Amplify Math

Grade 7

Volume 1: Units 1–4

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Cover illustration by Caroline Hadilaksono.

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Hello, curious mind!

Welcome to Grade 7. This grade is considered by some to be one of the wildest grades . . . at least in terms of the math.

This year, you'll scale skyscrapers, build your own brand, tame "rep-tiles," and even grapple with a gargantuan Godzilla. And that's just Unit 1!

We promise you, it's all possible. This year, you'll see not only that math makes sense, but it can be fun, too.

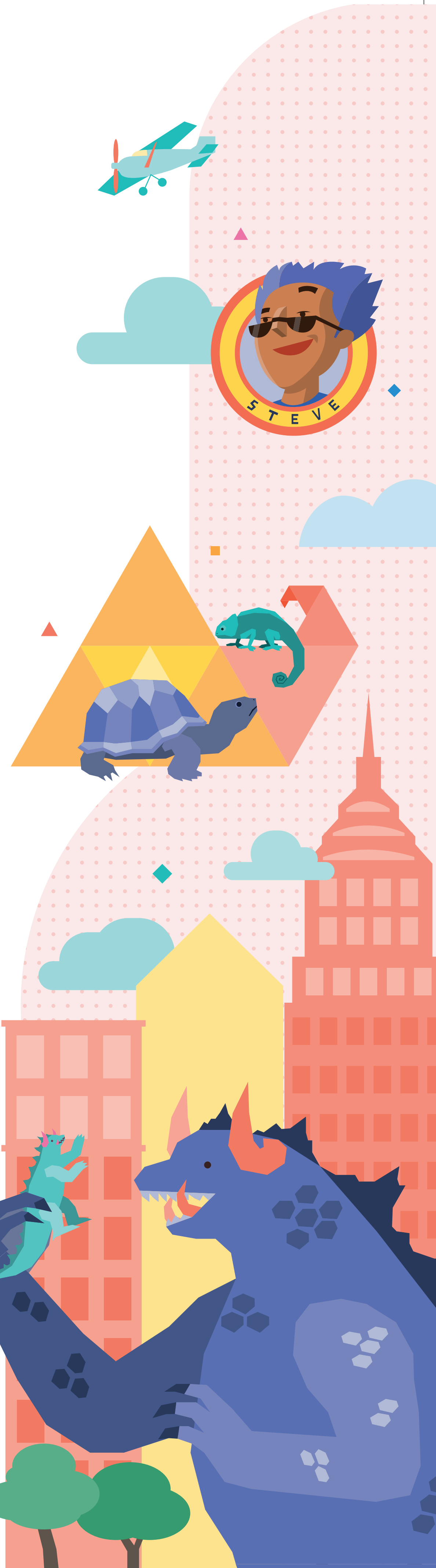
Before you dig in, we want you to know two things:

This book is special! It's where you'll record your thoughts, strategies, explanations, and, occasionally, any award-winning doodles that might come to mind.

When you go online, you won't be mindlessly plugging numbers into your device . . . You'll be pushing, pulling, crawling, teleporting, melting . . . , well, let's just say you'll be doing a lot of things, and you'll be teaming up with your classmates as you do.

Let's dig in!

Sincerely,
The Amplify Math Team



Volume 1

Unit 1 Scale Drawings

How did we come to understand what tiny things, like viruses, and humongous things, like Jupiter, look like? Scaling, of course! In this unit, we resize things — in very precise ways — to bring them into focus and make them more manageable to work with.

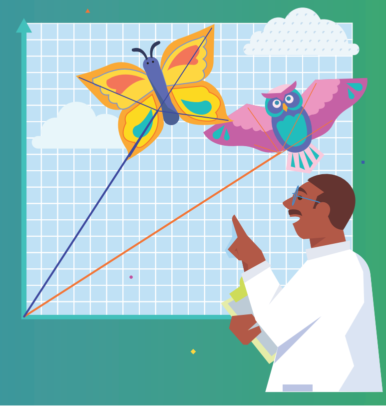
Unit Narrative:
Life in the Little Big City

How do you get the perfect fit?
If we are making a larger or smaller copy of something, it needs to look right. The key is the scale factor.

Who was the King of Monsters?
We use maps and other scale drawings to help simplify large, complex places. Interpreting them is about knowing the scale and how to measure.

Unit 2 Introducing Proportional Relationships

Unit Narrative: The World in Proportion



How did we come to understand what tiny things, like viruses, and humongous things, like Jupiter, look like? Scaling, of course! In this unit, we resize things — in very precise ways — to bring them into focus and make them more manageable to work with.

Who was the original globetrotter?
Tables help keep us organized, but equations tell an entire story with just a few symbols. We'll use both of them to represent proportional relationships.

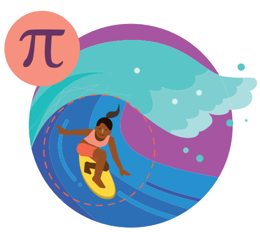
What good is a graph?
We turn to drawing, interpreting, and comparing proportional relationships in graphs, and notice what is particular to these types of graphs.

Unit 3 Measuring Circles

Some have said that the only place to find a perfect circle is inside the world of math. Explore the many circles and circular shapes both in and outside of this unit to reveal some of their mysteries.

Unit Narrative:
'Round and 'Round We Go

Why do aliens love circles?
Circles are famously difficult to measure precisely, but that won't stop us from trying. Let's see how close we can get.



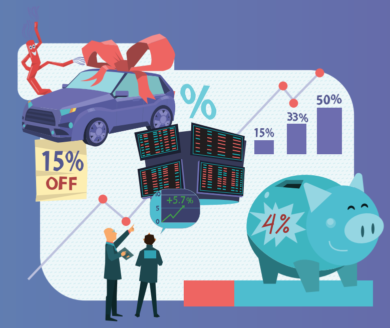
What makes a circle so perfect?
Squares and circles may not have much in common, but we'll need both to measure a circle's area.

Unit 4 Percentages

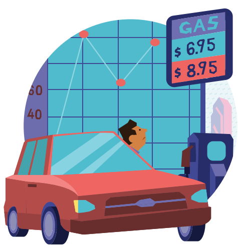
Unit Narrative:
Keepin' it 100

"Extra! Extra! 99% of adults don't remember numbers!*" Percents can be incredibly effective at communicating how much something has changed, but we must keep a watchful eye on what the numbers behind the percentages mean.

(*When asked to recite π to the 10th digit.)



Is there truth in numbers?
Numbers never lie, but should we always believe them? Percents can show how something changes — if we pay careful attention to the original amount.



Did a quarantined U.S. keep a healthy economy?
See why percentages are used to calculate taxes, tips, interest, and other amounts when spending or saving money.

Volume 2

Unit 5 Rational Number Arithmetic

Climb high, dig deep, and spend wisely as you explore the world using positive and negative numbers.

Unit Narrative:
A World of Opposites

What was Jeanne Baret's big secret?
Sure, you've probably been adding and subtracting for many years, but have you ever tried to take something away when you had less than zero to start with?

Who was the toughest Grandma to ever hike the Appalachian Trail?
Travel forwards and backwards in time to help make sense of multiplication and division of negative numbers.

How do you climb the world's most dangerous mountain?
Put it all together — adding, subtracting, multiplying, and dividing with rational numbers — while exercising your algebraic thinking muscles in a sneak preview of the next unit.

Unit 6 Expressions, Equations, and Inequalities

Unit Narrative:
Solving One Step at a Time

Numbers are great, but they won't get where we're going in this unit. It will take letters, symbols, and drawings to represent the varied and diverse mathematical ideas of algebraic thinking.

What are the first words you learn in "Caveman"?
Dog walking, tools of early civilization, and hangers all come together to help you explore new ways of solving equations.

Who were the VIPs of ancient Egypt?
Solving word problems is about making meaning of the quantities, and tape diagrams return to help.

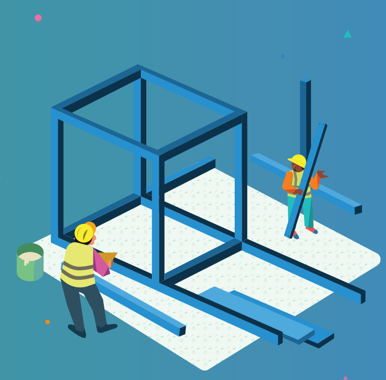
Did a member of the School of Night infiltrate your math class?
Expressions are not always equal, so we must reckon with inequalities. Thankfully, finding their solutions will feel familiar.

Which three blockheads did NASA send into space?
Find efficiencies for simplifying expressions like the Distributive Property and combining like terms.

Unit 7 Angles, Triangles, and Prisms

This unit is about the math of what can be seen and what can be held. Get ready to measure, build, and slice your way through an array of geometric figures.

Unit Narrative:
Journey to the Third Dimension



Did radio kill the aviation star?
As you'll see, some angles were just meant to go together. Here, you'll be introduced to complementary, supplementary, and vertical angles.

How did triangles help win a war?
In this Sub-Unit, you will find that constructing polygons with specific lengths and angle measures can have dramatically different results.

This machine will slice, but will it dice?
You've studied the surfaces of three-dimensional figures and the spaces inside them. Now, let's see what happens when we slice them open.

Unit 8 Probability and Sampling

Unit Narrative:
Winning Chance

It's impossible to see into the future, but that shouldn't stop us from trying, should it? Making predictions — taking limited information and making our best guess about what will happen — is all about knowing what's possible, what's impossible, and what's likely.

How did the women of Bletchley Park save the free world?
Welcome to probability, the math of games and chance. Discover how probability can reveal hidden information, even secret codes.

How did a blazing shoal bring the Philadelphia Convention Center to its feet?
When predicting the chances gets complicated, a simulation can help make predictions.

What's on your mind?
Not all data are created equal. It is important to know how to identify when a sample is representative of a population.

UNIT 1

Scale Drawings

How did we come to understand what tiny things, such as viruses, and humongous things, such as Jupiter, look like? Scaling, of course! In this unit you will resize objects — in very precise ways — to bring them into focus and make them more manageable to work with.

Essential Questions

- Why is it important to be precise when making scaled copies?
- Why are lengths and areas affected in different ways when creating scaled copies?
- How do scale models help you make sense of the world around you?
- *(By the way, how do you make a guy in a lizard suit taller than a skyscraper?)*

SUB-UNIT

1 Scaled Copies

Narrative: In the story *Alice in Wonderland*, Alice increased and decreased her size to help her solve problems.

You'll learn . . .

- what it means for two figures to be scaled copies of each other.
- how to identify and use the scale factor of two scaled copies.

SUB-UNIT

2 Scale Drawings

Narrative: Movies often use scale drawings and models to create the illusion of cities and objects, including monsters!

You'll learn . . .

- how to create scale drawings at different scales.
- how to use scale drawings to solve real-world problems.

You have a figure that you scale by a factor A. Then you scale that figure by a factor B. Finally, you scale that figure by a factor C, finding you have your original figure again. What is the product A • B • C?

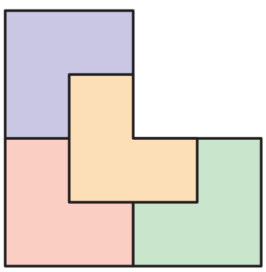
Scale-y Shapes

Let's see which shapes can be used to build larger copies of themselves.

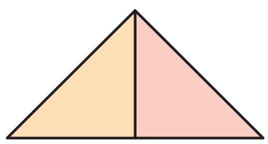
Warm-up Which One Doesn't Belong?

Study the images. Which image does not belong with the others? Explain your thinking.

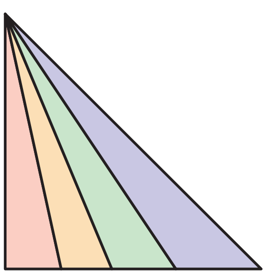
A.



C.



B.



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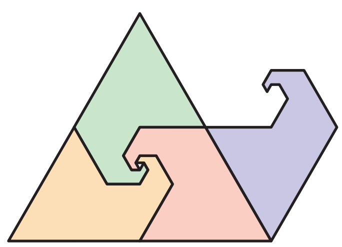


Collect and Display: As you explain your thinking, your teacher will collect the math language you use to add to a class display. You will continue to add and refer to this display throughout the unit.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Rep-tiles

You will be given pattern blocks.

1. Follow the directions to build each shape. Then use the space provided on this page to trace both the original shape and the shape that you built.

a. Using only triangles, build another triangle.

b. Using only rhombuses, build another rhombus.

c. Using only trapezoids, build another trapezoid.

Activity 1 Rep-tiles (continued)

2. Compare each original shape with the shapes you and your classmates built.

a What is the same about all of the triangles? What is different?

b What is the same about all of the rhombuses? What is different?

c What is the same about all of the trapezoids? What is different?

3. How many rhombuses are needed to build a rhombus that has side lengths twice as long as the original rhombus? Three times as long?

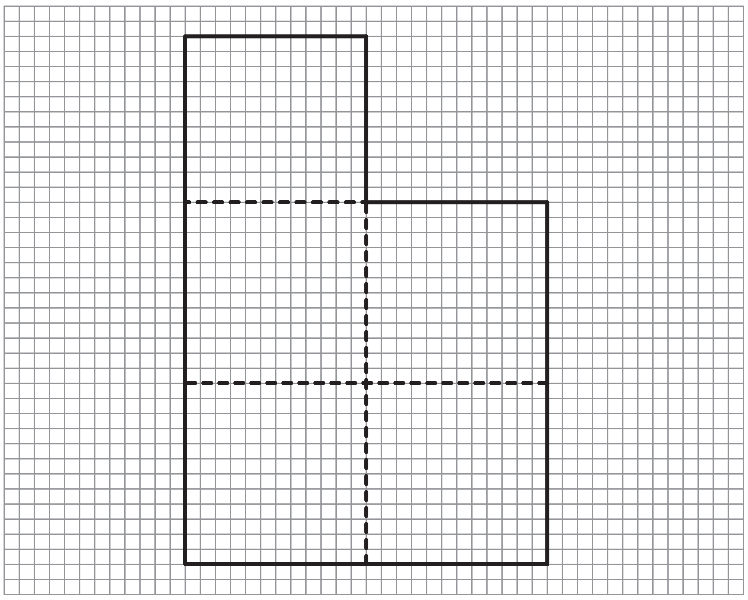
Are you ready for more?

Will four copies of the same shape always form a *rep-tile*? Explain your thinking.

Name: Date: Period:

Activity 2 Pentomino

Solomon Golomb, the creator of the *rep-tile*, had a real knack for creating games and giving them catchy names. Another of his creations, the *pentomino*, was named after he imagined what a domino would be called if, instead of two (*di-*) squares, it had five (*pent-*) squares. Thus, a pentomino is a polygon made of 5 equal-sized squares connected edge-to-edge. The pentomino shown also happens to be a rep-tile.



1. Use the grid to show how you can partition this shape into 4 smaller, equal-sized shapes that look like the larger shape.

2. Explain how you know your smaller equal-sized shapes are smaller copies of the larger shape.

Featured Mathematician

Solomon Golomb

Though he contributed much to the fields of communications, electrical engineering, and mathematics, Solomon Golomb will perhaps always be best remembered for the games he created. Take "Cheskers," for example, which combines chess and checkers, or his creation of "polyominoes," which inspired the arcade game Tetris. Golomb proved that doing serious math does not always require taking math very seriously.

STOP

Brendan Hoffman/Getty Images

Unit 1 Scale Drawings

Life in the Little Big City

For a first-time visitor, New York City can be an intimidating place. It's the most populous city in the U.S. As of 2020, it's home to 8.4 million people and spreads out over 302.6 square miles across five boroughs (neighborhoods), stitched together by bridges and tunnels. Between the massive skyscrapers, the snarling traffic, and the noise on the street, it's easy to get lost in all the confusion.

But with a quick subway ride, you might find yourself at the Queens Museum. There, within a quiet gallery, sits the "New York City Panorama" — a miniature model of one of the largest cities on Earth. With just one sweep of the head, a visitor can see the entire span of the city.

Here, the iconic 102-story Empire State Building is a quaint 15 in. tall. Central Park's sprawling 840 acres are now a more manageable 62 in^2. And the colossal Statue of Liberty, standing proudly over New York Harbor, barely reaches 4 in. tall.

Across the world, there is a tradition of taking objects and making them larger or smaller than their original size. Some are more straightforward to handle when they're smaller, while others can only be appreciated after they've been blown up to an impressive size.

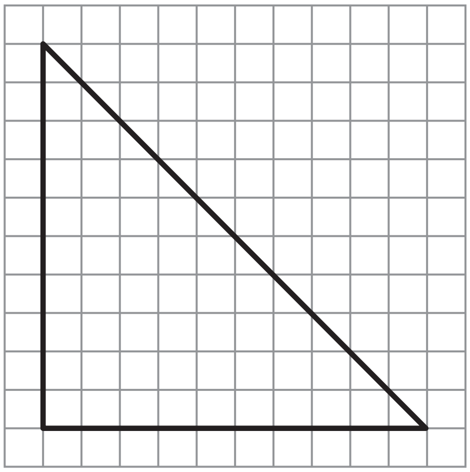
But we can't just shrink or enlarge objects willy-nilly. Precision is important, and for that we turn to the rules behind scaling.

Welcome to Unit 1.

Name: .... Date: .... Period: ....

Practice

1. Partition the triangle shown into 4 equal-sized triangles.



2. Each small square in the figures below has a side length of 1 unit and an area of 1 square unit.

Key: = 1 square unit

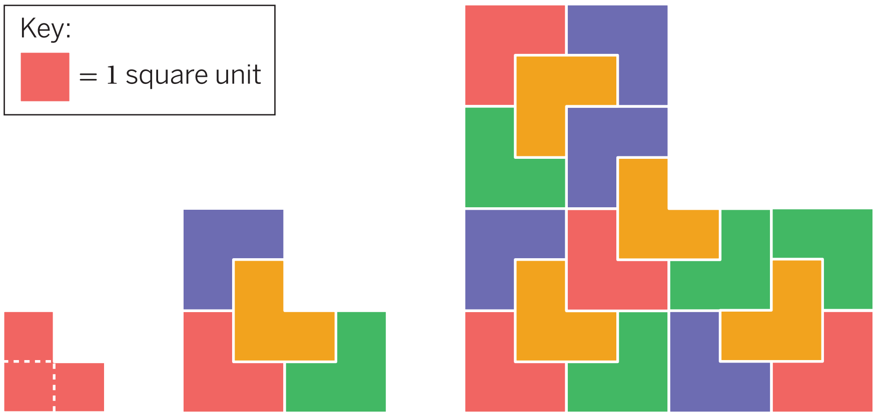


Figure 1 Figure 2 Figure 3

a Complete the table.

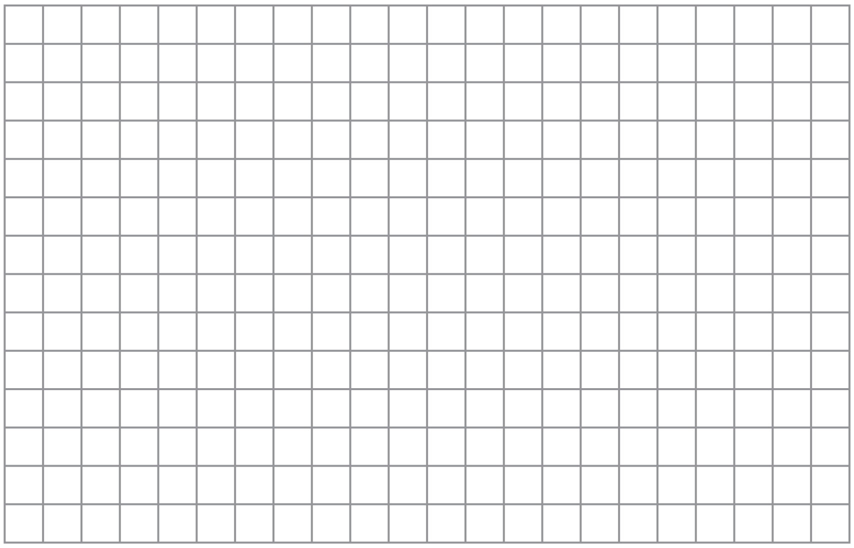
	Figure 1	Figure 2	Figure 3
Perimeter (in units)	8		
Area (in square units)	3		

b Describe any patterns you see.

Practice

Name: ______________________ Date: ____________ Period: ____________

3. Recall that a *pentomino* is a polygon composed of 5 equal-sized squares connected edge-to-edge. Use the grid to sketch as many unique pentominoes as possible.



4. Complete each equation with a number that makes it true.

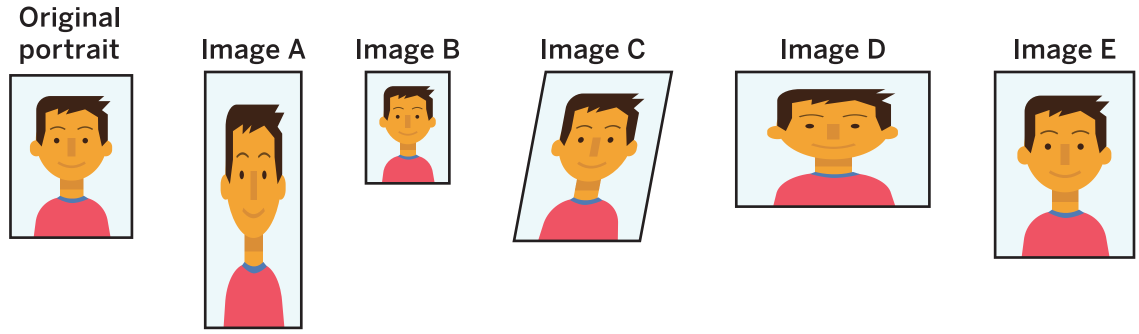
a $5 \cdot \square = 15$

b $6 \cdot \square = 9$

c $\square \cdot 4 = 32$

d $\square \cdot 12 = 3$

5. Andre's grandmother ordered school pictures. She thought she ordered the original portrait; however, the company sent her the following images. How is each image the same as or different from the original portrait of Andre?



SUB-UNIT

1 Scaled Copies

Narrative Connections

How do you get the perfect fit?

In Lewis Carroll's children's classic *Alice in Wonderland*, our hero finds herself too big to fit through a tiny door. That is, until she gulps down a bottle labeled "DRINK ME." Immediately, Alice shrinks down to the exact right size. But when she gets to the locked door, she realizes she left the key on the table, which is now too high to reach.

Luckily, Alice finds a small cake under the table, with the words "EAT ME" spelled out with berries. After gobbling it up, Alice shoots up to an incredible size and snatches the key.

Changing something's size is used in problem solving all the time.

Knowing the right size to fit a design to is key in the world of graphic design. For example, consider the postage stamp. With its tiny size, a designer can only fit a very small image inside its borders. That's perfectly fine for mailing a letter, but now imagine printing that same image on a billboard. The image would be practically invisible, unless its dimensions were significantly increased.

Like Alice's magic cake and bottle, we can increase or decrease something's size to help us solve problems. But in order for our models to make sense, there are a few rules we have to follow.

Unit 1 | Lesson 2

What Are Scaled Copies?

Let's explore scaled copies.

Warm-up Printing Copies

Odili Donald Odita is a Nigerian-American artist. The image shown is a portion of his laminated glass art installation, *Kaleidoscope*, on display in Brooklyn, New York.

Odili Donald Odita. 2012. Kaleidoscope [Laminated glass]. 20 Avenue Station, New York. Photo by MTA Arts for Transit and Urban Design. Attribution 2.0 Generic (CC BY 2.0)

A fashion designer is trying to incorporate Odita's artwork into his clothing line. He tried resizing and printing the art in different ways.

Design A Design B Design C

1. How is each design the same as or different from the original image?

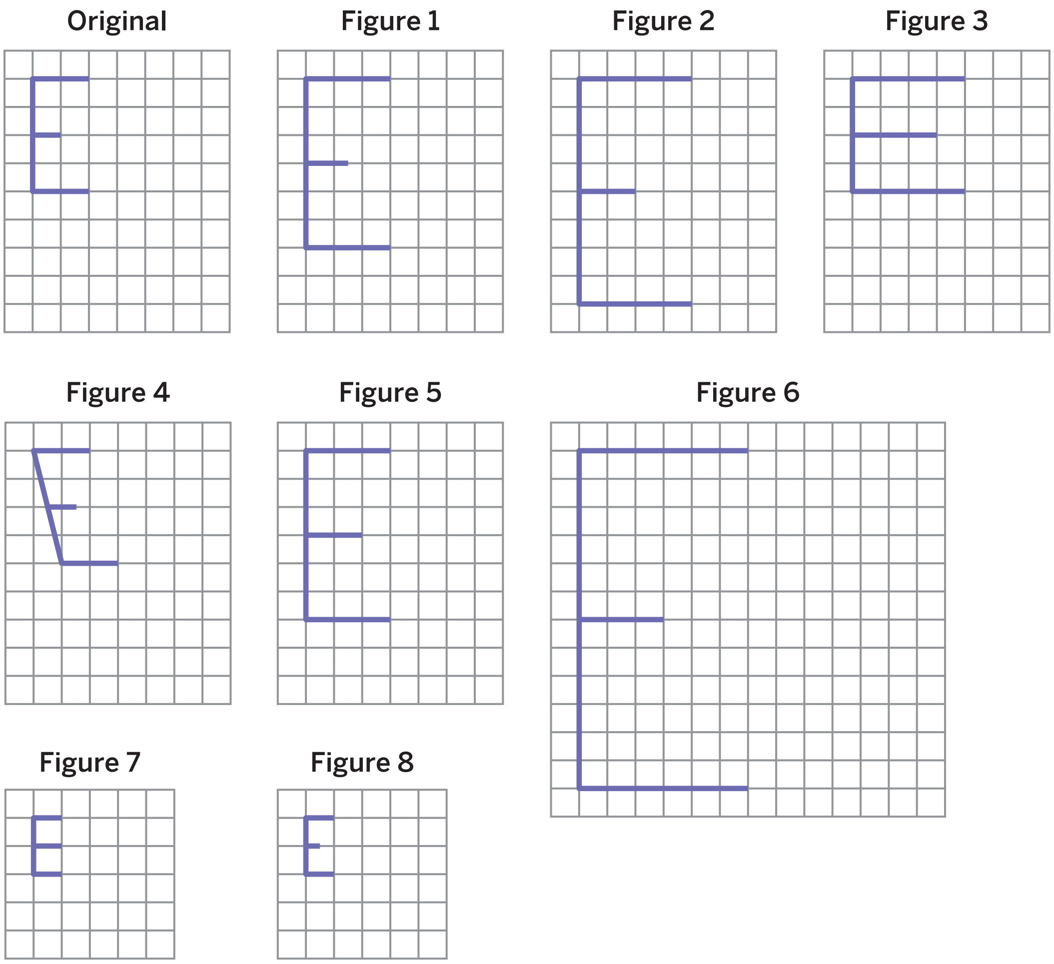
2. There was only one design the printer produced which the designer thought looked like the original artwork. Which one do you think that is? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Scaling E

Here is an original drawing of the letter E and some other drawings of the letter E.

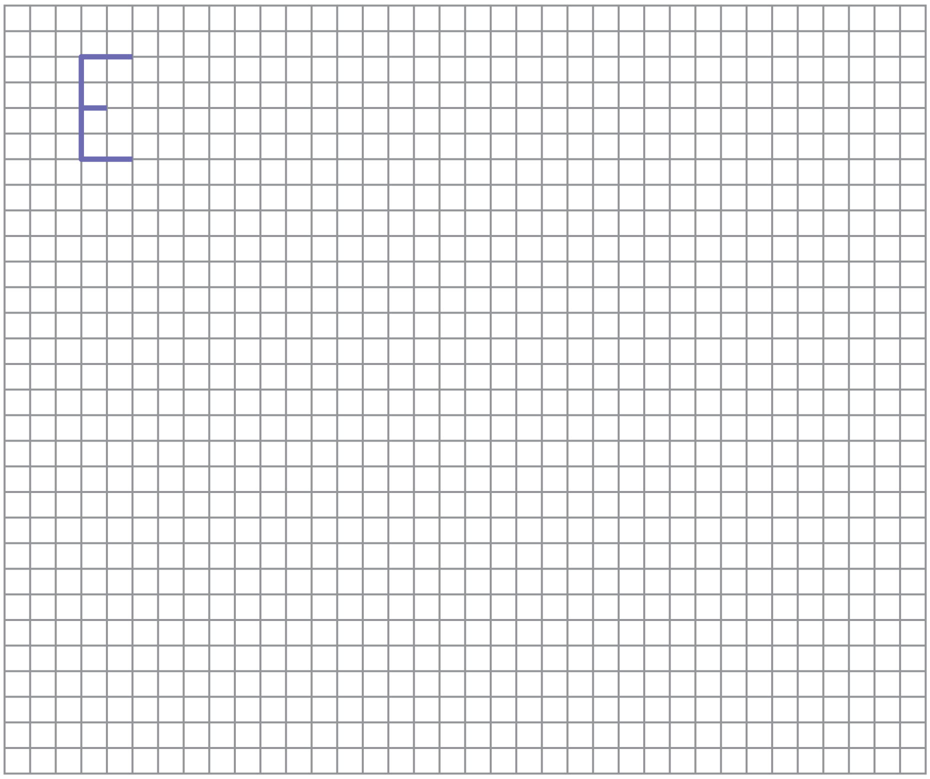


1. Identify all the drawings that are scaled copies of the original letter E drawing. Explain your thinking.

Activity 1 Scaling E (continued)

2. Examine the drawings you indicated as scaled copies more closely. Study the length of each segment that forms the letter E. What do you notice? How do they compare to the original letter E?

3. On the grid, draw a different scaled copy of the original letter E.



Name: .... Date: .... Period: ....

Activity 2 Card Sort: Pairs of Scaled Polygons

You will be given a set of 10 cards. Match pairs of polygons together which are scaled copies of each other. Record the matches here, and explain your thinking for each match.

Card .... and Card .... are scaled copies.

Reason:

Card .... and Card .... are scaled copies.

Reason:

Card .... and Card .... are scaled copies.

Reason:

Card .... and Card .... are scaled copies.

Reason:

Card .... and Card .... are scaled copies.

Reason:

Your teacher will tell you the class procedure you will use to check your responses to the card sort. Use this procedure to check your responses. Discuss and resolve any errors or disagreements.

Are you ready for more?

Is it possible to draw a polygon that is a scaled copy of both the polygon on Card 1 and Card 2? Either draw such a polygon, or explain how you know this is impossible.

STOP

Summary

In today's lesson . . .

You saw that figures which are different sizes, but otherwise appear identical, can be described in a more precise way. If each side length — or other dimension — of a figure is multiplied by the same value to create another figure, the two figures are ***scaled copies*** of one another.

Original Figure | Figure 1 | Figure 2

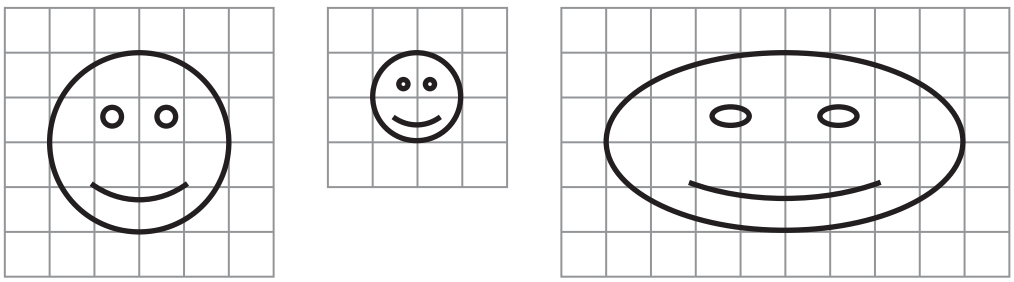


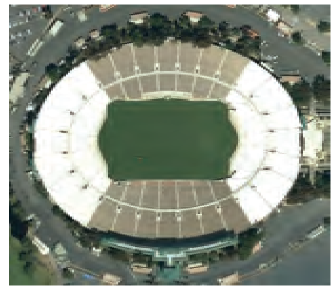
Figure 1 is a scaled copy of the original figure because it is narrower and shorter by the same multiplier. Figure 2 is *not* a scaled copy of the original figure because it has the same height but has been stretched wider than the original.

You will learn more about what it means for one figure to be a scaled copy of another in upcoming lessons.

Reflect:

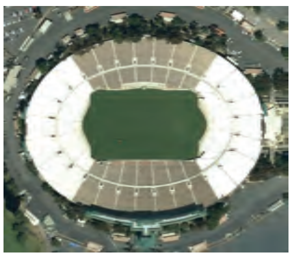
Name: .... Date: .... Period: ....

1. Refer to the image of the Rose Bowl in Pasadena, CA. Select *all* the images which appear to be scaled copies of the original. Then explain why the other images do not appear to be scaled copies.



NASA/WorldWind

A.



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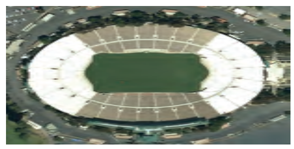

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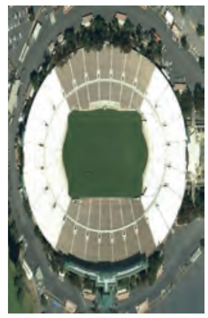

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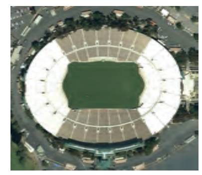


2. Study the drawings of the letter A shown. Which drawings are scaled copies of the original A? Explain your thinking.

Original

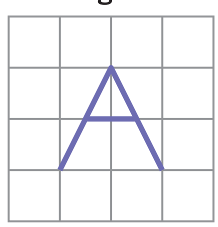


Figure 1

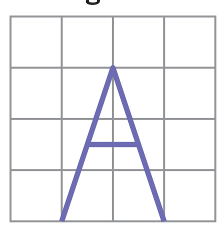


Figure 2

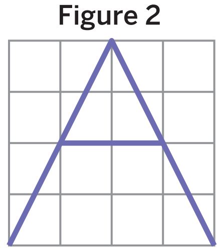


Figure 3

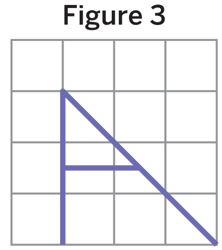
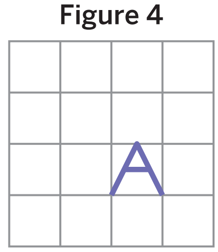


Figure 4



Practice

Name: .... Date: .... Period: ....

3. Tyler says that Figure 2 is a scaled copy of Figure 1 because it is half as tall. Do you agree with Tyler? Explain your thinking.

Figure 1

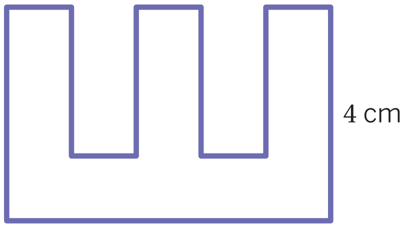


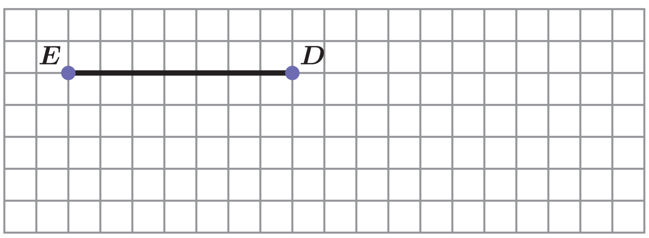
Figure 2

4. Evaluate each expression.

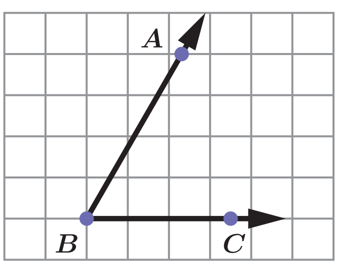
a $\frac{1}{4} \cdot 32$ **b** $\frac{1}{4} \cdot 5.6$ **c** $7.2 \cdot \frac{1}{9}$ **d** $2 \div \frac{1}{4}$

5. Examine the images shown. You will need a ruler and a protractor.

a Draw a segment twice the length of line segment ED.



b Determine the measure of angle ABC.



6. Find the missing value in each equation.

a $32 \cdot \boxed{} = 4$ **b** $24 \cdot \boxed{} = 3$ **c** $16 \cdot \boxed{} = 2$ **d** $8 \cdot \boxed{} = 1$

Name: ______________________ Date: __________ Period: __________

Unit 1 | Lesson 3

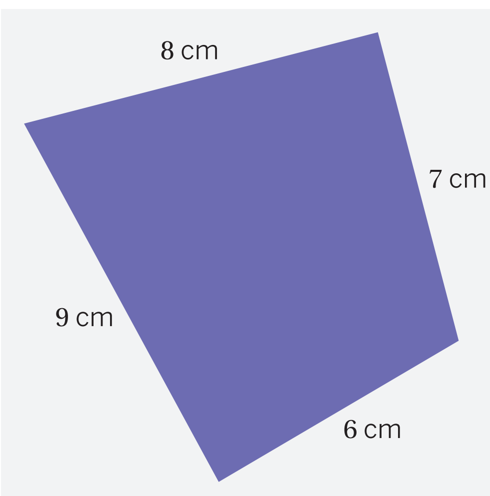
Corresponding Parts and Scale Factors

Let's describe the attributes of scaled copies.

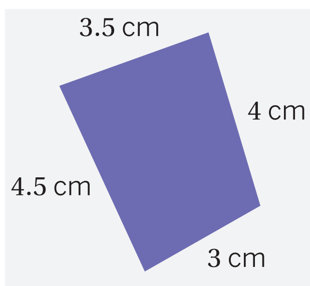
Warm-up Find and Fix

Clare attempted to create a smaller scaled copy of the original painting shown. Shawn noticed Clare made a mistake, so Shawn measured the side lengths of the purple quadrilateral in the center of each painting. Determine Clare's mistake and describe how she could fix her painting so that it is a smaller scaled copy of the original.

Original painting



Clare's painting



The figures may not be drawn to scale.

Log in to Amplify Math to complete this lesson online.

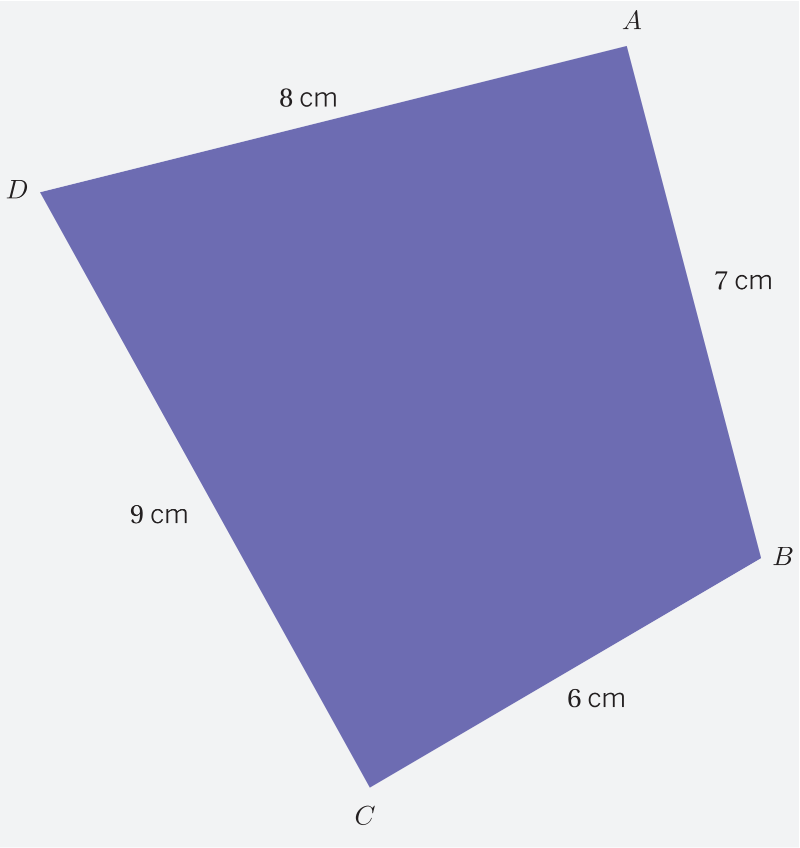
Activity 1 Checking the Angles

Plan ahead: What role will being precise have in this activity?

Shawn created some scaled copies of the painting from the Warm-up for Clare. The scaled copies are shown on this page and the next page.

1. Measure the angles in each painting.

a



The measure of angle A is ....

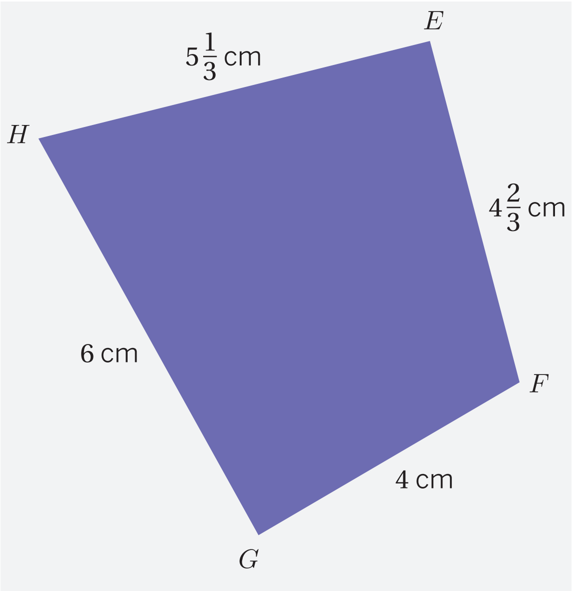
The measure of angle B is ....

The measure of angle C is ....

The measure of angle D is ....

Activity 1 Checking the Angles (continued)

b



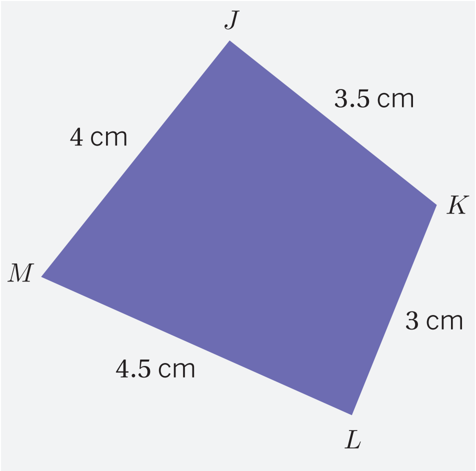
The measure of angle E is ______.

The measure of angle F is ______.

The measure of angle G is ______.

The measure of angle H is ______.

c



The measure of angle J is ______.

The measure of angle K is ______.

The measure of angle L is ______.

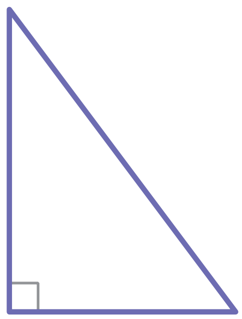
The measure of angle M is ______.

2. Make a conclusion about the angles in scaled copies based on the angles you have measured in this activity.

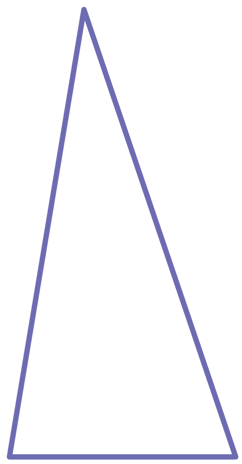
Activity 2 Scaled Triangles

Work with your group to identify *all* the scaled copies of Triangle O in the collection shown. Explain your thinking. If you disagree, discuss to reach an agreement.

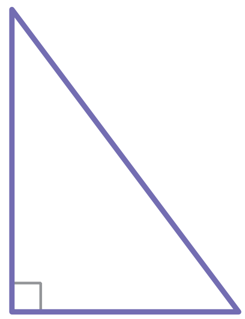
Triangle O



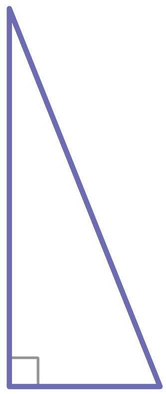
Triangle A



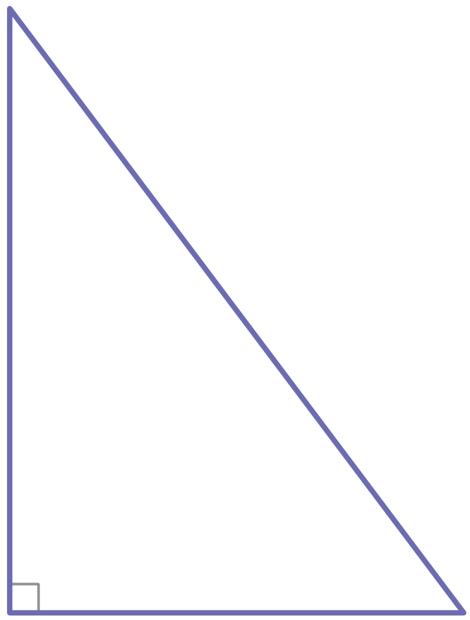
Triangle B



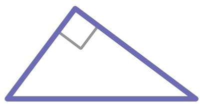
Triangle C



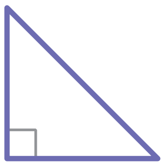
Triangle D



Triangle E



Triangle F



STOP

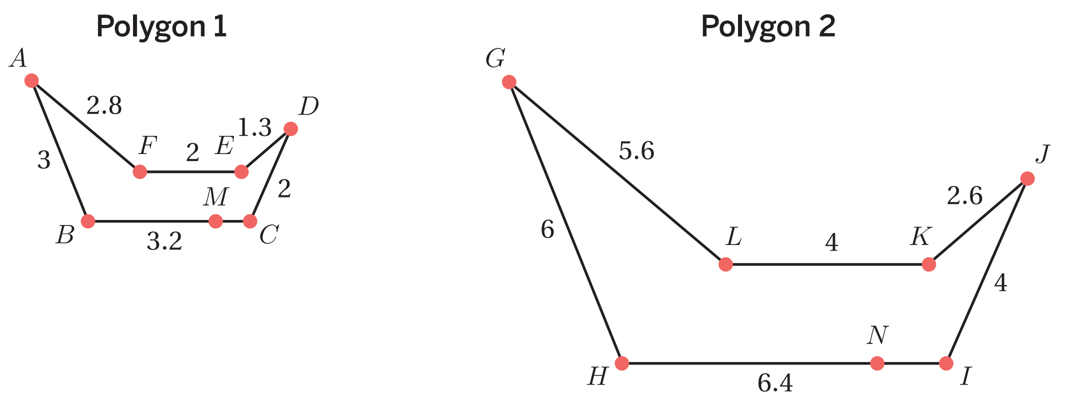
Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You noticed that a figure and its *scaled copy* have ***corresponding parts***, or parts that are in the same position in relation to the rest of each figure. These parts can be points, segments, or angles.

For example, Polygon 2 is a scaled copy of Polygon 1.



- Each point in Polygon 1 has a corresponding point in Polygon 2. For example, point B corresponds to point H and point M corresponds to point N.
- Each side in Polygon 1 has a corresponding side in Polygon 2. For example, side AF corresponds to side GL.
- Each angle in Polygon 1 also has a corresponding angle in Polygon 2. For example, angle BAF corresponds to angle HGL.

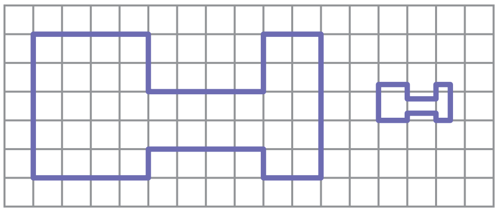
Because all the lengths in Polygon 2 are 2 times the corresponding lengths in Polygon 1, the ***scale factor*** that takes Polygon 1 to Polygon 2 is 2. The scale factor that takes Polygon 2 to Polygon 1 is $\frac{1}{2}$. The angle measures in Polygon 2 are the same as the corresponding angle measures in Polygon 1.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. The second H-shaped polygon is a scaled copy of the first.



   a. Highlight, color, or shade one pair of corresponding sides.

   b. What scale factor takes the original polygon to its smaller copy? Explain your thinking.

2. Suppose Figure B is a scaled copy of Figure A. Select *all* the statements that must be true.

   A. Figure B is larger than Figure A.

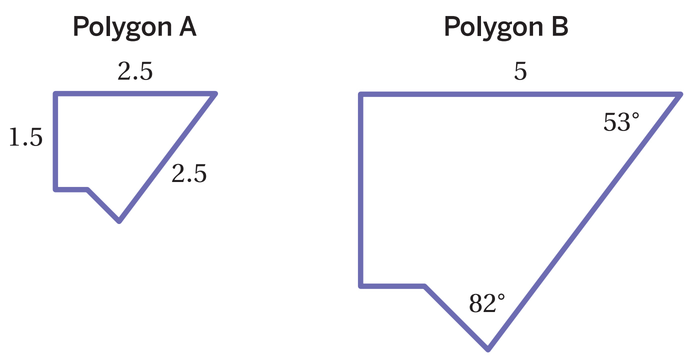
   B. Figure B has the same number of sides as Figure A.

   C. Figure B has the same perimeter as Figure A because the corresponding side lengths are the same between the figures.

   D. Figure B has the same number of angles as Figure A.

   E. Corresponding angles between Figure B and Figure A have the same measure.

3. Polygon B is a scaled copy of Polygon A.



   a. What is the scale factor that takes Polygon A to Polygon B? Explain your thinking.

   b. Determine and label the missing lengths of each of the corresponding sides in Polygon B.

   c. Determine and label the missing measures of each of the corresponding angles in Polygon A.

Name: ____________________ Date: __________ Period: __________

4. Draw a quadrilateral and a scaled copy of your quadrilateral on the grid.

5. Tyler gets his hair cut at the barber shop once every 4 weeks. How many trips will he make to the barber shop in 2 years? You may use the table to help, if needed. **Note:** There are 52 weeks in one year.

Trips to barber shop	Weeks
1	4
	52

6. Circle the phrase that makes each statement true.

a. The value of the expression $15 \cdot 0.75$ is . . .

less than 15 greater than 15

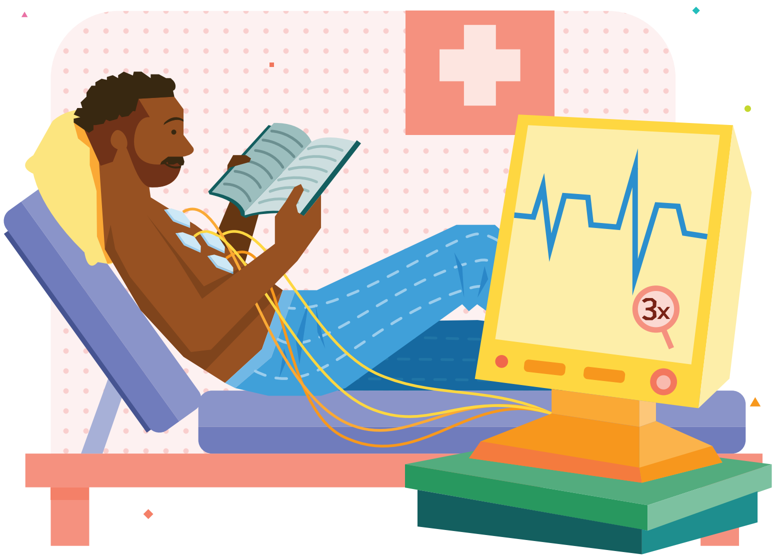
b. The value of the expression $15 \cdot 1.3$ is . . .

less than 15 greater than 15

Unit 1 | Lesson 4

Making Scaled Copies

Let's draw scaled copies.



Warm-up Number Talk

Circle the phrase that makes each statement true. Be prepared to explain your thinking.

1. The value of the expression $57 \cdot 0.83$ is . . .

   greater than 57   less than 57

2. The value of the expression $25 \cdot 1.23$ is . . .

   greater than 25   less than 25

3. The value of the expression $9.93 \cdot 0.984$ is . . .

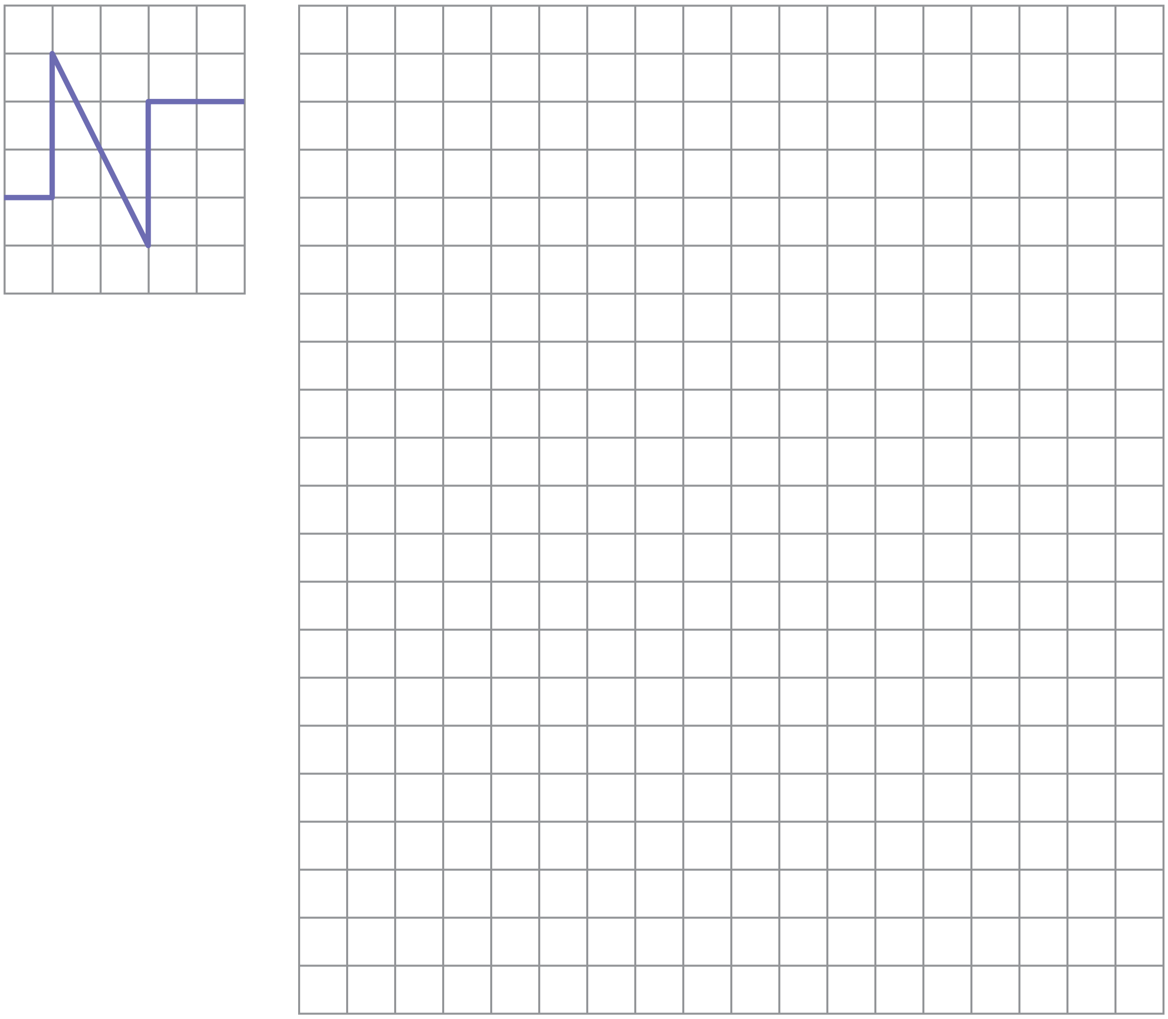
   greater than 10   less than 10

Log in to Amplify Math to complete this lesson online.

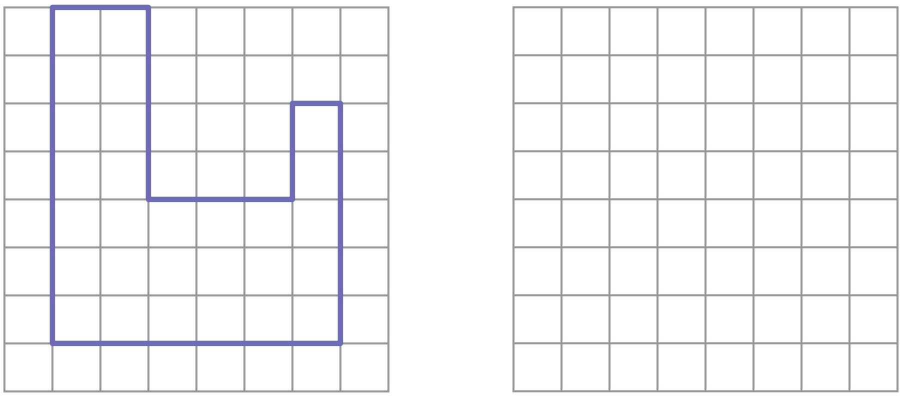
Name: ________________ Date: ________ Period: ________

Activity 1 Drawing Scaled Copies

1. Use the grid provided to draw a copy of the figure using a scale factor of 3.



2. Use the grid provided to draw a copy of the figure using a scale factor of $\frac{1}{2}$.

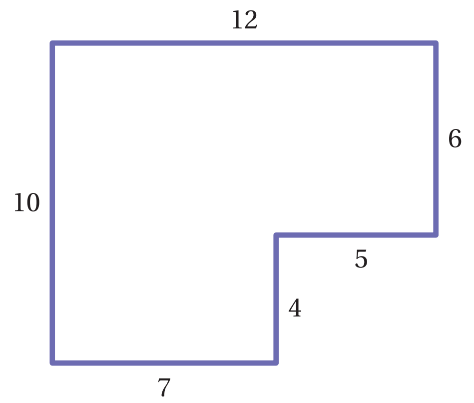


Activity 2 Which Operations?

Andre wants to scale a copy of Jada's drawing so the side corresponding to the 4-unit side in Jada's drawing is 8 units in his scaled copy. Andre says, "I wonder if I should add 4 units to the lengths of all the segments."

1. How would you respond to Andre? Show or explain your thinking.

Jada's drawing



2. Help Andre create his scaled copy by drawing it here. Consider using the edge of an index card or sheet of paper to measure the lengths needed to help with your thinking.

STOP

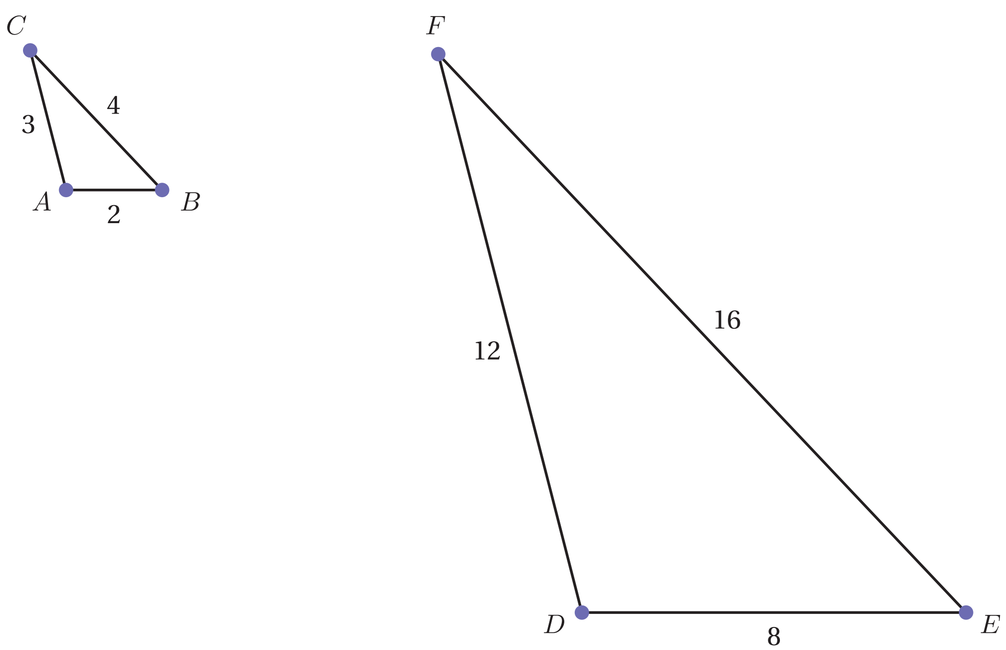
Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You saw how creating a scaled copy involves multiplying all of the lengths in the original figure by a scale factor.

For example, Triangle DEF is a scaled copy of Triangle ABC.



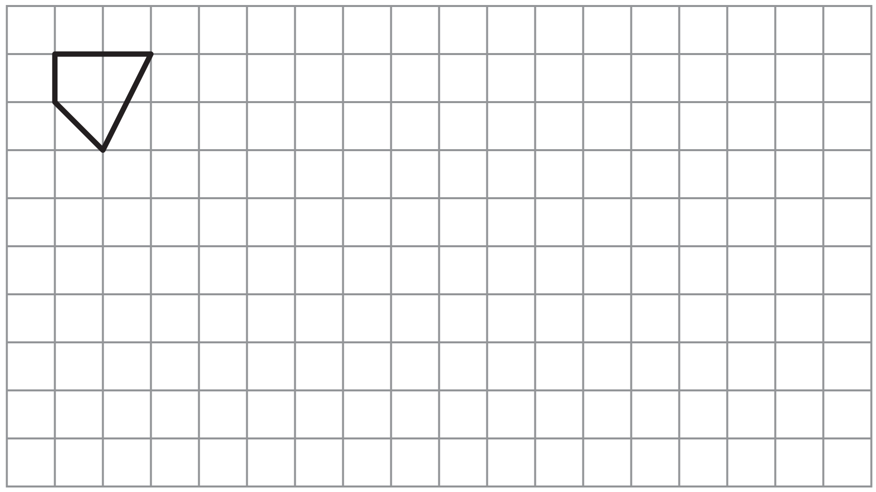
In Triangle DEF, each side is 4 times as long as its corresponding side in Triangle ABC.

- Segment DF corresponds to segment AC, $3 \cdot 4 = 12$, $DF = 12$.
- Segment FE corresponds to segment CB, $4 \cdot 4 = 16$, $FE = 16$.
- Segment DE corresponds to segment AB, $2 \cdot 4 = 8$, $DE = 8$.

Reflect:

Name: .... Date: .... Period: ....

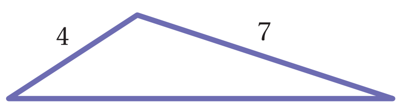
1. Create a scaled copy of the polygon shown using a scale factor of 4.



2. Quadrilateral A has side lengths of 6, 9, 9, and 12 units. Quadrilateral B is a scaled copy of Quadrilateral A, with its shortest side length measuring 2 units. What is the perimeter of Quadrilateral B?

3. Triangle Z is a scaled copy of Triangle M. Select *all* sets of values which could be the side lengths of Triangle Z.

Triangle M

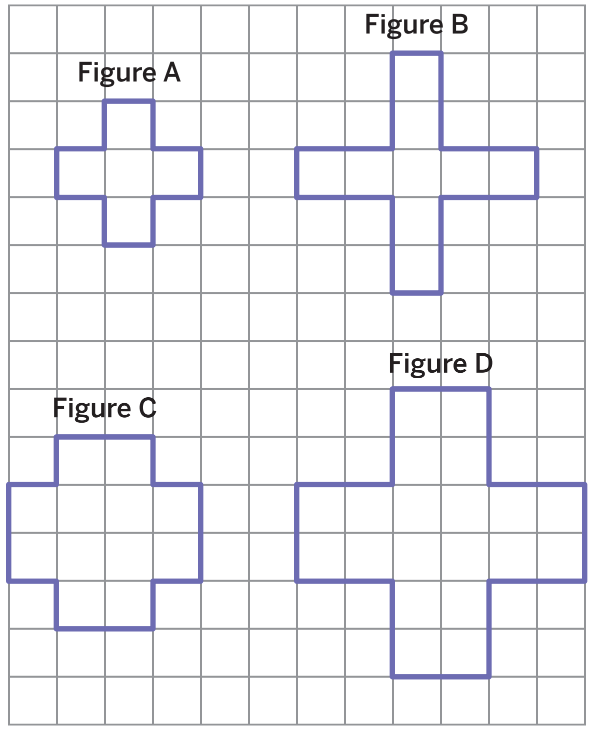


- **A.** 8, 11, and 14
- **B.** 10, 17.5, and 25
- **C.** 6, 9, and 11
- **D.** 6, 10.5, and 15
- **E.** 8, 14, and 20

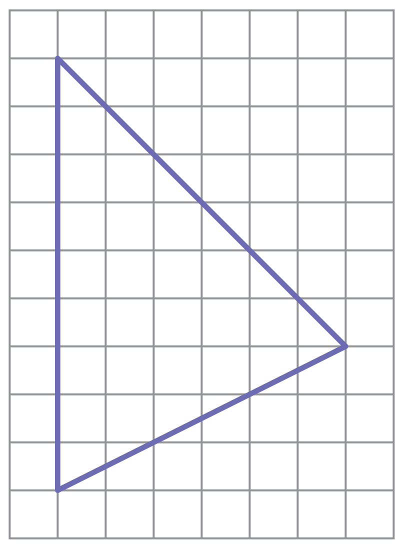
Name: .... Date: .... Period: ....

