Amplify Math

Grade 7

Volume 2: Units 5–8

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Hello, curious mind!

Welcome to Grade 7. This grade is considered by some to be one of the wildest grades . . . at least in terms of the math.

This year, you'll scale skyscrapers, build your own brand, tame "rep-tiles," and even grapple with a gargantuan Godzilla. And that's just Unit 1!

We promise you, it's all possible. This year, you'll see not only that math makes sense, but it can be fun, too.

Before you dig in, we want you to know two things:

This book is special! It's where you'll record your thoughts, strategies, explanations, and, occasionally, any award-winning doodles that might come to mind.

When you go online, you won't be mindlessly plugging numbers into your device . . . You'll be pushing, pulling, crawling, teleporting, melting . . . , well, let's just say you'll be doing a lot of things, and you'll be teaming up with your classmates as you do.

Let's dig in!

Sincerely,
The Amplify Math Team

Volume 1

Unit 1 Scale Drawings

How did we come to understand what tiny things, like viruses, and humongous things, like Jupiter, look like? Scaling, of course! In this unit, we resize things — in very precise ways — to bring them into focus and make them more manageable to work with.

Unit Narrative:
Life in the
Little Big City

Sub-Unit 1 Scaled Copies 11

How do you get the perfect fit?
If we are making a larger or smaller copy of something, it needs to look right. The key is the scale factor.

Sub-Unit 2 Scale Drawings 47

Who was the King of Monsters?
We use maps and other scale drawings to help simplify large, complex places. Interpreting them is about knowing the scale and how to measure.

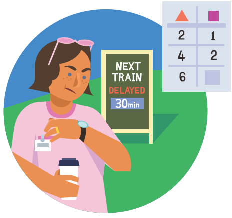
Unit 2 Introducing Proportional Relationships

Unit Narrative:
The World
in Proportion

How did we come to understand what tiny things, like viruses, and humongous things, like Jupiter, look like? Scaling, of course! In this unit, we resize things — in very precise ways — to bring them into focus and make them more manageable to work with.

Who was the original globetrotter?

Tables help keep us organized, but equations tell an entire story with just a few symbols. We'll use both of them to represent proportional relationships.



What good is a graph?

We turn to drawing, interpreting, and comparing proportional relationships in graphs, and notice what is particular to these types of graphs.

Unit 3 Measuring Circles

Some have said that the only place to find a perfect circle is inside the world of math. Explore the many circles and circular shapes both in and outside of this unit to reveal some of their mysteries.

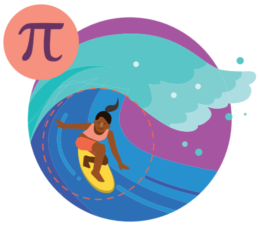
Unit Narrative:
'Round and
'Round We Go

LAUNCH

Sub-Unit 1 Circumference of Circles 219

Why do aliens love circles?

Circles are famously difficult to measure precisely, but that won't stop us from trying. Let's see how close we can get.



Sub-Unit 2 Area of Circles 261

What makes a circle so perfect?

Squares and circles may not have much in common, but we'll need both to measure a circle's area.

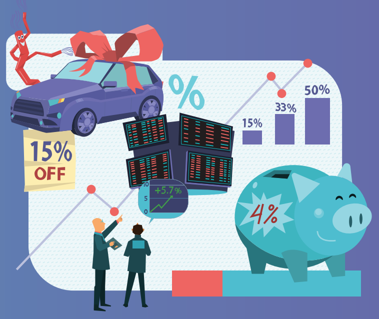
CAPSTONE

Unit 4 Percentages

“Extra! Extra! 99% of adults don’t remember numbers!*” Percents can be incredibly effective at communicating how much something has changed, but we must keep a watchful eye on what the numbers behind the percentages mean.

(*When asked to recite π to the 10th digit.)

Unit Narrative:
Keepin’ it 100



Is there truth in numbers?

Numbers never lie, but should we always believe them? Percents can show how something changes — if we pay careful attention to the original amount.

Did a quarantined U.S. keep a healthy economy?

See why percentages are used to calculate taxes, tips, interest, and other amounts when spending or saving money.

Volume 2

Unit 5 Rational Number Arithmetic

Climb high, dig deep, and spend wisely as you explore the world using positive *and* negative numbers.

Unit Narrative:
A World of Opposites

What was Jeanne Baret's big secret?

Sure, you've probably been adding and subtracting for many years, but have you ever tried to take something away when you had less than zero to start with?

Who was the toughest Grandma to ever hike the Appalachian Trail?

Travel forwards and backwards in time to help make sense of multiplication and division of negative numbers.

How do you climb the world's most dangerous mountain?

Put it all together — adding, subtracting, multiplying, and dividing with rational numbers — while exercising your algebraic thinking muscles in a sneak preview of the next unit.

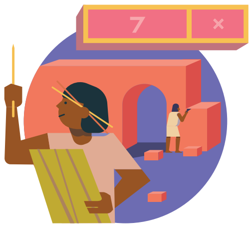
Unit 6 Expressions, Equations, and Inequalities

Unit Narrative:
Solving One
Step at a Time

Numbers are great, but they will not get us where we are going in this unit. It will take letters, symbols, and drawings to represent the varied and diverse mathematical ideas of algebraic thinking.

What are the first words you learn in "Caveman"?

Dog walking, tools of early civilization, and hangers all come together to help you explore new ways of solving equations.



Who were the VIPs of ancient Egypt?

Solving word problems is about making meaning of the quantities, and tape diagrams return to help.

Did a member of the School of Night infiltrate your math class?

Expressions are not always equal, so we must reckon with inequalities. Thankfully, finding their solutions will feel familiar.

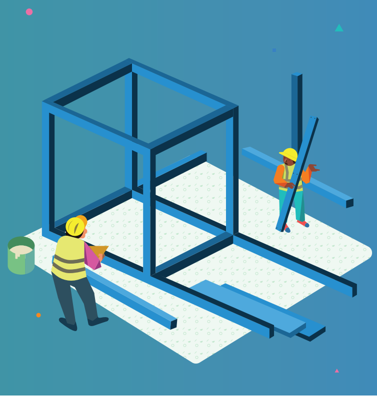
Which three blockheads did NASA send into space?

Find efficiencies for simplifying expressions like the Distributive Property and combining like terms.

Unit 7 Angles, Triangles, and Prisms

This unit is about the math of what can be seen and what can be held. Get ready to measure, build, and slice your way through an array of geometric figures.

Unit Narrative:
Journey to the Third Dimension



Did radio kill the aviation star?

As you'll see, some angles were just meant to go together. Here, you'll be introduced to complementary, supplementary, and vertical angles.

How did triangles help win a war?

In this Sub-Unit, you will find that constructing polygons with specific lengths and angle measures can have dramatically different results.

This machine will slice, but will it dice?

You've studied the surfaces of three-dimensional figures and the spaces inside them. Now, let's see what happens when we slice them open.

Unit 8 Probability and Sampling

Unit Narrative:
Winning Chance

It is impossible to see into the future, but that should not stop us from trying, should it? Making predictions — taking limited information and making our best guess about what will happen — is all about knowing what is possible, what is impossible, and what is likely.

How did the women of Bletchley Park save the free world?

Welcome to probability, the math of games and chance. Discover how probability can reveal hidden information, even secret codes.

How did a blazing shoal bring the Philadelphia Convention Center to its feet?

When predicting the chances gets complicated, a simulation can help make predictions.

What's on your mind?

Not all data are created equal. It is important to know how to identify when a sample is representative of a population.

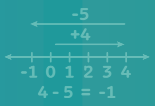
UNIT 5

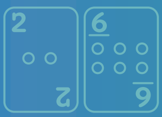
Rational Number Arithmetic

Climb high, dig deep, and spend wisely as you explore the world using positive *and* negative numbers.

Essential Questions

- How do you represent addition, subtraction, or multiplication of rational numbers on a number line?
- How is solving problems with rational numbers the same or different from solving problems with only non-negative rational numbers?
- How can rational numbers be used to represent real-world situations?
- *(By the way, do two negatives always make a positive?)*





$2 - (-3) = 2 + 3 = 5$

SUB-UNIT

1 Adding and Subtracting Rational Numbers

Narrative: From botany to temperature and elevation, rational numbers are everywhere!

You'll learn . . .

- what it means to add and subtract negative numbers.
- how to apply these operations to real-world contexts.

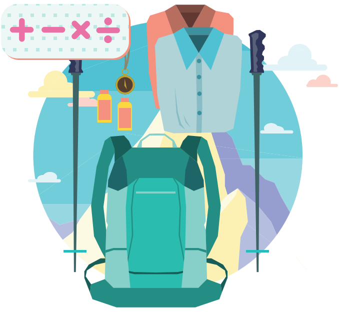
SUB-UNIT

2 Multiplying and Dividing Rational Numbers

Narrative: Discover how hiking the Appalachian trail relates to rational numbers.

You'll learn . . .

- how multiplication is related to position and direction.
- more about the relationship between multiplication and division.



SUB-UNIT

3 Four Operations With Rational Numbers

Narrative: Rational numbers can help you make preparations for climbing Mt. Everest.

You'll learn . . .

- how all four operations interact when working with rational numbers.
- how to apply the concepts you learned throughout the unit.

$2 \cdot (-\frac{3}{2}) = -3$

$|a - b| = |b - a|$

How many different values can you produce by placing parentheses in the following expression?

$$-1-1-1-1-1-1-1-1$$

$a \cdot b$

$(-a) \cdot (-b)$ $= ab$

$a \cdot (-b) = -ab$

$2 \cdot (-3) = -6$

$-6 \div 2 = -3$

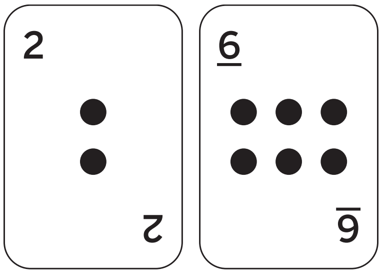
Target: Zero

Let's aim for zero in this card game.

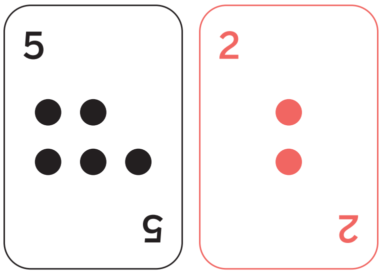
Warm-up Guess the Rule

Examine each set of cards and the score shown. Describe what you think is the rule for scoring each card set, and then predict the score of Set 4.

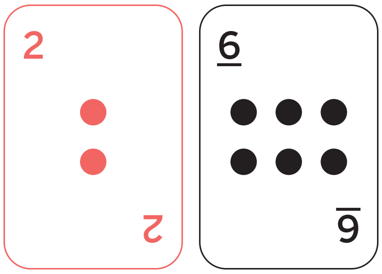
Set 1 Score: 8



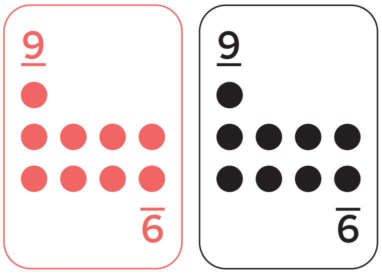
Set 2 Score: 3



Set 3 Score: 4



Set 4 Score: ....



What is the rule that gives the score for the card sets?

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Target: Zero, Part 1

Your group will be given a set of cards to play a game.

Players: 2–4

Goal: Combine your card values to produce a value as close to zero as possible.

Getting ready:

- Shuffle the set of cards and place them in a pile in the middle of the group.
- Choose one player to start the game.

For each round:

- Each player takes two cards from the pile and places them, face up, in front of them.
- When all players have their cards, the first player decides whether they want to take one additional card from the pile, or pass (not do anything). This continues until all players have had a turn.
- Complete the table for yourself at the end of each round, and compare your score to the other players. The player with the score closest to zero receives a check mark for winning the round.
- Reshuffle the cards for the next round.

	Cards	Score	Closest to 0?
Round 1			☐
Round 2			☐

1. How did you determine who was closest to zero after each round?

2. Consider the times when you decided whether to take an additional card.

   a. What thinking helped you to make your decision?

   b. If you had to change your strategy for the next round, how would you change it?

Activity 2 Target: Zero, Part 2

Continue playing the game, with the following updates to the rules:

- Instead of only taking one additional card, each player will have three chances to take an additional card.
- Keep your cards to yourself until the end of the round.
- After each player has taken all the cards they wish to take (a maximum of five cards), calculate your score.
- Show your cards to the other players and help each other confirm all scores are accurate. The player with the score closest to zero receives a check mark for winning the round.

	Cards	Score	Closest to 0?
Round 3			☐
Round 4			☐
Round 5			☐
Round 6			☐
Round 7			☐
Round 8			☐

Are you ready for more?

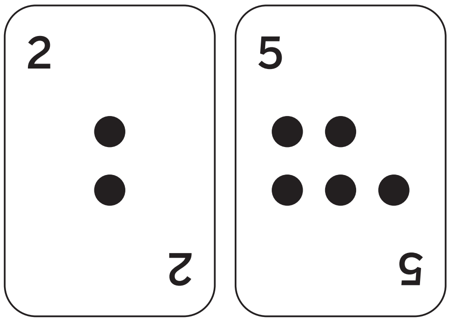
A Target: Zero player has a score of 1 during a round. If we know the player had two cards and neither card was greater than 6, how many different pairs of cards could the player have had?

Name: ________________ Date: ________ Period: ________

Activity 3 Playing Strategically

1. Remember that your goal is to combine your card values to produce a value as close to zero as possible. Which card would be ideal to add to your current cards? Draw it in the card provided.

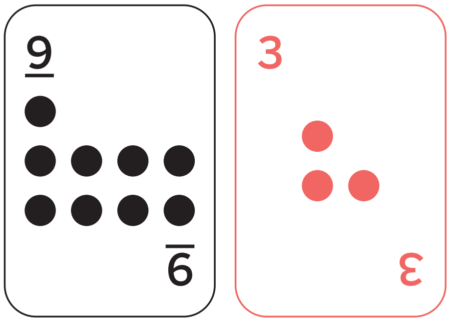
Current cards



New card

2. Which two cards would be ideal to add to your current cards? Draw them in the cards provided.

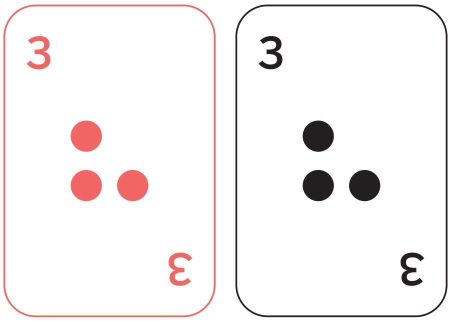
Current cards



New cards

3. Which three cards would be ideal to add to your current cards? Draw them in the cards provided.

Current cards



New cards

STOP

Narrative Connections

Unit 5 Rational Number Arithmetic

A World of Opposites

Opposites have played a major part in how we think about our world.

The philosopher Heraclitus believed that everything in the universe had an opposite. These opposites were bound together in a continuous cycle. Lao Tze, in the classic Chinese text “Tao Te Ching,” argued that every idea arises from its own opposite. Good cannot exist without evil. Beauty cannot exist without ugliness.

And in 1947, the physicist Neils Bohr designed his own coat-of-arms for the King of Denmark. Instead of the usual lions and crosses, Bohr drew a yin and yang symbol with the phrase “contraria sunt complementa.” In English, the phrase translates to “opposites are complementary.”

Around the globe, there are many kinds of opposites. There are the vast depths of the Mariana Trench and the breathtaking heights of Mount Everest. The lush, humid jungles of the Amazon Rainforest could not be more different than the arid winds of the Atacama Desert.

Throughout our planet, you can find opposites in temperatures, locations, heights, sizes, and even in measurements of time. In order to describe and understand all the characteristics of our world, we’ll need the full breadth of both the positive and the negative.

Welcome to Unit 5.

Name: ______________________ Date: __________ Period: ________

1. Complete each pattern by writing the missing values.

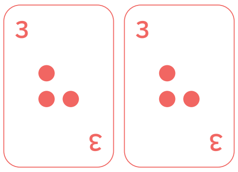
   a $5, 4, 3, 2,$ ________, ________, ________, $-2 \ldots$

   b $1, -1, 2,$ ________, $3, -3,$ ________, ________, ________ $\ldots$

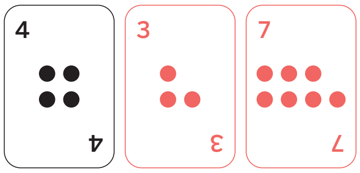
   c $8, 4,$ ________, $-4,$ ________, $-12,$ ________, ________ $\ldots$

2. Select *all* sets of cards that combine to produce a value of 0.

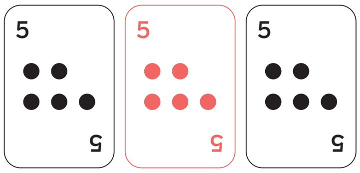
A.



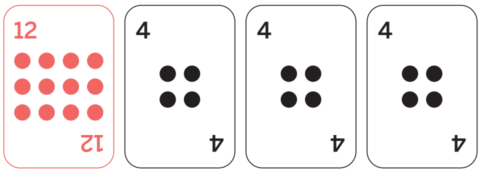
B.



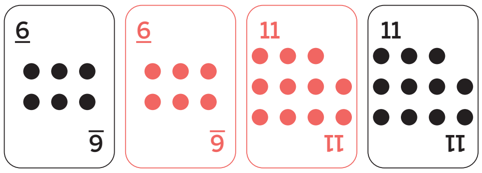
C.



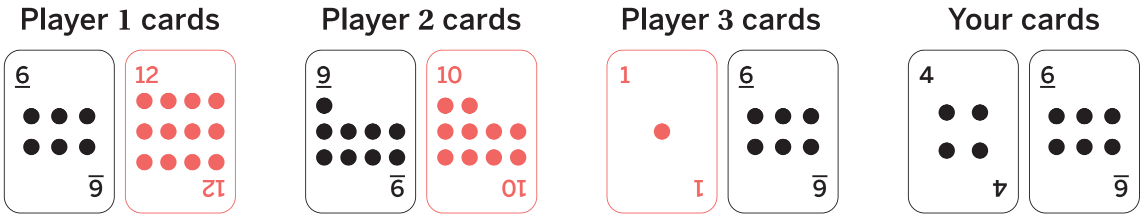
D.



E.



3. Imagine you are playing a game of Target: Zero and these are the cards for your group.



Which card would be ideal for you to select from the pile next?

Name: .... Date: .... Period: ....

4. Decide whether each table could represent a proportional relationship. If the relationship could be proportional, determine the constant of proportionality.

a

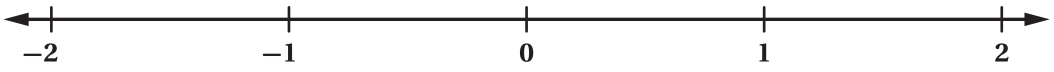
Number of buses	Number of wheels	Wheels per bus
5	30	
8	48	
10	60	
15	90	

b

Number of train cars	Number of wheels	Wheels per train car
20	184	
30	264	
40	344	
50	424	

5. Place the numbers in their approximate locations on the number line.

–1.5	0.5	0.11	1.1



SUB-UNIT

1

Adding and Subtracting Rational Numbers

Narrative Connections

What was Jeanne Baret's big secret?

In 1765, the botanist Philibert Commerçon was invited on an expedition to sail around the world. Commerçon wanted to bring his assistant and housekeeper, Jeanne Baret.

Baret had grown up in the French Loire Valley. There, she had become well versed in the native plant life, and was an herbal healer for her community. Baret would have been the perfect choice to join Commerçon. But there was a problem: Women were not allowed on French Navy ships. So Commerçon and Baret hatched a plan. Baret would disguise herself as a man, changing her name to "Jean."

For more than two years, "Jean" and Commerçon kept up their ruse. Together, they explored the rugged terrains Montevideo, Rio de Janeiro, and Tierra del Fuego. When Commerçon fell ill, Baret took over as chief botanist, collecting samples and lugging them through rough wilderness back to the ship for further study.

Imagine the sights she must have seen. She had grown up impoverished in the French countryside. Now, she was seeing sights no French woman had ever seen before: looming mountains, dense jungles, pebble beaches. Between Commerçon and Baret, the pair had collected over 6,000 specimens, many of which had never been seen before by a European.

For botanists like Baret, rational numbers are useful. They can be used to track the starting and ending points of one's journey, describe the differences of elevation of where different samples are collected, or compare the various climates of different locales. In these next lessons we will look at how to handle these rational numbers when adding and subtracting.

Unit 5 | Lesson 2

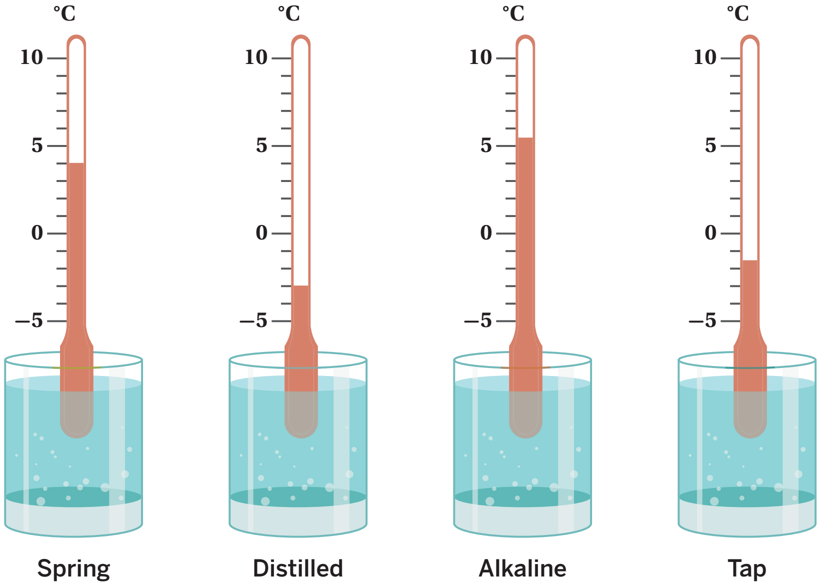
Interpreting Negative Numbers

Let's review what we know about negative numbers.

Warm-up Supercooling Liquids

The image shows the measured temperature of different kinds of water after being chilled.

1. What is the temperature shown on each thermometer?



Temperature (°C): ....

2. Which thermometer shows the greatest temperature?

3. Which thermometer shows the least temperature?

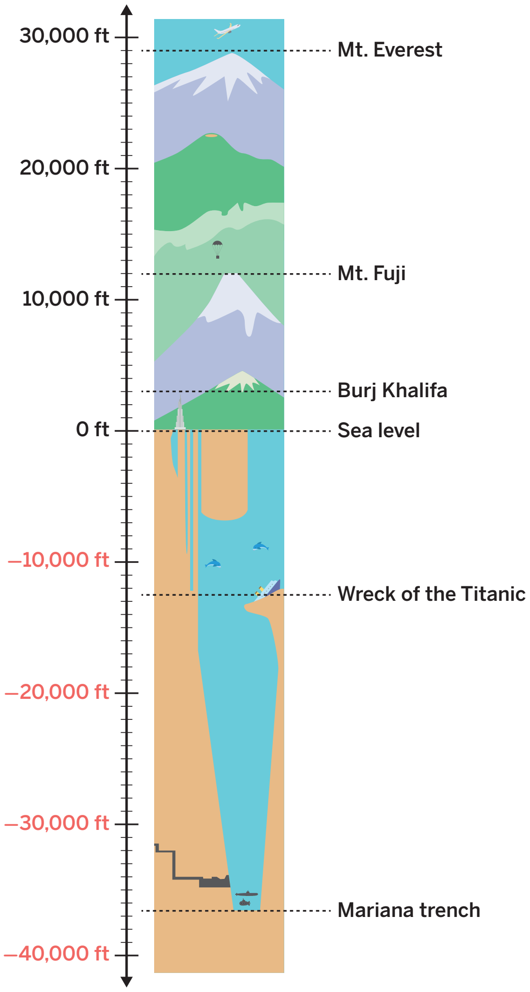
Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Exploring the Extremes

How high — or low — are you willing to go? Humans have now managed to explore both the highest and lowest points on Earth.

Compare the elevations of some of these extreme heights and depths.



1. About how far above or below sea level is each point?

- **a** Mt. Everest:
- **b** Mt. Fuji:
- **c** Burj Khalifa:
- **d** Wreck of the Titanic:
- **e** Mariana Trench:

2. A skydiver jumped out of a plane flying 12,000 ft above sea level. How does her starting vertical position compare to the height or depth of:

- **a** Mt. Fuji?
- **b** Mt. Everest?
- **c** The wreck of the Titanic?

3. A scuba diver is 100 ft below sea level. How does their vertical position compare to the height or depth of:

- **a** The Burj Khalifa?
- **b** The wreck of the Titanic?

4. The vertical distance of a butterfly from the starting point of the skydiver from Problem 2 is 500 ft. (Yes, they *do* fly that high!) What is the butterfly's distance from sea level?

Activity 2 Building a Number Line, Together

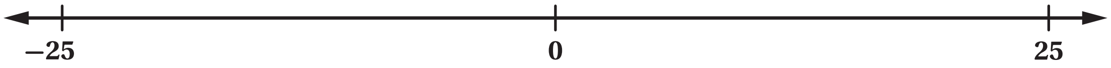
You will be provided with cards showing different numbers.

1. Discuss with your group how to order the numbers on your group's cards from least to greatest. Write the order you agree upon here.

Least **Greatest**

……………, ……………, ……………, ……………, ……………, ……………, ……………, ……………

2. This is a model of the number line in your classroom. Mark the number line with the approximate location of each number from your group's cards. (You may need to draw arrows to the location if you need more space to write your numbers.)



Wait for directions from your teacher about placing your cards on the class number line.

3. Select one of the cards your group placed on the number line. Describe the strategy you used for deciding where to place it on the class number line.

4. Are there any cards on the number line for which you do not agree with their placement? Explain your thinking.

STOP

Name: ______ Date: ______ Period: ______

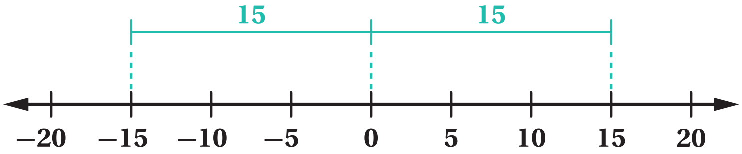
Summary

In today's lesson . . .

