your teacher. Each of you brings something different and valuable to the room. You bring your ideas, your curiosity, your creativity, your perseverance, your stories. These are the elements you need for a math class to work.

Whether it's the tans in a tangram, or a student in a class, understanding how things are put together starts with understanding the individual pieces.

Welcome to Unit 1.

Practice

Name: .... Date: .... Period: ....

1. Think about this upcoming year in your math class.

   a) Describe one goal you have for this year in math class.

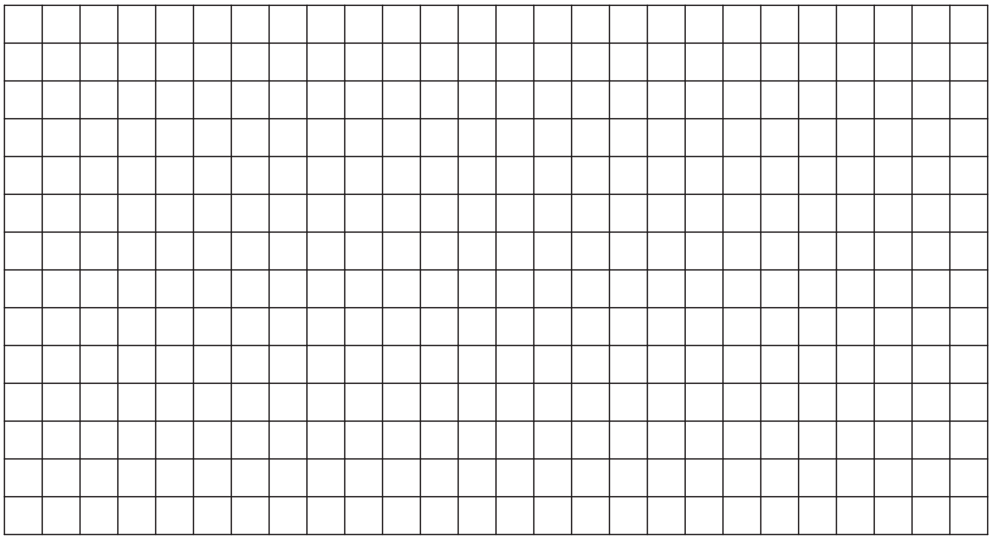
   b) What is:
      - one way you can help yourself reach your goal?
      - one way your teacher can help you reach your goal?
      - one way your peers can help you reach your goal?

2. What is most important to you about the cycle of giving and receiving constructive feedback?

3. What are the seven pieces that make up a tangram?

Name: .... Date: .... Period: ....

4. Each square in this grid has an area of 1 square unit. Draw three different quadrilaterals that each have an area of 12 square units.



5. For each statement about parallelograms, determine if it is *always* true, *sometimes* true, or *never* true.

a Opposite sides are parallel.

a All angles are right angles.

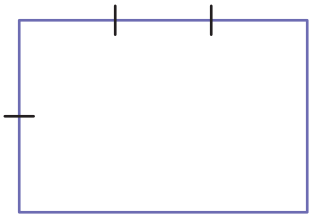
c Opposite angles are equal.

d Opposite sides are different lengths.

e A parallelogram is also a rectangle.

f A rectangle is also a parallelogram.

6. The side lengths of the rectangle shown are 3 cm and 2 cm. What is the area of the rectangle?



My Notes:

SUB-UNIT

1 Area of Special Polygons

Narrative Connections

Can a sum ever *really* be greater than its parts?

Nope. At least, not when it comes to geometry.

Most anything you can take apart, you can also put back together. But putting things together is the hard part!

You got a taste of this challenge with the tangrams you just saw. And if you've ever tried to assemble furniture, you know that things may not end up looking how they're supposed to, even when you use all the pieces.

So, while two different figures can be made up of exactly the same parts and take up the same amount of space, they can *still* look completely different.

When we're talking about two-dimensional figures, the amount of space a shape takes up is called its *area*.

Different shapes have their own relationships between their area and their dimensions — that is, their lengths. And if you know the area of one shape, you can figure out the area of any shape that can be made from it. As you'll see, when it comes to polygons, it always comes back to the humble triangle.

Unit 1 | Lesson 3

Tiling the Plane

Let’s look at tiling patterns and think about area.

Warm-up Exploring Your Geometry Toolkit

Take turns choosing tools from your geometry toolkit and discuss with your partner how you think each tool might be used in this geometry unit.

Record the names of the tools here, as you discuss them.

Log in to Amplify Math to complete this lesson online.

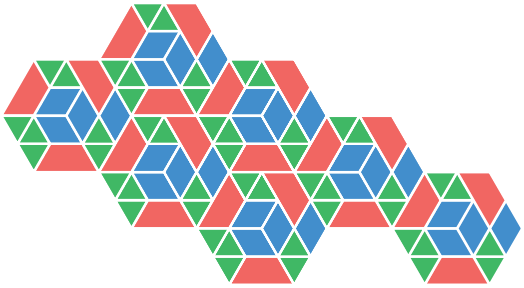
Name: ______ Date: ______ Period: ______

Activity 1 Tiling the Plane

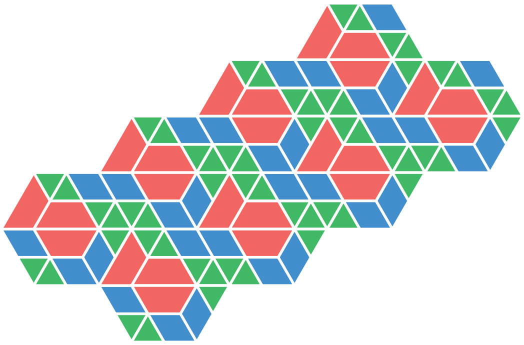
Filling spaces with tiles of different shapes and colors can be both decorative and beautiful. It is also very mathematical; mathematicians, such as Laura Escobar, have studied the connections between certain patterns of rhombus-shaped tiles and how letters can be ordered in different ways.

For now, you will be assigned either Pattern A or Pattern B. Determine which shape — rhombus, trapezoid, or triangle — covers more of the plane in your pattern. Be prepared to explain your thinking.

Pattern A



Pattern B



Collect and Display:
Your teacher will circulate and collect key terms and phrases to add to a class display as your group discusses the patterns. Refer to this display during future discussions.

Featured Mathematician

Laura Escobar

Hailing from Bogotá, Colombia, Laura Escobar is a professor of mathematics at Washington University in St. Louis, Missouri. Her research lies at the intersection of algebra, geometry, and combinatorics (the mathematics of how things are arranged, and how many ways there are to arrange them). In 2016, she was the lead author on a paper that explored connections between special tilings of rhombuses, combinations of letters, and what are known as "Bott-Samelson varieties."

STOP

Summary

In today's lesson . . .

You looked at copies of the same two-dimensional shapes being placed together in different ways, but always such that there were no gaps or overlaps. In thinking about which shapes make up more of a pattern or cover more of a region, you reasoned about *area*. Particularly, you revisited the idea of what it means for two shapes to have the same area.

This is just the start of the work you will do this year. You will use mathematics and the tools of mathematicians to answer questions, as well as ask and answer your *own* questions. You will continue this work and discover more flexible and efficient uses for the tools in your geometry toolkit, as you explore more about the concept of area in this unit.

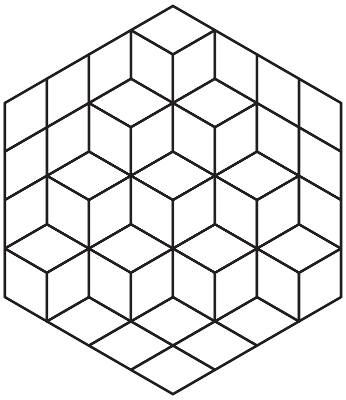
Just as important as understanding mathematical ideas for yourself, this lesson presented the first of many opportunities to practice speaking like a mathematician, by sharing your understanding and thinking with others. Also just like mathematicians, you worked together with partners and groups of classmates, as well as with your teacher, to help you arrive at your own understanding, while also considering the perspectives of others.

Reflect:

Name: .... Date: .... Period: ....

Practice

> **1.** What tool(s) could you use to help you visualize the shapes in this pattern?

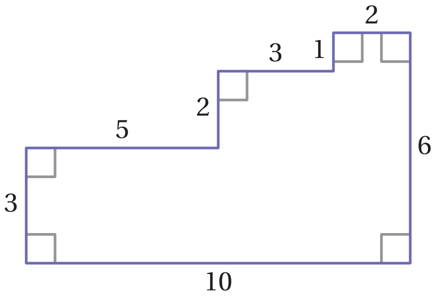


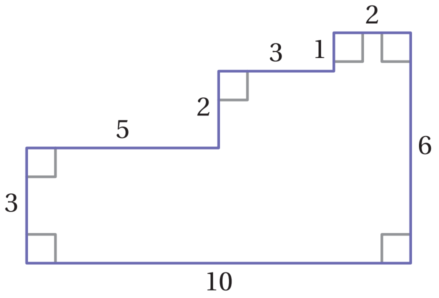
> **2.** Using only triangles, how many triangles would be needed to cover this pattern? Show or explain your thinking.

Practice

Name: .... Date: .... Period: ....

3. Here are two copies of the same shape. Show two different ways for determining the area of the shape. (Note. You don not need to calculate the actual area.) All angles are right angles.

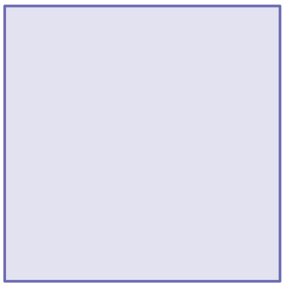




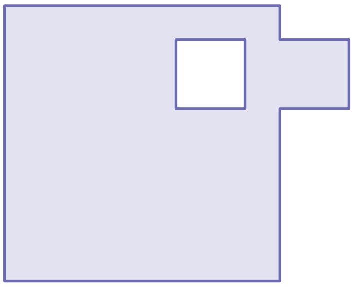
4. Which shape has a larger area: a rectangle that is 7 in. by $\frac{3}{4}$ in., or a square that has a side length of $2\frac{1}{2}$ in.? Show or explain your thinking.

5. Which shaded region covers more area? Show or explain your thinking.

Region A



Region B



Name: .... Date: .... Period: ....

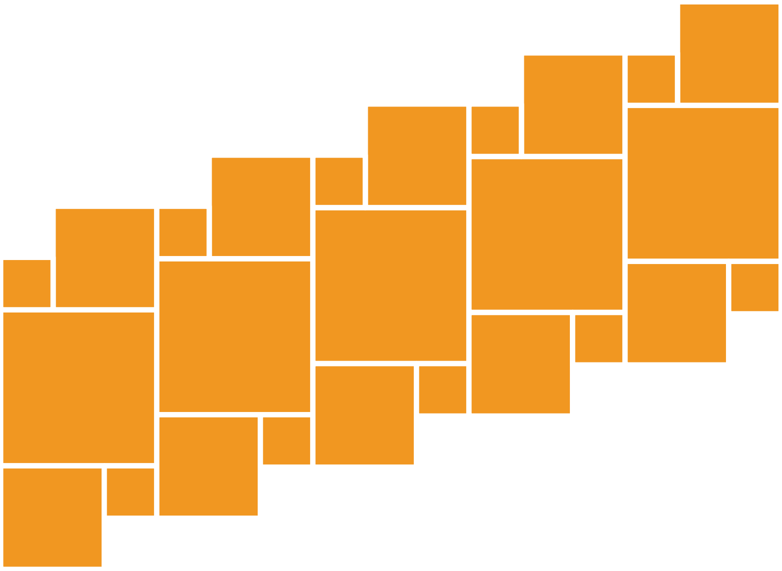
Unit 1 | Lesson 4

Composing and Rearranging to Determine Area

Let's create shapes and determine their areas.

Warm-up Comparing Regions

Which type of square — large, medium, or small — covers more area in this pattern? Consider using the tools in your geometry toolkit to help your thinking.



Reflect: Did you use any tools from your geometry toolkit to help you? Why or why not?

Log in to Amplify Math to complete this lesson online.

Activity 1 Composing and Rearranging Shapes

You will be given a square and several different-sized triangles. The area of the square is 1 square unit.

Part 1: Composing

1. What is the area of two small triangles when they are placed together? Be prepared to explain your thinking.

2. Use two or more pieces to create a new shape — that is not a square — with an area of 1 square unit.

a Draw your shape.

b What is the area of each piece?

3. Use as many of the pieces as you need to create a shape with an area of 2 square units.

a Draw your shape.

b What is the area of each *different* piece in your shape?

Name: ________________ Date: ________ Period: ________

Activity 1 Composing and Rearranging Shapes (continued)

Part 2: Rearranging

Use exactly the same pieces from Problem 3 to create a *different* shape than you created in Problem 3.

4. Draw your new shape.

5. What is the area of this new shape?

Part 3: Composing and Rearranging

Starting with the same pieces from Part 2, use additional pieces to add to your shape to create a new shape, now with an area of 4 square units.

6. Draw your shape.

7. What is the area of each *different* piece in your shape?

Are you ready for more?

Show how you can use all of your pieces to compose a single large square. What is the area of this large square?

STOP

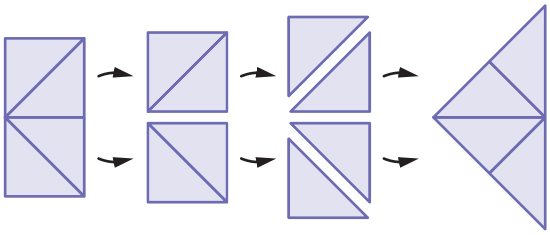
Summary

In today's lesson . . .

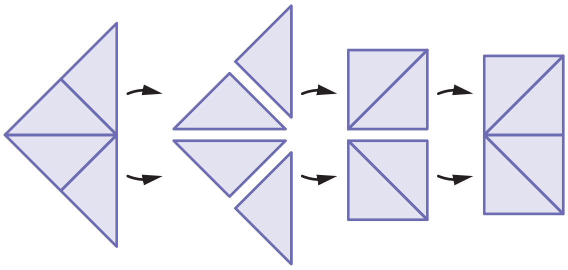
You used two important principles of area:

1. If two shapes can be placed one on top of the other, so that they match up exactly, with no gaps or overlaps, then they have the same *area*.
2. You can ***decompose*** (break it into pieces) a shape and ***rearrange*** (move them around) the pieces to form a new shape. The area of the original shape and the area of the new shape are the same.

Here are two illustrations of the second principle.



The rectangle can be decomposed into two squares, and then each square can be decomposed into two triangles. These four triangles can be rearranged to form the large triangle. So, the large triangle has the same area as the rectangle.



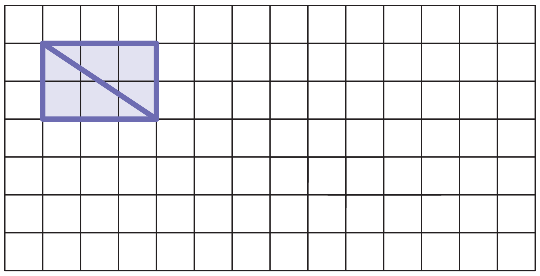
The large triangle can be decomposed into four smaller triangles. These four triangles can be rearranged to form two squares, which can be ***composed*** (placed together) to form the rectangle. So, the rectangle has the same area as the large triangle.

If each square in these illustrations has an area of 1 square unit, then the area of a small triangle is $\frac{1}{2}$ square unit, and the area of the large triangle is 2 square units.

Reflect:

Name: .... Date: .... Period: ....

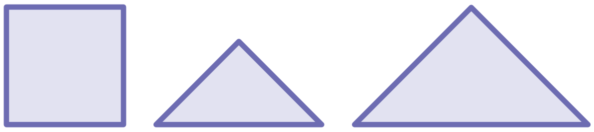
1. The figure shows a rectangle on a grid that is composed of two triangles.



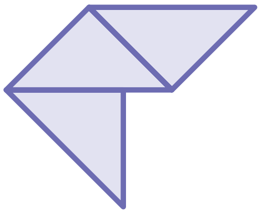
a Rearrange the two triangles to make a *different* shape.

b How does the area of this new shape compare to the area of the original rectangle? Explain your thinking.

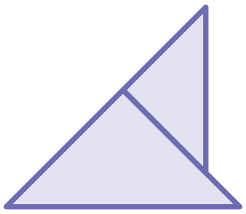
2. The area of the square is 1 square unit. Two copies of the smaller triangle can be arranged to form either of the other two shapes — the square or the larger triangle. Which figures have an area of $1\frac{1}{2}$ square units? Select *all* that apply.



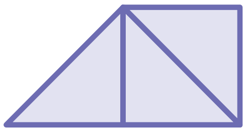
A.



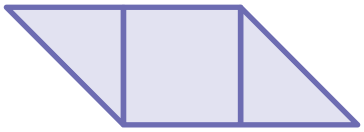
C.



B.

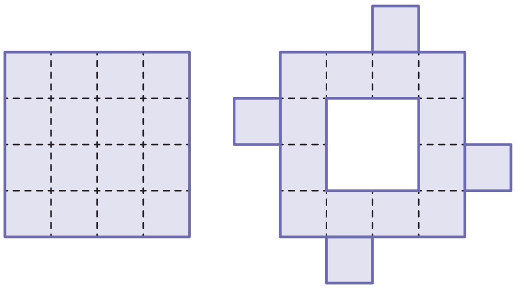


D.



Name: .... Date: .... Period: ....

3. Priya decomposed a square into 16 identical smaller squares. She then cut out 4 of the small squares and placed them around the outside of the original square to make a new shape, as shown. Which of these statements accurately describes how the area of her new shape compares to the area of the original square?



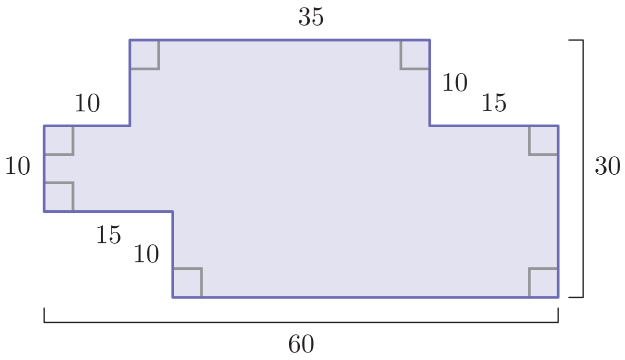
A. The area of the new shape is greater.

B. The two shapes have the same area.

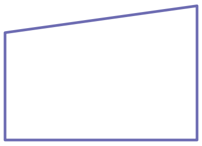
C. The area of the original square is greater.

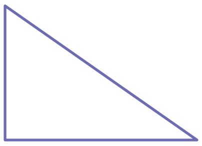
D. It is not possible to tell because neither the side length nor the area of the original square is known.

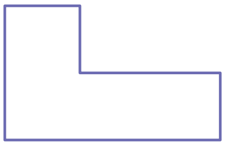
4. Tyler studied the figure shown and said, "I cannot determine the area because there are many different measurements, instead of just one length and one width that could be multiplied together." Explain why Tyler's statement is incorrect.



5. For each polygon, circle any angles that appear to be right angles.





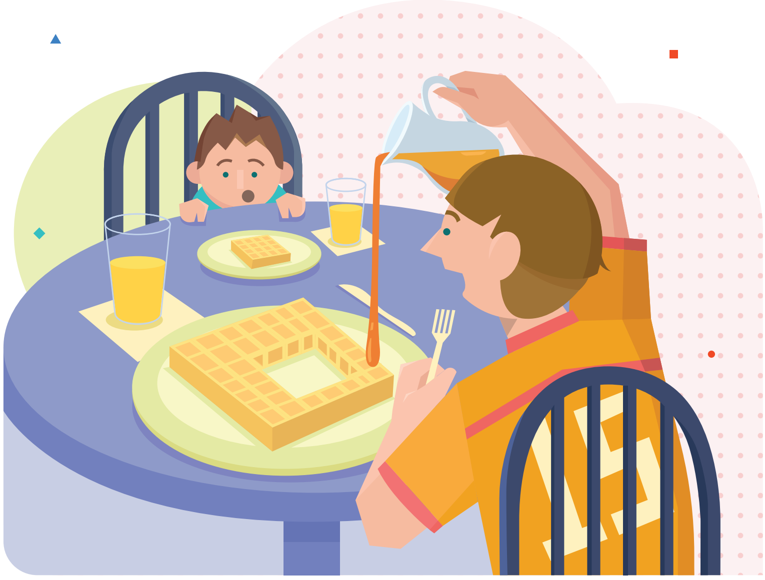


Name: ______________________ Date: ____________ Period: ____________

Unit 1 | Lesson 5

Reasoning to Determine Area

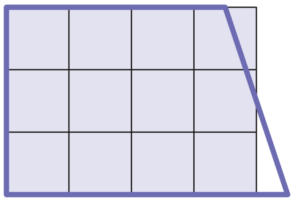
Let's use different strategies to determine the area of a shape.



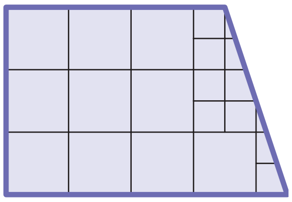
Warm-up What Is Area?

1. Which representation would you use to determine the area of the trapezoid? Be prepared to justify your choice.

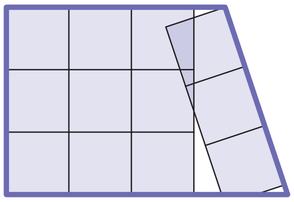
A.



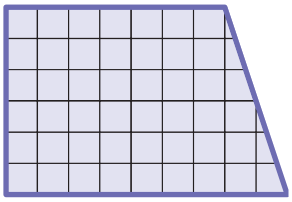
B.



C.



D.



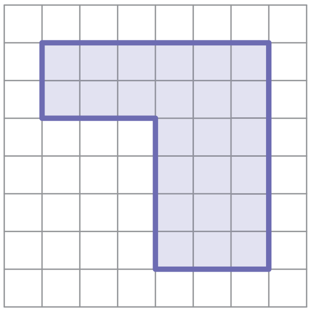
2. After the discussion, record your class definition of *area* here:

Log in to Amplify Math to complete this lesson online.

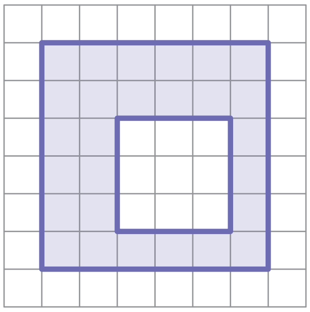
Activity 1 On the Grid

Each small square in these grids has an area of 1 square unit. Show or explain how to determine the total area, in square units, of each of the shaded regions without counting every square.

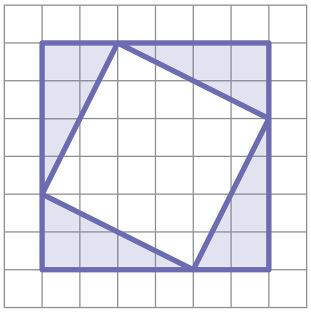
1.



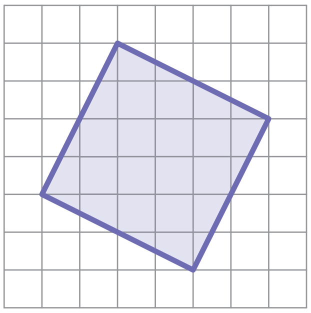
2.



3.



4.



Critique and Correct:
Your teacher will provide you with a sample response for Problem 3. Work with your partner to determine what the author meant and whether or not the response could be improved.

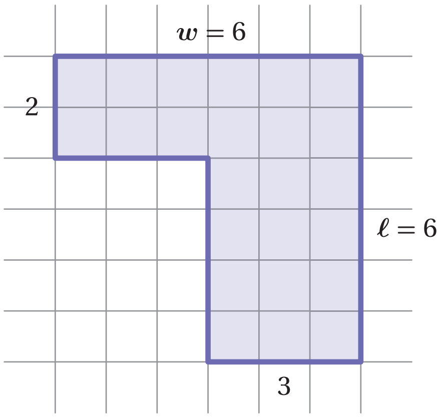
Name: .... Date: .... Period: ....

Activity 2 Off the Grid

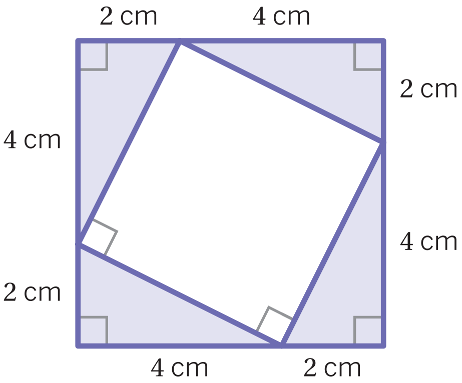
Plan ahead: How will you organize your thoughts so that you can clearly communicate them to others?

Determine the total area of all of the shaded regions in each figure. Explain or show your thinking.

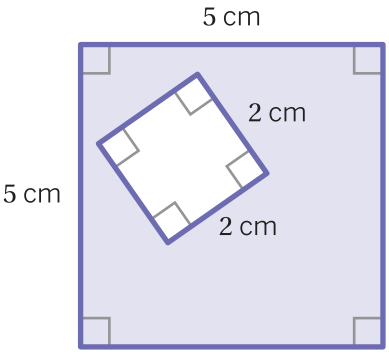
1.



2.



3.



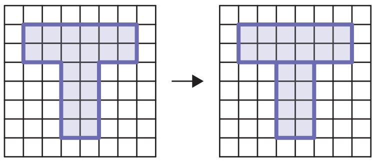
STOP

Summary

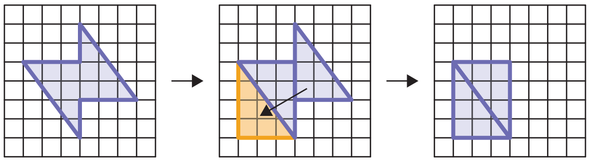
In today's lesson . . .

You saw there are several different strategies that can be used to determine the area of a shape. For instance, you can:

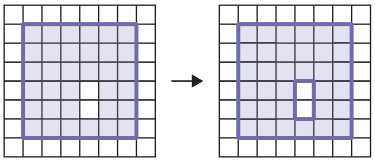
- Decompose the shape into two or more smaller shapes whose areas you know how to calculate, determine each of those areas, and then add them together.



- Decompose the shape and rearrange the pieces to form one or more other shapes whose areas you know how to calculate, determine each of those areas, and then add them together.



- Consider the shape as one with a missing piece, whose area is equal to the difference between the area of that shape and the area of the missing piece.



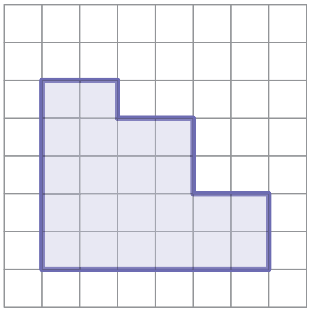
Area is always measured in square units. For example, when both side lengths of a rectangle are measured in centimeters, then the area is measured in square centimeters.

Reflect:

Name: .... Date: .... Period: ....

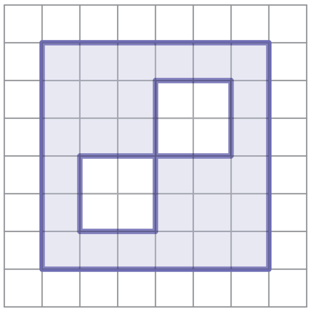
1. Each small square in these grids has an area of 1 square unit. Determine the total area of each of the shaded regions. Show or explain your thinking.

a



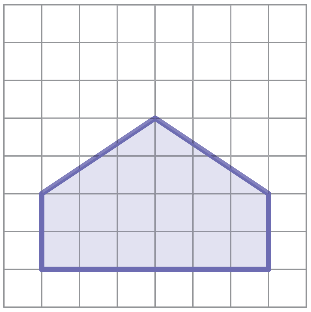
Area =

b



Area =

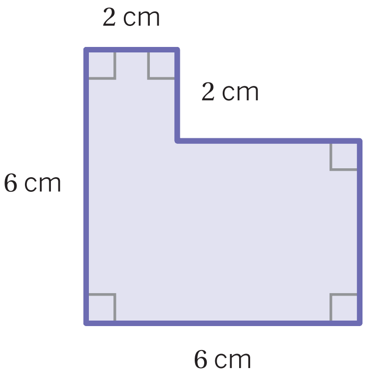
c



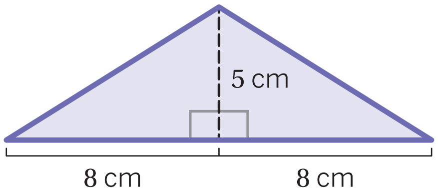
Area =

2. Determine the total area of each of the shaded regions. Show or explain your thinking.

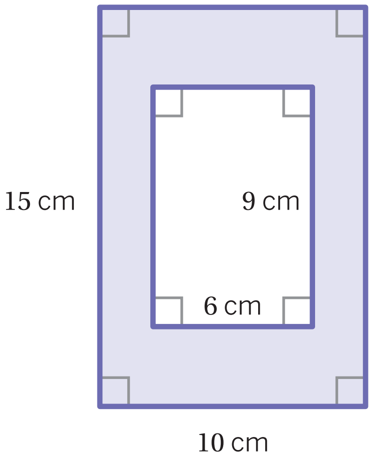
a



b



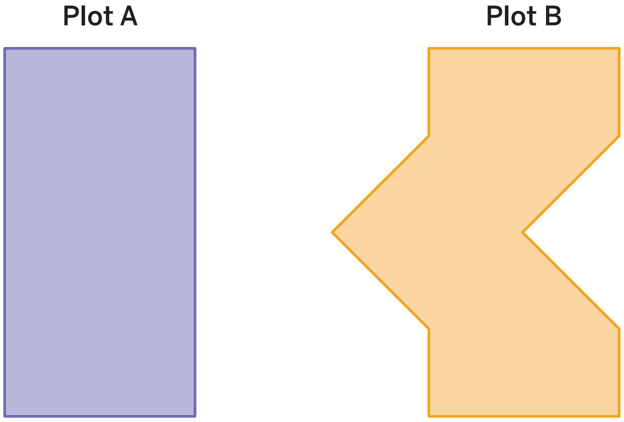
c



Practice

Name: ______________________ Date: ____________ Period: ____________

3. Two plots of land have very different shapes. Noah said that both plots of land have the same area. Do you agree with Noah? Explain your thinking.



4. A homeowner wants to fully cover a rectangular wall in her bathroom that measures 80 in. by 40 in. She will choose between square tiles with side lengths of either 8 in., 4 in., or 2 in. State whether you agree or disagree with each statement. Explain your thinking.

a Regardless of the chosen tile size, she will need the same number of tiles.

b Regardless of the chosen tile size, the area of the wall to be tiled remains the same.

c She will need two 2-in. tiles to cover the same area as one 4-in. tile.

d She will need four 4-in. tiles to cover the same area as one 8-in. tile.

e If she chooses the 8-in. tiles, she will need one fourth as many tiles as she would if she chooses the 2-in. tiles.

5. Draw two quadrilaterals that have at least one pair of sides that are *parallel*. Name each quadrilateral.

Name: ______ Date: ______ Period: ______

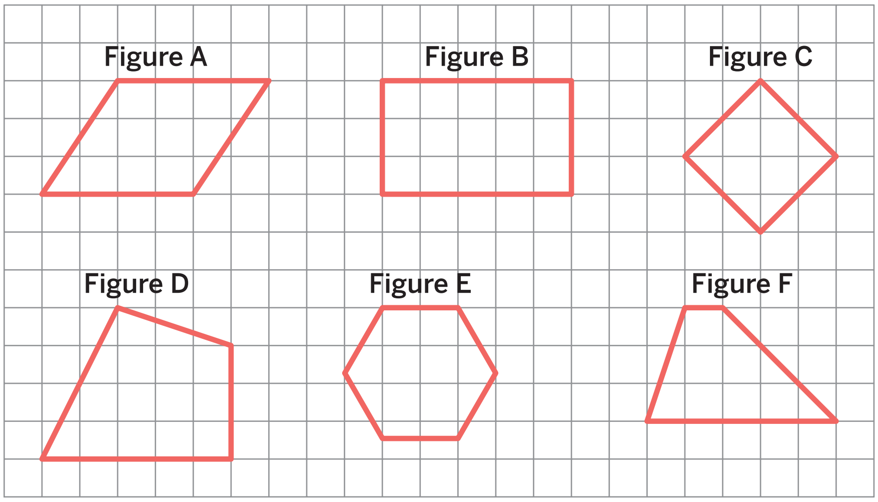
Unit 1 | Lesson 6

Parallelograms

Let's investigate the features and areas of parallelograms.

Warm-up Features of a Parallelogram

Figures A, B, and C are parallelograms. Figures D, E, and F are *not* parallelograms.



1. Study the examples and non-examples of parallelograms. What do you notice about:

   a. the number of sides the parallelograms have?

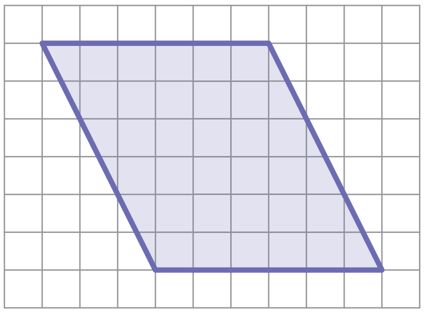
   b. the opposite sides of the parallelograms?

   c. the opposite angles of the parallelograms?

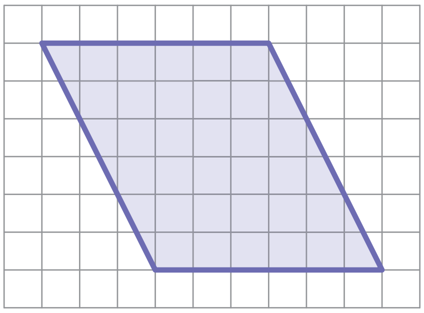
Log in to Amplify Math to complete this lesson online.

Activity 1 Decomposing and Rearranging Parallelograms

1. Refer to the parallelogram.

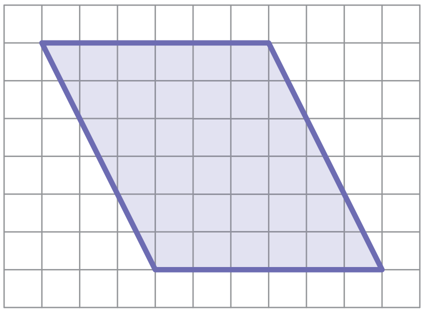


a Each small square in these grids has an area of 1 square unit. Determine the area of the parallelogram. Explain or use the grid to show the strategy you used.



b Compare answers with your group and take turns sharing your strategies.

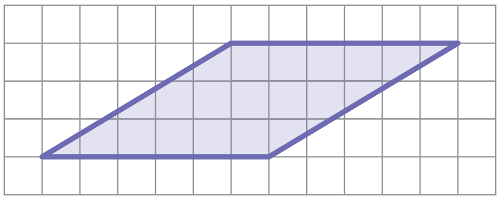
c Explain or show a different strategy than yours (used by someone else from your group) for determining the area of the parallelogram. If everyone in your group used the same strategy, work together to find a different strategy and explain or show that one.

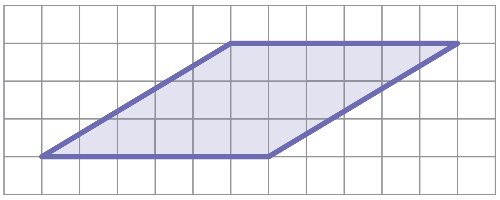


Name: Date: Period:

Activity 1 Decomposing and Rearranging Parallelograms (continued)

2. Here are two identical copies of the same parallelogram. Determine its area, and then justify your thinking by explaining or showing two different strategies.





Area:

Strategy 1:

Strategy 2:

Activity 2 Passing Parallelograms

You will be given a blank grid and a sheet containing a table. Draw any parallelogram you would like on the grid, but it *cannot* be a rectangle.

1. Determine the area of your parallelogram and record it in the table. When everyone in your group has finished, pass the drawing of your parallelogram to the person on your left.

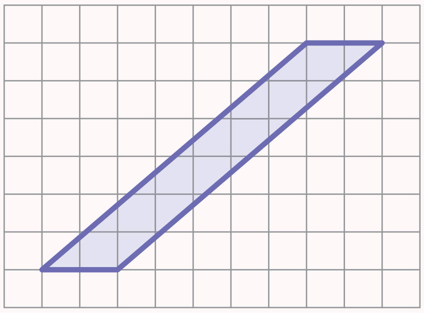
2. Determine the area of the parallelogram that was passed to you, *but do not draw on it*. Additional grids are available if you would like to redraw and mark up the parallelogram, or even cut it out. Record the area in the table alongside the name of the student who drew it.

3. Continue passing the parallelograms to the left, determining the area of each new parallelogram that is passed to you, and recording them in the table along with the name of the student who drew each one. **Note:** Each group member should see each parallelogram. Add rows to the table, as needed.

When your original parallelogram is returned to you, compare your responses and share your strategies with one another.

Are you ready for more?

Determine the area of this parallelogram.



STOP

Name: ______________________ Date: __________ Period: ________

Summary

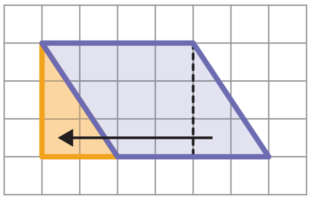
In today's lesson . . .

You revisited the defining properties of a ***parallelogram*** — a type of ***quadrilateral*** that has two pairs of parallel sides. In a parallelogram, each pair of opposite sides have the same length and each pair of opposite angles have the same measure. A *rectangle* is a special type of parallelogram in which all four angles are right angles.

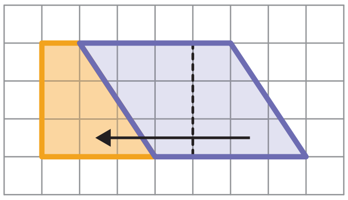
In order to determine the area of a parallelogram, you can decompose and rearrange it to calculate the area using a related rectangle:

- Decompose the parallelogram into two pieces and rearrange the pieces (using slides and flips) to form a rectangle that has the same area as the parallelogram.

Right triangle and trapezoid



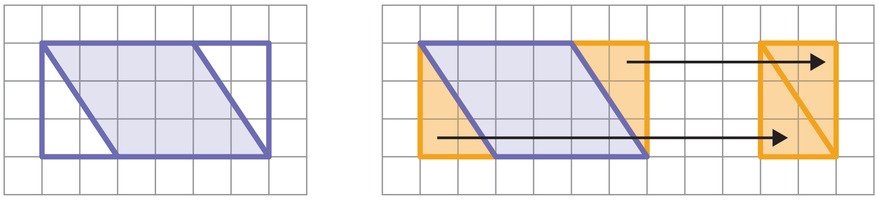
Two right trapezoids



The area of the related rectangle is $3 \cdot 4 = 12$ or 12 square units; therefore the area of the parallelogram is also 12 square units.

- Enclose the parallelogram in a rectangle, which is composed of two right triangles and a parallelogram. The two triangles can be composed to form a smaller rectangle, and the parallelogram's area is equal to the difference between the two rectangles' areas.

Enclose the parallelogram in a rectangle

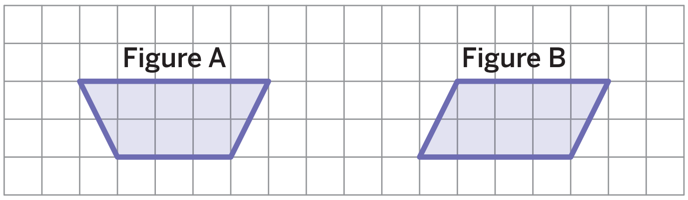


The area of the parallelogram is the difference of the two rectangles.
$(6 \cdot 3) - (2 \cdot 3) = 18 - 6 = 12$ or 12 square units.

Reflect:

Name: .... Date: .... Period: ....

1. Complete the table by deciding whether each figure is a parallelogram. For shapes that are *not* parallelograms, explain how you know they are not parallelograms.



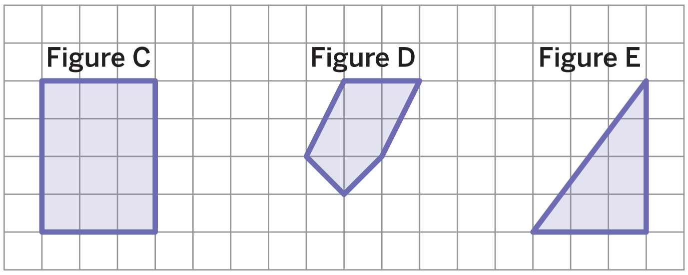
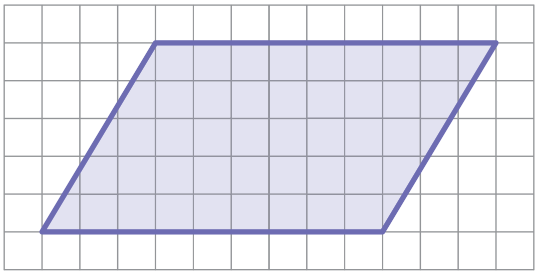


Figure	Parallelogram (Yes/No)	If not a parallelogram, how do you know?
A		
B		
C		
D		
E		

2. Refer to the parallelogram.

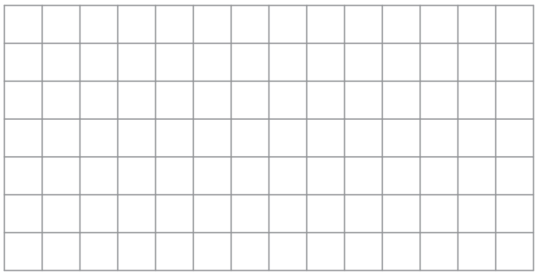
a Decompose and rearrange this parallelogram to form a rectangle.

b What is the area of the parallelogram? Explain or show the strategy you used.



Name: .... Date: .... Period: ....

3. Each small square in these grids has an area of 1 square unit. Draw a rectangle on the grid. Then decompose and rearrange the pieces of your rectangle to draw a parallelogram on the grid that has the same area. What is the area of each of your figures?



4. Determine which shape or shapes cover the greatest area and the least area of the plane in the pattern.

Greatest area:

Least area:

5. Use your geometry toolkit to draw each quadrilateral.

a Draw two quadrilaterals that have at least two sides that are perpendicular.

b Draw two quadrilaterals that have *no* sides that are perpendicular.

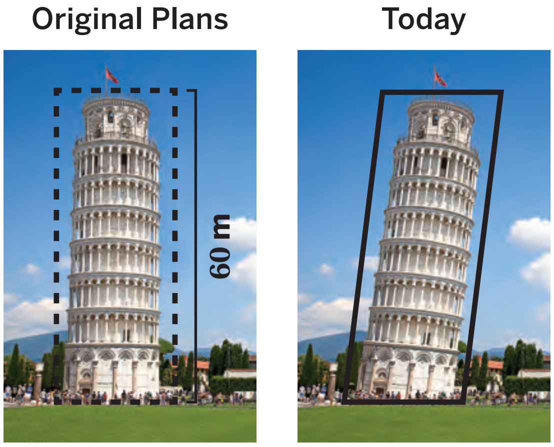
Unit 1 | Lesson 7

Bases and Heights of Parallelograms

Let's continue to investigate the areas of parallelograms.

Warm-up How Tall Is the Leaning Tower of Pisa?

The Leaning Tower of Pisa is a bell tower located in Pisa, Italy. Construction first began in 1173, but was halted multiple times due to wars, funding issues, and engineers trying to deal with the lean — which started after only three stories had been completed in 1178! Construction was finally completed in 1399, and the Leaning Tower of Pisa still stands today. It is not expected to fall for at least another 200 years, if ever.



Fedor Selivanov/Shutterstock.com

1. How would you determine how far above the ground someone would be, if they were standing on top of the tower today?

2. Define the terms *base* and *height* in your own words, and describe how each term relates to the two images of the tower.

Name: ________ Date: ________ Period: ________

Activity 1 The Right Height

1. Think about how the base of a parallelogram relates to its height.

a In Figures A, B, C, and D, the dotted lines *are* a corresponding height for the labeled base.

Figure A

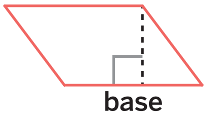


Figure B

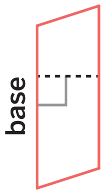


Figure C

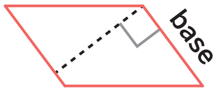
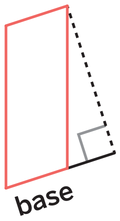


Figure D



b In Figures E, F, G, and H, the dotted lines *are not* a corresponding height for the labeled base.

Figure E

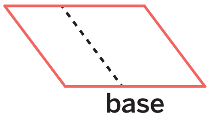


Figure F

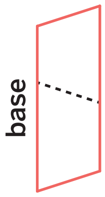


Figure G

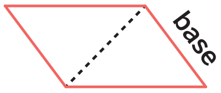
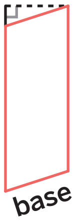


Figure H



c What must be true about a corresponding height for a given base in a parallelogram?

Discussion Support: As you share your responses, restate your classmates' reasoning to be sure you understand. Look for opportunities to challenge each other by respectfully agreeing or disagreeing.

Activity 1 The Right Height (continued)

2. Determine whether each statement is *true* or *false*. If a statement is false, write a related statement that is true.

Statement	True or false?	If false, make it true:
Only a horizontal side of a parallelogram can be a base.		
A base and its corresponding height must be perpendicular to each other.		
A height can only be drawn inside a parallelogram.		
A height can be drawn at any angle related to the side chosen as the base.		
For a given base, there is more than one way to draw a corresponding height.		

3. Each parallelogram is labeled to show a base b and a *potential* corresponding height h.

a Which parallelograms have a correctly labeled base and height pair?

b For each of the parallelograms that have an incorrectly labeled base and height pair, explain why the labels are not correct.

Figure J

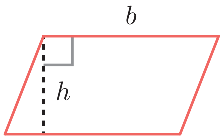


Figure K

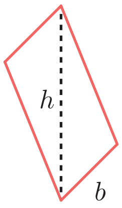


Figure L

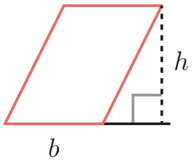


Figure M

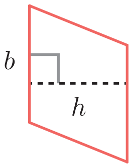
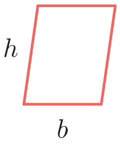


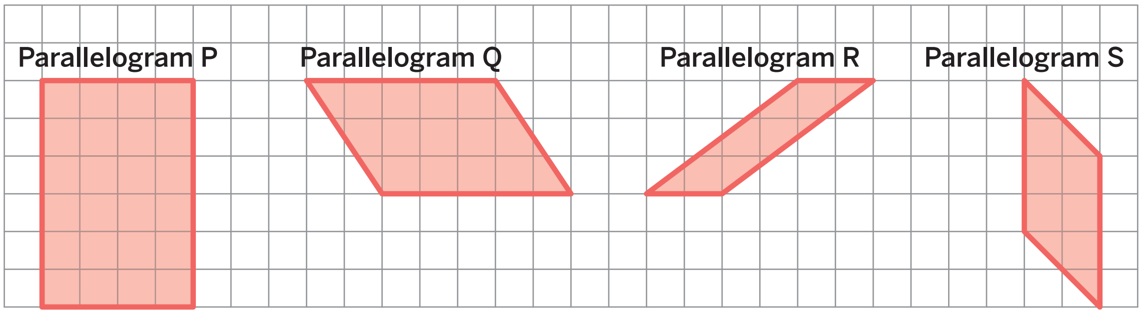
Figure N



Name: .... Date: .... Period: ....

Activity 2 A Formula for the Area of a Parallelogram

1. Complete the corresponding rows of the table for each parallelogram by:
   - identifying a base and a corresponding height and recording their lengths.
   - determining the area of each parallelogram.



Parallelogram	Base (units)	Height (units)	Area (square units)
P			
Q			
R			
S			
Any parallelogram	b	h	

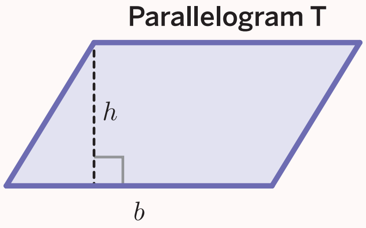
2. Complete the last row of the table by writing an expression that could be used to determine the area of *any* parallelogram, with base b and corresponding height h.

Are you ready for more?

What happens to the area of Parallelogram T if . . .

a the base is unchanged, but the height doubles? Triples? Is 100 times its original length?

b *both* the base and the height double? Triple? Each becomes 100 times their original length?



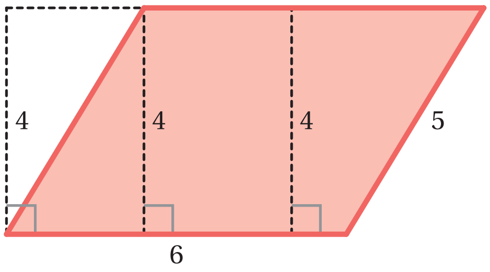
STOP

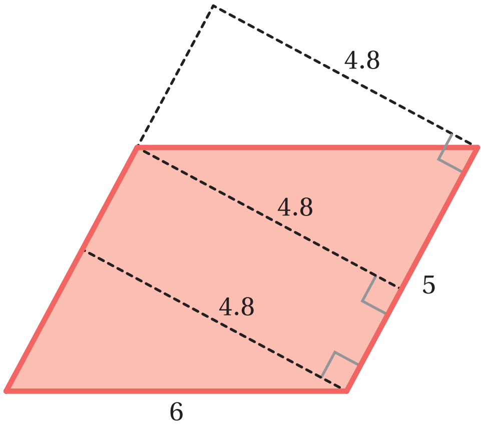
Summary

In today's lesson . . .

You saw that any of the four sides of a parallelogram can be chosen as the ***base***. Any perpendicular segment from a point on the base to the opposite side of the parallelogram represents the ***height***. There are infinitely many possible segments that can represent the height for a given base, including some that are drawn outside of the parallelogram.

These two figures show two possible bases for the same parallelogram, labeled with lengths of 6 and 5, and then three possible corresponding heights for each, labeled with lengths of 4 and 4.8.





$A = b \cdot h$

$A = 6 \cdot 4$

$A = 24$; The area is 24 square units

$A = b \cdot h$

$A = 5 \cdot 4.8$

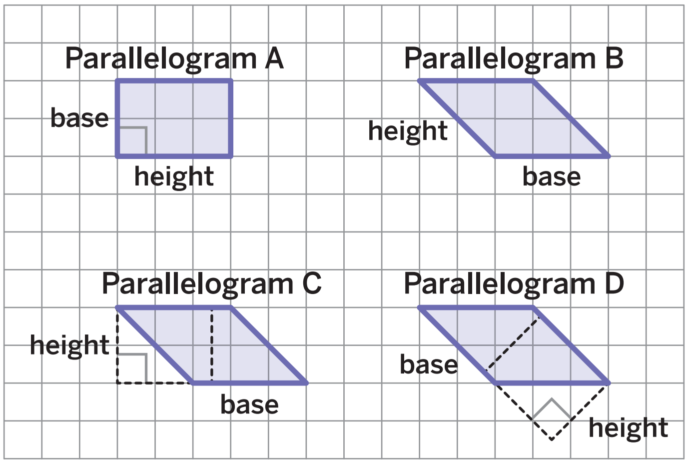
$A = 24$; The area is 24 square units

No matter which side of a parallelogram is chosen as the base, its area A is equal to the product of the length of the base b and the length of a corresponding height h.

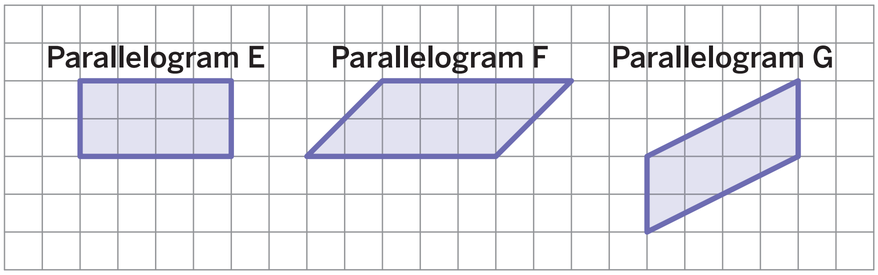
Reflect:

Name: .... Date: .... Period: ....

1. List the parallelograms that have a correct height labeled for the given base.



2. Each small square in these grids has an area of 1 square unit. Determine the area of each parallelogram.

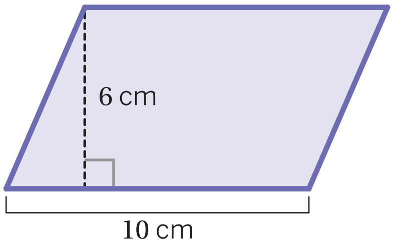


a Area of Parallelogram E:

b Area of Parallelogram F:

c Area of Parallelogram G:

3. Write an expression that can be used to calculate the area of the parallelogram shown. Then use your expression to calculate the area.



Area =

Practice

Name: .... Date: .... Period: ....

4. Determine whether each statement is *true* or *false*. If a statement is false, write a related statement that is true.

Statement	True or False?	If false, make it true:
A parallelogram has six sides.		
Opposite sides of a parallelogram are parallel.		
A parallelogram can have one pair or two pairs of parallel sides.		
All sides of a parallelogram must have the same length.		
All angles of a parallelogram must have the same measure.		

5. A square with an area of 1 m^2 was decomposed into nine identical smaller squares. Each smaller square was then decomposed into two identical triangles.

a What is the total area, in square meters, of six of the resulting triangles? Consider drawing a diagram to help with your thinking.

b How many of the resulting triangles would be needed to compose a shape that has an area of $1\frac{1}{2} \text{ m}^2$?

6. Write down as many things you know or remember about a rhombus.

Name: .... Date: .... Period: ....

Unit 1 | Lesson 8

Area of Parallelograms

Let's practice determining the area of parallelograms seen in the world.

Warm-up A Rhombus on the Road

Many road signs are simple geometric shapes — circles, triangles, quadrilaterals, and pentagons. This Pedestrian Crossing sign is a special type of parallelogram called a *rhombus*, which has four sides that are all the same length.

1. The sign also has four right angles. Besides being a parallelogram and a rhombus, what other shape(s) could describe the sign?

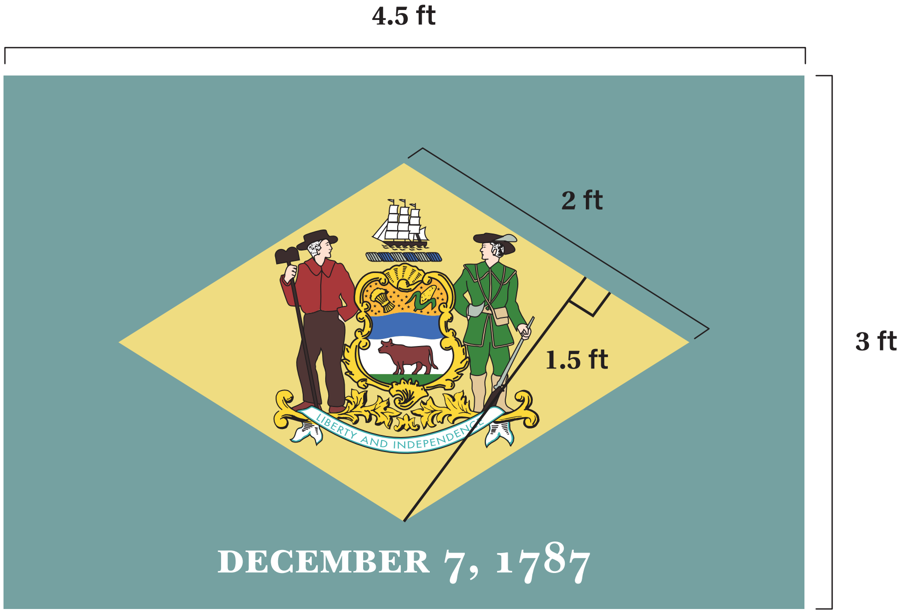
2. Each side of the sign measures 30 in. What is its area?

3. Draw another parallelogram that has the same area as the Pedestrian Crossing sign, but is neither a rhombus nor a rectangle. Be sure to label known lengths.

Log in to Amplify Math to complete this lesson online.

Activity 1 Parallelograms All Around

Delaware's state flag is shown. The official colors — colonial blue and buff yellow — represent a Revolutionary War uniform worn by General George Washington. The flag also contains the date on which Delaware became the first state to ratify the Constitution. The state's coat of arms, reading "Liberty and Independence," is displayed on top of a diamond because Delaware was once nicknamed the Diamond State. The yellow "diamond" in the center of the flag is actually a rhombus.



Public Domain

1. To create a proper rectangular flag that measures 3 ft by 4.5 ft, the rhombus would have side lengths of 2 ft and a perpendicular distance across of 1.5 ft. Determine how much of each color fabric is used to make the two main parts of the flag. Explain or show your thinking.

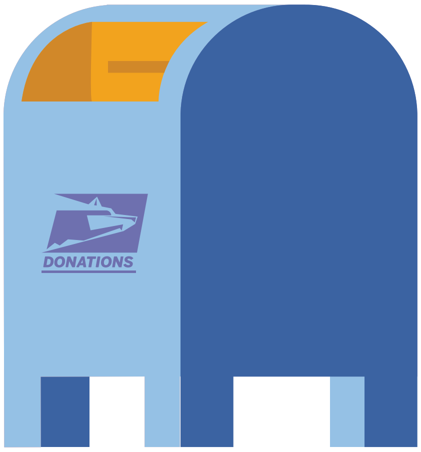
   a Yellow rhombus

   b Blue rectangle

Name: ________________ Date: ________ Period: ________

Activity 1 Parallelograms All Around (continued)

2. A local charity organization has placed drop boxes for donations around town, such as the one shown here.



a The base of the logo on a drop box measures approximately 25 cm, and the height measures approximately 15 cm. About how many square centimeters of space does the logo take up on the side of a drop box?

b A prototype for printing the logo on the side of a transport and delivery truck takes up about 735 in^2 of space, and measures about 35 in. horizontally across the bottom edge. What is the corresponding height of the logo for the truck?

3. Handicapped parking spaces are given extra clearance from the curb, and a "no parking" area is often marked in between to allow a wheelchair to enter and exit a vehicle safely. The slanted lines marking the "no parking" space shown here form 9 parallelograms and 2 right triangles (each of which is exactly half of one parallelogram).

Amy Sroka/Amplify

If the length of the parking space is 18 ft (the minimum required), and the "no parking" area covers 90 ft^2, how far is the right side of this handicapped parking space from the curb?

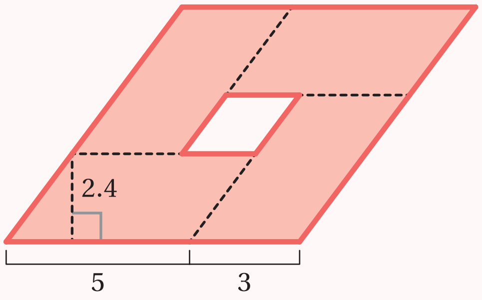
Activity 2 Drawing Parallelograms

Use the grid to draw two different parallelograms, labeled as P and Q, that meet the following criteria:

- **They both have the same area of 20 square units.**
- **Neither of the parallelograms is a rectangle.**
- **The two parallelograms do not have any side lengths in common with one other.**

Are you ready for more?

The shape shown is composed of smaller parallelograms. The shaded region is composed of four identical parallelograms. All lengths are in inches. What is the area of the unshaded parallelogram in the middle? Explain or show your thinking.



STOP

Name: ______________________ Date: ____________ Period: ____________

Summary

In today's lesson . . .

You saw examples of several different parallelograms in the real world. Given their dimensions, you calculated their areas, bases, and/or heights.

The formula for the area of a parallelogram can be used to determine *any* of the three measures involved, not just the area. For example:

- If you know the area and the length of the base, you can determine the length of a corresponding height.

Base	Height	Area
5 cm	?	60 cm^2

$$A = b \cdot h$$
$$60 = 5 \cdot ?$$
$$60 \div 5 = 12$$

The height is 12 cm.

- Similarly, if you know the area and the measure of a height, you can determine the length of the base.

Base	Height	Area
?	8 in	16 in^2

$$A = b \cdot h$$
$$16 = ? \cdot 8$$
$$16 \div 8 = 2$$

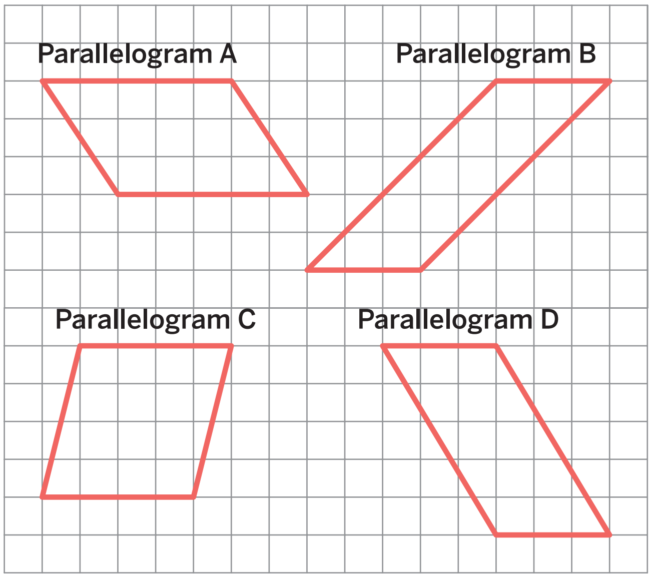
The base is 2 in.

Reflect:

Practice

Name: ________________ Date: ________ Period: ________

1. Three of these parallelograms have the same area. Which one has a *different* area than the others?



2. The base lengths b and corresponding heights h of four different parallelograms are listed. Which base-height pair represents the parallelogram with the greatest area?

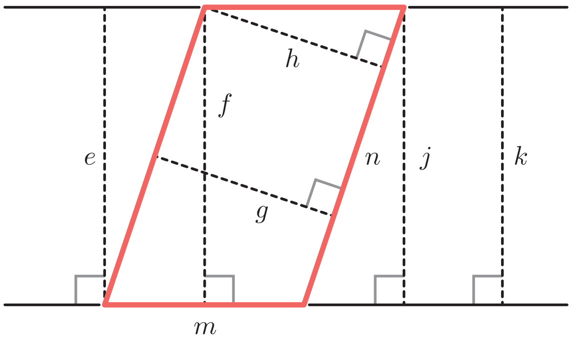
A. $b = 4, h = 3.5$
B. $b = 0.8, h = 20$
C. $b = 6, h = 2.25$
D. $b = 10, h = 1.4$

3. Two opposite faces of the Dockland Building in Hamburg, Germany are shaped like identical parallelograms. The length of the face of the building shown is approximately 86 m along the bottom, and its height is approximately 55 m from the bottom to the top. Using this information, estimate the area of this face of the building.

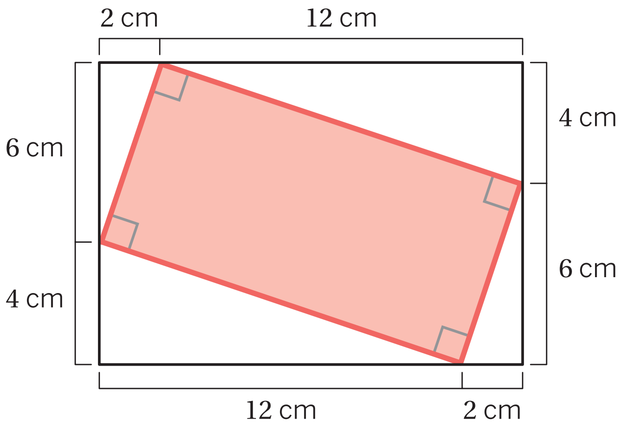
foto-select/Shutterstock.com

Name: .... Date: .... Period: ....

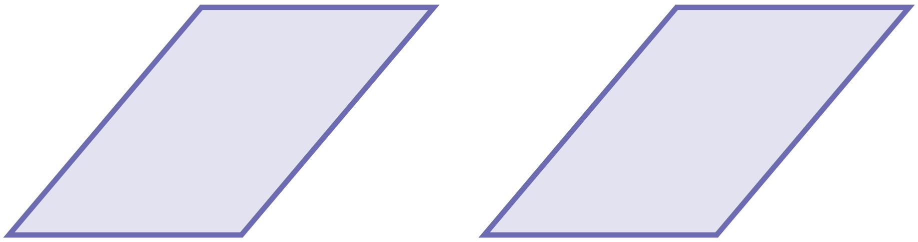
4. The parallelogram shown has side lengths m and n. List *all* of the lengths that represent a corresponding height for the base m.



5. Determine the area of the shaded region. Show or explain your thinking.



6. Draw a straight line on *each* copy of this quadrilateral, where each line partitions the shape into equal-sized halves. Show a different partition for each shape.



Unit 1 | Lesson 9

From Parallelograms to Triangles

Let's use what we know about parallelograms to determine the area of triangles on grids.

Warm-up Composing Parallelograms

You will be given two identical triangles on a grid. Create as many *different* parallelograms as you can using both triangles. Trace the perimeter of each different parallelogram on one of the grids. You might not use all the grids.

Log in to Amplify Math to complete this lesson online.

Name: ____________ Date: ____________ Period: ____________

Activity 1 Decomposing Parallelograms

Refer back to the Warm-up and label three of the parallelograms as A, B, and C.

1. Assume each grid square has an area of 1 square unit. Calculate the area of each parallelogram.

a Parallelogram A:

b Parallelogram B:

c Parallelogram C:

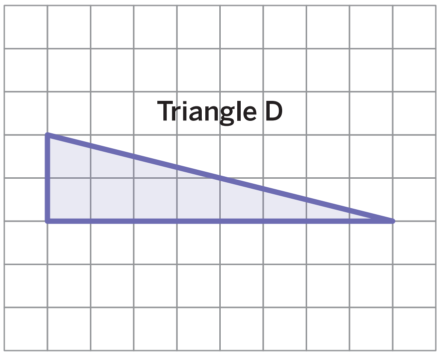
2. What is the area of one of the original triangles? Show or explain your thinking.

3. Draw a different triangle that has the same area as one of the triangles from the Warm-up. Show or explain how you know it has the same area.

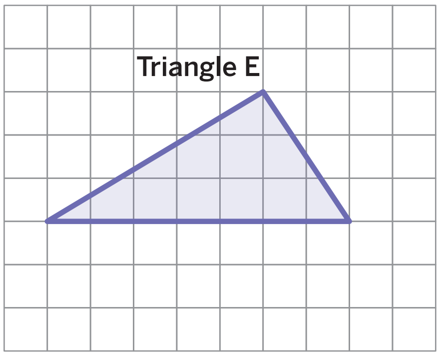
Activity 2 Determining the Area of Triangles

On this page and the next page, four different triangles, labeled D, E, F, and G, are shown on grids. Each small square in these grids has an area of 1 square unit. Choose *three* of the triangles and determine their areas. Use at least two different strategies altogether. Show or explain your thinking for each triangle you chose.

1.

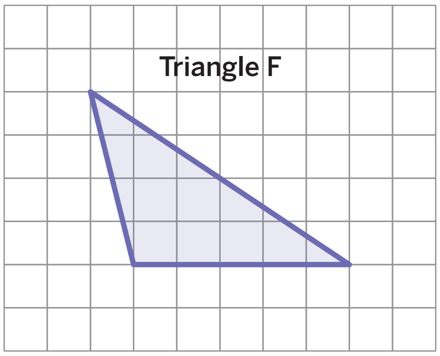


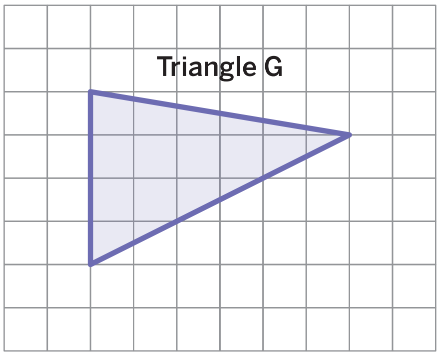
2.



Name: ______ Date: ______ Period: ______

Activity 2 Determining the Area of Triangles (continued)

3. 

4. 

Reflect: How did patterns provide a structure for finding the area of parallelograms and triangles?

STOP

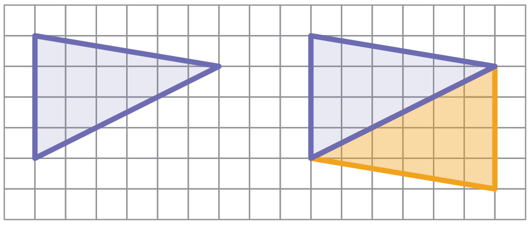
Summary

In today's lesson . . .

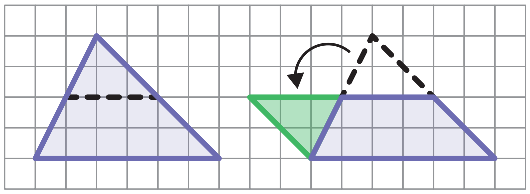
You saw several ways to reason about the area of a triangle using what you know about composition and decomposition of shapes and the area of parallelograms.

Here are three possible strategies to determine the area of a triangle on a grid:

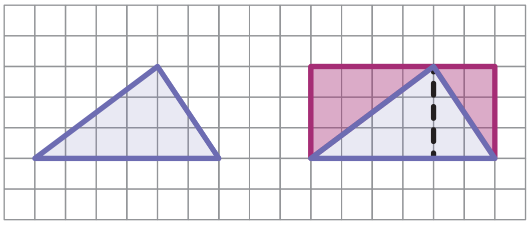
- Make a copy of the triangle and compose the two identical triangles to form a parallelogram — for a right triangle, this will be a rectangle. Because the two triangles have the same area, each triangle has an area that is exactly half the area of the parallelogram.



- Decompose the triangle and rearrange the pieces to form a parallelogram. Because the triangle and the parallelogram are made up of exactly the same pieces, their areas are equal.



- Enclose the triangle in a large rectangle that can be decomposed into two smaller rectangles. This also decomposes the triangle into two smaller triangles. Each of these smaller triangles has half the area of its enclosed rectangle. The sum of the two smaller triangles' areas is equal to the area of the original triangle.

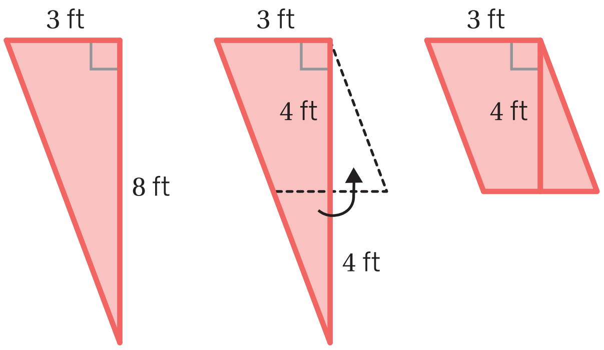


Reflect:

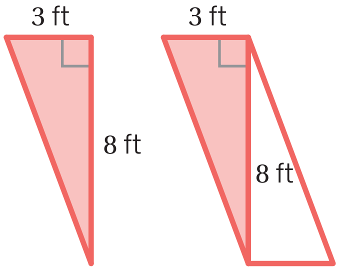
Name: .... Date: .... Period: ....

1. To determine the area of a given right triangle, Diego and Jada used different strategies.

a Diego drew a line through the midpoints of the two longer sides, which decomposed the triangle into a trapezoid and a smaller triangle. He then rearranged the two shapes to form a parallelogram. Explain how Diego could use his parallelogram to determine the area of the triangle.

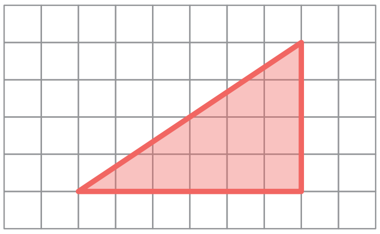


b Jada made an identical copy of the triangle and used the two identical copies to compose a different parallelogram. Explain how Jada could use her parallelogram to determine the area of the triangle.

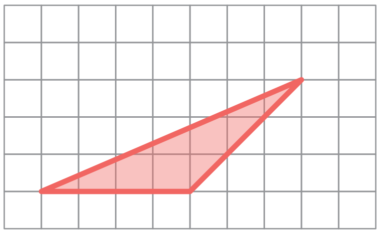


2. Each small square in these grids has an area of 1 square unit. Determine the area of each triangle.

a

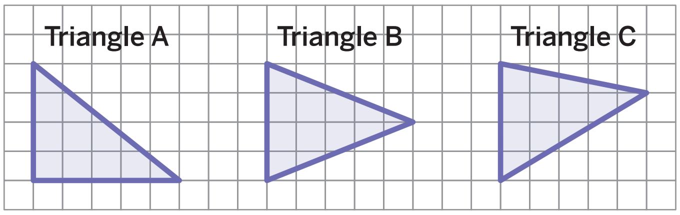


b



Name: .... Date: .... Period: ....

3. Which of these triangles has the greatest area? Show or explain your thinking.



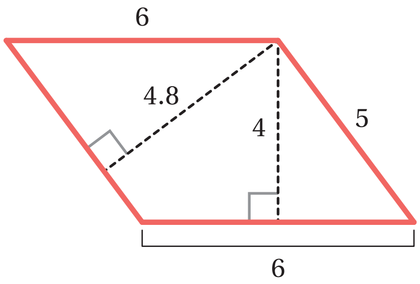
4. Solve the following problems involving the base, height, and area of parallelograms.

   a A parallelogram has a base of 3.5 units and a corresponding height of 2 units. What is its area?

   b A parallelogram has a base of 3 units and an area of 1.8 square units. What is the corresponding height for that base?

   c A parallelogram has an area of 20.4 square units. If the height that corresponds to a base is 4 units, what is the length of that base?

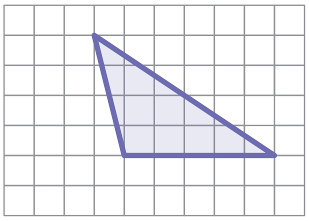
5. If the side that is 6 units long is the base of this parallelogram, what is its corresponding height?



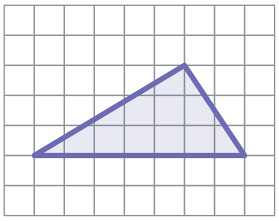
A. 6 units
B. 4.8 units
C. 4 units
D. 5 units

6. Using what you know about parallelograms, label what you think would be the base and height for each of these triangles.

a



b



Name: .... Date: .... Period: ....

Unit 1 | Lesson 10

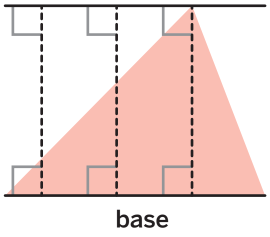
Bases and Heights of Triangles

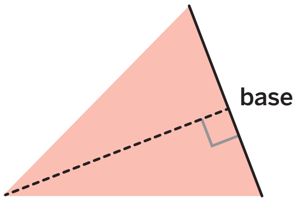
Let's find the bases and heights of triangles.

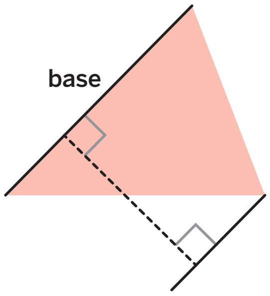
Warm-up Base and Height Pairs

Study the examples and non-examples of bases and heights in a triangle.

