Amplify Math

Grade 6

Volume 2: Units 5–8

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Cover illustration by Caroline Hadilaksono.

55 Washington Street, Suite 800,
Brooklyn, NY 11201
www.amplify.com

ISBN: 978-1-63643-001-0
Printed in [e.g., the United States of America]
[# of print run] [print vendor] [year of printing]

Hello, curious mind!

Welcome to Grade 6. For many students, it's a year of change and growth . . . and the same goes for what you'll be doing in math.

This year, you'll meet the Song dynasty of China and solve a few tangram puzzles they inspired, learn how a misplaced ruler led to the invention of the cardboard box, construct rhombicuboctahedrons (say that five times fast), and design a suspended tent for you and three friends to camp out in. And that's just Unit 1!

We promise you, it's all possible. This year, you'll see not only that math makes sense, but it can be fun, too.

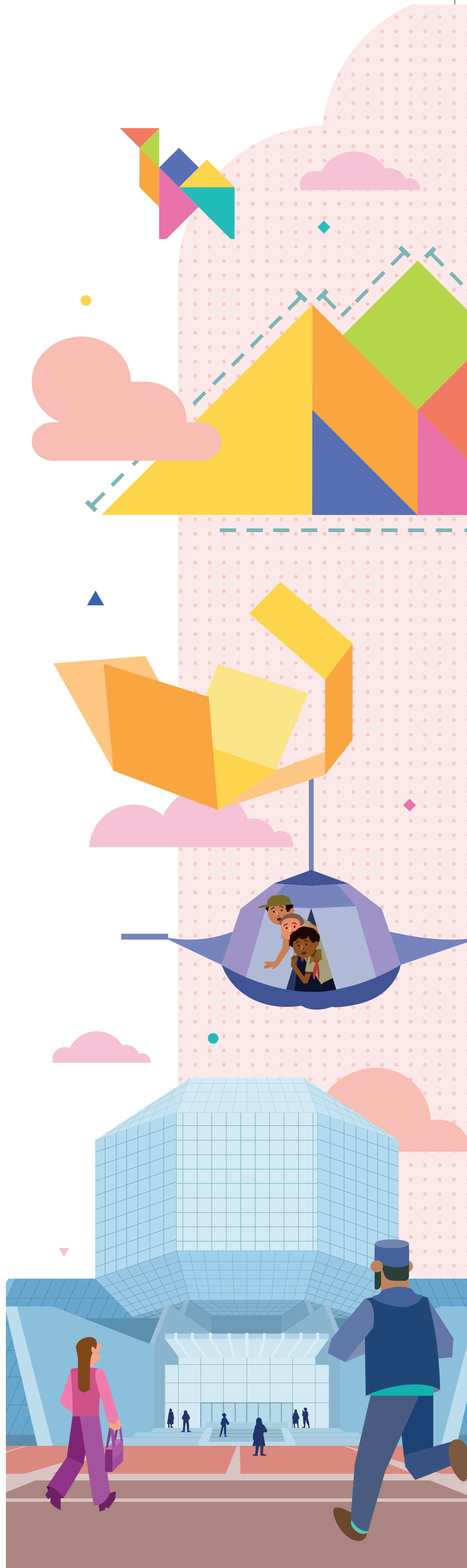
Before you dig in, we want you to know two things:

This book is special! It's where you'll record your thoughts, strategies, explanations, and, occasionally, any award-winning doodles that might come to mind.

When you go online, you won't be mindlessly plugging numbers into your device . . . You'll be pushing, pulling, crawling, teleporting, melting . . . , well, let's just say you'll be doing a lot of things, and you'll be teaming up with your classmates as you do.

Let's dig in!

Sincerely,
The Amplify Math Team

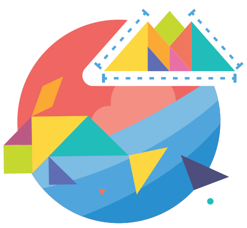


Volume 1

Unit 1 Area and Surface Area

Unit Narrative:
A Place for Space

Geometry is the mathematics of space and all the shapes and sizes within it, and even dimensions. You know the names of many special two-dimensional and three-dimensional figures, and have worked with the area of very basic shapes before. But, now, it is time to cover anything and everything, literally.



Can a sum ever really be greater than its parts?
Polygons are shapes whose sides are all line segments, and they can be decomposed and rearranged without changing their area.

How did a misplaced ruler change the way you shop?
Polyhedra are three-dimensional figures composed of polygon faces. Their surfaces can be decomposed.

Unit 2 Introducing Ratios

A little bit of this and a little bit of that. Well, maybe a lot of that? Wait, I think a ratio can help with this dilemma! Ratios help us see the relationship between one number and another, so when we make guacamole, it doesn't taste awful.

Unit Narrative:
Sensing a Ratio

Unit 3 Rates and Percentages

One black truffle costs how much?! A hummingbird flaps its wings how many times in one minute?! Unit rates — how much per one — are useful ratios. And sometimes how much per one hundred, a percentage, is useful too — if you want to know: Who should take the technical foul shot with no time on the clock? Do people really like dogs better than cats? Who won the election?

Unit Narrative:
Stand and Be Counted

How did student governments come to be?
Rates describe relationships between quantities like price and speed. Unit rates reveal which is a better deal or who is faster.

What can a corpse teach us about governing?
Percentages are rates per 100. They can compare relationships between parts and wholes, even when two quantities have different total amounts.

Unit 4 Dividing Fractions

Division can be used to solve equal-sized groups problems, including when the size of a group and even the number of groups are represented by fractions. See how you can apply what you already know about multiplication and division to follow the mysteries within Spöklik Furniture and fraction division.

Unit Narrative:
Crossing the Fractional Divide

Which item costs between 100 and 1,000 spök-bucks?
Multiplication and division are related, and the relationship between fractions and division can be used to estimate quotients.

How long is the bolt Samira needs?
To divide fractions, you can use multiplication, common denominators, or an algorithm. Apply these to determine the length of an oddly labeled bolt.

How can Maya fit Penny in the box?
When you know an area or volume, but not every side length, you will often divide fractions.

Volume 2

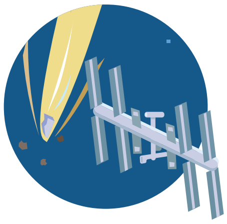
Unit 5 Arithmetic in Base Ten

Decimals embody the numerical language of precision. And, because we use a base ten number system, and the world is a messy place, decimals are everywhere. Being able to add, subtract, multiply, and divide any numbers with any number of decimal places can help you determine and make sense of some astonishing facts and human accomplishments that are world records.

Unit Narrative:
Making Moves With Decimals

How did a decimal decide an Olympic race?
Determine the results of high stakes competitions and identify record-setting moments by adding and subtracting decimals, as precisely as you need.

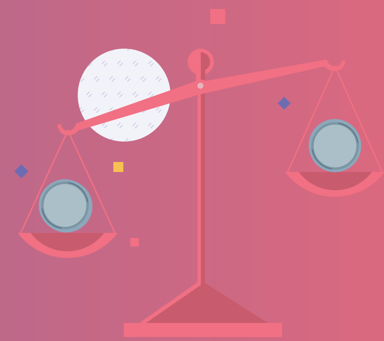
What happens when you make a small change to a big bridge?
To reproduce something at large or small scales so it looks the same, you need decimals and multiplication.



How do you dodge a piece of space junk?
Dividing whole numbers and decimals with many digits is the final set of operations you need to complete your trophy case.

Unit 6 Expressions and Equations

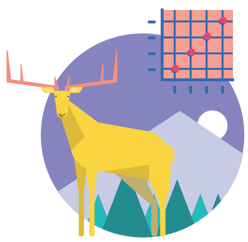
Unit Narrative:
The Power of Balance



Up until now, an equal sign meant you were being asked to calculate an answer. In this unit, you'll learn about its other meaning — balance. And when things are in balance, it becomes possible to know the unknown.

What's a bag of chips worth in Timbuktu?
Learn about the 14th century African salt trade, as you explore expressions and equations with tape diagrams and hanger diagrams.

How did a Welshman equalize England's upper crust with its common folk?
Extend the concept of equality as you investigate equivalent expressions, the all-important Distributive Property, and exponents.



What's more dangerous: a pack of wolves or a gang of elk?
Balance is everywhere, especially in ecosystems. You'll look at systems that are in and out of balance.

Unit 7 Rational Numbers

Think back to when you first learned about whole numbers and used them to count. Later, you saw there were numbers between them: fractions and decimals. Up until now, every number you've encountered has always been greater than 0. But no more. There is an entire set of numbers (just as many, in fact), lurking on the *other* side of every number line.

Unit Narrative:
Getting Where We're Going

What's the tallest mountain in the world?
Consider the most extreme locations on Earth as you discover negative numbers, which lend new meaning to positive numbers and zero.

How do you keep a quantity from wandering off?
A variable represents an unknown quantity. And sometimes it represents many possible values, which can be expressed as an inequality.

How did Greenland get so big?
Armed with the opposites of positive rational numbers, it's time you expanded your coordinate plane. Welcome to the four quadrants!

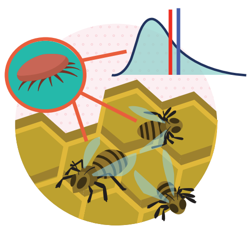
Unit 8 Data Sets and Distributions

Statistics is the science of collecting and analyzing data. It is one of the most relevant aspects of mathematics in everyday life. And it is also used by researchers in many fields, such as zoologists identifying new species and studying populations of endangered species. In all cases, knowing what is typical is critical to understanding what is not.

Unit Narrative:
Walk on the Wild Side with Data

How do you keep track of a disappearing animal?

When questions have more than one answer, it is helpful to visualize and describe a typical answer. For numbers, you can also identify the center and describe the spread of the numbers.



What's the buzz on honey bees?

For numerical data, you can summarize an entire data set by a single value representing the center of the distribution. The mean and the median represent two ways you can do this.

Where have the giant sea cows gone?

For numerical data, you can summarize an entire data set by a single value representing the variability of the distribution. The MAD, range, and IQR represent three ways you can do this.

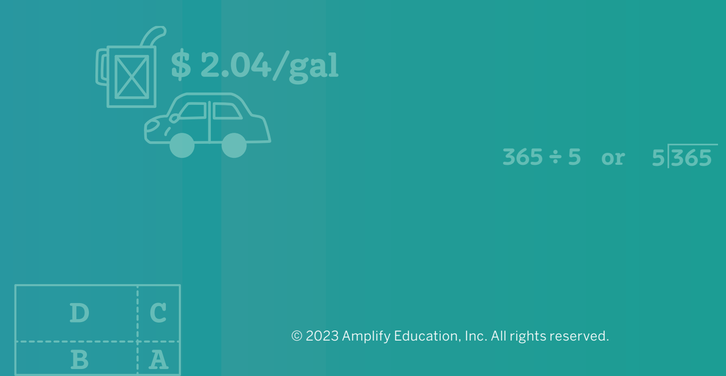
UNIT 5

Arithmetic in Base Ten

Decimals embody the numerical language of precision. And, because we use a base ten number system, and the world is a messy place, decimals are everywhere. Being able to add, subtract, multiply, and divide any numbers with any number of decimal places can help you determine and make sense of some astonishing facts and human accomplishments that are world records.

Essential Questions

- How can base ten units be composed and decomposed?
- How is the total number of decimal places in two decimal factors related to the total number of decimal places in their product?
- What is similar and different about using related whole number expressions when multiplying versus dividing decimals?
- *(By the way, what place value position could also be called the "oneths" place?)*



SUB-UNIT

1 | Adding and Subtracting Decimals

Narrative: Determine the results of high-stakes competitions and identify record-setting moments by adding and subtracting decimals.

You'll learn . . .

- methods for adding decimals.
- methods for subtracting decimals.

SUB-UNIT

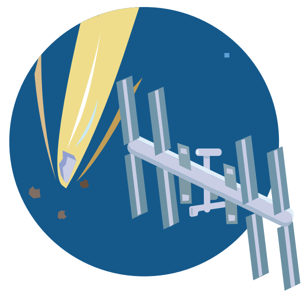
2 | Multiplying Decimals

Narrative: From tremendous bridges to tiny chess sets, multiplying decimals can help you make sense of engineering marvels in the world around you.

You'll learn . . .

- methods for multiplying decimals.
- how to represent decimal multiplication with diagrams.

SUB-UNIT



3 | Dividing Decimals

Narrative: Dividing decimals can tell you precisely how fast to go to dodge some space junk, and a whole lot more.

You'll learn . . .

- long division.
- other strategies for dividing decimals.

Start and end with the same number, going clockwise or counterclockwise, to get a whole number.

	−		
+	11.59	3.115	÷
	2.4	1.45	
	×		

Unit 5 | Lesson 1 – Launch

Precision and World Records

Let's look at the use of decimals in world records.

Warm-up Pin the Decimal on the Record

On March 25, 2016, Tom Wallisch had all eyes on him to break the world record for the longest rail grind on skis. To make this moment all that much more remarkable, an official Guinness World Records™ judge stood nearby, ready to verify Tom's accomplishment and place in history. After several unsuccessful attempts and two cold days later, the record-breaking slide happened.

Date: March 27, 2016
Location: Seven Springs, Pennsylvania, USA
Guinness World Record: Longest rail grind on skis
Distance: The number of meters recorded for the total distance included the following digits in order, 128656, but the decimal point is missing.

Where do you think the decimal point goes? As the decimal is "pinned," say the number aloud. Then complete the sentence.

Tom Wallish's record run was ______ meters.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 World's Longest Fingernails Challenge

At some point in 1979, Lee Redmond decided to let all of her fingernails continue to grow. Fast forward 30 years and they would measure, in total, a staggering 8.65 m (28 ft 4.5 in.). Unfortunately, in early 2009, Lee's nails all broke in an automobile accident, but she still holds the Guiness World Record for the "Longest fingernails on a pair of hands ever (female)."

But wait! There's more.

In 2014, the Guiness World Record for the "Longest fingernails on a single hand ever" was recorded by Shridhar Chillal of India. They measured 909.6 cm (29 ft 10.1 in.) in total. He still held the record as of 2020, but, after 66 years of not doing so, he did cut his nails on July 11, 2018.

Part 1

1. Whose fingernails are longer and by how much?

2. If Shridhar had let his nails grow to the exact same length on *both* hands, what would the total length of both hands have been?

Part 2

Your group will be assigned to recreate either Shridhar's fingernails or Lee's fingernails using strips of paper. Decide who in your group will wear the model. Tape strips of paper together and then tape the long strips to this person's nails for a truly visual experience of a world record.

Record the measurements your group uses and creates for each fingernail in the table.

	Thumb	Pointer	Middle	Ring	Pinky	Total
Shridhar						
Lee						

STOP

Narrative Connections

Unit 5 Arithmetic in Base Ten

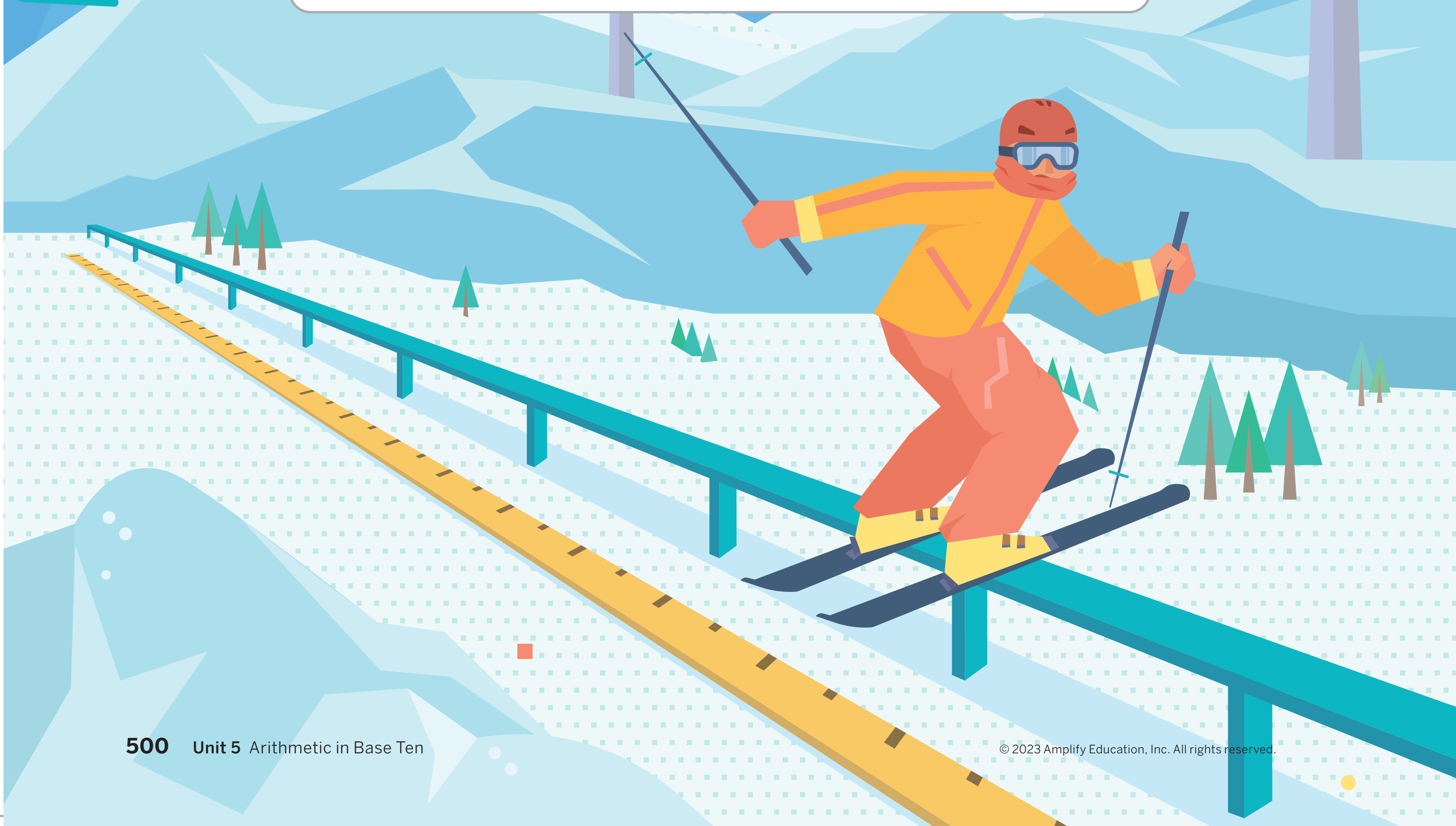
Making Moves With Decimals

From the strange to the inspiring, the Guinness World Records have dazzled people's imaginations for decades. Whether it's the world's longest egg noodle or the longest tongue, the organization has relied on decimals to determine whose names get to live on in their record books.

Decimals allow us to describe numbers with accuracy that is also easy to read. That's why we find decimals in so many different places. We use them to describe the prices of the things we buy at the supermarket. We use them to describe the weight of a package we are about to mail. They tick down on our stopwatches over the course of a race.

Anywhere we want to describe a quantity with precision, a decimal is sure to be in the mix. With just a quick glance, decimals let us know what is bigger or smaller, what is faster or slower, what is heavier or lighter, and what is nearer or farther. And given how often they show up, it is crucial to understand how to operate with them.

Welcome to Unit 5.



Practice

Name: ______________________ Date: __________ Period: __________

1. The Guinness World Record for the "Longest horns on a yak" belongs to Jericho the Yak from Welch, Minnesota, measuring at a total of 346.4 cm. A yak has two horns. If both horns were of equal length, what is the length of one horn?

2. Tickets to a show cost \$5.50 for adults and \$4.25 for students. A family is purchasing 2 adult tickets and 3 student tickets.

   a. Estimate the total cost.

   b. What is the exact cost?

   c. If the family gives the ticket attendant \$25, what is the exact amount of change they should receive?

3. Complete each expression with one of the following values so that the corresponding quotient fits correctly in each category. A number may be used more than once.

0.001	0.1	10	1,000

Close to $\frac{1}{100}$	Close to 1	Greater than 100
________ $\div 9$	________ $\div 0.12$	________ $\div \frac{1}{3}$
$12 \div$ ________	$\frac{1}{8} \div$ ________	$700.7 \div$ ________

Practice

Name: ______________ Date: ______________ Period: ______________

4. Determine each quotient. Show your thinking.

a $\frac{5}{6} \div \frac{1}{6}$

b $1\frac{1}{6} \div \frac{1}{12}$

c $\frac{10}{6} \div \frac{1}{24}$

5. One bottle of water holds $1\frac{1}{2}$ liters.

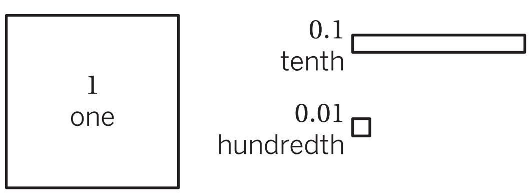
a How many $\frac{1}{5}$-liter glasses can be filled with one bottle of water?

b How many bottles of water are needed to fill a 16-liter jug?

6. How would you represent the number 3.4:

a In words?

b As a model using these blocks?



SUB-UNIT

1 Adding and Subtracting Decimals

Narrative Connections

How did a decimal decide an Olympic race?

Most of the time, whole numbers do a good job in telling us the outcome of any game or sport. With just a glance at a scoreboard, it should be clear which team is the champion. But while whole numbers are great for something like basketball, tennis, or competitive tuk tuk polo, there are some sports where you will want to be a bit more precise.

Take, for instance, the 1964 road cycling championship at the Tokyo Olympics. One hundred thirty-nine cyclists from 37 countries took to the race course, pelted by heavy rain. For much of the race, the peloton — the main group of cyclists — stayed clustered in a tight pack. Despite their best efforts, none of the cyclists could break out into a significant lead.

The race ended up having the tightest finish in history, with 51 cyclists all crossing the finish line in 4 hours 39 minutes and 51 seconds. But the race was not a tie. The winner, Italy's Mario Zanin finished first at 4:39:51.63, followed by Denmark's Kjell Rodian at 4:39:51.65 in second place, and Walter Godefroot from Belgium at 4:39:51.74 in third. The difference between the gold medalist Zanin and the cyclist who took 99th place was only 0.2 seconds!

As history has shown, a difference of a few hundredths of a second can make or break an Olympic championship. Understanding how to add and subtract these decimals can help you figure out who's in the lead and by how much.

Unit 5 | Lesson 2

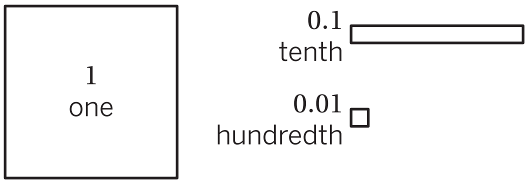
Speaking of Decimals . . .

Let's use the language of decimals.

Warm-up What's the Number?

1. Think of as many different ways as possible that you could say the number 0.23 aloud.

2. Show one way to represent 0.23 using these base ten blocks. If you can think of other ways, show as many ways as possible.



Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Representing Addition With Decimals

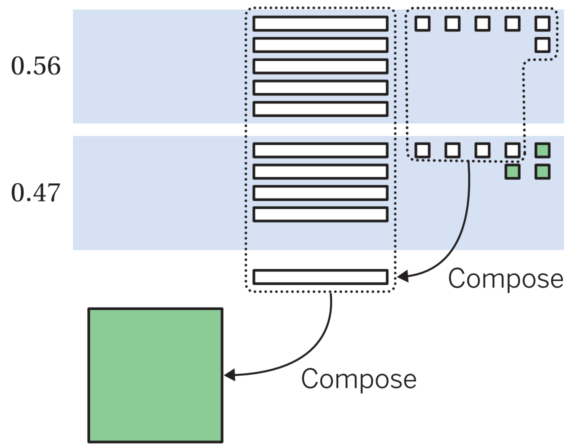
Part 1

Here are two methods you can use to calculate $0.56 + 0.47$. In the diagram, each large square represents 1, each rectangle represents 0.1, and each small square represents 0.01.

Vertical calculation:

$$\begin{array}{r} {\scriptstyle 1\ \ 1}\ \\ 0.56 \\ +\,0.47 \\ \hline 1.03 \end{array}$$

Base ten diagram:



1. Write an addition equation to represent this sum using both numbers and words.

2. Use what you know about base ten units and adding base ten numbers to respond to these questions.

   a. How can ten small squares be *composed* into a rectangle?

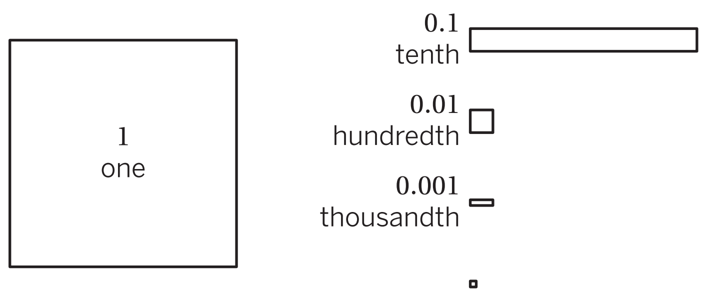
   b. How can ten rectangles be *composed* into a large square?

   c. How are the different types of *composition* represented in the vertical calculation?

Activity 1 Representing Addition With Decimals (continued)

Part 2

3. Each larger shape can be composed of 10 of the next smaller shape. What value does this new smallest shape represent?



4. Decide who will be Partner A and who will be Partner B. Evaluate each sum as indicated, using either a base ten diagram or a vertical calculation.

Partner A	Partner B
Calculate $6.038 + 0.091$.	Draw a diagram to represent $6.038 + 0.091$.
Draw a diagram to represent $2.5936 + 0.3132$.	Calculate $2.5936 + 0.3132$.

5. After you have both completed both of your problems, discuss the following questions with your partner.
   - How did the vertical calculation and the diagram show composing units?
   - Is one method of calculating more efficient than the other? If so, why?

Name: ______ Date: ______ Period: ______

Activity 2 Representing Subtraction With Decimals

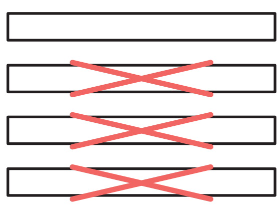
Common language and representations have always been important for sharing mathematical results.

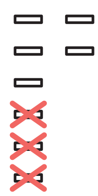
Two symbols that are important in communicating decimal operations are the digit 0 and the decimal point. Believe it or not, both of these are only about a thousand years old. In ancient China, they were introduced around 1247 CE. Qin Jiushao used the ○ symbol as the first digit in a number with a value less than 1. So by writing 096644 with that symbol as the 0, he was really indicating the number 0.96644.

Plan ahead: After evaluating your use of base ten blocks in Activity 1, how will you change your interactions for Activity 2?

Complete these problems using the same key from Activity 1 as a common language to interpret the values represented by each base ten diagram.

1. Here are diagrams representing three subtraction expressions. The pieces marked with "X" have been removed. With your partner, say the expressions represented by each diagram aloud, then evaluate the difference. Write the subtraction equation it represents using numbers.

   a 

   b 

   c 

Featured Mathematician

Qin Jiushao

Qin Jiushao was a 13th century Chinese mathematician, inventor, and politician. In *Shùshū Jiǔzhāng* ("Mathematical Treatise in Nine Sections") he published a general form of the Chinese Remainder Theorem and several new discoveries in algebra and geometry. Qin is also attributed with the first written use of a symbol for 0 and introducing an understanding of decimal fractions in China.

Activity 2 Representing Subtraction With Decimals (continued)

2. Determine each difference by drawing a base ten diagram *and* by using a vertical calculation. Compare your results with your partner. If you have different results, work together to discuss and resolve any differences until you both arrive at the same result.

a $0.05 - 0.02$

b $0.024 - 0.003$

c $1.26 - 0.14$

Compare and Connect: How do the base ten diagrams compare to the vertical calculations? Where do you see 0.024 in the base ten diagram for part b? Where do you see 0.003?

Are you ready for more?

A concession stand sells pretzels for $\$3.25$ each, drinks for $\$1.85$ each, and bags of popcorn for $\$0.99$ each. Clare purchased at least one of each item and spent no more than $\$10$. Could Clare have purchased each of the following combinations of items? Explain your thinking.

1. 2 pretzels, 2 drinks, and 2 bags of popcorn?

2. 1 pretzel, 1 drink, and 5 bags of popcorn?

STOP

Name: ________________ Date: ________ Period: ________

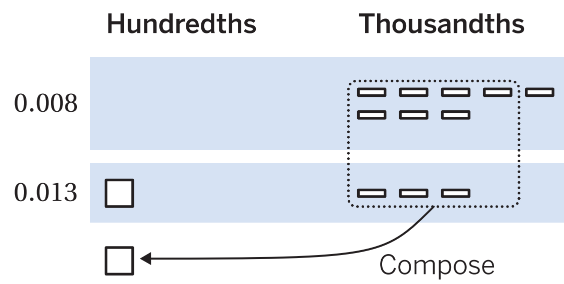
Summary

In today's lesson . . .

You saw that precise language is important when reading decimals, in order to clearly communicate the place value of the digits in a number. Place value and decimal language are also helpful when adding or subtracting decimals.

Base ten diagrams can be used to represent decimal numbers. A certain shape represents each place value, and exactly ten of that shape is the same size as the next larger shape. These diagrams are also helpful for representing and determining sums and differences of decimals.

Suppose you want to determine the sum of $0.008 + 0.013$. In this diagram, a small rectangle represents 0.001. You can group, or *compose*, 10 thousandths to make 1 hundredth. After composing these shapes, the diagram shows that the sum contains 2 hundredths and 1 thousandth, meaning that it is equal to 0.021.



You can also perform addition and subtraction by using a vertical calculation. The same sum, including the composing of thousandths, can be seen in the vertical calculation of $0.008 + 0.013$.

$$\begin{array}{r} \scriptstyle 1 \\ 0.013 \\ +\,0.008 \\ \hline 0.021 \end{array}$$

Reflect:

Practice

Name: ________________ Date: ________ Period: ________

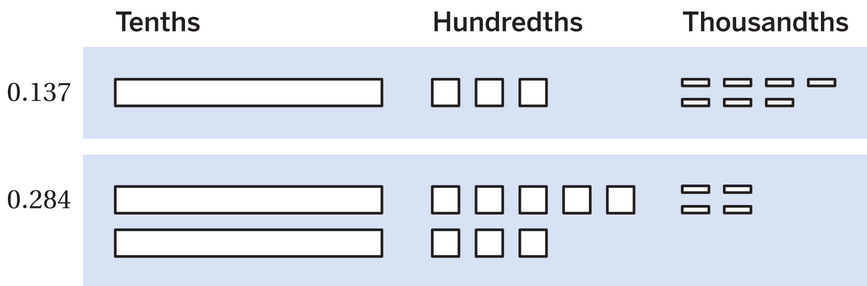
1. Use this key to complete these problems.

a What number does this diagram represent? 

b Draw a diagram that represents 0.216.

c Draw a diagram that represents 0.304.

2. The diagram represents the sum of 0.137 and 0.284.



a Use the diagram to determine $0.137 + 0.284$. Show your thinking.

b Show how to determine the sum using a vertical calculation.

$$\begin{array}{r} 0\,.\,1\,3\,7 \\ +\;0\,.\,2\,8\,4 \\ \hline \square\square\square\square\square \end{array}$$

c How are the methods similar in representing addition? How are they different?

3. Circle the vertical calculation in which the digits of the same place value are lined up correctly. Then complete that corresponding calculation.

a $3.25 - 1$

$$\begin{array}{r} 3.2\ \ 5 \\ -\ \ 1.0 \\ \hline \end{array} \qquad \begin{array}{l} \ \ 3.25 \\ -1.0 \\ \hline \end{array} \qquad \begin{array}{r} 3.25 \\ -\quad 1 \\ \hline \end{array}$$

b $0.5 + 1.15$

$$\begin{array}{l} \ \ 0.5 \\ +1.15 \\ \hline \end{array} \qquad \begin{array}{r} 0.5 \\ +1.1\ \ 5 \\ \hline \end{array} \qquad \begin{array}{r} 0.50 \\ +1.1\ \ 50 \\ \hline \end{array}$$

4. Calculate each sum or difference using a vertical calculation. Show your thinking.

a $10.6 + 1.7$

b $123.5 - 1.2$

5. Tyler's school is having a stair climbing challenge for "Better Health Week." He can now climb 135 stairs in 90 seconds.

a If Tyler climbs stairs at a constant rate, how many stairs can he climb per second?

b Shawn also participates in the challenge. Shawn can climb 75 stairs in 1 minute. Who climbs at a faster rate? Explain or show your thinking.

6. Show or explain how you would calculate the sum of $1.091 + 0.009$.

____

Unit 5 | Lesson 3

Adding and Subtracting Decimals

Let's add and subtract decimals.

Warm-up Do the Zeros Matter?

1. Mentally evaluate the expression $1.009 + 0.391$. Be prepared to explain your thinking.

2. Decide whether each equation is *true* or *false*. Be prepared to explain your thinking.

   a. $34.56000 = 34.56$

   b. $25 = 25.0$

   c. $2.405 = 2.45$

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Adding Decimals of Different Lengths

You have seen that the placement of the decimal point is important in communicating the value of a number. The Hindu-Arabic numerals we use today have been around since the 900s. During that time, it was Arab mathematician al-Uqlidisi who first introduced a unique symbol (′) for the decimal point.

Complete these problems, paying attention to the placement of the decimal point in numbers with decimals of different lengths.

1. Here are two calculations of $0.2 + 0.05$. One is correct and the other is incorrect.

$$\begin{array}{r} 0.2 \\ +\,0.05 \\ \hline 0.25 \end{array} \qquad \begin{array}{r} 0.2 \\ +\,0.05 \\ \hline 0.07 \end{array}$$

a. Which calculation is correct?

b. Why is the other calculation incorrect?

2. Calculate each sum. Consider setting up a vertical calculation or drawing base ten diagrams to help with your thinking.

a. $0.11 + 0.005$

b. $0.209 + 0.01$

c. $10.2 + 1.8456$

Featured Mathematician

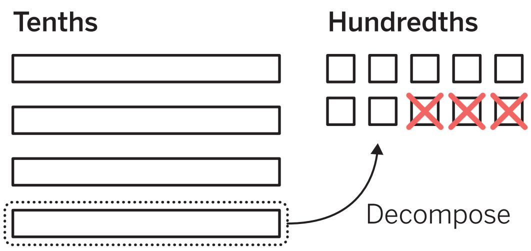
Abu'l Hasan Ahmad ibn Ibrahim al-Uqlidisi

Abu'l Hasan Ahmad ibn Ibrahim al-Uqlidisi was a mathematician who lived in the mid-to-late 10th century, most likely in Syria. In one of his two major works, the *Kitab al-fusul fi al-hisab al-Hindi*, Al-Uqlidisi introduced the Hindu-Arabic system of numerals, and in addressing decimal fractions, he suggested a ′ symbol for the decimal point. Al-Uqlidisi went on to show how previously known methods of arithmetic translated into this new number system, which turned out to be more useful for basic operations.

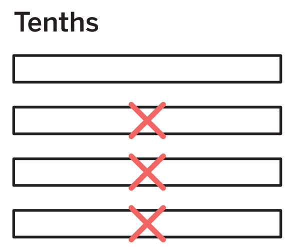
Activity 2 Subtracting Decimals of Different Lengths

To represent the expression $0.4 - 0.03$, Diego, Noah, and Elena each drew a different base ten diagram. In each diagram, one rectangle represents 0.1 and one square represents 0.01.

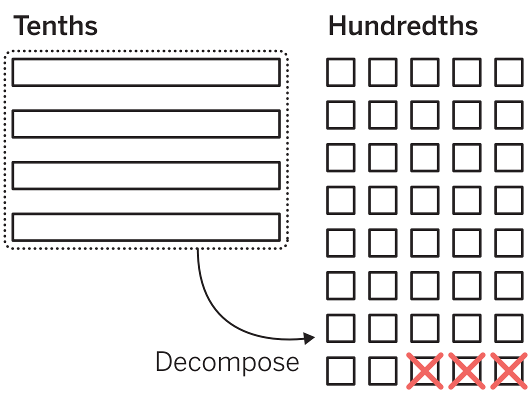
Diego's method:



Noah's method:



Elena's method:



1. With your partner, discuss how subtraction is shown in each student's diagram.

2. Does any diagram correctly represent $0.4 - 0.03$? If so, which one(s)? Explain your thinking.

Name: .... Date: .... Period: ....

Activity 2 Subtracting Decimals of Different Lengths (continued)

Show or explain your thinking for each of the following problems.

3. Evaluate each expression. Then compare your results with a partner. If you and your partner have different results, consider drawing a base ten diagram or using a vertical calculation to determine the correct solution.

a. $0.3 - 0.05$

b. $1.03 - 0.016$

c. $0.025 - 0.00735$

4. Refer to the table which shows the three medalists of the Skateboard Big Air event at the X Games in 2013.

X Games Barcelona 2013 — Skateboard Big Air

Final		
Rank	**Name**	**Score**
1	Bob Burnquist	93.165
2	Mitchie Brusco	90.995
3	Elliot Sloan	90.83

a. Circle the pair of medal-winning scores that were the closest. Explain your thinking.

Gold and Silver Silver and Bronze

b. What is the difference in the scores of the two closest medal winners?

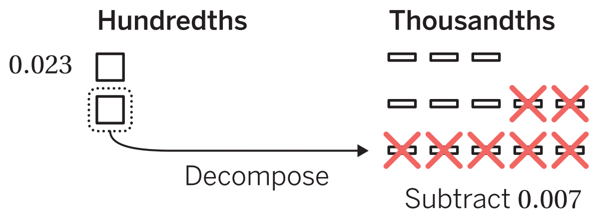
STOP

Summary

In today's lesson . . .

You continued exploring adding and subtracting decimals, but in cases where the two numbers in the given expressions had decimals of "different lengths" — meaning two different place values. It is important to correctly line up all place values for adding and subtracting decimals, including those any digits that are 0. Whenever a subtraction problem requires subtracting a greater digit from a lesser digit in one place value, then you need to *decompose* a value of 1 from the next greater place value to make 10 of the place value that you need.

Suppose you want to determine the difference of $0.023 - 0.007$. You need to subtract 7 thousandths (7 small rectangles in a base ten diagram) from 3 thousandths. While you might not think this is possible, you can actually decompose 1 hundredth into 10 thousandths, and then you can subtract 7 thousandths from a total of 13 thousandths (10 thousandths $+$ 3 thousandths).



Subtracting 7 thousandths from 13 thousandths leaves you with 6 thousandths. To complete the subtraction, you then have 1 hundredth left from 0.023, and you can subtract 0 hundredths. The difference is equal to 1 hundredth and 6 thousandths, which is written numerically as $0.023 - 0.007 = 0.016$.

$$
\begin{array}{r}
 \scriptstyle 1\ 13 \\
0.\,0\,\cancel{2}\,\cancel{3} \\
-\ 0.\,0\,0\,7 \\
\hline
0.\,0\,1\,6
\end{array}
$$

This difference can also be shown using a vertical calculation, with the same decomposing.

Reflect:

Practice

Name: ________________________ Date: ____________ Period: ____________

1. Determine each of the following sums. Consider drawing base ten diagrams to help with your thinking.

   a. $0.027 + 0.004$

   b. $0.203 + 0.01$

   c. $1.2 + 0.145$

2. Mai claims that 1.97 cannot be subtracted from 20 because 1.97 has two decimal places and 20 has none. Do you agree? Show or explain your thinking.

3. Complete each calculation to determine the correct difference.

   a.
$$\begin{array}{rrrcr} 1 & 4 & 2 & . & 6 \\ - & & 1 & . & 4 \\ \hline \boxed{} & \boxed{} & \boxed{} & . & \boxed{2} \end{array}$$

   b.
$$\begin{array}{rrcrr} 3 & 8 & . & 6 & 0 \\ - & 6 & . & 7 & 5 \\ \hline \boxed{} & \boxed{} & . & \boxed{} & \boxed{5} \end{array}$$

   c.
$$\begin{array}{rrrcrr} 2 & 4 & 1 & . & 7 & 6 \\ - & & 2 & . & 1 & 8 \\ \hline \boxed{} & \boxed{} & \boxed{} & . & \boxed{} & \boxed{8} \end{array}$$

Practice

Name: .... Date: .... Period: ....

4. A rectangular prism measures $7\frac{1}{2}$ cm by 12 cm by $15\frac{1}{2}$ cm.

a. Calculate the number of cubes with an edge length of $\frac{1}{2}$ cm that will fit into this prism.

b. What is the volume of the prism, in cubic centimeters? Show your thinking. **Hint:** Think about how many cubes with $\frac{1}{2}$-cm edge lengths fit into a cube with a volume of 1 cm^3.

5. A car travels 75 miles in 60 minutes at a constant speed. How far does the car travel in 18 minutes? Consider using the table to help with your thinking.

Minutes	Distance (miles)
60	75
6	
18	

6. Which calculation shows the correct way to determine the difference of $0.3 - 0.006$? Explain your thinking.

A. $\begin{array}{r} 0.3 \\ -\,0.006 \\ \hline 0.306 \end{array}$

B. $\begin{array}{r} 0.3 \\ -\,0.006 \\ \hline 0.097 \end{array}$

C. $\begin{array}{r} 0.30 \\ -\,0.006 \\ \hline 0.024 \end{array}$

D. $\begin{array}{r} 0.300 \\ -\,0.006 \\ \hline 0.294 \end{array}$

Name: ______________ Date: ______________ Period: ______________

Unit 5 | Lesson 4

X Games Medal Results

Let's practice adding and subtracting decimals.

Warm-up Notice and Wonder

Here are three ways to write a subtraction calculation. What do you notice? What do you wonder?

$$\begin{array}{r} 5 \\ -\,0.17 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ -\,0.17 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ -\,0.17 \\ \hline \end{array}$$

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Skateboard Big Air

In the fourth and final Summer X Games of 2013, in Los Angeles, California, the Skateboard Big Air Final came down to three athletes:

Athlete	Description
Bob Burnquist	Won gold at all three previous Summer X Games that same year. His event ended with a best score of 88.665 and a broken nose.
Elliot Sloan	Placed third in Barcelona. Placed second in the other two Summer X Games that year, in Munich and Fox do Iquaçu. On his fifth and final run, his total deductions were 9.835 from the base score of 100 points.
Tom Shaar	13-year old competitor who completed an amazing run and posted a leading score of 88.83.

Who won the gold medal and how much higher was his score compared to each of the other two competitors?

Are you ready for more?

In a *cryptarithmetic puzzle*, the digits 0–9 are represented using the first 10 letters of the alphabet, but they can be assigned in any order for a given puzzle. Determine the digits that correspond with the letters E, F, H, I, and J in this puzzle.

$$\begin{array}{r} \text{I H F . I J} \\ +\ \text{J I I . F I} \\ \hline \text{E J I . I E} \end{array}$$

Name: ______ Date: ______ Period: ______

Activity 2 Snowboard Cross

Now, let's look at some events from the Winter X Games from 2015, held in Aspen, Colorado. In particularly, let's look at the Snowboarder X Adaptive Finals.

Avid snowboarder, Keith Gabel, had his leg amputated after a horrible accident at the age of just 21. But that did not stop him from becoming a gold medalist at the X Games in 2015. His winning time was officially recorded as 0:57.168.

1. The results of the event are shown in the table. Calculate the finishing times of the silver and bronze medal runs.

Snowboarder X Adaptive

Final		
Rank	**Name**	**Time**
1	Keith Gabel	0:57.168
2	Carl Murphy	-0.281
3	Alex Massie	-4.127

Activity 2 Snowboard Cross (continued)

Let's look at the Women's Snowboarder X Finals. Lindsey Jacobellis was not just the winner of the Women's snowboarder X race in 2015, but she also holds the Guinness World Record for "Most Winter X Games medals (11) for Snowboarder X by a female athlete."

2. The table shows the results of the 2015 Women's Snowboarder X finals, which Jacobellis won.

X Games Aspen 2015 — Women's Snowboarder X

Final		
Rank	**Name**	**Time**
1	Lindsey Jacobellis	0:53.085
2	Dominique Maltais	0:53.234
3	Nelly Moenne Loccoz	0:53.384

Fun fact! This race was the first time a live drone was used for an aerial camera in a sporting event.

Calculate the time difference between the first and second places. Then calculate the difference between the first and third places. Be prepared to explain what the differences in time mean within the context of the race.

Name: .... Date: .... Period: ....

Activity 3 Missing Numbers

Determine and write the missing digits in each calculation so that the value of each sum or difference is correct. Be prepared to show or explain your thinking.

1.

$$\begin{array}{r} 0.404 \\ +\ \square.\square\square\square \\ \hline 1.000 \end{array}$$

2.

$$\begin{array}{r} 9.8765 \\ +\ \square\square.\square\square\square\square \\ \hline 10.0000 \end{array}$$

3.

$$\begin{array}{r} 0.7 \\ -\ \square.\square\square\square \\ \hline 0.012 \end{array}$$

4.

$$\begin{array}{r} 7 \\ -\ \square.\square\square\square\square \\ \hline 3.4567 \end{array}$$

5.

$$\begin{array}{r} 70 \\ -\ \square\square.\square\square\square\square \\ \hline 0.0089 \end{array}$$

STOP

Summary

In today's lesson . . .

You saw that for expressions having numbers with many non-zero digits, such as $0.25103 - 0.04671$, it would take a long time to draw a base ten diagram. With vertical calculations, you can determine the difference more efficiently. Even though you might not draw a base ten diagram, you can still think about base ten diagrams to help you make sense of the calculations, particularly with *composing* or *decomposing* units.

For example, drawing a base ten diagram to represent $0.25103 - 0.04671$ would require 29 base ten shapes and then 20 more to show the two sets of decomposed digits. Using a vertical calculation, as shown here, is more efficient.

$$
\begin{array}{rcccccc}
 & & & & & 10 & \\
 & & & 4 & \cancel{0} & 10 & \\
 & 0 & . & 2 & \cancel{5} & \cancel{1} & \cancel{0} & 3 \\
- & 0 & . & 0 & 4 & 6 & 7 & 1 \\
\hline
 & 0 & . & 2 & 0 & 4 & 3 & 2
\end{array}
$$

Reflect:

Practice

Name: .... Date: .... Period: ....

1. For each subtraction problem, circle the correct calculation.

a $7.2 - 3.67$

$$\begin{array}{r} 7.2 \\ -\,3.67 \\ \hline 3.05 \end{array} \qquad \begin{array}{r} 0\,7.2 \\ -\,3.67 \\ \hline 3.05 \end{array} \qquad \begin{array}{r} 7.20 \\ -\,3.67 \\ \hline 3.53 \end{array}$$

b $16 - 1.4$

$$\begin{array}{r} 1\,6 \\ -\,1.4 \\ \hline 0.2 \end{array} \qquad \begin{array}{r} 1\,6.0 \\ -\,1.40 \\ \hline 0.20 \end{array} \qquad \begin{array}{r} 16.0 \\ -\;\;1.4 \\ \hline 14.6 \end{array}$$

2. Explain how you could determine the difference between 1 and 0.1978. You do not need to actually calculate the difference.

3. The label on a bag of dried apples says it contains 0.384 lb of apples. The actual weight of one bag of apples is 0.3798 lb.

a Are the apples actually heavier or lighter than the weight stated on the label? Explain your thinking.

b How much heavier or lighter are the apples than stated on the label? Show your thinking.

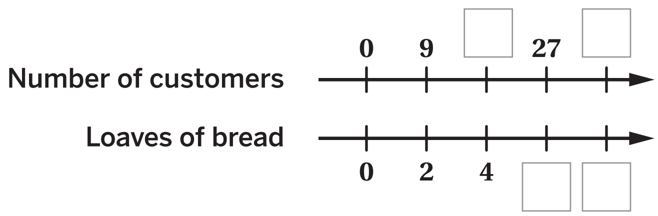
Practice

Name: ______________________ Date: __________ Period: __________

4. A shipping company is loading cube-shaped crates into a larger cube-shaped container. The smaller cubes have side lengths of $2\frac{1}{2}$ ft, and the larger shipping container has side lengths of 10 ft. How many crates will fit in the shipping container? Show your thinking.

5. For every 9 customers, a chef prepares 2 loaves of bread.

a The double number line and the table show varying numbers of customers and the number of loaves prepared by the chef. Complete each representation with the missing values.



Number of customers	Number of loaves
9	2
	4
27	
	14

b Use either representation to solve these problems.

- How many loaves are needed for 63 customers?
- If the chef prepares 20 loaves, how many customers would receive loaves?
- How much of a loaf is prepared for each customer?

6. Which of the following expressions are equal to 7.3? Select *all* that apply.

A. $73 \cdot \frac{1}{10}$

B. $73 \div \frac{1}{10}$

C. $73 \div 10$

D. $73 \cdot 10$

SUB-UNIT

2 Multiplying Decimals

Narrative Connections

What happens when you make a small change to a big bridge?

Since 2016, the Elizabeth Quay Bridge in Perth, Australia has been a landmark of the city. The bridge stands over the Swan River, suspended by two dramatic arches. Building it was a delicate job. Engineers had to install parts of the bridge, like the arch's stubs, even while the arches themselves were still being made.

To help with this, engineers needed to be accurate with their measurements. That way everything would match up once the finished arches were brought to the site. Bridges, buildings, and other structures often undergo changes as they move from design to the actual construction. Architects, engineers, and construction crews have to work together to accommodate any changes they might encounter.

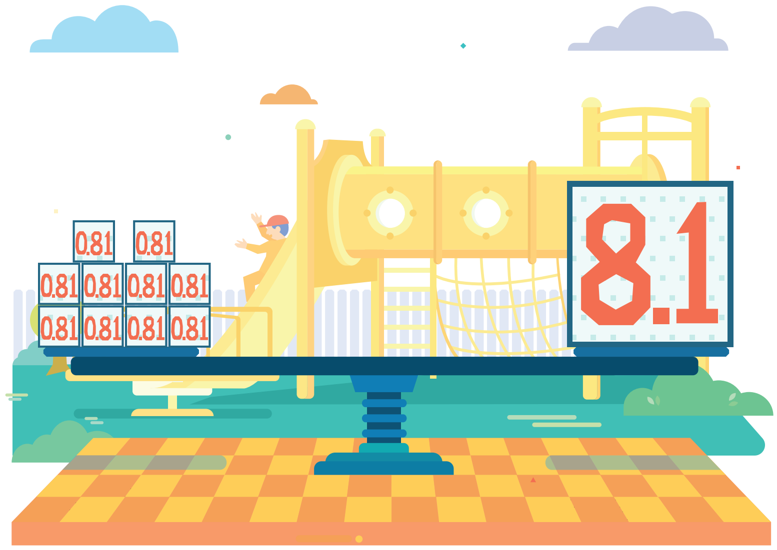
This kind of flexibility and precision are what decimals are made for. A designer might dream of a building using nice whole numbers. But reality may require that a few tenths or hundredths of a meter be shaved off or added. Even seemingly small adjustments can have a major impact for things like square footage and costs of materials.

To compute these, you have to understand how to multiply decimals. Before the first construction crane makes it to the work site, it is math that does the heavy lifting.

Unit 5 | Lesson 5

Decimal Points in Products

Let's look at products that are decimals.



Warm-up Multiplying by 10

1. Circle the equation for which the value of x is equal to 8.1. Be prepared to explain your thinking.

$x \cdot 10 = 810$ $\quad$ $x \cdot 10 = 81$ $\quad$ $x \cdot 10 = 8.1$ $\quad$ $x \cdot 10 = 0.81$

2. Compare 0.81 and 810. Which number is greater? How many times greater?

Discussion Support: What math terms can you use to describe how the structure of the equations in Problem 1 compare? Share those terms with your partner.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Fractions and Powers of 10

1. One of you will be Partner A and one of you will be Partner B. Determine the products and quotients for the problems in your assigned column. Then compare and discuss your responses, starting with part a.

Partner A	**Partner B**
a $250 \cdot \frac{1}{10}$	**a** $250 \div 10$
b $250 \cdot \frac{1}{100}$	**b** $250 \div 100$
c $48 \div 10$	**c** $48 \cdot \frac{1}{10}$
d $48 \div 100$	**d** $48 \cdot \frac{1}{100}$

2. Use your work from Problem 1 to determine the products of $720 \cdot (0.1)$ and $720 \cdot (0.01)$. Explain your thinking.

Activity 1 Fractions and Powers of 10 (continued)

3. Determine each product. Show your thinking.

   a. $36 \cdot 0.1$

   b. $24.5 \cdot 0.1$

   c. $1.8 \cdot 0.1$

   d. $54 \cdot 0.01$

   e. $9.2 \cdot 0.01$

4. Without calculating, determine how the values of the digits 7 and 5 change from the factor 750 to the product of $750 \cdot 0.001$. Explain your thinking.

Are you ready for more?

Ancient Romans wrote their numbers using symbols (or letters), now known as Roman numerals. They used the symbols I for 1, V for 5, X for 10, L for 50, C for 100, D for 500, and M for 1,000. Typically, a number was expressed by listing the necessary values from greatest to least, and readers understood that meant to add. For example, XVI represents $10 + 5 + 1 = 16$. But, for efficiency, they never repeated the same letter more than three times in a row.

1. What numbers are represented by IX, XLIV, and CMXCIV?
2. The film of the game for Super Bowl LIII has a copyright of MMXIX. What is the number of the Super Bowl, and in what year was it played?
3. What is the largest possible number you can write in Roman numerals using only these symbols and rules? Write it both as a number and in Roman numerals.

Name: ______ Date: ______ Period: ______

Activity 2 Fractions and Multiples of Powers of 10

1. Select *all* the expressions that are equivalent to $(0.6) \cdot (0.5)$. Be prepared to explain your thinking.

A. $6 \cdot 0.1 \cdot 5 \cdot 0.1$

B. $6 \cdot 0.01 \cdot 5 \cdot 0.1$

C. $6 \cdot 0.001 \cdot 5 \cdot 0.01$

D. $6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10}$

E. $6 \cdot \frac{1}{1000} \cdot 5 \cdot \frac{1}{100}$

F. $6 \cdot 5 \cdot \frac{1}{10} \cdot \frac{1}{10}$

G. $\frac{6}{10} \cdot \frac{5}{10}$

H. $6 \cdot 5 \cdot \frac{1}{100}$

2. Determine the value of $0.6 \cdot 0.5$. Show your thinking.

3. Determine the value of each product by writing and evaluating an equivalent expression that contains fractions.

a $0.3 \cdot 0.02$

b $1.5 \cdot 0.007$

STOP

Summary

In today's lesson . . .

You used the relationship between place value and unit fractions with denominators that are powers of 10 to reason about the location of the decimal point in products involving decimals. For example, to evaluate $24 \cdot (0.1)$, you can think of the expression in multiple ways, all of which are equivalent and result in a product of 2.4.

Reason about place value.	Multiply by the unit fraction.	Divide.
24 groups of 1 tenth, or 24 tenths.	$24 \cdot \frac{1}{10}$, because 0.1 is equivalent to $\frac{1}{10}$.	$24 \div 10$, because multiplying by $\frac{1}{10}$ has the same result as dividing by 10.

When you divide a number by 10, the digits will remain the same, but they will each have a value that is 10 times less. This means the decimal point should be located one place to the left.

Similar reasoning can be applied when evaluating expressions such as $(1.5) \cdot (0.43)$, which is equal to $\frac{15}{10} \cdot \frac{43}{100}$, or $\left(15 \cdot \frac{1}{10}\right) \cdot \left(43 \cdot \frac{1}{100}\right)$. Using the associative and commutative properties, this can be rewritten as $(15 \cdot 43) \cdot \left(\frac{1}{10} \cdot \frac{1}{100}\right)$. That essentially makes the equivalent product of $645 \cdot \frac{1}{1{,}000}$, which is equal to $645 \div 1{,}000$. So, the product of $(1.5) \cdot (0.43)$ has the same digits as 645, but the decimal point is located 3 places to the left, giving a final result of 0.645.

Reflect:

Name: ________ Date: ________ Period: ________

Practice

1. Determine the product. Use your work from parts a–d to complete part e.

a $122.1 \cdot \frac{1}{100}$

b $11.8 \cdot \frac{1}{100}$

c $1350.1 \cdot \frac{1}{100}$

d $1.704 \cdot \frac{1}{100}$

e What happens to the decimal point of the original number when you multiply it by $\frac{1}{100}$? Why do you think that is the case? Explain your thinking.

2. Which expressions have the same value as $0.06 \cdot 0.154$? Select *all* that apply.

A. $6 \cdot \frac{1}{100} \cdot 154 \cdot \frac{1}{1{,}000}$

B. $6 \cdot 154 \cdot \frac{1}{100{,}000}$

C. $6 \cdot 0.1 \cdot 154 \cdot 0.01$

D. $6 \cdot 154 \cdot 0.00001$

E. 0.00924

3. Calculate the value of each expression by first writing the decimal factors as fractions, and then writing their product as a decimal. Show your thinking.

a $0.1 \cdot 0.02$

b $0.3 \cdot 0.2$

c $1.2 \cdot 5$

d $0.9 \cdot 1.1$

Practice

Name: ______________________ Date: __________ Period: __________

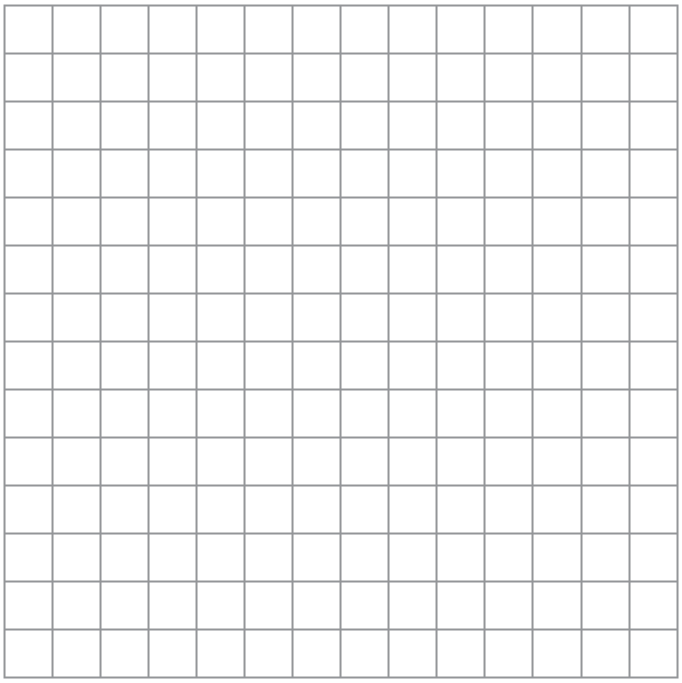
4. Evaluate each sum. Show your thinking.

a $33.1 + 1.95$

b $1.075 + 27.105$

c $0.401 + 9.28$

5. On the grid, draw a quadrilateral *that is not a rectangle* with an area of 18 square units. Show how you know the area is 18 square units.

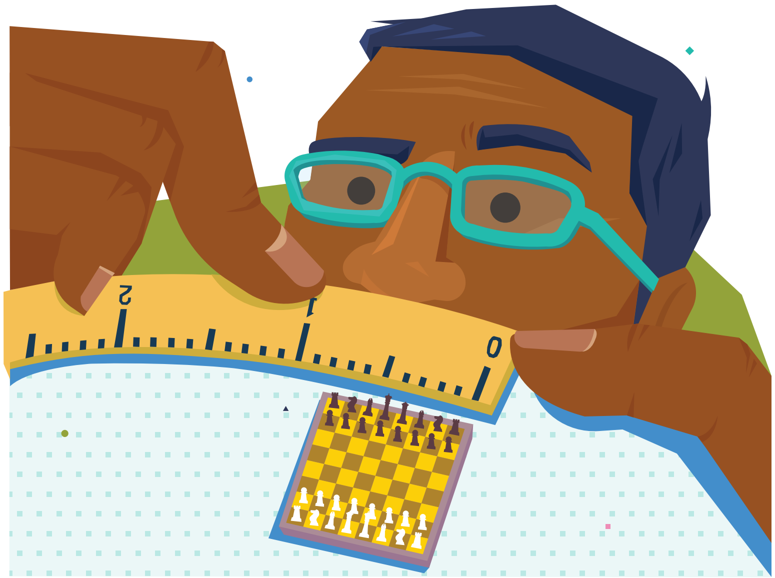


6. Write as many expressions using multiplication or division as you can think of that are equal to 6.

Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 6

Methods for Multiplying Decimals



Let's look at some ways to represent multiplication of decimals.

Warm-up Equivalent Expressions

Write as many expressions using multiplication or division as you can think of that are equal to 0.6.

Log in to Amplify Math to complete this lesson online.

Activity 1 The World's Smallest Chess Set

The Guiness World Record for "Smallest handmade chess set" has a top playing surface that measures 0.8 cm by 0.8 cm and was made in August 2020 by Ara Ghazaryan, an Armenian American artist.

1. Write an expression that determines the area of the world's smallest chess set. Do not evaluate your expression.

2. Write a related expression using two whole number factors. Then evaluate the expression.

3. How many times greater is the product of your related expression from Problem 2 than the product of your original expression from Problem 1? Why?

4. Explain how you can use your expression from Problem 2 to determine the area of the chessboard. Show your thinking.

Are you ready for more?

Like most Western chessboards, Ghazaryan's board is composed of 32 white squares and 32 black squares. What is the area of each square on Ghazaryan's board?

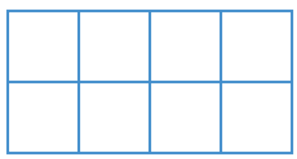
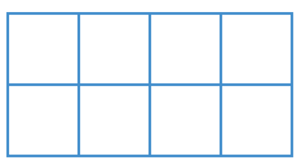
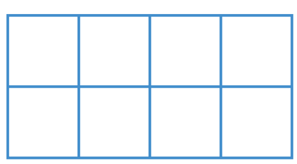
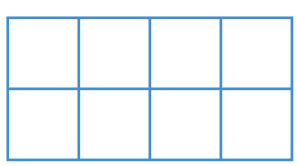
Name: .... Date: .... Period: ....

Activity 2 Related Expressions and Area Diagrams

1. Decide who will be Partner A and who will be Partner B. Partner A will complete Rows 1 and 3 in the following table. Partner B will complete Rows 2 and 4.

- Label the side length of each square in the diagram so that the area of the entire rectangle is represented by the expression.
- Determine the area of each *inner square*. Show your thinking.
- Determine the area of the *entire rectangle*. Show your thinking.

Then share and compare your responses.

Expression	Diagram	Area of each square (square units)	Area of the entire rectangle (square units)
$40 \cdot 20$			
$4 \cdot 2$			
$0.4 \cdot 0.2$			
$0.04 \cdot 0.02$			

Activity 2 Related Expressions and Area Diagrams (continued)

2. How does each of the products in the table relate to the equation $4 \cdot 2 = 8$? Show or explain your thinking.

3. Compute the product of $0.021 \cdot 4.7$ using the equation $21 \cdot 47 = 987$ and your thinking from Problem 2. Show or explain your thinking.

Reflect: In what ways did you show respect to your partner?

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw at least two other ways to think about and calculate a product of two decimals based on place value and the fact that decimals can be written as equivalent fractions — using related expressions with whole numbers and area diagrams.

Consider the product of $(0.04) \bullet (0.07)$. This expression can be rewritten as each of the following equivalent expressions.

$\frac{4}{100} \bullet \frac{7}{100}$ $\left(4 \bullet \frac{1}{100}\right) \bullet \left(7 \bullet \frac{1}{100}\right)$ $(4 \bullet 7) \bullet \left(\frac{1}{100} \bullet \frac{1}{100}\right)$

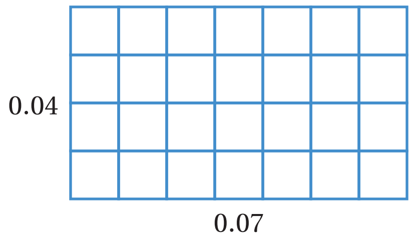
All of these expressions have a product of $\frac{28}{10{,}000}$, or 0.0028.

You can also multiply each factor by a power of 10 to create whole numbers, keeping track of how you changed the values, and then divide the product of those whole numbers by the product of both powers of 10.

- For $0.04 \bullet 0.07$, you could multiply both factors by 100.
- The expression $(0.04 \bullet 100) \bullet (0.07 \bullet 100)$ shows that is equal to $4 \bullet 7$.
- Because you multiplied by $100 \bullet 100$, you need to divide 28 by $10{,}000$ to get the actual product.

You can also use rectangular area models.

- For $0.04 \bullet 0.07$, you can represent $(4 \bullet 7) \bullet \left(\frac{1}{100} \bullet \frac{1}{100}\right)$ using unit squares that are $\frac{1}{100}$ by $\frac{1}{100}$, or 0.01 by 0.01.
- The rectangular area shown here looks the same as the rectangular area for multiplying $4 \bullet 7$.
- However, each square has an area of $\frac{1}{10{,}000}$ square units, instead of an area of 1 square unit.



Reflect:

Practice

Name: .... Date: .... Period: ....

1. Priya determines the value of the expression $1.05 \cdot 2.8$ by calculating $105 \cdot 28 = 2{,}940$ and then moving the decimal point three places to the left. She determines a product of 2.94 for the original expression.

   a. Why does Priya's method make sense?

   b. Use Priya's method to calculate $0.0015 \cdot 0.024$.

2. Determine each product. Show your thinking.

   a. $1.2 \cdot 0.11$

   b. $0.34 \cdot 0.02$

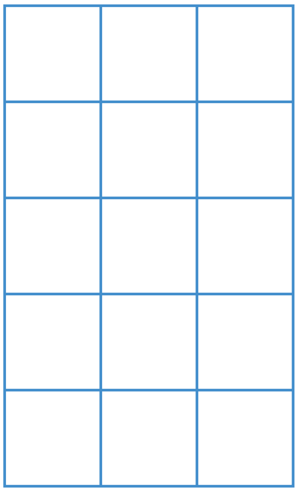
   c. $120 \cdot 0.002$

3. In this diagram, the area of the entire largest rectangle represents $0.3 \cdot 0.5$.

   a. Label the side lengths of each square forming the rectangle.

   b. What is the area represented by each square?

   c. What is the product of $0.3 \cdot 0.5$? Show or explain your thinking.

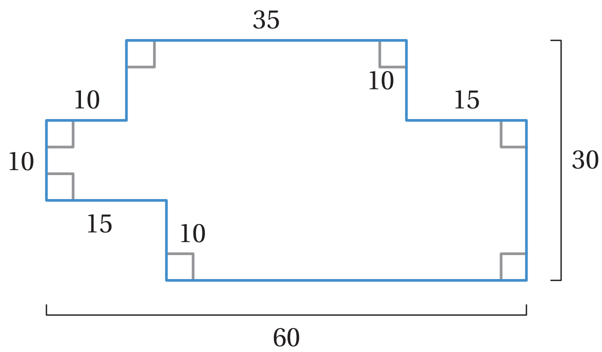


Name: .... Date: .... Period: ....

Practice

4. Evaluate $\frac{49}{50} \div \frac{7}{6}$. Show or explain your thinking.

5. Determine the area, in square units, of the figure. All angles are right angles. Show your thinking.



6. Evaluate $46 \cdot 32$. Show your thinking.

Unit 5 | Lesson 7

Representing Decimal Multiplication With Diagrams

Let's use area diagrams to determine products.

Warm-up Representing Multiplication

You will be given a table showing three strategies for representing and evaluating the expression $24 \cdot 13$.

1. How are the area models similar? How are they different?

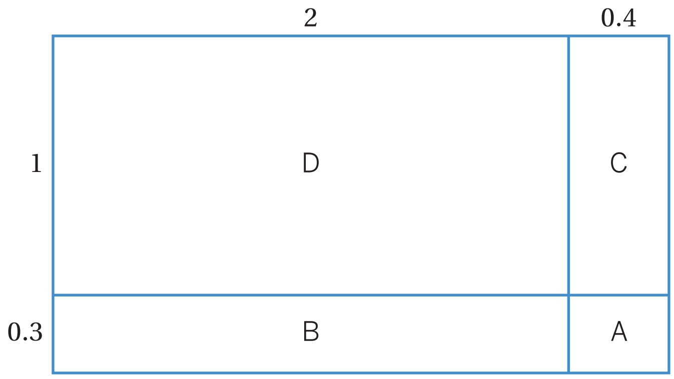
2. How are each of the area models related to their corresponding calculations?

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 Connecting Area Diagrams to Decimal Multiplication

Consider the area diagram.



1. What expression does the area diagram represent?

2. Label Regions A, B, C, and D with their areas. Show your thinking here.

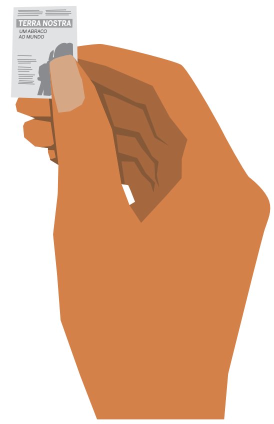
3. Determine the product that the area diagram represents. Show your thinking.

Are you ready for more?

Using the digits 0–9, write a multiplication expression that may be less efficient to solve with an area diagram than with another multiplication strategy. Both factors in your expression should be decimals that use each digit only once. Evaluate your expression.

Activity 2 The World’s Smallest Newspaper

The Guinness World Record for the “Smallest newspaper” was achieved in February 2012 by the Portuguese Nova Gráfica print shop. An exact replica of an issue of *Terra Nostra*, the miniature newspaper measured 0.72 in. by 0.99 in. and weighed 0.04 oz.



1. Determine the area of the newspaper’s front page by drawing and labeling an area diagram. Show your thinking.

2. Show how to calculate the area of the newspaper using vertical multiplication.

STOP

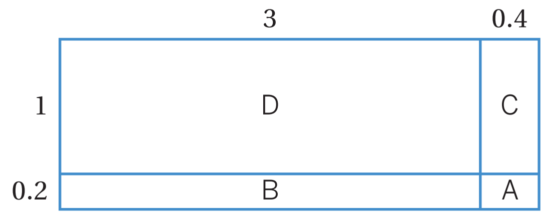
Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You used what you know about base ten numbers and determining the area of a rectangle to help you multiply two decimals.

The rectangle shown has side lengths of 3.4 units and 1.2 units. Each side length has been partitioned by place value units, so $3.4 = 3 + 0.4$ and $1.2 = 1 + 0.2$. The area of the rectangle, in square units, is the product of $3.4 \cdot 1.2$.



The total area is equal to the sum of the areas of the four smaller rectangles, which can be calculated in several ways. One way is using partial products — determining the area of each smaller rectangle by multiplying its length and width. Another is using the Distributive Property — multiplying the common length or width of two smaller rectangles by the sum of their other dimensions. Both of these can also be seen in a vertical calculation of the product, as shown here.

Partial Products

$$\begin{array}{rrl} & 3.4 & \\ \times & 1.2 & \\ \hline & 0.08 & \text{A} \\ & 0.6 & \text{B} \\ & 0.4 & \text{C} \\ + & 3 & \text{D} \\ \hline & 4.08 & \end{array}$$

Distributive Property

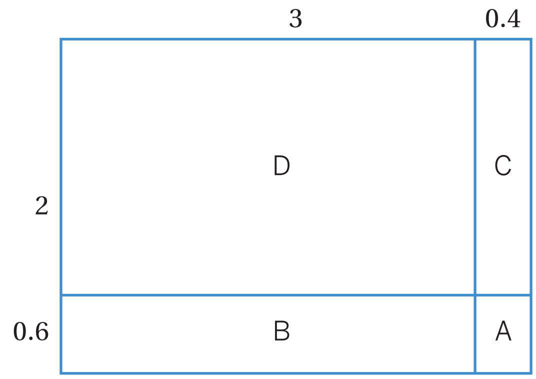
$$\begin{array}{rrl} & 3.4 & \\ \times & 1.2 & \\ \hline & 0.68 & \text{A + B} \\ + & 3.4 & \text{C + D} \\ \hline & 4.08 & \end{array}$$

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. The rectangle has an area in square units and has been partitioned into four smaller rectangles. For each expression, write the letter label of the smaller rectangle whose area matches each expression.



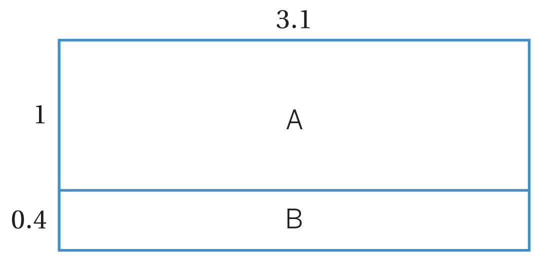
a. $3 \cdot 0.6$

b. $0.4 \cdot 2$

c. $0.4 \cdot 0.6$

d. $3 \cdot 2$

2. The area diagram represents $3.1 \cdot 1.4$.



a. Determine the areas of Rectangles A and B.

b. What is the sum of the areas?

3. Draw and use an area diagram and a corresponding vertical calculation to determine the product of $0.36 \cdot 0.53$.

Name: .... Date: .... Period: ....

4. Complete each calculation so that it shows the correct sum.

a

$$\begin{array}{r} 2\,.\,3\,\square \\ +\ \square\,.\,6\,4 \\ \hline 9\,.\,\square\,5 \end{array}$$

b

$$\begin{array}{r} 4\,.\,3\,\square \\ +\ \square\,.\,1\,5 \\ \hline 6\,.\,\square\,2 \end{array}$$

c

$$\begin{array}{r} 2\,.\,3\,\square \\ +\ \square\,.\,6\,4 \\ \hline 9\,.\,\square\,2 \end{array}$$

d

$$\begin{array}{r} 1\,.\,5\,\square \\ +\ \square\,.\,3\,8 \\ \hline 1\,.\,\square\,4 \end{array}$$

5. Diego bought 12 mini muffins for $\$4.20$.

a At this rate, how much would Diego pay for 4 mini muffins? Show your thinking.

b How many mini muffins could Diego buy with $\$3.00$? Show or explain your thinking.

6. Evaluate each expression.

a $2 \cdot 50$

b $2 \cdot 8$

c $2 \cdot 0.4$

d $2 \cdot 58.4$

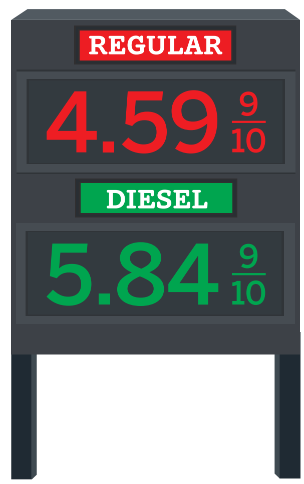
Unit 5 | Lesson 8

Calculating Products of Decimals

Let's multiply decimals.

Warm-up Number Talk

You need to buy 20 gallons of diesel gasoline. Mentally evaluate each expression. Be prepared to explain your thinking.



1. $20 \cdot 5$

2. $20 \cdot 0.8$

3. $20 \cdot 0.04$

4. $20 \cdot 5.84$

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 How Far Can One Tank of Gas Take You?

In 2011, Marko Tomac and Ivan Cvetkovic set the Guinness World Record for the "Greatest distance driven on a single tank of diesel fuel." They drove 1,581.88 miles through Croatia in a Volkswagen Passat 1.6 TDI BlueMotion, averaging 76.37 mpg.

1. In 2011, the average price, in U.S. dollars, for diesel fuel in Croatia was about $6.038 per gallon. At the same time, in the United States, the average price for diesel was 64% of what it cost in Croatia.

a. Estimate the cost of one gallon of diesel fuel in the United States in 2011. Explain your thinking.

b. Determine the actual price of a gallon of diesel in the United States in 2011, rounded to the nearest thousandth. Show your thinking.

Critique and Correct: Your teacher will present an incorrect response to Problem 1b. Be prepared to critique the response, correct it, and explain your thinking.

Activity 1 How Far Can One Tank of Gas Take You? (continued)

2. The Passat TDI's fuel tank can hold 18.5 gallons of diesel. How much less would it have cost Tomac and Cvetkovic to fill the tank in the United States compared to what they paid in Croatia?

   One partner should calculate the total cost in Croatia and the other partner should calculate the total cost in the United States. Then use both of your total costs to determine the price difference. Round your final response to the nearest cent. Show your thinking.

Are you ready for more?

1. Evaluate each of the following expressions. Write each result as a decimal.

   a. $1 - 0.1$ b. $1 - 0.1 + 10 - 0.01$ c. $1 - 0.1 + 10 - 0.01 + 100 - 0.001$

2. If the pattern in the expressions from Problem 1 repeated three more times so that the last number in the expression is 0.000001, what would be the value of the whole expression?

3. If all of the addition and subtraction symbols were multiplication symbols, how could you describe the final results for each expression in the pattern? Explain your thinking.

Name: .... Date: .... Period: ....

Activity 2 Going Wheelie Fast

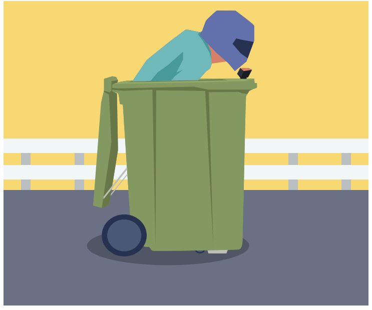
What happens when you put wheels and a motor on a toilet or a trash can? Well, you have a unique new way to get around town!

In September 2018, four 18-year-old British engineering students — Robert English, Thomas Ellis, Joe Summer, and William Beaty — achieved the Guinness World Record for "Fastest motorized toilet." Complete with a toilet brush as a steering stick, the HAWC (Highly Advanced Water Closet) Mk1 reached a top speed of 70.545 mph. And at the end of the race, the driver flushed the toilet to show that it actually worked!

In October 2020, Andy Jennings created the "World's fastest motorized trash can" using a garbage bin, motor bike engine, and handle bars from a child's scooter. He reached a top speed of approximately 69.2 kph.

Which traveled faster — the world's fastest motorized toilet or trash can? How much faster? Note: Use 1 km $= 0.6214$ mile and 1 mile $= 1.6093$ km.

Plan ahead: How will you control your impulses while working with these fun scenarios?



STOP

Summary

In today's lesson . . .

You used the relationship between place value and unit fractions with base ten denominators to generate a general process for multiplying any two decimals. Equivalent decimals and fractions such as $0.1 = \frac{1}{10}$ and $0.01 = \frac{1}{100}$, and so on, allow you to determine the product of two decimals.