You reviewed that you can use positive numbers and negative numbers to represent temperature and elevation.

Using a number line can help to compare ***rational numbers*** — especially when dealing with positive and negative numbers.

We use the term *absolute value* to describe how far a number is from 0.



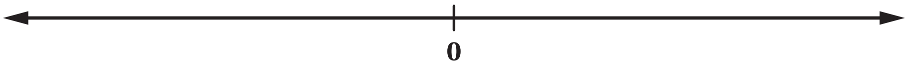
The numbers 15 and -15 are both 15 units from 0, so $|15| = 15$ and $|-15| = 15$. We call 15 and -15 *opposites*. They are on opposite sides of 0 on the number line, but are the same distance from 0.

Reflect:

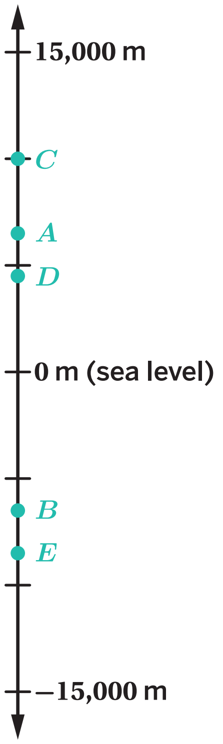
Name: ______________________ Date: ______________ Period: __________

1. Place the following numbers on the number line, and then circle the pair of values which are the same distance from 0.

2 3.5 -3.5 -4 1.75 $-1\frac{1}{2}$ -0.5



2. Match each point on the elevation chart with a reasonable elevation. Not all elevations will be used.



Elevation

__________ 4,500 m above sea level

__________ 4,500 m below sea level

__________ 6,500 m above sea level

__________ 6,500 m below sea level

__________ 8,500 m below sea level

__________ 10,000 m above sea level

__________ 11,500 m below sea level

3. Compare each pair of values using the symbols >, <, or =.

a 3 ______ -3

b 12 ______ 24

c -12 ______ -24

d 5 ______ $|-5|$

e 7.2 ______ 7

f -7.2 ______ $|7|$

g -1.5 ______ $-\frac{3}{2}$

h $-\frac{4}{5}$ ______ $-\frac{5}{4}$

i $-\frac{3}{5}$ ______ $-\frac{6}{10}$

j $-\frac{2}{3}$ ______ $\frac{1}{3}$

Name: .... Date: .... Period: ....

4. Han wants to buy a $\$30$ ticket to a basketball game, but the pre-order tickets are sold out. He knows there will be more tickets sold the day of the game, with a markup of 200%. How much should Han expect to pay for the ticket if he buys it the day of the game? Show or explain your thinking.

5. Decide whether or not each equation represents a proportional relationship. Show or explain your thinking.

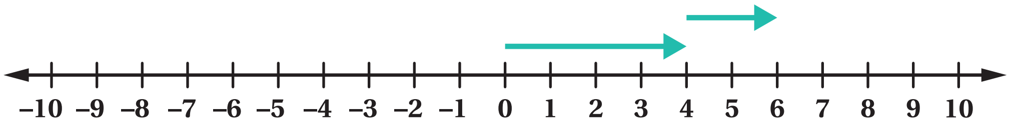
a Volume measured in cups, c, compared to the same volume measured in ounces, z: $c = \frac{1}{8}z$

b Area of a square, A, compared to the side length of the square, s: $A = s^2$

c Perimeter of an equilateral triangle, P, compared to the side length of the triangle, s: $3s = P$

d Length, L, compared to width, w, for a rectangle whose area is 60 square units: $L = \frac{60}{w}$

6. Which expression is modeled by this diagram?



A. $4 + 6$

B. $4 + 2$

C. $6 - 4$

D. $6 - 2$

Unit 5 | Lesson 3

Changing Temperatures

Let’s add positive and negative numbers.

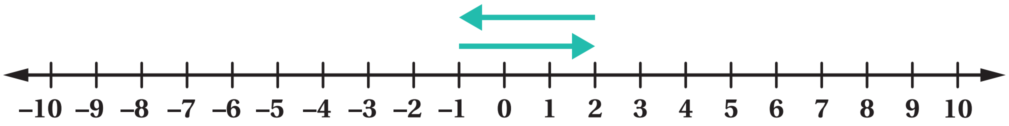
Warm-up Which One Doesn’t Belong?

Which number line does not belong? Be prepared to explain your thinking.

A.

B.

C.



D.

Log in to Amplify Math to complete this lesson online.

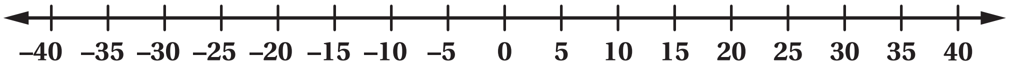
Name: .... Date: .... Period: ....

Activity 1 Warmer and Colder

Consider each starting temperature and how much it changes.
For each set:

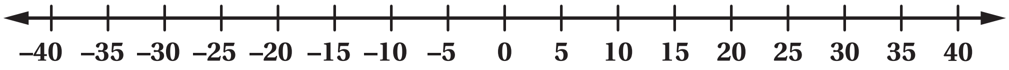
- **Represent the values on the number line.**
- **Determine the final temperature.**
- **Write an equation to represent the scenario.**

1. Start: 25°C Change: 10° warmer



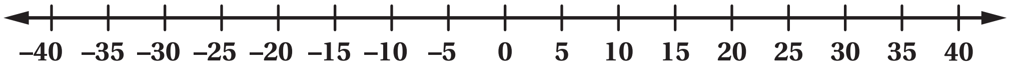
Equation: Final temperature:

2. Start: 25°C Change: 5° colder



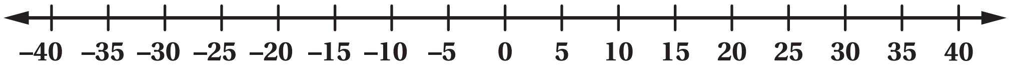
Equation: Final temperature:

3. Start: 25°C Change: 25° colder



Equation: Final temperature:

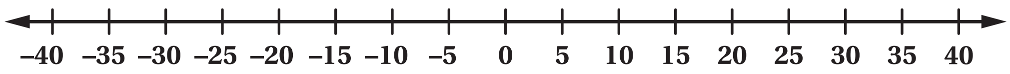
4. Start: 25°C Change: 50° colder



Equation: Final temperature:

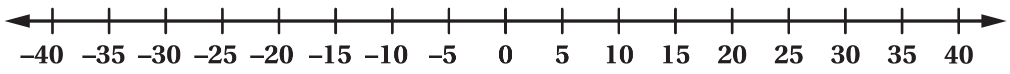
Activity 1 Warmer and Colder (continued)

5. Start: –20°C Change: 35° warmer



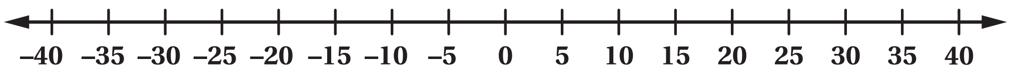
Equation: Final temperature:

6. Start: –20°C Change: 15° warmer



Equation: Final temperature:

7. Start: –20°C Change: 15° colder



Equation: Final temperature:

Are you ready for more?

You know the following things about three numbers, a, b, and c:

- a is less than b
- c is greater than b
- $|a|$ is greater than $|c|$

(Hint: It may be helpful to draw a number line and mark possible values for a, b, and c.)

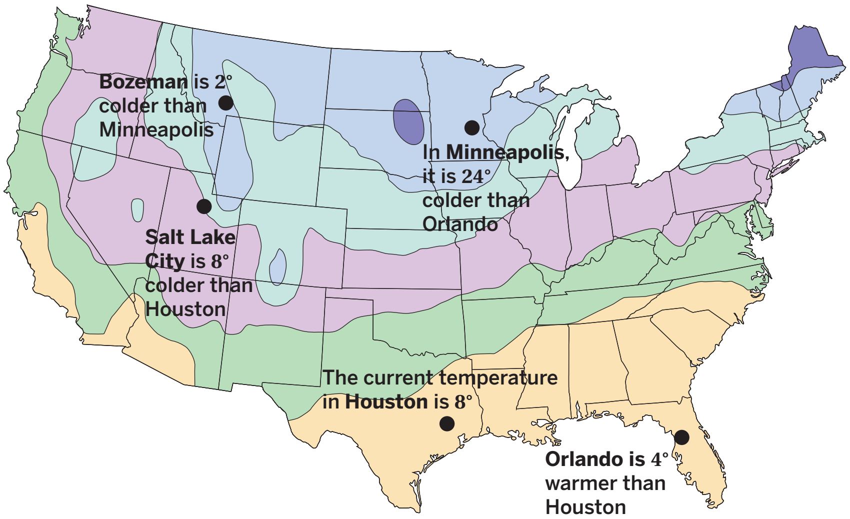
Mark each statement in the table as *always true*, *sometimes true*, or *never true*.

Statement	Always true	Sometimes true	Never true
$a + b < c$	☐	☐	☐
$\|a + b\| < c$	☐	☐	☐
$a + b = c$	☐	☐	☐
$a + b > c$	☐	☐	☐

Name: ______________________ Date: __________ Period: __________

Activity 2 Comparing Temperatures

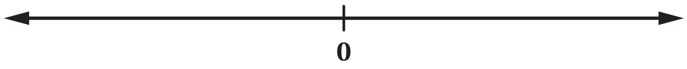
Refer to the map to complete the following problems. All temperatures shown are in °C.



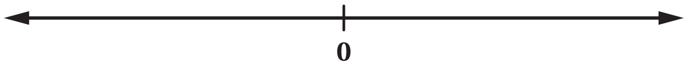
Co-craft Questions: Examine the map before beginning this activity. Work with your partner to write 2–3 questions you could ask about the information shown.

One city's current temperature is given. For each of the remaining cities, write an expression to represent the current temperature, and then use an arrow diagram to evaluate the expression.

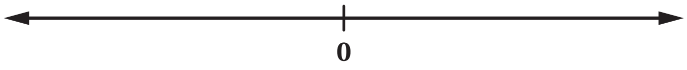
a City: Expression: Temp.:



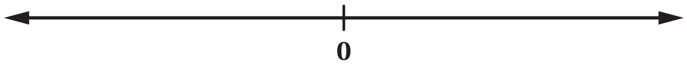
b City: Expression: Temp.:



c City: Expression: Temp.:



d City: Expression: Temp.:



Reflect: How well did you control your impulses and wait to draw conclusions at the end of the activity?

STOP

Summary

In today's lesson . . .

You learned that we can represent a change in temperature with a positive number if it increases and a negative number if it decreases.

We can also represent changing temperature using an ***arrow diagram***. The addition of positive numbers are represented with arrows pointing to the right and the addition of negative numbers are represented with arrows pointing to the left. When adding rational numbers, each arrow begins where the previous arrow ended.

This arrow diagram models the equation $7 + (-10) = -3$

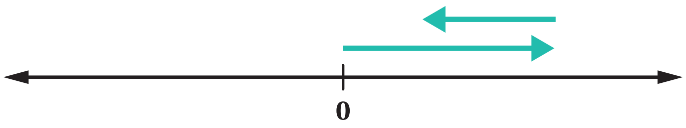
Reflect:

Name: .... Date: .... Period: ....

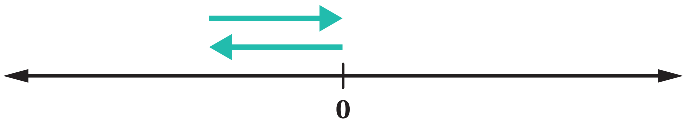
1. Match each arrow diagram with an expression it could represent.

Diagram

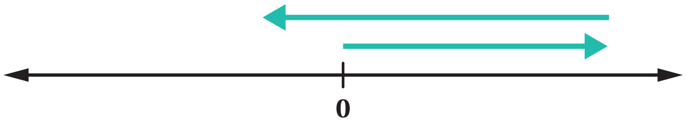
a



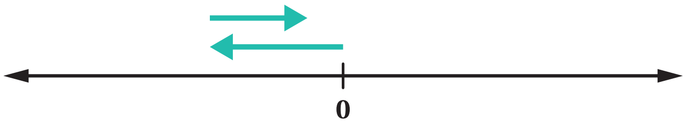
b



c



d



Expression

.... $-3 + (-3)$

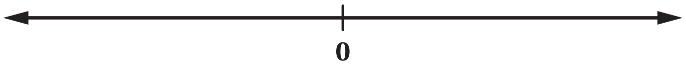
.... $5 + (-7)$

.... $3 + (-2)$

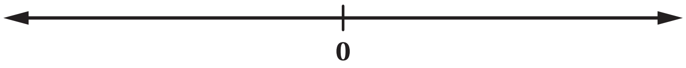
.... $-3 + 2$

2. Draw an arrow diagram to represent each of the given situations. Then write an addition equation that represents the change in temperature and the final temperature.

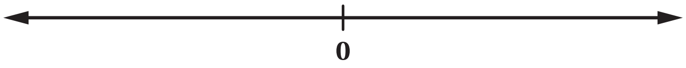
a The temperature was 80°F and then fell 20°F.



b The temperature was -13°F and then rose 9°F.



c The temperature was -5°F and then fell 8°F.



Name: .... Date: .... Period: ....

3. Draw an arrow diagram to represent each scenario and to determine the unknown value.

   a The temperature is $-2°C$. If the temperature rises by $15°C$, what would be the new temperature?

   b At midnight the temperature was $-6°C$. At midday the temperature is $9°C$. By how much did the temperature rise?

4. Last week, the price, in dollars, of a gallon of gasoline was g. This week, the price of gasoline per gallon increased by 5%. Which expressions represent this week's price, in dollars, of a gallon of gasoline? Select *all* that apply.

   A. $g + 0.05$

   B. $g + 0.05g$

   C. $1.05g$

   D. $0.05g$

   E. $(1 + 0.05)g$

5. Mai was assigned to make 64 bracelets for the craft sale. She made 125% of that number. 90% of the bracelets she made were sold. How many of Mai's bracelets were left after the craft sale?

6. Determine each sum.

   a $0.6 + 0.4$

   b $0.8 + 0.53$

   c $1.15 + 2.85$

Name: ________________ Date: ________ Period: ________

Unit 5 | Lesson 4

Adding Rational Numbers

Let's solve problems about adding rational numbers.

Warm-up What's the Opposite?

1. Describe the opposite of each action:

   a. Jumping down 10 steps.

   b. Taking the northbound train 6 stops.

   c. Running 150 m west.

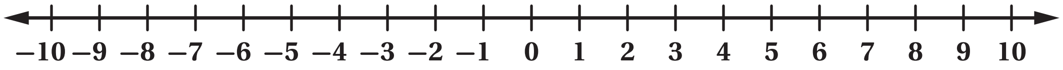
   d. Spending $5.50.

2. Write your own example with two opposite actions.

Log in to Amplify Math to complete this lesson online.

Activity 1 I Saw the Sign

Evaluate each expression shown in the table. You may use the number line to help reason about each expression.



Column 1
$-3 + 4$
$-3 + 3$
$-3 + 2$

Column 2
$4 + (-4)$
$4 + (-5)$
$4 + (-3)$

Column 3
$-3 + (-2)$
$-3 + (-3)$
$-3 + (-4)$

1. Examine Columns 1 and 2.

a When do you notice that the solution results in a positive sum?

b When do you notice that the solution results in a negative sum?

Name: ________________ Date: ________ Period: ________

Activity 1 I Saw the Sign (continued)

2. Now examine Column 3.

a When do you notice that the solution results in a positive sum?

b When do you notice that the solution results in a negative sum?

3. What rules could you write that are always true for adding positive and negative numbers?

Stronger and Clearer: Share your rules with 1–2 other pairs of students. Ask each other clarifying questions, such as, “How do you know your rules are *always* true?” Revise your responses based on their feedback.

Are you ready for more?

Fill in the boxes to make each equation true without using any number more than once.

$\square + (-\square) = -5$

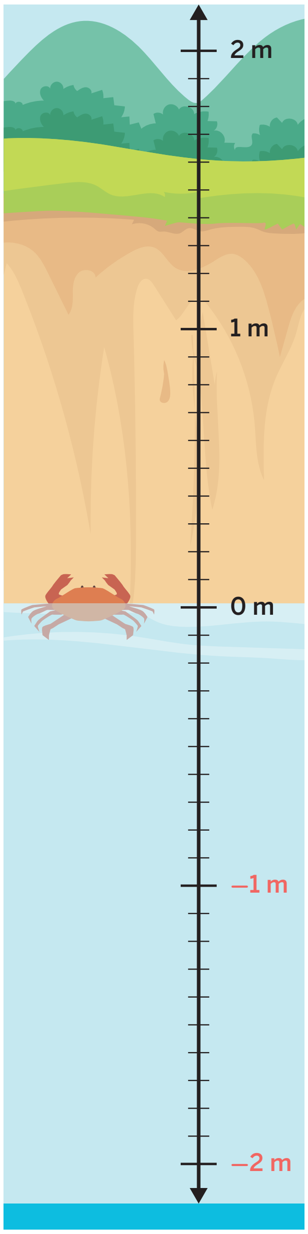
$\square + \square = -5$

$-\square + (-\square) = -5$

Activity 2 Above or Below the Water?

A rock crab climbs up and down a cliff by the sea, searching for food.

1. First, predict whether the crab will be above or below sea level. Then write an equation to check your prediction. **Note:** Assume the crab always starts at sea level, 0 m. The crab:



a Climbs up 1.2 m, and then climbs down 1.3 m.

b Climbs down 1.8 m, and then climbs down 0.2 m.

c Climbs down 1.23 m, and then climbs up 1.23 m.

d Climbs down 0.7 m, and then climbs up 0.65 m.

e Climbs up 1.3 m, and then climbs down 1.29 m.

2. How can you predict whether the crab will be above or below sea level without performing any calculations?

Name: .... Date: .... Period: ....

Activity 3 School Supply Number Line

1. Select two objects that are different lengths, and no longer than the number line shown here. (**Hint:** an eraser, marker cap, or paper clip might work.)

2. Let the length of the longer object be represented by a and the length of the shorter object be represented by b.

   Use the objects to measure and label each of the following points on the number line.

a	b	$2a$
$2b$	$a + b$	$-a$
$-b$	$a + (-b)$	$b + (-a)$

3. Complete each statement using <, >, or =. Use your number line to explain your reasoning.

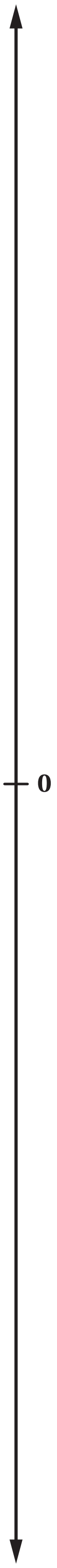
   a a .... b

   b $-a$ .... $-b$

   c $a + (-a)$ .... $b + (-b)$

   d $a + (-b)$ .... $b + (-a)$

   e $a + (-b)$ .... $-a + b$



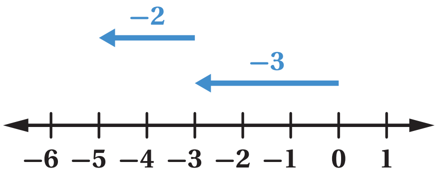
STOP

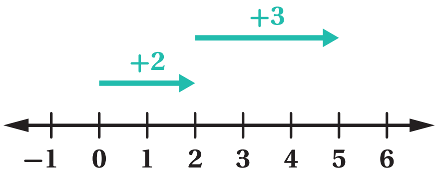
Summary

In today's lesson . . .

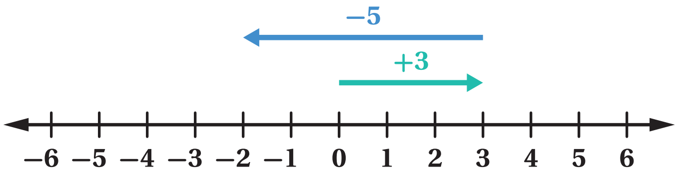
You formulated some rules for adding rational numbers:

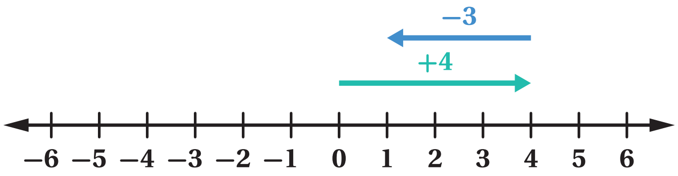
- To add two numbers with the same sign, determine the sum of the absolute value of each number, and then give the sum the same sign as the addends.



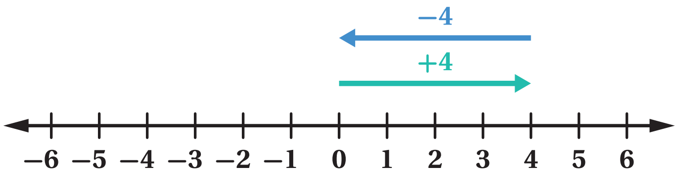


- To add two numbers with different signs, determine the difference of the absolute values, and then give the result the same sign as the number with the greater absolute value.





- When you add a number and its *opposite*, the sum is zero. These numbers are called ***additive inverses***.



Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. Determine the final elevation for each of the following:

a A bird starts at 20 m and rises 16 m.

b A fish starts at -9 m and descends 11 m.

c A diver starts at 5 m and descends 16 m.

d A whale starts at -9 m and rises 11 m.

2. Atoms are the smallest units of matter. One of the particles in an atom is called an *electron*. It has a charge of -1. Another particle in an atom is a *proton*. It has a charge of 1. The charge of an atom is the sum of the charges of the electrons and the protons. A carbon atom has an overall charge of 0, because it has 6 electrons and 6 protons and $-6 + 6 = 0$. Determine the overall charge for these other elements in the table.

	Charge from electrons	Charge from protons	Overall charge
Neon	-10	10	
Oxide	-10	8	
Copper	-27	29	
Tin	-50	50	

3. Determine each sum.

a $14.7 + 28.9$

b $-81.4 + (-12)$

c $-9.2 + 4.4$

d $51.8 + (-0.8)$

e $-\frac{1}{2} + \frac{3}{4}$

f $47\frac{2}{3} + \left(-47\frac{5}{6}\right)$

Name: .... Date: .... Period: ....

4. Last week, it rained g inches. This week, the amount of rain decreased by 5%. Which expressions represent the amount of rain that fell this week? Select *all* that apply.

A. $g - 0.05$

B. $g - 0.05g$

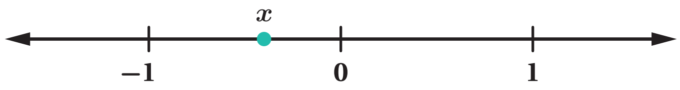
C. $0.95g$

D. $0.05g$

E. $(1 - 0.05)g$

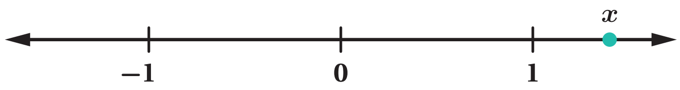
5. In each diagram, x represents a different value. For each diagram, write a statement or an equation that is *definitely* true about the value of x, and a statement or an equation that *could be* true about the value of x.

a



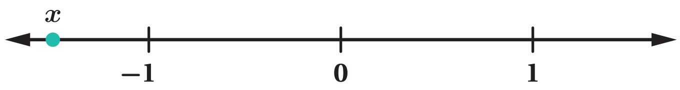
Definitely true: **Could be true:**

b



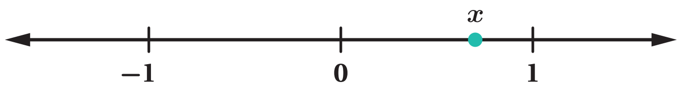
Definitely true: **Could be true:**

c



Definitely true: **Could be true:**

d



Definitely true: **Could be true:**

6. Write a positive or a negative value that represents each scenario.

a Andre spent \$12 at the movie theater.

b Jada earned \$42 mowing her neighbor's lawn.

c Lin lost \$2 in the wishing will.

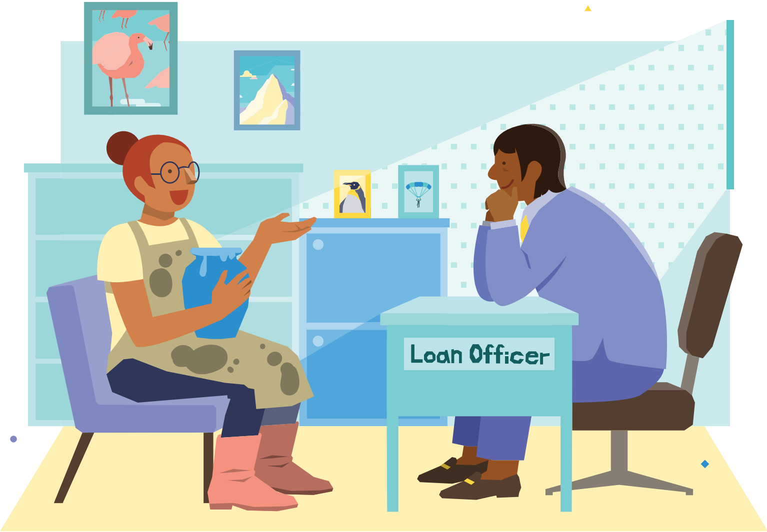
d Han was gifted \$13 by his grandmother.

Name: .... Date: .... Period: ....

Unit 5 | Lesson 5

Money and Debts

Let's apply what we know about
positive and negative numbers
to money.

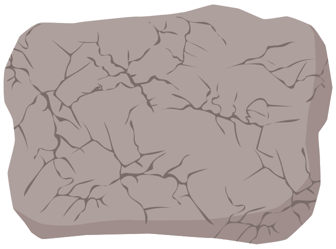


Warm-up Taking a Loan

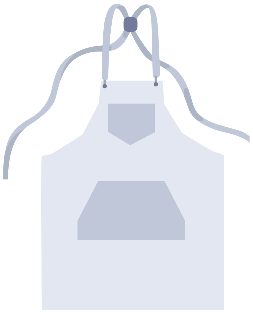
Priya makes and sells pottery. Sometimes she needs to borrow money from the bank to purchase supplies for making a batch of new pottery.