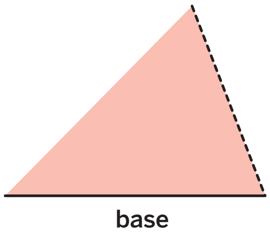
These dashed segments represent heights of the triangles.

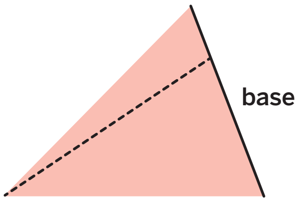


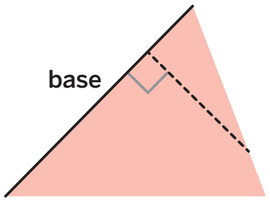




These dashed segments do *not* represent heights of the triangles.







Based on these examples and nonexamples, how would you define the *base* and *height* of a triangle?

Log in to Amplify Math to complete this lesson online.

Activity 1 The Truth About Bases and Heights

Refer to the examples and non-examples of bases and heights for triangles from the Warm-up.

1. Determine whether each statement is *true* or *false*. Place a check mark in the appropriate column.

Statement	True	False
Any side of a triangle can be the base.		
A height must always be one of the sides of a triangle.		
A height that corresponds to the base of a triangle can be drawn at any angle to the base.		
For a chosen base, there is only one possible height that can be drawn.		
A height must have an endpoint at a vertex of the triangle or along the line parallel to the base that extends from the opposite vertex.		

2. Choose one of the statements you identified as false, and explain why it is false.

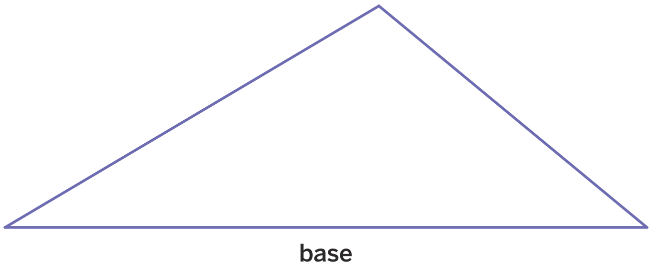
3. Using your chosen statement from Problem 2, alter the statement so that it is true. Rewrite the true statement here.

Name: .... Date: .... Period: ....

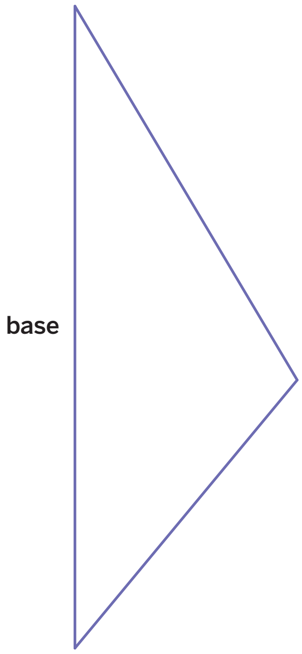
Activity 2 Hunting for Heights

1. Refer to Triangles A, B, C, and D. Draw a height that corresponds to each given base. Consider using an index card to help you.

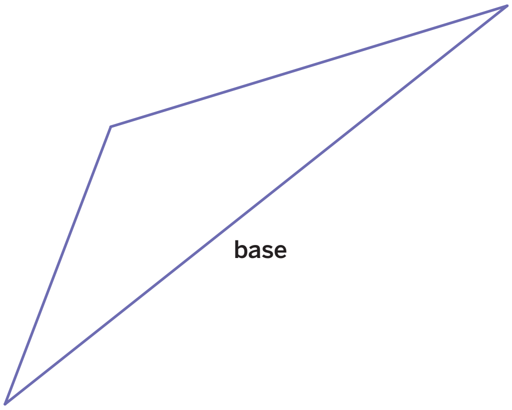
Triangle A



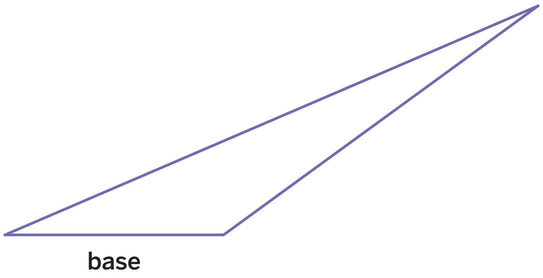
Triangle B



Triangle C



Triangle D

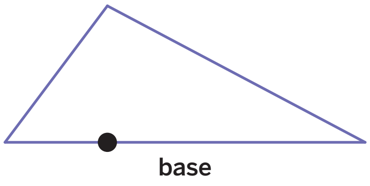


Activity 2 Hunting for Heights (continued)

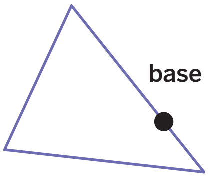
2. Refer to Triangles E–K. Continue to draw the heights for the given bases.

a For Triangles E–G, draw a height that intersects the base at the given point.

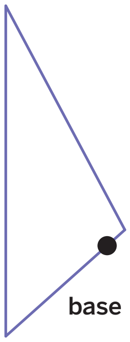
Triangle E



Triangle F

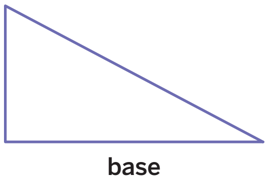


Triangle G

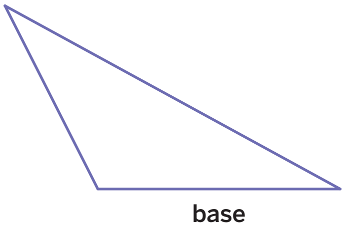


b For Triangles H–K, any corresponding height can be drawn.

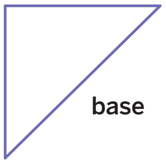
Triangle H



Triangle J



Triangle K



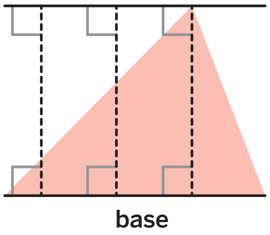
STOP

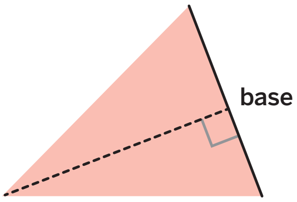
Name: ………………………………………… Date: ……………… Period: ………………

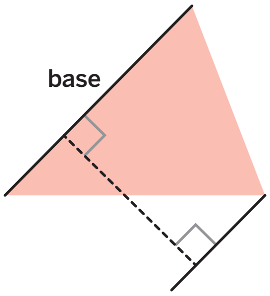
Summary

In today's lesson . . .

You saw that any side of a triangle can be chosen as the ***base*** of the triangle. Once the base has been identified, there are many possible segments that can be drawn for the ***height*** of the triangle. Most commonly this is the segment drawn perpendicular to the base from the *opposite vertex*. A height can also extend outside of the triangle – even entirely outside – from any point along the line containing the base to a line parallel to the base that contains the opposite vertex, intersecting both lines at right angles.







Even though any side of a triangle can be a base, some base and height pairs can be more easily determined than others, so it helps to choose strategically. For example:

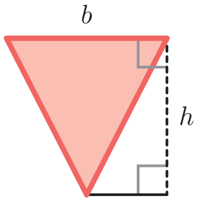
- When working with a right triangle, it often makes sense to use the two sides that form the right angle as the base and the height.
- When working on a grid, selecting a side that aligns to a horizontal or vertical grid line ensures that a perpendicular height can also be drawn along a grid line.

Reflect:

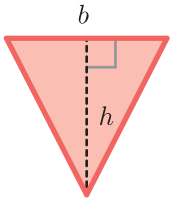
Name: .... Date: .... Period: ....

1. Name *all* the triangles for which a corresponding height h for the given base b is correctly identified.

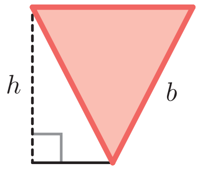
Triangle A



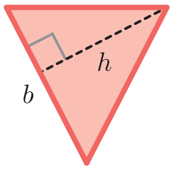
Triangle B



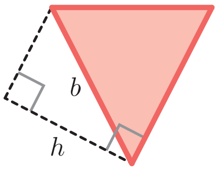
Triangle C



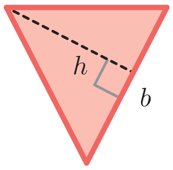
Triangle D



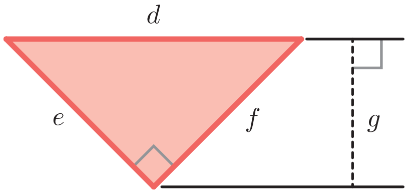
Triangle E



Triangle F



2. Refer to the right triangle shown. Name a corresponding height for each indicated base.

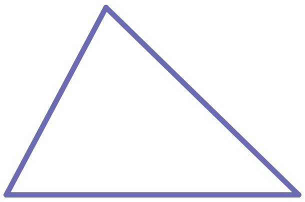


a Base d

b Base e

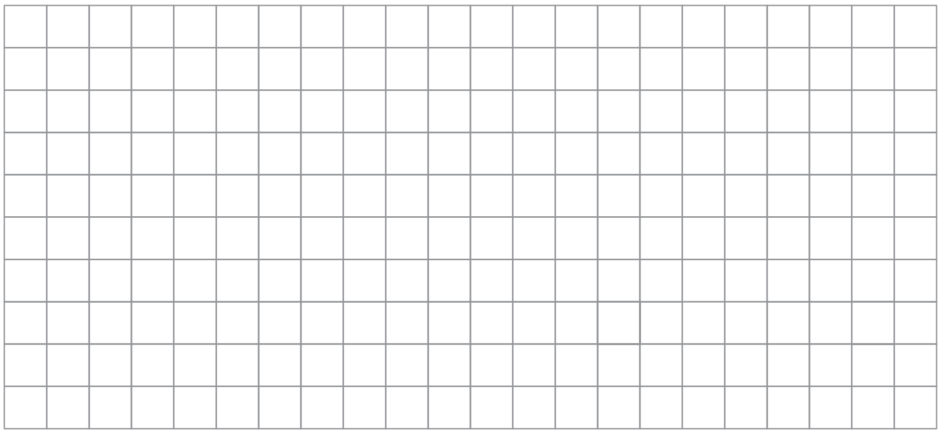
c Base f

3. Identify a base of the triangle and draw a corresponding height for your chosen base. Explain how to identify the base and how to draw the height.

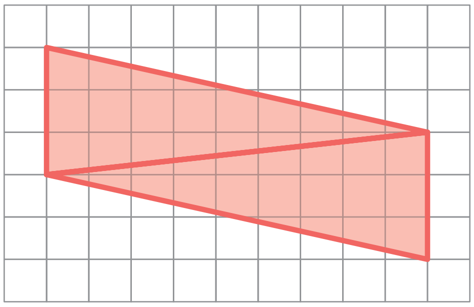


Name: .... Date: .... Period: ....

4. Draw two different parallelograms that have the same area. Label a corresponding base and height for each parallelogram and explain how you know the areas are the same.



5. Andre drew a diagonal segment connecting two opposite vertices of this parallelogram. Select *all* the true statements about the two triangles that are formed.



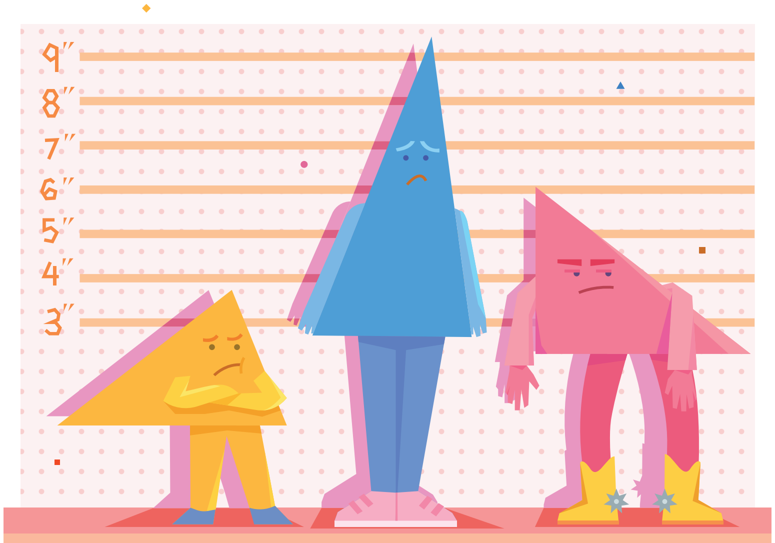
- **A.** Each triangle has two sides that are 3 units long.
- **B.** Each triangle has a side that is the same length as the diagonal segment.
- **C.** Each triangle has one side that is 3 units long.
- **D.** The triangles are not identical copies of one another.
- **E.** The triangles have the same area.
- **F.** Each triangle has an area that is half the area of the parallelogram.

6. How can drawing a triangle on a grid help you identify a valid base and height pair in the triangle?

Unit 1 | Lesson 11

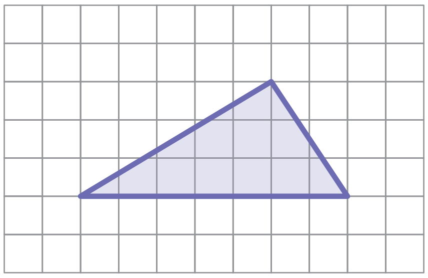
Formula for the Area of a Triangle

Let's write and use a formula to determine the area of any triangle.



Warm-up Same Bases, Same Heights

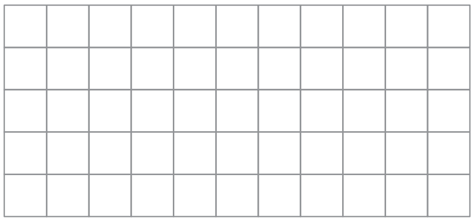
1. Label the base and draw a corresponding height for the triangle. Then write the measurement for each.



Base: .... units

Height: .... units

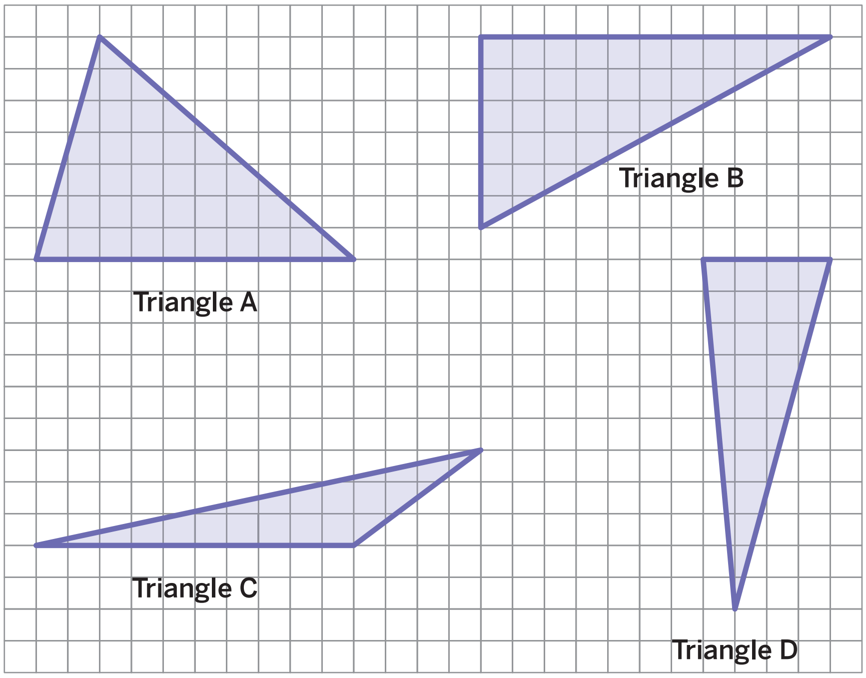
2. Draw a parallelogram with the same base and height measurements that you identified for the triangle in Problem 1.



Name: ______________________ Date: __________ Period: __________

Activity 1 The Formula for the Area of a Triangle

1. Label a base and corresponding height pair for each of the four triangles shown.



2. Complete the table by recording the following measurements of Triangles A, B, C, and D:

- The length of the base you identified
- The corresponding height
- The area

Triangle	Base (units)	Height (units)	Area (square units)
A			
B			
C			
D			
Any triangle	b	h	

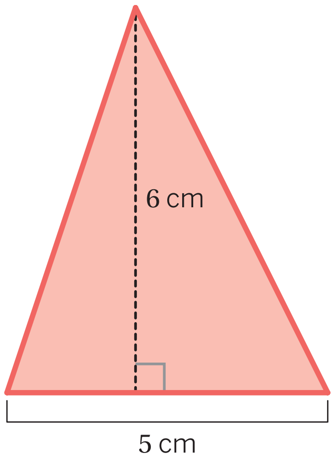
3. Complete the last row of the table by writing an expression for the area of any triangle, using b for the length of the base and h for the corresponding height.

Activity 2 Applying the Formula for the Area of a Triangle

Write the base and height measurements that can be used to calculate the area of each triangle. Then calculate the area and show or explain your thinking.

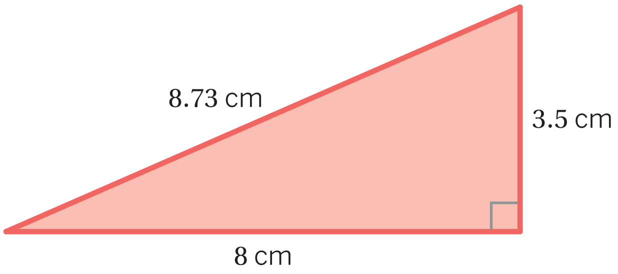
1. Base:
Height:
Area:
My thinking:

Triangle E



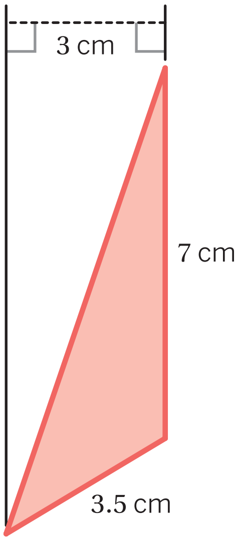
2. Base:
Height:
Area:
My thinking:

Triangle F



3. Base:
Height:
Area:
My thinking:

Triangle G



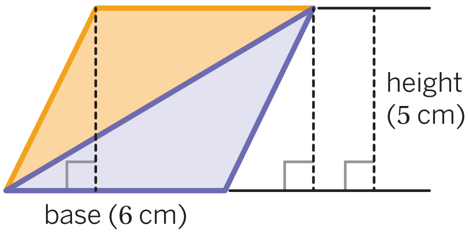
STOP

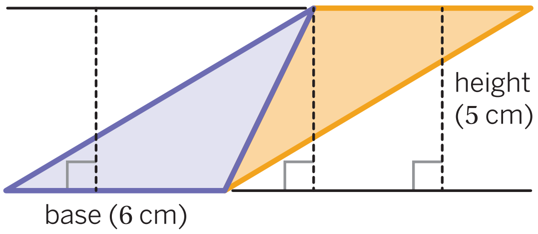
Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that the base and height pairs of a triangle are closely related to those of a parallelogram. Recall that two identical copies of a triangle can be composed to form a parallelogram. Regardless of how they are composed, the original triangle and the resulting parallelogram will have a common side length. Labeling these sides as the bases, the corresponding heights will be the same, no matter where those segments are drawn.





Using this relationship between triangles and parallelograms, the area of such a triangle will always be equal to exactly half the area of the corresponding parallelogram.

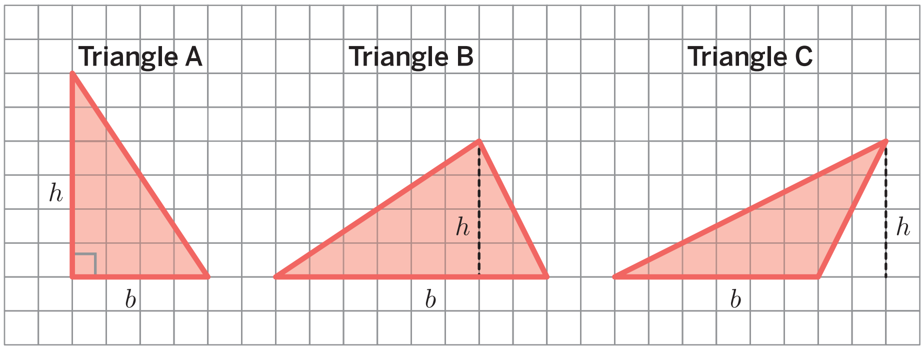
Area of Parallelogram	Area of Triangle
$A = b \bullet h$ $A = (6) \bullet (5)$ $A = 30$ The parallelogram has an area of 30 cm^2.	Half the area of the parallelogram, so; $A = \frac{1}{2}\ b \bullet h$ $A = \frac{1}{2} \bullet (6) \bullet (5)$ $A = 15$ The triangle has an area of 15 cm^2.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. For each triangle, a base b and its corresponding height h are labeled.



a Determine the area of each triangle.

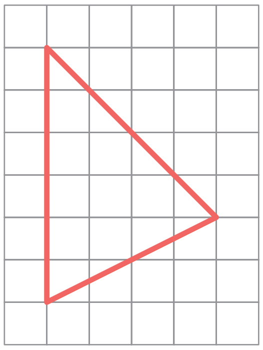
Triangle A:

Triangle B:

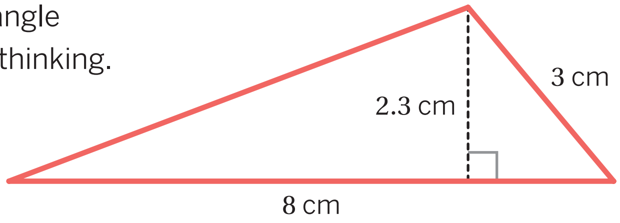
Triangle C:

b How is the area of *any* triangle related to the length of a chosen base and its corresponding height?

2. Determine the area of the triangle. Show or explain your thinking. To help with your thinking, carefully consider which side of the triangle to use as the base.

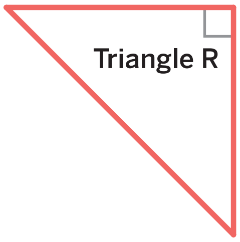


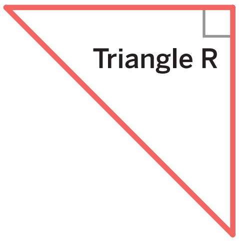
3. Determine the area of the triangle shown. Show or explain your thinking.



Name: .... Date: .... Period: ....

4. Triangle R is a right triangle. Is it possible to compose a parallelogram that is *not* a square using two identical copies of Triangle R? If so, show or explain how. If not, explain why not.





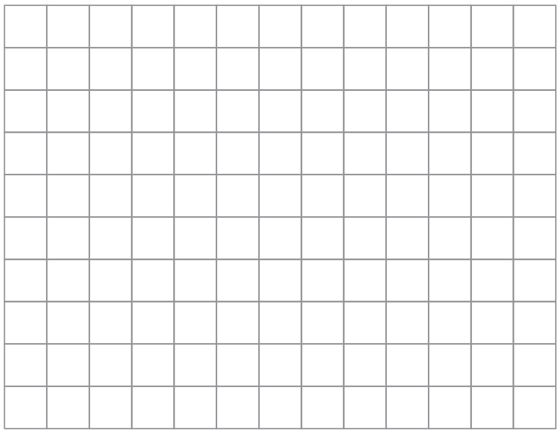
5. Determine the stated measurement for each parallelogram described.

a A parallelogram has a base of 9 units and a corresponding height of $\frac{2}{3}$ units. What is its area?

b A parallelogram has a base of 9 units and an area of 12 square units. What is the corresponding height for that base?

c A parallelogram has an area of 7 square units and the height corresponding to the chosen base is $\frac{1}{4}$ units. What is the length of the base?

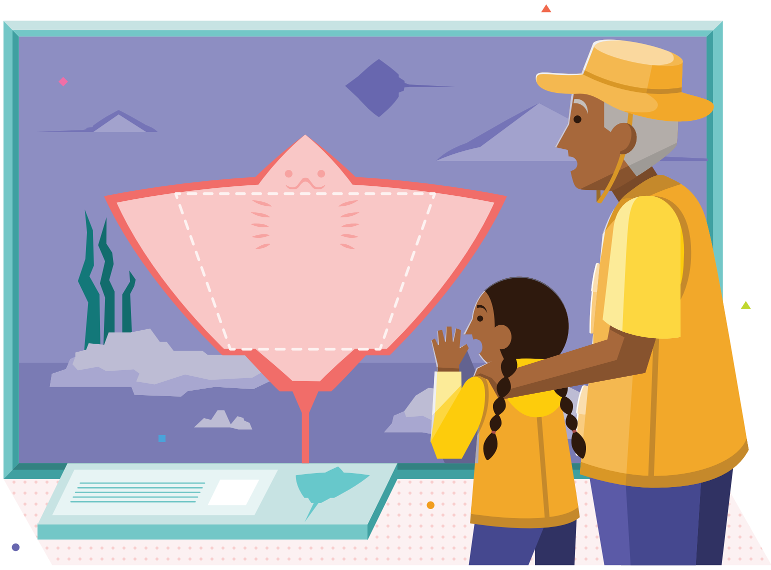
6. On the grid, show how a trapezoid can be composed using two or more parallelograms and/or triangles.



Unit 1 | **Lesson 12**

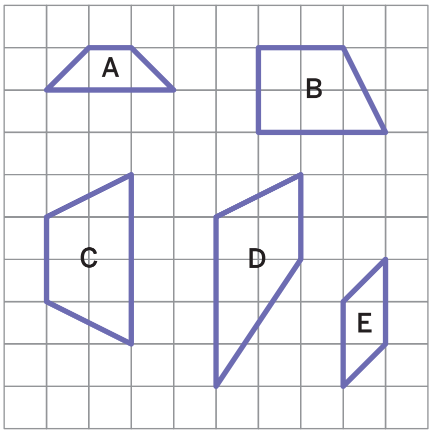
From Triangles to Trapezoids

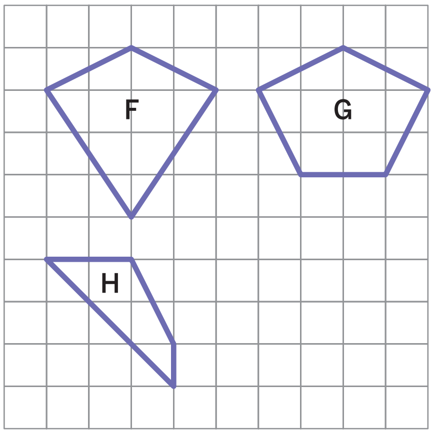
Let's apply what we know about triangles and parallelograms to trapezoids.



Warm-up Features of a Trapezoid

Study the figures on the grid. Figures A–E are trapezoids. Figures F–H are *not* trapezoids.





1. What do you notice about:
   - **a** the number of sides the trapezoids have?
   - **b** the opposite sides of the trapezoids?

2. Choose one shape from the non-examples and explain why it is *not* a trapezoid.

3. What do you notice about Figure E, compared to Figures A–D?

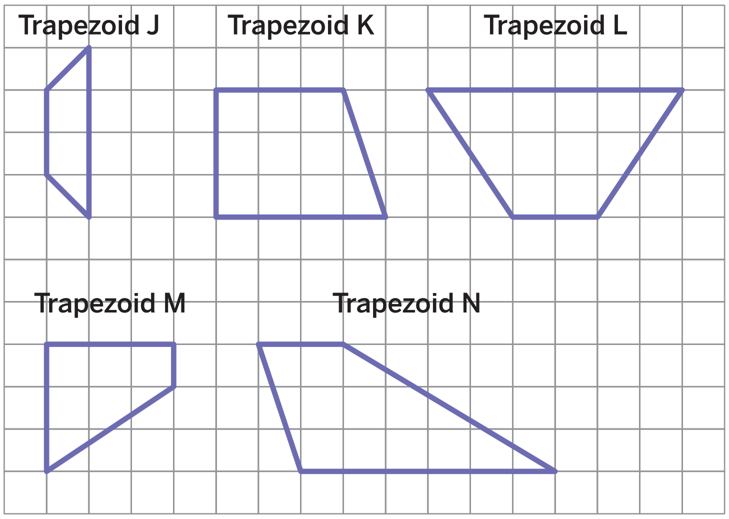
Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Decomposing Trapezoids

Even though they have just four sides, quadrilaterals are still studied by many mathematicians, including Santana Afton, who explores connections between quadrilaterals and prime numbers.

For now, refer to Trapezoids J–N. Show how each of the following decompositions can be created for at least one of the trapezoids by drawing partitioning lines. For each decomposition, write the letters of the corresponding trapezoids, based on your drawings.



1. Two triangles:
2. A parallelogram and a triangle:
3. A parallelogram and two identical triangles:
4. A parallelogram and two different triangles:

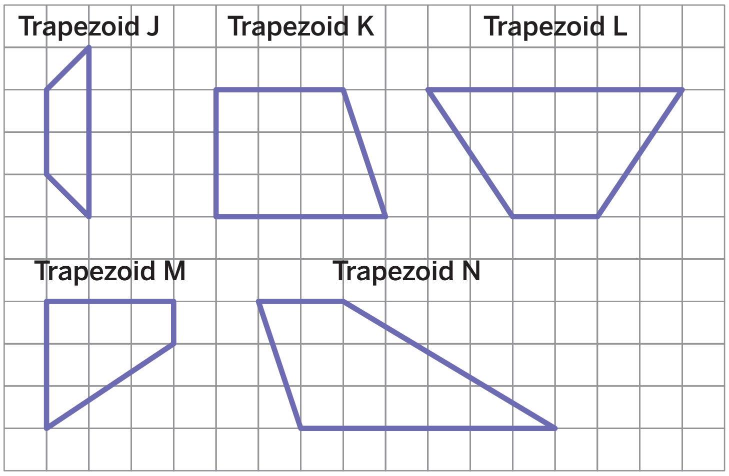
Featured Mathematician

Santana Afton

Santana Afton studies geometry and topology at Georgia Tech. As part of their research, Afton explores what are called "generalized quadrangles." These are structures that contain only quadrilaterals (that is, no triangles), and which can be described with two whole numbers. Beyond their research, Afton enjoys mentoring undergraduate and high school students interested in mathematics.

Activity 2 Area of Trapezoids

The same trapezoids from Activity 1 are shown. Determine the area of each trapezoid. Then complete the table by recording the base and height measurements for each.

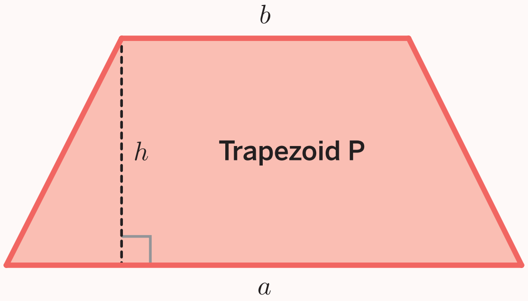


Trapezoid	Base 1 (units)	Base 2 (units)	Height (units)	Area (square units)
J				
K				
L				
M				
N				

Are you ready for more?

Trapezoid P shown here has bases with unknown lengths of a units and b units, and an unknown height of h units.

Add a row to your table, completing each of the first three columns with an appropriate letter and the last column with an expression. This expression will be the formula for the area of *any* trapezoid.



Hint: Consider looking for relationships or patterns among the values already in your table from Trapezoids J–N, or decompose Trapezoid P into shapes whose areas you can determine.

STOP

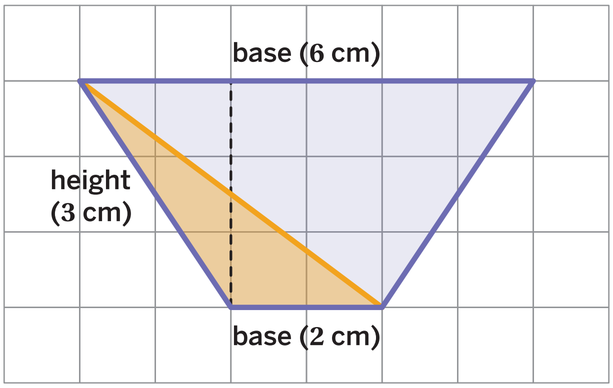
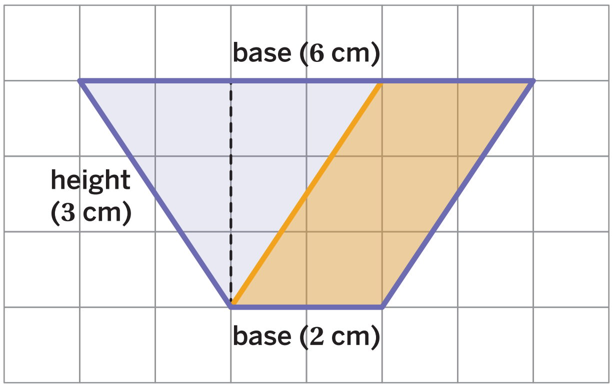
Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You reviewed the defining characteristics of a ***trapezoid*** — a quadrilateral that has at least one pair of parallel sides, which are called its bases.

You expanded on your understanding of the area of rectangles, parallelograms, and triangles, in order to decompose trapezoids and determine their areas. There are often multiple ways to decompose the same trapezoid.

Decomposition	Area
	Area is the sum of the area of two triangles. $A = \frac{1}{2}(2 \cdot 3) + \frac{1}{2}(6 \cdot 3)$ $A = \frac{1}{2}(6) + \frac{1}{2}(18)$ $A = 3 + 9$ $A = 12$ The area of the trapezoid is 12 cm^2.
	Area is the sum of the areas of the parallelogram and triangle. $A = (2 \cdot 3) + \frac{1}{2}(4 \cdot 3)$ $A = 6 + \frac{1}{2}(12)$ $A = 6 + 6$ $A = 12$ The area of the trapezoid is 12 cm^2.

Reflect:

Name: .... Date: .... Period: ....

1. Draw a trapezoid and the corresponding partition line(s) on each grid based on these descriptions. Then determine the area of each trapezoid.

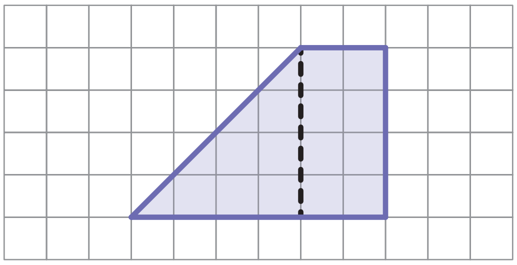
   a A trapezoid that can be decomposed into exactly one parallelogram and one triangle with a single partition line.

   Area:

   b A trapezoid that can be decomposed into exactly one parallelogram and two identical triangles using two partition lines.

   Area:

2. To help determine the area of this trapezoid, Clare decomposed it into two shapes by drawing the dashed line shown.

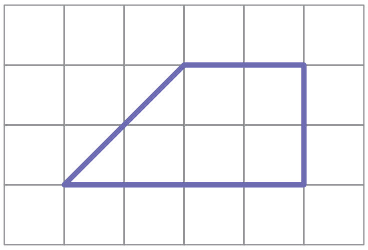


   a Clare thinks that the two resulting shapes have the same area. Do you agree? Explain your thinking.

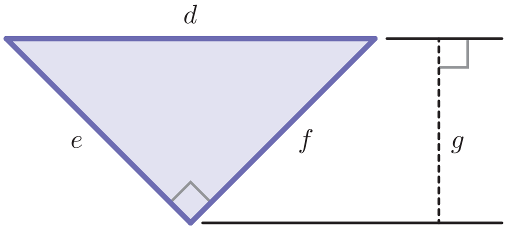
   b Did Clare decompose the trapezoid into two identical shapes? Explain your thinking.

Name: .... Date: .... Period: ....

3. Determine the area of the trapezoid. Each box is 1 square unit.

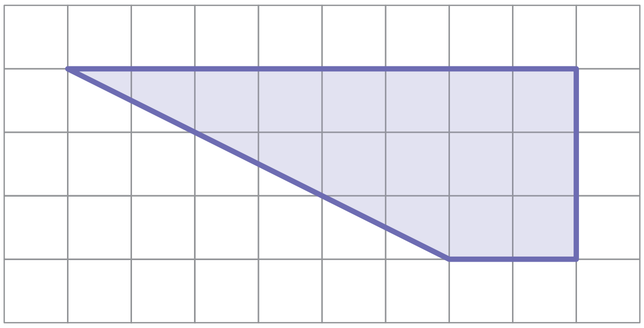


4. Can side d be a base of the triangle shown? If so, which length would be the corresponding height? If not, explain why not.

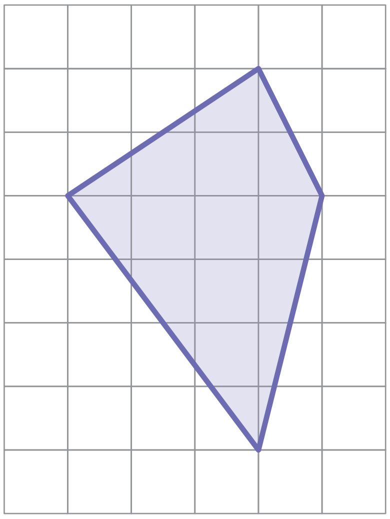


5. Decompose each shape into triangles.

a



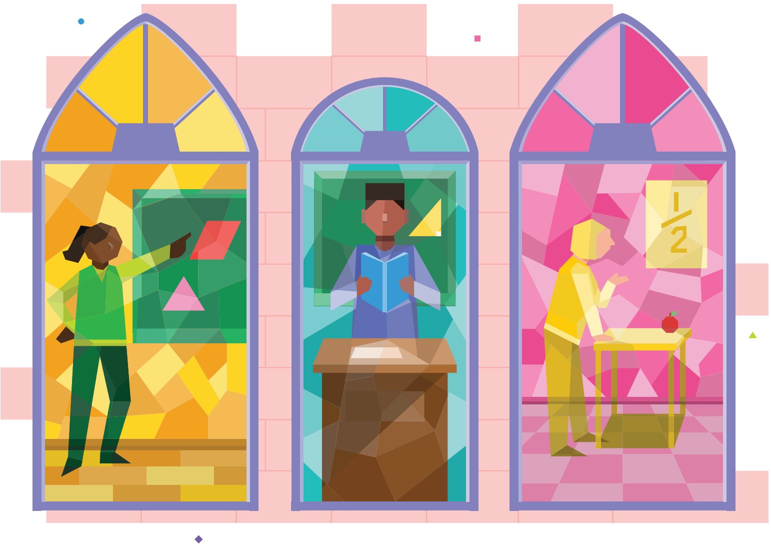
b



Unit 1 | Lesson 13

Polygons

Let's investigate polygons
and their areas.



Warm-up What Are Polygons?

You will be shown some examples and non-examples of polygons, as well as ten other figures, labeled L–U. Classify each figure as an example or a non-example of a polygon and record the figure letters in the appropriate section of the graphic organizer shown. Then describe the characteristics of a polygon based on features of the examples.

Examples:	**Non-examples:**
Characteristics:	**Definition:**

Polygons

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Stained Glass

You will be given a ruler and a calculator. You will create a design for a stained glass window that is composed of $8–12$ polygons, or panels, whose areas can all be determined. In the table, sketch each panel from your design and show how its area is calculated. You may use either inches or centimeters.

Polygon	Area

Activity 1 Stained Glass (continued)

Polygon	Area

Are you ready for more?

Determine the sum of the areas of your polygons. The total area of your polygons should equal the area of the piece of paper. Check your work to see if this holds true and explain why it is true.

Note: U.S. Letter size paper has dimensions of $8\frac{1}{2}$ in. by 11 in.

STOP

Name: ______ Date: ______ Period: ______

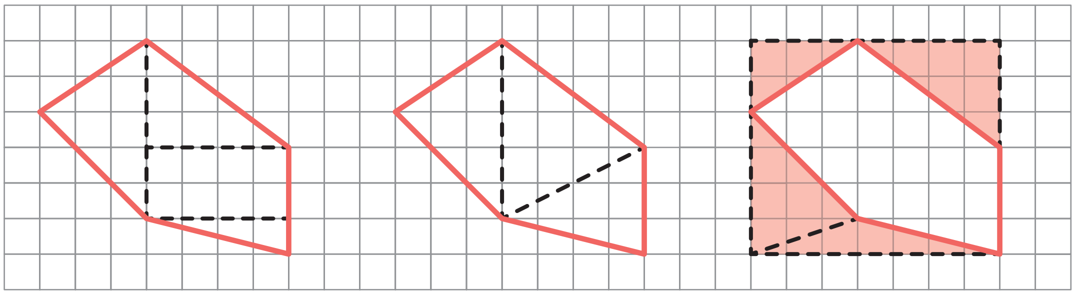
Summary

In today's lesson . . .

You worked with polygons, some familiar with known names and others that are similar in some ways, but different in other ways. A ***polygon*** is a two-dimensional shape composed of sides that are all line segments. For polygons:

- Line segments are straight, not curved.
- Each endpoint of every side connects to an endpoint of exactly one other side.
- The line segments of polygons do not cross each other. A point where two sides intersect is a *vertex* of the *polygon*. **Note:** The plural of *vertex* is *vertices*.
- There are always an equal number of vertices and sides for any polygon. For example, a polygon with 5 sides will have 5 vertices.

You can determine the area of *any* polygon using many familiar strategies, such as decomposing, rearranging, composing, or enclosing, to form shapes with known and well-defined areas — namely triangles and parallelograms. These areas can be determined using a grid or by using formulas, as long as the necessary lengths are known or can be determined, such as by measuring with a ruler.

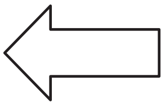


Reflect:

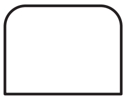
Name: .... Date: .... Period: ....

1. Select *all* the polygons.

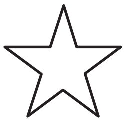
A.



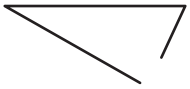
B.



C.

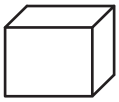


D.

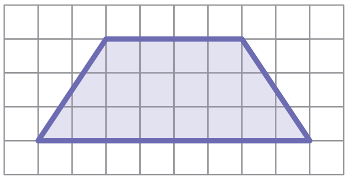


E.

F.

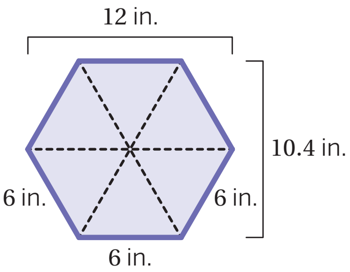


2. Determine the area of the trapezoid shown. Explain or show your strategy. Each square has an area of 1 square unit.

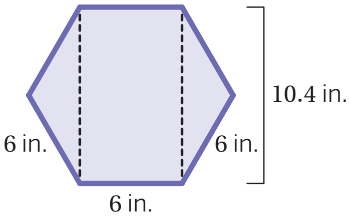


3. Lin and Andre used different methods to determine the area of the hexagon shown, where each side measures 6 in.

   - Lin decomposed the hexagon into six identical, equilateral triangles.
   - Andre decomposed the hexagon into a rectangle and two triangles.



Lin's method

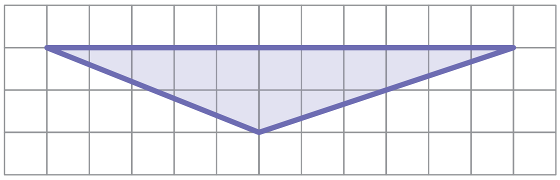


Andre's method

Show the calculations each person could have used to determine the area of the hexagon.

Name: .... Date: .... Period: ....

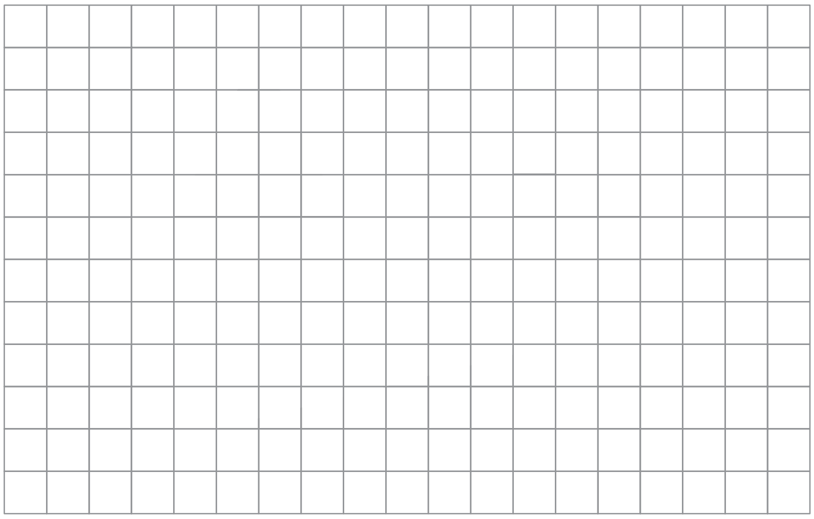
4. Identify a base and a corresponding height that can be used to determine the area of the triangle shown.



a Label the base b and the corresponding height h.

b Calculate the area of the triangle. Show your thinking.

5. On the grid, draw three different triangles with an area of 8 square units. Label the base and height of each triangle.



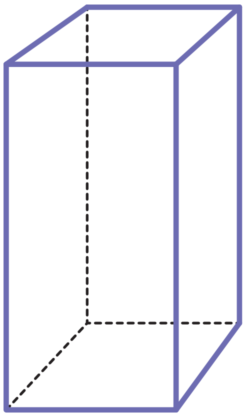
6. Consider Figure G.

a What makes this figure three-dimensional?

b How many polygons do you see along the surface of this prism? What type of polygons are they?

c What type of three-dimensional solid is this figure?

Figure G



My Notes:

SUB-UNIT

2 Nets and Surface Area

Narrative Connections

How did a misplaced ruler change the way you shop?

Not many people remember Robert Gair. But back in his day, he was a wealthy industrialist. In 1853, he came to New York from Scotland at the age of 14. And when the Civil War broke out just a few years later, he joined the Union army, earning the rank of captain before returning to New York with $10,000 to start a business manufacturing square-bottomed paper bags.

Then, on one fateful day in 1879, a ruler in one of Gair's bag-folding machines slipped out of place. Rather than folding the bags, it sliced through about 20,000 sheets of inventory, ruining them.

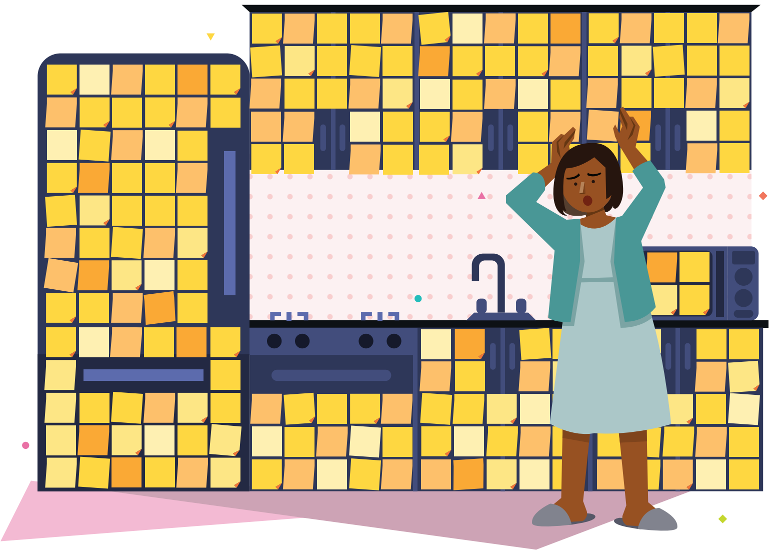
But where others might have simply seen destroyed inventory, Gair saw opportunity. It turned out that the slicing made the bags easier to fold up into containers.

And so the cardboard box was born!

That might not seem like a big deal to us in the 21st century, but cardboard boxes changed how we packaged things forever. Almost 150 years later, cardboard boxes are everywhere: in our kitchens, closets, and mailboxes. And practically everything that gets delivered these days comes in a cardboard box.

Gair's genius lay in his ability to see how the surface area of a simple flat figure could be manipulated to cover a three-dimensional space. It's time for you to think like Gair and take area into the third dimension.

Unit 1 | Lesson 14



What Is Surface Area?

Let's cover the surfaces of some three-dimensional objects.

Warm-up Notice and Wonder

In the next activity, you will watch a video of a cabinet being covered with sticky notes. Consider these images of moments captured from that video. What do you notice? What do you wonder?

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Covering the Cabinet

Plan ahead: In what ways can you show others respect when discussing different strategies used to complete the task?

Part 1

1. What information would you need to know in order to determine the total number of sticky notes it would take to cover the entire cabinet pictured in the Warm-up?

Part 2: Watch the first part of a video of the cabinet being covered with sticky notes.

2. Use the information from the video to calculate the total number of sticky notes needed to cover the cabinet entirely. Show or explain your thinking.

Are you ready for more?

How many sticky notes are needed to cover the outside of 3 cabinets pushed together (including the bottom of each cabinet)? Would the total number of sticky notes needed change if the cabinets were pushed together in different ways?

Activity 2 Building With Unit Cubes

You will be given 12 unit cubes. Each face of the cubes has an area of 1 square unit.

1. Use all 12 cubes to build a rectangular prism. Record the following dimensions of your prism.

   a. Length:

   b. Width:

   c. Height:

2. For your prism, determine each of the following. Show or explain your thinking.

   a. Volume, in cubic units:

   b. Surface area, in square units:

Are you ready for more?

Imagine you connected your rectangular prism to your partner's prism to form a new figure.

1. Would this new figure always be another rectangular prism?

2. If you started with identical prisms, would the total surface area of the new figure always be twice the surface area of your prism? Explain your thinking.

STOP

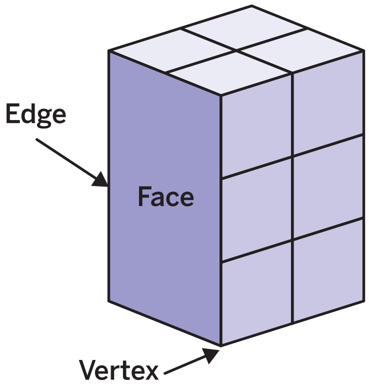
Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You explored some different attributes of a special type of three-dimensional solid, a *rectangular prism*, which is composed of six rectangular *faces*.

- A ***face*** of a three-dimensional solid is any one of the two-dimensional shapes that are joined to make the solid's outer surface.
- A shared side of two faces is called an ***edge***.
- The intersection point of two (or more) edges is called a ***vertex***.



In a rectangular prism, there will always be three pairs of identical (opposite) faces. Sometimes, two or more of the faces are identical. For example, in a cube, all six faces are identical squares.

Volume and surface area are two measurable attributes of all three-dimensional solids.

- ***Volume*** measures the number of unit cubes that can be packed into a figure without gaps or overlaps. Because volume is a *three-dimensional measure*, volume is expressed in *cubic units*.
- ***Surface area*** is the number of unit squares it takes to cover all of the faces of a solid without gaps or overlaps. Because surface area is a *two-dimensional measure*, it is expressed in *square units*. The surface area for any three-dimensional solid is equal to the total area (i.e., the sum of the areas) of all the individual faces.

Reflect:

Practice

Name: ________________ Date: ________ Period: ________

1. Which statement describes the surface area of this trunk?

A. The number of square inches that cover the top of the trunk.

B. The number of square feet that cover all the outside faces of the trunk.

C. The number of square inches of horizontal surface inside the trunk.

D. The number of cubic feet that can be packed inside the trunk.

Ttatty/Shutterstock.com

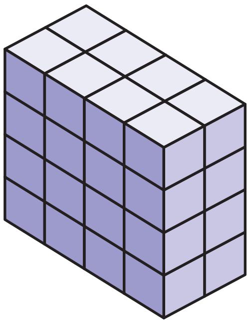
2. This rectangular prism is 4 units high, 4 units wide, and 2 units long. What is its surface area?

A. 16 square units

B. 32 square units

C. 48 square units

D. 64 square units



3. Compare the surface areas of Figure A and Figure B. Show or explain your thinking.

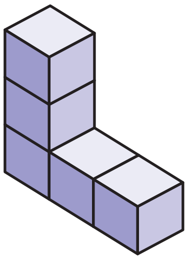


Figure A

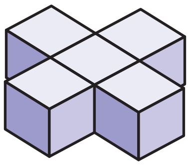
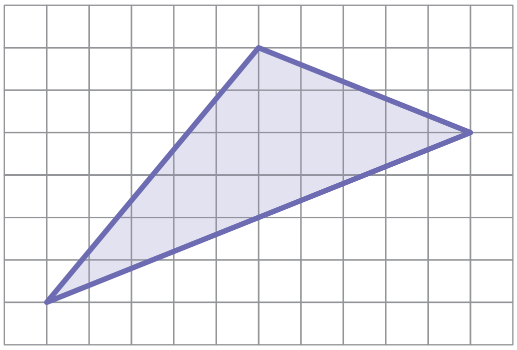


Figure B

Name: .... Date: .... Period: ....

4. Determine the area, in square units, of the shaded region. Show or explain your thinking.

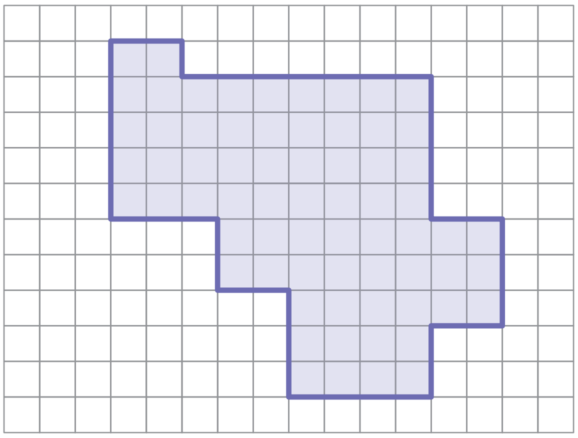


5. Determine the stated measure for each shape described.

a The length of the base of a parallelogram is 12 m and its corresponding height is 1.5 m. What is the area of the parallelogram?

b The length of the base of a triangle is 16 in. and its corresponding height is $\frac{1}{8}$ in. What is the area of the triangle?

6. Determine the area of the shaded figure. Show your thinking.



Unit 1 | Lesson 15

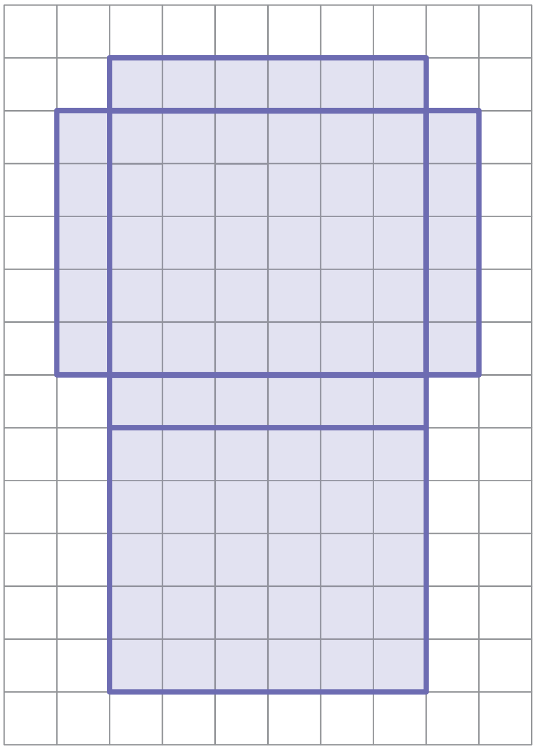
Nets and Surface Area of Rectangular Prisms

Let’s use nets to calculate the surface area of rectangular prisms.

Warm-up “Unfolding” a Rectangular Prism

You will watch a video of a rectangular prism being “unfolded.”

Here is a two-dimensional representation of the “unfolded” rectangular prism. Label the top, bottom, left, right, front, and back faces of the original three-dimensional figure.

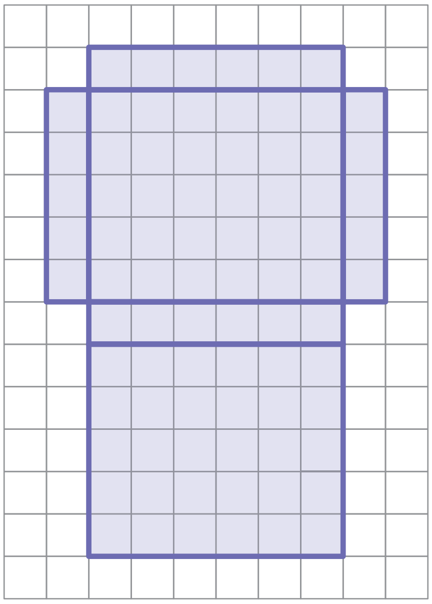


Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Using the Net of a Rectangular Prism

Here is the same net of the rectangular prism from the Warm-up.



1. Calculate the surface area of the rectangular prism in square units. Show or explain your thinking.

2. Determine whether each figure is a net of a rectangular prism. Be prepared to explain your thinking.

Figure A

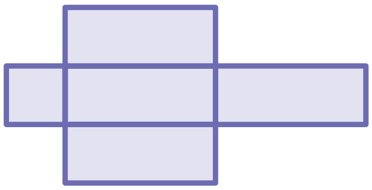


Figure B

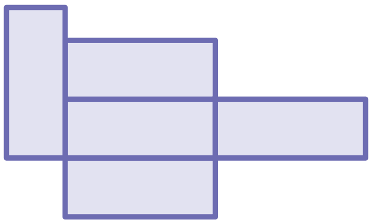


Figure C

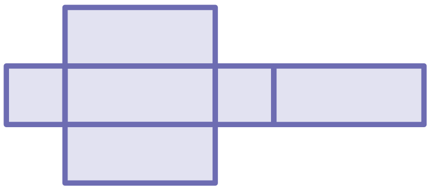
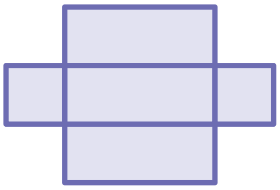


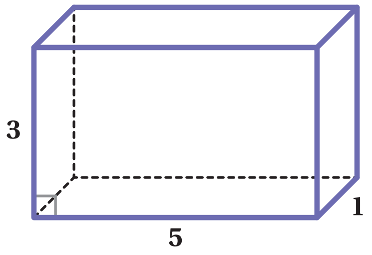
Figure D



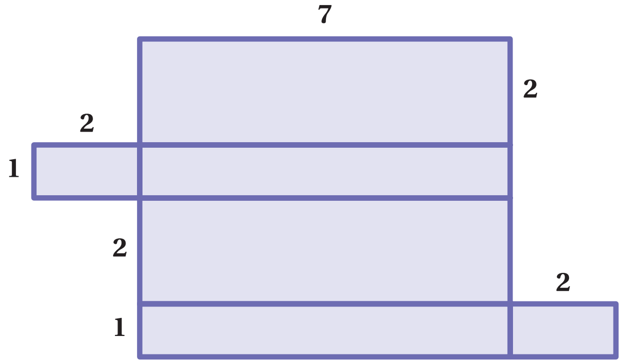
Activity 2 Comparing Boxes

Here is a model of one box and a net of another box. All lengths are in inches.

Box E



Box F



1. Draw a net for Box E on the grid.

2. Which box uses the least cardboard?

3. If each box was packed with 1-in. unit cubes, which box would be packed with more cubes? Show or explain your thinking.

STOP

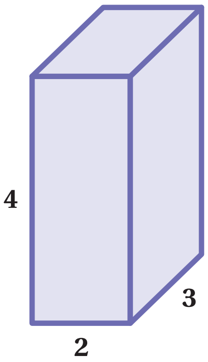
Name: ________________ Date: ________ Period: ________

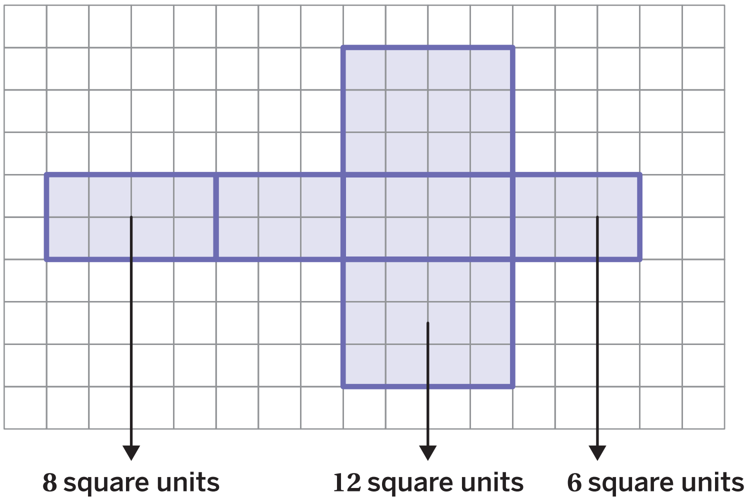
Summary

In today's lesson . . .

You saw that a ***net*** is a two-dimensional representation of a three-dimensional solid that shows the result of "unfolding" the solid such that all of the faces are clearly visible. A net can also be cut and folded to form a three-dimensional model of its corresponding solid.

Because nets show all of the polygons that form the faces of the solid, they are useful for calculating the solid's surface area. For example, the net of this rectangular prism shows three pairs of identical rectangles: 4 units by 2 units, 3 units by 2 units, and 4 units by 3 units. The surface area of the rectangular prism is 52 square units because the sum of the areas of all the faces is $8 + 8 + 6 + 6 + 12 + 12 = 52$.

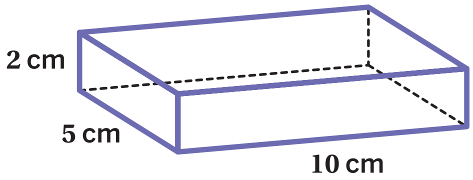




Reflect:

Name: .... Date: .... Period: ....

1. Refer to the rectangular prism shown.



a Use the grid to draw a net for the prism. The length of one grid square is 1 cm. Label the top, bottom, left, right, front, and back faces.

b Determine the surface area of the prism.

2. A rectangular prism is 4 units high, 2 units wide, and 6 units long. What is its surface area in square units? Show or explain your thinking.

Name: .... Date: .... Period: ....

3. Prism A and Prism B are both rectangular prisms. Prism A's dimensions are 3 in. by 2 in. by 1 in. Prism B's dimensions are 1 in. by 1 in. by 6 in. Select *all* of the statements that are true.

A. Prisms A and B have the same number of faces.

B. More 1-in. cubes can be packed into Prism A than into Prism B.

C. Prisms A and B have the same surface area.

D. The surface area of Prism B is greater than that of Prism A.

4. Select *all* of the units that would be appropriate to describe surface area.

A. Square meters

B. Feet

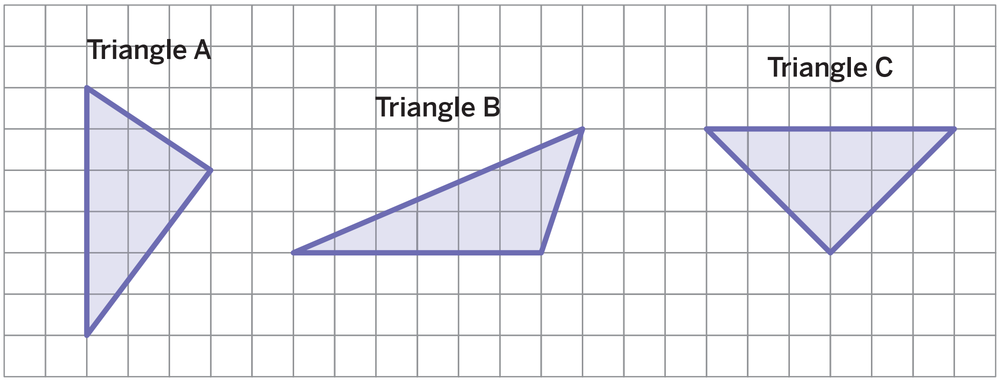
C. Centimeters

D. Cubic inches

E. Square inches

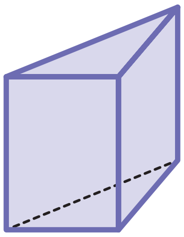
F. Square feet

5. Show how you know that each of these triangles has an area of 9 square units.

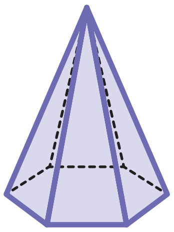


6. Name *all* the polygons that make up the faces of these three-dimensional figures.

a



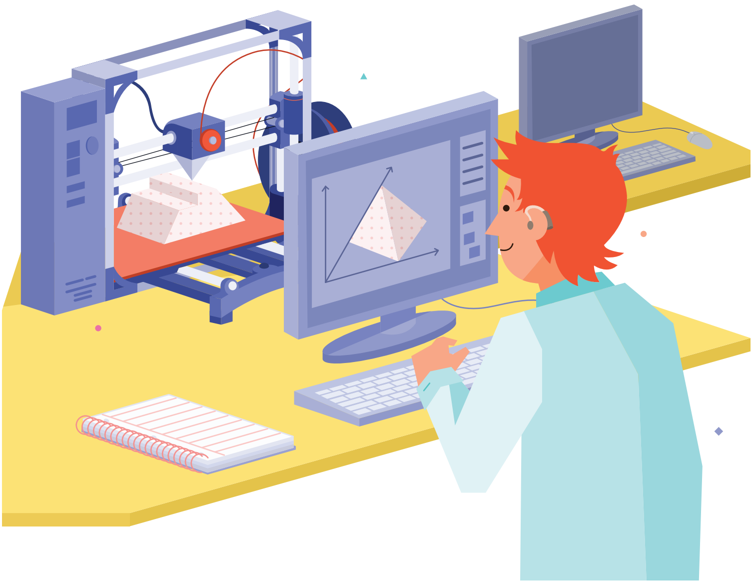
b



Unit 1 | Lesson 16

Nets and Surface Area of Prisms and Pyramids

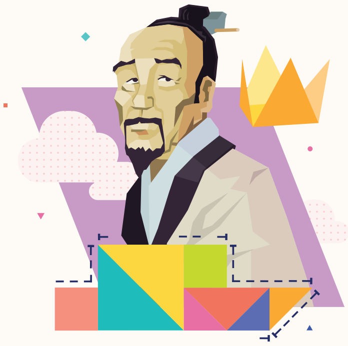
Let’s use nets to calculate the surface areas of other polyhedra.



Warm-up Card Sort: Three-Dimensional Figures

Third-century mathematician Liu Hui discovered and proved many relationships among three-dimensional solids with shared features and dimensions. You will be given a set of cards that show three-dimensional figures. Sort the figures into different groups of your choosing, and explain your thinking.

Featured Mathematician



Liu Hui

Liu Hui was born about 225 C.E. near what is now Zibo, China. He is most noted for writing commentaries on problems addressing number theory, geometry, algebra, and trigonometry, in the ancient text called “Nine Chapters on the Mathematical Art.”

One idea presented there, possibly for the first time anywhere, is known as Liu Hui’s Cube Puzzle. It shows how a cube can be decomposed into three solids with volumes of exactly $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$ the volume of the cube.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Using Nets to Calculate Surface Area

1. Nets of five polyhedra are shown. Which are prisms and which are pyramids? Be prepared to explain your thinking.

Figure G

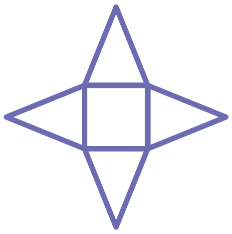


Figure H

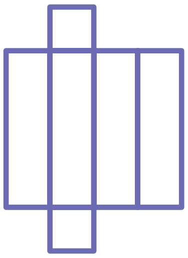


Figure J

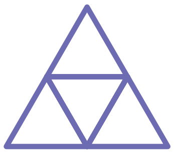


Figure K

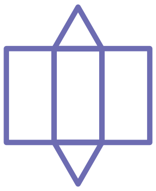
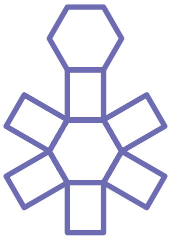


Figure L



2. The nets for Figures J and K are shown.

Figure J

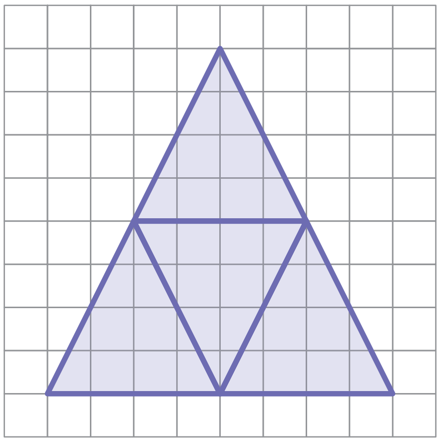
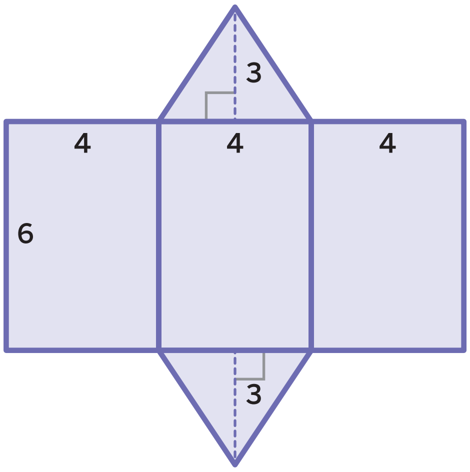


Figure K



a Label the base(s) of each three-dimensional figure.

b Name the type of polyhedron that each net would form when assembled.

c Determine the surface area of each polyhedron. Show your thinking.

Activity 2 Surface Area of Prisms and Pyramids

You will be given a two-dimensional drawing of a polyhedron measured in units.

1. Draw a net for your polyhedron on the grid. Each grid square represents 1 unit.

2. Calculate the surface area of your polyhedron. Show your thinking.

Historical Moment

Volume of an Incomplete Pyramid

The Rhind Mathematical Papyrus (~1650 B.C.E.) is one of the most famous surviving examples of ancient Egyptian mathematics. But an even older document, the Moscow Mathematical Papyrus (~1850 B.C.E.), has survived. Both works present a bunch of math problems and solutions — or in some cases, what were thought to be solutions at the time.

One example in the Moscow Mathematical Papyrus is how to determine the volume of a *frustum* — a pyramid with the top chopped off, or in other words an "incomplete" pyramid. For a pyramid with a square base that has area a, and where the top of the incomplete pyramid at height h is also a square with area b, the formula is: $\frac{1}{3} \cdot h \cdot (a^2 + ab + b^2)$.

Try drawing a frustum. What would you need to know to determine its surface area?

STOP

Name: ________________ Date: ________ Period: ________

Summary

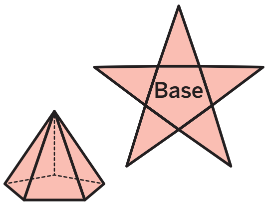
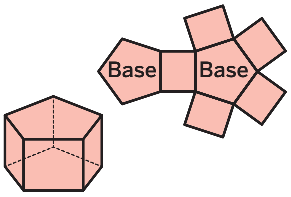
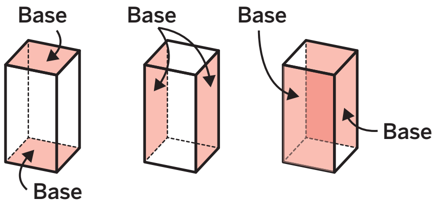
In today's lesson . . .

You worked with two types of solids — prisms and pyramids. Each of these is an example of a closed three-dimensional figure with flat faces that are all polygons, called a ***polyhedron***. (The plural of *polyhedron* is *polyhedra*.) A ***base*** (of a prism or pyramid) is a special face of a polyhedron, defined relative to the type of solid.

A ***pyramid*** has one *base*. All of the other faces are triangles meeting at a single vertex.

A ***prism*** has two *bases*, which are always parallel, identical copies of some polygon. All of the other faces are parallelograms (often rectangles). Because a rectangular prism has three pairs of parallel and identical rectangular faces, any of these pairs can represent the bases.

Both pyramids and prisms are named according to the shape of their bases.

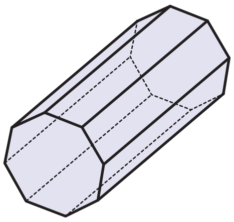
Pentagonal pyramid	Pentagonal prism	Rectangular prism
		

The surface area of a polyhedron is the sum of the areas of all its faces. Because a net shows every face of a polyhedron at once, it can be helpful in calculating surface area.

Reflect:

Name: .... Date: .... Period: ....

1. Refer to the polyhedron shown.



   a. Explain how you know the figure is a polyhedron.

   b. Is this polyhedron a prism, a pyramid, or neither? Explain your thinking.

   c. How many faces, edges, and vertices does it have?

2. Refer to each polyhedron and its corresponding net. Label all of the edges in each net with the correct lengths.

   a. **Figure A**

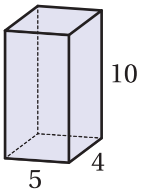
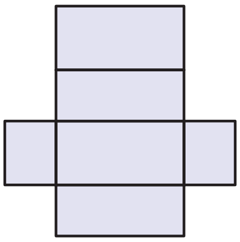


Figure A



   b. **Figure B**

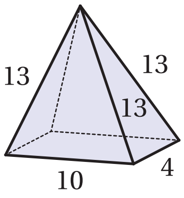
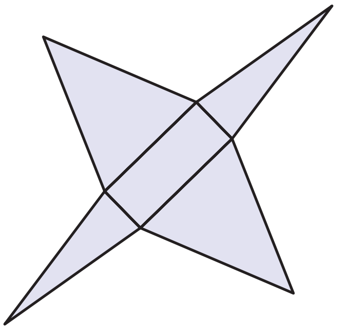
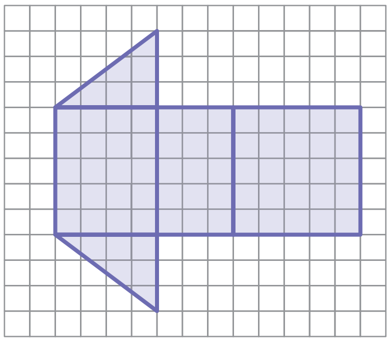


Figure B



3. Refer to the net.

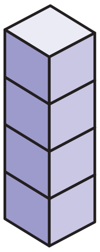


   a. Name the type of polyhedron that can be assembled from this net. Explain your thinking.

   b. Determine the surface area of this polyhedron. Show your thinking.

Name: .... Date: .... Period: ....

4. The figure shown is a representation of a rectangular prism built from unit cubes.

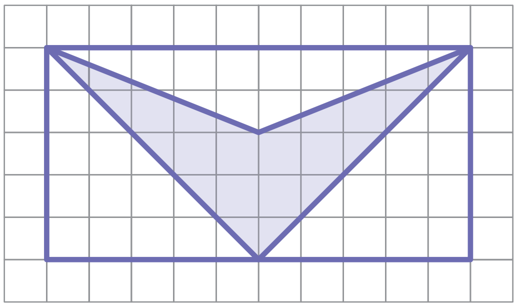


a Determine the volume in cubic units.

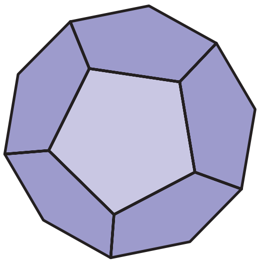
b Determine the surface area in square units.

c Determine whether the following statement is *true* or *false*. Show or explain your thinking. If you double the number of cubes and stack them all in the same way, *both* the volume and surface area will double.