For example, you can follow these steps to determine the product of 3.02 and 4.1

Step	Example
1. Write each decimal as the product of a whole number and a fraction.	$3.02 = 302 \cdot \frac{1}{100}$ $4.1 = 41 \cdot \frac{1}{10}$
2. Multiply the whole numbers.	$302 \cdot 41 = 12382$
3. Multiply the fractions.	$\frac{1}{100} \cdot \frac{1}{10} = \frac{1}{1000}$
4. Determine the the final product by: • Multiplying the product of the whole numbers by the product of the fractions; **or** • Divide product of the whole numbers by the reciprocal of the product of the fractions; **or** • Use the product of the fractions to determine the place value of the last digit, then locate the decimal point relative to its position at the end of the product of the whole numbers.	• $12382 \cdot \frac{1}{1000} = 12.382$; **or** • $12382 \div 1000 = 12.382$; **or** • $\frac{1}{1000}$ is the ten thousandths place, meaning the last digit will be three places after the decimal point. So, the product is 12.382

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. The Amazon rainforest covered 6.42 million km^2 in 1994. In 2014, it covered only $\frac{50}{59}$ as much. Which value is closest to the area of the Amazon forest in 2014? Explain how you know *without* calculating the exact area.

   A. 6.4 million km^2

   B. 5.4 million km^2

   C. 4.4 million km^2

   D. 3.4 million km^2

   E. 2.4 million km

2. Use vertical calculation to determine each product. Show your thinking.

   a. $5.4 \bullet 2.4$

   b. $1.67 \bullet 3.5$

3. A pound of blueberries costs $\$3.98$ and a pound of clementines costs $\$2.49$. What is the combined cost of 0.6 lb of blueberries and 1.8 lb of clementines? Round your answer to the nearest cent. Show your thinking.

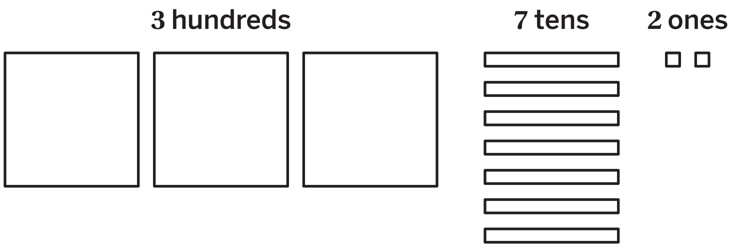
Practice

Name: .... Date: .... Period: ....

4. Which has a greater value: $7.4 - 0.0022$ or $7.39 - 0.0012$? Show your thinking.

5. Andre is planting saplings (baby trees). It takes him 30 minutes to plant 3 saplings. If each sapling takes the same amount of time to plant, how long will it take Andre to plant 14 saplings?

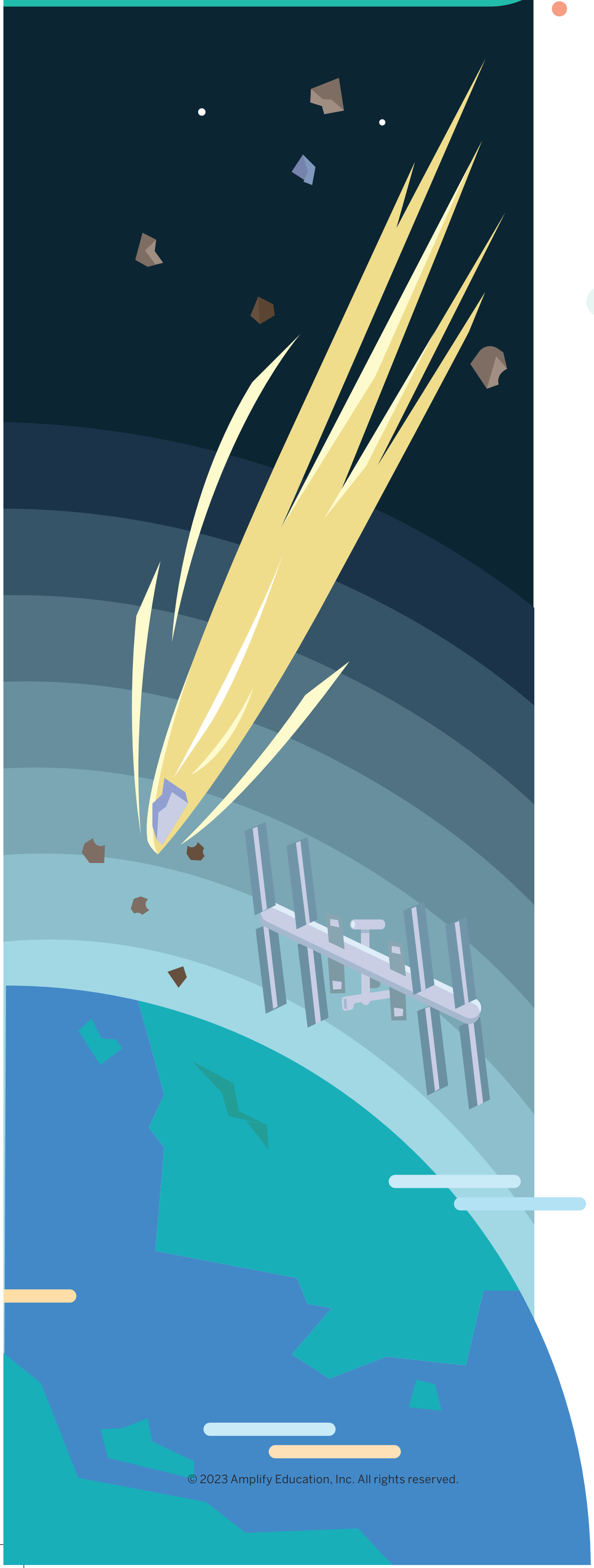
6. Elena used base ten diagrams to find $372 \div 3$. She started by representing 372 as shown here.



To show the division, she create 3 groups, each with 1 hundred. Then, she placed the tens and ones in each of the 3 groups. Draw what Elena's diagram might have looked like.

SUB-UNIT

3 Dividing Decimals



Narrative Connections

How do you dodge a piece of space junk?

The International Space Station is the largest human-made structure in space. Astronauts have been living in it since 2000, conducting scientific research while the station races around the Earth at more than 17,000 mph.

But, in 2015, all that could have come to an end when a piece of space junk was detected moving toward the station. The debris was only a few inches long. However, the combined speeds of the object and the space station meant it could have caused catastrophic damage.

Fortunately, there was enough time for the Space Station to execute an avoidance maneuver. It fired thrusters for 2 minutes and 20 seconds, changing its velocity by 0.3 m per second. This tiny change actually raised the station's orbit by about half a kilometer! Disaster avoided.

To date, there have been nearly 10,000 satellites launched into space. As these satellites break down, they add to the space junk constantly threatening other spacecraft orbiting around the Earth.

To avert catastrophe, things such as distance and velocity must be computed as accurately as possible. An understanding of how to divide whole numbers and decimals is the starting point to making these calculations. Without it, you could end up lost in space.

Unit 5 | Lesson 9

Exploring Division

Let's explore different ways to divide whole numbers.

Warm-up Number Talk

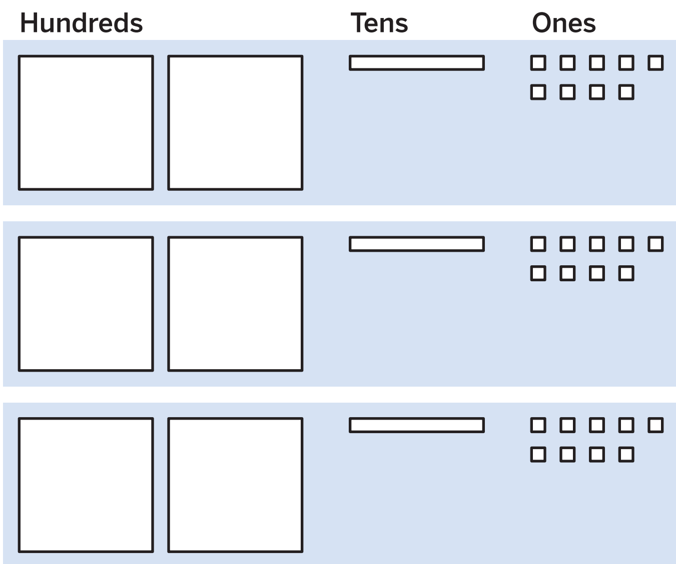
Mentally evaluate the quotient of $657 \div 3$. Be prepared to explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: ____________ Period: ________

Activity 1 Diagrams and Partial Quotients

To calculate $657 \div 3$, Jada used a base ten diagram and Andre used partial quotients.

Jada's Method	Andre's Method
	$\begin{array}{r} \boxed{219} \\ 9 \\ 10 \\ 200 \\ 3\overline{)657} \\ -\,600 \\ \hline 57 \\ -\,30 \\ \hline 27 \\ -\,27 \\ \hline 0 \end{array}$

1. Discuss with your partner how Jada's and Andre's methods are similar and different.

2. Calculate $896 \div 4$ using one of these two methods. Show your thinking, including how you know your solution is correct.

Activity 2 Using Long Division

Lin calculated the quotient of $657 \div 3$ using a method called *long division*. Her steps are shown and described here.

Lin arranged the numbers for vertical calculations. Her plan was to divide each digit of 657 into 3 groups, starting with the 6 hundreds. $3\overline{)657}$	There are 3 groups of 2 in 6, so Lin wrote 2 at the top and subtracted 6 from the 6, leaving 0. Then she brought down the 5 tens of 657. $\begin{array}{r} 2 \\ 3\overline{)657} \\ -6 \\ \hline 05 \end{array}$	There are 3 groups of 1 in 5, so she wrote 1 at the top and subtracted 3 from 5, which left a remainder of 2. $\begin{array}{r} 21 \\ 3\overline{)657} \\ -6 \\ \hline 5 \\ -3 \\ \hline 2 \end{array}$	She brought down the 7 ones of 657 and wrote it next to the 2, which made 27. There are 3 groups of 9 in 27, so she wrote 9 at the top and subtracted 27, leaving 0. $\begin{array}{r} 219 \\ 3\overline{)657} \\ -6 \\ \hline 5 \\ -3 \\ \hline 27 \\ -27 \\ \hline 0 \end{array}$

1. Discuss the following questions with your partner.

   a. How is Lin's method similar to Jada's and Andre's methods from Activity 1? How is it different?

   b. Explain why all three students were able to calculate the same solution.

Name: ______________________ Date: __________ Period: __________

Activity 2 Using Long Division (continued)

2. Work with your partner to determine the following quotients using long division.

a $1{,}332 \div 9$ **b** $1{,}816 \div 4$ **c** $4{,}352 \div 16$

3. Han used long division to divide $10{,}268 \div 4$. Using his work, determine the quotient.

$$
\begin{array}{r}
\square\,\square\,\square\,\square\,\square \\
4\overline{)1\,0\,2\,6\,8} \\
-\,8 \\
\hline
2\,2 \\
-\,2\,0 \\
\hline
2\,6 \\
-\,2\,4 \\
\hline
2\,8 \\
-\,2\,8 \\
\hline
0
\end{array}
$$

Are you ready for more?

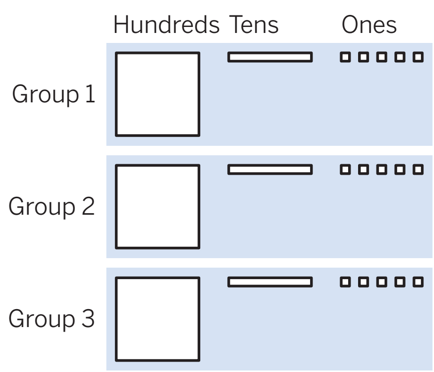
Which method would you use to calculate $3{,}902{,}718 \div 3$? Why? Determine the quotient. If you use the base ten method, include a key for the values.

STOP

Summary

In today's lesson . . .

You saw three ways to divide whole numbers. Each method is shown for $345 \div 3$.

Base ten diagrams	Partial quotients	Long division
Different shapes represent the hundreds, tens, and ones in 345, and then the shapes are partitioned into 3 equal-sized groups, starting with the hundreds. When there is a remainder of one unit, decompose it into ten of the next smaller unit.	Determine how many times 3 goes into the full value of each digit in each place value, starting with the hundreds place (300), and then subtract from 345 to see how much is left. As you move from place to place, you may eventually recognize a multiple ($3 \cdot 15 = 45$).	Similar to partial quotients, divide place by place, from left to right, but you should always remove the greatest possible amount each time relative only to the current place value. The digits in the quotient are determined and recorded one at a time, all as part of a single number (rather than multiple numbers that are added).
	$\begin{array}{r} \boxed{115} \\ 5 \\ 10 \\ 100 \\ 3\overline{)345} \\ -300 \\ \hline 45 \\ -30 \\ \hline 15 \\ -15 \\ \hline 0 \end{array}$ $\begin{array}{r} \boxed{115} \\ 15 \\ 100 \\ 3\overline{)345} \\ -300 \\ \hline 45 \\ -45 \\ \hline 0 \end{array}$	$\begin{array}{r} 115 \\ 3\overline{)345} \\ -3 \\ \hline 4 \\ -3 \\ \hline 15 \\ -15 \\ \hline 0 \end{array}$

Reflect:

Practice

Name: ______________________ Date: __________ Period: __________

1. Show how to determine $2{,}105 \div 5$ using the partial quotients method.

$$\begin{array}{r} 5\overline{)2105} \\ -2000 \\ \hline 105 \\ -100 \\ \hline 5 \\ -5 \\ \hline 0 \end{array}$$

2. Here is an incomplete calculation of $534 \div 6$. Complete the calculation.

$$\begin{array}{r} 8\,\boxed{} \\ 6\overline{)5\;\;3\;\;4} \\ -\boxed{}\boxed{}\phantom{\boxed{0}} \\ \hline \boxed{}\boxed{} \\ -\boxed{}\boxed{} \\ \hline 0 \end{array}$$

3. Why is the base ten diagram not an efficient way to evaluate $552 \div 46$? Explain your thinking by calculating the quotient using either the partial quotient or long division method.

Name: .... Date: .... Period: ....

4. Which of the polygons described has the greatest area? Explain your thinking

A. A rectangle that is 3.25 in. wide and 6.1 in. long.

B. A square with side length of 4.6 in.

C. A parallelogram with a base of 5.875 in. and a height of 3.5 in.

D. A triangle with a base of 7.18 in. and a height of 5.4 in.

5. A micrometer is equal to one millionth of a meter. A certain spider web is 4 micrometers thick. A fiber in a shirt is 1 hundred-thousandth of a meter thick.

a Is the spider web or the fiber wider? Explain your thinking.

b How many meters wider?

6. Explain how you would estimate the quotient of $485 \div 8$. Determine an estimate.

Name: .... Date: .... Period: ....

Unit 5 | Lesson 10

Using Long Division

Let's use long division.

Warm-up Number Talk

Mentally evaluate the quotient using estimation.

$328{,}624 \div 304$

Log in to Amplify Math to complete this lesson online.

Activity 1 Long Division in Action

Decide who will be Partner A and who will be Partner B. You will independently estimate the two quotients and then calculate them using long division. When you are both done, swap books and check your partner's results using multiplication.

1.

Partner A	Partner B
$7{,}680 \div 12$	$2{,}996 \div 14$
Estimate:	**Estimate:**
Calculate:	**Calculate:**
SWAP!	
Multiply to check:	**Multiply to check:**

Name: ______ Date: ______ Period: ______

Activity 1 Long Division in Action (continued)

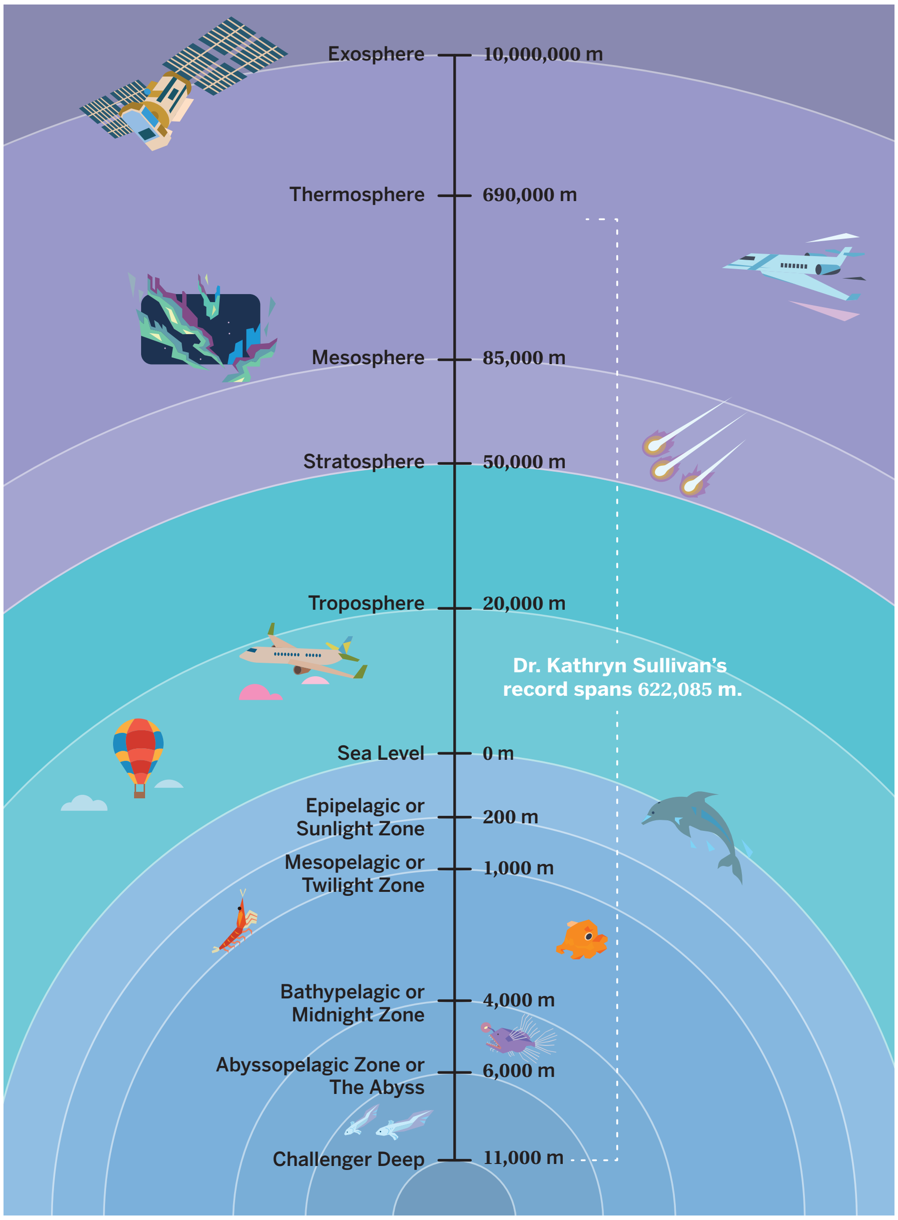
2.

Partner A	Partner B
$21{,}835 \div 11$	$27{,}768 \div 12$
Estimate.	**Estimate.**
Calculate.	**Calculate.**
SWAP!	
Multiply to check.	**Multiply to check.**

Activity 2 "The Most Vertical Woman in the World"

The current Guinness World Records holder for "Greatest vertical extent travelled by an individual (within Earth's exosphere)" is Dr. Kathryn Sullivan. She was the first person to visit both space and the deepest point on Earth. That's right — Dr. Sullivan has traveled to the thermosphere layer in outer space all the way down to the Challenger Deep in the Mariana Trench, for a grand total of 622,085 vertical meters.

This image shows all of the different layers of Earth's atmosphere and oceanic zones, and also the extent of Dr. Sullivan's full vertical travels.



Co-craft Questions:
Before you begin Problem 1, work with your partner to write 2–3 mathematical questions you have about this image.

Name: .... Date: .... Period: ....

Activity 2 "The Most Vertical Woman in the World" (continued)

Use the image on the previous page to help you complete Problems 1–3. Note the precise height Kathryn Sullivan reached in space was 611,151 m above sea level, and the precise depth she reached at the Mariana Trench was 10,934 m below sea level.

1. Dr. Sullivan is below sea level at a depth that is equal to her total distance traveled from sea level to the Mariana Trench divided by 14. Which zone is she in?

2. Dr. Sullivan is in the atmosphere, one third of the way to the farthest point into outer space that she reached. Which zone is she in?

3. Dr. Sullivan is above sea level, at an altitude that is equal to her total vertical distance traveled divided by 5. What object might Dr. Sullivan see when she looks around outside?

STOP

Summary

In today's lesson . . .

You practiced using long division to divide whole numbers with any number of digits. You also checked the results of long division using the inverse operation of multiplication, knowing that the product of the quotient and the divisor should be equal to the dividend. This can be done using the algorithm for multiplication you saw in the previous lessons.

For example, consider evaluating the expression $4896 \div 18$

Long division	Check
$\begin{array}{r} 0272 \\ 18\overline{)4896} \\ -\ 0 \\ \hline 48 \\ -\ 36 \\ \hline 129 \\ -\ 126 \\ \hline 36 \\ -\ 36 \\ \hline 0 \end{array}$	$\begin{array}{r} 272 \\ \times \quad 18 \\ \hline 2176 \\ +\ 272 \\ \hline 4896 \end{array}$

Reflect:

Name: ________________ Date: ________ Period: ________

Practice

1. Determine each quotient. Use multiplication to check your response. Show your thinking.

   a. $3\overline{)1107}$

   b. $7\overline{)16198}$

2. Use the long division calculation of $917 \div 7$ to complete these problems.

$$\begin{array}{r}
131 \\
7\overline{)917} \\
-7 \\
\hline
21 \\
-21 \\
\hline
07 \\
-7 \\
\hline
0
\end{array}$$

   a. There is a 7 under the 9 of 917. What does this 7 represent?

   b. What does the subtraction of 7 from 9 mean?

   c. Why is 1 written next to the 2 from $9 - 7$?

3. Han's calculation of $972 \div 9$ is shown here. His work is not correct.

$$\begin{array}{r}
180 \\
9\overline{)972} \\
-9 \\
\hline
72 \\
-72 \\
\hline
0 \\
-0 \\
\hline
0
\end{array}$$

   a. Identify and explain Han's mistake.

   b. Determine the correct quotient. Show your thinking.

Practice

Name: ________ Date: ________ Period: ________

4. One oz of yogurt contains 1.2 g of sugar. How many grams of sugar are in 14.25 oz of yogurt? Show your thinking.

A. 0.171 g

B. 1.71 g

C. 17.1 g

D. 171 g

5. The mass of one coin is 16.718 g. The mass of a second coin is 27.22 g. How much greater is the mass of the second coin than the first? Show your thinking.

6. Explain how to mentally calculate the quotient of $273 \div 3$, and then determine the quotient.

Name: ________________ Date: ________ Period: ________

Unit 5 | Lesson 11

Dividing Numbers That Result in Decimals

Let's determine quotients that are not whole numbers.

Warm-up Number Talk

Mentally evaluate each expression. Be prepared to explain your thinking.

1. $400 \div 8$

2. $80 \div 8$

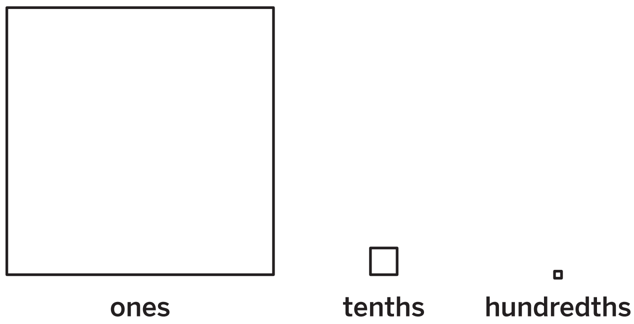
3. $16 \div 8$

4. $496 \div 8$

Log in to Amplify Math to complete this lesson online.

Activity 1 Keep Calm and Divide On

1. Decide who will be Partner A and who will be Partner B. Show how you know that each equations in your assigned column is true. For part a, draw base ten diagrams. For part b, use long division. Then share and discuss your work with your partner.



Partner A	Partner B
a $5 \div 4 = 1.25$	a $4 \div 5 = 0.8$
b $4 \div 5 = 0.8$	b $5 \div 4 = 1.25$

Name: .... Date: .... Period: ....

Activity 1 Keep Calm and Divide On (continued)

2. Use long division to evaluate each expression. Show your thinking and express each quotient as a decimal.

   a. $1 \div 8$

   b. $1 \div 25$

Reflect: How well did you and your partner communicate? What did you do to build your relationship?

Are you ready for more?

Use long division to evaluate $1 \div 3$. What do you notice?

Activity 2 Salinity Now

Over 96% of the water on Earth is saltwater. The amount of salt, measured as the *salinity* of the water, changes some properties of the water — and it varies quite a lot. Two bodies of water known to have extremely high salinity are the Dead Sea in Asia and Don Juan Pond in Antarctica. It is easy to float in the Dead Sea, and yet it is nearly impossible to swim because of the high salt content. Due to its high salt content, Don Juan Pond rarely freezes — even at temperatures of 50°C below the normal freezing point of water!

Many factors affect the exact salinity of a body of water, so scientists often take samples to measure the number of grams of salt per liter of water at different locations and at different times or under different conditions. For example, the salinity of the Great Salt Lake in Utah can fluctuate from 50 g per liter to 250 g per liter depending on the season, recent weather, and the exact location or depth of where the sample was taken.

You will be given two cards that each shows an amount of salt per water measured in samples taken from various sources. Use long division to determine the salinity of your two water sources and record this information in the table. Show your thinking and express any remainders as decimals.

Water source	Salt (g)	Water (liters)	Salinity (g/liter)

STOP

Name: ______________________ Date: __________ Period: __________

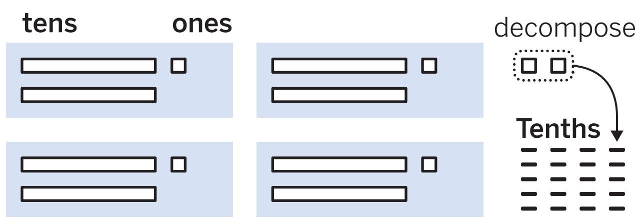
Summary

In today's lesson . . .

You used base ten diagrams and long division to divide two whole numbers whose quotient is not a whole number.

For example, to evaluate $86 \div 4$, you can think of dividing 86 into 4 equal groups. Whether you are using a base ten diagram or long division, the thought process is the same: there are 4 groups of 21 with 2 ones left over. The ones can be decomposed into 20 tenths and then distributed evenly to each of the 4 groups. Each group will have 2 tens, 1 one, and 5 tenths, so $86 \div 4 = 21.5$.

Base ten diagram



Long division

$$
\begin{array}{r}
21.5 \\
4\,\overline{)\,86.0} \\
-\,8 \\
\hline
6 \\
-\,4 \\
\hline
2\;0 \\
-\,2\;0 \\
\hline
0
\end{array}
$$

In the long division strategy, the remaining 2 ones are decomposed into 20 tenths by extending the dividend of 86 to an equivalent form of 86.0, which allows you to bring a 0 down to the right of the remainder of 2 (ones). That also means a decimal point is added to the right of the 1 (which is in the ones place) in the quotient, which shows that the resulting 5 is in the tenths place.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Use long division to show that the fraction and decimal in each pair are equal.

a $\frac{3}{4}$ and 0.75

b $\frac{3}{50}$ and 0.06

c $\frac{7}{25}$ and 0.28

2. Use long division to determine each quotient. Write the quotient as a decimal.

a $99 \div 12$

b $216 \div 5$

c $1988 \div 8$

Name: ______________ Date: ______________ Period: ______________

3. Tyler reasoned that $\frac{9}{25}$ is equivalent to $\frac{18}{50}$ and to $\frac{36}{100}$, so the decimal of $\frac{9}{25}$ is 0.36.

a Use long division to show that Tyler is correct.

b Use long division to determine whether the decimal of $\frac{18}{50}$ is also 0.36.

4. Complete each calculation so that the correct difference is shown.

a
$$\begin{array}{r} 5 \\ - \ \boxed{}\,\boxed{}\,\boxed{} \\ \hline 4.3\ 2\ 9 \end{array}$$

b
$$\begin{array}{r} 1 \\ - \ \boxed{}\,\boxed{}\,\boxed{} \\ \hline 0.0\ 1\ 5 \end{array}$$

c
$$\begin{array}{r} 1 \\ - \ \boxed{}\,\boxed{}\,\boxed{} \\ \hline 0.8\ 6\ 3 \end{array}$$

5. Use the equation $124 \cdot 15 = 1{,}860$ and what you know about fractions, decimals, and place value to explain where to place the decimal point as you compute $1.24 \cdot 0.15$.

6. Explain how to mentally calculate $0.8 \div 4$.

Unit 5 | Lesson 12

Using Related Expressions to Divide With Decimals

Let's use related expressions to divide with decimals.

Warm-up Number Talk

Mentally evaluate each expression. Be prepared to explain your thinking.

1. $80 \div 4$

2. $12 \div 4$

3. $1.2 \div 4$

4. $81.2 \div 4$

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Using Properties to Divide Decimals

Peggy Whitson, an American astronaut from Iowa, has held as many as five space-related Guinness World Records, four of which still stood as of 2020: "First woman to command the International Space Station," "Most spacewalks by a female," "Longest accumulated time on spacewalks by a female," and "Oldest female astronaut in space." Until December 2019, she also held the record for the "Longest continuous time in space by a female," spending about 289.2 straight days outside of Earth's atmosphere.

While many would say not a single minute in space feels like work, and, technically, there are regulations on official working hours, you can imagine that astronauts spend most of their time in space working. To put Whitson's accomplishment in perspective, consider the typical work week on Earth of 5 days, and think about how many "never-ending" work weeks she spent in space.

1. Explain how the expression $\left(2{,}892 \cdot \frac{1}{10}\right) \div 5$ represents the number of work weeks Whitson spent in space.

2. Show or explain how to evaluate $\left(2{,}892 \cdot \frac{1}{10}\right) \div 5$.

3. Use what you know about long division to evaluate $289.2 \div 5$ and determine how many work weeks Whitson was in space.

Activity 2 Related Expressions and Decimal Division

1. Without evaluating, explain why $100 \div 5$, $10 \div 0.5$, and $1 \div 0.05$ have the same quotient.

2. Calculations A–D all show incorrect attempts of using related expressions and long division to determine the quotient of $64 \div 1.6$. What mistake do you see in each calculation? Explain your thinking.

Calculation A

$$\begin{array}{r} 0.4 \\ 1.6\overline{)64} \\ -\ 0 \\ \hline 64 \\ -\ 64 \\ \hline 0 \end{array}$$

Calculation B

$$\begin{array}{r} 0.4 \\ 16\overline{)64.0} \\ -\ 64 \\ \hline 0\ 0 \\ -\ 0 \\ \hline 0 \end{array}$$

Calculation C

$$\begin{array}{r} 0.4 \\ 16\overline{)640} \\ -\ 64 \\ \hline 00 \\ -\ 0 \\ \hline 0 \end{array}$$

Calculation D

$$\begin{array}{r} 0.4 \\ 16\overline{)6.4} \\ -\ 0 \\ \hline 6\ 4 \\ -\ 6\ 4 \\ \hline 0 \end{array}$$

Name: .... Date: .... Period: ....

Activity 2 Related Expressions and Decimal Division (continued)

3. Use what you know about related expressions and long division to determine the quotient of $64 \div 1.6$.

Are you ready for more?

Using the digits 0–9, write a division expression that uses each digit once and has both numbers as decimals. Then evaluate it.

STOP

Summary

In today's lesson . . .

You used related expressions and long division to reason about division with decimals. You know that fractions such as $\frac{6}{4}$ and $\frac{60}{40}$ are equivalent because of the following.

- The numerator and denominator of $\frac{60}{40}$ are each 10 times those of $\frac{6}{4}$.
- Both fractions can be simplified to $\frac{3}{2}$.
- 60 divided by 40 is 1.5, and 6 divided by 4 is also 1.5.

That means that, as with fractions, division expressions can be equivalent, which can also be called related expressions. For example, the expressions $540 \div 90$ and $5.4 \div 0.9$ are both equivalent to $54 \div 9$ because of the following.

- They all have a quotient of 6.
- The dividend and the divisor in $540 \div 90$ are each 10 times the dividend and divisor in $54 \div 9$. Those in $5.4 \div 0.9$ are each $\frac{1}{10}$ of the dividend and divisor in $54 \div 9$. In both cases, the quotient does not change.

In general, multiplying a dividend and a divisor by the same number does not change the quotient. Multiplying by powers of 10 (e.g., 10, 100, $1{,}000$, etc.) can be particularly useful for dividing decimals, as you will see in an upcoming lesson.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Consider these four expressions:

$4.5 \div 0.09$ $45 \div 0.9$ $450 \div 9$ $4{,}500 \div 90$

a. Without evaluating, explain how you know all four expressions have the same value.

b. What is the quotient of $4.5 \div 0.09$?

2. Use long division to determine each quotient. Show your thinking.

a. $7.89 \div 2$ b. $39.54 \div 3$ c. $176 \div 0.5$

3. Kiran, Mai, Noah, and Clare set up a lemonade stand. At the end of one day they had made a profit of \$17.52. If the profit is divided equally, how much money would each student receive? Show or explain your thinking.

Practice

Name: ______________ Date: ______________ Period: ______________

4. At Bard's school, 460 of the students walk to school.

a The number of students who take public transit is 20% of the number of students who walk. How many students take public transit?

b The number of students who bike to school is 5% of the number of students who walk. How many students bike to school?

c The number of students who ride the school bus is 110% of the number of students who walk. How many students ride the school bus?

5. Calculate each difference. Show your thinking.

a $13.2 - 1.78$

b $23.11 - 0.376$

c $0.9 - 0.245$

6. Explain how to mentally calculate $720{,}000 \div 9{,}000$.

Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 13

Dividing Multi-digit Decimals

Let's divide any decimal by any other decimal.

Warm-up Same Quotients

Select *all* of the expressions that have the same quotient as $5.04 \div 7$. Be prepared to share your thinking.

A. $0.504 \div 700$

B. $5.04 \div 70$

C. $50.4 \div 70$

D. $504 \div 700$

E. $504{,}000 \div 700$

F. $504{,}000 \div 700{,}000$

Log in to Amplify Math to complete this lesson online.

Activity 1 Generalizing a Decimal Division Rule

Your small group, and then your whole class, will collectively write a general rule for division with decimals. You will test and revise your rules along the way.

Who	What
You	Evaluate the expression $3 \div 0.12$ in Problem 1.
Trios	**Discuss how you solved the problem.**
	Write a rule about how to divide decimals. This is your first attempt, and you will be able to revise it later. **First write:**
	Evaluate the expressions $7.5 \div 1.25$ and $1.8 \div 0.004$ in Problem 2.
	Discuss whether your rule holds true, and why or why not?
	Rewrite or revise your rule based on this discussion and record it here. This will be your group's final general rule. **Second write:**
Class	**Share group rules. Then, as a class, you will discuss and agree on a final rule.**
	Record your class rule here. **Third write (Class Rule):**
	Evaluate the expression $7.89 \div 2$ in Problem 3.
	Does the rule still hold true? Yes No

Name: ____________ Date: ____________ Period: ____________

Activity 1 Generalizing a Decimal Division Rule (continued)

Use this space for your calculations as you test your division rule for decimals.

1. $3 \div 0.12$

2. **a** $7.5 \div 1.25$ **b** $1.8 \div 0.004$

3. $7.89 \div 2$

Activity 2 Ham the Astrochimp

The first astronauts to travel into space were not actually human — they were primates! While not the first primate in space, Ham the astrochimp was the first to successfully pull levers while in space. His mission proved that it was physically possible for a human to pilot a spacecraft, paving the way for successful and safe human space exploration.

NASA

In 1961, when Ham was launched into space and successfully landed back on Earth, he weighed 37 lb, or about 16.7829 kg, on Earth. Weight is a measure of the pull of gravity between an object and the planet it is on. Because each planet has a different gravitational pull, objects — including Ham — do not weigh the same on every planet.

You will be given a card that shows Ham's weight, in kilograms, on a different planet. Write a division expression to determine how many times as much the gravitational pull of that planet is, as a factor relative to the gravitational pull of Earth. To prepare for the class *Gallery Tour*, record your results in the table and show any additional thinking that would help others understand.

Planet	Division expression	Gravitational pull relative to Earth (times as much)

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You developed a general rule and a process for dividing any two decimals, based on your work in the previous lessons with base ten numbers. Place value and related expressions can be used to transform any division problem into one where long division can be used.

When the divisor is a decimal, you can multiply it by a power of 10 to make it a whole number. But, remember, you must multiply the dividend by the same power of 10 for the expressions to be equivalent. You can also choose to multiply by a power of 10 that makes *both* the divisor and the dividend whole numbers. For example, $7.65 \div 1.2$ can be transformed into either of these related expressions.

$$\begin{array}{r} 6.375 \\ 1.2.\overline{)7.6.500} \\ -7\,2 \\ \hline 4\,5 \\ -3\,6 \\ \hline 90 \\ -84 \\ \hline 60 \\ -60 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 6.375 \\ 1.20.\overline{)7.65.000} \\ -7\,20 \\ \hline 45\,0 \\ -36\,0 \\ \hline 9\,00 \\ -8\,40 \\ \hline 600 \\ -600 \\ \hline 0 \end{array}$$

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Mai claims, "To determine the value of $109.2 \div 6$, I can divide $1{,}092$ by 60."

   a. Do you agree with Mai? Explain your reasoning.

   b. Calculate the quotient of $109.2 \div 6$. Show your thinking.

2. Write two different division expressions that have the same quotient as $61.12 \div 3.2$. Then evaluate $61.12 \div 3.2$. Show your thinking.

3. A bag of pennies weighs 5.1 kg. Each penny weighs 2.5 g. Which is the best estimate for the number of pennies in the bag? Show your thinking.

   A. 20 B. 200 C. $2{,}000$ D. $20{,}000$

Name: .... Date: .... Period: ....

Practice

4. Determine each difference. Show your thinking.

a $2.5 - 1.6$

b $0.72 - 0.4$

c $11.3 - 1.75$

d $73 - 1.3$

5. Plant B is $6\frac{2}{3}$ in. tall. Plant C is $4\frac{4}{15}$ in. tall. Complete the sentences and show your thinking.

a Plant C is .... times as tall as Plant B.

b Plant C is .... in. .... (taller or shorter) than Plant B.

6. One floor of a rectangular building measures 25 ft long, 50 ft wide, and 10 ft high.

a What is the square footage of the ground surface of the floor, the part on which people can stand?

b What would the total square footage be if the building contained 10 identical floors?

Unit 5 | Lesson 14 – Capstone

The So-called World's "Littlest Skyscraper"

Let's use what we know about decimals to explore a skyscraper of a scam.

Warm-up The So-called World's "Littlest Skyscraper"

There is no official definition, but It is generally agreed that a skyscraper is a building containing 10–20, or more, floors. In 1919, when J.D. McMahon drafted the blueprints for what is now known as the Newby-McMahon Building in the then-booming oil town of Wichita Falls, TX, investors were eager to contribute $200,000 for a 480-foot skyscraper.

But what happened? How did the "world's littlest skyscraper" end up containing only 4 floors (without any stairs!) and standing a mere 40 ft tall?

It turns out, according to local legend, that McMahon was a bit of a swindler. The investors had not examined the plans carefully when they approved the blueprints for a building with a listed height of 480″ (not 480′). In a lawsuit following the building's completion, the courts ultimately ruled in favor of McMahon, who, most likely with a large sum of the unspent investment still in his pockets, was never seen or heard from in the area again.

Newby-McMahon Building, c. 1919, also known as the "Worlds Littlest Skyscraper". Wichita Falls, Texas, by Travis K Witt, courtesy of Wikimedia Commons, is licensed under the Creative Commons Attribution-Share Alike 4.0 International license: https://creativecommons.org/licenses/by-sa/4.0/deed.en

What is the mathematical moral of this story?

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Then and Now: How Much Did It Cost?

Part 1

Refer to the Warm-up about the so-called world's "littlest skyscraper." Use the following facts, as needed, to evaluate each problem.

- **Investors thought the Newby-McMahon building would be 480 ft tall.**
- **The completed building measured 480 in., or 40 ft tall.**
- **Each of the 4 floors measured approximately 12 ft by 9 ft.**
- **$200,000 was invested to construct a 480-ft building in 1919.**
- **Inflation causes the prices of goods and services to rise over time. An item costing $1 in 1919 is estimated to have cost $14.78 in 2019.**

1. Assuming all relative costs have been adjusted at the same inflation rate, how much more would a 480-ft Newby-McMahon building cost to construct in 2019 than in 1919? Show your thinking.

2. You will be given a calculator to help with your computations. In 1919, how much more did investors pay, per square foot, than they thought they would be paying? Round to the nearest penny. Show each step of your work, using the calculator.

Activity 1 Then and Now: How Much Did It Cost? (continued)

Part 2

As the legend goes, McMahon was never seen or heard from again in Wichita Falls. Supposedly, he secretly fled to Switzerland in 1920 to run the exact same scam.

As a class, you will determine how much more investors paid in 1920, per square *meter,* than they thought they would be paying. But first, you and your partner will be assigned one of the following preliminary questions to answer. Circle your assigned question.

How much did the investors *think* they would be paying, per square meter?

How much did the investors *actually* pay, per square meter?

You will also be given a card showing conversion rates between feet and meters and U.S. dollars and Swiss francs, and you will have access to a calculator. Use these tools and your work from Activity 1 to answer your assigned question.

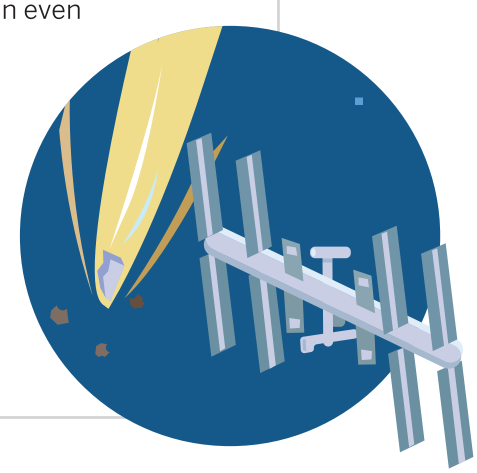
STOP

Unit Summary

Let's come to the point: when it comes to being precise, it's hard to beat a decimal. The world's record books are full of them. From fingernails to miles traveled on a tank of gas, decimals have been there to help declare a winner.

Of course, working with decimals isn't just about setting world records. From a butcher weighing cuts of meat, to an astronaut planning her flight through space the ability to work with decimals is crucial.

Thanks to our base ten numbering system, operating with decimals is not that different from operating with whole numbers. Just remember to pay attention to the place values and where the decimal point goes in each number. In fact, you can even work with decimals by operating with whole numbers first. Just remember to convert them back into the proper decimals at the end.



Now that you know place value, you can perform the four basic operations on whole numbers and decimals without breaking a sweat.

See you in Unit 6.

Practice

Name: .... Date: .... Period: ....

The Willis Tower in Chicago, IL, has the highest observation deck in the United States. The Skydeck is located on the 103rd floor at an elevation of $1{,}353$ ft.

1. The elevators reach the Skydeck in about 60 seconds. How many *meters* per second do the elevators travel? Use 1 ft $= 0.3$ m to evaluate. Show your thinking.

2. By how much would your answer change if you used 1 ft $= 0.3048$ m to evaluate Problem 1? Show your thinking.

3. Does the extra precision in Problem 2 make a difference, in the context of the problem? Explain your thinking.

Name: ______ Date: ______ Period: ______

4. Determine each difference. Show your thinking.

a. $3.572 - 2.6014$

b. $0.106 - 0.0315$

c. $0.151 - 0.028$

5. Write three numerical expressions that are equivalent to $0.0004 \cdot 0.005$.

6. Determine the following quotients. Show your thinking.

a. $24.2 \div 1.1$

b. $13.25 \div 0.4$

c. $170.28 \div 0.08$

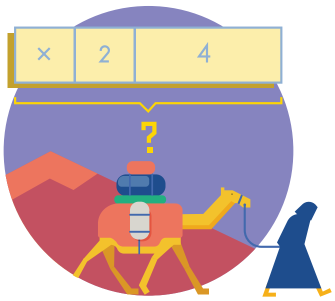
UNIT 6

Expressions and Equations

Up until now, an equal sign meant you were being asked to calculate an answer. In this unit, you'll learn about its other meaning — balance. And when things are in balance, it becomes possible to know the unknown.

Essential Questions

- What does it mean for an equation to be true? Can an equation be false?
- How can two quantities be equal when one is partially or totally unknown?
- What does it mean for two expressions to be equal? Equivalent? Is there a difference?
- *(By the way, what do a good yogi and a good accountant have in common?)*



SUB-UNIT

1 Expressions and Equations in One Variable

Narrative: Learn about the 14th century African salt trade, as you explore expressions and equations.

You'll learn . . .

- to model with tape and hanger diagrams.
- how to use the properties of equality to solve equations.

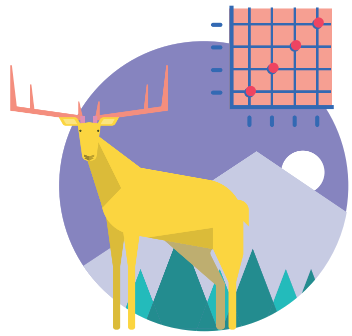
SUB-UNIT

2 Equivalent Expressions

Narrative: Extend the concept to equality to investigate equivalent expressions and exponents.

You'll learn . . .

- what makes expressions equivalent.
- about the Distributive Property.



SUB-UNIT

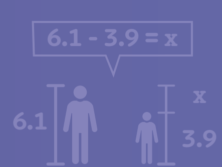
3 Relationships Between Quantities

Narrative: Balance is everywhere, from mathematical equations to ecosystems.

You'll learn . . .

- about independent and dependent variables.
- how to represent relationships in different ways.

This tape diagram has x segments, each of length x. The total length is 144. Solve for x.



Detecting Counterfeit Coins

How can a balance scale help you detect a counterfeit coin?

Warm-up Which One Doesn't Belong?

Which one doesn't belong? Be prepared to explain your thinking.

A. 

Nimon/Shutterstock.com

B. 

AndrewStarikov/Shutterstock.com

C. 

IgorNazarenko/Shutterstock.com

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Three Coins

Plan ahead: How will you organize your plan in order to manage your stress level?

There are three coins that look completely identical, but exactly one of the coins is counterfeit, and is slightly heavier than the two real coins. You cannot feel the difference in weight when you hold them. But you have a balance scale that is sensitive enough to detect it.

1. If you weigh two of the coins, you might get lucky and determine that one of them is heavier than the other. But you might not be so lucky. Will you always know which coin is the counterfeit coin after just one weighing? Why or why not?

2. Now imagine you again have three coins and one of them is counterfeit. But this time, you don't know if the counterfeit coin is slightly heavier or slightly lighter than the other two coins.

a Is it possible to know which coin is counterfeit after just one weighing?

b Will you *always* know which coin is the counterfeit after just one weighing?

c What is the smallest number of weighings you would need to perform so you could always say which coin is counterfeit?

Activity 2 More Coins

Many mathematicians LOVE good puzzles like these! Some solve them for fun. Some create them — often taking a known puzzle and tweaking it to make a new puzzle. And others, like Tanya Khovanova, study and research the deep structure and theory in recreational mathematics.

Try another. For each number of coins, assume one coin is counterfeit, and that the counterfeit coin is either slightly heavier or slightly lighter than the other coins. Determine the least number of weighings needed so that you could *always* say which coin is counterfeit.

1. Four coins

2. Eight coins

3. 12 coins

Stronger and Clearer: You will meet with another group to give and receive feedback on your response to Problem 1. Use this feedback to refine and improve your response.

Featured Mathematician

Tanya Khovanova

Born in Russia, Tanya Khovanova is an American mathematician. In high school she won silver and gold medals at the International Mathematical Olympiad (becoming only the second female gold medalist at the time), and then went on to earn a PhD in Mathematics from Moscow State University. Khovanova has found success teaching at the university level, coaching mathletes, contracting as a systems engineer and applied mathematician, and conducting research on quantum group theory and superstring theory. Her recent work explores combinatorics and recreational mathematics, creating and solving mathematical puzzles, including some related to the counterfeit coin problems in this lesson.

Rebecca Frankel

Name: Date: Period:

Activity 3 A Cruel Twist

Consider the same setup as the previous activities, with 13 total coins, except now you only know that either one coin is counterfeit (and could be slightly heavier or slightly lighter than the other coins) *or* all the coins are real.

1. What is the smallest number of weighings it would take so you could *always* say:

   a. Whether any of the coins is actually counterfeit?

   b. Which coin is counterfeit (if one of them is)?

STOP

Unit 6 Expressions and Equations

The Power of Balance

As long as there have been coins, there have been coin counterfeiters.

It got so bad in 375 BCE that the Greek city-state of Athens installed a *dokimastes* — an official coin tester — in the marketplace. The job of the dokimastes was to do what you just did — use a balance to sniff out the fakes.

Like counterfeit coins, not everything is always what it seems. In math, as in life, we encounter uncertainty and unknowns.

So, what do we do? Hide under our tunics?

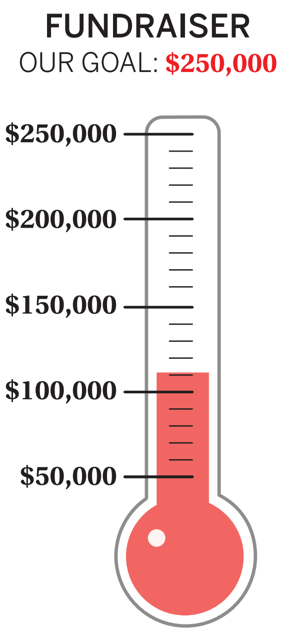
No! We do as the dokimastes did. Whether it's unknowns in trade, history, or different ecosystems, the method is still the same. With what we know about *balance*, we can strip away the mystery, step-by-step, until all that's left is the truth.

Welcome to Unit 6.

Practice

Name: .... Date: .... Period: ....

1. Write at least two different mathematical expressions or equations about the thermometer shown. Include either a fraction, a decimal number, or a percentage in each.



2. Determine each product.

a $21.2 \cdot 0.02$

b $2.05 \cdot 0.004$

Practice

Name: .... Date: .... Period: ....

3. Calculate $141.75 \div 2.5$ using a method of your choice. Explain your thinking.

4. There are 90 students in the band. Most band members rent their instruments, but the remaining 20% of the band members own their instruments.

   a. How many students in the band own their instruments?

   b. How many students in the band rent their instruments?

   c. What percent of students rent their instruments?

5. Mai brought 12 apples to school in the morning and then shared 6 apples with friends at lunch. Write an expression to represent the number of apples Mai had left after lunch.

SUB-UNIT

1 Expressions and Equations in One Variable

Narrative Connections

What's a bag of chips worth in Timbuktu?

Actually, quite a lot!

In the 14th century, Timbuktu was the heart of a massive trade network. Routes spanned the Sahara Desert, connecting Africa's northern coast with the grassy Western Sudan region and Akan forests. In Timbuktu, you could trade what you have for rare items, such as cloth, spices, or even ivory.

But only one prize was worth its weight in gold: salt!

It kept meat from spoiling, gave food its taste, and replenished any electrolytes the body sweated out in the intense heat. Salt was crucial for survival, and that made it precious.

Getting it though, was hard work. Slabs of salt had to be mined out from ancient sea beds. Then, they were strapped to camels and transported across the desert, traveling for weeks on end through dust storms and blistering heat.

The salt was so valuable that merchants would trade gold for it — sometimes pound for pound. So, depending on how salty those chips are, that bag could be worth a small fortune!

Trading is simplest when the values of goods are equal. You just match everything one-for-one.

But what happens when those values aren't equal? For that, we turn again to our old friend, balance! Whether it's gold, salt, or potato chips, once you know how two things balance against each other, you're on your way to a fair deal.

Keep that in mind next time you swap lunches!

Unit 6 | Lesson 2

Write Expressions Where Letters Stand for Numbers

Let's use expressions with variables to describe scenarios.

Warm-up The African Salt Trade

Write an expression to represent the underlined quantity from each statement. Use a letter to represent any unknown numbers.

Statement	Expression
The total distance a caravan traveled, if it traveled 5 km before sunrise, and 3 more km after sunrise.	
The total kilograms of salt a small caravan can carry, if each camel can carry a load of 9 kg, and there are 3 camels.	
The high temperature of the day, if it is 37 degrees warmer than the overnight low temperature.	
The total number of blocks of salt traded for bars of gold, if the number of blocks of salt is equal to one-half the number of bars of gold.	

Collect and Display: As you explain how you wrote your expressions, your teacher will add any math language you use to a class display. Continue to add to this display throughout this unit.

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 Known, Known, Unknown

1. There are 43 shops in the city market of Niani. The cities of Gao, Timbuktu, and Djenne each have more shops than Niani. Complete the table to show how many shops each city has.

Number of gold bars	Gao	Timbuktu	Djenne
How many more shops than Niani?	7	2	d
Number of shops			

2. A gold bar is worth the same as 12.5 kg of salt. Complete the table to show how much salt, in kilograms, is worth the same as each number of gold bars.

Number of gold bars	2	12	b
Salt (kg)			

3. Enoch had 31.25 g of salt on Monday. On Tuesday, he received x more grams of salt than he had on Monday. Write an expression to show how much salt Enoch had on Tuesday.

4. Two-fifths of the goods Enyonam is selling at her market stand are bolts of cloth. Write an expression to show the number of bolts of cloth if Enyonam's stand has:

   a. 20 goods.

   b. x goods.

Activity 2 Storytime

Think of a story that might be represented by each expression. For each, state what quantity x represents and what quantity the expression represents.

1. $30 + x$

a My story:

b In my story, x represents:

c In my story, the expression, $30 + x$, represents:

2. $12x$

a My story:

b In my story, x represents:

c In my story, the expression $12x$ represents:

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You wrote expressions where letters epresented numbers. We often use a letter, such as x or a, as a placeholder for an unknown number in expressions. This letter is called a ***variable***. For example, in the expression $u + 1$, the variable is u.

When a number is written "next to" a variable without an operation symbol, the number and the variable are multiplied. A number written next to a variable in this way is called a ***coefficient***. For example, $7x$ means the same as $7 \cdot x$. The variable is x, and the coefficient is 7.

If no coefficient is written next to the variable, the coefficient is 1. For example, in the expression $p + 3$, the coefficient of p is 1. You could also write $1p$, but it is not necessary because $1 \cdot p = p$.

Consider the following expressions:

Expression	Variable	Coefficient
$2x + 8$	x	2
$y - 4$	y	1

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Instructions for a craft project indicate that the length of a piece of red ribbon should be 7 in. less than the length of a piece of blue ribbon. Determine the length of the blue ribbon if the length of the red ribbon is:

   a. 10 in.

   b. 27 in.

   c. x in.

2. Gadisa has 3 times as many books as Folami. Determine the number of books that Gadisa has if Folami has:

   a. 15 books.

   b. 21 books.

   c. x books.

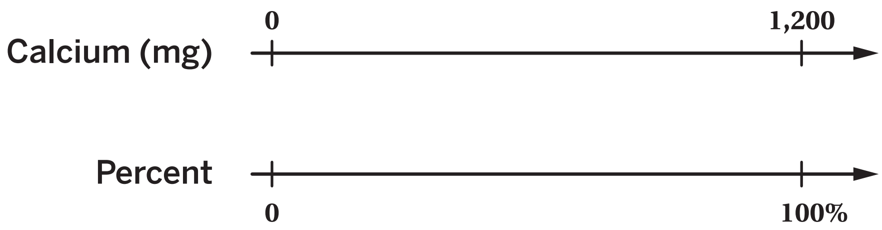
3. A bottle can hold 24 oz of water. It currently holds x oz of water.

   a. What does $24 - x$ represent in this situation?

   b. Write a question about this situation that has $24 - x$ as the answer.

Name: ______________ Date: ______________ Period: ______________

4. The daily recommended allowance of calcium for a sixth grader is 1,200 mg. One cup of milk has 25% of the recommended daily allowance of calcium. How many milligrams of calcium are in a cup of milk? Consider using the double number line to help with your thinking.



5. A trash bin has a capacity of 50 gallons. What percent of its capacity is each amount? Show your thinking.

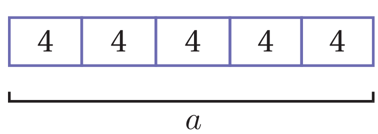
a 5 gallons

b 30 gallons

c 45 gallons

d 100 gallons

6. Lin, Shawn, and Tyler each write an equation to represent the given diagram.



Shawn's equation: $4 + 4 + 4 + 4 + 4 = a$

Lin's equation: $4 \cdot 5 = a$

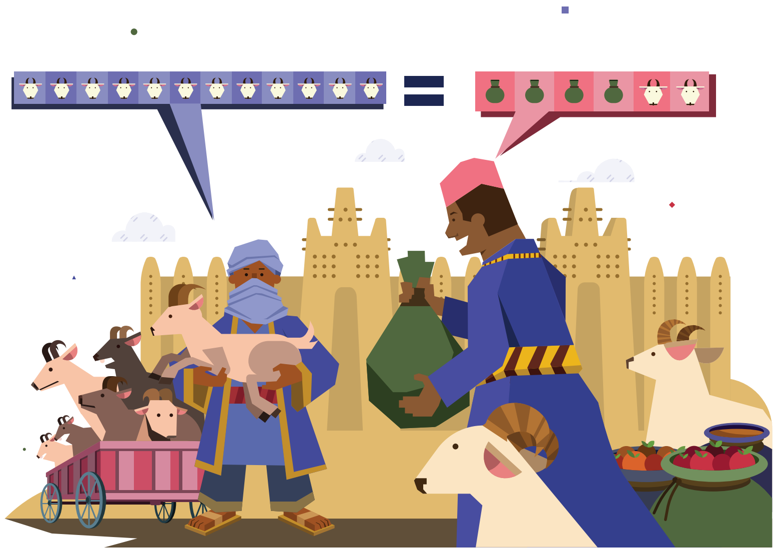
Tyler's equation: $5 + 4 = a$

Which student is correct? Explain your thinking.

Unit 6 | Lesson 3

Tape Diagrams and Equations

Let's see how tape diagrams and equations can show relationships between amounts.



Warm-up Moving From Expressions to Equations

1. Write an expression that could be represented by each tape diagram.

Tape diagram	Expression
a | 2 | 2 | 2 | 2 | 2 |	
a | 2 | 5 |	

2. Label the length of each tape diagram. Write as many equations as you can that could be represented by each tape diagram.

Tape diagram	Equations
a | 2 | 2 | 2 | 2 | 2 | $\underbrace{\qquad\qquad}_{\boxed{}}$	
b | 2 | 5 | $\underbrace{\qquad\qquad}_{\boxed{}}$	

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: ____________ Period: ____________

Activity 1 Tape Diagrams With Variables

Write as many different equations as you can that could be represented by each tape diagram.

Tape diagram	Equations
$\underbrace{\boxed{x}\boxed{x}\boxed{x}\boxed{x}}_{12}$	
$\underbrace{\boxed{4}\boxed{\quad x \quad}}_{12}$	

Reflect: How did you apply your strengths to write the equations?

Historical Moment

The history of variables . . . varies

Why letters? Why x? Depending who you ask, or what you read, you may hear quite different stories. And to some extent the only fact is, no one *really* knows.

The Ancient Greeks, such as Aristotle and Diophantus, used letters to represent numbers in the third and fourth centuries BCE. So did a smattering of Europeans from the 1200s CE all the way up to one name you might often hear as an answer to those questions – René Descartes (1596–1650 CE). He may have been the first to publish a widely read work, *La géometrie*, that clearly used variables consistently, to represent both knowns and unknowns, and yes x was one of them.

But symbolic algebra has also been traced to ancient India (c. 800–1000 CE), however much of that work has not been preserved. Whoever it was, or all of the whoevers, definitely contributed to the development of algebra as it is still studied today. And if you know a number, call it x, can be added to $1{,}928.37$, call it y, to make $9{,}182.73$, call it z, it is sure a relief to just simply write $x + y = z$. Of course, that doesn't get you the answer of what the value of x is. What is the value of x?

Activity 2 Storytime With Tape Diagrams

Part A

Draw a tape diagram to represent each story. Then use your drawings to determine which of the four equations best represents each story.

$x + 5 = 20$	$x = 20 + 5$	$5 \cdot 20 = x$	$5x = 20$

1. After Amara sold 5 kg of salt at the market in Gao on Friday, she had sold a total of 20 kg of salt for the week. She sold x kg before Friday.

Tape diagram	Equation

2. Kofi traded some salt for 20 gold bars at the market in Timbuktu, which is 5 times as many as he was able to trade in Djenne. He received x gold bars when he traded in Djenne.

Tape diagram	Equation

Part B

3. Choose one of the remaining equations *not* used in Part A and create your own story that represents the equation. Then draw a tape diagram for the equation.

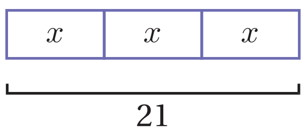
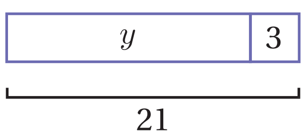
STOP

Name: ______________________ Date: __________ Period: ________

Summary

In today's lesson . . .

You used tape diagrams to help you visualize relationships between quantities. Here are two examples:

Tape diagram	Interpretations	Equations
	3 equal parts whose sum is 21	$x + x + x = 21$
	A number multiplied by 3, for which the product is 21	$3 \cdot x = 21$ $3x = 21$
	Two unequal parts, 3 and another number, whose sum is 21	$y + 3 = 21$
	3 taken away from the total of 21	$21 - 3 = y$
	Two unequal parts, 21 and another number, whose difference is 3	$21 - y = 3$

Reflect:

Practice

Name: ______________________ Date: ____________ Period: ____________

1. Consider the equation: $x + 4 = 17$.

   a. Draw a tape diagram to represent the equation. All three parts of the equation should be identified.

   b. How does your diagram show that $x + 4$ has the same value as 17?

2. Diego is trying to determine the value of x in the equation $5 \cdot x = 35$. He draws this diagram, but is not certain how to proceed.

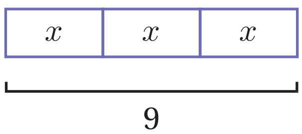
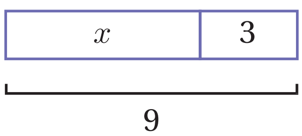
x	x	x	x	x

   a. Complete the tape diagram so it represents the equation $5 \cdot x = 35$.

   b. Determine the value of x.

3. For each tape diagram, write which of the following equation(s) could represent it. Write as many equations as you can.

$x + 3 = 9$	$3 \cdot x = 9$	$9 = 3 \cdot x$	$x + x + x = 9$
$3 + x = 9$	$x = 9 - 3$	$x = 9 \div 3$	

Tape diagram	Equation(s)
a 	
b 	

4. A shopper paid \$2.52 for 4.5 lb of potatoes, \$7.75 for 2.5 lb of broccoli, and \$2.45 for 2.5 lb of pears. What is the unit price of each item she bought? Explain your thinking.

5. A sports drink bottle contains 16.9 fluid ounces. Andre drank 80% of the bottle. How many fluid ounces did Andre drink? Explain your thinking.

6. Is the following equation true or false? Explain your thinking.

$a + 6 = 11$, when $a = 5$

Unit 6 | Lesson 4

Truth and Equations

Let's represent stories with equations, and see what makes them true or false.

Warm-up Making Equations True or False

1. The equation $a + b = c$ could be true or false.

   a. Determine values to replace a, b, and c that make the equation true. Draw a tape diagram that represents your equation to help you with your thinking.

   b. Determine values to replace a, b, and c that make the equation false.

2. The equation $x \cdot y = z$ could be true or false.

   a. Determine values to replace x, y, and z that make the equation true. Draw a tape diagram that represents your equation to help you with your thinking.

   b. Determine values to replace x, y, and z that make the equation false.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Revisiting the Market

Complete the following problems about the salt and gold trade in Mali.

1. Priya brought back bars of gold from the market, and each bar weighs 12.5 kg. The total weight of the gold she brought back is 75 kg. You want to determine how many bars of gold Priya brought back from the market.

   a. Write an equation for this scenario using g as the variable.

   b. If you substitute 5 or 6 as the value for g, does either value make the equation true? Be prepared to explain your thinking.

2. The average family in Timbuktu needs a minimum of 14.8 kg of salt to preserve a month's worth of food. Noah brought back 15.25 kg of salt from the market at the beginning of the month. You want to determine how much more he brought back than the minimum needed.

   a. Write an equation for this scenario, using s as the variable.

   b. If you substitute 0.45 or 0.5 as the value for s, does either value make the equation true? Be prepared to explain your thinking.

Activity 2 Two Lies and a Truth

Your group will be assigned one of the following equations containing a variable. Circle the equation you are assigned.

$18 = 3 + z$	$1000 - a = 400$	$h + \frac{3}{7} = 1$	$10c = 1$
$\frac{2}{3}d = \frac{10}{9}$	$10 = 0.5f$	$12.6 = b + 4.1$	$18 = 3q$

Determine a *solution to the equation* — a value that makes the equation true. Then determine two values that make the equation false. Record your group's equation and true/false values.

Equation	True	False

Complete the table as you visit each group's poster.

Equation	Solution to the equation	False values
$18 = 3 + z$		
$1{,}000 - a = 400$		
$h + \frac{3}{7} = 1$		
$10c = 1$		
$\frac{2}{3}d = \frac{10}{9}$		
$10 = 0.5f$		
$12.6 = b + 4.1$		
$18 = 3q$		

STOP

Name: Date: Period:

Summary

In today's lesson . . .

You saw that an equation can be true or false. An example of a true equation is $7 + 1 = 4 \cdot 2$ because both sides are equivalent to 8. An example of a false equation is $7 + 1 = 9$. Normally you would not write an equation that is false, especially when it only includes numbers. False equations more often appear when you substitute a value for a variable, giving you an equation involving only numbers.

An equation can also have variables in it, like $u + 1 = 8$. This equation is neither true nor false on its own. If u is equal to 3, the equation is false because $3 + 1$ does not equal 8. The equation is true if u is equal to 7 because $7 + 1 = 8$.

A ***solution to an equation*** is a number that can be substituted in place of the variable so that it makes the equation true. In $u + 1 = 8$, there is only one solution, which is 7.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. Select *all* the true equations.

A. $5 + 0 = 0$

B. $15 \cdot 0 = 0$

C. $1.4 + 2.7 = 4.1$

D. $\frac{2}{3} \cdot \frac{5}{9} = \frac{7}{12}$

E. $4\frac{2}{3} = 5 - \frac{1}{3}$

2. Match each equation with its solution.

Equation		Solution
a $k + \frac{3}{8} = 2$	______	$1\frac{5}{8}$
b $4.6 - m = 0.6$	______	2.3
c $2p = 4.6$	______	$\frac{8}{5}$
d $r \div \frac{8}{5} = 1$	______	4

3. Mai's water bottle had 24 oz in it. After she drank x oz of water, there were 10 oz left. Select *all* the equations that represent this scenario.

A. $24 \div 10 = x$

B. $24 + 10 = x$

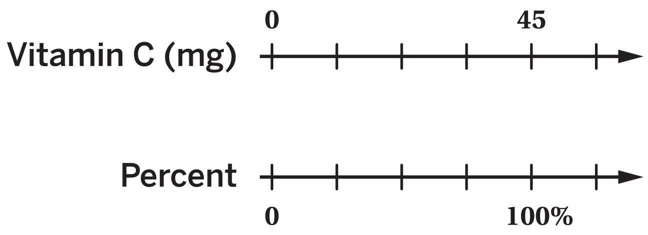
C. $24 - 10 = x$

D. $x + 10 = 24$

E. $10x = 24$

Name: .... Date: .... Period: ....

4. The daily recommended allowance of vitamin C for a sixth grader is 45 mg. One orange has about 75% of the recommended daily allowance of vitamin C. How many milligrams of vitamin C are in one orange? Consider using the double number line to help with your thinking.



5. Calculate each product.

a) $0.25 \cdot 1.4$

b) $0.061 \cdot 0.43$

c) $1.017 \cdot 0.072$

d) $5.226 \cdot 0.037$

6. What value would make the equation $5 + ? = 9$ true? Show or explain your thinking.

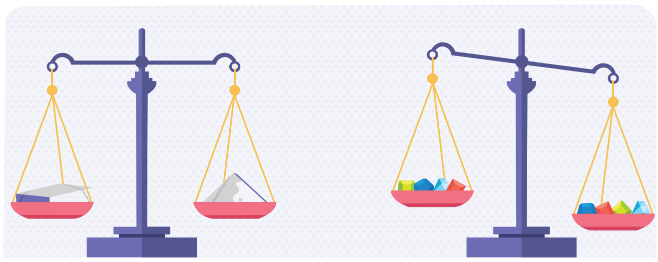
Unit 6 | Lesson 5

Staying in Balance

Let's use balanced hangers to help us write equations.

Warm-up Notice & Wonder

The people from Taghaza are ready to trade their salt for jewels. Refer to the picture. What do you notice? What do you wonder?



1. I notice . . .

2. I wonder . . .

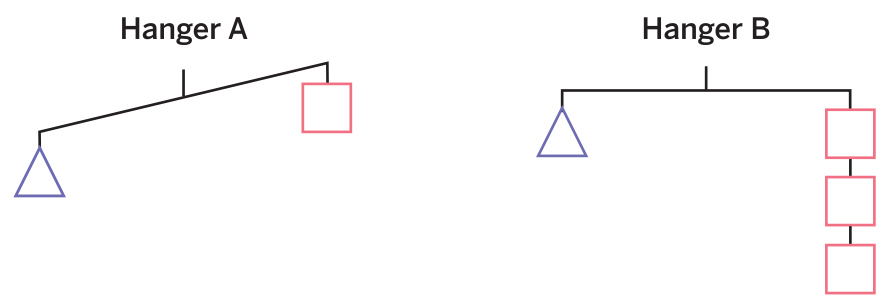
Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Akan Weights

In the ancient markets around what is now Mali, everyone trading goods would carry their own set of weights, often produced by Akan metalsmiths. These weights were known for being beautiful and accurate. Both the buyer and seller would take turns weighing the salt or gold, or whatever was being exchanged, using their own weights to ensure deals were fair.

The hanger diagrams show the relationship between a triangular weight and a square weight.



1. For each diagram, write a statement about the weights shown that *must* be true.

Hanger A	Hanger B

2. For each diagram, write a statement about the weights shown that *could* be either true or false.

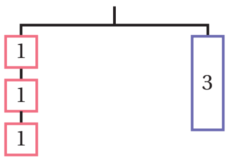
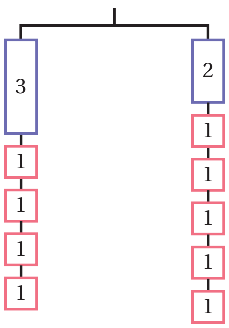
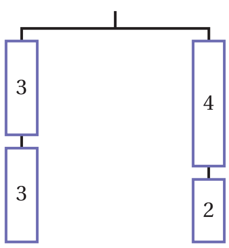
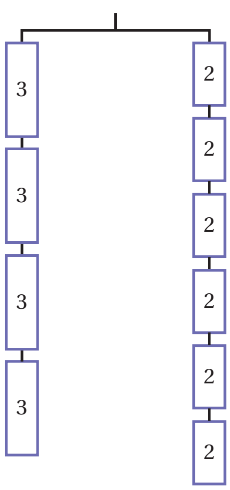
Hanger A	Hanger B

3. For each diagram, write a statement about the weights shown that *cannot possibly* be true.

Hanger A	Hanger B

Activity 2 Maintaining Balance

Hangers A, B, C, and D are balanced. Take turns using the given superpower to unbalance and then rebalance each hanger. Each superpower may be applied more than once. Be ready to explain how you knew what action would rebalance each hanger.

Balanced hanger	Superpower
Hanger A 	Add a weight of 1.
Hanger B 	Remove a weight of 1.
Hanger C 	Make a copy of the total weight on a side.
Hanger D 	You have two superpowers! Remove a weight of 3 from the left. Remove a weight of 2 from the right.

Name: ______________________ Date: ____________ Period: __________

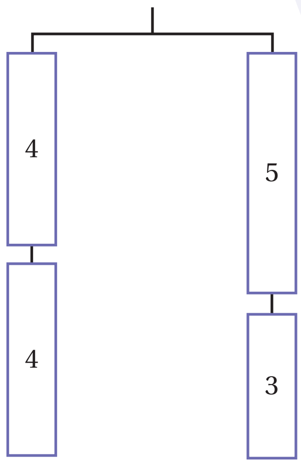
Activity 3 Keeping Equations True

Plan ahead: How does truth impact your decision making?

Hanger A is balanced and represents the equation $4 + 4 = 5 + 3$.

1. Each scenario in the table represents a change to Hanger A. Complete the table by describing how to rebalance the hanger. Then write an equation to represent the rebalanced hanger.

Hanger A



Change to left	Change to right	Equation
Add 2.		
	Subtract 6.	
Double the weight.		
	Subtract half of the weight.	

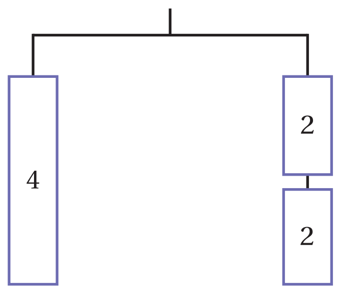
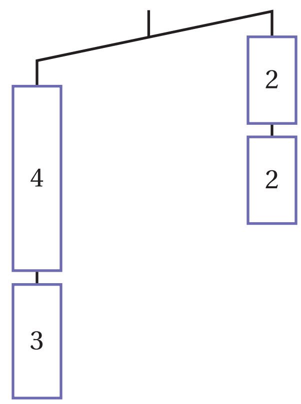
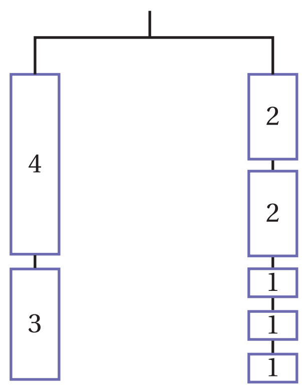
2. Choose one equation from the table above. Explain how you know the equation represents a balanced hanger.

STOP

Summary

In today's lesson . . .

You balanced hanger diagrams. A hanger is balanced when the total weights on both sides are equivalent. If you start with a balanced hanger and then change the weight on only one side, the hanger will no longer be balanced. But, if you change the weights on both sides in exactly the same way, the hanger will stay balanced.

Balanced hanger	Unbalanced hanger	Rebalanced hanger
Hanger A 	Hanger A 	Hanger A 

An equation can be used to represent a balanced hanger. When an equation is true, the expressions on both sides of the equal sign represent the same value. Just like changing the weights on a hanger, you can use operations to change both sides of an equation. The ***properties of equality*** tell you the equation will still be true if you perform the exact same operations (addition, subtraction, multiplication, or division) on both sides of the equal sign.

In the example above, the original balanced hanger can be represented using the equation $4 = 2 + 2$. The rebalanced hanger can be represented using the equation $4 + 3 = 2 + 2 + 1 + 1 + 1$. The hanger is rebalanced because the total weight on both sides is equal to 7.

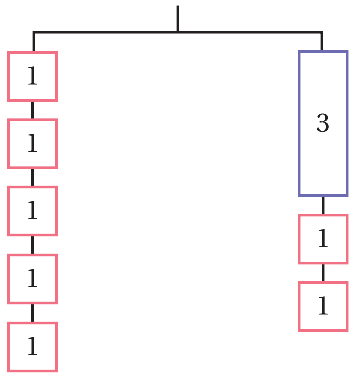
Reflect:

Practice

Name: ______ Date: ______ Period: ______

Use Hanger A to complete Problems 1 and 2.

Hanger A

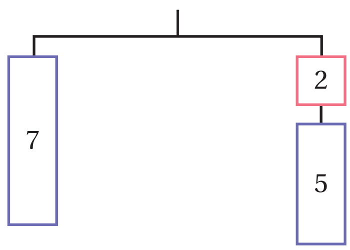


1. Hanger A is balanced. Write an equation that represents Hanger A.

2. Each scenario below presents a change to Hanger A. Describe how to rebalance the hanger. Then write an equation to represent the rebalanced hanger.

Change to left	Change to right	Equation
Add 3		
	Double the weight	

Hanger B



3. Hanger B is balanced. Izem added weights of 3 and 4 to the left side of Hanger B. Hodan then said it could be rebalanced by adding weights of 2 and 5 to the right side.

But Izem disagreed because the two weights added to each side would not be exactly the same. Do you agree with either Izem or Hodan? Explain your thinking.

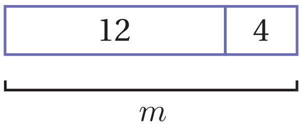
Name: ______ Date: ______ Period: ______

Practice

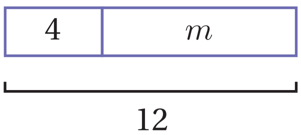
4. Match each equation with the diagram it represents.

Diagram

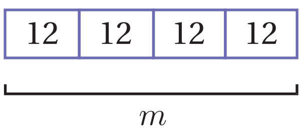
a



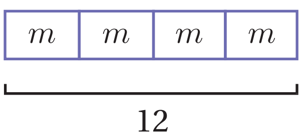
b



c



d



Equation

______ $12 - m = 4$

______ $12 = 4 \cdot m$

______ $m - 4 = 12$

______ $\frac{m}{4} = 12$

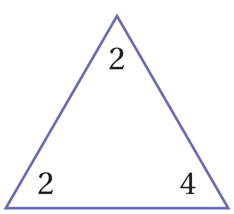
5. The area of a rectangle is 14 square units. It has side lengths x and y. Given each value for x, determine y.

a $x = 2\frac{1}{3}$

b $x = 4\frac{1}{5}$

c $x = \frac{7}{6}$

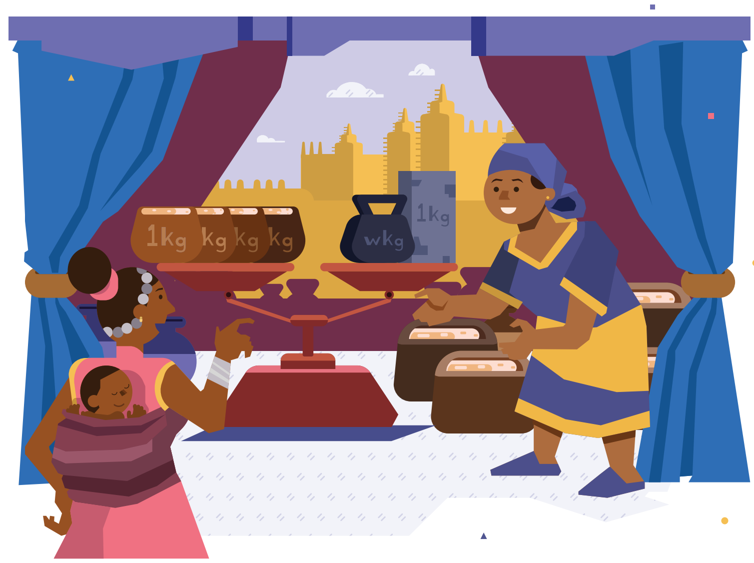
6. Write *all* the equations you can think of, for the fact family below.



Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 6

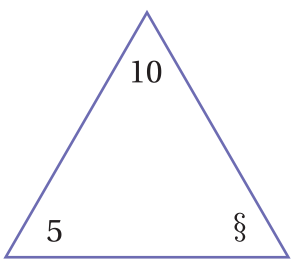
Staying in Balance With Variables



Let's use balanced hangers to help us write and solve equations with variables.

Warm-up Understanding an Ancient Sales Record

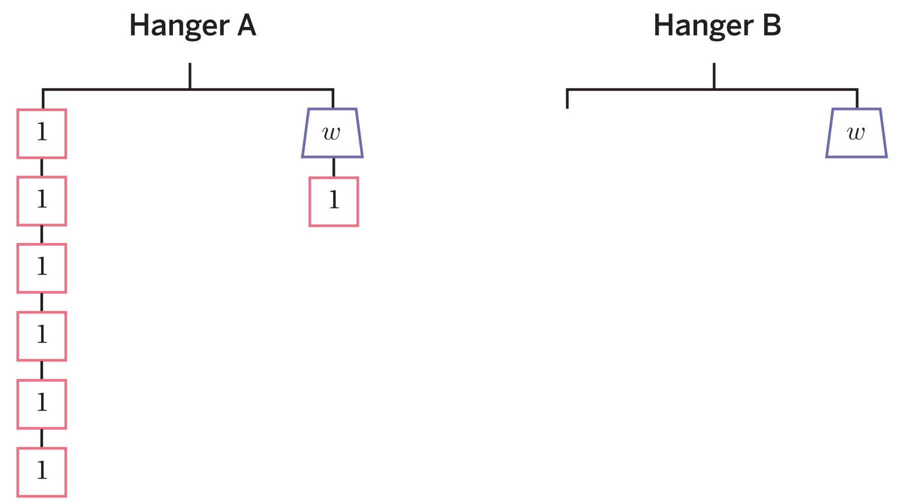
The diagram shown is a sales record from the most successful spice seller in Timbuktu. The seller has used numbers and a symbol to track the sales. Write *all* the equations you can think of to represent the relationship between 5, 10 and §. Leave the symbol in your equation.



Log in to Amplify Math to complete this lesson online.

Activity 1 Using Hangers to Write and Solve Equations With Variables

You are buying spices at the market in Timbuktu. The seller has placed several 1-kg bags of spices on the left side of a balance. The right side of the balance has a 1-kg weight and a weight labeled as w kg.



1. Hanger A represents how the seller balanced the spices and weights. Write an equation that represents Hanger A.

2. You want to know how much the unknown weight w weighs, to ensure a fair deal.

   a. Complete the left side of Hanger B so it is balanced.

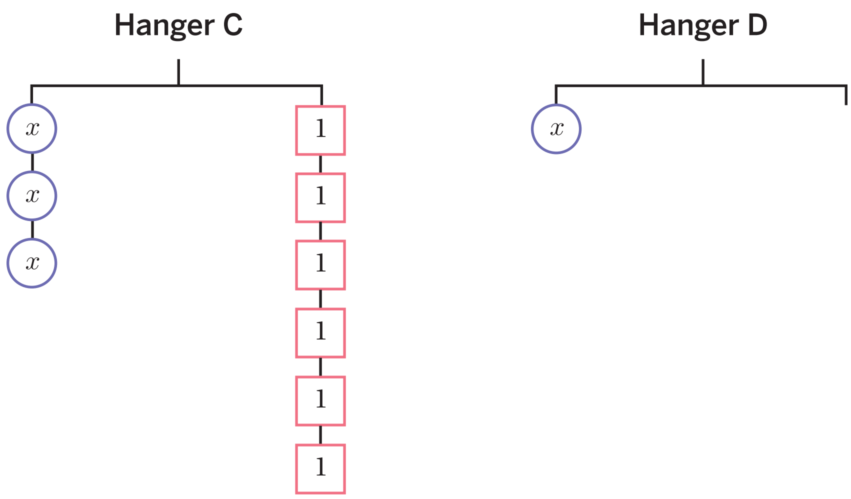
   b. What is the weight of w, in kilograms?

3. Take the weight of w, in kg, from Hanger B, and substitute it for the value of w in Hanger A's equation. Is Hanger A's equation true? Explain your thinking.

Name: ______ Date: ______ Period: ______

Activity 1 Using Hangers to Write and Solve Equations With Variables (continued)

Happy with your first purchase, you decide to buy some salt. The seller has placed several 1-kg blocks of salt on the right side of a balance. Three identical weights, each labeled as x kg, are on the left side.



4. Hanger C represents how the seller balanced the weights and salt. Write an equation that represents Hanger C.

5. You want to know how much the unknown weight x weighs, to ensure a fair deal.

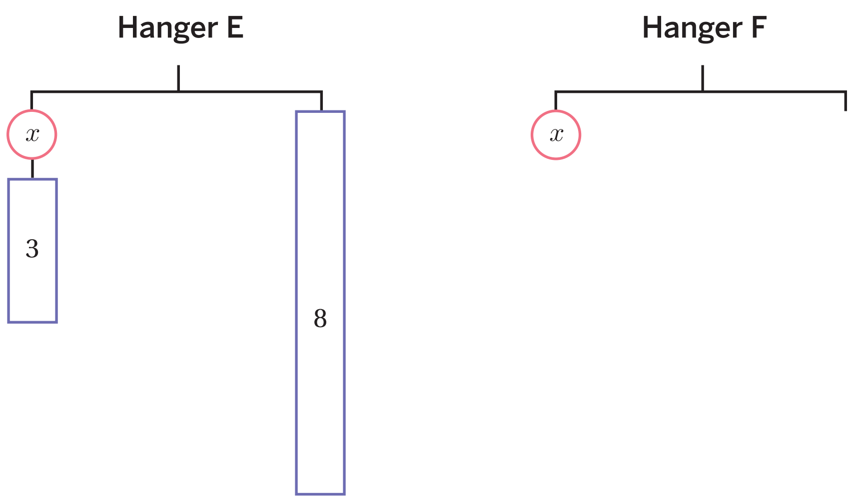
a Complete the right side of Hanger D so that it is balanced.

b What is the weight of each x, in kilograms?

6. Take the weight of x, in kilograms, from Hanger D, and substitute it for the value of x in Hanger C's equation. Is the equation true? Explain your thinking.

Activity 2 Writing and Solving Equations With Variables

Hanger E is balanced. Use it to complete the following problems.



1. Write an equation that represents Hanger E.

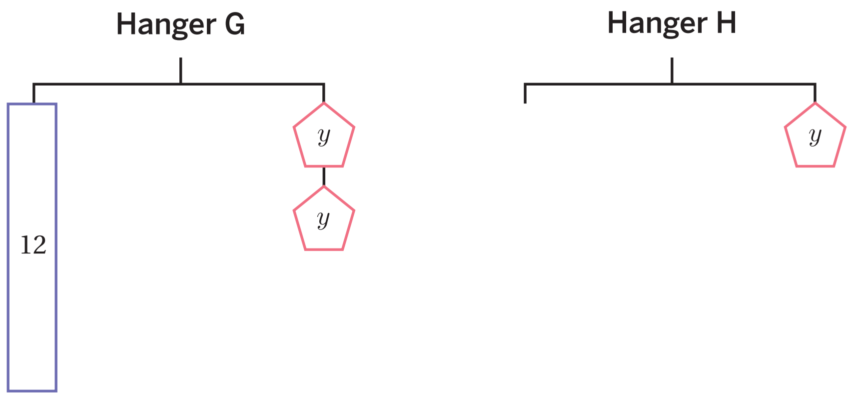
2. Complete the right side of Hanger F so that it is balanced. Then write an equation to represent the balanced hanger.

3. Explain how you know the value representing the weight of x is a solution to Hanger E's equation.

Name: ________ Date: ________ Period: ________

Activity 2 Writing and Solving Equations With Variables (continued)

Hanger G is balanced. Use it to complete the following problems.



4. Write an equation that represents Hanger G.

5. Complete the left side of Hanger H so that it is balanced. Then write an equation to represent the balanced hanger.

6. Explain how you know the value representing the weight of y is a solution to Hanger G's equation.

STOP

Summary

In today's lesson . . .

You used balanced hangers to write and solve equations. The table shows the four *properties of equality*, using the variables a, b, and c to represent any numbers.

Property of Equality	Formula	Equation
Addition Property of Equality	If $a = b$, then $a + 2 = b + 2$.	As $3 + 3 = 4 + 2$, then $3 + 3 + 2 = 4 + 2 + 2$
Subtraction Property of Equality	If $a = b$, then $a - 2 = b - 2$.	As $6 = 4 + 2$, then $6 - 2 = 4 + 2 - 2$
Multiplication Property of Equality	If $a = b$, then $a \bullet 2 = b \bullet 2$.	As $3 + 3 = 4 + 2$, then $2(3 + 3) = 2(4 + 2)$
Division Property of Equality	If $a = b$, then $a \div 2 = b \div 2$.	As $3 + 3 = 6$, then $(3 + 3) \div 2 = 6 \div 2$

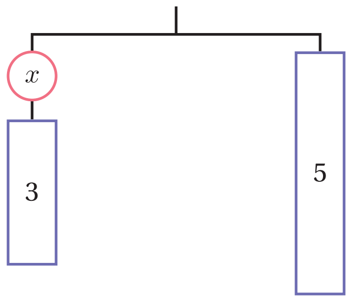
Reflect:

Practice

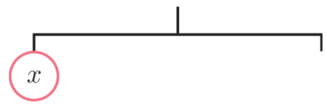
Name: ______________ Date: ______________ Period: ______________

Hangers A and C are balanced. Use them to complete Problems 1–3.

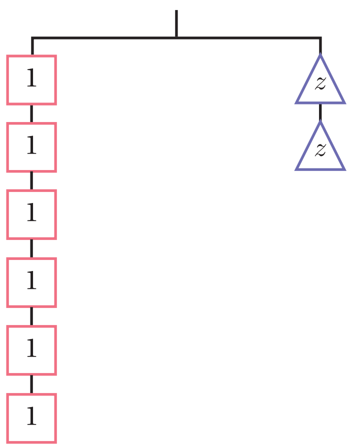
Hanger A



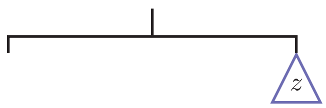
Hanger B



Hanger C



Hanger D



1. Write equations that represent Hanger A and Hanger C.

Hanger A:

Hanger C:

2. Balance Hangers B and D by completing the empty sides of the hangers. Then, write equations that represent Hanger B and Hanger D.

Hanger B:

Hanger D:

3. Explain how you know the value for x is a solution to the equation for Hanger A, and the value for z is a solution to the equation for Hanger C.

Practice

Name: ________ Date: ________ Period: ________

4. Lin needs to save up $20 for a new game. How much money has she saved at each percentage of her goal?

a. 25%

b. 75%

c. 125%

5. Calculate each quotient.

a. $0.009 \div 0.001$

b. $0.009 \div 0.002$

c. $0.0045 \div 0.001$

d. $0.0045 \div 0.002$

6. Evaluate each expression. Then, explain how you can use the first expression to evaluate the second expression.

a. $2 - 1$

b. $2 - 1.1$

Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 7

Practice Solving Equations

Let's solve equations by doing the same to each side.

Warm-up Subtracting From Five

Using mental math, determine the value of each expression.

1. $5 - 2$

2. $5 - 2.1$

3. $5 - 2.17$

4. $5 - 2\frac{7}{8}$

Compare and Connect: How are these expressions similar and different? Can you use your approach to Problem 1 to help you complete the other problems?

Log in to Amplify Math to complete this lesson online.

Activity 1 Solving Equations With a Partner

Work together to solve the following equations.

Equation	What I do to the variable side . . .		. . . I do to the other side.	Solve and check
$18 = 2x$	$\frac{2}{2}x$ Because this is multiplying x by 2, I need to divide by 2 to make it $1\ x$, or x.	$=$	$\frac{18}{2}$ I need to divide my side by 2, which equals 9.	**Solution:** $x = 9$ **Check:** $18 = 2 \cdot 9$
$36 = 4x$		$=$		**Solution:** **Check:**
$17 = x + 9$		$=$		**Solution:** **Check:**
$8x = 56$		$=$		**Solution:** **Check:**
$x + 3\frac{5}{6} = 8$		$=$		**Solution:** **Check:**

Name: ________________ Date: ________ Period: ________

Activity 1 Solving Equations With a Partner (continued)

Equation	What I do to the variable side . . .		. . . I do to the other side.	Solve and check
$21 = \frac{1}{4}x$		=		**Solution:** **Check:**
$2.17 + x = 5$		=		**Solution:** **Check:**
$10x = \frac{1}{3}$		=		**Solution:** **Check:**
$17.05 = 14.88 + x$		=		**Solution:** **Check:**

Activity 2 Representing Scenarios With Equations

Circle *all* the equations that represent each scenario. Then calculate a solution for each scenario. Consider drawing a diagram to help with your thinking.

1. Kofi has 8 fewer slabs of salt than Ime. If Ime has 26 slabs of salt, how many slabs of salt does Kofi have?

A. $26 - x = 8$

B. $x = 26 + 8$

C. $x + 8 = 26$

D. $26 - 8 = x$

$x =$ ____________

2. A market in Djenne has shops with 8 goats in each. There are 14 shops. How many total goats are there in the market?

A. $y = 14 \div 8$

B. $\frac{y}{8} = 14$

C. $\frac{1}{8}y = 14$

D. $y = 14 \cdot 8$

$y =$ ____________

3. A caravan bringing salt from Taghaza traveled 489 miles to the market in Timbuktu. If a second caravan from the gold mines of Lobi traveled 292 more miles to get to Timbuktu, how many miles did the second caravan travel?

A. $292 = 489 - z$

B. $z - 292 = 489$

C. $489 + 292 = z$

D. $292 = 489 + z$

$z =$ ____________

4. Amara traveled 27 miles last week from Gao to the Niger River, which was three times as far as Neela traveled. How far did Neela travel?

A. $3w = 27$

B. $w = \frac{1}{3} \cdot 27$

C. $w = 27 \div 3$

D. $w = 3 \cdot 27$

$w =$ ____________

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that writing and solving equations can help you answer questions about mathematical scenarios, which may involve whole numbers, fractions, or decimals. These equations can also be represented by tape diagrams or hanger diagrams, which may be helpful in solving them.

In some scenarios, the two quantities can be related by addition.

$$x + 1.5 = 3.25$$

You can keep the equality by subtracting the same value, 1.5, from both sides.

$$x + 1.5 - 1.5 = 3.25 - 1.5$$

This isolates the variable x on one side and reveals the solution.

$$x = 1.75$$

In other scenarios, the two quantities can be related by multiplication

$$\frac{3}{4}x = 54$$

You can keep the equality by dividing both sides by the same value $\frac{3}{4}$.

$$\frac{3}{4}x \div \frac{3}{4} = 54 \div \frac{3}{4}$$

This isolates the variable x on one side and reveals the solution.

$$x = 72$$

Reflect:

Practice

Name: ________________ Date: ________ Period: ________

For Problems 1 and 2, circle *all* the equations that represent each scenario. Then calculate the solution for each scenario.

1. Kiran's backpack weighs 3 lb less than Clare's backpack. Clare's backpack weighs 14 lb. How much does Kiran's backpack weigh?

A. $x + 3 = 14$

B. $3x = 14$ $x =$ ________

C. $x = 14 - 3$

D. $x = 14 \div 3$

2. Each notebook contains 60 sheets of paper. Andre has 5 notebooks. How many sheets of paper do Andre's notebooks contain in total?

A. $y = 60 \div 5$

B. $y = 5 \cdot 60$ $y =$ ________

C. $\frac{y}{5} = 60$

D. $5y = 60$

3. Solve each equation. Show your work. Be sure to check your solution by substituting it back into the original equation.

a $y + 1.8 = 14.7$

b $6 = \frac{1}{2}z$

c $3\frac{1}{4} = \frac{1}{2} + w$

d $2.5t = 10$

e $6x = 45$

4. Draw a tape diagram that represents each equation.

a $3 \cdot x = 18$

b $3 + x = 18$

c $17 - 6 = x$

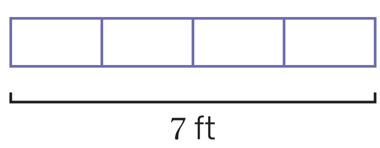
5. For a science experiment, students need to determine 25% of 60 grams.

- Jada says, "I can determine this by calculating $\frac{1}{4}$ of 60."
- Andre says, "25% of 60 means $\frac{25}{100} \cdot 60$."

Do you agree with either of them? Explain your thinking.

6. Lin is making a square planter for her garden. She has a 7 ft long piece of wood she is using for the sides. The marks where she is planning on cutting the wood are shown.

a Write an expression that can be used to determine the length of each section of wood:



b What is the length of each section of wood?

Unit 6 | Lesson 8

A New Way to Interpret a Over b

Let's investigate what a fraction means when the numerator and denominator are not whole numbers.

Warm-Up Division and Fractions

Determine a fraction that represents a solution for each scenario.

1. A buyer and seller agree that 4 blocks of salt weigh the same as 1 gold bar. If this is true, how many gold bars would weigh the same as 21 blocks of salt?

2. Kofi dug up 7 kg of salt in a mine in Taoudenni. He wants to transport equal amounts to 10 different markets. How many kilograms of salt should he transport to each market?

Log in to Amplify Math to complete this lesson online.

Name: ____________ Date: ________ Period: ________

Activity 1 Interpreting $\frac{a}{b}$

Solve each equation for x.

1. $35 = 7x$

2. $35 = 11x$

3. $7x = 7.7$

4. $0.3x = 2.1$

5. $\frac{2}{5} = \frac{1}{2}x$

Are you ready for more?

Solve the equation for x.

$$\frac{1}{6} \cdot \frac{3}{20} \cdot \frac{5}{42} \cdot \frac{7}{72} \cdot x = \frac{1}{384}$$

Activity 2 Storytime Again

Think of a story that could be represented by each equation. For each, state what quantity x represents, and the value for x that represents a solution.

1. $\frac{7}{10} + x = 1$

   a. In my story, x represents:

   b. A solution to the equation is:

2. $\frac{1}{4}x = \frac{3}{2}$

   a. In my story, x represents:

   b. A solution to the equation is:

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You investigated fractions in which the numerator and denominator were *not* whole numbers. In prior grades, you learned that a fraction, such as $\frac{4}{5}$, can be thought of in a few ways:

- $\frac{4}{5}$ is a number that you can locate on the number line by dividing the section between 0 and 1 into five equal parts and then counting four of those parts to the right of 0.
- $\frac{4}{5}$ is how much each person would receive if four wholes were shared equally among five people.
- $\frac{4}{5}$ is the quotient resulting from dividing $4 \div 5$.

You can extend this quotient meaning of a fraction to include numerators and denominators that are *not* whole numbers. For example, $\frac{1.5}{4.5} = 1.5 \div 4.5 = \frac{1}{3}$.

When you solve any equation using division, the solution can be written as a fraction. These fractions can also involve non-whole numbers in the numerator and denominator.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Select *all* the expressions that are equal to $\frac{3.15}{0.45}$.

   A. $3.15 \cdot 0.45$

   B. $3.15 \div 0.45$

   C. $3.15 \cdot \frac{1}{0.45}$

   D. $3.15 \div \frac{45}{100}$

   E. $3.15 \cdot \frac{100}{45}$

   F. $\frac{0.45}{3.15}$

2. Solve each equation. Show your thinking.

   a. $4a = 32$

   b. $4 = 32b$

   c. $10c = 26$

   d. $26 = 100d$

Name: ______________ Date: ______________ Period: ______________

Practice

3. For each equation, write a story problem represented by the equation and state what quantity x represents. Consider drawing a diagram to help with your thinking.

a $\frac{3}{4} + x = 2$

b $1.5x = 6$

4. In a lilac paint mixture, 40% of the mixture is white paint, 20% is blue, and the rest is red. There are 4 cups of blue paint used in a batch of lilac paint. Consider drawing a diagram to help with your thinking.

a How many cups of white paint are used?

b How many cups of red paint are used?

c How many cups of lilac paint will this batch yield?

5. Triangle P has a base of 12 in. and a corresponding height of 8 in. Triangle Q has a base of 15 in. and a corresponding height of 6.5 in. Which triangle has a greater area? Explain your thinking.

6. 20% of 50 is equivalent to which of the following?

A. 5

B. 10

C. 2

D. 20

Unit 6 | Lesson 9

Revisiting Percentages

Let's use equations to determine percentages.

Warm-up Number Talk

Mentally solve each problem. Be prepared to explain your thinking.

1. 50% of 10 equals what number?

2. 10 is 50% of what number?

3. 8 is what percent of 10?

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Representing and Solving Percentage Problems With Equations

You arrive at the market in Niani with 16 kg of cloth to trade.

1. Your first stop is the spice seller. You trade all of your cloth for rare and expensive spices. The spices weigh 25% of the weight of your cloth. How many kilograms of spices did you buy?

   a. Adaeze represents the scenario with the equation $\frac{25}{100} \cdot 16 = b$. Explain how her equation represents the scenario.

   b. Solve the equation to determine the number of kilograms of spices you bought.

2. Your next stop is the jeweler. You trade all your rare spices for diamonds. The diamonds weigh 150% of the weight of your spices. How many kilograms of diamonds did you buy?

   a. Write an equation to represent the scenario.

   b. Solve the equation to determine the number of kilograms of diamonds you bought.

3. Your final stop is the salt seller, where you buy 12 kg of salt. What percent is this of the original weight of cloth that you had when you arrived at the market?

   a. Write an equation to represent the scenario.

   b. Solve the equation to determine the percent.

Activity 2 Puppies Grow Up, Revisited

Aadan and Chinyelu visit the market in Gao to buy an Azawakh, a breed of dog that is popular for guarding ancient Mali villages.

1. When Aadan's dog was a puppy, it weighed 8 kg, which is 30% of its current adult weight. What is the current weight of Aadan's dog?

   a. Write an equation to represent the situation.

   b. Solve the equation to determine the adult weight of Aadan's dog.

2. When Chinyelu's dog was a puppy, it weighed 8 kg, which is 86% of its current adult weight. What is the current weight of Chinyelu's dog?

   a. Write an equation to represent the situation.

   b. Solve the equation to determine the adult weight of Chinyelu's dog.

3. How would your equations and solutions change if:

   a. Aadan's dog weighed 5.8 kg as a puppy, which is 30% of its current adult weight.

   b. 8 kg is 43% of the adult weight of Chinyelu's dog.

Are you ready for more?

Kofi wants to paint his room purple. He buys one gallon of a purple paint mixture that is 70% blue paint. Kofi wants to add more blue so that the mixture is 80% blue.

1. How much blue paint should Kofi add? Test these possibilities: 0.2 gallons, 0.3 gallons, 0.4 gallons, and 0.5 gallons.
2. Write an equation in which x represents the amount of paint Kofi should add.
3. Check that the amount of paint Kofi should add is a solution to your equation.

STOP

Name: ____________ Date: ____________ Period: ____________

Summary

In today's lesson . . .

You saw how equations can help you represent and solve problems in which one amount is a percentage of another amount. The phrase "n percent of w equals p," where n is the percent, w is the whole, and p is the part, can be represented by the equation $\frac{n}{100} \cdot w = p$.

You can substitute the known values into the equation $\frac{n}{100} \cdot w = p$, and use a variable to represent the unknown value. Your solution method will depend on which value is unknown — the percentage, the whole, or the part.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A crew has paved $\frac{3}{4}$ of a mile of road. If they have completed 50% of the work, how long is the road they are paving?

2. Consider the statement: 40% of x is 35.

   a. Write an equation that shows the relationship of 40%, x, and 35.

   b. Use your equation to determine x. Show your thinking.

3. Ime has completed 9 exam questions. This is 60% of the questions on the exam.

   a. Write an equation representing this situation. Explain the meaning of any variables you use.

   b. How many questions are on the exam? Show your thinking.

Name: .... Date: .... Period: ....

4. Consider the equation $2n - 3 = 7$.

a What is the variable?

b What is the coefficient of the variable?

c Which of these is the solution to the equation? $2, 3, 5, 7, n$

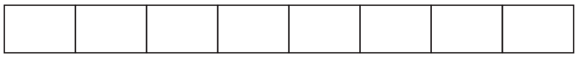
5. Which of these is a solution to the equation $\frac{1}{8} = \frac{2}{5} \cdot x$?

A. $\frac{2}{40}$ **B.** $\frac{5}{16}$ **C.** $\frac{11}{40}$ **D.** $\frac{17}{40}$

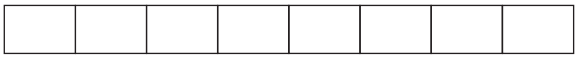
6. Draw tape diagrams to represent the following expressions:

a $4 + 2$

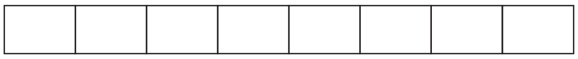
b $2 + 2 + 2 + 2$



c $2 \cdot 4$



d $4 \cdot 2$

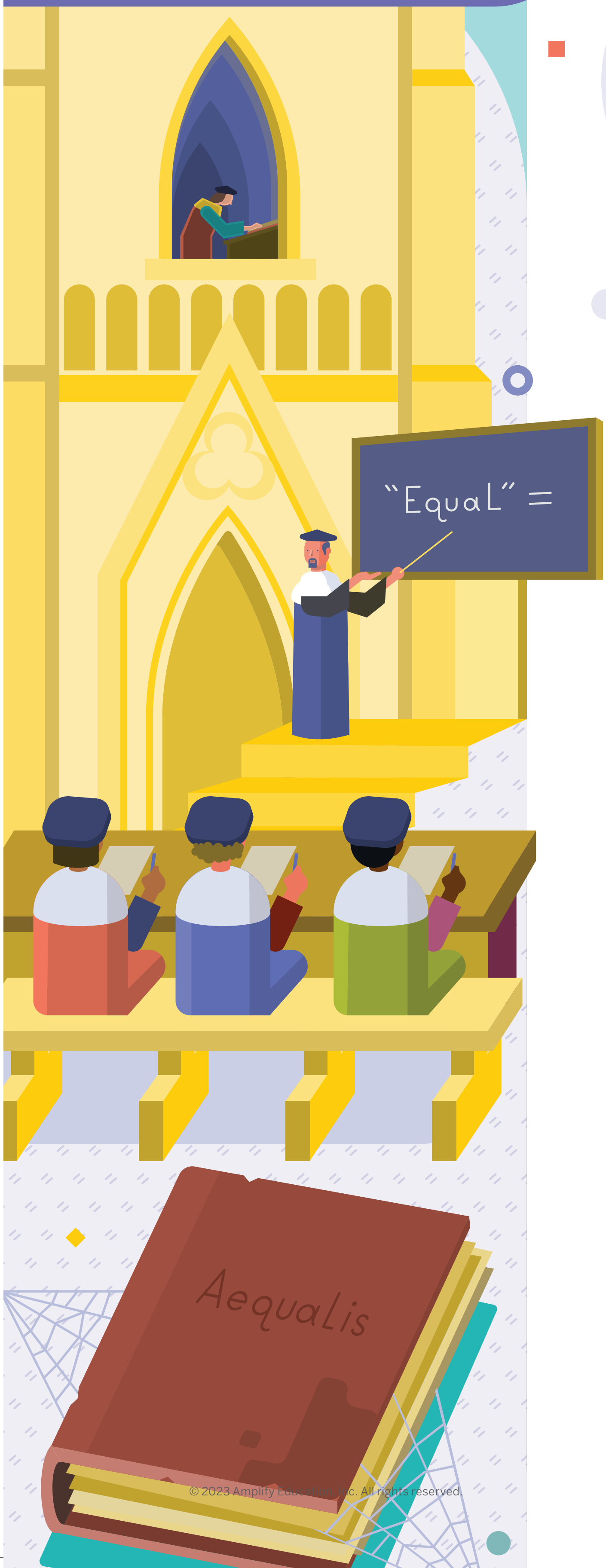


My Notes:

SUB-UNIT

2 Equivalent Expressions

Narrative Connections



How did a Welshman equalize England's upper crust with its common folk?

Quite literally, with an equal sign.

For this, let's go way back to the middle ages. Back then, in England, all math was written out in full sentences, and in Latin. To express that things were equal, English mathematicians used the word *aequalis*, which means "is equal to." But learning Latin was a luxury, and was only for the elite. The lower classes, who spoke only English, were effectively cut off, not just from math, but also from science, rhetoric, music, and philosophy.

Such was life until the mid-1550s, when a Welsh physician and teacher, Robert Recorde, set out to make learning math possible for everybody. Unlike mathematicians before him, Recorde wrote and taught in English, which gave commoners access to the mathematical understanding they were previously kept from.

In his 1557 book, *The Whetstone of Witte*, he simplified the process by inventing what we now call the equal sign:

"And to avoid the tedious repetition of these words, *is equal to*, I will set as I do often in work use, a pair of parallels . . . of one length, thus: =, because no two things can be more equal."

The two lines in his original equal sign (along with his plus sign) were much longer, as you can see in the following equation he wrote:

And the rest, as they say, is history.

Unit 6 | Lesson 10

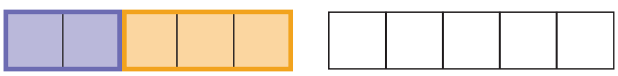
Equal and Equivalent (Part 1)

Let's use diagrams to determine which expressions are equivalent and which are just sometimes equal.

Warm-up Representing and Visualizing Mathematical Equality

1. Complete the right side of the diagram to represent the equation $2 + 3 = 3 + 2$. What do you notice?

$2 + 3 \quad = \quad 3 + 2$



2. Use the diagrams to represent the following expressions. What do you notice?

a $2 \cdot 3$

b $3 \cdot 2$

c $2 + 2 + 2$

d $3 + 3$

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Moving Toward Equivalence

Can you determine any values for x that make each of the following equations true?

You will be given a sheet with extra grids. Try drawing diagrams, such as those in the Warm-up, using the extra grids. Then copy one diagram for each equation here to show a value that makes the equations true.

1. $4x = 6 + 6$

2. $4x = x + 3 + 3$

3. $4x = x + x + x + x$

Activity 2 Detecting Equal and Equivalent Expressions

For each pair of expressions, determine which of the values 0, 1, and 3 make them equal. What does it mean if *all* values make them equal? Hint: You may want to draw diagrams to help with your thinking.

1. $a + 3$ and $9 - a$

2. $x + 2$ and $x + 3$

3. $b \div 3$ and $b \cdot \frac{1}{3}$

4. $a + a + a + a + a$ and $5a$

5. $2x$ and $3x$

Are you ready for more?

Are there any values that would make the following two expressions equal? Are they equivalent?

$3x + 1$ and $0x + 1$

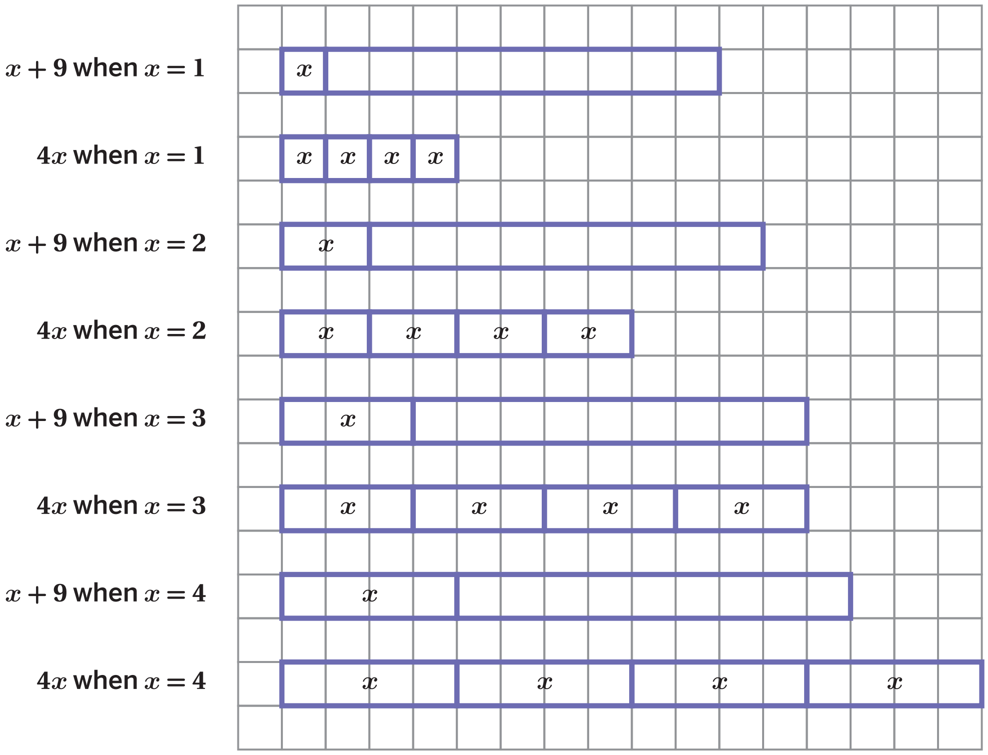
STOP

Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You used diagrams showing lengths of rectangles to see whether two expressions are equal for a particular value of the variable. For example, the expressions $x + 9$ and $4x$ are *equal* when x is 3, but they are not equal for any other values of x.



Other times, two expressions seem to be equal no matter what the value of the variable is, such as $3x + 4x$ and $5x + 2x$. When x is 3, both expressions equal 21. When x is 10, both expressions equal 70. In fact, these expressions will be equal for any value of x, because they are ***equivalent expressions***.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Use the grids below to draw a diagram of $x + 3$ and a diagram of $2x$ for each given value of x. Remember to line up the diagrams on one side.

a x is 1

b x is 2

c x is 3

d x is 4

e When are $x + 3$ and $2x$ equal? *Not* equal? Use your diagrams to explain.

Practice

Name: ______________ Date: ______ Period: ______

2. Select *all* the expressions that are *equivalent* to $3b$.

A. $b + b + b$

B. $b \div \frac{1}{3}$

C. $b \cdot b \cdot b$

D. $b + 3$

E. $2b + b$

3. Are the two expressions in the equation $2b + 5 + 1b = 3b + 5$ *equal* or *equivalent*? How do you know?

4. For each story problem, write an equation to represent the problem and then solve the equation. Be sure to explain the meaning of any variable you use.

a Jada's dog was $5\frac{1}{2}$ in. tall when it was a puppy. Now her dog is $14\frac{1}{2}$ in. taller. How tall is Jada's dog now?

b Lin picked $9\frac{3}{4}$ lb of apples, which was 3 times the weight of the apples Andre picked. How many pounds of apples did Andre pick?

5. Calculate each product.

a $2.3 \cdot 1.4$

b $1.72 \cdot 2.6$

c $18.2 \cdot 0.2$

d $15 \cdot 1.2$

6. Identify the property demonstrated by the equation $x + (3 + 5) = (x + 3) + 5$.

Name: ________ Date: ________ Period: ________

Unit 6 | Lesson 11

Equal and Equivalent (Part 2)

Let's use what we know about operations to decide whether two expressions are equivalent.

Warm-up Associative or Commutative Properties With Variables

Identify the property or properties shown in each equation.

a. $12 + x = x + 12$

b. $(3 \cdot 5) \cdot x = 3 \cdot (5 \cdot x)$

c. $3 + (x + 5) = x + 8$

d. $12 \cdot x = x \cdot 12$

e. $2b + 5 + 3b = 5 + 5b$

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Experimenting With Expressions and Values

1. Han was given the two expressions $5 + x$ and $x + 5$. He tried substituting 1, 2, 3, and $1{,}273{,}673$ for x into both expressions. He was shocked by what he saw. What did he see?

2. Han was then given another pair of expressions, $2y$ and $y + 3$. Again, he substituted 1, 2, and 3, and $1{,}273{,}673$ for y into both expressions. Did Han have the same reaction that he had in Problem 1?

3. You are now given the e$x$pressions $2z + 4z$ and $5z + z$. Choose some values to substitute into the expressions and record your findings. Is your reaction like Han's in Problems 1 or 2?

Activity 2 Making Equivalent Expressions

Write an equivalent expression for each given expression.

If I have	. . . you have
$2x \cdot 5$	
$(a \cdot 5) \cdot 2$	
$3b + 5b$	
$2q + 7 + q$	
$5y$	
$9 + (c + 3)$	
$5m + 4n + 3m$	

Reflect: Were you able to be optimistic when using the guess-and-check strategy to see if expressions are equivalent? Why or why not?

Are you ready for more?

For each question, decide whether you think the expressions are equivalent. Test your guess by choosing values for x and/or y.

1. $\frac{x \cdot x \cdot x \cdot x}{x}$ and $x \cdot x \cdot x$
2. $\frac{x + x + x + x}{x}$ and $x + x + x$
3. $2(x + y)$ and $2x + 2y$
4. $2xy$ and $2x \cdot 2y$

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You explored how to determine whether two expressions were equivalent. It would be impossible to test every possible value for a variable to know whether two expressions are always equal. So, how can you know for sure if two expressions are equivalent? One way is to use the meaning of operations and the properties of operations to show how the expressions are equivalent, because they can be rewritten or evaluated to look identical.

Property	Example
Associative Property of Addition	$(x + 4) + 6 = x + (4 + 6)$ $= x + 10$
Associative Property of Multiplication	$(x \cdot 4) \cdot 6 = x \cdot (4 \cdot 6) = 24x$
Commutative Property of Addition	$y + 4 = 4 + y$
Commutative Property of Multiplication	$y \cdot 4 = 4 \cdot y$

Reflect:

Practice

Name: Date: Period:

1. a Do $4x$ and $15 + x$ have the same value when x is 5?

   b Are $4x$ and $15 + x$ equivalent expressions? Explain your thinking.

2. a Check that $2b + b$ and $3b$ have the same value when b is 1, 2, and 3.

   b Do $2b + b$ and $3b$ have the same value for all the given values of b?

   c Are $2b + b$ and $3b$ equivalent expressions? Explain your thinking.

3. Create your own equivalent expressions. What property helps you know that they are equivalent?

   a

   Property:

   b

   Property:

Name: ______ Date: ______ Period: ______

Practice

4. Clare said the value of g is the same in the following statements because they both include 30%, 90, and g. Do you agree or disagree? Explain your thinking.

30% of 90 is equal to g. 30% of g is equal to 90.

5. A television has a length of ℓ in., a width of 28 in., and an area of $1{,}358$ in^2. Select *all* the equations that represent the relationship between the side lengths and area of the television.

A. $\ell \bullet 1{,}358 = 28$

B. $28 \bullet \ell = 1{,}358$

C. $1{,}358 \div 28 = \ell$

D. $28 \bullet 1{,}358 = \ell$

E. $1{,}358 \div \ell = 28$

6. Explain how evaluating the expression $4(200 + 90 + 2)$ can help you to mentally calculate the product of $4 \bullet 292$.

Name: .... Date: .... Period: ....

Unit 6 | Lesson 12

The Distributive Property (Part 1)

Let's use rectangles to understand the Distributive Property with variables.

Warm-up Number Talk

Calculate the product mentally. Be prepared to explain your thinking.

$5 \cdot 98$

Log in to Amplify Math to complete this lesson online.

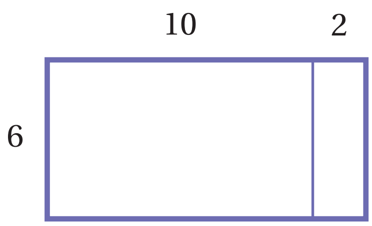
Name: ________ Date: ________ Period: ________

Activity 1 Representing the Area of Joined Rectangles

Part 1

Write an expression that represents the area of the largest, outlined rectangle in Figure A. Explain your thinking.

Figure A



Expression: ________

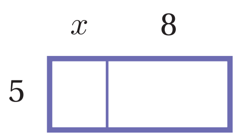
Compare and Connect: You will compare your expressions and approaches with another pair of students. Be ready to explain how you wrote your expression.

Activity 1 Representing the Area of Joined Rectangles (continued)

Part 2

Write as many expressions as you can for the total area of the largest, outlined rectangle in figure B. Explain your thinking.

Figure B

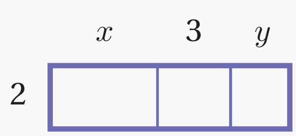


Expression: ....

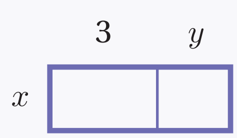
Are you ready for more?

Write an expression to represent the total area of each rectangle.

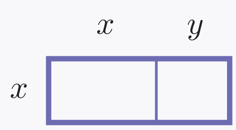
a



b



c

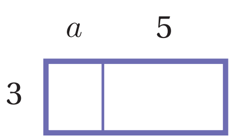


Name: ______ Date: ______ Period: ______

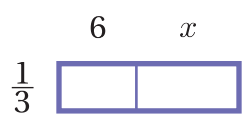
Activity 2 Determining Areas of Partitioned Rectangles

For each rectangle, write expressions for the length and width, and two expressions for the total area of the largest, outlined rectangle. Record them in the table.

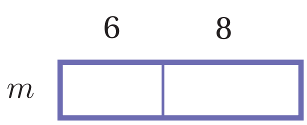
Rectangle A



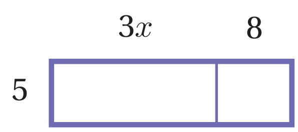
Rectangle B



Rectangle C



Rectangle D



Rectangle	Length	Width	Area as a product of width and sum of the lengths	Area as the sum of the areas of the smaller rectangles
A				
B				
C				
D				

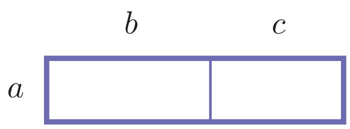
Choose one rectangle. Assign a value to the variable, and evaluate both expressions to determine the total area. Repeat with a new value for the variable. What do you notice?

STOP

Summary

In today's lesson . . .

You explored how to determine the total area of a partitioned rectangle, by using the Distributive Property to write and evaluate two different expressions: $a(b + c)$ and $ab + ac$.



- $a(b + c)$ represents the total area by multiplying the width, a, by the sum of the lengths, $b + c$.
- $ab + ac$ represents the total area by adding the areas of the smaller, partitioned rectangles.

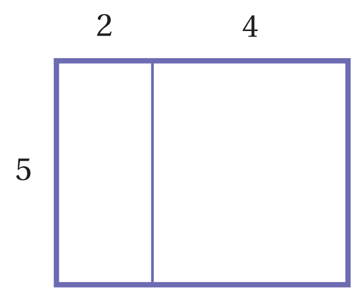
The expressions are equivalent because they refer to the same area. No matter what value you substitute for a variable, the total area is the same.

Reflect:

Name: ______________ Date: ______________ Period: ______________

1. Select *all* of the expressions that represent the area of the largest, outlined rectangle in Figure A.

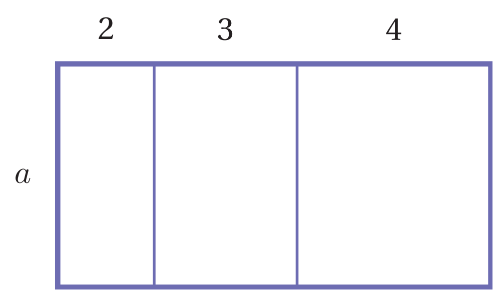
Figure A



- **A.** $5(2 + 4)$
- **B.** $5 \cdot 2 + 4$
- **C.** $5 \cdot 2 + 5 \cdot 4$
- **D.** $5 \cdot 2 \cdot 4$
- **E.** $5 + 2 + 4$
- **F.** $5 \cdot 6$

2. Refer to Figure B.

Figure B

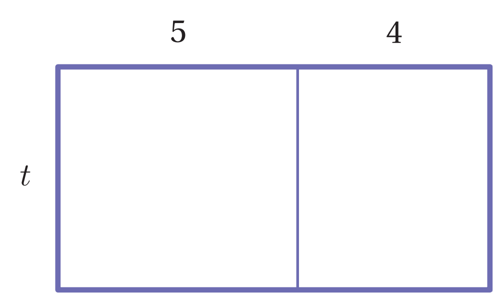


a Explain why the area of the larger, bold rectangle is $2a + 3a + 4a$.

b Explain why the area of the larger, bold rectangle is $(2 + 3 + 4)a$.

3. Choose the expressions that do *not* represent the total area of the larger, bold rectangle in Figure C. Select *all* that apply.

Figure C



- **A.** $5t + 4t$
- **B.** $t + 5 + 4$
- **C.** $9t$
- **D.** $4 \cdot 5 \cdot t$
- **E.** $t(5 + 4)$

Name: .... Date: .... Period: ....

4. Consider the statement: 120% of x is equal to 78.

a Write an equation that shows the relationships between 120%, x, and 78.

b Use your equation to determine x.

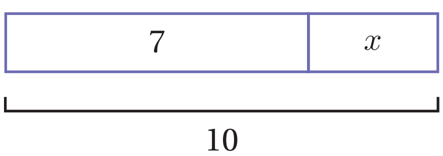
5. Kiran's aunt is 17 years older than Kiran.

a How old will Kiran's aunt be when Kiran is 15 years old?

b How old will Kiran's aunt be when Kiran is x years old?

c How old will Kiran be when his aunt is 60 years old?

6. What is the value of x in the rectangle shown? Explain your thinking.



Name: ________ Date: ________ Period: ________

Unit 6 | Lesson 13

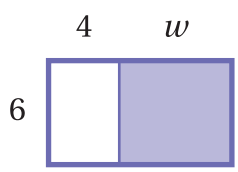
The Distributive Property (Part 2)

Let's practice writing equivalent expressions using the Distributive Property.

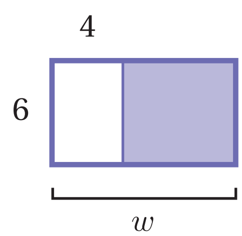
Warm-up Notice and Wonder

Refer to these rectangles. What do you notice? What do you wonder?

Rectangle A



Rectangle B



1. I notice . . .

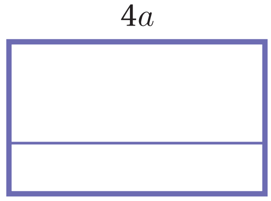
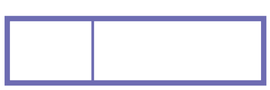
2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Drawing Partitioned Rectangles and Writing Equivalent Expressions

Complete the table. Each row should have:

- **An expression that shows the length and width of its partitioned rectangle.**
- **A partitioned rectangle whose area and labels match the expression. If the rectangle has been started, label any missing dimensions.**
- **An equivalent expression for the rectangle's total area.**

Expression	Rectangle	Equivalent expression
$2(12 + 4)$		
	8 	$24 + 8$
$2(2a + 10)$		
	5 	$4a + 20$
$a(4 + 8)$		
$4a + 8a$	$4a$ 2 1 	
$3(2a + 4b)$		
	6 	$6a + 12b$

Name: ________________ Date: ________ Period: ________

Activity 2 Writing Equivalent Expressions Using the Distributive Property

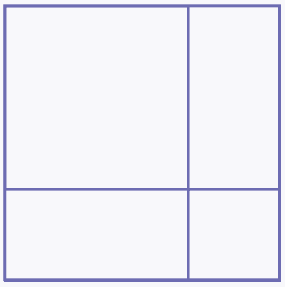
With a partner, use the Distributive Property to write an equivalent expression for each row, and explain your thinking to your partner. Take turns selecting which row to work on next. Consider drawing a diagram to help with your thinking.

Sum or difference	Product
	$(5 + 6)b$
$10x - 5$	
$4x + 7x$	
$\frac{1}{2}x - 3$	
$6c + 2c$	
$20p - 10z$	
$x + 2x + 3x$	
	$y(3x + 4z)$

Are you ready for more?

The total area of the largest, outlined rectangle shown can be written as $(x + 2)^2$, where the length is $x + 2$, and the width is also $x + 2$.

1. Label the length and width of the largest, outlined rectangle.
2. Determine the area of each smaller rectangle.
3. Using the areas of the smaller rectangles, write another expression to represent the total area of the largest, outlined rectangle.



STOP

Summary

In today's lesson . . .

You saw how the Distributive Property can be used to write a sum or difference as a product: $a(b + c)$ or $a(b - c)$. It can also be used to write a product as a sum or difference: $ab + ac$ or $ab - ac$.

When you are given an expression in one form, you can write an equivalent expression in the other form by drawing a partitioned rectangle, where a is the width, and b and c are the lengths.

To write an equivalent expression for $a(b + c)$, multiply a by both b and c to get $ab + ac$. This is known as distributing a to both b and c. For example, to write an equivalent expression for $2(3 + 4)$, multiply 2 by both 3 and 4 to get $(2 \cdot 3) + (2 \cdot 4)$, or $6 + 8$.

To write an equivalent expression for $ab + ac$, divide ab and ac by their common factor, a. The equivalent expression is $a(b + c)$. For example, to write an equivalent expression for $(3 \cdot 4) + (3 \cdot 5)$, which is $12 + 15$, divide 12 and 15 by the common factor 3. The equivalent expression is $3(4 + 5)$.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. For each expression, use the Distributive Property to write an equivalent expression.

   a. $4(x + 2)$

   b. $(6 - 8) \cdot x$

   c. $8x + 12$

   d. $6x - 9y$

   e. $6x - 4y - 2z$

2. Select *all* the expressions that are equivalent to $16x + 36$.

   A. $16(x + 20)$

   B. $x(16 + 36)$

   C. $4(4x + 9)$

   D. $2(8x + 18)$

   E. $2(8x + 36)$

3. Priya rewrites the expression $8y - 24$ as $8(y - 3)$. Han rewrites $8y - 24$ as $2(4y - 12)$. Are Priya and Han's expressions both equivalent to $8y - 24$? Explain your thinking.

Practice

Name: ________ Date: ________ Period: ________

4. Solve each equation. Show your thinking.

a. $10 = 4a$

b. $5b = 17.5$

c. $1.036 = 10c$

d. $0.6d = 1.8$

e. $15 = 0.1e$

5. Select *all* the expressions that are equivalent to the expression $\frac{1}{2}z$.

A. $z + z$

B. $z \div 2$

C. $z \cdot z$

D. $\frac{1}{4}z + \frac{1}{4}z$

E. $2z$

6. Clare said that both of the following expressions can be represented by writing 10^3. Do you agree or disagree? Explain your thinking.

$$10 + 10 + 10 \qquad 10 \cdot 10 \cdot 10$$

Name: ______________________ Date: __________ Period: __________

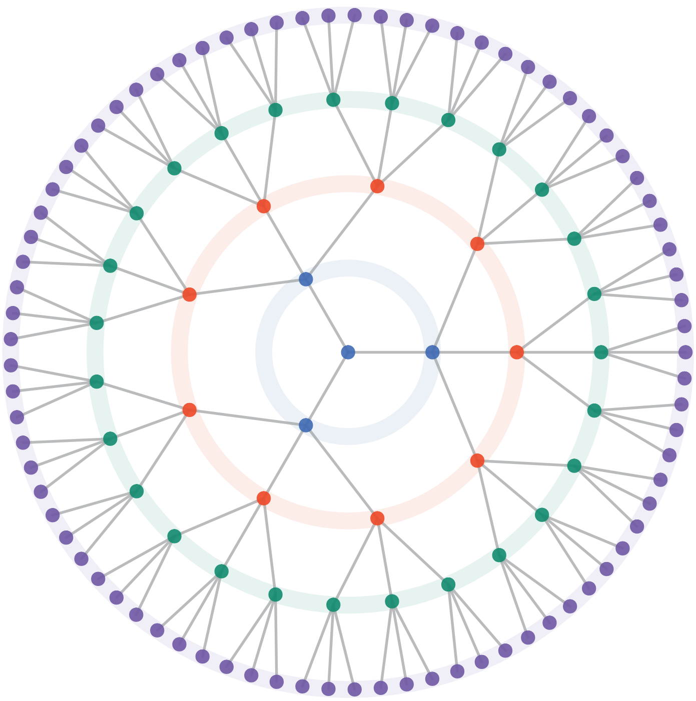
Unit 6 | Lesson 14

Meaning of Exponents

Let's see how exponents show repeated multiplication.

Warm-up Notice and Wonder

Consider the image. What do you notice? What do you wonder?



1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Fundraising Outreach for the Animal Shelter

You want to raise money for the local animal shelter. Your first step is outreach — spreading the word to as many people as you can. You create three outreach options:

- **Option 1:** Post one message on social media that will instantly reach 50,000 people.
- **Option 2:** Email 2 different people each day for 28 days.
- **Option 3:** Start a chain email, in which each person forwards your email to 2 new people. On the first day, you email 2 people. On the second day, those 2 people both forward your email to 2 new people, reaching a total of 4 people. This will continue for 28 days.

Plan ahead: How does conducting a fundraiser demonstrate empathy for others?

1. Which outreach option would you choose? Explain your thinking.

2. Priya claims you can use the expression $2 + 2 + 2 + 2 + 2$ to determine the total number of people who have received the chain email on the fifth day. Do you agree or disagree? Explain your thinking.

3. Write an expression to represent the number of new people the chain email will reach on Day 28. Do not evaluate. Explain your thinking.

Name: .... Date: .... Period: ....

Activity 2 Fundraising Success?

The animal shelter wants to raise $\$50{,}000$ in 10 days. Having seen how powerful a chain email can be, you decide to send an email to three friends, asking them each to donate $\$1$, and then forward the email and donation request to three more friends, and so on. Assume that each friend donates the requested amount.

1. The number of dollars raised on the third day will be $3 \cdot 3 \cdot 3$.

a. Write an equivalent expression using exponents.

b. Evaluate the expression. What does this value mean?

2. You will have raised $\$81$ on the fourth day. Show or explain how you can use this information to calculate the amount of money raised on the fifth day.

3. If you have raised $\$2{,}187$ on one day, what day was it? Write an expression with an exponent to help explain your thinking.

4. Will you reach the $\$50{,}000$ fundraising goal by the tenth day? Show or explain your thinking.

STOP

Summary

In today's lesson . . .

You saw how exponents represent repeated multiplication. When you write an expression, such as 2^n, 2 is the base, and n is the exponent. If n is a positive whole number, it tells you how many factors of 2 you should multiply to determine the value of the expression. For example, $2^1 = 2$, and $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$.

There are different ways to say 2^5. You can say "two raised to the power of five," "two to the fifth power," or just "two to the fifth."

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. Select *all* the expressions that are equivalent to 64.

   A. 2^6

   B. 2^8

   C. 4^3

   D. 8^2

   E. 16^4

   F. 32^2

2. Select *all* the expressions that are equivalent to 3^4.

   A. 7

   B. 4^3

   C. 12

   D. 81

   E. 64

   F. 9^2

   G. $3 + 3 + 3 + 3$

   H. $3 \cdot 3 \cdot 3 \cdot 3$

3. 4^5 is equal to $1,024$. Use this to evaluate each expression.

   a. 4^6

   b. 4^4

   c. $4^3 \cdot 4^2$

Practice

Name: ________ Date: ________ Period: ________

4. Solve each equation. Show your thinking.

a $a - 2.01 = 5.5$

b $b + 2.01 = 5.5$

c $10c = 13.71$

d $100d = 13.71$

5. Which expressions represent the total area of the largest, outlined rectangle? Select *all* that apply.

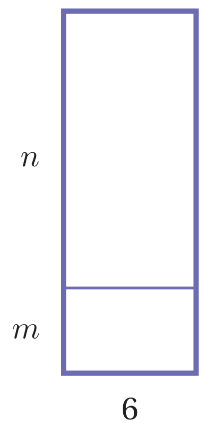
A. $6(m + n)$

B. $6n + m$

C. $6n + 6m$

D. $6mn$

E. $(n + m) \cdot 6$



6. Using what you know about the properties of operations, rewrite each expression using a different operation. Do not evaluate.

a $3 + 3$

b $3 \cdot 2$

c 3^2

Name: ________ Date: ________ Period: ________

Unit 6 | Lesson 15

Evaluating Expressions With Exponents

Let's determine the values of expressions with exponents.

Warm-up Is the Equation True?

Without evaluating, decide whether each equation is true or false. Be prepared to explain your thinking.

a. $2^4 = 2 \cdot 4$

b. $3 + 3 + 3 + 3 + 3 = 3^5$

c. $5^3 = 5 \cdot 5 \cdot 5$

d. $2^3 = 3^2$

e. $14^1 = 7^2$

Log in to Amplify Math to complete this lesson online.

Activity 1 Revisiting Your Fundraising Strategy

A local radio station was so impressed with your efforts to raise money for the local animal shelter, they offered to help in one of four possible ways:

- **Offer 1:** Donate $2 on the first day and then double the amount every day for 6 days. On the seventh day, they will give you an extra $100.
- **Offer 2:** Donate $2 on the first day and then double the amount every day for 9 days. On the tenth day, they will give $100 from the money they raised to a listener in a raffle.
- **Offer 3:** Donate $2 on the first day and then double the amount every day for 7 days. On the eighth day, they will triple the total amount up to that point.
- **Offer 4:** Donate $50 on the first day and and then donate $2 every day, for 20 days.

Which offer would you choose? Use expressions or equations to explain your thinking.

Name: ________ Date: ________ Period: ________

Activity 2 Are They Equal?

Determine whether the expressions in each row are equivalent. Be prepared to share your thinking with your partner.

Column A	Column B	Equivalent?
$5^2 + 4$	$2^2 + 25$	
$2^4 \cdot 5$	$2^3 \cdot 10$	
$3 \cdot 4^2$	$12 \cdot 2^1$	
$5 + 4^3$	9^3	
$9 \cdot 2^1$	$3 \cdot 6^1$	
$(6 + 2)^2$	$6^2 + 2^2$	

STOP

Summary

In today's lesson . . .

You explored evaluating expressions with exponents. To ensure everyone evaluates expressions, such as $6 \cdot 4^2$, in the same way, and gets the same answer, the convention is to evaluate the part of the expression with the exponent first.

$$\begin{aligned} 6 \cdot 4^2 &= 6(4 \cdot 4) \\ &= 6 \cdot 16 \\ &= 96 \end{aligned}$$

If you want to communicate that 6 and 4 should be multiplied first, and then squared, you should use parentheses to group parts together: $(6 \cdot 4)^2$.

$$\begin{aligned} (6 \cdot 4)^2 &= 24^2 \\ &= 24 \cdot 24 \\ &= 576 \end{aligned}$$

Reflect:

Name: ____________ Date: ________ Period: ________

Practice

1. Evaluate each expression. Show your thinking.

a $7 + 2^3$

b $9 \cdot 3^1$

c $20 - 2^4$

d $2 \cdot 6^2$

e $8 \cdot \left(\frac{1}{2}\right)^2$

f $\left(\frac{1}{5} \cdot 5\right)^5$

2. Lin says, "I took the number 8, and then multiplied it by the square of 3." Select *all* the expressions that are equal to Lin's result.

A. $8 \cdot 3^2$

B. $(8 \cdot 3)^2$

C. $8 \cdot 2^3$

D. $3^2 \cdot 8$

E. 24^2

F. 72

Practice

Name: ______ Date: ______ Period: ______

3. Andre says, "I multiplied 4 by 5 and then cubed the result." Select *all* the expressions that are equal to Andre's result.

   A. $4 \cdot 5^3$
   B. $(4 \cdot 5)^3$
   C. $(4 \cdot 5)^2$
   D. $5^3 \cdot 4$
   E. 20^3
   F. 500
   G. $8{,}000$

4. Solve each problem. Show or explain your thinking.

   a. 125% of e is 30. What is e?

   b. 35% of f is 14. What is f?

5. Which expressions have values that are solutions to the equation $2.4y = 13.75$? Select *all* that apply.

   A. $13.75 - 2.4$
   B. $13.75 \cdot 2.4$
   C. $13.75 \div 2.4$
   D. $\frac{13.75}{2.4}$
   E. $2.4 \div 13.75$

6. Evaluate each expression when $x = 2$.

   a. $3 + x$

   b. $3 \cdot x$

   c. x^3

   d. 3^x

Name: ____________ Date: ________ Period: ________

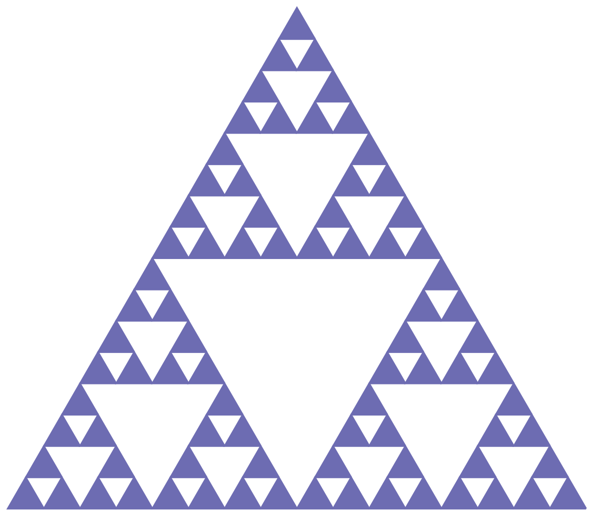
Unit 6 | Lesson 16

Analyzing Exponential Expressions and Equations

Let's investigate expressions and equations with variables and exponents.

Warm-up The Sierpiński Triangle

Complete the column in the table for x^3 using the different values of x. Then, as you watch the animation, follow along to complete the column for 3^x, which represents the number of triangles in the animation. What do you notice about the two columns?



x	x^3	3^x
1		
2		
3		
4		

Log in to Amplify Math to complete this lesson online.

Activity 1 Twenty Questions: Evaluating Expressions With Variables and Exponents

You will given two sets of cards – Expression Cards and Value Cards. With your partner, determine who will be Partner A and Partner B. Partner A starts with an expression card, and Partner B starts with a value card. Do not show or read your card to your partner. The partner with the value card will ask questions to determine the expression. Then, if possible, evaluate the expression together by substituting the information from the value card.

Information gathered	Expression	Solution		
		Value card:		
		Value card:		
		Value card:		
		Value card:		
		Value card:		
		Value card:		
		Value card:		

Sample Questions Bank:
What is the coefficient of the variable term?
Is something added to the variable?
Is the variable an exponent, a base, or neither?

Name: ______________ Date: ______ Period: ______

Activity 2 Experimenting With Exponents in Equations

Determine a solution to each equation from the list of numbers. (Numbers in the list may be a solution to more than one equation, and not all numbers in the list will be used.)

2	4	5	$\frac{4}{3}$	6	8	$\frac{8}{125}$	3	$\frac{8}{9}$

1. $64 = x^2$

2. $64 = 4^x$

3. $2^x = 32$

4. $x = \left(\frac{2}{5}\right)^3$

5. $\frac{16}{9} = x^2$

6. $2 \cdot 2^5 = 2^x$

7. $2x = 2^4$

8. $4^3 = 8^x$

Critique and Correct: Describe and correct any errors in this statement: "The solution to the equation $x = \left(\frac{2}{5}\right)^3$ is $\frac{6}{15}$ because $\left(\frac{2}{5}\right)^3$ is equal to $\frac{2}{5} \cdot 3$."

STOP

Summary

In today's lesson . . .

You saw expressions with exponents that had a variable as either the base or the exponent. You evaluated these expressions using the *order of operations,* which meant evaluating the exponent term before any other operations outside of any parentheses. You also used this order of operations when evaluating the expressions on one or both sides of an equation with a variable, such as x. This allowed you to determine a *solution* — what value of x made the equation true. For example, to evaluate the expression $2x^3$ when x is 5, you replace the letter x with 5 to get $2 \cdot 5^3$. This is equal to $2 \cdot 125$ or 250.

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. Evaluate each expression for the given value of each variable.

a. $2 + x^3$, when x is 3

b. x^2, when x is $\frac{1}{2}$

c. $3x^2 + y$, when x is 5 and y is 3

d. $10y + x^2$, when x is 6 and y is 4

2. Decide whether the expressions have the same value. If not, determine which expression has the greater value.

a. 2^3 and 3^2

b. 1^{31} and 31^1

c. 4^2 and 2^4

d. $\left(\frac{1}{2}\right)^3$ and $\left(\frac{1}{3}\right)^2$

3. Match each equation with its solution.

a. $7 + x^2 = 16$ ______ $x = 1$

b. $5 - x^2 = 1$ ______ $x = 2$

c. $2 \cdot 2^3 = 2^x$ ______ $x = 3$

d. $\frac{3^4}{3^x} = 27$ ______ $x = 4$

Name: .... Date: .... Period: ....

4. An adult pass at the amusement park costs 1.6 times as much as a child's pass.

a Complete the table.

Child's pass ($)	5	10	w
Adult pass ($)			

b If a child's pass costs $\$15$, how much does an adult pass cost?

5. Jada reads 5 pages every 20 minutes. At this rate, how many pages can she read in 1 hour? Show your work using the double number line and the table.

Double number line:

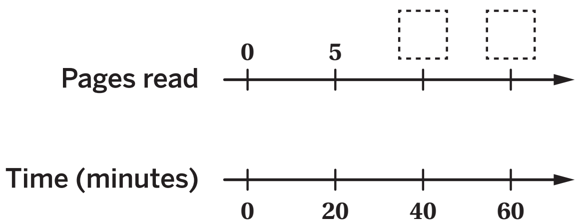


Table:

Pages read	Time (minutes)
5	20

Which strategy do you think is better, and why?

6. Han works at a grocery store. Last week, he worked 8 hours and earned 84.

a Han always earns the same amount per hour worked. Determine the missing values in the table.

Hours	8	2	
Pay ($)	84		105

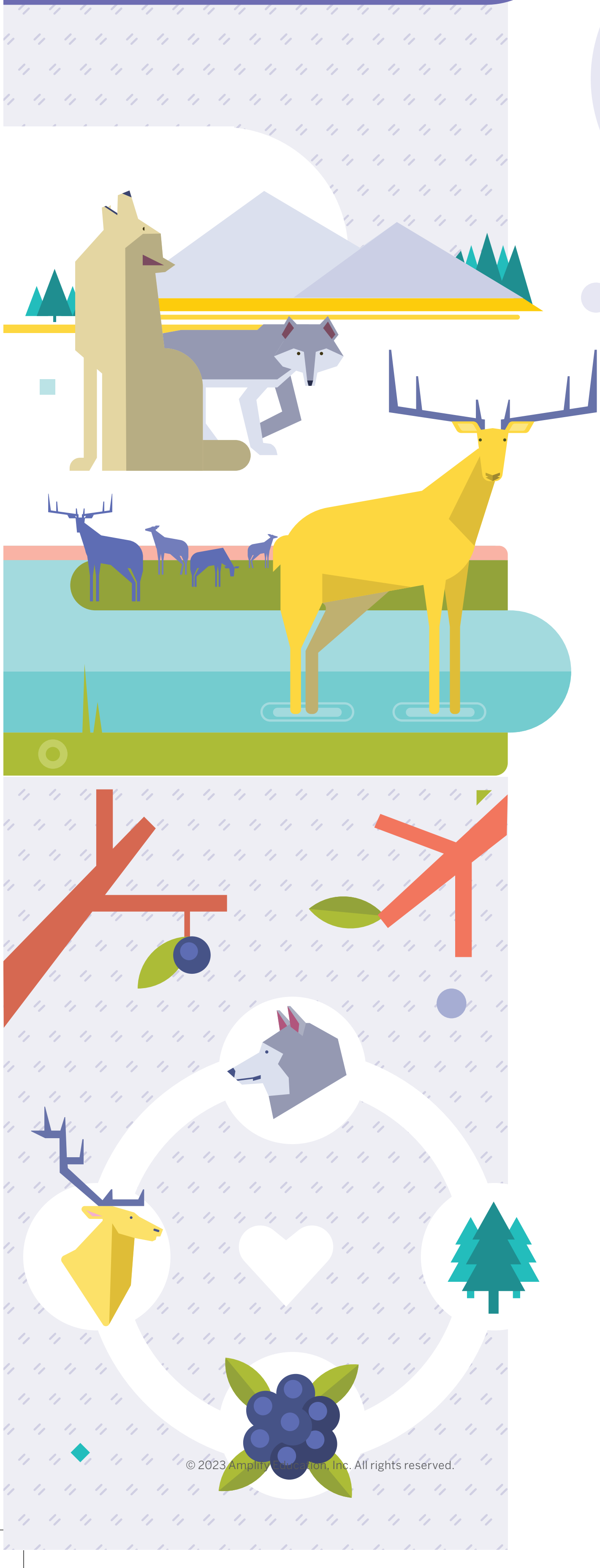
b What is the relationship between the number of hours worked and Han's pay?

SUB-UNIT

3 Relationships Between Quantities

Narrative Connections

What's more dangerous: a pack of wolves or a gang of elk?



Let's go back to the early 20th century, when the U.S. government allowed hunting and killing of "undesirable predators," namely wolves, in its national parks. So many wolves were killed, that, by 1926, there wasn't a single one left in Yellowstone National Park.

So, where do elk come in?

As it turns out, the elk's greatest predator was the gray wolf. With the wolves gone, the elk were free to multiply. And multiply. And multiply. Soon, Yellowstone had more elk than you could shake an antler at.

The elk ate everything in sight — grass, shrubs, and saplings. This meant no grazeland for bison, no berries for grizzlies, no trees for beavers or birds. The whole ecosystem was out of whack.

Finally, in 1995, the National Parks Service reintroduced wolves back into the park. Since then, the park has seen remarkable changes. Willow, aspen, and cottonwood trees have grown back, and animal populations are rebounding.

A healthy ecosystem is a balanced ecosystem — its different pieces pushing and pulling on each other: the predators, the prey, the fruiting plants, the shade-giving trees . . . When we understand how these pieces work together, we can make meaningful choices that support the system — deciding what to plant, what to nourish, and what species to protect.

Unit 6 | Lesson 17

Two Related Quantities (Part 1)

Let's use equations and graphs to describe relationships with ratios.

Warm-up Increases and Decreases

1. What are two real-world quantities you think might increase together or decrease together? In other words, as the amount of one goes up, the amount of the other also goes up; or both amounts go down.

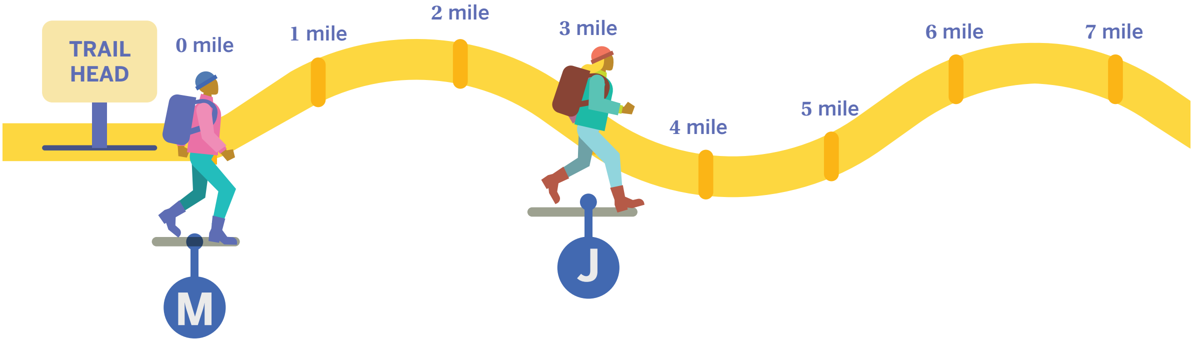
2. What are two real-world quantities you think might increase and decrease opposite of one another? In other words, as the amount of one goes up, the amount of the other goes down.

Log in to Amplify Math to complete this lesson online.

Name: ____________ Date: ____________ Period: ____________

Activity 1 Introducing Equations With Two Variables

Jada and Mai hiked the same trail in Yellowstone National Park. They walked at the same speed, but started at different locations; Jada started at mile marker 3, while Mai started at the beginning of the trail (mile marker 0).



1. If Jada's location on the trail is represented by J, and Mai's location on the trail is represented by M, what are two equations that can represent this relationship?

   Equation 1: $J =$ ____________

   Equation 2: $M =$ ____________

2. Complete these sentences. Be prepared to explain your thinking.

   a In Equation 1, you need to know the value of ______ to determine the value of ______, so ______ is called the ____________ variable. ______ is called the ____________ variable because it is used to calculate the value of ______.

   b In Equation 2, which is the ***independent variable*** — the variable you use to perform the calculation? ______

   Which is the ***dependent variable*** — the variable you determine after performing the calculation? ______

Are you ready for more?

If you did not know the mile marker where Jada started hiking, the equation could be written as $M + s = J$, where s represents where Jada started. Solve the equation to determine where Jada started, if she was at mile marker 15 when Mai was at mile marker 7.

Activity 2 Bear Populations In Yellowstone National Park

Did you know that the Greater Yellowstone Ecosystem is one of only a few areas in North America where both black bears and grizzly bears coexist? Their current populations are approximately 600 black bears and 700 grizzly bears.

1. Assuming the ratio of black bears to grizzly bears is always the same, complete the table.

Black bears, (b)	Grizzly bears, (g)	Total bears, (t)
6		
		39
		52
	49	
48		
		130
	700	
		1,950

2. Refer to the table.

   a. Write a fraction that represents the ratio of black bears to total bears.

   b. Write an equation that represents the relationship between the number of black bears b as the dependent variable and the total number of bears t as the independent variable.

3. Write an equation that will always describe the relationship between b and g, where g is the independent variable.

4. Write an equation that will always describe the relationship between b and g, where b is the independent variable.

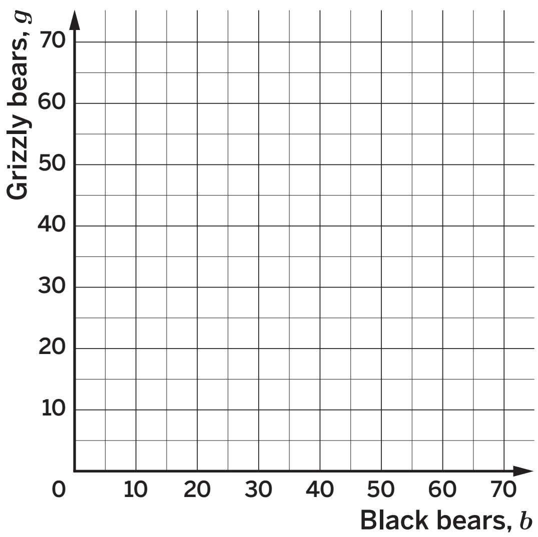
Name: ________ Date: ________ Period: ________

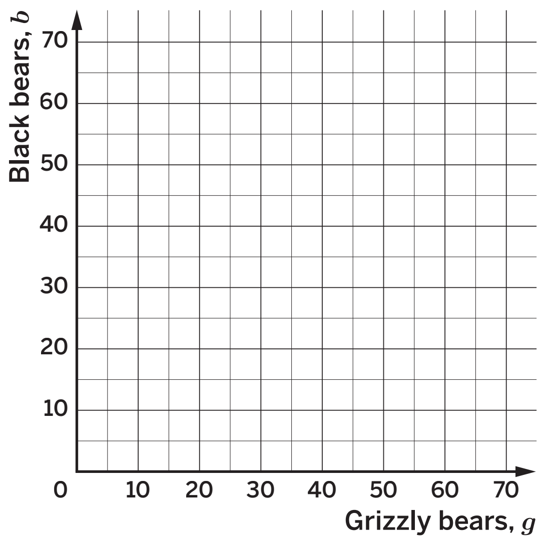
Activity 2 Bear Populations In Yellowstone National Park (continued)

Visual mathematical representations, such as graphs, that model the relationship between two quantities are important tools. They can aid in understanding relationships more deeply but also make it easier to show and explain them to a wider audience. Not all relationships are simple, or happy. For instance Rae Wynn-Grant has studied and modeled many aspects of the Greater Yellowstone Ecosystem, including the mortality risk of bears living in areas near human populations.

Even with the same values, a graph can show different things, depending on choices a researcher makes. You will see a small example of this as you complete Problem 5.

5. Use the points in the table to create two graphs that show the relationship between b and g. Match each relationship to one of the equations you wrote.





Featured Mathematician

Rae Wynn-Grant

Wildlife ecologist Rae Wynn-Grant was born in California and holds bachelor's and master's degrees in Environmental Studies and a PhD in Ecology and Evolution. She is a Research Faculty member at the Bren School of Environmental Science and Management where she conducts research on carnivore behavior and ecology. She has also studied and published work focused on the Greater Yellowstone Ecosystem and carnivores there, such as wolves and bears.

STOP

Summary

In today's lesson . . .

You revisited representing equivalent ratios with tables, and connected those tables to writing equations and graphing the points. You did this with the populations of black bears and grizzly bears in Yellowstone National Park.

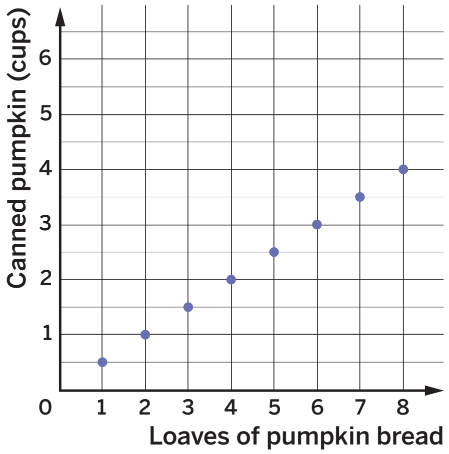
In an equation, the ***dependent variable*** is the one resulting from a calculation, and the ***independent variable*** is the one used to calculate the value of the dependent variable.

When you previously studied equivalent ratios, you used double number lines and tables. Graphs and equations provide other tools for working with equivalent ratios.

Reflect:

Name: ____________ Date: ________ Period: ________

1. This graph shows some values for the number of cups of canned pumpkin p needed to make b loaves of pumpkin bread.



a What does the point $(8, 4)$ mean in terms of the amount of canned pumpkin and number of loaves of bread?

b Complete the table so that the numbers in each column represent the coordinates of a point on the graph.

b	1	2	3	4	5	6	7
p							

c Write an equation that gives the number of cups of canned pumpkin in terms of the number of loaves of bread.

2. Each serving of a certain fruit snack contains 90 calories.

a Han wants to know how many calories he gets from the fruit snacks. Write an equation that shows the number of calories c in terms of the number of servings n.

b Tyler needs some extra calories each day during his sports season. He wants to know how many servings he can have each day if all the extra calories come from the fruit snack. Write an equation that shows the number of servings n in terms of the number of calories c.