Priya's current cash: $150

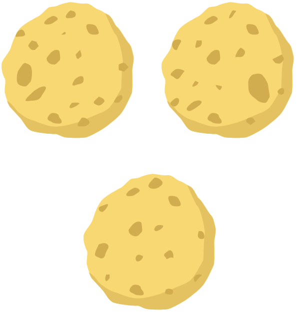
Needed supplies for next batch of pottery: 100 lb of clay, a new apron, and 3 sponges



$1.25 per pound



$25



$2 per sponge

How much does Priya need to borrow to pay for her supplies? Show or explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Dealing With Debt

During the 10th century CE, many artisans in the Middle East were paid to craft ceramic tiles used in decorating important buildings, such as mosques and palaces. Helpfully, at nearly the same time, the Persian mathematician Abu al-Wafa’ Buzjani was busy writing a book showing how people could use negative numbers to keep track of debts in their business dealings.

We could imagine an artisan buying supplies for crafting the tiles and needing to go into debt over $1{,}000$ years ago, just as today.

The artisan keeps track of their account balance by scratching the number of silver coins they spend and earn into pieces of clay.

Write and evaluate an expression to represent the artisan’s balance in coins.

Featured Mathematician

Abu al-Wafa’ Buzjani

Abu al-Wafa’ Buzjani should have been quite popular in his day, owing to the fact that he wrote about mathematics for several different audiences. In 10th century Iraq, astronomers, craftspeople, businesses, and accountants all could have picked up one of his “how-to” books to help them understand how math could help them in their field.

Abu al-Wafa’s “Book on What Is Necessary from the Science of Arithmetic for Scribes and Businessmen” is apparently the first and only place where negative numbers were used in Arabic texts during this time period.

“Buzjani, Persian mathematician and astronomer.” Public Domain via Wikimedia Commons

Name: .... Date: .... Period: ....

Activity 2 Energy Supply

Homes with solar panels may actually produce more energy than they use, depending on the amount of sunlight they get. If a solar panel system produces more energy than a household can consume over a period of time, the excess energy is sent back into the electric grid.

Bard's home has a rooftop solar panel system. The statement from the local energy company shows Bard's family whether they consumed or produced more energy. Unfortunately, it seems that the printer ran out of ink as it was printing the statement!

1. Bard knows that the cost for energy is $0.25 per kilowatt hour (kWh). Complete the missing information in the statement.

	Details	Charges ($)	Credits ($)	Balance ($)
1/31	Consumed: 500 kwH Produced: 480 kwH	5.00		5.00
2/28	Consumed: 525 kwH Produced: 490 kwH	8.75		13.75
3/31	Consumed: 497 kwH Produced: 500 kwH		0.75	13.00
4/30	Consumed: 482 kwH Produced: 550 kwH			
5/31	Consumed: 470 kwH Produced: 520 kwH			
6/30	Consumed: 515 kwH Produced: 515 kwH			
7/31	Consumed: 545 kwH Produced: 530 kwH			
Totals				

2. How much does the energy company need to pay Bard's family to result in a balance of $0?

STOP

Summary

In today's lesson . . .

You saw that it is sometimes necessary to borrow money to cover costs that you will pay back in the future. Though there are different names for what we call this owed money (e.g. *debt*, *balance*), we typically represent these amounts as negative values. This makes sense — if the money we have can be represented as a positive number, the money we owe may rightly be referred to as negative.

We can use positive numbers to represent payments into a bank account (***deposits***) and negative numbers to represent money taken out of an account (***withdrawals***). We can also use a negative balance to represent ***debt*** (owing money). The *additive inverse* of a number can be used to determine how much money is needed to reach a ***balance*** of zero.

Reflect:

Name: ______________________ Date: __________ Period: ________

1. A baker has an account with her flour supplier. Each month, she purchases flour as needed to bake the bread she sells at her store. She tracks her spending and how much she earns from sales in this table.

Spent \$45.50	Sold \$37.55	Spent \$40.00
Sold \$40.00	Sold \$75.00	Spent \$37.55

Shawn and Priya are helping the baker determine whether she will be in debt.

Shawn's strategy	Priya's strategy
$-45.50 + (-40.00) + (-37.55) = -123.05$ $37.55 + 40.00 + 75.00 = 152.55$ $152.55 + (-123.05) = 29.50$	$-40.00 + 40.00 + (-37.55) + 37.55 + (-45.50) + 75.00$ $= 0 + 0 + (-45.50) + 75.00$ $= -45.50 + 75.00$ $= 29.50$

Both Shawn and Priya agree the baker will not be in debt. She will have \$29.50 left over after paying her flour supplier.

a Describe Shawn and Priya's strategies.

b Whose strategy do you prefer? Why do you prefer this strategy?

2. The table shows five transactions and the resulting account balance in a bank account, but some values are missing. Complete the table with the missing information.

	Amount (\$)	Balance (\$)
Transaction 1	200	200
Transaction 2	-147	53
Transaction 3	90	
Transaction 4	-229	
Transaction 5		0

Name: .... Date: .... Period: ....

3. Solve each problem. Show your thinking.

a Mai has $54 in her bank account. A store credits her account with a $10 refund, which means the store gave her money back. How much does she now have in the bank?

b Priya's bank account is overdrawn by $60, which means her balance is −$60. She gets $85 for her birthday and deposits it into her account. How much does she now have in the bank?

c Diego is overdrawn at the bank by $180. He gets $70 for his birthday and deposits it. What is his account balance now?

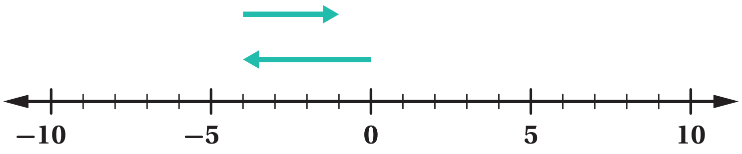
d Han has $37 in his bank account and writes a check for $87. After the check has been cashed, what will the bank balance show?

4. Evaluate the following expression.

$$1\frac{4}{5} + \left(-2\frac{4}{5}\right)$$

5. Write an equation to represent the sum shown on each arrow diagram.

a



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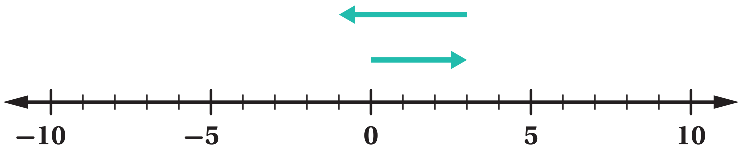


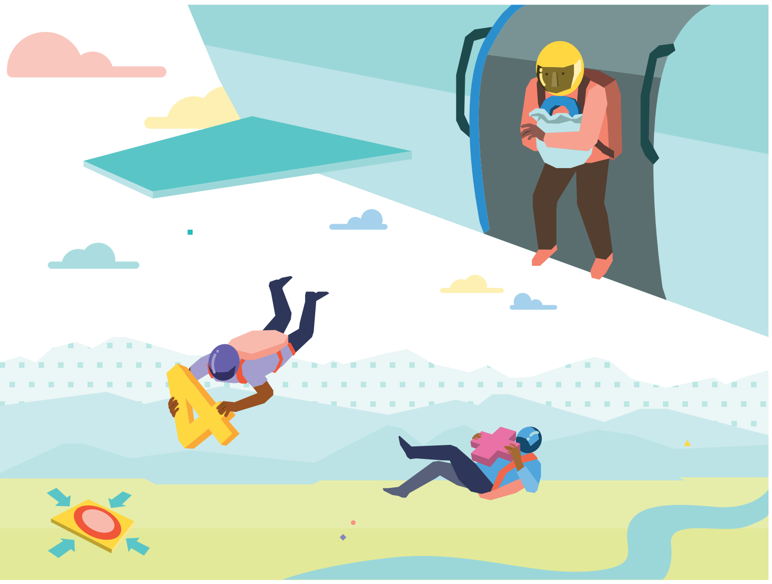
c

d

Name: ______________________ Date: __________ Period: __________

Unit 5 | Lesson 6

Representing Subtraction

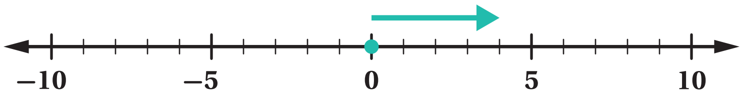


Let's subtract rational numbers.

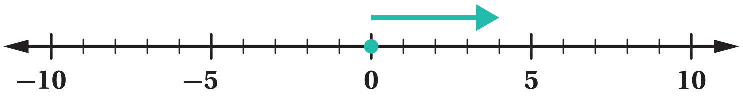
Warm-up Getting to Zero

Determine the missing value that would make each equation true. Add an arrow to each diagram representing the value being added or subtracted.

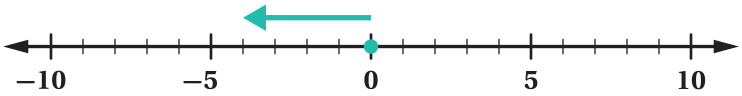
1. $4 + \boxed{} = 0$



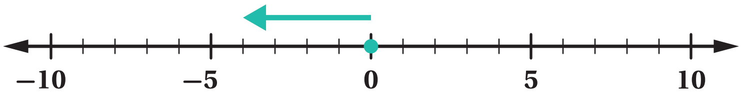
2. $4 - \boxed{} = 0$



3. $-4 + \boxed{} = 0$



4. $-4 - \boxed{} = 0$



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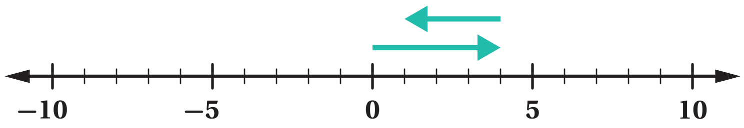
Activity 1 Addition or Subtraction?

1. Match each number line with the expression that best represents it. Then determine the sum or difference. A number line may be used more than once.

a



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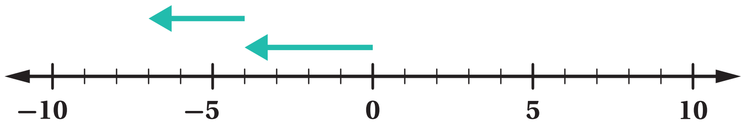

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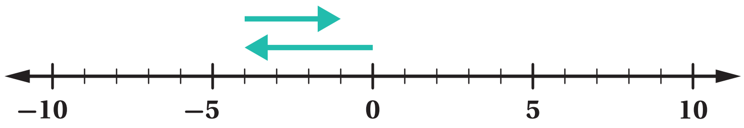


.... $4 + (-3) =$ ....

.... $-4 - 3 =$ ....

.... $-4 + 3 =$ ....

.... $-4 + (-3) =$ ....

.... $4 - 3 =$ ....

.... $-4 - (-3) =$ ....

2. Complete the table below based on your responses in Problem 1.

	Subtraction expression	Addition expression
Number line a		
Number line b		
Number line c		

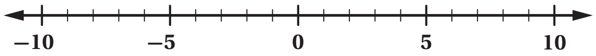
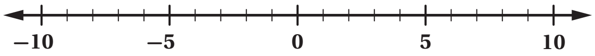
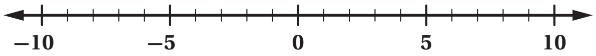
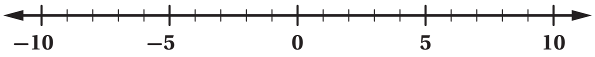
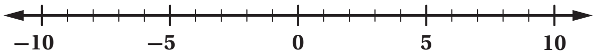
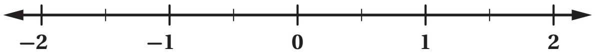
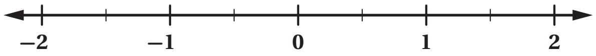
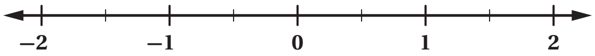
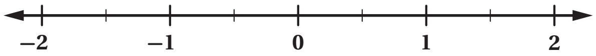
3. What do you notice?

Name: ______________________ Date: ____________ Period: __________

Activity 2 Partner Problems

With your partner, decide who will complete Column A and who will complete Column B. After each row, share your responses with each other. Your responses should be the same. If they are not the same, work together to correct any errors or resolve any disagreements.

Use the number lines provided, if helpful.

Column A	Column B
1. $-5 + (-2)$ 	$-5 - 2$ 
2. $-8 - (-1)$ 	$-8 + 1$ 
3. $6 + 3$ 	$6 - (-3)$ 
4. $\frac{1}{4} + \left(-\frac{1}{2}\right)$ 	$\frac{1}{4} - \frac{1}{2}$ 
5. $-1.4 - (-0.8)$ 	$-1.4 + 0.8$ 

STOP

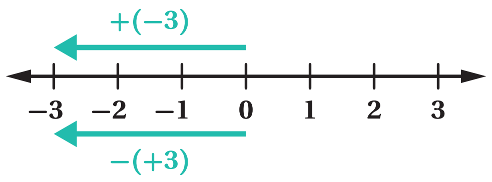
Summary

In today's lesson . . .

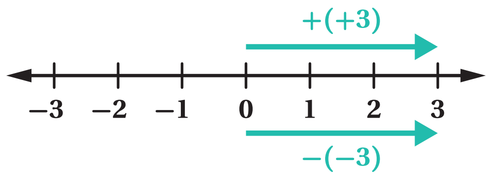
You reasoned that subtraction expressions can be expressed as equivalent addition expressions.

In general, when looking at an arrow on a number line, it can represent either addition or subtraction.

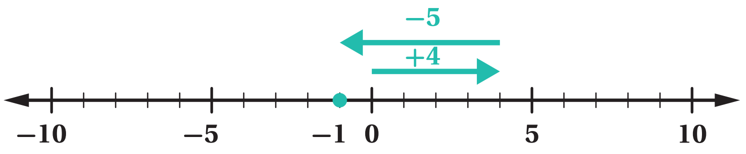
Left facing arrow



Right facing arrow

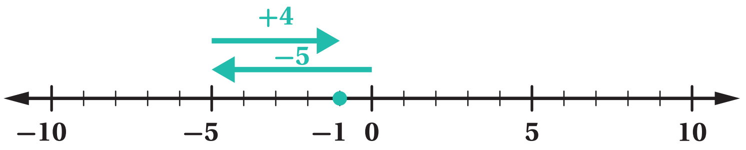


Using this representation, you found that you could determine the difference of two values by rewriting the expression as an addition expression using the *additive inverse* of the second value.



$4 - 5 = -1$

$4 + (-5) = -1$



$-5 - (-4) = -1$

$-5 + 4 = -1$

Reflect:

Name: .... Date: .... Period: ....

1. Match each subtraction expression with its corresponding equivalent addition expression.

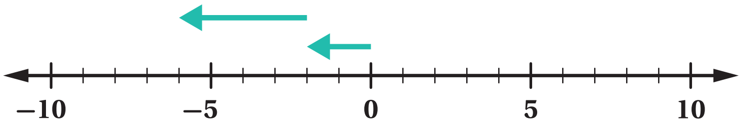
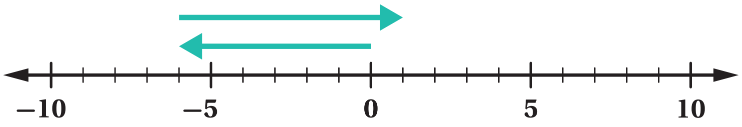
a $-3 - 2$ .... $3 + (-2)$

b $-3 - (-2)$ .... $3 + 2$

c $3 - 2$ .... $-3 + 2$

d $3 - (-2)$ .... $-3 + (-2)$

2. For each number line, write an expression using addition and an equivalent expression using subtraction.

Number line	Addition	Subtraction
		
		
		
		

3. Determine each difference. Use the number line, if helpful.

a $6 - 4$

b $6 - (-4)$

c $4 - 6$

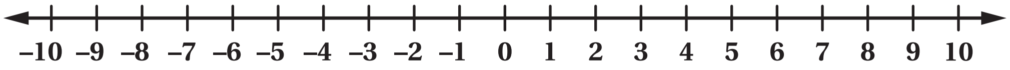
d $4 - (-6)$

e $-6 - 4$

f $-6 - (-4)$

g $-4 - 6$

h $-4 - (-6)$



Practice

Name: ______ Date: ______ Period: ______

4. A restaurant bill is $59 and you pay $72 including gratuity. What percentage represents the amount of gratuity paid? Show your thinking.

5. A company produces screens of different sizes. Based on the table, could there be a proportional relationship between the number of pixels and the area of the screen? If so, write an equation representing the relationship. If not, explain your thinking.

Area of screen (in^2)	Number of pixels
6	31,104
72	373,248
105	544,320
300	1,555,200

6. Determine each sum or difference. Use a number line, if helpful.

a $6 + (-4)$

b $-10 + 5$

c $-3 + (-2)$

d $-2 - 4$

e $5 - 10$

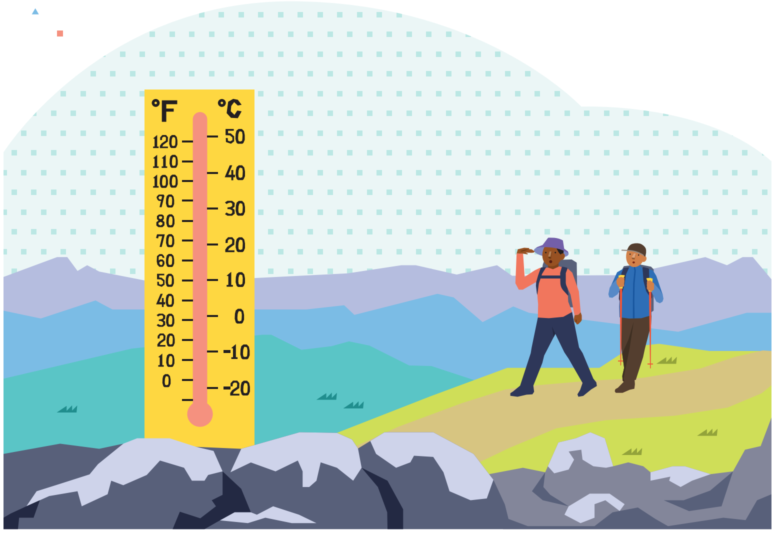
f $3 - (-2)$

Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 7

Subtracting Rational Numbers (Part 1)

Let's determine the difference of rational numbers.



Warm-up Number Talk

Determine each sum.

Column A	Column B
$3 + 2$	$2 + 3$
$5 + (-9)$	$-9 + 5$
$-11 + 2$	$2 + (-11)$
$-6 + (-3)$	$-3 + (-6)$
$-1.2 + (-3.6)$	$-3.6 + (-1.2)$
$-2\frac{1}{2} + \left(-3\frac{1}{2}\right)$	$-3\frac{1}{2} + \left(-2\frac{1}{2}\right)$

What do you notice about the expressions in Column A compared to Column B?
What do you notice about the sums in Column A and Column B?

Log in to Amplify Math to complete this lesson online.

Activity 1 Partner Problems

With your partner, decide who will complete Column A and who will complete Column B. After each row, share your responses with your partner. Compare your responses, and decide if they should be the same or not.

1. Determine each difference.

Column A	Column B
$3 - 2$	$2 - 3$
$5 - (-9)$	$-9 - 5$
$-11 - 2$	$2 - (-11)$
$-6 - (-3)$	$-3 - (-6)$
$-1.2 - (-3.6)$	$-3.6 - (-1.2)$
$-2\frac{1}{2} - \left(-3\frac{1}{2}\right)$	$-3\frac{1}{2} - \left(-2\frac{1}{2}\right)$

2. What do you notice about the expressions in Column A compared to Column B? What do you notice about their values?

Are you ready for more?

Complete this table so that every row and every column has a sum of 0. Can you determine another way to solve this puzzle?

	-12	0		5
0			-18	25
25		-18	5	-12
-12				-18
	-18	25	-12	

Name: .... Date: .... Period: ....

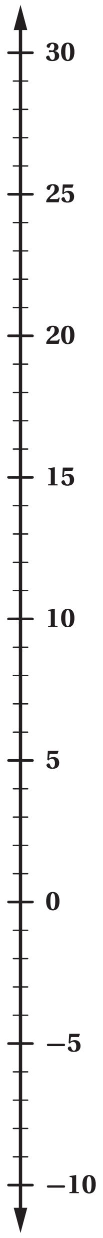
Activity 2 Expressions With Temperature

Tyler and Lin took a hike from Death Valley to Telescope Peak and back. They began their hike at Shorty's Well and ended at Badwater. They recorded the temperature throughout their hike. The temperature of each location was recorded in the table shown.

Using the table, write an expression, and then determine the change in temperature between each location. The first row has been completed for you. Consider using the number line to help with your thinking.

Location	Temperature (°C)
Shorty's Well	25
Hanaupah Spring	20
Telescope Peak	−3
South Fork	20
Badwater	22.8

Expression	Change (°C)
20 − 25	−5



STOP

Summary

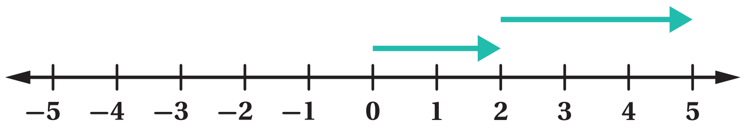
In today's lesson . . .

You saw that when we talk about the difference of two numbers, we mean, "subtract them." But unlike addition the order of the numbers in the subtraction expression affects the result.

Consider the following examples:

$$-3 - 2 = -3 + (-2) = -5$$

$$2 - (-3) = 2 + 3 = 5$$



The difference can be positive or negative, depending on the order.

Reflect:

Name: .... Date: .... Period: ....

1. Write an expression to model each scenario, and then solve the problem.

a How much warmer is 82°F than 40°F?

b How much warmer is 82°F than -40°F?

2. Diego measured the temperature when he left for school in the morning, and then again when he got home in the afternoon. He said the change in temperature was -3°F.

a What does it mean for the change in temperature to be -3°F?

b Identify a pair of values that could represent the temperature when he left for school versus when he arrived home in the afternoon.

3. After their hike, Lin and Tyler researched their elevation at each of the locations at which they had temperature recordings. Write an expression then determine the change in elevation between each location from Shorty's Well to Badwater.

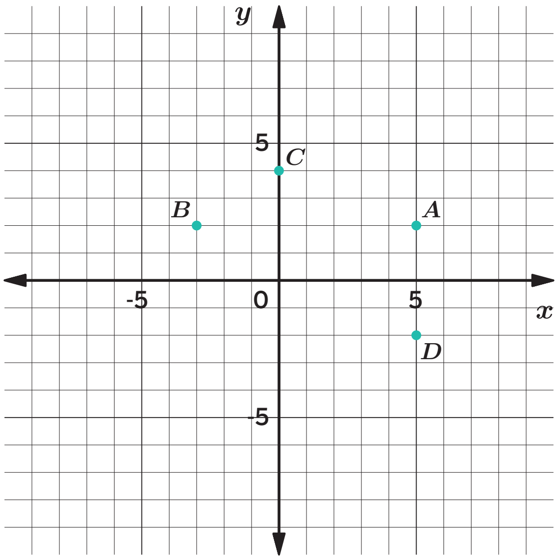
Location	Elevation (ft)
Shorty's Well	-262
Hanaupah Spring	$4,003$
Telescope Peak	$11,049$
South Fork	$3,700$
Badwater	-289

Expression	Change (ft)

Practice

Name: .... Date: .... Period: ....

4. Consider the 4 points shown on the coordinate plane.



a Determine the coordinates of each point.

Point A:

Point B:

Point C:

Point D:

b Which point is 8 units from point A?

c Plot two points that are 3 units from point B, and label them E and F.

5. A family goes to a restaurant. When the bill is delivered by the server, the gratuity chart shown is provided.

How much was the price of the meal? Show or explain your thinking.

Gratuity Guide For Your Convenience:

15% would be $4.89

18% would be $5.87

20% would be $6.52

6. Plot and label each point on the coordinate plane.

A: $(0, 2)$

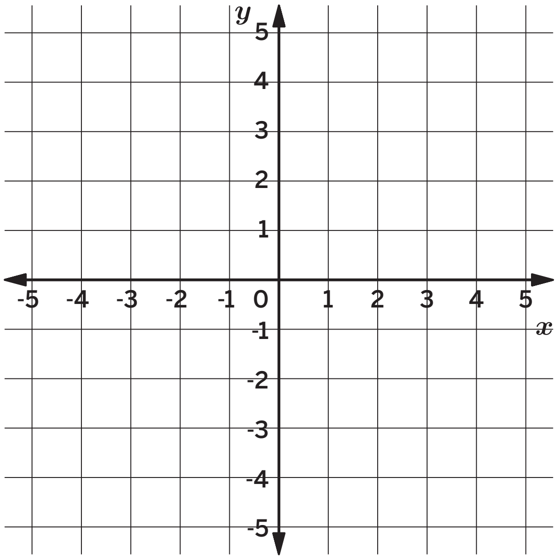
B: $(-3, 0)$

C: $(1, 4)$

D: $(-2, -3)$

E: $(3, -4)$

F: $(-2, 1)$



Name: ________ Date: ________ Period: ________

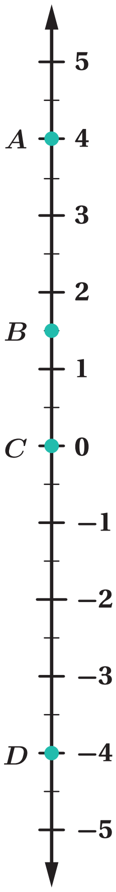
Unit 5 | Lesson 8

Subtracting Rational Numbers (Part 2)

Let's get more practice subtracting rational numbers.

Warm-up Determining Distance

Determine the distance between each set of points, based on the given number line.



1. A and C

2. D and C

3. A and D

4. B and C

5. B and A

6. B and D

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Activity 1 Differences and Distances

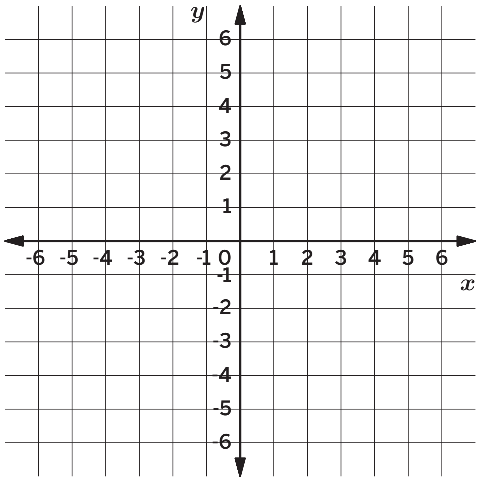
1. Plot these points on the coordinate plane:

   $A\ (5, 4)$

   $B\ (5, -2)$

   $C\ (-3, -2)$

   $D\ (-3, 4)$



2. Connect the dots in order. What shape is created?

3. What are the side lengths of Figure $ABCD$?

4. What is the difference between the y-coordinates of A and B? Show your thinking.

5. What is the difference between the y-coordinates of B and A? Show your thinking.

6. How do the differences of the coordinates relate to the distance between the two points?

Discussion Support: What math language can you use in your response to Problem 6? How can you represent this idea using both words and symbols?