5. Determine the area of the shaded figure shown. Show your thinking.



6. This image shows exactly half of a polyhedron called a *dodecahedron*. What polygons make up the faces of a dodecahedron? How many faces does it have?



Unit 1 | **Lesson 17**

Constructing a Rhombicuboctahedron

Let's use nets to construct
a rhombicuboctahedron.

Warm-up Notice and Wonder

Consider these images of the National Library of Belarus. What do you notice? What do you wonder?

webhobbit/Shutterstock.com

Grisha Bruev/Shutterstock.com

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Constructing a Model of the Library

The National Library of Belarus is a *rhombicuboctahedron*, a polyhedron composed of eighteen squares and eight triangles. Each square face has an edge length of 24 m, and each triangular face has a height of approximately 20.8 m.

1. Write an expression to represent the surface area of the National Library of Belarus. Then evaluate your expression to determine its surface area, in square meters.

2. You will be given a copy of a net for the library, a pair of scissors, and some glue or tape. Use these to assemble a model of the National Library of Belarus.

Are you ready for more?

The exterior of the National Library of Belarus is completely covered with glass windows. The total surface area represents approximately the total amount of glass, in square meters, that is needed to cover the exterior of the library.

1. How well do you think your response to Problem 1 in Activity 1 represents the actual amount of glass that was used to build the library? Do you think it is more likely to be an overestimate or an underestimate? Explain your thinking.

2. Show some calculations that could be used to estimate the difference between your original calculation and the actual surface area covered by glass.

Reflect: How did you remain positive and confident while constructing the model?

STOP

Summary

In today's lesson . . .

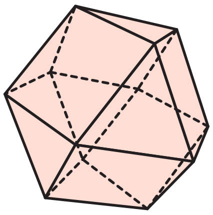
You applied key concepts of nets and surface area to create a three-dimensional model of the Belarus National Library, which is a polyhedron called a *rhombicuboctahedron*.

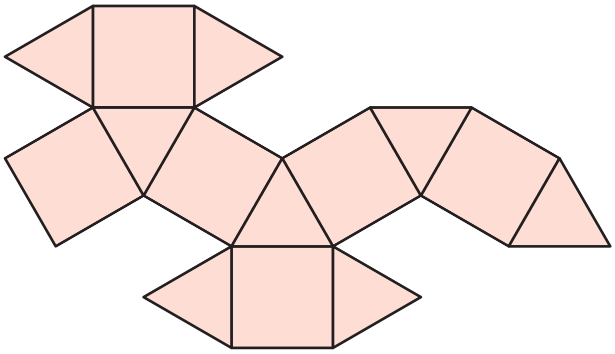
The surface area of any polyhedron is the total area (i.e., the sum of the areas) of all the individual faces. To simplify calculations, you can group faces that are identical copies of one another. For example, because a rhombicuboctahedron is composed of eighteen identical squares and eight identical triangles, you can multiply the area of one square by 18 and the area of one triangle by 8, and then add these areas.

> **Reflect:**

Name: ______________ Date: ______________ Period: ______________

Use the figures showing a cuboctahedron and a possible net to complete Problems 1–3.





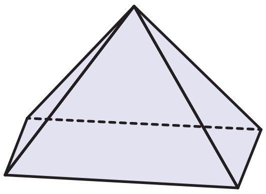
1. How many faces does a cuboctahedron have? How many of the faces are squares? How many of the faces are triangles?

2. Is a cuboctahedron a polyhedron? How do you know?

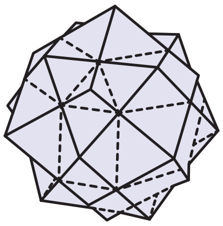
3. Each square face has an edge length of 3 in., and each triangular face has a height of approximately 2.6 in. Calculate the surface area of the cuboctahedron.

4. Select *all* of the figures that are polyhedra.

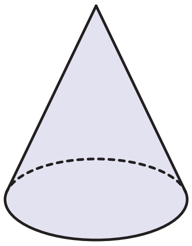
A.



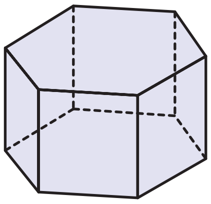
B.



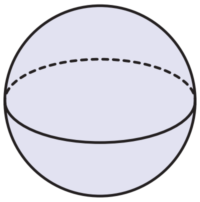
C.



D.



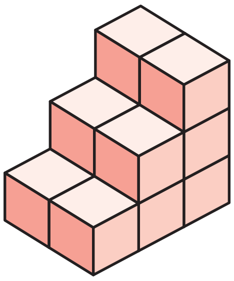
E.



Practice

Name: .... Date: .... Period: ....

5. This figure is composed of 12 unit cubes.



a What is the surface area of the figure? Show or explain your thinking.

b How would its surface area change if the top two cubes are removed?

6. Complete the missing value in each row of the table. The first row has been completed for you:

	Power	Expanded	Product
	3^4	$3 \cdot 3 \cdot 3 \cdot 3$	81
a	5^2		
b		$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	
c		$4 \cdot 4 \cdot 4$	
d			36

Name: ______________________ Date: __________ Period: __________

Unit 1 | Lesson 18

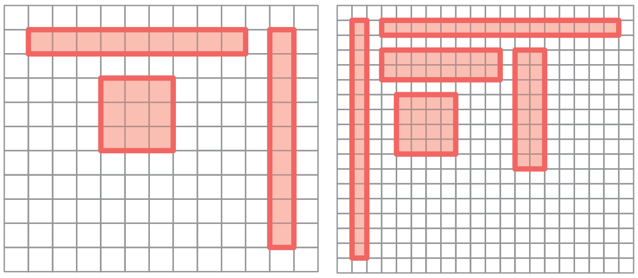
Simplifying Expressions for Squares and Cubes

Let's write expressions for the attributes of squares and cubes.

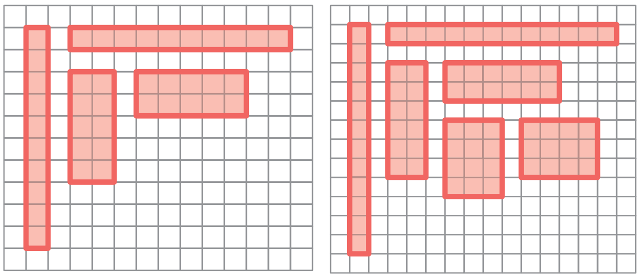
Warm-up How Do They Compare?

Explain how Group A and Group B are similar and how they are different.

Group A



Group B



1. Group A and Group B are similar because . . .

2. Group A and Group B are different because . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Building "Perfect" Cubes

You will be given 32 unit cubes.

1. Build the largest cube possible, using any or all of your 32 unit cubes. How many unit cubes did you use?

2. What do you notice about the edge lengths in the cube you built?

3. Determine each of the following for your cube. Show your thinking and include appropriate units.

   a Area of each face:

   b Surface area:

   c Volume:

4. Could you build a cube using exactly 20 unit cubes? Explain your thinking.

5. How many different-sized cubes are possible if you can use *up* to 32 unit cubes for each cube?

Are you ready for more?

Imagine you teamed up with another group and now have 64 unit cubes to use to build cubes.

1. What is the largest cube you could make? Explain your thinking.

2. Calculate the volume and surface area of the perfect cube you described in Problem 1.

   a Volume:

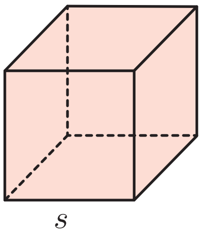
   b Surface Area:

3. How many groups would you need to team up with in order to have enough unit cubes to build a cube with a height of 10 units? Assume each group has 64 unit cubes.

Name: ________________ Date: ________ Period: ________

Activity 2 Writing Expressions for the Attributes of Cubes

Consider the cube with edge length s.



1. Draw a net of the cube.

2. Write an expression to represent each of the following for a cube with side length s. Include the appropriate units.

   a. Area of each face:

   b. Surface area:

   c. Volume:

Are you ready for more?

The number $15{,}625$ is a *perfect square* because it is equal to $125 \cdot 125$. It is also a *perfect cube* because it is equal to $25 \cdot 25 \cdot 25$. Find another number that is both a perfect square and a perfect cube. How many of these can you find?

STOP

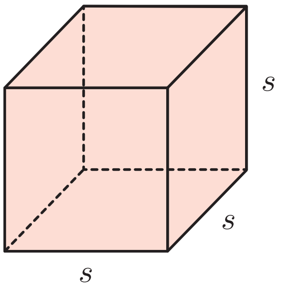
Summary

In today's lesson . . .

You explored perfect squares and perfect cubes. A ***perfect square*** is the product of a factor and itself. The number 16 is a perfect square because $4 \cdot 4 = 16$. A ***perfect cube*** is the product of a factor multiplied by itself three times. The number 27 is a perfect cube because $3 \cdot 3 \cdot 3 = 27$.

A perfect square can be represented geometrically as the area of a square with whole number side lengths because its sides are all identical copies of one another. A perfect cube can be represented geometrically as the volume of a cube with whole number edge lengths because its faces are all identical squares.

Consider the cube with edge length s units. When you substitute s into the known formulas for area of a parallelogram (a face), and surface area and volume of rectangular prisms, the resulting expressions can be simplified. And those simplified expressions can be used to make calculations more efficient when working with cubes.



- **Area:** The area of each square face is equal to $s \cdot s$ square units.
- **Surface area:** The sum of the areas of all six faces, $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$, or $6 \cdot (s \cdot s)$ square units.
- **Volume:** The volume is equal to $s \cdot s \cdot s$ cubic units.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Decide whether each number is a perfect square, a perfect cube, *both* a perfect square and a perfect cube, or *neither*. Show or explain your thinking.

a 1

b 3

c 8

d 16

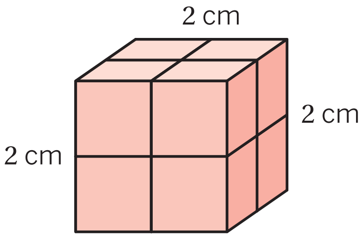
e 20

f 64

g 100

h 1,000

2. For this cube, calculate each of the following measurements. Be sure to include appropriate units.



a Area of each face:

b Surface area:

c Volume:

Name: .... Date: .... Period: ....

3. Determine the stated measure(s) in each scenario, using appropriate units.

a A square has side length 4 cm. What is its area?

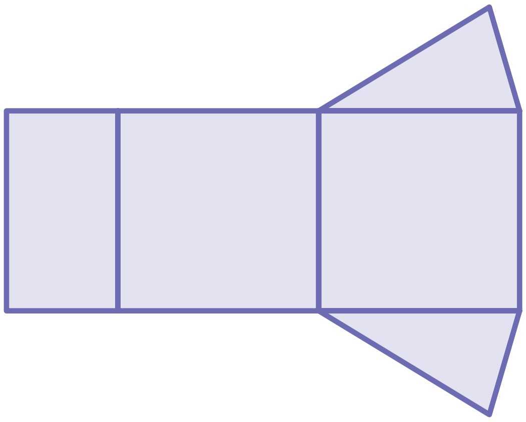
b The area of a square is 49 square meters. What is its side length?

c The edge length of a cube is 3 in. What is its volume? surface area?

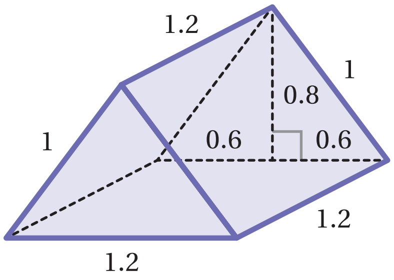
4. Refer to the net.

a What type of polyhedron can be assembled from this net?

b Label the dimensions or measures you would need to know in order to calculate the surface area.



5. Calculate the surface area of the triangular prism shown. All measurements are in meters.



6. Evaluate each expression.

a $10 + 10$

b $10 \cdot 10$

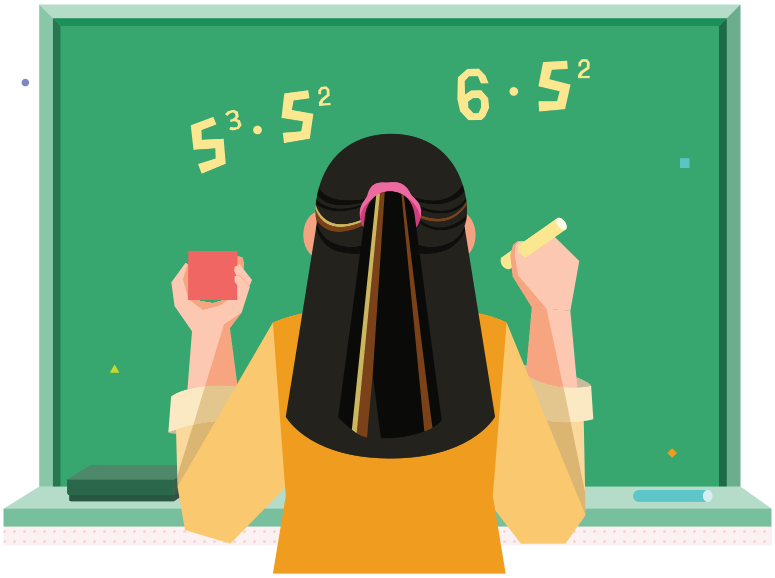
c 10^2

d 10^3

Name: .... Date: .... Period: ....

Unit 1 | Lesson 19

Simplifying Expressions Even More Using Exponents



Let's write expressions with exponents to represent the volume and surface area of cubes.

Warm-up Which Is Greater?

Without calculating the value of each expression, use what you know about operations to determine which expression in each pair represents the greater value. Be prepared to explain your thinking.

1. **Expression A:** $10 \cdot 3$ **Expression B:** 10^3

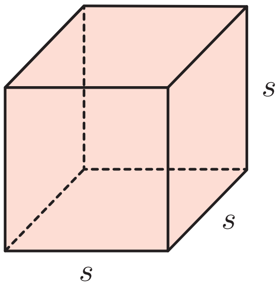
2. **Expression A:** 10^2 **Expression B:** $9 \cdot 9$

3. **Expression A:** $10 + 10 + 10 + 10 + 10 + 10$ **Expression B:** $5 \cdot 10$

Log in to Amplify Math to complete this lesson online.

Activity 1 Card Sort: Sorting Expressions and Units

You will be given a set of cards that contain expressions and related units for this cube with side length s cm.



1. Sort the cards into four groups, expressions, and units that represent:
   - The area of each face
   - The surface area
   - The volume
   - None of these

   Use the table to record how you sorted the cards.

Area of each face	Surface area	Volume	None

2. Explain why the cards in the *None* group did not belong in any other group.

Name: .... Date: .... Period: ....

Activity 2 Using Exponents to Express Attributes of Cubes

1. A cube has an edge length of 7 in. Determine whether each statement is *true* or *false*. Explain your thinking.

a The area of each face is 14 in^2 because $7^2 = 14$.

b You can calculate the volume of the cube by evaluating 7^3.

c The surface area is 84 in^2 because $6 \bullet (7 \bullet 2) = 84$.

d You can use in^3 as units to represent volume.

2. A cube has a volume of 125 cubic units. What is its surface area? Show or explain your thinking.

Stronger and Clearer: Share your responses with 2–3 partners to get feedback on your clarity and reasoning. After receiving feedback, revise your responses.

Are you ready for more?

1. Can a cube have the same numeric value for both its surface area and volume? Explain your thinking.

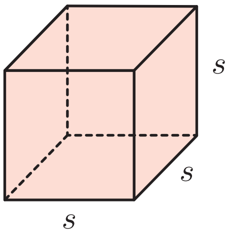
2. Without calculating, how can you determine whether a cube's volume or surface area is greater?

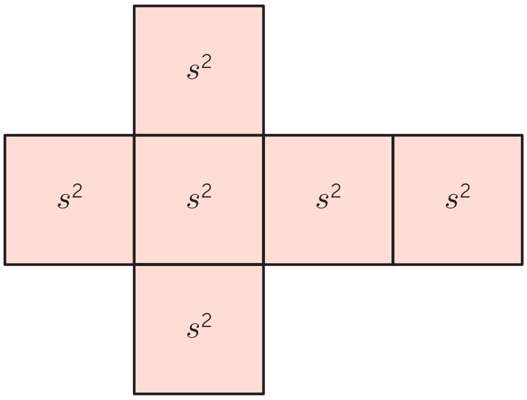
STOP

Summary

In today's lesson . . .

You saw how the formulas for surface area and volume of a cube can be simplified using exponents. Consider a cube with edge length s units and its net.





To calculate the . . .

- *Area of each face*: The expression $s \cdot s$ can be written as s^2. This expression is read as "s **squared**." The **exponent** 2 tells you how many times to multiply the repeated factor s by itself.
- *Surface area*: The expressions $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$ or $6 \cdot (s \cdot s)$ can be written as $s^2 + s^2 + s^2 + s^2 + s^2 + s^2$ or $6 \cdot s^2$.
- *Volume*: The expression $s \cdot s \cdot s$ can be written as s^3. This expression is read as "s **cubed**."

Exponents are also used to represent the appropriate units for each measurement. For example, if the edge length of a cube was s in., then:

- The area of each face and the surface area would both have units of square inches, which can be written as "in^2."
- The volume would have units of cubic inches, which can be written as "in^3."

Reflect:

Name: ______________________ Date: __________ Period: __________

1. A cube has an edge length of x cm. Write an expression for each of the following measures of the cube:

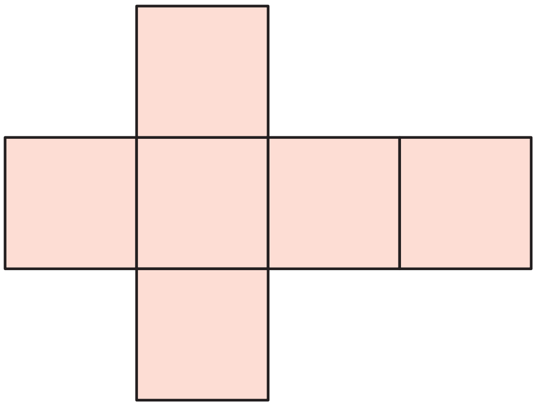
a Surface area:

b Volume:

2. This net is composed of square faces.

a Name the type of polyhedron that can be assembled from this net.

b If each square has a side length of 61 cm, write expressions for the surface area and the volume of the polyhedron.



3. Determine each stated measure using appropriate units. Show your thinking.

a The surface area of a cube with edge length 8 in.

b The volume of a cube with edge length $\frac{1}{3}$ cm.

c The edge length of a cube that has a volume of 8 ft^3.

Name: .... Date: .... Period: ....

4. Refer to Figures A and B. State whether each figure is a polyhedron. Explain your thinking.

Figure A

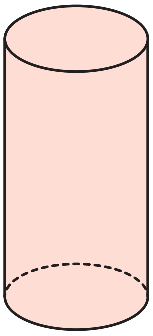
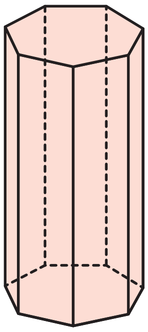
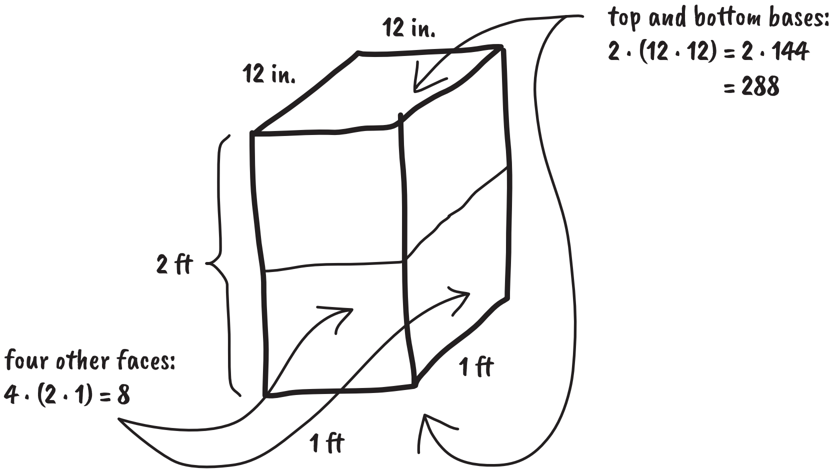


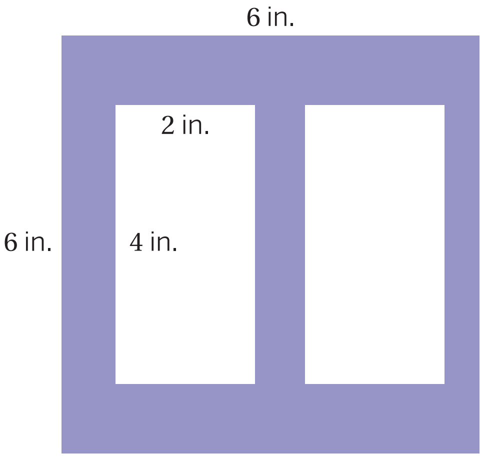
Figure B



5. Here is Elena's work for calculating the surface area of a rectangular prism with dimensions 1 ft by 1 ft by 2 ft. She concluded that the surface area of the prism is 296 ft^2. Do you agree or disagree? Show or explain your thinking.



6. Determine the area of the shaded region. The unshaded rectangles are identical.



Name: ______ Date: ______ Period: ______

Unit 1 | Lesson 20 – Capstone

Designing a Suspended Tent

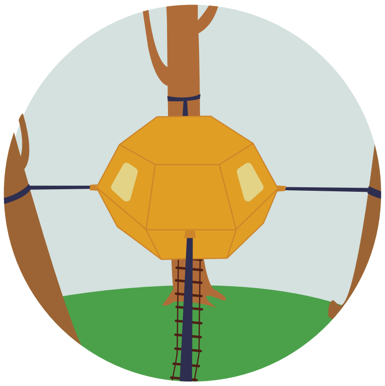
Let's design a tent that can hang from trees.

Warm-up Camping Out . . . and Up

Have you ever been camping?

You might know that tents come in a variety of shapes and sizes, but did you know that some can be suspended in trees?

Study these examples of suspended tents.



In your group, discuss:

1. The similarities and differences among these tents.

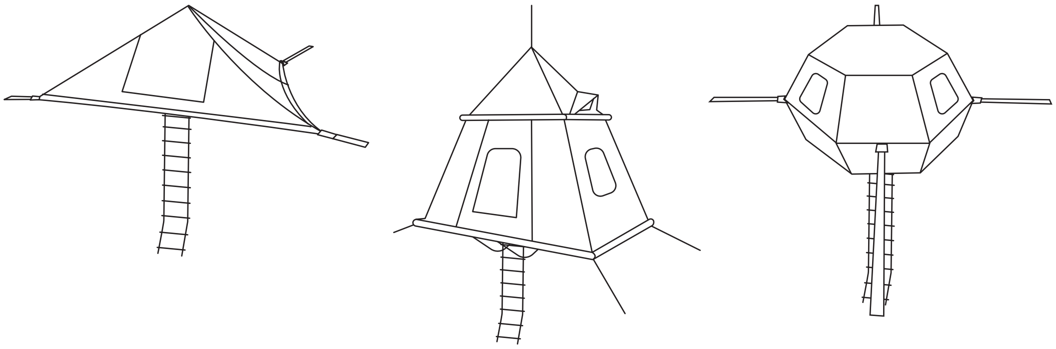
2. The pros and cons of the various designs.

Log in to Amplify Math to complete this lesson online.

Activity 1 Suspended Tent Design

Most tents are made to accommodate adults, but your task is to design a suspended tent to accommodate up to three people that are about your age.

Here are some examples of popular designs.

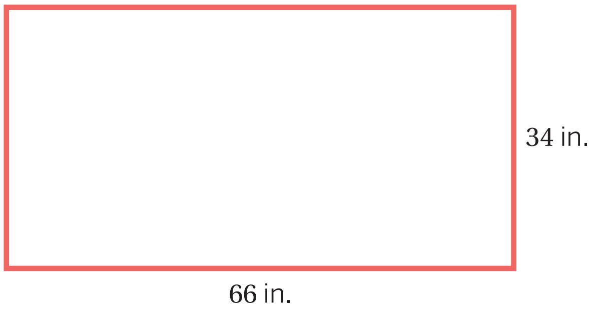


Develop and sketch a design for your suspended tent. It can look like one of these or can be anything else you come up with. But the tent must include a floor because the ground is not an option! You also need to be able to estimate and justify mathematically the total amount of fabric it will take to construct your tent.

Consider the following specifications to help with your designs.

Height description	Height of tent (ft)	Notes
Sitting height	3	Campers are able to sit, lie, or crawl inside the tent.
Kneeling height	4	Campers are able to kneel inside the tent. Found mainly in 3–4 person tents.
Standing height	5.5	Most campers are able to stand upright.

Sleeping bag measurements for 10–12 year olds:



Name: .... Date: .... Period: ....

Activity 1 Suspended Tent Design (continued)

After the Gallery Tour, discuss the following questions with your group. Record your groups' agreed-upon responses here.

1. Which tent design used the least fabric?

2. Which tent design used the most fabric?

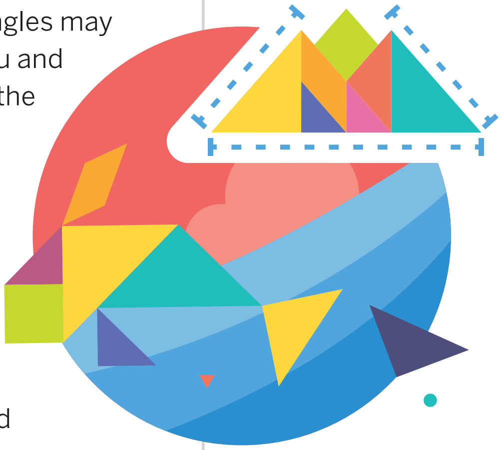
3. Which difference(s) in the designs have the greatest impact on the amount of fabric needed for the tent? Explain your thinking.

Are you ready for more?

Estimate how much extra floor space would there be if three sleeping bags are placed on the floor, without overlapping. Show and explain your thinking by drawing a sketch of the interior floor space along with your calculations.

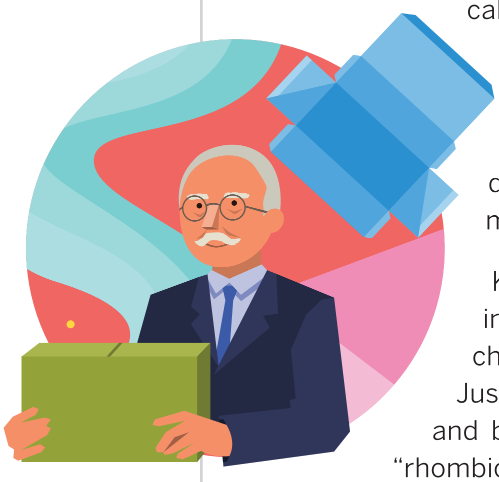
STOP

Unit Summary



Measuring the areas of shapes like squares and rectangles may not seem very complicated. But take a look around you and you'll find that things are a *bit* more complicated than the "perfect" shapes laid out on a sheet of paper.

Take something like the National Library of Belarus, which is shaped like a giant rhombicuboctahedron. Its astounding, gemstone-like structure reflects the treasure of knowledge stored within. And yet, to make this building a reality, architects Viktor Kramarenko and Mikhail Vinogradov needed ways to calculate precisely how much glass and steel they would need.



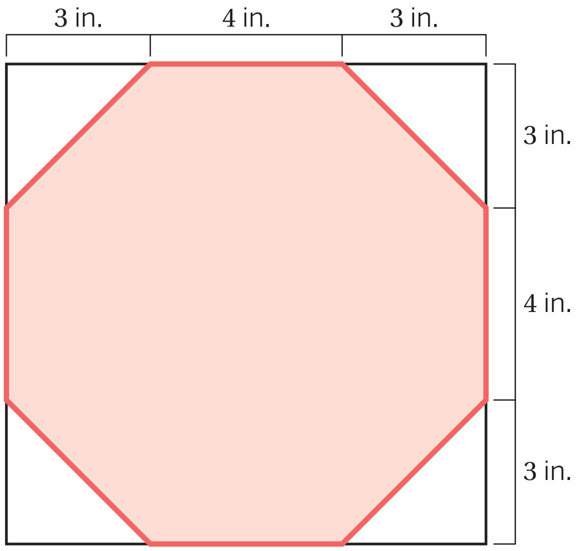
So where to start? Well, like most complex tasks in life, they can be broken down—or decomposed—into smaller, more manageable parts.

Keep that in mind in the days ahead. Both in and out of math class, you'll face many challenges that might seem overwhelming. Just remember to slow down, take your time, and breathe. Something as scary sounding as a "rhombicuboctahedron" might be nothing more than a few squares and triangles.

See you in Unit 2.

Name: .... Date: .... Period: ....

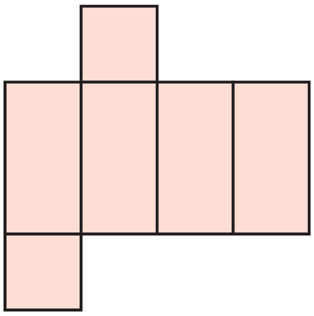
1. Refer to the octagon shown.
Note: The diagonal sides of the octagon are *not* 4 in. long.



a While estimating the area of the octagon, Lin reasoned that it must be less than 100 in^2. Do you agree? Explain your thinking

b Find the exact area of the octagon. Show your thinking.

2. Tyler said that the net shown *cannot* be a net for a square prism because not all the faces are squares. Do you agree with Tyler? Explain your thinking.



Name: .... Date: .... Period: ....

3. Which of these five polyhedra are prisms? Which are pyramids?

Figure A

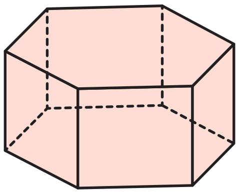


Figure B

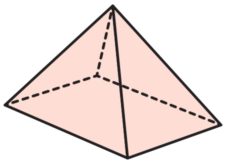


Figure C

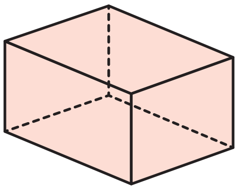


Figure D

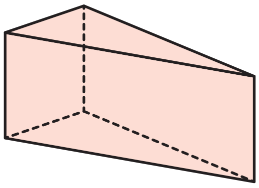
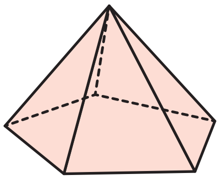


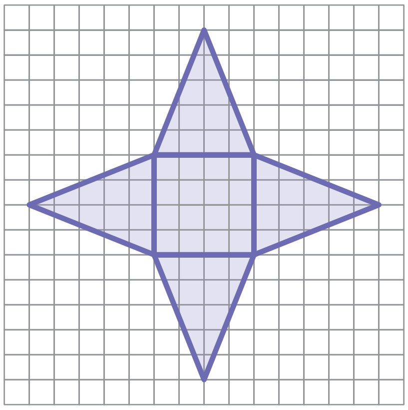
Figure E



Prisms:

Pyramids:

4. Refer to the net shown.



   a. What three-dimensional figure can be assembled from the net?

   b. What is the surface area of the figure? **Note:** One grid square represents 1 square unit.

5. Match each quantity with an appropriate unit of measurement.

a	The surface area of a tissue box	.... square meters
b	The amount of soil in a planter box	.... yards
c	The area of a parking lot	.... cubic inches
d	The length of a soccer field	.... cubic feet
e	The volume of a fish tank	.... square centimeters

My Notes:

UNIT 2

Introducing Ratios

A little bit of this and a little bit of that. Well, maybe a lot of that? Wait, I think a ratio can help with this dilemma! Ratios help us see the relationship between one number and another, so when we make guacamole, it doesn't taste awful.

Essential Questions

- What does a ratio say about the relationship between quantities?
- How can ratios reflect fairness?
- How can ratios help you estimate solutions to seemingly impossible real-world problems?
- *(By the way, is it possible to have too much cowbell?)*

SUB-UNIT

1 What are Ratios?

Narrative: Whether it's colors or tiles, having the right amount of each part is the key.

You'll learn . . .

- how ratios represent comparison.
- connections between ratios, multiplication, and division.

SUB-UNIT

2 Equivalent Ratios

Narrative: Ratios can help you keep a rhythm and balance the sounds of music.

You'll learn . . .

- when two ratios are equivalent.
- about common factors and common multiples.

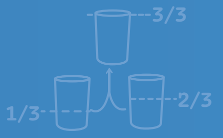
SUB-UNIT

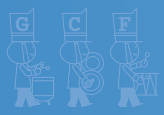
3 Solving Ratio Problems

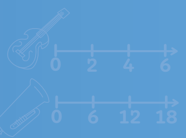
Narrative: Every good cook knows that ratios are an important ingredient of any recipe.

You'll learn . . .

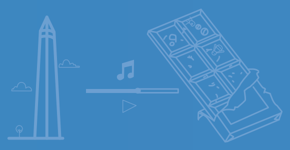
- how to use ratios to find missing values.
- to apply ratios to real-world problems.

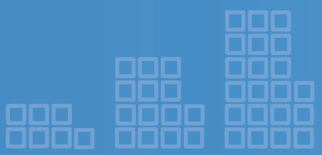






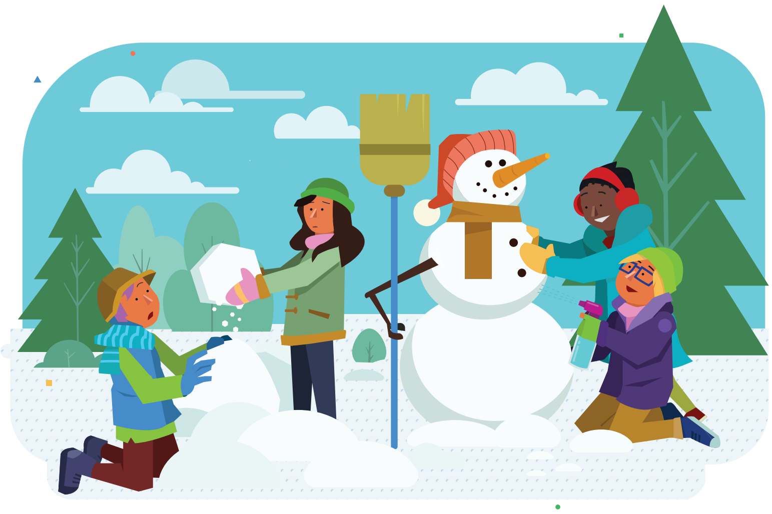
The ratio of the different edge lengths of a rectangular prism is 1:2:3. If its volume is 1296 units^3, what is the length of its longest edge?





Fermi Problems

Let's explore Fermi problems.



Warm-up Cardiac Rhythm

Describe how you could make a rough estimate to solve this problem: "How many times does your heart beat in a year?" Include any information you would need to know.

Co-craft Questions: Work with your partner to come up with 2–3 questions you might have about the information needed to solve this problem.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 The Fermi Carousel

Enrico Fermi was an Italian scientist born in Rome in 1901. Immediately after receiving the Nobel Prize for Physics in 1938, he and his family immigrated to the United States — "immediately" because Italy's close association with Nazi Germany was unsettling given that Fermi's wife was Jewish. While in the U.S., Fermi became known for his uncanny ability to quickly "guesstimate" solutions to seemingly impossible-to-answer mathematical problems by working with reasonable information and approximations to make back-of-the-envelope calculations. He made a habit of challenging his students and fellow scientists with these types of questions.

The Fermi family arriving in America, Jan. 1939.
AIP Emilio Segrè Visual Archives, Wheeler Collection

Part 1

You and your group will rotate around the room to various stations where Fermi problems have been placed. At each station, you will have a limited amount of time to think about and write down one of three things: assumptions you would have to make, related questions, or approximate answers to any questions from previous groups.

1. How long would it take to read the dictionary?

2. How many balloons could you fit in your classroom?

3. How many hours of television does a 6th grader watch in a year?

4. How long would it take to paddle across the Pacific Ocean?

5. How many liters of water does the school use each week?

6. How many times could you say the alphabet in 24 hours?

7. How many single strands of hair are on your head?

Activity 1 The Fermi Carousel (continued)

8. How many blades of grass are there on a football field?

9. How many grand pianos could you fit in the cafeteria?

10. How many pieces of cooked spaghetti would you need to wrap around the perimeter of your school?

11. How much pudding would it take to fill a swimming pool?

12. If all the books in the school were stacked on top of each other in one pile, how tall would the pile be?

Part 2

You now have one of the Fermi problems to try to solve as a group. Identify the questions, answers, and assumptions that are helpful. Work together to come up with a way you might solve your problem, adding more assumptions and related questions as necessary.

You will be given a separate sheet to illustrate how your group interpreted the information to solve the Fermi problem for others to see. Be sure to include diagrams (or pictures), numbers, and words.

Are you ready for more?

Think about information that would be needed to work through this Fermi problem. Provide a plan for how you would solve the problem.

Some research has shown that it takes 10,000 hours of practice for a person to achieve the highest level of performance in any field — sports, music, art, chess, programming, etc. If you aspire to be a top performer in a field you love, such as Michael Jordan in basketball, Frida Khalo in painting, or Maya Angelou in literature, how many years would it take you to meet that 10,000-hour benchmark if you start now? How old would you be?

STOP

Unit 2 Unit Title

Sensing a Ratio

Nothing says winter like a snowman: three massive snowballs for the head and body; twigs for arms; button eyes; and of course, the carrot nose.

The art of building snowmen has been around since the Middle Ages. In the winter of 1511, the peasants of Brussels protested the ruling class by filling their city with 110 snowmen, posed in shocking and embarrassing positions. In 1690, the village of Schenectady, New York posted two snowmen to guard their town, leaving them vulnerable to a raid led by neighboring French Canadians. In 1950, students from the city of Sapporo, Japan built six snow statues in Odori Park. This kicked off what would become Sapporo's world-famous Snow Festival, which today attracts more than 2 million attendees to its massive snow sculpture competition.

It seems like anywhere you can find snow, *someone* has built a snowman nearby. Not all snow is created equal though. Sometimes it's nice and fluffy. But sometimes it's icy and hard, or watery and slushy. For building snowmen, you need snow that's strong and solid, but still easy to shape. Getting that consistency requires the perfect mix of ice and water.

Even with limited experience, you can estimate the right mix by how the snow feels in your hands—how much give it has, the way it holds together. But this relationship between water and ice can also be expressed using numbers, to let you know when it's a good day to build that perfect snowman. And those numbers can even help you estimate whether you have enough snow to make it five feet tall, or five hundred feet tall!

Welcome to Unit 2.

Practice

Name: .... Date: .... Period: ....

Circle the questions or information that would be helpful to solve each Fermi problem.

1. How much food does a school throw out in a month?

A. There are 2 bones in a chicken wing.

B. How many garbage bags are filled after each lunch period?

C. How many bags of recyclables are used after each lunch period?

D. The school uses 5 buckets of compostable materials for the school garden.

2. How many plastic flamingos are on people's lawns in the United States?

A. For every 3 people living in apartments, there are 6 people living in a house.

B. How many people live in the United States of America?

C. There are 400 million blades of grass per person in the world.

D. How many flamingos are at the Columbus Zoo and Aquarium?

3. Complete each equation to make it true.

a $3 \cdot \frac{1}{3} =$ ....

b $10 \cdot \frac{1}{10} =$ ....

c $19 \cdot \frac{1}{19} =$ ....

d $a \cdot \frac{1}{a} =$ .... (as long as a does not equal 0)

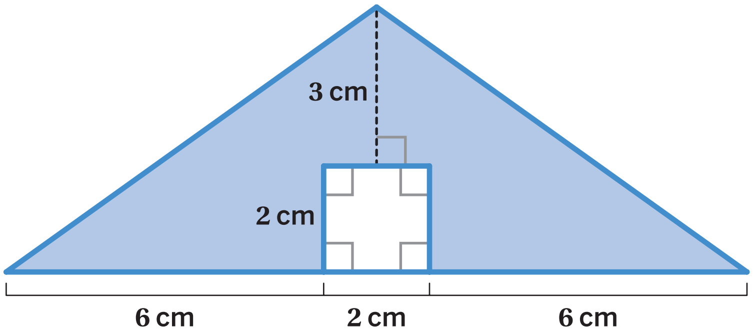
e $5 \cdot$ .... $= 1$

f $17 \cdot$ .... $= 1$

g $b \cdot$ .... $= 1$

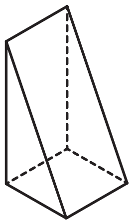
Name: .... Date: .... Period: ....

4. Find the area of the shaded region. Show or explain your thinking.

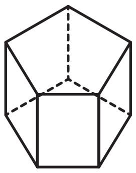


5. Select *all* the figures that are triangular prisms.

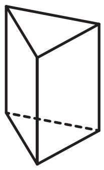
A.



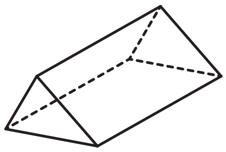
B.



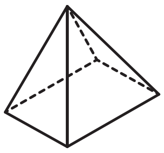
C.



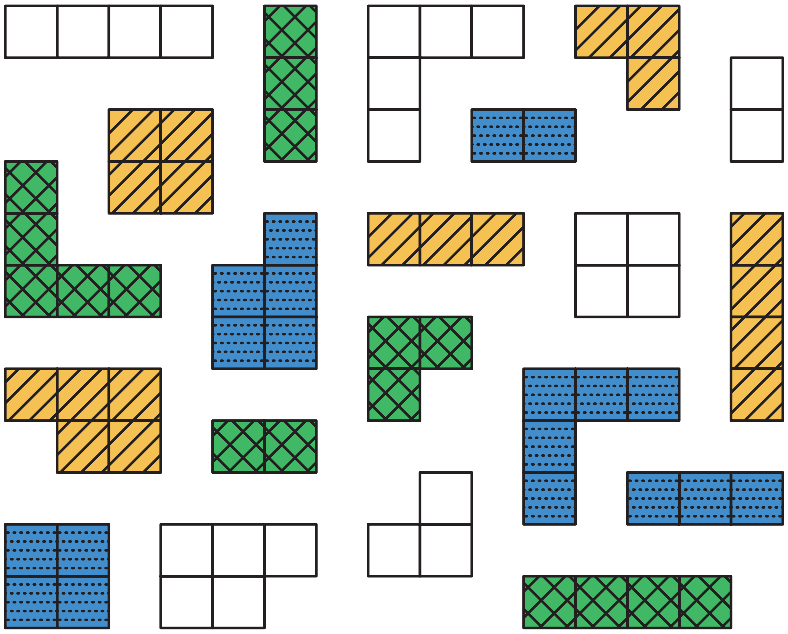
D.



E.



6. Think of different ways you could sort these figures. What categories could you use?



My Notes:

SUB-UNIT

1 What Are Ratios?

Narrative Connections

How does an eggplant become a plum?

Few things are more important to painters than color.

“Color in painting,” according to Vincent Van Gogh, “is like enthusiasm in life.” In her diaries, Mexican painter Frida Kahlo described herself as “CHROMOPHORE—the one who gives color.” Meanwhile French impressionist Claude Monet described color as his “day-long obsession, joy and torment.” It is through color that artists give their paintings a sense of life and motion, enabling them—in the words of Georgia O’Keefe—to “say things . . . [they] couldn’t say any other way.”

In painting, there are three primary colors: red, yellow, and blue. They’re called “primary” because they can’t be mixed from the *other* colors. They can, however, be combined to create a wide range of other colors. Green, purple, orange, and the shades in between are made from these primary colors. With the right combinations, artists can create light and shadow, or give an image depth. They can draw attention to certain areas of the canvas, or evoke a particular emotion within us.

One of the greatest innovations in color came not from an artist, but from the mathematician Isaac Newton. Newton bent a sunbeam through a prism, creating a rainbow spectrum. He arranged the resulting colors onto a wheel. Over time, this color wheel would evolve into a useful tool to help artists choose meaningful color combinations.

Painters must use these combinations of primary colors in just the right amounts. The wrong combination could mean the difference between blush and cerise, or eggplant and plum! To get the exact right shade, we need a way to express the relationship between the amounts of different pigments.

Unit 2 | Lesson 2

Introducing Ratios and Ratio Language

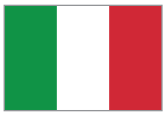
Let's use visuals to describe how quantities relate to each other.

Warm-up Categorizing Flags of the World

Think about how you could group these flags into two or more categories. Be prepared to explain your thinking.


Bosnia and Herzegovina

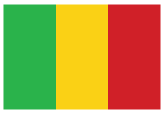

Cambodia


Italy


Kiribati


United Arab Emirates


Bhutan

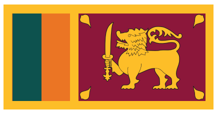

Mali

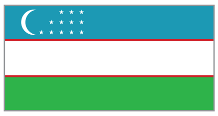

Belgium


Jordan


Papua New Guinea


Uruguay


Sri Lanka


Uzbekistan


Namibia


Romania


Honduras

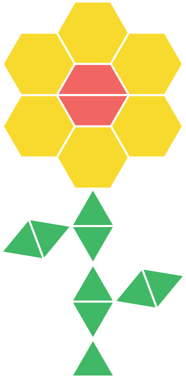
Public Domain

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Ratios and Ratio Language in Flower Patterns

Refer to the flower constructed from pattern blocks.



1. Record the number of each pattern block shape used to make this flower.

   Trapezoids: ....

   Hexagons: ....

   Triangles: ....

2. Complete each statement comparing the number of trapezoids and hexagons.

   a There are .... trapezoids for every .... hexagons.

   b The statement in part a describes the *ratio* of trapezoids to hexagons, which is .... to ....

   c What is the ratio of hexagons to trapezoids? .... to ....

   d Write another sentence that describes the ratio of hexagons to trapezoids.

3. Complete each statement comparing the number of triangles and hexagons.

   a The ratio of triangles to hexagons is .... to ....

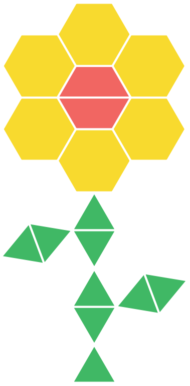
   b There are .... triangles for every .... hexagons.

   c What is the ratio of hexagons to triangles? .... to ....

   d Write another sentence that describes the ratio of hexagons to triangles.

Activity 2 Ratios and Ratio Language in Flower Gardens

Refer to the flowers constructed from pattern blocks.

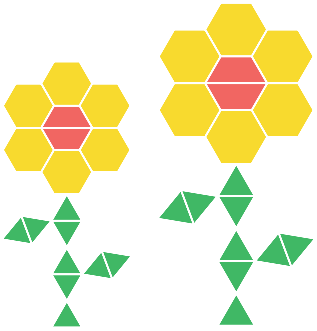


1. Record the number of each pattern block shape used to make all three flowers.

Trapezoids: ....

Hexagons: ....

Triangles: ....



2. Write as many sentences as you can think of that describe ratios in this flower garden.

Hint: There are at least 36 sentences you can write!

Collect and Display: Your teacher will walk around and collect language you use to describe the ratios. This language will be added to a class display for your reference.

3. What do you notice about the ratios that describe a single flower and the ratios that describe the entire flower garden?

Name: .... Date: .... Period: ....

Activity 3 Two Truths and a Lie

You will be given a set of instructions to follow for creating a design by using pattern blocks.

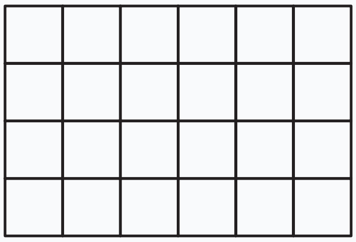
As you *Mix and Mingle* with other groups, record which statement is the lie for each group. Briefly explain why it is a lie.

Group	Lie	Explanation
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Reflect: How did you determine whether a statement was a truth or a lie?

Are you ready for more?

1. Use two colors to shade the rectangle so that there are 2 square units of one color for every 1 square unit of the other color.
2. The rectangle you just shaded has an area of 24 square units. Draw a different shape that does not have an area of 24 square units, but that can also be shaded with two colors in a 2 : 1 ratio. Shade your new shape by using two colors.



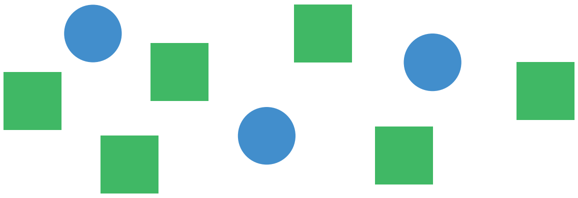
STOP

Summary

In today's lesson . . .

You began to investigate a specific type of relationship between two or more quantities called a ***ratio relationship***.

There are many ways you can describe a situation using *ratio language*. For example, consider this set of squares and circles:



Some statements that describe the relationship between squares and circles using ratio language are:

- The ratio of circles to squares is 3 to 6.
- There are 6 squares for every 3 circles.
- The ratio of circles to squares is $3 : 6$.
- There are 2 times as many squares as there are circles.

Reflect:

Name: .... Date: .... Period: ....

1. In a fruit basket, there are 9 bananas, 4 apples, and 3 plums.

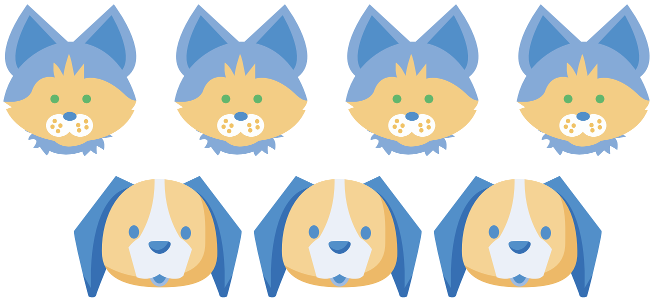
   a The ratio of bananas to apples is .... : ....

   b The ratio of plums to apples is .... to ....

   c For every .... apples, there are .... plums.

   d For every 3 bananas, there is 1 ....

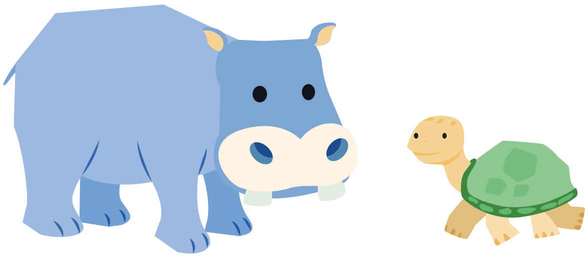
2. Complete the sentences to describe a ratio relationship between the two types of animals in this collection of cats and dogs.



   a The ratio of dogs to cats is ....

   b For every .... dogs, there are .... cats.

3. Write two different sentences that use ratios to relate the number of eyes to the number of legs in this picture.



Name: .... Date: .... Period: ....

4. Choose an appropriate unit of measurement for each quantity: cm, cm^2, or cm^3.

a Area of a rectangle

b Volume of a prism

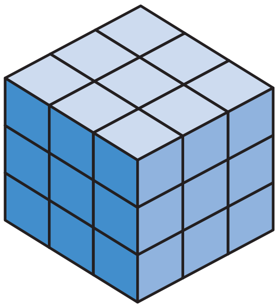
c Side of a square

d Area of a square

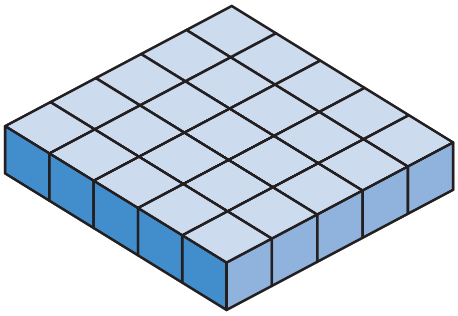
e Volume of a cube

5. Determine the volume and surface area of each prism.

a **Prism A:** 3 cm by 3 cm by 3 cm



b **Prism B:** 5 cm by 5 cm by 1 cm



c Compare the volumes of the prisms and then their surface areas. Does the prism with the greater volume also have the greater surface area?

6. Show at least three different ways you could represent the number 18 using an area model.

Name: ________ Date: ________ Period: ________

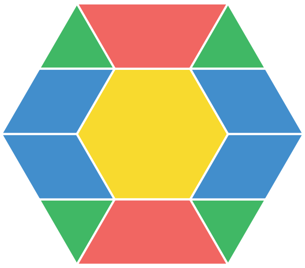
Unit 2 | Lesson 3

Representing Ratios With Diagrams

Let’s use diagrams to represent ratios.

Warm-up Pattern Blocks and Ratios

Refer to this design made up of pattern block shapes.



1. Choose two of the shapes in the design, and draw a diagram to represent the ratio of those shapes.

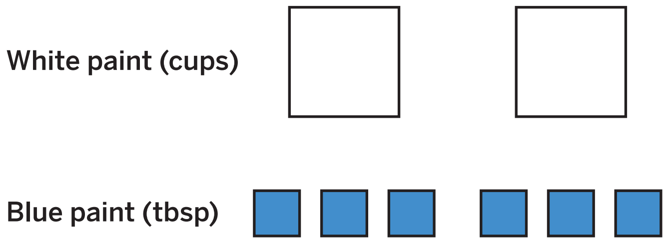
2. Trade books with a partner. On their page, write a sentence to describe a ratio shown in their diagram. Your partner will do the same for your diagram.

3. Return your partner’s book. Read the sentence written on your page. If you disagree with the statement, try to rewrite it and explain your thinking to your partner.

Log in to Amplify Math to complete this lesson online.

Activity 1 Mixing Paint

To create a light blue paint, Elena mixed 2 cups of white paint with 6 tablespoons (tbsp) of blue paint.



1. Discuss each statement, and circle *all* those that correctly describe this situation. Make sure that both you and your partner agree with each circled response.

A. The ratio of cups of white paint to tablespoons of blue paint is 2 : 6.

B. There is 1 cup of white paint for every 3 tbsp of blue paint.

C. There are 3 tbsp of blue paint for every cup of white paint.

D. For every tablespoon of blue paint, there are 3 cups of white paint.

E. For every 6 tbsp of blue paint, there are 2 cups of white paint.

F. For every 3 cups of white paint, there are 7 tbsp of blue paint.

2. Jada also made a light blue paint for an art project by mixing 3 cups of white paint with 9 tablespoons of blue paint.

a Draw a diagram that represents Jada's light blue paint.

b Write at least two sentences describing the ratio of white paint and blue paint that Jada mixed.

Name: .... Date: .... Period: ....

Activity 2 Card Sort: Representing Ratios

You will be given a set of cards describing different amounts of ingredients used in a recipe for guacamole.

1. Take turns with your partner selecting a sentence and matching it with a diagram.

- Explain to your partner how you know the sentence and the diagram match.
- If you disagree with a match your partner presents, explain your thinking and discuss until you reach an agreement.
- Record the number of the sentence that matches each diagram in the table. More than one sentence may match a given diagram.

Diagram	Sentence number
A	
B	
C	
D	
E	
F	

2. After you and your partner have agreed on a match for all of the sentences, compare your matches with the answer key. Discuss any mismatches and update your table with the correct matches.

3. Would guacamole made by using the ratios in Diagrams E and F taste the same? Why or why not?

4. Select one of Diagrams A–D and write another sentence that describes the ratio shown.

Are you ready for more?

If guacamole was made using 4 cloves of garlic, 6 limes, and 12 avocados, would it taste the same as the recipe shown in Diagram F? If not, describe the difference in taste.

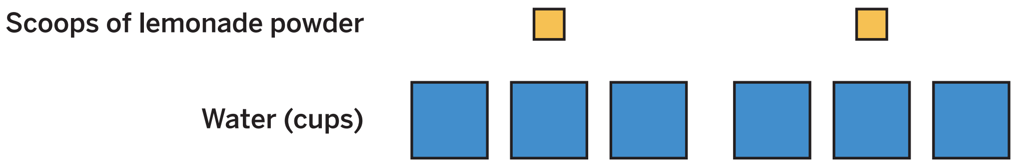
STOP

Summary

In today's lesson . . .

You saw that ratio relationships between quantities can be described using ratio language and can also be represented using diagrams.

For example, a recipe for lemonade, "mix 2 scoops of lemonade powder with 6 cups of water" can be represented using the diagram:



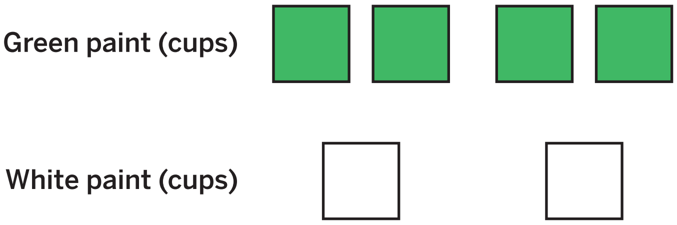
The ratio of scoops of lemonade powder to cups of water is 2 to 6, which can be written as $2 : 6$.

You used diagrams to reason about other ways the relationship between two quantities can be described. For example, you could also say that every scoop of lemonade powder corresponds to 3 cups of water, which can be written as the ratio $1 : 3$.

Reflect:

Name: ________ Date: ________ Period: ________

1. The diagram represents the cups of green paint and cups of white paint in a paint mixture. Select *all* the statements that correctly describe the relationship between green paint and white paint.



A. The ratio of cups of white paint to cups of green paint is 2 to 4.

B. For every cup of green paint, there are 2 cups of white paint.

C. The ratio of cups of green paint to cups of white paint is $4 : 2$.

D. For every cup of white paint, there are 2 cups of green paint.

E. The ratio of cups of green paint to cups of white paint is $2 : 4$.

2. A recipe for snack mix says to combine 2 cups of raisins with 4 cups of pretzels and 6 cups of almonds.

a Create a diagram to represent the amounts of each ingredient in this recipe.

b Use your diagram to complete each sentence.

The ratio of pretzels to almonds is ________ : ________.

There are ________ cups of pretzels for every 1 cup of raisins.

There are ________ cups of almonds for every 1 cup of raisins.

Name: .... Date: .... Period: ....

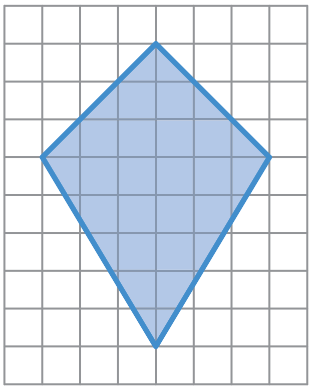
3. Determine the stated measurements for the squares described. Include the appropriate units.

a A square is 3 in. by 3 in. What is its area?

b A square has a side length of 5 ft. What is its area?

c The area of a square is 36 cm^2. What is the length of each side of the square?

4. Determine the area of this quadrilateral. Show or explain your strategy.



5. Evaluate each expression.

a $\frac{1}{8} \cdot 8$

b $\frac{1}{8} \cdot 7$

c $\frac{3}{8} \cdot 8$

d $\frac{3}{8} \cdot 7$

6. Mai is creating goodie bags for her birthday party. The table shows the number of items in one goodie bag.

Bracelets	Animal erasers	Stickers
1	3	12

a Determine the number of animal erasers Mai would need to make 8 goodie bags.

b Determine the number of bags Mai made if she used 72 stickers.

Name: ______________________ Date: __________ Period: ________

Unit 2 | Lesson 4

A Recipe for Purple Oobleck

Let’s explore ratios in recipes.

Warm-up Number Talk

Mentally evaluate each expression. Be prepared to explain your thinking.

1. $6 \cdot 15$

2. $12 \cdot 15$

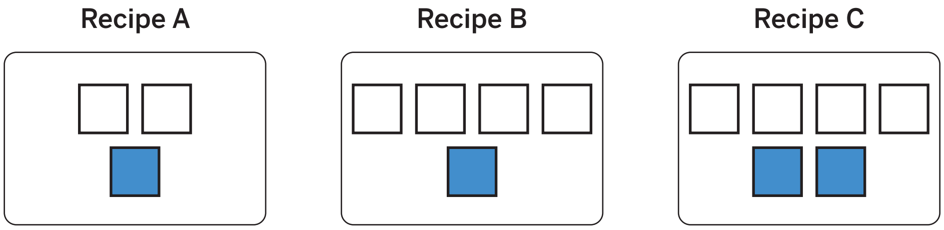
3. $6 \cdot 45$

4. $13 \cdot 45$

Log in to Amplify Math to complete this lesson online.

Activity 1 Making Oobleck

Oobleck **is a substance called a suspension, which can mimic the qualities of both a solid and a liquid. Here are diagrams representing three possible recipes for making oobleck using cornstarch and water.**



Key:

□ = 1 **cup of cornstarch** 🟦 = 1 **cup of water**

1. How might the texture of oobleck made from Recipe A compare to the texture of oobleck made from Recipe B?

2. Use the diagrams to complete each pair of statements.

   a Recipe A uses .... cup(s) of cornstarch and .... cup(s) of water.
   The ratio of cups of cornstarch to cups of water in Recipe A is ....

   b Recipe C uses .... cup(s) of cornstarch and .... cup(s) of water.
   The ratio of cups of cornstarch to cups of water in Recipe C is ....

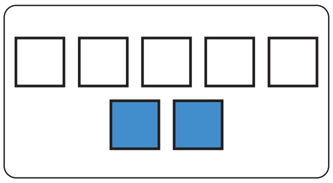
3. How might the texture of oobleck made from Recipe A compare to the texture of oobleck made from Recipe C?

Name: ______________________ Date: ____________ Period: ________

Activity 1 Making Oobleck (continued)

4. Refer to Recipe D shown here.

Recipe D



Key:

☐ = 1 **cup of cornstarch**

🟦 = 1 **cup of water**

a Write the ratio of cornstarch to water in Recipe D.

b Describe the consistency of oobleck made from Recipe D.

c What could be done to “fix” Recipe D so that the oobleck made will have the same consistency as oobleck made from Recipe A? **Note:** You cannot remove any ingredient that is already added to the mixture. You can only add ingredients.

d Using your fix, write a ratio for a new Recipe E so that oobleck made from Recipe E has the same consistency as oobleck made from Recipe A.

5. What do you notice about the ratios for Recipes A, C, and E?

Activity 2 Coloring Your Oobleck

When mixing colors, ratios can tell you when two results should be the same. However, not everyone sees colors the same way. There are several reasons for this – one reason is that most people (called *trichromats*) have three types of retinal cone cells, while some (called *tetrachromats*) have four types. Trichromats can see around 1 million different colors, while tetrachromats can see as many as 100 million colors!

Researchers like Dr. Kimberly A. Jameson study how people experience colors differently by presenting different ratios of colors mixed together for subjects to identify and categorize.

Now imagine you are running color-matching experiments of your own, using dyed oobleck. To color one batch of oobleck purple, you can add 2 red drops and 5 blue drops of food coloring to water.

1. What is the ratio of red drops to blue drops of food coloring for one batch?

2. Draw a diagram showing the number of red drops related to the number of blue drops that would make *double* the amount of food coloring. Then write these amounts as a ratio.

3. How do you know that this will make the exact same purple?

Featured Mathematician

Kimberly A. Jameson

Kimberly A. Jameson is a Project Scientist at UC Irvine's Institute for Mathematical Behavioral Sciences. She has conducted numerous research studies on the perception of color, human tetrachromacy, and why individuals "see" colors differently.

Kimberly A. Jameson

Name: .... Date: .... Period: ....

Activity 2 Coloring Your Oobleck (continued)

4. Write the ratio of the number of red drops to the number of blue drops of food coloring that are needed to *triple* the mixture of food coloring. Explain your thinking.

5. How many batches of oobleck can you color with 10 drops of red food coloring and 25 drops of blue food coloring?

6. Find another ratio of red drops to blue drops that would produce the same purple color.

7. How many batches of oobleck can you color using this new ratio of red to blue drops?

Are you ready for more?

Sports drinks use sodium (better known as salt) to help people replenish electrolytes. Here are the nutrition labels of two sports drinks.

Sports drink A

Nutrition Facts
Serving Size 8 fl oz (240 mL)
Serving Per Container 4

Amount Per Serving
Calories 50

		% Daily Value*
Total Fat	0 g	0%
Sodium	110 mg	5%
Potassium	30 mg	1%
Total Carbohydrate	14 g	5%
Sugars	14 g	
Protein	0 g	

% Daily Value are based on a 2,000 calorie diet.

Sports drink B

Nutrition Facts
Serving Size 12 fl oz (355 mL)
Serving Per Container about 2.5

Amount Per Serving
Calories 80

		% Daily Value*
Total Fat	0 g	0%
Sodium	150 mg	6%
Potassium	35 mg	1%
Total Carbohydrate	21 g	7%
Sugars	20 g	
Protein	0 g	

% Daily Value are based on a 2,000 calorie diet.

1. Which of these drinks is saltier? Explain your thinking.

2. If you wanted to make sure a sports drink was less salty than both of these drinks shown here, what ratio of sodium to water would you use?

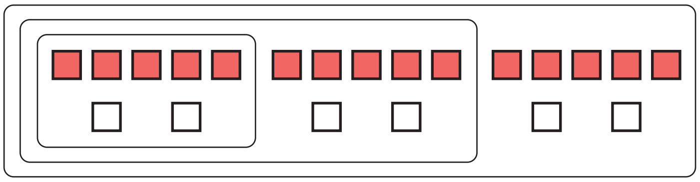
STOP

Summary

In today's lesson . . .

You explored different combinations of cornstarch and water, and red or blue food coloring. You were able to create different textures and different colors. You realized that some combinations created the same texture or color and compared the *ratio* of their ingredients using diagrams and numeric values. A ***ratio*** is a comparison of two quantities, such that for every a units of one quantity, there are b units of another quantity.

The diagram shows the *ratio* of red paint to white paint in a single batch, double batch, and triple batch of a recipe.



Single batch: $5 : 2$.

Double batch: $10 : 4$

Triple batch: $15 : 6$.

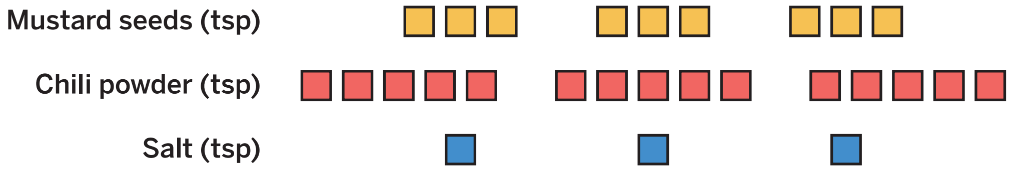
These ratios are ***equivalent*** because they all represent the same pink color (or the same ratio of red paint to white paint).

Reflect:

Name: .... Date: .... Period: ....

1. Priya makes chocolate milk by mixing 2 cups of milk and 5 tbsp of cocoa powder. Draw a diagram that clearly represents doubling her recipe for chocolate milk.

2. A recipe for 1 batch of spice mix says, “Combine 3 tsp of ground mustard seeds, 5 tsp of chili powder, and 1 tsp of salt.” How many batches are represented by the diagram? Show or explain your thinking.



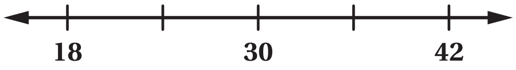
3. In a recipe for sparkling grape juice, the ratio of cups of sparkling water to cups of grape juice concentrate is 3 to 1.

- **a** Find two more ratios of cups of sparkling water to cups of juice concentrate that would make larger amounts of sparkling grape juice that each tastes the same as this recipe.

- **b** Write a ratio of sparkling water and grape juice that would *not* taste the same as this recipe. Then describe how it would taste different.

Name: .... Date: .... Period: ....

4. The tick marks on this number line are equally spaced. Write the missing numbers under the two unlabeled tick marks on the number line.



5. At the kennel, there are 6 dogs for every 5 cats.

a The ratio of dogs to cats is .... to ....

b For every .... cats there are .... dogs.

c Would the ratio $6 : 5$ represent the relationship between the total number of dogs and the total number of cats? Explain your thinking.

d Would the ratio $6 : 5$ represent the relationship between the total number of cats and the total number of dogs? Explain your thinking.

6. What is $\frac{1}{4}$ of 100? Show or explain your thinking.

Name: ________ Date: ________ Period: ________

Unit 2 | Lesson 5

Kapa Dyes

Let's see how mixing colors relates to ratios.

Warm-up Number Talk

Mentally evaluate each expression.

1. $24 \div 4$

2. $\frac{1}{4} \cdot 24$

3. $24 \div \frac{4}{1}$

4. $5 \div 4$

Log in to Amplify Math to complete this lesson online.

Activity 1 'Uki'uki/Ma'o Dye

Artist Dalani Tanahy specializes in *kapa*, the traditional native Hawaiian technique for making cloth using tree bark. An important part of the process is decorating kapa with designs and patterns using natural dyes. Specific ratios of two dyes are mixed to create new colors. For example, greens from the maʻo plant and blues from the 'uki'uki berry combine to make a teal dye.

Combining blue and green food coloring or paint can replicate the specific teal color — the 'uki'uki/ma'o dye — shown on the wheel.

Photo courtesy of Dalani Tanahy

1. A mixture for 'uki'uki/ma'o dye uses a ratio of green to blue of $15 : 45$. Draw a diagram to represent this mixture.

Plan ahead: In what ways will you show respect for other cultures as you complete this activity?

2. Think about making a smaller amount of the same color of the 'uki'uki/ma'o dye.

a What is one way you could make a smaller amount that is the same color? Show or explain your thinking.

b Is the ratio in your mixture from Problem 2a the same as the ratio in the original mixture? Explain your thinking using ratios and ratio language.

Name: ________________ Date: ________ Period: ________

Activity 1 'Uki'uki/Ma'o Dye (continued)

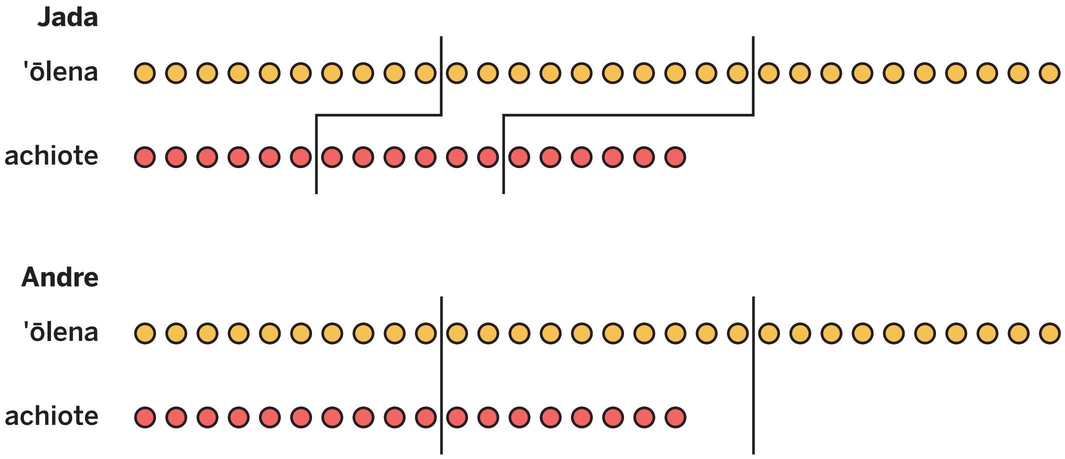
3. Write the ratios from Problems 1 and 2 in the table. Describe any patterns you notice.

Ma'o green	'Uki 'uki blue

Activity 2 'Ōlena/Achiote Dye

The orange dye seen on the color wheel is traditionally made by combining yellow from the 'ōlena (turmeric) plant and red from the seeds of the achiote plant. A mixture for 'ōlena/achiote dye calls for 30 ml of 'ōlena yellow with 18 ml of achiote red.

1. Jada and Andre each attempted to make a smaller amount of the same 'ōlena/achiote color using food coloring. Jada mixed 10 ml of 'ōlena yellow with 6 ml of achiote red. Andre mixed 5 ml of 'olena yellow with 5 ml of achiote red. Diagrams that represent their color mixtures are shown.



a Does either person's color mixture make the same color orange as the known 'ōlena/achiote mixture? Explain your thinking.

b If either person's mixture did not produce the same color orange, what might they have done incorrectly?

Name: ________________ Date: ________ Period: ________

Activity 2 'Ōlena/Achiote Dye (continued)

2. Describe one other way you could combine different amounts of 'ōlena yellow and achiote red that would result in the same orange color as the original mixture but produce a *smaller amount*. Show or explain your thinking.

3. Complete the table with the possible ratios for making the known 'ōlena/achiote dye.

'Ōlena yellow	Achiote red
30	18

STOP

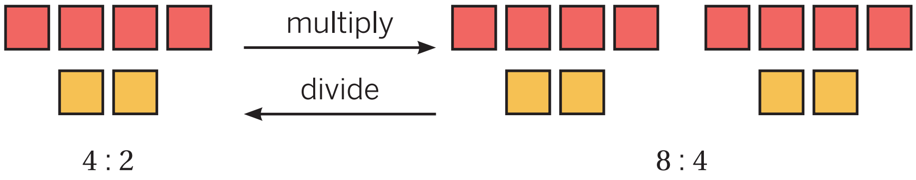
Summary

In today's lesson . . .

You saw again that when mixing colors, you can use *ratios* to determine different amounts of each color that can be combined to create the same color.

To make *larger* amounts, you can always *multiply* the amount of each color by the same number (greater than 1) and the color will be the same.

To make smaller amounts, you can always *divide* the amount of each color by the same number (or multiply by the same fraction), and the color will be the same.



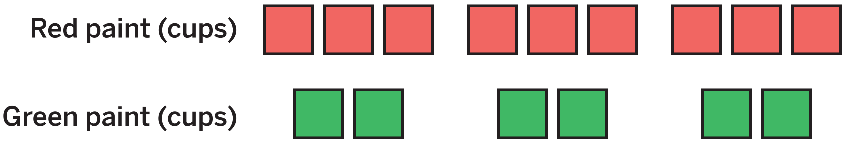
Both groups represent a ratio of $4 : 2$ and makes the same color orange paint. Ratios $4 : 2$ and $8 : 4$ are *equivalent* because in each ratio the first value is double the second value.

Reflect:

Practice

Name: ________________ Date: ________ Period: ________

1. The diagram shows a mixture of red paint and green paint needed for 3 batches of a particular brown paint. How could you show 1 batch of the same brown paint? What is the ratio of red paint to green paint, for 1 batch?



2. Diego makes green paint by mixing 10 tbsp of yellow paint and 2 tbsp of blue paint. Which of these mixtures produce the same green paint as Diego's mixture, but in a smaller amount? Select *all* that apply.

   A. For every 5 tbsp of blue paint, mix 1 tbsp of yellow paint.

   B. Mix tablespoons of yellow paint and blue paint in the ratio $5:1$.

   C. Mix tablespoons of yellow paint and blue paint in the ratio 15 to 3.

   D. Mix 11 tbsp of yellow paint and 3 tbsp of blue paint.

   E. For every tablespoon of blue paint, mix 5 tbsp of yellow paint.

3. To make 1 batch of sky blue paint, Clare mixes 2 cups of blue paint with 1 gallon of white paint.

   a. Clare only needs half the amount of sky blue paint. What ratio would represent half the recipe?

   b. Explain how to make a mixture that is a darker tint of blue than the sky blue.

   c. Explain how to make a mixture that is a lighter tint of blue than the sky blue.

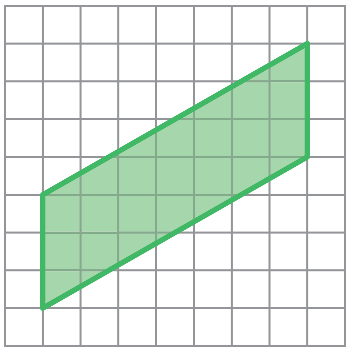
Name: .... Date: .... Period: ....