Practice

Name: ______________________ Date: ____________ Period: ____________

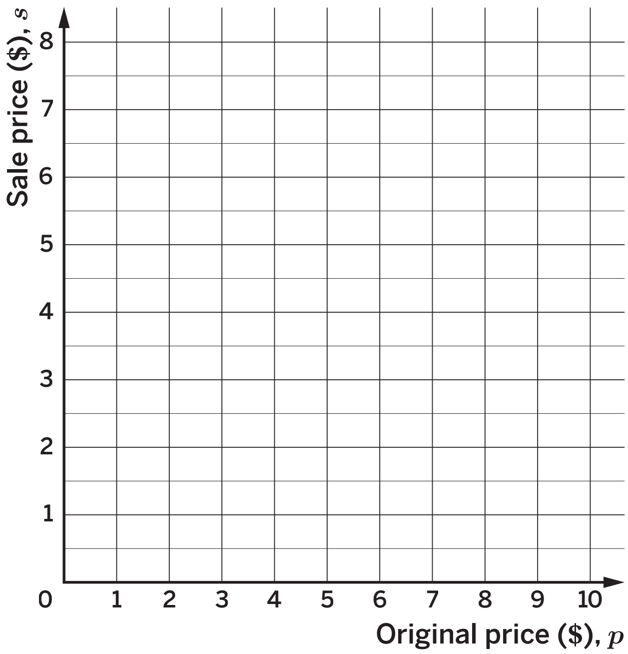
3. Kiran shops for books during a 20% off sale.

a What percent of the original price of a book does Kiran pay during the sale?

b Complete the table to show how much Kiran pays for books during the sale. Write an equation that relates the sale price s to the original price p.

c Plot the points on the graph to show the relationship between the sale price and the original price.

Original price, p ($)	Sale price, s ($)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	



4. Calculate $12.34 \cdot 0.7$. Show your thinking.

5. Han is planning to ride his bike 24 miles. How long will it take him if he rides at a constant speed of 3 mph? Write an equation to represent this scenario, and then solve the equation.

Name: .... Date: .... Period: ....

Unit 6 | Lesson 18

Two Related Quantities (Part 2)

Let's use equations and graphs to describe stories with constant rates.

Warm-up Hiking Around Old Faithful

Lin and Priya each start their hike at 9:00 a.m. They each hike at a steady rate. About what time do they finish their hike?

1. Do you have enough information to solve this problem? If not, what other information would be useful?

2. Lin and Priya are hiking the Upper Geyser Basin and Observation Point Loop, which is a 4.5-mile trail. Lin can hike 10 miles in 5 hours, and Priya can hike 23 miles in 10 hours. If they each leave the trailhead at 9:00 a.m. and hike at a steady rate, at about what times do they finish the trail?

Log in to Amplify Math to complete this lesson online.

Activity 1 Reintroduction of the Grey Wolf

In 1995, the grey wolf was reintroduced into Yellowstone National Park. The effects of the reintroduction amazed scientists, who recorded increases in the populations of everything from vegetation and migrating birds, to other animals, such as beavers, otters, and bears. As a result, the whole landscape of Yellowstone has dramatically changed.

On January 12, 1995, the first of 31 grey wolves were released into the park. That same year, there was only one colony of beavers left (a colony has, on average, 6 beavers), and most willow trees were nothing more than shrubs.

The following table represents how the reintroduction of the grey wolf could result in positive changes for beaver populations and willow tree growth.

1. Complete the table representing the different animal populations and the average height of willow trees at the end of each year, assuming that all the rates of increase are constant each year.

Year	Wolves	Beavers	Average Willow Tree Height (ft)
1		6	
2	62		7
3			
6	186	36	21
8			

2. How much did each population and the average height of willow trees increase per year?

Wolves:
Beavers:
Average height of willow trees:

3. How long does it take for each animal population to increase by 1 animal and for the willow tree height to increase by 1 ft?

Wolves:

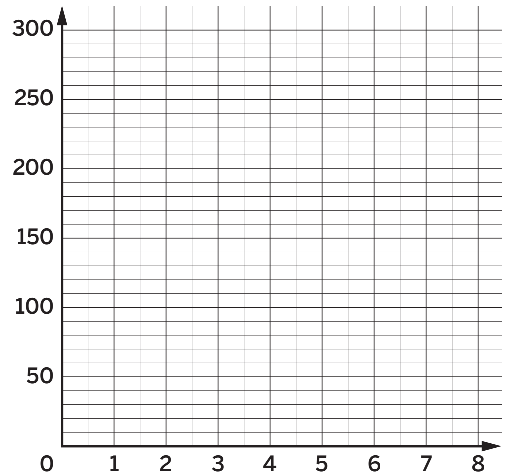
Beavers:

Willow trees:

Name: ______ Date: ______ Period: ______

Activity 1 Reintroduction of the Grey Wolf (continued)

4. Graph the populations or average heights for the wolf, beaver, and willow trees, through 2002, where 1995 corresponds to Year 1 of the program. Use a different color for each of the three quantities.



5. Diego says that the equation $w = 31y$ represents the relationship between the years of the program and the wolf population, where w represents the number of wolves and y represents the number of years.

   a. Explain why the equation $w = 31y$ relates the wolf population to the number of years.

   b. Write equations that relate the beaver population b and the growth of the willow tree t to years of the program y.

   c. Is the independent variable the same in all three equations?

6. Use the equations you wrote to predict the animal and plant growth, if they continued to increase at the same rates, 12 years into the program.

   Wolves:

   Beavers:

   Willow trees:

STOP

Summary

In today's lesson . . .

You used equations and graphs to represent stories that have constant rates. Equations can be used to represent the relationship between two quantities more generally, such as time and distance traveled.

Equations can also help you answer questions like: "How far can a boat travel in 3.25 hours if it is traveling at a constant speed of 25 mph?" or, "How long does it take for the boat to travel 60 miles?" Once you have an equation, if you know the value of one quantity, you can substitute and evaluate to solve for the value of the other quantity.

For example, you can use t to represent the time in hours and d to represent the distance in miles that the boat travels.

When you know the time and want to determine the distance:	When you know the distance and want to determine the time:
You can write $d = 25t$. In this equation, if t changes, d is affected by the change, so t is the independent variable and d is the dependent variable. This equation can help you determine d when you have any value of t. In 3.25 hours, the boat can travel $25(3.25)$ or 81.25 miles.	You can write $t = \frac{d}{25}$. In this equation, if d changes, t is affected by the change so d is the independent variable and t is the dependent variable. This equation can help you determine t for any value of d. To travel 60 miles, it will take $\frac{60}{25}$ or $2\frac{2}{5}$ hours.

Reflect:

Name: .... Date: .... Period: ....

1. A car is traveling down a road at a constant speed of 50 mph.

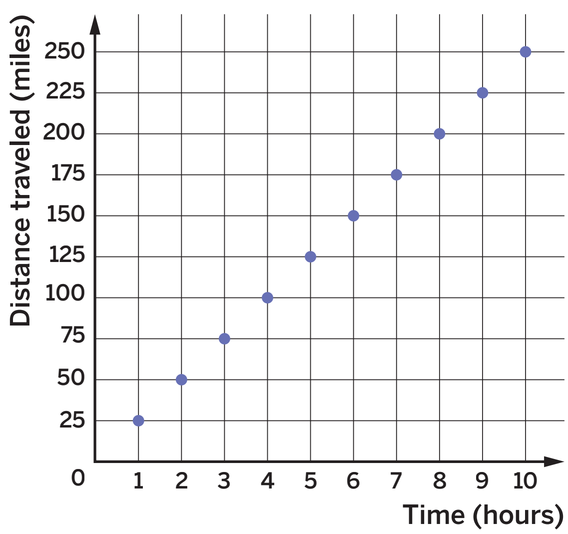
a Complete the table with the amounts of time it takes the car to travel certain distances, or the distances traveled for certain amounts of time.

Time (hours)	Distance (miles)
2	
1.5	
t	
	50
	300
	d

b Write an equation that represents the distance d, traveled by the car in miles, given the time t, in hours.

c In your equation, which is the dependent variable and which is the independent variable?

2. The graph represents the amount of time, in hours, it takes a ship to travel various distances in miles.



a Write the coordinates for one point on the graph. What does the point represent?

b What is the speed of the ship in miles per hour?

c Write an equation that relates the time t it takes to travel a given distance d.

3. Han is planning to ride his bike 24 miles.

a How long will it take if he rides at a rate of 4 mph? 6 mph?

b Write an equation that Han can use to determine the time t will take to ride 24 miles, if his rate in miles per hour is represented by r.

Practice

Name: ______ Date: ______ Period: ______

4. Determine a solution to each equation. Show or explain your thinking.

a $2^x = 8$

b $2^x \cdot 2^3 = 2^7$

c $\frac{2^x}{2^3} = 2^5$

5. Select *all* the expressions that are equivalent to $5x + 30x - 15x$.

A. $5(x + 6x - 3x)$

B. $(5 + 30 - 15) \cdot x$

C. $x(5 + 30x - 15x)$

D. $5x(1 + 6 - 3)$

E. $5(x + 30x - 15x)$

6. Create an example of a balanced hangar diagram using the given figures. You may use each figure more than once. Then write and solve an equation that matches your diagram.

$\boxed{1}$ x

Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 19 – Capstone

Creating a Class Mobile

Let's make a class mobile that represents our class and our unit on expressions and equations.

Warm-up Notice & Wonder

What do you notice about this mobile? What do you wonder?

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Making a Mobile Ornament

Write down a word or brief description of something that represents your identity. Using that as inspiration, sketch an ornament for the class mobile in the space provided. Various materials will be made available to you so that you can create a model of your ornament.

Something that represents me is:

Once you have completed creating your ornament, it will be assigned a variable. Record your variable here.

Name: .... Date: .... Period: ....

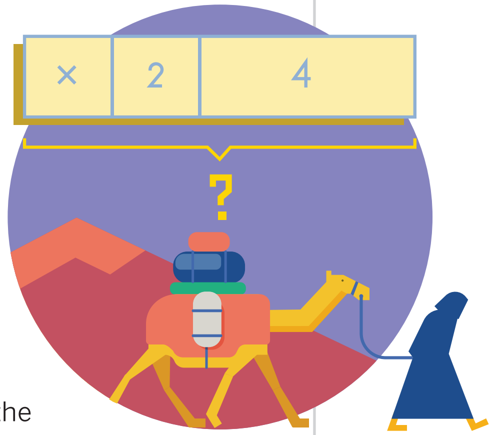
Activity 2 Assembling the Class Mobile

As we build the mobile, make mental notes of when you apply the thinking you learned about expressions and equations in this unit.

Sketch the finished mobile that your class created in the space provided.

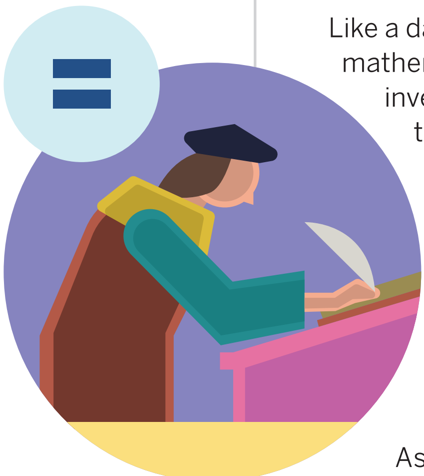
STOP

Unit Summary



When you see it in action, balance can be a work of art. Balance factors into so much of our world.

It provides the basis for how we value what we buy and sell, such as the salt trade in Trans-Saharan Africa. Balance also helps us protect our ecosystems. In fragile habitats, like Yellowstone National Park, the overabundance of one species can have dramatic consequences for everything else that shares that environment. It's only when the plant and animals exist in balance with each other that ecosystems can thrive.

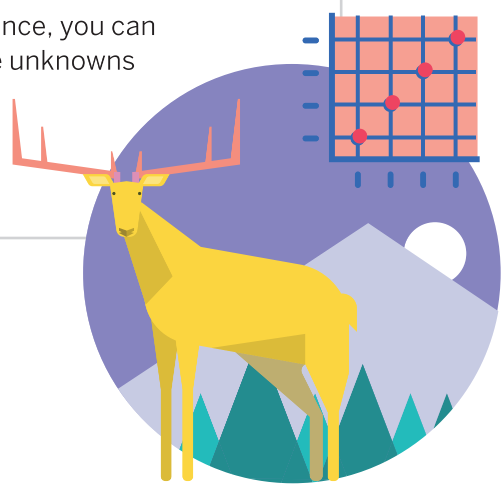


Like a dancer who balances on the tips of their toes, the mathematician balances on the equal sign. It is this very sign, invented by Robert Recorde in the 1550s, that tells us when the two values of an equation are equal.

And it is this "equalness" that gives an equation its balance. Understanding this balance gives you a powerful tool for solving for an equation's unknown values, or variables. All you have to do is remember to maintain that balance. That means anything you do to one side of the equal sign, you have to do to the other side. (Just as you saw with the mobile!)

As long as you keep both sides in balance, you can clear up what you don't need, leaving the unknowns completely revealed.

See you in Unit 7.



Name: .... Date: .... Period: ....

Practice

1. Select *all* the expressions that are equivalent to $4b$.

A. $b + b + b + b$

B. $b + 4$

C. $2b + 2b$

D. $b \cdot b \cdot b \cdot b$

E. $b \cdot \frac{1}{4}$

2. Complete each problem. Show or explain your thinking.

a. 20% of a is 11. What is a?

b. 75% of b is 12. What is b?

c. 80% of c is 20. What is c?

d. 200% of d is 18. What is d?

3. Solve each equation. Show your thinking.

a. $111 = 14a$

b. $13.65 = b + 4.88$

c. $c + \frac{1}{3} = 5\frac{1}{8}$

d. $\frac{2}{5}d = \frac{17}{4}$

e. $5.16 = 4e$

Practice

Name: ______ Date: ______ Period: ______

4. Andre ran $5\frac{1}{2}$ laps around a track in 8 minutes, at a constant speed. It took Andre x minutes to run each lap. Select *all* the equations that represent this situation.

A. $\left(5\frac{1}{2}\right)x = 8$

B. $5\frac{1}{2} + x = 8$

C. $5\frac{1}{2} - x = 8$

D. $5\frac{1}{2} \div x = 8$

E. $x = 8 \div \left(5\frac{1}{2}\right)$

F. $x = \left(5\frac{1}{2}\right) \div 8$

5. Select *all* the expressions that are equal to $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$.

A. $3 \cdot 5$

B. 3^5

C. $3^4 \cdot 3$

D. $5 \cdot 3$

E. 5^3

6. Evaluate each expression. Show your thinking.

a $5 + 3^2$

b $4 \cdot 5^1$

c $15 - 2^3$

d $5 \cdot \left(\frac{1}{2}\right)^3$

e $\left(\frac{1}{4} \cdot 4\right)^4$

My Notes:

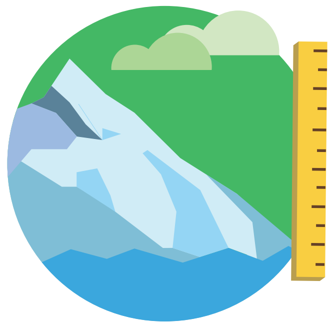
UNIT 7

Rational Numbers

Think back to when you first learned about whole numbers and used them to count. Later, you saw there were numbers between them: fractions and decimals. Up until now, every number you've encountered has always been greater than 0. But no more. There is an entire set of numbers (just as many, in fact), lurking on the *other* side of every number line.

Essential Questions

- What does it mean for a value to be less than zero?
- How can a number be closer to zero and have a greater value?
- How can two objects move the same distance, but end up in different places?
- *(By the way, how many lefts make a right?)*



SUB-UNIT

1 Negative Numbers and Absolute Value

Narrative: Knowing the tallest mountain on Earth depends on whether its height is measured *from sea level* or *from Earth's center.*

You'll learn . . .

- that numbers can be positive or negative.
- about the absolute value of numbers.

SUB-UNIT

2 Inequalities

Narrative: Inequalities, such as $<$, $\leq$, $>$, and $\geq$, can describe real-world scenarios with precision.

You'll learn . . .

- how to write inequality statements to represent real-world scenarios.
- whether negative solutions make sense within context.

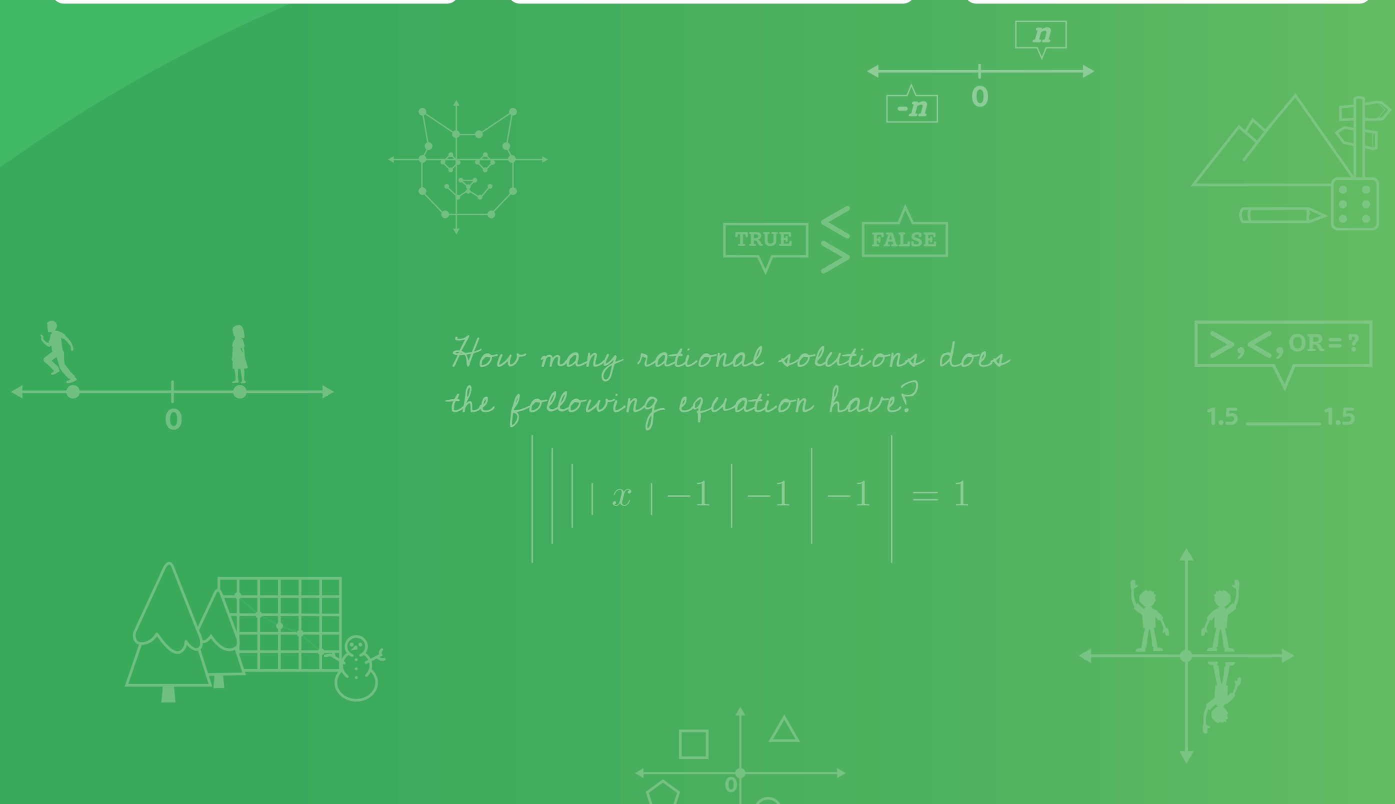
SUB-UNIT

3 The Coordinate Plane

Narrative: Gerardus Mercator introduced the world to using parallel and perpendicular lines plot locations on a map grid.

You'll learn . . .

- about the four quadrants of the coordinate plane.
- how to use absolute value to determine the distance between points.



Unit 7 | Lesson 1 – Launch

How Far? Which Way?

Let's think about magnitude and direction as we move around a flat surface.

Warm-up Hot 100

In small groups, take turns writing down as many words or phrases as you can think of that are related to *magnitude* and *direction*.

- While one person writes, the person to their right will continue to roll a die until a 6 is rolled.
- At that time, the person to their left begins to write. The previous writer will roll the die.
- Continue until your group has 100 words or phrases, or time is called.

Magnitude	Direction

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Ship in the Fog Scavenger Hunt

In your group, decide which person will be the hunter. The remaining group members will be the callers. The callers will be given a card that identifies an object to be located in the classroom.

One at a time, cycling through the same order, each caller will give *one* verbal instruction to guide the hunter toward the object. The hunter can only move in a *vertical* or *horizontal* direction — no diagonal steps!

As time permits, take turns being the hunter. The new callers will be given another card that identifies a new object to hunt.

After each round, record any new words representing either magnitude or direction that your group used to help guide the hunters.

Plan ahead: How can thinking about the hunter's perspective help you give better directions?

Magnitude	Direction

Activity 2 Blindfold Mazes

You and your partner will be given a maze. Keep the maze face down until you are told to turn it over. At that time, place the blindfolded partner's pencil at the Start of the maze and give them directions to help them move through the maze to the Exit. Record the time it takes to reach the Exit.

1. **Maze 1**

   Time: .... *min* .... *sec*

   Once you have successfully completed the first maze, switch roles and repeat with a new maze.

2. **Maze 2**

   Time: .... *min* .... *sec*

STOP

Unit 7 Rational Numbers

Getting Where We're Going

Believe it or not, there are people to this day who still play chess by mail. There are lots of reasons you might want to play by mail. Maybe you prefer having time to think about your move. Maybe you do not live near others who like to play chess. Maybe your opponent happens to have breath like pickled gym socks.

Whatever the reason, these players need a way to coordinate with each other. Thankfully, chess boards are arranged in grids, using letters and numbers. So, to talk about any location on the board, players just use the right letters and numbers.

Grids are fantastic for helping us communicate about where things are. But to get there, you're going to need two things: magnitude and direction.

Magnitude tells us *how far* to travel, while direction tells us *which way* to travel.

By now, you noticed numbers are great for showing magnitude (in miles, in blocks, in captured pawns, in postage stamps, etc.), but is there a way to use numbers to talk about direction as well?

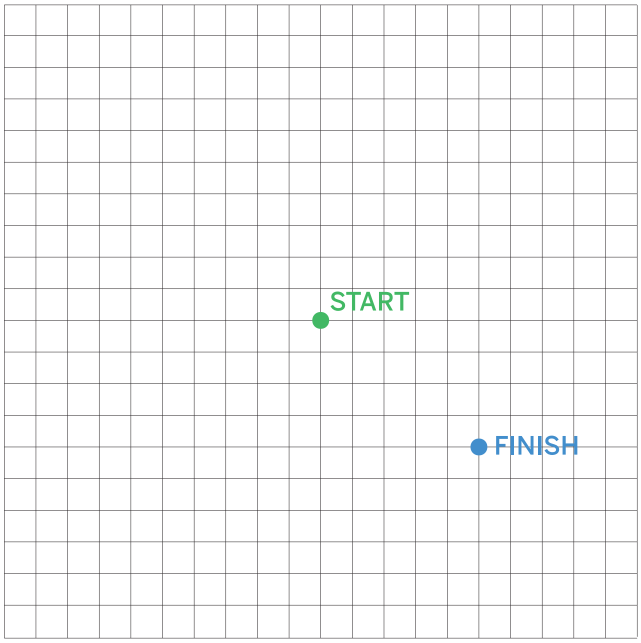
This is the question you will look at in these next lessons. Get on your hiking shoes! From the highest peaks to far off lands, we'll look at how we can use magnitude and direction to get us anywhere.

Welcome to Unit 7.

Practice

Name: .... Date: .... Period: ....

1. Use the grid and points labeled START and FINISH to answer parts a–c.



a From the point labeled START, move two spaces to the left and three spaces up. Plot a point at this location and label it A.

b From the point labeled START, move three spaces down and four spaces to the right. Plot a point at this location and label it B.

c Describe how to move from point A to the point labeled FINISH.

2. Use the symbols >, <, or = to compare each pair of numbers.

a 0.4 .... $\frac{10}{40}$

b 12.030 .... 12.03

c $\frac{16}{17}$ .... $\frac{11}{12}$

d $\frac{13}{44}$ .... $\frac{13}{43}$

e 21 .... 2.1

f 31.2 .... 31.02

3. The total force of a tug-of-war team is equal to 4.5 Newtons per team member. Complete the table to show the total force for each team size.

Team size (number of members)	8	23	n
Total force (Newtons)			

Name: ________ Date: ________ Period: ________

4. Which expressions are solutions to the equation $\frac{3}{4}x = 15$? Select *all* that apply.

A. $\frac{15}{\frac{3}{4}}$

B. $\frac{15}{\frac{4}{3}}$

C. $\frac{4}{3} \cdot 15$

D. $\frac{3}{4} \cdot 15$

E. $15 \div \frac{3}{4}$

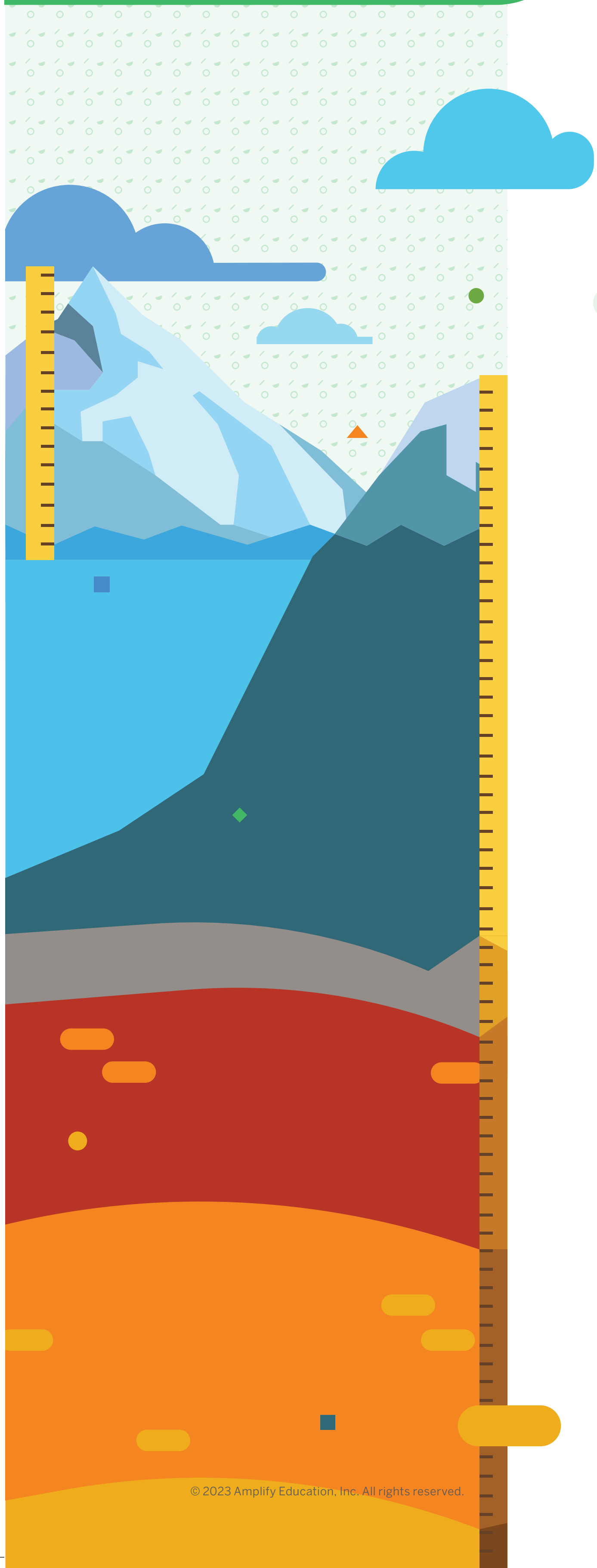
5. Andre and Clare's teacher asked them to draw a number line from 0 to 10. Andre drew his number line horizontally, and Clare drew hers vertically.

a Draw both of their number lines. Then plot and label the locations of 3.5 and $\frac{13}{2}$.

b Describe a real-world context for which Andre's number line might be the best representation.

c Describe a real-world context for which Clare's number line might be the best representation.

My Notes:



What's the tallest mountain in the world?

If you said "Mount Everest," you're only *kind* of right.

Everest is certainly the tallest mountain *above sea level*. From its peak to where the ocean meets the land, Everest is roughly 29,035 ft tall.

As a surface-dwelling human, it figures you'd start at sea level. But if you were a devil worm nematode – a creature that spends its life more than a mile underground – you might suggest a different contender.

Meet Chimborazo of Ecuador, the tallest mountain measured *from the center of the Earth*.

You might think that title would go to Everest again. After all, if you're the farthest from sea level, you'd also have to be the farthest from the Earth's center...right?

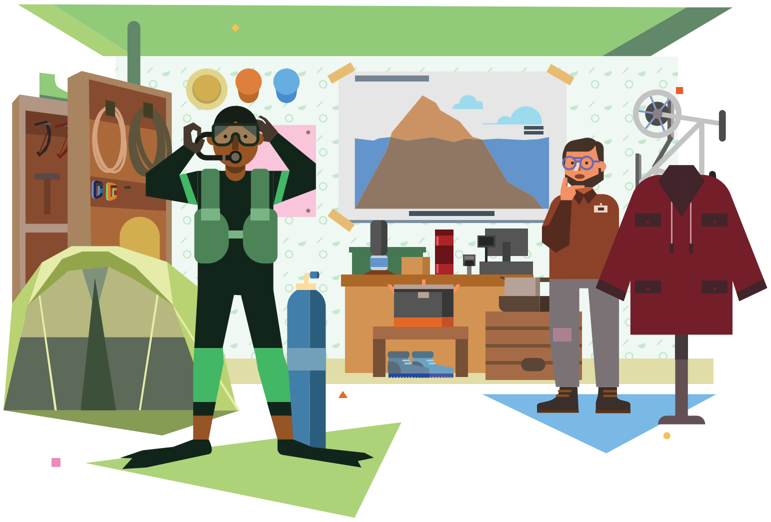
While that would be true if the Earth were a perfect sphere, in reality, the Earth is more like a *slightly* squashed tomato (technically called an *oblate spheroid*). The Earth is a little flatter at its poles, and it bulges out at the equator. And since Chimborazo sits on that bulge, it's about 1.3 miles taller than Everest, putting it at a whopping 3,967.1 miles from the Earth's center.

All this goes to show that we should be careful when we talk about measuring distances. It's not always as simple as breaking out a really long measuring tape. Sometimes the first thing we have to do is decide *where we're measuring from*.

Unit 7 | Lesson 2

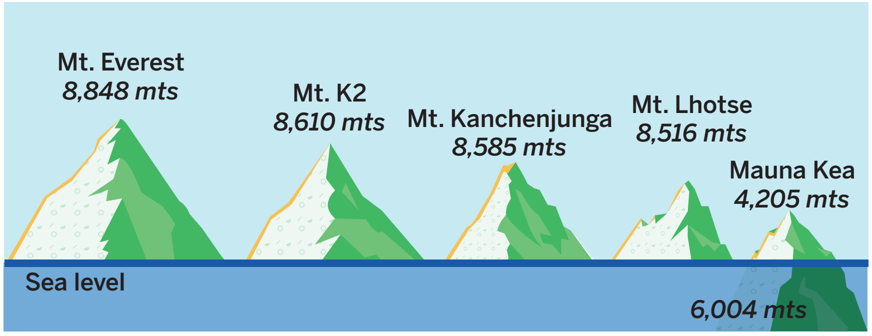
Positive and Negative Numbers

Let's explore how we represent elevations.



Warm-up Notice and Wonder

Consider the image showing some of the tallest mountains in the world and their heights. What do you notice? What do you wonder?



1. I notice . . .

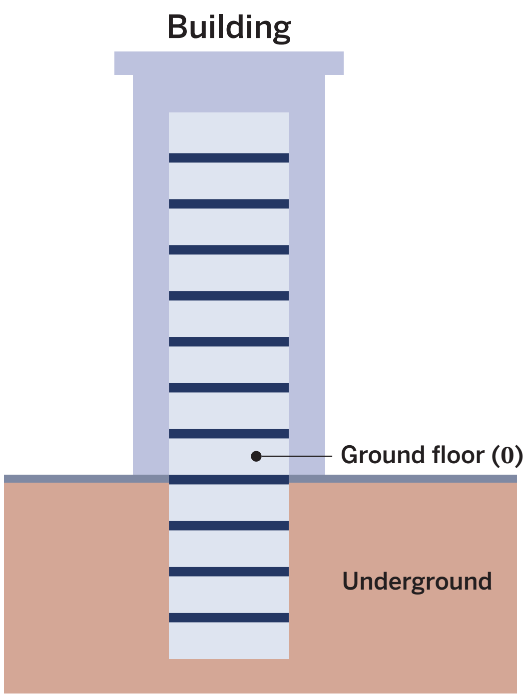
2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: Date: Period:

Activity 1 The Apartment Building

An apartment building has a total of 12 floors. The ground floor is Floor 0, and the floor above that is Floor 1.



1. Priya lives on Floor 2 and wants to visit her friend Mai, who lives on Floor 7. How many floors up does Priya need to go?

2. Priya and Mai are now both on Floor 7 and decide to race down 9 floors. On what floor will they end up?

3. Priya and Mai now want to exit the building from the ground floor. How many floors away is the ground floor? In which direction do they need to go?

4. How did you name the resulting floor in each situation?

5. What does it mean when a floor is above Floor 0? Below Floor 0?

6. Do numbers below 0 make sense outside of the context of an apartment building? Give at least one example to support your thinking.

Activity 2 High Places, Low Places

1. The table shows the elevations of several U.S. cities.

City	Elevation (ft)
Harrisburg, PA	320
Bethel, IN	1,211
Denver, CO	5,280
Coachella, CA	-22
Death Valley, CA	-282
New York City, NY	33
Miami, FL	0

a. Of the cities in the table, which has the second highest elevation?

b. How would you describe the elevation of Coachella, CA, in relation to sea level?

c. How would you describe the elevation of Death Valley, CA, in relation to sea level?

d. How would you describe the elevation of Miami, FL?

2. A city not listed in the table has a higher elevation than Coachella, CA. Select *all* numbers that could represent the city's elevation. Be prepared to explain your thinking.

A. -11 ft

B. -35 ft

C. 4 ft

D. -8 ft

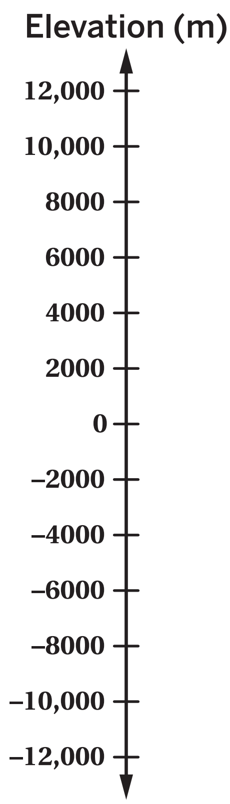
E. 0 ft

Name: ______ Date: ______ Period: ______

Activity 2 High Places, Low Places (continued)

The table shows the elevations of several geological landmarks, representing some of the highest points on land and lowest points in the oceans.

Label	Landmark	Location	Elevation (m)
A	Mount Everest	Nepal, Asia	8,848
B	Puerto Rico Trench	Atlantic Ocean	−8,600
C	Denali	United States, North America	6,168
D	Pichu Pichu	Peru, South America	5,664
E	Tonga Trench	Pacific Ocean	−10,882
F	Mount Kilimanjaro	Tanzania, Africa	5,895
G	Sunda Trench	Indian Ocean	−7,725
H	Mariana Trench	Pacific Ocean	−11,033



3. Refer to the table.

a Plot a point on the vertical number line for each location, and label it using the corresponding capital letter from the table.

b Which landmark is the lowest? What is its elevation?

c Which landmark is the highest? What is its elevation?

d What would a point at 0 represent on your vertical number line? What do points above 0 represent? Points below 0?

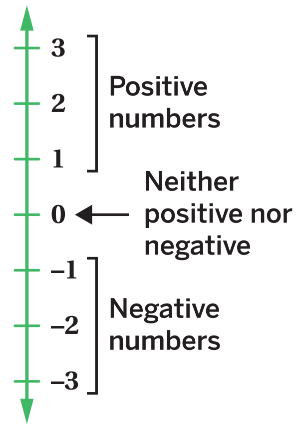
e Which is farther from sea level: the bottom of the Mariana Trench or the top of Mount Everest? Explain.

STOP

Summary

In today's lesson . . .

You have seen that numbers can be less than zero, such as numbers representing elevations below sea level. Until now, you have only been working with positive numbers and zero, which include whole numbers, fractions, and decimals. ***Positive numbers*** are numbers that are greater than zero. ***Negative numbers*** are numbers that are less than zero. Zero is *neither* positive *nor* negative. The meaning of a negative number in a context depends on the meaning of zero in that context.



For example, elevation describes the height or depth of a location or object relative to sea level. An elevation of 0 represents *sea level*. So, a *positive* elevation is *above* sea level, and a *negative* elevation is *below* sea level.

Reflect:

Name: ______________________ Date: ____________ Period: ________

Practice

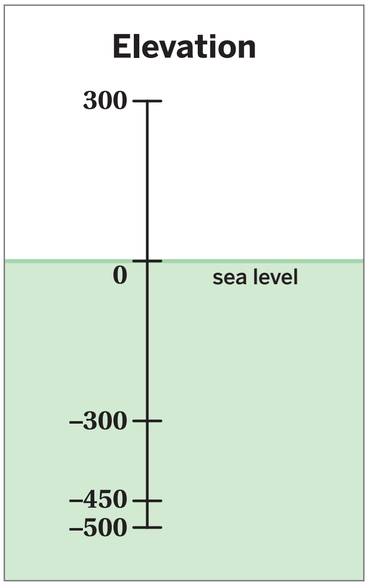
1. The statements in parts a–d describe the movements of a whale swimming in the ocean. Each statement starts from the whale's elevation in the previous statement.

   a. A whale is at the surface of the ocean to breathe. What is the whale's elevation?

   b. The whale swims down 300 ft to feed. What is the whale's elevation now?

   c. The whale swims down 150 ft more to rest. What is the whale's elevation now?

   d. Plot and label the three elevations as points on the vertical number line.



2. Using what you know about positive and negative numbers, answer these questions about sea level and elevation.

   a. Is an elevation of -10 ft closer to sea level or farther from sea level than an elevation of -8 ft?

   b. Is an elevation of 3 m closer to sea level or farther from sea level than an elevation of -4 m?

3. Complete the table for each elevation with the correct sign and its relation to sea level. If you have time, add in extra rows to think of your own examples.

Elevation	Corresponding sign (+/–/no sign)	Relation to sea level (above/below/at)
0 miles		
-10 ft		
12 m		

Name: .... Date: .... Period: ....

4. Write an equation to represent each situation. Then solve the equation.

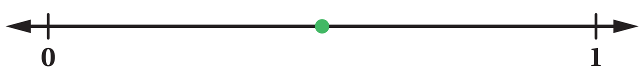
a Andre drinks 15 oz of water, which is $\frac{3}{5}$ of a bottle. How much does the bottle hold? Use x to represent the number of ounces of water the bottle holds.

b A bottle holds 15 oz of water. Jada drank 8.5 oz of water. How many ounces of water are left in the bottle? Use y to represent the number of ounces of water left in the bottle.

c A bottle holds z ounces of water. A second bottle holds 16 oz, which is $\frac{8}{5}$ times as much water as the first bottle. How much does the first bottle hold?

5. A rectangle has an area of 24 square units, and one side has a length of $2\frac{3}{4}$ units. Determine the length of the other side of the rectangle. Show your thinking.

6. Estimate the value of the point plotted on the number line as a fraction and as a decimal.



a Fraction: ....

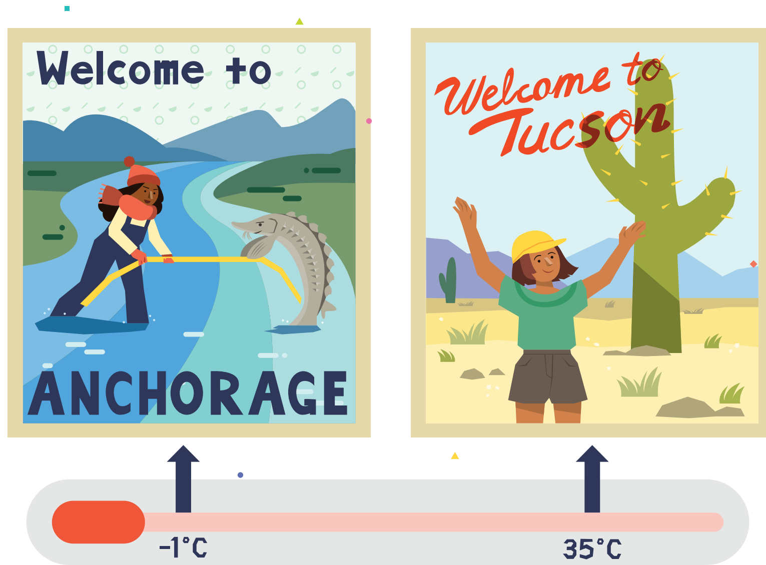
b Decimal: ....

Name: ______ Date: ______ Period: ______

Unit 7 | Lesson 3

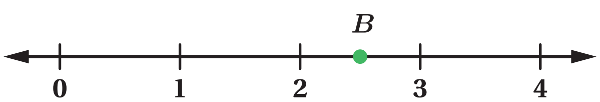
Points on the Number Line

Let's plot positive and negative numbers on the number line.



Warm-up A Point on the Number Line

Select *all* of the following numbers that *could* be the value of B.



A. 2.5

B. $\frac{2}{5}$

C. $\frac{5}{2}$

D. 2.01

E. $\frac{25}{10}$

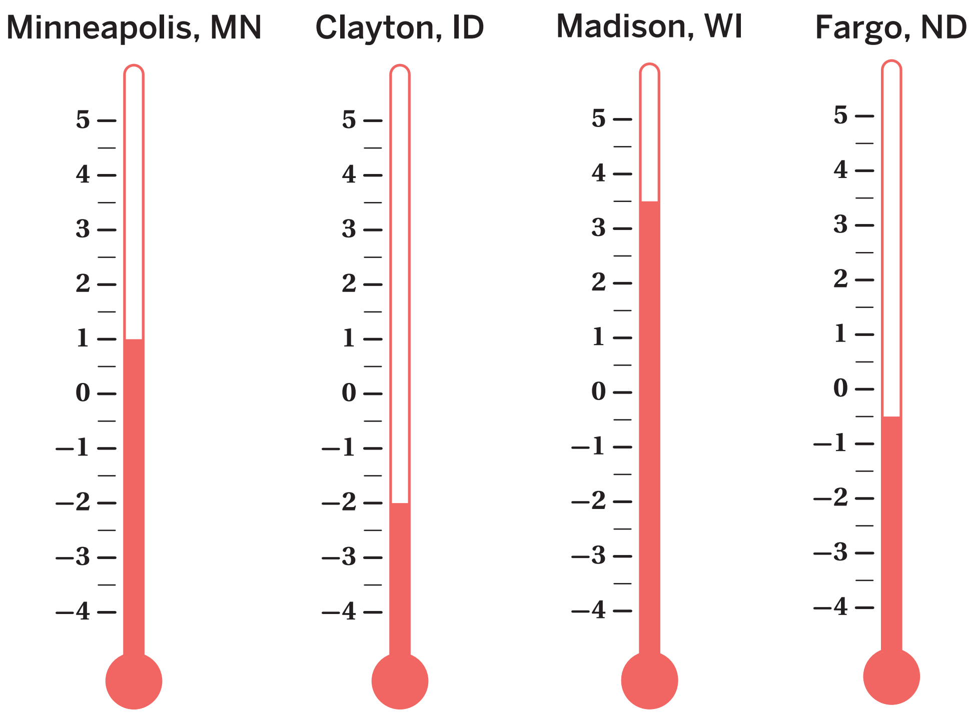
F. 2.49

Log in to Amplify Math to complete this lesson online.

Activity 1 What is the Temperature?

1. Each of the four thermometers shows the temperature, in degrees Celsius, in a different city at the same time of day on March 2, 2020. Complete the table by writing the temperature in each city.

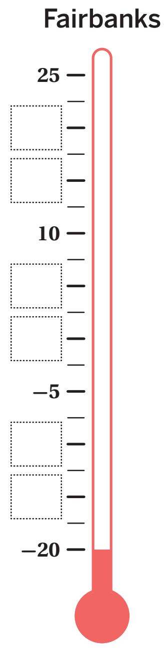
City	Temperature (°C)
Minneapolis	
Clayton	
Madison	
Fargo	



Name: ________________ Date: ________ Period: ________

Activity 1 What is the Temperature? (continued)

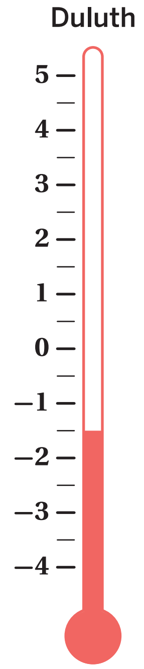
2. One thermometer shows the temperature, in degrees Celsius, in Fairbanks, Alaska on March 3, 2020.



a. What was the temperature?

b. The thermometer is missing some labels. Write the missing numbers in the boxes on the thermometer.

3. Elena says that the thermometer for Duluth, Minnesota reads $-2.5°C$ because the top of the liquid rises above $-2°C$. Jada disagrees and says that it reads $-1.5°C$ because the top of the liquid is below $-1°C$. Do you agree with either of them? Explain your thinking.



4. The temperatures in Phoenix, Arizona, and Portland, Maine, are rarely the same.

a. One morning, the temperature in Phoenix was $8°C$. That same morning in Portland, it was $12°C$ cooler than it was in Phoenix. What was the temperature in Portland?

b. At noon on another day, Portland and Phoenix each had temperatures that measured the same distance from zero on the thermometer. Portland had a negative temperature and Phoenix had a positive temperature. If the temperature in Portland was $18°C$ colder than the temperature in Phoenix, what was the temperature, in Celsius, in each city? Explain your thinking.

Activity 2 Folded Number Lines

You will be given a sheet of graph paper and a straightedge.

1. Follow the steps to create your own number line.

- Use the straightedge to draw a horizontal line with arrows on each end. Mark a point in the middle of the line and label it 0.
- To the right of 0, draw tick marks at every vertical grid line on your graph paper. Label the tick marks $1, 2, 3, \ldots, 10$. This is the positive side of your number line.
- Fold your paper so that a vertical crease goes across the number line through 0 and the two sides of the number line match up perfectly.
- Use the fold to help you trace the tick marks that you already made onto the other side of the number line. Unfold and label the tick marks $-1, -2, -3, \ldots, -10$. This is the negative side of your number line.

2. Use your number line to respond to the following.

a Which number is the same distance from 0 as the number 4?

b Which number is the same distance from 0 as the number -7?

c Two numbers that are the same distance from 0 on the number line are called ***opposites***. Both of the pairs 7 and -7, and -4 and 4 are opposites. What is another pair of opposites?

d Determine how far the number 5 is from 0 on the number line. Then determine both a positive number and a negative number that are each farther from 0 on the number line than the number 5.

STOP

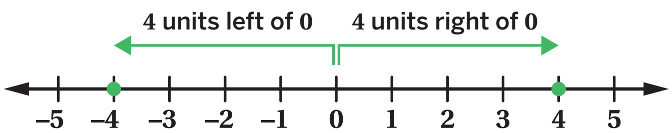
Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

Just as you can extend a number line to the right of 0 to show positive numbers, the number line can also be extended to the left of 0 to show negative numbers.

The number 4 is positive, so its location is 4 units to the *right* of 0 on the number line. The number -4 is negative, so its location is 4 units to the *left* of 0 on the number line.



Two numbers are ***opposites*** if they are the same distance from zero, but on different sides, on the number line. Therefore, -4 and 4 are opposites. Every number has an *opposite,* including fractions and decimals. In the case of 0, 0 is its own opposite.

All of the positive numbers you have ever seen — whole numbers, fractions, and decimals — can all be written as fractions, and can all be located precisely on the number line.

- All of the whole numbers and their *opposites,* including 0, are ***integers***.
- All of the numbers that can be written as fractions, including whole numbers and decimals, and their opposites are called ***rational numbers***.

The numbers 2 and -2 are both integers, and they are both rational numbers. The numbers 8.3, -8.3, $\frac{3}{2}$, and $-\frac{3}{2}$ are all rational numbers, but none are integers.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. For each number, name its opposite.

a -5

b 0.875

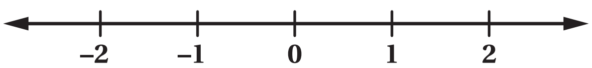
c $\frac{4}{5}$

d 0

e -10.4

f $-8{,}003$

2. Plot and label each point at the correct location on the number line.



a Point A is located at the opposite of -2.

b Point B is located at the opposite of 0.5.

c Point C is located at -2.

d Point D is located at $\frac{3}{2}$.

e Point E is located at -1.5.

f Point F is located at $-\frac{4}{3}$.

3. Where would the temperature -5.5°F be located on a thermometer? Select *all* that apply.

A. Between -5°F and -4°F

B. Between -5°F and -6°F

C. Between 5°F and 10°F

D. Between -10°F and -5°F

E. Between 0°F and -5°F

Practice

Name: .... Date: .... Period: ....

4. Solve each equation for x.

a. $8x = \frac{2}{3}$

b. $1\frac{1}{2} = 2x$

c. $5x = \frac{2}{7}$

d. $\frac{1}{4}x = 5$

e. $\frac{1}{5} = \frac{2}{3}x$

5. There are 15.24 cm in 6 in. How many centimeters are in each of the following?

a. 1 ft

b. 1 yd

6. The elevation of Memphis, TN, is 338 ft. The elevation of Nashville, TN, is 597 ft.

a. Which city has a higher elevation?

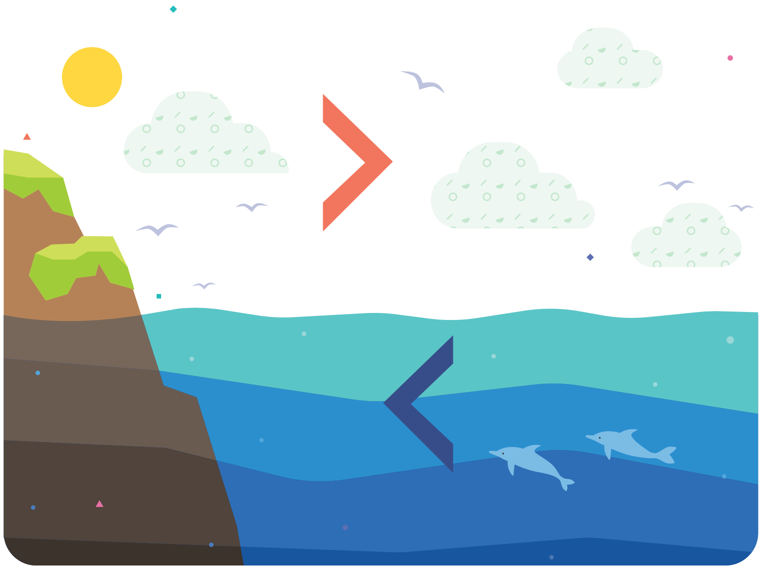
b. Use the correct equality symbol ($<$, $>$, $=$) to complete the comparison statement.

338 597

Unit 7 | Lesson 4

Comparing Integers

Let's compare integers on the number line.



Warm-up True or False

Determine whether each inequality is true or false. Be prepared to explain your thinking.

1. $\frac{5}{4} < 2$

2. $8.5 > 0.95$

3. $8.5 < 7$

4. $10.00 < 100$

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Human Number Line

You will be given a number card. Pair up with the classmate whose number card is the opposite of your number. Then discuss where your numbers would each be located on the number line.

1. Prepare for the human number line. Complete the sentences.

a My number is ________

b The opposite of my number is ________

2. Reflect on the activity and the human number line that was created. Write one statement about comparing positive and negative integers that is always true.

Compare and Connect:
As you respond to Problem 2, think about how the number line shows both magnitude and direction. Can you use the term *magnitude* in your response?

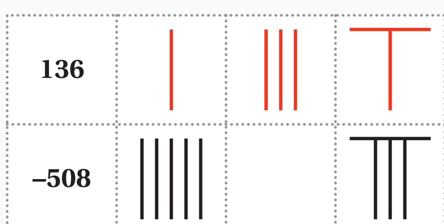
Historical Moment

Chinese Number Rods

One of the first known records of negative numbers being used comes from the Chinese number rod system seen here, which dates to around 200 BCE. The need for positive and negative numbers came from taxes — receiving and paying currency.

Use the image to decipher and determine how to write each of the following numbers using Chinese number rods.

1. 57,193
2. −60,428



Activity 2 Comparing Elevations

The elevations of several cities around the world are shown in the table.

Location	Tripoli, Libya	San Juan, Puerto Rico	New Orleans, Louisiana	Amsterdam, Netherlands	Jakarta, Indonesia	Taipei, Taiwan	Tunis, Tunisia
Elevation (ft)	6	-4	-7	-2	3	5	0

1. What do the following numbers mean in this context?

   a) 0

   b) -7

2. Plot all of the elevations on a number line.

Name: ______ Date: ______ Period: ______

Activity 2 Comparing Elevations (continued)

3. Decide whether each inequality statement is true or false. Be prepared to explain what each statement means in the context of the elevation of the cities.

   a. $-2 < 6$

   b. $-4 < -7$

   c. $3 > -4$

   d. $-7 > 5$

4. Jada said, "I know that 2 is less than 4, so -2 must be less than -4. This means Amsterdam has a lower elevation than San Juan." Do you agree? Explain your thinking.

5. Andre said, "3 is less than -7 because 3 is closer to 0 on the number line. This means Jakarta has a lower elevation than New Orleans." Do you agree? Explain your thinking.

6. The shore of the Dead Sea has an elevation of $1{,}419$ ft below sea level. The Challenger Deep, part of the Mariana Trench in the western Pacific Ocean, has a depth of $36{,}201$ ft.

   a. Write the elevations of the Dead Sea and Challenger Deep.

      Dead Sea: ______ **Challenger Deep:** ______

   b. Which one has the lower elevation? Explain your thinking.

   c. Write an inequality that compares the elevations.

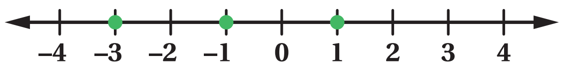
STOP

Summary

In today's lesson . . .

You compared positive and negative integers on the number line. The ***sign*** of a number tells you whether it is positive or negative. For example you read the number -3 as "negative three". The symbol '–' tells you that the sign of-3 is negative. Although 0 is not written with a negative sign and looks to be written the same way we write positive numbers, it has *no sign* because it is *neither* positive *nor* negative.

You use the words *greater than* and *less than* to compare numbers when you refer to their values or corresponding positions on the number line. This is true for negative integers as well.



- The integer -3 is to the *left* of -1, so -3 is *less than* -1. You can write this as the inequality statement $-3 < -1$.
- The integer -1 is to the *right* of -3, so -1 is *greater than* -3. You can write this as the inequality statement $-1 > -3$.
- You can also write $1 > -1$ and $1 > -3$ because 1 is to the right of both of the other integers.

Any number that is located to the *left* of a number n is *less than* n. And any number that is located to the *right* of a number n is *greater than* n. This means that any positive number is *greater than* any negative number, and any negative number is *less than* any positive number.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. Write four inequality statements using the integers -2, 4, -7, and 10.

2. Determine whether each inequality statement is true or false. Explain your thinking.

a $-5 < 2$

b $3 > -8$

c $-12 > -15$

d $-13 > -12$

3. Here is a true statement: $-8 < -7$. Select *all* the statements that are equivalent to $-8 < -7$.

A. -8 is farther to the right on the number line than -7.

B. -8 is farther to the left on the number line than -7.

C. -8 is less than -7.

D. -8 is greater than -7.

E. -7 is less than -8.

F. -7 is greater than -8.

Practice

Name: .... Date: .... Period: ....

4. Each lap around a track is 400 m.

a How many meters did Kiran run if he ran:

2 laps?

5 laps?

x laps?

b If Noah ran 14 laps, how many meters did he run?

c If Jada ran $7{,}600$ m, how many laps did she run?

5. A stadium has a total capacity of $16{,}000$ people.

a If there are $13{,}920$ people in the stadium, what percent of its total capacity is filled? Show your thinking.

b What percent of its total capacity is *not* filled?

6. Complete the table.

Inequality	Description in words	True or false
$\frac{3}{2} > 2$		
	Two and three hundredths is less than two and three tenths.	
$\frac{2}{3} < 0.5$		

Name: ________ Date: ________ Period: ________

Unit 7 | Lesson 5

Comparing and Ordering Rational Numbers

Let's compare and order rational numbers.

Warm-up How Do They Compare?

Use the symbols >, <, = to compare each pair of numbers. Be prepared to explain your thinking.

1. 15 ______ 1.5
2. 6.050 ______ 6.05
3. $\frac{19}{24}$ ______ $\frac{19}{21}$
4. 15 ______ −15
5. 9.02 ______ 9.2
6. 0.4 ______ $\frac{3}{5}$
7. −9 ______ −10
8. $\frac{24}{20}$ ______ 1.2

Log in to Amplify Math to complete this lesson online.

Activity 1 Human Number Line, Revisited

You will be given a number card. Pair up with the classmate whose number card is opposite of your number. Then discuss where your numbers would each be located on the number line.

1. Prepare for the human number line. Complete the sentences.

a My number is ______.

b The opposite of my number is ______.

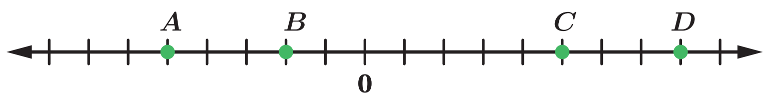
2. Reflect on the activity and the human number line that was created. Write one statement about ordering rational numbers that is always true.

Stronger and Clearer: Share the reflection you wrote about rational numbers in Problem 2 with a partner to give and receive feedback. Revise your response based on the feedback you receive.

Name: ______ Date: ______ Period: ______

Activity 2 Comparing and Ordering Points on a Line

The number line shows the points A, B, C, D, and 0. Copies of the number line can be provided if you need them.



1. If the tick marks on the number line increase by 1s, what is the position of each letter?

   A: ______ B: ______ C: ______ D: ______

2. If the tick marks on the number line do *not* increase by 1s, and D represents the location of 2, what do A, B, and C represent?

   A: ______ B: ______ C: ______

3. Suppose A represents $-2\frac{1}{2}$. Clare says that means D represents $2\frac{1}{2}$. Is she correct? Explain your thinking.

4. Suppose the letters represent the elevations of four cities in Florida. Use the letters to list the cities in order of their elevations, from:

   a. Least to greatest.

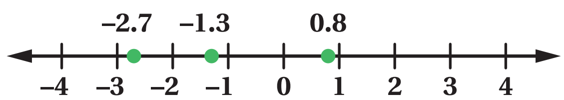
   b. Greatest to least.

STOP

Summary

In today's lesson . . .

You used a number line to compare and order positive and negative rational numbers, just as you previously did for integers. Any number to the *left* on the number line is *less than* a number to its right, and any number to the *right* on the number line is *greater than* a number to its left.



When ordering three or more rational numbers from *least to greatest*, list them in the order they appear on the number line going from *left to right*.

For example, the numbers -2.7, -1.3, and 0.8 are listed from least to greatest because of the order they appear on the number line. You can also represent this using a *compound inequality* statement $-2.7 < -1.3 < 0.8$.

When ordering rational numbers from *greatest to least*, list them in the order they appear on the number line going from *right to left*.

For example, 0.8, -1.3, -2.7 are listed from greatest to least, and you can write the compound inequality statement $0.8 > -1.3 > -2.7$.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Select *all* the numbers that are greater than -5.

A. 1.3

B. -6

C. -12

D. $\frac{1}{7}$

E. -1

F. -4

2. Order these numbers from least to greatest: $\frac{1}{2}$, 0, 1 $-1\frac{1}{2}$, $-\frac{1}{2}$, -1.

3. The table shows the boiling points of certain elements in degrees Celsius. List the elements in the order of their boiling points, from *least* to *greatest*.

Element	Boiling point (°C)
Argon	-185.8
Chlorine	-34
Fluorine	-188.1
Hydrogen	-252.87
Krypton	-153.2

4. Explain why zero is considered its own opposite.

Practice

Name: ________ Date: ________ Period: ________

5. Evaluate each quotient.

a $\frac{1}{2} \div 2$

b $2 \div 2$

c $\frac{1}{2} \div \frac{1}{2}$

d $\frac{38}{79} \div \frac{38}{79}$

6. Each student starts with $10. Write an expression to represent how much money each student has after the event described. *Do not evaluate the expressions.*

a Elena buys $3 worth of stickers.

b Tyler earns $5 walking dogs.

c Diego is paid $4 for fixing a computer.

d Lin pays $8 for a new app.

e Priya sells a t-shirt for $9.

Name: ______________________ Date: __________ Period: __________

Unit 7 | Lesson 6

Using Negative Numbers to Make Sense of Contexts

Let's make sense of negative amounts of money.

Warm-up Notice and Wonder

Consider the following table. What do you notice? What do you wonder?

Activity	Amount ($)
Mow my neighbor's lawn	30.00
Babysit my cousin	45.00
Buy my lunch	–10.80
Sell newspapers	15.00
Purchase a shirt	–18.69
Brush my teeth	0.00
Pay a library fine	–1.25

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Managing the School Store

As manager of the school store, Elena keeps records of all the items purchased to stock the store, and all the items sold to students. The table shows her records for Tuesday.

Item	Quantity	Amount ($)
Pack of pencils	–15	18.75
Erasers	30	–22.50
Erasers	–20	15.00
Notebooks	22	–29.70
Packs of markers	–25	62.50
Bags of pretzels	–22	16.50

1. For each column, identify what the following represents:
   a. A positive number.
   b. A negative number.
   c. 0

2. Which item did Elena sell the most? Explain your thinking.

3. Mai said there was a greater change in the number of notebooks than in the number of bags of pretzels because 22 > –22. Do you agree or disagree? Explain your thinking.

4. Priya pays $1.25 for a pack of pencils. Represent the value of the pencils from both Priya's perspective and Elena's perspective.

5. Draw a number line to represent the "Quantity" column.

Name: ______ Date: ______ Period: ______

Activity 2 Owning Your Own Business

You and your partner co-own a business.

1. What is the name of your business?

2. Identify three products that your business sells, and set their unit prices.

Product	Product Name	Unit Price ($)
A		
B		
C		

3. Determine whether each scenario represents a *credit* or a *debit* for your business. Complete the table with the amount of money your business has gained or lost.

Scenario	Amount ($)
You buy $250.46 worth of supplies.	
A customer buys 100 of Product *A*.	
A flood destroys $350 worth of products.	
You pay your $124.79 electricity bill.	
You sell 50 of Product *B*, and 30 of product *C*.	
You gain 5 new followers on social media.	

4. Has your business made a profit, or is it operating at a loss? Explain your thinking.

Reflect: How can goal-setting help prepare you for success?

STOP

Summary

In today's lesson . . .

You explored the meaning of negative numbers, given real-world contexts. Sometimes you use positive and negative numbers to represent *changes* in quantity. If the quantity *increases*, the change is *positive*. If the quantity *decreases*, the change is *negative*. This is especially common when representing money received (positive numbers) and money spent (negative numbers).

Whether a number is considered positive or negative depends on a person's perspective. For example, suppose Clare's grandmother gives her $\$20$ for her birthday. Clare could view this as positive 20, which might be written as $+20$, because her amount of money increased. But her grandmother could view this as negative 20, which would be written as -20, because her amount of money decreased.

When using positive and negative numbers to represent changes, you have to be very clear about what it means when the change is positive and what it means when the change is negative. You also need to consider whose perspective you want to represent.

Reflect:

Practice

Name: ____________ Date: ____________ Period: ____________

1. Write a positive or negative number to represent the change for each scenario.

   a. Tuesday's high temperature was 4 degrees colder than Monday's high temperature.

   b. Wednesday's high temperature was 3.5 degrees warmer than Tuesday's high temperature.

   c. I earned \$6.50 for babysitting my brother.

   d. I spent \$2 on a gift for a friend.

2. Decide whether the quantity in each scenario is positive or negative, from the perspective of the person or object indicated. Then describe a scenario, in the same context, that would have the opposite sign.

   a. Tyler's puppy gained 5 lb.

   b. The aquarium leaked 2 gallons of water.

   c. Andre received a gift of \$10.

   d. Kiran gave a gift of \$10.

   e. A climber descended 550 ft.

3. For the integers -15, 28, and 0, complete the following tasks.

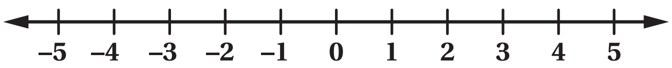
   a. Describe a real-world scenario where each integer represents a quantity.

   b. Describe a real-world scenario where each integer represents a *change* in a quantity. State whose perspective matches the sign of the number.

Practice

Name: .... Date: .... Period: ....

4. Refer to the number line.



a Label the points that are 4 units from 0. What would happen to these points if you folded the number line so that a vertical crease goes through 0?

b Label the points that are $\frac{5}{2}$ units from 0. What is the distance between these points?

5. Evaluate each expression.

a $2^3 \cdot 3$

b $\frac{4^2}{2}$

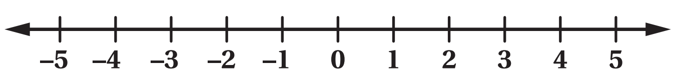
c 3^1

d $6^2 \div 4$

e $2^3 - 2$

f $10^2 + 5^2$

6. Han and Noah both leave the park and walk in opposite directions towards their homes. They both live the same distance away from the park. Let 0 represent the location of the park. If the location of Han's house can be represented by 3 on the number line, mark the location of Noah's home.

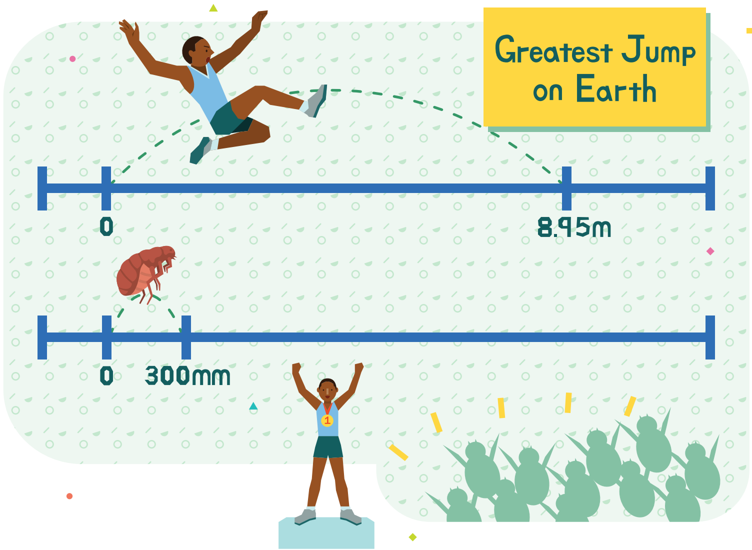


Name: .... Date: .... Period: ....

Unit 7 | Lesson 7

Absolute Value of Numbers

Let's explore distances from zero more closely.



Warm-up Number Talk

For each pair of numbers, mentally decide which one has a value that is closer to 0.

1. $\frac{9}{11}$ or $\frac{15}{11}$

2. $\frac{1}{5}$ or $\frac{1}{9}$

3. 1.25 or $\frac{5}{4}$

4. 5 or -2

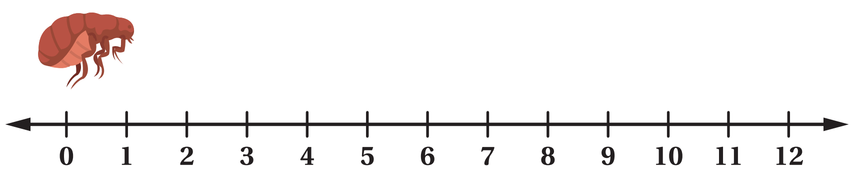
5. -0.33 or -0.25

Log in to Amplify Math to complete this lesson online.

Activity 1 Jumping Fleas

The current world record for long jump is held by American Mike Powell, who jumped a distance of 8.95 m at the 1991 World Championships in Athletics. But the greatest jumper in the animal kingdom, relative to its size, is actually the flea! Fleas can jump over 200 times their own body length. Imagine a little 1.5-mm flea in a long jump competition jumping an impressive 300 mm (that's 0.3 m, or about 12 in.). Well, impressive for a flea — it's all relative!

A flea is jumping around on a number line, where each tick mark represents 1 in. You will be given the cut-out of a flea and a number line to record the flea's jumps. Use your flea to help you complete this activity. Each jump should be 12 in.



1. The flea starts at 0 and jumps once.

   a. Where might it land?

   b. Could it land somewhere else? If so, where? If not, why not?

2. Suppose you do not know where the flea starts, but it jumps twice.

   a. If the flea lands at 0, where could it have started?

   b. Could the flea have started from a different point and still ended up at 0?

3. Now there are two fleas, both starting at 1.

   a. The first flea jumps once to the right, and the second flea jumps once to the left. Where does each flea land?

      First flea: .... **Second flea:** ....

   b. How far away from 0 does each flea land?

      First flea: .... **Second flea:** ....

Name: .... Date: .... Period: ....

Activity 2 Absolute Value With Jumping Fleas

The *absolute value* of a number represents its distance from 0. To represent the absolute value of a number n, use the notation $|n|$, which can also be interpreted as the distance from n to 0 on the number line.

Let's think about some more fleas jumping around on a number line.

1. If a flea is 6 units to the left of 0 on the number line, how do you write its distance from 0 as an absolute value expression?

2. If a flea is to the right of 0 and the absolute value of its location is equal to 12.5, where on the number line is the flea located?

3. What does $|-7|$ represent? What is its value?

4. What does $|1.8|$ represent? What is its value?

Are you ready for more?

Consider a flea that does not jump the maximum possible distance of 12 in. It can start anywhere on the number line and must jump at least 1 in., but always toward 0. List three possible combinations of starting points and distances jumped, in inches. Then determine the flea's ending point for each, and how far this final distance is from 0.

Starting point	Distance jumped (in.)	Ending point	Final distance from 0

Activity 3 Absolute Value With Elevation and Temperature

1. Part of the city of New Orleans is 6 ft below sea level. You can use -6 ft to describe its elevation, and $|-6|$ ft to describe its distance from sea level. In the context of elevation, what would each of the following describe?

   a. 25 ft

   b. $|25|$ ft

   c. -8 ft

   d. $|-8|$ ft

2. You can use $-5°C$ to describe a temperature that is 5 degrees below the freezing point, $0°C$, and you can use $5°C$ to describe a temperature that is 5 degrees above the freezing point. In the context of temperature, in degrees Celsius, what do each of the following describe?

   a. 1

   b. -4

   c. $|12|$

   d. $|-7|$

Are you ready for more?

At a certain time, the difference between the temperatures in New York City and Boston was $7°C$. At that same time, the difference between the temperatures in Boston and Chicago was also $7°C$. Could the temperatures in New York City and Chicago be the same? Is it possible for one to be above freezing and the other below? Explain your thinking.

STOP

Name: .... Date: .... Period: ....

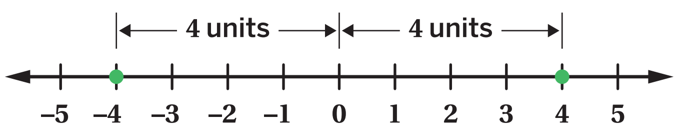
Summary

In today's lesson . . .

You explored how numbers are related to zero. In contexts such as elevation and temperature, it is important to understand what 0 represents. For elevation, 0 ft represents sea level. For temperature, $0°$ Celsius is the freezing point of water. However, this is *not* the freezing point for degrees Fahrenheit.

The ***absolute value*** of a number represents its distance from 0.

- For example, the absolute value of -4 is 4, because -4 is 4 units to the left of 0.
- The absolute value of 4 is also 4, because 4 is 4 units to the right of 0.
- Opposites always have the *same* absolute value because they are both located the same distance from 0, just in opposite directions.



The absolute value of any number is always a non-negative value because it represents distance, and distance is always non-negative.

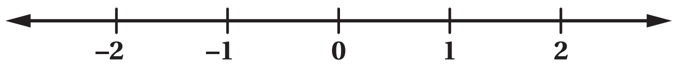
- To represent the absolute value of 4, use the notation $|4|$, which equals 4.
- To represent the absolute value of -8, use the notation $|-8|$, which equals 8.
- Opposite values have the same absolute value since they are the same distance from 0; $|-6| = |6| = 6$

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Plot and label *all* the numbers that have an absolute value of $\frac{3}{2}$ on the number line.



2. The temperature at dawn was $6°C$ away from $0°C$. Select *all* the temperatures that could have been the temperature at dawn.

A. $-12°C$

B. $-6°C$

C. $0°C$

D. $6°C$

E. $12°C$

3. Order the values from least to greatest.

$|2.7|$ $\quad 0 \quad 1.3 \quad |-1| \quad 2$

4. Complete each sentence with a number (or an expression with an exponent) that makes it true.

a 3^4 is .... times greater than 3^3.

b 5^3 is .... times greater than 5^2.

c 7^{10} is .... times greater than 7^8.

d 17^6 is .... times greater than 17^4.

e 5^{10} is .... times greater than 5^4.

Name: ______________________ Date: __________ Period: __________

5. Elena donates some money to charity each time she earns money as a babysitter. The table shows how much money in dollars d she donates for different amounts of money m that she earns.

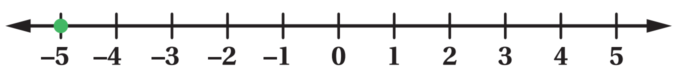
d (\$)	4.44	1.80	3.12	3.69	2.16
m	37	15	26	30	18

a What percent of her money earned does Elena donate to charity? Show your thinking.

b Which quantity, m or d, would be the better choice for the dependent variable in an equation describing the relationship between m and d? Explain your thinking.

c Use your choice from part b to write an equation that relates m and d.

6. Plot a point at the opposite of –5 on the number line.



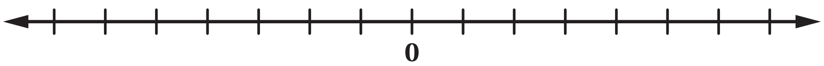
Unit 7 | Lesson 8

Comparing Numbers and Distances From Zero

Let's use absolute value and negative numbers to think about elevation.

Warm-up Opposites

1. Suppose n is a rational number. Choose a value for n and plot it on the number line.



2. Refer to your value for n.

   a. Plot $-n$ on the number line.

   b. What is the value of $-n$?

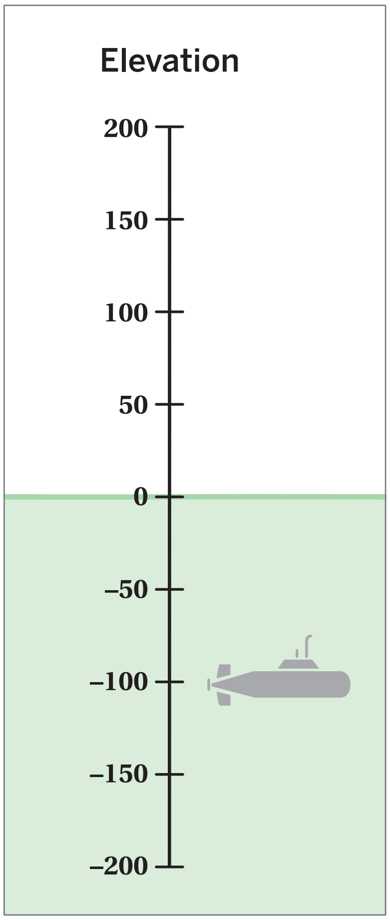
Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Comparing Elevations and Distances From Sea Level

The connection between absolute value and distance will return in more advanced mathematics. Absolute values and similar ideas are commonly used, even in recent mathematical research, such as the work of Mary Deconge-Watson.

Take a look at these examples of distance. A submarine is at an elevation of -100 ft, as shown on the vertical number line. Compare the elevations of the seagull, giant tube worm, flying fish, and coral reef to the elevation of the submarine.



1. Use the following information to plot and label a possible location where the seagull, giant tube worm, flying fish, and coral reef could each be found.

a A seagull is located at an elevation, S, that is greater than the elevation of the submarine. It is farther away from sea level than the submarine.

b A giant tube worm is located at an elevation, G, that is less than the elevation of the submarine.

c A flying fish is located at an elevation, F, that is greater than the elevation of the submarine. It is closer to sea level than the submarine.

d A coral reef is located at an elevation, C, that is the same distance from sea level as the submarine, but not at the same location.

Activity 1 Comparing Elevations and Distances From Sea Level (continued)

2. Complete the table as follows:

a Write a possible elevation for each animal.

b Use < , >, or = to compare the elevation of each animal to the elevation of the submarine.

c Use absolute value to represent the distance from sea level to each animal.

	Possible elevation	Compare to submarine	Distance from sea level
Submarine	-100 ft	$-100 = -100$	$\lvert -100 \rvert$ or 100 ft
Seagull			
Giant tube worm			
Flying fish			
Coral reef			

3. Priya says the elevation of a sea turtle is less than the submarine's and closer to sea level. Is this possible? Explain your thinking.

Featured Mathematician

Mary Deconge-Watson

Born in Louisiana in 1933, Deconge-Watson became the 15th African-American woman ever to earn a Ph.D. in mathematics. Her doctoral thesis is entitled *2-Normed Lattices and 2-Metric Spaces*. Mathematically, "norming" something is very much like taking its absolute value or determining its distance from zero. She is also known for her work related to the Cauchy problem for parabolic equations, which goes well beyond absolute values.

Name: ________________ Date: ________ Period: ________

Activity 2 Inequality Mix and Match

Here are some numbers, expressions, and symbols. Work with your partner to write three true comparison statements and corresponding sentences in the table.