Practice

4. Priya and Tyler are discussing the figures shown. Priya thinks that Figures B, C, and D are scaled copies of Figure A. Tyler says Figures B and D are scaled copies of Figure A. Do you agree with Priya or Tyler? Explain your thinking.



5. Calculate the area of the triangle.
Note: Think about which side to use as the base.



6. Read each scenario to determine who earned the most points. Explain your thinking.

a. Jada scored $\frac{5}{4}$ the number of points that Bard earned.

b. Priya scored $\frac{2}{3}$ the number of points that Andre earned.

Unit 1 | Lesson 5

The Size of the Scale Factor

Let's observe the effects of different scale factors.

Warm-up Number Talk

1. Mentally evaluate each expression. Record each value in the table.

Set A	Set B	Set C
$12 \cdot 2$	$12 \cdot 1$	$12 \cdot \frac{1}{2}$
$12 \cdot 3$	$12 \cdot \frac{2}{2}$	$12 \cdot 0.25$
$12 \cdot 2.5$		$12 \cdot \frac{2}{3}$
$12 \cdot \frac{3}{2}$		

2. What do you notice about the second factors and the values of the expressions in Set A? Set B? Set C?

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Card Sort: Scaled Copies

You will be given a set of cards. On each card, Figure A is the original figure and Figure B is the scaled copy. Sort the cards into *at least* three categories, but *no more than* five categories. Describe each of your categories.

Category 1 Description:

Cards:

Category 2 Description:

Cards:

Category 3 Description:

Cards:

Category 4 Description:

Cards:

Category 5 Description:

Cards:

Are you ready for more?

Suppose Triangle B is a scaled copy of Triangle A using the scale factor $\frac{1}{2}$.

1. The side lengths of Triangle B are how many times the length of the corresponding sides of Triangle A?

2. Imagine you scale Triangle B by a scale factor of $\frac{1}{2}$ to create Triangle C. The side lengths of Triangle C are how many times the length of the corresponding sides of Triangle A?

3. Triangle B has been scaled once. Triangle C has been scaled twice. Imagine you scale Triangle A n times to create Triangle N, each time using a scale factor of $\frac{1}{2}$. The side lengths of Triangle N are how many times the lengths of the corresponding sides of Triangle A?

Activity 2 Determining Scale Factors

You will need the same cards from Activity 1. For each card, determine the scale factor that was used to take Figure A to Figure B. Show or explain your thinking.

	Scale factor	Your thinking . . .
Card 1		
Card 2		
Card 3		
Card 4		
Card 5		
Card 6		
Card 7		
Card 8		
Card 9		
Card 10		
Card 11		
Card 12		
Card 13		

1. Which scale factors produced larger figures? Smaller figures?

2. Examine Cards 8 and 12. What do you notice about the figures? The scale factors?

3. Examine Cards 1 and 7 and then Cards 10 and 13 more closely. What do you notice about the shapes and sizes of the figures? The scale factors?

Name: ________________ Date: ________ Period: ________

Activity 3 Scaling a Puzzle

Your group will be given a set of puzzle pieces. You will also be given one square for each puzzle piece. Like the Polymath Project, you might find that you can solve this puzzle faster with many minds instead of just one.

1. If you were to draw scaled copies of each puzzle piece using a scale factor of $\frac{1}{2}$, would they be larger or smaller than the original piece? How do you know?

2. Each group member should select at least one puzzle piece. Create a scaled copy of each puzzle piece on the blank square you were given, using a scale factor of $\frac{1}{2}$.

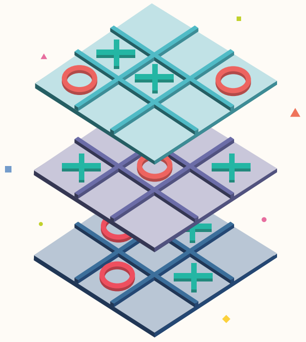
3. Arrange all six of the original puzzle pieces together as shown. Then arrange all six of your scaled copies together. Compare your scaled puzzle with the original puzzle. Which parts seem to be scaled correctly and which seem "off"? What might have caused those parts to be scaled incorrectly?

1	2	3
4	5	6

4. Revise any of the scaled puzzle pieces which may have been drawn incorrectly.

5. If you were to lose one of the pieces of the original puzzle, but still had its scaled copy, how could you recreate the lost piece?

Featured Mathematician



Polymath Project

How many people does it take to figure out whether a four-dimensional game of tic-tac-toe will end in a tie? More than one, apparently. The Polymath Project, started in 2009 by Timothy Gowers, imagined that the brains of many people working together would help to solve this problem faster than each person working independently. After seven weeks and the contribution of more than 40 people, this famous problem, known as the Hales–Jewett theorem, was solved. Keep the Polymath Project in mind as you and your classmates work together this year.

STOP

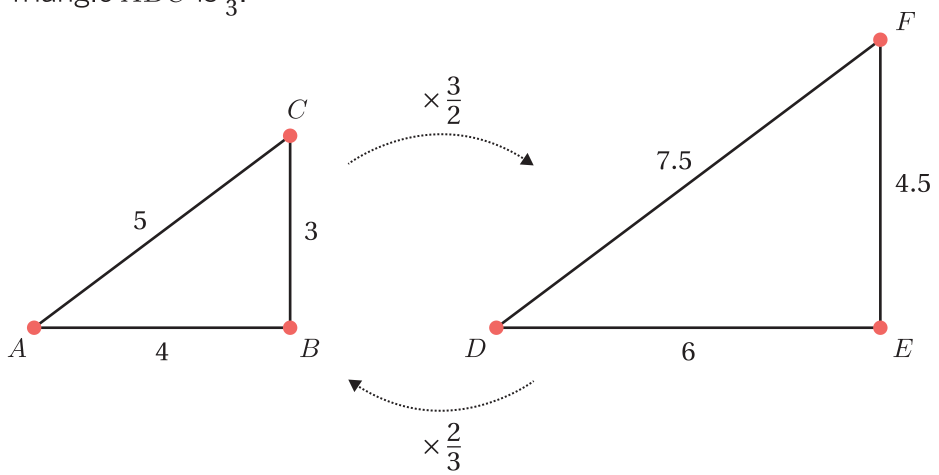
Summary

In today's lesson . . .

You saw how the size of the scale factor affects the size of the scaled copy.

- When the scale factor is greater than 1, the scaled copy is larger than the original figure.
- When the scale factor is less than 1, the scaled copy is smaller than the original figure.
- When the scale factor is equal to 1, the scaled copy is the same size as the original figure.

Triangle DEF is a larger scaled copy of Triangle ABC, because the scale factor that takes Triangle ABC to Triangle DEF is $\frac{3}{2}$. Likewise, Triangle ABC is a smaller scaled copy of Triangle DEF, because the scale factor that takes Triangle DEF to Triangle ABC is $\frac{2}{3}$.



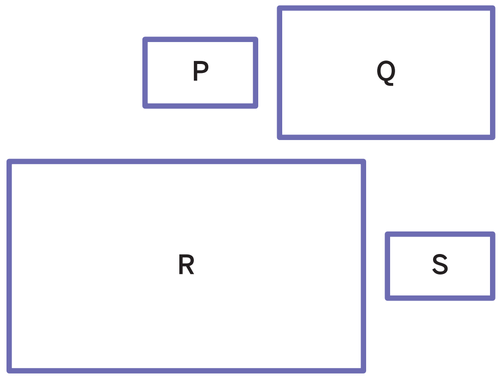
This means that Triangles ABC and DEF are scaled copies of each other. It also shows that scaling can be reversed using ***reciprocal*** scale factors, such as $\frac{2}{3}$ and $\frac{3}{2}$.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Rectangles P, Q, R, and S are scaled copies of one another. For each pair, decide if the scale factor that takes one figure to another is greater than 1, equal to 1, or less than 1.



a from Rectangle P to Rectangle Q

b from Rectangle P to Rectangle R

c from Rectangle Q to Rectangle S

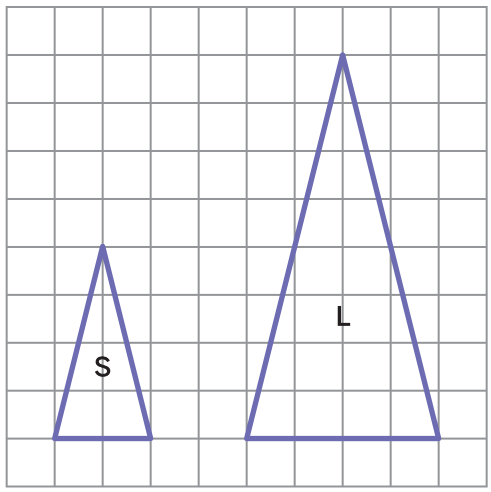
d from Rectangle Q to Rectangle R

e from Rectangle S to Rectangle P

f from Rectangle R to Rectangle P

g From Rectangle P to Rectangle S

2. Triangle S and Triangle L are scaled copies of one another.



a What is the scale factor that takes Triangle S to Triangle L?

b What is the scale factor that takes Triangle L to Triangle S?

c Triangle M (not drawn) is also a scaled copy of Triangle S. The scale factor that takes Triangle S to Triangle M is $\frac{3}{2}$. What is the scale factor that takes Triangle M to Triangle S?

3. Will any two squares always be scaled copies of one another? Explain your thinking.

Name: ______________________ Date: __________ Period: __________

4. Quadrilateral A has side lengths 2, 3, 5, and 6 units. Quadrilateral B has side lengths 4, 5, 8, and 10 units. Could one of the quadrilaterals be a scaled copy of the other? Explain your thinking.

5. Select *all* the ratios that are equivalent to $12 : 3$.

A. $6 : 1$

B. $1 : 4$

C. $4 : 1$

D. $24 : 6$

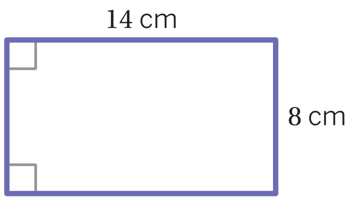
E. $15 : 6$

F. $1{,}200 : 300$

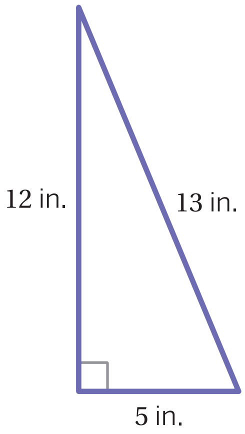
G. $112 : 13$

6. Calculate the area and perimeter of each shape.

a



b



Name: ____________ Date: ____________ Period: ____________

Unit 1 | Lesson 6

Scaling Area

Let's investigate the area of scaled copies.

Warm-up Sharing a Vegetable Lasagna

Elena's father baked a rectangular, deep-dish vegetable lasagna for his 4 children. He cut 3 equal-sized pieces and served each one to 3 of his children.

He noticed that *both* the length and width of the original lasagna had been halved. Can he give his fourth child a piece that is the same size as the others? Show or explain your thinking.

Compare and Connect: Create a display to show your thinking. Then compare your display with a partner and make connections between your different approaches.

Log in to Amplify Math to complete this lesson online.

Activity 1 Scaling Perimeter and Area

A rectangle, with dimensions of 12 cm by 6 cm, is scaled by the different scale factors shown in the table.

1. Complete the table using the given scale factors.

Scale factor	Length (cm)	Width (cm)	Perimeter (cm)	Area (cm^2)
1	12	6		
2				
3				
$\frac{1}{2}$				
$\frac{1}{4}$				
$\frac{2}{3}$				

2. How do the perimeters of a figure and its scaled copy compare when the scale factor is 2? Explain your thinking.

3. How do the perimeters of a figure and its scaled copy compare when the scale factor is $\frac{1}{4}$? Explain your thinking.

4. Study the other scale factors and perimeters. Make a conjecture of how scaling a figure by a scale factor of x affects the perimeter.

5. Choose a new scale factor which reduces the size of the rectangle. Test your conjecture from Problem 4 by determining the perimeter of the scaled copy. If necessary, revise your response to Problem 4, retesting as needed.

Name: ________ Date: ________ Period: ________

Activity 1 Scaling Perimeter and Area (continued)

6. How do the areas of a figure and its scaled copy compare when the scale factor is 2? Explain your thinking.

7. How do the areas of a figure and its scaled copy compare when the scale factor is $\frac{1}{4}$? Explain your thinking.

8. Study the other scale factors and areas. Make a conjecture of how scaling a figure by a scale factor of y affects the area.

9. Choose a new scale factor which reduces the size of the rectangle. Test your conjecture from Problem 8 by determining the area of the scaled copy. If necessary, revise your response to Problem 4, retesting as needed.

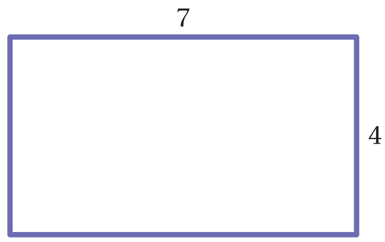
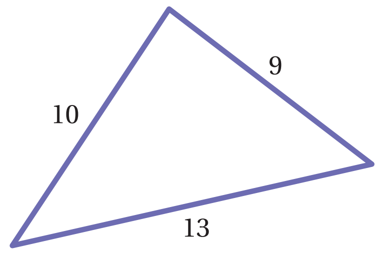
Are you ready for more?

If a rectangular prism is scaled by a factor of 2, how do you think the volume would change? Explain your thinking. (Note: Does this help you understand why the largest animals on Earth are found in the water?)

Reflect: How did you ask for help during the activity? How did you provide help?

Activity 2 Partner Problems: Scale Factor

With your partner, decide who will complete Column A and who will complete Column B. After each row, share your responses with your partner. Although the problems in each row are different, your responses should be the same. If they are not the same, work together to correct any errors or resolve any disagreements.

Column A	Column B
1. The rectangle shown is scaled by a factor of 2. How many times greater is the area of the scaled copy than the area of the original rectangle? 	The triangle shown is scaled by a factor of 4. How many times greater is the perimeter of the scaled copy than the original triangle? 
2. A hexagon has a perimeter of 100 units and is scaled by a factor of $\frac{1}{4}$. What is the perimeter, in units, of the scaled copy?	A pentagon is scaled by a factor of 5. How many times greater is the area of the scaled copy than the original pentagon?

Are you ready for more?

Continue the Partner Problems routine.

1. A map with dimensions 3 ft by 9 ft is scaled by a factor of one third. How many times as great is the area of the new map than the original?	A painting with dimensions 36 ft by 18 ft is scaled so that the new dimensions are 4 ft by 2 ft. What is the scale factor?
2. An image has dimensions of $\frac{1}{2}$ in. by 2 in. and is scaled so that the new dimensions are 8 in. by 32 in. What is the scale factor?	A picture with dimensions 4 in. by 6 in. is scaled by a factor of 4. How much greater is the area of the new picture?
3. The area of a rectangle is 24 cm^2. The rectangle is scaled so that the new dimensions are 2 cm by 3 cm. What is the scale factor?	The dimensions of a rectangle are 6 in. by 8 in. The rectangle is scaled so that the new area is 12 in^2. What is the scale factor?

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You saw how creating scaled copies of figures affects the perimeter and area of the figure in different ways.

Side length	Perimeter	Area
The length of a side in the scaled copy is the product of the corresponding side length in the original figure and the scale factor.	The perimeter of the scaled copy is the product of the perimeter of the original figure and the scale factor.	The area of the scaled copy is the product of the area of the original figure and the square of the scale factor, or $(\text{scale factor})^2$.

Reflect:

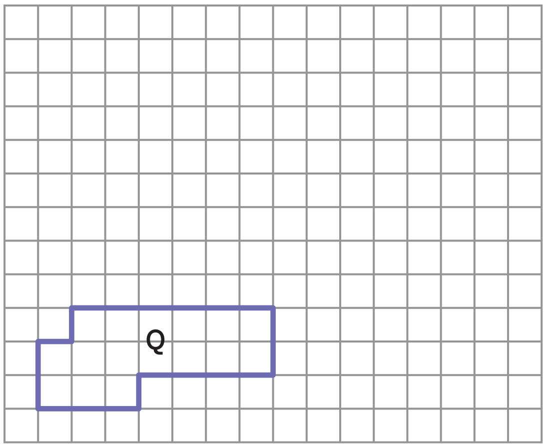
Name: .... Date: .... Period: ....

1. Consider Polygon Q shown on the grid.

   a. On the grid, draw a scaled copy of Polygon Q using a scale factor of 2.

   b. Compare the perimeters of Polygon Q and its scaled copy.

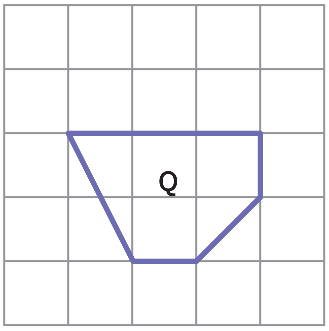
   c. Compare the areas of Polygon Q and its scaled copy.



2. Suppose a right triangle has an area of 36 square units. If you draw scaled copies of this triangle using the scale factors in the table, what will the areas of these scaled copies be? Complete the table and explain your thinking.

Scale factor	Area (square units)
1	36
2	
3	
5	
$\frac{1}{2}$	
$\frac{2}{3}$	

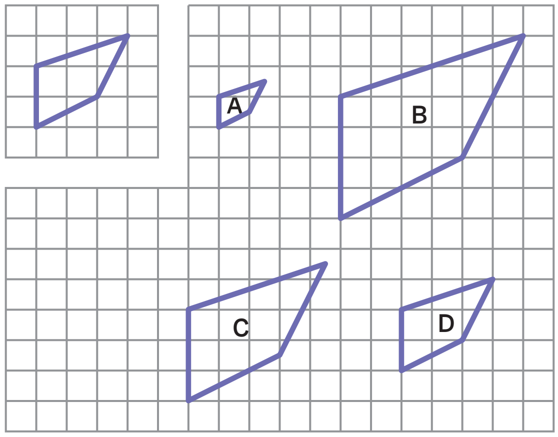
3. Diego drew a scaled version of a Polygon P and labeled it Q. If the area of Polygon P is 72 square units, what scale factor did Diego use to take Polygon P to Polygon Q? Explain your thinking.



Name: ______ Date: ______ Period: ______

Practice

4. An unlabeled polygon is shown, along with its scaled copies. For each copy, determine the scale factor. Explain your thinking.



a Polygon A:

b Polygon B:

c Polygon C:

d Polygon D:

5. Solve each equation. Show your thinking.

a $\frac{1}{7} \cdot x = 1$

b $x \cdot \frac{1}{11} = 1$

c $1 \div \frac{1}{5} = x$

6. In the video game CastleDay, you must collect materials to build your castle. To build each castle wall, you need 10 pieces of lumber and 12 stones. Complete the table to calculate the amount of material needed to build the indicated number of walls.

Number of walls	Pieces of lumber	Stones
1	10	12
2		
3		
4		
10		

My Notes:

SUB-UNIT 2

Scale Drawings

Narrative Connections

Who was the King of Monsters?

Before CGI, there was Haruo Nakajima.

You might not recognize his face, but for years Nakajima played the most recognizable monster in movie history: Godzilla!

Born in 1929 in Yamagata, Japan, Nakajima began his career as a stunt actor in samurai films. When a director noticed the energy Nakajima put into his stunt work, he was chosen to play the 15-story-tall monster.

It was hard work. The movie's special effects team put Nakajima in a suit made from mixed-concrete. Under the hot studio lights, Nakajima had to destroy a miniature scale model of Tokyo that the team had meticulously built.

A single set could take weeks for the team to build. The special effects director was Eiji Tsubaraya. Under his leadership, his team built their models at $\frac{1}{25}$th-to-$\frac{1}{50}$th the scale of their real life counterparts. At these scales, the team's attention to detail made the scenes look realistic. And once the sets were built, it was up to Nakajima to knock them down!

Rather than build a gigantic monster, which would have been costly and impractical, the special effects team used mathematically scaled models. These models helped create the illusion of a very large, very real Godzilla. With Nakajima's fearsome acting, as well as a few camera tricks, the monster would go on to terrify movie-goers across the globe.

Unit 1 | Lesson 7

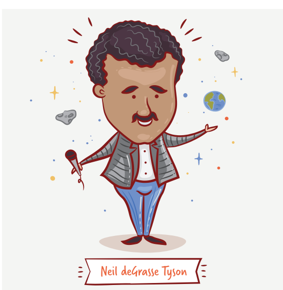
Scale Drawings

Let's explore scale drawings.

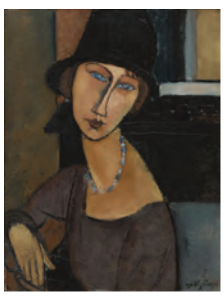
Warm-up What is a Scale Drawing?

Some images of the astrophysicist Neil deGrasse Tyson, the poet Anna Akhmatova, and a map of Paris are shown. Study the two groups of images. What is unique about each group? What is similar? Explain your thinking.

Group 1



Yeti Crab/Shutterstock.com



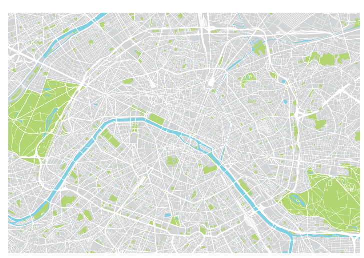
Amedeo Modigliani. 1917. Jeanne hébuterne (Au chapeau) [oil on panel]. Photo by Sotheby's NY.

Kursat Unsal/Shutterstock.com

Group 2

Anait/Shutterstock.com

Public Domain



Ink Drop/Shutterstock.com

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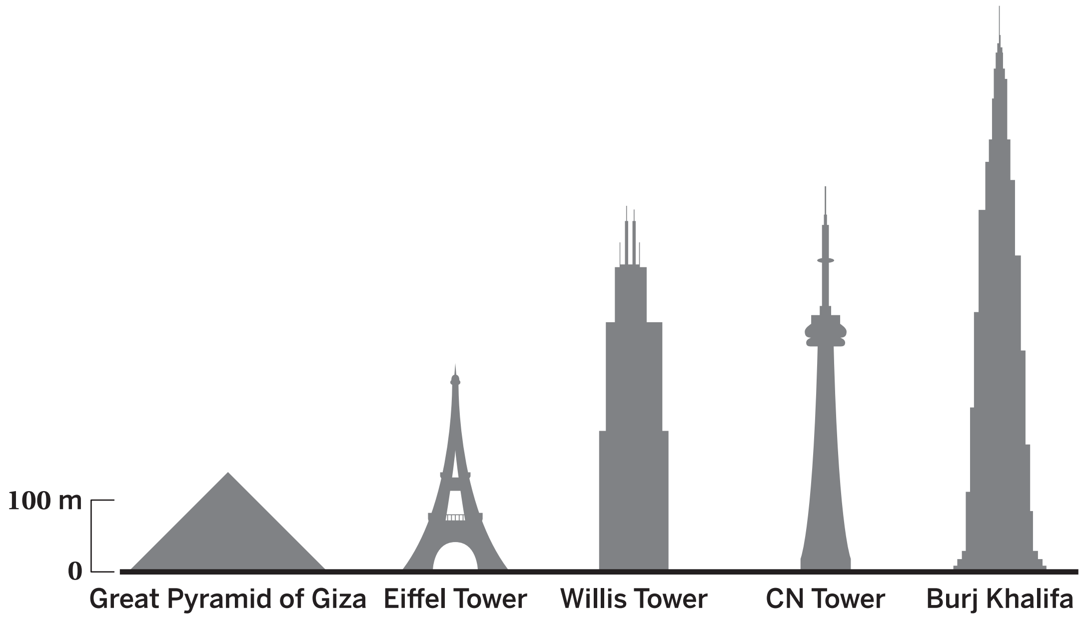
Name: ______________________ Date: ____________ Period: ____________

Activity 1 Tall Structures

Refer to the scale drawing of some of the world's tallest structures.



1. How tall is the Willis Tower? How tall is the Great Pyramid of Giza? Show or explain your thinking.

2. About how much taller is the Burj Khalifa than the Eiffel Tower? Show or explain your thinking.

3. Measure the line segment that shows the scale to the *nearest tenth* of a centimeter. Write the scale of the drawing using numbers and words.

Are you ready for more?

The tallest mountain in the world is Mt. Everest, rising 8,848 m above sea level. Estimate how many sheets of paper you would need to draw the height of Mt. Everest using the same scale from this activity. Explain your thinking.

Activity 2 Sizing up a Basketball Court

You will be given a scale drawing of a basketball court. The drawing does not have any measurements labeled, but it states that 1 cm on the scale drawing represents 2 m on the actual basketball court.

1. Measure the distances on the scale drawing that are labeled a–d to the *nearest tenth* of a centimeter. Record your results in the table.

2. Using the information that 1 cm represents 2 m, complete the table to show how long each measurement on the scaled court would be on the actual basketball court.

	Actual length (m)	Scaled length (cm)
Length of court, a		
Width of court, b		
Hoop to hoop, c		
3-point line to sideline, d		

Are you ready for more?

A contractor needs to finish the court's surface and the gray bench areas with a clear layer of polyurethane — a type of resin or varnish commonly used on the surfaces of basketball courts. What is the area, in square meters that needs to be covered?

STOP

Name: ______________ Date: ______________ Period: ______________

Summary

In today's lesson . . .

You saw that ***scale drawings*** are two-dimensional representations of actual objects or places. Floor plans and maps are some examples of scale drawings.

On a scale drawing:

- Every part or section corresponds to a part or section in the actual object.
- Lengths on the drawing are enlarged or reduced by the same scale factor.
- A ***scale*** tells you how actual measurements are represented on the drawing. For example, if a map has a scale of "1 in. to 5 miles," then a 0.5-in. line segment on that map would represent an actual distance of 2.5 miles.

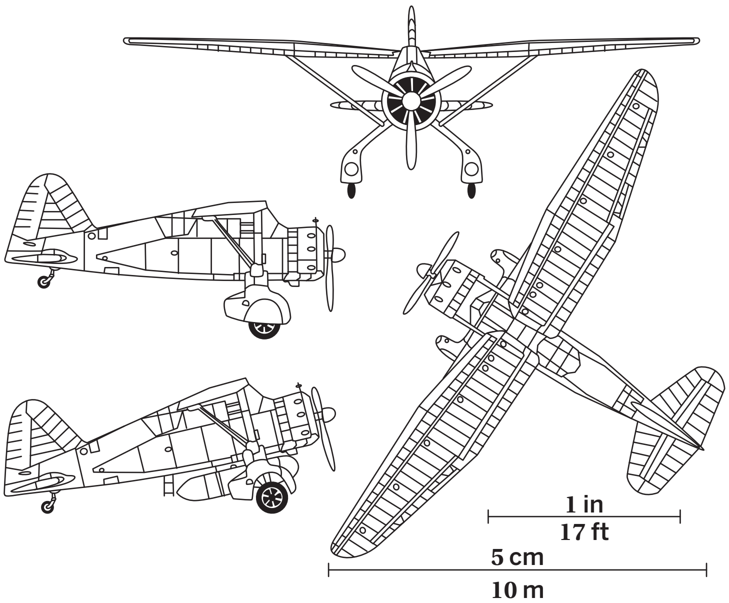
A scale drawing may not show every detail of the actual object. However, the features that are shown correspond to the actual object and follow the specified scale.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. The Westland Lysander was an aircraft used by the British Royal Air Force in the 1930s. Here are some scale drawings that show the top, side, and front views of the Lysander.



Use the scales and scale drawings to approximate the actual lengths of the following. Show or explain your thinking.

a. The wingspan of the plane, to the nearest foot.

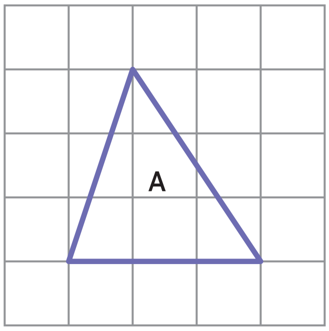
b. The height of the plane, to the nearest foot.

c. The length of the Lysander Mk.I, to the nearest meter.

2. A blueprint for a building includes a rectangular room that measures 3 in. long and 5.5 in. wide. The scale for the blueprint says that 1 in. on the blueprint is equivalent to 10 ft on the actual building. What are the actual building dimensions of this rectangular room? Use the table to support your thinking.

	Given scale	Length	Width
Original	10 ft		
Blueprint	1 in.	3 in.	5.5 in.

3. Refer to Triangle A. Tyler created a scaled copy of Triangle A with an area of 72 square units.



a How many times greater is the area of the scaled copy compared to that of Triangle A?

b What scale factor did Tyler apply to Triangle A to create the scaled copy?

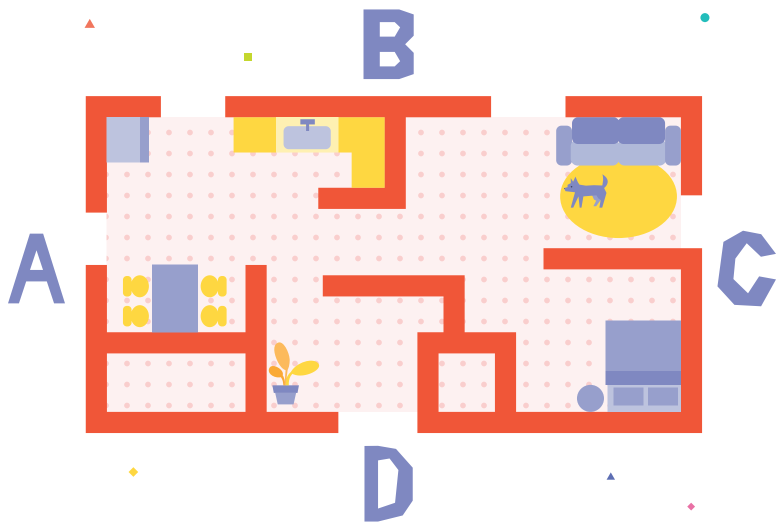
c What is the length of the side of the scaled copy that corresponds with the horizontal side shown in Triangle A?

4. Complete the table so that the two values in each row are related by the same ratio.

3	12
1.5	
4	16
	1
	3

Creating Scale Drawings

Let's create our own scale drawings.



Warm-up Number Talk

Without calculating, decide which expression has a greater quotient. Circle the expression that has the greater quotient.

1. $11 \div 20$ or $25 \div 20$

2. $9.3 \div 3$ or $9.3 \div 5$

3. $7 \div \frac{2}{3}$ or $7 \div \frac{3}{4}$

4. $18 \div 7$ or $15 \div 9$

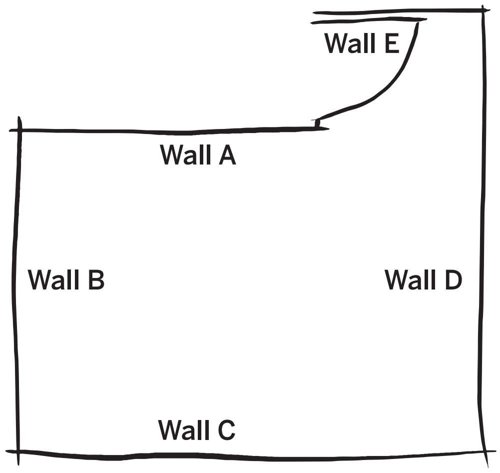
Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Bedroom Floor Plan

Here is a rough sketch of Noah's bedroom. Note: The sketch is *not* a scale drawing.

1. The actual length of Wall C is 4 m. To represent Wall C, Noah draws a segment 16 cm long. What scale is he using? Show your thinking.



2. Use the scale from Problem 1 to complete the table with the missing actual or scaled lengths.

Wall	Actual length (m)	Scaled length (cm)	Ratio of scaled length to actual length
A	2.5		
B		11	
C	4	16	
D	3.75		
E		6	

3. Complete the table to find the ratio of each scaled length to its corresponding actual length. What do you notice? Discuss your thinking with your partner.

4. In Problem 3, you found one way to represent the scale. Find another way to represent the scale.

Are you ready for more?

If Noah wanted to draw another floor plan on which Wall C was 20 cm, would 5 m to 1 cm be the right scale to use? Explain your thinking.

Activity 2 Two Maps of Utah

If a rectangle is drawn around the state of Utah, the rectangle would be about 270 miles wide and about 350 miles tall. The missing upper right corner would be about 110 miles wide and about 70 miles tall. You will create a scale drawing of Utah where 1 cm represents 50 miles.

1. How will you find the lengths needed for the scale drawing?

2. Create the scale drawing in the space provided. Show any necessary work or calculations. Label the dimensions of your scale drawing.

Name: .... Date: .... Period: ....

Activity 2 Two Maps of Utah (continued)

Now, you will create a second scale drawing of Utah where the 270-mile side is drawn as 3.6 cm.

3. What is the scale for this drawing? Explain your thinking.

4. Create the scale drawing in the space provided. Show any necessary work or calculations. Label the dimensions of your scale drawing.

5. How do your two scale drawings compare? How does the choice of scale influence the drawing?

STOP

Summary

In today's lesson . . .

You analyzed scales and scale drawings. Suppose you want to create a scale drawing of a room's floor plan that has the scale "1 in. on the drawing is equal to 4 ft in the room." You can divide the actual lengths in the room (in feet) by 4 to find the corresponding lengths (in inches) for your drawing.

Suppose the longest wall is 15 ft long. Because $15 \div 4 = 3.75$, your drawing should include a line that is 3.75 in. long to represent this wall.

There is more than one way to express this scale. The following three scales are all equivalent, because they represent the same relationship between lengths on a drawing and actual lengths.

Equivalent scales

1 in. to 4 ft	$\frac{1}{2}$ in. to 2 ft	$\frac{1}{4}$ in. to 1 ft

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. A book measures 6 in. wide and 9 in. tall. The publisher wants to display an image of the book on a billboard. The width of the book on the billboard is 36 in. Answer each of the following questions. Show or explain your thinking.

   a. What scale is used for the image on the billboard?

   b. How tall is the book on the billboard?

2. A version of the flag of Colombia is a rectangle that measures 6 ft long with three horizontal stripes.

   - The top yellow stripe is 2 ft tall.
   - The middle blue stripe is 1 ft tall.
   - The bottom red stripe is 1 ft tall.

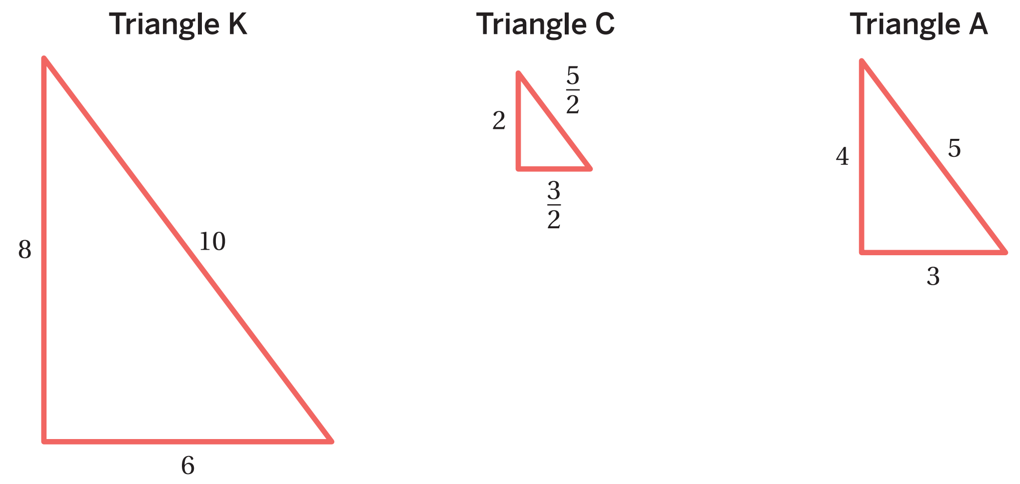
   a. Create a scale drawing of the flag so the top yellow stripe is 3 cm tall. Label the dimensions.

   b. Create a scale drawing of the flag with a scale of 1 cm to 2 ft. Label the dimensions.

Name: ______ Date: ______ Period: ______

Practice

3. These triangles are scaled copies of each other.



For each pair of triangles indicated, determine how many times greater the area of the second triangle is than the area of the first triangle. Show or explain your thinking.

a. Triangle A and Triangle K

b. Triangle C and Triangle K

c. Triangle A and Triangle C

4. There are 2.54 cm in 1 in. and 36 in. in 1 yd. How many centimeters are in 1 yd?

5. Complete the blanks so that the following problem has an answer of 150 ft per minute:

Lin walked from her school to the candy store. The store is ______ ft away from her school. It took her ______ minutes to get there. How fast was Lin walking, in feet per minute?

Name: ______ Date: ______ Period: ______

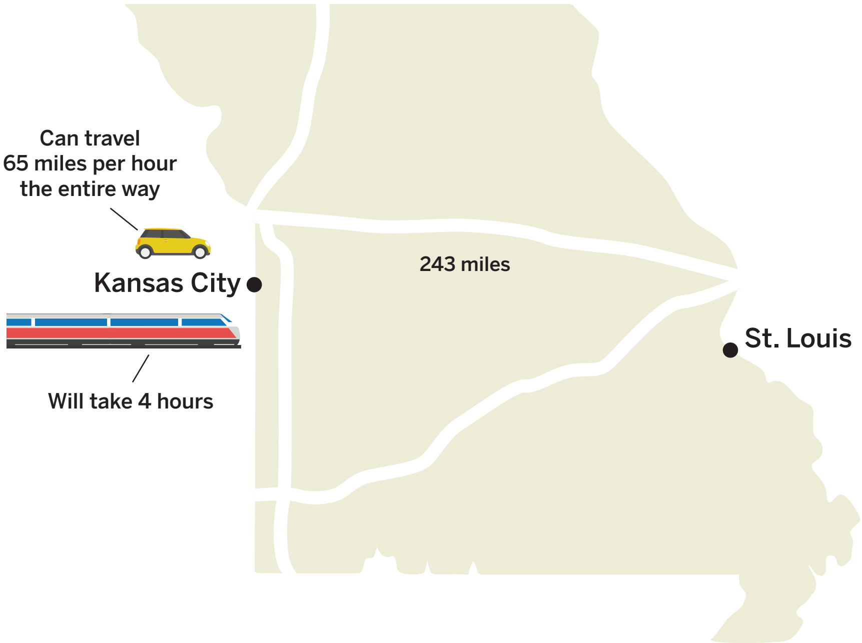
Unit 1 | Lesson 9

Scale Drawings and Maps

Let's use scale drawings to solve problems.

Warm-up In a Rush

Elena and Lin need to travel from Kansas City to St. Louis as quickly as possible. Should they drive or take the train? Explain your thinking.



Log in to Amplify Math to complete this lesson online.

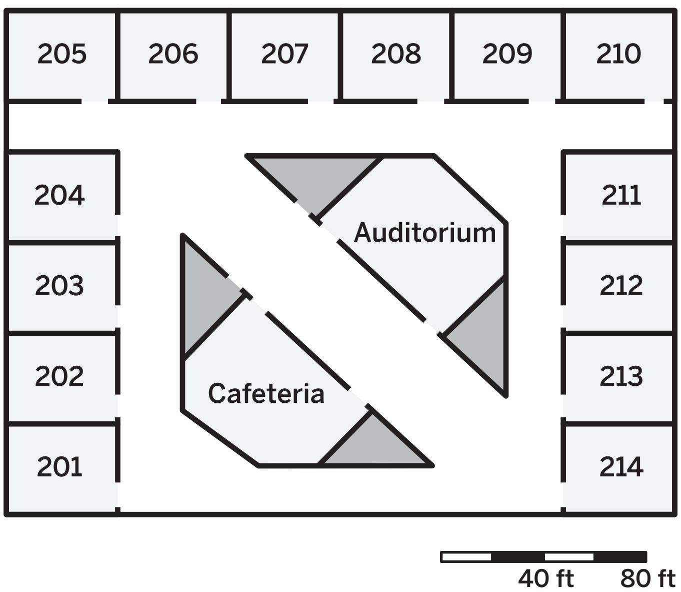
Activity 1 Speed Limit

Plan ahead: How do you feel about the activity? How will self-confidence help you be successful?

Bard thought that the students at school were walking too fast in the hallways and noticed students were bumping into other students between classes. So, Bard created some speed limit signs and posted them in the hallways. But Bard wondered if the speed limit was reasonable and decided to test it out.

It took Bard 1 minute to walk from 2nd period class in Room 205 to 3rd period class in Room 214. Did Bard follow the hallway speed limit rule?

1. Without measuring or calculating, make a guess about whether Bard followed the speed limit. Explain your thinking.



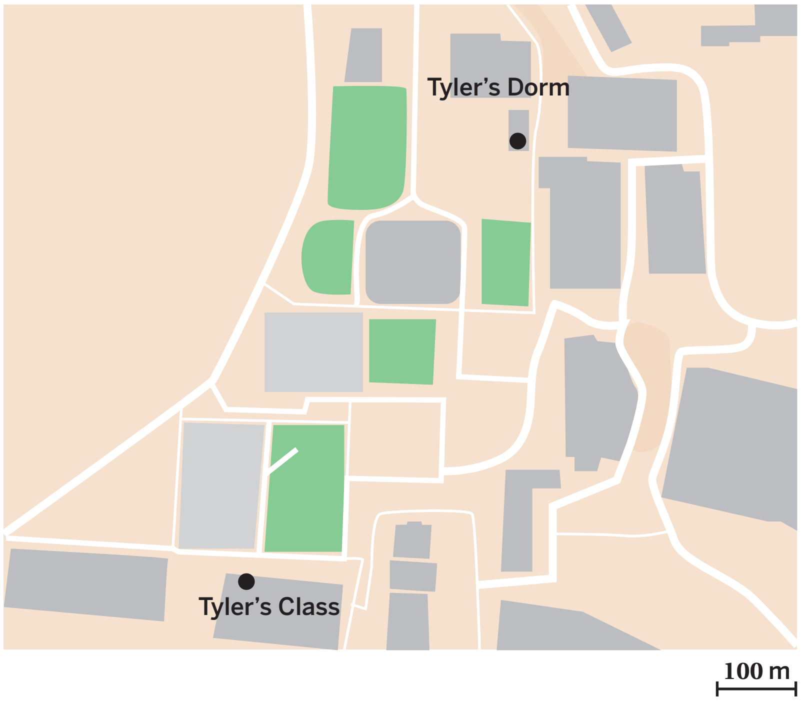
2. Determine whether Bard followed the hallway speed limit. Explain your thinking.

Name: ______ Date: ______ Period: ______

Activity 2 Late to Class?

Tyler just woke up in his dorm! His class begins at 9:45 a.m. He plans to take his scooter, which travels at an average speed of 4 m per second. If he leaves at 9:40 a.m., he wants to know if he will make it to class on time.

1. Without measuring or calculating, make a guess as to whether Tyler will make it to class on time. Explain your thinking.



2. Use mathematical calculations to determine if your guess in Problem 1 is reasonable. Explain your thinking.

STOP

Summary

In today's lesson . . .

You discovered that a map with a scale helps to estimate the distance between two locations by measuring the distance on the map and using the scale to find the actual distance. Once the distance between the two locations is known:

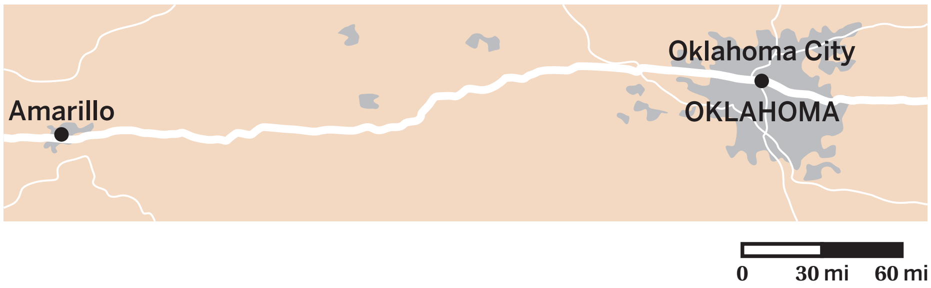
You can calculate . . .	By . . .
The average speed	Finding the quotient of the distance and the time, if you know how long the trip takes. **Average speed = Distance ÷ Time**
How long the trip takes	Finding the quotient of the distance and the average speed, if you know the average speed. **Time = Distance ÷ Average speed**

Reflect:

Name: .... Date: .... Period: ....

Practice

1. This map shows parts of Texas and Oklahoma.



a Approximately how far is it from Amarillo, Texas, to Oklahoma City, Oklahoma?

b Driving at an average speed of 70 miles per hour, will it be possible to make this trip in 3 hours? Explain your thinking.

2. Wyoming is one of two states that is almost perfectly rectangular. Suppose a wall map of Wyoming is made with the scale 1 in. to 10 miles.

a If the northern border of Wyoming on the wall map has a length of 2 ft, 10 in., how long is the actual northern border of Wyoming? Show or explain your thinking.

b If a straight road in Wyoming is 240 miles long, how long would the road be when represented on the map?

Practice

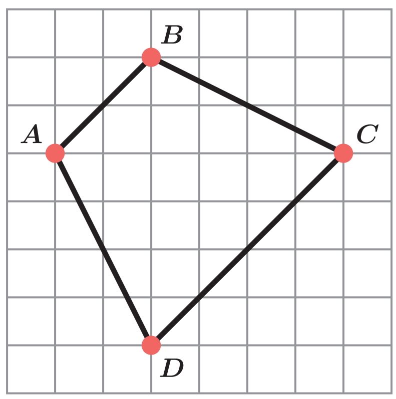
Name: ________________ Date: ________ Period: ________

3. Imagine you live in Brasilia, Brazil. You want to take a 24-hour motorcycle road trip. If your motorcycle can travel at an average speed of 50 miles per hour, what are some other countries you could visit?

4. Quadrilateral $PQRS$ is a scaled copy of Quadrilateral $ABCD$.

- Point P corresponds to point A.
- Point Q corresponds to point B.
- Point R corresponds to point C.
- Point S corresponds to point D.

If the distance between points P and R is 3 units, what is the distance between points Q and S? Explain your thinking.



5. Suppose the scale on a scale drawing reads "1 cm represents 2 m". If the area of the scale drawing is 4 cm^2, what is the area of the actual object the drawing represents? Explain your thinking.

Name: ______ Date: ______ Period: ______

Unit 1 | Lesson 10

Changing Scales in Scale Drawings

Let's explore different scale drawings of the same object or location.

Warm-up Estimating Measurements

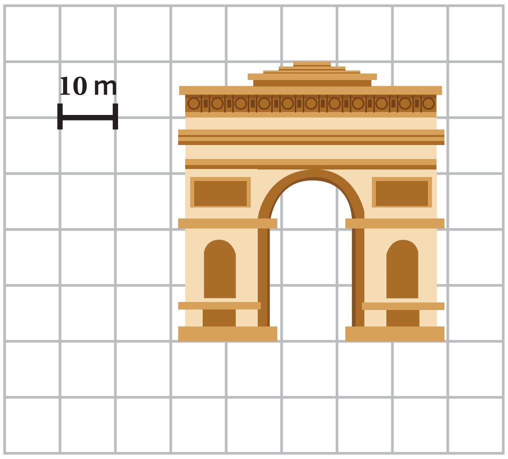
Here is a scale drawing of an average seventh grader's foot next to a scale drawing of the largest human foot in the world. Estimate the length of the larger foot.

Log in to Amplify Math to complete this lesson online.

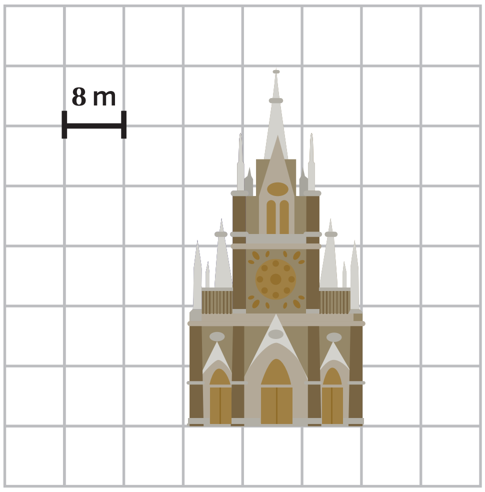
Activity 1 Different Scales

Consider the following scale drawings.

1. A scale drawing of the Arc de Triomphe in Paris, France is shown. If you created a new scale drawing so that each side length of a square represents 2 m, what would be the height — in squares — of your new scale drawing? Explain your thinking.



2. Here is a scale drawing of Las Lajas Sanctuary in Nariño, Colombia. If you created a new scale drawing so that each side length of a square represents 12 m, what would be the height — in squares — of your new scale drawing? Explain your thinking.

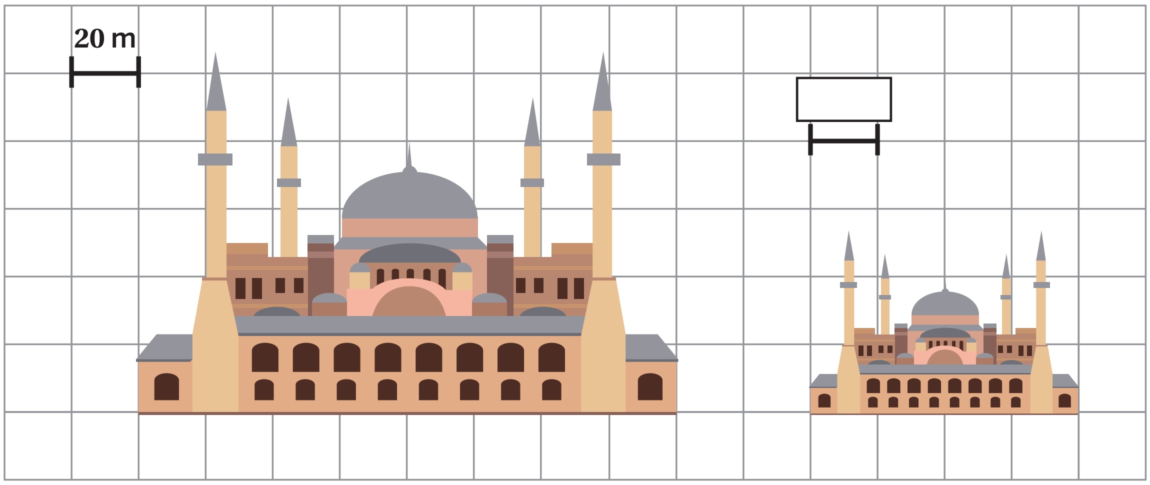


Stronger and Clearer: After drafting your responses to Problems 1 and 2, have 2–3 partners review and ask clarifying questions about your response. Use their feedback and borrow words/phrases you heard from others to add to and revise your responses.

Name: ________ Date: ________ Period: ________

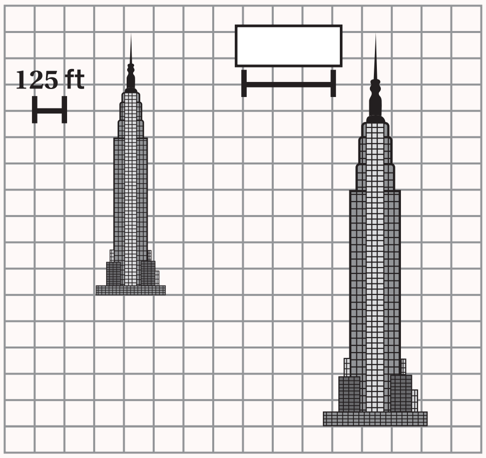
Activity 1 Different Scales (continued)

3. Two scale drawings of the Hagia Sophia in Istanbul, Turkey, are shown. Determine the missing scale on the smaller scale drawing. Explain your thinking.



Are you ready for more?

Two scale drawings of the Empire State Building in New York City, New York are shown. Determine the missing scale on the larger scale drawing. Explain your thinking.



Activity 2 Same Plot, Different Drawings

The map shows a part of Philadelphia, Pennsylvania, near Logan Square. The Benjamin Franklin Parkway partitions a plot of land into a right triangle. You will be given a scale to use and grid paper.

1. Create a scale drawing of the plot of land on the grid paper. Make sure to label your scale and the dimensions of the triangle on your drawing.

2. What is the area of the triangular plot on land on your scale drawing? Show or explain your thinking.

3. How many square meters are represented by 1 cm^2 in your scale drawing?

Pause here until everyone in your group has completed their scale drawings.

4. Order your scale drawings from the largest drawing to the smallest drawing. What do you notice about the scales when your drawings are placed in this order?

Are you ready for more?

What is the scale factor representing the actual plot of land to your scale drawing?

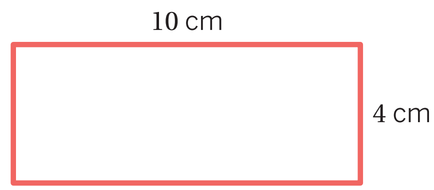
STOP

Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that you can change scales of drawings. You can think of changing scales in two ways, or using one of two methods shown.



Suppose a scale drawing of a rectangle was created using a scale of 1 cm to 90 m. The dimensions of the scale drawing are 4 cm by 10 cm. Suppose you want to create a new scale drawing of the same rectangle using a scale of 1 cm to 30 m. How can you determine the dimensions of the new scale drawing?

Method 1

- Use the original scale to find the dimensions of the actual rectangle.
- Then use these dimensions and the new scale to determine the dimensions of the new scale drawing.

Method 2

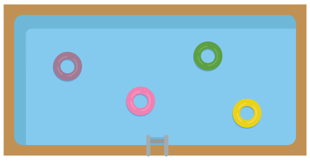
- Think about how the two different scales are related to each other.
- Because $90 \div 30 = 3$, each length in the new scale drawing should be 3 times as long as it was in the original drawing.
- The dimensions of the new scale drawing should be 12 cm by 30 cm, because $4 \cdot 3 = 12$ and $10 \cdot 3 = 30$.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. On a scale drawing of a rectangular swimming pool, 1 cm represents 1 m.



a. What are the dimensions of the actual swimming pool?

b. Will a new scale drawing of the same swimming pool, where 1 cm represents 2 m, be larger or smaller than the original drawing? Check your thinking by creating a scale drawing of the swimming pool where 1 cm represents 2 m.

2. A map of a park shows a scale of 1 in. to $1{,}000$ ft. Another map of the same park shows a scale of 1 in. to 500 ft. Which map is larger? Show or explain your thinking.

3. The floor plan of a restaurant shows a scale of 1 in. to 12 ft. The floor plan shows the area of the restaurant is 60 in^2. Han says that the actual area of the restaurant is 720 ft^2. Do you agree or disagree? Explain your thinking.

Practice

Name: .... Date: .... Period: ....

4. Simplify each expression.

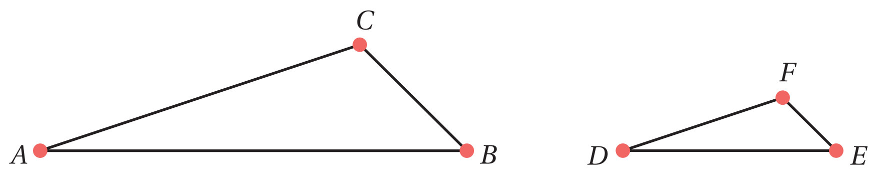
a $6 \div \frac{2}{3}$

b $\frac{4}{9} \div 12$

c $\frac{5}{8} \div \frac{15}{16}$

d $2\frac{4}{9} \div \frac{11}{12}$

5. Triangle DEF is a scaled copy of Triangle ABC. For each of the indicated parts of Triangle ABC, identify the corresponding part of Triangle DEF.



a Angle ABC

b Angle BCA

c Side AC

d Side BA

6. Match the equivalent measurements.

a 1 cm .... 1 in

b 2.54 cm .... 1 ft

c $\frac{1}{3}$ yd .... 3 ft

d 1 yd .... $\frac{1}{100}$ m

Scales Without Units

Let's explore a different way to express scales.

Warm-up Which One Doesn't Belong?

Which scale doesn't belong with the others? Be prepared to explain your thinking.

A. 1 to 100

B. 1 cm to 10 cm

C. 30 m to 3 km

D. 1 in. to 1 ft

Log in to Amplify Math to complete this lesson online.

Name: ____________ Date: ____________ Period: ____________

Activity 1 Godzilla

Review the Sub-Unit opener about the first Godzilla film. During the filming of the first Godzilla, released in 1954, the city of Tokyo was replicated using the scale of 1 to 25 and Godzilla appeared to be 50 m tall. However, the creators of the 1984 film, *The Return of Godzilla*, increased the height of Godzilla to 80 m and used a scale of 1 to 40 to create the buildings.

Keio Plaza Hotel

Osugi/Shutterstock.com

1. Keio Plaza Hotel was partially destroyed in the film, *The Return of Godzilla*. The height of the hotel on the movie set was approximately 4.5 m tall. Using the scale from the 1984 movie, how tall is the actual building in Tokyo? Show or explain your thinking.

2. In *The Return of Godzilla*, Godzilla also destroyed the Shinjuku Sumitomo Building which has a height of approximately 210 m. How tall did the special effects team make the building on set? Show or explain your thinking.

Shinjuku Sumitomo Building

Osugi/Shutterstock.com

3. A bus measures 40 ft long, 8 ft wide, and 10 ft tall. What would the dimensions of the scale model be on the set of *The Return of Godzilla*? Show or explain your thinking.

Are you ready for more?

Haruo Nakajima, the actor playing Godzilla in the original film, was approximately 1.67 m tall but portrayed a monster 50 m tall. Suppose you portrayed a monster. Using this same scale and your height, how tall would your monster be? Explain your thinking.

Activity 2 Same Drawing, Different Scales

A rectangular parking lot is 120 ft long and 75 ft wide.

- **Lin created a scale drawing of the parking lot at a scale of 1 in. to 15 ft. The drawing she created measures 8 in. by 5 in.**
- **Diego created a different scale drawing of the parking lot at a scale of 1 to 180. The drawing he created also measures 8 in. by 5 in.**

1. Explain or show how each scale would create a drawing that measures 8 in. by 5 in.

2. Use a separate sheet of paper to create your own scale drawing of the same parking lot at a scale of 1 in. to 20 ft. Be prepared to explain your thinking.

3. Express the scale of 1 in. to 20 ft as a scale without units. Explain your thinking.

STOP

Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw a scale with units can be expressed as a scale without units by converting one measurement in the scale to the same unit as the other. For example, the following scales are equivalent:

1 in. to 200 ft 1 in. to $2{,}400$ in. (because there are 12 in. in 1 ft) 1 to $2{,}400$

The scale 1 to $2{,}400$ tells you the actual distances are $2{,}400$ times their corresponding distances on the drawing. This scale also tells you that the distances on the drawing are $\frac{1}{2400}$ times the actual distances they represent. Remember, this is known as the *scale factor*.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Three scale drawings of a car are drawn, each using one of the three scales shown. Order the scale drawings from smallest to largest.
Note: There are about 1.1 yd in 1 meter, and 2.54 cm in 1 in.

Scale drawing A	1 in. to 1 ft
Scale drawing B	1 in. to 1 m
Scale drawing C	1 in. to 1 yd

Smallest **Largest**

2. Which scales are equivalent to the scale 1 in. to 1 ft? Select *all* that apply.

A. 1 to 12
B. $\frac{1}{12}$ to 1
C. 100 to 0.12
D. 5 to 60
E. 36 to 3
F. 9 to 108

3. A model airplane is created at a scale of 1 to 72. If the model plane is 8 in. long, how long is the actual airplane in feet? Explain your thinking.

4. Quadrilateral A has side lengths 3, 6, 6, and 9. Quadrilateral B is a scaled copy of Quadrilateral A, with the shortest side length equal to 2. Jada says, "Because all of the side lengths decrease by 1 using this scale, the perimeter decreases by 4 as there are 4 sides." Do you agree with Jada? Explain your thinking.

Name: ________ Date: ________ Period: ________

Practice

5. $5\frac{5}{8}$ cups of water fill $4\frac{1}{2}$ identical water bottles. How many cups fill each bottle? Show or explain your thinking.

6. Use the table of conversions to calculate how many inches are in 1 mile. Explain which conversion(s) you used.

Customary units	Metric units	Equivalent lengths in different systems
1 ft = 12 in.	1 m = 1,000 mm	1 in. = 2.54 cm
1 yd = 36 in.	1 m = 100 cm	1 ft ≈ 0.30 m
1 yd = 3 ft	1 km = 1000 m	1 mile ≈ 1.61 km
1 mile = 5,280 ft		1 cm ≈ 0.39 in.
		1 m ≈ 39.37 in.
		1 km ≈ 0.62 miles

Unit 1 | Lesson 12

Units in Scale Drawings

Let's see how different scales can describe the same relationship.

Warm-up Centimeters in a Mile

There are 2.54 cm in 1 in., 12 in. in 1 ft, and 5,280 ft in 1 mile.

1. Choose the expression that gives the number of centimeters in 1 mile. Explain your thinking.

   A. $\dfrac{2.54}{12 \cdot 5280}$

   B. $5280 \cdot 12 \cdot 2.54$

   C. $\dfrac{1}{5280 \cdot 12 \cdot 2.54}$

   D. $5280 + 12 + 2.54$

   E. $\dfrac{5280 \cdot 12}{2.54}$

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Equivalent Scales Bingo

1. You will be given a bingo card. Read these directions for how to play.

- Your teacher will read a scale and display it.
- Check your bingo card to see if you have an equivalent scale (or scales) to the one shown.
- Use the table of unit conversions provided here to help you determine equivalent scales.
- You may use all available space on this page for your calculations.
- Play until you or one of your classmates mark 5 boxes in one row, column, or diagonal.

Customary units	Metric units	Equal lengths in different systems
1 ft = 12 in.	1 m = 1,000 mm	1 in. = 2.54 cm
1 yd = 36 in.	1 m = 100 cm	1 ft ≈ 0.30 m
1 yd = 3 ft	1 km = 1,000 m	1 mile ≈ 1.61 km
1 mile = 5,280 ft		1 cm ≈ 0.39 in.
		1 m ≈ 39.37 in.
		1 km ≈ 0.62 mile

Activity 1 Equivalent Scales Bingo (continued)

2. Tyler was asked to determine whether the scales 1 cm to 5 m and 1 in. to 5 ft are equivalent. His work and explanation are shown.

Tyler's work:
Yes, they are equivalent.
1 to 5 is equal to 1 to 5

Tyler's explanation:
To find whether the two scales are equivalent, just make the second part of the scale have the same unit as the first part. Then you can ignore the units and write a scale without units. The scales without units are the same, so they are equivalent scales.

Tyler has a misconception about scales with and without units. Correct his work and rewrite his explanation so that it is accurate and clearer.

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that scales can be expressed in many different ways, including using different units or not using any units at all. For example:

- You can express the scale 1 in. to 5 miles as 1 in. to $316,800$ in. or 1 to $316,800$.

Some scales are equivalent. For example:

- The scale 1 mm to 1 m can be expressed as 1 to $1,000$.
- The scale 1 m to 1 km can also be expressed as 1 to $1,000$.

These are referred to as ***equivalent scales***.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. The Empire State Building in New York City, New York, is about 1,450 ft tall — including the antenna at the top — and 400 ft wide. Andre wants to make a scale drawing of the front view of the Empire State Building on an $8\frac{1}{2}$-in. by 11-in. sheet of paper. Select the scale that you think is the *most appropriate* for the scale drawing. Explain your thinking.

A. 1 in. to 1 ft

B. 1 in. to 100 ft

C. 1 in. to 1 mile

D. 1 cm to 1 m

E. 1 cm to 50 m

F. 1 cm to 1 km

2. Select *all* of the scales that are equivalent to 3 cm to 15 m. Explain your thinking.

A. 3 in. to 15 in.

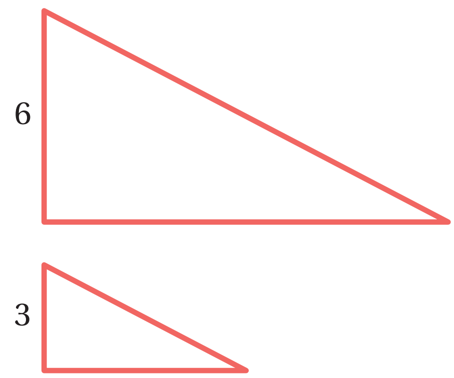
B. 1 cm to 5 m

C. 3 m to 15 cm

D. 4 mm to 2 m

E. 1 in. to 5 ft

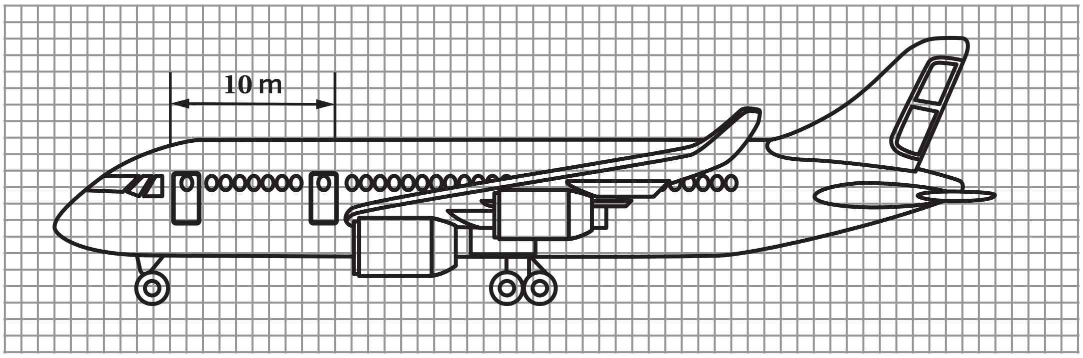
3. The larger triangle shown is a scaled copy of the smaller triangle shown. The labeled side lengths are corresponding sides. The area of the smaller triangle is 9 square units. What is the area of the larger triangle? Show or explain your thinking.