4. A smoothie recipe calls for 3 cups of milk, 2 frozen bananas and 1 tbsp of cocoa powder.

a Create a diagram to represent the quantities of each ingredient in the recipe.

b Write *three* different sentences that use ratio language to describe the recipe.

5. Determine the area of the parallelogram. Show your thinking.



6. Evaluate each product.

a $3 \cdot \frac{2}{3}$

b $\frac{7}{5} \cdot 5$

c $2 \cdot \frac{3}{4}$

SUB-UNIT

2 Equivalent Ratios

Narrative Connections

How do you put your music where your mouth is?

Antoinette Clinton was just 20 years old when she took the stage in Leipzig, Germany. Better known by her stage name, Butterscotch, she was born in Sacramento, California to a musical family. Her mother was a piano teacher. Her siblings played trumpet, cello, clarinet, and trombone. But tonight was the night of the first Beatbox Battle World Championship. She had come to showcase a different musical instrument: herself!

Beatboxing has long been a core element of hip-hop. Pioneered by artists like Doug E. Fresh, Biz Markie, and Darrell “Buffy” Robinson, performers use their mouth, throat, and nose to imitate a drum kit. MC’s would then rap over their beats.

More than 20 years later, beatboxing re-emerged as an international phenomenon. In 2005, Butterscotch was crowned the first Individual Female Beatbox Battle World Champion. Two years later, she beat out 18 men to become the West Coast beatboxing champion.

To be a champion beatboxer, you need a strong sense of timing. An artist needs to know the length of each of their “hits”, as well as how many “hits” they can fit into a measure of music. Ratios give performers a way to conceptualize and map those hits so that they never miss a beat.

Unit 2 | Lesson 6

Defining Equivalent Ratios

Let's investigate equivalent ratios.

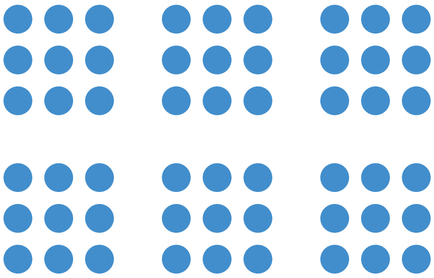
Warm-up Dots and Half Dots

Determine the number of dots in each image. Be prepared to explain your thinking.

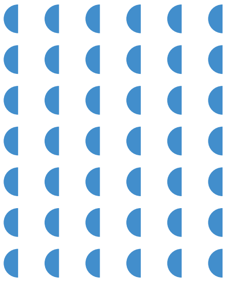
Key:

● = 1

Dot Pattern 1



Dot Pattern 2



Name: .... Date: .... Period: ....

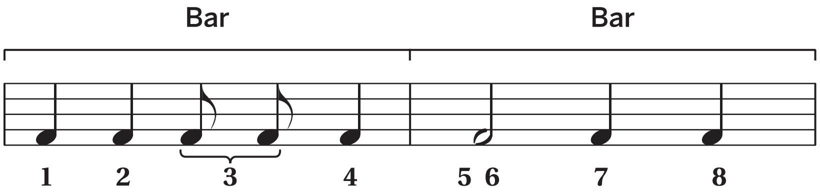
Activity 1 Clapping a Rhythm

Notes are used in music to make systematic arrangements of sound, called *rhythms*. Different notes indicate how long a sound is played — the number of *counts* for which a note is held. Some notes are shorter and some notes are longer. Several shorter notes create a faster sounding rhythm, while longer notes create a slower sounding rhythm.

Here are the notations for representing three types of notes in a musical composition.

Eighth Note	Quarter Note	Half Note
♪ $= \frac{1}{2}$ count	♩ $= 1$ count	𝅗𝅥 $= 2$ counts

The composition of notes shown here has two sections, called *bars*. Your group will be assigned a count 1, 2, 3 or 4. When directed, follow the counts and clap your part according to the notes assigned to your count in each bar.



1. How many notes are in each bar?

2. How many counts are in each bar?

3. How many counts would you expect to be in a third bar?

4. Complete three more rows of this table showing the number of counts for different numbers of bars.

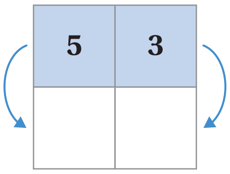
5. Do the counts and bars represent a ratio relationship? Explain or show your thinking.

Compare and Connect: Compare with your group how you generated the values in your table, paying close attention to your reasoning.

Counts	Bars
4	1

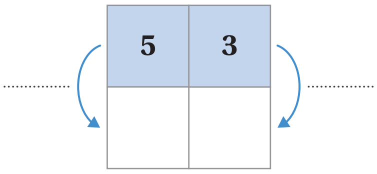
Activity 2 What Are Equivalent Ratios?

The ratios $5:3$ and $10:6$ are *equivalent ratios* because they describe the same ratio relationship. Complete the ratio box to show this is true.

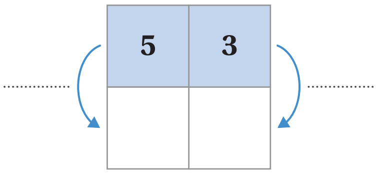


1. Determine whether each ratio is also equivalent to $5:3$ and $10:6$. Show or explain your thinking using a ratio box.

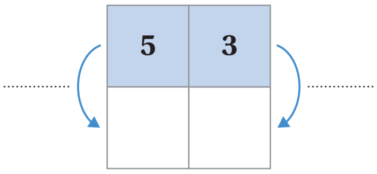
a $15:12$

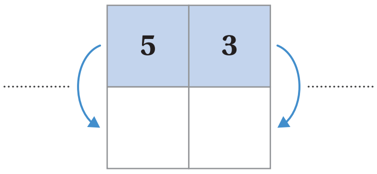


b $30:18$



2. Determine two additional ratios that are equivalent to $5:3$. Show your thinking by using ratio boxes.





3. Write a definition for *equivalent ratios*.

4. How do you know when two ratios are *not* equivalent?

STOP

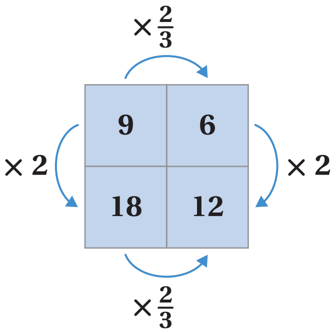
Name: ____________________ Date: __________ Period: __________

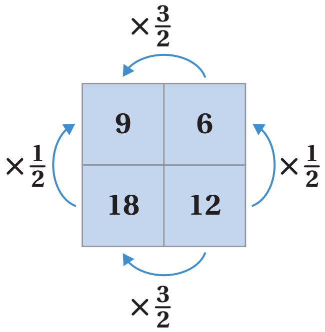
Summary

In today's lesson . . .

You saw that ***equivalent ratios*** are two ratios where the values for one quantity in each ratio can be multiplied or divided by the same number to generate the values for the second quantity in each ratio. For example, the ratios $9:6$ and $18:12$ are equivalent because $9 \cdot \frac{2}{3} = 6$ and $18 \cdot \frac{2}{3} = 12$.

You used a *ratio box* to show and generate *equivalent ratios*.





Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Here are four pairs of equivalent ratios. Explain or show, such as by drawing a ratio box, how you know each pair of ratios is equivalent.

a $4 : 5$ and $8 : 10$

b $18 : 3$ and $6 : 1$

c $2 : 7$ and $10{,}000 : 35{,}000$

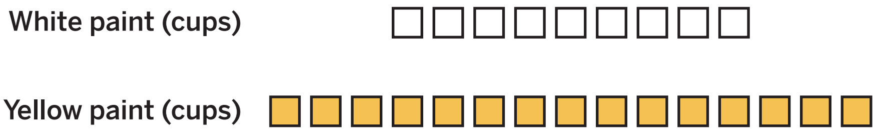
d $24 : 18$ and $8 : 6$

2. Explain why $6 : 4$ and $18 : 8$ are *not* equivalent ratios.

3. Do the ratios $3 : 6$ and $6 : 3$ describe the same relationship? Why or why not?

Name: ________________ Date: ________ Period: ________

4. This diagram represents 3 batches of light yellow paint. Draw a diagram that represents 1 batch of the same shade of light yellow paint.



5. In a fruit bowl there are 6 bananas, 4 apples, and 3 oranges. Complete the statements about the ratios of types of fruit in the bowl.

a For every 4 ________________, there are 3 ________________.

b The ratio of ________________ to ________________ is $6 : 3$.

c The ratio of ________________ to ________________ is $4 : 6$.

d For every 1 orange, there are ________________ bananas.

6. The table shows the number of cups of flour needed to make two batches of a recipe for banana bread. Complete the table by adding three more rows of ratios that are equivalent to $4 : 2$.

Flour	Batches
4	2

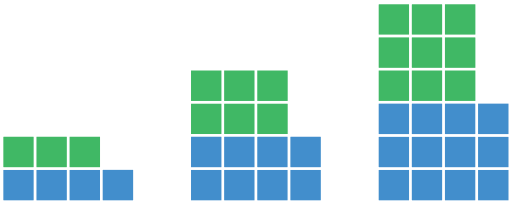
Unit 2 | Lesson 7

Representing Equivalent Ratios With Tables

Let's use tables to represent equivalent ratios.

Warm-up How Is It Growing?

Look for a pattern in the set of figures shown.



1. How many total squares will be in:
   - **a** Figure 4?
   - **b** Figure 5?
   - **c** Figure 10?

2. Describe how you see the pattern growing.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Jazz Rhythm and Horn Sections

In April 2020, the jazz world lost an icon when Ellis Marsalis, Jr. passed away from COVID-19. Ellis was the patriarch of the legendary Marsalis family, and he left behind a tremendous legacy, from his recordings, to the Ellis Marsalis Center for Music in his beloved hometown of New Orleans, to his teachings and many former students. Perhaps the most noteworthy students being several of his sons, accomplished jazz musicians in their own rights – with multiple Grammy Awards. Ellis (piano) and four of his six sons, Branford (saxophone), Wynton (trumpet), Delfeayo (trombone), and Jason (drums) are considered the "first family of jazz" for good reason.

Jim Katz

A jazz orchestra, also called a big band, typically consists of a horn section made up of 5 saxophones, 4 trumpets, 4 trombones, and a rhythm section featuring a piano, a guitar, a bass, and drums.

1. What is the ratio of the number of members of the horn section to the number of members of the rhythm section in a typical jazz orchestra?

2. Use a ratio box to show how you know whether the ratio of the number of horn players to the number of rhythm players in the Marsalis family is equivalent to that in a typical jazz orchestra.

3. Imagine several jazz orchestras get together for a concert. Complete this ratio table with different possible equivalent ratios of the number or horn players to the number of rhythm players for three different-sized orchestras.

Horn players	Rhythm players
13	4

Activity 2 Beignet Recipe

A large family tripled a beignet recipe and used 3 cups of evaporated milk, 21 cups of flour, a half dozen eggs, $4\frac{1}{2}$ cups of warm water, $1\frac{1}{2}$ cups of sugar, $\frac{3}{4}$ of a cup of shortening, 6 tsp of active yeast, and 3 tsp of salt.

1. Determine four equivalent ratios for the amounts of flour and milk needed to make different-sized batches of the same beignet recipe: two that use *more* flour and milk, and two that use *less* flour and milk.

2. What method(s) did you use to determine the equivalent ratios using more ingredients? Less ingredients?

3. How do you know that each row shows a ratio that is equivalent to your original ratio? Show or explain your thinking.

Are you ready for more?

You have created a best-selling recipe for lemon scones. The ratio of sugar to flour is $2:5$. Use a separate sheet of paper to create a table in which each entry represents amounts of sugar and flour that might be used at the same time in your recipe.

- One entry should have amounts where you have fewer than 25 cups of flour.
- One entry should have amounts where you have between 20–30 cups of sugar.
- One entry can have any amount using more than 500 cups of flour.

STOP

Name: ________________ Date: ________ Period: ________

Summary

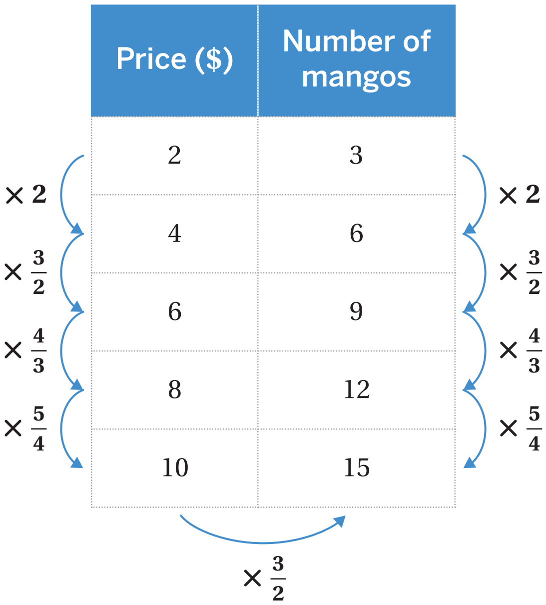
In today's lesson . . .

You saw that you can add column headers to a *ratio box* and extend it down by adding more rows to represent multiple sets of *equivalent ratios*. This is then called a ***ratio table***.

You can use a ratio table in the same ways you used a ratio box — to determine or verify *equivalent ratios*.

This table shows the price of different number of mangos.

The values in each row can be determined by multiplying the corresponding values in each previous row by some same number.



Price ($)	Number of mangos
2	3
4	6
6	9
8	12
10	15

Notice that each row in the table shows that the ratio of number of mangos to the total cost is 3:2, which means that each value in the number of mangos column is 1.5 (or $\frac{3}{2}$) times the corresponding cost in dollars from the same row.

Reflect:

Practice

Name: ______________________ Date: __________ Period: __________

1. A particular orange paint is made by mixing 5 parts of yellow paint with 6 parts of red paint.

   a Complete the table with the amounts of yellow paint and red paint needed to make different amounts of the same shade of orange paint.

Yellow paint (parts)	Red paint (parts)
5	6

   b Explain how you know that these amounts of yellow paint and red paint will make the same shade of orange paint.

2. A car travels at a constant speed and its distance traveled in 1, 2, and 3 hours is shown in the table. How far does the car travel in 12 hours? Explain or show your thinking.

Time (hours)	Distance (km)
1	70
2	140
3	210

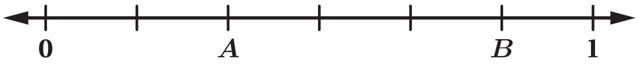
Name: .... Date: .... Period: ....

3. In a recipe for scones, there is 1 cup of milk for every 3 cups of flour. A baker needs to make 5 batches of scones. Determine how much of each ingredient the baker will need. Consider using a table to help with your thinking.

   a How many cups of milk are needed to make 5 batches of scones?

   b How many cups of flour are needed to make 5 batches of scones?

4. The tick marks on the number line are equally-spaced apart. Write fractions to represent the values of locations A and B on the number line.



5. Noah's recipe for sparkling orange juice uses 4 liters of orange juice and 5 liters of soda water. Determine *two* more equivalent ratios of liters of orange juice to liters of soda water that would make sparkling orange juice that tastes the same as Noah's recipe.

6. List *all* the factors of 20.

Unit 2 | Lesson 8

Reasoning With Multiplication and Division

Let's use multiplication and division to go from a given number to any other number.

Warm-up Problem Strings

Mentally determine each product and quotient.

Expression	Product
7 • 3	
7 • 6	
7 • 30	
7 • 15	
7 • 18	
14 • 12	
14 • 48 • 25	

Expression	Quotient
210 ÷ 21	
210 ÷ 42	
1,260 ÷ 84	
1,260 ÷ 7	
630 ÷ 35	
210 ÷ 35	
210 ÷ 7 ÷ 5 ÷ 3	

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 Divide and Multiply, Multiply and Divide

Write a sequence of operations that connects each starting number to the corresponding target number by using only multiplication and division. An example is provided in the first row of the table.

Starting number	Sequence of operations	Target number
6	$\div 3 \cdot 2$	4
12		20
60		50
24		9
5		8

Are you ready for more?

Write a sequence of operations that connects the starting number $\frac{2}{3}$ to the target number $\frac{1}{4}$ by using only multiplication and division.

Activity 2 Two Operations, One Operation

For Problems 1 and 2, refer to the table that shows several starting numbers and their corresponding target numbers.

1. Write a sequence of *exactly two operations* that connects each starting number to the corresponding target number in the table by using only multiplication and division.

2. Write *one operation*, by using either multiplication or division, that connects each starting number to the corresponding target number in the table.

Starting number	Two operations	One operation	Target number
5			4
12			17
123			987
848			484

Are you ready for more?

Think about any two numbers, calling the starting number a and the target number b.

1. Write an expression representing a sequence of two multiplication and division operations that connects a to b.

2. Write one multiplication or division operation that also connects a to b.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You revisited important relationships between multiplication and division and how they relate to fractions. You reasoned that when completing one division step and one multiplication step they can be completed in either order, or even be rewritten using one operation.

You applied this understanding to reason about how to apply division and multiplication to connect two values. You determined that to get from the first value to the second value you can divide by the first value then multiply by the second value. You can simplify this into a single multiplication expression using the relationship between division and fractions.

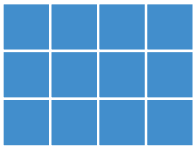
Starting value	Two operations	One operation	Target value
9	$\div 9 \cdot 23$	$\cdot \frac{9}{23}$	23

Reflect:

Practice

Name: .... Date: .... Period: ....

1. This area model can be used to represent the product $3 \cdot 4$ and also the quotient $12 \div 4$.



a How could the model be adjusted to represent the product $6 \cdot 4$?

b How could the model be adjusted to represent the quotient $6 \div 4$?

2. Write an expression to represent each statement.

a Multiply 11 by the quotient of 5 and 6.

b Divide 20 by the product of 3 and 7.

3. Determine whether each expression is greater than, less than, or equal to $\frac{4}{7}$. Explain or show your thinking.

a $\frac{9}{10} \cdot \frac{4}{7}$

b $\frac{4}{7} \cdot \frac{10}{9}$

c $\frac{8}{14}$

Name: ______________________ Date: __________ Period: __________

4. The surface area of a cube is equal to $\frac{90}{41}$ cm^2. What is the area of one face of the cube? Explain or show your thinking.

5. Complete the table to determine at least two equivalent ratios to $42 : 30$ with lesser values and at least two equivalent ratios with greater values.

42	30

6. Label each number as *prime* or *composite*. If the number is *composite*, list as many factors as you can. If the number is *prime*, explain your thinking.

a 24

b 31

c 93

d 2

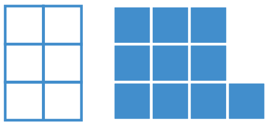
Unit 2 | Lesson 9

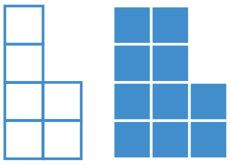
Common Factors

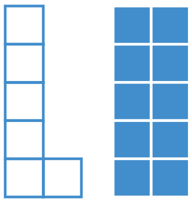
Let's use factors to solve problems.

Warm-up Figures Made of Squares

Study these four pairs of figures. How are the pairs of figures similar? How are they different?







Similarities	Differences

Name: .... Date: .... Period: ....

Activity 1 Percussion Camp

The percussion section of a university marching band consists of 12 snares, 10 cymbals, 8 bass drums, 4 timpani drums, and 4 tenor drums (also called quads). During summer practice, smaller groups will break off to rehearse.

1. How could the snares and bass drums be grouped so there is the same number of each instrument in every group?

2. How could the cymbals and timpani drums be grouped so that there is the same number of each instrument in every group?

3. Could the entire percussion section be placed into smaller groups so that each group includes the same number of each instrument? If so, how?

Are you ready for more?

Several percussion sections are getting together to practice a song for a parade. There are 24 gong players and 16 triangle players.

What is the greatest number of smaller groups that they could be arranged into where each group has the same number of gong players and the same number of triangle players?

Activity 2 Greatest Common Factor

Not all musicians think about their music as being related to math, even though it likely is. Jazz drummer Clayton Cameron on the other hand once heard his music referred to as "some beautiful numbers" (as in musical numbers) and ever since he has not stopped thinking about that relationship. He has even coined the term *a-rhythm-etic* to describe the "cycles and groupings of numbers and how they feel." Musical cycles are closely related to greatest common factor.

1. The "greatest common factor" of 30 and 18 is 6. What do you think the term *greatest common factor* means?

2. Determine *all* of the common factors of 21 and 6. Then identify the greatest common factor.

3. Determine the greatest common factor of each pair of numbers.

a 28 and 12

b 35 and 96

Featured Mathematician

Clayton Cameron

Clayton Cameron is a native of Los Angeles and is a lecturer on Global Jazz Studies at the UCLA Herb Alpert School of Music. After receiving a degree in music from California State University at Northridge, Cameron became a rising star in the music industry, performing as a percussionist with countless award-winning acts. He is particularly known for perfecting "the art of the brush technique," which he did by treating it more as a science of numbers.

Name: .... Date: .... Period: ....

Activity 2 Greatest Common Factor (continued)

4. A small rectangular bulletin board is 12 in. tall and 27 in. wide. Elena plans to cover it with squares of colored paper that are all the same size. The paper squares come in different sizes; all of them have whole-number inches for their side lengths.

a What is the side length of the largest square that Elena could use to cover the bulletin board completely without gaps and overlaps? Explain or show your thinking.

b How is the solution to this problem related to the greatest common factor?

Are you ready for more?

A school has 1,000 lockers, all lined up in a hallway. Each locker is closed. Then . . .

- One student goes down the hall and opens each locker.
- A second student goes down the hall and closes every second locker: lockers 2, 4, 6, and so on.
- A third student goes down the hall and changes every third locker. If a locker is open, he closes it. If a locker is closed, he opens it.
- A fourth student goes down the hall and changes every fourth locker.

This process continues up to the thousandth student! At the end of the process, which lockers will be open?

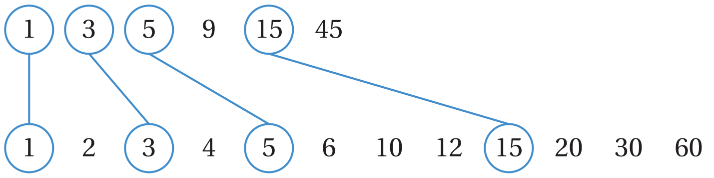
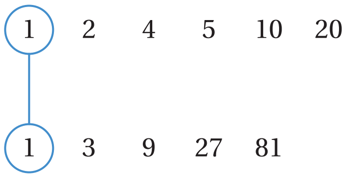
STOP

Summary

In today's lesson . . .

You reviewed that a *factor* of a whole number is another whole number that divides into the given number evenly (with no remainder). Given any two whole numbers, you reasoned that you could determine their ***common factors*** and their ***greatest common factor*** (GCF).

Two whole numbers can have one or many common factors, but will only ever have one GCF.

Numbers	Factors	Greatest common factor
45 60	 1 3 5 9 15 45 1 2 3 4 5 6 10 12 15 20 30 60	15
20 81	 1 2 4 5 10 20 1 3 9 27 81	1

Reflect:

Name: .... Date: .... Period: ....

1. In your own words, what does the term *greatest common factor* mean? Describe a process for determining the greatest common factor of two numbers.

2. A teacher is making gift bags. Each bag is to be filled with pencils and stickers. The teacher has 24 pencils and 36 stickers to use. Each bag will have the same number of each item, with no items left over. For example, she could make 2 bags with 12 pencils and 18 stickers each. What are some other possibilities? Explain or show your thinking.

3. A school chorus has 90 sixth-grade students and 75 seventh-grade students. The music director wants to make groups of performers, with the same combination of sixth- and seventh-grade students in each group. She wants to form as many groups as possible.

   a What is the greatest number of groups that could be formed? Explain or show your thinking.

   b Using your answer from Problem 3a, how many students of each grade would be in each group?

Practice

Name: .... Date: .... Period: ....

4. Complete each statement about a class that has 4 boys for every 3 girls.

a The ratio of boys to girls is .... to ....

b The ratio of girls to boys is .... to ....

c For every .... boys there are .... girls.

d The ratio of girls to boys is .... : ....

5. Clare makes purple paint by mixing 6 tbsp of blue paint and 4 tbsp of red paint. Which of these mixtures produces the same purple paint as Clare's mixture? Select all that apply.

A. Mix tablespoons of blue paint and red paint in the ratio of $3 : 2$.

B. For every 3 tbsp of red paint, mix 2 tbsp of blue paint.

C. Mix tablespoons of blue paint and red paint in the ratio of $9 : 6$.

D. For every 2 tbsp of red paint, mix 3 tbsp of blue paint.

E. Mix 7 tbsp of blue paint and 5 tbsp of red paint.

6. List all of the multiples of 5 less than or equal to 50.

Name: .... Date: .... Period: ....

Unit 2 | Lesson 10

Common Multiples

Let's use multiples to solve problems.

Warm-up Keeping a Steady Beat

Part 1

You will be given instructions for making a rhythm as a class. As you play your part, think about this question.

1. When will the two sounds happen at the same time?

Part 2

You will be given new instructions for making a different rhythm. As you play your part, think about these questions.

2. When will the two sounds happen at the same time?

3. What would happen if you kept on playing the rhythm?

Log in to Amplify Math to complete this lesson online.

Activity 1 A New Rhythm

Plan ahead: What choices will you make to control your impulses as you participate in making a rhythm?

Part 1

You will be given instructions for another new rhythm. As you play your part, think about these questions.

1. When will the three sounds happen at the same time (up to 36)?

2. When is the first time that the two sounds happen at the same time?

Part 2

Let's explore common multiples some more.

Suppose in a new rhythm, you clap every 4 beats, stomp every 8 beats, and say "yeah" every 12 beats.

3. When will the three sounds happen at the same time (up to 48 beats)?

4. When is the first time that the two sounds happen at the same time?

5. Explain the patterns in the beats where multiple sounds are happening at the same time using the language of multiples and common multiples.

Name: ________ Date: ________ Period: ________

Activity 2 Least Common Multiple

1. The “least common multiple” of 6 and 8 is 24. What do you think the term *least common multiple* means?

2. Determine *all* the multiples of 10 and 8 that are less than 100. Then identify the least common multiple.

3. What is the least common multiple of 7 and 9?

Are you ready for more?

1. For two given numbers, can the least common multiple be one of the two numbers? Show or explain your thinking, and provide an example.

2. Can the greatest common factor be one of the two numbers? Show or explain your thinking, and provide an example.

3. Can the least common multiple and the greatest common factor of two different numbers be the same? Show or explain your thinking, and provide an example.

STOP

Summary

In today's lesson . . .

You reviewed that a *multiple* of a whole number is the product of that whole number and another whole number. Given any two whole numbers, you reasoned that you could determine their ***common multiples*** and their ***least common multiple*** (LCM).

Two whole numbers have infinite common multiples, but will only ever have one LCM.

Numbers	Multiples	Least common multiple
4	4 8 ⑫ 16 20 ㉔ 28 32 ㊱ …	12
6	6 ⑫ 18 ㉔ 30 ㊱ 42 48 54 …	
2	2 4 6 ⑧ 10 12 14 ⑯ 18 …	8
8	⑧ ⑯ 24 32 40 48 56 48 54 …	

Reflect:

Name: ____________ Date: ________ Period: ________

Practice

1. Consider different-colored lights that each blink at certain intervals of seconds.

   a A green light blinks every 4 seconds and a yellow light blinks every 5 seconds. When will both lights blink at the same time?

   b A red light blinks every 12 seconds and a blue light blinks every 9 seconds. When will both lights blink at the same time?

   c Explain how to determine when any two lights will blink at the same time.

2. Think about the multiples of 10 and 15.

   a List *all* the multiples of 10, up to 100.

   b List *all* the multiples of 15, up to 100.

   c What is the least common multiple of 10 and 15?

3. At a local store, cups are sold in packages of 8. Napkins are sold in packages of 12.

   a What is the fewest number of packages of cups and the fewest number of packages of napkins that can be purchased so there will be the same number of cups as napkins?

   b How many sets of individual cups and napkins will there be?

Name: .... Date: .... Period: ....

4. One batch of light green paint uses 2 cups of green paint and 7 cups of white paint. Jada made a large amount of light green paint by using 10 cups of green paint.

a The amount of light green paint she made is equivalent to how many batches?

b How many cups of white paint did she use?

5. What are *three* different ratios that are equivalent to the ratio $3 : 12$? Explain how you know your ratios are equivalent.

6. Think about the number 12.

a What are *all* the factors of 12?

b What are some multiples of 12?

Name: ______________________ Date: __________ Period: __________

Unit 2 | Lesson 11

Navigating a Table of Equivalent Ratios

Let’s use a table of equivalent ratios.

Warm-up Number Talk

Mentally determine each product.

1. $\frac{1}{3} \cdot 21$

2. $\frac{1}{6} \cdot 21$

3. $5.6 \cdot \frac{1}{8}$

4. $\frac{1}{4} \cdot 5.6$

Log in to Amplify Math to complete this lesson online.

Activity 1 Concert Ticket Prices

Noah's and Jada's families purchased tickets to a Chicago Sinfonietta concert. Complete the table to help with your thinking as you complete the problems.

Number of tickets	Price ($)

1. Noah bought 4 tickets and paid $\$103$. What was the cost per ticket? Show or explain your thinking.

2. Jada's family bought 9 tickets for her family to attend and paid $\$231.75$. Did Jada and Noah pay the same price per ticket? If not, who paid more? Show or explain your thinking.

3. How much would Jada have spent in total if she bought 1 more ticket for the same price?

Name: .... Date: .... Period: ....

Activity 2 Chicago Deep-Dish Pizza

After attending the concert, Jada's family heads to a local restaurant to enjoy some famous Chicago deep dish pizza. The crust for an extra large pizza uses 12 tbsp of cornmeal and 16 tbsp of butter. Noah's family is going to make their own deep dish pizza at home, but they don't need nearly as large of a pie.

1. How many tablespoons of cornmeal and butter are needed for the smallest pizza with a crust that has the same consistency and tastes the same, but that can be made using whole numbers of tablespoons of each ingredient? Consider completing the table to help organize your thinking.

Cornmeal (tbsp)	Butter (tbsp)

2. How many other sizes of pizza that are smaller than the restaurant's extra large size can be made using only whole tablespoons of both of those ingredients? What are the ratios of cornmeal to butter?

3. What are the two ratios containing a 1 of cornmeal and butter in the crust recipe?

a Ratio of cornmeal to butter for 1 tbsp cornmeal:

b Ratio of cornmeal to butter for 1 tbsp butter:

STOP

Summary

In today's lesson . . .

You used a ratio table to determine some special equivalent ratios that you have seen before, but now in the context of different scenarios.

One such type of special ratio is when the value for one of the two quantities is equal to 1. These ratios tell you the exact amount of a quantity that corresponds to precisely 1 unit of another quantity. You can see in the Granola-to-Price table that there is a ratio for the cost of 1 pound of granola or the amount of granola for \$1. This is often read as the "price *per* pound," or the "unit price," because the word *per* means "for each," or, more specifically, "for each 1."

Another type of special ratio is when the values share a common factor. An equivalent ratio can always be determined by dividing both quantities by the greatest common factor of the numbers in any equivalent ratio. From the table, 16 : 20 shares the factor of 4.

These types of special ratios are useful for generating equivalent ratios in a ratio table.

	Granola (lb)	Price (\$)	
$\div 4$	16	20.00	$\div 4$
$\times \frac{1}{5}$	4	5.00	$\times \frac{1}{5}$
	1	1.25	
$\times 3$	0.8	1.00	$\times 3$
	2.4	3.00	
	62	77.50	

Reflect:

Name: .... Date: .... Period: ....

1. Kiran reads 22 pages in 44 minutes. He spends the same amount of time per page. Consider using the table to help with your thinking as you solve each of the following problems.

   a How long does it take Kiran to read 1 page?

   b How many pages can he read in 1 minute?

Time in minutes	Number of pages
44	22

2. Mai is making personal pizzas. For 4 pizzas, she uses 10 oz of cheese.

   a How much cheese does Mai use for each pizza?

   b At this same rate, how much cheese will she need to make 10 pizzas?

Number of pizzas	Ounces of cheese
4	10

3. A triple batch recipe of peanut butter granola bars contains 3 cups of peanut butter and 6 cups of oats. How much peanut butter and oats are in one batch of granola bars? Explain or show your thinking.

Name: ________________ Date: ________ Period: ________

4. Each of these is a pair of equivalent ratios. For each pair, explain how you know they are equivalent ratios.

   a. $600 : 450$ and $60 : 45$

   b. $60 : 45$ and $4 : 3$

   c. $600 : 450$ and $4 : 3$

5. Complete the table to show the amounts of flour and milk needed for a pancake recipe in 3 different-sized batches.

Flour (cups)	Milk (cups)
4	2

6. Plot the following numbers on the number line: $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{3}{6}$. Explain how you determined where to plot them.

Name: ________________ Date: ________ Period: ________

Unit 2 | Lesson 12

Tables and Double Number Line Diagrams

Let’s use double number lines to represent equivalent ratios.

Warm-up Constant Dividend

1. Mentally determine each quotient.

   a $150 \div 2$

   b $150 \div 4$

   c $150 \div 8$

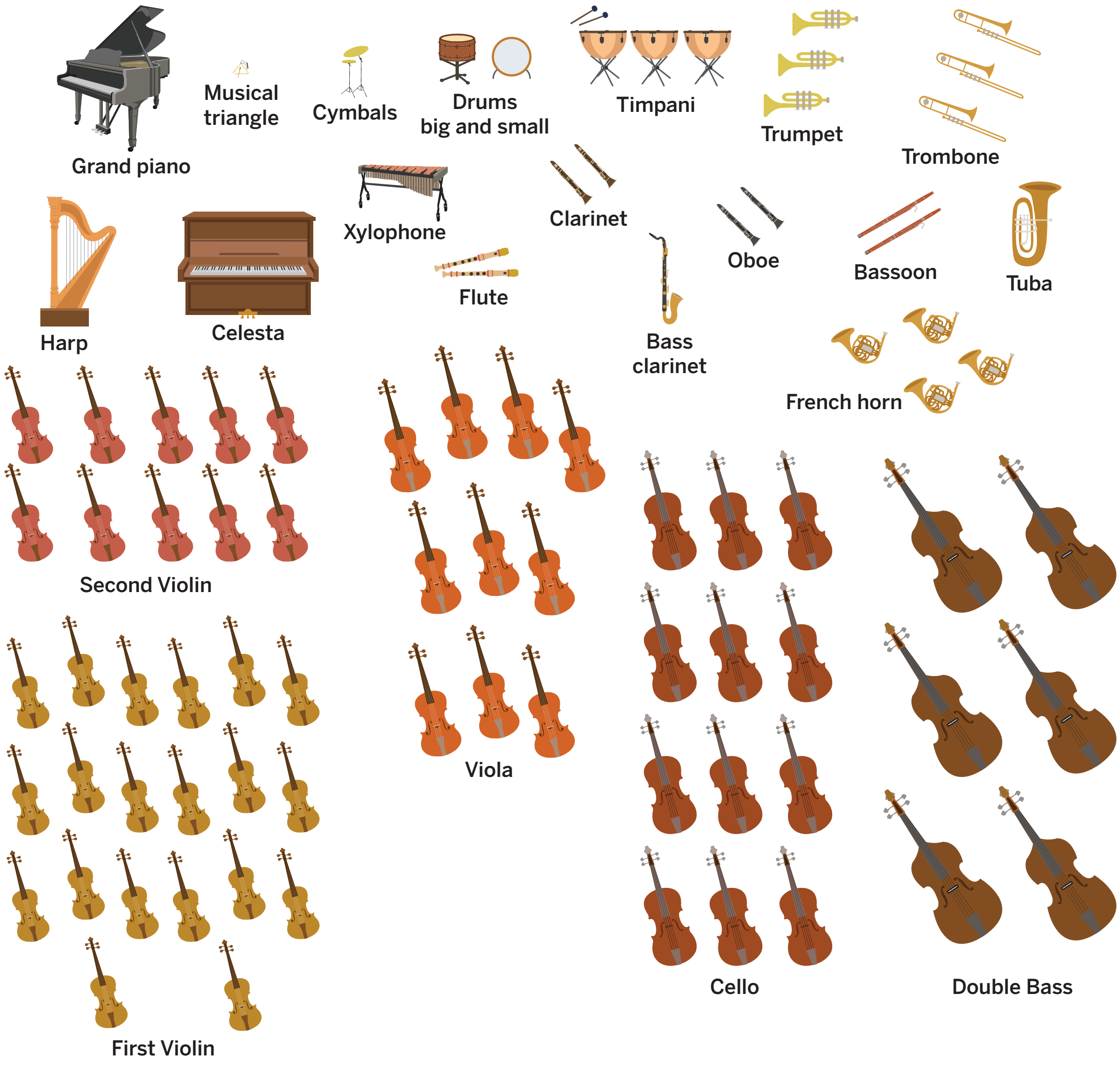
2. Locate and label *all* of the quotients from Problem 1 on this number line.

Log in to Amplify Math to complete this lesson online.

Activity 1 A Larger Orchestra

In 2019, a new Guinness World Record for the largest orchestra was set when 8,097 musicians came together in Saint Petersburg, Russia. When multiple orchestras get together to create a larger orchestra, they want to keep the ratios of instruments equivalent, so the overall sound is balanced in the same ways.

Most orchestras consist of at least 90 instruments, and this image shows the number of each type of instrument in an example orchestra.



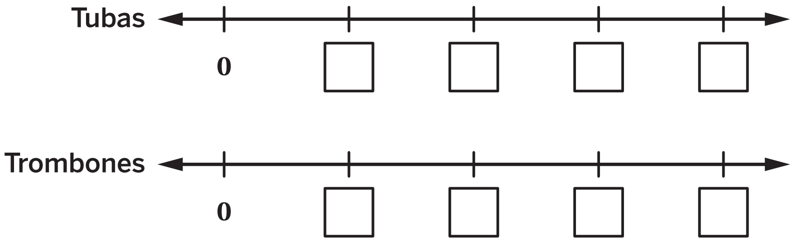
Name: .... Date: .... Period: ....

Activity 1 A Larger Orchestra (continued)

Suppose you need to organize larger orchestras that have the same balance of sound by keeping the ratios of instruments equivalent to those in a typical orchestra.

1. Consider the balance between the tubas and trombones.

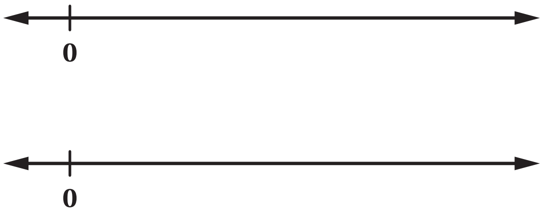
a Complete the double number line to show possible numbers of tubas and trombones in different-sized orchestras.



b Choose two of your ratios and explain how you know they are equivalent.

2. Consider the balance between the French horns and double basses.

a Complete the double number line to show possible numbers of French horns and double basses in different-sized orchestras.



b Choose two of your ratios and explain how you know they are equivalent.

c For each ratio you chose in part b, determine how many total instruments would be in each of those orchestras. Explain or show your thinking.

Activity 2 Tables and Double Number Lines

You want to create other possible numbers of these instruments in larger orchestras, making sure that the ratios are equivalent so that the overall sound stays balanced and sounds the same.

1. Choose two pairs of two different instruments from a typical orchestra.

Instrument pair 1: .... and ....

Instrument pair 2: .... and ....

2. You will create a double number line for one pair of instruments and a table for the other. Your partner will create the opposite representations for the opposite pairs of instruments.

a Create your double number line and table here. Label each representation clearly to show which pair of instruments corresponds to each.

Name: ________________ Date: ________ Period: ________

Activity 2 Tables and Double Number Lines (continued)

b Discuss both representations for both pairs of instruments with your partner. Once you agree on all of the information being presented, modify your representations as needed and copy your partner's representations here.

3. Compare and contrast the pros and cons of using a table versus a double number line to determine and represent equivalent ratios.

Historical Moment

Friendly Numbers

Pairs of related numbers have all sorts of fun names! There are amicable numbers (identified even earlier than 850 CE, and notably worked on by the Iraqi mathematician Thābit ibn Qurra). There are friendly numbers — the ratio of the sum of the factors of each number to itself, called its abundancy index, is the same for both numbers. And then of course there are solitary numbers — basically, not friendly numbers, meaning no other number shares its ratio.

For some numbers, it is not yet known whether they are friendly or solitary. As of 2021, the smallest, and arguably craziest, example is 10. A group of students at Clarkson University published a paper in 2006 that partly proved what must and must not be true about a friend of 10, if it exists. No computer program has found a friend of 10 just yet, but we know any friend must have at least 31 digits!

1. Show that 6 and 28 are a friendly pair.

2. What is the abundancy index of 10, written as a ratio?

STOP

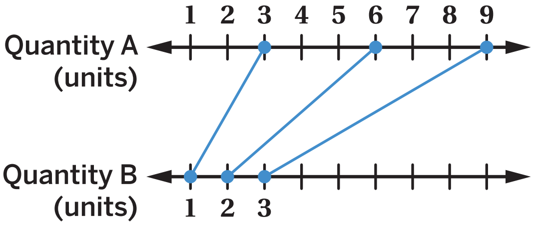
Summary

In today's lesson . . .

You saw a new way to represent equivalent ratios, using *double number line diagrams*.

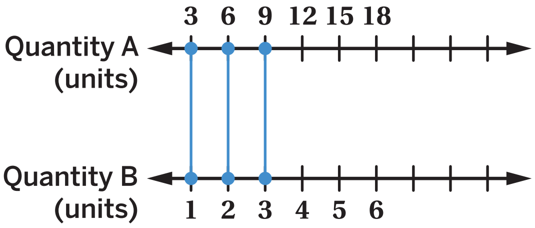
You can choose the scales for the number lines in either of two ways:

Using the *same scale* (such as by 1s).



This is helpful for seeing how much more of one quantity there is than the other, and how the pattern grows.

Using *different scales* (such as by 1s and 3s).



This is helpful for identifying equivalent ratios at a glance.

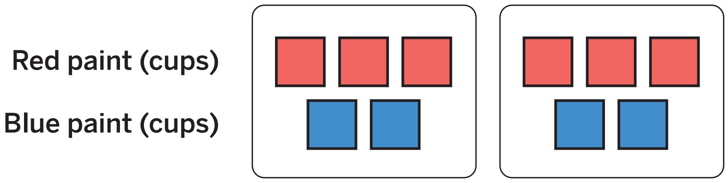
Double number lines and tables are two different representations that can both be used to help generate and identify equivalent ratios. In both representations, you should include labels and units for each quantity. On a double number line, the numbers are always listed in order. In a table, you can write the equivalent ratios in any order.

Reflect:

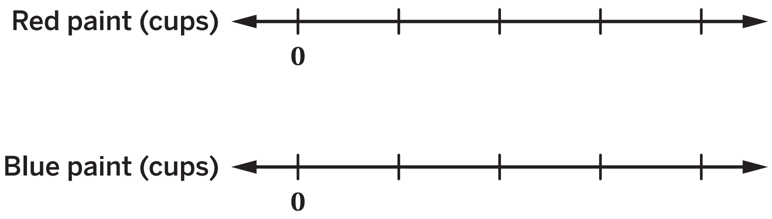
Name: ________________ Date: ________ Period: ________

1. In an orchestra, the ratio of clarinets to cello is $2 : 12$. Multiple orchestras are planning to combine to create a larger orchestra and they want to keep the ratio of instruments equivalent so the sound is balanced the same. Create a table to show how many instruments would be needed in three other possible sizes of orchestras.

2. The diagram shows the amounts of red and blue paint that make 2 batches of a purple paint.



a Complete the double number line representing the amounts of red and blue needed to make the same purple paint. Label the tick marks to show the different amounts of red and blue paint that can be used to make different total amounts of this same purple paint. Equivalent ratios should be aligned vertically.

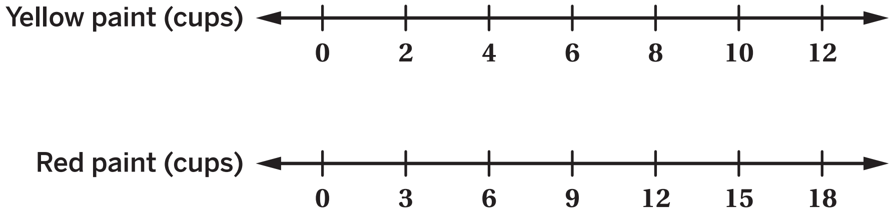


b Making the same shade of purple paint by using 12 cups of red paint is equivalent to making how many batches?

c Making the same shade of purple paint by using 6 cups of blue paint is equivalent to making how many batches?

Name: .... Date: .... Period: ....

3. One batch of a particular orange paint is made with 2 cups of yellow paint for every 3 cups of red paint. On the double number line, circle the numbers of cups of yellow and red paint needed for 3 batches of this same shade of orange paint.



4. Diego estimates that there will need to be 3 pizzas for every 7 kids at his party. Select *all* the statements that represent this ratio.

A. The ratio of kids to pizzas is 7 : 3.

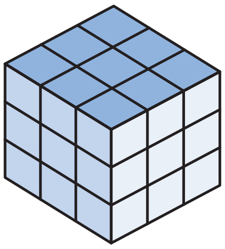
B. The ratio of pizzas to kids is 3 to 7.

C. The ratio of kids to pizzas is 3 : 7.

D. The ratio of pizzas to kids is 7 to 3.

E. For every 7 kids, there needs to be 3 pizzas.

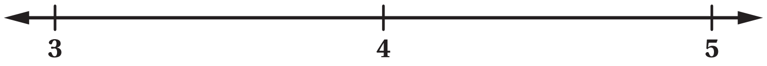
5. In this cube, each small square has a side length of 1 unit.



a What is the surface area of this cube?

b What is the volume of this cube?

6. Plot $3\frac{1}{4}$ and 4.75 at their correct locations on the number line.



Name: .... Date: .... Period: ....

Unit 2 | Lesson 13

Tempo and Double Number Lines

Let's look at song tempos and draw more double number line diagrams.

Warm-up Ordering on a Number Line

1. Locate and label the following numbers on the number line.

$\frac{1}{2}$	$1\frac{3}{4}$	$\frac{1}{4}$	1.5	1.75

2. Write one fraction and one decimal that are not equivalent to each other or to any of the numbers plotted on the number line. Plot and label your two numbers on the number line.

Log in to Amplify Math to complete this lesson online.

Activity 1 Song Tempos

Tempo **is the speed or pace at which a song is played. In Western classical music, Italian words are used to describe different tempo markings, which correspond to different ranges of beats per minute (bpm). These tempo markings can also be used to describe how to dance to such a song. Refer to the table of different tempo markings and their corresponding beats per minute.**

Tempo marking	Common definition (bpm)
Prestissimo	Very very fast (> 200 bpm)
Presto	Very fast (169–200 bpm)
Allegro	Fast (121–168 bpm)
Moderato	Moderate (109–120 bpm)
Andante	Walking pace (76–108 bpm)
Adagio	Slow and stately (66–75 bpm)
Lento/Largo	Very slow (41–65 bpm)
Grave (grah•vey)	Slow and solemn (20–40 bpm)

1. Think of two songs that you know. One should be a faster song and one should be a slower song. Mark where you think their tempos would be on the line.

2. Assuming all of the songs described are played at the same tempo throughout, determine the tempo marking of each song.

a A 5-minute song containing 750 beats.

b A 5-minute song containing 225 beats.

c A 3-minute song containing 276 beats.

d A 4-minute song containing 460 beats.

Name: .... Date: .... Period: ....

Activity 1 Song Tempos (continued)

3. Choose a tempo marking from the table and number of minutes for the length of a song.

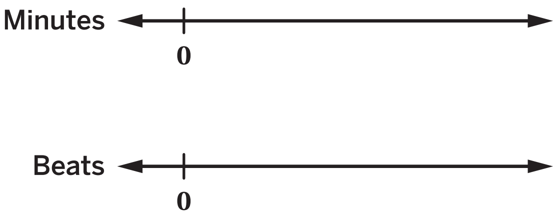
Tempo: ....

Length of song (minutes): ....

a. How many beats per minute could the song have?

b. Using your answer from Problem 3a, how many total beats would the song have?

c. Complete the double number line to show the beats for each passing minute.



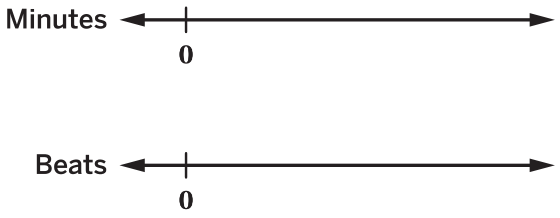
Stronger and Clearer: After you complete Problem 3, your teacher will provide you some time to work with your partner to clarify and revise your thinking.

Activity 2 Faster and Slower Tempos

1. Frederick Chopin's Waltz no. 10 in B minor is played at a moderato tempo.

   a What could be a possible number of beats per minute for this song?

   b The song is 3 minutes and 30 seconds long. Complete the double number line to show the number of beats for *each passing minute*.



2. Choose another song from the list with a different tempo than in Problem 1.

   a Write your chosen song title here.

   b What could be a possible number of beats per minute for this song?

   c Create a double number line showing the number of beats for *each passing 30 seconds* of the song (or up to 5 minutes).

3. Which song is being played at a faster tempo? How do your double number lines for Problems 1 and 2 show this?

STOP

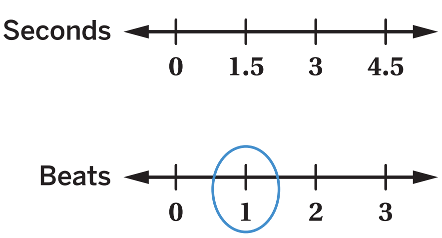
Name: .... Date: .... Period: ....

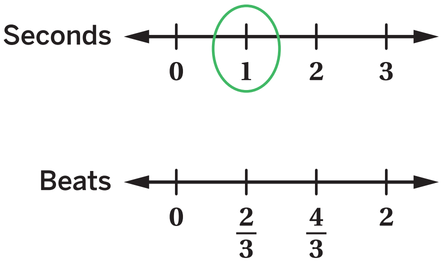
Summary

In today's lesson . . .

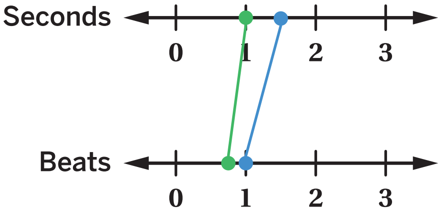
You saw that different songs are played at different tempos, which can be represented as the ratio of beats to seconds.

You can represent the ratio for 1 beat and the ratio for 1 second using two number line diagrams:





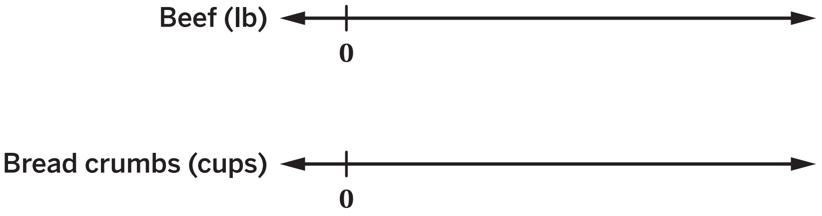
This can also be done with one diagram, using the same scale of 1 on both number lines:



Reflect:

Name: .... Date: .... Period: ....

1. One batch of meatloaf contains 2 lb of beef and $\frac{1}{2}$ cup of breadcrumbs. Complete the double number line to show the amounts of beef and breadcrumbs needed for 1, 2, 3, and 4 batches of meatloaf.



2. The song *Perfect* by Ed Sheeran is 4 minutes and 23 seconds long and is played at a lento/largo 63 bpm. Create a double number line to show the number of beats for each passing minute up to 4 minutes.

3. A recipe for tropical fruit punch says, “Combine 4 cups of pineapple juice with 5 cups of orange juice.”

   a Create a double number to show the amount of each type of juice in 1, 2, 3, and 4 batches of the recipe.

   b The recipe also calls for $\frac{1}{3}$ cups of lime juice for every 5 cups of orange juice. Add a third number line to your diagram to represent the amount of lime juice in each number of batches of tropical fruit punch.

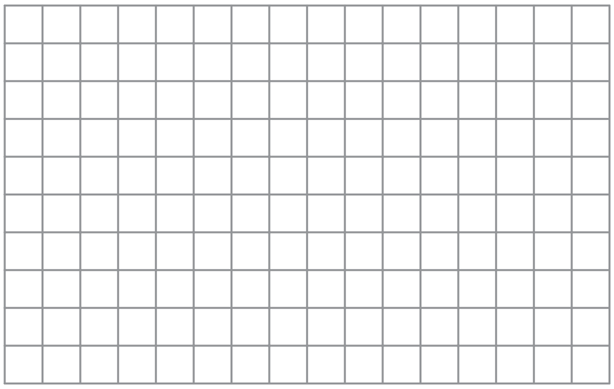
   c If 12 cups of pineapple juice are used with 20 cups of orange juice, will the recipe taste the same? Explain your reasoning.

Name: ________________ Date: ________ Period: ________

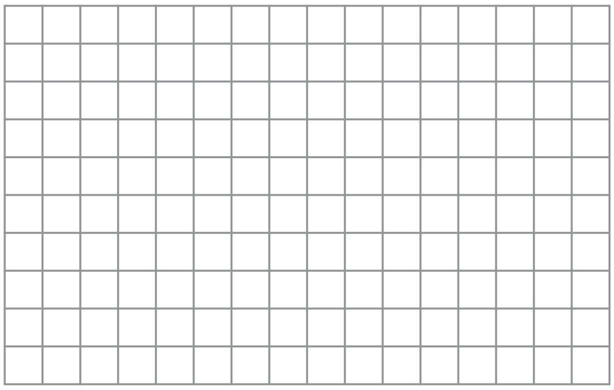
4. Determine three different ratios that are equivalent to $3 : 11$. Explain how you know all of your ratios are equivalent.

5. Draw each of the indicated figures using the grids shown.

a Draw a parallelogram that has an area of 24 square units, but is not a rectangle. Explain or show how you know the area is 24 square units.



b Draw a triangle that has an area of 24 square units. Explain or show how you know the area is 24 square units.



6. Noah bought 7 boxes of pasta. Each box was 12 in. tall. How many pounds of pasta did Noah buy in all?

a What given information do you need to use to solve the problem?

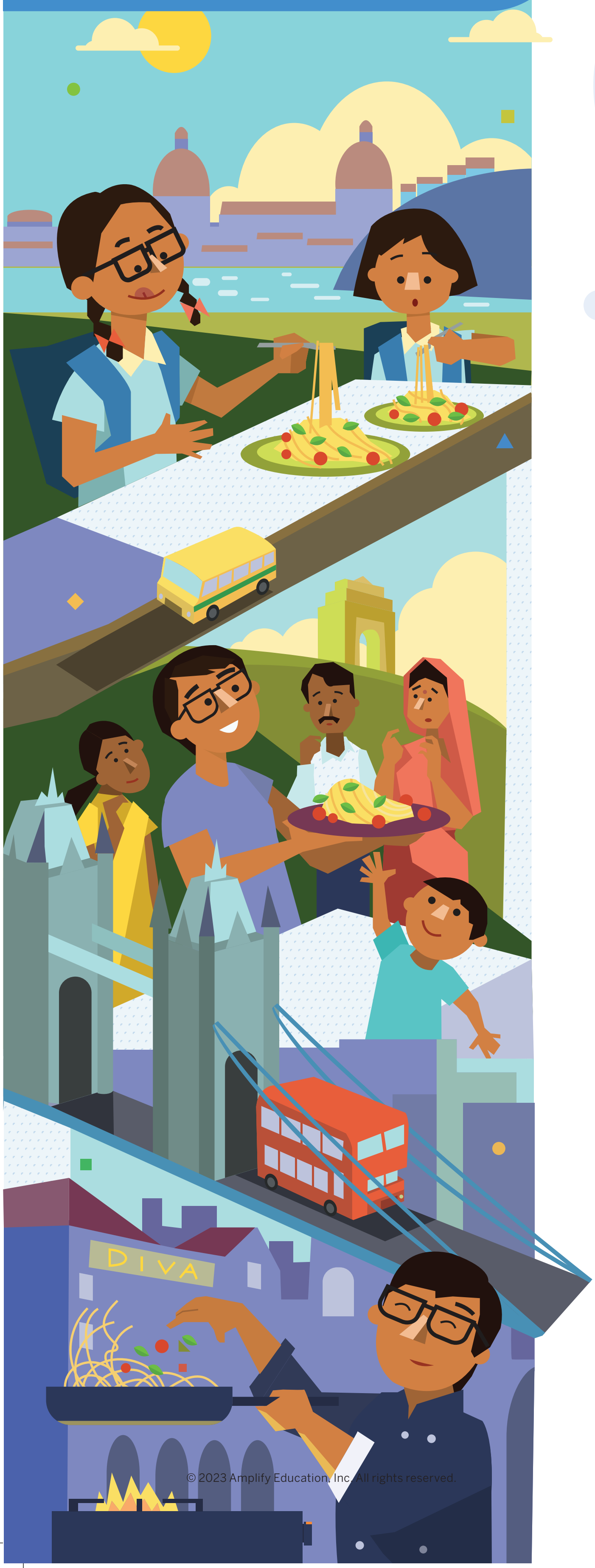
b What given information do you *not* need to use to solve the problem?

c What information is missing that you would need to know in order to solve the problem?

My Notes:

SUB-UNIT

3 Solving Ratio Problems



Narrative Connections

Who brought Italy to India and back again?

In the 1980s, Italian cuisine was rare in Kolkata, India. And yet, for 10-year-old Ritu Dalmia, there was nothing better. She had gotten a taste for it after a school trip to Italy. For a month, she and her classmates sampled dishes like spaghetti pomodoro. For Dalmia, it was love-at-first-taste.

This love would start her on a journey many decades long, spanning multiple countries.

She opened MezzaLuna, one of Delhi's first Italian restaurants. Two years later, Dalmia headed to London to open Vama, a successful, high-end Indian restaurant. Five years after that, she returned to India to open another Italian restaurant — Diva. Diva was so successful that offshoots sprouted up, including Diva Cafe, DIVA Piccola, and Latitude 28. Not one to rest on her laurels, Dalmia returned to the source — Italy — to open Cittamani. This exciting new restaurant fused Indian cuisine with Italian ingredients.

Dalmia's passion has brought new tastes and flavors to those who might not otherwise have the opportunity to try them. Whether you're a home cook or a globe-hopping celebrity chef, the right ingredients in the right amounts are important to executing a meal. But to get the recipe exactly right, ratios are the key ingredient!

Solving Equivalent Ratio Problems

Let's practice identifying needed information to solve ratio problems.

Warm-up What Do You Want to Know?

You know that a red car and a blue car both entered the same highway at the same time and both have been traveling at constant speeds. You want to know how far apart they are after 4 hours.

What information would you need to know in order to determine how far apart they are after 4 hours? Be prepared to explain *why* you need that information.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Info Gap: Selling Hot Chocolate

Jada and Noah are going to sell hot chocolate in the cafeteria during lunch. Noah will make the hot chocolate, and Jada will make the signs to advertise their new business.

You will receive either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given the *problem card*:	If you are given the *data card*:
1. Silently read your card and think about what information you need to be able to solve the problem.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner "What specific information do you need?" and wait for them to *ask* for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask "Why do you need that information?" Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently, using a representation of your choice.	**4.** Read the *problem card* and solve the problem independently, using a representation of your choice.
5. Read the *data card*. Share your representation, and discuss your thinking.	**5.** Share the *data card* and your representation, and discuss your thinking.

Are you ready for more?

Noah accidentally added 5 tbsp of cocoa to the hot chocolate mix, and the ratio of cocoa to milk is now $9 : 11$. How many tablespoons of cocoa and cups of milk were in the original mix?

STOP

Summary

In today's lesson . . .

You solved problems involving equivalent ratios by using three given pieces of information:

- Two values that allow you to write a ratio describing the relationship between the two quantities involved.
- A third value that gives a different amount of one of the quantities, which indicates you are interested in determining a corresponding fourth value to make an equivalent ratio.

Suppose you wanted to determine the missing value in the given ratio table, there are multiple methods to consider:

	Time	Length	
	5	2	
× 8	?	16	× 8

To go from 2 to 16 multiply by 8.

$$5 \cdot 8 = ?$$

The missing value is 40.

Time	Length
5	2
?	16

To go from 2 to 5 multiply by $\frac{5}{2}$.

$$16 \cdot \frac{5}{2} = ?$$

The missing value is 40

Reflect:

Name: .... Date: .... Period: ....

1. A chef is making pickles. He needs 15 gallons of vinegar. The store sells 2 gallons of vinegar for \$3.00, but allows customers to buy any amount of vinegar. Which of the following ratios correctly represents the price of the vinegar? Select *all* that apply.

A. 4 gallons to \$3.00

B. 1 gallon to \$1.50

C. 30 gallons to \$45.00

D. \$2.00 to 30 gallons

E. \$1.00 to $\frac{2}{3}$ gallons

2. A caterer needs to buy 21 lb of pasta for a wedding. A local store sells handmade pasta by the pound. It costs \$12 for 8 lb of pasta. Consider this question: If all pasta is sold at this same rate, how much will the caterer pay for the pasta they need?

a Write a ratio for the given information about the cost of pasta.

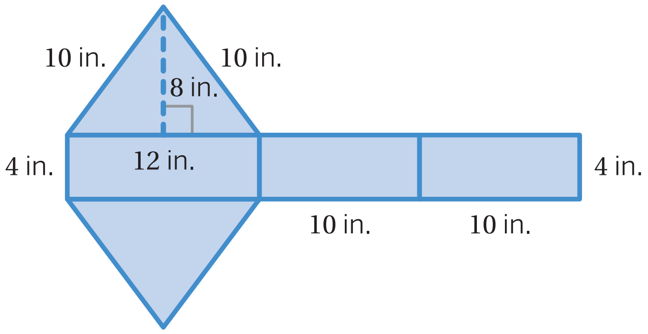
b To answer the question, would it be more helpful to write an equivalent ratio using 1 lb of pasta or \$1? Explain your thinking, and then write that equivalent ratio.

c Calculate the answer to the question and show or explain your thinking.

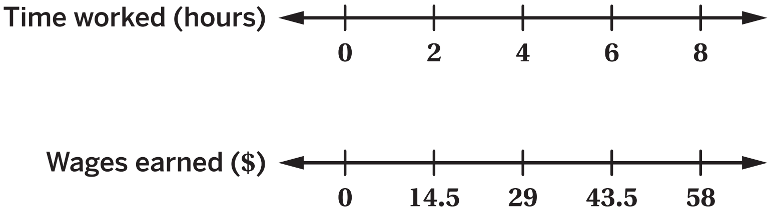
3. Lin is reading a 47-page book. She read the first 20 pages in 35 minutes. If she continues to read at the same rate, will she be able to complete this book in less than 1 hour? Show or explain your thinking.

Name: .... Date: .... Period: ....

4. Determine the surface area of the polyhedron that can be assembled from this net. Show your thinking.



5. A cashier worked an 8-hour day and earned $58.00. The double number line shows the amount she earned for working different numbers of hours. For each question, explain your thinking.



a How much does the cashier earn per hour?

b How much will the cashier earn if she works 3 hours?

6. An art teacher is making three different mixtures of orange paint. Identify the mixture, or mixtures, that satisfy each condition.

- **Mixture A:** 4 ml red paint and 3 ml yellow paint
- **Mixture B:** 4 ml red paint and 2 ml yellow paint
- **Mixture C:** 5 ml red paint and 3 ml yellow paint

a The most amount of paint is made.

b The most amount of yellow paint is used.

c The paint that looks the most yellow.

Name: ________ Date: ________ Period: ________

Unit 2 | Lesson 15

Part-Part-Whole Ratios

Let’s look at situations where you can add the quantities in a ratio together.

Warm-up Sparkling Orange Juice

Do you love fizzy drinks, but want to limit the amount of artificial sugar you drink? Are you bored with plain old juice for breakfast? Try this easy recipe! To make sparkling juice, mix 4 parts juice with 3 parts soda water.

Write as many ratios as you can that involve orange juice, soda water, and total sparkling orange juice. Include units in your ratios, and be prepared to explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Making All-Natural Food Coloring

Did you know that you can create all-natural food coloring by using spices, fruits, and vegetables? Not only can these foods be used to create a beautiful array of colors, they also provide a boost of vitamins and nutrients to any recipe. Note: While spices can be mixed directly with water, fruits and vegetables should first be pressed and juiced to avoid having skins in the final mixture.

1. A recipe for purple food coloring calls for 5 tsp of blueberry juice and 3 tsp of water.

a How many teaspoons of food coloring would this recipe make?

b Shawn needs a batch of 32 tsp of food coloring. How much of each ingredient should Shawn use? Show or explain your thinking.

c How *many times* smaller is the original recipe than Shawn's batch?

2. A red food coloring recipe says, "Mix 4 tbsp raspberry juice with 3 tbsp of strawberry juice and 2 tbsp of water." Kiran wants to make 45 tbsp of red food coloring. He has plenty of water, but he only has 24 tbsp of raspberry juice and 21 tbsp of strawberry juice.

a Does Kiran have enough ingredients to make 45 tbsp of red food coloring? Show or explain your thinking.

Name: .... Date: .... Period: ....

Activity 1 Making All-Natural Food Coloring (continued)

b What is the most food coloring Kiran can make using the ingredients he has? Show or explain your thinking.

Are you ready for more?

Use all of the digits 1 through 9 to create three equivalent ratios. Use each digit only one time.

☐ : ☐ **is equivalent to** ☐ ☐ : ☐ **and** ☐ ☐ : ☐ ☐

Activity 2 Buying Supplies

Han is excited to experiment with flavored sparkling water and natural food colorings. He wants to buy some supplies by using the nickels, dimes, and quarters he has saved up in his piggy bank. For every 2 nickels, there are 3 dimes. For every 2 dimes, there are 5 quarters. There are 500 coins total. How much money does Han have to buy supplies? Show or explain your thinking.

STOP

Name: ____________ Date: ____________ Period: ____________

Summary

In today’s lesson . . .

You worked with ratios that describe a relationship among two or more quantities that have the same units and can be combined (or added together) to make a *total* amount of some other quantity. The total can also be represented in ratios, and you can use equivalent ratios to solve problems with one *or more* unknown quantities.

For example, mixing 3 cups of yellow paint with 2 cups of blue paint produces a total of 5 cups of green paint. If you need to make 15 cups of green paint, you can use the ratio of $3:2:5$ for blue to yellow to green (total) paint to determine how much yellow *and* blue are needed.

$\times 3$

Yellow (cups)	Blue (cups)	Green (cups)
3	2	5
?	?	15

$\times 3$

$3 \cdot 3 = ?$
9 cups of yellow paint

$2 \cdot 3 = ?$
6 cups of blue paint

Ratios can also represent relationships among quantities when the specific units are not known. For example, 3 parts of yellow paint for every 2 parts of blue paint will still produce 5 parts of the same green paint. Any appropriate unit, such as teaspoons or cups or gallons, can be used in place of “parts” without changing the ratio of $3:2$.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. The ratio of cats to dogs at a boarding facility one weekend is $4 : 5$. There are 27 dogs and cats staying for the weekend in all. How many dogs are there? How many cats are there? Show your thinking.

2. Last month, there were 4 sunny days for every rainy day. If there were 30 days in the month, how many days were rainy? Show or explain your thinking.

3. A teacher is planning a class trip to the aquarium. The aquarium requires 2 chaperones for every 15 students. If the teacher orders 85 tickets, how many tickets are for chaperones, and how many are for students? Show or explain your thinking.

Name: .... Date: .... Period: ....

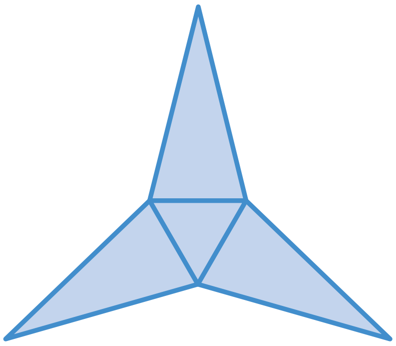
4. In a triple batch of a spice mix, there are 6 tsp of garlic powder and 15 tsp of salt.

a How much garlic powder should be used to make the same spice mix with 5 tsp of salt?

b How much salt should be used to make the same spice mix with 8 tsp of garlic powder?

c If there are 14 tsp of this spice mix, how much salt is in it?

5. Which type of polyhedron can be assembled from this net?



A. Triangular pyramid

B. Trapezoidal prism

C. Rectangular pyramid

D. Triangular prism

6. Usain Bolt is a Jamaican sprinter who won gold medals in three consecutive Olympic Games. His top speed has been measured at 27 miles per hour. Select *all* of the animals whose top speed is *slower* than Usain Bolt's.

A. Elephant: 15 miles per hour

B. Lion: 25 miles per half hour

C. Squirrel: 3 miles per 20 minutes

D. Roadrunner: 5 miles per 15 minutes

Unit 2 | Lesson 16

Comparing Ratios

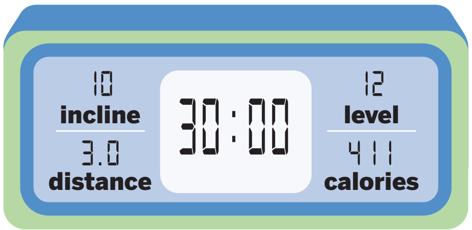
Let's compare ratios.

Warm-up Notice and Wonder

Mai and Jada each completed a run on a treadmill. Their treadmill displays are shown. What do you notice? What do you wonder?

Mai's treadmill display.

Jada's treadmill display.



1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Comparing Chili Peppers

Have you ever taken a bite of a chili pepper and felt like your mouth was on fire? Blame it on the capsaicin (kap·sei·sn), a natural chemical found in most varieties of peppers. The more capsaicin, the spicier the pepper. The level of spiciness is measured on a scale of Scoville Heat Units (SHU). It seems that usually the spicier the pepper, the more expensive it is. In fact, pure capsaicin can measure up to 16,000,000 SHU and can cost as much as $49 for one ounce!

Andre bought ground chili powders of six different kinds of peppers.
He paid:

- $40 for 8 oz of Trinidad Scorpion
- $5 for 4 oz of Jalapeño
- $18 for 2 oz of Carolina Reaper
- $12 for 3 oz of Ghost Pepper
- $20 for 16 oz of Chipotle
- $20 for 10 oz of Habanero

List the six chili powders in order from most to least expensive, and include their unit prices (price per ounce). Show or explain your thinking.

Activity 2 All-Natural Flavoring

You will each design a recipe for your own all-natural flavoring using ingredients from around the world.

- **Your group will be given a set of six flavor cards. Sort the cards into a *sweet* pile and a *sour* pile.**
- **Take turns drawing cards. Each person should select one sweet card and one sour card.**

1. Assign a different number of parts, anywhere from 2 to 20, for each flavor.

a Write the ratio of your ingredients, including the units.

b Describe the flavor of your mix. Be sure to include whether the overall flavor is more sweet or more sour.

2. Compare the sweetness *and* the sourness of the flavor mixtures each member of your group concocted. Show or explain your thinking.

Are you ready for more?

Choose two recipes from your group. How can you make both soda waters taste the same — the same sweetness and sourness? Change as little as possible, and only by adding.

STOP

Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You compared scenarios involving ratios by checking if the scenarios represent something happening at the *same rate*. You created an equivalent ratio for one or both scenarios so that the value (and units) for one quantity in each ratio is the same.

For example, let's compare the price for blue and red paints. 6 liters of red paint costs $8, and 2 liters of blue paint cost $3. Which color paint is more expensive (costs more per liter)?

There are multiple methods to consider:

- Making the number of liters the same and comparing the price.
- Making the price the same and comparing the amount of paint.
- Comparing the price for 1 liter for both paints.

Red Paint

Strategy	Liters	Price ($)
Same liters	6	8
Same price	18	24
Unit price	1	1.33

Blue Paint

Strategy	Liters	Price ($)
Same liters	6	9
Same price	16	24
Unit price	1	1.5

- Price for 6 liters is higher for blue paint than for red paint.
- For the same price of $24 you can buy less blue paint than red paint.
- One liter of blue paint costs more than one liter of red paint.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A slug travels 3 cm in 3 seconds. A snail travels 6 cm in 6 seconds. Both travel at constant speeds. Mai says, “The snail was traveling faster because it went a greater distance.” Do you agree with Mai? Show or explain your thinking.

2. If you blend 2 scoops of chocolate frozen yogurt with 1 cup of milk, you will make a milkshake with a stronger chocolate flavor than if you blended 3 scoops of chocolate frozen yogurt with 2 cups of milk. Show or explain why this is true.

3. There are 2 mixtures of light purple paint.
   - Mixture A is made with 5 cups of purple paint and 2 cups of white paint.
   - Mixture B is made with 15 cups of purple paint and 8 cups of white paint.

   Which mixture is a lighter tint of purple? Explain your thinking.

Name: .... Date: .... Period: ....

4. Diego can type 140 words in 4 minutes.

a At this same rate, how long will it take him to type 385 words?

b At this same rate, how many words can he type in 15 minutes?

5. Andre and Han are moving boxes. Andre can move 4 boxes every half hour. Han can move 5 boxes every half hour. How long will it take Andre and Han to move a total of 72 boxes?

6. Here are two lemonade recipes.

Recipe A: Mix 3 cups of lemon juice with 2 cups of water.

Recipe B: Mix 3 cups of lemon juice with 3 cups of water.

a What fraction of Recipe A is lemon juice?

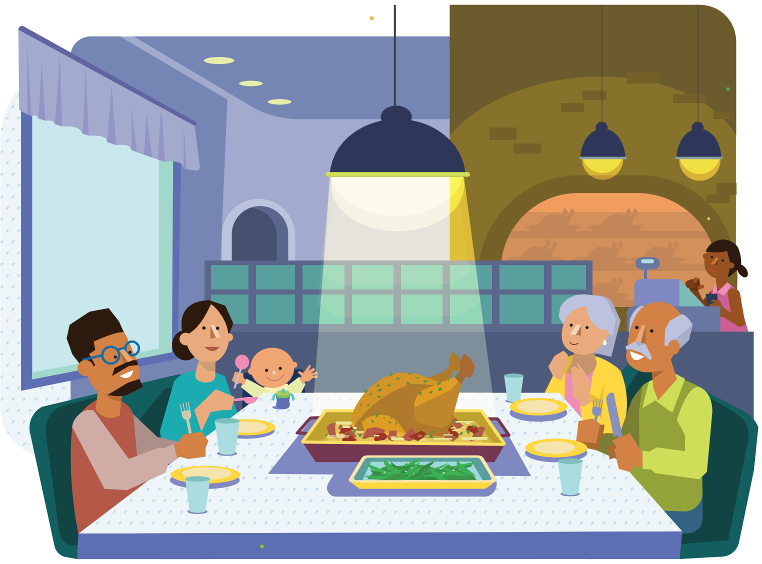
b What fraction of Recipe B is lemon juice?

c Which recipe is more tart (stronger lemon flavor)? Explain your thinking.

Unit 2 | Lesson 17

More Comparing and Solving

Let's practice using ratios
to solve more problems.



Warm-up Is First Always Fastest?

Tyler and Shawn wanted to finish their books today. They both started reading at the same time. Tyler finished the last 5 pages of his book in 10 minutes. Shawn finished the last 10 pages in 15 minutes.