-0.7	$-\frac{3}{5}$	1	4	$\lvert -8 \rvert$	$<$
$-\frac{6}{3}$	-2.5	2.5	8	$\lvert 0.7 \rvert$	$=$
-4	0	$\frac{7}{2}$	$\lvert 3 \rvert$	$\left\lvert -\frac{5}{2} \right\rvert$	$>$

1. Each partner selects a number or expression. *At least* one should be a decimal, fraction, negative number, or absolute value expression. Decide which equality or inequality symbol would make a *true* comparison statement.
2. Once you agree upon and record your comparison statement, write a sentence (in words) to match the statement, using one or more of the following phrases:
   - is equal to
   - is the absolute value of
   - is greater than
   - is less than
3. Across your three comparisons, there must be *at least*:
   - two negative numbers
   - two absolute value expressions
   - one decimal
   - one fraction

Comparison statement	Sentence

STOP

Summary

In today's lesson . . .

You used absolute value and negative numbers to think about elevation. The context of elevation can help you compare two rational numbers or two absolute values, and understand the difference between these comparisons.

For example, suppose an anchor has an elevation of -10 m and a house has an elevation of 12 m.

- To compare their *elevations* and describe the anchor as having a lower elevation than the house, you can write $-10 < 12$, which means -10 is less than 12.
- To compare their *distances from sea level* and describe the anchor as being closer to sea level than the house, you can write $|-10| < |12|$, which means the distance from -10 to 0 is less than the distance from 12 to 0.

You can compare any rational numbers and their absolute values outside of the context of elevation. For example:

- To compare the distances that -47.5 and 5.2 are each from 0, you can say $|-47.5|$ is 47.5 units from 0 and $|5.2|$ is 5.2 units from 0, so $|-47.5| > |5.2|$.
- The expression $|-18| > 4$ means that the absolute value of -18 is greater than 4, which means that -18 units is more than 4 units from 0 on the number line.

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. In the context of elevation, what would $|-7|$ ft mean?

2. Match each verbal statement with an equivalent mathematical statement.

Verbal statement

a. The number -4 is a distance of 4 units from 0 on the number line.

b. The number -63 is a distance of more than 4 units from 0 on the number line.

c. The number 4 is greater than the number -4.

d. The numbers 4 and -4 are the same distance from 0 on the number line.

e. The number -63 is less than the number 4.

f. The number -63 is farther away from 0 than the number 4 on the number line.

Mathematical statement

______ $|-63| > 4$

______ $-63 < 4$

______ $|-63| > |4|$

______ $|-4| = 4$

______ $4 > -4$

______ $|4| = |-4|$

3. Use the symbols $>$, $<$, or $=$ to make a true comparison statement for each pair of values.

a. -32 ______ $|15|$

b. $|-32|$ ______ $|15|$

c. 5 ______ -5

d. $|5|$ ______ $|-5|$

e. 2 ______ -17

f. 2 ______ $|-17|$

g. $|-27|$ ______ $|-45|$

h. $|-27|$ ______ -45

Practice

Name: ________ Date: ________ Period: ________

4. Mai received and spent money in the following ways last month. For each of the following, write either a positive or negative number to represent the change in money from her perspective.

a. Her grandmother gave her $\$25$ in a birthday card.

b. She received $\$14$ for babysitting.

c. She spent $\$10$ on a ticket to the concert.

d. She donated $\$3$ to a local charity.

e. She earned $\$2$ interest on money that was in her savings account.

5. The following list shows the record low temperatures in five U.S. cities.

- Death Valley, CA: -45°F (January, 1937)
- Danbury, CT: -37°F (February, 1943)
- Monticello, FL: -2°F (February, 1899)
- East Saint Louis, IL: -36°F (January, 1999)
- Greenville, GA: -17°F (January, 1940)

a. Which of these cities had the lowest record temperature?

b. Which city had a lower record temperature: Monticello, FL, or Greenville, GA?

c. Which city had a lower record temperature: Danbury, CT, or East Saint Louis, IL?

d. How many more degrees colder is the record low temperature in Greenville, GA than the record low temperature in Monticello, FL?

6. Write an expression that matches each phrase.

a. 3 more than g

b. Twice of h

SUB-UNIT

2 Inequalities

How do you keep a quantity from wandering off?

Place it in bounds!

Most of the time, we use inequalities to compare values — such as when we write $5 > 4$. But inequalities can also be used as constraints, or *bounds,* for variables that can take on different values. If there are x people in a movie theater, and the fire marshal will shut the theater down if it exceeds a maximum capacity of 500 people, then you had better hope that $x < 500$. Otherwise, you can forget about catching the movie!

But what if there were *exactly* 500 people in the theater? That's the maximum capacity, which means it's still technically safe, and the fire marshal shouldn't be crashing the movie. So, saying $x < 500$ doesn't quite cut it, because it doesn't tell you that it's okay if x happens to equal 500 on the nose.

To express this mathematically, we'll need to add two more symbols to your arsenal: $\geq$ and $\leq$.

These are the "greater than or equal to" and the "less than or equal to" symbols. They're a mouthful to say, but they're fantastic for *including* a value within a bound.

In these next few lessons, you'll see there are *many* ways to describe inequalities: *less than*, *no more than*, *at least*, *at most*, *up to*, *between*, *betwixt* — well, maybe no one says *betwixt* anymore. But with the $<$, $>$, $\leq$, and $\geq$ symbols, you'll have everything you need to keep those unruly quantities in check.

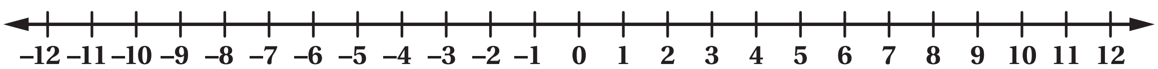
Unit 7 | Lesson 9

Writing Inequalities

Let's write some inequalities.

Warm-up Guess My Number

Each new guess should take into account all of the clues you have heard up to that point. Consider plotting your guesses on the number line to help your thinking.



Clue	Guess

My *final* mystery number guess is: ....

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Stories About 9

You will be given a set of cards.

1. For each scenario, match the appropriate description of all possible solutions with the corresponding inequality statement.
2. Write down one number that is a possible solution to each inequality.

Scenario	Description	Inequality	Possible solution to the inequality

Compare and Connect: What do each of the phrases *more than, no more than, at least,* and *less than* mean? How do they relate to the inequality symbols? Discuss with your partner.

Activity 2 How High, How Low?

The picture shows an adult giraffe and a twelve-year old girl. Refer to this picture to help you complete the activity to estimate the height of the giraffe.

1. Complete the sentences.

a What do you think is a good estimate for the *minimum* height of the giraffe? Does this mean the giraffe cannot be *shorter* or taller than this height?

b What do you think is a good estimate for the *maximum* height of the giraffe? Does this mean the giraffe cannot be shorter or *taller* than this height?

2. Write one inequality statement to represent your estimate for the *minimum* height, in feet, of the giraffe. Then write another inequality statement to represent your estimate for its *maximum* height, in feet. Use the variable h to represent the unknown height.

3. Determine an estimate of h. Use your inequalities for maximum and minimum heights to explain how you know your estimate is reasonable.

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You wrote inequalities to represent real-world contexts. When two numbers are not equal, you can use an inequality to compare the numbers. For example, you can write $3 < 4$ to show that the value of 3 is less than the value of 4.

A ***solution to an inequality*** is any value that makes the inequality true. For example, you could say that 5 is a solution to the inequality $c < 10$ because $5 < 10$ is a true statement. You could also say that 12 is *not* a solution because $12 < 10$ is not a true statement. These types of inequalities with a variable have an infinite number of possible solutions — also called the *solution set* — because there is always another number that is greater/less than the last.

Inequalities can be categorized into two categories, base on whether or not their solution set includes the *boundary value* or not:

Strict inequalities		Non-strict inequalities	
$<$	$>$	$\leq$	$\geq$
less than	greater than	less than or equal to	greater than or equal to
Solution set does *not* include the boundary value.		Solution set *does* include the boundary value.	

Reflect:

Practice

Name: .... Date: .... Period: ....

1. At a book sale, all books cost less than $\$5$.

   a. What is the most expensive price a book could be?

   b. Write an inequality to represent the possible costs of books, in dollars, at the sale.

2. Kiran started his homework before $7:00$ p.m. and finished his homework after $8:00$ p.m. Let h represent the number of hours Kiran spent on his homework. Decide whether each statement is *definitely true*, *definitely not true*, or *possibly true*.

   a. $h > 1$

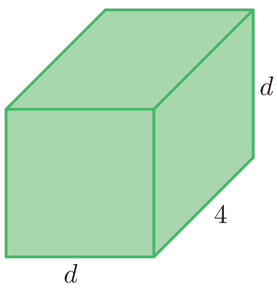
   b. $h > 2$

   c. $h < 1$

   d. $h < 2$

3. Clare tries to drink 13 cups of water every day, but many days she drinks less. Write an inequality to represent the possible cups of water c that Clare drinks on any given day.

4. Consider a rectangular prism with length 4 units, width d units, and height d units.



   a. Write an expression for the volume of the prism, v, in terms of d.

   b. Determine the volume of the prism when $d = 1$, when $d = 2$, and when $d = \frac{1}{2}$.

Name: .... Date: .... Period: ....

Practice

5. Match each statement with its corresponding inequality. All of the statements are true.

Statement

a. The number -15 is farther from 0 than the number -12 on the number line.

b. The number -12 is a distance of 12 units from 0 on the number line.

c. The distance between -12 and 0 on the number line is greater than -15.

d. The numbers 12 and -12 are the same distance from 0 on the number line.

e. The number -15 is less than the number -12.

f. The number 12 is greater than the number -12.

Inequality

.... $|-12| > -15$

.... $-15 < -12$

.... $|-15| > |-12|$

.... $|-12| = 12$

.... $12 > -12$

.... $|12| = |-12|$

6. What is the difference between the inequality statements $n \leq 8$ and $n < 8$? Write a real-world scenario that could be represented by each inequality.

Unit 7 | Lesson 10

Graphing Inequalities

Let's graph some inequalities.

Warm-up Thinking About Limits

1. You are riding in the car on a highway and pass a speed limit sign exactly like the one shown.

   a. What is the fastest, in mph, you could go without breaking the speed limit?

   b. Write an inequality to match your response that represents all the acceptable speeds, using s as the variable to represent speed, in miles per hour.

2. Water freezes and becomes ice when it reaches a temperature of $32°\text{F}$.

   a. What are some temperatures at which water will *not* freeze?

   b. Write an inequality to represent all of the possible temperatures, in degrees Fahrenheit, at which water does not freeze. Use the variable d to represent temperature, in degrees Fahrenheit.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

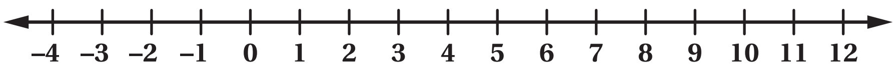
Activity 1 Stories About 9, Revisited

Refer to these scenarios from Lesson 9, Activity 1.

1. **Scenario 1:** Priya's mother wants to live less than 9 miles from her work.

   a. Write an inequality to represent this scenario using m as the variable.

   b. Use the number line to plot all the possible solutions to your inequality.



   c. Write three possible values for m.

2. **Scenario 2:** Tyler can carry no more than 9 lb of apples.

   a. Write an inequality to represent this scenario using p as the variable.

   b. Use the number line to plot all the possible solutions to your inequality.

   c. Write three possible values for p.

3. How are the number lines from Scenario 1 and Scenario 2 similar? How are they different?

Activity 2 Match It Up

You will be given a set of cards showing inequalities, number lines, and scenarios. Match each inequality with the corresponding number line showing all possible solutions. Then list one possible solution. If you have time, try matching each inequality with a scenario.

Inequality	Number line	Possible solution	Scenario (optional)

STOP

Name: ______________________ Date: ____________ Period: ____________

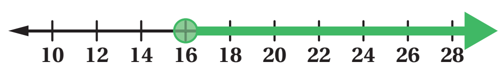
Summary

In today's lesson . . .

You explored how a scenario involving an unknown quantity and a given boundary value, or *constraint*, can be represented by an inequality which has infinite possible solutions.

To represent this on a number line, shade part of the number line to indicate that every possible point you could plot in the shaded region is a solution. This means every rational number in the shaded region is a solution. Then bold the arrow on one end of the number line to show that the possible solutions continue on forever in that direction.

To represent the boundary value, you first must consider which type of inequality symbol is used:

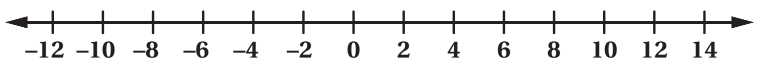
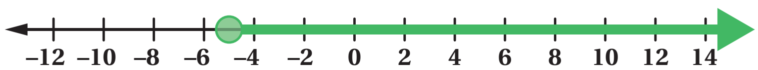
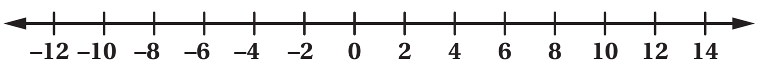
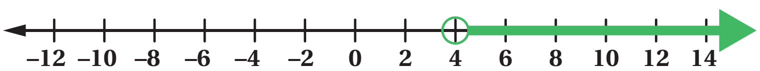
Strict inequalities		Non-strict inequalities	
$<$	$>$	$\leq$	$\geq$
The boundary value is *not* a solution, so an open (empty) circle is used on the boundary value.		The boundary value *is* a solution, so a closed (filled in) circle is used on the boundary value.	
ex: Graph the solution set for $t < 3$ 		ex: Graph the solution set for $p \geq 16$ 	

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Complete the table to show the inequality statements paired with their corresponding number line graphs.

Inequality statement	Number line graph
$s < 7$	
	
$s < 10$	
	

2. A sign on the road reads: "Speed limit, 60 mph."

a Let s represent the speed, in miles per hour, of a car. Write an inequality that represents the information on the sign.

b Draw a number line graph to represent the solutions to the inequality.

c Could 60 be a value of s? Explain your thinking.

3. Consider the inequality $k > 5$.

a Which of these numbers are solutions to the inequality: $4, 5, 6, 5.2, 5.01$? List *all* that apply.

b Draw a number line graph to represent the inequality.

Name: ______ Date: ______ Period: ______

Practice

4. Suppose the price of a cell phone at one store is $\$250$.

a. Elena's mom buys one of these cell phones on sale for $\$150$. What percent of the price did she pay?

b. Elena's dad also buys one of these cell phones from a different store, and he pays 75% of the price. How much did he pay?

5. Here are five expressions that each show a sum or a difference. Use the Distributive Property to write an equivalent expression for each that is a product with two factors.

a. $2a + 7a$

b. $5z - 10$

c. $c - 2cd$

d. $r + r + r + r + r$

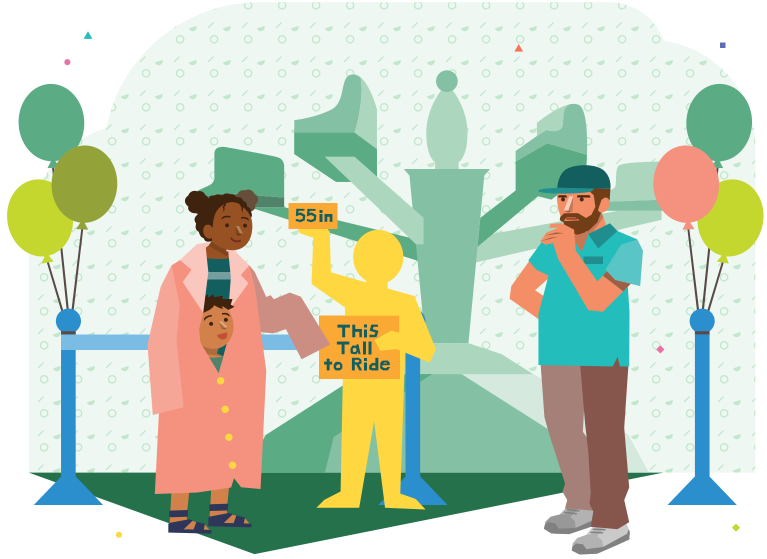
e. $2x - \frac{1}{2}$

6. Han says he is thinking of a number that is greater than -3. Could -3 be the number? Explain your thinking.

Unit 7 | Lesson 11

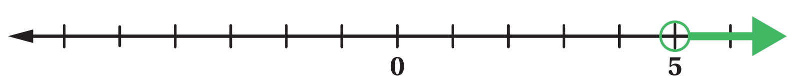
Solutions to One or More Inequalities

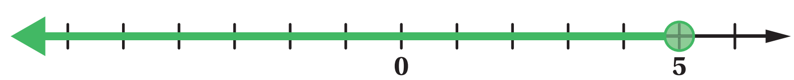
Let's think about the solutions to inequalities.



Warm-up Is Five a Solution?

Determine whether 5 is a solution to each inequality. If 5 is *not* a solution, provide at least one example of a solution. Be prepared to explain your thinking.

1. 

2. 

3. $n \geq 5$

4. $n < 5$

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 Amusement Park Rides

The brochure for an amusement park includes this table showing the height requirements for some of the rides.

To ride the . . .	You must be . . .
High Bounce	between 55 and 72 in. tall
Climb-A-Thon	under 60 in. tall
Twirl-O-Coaster	58 in. minimum
Tilt-A-Round	58 in. maximum

1. Jada is 58 in. tall. Kiran is 60 in. tall. What rides can they ride together? Use the number lines to support your response.

High Bounce:

Climb-A-Thon:

Twirl-O-Coaster:

Tilt-A-Round:

Activity 1 Amusement Park Rides (continued)

2. Han's cousin is 55 in. tall. Han says she is not tall enough to ride the High Bounce, but Tyler says that she is tall enough. Do you agree with either Han or Tyler? Explain your thinking.

3. Priya can ride the Climb-A-Thon, but she cannot ride either the High Bounce or the Twirl-O-Coaster. Select *all* the heights that could represent Priya's height. Be prepared to explain your thinking.

A. 59 in.

B. 53 in.

C. 56 in.

D. 54.9 in.

4. Write at least one inequality for each of the height requirements. Use h for the unknown height, in inches.

a High Bounce: ....

b Climb-A-Thon: ....

c Twirl-O-Coaster: ....

d Tilt-A-Whirl: ....

5. The inequalities $h < 75$ and $h > 64$ represent the height restrictions, in inches, for the Roller Ride. If Diego is tall enough to ride the Roller Ride, how tall could he be? List at least 3 possible heights, in inches. Be prepared to explain your thinking.

Name: .... Date: .... Period: ....

Activity 2 What Number Am I?

Your group will be given three sets of cards — one set shows inequalities, another set shows number lines, and the third set shows rational numbers. Place the inequality and number line cards face up where everyone can see them. Shuffle the number cards and stack them face down.

Plan ahead: How can being prepared to justify your conclusions help your group be successful in the activity?

To play:

- One person in your group is the *detective*. The others will give clues.
- Select one number card from the stack — the *mystery number* — and show it to everyone except the detective.
- The people giving clues each choose either an inequality card or a number line card that will help the detective identify the mystery number.
- The detective studies the selected cards — inequalities and/or number lines — and then makes a guess of what the mystery number might be.
  - » If the detective does not guess the number, everyone else chooses another card, which is again either an inequality card or number line card. All the previously selected cards should remain face up.
  - » The game ends when the detective has either identified the mystery number (wins), or made three incorrect guesses (loses).
- Repeat the game until everyone has a turn being the detective. Be sure to select a new mystery number for each game.

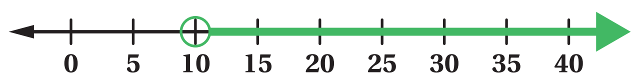
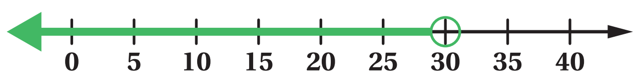
STOP

Summary

In today's lesson . . .

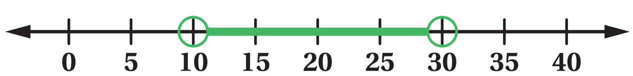
Sometimes a scenario involves both upper and lower bounds, or both a maximum and minimum possible value. To represent these types of situations, you can write two inequalities — one for the upper bound and one for the lower bound.

- For example, if you know that it rained for more than 10 minutes, but less than 30 minutes, you write two separate inequalities and graph them on two separate number lines.

Inequality	Number line
$r > 10$	
$r < 30$	

Any number greater than 10 is a solution to $r > 10$, and any number less than 30 is a solution to $r < 30$. But to meet *both* conditions — more than 10 *and* less than 30 — a solution must be a number that makes both inequalities true. By substituting the same value for r into each inequality, you can determine whether the value is a solution to both inequalities.

You can also represent all the possible solutions that satisfy both inequalities by graphing the two inequalities on the same number line, but only shading the values that overlap. The values that overlap are solutions to both inequalities — as shown.



Reflect:

Practice

Name: .... Date: .... Period: ....

1. An amusement park ride has the following restrictions on height h, in inches: $h > 48$ and $h < 65$.

   a. Draw one number line to represent all the possible values that satisfy both height restrictions.

   b. Mai is 48 in. tall, and Clare is 65 in. tall. Can either of them get on this ride? Explain your thinking.

2. Select *all* the numbers that are solutions to both $c < 5$ and $c > -1$.

   A. -1

   B. -1.5

   C. 0

   D. 5

   E. 5.05

   F. 4

   G. -0.5

3. One day in Boston, MA, the high temperature was $60°\text{F}$ and the low temperature was $52°\text{F}$.

   a. Write two inequalities to describe all the possible temperatures t in Boston, MA on that day.

   b. Represent all the possible temperatures on one number line.

Name: .... Date: .... Period: ....

4. Select *all* the true statements.

A. $-5 < |-5|$

B. $|-6| < -5$

C. $|-6| < 3$

D. $4 < |-7|$

E. $|-7| < |-8|$

5. Match each equation with its solution.

Equation		Solution
a $x^4 = 81$	....	2
b $x^2 = 100$	....	3
c $x^3 = 64$	....	4
d $x^5 = 32$	....	10

6. A number n is greater than -5, but less than 5.

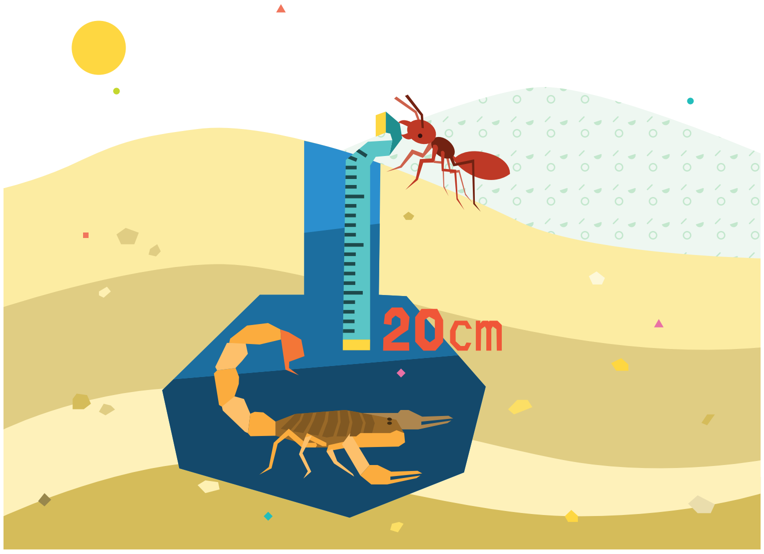
a Represent *all* the solutions that are negative numbers on a number line.

b Represent *all* the solutions that are non-negative numbers on a number line.

Name: ________ Date: ________ Period: ________

Unit 7 | Lesson 12

Interpreting Inequalities



Let's examine what inequalities can tell us.

Warm-up Turtle Clutch

On average, sea turtles lay n eggs in a *clutch* (a group of eggs).

1. What does $50 \leq n$ mean in this scenario?

2. What does $n \leq 110$ mean in this scenario?

3. Draw two number lines to represent the solutions to each of the two inequalities.

4. Name a possible value for n that is a solution to both inequalities.

Log in to Amplify Math to complete this lesson online.

Activity 1 Extreme Elevations

You will compare the elevations where some animals, including humans, live on Earth. Throughout this activity, be prepared to explain your thinking.

Part 1

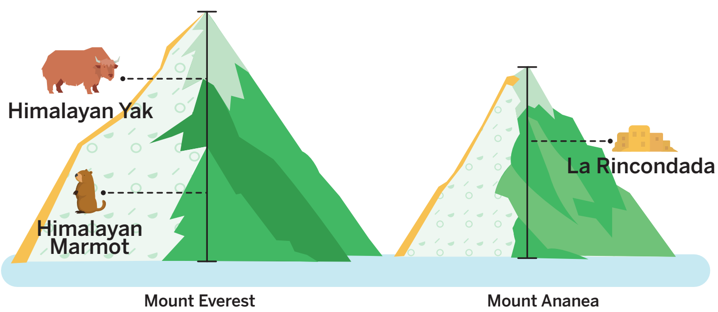
The deathstalker scorpion and Sahara desert ant live at the same elevation in the Sahara desert.

1. Write an equation to represent this scenario. Use d to represent the elevation of the deathstalker scorpion and a to represent the elevation of the Sahara desert ant.

2. To escape the scorching heat of the desert, the deathstalker scorpion spends most of the day buried 20 cm below the ground. Write an inequality to compare the elevations of a buried deathstalker scorpion and a Sahara desert ant.

Part 2

The image shows Mount Everest, shared by Nepal and China, and Mount Ananea in Peru.



3. Write an inequality to compare the Himalayan yak's elevation y to the Himalayan marmot's elevation m.

4. The town of La Rinconada is home to 50,000 people living at the extreme elevation of 16,700 ft on Mount Ananea.

   a. Write an inequality to compare the marmot's elevation m to the elevation of La Rinconada.

   b. What is one possible elevation for the Himalayan yak?

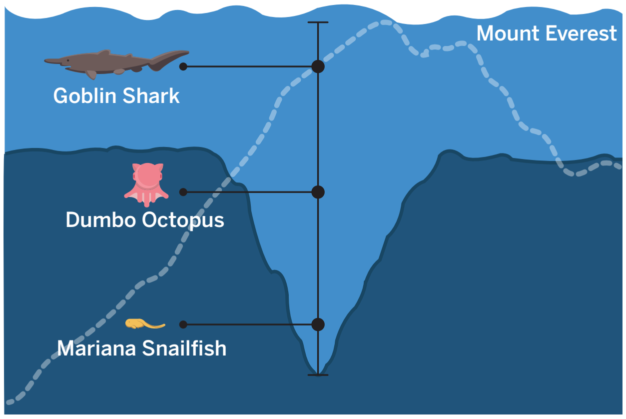
   c. What is one possible elevation for the Himalayan marmot?

Name: ______ Date: ______ Period: ______

Activity 1 Extreme Elevations (continued)

Part 3

The maximum depth of the Mariana Trench is 36,201 ft, which is more than 7,100 ft farther below sea level than the distance that Mount Everest rises above sea level! The image shows some creatures that live at different depths in and around the Mariana trench.



5. Mariana snailfish have been found as deep as 26,716 ft below sea level. Write an inequality comparing this location to either of the locations of the Dumbo octopus o or the Goblin shark g.

6. The Dumbo octopus is the deepest dwelling octopus, living around 11,000 ft below sea level.

   a. Write an inequality comparing the locations of the goblin shark and the Dumbo octopus.

   b. List two possible locations where the Goblin shark could be found swimming, if it does not swim more than 3,200 ft below sea level.

   c. Tyler says that the Goblin shark could be found at an elevation of 2,000 ft because $2,000 > -11,000$? Do you agree or disagree with Tyler? Explain your thinking.

7. Another sea creature living in the Mariana trench is the fangtooth fish. It can be found swimming between 1,640 and 16,400 ft, inclusive, below sea level.

   a. Write two inequalities that represent the possible locations where the fangtooth fish could be found swimming.

   b. Complete the following inequality statements, about the locations where the Dumbo octopus and fangtooth fish live, to make them true. Use *each* <, >, ≤, ≥ *symbol only once.*

   $-1,640 \square\, o \quad -16,400 \square\, o \quad -11,000 \square\, f \quad -11,000 \square\, f$

Activity 2 Extreme Temperatures

Usually extreme elevations mean extreme temperatures. You will be given a map showing the extreme temperatures in which some animals live.

1. The Himalayan yak and the people of La Rinconada live at extreme elevations above sea level. Let's look at the temperatures in which they can survive.

a Write *two* inequalities that represent the temperatures, in degrees Fahrenheit, at which the Himalayan yak can survive.

b What is an example of a negative temperature in which the Himalayan yak could survive?

c Write *two* inequalities that represent the temperatures, in degrees Fahrenheit, seen in La Rinconada.

d What is an example of a temperature that might be observed during the summer months in La Rinconada?

2. The animals living in the Mariana Trench live at extreme elevations below sea level, but the water temperature around the habitat of the Dumbo octopus does not fluctuate much. What are all the possible whole number temperatures of the water where it lives?

3. What is an example of a temperature in which the Sahara desert ant could survive, but the deathstalker scorpion could not?

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You used inequalities to represent possible elevations and temperatures in which animals on Earth can survive. When you are determining the ***solutions to an inequality*** that represent a real-world scenario, you need to consider what values are reasonable in the context. Some numbers that are solutions to an inequality outside of a context may not make sense when you consider the context.

Scenario	Noah scored less than 25 points in a basketball game.	The temperature in the summer is between 65°F and 103°F.
Reasonable values	$24, 22, 8$	$65.8, 93, 103$
Inequality or Inequalities	$p < 25$ and $p \geq 0$	$t \geq 65$ and $t \leq 103$
Discrete or Continuous?	Discrete Only whole-number values of points can be scored in basketball, so not *all* values between 0 and 25 points are solutions.	Continuous Temperature can be measured in decimal and fraction values.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. There is a closed carton of eggs in Mai's refrigerator. The carton contains e eggs, and it can hold up to 12 eggs.

a What does the inequality $e < 12$ mean in this context?

b What does the inequality $e > 0$ mean in this context?

c What are some possible realistic values of e that will make both $e < 12$ and $e > 0$ true?

2. Tyler has more than $\$10$. Elena has more money than Tyler. Mai has more money than Elena. Let t be the amount of money that Tyler has, e be the amount of money that Elena has, and m be the amount of money that Mai has. Select *all* the statements that must be true.

A. $m = e$

B. $m > 10$

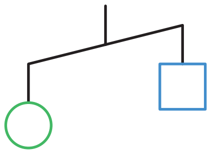
C. $e > 10$

D. $t > 10$

E. $e > m$

F. $t < e$

3. Here is a diagram of an unbalanced hanger.



a Write an inequality to represent the relationship between the two weights. Use s to represent the weight, in grams, of the square, and use c to represent the weight, in grams, of the circle.

b One circle weighs 12 grams. Write an inequality to represent the weight of one square.

c Could 0 be a value of s? Explain your thinking.

Name: .... Date: .... Period: ....

Practice

4. Write an inequality statement to represent each scenario.

a. Jada is taller than Diego. Diego is 54 in. tall. Write an inequality that compares Jada's height j, in inches, to Diego's height.

b. Jada is shorter than Elena. Elena is 5 ft tall. Write an inequality that compares Jada's height j, in inches, to Elena's height.

5. Select *all* the expressions that are equivalent to $\left(\frac{1}{2}\right)^3$.

A. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

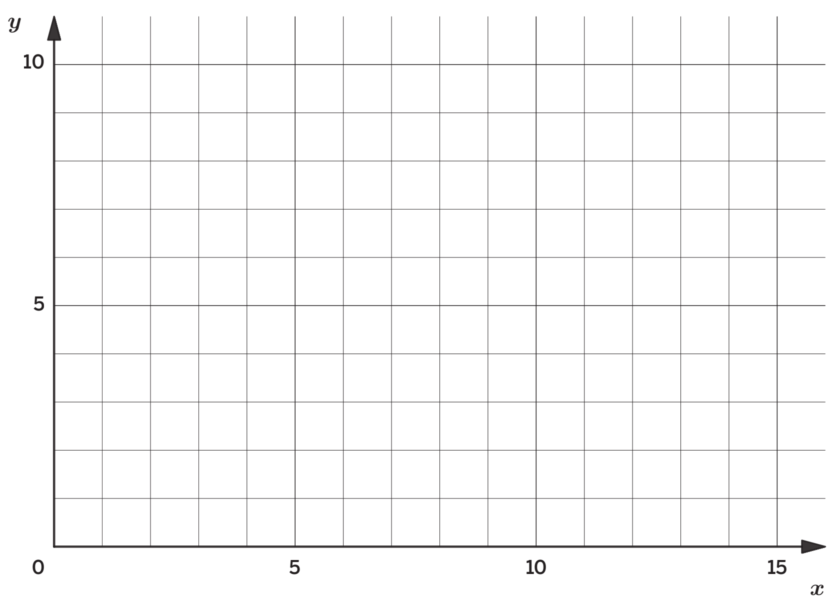
B. $\left(\frac{1}{3}\right)^2$

C. $\frac{1 \cdot 1 \cdot 1}{2 \cdot 2 \cdot 2}$

D. $\frac{1}{6}$

E. $\frac{1}{8}$

6. Select four points and plot them on the coordinate plane. Label them as ordered pairs.



My Notes:

SUB-UNIT

3 The Coordinate Plane

Narrative Connections

How did Greenland get so big?

Maybe it's not as big as you think . . .

If you remember your geography, Greenland is that big, chunky landmass, just northeast of Canada. It's almost as big as Africa! Except . . . it isn't.

Greenland is actually *14 times smaller* than Africa. So why does it look so distorted?

For that, we can thank the mapmaker Gerardus Mercator. In 1569, he invented a novel way of visualizing maps that we still use today, called the "Mercator Projection."

You see, longitude and latitude (the horizontal and vertical lines on maps) aren't really straight lines at all. They're arcs that trace a path along the curved surface of the Earth.

These circles are perfectly sensible on a 3D globe. But for 2D maps, they're a nightmare, especially for sailors. Figuring out longitude, in particular, required precise measurements of time, speed, and bearing, just to even *approximate* where you were.

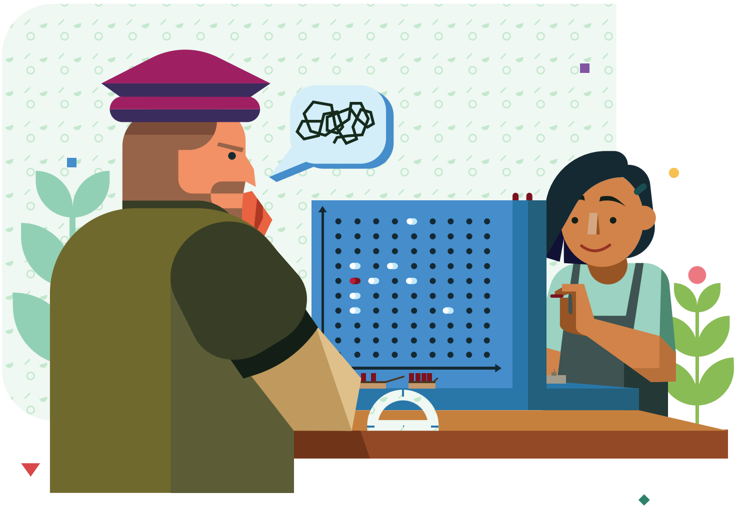
Before Mercator, mapmakers (sensibly) drew latitude and longitude as curves. But that rebel Mercator, depicted them as straight, perpendicular lines. This allowed ships to follow one bearing, without having to make too many little adjustments.

However, straightening these lines meant having to stretch and distort the continents, especially closer to the poles. And that's why places like Greenland and Antarctica look humongous — a tradeoff sailors were happy to make. By using parallel and perpendicular lines, Mercator effectively placed a grid over the world, making it simpler to locate things and plot an effective course to any spot on the map.

Unit 7 | Lesson 13

Extending the Coordinate Plane

Let's explore and extend the coordinate plane.



Warm-up Ship Versus Ship

You will be given two coordinate grids, one labeled MINE and one labeled THEIRS, and four cut-out ships.

1. You have two "ships" that both have a length of 4 units. On your grid labeled MINE, place Ship A along a horizontal line and Ship B along a vertical line. Both ships should start and end at an intersection point with whole number coordinates. In the table, record the four points with whole number coordinates that Ship A and Ship B each cover.

Ship A	Ship B

2. Play the game as follows:
   - Take turns guessing the locations of your partner's two ships by naming coordinates.
   - If your partner's guess is a point occupied by one of your ships, say "hit." If it is not, say "miss."
   - On your paper, record the results of your guesses on the THEIRS grid, and your partner's guesses on the MINE grid. Place an "X" on points that are hits and an "O" on points that are misses.
   - Once all four points of a ship have been identified, that ship is sunk.
   - The first person to sink both of their partner's ships wins.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Double-Folded Coordinate Planes

You will be given a straightedge and a coordinate plane, labeled with horizontal axis x and vertical axis y.

Part 1

1. Follow the steps to your coordinate plane in one direction.
   - Carefully fold your paper along the y-axis and mark a crease there.
   - Use the straightedge to draw a vertical line along the crease that extends the y-axis downward. Draw an arrow at the end of the line.
   - Use what you know about number lines to label the new tick marks on the y-axis.

2. Plot a point in the space that is below the x-axis and to the right of the y-axis. How can you identify the location of this point using coordinates?

Part 2

3. Follow the steps to extend your coordinate plane in the other direction.
   - Carefully fold your paper along the x-axis and mark a crease there.
   - Use the straightedge to draw a horizontal line along the crease that extends the x-axis to the left. Draw an arrow at the end of the line.
   - Use what you know about number lines to label the new tick marks on the x-axis.

4. Plot a point in each of the spaces that are to the left of the y-axis: one above the x-axis, and one below it. How can you identify the locations of these points using coordinates?

5. Study your coordinate plane.

   a. How many distinct areas are there on your coordinate plane now?

   b. What do you think a good name for each of these areas might be?

Activity 2 Ultimate Ship Versus Ship

You will be given two more coordinate grids, one labeled MINE and the other labeled THEIRS. Now, there will be four *quadrants* in each grid. You will need the same four cut-out ships you used in the Warm-up.

Plan ahead: How will you apply your previous knowledge of the game structure to better your chances of winning?

1. You have four "ships" that each have a length of 4 units. On the grid labeled MINE, place each ship along either a horizontal or vertical line. You can only place one ship in each quadrant. Each ship should start and end at an intersection point with whole number coordinates. In the table, record the four points with integer coordinates that each of your ships cover.

Ship A	Ship B	Ship C	Ship D

2. Play the game just like in the Warm-up:
   - Take turns guessing the locations of your partner's ships by naming coordinates.
   - If your partner's guess is a point occupied by one of your ships, say "hit." If it is not, say "miss."
   - On your paper, record the results of your guesses on the THEIRS grid, and your partner's guesses on the MINE grid. Place an "X" on the points that are hits and an "O" on points that are misses.
   - Once all four points of a ship have been identified, that ship is sunk.
   - The first person to sink all four of their partner's ships wins.

STOP

Name: .... Date: .... Period: ....

Summary

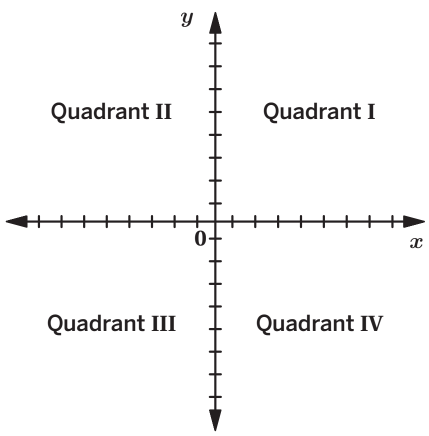
In today's lesson . . .

You extended the coordinate plane to include negative numbers. Just as a number line can be extended to the left (horizontally) or down (vertically) to include negative numbers, the x- and y-axes of the coordinate plane can also be extended to include negative numbers. In doing so, the axes cross at the origin, $(0, 0)$, to create four regions, called ***quadrants***. The names of these quadrants use Roman numerals, so that we can reference them easily.

- For example, "a point in quadrant III" indicates a point that is located in the lower left quadrant, left of the y-axis and below the x-axis.

Points represented by ordered pairs can have negative x- and y-coordinates.

This coordinate plane shows the locations of quadrants I, II, III, and IV. The table shows the signs of the x- and y-coordinates of any point located in each quadrant.



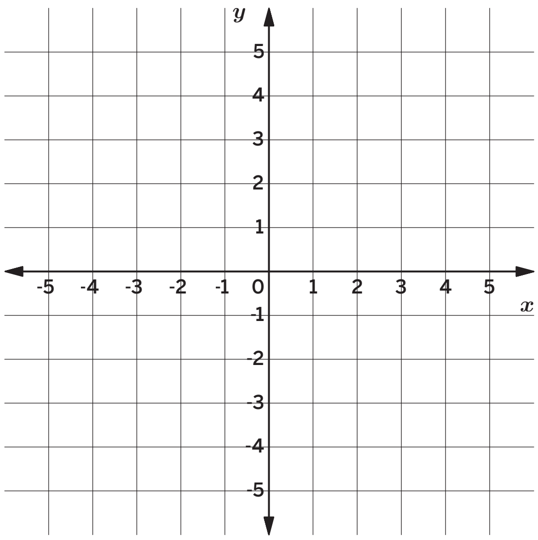
Quadrant	x-coordinate	y-coordinate
I	positive	positive
II	negative	positive
III	negative	negative
IV	positive	negative

Reflect:

Practice

Name: ________ Date: ________ Period: ________

Use the coordinate plane to plot points for Problems 1 and 2.



1. Plot and label the following four points: $A(-2, 3)$, $B(2, 3)$, $C(-2, -3)$, and $D(2, -3)$.

2. Plot a point on one of the axes and label it E. What are the coordinates of point E?

3. These three coordinates form a line: $(-3, 4)$, $(0, 4)$, and $(6, 4)$.

a Is the line vertical or horizontal? Explain how you know.

b Write the coordinates for two other points that are also on this same line.

4. Lin ran 29 m in 10 seconds. She ran at a constant speed.

a How far did Lin run every second?

b At this rate, how far can she run in 1 minute?

Name: .... Date: .... Period: ....

5. One night, it was $24°C$ degrees warmer in Tucson, AZ, than it was in Minneapolis, MN. If the temperatures in Tucson and Minneapolis were opposites, what was the temperature in Tucson?

A. $-24°C$

B. $-12°C$

C. $12°C$

D. $24°C$

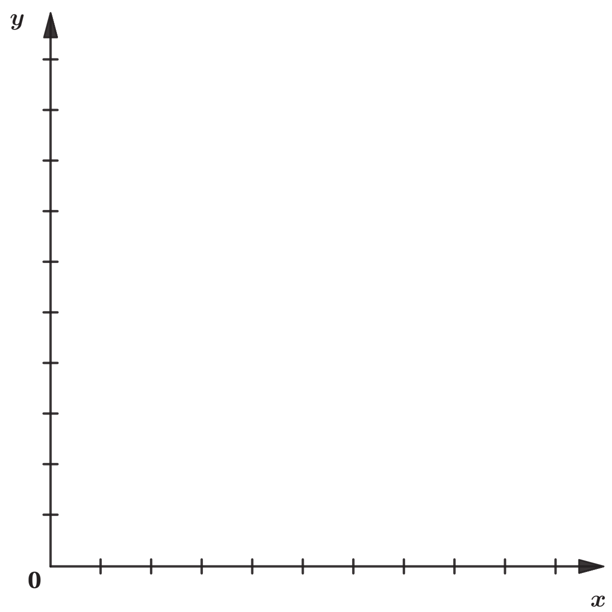
6. Refer to the coordinate plane showing the first quadrant. If the intervals of the ticks on the x- and y-axes increased by 1s, then a point that is plotted exactly halfway between 0 and the first tick on either axis would have a coordinate value of 0.5.

What would be the coordinate value of a point that is plotted exactly halfway between 0 and the first tick on either axis if the intervals increased by:

a 2s

b 10s

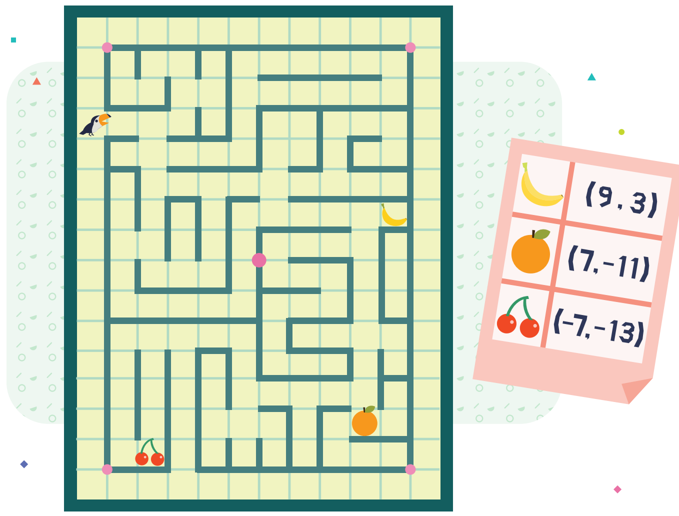
c 0.5s



Unit 7 | Lesson 14

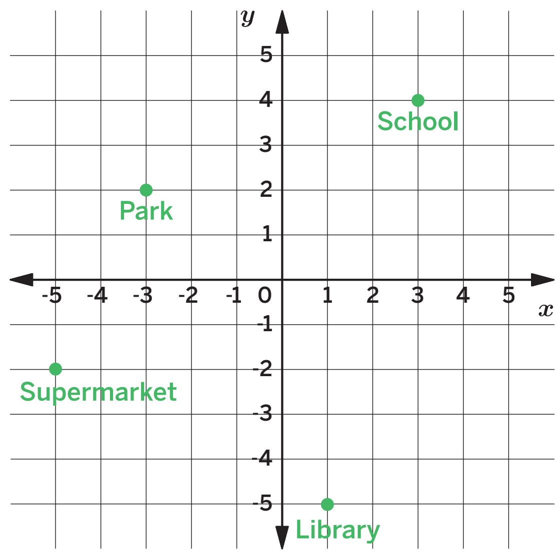
Points on the Coordinate Plane

Let's plot points between the tick marks on a coordinate plane.



Warm-up A Map of the Town

Use the map to complete the table indicating the place, quadrant, and coordinates of each location in the town.



1.

Place	Quadrant	Coordinates
School	I	(3, 4)
	IV	(1, −5)
Supermarket		

2. Think of another place to include in the town. Plot its location on the town map, and then write down the quadrant in which it is located, and its coordinates.

Place:

Quadrant:

Coordinates:

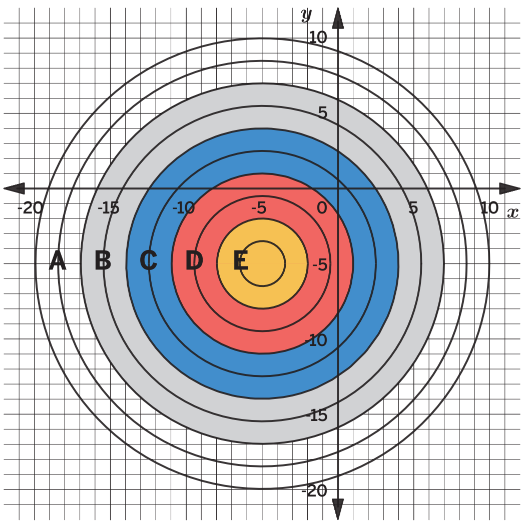
Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Coordinated Archery

Part 1

A single-spot target like this one is commonly used in many archery competitions, and the scoring system is shown in the table.



	Outer ring (points)	Inner ring (points)
White ring (A)	1	2
Grey ring (B)	3	4
Blue ring (C)	5	6
Red ring (D)	7	8
Yellow ring (E)	9	10

1. How many points would you score if you hit the target at each of these coordinates?

   a. $(-5, -2)$

   b. $(-9.5, 1)$

2. Write an ordered pair for a hit that would score 2 points.

3. Clare hit the target four times, but each arrow hit the target in a different quadrant. Write possible coordinates for each hit in the table and determine her total score.

   Clare's total score would be .... points.

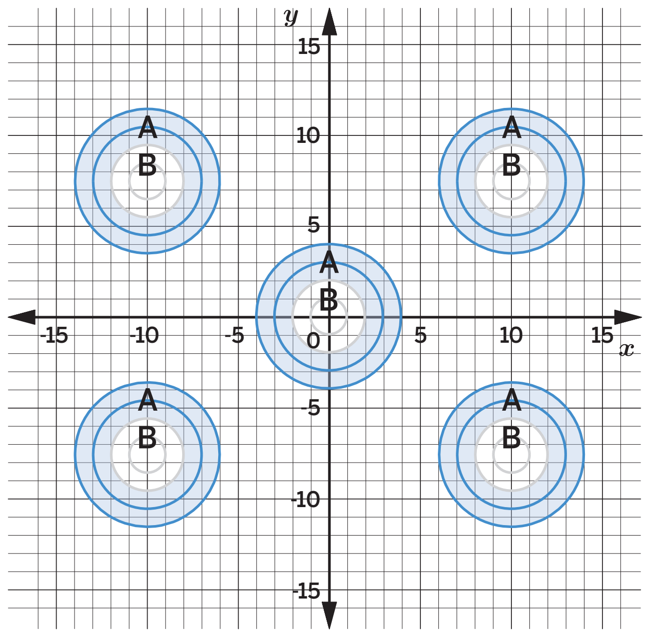
	Coordinates	Point(s)
Quadrant I		
Quadrant II		
Quadrant III		
Quadrant IV		

Activity 1 Coordinated Archery (continued)

Part 2

A five-spot target like the one shown is used in some other archery competitions, and has the following scoring system:

- **4 points for any blue ring (A)**
- **5 points for any white ring (B)**



4. Choose either 16 or 20 points as your goal. You have four arrows to hit the target and score *exactly* the chosen point total. Each arrow must hit the target in a different quadrant. Write the coordinates of four possible hits that allow you to achieve your goal.

My Goal: 20 points or 16 points (circle one)

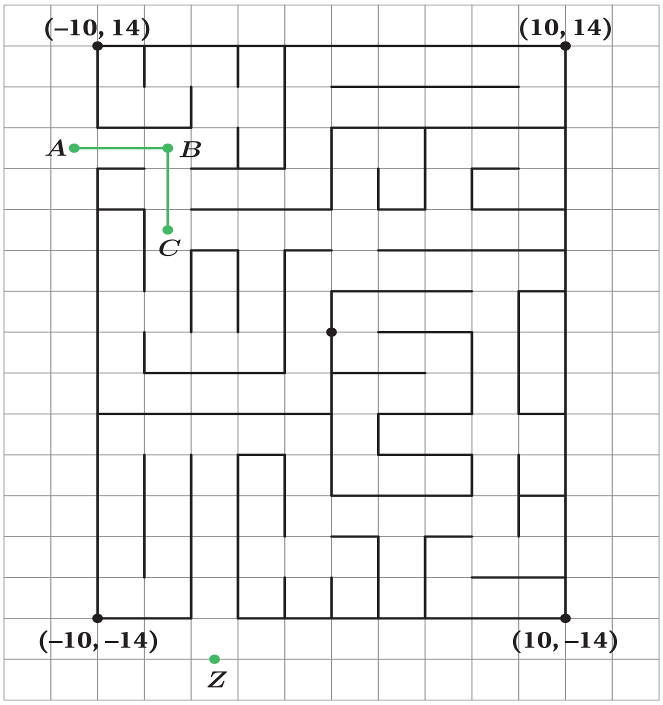
	Coordinates
Quadrant I	
Quadrant II	
Quadrant III	
Quadrant IV	

Name: .... Date: .... Period: ....

Activity 2 A Coordinate Maze

Refer to the maze. The point in the center is located at (0, 0), and the side of each grid square is 2 units long.

1. You enter the maze by traveling from point A to point B. You turn at point B and travel to point C. Continue drawing line segments to show your way through the maze, all the way to the exit to reach point Z. As you work, label each turning point with a letter and write its coordinates in the table.



Point	Coordinates
A	(–11, 9)
B	(–7, 9)
C	(–7, 5)

2. Choose any two turning points from your path that share the same line segment. What is the same about their coordinates? Explain why they share that feature.

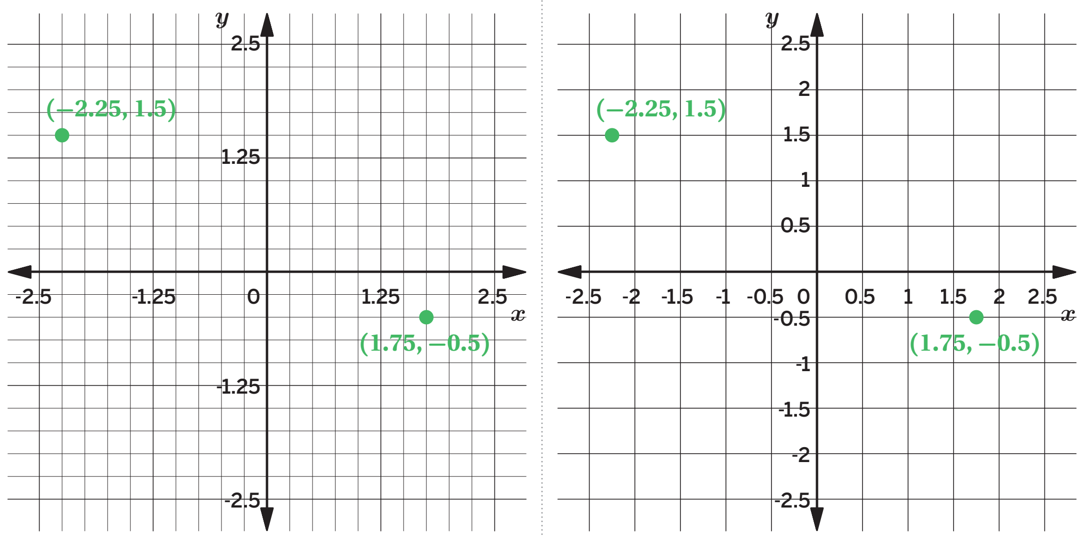
STOP

Summary

In today's lesson . . .

You worked with points that are plotted between the tick marks on a coordinate plane. The coordinate plane can be used to represent ordered pairs of numbers. When plotting or interpreting points on the coordinate plane, you need to pay attention to the scales of the axes — the intervals between the tick marks. A scale of 1 is often used, but sometimes different scales are more advantageous to show a set of really large or really small values. No matter the scale, every possible combination of values can be plotted, but some scales help you to more efficiently read and interpret coordinates.

For example, consider the coordinates of the points $(1.75, -0.5)$ and $(-2.25, 1.5)$.



- A scale of 0.25 is used on both axes.
- Both sets of coordinates are divisible by 0.25.
- Both points are plotted at intersection points on the grid.

- A scale of 0.5 is used on both axes.
- -2.25 and 1.75 are not multiples of 0.5, so neither point is plotted at an intersection point on the grid.
- 1.75 is plotted halfway between 1.5 and 2.
- -2.25 is plotted halfway between -2.5 and -2.

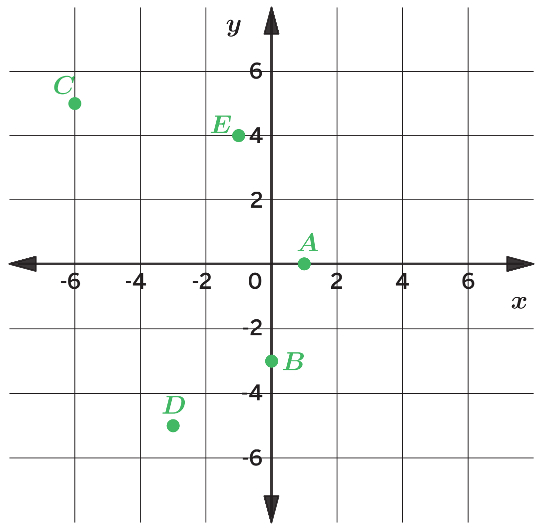
Reflect:

Name: .... Date: .... Period: ....

Practice

1. Write the coordinates of each point.

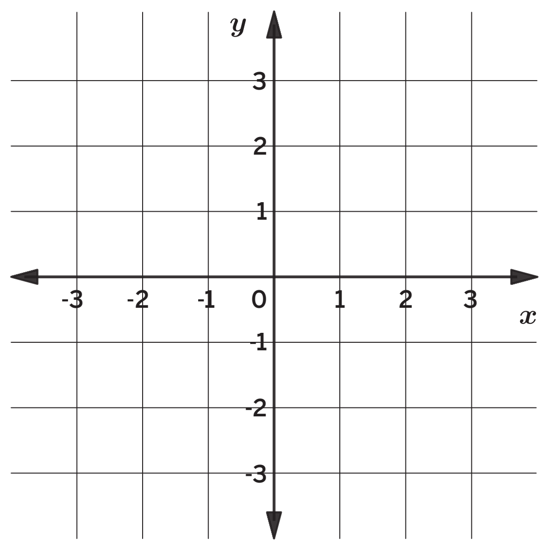
Point	Coordinates
A	
B	
C	
D	
E	



2. Refer to the coordinate plane.

a. Name four points with non-integer coordinates that would form a square with the origin at its center.

b. Plot these points on the coordinate plane to verify that they form a square.



3. Refer to the coordinate plane.

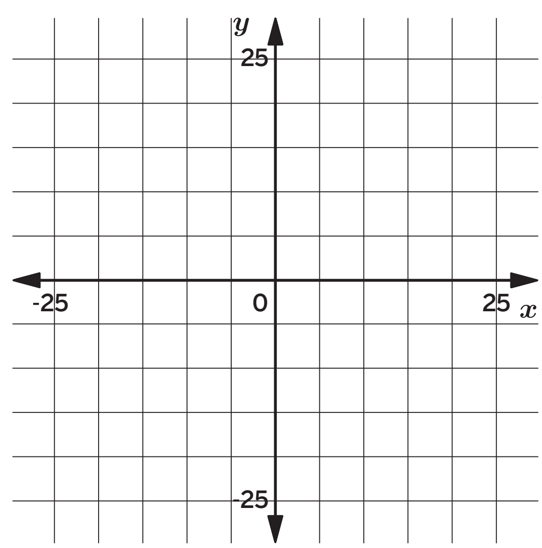
a. What is the scale for this coordinate plane?

b. Plot and label points A, B and C on the coordinate plane, approximating as needed.

$A(-10, 11)$

$B(-7.5, -12.5)$

$C(21, 24)$



Practice

Name: .... Date: .... Period: ....

4. Which of the following changes would you represent using a negative number? Explain what a positive number would represent in that same context.

A. a loss of 4 points

B. a gain of 50 yards

C. a loss of $10

D. an elevation above sea level

5. A corn field has an area of 28.6 acres. It requires about 15,000,000 gal of water. Approximately how many gallons of water does it require per acre?

A. 5,000 gallons

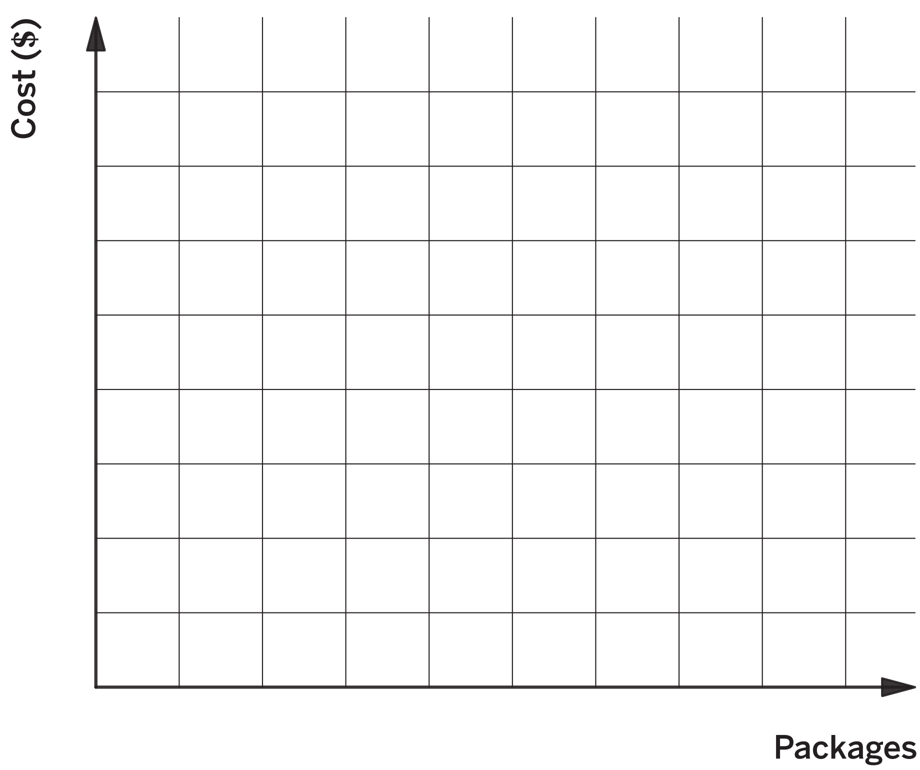
B. 50,000 gallons

C. 500,000 gallons

D. 5,000,000 gallons

6. Shawn went to the post office twice in the last week. The first time, Shawn sent 3 packages for a total of 25. The second time, Shawn sent 5 packages for a total of 40.

Label the tick marks on the x- and y-axis, and then plot two points to represent the relationship between packages sent and money spent.



Name: ______ Date: ______ Period: ______

Unit 7 | Lesson 15

Interpreting Points on the Coordinate Plane

Let's examine what points on the coordinate plane can tell us.

Warm-up English Winter

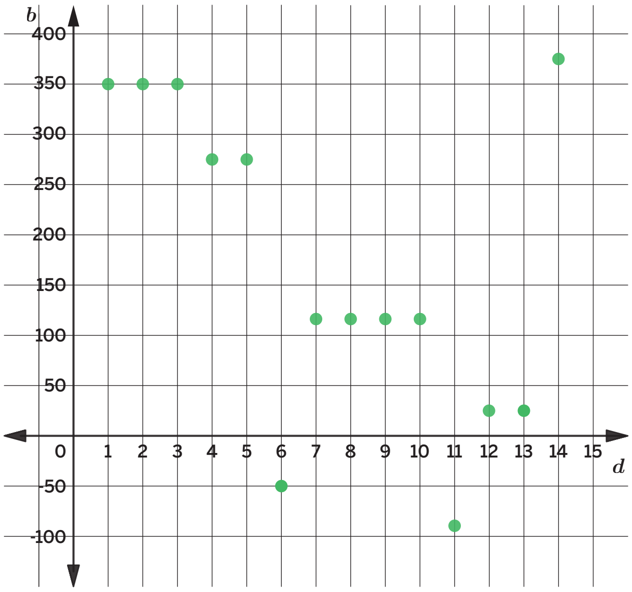
The following data were collected over one December afternoon in England. Draw and label an appropriate pair of axes, with time on the horizontal axis and temperature on the vertical axis. Then plot the points.

Time after noon (hours)	0	2.5	4	6	7	8
Temperature (°C)	5	3	1	$-\frac{7}{2}$	-4	-4

Log in to Amplify Math to complete this lesson online.

Activity 1 Interpreting Account Balances

The graph shows the balance in a bank account over a period of 14 days. The axis labeled b represents the account balance in dollars, and the axis labeled d represents the day.



1. Estimate the greatest account balance. On which day did it occur?

2. Estimate the least account balance. On which day did it occur?

3. What does the point $(6, -50)$ tell you about the account balance?

4. How can you interpret $|-50|$ within this context?

5. Write two inequalities to describe the account balance b in dollars over the entire 14-day period.

Name: ______ Date: ______ Period: ______

Activity 2 Elevation and Temperature on Mauna Kea

Mauna Kea, the tallest mountain in the world, has a total height of 10,210 m. Its height extends from its base that is 6,005 m below sea level to its peak that is 4,205 m above sea level. The following data were collected one afternoon at different elevations on Mauna Kea.

Temperature (°C)	Elevation (m)
-1	4,205
2	3,500
16	1,500
25	0
20	-200
10	-700
4	$-1,000$
0	$-6,005$

1. On a piece of graph paper, draw and label an appropriate pair of axes, with temperature on the x-axis and the elevation on the y-axis. Then plot the data.

2. What does a point in quadrant III represent in this context?

3. Write two inequalities to describe all the possible elevations e from the base of Mauna Kea to its peak.

4. Write two inequalities to describe the temperatures t recorded on Mauna Kea during that afternoon.

Critique and Correct:
Your teacher will display two incorrect inequalities. Work with your partner to critique the inequalities, correct them, and clarify why and how you corrected them.

STOP

Summary

In today's lesson . . .

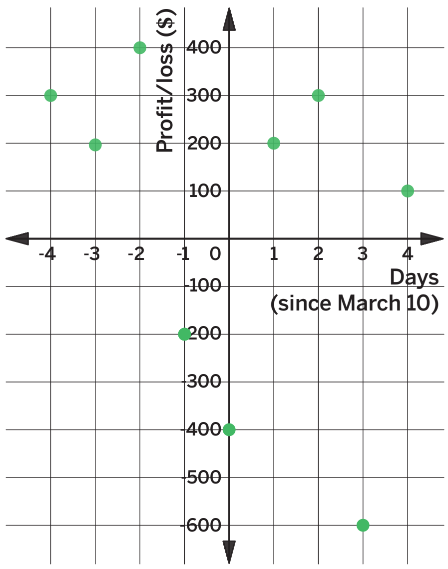
You interpreted points plotted on the coordinate plane. The coordinate plane can be used to show relationships between two quantities that are not just horizontal and vertical locations. Each axis can represent a different quantity, and in order to plot all the values, you can change the scales along the axes of a coordinate plane by selecting appropriate:

- maximum and minimum coordinates, which may need to be different on each axis, and
- intervals between tick marks or grid lines — the scale of the axis — which also may need to be different on each axis.

The origin is always the point on the coordinate plane where the x- and y-axes cross, but it does not always have to be shown as the center of a graph.

Points on a coordinate plane can also be used to represent and interpret information about a given scenario that involves two related quantities.

- For example, this graph shows a company's daily profits or losses recorded on different days around March 10 of one year.



Knowing what quantities the x- and y-axes each represent, you can interpret both the meaning of individual coordinates as well as the meaning of the ordered pair as a whole.

- For example, on the graph shown, an x-coordinate of -4 represents the day 4 days *before* March 10 (or March 6), and a y-coordinate of 300 represents a profit of $\$300$. Together, the point $(-4, 300)$ represents that on March 6, the company made a profit of $\$300$.

Reflect:

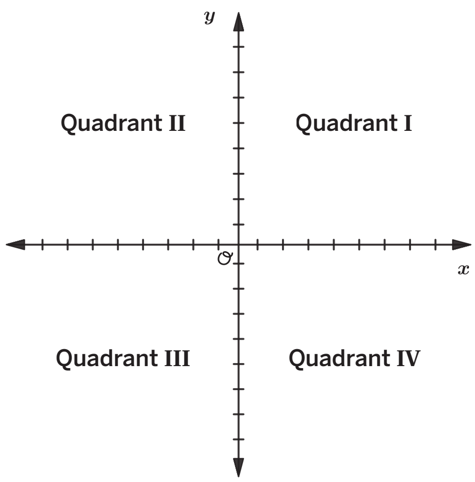
Practice

Name: ________ Date: ________ Period: ________

1. The elevation of a submarine at different times is shown in the table. Draw and label an appropriate pair of axes. Then plot the points.

Time after noon (hours)	Elevation (m)
0	-567
1	-892
2	$-1{,}606$
3	$-1{,}289$
4	-365

2. The x-axis represents the number of hours from noon (12 p.m.), and the y-axis represents the temperature in degrees Celsius.



a. At 9 a.m., it was below freezing. In which quadrant would this point be plotted?

b. At 11 a.m., it was 10°C. In which quadrant would this point be plotted?

c. Choose another time and temperature. Then determine the quadrant in which your point would be plotted.

d. What does the point $(0, 0)$ represent in this context?

Name: ______________ Date: ______________ Period: ______________

3. Diego will plot these points: $(-50, 0)$, $(150, 100)$, $(200, -100)$, $(350, 50)$, $(-250, 0)$. What interval could he use for the tick marks on each axis? Explain your thinking.

4. The inequalities $h > 42$ and $h < 60$ represent the height requirements for an amusement park ride, where h represents a person's height in inches. Write a sentence or draw a sign that describes these rules as clearly as possible.

5. Solve each equation.

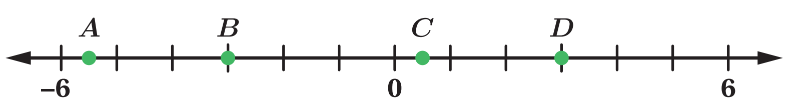
a $3a = 12$

b $b + 3.3 = 8.9$

c $1 = \frac{1}{4}c$

d $5\frac{1}{4} = d + \frac{1}{4}$

6. Refer to the number line.



a For each point, determine the distance from zero.

b What is the distance from B to D?

Name: ____________ Date: ________ Period: ________

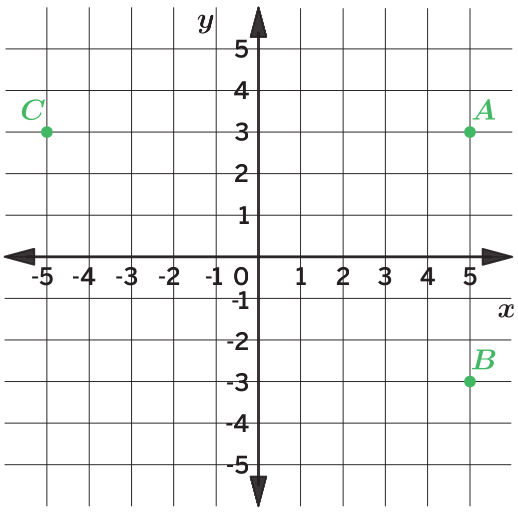
Unit 7 | Lesson 16

Distances on the Coordinate Plane

Let's explore distance on the coordinate plane.

Warm-up Reflections on the Coordinate Plane

Refer to the coordinate plane.



1. Label points A, B, and C with their coordinates.

2. Which point would line up with point A if you folded the graph along the y-axis? How far away are each of these points from the y-axis?

3. Which other point would line up with point A if you folded the graph along the x-axis? How far away are each of these points from the x-axis?

4. What should the coordinates be for a point D in quadrant III, that would complete this set of "similar" points? Explain how you determined those coordinates.

Log in to Amplify Math to complete this lesson online.

Activity 1 Crossing an Axis

You will be given a blank coordinate plane.

1. Point E is 2 units from the x-axis and 6 units from the y-axis. Plot point E on your graph and write its coordinates here.

2. Point F has the same coordinates as point E, except its y-coordinate has the opposite sign.

a Plot point F on your graph and write its coordinates here.

b In which quadrant is point F? To travel from point E to point F, which axis do you cross?

c How many units apart are points E and F?

3. Point G has the same coordinates as point E, except its x-coordinate has the opposite sign.

a Plot point G on your graph and write its coordinates here.

b In which quadrant is point G? To travel from point E to point G, which axis do you cross?

c How many units apart are points E and G?

4. Think about the signs of the coordinates of points in different quadrants of the coordinate plane.

a When two points are directly across the x-axis from each other, what is true about the y-coordinates of these points?

b When two points are directly across the y-axis from each other, what is true about the y-coordinates of these points?

Reflect: How well do you understand the material? How did repetition help your self-confidence?

Name: ______ Date: ______ Period: ______

Activity 2 Determining Distances on a Map

You will be given a map of a town represented on a coordinate plane. Each unit represents one block.

Place	Coordinates
Priya's house	
Supermarket	
Post Office	
School	$(3.5, -3)$
Library	
Town Hall	
Diego's house	

1. Label each place in town with its coordinates.

2. Determine the number of blocks Priya and Diego walk as they walk around the town.
   - a Priya walks from her house to the supermarket.
   - b Priya then walks from the supermarket to school.
   - c Diego walks from his house to the library.
   - d Diego then walks to the supermarket.
   - e How far did Priya walk all together? Diego?

3. If Priya stops at the post office after the supermarket, how could you determine the distance between the post office and the supermarket *without* counting? **Hint:** Study the coordinates of the supermarket and the post office.

4. The movie theater is located outside of town at $(32, 2)$. What is the distance between Priya's house and the movie theater? Explain your thinking.

5. Which place in town is located 2 blocks from Town Hall?

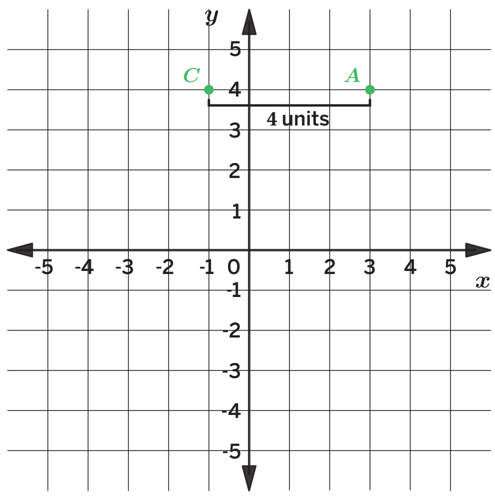
6. Noah's house is located in quadrant III. One coordinate of this point is the opposite of the coordinate of the point representing the school.
   - a What are the coordinates of Noah's house? Plot and label Noah's house on the map.
   - b How many blocks does Noah walk to school?

STOP

Summary

In today's lesson . . .

You discovered that for any two points that lie on the same horizontal or vertical line, you can determine the distance between them. Specifically you explored the following strategies for determining the distance between two points:

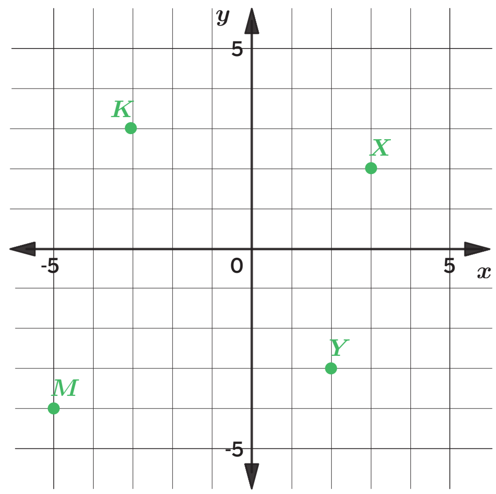
Strategy	Example
Count the units between them.	
If they are in the same quadrant, subtract the coordinates that are different.	For $(6, -2)$ and $(3, -2)$, the distance between them is: $6 - 3 = 3$ 3 units
If they are in different quadrants, use the absolute value to determine the distance each point is from the axes between them.	For $(-6, -2)$ and $(12, -2)$, first determine the distance that -6 and 12 are from the y-axis. $\|(-6)\| = 6$ $\|12\| = 12$ The distance is the sum of their respective distances. $6 + 12 = 18$ 18 units

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Points K, M, X and Y are plotted and labeled on the coordinate plane.

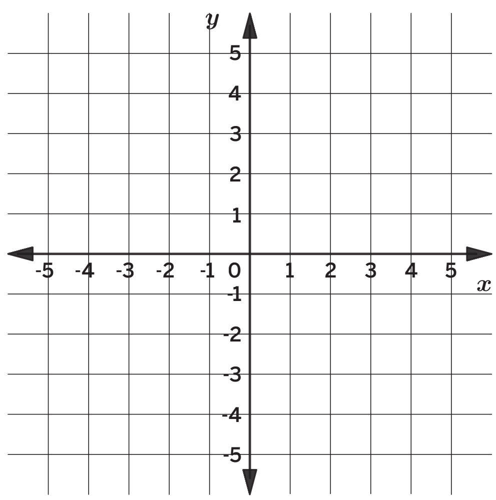


a Write the coordinates of each point.

b Plot a point that is located a distance of 3 units from point K. Label it P. What are its coordinates?

c Plot a point that is located a distance of 2 units from point M. Label it W. What are its coordinates?

2. Plot and label four points on the coordinate plane that are each located a distance of 3 units from point P at $(-2, -1)$. Write the coordinates of each of your four points.



3. Each pair of points is connected to form a line segment in the coordinate plane. What is the length of each line segment?

a $A(3, 5)$ and $B(3, 6)$

b $C(-2, -3)$ and $D(-2, -6)$

c $E(-3, 1)$ and $F(-3, -1)$

Practice

Name: ________ Date: ________ Period: ________

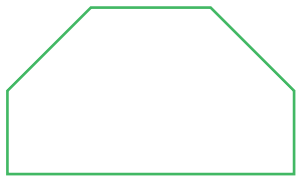
4. Noah's recipe for sparkling orange juice uses 4 liters of orange juice and 5 liters of soda water.

   a. Noah prepares large batches of sparkling orange juice for school parties. He usually knows the total number of liters t that he needs to prepare. Write an equation that shows how Noah can determine the number of liters s of soda water he needs if he knows t.

   b. Sometimes the school purchases a certain number of liters j of orange juice and Noah needs to determine how much sparkling orange juice he can make. Write an equation that Noah can use to determine t if he knows j.

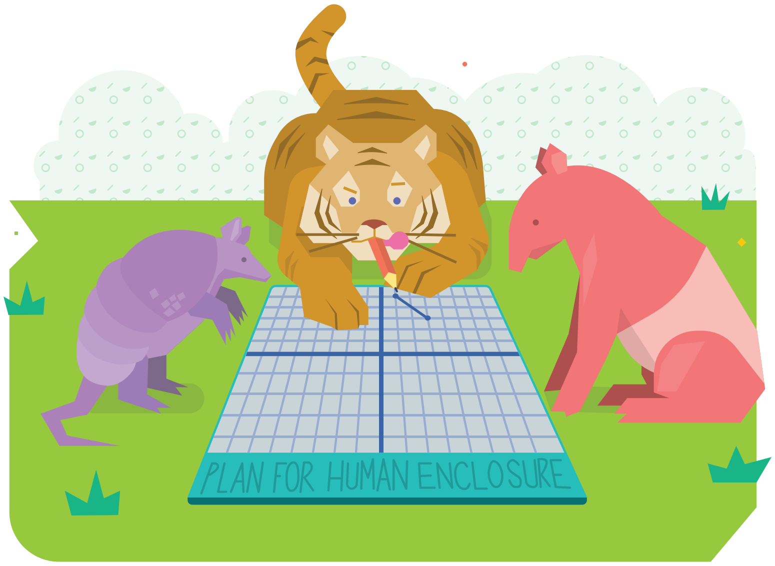
5. For a suitcase to be checked on a flight (instead of carried by hand), it can weigh at most 50 lb. Andre's suitcase weighs 23 kg. Can Andre check his suitcase? Explain or show your thinking. **Hint:** 10 kg ≈ 22 lb.

6. Describe the properties of the figure shown. Be sure to use the following words to describe the figure: *vertices*, *parallel lines*, *angles*. Is it a polygon?