Name: .... Date: .... Period: ....

Practice

4. A blueprint of an airplane is shown. Use the scale to determine the length of the plane, to the nearest meter. Show or explain your thinking.



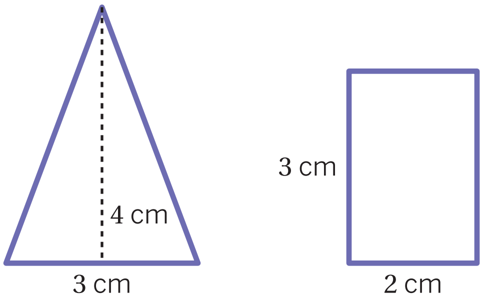
5. Water costs $1.25 per bottle. At this rate, calculate the cost of each of the following. Show or explain your thinking.

a. 10 bottles:

b. 20 bottles:

c. 50 bottles:

6. A triangle and a rectangle are shown. Find a scale factor that allows for the triangle to fit entirely within the rectangle, taking up as much space as possible. Explain your thinking.

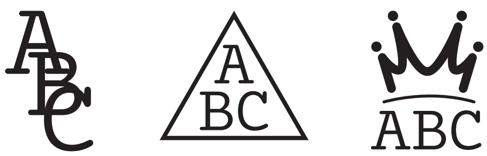


Build Your Brand

Let's design a logo for your personal brand.

Warm-up Sketch Your Logo

An important step in building your personal brand is designing an iconic logo.



1. Sketch at least two ideas for your logo. Your logo must include your initials, but you may choose how to arrange them. You may also choose to include any design elements.

2. Which design is your top choice? Explain your thinking.

Name: Date: Period:

Activity 1 Large- and Small-Scale

You will be given a ruler and a chart with several promotional items.

1. Draw your selected logo design to fill as much space in the grid as possible.

2. Label and measure as many lengths of your logo design as you can. Keep in mind that the more measurements you have, the greater the precision in your scaled copies.

3. Create your scale drawings.
   - a Choose at least 2 promotional items from the chart of promotional items.
   - b Decide on an appropriate scale factor to fit the space allowed for each item.
   - c Draw scaled copies of your logo here and on the next page, using the appropriate scale factors.

Promotional item first choice: **Scale factor:**

Activity 1 Large- and Small-Scale (continued)

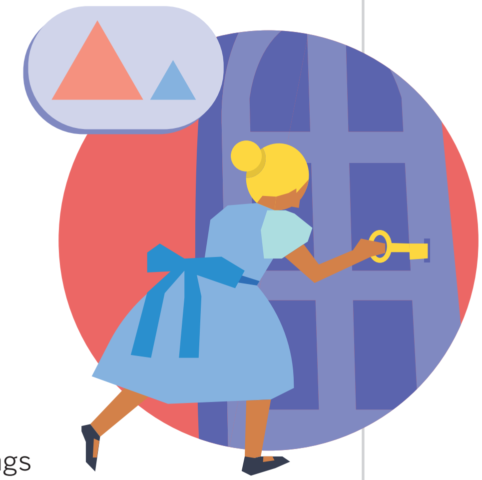
Promotional item second choice:

Scale factor:

STOP

Reflect: How did you show appreciation for the logos of other students in your class?

Unit Summary



Whether it's designing a logo for a personal brand or building a destructible city for a blockbuster movie, objects aren't always the size we need them to be. But thanks to the wonders of scaling, we can change those sizes to fit our exact needs.

There are many reasons to change something's size. When we scale things up, we can see smaller things in finer detail—like a bug blown up under a magnifying glass. Meanwhile, scaling things down lets us take large, complicated things like geographical features and turn them into something that's easier to handle, like on a map.

We scale a length through multiplication or division using a scale factor. But in order for these changes to be meaningful, they also have to be uniform. That means applying that scale factor across all dimensions. Otherwise, you could end up with something squashed down or stretched out.

Scaling lets us imagine the world in new ways. It shows us perspectives we might not have considered before. From bustling cities to colossal movie icons, scaling lets us experience it all.

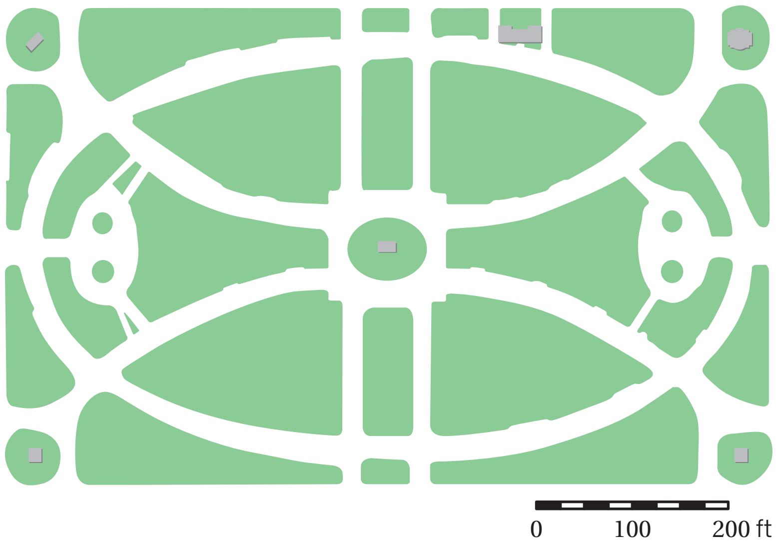
See you in Unit 2.

Practice

Name: ________________ Date: ________ Period: ________

1. A scale map of Lafayette Square in Washington, D.C., is shown.

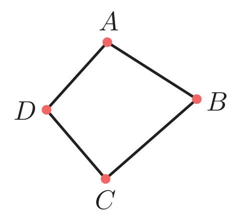
Lafayette Square

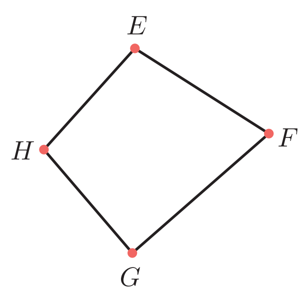


a. Find the actual side lengths, in feet, of Lafayette Square.

b. Use an inch ruler to measure the line segment of the scale. About how many feet does 1 in. represent on this map?

2. Quadrilateral $EFGH$ is a scaled copy of Quadrilateral $ABCD$. Match each part of Quadrilateral $ABCD$ with its corresponding part on Quadrilateral $EFGH$.



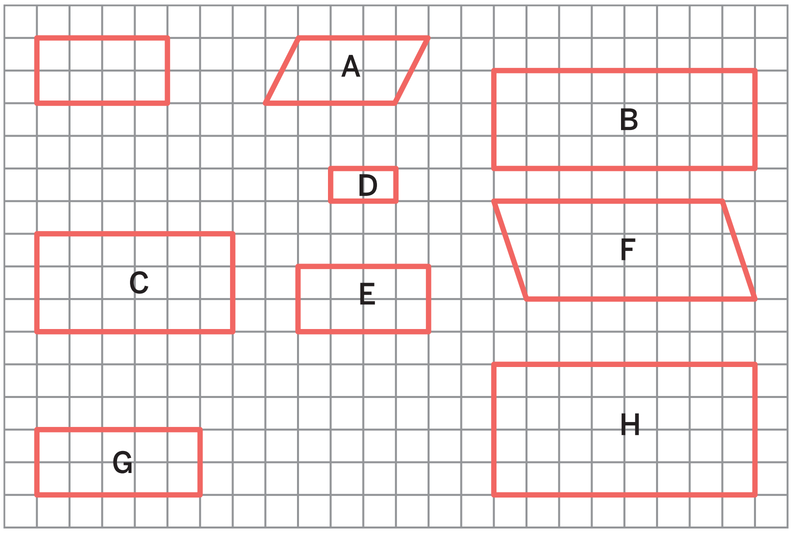


Quadrilateral $ABCD$	Quadrilateral $EFGH$
a. Angle D	______ Side HE
b. Side AB	______ Angle H
c. Side DA	______ Side GH
d. Angle A	______ Side EF
e. Angle B	______ Angle G
f. Side CD	______ Side FG
g. Angle C	______ Angle E
h. Side BC	______ Angle F

Name: ________ Date: ________ Period: ________

Practice

3. An unlabeled rectangle is shown, along with several quadrilaterals that are labeled.



Place a check mark in the table to indicate which quadrilaterals are scaled copies of the unlabeled rectangle. For each scaled copy, write the scale factor used to create the scaled copy.

	A	B	C	D	E	F	G	H
Scaled copy								
Scale factor								

4. Provide the dimensions of two different rectangles that satisfy the given criteria.

- The two rectangles are scaled copies.
- Their combined perimeters are less than 100 in.
- Their combined areas are greater than 100 in^2.

UNIT 2

Introducing Proportional Relationships

Across the world and across cultures, people appreciate when things are fair. Proportional relationships represent situations when two things remain fair and balanced as they grow.

Essential Questions

- What does it mean for two things to be proportionally related? How can you tell?
- What are the different ways you can represent proportional relationships? How are the representations related?
- *(By the way, how can fractions impact the way you feel about something?)*

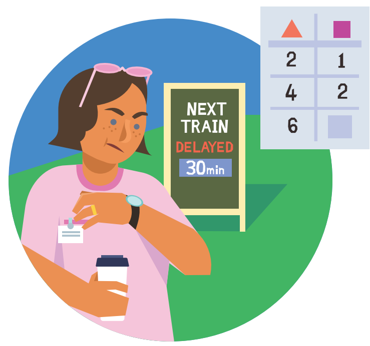
SUB-UNIT

1 Representing Proportional Relationships With Tables and Equations

Narrative: Explorer Ibn Battuta may have used proportionality to navigate different currencies and units.

You'll learn . . .

- the meaning of a proportional relationship and the constant of proportionality.
- how to represent proportional relationships using tables and equations.



SUB-UNIT

2 Representing Proportional Relationships With Graphs

Narrative: Using a graph is an intuitive way to analyze proportionality and understand more about a relationship.

You'll learn . . .

- what the graph of a proportional relationship looks like.
- the connections between different representations of proportional relationships.

$y = kx$

One-and-a-half workers can dig one-and-a-half holes in one-and-a-half hours. How many holes can one worker dig in three hours?

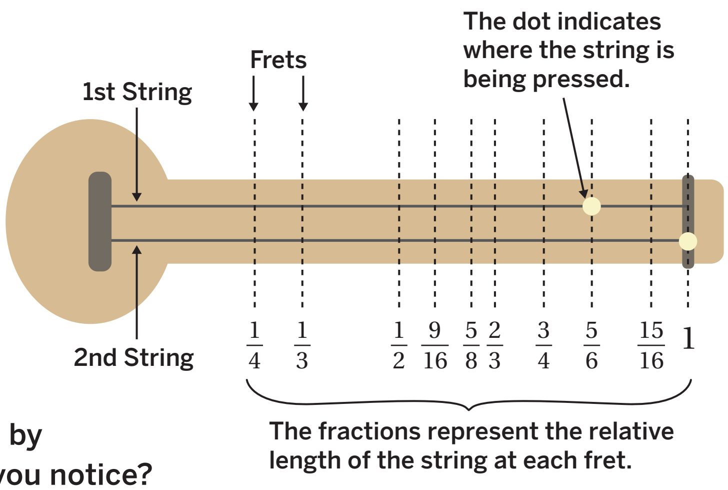
Unit 2 | Lesson 1 – Launch

Making Music

Let's explore ratios on a stringed instrument.

Warm-up Notice and Wonder

Throughout history, various civilizations have built and played two-stringed instruments. Examples include the erhu, originating in China, the morin khuur, from Mongolia, and the dutar out of Persia. You will be provided with a digital two-stringed instrument.



Explore the sounds you can make by playing the instrument. What do you notice? What do you wonder?

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 How Strings Make Music

You will watch a video about stringed instruments, music, and ratios. Before watching the video, complete the "Know" and the "Want to know" sections of the KWL chart on stringed instruments. After watching the video, complete the "Learned" section.

Note: KWL stands for What I *Know*, What I *Want to Know*, and What I *Learned*.

Stringed instruments		
Know	Want to know	Learned

Activity 2 Playing With Ratios

You will be provided with a digital two-stringed instrument to explore combinations of sounds. Eugenia Cheng, a mathematician and musician, has found that mathematical patterns help explain why certain musical combinations are used more often than others.

1. Move the fingers on the strings of the instrument. Complete the table for each pair of notes you play.

	Length (1st string)	Length (2nd string)	Ratio (2nd : 1st)	Simplified ratio	How would you describe the pair of notes? A B C
1st pair					A B C
2nd pair					A B C
3rd pair					A B C
4th pair					A B C
5th pair					A B C
6th pair					A B C
7th pair					A B C
8th pair					A B C

Name: ______________ Date: ______________ Period: ______________

Activity 2 Playing With Ratios (continued)

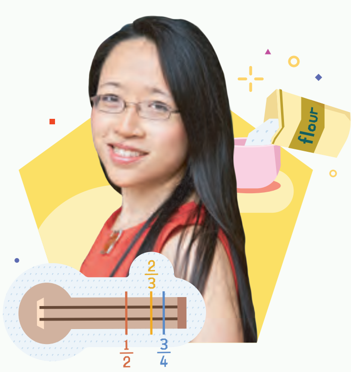
2. Look at the simplified ratios where you circled A. What do you notice?

3. Look at the simplified ratios where you circled C. What do you notice?

4. Compare your choices (A, B, and C) with a partner. What do you have in common? What is different?

5. Work together to write a conjecture that describes the simplified ratios for which you circled A compared to those for which you circled C.

Featured Mathematician



Eugenia Cheng

"If $a \times b = b \times a$, then ♫ × ♬ = ♬ × ♫." Or, at least that's how Eugenia Cheng might see it. As both an accomplished musician and mathematician, Cheng has studied and worked to explain the many ways math has influenced music. Her ability to make connections between different spheres of life has not stopped there, however. You can also find her explaining the connections between math and cooking, as well as how math can be used to study how people relate to one another more equitably.

STOP

Paul Crisanti

Unit 2 Introducing Proportional Relationships

The World in Proportion

In the early 1900s, Wuxin was an up-and-coming city in China. It budded with smokestacks, textile factories — even a new railway. And walking through its streets you might hear the musician Abing, playing a mournful song.

Abing had a hard life. Born in 1893, he was orphaned and sent to live at the local daoist temple. As an adult, he lost the use of his eyes and wandered the city, playing the erhu for money.

The erhu is a two-stringed fiddle originating from the nomadic peoples of Central Asia. When a bow is drawn across the strings, it makes a distinctive, sad sound.

But his fortunes changed in 1950. Musicologists traveled to Wuxin, and there discovered Abing. They recorded three of his songs, including “The Moon’s Reflection on the Erquan Spring.” This song — as well as Abing’s story — would become well known across China, influencing the country’s distinctive musical character.

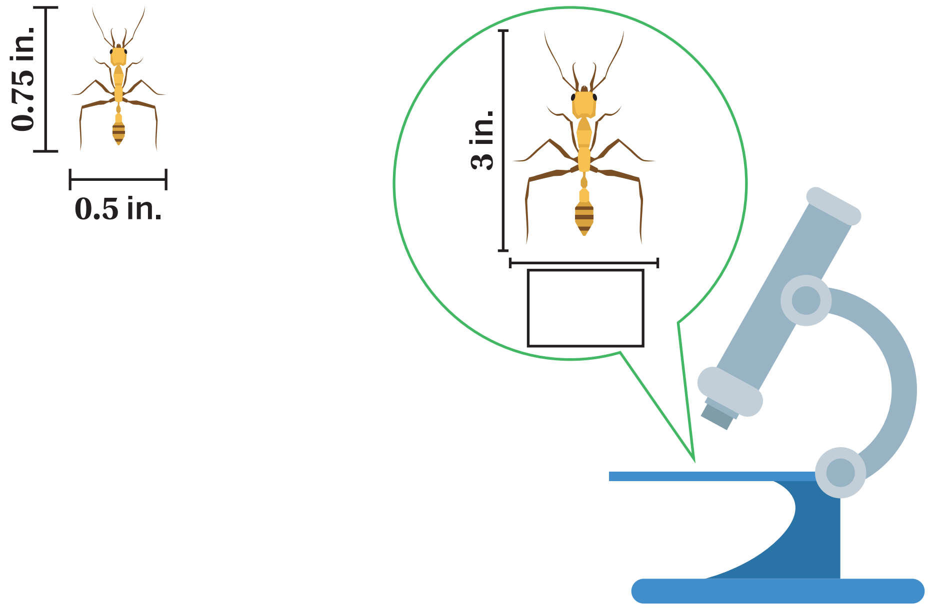
In this unit, you’ll look at some different ways people have exchanged cultural ideas, such as the music of the erhu. This sort of communication can be difficult at times, due to differences in language, customs, and even units of measurements. Math, and ratios in particular, gives us a way to line those measurements up. Once lined up, we open ways to exchange information, from weights and measures to the precise tuning of an erhu.

Welcome to Unit 2.

Name: ______ Date: ______ Period: ______

Practice

1. The bulldog ant, known for its impressive leaping ability, is more easily studied under a microscope. Determine the missing width of the magnified bulldog ant.



2. Match each object with an appropriate scale for a drawing that would fit on a regular sheet of paper. Not all of the scales in the list will be used.
Note: A regular sheet of paper measures 8.5 in. by 11 in.

Object	Scale
a A person	______ 1 in. : 1 ft
b A football field (120 yd by 53 yd)	______ 1 cm : 1 m
c The state of Washington (about 240 miles by 360 miles)	______ 1 : 1,000
d The floor plan of a house	______ 1 ft : 1 mile
e A rectangular farm (6 miles by 2 miles)	______ 1 : 100,000
	______ 1 mm : 1 km
	______ 1 : 10,000,000

Practice

Name: .... Date: .... Period: ....

3. Study the image. Which fish costs the least, per pound? Explain your thinking.

4. At snack time, two classmates establish a rate for trading veggie straws and pretzels. Help them complete the table below to determine all of the equivalent ratios for trading amounts of veggie straws and pretzels. Explain how you found how many pretzels should be traded for 1 veggie straw.

Veggie straws	Pretzels
1	
2	3
4	
10	

SUB-UNIT

1 Representing Proportional Relationships With Tables and Equations

Narrative Connections

Who was the original globetrotter?

Put yourself in the well-worn shoes of Abu Abdullah Muhammad ibn Battuta.

Born in the 14th century in Tangier in Morocco, Ibn Battuta was a scholar. When he was 20, he made a pilgrimage to Mecca. After reaching Mecca, he continued on. His travels took him through Persia (now modern-day Iran), Yemen, Somalia, Kenya, Tanzania, Turkey, India, Sri Lanka, and China.

All told, Ibn Battuta traveled nearly 75,000 miles over the course of 30 years. This is 60,000 miles more than Marco Polo!

Many travelers have wanted to see new places and meet new people. Like Ibn Battuta, those who journey abroad still need a way of understanding the customs of foreign lands.

Travelers might encounter differences in the local currencies and units of measurement. What is the exchange rate between a dirham and a guider? How many cubits are in a furlong? How many drams of flour do you need to make a six-talent casserole?

To answer these kinds of questions, we have to understand the relationship between these units and units we are already familiar with. One way to visualize and describe those relationships is with tables and equations.

Unit 2 | Lesson 2

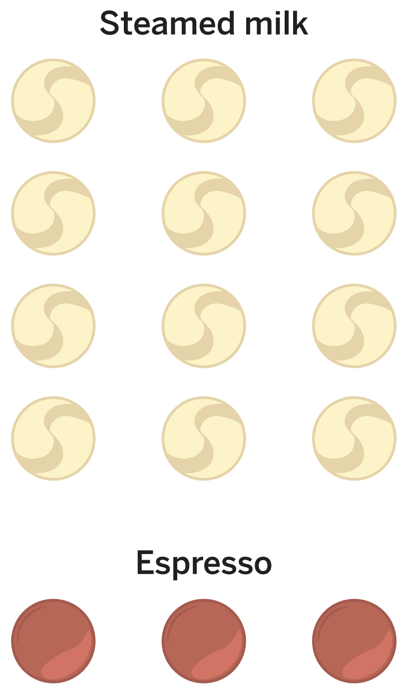
Introducing Proportional Relationships With Tables

Let's solve problems involving proportional relationships by using tables.

Warm-up The Right Ratio

Kiran, a barista, makes a latte with a particular ratio of espresso and steamed milk. His customers have told him that this ratio is a good balance — not too "milky" or too "espresso-y." Complete the table for the different-sized cups at his shop.

Size	Espresso (fluid oz)	Steamed milk (fluid oz)
Mini	0.5	
Small	1	
Medium	3	12
Large	5	
Extra large		24



Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Making Coffee for the Masses

Kiran earned enough money from his job to open up his own diner — a lifelong dream of his. Each morning, he needs to make a large amount of coffee for the daily expected customers.

Some days are busier than others, so Kiran changes how much coffee he makes. This table tracks how much of each ingredient to use each day.

Note: The diner is closed on Mondays.

Day	Coffee beans (oz)	Water (fluid oz)
Tuesday	40	50
Wednesday	16	20
Thursday	25	$31\frac{1}{4}$
Friday	48	60
Saturday	80	100
Sunday	60	75

1. Will the coffee taste the same each day? Explain your thinking.

2. How can you tell whether the water and coffee beans are in a ***proportional relationship?***

Activity 2 Pence and Wampum

Several Native American tribes used wampum — strings of purple and white shell beads — to represent value and importance long before British and Dutch colonists arrived. It became the first currency used in the Massachusetts Bay Colony in 1630.

A steady exchange rate from pence — the coins used by the British — to wampum beads was established: 4 pence were equal in value to 10 purple wampum beads.

Number of pence	Number of purple wampum beads
1	
4	10
6	
8	
14	

1. Complete the table. What is the unit rate of purple wampum beads to pence? Show or explain your thinking.

2. How can you tell whether the relationship between pence and purple wampum beads is proportional? Explain your thinking.

3. If you know the number of pence, what is the ***constant of proportionality*** that gives the number of purple wampum beads?
Explain what this value represents in the context of the two currencies.

Are you ready for more?

It currently costs more to make a penny — about 1.75 cents per penny — than it is worth. If we changed the value of a penny to match what it costs to make, about how many pennies would be equal to a quarter?

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You noticed that in a ***proportional relationship***, the values for one quantity are each multiplied by the same number to get the values for the other quantity.

This table shows the costs of different amounts of soybeans. Notice that each row in the table shows that the ratio of soybeans to total cost is $1 : 2$.

Soybeans (lb)	Cost ($)
1	2
2	4
8	16
$\frac{1}{2}$	1
$\frac{1}{4}$	0.50

You can multiply any value in the soybeans column by 2 to get the value in the cost column. This value, 2, is called a *unit rate* because 2 dollars are needed to buy 1 lb of soybeans.

We also say that 2 is the ***constant of proportionality*** that gives the total cost if you know the number of pounds of soybeans. This means that the ratio of total cost to pounds of soybeans remains *constant*, no matter how many pounds of soybeans there are.

Any proportional relationship will have a constant of proportionality, which has the same value as the unit rate that represents the relationship.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. Which ratios are equivalent? Select *all* that apply.

   A. $4 : 7$
   B. $8 : 15$
   C. $\frac{1}{7} : \frac{1}{4}$
   D. $2 : 3$
   E. $20 : 35$

2. When Han makes a strawberry smoothie, he mixes 2 cups of strawberries with 3 cups of milk. The table shows the number of cups of each ingredient that are needed to make different batches. Use the information in the table and bank provided here to complete each statement.

Word or number bank

cups of strawberries	4
$\frac{3}{2}$	cups of milk

Strawberries (cups)	Milk (cups)
2	3
8	12
1	$\frac{3}{2}$
10	15

   a. The table shows a proportional relationship between ________ and ________.

   b. The scale factor from the first row of the table to the second row is ________.

   c. The constant of proportionality that gives the number of cups of milk if you know the number of cups of strawberries is ________.

3. A certain shade of pink is created by adding 3 cups of red paint to 7 cups of white paint.

   a. How many cups of red paint should be added to 1 cup of white paint to create the same shade of pink? Complete the table.

   b. What is the constant of proportionality? Explain your thinking.

White paint (cups)	Red paint (cups)
1	
7	3

Name: ______ Date: ______ Period: ______

Practice

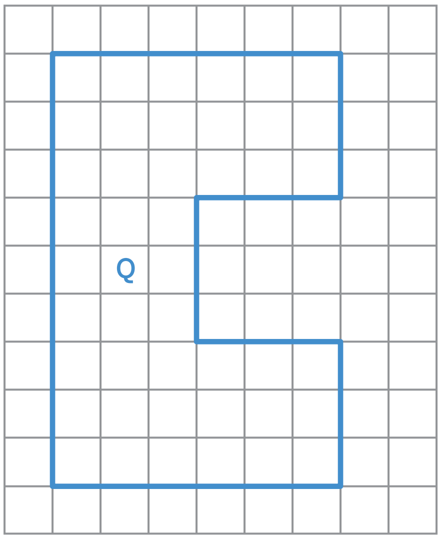
4. Solve each equation.

a $x + 2\frac{2}{3} = 6\frac{2}{3}$

b $\frac{1}{2}x = 12$

c $3\frac{3}{4}x = 2$

5. Noah drew a scaled copy of Polygon P and labeled it Polygon Q. Polygon Q is shown. If the area of Polygon P is 5 square units, what scale factor did Noah apply to Polygon P to create Polygon Q? Explain your thinking.



6. Jada and Lin are discussing the relationship between feet and yards. Jada says the constant of proportionality is 3, while Lin says the constant of proportionality is $\frac{1}{3}$.

a In what situation would Jada be correct?

b In what situation would Lin be correct?

Unit 2 | Lesson 3

More About the Constant of Proportionality

Let's solve more problems involving proportional relationships by using tables.

Warm-up Currency Exchange

Different countries around the world use different currencies (types of money). If you know the amount of money in one currency, you can use exchange rates to determine its equivalent amount in another currency. In 2020, the exchange rate from Chinese yuan (¥) to Philippine pesos (₱) was approximately 1 to 7.

1. Complete the table to determine the costs of several items in each currency.

	Chinese yuan (¥)	Philippine pesos (₱)
	1	7
Steamed dumplings	8	
Bottle of water	2	
Short taxi ride		35
Milk tea		119

2. Choose one item for which you found the cost. How did you find the cost? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: ____________ Period: ____________

Activity 1 Using Tables to Convert Currency

There is a proportional relationship between U.S. dollars (USD, \$) and British pounds (GBP, £). There are two ways you can think about this proportional relationship. The table shows the number of British pounds that are equivalent to 4 U.S. dollars.
Note: Exchange rates are from 2020.

1. If you know the cost of an item in USD, you can calculate the cost in GBP.

a. Complete the table.

b. What is the constant of proportionality that gives the cost in GBP if you know the cost in USD? Express this value as a fraction.

USD (\$)	GBP (£)
4.00	3.00
2.00	
1.00	
100.20	

2. If you know the cost of an item in GBP, you can calculate the cost in USD.

a. Complete the table.

b. If you know the cost in GBP, what is the constant of proportionality that gives the cost in USD? Express this value as a fraction.

GBP (£)	USD (\$)
3.00	4.00
27.00	
1.50	
90.75	

3. How are the two constants of proportionality related to each other?

4. Complete each sentence.

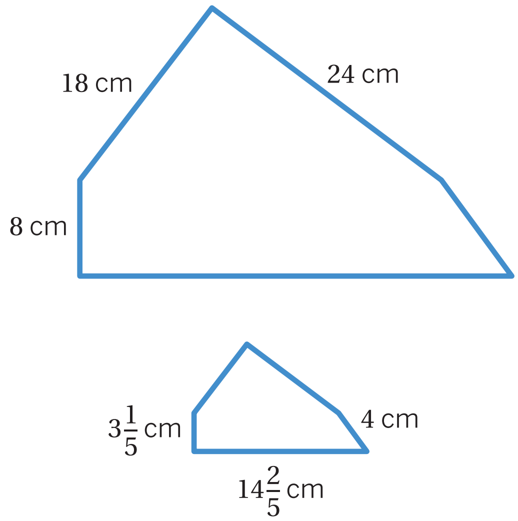
a. To convert from USD to GBP, I can . . .

b. To convert from GBP to USD, I can . . .

Activity 2 Scaled Figures, Revisited

These two pentagons shown are scaled figures. Determine the missing corresponding side lengths and complete the table.

Side length of large figure (cm)	Corresponding side length of small figure (cm)
8	$3\frac{1}{5}$
18	
24	
	4
	$14\frac{2}{5}$



The figures may not be drawn to scale.

1. What is the scale factor that takes the large figure to the small figure?

2. If you know the corresponding side length of the large figure, what is the constant of proportionality that gives the side length of the small figure?

3. Explain how the scale factor and the constant of proportionality are related.

4. What is the constant of proportionality that gives the side length of the large figure if you know the corresponding side length of the small figure?

5. Complete each sentence:

   a. To calculate the side lengths of the small figure when I know the corresponding side lengths of the large figure, I can . . .

   b. To calculate the side lengths of the large figure when I know the corresponding side lengths of the small figure, I can . . .

Collect and Display: Your teacher will walk around and collect language you use to describe the scale factors. This language will be added to a class display for your reference.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that every proportional relationship has two constants of proportionality, depending on which quantity you know and which one you want to find.

The tables show the relationship between the cost and weight of soybeans.

Weight (lb)	Cost ($)
$\frac{1}{2}$	1.00
1	2.00
$13\frac{3}{4}$	27.50

Constant of proportionality: 2

- To determine the cost from a known weight, you can multiply the number of pounds by 2.
- This means the cost is proportional to the weight, with a constant of proportionality of 2.

Cost ($)	Weight (lb)
1.00	$\frac{1}{2}$
2.00	1
27.50	$13\frac{3}{4}$

Constant of proportionality: $\frac{1}{2}$

- To determine the weight from a given cost, you can divide the cost by 2 or multiply by $\frac{1}{2}$.
- This means the weight is proportional to the cost, with a constant of proportionality of $\frac{1}{2}$.

Notice that 2 and $\frac{1}{2}$ are reciprocals. When two quantities are in a proportional relationship, there are two constants of proportionality, which are *reciprocals* of each other.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Triangle A and Triangle B are scaled figures. Use the table to compare lengths of corresponding sides.

Side length of Triangle A (in.)	Corresponding side length of Triangle B (in.)
1	6
$\frac{1}{2}$	
	8

a Complete the table to determine the missing side lengths of each triangle.

b What is the constant of proportionality that gives the side length of Triangle A given its corresponding side length of Triangle B?

c What is the constant of proportionality that gives the side length of Triangle B given its corresponding side length of Triangle A?

2. Consider 1 km is equal to 1,000 m.

a Complete each table to show equivalent distances. Then complete the sentences to interpret the constant of proportionality for each table.

Distance (kilometers)	Equivalent distance (meters)
1	1,000
5	
20	
0.3	

Distance (meters)	Equivalent distance (kilometers)
1,000	1
250	
12	
1	

The constant of proportionality tells me that:

The constant of proportionality tells me that:

b What is the relationship between the two constants of proportionality?

Name: ______ Date: ______ Period: ______

Practice

3. Jada and Lin are comparing inches and feet. Jada says the constant of proportionality is 12. Lin says it is $\frac{1}{12}$. Do you agree with either of them? Explain your thinking.

4. Solve each equation. Show your thinking.

a. $\frac{1}{2} + x = 2$

b. $\frac{2}{3}y = 6$

c. $3 = \frac{1}{4}b$

5. Which of the following scales are equivalent to the scale 1 cm to 5 km? Select *all* that apply.

A. $\frac{1}{2}$ cm to $2\frac{1}{2}$ km

B. 1 mm to 150 km

C. 5 cm to 1 km

D. 5 mm to 2.5 km

E. 1 mm to 500 m

6. A bakery charges $\$4.80$ for 16 oz of bubble tea.

a. Complete the table so that it shows a proportional relationship between ounces of tea and cost.

Tea (oz)	Cost ($)
16	4.80
20	
	7.20

b. Complete the table so that it shows a relationship that is *not* proportional between ounces of tea and cost.

Tea (oz)	Cost ($)
16	4.80
20	
	7.20

Unit 2 | Lesson 4

Comparing Relationships With Tables

Let's explore how proportional relationships are different from other relationships.

Warm-up Adjusting a Recipe

***Agua fresca* is a traditional juice drink served throughout Mexico. One agua fresca recipe lists the following ingredients:**

- **4 cups of chopped fruit (e.g., watermelon, papaya, or tamarind)**
- **3 cups of water**
- **$\frac{1}{3}$ cup of sugar**

SALMONNEGRO-STOCK/Shutterstock.com

Determine the amounts of the ingredients needed for the following four new versions of this agua fresca recipe.

1. A version that would make more agua fresca and taste the same as the original recipe.

2. A version that would make less agua fresca and taste the same as the original recipe.

3. A version that would have a stronger fruit taste than the original recipe.

4. A version that would have a weaker fruit taste than the original recipe.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Visiting the State Park

The entrance to a state park costs $6.00 per vehicle plus $2.00 per person in the vehicle.

1. How much does it cost for a vehicle with 2 people to enter the park? 4 people? 5 people? Record your responses in the table.

2. Each person in the vehicle splits the entrance cost equally. For each row in the table, how much does each person pay?

Number of people in the vehicle	Total entrance cost ($)
2	
4	
5	

3. A van carrying 15 people arrived at the park.

   a. What is the total entrance cost for the van? Explain your thinking.

   b. What is the cost per person? Explain your thinking.

4. Does the relationship between the number of people and the total entrance cost represent a proportional relationship? Explain your thinking.

Are you ready for more?

1. What equation could be used to calculate the total entrance cost c for a vehicle with *any* number of people p?

2. What expression could be used to calculate the total entrance cost per person?

Activity 2 Measuring Snow

Han and his cousin live on opposite sides of the Niagara River. Han lives in Fort Erie, Ontario, Canada, while his cousin lives in Buffalo, New York, USA. During a snowstorm, Han and his cousin decided to track the snowfall at each of their homes to see if they were getting the same amount of snow.

Han's house:

Time passed (h)	Snowfall (cm)
0.5	2
1	4
2	13
4	24

His cousin's house:

Time passed (h)	Snowfall (cm)
0.5	3
1	6
3	18
4	24

1. Based on the information collected, did the snow appear to fall at a constant rate at Han's house (is there a constant of proportionality)? Explain your thinking.

2. Based on the information collected, did the snow appear to fall at a constant rate at his cousin's house (is there a constant of proportionality)? Explain your thinking.

3. Does either table show a nonproportional relationship? Explain your thinking.

4. Does either table show a proportional relationship? Explain your thinking.

Reflect: How did you display confidence in your abilities today? How did self-confidence affect your optimism?

Name: .... Date: .... Period: ....

Activity 3 Card Sort: Tables of Proportional Relationships

You will be given a set of cards.

1. Sort the cards into two categories: *possible proportional relationships* and *nonproportional relationships.* List the cards in the appropriate category in the table.

Possible proportional relationships	Nonproportional relationships

2. For each card that could represent a proportional relationship, determine the constant of proportionality. Explain its meaning in context.

STOP

Summary

In today's lesson . . .

You saw examples of ***nonproportional relationships***, where the ratios of the two quantities in a table of values are not equivalent.

These tables show the cost for soybeans at two different stores.

Store A

Weight (lb)	Cost ($)	Cost per pound ($/lb)
1	2	2
2	4	2
5	10	2
10	20	2

- At Store A, the cost is $2 per pound regardless of the number of pounds of soybeans purchased.
- Based on the table for Store A, there could be a proportional relationship between the pounds of soybeans and the cost with a constant of proportionality of 2.

Store B

Weight (lb)	Cost ($)	Cost per pound ($/lb)
1	2	2
2	3.50	1.75
5	8	1.60
10	15	1.50

- At Store B, the cost per pound changes. There is no constant of proportionality, so this is a nonproportional relationship.

Reflect:

Practice

Name: ______________________ Date: __________ Period: __________

1. Each table below shows a relationship between two quantities. Determine whether each table shows a proportional or nonproportional relationship. If the relationship could be proportional, determine the constant of proportionality.

a. Distance between a sound and the listener

Distance (ft)	Sound level (dB)
5	85
10	79
20	73
40	67

b. The cost of a fountain drink at Sandwich Hut

Drink (oz)	Cost ($)
16	1.49
20	1.59
24	1.69

2. There's an old fable called *The Tortoise and the Hare*, in which a rabbit (hare) and a turtle (tortoise) are in a race. The two tables show the distances each traveled after certain times, based on the events of the story. For each character, determine whether there is a proportional relationship between the distance traveled and time? If so, determine the constant of proportionality.

Turtle's run:

Distance (m)	Time (minutes)
108	2
405	7.5
540	10
1,768.5	32.75

Rabbit's run:

Distance (m)	Time (minutes)
800	1
900	5
1,1075.5	20
1,524	32.5

Practice

Name: ______________ Date: ______________ Period: ______________

3. A taxi company charges $\$1.00$ for the first $\frac{1}{10}$ mile and then $\$0.10$ for each additional $\frac{1}{10}$ mile.

a Is there a proportional relationship between the distance traveled and the total cost? Explain your thinking.

Distance (miles)	Cost ($)
$\frac{1}{10}$	1.00
$\frac{2}{10}$	1.10
$\frac{9}{10}$	
$3\frac{1}{10}$	

b Complete the table with the missing information to verify your response from part a.

4. Kiran and Mai are standing at one corner of a rectangular field of grass looking at the opposite corner along the diagonal. Kiran says that if the field were twice as long and twice as wide, then the diagonal will be twice as long. Mai says the diagonal will be more than twice as long because the diagonal is longer than the side lengths. Do you agree with either of them? Explain your thinking.

5. If $y = \frac{2}{3}x$, complete the table.

x	y
12	
	16

Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 5

Proportional Relationships and Equations

Let's write equations describing proportional relationships.

Warm-up Feeding a Crowd

The national dish of El Salvador is a stuffed griddle cake called *pupusa*. It is traditionally served with a pickled cabbage side dish called *curtido*, which is similar to kimchi or sauerkraut.

A recipe indicates that 2 cups of curtido will be enough to serve with 6 pupusas. Complete the table and these problems.

Megan Betteridge/Shutterstock.com

1. How many pupusas could be served with 1 cup of curtido? Explain your thinking.

2. How many pupusas could be served with 3 cups of curtido? 12 cups? 43 cups? Explain your thinking.

3. How many pupusas can be served with x cups of curtido?

Cups of curtido	Number of pupusas
1	
2	6
3	
12	
$\frac{3}{5}$	
x	

Log in to Amplify Math to complete this lesson online.

Activity 1 Teaspoons and Tablespoons

When baking, it could be helpful to know that 3 teaspoons (tsp) is equivalent to 1 tablespoon (tbsp).

1. How many tablespoons are equivalent to 1 tsp?

2. Let y represent the number of tablespoons and let x represent the number of teaspoons. Write an equation that gives the number of tablespoons y given the number of teaspoons x.

3. You will now use either a table or the equation you wrote in Problem 2 to answer the following questions. With your partner, determine who will use a table and who will use the equation. Record your work in the space provided at the bottom of this page.

   How many tablespoons are equivalent to 1 tsp? $1\frac{1}{2}$ tsp? $\frac{1}{2}$ tsp? 5 tsp?

Equation:

Table:

Number of tsp	Number of tbsp
$\frac{1}{2}$	
1	
$1\frac{1}{2}$	
5	

Name: ______ Date: ______ Period: ______

Activity 2 Baking Bread

A bakery uses 8 tbsp of honey for every 10 cups of flour to make bread dough. Some days there are bigger batches and some days there are smaller batches. However, the bakery always uses the same ratio of honey and flour. Use the table, if needed, to complete these problems.

1. Let f represent the number of cups of flour needed for h tablespoons of honey. Write an equation that determines the number of cups of flour when the amount of honey is given. Explain your thinking.

Honey (tbsp)	Flour (cups)

2. Determine the number of cups of flour needed for the following amounts of honey.

a. 15 tbsp

b. 17 tbsp

c. $7\frac{1}{2}$ tbsp

Are you ready for more?

How many tablespoons of honey are needed to make a batch of dough with 8 cups of flour? Explain your thinking.

STOP

Summary

In today's lesson . . .

You saw that proportional relationships can be represented by using the equation $y = kx$, where k is the constant of proportionality.

For example, the table shows the proportional relationship between the cost and the number of pounds of soybeans at a certain store.

The cost of the soybeans is proportional to the weight, in pounds, with a constant of proportionality of 2.

If c represents the cost and p represents the weight, in pounds, of soybeans, then you can represent the proportional relationship with the equation $c = 2p$.

Weight (lb)	Cost ($)
$\frac{1}{2}$	1.00
1	2.00
2	4.00
p	$2p$

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. The ceilings in many basements are made up of rectangular tiles. Suppose that for one basement, each square meter of ceiling requires 10.75 tiles.

Area of ceiling (m^2)	Number of tiles
1	
10	
	53.75
x	

a Complete the table with the missing values.

b Write an equation to represent the number of tiles y that are needed to tile a ceiling with an area of x m^2.

2. Each of the following tables represents a proportional relationship. For each table, determine the constant of proportionality and write an equation that represents the relationship.

a

s	P
2	8
3	12
5	20
10	40

Constant of proportionality: ________

Equation: $P =$ ________

b

d	C
2	6.28
3	9.42
5	15.7
10	31.4

Constant of proportionality: ________

Equation: $C =$ ________

3. In October of 2019, Mai received 325 Norwegian kroner (kr) in exchange for $\$50$ (Australian dollars). Use the table, if needed, to complete these problems.

Australian dollars (\$)	Norwegian kroner (kr)

a How many kroner would Mai have received in exchange for $\$1$?

b Write an equation modeling the amount of kroner k received in exchange for a dollars in October of 2019.

c Determine the number of kroner Mai would receive in exchange for $\$120$.

Practice

Name: ____________ Date: ____________ Period: ____________

4. A map of Colorado shows a scale of 1 in. to 20 miles, or 1 to $1{,}267{,}200$. Given that 1 mile is equivalent to $5{,}280$ ft, are these two ways of reporting the scale the same? Explain your thinking.

5. A bicycle travels 21 m in 3 seconds.

a Complete the table of values.

b What is the constant of proportionality? What does it represent in context?

Time (s)	Distance (m)
3	21
$1\frac{1}{2}$	
	$6\frac{3}{10}$

6. An ant crawled the length of a classroom floor in 50 seconds at a constant rate. If the length of the classroom floor was 20 ft, what was the ant's speed, in feet per second? Show or explain your thinking.

Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 6

Speed and Equations

Let's write equations describing proportional relationships involving speed.

Warm-up Number Talk

Mentally evaluate each expression. Be prepared to explain your thinking.

1. $\frac{2}{3} \cdot \frac{1}{2}$

2. $\frac{4}{3} \cdot \frac{1}{4}$

3. $\frac{3}{2} \div \frac{1}{2}$

4. $\frac{9}{6} \div \frac{1}{2}$

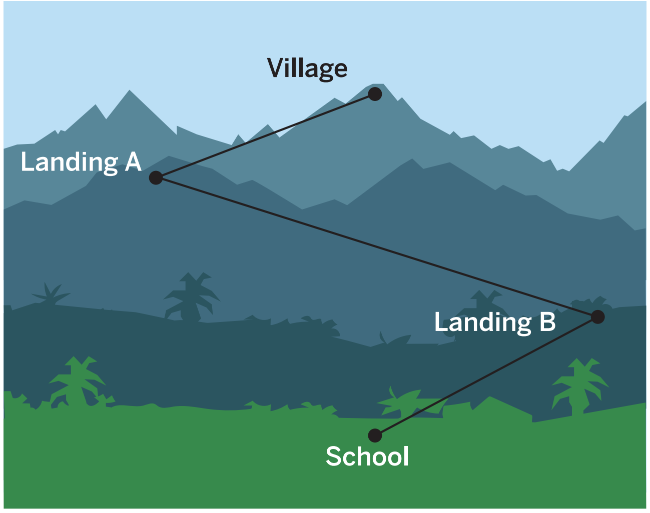
Log in to Amplify Math to complete this lesson online.

Activity 1 Zipping Along

In a remote village in the mountains of Colombia, the only way to cross the river valley is by zipline.

Some students take a zipline to get to school. The table shows the time it takes a student on the zipline to travel from one landing to another. The traveling speed on all the ziplines is constant.

Complete the table as you solve each problem. Be prepared to explain your thinking.



Segment	Time	Distance (m)	Speed (m per minute)
Village to Landing A	1 minute	600	
Landing A to Landing B	1 minute 40 seconds		
Landing B to school		500	

1. At what speed do the students travel on the zipline?

2. What is the distance between Landing A and Landing B?

3. How many seconds does it take a student to travel from Landing B to school?

4. Priya says the constant of proportionality that gives the distance, in meters, if you know the time, in minutes, is 600. Shawn says the constant of proportionality is $\frac{1}{600}$. Do you agree with either of them? Explain your thinking.

Name: .... Date: .... Period: ....

Activity 2 Paddling to School

In Nepal, some students paddle a kayak to get to school. Suppose it takes one student $\frac{1}{4}$ hour to paddle $\frac{1}{2}$ miles.

1. How fast does the student paddle in miles per hour?

2. How far would the student paddle in t hours at this speed? Explain your thinking.

3. If d represents the distance that the student paddles at this speed for t hours, write an equation that gives the value of d if you know the value of t.

4. How far would the student paddle in 3 hours at this speed? In $3\frac{1}{2}$ hours? Show or explain your thinking.

STOP

Summary

In today's lesson . . .

You noticed that when a person, animal, or object is traveling at a constant speed, there is a proportional relationship between the time traveled and the distance traveled.

The table shows the distance, in meters, traveled for certain periods of time, in seconds. The table also shows the speed, in meters per second.

Time (seconds)	Distance traveled (m)	Speed (m per second)
1	$\frac{3}{2}$	$\frac{3}{2}$
$\frac{2}{3}$	1	$\frac{3}{2}$
2	3	$\frac{3}{2}$
t	$\frac{3}{2}t$	$\frac{3}{2}$

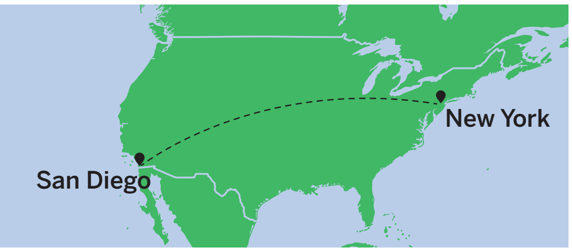
- The last row in the table indicates that, if you know the amount of time, t, you can always multiply it by $\frac{3}{2}$ to determine the distance d traveled.
- The equation $d = \frac{3}{2}t$ represents this relationship more succinctly.
- If you know the amount of time traveled, the speed, or rate of travel, is the constant of proportionality in the proportional relationship that gives the distance traveled.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. On its way from New York to San Diego, a plane flew at a constant speed over Pittsburgh, Saint Louis, Albuquerque, and Phoenix.



a. The table shows the flight time and distance traveled for each segment of the flight. Complete the table.

Segment	Time	Distance (miles)	Speed (mph)
Pittsburgh to Saint Louis	1 hour	550	
Saint Louis to Albuquerque	1 hour 42 minutes		
Albuquerque to Phoenix		330	

b. Let t represent the time in hours and d represent the distance in miles. Write an equation that represents the distance traveled for t hours.

2. A car is traveling a highway at a constant speed. The equation that represents the distance d, in miles, traveled by this this car for t hours is $d = 65t$

a. What does the value 65 represent in this situation?

b. At this rate, how many miles will the car travel in 1.5 hours?

Practice

Name: .... Date: .... Period: ....

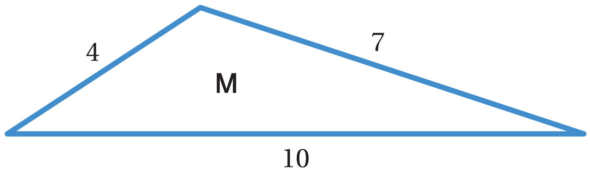
3. A train travels at a constant speed between Springfield and Chicago. The train travels $100\frac{1}{2}$ miles in $\frac{3}{4}$ hours.

a How far does the train travel in one hour?

b How far will the train travel in t hours, at this same speed?

c If d represents the distance that the train travels at this speed for t hours, write an equation relating t and d.

4. Triangle Z is a scaled copy of Triangle M. Which sets of values could represent the side lengths of Triangle Z? Select *all* that apply.



A. $8, 11, 14$

B. $10,\ 17.5,\ 25$

C. $6, 9, 11$

D. $6,\ 10.5,\ 15$

E. $8,\ 14,\ 20$

5. Lin has already read 36 pages of the 200 pages of her book in 3 days. Her teacher has asked her when she will be able to return the book to the class library. Which is better for Lin to know:
How many pages does she read per day?
How many days does it take her to read per page?
Explain your thinking.

Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 7

Two Equations for Each Relationship

Let's investigate the equations that represent proportional relationships.

Warm-up Meters and Centimeters

There are 100 centimeters in 1 meters.

1. Complete each table.

Number of meters	Number of centimeters
1	
0.94	
1.67	
57.24	
x	

Number of centimeters	Number of meters
100	
250	
78.2	
123.9	
y	

2. Complete the following statements.

   a. The constant of proportionality for the first table is ______.

   b. The constant of proportionality for the second table is ______.

Discussion Support: During the discussion, your teacher will display a partially-complete sentence. Use the math language you are learning to complete the sentence.

Log in to Amplify Math to complete this lesson online.

Activity 1 Taking a Road Trip

Clare, Andre, Shawn, and Kiran are taking a road trip to go camping. As they pull over to a rest area, they wonder if they will have enough gas to make it to their destination. They record that it took 3 gallons of gasoline to travel the first 80 miles. The four friends each write an equation relating the number of gallons of gasoline and the number of miles. They let g represent the number of gallons of gasoline used for m miles.

You will be given a card with an equation on it.

1. Consider whether the equation on your card represents the situation. Discuss your ideas with the other members of your group. Explain your thinking.

Clare's equation: **Explanation:**	**Andre's equation:** **Explanation:**
Shawn's equation: **Explanation**	**Kiran's equation:** **Explanation:**

2. There are 5 gallons of gasoline left in the tank and they have 130 miles left to travel. Will they make it? Explain your thinking.

Name: ________ Date: ________ Period: ________

Activity 2 Trading Gold for Salt

In earlier civilizations, salt was incredibly valuable as a way to preserve food (in the absence of refrigerators and ice). In Africa, a substantial trade economy developed between the Ghana Empire and North African states, who often traded salt for gold.

Imagine yourself in a market in Koumbi Saleh, the capital of the Ghana Empire, in the year 1000 CE. As you walk around, you witness a transaction where one trader receives $\frac{2}{5}$ oz of gold in exchange for $\frac{3}{2}$ oz of salt.

1. How many ounces of salt would be needed to trade for one ounce of gold?

2. Complete the table to show how many ounces of salt can be traded for different amounts of gold.

Gold (oz)	Salt (oz)
1	
7	
30	

3. What is the constant of proportionality? What does it represent?

4. If the columns are switched in the table, what is the constant of proportionality? Explain your thinking.

Activity 2 Trading Gold for Salt (continued)

5. Let g represent the number of ounces of gold and let s represent the number of ounces of salt. Write *two* equations that represent the relationship between g and s.

6. Here is a list of the items some traders brought to trade. Calculate the value of what they received after the trade.

a. Trader 1 brought 5.5 oz of gold and received . . .

b. Trader 2 brought 2 oz of salt and received . . .

c. Trader 3 brought 0.5 oz of salt and received . . .

d. Trader 4 brought 100 oz of gold and received . . .

Are you ready for more?

In 2020, one oz of gold was worth about $\$1{,}900$. If a teaspoon of salt weighs about $\frac{1}{5}$ oz, how much would a teaspoon of salt be worth, in today's dollars, at the Koumbi Saleh market?

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You saw that when two quantities x and y are in a proportional relationship, you can write the equation $y = kx$ and say, "y is proportional to x." In this case, the number k is the corresponding constant of proportionality.

You can also write the equation $x = \frac{1}{k} y$ and say, "x is proportional to y." In this case, the number $\frac{1}{k}$ is the corresponding *constant of proportionality*. Each equation can be useful, depending on the information you have and the quantity you are trying to calculate.

For example, if one pound of soybeans costs $\$2.00$, you can say . . .

- The cost c is proportional to the weight w. The equation $c = 2w$ represents this situation.
- The weight w is proportional to the cost c. The equation $w = \frac{1}{2} c$ represents this situation. This shows you can purchase $\frac{1}{2}$ of a pound of soybeans for $\$1$.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. The table represents the relationship between a set of measurements in inches and the same measurements converted to feet.

   a. Complete the table.

   b. Let x represent the number of inches and let y represent the number of feet. Write an equation that converts inches to feet.

Number of inches	Number of feet
12	1
420	
	$\frac{1}{2}$
75	
1	
x	

2. A concrete building block weighs 28 lb. If b represents the number of concrete blocks and w represents the total weight, write two equations relating b and w.

   $w =$ .... $b =$ ....

3. A store sells rope by the meter. The equation $p = 0.8L$ represents the price p, in dollars, of a nylon rope that is L m long.

   a. What is the cost per meter of the nylon rope?

   b. How long is a piece of nylon rope that costs $\$1.00$?

Practice

Name: ______ Date: ______ Period: ______

4. The table represents a proportional relationship. Determine the constant of proportionality and write an equation to represent the relationship.

a	y
$2\frac{2}{3}$	$\frac{2}{3}$
$5\frac{3}{5}$	$1\frac{2}{5}$
$\frac{40}{3}$	$\frac{10}{3}$
$\frac{6}{5}$	$\frac{3}{10}$

a **Constant of proportionality:**

b **Equation:**

5. On a scale drawing of a bacteria, 1 cm represents 1 micron. Select *all* the statements that express the same scale. **Note:** 1 micron is $\frac{1}{10000}$ cm.

A. 1 cm on the drawing represents $\frac{1}{1000}$ mm on the bacteria.

B. 1 mm on the drawing represents $10{,}000$ cm on the bacteria.

C. 1 micron on the bacteria is represented by 10 mm on the drawing.

D. 10 microns on the bacteria is represented by 0.1 m on the drawing.

6. Solve each equation. Show your thinking:

a $\frac{2}{3}x = 18$

b $\frac{3}{2}x = 18$

c $x + \frac{2}{3} = \frac{8}{3}$

d $x + \frac{3}{2} = \frac{8}{3}$

Unit 2 | Lesson 8

Using Equations to Solve Problems

Let's use equations to solve problems involving proportional relationships.

Warm-up Decimal Placement

Place the decimal point in the appropriate location for each quotient.

1. $42.6 \div 7 = 608571$

2. $426 \div 7 = 608571$

3. $426 \div 70 = 608571$

4. $4.26 \div 7 = 608571$

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Concert Tickets

A performer expects to sell $5{,}000$ tickets for an upcoming concert. They will make a total of $\$311{,}000$ in sales from these tickets.

1. If all tickets have the same price, what is the price of one ticket?

2. How much will the performer make when $7{,}000$ tickets are sold?

3. How much will the performer make if they sell . . .

   a. $10{,}000$ tickets?

   b. $50{,}000$ tickets?

   c. $120{,}000$ tickets?

   d. a million tickets?

   e. x tickets?

4. If the performer makes $\$404{,}300$, how many tickets were sold? Let x represent the number of tickets sold. Write and solve an equation.

5. How many tickets will they have to sell to make $\$5{,}000{,}000$? Let x represent the number of tickets sold. Write and solve an equation.

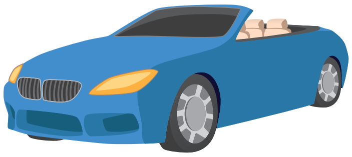
Activity 2 Miles per Gallon

You and your family are going to travel 600 miles for a trip to visit a friend who recently moved to a new city. At the rental car agency, your family allows you to choose from three possible vehicles.

Sedan

33 mpg

Convertible



27 mpg

SUV

23 mpg

Select a car and then complete these problems.

1. Which vehicle did you choose? Explain your thinking.

Let m represent the number of miles and let g represent the number of gallons of gas.

2. Write an equation that gives the number of miles your car will travel with a certain number of gallons of gas.

3. Write an equation that gives the number of gallons of gas needed to travel a certain number of miles.

4. If gas currently costs $2.59 per gallon, how much will the entire trip cost?

STOP

Reflect: How did you make sure that you achieved the personal and academic goals you had set for yourself?

Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You recalled that if there is a proportional relationship between two quantities, their relationship can be represented by an equation of the form $y = kx$. Sometimes writing an equation is the most efficient way to solve a problem.

For example, the highest mountain peak in North America, Denali, is $20{,}310$ ft above sea level. How many miles is that?

There are $5{,}280$ ft in 1 mile.

Let f represent a distance measured in feet and m represent the same distance measured in miles.

$$f = 5280m$$

If Denali's height is $20{,}310$ ft, then

$$20310 = 5280m$$
$$20310 \div 5280 = 5280m \div 5280$$
$$m \approx 3.85$$

So, m is approximately 3.85 miles. This means that Denali is approximately 3.85 miles above sea level.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A skateboard travels down a sidewalk at a constant speed as described by the equation $d = 9\frac{1}{3}t$ where d represents the distance in miles that the skateboard travels at this speed in t hours.

   a. What does the value $9\frac{1}{3}$ represent?

   b. Use the equation to calculate the number of miles the skateboard travels in $\frac{1}{4}$ hours.

   c. How long does it take the skateboarder to travel $4\frac{2}{3}$ miles?

2. Elena has bottles of water in which each bottle holds 17 fluid ounces.

   a. Write an equation that gives the volume of water w, in fluid ounces, for any number of bottles b.

   b. Use your equation from Part a to calculate the volume of water in 51 bottles.

   c. How many bottles are needed to hold 51 fluid ounces of water?

3. There are 2.54 cm in 1 in. Let y represent a distance measured in centimeters and x represent the same distance measured in inches. Write and solve an equation that shows the number of centimeters that are equivalent to 58 in.

Name: .... Date: .... Period: ....

Practice

4. Each table represents a proportional relationship. Complete each table by finding the missing values. Draw a circle around each constant of proportionality.

a

x	y
2	10
	15
7	
1	

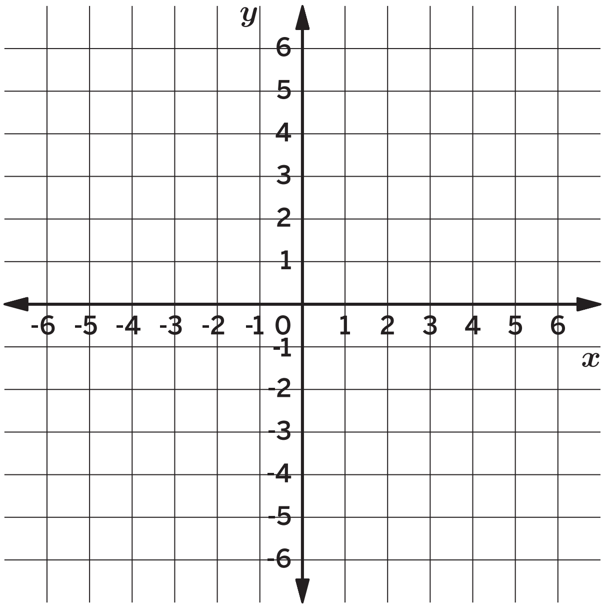
b

a	b
12	3
20	
	10
1	

c

m	n
5	3
10	
	18
1	

5. A rectangle has vertices located at (2, 2), (2, –3), (–4, –3), and (–4, 2). Calculate the perimeter of the rectangle. Use the coordinate plane to help you.



6. Complete each ratio table so that each row shows a ratio that is equivalent to the top row of the table.

a

1	3
3	
6	
	30

b

2	1
4	
5	
	17

c

4	3
1	
2	
	6

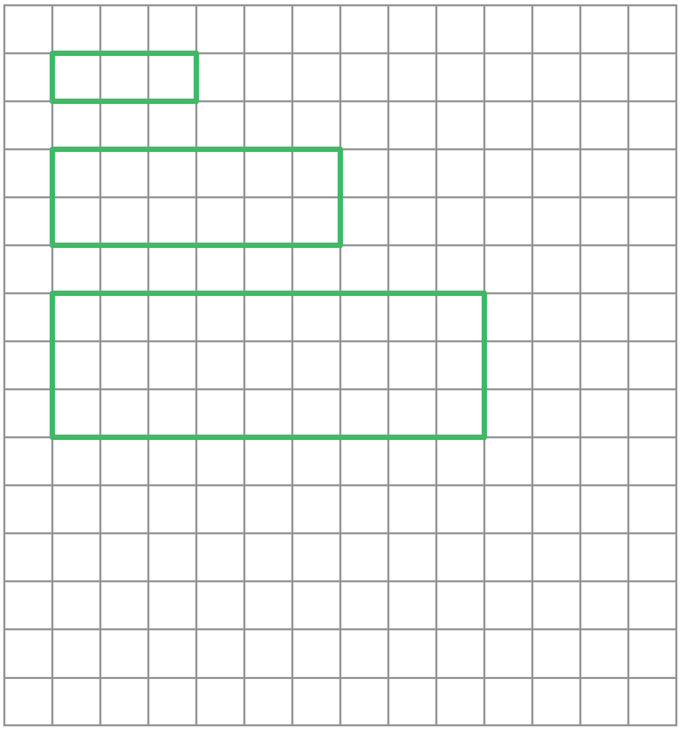
Unit 2 | Lesson 9

Comparing Relationships With Equations

Let's develop methods for deciding whether a relationship is proportional.

Warm-up Patterns With Rectangles

Consider the rectangles shown.



1. What pattern(s) do you notice?

2. If the pattern continues, sketch the next rectangle in the pattern.

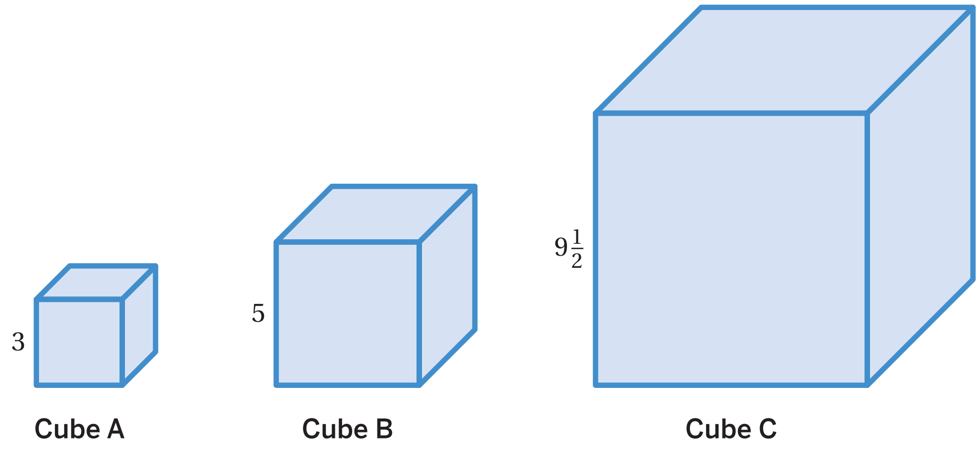
3. Do you see any quantities that are in a proportional relationship? If so, what is the constant of proportionality? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Total Edge Length, Surface Area, and Volume

Three cubes with different side lengths are shown.



1. How long is the total edge length of each cube?

Cube	Side length	Total edge length
A	3	
B	5	
C	$9\frac{1}{2}$	
Any cube with side length s	s	

2. What is the surface area of each cube?

Cube	Side length	Surface area
A	3	
B	5	
C	$9\frac{1}{2}$	
Any cube with side length s	s	

Activity 1 Total Edge Length, Surface Area, and Volume (continued)

3. What is the volume of each cube?

Cube	Side length	Volume
A	3	
B	5	
C	$9\frac{1}{2}$	
Any cube with side length s	s	

4. Which of these relationships is proportional? Explain your thinking.

5. Let s represent the side length of a cube. Write three equations for the cube that give:

a The total edge length E.

b The total surface area A.

c The volume V.

Name: ______________________ Date: __________ Period: __________

Activity 2 All Kinds of Equations

Consider these equations.

$y = 4 + x$ $\quad y = 4x$ $\quad y = \frac{4}{x}$ $\quad y = \frac{x}{4}$ $\quad y = x^4$ $\quad y = 4^x$

1. Predict which of the equations represent a proportional relationship between the variables and circle these equations.

2. Complete each table for the first four equations.

$y = 4 + x$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$y = 4x$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$y = \frac{4}{x}$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$y = \frac{x}{4}$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

Activity 2 All Kinds of Equations (continued)

3. Based on the results of your tables, which equations are actually proportional? Was your prediction accurate? Explain your thinking.

4. What do the equations of the proportional relationships have in common?

Are you ready for more?

Complete the table for the remaining two equations. Determine whether they represent proportional relationships. Explain your thinking.

$y = x^4$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$y = 4^x$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw many equations, but only a few represented proportional relationships. You can determine whether a relationship is proportional by calculating the ratio of each value of y with its corresponding value of x. If the ratios are the same for each corresponding value of x and y, the relationship is proportional and that ratio is the constant of proportionality k.

The equation for a proportional relationship is written as $y = kx$. If an equation cannot be written in this form, then the equation represents a nonproportional relationship.

The table shows a proportional relationship.