1. Who finished their book first? Be prepared to explain your thinking.

2. Who read the fastest? Be prepared to explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Catering an Event

A local restaurant is catering a large event. The main course is baked chicken with a garlic parmesan sauce, roasted red potatoes, and sautéed green beans. Here is the progress of each sous chef.

- **Lin needed to prepare 108 cups of garlic parmesan sauce. She has made the first 54 cups in 3 hours.**
- **Diego roasted the first 100 cups of red potatoes in 4 hours. He still needs to prepare 45 more cups.**
- **Clare sautéed 160 cups of green beans in 5 hours. She needs to prepare a total of 192 cups.**

1. If each sous chef started at the same time and continues to work at these same constant rates, in what order will the dishes be completed? Show or explain your thinking.

2. Order how quickly the sous chefs worked from fastest to slowest. Explain your thinking.

Fastest **Slowest**

Activity 2 The Bliss Point

Have you ever wondered why you crave certain foods? Well, ratios (and science) can explain that. Foods are most desirable when they hit the *bliss point* — the “just right” ratio of salt to sugar to fat. When combined, these nutrients activate the reward centers in your brain, causing you to want more, more, more!

The test lab of a food company is experimenting with three new flavors of flavored pretzels by altering the salt : sugar : fat ratio in the seasoning mixes.

Recipe	Salt (parts)	Sugar (parts)	Fat (parts)
A	3	2	1
B	3	7	2
C	3	6	1

1. Describe the flavor of each recipe.

2. Order the recipes from most salty to least salty, most sweet to least sweet, and most rich (fat content) to least rich (fat content). Show your thinking.

a

Most salty **Least salty**

b

Most sweet **Least sweet**

c

Most rich **Least rich**

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You expanded on your understanding of equivalent ratios to compare ratios to determine which is happening at a greater or lesser rate.

For example, consider two recipes for sweet and sour sauce using sweet honey and sour pineapple juice to determine which is more sour. In Recipe A the ratio of honey to pineapple is $5 : 11$, in Recipe B the ratio is $9 : 23$

You can compare the amount of sour pineapple juice to the total amount of parts in each recipe.

Recipe A

Pineapple	Total
11	16
22	32

× 2

× 2

Recipe B

Pineapple	Total
23	32

By making the total equivalent in each recipe, you can see that Recipe B is more sour than Recipe A since $23 > 22$.

Reflect:

Practice

Name: .... Date: .... Period: ....

Jada is making hot chocolate. She has 3 different powdered flavor packets that she can add to the milk. Each ingredient is measured in ounces. Use this table to complete Problems 1–3.

Packet	Chocolate	Vanilla	Cinnamon
A	3	2	3
B	3	1	2
C	5	2	5

1. Jada says that Packet C will have the strongest chocolate flavor. Do you agree or disagree? Show or explain your thinking.

2. Compare the flavor of each recipe.

3. Jada mixed several of the same flavor packets together in a bowl. Her mixture has a ratio of chocolate to vanilla to cinnamon of $36 : 12 : 24$.

a Which flavor packet did she mix together? Explain your thinking.

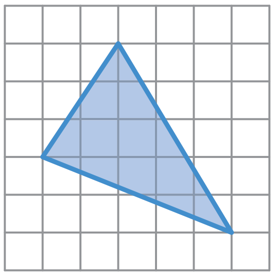
b How many of the packets did she mix together? Explain your thinking.

Name: .... Date: .... Period: ....

Practice

4. The ratio of students wearing sneakers to those wearing boots is 5 to 6. If there are 33 students in the class, and all of them are wearing either sneakers or boots, how many of them are wearing sneakers? Show your thinking.

5. Determine the area of the triangle. Show your thinking.



6. Identify a unit of measurement that can be used to measure:

a The length of a neighborhood road.

b The volume of a car's gas tank.

c The weight of a barbell.

Unit 2 | Lesson 18

Measuring With Different-Sized Units

Let's measure the length, volume, or weight of an object by using different units.

Warm-up Matching Units to Attributes

Write each unit in the appropriate column of the table for the attribute of an object it can be used to measure.

centimeter (cm)	cup (c)	inch (in.)	gram (g)
kilogram (kg)	kilometer (km)	liter (l)	meter (m)
ounce (oz)	pound (lb)	quart (qt)	yard (yd)

Length	Volume	Weight

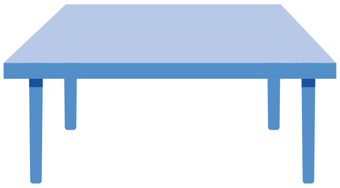
Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Units in the Real World

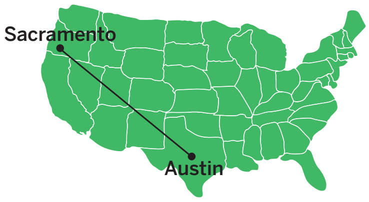
For each of the images shown, write an attribute (length, volume, or weight) you could measure. Then write an appropriate unit of measurement for the chosen attribute. You should use each attribute of length, volume, or weight at least once.

Attribute:
Unit of measurement:



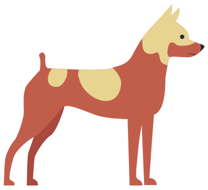
Attribute:
Unit of measurement:

Attribute:
Unit of measurement:

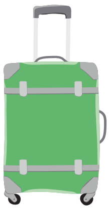


Attribute:
Unit of measurement:

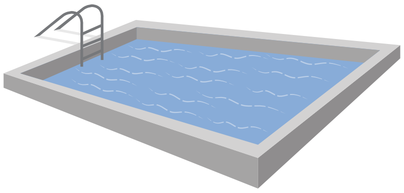
Attribute:
Unit of measurement:



Attribute:
Unit of measurement:



Attribute:
Unit of measurement:



Attribute:
Unit of measurement:

Activity 2 Measurement Stations

Station 1: Length

1. You will be given two different objects to measure. Estimate what you think the measurement would be for each item.

Item 1: ________ ________ in. ________ cm

Item 2: ________ ________ in. ________ cm

2. Use a ruler to measure each object to the nearest whole number in both inches and centimeters. Record your measurements in the table.

Length of:	Inches	Centimeters

3. Did it take more inches or centimeters to measure the indicated length? Why?

Station 2: Weight

1. You will be given two different objects to measure on the scale. Estimate what you think the weight is of each object.

Item 1: ________ ________ oz ________ lb ________ g ________ kg

Item 2: ________ ________ oz ________ lb ________ g ________ kg

2. Use a scale to weigh each object with as many different units as possible. Record your measurements in the table.

Object	Ounces	Pounds	Grams	Kilograms

Name: ________________ Date: ________ Period: ________

Activity 2 Measurement Stations (continued)

3. Did it take more ounces or grams to weigh the indicated object? Why?

Station 3: Volume

1. Look at the one-gallon jug of water. Estimate how many quart and liter bottles it will fill. Use decimals as needed in your estimates.

A gallon of water: ________ quarts ________ liters

2. You will be given materials to conduct the following experiment (or will watch a video of the experiment) to measure the volume in both quarts and liters. Record your measurements, estimating when necessary, in the table.

- Empty the gallon of water into the quart bottles, making sure to fill each bottle fully. How many quarts can be filled from the gallon jug? Record your response in the table.
- Refill the gallon jug and repeat the process of emptying it into the liter bottles. How many liters can be filled from the gallon jug? Estimate to the nearest tenth. Record your response in the table.

	Quarts	Liters
1 gallon		

3. Which is the larger unit, a quart or a liter? Explain your thinking.

STOP

Summary

In today's lesson . . .

You reviewed some standard measurement units for the attributes of length, volume, and weight. By experimenting with everyday objects, you saw that the size of the unit you use to measure something affects the measurement.

If you measure the same quantity with different units, it will take more of the smaller unit and fewer of the larger unit to express the measurement.

- For example, a room that measures 4 yd in length will also measure 12 ft in length. This makes sense based on the sizes of those two different units because a yard is longer than a foot.
- A similar relationship is true when weighing an object in pounds and then in ounces; or measuring the volume of a container in gallons and then in quarts.

The size of the object relative to the attribute you are measuring, and the amount of precision you need for your measurement, can help you determine the best unit of measurement.

Reflect:

Name: .... Date: .... Period: ....

1. Determine whether each pair of units measures length, volume, or weight and place a check mark in the appropriate column in the table. Then circle or underline the larger unit in each pair.

	Length	Volume	Weight
yard or foot			
quart or gallon			
meter or kilometer			
pound or ounce			
gram or kilogram			

2. Clare says, "This classroom is 11 m long. A meter is longer than a yard, so if I measure the length of this classroom in yards, I will get less than 11 yd." Do you agree or disagree with Clare? Explain your thinking.

3. Tyler wants to mail a package that weighs $4\frac{1}{2}$ lb. Which of the following could be the weight of the package in kilograms?

A. 2.04 kg

B. 4.5 kg

C. 9.92 kg

D. 4,500 kg

Practice

Name: .... Date: .... Period: ....

4. Elena mixes 5 cups of apple juice with 2 cups of sparkling water to make sparkling apple juice. She wants to make 35 cups of sparkling apple juice for a party. How much of each ingredient should Elena use? Show or explain your thinking.

5. Lin bought 3 hats for \$22.50. At this same rate, how many hats could she buy with \$60.00? Use the table to help with your thinking.

Number of hats	Price (\$)

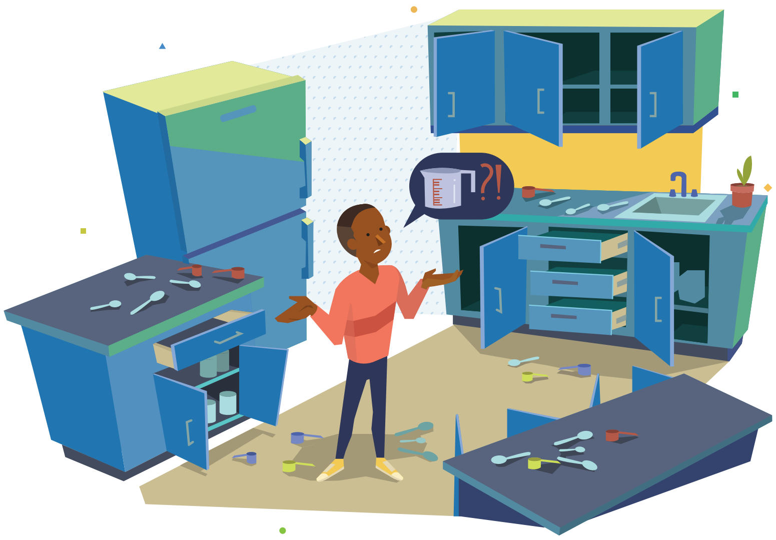
6. In one minute, Han runs 500 ft and Lin runs 750 ft.

a If they each run at those same rates, how far would each run in 20 minutes?

b In 20 minutes, how many *times farther* does Lin run than Han?

Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 19



Converting Units

Let's convert measurements to different units.

Warm-up Road Trip

Elena and her mom are visiting England from the United States. One day, Elena's mom was driving on the highway at a speed of 60 miles per hour when she got pulled over by the police for speeding. Outside the car, Elena noticed this road sign.

1. What do you think happened?

2. Where else might Elena and her mom run into similar issues while they are exploring England?

Log in to Amplify Math to complete this lesson online.

Activity 1 Cooking With a Tablespoon

Noah wants to make apple crisp using the following recipe, but he cannot find any measuring cups! He only has a tablespoon (tbsp) for measuring. Luckily, in the cookbook it says that 1 cup is equivalent to 16 tbsp, and 1 tbsp is equivalent to 3 teaspoons (tsp).

Apple crisp recipe

4 medium-size apples, chopped

$\frac{3}{8}$ cup brown sugar

$\frac{3}{4}$ cup oats

$\frac{1}{4}$ cup butter

$\frac{1}{2}$ cup chopped pecans

2 tsp cinnamon

1 tsp vanilla extract

1. Complete the table to help Noah adjust the recipe so that all measurements are in tablespoons.

Apple crisp recipe

4 medium-size apples, chopped

________ tbsp brown sugar

________ tbsp oats

________ tbsp butter

________ tbsp chopped pecans

________ tbsp cinnamon

________ tbsp vanilla extract

Critique and Correct: After you complete Problem 2, your teacher will provide you with an incorrect statement about this situation. Work with your partner to identify and analyze the error(s) and write a correct statement.

2. Noah decides to add in some dried cranberries to the recipe, and measures 10 tbsp. As he updates the original recipe he writes $\frac{2}{3}$ cup of cranberries. Did he write the correct amount? Show or explain your thinking by using a double number line diagram, a table, or other representation.

Name: ________________ Date: ________ Period: ________

Activity 2 Metric Recipes

You found a recipe for Chicken and Mushroom Pie online, but all the measurements are in metric units (milliliters and grams). Your measuring cup only shows cups and fractions of cups, and your scale only displays weight in ounces. In order to make the recipe with the tools you have, you need to convert the amounts from metric to U.S. Customary units.

Chicken and Mushroom Pie
25 ml canola oil
420 g skinless boneless chicken thighs
110 g chopped onion
250 g mushrooms
42 g flour
360 ml chicken stock
200 ml milk
1 package of puff pastry
1 egg

Approximate Conversions:
237 ml $\approx$ 1 cup
28 g $\approx$ 1 oz

You will be given two sets of cards:

- One with the amount of each ingredient from the recipe (except the puff pastry and egg) in metric units.
- One with the same amounts converted to U.S. Customary units (but the units have been left off).

1. Work with your partner to match one card from each set for each ingredient using the approximate conversions provided. You may use a calculator to perform the conversions. Round each conversion to the nearest tenth.

2. Complete the table on the next page.

- Paste or copy the recipe amount in metric units in the first column.
- Paste or copy the corresponding amount converted into U.S. Customary units in the second column. Be sure to write in the appropriate units: cups or ounces.
- Explain or show your thinking in the third column.

Activity 2 Metric Recipes (continued)

Recipe amount (metric units)	Converted amount (U.S. Customary units)	Explain or show your thinking:

STOP

Name: .... Date: .... Period: ....

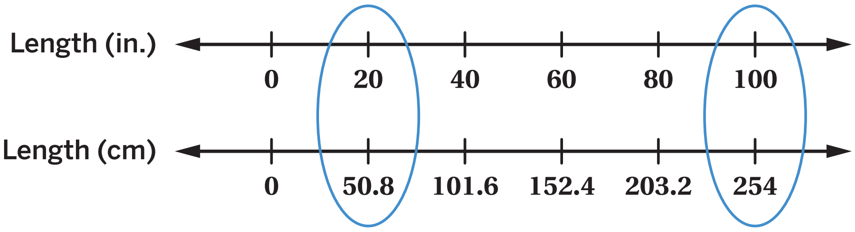
Summary

In today's lesson . . .

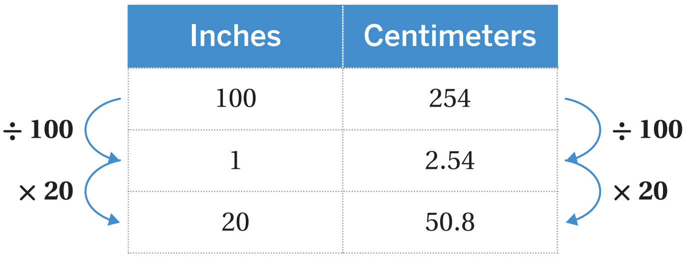
You saw that when you measure the same attribute of two or more objects by using the same two different units, the pairs of measurements are equivalent ratios. You can reason with these equivalent ratios to convert measurements from one unit to another.

Suppose you measure the side of a table to have a length of 20 in. You want to know this length in centimeters. Given that 100 in. is equal to 254 cm, you can use the ratio of inches to centimeters of 100 : 254 to determine an equivalent ratio for 20 in. This can be done and represented in several ways.

Using a double number line diagram:



Using a ratio box or a table:



Inches	Centimeters
100	254
1	2.54
20	50.8

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Priya's family exchanged 250 dollars for 4,250 pesos. Complete the table to determine the conversions between pesos and dollars.

Pesos	Dollars
4,250	250
	25
	1
	3
510	

2. There are 3,785 ml in 1 gallon, and there are 4 qt in 1 gallon.

a How many milliliters are in 3 gallons? Show or explain your thinking.

b How many milliliters are in 1 quart? Show or explain your thinking.

3. Tyler is making a soup that calls for 28 oz of potatoes. Determine the approximate weight of potatoes needed in both kilograms and grams. **Note:** 1 kg is approximately 35 oz.

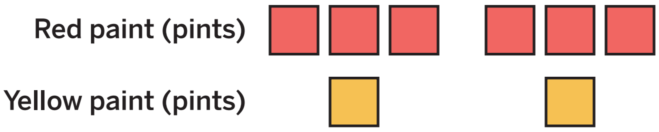
4. A simple trail mix uses only 7 oz of almonds for every 5 oz of raisins. How many ounces of almonds would be in a one-lb bag (16 oz) of this trail mix? Show or explain your thinking.

Name: .... Date: .... Period: ....

5. Identify whether each unit measures length, volume, or weight by placing a check mark in the appropriate column. Then circle the largest unit from each category.

Unit	Length	Volume	Weight
mile			
cup			
pound			
milliliter			
yard			
gram			
kilogram			
pint			
liter			
teaspoon			
centimeter			

6. The diagram represents the pints of red and yellow paint in a mixture. Select *all* statements that accurately describe the diagram.



A. The ratio of yellow paint to red paint is 2 to 6.

B. For every 3 pt of red paint, there is 1 pt of yellow paint.

C. For every pint of yellow paint, there are 3 pt of red paint.

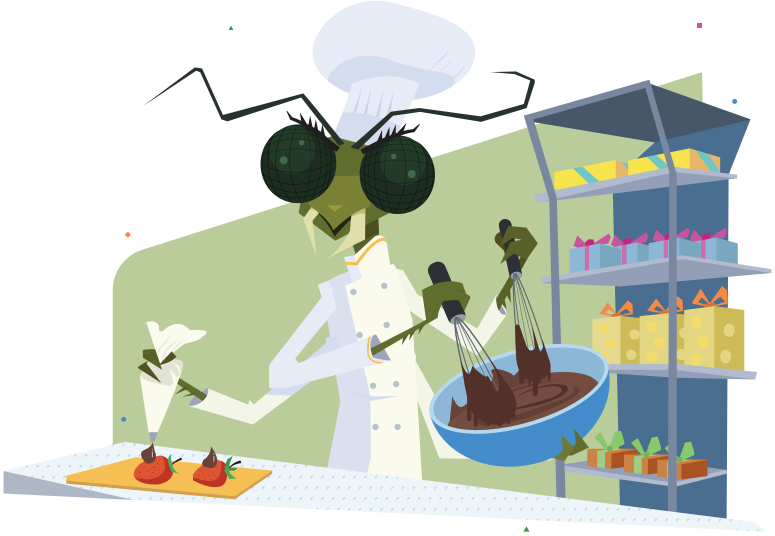
D. For every pint of yellow paint there are 6 pt of red paint.

E. The ratio of red paint to yellow paint is 6 : 2.

Unit 2 | Lesson 20 – Capstone

More Fermi Problems

Let's solve a Fermi problem.



Warm-up Making Guesses

1. Choose one of these Fermi problems that you would like to answer.
   - How many sticky notes will it take to cover the Washington Monument?
   - How many insect fragments are allowed to be in the 2020 world's largest chocolate bar that weighed approximately 12,770 lb?
   - If a radio station played your favorite song non-stop for the rest of your life, how many times would you hear it?

2. Use the following structure to make your best first guess (without any calculations) for the answer to your problem. Be prepared to explain your thinking.

   a. A number that is probably too small.

   b. A number that is probably too big.

   c. Your best first guess.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Educated Guesses and Calculating

Part 1: Educated Guesses

1. List any questions you would need to answer first in order to arrive at a more likely solution to your chosen Fermi problem from the Warm-up.

2. Write your best guesses for each question on your list.

3. Based on your guesses, what do you think is the answer to your Fermi problem? Show or explain your thinking.

Activity 1 Educated Guesses and Calculating (continued)

Part 2: Gathering Data and Calculating

4. You will be given a data card, or time to conduct your own research. Use the information gathered to first answer the questions on your list more precisely, adding or refining questions as necessary. Then use the information to calculate an answer to your Fermi problem. Show or explain your thinking.

5. Create a poster that will be displayed for a Gallery Tour. Your poster should clearly show your classmates not only the answer you came up with but also how you worked through the Fermi process. Be sure to include:

- The Fermi problem.
- Your first “wild” guesses.
- Your educated guess.
- Assumptions and estimations you made.
- Your calculations.
- One or two sentences stating your final answer and any other conclusions.

Name: .... Date: .... Period: ....

Activity 2 Posing and Answering a Fermi Question

1. Write another Fermi problem related to the same context as the question you answered in Activity 1. Then write down your best first guess.

2. What information from Activity 1 can you use to solve your new question?

3. What new information do you need?

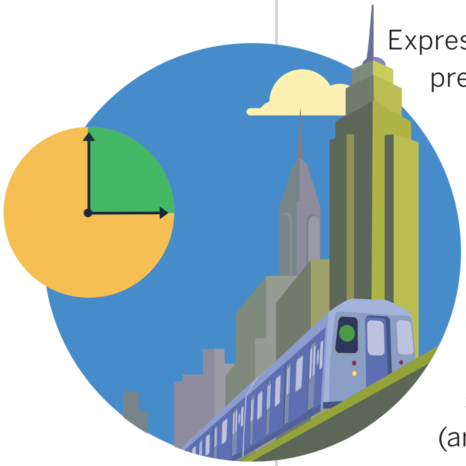
4. Conduct some research to gather the new information you need, and then determine an answer to your Fermi problem. Show or explain your thinking.

STOP

Unit Summary

Ratios are everywhere. They are in the paint on the walls, the food on your plate, even in the rhythms of your music. They give things their consistency and balance. The wrong ratio of red to blue can mean the difference between fuchsia and mauve. A band that's off-rhythm can cause any dancer to stumble.

Much like you saw with Oobleck, the ratios in paint colors, song tempos, and recipes are a constant dance between quantities. In many ways, you've known these quantities all your life. You can taste when soup has too much salt. You can see when the shade is off when mixing colors.



Expressing these ratios mathematically lets you be more precise than with "feel" alone.

While the problems you explored throughout this unit have been light-hearted, your process mirrored that of mathematicians, scientists, business leaders, and policy experts who are working on some of the world's most complex problems. From managing the world's ecosystems and preparing disaster relief efforts to starting a new business and launching spaceships to Mars, ratios play a key role in the world (and Universe) around us.

See you in Unit 3.

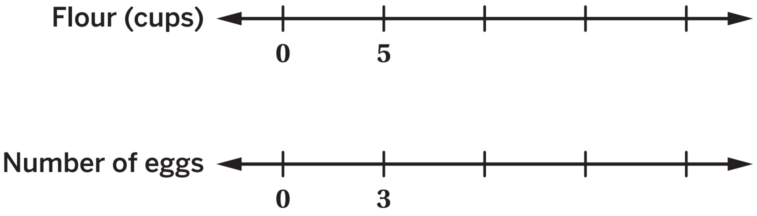
Name: .... Date: .... Period: ....

1. Jada wants to determine how many cups of vanilla pudding it would take to fill an Olympic-sized swimming pool.

a What information does she need to solve this problem?

b How can she use the information to solve the problem?

2. This double number line diagram shows the amount of flour and eggs needed for one batch of almond scones.



a Complete the diagram to show the amount of flour and eggs needed for 2, 3, and 4 batches of almond scones.

b What is the ratio of cups of flour to eggs?

c How much flour and how many eggs are used in 4 batches of almond scones?

d How much flour is used with 6 eggs?

e How many eggs are used with 15 cups of flour?

3. One batch of pink paint uses 2 cups of red paint and 7 cups of white paint. Mai made a large amount of the same color pink paint using 14 cups of red paint.

a How many batches of pink paint did she make?

b How many cups of white paint did she use?

Practice

Name: .... Date: .... Period: ....

4. Train A travels 30 miles in $\frac{1}{3}$ hour, and Train B travels 20 miles in $\frac{1}{2}$ hour. If both trains travel at a constant speed, explain how you know that Train A is traveling faster than Train B.

5. Diego has 48 strawberry breakfast bars, 64 blueberry breakfast bars, and 100 lemon breakfast bars for a bake sale. He wants to make bags that have all three types of breakfast bars and the same number of each type in each bag.

a How many bags can he make without having any breakfast bars left over?

b Is there another possible solution? If so, what is another solution?

6. Tyler's height is 57 in. Which of the following could reasonably represent his height in centimeters?

A. 22.4 cm

B. 57 cm

C. 144.8 cm

D. 3,551 cm

My Notes:

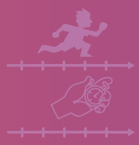
UNIT 3

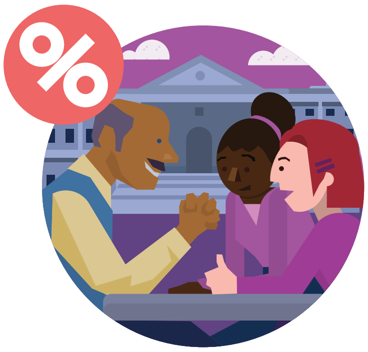
Rates and Percentages

One black truffle costs how much?! A hummingbird flaps its wings how many times in one minute?! Unit rates — how much per one — are useful ratios. And sometimes how much per one hundred, a percentage, is useful too — if you want to know: Who should take the technical foul shot with no time on the clock? Do people really like dogs better than cats? Who won the election?

Essential Questions

- How are the terms *same rate*, *constant rate*, and *unit rate* similar and different?
- What is the relationship between unit rates and percentages?
- How are percentages used to estimate and compare quantities?
- *(By the way, if you're a day late and a dollar short, do you need more time or more money?)*





SUB-UNIT

1 Rates

Narrative: From planning a school event to running a race, rates are a great tool for measuring and comparing things.

You'll learn . . .

- about unit rates.
- to apply rates to problems about price and speed.

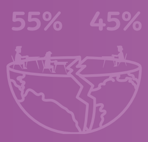
SUB-UNIT

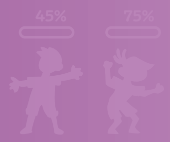
2 Percentages

Narrative: Percentage is a helpful way to compare values and understand populations, and even make decisions.

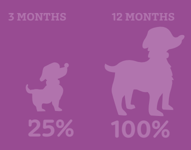
You'll learn . . .

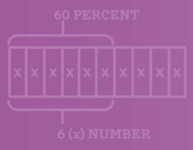
- how percentages are connected to rates and then number 100.
- to compare values using percentages.





50% of 600% of a positive number equals that number's square. What is the number?





Unit 3 | Lesson 1 – Launch

Choosing Representation for Student Council

Let's think about representation and what is fair.

Warm-up Notice and Wonder

The students at a middle school are preparing to vote for who will represent the student population on the Student Council. The school's administration has always used the following system to determine who can be elected to each office. What do you notice? What do you wonder?

Office	Grade
President	8
Vice President	8
Secretary	7
Treasurer	7
Historian	6
Spirit Commissioner	6

1. I notice . . .

2. I wonder . . .

Co-craft Questions: Share your questions with a partner. Together, come up with 1–2 questions you could ask related to fairness and representation.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Who Disagrees More?

Students in one class from each grade were asked whether they agreed or disagreed with the following statement: "The current structure of the Student Council is fair."

	Agree	Disagree
Class A (Grade 8)	14	10
Class B (Grade 7)	9	15
Class C (Grade 6)	12	18

> **1.** Rank the classes in order of disagreement — the class that disagrees most strongly to the class that disagrees least strongly. Show or explain your thinking.

Class	
	Disagrees most strongly
	Disagrees least strongly

The principal of the school says, "Majority rules — 100 students from each grade will be allowed to vote on whether to change the structure of the Student Council. If more than half of the students vote to change the structure, then it will be changed."

Refer to the table at the top of this page. Suppose the votes of each class (Class A, Class B, and Class C) represent the portion of the 100 votes from that grade who agree or disagree with the current structure.

Activity 1 Who Disagrees More? (continued)

2. Based on this, how many students in each grade will vote to change the structure? How many students will vote to keep the structure? Show or explain your thinking.

	Keep the structure	Change the structure
Grade 8		
Grade 7		
Grade 6		

3. Based on this data, will the structure of the Student Council be changed? Explain your thinking.

Are you ready for more?

The principal reconsiders how many students will be given a vote. The same number of students from each grade will be given a vote. The class from each grade (Class A, Class B, or Class C) represents how all of these students from that grade will vote.

What is the fewest number of students who can vote from each grade so that the number of votes for keeping or changing the structure are whole numbers? Show or explain your thinking.

Name: .... Date: .... Period: ....

Activity 2 A Fairer Representation

Grade	Students
8	250
7	285
6	320

The current Student Council and the school's administration met to determine how to restructure the Student Council for future elections. They decided to keep the six existing offices and add four more offices, each with the title of Representative. The table shows the school's current enrollment by grade.

Any student can be elected to any of the 10 positions. How should the offices be filled so that they provide the most fair representation of the entire student population? Show or explain your thinking.

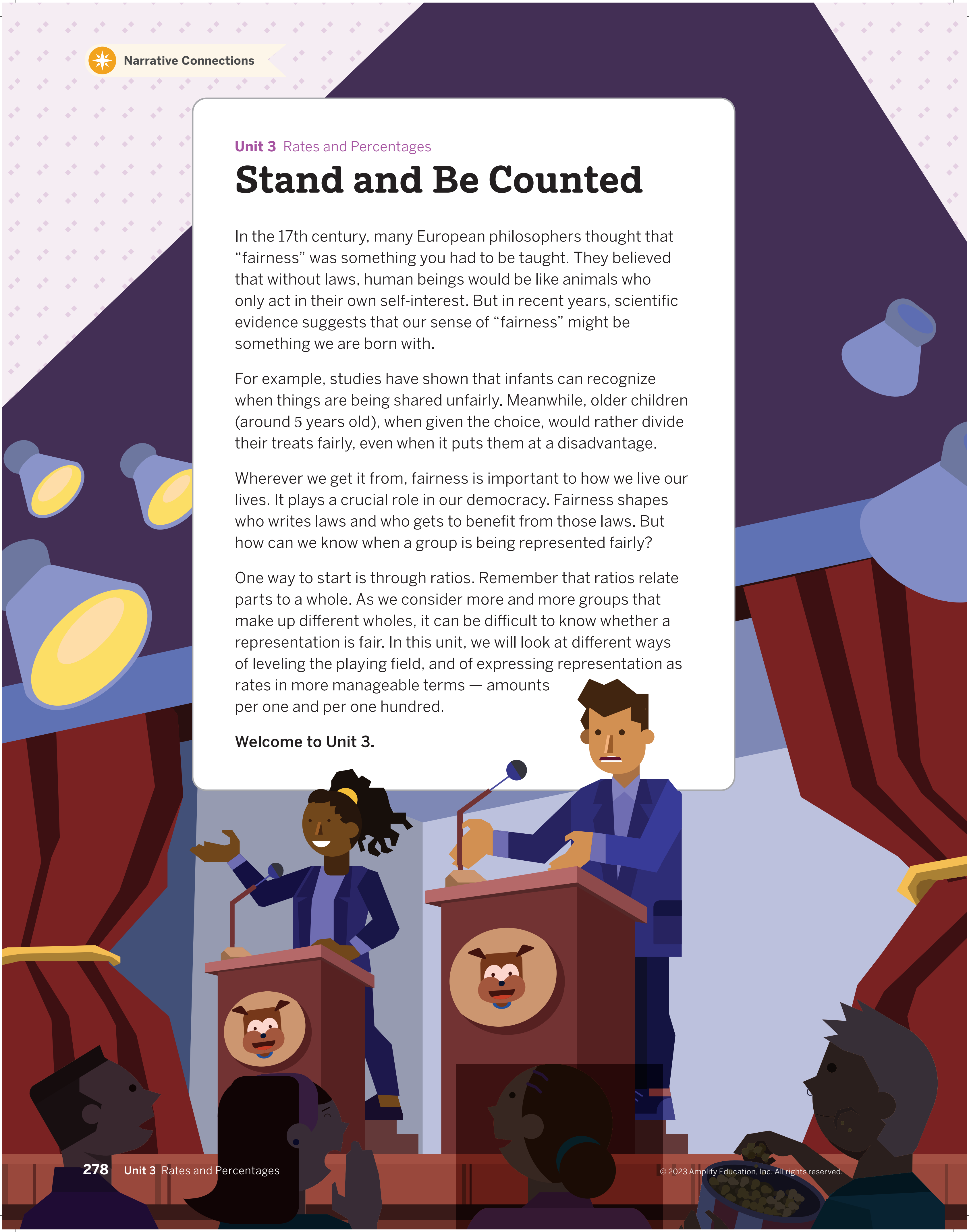
Are you ready for more?

Imagine that the role of a Representative is to serve as "the voice" of a selected group of students from all three grades. Each Representative will be responsible for voicing the concerns and opinions of exactly the same total number of students and also the same number of students from each grade.

1. What is the greatest number of Representatives there can be on the Student Council? Show or explain your thinking.

2. How many students from each grade will each Representative represent? Show or explain your thinking.

STOP



Unit 3 Rates and Percentages

Stand and Be Counted

In the 17th century, many European philosophers thought that “fairness” was something you had to be taught. They believed that without laws, human beings would be like animals who only act in their own self-interest. But in recent years, scientific evidence suggests that our sense of “fairness” might be something we are born with.

For example, studies have shown that infants can recognize when things are being shared unfairly. Meanwhile, older children (around 5 years old), when given the choice, would rather divide their treats fairly, even when it puts them at a disadvantage.

Wherever we get it from, fairness is important to how we live our lives. It plays a crucial role in our democracy. Fairness shapes who writes laws and who gets to benefit from those laws. But how can we know when a group is being represented fairly?

One way to start is through ratios. Remember that ratios relate parts to a whole. As we consider more and more groups that make up different wholes, it can be difficult to know whether a representation is fair. In this unit, we will look at different ways of leveling the playing field, and of expressing representation as rates in more manageable terms — amounts per one and per one hundred.

Welcome to Unit 3.

Practice

Name: ____________ Date: ____________ Period: ____________

1. Students in two schools were asked whether they agreed with a rule that limits each student to no more than one snack from the vending machine per day. The results are shown in the table. State at least two conclusions you can make from the results.

	Agree	Disagree
School A	230	425
School B	175	215

2. At a local middle school, Grade 6 has more Student Council representation than Grade 7 because there are more sixth graders than seventh graders. All of the students in both grades were asked whether they agreed with this representation. This table shows their responses. Tyler claims, "The students in both grades agreed equally." Is Tyler correct? Show or explain your thinking.

	Agree	Disagree
Grade 6	300	210
Grade 7	200	140

Name: .... Date: .... Period: ....

3. A recipe for one batch of bran muffins says to use 3 cups of flour and 2 cups of wheat bran. Create the double number line diagram to show the amounts of flour and wheat bran needed to make 3, 4, and 5 batches.

4. Evaluate each expression. Write each value as both a fraction and a decimal.

a $164 \div 8$

b $183 \div 6$

c $151.5 \div 5$

SUB-UNIT

1 Rates

Narrative Connections

How did student governments come to be?

Back in the 18th century, colleges were very different from how they are today. Schools saw themselves as parents and students as their children. It wasn't just their duty to teach, but to manage a student's entire upbringing — both academically and morally. They tightly regulated everything about a student's life: not just their studies, but what they did both in and out of class.

But this tight hold led to student unrest. Many gathered in violent demonstrations and destroyed school property. These students saw themselves as adults, not children to be looked after. Informal student groups began to form, looking to make changes on campus by communicating with the school administration.

By the early 1900s, reforms were happening across the country. Journalists were exposing the difficult conditions of immigrants, factory workers, and America's poor. As more students entered college, they brought the spirit of reform with them. These students were interested in championing students' interests through a democratic political process. And so modern student governments were born.

The amount of power and responsibility each student government has differs from one school to another, but every student government acts as a voice for the whole student body. They help organize events like fundraisers, rallies, and food drives, and raise money for clubs. To represent the needs of their school and its student population, it is important for student governments to understand the issues students face and the ratios and rates at which students are affected.

Unit 3 | Lesson 2

How Much for One?

Let's use ratios to describe how much items cost.

Warm-up Number Talk

Mentally evaluate the quotient. Be prepared to explain your thinking.

246 ÷ 12

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 Shopping for School Spirit Week

The Student Council at your school is starting to plan for School Spirit Week. The council members are looking into the cost of some different items that can be customized with the school logo.

1. 20 custom baseball caps cost $70. Each item costs the same amount.

a How much will 40 baseball caps cost?

b What is the cost per baseball cap?

c At this rate, how much will 11 baseball caps cost?

2. 12 reusable water bottles with the school logo costs $9. Each item costs the same amount.

a What is the cost per bottle?

b At this rate, how much will 7 water bottles cost?

c How many bottles can you buy for $3. Show or explain your thinking.

Are you ready for more?

Glow bracelets imprinted with your school name cost $415 for a package of 500 bracelets, $810 for 1,000 bracelets, or $1,600 for 2,000 bracelets. Which is the best deal?

Activity 2 Profits From School Spirit Sales

After the first week of the school spirit sale, the Treasurer is looking into the profits made from the different items sold by the Student Council. Refer back to the costs per item you calculated in Activity 1.

1. 200 custom t-shirts were purchased. Each t-shirt is sold for $12.

a What is the profit per t-shirt?

b At this rate, what will the profit be after all 200 t-shirts are sold? Show or explain your thinking.

2. 150 baseball caps were purchased. Each cap is sold for $5.

a What is the profit per cap?

b At this rate, what will the profit be after all 150 caps are sold? Show or explain your thinking.

3. 150 plastic bottles were purchased. Each bottle is sold for $1.50.

a What is the fewest number of bottles that need to be sold to make a minimum profit of $80? Show or explain your thinking.

b If the profit from the bottles after the first week was $112.50, did they sell all the bottles? Show or explain your thinking.

4. Show or explain any process that you repeated over and over to find the profit for each item using diagrams, words, and/or numbers.

Reflect: How did you evaluate the reasonableness of your results?

STOP

Name: ______________________ Date: __________ Period: ________

Summary

In today's lesson . . .

You saw that a ***rate***, like a ratio, is a comparison of how two values change together. In a *rate,* the two values being compared always have different units. Some examples of rates are:

- $3.00 for 12 bananas.
- 50 miles in 2 hours.
- 18 fish for 6 penguins.

Typically, a rate is given as a ***unit rate***, where the second value in the comparison is 1 and is written as "how much of A per one quantity of B" and can be represented as a single number. Common unit rates are:

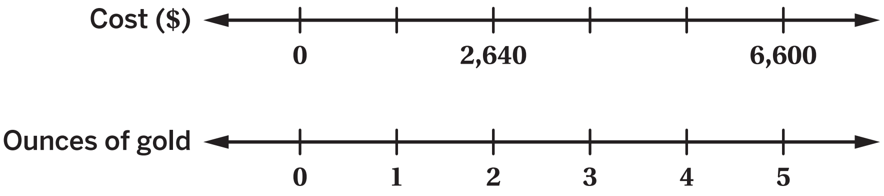
- Unit prices: $0.25 per banana, $5 per ticket.
- Speeds: 25 miles per hour, 3 kilometers per minute.
- Pay: $15.00 per hour, $45,000 per year.

Knowing the unit rate can aid in solving problems. Consider the unit rate of $5 per ticket. This means that every ticket corresponds to an increase in the cost of $5. The total cost of the tickets will always be the product of the unit price and the number of tickets. To determine the cost of 10 tickets, you would evaluate $5 \cdot 10$, for a total cost of $50.

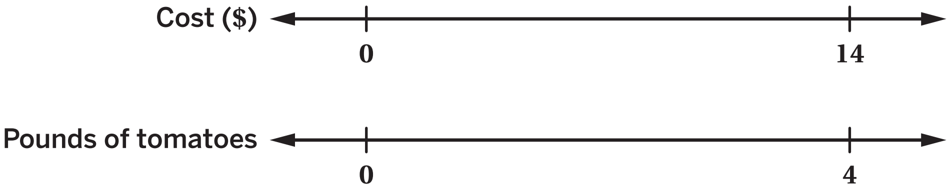
Reflect:

Name: .... Date: .... Period: ....

1. In 2016, the cost of 2 oz of pure gold was $2,640. Complete the double number line to show the cost for 1, 3, and 4 oz of gold at this rate.



2. The double number line shows that 4 lb of tomatoes cost $14. Add tick marks and labels to the diagram to represent the prices of 1, 2, and 3 lb of tomatoes at this same rate.



3. Priya bought these items at the grocery store. Determine the unit price of each item.

   a. 12 eggs for $3. What is the cost per egg?

   b. $7.50 for 3 lb of peanuts. What is the cost per pound?

   c. 4 rolls of paper towels for $2. What is the cost per roll?

   d. $3.50 for 10 apples. What is the cost per apple?

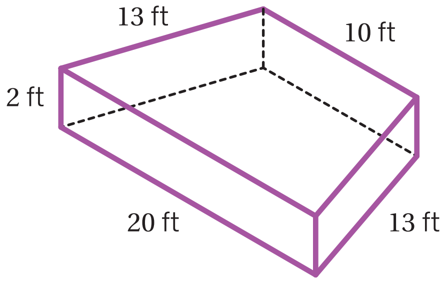
4. Clare made a smoothie with 1 cup of yogurt, 3 tbsp of peanut butter, 2 tsp of chocolate syrup, and 2 cups of crushed ice.

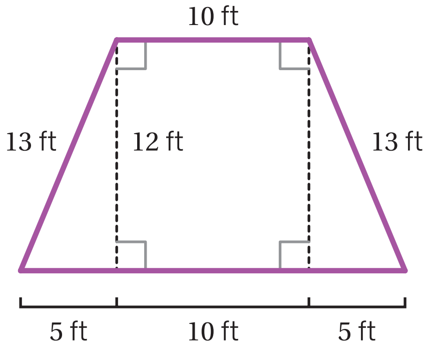
   a. Kiran tried to make a larger batch of this recipe. He used 2 cups of yogurt, 6 tbsp of peanut butter, 5 tsp of chocolate syrup, and 4 cups of crushed ice. He did not think it tasted right. Describe how the flavor of Kiran's recipe compares to Clare's recipe.

   b. How could Kiran change the quantities that he used so that his smoothie tastes just like Clare's?

Name: .... Date: .... Period: ....

5. A drama club is building a wooden stage in the shape of a trapezoidal prism. The height of the stage is 2 ft. Some measurements of the stage are shown.





a What is the surface area of the stage? Consider drawing a net to help with your thinking.

b If every face of the stage needs to be painted except the bottom, what is the total area that will need to be painted?

6. Lin and Han both walk at constant speeds to get to school.

a Lin traveled 8 blocks in 10 min. What is her speed in blocks per minute?

b Han traveled 6 blocks in 8 min. What is his speed in blocks per minute?

c Who is a faster walker? Explain your thinking.

Unit 3 | Lesson 3

Constant Speed

Let's use ratios to determine how fast objects or people move.

Warm-up Number String

Mentally calculate each quotient.

1. $30 \div 10$

2. $34 \div 10$

3. $3.4 \div 10$

4. $34 \div 100$

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Moving 5 Meters

Your school is planning a 5K walkathon fundraiser! You and your classmates want to determine approximately how long it would take to walk from the start line to the finish line.

Decide in your group who will be the "mover" — the person being timed — and who will be the "timer" — the person using the stopwatch. The timer should have the mover's Student Edition book in order to record the times.

1. Follow these steps to collect the data.

Round 1:

- The mover stands at the warm-up line (before the start line).
  The timer stands at the finish line, 5 m away.
- The mover starts walking *at a slow, steady speed* along the path.
- When the mover reaches the start line, they say, "Start!" and the timer starts the stopwatch.
- The mover keeps moving at this same speed along the path.
- When the mover reaches the finish line, they say, "Stop!" The timer stops the stopwatch and records the time, rounded to the nearest second, in the table of the mover's book.

Round 2:

- The mover follows the same instructions, but this time, walking *at a fast, steady speed*.
- The mover travels along the path and the timer records the time in the same way.

Repeat these steps until each person in the group has a chance to be the mover, walking along the path twice: once at a slow, steady speed, and once at a fast, steady speed.

Your slow moving time (seconds)	Your fast moving time (seconds)

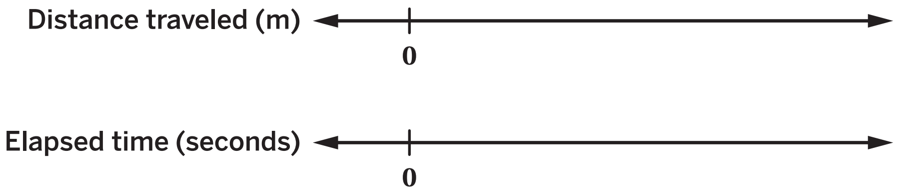
Are you ready for more?

Use your data for the amount of time to walk 5 m to determine how long it would take you to run a marathon at both speeds. Note: A marathon has a distance of 26.2 miles (which is about 42,165 m).

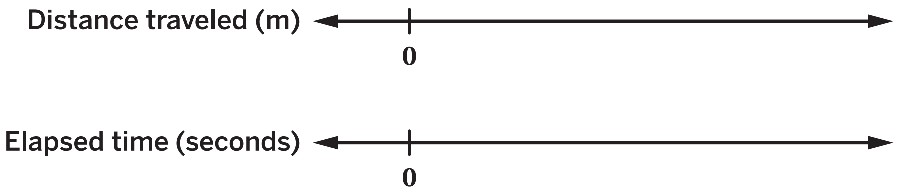
Activity 1 Moving 5 Meters (continued)

2. Use the double number line diagrams and your recorded times in the table on the previous page to complete these problems.

a What was the distance, in meters, you traveled in 1 second when walking at a slow, steady speed?



b What was the distance, in meters, you traveled in 1 second when walking at a fast, steady speed?



c How could this data help you estimate the total amount of time it would take you to complete the 5K walkathon?

Are you ready for more?

In 2011, a professional climber scaled the outside of the tallest building in the world, the Burj Khalifa in Dubai, making it all the way to 828 m (the highest point on which a person can stand) in 6 hours.

Assuming they climbed at the same rate the whole way:

1. How far did they climb in 2 hours? In 5 hours?

2. How far did they climb in 15 minutes?

Name: Date: Period:

Activity 2 Moving for 10 Seconds

Lin and Diego each walked at constant speeds for 10 seconds.
Lin walked 11 m and Diego walked 14 m.

1. Determine the rates at which Lin and Diego walked in meters per second.

2. How do these rates tell you who walked faster?

3. Han also walked at a constant speed. He walked 22 m in 20 seconds. Compare Han's speed to:

- **a** Lin's speed
- **b** Diego's speed

4. Use the data from your faster speed in Activity 1.

- **a** Estimate how far *you* could walk in 10 seconds at that rate.
- **b** Compare Han's speed to your faster speed.

Are you ready for more?

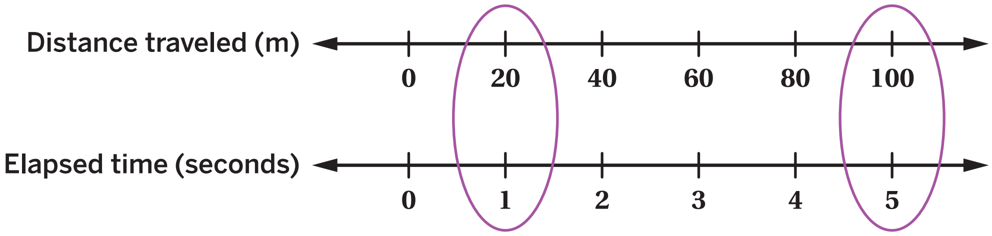
Lin and Diego want to walk a race in which they will both finish when the timer reads exactly 30 seconds. Who should get a head start, and how long should the head start be?

STOP

Summary

In today's lesson . . .

You explored unit rate in the context of speed — that is, how much of something occurs for every one unit of time. For example, suppose a train traveling at a constant speed traveled a distance of 100 m in 5 seconds. You can use a table of equivalent ratios or create a double number line to determine the unit rate that represents its speed, which is 20 m per second.



When you know the rate at which an object or person is traveling, which is its speed, then you can also use this to answer other questions about the situation. For example, using the unit rate of 20 m per second, you can:

- Multiply to get $20 \cdot 30 = 600$, to determine that the train would travel 600 m in 30 seconds.
- Determine the number that when multiplied by 20 gives a product of 480, which is $480 \div 20 = 24$. This tells you it would take the train 24 seconds to travel 480 m.

Reflect:

Name: .... Date: .... Period: ....

1. Han ran 10 m in 2.7 seconds. Priya ran 10 m in 2.4 seconds.

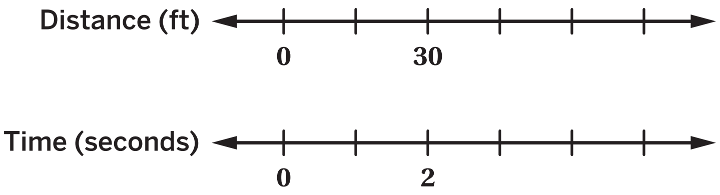
a Who ran faster? Explain your thinking.

b At this rate, how long would it take each person to run 50 m? Show or explain your thinking.

2. A scooter traveled 30 ft in 2 seconds at a constant speed.

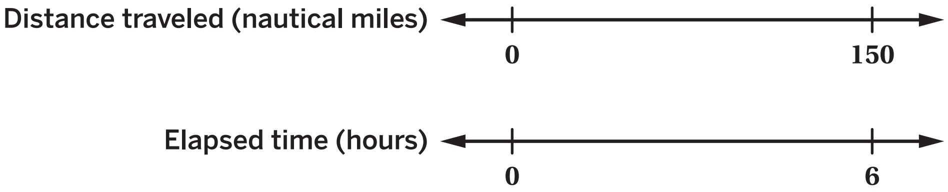
a What was the speed of the scooter in feet per second?

b Complete the double number line to show the distance the scooter could travel after 1, 3, 4, and 5 seconds, at this same rate.



c A skateboard traveled 55 ft in 4 seconds at a constant speed. Compare the speeds of the skateboard and the scooter.

3. A cargo ship traveled 150 nautical miles in 6 hours at a constant speed. How far did the cargo ship travel in one hour? Use the double number line to help with your thinking.



Name: .... Date: .... Period: ....

4. A recipe for pasta dough states, "Use 150 grams of flour per large egg."

a How much flour is needed if 6 large eggs are used?

b How many eggs are needed if 450 grams of flour are used?

5. Each of the following is a pair of equivalent ratios. For each pair, show or explain how you know the ratios are equivalent.

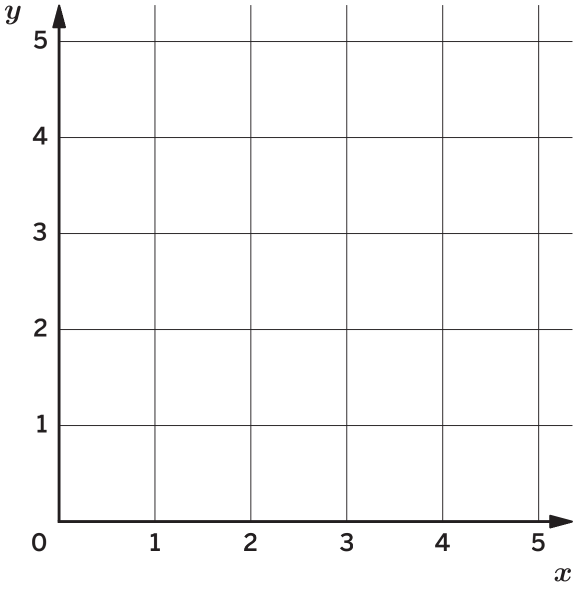
a $5 : 1$ and $15 : 3$

b $25 : 5$ and $10 : 2$

c $198 : 1{,}287$ and $2 : 13$

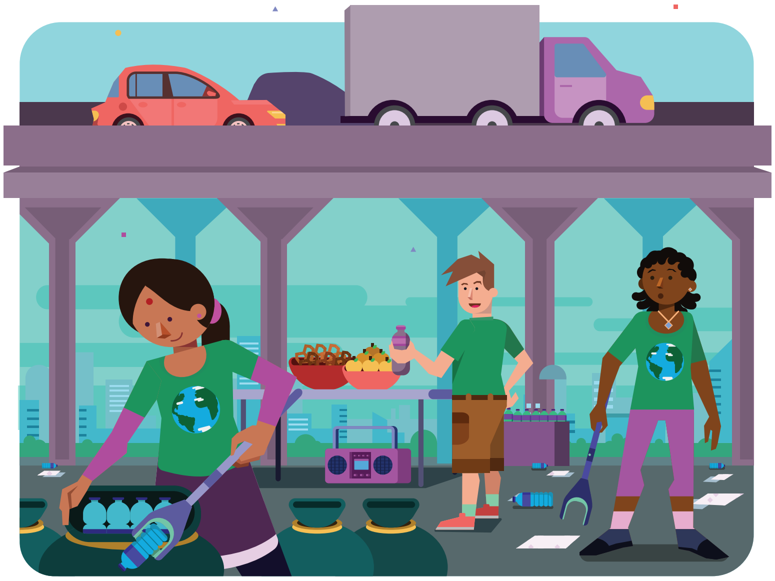
6. Plot and label the following points on the coordinate plane.

$A(1, 3)$, $B(2, 2)$, $C(3, 1)$, $D(4, 3)$



Name: .... Date: .... Period: ....

Unit 3 | Lesson 4



Comparing Speeds

Let's use graphs to represent ratios and to compare speeds.

Warm-up Which One Doesn't Belong?

Study the rates. Which rate does not belong with the others? Be prepared to explain your thinking.

A. 5 miles in 15 minutes

B. 20 miles per 1 hour

C. 3 minutes per mile

D. 32 km per 1 hour

Collect and Display: Your teacher will collect words and phrases you use as you explain your thinking. This language will be added to a class display for your reference.

Log in to Amplify Math to complete this lesson online.

Activity 1 Sweep-A-Street

For community service, Lin's school has decided to participate in the Sweep-A-Street program. Each grade level will be responsible for maintaining 3,220 m of road.

Part 1

The table shows how long it took each grade to pick up litter along one side of their stretch of road. What do you notice? What do you wonder?

Grade	Time (minutes)	Distance (m)
6	30	2,100
7	30	3,000
8	45	3,150

1. I notice . . .

2. I wonder . . .

Historical Moment

Fair Taxes

The book *Jiuzhang Suanshu* ("Nine Chapters on the Mathematical Art") by unknown authors from China, dating to around 200 BCE, provides insight into both how mathematics developed in that region and what life and their feudal society looked like at the time. The sixth chapter contained twenty-eight problems mostly related to distributing and transporting grain, which was used as currency to pay taxes. The problems indicated that tax rates were determined by combinations of local population, distance to the central bureau, and the current value of the grain.

Determine the solution to this problem from the text:

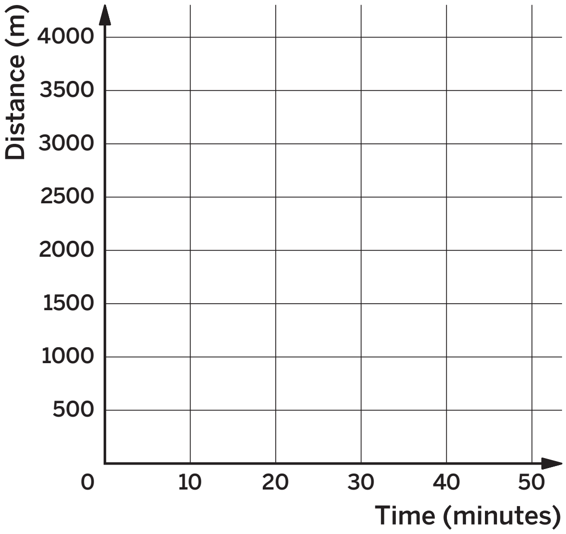
"An unloaded cart travels 70 li a day and a loaded cart travels 50 li a day. Transporting millet from the National Granary to Shanglin, one makes 3 round trips in 5 days, how far is the distance between these locations?"

Name: .... Date: .... Period: ....

Activity 1 Sweep-A-Street (continued)

Part 2

3. Use the table from Part 1 to plot points on the graph to represent each grade's time and distance.



4. Refer to your graph. What do you notice? What do you wonder?

   a I notice . . .

   b I wonder . . .

5. At what rate was each grade working to clean up their stretch of road?

6. The eighth grade students worked at the same rate for the entire time.

   a How many meters of road did the eighth grade students clean up in the first 30 minutes?

   b Use your graph to explain how you know your answer to part a is correct.

Activity 2 Revisiting the Sweep-A-Street Project

The next month, the eighth grade students decided to split up into two equal groups. Group A picks up litter on the left side of the street and Group B picks up litter on the right side of the street. After the first 12 minutes, Group A has covered 600 m. After the first 24 minutes, Group B has covered 1,080 m.

> **1.** Which group is working at a faster rate? How much faster? Show or explain your thinking.

> **2.** Group A and Group B both started on their own side of the road at the same place along the eighth grade stretch and worked in the same direction. Complete the tables to represent the amount of time it takes each group to cover different distances. Use your tables to complete the following problems.

Group A

Time (minutes)	Distance (meters)

Group B

Time (minutes)	Distance (meters)

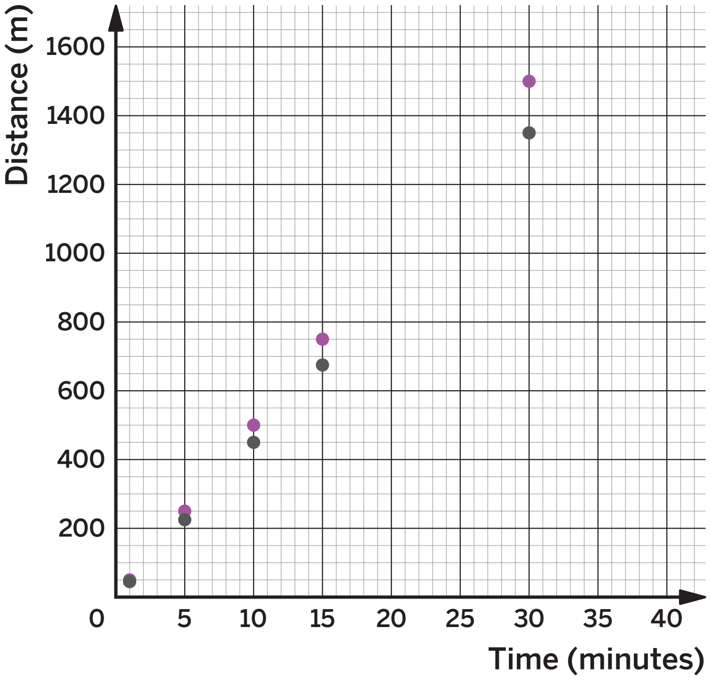
a How far apart were the two groups after 15 minutes?

b How many minutes did it take for them to end up 150 m apart?

Name: .... Date: .... Period: ....

Activity 2 Revisiting the Sweep-A-Street Project (continued)

3. Plot at least three points for each group on the same coordinate plane by using your tables of values from Problem 2.



4. Explain how the graph can be used to show your responses to Problems 1 and 2.

STOP

Summary

In today's lesson . . .

You applied your understanding of *unit rate* to compare ratios and determine if they were equivalent. You reasoned that if two scenarios involving rates, like speeds, have the same unit rate, then they are equivalent ratios.

You expanded your use of tables to represent ratio relationships, to create the corresponding graph, where each ratio "$a : b$" represents the point (a, b) on the coordinate plane.

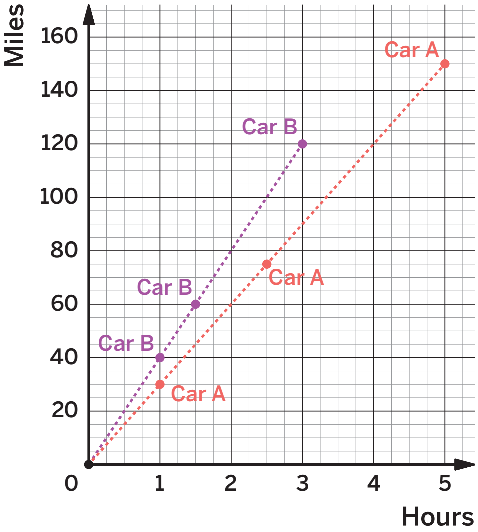
These tables and this graph represent some distances and times of two cars traveling at constant speeds.

Car A

Hours	Miles
1	30
2.5	75
5	150

Car B

Hours	Miles
1	40
1.5	60
3	120



Notice:

- Each ratio for Car A is equivalent to the unit rate 30 mph, and together they form a straight line through $(0, 0)$.
- Each ratio for Car B is equivalent to the unit rate 40 mph, and together they form a different straight line through $(0, 0)$.
- The unit rates for Cars A and B are *not* equivalent so the rates form two distinct lines.

Reflect:

Name: .... Date: .... Period: ....

1. Andre types 208 words in 4 minutes. Noah types 342 words in 6 minutes. Both type at a constant rate. Who types faster?

2. A corn vendor at a farmers market is selling a bag of 8 ears of corn for $\$2.56$. Another vendor is selling a bag of 12 ears of corn for $\$4.32$. Which bag is the better deal? Show or explain your thinking.

3. A kangaroo hops 2 km in 3 minutes. At this rate:

a How long will it take the kangaroo to hop 5 km?

b How far will the kangaroo hop in 2 minutes?

Name: ________________ Date: ________ Period: ________

4. 4 movie tickets cost $48. At this rate, what is the cost of:

a 5 movie tickets?

b 11 movie tickets?

5. A grocery store is having a sale on frozen vegetables. Four bags are sold for $11.96. At this rate, what is the cost of:

a 1 bag?

b 9 bags?

6. Write an expression for each scenario.

a A group of 4 friends fairly shares the $126 they earned doing yard work over the weekend. Write an expression to represent how much money each friend received, then evaluate your expression.

b A colony of 126 ants fairly shares 4 large leaves. Write an expression to represent how many leaves each ant receives then evaluate your expression.

Name: .... Date: .... Period: ....

Unit 3 | Lesson 5

Interpreting Rates

Let's explore unit rates and their graphs some more.

Warm-up Something per Something

1. Think of two "somethings" you have heard described in terms of "something *per* something."

2. Share your ideas with your group, and listen to everyone else's ideas. Make a list of all of your group's unique ideas. Be prepared to share these with the class.

Log in to Amplify Math to complete this lesson online.

Activity 1 Dog Biscuits

Plan ahead: How can you communicate your thinking clearly? What tools will you use?

In honor of National Pet Adoption Month in June, students are making dog biscuits for a local animal shelter. The instructions for a large batch say, "Mix 40 cups of whole wheat flour with 20 eggs and 40 tbsp of canola oil. Then add 5 cups of peanut butter."

1. Priya and Han create their own strategy for thinking about the ingredients they need to purchase. Complete the table to show each of their results if:
   - **a** Priya determines how much flour they need to buy per cup of peanut butter they buy.
   - **b** Han determines how much peanut butter they need to buy per cup of flour they buy.

Flour (cups)	Peanut butter (cups)
40	5

2. Before they go to the store, the culinary arts teacher says they can use whatever they find in the pantry. They find a bag containing 24 cups of flour. Which unit rate would be most efficient to determine how much peanut butter is needed to use *all* the flour? Explain your thinking.

3. They find 4 cups of peanut butter. What unit rate would be most efficient to determine how much flour is needed to use *all* the peanut butter? Explain your thinking.

4. Priya and Han decide to take exactly the largest possible amounts of whole cups of flour and peanut butter from the pantry that allow them to make a batch that still tastes the same, and then buy any remaining needed amounts to make a full larger batch. How much of each ingredient do they need to buy? Show or explain how your response relates to one of the unit rates.

Name: ________________ Date: ________ Period: ________

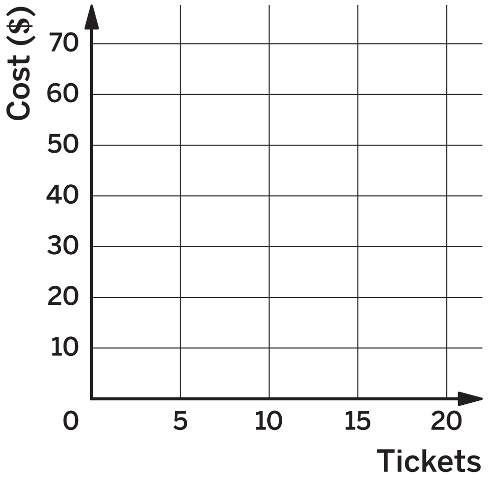
Activity 2 Raffle Tickets

The animal shelter is holding a raffle to help cover expenses like food and supplies for the animals. Tyler paid $20 for 5 raffle tickets.

1. Complete the table to show different numbers of tickets that can be purchased for different dollar amounts at this same rate. Be prepared to explain your thinking.

Tickets	Cost ($)	Cost per ticket ($)
5	20	
1		
10		
	48	
	64	
250		

2. Plot points on the graph to represent each pair of numbers of tickets and cost, in dollars, from the table. **Note:** You only need to include values that can be shown.



Activity 2 Raffle Tickets (continued)

3. Explain how your graph represents the unit rate of dollars per ticket.

4. Explain how your graph represents the unit rate of tickets per dollar.

Are you ready for more?

What “deal” on tickets for Tyler’s raffle might sound like a good deal, but is actually a little worse than buying tickets at the normal price?

STOP

Name: ______________________ Date: __________ Period: ________

Summary

In today's lesson . . .

You saw that, for a ratio $A : B$, there are two corresponding unit rates — how much of Quantity A per 1 of Quantity B, and how much of Quantity B per 1 of Quantity A.

In a ratio table, the unit rates represent the constant factors by which the values for one quantity can each be multiplied to determine the corresponding value for the other quantity. Consider a constant rate when 4 lb of apples costs $\$10$.

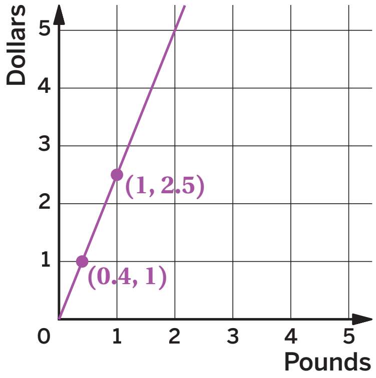
$\div 0.4$

$\cdot 2.50$

Pounds	Dollars	Dollars per pounds	Pounds per dollars
4	10	$10 \div 4 = 2.50$	$4 \div 10 = 0.4$
2	5	$5 \div 2 = 2.50$	$2 \div 5 = 0.4$
1	2.5	$2.50 \div 1 = 2.50$	$1 \div 2.50 = 0.4$
0.4	1	$1 \div 0.4 = 2.50$	$0.4 \div 1 = 0.4$

$\div 2.5$

$\cdot 0.4$



On a graph of ratios equivalent to A : B, both unit rates can be represented by points that will fit the same straight-line pattern.

- One unit rate is located where the horizontal coordinate A is 1, at the point $\left(1, \frac{B}{A}\right)$.
- The other unit rate is located where the vertical coordinate B is 1, at the point $\left(\frac{A}{B}, 1\right)$.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A pink paint mixture uses 4 cups of white paint for every 3 cups of red paint. Complete the table to show different quantities of red and white paint for the same tint of pink.

White paint (cups)	Red paint (cups)
4	3
	1
1	
	4
5	

2. A farm lets you pick 3 pt of raspberries for $\$12.00$.

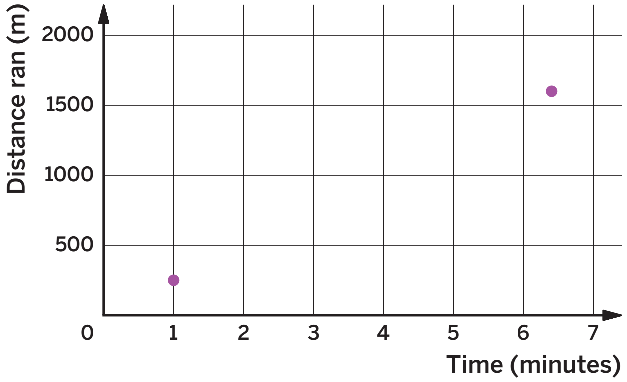
a. What is the cost per pint?

b. How many pints can you buy per dollar?

c. At this rate, how many pints can you afford for $\$20.00$?

d. At this rate, how much will 8 pt of raspberries cost?

3. Mai runs laps around a 400-m track at a constant speed of 250 m per minute. How many minutes does it take Mai to complete 4 laps of the track? Show or explain your thinking using the graph.



Name: .... Date: .... Period: ....

4. Han and Tyler are both following the same polenta recipe that calls for 5 cups of water for every 2 cups of cornmeal.

- Han says, "I am using 3 cups of water. I will need $1\frac{1}{5}$ cups of cornmeal."
- Tyler says, "I am using 3 cups of cornmeal. I will need $7\frac{1}{2}$ cups of water."

Do you agree with either of them? Show or explain your thinking.

5. At 10:00 a.m., Han and Tyler both started running toward each other from opposite ends of a 10-mile path along a river. Han runs at a pace of 12 minutes per mile. Tyler runs at a pace of 15 minutes per mile.

a How far does Han run after half an hour? After an hour?

b Do Han and Tyler meet on the path within 1 hour? Show or explain your thinking.

6. Evaluate each expression. Write the value of each quotient as a fraction, then as a decimal.

a $4 \div 5$

b $5 \div 4$

Unit 3 | Lesson 6

Comparing Rates

Let's use graphs to compare rates.

Warm-up Equivalent Ratios

Circle *all* the ratios that are equivalent to $12:4$. Be prepared to explain your thinking.

A. $3:1$

B. $1:\frac{1}{4}$

C. $2:\frac{2}{3}$

D. $24:12$

Log in to Amplify Math to complete this lesson online.

Name: ______________ Date: ______________ Period: ______________

Activity 1 Planning a Celebration

The Teacher of the Year award recipient will be announced at the next pep rally. To honor the winner, the Student Council is planning to have balloons, streamers, or confetti drop onto the audience. The table shows the cost *c*, in dollars, for *b* packs of each item.

	Number of packs	Cost ($)	Unit price ($ per pack)
Balloons	b	c	
Streamers	$2 \bullet b$	$2 \bullet c$	
Confetti	$4 \bullet b$	$3 \bullet c$	

1. Determine the unit price of each item. Record your responses in the table.

2. Order the items from least expensive to most expensive.
Explain your thinking.

Stronger and Clearer: You will share your response to Problem 2 with your classmates to get feedback on your clarity and reasoning. After receiving feedback, revise your response.

Activity 2 Using Graphs to Compare

Part 1

You will use your completed table from Activity 1.

1. Each person in your group will work with *one* item from the table. Write the name of the item that you have chosen.

My item:

2. You will be assigned values for b and c. Use these values to determine the unit price of your item.

$b =$

$c =$

Unit price:

3. Complete the table for the first three values. Choose two more values for b, and determine the corresponding value for c.

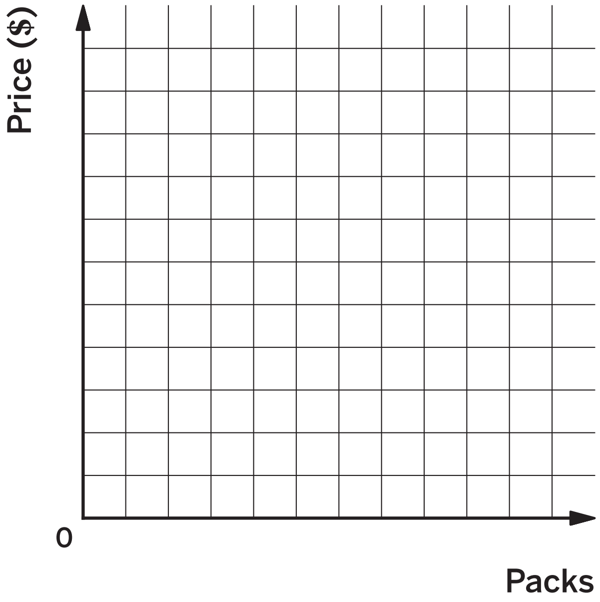
Packs	Cost ($)
1	
2	
3	

Name: ________________ Date: ________ Period: ______

Activity 2 Using Graphs to Compare (continued)

Part 2

4. Graph your points and connect them with a line that passes through the origin. Be sure to label the scales on the axes.



5. Add your partners' lines to your graph.

6. Compare all of the lines created by your group. How does the graph support your conclusions from Activity 1 about which items were the least expensive? Most expensive? Same price?

STOP

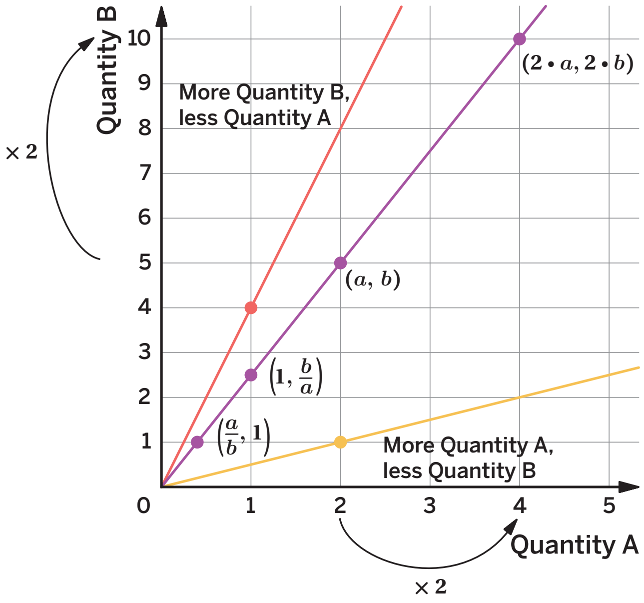
Summary

In today's lesson . . .

You saw that when two ratios have the same unit rates, they are equivalent ratios. When the values for both quantities in a ratio relationship are multiplied by the same factor, the result is an equivalent ratio.

For example, consider the ratio $2 : 5$. It has the same unit rate as $2 \bullet 2 : 5 \bullet 2$. Therefore, $2 : 5$ is equivalent to $4 : 10$.

You have seen how to compare two rates by using values in a table or by identifying ratios with shared values in a graph. Another way to compare rates by using the graph is to look at the "steepness" of the lines that connect each set of equivalent ratios. The rate that indicates "more" of the quantity along the vertical axis has a line that is steeper, while the rate that indicates "less" has a line that is less steep.



Reflect:

Name: ________________ Date: ________ Period: ________

1. Mai and Priya rode their scooters around the neighborhood. Mai traveled 15 m in 6 seconds. Priya traveled 22 m in 10 seconds. Who traveled faster? Show or explain your thinking.

2. Here are the prices of same-sized bottles of a brand of juice at different stores. Which store offers the best deal per bottle? Show or explain your thinking.
   - **Store X:** 4 bottles for $2.48
   - **Store Y:** 5 bottles for $3.00
   - **Store Z:** 59 cents per bottle

3. Two planes travel at their top constant speed. Plane A can travel 2,800 miles in 5 hours. Plane B can travel 3,885 miles in 7 hours. Which plane has a faster top speed? Explain your thinking.

Name: ______________________ Date: ____________ Period: ________

4. A box of cereal weighs 600 g. How many pounds does the box of cereal weigh? Show or explain your thinking. **Note:** 1 kg is approximately 2.2 lb.

5. Refer to the two three-dimensional figures shown. Determine whether each statement describes *Figure A*, *Figure B*, *both*, or *neither*.