Name: ______ Date: ______ Period: ______

Unit 7 | Lesson 17



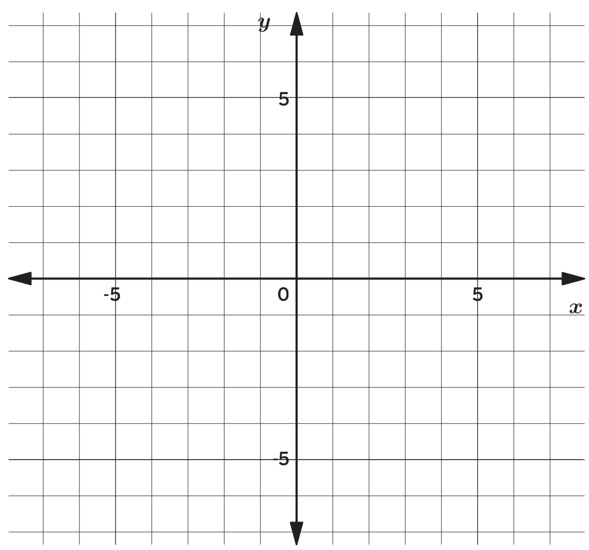
Shapes on the Coordinate Plane

Let's use the coordinate plane to solve problems and determine perimeters.

Warm-up Plotting Polygons

1. Draw a figure on the coordinate plane with at least three of the following properties:

- 6 vertices
- 1 pair of parallel sides
- 1 or more right angles
- 2 sides of the same length



2. Is your figure a polygon? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Polygons and Perimeter

The coordinate plane is a place where numbers and shapes come together. The Persian mathematician Omar Khayyam made these connections almost a thousand years ago, and the French mathematician René Descartes built on Khayyam's work to give us the coordinate plane we know and love today.

Take a look at these three polygons on the coordinate plane. Here are their vertices.

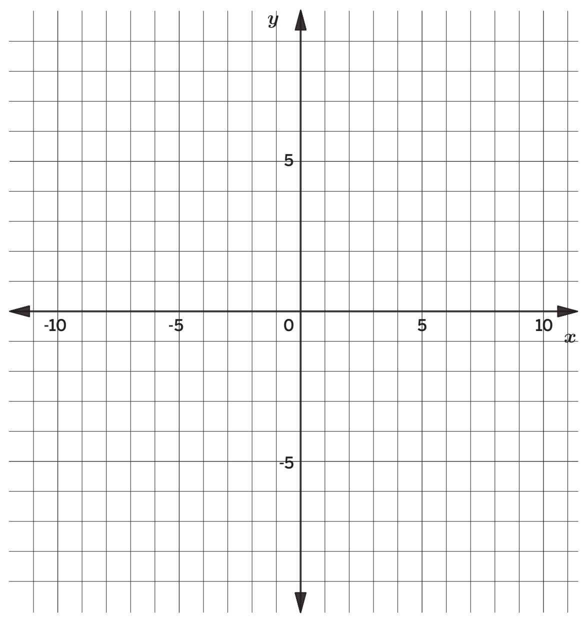
Polygon A: $(-7, 4)$, $(-8, 5)$, $(-8, 6)$, $(-7, 7)$, $(-5, 7)$, $(-5, 5)$

Polygon B: $(4, 3)$, $(3, 3)$, $(2, 2)$, $(2, 1)$, $(3, 0)$, $(4, 0)$, $(5, 1)$, $(5, 2)$

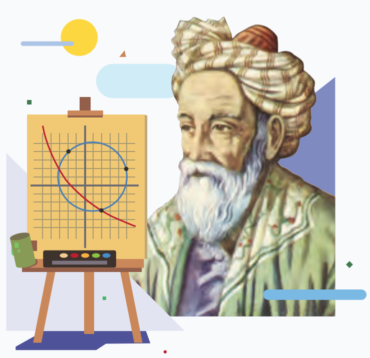
Polygon C: $(-5, -5)$, $(-5, -8)$, $(5, -8)$, $(5, -5)$

1. Plot the polygons on the coordinate plane, connecting the points in the order that they are listed. Label the polygons as A, B, and C.

2. Can you determine the perimeter of any of the polygons? If yes, which one(s), and what are their perimeters? If no, why not?



Featured Mathematician



Omar Khayyam

Khayyam was born in 1048 AD in the city of Nishapur in what is now Iran. An astronomer, he led a team that developed the highly accurate Jalali calendar, which would be used by much of the world for hundreds of years. And as a mathematician, he worked at the *intersection of algebra and geometry*, studying curves and parallel lines, and helping to unify these two branches of mathematics.

Name: ______ Date: ______ Period: ______

Activity 2 Fencing for a Wildlife Refuge

The coordinate plane on the next page represents all the land on a wildlife refuge. The locations of an existing observation tower and an elevated walkway are shown. You have been tasked with designing the size and locations of the enclosures for three endangered animals from the Amazon Rainforest that will live at the refuge: South American tapirs, giant armadillos, and tiger cats.

Your enclosures must meet the following criteria:

- The tiger cats need the most space to run, followed by the tapirs. The armadillos need the least space.
- None of the animal enclosures can share any fence.
- There should be at least one foot between the observation tower and any enclosure. *Note: this does not include the walkway.*

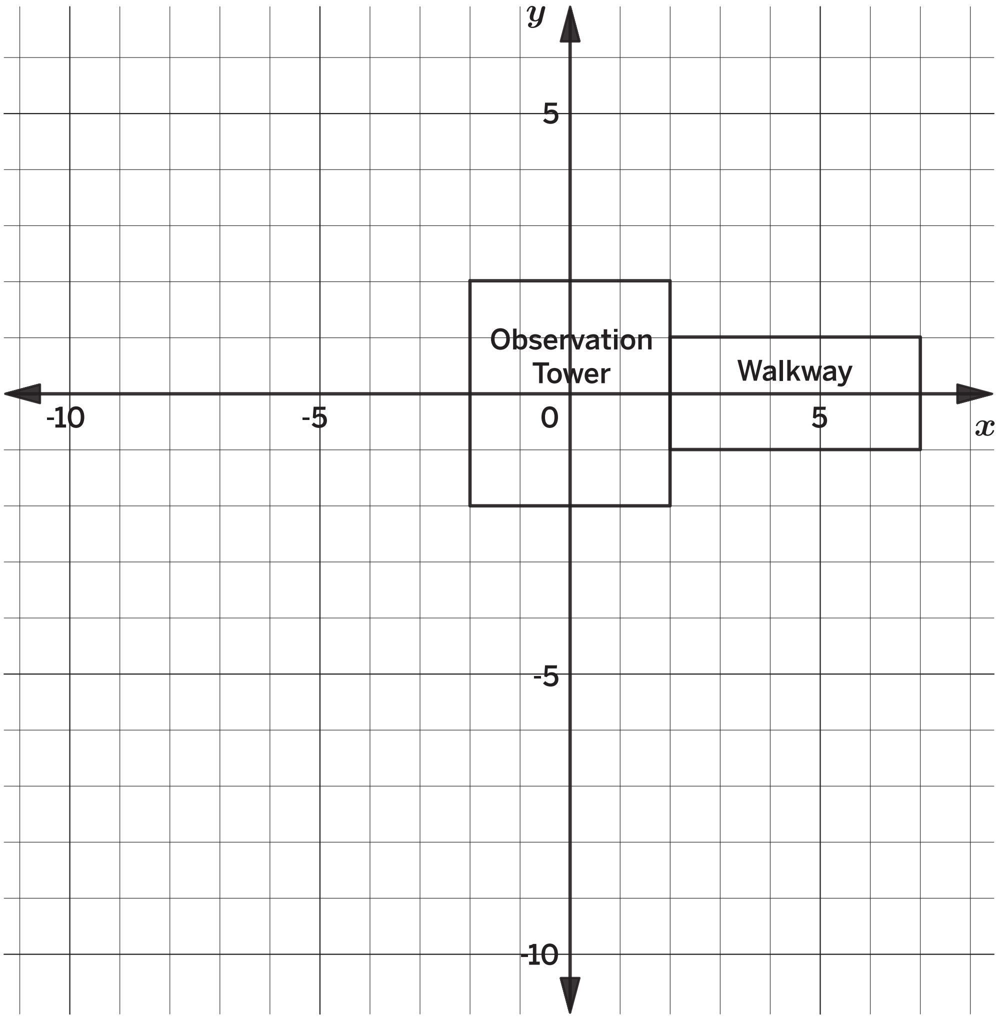
1. Plot your three enclosures on the coordinate plane on the next page. Record the coordinates of each "corner" in the table.

2. Determine how much fencing, in feet, will you need for each enclosure, and record those values in the table.

Animal	Enclosure coordinates	Fencing needed (ft)
Tiger cats		
Tapirs		
Armadillos		

Activity 2 Fencing for a Wildlife Refuge (continued)

Hint: One unit = 1 ft.



Are you ready for more?

Determine the area of each of your enclosures.

Animal	Area (ft^2)
Tiger cat	
Tapir	
Armadillo	

STOP

Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You can construct a polygon in the coordinate plane by plotting points that represent the vertices, and then connecting those points with line segments to form the sides. You can use the coordinate plane to identify certain properties of polygons. Suppose a polygon is graphed in the coordinate plane. You can determine:

- The number of sides, and in some cases, the lengths of those sides.
- Whether two sides are parallel or perpendicular.

This information can help you identify and describe shapes based on the points that form their vertices. One way you can use information like this is to determine the lengths of the line segments that form the sides of a polygon, and add them together to calculate its perimeter.

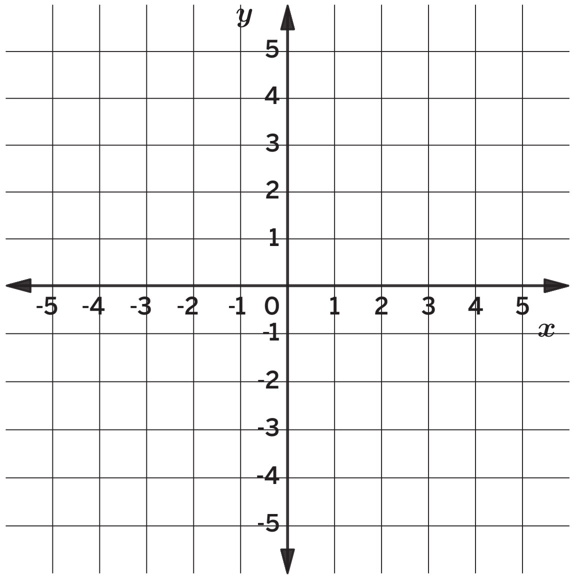
Reflect:

Practice

Name: .... Date: .... Period: ....

1. Refer to the coordinate plane.

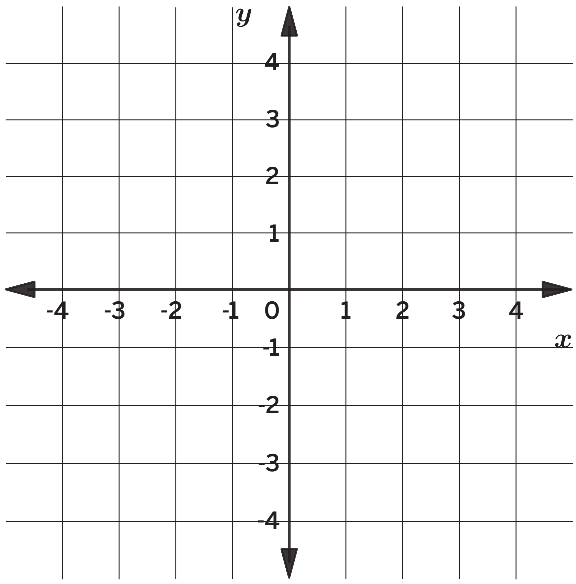
a. Draw a square with a perimeter of 20 units and one vertex at the point $(-3, 5)$.



b. Write the coordinates of the other vertices.

2. Refer to the coordinate plane.

a. Plot and connect the following points in the order they are listed to form a polygon: $(-3, 2)$, $(2, 2)$, $(2, -4)$, $(-1, -4)$, $(-1, -2)$, $(-3, -2)$.



b. What is the perimeter of the polygon?

Name: .... Date: .... Period: ....

Practice

3. The coordinates of a rectangle are $(3, 0)$, $(3, -5)$, $(-4, 0)$ and $(-4, -5)$.

a What are the length and width of the rectangle?

b What is the perimeter of the rectangle?

c What is the area of the rectangle?

4. Circle *all* the equations that could represent each scenario. Then determine the solution for each scenario using one of the selected equations.

a Jada's cat weighs 3.45 kg. Andre's cat weighs 1.2 kg more than Jada's cat. How much does Andre's cat weigh?

$x = 3.45 + 1.2$ $\quad x = 3.45 - 1.2$ $\quad x + 1.2 = 3.45$ $\quad x = 1.2 + 3.45$

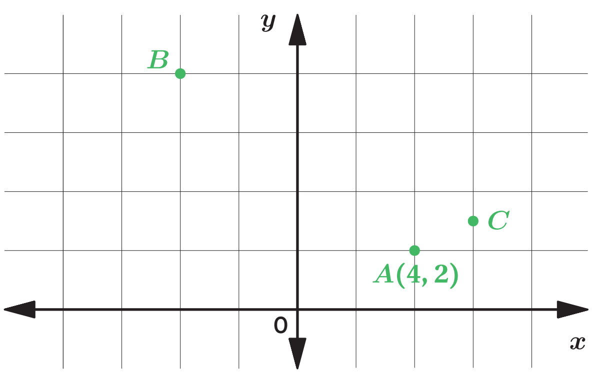
Solution:

b Apples cost $\$1.60$ per pound at the farmer's market. They cost 1.5 times as much at the grocery store. How much do apples cost per pound at the grocery store?

$y = 1.5 \cdot 1.60$ $\quad y = 1.60 \div 5$ $\quad 1.5y = 1.60$ $\quad \frac{y}{1.5} = 1.60$

Solution:

5. Use the coordinates of point A to help you determine the coordinates of points B and C. Explain your thinking.



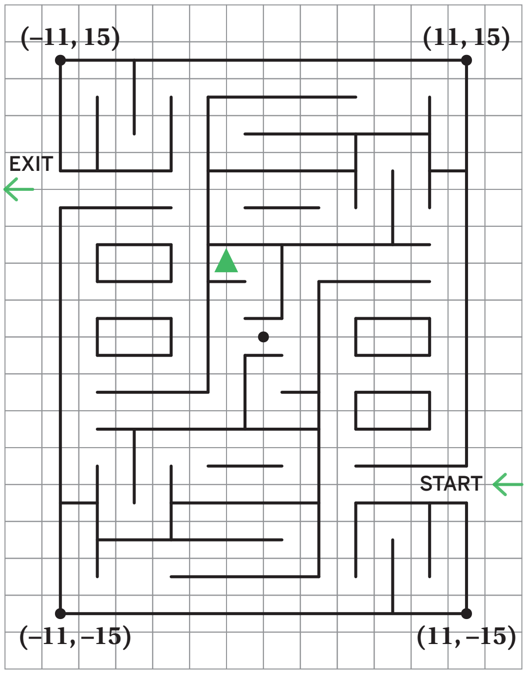
Unit 7 | Lesson 18

Lost and Found Puzzles

Let's use the coordinate plane to solve problems and puzzles.

Warm-up Lost in the Maize Maze

Noah got lost in the maize maze. The triangle represents his stopping point. Help him find his way out.



1. Write the coordinates that represent the point at which Noah stopped.

2. Plot and label the points where Noah needs to turn to make his way out of the maze, starting from the point where he realized he was lost.

Name: ________ Date: ________ Period: ________

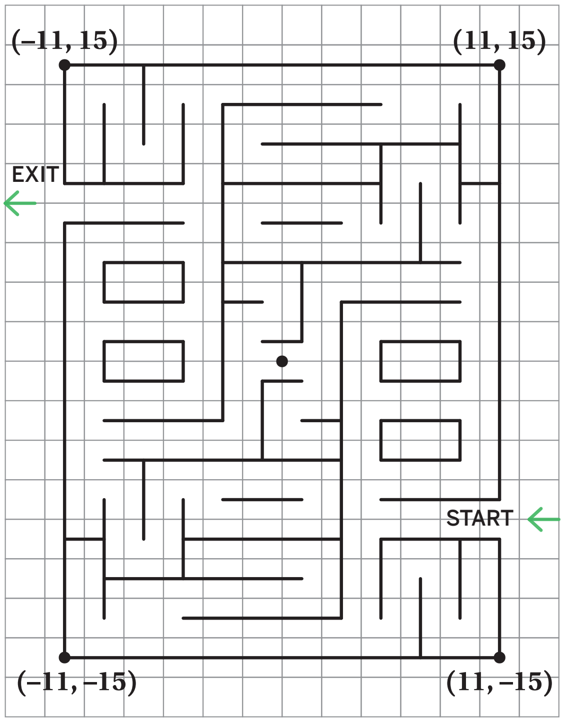
Activity 1 Found in the Maize Maze?

Han entered the maze at the Exit to help Noah. He did not know that Noah was already making his way out (following the path you mapped out in the Warm-up).

Co-craft Questions:
Before you begin the activity, study this maze. Work with your partner to write 2–3 mathematical questions you have about this maze or scenario.

1. Han stopped at $(-7, -2)$. Plot where Han stopped.

2. Do you think Han was able to find Noah? Explain your thinking.



3. What is the shortest distance Han could have walked before stopping? Record the ordered pairs of each line segment representing Han's path, and the distance between each pair of coordinates. The first part of his route has been completed in the table.

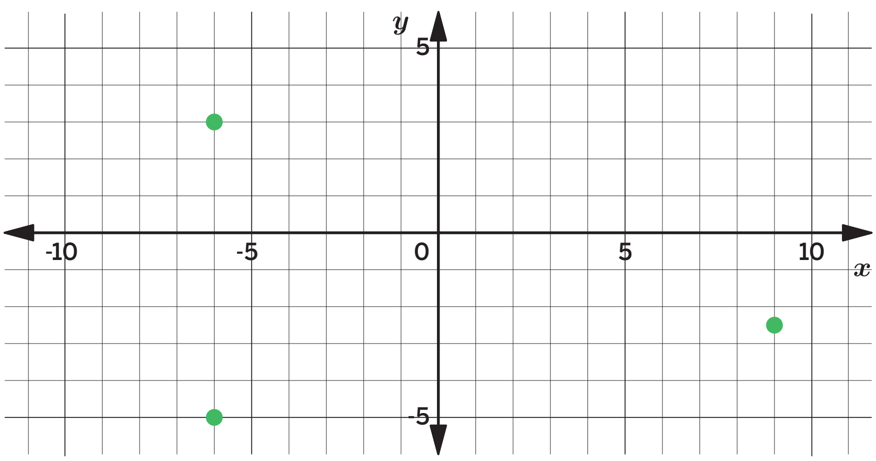
Line segment	Ordered pairs	Distance (units)
1	$(-12, 8)$ and $(-4, 8)$	8
2		
3		
	Total Distance:	

Activity 2 Lost Treasures

Can you determine the mystery locations of all the lost treasures? The location of three of the treasures are marked on the coordinate grid. Use the clues and the given points to determine which treasure is at each location as well as the coordinates of the final treasure.

- The places where the necklace and puppy can be found are points that can be seen as a reflection across one axis on the coordinate plane.
- The puppy ran out from under a bed and ran 5.5 blocks north and 3 blocks west to a friend's house.
- The sports gear can be found at the point farthest away from the y-axis.
- The lost treasure under the bed is located at the point closest to the x-axis.
- The person who lost the granola bar also lost a different treasure. They found the treasure that was in the locker first. Then they walked 8 blocks to find the granola bar.
- That same person was given a look of disgust by someone who works where one of the treasures was found, but thought "Well, at least I only walked 8 blocks to come get it."

You can use the coordinate plane, the table, or both, to help track the given information and solve the mystery.



Name: ______ Date: ______ Period: ______

Activity 2 Lost Treasures (continued)

		Lost treasures				Ordered pairs			
		Gold necklace	Granola bar	Sports gear	Puppy	$(9, -2.5)$	?	$(-6, -5)$	$(-6, 3)$
Locations	Locker at school								
	Under the bed								
	Friend's house								
	Gym								
Ordered pairs	$(9, -2.5)$								
	?								
	$(-6, -5)$								
	$(-6, 3)$								

Lost treasure	Location	Ordered pairs
Gold necklace		
Granola bar		
Sports gear		
Puppy		

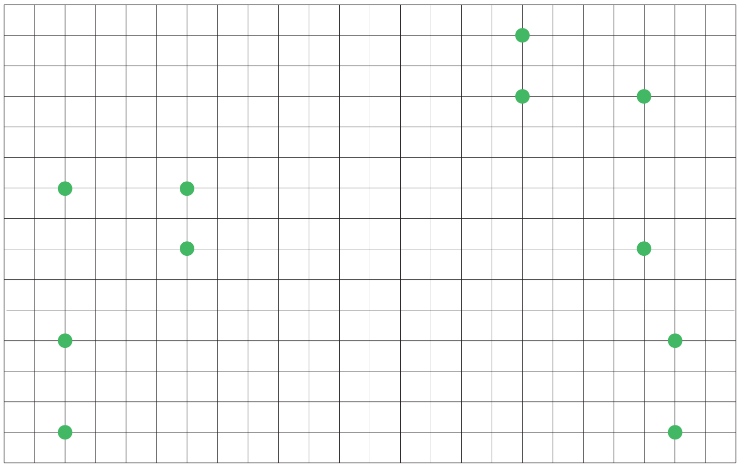
Activity 3 Mystery Maze Design Challenge

You will be given a blank grid.

1. Work with a partner to create a path that goes through all the given points on the coordinate plane, connected in any order, but the path *cannot* cross itself. This will be a *solution path* to your maze. Then design a maze around that path.

You must:

- Plot and label the location of the origin. It can be anywhere on the grid, but there has to be at least one turning point in each of the four quadrants.
- Identify the scale of your grid, as going by either 1s, 2s, or 3s.
- Add two more turning points, but they cannot be in the same quadrant.
- Label one point as the *Entrance* and label one point as the *Exit*, but they cannot be in the same quadrant.
- Label each line segment using capital letters starting with A representing the segment from the *Entrance* to the next point on the solution path. Even if two line segments are on the same line, they should be labeled separately.



Name: ________ Date: ________ Period: ________

Activity 3 Mystery Maze Design Challenge (continued)

2. Record the coordinates of each line segment and its distance.

Line segment	Ordered pairs	Distance
A		
B		

3. When your maze is complete, you will switch partners and challenge each other to solve your mazes, using the information in the table.

STOP

Summary

In today's lesson . . .

You solved puzzles using the coordinate plane. The coordinate plane is a useful tool for modeling the positions or locations of places and things, such as on a map. It also makes it possible to determine distances, which means it can be used to describe situations involving movement as well — between two locations, or traveling a certain distance in a particular direction from one location.

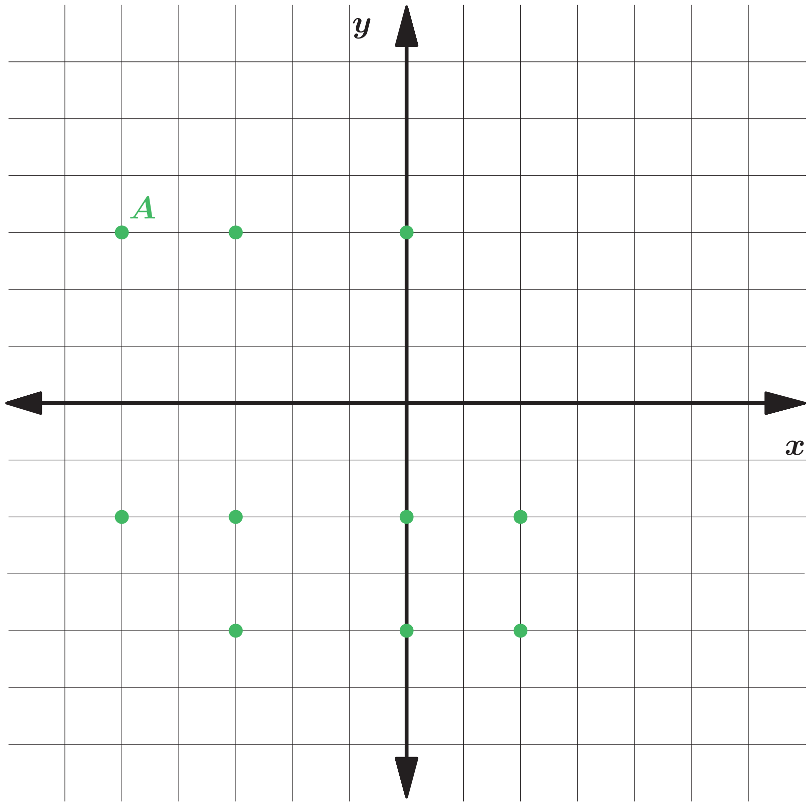
To move between two fixed points, the distance is the same no matter which path you choose. But depending on where you start, the directions and movements needed to get from one to the other will be different. As more points are plotted, or more locations represented on a map, the structure of the coordinate plane provides a shared perspective for you to get to where you are going, be it an exit to a maize maze, a friend's house, or a school.

Reflect:

Name: ____________ Date: ____________ Period: ____________

Practice

Use the points plotted in this coordinate plane for Problems 1–3. The coordinates of point A are $(-15, 9)$.



1. What is the scale of the axes? Explain your thinking.

2. Connect the points to create a maze-like path starting from point A. The line segments should not intersect each other.

3. What is the total distance along your path from point A to the end of the path?

4. Explain how to calculate a number that is equivalent to $\frac{2.1}{1.5}$.

Practice

Name: .... Date: .... Period: ....

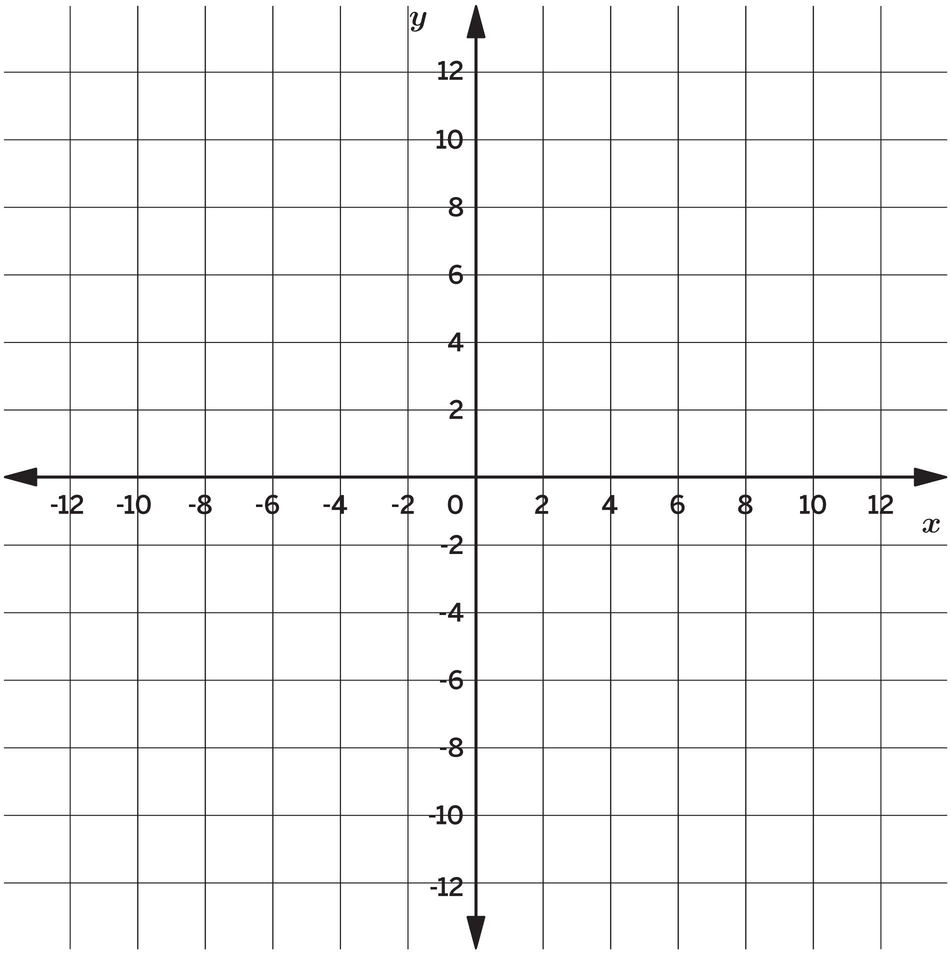
5. Lin's family needs to travel 325 miles to reach her grandmother's house.

a. After they have traveled 26 miles, what percent of the distance to her grandmother's house have they completed?

b. How far will they have they traveled when they have completed 72% of the distance to her grandmother's house?

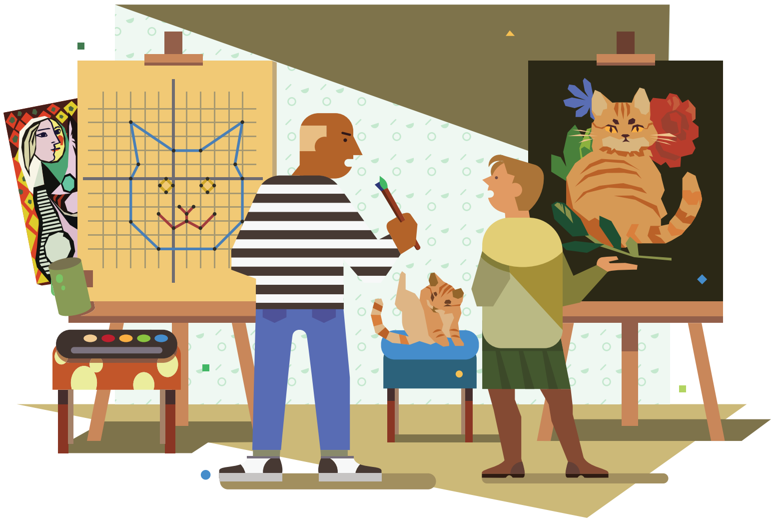
c. Suppose they traveled 377 miles. What percent of the distance to her grandmother's house have they completed?

6. Plot the following points on the coordinate plane: $(3, 5)$, $(3, -2.5)$, $(-1, -2.5)$.



Name: ______ Date: ______ Period: ______

Unit 7 | Lesson 19 – Capstone

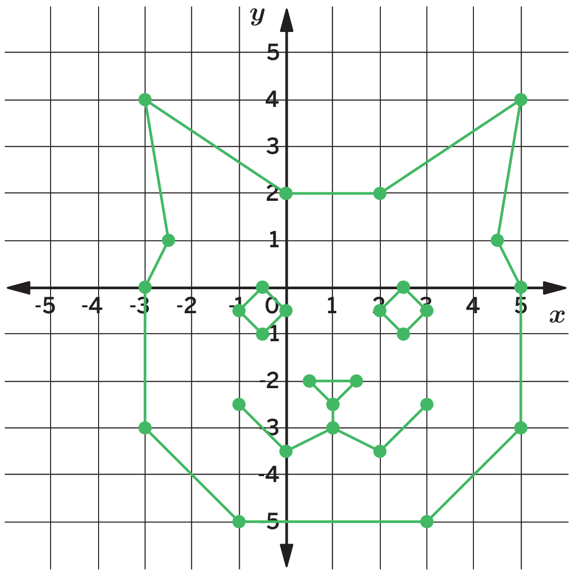


Drawing on the Coordinate Plane

Let's draw on the coordinate plane.

Warm-up Cat Pictures

Refer to the image of the cat drawn on the coordinate plane.



1. List the coordinates of the points plotted that form the cat's eyes.

2. Add more detail to the image, such as whiskers, the inside of the ears, a bow, or whatever you would like — but you must add at least three new points. Then add the coordinates to your list.

Log in to Amplify Math to complete this lesson online.

Activity 1 Image Race

Part 1

With your partner, draw a recognizable image on a piece of graph paper. Your picture must include the following

- A vertical and a horizontal axis
- At least four points in each quadrant of your coordinate plane
- A maximum of 20 points

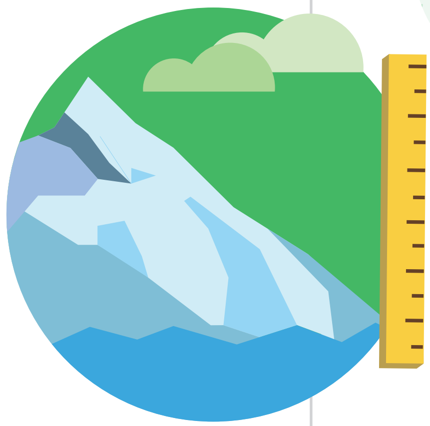
Part 2

Pair up with another group to compete in a drawing game using each other's images. Play the game as follows:

- With your partner, choose who will be the artist, and who will be the caller.
- The callers on each team exchange images. Do not let the artists see the drawing.
- The callers direct the artists, but are only allowed to give the following types of directions:
    - Where to draw a vertical or horizontal axis
    - The scale to label the ticks on an axis
    - The coordinates of a point to be plotted
    - Whether to connect two points
- As they draw, the artists try to guess what the image is.
- The first team to guess correctly wins.

STOP

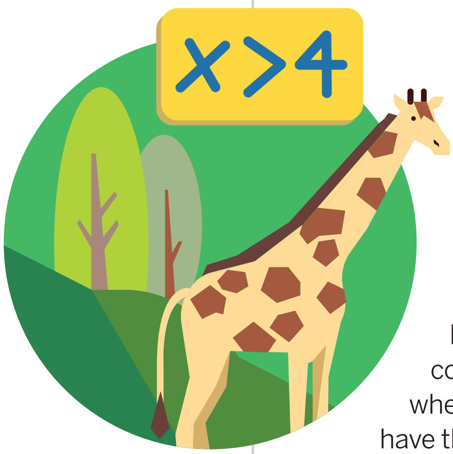
Unit Summary



Every journey begins with a single step. But you won't get far if you don't know where you're going. Whether it's a ship crossing the Atlantic, or a piece on a chessboard, it's not enough to know just *how far* to go. You also need to know what *direction* to head in.

Imagine you are helping a stranger find their way. You might use words like left and right, or North and South.

We can do the same with positive and negative numbers. We see this all the time with temperature and elevation. When it's bitingly cold, we say it's 10°C below zero, or -10°C. And when you're in the deepest parts of the Mariana Trench, you're $36{,}000$ feet below sea level, or at $-36{,}000$ feet.



The farther a number is from zero, the greater its absolute value. But while the absolute value of -10 is greater than the absolute value of -5, -5 is the greater number. After all, an elevation of $-36{,}000$ feet is still higher up than an elevation of $-36{,}001$ feet.

Knowing that numbers can be positive or negative, and combining that with a coordinate plane lets us describe where things are in two dimensions. On one dimension we have the x-axis (what you might call "left" and "right"). On the other we have the y-axis (what you might call "up" and "down"). Together they form a grid, like the lines on a map.

Now, with positive and negative numbers under our belt, and the ability to read points on a coordinate plane, we're well on our way to getting where we're going!

See you in Unit 8.

Practice

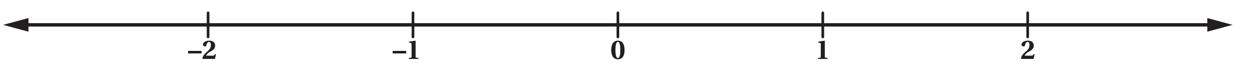
Name: ______ Date: ______ Period: ______

1. Order the following temperatures, in degrees Fahrenheit, from coldest to warmest:

- 5 degrees above zero
- 3 degrees below zero
- 6 degrees above zero
- $2\frac{3}{4}$ degrees below zero

2. Plot each of the following numbers on the number line. Label each point with its numeric value.

0.4	-1.5	$-1\frac{7}{10}$	$-\frac{11}{10}$



3. Which value is greater: $-\frac{9}{20}$ or -0.5? Show your thinking. Consider plotting the numbers on a number line to help with your thinking.

Name: .... Date: .... Period: ....

4. Write a number that matches each description.

a A number whose value is equal to the absolute value of -12.9.

b A number whose absolute value is equal to 5.

c A positive number whose value is less than $|4.7|$.

d A negative number whose absolute value is greater than $|-2.6|$.

5. Noah said, "If a is a rational number, $-a$ will always be a negative number." Do you agree with Noah? Explain your thinking.

6. Draw and label an appropriate pair of axes and plot the points.

$\left(\frac{1}{5}, \frac{4}{5}\right)$	$\left(-\frac{3}{5}, \frac{2}{5}\right)$	$\left(-1\frac{1}{5}, -\frac{4}{5}\right)$	$\left(\frac{1}{5}, -\frac{3}{5}\right)$

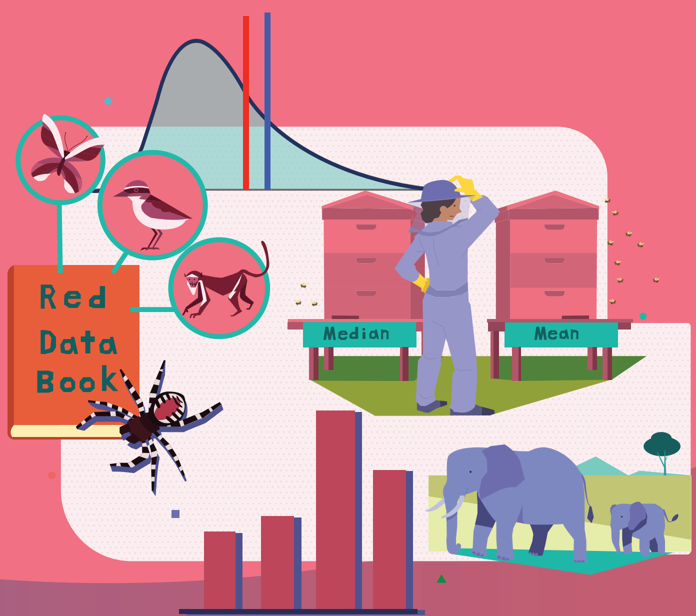
UNIT 8

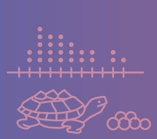
Data Sets and Distributions

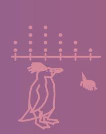
Statistics is the science of collecting and analyzing data. It is one of the most relevant aspects of mathematics in everyday life. And it is also used by researchers in many fields, such as zoologists identifying new species and studying populations of endangered species. In all cases, knowing what is typical is critical to understanding what is not.

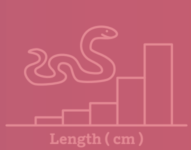
Essential Questions

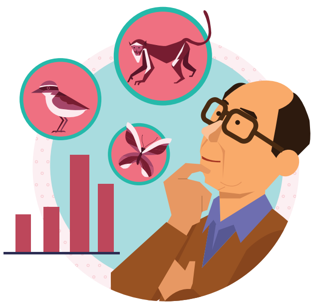
- What makes a question statistical?
- What does a measure of center tell you about a distribution?
- What does a measure of variability tell you about a distribution?
- *(By the way, if mean people get MAD, do median people get IQR?)*











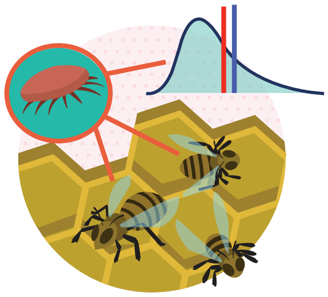
SUB-UNIT

1 Statistical Questions and Representing Data

Narrative: Collecting data on the populations of animals can help us understand if they are at risk for extinction.

You'll learn . . .

- about questions that don't have a single answer.
- how to use numbers to describe a data set.



SUB-UNIT

2 Measures of Center

Narrative: To understand the issues facing honey bees, we need precise numbers to describe them.

You'll learn . . .

- about "typical values".
- How to summarize a set of data using the mean or median.

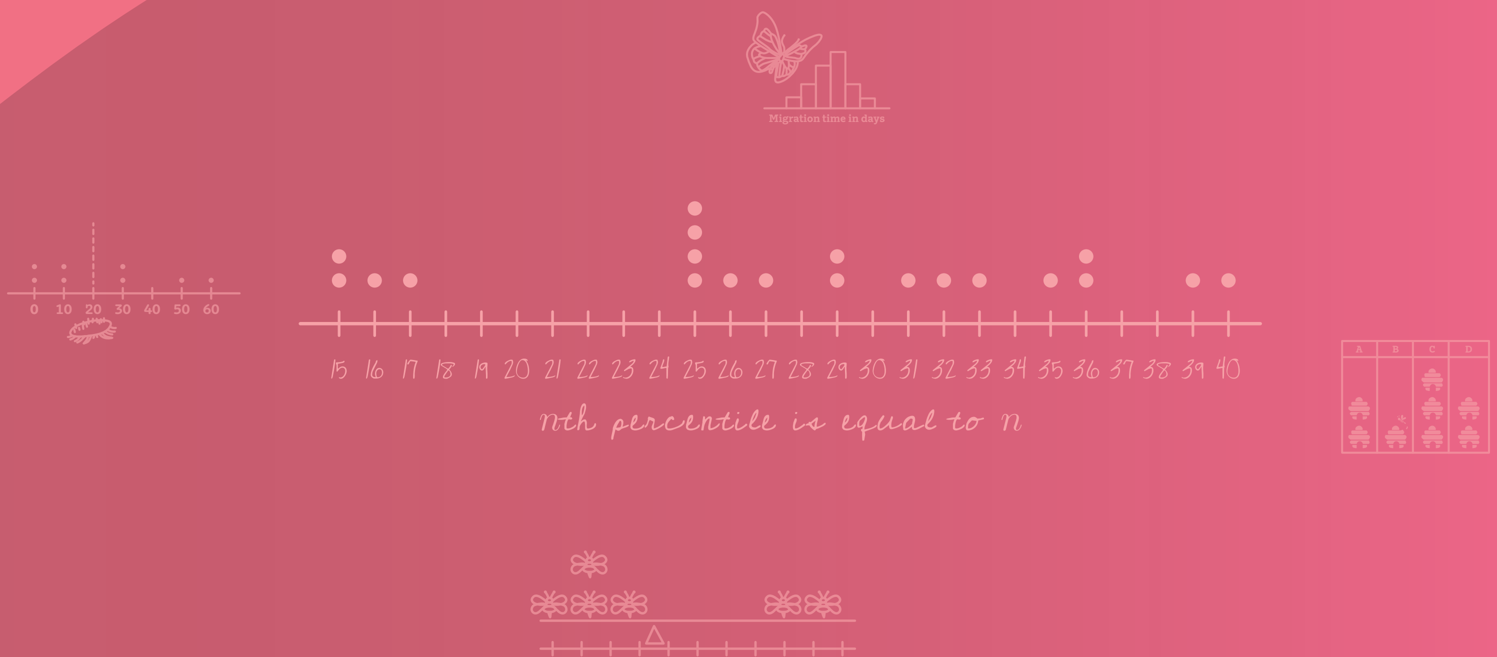
SUB-UNIT

3 Measures of Variability

Narrative: Measures of variability in data help us understand even more about the manatee population.

You'll learn . . .

- about variability in data.
- how to summarize a data set using the mean absolute deviation or interquartile range.



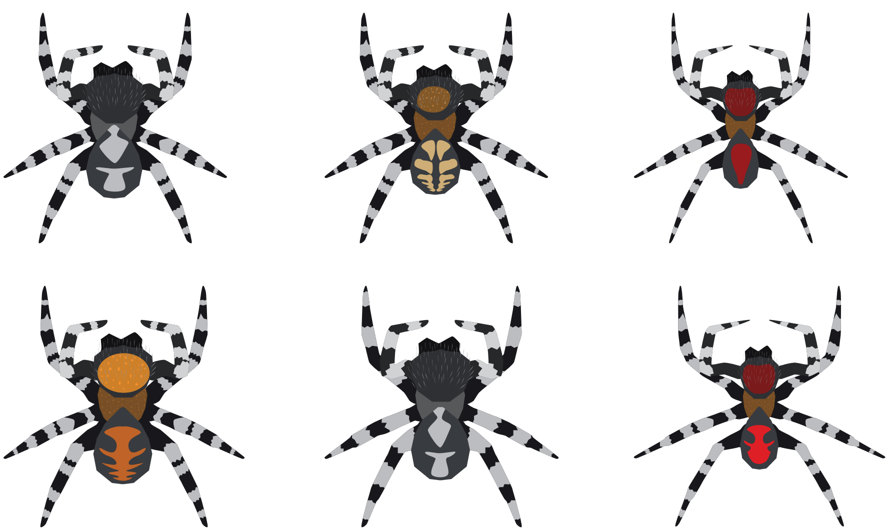
Unit 8 | Lesson 1 – Launch

Plausible Variation or New Species?

Let's determine whether a spider is just a rare variation or a new species.

Warm-up Genus Variation

Analyze the images of 6 specimens of male spiders belonging to the *Loureedia* genus. Briefly describe what you notice and think might be true about the sizes, shapes, colors, markings, and any other features of this genus of spiders. Use words such as: *mostly, generally, typically, normally, sometimes, rarely, never*, etc.

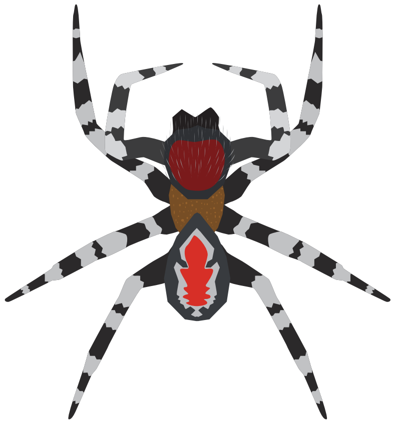


Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Beyond a Reasonable Doubt?

You will be given fact sheets for 3 known species of spiders belonging to the *Loureedia* genus and one unclassified specimen, as seen here.



Part 1

With your group, decide whether you believe the unclassified specimen belongs to any of the three known species or is a new, previously unidentified species. Record your observations and the scientific or mathematical evidence you would use to support your claim.

Part 2

1. You will now jigsaw with other groups and take turns sharing your claims and the related evidence, one at a time. For each claim that does not match yours, take turns providing skeptical counterarguments. For example, you could say, "As a skeptic, I would say the specimen is not as long as the average *Loureedia annulipes*, but it may not be fully grown."

2. With your new group, write two to three questions that would help you come to a consensus about the classification. These can be questions you already answered in making your claims, new questions that could also be answered using the data available to you, or even questions that would require additional information. If a question requires more information, be sure to state that extra information, as well.

Co-craft Questions:
You will share your questions with another group. Work together to decide which questions might be the most helpful. You may decide to write new questions.

Activity 2 Further Microscopic Evidence

You will be given another data sheet that provides some measurements of the *pedipalps* – the antenna-like sensory organ on a spider's head – for the unclassified specimen and the three known species.

Analyze the measurements. Using that analysis and previously gathered information, investigate and make one final claim: Which species does the unclassified specimen belong to? Or, do you believe it is a new, previously unidentified species?

Be prepared to share all of the questions you asked and the evidence you used to answer them in support of your claim.

STOP

Unit 8 Data Sets and Distributions

Walk on the Wild Side With Data

In 2012, a team of international scientists journeyed to Israel. It was part of a broad study on a family of *Eresidae* spiders, or velvet spiders. It was there, in Israel's sandy terrain, that the team identified a new type of a velvet spider. These spiders are small, with velvety hairs, and live in underground burrows. Inspired, the researchers named them *Loureedia* after the 1960s rock icon Lou Reed and his band the Velvet Underground.

Much of what we know about the world's diverse species is thanks to the scientists that study and catalogue them. But what exactly is a *species*? Put simply, a species is a category of living things that share common characteristics and can breed together.

To date, there are only four species of *Loureedia* spiders in existence. Since their discovery, many of them have been sold in an illegal pet trade, putting them at risk of becoming endangered.

Deciding what counts as being part of one species rather than another can get complicated. But it is important for understanding when a species is at risk of extinction. That is why specialists monitor the populations of different species, sometimes enlisting the help of local observers and animal enthusiasts.

With reliable data, researchers can encourage humans to be more responsible, and protect all wildlife, no matter the number of legs.

Welcome to Unit 8.

Practice

Name: .... Date: .... Period: ....

1. You see a herd of deer and want to know whether they are mule deer.

   a. Write two questions you could ask that would result in data you can observe.

   b. Write two questions you could ask that would result in data you need to measure.

2. Darwin's bark spiders display an extreme example of dimorphism, with females being much larger than males. The *average* total body lengths can be represented as 0.8 +/– 0.1 in. for females, and 0.2 +/– 0.05 in. for males. The lengths listed here each represent a spider. For each length, state whether you believe it could reasonably be a female Darwin's bark spider (F), a male Darwin's bark spider (M), either a female or a male Darwin's bark spider (E), or is most likely not a Darwin's bark spider (N).

   a. 0.88 in.

   b. 0.16 in.

   c. 0.70 in.

   d. 0.11 in.

   e. 0.39 in.

   f. 0.62 in.

3. The largest known species of jumping spider is *Hyllus giganteus*. The average lengths of these spiders are 2.15 +/– 0.35 cm. Consider a *Hyllus giganteus* whose length matches the shortest possible average length based on this. If that spider recorded a jump covering a horizontal distance of 14.4 cm, then:

   a. What would be the horizontal distance for a similar jump by a *Hyllus giganteus* whose length matches the longest possible average length?

   b. For fun, how far of a horizontal distance could you jump similarly, if you were a *Hyllus giganteus*? **Hint:** Use your height.

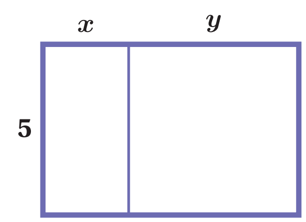
Name: ______________________ Date: __________ Period: ________

4. Order the following numbers from least to greatest.

$-4, \frac{1}{4}, 0, 4, -3\frac{1}{2}, \frac{7}{4}, -\frac{5}{4}$

Least **Greatest**

5. Select *all* the expressions that represent the total area of the largest, outlined rectangle.



A. $5(x + y)$

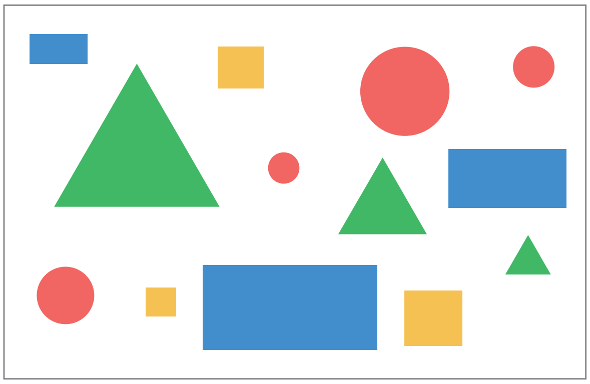
B. $5 + xy$

C. $5x + 5y$

D. $2(5 + x + y)$

E. $5xy$

6. Describe how you would sort the items in the group into two or more categories. Then write how many items would belong in each category.



My Notes:

SUB-UNIT

1 Statistical Questions and Representing Data

Narrative Connections

How do you keep track of a disappearing animal?

Imagine if a species of bird that normally visits your neighborhood suddenly disappeared. How would you know if they went south for the winter, or went extinct?

This was the problem Sir Peter Scott pondered in the 1950s and 60s. Scott was an avid ornithologist and a member of the International Union for Conservation of Nature (IUCN). Under the IUCN, he worked on a system that tracked different species in danger of extinction. Using reliable data, this system would document the populations of different species across the globe.

Finally, in 1966, Scott debuted the “Red Data Books.” These books drew on data from observers and scientists around the world. The Red Data Books would eventually become the IUCN Red List still used today. These lists provide information on different species’ population distributions. They categorize a species’ extinction risk and provide a baseline against which a species’ population can be measured.

With more and more species under threat, it is important to use every resource available. Data and statistics can be a powerful tool for understanding conservation issues. With these skills, conservationists can persuade people to act and ultimately help to save the natural world from going extinct.

Unit 8 | Lesson 2

Statistical Questions

Let's explore different kinds of data and the questions they can help answer.

Warm-up Sorting Questions

In 2016, the American bison (*Bison bison*) was named the national mammal of the United States. Despite this honor, the International Union for Conservation of Nature (IUCN) lists it as Near Threatened, because its survival is heavily dependent upon conservation efforts.

O.S. Fisher/Shutterstock.com

You will be given a set of cards with different questions about the bison on each card. Sort the questions into two or more categories, and name your categories. Be prepared to explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Matching Questions to Their Data

Part 1

Ten conservationists each answered all of the *numerical* questions from the Warm-up. You will be given another set of cards that each contain the responses of the ten conservationists to one of the questions. Match each data set with the one question to which it most likely corresponds.

Part 2

Sort the four pairs of cards (questions and corresponding data) from Part 1 into two or more categories based on the data sets. Be prepared to explain your thinking.

Activity 1 Matching Questions to Their Data (continued)

Part 3

With your partner, write one new question about the American bison for each category.

	Categorical	Numerical
Data *without* variability		
Data *with* variability		

Collect and Display:
Be prepared to share how you wrote new questions for each category. Your teacher will add the language you use to a class display that you can refer to during this unit.

Are you ready for more?

Tyler and Han want to collect data for the question, "Which sixth grader lives the farthest from school?"

- Tyler says, "The data that we collect will not have variability because only one person lives the farthest from school."
- Han says, "There will be variability in the data we collect. We would not actually be asking everyone, 'Which sixth grader lives the farthest from school?' Instead, we can ask, "How far do you live from school?' Responses to that question are expected to have variability."

Do you agree with either one of them? Explain your reasoning.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that you can conduct surveys or take measurements to collect data to answer questions about a topic. The data collected are called a *data set*, and the data can be either categorical or numerical. ***Categorical data*** can be sorted into categories, such as *food bison eat* or *color of a calf*. ***Numerical data*** are numbers, quantities, or measurements that can be meaningfully compared. For example, the weights of bison can be measured and then compared to determine a typical weight.

Some data with numbers are categorical because the numbers are not quantities or measurements. For example, telephone area codes are categorical data because the numbers are labels, rather than quantities or measurements that can be meaningfully compared.

Both numerical data and categorical data can show *variability*, which means the data values are expected to contain more than one value. A question that can be answered by using data that has variability is called a ***statistical question***. The question, "Which classroom in your school has the most books?" is a statistical question because, in order to answer it, you need to count all of the books in each classroom of your school. The data will likely show variability because you would expect each classroom to have a different number of books.

In order to answer that question, you may also poll or survey all of the classrooms in your school by asking, "How many books are in your classroom?" This is not a statistical question, because it would be expected that every person in each classroom could count their books and would give the same answer.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Sixth graders were asked, “What grade are you in?” Explain why this is *not* a statistical question.

2. Lin and her friends went out for muffins after school. The following questions came up during their trip. Select *all* the questions that are statistical questions.

   A. How far is the bakery?

   B. What is the most popular muffin this week?

   C. What does a group of 4 people typically spend on muffins at this shop?

   D. Do kids usually prefer to get one large muffin or two mini muffins?

   E. How many flavors are there to choose from?

3. Here is a list of statistical questions. What data would you collect and analyze to answer each question? For numerical data, include the unit of measurement that you would use.

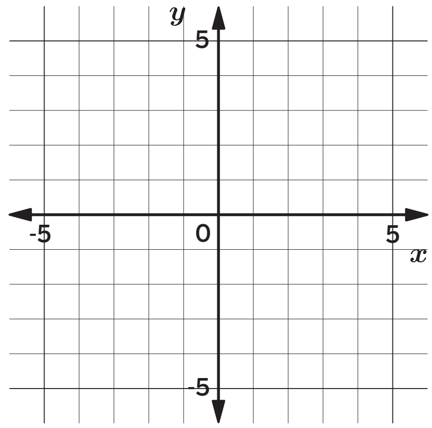
   a What is a typical height of female athletes on the U.S. Olympic Gymnastics team?

   b Are most adults who work at the school football fans?

   c How long do drivers generally need to wait at a red light in Washington, D.C.?

Name: .... Date: .... Period: ....

4. Triangle DEF has vertices $D(-4, -4)$, $E(-2, -4)$, and $F(-3, -1)$.



a Plot and connect the vertices of the triangle in the coordinate plane. Label the vertices.

b Name the coordinates of 3 points that lie inside the triangle.

c What is the area of the triangle? Show or explain your thinking.

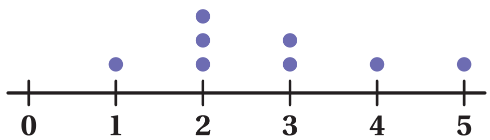
5. Order these numbers from least to greatest:

$\lvert 17 \rvert$	$\lvert -18 \rvert$	-18	$\lvert 19 \rvert$	20

Least **Greatest**

6. Are the set of data and the line plot showing the same information? Explain your thinking.

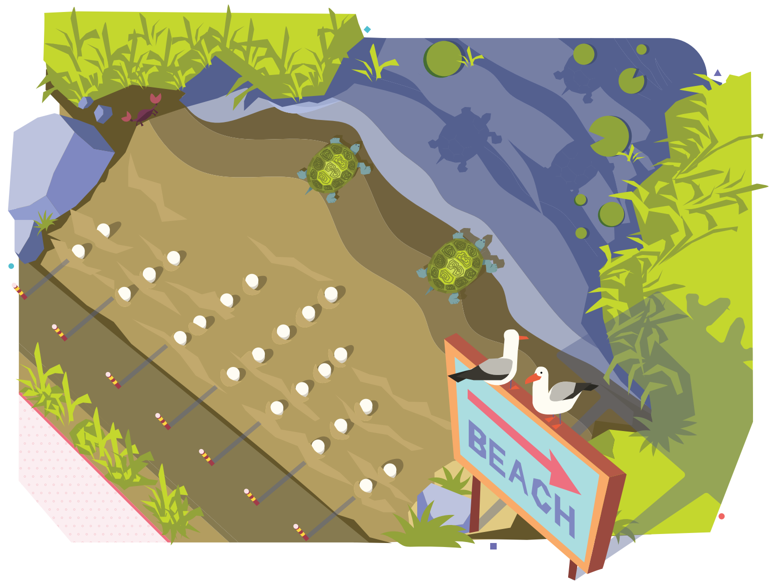
2	1	5	2
3	4	2	4



Unit 8 | Lesson 3

Interpreting Dot Plots

Let's represent and interpret data with dot plots.



Warm-up Counting Diamondback Terrapin Eggs

The IUCN lists the Diamondback Terrapin (*Malaclemys terrapin*) as Vulnerable, meaning it faces a high risk of extinction in the wild. The decline in the terrapin population is attributed to two threats: crabbing and the loss of nesting lands caused by human development.

Jay Ondreicka/Shutterstock.com

A team of conservationists are studying the breeding patterns of terrapins in the Chesapeake Bay. Here are the data representing the number of eggs found in 25 different clutches (nests).

13, 11, 7, 9, 6, 6, 5, 8, 10, 7, 14, 11, 9, 6, 7, 8, 10, 6, 9, 8, 7, 6, 13, 5, 8

What was the most common number of eggs found in the clutches? Show or explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Organizing Data With Frequency Tables

Data are often collected and analyzed to identify what is "typical," or expected, of that data. As a class, you will organize the data of turtle eggs from the Warm-up in a frequency table, and then analyze it to identify typical values.

Number of eggs	Frequency

1. Revisit your work and response from the Warm-up question. How might a frequency table make it more efficient to determine the most common number of eggs in a clutch? Explain your thinking.

2. The lead conservationist discovers another clutch in the marsh.

   a. Would you expect 10 or more eggs or less than 10 eggs in this clutch? Explain your thinking.

   b. How many eggs would you typically expect there to be in any other new clutch that is discovered? Explain your thinking.

Activity 2 Using Dot Plots to Represent and Describe Data

The IUCN is made up of over 1,400 organizations and the collective 17,000 experts working in them. The employees of IUCN, such as Programme Officers, oversee the work. In their role, Programme Officers contribute to, edit, and publish reports, which involve reviewing data and information gathered, and then interpreting it to share out. They also make recommendations for future actions and efforts.

Laura Máiz-Tomé served several years as the Programme Officer for the Freshwater Biodiversity Unit at IUCN, focusing on the protection and conservation of wetland-dependent species, which would include the Diamondback terrapin.

Represent the terrapin egg data from Activity 1 and think about how a dot plot and a frequency table could each be used to share the information the data represents.

1. Construct a dot plot representing the number of eggs in each clutch.

2. How are the frequency table from Activity 1 and your dot plot similar? How are they different?

Featured Mathematician

Laura Máiz-Tomé

Laura Máiz-Tomé is a Spanish political-ecologist who has a bachelor's degree in Environmental Policy and two master's degrees in Natural Protected Areas, and Environmental Assessment and Management. As the Programme Officer for the Freshwater Biodiversity Unit at IUCN, she led large-scale biodiversity assessments, including species extinction risk assessments for the IUCN Red List and the identification of Freshwater Key Biodiversity Areas. She served a similar role in the Ecosystem Assessment and Policy Support Unit at the Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services (IPBES), and continues this work as a consultant.

Name: .... Date: .... Period: ....

Activity 2 Using Dot Plots to Represent and Describe Data (continued)

The *distribution* of a data set refers to the combination of all of the values in the data set and their frequencies, which can be described by features of the overall shape of the data when represented visually, such as being arranged in a dot plot.

3. Use your dot plot of the turtle egg data to describe the distribution of the data set.

4. What percent of clutches had:

a More than 6 eggs in them?

b 8 or fewer eggs in them?

5. The lead conservationist learns that a 26th clutch has been discovered in the marsh. She says, "There are probably 20 eggs in that clutch!" Is her statement reasonable? Explain your thinking.

6. Revisit your work in Problem 2b from Activity 1. How might a dot plot be more efficient for determining a typical number of eggs in a clutch? Explain your thinking.

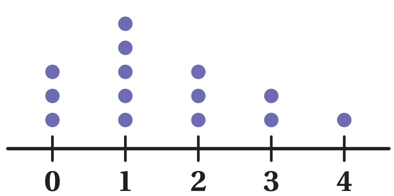
STOP

Summary

In today's lesson . . .

You saw that the term ***frequency*** refers to the number of times a "value" occurs in a data set. The most frequently occurring value is called the ***mode***. You can also represent and analyze the ***distribution*** of a data set – that is, information that describes all the data values and their frequencies. One way to describe a distribution is by identifying *typical values* – those that would be expected, based on the other values observed.

Organizing data is helpful for describing a data set and answering both non-statistical and statistical questions about it. For example, consider the data about number of siblings a group of sixth graders have:

Data list	0, 0, 0, 1, 1, 1, 1, 1, 2, 2, 2, 3, 3, 4,	Allows you to see all of the data
Frequency Table	Siblings: 0, 1, 2, 3, 4 Frequency: 3, 5, 3, 2, 1	Allows you to compare the frequency of each response (for categorical or numerical data)
Dot Plot		Allows you to compare frequencies in numerical data as well as provides a visual of the data.

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. Here are descriptions of data sets. Select *all* the descriptions that could be graphed as dot plots.

   A. Class size for the classes at an elementary school.

   B. Colors of cars in a parking lot.

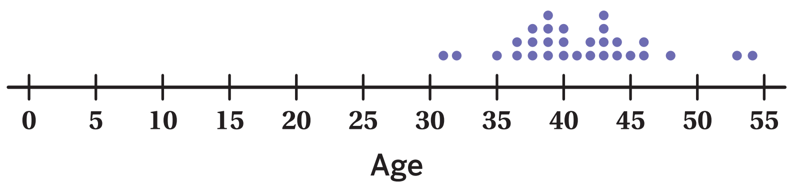
   C. Favorite sport of each student in a sixth grade class.

   D. Birth weights for the babies born during October at a hospital.

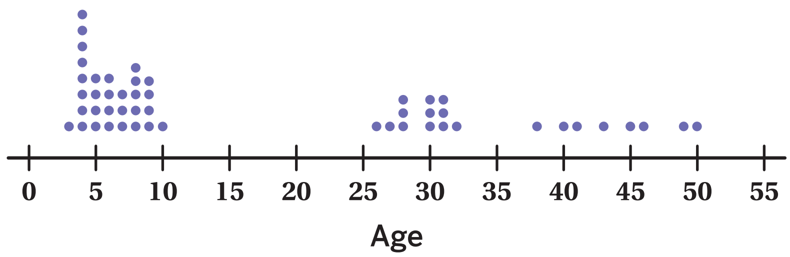
   E. Number of goals scored in each of 20 games played by a soccer team.

2. A movie theater is showing three different movies. These dot plots represent the ages of the people who were at the Saturday afternoon matinee of each of these movies.

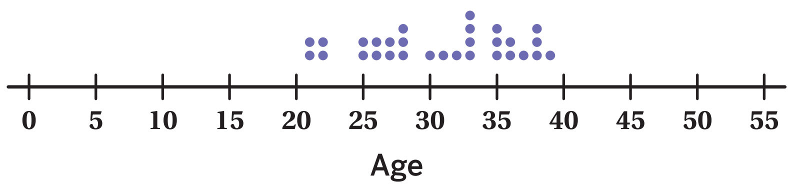
Movie A



Movie B



Movie C



a One of these movies was an animated movie rated G for general audiences. Do you think it was movie A, B, or C? Explain your thinking.

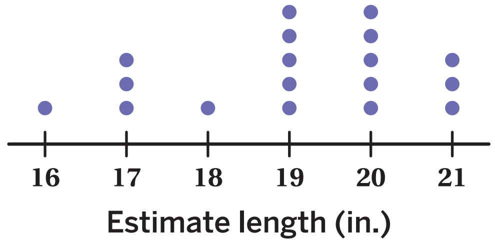
b For which movie was the typical audience member about 30 years old?

c What was a typical age for an audience member seeing Movie A?

Practice

Name: .... Date: .... Period: ....

3. A teacher drew a line segment that was 20 in. long on the blackboard. She asked each of her students to estimate the length of the segment and used all of their estimates to construct this dot plot.



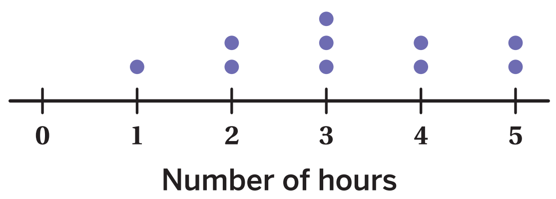
a. How many students are in the class?

b. Were students generally accurate in their estimates of the length of the line segment the teacher drew? Explain your thinking.

4. Determine the value of each expression.

a. $3.727 + 1.384$ b. $3.727 - 1.384$ c. $5.01 \cdot 4.8$ d. $5.01 \div 4.8$

5. The dot plot shows the numbers of hours spent last night on homework by a group of students.



a. What does each value on the number line represent?

b. What does each dot represent?

Name: ____________ Date: ________ Period: ________

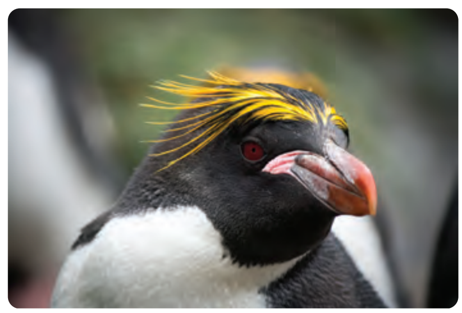
Unit 8 | Lesson 4

Using Dot Plots to Answer Statistical Questions

Let's use dot plots to answer statistical questions.

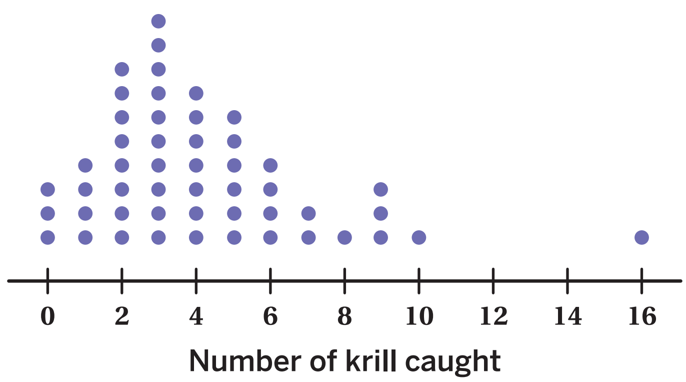
Warm-up The Macaroni Penguin

The IUCN classifies the Macaroni penguin (*Eudyptes chrysolophus*) as Vulnerable. The overall global population of Macaroni penguins has experienced a 47% decline over three generations, and they still face ongoing threats from climate change, commercial fishing, and food competition with the increasing fur seal population.



james_stone76/Shutterstock.com

A scientist is studying the hunting and feeding habits of 50 Macaroni penguins on Bird Island, South Georgia, Antarctica. The dot plot shows the number of krill that each of several penguins caught on their first dive one day.



What is a typical number of krill that one of these 50 penguins caught on their first dive? Be prepared to explain your thinking.

Activity 1 The Hunt for Red Krill

The scientist knows that, during their winter migration, Macaroni penguins spend most of their day foraging for food. She notices that the males tend to stay underwater longer than the females, so she poses the question: "Do male and female penguins typically hunt at the same depth?" To help answer this question, she put trackers on 25 male penguins and 25 female penguins to determine how deep each penguin dives to hunt.

You and your partner will each be given a dot plot. One partner will examine the data for the male penguins and the other will examine the data for the female penguins.

Part 1

Work together with your partner to complete these problems.

1. Is the scientist's question a statistical question? Explain your thinking.

2. How would you interpret one dot above 0 on the dot plot in this context?

Part 2

Use your dot plot to complete these problems.

3. How would you describe the *center* of the data? What might that tell you in context?

4. How could you use the center to describe a typical hunting depth for this group of penguins? Explain your thinking.

Name: .... Date: .... Period: ....

Activity 1 The Hunt for Red Krill (continued)

Part 3

Share your work from Part 2 with your partner. Then work together to complete these problems. Be prepared to explain your thinking.

5. Do male and female penguins typically hunt at the same depth?

6. Overall, which group – male penguins or female penguins – are more alike in the depth at which they hunt? Explain your thinking.

Are you ready for more?

Consider this data set: 20, 20, 21, 23, 23, 26, 26, 26, 26, 27, 27, 27, 28, 28, 29, 29, 29, 30, 32, 34.

1. Describe the center and spread of the data set.

2. Describe how the center and spread would change (or not) if:

   a. Each value is increased by 5.

   b. Each value is decreased by 5.

   c. All values equal to the center you named in Problem 1 remains the same. Each value less than the center is increased by 5, and each value greater than the center is decreased by 5.

Activity 2 Seasonal Hunting Patterns

The scientist then became curious whether the hunting behaviors of the Macaroni Penguin is consistent throughout the year. She posed the question, "Do the penguins' dives last the same amount of time during the summer (breeding season) and the winter (migration)?" She recorded the average duration of 25 penguins' dives in both the summer and the winter.

You will be given two dot plots of these two data sets. Based on the dot plots, state whether you agree or disagree with each of the following statements about this group of penguins. Be prepared to explain your thinking.

1. 72 seconds is a good description of the center of the data for how long the penguins spend per dive in the winter.

2. In general, 72 seconds is a good estimate for how long the penguins typically spend per dive in the summer.

3. Overall, the penguins' dives typically lasted the same amount of time in both the summer and the winter.

4. The penguins' dive times were more alike in the winter.

STOP

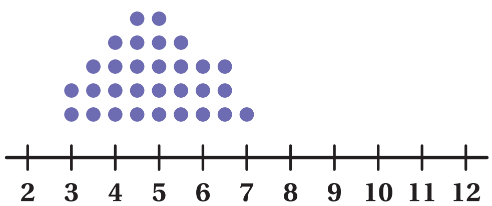
Name: ______ Date: ______ Period: ______

Summary

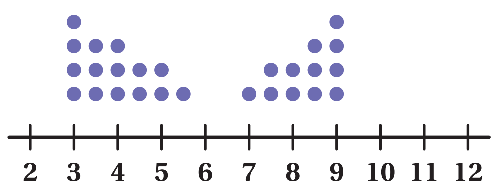
In today's lesson . . .

You saw that one way to describe or compare what is typical or characteristic for a numerical data set is by looking at the *center* and *spread* of its distribution. The ***center*** is a single value in the middle of a data set that represents a typical value. The ***spread*** describes how alike or different the values in a distribution are, often in relationship to the center. Representations other than lists or frequency tables, such as a *dot plot*, are more helpful for identifying and describing these aspects of a distribution.

For example, here are two dot plots showing the distributions of weights of several dogs and cats.



Cat weight (kg)



Dog weight (kg)

- The data is *symmetric*
- There are no ***gaps*** in the data
- The data is ***clustered*** around a ***peak*** between 4.5 and 5 kg.
- The *typical* weight for these cats is between 4 and 5.5 kg.
- The data is *symmetric*

- There is a ***gap*** in the data between $5\frac{1}{2}$ and 7 kg.
- The data is in two ***clusters*** with a ***peak*** at 3 kg and another peak at 9 kg.
- These dogs have a *typical* weight between 3 and 4 kg or between $8\frac{1}{2}$ and 9 kg.

Reflect:

Practice

Name: .... Date: .... Period: ....

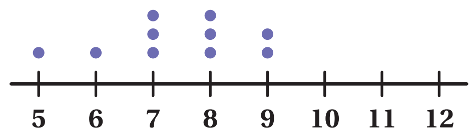
1. Three sets of data about ten sixth graders were used to make three unlabeled dot plots. Match each dot plot with the most appropriate label.

   a. Ages in years

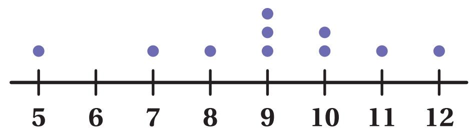
   b. Hours of sleep on school nights

   c. Hours spent doing chores each week

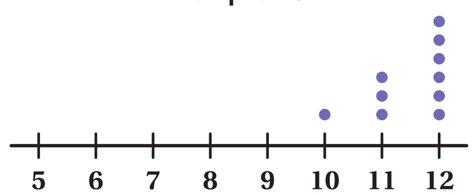
Dot plot 1



Dot plot 2



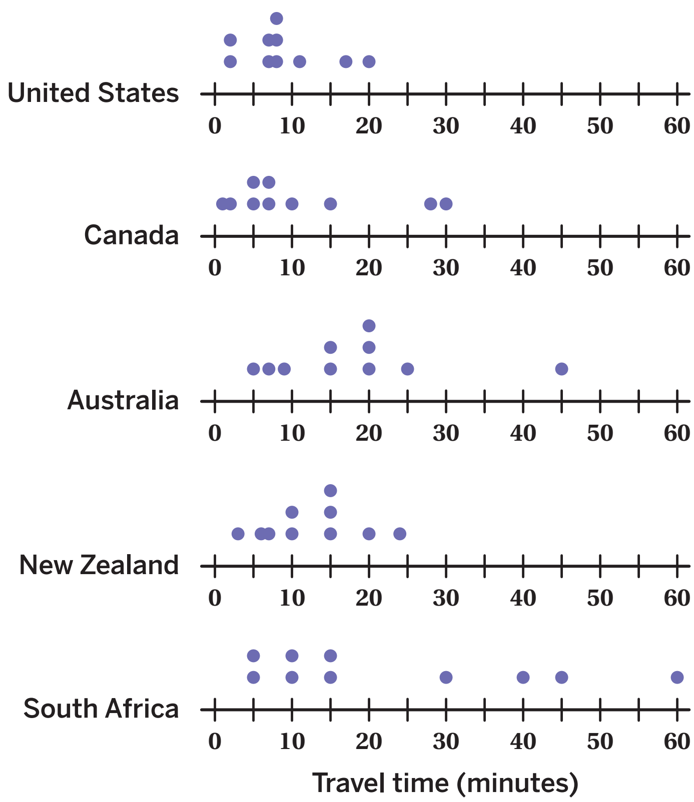
Dot plot 3



2. These dot plots represent the time it takes for ten sixth graders from the United States, Canada, Australia, New Zealand, and South Africa to get to school.

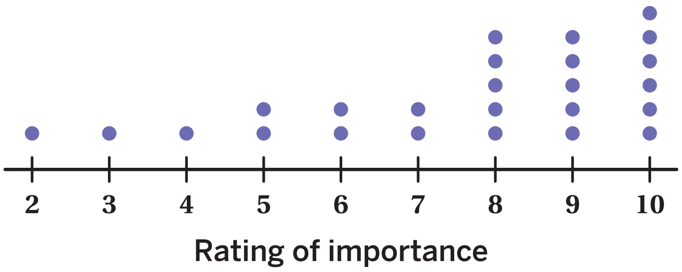
   a. List the countries in order of typical travel times, from least to greatest.

   b. Which country has the least spread in travel times? The greatest?

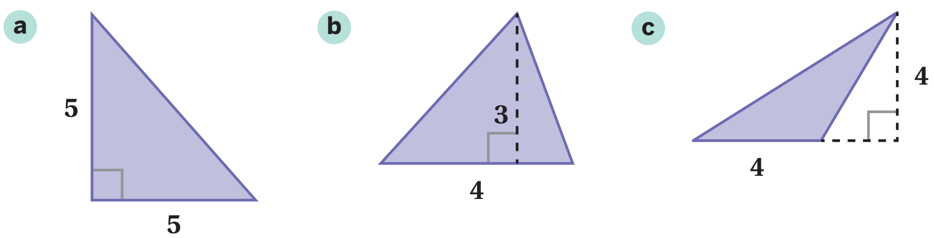


Name: .... Date: .... Period: ....

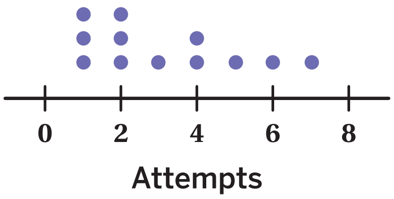
3. Twenty-five students were asked to rate, on a scale of 0 to 10, how important it is to reduce pollution. A rating of 0 means "not at all important" and a rating of 10 means "very important." The dot plot shows their responses. Explain why a rating of 6 is *not* a good description of the center of this data set.



4. Determine the area, in square units, of each triangle.



5. Priya created a dot plot of the number of attempts it took each of 12 of her classmates to successfully throw a ball into a basket. Write a question for which the answer would be:



a. More than half the classmates.

b. 3.

Unit 8 | Lesson 5

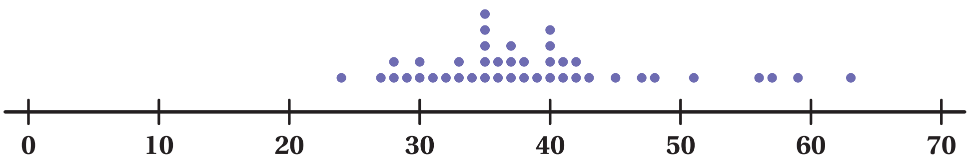
Interpreting Histograms

Let's explore how histograms represent data sets.