- For any proportional relationship where $y = kx$, you can find the constant of proportionality k by using the equation $k = \frac{y}{x}$, when x does not equal 0.
- In this example, $k = 5$. So, the equation of the proportional relationship is $y = 5x$.

x	y	$\frac{y}{x}$
20	100	5
3	15	5
11	55	5
1	5	5

Reflect:

Practice

Name: .... Date: .... Period: ....

1. The relationship between a distance in yards y and the same distance in miles m is given by the equation $y = 1760m$.

a Complete the table.

b Is there a proportional relationship between a distance in yards and the same distance in miles? Explain your thinking.

Distance (miles), m	Distance (yd), y
1	
5	
	3,520
	17,600

2. Determine whether each equation represents a proportional relationship. Place a check mark in the appropriate box.

	Proportional	Nonproportional
a The remaining length L of a 120-in. rope after x in. have been cut off: $120 - x = L$.	☐	☐
b The total cost t after 8% sales tax is added to an item's price p: $1.08p = t$.	☐	☐
c The number of marbles x each sister gets after m marbles are shared equally among four sisters: $x = \frac{m}{4}$.	☐	☐
d The volume V of a rectangular prism whose height is 12 cm and whose base is a square with side lengths of s cm: $V = 12s^2$.	☐	☐

3. For each representation, determine whether it could represent a proportional relationship. Explain your thinking.

a

x	y
2	5
3	7.5
6	15

b $y = 3.2x + 5$

Name: .... Date: .... Period: ....

Practice

4. To transmit information on the internet, large files are broken into packets of smaller sizes. Each packet has $1{,}500$ bytes of information. If p represents the number of packets and b represents the number of bytes of information, an equation relating packets to bytes of information is given by $b = 1500p$.

a. How many packets would be needed to transmit $30{,}000$ bytes of information?

b. How much information could be transmitted in $30{,}000$ packets?

c. Each byte contains 8 bits of information. Write an equation representing the relationship between the number of packets and the number of bits. Remember to define your variables.

5. For each point, name the quadrant in which it is located or the axis on which it is located.

a. $(4, 6)$

b. $(4, -6)$

c. $(-4, -6)$

d. $(-4, 6)$

e. $(4, 0)$

f. $(0, -6)$

6. Based on the information you are given, determine whether each situation below could be described as a proportional relationship. Explain your thinking.

	Proportional? (Yes/No)	Explain your thinking.
In 1 hour it rained 2 cm, and in 3 hours it rained 6 cm.		
The weight, w, of s soup cans can be modeled by the equation $w = 14s$.		
The height of a tower of blocks and the number of various-sized blocks used to build the tower.		

Unit 2 | Lesson 10

Solving Problems About Proportional Relationships

Let's solve problems about proportional relationships.

Warm-up What Do You Want to Know?

Consider the problem: A train is traveling at a constant speed from Barcelona, Spain, to Paris, France. What time will the train arrive in Paris?

What information do you need to know in order to solve the problem?

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Info Gap: Biking and Rain

You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given a *problem card*:	If you are given a *data card*:
1. Silently read your card and think about what information you need to be able to solve the problem.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner, *"What specific information do you need?"* and wait for them to ask for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask *"Why do you need that information?"* Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently in the space provided at the bottom of this page.	**4.** Read the *problem card* and solve the problem independently in the space provided at the bottom of this page.
5. Read the *data card* and discuss your thinking.	**5.** Share the *data card* and discuss your thinking.

Pause here so your teacher can review your work. You will be given a new set of cards. Repeat the activity, trading roles with your partner.

Problem 1 Work	Problem 2 Work
Let M be the distance Mai traveled in meters, N be the distance Noah traveled in minutes, and t be the time in minutes.	Let r be the centimeters of rain, and h be the number of hours.

Activity 2 Card Sort: Determining Proportionality

You will be given a set of cards.

1. Based on the information in each table or scenario, sort the cards into two categories: situations that could represent *proportional relationships* and those that represent *nonproportional relationships*. List the cards in the appropriate category.

Proportional relationships	Nonproportional relationships

2. For each card that could represent proportional relationships, determine the constant of proportionality and write an equation to represent the relationship.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that when a situation is proportional, it involves a constant rate (*the constant of proportionality*). In order to determine if situations represent proportional relationships, you can check the following:

- Are the ratios of corresponding values always the same? If you find at least one ratio that is not the same, then you know it is not a proportional relationship.
- Is there a single value that you can always multiply one quantity by to get the other quantity? If so, it is most likely a proportional relationship.

You can describe proportional relationships with words, model them with tables, and represent them using equations. Once you have established that a relationship is proportional, you can represent it algebraically by writing an equation of the form $y = kx$. If you know any two values in this equation, you can use the equation to efficiently solve for the unknown value.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. For each situation, explain whether the relationship is proportional or nonproportional. Explain your thinking.

a) The weight of a stack of standard 8.5 in. by 11 in. copier paper and the number of sheets of paper.

b) The weight of a stack of different-sized books, where each book weighs a different amount, and the number of books in a stack.

2. Each package of a popular toy includes 2 batteries.

a) Are the number of toys and the number of batteries in a proportional relationship? Is so, what are the two constants of proportionality? If not, explain your thinking.

b) Use t for the number of toys, and b for the number of batteries. Write two equations relating the two variables.

$b =$ .... $t =$ ....

3. Lin and her brother were born on the same date, yet in different years. Lin was 5 years old when her brother was 2 years old.

a) Complete the table to find their corresponding ages for different years.

b) Is there a proportional relationship between Lin's age and her brother's age? Explain your thinking.

Lin's age	Her brother's age
5	2
6	
15	
	25

4. A student claims that the equation $y = \frac{x}{9}$ does not represent a proportional relationship between x and y because it shows that the variable x is divided by a constant to obtain the variable y and not multiplied, and proportional relationships involve multiplication by a constant. Do you agree or disagree with this reasoning? Explain your thinking.

Name: .... Date: .... Period: ....

5. Quadrilateral A has side lengths 3 units, 4 units, 5 units, and 6 units. Quadrilateral B is a scaled copy of Quadrilateral A with a scale factor 2. Select *all* of the following that could be the side lengths of Quadrilateral B.

   A. 5 units

   B. 6 units

   C. 7 units

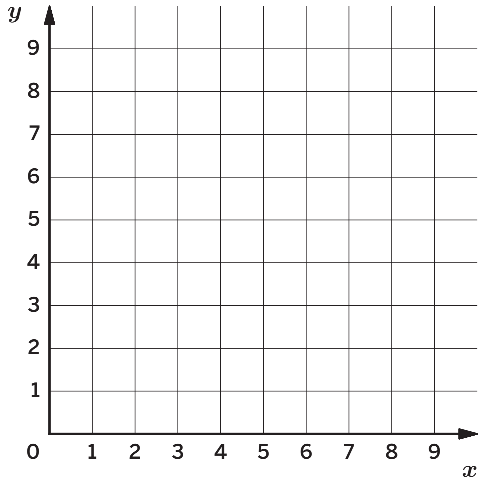
   D. 8 units

   E. 9 units

6. Consider the set of ordered pairs shown.

$(0, 0)$	$(2, 2)$	$(3, 3)$	$(6, 6)$

a Plot the points represented by these ordered pairs on the coordinate plane.



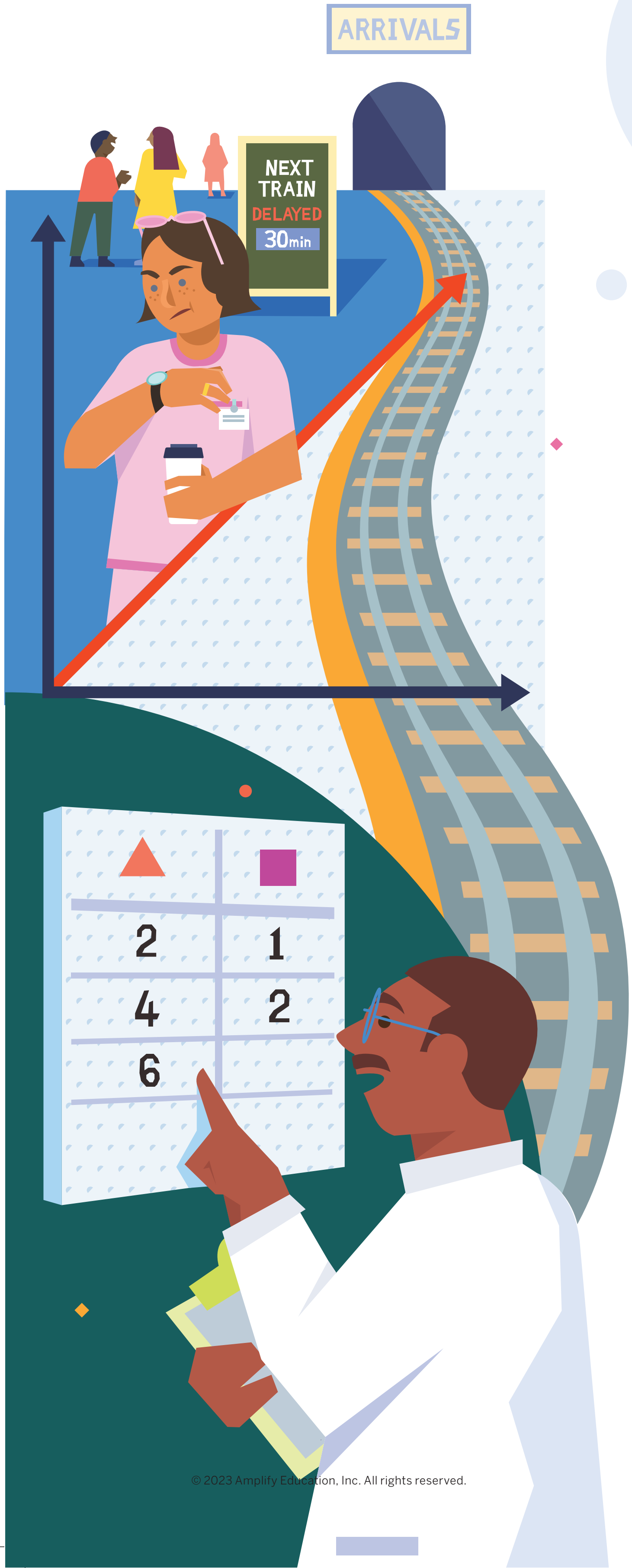
b What do you notice about the points you plotted?

c Name another point which will follow the same pattern.

My Notes:

SUB-UNIT

2 Representing Proportional Relationships With Graphs



Narrative Connections

What good is a graph?

In the previous lessons, you saw how tables and equations were useful for representing proportional relationships. Tables let you organize your data so you can find the **constant of proportionality**.

Meanwhile, the equation $y = kx$ gave you a way to express *any* proportional relationship.

These representations each have their own pros and cons. Tables offer a practical way to collect and organize data, but checking every entry for proportionality can be tedious.

Meanwhile, an equation may reveal the rules underpinning a proportional relationship, but getting useful data requires applying the equation several times.

In this next set of lessons you will learn about another kind of representation for proportional relationships: graphs!

Graphs can be a fast, intuitive way to understand and appreciate the data and mathematics underlying proportional relationships. With just one look, you will be able to tell if a relationship is proportional; find its constant of proportionality; and see all the solutions for that relationship.

Unit 2 | Lesson 11

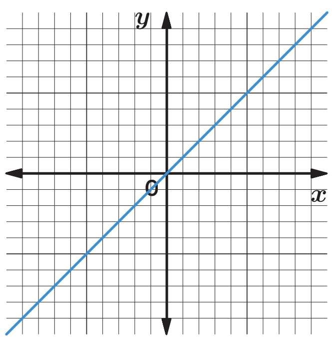
Introducing Graphs of Proportional Relationships

Let's see how graphs of proportional relationships differ from graphs of other relationships.

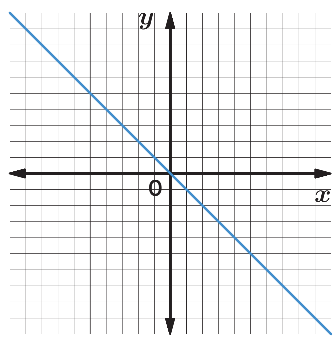
Warm-up Which One Doesn't Belong?

Four graphs are shown. Which graph does not belong with the others? Explain your thinking.

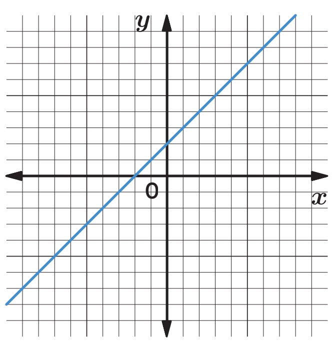
A.



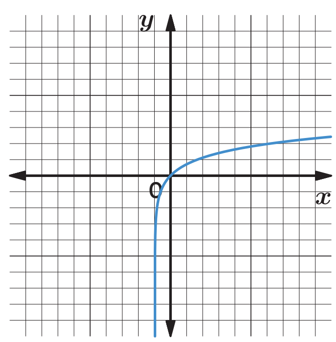
B.



C.



D.



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Name: ________ Date: ________ Period: ________

Activity 1 T-shirts for Sale

Plan ahead: How can you make everyone in your group feel valuable?

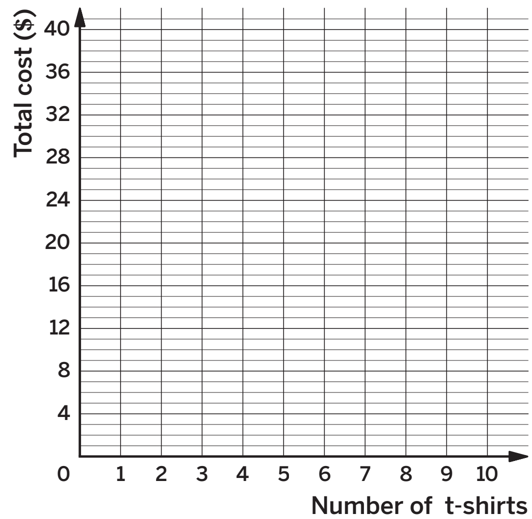
A store is having a sale on their t-shirts. Have each member of your group choose a different type of t-shirt. Circle the one you chose.

- **Screen printed t-shirts are on sale for $8.00 each.**
- **Tie-dyed t-shirts are on sale for $5.00 each.**
- **Last season's t-shirts are on sale for $2.50 each.**
- **Plain t-shirts are on sale for $4.00 each.**

1. For your selected t-shirt type, complete the table.

Number of t-shirts	Total cost ($)
0	
1	
2	
5	
8	

2. Plot the ordered pairs from the table on the coordinate plane.



3. Compare your graph to your group members' graphs. What is similar? What is different?

4. For your selected t-shirt type, what is the cost of 0 t-shirts? Is this the same for all of your group members' selected t-shirt types?

5. The graphs you created in Problem 1 are examples of proportional relationships. What are some characteristics of the graphs of proportional relationships?

Activity 2 Card Sort: Graphs of Proportional Relationships

You will be given a set of cards.

1. Sort the cards into two categories: *proportional relationships* and *nonproportional relationships*. List the cards in the appropriate category.

Proportional relationships	Nonproportional relationships

2. For each card in the *nonproportional* category, explain why you think the relationship is not proportional.

Are you ready for more?

For the graphs you sorted as *proportional*, find the constant of proportionality.

STOP

Name: ______ Date: ______ Period: ______

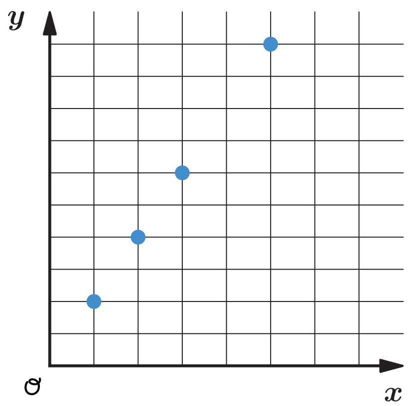
Summary

In today's lesson . . .

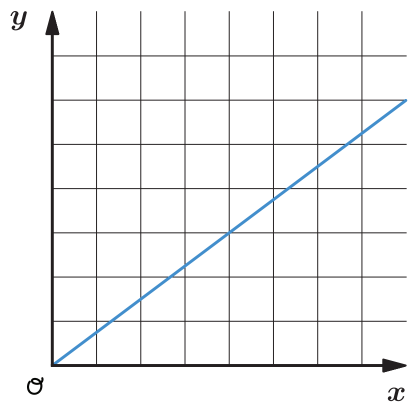
You saw many graphs of proportional and nonproportional relationships. Graphs of proportional relationships are lines which pass through the origin, $(0, 0)$.

Here are some examples of graphs of proportional relationships.

- The points form a line that, if connected, would pass through the origin.

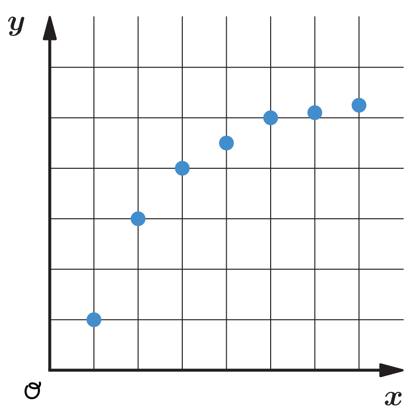


- The relationship is a line that passes through the origin.

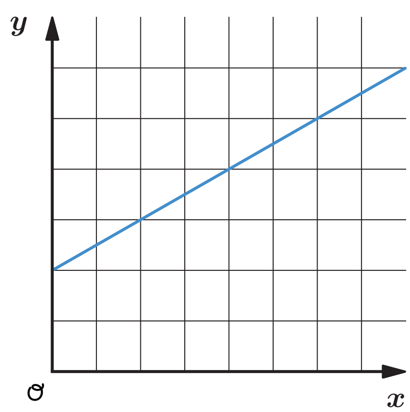


Here are some examples of graphs of nonproportional relationships.

- The points form a curve, not a line.



- The line does not pass through the origin.



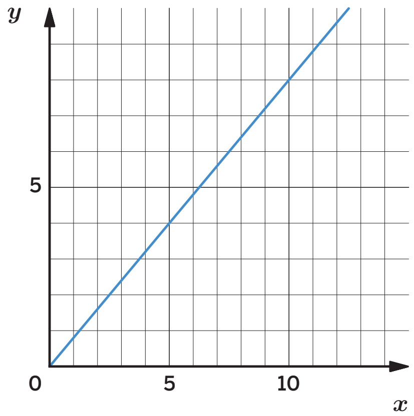
Reflect:

Practice

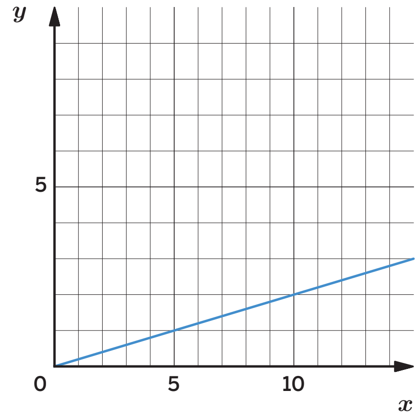
Name: ______________ Date: ______________ Period: ______________

1. Which graphs represent a proportional relationship? Explain your thinking.

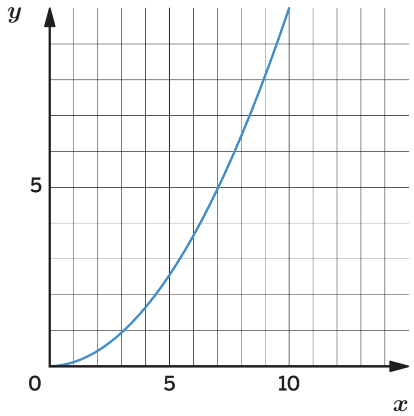
A.



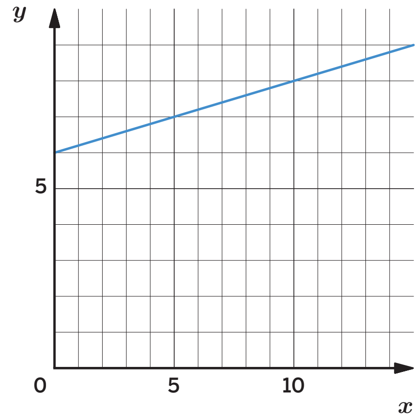
C.



B.



D.

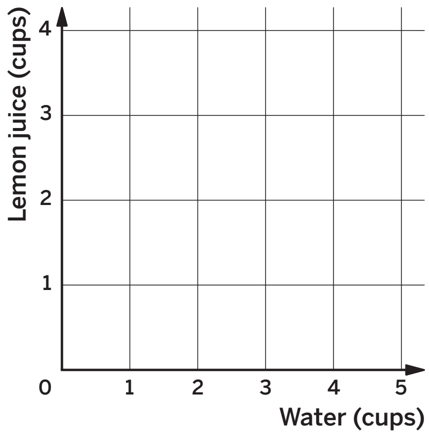


2. A lemonade recipe calls for $\frac{1}{4}$ cup of lemon juice for every cup of water. The table shows some values.

a Graph the ordered pairs.

Water (cups)	Lemon juice (cups)
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1

b Determine whether there is a proportional relationship using the graph.



Name: ______ Date: ______ Period: ______

Practice

3. A turtle is walking away from a rock. Let x represent the time in minutes that the turtle is walking. Let y represent the distance in meters between the rock and the turtle. If x and y are in a proportional relationship, select *all* of the true statements.

A. The equation $y = 3x$ could represent the distance that the turtle walks.

B. The turtle walks for a bit and then stops for a minute before walking again.

C. The turtle walks away from the rock at a constant rate.

D. The equation $y = x + 3$ could represent the distance that the turtle walks.

E. After 6 minutes, the turtle walks 18 m and after 10 minutes, the turtle walks 20 m.

4. Decide whether each table represents a proportional relationship. If the relationship is proportional, what is the constant of proportionality?

a The width and height of a photo

Width of photo (in.)	Height of photo (in.)
2	3
4	6
5	7
8	10

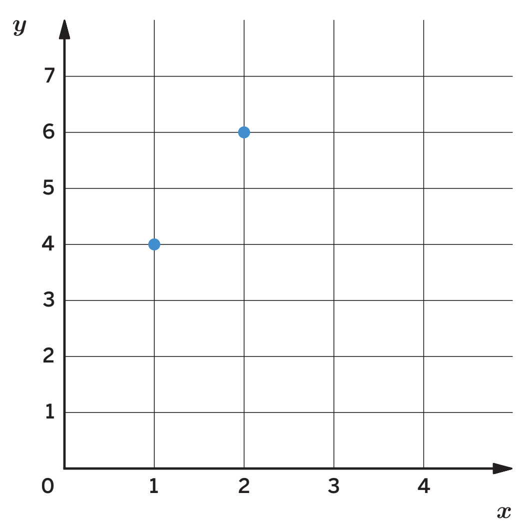
b The distance from which a lighthouse can be seen

Height of a lighthouse (ft)	Distance it can be seen (miles)
20	6
45	9
70	11
95	13

5. Elena and Diego are studying the graph.

- Elena claims the graph is proportional because the points form a line.
- Diego says the relationship is nonproportional because the line would not pass through the origin.

Who is correct? Explain your thinking.



Unit 2 | Lesson 12

Interpreting Graphs of Proportional Relationships

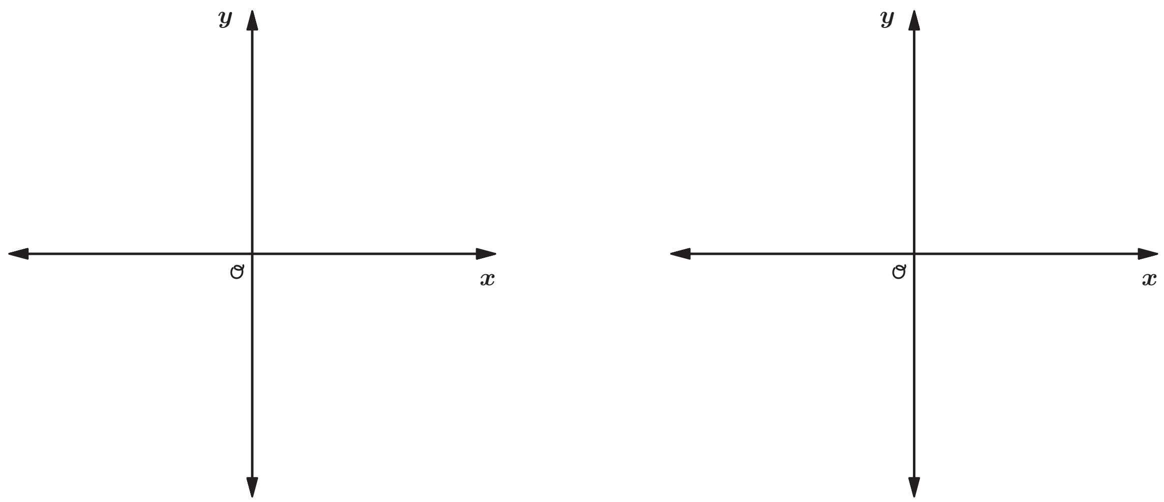
Let's read stories from the graphs of proportional relationships.

Warm-up Graph It

Sketch a graph that could represent each type of relationship.

1. Proportional relationship

2. Nonproportional relationship



Log in to Amplify Math to complete this lesson online.

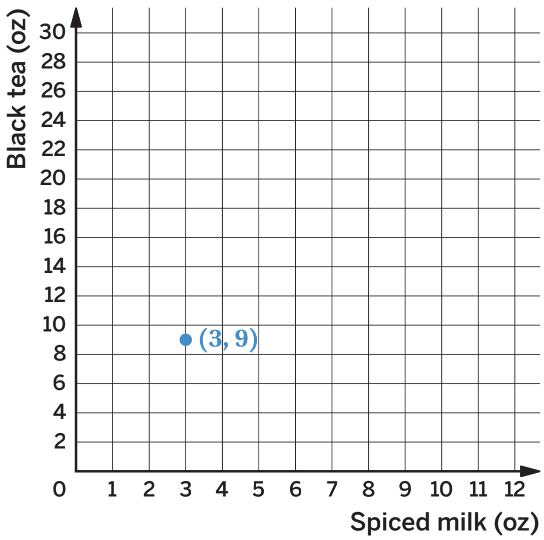
Name: ______________________ Date: ____________ Period: ____________

Activity 1 Making Tea With a Chai Wallah

Chai is a hot drink made with a combination of black tea and spiced milk. It is so popular that you can find a chai wallah making and selling it on just about every street corner in India. The ingredients in chai are always proportional so that it tastes the same each time. Let's see how we can represent this relationship in different ways.

Dietmar Temps/Shutterstock.com

1. The coordinates $(3, 9)$ are plotted on the graph. What does the point $(3, 9)$ represent?



2. Complete the table for other quantities which fit this proportional relationship.

Spiced milk (oz)	Black tea (oz)
3	
8	
2	
5	

3. Plot each ordered pair from the table on the graph. Connect the points with a line.

4. Does your line pass through the origin? What does the origin represent in this scenario?

5. What is the value of y when the value of x is 1? Plot and label this point. What does this point represent in this scenario?

Activity 1 Making Tea With a Chai Wallah (continued)

6. What is the constant of proportionality for this scenario? How can you find this on the graph?

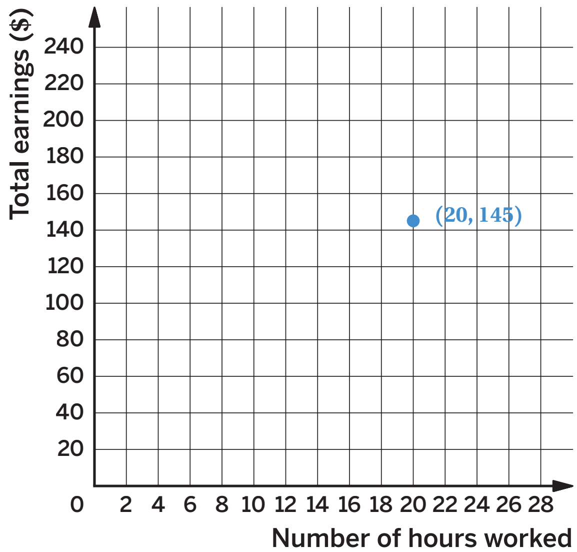
7. How many ounces of spiced milk are mixed with 13.5 oz of black tea? How can you find this on the graph?

8. Write an equation for this scenario of the form $y = kx$, where k is the constant of proportionality

Name: ________ Date: ________ Period: ________

Activity 2 Tyler's Job

Tyler works as a camp counselor and earns the federal minimum wage, as of the year 2020.



1. The point on the graph shows Tyler's earnings after working at the camp. What do the coordinates of the point tell you about the scenario?

2. Draw a line representing this proportional relationship.

3. Does your line pass through the point $(0, 0)$? Explain the meaning of this point in the context of this scenario.

4. What is the constant of proportionality for this relationship? What does it tell you about Tyler's earnings? Plot the point that shows the constant of proportionality and label it.

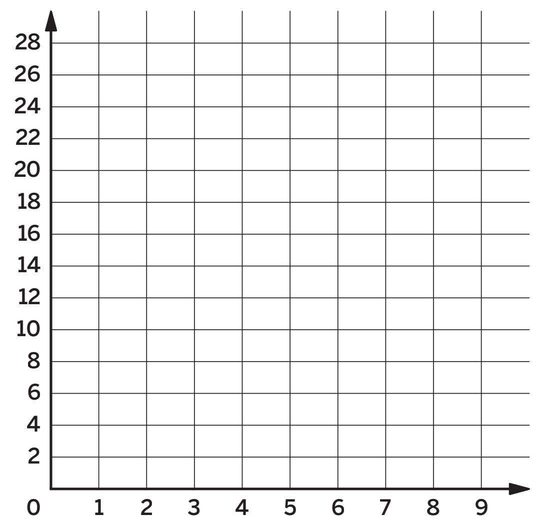
5. Let k represent the constant of proportionality for the number of hours x that Tyler works and his earnings. Write an equation of the form $y = kx$.

6. Use your equation from Problem 5 to determine how much money Tyler would make for working 16 hours. Plot this point on the graph and label it.

Activity 3 Building Your Own Proportional Relationship

In this activity, you will build your own proportional relationship and represent it in different ways.

1. Plot a point on the graph and label the coordinates.



2. Draw a line that passes through this point and that shows a proportional relationship.

3. Create a scenario representing a proportional relationship that includes the point you plotted.

4. Label the axes with your variables.

5. What is the constant of proportionality? What does it mean in this scenario?

6. How is your constant of proportionality shown on your graph?

7. Write an equation of the form $y = kx$, where k is the constant of proportionality, to represent your scenario.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You analyzed graphs of proportional relationships. Each point on a graph tells a story using the quantities represented by x and y. The constant of proportionality is found on the graph of a proportional relationship by . . .

- Finding the value of y when x is equal to 1.
- Finding the ratio $\frac{y}{x}$ for a given ordered pair.

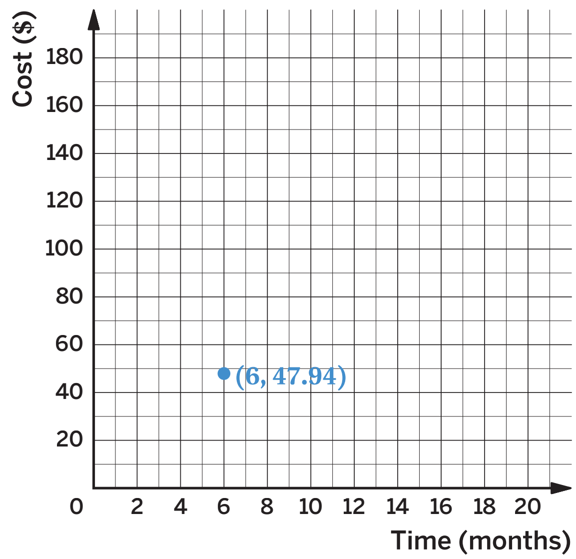
Note that the relationship must be proportional for the constant of proportionality to exist and to be able to be found by using these strategies.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. There is a proportional relationship between the number of months m, during which a person has a streaming movie subscription and the total cost c of the subscription. The cost for 6 months is $\$47.94$. The point $(6, 47.94)$ is shown on the graph.



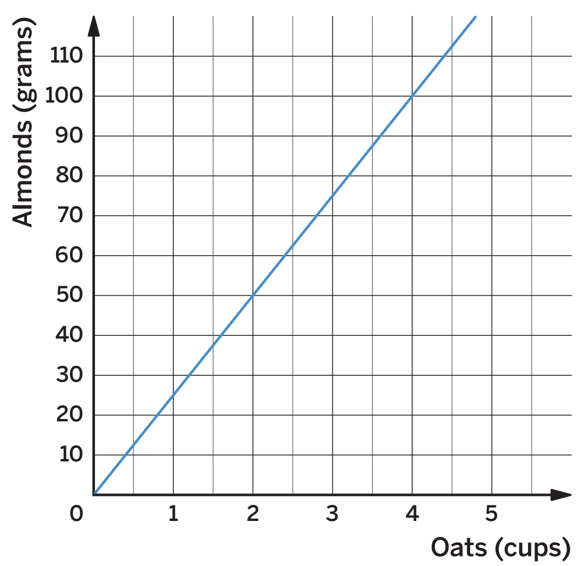
a. What is the constant of proportionality?

b. What does the constant of proportionality represent in this context?

c. Plot at least three more points showing the same relationship on the graph, and label the points with their coordinates.

d. Using the constant of proportionality from part a, write an equation that represents the relationship between c and m.

2. The graph shows the amounts of almonds, in grams, for different amounts of oats, in cups, in a granola mix.



a. Determine the value of k, the constant of proportionality, and explain its meaning.

b. Label the point $(1, k)$ on the graph.

Name: .... Date: .... Period: ....

Practice

3. What information do you need to know to write an equation relating two quantities in a proportional relationship?

4. Han and Clare were running laps around the track. The coach recorded their times at the end of laps 2, 4, 6, and 8. Complete the table, if needed, to help with your thinking.

Han's run:

Distance (laps)	Time (minutes)	Minutes per lap
2	4	
4	9	
6	15	
8	23	

Clare's run:

Distance (laps)	Time (minutes)	Minutes per lap
2	5	
4	10	
6	15	
8	20	

a Is Han running at a constant speed? Is Clare? Explain your thinking.

b If you found that one or both friends are running at a constant speed, write an equation that represents the relationship. Remember to define your variables.

5. The cost paid for two different varieties of apples are shown on the graph. Which variety has a higher unit cost? Explain your thinking.

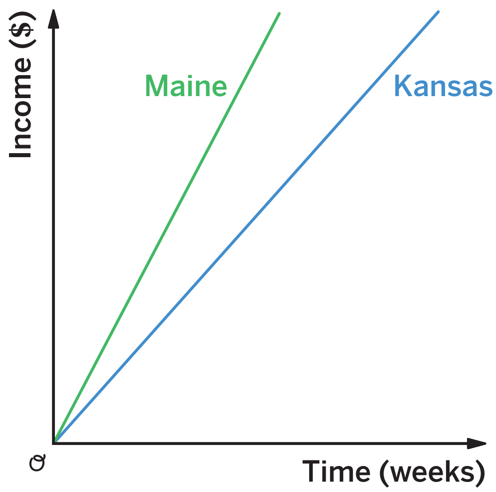
Unit 2 | Lesson 13

Using Graphs to Compare Relationships

Let's graph more than one relationship on the same coordinate plane.

Warm-up Notice and Wonder

The graph shows the earnings of two full-time workers earning minimum wage in Kansas and in Maine. What do you notice? What do you wonder?



1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Race to $\$1{,}000$

Diego, Lin, and Tyler all earn minimum wage, as of the year 2020, working as camp counselors in their respective cities. Recall from Lesson 12 that Tyler earned $\$145$ after working 20 hrs, and his earnings can be represented by the equation $y = 7.25x$, where x represents the number of hours worked and y represents the total earnings, in dollars.

1. Complete the table of values and write an equation to represent each of their earnings, based on these descriptions.

a Diego lives in Washington, D.C. Last week, he worked for 4 hours and earned $\$60$.

Equation:

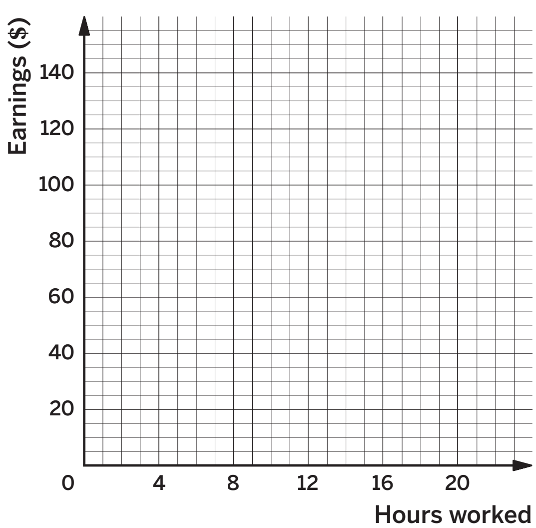
Diego's hours worked, x	Diego's earnings ($), y
0	
4	60
1	

b Lin lives in Providence, Rhode Island. Last week, she worked for 10 hours and earned $\$105$.

Equation:

Lin's hours worked, x	Lin's earnings ($), y
	0
10	105
1	

2. Draw a graph representing Diego, Lin, and Tyler's earnings on the same coordinate plane. Label each line with the appropriate name.



Compare and Connect: Compare with your partner how the earnings relate to the constant of proportionality, paying close attention to the steepness of the graph.

Activity 1 Race to $1,000 (continued)

3. Which person has the greatest hourly wage?

  a How is this reflected in the equation or in the table?

  b How is this reflected on the graph?

4. Which person will need to work the most number of hours to earn $1,000? Explain your thinking.

5. Diego, Lin, and Tyler all attend the same university. The cost, including tuition and fees, for one year is $24,360.

  a How many hours would each student need to work as a camp counselor to earn enough money for tuition and fees?

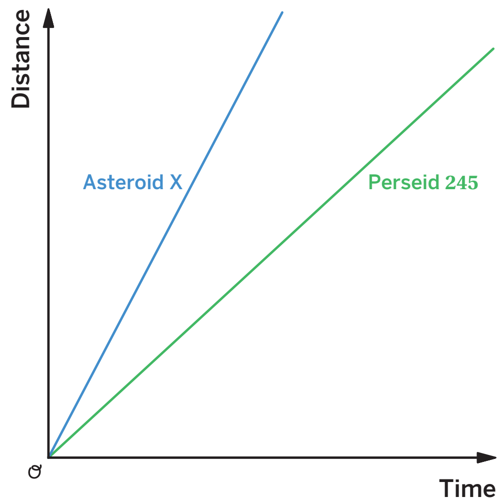
  b Assuming they each work full-time (40 hours per week), how many weeks would each have to work?

  c Assuming they only work during their summer vacation (15 weeks), how many summers would each need to work?

Name: ________ Date: ________ Period: ________

Activity 2 Space Rocks!

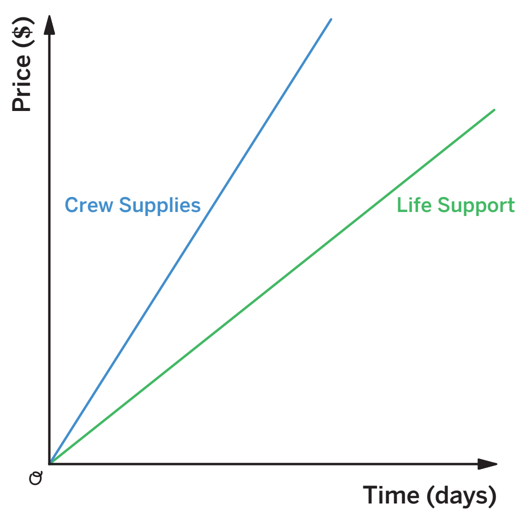
1. Meteoroid Perseid 245 and Asteroid X travel through the solar system. The graph shows their distance traveled as time passes.



   a. Which object will take longer to travel 3,000,000 miles? Explain your thinking.

   b. Does Asteroid X travel faster or slower than Perseid 245? Explain your thinking.

2. The cost of having a crew on the International Space Station can be separated into multiple categories. Two such categories are Life Support and Crew Supplies. The graph shows the relationship between the cost of these two categories and the number of days spent in space. If a crew spends 4 days in space, which category costs less? Explain your thinking.



STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You compared the graphs of proportional relationships on the same coordinate plane and saw that the steeper the line, the greater the constant of proportionality.

For example, the graph shows the cost of soybeans at two different stores.

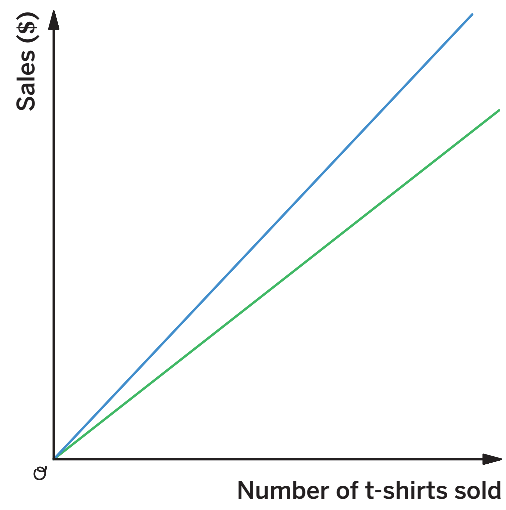
- On the graph, you can see that Store A charges more than Store B because its line is steeper.
- You can also compare the graphs' constants of proportionality.
- Store A charges \$2 per lb ($k = 2$), while Store B charges \$1 per lb ($k = 1$). Store A has a greater constant of proportionality than Store B.

Reflect:

Name: ____________ Date: ________ Period: ________

Practice

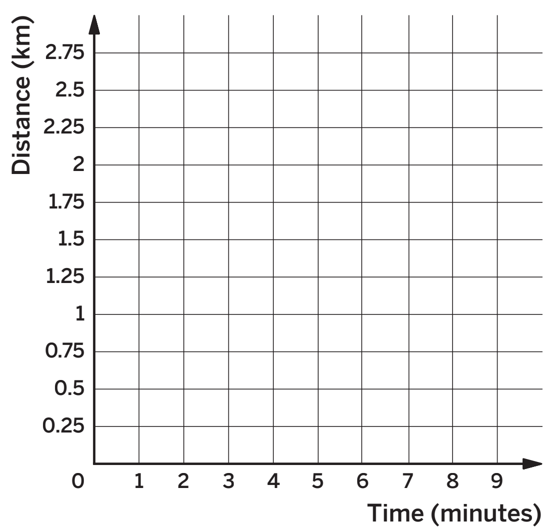
1. Shawn and Elena both sell t-shirts online with original designs. Shawn sold 8 t-shirts at a certain price per t-shirt and made $\$96$. Elena sold 12 t-shirts at another price per t-shirt and made $\$120$. The graph shows two lines that represent the relationship between sales and number of t-shirts sold. Determine the line that represents the amount of sales for each person. Label each line with the appropriate one name. Explain your thinking.



2. Lin and Andre biked home from school, each at a constant rate. Lin biked 1.5 km in 5 minutes. Andre bike 2 km in 8 minutes.

a. Graph the two lines that represent the bike rides of Lin and Andre. Label each line with the appropriate name.

b. For each line, label the point with coordinates $(1, k)$ and determine the value of k.

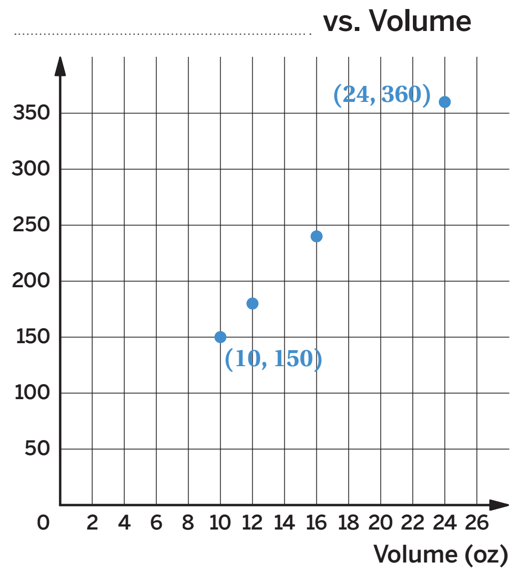


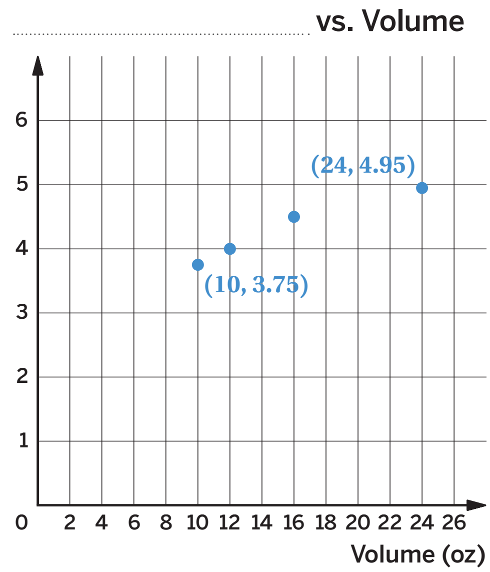
c. Who biked faster? Explain your thinking.

Practice

Name: .... Date: .... Period: ....

3. The graphs show some data from a coffee shop menu. One graph shows the relationship between cost, in dollars, and drink volume, in ounces. The other graph shows the relationship between the number of calories and drink volume, in ounces.



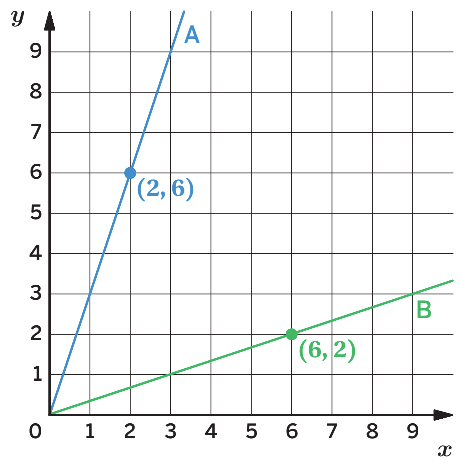


a. Which relationship is modeled by each graph? Complete the title of each graph. Explain your thinking.

b. Which graph appears to represent a proportional relationship? Explain your thinking.

c. For the proportional relationship, determine the constant of proportionality. What does this value mean in the context of this problem?

4. Determine the constant of proportionality for each line. Then write the equation that represents each line.



Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 14

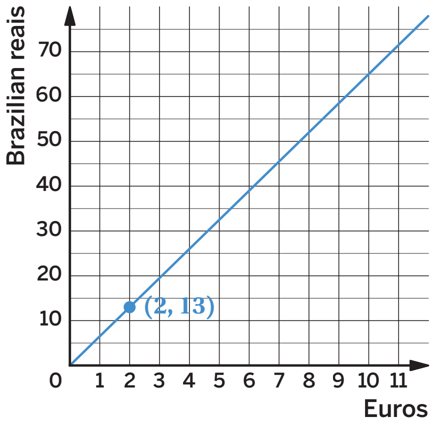
Two Graphs for Each Relationship

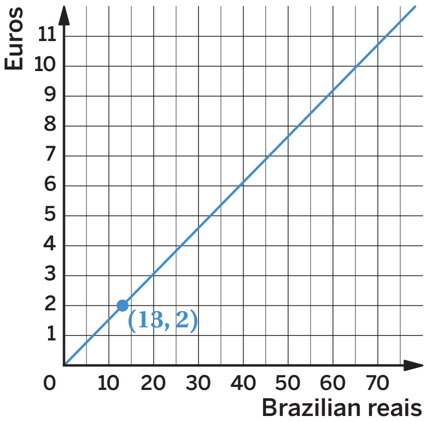
Let's use tables, equations, and graphs to solve problems about proportional relationships.

Warm-up Notice and Wonder

The two graphs show the relationship between the value of the *euro*, the currency of the European Union, and the value of the Brazilian *real*, the currency of Brazil. The exchange rates are from 2020. What do you notice? What do you wonder?

Note: The plural of a Brazilian real is *reais*.





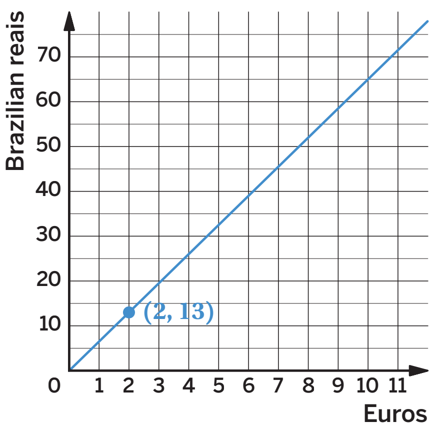
1. I notice . . .

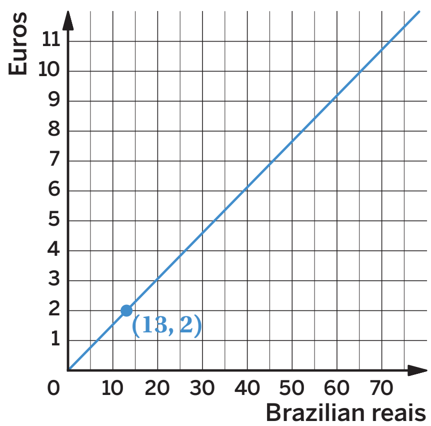
2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Traveling from Brazil to Europe and Back

Refer to the graphs from the Warm-up.





1. What is the constant of proportionality in Graph A? What does this tell you?

2. What is the constant of proportionality in Graph B? What does this tell you?

3. Jada is traveling from Germany to Brazil. Germany uses the euro.

a Which graph would be most helpful for her to use? Why?

b What is the equation for this relationship? Remember to define your variables.

4. Elena is traveling from Brazil to Portugal.

a Which graph would be most helpful for her to use? Why?

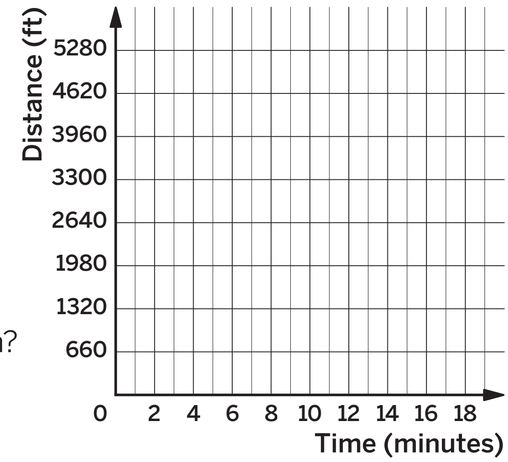
b What is the equation for this relationship? Remember to define your variables.

5. What is similar about these two relationships? What is different?

Name: ______ Date: ______ Period: ______

Activity 2 One Relationship With Two Graphs

Shawn is running 1 mile ($5{,}280$ ft) for gym class at a steady pace of 440 ft per minute.

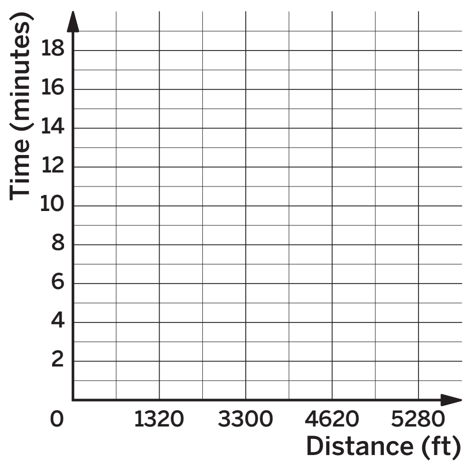


1. Write an equation for Shawn's run. Remember to define your variables.

2. If Shawn runs for 3 minutes, how far will Shawn run? Show or explain your thinking.

3. What is the ordered pair you found in Problem 2? Plot this point on the graph and draw the line representing the proportional relationship.

Let's represent Shawn's run another way.



4. Which variable is on the x-axis of the graph? On the y-axis?

5. Recall that Shawn runs 440 ft per minute. How long does it take Shawn to run 1 ft? Explain your thinking.

6. Using the same variables from Problem 2, write an equation for this representation of Shawn's run. How long does it take Shawn to run $2{,}640$ ft? Show or explain your thinking.

7. What is the ordered pair you found in Problem 6? Plot this point on the graph and draw the line representing the proportional relationship.

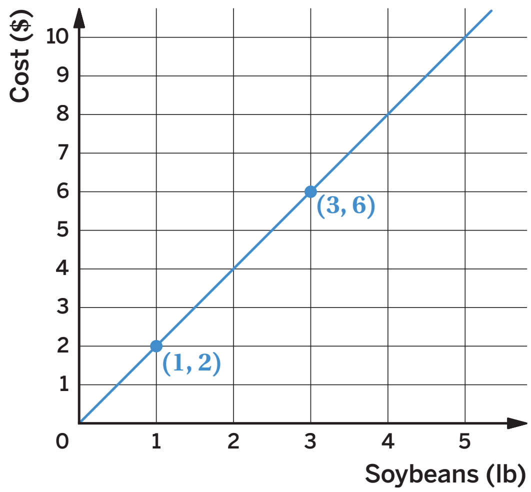
STOP

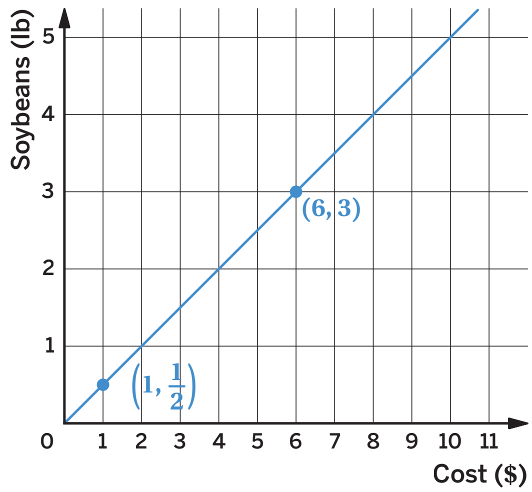
Summary

In today's lesson . . .

You again noticed the two different proportional relationships between variables from graphs.

Here are two graphs showing the proportional relationships between the weight of soybeans in pounds w and the total cost c, in dollars.





Constant of proportionality:
This graph contains the point $(1, 2)$, so $k = 2$.

Equation: $c = 2w$

Constant of proportionality:
This graph contains the point $\left(1, \frac{1}{2}\right)$, so $k = \frac{1}{2}$.

Equation: $w = \frac{1}{2}c$

Even though the relationship between weight and cost are the same, you are making a different choice in each case about the variables to place on the x- and y-axes. The graph on the left shows weight as the x-variable, and the graph on the right shows cost as the x-variable.

Reflect:

Name: ______ Date: ______ Period: ______

Practice

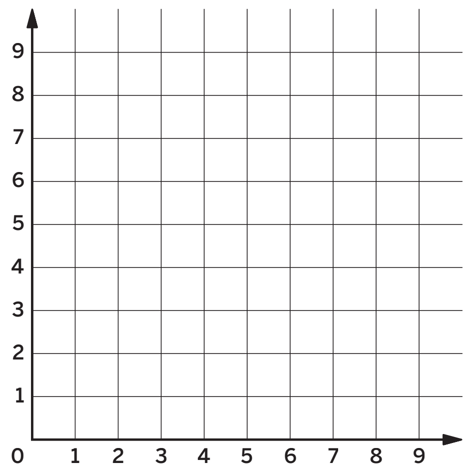
1. To help protect the environment, some supermarkets allow people to fill their own honey container. A customer buys 12 oz of honey for $\$5.40$.

a How much does the honey cost per ounce?

b How much honey can be bought per dollar?

c Write two different equations representing this scenario. Remember to define your variables.

d Choose one equation and draw its graph on the coordinate plane. Be sure to label the axes.

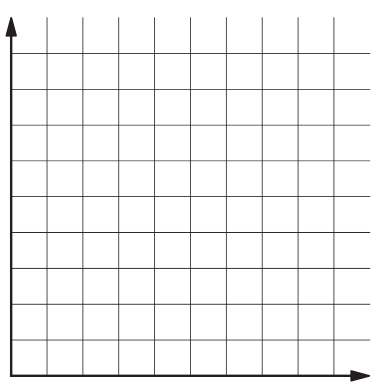


2. A trail mix recipe lists 4 cups of raisins for every 6 cups of peanuts. There is a proportional relationship between the number of cups of raisins r and the number of cups of peanuts p.

a Write the equation for the relationship in which the constant of proportionality is greater than 1. Label and scale the axes and then graph the relationship.

Equation: ______

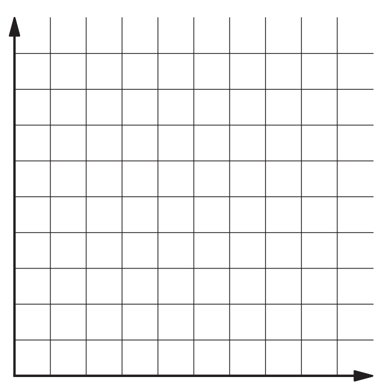
Graph:



b Write the equation for the relationship in which the constant of proportionality is less than 1. Label and scale the axes and graph the relationship.

Equation: ______

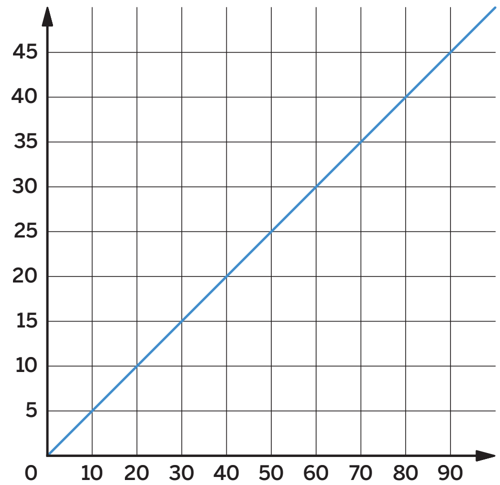
Graph:



Practice

Name: ______ Date: ______ Period: ______

3. The graph of a proportional relationship is shown. Write a scenario that could be represented by this graph. Label the axes with the quantities in your scenario. Then choose a point on the graph and describe what the coordinates represents in your scenario.



4. Match each equation with its graph.

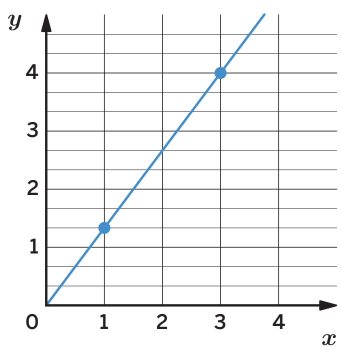
a $y = \frac{1}{4}x$ ______

b $y = \frac{3}{2}x$ ______

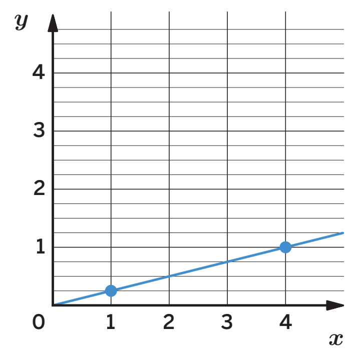
c $y = 2x$ ______

d $y = \frac{4}{3}x$ ______

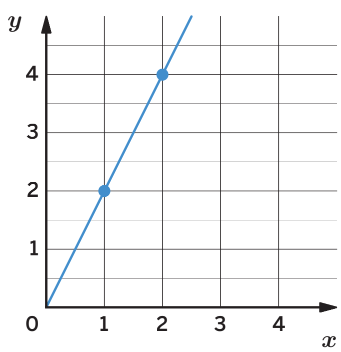
Graph 1



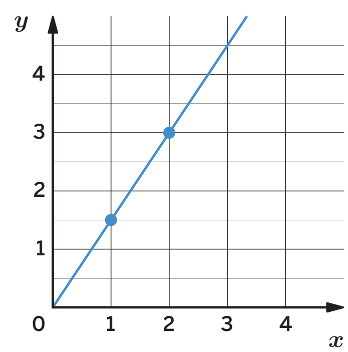
Graph 2



Graph 3



Graph 4



5. Which three ordered pairs represent the same proportional relationship? Explain your thinking.

A. $(1, 4)$

B. $(3, 12)$

C. $(9, 36)$

D. $(5, 24)$

Name: ______________ Date: ______________ Period: ______________

Unit 2 | Lesson 15

Four Ways to Tell One Story (Part 1)

Let's find the constant of proportionality from different representations.

Warm-up True or False?

Determine whether each equation is *true* or *false*.
Place a check mark in the appropriate box. Be prepared to explain your thinking.

	True	False
1. $\frac{3}{2} \cdot 16 = 3 \cdot 8$	☐	☐
2. $\frac{3}{4} \div \frac{1}{2} = \frac{6}{4} \div \frac{1}{4}$	☐	☐
3. $2.8 \cdot 13 = 0.7 \cdot 52$	☐	☐

Log in to Amplify Math to complete this lesson online.

Activity 1 Tables, Graphs, and Equations

Plan ahead: What is your plan for resolving conflict constructively and effectively?

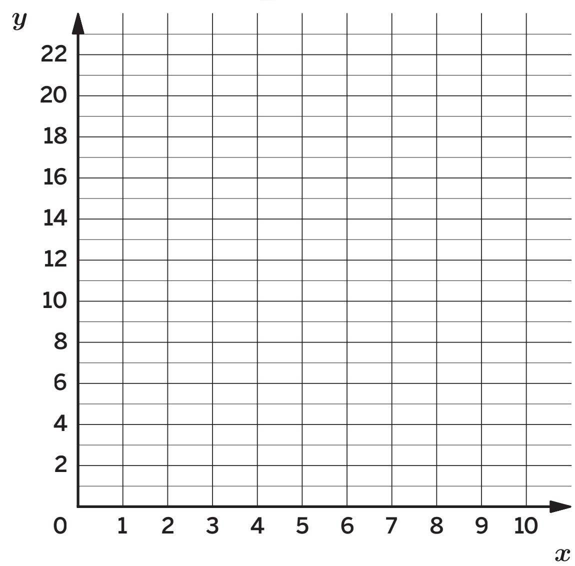
You will be given one point.
Write the coordinates here: ______________.

1. Plot the point on the coordinate plane.

2. Use a ruler to line up your point with the origin. Draw a line starting at the origin, passing through your point, and continuing to the edge of the graph.

3. Complete the table by using ordered pairs on your graph. Use a fraction to represent any value that is not a whole number.

4. Write an equation representing the relationship between x and y.



5. What is the y-coordinate of your graph when the x-coordinate is 1?

   a Plot and label this point on your graph.

   b Circle where you see this value in the table.

   c Circle where you see this value in your equation.

x	y
0	
1	
2	
3	
4	
5	

x	y
6	
7	
8	
9	
10	

6. Describe any connections you see between the table, the graph, and the equation.

Name: ______________ Date: ______________ Period: ______________

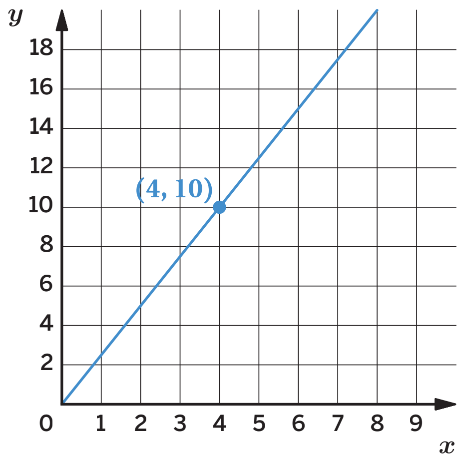
Activity 2 Finding the Constant of Proportionality

Part 1 The following representations are from the same proportional relationship. Have each group member choose a, b, or c. Circle the one you choose.

1. Find the constant of proportionality for your chosen representation. Explain your thinking.

a Clare walked 15 m in 6 seconds.

b



c

x	y
0	0
5	12.5
8	20

2. Compare your constant of proportionality with your group members. Resolve any discrepancies or differences among your responses. What does the constant of proportionality tell you about the scenario?

3. Write an equation representing the scenario. Remember to define your variables.

4. Does the ordered pair $(16, 40)$ fit the relationship? Explain your thinking. Include the representation(s) you used to determine your response.

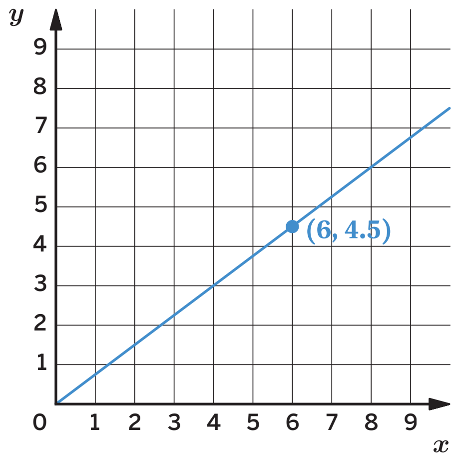
5. Does the ordered pair $(7, 14)$ fit the relationship? Explain your thinking. Include the representation(s) you used to determine your response.

Activity 2 Finding the Constant of Proportionality (continued)

Part 2 The following representations are from the same proportional relationship. Use the same letter — a, b, or c — that you chose in Part 1.

6. Find the constant of proportionality for your chosen representation. Show or explain your thinking.

a



b

x	y
0	0
3	2.25
8	6

c Han read $\frac{2}{3}$ page in $\frac{1}{2}$ minute.