Figure A

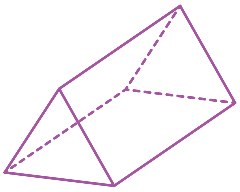
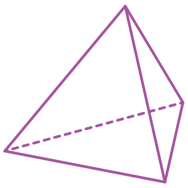


Figure B



a This figure is a polyhedron.

b This figure has triangular faces.

c There are more vertices than edges in this figure.

d This figure has rectangular faces.

e This figure is a pyramid.

f There is exactly one face that can represent the base for this figure.

g The base of this figure is a triangle.

h This figure has two identical and parallel faces that can represent the base.

6. Match each fraction with its decimal equivalent.

Fraction	**Decimal**
a $\frac{1}{2}$	________ 0.25
b $\frac{3}{12}$	________ 0.75
c $\frac{1}{20}$	________ 0.50
d $\frac{3}{4}$	________ 0.8
e $\frac{4}{5}$	________ 0.05

Name: ______________________ Date: __________ Period: __________

Unit 3 | Lesson 7

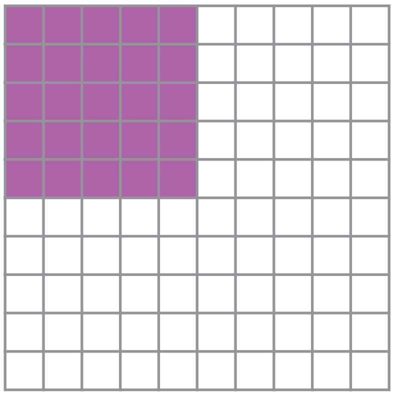
Solving Rate Problems

Let's use unit rates to compare constant speeds and prices.

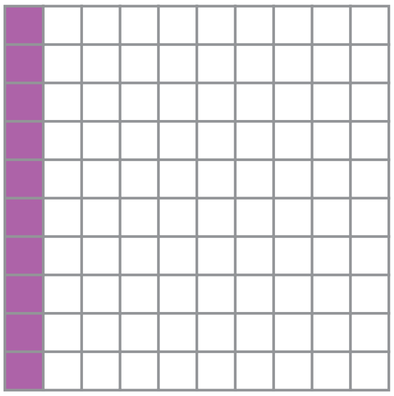
Warm-up Grids of 100

How much of each grid is shaded?

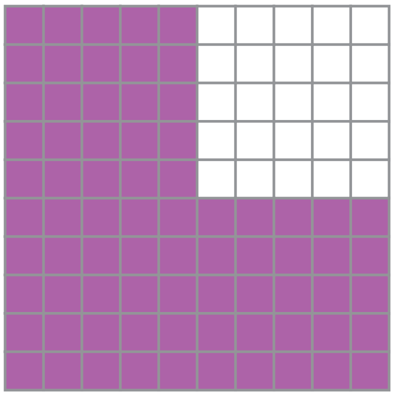
Grid A



Grid B



Grid C



Log in to Amplify Math to complete this lesson online.

Activity 1 The Fastest of All

Babylonian astronomy in the first millennium BCE, led by Mesopotamian astronomers, focused on observations and predictions — many of which have since proved to be quite accurate. Without yet knowing that the Earth orbited the sun in an ellipse, they determined that the orbital speed was not constant — it is slower in spring and faster in autumn. How? By looking at the rate at which the sun passed through the sky as a ratio of distance to time, and noting that it changed throughout the year.

Today's scientists similarly use observations and calculations to describe animal's top speeds. Here is a table showing how far some animals can sprint at their top constant speed for one minute. Order these animals from fastest to slowest.

Note: 1 in. = 2.54 cm

Animal	Sprint distance
Cougar	1,408 yd
Antelope	1 mile
Hare	49,632 in.
Kangaroo	1,073 m
Ostrich	1.15 km
Coyote	3,773 ft

Featured Mathematician

Mesopotamian astronomers

Mesopotamian astronomers of the Neo-Babylonian Empire and beyond, such as Naburimannu (c. unknown, 6th–3rd century BCE), Kidinnu (c. 4th century BCE), and Berossus and Sudines (c. 3rd century BCE), made significant contributions to our early understandings of cyclical events – planetary motion and orbits. They looked for constant rates, but also made sense of non-constant rates. To some extent, their observations and calculations set the stage for leap years, daylight savings time, and even your daily horoscope.

Name: ______ Date: ______ Period: ______

Activity 2 Card Sort: Who Is Offering a Better Deal?

The Student Council is building care packages for members of the community in need. You will be given a set of cards showing differently advertised offers for the care package items.

1. Discuss Card A with your partner and decide which store offers the better deal. Show or explain your thinking.

2. Each partner should then take two of the remaining cards (B–E).

- **a** Decide, by yourself, which store offers the better deal on your two cards. Be prepared to explain your thinking.
- **b** Take turns with your partner explaining which store offers the better deal for each of your cards. Listen to your partner's explanations for their cards. If you disagree, explain your thinking.
- **c** Revise any decisions about the deals on your cards based on the feedback from your partner.
- **d** Group the cards into two sets: the cards with the better deal at Store A in one set, and the cards with better deal at Store B in the other set.

Are you ready for more?

Create your own deal for the soap on Card F.

1. Your deal should describe the number of bars of soap and the total cost, in dollars.

2. Compare your deal with your partner's deal. Which is a better deal? Show or explain your thinking.

STOP

Summary

In today's lesson . . .

You solved constant rate problems, using one or both unit rates to calculate equivalent ratios or compare scenarios.

For example, suppose an 8 oz bag of shredded cheese is on sale for $\$2$, and a 2 kg bag of the same cheese is normally sold for $\$16$. There are at least two different ways to determine which is the better price per weight of cheese.

Compare the unit rates of dollars per kilogram to see that the large bag is a better deal because it costs less money for the same amount of cheese.

- The large bag costs $\$8$ per kg, because $16 \div 2 = 8$.
- The small bag holds $\frac{1}{2}$ lb of cheese because there are 16 oz in 1 lb, so it costs $\$4$ per lb. This is about $\$8.80$ per kg because there are about 2.2 lb in 1 kg and $4.00 \cdot 2.2 = 8.80$.

Compare the unit rates of ounces per dollar to see that the large bag is a better deal because you get more cheese for the same amount of money.

- With the small bag, you get 4 oz per dollar, because $8 \div 2 = 4$.
- The large bag holds $2{,}000$ g of cheese because there are $1{,}000$ g in 1 kg. So, you get 125 g per dollar, because $2{,}000 \div 16 = 125$. This is about 4.4 oz per dollar because there are about 28.35 g in 1 oz, and $125 \div 28.35 \approx 4.4$.

Another way to solve the problem would be to compare the unit prices of each bag in dollars per ounce. Try it!

Reflect:

Name: .... Date: .... Period: ....

Practice

1. This package of sliced cheese costs $2.97. How much would a package with 18 slices cost at the same price per slice? Show or explain your thinking.

2. A company claims that its newest copy machine can print 120 pages per minute. A teacher printed 700 pages in 6 minutes. Is the company's claim true? Show or explain your thinking.

3. Order these objects from heaviest to lightest.
Note: 1 lb = 16 oz, 1 kg ≈ 2.2 lb, and 1 ton = 2,000 lb.

	Heaviest
	Lightest

Item	Weight
School bus	9 tons
Horse	1,100 lb
Elephant	5,500 kg
Grand piano	15,840 oz

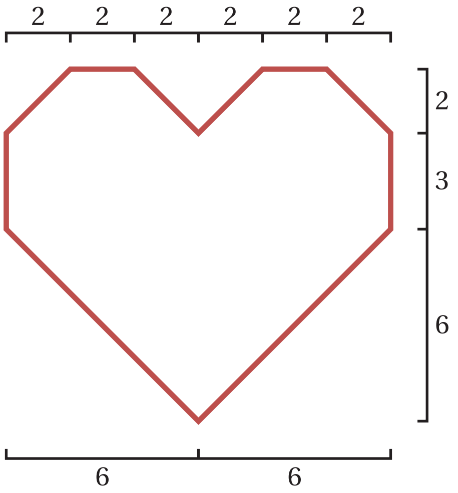
Name: .... Date: .... Period: ....

4. Andre sometimes mows lawns on the weekend to earn extra money.

- Two weeks ago, he mowed a neighbor's lawn for $\frac{1}{2}$ hour and earned $10.
- Last week, he mowed his uncle's lawn for 1.5 hours and earned $30.
- This week, he mowed the lawn of a community center for 2 hours and earned $30.

Which jobs paid better than others? Explain your thinking.

5. Calculate the area of this polygon. Show your thinking. All measurements are in centimeters.



6. Match each fractions or decimal with its equivalent fraction with a denominator of 100.

.... $\frac{6}{10}$

.... $\frac{6}{5}$

.... 3.5

.... $\frac{4}{10}$

.... 0.2

.... 0.02

a $\frac{350}{100}$

b $\frac{20}{100}$

c $\frac{60}{100}$

d $\frac{2}{100}$

e $\frac{40}{100}$

f $\frac{120}{100}$

SUB-UNIT

2 Percentages

Narrative Connections

What can a corpse teach us about governing?

Wander the streets of Central London long enough and you will come across University College London. Within its walls, you'll find a curious sight at the Student Center: the preserved corpse of Jeremy Bentham!

Born in 1748, Bentham worked as an attorney and law critic. In many ways he was ahead of his time. He wrote extensively, pushing for prison reform, universal suffrage, and the decriminalization of homosexuality. But Bentham's greatest claim to fame is as the father of "utilitarianism."

Utilitarianism is a philosophy that argues that society should act to create the greatest amount of happiness for the greatest number of people. This was a startling revelation in 19th century Europe. European society was organized in a strict class structure. Bentham suggested that a pauper's happiness was worth the same as that of a noble or a prince.

This philosophy helped justify democracy as a form of government. It argued for governments to be responsible for the well-being of all its people.

While unit rates are useful for representing things that change, describing the change per one unit is less practical when looking at groups of a population. For example, to identify an issue's supporters and critics, there is another kind of ratio we can use—per one hundred. In the next lessons, you will see how this does a better job at representing the different individuals within society.

Unit 3 | Lesson 8

What Are Percentages?

Let's learn about percentages.

Warm-up Dollars and Cents

Mentally solve each problem. Be prepared to explain your thinking.

1. How many cents are in one dollar?

2. How many dollars are in one cent?

3. A sticker costs 25 cents. How many dollars is that?

4. A pen costs 1.5 dollars. How many cents is that?

Log in to Amplify Math to complete this lesson online.

Name: ____________ Date: ____________ Period: ____________

Activity 1 Coins

1. Complete the table to show the values of these U.S. coins.

	Penny	Nickel	Dime	Quarter	Half-dollar	Dollar
Fraction of $1	$\frac{1}{100}$					
Value ($)	0.01					
Value (cents)	1					

A quarter is worth 25 cents, and a dollar is worth 100 cents. You can also say that a quarter is worth 25% of a dollar. The % symbol is read as "percent."

2. Complete each statement with the missing percent value.

   a. A penny is ________ % of a dollar.
   b. A nickel is ________ % of a dollar
   c. A dollar is ________ % of a dollar

3. Write the name of the coin that matches each expression.

   a. 10% of a dollar ____________
   b. 50% of a dollar ____________

4. 5 nickels is what percent of a dollar? Explain your thinking.

Are you ready for more?

Determine how to make 120% of a dollar by using:

1. the fewest number of coins

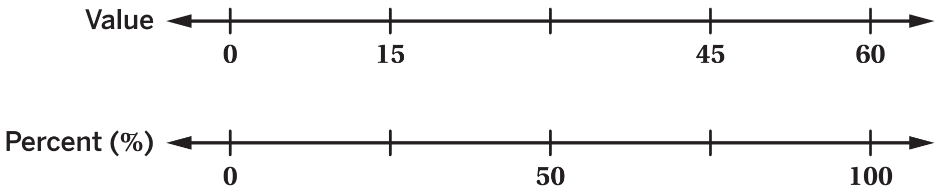
2. the most coins

3. two sets of coins, where the same type of coin is not in both sets

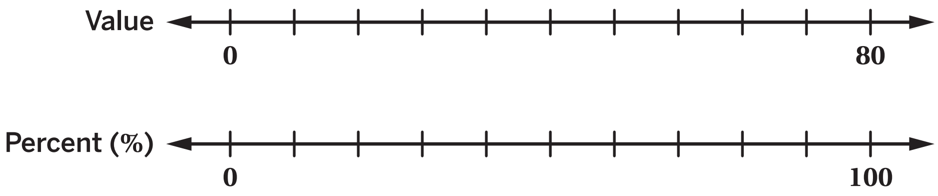
Activity 2 Using Double Number Lines

1. Complete the double number lines by filling in the missing values.

a



b



2. Use either double number line from Problem 1 to write two complete percentage statements.

a .... is .... % of ....

b .... % of .... is ....

3. Use your double number lines to determine a reasonable estimate for each statement. Explain your thinking.

a 35% of 60

b 35% of 80

STOP

Name: .... Date: .... Period: ....

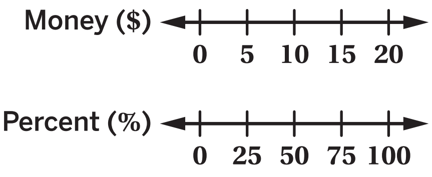
Summary

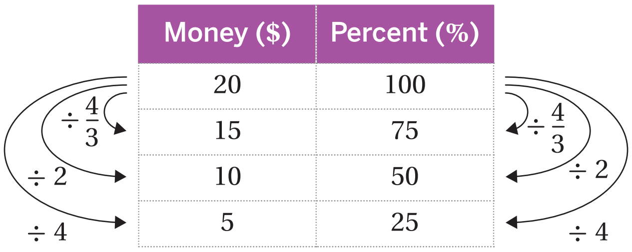
In today's lesson . . .

You saw that a ***percentage*** is a rate per 100. A percentage tells you how much of a quantity you have in comparison to a fixed amount, often a whole or total. For example, if you take a total of \$20 to the mall, then you say that this amount is 100 percent of your spending money. This can also be written as 100%.

If you spend some money and have \$10 left, then you can determine the percentage left of the original total by determining an equivalent ratio to $10 : 20$ in the form $x : 100$. The equivalent ratio is $50 : 100$, which means you have 50% of your money left. Notice this is similar to determining the unit rate (the rate per 1), which is the equivalent ratio that looks like $y : 1$ (and in this example, would be $\frac{1}{2} : 1$).

The double number line diagram and table shown here both include some other percentages of \$20. Notice that 20 dollars and 100 percent are aligned, so you can also see that $20 : 100$ is equivalent to $10 : 50$. This means that if \$20 is 100%, then \$10 is 50%.





Money (\$)	Percent (%)
20	100
15	75
10	50
5	25

Reflect:

Name: .... Date: .... Period: ....

1. What percent of a dollar is represented by the value of each combination of coins?

   a 4 dimes

   b 1 nickel and 5 pennies

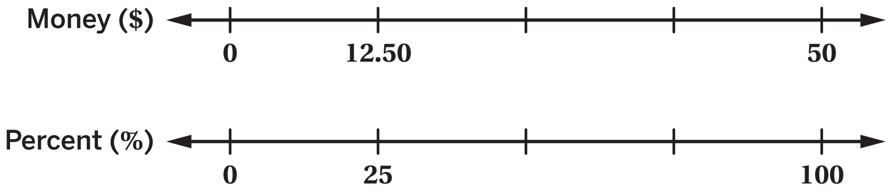
   c 2 quarters and 1 dime

2. List *two* different combinations of coins that make each percent of a dollar.

   a 30% of a dollar

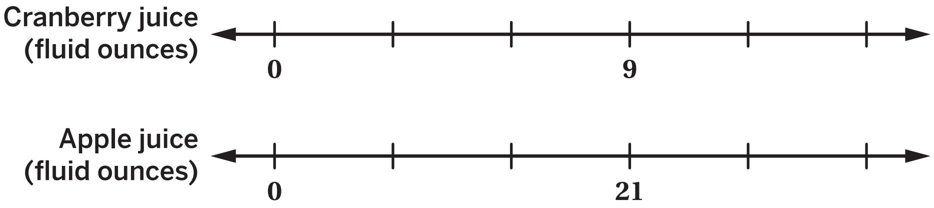
   b 70% of a dollar

3. Complete the double number line to show dollar amounts corresponding to different percents of $50.

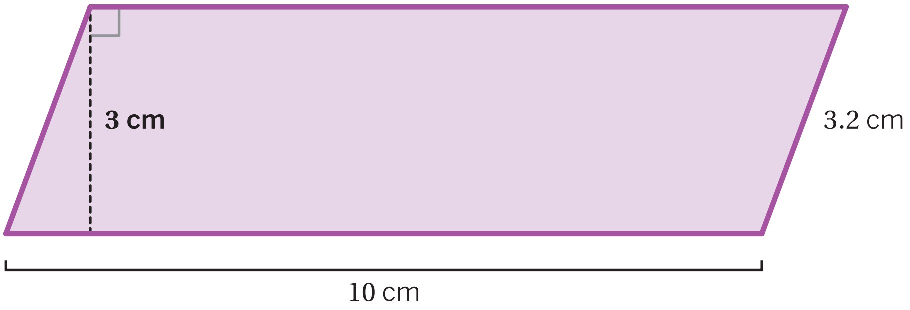


Name: ______________________ Date: __________ Period: ________

4. Tyler makes his favorite juice blend by mixing cranberry juice with apple juice in the ratio shown by the double number line. Complete the double number line to show smaller and larger batches that would taste the same as Tyler's juice blend.



5. Determine the area of the parallelogram.



6. Each word names a type of denominator. For each, write three fractions with the given denominator:

- a fraction that is less than 1
- a fraction that is equal to 1
- a fraction that is greater than 1

a Thirds

b Eighths

c Hundredths

Unit 3 | Lesson 9

Determining Percentages

Let's determine percentages in general.

Warm-up Comparing Snowfall

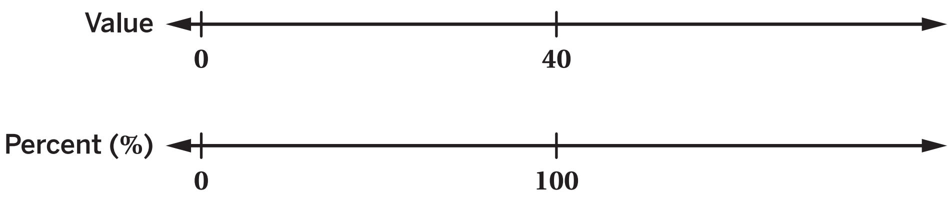
Vail, Colorado, normally gets an average of 189 in. of snow per year. During the 2019–2020 ski season, 229 in. of snow fell in Vail. Use percentages to describe how the 2019–2020 total snowfall compares to the average snowfall.

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 Determining the Percentage

1. To complete parts a–c, compare each value to 40 by using percentages. Use the double number line to show your thinking.

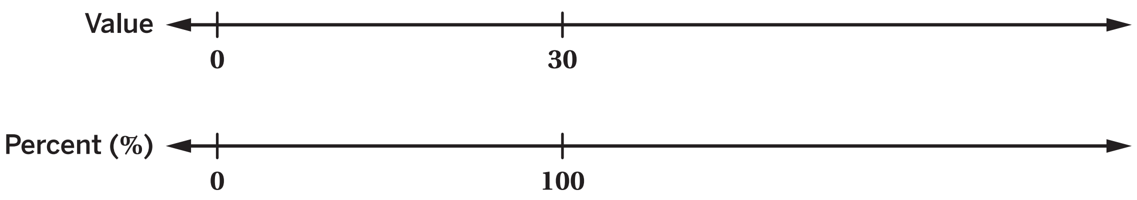


a 20 is what percent of 40?

b 4 is what percent of 40?

c 80 is what percent of 40?

2. To complete parts a–c, compare each value to 30 by using percentages. Use the double number line to show your thinking.



a 3 is what percent of 30?

b 75 is what percent of 30?

c 3.3 is what percent of 30?

3. Find the missing fractions for parts a–b. Then complete part c.

a 12 is what fraction of 18?

b 18 is what fraction of 12?

c Consider comparing each pair of numbers from parts a and b by using percentages. Would you expect the percentage to be less than, greater than, or equal to 100%? Explain your thinking.

Activity 2 Dance Marathon

Plan ahead: How will you motivate yourself to persist even if a pattern is not immediately evident?

The Student Council hosted a Dance Marathon to raise money for the local public library. The number of hours each of four students spent dancing is shown in the table.

1. What percent of Diego's dancing time did each student dance? Complete the table.

	Time spent dancing (hours)	Fraction of Diego's time	Fraction of Diego's time as division	Fraction written as a decimal	Percent (%) of Diego's time
Diego	20	$\frac{20}{20}$	$20 \div 20$	1.00	100%
Jada	15				
Lin	24				
Noah	9				

2. What patterns do you notice?

3. Write an expression to show how to calculate what percent c hours is of Diego's time.

4. What percent of Jada's time did Lin dance? Show or explain your thinking.

Are you ready for more?

1. When is 8 less than 100%? More than 100%?

2. When is 8 less than 50%? More than 50%?

STOP

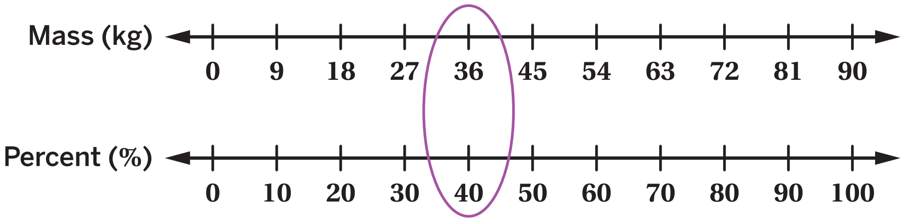
Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You applied your understanding of ratios to determine what percent one amount is relative to another amount. For example, suppose an adult weighs 90 kg and a child weighs 36 kg. To determine the child's weight as a percent of the adult weight, you can use multiple methods:

Double number lines



Ratio tables

	Mass (kg)	Percent (%)	
	90	100	
$\times \frac{1}{90}$	1	$\frac{1}{90} \times 100$	$\times \frac{1}{90}$
$\times 36$	36	$\frac{36}{90} \times 100$	$\times 36$

- Determine the unit rate (what percent matches 1 kg).
- Use the unit rate to determine the percent that corresponds with 36 kg.

Expressions

$$36 \div 90 \cdot 100 = \frac{36}{90} \cdot 100 = 40$$

Evaluate $\frac{p}{w} \cdot 100$, to determine the percent that one value p is of another value w.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A sign in front of a roller coaster says, “You must be 40 in. tall to ride.” What percent of this height is:

   a 34 in.?

   b 54 in.?

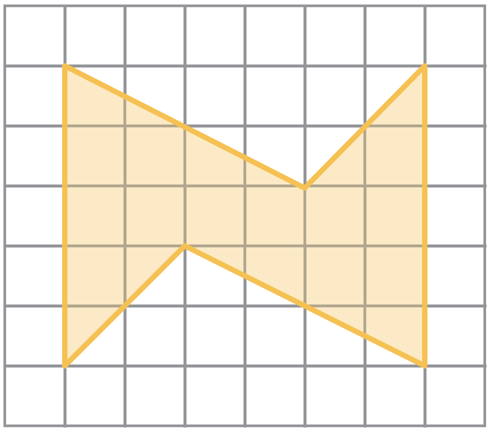
2. At a hardware store, a tool set normally costs \$80. During a sale this week, the tool set costs \$12 less than normal. What percent of the normal price represents the savings? Show or explain your thinking.

3. A bathtub can hold 80 gallons of water. The faucet flows at a rate of 4 gallons per minute. What percent of the bathtub’s capacity will be filled after 6 minutes? Show or explain your thinking.

Name: .... Date: .... Period: ....

Practice

4. Determine the area of this shape. Show your thinking.



5. Elena is 56 in. tall. **Note:** 100 in. = 254 cm.

a What is her height in centimeters? Show or explain your thinking.

b What is her height in meters? Show or explain your thinking.

6. Evaluate each product.

a $\frac{1}{4} \cdot 4$

b $0.25 \cdot 40$

c $\frac{3}{4} \cdot 400$

d $0.10 \cdot 444$

Unit 3 | Lesson 10

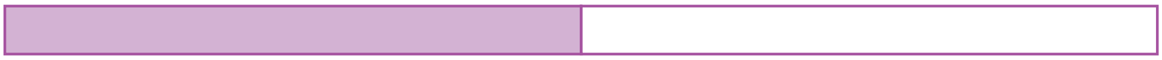
Benchmark Percentages

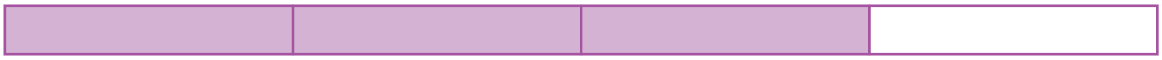
Let’s use fractions to make sense of some common percentages.

Warm-up What Percentage Is Shaded?

What percent of each diagram is shaded? Be prepared to explain your thinking.

1. 

2. 

3. 

Name: ________________ Date: ________ Period: ________

Activity 1 Using Benchmark Percentages

Think about the strategies you can use to solve Problems 1 and 2. Discuss the strategies with your partner, and when you agree, solve each problem. Be prepared to explain your thinking.

1. Determine each percent of 900.

- **a** 50%
- **b** 5%
- **c** 1%
- **d** $\frac{1}{2}\%$
- **e** 63%

2. Determine each percent of 50.

- **a** 50%
- **b** 5%
- **c** $\frac{1}{5}\%$
- **d** 110%

3. How can you determine 115% of any number?

4. Is $\frac{3}{5}$ the same as $\frac{3}{5}\%$? Explain your thinking.

Are you ready for more?

Is the following statement true or false? Explain your thinking.

If $21 : 28$ is equivalent to $x : 100$, and $30 : 40$ is equivalent to $y : 100$, then x is equal to y.

Activity 2 Student Pet Owners

Tyler surveyed all 400 students in his school to determine how many students own different types of pets. Some students own more than one pet. Here are the results of the survey.

- **4 students own a reptile.**
- **20 students own a bird.**
- **40 students own another type of small animal (gerbil, rabbit, etc.).**
- **50 students own a fish.**
- **100 students own a cat.**
- **200 students own a dog.**

1. What percent of students own each type of pet? Show your thinking.

	Reptile	Bird	Small animal	Fish	Cat	Dog
Percent (%)						

Name: ______ Date: ______ Period: ______

Activity 2 Student Pet Owners (continued)

2. Assume the school's percentages are representative of all middle schoolers in the entire school district, meaning pet ownership occurs at the same rates. If there are 1,100 middle school students in the district, how many students are expected to own each type of pet? Show your thinking.

	Reptile	Bird	Small animal	Fish	Cat	Dog
Number of Students						

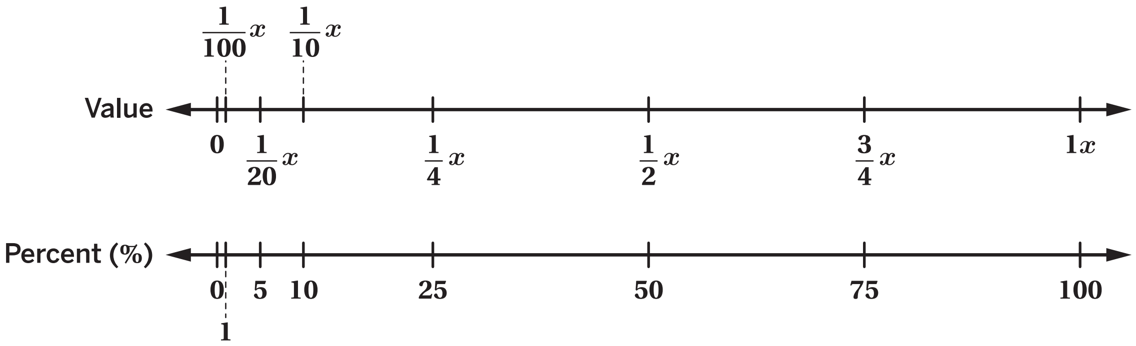
STOP

Summary

In today's lesson . . .

You extended your understanding of benchmark fractions to represent *benchmark percentages*. Applying ratio thinking to benchmark percentages and fractions can help you to estimate and calculate with percentages.

For example, if x represents a number, then it has a value equal to 100%. This double number line shows the relationship between some benchmark fractions and percentages.



Any non-unit fraction with those denominators can be related to multiples of those percentages, such as $\frac{7}{10}$ and 70%, or $\frac{3}{20}$ and 15%. Percents of the same whole can also be added. For example, 15% is both $3 \cdot 5\%$ and $5\% + 5\% + 5\%$.

In general, any whole number percent of a number can be determined because it is just a multiple of 1% of that number.

Reflect:

Name: .... Date: .... Period: ....

1. Explain how you could mentally calculate each quantity described.

a 25% of any number

b Andre lives 1.6 km from school. What is 10% of 1.6 km?

c Diego lives $\frac{1}{2}$ miles from school. What is 50% of $\frac{1}{2}$ miles?

2. Explain how you could mentally calculate each quantity described.

a 15 is what percent of 30?

b 3 is what percent of 12?

c 6 is what percent of 10?

3. Noah says that, to determine 20% of a number, divide the number by 5.
For example, he says that 20% of 60 is 12 because $60 \div 5 = 12$.
Does Noah's method always work? Explain your thinking.

Practice

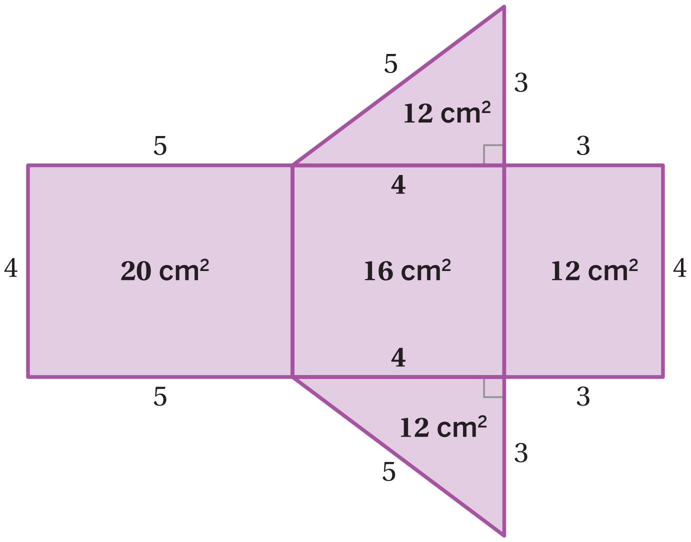
Name: .... Date: .... Period: ....

4. Andre paid $13 for 3 books. Diego bought 12 books priced at the same rate. How much did Diego pay for the 12 books? Show or explain your thinking.

5. Jada drew a net for a polyhedron and determined the area of each face. Her work is shown.

a What polyhedron can be assembled from this net?

b Jada made some mistakes in her area calculation. What were the mistakes?



c Determine the surface area of the polyhedron. Show your thinking.

6. Determine whether each product will be *less than*, *greater than*, or *equal* to 40.

a $\left(\frac{6}{4}\right) \cdot 40$

b $\left(\frac{8}{8}\right) \cdot 40$

c $\left(\frac{1}{2}\right) \cdot 40$

Name: .... Date: .... Period: ....

Unit 3 | Lesson 11

This Percent of That

Let’s determine any percent of any number.

Warm-up Fundraising Goals

Three friends — Lin, Jada, and Andre — each had the goal of raising $40 for the animal shelter. How much money did each person raise? Be prepared to explain your thinking.

1. Lin raised 100% of her goal.

2. Jada raised 50% of her goal.

3. Andre raised 150% of his goal.

Log in to Amplify Math to complete this lesson online.

Activity 1 Puppies Grow Up

1. Jada adopted a new 3-month-old puppy from the shelter. The vet says that the puppy will grow to weigh 45 lb as an adult. Refer to the chart and determine a possible weight for Jada's puppy at each of the given ages.

Age (months)	Percent of adult weight (%)	Possible weight (lb)
3	21–24	
4	32–37	
6	48–55	
10	87–93	
12	100	45

2. Andre also adopted a 3-month-old puppy from the shelter and it weighs 9 lb. The vet says that this puppy is now at about 30% of its adult weight. Refer to the chart and determine how much Andre's puppy weighed at each of the given ages. Record the weights as fractions or decimals to the nearest tenth of a pound.

Age (months)	Percent of adult weight (%)	Weight (lb)
3	30	9
6	43	
10	95	
12	100	30

3. Did either puppy grow at a constant rate of weight per month? Explain your thinking.

Name: ________ Date: ________ Period: ________

Activity 2 What School Does Everyone Come From?

Elena's middle school is attended by students who attended three different elementary schools. She wants to know the percentages of students that attended each elementary school. This table shows how many students attended each elementary school.

Elementary school	Number of students	Percent of students (%)
Susan B. Anthony ES	143	
Annie E. Harper ES	286	
Miguel Trujillo ES	121	
All	550	100

1. Complete the table to show the percent of students that attended each of the three elementary schools.

2. This tape diagram represents all 550 students at Elena's middle school. Partition and label the tape diagram to show the percent of students from each of the elementary schools. The size of each part can be approximate, but should accurately reflect the relative amounts of students.

3. There are 850 students in the high school who also attended Elena's middle school. The percents of those students who attended each of the elementary schools are the same as those you determined in Problem 1. Complete the table to show how many high school students attended each of the elementary schools.

Elementary school	Number of students	Percent of students (%)
Susan B. Anthony ES		
Annie E. Harper ES		
Miguel Trujillo ES		
All	850	100

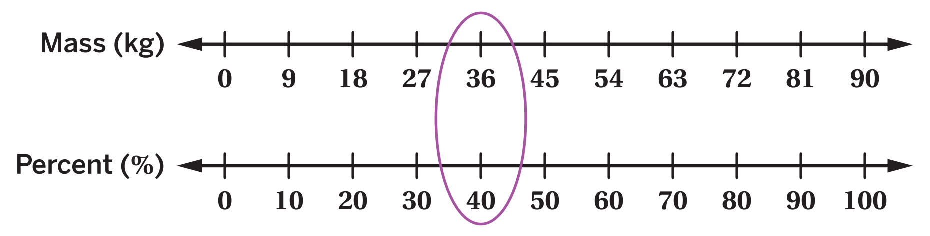
STOP

Summary

In today's lesson . . .

You applied your understanding of ratios and percentages to determine what part of a whole, or total, corresponds to a given percentage. For example, suppose an adult weighs 90 kg and a child weighs 40% of the adult's weight. To determine the child's weight, you can use multiple methods:

Double number lines



Ratio tables

	Weight	Percent	
	90	100	
$\times \frac{1}{100}$	$\frac{90}{100}$	1	$\times \frac{1}{100}$
$\times 40$	$\left(\frac{90}{100}\right) \times 40$	40	$\times 40$

- Determine what value corresponds with 1%.
- Multiply the corresponding value by the given percentage.

Expressions

$\frac{40}{100} \cdot 90 = 36$

Evaluate $\frac{n}{100} \cdot w$, to determine $n\%$ of w.

Reflect:

Name: .... Date: .... Period: ....

1. Solve each problem. Show or explain your thinking.

a During basketball practice on Monday, Mai attempted 40 free throws and she made 25% of them. How many free throws did she make?

b On Tuesday, Priya made 12 free throws in practice. On Wednesday, she made 150% as many free throws in practice as she made on Tuesday. How many free throws did Priya make in practice on Wednesday?

2. A 16-oz bottle of orange juice says it contains 200 mg of vitamin C, which is 250% of the daily recommended allowance of vitamin C for adults. What is 100% of the daily recommended allowance of vitamin C for adults? Show or explain your thinking.

3. Select *all* of the expressions that could be used to determine 80% of x.

A. $\frac{8}{100} \cdot x$

B. $\frac{8}{10} \cdot x$

C. $\frac{8}{5} \cdot x$

D. $80 \cdot x$

E. $(0.8) \cdot x$

F. $\frac{80}{100} \cdot x$

G. $\frac{4}{5} \cdot x$

H. $8 \cdot x$

I. $\frac{4}{10} \cdot x$

J. $(0.08) \cdot x$

Name: .... Date: .... Period: ....

4. Diego owns a skateboard, a scooter, a bicycle, and a go-cart. He wants to know which vehicle travels the fastest. A friend records how far Diego travels while riding each vehicle as fast as he can along a straight, level path for 5 seconds. The table shows the results.

Vehicle	Distance traveled
Skateboard	90 ft
Scooter	1,020 in.
Bicycle	48 m
Go-cart	0.3 km

a What is the distance he traveled, in centimeters, while riding each vehicle for 5 seconds?

b List the vehicles in order from fastest to slowest.

5. It takes 10 lb of raw potatoes to make 12 lb of mashed potatoes. At this same rate:

a How many pounds of mashed potatoes can be made with 15 lb of raw potatoes? Show or explain your thinking.

b How many pounds of raw potatoes would be needed to make 60 lb of mashed potatoes? Show or explain your thinking.

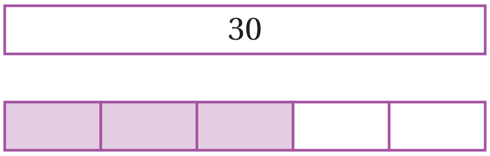
6. Match each expression with its corresponding tape diagram.

.... $\frac{3}{5} \cdot 30$

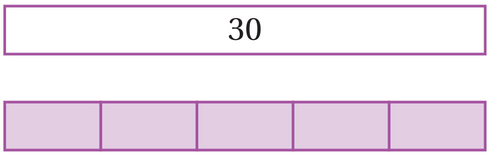
.... $\frac{1}{3} \cdot 5$

.... $\frac{5}{3} \cdot 30$

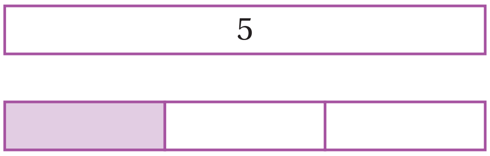
a



b



c



Name: ______________ Date: ______________ Period: ______________

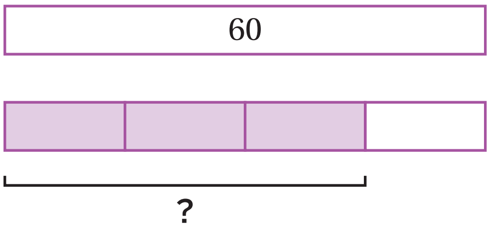
Unit 3 | Lesson 12

This Percent of What

Let’s use tape diagrams to represent percentages and to determine an unknown whole.

Warm-up Notice and Wonder

What do you notice? What do you wonder?



1. I notice . . .

2. I wonder . . .

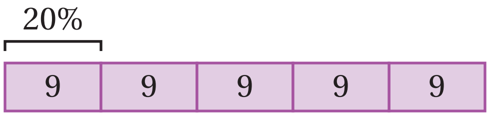
Log in to Amplify Math to complete this lesson online.

Activity 1 Actual and Predicted Weights

Lin has a new puppy that weighs 9 lb. It currently weighs about 20% of its predicted adult weight. Noah has a dog that currently weighs 90 lb. Its predicted adult weight is 72 lb.

	Current weight (lb)	Predicted adult weight (lb)
Lin's puppy	9	?
Noah's dog	90	108

1. Lin drew the following diagram to represent the weight of her puppy.



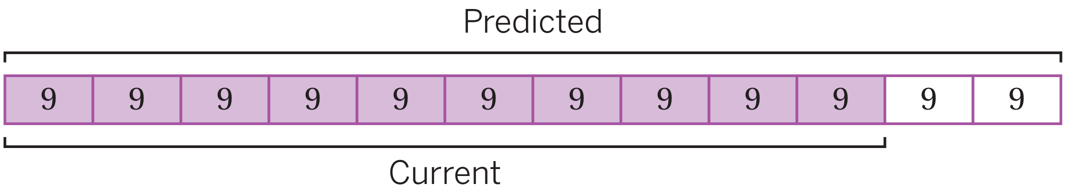
a. What is the predicted adult weight of Lin's puppy? How can you see that in the diagram?

b. What fraction of its predicted adult weight will Lin's puppy be when it weighs 27 lb? How can you see that in the diagram?

Name: .... Date: .... Period: ....

Activity 1 Actual and Predicted Weights (continued)

2. Noah drew the following diagram to represent the weight of his dog.



a What percent of Noah's dog's current weight is his dog's predicted weight? How can you see that in the diagram?

b Use percentages to compare the current weights of Noah's dog and Lin's puppy. How does the diagram show that comparison?

c Use percentages to compare the predicted adult weights of Noah's dog and Lin's puppy. How does the diagram show that comparison?

Activity 2 Multilingual Middle-Schoolers

Some middle-school students wanted to investigate how many of their classmates speak a language *other* than English outside of school. A survey was given to all of the sixth, seventh, and eighth grade students in the school.

1. Of the surveys returned by eighth graders, 54 responses indicated that they spoke a language other than English outside of school.

a If this represents 60% of the eighth grader's responses, how many eighth graders responded to the survey? Show or explain your thinking.

b If 45% of all the eighth graders responded to the survey, how many eighth graders in total are in the school? Show or explain your thinking.

2. Of the surveys returned by seventh graders, 48 responses indicated that they spoke a language other than English outside of school.

a If this represents 64% of the seventh grader's responses, how many seventh graders responded to the survey? Show or explain your thinking.

Name: ________________ Date: ________ Period: ________

Activity 2 Multilingual Middle-Schoolers (continued)

b If 30% of all the seventh graders responded to the survey, how many seventh graders in total are there in the school? Show or explain your thinking.

3. Of the surveys returned by sixth graders, 32 responses indicated that they spoke a language other than English outside of school.

a If this represents 40% of the sixth grader's responses, how many sixth graders responded to the survey? Show or explain your thinking.

b If 32% of all the sixth graders responded to the survey, how many sixth graders in total are there in the school? Show or explain your thinking.

Critique and Correct: Your teacher will present an incorrect statement about this situation. With a partner, determine why it is incorrect and then correct it.

Are you ready for more?

Andre is planning to go on a hike with his dog. Decide whether each scenario is possible.

1. Andre plans to bring 150% as much water as he brought on his last hike.
2. Andre plans to drink 150% of the water he brought on the hike.

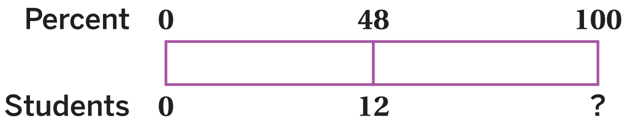
STOP

Summary

In today's lesson . . .

You used tape diagrams to help you to reason about scenarios involving percentages and to determine the unknown "whole" amount when given the percentage and the part.

For example, if you learn that 48% of students in your class packed lunch and that corresponds to 12 students, you can use what you know about ratio relationships to determine how many students are in the class (the value that corresponds with 100%).



The ratio of percent to students is $48 : 12$. To go from the first value (48) to the second value (12), you can multiply by $\frac{12}{48}$.

To get the value that corresponds with 100, you evaluated:

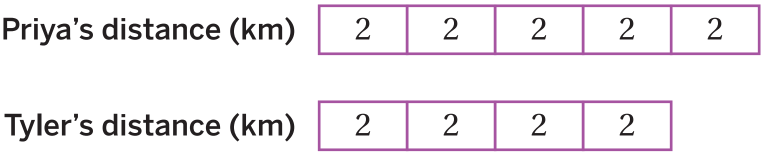
$$100 \cdot \frac{12}{48} = 25$$

In general, to go from the percent n to the corresponding part p, you multiply by $\frac{p}{n}$, so to determine the amount that corresponds to 100% (the whole, w), you can use the relationship $\frac{p}{n} \cdot 100 = w$.

Reflect:

Name: ________ Date: ________ Period: ________

1. This tape diagram shows how far two students walked.



a What percent of Priya's distance did Tyler walk?

b What percent of Tyler's distance did Priya walk?

2. A bakery makes 40 different varieties of muffins. 25% of the flavors have cinnamon as one of the ingredients. Draw a tape diagram to show how many varieties have cinnamon and how many do not have cinnamon.

3. There are 28 sixth graders and 14 seventh graders in the middle school band. The sixth graders make up 40% of the band members, the seventh graders make up 20% of the band members, and the rest of the band members are eighth graders.

a What percent of the band members are eighth graders? Show or explain your thinking.

b How many total members are there in the middle school band? Show or explain your thinking.

Name: ________________ Date: ________ Period: ________

4. Which is a better deal per ticket: 5 tickets for $\$12.50$ or 8 tickets for $\$20.16$? Show or explain your thinking.

5. An athlete runs 8 miles in 50 minutes on a treadmill. At this rate:

   a How long will it take the athlete to run 9 miles? Show or explain your thinking.

   b How far can the athlete run in 1 hour? Show or explain your thinking.

6. Evaluate each product or quotient.

   a $4.5 \cdot 10$

   b $4.5 \div 10$

Name: .... Date: .... Period: ....

Unit 3 | Lesson 13

Solving Percentage Problems

Let’s solve more percentage problems.

Warm-up Number Talk

Mentally evaluate each expression.

1. $0.23 \cdot 100$

2. $50 \div 100$

3. $145 \cdot \frac{1}{100}$

4. $0.07 \cdot 100$

Log in to Amplify Math to complete this lesson online.

Activity 1 Reporting on Audience Size

You are a reporter for your school newspaper, writing a series of articles on attendance of different school events. You know that the most recent music concert was attended by 288 people. Use the attendance information gathered by sources for three other events to respond to each of the editor's requests and then write a headline for an article about each event.

1. **Source:** "Concert attendance was about 70% of the basketball game."
   Editor: "How many people could have attended the basketball game?"

   Headline: ....

2. **Source:** "Attendance for the drama play was 360."
   Editor: "Compare the drama play's attendance to the music concert's attendance by using a percentage."

   Headline: ....

3. **Source:** "Attendance for literacy night was 75% of the attendance for the drama play."
   Editor: "How many people attended literacy night?"

   Headline: ....

Reflect: How do you evaluate your use of percentages in the headlines? How did your headlines reflect ethical responsibility?

Name: ____________ Date: ________ Period: ________

Activity 2 What's the Better Deal?

You and your partner are contestants on a new game show. In each of four rounds, you will be presented with two options describing different deals on the same item.

Your goal is to choose the option that is the better deal. Once you come to a decision together, you must *explain* your choice to the host (while riding a unicycle backwards across a tightrope and juggling blobs of oobleck). The host will then award you a prize card based on your explanation and choice.

After you complete all four rounds, your final prize will be revealed!

	Option 1	Option 2	Which would you choose?
1.	An item costs $99.95 at Store A. There is a coupon for 25% off the price of the item.	The same item costs $109.95 at Store B. There is a coupon for 30% off the price of the item.	☐ Store A ☐ Store B
2.	An item normally costs $375, but due to a generous donation from a nearby middle school, the cost is reduced to $75. What percent is $75 of the original cost?	An item costs $25 at a store. The sale price is $22.50 What percent is the sale price of the original cost?	☐ Price reduction ☐ Sale

Activity 2 What's the Better Deal? (continued)

	Option 1	Option 2	Which would you choose?
3.	An item costs $30 at Store C. There is a sale for "Buy 1, Get 1 Half-Off." Two items are bought.	A similar item costs $32 at Store D. There is a sale for, "Buy two, get 33% off." Two items are bought.	☐ BOGO Half Off ☐ Percent-Off Sale
4.	If a 6-month supply of an item is bought at a store, there is a $20 mail-in rebate. The price for one month is $11.33.	The online price of one month's supply of the same item is $19.24. If you buy 6, you receive 50% off.	☐ Mail-in Rebate ☐ Online

Are you ready for more?

You want to repaint all the walls in a room. All corners are right angles. The east wall is 3 yd long. The south wall is 10 ft long, but has a window, 5 ft by 3 ft, that will not be painted. The west wall is 3 yd long, but has a door, 7 ft tall by 3 ft wide, that will not be painted. The north wall includes a closet, 6.5 ft wide, with floor-to-ceiling mirrored doors that will not be painted. The ceiling is 8 ft high.

1. If you paint all the walls in the room, how many square feet do you need to cover?
2. 2 qt of paint will cover 175 ft^2. You need to apply 2 coats of paint. How much paint will you need to buy?
3. Paint can only be purchased in 1-qt or 1-gallon containers. How much will the paint cost if it costs $10.90 per quart and $34.90 per gallon?
4. You have a coupon for 20% off all quart-sized paint cans. How does that affect the cost of the project?

STOP

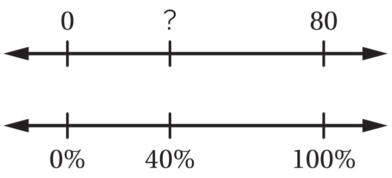
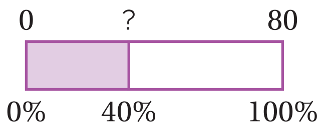
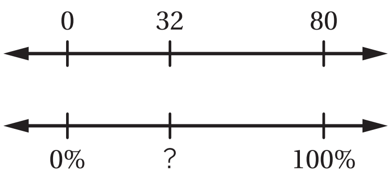
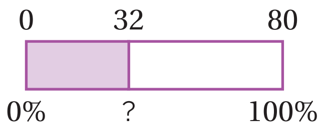
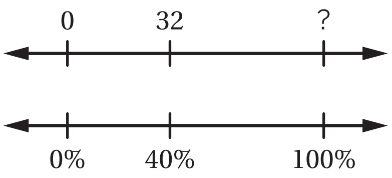
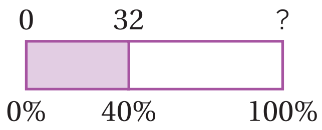
Name: ______________________ Date: __________ Period: ________

Summary

In today's lesson . . .

You solved three different types of percentage problems: determining a missing part, a whole, or a percentage. In all these cases, you applied your understanding of equivalent ratios, where $part : whole$ is equivalent to $percent : 100$.

To solve such problems you can use double number lines, tape diagrams, or algorithms to determine the missing value.

	Double number line	Tape diagram	Algorithm
Part			$80 \cdot \frac{40}{100} = 32$
Percent			$\frac{32}{80} \cdot 100 = 40$
Whole			$\frac{32}{40} \cdot 100 = 80$

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Determine each missing value. Show or explain your thinking.

a 160 is what percent of 40?

b 40 is 160% of what number?

c What number is 40% of 160?

2. A store is having a 20%-off sale on all merchandise. If Mai buys one item and saves $\$13$, what was the original price of her purchase? Show or explain your thinking.

3. To determine what number is 40% of 75, Priya calculates $\frac{2}{5} \bullet 75$.

a Does Priya's calculation give the correct value for 40% of 75? Show or explain your thinking.

b If x represents a number, does $\frac{2}{5} \bullet x$ always represent 40% of that number? Explain your thinking.

Name: .... Date: .... Period: ....

4. An ant travels at a constant rate. It can travel 30 cm every 2 minutes.

a At what rate does the ant travel per centimeter? Show or explain your thinking.

b At what rate does the ant travel per minute?

5. Use the diagrams of these parallelograms and their given areas to determine the missing length (labeled with a “?”) indicated on each parallelogram.

Figure A

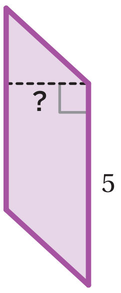


Figure B

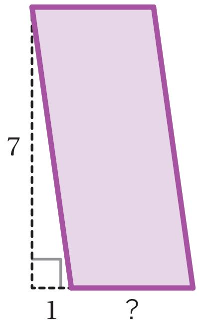
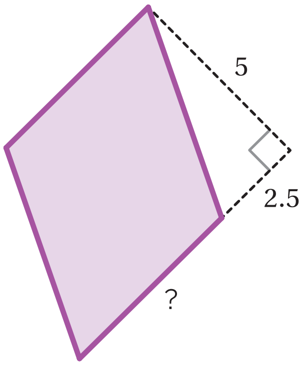


Figure C



a Figure A has an area of 10 square units.

b Figure B has an area of 21 square units.

c Figure C has an area of 25 square units.

6. Consider the numerals in the number 1.5 million.

a What value does the 1 represent?

b What value does the 5 represent?

c Write 1.5 million in standard form.

Unit 3 | Lesson 14

If Our Class Were the World

Let's use percentages to better understand our world.

Warm-up Number String

1. Write 7.8 billion in standard form.

2. Write 7.8 billion in expanded form.

3. Write 0.012 billion in expanded form.

4. Write 7.812 billion in standard form.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 All 7.8 Billion of Us

As of August 2020, there were approximately 7.8 billion people in the world.

If the whole world were represented by a 30-person class:

- 14 people would eat rice as their main food.
- 12 people would be under the age of 20.
- 5 people would live in Africa.

1. What percent of the people in the class would *not* eat rice?

2. What percent of the people in the world would be under the age of 20?

3. Based on the number of people in the class representing people that live in Africa, how many people in the world can be predicted to live in Africa? Show or explain your thinking.

Are you ready for more?

In 1850, there were approximately 1.2 billion people in the world. In 1950, there were approximately 2.6 billion people in the world. The projected world population in 2050 is 9.7 billion. How many people would 1 person in a class of 30 represent in the years 1850, 1950, and 2025?

Activity 2 If Our Class Were the World

Study the tables shown. Each table includes information about one characteristic of the world population.

Suppose your class represents all of the people in the world. In this activity, you will calculate the number of students in your class who would have the same characteristics shown.

Your group will then be assigned one characteristic to represent in a visual display with your choice of diagram(s). Give your display the title "If Our Class Were the World" and include the characteristic.

1. Handedness

	World (in billions)	Percent (%)	Class
Left-handed	0.702		
Right-handed	7.02		
Ambidextrous	0.078		

2. Age

	World (in billions)	Percent (%)	Class
14 and under		25.33	
15–24		15.42	
25–54		40.67	
55 and over		18.58	

3. Home setting

	World (in billions)	Percent (%)	Class
Rural	3.4554		
Urban	4.3446		

Name: ______________________ Date: __________ Period: ________

Activity 2 If Our Class Were the World (continued)

4. Continent

	World (in billions)	Percent (%)	Class
Europe		9.59	
Asia		59.54	
Africa		17.20	
North America		7.60	
South America		5.53	
Oceania		0.55	
Antarctica		0	

5. Most spoken language

	World (in billions)	Percent (%)	Class
English		16.5	
Hindi		8.3	
Mandarin Chinese		14.6	
Spanish		7	
French		3.6	
Arabic		3.6	
Other		46.4	

STOP

Summary

In today's lesson . . .

You looked at several characteristics of the world's population of about 7.8 billion people. Each of the categories of people for each characteristic can be represented by the actual number of people that it describes. Not only are those numbers often very large, sometimes they are very far apart. When this happens, it is more difficult to compare, or get a sense of, the real differences in the population.

The math that you have studied in the last two units can be helpful when comparing differences in numbers that are large and/or far apart. Ratios, rates, and percentages serve two important purposes:

- They allow you to compare quantities that are on different scales because they describe things in terms of multiplying and dividing, instead of adding and subtracting.
- Rates and percentages, bring everything to the same scale, most commonly with a reference point of either 1 or 100, which makes comparing numbers more straightforward.

To summarize, instead of having to use very large and far apart numbers for different populations in the world, you were able to determine percentages to help you compare the different groups and see what the distribution of people really looks like. You were also able to determine even smaller numbers with a true sense of personal reality behind them, with either equivalent ratios, unit rates, or percentages. This allowed you to see exactly what your class would look like if the ratios of all different types of people were equivalent to those of the world's population.

Reflect:

Name: .... Date: .... Period: ....

1. On a field trip, there are 3 chaperones for every 20 students. There are 92 people on the trip.

   a How many chaperones are on the field trip?

   b How many students are on the field trip?

   c What percent are chaperones?

   d What percent are students?

2. Last Sunday, 1,575 people visited an amusement park. 56% of the visitors were adults, 16% were teenagers, and 28% were children (ages 12 and under). Determine the numbers of adults, teenagers, and children who visited the park last Sunday.

3. Complete each percentage statement.

   a 20% of 60 is ....

   b 25% of .... is 6.

   c .... % of 100 is 14.

   d 50% of 90 is ....

   e 10% of .... is 7.

   f 30% of 70 is ....

Practice

Name: .... Date: .... Period: ....

4. Select *all* of the expressions whose value is greater than or equal to 100.

   A. 120% of 100

   B. 50% of 150

   C. 150% of 50

   D. 20% of 800

   E. 200% of 30

   F. 500% of 400

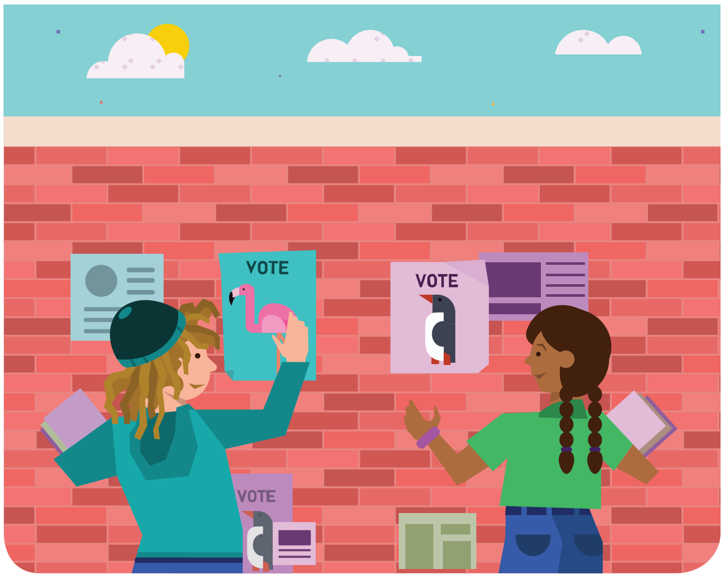
   G. 1% of 1,000

5. Kiran knows that there are 4 qt in 1 gallon. He wants to convert 6 qt to gallons, but cannot remember whether he should multiply 6 by 4 or divide 6 by 4. What should he do? Show or explain your thinking.

6. If there are 5,000 voters, what would be the least number of votes needed for a majority? Explain your thinking.

Name: ______________________ Date: __________ Period: __________

Unit 3 | Lesson 15 – Capstone



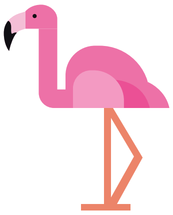
Voting for a School Mascot

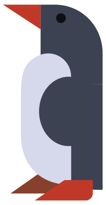
Let's think about different ways of voting.

Warm-up Flamingos and Penguins

After conducting some survey research, a student council worked with the school administration to narrow down their list of possible new school mascots to Flamingos and Penguins.

1. Which mascot would you vote for? Circle one.





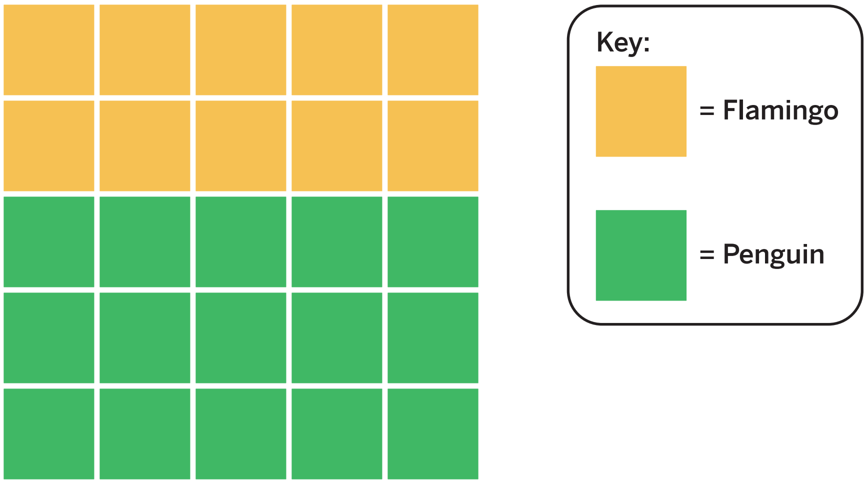
2. Think about how ratios, rates, or percentages could be useful for deciding the winning mascot based on votes. Be prepared to explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 A Game of Zones

The student body will vote next week during homeroom. Each classroom will vote as a group. If most of the students in the classroom vote for the flamingo, then the classroom vote goes to the flamingo. If most of the students vote for the penguin, then the classroom vote goes to the penguin.

The student council is running some test scenarios. The following diagram shows how a sample of 25 classrooms will vote, arranged by their locations in the building.



1. Which mascot will receive the most classroom votes? What percent of classrooms will be voting for this mascot?

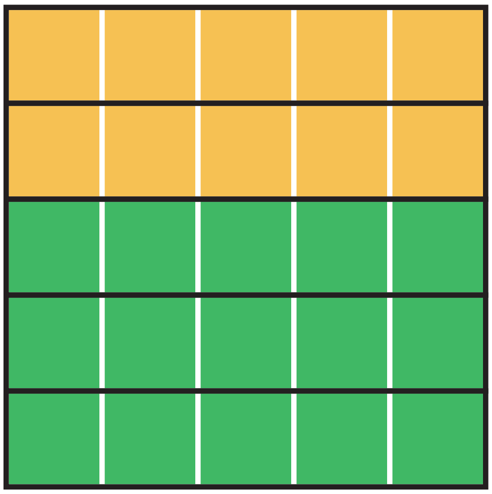
Name: .... Date: .... Period: ....

Activity 1 A Game of Zones (continued)

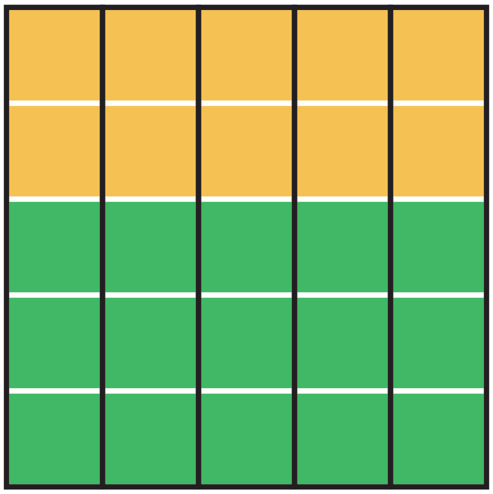
Instead of counting the results of all 25 classrooms, someone suggests that the classrooms can be arranged into 5 zones, with 5 classrooms in each zone. Whichever mascot wins the most classrooms in each zone wins that zone, and the mascot that wins the most zones wins the election.

2. Here are two ways the 5 zones could be defined. For each scenario, determine which mascot would win. Explain your thinking.

a

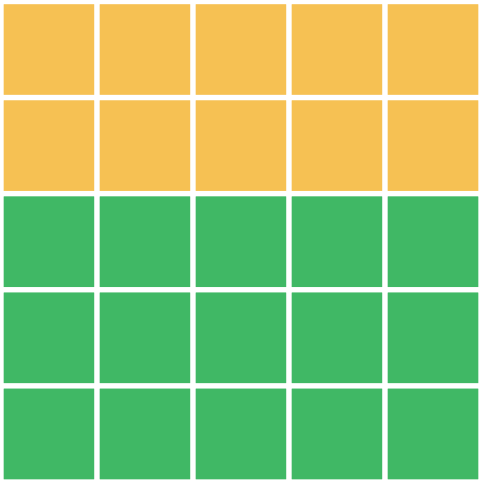


b



3. Show how 5 zones could be defined differently so that each zone still has 5 classrooms but *the Flamingo wins*.

Hint: Each classroom in a zone must be next to each other without any gaps or holes between them.

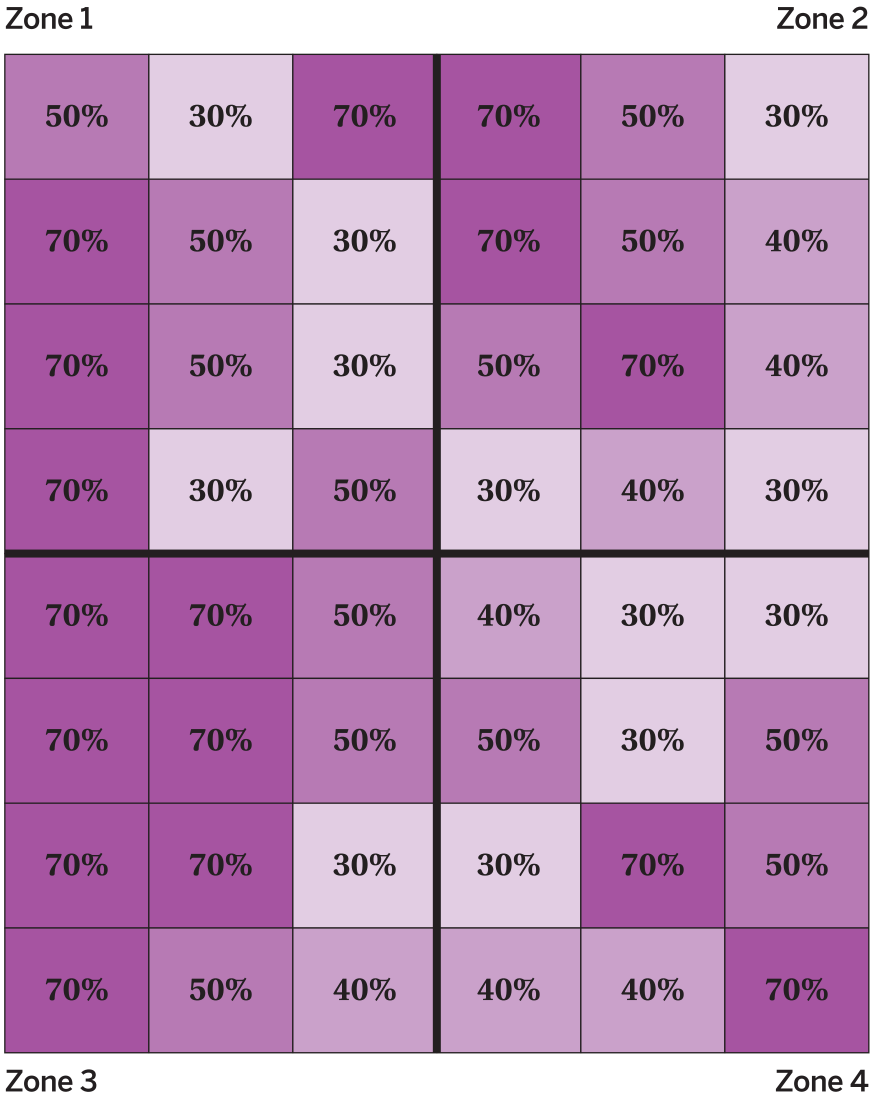


Activity 2 The Mascot Vote

The grid shows the results from the school vote.

- The school was organized into four zones, shown by the heavy black lines on the grid. Each zone gets one vote, based on the individual student votes (*not* the classroom votes) in that zone.
- Every small square on the grid represents a classroom with exactly 30 students who all voted.
- The percentages of the 30 students in each classroom who voted for the Flamingo are shown.

Zoned Voting Grid



50%	30%	70%	70%	50%	30%
70%	50%	30%	70%	50%	40%
70%	50%	30%	50%	70%	40%
70%	30%	50%	30%	40%	30%
70%	70%	50%	40%	30%	30%
70%	70%	50%	50%	30%	50%
70%	70%	30%	30%	70%	50%
70%	50%	40%	40%	40%	70%

Name: ______________________ Date: __________ Period: ________

Activity 2 The Mascot Vote (continued)

Researchers, such as Moon Duchin, use ratios and geometry to understand how arrangements of zones can affect the outcomes of votes.

Next, you will analyze the grid of classroom votes.

1. Use the grid to determine which mascot had more students voting for it. Explain your thinking.

2. Which mascot won more zones? Explain your thinking.

3. You will be given another copy of the grid, but without boundary lines between zones. Using the following Zoning Rules, draw new boundary lines so that the mascot that won the vote in Problem 2 would now lose the vote.

Zoning Rules

1. Each zone must have at least 250 students.
2. There must be exactly four zones.
3. The boundary of each zone must be continuous, with no breaks in the boundary, but the boundary need not be a single straight line.
4. Each classroom must be in one, and only one, zone.

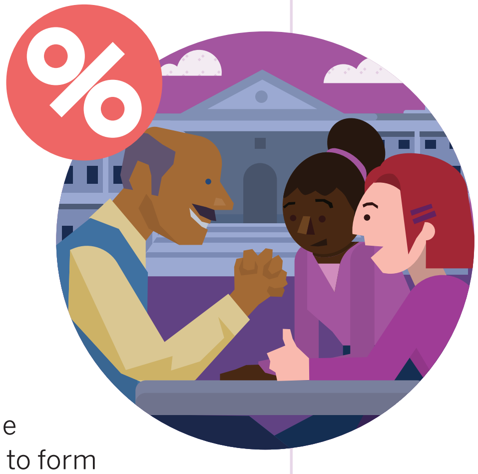
Featured Mathematician

Moon Duchin

Born in Connecticut, Duchin earned a doctorate in mathematics from the University of Chicago. Her research focuses on geometric group theory, low-dimensional topology, and dynamics. Duchin studies applications of geometry and computing to U.S. redistricting, looking at how the shapes of districts (or zones) can affect the outcomes of elections.

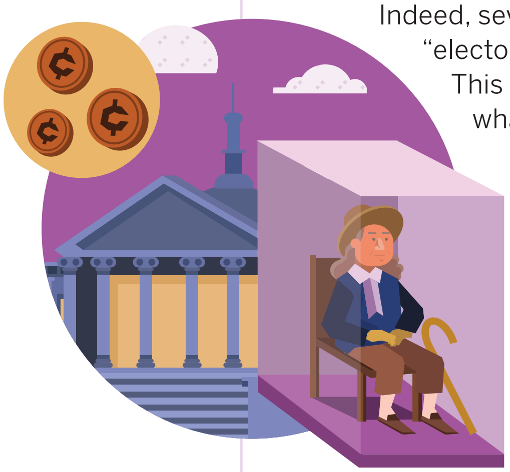
STOP

Unit Summary



Throughout this unit, you have seen how rates and percentages can be used to represent the characteristics and beliefs of populations. For example, writing votes as a percentage can make it clearer which mascot — the Flamingo or the Penguin — won a school-wide vote.

But as you just saw, *how* these votes are counted is also mathematically important. Should every individual's vote count the same? Sometimes, for practical or historic reasons, the votes of many individuals are grouped together to form a larger "zone." The zone is then counted as just one vote. This can empower some voices, but can also turn a vote on its head.



Indeed, several U.S. presidents have won the "electoral vote" while losing the "popular vote." This outcome is mathematically similar to what you saw with the mascot vote. In 1876, 51% of America's voting population cast ballots for Samuel Tilden, while just 48% voted for Rutherford B. Hayes. (If you counted carefully, you might wonder what happened to the other 1%. There happened to be a third candidate, Peter Cooper.) Nevertheless, Hayes won the election, receiving 185 electoral votes to Tilden's 184.

When it comes to representation, rates and percentages are an important part of the story. But they are just part of the story. To see the bigger picture, you must also understand how society uses these numbers to make decisions.

See you in Unit 4.

Practice

Name: .... Date: .... Period: ....

A school is deciding on a school mascot. They have narrowed the choices down to the Banana Slug or the Sea Lion. The principal's plan for deciding is that each of three randomly selected classes will get one vote.

Each class held an election, and the winning choice was the one vote for the whole class. This table shows how the three classes voted.

	Banana Slug	Sea Lion	Class vote
Class A	21	6	Banana Slug
Class B	14	10	
Class C	6	30	

1. Complete the table.

2. Which mascot won, according to the principal's plan? What percent of the class votes did the winning mascot get under this voting method? Show or explain your thinking.

3. Which mascot received the most individual student votes overall? What percentage of all the individual student votes did this mascot receive?

Practice

Name: .... Date: .... Period: ....

4. Han spent 75 minutes practicing the piano over the weekend. Complete each problem and show or explain your thinking.

a Priya practiced the violin for 152% as much time as Han practiced the piano. How long did Priya practice?

b Tyler practiced the clarinet for 64% as much time as Han practiced the piano. How long did Tyler practice?

5. Determine the missing value for each of the following. Then order the values from greatest to least. Show your thinking.

a 55% of what number is 99?

b 300% of what number is 78?

c 12% of what number is 84?

6. A restaurant posts a sign by the front door that states, "Maximum occupancy: 75 people." Determine each percentage and show or explain your thinking.

a What percent of the maximum occupancy is 9 people?

b What percent of the maximum occupancy is 51 people?

c What percent of the maximum occupancy is 84 people?

My Notes:

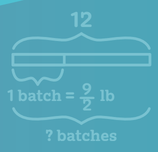
UNIT 4

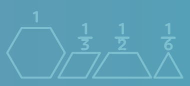
Dividing Fractions

Division can be used to solve equal-sized groups problems, including when the size of a group and even the number of groups are represented by fractions. See how you can apply what you already know about multiplication and division to follow the mysteries within Spöklik Furniture and fraction division.

Essential Questions

- How can dividing by the same fraction be interpreted in two different ways?
- How is dividing by a fraction related to multiplying fractions?
- What does it mean when a quantity represents a fractional number of equal-sized groups?
- *(By the way, how many tanks could a fish tank fill if a fish tank could fill tanks?)*





$$\frac{5}{2} \div \frac{3}{4} = \frac{5}{2} \cdot \frac{4}{3}$$

SUB-UNIT

1

Interpreting Division Scenarios

Narrative: Find a missing friend at Spöklik Furniture, where not everything is what it seems.

You'll learn . . .
- two ways to think about division.
- connections between multiplication, division, and fractions.

SUB-UNIT

2

Division With Fractions

Narrative: Use fractions to build furniture in the Spöklik Showroom with some ghostly companions.

You'll learn . . .
- strategies for dividing fractions.
- how and why these strategies work.

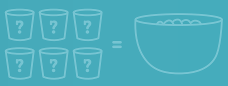
SUB-UNIT

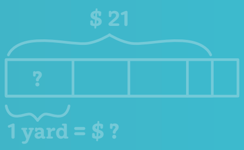
3

Fractions in Lengths, Areas, and Volumes

Narrative: Make your way out of Spöklik Furniture, and measure with fractions as you go.

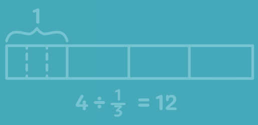
You'll learn . . .
- about fractional measurements.
- to solve measurement problems by dividing fractions.

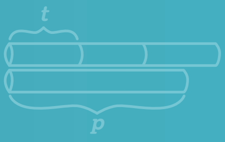


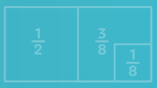


Evaluate:

$$\frac{1}{2} \div \frac{2}{3} \div \frac{3}{4} \div \ . \ . \ . \ \div \frac{59}{60}$$







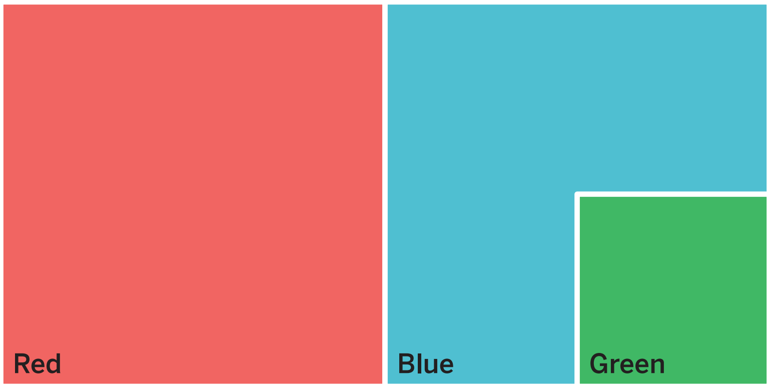
Unit 4 | Lesson 1 – Launch

Seeing Fractions

Let's look for fractions in different patterns.