Warm-up Chimpanzee Lifespans (Part 1)

The chimpanzee (*Pan troglodytes*) is listed as Endangered by IUCN and its population is decreasing. Here is a dot plot showing the life spans of 40 chimpanzees that lived in the wild.

Helen J Davies/Shutterstock.com



1. Write two statistical questions that can be answered using the dot plot.

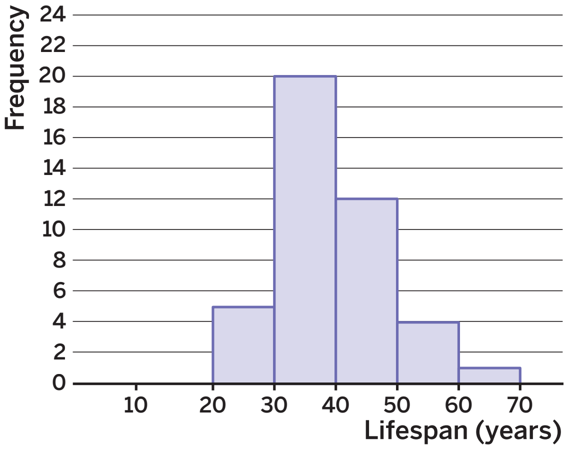
2. What would you consider to be a typical lifespan for a chimpanzee in the wild? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Chimpanzee Lifespans (Part 2)

In a histogram, each bar includes the left boundary value but not the right boundary value. For example, in this histogram showing the lifespans of the same 40 chimpanzees from the Warm-up, the first bar includes those that lived from 20 to 29.99999 . . . years, but not 30 years.



1. Refer to the histogram.

a How many chimpanzees lived at least 40 years?

b How many chimpanzees lived exactly 30 years?

c How many chimpanzees lived at least 50 years and less than 70 years?

d What was the longest a chimpanzee lived?

e Refer back to Problem 2 from the Warm-up. Would your answer be different based on this histogram of the data? Explain your thinking.

2. Discuss these questions with a partner and record your responses:

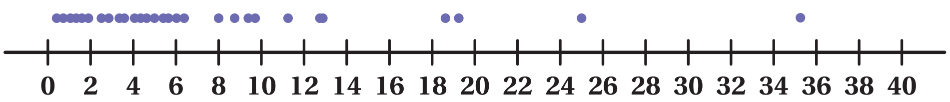
a If you used the dot plot from the Warm-up to answer the questions in Problems 1a–1d, how might your answers be different?

b How are the histogram and the dot plot alike? How are they different?

Activity 2 Populations of U.S. States and D.C.

Part 1

Every ten years, the United States conducts a census, which is an effort to count its entire human (*Homo sapiens*) population. The dot plot shows the population data from the 2010 census for each of the fifty states and the District of Columbia (D.C.).



Population of states (in millions)

1. Some statistical questions are shown about the populations of these 50 states and D.C. Decide whether it would be *possible* to answer each question from the dot plot. For those that would be possible, also decide whether the answer, for this data, is *clearly shown on the dot plot*. Be prepared to explain your thinking.

In the middle column of this table (Dot plot), mark your decision for each question by writing: C (clearly shown), P (possible to answer, but not clearly shown), or NP (not possible to answer).

Statistical question	Dot plot	Histogram
a How many states have populations greater than 15 million?		
b Which states have populations greater than 15 million?		
c How many states have populations less than 5 million?		
d What is a typical state population?		
e Are there more states with fewer than 5 million people or more states with between 5 and 10 million people?		
f How would you describe the distribution of the state populations?		

Name: ______________________ Date: __________ Period: __________

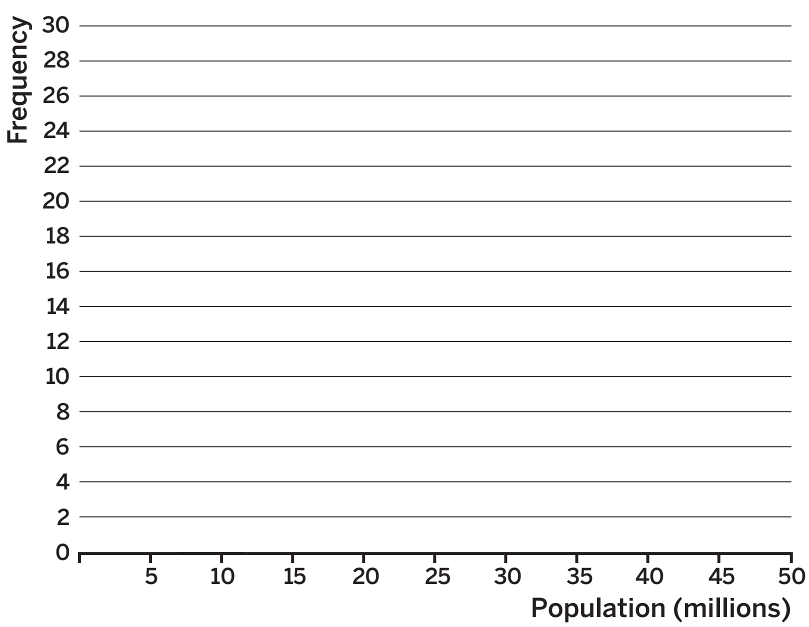
Activity 2 Populations of U.S. States and D.C. (continued)

Part 2

You will now be given the population data for each of the 50 states and the District of Columbia from the 2010 census.

2. Using the population data for each country, complete the table. Then use the grid and the information in your table to construct a histogram.

Population (millions)	0 to less than 5	5 to less than 10	10 to less than 15	15 to less than 20	20 to less than 25	25 to less than 30	30 to less than 35	35 to less than 40
Frequency								



3. Revisit the questions in the table in Problem 1, and complete the same exercise as you did with the dot plot. This time, make your decisions for each question about whether it is possible to answer and if the answer is clearly shown *in the histogram*. In the last column of the table on the previous page, mark these decisions by again writing: C (clearly shown), P (possible to answer, but not clearly shown), or NP (not possible to answer). Be prepared to explain your thinking.

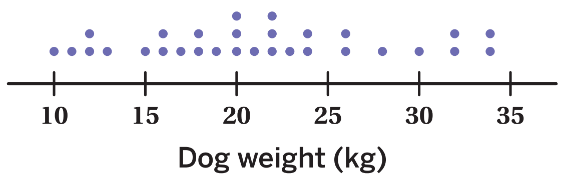
STOP

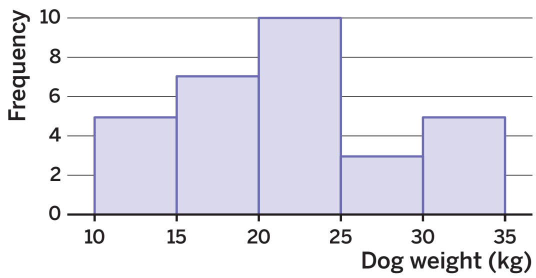
Summary

In today's lesson . . .

You saw that in addition to using dot plots, distributions of numerical data can be represented using ***histograms***.

The dot plot and histogram represent the same data set of the weights, in kilograms, of 30 dogs.





In a histogram, data values are grouped into *bins* that cover a range of values, and each *bin* has the same width. The total frequency of all values in that range is represented by a bar, including the left boundary (least value) but excluding the right boundary (greatest value). For example, the height of the tallest bar, between 20 and 25, represents weights of 20 kg up to (but not including) 25 kg.

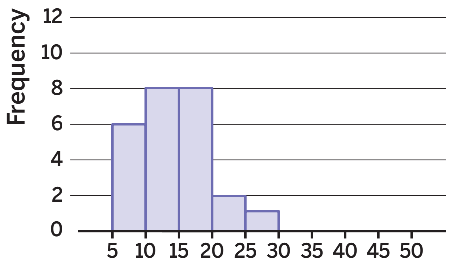
Notice that the histogram and the dot plot have similar shapes – roughly symmetric around the center between 20 kg and 25 kg, with a spread going from about 10 kg less than that to about 10 kg more. One advantage of a dot plot is that it always shows all of the data values. A histogram generally never shows individual values, and its shape can change based on the chosen bin size. However, histograms may be more efficient to construct and interpret for large data sets, when there is a wider range of values, or when very few data values are the same.

Reflect:

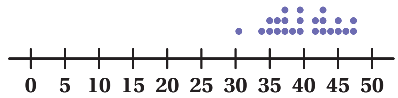
Name: .... Date: .... Period: ....

1. Match each histogram with its dot plot.

Histogram A

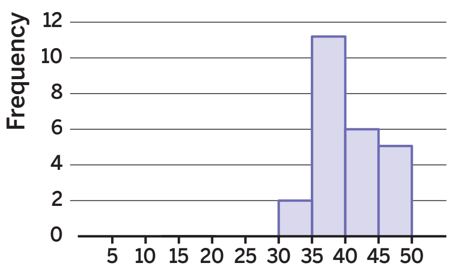


Dot plot 1

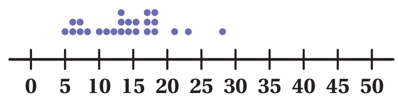


....

Histogram B

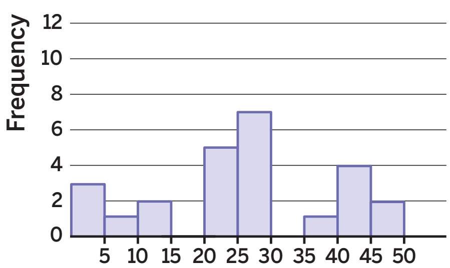


Dot plot 2

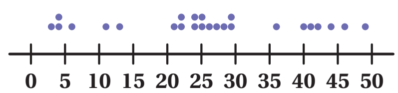


....

Histogram C



Dot plot 3



....

2. Here is a histogram that summarizes the lengths, in feet, of a group of adult female Great White Shark (*Carcharodon carcharias*). Select *all* the statements that are true about the histogram.

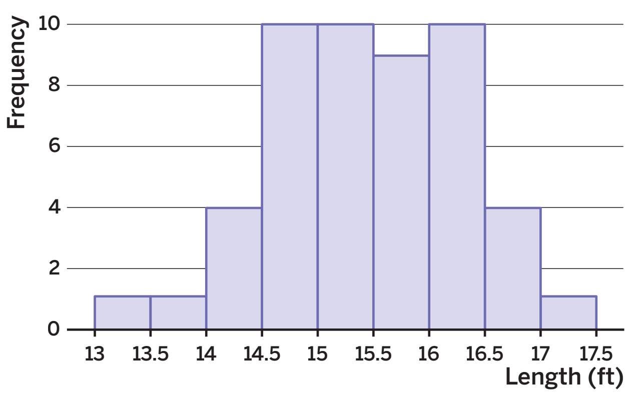
A. A total of 9 sharks were measured.

B. A total of 50 sharks were measured.

C. The longest shark was 10 ft long.

D. Most of the sharks were over 17 ft long.

E. Two sharks were less than 14 ft long.



Practice

Name: ______ Date: ______ Period: ______

3. This frequency table shows the times, in minutes, it took 40 sixth grade students to run 1 mile. Use the frequency table to draw a histogram of the data.

Time (minutes)	Frequency
4 to less than 6	1
6 to less than 8	7
8 to less than 10	13
10 to less than 12	12
12 to less than 14	7
14 to less than 16	2

4. The point $(-2, 3)$ is one vertex of a square on a coordinate plane. Name three points that could be the other vertices.

5. How are these questions similar or different? How would you expect the answers to these questions to be similar or different?
- What is the height of the tallest and lowest loop of the 5-loop roller coaster?
- What is the minimum and maximum speed of the roller coaster?

Name: ______________ Date: ______________ Period: ______________

Unit 8 | Lesson 6

Using Histograms to Interpret Statistical Data

Let's draw histograms and use them to answer questions.

Warm-up Which One Doesn't Belong?

Four questions about the population of Alaska are shown. Which question does not belong? Be prepared to explain your thinking.

a. At what age do Alaska residents generally retire?

b. At what age can Alaskans vote?

c. What is the age difference between the youngest and oldest Alaska residents with a full-time job?

d. Which age group makes up the largest percent of the population: under 18 years, 18–24 years, 25–34 years, 35–44 years, 45–54 years, 55–64 years, or 65 years or older?

Log in to Amplify Math to complete this lesson online.

Activity 1 Measuring Antiguan Racers

The Antiguan racer (*Alsophis antiguae*) is one of the rarest snakes in the world, listed as Critically Endangered by IUCN, only living only on islands near Antigua and Barbuda. It is so rare, in fact, that it has been thought to have been extinct two separate times – in 1936 and 2009. Most of the Antiguan racers alive today have been microchipped to monitor their growth and survival.

Public Domain

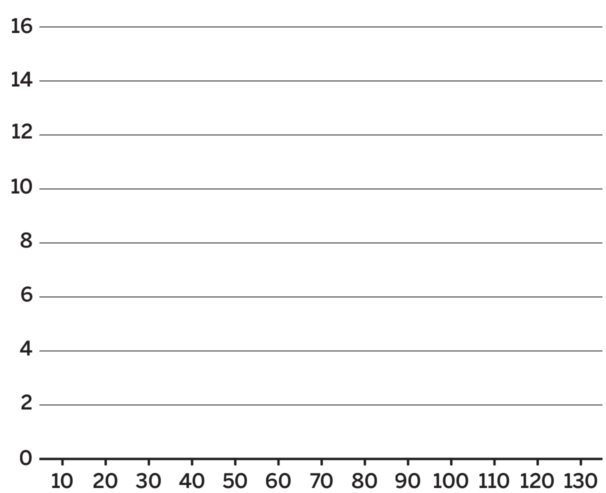
The table shows the lengths, in centimeters, of 25 Antiguan racers.

39	41.3	59.9	67	72.5	79.8	89	96.1	97.8	98	98.6	99.8	99.9
100	100	100.3	100.7	102.8	103.9	104	104	104.1	104.7	106.7	115.2	

1. Complete the frequency table representing the lengths of these 25 Antiguan racers.

Length (cm)	Frequency
0 to less than 20	
20 to less than 40	
40 to less than 60	
60 to less than 80	
80 to less than 100	
100 to less than 120	

2. Use the grid and the information in the table to draw a histogram for the data of snake lengths. Be sure to label the axes of your histogram.



Name: .... Date: .... Period: ....

Activity 1 Measuring Antiguan Racers (continued)

3. Based on your histogram, describe a typical length for an Antiguan racer? Explain your thinking.

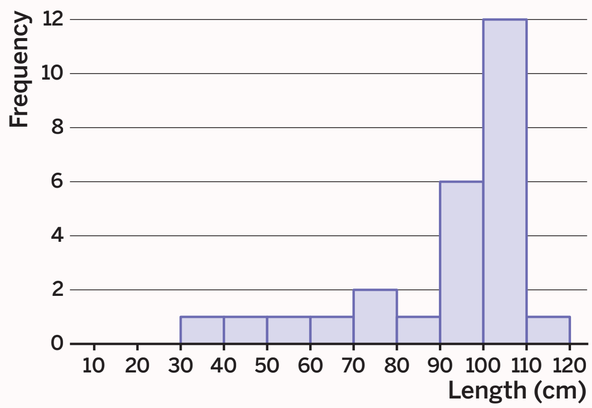
4. Based on the data, your histogram and your description of a typical length in Problem 3, describe the distribution of the data.

Are you ready for more?

Here is a histogram representing the Antiguan racer measurement data. In this histogram, the measurements are given in different bins.

1. Based on this histogram, what would you estimate is a typical length for the 25 Antiguan racers?

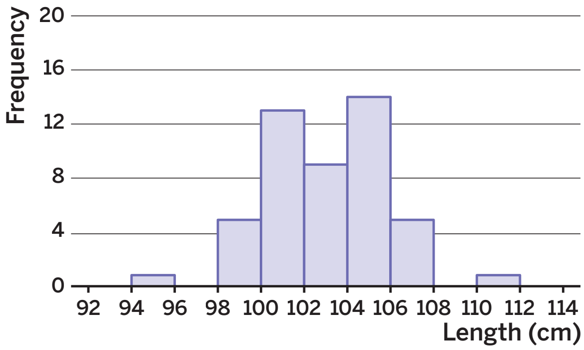
2. Compare this histogram with the one you created in Problem 2 of the activity. How are the distributions of data summarized in the two histograms the same? How are they different?



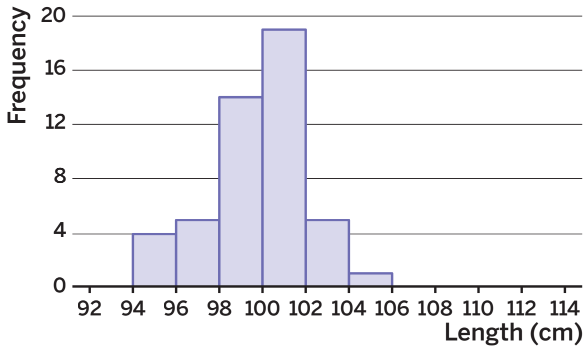
Activity 2 Match the Histogram

Here are two histograms representing the lengths of 50 adult male and 50 adult female Antiguan racers.

Histogram A



Histogram B



1. Describe the distribution of the lengths of the Antiguan racers represented in Histogram A in terms of the center and spread of the data.

2. Describe the distribution of the lengths of the Antiguan racers represented in Histogram B, in terms of the center and spread of the data.

3. If the female Antiguan racer is typically longer than the male, which histogram do you think represents the lengths of female racers? About how much longer would you say a typical adult female is compared to a typical adult male?

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You learned how to construct a histogram, using a list or table of data.

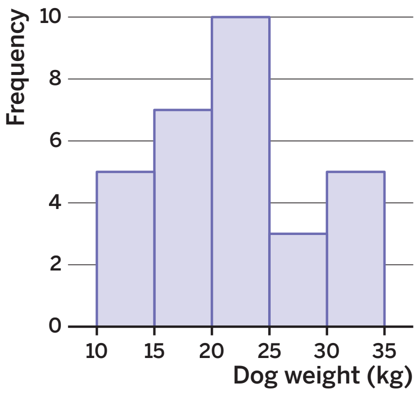
- First, you construct a number line that includes both the ***minimum*** (least value) and ***maximum*** (greatest value) from the data set.
- Next, you use that *range* of values to determine a reasonable bin size, remembering that each bin should have the same width. The number of bins impacts the shape of the distribution that is represented, so too many or too few could also impact how the data are interpreted.
- Finally, you need to draw the bars so that the heights represent the total number of data values corresponding to the range for each bin. Remember that each bin includes the left boundary value, but not the right boundary value.

For example, consider this ordered list of the weights, in kilograms, of 30 dogs:

10	11	12	12	13	15	16	16	17	18	18	19	20	20	20
21	22	22	22	23	24	24	26	26	28	30	32	32	34	34

Using the data, you can see that the range of values goes from 10 to 34 kg, so a reasonable bin size could be 5 kg, which means your number line should have values from 10 to 35, as shown here.

The histogram shows that a typical weight for this group of dogs is between 20 and 25 kg.



Reflect:

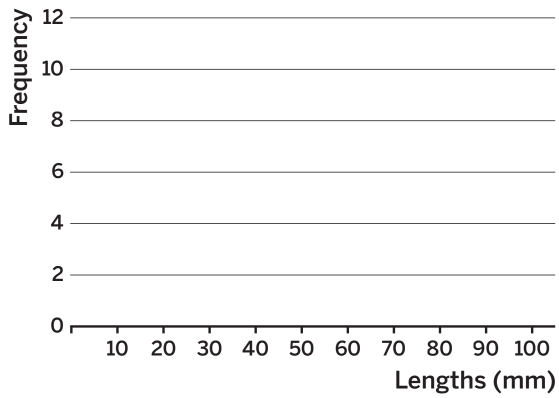
Name: .... Date: .... Period: ....

1. The table shows the lengths, in millimeters, of 25 earthworms.

6	11	18	19	20	23	23	25	25	26	27	27	28
29	32	33	41	42	48	52	54	59	60	77	93	

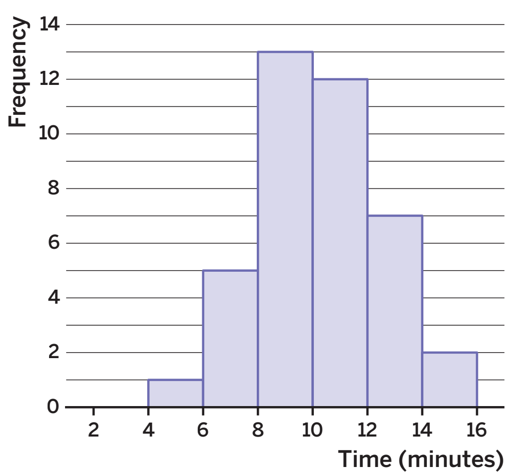
a Complete the frequency table for the lengths of the earthworms. Then use the grid to draw a histogram. Be sure to label the horizontal axis of your histogram.

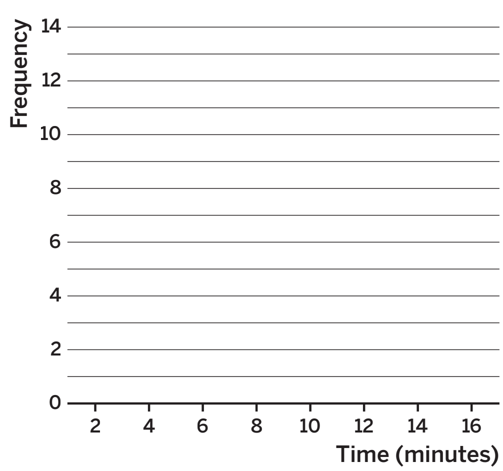
Length (mm)	Frequency
0 to less than 20	
20 to less than 40	
40 to less than 60	
60 to less than 80	
80 to less than 100	



2. Forty sixth graders ran 1 mile. The histogram summarizes their times, in minutes. The center of the distribution is approximately 10 minutes. On the blank axes, draw a second histogram that has:

- A distribution of times for a different group of 40 sixth graders.
- A center at 10 minutes.
- Less variability than the distribution shown in the first histogram.

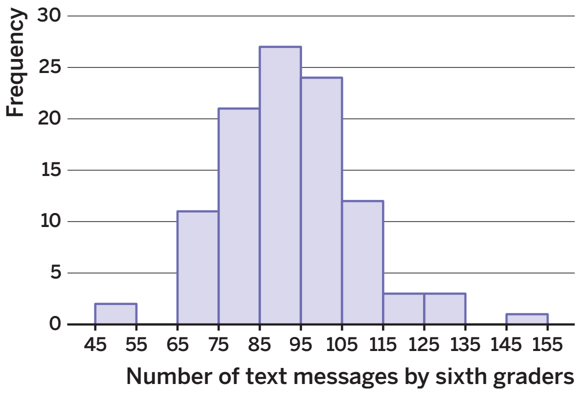


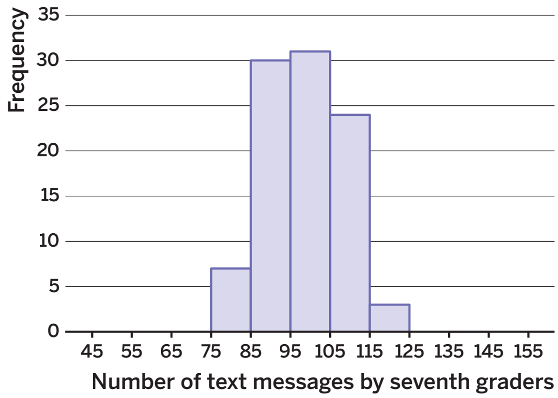


Practice

Name: ______________________ Date: __________ Period: __________

3. These two histograms show the number of text messages sent in one week by 2 groups of 100 students. The first histogram summarizes the data for sixth graders. The second histogram summarizes the data for seventh graders.





a. Do the two data sets have approximately the same center and spread? Explain your thinking.

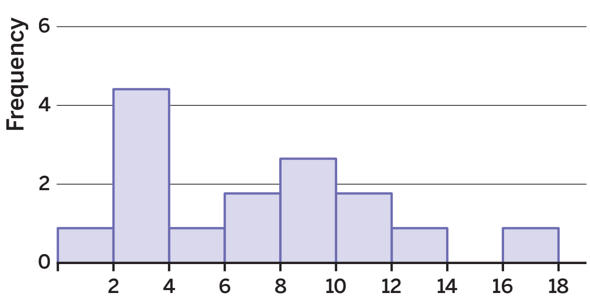
b. Overall, which group of students — sixth or seventh grader — typically sends more text messages? Explain your thinking.

4. Jada has d dimes, and no other money. She has more than 30 cents but less than a dollar.

a. Write two inequalities that represent how many dimes Jada has.

b. How many possible solutions make both inequalities true? If possible, describe or list the solutions.

5. How would you describe this histogram? Be sure to use all of the relevant language from this unit, such as: *peak, gap, cluster, spread, center*, and *typical*.



Unit 8 | Lesson 7

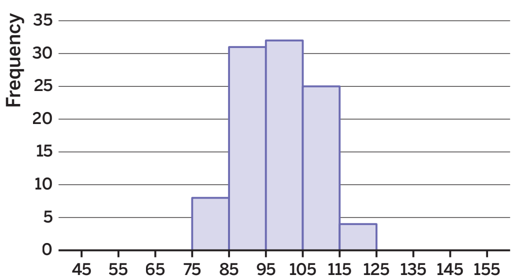
Describing Distributions With Histograms

Let's describe distributions by shapes and features displayed on histograms.

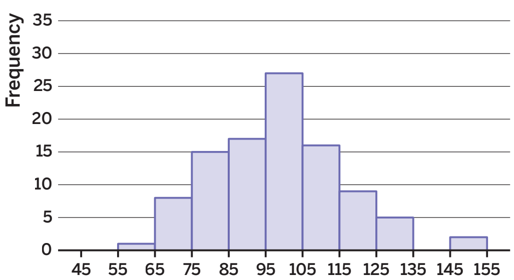
Warm-up Which One Doesn't Belong?

Which histogram does not belong? Be prepared to explain your thinking.

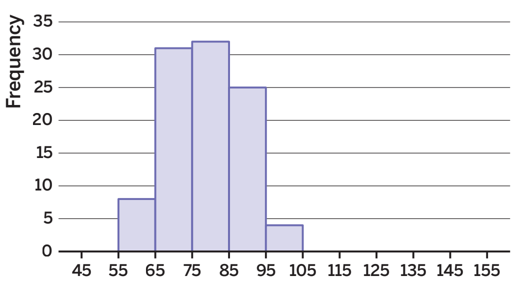
Histogram A



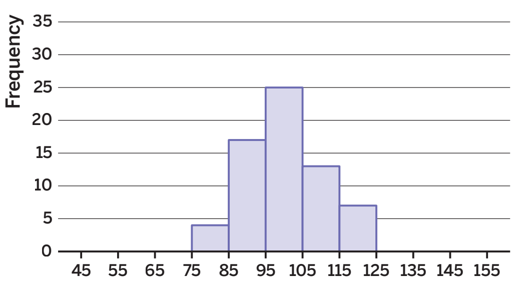
Histogram C



Histogram B



Histogram D



Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Sorting Histograms

Any diagram of a distribution — dot plot or histogram — is described as *symmetric* if you can draw a line on the diagram so that the parts on one side of the line mirror the parts on the other side. It is actually quite rare for distributions of real-world data to be perfectly symmetric, but many are close enough, and the distributions are called *approximately symmetric*.

1. Your group will be given a set of histogram cards. Sort them into two piles — one for histograms that are approximately symmetric, and another for those that are not.

2. *Discuss* your sorting decisions *with another group*. Resolve any disagreements and record your final decisions.

Approximately symmetric:
Not symmetric:

3. Histograms are also described by how many peaks they have. Histogram A is an example of a distribution with at least one peak. Which other histograms have this feature?

4. Histogram A is also an example of a histogram with a *gap*, which is an interval between two bars where there are no data values. Which other histograms have this feature?

5. Sometimes there are a few data values in a data set that are *far* from the center. Histogram A is an example of a distribution with this feature, in the 80 to 85 interval. Which other histograms have this feature?

Are you ready for more?

Create a scenario that Histogram E could represent.

Activity 2 Monarch Butterfly Migration

In the 1980s, the IUCN created a new threat level of Not Evaluated as a result of the decrease in monarch butterfly (*Danaus plexippus*) populations counted during their migration, calling this a "threatened phenomenon." The Not Evaluated status acknowledges that the species needs to be assessed because there are concerns about its population, but not enough data to make another classification. The IUCN's Special Survival Commission Butterfly Specialist Group is currently assessing three butterfly groups, one of which is the monarch.

Sue Bishop/Shutterstock.com

You will be given information related to the migration of monarch butterflies from as far north as Canada down to their wintering place in Mexico. One partner will create the histogram for Problem 1 and the other will create the bar graph for Problem 3; then you will work on Problems 2, 4, and 5 together.

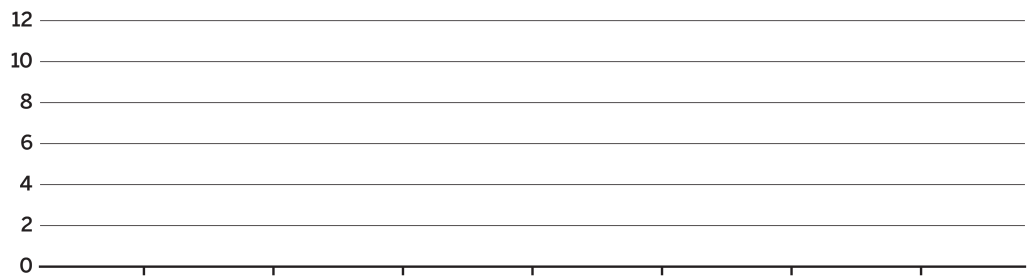
1. Use the data to create a histogram that shows how many days it took 75 tagged monarch butterflies to migrate from as far north as Canada down to Mexico.

2. Describe the distribution of the migration times. Comment on the center and spread of the data, as well as the shape and features.

Name: ______________________ Date: ____________ Period: ____________

Activity 2 Monarch Butterfly Migration (continued)

3. Use the data to create a *bar graph* on the dangers monarch butterflies face during migration that prevent them from reaching their destination. Include labels for the vertical and horizontal axes.



4. Describe what you learned about the dangers to monarch butterflies during their migration. Comment on any patterns you noticed.

5. Think about histograms and the bar graphs. How are they the same? How are they different?

Compare and Connect: As you respond to Problem 5, what math language can you use to compare histograms and bar graphs? What kind of data does each graph show?

Are you ready for more?

Research one of these data sets (or choose your own) to create a histogram. Then describe the distribution.

- Heights of 30 athletes from multiple sports or the same sport.
- High temperatures for each day of the last month in a city you would like to visit.
- Prices for all the menu items at a local restaurant.

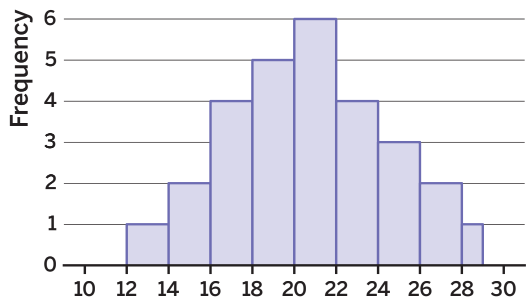
STOP

Summary

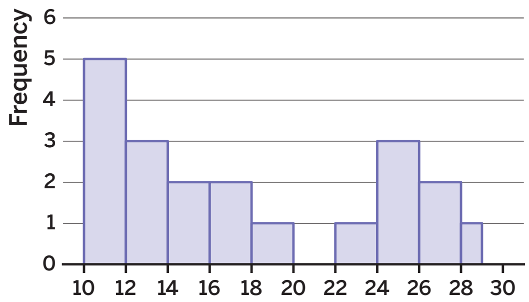
In today's lesson . . .

You continued to explore ways to describe the shape and features of a distribution represented by a histogram. Here are two distributions with very different shapes and features.

Histogram A



Histogram B



- Histogram A is symmetric and has a peak near 21. Histogram B is not symmetric and has two peaks, one near 11 and one near 25.
- Histogram B has two clusters.
- Histogram B also has a gap between 20 and 22.

Bar graphs and histograms may seem alike, but they are very different.

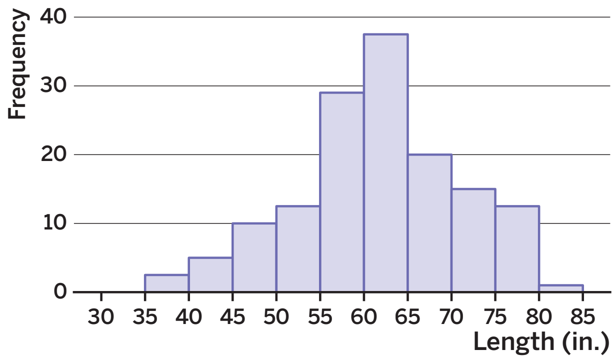
- Bar graphs represent categorical data. Histograms represent numerical data.
- Bar graphs have spaces between the bars. Histograms show a space between bars *only* when there is a gap and no data values fall between the bars.
- Bars in a bar graph can be in any order, and the number of bars depends on the number of categories. Bars in a histogram always represent ranges of values in numerical order, and you can choose how many bars to use.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. The histogram summarizes the data on the body lengths of 143 black bears (*Ursus americanus*). Describe the distribution of body lengths. Be sure to comment on the overall shape, center, and spread.



2. Which data set is more likely to produce a histogram with a symmetric distribution? Explain your thinking.

   a. Data on the length of songs, in seconds, on a pop album.

   b. Data on the time, in seconds, spent talking on the phone yesterday by every person in the school.

3. Select *all* of the data sets that would likely have one or more gaps when represented by a histogram. For each data set that may have a gap, briefly describe or give an example of why.

   A. The ages of students enrolled in a sixth grade class.

   B. The ages of people in an elementary school building on any given day.

   C. The ages of people eating in a family restaurant.

   D. The ages of runners in a marathon.

Practice

Name: ________ Date: ________ Period: ________

4. Evaluate the expression $4x^3$ for each value of x.

a 1

b 2

c $\frac{1}{2}$

5. Jada drank 12 oz of water from her bottle. This is 60% of the water the bottle holds.

a Write an equation to represent this situation. Explain the meaning of any variables you use.

b How much water does the bottle hold?

6. Determine each quotient.

a $128 \div 8$

b $228 \div 6$

c $12.4 \div 8$

d $0.99 \div 0.3$

SUB-UNIT 2 Measures of Center

Narrative Connections

What's the buzz on honey bees?

You've seen bees before. They buzz around our gardens, busy as — well, a bee! Studying one bee can tell you a lot about it, but it is only a small part of the bigger picture. Each bee is part of a collective called a hive. This hive includes workers, drones, and, most notably, the queen. Beekeepers and scientists the world over are working to make sure that honey bee hives increase in health and number.

While honey bees pollinate about 75% of the world's food production, their numbers have been declining. Colony Collapse Disorder (CCD) is the largest cause of this decline. CCD occurs when worker bees disappear, leaving their queen behind. There are many theories behind what causes CCD. Some possible explanations include climate change, mite infestation, and pesticide use.

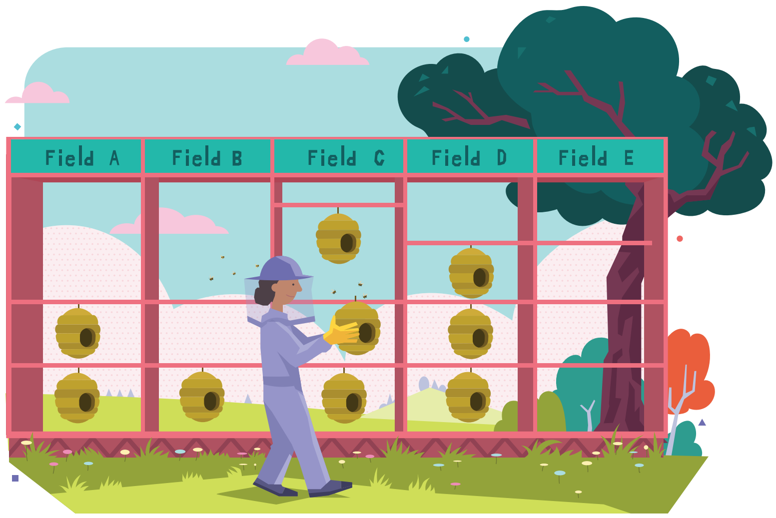
As biologists continue to study the causes of CCD, the Environmental Protection Agency has banned pesticides harmful to bee habitats in the U.S. Even tech companies have gotten involved. They have developed devices that alert beekeepers to problems in their hives.

While these are good first steps, there is still more work to be done. After all, honey bees are just one species among millions. Prioritizing this one species can create imbalance and do harm to another. To get a precise picture of the issues bees face, we need a precise way to describe their numbers. By studying the mean and median values in available data sets, we not only learn about the honey bees, but also their role in our ecosystem.

Unit 8 | Lesson 8

Mean as a Fair Share

Let's explore the mean of a data set and what it tells us.



Warm-up Four for Four

Complete the equation using four *different* single digit addends from 0–9. Show your thinking

$(\square + \square + \square + \square) \div 4 = 4$

Log in to Amplify Math to complete this lesson online.

Name: ______________ Date: ______________ Period: ______________

Activity 1 Relocating Honey Bee Hives

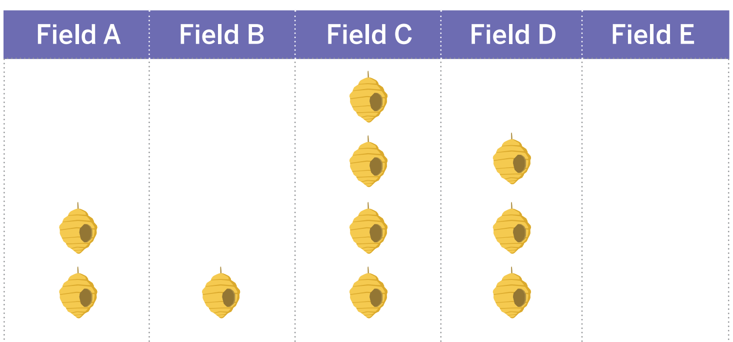
Pollinators like bees, birds, bats, beetles, and butterflies are crucial to life on Earth. Without them, many species of flora (flowers, plants, and trees) would die, limiting the diversity of food available for animals and humans.

Daniel Prudek/Shutterstock.com

The honey bee (*Apis mellifera*) is one of the most prolific pollinators in the world. Farmers use these bees to increase crop production, moving hives to support different crops that grow at different times of the year. However, moving bees can be disruptive to them and, thus, to their ability to pollinate. Because bees map efficient routes between their colonies and the surrounding flora, farmers use the “rule of three” when relocating a hive. Either move the hive no more than 3 ft, placing them close enough to be familiar with the surroundings, or move it at least 3 miles, which is far enough for them to know the hive is at a completely different location.

Part 1

1. Consider a farmer with 5 fields and 10 honey bee hives. This table shows where the hives are currently located. For the upcoming growing season, the farmer wants the hives distributed equally among all 5 fields. How many hives will end up in each field? Use the table to show how it could be done, and explain your thinking.



The number of hives in each field after they are equally distributed represents the *mean* or the *average* number of hives per field given the total, regardless of where they are located.

2. Write an expression for determining the *mean* number of hives the farmer has for each of the 5 fields. Show or explain your thinking.

Activity 1 Relocating Honey Bee Hives (continued)

Part 2

3. Another farmer has 6 fields where hives could be located. Every field has a different number of hives, and there is an *average* of 3 hives per field. Complete the table to show two different possible arrangements of hives that match this description.

	Field A	Field B	Field C	Field D	Field E	Field F
Arrangement 1						
Arrangement 2						

4. A third farmer has 5 fields where hives are located, and the *mean* number of hives per field is 11.

   a. By simply looking at the data sets, and without calculating, describe how you could identify which of the following data sets could possibly represent the hives in each of the farmer's fields, and which could not. Explain your thinking.

   Data set A: 11, 8, 7, 9, 8

   Data set B: 12, 7, 13, 9, 14

   Data set C: 11, 20, 6, 9, 10

   Data set D: 8, 10, 9, 11, 11

   b. Determine which data set is the farmer's. Explain how you know.

Name: ______ Date: ______ Period: ______

Activity 2 Creating a Consistent Schedule

An apple orchard owner has 50 honey bee hives. He hires a beekeeper (also called an *apiarist*) to tend the bees and collect the honey they produce to sell alongside his apples. This table shows the number of hours that the apiarist worked last week.

Monday	Tuesday	Wednesday	Thursday	Friday
3	6	11	7	4

1. How many hours did the apiarist typically work each day?

2. The apiarist knows the amount of work will be the same next week, but wants to evenly distribute his hours throughout the week – Monday and Friday were nice, but he was exhausted after Wednesday! How long would he typically work each day next week if the hours were evenly distributed? Show or explain your thinking.

3. The following list shows the amounts of honey, in pounds, the apiarist collected Monday through Thursday last week.
   amount of honey (in pounds): 15, 9, 15, 12

   After finishing work on Friday, the apiarist claimed that he had collected an average of 15 lb of honey per day that week. If he is correct, then how many pounds of honey did he collect on Friday? Show or explain your thinking.

Are you ready for more?

Starting next month, the apiarist is going to need to work 7 more hours each week. If he decides to work the same number of hours each day from Monday through Saturday, should he expect the typical number of hours worked each day to increase or decrease from the typical value you found in Problem 2? Explain your thinking.

STOP

Summary

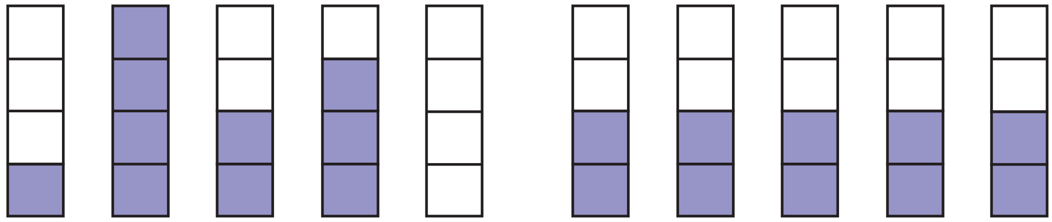
In today's lesson . . .

You saw that you can summarize a data set in terms of typical values using a single number, called a ***measure of center***. One way to *measure the center* of a data set is to think about it as "determining a fair share," or "leveling out the distribution," so that each value would have the same frequency. And one way to calculate that value is to determine the ***average*** value of all the data values.

For example, suppose this data set represents the numbers of liters of water in 5 bottles: $1, 4, 2, 3, 0$.

To calculate an *average*, you first add up all of the values to determine the total (10 liters). Then you divide that sum by the number of values (5 bottles), which in this example can be represented by the expression $(1 + 4 + 2 + 3 + 0) \div 5$, or $10 \div 5$. So, the average liters of water in the 5 bottles is 2 liters (per bottle).

This average value also represents the "fair share," which can be determined by redistributing equal amounts to the original number of "groups" (the 5 bottles), as shown in this diagram.



As a *measure of center* for a data set, the average of all the values in the data set is called the ***mean***.

Reflect:

Name: ______________________ Date: ____________ Period: __________

Practice

1. A preschool teacher is rearranging four boxes of playing blocks so that each box contains an equal number of blocks. Currently Box A has 32 blocks, Box B has 18, Box C has 41, and Box D has 9 blocks. Select *all* the ways he could make each box have the same number of blocks.

   A. Remove all the blocks and make four equal piles, which will contain 25 blocks. Then put each pile in one of the boxes.

   B. Remove 7 blocks from Box A and place them in Box B.

   C. Remove 23 blocks from Box A, 9 blocks from Box B, and 32 blocks from Box C. Redistribute the 64 blocks that were removed, putting 16 in each box.

   D. Remove 7 blocks from Box A and place them in Box B. Remove 21 blocks from Box C and place them in Box D.

   E. Remove 7 blocks from Box A and place them in Box B. Remove 16 blocks from Box C and place them in Box D.

2. In a round of mini-golf, Clare records the number of putts she takes on each hole, as shown in the table. She claims that her mean number of putts per hole is 3. Explain why Clare is correct.

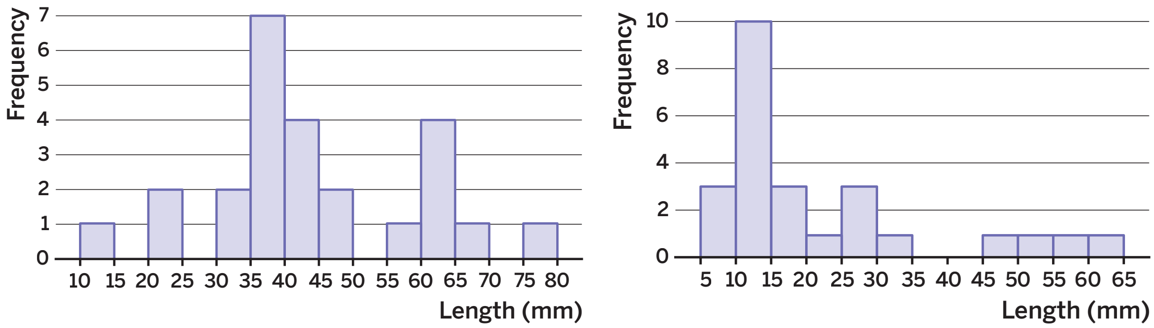
2	3	1	4	5	2	3	4	3

3. In her English class, Mai's teacher gives 4 quizzes, each worth 5 points. After 3 quizzes, Mai has scores of 4, 3, and 4. How many points on the last quiz are needed for Mai to have an average score of 4? Show or explain your thinking.

Practice

Name: .... Date: .... Period: ....

4. An earthworm farmer examined two containers of a certain species of earthworms so that he could learn about their lengths. He measured 25 earthworms in each container and recorded their lengths in millimeters. The histograms of the lengths for each container are shown.



a. Which container typically has longer worms than the other container?

b. For which container would 15 mm be a reasonable description of a typical length of the worms in the container?

c. If length is related to age, which container had the younger worms?

5. Diego thinks that $x = 3$ is a solution to the equation $x^2 = 16$. Do you agree? Show or explain your thinking.

6. Identify at least two ways in which the expressions are alike.

$$\frac{3+3+3}{3} \qquad \frac{3+3+3+3}{4}$$

Name: ______________________ Date: __________ Period: __________

Unit 8 | Lesson 9

Mean as the Balance Point

Let's look at another way to understand the mean of a data set.

Warm-up Which One Doesn't Belong?

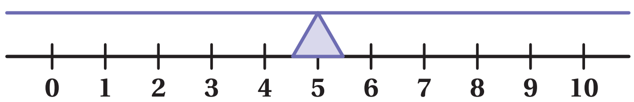
Which expression does not belong? Be prepared to explain your thinking.

Expression A	Expression B	Expression C	Expression D
$\frac{8+8+4+4}{4}$	$\frac{10+10+4}{4}$	$\frac{9+9+5+5}{4}$	$\frac{6+6+6+6+6}{5}$

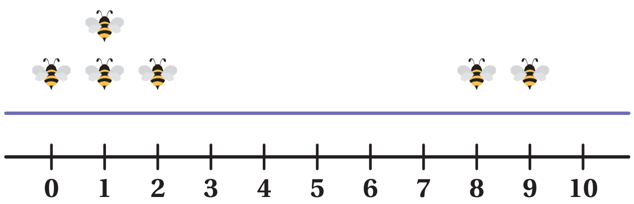
Log in to Amplify Math to complete this lesson online.

Activity 1 Balancing Bumble Bees

1. Here is a beam and a balance point. You will be given 5 identical bees — they all weigh the same. Determine at least two ways you could place all of the bees on the beam to make them balance. Record your favorite placement on the beam below. Be prepared to explain your thinking.



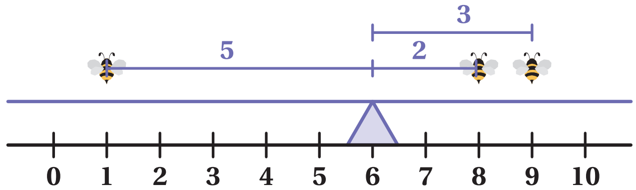
2. Place the triangle under the beam to show you you could balance the bees. Explain your thinking.



Name: ________ Date: ________ Period: ________

Activity 1 Balancing Bumble Bees (continued)

3. Here is Mai's strategy to balance three bees.



a Are the bees balanced? Explain your thinking.

b If the bees are balanced, do you think Mai's strategy will always work? If not, explain how Mai might not have been thinking about it correctly.

4. You will be given four cards that show bees on beams. Determine which sets of bees, if any, are balanced. If they are balanced, explain why. If they are not balanced, explain why and which direction — left or right — the balance (triangle) would need to move to balance the bees.

5. Determine the means for each set of bees in Problem 4. How do the means relate to your responses from Problem 4?

Activity 2 Threats to the Honey Bees

The IUCN has not yet classified honey bees on their Red List. However, many are worried about decades of steep declines in the insect's population. The National Agricultural Statistics Service (NASS) is one of the groups within the U.S. Department of Agriculture that monitors honey bee populations. Cynthia Clark, an American statistician, has been recognized for her work improving the quality of data that the NASS and other organizations, such as the United States Census Bureau, collect and use.

Mirko Graul/Shutterstock.com

Two of the biggest threats to honey bees are varroa mite infestations and Colony Collapse Disorder (CCD). Varroa mites transmit varroosis and other deadly diseases that spread from hive to hive. CCD occurs when the majority of adult worker bees disappear from the hives at the same time, leaving too few adult bees to care for all of the young bees. This causes entire colonies to die.

You will be given two dot plots that show the number of colonies lost to varroa mites and CCD in ten months at a Mississippi apiary. On average, which threat — varroa mites or CCD — killed more honey bee hives? Show or explain your thinking.

Featured Mathematician

Cynthia Clark

Cynthia Clark is a statistician who was born in Colorado. She holds both a BS and MS in Mathematics, and went on to pursue a PhD in Mathematics at Iowa State University. However, she graduated with a PhD in Statistics instead. Throughout her career, Clark was known for her work improving the quality of data, and she served in positions for the Office of Federal Statistical Policy, the Office of Management and Budget, the National Agricultural Statistics Service, and the United States Census Bureau. In 2011, she was given the Presidential Meritorious Rank Award.

STOP

Name: ________________ Date: ________ Period: ________

Summary

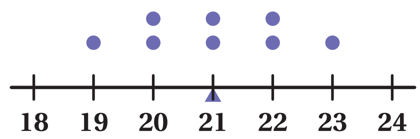
In today's lesson . . .

You saw another interpretation of the *mean* as a measure of center for a distribution, which is the "balance point" for the distribution.

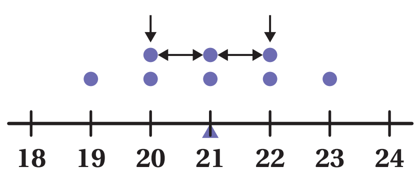
Consider this data set and its corresponding dot plot, which has a distribution that is completely symmetric, and a mean of 21 (located on the dot plot by the triangle).

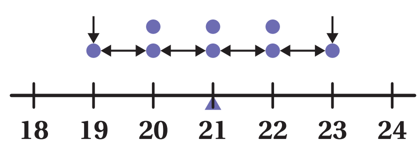
19, 20, 20, 21,

21, 22, 22, 23



Now think of the number line as a seesaw, balancing on the triangle representing the mean. If each dot were a real object and they all weighed the same, then two dots that are located the same distance from the mean but on opposite sides would balance. For example, the points at 20 and the points at 22 are all 1 away from 21 but on opposite sides, so they would balance each other around 21. Similarly, the points at 19 and 23 are both 2 away, but on opposite sides, and would balance each other. The points at 21 itself would have no effect on the balance.





Even when a distribution is not completely symmetric, the mean represents the balance point of the sum of all the distances of the values less than the mean, and the sum of all the distances of the values greater than the mean.

Reflect:

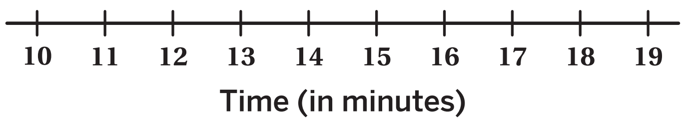
Practice

Name: ______________ Date: ______________ Period: ______________

1. The data set shows 5 times, in minutes, for Kiran's walks on 5 school days.

16	11	18	12	13

a Create a dot plot for Kiran's data.



b Without calculating, decide whether 15 minutes would be a good estimate of the mean. If you think it is a good estimate, explain your thinking. If not, give a better estimate and explain your thinking.

c Calculate the mean for Kiran's data. Show your thinking.

d In the table, record the distance of each data value from the mean and its location relative to the mean.

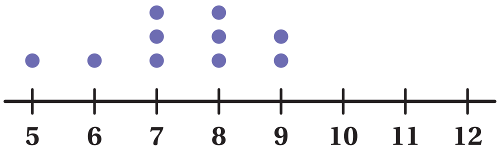
Time (minutes)	Distance from the mean	Greater or less than the mean?
16		
11		
18		
12		
13		

e Calculate the sum of all distances to the left of the mean, and then calculate the sum of distances to the right of the mean. Explain how these sums show that the mean is a balance point for the values in the data set.

Name: ________ Date: ________ Period: ________

Practice

2. This dot plot shows the hours of sleep received by ten sixth grade students last night.



Elena estimated the mean number of hours of sleep to be 8.5 hours, Andre estimated it to be 7.5 hours, and Noah estimated it to be 6.5 hours. Which estimate do you think is best? Explain your thinking.

3. In a basketball game, Shawn scored 20 points and Bard scored 30 points. The mean number of points scored by Shawn, Bard, and Tyler for that game was 40 points. How many points did Tyler score in that game? Show or explain your thinking.

4. Evaluate each quotient.

a $\frac{2}{5} \div 2$

b $\frac{2}{5} \div 5$

c $2 \div \frac{2}{5}$

d $5 \div \frac{2}{5}$

5. The data set shows how many hours 10 students spent playing outside last week.

5	3	4	2	2	4	5	2	1	3

What is the middle of the data?

Unit 8 | Lesson 10

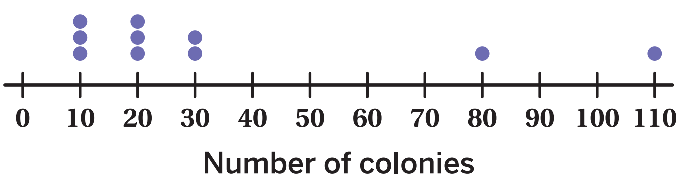
Median

Let's explore the median of a data set and what it tells us.

Warm-up Monitoring Infestation Rates

You have seen that varroa mites pose a serious threat to honey bees' survival. However, the mere presence of these parasites does not guarantee a death sentence for a bee or its colony. If the parasite population is kept in check, entire colonies can remain healthy. The general consensus is that an infestation rate of less than 3% is safe. This is calculated using a sugar roll test.

This dot plot shows the number of colonies with infestation rates greater than 3% at ten North Dakota apiaries.



1. Use the dot plot to think about what a typical number of colonies with infestation rates greater than 3% is for these ten apiaries. Then without making any calculations, estimate the *center* of the data, and mark its location on the dot plot.

2. Determine the *mean* for the data, and mark its location on the dot plot with a triangle.

3. How does the mean compare to the value that you identified as the center of the data? Why might that be?

Log in to Amplify Math to complete this lesson online.

Name: ______________ Date: ______________ Period: ______________

Activity 1 Determining the Middle

Part 1

Your teacher will give you an index card. Write your first and last name on the card and record the total number of letters in your name. As a class, you will determine the middle of the data set, which is another measure of center called the ***median***.

Part 2

1. How did you determine the median?

2. How does the process for determining the median change depending on the number of data values?

3. In the context of what the values in the data set represent, what does the median tell you?

Stronger and Clearer: Share your responses to Problems 1–3 with another pair. Ask clarifying questions, such as "If you have a data set with 10 values, is the median the 5th value? Why or why not?" Revise your responses, as needed.

Activity 2 Varroa Mites, Bee Gone!

1. Here is the data set from the Warm-up representing the number of colonies with varroa mite infestation rates greater than 3% in 10 North Dakota apiaries.

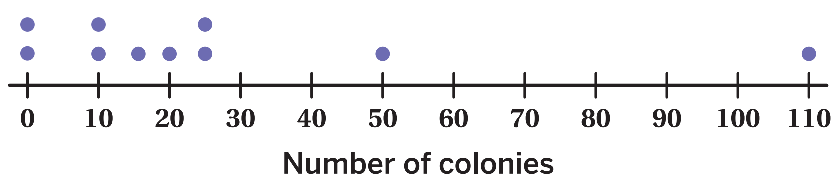
20	10	30	20	80	10	30	10	20	110

a Determine the median of the data. Show your thinking.

b In the context of what the data set represents, what does the median tell you?

c How does the median represent the center of the data differently than the mean?

2. All 10 apiaries tried to control the varroa mite populations by using standard intervention techniques. This dot plot shows the number of colonies with infestation rates still greater than 3% *after* the interventions.



a Determine the median number of colonies with infestation rates still greater than 3%. Mark the location of the median on your dot plot with a rhombus. Show your thinking.

b In general, were the interventions successful? Use your work to explain your thinking.

Are you ready for more?

Create a data set with a mean that is much less than what you would consider to be a typical value for the data set you invented.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You determined another measure of center for a distribution called the ***median***. The *median* is literally the "middle" value in a data set when the values are listed in order from least to greatest (or greatest to least). Half of the data values have values less than or equal to the median, and half of the data values have values greater than or equal to the median.

To determine the median from an ordered representation of the data, such as a list or a dot plot, you repeat a process of eliminating the pairs of least and greatest values.

Odd number of values	$\cancel{0}$ $\cancel{1}$ $\cancel{1}$ 2 $\cancel{2}$ $\cancel{4}$ $\cancel{5}$ Once all pairs have been eliminated, only one value remains in the middle, making it the *mean*. Mean: 2
Even number of values	$\cancel{0}$ $\cancel{1}$ $\cancel{1}$ 1 2 $\cancel{2}$ $\cancel{4}$ $\cancel{5}$ Once all pairs have been eliminated, two values remain. Their average is the mean. $(1 + 2) \div 2 = 1.5$ Mean: 1.5

Reflect:

Name: .... Date: .... Period: ....

1. Priya attempted the first level of a video game 10 times. Her scores are shown. What is her median score?

130	150	120	170	130
120	160	160	190	140

A. 125

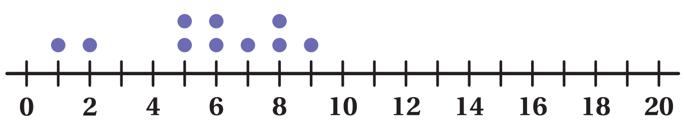
B. 145

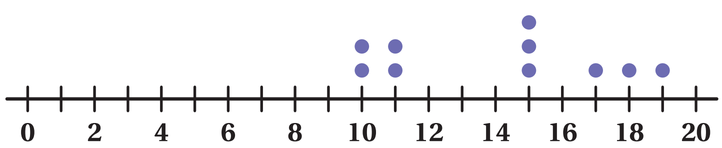
C. 147

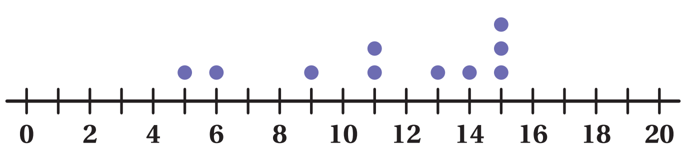
D. 150

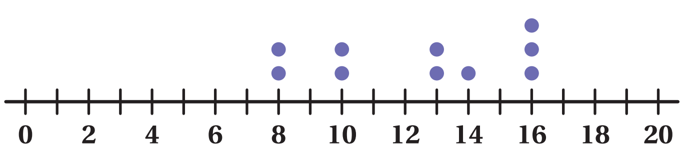
2. A teacher sorted a class's scores on their last test and noticed that 12 students scored higher than Clare, 12 students scored lower than Clare, and no one scored the same as Clare. Does this mean that Clare's score on the test is the median? Explain your thinking.

3. Match each dot plot with its median.

a


b


c


d


.... 15

.... 12

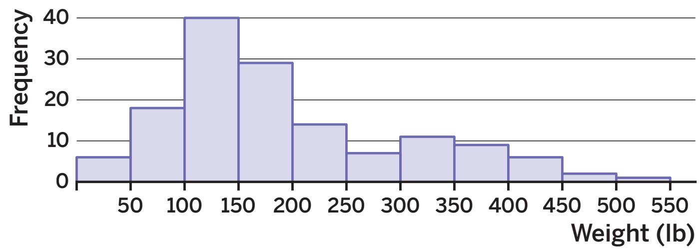
.... 13

.... 6

Name: .... Date: .... Period: ....

Practice

4. This histogram shows the weights, in pounds, of 143 wild bears. The ages of the bears ranged from newborn to 15 years old.



a. What can you say about the heaviest bear in this group?

b. What is a typical weight for the bears in this group?

c. Do more than half of the bears in this group weigh less than 250 lb?

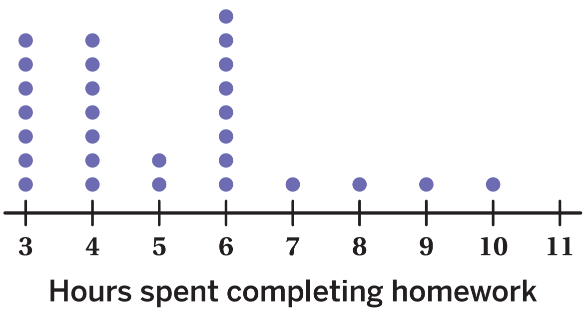
d. If weight is related to age, with older bears tending to have greater body weights, would you say that there were more old bears or more young bears in the group? Explain your thinking.

5. Noah's water bottle contains more than 1 qt of water but less than $1\frac{1}{2}$ qt. Let w represent the number of quarts that the bottle can hold. Select *all* the true statements.

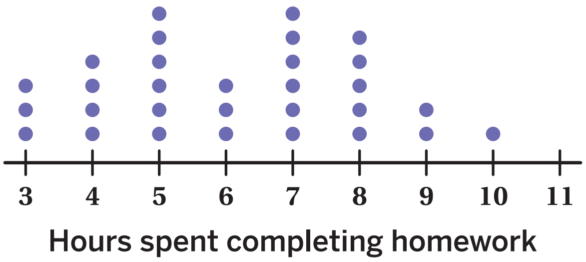
A. w could be $\frac{3}{4}$.
B. w could be 1.
C. $w > 1$
D. w could be $\frac{4}{3}$.
E. w could be $\frac{5}{4}$.
F. w could be $\frac{5}{3}$.
G. $w > 1.5$

6. In one class, students typically spent 6 hours per week completing homework. Without calculating, which dot plot best represents this class?

A.



B.



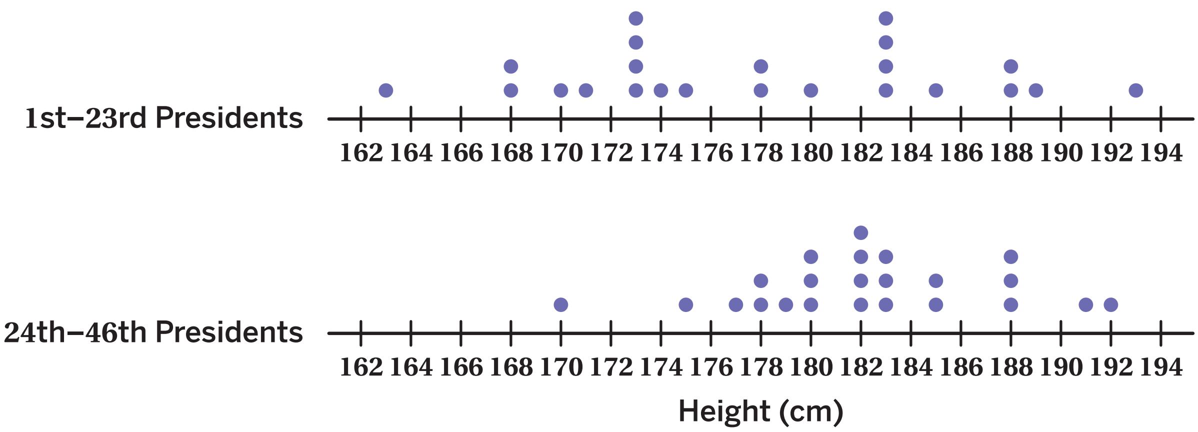
Unit 8 | Lesson 11

Comparing Mean and Median

Let's compare the mean and median of data sets.

Warm-up Heights of the Presidents

Two dot plots are shown. The first dot plot shows the heights of the first 23 U.S. presidents who served from 1789–1893. The second dot plot shows the heights of the next 23 presidents who served from 1893–2021. All heights are rounded to the nearest centimeter.



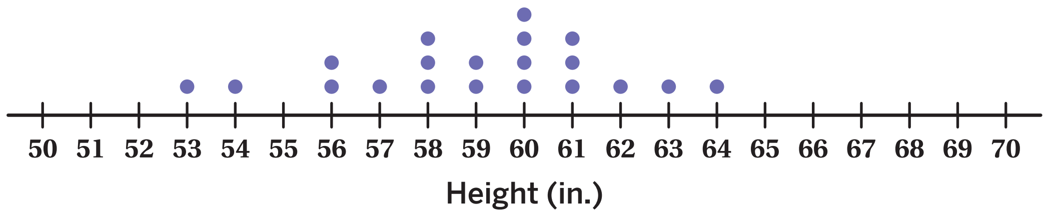
Without calculating, use the dot plots to compare the heights of the first 23 presidents to the heights of the second 23 presidents. Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Moving the Middle

The dot plot shows the heights, in inches, of 20 players on Shawn's soccer team.



1. Determine one number to describe a typical height, in inches, of a player on the team by calculating:

a The mean height.

b The median height.

2. A new player joins Shawn's soccer team. For each description of how that player's height impacts the mean of the whole team when the new player is included, determine:

- How tall the new player might be, without calculating.
- What happens to the median.

a The mean height remains the same.

b The mean height increases.

3. Would it be possible for one new player to join Shawn's team and for both the mean and median to remain the same as for the original data? Explain your thinking.

Activity 1 Moving the Middle (continued)

4. Two players join the soccer team. For each description of how those two players' heights impact the mean and median of the whole team when they are included, describe what you would know about the heights of the two players. Do not calculate. Be prepared to explain your thinking.

   a. The mean remains the same but the median decreases slightly.

   b. The mean might change but the median does not change.

5. Determine whether each scenario is possible when two new players join Shawn's team. Explain your thinking.

   a. Both the mean and median remain the same as in the original data.

   b. The mean and median end up a lot farther apart than in the original data.

Are you ready for more?

Create a data set with five numbers that has a mean of 10 and a median of 12. Show or explain your thinking.

Name: ________ Date: ________ Period: ________

Activity 2 Mean or Median?

Despite the serious threats that honey bees face, there is some evidence that their situation is improving, in part because apiarists are working to renovate existing colonies by introducing a new queen or other new bees.

Part 1

You will be given six cards. Each card contains information about the number of honey bee colonies renovated across several apiaries in the state listed. Each card also includes either a dot plot or a histogram, as well as both the mean and the median.

1. Sort the cards into *two* groups based on the distributions and measures of center shown. Be prepared to explain your thinking.

2. Discuss your sorting decisions with another group. Resolve any disagreements, and record your final decisions.

Part 2

You will be assigned one state. For your state:

3. Explain the pros and cons of using each measure of center to summarize the data.

4. Make a case for which measure of center *best* summarizes the data. Explain your thinking.

STOP

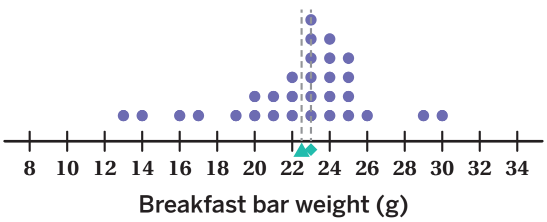
Summary

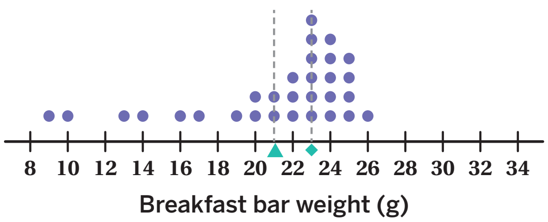
In today's lesson . . .

You compared how well the mean and median each describe or summarize the center of a distribution. Each measure of center tells you slightly different things, and one may be more appropriate depending on the distribution of the data and the context.

In general, when a distribution is symmetric, or approximately symmetric, the mean and median values are close. Less symmetric distributions, such as those with *outliers* (values much less or greater than typical values in the distribution), tend to have the mean and median farther apart. The mean is calculated using the values of every data value, so it shifts away from other typical values when there are outliers. However, the median counts all values the same (values of 1 less or 100 less than typical are both just one point), so it does not generally shift away from other typical values when there are outliers.

Consider these two dot plots showing the weights, in grams, of two different batches of 30 breakfast bars. The means (triangles) and medians (rhombuses) have been marked.





The mean is a good measure of center for the first batch because it represents a balance point and the data are symmetric, but it could be misleading for the second batch because the data are not symmetric and most values are greater.

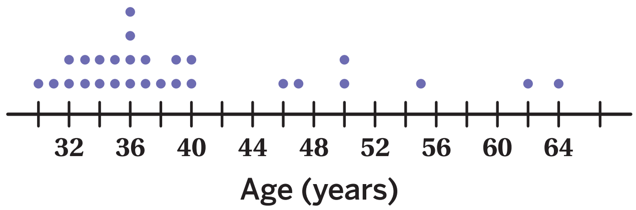
The median is a good measure of center for both the first batch and the second batch, but it could be misleading to someone who does not know what the distribution looks like if they assume the data are symmetric.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Here is a dot plot that shows the ages of teachers at a school. Which of these statements is true of the data set shown in the dot plot?



A. The mean is less than the median.

B. The mean is approximately equal to the median.

C. The mean is greater than the median.

D. The mean cannot be determined.

2. Bard asked each of five friends to attempt to throw a ball in a trash can until they succeeded. Bard recorded the number of unsuccessful attempts made by each friend as: 1, 8, 6, 2, 4. However, Bard made an error in recording. The 8 in the data set should have been 18. How would changing the 8 to 18 affect the mean and median of the data set?

A. The mean would decrease and the median would not change.

B. The mean would increase and the median would not change.

C. The mean would decrease and the median would increase.

D. The mean would increase and the median would increase.

3. For history class, Han's homework scores are shown below.

100	100	100	100	95	100	90	100	0

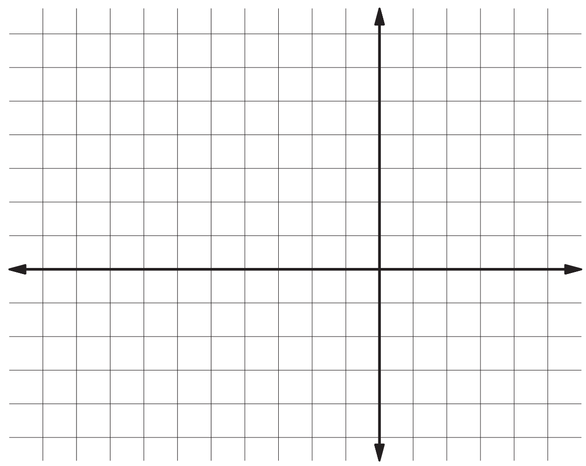
The history teacher uses the mean to calculate an overall grade for homework. Write an argument for Han to explain why median would be a better measure to use for his homework grades.

Practice

Name: ______ Date: ______ Period: ______

4. Zookeepers recorded the age, weight, sex, and height of the 10 pandas at their zoo. Write two statistical questions that could be answered using these data sets.

5. Here is a set of coordinates. Draw and label an appropriate pair of axes and plot the points.
$A(1, 0)$, $B(0, -0.5)$, $C(-4, 3.5)$, $D(-1.5, -0.5)$



6. Elena and Kiran have been practicing their basketball free throws during recess. They record the number of baskets they make out of 10 attempts. Here are their data sets for 12 school days.

Elena:

4	5	1	6	9	7	2	8	3	3	5	7

Kiran:

2	4	5	4	6	6	4	7	3	4	8	7

a Calculate the mean number of baskets each player made, and then compare the means.

b What do the means tell you about each player's free-throw shooting ability in this context?

SUB-UNIT 3 Measures of Variability

Where have the giant sea cows gone?

Look off the Florida coast and in the shallows, you might see a rush of bubbles followed by a whoosh of water and air. These are the tell-tale signs of a manatee. Also known as "sea cows," manatees are the gentle giants of the ocean. These aquatic mammals are huge. They average about ten ft in length and weigh around 1,200 lb.

To date there are only three known species of manatees in existence — the West Indian manatee, the Amazonian manatee, and the African manatee. And, yet, despite having only a few natural predators, manatees worldwide are in danger of extinction.

The reason? Humans.

Every year, hundreds of manatees are injured or killed by boaters. In the winter season, manatees congregate in warm shallow waters, like those off the Florida coast. Boat drivers, unable to see them, often run them over. Others have been trapped and killed in manmade structures like floodgates and canal locks. Others still get tangled up in fishing lines. Meanwhile climate change has changed ocean temperatures, spurring on habitat loss. Manatees suffer from overcrowding and exposure to cold stress that their sensitive bodies can't withstand.

The manatees' survival depends on us making smart choices to protect their environment. To do that, we have to be well-informed about where they live, their migration patterns, and the habits in their lifecycle. Through statistics, scientists can account for variabilities in the data and deepen their understanding of how to help manatee populations thrive.

Unit 8 | Lesson 12

Describing Variability

Let's study distances between data values and the mean or median.

Warm-up Spotting Manatees

Florida manatees (*Trichechus manatus latirostris*) are listed as Endangered by the IUCN, but, interestingly, the US Fish and Wildlife Service (USFWS) took Florida manatees off their endangered species list in 2017.

Alex Couto/Shutterstock.com

Crystal River, Homosassa River, and Chassahowitzka River are three locations in Citrus County, Florida, where aerial surveys are conducted to count and monitor Florida manatee populations. Given the competing labeling between the IUCN and USFWS, keeping track of their populations is very important.

Your teacher will show you data sets from these 3 locations showing the number of manatees sighted on 12 different days.

Without calculating, predict which location has the greatest mean, median, and spread.

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 Counting Manatees

Refer to the dot plots of the same sightings data from the Warm-up, showing the mean and median.

1. Does either the mean or the median describe the variability in the data for each location?

2. Describe the variability of the three locations based on their mean and median, thinking about how they are the same or different.

3. Use the dot plots, the data sets, and their measures of center (if you think they are useful), to compare the variability of the three data sets.

4. Which location would you choose to go to if you wanted to see at least 4 manatees? Explain your thinking.

Are you ready for more?

Think about some issues that scientists and volunteers who monitor and count manatees may face in trying to gather accurate data.

1. How could each of those issues impact the counts and data available?

2. How could that also impact how the data is interpreted by different groups, such as IUCN and USFWS?

Activity 2 Variability at Chassahowitzka River

Refer to the aerial survey data from Chassahowitzka River to answer the problems.

1. Complete the table by determining the difference between each value and the:
   - Mean.
   - Median.

Chassahowitzka River	3	5	12	2	1	2	3	0	3	15	5	9
Distance from the mean, 5	2											4
Distance from the median, 3	0											

2. Describe what the values in each row of your completed table might tell you about the *variability* of the data.

3. Select the measure of center — mean or median — that you think could best be used to help summarize the *variability* of the data set with one number. Use the values in the corresponding row in the table to help you determine one number to describe the variability of the original data set. Explain your thinking.

Measure of center	One number for variability	Explanation

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You shifted away from focusing on how the values in a data set can be used to determine one or more numbers that describe the center of a distribution. You began to explore these same ideas of using the values in a data set to determine one or more numbers that instead describe the *spread* of a distribution. With spread, this can be a little more involved and can be interpreted a few different ways.

One way to think about it would be to consider how far apart the values are overall. Another way to think about it would be to consider how far apart the values are relative to one other number, such as the center. Yet another way to think about it would be to consider how far apart each number is from each and every one of the other numbers. And there are probably even more possibilities.

All of these possibilities have one thing in common — they are attempting to describe the amount of *variability* in a data set. The ***variability*** of a data set is a description of how far away, or how spread out, data values generally are from the center.

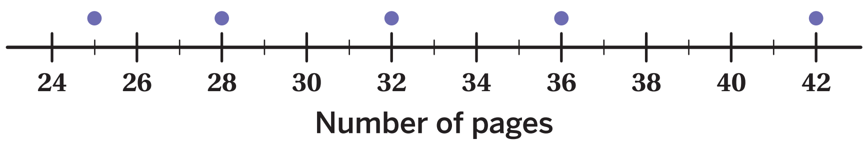
In the next lessons, you will see some commonly used *measures of variability* that relate to the measures of center you already know — mean and median.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Han recorded the number of pages that he read each day for five days. This dot plot shows his data.



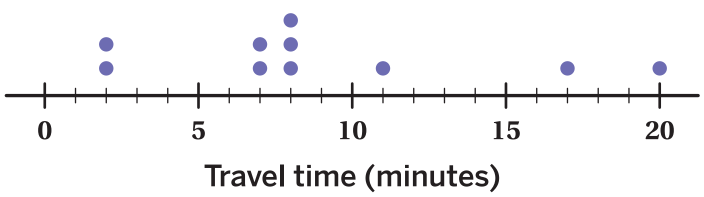
a. Is 30 pages a good estimate for the mean number of pages that Han read each day? Explain your thinking.

b. Determine the mean number of pages that Han read during the five days. Draw a triangle to mark the mean on the dot plot.

c. Use the dot plot and the mean to complete the table.

Number of pages	Distance from mean	Left or right of mean
25		left
28		
32		
36		
42		

2. Ten sixth graders recorded the amounts of time each of them took to travel to school. The dot plot shows their travel times.



a. What is a typical amount of travel time for the 10 sixth graders? Explain your thinking.

b. Jada believes that travel times between 5 and 13 minutes are common for this group. Do you agree? Explain your thinking.

Name: .... Date: .... Period: ....

Practice

3. In an archery competition, scores for each round are calculated by averaging the distances of 3 arrows from the center of the target.

An archer's three arrows were 1.6 in., 0.5 in., and 2.9 in. from the center in the first round. In the second round, the archer's arrows were farther from the center *on average*, but more consistent. What values for the distances in the second round could fit this description? Explain your thinking.