7. Compare your constant of proportionality with your group members. Resolve any discrepancies or differences among your responses. What does the constant of proportionality tell you about the scenario?

8. Write an equation representing the scenario. Remember to define your variables.

9. Does the ordered pair (35, 26.25) fit the relationship? Explain your thinking. Include the representation(s) you used to determine your response.

10. Name another ordered pair which will fit the relationship. Explain your thinking. Include the representation(s) you used to determine your response.

STOP

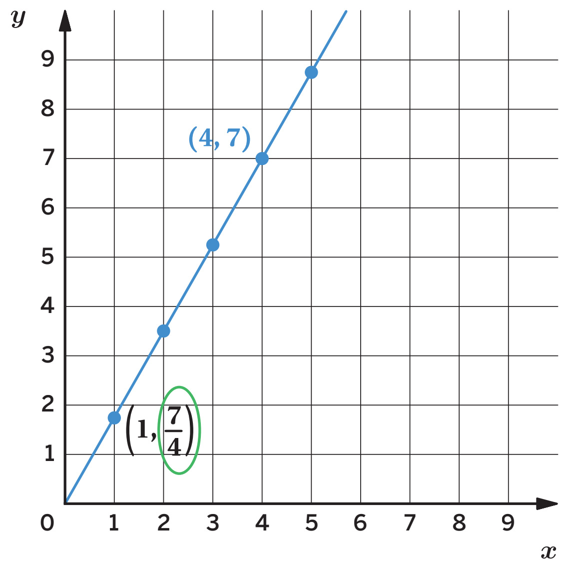
Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You found the constant of proportionality k using various representations.

Graph:



Equation:

$y = \frac{7}{4} x$

Table:

x	y
0	0
1	$\frac{7}{4}$
2	$\frac{7}{2}$
3	$\frac{21}{4}$
4	7

- You can find the value of k by looking for the corresponding value of y for when the value of x is 1.
- If you know an ordered pair (x, y), then $k = \frac{y}{x}$.
- If you have the equation of a proportional relationship of the form $y = kx$, the coefficient of x is the constant of proportionality.

Reflect:

Practice

Name: ______________________ Date: ____________ Period: ____________

1. The point $\left(\frac{3}{2}, \frac{6}{10}\right)$ lies on a graph representing a proportional relationship. Which of the following points also lie on the same graph? Select *all* that apply.

A. $(0.4, 1)$

B. $\left(1.5, \frac{6}{10}\right)$

C. $\left(\frac{6}{5}, 3\right)$

D. $\left(4, \frac{22}{5}\right)$

E. $(15, 6)$

2. Find the constant of proportionality for each representation. Show your work or explain your thinking.

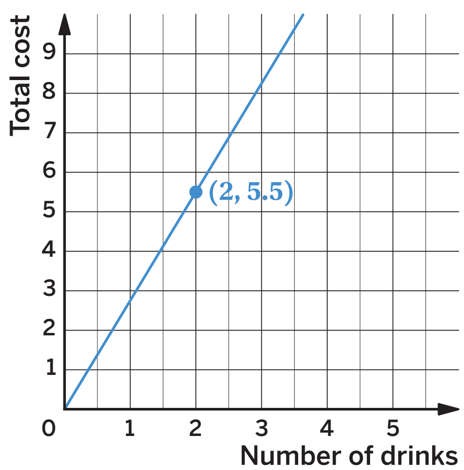
a. Bard walked 15 ft in 3 seconds.

b. $C = 4.8n$

c.

x	y
0	0
$\frac{7}{10}$	$\frac{28}{15}$

d.



3. Diego bought 4 lb of almonds for \$23.96. The total cost of the almonds is proportional to the number of pounds bought. How much would Diego spend on 6 lb of almonds? Explain your thinking.

Name: ______________________ Date: __________ Period: __________

4. On a flight from New York to London, an airplane travels at a constant speed. Let d represent the distance traveled in miles and let t represent the number of hours. An equation relating d and t is $t = \frac{1}{500}d$. How long will it take the plane to travel 800 miles?

5. In Canadian coins, 16 quarters is equal in value to 2 toonies.

a Complete the table.

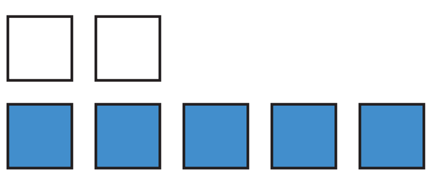
b What does the value for the number of toonies in the first row of the table mean in this situation?

Number of quarters	Number of toonies
1	
16	2
20	
24	

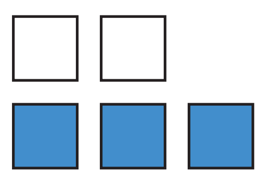
c What is the constant or proportionality which helps you find the number of quarters for an equivalent amount of toonies?

6. What is the ratio of shaded to unshaded squares in each diagram?

a



b



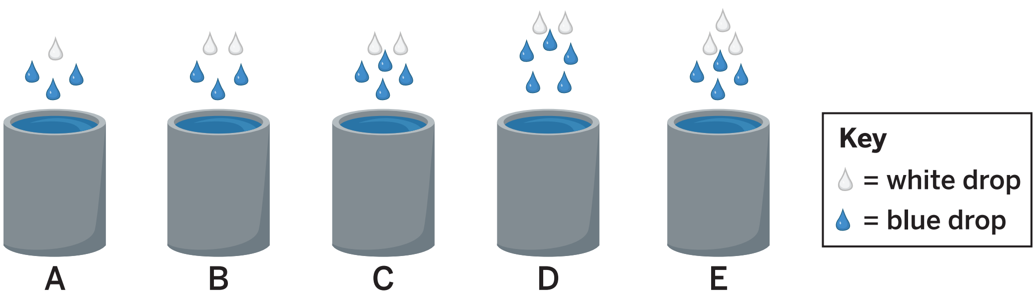
Unit 2 | Lesson 16

Four Ways to Tell One Story (Part 2)

Let's compare relationships that are and are not proportional in four different ways.

Warm-up Which is the Bluest?

Consider the paint cans.



1. If the drops of paint were mixed, which would be the *bluest* (most blue)? Explain your thinking.

2. List the cans of paint in order from least blue to bluest. Show or explain your thinking.

Least blue **Bluest**

Log in to Amplify Math to complete this lesson online.

Name: ____________ Date: ________ Period: ________

Activity 1 Card Sort: Four Representations

You will be given a card. Find the three other representations matching your card, and complete the information in the chart.

Define the variables:	
Verbal description: (copy the verbal description from the card)	**Table:** (write the card number) ________ **Graph:** (write the card number) ________ **Equation:** ________
Explain how you know the relationship is or is not proportional. Give as many reasons as you can.	Explain what each number and letter in the equation represent.

Pause here and wait for instructions.

For the new set of cards, complete the information in the chart.

Define the variables:	
Verbal description: (copy the verbal description from the card)	**Table:** (write the card number) ________ **Graph:** (write the card number) ________ **Equation:** ________
Explain how you know the relationship is or is not proportional. Give as many reasons as you can.	Explain what each number and letter in the equation represent.

Activity 1 Card Sort: Four Representations (continued)

Pause here and wait for instructions.

For the new set of cards, complete the information in the chart.

Define the variables:	
Verbal description: (copy the verbal description from the card)	**Table:** (write the card number) .... **Graph:** (write the card number) .... **Equation:** ....
Explain how you know the relationship is or is not proportional. Give as many reasons as you can.	Explain what each number and letter in the equation represent.

Pause here and wait for instructions.

For the new set of cards, complete the information in the chart.

Define the variables:	
Verbal description: (copy the verbal description from the card)	**Table:** (write the card number) .... **Graph:** (write the card number) .... **Equation:** ....
Explain how you know the relationship is or is not proportional. Give as many reasons as you can.	Explain what each number and letter in the equation represent.

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You matched different representations of the same relationship. Some were proportional and some were not. The following are ways to determine whether a relationship is proportional:

- A table must have a constant ratio between each value of y and its corresponding value of x.
- A verbal description must have a constant ratio between the value of y and its corresponding value of x. The description may use words such as *constant rate*, *each*, *every*, or *per*.
- An equation must be of the form $y = kx$, where k represents the constant of proportionality.
- A graph must be a line (or points that would fall on a line) that passes through the origin.

Some relationships are not proportional. The following are ways to determine whether a relationship is nonproportional:

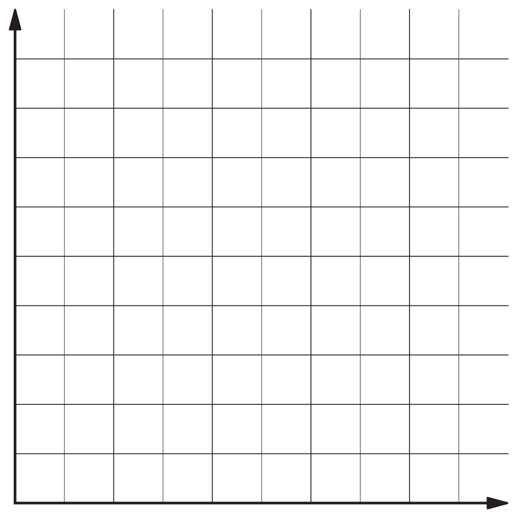
- The graph of a relationship is not a straight line that passes through the origin.
- The equation cannot be expressed in the form $y = kx$.
- The table does not have a constant of proportionality that you can multiply by any number in the first column to get the associated number in the second column.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Let c represent the cost in dollars and g represent the number of gallons of gas purchased at a neighborhood gas station. The equation $c = 2.95g$ gives the cost of gas on a particular day.



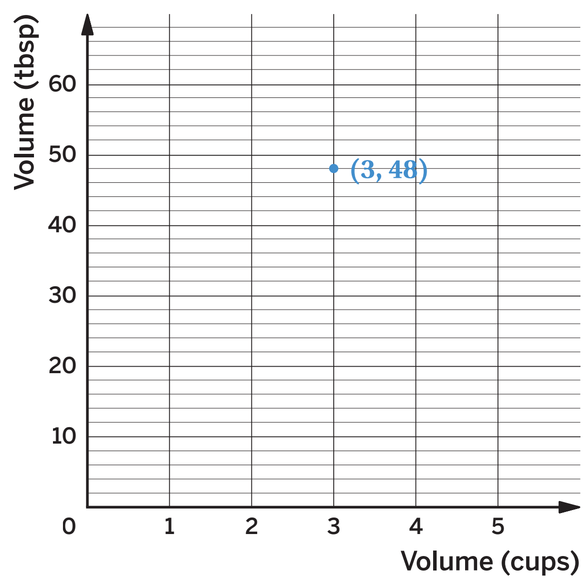
a Write four ordered pairs (gallons of gas, cost) that fit this relationship.

b Create a graph of the relationship. Label and scale your axes.

c What does the value 2.95 represent in this situation?

d Jada's mom remarks, "You can get about a third of a gallon of gas for a dollar." Is she correct? Explain your thinking.

2. There is a proportional relationship between the volume measured in cups and the same volume measured in tablespoons. The graph shows that 3 cups are equivalent to 48 tablespoons.



a Plot and label at least two more points that fit the relationship then draw a line to represent this relationship.

b For which value of y is the point $(1, y)$ on the line?

c What is the constant of proportionality for this relationship?

d Let c represent the number of cups and t represent the number of tablespoons. Write an equation representing this relationship.

Name: .... Date: .... Period: ....

Practice

3. Consider these equations and graphs.

a Match each equation with its graph.

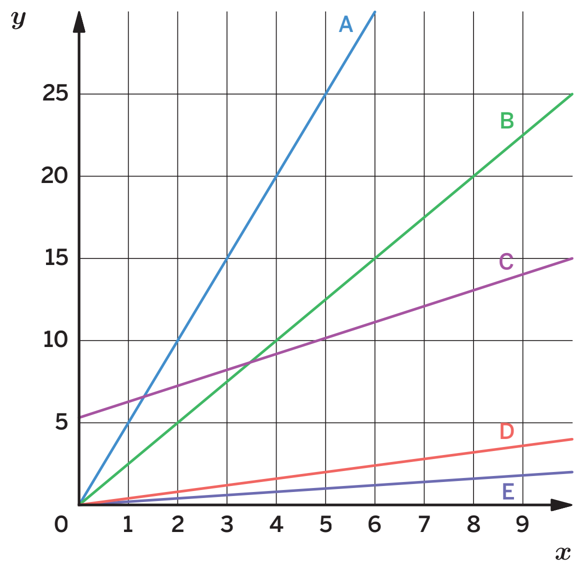
.... $y = \frac{2}{5}x$

.... $y = 5x$

.... $y = \frac{5}{2}x$

.... $y = \frac{1}{5}x$

.... $y = x + 5$



b How did you determine your matches? Explain your thinking.

4. Before your next math class, think about a restaurant where you love to eat in your area. Research some of the prices of a few of your favorite foods to order and bring this information to class with you.

5. In the fall of 2021, the 1 USD was equivalent to 1.25 CAD. Write two equations showing the relationship between US dollars, u, and Canadian dollars, c.

Welcoming Committee

Let's help a new student from another country feel welcomed and acquainted with your area.

Warm-up Pick your Partner

Several new students have just moved from other countries to your area and attend your school. Your teachers have asked you to partner up with one of these students to help them get to know your area better.

			
Name	Kanchana	Ismat	Emilia
Country	Thailand	Pakistan	Slovakia
Currency exchange rate	1 U.S. dollar = 31.32 baht	1 U.S. dollar = 165 rupees	1 U.S. dollar = 0.85 euros
Interests	playing sports, video games	swimming, reading, nature	meeting new people, fashion, art

POP-THAILAND/Shutterstock.com; Watcharin panyawutso/iStock; MarianVejcik/iStock

1. Select a student to partner with and write their name here: ______

2. How many U.S. dollars are equal to 1 of their currency?

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 The Best Place to Eat

Think of a place to eat near your home, perhaps your favorite restaurant. In this activity, you will introduce your partner to some of your favorite menu items.

Create a sample order for your partner with at least three items from the menu. Include prices in U.S. dollars and in their currency for each item, so they know how much to expect to spend.

1. Write an equation you can use to convert the price in U.S. dollars to the price in your partner's currency.

2. Complete the table with at least three items in the sample order.

Food, drink, or dessert item	Cost (U.S. dollars)	Cost (partner's currency)
Totals:		

Are you ready for more?

Though it is not customary in every culture, in the United States, it is expected that customers include a tip when served food at a restaurant. 20% of your order's total is the recommended tipping percentage for wait service at a restaurant. At this rate, how much should the tip be for your sample order in your partner's currency?

Activity 2 The Best Things to Do

Use the grid provided to draw a simplified map of the area where you live. Be sure to include a scale for your map.

1. Make a plan to visit at least three places on your map. Write your plan here.

Compare and Connect: During the Gallery Tour at the end of this activity, look for similarities in how your classmates used the math of this unit to create a schedule.

Name: ______ Date: ______ Period: ______

Activity 2 The Best Things to Do (continued)

While John Urschel was pursuing his Ph.D. in mathematics at Massachusetts Institute of Technology (MIT), he studied a version of the "traveling salesman problem." This problem involves minimizing the time it takes to travel between various locations, which you will explore in Problem 2.

2. Assume that you and your partner walk at an average speed of 1.4 yd per second. Using your plan from Problem 1, create a schedule for your day with accurate estimates for time spent doing activities and time spent traveling between activities. Show or explain your thinking.

Featured Mathematician

John Urschel

Yes, that is a photograph of a professional American football player. Meet John Urschel, a former NFL offensive lineman *and* mathematician. Urschel's research interests include data science and machine learning. He is also interested in changing how students learn math and in increasing African American and female representation in university math departments.

"John Urschel, a player on the National Football League." by Jeffrey Beall, courtesy of Wikimedia Commons (CCBY3.0)

STOP

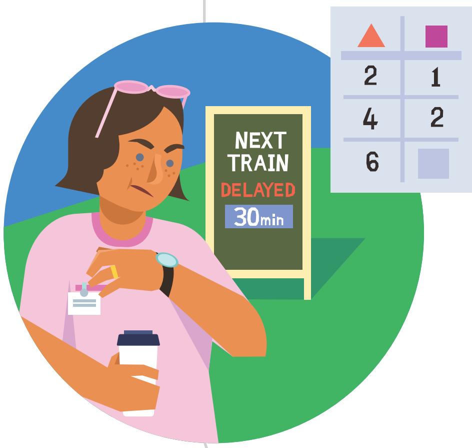
Narrative Connections

Unit Summary

It's not easy meeting someone for the first time. You might offer your name, or shake hands, but after that what should a person do?

Many people start by trying to find some commonalities. They might chat about the weather, or discuss their interests, hoping to find something to bond over. Maybe books or movies. Maybe a shared love for erhu music!

Whether it's welcoming a new kid at school or getting around in a foreign country, finding common ground is often a good way to initiate conversation.



When it comes to quantities, you can achieve that common ground through proportional relationships. When you multiply by a **constant of proportionality**, you can see how two quantities—be they pesos or dollars, grams or ounces—relate to each other.

By understanding first what we have in common, we put ourselves in a position to learn, understand, and relate.

See you in unit 3.

Practice

Name: .... Date: .... Period: ....

1. In which country is a pound of bananas cheaper? Explain your thinking.

Peru:

Cost	2 lb for 2,000 Peruvian soles
Exchange rate	1 U.S. dollar for 1,550 Peruvian soles

Chile:

Cost	3 lb for 1,800 Chilean pesos
Exchange rate	1 U.S. dollar for 600 Chilean pesos

2. If a person walks at an average speed of 1.4 m per second, is it reasonable to expect them to walk 1,000 m between 9:45 a.m. and 10:10 a.m.? Explain your thinking.

3. Write an equation for the relationship represented in each table.

a

x	y
2	14
5	35
9	63
$\frac{1}{3}$	$\frac{7}{3}$

b

x	y
3	360
5	600
8	960
12	1,440

c

x	y
75	3
200	8
1,525	61
10	0.4

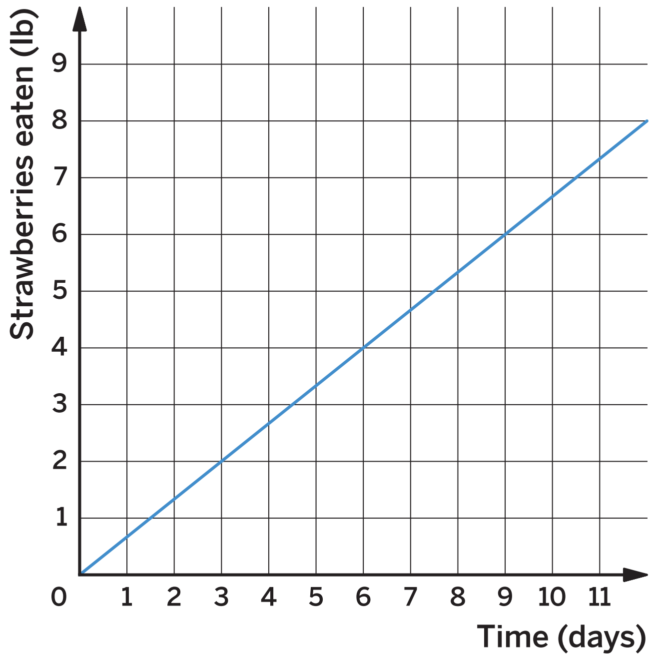
d

x	y
4	10
6	15
22	55
3	$7\frac{1}{2}$

Practice

Name: ______________________ Date: __________ Period: __________

4. Represent the proportional relationship shown in the graph in different ways.



a In a table:

Time (days)	Strawberries eaten (lb)

b With an equation:

c With a story:

5. On its way from Los Angeles to Boston, a plane traveling at a constant speed flew over Las Vegas, Denver, Cedar Rapids, and Detroit. Complete the table.

Segment	Time	Distance (miles)	Speed (mph)
Las Vegas to Denver	1 hour	575	
Denver to Cedar Rapids	1 hour 12 minutes		
Cedar Rapids to Detroit		460	

My Notes:

UNIT 3

Measuring Circles

Some have said that the only place to find a perfect circle is inside the world of math. Explore the many circles and circular shapes both in *and* outside of this unit to reveal some of their mysteries.

Essential Questions

- How do we measure circles when all of our tools are straight?
- What is π and what does it have to do with circles?
- How can squares help you measure the space inside circles?
- *(By the way, does π really go on forever?)*

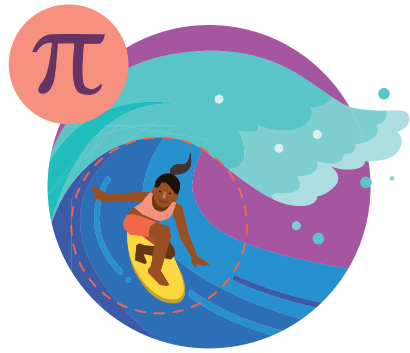
SUB-UNIT

1 | Circumference of Circles

Narrative: Explore ways to measure a circle — a shape that has fascinated humans for thousands of years.

You'll learn . . .

- how some measures of a circle are proportional.
- how to approximate the distance around a circle.



SUB-UNIT

2 | Area of Circles

Narrative: Discover how one of the most fascinating constants of proportionality, π, is related to the area of a circle.

You'll learn . . .

- the relationship between a circle's circumference and area.
- how to approximate the area of a circle.

The area of the clock is 42 square units. What is the combined areas of the shaded regions?

Unit 3 | Lesson 1 – Launch

The Wandering Goat

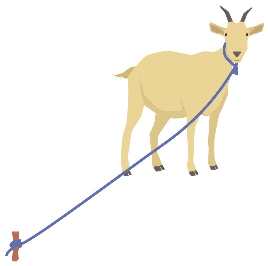
Let's explore how far a goat on a rope can roam.

Warm-up The Goat Problem (Part 1)

A famous recreational math problem — known as "The Goat Problem" — was first published in *The Ladies' Diary* in 1748. A variation of this problem asks the question, "If a goat were tethered by a rope to a peg, what space could the goat occupy (where could it roam)?"

1. Choose one person to act as the goat and one person to act as the peg. Using the string provided by your teacher, explore what space the goat could roam in while it is attached to the peg.

2. Sketch the shape of the space where the goat would be able to roam while tethered to a rope that is attached to the peg.



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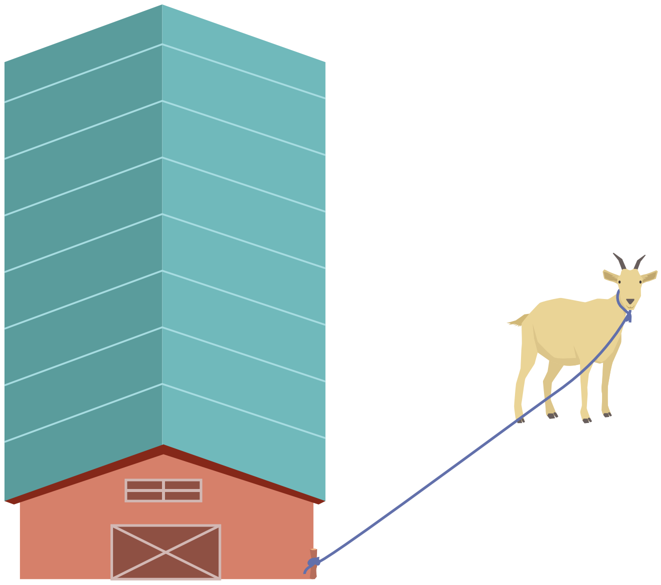
Name: .... Date: .... Period: ....

Activity 1 The Goat Problem (Part 2)

Now imagine that the goat is tethered to a rope that is attached to the corner of a rectangular barn. Assume the rope is shorter than both the length and the width of the barn. You will be given the materials needed for this activity.

1. Using the materials provided and objects around the classroom, come up with a plan to model this situation. Describe your plan in the space provided.

2. Sketch the shape of the space in which the goat would be able to roam while tethered to a rope that is attached to the corner of the barn.

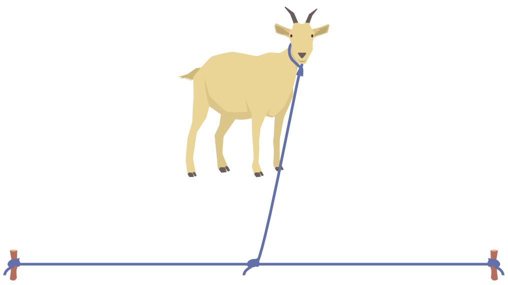


Activity 2 The Goat Problem (Part 3)

Now imagine that two pegs are in an open field and are connected by a bar. The goat's rope is attached by making a loop so that it can slide along the bar attaching the two pegs. You will have access to the same materials used in the previous activity.

1. Using the materials provided and objects around the classroom, come up with a plan to model this situation. Describe your plan in the space provided.

2. Sketch the shape of the space in which the goat would be able to roam, while tethered to the rope between the two pegs.



STOP

Unit 3 Measuring Circles

‘Round and ‘Round We Go

Circles appear in countless places in the world around us, from the very large to the very small. When pioneering American astronauts like John Glenn and Sally Ride looked out of their spacecraft, their view was dominated by Earth’s circular profile. And when working with some of the earliest computers, mathematicians from John von Neumann to Edith Windsor were surrounded by the circular outlines of vacuum tubes, resistors, capacitors, and more.

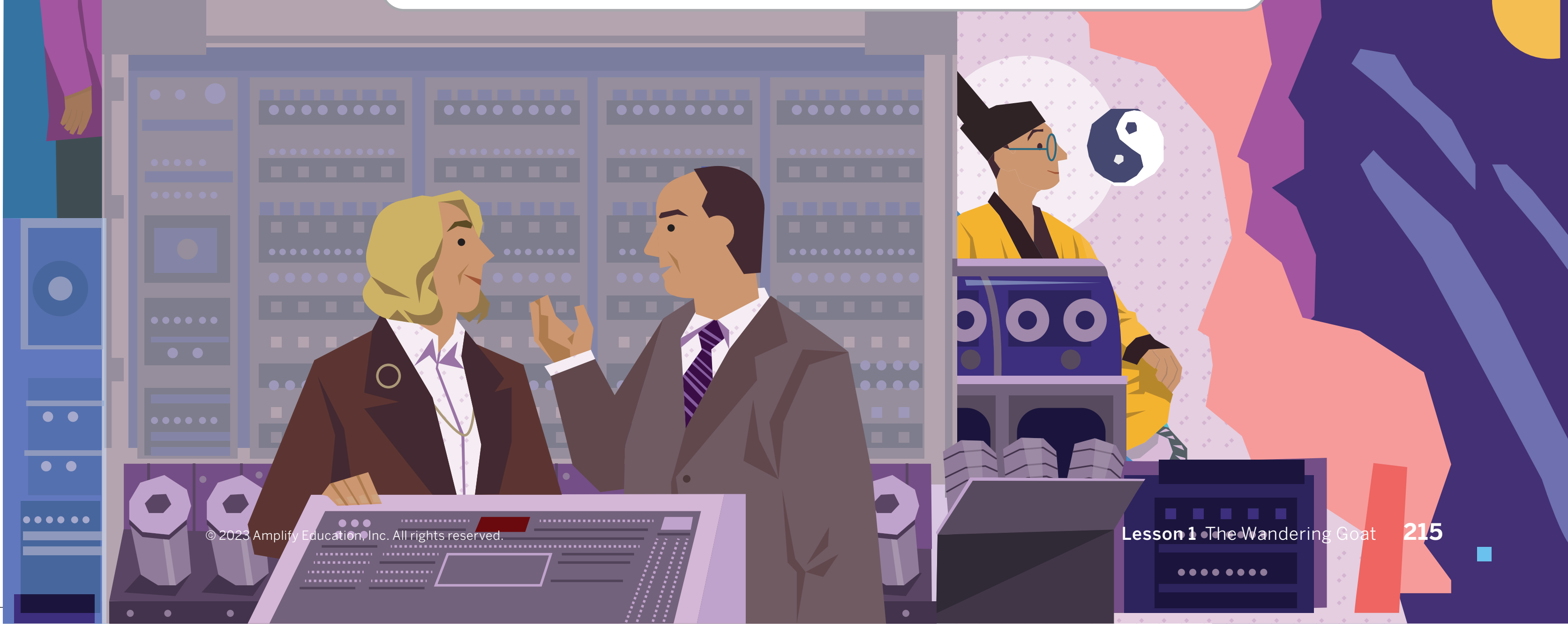
At the same time, no shape is more elegant, more simple, or more symmetric than the circle.

Among indigenous peoples of the Great Plains of North America, there is the “sacred hoop” — a circle with spokes pointing in the four cardinal directions, representing the cycles of nature. In China, the Taoist concept of “yin” and “yang” — two opposing, but interconnected forces — come together to form a circle. And across the United States, circles can represent the eternity of marriage, from engagement rings to circular diamond pins.

Given the circle’s beauty and near-universal nature, it may be surprising that the mathematical systems you have seen (so far) don’t work well with circles. For example, it is far easier to draw a rectangle on graph paper than it is to draw a circle. And yet, from wheels to planetary orbits (but more on shapes called “ellipses” in high school!), circular shapes are central to humankind.

So, without further ado, let’s find out what, exactly, makes this shape go ‘round.

Welcome to Unit 3.

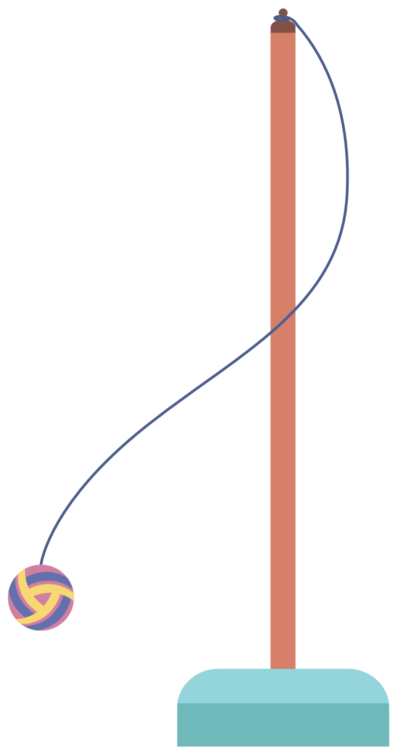


Practice

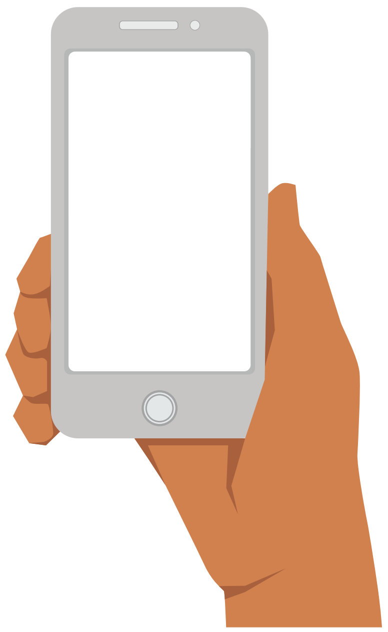
Name: .... Date: .... Period: ....

1. Tetherball is a game played between two players who stand across from each other on opposite sides of a tall pole. There is a ball tethered to the pole by a chain, and players hit the ball back and forth in opposite directions. During the game the players must stay within the tetherball court.

   The court encompasses the space in which the ball could travel during the game. Sketch the shape of the court.



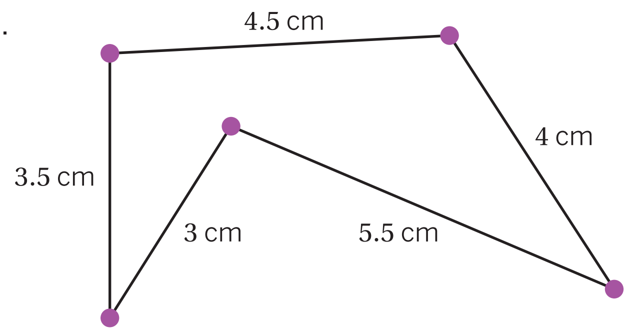
2. Mark the space on the smartphone that could be reached by the person's thumb without moving the rest of their hand. Explain your thinking.



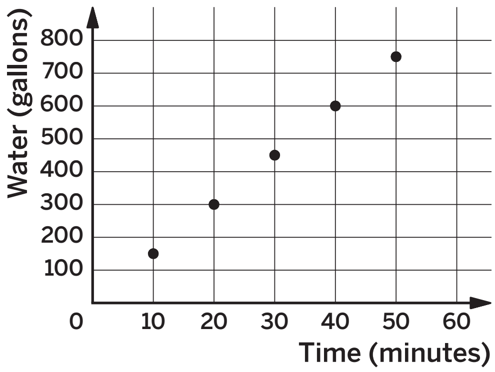
Name: .... Date: .... Period: ....

Practice

3. Determine the perimeter of the polygon.



4. The graph shows that while it was being filled, the number of gallons of water in a swimming pool was approximately proportional to the time that had passed in minutes.



a. Estimate the constant of proportionality that gives the number of gallons of water per minute.

b. About how much water was in the pool after 25 minutes?

c. Approximately how many minutes have passed when there were 500 gallons of water in the pool?

5. Go through your home in search of each of the following. List your responses and explain your thinking for each.

a. Two objects that you would describe as round, but not circular.

b. Two objects that are round and circular.

My Notes:

SUB-UNIT

1 Circumference of Circles

Narrative Connections

Why do aliens love circles?

In the United Kingdom, farmers have woken up to find portions of their field flattened into circular patterns. These patterns range anywhere from a single circle approximately 6 feet in diameter, to patterns with more than 400 circles spanning 1,000 feet.

These strange patterns are called crop circles. For decades, they have stirred up people's imaginations. Some consider them evidence of alien life. Others call them an elaborate hoax.

In 1991, artists Doug Bower and Dave Chorley took responsibility for at least 200 British crop circles dating back to 1978. But that still left thousands of circles unaccounted for around the world.

Most observers attribute these circles to copycat artists. Others still insist that aliens are among us!

Whoever the makers of intricate patterns are, circles have fascinated people—and any other intelligent beings that happen to be on this planet—for centuries.

Unit 3 | Lesson 2

Exploring Circles

Let's explore circles.

Warm-up A Perfect Circle

Without tracing an object, draw the best circle you can on a separate piece of paper. Write your name on the back of your paper.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Card Sort: Round Objects

You will be given a set of cards.

1. Study the image on each card. Sort the cards into groups of objects that are either circles or not circles.

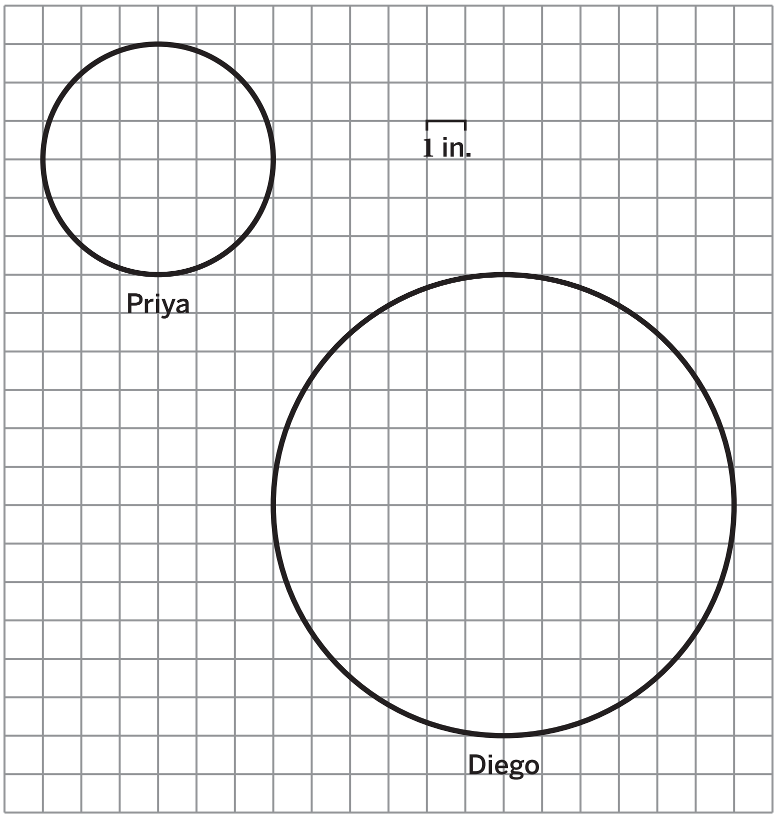
Circles	Not circles

2. For each card that you classified as *not a circle*, explain your thinking.

3. For the group of cards that you classified as circles, determine what characteristics they have in common. Think about what was "wrong" with the images on the cards you described as *not circles* in Problem 2.

Activity 2 Measuring Circles

Priya and Diego each claimed that they drew a 6 in. circle. What do you think they each meant? Explain your thinking.



Historical Moment

Why do we say "radius" and "diameter"?

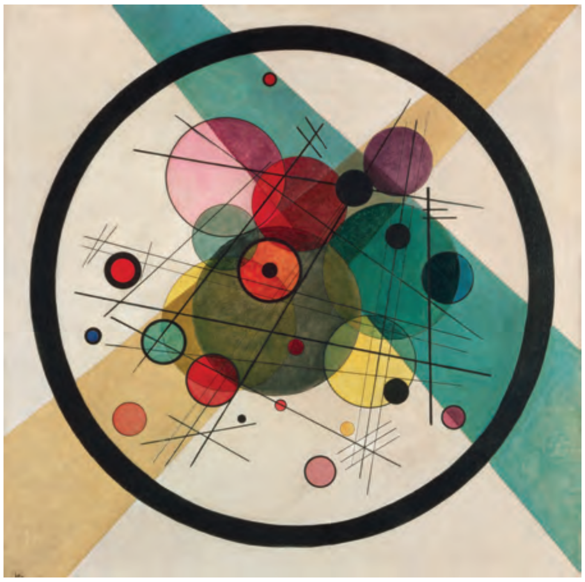
Today, you learned that the *radius* of a circle is the line segment, or the distance, from the center of a circle to a point on the circle. The term *radius* has Latin roots and originally referred to both a ray and to the spoke of a chariot's wheel. Due to its Latin roots, the plural of radius is *radii* (although, sometimes English speakers say "radiuses").

The term *diameter*, on the other hand, has Greek roots. It is composed from the words *dia* meaning "across or through," and *metron*, meaning "measure." Literally, it can be translated as "measure across," which is exactly what the diameter allows us to do on a circle.

Name: .... Date: .... Period: ....

Activity 3 Art of the Circle

Vasily Kandinsky was a 20th-century Russian abstract painter. Among his works is a painting titled *Circles in a Circle* which consists of 26 overlapping circles contained within one larger black circle. In order to create this geometric design, Kandinsky used both a ruler and a compass to aid him in his composition.



Vasily Kandinsky, Circles in a Circle, 1923. Oil on canvas. 38 7/8 × 37 5/8 inches. The Louise and Walter Arensberg Collection, 1950

Your goal is to create your own Kandinsky-inspired piece using a compass, ruler, and pencil that meets the following criteria. Complete the labels as you complete your drawing.

Criteria:

- The largest circle should start in the center of the page (marked with a dot) and have a diameter of 7 in. Label this circle as Circle A.
- Within the large circle, you should have four additional circles. Label these circles as Circles B–E.
- One of the interior circles should not overlap with any other circle. This is labeled as Circle .... on my drawing.
- Two of the interior circles *must* overlap. These are labeled as Circles .... and .... on my drawing.
- One circle should have a radius of 1.5 in. This is labeled as Circle .... on my drawing.
- One circle should have a diameter of 2 in. This is labeled as Circle .... on my drawing.
- Two of the circles should have the same center. These are labeled as Circles .... and .... on my drawing.

Collect and Display: At the end of this activity, you will describe how your class' art pieces are similar and different. Your teacher will add the math language you use to a class display that you can refer to throughout this unit.

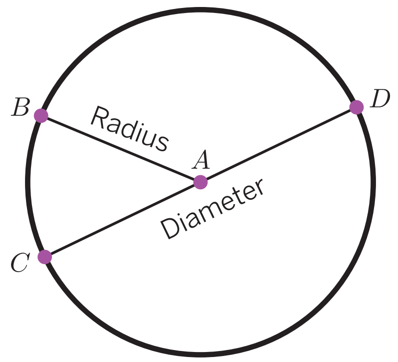
STOP

Summary

In today's lesson . . .

You used your background knowledge about circular objects to define a ***circle*** as a shape that is made up of all the points that are the same distance from the ***center of the circle***. This distance has a special name: ***radius***.

Another way to describe the size of a circle is to measure the distance across the middle. This distance is called the ***diameter***.



A radius can refer to:

- A line segment that connects the center of a circle with a point on the circle.
- The length of this segment.

Segments AB, AC, and AD are all radii of Circle A.

A diameter can refer to:

- A line segment with endpoints on the circle, that passes through its center.
- The length of this segment.

Segment CD is the diameter of Circle A.

For any circle, all radii are equal length and the diameter is twice the length of the radius.

Reflect:

Name: ________ Date: ________ Period: ________

Practice

1. Use a compass to draw a circle in the space provided.

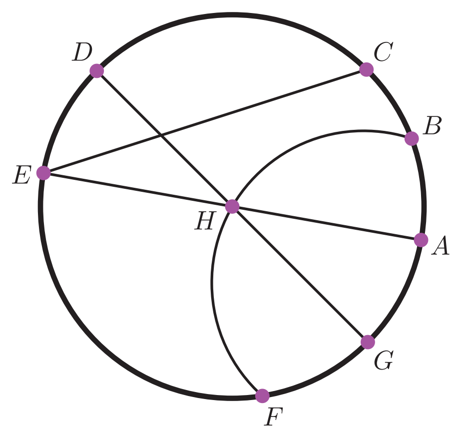
   a. Draw and label a radius on your circle.

   b. What is the length of the radius of your circle?

   c. Draw and label a diameter on your circle.

   d. What is the length of the diameter of your circle?

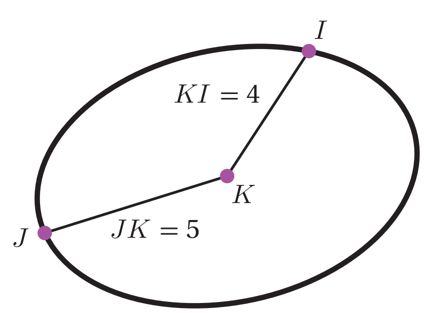
2. A circle with a center at point H is shown. Use the diagram for parts a and b.



   a. Identify *all* the diameters. Explain your thinking.

   b. Identify *all* the radii. Explain your thinking.

3. Lin was asked by her teacher to create a circle and draw a diameter. Her drawing is shown. Identify and describe her mistakes.



Practice

Name: ______ Date: ______ Period: ______

4. A small batch of lemonade is made by mixing $\frac{1}{4}$ cup of sugar, 1 cup of water, and $\frac{1}{3}$ cup of lemon juice. After confirming it tastes good, a larger batch will be made using the same ratio of ingredients.

a Using 10 cups of water, how much sugar should be added so that the larger batch tastes the same as the smaller batch?

b Using $3\frac{1}{2}$ cups of water, how much sugar should be added so that the larger batch tastes the same as the smaller batch?

5. The graph of a proportional relationship contains the point $(3, 12)$. What is the constant of proportionality of the relationship?

6. The scale factor that maps Polygon A onto Polygon B is 3.

a The perimeter of Polygon B is 90 cm, what is the perimeter of polygon A? Show or explain your thinking.

b Polygons A and B are both regular hexagons. Determine the side length of one side in each polygon. Show or explain your thinking.

c If the area of Polygon A is about 65 cm^2, determine the approximate area of Pentagon B. Show or explain your thinking.

Name: ________________ Date: ________ Period: ________

Unit 3 | Lesson 3

How Well Can You Measure?

Let's see how accurately you can measure.

Warm-up Measuring a Square

You will be given a card with a square and measuring tools. Complete the table for the square on your card. Check all measurements with your group before finalizing your responses.

	Side length (cm)	Diagonal (cm)	Perimeter (cm)
Card 1			
Card 2			
Card 3			
Card 4			
Card 5			
Card 6			
Card 7			
Card 8			

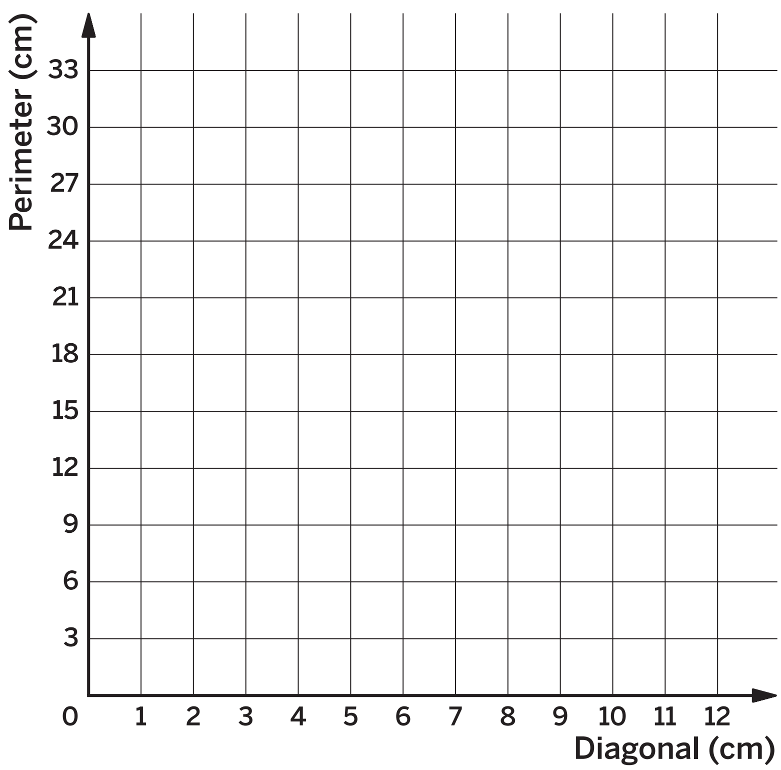
Log in to Amplify Math to complete this lesson online.

Activity 1 Diagonal and Perimeter

Refer to the table of data collected during the Warm-up as you complete this activity.

1. Is there a constant of proportionality for the relationship between the perimeter and the diagonal of each square? Explain your thinking. Complete your calculations in the space provided or next to your table in the Warm-up. Round all values to the nearest hundredth.

2. Plot the relationship between the length of the diagonal and the perimeter of each square as an ordered pair (diagonal, perimeter).



Name: ______ Date: ______ Period: ______

Activity 1 Diagonal and Perimeter (continued)

3. What do you notice about the graph?

4. Based on your responses from Problems 1 through 3, is the relationship between the perimeter and diagonal of a square proportional? Explain your thinking.

Reflect: How were you able to control your impulses and discipline yourself to work toward your goals?

Are you ready for more?

Consider this question: Is there a proportional relationship between the area of a square and the length of its diagonal?

1. Make a prediction. Explain your thinking.

2. Test your prediction by calculating the area of three different squares from the Warm-up and the ratio of the area of each square to the length of its diagonal.

3. Was your prediction correct? Explain your thinking.

Activity 2 Missing Parts of a Square

Use the relationship between a square's side length, the length of its diagonal, and its perimeter to respond to these problems. Round your values to the nearest hundredth, unless otherwise indicated.

1. What is the constant of proportionality that gives the:
   a. Perimeter, when the diagonal is known, based on Activity 1?
   b. Side length, when the perimeter is known?

Diagonal	Perimeter	Side length
5 cm		
	60 ft	
		3 in.

2. Complete the missing values in the table.

3. Based on the information in the table, what is the approximate constant of proportionality that gives the side length of a square if you know the diagonal? Round your response to the nearest hundredth.

4. A certain square has a diagonal of 12 units. Two methods are shown for calculating the side length. Explain why *both* methods are correct even though they result in slightly different values.

Method 1	Method 2
$12 \cdot 2.83 = 33.96$ $33.96 \cdot 0.25 = 8.49$ The side length is 8.49 units.	$12 \cdot 0.71 = 8.52$ The side length is 8.52 units.

STOP

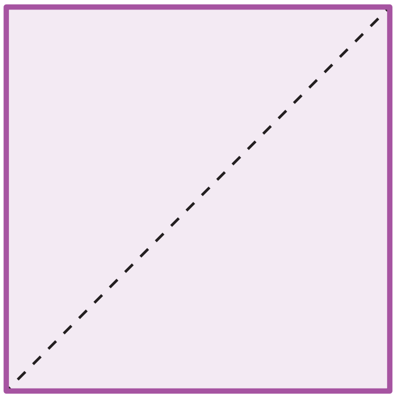
Name: ____________ Date: ____________ Period: ____________

Summary

In today's lesson . . .

You used your prior experience in working with scaled figures and proportional relationships to approximate the constant of proportionality between the perimeter and the diagonal length of any square to be around 2.83. By graphing the measurements of a variety of square side lengths, you saw that coordinate pairs (diagonal, perimeter) form a line that passes through the origin. You determined that due to inaccuracies in measurement, this was sufficient evidence to conclude proportionality.

You used your approximation to reason about the lengths of the sides, the diagonal, and the perimeter of different squares. For example, if you know that the perimeter of this square is 200 units, you can approximate the diagonal and calculate the side length:



Diagonal:

$$200 \div 2.83 = d$$
$$70.67 \approx d$$

Side length:

$$200 \div 4 = s$$
$$50 = s$$

The diagonal is approximately 70.67 units and the side length is 50 units.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

Round all responses to the nearest hundredth.

1. Estimate the side length of a square that has a diagonal of 9 cm.

2. Select *all* of the relationships that are proportional.

A. The side length and the perimeter of a square.

B. The area and the perimeter of a square.

C. The side length and the diagonal length of squares.

D. The perimeter and the diagonal length of a square.

E. The diagonal length and the area of a square.

3. Diego created a graph of two quantities that he measured and said, "The points all lie on a line except one, which is a little bit above the line. This means that the quantities cannot be proportional." Do you agree or disagree with Diego? Explain your thinking.

4. Here are several recipes for making sparkling lemonade. For each recipe, describe how many tablespoons of lemonade mix are required per cup of sparkling water.

a 4 tbsp of lemonade mix and 12 cups of sparkling water

b 4 tbsp of lemonade mix and 6 cups of sparkling water

c 3 tbsp of lemonade mix and 5 cups of sparkling water

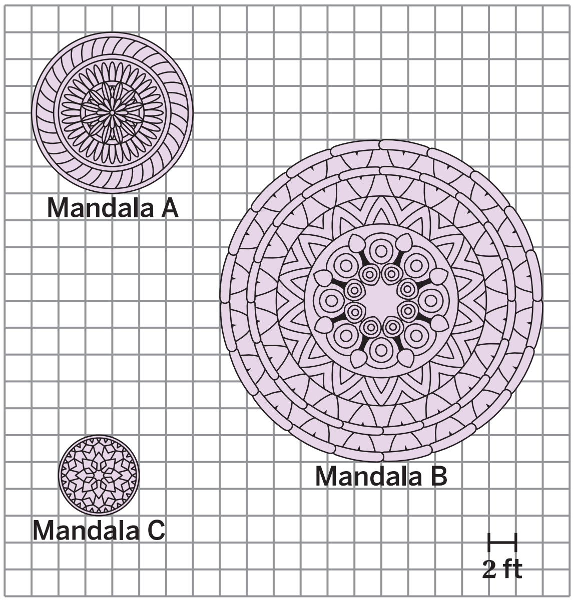
d $\frac{1}{2}$ tbsp of lemonade mix and $\frac{3}{4}$ cups of sparkling water

Practice

Name: ________ Date: ________ Period: ________

5. A mandala is a geometric figure that has spiritual relevance in many religions, including Hinduism and Buddhism. The word, *mandala*, is Sanskrit for *circle*. Priya recently designed the three circular mandalas shown. Determine the diameter and radius for each mandala.

	Radius (ft)	Diameter (ft)
A		
B		
C		



6. Complete the table that gives the side lengths and related perimeters for several equilateral triangles.

Side length	Perimeter
3	
10	
	81
	315

a. Explain why the relationship between the perimeter and side length for an equilateral triangle is proportional.

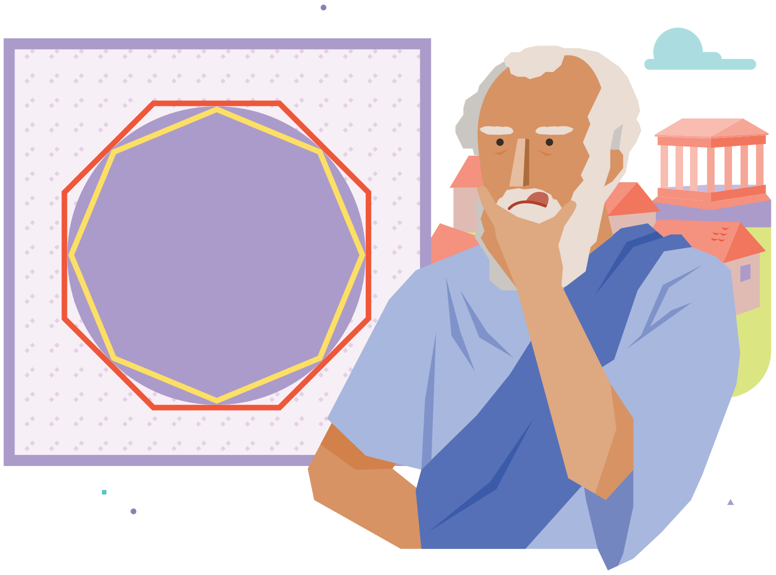
b. What is the constant of proportionality of the relationship?

c. Write an equation for determining the perimeter P of an equilateral triangle that has a side length of s.

Unit 3 | Lesson 4

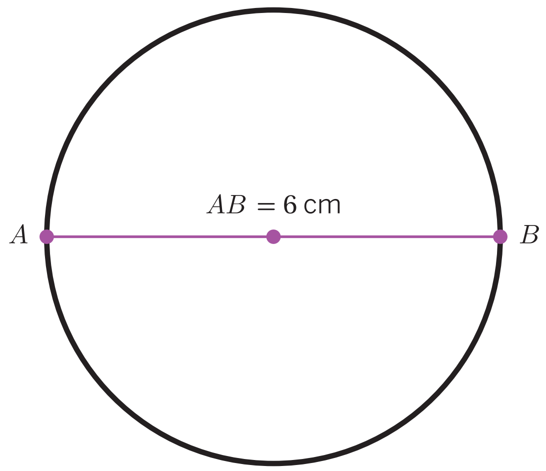
Exploring Circumference

Let's explore the distance around a circle.



Warm-up Measuring a Circle

Use the diagram to respond to these questions.



1. What is the diameter of the circle?

2. What is the radius of the circle?

3. What would you estimate the distance around the circle to be? **Note:** It is okay to guess.

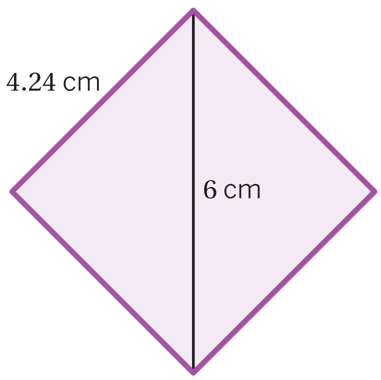
Name: ________________ Date: ________ Period: ________

Activity 1 Refining Your Estimation

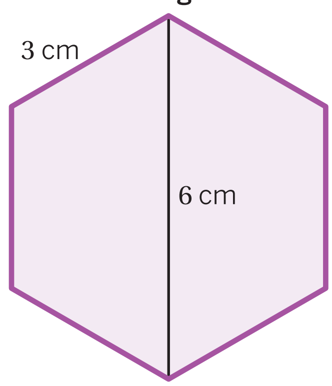
Around the year 250 BCE, Archimedes calculated the perimeter of regular polygons with the same diagonal length to help him approximate the *circumference* of a circle. Let's see how you might refine your estimation for the circumference of the circle in the Warm-up using Archimedes' method.

1. Each of the following regular polygons has a diagonal that measures 6 cm. Complete the table of values using the information from each diagram. Round to the nearest hundredth, if necessary.

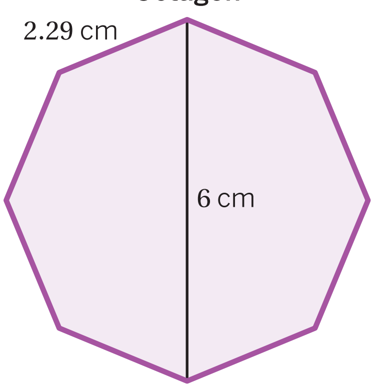
Square



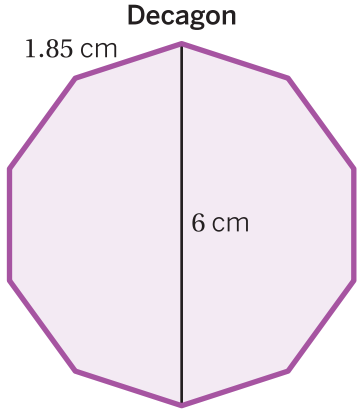
Hexagon



Octagon



Decagon



	Diagonal (cm)	Perimeter (cm)	$\frac{\text{Perimeter}}{\text{Diagonal}}$
Square	6		
Hexagon	6		
Octagon	6		
Decagon	6		

Co-craft Questions: Pause here and work with your group members to write 2–3 questions you have about the values in this table.

Activity 1 Refining Your Estimation (continued)

2. What do you notice about the values in the table?

3. Refine your prediction for the circumference of the circle that you made during the Warm-up. Record your new prediction in the table along with the ratio of your predicted circumference to the diameter. Explain your thinking.

Diameter (cm)	Circumference (cm)	$\frac{\text{Circumference}}{\text{Diameter}}$
6		

Featured Mathematician

Archimedes

Archimedes of Syracuse was a Greek mathematician (as well as an inventor, astronomer, and engineer!) who lived in the 3rd century BCE. He wrote "Measurements of a Circle," a treatise that discussed the relationship between the area, diameter, and circumference of a circle. He used the perimeters of two 96-sided regular polygons — one surrounding a circle and the other surrounded *by* the circle — to conclude that π must be between $3\frac{10}{71}$ and $3\frac{1}{7}$. You will learn more about π in the next activity.

Fetti, Domenico. "Archimedes" 1620. Oil on Canvas. Alte Meister Museum, Dresden. Courtesy of Wikimedia Commons, (https://commons.wikimedia.org/wiki/File:Domenico-Fetti_Archimedes_1620.jpg). Public Domain.

Name: ______ Date: ______ Period: ______

Activity 2 Measuring Circumference and Diameter

You will be given several circular objects.

1. Examine your circular objects. What tools or methods could you use to help you measure the circumference and diameter of each object?

2. What are some challenges that you might encounter in trying to measure your object?

Pause here and wait for further instructions from your teacher.

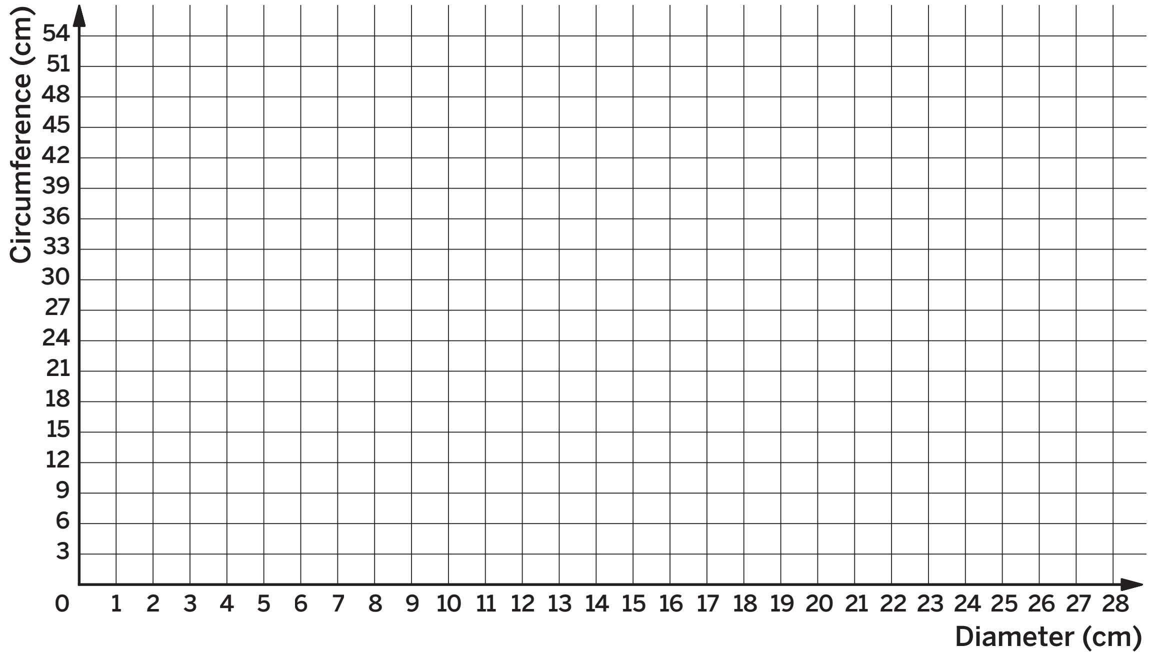
3. Using the digital tool, or the objects provided by your teacher, complete the table for four circles.

	Diameter (cm)	Circumference (cm)
Circle 1		
Circle 2		
Circle 3		
Circle 4		

4. What do you predict a graph of your table of values would look like? Explain your thinking.

Activity 2 Measuring Circumference and Diameter (continued)

5. Plot the ordered pairs (diameter, circumference) from your table on the coordinate plane.



6. Does your graph match the prediction you made in Problem 4? Explain your thinking.

7. Calculate the ratio of each circle's circumference to its diameter, rounding to the nearest thousandth. What would you approximate the constant of proportionality that gives the circumference if you know the diameter?

8. Use your response from Problem 7 to refine your estimation for the circumference of a circle with a diameter of 6 cm.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that the distance around a circle is called its ***circumference***. In the same way that there is a proportional relationship between the diagonal of a square and its perimeter, there is also a proportional relationship between the diameter of a circle and its circumference.

For any diameter d, you can calculate the circumference C using the formula $C = kd$, where k represents the constant of proportionality. For this relationship, k is a value somewhat greater than 3 and is represented by the Greek letter ***π (pi)***. Thus, the relationship is represented by the equation $C = \pi d$.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Diego measured the diameter and circumference of several circular objects and recorded his measurements in the table. One of his pairs of measurements is impossible. Which object has the impossible measurements? Explain your thinking.

	Diameter (cm)	Circumference (cm)
Half dollar coin	3	10
Flying disc	23	27
Jar lid	8	25
Flower pot	15	48

2. Explain whether each pair of measurements could be a reasonable approximation for the diameter and circumference of a circle. Explain your thinking.

a 5 m and 22 m

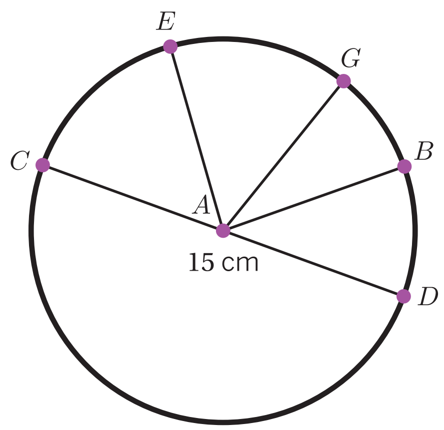
b 19 in. and 60 in.

c 33 cm and 80 cm

3. Point A is the center of the circle, and the length of segment CD is 15 cm.

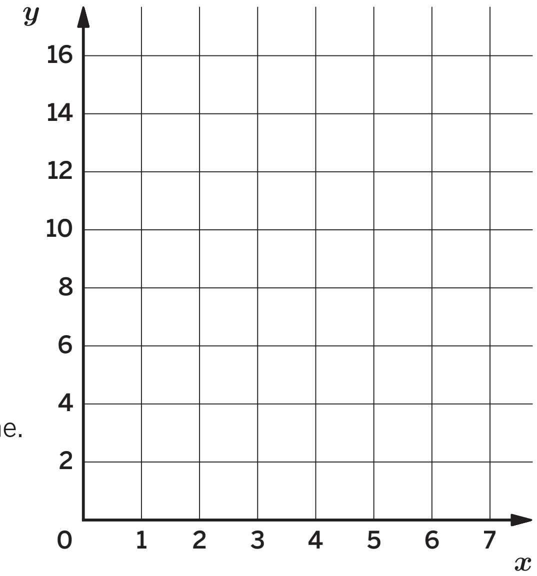
a Name a segment that is a radius. What is its length?

b Name a segment that is a diameter. What is its length?



Name: ______________________ Date: __________ Period: __________

4. Consider the equation $y = 1.5x + 2$.

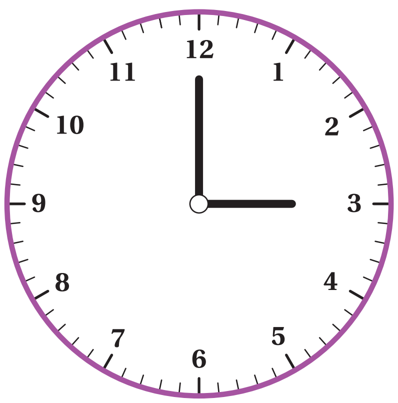


a Determine four pairs of x- and y-values that make the equation true.

b For the values you determined in part a, plot the ordered pairs (x, y) on the coordinate plane.

c Based on the graph, can this be a proportional relationship? Why or why not?