Warm-up Identifying Fractions

Consider the image. What fractions do you see?



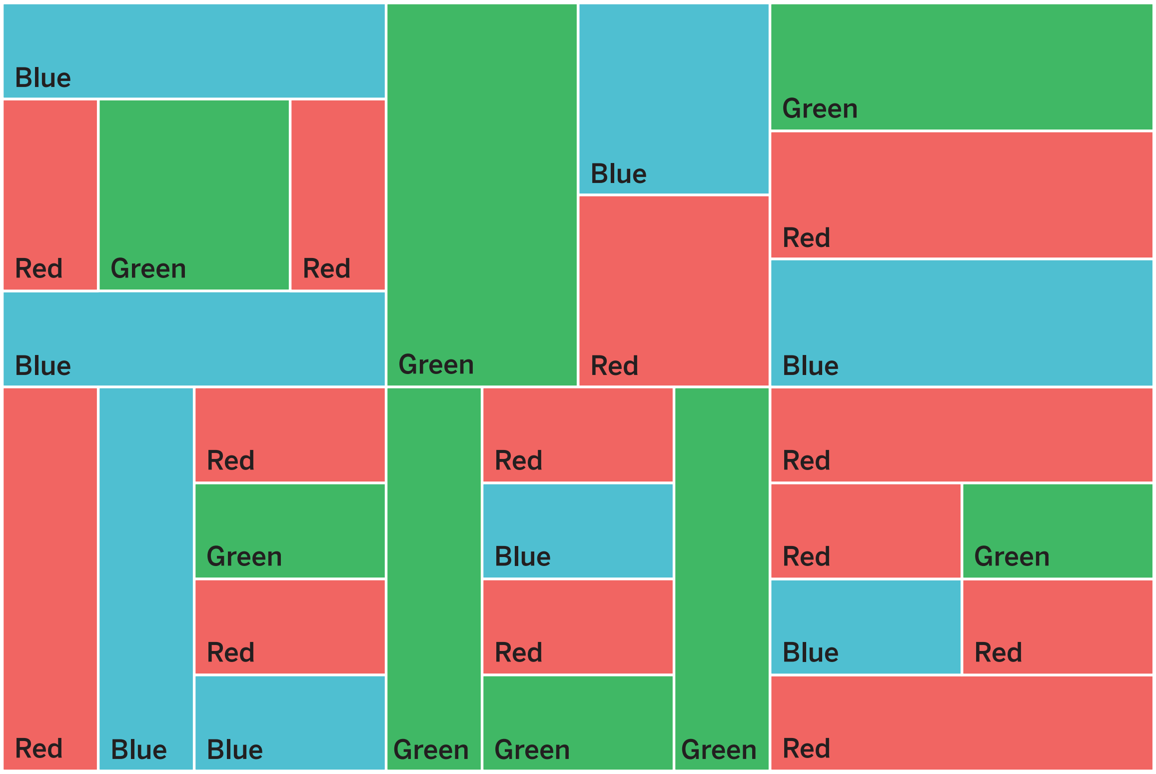
Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 What Fractions Do You See?

Part 1

Consider the image.



1. You will be assigned one of the two fractions below. How many ways can you see the fraction in the image?

a $\frac{1}{12}$

b $\frac{1}{8}$

Activity 1 What Fractions Do You See? (continued)

Part 2

2. You will be given a recording sheet. As a group, identify as many fractions and their corresponding wholes as possible before time is called.

3. Give your team 1 point for every fraction on your list. You may count the same fraction more than once as long as it refers to a different whole.

Points: ....

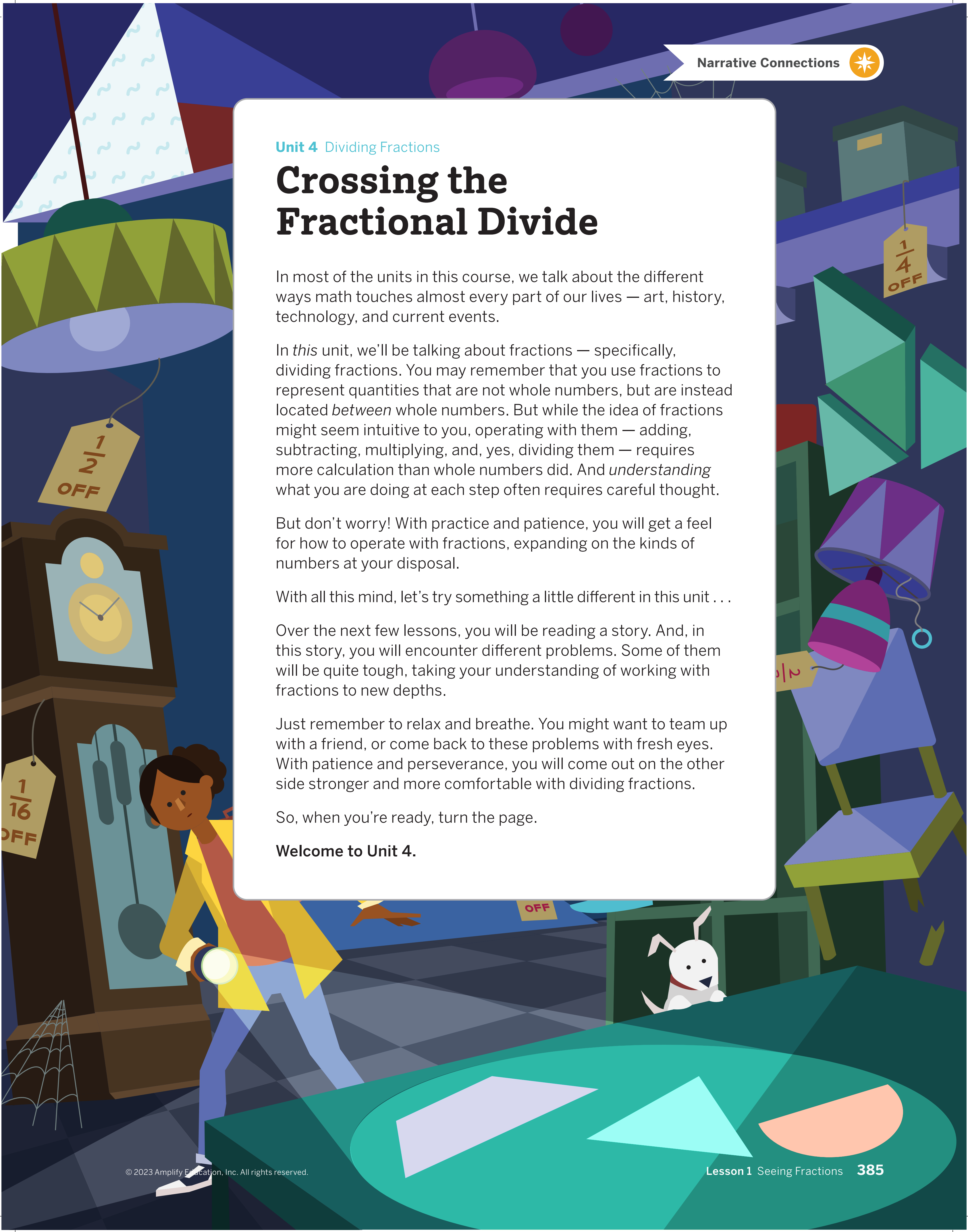
Part 3

4. Compare your list against another group's list. You will earn 1 bonus point for every fraction on *your* list that is *not* on their list. To earn that bonus point, you must explain your thinking to the other group, and they must agree with you.

Bonus points: .... **Total points:** ....

Reflect: In what ways did you demonstrate confidence as you looked for fractions?

STOP



Unit 4 Dividing Fractions

Crossing the Fractional Divide

In most of the units in this course, we talk about the different ways math touches almost every part of our lives — art, history, technology, and current events.

In *this* unit, we'll be talking about fractions — specifically, dividing fractions. You may remember that you use fractions to represent quantities that are not whole numbers, but are instead located *between* whole numbers. But while the idea of fractions might seem intuitive to you, operating with them — adding, subtracting, multiplying, and, yes, dividing them — requires more calculation than whole numbers did. And *understanding* what you are doing at each step often requires careful thought.

But don't worry! With practice and patience, you will get a feel for how to operate with fractions, expanding on the kinds of numbers at your disposal.

With all this mind, let's try something a little different in this unit . . .

Over the next few lessons, you will be reading a story. And, in this story, you will encounter different problems. Some of them will be quite tough, taking your understanding of working with fractions to new depths.

Just remember to relax and breathe. You might want to team up with a friend, or come back to these problems with fresh eyes. With patience and perseverance, you will come out on the other side stronger and more comfortable with dividing fractions.

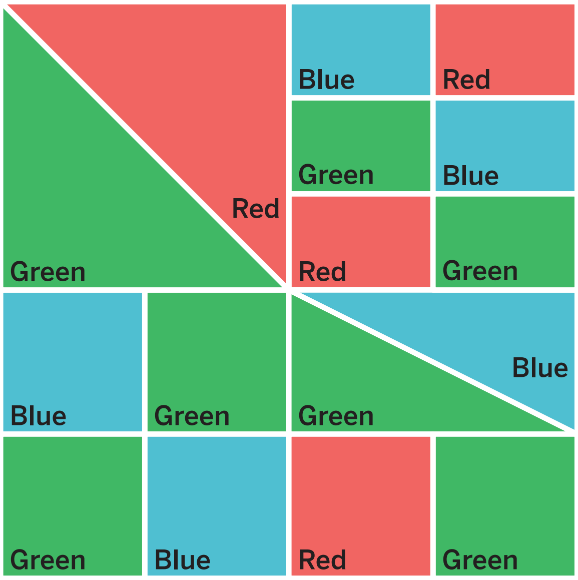
So, when you're ready, turn the page.

Welcome to Unit 4.

Practice

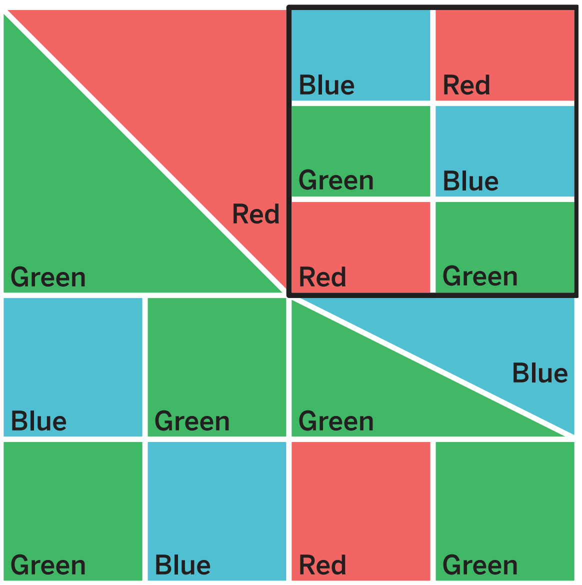
Name: .... Date: .... Period: ....

The square is composed of different shapes in three different colors.



1. Identify *at least one* way in which you see $\frac{1}{2}$ represented in the square.

2. The large square can also be broken into four smaller squares. One of these smaller squares is highlighted in the image shown. What fraction of one of these sections does each color cover?



3. Han, Shawn, and Bard are considering the area covered by each color in the top right square. Han says that each color covers $\frac{1}{3}$. Shawn says each color covers $\frac{1}{6}$. Bard says each color covers $\frac{1}{12}$. Who is correct? Explain your thinking.

Name: .... Date: .... Period: ....

4. A pound of ground beef costs $5. At this rate, what is the cost of:

a 3 lb?

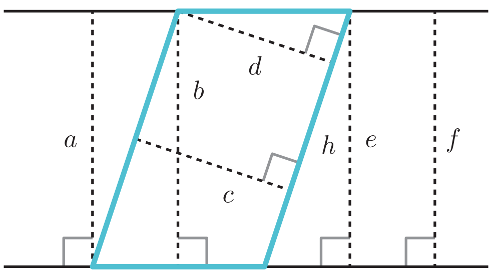
b $\frac{1}{2}$ lb?

c $\frac{1}{4}$ lb?

d $\frac{3}{4}$ lb?

e $3\frac{3}{4}$ lb?

5. In the figure, side h is the base of the parallelogram. Select *all* the segments that could represent the height of the parallelogram.



A. Segment a

B. Segment b

C. Segment c

D. Segment d

E. Segment e

F. Segment f

G. Segment g

H. Segment h

6. A chef has a 12-lb bag of rice. Each day she uses $\frac{1}{2}$ lb of rice for various dishes served at her restaurant. Which of the following equations will help find the number of days that the bag of rice will last? Select *all* that apply.

A. $12 - \frac{1}{2} = ?$

B. $12 \cdot ? = \frac{1}{2}$

C. $? \cdot \frac{1}{2} = 12$

D. $\frac{1}{2} \cdot ? = 12$

E. $12 \div \frac{1}{2} = ?$

F. $12 \div ? = \frac{1}{2}$

G. $? \div 12 = \frac{1}{2}$

H. $? \div \frac{1}{2} = 12$

My Notes:

SUB-UNIT

1 Interpreting Division Scenarios

Narrative Connections

Welcome to Spöklik Furniture

An eeriness settles over Spöklik Furniture. Once upon a time, people came here for new couches, fine china, and silver serving spoons. Now, it's nothing but an abandoned old warehouse — full of cobwebs and unsold furniture . . . So, what was your friend Maya doing out here? She texted, asking to meet here. Now, you find her phone lying by the entrance, battery dead. You walk past the shopping carts to the store's first section: Housewares.

Clicking on your flashlight, you start your search. After a few minutes of wandering, the place starts to feel like a maze! You try to trace your steps back to the entrance, but end up going in circles. Suddenly, you hear a pair of voices:

"Martha! We can't afford *that*!"

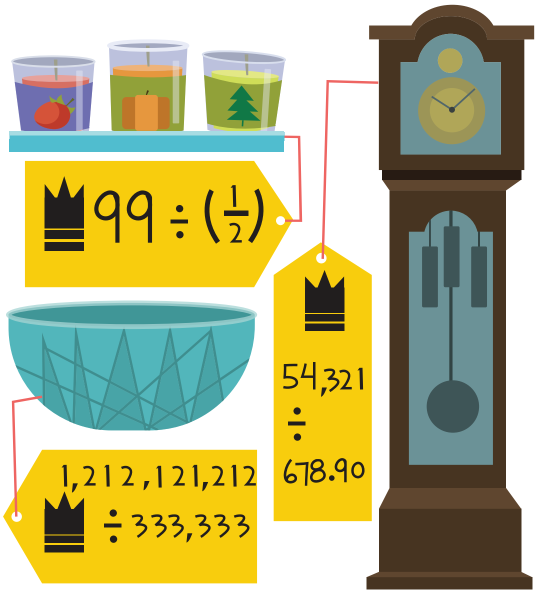
"Oh, live a little, George! You don't want the Albees calling us cheap, do you?"

Two figures appear, pushing a shopping cart. Martha cradles a crystal salad bowl in her arms. "You there!" she coos, gesturing toward you. "Be a darling and lend us a hand."

"What's the problem?" you ask.

"We're buying a house warming gift for our friends, the Albees. The trouble is, we can't decide what to get. If the gift is too cheap, it'll be a scandal. But if it's too expensive, George will have a fit. **We need an item that costs between 100 and 1,000 spök-bucks**, but these prices are *so* confusing . . ."

Looking at their cart, you see that, instead of showing the price, each tag is printed with a strange division problem. George and Martha look at you with imploring eyes. Maybe if you help them, they can help you get out of this place . . .



Unit 4 | Lesson 2

Meanings of Division

Let’s explore ways to think about division.

Warm-up One Expression, Two Interpretations

Consider the division expression $20 \div 4$. Try to think of at least two meanings it could have.

You will be given two index cards. Write one story problem on each card. Both stories should have the same context, but each should represent a different meaning of the division expression. Be sure to include a question at the end.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Representing and Interpreting Division

Part 1

Consider the expression $12 \div 6$. Write two different story problems that the expression could represent, using the same context for both.

- **One problem should include a question about "how many in each group" and the other should include a question about "how many groups."**
- **Draw two different diagrams to represent each of your problems, and be prepared to explain your thinking. One diagram for each problem should be a tape diagram.**

1. **Story Problem 1:** "How many in each group?"

   Scenario:

   Tape Diagram: **Other Diagram:**

2. **Story Problem 2:** "How many groups?"

   Scenario:

   Tape Diagram: **Other Diagram:**

Activity 1 Representing and Interpreting Division (continued)

Part 2

Use your same context from Part 1 to think about each of the following division expressions.

- Write one story problem that each expression could represent. Be sure to include a question.
- Draw one diagram that shows the expression in terms of equal-sized groups.
- State the answer to the question in each of your problems by using a complete sentence.

3. $12 \div 4$

Story Problem:

Diagram:

Solution:

4. $12 \div 2$

Story Problem:

Diagram:

Solution:

5. $12 \div \frac{1}{2}$

Story Problem:

Diagram:

Solution:

STOP

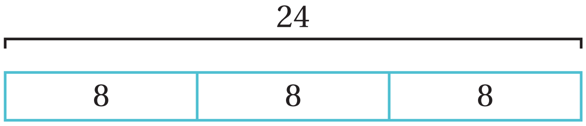
Name: .... Date: .... Period: ....

Summary

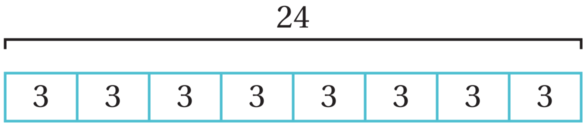
In today's lesson . . .

You wrote story problems to represent two different interpretations of division. Both interpretations involve thinking about equal-sized groups. One is associated with questions about "how many groups" and the other is associated with questions about "how many in each group."

Suppose 24 bagels are being distributed into boxes. The expression $24 \div 3$ could be understood in two ways:



24 bagels are distributed equally into 3 boxes.



24 bagels are distributed into boxes with 3 bagels in each box.

In both interpretations, the quotient is the same $(24 \div 3 = 8)$, but it has different meanings. For the example with the bagels:

- When there are 3 boxes, 8 represents *the size of a group* (the number of bagels in each box).
- When there are 3 bagels in each box, 8 represents *the number of groups* (the number of boxes with 3 bagels in each).

Reflect:

Practice

Name: ______________________ Date: __________ Period: __________

1. 20 cups of orange juice are being divided to make smoothies. The equation $20 \div \frac{1}{4} = 80$ represents how the cups of orange juice are used.

a If $\frac{1}{4}$ represents the fraction of smoothie that can be made, what does 80 represent?

b If 80 represents the number of smoothies that can be made, what does $\frac{1}{4}$ represent?

c Which interpretation of the equation makes the most sense in this scenario?

2. A sixth-grade science club needs \$180 to pay for tickets to visit a science museum. All tickets cost the same amount.

a Describe two different meanings for $180 \div 15$ in this scenario.

b Determine the quotient and explain what it represents in each meaning.

3. Write a division expression, and then draw two different diagrams to show how you can think about your expression in two different ways.

Name: .... Date: .... Period: ....

4. Complete the table.

Fraction	Decimal	Percentage of 1
$\frac{1}{4}$	0.25	25%
	0.1	
		75%
$\frac{2}{10}$ or $\frac{1}{5}$		
	1.5	
		140%

5. Jada walks at a speed of 3 mph. Elena walks at a speed of 2.8 mph. If they both begin walking along a trail at the same time, how much farther will Jada have walked after 3 hours? Explain your thinking.

6. Solve each problem.

a Using the digits 3, 4, and 12, write one multiplication equation and one division equation.

b Fill in the blanks in the equations by using these terms. A term may be used more than once.

dividend	**divisor**	**factor**	**product**	**quotient**

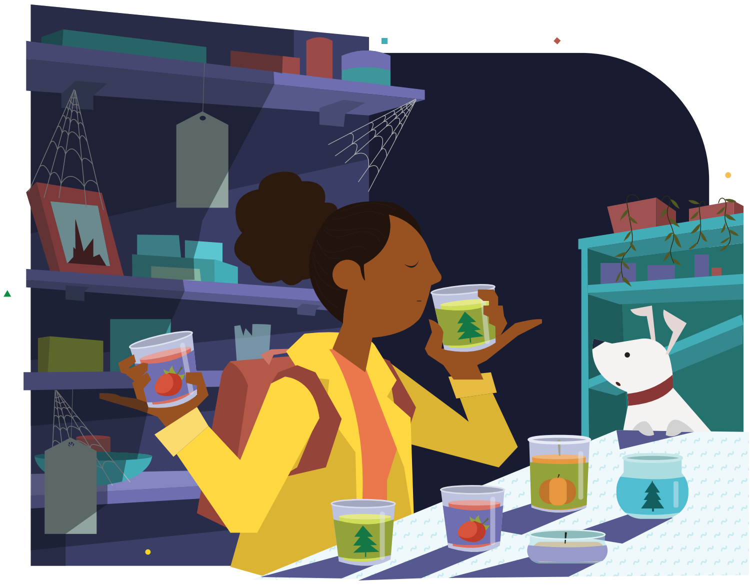
.... • .... = ...

.... ÷ .... = ...

Unit 4 | Lesson 3

Relating Division and Multiplication

Let's review how division and multiplication are related.



Warm-up Fact Families

Complete each column header with a third number to form a fact family. Then write the two multiplication equations and two division equations that correspond to each fact family in the blank rows of that column.

5, 20, ...	**12, 60,**	$\frac{1}{2}$, **10,** ....

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Multiplication or Division?

Some Spöklik employees are making scented jar candles. They each use melted wax to fill their jars in different ways. For each problem:

- **Choose an operation that could be used to solve the problem.**
- **Write an equation with your chosen operation, using a question mark for the unknown.**
- **Draw a diagram to help you solve for the unknown in your equation.**
- **Write the solution to the problem in a complete sentence.**

1. Mai has 4 jars, and she puts $\frac{1}{2}$ cup of melted cinnamon-toast- scented wax in each jar. How many cups of melted wax does Mai use?

Operation:

Diagram:

Equation:

Solution:

2. Priya has $\frac{1}{2}$ cup of pumpkin-frost-scented wax. She puts an equal amount of the melted wax into 4 jars. How many cups of wax are in each jar?

Operation:

Diagram:

Equation:

Solution:

3. Han has 4 cups of pine-scented wax to put into jars. If he puts $\frac{1}{2}$ cup of wax in each jar, how many jars can he fill?

Operation:

Diagram:

Equation:

Solution:

Activity 2 Multiplication and Division

For each problem:

- **Write a multiplication equation *and* a division equation. Use a question mark to represent the unknown in each equation.**
- **Estimate a solution to the problem.**
- **Draw a diagram to represent and to solve the problem. Explain your thinking.**
- **Write your solution in a complete sentence.**

1. Lin filled 5 jars, using a total of $7\frac{1}{2}$ cups of strawberry jam. How many cups of jam are in each jar?

Multiplication equation: **Division equation:** **Estimated solution:**

Diagram:

Solution and explanation:

2. Diego had some jars. He put $\frac{3}{4}$ cup of grape jam in each jar, using a total of $6\frac{3}{4}$ cups. How many jars did he fill?

Multiplication equation: **Division equation:** **Estimated solution:**

Diagram:

Solution and explanation:

Are you ready for more?

Using the numbers $2\frac{2}{3}$ and 8, write all of the possible multiplication and division expressions. Then order the expressions, based on their values, from least to greatest.

STOP

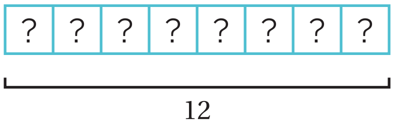
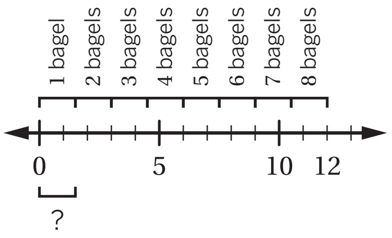
Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You revisited the relationship between the operations of multiplication and division to write related equations to determine the unknown values in a scenario.

For example, consider a scenario where 12 oz of cream cheese is being divided amongst 8 bagels. To determine the amount of cream cheese per bagel, you can represent the scenario using diagrams, a multiplication equation or division equation.

Tape diagram	Number line
	

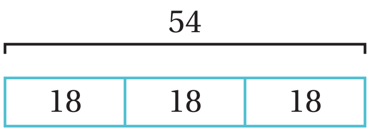
Multiplication expression	Division expression
$8 \bullet ? = 12$	$12 \div 8 = ?$

In each representation, the missing value is $1\frac{1}{2}$, which can be interpreted as "$1\frac{1}{2}$ oz of cream cheese on each of the 8 bagels for a total of 12 oz" or "12 oz of cream cheese divided evenly onto 8 bagels is $1\frac{1}{2}$ oz on each bagel."

Reflect:

Name: ______________________ Date: __________ Period: __________

1. Write a multiplication equation and a division equation that could be represented by the diagram shown.



2. Mai has $36 to spend on movie tickets. Each ticket costs $4.50.

a Write a multiplication equation and a division equation that could be used to determine how many tickets Mai can buy.

b Draw a diagram to determine the number of tickets Mai can buy.

c How many tickets can Mai buy?

3. Write a real-world problem that could be represented by the equation.

$4 \div 1\frac{1}{3} = ?$

4. Mini scones cost $3.00 per dozen.

- Andre says, "I have $2.00, so I can afford 8 mini scones."
- Elena says, "I want to get 16 mini scones, so I will pay $4.00."

Do you agree with one, both, or neither of them? Explain your thinking.

Name: ________________ Date: ________ Period: ________

5. A family has a monthly budget of $\$2{,}400$. Use the percentages to determine how much money the family spends on each category. Show or explain your thinking.

   a 44% is spent on housing.

   b 23% is spent on food.

   c 6% is spent on clothing.

   d 17% is spent on transportation.

   e The rest is put into savings.

6. Noah is making a craft in which he needs $\frac{1}{2}$ m long pieces of rope. If his grandfather gives him a piece of rope that 5 m long, how many pieces of $\frac{1}{2}$ m long rope can he cut?

Unit 4 | Lesson $\frac{16}{3} \div$ 🕯

Size of Divisor and Size of Quotient

Let’s explore quotients of different sizes.

Warm-up Number Talk

Mentally evaluate each expression.

1. $5{,}000 \div 5$

2. $5{,}000 \div 2{,}500$

3. $5{,}000 \div 10{,}000$

4. $5{,}000 \div 500{,}000$

Discussion Support: Think about how your quotient changes as the divisor increases. Be prepared to share your thinking with your classmates.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 How Many Does It Take?

Two clerks are working on a display in the Spöklik kitchen and dining sections. To help show the differences in the heights of different tables, drawers, and shelves, they plan to position towers of cube-shaped objects in, on, or under the showroom pieces.

1. Estimate how many of each object it would take to build a tower from the floor to the bottom of a breakfast table that is 3 ft tall. Be prepared to explain your thinking.

a Milk crates

Kari Marttila/Shutterstock.com

b ABC blocks

MidoSemsem/Shutterstock.com

c Dice

xpixel/Shutterstock.com

d Puzzle cubes

gd_project/Shutterstock.com

2. You want to stack cookbooks vertically to fill a shelf with a height of 72 cm.

a If the spine of each cookbook is 2 cm thick, write and evaluate an expression to determine how many cookbooks you need to fill the shelf.

b If the spine of each cookbook is $1\frac{1}{2}$ cm thick, write and evaluate an expression to determine how many cookbooks you need to fill the shelf.

Activity 2 Estimating and Ordering Quotients

1. You will be given a set of division expressions. Order the values of the quotients from least to greatest by estimating, without actually carrying out the division.

   Set 1:

   Set 2:

2. Without calculating, estimate whether each quotient is close to 0, close to 1, or much larger than 1. Write each expression in the corresponding column of the table. Be prepared to explain your thinking.

$30 \div \frac{1}{2}$	$18 \div 19\frac{1}{3}$	$30 \div 0.45$	$18 \div 0.18$
$9 \div 10$	$15{,}000 \div 1{,}500{,}000$	$\frac{5}{9} \div 10{,}000$	$15{,}000 \div 14{,}500$

Close to 0	Close to 1	Much greater than 1

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You approximated the quotient of two values using estimation, approximation, and by comparing the relative sizes of the dividend and the divisor.

In general . . .

- **When the dividend is greater than the divisor, the quotient is greater than 1.**

  For example, $4 \div 1\frac{2}{3}$ can be approximated by $4 \div 2 = 2$, so the quotient is greater than 1.

- **When the dividend and the divisor are approximately equal, the quotient is close to 1.**

  For example, $4\frac{1}{8} \div 4\frac{2}{3}$ can be approximated by $4 \div 4 = 1$, so the quotient is about 1.

- **When the dividend is greater than the divisor, the quotient is greater than 1.**

  For example, $1\frac{2}{3} \div 4$ can be approximated by $2 \div 4 = \frac{1}{2}$, so the quotient is less than 1.

Reflect:

Name: ________ Date: ________ Period: ________

1. Consider these descriptions of objects. Order the *number* of objects described from least to greatest.

Description A:
Canned vegetables in a stack that is 1 ft high.

Description B:
Dictionaries in a stack that is 1 ft high.

Description C:
Dollar bills in a stack that is 1 ft high.

Description D:
Slices of bread in a stack that is 1 ft high.

Least number of objects **Greatest number of objects**

2. Complete the sentences with the numbers shown. Use each number only once.

4	40	$4,000$

a The value of ________ $\div$ 40.01 is close to 1.

b The value of ________ $\div$ 40.01 is much less than 1.

c The value of ________ $\div$ 40.01 is much greater than 1.

3. Complete each statement with the given words. A word *may* be used more than once.

dividend	divisor	greater	lesser	quotient

a For the same dividend, the greater the divisor, the ________ the ________.

b For the same dividend, the ________ the ________, the greater the quotient.

c For the same divisor, the ________ the ________, the lesser the quotient.

d For the same divisor, the greater the dividend, the ________ the ________.

Name: ________ Date: ________ Period: ________

Practice

4. A rocking horse has a weight limit of 60 lb.

a What percentage of the weight limit is 33 lb?

b What percentage of the weight limit is 114 lb?

c What weight is 95% of the weight limit?

5. Diego has 90 songs on his playlist with the following percentages of various genres. How many songs are there for each genre?

a 40% rock

b 10% country

c 30% hip-hop

d The rest of the playlist is electronica.

6. Evaluate each expression.

a $\frac{1}{2} \bullet \frac{2}{3}$

b $\frac{1}{12} \bullet \frac{9}{8}$

c $\frac{1}{8} + \frac{5}{8}$

d $\frac{5}{8} - \frac{1}{4}$

My Notes:

SUB-UNIT

2

Division With Fractions

Narrative Connections

Spöklik Furniture: The Showroom

“Excellent!,” Martha says. “The Albees will *adore* this. You’ve been so wonderful — much more helpful than that girl with the ghastly yellow jacket!” *Yellow jacket . . .? Maya’s jacket is yellow*!

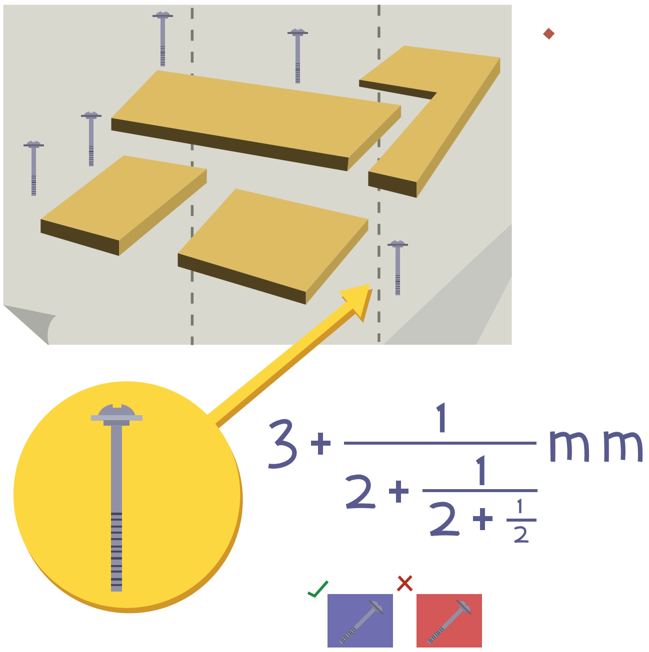
“We passed her in the showroom,” George says. “Just through there.” He points to a set of doors. *Odd. That wasn’t there before . . .* You race for the door.

Spöklik’s showroom is as grand as it is confusing: a massive warehouse with model living rooms, bedrooms, kitchens, and bathrooms. Once, shoppers wandered through this maze of displays, looking for furniture or decoration ideas. Now, the place has a strange, lonely quality. Lamplight from the model rooms spills out into the wide aisles as you step quietly, searching for Maya.

Suddenly, there is a crash! Turning the corner, you find a ghostly woman in coveralls sitting in the aisle, surrounded by tools and furniture parts. Her name tag reads: SAMIRA, SPÖKLIK TEAM MEMBER.

‘PICKLES!’, she swears, scattering a pile of dowels with a kick. She looks up, noticing you. “Sorry. Didn’t mean to scare you. It’s just that I’ve been building this thing for a *lifetime* now . . .” She gestures to a loose pile of boards and you realize you have no idea what it’s supposed to be — a bookshelf? A dresser? A bed? “Maybe if we put our heads together, I can finally get this thing built. All I need is a Number 42 serrated flange bolt with a struntprat stem.”

Samira points to the picture on her instructions. “See? The stem needs to be *this* long. The problem is I can’t make heads or tails of this number!”

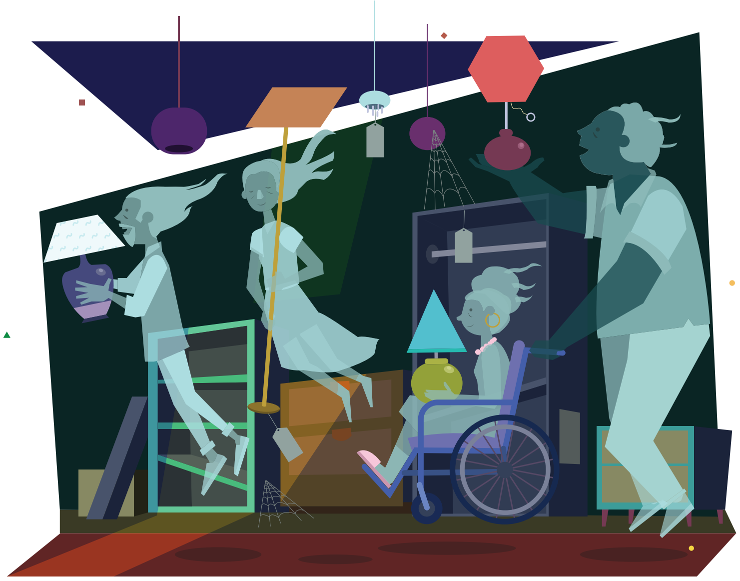


How long is the bolt Samira needs?

Unit 4 | **Lesson 5**

How Many Groups?

Let's use blocks and diagrams to think about division with fractions.



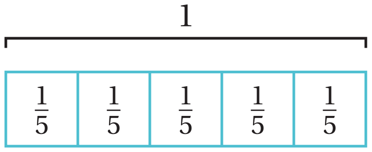
Warm-up Equal-sized Groups

Write a multiplication equation and a division equation to represent each statement or diagram.

1. Eight \$5 bills are worth \$40.

2. There are 9 thirds in 3 ones.

3.



Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Reasoning With Pattern Blocks

Use the pattern blocks to solve the problems in Parts 1 and 2.

Part 1

If a hexagon represents 1 whole, what fractions of a whole does each of the following shapes or combinations of shapes represent? Show or explain your thinking.

1. 1 triangle

2. 1 rhombus

3. 1 trapezoid

4. 4 triangles

5. 2 hexagons and 1 rhombus

Part 2

You will be given a sheet that defines a different shape as representing 1 whole. Use the pattern blocks to determine the value represented by each shape or combination of shapes. Be prepared to explain your thinking.

Activity 2 When the Size of the Group Is a Fraction

1. A hexagon represents 1 whole. Draw a diagram by using pattern block shapes to represent each multiplication equation.

a $3 \cdot \frac{1}{6} = \frac{1}{2}$

b $2 \cdot \frac{3}{2} = 3$

Plan ahead: How will knowing the shape that represents 1 whole help you use an organized approach to the activity?

2. Write an equation that could be used to represent each question. Use a ? for the unknown. Then solve the equation.

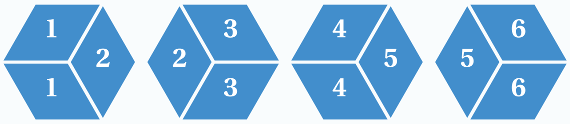
a How many $\frac{1}{2}$s are in 4?

b How many $\frac{2}{3}$s are in 2?

c How many $\frac{1}{6}$s are in $1\frac{1}{2}$?

Are you ready for more?

Which of the following can be represented by these pattern blocks? Select *all* that apply.



A. How many $\frac{2}{3}$s are in 4?

B. How many 4s are in $\frac{2}{3}$?

C. $? \cdot \frac{2}{3} = 4$

D. $4 \div \frac{2}{3} = ?$

E. How many $\frac{1}{2}$s are in 4?

STOP

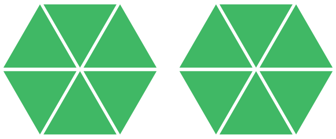
Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You looked at equal-sized groups problems where the size of each group was known, but was not a whole number. One such problem is, "How many $\frac{1}{6}$s are in 2?" To answer this question, you can write division and multiplication equations, such as $2 \div \frac{1}{6} = ?$ and $? \cdot \frac{1}{6} = 2$. You can also represent such problems by using pattern blocks, such as the ones shown.

If the hexagon represents 1 whole, then a triangle must represent $\frac{1}{6}$ because 6 triangles make 1 hexagon, which also represents $\frac{6}{6}$. So, answering the question, "How many $\frac{1}{6}$s are in 2?," is the same as answering, "How many triangles make two hexagons?"



The value 12 makes both equations true: $2 \div \frac{1}{6} = 12$ and $12 \cdot \frac{1}{6} = 2$. In terms of equal-sized groups, the size of each group is $\frac{1}{6}$ and 12 of them make 2.

This also works when the total is not a whole number, such as with $\frac{3}{2} \div \frac{1}{6} = ?$, which is the same as asking, "How many triangles make three trapezoids?" Either way, the answer is 9.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. In the figure, the hexagon represents 1 whole.

Determine how many $\frac{1}{3}$s are in $1\frac{2}{3}$. Show or explain your thinking.

2. A shopper buys cat food in 3-lb bags. Her cat eats $\frac{3}{4}$ lb each week. How many weeks does one bag last?

a Draw a diagram to represent the scenario. Label your diagram.

b Write a multiplication or division equation to represent the scenario.

c Determine how many weeks one bag lasts. Explain your thinking.

Name: .... Date: .... Period: ....

3. Which question can be represented by the equation $? \cdot \frac{1}{8} = 3$?

A. How many 3s are in $\frac{1}{8}$?

B. What is 3 groups of $\frac{1}{8}$?

C. How many $\frac{1}{8}$s are in 3?

D. What is $\frac{1}{8}$ of 3?

4. Noah and his friends are going to an amusement park. The total cost for 8 admission tickets is \$100, and each person pays the same admission price. Noah brought \$13. Did he bring enough money for an admission ticket to the park? Show or explain your thinking.

5. Write a division expression with a quotient that is:

a Greater than $8 \div 0.001$.

b Less than $8 \div 0.001$.

c Between $8 \div 0.001$ and $8 \div \frac{1}{10}$.

6. Write a division equation that could be represented by this tape diagram.

7	7	7	7	7	7	7	7

Unit 4 | Lesson 6

Using Diagrams to Determine the Number of Groups

Let's use blocks and diagrams to understand more about division with fractions.

Warm-up Reasoning With Fraction Strips

You will be given a set of fraction strips. Write the fraction or whole number that answers each question. Be prepared to share your thinking.

1. How many $\frac{1}{2}$s are in 2?

2. How many $\frac{1}{5}$s are in 3?

3. How many $\frac{1}{8}$s are in $1\frac{1}{4}$?

4. $1 \div \frac{2}{6} = ?$

5. $6 \div \frac{2}{5} = ?$

6. $4 \div \frac{2}{10} = ?$

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 More Reasoning With Pattern Blocks

Use the pattern blocks to complete Problems 1–4.

1. How many rhombuses are in one trapezoid? Show or explain your thinking.

2. If a triangle represents 1 whole, write at least two equations that could be used to determine how many rhombuses are in a trapezoid. Use a ? to represent the unknown.

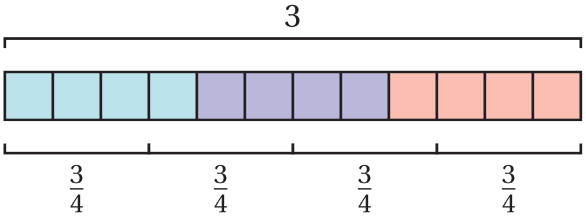
3. If a rhombus represents 1 whole, write at least two equations that could be used to determine how many rhombuses are in a trapezoid. Use a ? to represent the unknown.

4. If a trapezoid represents 1 whole, write at least two equations that could be used to determine how many rhombuses are in a trapezoid. Use a ? to represent the unknown.

Activity 2 Representing Fractional-Sized Groups

Fractional units have been used as a way of calculating with more and more precision for centuries. In the 12th century, Indian mathematician Bhāskara II used these ideas to calculate the “instantaneous” motion of a planet – how fast it traveled over very short intervals of time, which he determined could be calculated to at least 1 truti $\left(\frac{1}{33{,}750}\text{ seconds}\right)$. Knowing how many of those intervals fit into another time period, the planet’s next location could be determined.

Consider how this diagram could represent time intervals of $\frac{3}{4}$ second in 3 seconds.



Part 1

Write a different story problem (that does not need to be related to planets or time) to represent the diagram, and include a question to find an unknown in the problem. Write a multiplication equation and a division equation that represents the diagram and that could be used to answer the question from your story problem. You may use a ? to represent the unknown in the equations.

Featured Mathematician

Bhāskara II

Indian mathematician and astronomer Bhāskara II, also known as “Bhāskara, the teacher” (c. 1114–1185 CE), was a major contributor to early Indian mathematics. His work covered a variety of topics, including differential calculus – the study of rates of change between two quantities, and particularly at a single instant. He did this work nearly 500 years before many European mathematicians commonly credited for similar discoveries were even born.

Name: .... Date: .... Period: ....

Activity 2 Representing Fractional-Sized Groups (continued)

Part 2

For Problems 1–3, write a multiplication equation and a division equation that can be used to answer the question. Then draw a tape diagram to represent the problem, and determine the solution.

1. How many $\frac{3}{4}$s are in 1?

Multiplication equation:

Division equation:

Tape diagram:

Solution:

2. How many $\frac{2}{3}$s are in 3?

Multiplication equation:

Division equation:

Tape diagram:

Solution:

3. How many $\frac{3}{2}$s are in $\frac{9}{2}$?

Multiplication equation:

Division equation:

Tape diagram:

Solution:

STOP

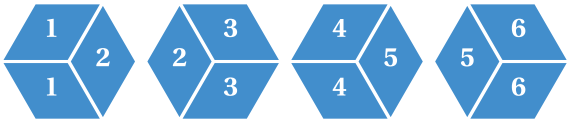
Summary

In today's lesson . . .

You continued to use representations to solve equal-sized groups problems where the size of each group was not a whole number. For example, suppose one batch of biscuits requires $\frac{2}{3}$ of a cup of flour and you want to know, "How many batches can be made with 4 cups of flour?" The size of each group is $\frac{2}{3}$ and you want to know how many groups are needed to make 4. This can be represented by the equations $4 \div \frac{2}{3} = ?$ and $? \cdot \frac{2}{3} = 4$.

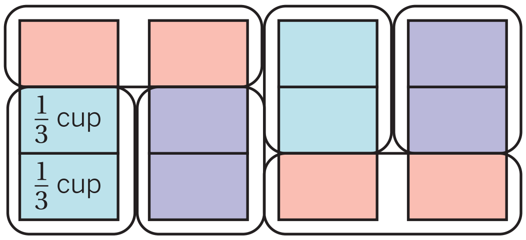
With pattern blocks, there is no single shape that represents $\frac{2}{3}$. However, because 3 rhombuses make a hexagon, 1 rhombus represents $\frac{1}{3}$, and your groups of $\frac{2}{3}$ can be represented by two rhombuses.

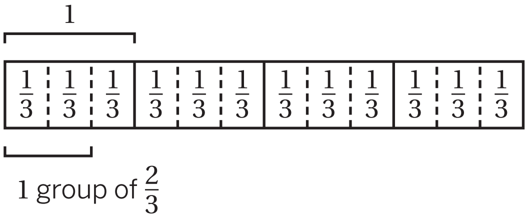
You can see that 6 pairs of rhombuses make 4 hexagons, so there are 6 groups of $\frac{2}{3}$ in 4, and that means $4 \div \frac{2}{3} = 6$.



Unfortunately, pattern blocks are limited to fractions with certain denominators. But there are plenty of other kinds of diagrams that can also help you reason about equal-sized groups involving fractions, such as equipartitioned rectangles, fraction strips, and tape diagrams.

Each of these diagrams shows $4 \div \frac{2}{3} = 6$ in different ways.





Reflect:

Name: .... Date: .... Period: ....

1. The expression $3 \div \frac{1}{4}$ can be used to represent the problem "How many groups of $\frac{1}{4}$ are in 3?" Draw a tape diagram to represent this problem. Then determine the solution.

2. Describe how to draw a tape diagram to represent and solve the equation $3 \div \frac{1}{5} = ?$ You do not have to actually draw a diagram, but you may if it helps with your thinking and explanation.

Practice

Name: .... Date: .... Period: ....

3. Consider the problem: “How many $\frac{1}{2}$ days are there in 1 week?”

   a Write either a multiplication equation or a division equation to represent the problem.

   b Draw a tape diagram to represent your equation and then solve the problem.

4. At a farmer’s market, two vendors sell fresh milk. One vendor sells 2 liters for $\$3.80$, and another vendor sells 1.5 liters for $\$2.70$. Which is the better deal? Show or explain your thinking.

5. Calculate each percentage.

   a What is 10% of 70?

   b What is 10% of 110?

   c What is 25% of 160?

   d What is 50% of 90?

6. Evaluate each expression.

   a $\frac{1}{2} \cdot \frac{3}{4}$

   b $1\frac{2}{3} \cdot \frac{3}{4}$

Name: ________ Date: ________ Period: ________

Unit 4 | Lesson 7

Dividing With Common Denominators

Let's think about dividing things into groups using common denominators.

Warm-up Estimating a Fraction of a Number

Write a multiplication expression that could be used to solve each problem. Then use your expression to *estimate* the solution.

1. What is $\frac{1}{3}$ of 7?

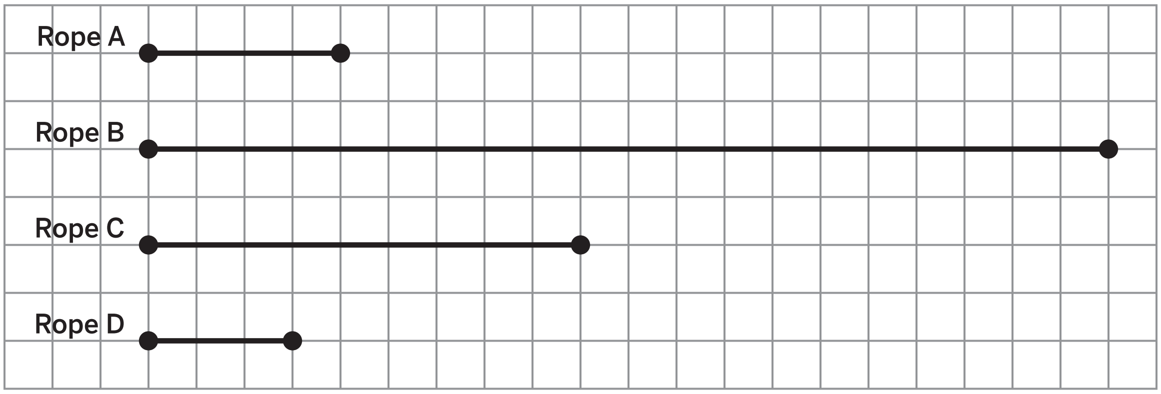
2. What is $\frac{4}{5}$ of $9\frac{2}{3}$?

3. What is $2\frac{4}{7}$ of $10\frac{1}{9}$?

Log in to Amplify Math to complete this lesson online.

Activity 1 Fractions of Ropes

These segments represent four different lengths of rope.



For each problem, write a multiplication equation and a division equation that can be used to complete the sentences comparing the lengths of the two ropes.

1. Each grid square has a length of 1 unit.

   a Rope B is .... times as long as rope A.

   Equations:

   b Rope C is .... times as long as rope A.

   Equations:

   c Rope D is .... times as long as rope A.

   Equations:

2. Each square represents $\frac{1}{3}$ unit.

   a Rope B is .... times as long as rope A.

   Equations:

   b Rope C is .... times as long as rope A.

   Equations:

   c Rope D is .... times as long as rope A.

   Equations:

Name: ______________ Date: ______________ Period: ______________

Activity 2 Fractional Batches of Mashed Potatoes

One batch of mashed potatoes uses $4\frac{1}{2}$ lb of potatoes. A chef made different-sized batches on different days. The table shows the amounts of potatoes she used each day.

Tuesday	Wednesday	Thursday	Friday
12 lb	$7\frac{1}{2}$ lb	$6\frac{3}{4}$ lb	$1\frac{2}{3}$ lb

Three Reads: To make sense of this information, you will read this text three times. Your teacher will instruct you on what to focus for each read.

1. Write a division equation and draw a tape diagram for each day. Use both to determine how many batches of mashed potatoes she made.

a Tuesday

b Wednesday

c Thursday

d Friday

Activity 2 Fractional Batches of Mashed Potatoes (continued)

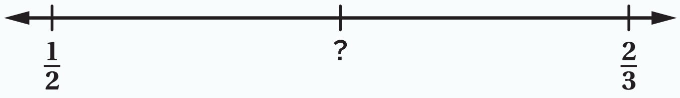
After several complaints about cold and dried-out mashed potatoes, the chef has decided to start making fresh single *servings* for every order, each using $\frac{1}{2}$ lb of potatoes. But at the end of the next week, she only has $\frac{1}{3}$ lb of potatoes left.

2. Write a division equation using common denominators and draw a diagram to represent how much of a serving she can make.

3. Imagine that she had $\frac{2}{3}$ lb of potatoes left instead. Write a division equation using common denominators to represent the situation, and then determine how much of a serving she can make.

Are you ready for more?

Determine the missing value. Show or explain your thinking.



Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You saw that common denominators can be used to help you evaluate quotients involving fractions. This is helpful for both determining the quotients and also estimating and interpreting the results. For example, $\frac{1}{4} \div \frac{1}{3}$ is asking, "How many groups of $\frac{1}{3}$ make $\frac{1}{4}$?" Rewriting that as $\frac{3}{12} \div \frac{4}{12}$ gives you the insight that the answer will be less than one whole group! And so the question could be rephrased several ways:

- "How much of a group of $\frac{1}{3}$ makes $\frac{1}{4}$?"
- "How many times as large as $\frac{1}{3}$ is $\frac{1}{4}$?"
- "What fraction of $\frac{1}{3}$ is $\frac{1}{4}$?"

The common denominator, 12 in this example, also represents a unit fraction that divides evenly into both given fractions. So, from the example, there are 4 twelfths in one third, and there are 3 twelfths in one fourth. And this means both quantities are being measured using the same units (twelfths), which means the expression can be interpreted using whole numbers. However, many times 4 ones goes into 3 ones is the same as the number of times 4 twelfths goes into 3 twelfths, or even 4 fifty-ninths goes into 3 fifty-ninths. The quotient is always equal to $3 \div 4$, which can be written as a fraction, $\frac{3}{4}$.

In general, once you have common denominators, the quotient of $\frac{a}{c} \div \frac{b}{c}$ is equal to the numerator of the dividend divided by the numerator of the divisor, $a \div b$. And better yet, that can always be written as the fraction $\frac{a}{b}$.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A recipe calls for $\frac{1}{2}$ lb of flour for 1 batch. How many batches can be made with each of these amounts? Show or explain your thinking.

   a 1 lb

   b $\frac{3}{4}$ lb

   c $\frac{1}{4}$ lb

2. Whiskers, the cat, weighs $2\frac{2}{3}$ kg. Piglio weighs 4 kg. For each problem, write a multiplication and division equation and decide whether the solution is greater than 1 or less than 1. Then determine the solution.

   a How many times as heavy as Piglio is Whiskers?

   b How many times as heavy as Whiskers is Piglio?

3. Draw a tape diagram to represent the problem: What fraction of $2\frac{1}{2}$ is $\frac{4}{5}$? Then determine the solution.

Name: .... Date: .... Period: ....

4. How many groups of $\frac{3}{4}$ are in each of these quantities? Show or explain your thinking.

a $\frac{11}{4}$

b $6\frac{1}{2}$

5. Which problem can be represented by the equation $4 \div \frac{2}{7} = ?$

A. What are 4 groups of $\frac{2}{7}$?

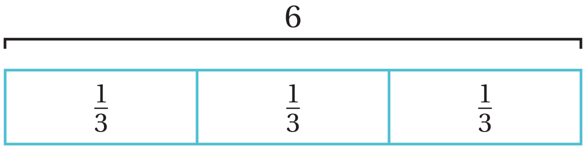
B. How many $\frac{2}{7}$s are in 4?

C. What is $\frac{2}{7}$ of 4?

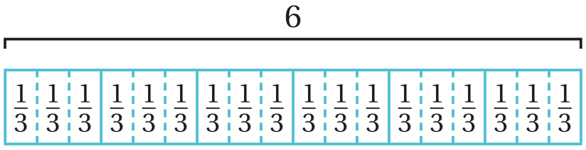
D. How many 4s are in $\frac{2}{7}$?

6. Which of these tape diagrams represent the expression $6 \div \frac{1}{3}$?

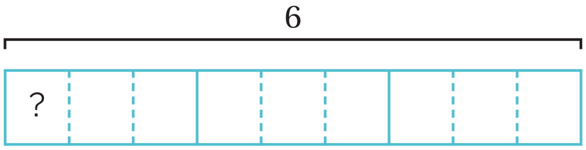
A.



B.



C.



Unit 4 | Lesson 8

How Much in Each Group? (Part 1)

Let's look at division problems that help to determine the size of one group.

Warm-up Write a Scenario

1. Write a scenario that can be represented by the equation $6 \div \frac{2}{3} = ?$ Include a question at the end.

2. Trade scenarios with your partner, and answer your partner's question.

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

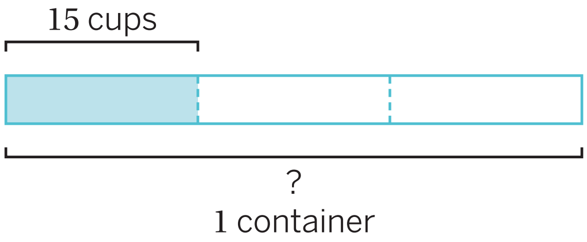
Activity 1 What Is the Group?

Part 1

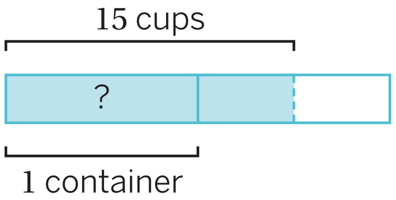
Match each scenario with a corresponding tape diagram that can be used to represent it. Be prepared to explain your thinking.

Scenario **Tape diagram**

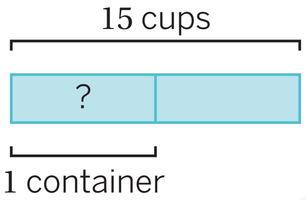
1. Tyler poured 15 cups of water into 2 equal-sized containers. He filled each container. __________



2. Kiran poured 15 cups of water into 2 equal-sized containers. He filled $1\frac{1}{2}$ containers. __________



3. Mai poured 15 cups of water into 1 container. The container is only $\frac{1}{3}$ full. __________



Compare and Connect: What connections do you see between the words and their related diagrams? Why is it that only one diagram has 15 cups representing its total length?

Activity 1 What Is the Group? (continued)

Part 2

Using your matches from Part 1, complete the table.

- Write a question that each diagram could be used to answer.
- Write a multiplication equation and a division equation that could be used to answer each question.
- Determine the answer to each question and write it in a complete sentence.

Scenario 1:

Question:	**Equations:**
Answer:	

Scenario 2:

Question:	**Equations:**
Answer:	

Scenario 3:

Question:	**Equations:**
Answer:	

Name: .... Date: .... Period: ....

Activity 2 An Adopted Highway

Three sixth grade classes adopted different sections of a highway to keep clean. Represent each scenario with a tape diagram, a division equation, and a multiplication equation. Then determine how long of a highway section each class adopted.

1. Priya's class adopted two equal-sized sections of the highway. The combined length of the two sections is $\frac{3}{4}$ mile long. How long is each section?

a **Tape diagram:**

b **Division and multiplication equations:**

c **Solution:**

2. Han's class adopted one section of the highway. The length of $\frac{1}{3}$ of the section is $\frac{3}{4}$ mile long. How long is the whole section?

a **Tape diagram:**

b **Division and multiplication equations:**

c **Solution:**

3. Lin's class adopted some equal-sized sections of the highway. The combined length of $1\frac{1}{2}$ sections is $\frac{3}{4}$ mile long. How long is each section?

a **Tape diagram:**

b **Division and multiplication equations:**

c **Solution:**

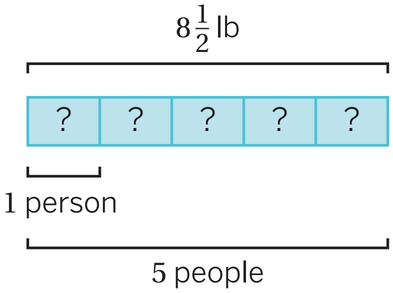
STOP

Summary

In today's lesson . . .

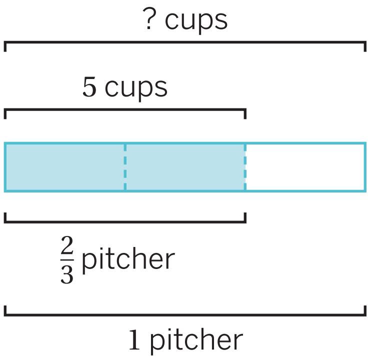
You looked at equal-sized groups problems where the total and number of groups were known, but the size of each group was unknown. These types of scenarios are probably familiar from working on fair sharing problems previously, and they can also still be represented by both division and multiplication equations.

- For example, if 5 people are sharing $8\frac{1}{2}$ lb of cherries equally and you want to know how many pounds of cherries each receives, you can write division and multiplication equations, $8\frac{1}{2} \div 5 = ?$ and $5 \cdot ? = 8\frac{1}{2}$, or draw a tape diagram, such as the one shown.



- Sometimes you know the amount for a fraction of a group, but not the amount for one *whole* group. For example, if 5 cups of water was poured into a pitcher and filled $\frac{2}{3}$ of the pitcher, you could determine how many total cups the entire pitcher holds by writing a division equation, $5 \div \frac{2}{3} = ?$, or drawing a tape diagram, such as the one shown.

The second diagram can aid your thinking, showing that, if $\frac{2}{3}$ of the pitcher is 5 cups, then $\frac{1}{3}$ is *half* of 5 cups, or $\frac{5}{2}$. And 3 times that is 1 whole, so $3 \cdot \frac{5}{2}$, or $\frac{15}{2}$ cups. You can check your answer by multiplying: $\frac{2}{3} \cdot \frac{15}{2} = \frac{30}{6} = 5$.



Reflect:

Name: .... Date: .... Period: ....

1. For each situation, complete the tape diagram to represent and solve the problem.

a Mai picked 1 cup of strawberries, which is enough for $\frac{3}{4}$ of a pan of strawberry oatmeal bars. How many cups does she need for the whole pan?

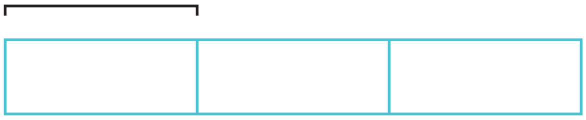
b Priya picked $1\frac{1}{2}$ cups of raspberries, which is enough for $\frac{3}{4}$ of a raspberry bread. How many cups does she need for the whole bread loaf?

2. Tyler painted $\frac{9}{2}$ yd^2 of wall area with 3 gallons of paint. How many gallons of paint does it take to paint each square yard of wall?

3. After walking $\frac{1}{4}$ mile from home, Han is $\frac{1}{3}$ of his way to school. What is the distance between his home and school?

a Write a multiplication equation and a division equation to represent this situation.

b Complete the diagram to represent the scenario.



c Determine the solution.

Practice

Name: .... Date: .... Period: ....

4. Consider these three expressions.

$56 \div 8$	$56 \div 8{,}000{,}000$	$56 \div 0.000008$

a Without calculating, order the quotients from least to greatest.

Least **Greatest**

b Explain how you ordered the three quotients.

5. Consider the division equation: $\frac{4}{5} \div \frac{2}{3} = ?$

a Write a story (including a question) that would represent the equation.

b Determine the answer to the question.

6. Use the numbers 20, 5, and 4 to write a division expression with a quotient that is:

a Greater than 1.

b Less than 1.

c Close to 1, but not equal to 1.

Name: .... Date: .... Period: ....

Unit 4 | Lesson 9

How Much in Each Group? (Part 2)

Let's practice dividing fractions in real-world scenarios.

Warm-up Relating Dividends, Divisors, and Quotients

1. Without calculating, decide whether each quotient is greater than 1 or less than 1. Be prepared to explain your thinking.

a $\frac{1}{2} \div \frac{1}{4}$

b $1 \div \frac{3}{4}$

c $\frac{2}{3} \div \frac{7}{8}$

d $2\frac{7}{8} \div 2\frac{3}{5}$

2. Write four other division expressions that include at least one fraction each. Two should have a quotient that is less than 1, and two should have a quotient that is greater than 1.

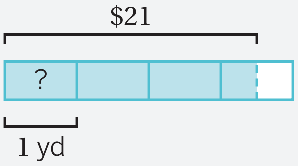
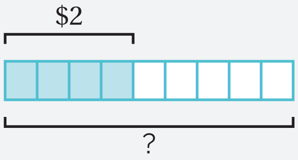
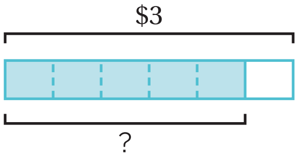
Quotient less than 1	Quotient greater than 1

Log in to Amplify Math to complete this lesson online.

Activity 1 Reupholstering a Chair

George and Martha loved the style of one particular decorative chair that was on clearance at Spöklik. However, the chair was on clearance for a reason — the fabric was hideous! They decided to buy the chair anyway and pay for the premium upgrade of custom reupholstering. Because they could not agree on a color or pattern, they decided to only consider the cost.

To help understand the odd ways that prices and measurements are done at Spöklik, George and Martha started to create this table, but have not completed it. Use the given information to complete the table.

Scenario	Estimated solution	Tape diagram	Division equation	Solution
$3\frac{1}{2}$ yd of purple-and-white-striped fabric costs $21. How much does 1 yd cost?	$7		$21 \div 3\frac{1}{2} = ?$	1 yd costs $6.
$\frac{4}{9}$ yd of leopard-print fabric costs $2. How much does 1 yd cost?			$2 \div \frac{4}{9} = ?$	
$1\frac{1}{5}$ yd of blue-velvet fabric costs $3. How much does 1 yd cost?				1 yd costs $2.50.
$\frac{3}{8}$ yd of fabric is enough to reupholster $\frac{1}{3}$ of the chair. How much fabric is needed for the entire chair?			$\frac{3}{8} \div \frac{1}{3} = ?$	$1\frac{1}{8}$ yd of fabric is needed to reupholster the entire chair.

Name: ______ Date: ______ Period: ______

Activity 2 A Scenario of Your Own

Consider the division expression: $1\frac{1}{2} \div \frac{2}{5} = ?$

- You and your partner will each write a scenario that this equation could represent. Your scenario should include a question. One partner's scenario should include a question about "how many groups," and the other partner's scenario should include a question about "how much in one group."
- Exchange scenarios with your partner.
- Draw a model, such as a tape diagram, to represent your partner's scenario.
- Solve the equation, and write your solution to your partner's scenario in a complete sentence.
- Take turns sharing and discussing your models and your thinking. Record each other's work so your table is complete.

	How many groups?	How much in one group?
Scenario		
Model		
Solution		

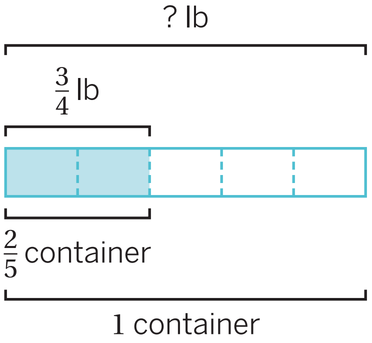
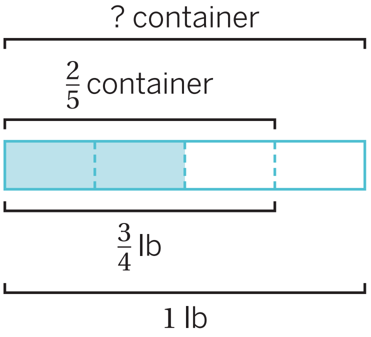
STOP

Summary

In today's lesson . . .

You continued to look at equal-sized groups problems in which the size of a group was unknown, but given the amount in a fraction of a group. Depending on the context, it is important to think about which quantity represents the groups.

For example, consider this scenario: $\frac{3}{4}$ lb of rice fills $\frac{2}{5}$ of a container. There are two possible groups: pounds or containers. So, there are two different questions you could ask, and each requires different equations and diagrams.

How many pounds in 1 container?	How many containers for 1 lb?
$\frac{2}{5} \cdot ? = \frac{3}{4}$ $\qquad$ $\frac{3}{4} \div \frac{2}{5} = ?$	$\frac{3}{4} \cdot ? = \frac{2}{5}$ $\qquad$ $\frac{2}{5} \div \frac{3}{4} = ?$
	
Because $\frac{2}{5}$ of a container can be filled with $\frac{3}{4}$ lb of rice, then $\frac{1}{5}$ of a container could be filled with half of that, or $\frac{3}{8}$ lb. This means the amount in one whole container is equal to $5 \cdot \frac{3}{8}$, or $\frac{15}{8}$ lb.	Because $\frac{3}{4}$ lb can fill $\frac{2}{5}$ of a container, then $\frac{1}{4}$ lb could fill $\frac{1}{3}$ of $\frac{2}{5}$, or $\frac{2}{15}$ of a container. This means one whole pound could fill $4 \cdot \frac{2}{5}$, or $\frac{8}{15}$ of a container.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A group of friends are equally sharing $2\frac{1}{2}$ lb of berries.

a If each friend receives $\frac{5}{4}$ lb of berries, how many friends are sharing the berries?

b If 5 friends are sharing the berries, how many pounds of berries does each friend receive?

2. $\frac{2}{5}$ kg of soil fills $\frac{1}{3}$ of a container. Can 1 kg of soil fit in the container? Show or explain your thinking.

3. After it rained for $\frac{3}{4}$ of an hour, a rain gauge is $\frac{2}{5}$ of the way filled. If it continues to rain at that same rate for 15 more minutes, what fraction of the rain gauge will be filled?

a To help solve this problem, Diego wrote the equation $\frac{3}{4} \div \frac{2}{5} = ?$ Explain why this equation does *not* represent the problem.

Vadym Zaitsev/Shutterstock.com

b Write a multiplication equation and a division equation that represents this problem.

c Use your equations to solve the problem.

Name: .... Date: .... Period: ....

4. 3 tickets to a museum cost $12.75. At this same rate, what is the cost of:

a 1 ticket?

b 5 tickets?

5. The first row in this table shows a recipe for 1 batch of trail mix. Complete the table to show the amounts of each ingredient needed for 2, 3, and 4 batches of trail mix to be made using the same recipe.

Number of batches	Cereal (cups)	Almonds (cups)	Raisins (cups)
1	2	$\frac{1}{3}$	$\frac{1}{4}$
2			
3			
4			

6. For each statement, write a corresponding multiplication expression or division expression. Then evaluate your expression.

a The product of three fourths and one third.

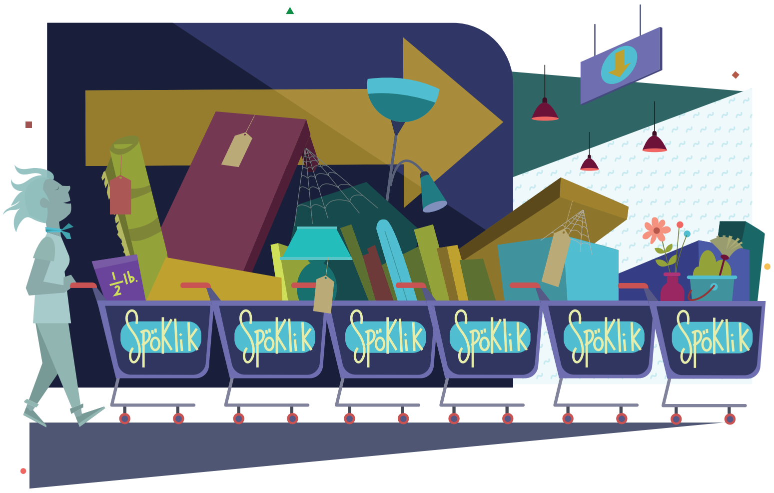
b The quotient of 5 and $\frac{1}{3}$.

Name: ____________ Date: ____________ Period: ____________

Unit 4 | Lesson 10

Dividing by Unit and Non-Unit Fractions

Let’s look for patterns when we divide by fractions.



Warm-up Dividing by a Whole Number

You and your partner will each work with the problems in one column. For each of your problems, write an equation that could be used to solve the problem with the indicated operation. Then draw a diagram to show your thinking and determine the solution.

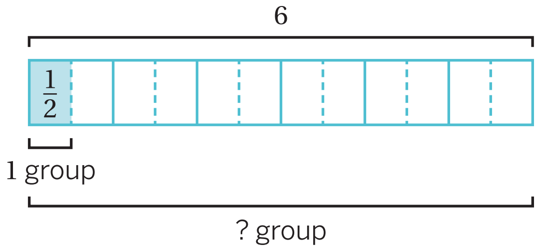
Partner A	**Partner B**
How many 3s are in 12?	**How much is 12 groups of $\frac{1}{3}$?**
Division equation:	Multiplication equation:
How many 4s are in 12?	**How much is 12 groups of $\frac{1}{4}$?**
Division equation:	Multiplication equation:

Log in to Amplify Math to complete this lesson online.

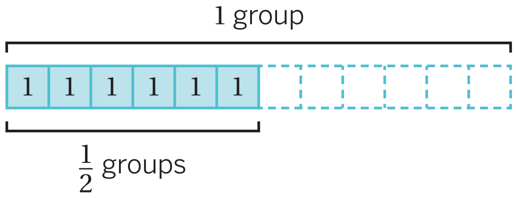
Activity 1 Dividing Whole Numbers by Fractions

Elena and Diego are trying to determine the quotient of the expression $6 \div \frac{1}{2}$.

Elena thought of the problem as asking, "How many $\frac{1}{2}$s are in 6?" and she drew this tape diagram.



Diego thought of the problem as asking, "If there are 6 in $\frac{1}{2}$ of a group, how much is in 1 group?" and he drew this tape diagram.



Choose *either* Elena's method or Diego's method and use it for Problems 1–4.

1. For each division expression, draw a diagram to represent the quotient. Then determine the value of the quotient.

a $6 \div \frac{1}{3}$

Value of the expression: ....

b $6 \div \frac{2}{3}$

Value of the expression: ....

Name: .... Date: .... Period: ....

Activity 1 Dividing Whole Numbers by Fractions (continued)

2. For each division expression, draw the diagram to represent the quotient. Then determine the value of the quotient.

a $6 \div \frac{1}{4}$

Value of the expression: ....

b $6 \div \frac{3}{4}$

Value of the expression: ....

3. Examine the expressions, diagrams, and quotients from Problems 1 and 2. Look for any patterns. Describe what you notice.

4. Choose the correct word for each blank to make a true statement.

numerator	denominator

Dividing a number by a fraction is the same as multiplying by the .... of the fraction and dividing by the .... of the fraction.

Activity 2 Dividing Fractions by Fractions

Plan ahead: How can you use diagrams to help more clearly communicate your thinking?

Choose one of the methods for dividing whole numbers by fractions from Activity 1 to determine whether it still works when the dividend is *not* a whole number. For each division expression, draw a diagram and then determine the quotient. Be prepared to explain your thinking.

1. $\frac{8}{9} \div \frac{2}{3}$

2. $\frac{7}{8} \div \frac{5}{4}$

STOP

Name: ______________ Date: ________ Period: ________

Summary

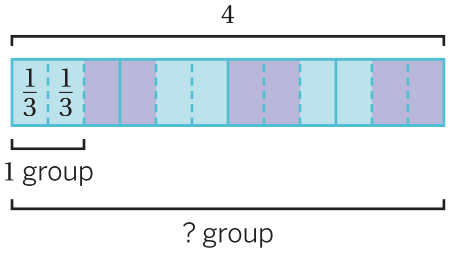
In today's lesson . . .

You compared the similarities and differences in the process of solving problems such as, "How many $\frac{1}{3}$s are in 4?" and "What is $4 \div \frac{1}{3}$?"

You can reason that there are 3 thirds in 1, so there are $(4 \cdot 3)$ thirds in 4. In other words, dividing 4 by $\frac{1}{3}$ has the same result as multiplying 4 by 3.

In general, dividing a number by a unit fraction $\frac{1}{b}$ is the same as multiplying the number by b, which is the ***reciprocal*** of $\frac{1}{b}$.

How can you reason about $4 \div \frac{2}{3}$? You already know that there are $(4 \cdot 3)$, or 12, groups of $\frac{1}{3}$s in 4. To determine how many $\frac{2}{3}$s are in 4, you need to place every 2 of the $\frac{1}{3}$s into a group. Doing this results in half as many groups, which is 6 groups. In other words:



$$4 \div \frac{2}{3} = (4 \cdot 3) \div 2$$

or

$$4 \div \frac{2}{3} = (4 \cdot 3) \cdot \frac{1}{2}$$

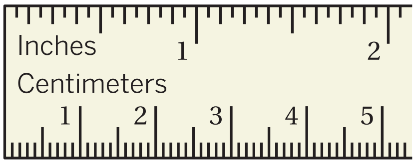
In general, dividing a number by $\frac{a}{b}$, is the same as multiplying the number by b and then dividing by a, or multiplying the number first by b and then by $\frac{1}{a}$.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A centimeter ruler is shown.



a Use the ruler to determine the quotients of $1 \div \frac{1}{10}$ and $4 \div \frac{1}{10}$.

b What calculation did you use each time?

c Use this pattern to determine $18 \div \frac{1}{10}$.

d Explain how you could determine the quotients of $4 \div \frac{2}{10}$ and $4 \div \frac{8}{10}$.

2. Determine each quotient.

a $5 \div \frac{1}{10}$

b $5 \div \frac{3}{10}$

c $5 \div \frac{9}{10}$

3. Use the equation $2\frac{1}{2} \div \frac{1}{8} = 20$ to determine $2\frac{1}{2} \div \frac{5}{8}$. Show or explain your thinking.