Name: ________________ Date: ________ Period: ________

Activity 2 My Lifetime Timeline

John Denver once sang, “Today is the first day of the rest of my life.” Imagine today is a fresh start, and create a number line of the life you’ve had so far and the life you hope to have in the future. Your teacher will provide you a large number line for this activity.

1. Including the day that you were born, think of three important events that have happened so far in your lifetime. Write down how old you were when each event happened, to the nearest month. (For example, Mai was 2 years and 1 month old the day her brother was born, so she would say that she was $2\frac{1}{12}$ years old.)

a The day I was born. I was 0 years old.

b ________________

c ________________

2. If today is represented by 0, write and simplify and an expression to determine what number on your timeline would represent each event from Problem 1. Write each value as a mixed number to the nearest month. (For example, Mai is currently 12 years and 3 months old, so her day of birth would be represented by $-12\frac{1}{4}$.)

a ________________

b ________________

c ________________

3. Add each event from Problem 1 to the number line. Use the labels a, b, and c.

Activity 2 My Lifetime Timeline (continued)

4. Think of three goals you would like to accomplish in the next 15 years. Describe them in the space provided. Next to each goal, identify what age you hope to be when you reach that goal. (For example, Mai has a goal of graduating from high school with her class. She would be 17 years and 11 months old.)

d ....

e ....

f ....

5. If today is represented by 0, write and simplify and an expression to determine what number would represent each event from Problem 4. Write each value as a mixed number to the nearest month. (For example, Mai's high school graduation would be represented by the expression $17\frac{11}{12} - 12\frac{1}{4} = 5\frac{2}{3}$.)

d ....

e ....

f ....

6. Add each event from Problem 4 to your timeline. Use the labels d, e, and f.

STOP

Name: ________________ Date: ________ Period: ________

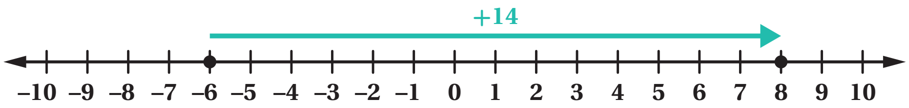
Summary

In today's lesson . . .

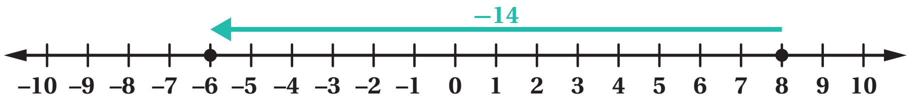
You continued your study of subtracting numbers, recognizing that whether you are determining the "difference of" or "distance between" two values you are determining how far apart they are.

When determining the *difference*, the order of the values in an expression matters. A positive or negative difference indicates whether the second value is greater than or less than the first value.

$8 - (-6)$

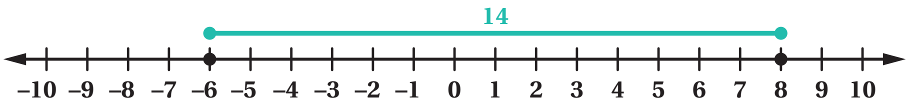


$-6 - 8$



When determining the ***distance***, the order does not matter. For any two values, a and b , $|a - b| = |b - a|$.

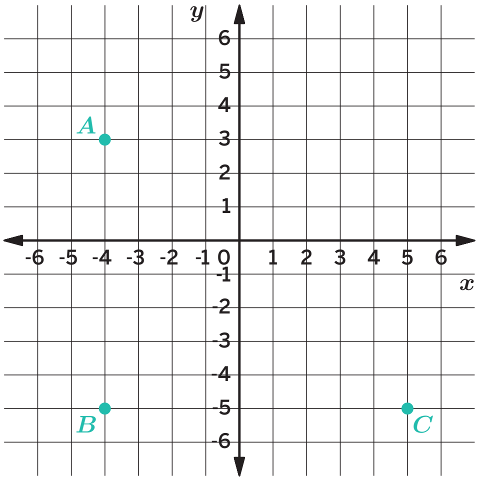
$|8 - (-6)| =$
$|-6 - 8|$



Reflect:

Name: .... Date: .... Period: ....

1. Use the graph to answer each problem.



a What is the distance between points A and B?

b What is the difference of the y-coordinates of points B and A?

c What is the distance between points B and C?

d What is the difference of the x-coordinates of points C and B?

2. Tyler started practicing karate $3\frac{1}{2}$ years ago and started playing piano $4\frac{1}{3}$ years ago. How much time passed between when he started playing piano and started practicing karate? Show or explain your work.

3. Determine a missing value that would make each equation true.

a $-6 - (-10) = \boxed{}$

b $|4 - 7| = \boxed{}$

c $\boxed{} - (-5) = 16$

d $|\boxed{} - 3| = 9$

Name: .... Date: .... Period: ....

4. Complete each statement with a number that makes the statement true.

a .... $< 7°\text{C}$

b .... $< -3°\text{C}$

c $-0.8°\text{C} <$ .... $< -0.1°\text{C}$

d .... $> -2°\text{C}$

5. Determine each difference.

a $-5 - 6$

b $35 - (-8)$

c $\frac{2}{2} - \frac{3}{5}$

d $-4\frac{3}{8} - \left(-1\frac{1}{4}\right)$

6. Use the rules for addition and subtraction to determine whether the sum or difference is a positive or negative value and explain your thinking. You do not need to evaluate each expression:

Expression	Positive	Negative	Explanation
$9 + (-7)$	☐	☐	
$9 - (-7)$	☐	☐	
$-10 + (-4)$	☐	☐	
$-3 - 6$	☐	☐	

Unit 5 | Lesson 9

Adding and Subtracting Rational Numbers

Let's determine the sum and difference of rational numbers.

Warm-up Positive or Negative?

Without performing any calculations, determine whether each scenario could be represented by a positive or negative value. Be prepared to explain your thinking

Scenario	Positive	Negative
Scenario 1: How much greater is $4\frac{1}{5}$ than $-12\frac{3}{8}$?	☐	☐
Scenario 2: A company had \$5,350 of sales during the first week of the month. They spent \$6,432 on salary, rent, and supplies during that same week. What is their total earnings?	☐	☐

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 Writing Expressions

Antarctica has some of the most extreme conditions on Earth, holding the record for both the coldest and driest location. Approximately 98% of Antarctica is covered with ice, but, due to climate change, the amount of ice is shrinking each year. One scientist working to study this phenomena is NASA's Sheila Wall. She served as the lead structural analyst on the Ice, Cloud and Land Elevation Satellite-2 (ICESat-2) mission which investigates the changes of the polar ice sheets and how they affect rising sea levels.

Volodymyr Goinyk/Shutterstock.com

Three Reads: Read each scenario three times.

1. Make sense of the scenario.
2. What quantities are given?
3. Will the solution be positive or negative?

1. Write an expression that represents each scenario. Without calculating, determine whether each expression would result in a positive or negative value.

Scenario	Expression	Positive	Negative
a The highest temperature ever recorded in Antarctica was 69.3°F. The lowest temperature was −135.8°F. What is the difference between these temperatures?		☐	☐
b At the South Pole, the average maximum temperature in July is −67°F. This is 52°F lower than the average maximum temperature in January. What is the average maximum temperature in January?		☐	☐
c The highest point in Antarctica is the top of Vinson Massif Mountain at 16,050 ft above sea level. The lowest point is within the Denman Glacier, reaching 11,500 ft below sea level. What is the distance between the highest and lowest point of Antarctica?		☐	☐

Activity 1 Writing Expression (continued)

2. After coming to a consensus on which expressions are positive or negative, determine the solution to each scenario. Show or explain your thinking.

   a. The difference between the lowest and highest temperature is ________.

   b. The average maximum temperature in January is ________.

   c. The distance between the highest and lowest point in Antarctica is ________.

Featured Mathematician

Sheila M. Wall

As an aeronautical engineer, Sheila Wall uses mathematics to create models of proposed designs and evaluate the structural integrity of each design. As she described, “Creating my mathematical models, 3D numerical representations of the instruments, is one of my favorite aspects of my job. I feel like I’m creating my very own jigsaw puzzle or elaborate LEGO set.” Along with working on the ICESAT-2 mission, Sheila Wall has also worked on the Lunar Reconnaissance Orbiter mission and the Lucy mission to five Jupiter Trojans.

Name: .... Date: .... Period: ....

Activity 2 Writing Scenarios

You will write a scenario about one of these expressions. Choose your expression and circle it. Do not share which expression you chose with your partner!

$-3.5 - 2.8$	$2.8 - (-3.5)$	$\lvert -3.5 - 2.8 \rvert$	$-3.5 + 2.8$

1. Write a real-world scenario that can be represented by the expression. Be creative!

Stop here and wait for your partner to finish. Cover the top of your paper so that your partner cannot see which expression you chose, but can read your scenario. Once you are both ready, swap pages with your partner, and respond to Problems 2 and 3 on *their paper*.

2. Which expression matches your partner's scenario? Circle the expression.

$-3.5 - 2.8$	$2.8 - (-3.5)$	$\lvert -3.5 - 2.8 \rvert$	$-3.5 + 2.8$

3. Uncover the top of your paper. Does the expression you chose match the expression your partner chose? If not, determine where the misunderstanding occurred.

Plan ahead: How will you show respect towards your group members as they share their strategies?

Activity 3 Different Methods

Tyler evaluated the expression $-4 + 5 - 6 + (-9) - (-3) + 11$ and determined that it was equal to 0.

1. Without discussing with your group, verify Tyler's value.

2. Compare your method of evaluating the expression with your group.

   a. Do you all agree with Tyler? If not, check each person's work.

   b. Did you all approach the problem the same way? If not, what were some differences?

   c. Did anyone in your group use a method that you may want to try in a similar problem? Explain the method.

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You solidified your understanding of adding and subtracting with rational values. You reasoned about which operation is most appropriate to represent a variety of real-world scenarios and applied your understanding of addition and subtraction to reason about whether sums or differences would be positive or negative prior to completing any computations. You then applied all of your strategies for addition and subtraction to simplify expressions involving both operations, noting that there are multiple paths to the same solution.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A fish dives $2\frac{1}{3}$ ft. If its initial location was $5\frac{1}{5}$ ft below sea level. What is its current location? Show your thinking.

2. Determine whether each expression has a positive or negative sum or difference without completing any calculations. Explain your thinking for each expression.

Expression	Positive	Negative
$\lvert -12.854 + 89.998\rvert$	☐	☐
Explain your thinking.		
$13.11 - 15.83$	☐	☐
Explain your thinking.		
$-12\frac{4}{5} - \left(-2\frac{1}{3}\right)$	☐	☐
Explain your thinking.		

3. Bard's work for simplifying $-4 - (-3) + 8$ is shown. Explain why Bard is incorrect, and then simplify the expression correctly.

I know that addition comes before
subtraction in the order of operations,
so, $-4 - (-3) + 8 = -4 - 5$
because $(-3) + 8 = 5$.
$-4 - 5 = -9$, so
$-4 - (-3) + 8 = -9$.

Name: .... Date: .... Period: ....

4. Determine the solution to each problem. Show your thinking.

- **a** How much higher is 500 m than 400 m?
- **b** How much higher is 500 m than -400 m?
- **c** What is the change in elevation from $3{,}400$ m and $8{,}500$ m?
- **d** What is the change in elevation from -300 m and $8{,}500$ m?
- **e** How much higher is -200 m than -450 m?

5. Tyler orders a meal that costs \$15.

- **a** If the tax rate is 6.6%, how much will the sales tax be on Tyler's meal? Show your thinking.
- **b** Tyler also wants to leave a tip for the server. How much do you think he should pay in all? Explain your thinking.

6. A truck is traveling a constant rate down a section of the highway. It took 8 seconds to travel 704 ft. Show your thinking for each part.

- **a** What is the speed of the truck in feet per second?
- **b** How far will the truck travel in 5 seconds?
- **c** If the truck traveled $5{,}280$ ft (1 mile), how long would that take?

My Notes:

SUB-UNIT

2 Multiplying and Dividing Rational Numbers

Narrative Connections

How many Mt. Everests can a grandma climb between Georgia and Maine?

Emma Gatewood stood atop Mt. Oglethorpe. The Georgia wilderness lay before the 67-year-old great-grandmother — hundreds of mountains, trees, rivers, and streams. She had told her family back in Ohio that she was going for a walk. What she didn't tell them was that the walk was going to cover 2,168 miles along the length of the Appalachian Trail.

The Appalachian Trail follows the Appalachian Mountains across nearly a dozen states. Established in 1937, the trail was a place anyone could go to reconnect with nature. The trail linked work, study, and farming camps that were set up along the mountain range. Every year, "thru-hikers" like Gatewood take up the challenge of hiking its total length.

In the summer of 1955, equipped with barely any supplies and a pair of tennis shoes, Gatewood set to conquer the trail. Over the course of a season, she faced rocky passes, dizzying heights, floods, storms, and rattlesnakes. She ate off the land and slept rough in improvised shelters. Soon, newspapers caught wind of her story, and strangers started offering her food and a place to stay. Finally, after 146 days, across 14 states, she emerged from the trail at Maine's Mount Katahdin.

By the time her trip was done, she had trekked the equivalent of climbing up and down Mt. Everest 16 times. Not only was she the oldest hiker to thru-hike the trail, she was also the first woman. When reporters asked her why she set out on this task, she simply answered, "For the heck of it!"

To determine that Gatewood hiked the equivalent of 16 times the height of Mt. Everest (up and down), we need to understand how to multiply and divide rational numbers.

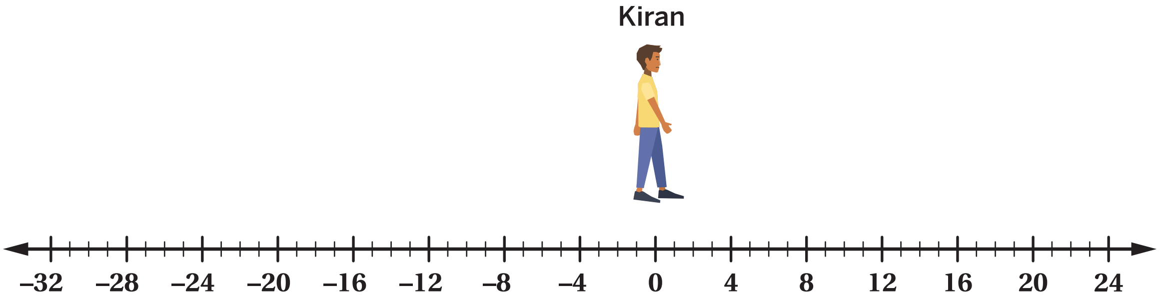
Unit 5 | Lesson 10

Position, Speed, and Time

Let’s use rational numbers to represent time and movement.

Warm-up Before and After

Kiran is walking at a constant speed of 4 ft/second. His position at this moment is indicated by 0 on the number line shown.



For each description, identify and mark his location on the number line. Be prepared to explain your thinking.

- **a** Where will Kiran be in 1 second?
- **b** Where will Kiran be in 5 seconds?
- **c** Where was Kiran 1 second ago?
- **d** Where was Kiran 5 seconds ago?

Compare and Connect:
What do you notice about the locations on the number line that represent Kiran’s position *before* his current position? *After* his current position?

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Backward and Forward in Time

The Appalachian Trail runs from Georgia to Maine, and it is the longest hiking-only footpath in the world. One of the most scenic sections of the path is Max Patch in North Carolina. A photographer has set up a camera to capture the vista in different types of light. Hikers pass the camera throughout the day while it is set up on the trail.

Russell_Martin/Shutterstock.com

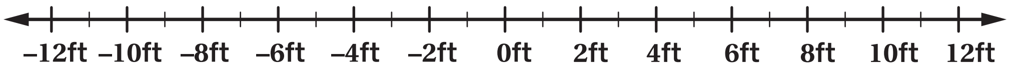
1. Here are some positions and times for one of the hikers:

Time (s)	−3	−2	−1	0	1	2
Position (ft)				0	4	8

a Where was the hiker at time 0?

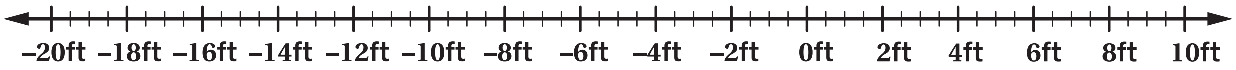
b What is the speed of the hiker?

c Complete the table. Use the number line to help you.



2. Use the number line to help you to complete the table to determine position of each hiker if the hiker is traveling at a constant speed for the indicated time period.

Speed (ft/second)	Time (seconds)	Expression describing ending position	Ending position
3	−3	$3 \bullet (-3)$	−9
2	3		
3.5	−2		
4.5	−4		



3. What do you notice about the relationship between the expressions and the sign of the ending position?

Activity 2 Multiplication or Addition?

1. Complete the missing expressions and values in the table.

Expression as a product	Expression as a sum	Value of the expressions
$4 \cdot (-3)$	$(-3) + (-3) + (-3) + (-3)$	-12
$5 \cdot (-2)$		
	$\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)$ $+ \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)$	
$3 \cdot 0.8$		
	$(-1.6) + (-1.6)$	
$5 \cdot (-1)$		
		-24

2. Complete each sentence. Be prepared to explain your thinking.

a The sign of a positive value multiplied by a positive value is always ______________ .

b The sign of a positive value multiplied by a negative values is always ______________ .

STOP

Name: ______________________ Date: __________ Period: ________

Summary

In today's lesson . . .

You saw that multiplying rational numbers is very similar to multiplying positive numbers. In fact, the product of a positive and a negative, is the same as the product of two positive numbers with a change of sign.

To determine the product of a positive number and a negative number, first determine the product of the absolute value of the two numbers, and then make the product negative.

For example, to determine the product of $2 \cdot \left(-\frac{3}{2}\right)$, think about the product of $2 \cdot \frac{3}{2}$.

$2 \cdot \frac{3}{2} = 3$

So, $2 \cdot \left(-\frac{3}{2}\right) = -3$.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Match each multiplication expression with its equivalent addition expression.

a	$5 \cdot (-2)$	....	$-2 + (-2) + (-2) + (-2) + (-2)$
b	$6 \cdot (-1)$	....	$-5 + (-5)$
c	$3 \cdot (-8)$	....	$-1 + (-1) + (-1) + (-1) + (-1) + (-1)$
d	$1 \cdot (-6)$	....	-6
e	$2 \cdot (-5)$	....	$-3 + (-3) + (-3) + (-3) + (-3) + (-3) + (-3) + (-3)$
f	$8 \cdot (-3)$	....	$-8 + (-8) + (-8)$

2. Determine whether each equation is *true* or *false*, and place a mark in the appropriate box. If the equation is false, change one value of the equation to make it true, and write the altered equation on the line.

		True 👍	**False** 👎	**Altered Equation**
a	$3 \cdot (-6) = -18$	☐	☐	....
b	$5 \cdot (-2) = 10$	☐	☐	....
c	$\frac{2}{3} \cdot (-3) = -2$	☐	☐	....
d	$1 \cdot (-0.03) = 0.03$	☐	☐	....

3. Shawn and Jada are walking in the same direction on a path through the park. They walk by the fountain on the side of the path at the same time. When they pass the fountain, Jada is walking at a speed of 5 ft/second and Shawn is walking at a speed of 3.5 ft/second.

a If 0 represents the location of the fountain, what values represent the location of each person in 5 seconds? Show or explain your thinking.

b 8 seconds before arriving at the fountain, how many feet in front of Jada was Shawn? Show or explain your thinking.

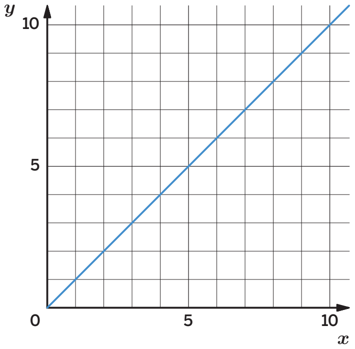
Name: .... Date: .... Period: ....

4. For each equation, write two more equations using the same numbers that express the same relationship in a different way.

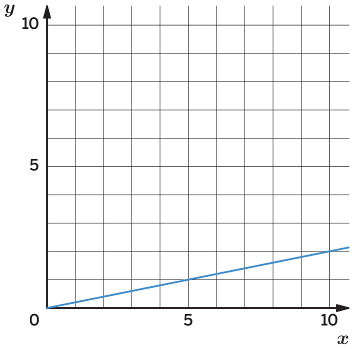
Original equation	Equation 1	Equation 2
$3 + 2 = 5$		
$7.1 + 3.4 = 10.5$		
$15 - 8 = 7$		
$\frac{3}{2} + \frac{9}{5} = \frac{33}{10}$		

5. Which of the following graphs could *not* represent a proportional relationship? Select *all* that apply. Explain your thinking.

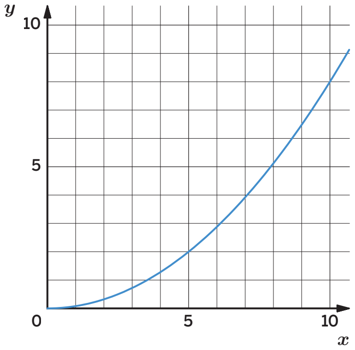
A.



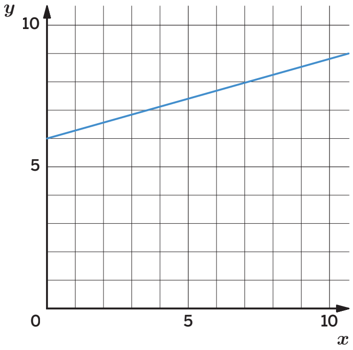
C.



B.



D.



6. An elevator in a building with multiple levels of parking under it, travels at a speed of 10 ft/second. If it is currently on the ground floor (0 ft), and is traveling up and not making stops, determine its location at each given time.

a After 5 seconds have passed.

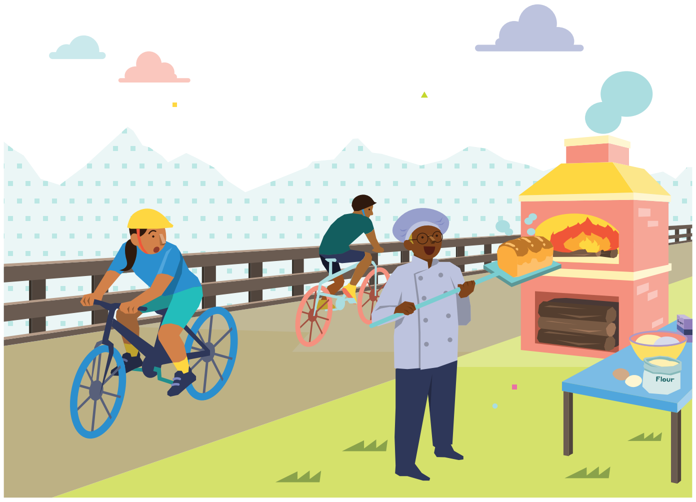
b After 12 seconds have passed.

c 4 seconds before it made it to the ground floor.

Unit 5 | Lesson 11

Multiplying Rational Numbers

Let's multiply rational numbers.



Warm-up Going Up or Down?

An elevator in a certain building travels at an average speed of 20 ft per second.

If the manager at a hotel wants to be able to predict the location of the elevator after t seconds, how could she use mathematics to model the difference between the elevator going up and the elevator going down?

Log in to Amplify Math to complete this lesson online.

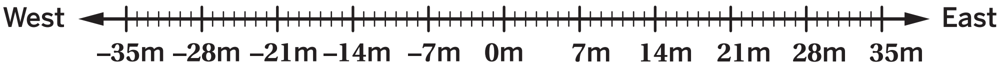
Name: .... Date: .... Period: ....

Activity 1 Velocity and Time

The longest recreational trail in the world (open for hikers, bikers, horseback riders, and snowmobilers) is 14,913 miles long. It stretches from the Pacific to the Atlantic Ocean, across 13 provinces in Canada, and is called The Great Trail. A section of the trail, named the Kettle Valley Rail Trail, was built on a 1915 railway and has the largest collection of stone ovens in North America, built by masons during the construction of the original railway.

Anna Dunlop/Shutterstock.com

1. Two bikers pass one of the stone ovens at the same time. Bicycle A is traveling east at a speed 7 m/second, while Bicycle B is traveling west at a speed of 7 m/second. Use the number line shown to help you.



a What is the *velocity* of Bicycle A?

b Use the number line to help you complete the table for the location of Bicycle A at the given times.

Time (seconds)	–3	–2	–1	0	1	2
Position (m)				0		

c What is the *velocity* of Bicycle B?

d Use the number line to help you complete the table for the location of Bicycle B at the given times.

Time (seconds)	–3	–2	–1	0	1	2
Position (m)				0		

2. What is the relationship between the location of Bicycle A and Bicycle B at any time?

Activity 1 Velocity and Time (continued)

3. Complete the table for several different bikers passing the oven.

	Velocity (m/second)	Time (s)	Expression	Position (m)
Biker A	5	10	$5 \cdot 10$	50
Biker B	–2	30		
Biker C	3	–40		
Biker D	–10	–20		
Biker E	–1.5	–8		

4. Complete each sentence. Be prepared to explain your thinking.

a The sign of a positive number multiplied by a positive number is ________ .

b The sign of a positive number multiplied by a negative number is ________ .

c The sign of a negative number multiplied by a positive number is ________ .

d The sign of a negative number multiplied by a negative number is ________ .

Name: ________________ Date: ________ Period: ________

Activity 2 Distributing With Negatives

Analyze both methods for evaluating each expression. Use what you know about multiplying rational numbers to determine each missing value.

1. Study the work shown.

Order of operations	Distributive Property
$(14 + (-4)) \cdot (-3) = (10) \cdot (-3)$ $= -30$	$(14 + (-4)) \cdot (-3) = 14 \cdot (-3) + (-4) \cdot (-3)$ $= -42 + \boxed{}$

a What is the sign of the missing value? Explain your thinking.

b What must the product of -4 and -3 be to make the last line true?