5. The minute hand of a circular clock measures 4 in. The distance from the end of the minute hand to the outer edge of the clock is 2 in.



a What is the radius of the clock?

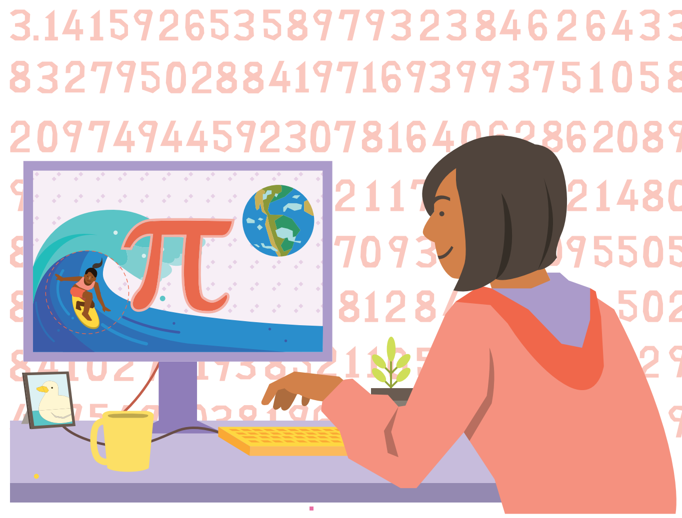
b What is the diameter of the clock?

c What is the approximate circumference of the clock?

Unit 3 | Lesson 5

Understanding π

Let's explore different approximations of π.



Warm-up Reasonable or Unreasonable?

Suppose a circle has a radius of 5 cm. Which of the following could be a reasonable approximation of the circumference of the circle? Select *all* that apply.

A. 7.8 cm

B. 15.5 cm

C. 10 cm

D. 31.4 cm

E. 5π cm

F. 10π cm

Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Approximating π

Throughout history, various civilizations have chased the most accurate approximation of π. You will be given a card with one of these approximations.

1. Use the historical approximation of π on your card to complete the missing values in the table. Round each value to the nearest thousandth.

   Card ________

Radius	Diameter	Circumference
3		
110		
	1,374	

Pause here and wait for further instructions from your teacher.

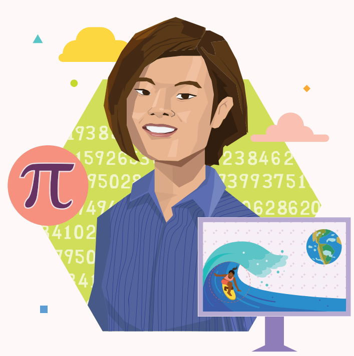
In 2019, Emma Haruka Iwao, became the first person to use cloud computing to break a world record for calculating the digits of π, computing over 31.4 trillion digits.

2. Using the π button on your calculator, determine a more accurate approximation for each missing value in the table. Round to the nearest thousandth.

Radius	Diameter	Circumference
3		
110		
	1,374	

3. Compare your tables from Problem 1 as a group. Which of the approximations discussed seems to be the closest approximation of π?

Featured Mathematician



Emma Haruka Iwao

At age 12, Emma Haruka Iwao became fascinated with π and computers' abilities to calculate billions of digits of π. In 2019, Iwao smashed the previous record for computing digits of π by 9 trillion digits. She became the first woman to hold the Guinness World Record™ for calculating digits of π, with over 31.4 trillion digits (31,415,926,535,897, to be exact). She used an application called "y-cruncher" and 25 virtual computers. The calculations took 121 days and 170 terabytes of data to complete.

Activity 2 Approximating Circumference

NASA uses 15 digits in their approximation of π for calculations involving interplanetary travel. For most earthly calculations, approximations of 3, 3.14, or $\frac{22}{7}$ are usually sufficient. For each problem, determine which approximation of π you think is most appropriate before approximating the circumference.

1. A satellite follows a circular path around Earth. The distance between the center of Earth and the satellite is approximately $42{,}000$ km. Rounded to the nearest ten-thousand kilometers, about how far does the satellite travel to complete one rotation around Earth?

a Choose an approximation of π: 3, 3.14, or $\frac{22}{7}$ Explain your choice.

b About how far does the satellite travel? Round to the nearest ten-thousand kilometers.

2. In the Contemplative Court at the Smithsonian's National Museum of African American History and Culture, there is a waterfall in which water falls 30 ft from a circular opening in the ceiling. The circular opening in the ceiling is 20 ft wide. What is the approximate circumference of the opening?


Lewis Tse Pui Lung/Shutterstock.com

a Choose an approximation of π: 3, 3.14, or $\frac{22}{7}$. Explain your choice.

b What is the approximate circumference of the opening? Round to the nearest foot.

STOP

Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You explored the different historical approximations of π. While the exact value of π is a non-ending, non-repeating decimal, you can use approximations such as 3.14 or $\frac{22}{7}$ to solve problems about the relationships between a circle's diameter, or radius, and its circumference.

For example, if a circle has a diameter of 10 cm, the most accurate measurement of its circumference is 10π cm. Writing the circumference in terms of π is actually referred to as the "exact value" of the circumference. To approximate this value as a decimal, you can calculate $10 \cdot 3.14$ and say that the circumference is approximately 31.4 cm.

Reflect:

Practice

Name: .... Date: .... Period: ....

Unless otherwise noted, round all responses to the nearest hundredth.
Note: You will need a compass for Problem 4.

1. Complete the table using one of the approximations of π you learned in this lesson. Circle the approximation you chose. Round to the nearest hundredth.

3.14 3 $\frac{22}{7}$

	Diameter	Circumference
Hula hoop	35 in.	
Circular pond	177.07 ft	
Magnifying glass	5.2 cm	
Car tire	22.8 in.	

2. Complete the table using the π button on your calculator. Round to the nearest hundredth, if necessary.

	Radius	Diameter	Circumference
Wagon wheel		3 ft	
Airplane propeller	24 in.		
Orange slice		6.5 cm	

3. A decorative border around a watch face measures 10 cm. Select *all* of the following that could represent a possible length of the minute hand on the watch. Explain your thinking.

A. 3 cm

B. 1.5 cm

C. 1.67 cm

D. 1 cm

E. 2 cm

Practice

Name: ____ Date: ____ Period: ____

4. A cyclist rode 6.75 miles in 0.9 hours.

   a. How fast did she travel, in miles per hour?

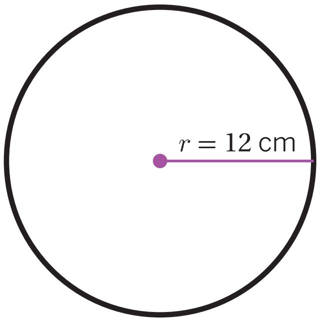
   b. If she continued to travel at the same speed, how long would it take her to travel 30 miles?

5. Select *all* of the expressions that are equivalent to the expression $x \bullet x \bullet x \bullet x \bullet x$.

   A. $5x$
   B. $x^2 \bullet x^3$
   C. x^5
   D. $2x \bullet x \bullet x \bullet x$
   E. $x^2 \bullet x^2 \bullet x$
   F. $5x^5$

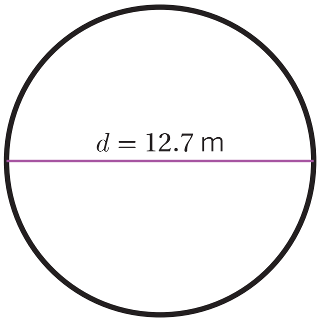
6. Determine the radius, diameter, and circumference of each circle.
   Note: The figures may not be drawn to scale.

   a.



   Radius: ____
   Diameter: ____
   Circumference: ____

   b.

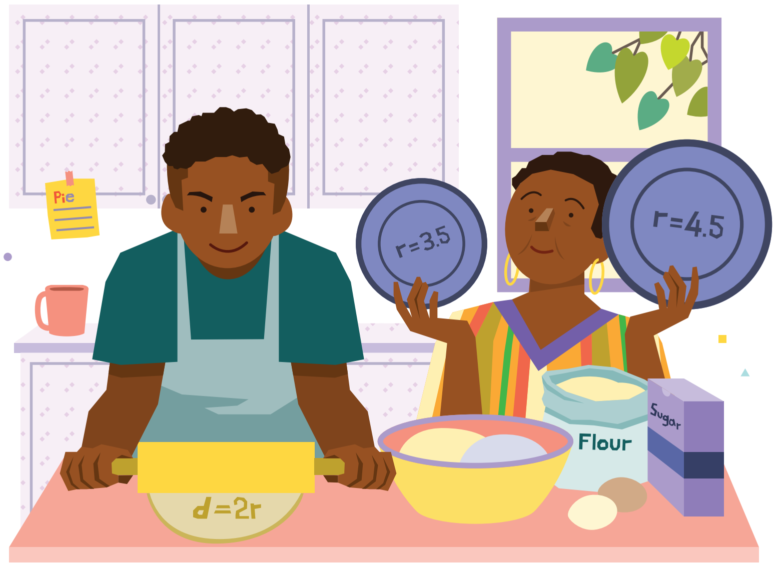


   Radius: ____
   Diameter: ____
   Circumference: ____

Unit 3 | Lesson 6

Applying Circumference

Let's use π to solve problems.



Warm-up Measured Exactly

Priya's grandfather keeps mowing over her grandmother's flower beds! She wants to build a stone border around each one to protect them from the blades of the lawnmower. Determine the number of feet of stone needed to surround each flower bed. Show your thinking.

1.

2.

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Practice With π

For each table, use the given information to determine the missing lengths. Leave your response for each length in terms of π — the most accurate measurement.

1.

Circumference	Diameter	Radius
80π		

Show your thinking:

2.

Circumference	Diameter	Radius
80		

Show your thinking:

3. What was similar about the values in the tables? What was different?

Historical Moment

But why π?

Much like we use just a few letters to represent entire phrases when we type and text today, mathematicians have been using letters and symbols to represent ideas for centuries (OMG, that's crazy, right?!). As early as 1647 CE, π was used to represent the Greek word for *periphery*, or circumference, of a circle. It wasn't until 1706 CE, that Welsh mathematician William Jones published the first mathematical paper using π to represent the ratio of the circumference to the diameter of a circle. It wasn't adopted widely by mathematicians until the end of the 1700s after it was popularized by German mathematician Leonard Euler (pronounced "Oy-ler").

Activity 2 Card Sort: Radius, Diameter, or Circumference

You will be given a set of cards.

1. Read the scenario described on each card. Determine whether the problem is asking you to determine the length of the radius, diameter, or circumference. Record the card numbers in the table here.

Radius	Diameter	Circumference

2. Choose one card from each category and solve the problem on the card. Use the π button on your calculator and round your responses to the nearest hundredth.

Radius: Card ________

Diameter: Card ________

Circumference: Card ________

STOP

Name: ______________________ Date: __________ Period: __________

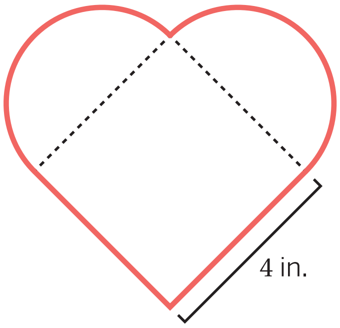
Summary

In today's lesson . . .

You saw that in order to give an exact value for problems involving radius, diameter, or circumference, you can use the symbol π in your response. Even using the π button on the calculator is technically an approximation of the non-ending decimal, so it is not as accurate as leaving your response in terms of π.

You gained fluency in using the formulas, $C = \pi d$, $d = 2r$, and $C = 2\pi r$ by analyzing and solving problems involving the radius, diameter, and circumference of circles in real-world situations, including determining the perimeter of shapes involving circular pieces.

For example, this heart-shaped box consists of two semicircles and a square. To determine the exact perimeter of the box, you would add the circumference of the circle to the two exterior sides of the square.



$P = \pi d + 2s$

$P = \pi \cdot 4 + 2 \cdot 4$

$P = 4\pi + 8$

The exact perimeter is $4\pi + 8$ in.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Determine the *exact* measurements based on the information given.

a

Circumference	Diameter	Radius
5π		

Show your thinking:

b

Circumference	Diameter	Radius
12		

Show your thinking:

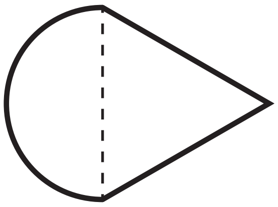
2. For each measurement, determine whether it represents the *radius*, *diameter*, or *circumference*. Place a check mark in the appropriate column and record the measurement in that cell. Then determine the *exact* lengths for the other two measurements of the circle.

	Radius	Diameter	Circumference
The tires of a mining truck are 14 ft tall.			
The fence around a circular pool is 76 ft long.			
The center to the edge of a DVD measures 60 mm.			

Practice

Name: Date: Period:

3. A semicircle is joined to an equilateral triangle with side lengths of 12 units. Determine and correct the mistake that Kiran made in his work to determine the perimeter of the resulting shape.



Kiran's work:
Semicircle + Triangle

$$\frac{1}{2}\pi \cdot d + 3(s)$$
$$= \frac{1}{2}\pi(12) + 3(12)$$
$$= 6\pi + 36$$

4. Circle A has a diameter of 1 ft. Circle B has a circumference of 1 m. Which circle is larger? Explain your thinking. **Note:** 1 in. $= 2.54$ cm

5. Evaluate each expression if $x = 3$.

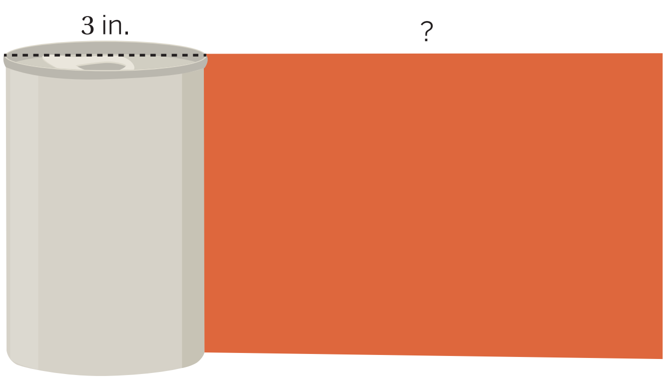
a) x^2

c) $\left(\frac{1}{3}\right)^x$

b) $3x^2$

d) $2x^3$

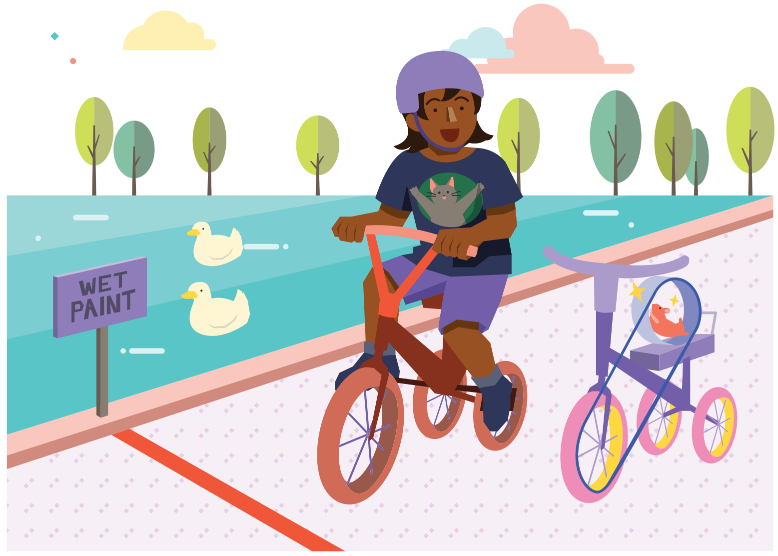
6. A soup can with a diameter of 3 in. has a wrapper that wraps exactly once around its outside. If the wrapper is rolled out, what is its approximate length?



Unit 3 | Lesson 7

Circumference and Wheels

Let's explore how far different wheels roll.



Warm-up Tricycle Tracks

A child riding her tricycle is about to ride through some wet paint. Sketch your prediction of what the paint tracks will look like as she travels along the sidewalk.

1. Compare your sketch with a classmate. What is similar? What is different?

2. What assumptions did you need to make when creating your sketch?

3. What information would you need to make a more accurate prediction?

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Name: ________ Date: ________ Period: ________

Activity 1 Things That Roll

Plan ahead: What steps will you take during the activity to ensure the safety of you and your peers?

You will be given a circular object and a sheet of paper.

Your task is to follow these instructions to create a drawing:

- On a separate sheet of paper, use a ruler to draw a line across the longest part of the page.
- Roll your object along the line and mark where it completes one rotation.
- Along the line, draw marks that are spaced the same distance apart as the diameter of your object.

1. Use your ruler to measure each distance. Record these values in the table.

Object	Diameter	Distance traveled in one rotation

2. Share your results with your group and record their measurements in the table.

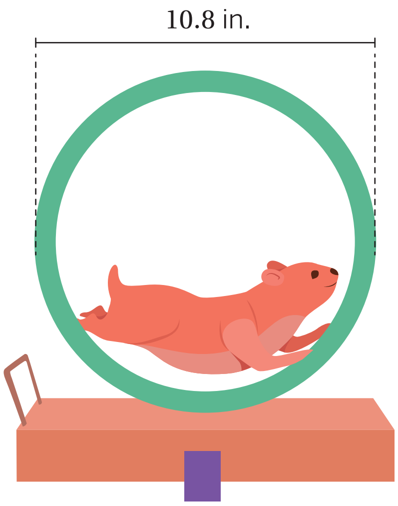
3. What is the relationship between the diameter of the object and the distance traveled in one rotation?

Are you ready for more?

How many rotations would it take for your object to roll 1 m? 1 km?

Activity 2 Rotations and Distance

Andre's pet hamster loves to run on her exercise wheel.



1. How far does the hamster run in . . .

   a 1 rotation? Round to the nearest tenth of an inch.

   b 30 rotations? Round to the nearest inch.

2. Write an equation relating the distance the hamster runs in inches to the number of wheel rotations.

3. How many rotations does the hamster wheel make if the hamster could run 1 mile? Explain your thinking. **Note:** 12 in. $= 1$ ft; $5{,}280$ ft $= 1$ mile.

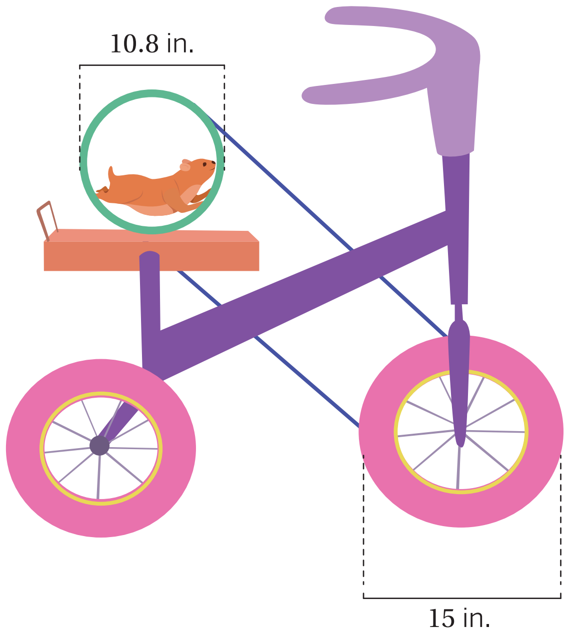
Are you ready for more?

If the length of the diameter of the hamster wheel was doubled, how would that affect the number of rotations needed to run a certain distance?

Name: .... Date: .... Period: ....

Activity 3 Rotations and Speed

Andre's hamster loves to feel the breeze in her fur, so he modified a tricycle for her to ride around the neighborhood. To make the vehicle, Andre wrapped a chain around her exercise wheel and the front wheel of a tricycle, as shown in the diagram.



1. How many times does the hamster wheel need to rotate to turn the tricycle wheel 1 full rotation? Round to the nearest tenth of a rotation.

2. How many times does the hamster wheel need to rotate to make the tricycle travel 1 mile?

3. How fast does the hamster wheel need to rotate for the tricycle to travel 5 miles per hour?

STOP

Summary

In today's lesson . . .

You saw that the circumference of a circle has the same value as the distance a wheel will travel during a single complete rotation.

You also saw how a wheel's circumference is related to the distance traveled by the wheel and its number of rotations. This is how vehicles with wheels are able to track and display their miles traveled, current speed, gas mileage, and other data.

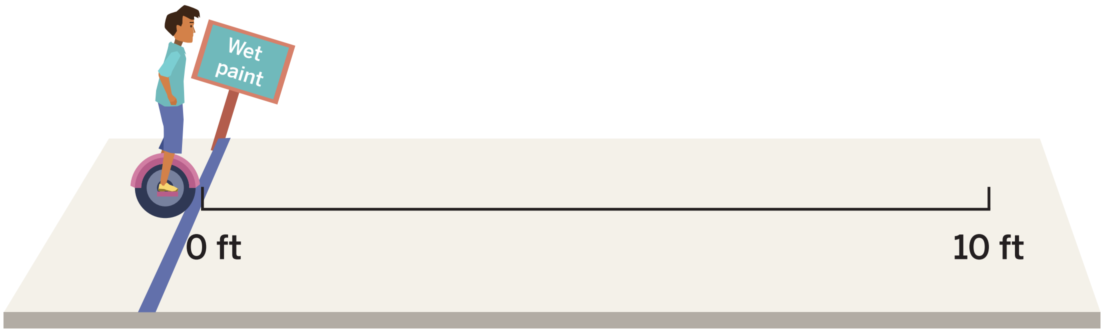
When making calculations with distance and time, it is sometimes appropriate to convert measurements to different units.

Reflect:

Practice

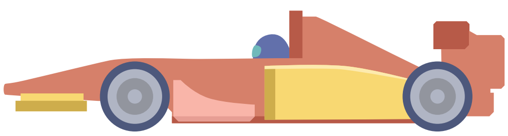
Name: .... Date: .... Period: ....

1. A unicycle rolls through a paint spill. If the circumference of the unicycle wheel is 30 in., mark where the wheel will leave paint tracks.



2. The diameter of a bike wheel is 27 in. If the wheel makes 15 complete rotations, how many feet does the bike travel?

3. The diameter of each wheel on a Formula One race car is 26 in. If the tires must be changed after 150,000 rotations, how many miles will the race car travel on 1 set of tires?
Note: 12 in. = 1 ft; 5280 ft = 1 mile



Practice

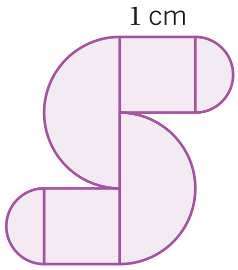
Name: ______ Date: ______ Period: ______

4. The list of numbers shown represent the measurements of the radius, diameter, and circumference of Circles A and B. Circle A is smaller than Circle B. Complete the table with the appropriate quantities.

2.5	5	7.6	15.2	15.7	47.8

	Radius	Diameter	Circumference
Circle A			
Circle B			

5. The figure shown is composed of two squares, each with side lengths of 1 cm, two larger semicircles, and two smaller semicircles. Select *all* the expressions that correctly calculate the perimeter of the shape, in centimeters. Explain your thinking.



A. 7

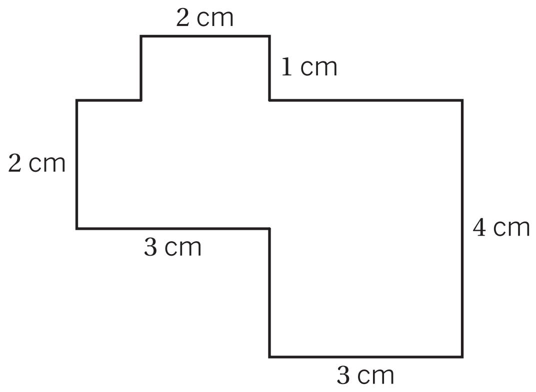
B. 7π

C. $4 + \pi + 2\pi$

D. $4 + 3\pi$

E. $4 + 7\pi$

6. Determine the area of the polygon. Show your thinking.



SUB-UNIT

2

Area of Circles

Narrative Connections

What makes a circle so perfect?

People have admired circles for thousands of years. Ancient Babylonian mathematicians, as well as Chinese, Egyptian, and Greek mathematicians, further sought to understand the proportions of circles. They examined the ratio between the distance around a circle and the length across its middle. They found the ratio was always exactly the same number. A number so cool, it got its own name: pi (or in Greek: π)

But what is pi exactly? It's a little greater than three. To be precise, it's a little greater than 3.14 but a little less than 3.15. To be even more precise, it's close to 3.14159265359, but that's not quite it either. In fact, the digits after the decimal point go on forever, never terminating or settling into a repeating pattern.

Every circle you see contains this number within its proportions, whether it's a nickel or the Earth's equator. (Neither of which is a perfect circle, but they are both pretty close!)

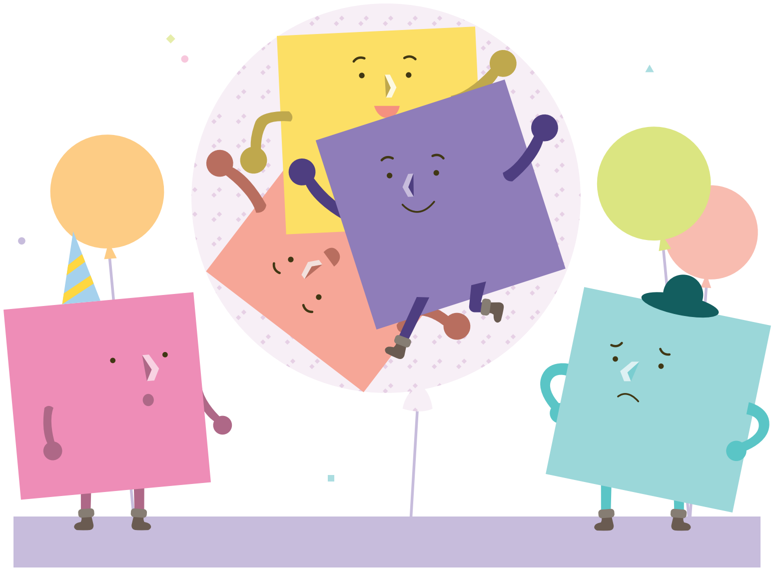
People have gone to great lengths to show their affection for this special number. Retired engineer Akira Haraguchi memorized over 110,000 digits of pi. It took him more than half a day to recite them. In 2019, computer scientist Emma Haruka Iwao computed over 31 trillion digits of pi. The computation took 25 virtual machines four months to complete, and made Iwao a world-record holder. Of course, you'll never need that many digits of pi to actually do anything—the first few digits usually suffice.

Next time you're jingling coins in your pocket, or surfing through a cylindrical ocean wave, think of the beauty of circles, and that special ratio pi.

Unit 3 | Lesson 8

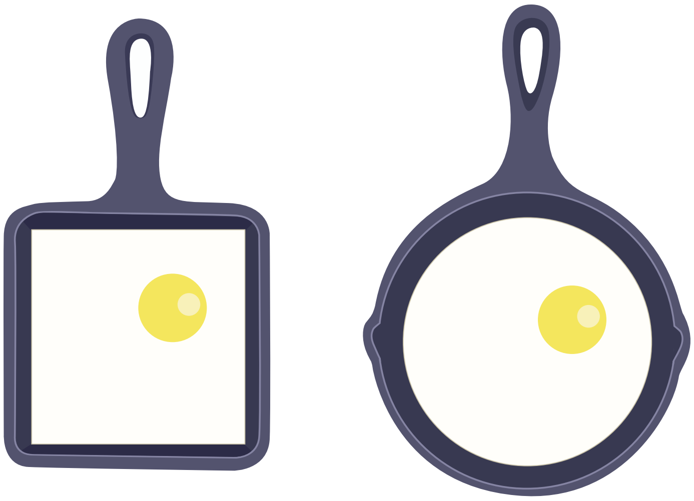
Exploring the Area of a Circle

Let's investigate the areas of circles.



Warm-up Comparing Areas

Sometimes, a cracked egg will fill a frying pan to its edge. Compare the areas of the fried eggs. Which is greater? How do you know?



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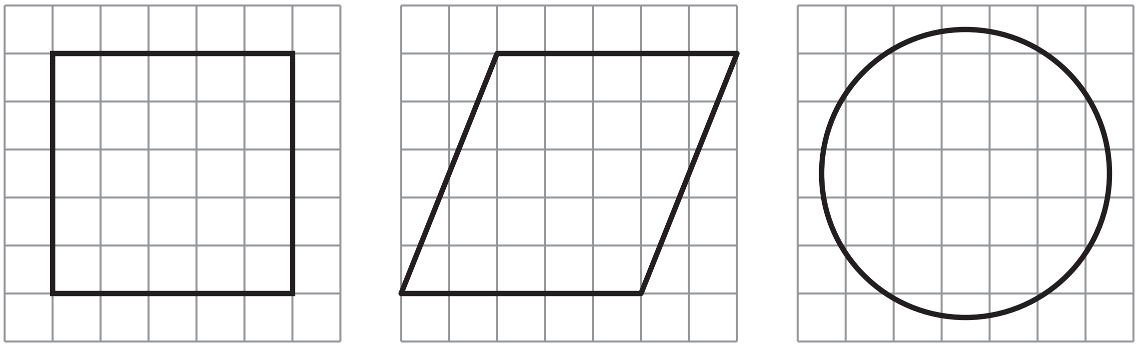
Name: ________ Date: ________ Period: ________

Activity 1 Estimating Areas

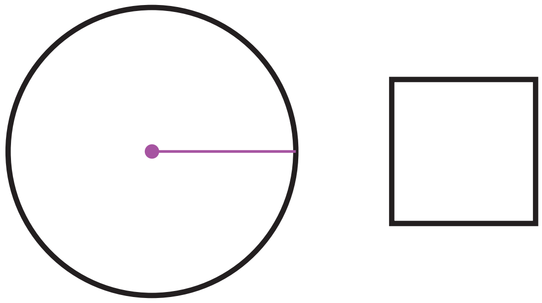
Decide which figure has the greatest area. Describe or show how you determined which figure has the greatest area.

Figure A **Figure B** **Figure C**



Activity 2 Covering a Circle

The side length of the square is the same as the radius of the circle.



1. Estimate how many of these squares would be needed to cover the circle entirely by responding to the following questions. Assume that you can break the square up into smaller pieces, if needed.

- **a** What number of squares do you think would be *too high*?
- **b** What number of squares do you think would be *too low*?
- **c** What number of squares do you think would be *just right*?

Pause here and wait for further instructions from your teacher.

2. You will be given some materials.

- **a** Cut the squares out and arrange them so that they fill the space inside your circle, without overlapping each other.
- **b** You may cut the squares into smaller pieces to help them fit. Keep track of how many squares you use to fill your circle.

3. You will meet with a group that was given a different-sized circle. Discuss what you noticed about your circles. What is the same? What is different?

STOP

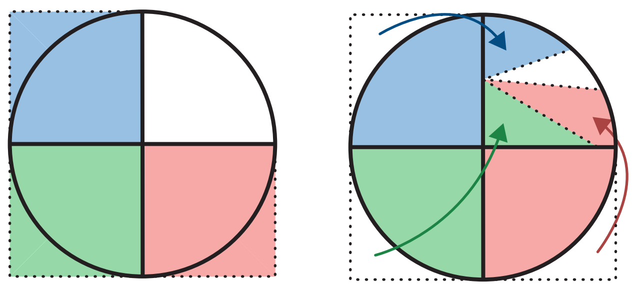
Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that because the area of polygons can be measured using unit squares, it is also possible to use this method to measure the area of a circle. However, you may have noticed that determining the precise amount of squares needed to cover a circle can be challenging.

For now, you can estimate that the area of a circle is equal to about 3 squares, where each square has a side length equal to the radius of the circle. This can be expressed using the formula $A \approx 3r^2$. Because you know how to find the area of a square with a certain side length, you can use this relationship to estimate the area of the related circle.



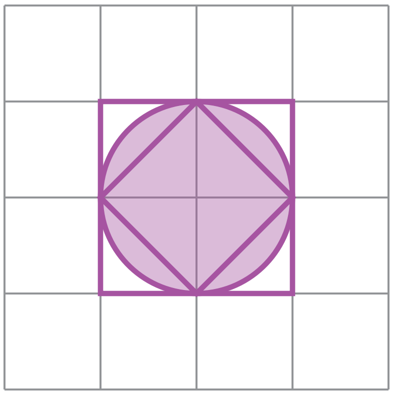
Unlike the circumference, the area is *not proportional* to the radius. You will investigate and refine the relationship between the area and the radius of a circle in the upcoming lessons.

Reflect:

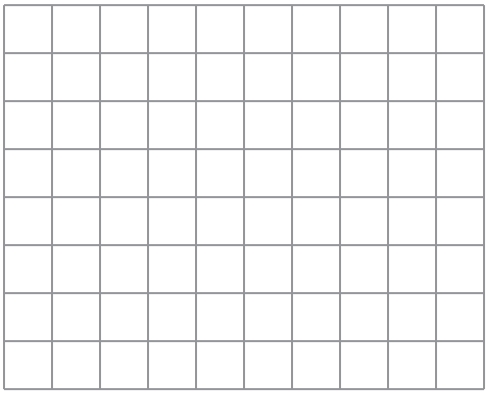
Practice

Name: .... Date: .... Period: ....

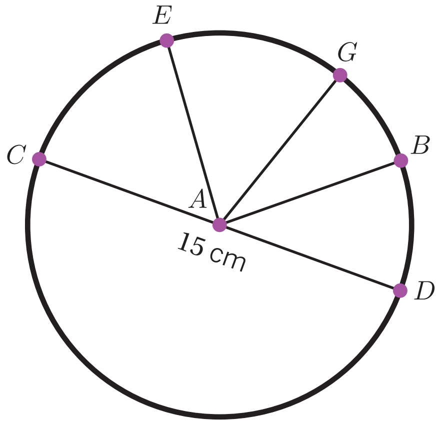
1. Refer to the image that shows two squares and a circle. Use the image to explain why the area of the circle is greater than 2 square units, but less than 4 square units.



2. Use the grid to draw the best freehand circle you can with an area of about 12 square units. You may need to make a few attempts. Explain your thinking.



3. Point A represents the center of the circle, and the length of segment CD is 15 cm. Determine the exact circumference of the circle. Your response should be written in terms of π.



Name: ________________ Date: ________ Period: ________

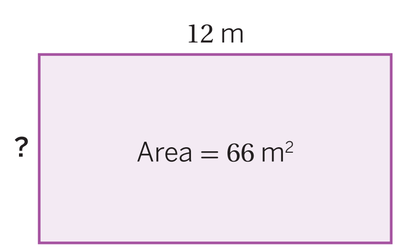
4. Lin's bike travels 100 m when her wheels rotate 55 times. What is the circumference of her wheels? Explain your thinking.

5. Bard has been making homemade birdseed for the birdfeeder outside the window. Bard has found that the birds seem to best like the recipe that uses $1\frac{1}{2}$ cups of sunflower seeds and $\frac{3}{4}$ dried corn kernels.

a How many sunflower seeds are needed to mix with 9 cups of dried corn kernels? Show or explain your thinking.

b How many dried corn kernels are needed to mix with $\frac{15}{4}$ cups of sunflower seeds? Show or explain your thinking.

6. Find the missing width of the rectangle.



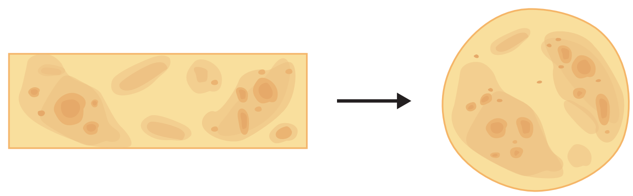
Unit 3 | Lesson 9

Relating Area to Circumference

Let's rearrange circles to calculate their areas.

Warm-up A Different Way to Make a Circle

The employees at the tortilla factory Tortilleria Azteca have misplaced their circular dough-cutter. In the meantime, all they have available is a rectangular cutter.



How can they slice and rearrange their rectangular shaped dough into a circular shape, without wasting any, before they bake it?

You will be given the materials for this activity.

1. Cut out the triangular slices of your rectangle. (The half-triangle on each end can be combined into a whole triangle.)

2. Rearrange the pieces to form a circular shape.

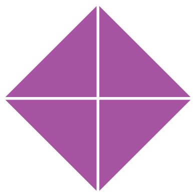
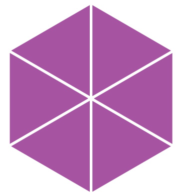
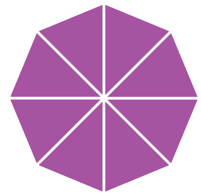
3. Compare your group's circular shape to another group's. Describe and explain the differences.

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: ____________ Period: __________

Activity 1 Approximating a Circle

Each of the regular polygons shown is made from slicing a rectangle into triangles, similarly to how you did in the Warm-up. They have each been rearranged into a regular polygon to approximate a circle.

Rectangle			
Regular polygon			

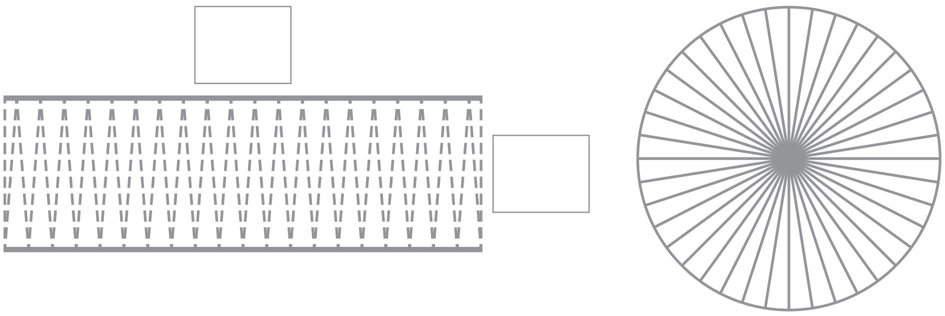
1. Imagine another rectangle is cut into 12 slices. Describe or sketch what the arrangement of triangles into a regular polygon would look like.

2. How would the regular polygon look if you cut the rectangle into even more triangular slices?

3. How is the area of the rectangle related to the area of the arrangements you made for each regular polygon? Explain your thinking.

Activity 2 Relating Area to Circumference

In this activity, you will refer to the following diagram that shows a rectangle and a circle.



1. Consider how the dimensions of the rectangles and regular polygons in Activity 1 were related.

   a. Refer to the dashed lines in the diagram. Each dashed line represents the approximate width of the rectangle. Where are these lines represented on the circle? Trace one pair of corresponding parts on the rectangle and on the circle with the same color.

   b. Refer to the top solid line. This line represents the length of the rectangle. Where is this line represented on the circle? Trace the corresponding parts on the rectangle and on the circle with a second color.

   c. Refer to the bottom solid line. This line also represents the length of the rectangle. Where is this line represented on the circle? Trace the corresponding parts on the rectangle and on the circle with a third color.

2. Let C represent the circumference of the circle and let r represent the radius, label the rectangle with its length and width, in terms of C and r.

3. Write an expression that represents the area of the rectangle, in terms of C and r.

4. How does the area of the rectangle relate to the area of the circle?

5. Refer to the expression you wrote in Problem 3. Does this expression also represent the area of the circle? Explain your thinking and write 1–2 sentences that describe how the area of a circle relates to its circumference.

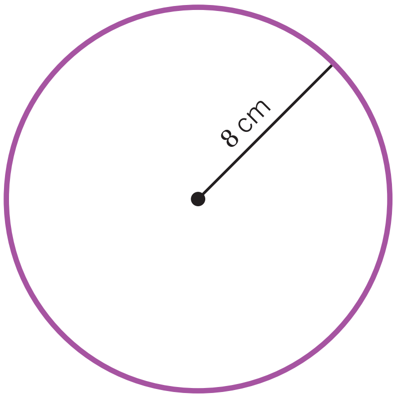
Stronger and Clearer: Exchange your responses to Problem 5 with 2–3 classmates. Ask each other clarifying questions and offer suggestions for improvement. Then revise your original draft based on their feedback.

Name: .... Date: .... Period: ....

Activity 3 Finding the Area, Given Different Information

The formula for calculating the area of a circle is often written as $A = \pi r^2$. Use this formula to solve the following problems related to the area of a circle.

1. Determine the approximate area of the circle. Round to the nearest centimeter.



2. A circle has a circumference of 31.4 in. Determine the approximate area of the circle to the nearest tenth of a square inch. Use 3.14 as the approximation of π.

STOP

Summary

In today's lesson . . .

You saw how you can use what you know about the area of a rectangle to reason about the formula for the area of a circle.

If C represents a circle's circumference and r represents its radius, then $C = 2\pi r$. The area of a circle can then be determined by taking the product of half the circumference and the radius.

Let A represent the area of the circle.

$A = \frac{1}{2}C \bullet r$	The area is the product of half the circumference and the radius.
$A = \frac{1}{2}(2\pi r) \bullet r$	The circumference is equal to $2\pi r$.
$A = \pi r^2$	Simplify. The product of $\frac{1}{2}$ and 2 is 1.

Remember that the expression $r \bullet r$ can be written as r^2, and this is read as "r squared."

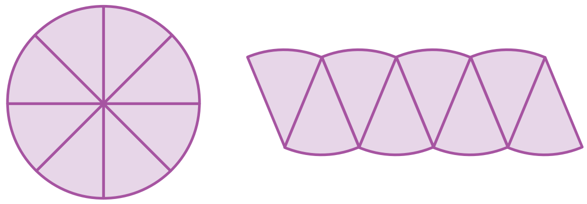
This means that if you know the radius, diameter, or circumference of a circle, you can determine the circle's area.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. The circle shown is divided into 8 equal wedges which are rearranged. Let r represent the radius of the circle. The circle's circumference is represented by the expression $2\pi r$. How does the image help to explain why the area of the circle is represented by the expression πr^2?



2. Jada paints a circular table that has a diameter of 37 in. What is the *exact* area of the table, in terms of π? Show your thinking.

3. A circle's circumference is approximately 76 cm. Estimate the radius, diameter, and area of the circle to the nearest tenth. Show your thinking.

Practice

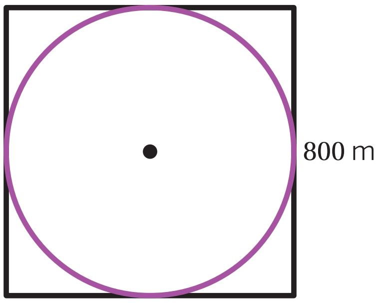
Name: .... Date: .... Period: ....

4. The Carousel on the National Mall in Washington, DC, has 4 rings of horses. Andre is riding on the inner ring, which has a radius of 9 ft. Lin is riding on the outer ring, which is 8 ft farther from the center than the inner ring.

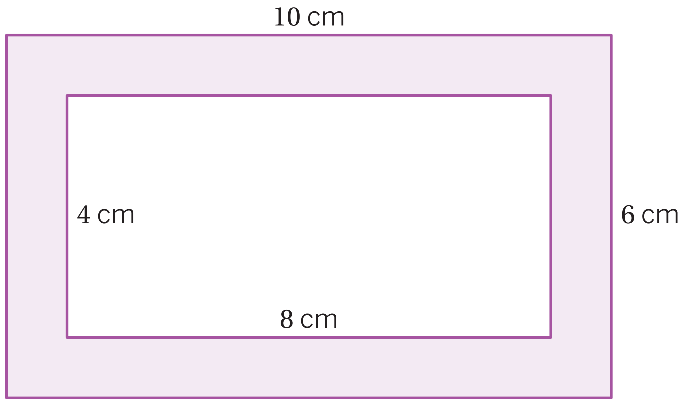
a. In one rotation of the carousel, how much farther does Lin travel than Andre?

b. One rotation of the carousel lasts 12 seconds. How much faster does Lin travel than Andre?

5. Determine the diameter of the circle. Explain your thinking.



6. Determine the area of the shaded region. Show your thinking.



Name: ______ Date: ______ Period: ______

Unit 3 | Lesson 10

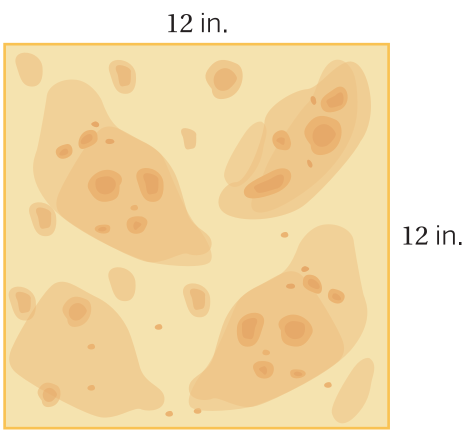
Applying Area of Circles

Let's find the areas of shapes that are made up of circles.

Warm-up How Many Tortillas Fit?

At the Oaxaca Tortilla Factory, the operations team is trying to find the best arrangement for cutting four tortillas of equal size from a square sheet of dough. Their goal is to have as little dough left over as possible after the tortillas are cut out.

1. Sketch an arrangement of four circles of equal size that could be cut from the following sheet of dough.

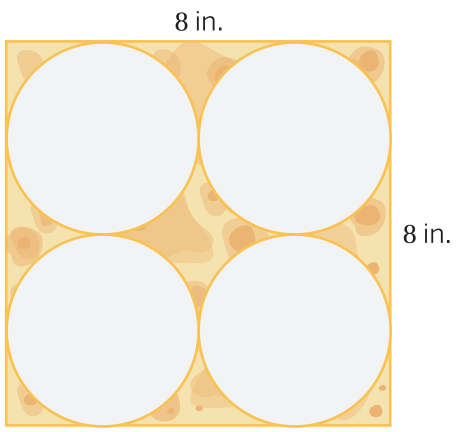


2. How large is each tortilla? Explain your thinking.

Activity 1 Making Use of the Leftovers

The Oaxaca Tortilla factory strives to be as economical as possible. After cutting the circular tortillas from their square sheets of dough, they collect all the remaining dough and re-roll it to the same thickness.

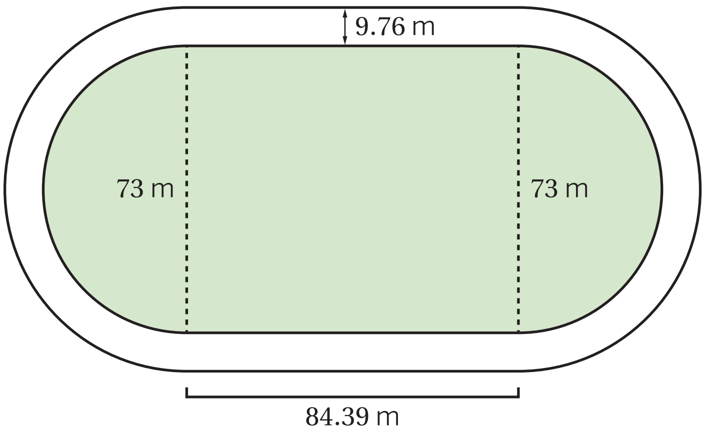
Four medium tortillas have been cut from this square sheet of dough. Is there enough dough left to make an additional medium tortilla? Show or explain your thinking.



Name: ______ Date: ______ Period: ______

Activity 2 The Running Track

The field inside a running track is made up of a rectangle 84.39 m long and 73 m wide with a semicircle at each end. The running lanes are 9.76 m wide all the way around the track.



What is the *exact* area of the running track that goes around the field? Write your response in terms of π. Explain your thinking.

STOP

Summary

In today's lesson . . .

You solved problems that required you to find the area of spaces inside, outside, and around circles and rectangles.

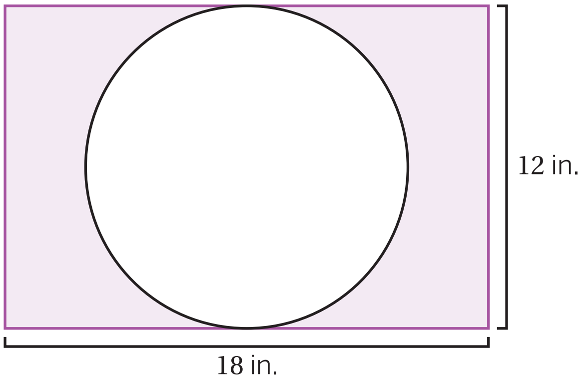
Strategizing about and solving complex problems like these, where the path to the solution is not obvious, are important to your growth as a mathematical thinker.

Reflect:

Practice

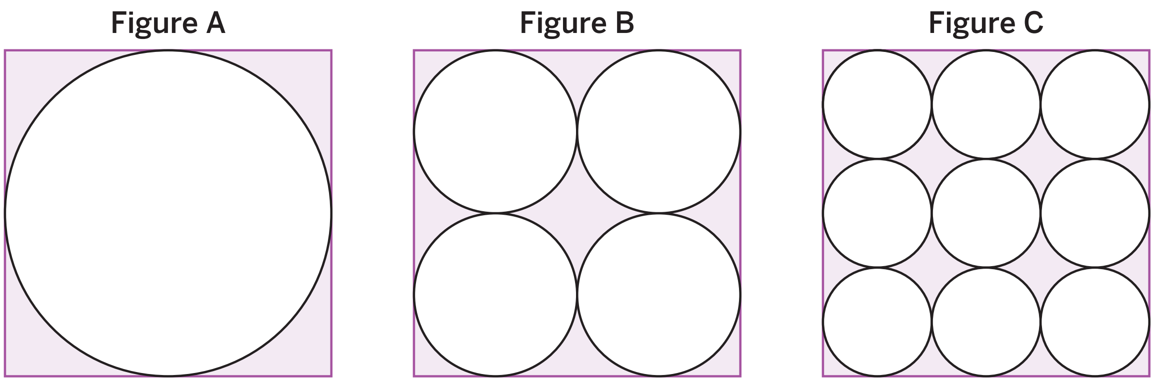
Name: ________ Date: ________ Period: ________

1. Find the exact area of the shaded region. Express your answer in terms of π.



2. A circle with a 12-in. diameter is folded in half and then folded in half again. What is the *exact* area of the resulting shape? Express your answer in terms of π.

3. Each square represented has a side length of 12 units. Compare the areas of the shaded regions in these three figures. Which figure, if any, has the largest shaded region? Show or explain your thinking.



Practice

Name: .... Date: .... Period: ....

4. Select *all* the pairs of quantities that are proportional to each other. For the quantities that are proportional, write an equation that relates them.

   A. The radius and diameter of a circle.

   B. The radius and circumference of a circle.

   C. The radius and area of a circle.

   D. The diameter and circumference of a circle.

   E. The diameter and area of a circle.

5. A graffiti artist is spray painting a mural of the Moon onto a wall. The mural will have a diameter of 10 ft. Each can of spray paint holds 12 oz, and 1 oz of spray paint will cover about 2 ft^2. How many cans of spray paint will be needed to create the mural of the Moon?

6. Describe a real-world situation for which knowing either the *circumference* or *area* of a circular object is important.

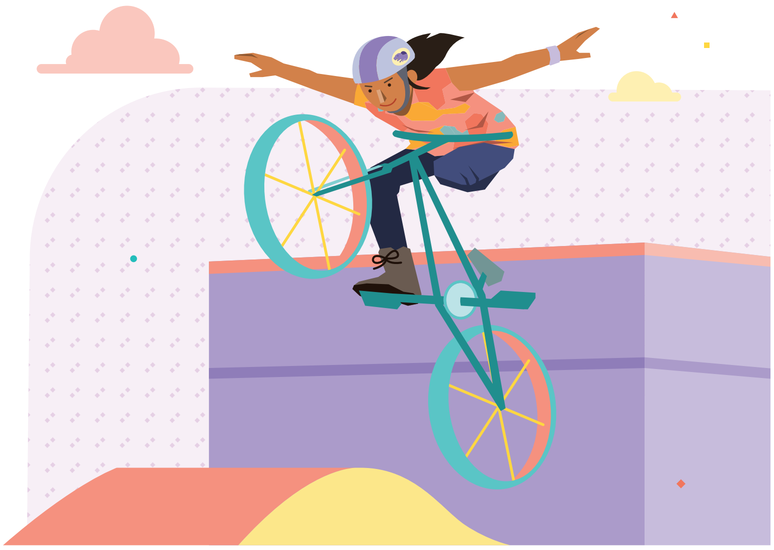
   a circumference

   b area

Name: ______ Date: ______ Period: ______

Unit 3 | Lesson 11

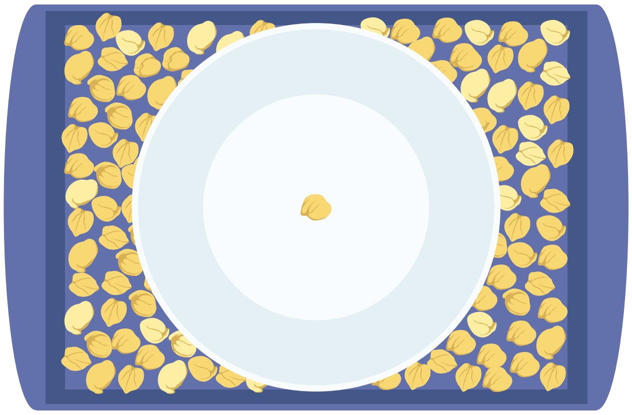
Distinguishing Circumference and Area



Let's contrast circumference and area.

Warm-up Filling the Plate

Study the image of the plate on top of a pan filled with chickpeas.



1. About how many chickpeas could fit on the plate in a single layer? Explain your thinking

2. About how many chickpeas would fit around the edge of the plate? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Card Sort: Circle Problems

You will be given a set of cards. Each card contains a problem and is folded over so that only the original problem is revealed. Do not open the folded part while you are working on Problems 1 and 2.

1. Sort the cards into two groups based on whether you would use the *circumference* or the *area* of the circle to solve the problem.

Circumference	Area

Pause here so your teacher can review your work.

2. Your teacher will assign you a card to examine more closely. Estimate the solution to the problem on the card. Explain your thinking.

   My Card: ____________________

3. Open the folded part of your card to reveal some new information. Use the information to calculate the solution to the problem.

Name: .... Date: .... Period: ....

Activity 2 Analyzing Circle Claims

Here are two students' responses to some problems similar to those you worked on in Activity 1. Do you agree with either of them? Show or explain your thinking.

1. How many feet are traveled by this child riding once around the merry-go-round?

- **Clare:** "The radius of the merry-go-round is about 4 ft, so the distance around the edge is about 8π ft."
- **Andre:** "The diameter of the merry-go-round is about 4 ft, so the distance around the edge is about 4π ft."

PeopleImages/iStock

2. How much room is there to spread peanut butter on the cracker shown?

- **Clare:** "The radius of the cracker is about 3 cm, so the space for peanut butter is about 6π cm^2."
- **Andre:** "The diameter of the cracker is about 3 in., so the space for peanut butter is about 2.25 in^2."

Pxfuel.com

3. Professional BMX rider Julian Molina needs to ride about 20 m to get enough speed to complete a BMX trick over an obstacle. How many times do his bike wheels rotate while traveling this distance?

- **Clare:** "The diameter of the bike wheel is about 0.5 m. In 20 ft, the wheels will rotate completely about 12.7π times."
- **Andre:** "I agree with Clare's estimate of the diameter, but the bike wheels will rotate completely about 6.4 times."

Reflect: How well were you able to communicate whether you agreed or disagreed with your classmates?

STOP

Summary

In today's lesson . . .

You saw that sometimes you need to find the circumference of a circle, and sometimes you need to find the area. Here are some examples of quantities related to either the circumference or the area of a circle:

Circumference

- The length of a circular path.
- The distance a wheel will travel after one complete rotation.
- The length of a piece of rope coiled in a circle.

Area

- The amount of land that is cultivated on a circular field.
- The amount of paint needed for a mural of the Sun.
- The number of tiles needed to cover a round table.

In both cases, the radius (or diameter) of the circle is all that is needed to perform the necessary calculation.

- The circumference of a circle with radius r is represented by the expression $2\pi r$, while its area is represented by the expression πr^2.
- Circumference is measured in linear units (such as cm, in., km) while area is measured in square units (such as cm^2, in^2, km^2).

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. For each problem, decide whether the *circumference* of the circle or the *area* of the circle is most useful for solving the problem. Explain your thinking.

a A car's wheels rotate at a rate of 1,000 revolutions per minute. The diameter of each wheel is 23 in. You want to know how fast the car is traveling.

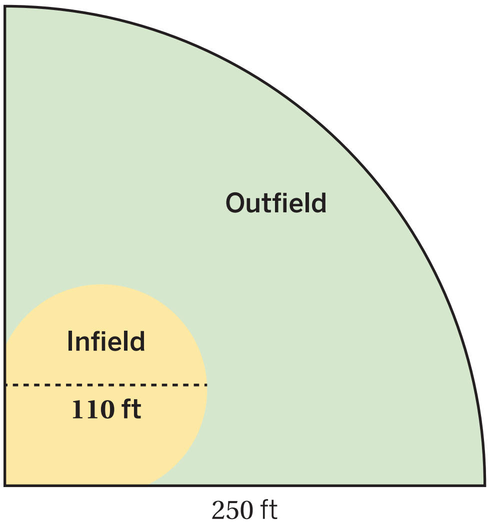
b A circular kitchen table has a diameter of 60 in. You want to know how much fabric is needed to cover the top of the table.

c A circular puzzle measures 20 in. in diameter. All of the pieces are about the same size. You want to know about how many pieces are in the puzzle.

d There is some free time in your schedule to exercise before you have to start your homework. You want to know about how long it takes to walk around a circular pond near your house.

2. The diagram of a softball field is shown.

a Estimate the length of the fence around the field.

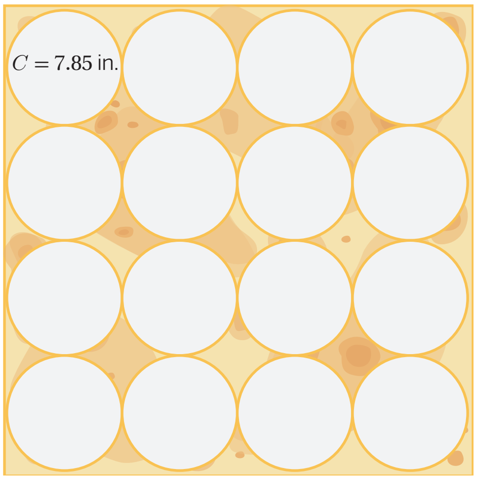


b Estimate the area of the outfield.

Practice

Name: ______________________ Date: ____________ Period: ____________

3. 16 small tortillas have been cut from this square sheet of dough. The circumference of each small tortilla is 7.85 in. How many small tortillas can be made from the leftover dough? Show your thinking.

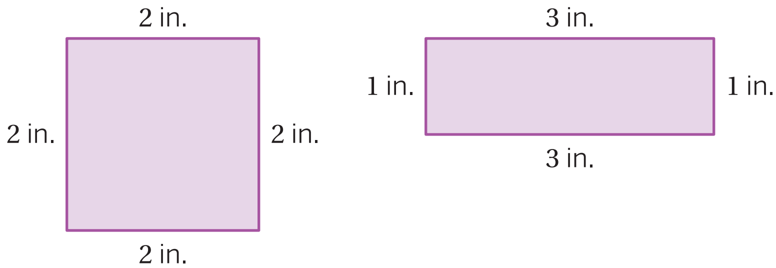


4. While in math class, Priya and Kiran come up with two ways of thinking about the proportional relationship shown in the table. Both students agree that they can solve the equation $5k = 1750$ to find the constant of proportionality. Who is correct? Explain your thinking.

x	y
2	
5	1750

- **Kiran:** "I can solve this equation by dividing 1750 by 5."
- **Priya:** "I can solve this equation by multiplying 1750 by $\frac{1}{5}$."

5. The figures shown have equal perimeters. Do they have equal areas? Show or explain your thinking.



Name: ________ Date: ________ Period: ________

Unit 3 | Lesson 12 – Capstone

Capturing Space

Let's find out which shape captures the most area.

Warm-up Same String, Different Shapes

You will be given an 8-in. length of string. Tie the ends together in a knot to make a loop.

1. What are some different shapes you can make with your loop of string? Trace each shape here.

2. What do your shapes have in common? What is different?

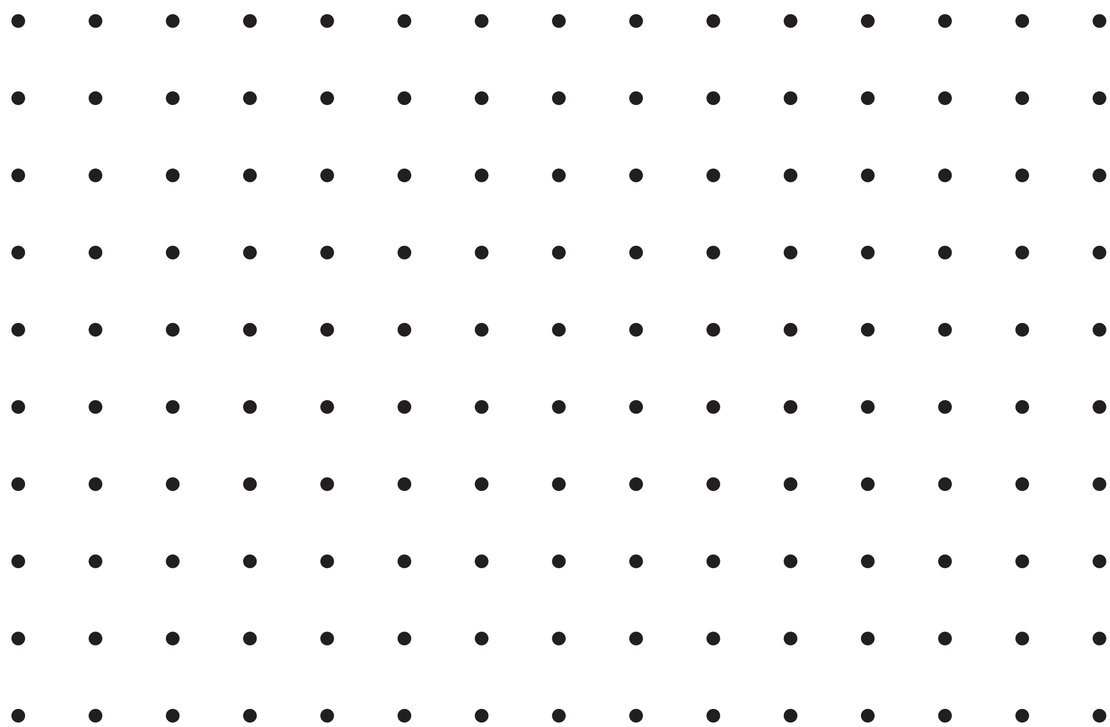
Log in to Amplify Math to complete this lesson online.

Activity 1 Capture the Dots Game

You and your partner will play the following game.

Goal: Capture as many dots by enclosing them inside of a shape made with your string.
For each turn:

- Form your loop of string into a shape — any shape — to surround any number of free dots. **Note:** You cannot overlap with other shapes that have already been drawn.
- Draw the shape of your string on the grid.
- Tally the number of the dots captured inside of the shape, and write your initials inside the shape.
- Take turns until there are no dots left. The winner has captured the most dots!



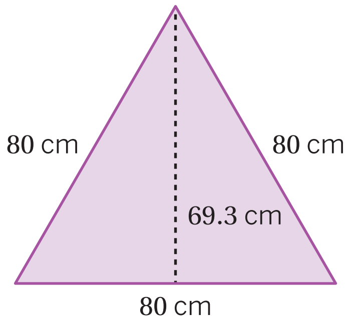
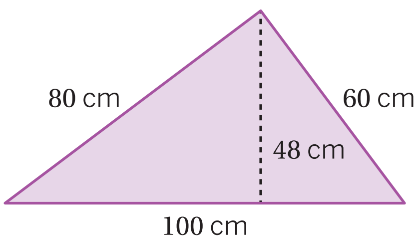
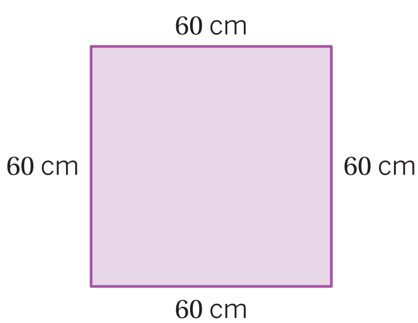
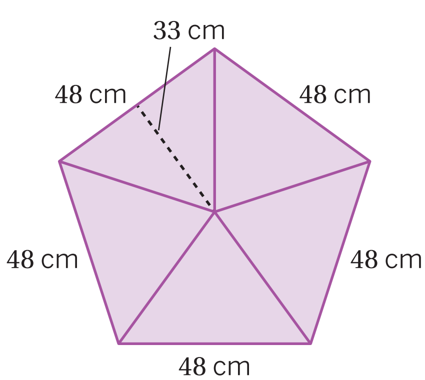
Player 1 tally	Player 2 tally

What strategy or strategies did you use? If you were to play the game again, would your strategy change? How?