4. Draw a number line.

a Plot and label three numbers between -2 and -8 (not including -2 or -8).

b Use the numbers you plotted and the symbols $<$ and $>$ to write three inequality statements.

5. Jada earns money from babysitting, walking her neighbor's dogs, and running errands for her aunt. Every four weeks, she combines her earnings and divides them into three equal parts — one for spending, one for saving, and one for donating to a charity. Jada donated $\$26.00$ of her earnings from the past four weeks to charity.

Make two different lists showing possible amounts she could have earned from each job in the last four weeks, if she did not earn the same amount from any two jobs.

6. Determine each sum or difference:

a $3.3 + 2.1$

b $33 + 2.1$

c $3.3 - 2.1$

d $33 - 2.1$

Unit 8 | Lesson 13

Variability and MAD

Let's use the mean and mean absolute deviation (MAD) to describe variability.

Warm-up Number Talk

Mentally evaluate each expression. Be prepared to explain your thinking.

$42 \div 12$

$2.4 \div 12$

$44.4 \div 12$

$46.8 \div 12$

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Manatee Scars

Many manatees bear scars from having narrowly escaped encounters with one of their biggest threats – boats in shallow water. The tables show the number of manatees (out of every 10) spotted with scars over 12 days.

gary powell/Shutterstock.com

1. Record the distance between each data value and the mean at Crystal River.

Manatees with scars (out of every 10)	9	4	5	1	6	7	2	8	3	3	5	7
Distance from mean (5)	4	1										

Calculate the *average of the distances from the mean* in the table. This value is called the ***mean absolute deviation (MAD)*** of the data set. Show your thinking and round to the nearest tenth.

Crystal River's MAD: ______

2. Determine the mean absolute deviation of the data from Homosassa River. Show your thinking and round to the nearest tenth.

Number with scars (out of every 10)	2	4	5	4	6	6	4	7	3	4	8	7
Distance from mean (5)												

Homosassa River's MAD: ______

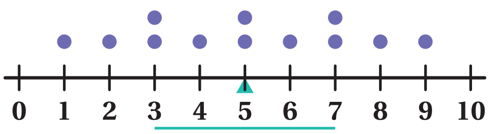
3. Determine the mean absolute deviation of the data from Chassahowitzka River. Show your thinking and round to the nearest tenth.

Number with scars (out of every 10)	7	8	9	0	1	2	1	3	2	10	9	8
Distance from mean (5)												

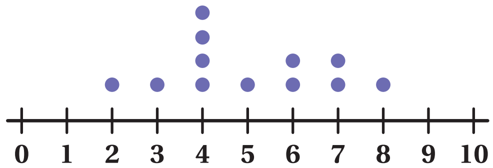
Chassahowitzka River's MAD: ______

Activity 1 Manatee Scars (continued)

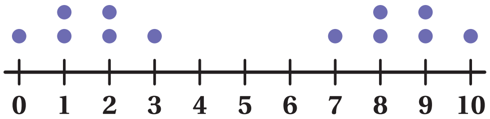
Here are dot plots representing the same data for the scarred manatees. For Crystal River, the mean is represented by a triangle, and the segment below the triangle represents the span of values between one mean absolute deviation less than the mean and one mean absolute deviation more than the mean.



Crystal River



Homosassa River



Chassahowitzka River

4. Add similar marks to the dot plots for Homosassa River and Chassahowitzka River to represent the mean (triangle) and the MAD (segment).

5. Use the mean and MAD to describe and compare the distributions at the three locations.

6. Use the mean and the MAD to complete the statements describing typical values for each location:

   a. At Crystal River, there were typically between .... and .... manatees with scars sighted each day.

   b. At Homosassa River, there were typically between .... and .... manatees with scars sighted each day.

   c. At Chassahowitzka River, there were typically between .... and .... manatees with scars sighted each day.

7. If you were to observe and count manatees with scars at one sight for one day, at which location do you think you would typically see 7 out of the first 10 manatees with scars? Explain your thinking.

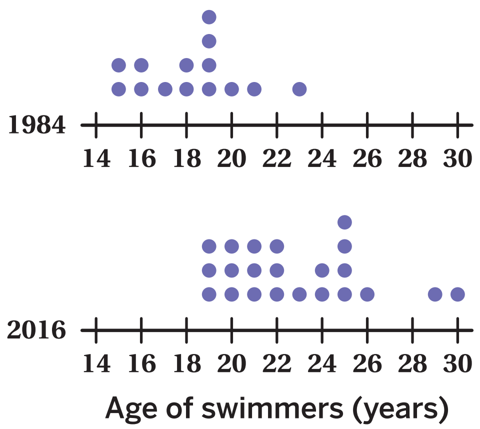
Name: ____________ Date: ________ Period: ________

Activity 2 Human Swimmers

In 1984, the mean age of swimmers on the U.S. women's swimming team was 18.2 years and the MAD was 2.2 years. In 2016, the mean age of the swimmers was 22.8 years, and the MAD was 3 years.

1. Are the swimmers on the 1984 team closer in age to one another than the swimmers on the 2016 team are to one another? Explain your thinking.

2. Here are dot plots showing the ages of the women on the U.S. swimming team in 1984 and in 2016. For both parts, round to the nearest tenth.



   a. What percent of the data is within the MAD on both sides of the mean in 1984?

   b. What percent of the data is within the MAD on both sides of the mean in 2016?

3. If the 29- and 30-year old swimmers were not on the 2016 team, the mean age of swimmers for that year would be 22.2 years old and the MAD would be 1.9 years. What impact would that have on your answer to Problem 1?

STOP

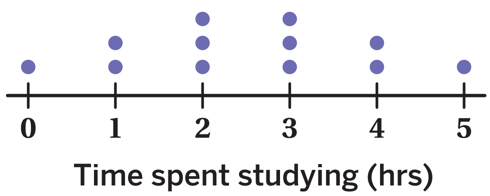
Summary

In today's lesson . . .

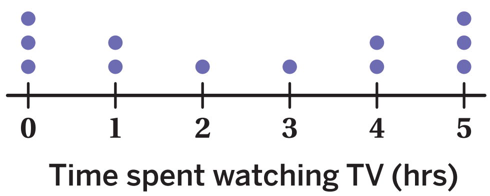
You saw that the values in a data set can be used to describe its variability or spread. A ***measure of variability*** is a way to summarize how the values in a data set vary. In other words, a single number that describes how spread out the distribution is around typical values or its center.

For example, the mean represents the center and you can determine the amount of variability around the mean – how far away, or how spread out, a typical data value is from the mean. One way to do that is to determine the distance between each value in the data set and the mean. Then you can use those to determine the average (or mean) distance between all the data values and the mean. The result is a measure of variability called the ***mean absolute deviation (MAD)***.

Consider the following distributions:

<table>
<tr><td>
</td><td>Mean: 2.5
MAD: 1.17

The average student spends 2.5 hrs studying.

Students typically spend between 1.33 hr and 3.67 hrs studying
$(2,5 \pm 1.17)$</td></tr>
<tr><td>
</td><td>Mean: 2.5
MAD: 1.83

The average student spends 2.5 hrs studying.

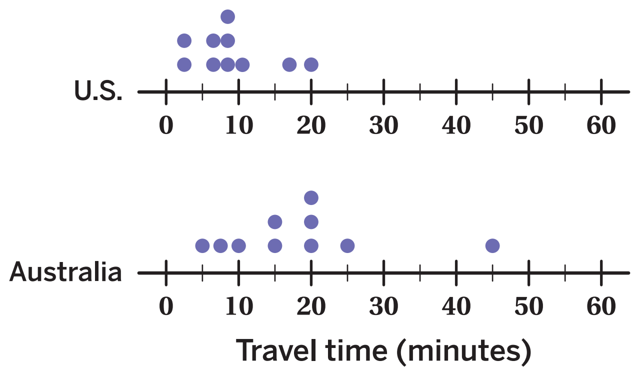
Students typically spend between 0.67 hr and 4.33 hrs studying
(2.5 ± 1.83)</td></tr>
</table>

Reflect:

Name: ________________ Date: ________ Period: ________

Practice

1. The dot plots show the amounts of time that 10 U.S. students and 10 Australian students took to get to school. Which statement is true about the MAD of the Australian data set?



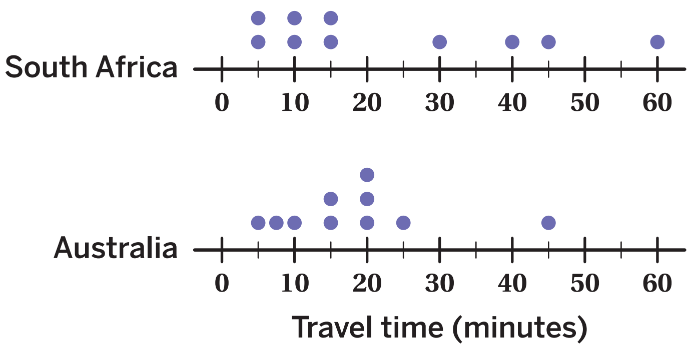
A. It is significantly less than the MAD of the U.S. data set.

B. It is exactly equal to the MAD of the U.S. data set.

C. It is approximately equal to the MAD of the U.S. data set.

D. It is significantly greater than the MAD of the U.S. data set.

2. The dot plots show the amounts of time that 10 South African students and 10 Australian students took to get to school. Without calculating, solve the problems.



a. Which data set has the lesser mean? Explain your thinking.

b. Which data set has the lesser MAD? Explain your thinking.

c. What does a lesser mean tell you in this context?

d. What does a lesser MAD tell you in this context?

Practice

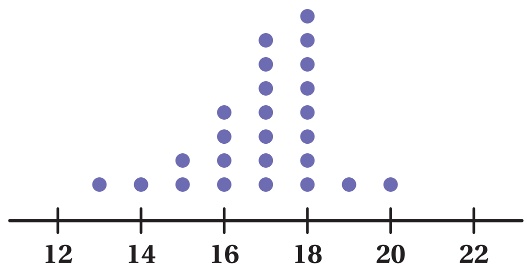
Name: ______ Date: ______ Period: ______

3. Two high school basketball teams have identical records of 15 wins and 2 losses. Sunnyside High School's mean number of points per game is 50 points, with a MAD of 4 points. Shadyside High School's mean number of points per game is 60 points, with a MAD of 15 points.

Which team's scores in all 17 games were the most consistent? Explain your thinking.

4. Draw and label an appropriate pair of axes. Then plot and label the points $A(10, 50)$, $B(30, 25)$, $C(0, 30)$, $D(20, 35)$.

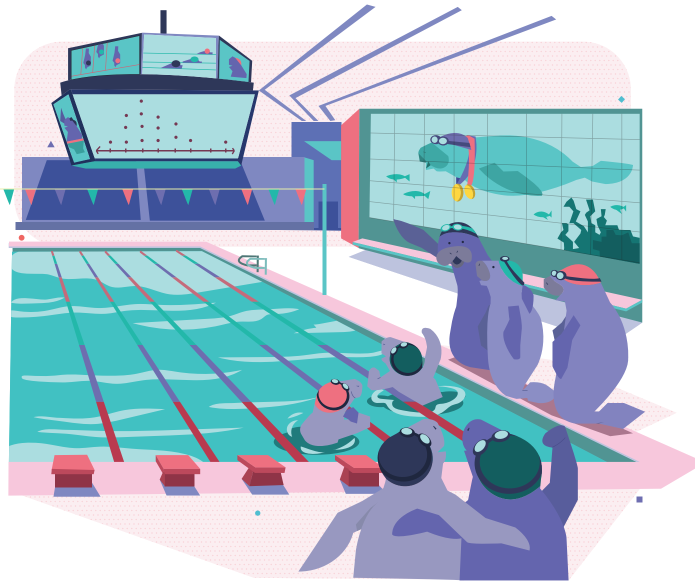
5. The dot plot shows the birth weights, in ounces, of all the puppies born at a kennel in the past month. Determine the median using only the dot plot. Show or explain your thinking.



Name: ______ Date: ______ Period: ______

Unit 8 | Lesson 14

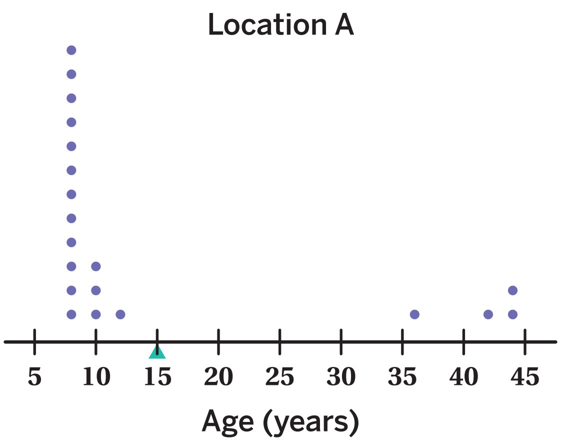
Variability and IQR

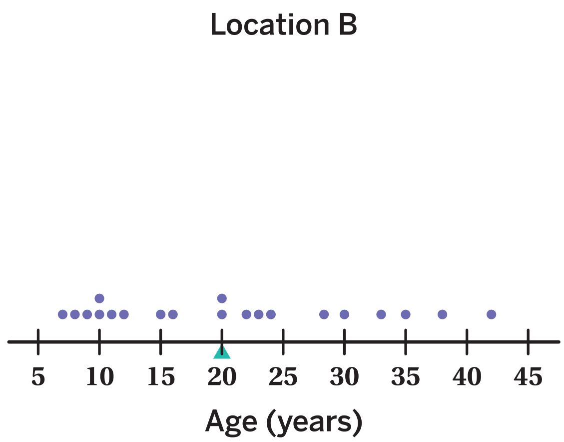


Let's use the median and the interquartile range (IQR) to describe variability.

Warm-up Notice and Wonder

The two dot plots show the ages of 20 manatees from 2 different locations. The mean of each data set is marked with a triangle.





1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 The Five-Number Summary

You have seen data sets that are not symmetric, have a wide spread, or have outliers, and, for those, the median is a good choice of center. But because the MAD corresponds to mean, what about describing and summarizing variability for those types of distributions? And what about the different reasons behind why the data may look like that?

Statisticians deal with these questions and issues all the time, because the reality is, reality is messy! Statistician Mary C. Christman, who has served as an advisor to the Florida Fish and Wildlife Commission Research Institute, has spent part of her career addressing exactly that. Collecting environmental data, such as about manatees, is not easy and has many challenges.

Here are the ages of twenty manatees from Location B, ordered from least to greatest. Use the data set to complete the problems, and think about how your work is dealing with the relatively wide spread of values in this data set.

7 8 9 10 10 11 12 15 16 20 20 22 23 24 28 30 33 35 38 42

1. Circle the least data value and label it *Minimum*. Then circle the greatest data value and label it *Maximum*.

2. Determine the following values. Mark the position of each value in the data and label each as indicated.

	Value	Mark	Label
Median		◆	Q2
Middle value of the *lower half* of the data		↑	Q1
Middle value of the *upper half* of the data		↑	Q3

Featured Mathematician

Mary C. Christman

Mary C. Christman holds a BS in Biology from the University of Pennsylvania, an MS in Marine Biology and Physical Oceanography from the University of Delaware, and a PhD in Mathematical Statistics from George Washington University. She is currently the owner of MCC Statistical Consulting, which specializes in collecting and representing environmental and ecological data. She has advised the Florida Fish and Wildlife Commission Research Institute on the coastal ecosystems of Florida and the effects of the "red tide" phenomenon on both humans and sea life, including manatees.

Name: ______ Date: ______ Period: ______

Activity 1 The Five-Number Summary (continued)

Look back at the ordered list of data on the previous page, now with the marks and labels. The data set has been split into four equal parts from the minimum to the maximum. The three values labeled Q1, Q2, and Q3 that divide the data are called *quartiles*.

- The *first quartile* represents an upper bound for the lowest 25% of the data, which is always contained between the minimum and Q1. It is also referred to as the *25th percentile*. Q1 is also a lower bound for the highest 75% of the data.
- The *second quartile* corresponds to the *median*, and it represents an upper bound for the lowest 50% of the data, which is always contained between the minimum and Q2. It is also referred to as the *50th percentile*. Q2 is also a lower bound for the highest 50% of the data.
- The *third quartile* represents an upper bound for the lowest 75% of the data, which is always contained between the minimum and Q3. It is also referred to as the *75th percentile*. Q3 is also a lower bound for the highest 25% of the data.

Together, these five numbers — minimum, Q1, Q2, Q3, maximum — make up what is called the *five-number summary* for a data set.

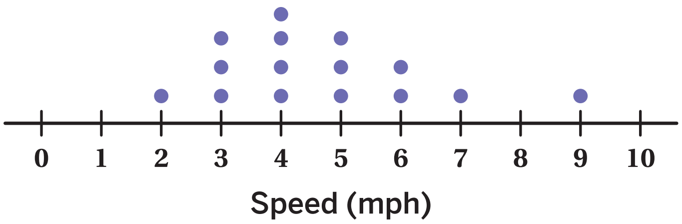
3. Record the *five-number summary* for data representing the ages of the manatees.

Minimum: ______ **Q1:** ______ **Q2:** ______ **Q3:** ______ **Maximum:** ______

4. What does the value of the third quartile (Q3) tell you about the ages of the manatees at this location?

Activity 2 Range and Interquartile Range

Here is a dot plot that shows 15 recorded speeds of a manatee, in miles per hour.



1. Write the five-number summary for this data set. Show your thinking.

   Minimum: .... **Q1:** .... **Q2:** .... **Q3:** .... **Maximum:** ....

2. One way to describe the *spread* of values in a data set is to look at the difference between the maximum and minimum values. This is called the ***range***. What is the range of the speeds of the manatee?

3. Another way to describe the spread of values in a data set is to look at the difference between the upper quartile (Q3) and the lower quartile (Q1). This is called the ***interquartile range (IQR)***.

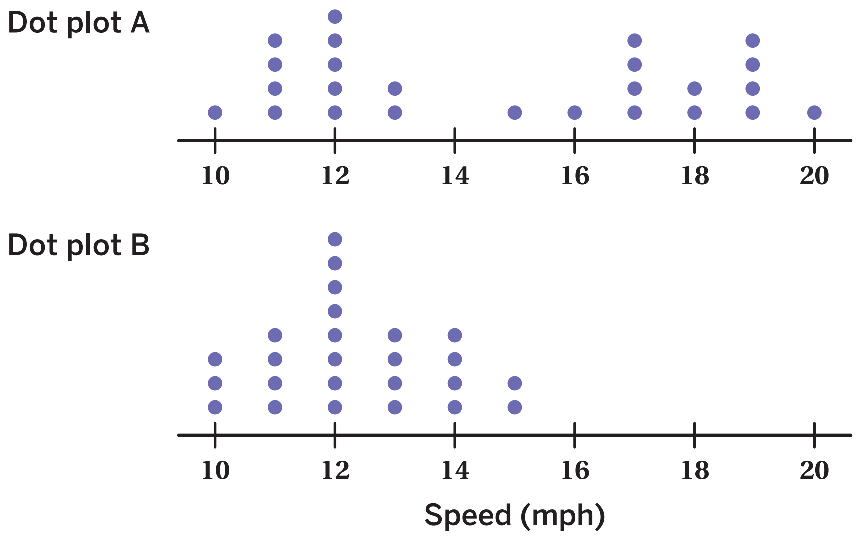
   a What is the interquartile range (IQR) of this manatee's speeds?

   b How does the IQR relate to typical values?

Name: ______ Date: ______ Period: ______

Activity 2 Range and Interquartile Range (continued)

4. Here are two more dot plots showing the speeds of two other manatees.



Without doing any calculations, predict:

a Which data set has the lesser range?

b Which data set has the lesser IQR?

5. Check your predictions by calculating the ranges and IQRs for the data in each dot plot.

Data set A:

Range: ______ **IQR:** ______

Data set B:

Range: ______ **IQR:** ______

STOP

Summary

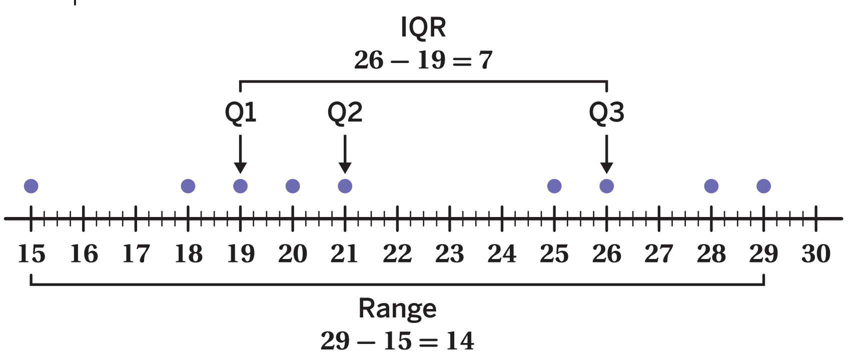
In today's lesson . . .

You saw how to calculate the ***five-number summary*** for a data set, which can be used to summarize its distribution. The *five-number summary* consists of the minimum, maximum, and the three quartiles, Q1, Q2, and Q3.

The ***first quartile (Q1)*** is the median of the lower half of the data.	The ***second quartile (Q2)*** is the median of the entire data set.	The ***third quartile (Q3)*** is the median of the upper half of the data.

You used the *five-number summary* to calculate two measures of ***variability*** that can be used to describe the distribution of a data set in terms of its spread.

- One measure, the ***range***, represents the difference between the maximum and minimum values of a data set.
  - The range gives you a basic overall sense of how spread out the data is, but it does not tell you about variability and how it is distributed between the minimum and maximum values.
- The other measure is called the ***interquartile range (IQR)***, which represents the range of the middle 50% of the data.
  - A greater IQR indicates more variability because the middle 50% of the data is more spread out.

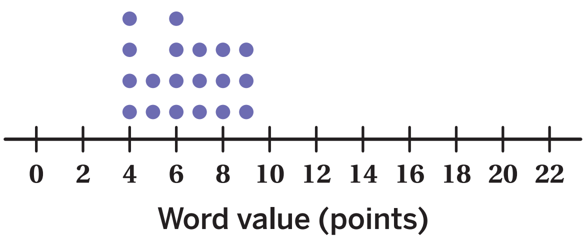


Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. In a word game, 1 letter is worth 1 point. This dot plot shows the scores for 20 words.



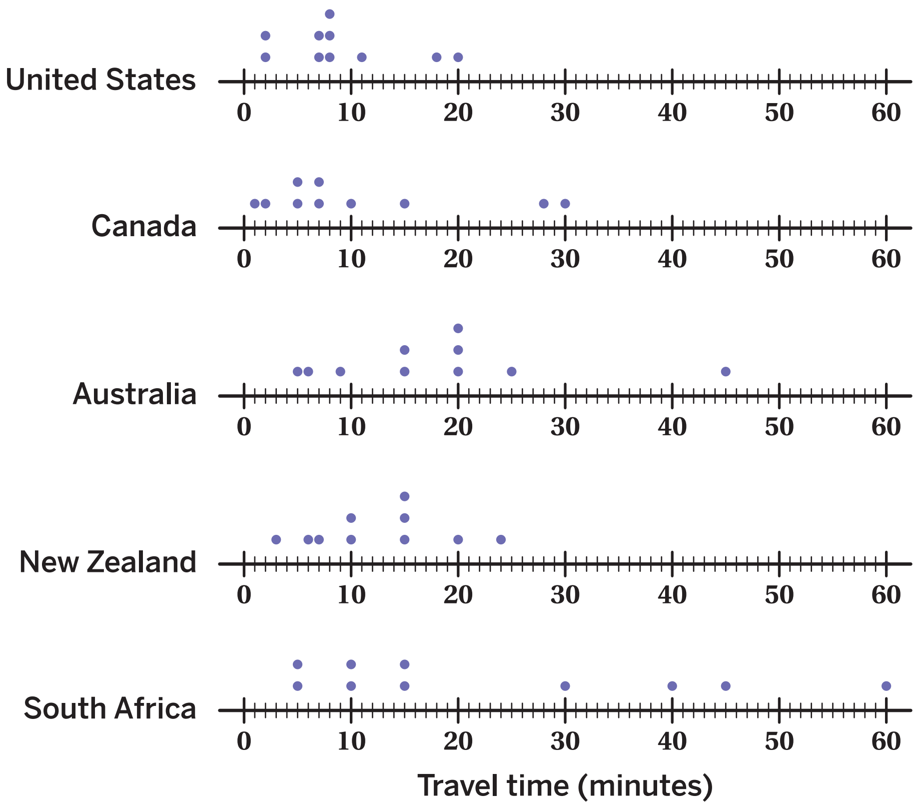
a What is the median score?

b What is the first quartile (Q1)?

c What is the third quartile (Q3)?

d What is the interquartile range (IQR)?

2. The 5 dot plots show the travel times to school of 10 sixth graders in 5 countries. Match each dot plot with the correct median and IQR by writing the country's name next to each set of statistics.



a Median: 17.5, IQR: 11

b Median: 15, IQR: 30

c Median: 8, IQR: 4

d Median: 7, IQR: 10

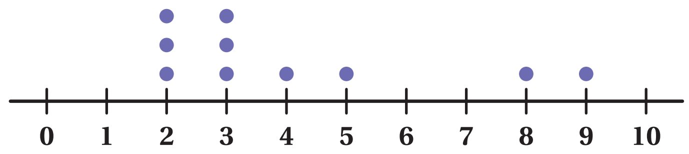
e Median: 12.5, IQR: 8

Practice

Name: .... Date: .... Period: ....

3. Mai and Priya each bowled 10 games and recorded their scores. Mai's median score was 120, with an IQR of 5. Priya's median score was 118, with an IQR of 15. Whose scores had less variability? Explain how you know.

4. Here is a dot plot that represents a data set. Explain why the mean of the data set is greater than its median.



5. Two TV shows each asked 100 viewers for their ages. For Show X, the mean age of the viewers was 25 years old, with a MAD of 15 years. For Show Y, the mean age of the viewers was 20 years old, with a MAD of 5 years. A sixth grader watches one of the two shows. Which show do you think the student most likely watches? Explain your thinking.

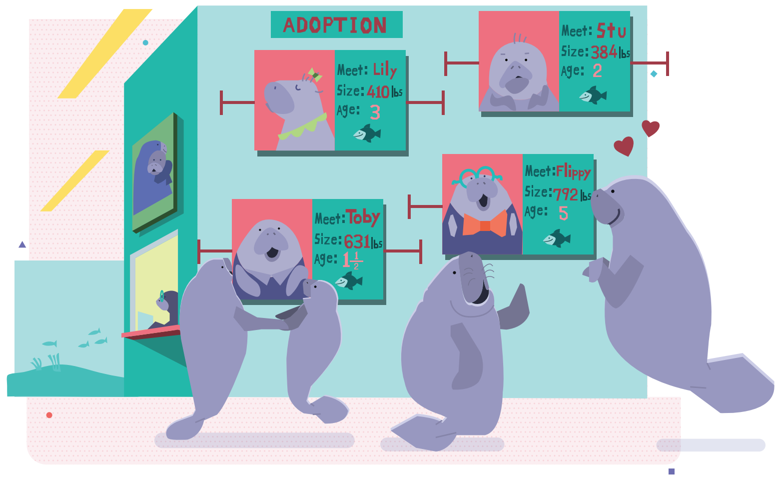
6. Write a scenario that could be represented by this number summary.

Minimum: 5 **Median:** 7.5 **Maximum:** 9

Name: ______ Date: ______ Period: ______

Unit 8 | Lesson 15

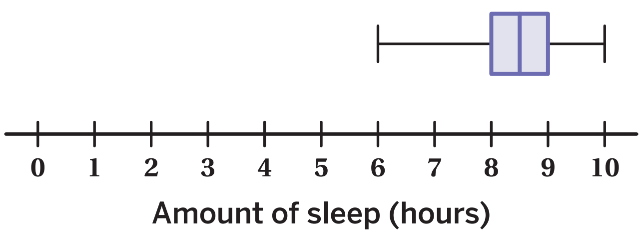
Box Plots



Let’s explore how box plots can summarize distributions.

Warm-up Notice and Wonder

Several students were asked how much sleep, in hours, they usually receive on a school night. Here is a representation of the data collected. What do you notice? What do you wonder?



1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Living Box Plot

Conservation groups have been monitoring manatees that return to the same locations year after year, noting migration habits, births, and deaths. Many of these returning visitors have been given names, and they can be identified by their markings as well as their personalities (one named Howie apparently likes to tip over researchers' canoes).

You will be given the names of 24 manatees from Blue Spring State Park that are up for "adoption," and an ordered list of values representing the lengths of their names.

1. Determine the five-number summary for the data set representing the lengths of the manatees' names.

   Minimum: .... **Q1:** .... **Q2:** .... **Q3:** .... **Maximum:** ....

Your class will make a ***box plot*** (sometimes called a *box-and-whisker plot*) to represent the distribution of the lengths of the manatees' names.

2. Record the completed box plot here.

Length of manatee names

3. Using your box plot to represent the length of manatee names data, determine the percentages of the data represented by each of these elements of the box plot.

   a The left whisker: ....

   b The box: ....

   c The right whisker: ....

Name: ______ Date: ______ Period: ______

Activity 2 Sea Turtles

Here are the data collected from weighing, in pounds, 11 different specimens for each of 3 different species of sea turtles that nest at the Outer Banks of North Carolina.

Chatchantharangsee/Shutterstock.com

Hawksbill (*Eretmochelys imbricata*; IUCN Critically Endangered)

210.75	155.5	100	120	235	180	190.25	198	200	145	165

Olive Ridley (*Lepidochelys olivacea*; IUCN Vulnerable)

98	87.5	85	90	90	90	100	92.75	95	90.25	91

Loggerhead (*Caretta caretta*; IUCN Vulnerable)

150	295	180	185	200	185.5	215	220.25	225	245	160

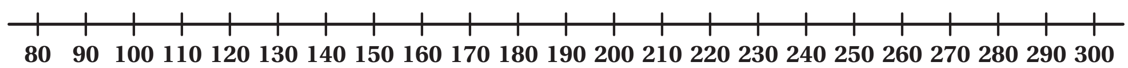
1. As a group, decide which turtle each of you will focus on. Write the five-number summary for the data for that turtle and then calculate the interquartile range (IQR).

	Minimum	Q1	Q2	Q3	Maximum	IQR
Hawksbill						
Olive Ridley						
Loggerhead						

2. Share your data with your group and write 3 sentences explaining what the IQRs tell you about the weights of these three species of sea turtles.

Activity 2 Sea Turtles (continued)

3. Draw the dot plot for your turtle, and then share with your group. Draw the box plot for each sea turtle, at different heights along this one number line. Label each box plot with the name of the species of turtles' weights it represents.



Turtle weights (lb)

4. How are the weights of the Hawksbill and the Loggerhead the similar? How are they different?

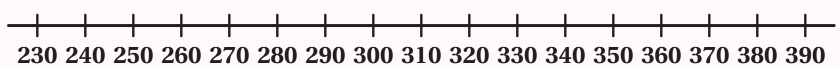
5. How are the weights of the Loggerhead and the Olive Ridley similar? How are they different?

Are you ready for more?

Weights of a set of Green Sea turtles were also recorded.

- **The minimum weight was the same as the maximum weight of the Loggerheads.**
- **The maximum weight was 3.8 times greater than the maximum weight of the Olive Ridleys.**
- **The IQR is equal to the sum of the IQRs of the Olive Ridley and Loggerhead.**

1. Draw a box plot that could represent the Green Sea Turtle's data.



2. Can you estimate the median weight for the Green Sea Turtles? If so, how? If not, why not?

STOP

Name: ________ Date: ________ Period: ________

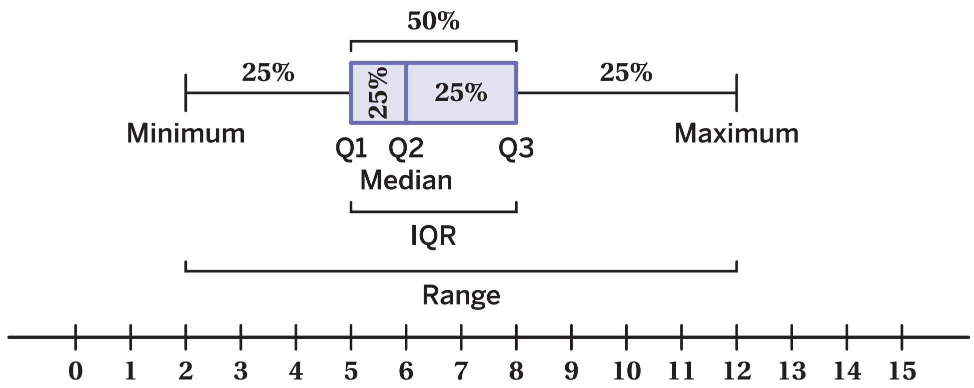
Summary

In today's lesson . . .

You saw how a ***box plot*** can be used to represent the five-number summary of a data set.

For example:

Minimum	Q1	Q2 or median	Q3	Maximum
2	5	6	8	12



Box plots on their own, however, do not show you how many data points were in each set, or the values of any individual data points, except the minimum and maximum. They do, however, make it possible to compare minimums, maximums, medians, quartiles, ranges, and IQRs for two or more sets of data at a glance.

Reflect:

Practice

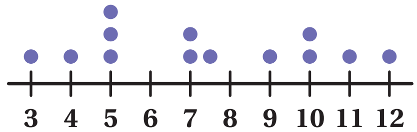
Name: .... Date: .... Period: ....

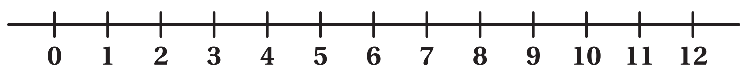
1. The data show the number of hours per week sixth and seventh graders spent doing homework. Create two box plots above the same number line to represent both sets of data.

Sixth graders:

Minimum: 1.5 **Q1**: 2 **Median:** 3.5 **Q3:** 5 **Maximum:** 7

Seventh graders:





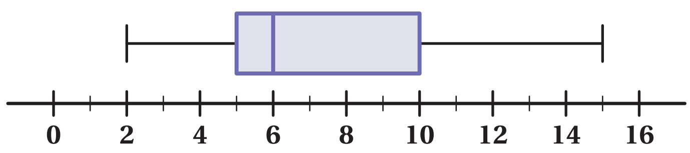
a How are the distributions of the time spent doing homework by the students in the two grades most alike? Explain your thinking.

b Which grade level of students corresponds to the data with the greatest variability in time spent on homework? Explain your thinking with measures of data.

Name: .... Date: .... Period: ....

Practice

2. Each student in a class recorded how many books they read during the summer. The box plot summarizes their data.



Number of books

a. What is the greatest number of books read by a student in this group?

b. What is the median number of books read by the students?

c. What is the interquartile range (IQR) for the number of books read by the students?

3. There are 20 pennies in a jar. If 16% of the coins in the jar are pennies, how many coins are there in the jar?

4. Here is a list of questions about the students and teachers at a school. Select *all* the questions that are statistical questions.

A. What is the most popular lunch choice?

B. What school do these students attend?

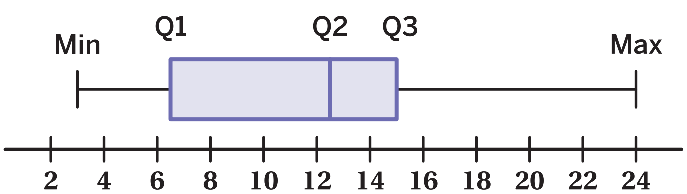
C. How many math teachers are in the school?

D. What is a common age for the teachers at the school?

E. About how many hours of sleep do students generally get on a school night?

F. How do students usually travel from home to school?

5. Scientists believe people blink their eyes to keep the surface of the eye moist and also to give the brain a brief rest. Write two statistical questions that could be answered using the box plot.



Number of blinks in 1 minute

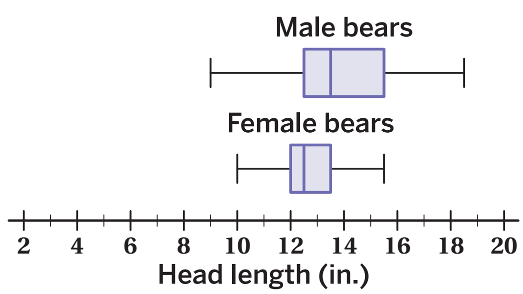
Minimum: 3 **Q1:** 6.5 **Q2:** 12.25 **Q3:** 15 **Maximum:** 24

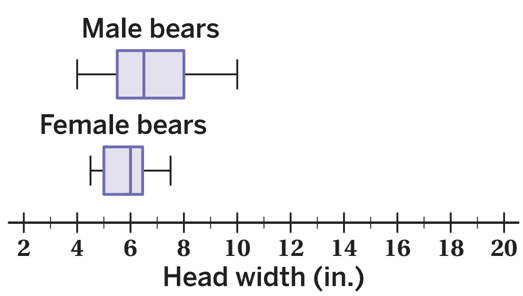
Comparing MAD and IQR

Let's compare data sets using visual displays.

Warm-up Grizzly Data

In one study on 143 Grizzly bears (*Ursus arctos*), researchers measured the head lengths and head widths of the bears. The ages of the bears ranged from newborns (0 years) to 15 years. These box plots summarize the data.





1. Write 4 statistical questions that could be answered using the box plots — 2 questions about the head length and 2 questions about the head width.

2. Trade questions with your partner. For each question, state whether you agree that it is a statistical question. Explain your thinking. Use the box plots to answer each of your partner's questions.

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Will the Yellow Perch Survive?

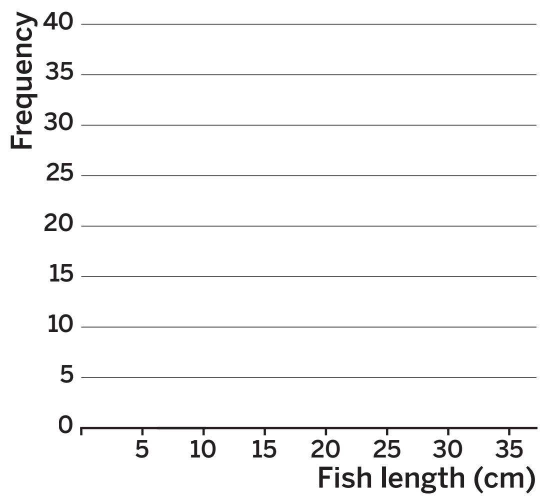
You are a scientist studying the yellow perch (*Perca flavescens*), a species of fish. You and your fellow scientists know that, in general, the length of a fish is related to its age. This means that the longer the fish, the older it is. Adult yellow perch vary in size, but they typically measure between 10 and 25 cm long.

RLS Photo/Shutterstock.com

The Great Lakes Water Institute caught, measured, and released yellow perch at several locations in Lake Michigan. You and your colleagues have been asked to evaluate their data.

Part 1

1. Use the data to construct a **histogram** that shows the lengths of the captured yellow perch. Each bar should contain the lengths shown in each row in the table.



2. How many fish were measured? How do you know?

3. How would you describe the shape of the distribution?

4. Estimate the median length for this sample. Describe how you made this estimate.

5. Predict whether the mean length of this sample is *greater than*, *less than*, or *nearly equal* to the median length for this sample of fish? Explain your prediction.

Activity 1 Will the Yellow Perch Survive? (continued)

Part 2

6. Using the data and other representations provided on the data sheet, interpret the mean and MAD. What do they tell you about the lengths and ages of the yellow perch?

7. Interpret the median and IQR. What do they tell you about the lengths and ages of the yellow perch?

8. Which pair of measures of center and variability — mean and MAD, or median and IQR — do you think summarize the distribution of the lengths of the yellow perch better? Explain your thinking.

Part 3

9. Based on your data analysis of this sample and the following classification: young (< 10 cm), adult (≥ 10 and < 25), and old (≥ 25), which graph best represents a typical age for the yellow perch — the histogram, dot plot, or box plot? Explain your thinking.

10. Some researchers are concerned about the survival of the yellow perch. Do you think the lengths (or the ages) of the fish in this sample are something to worry about? Explain your thinking.

STOP

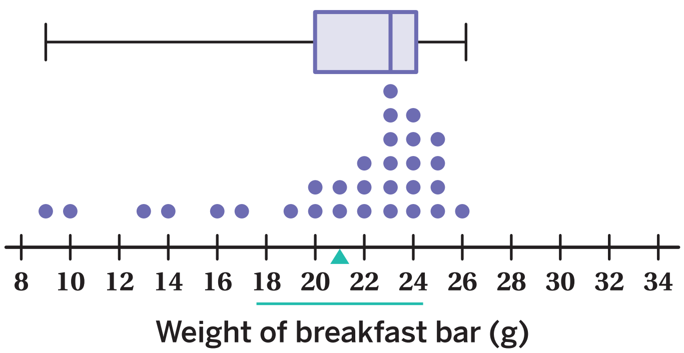
Name: Date: Period:

Summary

In today's lesson . . .

You compared how well the MAD and IQR each describe or summarize the variability in a distribution. Similar to the differences between mean and median, each measure of variability tells you slightly different things, and one may be more appropriate. It is still true that when a distribution is not symmetric, the median and IQR are often more representative measures of center and spread than the mean and MAD. However, the decision on which pair of measures to use depends on what you want to know about the data or the question you are answering.

Here is the dot plot from the example you saw in Lesson 11, and also a box plot for the same data representing the weights of 30 different breakfast bars.



The mean is 21 g and the MAD is 3.4 g. This suggests that a breakfast bar typically weighs between 17.6 g and 24.4 g, as represented by the triangle and horizontal segment around it. The median bar weight is 23 g and the IQR is 4 g. This suggests that a breakfast bar typically weighs between 20 g and 24 g, as represented by the box.

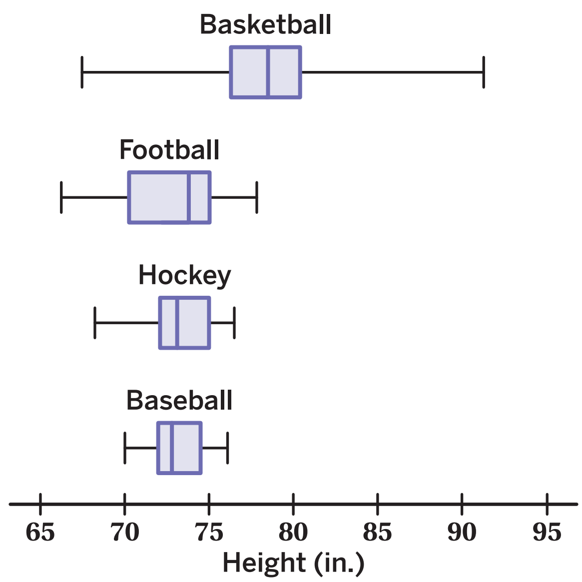
These two pairs of measures paint very different pictures of the variability of the weights of the breakfast bars. But this is not because the MAD and IQR vary a lot. Rather it is the combination of differences in both the measures of center and the measures of variability together that make up the difference. Because the distribution is not symmetric, the median is closer to the middle of the largest cluster of values. If you were to ignore the lighter weights, the median and IQR would give a more accurate picture of how much a breakfast bar typically weighs.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

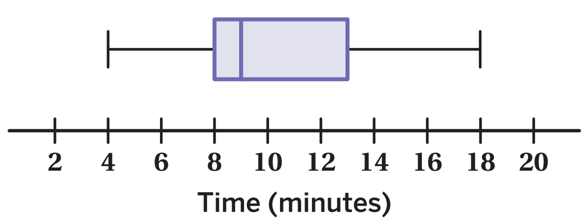
1. The box plots summarize the heights of 20 professional male athletes in basketball, football, hockey, and baseball.



a In which two sports are the players' height distributions most alike? Explain your thinking.

b Which sport shows the greatest variability in players' heights? Which sport shows the least variability?

2. Here is a box plot that summarizes data for the time, in minutes, that a fire department took to respond to 100 emergency calls. Select *all* the statements that must be true based on the box plot.



A. About 75% of the response times were 8 minutes or less.

B. At least half of the response times were 11 minutes or longer.

C. Most of the response times were under 13 minutes.

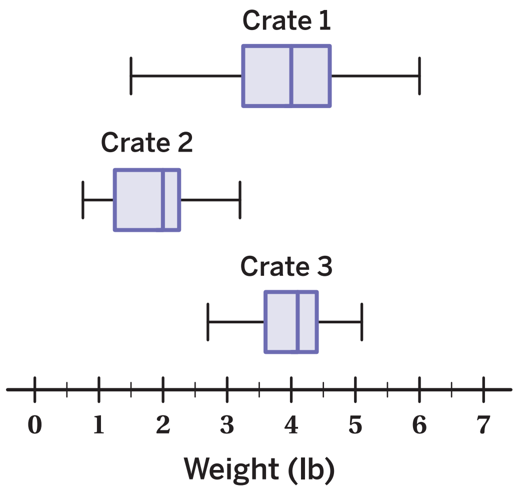
D. Fewer than 30 of the response times were over 13 minutes.

E. There were more response times that were greater than 13 minutes than those that were less than 9 minutes.

Name: .... Date: .... Period: ....

Practice

3. Pineapples were packed in 3 large crates. For each crate, the weight of every pineapple in the crate was recorded. Here are three box plots that summarize the weights of the pineapples in each crate. Select *all* of the statements that are true, according to the box plots.



A. The weights of the pineapples in Crate 1 were the most varied.

B. The heaviest pineapple was in Crate 1.

C. The lightest pineapple was in Crate 1.

D. Crate 3 had the greatest median weight and the greatest IQR.

E. More than half the pineapples in Crate 1 and Crate 3 were heavier than the heaviest pineapple in Crate 2.

4. Solve each equation.

a $9v = 1$

b $1.37w = 0$

c $1 = \frac{7}{10}x$

d $12.1 = 12.1 + y$

e $\frac{3}{5} + z = 1$

5. When might it be acceptable to ignore a data value and leave it out of calculations for measures of center and variability? Would it ever be acceptable? Explain your thinking.

Unit 8 | Lesson 17 – Capstone

Asian Elephant Populations

Let's look at a real-world scenario involving messy data and competing interests.

Warm-up Looking at Messy Data

The IUCN lists the Asian elephant (*Elephas maximus*) as Endangered, citing a general decline in population that has most likely been occurring for centuries. You will be given the population data used by IUCN in their 2018 survey assessment to complete these problems.

Mogens Trolle/Shutterstock.com

1. What questions do you have about how this data is represented by country or region?

2. How are the data different from the data you have seen throughout the unit?

3. Why do you think the data are different in these ways?

4. Can you trust the data?

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Population Perspectives

Part 1: **Be the Person**

Your group will be given a persona. Use that persona to respond *as that person would* to the following prompts.

1. What is your goal and how does the population of Asian elephants impact you and your goals?

2. Consider how the data on elephant populations from the Warm-up can support your goals. How will you choose the population value to use for each country? Explain your thinking.

Activity 1 Population Perspectives (continued)

Part 2: Be the Statistician

You will be given measures of center and variability that were each calculated for the data using different values. Choose the measure of center and variability calculated with the data values that best support your perspective. Explain your thinking.

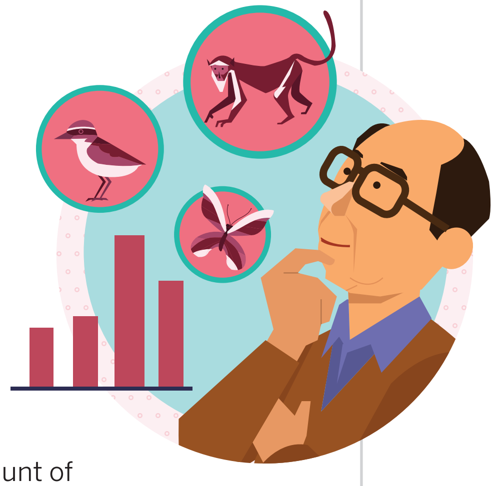
Part 3: Be the Voice

You now need to convince IUCN officials of your position to either keep the Asian Elephant as *Endangered*, downgrade it to *Vulnerable*, or reclassify it as *Critically Endangered*. Include any data analysis and visuals that support your claims.

STOP

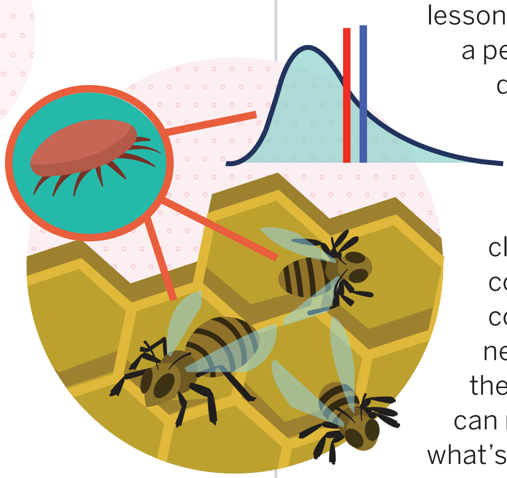
Narrative Connections

Unit Summary

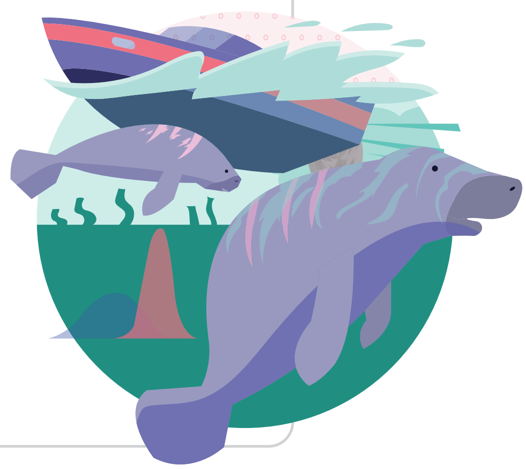


It is a big planet. We share it with all kinds of living things. But no creature has flourished quite like *Homo sapiens*. Our presence has dramatically affected the ecosystems of countless species — from the humble honey bee to the massive manatee. So it's only fair that we try to put things right.

To do that, we must understand what the data are telling us. Much of the time, data are messy. It can suggest multiple answers and show a certain amount of variation. On top of that, how data are presented can also be misleading. You saw as much yourself in this last lesson. While the data on Asian elephants remained fixed, a person's agenda could affect how they present that data. At the extreme, data can even be manipulated to support claims that are not necessarily true.



But, thanks to statistics, we have ways to manage that messiness and sniff out misleading claims. This is especially important in wildlife conservation. In nature, animal populations are constantly changing. But conservationists still need ways to confidently measure and report these numbers. With the mean and median, you can measure the center of a data set. This will tell you what's typical of a group. Meanwhile, a data set's mean absolute deviation (MAD) and interquartile range (IQR) will help describe how much variability is within that data set.



Just as the scientists, conservationists, and policy makers can use this type of data analysis to direct their energy toward helping the species that need it most, you can too — toward anything you are interested in or passionate about. How could you spend some time making sense of the data to help your community between now and next school year?

See you next year.

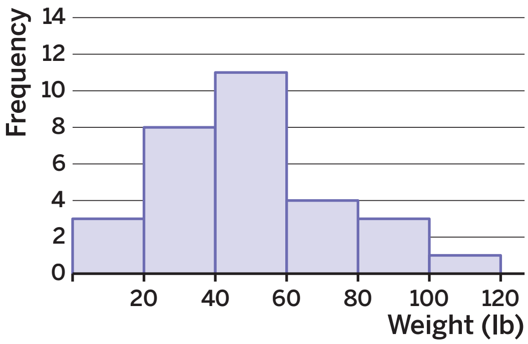
Practice

Name: .... Date: .... Period: ....

1. In this final unit, you answered statistical questions by representing and interpreting data. A common saying is "Data doesn't lie." Do you agree with this statement? Explain your thinking.

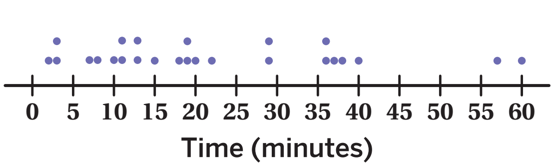
2. Use the data and histogram of the weights of dogs at an adoption shelter. Is the histogram a good representation of the data? Why or why not?

17	17	18	20	25	26	27	29	32	37	39	41	45	46	47
48	52	52	53	57	58	59	62	70	73	78	82	89	95	101

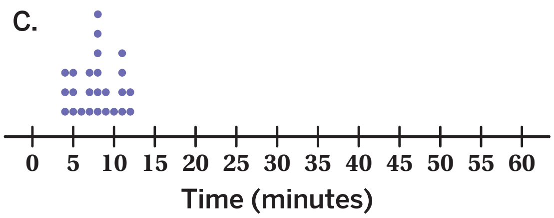


3. The dot plots show how much time, in minutes, students in a class took to complete each of five different tasks. Select *all* the dot plots of tasks for which the mean time is approximately equal to the median time.

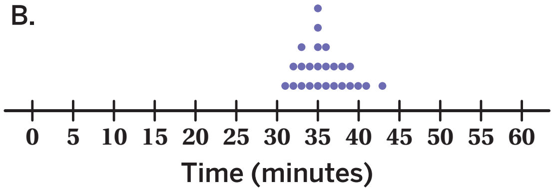
A.



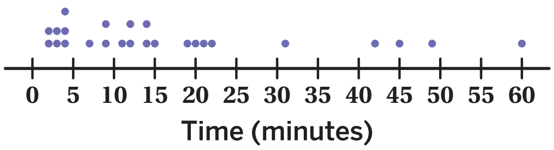
C.



B.

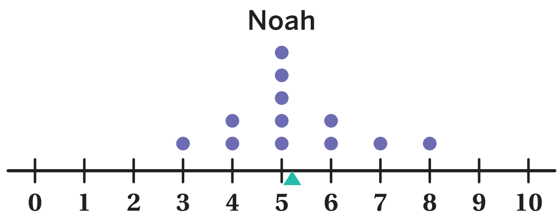


D.



Name: ________________ Date: ________ Period: ________

4. Noah and Lin recorded their basketball scores the same way: the number of baskets made out of 10 attempts. Each collected 12 data values. Noah's mean number of baskets was 5.25, and his MAD was 1. Lin's mean number of baskets was 6.



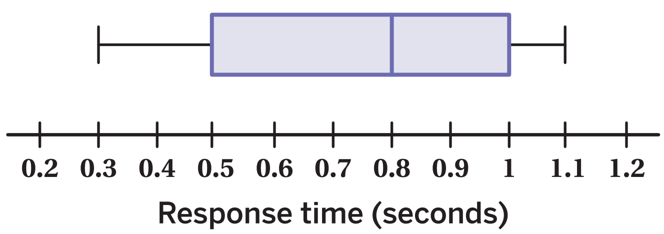
Lin	6	5	4	7	6	5	7	8	5	6	5	8
Distance from 6												

a. Calculate Lin's MAD and compare it to Noah's. What do you notice?

b. What do you notice about the distributions? Draw a dot plot if necessary.

c. What can you say about the two players' shooting accuracy and consistency?

5. The box plot displays the data on the response times of 100 mice to seeing a flash of light. How many mice are represented by the rectangle between 0.5 and 1 second?



Glossary/Glosario

English | **Español**

A

absolute value The value that represents the distance between a number and zero. For example, because the distance between -3 and 0 is 3, the absolute value of -3 is 3, or $|-3| = 3$.

Addition Property of Equality A property stating that if $a = b$, then $a + c = b + c$.

area The number of unit squares needed to fill a two-dimensional shape without gaps or overlaps.

average The average of a set of values is their sum divided by the number of values in the set. The average represents a fair share, or a leveling out of the distribution, so that each value in the set has the same frequency.

valor absoluto Valor que representa la distancia entre un número y cero. Por ejemplo, dado que la distancia entre -3 y 0 es 3, el valor absoluto de -3 es 3, o $|-3| = 3$.

Propiedad de igualdad en la suma Propiedad que establece que si $a = b$, entonces $a + c = b + c$.

área Número de unidades cuadradadas necesario para llenar una forma bidimensional sin dejar espacios vacíos ni superposiciones.

promedio El promedio de una serie de valores es su suma dividida por la cantidad de valores en el conjunto. El promedio representa una repartición justa, o igualada, de la distribución, de manera que cada valor del conjunto tenga la misma frecuencia.

B

base (of an exponential expression) The number that is raised to a power. The power indicates the number of times the base is multiplied by itself.

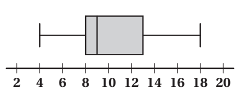
base (of a parallelogram) Any chosen side of the parallelogram.

base (of a prism) Either of the two identical, parallel faces of a prism that are connected by a set of rectangular faces.

base (of a pyramid) The face of a pyramid that is opposite from the vertex, where all the triangular faces connect.

base (of a triangle) Any chosen side of the triangle.

box plot A visual representation of the five-number summary for a numerical data set.



base (de una expresión exponencial) Número que se eleva a una potencia. La potencia indica el número de veces que la base debe multiplicarse por sí mismo.

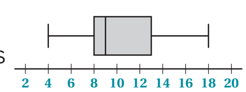
base (de un paralelogramo) Cualquier lado escogido del paralelogramo.

base (de un prisma) Cualquiera de las dos caras idénticas y paralelas de un prisma que están conectadas por un conjunto de caras rectangulares.

base (de una pirámide) La cara de una pirámide que se opone al vértice, donde todas las caras triangulares se conectan.

base (de un triángulo) Cualquier lado escogido del triángulo.

diagrama de cajas Representación visual del resumen de cinco números de un conjunto de datos numéricos.



C

categorical data Data that can be sorted into categories rather than counted, such as the different types of food bison eat or the colors of the rainbow.

center A value that represents the typical value of a data set.

coefficient A number that is multiplied by a variable, typically written in front of or "next to" the variable, often without a multiplication symbol.

datos categóricos Datos que pueden ser clasificados en categorías en vez de ser contados, como por ejemplo los diferentes tipos de comida que come un bisonte o los colores del arcoíris.

centro Valor que representa el valor típico de un conjunto de datos.

coeficiente Número por el cual una variable es multiplicada, escrito comúnmente frente o junto a la variable sin un símbolo de multiplicación.

Glossary/Glosario

English

common factor A number that divides evenly into each of two or more given numbers.

common multiple A number that is a multiple of two or more given numbers.

compose To place together shapes or numbers, or to combine them.

coordinate plane A two-dimensional plane that represents all the ordered pairs (x, y), where x and y can both take on values that are positive, negative, or zero.

cubed The raising of a number to the third power (with an exponent of 3). This is read as that number, "cubed."

D

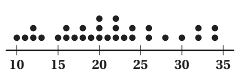
decompose To take apart a shape or number.

dependent variable In a relationship between two variables, the dependent variable represents the output values. The output values are unknown until the indicated calculations are performed on the independent variable.

distribution A collection of all of the data values and their frequencies. A distribution can be described by its features when represented visually, such as in a dot plot.

Division Property of Equality A property stating that if $a = b$ and c does not equal 0, then $a \div c = b \div c$.

dot plot A representation of data in which the frequency of each value is shown by the number of dots drawn above that value on a horizontal number line. A dot plot can only be used to represent numerical data.



Español

factor común Número que divide en partes iguales cada número de entre dos o más números dados.

múltiplo común Número que es múltiplo de dos o más números dados.

componer Unir formas o números, o combinarlos.

plano de coordenadas Plano bidimensional que representa todos los pares ordenados (x, y), donde tanto x como y pueden representar valores positivos, negativos o cero.

al cubo Un número elevado a la tercera potencia (con un exponente de 3) se lee como ese número "al cubo".

D

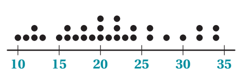
descomponer Desmontar una forma o un número.

variable dependiente En una relación entre dos variables, la variable dependiente representa los valores de salida. Los valores de salida son desconocidos hasta que se realizan los cálculos indicados sobre la variable independiente.

distribución Una colección de todos los valores de datos y sus frecuencias. Una distribución puede ser descrita según sus características cuando es representada en forma visual, como por ejemplo en un diagrama de puntos.

Propiedad de igualdad en la división Propiedad que establece que si $a = b$ y c no equivale a 0, entonces $a \div c = b \div c$.

diagrama de puntos Representación de datos en la cual la frecuencia de cada valor es equivalente al número de puntos que aparecen sobre dicho valor en una línea numérica horizontal. Un diagrama de puntos solo se puede usar para representar datos numéricos.



English	Español
E	
edge A line segment where two faces of a three-dimensional figure meet. The term *edge* can also refer to the side of a two-dimensional shape.	**arista** Segmento de una línea donde se encuentran dos caras de una figura tridimensional. *Arista* puede también referirse al lado de una forma bidimensional.
equation Two expressions with an equal sign between them. When the two expressions are equal, the equation is true. An equation can also be false when the values of the two expressions are not equal.	**ecuación** Dos expresiones con un signo de igual entre ellas. Cuando las dos expresiones son iguales, la ecuación es verdadera. Una ecuación también puede ser falsa, cuando los valores de las dos expresiones no son iguales.
equivalent If two mathematical quantities (especially fractions, ratios, or expressions) are equal in any form, then they are *equivalent*.	**equivalente** Si dos objetos matemáticos (especialmente fracciones, razones o expresiones) son iguales de cualquier manera, entonces son *equivalentes*.
equivalent expressions Two expressions whose values are equal when the same value is substituted into the variable for each expression.	**expresiones equivalentes** Dos expresiones cuyos valores son iguales cuando se sustituye el mismo valor en la variable de cada expresión.
equivalent fractions Two fractions that represent the same value or location on the number line.	**fracciones equivalentes** Dos fracciones que representan el mismo valor o la misma ubicación en la línea numérica.
equivalent ratios Any two ratios where the values for one quantity in each ratio can be multiplied or divided by the same number to get the values for the other quantity in each ratio.	**razones equivalentes** Dos razones para las cuales los valores de una cantidad en cada razón pueden ser multiplicados o divididos por el mismo número para obtener así los valores de la otra cantidad en cada razón.
exponent The number of times a factor is multiplied by itself.	**exponente** Número de veces que un factor es multiplicado por sí mismo.
expression A set of numbers, letters, operations, and grouping symbols that represent a quantity that can be calculated.	**expresión** Conjunto de números, letras, operaciones y símbolos de agrupamiento que representa una cantidad que puede ser calculada.
F	
face One of many two-dimensional shapes that form the outer surface of a three-dimensional figure.	**cara** Una de muchas formas bidimensionales que forman la superficie externa de una figura tridimensional.
factor A number that divides evenly into a given whole number. For example, the factors of 15 are $1, 3, 5$, and 15.	**factor** Número que divide de manera exacta a otro número dado. Por ejemplo, los factores de 15 son $1, 3$, 5 y 15.
five-number summary The minimum, first quartile, median, third quartile, and maximum values of a data distribution.	**resumen de cinco números** El mínimo, el primer cuartil, la mediana, el tercer cuartil y los valores máximos de una distribución de datos.
frequency The number of times a value occurs in a data set.	**frecuencia** Número de veces que un valor está presente en un conjunto de datos.

Glossary/Glosario

English

Español

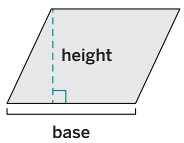
G

greatest common factor The common factor of two or more given whole numbers whose value is the greatest (often abbreviated as "GCF").

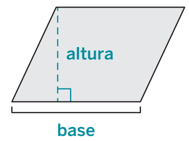
máximo factor común Factor común de dos o más números enteros dados, cuyo valor es el mayor (comúnmente abreviado como "MFC").

H

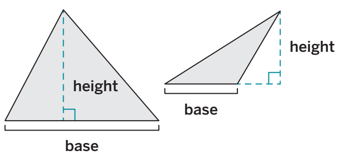
height (of a parallelogram) A segment measuring the shortest distance from the chosen base to the opposite side.



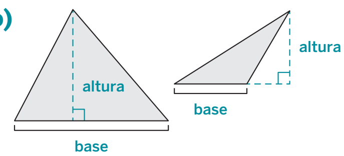
altura (de un paralelogramo) Segmento que mide la distancia más corta desde la base escogida hasta el lado opuesto.



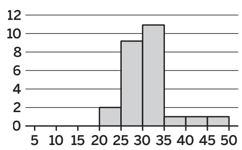
height (of a triangle) A segment representing the distance between the base and the opposite vertex.



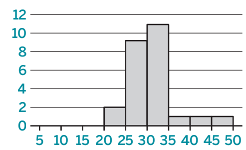
altura (de un triángulo) Segmento que representa la distancia entre la base y el vértice opuesto.



histogram A visual way to represent frequencies of numerical data values that have been grouped into intervals, called bins, along a number line. Bars are drawn above the bins where data exists, and the height of each bar reflects the frequency of the data values in that interval.



histograma Forma visual de representar frecuencias de valores de datos que han sido agrupados en intervalos, llamados contenedores, a lo largo de una línea numérica. Se dibujan barras sobre los contenedores donde existen los datos, y la altura de cada barra refleja la frecuencia de los valores de datos en ese intervalo.



I

independent variable In a relationship between two variables, the independent variable represents the input values. Calculations are performed on the input values to determine the values of the dependent variable.

variable independiente En una relación entre dos variables, la variable independiente representa los valores de entrada. Se realizan cálculos con los valores de entrada para determinar los valores de la variable dependiente.

integers Whole numbers and their opposites.

enteros Números completos y sus opuestos.

interquartile range (IQR) A measure of spread (or variability) that is calculated as the difference between the third quartile (Q3) and the first quartile (Q1).

rango intercuartil (RIC) Medida de dispersión (es decir, de variabilidad) que es calculada mediante la diferencia entre el tercer cuartil (C3) y el primer cuartil (C1).

L

least common multiple The common multiple of two or more given whole numbers whose value is the least (often abbreviated as "LCM").

mínimo común múltiplo Múltiplo común de dos o más números enteros dados, cuyo valor es el menor (comúnmente abreviado como "MCM").

long division A method that shows the steps for dividing base ten whole numbers and decimals, dividing one digit at a time, from left to right.

$$\begin{array}{r} 219 \\ 3\overline{)657} \\ \underline{-6} \\ 5 \\ \underline{-3} \\ 27 \\ \underline{-27} \\ 0 \end{array}$$

división larga Método que muestra los pasos necesarios para dividir números enteros en base diez y decimales, por medio de la división de un dígito a la vez, de izquierda a derecha.

$$\begin{array}{r} 219 \\ 3\overline{)657} \\ \underline{-6} \\ 5 \\ \underline{-3} \\ 27 \\ \underline{-27} \\ 0 \end{array}$$

English | Español

M

magnitude (of a number) The absolute value of a number, or the distance of a number from 0 on the number line.

maximum The value in a data set that is the greatest.

mean A measure of center that represents the average of all values in a data set. The mean represents a fair share distribution or a balancing point of all of the values in the data set.

mean absolute deviation (MAD) A measure of spread (or variability) calculated by determining the average of the distances between each data value and the mean.

measure of center A single number used to summarize the typical value of a data set.

measure of variability A single number used to summarize how the values in a data set vary.

median The middle value in the data set when the values are listed in order from least to greatest. When there is an even number of data points, the median is the average of the two middle values.

minimum The value in a data set that is the least.

mode The most frequently occurring value in a data set. A data set may have no mode, one mode, or more than one mode.

multiple A number that is the product of a given number and a whole number. For example, multiples of 7 include 7, 14, and 21.

Multiplication Property of Equality A property stating that, if $a = b$, then $a \cdot c = b \cdot c$.

magnitud (de un número) Valor absoluto de un número, o la distancia de un número con respecto al 0 en la línea numérica.

máximo El valor más grande en un conjunto de datos.

media Medida del centro que representa el promedio de todos los valores de un conjunto de datos. La media representa una distribución equitativa o un punto de equilibrio entre todos los puntos del conjunto de datos.

desviación absoluta media (DAM) Medida de dispersión (o variabilidad) que se calcula mediante la obtención del promedio de la distancia entre cada valor de datos y la media.

medida de centro Número individual que se utiliza para resumir el valor típico en un conjunto de datos.

medida de variabilidad Número individual que se utiliza para resumir cómo varían los valores en un conjunto de datos.

mediana Valor medio de un conjunto de datos cuando sus valores están ordenados de menor a mayor. Cuando la cantidad de puntos de datos es par la mediana es el promedio de los dos valores medios.

mínimo Valor que es el menor de un conjunto de datos.

modo Valor que aparece con mayor frecuencia en un conjunto de datos. Un conjunto de datos puede tener un modo, más de un modo o ningún modo.

múltiplo Número que es el producto de un número dado y un número entero. Por ejemplo, entre los múltiplos de 7 se incluyen 7, 14 y 21.

Propiedad de igualdad en la multiplicación Propiedad que establece que si $a = b$, entonces $a \cdot c = b \cdot c$.

Glossary/Glosario

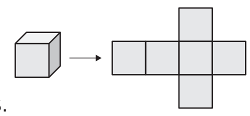
English | **Español**

N

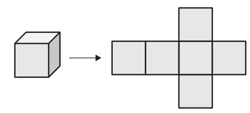
negative number A number whose value is less than zero.

número negativo Número cuyo valor es menor que cero.

net A two-dimensional representation, or "flattening," of a three-dimensional solid's surface that shows all of its faces.



red Representación bidimensional, o "aplanamiento", de la superficie de un sólido tridimensional, para mostrar todas sus caras.



numerical data Numbers, quantities, or measurements that can be meaningfully compared.

datos numéricos Números, cantidades o medidas que pueden ser comparadas de manera significativa.

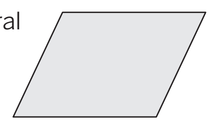
O

opposite numbers Two numbers that are the same distance from 0, but are on different sides of the number line.

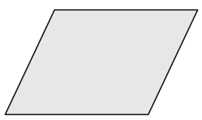
números opuestos Dos números que están a la misma distancia de 0, pero que están en lados diferentes de la línea numérica.

P

parallelogram A type of quadrilateral with two pairs of parallel sides.



paralelogramo Tipo de cuadrilátero con dos pares de lados paralelos.



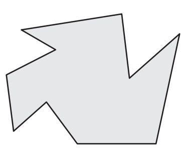
per For each.

por Por cada uno de los elementos.

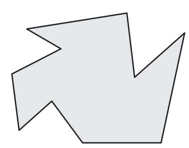
percentage A rate per 100. (A specific *percentage* is also called a *percent*, such as "70 percent.")

porcentaje Tasa por cada 100. (Un *porcentaje* específico también es llamado *por ciento*, como por ejemplo "70 por ciento".)

polygon A closed, two-dimensional shape with straight sides that do not cross each other.



polígono Forma cerrada y bidimensional de lados rectos que no se entrecruzan.



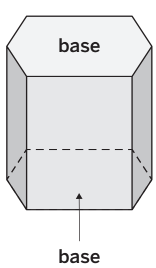
polyhedron A closed, three-dimensional shape with flat sides. (The plural of *polyhedron* is *polyhedra*.)

poliedro Forma cerrada y tridimensional de lados planos.

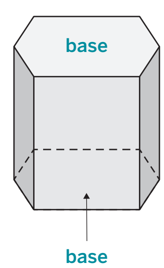
positive number A number whose value is greater than zero.

número positivo Número cuyo valor es mayor que cero.

prism A three-dimensional figure with two parallel, identical faces (called *bases*) that are connected by a set of rectangular faces.



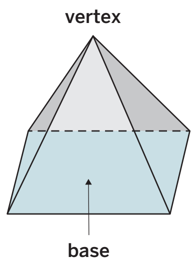
prisma Figura tridimensional con dos caras iguales y paralelas (llamadas *bases*) que se conectan entre sí a través de un conjunto de caras rectangulares.



English

properties of equality Rules that apply to all equations. These include properties of addition, subtraction, multiplication, and division, which state that if an equation is true, then performing the same operation to both sides will result in an equivalent equation.

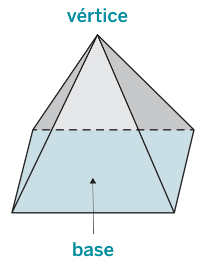
pyramid A three-dimensional figure with one base and a set of triangular faces that meet at a singular vertex.



Español

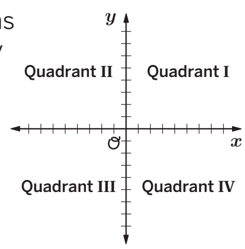
propiedades de igualdad Reglas que son aplicables a todas las ecuaciones. Incluyen las propiedades de suma, resta, multiplicación y división, las cuales señalan que si una ecuación es verdadera, al realizar la misma operación en ambos lados se obtendrá una ecuación equivalente.

pirámide Figura tridimensional con una base y un conjunto de caras triangulares que se conectan en un solo vértice.

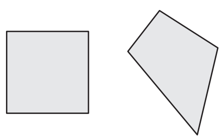


Q

quadrant Each of the four regions of the coordinate plane formed by the vertical and horizontal axes. The quadrants are labeled counterclockwise from top right to bottom right as I, II, III, IV.

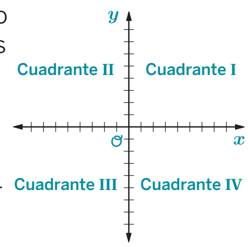


quadrilateral A polygon with exactly four sides.

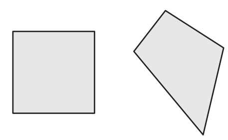


quartile One of three numbers (Q1, Q2, Q3) that divide an ordered data set into four sections so that each section contains 25% of data points.

cuadrante Cada una de las cuatro regiones del plano de coordenadas formado por los ejes vertical y horizontal. Los cuadrantes se identifican en sentido contrario a las agujas del reloj, desde la parte superior derecha a la parte inferior derecha, como I, II, III y IV.



cuadrilátero Polígono de exactamente cuatro lados.



cuartil Uno de los tres números (C1, C2, C3) que dividen un conjunto ordenado de datos en cuatro secciones, de manera que cada sección contenga el 25% de los puntos de datos.

Glossary/Glosario

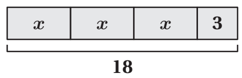
English	Español
R	
range A measure of spread (or variability) that is calculated as the difference between the maximum and minimum values in the data set.	**rango** Medida de dispersión (o variabilidad) que es calculada mediante la diferencia entre los valores máximos y mínimos de un conjunto de datos.
rate A comparison of how two quantities change together.	**tasa** Comparación de cuánto cambian dos cantidades en conjunto.
ratio A comparison of two quantities, such that for every a units of one quantity, there are b units of another quantity.	**razón** Una comparación entre dos cantidades, de modo tal que por cada a unidades de una cantidad, hay b unidades de la otra cantidad.
rational numbers The set of all the numbers that can be written as positive or negative fractions.	**números racionales** Conjunto que consta de todos los números que pueden ser escritos como fracciones positivas o negativas.
ratio relationship A relationship between quantities that establishes that the values for each quantity will always change together in the same way.	**relación de razón** Relación entre cantidades que establece que los valores para cada cantidad siempre cambiarán en conjunto de la misma manera.
ratio table A table of values organized in columns and rows that contains equivalent ratios.	**tabla de razones** Tabla de valores organizada en columnas y filas que contiene razones equivalentes.
reciprocal Two numbers whose product is 1 are *reciprocals* of each other. (When written in simplest fraction form, the numerator of each number corresponds to the denominator of the other number. For example, $\frac{3}{5}$ and $\frac{5}{3}$ are reciprocals.)	**recíproco/a** Dos números cuyo producto es 1 son *recíprocos* entre sí. (Al escribirlo en la forma de fracción más simple, el numerador de cada número corresponde al denominador del otro número. Por ejemplo, $\frac{3}{5}$ y $\frac{5}{3}$ son recíprocos.)
region The space inside a shape or figure.	**región** Espacio al interior de una forma o figura.
S	
sign (of a number) Indication of whether a number is positive or negative.	**signo (de un número)** Indicación de si un número es positivo o negativo.
solution to an equation A number that can be substituted in place of a variable to make an equation true.	**solución a una ecuación** Número que puede sustituir una variable para volver verdadera una ecuación.
solution to an inequality Any number that can be substituted in place of a variable to make an inequality true.	**solución a una desigualdad** Cualquier número que puede sustituir una variable para volver verdadera una desigualdad.
spread The variability of a distribution. A description of how the data values in the distribution vary from the center of the distribution.	**dispersión** Variabilidad de una distribución. Descripción de cuánto varían los valores en comparación con el centro de una distribución de datos.
squared The raising of a number to the second power (with an exponent of 2). This is read as that number, "squared."	**al cuadrado** Un número elevado a la segunda potencia (con un exponente de 2) se lee como ese número "al cuadrado".
statistical question A question that anticipates variability and can be answered by collecting data.	**pregunta estadística** Pregunta que anticipa variabilidad y que se puede responder mediante la recolección de datos.

English

Subtraction Property of Equality For rational numbers a, b, and c, if $a = b$, then $a - c = b - c$.

surface area The number of unit squares needed to cover all of the faces of a three-dimensional figure without gaps or overlaps.

T



tape diagram A model in which quantities are represented as lengths (of tape) placed end-to-end, and which can be used to show addition, subtraction, multiplication, or division.

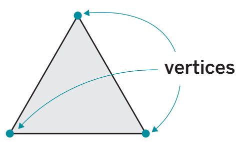
U

unit rate How much one quantity changes when the other changes by 1.

V

variability The spread of a distribution. A description of how the data values in the distribution vary from the center of the distribution.

variable A letter that represents an unknown number in an expression or equation.



vertex A point where two sides of a two-dimensional shape or two or more edges of a three-dimensional figure intersect. (The plural of *vertex* is *vertices*.)

volume The number of unit cubes needed to fill a three-dimensional figure without gaps or overlaps.

Español

Propiedad de igualdad en la resta Para los números racionales a, b y c, if $a = b$, entonces $a - c = b - c$.

área de superficie Número de unidades cuadradas necesarias para cubrir todas las caras de una figura tridimensional sin dejar espacios vacíos ni superposiciones.

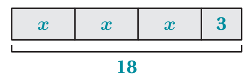
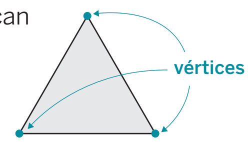


diagrama de cinta Modelo en el cual las cantidades están representadas como longitudes (de una cinta) colocadas de forma continua, y que pueden ser usadas para mostrar suma, resta, multiplicación y división.

tasa unitaria Cuánto cambia cierta cantidad cuando la otra cambia por un factor de 1.

variabilidad La dispersión de una distribución. Descripción de cuánto varían los valores en comparación con el centro de una distribución de datos.

variable Letra que representa un número desconocido en una expresión o ecuación.



vértice Punto donde se intersecan dos lados de una forma bidimensional, o dos o más aristas de una figura tridimensional.

volumen Número de unidades cúbicas necesario para llenar una figura tridimensional sin dejar espacios vacíos ni superposiciones.

Index

Index

Index

I

J

K

L

M

Index

Index

Index

Y

Z