2. Study the work shown.

Order of operations	Distributive Property
$-4 \cdot (-3 + 2) = -4 \cdot (-1)$ $= 4$	$-4 \cdot (-3 + 2) = -4 \cdot (-3) + (-4) \cdot (2)$ $= 12 + \boxed{}$

c What is the sign of the missing value? Explain your thinking.

d What must the product of -4 and 2 be to make the last line true?

3. Simplify the expression $(-6 + 16) \cdot (-10)$ using the Distributive Property. Check your response by using the order of operations.

STOP

Summary

In today's lesson . . .

You noticed that multiplying rational numbers, no matter the order, is very similar to multiplying positive numbers. You formulated the following rules:

- The product of two numbers with the same sign is positive.
- The product of two numbers with different signs is negative.

Once you have determined the sign of the product, multiply the magnitudes of the numbers as you would when multiplying two positive numbers. These rules work for all rational numbers, including integers and rational numbers.

Reflect:

Name: .... Date: .... Period: ....

1. Determine whether each expression has a positive or negative product. Place a checkmark in the appropriate column. Explain your thinking.

Expression	Positive	Negative
$4.5 \cdot (-3.085)$	☐	☐
$-\frac{1}{8} \cdot \left(\frac{-7}{53}\right)$	☐	☐
$-4.62 \cdot \frac{4}{5}$	☐	☐
$(-698) \cdot (-4.506)$	☐	☐

2. Determine the missing value in each equation.

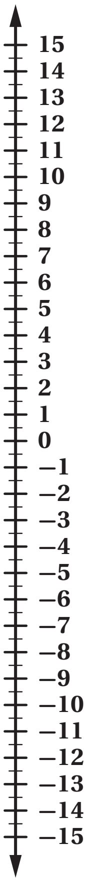
a $-2 \cdot (-4.5) = \square$

b $-8.7 \cdot (-10) = \square$

c $-7 \cdot \square = 14$

d $\square \cdot (-10) = 90$

3. A weather station reports that the temperature is currently 0°C and has been falling at a constant rate of 3°C per hour. If it continues to fall at this rate, determine each indicated temperature. Show or explain your thinking.



a What will the temperature be in 2 hours?

b What will the temperature be in 5 hours?

c What will the temperature be in a $\frac{1}{2}$ hour?

d What was the temperature 1 hour ago?

e What was the temperature 3 hours ago?

f What was the temperature 4.5 hours ago?

Name: .... Date: .... Period: ....

4. To make a specific hair dye, a hair stylist uses a ratio of $1\frac{1}{8}$ oz of red tone, $\frac{3}{4}$ oz of gray tone, and $\frac{5}{8}$ oz of brown tone.

a If the stylist needs to make 20 oz of dye, how much of each dye color is needed? Show or explain your thinking.

b If the stylist needs to make 100 oz of dye, how much of each dye color is needed? Show or explain your thinking.

5. The vertices of Rectangle $FROG$ are at coordinates $(–2, 5)$, $(–2, 1)$, $(6, 1)$, and $(6, 5)$.

a Determine the perimeter of the rectangle. Plot the points on a coordinate plane if you need help. Show or explain your thinking.

b Determine the area of Rectangle $FROG$. Show your thinking.

c The coordinates of the vertices of Rectangle $PLAY$ are $(-11, 20)$, $(-11, -3)$, $(-1, -3)$, and $(-1, 20)$. Determine the perimeter and area of this rectangle. Show or explain your thinking.

6. Evaluate each expression. Show your thinking.

a $3 \div 1.5 \bullet 2$

b $3 - 2 + 4 \bullet \frac{1}{4}$

Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 12

Multiply!

Let’s get more
practice multiplying
rational numbers.

Warm-up Math Talk

Determine whether each missing value is positive or negative. Be prepared to explain your thinking.

1. $\square \cdot 12 = -48$

2. $-3 \cdot \square = -81$

3. $-\frac{1}{3} \cdot \frac{2}{5} = \square$

4. $-6.82 \cdot \square = 3.8$

5. $-3\frac{1}{2} \cdot \left(-6\frac{2}{9}\right) = \square$

Log in to Amplify Math to complete this lesson online.

Activity 1 Greatest Product

Your group will be given a set of cards to play a game.

Players: 2–4

Goal: Produce the greatest product when the values of the cards are multiplied.

Getting ready:

- Shuffle the set of cards and place them in a pile in the middle of the group.
- Choose one player to start the game.
- Black cards represent positive values. Red cards represent negative values.
  Note: Cards printed in black and white will show the red cards as light gray.

For each round, follow these steps

Step 1 Each player selects two cards from the pile. Do not show your cards to other players.

Step 2 When all players have their cards, the first player decides whether they want to select one more card from the pile or to pass (not do anything). This continues until all players have had a turn.

Step 3 Repeat Step 2. Each player can have a maximum of four cards. Once all players have selected their cards, they calculate their score and complete the first two columns of the table.

Step 4 All players present their cards to their group and come to a consensus on the winner.

Step 5 Reshuffle cards for the next round.

	Cards	Score	Greatest product?
Round 1			☐
Round 2			☐
Round 3			☐

1. Consider the times when you decided whether to take an additional card.

a What thinking helped you to make your decision?

b If you had to change your strategy for the next round, how would you change it?

Name: ________ Date: ________ Period: ________

Activity 2 Partner Problems

With your partner, decide who will complete Column A and who will complete Column B. After each row, share your responses with your partner. Compare your responses, and discuss and resolve any differences.

Column A	Column B
$-(-0.5)$	$(-1)(-1) \bullet 0.5$
$-\frac{1}{2} \bullet 12 \bullet \left(-\frac{2}{3}\right)$	$\frac{1}{2} \bullet (-12) \bullet \left(-\frac{2}{3}\right)$
$-8 \bullet (-4) \bullet \left(-\frac{1}{2}\right)$	$-\frac{1}{2} \bullet (-8) \bullet (-4)$
$\left(\frac{2}{3} \bullet (-3)\right) \bullet (-6)$	$\frac{2}{3} \bullet (-3 \bullet (-6))$

Are you ready for more?

Continue the Partner Problem routine. Explain why each pair of expressions are equivalent.

Column A	Column B
$-2\,(0.35 + 2.15)$	$2(-0.35 + (-2.15))$
$-\frac{1}{4}(6 - 2)$	$(2 - 6) \bullet \frac{1}{4}$

STOP

Summary

In today's lesson . . .

You reasoned that rules for multiplying rational numbers extend beyond integer values to all rational numbers. In general, for any pair of rational numbers, if the two numbers have the same sign their product is positive, and if the two numbers have different signs, their product is negative.

This rule extends to the product of more than two rational numbers.

- If the number of negative factors is even, then the product is positive.
- If the number of negative factors is odd, then the product is negative.

Reflect:

Name: .... Date: .... Period: ....

1. Evaluate each expression.

a $-12 \cdot \frac{1}{3}$

b $-12 \cdot \left(-\frac{1}{3}\right)$

c $12 \cdot \left(-\frac{5}{4}\right)$

d $-12 \cdot \left(-\frac{5}{4}\right)$

2. Evaluate each expression.

a $-1 \cdot 2 \cdot 3 \cdot \frac{1}{6}$

b $-1 \cdot (-2) \cdot 3 \cdot \frac{1}{6}$

c $-1 \cdot (-2) \cdot 3 \cdot \left(-\frac{1}{6}\right)$

d $-1 \cdot (-2) \cdot (-3) \cdot \frac{1}{6}$

e $-(-2) \cdot (-3) \cdot \left(-\frac{1}{6}\right)$

3. Match each expression with an equivalent expression.

a $3 \cdot (-4)$

b $-1 \cdot (-1) \cdot 5$

c $-(-(-3))$

d $(-1 \cdot (-3)) \cdot (2 \cdot 2)$

e $-\frac{1}{3} \cdot (-12) \cdot \left(\frac{3}{4}\right)$

f $-1 \cdot 5$

.... $-1 \cdot (-3 \cdot 2) \cdot 2$

.... $-(-(-5))$

.... $6 \cdot 6 \cdot \left(-\frac{1}{3}\right)$

.... $-15 \cdot (-1) \cdot \left(\frac{1}{3}\right)$

.... $-12 \cdot \left(-\frac{1}{3}\right) \cdot \left(-\frac{3}{4}\right)$

.... $\left(-\frac{2}{3}\right) \cdot (-3) \cdot \left(\frac{3}{2}\right)$

Practice

Name: .... Date: .... Period: ....

4. Consider the equation $30 + (-30) = 0$.

a Write another sum of two numbers that equals 0.

b Write a sum of three numbers that equals 0.

c Write a sum of four numbers that equals 0, none of which are opposites.

5. Clare and Han are both riding bikes and pass each other on a trail.

a Clare is cycling at a velocity of 12 miles per hour. If the position when she passes Han is 0, what will her position be after 45 minutes? Show or explain your thinking.

b Han is cycling at a velocity of -8 miles per hour. If the position when he passes Clare is 0, what will his position be after 45 minutes?

6. Determine each quotient.

a $8 \div 24$

b $8 \div \frac{1}{3}$

c $1\frac{1}{4} \div \frac{1}{2}$

Name: ______________________ Date: __________ Period: __________

Unit 5 | Lesson 13

Dividing Rational Numbers

Let's divide rational numbers.

Warm-up Thinking About the Sign

For each equation, determine which number, A or B, makes the equation true. Circle the correct number and be prepared to share your thinking.

Equation	Number A	Number B
$12x = -36$	3	-3
$-5x = -35$	7	-7
$6x = 48$	8	-8
$-8x = 72$	9	-9

Log in to Amplify Math to complete this lesson online.

Activity 1 Equation Families

Refer to this fact family of multiplication and division equations.

$3 \bullet 4 = 12$ $4 \bullet 3 = 12$ $12 \div 3 = 4$ $12 \div 4 = 3$

1. Complete the missing parts of the equation in each fact family.

a $-3 \bullet 4 = -12$ $4 \bullet \square = -12$ $-12 \div \square = 4$ $\square \div \square = -3$

b $\square \bullet (-3) = -9$ $3 \bullet \square = -9$ $\square \div 3 = -3$ $-9 \div \square = \square$

c $\square \bullet \square = 42$ $-7 \bullet \square = 42$ $42 \div \square = -6$ $\square \div \square = \square$

2. Create a different fact family of multiplication and division equations, using at least one negative number.

3. Complete each sentence. Be prepared to explain your reasoning.

a The sign of a positive number divided by a positive number is ....

b The sign of a positive number divided by a negative number is ....

c The sign of a negative number divided by a positive number is ....

d The sign of a negative number divided by a negative number is ....

Name: ________ Date: ________ Period: ________

Activity 2 How Close Can You Get?

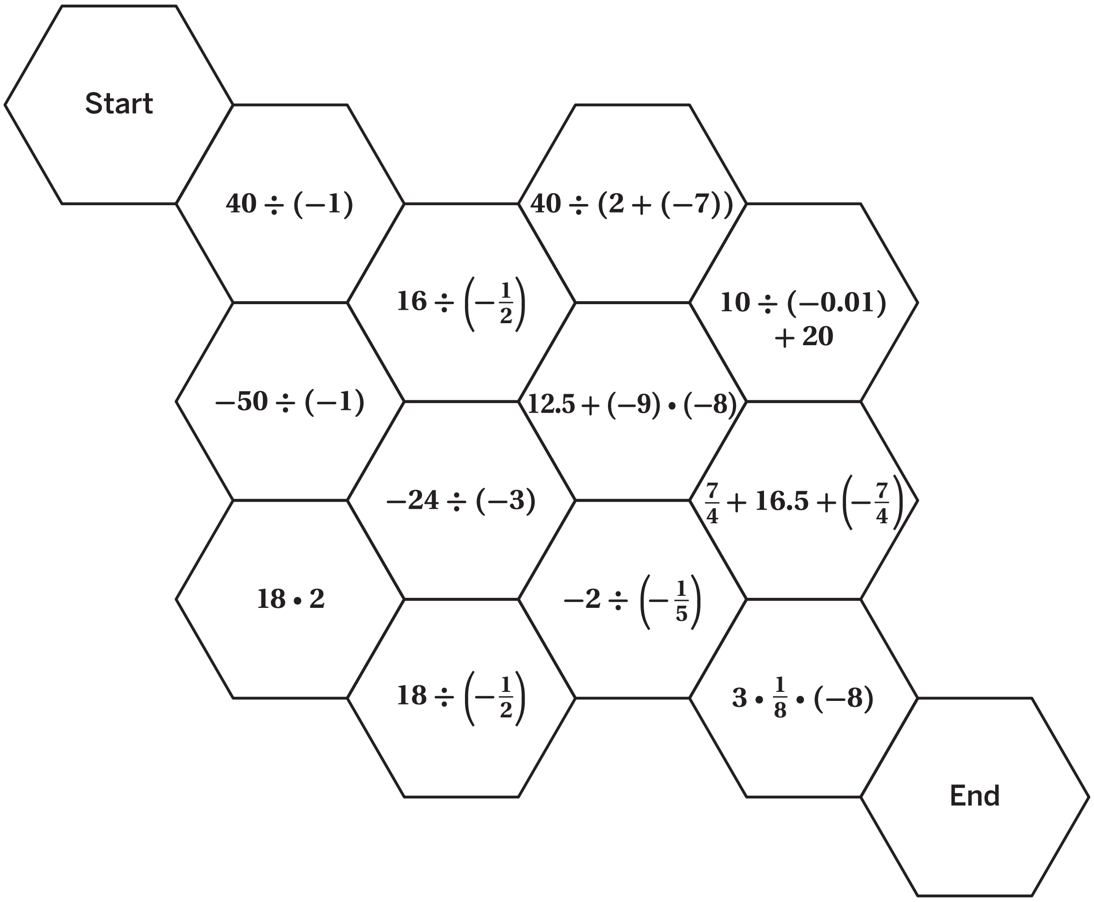
Your goal is to reach the End hexagon of the game board with a score as close to the target score as possible.

Write your target score here. **Target score:** ________

Beginning at the start, move from one hexagon to an adjacent hexagon. Evaluate the expression. The value you produce is your score for that hexagon. As you move, add your scores together to determine your final score.

Rules:

- You can only move to an adjacent hexagon.
- You may not move back to a hexagon you have already visited.



Final score:

STOP

Summary

In today's lesson . . .

You saw that every multiplication equation belongs to a family of equations that includes a division equation. Because of this, every multiplication problem can be rewritten as a division problem:

$6 \div 2 = 3$ because $2 \cdot 3 = 6$.

$-6 \div 2 = -3$ because $2 \cdot (-3) = -6$.

$6 \div (-2) = -3$ because $-2 \cdot (-3) = 6$.

$-6 \div (-2) = 3$ because $-2 \cdot 3 = -6$.

Because you know how to reason about signs when multiplying rational numbers, you also know about the signs when dividing them.

- The sign of the quotient of a positive number divided by a negative number is always negative.
- The sign of the quotient of a negative number divided by a positive number is always negative.
- The sign of the quotient of a negative number divided by a negative number is always positive.

Once you have determined the sign of the quotient, divide the magnitudes of the numbers as you would when dividing two positive numbers.

Reflect:

Name: .... Date: .... Period: ....

1. Determine whether the solution of each equation is positive or negative.

a $2 \cdot x = 6$

b $-2 \cdot x = 6.1$

c $2.9 \cdot x = -6.04$

d $-2.473 \cdot x = -6.859$

e $-2\frac{4}{7} \div x = 6\frac{2}{3}$

f $x \div 0.2 = -0.6$

2. Determine each quotient.

a $24 \div (-6)$

b $-15 \div (0.3)$

c $-4 \div (-20)$

d $0.015 \div (-\,0.3)$

e $\frac{2}{5} \div \frac{3}{4}$

f $\frac{9}{4} \div \left(-\frac{3}{4}\right)$

g $-\frac{5}{7} \div \left(-\frac{1}{3}\right)$

h $-\frac{5}{3} \div \frac{1}{6}$

3. Evaluate each expression.

a $8.61 + 7.39 + (-\,7.39) + 7.39$

b $6 \cdot \left(-\frac{1}{6}\right) \div \left(-\frac{1}{10}\right)$

4. In order to make a specific shade of green paint, a painter mixes $1\frac{1}{2}$ quarts of blue paint, 2 cups of green paint, and $\frac{1}{2}$ gallon of white paint. How much of each color is needed to make 100 cups of this shade of green paint?

Practice

Name: .... Date: .... Period: ....

5. The table shows the highest and lowest elevations on each continent. Complete the table to determine the difference between the highest and lowest elevations for each continent.

	Highest point (m)	Lowest point (m)	Difference
Africa	5,895	-155	
Antarctica	4,892	-50	
Asia	8,848	-427	
Australia	4,884	-15	
Europe	4,810	-28	
North America	6,198	-86	
South America	6,960	-105	

Which continent has the greatest difference in elevation? The least?

6. Place a checkmark in the appropriate column that describes the solution for each scenario.

Scenario	Less than 100	Exactly 100	More than 100
A car travels for 2 hours at 40 mph. How many miles did it travel?	☐	☐	☐
An employee earns $15 per hour and works 8 hours. How much money did they earn?	☐	☐	☐
A sprinter runs 100 m in 9.8 seconds. How fast were they running, in meters per second?	☐	☐	☐

Name: .... Date: .... Period: ....

Unit 5 | Lesson 14

Negative Rates

Let's apply what we know about rational numbers.

Warm-up Water Consumption

Complete each sentence.

1. If a laundry machine used 9 gallons of water per load for 3 loads of laundry, it will have used .... for 3 loads.

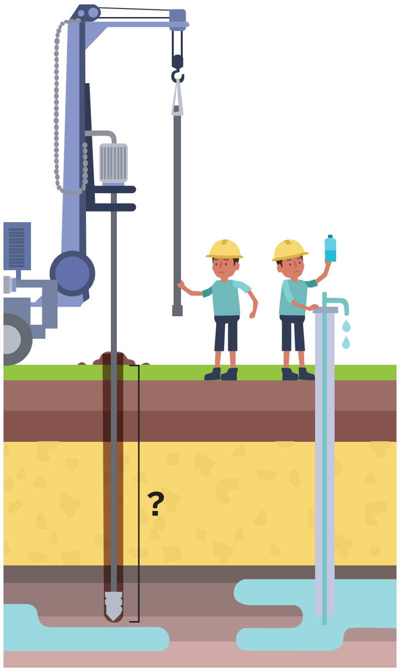
2. If you drink .... of water per day for 8 days, you will drink .... altogether.

3. If you drink $\frac{1}{2}$ gallon of water per day, it will take you .... to drink 5 gallons of water.

Log in to Amplify Math to complete this lesson online.

Activity 1 Drilling a Well

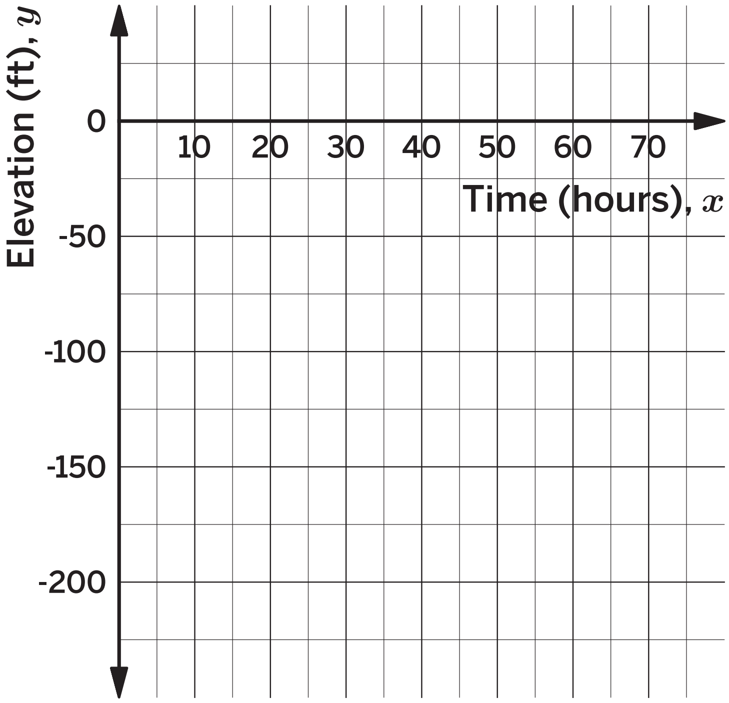
A water well drilling rig has dug to an elevation of -20 ft after 8 hours of continuous use.



1. Assuming the rig drilled at a constant rate, what was the elevation of the drill after 5 hours?

2. If the rig has been running constantly and is currently at an elevation of -147.5 ft, for how long has the rig been running?

3. Plot and label the points, relating time in hours to the height of the drill, from your solutions to Problems 1 and 2 on the graph shown. Draw a line through the points.



Name: .... Date: .... Period: ....

Activity 2 Diving With the Ama

The *ama* are Japanese female divers who free dive — without using oxygen tanks — to collect food and pearls from the bottom of the cold sea. Ama typically dive to 30 ft below the surface to reach the seafloor.

Nakarin_thailand/iStock

1. An ama dives at a rate of -1.9 ft per second.

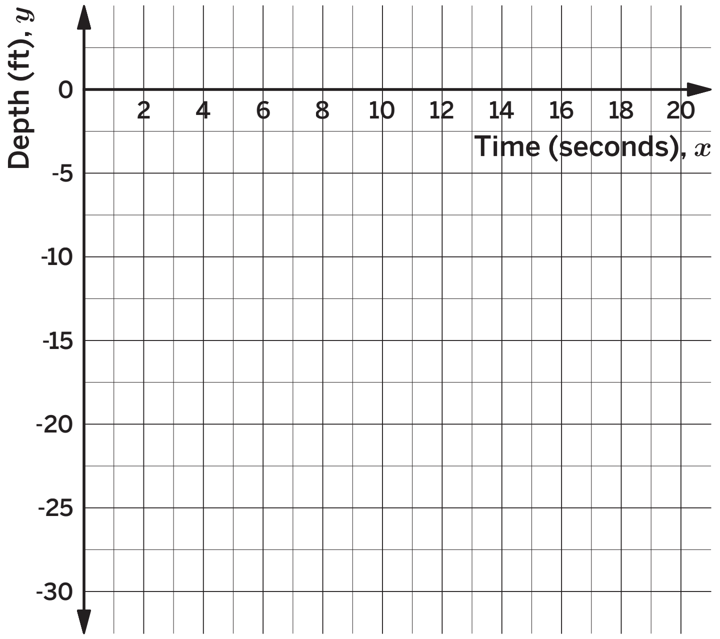
a Complete the table to find the depths of the ama at different times.

Time (seconds), x	0	1	10	15	x
Depth (ft), y					

b Write an equation to model the relationship for the depth in feet y of the ama, if you know the time in seconds, x.

Activity 2 Diving With the Ama (continued)

c Plot and label the depth of the ama on the graph for 3 different times. Draw a line through the points.



d How long did it take the ama to reach the seafloor, located at -30 ft? Show or explain your thinking.

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that you can have a negative rate of change when you need to describe a relationship where something is decreasing. You can probably already think of several contexts where a negative rate might be useful. In this lesson, both drilling a well and diving to the seafloor illustrated a decreasing height.

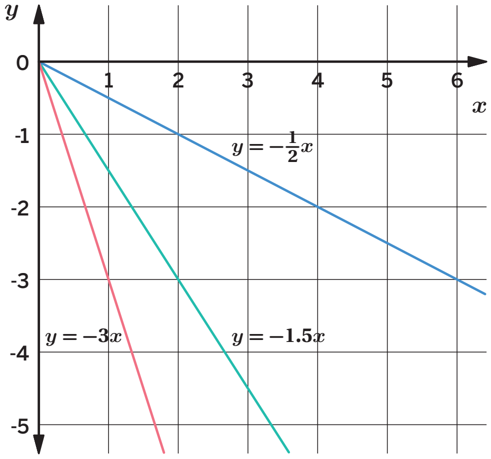
When writing an equation for a negative rate in the form of $y = kx$, the rate k will be a negative number. These equations are all examples of relationships with negative rates:

$y = -3x$

$y = -\frac{1}{2}x$

$y = -1.5x$

The graph of a relationship with a negative constant rate will be a line that slopes downward as you read the coordinate plane from left to right.



Reflect:

Practice

Name: .... Date: .... Period: ....

1. Describe a real-world situation where each of the following quantities might be used to describe a rate of change.

   a -20 gallons per hour

   b -10 feet per minute

   c -0.1 kilograms per minute

2. A submarine is only allowed to change its depth by rising toward the surface in increments of 60 m. It starts off at -340 m.

   a At what depth is it after:

   1 stage? 2 stages? 3 stages?

   b How many stages will it take to return to the surface?

3. A submarine was testing its buoyancy system and was diving and rising below the surface of the ocean.

   a The submarine was traveling -3.4 m per minute for 7.5 minutes. How far did it go?

   b The submarine traveled -1.5 m in 0.3 minutes. What was its velocity?

   c What do you think that negative values for distance and velocity could mean in this situation?

Name: ____________ Date: ____________ Period: ____________

4. Evaluate each expression.

a $-9.2 + (-7)$

b $-4\frac{3}{8} - \left(-1\frac{1}{4}\right)$

c $-24 \cdot \left(-\frac{7}{6}\right)$

d $-\frac{4}{3} \div (-24)$

5. A shopper bought a watermelon, a pack of napkins, and some paper plates. In his state, there is no tax on food. The tax rate on non-food items is 5%. The total for the three items he bought was \$8.25 before tax, and he paid \$0.19 in tax. How much did the watermelon cost? Show or example your thinking.

6. Evaluate each expression.

a $1 \cdot \frac{1}{4}$

b $1 \div 4$

c $1 \div \frac{1}{4}$

d $4 \div \frac{1}{4}$

My Notes:

SUB-UNIT

3 Four Operations With Rational Numbers

Narrative Connections

How do you climb the world’s most dangerous mountain?

For decades, no one thought you *could* climb Mount Everest. Let alone the fact that its summit is more than 29,000 ft above sea level. And forget the freezing air of its infamous “Death Zone,” where just a moment of exposed skin can end in frostbite. No, the real problem is pressure.

The air pressure at Everest’s summit is about $\frac{1}{3}$ what it is at sea level. That means our lungs have to work much harder to breathe. Oxygen tanks help, but climbers must also gradually let their lungs adjust to the difference in pressure. It’s the same concept scuba divers use to reach extreme ocean depths. But to adjust properly, you must spend extra days on the mountain. And every moment in the “Death Zone,” where the weather can change quickly, is a moment spent in danger.