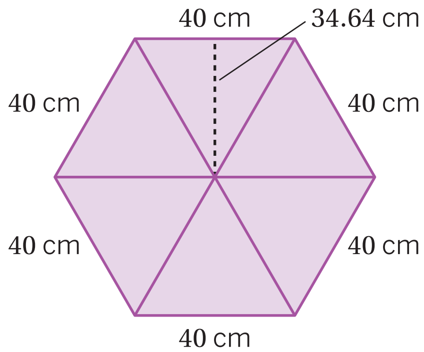
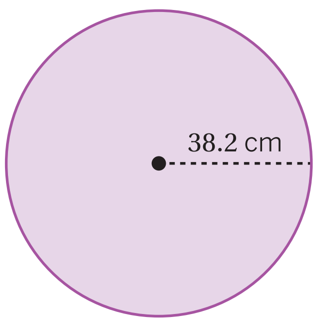
Name: ______________________ Date: __________ Period: ________

Activity 2 Enclosing the Largest Space

1. Determine the perimeter and area of each figure shown in the table. Show or explain your thinking in the space provided in the table.

Figure	Perimeter	Area
		
		
		
		

Activity 2 Enclosing the Largest Space (continued)

Shape	Perimeter	Area
		
		

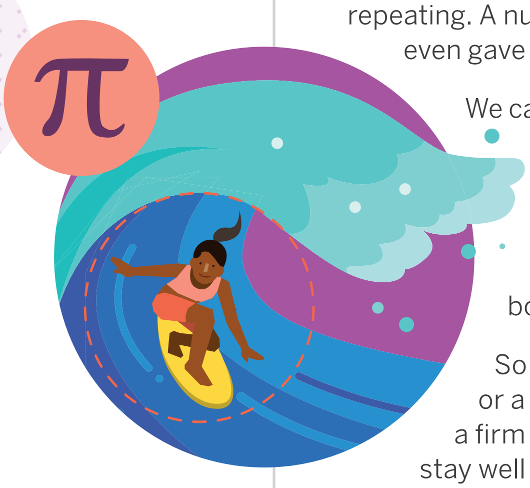
2. What conclusions can you make about the areas of different shapes, compared to their perimeters?

STOP

Unit Summary

Where would we be without circles? They form the basis for the gears and wheels of our bikes. They help define the shape of our Sun and our Moon. From sacred hoops to Taoists symbols, to even our modern day wedding bands — circles have a special significance all over the world.

The circle fascinated the world's ancient thinkers. Scholars from Ancient Mesopotamia, China, Egypt, and Greece all studied this shape and uncovered a special number found in every circle. A number that — in its decimal form — goes on forever without repeating. A number so special that mathematicians even gave it its own symbol.



We call this number pi. It is represented by the symbol π.

This constant is defined as the ratio of a circle's circumference (the distance around the circle) to its diameter (the distance across a circle, through its center). Using π, you can calculate both the area and circumference of any circle.

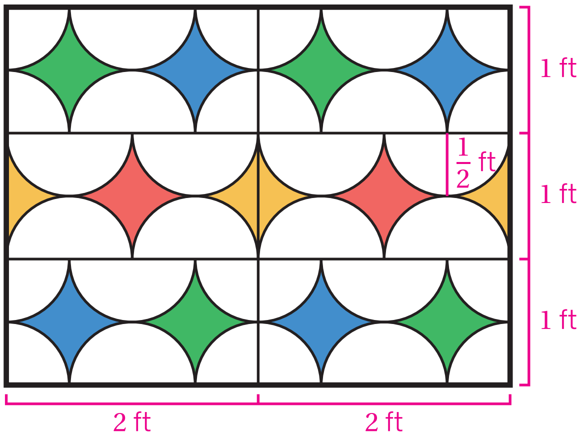
So whether you're an alien visitor leaving your mark or a computer scientist breaking a world record, with a firm understanding of circles and π, you're sure to stay well ahead of the curve.

See you in Unit 4.

Practice

Name: ________ Date: ________ Period: ________

1. The students in art class are designing a stained-glass window to hang in the school entryway. The window will be 3 ft tall and 4 ft wide, containing 6 rectangles of the same size. Their design is shown.



They have raised $100 for the project.

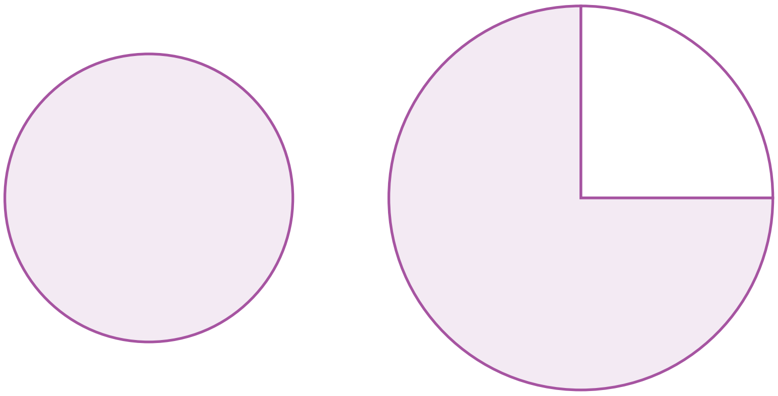
The colored glass costs $5 per square foot and the clear glass (white space) costs $2 per square foot. The material they need to join the pieces of glass together costs 10 cents per foot and the frame around the window costs $4 per foot.

Do they have enough money to cover the cost of making the window? Show or explain your thinking.

Name: ______ Date: ______ Period: ______

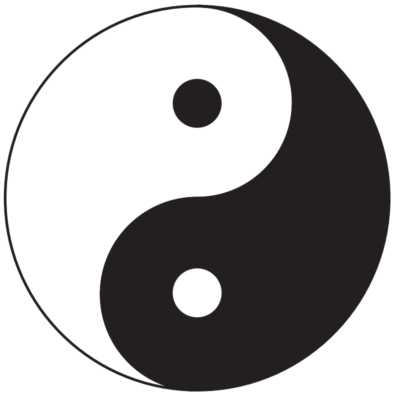
Practice

2. Find a value for each radius that would make the areas of the shaded regions equal. Show or explain your thinking.



The figures may not be drawn to scale.

3. The diameter of this circular symbol, known as yin and yang, is 2 cm. What is the *exact* area of the black space?



A. $\frac{\pi}{2}$ cm^2

B. π cm^2

C. 2π cm^2

D. 4π cm^2

UNIT 4

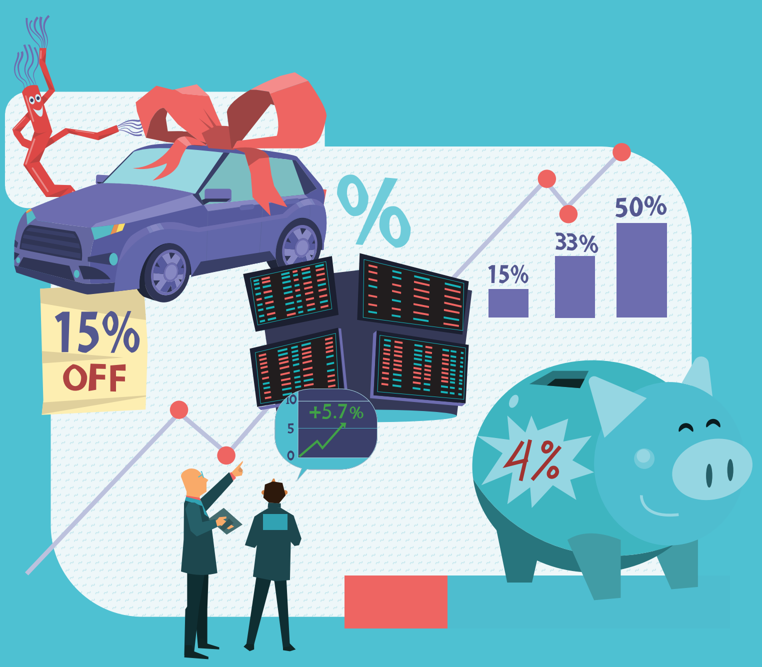
Percentages

"Extra! Extra! 99% of adults don't remember numbers!*" Percents can be incredibly effective at communicating how much something has changed, but we must keep a watchful eye on what the numbers behind the percentages mean.

(*When asked to recite π to the 10th digit.)

Essential Questions

- How are percentages related to proportional relationships?
- How are percentages used to represent change?
- When is it most helpful to use percentages?
- *(By the way, how come the price you see isn't always the price you pay?)*

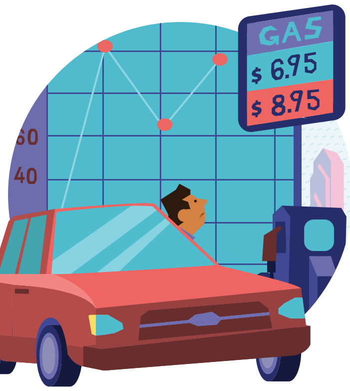


SUB-UNIT

1 Percent Increase and Decrease

Narrative: Having a solid understanding of percentages can help you spot misleading news headlines.

You'll learn . . .
- how tape diagrams can model percent change.
- how to use expressions and equations to solve problems involving percent change.



SUB-UNIT

2 Applying Percentages

Narrative: Understand the importance of using precision when communicating financial aspects of percent change.

You'll learn . . .
- how percent change is used in financial contexts.
- how to solve problems involving percent change and money.

100 ± 1.5% √

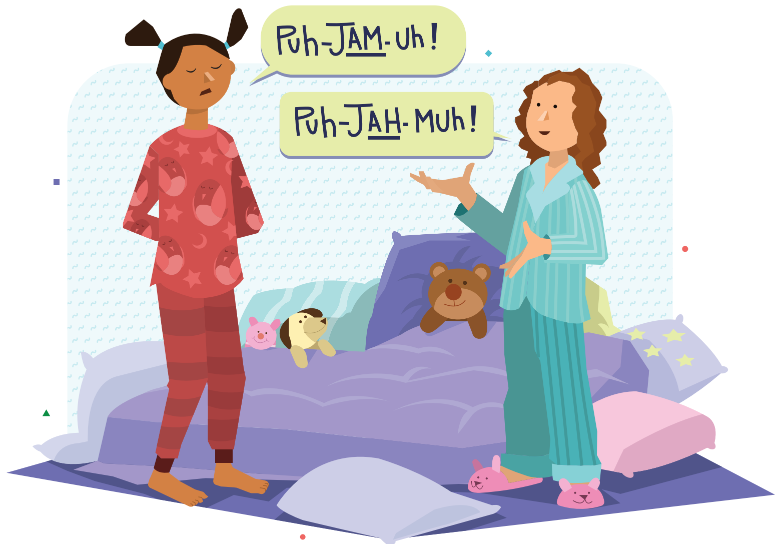
A watermelon weighs 6 lb and is 99% water. After sitting out in the sun for a few days, the watermelon is 98% water. How much does the watermelon weigh now?

HEADLINES

Unit 4 | Lesson 1 – Launch

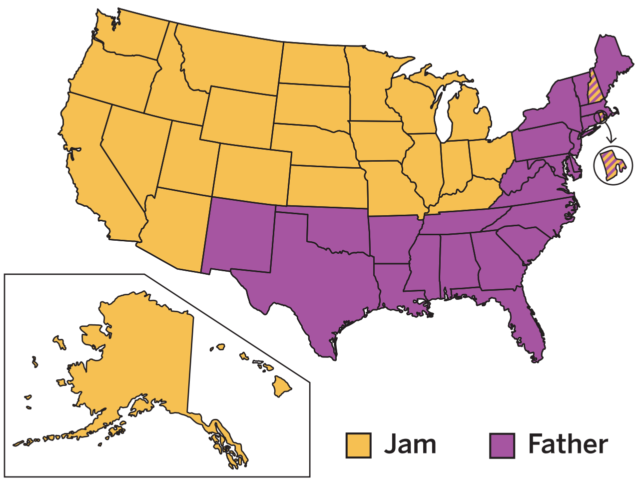
(Re)Presenting the United States

Let's use percentages to represent the United States.



Warm-up Notice and Wonder

In 2003, the "Harvard Dialect Survey" was given to a group of people from each state to explore the different ways people speak across the United States. This map shows how the majority of people in each state pronounced the second vowel in the word *pajamas*.



1. What do you notice?

2. What do you wonder?

Name: .... Date: .... Period: ....

Activity 1 Analyzing State Data

Your teacher will give you two cards with information about a state. Use your cards to complete the following problems. As a class, you will combine the data on your cards to represent information from the Warm-up about the pronunciation of *pajamas* in the U.S. — by area and by population — on two 10-by-10 grids.

First Card:

1. State: ....

2. What percentage of the total land area of the U.S. does your state occupy? Show or explain your thinking. Round your percentage to the nearest hundredth.

3. Based on the information in the map from the Warm-up and the percentage of the land area your state occupies, how many squares (or partial squares) out of 100 should you shade to represent your state in each color?

   a. Yellow: .... b. Purple: ....

   Explain your thinking.

4. What percentage of the population of the U.S. does your state contain? Show or explain your thinking. Round to the nearest hundredth.

5. Based on the information in the map from the Warm-up and the percentage of the population your state contains, how many squares (or partial squares) out of 100 should you shade to represent your state in each color?

   a. Yellow: .... b. Purple: ....

6. Explain your thinking.

Pause here. Before moving on to your second card, add the information for your first state to the classroom grids.

Activity 1 Analyzing State Data (continued)

Second Card:

1. State: ....

2. What percentage of the total land area of the U.S. does your state occupy? Show or explain your thinking. Round to the nearest hundredth.

3. Based on the information in the map from the Warm-up and the percentage of the land area your state occupies, how many squares (or partial squares) out of 100 should you shade to represent your state in each color?

   a. Yellow: ....

   b. Purple: ....

4. Explain your thinking.

5. What percentage of the population of the U.S. does your state contain? Show or explain your thinking. Round to the nearest hundredth.

6. Based on the information in the map from the Warm-up, and the percentage of the population your state contains, how many squares (or partial squares) out of 100 should you shade to represent your state in each color?

   a. Yellow: ....

   b. Purple: ....

7. Explain your thinking.

Add the information for your second state to the classroom grids.

Name: ______ Date: ______ Period: ______

Activity 2 Comparing Representations

As the U.S. Chief Statistician for almost 25 years, Katherine Wallman oversaw 13 federal agencies that collected and analyzed data about the U.S. and the hundreds of millions of people who live here.

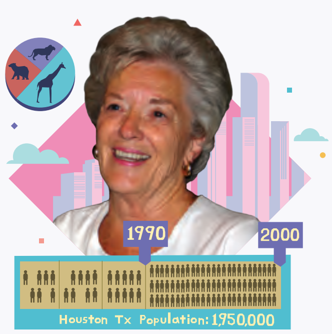
Your data set from Activity 1 may not have been collected from hundreds of millions of people, but you can still use the representations your class created in the previous activity to analyze how people in the U.S. pronounce the word *pajamas*.

1. What information are you able to see more clearly on the original map than on the grid of the land area?

2. What information are you able to see more clearly on the grid of the land area than on the original map?

3. Looking at both grids, would it be appropriate to say that most people in the U.S. pronounce the second vowel in *pajamas* like "jam" or "father"? Explain your thinking

Featured Mathematician



Katherine Wallman

Imagine being in charge of making sure that every single person in the U.S. is counted. Now imagine this task being only $\frac{1}{13}$th of your responsibilities. As Chief Statistician from 1992 to 2017, Katherine Wallman oversaw the data collection of thirteen different federal agencies including the Census Bureau, the National Center for Health Statistics, and the National Center for Education Statistics. She helped the U.S. government make sense of the immense amount of data they collected to inform decisions and improve policies.

STOP

Eric Sampson, from the ASA, 2007

Unit 4 Percentages

Keepin' it 100

The verdict is in: People love percents!

We use them in everything — on our coupons, in department stores, and on our nutritional labels. Governments use them to describe tax rates, and the very clothing on your back is labeled with the percentages of different fabric types.

And as far as ratios go, percentages are *great* for communicating the way we feel. If a coach tells you to give 110% on the football field, it motivates you more than if they had asked you to give 22 out of 20. Percentages are also great for making comparisons. It's a lot quicker, for example, to choose a jar of peanut butter that contains 5% of your daily recommended sodium over a jar that contains 22% (blech!).

There's something comforting about seeing a value represented as the number of parts per one hundred. But things can get tricky once we start using percentages to describe *how things change*.

Our intuition for what those percentages mean start to get twisted. What does it mean for an item to be 20% off after a $15 rebate? How much more are we getting if a cereal box is now 15% larger?

Given how often we use percentages — in our current events, in our scientific research, in our reports on the economy, and as we describe changes in price — having a strong grasp of how to perform calculations and communicate with percents is key to being an informed decision-maker.

But don't worry. Stick with us, and we'll help you keep it 100.

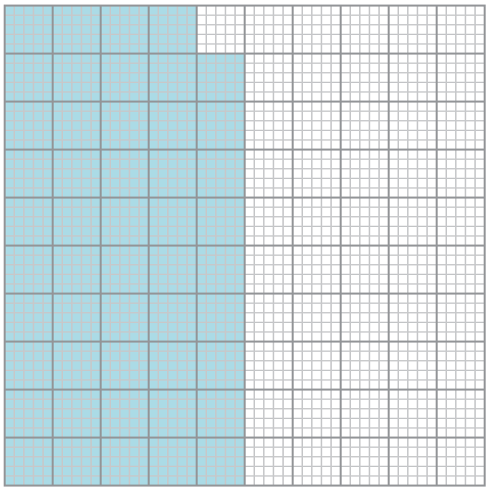
Welcome to Unit 4.

Name: ________ Date: ________ Period: ________

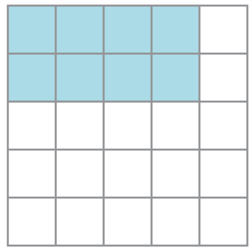
Practice

1. Determine the percent of each grid that is shaded.

a



b



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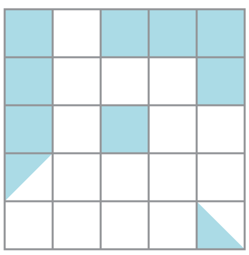
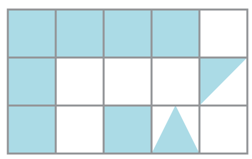
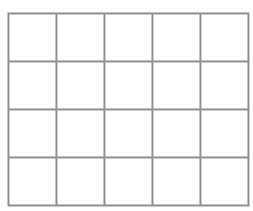


d



2. Shade 42% of the grid. Show or explain your thinking.



Practice

Name: ______________________ Date: ____________ Period: ____________

3. Crater Lake in Oregon, is shaped like a circle with a diameter of about 5.5 miles.

   a. How far is it around the perimeter of Crater Lake? Round to the nearest hundredth. Show or explain your thinking.

   b. What is the area of the surface of Crater Lake? Round to the nearest hundredth. Show or explain your thinking.

4. Ants have 6 legs. Elena and Andre write equations showing the proportional relationship between the number of ants a and the number of ant legs g. Elena writes $a = 6 \cdot g$ and Andre writes $g = \frac{1}{6} \cdot a$. Do you agree with either of the equations? Explain your thinking.

5. Lin has a scale model of a modern train. The model is created at a scale of 1 to 48.

   a. The height of the model train is 102 mm. What is the actual height, in meters, of the train? Explain your thinking.

   b. On the scale model, the distance between the wheels on the left and the wheels on the right is $1\frac{1}{4}$ in. The state of Wyoming has some old railroad tracks that are 4.5 ft apart. Can the modern train travel on these tracks? Explain your thinking.

6. Evaluate each expression:

   a. $30 \cdot 0.2$

   b. $30 \cdot 0.02$

   c. $30 \div 0.2$

   d. $30 \div 0.02$

SUB-UNIT

1 Percent Increase and Decrease

Narrative Connections

Is there truth in numbers?

In October 2018, the World Wildlife Fund (WWF) released its annual Living Planet report. The next day, newspapers reported headlines, such as: "Humanity has wiped out 60% of animal populations since 1970," "60% of world's wildlife has been wiped out since 1970," and "Humans have killed off more than half the world's wildlife populations."

If this dramatic destruction of animal life makes you ball up your fists in fury, you are not alone. People flew to social media to express their outrage, anger, and grief.

The problem? These headlines weren't entirely true . . .

The original report from the WWF described that — *of the animal populations they studied* — populations had declined by 60%, *on average*. That meant some groups' numbers went up. Some went down (some by much more than 60%).

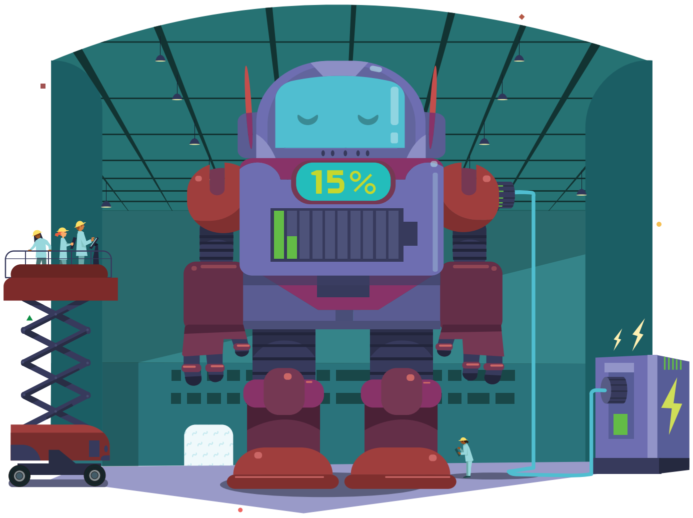
Whenever we talk about percent change, we have to be careful. Percentages are powerful numbers. They make us feel things, especially when used in the news media. We trust the news to report what's true so that we can understand what's happening in the world. But that trust can be abused, whether intentionally or unintentionally.

So, how can we protect ourselves from misleading percentages in the headlines? The first thing to do is to keep a critical eye. Look closely at the actual data and not just on how the headlines make you feel. Let's roll up our sleeves and dig into how changes in percentage work.

Unit 4 | Lesson 2

Understanding Percentages Involving Decimals

Let's explore percentages that are not whole numbers.



Warm-up Comparing Coupons

Mai and Noah both went shopping.
Mai used a 25% off coupon on her purchase.
Noah used a 20% off coupon on his purchase.

1. What conclusion(s) are you able to make about how much money each person saved on their purchase?

2. Noah claims that he saved more money than Mai. Is it possible for him to be correct? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Percentage of 60

1. Match each verbal description with the expression that represents it. Not all expressions will match with a verbal description.

   a. 30% of 60
   b. 3% of 60
   c. 300% of 60
   d. 0.3% of 60

   ______ $300 \cdot 60$
   ______ $30 \cdot 60$
   ______ $3 \cdot 60$
   ______ $0.3 \cdot 60$
   ______ $0.03 \cdot 60$
   ______ $0.003 \cdot 60$

2. Evaluate each expression to determine the value of each percentage in Problem 1. What do you notice?

   a. 30% of 60
   b. 3% of 60
   c. 300% of 60
   d. 0.3% of 60

3. Use your responses from Problem 2 to determine the following percentages of 60. Show or explain your thinking.

   a. 33% of 60
   b. 30.3% of 60
   c. 0.6% of 60
   d. 600.03% of 60

Activity 2 Tape Diagrams and Percentages

1. Match each statement with the tape diagram that can be used to represent it.

a 4.5% of 90 is what value? ....

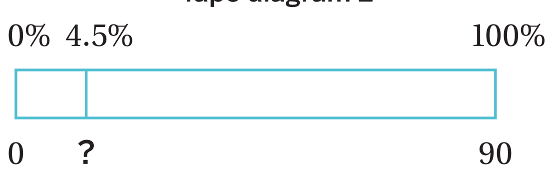
b 90 is 4.5% of what value? ....

c 4.5 is what percent of 90? ....

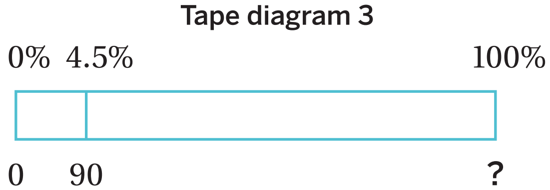
2. Determine the unknown value in each scenario. Show or explain your thinking.

Tape diagram 1

Tape diagram 2



Tape diagram 3



STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You discovered that you can work with percentages that are not whole numbers.

In Grade 6, you learned that to determine 30% of a quantity, you multiply by 30 and then divide by 100, or multiply by 0.30. The same method works for percentages that are not whole numbers, such as 7.8% or 2.5%. To determine 2.5% of a quantity, you can multiply the quantity by 0.025.

You can also use mental math to help determine percentages, such as 2.5%. For example, in order to determine 2.5% of 80, you could first determine 25% of 80, which is 20, and then divide by 10, which is 2.

In Grade 6, you used tape diagrams to help make sense of problems involving whole number percentages. You can also use tape diagrams to help make sense of problems involving percentages that are not whole numbers.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Without the use of a calculator, determine each percentage of 75. Show or explain your thinking.

   a. What is 10% of 75?

   b. What is 1% of 75

   c. What is 0.1% of 75

   d. What is 0.5% of 75?

   e. What is 500.05% of 75?

2. Clare, Elena, and Bard are each working to determine the unknown value in the tape diagram. Their work is shown. Study their work and explain what process(es) they each used.

Clare's work:	Elena's work:	Bard's work:
1% of 80 is 0.8 $6.4 \div 0.8 = 8$ 8%	$\frac{6.4}{80} \cdot 100 = 8$ 8%	If 80 is 100%, then 8 is 10%, and 0.8 is 1%. $8 - 2 \cdot 0.8 = 6.4$, so it is equivalent to $10\% - 2 \cdot 1\%$, which is 8%.

Name: ____________ Date: ________ Period: ________

3. Select *all* of the options that have the same value as $3\frac{1}{2}\%$ of 20.

A. 3.5% of 20

B. $3\frac{1}{2} \cdot 20$

C. $0.35 \cdot 20$

D. $0.035 \cdot 20$

E. 7% of 10

4. A 50 cm piece of wire is bent into a circle. What is the approximate area of this circle?

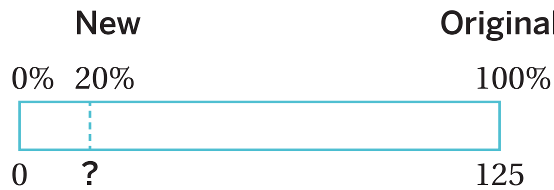
5. Match verbal description with the tape diagram that can be used to represent it. Then determine the unknown value.

a. 125% of 80

b. 25% of 80

c. 80% of 125

d. 20% of 125

____


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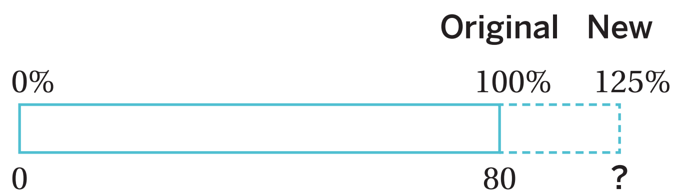

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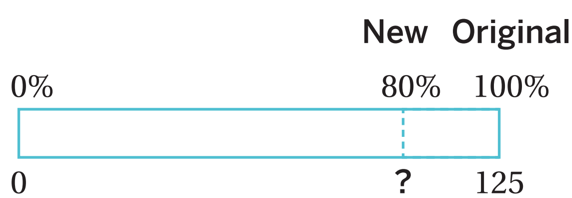

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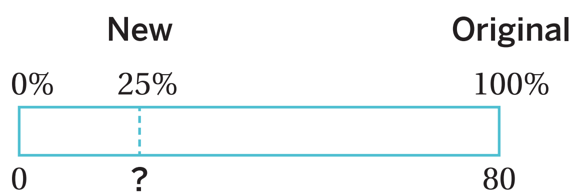


Unit 4 | Lesson 3

Percent Increase and Decrease

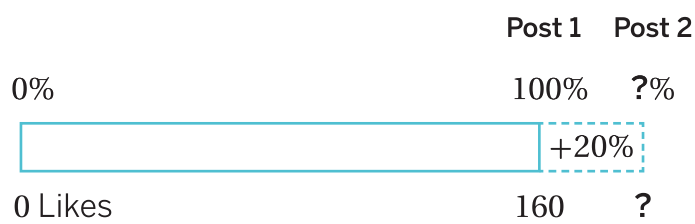
Let's use percentages to describe increases and decreases.

Warm-up Understanding 20%

Three students tracked their "likes" on Instapix between two posts. Each student's first post had 160 likes. Match each scenario with the corresponding tape diagram that can be used to determine the number of likes on the second post.

Scenario | **Tape diagram**

a Han had 20% of the "likes" on his second post that he had on his first post. ________



b Mai had 20% more "likes" on her second post than on her first post. ________

c Shawn had 20% less "likes" on the second post than on the first post. ________

Log in to Amplify Math to complete this lesson online.

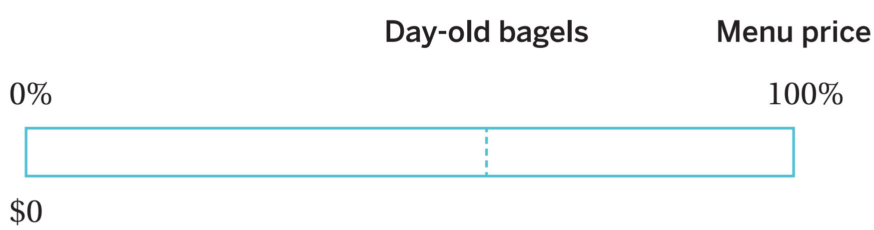
Name: ______ Date: ______ Period: ______

Activity 1 Increase or Decrease

Andre owns a café. Use what you know about percentages to determine the price of different items on his menu.

1. In order to set the price of a smoothie on his menu, Andre adds a 225% percent increase onto the cost of the ingredients, which is $1.50. Use the tape diagram to help you to determine the price of a smoothie shown on his menu.

2. Andre reduces the price of day-old bagels by 40% from the menu price. Use the tape diagram to determine the price of a day-old bagel, if the menu price of a bagel is $0.95.



Activity 2 Markup and Markdown

1. It costs Stores A, B, and C $13.50 to buy each t-shirt that they then sell in their store. In order to make a ***profit***, they add a ***markup*** to the cost of each t-shirt. For each store, complete the tape diagram representing the markup. Then determine the ***retail price*** for each store. Round all responses to the nearest hundredth.

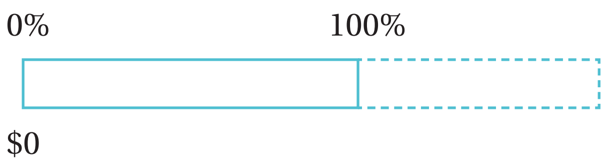
a **Store A:** 40% markup

Tape diagram:

Show your thinking.

b **Store B:** 72% markup

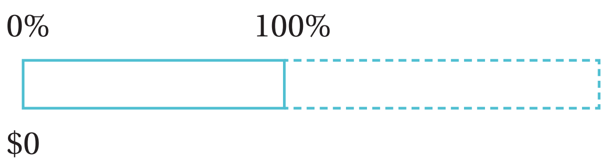
Tape diagram:



Show your thinking.

c **Store C:** 120.5% markup

Tape diagram:



Show your thinking.

Name: ________ Date: ________ Period: ________

Activity 2 Markup and Markdown (continued)

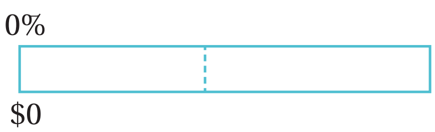
2. Stores A, B, and C are each having an end of season sale on the same brand and style of sneakers. The table gives the information on the sales at each store.

	Store A	Store B	Store C
Retail price ($)	80.00	53.97	41.14
Markdown (%)	55	33.3	12.5

a Without performing any calculations, which store do you think will have the best price? Explain your thinking.

b Complete each tape diagram to determine the store that will have the best price.

Store A tape diagram:



Show your thinking:

Store B tape diagram:

Show your thinking:

Store C tape diagram:

Show your thinking:

Reflect: How well were you able to communicate which sneakers had the best price?

STOP

Summary

In today's lesson . . .

You used tape diagrams to make sense of problems involving ***percent increase*** and ***percent decrease***. You reasoned that in order to solve these types of problems, you can start with the original 100% and either add or subtract the percentage of increase or decrease. Specifically, you explored the concept of ***markups*** and ***markdowns***. You learned that markups are used to determine ***retail prices*** that ensure that companies make a ***profit*** on goods that they sell.

Consider these examples:

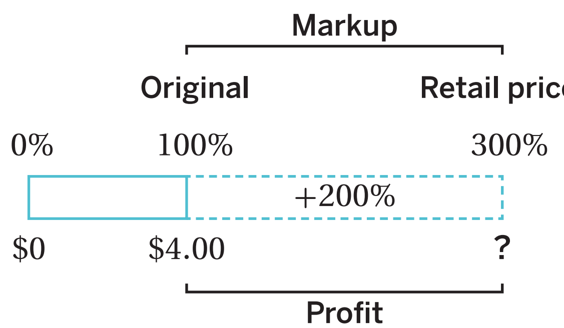
A store sells a $4.00 t-shirt with a markup of 200%.

Profit ($):

$2 \cdot 4.00 = 8.00$

Retail price ($):

$3 \cdot 4.00 = 12.00$



The retail price of a box of cereal is $5.00. It is being sold with a markdown of 30%.

Discount ($):

$0.3 \cdot 5.00 = 1.50$

Sale price ($):

$0.7 \cdot 5.00 = 3.50$

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. Write each percent increase or decrease as a percentage of the original amount. The first one is completed for you. Consider drawing a diagram to help with your thinking.

a This year, there was 40% more snow than last year.
The amount of snow this year is 140% of the amount of snow last year.

b This year, there were 25% fewer sunny days than last year.

c A restaurant adds a 250% markup to the price of the ingredients to set the menu price.

d The sales price of a pair of earrings is 10% less than the retail price.

2. Label the diagram to represent each of the following situations.

a The amount of flour that a bakery used this month was 50% more than the amount of flour used last month. Let f represent the amount of flour last month.

b The amount of milk that the baker used this month was 75% less than the amount of milk used last month. Let m represent the amount of milk last month.

Practice

Name: ________ Date: ________ Period: ________

3. Priya is trying to determine the cost of a new pair of boots after applying a 40% off coupon. The original cost of the boots is $\$84.00$. Circle and correct the mistake in her work.

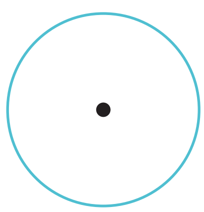
$0.4 \cdot 84.00 = 33.60$

The boots will cost $\$33.60$.

4. To make a shade of paint called jasper green, 4 cups of green paint are mixed with $\frac{2}{3}$ cups of black paint. How much green paint should be mixed with 4 cups of black paint to make jasper green? Show or explain your thinking.

5. Refer to the circle in part a.

a. Draw a scaled copy the circle, using a scale factor of 2.



b. How does the circumference of the scaled copy compare to the circumference of the original circle?

c. How does the area of the scaled copy compare to the area of the original circle?

6. Use what you know about percentages to complete these problems. Show your thinking.

a. What is $\$40\%$ of 12?

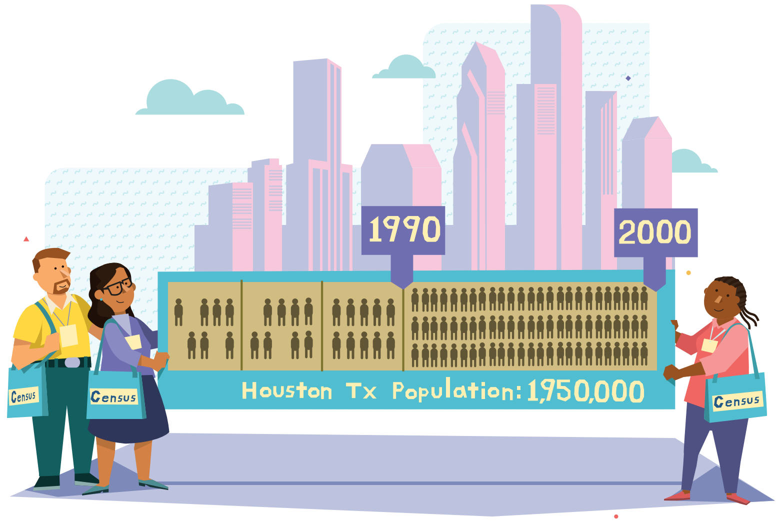
b. 12 is 40% of what number?

c. What is 114% of 21?

d. 21 is 114% of what number?

Name: ______ Date: ______ Period: ______

Unit 4 | Lesson 4



Determining 100%

Let's solve more problems about percent increase and decrease.

Warm-up Matching Scenarios

Match each scenario with the corresponding tape diagram that could be used to represent it.

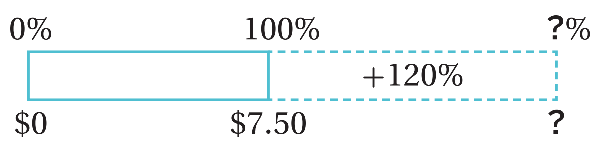
a What is 120% of 7.50?

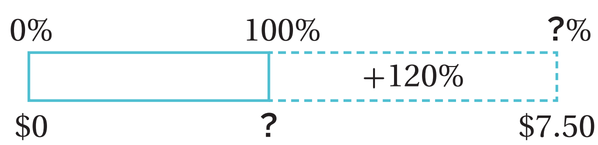
b 7.50 is 120% of what number?

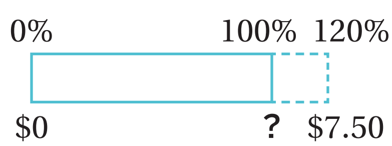
c What is the retail price of a pair of pants including a 120% markup on the purchase price of $7.50?

d What is the cost of a pair of earrings if the retail price, including a 120% markup, is $7.50?

______ 

______ 

______ 

______ 

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Activity 1 Population Growth and Decline

1. In the year 2000, the population of Houston, TX, was approximately $1{,}950{,}000$. This was after an approximate 20% increase in population from 1990. What was the population of the city in 1990? Draw a tape diagram that can be used to represent this situation. Then determine the population.

Tape diagram:

0% 100%

0

Show your thinking.

2. In the year 2000, the population of Detroit, MI, was approximately $950{,}000$. This was after an approximate 7.5% decrease in population from 1990. What was the population of the city in 1990? Draw a tape diagram that can be used to represent this situation. Then determine the population.

Tape diagram:

0% 100%

0

Show your thinking.

Are you ready for more?

From 1970 to 2000, the population of San Antonio, TX, increased by about 20% each decade.

1. If the population in 2000 was $1{,}145{,}000$, determine the approximate population in:
   a. 1990? **b.** 1980? **c.** 1970?

2. Based on your calculations, did the population increase by 30% from 1970 to 2000? Show or explain your thinking.

Name: .... Date: .... Period: ....

Activity 2 Card Sort: New or Original?

You will be given a set of cards.

1. Read the scenario on each card. Determine whether you are being asked to determine the *new value* or the *original value*.

New value	Original value

2. Choose two scenarios from each category and solve the problem on the card. Show your thinking. Use a tape diagram, if needed, to help you make sense of the scenario.

New value	
Card	Card ....

Original value	
Card	Card ....

STOP

Summary

In today's lesson . . .

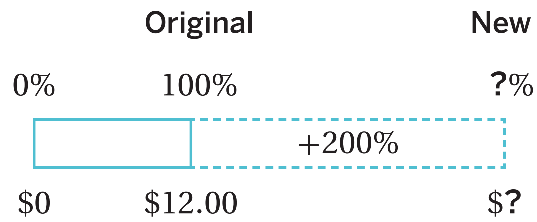
You determined that when solving problems about percent increase and percent decrease, it is important to start by asking yourself, "What does 100% represent in this situation?" You can then work to determine a percentage of that amount.

Consider these examples:

What is $12.00 increased by 200%?

So, $12.00 increased by 200% is $36.00.

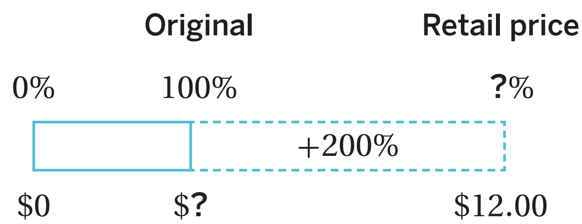
1. Add the percent change to 100%.
   100% + 200% = 300%
2. Multiply 12 by the decimal value that represents 300%. $12 \cdot 3 = 36$



What number increased by 200% is $12.00?

So, $4.00 increased by 200% is $12.00.

1. Add the percent change to 100%.
   100% + 200% = 300%
2. Divide 12 by the decimal value that represents 300%.
   $12 \div 3 = 4$



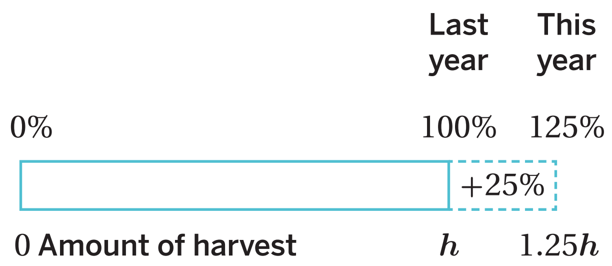
Reflect:

Name: .... Date: .... Period: ....

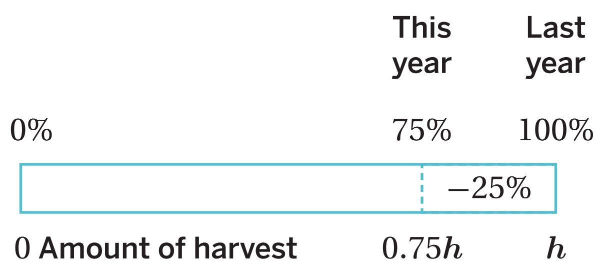
1. The number of fish in a lake decreased by 15% between last year and this year. This year, there were 51 fish in the lake. What was the population last year? Consider drawing a tape diagram, to help you make sense of the situation.

2. Match each tape diagram with its corresponding scenario. It is possible that each diagram can be matched with more than one scenario.

a



b



.... This year's strawberry harvest is 25% more than last year's.

.... This year's blueberry harvest is 75% of last year's.

.... This year's peach harvest is 25% less than last year's.

.... This year's plum harvest is 125% of last year's.

3. Noah thinks the solutions to these two problems are the same. Do you agree with him? Explain your thinking.

This year, a herd of bison had a 10% increase in population. If there were 550 bison in the herd last year, how many are in the herd this year?

This year, another herd of bison had a 10% decrease in population. If there are 550 bison in the herd this year, how many were in the herd last year?

Practice

Name: .... Date: .... Period: ....

4. Jada is creating circular birthday invitations for her friends. The diameter of the circle is 12 cm. She bought 180 cm of ribbon to glue around the edge of each invitation. How many invitations can she create using this ribbon? Show or explain your thinking.

5. A certain type of car has room for 4 passengers to be seated.

a Write an equation relating the number of cars n to the number of passengers p.

b How many passengers could be seated in 78 cars? Show your thinking.

c How many cars would be needed to seat 78 passengers? Show your thinking.

6. Determine each of the following percentages. Show your thinking. Round to the nearest hundredth, if necessary.

a 50 is what percent of 10?

b 5 is what percent of 12?

c 12 is what percent of 42?

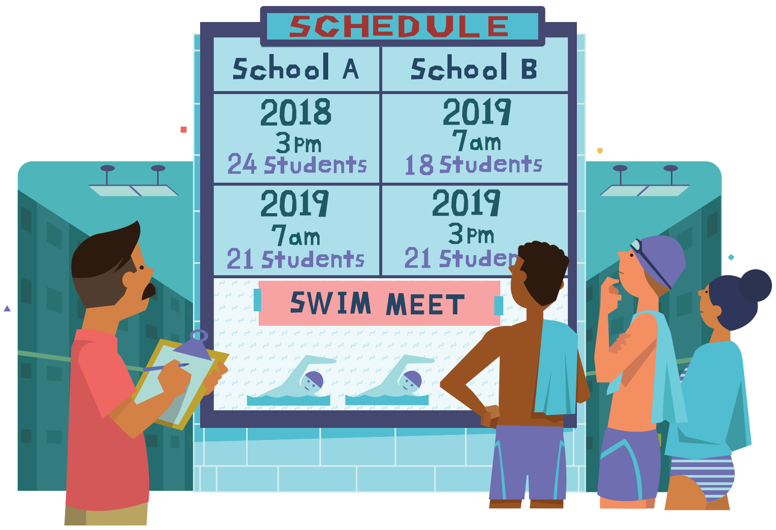
d 42 is what percent of 12?

Name: ____________ Date: ________ Period: ________

Unit 4 | Lesson 5

Determining Percent Change

Let's determine the increase or decrease of a quantity as a percent.



Warm-up Who Is Correct?

Han and Mai are both sports reporters for their school newspaper. This week, the school's basketball team played two games. During the first game, they scored 80 points and during the second game, they scored 100 points.

- Han says that the team increased their points scored by 20% because 20% of 100 points is 20 points.
- Mai says that the team increased their points scored by 25% because 25% of 80 points is 20 points.

Who is correct? Show or explain your thinking.

Three Reads: Read the introductory information three times.

1. Make sense of the scenario.
2. What mathematical quantities are given?
3. Brainstorm strategies to determine who is correct.

Log in to Amplify Math to complete this lesson online.

Activity 1 Changing Swimming Time

Two schools, School A and School B, have competitive swim teams. In 2020, School A had practice after school, but in 2021, it was moved before school. School B had practice before school in 2020, but moved it after school in 2021. Both schools are interested in seeing whether changing the time of practice affected the number of students on the swim team. The table shows the number of students on the swim team for the two schools for 2020 and 2021.

	School A	School B
2020	24	18
2021	21	21

1. A local newspaper reported that changing the practice time affected the number of students on the swim team at each school by the same amount.

   a. What was the change in the number of students on the swim team at School A?

   b. What was the change in the number of students on the swim team at School B?

2. The same newspaper claimed that, because the schools had the same change in the number of students, the size of the swim team in each school changed by the same percent.

   a. Without performing any calculations, do you agree or disagree with the newspaper's claim? Explain your thinking.

   b. What is the percent decrease in the size of the swim team of School A? Show or explain your thinking.

Name: ______ Date: ______ Period: ______

Activity 1 Changing Swimming Time (continued)

c What is the percent increase in the size of the swim team of School B? Show or explain your thinking.

d Do your results in parts b and c support the newspaper's claim?

Activity 2 Comparing Methods

School B was so encouraged by the impact of changing the practice from the morning to the afternoon for the swim team, they decided to move the start time of school 30 min later in hopes of decreasing the number of late arrivals each morning. Kiran and Tyler were both asked to calculate the percent decrease in the number of students that showed up late to first period before and after the start time was changed. The data is shown in the table.

Original start time	Later start time
48	30

1. Draw a diagram that can be used to represent this scenario.

2. Compare and contrast Kiran's method and Tyler's method.

Kiran's method	Tyler's method
$\frac{30}{48} \cdot 100 = 62.5$ 30 is 62.5% of 48 and 48 represents 100%. $100\% - 62.5\% = 37.5\%$ There is a decrease of 37.5% in students arriving late.	$48 - 30 = 18$ The number of students arriving late decreased by 18 students. $\frac{18}{48} \cdot 100 = 37.5$ 18 is 37.5% of 48, so there was a 37.5% decrease in students arriving late.

a How are they similar?

b How are they different?

Name: ______ Date: ______ Period: ______

Activity 3 Card Sort: Comparing Values

You will be given a set of cards.

1. Match each scenario to what you are being asked to determine: the original amount, the new amount, or the percent change.

Original value	New value	Percent change

2. Choose one of each type of problem to solve. Identify which card your work matches. Consider using a tape diagram to help with your thinking.

Original value: Card ______

New value: Card ______

Percent change: Card ______

STOP

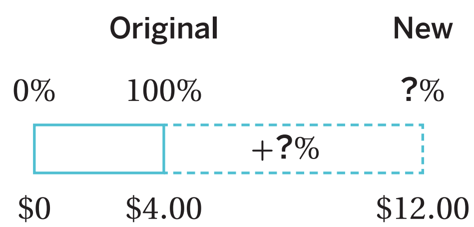
Summary

In today's lesson . . .

You identified the ***percent change*** when given two values. As in the previous lesson, you determined that it was important to start by asking yourself, "What does 100% represent in this situation?"

Consider these examples:

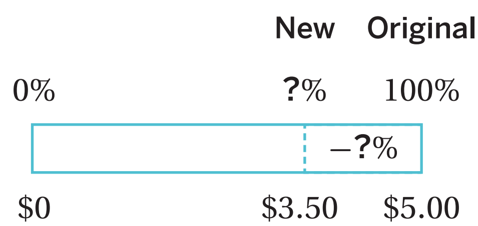
An item costs $\$4.00$ and a store sells it for $\$12.00$. What is the percent change?



The price increased by 200%.

1. The percent that corresponds with $\$12.00$ is $\frac{12}{4} \cdot 100 = 300$ or 300%.
2. The percent change is $300\% - 100\% = 200\%$.

A store changes the price of an item from $\$5.00$ to $\$3.50$. What is the percent change?



The cost decreased by 30%.

1. The percent that corresponds with $\$3.50$ is $\frac{3.50}{5.00} \cdot 100 = 70$ or 70%.
2. The percent change is $100\% - 70\% = 30\%$.

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. Determine each percent change. Show or explain your thinking. Consider drawing a tape diagram to help with your thinking.

   a. **Original Price:** $\$6.00$
   Markup Price: $\$7.20$

   b. **Original Price:** $\$7.20$
   Markdown Price: $\$6.00$

2. A certain small town had a population of $2{,}000$ in 1990.

   a. By 2000, the population increased to $2{,}500$. What is the percent change from 1990 to 2000? Show or explain your thinking.

   b. In 2010, the population decreased to $2{,}000$. What is the percent change from 2000 to 2010? Show or explain your thinking.

   c. Why is the percent change in part a not equal to the percent change from part b? Explain your thinking.

3. Without determining the actual percent increase, explain why it is not reasonable to say that, if the price of a bag of clementines increases from $\$4.00$ to $\$5.00$, it represents a 125% increase.

Practice

Name: ____________ Date: ____________ Period: ____________

4. A person's resting heart rate is typically between 60 and 100 beats per minute. Noah looks at his watch, and counts 8 heartbeats in 10 seconds.

a Is his heart rate typical? Explain your thinking.

b Write an equation that represents h, the number of times Noah's heart beats (at this rate) in m minutes.

5. Elena walked 12 miles. Then she walked $\frac{1}{4}$ of that distance. How many miles did she walk altogether? Select *all* that apply.

A. $12 + \frac{1}{4}$

B. $12 \cdot \frac{1}{4}$

C. $12 + 12 \cdot \frac{1}{4}$

D. $12\left(1 + \frac{1}{4}\right)$

E. $12 \cdot \frac{3}{4}$

F. $12 \cdot \frac{5}{4}$

6. Kiran and Elena are both simplifying the expression $4(2x + 8x)$. Each person's first step is shown:

Kiran	Elena
$4 \cdot 2x + 4 \cdot 8x$	$4(10x)$

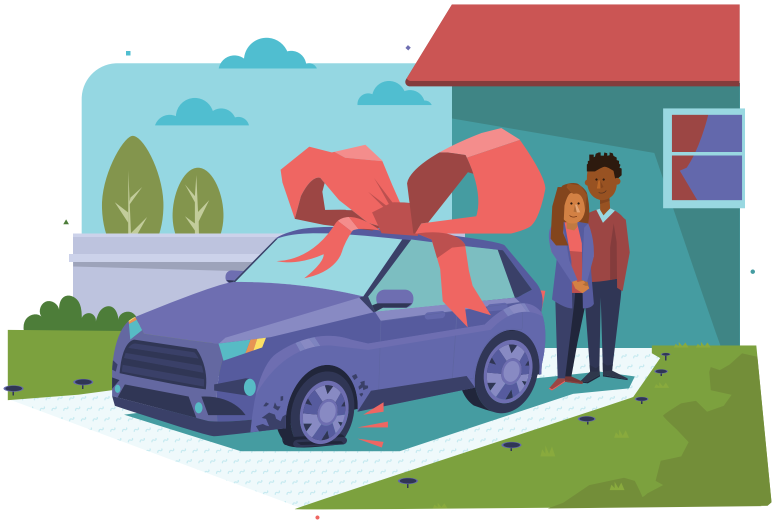
a Which student used the Distributive property as their first step?

b Finish simplifying each student's work to show that they are equal.

Name: ______ Date: ______ Period: ______

Unit 4 | Lesson 6

Percent Increase and Decrease With Equations



Let's use equations to represent scenarios involving percent increase and decrease.

Warm-up Analyzing Increase

Three news stations are reporting that the number of people who exercise at least twice a week in a certain town has increased from 1,600 people last year to 1,920 people this year.

- **News Town reports that this represents a 20% increase.**
- **Local News Network reports that this represents a 120% increase.**
- **News 4 You reports that this represents 120% of the number of people from the previous year.**

Which station (or stations) is correct? Show or explain your thinking. Consider drawing a tape diagram to help with your thinking.

Activity 1 More Markup and Markdown

As you solve each problem, consider drawing a tape diagram to help with your thinking and make sense of each scenario.

1. A café adds a 225% markup on the cost of ingredients for each dish they sell to determine the menu price. Determine each menu price given the cost of ingredients shown.

Cost of ingredients, ($)	Show your thinking	Menu price, ($)
1.00		
3.50		
5.70		
x		

2. A grocery store offers a markdown of 15% on all items that are within a week of their expiration date. Determine the sale price on each item, given its original cost.

Cost of item, ($)	Show your thinking	Sale price, ($)
1.00		
5.00		
10.20		
x		

Name: ______ Date: ______ Period: ______

Activity 2 Who Is Correct?

Typically, the value of a new car decreases by 18% by the end of the first year. Let c represent the cost of a car when it is new and v represent the value of the car by the end of the first year.

1. Complete the tape diagram to represent this scenario.

2. Five students wrote equations that they think represent this scenario and tape diagram. Study these equations. Which of the students' equations are correct? Explain your thinking.

Student	Equation
Priya	$0.18c = v$
Diego	$c - 0.18c = v$
Han	$(1 - 0.18)c = v$
Clare	$c + 0.18c = v$
Shawn	$0.82c = v$

Reflect: Why did you need to consider the person's perspective in order to determine which equations were correct?

Activity 3 Representing Percent Change With Equations

For each scenario, write and solve an equation to determine the unknown values. Consider drawing a tape diagram to help with your thinking and make sense of each scenario.

1. From last year to this year, the cost of a popular cell phone increased by 30%.

a. Write an equation representing the cost of the cell phone this year y, given its cost last year x.

b. Determine the price of the phone last year, if, this year, it cost $\$650$.

c. Determine the price this year, if, last year, the cell phone cost $\$900$.

2. You have a coupon for 28% off any item in a store.

a. Write an equation representing the sale price y of any item, given the retail price of x.

b. If the sale price of an item is $\$18.00$, what was the retail price?

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You reasoned about the different ways you can represent percent change problems using equations.

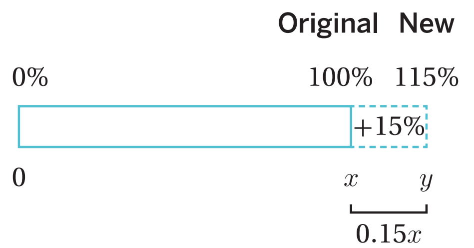
For example, suppose y is 15% more than x.

These three equations can be written to model the relationship between x and y:

$y = x + 0.15x$

$y = (1 + 0.15)x$

$y = 1.15x$



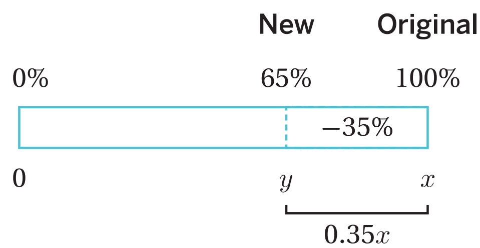
For example, suppose y is 35% less than x.

These three equations can be written to model the relationship between x and y:

$y = x - 0.35x$

$y = (1 - 0.35)x$

$y = 0.65x$



Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. While playing a video game together, Clare scored 50% more points than Tyler. If c is the number of points that Clare scored and t is the number of points that Tyler scored, which equations represent the relationship between Clare's score and Tyler's score? Select *all* that apply.

A. $c = 1.5t$

B. $c = t + 0.50$

C. $c = t + 0.50t$

D. $c = t + 50$

E. $c = (1 + 0.50)t$

2. Write an equation to represent each problem and then solve the problem.

a. On the day it is released, a new video game costs $\$60$. Within a few weeks, the price of new games generally drops by about 12%. What will the sale price of the game be a few weeks after release?

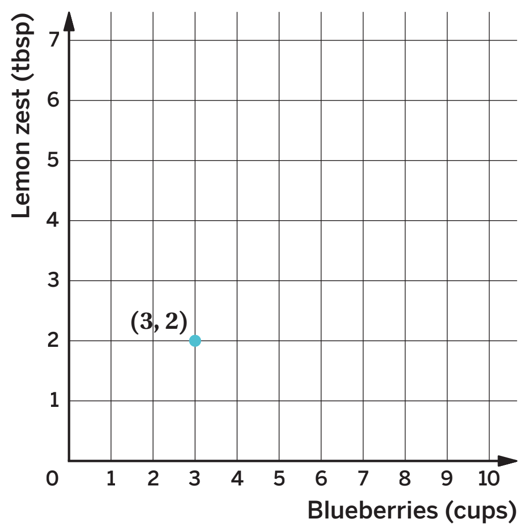
b. Elena's aunt bought her a savings bond when she was born. When Elena is 12 years old, the bond had increased in value by 105% and is worth $\$307.50$. How much was the bond worth when Elena's aunt bought it?

3. Shawn wrote the equation $0.80x = y$ to represent the cost y of an item that is sold at an 80% discount, where x represents the original price. Explain the mistake that Shawn made. Then write the correct equation.

Name: ________ Date: ________ Period: ________

Practice

4. The relationship between the cups of blueberries and the amount of lemon zest used to create a blueberry glaze is shown on the graph.



a Plot another point on the graph that keeps the ratio of blueberries to lemon zest consistent.

b What is the constant of proportionality of the relationship shown in the graph?

c What does the constant of proportionality represent?

d Write two equations that represent the relationship between the number of cups of blueberries b and the number of tablespoons of lemon zest z.

5. Could 7.2 in. and 28 in. be the diameter and circumference of the same circle? Explain your thinking.

6. Write an equation to represent each scenario.

a What is the distance d traveled after t hours driving at 50 mph?

b A t-shirt costs $\$18.00$. What is the total cost c of buying t t-shirts?

c A worker is paid minimum wage in their state of $\$10.00$ per hour. What is their total earnings e for h hours worked?

d What is the value of y if it represents 40% of a given value x?

Unit 4 | Lesson 7

Using Equations to Solve Percent Problems

Let's use equations to solve problems about percent change.

Warm-up Number Talk

Determine the value that makes each equation true. Be prepared to explain your thinking.

1. $100 \cdot \boxed{} = 106$

2. $100 \cdot \boxed{} = 90$

3. $90 \div \boxed{} = 100$

4. $106 \div \boxed{} = 100$

Compare and Connect: Share your strategies with a partner and discuss what is similar and different. You will see multiple strategies in this lesson to solve problems involving percent change. Continue to compare these strategies.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Analyzing Increase, Revisited

Three news stations report that the number of people who exercise at least twice a week in a certain town has increased from 1,600 people last year to 1,920 people this year.

Elena and Jonah both wrote an equation to make sense of this situation.

1. Elena wrote the equation $1600 \cdot x = 1920$.

a. Identify what each value in her equation represents:

1,600 represents . . .

1,920 represents . . .

x represents . . .

b. Solve the equation for x. Show your thinking.

c. What does the solution represent in terms of the situation?

2. Jonah wrote the equation $1600y = 320$.

a. Identify what each value in his equation represents:

1,600 represents . . .

320 represents . . .

y represents . . .

b. Solve the equation for y. Show your thinking.

c. What does the solution represent in terms of the situation?

Activity 2 Partner Problems: Percent Change Equations

With your partner, decide who will use Elena's method and who will use Jonah's method to determine the percent change for the following scenario. Use your designated method, and then compare your response with your partner. If your responses are not the same, work together to correct any errors or resolve any disagreements. Consider drawing a tape diagram to help with your thinking and make sense of the scenario.

A school started a peer tutoring program. They asked students prior to the program to rate how nervous they felt about learning new math concepts, and then again after the program. A rating of 0 meant they did not feel nervous, and 4 meant they felt very nervous.

- Prior to the peer tutoring program, the average rating was 2.24.
- After having peer tutoring, the average rating was 1.81.

Write and solve an equation to determine the percent change in the ratings.

Elena's method (*percent of* method)	Jonah's method (*percent change* method)
Let .... represent . . . Show your thinking: Solution:	Let .... represent . . . Show your thinking: Solution:

Name: ________ Date: ________ Period: ________

Activity 3 Using Equations to Solve Percent Problems

1. Match each scenario with the *two* equations that could be used to help determine the unknown value.

   a The number of students on honor roll increased by 130%. Most recently, there were 150 students on honor roll. How many students were originally on the honor roll?

   b The number of students playing on more than one sports team decreased from 150 students to 130 students. What was the percent change?

   c The number of students who eat school lunch increased by 130%. Last year, there were 150 students who ate school lunch. How many eat school lunch this year?

   ______ $150x = 130$

   ______ $150 + 150 \cdot 1.30 = x$

   ______ $x + 1.30x = 150$

   ______ $150 \cdot 2.30 = x$

   ______ $150x = 20$

   ______ $2.3x = 150$

2. For each scenario, choose one equation to analyze. Then use the equation to determine the unknown value.

	Scenario A	Scenario B	Scenario C
Equation			
What does x represent?			
What is the solution? Show your thinking.			

STOP

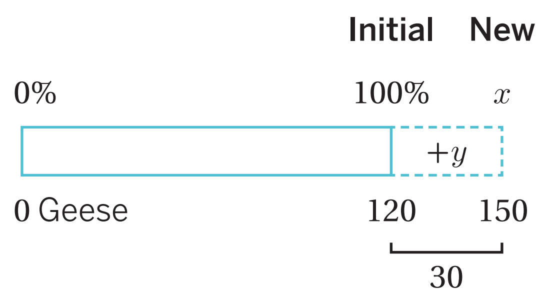
Summary

In today's lesson . . .

You reasoned about how to use equations to model the *percent of* and the *percent change* between two values. You also determined that once you have calculated one of the values — percent of or percent change — you have sufficient information to determine the other.

For example, suppose the number of geese landing at a certain pond changed from 120 in Week 1 to 150 in Week 2.

These two equations can be written to model the relationship between the original value and the new value of geese, where x represents the *percent of* the original number of geese and y represents the *percent change* from the initial number of geese. You can solve each equation, as follows.



$120x = 150$

$120y = 30$

$$\begin{aligned} 120x &= 150 \\ 120x \div 120 &= 150 \div 120 \\ x &= 1.25 \end{aligned}$$

$$\begin{aligned} 120y &= 30 \\ 120y \div 120 &= 30 \div 120 \\ y &= 0.25 \end{aligned}$$

The solution $x = 1.25$ means that 150 geese is 125% of the original value of 120 geese.

The solution $y = 0.25$ means that 150 geese is a 25% increase from the original value of 120 geese.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. After an adoption event, the number of dogs at a local pound has decreased from 35 dogs to 14 dogs. Tyler wrote the equation $35d = 21$ to determine the percent decrease in the number of dogs at the pound.

   a. What does each value in the equation represent?
   35 represents . . .

   d represents . . .

   21 represents . . .

   b. What is the percent decrease in the number of dogs at the pound?

2. The number of people who "liked" Han's last two photos on Instapix increased from 90 to 108. Write and solve an equation to determine the percent change in the number of likes he received.

3. Match each scenario with the equation(s) that could be used to determine the unknown value.

   a. The cost of a laptop case increased from \$20 to \$22. What is the percent increase in cost?

   b. The number of people attending a picnic increased by 20%. If 22 people originally attended, what is the new attendance?

   c. The number of kittens up for adoption decreased from 22 kittens to 20 kittens. What is the percent decrease?

   .... $22x = 2$

   .... $22 + 0.20 \cdot 22 = x$

   .... $1.20 \cdot 22 = x$

   .... $20x = 2$

   .... $(1 + 0.20) \cdot 22 = x$

Practice

Name: .... Date: .... Period: ....

4. It takes an ant farm 3 days to consume $\frac{1}{2}$ of an apple. At this rate, in how many days will the ant farm consume 3 apples?

5. If x represents a positive number, select *all* of the expressions whose value is greater than x.

A. $\left(1-\frac{1}{4}\right)x$

B. $\left(1+\frac{1}{4}\right)x$

C. $\frac{7}{8}x$

D. $\frac{9}{8}x$

E. $x+1$

F. $x-1$

6. Determine the unknown values in the table.

x	10.00	1	
y	10.45		6.27

SUB-UNIT

2 Applying Percentages

Narrative Connections

Did a quarantined U.S. keep a healthy economy?

In October 2020, the U.S. Commerce Department reported the nation's economy had a record growth rate of 33.1% in the third quarter (i.e., July through September) of 2020. For many Americans, this seemed like good news. Yet, with the country locked down and under quarantine from COVID-19, America's businesses — especially brick-and-mortar stores — were suffering.

To many, a number like 33.1% might feel like a sign that the tide had turned.

But if we take a step back, we'd see that in the country's *second* quarter (April through June), the economy had declined by a historic 31.4%. This decline set the calculations for the third quarter's growth rate at a much lower baseline.