Name: .... Date: .... Period: ....

4. A box contains $1\frac{3}{4}$ lb of pancake mix. Jada used $\frac{7}{8}$ lb for a recipe. What fraction of the pancake mix in the box did she use? Show or explain your thinking.

5. Calculate each percentage.

- **a** 25% of 400
- **b** 50% of 90
- **c** 75% of 200
- **d** 10% of $8,000$
- **e** 5% of 20
- **f** 20% of 100

6. Determine the fractional value of each division problem.

- **a** $5 \div 9$
- **b** $5 \div 2$
- **c** $2 \div 10$

Unit 4 | Lesson 11

Using an Algorithm to Divide Fractions

Let's divide fractions by using the rule we learned.

Warm-up Multiplying Fractions

Evaluate each expression. Be prepared to explain your thinking.

1. $\frac{2}{3} \cdot 27$

2. $\frac{1}{2} \cdot \frac{2}{3}$

3. $\frac{2}{9} \cdot \frac{3}{5}$

4. $\frac{27}{100} \cdot \frac{200}{9}$

5. $1\frac{3}{4} \cdot \frac{5}{7}$

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 Exploring the Fraction Division Algorithm

Consider this statement from Lesson 10: “In general, dividing a number by $\frac{a}{b}$, is the same as multiplying the number by b and then dividing by a, or multiplying the number first by b and then by $\frac{1}{a}$.

1. Select *all* of the expressions that represent the same value as $n \div \frac{a}{b}$.

A. $n \bullet \frac{a}{b}$

B. $n \bullet a \div b$

C. $n \bullet b \div a$

D. $n \div a \bullet b$

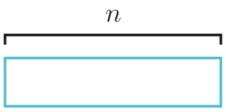
E. $n \div b \bullet a$

F. $n \bullet \frac{b}{a}$

G. $n \div \frac{b}{a}$

H. $\frac{b}{a} \bullet n$

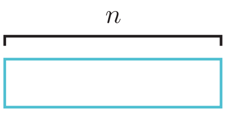
2. This tape diagram represents a number n.



a Explain how you would use the tape diagram to show $n \div \frac{a}{b}$.

Stronger and Clearer: You’ll meet with 2–3 partners to give and receive feedback on your responses to Problem 2. Use this feedback to improve your response.

b Use the tape diagram to show $n \div \frac{3}{4}$.



Activity 1 Exploring the Fraction Division Algorithm (continued)

3. Select *all* of the expressions that *always* have the same value as $\frac{c}{d} \div \frac{a}{b}$.

A. $\frac{c}{d} \bullet \frac{a}{b}$

B. $\frac{c}{d} \bullet a \div b$

C. $\frac{c}{d} \bullet b \div a$

D. $\frac{c}{d} \div a \bullet b$

E. $\frac{c}{d} \div b \bullet a$

F. $\frac{c}{d} \bullet \frac{b}{a}$

G. $\frac{c}{d} \div \frac{b}{a}$

H. $\frac{a}{b} \bullet \frac{c}{d}$

I. $\frac{a}{b} \div \frac{c}{d}$

J. $\frac{b}{a} \div \frac{d}{c}$

K. $c \div d \bullet a \div b$

L. $c \div d \bullet b \div a$

M. $c \div d \div a \bullet b$

N. $c \div d \div b \bullet a$

Name: .... Date: .... Period: ....

Activity 2 Practice Dividing Fractions

Recall from Lesson 6 that Bhāskara II used fractions in developing notions of differential calculus in 12th century India. Nearly 800 years later, those two topics are still actively being used by mathematicians, such as Ron Buckmire. Buckmire has been working on a model for predicting what fraction of a film's total earnings (or "gross") comes after its opening weekend. Given two films, how could you compare which will perform better? By dividing their fractions, of course.

Here are several division expressions that could represent any two quantities you might want to compare. Evaluate each expression by dividing the fractions.

1. $\frac{1}{2} \div \frac{2}{3}$

2. $\frac{2}{5} \div \frac{1}{3}$

3. $1\frac{1}{4} \div \frac{2}{5}$

4. $\frac{9}{10} \div 1\frac{2}{9}$

Featured Mathematician

Ron Buckmire

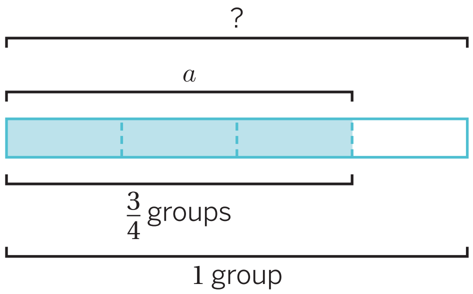
Born in Grenada, Ron Buckmire is a Professor of Mathematics and the Associate Dean for Curricular Affairs and Director of the Core Program at Occidental College. He is also a co-founder of the Barbara Jordan/Bayard Rustin Coalition, a civil rights organization. Buckmire's mathematical research focuses on numerical analysis and applied mathematics, including mathematical modeling. For example, he applied ordinary differential equations to develop a model for predicting the time evolution of theatrical film grosses.

STOP

Summary

In today's lesson . . .

You saw that the division equation $a \div \frac{3}{4} = ?$ is equivalent to the multiplication equation $\frac{3}{4} \bullet ? = a$, so you can think of it as meaning "$\frac{3}{4}$ of what number is a?" and represent it with a diagram as shown. The length of the entire diagram represents the unknown number.



If $\frac{3}{4}$ of a number is a, then you can first divide a by 3 to determine $\frac{1}{4}$ of the number. Then you multiply the result by 4 to determine the number.

The steps above can be written as: $a \div 3 \bullet 4$. Dividing by 3 is the same as multiplying by $\frac{1}{3}$, so you can also find the number by using the expression: $a \bullet \frac{1}{3} \bullet 4$.

In other words, because $a \div 3 \bullet 4 = a \bullet \frac{1}{3} \bullet 4$ and $a \bullet \frac{1}{3} \bullet 4 = a \bullet \frac{4}{3}$, you have the result: $a \div \frac{3}{4} = a \bullet \frac{4}{3}$. In general, dividing a number by a fraction $\frac{c}{d}$ is the same as multiplying the number by $\frac{d}{c}$, which is the ***reciprocal*** of the fraction.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Select *all* the statements that provide the correct steps for evaluating the expression $\frac{14}{15} \div \frac{7}{5}$.

A. Multiply $\frac{14}{15}$ by 5, and then multiply by $\frac{1}{7}$.

B. Divide $\frac{14}{15}$ by 5, and then multiply by $\frac{1}{7}$.

C. Multiply $\frac{14}{15}$ by 7, and then multiply by $\frac{1}{5}$.

D. Multiply $\frac{14}{15}$ by 5, and then divide by 7.

2. Claire said that $\frac{4}{3} \div \frac{5}{2}$ is $\frac{10}{3}$ because $\frac{4}{3} \bullet 5 = \frac{20}{3}$ and $\frac{20}{3} \div 2 = \frac{10}{3}$. Explain why Clare's quotient and reasoning are incorrect. Determine the correct quotient.

3. Determine the value of each of the following.

a $\frac{8}{9} \div 4$

b $\frac{3}{4} \div \frac{1}{2}$

c $\frac{9}{2} \div \frac{3}{8}$

d $3\frac{1}{3} \div \frac{2}{9}$

Practice

Name: ______________________ Date: ____________ Period: ____________

4. Consider the problem: After charging for $\frac{1}{3}$ of an hour, a phone is at $\frac{2}{5}$ of its full power. How long will it take the phone to charge completely? Decide whether each equation can represent the situation. Write *yes* or *no*.

a $\frac{1}{3} \cdot ? = \frac{2}{5}$

b $\frac{1}{3} \div \frac{2}{5} = ?$

c $\frac{2}{5} \div \frac{1}{3} = ?$

d $\frac{2}{5} \cdot ? = \frac{1}{3}$

5. Elena and Noah are each filling a bucket with water. Noah's bucket is $\frac{2}{5}$ full and the water weighs $2\frac{1}{2}$ lb. How much does Elena's water weigh if her bucket is full and her bucket is identical to Noah's?

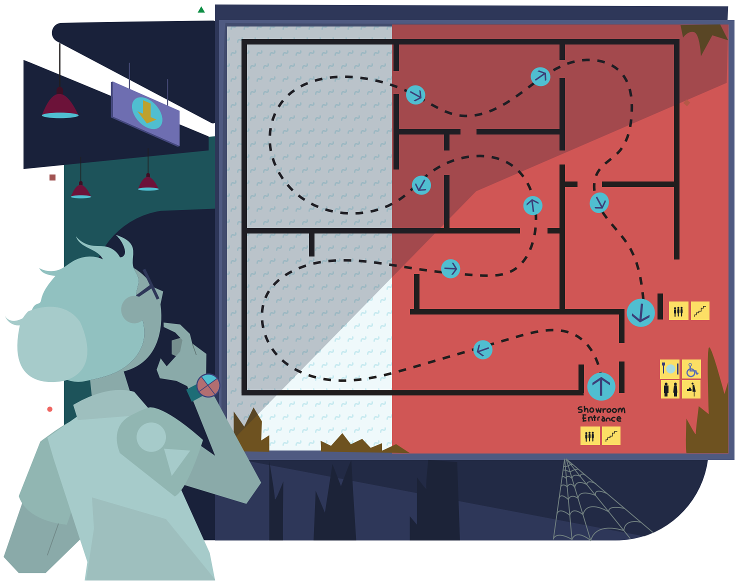
a Write a multiplication and a division equation to represent the scenario.

b Draw a diagram to show the relationship between the quantities and determine the answer.

6. Without calculating, determine how the expressions $98 \cdot 25$ and $(100 \cdot 25) - (2 \cdot 25)$ are related. Explain your thinking.

Name: ________________ Date: ________ Period: ________

Unit 4 | **Lesson 6** ÷ 



Related Quotients

Let's solve division problems by using related quotients.

Warm-up Number Talk

Mentally determine the product: 19 • 14. Be prepared to explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Related Division Expressions

1. Write a division expression that can help answer each of these questions. Then show your work for evaluating each expression to determine a solution, and write your solution as a complete sentence.

a How many groups of $\frac{3}{8}$ are in 6?

Expression: **Solution:**

b How many groups of $\frac{3}{4}$ are in 12?

Expression: **Solution:**

c How many groups of $\frac{3}{2}$ are in 24?

Expression: **Solution:**

d How many groups of 3 are in 48?

Expression: **Solution:**

e How many groups of $\frac{1}{4}$ are in 4?

Expression: **Solution:**

Name: .... Date: .... Period: ....

Activity 1 Related Division Expressions (continued)

2. Consider your solutions for Problems 1a–1e.

a How are your quotients related?

b How are your expressions related? Explain your thinking.

3. Write a division expression that would result in the same quotient as $3 \div \frac{3}{16}$, where both the dividend and divisor are *whole numbers*. Explain your thinking.

Activity 2 Using Related Quotients

A Spöklik security guard is investigating a strange noise reported in the Furniture Department. Looking at the store map, she estimates that she has covered about $\frac{2}{3}$ of the department in roughly $\frac{3}{4}$ of an hour. Will she complete her investigation of the entire department in one hour?

1. Write an expression to represent the problem and then evaluate your expression.

Expression: **Evaluation:**

2. Explain how your expression and its quotient help you to determine whether the security guard will finish investigating the entire Furniture Department in one hour.

3. Write a related division expression that results in the same quotient, but with *either* the dividend *or* the divisor written as a unit fraction. Explain how you created your related expression.

4. Explain what the dividend and divisor mean in context. Use a strategy other than the algorithm to show how the quotient gives you the same information as Problem 1.

Name: ______________________ Date: __________ Period: ________

Activity 3 Connecting to Ratios and Unit Rates

Here is how Elena and Andre each solved the problem in Activity 2.

Elena's Method

	Area investigated	Hours	
$\times\frac{3}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\times\frac{3}{2}$
	1	$\frac{9}{8}$	

Andre's Method

	Area investigated	Hours	
$\times\frac{4}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\times\frac{4}{3}$
	$\frac{8}{9}$	1	

1. How does each method represent a division of fractions?

2. Use a ratio strategy, such as either Elena or Andre did, to determine each quotient.

a $4\frac{3}{5} \div \frac{2}{7}$

$4\frac{3}{5}$	$\frac{2}{7}$

b $\frac{2}{7} \div 4\frac{3}{5}$

$\frac{2}{7}$	$4\frac{3}{5}$

STOP

Summary

In today's lesson . . .

You saw that you can solve division problems by using related expressions. When you multiply or divide both the dividend and the divisor by the same number, the result is a related expression with the same quotient. This is similar to creating equivalent ratios by multiplying or dividing both quantities by the same number!

Related division expressions that feature two whole numbers or a unit fraction are particularly useful. For example, to evaluate $20 \div \frac{4}{3}$, you could multiply both the dividend and divisor by 3 to make the related expression $60 \div 4$, which equals 15. You could also divide both the dividend and the divisor by 4 to make the related expression $5 \div \frac{1}{3}$.

You can evaluate any of these quotients by using the algorithm or ratio thinking.

Algorithm	Ratio Thinking
$20 \div \frac{4}{3}$ $20 \div \frac{4}{3} = \frac{20}{1} \div \frac{4}{3}$ $= \frac{20}{1} \cdot \frac{3}{4}$ $= \frac{60}{4}$ or 15	$5 \div \frac{1}{3}$ • There are 3 groups of $\frac{1}{3}$ in 1. • There are 6 groups in 2, 9 groups in 3, 12 groups in 4, and 15 groups in 5. • There are 15 groups of $\frac{1}{3}$ in 5.

All such related division expressions will result in the same quotient of 15.

Reflect:

Name: ________ Date: ________ Period: ________

Practice

1. Write a division expression to represent each problem. Then evaluate your expression to solve each problem. Explain your thinking.

a How many groups of $\frac{2}{5}$ are in 4?

b How many groups of $\frac{4}{5}$ are in 8?

c How many groups of $\frac{6}{5}$ are in 12?

2. What fraction of $3\frac{1}{2}$ is $\frac{3}{4}$?

a Write a division expression to represent the problem. Then evaluate your expression.

Expression: **Evaluate:**

b Write a related division expression that results in the same quotient, but where both the dividend and divisor are *whole numbers*. Explain your thinking.

3. Bard is walking their dog on a path that is $\frac{4}{5}$ mile and has already walked $\frac{2}{3}$ mile. What fraction of the path has Bard already walked? Show or explain your thinking by using a method other than the algorithm.

Name: .... Date: .... Period: ....

4. The drama club sold 300 shirts. 31% were sold to fifth graders, 52% were sold to sixth graders, and the rest were sold to teachers. How many shirts were sold to each group — fifth graders, sixth graders, and teachers? Show or explain your thinking.

5. Mai has some pennies and dimes. She does not have any other coins. The ratio of Mai's pennies to dimes is 2 to 3.

a From the information given, can you determine how many coins Mai has?

b If Mai has 55 coins, how many of each kind of coin does she have? Explain your thinking.

c How much are her 55 coins worth?

6. Kiran and Clare are comparing the numbers $1{,}000$ and 10. Kiran says that $1{,}000$ is 100 times as large as 10. Clare says that 10 is $\frac{1}{100}$ times as large as $1{,}000$. Who is correct? Explain your thinking.

SUB-UNIT

3 Fractions in Lengths, Areas, and Volumes

Narrative Connections

Spöklik Furniture: Checking Out

“Is this what you were looking for?” a voice asks. You look up and your heart almost leaps out of your chest. You see a girl in a yellow jacket: Maya!

“Ah, perfect!” Samira says. Samira takes the bolt from Maya’s outstretched hand. As Samira continues working, Maya throws her arms around you. Her dog, Penny, barks happily behind her.

“You two have been super helpful,” Samira says. “Thanks for everything!”

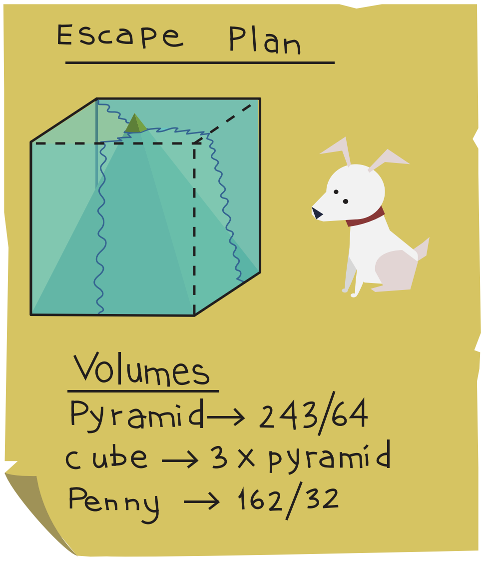
“No problem,” Maya says, “We’d better be going now!” Maya takes you by your wrist and, together, you make your way out through a stairwell.

“I’m so glad I found you. There’s an exit through the checkout section, but the guards won’t let me through with Penny. They’re convinced she belongs in the store!”

At the bottom of the stairs, you step out into a massive room. Spectral shoppers hover in line, waiting to check out. Beyond the row of registers are the exit doors, watched over by the security guards Maya warned you about.

Suddenly, you spot a shopper toward the back of one of the lines. This shopper seems distracted by a set of tea towels. In their cart, you see a large cardboard box. Suddenly, an idea occurs to you: You can sneak Penny through by hiding her in the box!

You, Maya, and Penny quietly creep behind the cart. Opening the box, you find a strange sight: a jade statue shaped like a pyramid, surrounded by 3 squishy foam packets. **“We’ll have to get rid of some of these packets for Penny to fit,” Maya whispers. “But how many?”**



Escape Plan

Volumes

Pyramid → $243/64$

cube → 3 x pyramid

Penny → $162/32$

Unit 4 | Lesson 13

Fractional Lengths

Let's solve problems about fractional lengths.

Warm-up Comparing Paper Rolls

The image shows bath tissue rolls and a paper towel roll. Let b represent the length of a bath tissue roll and let p represent the length of a paper towel roll. Write a multiplication equation and a division equation that represents the length of one roll in terms of the other.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 How Many Times as Tall or as Long?

1. A security guard at Spöklik's self-checkout likes to hide behind a potted plant while monitoring the area for theft. The plant is 4 ft tall, and the security guard is $5\frac{2}{3}$ ft tall. Write a division expression that could be used to answer each question. Then evaluate your expressions and determine the solutions.

   a. How many times as tall as the plant is the guard?

   Expression: **Evaluate:**

   Solution:

   b. What fraction of the guard's height is the plant's height?

   Expression: **Evaluate:**

   Solution:

Activity 1 How Many Times as Tall or as Long? (continued)

2. The security guard works $9\frac{1}{2}$-hour long shifts. At one point during a shift, the guard looked at the clock and realized it had been $3\frac{3}{4}$ hours since the shift started.

a Without calculating, determine if the guard has worked *at least* half of the shift. Explain your thinking.

b Calculate exactly how much of the shift the guard has worked. Show your thinking.

c Is your answer to part b reasonable based on your answer to part a? Explain your thinking.

Are you ready for more?

An envelope has a perimeter of $18\frac{1}{3}$ in., and its width is $\frac{2}{3}$ as long as its length. What is the area of the envelope?

Name: ________________ Date: ________ Period: ________

Activity 2 Info Gap: Decorating Notebooks

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card:*	If your teacher gives you the *data card:*
1. Silently read your card. Think about what information you need to be able to answer the question.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information. If your partner asks for information that is not on the card, tell them you do not have that information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask *"Why do you need that information?"* Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently.	**4.** Read the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.	**5.** Share the *data card* and discuss your reasoning.

STOP

Summary

In today's lesson . . .

You saw that division can help to solve comparison problems in which you determine how many times as large one quantity is compared to another.

For example, consider the lengths of two songs from a sixth grade chorus concert. The first song is $1\frac{1}{2}$ minutes long and the second song is $3\frac{3}{4}$ minutes long. You can compare the lengths of the two songs by asking either of two different questions, as shown in the table.

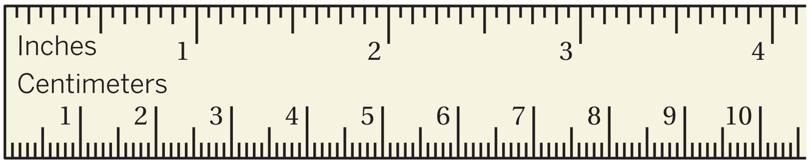
How many times as long as the first song is the second song?	What fraction of the second song is the first song?
$? \bullet 1\frac{1}{2} = 3\frac{3}{4}$ $3\frac{3}{4} \div 1\frac{1}{2} = ?$ $= \frac{15}{4} \div \frac{3}{2}$ $= \frac{15}{4} \bullet \frac{2}{3}$ $= \frac{30}{12}$ or $\frac{5}{2}$ or $2\frac{1}{2}$ The second song is $2\frac{1}{2}$ times as long as the first song.	$? \bullet 3\frac{3}{4} = 1\frac{1}{2}$ $1\frac{1}{2} \div 3\frac{3}{4} = ?$ $= \frac{3}{2} \div \frac{15}{4}$ $= \frac{6}{4} \div \frac{15}{4}$ $= \frac{6}{15}$ or $\frac{2}{5}$ The first song is $\frac{2}{5}$ as long as the second song.

Both questions can be represented by using different pairs of multiplication and division equations, and both can be answered by using any of the strategies you have seen for division.

Reflect:

Name: .... Date: .... Period: ....

1. One in. is about the same length as $2\frac{27}{50}$ cm.



a About how many centimeters long is 3 in.? Show your thinking.

b Using this approximation, what fraction of 1 in. is 1 cm? Show your thinking.

c What question can be answered by determining $10 \div 2\frac{27}{50}$ in this context?

2. A zookeeper is $6\frac{1}{4}$ ft tall. A young giraffe is $9\frac{3}{8}$ ft tall.

a What fraction of the giraffe's height is the zookeeper? Show your thinking.

b How many times as tall is the giraffe than the zookeeper? Show your thinking.

3. A rectangular bathroom floor is covered with square tiles that have side lengths of $1\frac{1}{2}$ ft. The length of the bathroom floor is $10\frac{1}{2}$ ft and the width is $6\frac{1}{2}$ ft. How many tiles does it take to cover the:

a Length of the floor?

b Width of the floor?

Practice

Name: .... Date: .... Period: ....

4. $\frac{3}{4}$ cup of oatmeal has $\frac{1}{10}$ of the recommended daily value of iron.
What fraction of the daily recommended value of iron is in 1 cup of oatmeal?

a Write a multiplication and a division equation to represent the scenario.

b Determine the solution to the problem. Show your thinking.

5. Clare says, "There are $2\frac{1}{2}$ groups of $\frac{4}{5}$ in 2." Do you agree with her? Show your thinking.

6. A rectangle has a length of 4 in. and an area of 14 in^2.

a What is the width of the rectangle? Show or explain your thinking.

b Write an equation that represents the area of the rectangle using the length of the width you calculated in part a, that shows the Commutative Property of Multiplication.

Name: ________ Date: ________ Period: ________

Unit 4 | Lesson 14

Area With Fractional Side Lengths

Let's explore the area of rectangles and triangles with fractional side lengths.

Warm-up Area Match Up

Figures A–D are rectangles with different areas, but all of their shaded regions have the *same* area.

Figure A

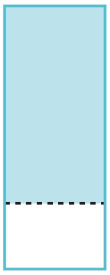


Figure B

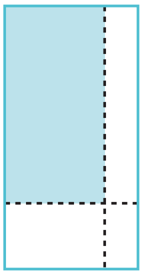


Figure C

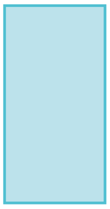
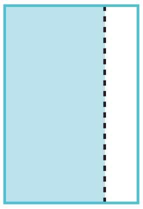


Figure D



1. Each of these expressions represents the entire area of one of the figures. Match each expression to the correct figure letter. Be prepared to explain your thinking.

$2 \cdot 4$ ________

$2\frac{1}{2} \cdot 4$ ________

$2 \cdot 4\frac{3}{4}$ ________

$2\frac{1}{2} \cdot 4\frac{3}{4}$ ________

2. Use the rectangle whose area is $2\frac{1}{2} \cdot 4\frac{3}{4}$ to show that the value of $2\frac{1}{2} \cdot 4\frac{3}{4}$ is $11\frac{7}{8}$.

Log in to Amplify Math to complete this lesson online.

Activity 1 How Many Would it Take?

Noah would like to cover a rectangular tray by using rectangular tiles with no gaps or overlaps. The tray has a width of $11\frac{1}{4}$ in. and an area of $50\frac{5}{8}$ in^2.

1. Let ℓ represent the length of the tray. Write an equation that represents the scenario and determine the length of the tray in inches. Show your thinking.

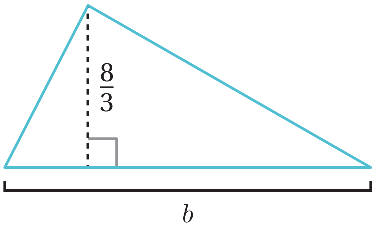
2. If each tile measures $\frac{3}{4}$ in. by $\frac{9}{16}$ in., how many tiles would Noah need to cover the tray completely? Would he need to cut or break any of the tiles? If so, what is the least number he would have to cut? Draw a diagram to show your thinking.

Name: .... Date: .... Period: ....

Activity 2 Areas of Triangles With Fractional Lengths

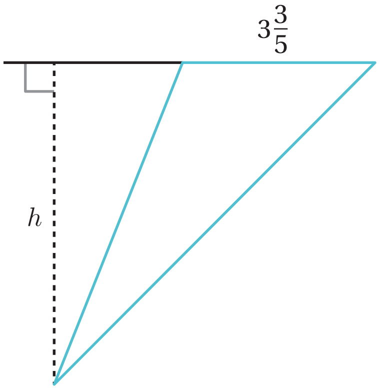
1. The area of Triangle A is 8 square units. What is the missing length b? Show your thinking.

Triangle A



2. Shawn said the missing length h of Triangle B could be determined by solving the equation $3\frac{3}{5} = \frac{1}{2} \cdot 10\frac{4}{5} \cdot h$. Do you agree or disagree? If you agree, use Shawn's equation to solve for h. If you disagree, write an equation that would solve for h and then solve your equation.

Triangle B



Area $= 10\frac{4}{5}$ square units

STOP

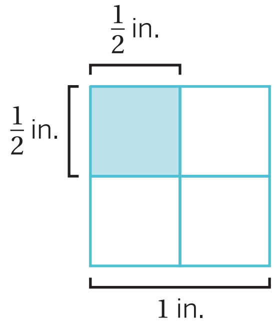
Summary

In today's lesson . . .

You applied the area formulas for parallelograms and triangles you learned in a previous unit to determine missing values when the measurements of a rectangle or triangle included fractional side lengths.

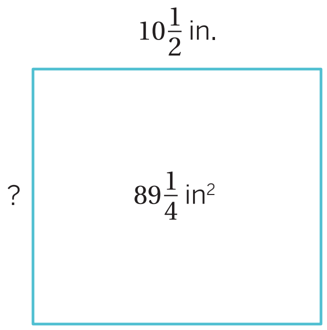
Recall that rectangles and squares are special examples of parallelograms, and for a rectangle with side lengths a units and b units, its area is equal to $a \bullet b$ square units.

This diagram shows how the formula applies to a square with a fractional side length of $\frac{1}{2}$ in. Its area is equal to the product $\frac{1}{2} \bullet \frac{1}{2}$, which means its area is $\frac{1}{4}$ in^2.



As with whole numbers, you can also use these area formulas to determine an unknown length. If you know the *area* and *one side length* of a rectangle, then you can divide to determine the *other* side length.

For example, the equation $10\frac{1}{2} \bullet ? = 89\frac{1}{4}$ shows the relationship between the area and the given side length of this rectangle. To determine the missing side length, you can divide: $89\frac{1}{4} \div 10\frac{1}{2} = ?$



And all of this also still works for a triangle with base b and height h. When one or both of those values are fractions, the area is still equal to $\frac{1}{2} \bullet b \bullet h$.

Reflect:

Name: .... Date: .... Period: ....

1. A worker is tiling the floor of a rectangular room that is 12 ft by 15 ft. The tiles are squares with a side length of $1\frac{1}{3}$ ft. How many tiles are needed to cover the entire floor? Show or explain your thinking.

2. A television screen has a length of $16\frac{1}{2}$ in., a width of w inches, and an area of 462 in^2. Select *all* the equations that represent the relationship between the dimensions of the television.

A. $w \bullet 462 = 16\frac{1}{2}$

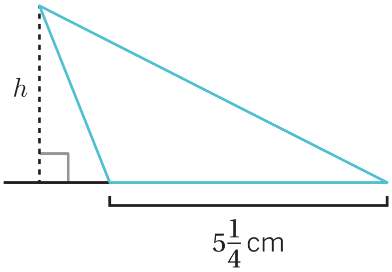
B. $16\frac{1}{2} \bullet w = 462$

C. $462 \div 16\frac{1}{2} = w$

D. $462 \div w = 16\frac{1}{2}$

E. $16\frac{1}{2} \bullet 462 = w$

3. The triangle has an area of $7\frac{7}{8}$ cm^2. What is the length of h? Explain your thinking.



Practice

Name: ________ Date: ________ Period: ________

4. A bookshelf is 42 in. long.

a If books are lined up with a width of $1\frac{1}{2}$ in., how many will fit on the bookshelf? Explain your thinking.

b A bookcase has five of these bookshelves. How many total feet of shelf space does the bookcase have? Explain your thinking.

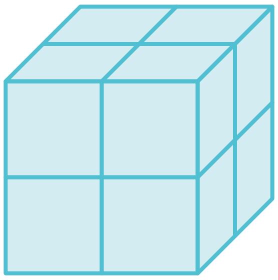
5. How many groups of $1\frac{2}{3}$ are in each of these quantities?

a $1\frac{5}{6}$

b $4\frac{1}{3}$

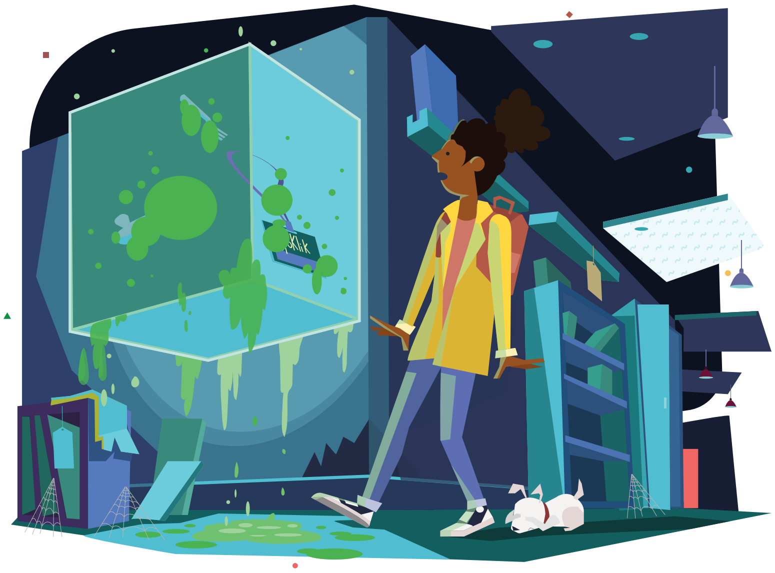
c $\frac{5}{6}$

6. The figure shows a larger cube where the side length of each smaller cube is 1 in. What is the volume of the larger cube? Show or explain your thinking.



Name: .... Date: .... Period: ....

Unit 4 | Lesson 15



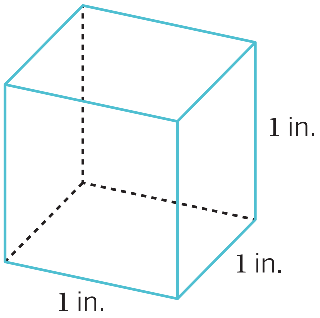
Volume of Prisms

Let's explore the volume of prisms with fractional lengths.

Warm-up Cubes in a Cube

The figure shows a cube with edge lengths of 1 in.

1. How many cubes with edge lengths of $\frac{1}{2}$ in. are needed to fill this cube?



2. What fraction of the 1-in. cube is filled by one $\frac{1}{2}$-in. cube?

Log in to Amplify Math to complete this lesson online.

Activity 1 Volume of Cubes and Prisms

1. You will be given cubes with an edge length of $\frac{1}{2}$ in. to build prisms with the lengths, widths, and heights shown in the table.

a For each prism, use the table to record how many $\frac{1}{2}$-in. cubes can be packed into the prism. Then determine the volume of the prism.

Prism length (in.)	Prism width (in.)	Prism height (in.)	Number of $\frac{1}{2}$-in. cubes in prism	Volume of prism (in^3)
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$		
1	1	$\frac{1}{2}$		
2	1	$\frac{1}{2}$		
	2	1		4
4		$\frac{3}{2}$		12
5	4			40
5	4			

b Examine the values in the table. What is the relationship between the volume and the number of $\frac{1}{2}$-in. cubes?

Are you ready for more?

Determine three unit fractions whose sum is $\frac{1}{2}$. An example is $\frac{1}{8}+\frac{1}{8}+\frac{1}{4}=\frac{1}{2}$. How many examples like this can you find?

Name: .... Date: .... Period: ....

Activity 2 Cubes in Prisms

1. Diego says that 108 cubes, each with an edge length of $\frac{1}{3}$ in., are needed to fill a rectangular prism that is 3 in. by 1 in. by $1\frac{1}{3}$ in. Do you agree or disagree with Diego? Show or explain your thinking.

2. Lin and Noah are packing small cubes into a larger cube with an edge length of $1\frac{1}{2}$ in. Lin is using cubes with an edge length of $\frac{1}{2}$ in., and Noah is using cubes with an edge length of $\frac{1}{4}$ in.

   a Will Lin and Noah need the same number of cubes? If not, who would need more? Show or explain your thinking.

   b If Lin and Noah each use their small cubes to calculate the volume of the larger cube with $1\frac{1}{2}$-in. edges, will they get the same answer? Show or explain your thinking.

Are you ready for more?

1. Determine the area of a rectangle with side lengths $\frac{1}{2}$ and $\frac{2}{3}$ units.

2. Determine the volume of a rectangular prism with side lengths $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ units.

3. What happens if you keep multiplying these fractions in this pattern, $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdot \frac{5}{6} \cdot \ldots$?

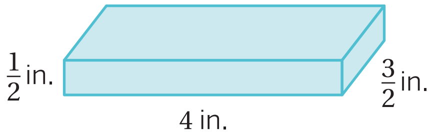
STOP

Summary

In today's lesson . . .

You saw that, to determine the volume of a rectangular prism with fractional edge lengths, you can think of the prism as being built of cubes that have a unit fraction for their edge length.

For example, consider a prism that has a height of $\frac{1}{2}$ in., a width of $\frac{3}{2}$ in., and a length of 4 in.



- The volume of the prism is 3 in^3, as determined by multiplying the fractional edge lengths given in inches: $\frac{1}{2} \cdot \frac{3}{2} \cdot 4 = 3$.
- You can also find the volume by building that same prism using cubes with $\frac{1}{2}$-in. edge lengths. The prism would be:
  - » 1 cube high, because $1 \cdot \frac{1}{2} = \frac{1}{2}$.
  - » 3 cubes wide, because $3 \cdot \frac{1}{2} = \frac{3}{2}$.
  - » 8 cubes across, because $8 \cdot \frac{1}{2} = 4$.

The volume of the prism is equal to $1 \cdot 3 \cdot 8$, or 24 of the $\frac{1}{2}$-in. cubes.

Because each cube has an edge length of $\frac{1}{2}$ in., then each cube has a volume of $\frac{1}{8} \text{ in}^3$ because $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$. Therefore, the prism that can be filled with 24 of these cubes has a volume of $24 \cdot \frac{1}{8}$, or 3 in^3.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Clare is using small wooden cubes with edge length of $\frac{1}{2}$ in. to build a larger cube that has edge length 4 in. How many small cubes does she need? Explain your thinking.

2. Consider a prism that is 5 units by 5 units by 8 units.

   a. Which expression can be used to determine how many cubes with an edge length of $\frac{1}{3}$ units are needed to fill the prism?

   A. $5\frac{1}{3} \cdot 5\frac{1}{3} \cdot 8\frac{1}{3}$

   B. $5 \cdot 5 \cdot 8$

   C. $(5 \cdot 3) \cdot (5 \cdot 3) \cdot (8 \cdot 3)$

   D. $(5 \cdot 5 \cdot 8) \cdot \left(\frac{1}{3}\right)$

   b. Mai says that she can determine the number of cubes by multiplying the edge lengths of the prism and then multiplying the result by 27. Do you agree with Mai? Explain your thinking.

3. A rectangular prism measures $2\frac{2}{5}$ in. by $3\frac{1}{5}$ in. by 2 in.

   a. Andre says, "It takes more cubes with edge length of $\frac{2}{5}$ in. than cubes with edge length of $\frac{1}{5}$ in. to pack the prism." Do you agree with Andre? Explain your thinking.

   b. How many cubes with edge length of $\frac{1}{5}$ in. would pack the prism? Explain your thinking.

   c. Show or explain how you can use your answer from part b to determine the volume of the prism in cubic inches.

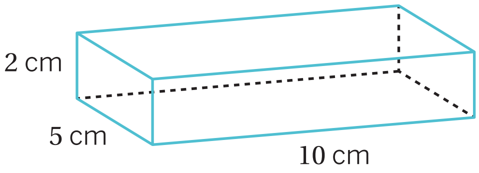
Name: .... Date: .... Period: ....

4. It takes $1\frac{1}{4}$ minutes to fill a 3-gallon bucket of water with a hose. At this same rate, how long does it take to fill a 50-gallon tub? Show or explain your thinking.

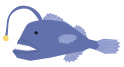
5. A teacher wants to make an art paste. The table shows the ratio of the number of cups of flour to the number of cups of water needed. Complete the table to show the other equivalent ratios.

Flour (cups)	Water (cups)
1	$\frac{1}{2}$
4	
	3
$\frac{1}{2}$	

6. Priya claimed that the base of this figure has an area of 50 cm^2 and the volume is 100 cm^3. Bard claimed that the the base has an area of 10 cm^2, but agreed that the volume is 100 cm^3. Can both Priya and Bard be correct? Show or explain your thinking.

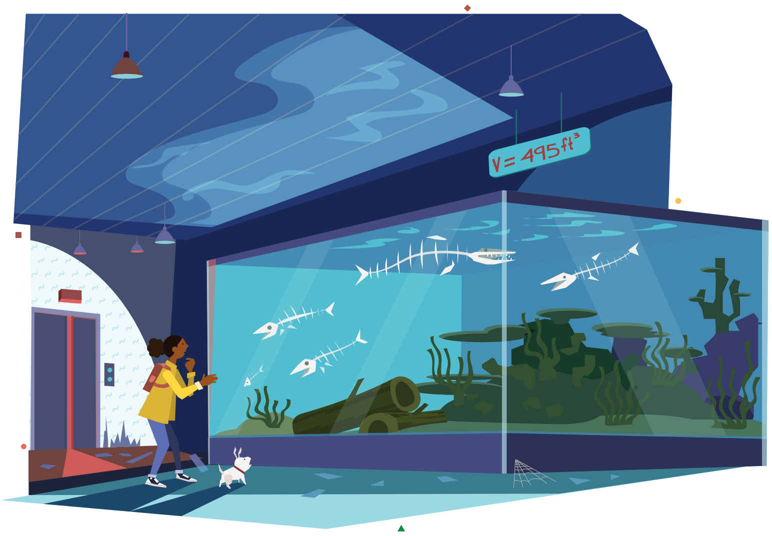


Name: ______________________ Date: __________ Period: __________

Unit 4 | Lesson $\frac{4}{5} \div$ 

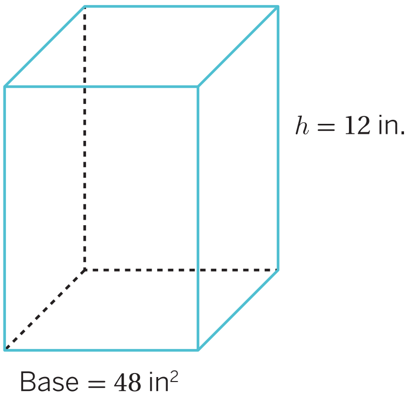
Fish Tanks Inside of Fish Tanks

Let’s look at the volume of some more prisms with fractional measurements.



Warm-up Clare’s Fish Tank

The figure shows Clare’s fish tank. Use the figure to complete Problems 1–2. Be prepared to explain your thinking.



1. Assume that the length and width of the base are whole numbers. How many cubes with an edge length of 1 in. could be packed into Clare’s fish tank?

2. How is the number of 1-in. cubes related to the volume of the prism?

Activity 1 Spöklik's Fish Tank

1. In the checkout area of Spöklik Furniture Store, there is a fish tank in the shape of a rectangular prism. It has a volume of 495 ft^3. The length of the tank is 10 ft and the width is $8\frac{1}{4}$ ft. What is the height of the tank? Show or explain your thinking.

2. As Clare stood guard by Spöklik's exit doors, she was reminded of the fish tank she used to have and she asked the two questions shown. You will work with a partner, Partner A should respond to Question A and Partner B responds to Question B. Compare and discuss your responses.
Hint: The dimensions of Clare's fish tank were 8 in. by 6 in. by 12 in.

Question A	Question B
How many times would Clare need to fill her fish tank in order to fill the Spöklik fish tank?	How many of Clare's fish tanks could fit *whole* in the Spöklik fish tank?

Are you ready for more?

1. If the number of Clare's fish tanks that could fit in the Spöklik fish tank from Question B cannot include any fractions of a tank, what is the difference in the number of fish tanks?

2. To fill the unfilled space using one other tank, what would the dimensions of this extra tank be?

Name: ________ Date: ________ Period: ________

Activity 2 The Ocean Voyager Exhibit

The Ocean Voyager exhibit at the Georgia Aquarium in Atlanta, GA, is home to four whale sharks named Trixie, Alice, Yushan, and Taroko. This exhibit was designed *especially* for these four massive fish (Trixie herself is over 27 ft long!) and holds over 6.3 million gallons of seawater. These whale sharks, along with more than 50 other species, can be seen behind one of the largest aquarium viewing windows in the world, which measures 61 ft long by 23 ft high.

Arvind Balaraman/Shutterstock.com

1. Partner A from Activity 1 should solve Problem A, and Partner B should solve Problem B.

Problem A	Problem B
How many of Clare's fish tanks could fit directly in the view from this window?	How many times would Clare need to fill her tank to fill the view of the Ocean Voyager viewing window?

2. If the depth of this aquarium is 30 ft, how many of the Spöklik fish tanks could fit in the tank directly in view from this window?

Are you ready for more?

Search for a live webcam of the Ocean Voyager exhibit, or any other similar exhibit that has one, and take a virtual visit to an aquarium. As you watch the exhibit, estimate how many fish you see. Explain your thinking.

STOP

Summary

In today's lesson . . .

You put tanks inside tanks inside tanks. **Note:** No fish, large or small, were harmed in the making of this lesson.

Fractions are an important and widely useful aspect of mathematics, and your journey with fractions, that likely began in third grade, is, in some ways, now complete. You can locate them on a number line, use them to compare parts to wholes and fractions to fractions, and, now with division under your belt, you can perform all four operations with fractions. Take a moment to review what you know about fractions and operations:

- To add or subtract fractions, determine equivalent fractions with a common denominator, so the parts involved are the same size. Then simply add or subtract the numbers of those parts — the numerators. For example:

  $\frac{3}{2} - \frac{4}{5} = \frac{15}{10} - \frac{8}{10} = \frac{7}{10}$

- To multiply fractions, multiply the denominators as another way of determining a common denominator and making same-sized parts. Then multiply the numerators to determine how many of those parts are there. For example:

  $\frac{3}{8} \bullet \frac{5}{9} = \frac{3 \bullet 5}{8 \bullet 9} = \frac{15}{72}$, which can be simplified to $\frac{5}{24}$.

- To divide a number by a fraction $\frac{a}{b}$, multiply the dividend by the reciprocal $\frac{b}{a}$. For example:

  $\frac{4}{7} \div \frac{5}{3} = \frac{4}{7} \bullet \frac{3}{5} = \frac{12}{35}$

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A pool in the shape of a rectangular prism is being filled with water. The length of the pool is 24 ft and its width is 15 ft. When the height of the water in the pool is $1\frac{1}{3}$ ft, what is the volume of the amount of water in the pool?

2. To give their animals essential minerals and nutrients, farmers and ranchers often have a block of salt available for their animals.

AJSTUDIO PHOTOGRAPHY/
Shutterstock.com

   a A rancher is ordering cube-shaped blocks of salt. The edge lengths of each block are $\frac{5}{12}$ ft. Is the volume of one block greater than or less than 1 ft^3? Explain your thinking.

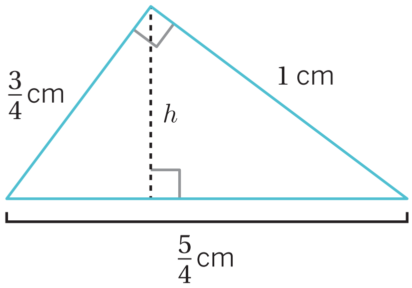
   b The box that contains the blocks of salt ordered measures $1\frac{1}{4}$ ft by $1\frac{2}{3}$ ft by $\frac{5}{6}$ ft. How many cubes-shaped blocks of salt can fit in the box? Show or explain your thinking.

3. Before refrigerators existed, some people had blocks of ice delivered to their homes. The delivery wagon contained a storage box that was a rectangular prism. Consider such a storage box that measures $7\frac{1}{2}$ ft by 6 ft by 6 ft. If the blocks of ice stored in the box were cubes with an edge length of $1\frac{1}{2}$ ft, how many blocks of ice could fit in the storage box?

   A. 270
   B. $3\frac{3}{4}$
   C. 80
   D. 180

Name: .... Date: .... Period: ....

4. Consider the right triangle.



a What is the area of the triangle? Show your thinking.

b What is the height h, in centimeters, for the base that is $\frac{5}{4}$ cm long? Show your thinking.

5. Consider a bucket that contains $11\frac{2}{3}$ gallons of water and is $\frac{5}{6}$ full.

a Write a multiplication equation *and* a division equation that could be used to determine how many gallons of water the bucket can hold when it is full.

b How many gallons of water would be in the bucket when it is full?

6. Priya's cat weighs $5\frac{1}{2}$ lb and her dog weighs $8\frac{1}{4}$ lb. For each part, estimate a number that would complete each comparison statement. Then find an exact solution. If any of your estimates were not close to the solution, explain why that may have happened.

a The cat is ___ as heavy as the dog.

Estimate: **Calculate:**

Explain:

b The dog is ___ lb heavier than the cat.

Estimate: **Calculate:**

Explain:

Name: ________________ Date: ________ Period: ________

Unit 4 | Lesson 17 – Capstone

Now, Where Was That Bus?

Let's solve a location mystery by using fractions.

Warm-up Hunting for Clues

After smuggling Penny past the guards — which was not too difficult because one of them seemed to be unusually preoccupied with a large fish tank — Maya ran to the elevator of the 12-floor parking garage, to catch a shuttle bus home, away from Spöklik. As she entered the elevator, a voice mysteriously called out, "Please enter your floor, followed by your row, and then your platform."

But, instead of numbers, the elevator buttons showed pictures of items from Spöklik's different departments. Maya reached into her pocket, hoping her bus ticket was still there. It was! Printed on the ticket was the following information:

There is no WAY Maya is going back into Spöklik for the information! However, there are three clues in this unit that could help. Can you help Maya by first finding the clues?

Lesson	Clue

TICKET

Location $7\frac{1}{2}$:

Each floor is divided by rows, and then divided by platforms. Visit our Housewares, Showroom and Check-out sections for more information!

(Lost values are not the responsibility of Spöklik or its employees.)

Log in to Amplify Math to complete this lesson online.

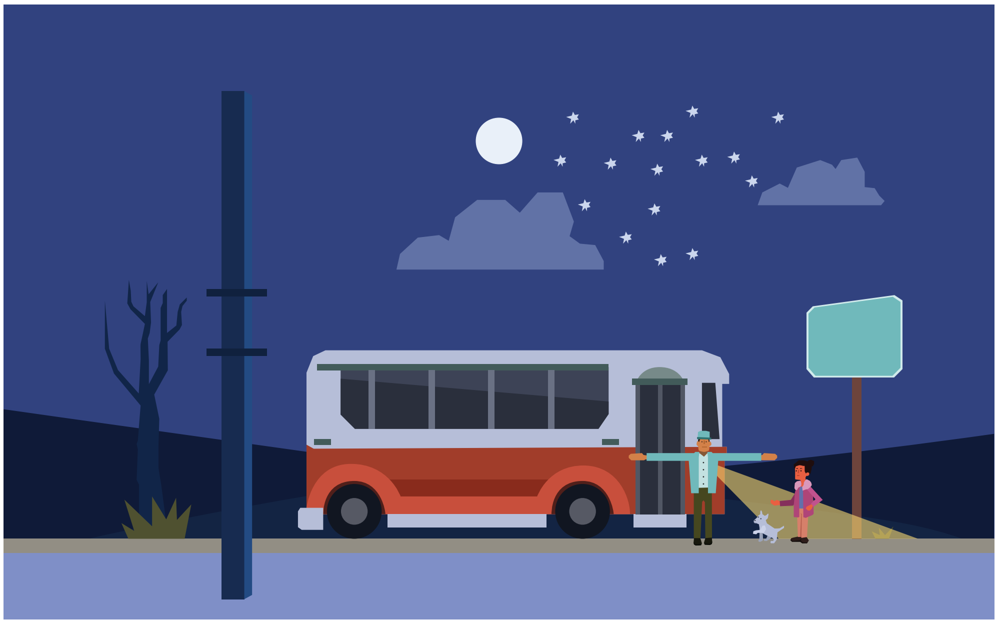
Activity 1 Determining the Right Combination

With the information from the Warm-up, determine the combination of buttons Maya needs to press in order for the elevator to take her to the correct floor, row, and platform that matches the location of the bus stop. Hint: Read the bus ticket closely.

Bus Stop Location:

Floor: .... **Row:** .... **Platform:** ....

Elevator Button Combination:



STOP

Unit Summary

Now that Maya and Penny are on their way home, it's time to set aside any fear you may have had of dividing fractions.

In your math studies, you are bound to run into concepts that might seem a little mysterious at first. But never forget that while math can come off as serious, it also has a playful side. Good mathematicians know how to have fun. They try things out, notice patterns, and even turn their problems upside down (sometimes literally!).

But like any game, there are rules. And when you know how to play with these rules, surprising things can happen. In this unit, you saw how playing with a numerator or denominator affects a quotient. You also saw how dividing and multiplying are two sides of the same coin. You even found new ways of thinking about what dividing fractions even mean: either splitting a quantity into a certain number of equal groups and asking how large they are, or splitting a quantity into groups of a certain size and asking how many there are.

These insights are possible when you are flexible and imaginative. So the next time you find yourself staring down a tough math problem, it never hurts to crack a smile and treat it like a game.

See you in Unit 5.

Name: .... Date: .... Period: ....

Reflect upon your personal and social learning in the context of this math course over these first four units.

1. Refer back to Unit 1, Lesson 1, Practice Problem 1. You were asked to identify two of your strengths as a math student and two areas in which you would like to grow or improve as a math student.

a Thinking about the areas for growth, how do you believe you have progressed in these areas?

b What do you feel you have done to make that progress?

c Is there a new strength you have developed over the past 4 units?

d Is there a new area for growth that you have identified for yourself?

2. Identify an internal motivator for you to do your best. An *internal motivator* is one that you use to motivate yourself, whereas an *external motivator* is something others do to help you want to do your best.

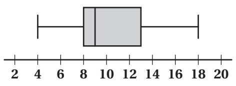
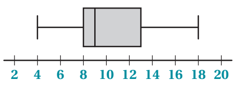
Name: ________ Date: ________ Period: ________

3. Describe a time when you were either supported by a math peer or when you supported a math peer.

4. How has the math up to this point made you rethink your place in your community? Your community can refer to your class, school, or local community. Maybe even your country or global community.

5. Describe how you have seen a peer grow as a mathematician so far this year.

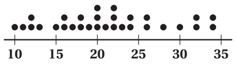
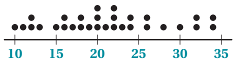
Glossary/Glosario

English	Español
A	
absolute value The value that represents the distance between a number and zero. For example, because the distance between -3 and 0 is 3, the absolute value of -3 is 3, or $\lvert -3 \rvert = 3$.	**valor absoluto** Valor que representa la distancia entre un número y cero. Por ejemplo, dado que la distancia entre -3 y 0 es 3, el valor absoluto de -3 es 3, o $\lvert -3 \rvert = 3$.
Addition Property of Equality A property stating that if $a = b$, then $a + c = b + c$.	**Propiedad de igualdad en la suma** Propiedad que establece que si $a = b$, entonces $a + c = b + c$.
area The number of unit squares needed to fill a two-dimensional shape without gaps or overlaps.	**área** Número de unidades cuadradadas necesario para llenar una forma bidimensional sin dejar espacios vacíos ni superposiciones.
average The average of a set of values is their sum divided by the number of values in the set. The average represents a fair share, or a leveling out of the distribution, so that each value in the set has the same frequency.	**promedio** El promedio de una serie de valores es su suma dividida por la cantidad de valores en el conjunto. El promedio representa una repartición justa, o igualada, de la distribución, de manera que cada valor del conjunto tenga la misma frecuencia.
B	
base (of an exponential expression) The number that is raised to a power. The power indicates the number of times the base is multiplied by itself.	**base (de una expresión exponencial)** Número que se eleva a una potencia. La potencia indica el número de veces que la base debe multiplicarse por sí mismo.
base (of a parallelogram) Any chosen side of the parallelogram.	**base (de un paralelogramo)** Cualquier lado escogido del paralelogramo.
base (of a prism) Either of the two identical, parallel faces of a prism that are connected by a set of rectangular faces.	**base (de un prisma)** Cualquiera de las dos caras idénticas y paralelas de un prisma que están conectadas por un conjunto de caras rectangulares.
base (of a pyramid) The face of a pyramid that is opposite from the vertex, where all the triangular faces connect.	**base (de una pirámide)** La cara de una pirámide que se opone al vértice, donde todas las caras triangulares se conectan.
base (of a triangle) Any chosen side of the triangle.	**base (de un triángulo)** Cualquier lado escogido del triángulo.
box plot A visual representation of the five-number summary for a numerical data set. 	**diagrama de cajas** Representación visual del resumen de cinco números de un conjunto de datos numéricos. 
C	
categorical data Data that can be sorted into categories rather than counted, such as the different types of food bison eat or the colors of the rainbow.	**datos categóricos** Datos que pueden ser clasificados en categorías en vez de ser contados, como por ejemplo los diferentes tipos de comida que come un bisonte o los colores del arcoíris.
center A value that represents the typical value of a data set.	**centro** Valor que representa el valor típico de un conjunto de datos.
coefficient A number that is multiplied by a variable, typically written in front of or "next to" the variable, often without a multiplication symbol.	**coeficiente** Número por el cual una variable es multiplicada, escrito comúnmente frente o junto a la variable sin un símbolo de multiplicación.

Glossary/Glosario

English	Español
common factor A number that divides evenly into each of two or more given numbers.	**factor común** Número que divide en partes iguales cada número de entre dos o más números dados.
common multiple A number that is a multiple of two or more given numbers.	**múltiplo común** Número que es múltiplo de dos o más números dados.
compose To place together shapes or numbers, or to combine them.	**componer** Unir formas o números, o combinarlos.
coordinate plane A two-dimensional plane that represents all the ordered pairs (x, y), where x and y can both take on values that are positive, negative, or zero.	**plano de coordenadas** Plano bidimensional que representa todos los pares ordenados (x, y), donde tanto x como y pueden representar valores positivos, negativos o cero.
cubed The raising of a number to the third power (with an exponent of 3). This is read as that number, "cubed."	**al cubo** Un número elevado a la tercera potencia (con un exponente de 3) se lee como ese número "al cubo".

D

decompose To take apart a shape or number.	**descomponer** Desmontar una forma o un número.
dependent variable In a relationship between two variables, the dependent variable represents the output values. The output values are unknown until the indicated calculations are performed on the independent variable.	**variable dependiente** En una relación entre dos variables, la variable dependiente representa los valores de salida. Los valores de salida son desconocidos hasta que se realizan los cálculos indicados sobre la variable independiente.
distribution A collection of all of the data values and their frequencies. A distribution can be described by its features when represented visually, such as in a dot plot.	**distribución** Una colección de todos los valores de datos y sus frecuencias. Una distribución puede ser descrita según sus características cuando es representada en forma visual, como por ejemplo en un diagrama de puntos.
Division Property of Equality A property stating that if $a = b$ and c does not equal 0, then $a \div c = b \div c$.	**Propiedad de igualdad en la división** Propiedad que establece que si $a = b$ y c no equivale a 0, entonces $a \div c = b \div c$.
dot plot A representation of data in which the frequency of each value is shown by the number of dots drawn above that value on a horizontal number line. A dot plot can only be used to represent numerical data. 	**diagrama de puntos** Representación de datos en la cual la frecuencia de cada valor es equivalente al número de puntos que aparecen sobre dicho valor en una línea numérica horizontal. Un diagrama de puntos solo se puede usar para representar datos numéricos. 

English	Español
E	
edge A line segment where two faces of a three-dimensional figure meet. The term *edge* can also refer to the side of a two-dimensional shape.	**arista** Segmento de una línea donde se encuentran dos caras de una figura tridimensional. *Arista* puede también referirse al lado de una forma bidimensional.
equation Two expressions with an equal sign between them. When the two expressions are equal, the equation is true. An equation can also be false when the values of the two expressions are not equal.	**ecuación** Dos expresiones con un signo de igual entre ellas. Cuando las dos expresiones son iguales, la ecuación es verdadera. Una ecuación también puede ser falsa, cuando los valores de las dos expresiones no son iguales.
equivalent If two mathematical quantities (especially fractions, ratios, or expressions) are equal in any form, then they are *equivalent*.	**equivalente** Si dos objetos matemáticos (especialmente fracciones, razones o expresiones) son iguales de cualquier manera, entonces son *equivalentes*.
equivalent expressions Two expressions whose values are equal when the same value is substituted into the variable for each expression.	**expresiones equivalentes** Dos expresiones cuyos valores son iguales cuando se sustituye el mismo valor en la variable de cada expresión.
equivalent fractions Two fractions that represent the same value or location on the number line.	**fracciones equivalentes** Dos fracciones que representan el mismo valor o la misma ubicación en la línea numérica.
equivalent ratios Any two ratios where the values for one quantity in each ratio can be multiplied or divided by the same number to get the values for the other quantity in each ratio.	**razones equivalentes** Dos razones para las cuales los valores de una cantidad en cada razón pueden ser multiplicados o divididos por el mismo número para obtener así los valores de la otra cantidad en cada razón.
exponent The number of times a factor is multiplied by itself.	**exponente** Número de veces que un factor es multiplicado por sí mismo.
expression A set of numbers, letters, operations, and grouping symbols that represent a quantity that can be calculated.	**expresión** Conjunto de números, letras, operaciones y símbolos de agrupamiento que representa una cantidad que puede ser calculada.
F	
face One of many two-dimensional shapes that form the outer surface of a three-dimensional figure.	**cara** Una de muchas formas bidimensionales que forman la superficie externa de una figura tridimensional.
factor A number that divides evenly into a given whole number. For example, the factors of 15 are 1, 3, 5, and 15.	**factor** Número que divide de manera exacta a otro número dado. Por ejemplo, los factores de 15 son 1, 3, 5 y 15.
five-number summary The minimum, first quartile, median, third quartile, and maximum values of a data distribution.	**resumen de cinco números** El mínimo, el primer cuartil, la mediana, el tercer cuartil y los valores máximos de una distribución de datos.
frequency The number of times a value occurs in a data set.	**frecuencia** Número de veces que un valor está presente en un conjunto de datos.

Glossary/Glosario

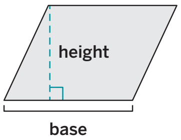
English

G

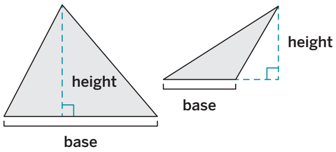
greatest common factor The common factor of two or more given whole numbers whose value is the greatest (often abbreviated as "GCF").

H

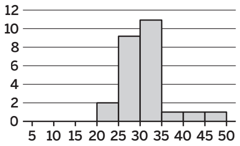
height (of a parallelogram) A segment measuring the shortest distance from the chosen base to the opposite side.



height (of a triangle) A segment representing the distance between the base and the opposite vertex.



histogram A visual way to represent frequencies of numerical data values that have been grouped into intervals, called bins, along a number line. Bars are drawn above the bins where data exists, and the height of each bar reflects the frequency of the data values in that interval.



I

independent variable In a relationship between two variables, the independent variable represents the input values. Calculations are performed on the input values to determine the values of the dependent variable.

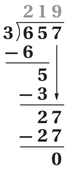
integers Whole numbers and their opposites.

interquartile range (IQR) A measure of spread (or variability) that is calculated as the difference between the third quartile (Q3) and the first quartile (Q1).

L

least common multiple The common multiple of two or more given whole numbers whose value is the least (often abbreviated as "LCM").

long division A method that shows the steps for dividing base ten whole numbers and decimals, dividing one digit at a time, from left to right.



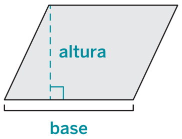
Español

G

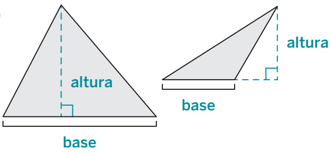
máximo factor común Factor común de dos o más números enteros dados, cuyo valor es el mayor (comúnmente abreviado como "MFC").

H

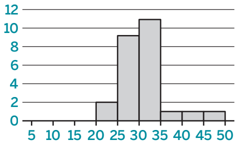
altura (de un paralelogramo) Segmento que mide la distancia más corta desde la base escogida hasta el lado opuesto.



altura (de un triángulo) Segmento que representa la distancia entre la base y el vértice opuesto.



histograma Forma visual de representar frecuencias de valores de datos que han sido agrupados en intervalos, llamados contenedores, a lo largo de una línea numérica. Se dibujan barras sobre los contenedores donde existen los datos, y la altura de cada barra refleja la frecuencia de los valores de datos en ese intervalo.



I

variable independiente En una relación entre dos variables, la variable independiente representa los valores de entrada. Se realizan cálculos con los valores de entrada para determinar los valores de la variable dependiente.

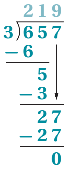
enteros Números completos y sus opuestos.

rango intercuartil (RIC) Medida de dispersión (es decir, de variabilidad) que es calculada mediante la diferencia entre el tercer cuartil (C3) y el primer cuartil (C1).

L

mínimo común múltiplo Múltiplo común de dos o más números enteros dados, cuyo valor es el menor (comúnmente abreviado como "MCM").

división larga Método que muestra los pasos necesarios para dividir números enteros en base diez y decimales, por medio de la división de un dígito a la vez, de izquierda a derecha.

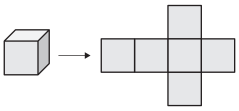
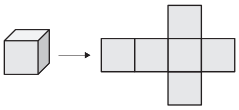


M

English	Español
magnitude (of a number) The absolute value of a number, or the distance of a number from 0 on the number line.	**magnitud (de un número)** Valor absoluto de un número, o la distancia de un número con respecto al 0 en la línea numérica.
maximum The value in a data set that is the greatest.	**máximo** El valor más grande en un conjunto de datos.
mean A measure of center that represents the average of all values in a data set. The mean represents a fair share distribution or a balancing point of all of the values in the data set.	**media** Medida del centro que representa el promedio de todos los valores de un conjunto de datos. La media representa una distribución equitativa o un punto de equilibrio entre todos los puntos del conjunto de datos.
mean absolute deviation (MAD) A measure of spread (or variability) calculated by determining the average of the distances between each data value and the mean.	**desviación absoluta media (DAM)** Medida de dispersión (o variabilidad) que se calcula mediante la obtención del promedio de la distancia entre cada valor de datos y la media.
measure of center A single number used to summarize the typical value of a data set.	**medida de centro** Número individual que se utiliza para resumir el valor típico en un conjunto de datos.
measure of variability A single number used to summarize how the values in a data set vary.	**medida de variabilidad** Número individual que se utiliza para resumir cómo varían los valores en un conjunto de datos.
median The middle value in the data set when the values are listed in order from least to greatest. When there is an even number of data points, the median is the average of the two middle values.	**mediana** Valor medio de un conjunto de datos cuando sus valores están ordenados de menor a mayor. Cuando la cantidad de puntos de datos es par la mediana es el promedio de los dos valores medios.
minimum The value in a data set that is the least.	**mínimo** Valor que es el menor de un conjunto de datos.
mode The most frequently occurring value in a data set. A data set may have no mode, one mode, or more than one mode.	**modo** Valor que aparece con mayor frecuencia en un conjunto de datos. Un conjunto de datos puede tener un modo, más de un modo o ningún modo.
multiple A number that is the product of a given number and a whole number. For example, multiples of 7 include 7, 14, and 21.	**múltiplo** Número que es el producto de un número dado y un número entero. Por ejemplo, entre los múltiplos de 7 se incluyen 7, 14 y 21.
Multiplication Property of Equality A property stating that, if $a = b$, then $a \bullet c = b \bullet c$.	**Propiedad de igualdad en la multiplicación** Propiedad que establece que si $a = b$, entonces $a \bullet c = b \bullet c$.

Glossary/Glosario

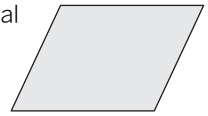
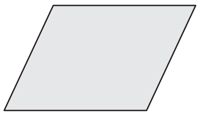
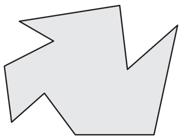
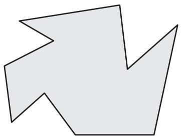
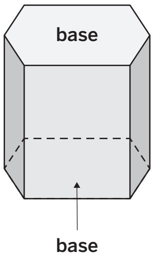
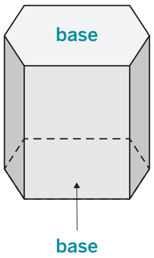
N

English	Español
negative number A number whose value is less than zero.	**número negativo** Número cuyo valor es menor que cero.
net A two-dimensional representation, or "flattening," of a three-dimensional solid's surface that shows all of its faces. 	**red** Representación bidimensional, o "aplanamiento", de la superficie de un sólido tridimensional, para mostrar todas sus caras. 
numerical data Numbers, quantities, or measurements that can be meaningfully compared.	**datos numéricos** Números, cantidades o medidas que pueden ser comparadas de manera significativa.

O

English	Español
opposite numbers Two numbers that are the same distance from 0, but are on different sides of the number line.	**números opuestos** Dos números que están a la misma distancia de 0, pero que están en lados diferentes de la línea numérica.

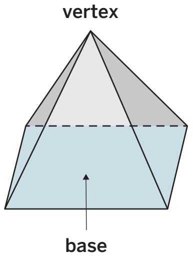
P

English	Español
parallelogram A type of quadrilateral with two pairs of parallel sides. 	**paralelogramo** Tipo de cuadrilátero con dos pares de lados paralelos. 
per For each.	**por** Por cada uno de los elementos.
percentage A rate per 100. (A specific *percentage* is also called a *percent*, such as "70 percent.")	**porcentaje** Tasa por cada 100. (Un *porcentaje* específico también es llamado *por ciento*, como por ejemplo "70 por ciento".)
polygon A closed, two-dimensional shape with straight sides that do not cross each other. 	**polígono** Forma cerrada y bidimensional de lados rectos que no se entrecruzan. 
polyhedron A closed, three-dimensional shape with flat sides. (The plural of *polyhedron* is *polyhedra*.)	**poliedro** Forma cerrada y tridimensional de lados planos.
positive number A number whose value is greater than zero.	**número positivo** Número cuyo valor es mayor que cero.
prism A three-dimensional figure with two parallel, identical faces (called *bases*) that are connected by a set of rectangular faces. 	**prisma** Figura tridimensional con dos caras iguales y paralelas (llamadas *bases*) que se conectan entre sí a través de un conjunto de caras rectangulares. 

English

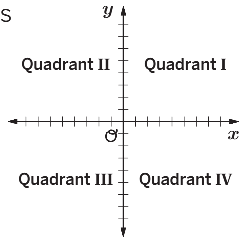
properties of equality Rules that apply to all equations. These include properties of addition, subtraction, multiplication, and division, which state that if an equation is true, then performing the same operation to both sides will result in an equivalent equation.

pyramid A three-dimensional figure with one base and a set of triangular faces that meet at a singular vertex.

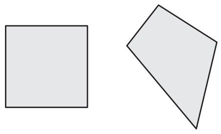


Q

quadrant Each of the four regions of the coordinate plane formed by the vertical and horizontal axes. The quadrants are labeled counterclockwise from top right to bottom right as I, II, III, IV.



quadrilateral A polygon with exactly four sides.

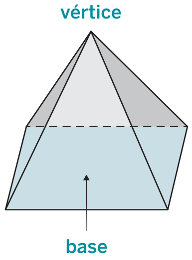


quartile One of three numbers (Q1, Q2, Q3) that divide an ordered data set into four sections so that each section contains 25% of data points.

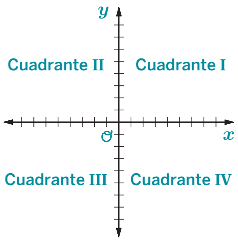
Español

propiedades de igualdad Reglas que son aplicables a todas las ecuaciones. Incluyen las propiedades de suma, resta, multiplicación y división, las cuales señalan que si una ecuación es verdadera, al realizar la misma operación en ambos lados se obtendrá una ecuación equivalente.

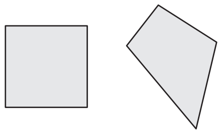
pirámide Figura tridimensional con una base y un conjunto de caras triangulares que se conectan en un solo vértice.



cuadrante Cada una de las cuatro regiones del plano de coordenadas formado por los ejes vertical y horizontal. Los cuadrantes se identifican en sentido contrario a las agujas del reloj, desde la parte superior derecha a la parte inferior derecha, como I, II, III y IV.



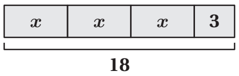
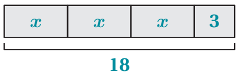
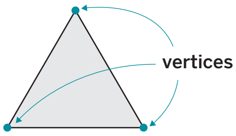
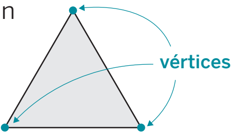
cuadrilátero Polígono de exactamente cuatro lados.



cuartil Uno de los tres números (C1, C2, C3) que dividen un conjunto ordenado de datos en cuatro secciones, de manera que cada sección contenga el 25% de los puntos de datos.

Glossary/Glosario

English	Español
R	**R**
range A measure of spread (or variability) that is calculated as the difference between the maximum and minimum values in the data set.	**rango** Medida de dispersión (o variabilidad) que es calculada mediante la diferencia entre los valores máximos y mínimos de un conjunto de datos.
rate A comparison of how two quantities change together.	**tasa** Comparación de cuánto cambian dos cantidades en conjunto.
ratio A comparison of two quantities, such that for every a units of one quantity, there are b units of another quantity.	**razón** Una comparación entre dos cantidades, de modo tal que por cada a unidades de una cantidad, hay b unidades de la otra cantidad.
rational numbers The set of all the numbers that can be written as positive or negative fractions.	**números racionales** Conjunto que consta de todos los números que pueden ser escritos como fracciones positivas o negativas.
ratio relationship A relationship between quantities that establishes that the values for each quantity will always change together in the same way.	**relación de razón** Relación entre cantidades que establece que los valores para cada cantidad siempre cambiarán en conjunto de la misma manera.
ratio table A table of values organized in columns and rows that contains equivalent ratios.	**tabla de razones** Tabla de valores organizada en columnas y filas que contiene razones equivalentes.
reciprocal Two numbers whose product is 1 are *reciprocals* of each other. (When written in simplest fraction form, the numerator of each number corresponds to the denominator of the other number. For example, $\frac{3}{5}$ and $\frac{5}{3}$ are reciprocals.)	**recíproco/a** Dos números cuyo producto es 1 son *recíprocos* entre sí. (Al escribirlo en la forma de fracción más simple, el numerador de cada número corresponde al denominador del otro número. Por ejemplo, $\frac{3}{5}$ y $\frac{5}{3}$ son recíprocos.)
region The space inside a shape or figure.	**región** Espacio al interior de una forma o figura.
S	**S**
sign (of a number) Indication of whether a number is positive or negative.	**signo (de un número)** Indicación de si un número es positivo o negativo.
solution to an equation A number that can be substituted in place of a variable to make an equation true.	**solución a una ecuación** Número que puede sustituir una variable para volver verdadera una ecuación.
solution to an inequality Any number that can be substituted in place of a variable to make an inequality true.	**solución a una desigualdad** Cualquier número que puede sustituir una variable para volver verdadera una desigualdad.
spread The variability of a distribution. A description of how the data values in the distribution vary from the center of the distribution.	**dispersión** Variabilidad de una distribución. Descripción de cuánto varían los valores en comparación con el centro de una distribución de datos.
squared The raising of a number to the second power (with an exponent of 2). This is read as that number, "squared."	**al cuadrado** Un número elevado a la segunda potencia (con un exponente de 2) se lee como ese número "al cuadrado".
statistical question A question that anticipates variability and can be answered by collecting data.	**pregunta estadística** Pregunta que anticipa variabilidad y que se puede responder mediante la recolección de datos.

English	Español
Subtraction Property of Equality For rational numbers a, b, and c, if $a = b$, then $a - c = b - c$.	**Propiedad de igualdad en la resta** Para los números racionales a, b y c, if $a = b$, entonces $a - c = b - c$.
surface area The number of unit squares needed to cover all of the faces of a three-dimensional figure without gaps or overlaps.	**área de superficie** Número de unidades cuadradas necesarias para cubrir todas las caras de una figura tridimensional sin dejar espacios vacíos ni superposiciones.
T	
tape diagram A model in which quantities are represented as lengths (of tape) placed end-to-end, and which can be used to show addition, subtraction, multiplication, or division.  	**diagrama de cinta** Modelo en el cual las cantidades están representadas como longitudes (de una cinta) colocadas de forma continua, y que pueden ser usadas para mostrar suma, resta, multiplicación y división. 
U	
unit rate How much one quantity changes when the other changes by 1.	**tasa unitaria** Cuánto cambia cierta cantidad cuando la otra cambia por un factor de 1.
V	
variability The spread of a distribution. A description of how the data values in the distribution vary from the center of the distribution.	**variabilidad** La dispersión de una distribución. Descripción de cuánto varían los valores en comparación con el centro de una distribución de datos.
variable A letter that represents an unknown number in an expression or equation.	**variable** Letra que representa un número desconocido en una expresión o ecuación.
vertex A point where two sides of a two-dimensional shape or two or more edges of a three-dimensional figure intersect. (The plural of *vertex* is *vertices*.)  	**vértice** Punto donde se intersecan dos lados de una forma bidimensional, o dos o más aristas de una figura tridimensional. 
volume The number of unit cubes needed to fill a three-dimensional figure without gaps or overlaps.	**volumen** Número de unidades cúbicas necesario para llenar una figura tridimensional sin dejar espacios vacíos ni superposiciones.

Index

Index

Index

Index

Index

Index