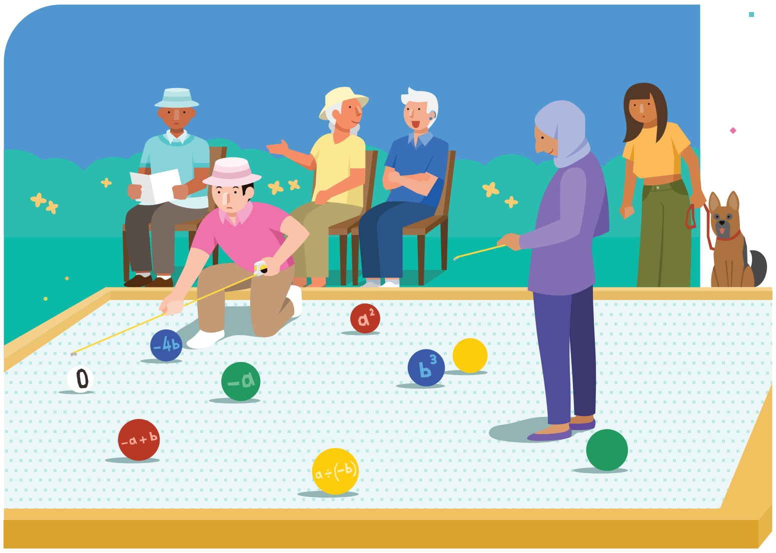
In other words, to succeed in climbing Everest, you must be very good at making decisions and understanding tradeoffs. One day of rest to acclimatize may save you some oxygen, but it will cost you time. And if you fill your pack with spare oxygen, you cannot fill it with warm clothes to protect you from the air. Every part of the preparation has to be kept carefully in balance.

Rational numbers, both positive and negative, are a powerful tool for modeling these difficult decisions climbers must make. They are some of the tools climbers use to help ascend straight through the Death Zone and all the way to the summit.

Unit 5 | Lesson 15

Expressions With Rational Numbers

Let's develop our number sense with rational numbers.



Warm-up True or False?

Decide whether each statement is true or false, and then place a checkmark in the appropriate box. Be prepared to explain your thinking.

	True	False
$(-38.76)\ (-15.6) < 0$	☐	☐
$10{,}000 - 99{,}999 < 0$	☐	☐
$\left(\frac{3}{4}\right)\left(-\frac{4}{3}\right) = 0$	☐	☐
$30(-80) - 50 = 50 - 30(-80)$	☐	☐
$-\frac{1}{2} = \frac{-1}{2} = \frac{1}{-2} = -\left(\frac{1}{2}\right)$	☐	☐

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Card Sort: The Same, but Different

You will be given a set of cards. Group them into pairs of expressions that have the same value.

1. Write the pairs of matching expressions in the table so that each row contains a matching pair of expressions.

2. Write an expression that is equivalent to each expression.

a $-\frac{1}{3} \div (-7)$

b $-1.91 - (-1.91)$

c $-\frac{1}{4} \cdot \left(-\frac{1}{4}\right)$

Reflect: How did your confidence level about equivalent expressions change throughout this activity?

Activity 2 Near and Far From Zero

1. For each set of values for a and b, evaluate the given expressions — on a separate piece of paper when necessary — and record your solutions in the table.

a	b	$-a$	$-4b$	$-a+b$	$a \div (-b)$	a^2	b^3
$-\frac{1}{2}$	6						
$\frac{1}{2}$	-6						
-6	$-\frac{1}{2}$						

2. When $a = -\frac{1}{2}$ and $b = 6$, which expression:

a Has the greatest value?

b Has the least value?

c Is the closest to zero?

3. When $a = \frac{1}{2}$ and $b = -6$, which expression:

a Has the greatest value?

b Has the least value?

c Is the closest to zero?

4. When $a{-}6$ and $b = -\frac{1}{2}$, which expression:

a Has the greatest value?

b Has the least value?

c Is the closest to zero?

Are you ready for more?

Are there any values could you use for a and b that would make all of these expressions have the same value? Explain your reasoning.

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You saw that making sense of rational numbers requires some flexible thinking. For instance, the value of the number $-x$ is not always negative! If the value of x is $-3\frac{7}{8}$, then:

$$-x = -\left(-3\frac{7}{8}\right) = 3\frac{7}{8}$$

When working with rational numbers, reasoning about the signs is one of the most important considerations. There are some generalizations you have been able to make and can continue to use:

- When adding rational numbers, the sum will have the same sign as the addend with the greater absolute value.
- Subtracting a number is the same as adding its opposite, or additive inverse.
    - An even amount of negative numbers will give a positive result.
    - An odd amount of negative numbers will give a negative result.
- When multiplying or dividing rational numbers.
- Dividing by a number is the same as multiplying by that number's reciprocal, or multiplicative inverse.

Reflect:

Name: .... Date: .... Period: ....

1. Evaluate each expression.

a $-22 + 5$

b $-22 - (-5)$

c $(-22)(-5)$

d $-22 \div 5$

2. Evaluate each expression when x is $\frac{2}{5}$, y is -4, and z is -0.2.

a $x + y$

b $2x - z$

c $x + y + z$

d $y \bullet z$

3. Order the expressions from least to greatest based on their values when x is $-\frac{1}{4}$.

x $\quad$ $1 - x$ $\quad$ $x - 1$ $\quad$ $-1 \div x$

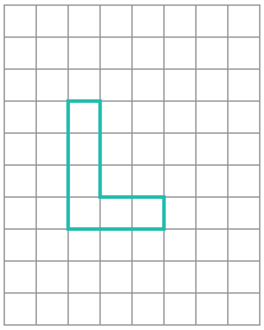
Least **Greatest**

Name: ______________________ Date: __________ Period: __________

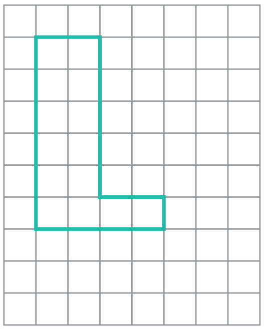
4. The price of an ice cream cone is $\$3.25$, but it costs $\$3.51$ with tax. What is the sales tax rate?

5. Which is a scaled copy of Polygon A? Identify a pair of corresponding sides and a pair of corresponding angles. Compare the areas of the scaled copies.

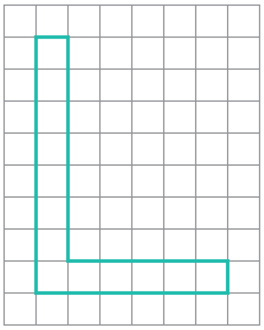
Polygon A



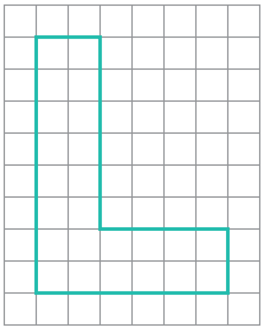
Polygon B



Polygon C



Polygon D



6. Use long division divide each of the following. Show your thinking.

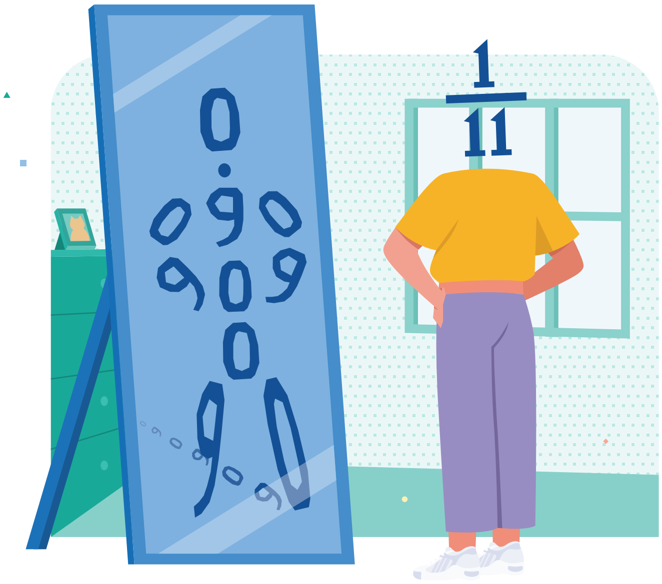
a Divide 496 by 4.

b Determine the quotient of 3.8 and 0.004.

Unit 5 | Lesson 16

Say It With Decimals

Let's represent fractions with decimals.



Warm-up Notice and Wonder

A calculator gives the following decimal representations for some unit fractions.

$\frac{1}{2} = 0.5$

$\frac{1}{3} = 0.333333333$

$\frac{1}{4} = 0.25$

$\frac{1}{5} = 0.2$

$\frac{1}{6} = 0.166666667$

$\frac{1}{7} = 0.142857143$

$\frac{1}{8} = 0.125$

$\frac{1}{9} = 0.111111111$

$\frac{1}{10} = 0.1$

$\frac{1}{11} = 0.090909091$

What do you notice? What do you wonder?

1. I notice . . .

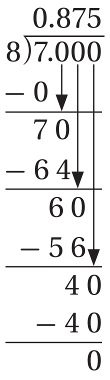
2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Repeating Decimals

Let's review how to write $\frac{7}{8}$ as a decimal using long division.



$\frac{7}{8} = 0.875$

1. Determine the decimal value of $\frac{9}{25}$ using long division.

2. Determine the decimal value of $\frac{2}{3}$ using long division.

3. What did you notice in Problem 2?

Activity 2 Practice With Repeating Decimals

Use long division to express each fraction as a decimal. If you recognize repetition, stop and write the decimal using repeating bar notation.

1. $\frac{2}{5}$

2. $\frac{4}{9}$

3. $\frac{5}{6}$

4. $\frac{4}{11}$

5. $\frac{7}{12}$

6. $\frac{3}{8}$

Are you ready for more?

As you saw in a previous lesson, $\frac{22}{7}$ is used as an approximation for π. Express this fraction as a decimal. How does this approximation compare to 3.14?

Name: .... Date: .... Period: ....

Activity 3 Complex Fractions

1. Review the two expressions shown. Do you think they have the same or different values? Explain your thinking.

a $\dfrac{2}{\frac{3}{4}}$

b $\dfrac{\frac{2}{3}}{4}$

2. Determine the value of each complex fraction. Write your response as a fraction in simplest form.

a $\dfrac{2}{\frac{3}{4}}$

b $\dfrac{\frac{2}{3}}{4}$

c $\dfrac{\frac{2}{3}}{\frac{4}{3}}$

3. Express each of your responses in Problem 2 as a decimal. Show your thinking.

a ....

b ....

c ....

STOP

Summary

In today's lesson . . .

You used long division to represent fractions as decimals. Sometimes the decimal ***terminates*** (ends) and sometimes it is a ***repeating decimal*** where the decimal is non-terminating and repeats. This can be written using ***bar notation*** over the digits which repeat or with the ellipses (. . .) at the end.

$$\frac{1}{3} = 0.333\ldots = 0.\overline{3}$$

$$\frac{14}{99} = 0.14141414\ldots = 0.\overline{14}$$

Remember to put the bar *only* over the repeating digits.

$$\frac{53}{90} = 0.5888\ldots = 0.5\overline{8}$$

Reflect:

Name: .... Date: .... Period: ....

1. Diego was writing fractions as repeating decimals. He used long division and determined $\frac{52}{225} = 0.23111111$. He wrote his answer as $0.\overline{231}$. Is he correct? Explain your thinking.

2. Write each fraction as a decimal. If you recognize repetition, stop and write the decimal using bar notation.

a $\frac{9}{11}$

b $\frac{73}{90}$

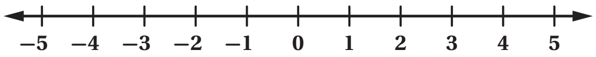
3. Determine the value of each complex fraction. Write your response as a fraction in simplest form. Show your thinking.

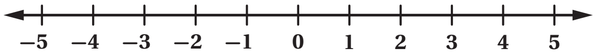
a $\dfrac{4}{\frac{8}{9}}$

b $\dfrac{\frac{4}{8}}{9}$

Name: .... Date: .... Period: ....

4. Bard says that because $4 + (-3)$ is equivalent to $(-3) + 4$, this means that $4 - (-3)$ must be equal to $(-3) - 4$. Explain why Bard is incorrect. Use the provided diagrams in your explanation, if helpful.





5. Each of the expressions shown has a value of $-\frac{1}{2}$.

$-\frac{1}{4} + \left(-\frac{1}{4}\right)$	$\frac{1}{2} - 1$	$-2 \cdot \frac{1}{4}$	$-1 \div 2$

Write five expressions of your own: a sum, a difference, a product, a quotient, and one using at least two operations that all have the value of $-\frac{3}{4}$.

6. Evaluate each expression.

a $-5 - 8$

b $-12 + (-3)$

c $-5 \cdot 8$

d $16 \div \left(-\frac{1}{2}\right)$

e $-3 + 2 \cdot (-8) \cdot (-10)$

f $-6 \div (-2) \cdot (-3)$

Name: ________ Date: ________ Period: ________

Unit 5 | Lesson 17

Solving Problems With Rational Numbers

Let's use all four operations with rational numbers to solve problems.

Warm-up Which One Doesn't Belong?

Four equations are shown. Which expression does not belong with the others? Explain your thinking.

A. $-2(3+1) = -2 \cdot 3 + (-2) \cdot 1$

B. $-2 \cdot 4 = 4 \cdot (-2)$

C. $-2 + 10 = 10 + (-2)$

D. $2 + (-2 + (-8)) = (2 + (-2)) + (-8)$

Log in to Amplify Math to complete this lesson online.

Activity 1 Energy Supply, Revisited

Bard's family decided to change their energy company and they want to better understand their new rates for energy consumption. The statement from the energy company shows their usages, charges, and credits for the first three months of the new year.

	Details	Charges ($)	Credits ($)	Balance
1/31	Consumed: 515 kwH Produced: 425 kwH	20.25		20.25
2/28	Consumed: 484 kwH Produced: 456 kwH	6.30		
3/31	Consumed: 515 kwH Produced: 533 kwH		4.05	
Totals				

1. Complete the missing information in the statement.

2. What is the new rate for the energy company in dollars per kilowatt-hours? Show your thinking.

3. During the previous year, Bard's family consumed 484 kwh of power and produced 520 kwh of power in April. Assuming they use and produce the same amount of power this year, what charges or credits should they expect on their April bill? Show your thinking.

Are you ready for more?

1. Determine the value of the expression without a calculator.
   $2(-30) + (-3)(-20) + (-6)(-10) - 2 \cdot 3 \cdot 10$

2. Write an equivalent expression using addition, subtraction, multiplication, *and* division and only negative numbers that have the same value.

Name: ________________ Date: ________ Period: ________

Activity 2 Deep Ocean Exploration

According to the National Oceanic and Atmospheric Administration (NOAA), over 80% of the world's oceans remain unexplored. As part of the international goal of mapping the entire ocean floor by 2030, the NOAA ship *Okeanos Explorer* completed research in the North Atlantic Ocean. During their expedition they tested the autonomous underwater vehicle *Orpheus*.

National Oceanic and Atmospheric Administration

1. *Orpheus* can descend at a constant rate of 30 m per minute. A sensor on the vehicle begins gathering data when *Orpheus* is 250 m below sea level. Complete the table that shows *Orpheus*'s location for the given times.

Time (minutes)	Change (m)	Expression	Location (m)
0	0	–250 + 0 • (–30)	–250
1	–30	–250 + 1 • (–30)	–280
5	–150		
10			

2. Using the data provided, determine the location of *Orpheus* prior to the sensor being activated.

Time (minutes)	Change (m)	Expression	Location (m)
1	–30	–250 + 1 • (–30)	–280
0	0	–250 + 0 • (–30)	–250
–1	30	–250 + (–1) • (–30)	–220
–2	60		
–3			
–5			

Activity 2 Deep Ocean Exploration (continued)

3. How many minutes was *Orpheus* descending before the sensor turned on?

Are you ready for more?

Evaluate each of the following expressions.

1. $1 - 2 + 3 - 4 + 5 - 6$

2. $1 - 2 + 3 - 4 + \ldots + 99 - 100$

3. $1 - 3 + 5 - 7 + \ldots + 97 - 99$

4. $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2$

5. $1^2 - 2^2 + 3^2 - 4^2 + \ldots + 99^2 - 100^2$

STOP

Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You solved problems involving all four operations with rational numbers. You saw that each operation could be used to determine different information, such as in the activity about electricity. In other scenarios, such as the activity about *Orpheus*, you have to use multiple operations together to model a situation.

No matter the scenario, the order of operations and rules for adding, subtracting, multiplying, and dividing still hold true.

Remember, when working with *two* rational numbers:

	Same sign	Different signs
Addition	Add their magnitudes and keep the same sign for the sum.	Subtract their magnitudes and use the sign of the number with the greater magnitude for the sum.
Subtraction	Rewrite the difference as the sum of the additive inverse, and then follow the rules for addition.	
Multiplication or Division	Multiply or divide as you would with two positive values. The product or quotient is positive.	Multiply or divide as you would with two positive values. The product or quotient is negative.

Reflect:

Name: .... Date: .... Period: ....

1. A bank charges $7.50 per month for a checking account. A bank account has a balance of $85.00. No money is withdrawn or deposited, except the service charge.

a How much money will be in the account after five months?
Show your thinking.

b How many months until the account balance is negative? Show your thinking.

2. Each table shows the transactions, in dollars, in a checking account for one month.

January	February	March	April
−38.50	250.00	−14.00	−86.80
126.30	−135.20	99.90	−570.00
429.40	35.50	−82.70	100.00
−265.00	−62.30	−1.50	−280.10

a Determine the total of the transactions for each month. Show your thinking.

b Determine the mean (average) total for the four months. Show your thinking.

Name: .... Date: .... Period: ....

3. A large aquarium of water is being filled with a hose. Due to a problem, the sensor does not start working until some time into the filling process. The sensor starts its recording at the time zero minutes. The sensor initially detects the tank has 225 liters of water in it.

a The hose fills the aquarium at a constant rate of 15 liters per minute. What will the sensor read at the time 5 minutes?

b Later, someone wants to use the data to find the amount of water at times before the sensor started. What should the sensor have read at the time -7 minutes?

4. A furniture store pays a wholesale price for a mattress. Then the store marks up the retail price to 150% of the wholesale price. Later they put the mattress on sale for 50% off the retail price. A customer just bought the mattress on sale and paid $\$1{,}200$.

a What was the retail price of the mattress, before the discount? Show or explain your thinking.

b What was the wholesale price, before the markup? Show or explain your thinking.

5. Solve each equation. Show your thinking.

a $2\frac{1}{2}x = 3\frac{3}{4}$

b $1\frac{3}{4} + y = 2\frac{1}{2}$

c $8.4 = 24b$

d $9.03 = z + 0.82$

Unit 5 | Lesson 18

Solving Equations With Rational Numbers

Let's solve equations that include negative numbers.

Warm-up Number Talk

The variables a through h all represent different numbers. Mentally determine the number(s) that make each equation true.

1. $-6 + 6 = a$

2. $11 + b = 0$

3. $c + d = 0$

4. $\frac{3}{5} \cdot \frac{5}{3} = e$

5. $7 \cdot f = 1$

6. $g \cdot h = 1$

Critique and Correct: Your teacher will display an incorrect statement about one of these equations. Work with a partner to critique the statement, correct it, and explain why the statement was incorrect.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Matching Solutions

For each equation, determine which solution, A or B, is correct. Circle the correct solution and be prepared to share your thinking.

Equation	Solution A	Solution B
$-\frac{1}{2}x = -5$	$x = 10$	$x = -4.5$
$x + (-2) = -6.5$	$x = -8.5$	$x = -4.5$
$-2 + x = \frac{1}{2}$	$x = 2\frac{1}{2}$	$x = -1\frac{1}{2}$
$-2x = -9$	$x = 4.5$	$x = -7$

Activity 2 Matching Equations

Analyze the four equations shown. Match the equations that have the same solution. You should have two pairs of equations.

$x + 2 = 8$	$x - 2 = 8$	$x + (-2) = 8$	$x - (-2) = 8$

1. My first pair of equations with the same solution are

.... and ....

a Explain your thinking for why the two equations have the same solution.

b Solve the addition equation. Show your thinking.

c Solve the subtraction equation using the same inverse operation you used in part b. What do you notice?

d Are there any other methods for solving this pair of equations? Show or explain your thinking.

Name: .... Date: .... Period: ....

Activity 2 Matching Equations (continued)

2. My second pair of equations with the same solution are

.... and ....

a Explain your thinking for why the two equations have the same solution.

b Solve the addition equation. Show your thinking.

c Solve the subtraction equation using the same inverse operation you used in part b. What do you notice?

d Are there any other methods for solving this pair of equations? Show or explain your thinking.

Activity 3 Equations and Solutions

For each equation, choose the operation that would be used to solve the problem. You should not use the same operation for more than one equation. Not all of the operations will be used.

$+\left(\frac{2}{3}\right)$	$-\left(\frac{2}{3}\right)$	$\cdot\left(\frac{2}{3}\right)$	$\cdot\left(-\frac{2}{3}\right)$	$\cdot\left(\frac{3}{2}\right)$	$\cdot\left(-\frac{3}{2}\right)$

Equation	Operation	Work
$-\frac{2}{3}x = \frac{4}{9}$		
$x - \left(-\frac{2}{3}\right) = 1\frac{1}{6}$		
$x + \left(-\frac{2}{3}\right) = -3\frac{1}{6}$		
$-\frac{3}{2}x = -3\frac{1}{2}$		

STOP

Name: ______________________ Date: ____________ Period: __________

Summary

In today's lesson . . .

You reasoned that you could use the same methods for solving equations with rational numbers that you did with positive numbers. You related your understanding of equivalent expressions with rational numbers to solving equations with rational numbers.

Because subtracting in the same as adding the additive inverse, you reasoned that there are two ways to solve equations of the form $x + p = q$.

For example, consider these two methods for solving $x + (-2) = -5$:

$$\begin{array}{ccc} x + (-2) = -5 & \text{or} & x + (-2) = -5 \\ x + (-2) - (-2) = -5 - (-2) & & x - 2 = -5 \\ x = -5 + 2 & & x - 2 + 2 = -5 + 2 \\ x = -3 & & x = -3 \end{array}$$

Both strategies are equivalent because subtracting -2 is the same as adding 2.

To solve equations of the form $px = q$, you also determined that dividing by a number is the same as multiplying by its reciprocal.

For example, consider these two methods for solving $-\frac{4}{5}x = -\frac{2}{3}$:

$$\begin{array}{ccc} -\frac{4}{5}x = -\frac{2}{3} & \text{or} & -\frac{4}{5}x = -\frac{2}{3} \\ -\frac{4}{5}x \div \left(-\frac{4}{5}\right) = -\frac{2}{3} \div \left(-\frac{4}{5}\right) & & -\frac{4}{5}x \bullet \left(-\frac{5}{4}\right) = -\frac{2}{3} \bullet \left(-\frac{5}{4}\right) \\ x = \frac{5}{6} & & x = \frac{5}{6} \end{array}$$

Both strategies are equivalent because dividing by $-\frac{4}{5}$ is the same as multiplying by $-\frac{5}{4}$.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Match each equation with the operation that could be used to solve it.

a $x + (-10) = 5$ .... $\cdot\ (-10)$

b $-10x = 5$ .... $+\ 10$

c $-\frac{1}{10}x = 5$ .... $\cdot\ 10$

d $x - (-10) = 5$ .... $-\ 10$

e $\frac{1}{10}x = 5$ .... $\cdot \left(-\frac{1}{10}\right)$

2. Identify the mistake in solving each equation, and then solve the equation correctly.

	Incorrect work	Explain the mistake	Correct work
$-3x = 6$	$-3x = 6$ $-3x + 3 = 6 + 3$ $x = 9$		
$x - (-2) = 12$	$x - (-2) = 12$ $x - (-2) + 2 = 12 + 2$ $x = 14$		
$\frac{4}{5}x = -\frac{9}{10}$	$\frac{4}{5}x = -\frac{9}{10}$ $\frac{4}{5}x \div \frac{5}{4} = -\frac{9}{10} \div \frac{5}{4}$ $x = -\frac{18}{25}$		

3. Solve each equation. Show your thinking.

a $-4.5 = a - 8$

b $-12 = -3y$

c $\frac{2}{5}x = -\frac{8}{15}$

Name: .... Date: .... Period: ....

4. Write an equation that satisfies each situation. Explain your thinking.

a A number is added to a variable. The solution of the equation is -8.

b A number is multiplied by a variable. The solution of the equation is $-\frac{4}{5}$.

5. In 2012, James Cameron descended to the bottom of Challenger Deep in the Mariana Trench, the deepest point in the ocean. The vessel he rode in was called DeepSea Challenger. Challenger Deep is $35{,}814$ ft at its lowest point.

a DeepSea Challenger's descent was a change in depth of -4 ft per second. Use the equation $y = -4x$ to model the relationship where y is the depth and x is the time in seconds that have passed. How many seconds does this model suggest it would take for DeepSea Challenger to reach the bottom?

b The DeepSea Challenger made a one-hour ascent to the surface. How many seconds is this?

c The ascent can be modeled by a different proportional relationship, $y = kx$. What is the value of k in this scenario?

6. Kiran drinks 6.4 oz of milk each morning. How many days does it take him to finish a 32 oz container of milk.

a Write and solve an equation for this situation.

b What does the variable represent?

Unit 5 | Lesson 19

Representing Contexts With Equations

Let's write equations that represent scenarios.

Warm-up Don't Solve It!

Without solving, determine whether the solution of each equation is positive or negative, and place a check mark in the appropriate box. Be prepared to explain your thinking.

Equation	Positive	Negative
$-8.7a = -1.4$	☐	☐
$-8.7b = 1.4$	☐	☐
$-8.7 + c = -1.4$	☐	☐
$-8.7 - d = -1.4$	☐	☐

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Warmer or Colder Than Before

For each scenario:

- **Determine the *two* equations that could represent the scenario from the given bank of equations.**
- **Explain what the variable v represents in the context of the scenario.**
- **Determine the value of v that makes both equations true. Show or explain your thinking.**

$-4v = -16$	$v + 16 = -4$	$-16v = -4$
$v = -4 - 16$	$-4 + v = -16$	$v = -16 + 4$
$v - 16 = -4$	$4v = -16$	$v = -16 \bullet \left(-\frac{1}{4}\right)$

	At midnight, the temperature was 0°F and dropping 4° per hour. At a certain time, the temperature is -16°F.	Between 6 a.m. and noon, the temperature rose 16°F to be -4°F.	At midnight, the temperature was -4°F. By 4 a.m., the temperature had fallen to -16°F degrees.
Equation 1			
Equation 2			
v represents . . .			
My thinking . . .			