Even with an increase of 33.1%, this growth still wasn't enough to off-set the second quarter's dramatic decline. So, while the third quarter definitely saw *some* growth, the economy was still languishing.

Having an accurate sense of the economy is important because it affects your daily life. A depressed economy can result in increased costs of goods, less reliable public transportation and healthcare, and increased challenges finding work. Because so much of our language around the economy and finances is communicated through percentages, it is crucial to understand exactly what these percentages are saying.

Unit 4 | Lesson 8

Tax and Tip

Let's learn about sales tax and tips.

Warm-up Notice and Wonder

At the bookstore, Shawn is surprised by the amount that shows up on the cash register that is required to purchase a certain book

What do you notice? What do you wonder?

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ____________ Date: ____________ Period: ____________

Activity 1 Third Place Books

Noah's grandmother owns and manages two bookstores in different counties in the Kansas City area —one on the Kansas side, and one on the Missouri side.

Each county has its own sales tax rate.

Plan ahead: What goals do you have for this activity and how will you achieve them?

1. Complete each table shown by determining the sales tax and total cost of each book.

Kansas store

Title	Price ($)	Sales tax ($)	Total cost ($)
The House on Mango Street	8.00	0.48	8.48
The Skin I'm In	25.00	1.50	
A Wrinkle in Time	1.00		
Brown Girl Dreaming	12.00		
	x		

Missouri store

Title	Price ($)	Sales tax ($)	Total cost ($)
The House on Mango Street	8.00	0.60	8.60
The Skin I'm In	25.00	1.88	
A Wrinkle in Time	1.00		
Brown Girl Dreaming	12.00		
	x		

2. Which book will cost less, including sales tax: a $20.99 book at the Kansas store, or a $20.49 book at Missouri store? Consider drawing tape diagrams to help with your thinking.

Activity 2 Tipping

Mai is a server at a diner. In the United States, most servers at restaurants earn an hourly wage plus tips from customers they serve. Refer to the check shown from one of Mai's recent customers.

Date: Sept. 12th
Time: 6:55 PM
Server: Mai

Chicken Parm	**15.50**
Eggplant Parm	**12.50**
Lemon Soda	**2.00**
Tea	**3.00**
Subtotal	**32.00**
Sales tax (9.5%)	**3.04**
Total	**35.04**

1. At Mai's diner, customers usually tip between 15% and 25% of the cost of their meal.

a Mai tells her colleague that her customers appeared happy with their meal, so she predicts that they will leave a tip that is 20% of the total (including sales tax). What is Mai's prediction, in dollars?

b Mai's colleague, Tyler, predicts the customers will leave a $8.00 tip. Write the tip as a percentage of the total cost. Round to the nearest tenth of a percent.

2. The customers actually leave $43.00, and tell Mai that the extra money is for her tip.

a What percentage of the total did the customers tip, to the nearest tenth of a percent?

b Whose estimate was more accurate: Mai's or Tyler's?

Are you ready for more?

Clare was the customer that paid for the meal in this activity. She noticed that the sales tax of 9.5% and the tip of 22.7% would combine to be 32.2%, but that the $43 she paid was actually 34.4% more than the cost of the meal. She wonders why these two percents are not the same. Explain to Clare why they are not the same.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You calculated percentages related to sales tax and tips. ***Sales tax*** is an amount of money that a government agency collects on the sale of certain items. Often, the tax rate is given as a percentage of the cost.

Fractional percentages often arise when a state or city charges a sales tax on a purchase. For example, the sales tax in Arizona is 7.5%. This means that when someone purchases an item, the total cost of that item is actually increased by 0.075 times the amount on the price tag.

The total cost to the customer is the price tag cost plus the sales tax. This is a *percent increase*. For example, in Arizona, the total cost to a customer is 107.5% of the price listed on the tag.

A ***tip*** is an amount of money that a person gives someone who provides a service, such as restaurant servers, hairdressers, and delivery drivers. It is customary in many restaurants to tip the server about 20% of the cost of the meal. If a person plans to leave a 20% tip on a meal, then the total cost will be 120% of the cost of the meal.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. In a city in Ohio, the sales tax rate is 7.25%. Complete the table to show the sales tax and the total cost, including tax, for each item.

Item	Price before tax ($)	Sales tax ($)	Total cost ($)
Drum sticks	8.00		
Maracas	22.00		
Music stand	14.50		

2. The sales tax rate in New Mexico is 5.125%.

a. Select *all* the expressions that represent the sales tax you would pay in New Mexico for an item that costs c.

A. $5.125c$

B. $0.5125c$

C. $0.05125c$

D. $c \div 0.05125$

E. $\frac{5.125}{100}c$

b. Write an expression to represent the total cost, including sales tax, for an item that costs c in New Mexico.

3. The table shows the cost of certain items and the amount of sales tax charged on each in Nevada.

Cost of item ($)	Sales tax ($)
10	0.46
50	2.30
5	0.23

a. What is the sales tax rate in Nevada?

b. Write an expression that represents the amount of sales tax charged, in dollars, on an item that costs c dollars.

c. Write an expression to represent the total cost, including sales tax, for an item that costs c.

Name: ______ Date: ______ Period: ______

Practice

4. Determine the following values using what you know about percentages. Show your thinking.

a) 3.8% of 25

b) 0.2% of 50

c) 180.5% of 99

5. A hiking trail is $\frac{7}{8}$ of a mile long. Jada and Kiran hike the trail together, completing it in $\frac{1}{4}$ of an hour. What was Jada and Kiran's average speed? Show your thinking.

6. Tyler bought a backpack that was on sale for 35% off. If the sale price of the backpack was $\$26$, what was the original price of the backpack? Show your thinking.

Unit 4 | Lesson 9

Percentage Contexts

Let's learn about more situations that involve percentages.

Warm-up Matching Expressions

The following expressions might represent leaving a 15% tip on a $20 meal. Match the expressions with their corresponding description. Not all expressions will be matched. It is possible that more than one expression will match with the same description.

Expression

a $15 \cdot 20$

b $20 + (0.15 \cdot 20)$

c $1.15 \cdot 20$

d $\frac{15}{100} \cdot 20$

e $15 + 20$

Description

.... The amount of the tip.

.... The total amount, including the tip.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Commission at the Barbershop

Tiki's Barbershop offers several haircutting services. Some services are discounted when you purchase both at the same time.

At Tiki's, all barbers earn their wages from *commission*, which is a percentage of the cost of the service that a business pays to the employee.

Tiki's Barbershop

Haircut	\$20
Shave	\$10
Haircut and shave	\$30
Beard trim/lineup	\$9
Designs	\$10+

1. For each haircut, the barber keeps \$12 and the barbershop owner receives \$8. What is the barber's commission, as a percentage?

2. If the commission percentage remains the same, how much will the barber earn in commission for a haircut and shave?

3. Is a higher commission percentage better for the barber or the owner of the barbershop? Explain your thinking.

4. If a barber wants to earn \$150 a day, what is the total cost of the services they need to provide?

Activity 2 In Whose Interest Is Simple Interest?

Having a bank account can be very helpful for saving money, even at a young age, according to economics professor Ebonya Washington. When you deposit money into a savings account, a bank will pay you what is called *interest* based on the amount of money you have in your account. Because the bank is able to borrow your money while they hold it, they agree to give you a certain amount in return, known as interest.

***Simple interest* is calculated with the following formula:**

simple interest = principal × rate × time, where *principal* represents the amount of money that was borrowed.

The formula can be expressed as $I = prt$, where I represents the simple interest earned, p represents the principal, r represents the annual interest rate, and t represents the time, in years.

1. Diego was just hired for his first job, and decides to buy a car to help him get to work. He borrows $\$10,000$ from his bank to purchase the car. The bank charges a rate of 3.5% simple interest per year for their car loans.

a If Diego pays the loan back consistently over the course of 10 years, how much total interest will he pay?

b Including the amount of the loan and the amount of interest, how much will Diego pay for his car?

Name: ______ Date: ______ Period: ______

Activity 2 In Whose Interest Is Simple Interest? (continued)

2. Diego has a savings account at the same bank, where he keeps some of the extra money he earns from his job. He deposits $275 in his bank account. If the bank pays Diego 8% per year for holding his money, after how many years will Diego have at least $400 in his bank account?

Featured Mathematician

Ebonya Washington

Ebonya Washington is an economics professor who takes her research cues from the everyday world around her. A recent idea came while standing in line at the grocery store: Another person in line expressed their frustration about how the grocery store had raised prices the day food stamps came out, prompting Washington to research and write a paper on the subject. Washington also works to promote greater diversity in the field of economics.

Activity 3 Card Sort: Percentage Situations

You will be given a set of cards.

Take turns with your partner matching each situation with a percentage type. For each match, explain your thinking. If you and your partner disagree, work together to resolve any differences and reach an agreement.

Situation card	Percentage type	Explain your thinking.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that there are some everyday situations where a percentage is added to or subtracted from a given amount, in order to be paid to another person or organization who is providing a service.

	Paid to . . .	How it works
Sales tax	The government.	Added to the price of the item(s).
Tip (gratuity)	The server.	Added to the cost of the meal.
Interest	The lender (or account holder).	Added to the balance of a loan, credit card, or bank account.
Markup	The seller.	Added to the price of an item so the seller can make a profit.
Markdown (discount)	The customer.	Subtracted from the price of an item to encourage the customer to buy it.
Commission	The salesperson.	Subtracted from the payment that is collected at a business.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A car dealership pays a wholesale price of $\$12{,}000$ to purchase a vehicle. The car dealership intends to make a 32% profit.

   a. By how much will they mark up the price of the vehicle?

   b. After the markup, what will the retail price of the vehicle be?

   c. The salesperson will earn a 6.5% commission on the sale. How much will their commission be? Show or explain your thinking.

2. Lin is shopping for a couch with her dad and hears him ask the salesperson, "How much is your commission?" The salesperson says that her commission is $5\frac{1}{2}\%$ of the selling price.

   a. How much commission will the salesperson earn by selling a couch for $\$495$?

   b. How much money will the store get from the sale of the couch?

3. A college student takes out a $\$7{,}500$ loan from a bank. Assuming no payments have been made, what will be the balance of the loan after 1 year?

   a. If the bank charges 3.8% interest each year?

   b. If the bank charges 5.3% interest each year?

Name: .... Date: .... Period: ....

4. A store is having a 20% off sale.

a With this discount, the price of one pair of pants before tax is $\$15.20$. What was the original price of the pants?

b A pair of shoes at the same store originally cost $\$49.99$. What will the shoes cost after the discount?

5. Match each value with its corresponding point on the number line:

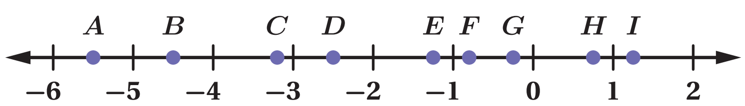
.... -4.5

.... -3.2

.... -2.5

.... $-\frac{4}{5}$

.... $-\frac{5}{4}$



6. The tape diagram represents a situation involving the discounted price of an item. Identify each of the following:

a The original price.

b The sale price.

c The discount amount, in dollars.

d The percent of the discount.

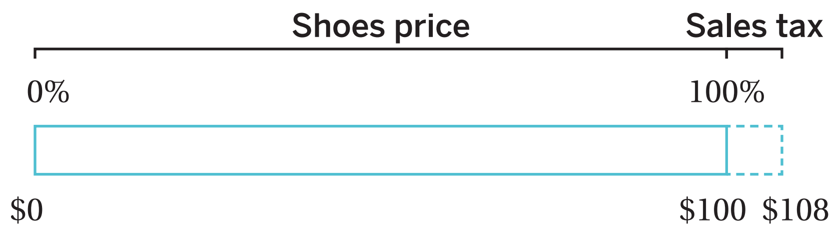
Determining the Percentage

Let's determine unknown percentages.

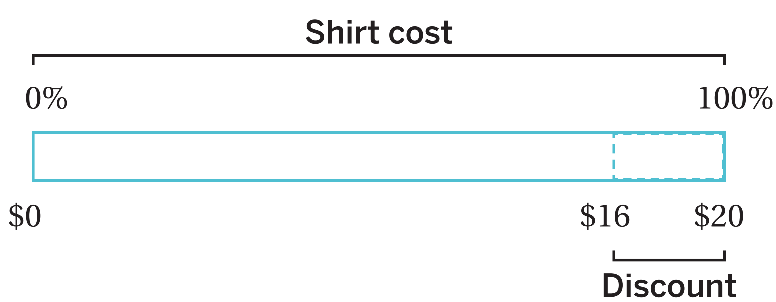
Warm-up Percentages in Context

Refer to the tape diagram to solve each problem.

1. What percent of the price of the shoes is the sales tax?



2. What percent of the shirt cost is the discount?



Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 What Is the Percentage?

1. According to the U.S. Department of Labor in 2015 and 2016, a farmworker earned about $0.36 for every 30 lb of tomatoes picked. In 2016, the price of 30 lb of tomatoes at a grocery store was about $60. What percent of the cost of tomatoes at the grocery store did a farmworker earn?

2. The bill for a meal was $33.75. The customer left $40.00. What percent of the bill was the tip?

Activity 1 What Is the Percentage? (continued)

3. The original price of a bicycle was \$375. Now it is on sale for \$295. What percent of the original price was the markdown?

Are you ready for more?

Earlier in this activity, you read that a farmworker earns about \$0.36 for every 30 lb of tomatoes picked.

1. How many pounds of tomatoes must be picked per hour in order to earn the U.S. federal minimum wage of \$7.25 (as of 2020)?

2. A typical farmworker picks 875 lb of tomatoes every hour. What wage does this typical farmworker earn?

sunlover/Shutterstock.com

Name: ______________________ Date: __________ Period: __________

Activity 2 Info Gap: Fair Trade Produce

Diego and Kiran did some research and learned that buying fair trade produce can help to ensure that a greater percentage of the cost of produce goes to these workers.

You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given a *problem card*:	If you are given a *data card*:
1. Silently read your card and think about what information you need to be able to solve the problem.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner, *"What specific information do you need?"* and wait for them to ask for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask *"Why do you need that information?"* Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently in the space provided.	**4.** Read the *problem card* and solve the problem independently in the space provided.
5. Read the *data card* and discuss your reasoning.	**5.** Share the *data card* and discuss your reasoning.

Pause after Problem 1 so your teacher can review your work. You will be given a new set of cards. Repeat the activity by trading roles with your partner.

Problem 1 Work	Problem 2 Work

STOP

Summary

In today's lesson . . .

You saw how percent change can be applied to contexts involving tax, tip, commission, and other contexts involving the exchange of money. It can be especially important to pay careful attention to vocabulary in problems involving percentage contexts.

As problems involving percentages become more complicated, it is also important to have a plan; keep track of what you have already determined and what you still need to determine. This will help you as you work through multiple steps on your way to the solution to the problem.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. A music store marks up the instruments it sells by 30%. Consider drawing a tape diagram to help with your thinking.

   a. If the store bought a guitar for \$45, what will be its retail price?

   b. If the price tag on a trumpet shows \$104, how much did the store pay for it?

   c. If the store paid \$75 for a clarinet and sold it for \$100, did the store mark up the price by 30%?

2. A family eats at a restaurant. The bill is \$42. The family leaves a tip and spends \$49.77.

   a. What was the amount of the tip?

   b. How much was the tip as a percent of the bill?

3. The price of gold is often reported per ounce. At the end of 2015, it was \$1,060. At the end of 2020, it was \$1,895. By what percent did the price per ounce of gold increase?

Practice

Name: ________ Date: ________ Period: ________

4. A phone keeps track of the number of steps a person has taken and the distance traveled. Based on the information in the table, is there a proportional relationship between the two quantities? Explain your thinking.

Number of steps	Distance (km)
950	1
2,852	3
4,845	5.1

5. Solve each equation. Show your thinking.

a. $\frac{2}{3}x = \frac{8}{15}$

b. $1.8 + x = 7.2$

c. $5\frac{4}{5} = 3\frac{2}{3} + x$

6. A radar gun measured the speed of a baseball at 101.2 mph. If the baseball was actually traveling at a speed of 99.8 mph, how much was the radar gun off by?

Name: Date: Period:

Unit 4 | Lesson 11

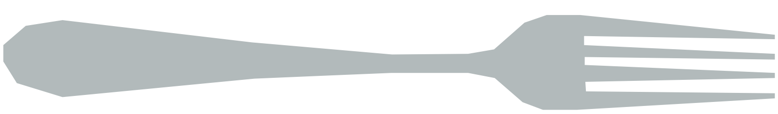
Measurement Error

Let's use percentages to describe how accurately we can measure.

Warm-up Measuring With Different Rulers

The Fickle Flatware company makes eating utensils. They have asked for consultants from your company, Accuracy Associates, to visit their factory and measure their products.

You will be given two rulers — the Fickle Flatware ruler and your company's ruler. Measure and record the lengths of each utensil to the nearest marking on each ruler.



Measurements

Fickle Flatware ruler	Accuracy Associates ruler

Log in to Amplify Math to complete this lesson online.

Activity 1 Comparing Right and Wrong

The Fickle Flatware company has been facing challenges checking that all of their utensils are the correct length. They want to know how incorrect the measurements might be, using their rulers.

1. Using the measurements from the Warm-Up for each ruler, determine the amount of difference in your measurement. Record your measurements and this difference in the table.

2. Determine the amount of this difference as a percent of the actual length, to the nearest tenth of a percent.

	Fickle Flatware ruler measurement	Accuracy Associates ruler measurement	Difference	Difference, as a percent
Spoon				
Fork				
Knife				

Are you ready for more?

A micrometer screw gauge is an instrument that can measure lengths to the nearest micron. One micron is one millionth of 1 m. Would this instrument be useful for measuring any of the following things? If so, what would the largest percent error be?

a. The thickness of an eyelash, which is typically about 0.1 mm.

b. The diameter of a red blood cell, which is typically about 8 microns.

c. The diameter of a hydrogen atom, which is about 100 picometers. One picometer is one trillionth of 1 m.

Name: .... Date: .... Period: ....

Activity 2 Partner Problems: Percent Error

With your partner, decide who will complete Column A and who will complete Column B. For each row, compare your response with your partner. Although the problems in each row are different, your solutions should be the same. If they are not the same, discuss and resolve any differences.

Column A	Column B
1. A meteorologist predicted that a region would receive 10 in. of snow accumulation. The actual amount of snow accumulation was 11 in. What is the percent error?	The crowd at a sporting event is estimated to be 3,000 people. The exact attendance is 2,751 people. What is the percent error?
2. The pressure in a bicycle tire is 63 psi. This is 5% too high, compared to what the manual says is the correct pressure. What is the correct pressure?	A cash register has 0.5% more money than it should, based on receipts. If the register has \$60.30 in it, how much should it have?

STOP

Summary

In today's lesson . . .

You saw that ***percent error*** can be used to describe any situation where there is a correct value and an incorrect value, and you want to describe the relative difference between them. For example, suppose a milk carton manufactured by a company is supposed to contain 16 fluid ounces, but it only contains 15 fluid ounces:

- The measurement error is 1 fluid ounce.
- The percent error is 6.25% because $\frac{1}{16} \cdot 100 = 6.25$.

It is important to remember that the amount of the error is always compared to the actual or correct value to determine the percent error. You can use the following formula.

$$\text{percent error} = \frac{\text{(difference between correct and incorrect value)}}{\text{correct value}} \cdot 100$$

Reflect:

Name: ______________ Date: ______________ Period: ______________

Practice

1. The depth of a lake is 15.8 m.

   a. Jada accurately measured the depth of the lake to the nearest meter. What was Jada's measurement?

   b. By how many meters does Jada's measured depth differ from the actual depth?

   c. Express the measurement error as a percent of the actual depth.

2. A watermelon weighs $8{,}475$ g. A scale measured the weight with an error of 12% less than the actual weight. What was the measured weight?

3. Noah's oven thermometer gives a reading that is 2% greater than the actual temperature.

   a. If the actual temperature is $325°F$, what will the thermometer reading be?

   b. If the thermometer reading is $78°F$, what is the actual temperature?

Practice

Name: .... Date: .... Period: ....

4. Shawn and Priya are on the same basketball team, and have a friendly competition with each other to see who can score more points each game. In one game, Shawn scored 10% more points than Priya. Shawn wrote the equation $s = (1 + 0.10)p$ to represent the relationship between the number of points s Shawn scored and the number of points p Priya scored. Write at least two other equations that can also represent this relationship.

5. Complete each blank using the symbols $>$, $<$, or $=$.

a -201 5

b -5 .... -8

c 3 .... $|-3|$

d -6 .... $-(-6)$

e $-\frac{3}{4}$ .... $\frac{4}{3}$

f 3.24 .... $-(-3.24)$

g $-\frac{2}{3}$ .... $-\frac{5}{6}$

h $-|4|$ .... $-|-4|$

6. Commercial planes typically fly at a maximum altitude of $38{,}000$ ft and a minimum altitude of $31{,}000$ ft. This range allows them to avoid other aircraft and weather conditions during flight.

a Write one or more inequalities to describe the altitude at which a plane typically flies.

b Draw and label a number line showing the possible altitudes.

Name: ______________________ Date: ____________ Period: ____________

Unit 4 | Lesson 12

Error Intervals

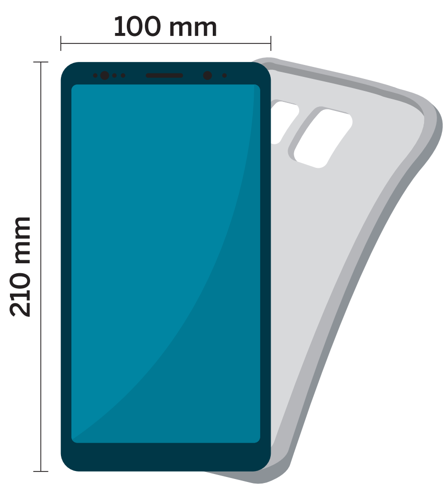
Let's determine how much error is acceptable.

Warm-up Acceptable Error

The Soft Shield company makes flexible phone cases.

The manufactured size of a case can be up to 1% off and still fit the phone.

Determine one set of acceptable dimensions for a flexible phone case for the phone shown.



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Activity 1 Budgeting Tolerance

The City of Burlington accepts 1.5% error in spending compared to the budget for any City Department. If the actual spending is off by more than this amount of error, the Mayor conducts a careful review of all money spent by that department.

1. The Department of Health had a budget of $\$90{,}000$.

a List some acceptable amounts for their spending.

b List some unacceptable amounts for their spending.

2. Complete the table with possible values for each of the empty boxes.

	Budget ($)	Spending ($)	Acceptable?
Parks and Recreation	30,000	31,000	
Transportation		45,000	✓
Sanitation			✓

3. How did you choose your values for the Department of Sanitation? Explain your thinking.

Name: ______________________ Date: __________ Period: __________

Activity 2 Info Gap: Quality Control

You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given a *problem card*:	If you are given a *data card*:
1. Silently read your card and think about what information you need to be able to solve the problem.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner, *"What specific information do you need?"* and wait for them to *ask* for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask *"Why do you need that information?"* Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently in the space provided.	**4.** Read the *problem card* and solve the problem independently in the space provided.
5. Read the *data card* and discuss your reasoning.	**5.** Share the *data card* and discuss your reasoning.

Pause after Problem 1 so your teacher can review your work. You will be given a new set of cards. Repeat the activity by trading roles with your partner.

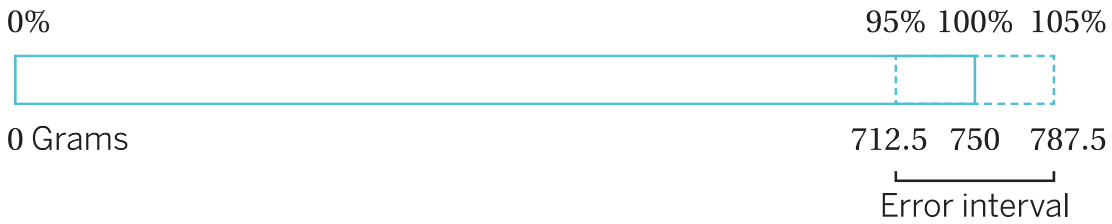
Problem 1 work	Problem 2 work

STOP

Summary

In today's lesson . . .

You saw that percent error is often used to express a range of acceptable values. For example, if a box of cereal is guaranteed to have 750 g of cereal, with a margin of error of less than 5%, what are possible values for the actual number of grams of cereal in the box? The error could be as great as $0.05 \cdot 750 = 37.5$ and could be either less than or greater than the guaranteed amount.



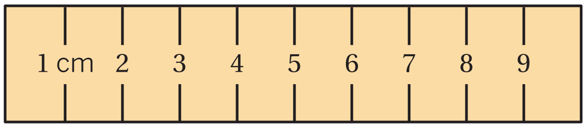
Therefore, the box can have anywhere between 712.5 and 787.5 g of cereal in it, but it should not have 700 g or 800 g, because both of those are more than 37.5 g away from 750 g. This can be represented with the expressions $712.5 \leq x$ and $x \leq 787.5$, where x represents acceptable amounts of the number of grams of cereal. This set of acceptable values is called the ***error interval***.

Reflect:

Name: ______________________ Date: ____________ Period: ________

Practice

1. Jada measured the height of a plant in a science experiment using the ruler shown and determined that it was 5 cm.



a. What is the greatest actual height that the plant could be? Explain your thinking.

b. What is the least actual height that the plant could be? Explain your thinking.

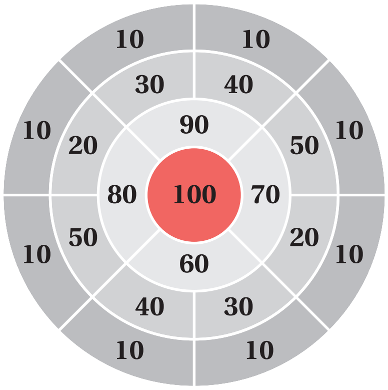
c. How great could the percent error in Jada's measurement be?

2. The reading on a car's speedometer has 1.6% maximum error. The speed limit on a road is 65 mph.

a. The speedometer reads 64 mph. Is it possible that the car is going over the speed limit?

b. The speedometer reads 66 mph. Is the car definitely going over the speed limit?

3. Tyler's darts club team wants Tyler's score to be within 10% of 140 points on his next turn. He will throw 3 darts. Which zones should he aim for to meet his goal?



Practice

Name: .... Date: .... Period: ....

4. Match each scenario with the equation(s) that could be used to determine the unknown value. A scenario may match with multiple equations.

a A customer at a cafe left $12.50 for a meal that cost $10, including tax. What was the percent of tip they left?

b A book cost $10 after sales tax of 1.25% was added to the original price. What was the original cost of the book?

c A hockey rink usually charges $12.50 per person to skate. They discount the price on Tuesdays to $10 per person. What is the percent discount?

.... $1.0125x = 10$

.... $10 \div 1.0125 = x$

.... $\frac{2.50}{10} \cdot 100 = x$

.... $\frac{2.50}{12.50} \cdot 100 = x$

5. Evaluate each expression.

a $2(6 - 1)$

b $6 \div 3 \cdot 2$

c $4 \div 2^2$

d $3 - 1 + 2$

e $4.2 + 8.83 - 1.2$

f $10 \cdot 5.1 \div 0.3$

g $2 + 0.2^2$

h $3^2 + (0.75 + 1.25)^2$

6. Two reporters, Noah and Clare wrote articles about how the restaurant industry includes markups on the cost of the food they sell. An excerpt from each article is shown. Is it possible for both of them to be correct? Show or explain your thinking.

Noah's article:	Clare's article:
"Generally, restaurants triple the cost of ingredients to determine the menu price of a dish."	"In the restaurant industry, it is standard practice for the cost of each dish to include a 200% markup on the cost of ingredients."

Name: ______________________ Date: ______________ Period: ____________

Unit 4 | Lesson 13 – Capstone

Writing Better Headlines

Let's write responsible and accurate headlines.

Warm-up Choosing a Headline

Headlines play a powerful role in communicating information. Sometimes, readers choose to read *only* the headlines of certain articles.

Read the headline and article excerpt shown. Select a different headline from the given choices that could be used for this article. Explain your choice.

Burlington Bulletin
@burlingtonNews

MAYOR PLEDGES $500 TO FIX PLAYGROUND

Burlington Bulletin @burlingtonNews

The Mayor of Burlington promised hundreds of dollars to help fix the playground on Hopper St.Resident Emmy Cartwright expressed disappointment with the news, "The playground needs at least $10,000 to fix all of the broken equipment and unsafe surfaces. I sure hope the Mayor reconsiders the budget."

A. "Money for playground falls short of the goal"

B. "Mayor pledges only 5% of what's needed for playground"

C. "Mayor pledges $9,500 less than what playground needs"

D. Write your own:

Log in to Amplify Math to complete this lesson online.

Activity 1 Editing Headlines

Plan ahead: How will you analyze the headlines so that you can more accurately represent the situation?

When percentages are used appropriately and proper context is given, they can help highlight important information. However, percentages can also be used to mislead the audience, if they are not familiar with the calculations used.

For each problem, read the headline and examine the data. Then determine whether the headline is appropriate.

1. **Headline:** Stock index up 6.3% in June!

	Jan 2020	Feb 2020	Mar 2020	Apr 2020	May 2020	Jun 2020
Stock index price ($)	3,278	3,277	2,652	2,761	2,919	3,104

a. Is the headline mathematically accurate? Explain your thinking.

b. Does the headline highlight the *most* important information? Explain your thinking.

c. Write an alternate appropriate headline for the data. Be sure to include a percentage.

Name: ______ Date: ______ Period: ______

Activity 1 Editing Headlines (continued)

2. Headline: Shark attacks at area beaches increase 100%!

	Number of beachgoers	Attacks	Injuries	Deaths
2018	9,000,000	1	1	0
2019	10,900,000	2	1	0

a. Is the headline mathematically accurate? Explain your thinking.

b. Does the headline highlight the *most* important information? Explain your thinking.

c. Write an alternate appropriate headline for the data. Be sure to include a percentage.

Activity 2 Reporting Responsibly

You will be given some data sets. Choose one to analyze in order to create an appropriate headline.

1. Select a data set and read through the information carefully.

   a. What do you notice about the data?

   b. What questions could be answered by analyzing the data set? Write at least two. Decide for which one you will write a headline and place a checkmark next to it.

2. Write a headline that responsibly reports the selected story. Be sure to include a percentage in your headline.

   Show your thinking and calculations here:

   Headline:

   Why this headline summarizes the story:

Stronger and Clearer: During the *Gallery Tour*, you will give feedback on your peers' work and headlines. Use the feedback you receive to refine and improve your headline and explanation.

STOP

Unit Summary

Percentages make information pop! Whether a store has a sale, a teacher grades a test, or a sports fan is evaluating their players' stats, percentages can help make their message clear and to the point. Look through a newspaper and notice how many times percentages show up to help explain what's happening in the world, whether it's the economy, the climate, or public health.

But be careful! While percentages can make information clearer, *percent change* can be trickier to untangle.

The key is remembering that every percentage is a ratio that's been scaled to be out of 100. Asking yourself, "What does 100% represent in this situation?" will help make it clear what the different numbers are supposed to represent. Once that's clear, you'll have everything you need to know when percentages are being used effectively, or when they're misleading.

Learning this helps you better understand what's happening in the world, whether it's a surcharge in a restaurant bill or the effects of a new government policy. With percentages now securely a part of your growing math vocabulary, you're 100% ready to take on what the headlines throw at you.

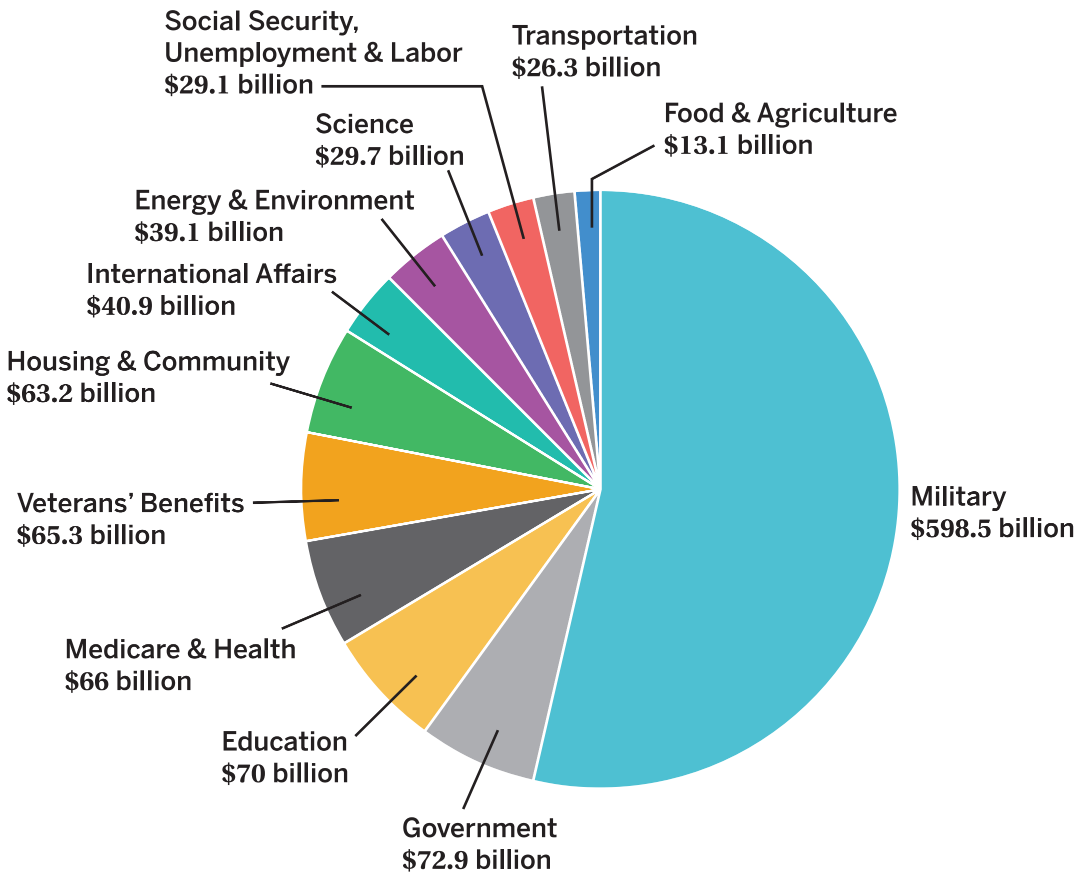
See you in Unit 5.

Practice

Name: ________ Date: ________ Period: ________

1. The diagram shows a pie chart of discretionary spending by the U.S. federal government in 2015. In 2015, discretionary spending represented about 29% of the total budget. Write an appropriate headline that includes a percentage using the information in the diagram.

Total U.S. federal government discretionary spending in 2015: $1.11 trillion



2. Order each ratio from the least percentage to the greatest percentage. Show your thinking.

Least percentage **Greatest percentage**

Ratio A	63 out of 72
Ratio B	18 out of 20
Ratio C	132 out of 150

Name: .... Date: .... Period: ....

Practice

3. A bakery uses 30% more flour this month than last month. If the bakery used 560 kg of flour last month, how much did it use this month?

4. On December 29, a certain stock price was $530. On December 30, it dropped 1%. On December 31, it rose 3%. By what percent did the price change from December 29 to December 31?

5. A grocery store allows you to use multiple coupons when checking out. You have a $5 off coupon and a 10% off coupon. The register will calculate the new price after each coupon is used. Does the order you use the coupons make a difference? Explain your thinking.

6. A clothing store has a rule that you can only use one coupon for a purchase. Which of these coupons will give you the greatest discount? Explain your thinking.

Take 10% OFF the regular price, then take an additional 15% OFF the sale price!	**Take 25% OFF the regular price!**

Glossary/Glosario

A

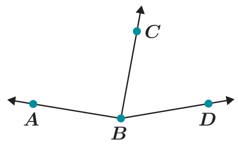
English

absolute value The value that represents the distance between a number and zero. For example, because the distance between -3 and 0 is 3, the absolute value of -3 is 3, or $|-3| = 3$.

Addition Property of Equality A property stating that, if $a = b$, then $a + c = b + c$.

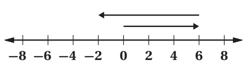
additive inverse The additive inverse of a number a is the number that, when added to a, gives a sum of zero. It is the number's opposite.

adjacent angles Angles that share a common side and vertex. For example, $\angle ABC$ and $\angle CBD$ are adjacent angles.



area The number of unit squares needed to fill a two-dimensional figure without gaps or overlaps.

arrow diagram A model used in combination with a number line to show positive and negative numbers and operations on them.



Associative Property of Addition A property stating that how addends are grouped does not change the result. For example, $(a + b) + c = a + (b + c)$.

Associative Property of Multiplication A property stating that how factors are grouped in multiplication does not change the product. For example, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

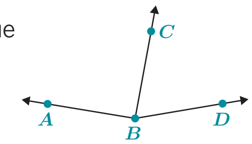
Español

valor absoluto Valor que representa la distancia entre un número y cero. Por ejemplo, dado que la distancia entre -3 y 0 es 3, el valor absoluto de -3 es 3, o $|-3| = 3$.

Propiedad de igualdad en la suma Propiedad que establece que si $a = b$, entonces $a + c = b + c$.

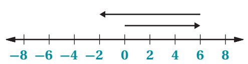
inverso aditivo El inverso aditivo de un número a es el número que, cuando se suma a a, resulta en cero. Es el opuesto del número.

ángulos adyacentes Ángulos que comparten un lado y un vértice. Por ejemplo, $\angle ABC$ y $\angle CBD$ son ángulos adyacentes.



área Número de unidades cuadradadas necesario para llenar una figura bidimensional sin dejar espacios vacíos ni superposiciones.

diagrama de flechas Modelo que se utiliza en combinación con una línea numérica para mostrar números positivos y negativos, y operaciones sobre estos.



Propiedad asociativa de la suma Propiedad que establece que la forma en que se agrupan los sumandos en una suma no cambia el resultado. Por ejemplo, $(a + b) + c = a + (b + c)$.

Propiedad asociativa de la multiplicación Propiedad que establece que la forma en que se agrupan los factores en una multiplicación no cambia el producto. Por ejemplo, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

B

English

balance The amount that represents the difference between positive and negative amounts of money in an account.

bar notation Notation that indicates the repeated part of a repeating decimal. For example, $0.\overline{6} = 0.66666\ldots$.

base (of a prism) Either of the two identical, parallel faces of a prism that are connected by a set of rectangular faces.

base (of a pyramid) The face of a pyramid that is opposite from the vertex, where all the triangular faces connect.

Español

balance Cantidad que representa la diferencia entre cantidades positivas y negativas de dinero en una cuenta bancaria.

notación de barras Notación que indica la parte repetida de un número decimal periódico. Por ejemplo, $0.\overline{6} = 0.66666\ldots$.

base (de un prisma) Cualquiera de las dos caras idénticas y paralelas de un prisma que están conectadas por un conjunto de caras rectangulares.

base (de una pirámide) La cara de una pirámide que se opone al vértice, donde todas las caras triangulares se conectan.

Glossary/Glosario

English

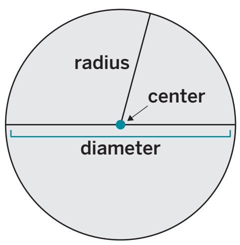
C

center of a circle The point that is the same distance from all points on the circle.

certain A certain event is an event that is sure to happen. (The probability of the event happening is 1.)

chance experiment An experiment that can be performed multiple times, in which the outcome may be different each time.

circle A shape that is made up of all of the points that are the same distance from a given point.



circumference The distance around a circle.

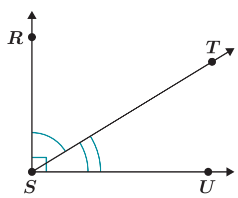
coefficient A number that is multiplied by a variable, typically written in front of or "next to" the variable, often without a multiplication symbol.

commission A fee paid for services, usually as a percentage of the total cost.

common factor A number that divides evenly into each of two or more given numbers.

commutative property Changing the order in which numbers are either added or multiplied does not change the value of the sum or product.

complementary angles Two angles whose measures add up to 90 degrees. For example, $\angle RST$ and $\angle TSU$ are complementary angles.



constant of proportionality The number in a proportional relationship by which the value of one quantity is multiplied to get the value of the other quantity.

coordinate plane A two-dimensional plane that represents all the ordered pairs (x, y), where x and y can both represent values that are positive, negative, or zero.

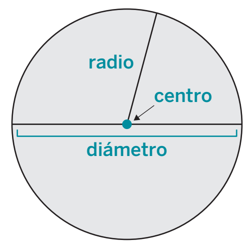
Español

centro de un círculo Punto que está a la misma distancia de todos los puntos del círculo.

seguro Un evento seguro es un evento que ocurrirá con certeza. (La probabilidad de que el evento ocurra es 1.)

experimento aleatorio Experimento que puede ser llevado a cabo muchas veces, en cada una de las cuales el resultado será diferente.

círculo Forma compuesta de todos los puntos que están a la misma distancia de un punto dado.



circunferencia Distancia alrededor de un círculo.

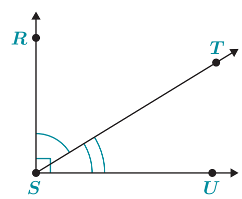
coeficiente Número por el cual una variable es multiplicada, escrito comúnmente frente o junto a la variable.

comisión Pago realizado a cambio de algún servicio, usualmente como porcentaje del costo total.

factor común Número que divide en partes iguales cada número de entre dos o más números dados.

propiedad conmutativa Cambiar el orden de los operandos en una suma o multiplicación no cambia el valor final de la suma o el producto.

ángulos complementarios Dos ángulos cuyas medidas suman 90 grados. Por ejemplo, $\angle RST$ y $\angle TSU$ son ángulos complementarios.

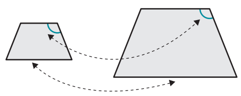


constante de proporcionalidad En una relación proporcional, el número por el cual el valor de una cantidad es multiplicado para obtener el valor de otra cantidad.

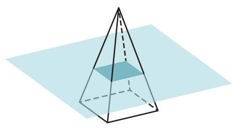
plano de coordenadas Plano bidimensional que representa todos los pares ordenados (x, y), donde tanto x como y pueden representar valores positivos, negativos o cero.

English

corresponding parts Parts of two scaled copies that match up, or "correspond" with each other. These corresponding parts could be points, segments, angles, or lengths.

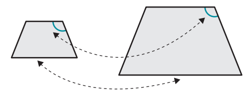


cross section A cross section is the new face seen when slicing through a three-dimensional figure. For example, a rectangular pyramid that is sliced parallel to the base has a smaller rectangle as the cross section.

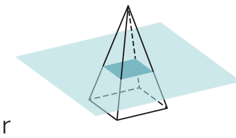


Español

partes correspondientes Partes de dos copias a escala que coinciden, o "se corresponden" entre sí. Estas partes correspondientes pueden ser puntos, segmentos, ángulos o longitudes.



corte transversal Un corte transversal es la nueva cara que aparece cuando una figura tridimensional es rebanada. Por ejemplo, una pirámide rectangular que es rebanada en forma paralela a la base tiene un rectángulo más pequeño como corte transversal.

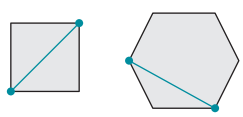


D

debt Amount of money that has been borrowed and owed to the person or bank from which it was borrowed.

deposit Money put into an account.

diagonal A line segment connecting two vertices on different sides of a polygon. The *diagonal* of a square connects opposite vertices.



diameter The distance across a circle through its center. The line segment with endpoints on the circle, that passes through its center. (See also *circle*.)

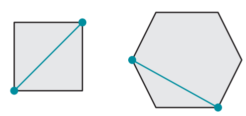
discount A reduction in the price of an item, typically due to a sale.

Distributive Property A property that states the product of a number and a sum of numbers is equal to the sum of two products: $a\,(b + c) = ab + ac$.

deuda Cantidad de dinero que ha sido pedida prestada y se le debe a la persona o al banco que la prestó.

depósito Dinero colocado en una cuenta.

diagonal Segmento de una línea que conecta dos vértices en lados diferentes de un polígono. La *diagonal* de un cuadrado conecta vértices opuestos.



diámetro Distancia a través de un círculo que atraviesa su centro. Segmento de línea cuyos extremos limitan con el círculo y que pasa por su centro. (Ver también *círculo*.)

descuento Reducción del precio de un artículo, usualmente debido a una venta de rebaja.

Propiedad distributiva Propiedad que establece que el producto de un número y una suma de números es igual a la suma de dos productos: $a\,(b + c) = ab + ac$.

E

equally likely as not An event that has equal chances of occurring and not occurring. (The probability of the event happening is exactly $\frac{1}{2}$.)

equation Two expressions with an equal sign between them. When the two expressions are equal, the equation is true. An equation can also be false, when the values of the two expressions are not equal.

tan probable como improbable Evento que tiene las mismas posibilidades de ocurrir que de no ocurrir. (La probabilidad de que ocurra es exactamente $\frac{1}{2}$.)

ecuación Dos expresiones con un signo igual entre sí. Cuando las dos expresiones son iguales, la ecuación es verdadera. Una ecuación también puede ser falsa, cuando los valores de las dos expresiones no son iguales.

Glossary/Glosario

English	Español
equivalent equations Equations that have the same solution.	**ecuaciones equivalentes** Ecuaciones que tienen la misma solución.
equivalent expressions Two expressions whose values are equal when the same value is substituted into the variable for each expression.	**expresiones equivalentes** Dos expresiones cuyos valores son iguales cuando se sustituye el mismo valor en la variable de cada expresión.
equivalent ratios Any two ratios in which the values for one quantity in each ratio can be multiplied or divided by the same number to get the values for the other quantity in each ratio.	**razones equivalentes** Dos razones entre las cuales los valores de una cantidad en cada razón pueden ser multiplicados o divididos por el mismo número para obtener así los valores de la otra cantidad en cada razón.
equivalent scales Different scales (relating scaled and actual measurements) that have the same scale factor.	**escalas equivalentes** Diferentes escalas (que relacionan medidas a escala y reales) que tienen el mismo factor de escala.
error interval A range of values above and below an exact value, expressed as a percentage.	**intervalo de error** Rango de valores por sobre y por debajo de un valor exacto, expresado como porcentaje.
event A set of one or more outcomes in a chance experiment.	**evento** Conjunto de uno o más resultados de un experimento aleatorio.
expand To expand an expression means to use the Distributive Property to rewrite a product as a sum. The new expression is equivalent to the original expression.	**expandir** Expandir una expresión significa usar la Propiedad distributiva para volver a escribir un producto como una suma. La nueva expresión es equivalente a la expresión original.

F

English	Español
factor To factor an expression means to use the Distributive Property to rewrite a sum as a product. The new expression is equivalent to the original expression.	**factorizar** Factorizar una expresión significa usar la Propiedad distributiva para volver a escribir una suma como un producto. La nueva expresión es equivalente a la expresión original.

G

English	Español
gratuity See the definition for *tip*.	**gratificación** Ver *propina*.
greater than or equal to $x \geq a$, x is greater than a or x is equal to a.	**mayor o igual a** $x \geq a$, x es mayor que a o x es igual a a.

H

English	Español
hanger diagram A model in which quantities are represented as weights attached to either side of a hanger. When the hanger is balanced, the sum of the quantities on either side must be equal.	**diagrama de colgador** Modelo en el cual ciertas cantidades son representadas como pesos sujetos a cada lado de un colgador. Cuando el colgador está en equilibrio, la suma de las cantidades en cualquiera de los lados debe ser igual.

English

I

impossible An impossible event is an event that has no chance of occurring. The probability of the event happening is 0.

inequality A statement relating two numbers or expressions that are not equal. The phrases *less than*, *less than or equal to*, *greater than*, and *greater than* or *equal to* describe inequalities.

integers Whole numbers and their opposites.

inverse operations Operations that "undo" each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.

L

less than or equal to $x \leq a$, x is less than a or x is equal to a.

like terms Terms in an expression that have the same variables and can be combined, such as $7x$ and $9x$.

likely A likely event is an event that has a greater chance of occurring than not occurring. (The probability of happening is more than $\frac{1}{2}$.)

long division A method that shows the steps for dividing base ten whole numbers and decimals, dividing one digit at a time, from left to right.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\ 4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

M

magnitude The absolute value of a number, or the distance of a number from 0 on the number line.

markdown An amount, expressed as a percentage, subtracted from the cost of an item.

markup An amount, expressed as a percentage, added to the cost of an item.

multi-step event When an experiment consists of two or more events, it is called a multi-step event.

multiplicative inverse Another name for the reciprocal of a number; The multiplicative inverse of a number a is the number that, when multiplied by a, gives a product of 1.

Español

I

imposible Un evento imposible es un evento que no tiene posibilidad de que ocurra. La probabilidad de que ocurra es 0.

desigualdad Enunciado que relaciona dos números o expresiones que no son iguales. Las expresiones "menor que", "menor o igual a", "mayor que" o "mayor o igual a" describen desigualdades.

enteros Números completos y sus opuestos.

operaciones inversas Operaciones que se cancelan entre sí. La suma y la resta son operaciones inversas. La multiplicación y la división son operaciones inversas.

L

menor o igual a $x \leq a$, x es menor que a o x es igual a a.

términos semejantes Partes de una expresión que tiene la misma variable y que pueden ser sumadas, tales como $7x$ and $9x$.

probable Un evento probable es un evento que tiene más posibilidad de ocurrir que de no ocurrir. (La probabilidad de que ocurra es mayor que $\frac{1}{2}$.)

división larga Método que muestra los pasos necesarios para dividir números enteros en base diez y decimales, por medio de la división de un dígito a la vez, de izquierda a derecha.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\ 4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

M

magnitud Valor absoluto de un número, o distancia de un número con respecto a 0 en la línea numérica.

descuento Monto, expresado como porcentaje, que se resta al costo de un producto.

sobreprecio Monto, expresado como porcentaje, que se agrega al costo de un producto.

evento de varios pasos Cuando un experimento consiste en dos o más eventos, es llamado un evento de varios pasos.

inverso multiplicativo Otro nombre para el recíproco de un número. El inverso multiplicativo de un número a es el número que, cuando se multiplica por a, tiene como producto 1.

Glossary/Glosario

English | **Español**

N

negative numbers Numbers whose values are less than zero.

nonproportional relationship A relationship between two quantities in which the corresponding values do not have a constant ratio. (In other words, a relationship that is *not* a proportional relationship.)

números negativos Números cuyos valores son menores que cero.

relación no proporcional Relación entre dos cantidades, en la cual los valores correspondientes no tienen una razón constante. (En otras palabras, una relación que *no* es una relación proporcional.)

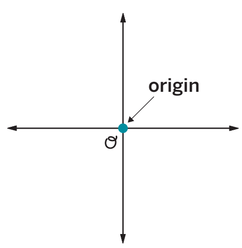
O

opposites Two numbers that are the same distance from 0, but are on different sides of the number line.

order of operations When an expression has multiple operations, they are applied in a consistent order (the "order of operations") so that the expression is evaluated the same way by everyone.

ordered pair Two values, written as (x, y), that represent a point on the coordinate plane.

origin The point represented by the ordered pair $(0, 0)$ on the coordinate plane. The *origin* is where the x- and y-axes intersect.



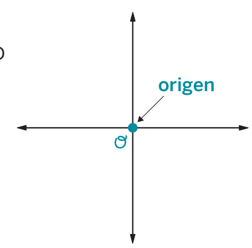
outcome One of the possible results that can happen when an experiment is performed. For example, the possible outcomes of tossing a coin are heads and tails.

opuestos Dos números que están a la misma distancia de 0, pero que están en lados diferentes de la línea numérica.

orden de las operaciones Cuando una expresión contiene múltiples operaciones, estas se aplican en cierto orden consistente (el "orden de las operaciones") de forma que la expresión sea evaluada de la misma manera por todas las personas.

par ordenado Dos valores, escritos como (x, y), que representan un punto en el plano de coordenadas.

origen Punto representado por el par ordenado $(0, 0)$ en el plano de coordenadas. El *origen* es donde los ejes x y y se intersecan.



resultado El resultado de un experimento aleatorio es una de las cosas que pueden ocurrir cuando se realiza el experimento. Por ejemplo, los posibles resultados de tirar una moneda al aire son cara o cruz.

P

percent change How much a quantity changed (increased or decreased), expressed as a percentage of the original amount.

percent decrease The amount a value has gone down, expressed as a percentage of the original amount.

percent error The difference between approximate and exact values, as a percentage of the exact value.

cambio porcentual Cuánto ha cambiado una cantidad (aumentado o disminuido), expresado en un porcentaje del monto original.

disminución porcentual Cantidad en que un valor ha disminuido, expresada como porcentaje del monto original.

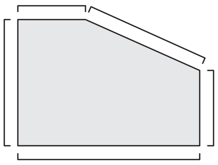
error porcentual Diferencia entre valores aproximados y valores exactos, expresada como porcentaje del valor exacto.

English

percent increase The amount a value has gone up, expressed as a percentage of the original amount.

percentage A rate per 100. (A specific *percentage* is also called a *percent*, such as “70 percent.”)

perimeter The total distance around the sides of a two-dimensional figure.



pi, or π The ratio between the circumference and the diameter of a circle.

polygon A closed, two-dimensional shape with straight sides that do not cross each other.

population A set of people or objects that are to be studied. For example, if the heights of people on different sports teams are studied, the population would be all the people on the teams.

population proportion A number in statistics, between 0 and 1 that represents the fraction of the data that fits into the desired category.

positive numbers Numbers whose values are greater than zero.

prism A three-dimensional figure with two parallel, identical faces (called *bases*) that are connected by a set of rectangular faces.

probability The ratio of the number of favorable outcomes to the total possible number of outcomes. A probability of 1 means the event will always happen. A probability of 0 means the event will never happen.

profit The amount of money earned, minus expenses.

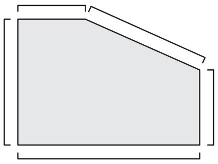
properties of equality Rules that apply to all equations. These include properties of addition, subtraction, multiplication, and division, which state that, if an equation is true, then applying the same operation to both sides will give a new equation that is also true.

Español

aumento porcentual Monto en que un valor ha incrementado, expresado como porcentaje del monto original.

porcentaje Tasa por cada 100. (Un *porcentaje* específico también es llamado *por ciento*, como por ejemplo “70 por ciento.”)

perímetro Distancia total alrededor de los lados de una forma bidimensional.



pi, o π Razón entre la circunferencia y el diámetro de un círculo.

porcentaje Tasa por cada 100. (Un porcentaje específico también es llamado “por ciento”, como por ejemplo “70 por ciento”.)

población Una población es un conjunto de personas o cosas por estudiar. Por ejemplo, si se estudia la altura de las personas en diferentes equipos deportivos, la población constaría de todas las personas que conforman los equipos.

proporción de la población En estadística, número entre 0 y 1 que representa la fracción de los datos que cabe en la categoría deseada.

números positivos Números cuyos valores son mayores que cero.

prisma Forma tridimensional con dos caras iguales y paralelas (llamadas *bases*) que se conectan entre sí a través de un conjunto de caras rectangulares.

probabilidad La razón entre el número de resultados favorables y el número total posible de resultados. Una probabilidad de 1 significa que el evento siempre ocurrirá. Una probabilidad de 0 significa que el evento nunca va a ocurrir.

ganancia Monto del dinero obtenido, menos los gastos.

propiedades de igualdad Reglas que son aplicables a todas las ecuaciones. Incluyen las propiedades de suma, resta, multiplicación y división, las cuales señalan que si una ecuación es verdadera, al aplicar la misma operación a ambos lados se obtendrá una nueva ecuación que también es verdadera.

Glossary/Glosario

English

proportional relationship A relationship in which the values for one quantity are each multiplied by the same number (the *constant of proprtionality*) to get the values for the other quantity.

pyramid A three-dimensional figure with one base and a set of triangular faces that meet at a singular vertex.

Español

relación proporcional Relación en la que los valores de una cantidad se multiplican cada uno por el mismo número (la *constante de proporcionalidad*) para encontrar los valores de la otra cantidad.

pirámide Forma tridimensional con una base y un conjunto de caras triangulares que se intersecan en un solo vértice.

R

radius A line segment that connects the center of a circle with a point on the circle. The term *radius* can also refer to the length of this segment. (See also *circle*.)

random sample A sample that has an equal chance of being selected from the population as any sample of the same size

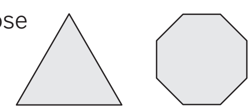
rate A comparison of how two quantities change together.

ratio A comparison of two quantities by multiplication or division.

rational numbers The set of all numbers, positive and negative, that can be written as fractions. For example, any whole number is a rational number.

reciprocal Two numbers whose product is 1 are *reciprocals* of each other. (For example, $\frac{3}{5}$ and $\frac{5}{3}$ are reciprocals.)

regular polygon A polygon whose sides all have the same length and whose angles all have the same measure.



relative frequency The relative frequency is the ratio of the number of times an outcome occurs in a set of data. The relative frequency can be written as a fraction, a decimal, or a percentage.

repeating decimal A decimal in which there is a sequence of non-zero digits that repeat indefinitely.

representative sample A sample is representative of a population if its distribution resembles the population's distribution in center, shape, and spread.

radio Segmento de una línea que conecta el centro de un círculo con un punto del círculo. *Radio* también puede referirse a la longitud de este segmento. (Ver también *círculo*.)

muestra al azar Muestra que tiene la misma posibilidad de ser seleccionada de entre la población que cualquier otra muestra del mismo tamaño.

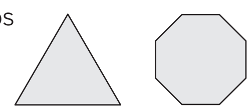
tasa Comparación de cuánto cambian dos cantidades en conjunto.

razón Comparación de dos cantidades a través de la multiplicación o la división.

números racionales Conjunto de todos los números positivos y negativos que pueden ser escritos como fracciones. Por ejemplo, todo número entero es un número racional.

recíproco/a Dos números cuyo producto es 1 son *recíprocos* entre sí. (Por ejemplo, $\frac{3}{5}$ y $\frac{5}{3}$ son recíprocos.)

polígono regular Polígono cuyos lados tienen todos la misma longitud y cuyos ángulos tienen todos la misma medida.



frecuencia relativa La frecuencia relativa es la razón del número de veces que ocurre un resultado en un conjunto de datos. Se puede escribir como una fracción, un decimal o un porcentaje.

número decimal periódico Decimal que tiene una secuencia de dígitos distintos de cero que se repite de manera indefinida.

muestra representativa Una muestra es representativa de una población si su distribución asemeja la distribución de la población en centro, forma y extensión.

English

retail price The price a store typically charges for an item

right angle An angle whose measure is 90 degrees.

S

sales tax An additional cost, as a rate to the cost of certain goods and services, applied by the government.

sample Part of a population. For example, a population could be all the seventh graders at one school. One sample of that population is all the seventh graders who are in band.

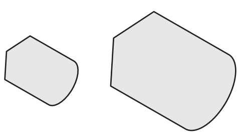
sample space A list of every possible outcome for a chance experiment.

scale A ratio, sometimes shown as a segment, that indicates how the measurements in a scale drawing represent the actual measurements of the object shown.

scale drawing A drawing that represents an actual place, object, or person. All of the measurements in the scale drawing correspond to the measurements of the actual object by the same scale.

scale factor The value that side lengths are multiplied by to produce a certain scaled copy.

scaled copy A copy of a figure where every length in the original figure is multiplied by the same value to produce corresponding lengths in the copy.



simple interest An amount of money that is added on to an original amount, usually paid to the holder of a bank savings account.

simulation An experiment that is used to estimate the probability of a real-world event.

solution to an equation A value that will make an equation true when substituted into the equation.

solution to an inequality A value that will make an inequality a true statement when substituted into the inequality.

straight angle An angle whose measure is 180 degrees.
For example, $\angle EFH$ is a straight angle.

Español

precio de venta al público Precio que una tienda comercial usualmente cobra por un producto.

ángulo recto Ángulo cuya medida es de 90 grados.

impuesto de venta Costo adicional, como una tasa del costo de ciertos bienes y servicios, aplicado por el gobierno.

interés simple Monto de dinero que se agrega a un monto original, usualmente pagado al titular o a la titular de una cuenta bancaria de ahorros.

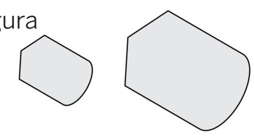
espacio de muestra Lista de cada resultado posible de un experimento aleatorio.

escala Razón, a veces mostrada como segmento, que indica de qué forma las medidas de un dibujo a escala representan las verdaderas medidas del objeto mostrado.

dibujo a escala Dibujo que representa un lugar, objeto o persona real. Todas las medidas en el dibujo a escala corresponden en la misma escala a las medidas del objeto real.

factor de escala Valor por el cual las longitudes de cada lado se multiplican para producir cierta copia a escala.

copia a escala Copia de una figura donde cada longitud de la figura original es multiplicada por el mismo valor, para producir longitudes correspondientes en la copia.



muestra Una muestra es una parte de la población. Por ejemplo, una población podría ser todos/as los/as estudiantes de séptimo grado en una escuela. Una muestra de esa población son todos/as los/as estudiantes de séptimo grado que están en una banda.

simulación Un experimento que es utilizado para estimar la probabilidad de un evento en el mundo real.

solución a una ecuación Número que puede sustituir una variable para volver verdadera una ecuación.

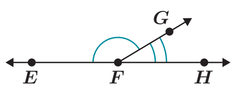
solución a una desigualdad Cualquier número que puede sustituir una variable para volver verdadera una desigualdad.

ángulo llano Ángulo cuya medida es de 180 grados.
Por ejemplo, $\angle EFH$ es un ángulo llano.

Glossary/Glosario

English

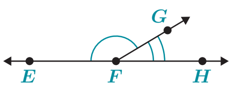
supplementary angles Two angles whose measures add up to 180 degrees. For example, $\angle EFG$ and $\angle GFH$ are supplementary angles.



surface area The number of unit squares needed to cover all of the faces of a three-dimensional figure without gaps or overlaps.

Español

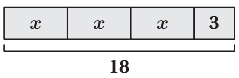
ángulos suplementarios Dos ángulos cuyas medidas suman 180 grados. Por ejemplo, $\angle EFG$ y $\angle GFH$ son ángulos suplementarios.



área de superficie Número de unidades cuadradas necesarias para cubrir todas las caras de una figura tridimensional sin dejar espacios vacíos ni superposiciones.

T

tape diagram A model in which quantities are represented as lengths (of tape) placed end-to-end, and which can be used to show addition, subtraction, multiplication, and division.



term A term is a part of an expression. It can be a number, a variable, or a product of a number and a variable.

terminating decimal A decimal that ends at a specific place value.

tip An amount given to a server at a restaurant (or other service provider) that is calculated as a percentage of the bill.

tree diagram A diagram that represent all the possible outcomes in an experiment.

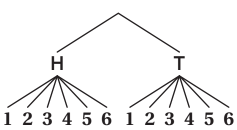
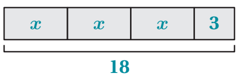


diagrama de cinta Modelo en el cual las cantidades están representadas como longitudes (de cinta) colocadas de forma continua, y que pueden ser usadas para mostrar suma, resta, multiplicación y división.

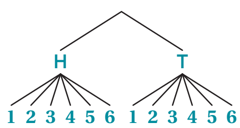


término Un término es una parte de una expresión. Puede ser un número individual, una variable o el producto de un número y una variable.

decimal exacto Un decimal que termina en un valor posicional específico.

propina Cantidad dada a un mesero o mesera en un restaurante (o a una persona que presta cualquier otro servicio) que se calcula como porcentaje de la cuenta.

diagrama de árbol Diagrama que representa todos los resultados posibles.



U

unit rate How much one quantity changes when the other changes by 1.

unlikely An unlikely event is an event that has small chance of occurring. (The probablity of the event happening is less than $\frac{1}{2}$.)

tasa unitaria Cuánto cambia cierta cantidad cuando la otra cambia por un factor de 1.

improbable Un evento improbable es un evento que tiene pocas posibilidades de ocurrir. (La probabilidad de que ocurra es menor que $\frac{1}{2}$.)

English | Español

V

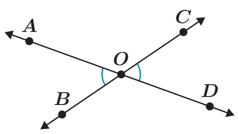
variable A letter that represents an unknown number in an expression or equation.

variable Letra que representa un número desconocido en una expresión o ecuación.

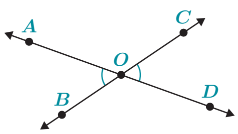
velocity A quantity that represents the speed and the direction of motion. In general, speed, like distance, is always positive, but velocity can be either positive or negative.

velocidad Cantidad que representa la rapidez y la dirección de un movimiento. En general, la rapidez, como la distancia, es siempre positiva, pero la velocidad puede ser tanto positiva como negativa.

vertical angles Opposite angles that share the same vertex. They are formed by a pair of intersecting lines. Their angle measures are equal. For example, $\angle AOB$ and $\angle COD$ are vertical angles.



ángulos verticales Ángulos opuestos que comparten el mismo vértice. Están compuestos de un par de líneas que se intersecan. Sus medidas de ángulo son iguales. Por ejemplo, $\angle AOB$ y $\angle COD$ son ángulos verticales.



volume The number of unit cubes needed to fill a three-dimensional figure without gaps or overlaps.

volumen Número de unidades cúbicas necesario para llenar una figura tridimensional sin dejar espacios vacíos ni superposiciones.

W

withdrawal Money taken out of an account.

retiro Dinero que es extraído de una cuenta.

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