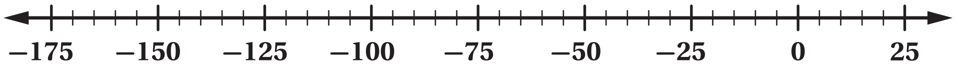
Activity 2 Changing Elevation

Hang Son Doong ("Mountain River Cave") in Vietnam, is the largest cave in the world. It is so massive, it can fit an entire Manhattan city block, including 40-story buildings, or have a 747 plane fly through it without the wings touching sides. Not discovered until 1990, and not first explored until 2009, there is much still to be discovered about this incredible wonder.

Vietnam Stock Images/Shutterstock.com

1. In 2019, multiple members of the diving team were given the opportunity to explore a new underground tunnel in Son Doong Cave. They dove as far as they could below sea level, then dropped a weighted rope 42 m down, reaching 120 m below sea level. How deep was the team when they dropped the rope?

a Draw an arrow diagram on the number line that represents the problem.



b Write an equation to represent the scenario. Make sure that you define your variable.

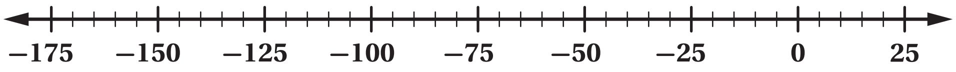
c Solve your equation to determine the unknown value. Show your thinking.

Name: ________________ Date: ________ Period: ________

Activity 2 Changing Elevation (continued)

2. If the team of divers descended at a rate of 20 m/minute, how long did it take them to reach their maximum depth (the depth at which they dropped their rope)?

a Draw an arrow diagram on the number line that represents the scenario.



b Write an equation to represent the scenario. Make sure that you define your variable.

c Solve your equation to determine the unknown value. Show your thinking.

Are you ready for more?

To ascend, divers travel at a maximum rate of 9 m per minute. They also need to pause for safety stops to allow for decompression during the ascent. The first safety stop, called a “deep stop” should be made at 50% of the total depth for 60 seconds.
The second should be taken at 5 m below the surface for at least 3 minutes.

What is the minimum time, to the nearest minute, it took the divers to ascend from their dive? Show or explain your thinking.

Activity 3 Equations Tell a Story

Your teacher will provide your group with a scenario.
Create a visual display about your statement that includes:

- **An equation that represents the scenario.**
- **What your variable *and* each value in your equation represent.**
- **How the operation(s) in your equation represent the relationship in the scenario.**
- **How to use inverse operations to solve for the unknown quantity.**
- **The solution to your equation.**

You can use the grid below to help you organize your visual display, if helpful.

Card ……	**Equation:** ……
…… represents . . . …… represents . . . …… represents . . .	We used …… as the operation in our equation because . . .
To solve our equation . . .	**Solution:**

STOP

Name: .... Date: .... Period: ....

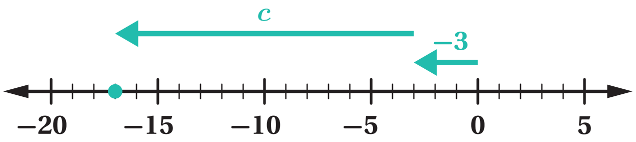
Summary

In today's lesson . . .

You connected your understanding of solving problems involving rational numbers on the number line to equations. You used variables and equations to represent and answer questions about scenarios.

For example:

- If the temperature is -3°C at 6 p.m., and at 3 a.m. it is -17°C, you can write and solve an equation to determine the change c in temperature.

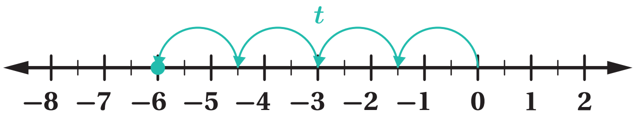


$$-3 + c = -17$$

$$-3 + c + 3 = -17 + 3$$

$$c = -14$$

- If a starfish is descending at a rate of $\frac{3}{2}$ ft per hour, you can write and solve an equation to determine how many hours t it will take the starfish to descend 6 ft.



$$-\frac{3}{2}t = -6$$

$$-\frac{3}{2}t \bullet \left(-\frac{2}{3}\right) = -6 \bullet \left(-\frac{2}{3}\right)$$

$$t = 4$$

Reflect:

Name: .... Date: .... Period: ....

1. Match each equation with the next step for solving it.

a $-\frac{1}{2}x = 2$

b $x - (-2) = 2$

c $-2x = 2$

d $x + (-2) = 2$

.... $x = 2 \cdot (-2)$

.... $x = 2 + 2$

.... $x = 2 \cdot \left(-\frac{1}{2}\right)$

.... $x = 2 - 2$

2. Match each situation with an equation that represents it.

a A whale is diving at a rate of 2 m/second. How long will it take for the whale to get from the surface of the ocean to elevation of -12 m?

b A diver descended below the surface of the ocean. After 2 minutes, she was 12 ft below the surface. At what rate was she diving?

c The temperature was -12°C and changed to 2°C. What was the change in temperature?

d The temperature was 2°C and changed to -12°C. What was the change in temperature?

.... $2c = -12$

.... $-12 + c = 2$

.... $-2c = -12$

.... $2 + c = -12$

3. Starting at noon, the temperature dropped steadily at a rate of 0.8°C per hour. For each scenario, write and solve an equation and describe what the variable represents.

a How many hours did it take for the temperature to decrease by 4.4°C?

b If the temperature after the 4.4°C drop was -2.5°C, what was the temperature at noon?

Name: .... Date: .... Period: ....

4. Determine the value of each expression.

a $12 + (-10) \cdot 2$

b $-5 - 6 + 3$

c $-42 \div (-6) + 17 \cdot (-1)$

d $35 - (-8 + 5)$

e $-4 \cdot \left(\frac{1}{2} + 3\right)$

5. The tick marks on the number line are evenly spaced. Label the missing values.

6. Diego spends $\$4.50$ each day that he buys lunch.

a Which value 4.5 or -4.5 represents Diego's change in money each day from lunch? Explain your thinking.

b Write an equation representing Diego's change in money c after d days.

c If Diego gives himself a budget of $\$90$ for lunches this month, how many days will he be able to buy his lunch? Show or explain your thinking.

Unit 5 | Lesson 20 – Capstone

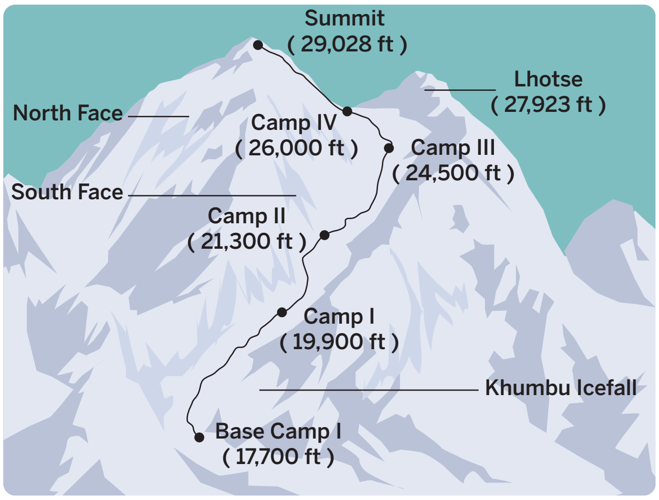
Summiting Everest

Let's play a game to imagine what it's like to prepare to summit Mt. Everest.

Warm-up Understanding the Route

Though there is more than one route to the peak of Mt. Everest, the most common route is along the South face. Climbers hike from camp to camp, spending time at each camp acclimatizing (getting used to the thinner air, which has less oxygen as the elevation increases).

Which section between two camps do you think would take the longest amount of time to climb? Why do you think that?



Historical Moment

Climbing Mt. Everest

Climbing Everest is no easy feat, and no one knows it better than the Sherpas. Almost every expedition up the mountain is led by Sherpas, the Nepalese mountaineers who have lived in and climbed the Himalayas for centuries. Sherpa Tenzing Norgay was one of the first to summit Everest in 1953 (along with Sir Edmund Hillary). Due to the grueling nature of the climb and the amount of careful coordination needed, arranging an expedition led by a Sherpa can cost over $50,000 per person today.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Making Preparations

To climb Mt. Everest, you must be prepared. Most people who attempt the climb spend months conditioning their body and their mind, and ensuring they have the appropriate gear. Your preparation in this activity will impact your success in the game in Activity 2.

You have 4 weeks to prepare for your 2-week expedition up Mt. Everest. To help guide your preparations, you should know that for each day spent on the mountain, you will:

- Lose 1 strength point.
- Lose 1 perseverance point.
- Use 1 liter of oxygen.
- Lose $0.3°\text{F}$ of body temperature.

Assign each day on the calendar one type of preparation.

Physical training (P)	**Mental conditioning (M)**	**Gear upgrades (G)**
For each day of preparation: • You gain 1.5 Strength points. • You use 0.01 liter less of oxygen each day.	For each day of preparation: • You gain 1 Perseverance point. • You use 0.01 liter less of oxygen each day. • You lose 0.5 Strength point.	For each day of preparation: • You lose $0.05°\text{F}$ less of your body temperature each day.

Total points from preparations:

Strength	Perseverance

Preparation Calender

Week	Activity						
1							
2							
3							
4							

Total rate savings:

Oxygen use savings	Body temperature savings

Reflect: How did you encourage your group members as they spoke?

Activity 2 The Summit Attempt

Even for those who are fully prepared, reaching the summit of Mt. Everest is not guaranteed. Weather conditions, time, and your health are all important factors that are not under your control.

You will be given a pair of number cubes.

Your Goal: Reach the summit before your run out time or resources. You will have 14 days to reach the summit.

How to play:

Getting set up:

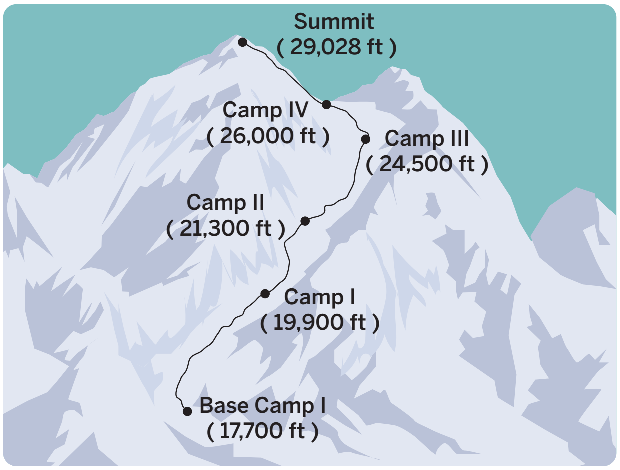
- Enter your total points from your preparations in your log book. Be sure to adjust your rate for oxygen and body temperature based on the total rate savings from your preparations.
- Start at Base Camp. Note: You will never go lower than Base Camp.

For each turn:

- Roll both number cubes and determine their sum. (Each roll will affect everyone on your team in the same way.)
- Each roll represents the conditions for 1 day. Follow the directions for your sum.
- In the table, record the new values for yourself.
- Continue taking turns rolling until you have either reached the summit, run out of a resource, have a body temperature of less than 95 degrees, or run out of time.

If you roll a sum of:

2–7	8–9	10	11	12
Good weather. Move up one camp.	Poor weather. Remain at current camp.	Extremely bad weather. Move down one camp *and* lose 1 Strength point.	Extreme cold. Lose 0.2° F from body temperature *and* 1 Perseverance point. Remain at current camp.	Altitude sickness. Lose 2 Strength points *and* 1 Perseverance point. Move down one camp.



Name: .... Date: .... Period: ....

Activity 2 The Summit Attempt (continued)

Log book:

		Strength	Perseverance	Oxygen remaining (liters)	Body temperature (°F)	Current camp
Starting values:				10	98.6	Base Camp
Change per day:		−1	−1	−1	−0.3	
Day	**Sum:**					
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						

STOP

Unit Summary

It's easy to forget how large our planet is. Beyond the comforts of home, there are towering cliffs, wide oceans, and miles and miles of rocky terrain. These hard-to-reach places have tempted adventurers for centuries. From the breath-stealing heights of Mt. Everest to the sub-zero temperatures of Antarctica, there's no shortage of places for people to test their strength and resolve.

Many of these challenges can be described using rational numbers. Rational numbers show not only *how much* of something there is, but also in what *direction*. This is especially useful when discussing elevation and temperature, where quantities can be above or below the starting point of zero.

But we can use rational numbers for more than just position. Rational numbers open a world where we acknowledge what it means to lose something, to owe something, and to go below where we started. In the fields of accounting and finance, positive and negative numbers show up as credit and debt. Operating with these numbers can give you the big picture on your total gains and losses.

While addition and subtraction may seem relatively simple with rational numbers, be careful when multiplying or dividing. A misplaced sign can mean the difference between climbing a triumphant peak or plummeting into the deepest depths.

See you in Unit 6.

Name: .... Date: .... Period: ....

Practice

1. Evaluate each expression.

a $1.4 + (-2.4)$

b $-0.6 + 0.8$

c $\frac{1}{4} - \left(-\frac{3}{4}\right)$

d $-\frac{7}{8} + \left(-\frac{1}{4}\right)$

e $-\frac{3}{4} - (-0.85)$

f $3\frac{4}{5} - 4\frac{1}{4}$

2. Complete the equations using the values in the table. You may only use each at most once.

12	-12	8	-8
6	-6	4	-4
2	-2	1	-1
$\frac{3}{4}$	$-\frac{3}{4}$	$\frac{2}{3}$	$-\frac{2}{3}$

a .... × .... = ...

b .... ÷ .... = ...

c .... + ... = ...

d .... − .... = ...

Name: .... Date: .... Period: ....

3. The table shows some information from the stock market. Complete the table.

Company	Value on Day 1 ($)	Value on Day 2 ($)	Change in value ($)	Change in value as a percentage of Day 1 value
Tech company	107.95	111.77	3.82	3.54%
Appliance manufacturer		114.03	2.43	2.18%
Oil corporation	26.14	25.14		−3.83%
Department store	7.38	7.17		
Jewelry company		70.30		2.27%

4. Noah says that speed is related to velocity in the same way that distance is related to difference. Explain what he means by this statement.

5. Using the given values, write an expression applying only addition and subtraction whose value is zero.

−3	−10	−7

My Notes:

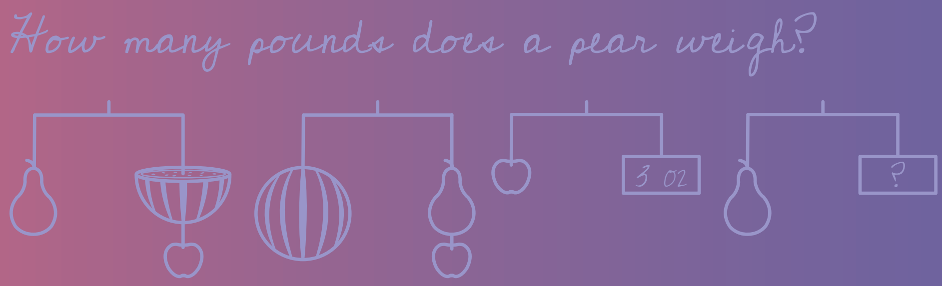
UNIT 6

Expressions, Equations, and Inequalities

Numbers are great, but they will not get us where we are going in this unit. It will take letters, symbols, and drawings to represent the varied and diverse mathematical ideas of algebraic thinking.

Essential Questions

- Which representations best help you make sense of certain mathematical scenarios?
- Which strategies that worked for solving simple equations or inequalities can be put to use when solving more complex ones?
- How can you increase your efficiency when solving mathematical problems?
- *(By the way, what does dog walking have to do with mathematics?)*



SUB-UNIT

1

Solving Two-Step Equations

Narrative: Symbols and diagrams can help you to make sense of equations, and even to solve them.

You'll learn . . .

- different ways to model equations.
- powerful strategies for solving equations.

SUB-UNIT

2

Solving Real-World Problems Using Two-Step Equations

Narrative: From ancient Egypt to the modern world, solving equations can help you solve problems.

You'll learn . . .

- how tape diagrams can represent equations.
- how equations can, in turn, represent real-world problems.

SUB-UNIT

3

Inequalities

Narrative: Inequalities are more than symbols. And you already have the tools to solve them.

You'll learn . . .

- how solving inequalities is very similar to solving equations.
- how to use inequalities to model and solve problems.

SUB-UNIT

4

Equivalent Expressions

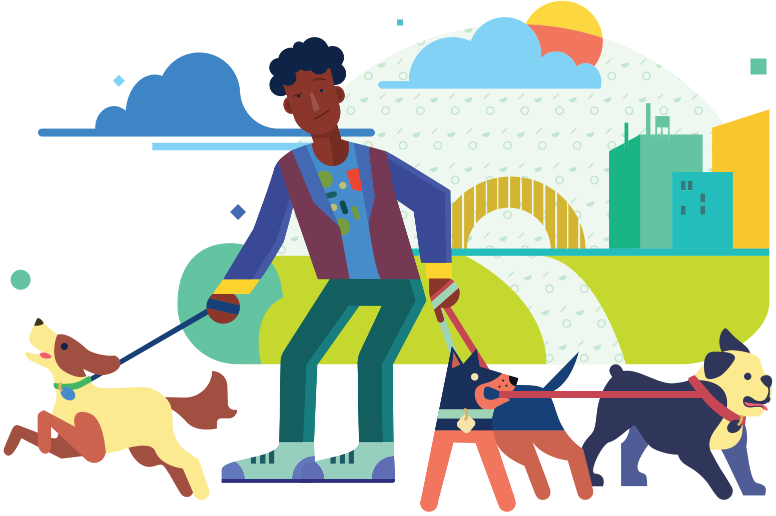
Narrative: Technology can make life a little easier. So can changing the structure of a mathematical expression.

You'll learn . . .

- how to expand and factor expressions.
- how to combine like terms to shorten expressions.

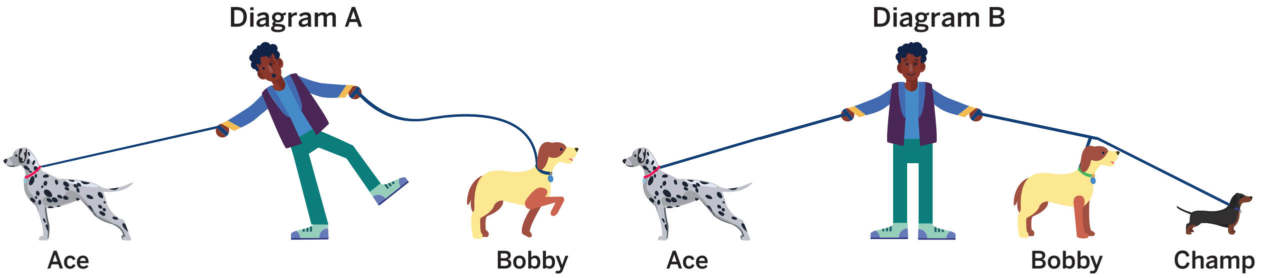
Keeping the Balance

Let's walk some dogs.



Warm-up Notice and Wonder

Examine these diagrams. What do you notice? What do you wonder?



1. I notice . . .

2. I wonder . . .

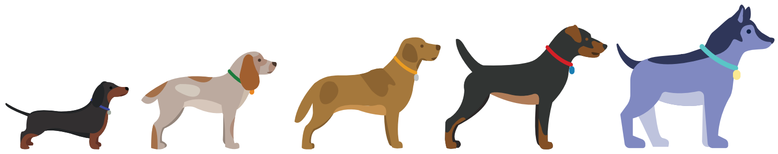
Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Walking Dogs Like a Pro

Welcome to Pawston University! We have produced some of the nation's finest dog walkers. Your first challenge is to balance some leashes.

How can you balance all five dogs on two leashes, holding one leash in each hand?



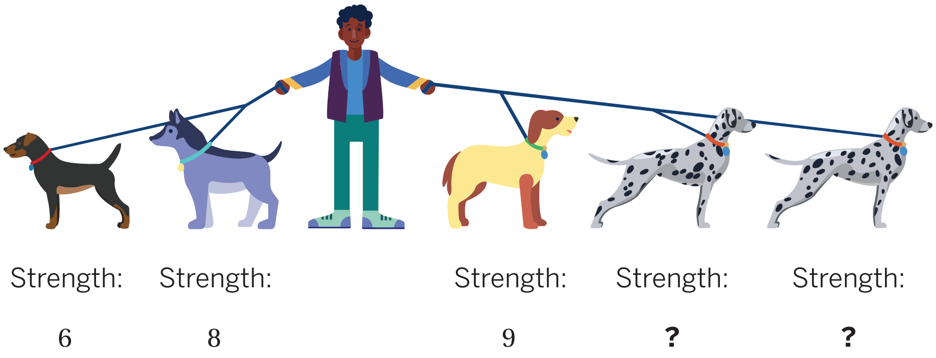
Dale	**Eartha**	**Fifi**	**Greg**	**Harriet**
Strength: 1	Strength: 2	Strength: 3	Strength: 6	Strength: 8

Write the names and strengths of the dogs you would place on the left side and the right side.

Activity 2 Two New Customers

Congratulations! You have learned how to balance your leashes. But what happens when you do not know the strength of a dog?

Let's figure out the strength of the new dogs in the diagram — whose strengths have not been labeled. Assume that the dog walker feels an equal pull in both directions, and that the dogs who look the same have the same strength.



What is the strength of each new dog? Explain your thinking.

Are you ready for more?

Complete in each box with a single value that makes each equation true. For Problem 3, use the same value for both boxes.

1. $9 + 14 = 7 + 2 \cdot \square$
2. $5\,(\square + 8) = 9 \cdot 7 - 3$
3. $4 + \square = 3 \cdot \square$

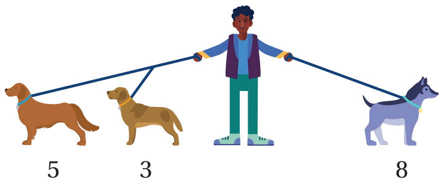
Name: ________ Date: ________ Period: ________

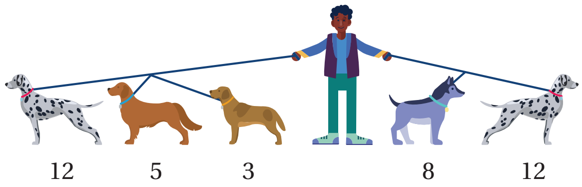
Activity 3 Inventing Your Own Terminology

Emily Riehl is a mathematician who prefers to write about her work in higher category theory in a way that makes the technical language more understandable.

Rename the dog-walker's balancing techniques using language that makes the most sense to you.

a

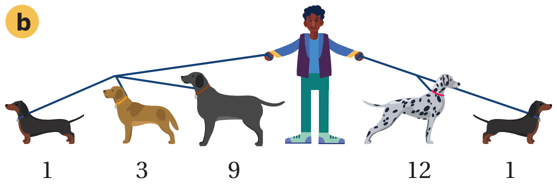




Old name: Addition Property of Equality

New name:

b

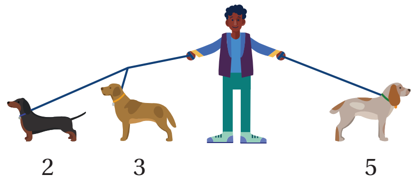


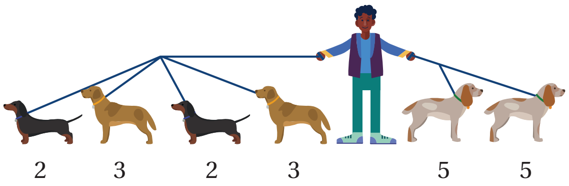
Old name: Subtraction Property of Equality

New name:

Activity 3 Inventing Your Own Terminology (continued)

c

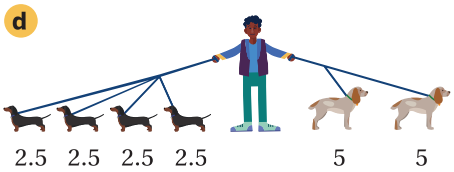


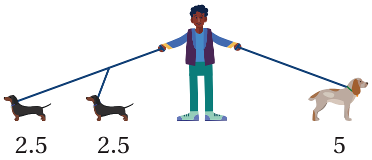


Old name: Multiplication Property of Equality

New name:

d





Old name: Division Property of Equality

New name:

Featured Mathematician

Emily Riehl

Emily Riehl is the kind of mathematician who wants to open up the world of math to more people, rather than hiding it behind hard-to-understand language and obscure symbols. In addition to studying "Higher Category Theory", Riehl writes about math for an audience that does not just include other professors and academics — which is rare in the world of university-level mathematics. You might even catch her using music or sports as metaphors for the math she studies.

STOP

Mona Merling

Unit 6 Expressions, Equations, and Inequalities

Solving One Step at a Time

OK, let's be real. When you see someone walking their dog, chances are you are not thinking, "Oh, what an adorable math problem!"

You are thinking, "Look at the *pwetty puppers!*"

. . . or "Who's a good boy?"

. . . or "Someone better clean that up."

But, believe it or not, that dog is made up of numbers. Numbers are inside how that dog walks down the street, how it wags its tail, how it chows down on its kibble, and how it chases squirrels.

In fact, *everything* we observe can be expressed in the language of mathematics.

Like Mandarin, English, and Hindustani, we can use math to describe how things are and how they work. While spoken language uses words, math uses numbers, symbols, operations, diagrams, equations, and inequalities. From getting tugged by a Pit Bull to building the pyramids, many stories of the human experience can be told and many problems can be solved, one mathematical step at a time.

Welcome to Unit 6.

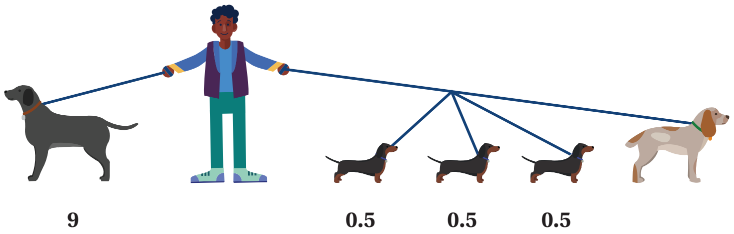
Name: .... Date: .... Period: ....

1. Using each of the numbers from 0 to 9 at most once, find as many different ways as you can to make the equation true.

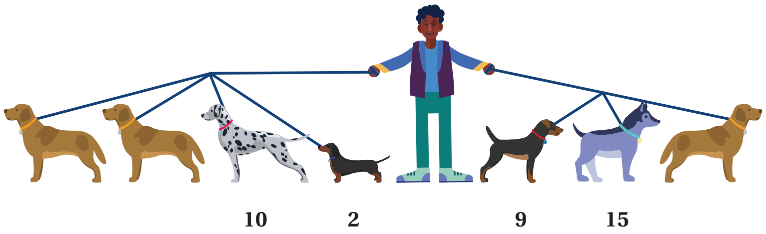
$$\square + \square = \square + \square + \square$$

2. Each balanced diagram has some dogs with unknown strengths. Determine the strength of each unknown dog. Assume dogs that look alike have the same strength.

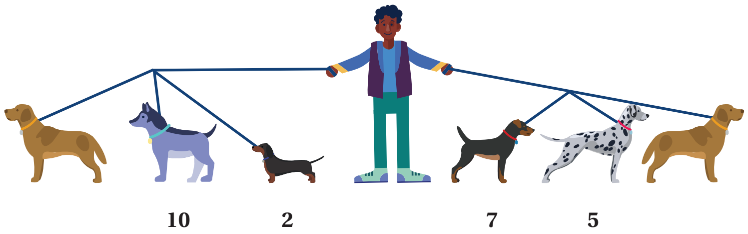
a



b



c



3. Create a list of four dogs, including their strengths, that *cannot* be balanced in any way on opposite sides of the dog walker.

Name: .... Date: .... Period: ....

4. Which problem *cannot* be answered by the solution to the equation $3x = 27$?

A. Elena read three times as many pages as Noah. She read 27 pages. How many pages did Noah read?

B. Lin has 27 stickers. She gives 3 stickers to each of her friends. With how many friends did Lin share her stickers?

C. Diego paid $27 for a concert ticket. What is the cost of 3 of these tickets?

D. The coach splits a team of 27 students into 3 groups to practice skills. How many students are in each group?

5. Solve each equation. Show your thinking.

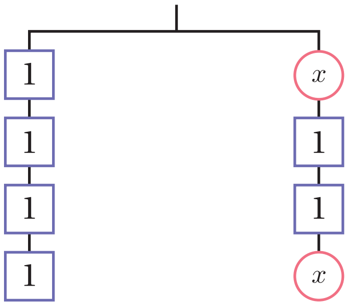
a $8.5 \cdot 3 = a$

b $7 + b = 11$

c $c - 3 = 15$

d $d \cdot 4 = 32$

6. Write an equation that represents the hanger diagram.



My Notes:

SUB-UNIT

1 Solving Two-Step Equations

What are the first words you learn in “Caveman”?

Strictly speaking, there is no language called “Caveman.” In fact, scientists are pretty sure our cave-dwelling ancestors did not speak any language at all. But that does not mean early humans had nothing to say!

Archaeologists have discovered that many ancient cave paintings were drawn near “acoustic hotspots." These hotspots made anyone standing in that spot, loud and clear for others in the cave to hear. Scientists theorize that early humans connected the sounds made in those hotspots to the paintings on the walls. So, if one of our cave-dwelling ancestors painted a horse in full gallop or a trumpeting mammoth, these images may have represented similar sounds they heard in that very spot.

We do this kind of symbolic thinking every time we use letters (“H-O-R-S-E”) to represent a word (“horse”). This kind of thinking, they believe, helped bring about language as we know it today.

For thousands of years, our species has used pictures and symbols to express how we see the world. As a species we have come a long way from cave drawings. Today we can use words, emojis, pictures, and diagrams to express ourselves. These symbols help us connect, whether we are in the same classroom or on the other side of human history. And as you’ll see, they can also help us to express and understand mathematical thinking.

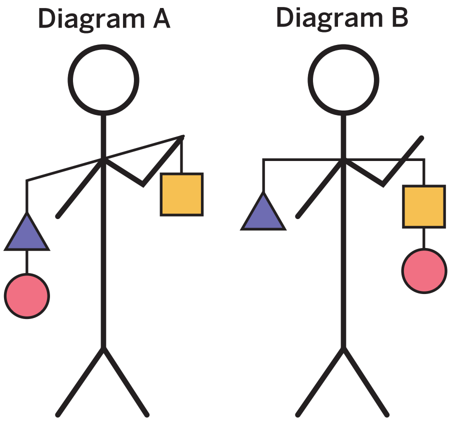
Unit 6 | Lesson 2

Balanced and Unbalanced

Let's see how hanger diagrams can represent balanced relationships.

Warm-up Carry On

Since early civilization, humans have used carrying poles to transport heavy loads over long distances. Refer to the diagrams.



1. What do Diagrams A and B represent?

2. If you had to carry a heavy load, would you rather carry it as shown in Diagram A or Diagram B? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Hanging Out

Co-craft Questions: Work with your partner to write 2–3 mathematical questions you could ask about these hanger diagrams before beginning this activity.

In the two hanger diagrams, all the triangles weigh the same as one another, and all the squares weigh the same as one another.

Diagram A

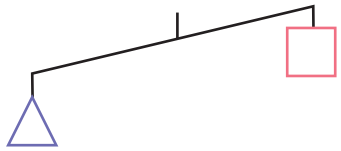
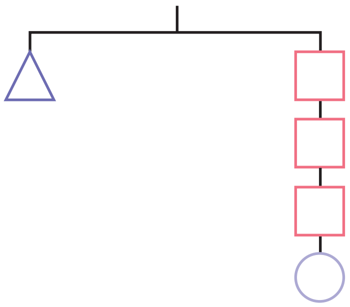


Diagram B



Based on the diagrams, what is . . .

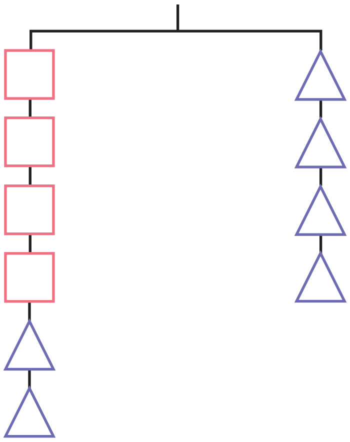
1. One thing that *must* be true?

2. One thing that *could* be true?

3. One thing that *cannot* possibly be true?

Activity 2 Manipulating a Hanger Diagram (Part 1)

The hanger diagram shown is balanced because the weight on both sides is the same.



1. Which weights can be removed so that the hanger remains balanced? Determine as many strategies as possible.

2. If a triangle weighs 4 g, how much does a square weigh? Explain your thinking.

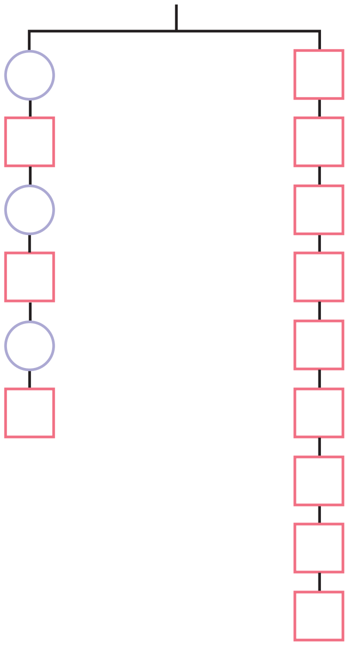
Are you ready for more?

If the weight of a square is x g and the weight of a triangle is 4 g, what equation could represent this hanger diagram?

Name: .... Date: .... Period: ....

Activity 3 Manipulating a Hanger Diagram (Part 2)

The hanger diagram shown is balanced because the weight on both sides is the same.



1. Which weights can be removed so that the hanger diagram remains balanced? Determine as many responses as possible.

2. If a square weighs $\frac{1}{2}$ lb, how much does a circle weigh? Explain your thinking.

Are you ready for more?

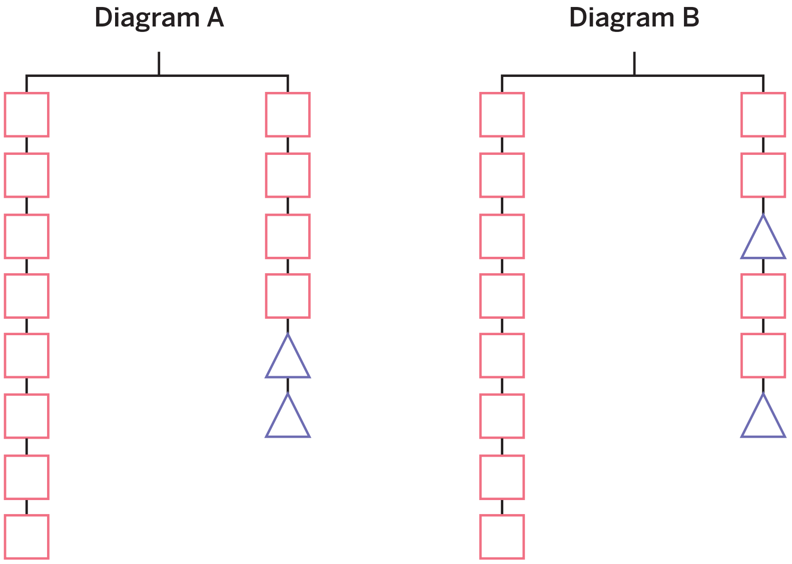
If the weight of a circle is x lb and the weight of a square is $\frac{1}{2}$ lb, what equation could represent this hanger diagram?

STOP

Summary

In today's lesson . . .

You used hanger diagrams to represent balanced relationships. You saw that you could use properties of equality to reason with and manipulate different hanger diagrams to determine the weight of different shapes on the diagram. Consider Diagrams A and B.

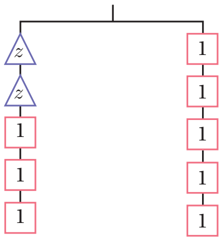
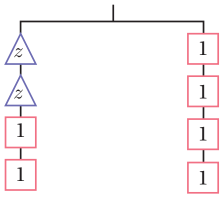
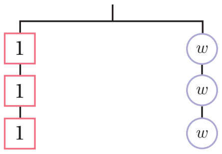
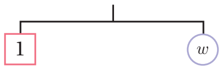
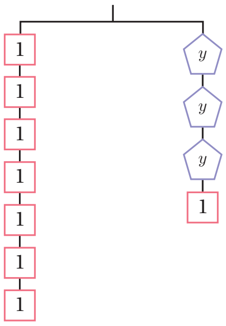
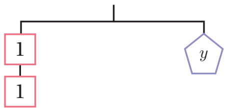


Although the diagrams have different arrangements, they are modeling the same relationship between triangles and squares and can be used to determine the relationship between the weight of a triangle when compared to the weight of squares.

Reflect:

Name: .... Date: .... Period: ....

1. Refer to the table. Determine what happened to the first hanger diagram that resulted in the second hanger diagram. Then, name the property (or properties) of equality that tell you that if the first hanger is balanced, then so is the second hanger diagram.

First hanger	Second hanger	What happened?	What property?
a 			
b 			
c 			

2. Each hanger diagram from Problem 1 is balanced. Determine the weight of each lettered shape, and explain your thinking. You may draw on the diagrams to help with your thinking.

a $z =$ ....

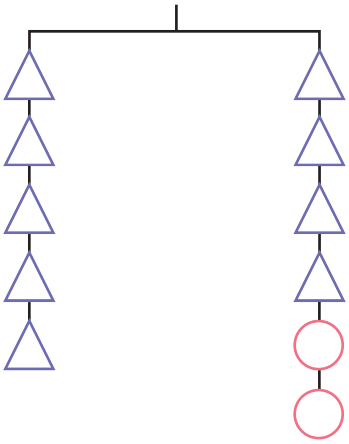
b $w =$ ....

c $y =$ ....

Name: .... Date: .... Period: ....

3. Refer to the balanced hanger diagram.

Determine the weight of a circle if each triangle weighs:



a 2 lb.

b 1 lb.

c 0.5 lb.

4. A car is traveling at a constant speed. Determine the number of miles the car would travel in 1 hour at each given rate.

a 135 miles in 3 hours

b 22 miles in $\frac{1}{2}$ hours

c 7.5 miles in $\frac{1}{4}$ hours

d $\frac{100}{3}$ miles in $\frac{2}{3}$ hours

5. Solve each equation. Show your work or explain your thinking.

a $21 = x + 9$

b $3x = 57$

c $x - 7.5 = 18.5$

d $15 = \frac{5}{8}x$

Name: .... Date: .... Period: ....

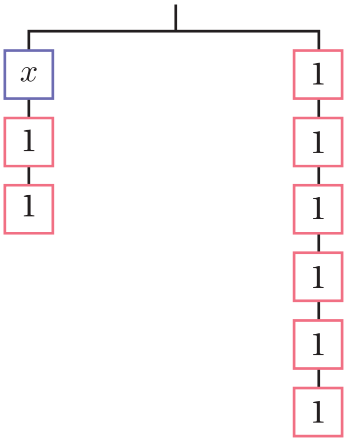
Unit 6 | Lesson 3

Reasoning About Solving Equations (Part 1)

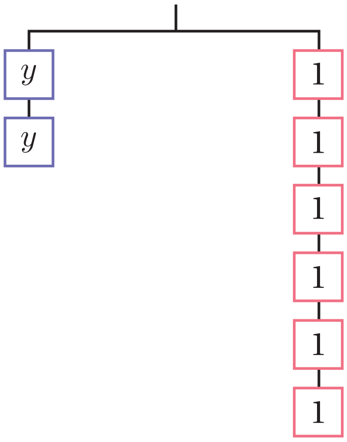
Let’s see how a balanced hanger diagram is like an equation, and how moving its weights is like solving an equation.

Warm-up Hanger Diagrams and Equations

Refer to the hanger diagrams and the equations that represent them.



$x + 2 = 6$



$2y = 6$

1. Explain how you can use the diagrams to determine the values of x and y.

2. Explain how you can use the equations to determine the values of x and y.

Log in to Amplify Math to complete this lesson online.

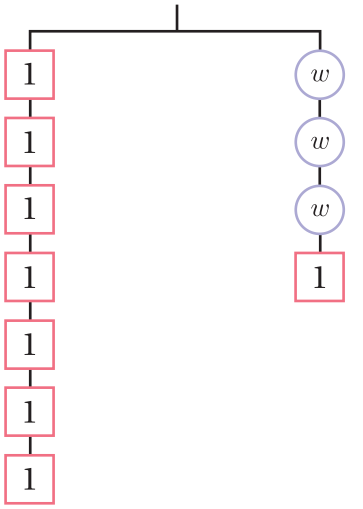
Activity 1 Matching Hanger Diagrams and Equations

1. Each of these equations represents one of the following hanger diagrams.

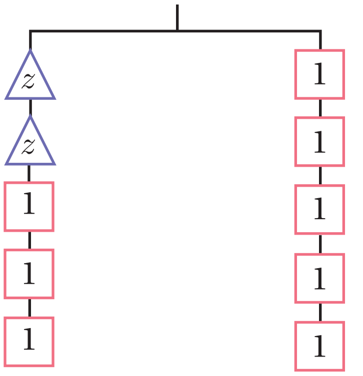
$2\boxed{} + 3 = 5$ $\qquad 6 = 2\boxed{} + 3$ $\qquad 7 = 3\boxed{} + 1$ $\qquad 3\boxed{} + 2 = 3$

Write the equation below its matching hanger diagram.
Replace the box in each equation with either w, x, y, or z.

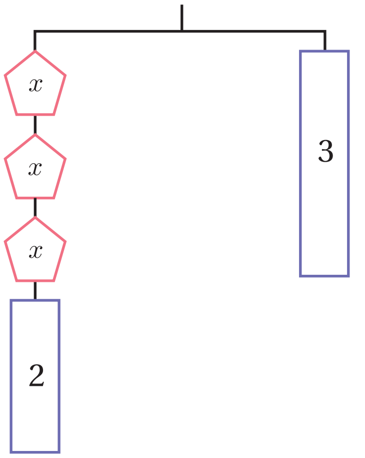
a



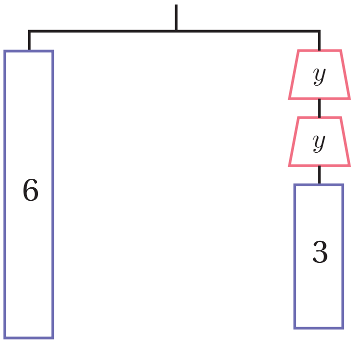
b



c



d



2. Use the hanger diagrams to help you solve each equation.

a ……………………

b ……………………

c ……………………

d ……………………

Name: ________________ Date: ________ Period: ________

Activity 2 Solving Equations

Solve each equation. Show all work. Draw a hanger diagram, if needed.

1. $3x + 1 = 7$

2. $4w + \frac{3}{2} = \frac{17}{2}$

Are you ready for more?

Solve each equation without using a hanger diagram.

1. $2.3z + 2.2 = 6.8$

2. $\frac{3}{4}w + \frac{1}{4} = \frac{19}{4}$

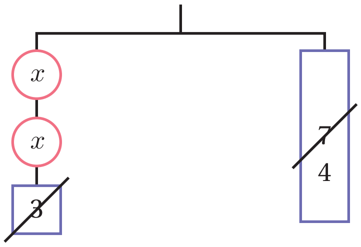
STOP

Summary

In today's lesson . . .

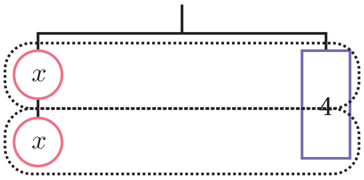
You showed that two amounts are equal using hanger diagrams and equations. You can use a hanger diagram to reason about how to find an unknown amount in an equation. You can also write the steps for finding an unknown amount in an equation, without using a hanger diagram. For example, you can solve the equation $2x + 3 = 7$ using these steps:

Remove 3 from both sides.



$$2x + 3 = 7$$
$$2x + 3 - 3 = 7 - 3$$

Divide into two equal groups.

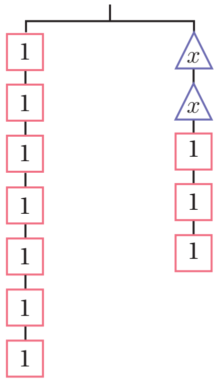


$$2x = 4$$
$$2x \div 2 = 4 \div 2$$
$$x = 2$$

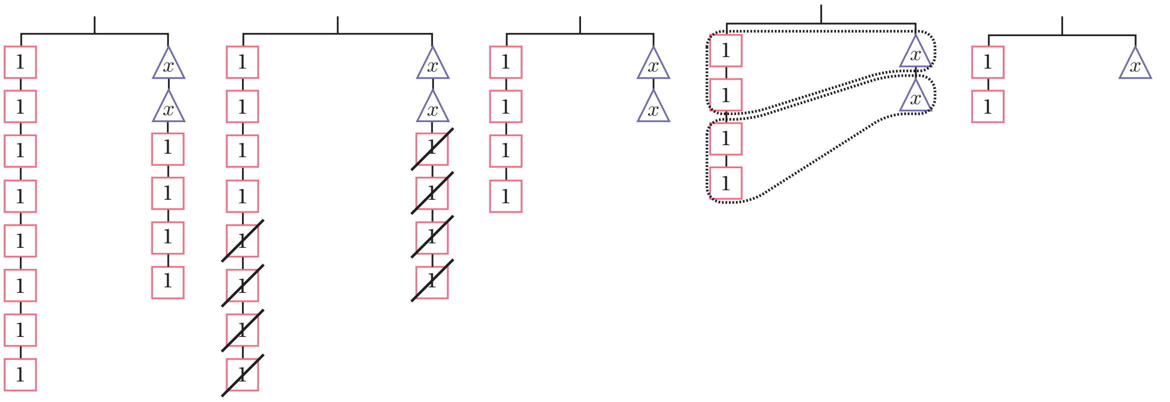
Reflect:

Name: .... Date: .... Period: ....

1. Explain how the parts of the hanger diagram compare to the parts of the equation $7 = 2x + 3$.



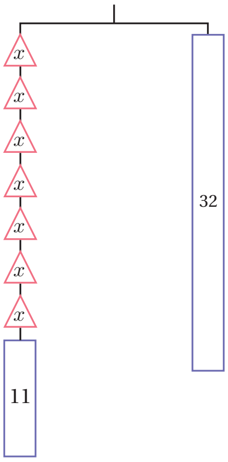
2. Shawn used a hanger diagram to solve the equation $8 = 2x + 4$. Write an equation to represent each step.



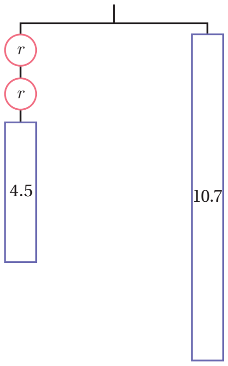
$8 = 2x + 4$ ....

3. Solve each equation. Use the hanger to help with your thinking.

a $7x + 11 = 32$



b $2r + 4.5 = 10.7$



Name: .... Date: .... Period: ....

4. Lin and Tyler are drawing circles. Tyler's circle has twice the radius of Lin's circle. Tyler thinks that his circle will have twice the area of Lin's circle as well. Do you agree with Tyler? Explain your thinking.

5. Jada and Priya are working together to solve the equation $\frac{2}{3} + x = 4$.

- Jada says, "I think we should multiply each side by $\frac{3}{2}$, because that is the reciprocal of $\frac{2}{3}$."
- Priya says, "I think we should add $-\frac{2}{3}$ to each side because that is the opposite of $\frac{2}{3}$."

a Which person's strategy should they use? Why?

b Write a different equation that can be solved using the other person's strategy.

6. For each expression, use the Distributive Property to determine an equivalent expression.

a $4(x + 9)$ ....

b $8(y - 2z)$ ....

Name: .... Date: .... Period: ....

Unit 6 | Lesson 4

Reasoning About Solving Equations (Part 2)

Let’s use hangers to understand two different ways of solving equations with parentheses.

Warm-up Algebra Talk: Seeing Structure

Mentally solve each equation, and record its solution.

a $x + 1 = 5$

b $2(x + 1) = 10$

c $3(x + 1) = 15$

d $500 = 100(x + 1)$

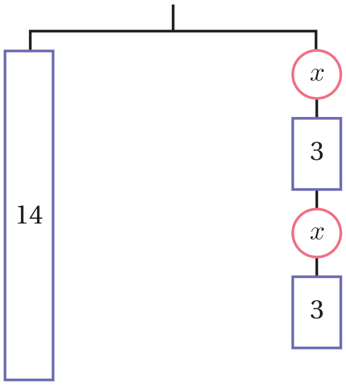
Are you ready for more?

Create your own equation, using parentheses, which has the same solution as one of the equations from the Warm-up.

Log in to Amplify Math to complete this lesson online.

Activity 1 Either/Or

Analyze the following hanger diagram. Be prepared to share your thoughts with a partner.

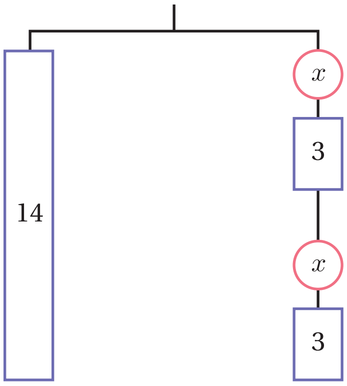


1. Explain why the equation $14 = 2(x + 3)$ could represent the hanger diagram.

2. Explain why the equation $14 = 2x + 6$ could represent the hanger diagram.

3. Determine the value of x. Use the hanger diagram to support your thinking.

$x =$ ....



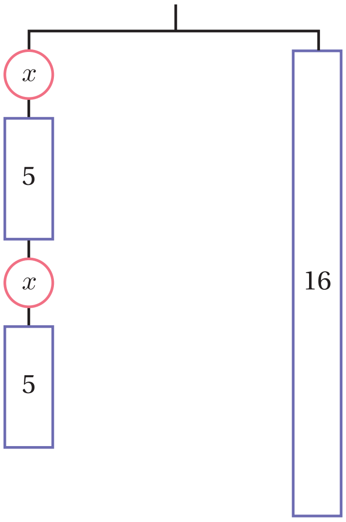
Reflect: How well did you manage your stress level by staying organized?

Name: ________________ Date: ________ Period: ________

Activity 2 Using Hangers to Solve Equations

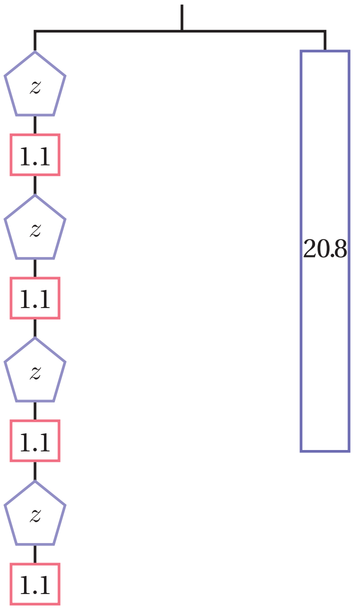
Solve each equation. Show all work. Use the hanger diagram to support your reasoning.

1.



$2(x + 5) = 16$

2.



$4(z + 1.1) = 20.8$

Activity 3 Now You Try

Solve each equation. Draw a hanger diagram, if needed.

1. $3(x+9)=30$

2. $3000=3(y+200)$

3. $\frac{1}{2}\left(x+\frac{2}{3}\right)=\frac{20}{3}$

Are you ready for more?

Solve the equation $\frac{1}{3}(w+4)=\frac{10}{3}$. Show your thinking.

STOP

Name: ______________________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You expanded your equation-solving capabilities to solve equations with quantities grouped in parentheses. You analyzed the equations $14 = 2x + 6$ and $14 = 2(x + 3)$ and saw that these equations are equivalent because of the Distributive Property.

You determined that you could solve equations of the form $p(x + r) = q$ in two ways: using the Distributive Property, or not.

For example, consider the equation $3(x + 1) = 9$.

Using Distributive Property

$$\begin{aligned} 3(x + 1) &= 9 \\ 3x + 3 &= 9 \\ 3x + 3 - 3 &= 9 - 3 \\ 3x &= 6 \\ 3x \div 3 &= 6 \div 3 \\ x &= 2 \end{aligned}$$

Without Using Distributive Property

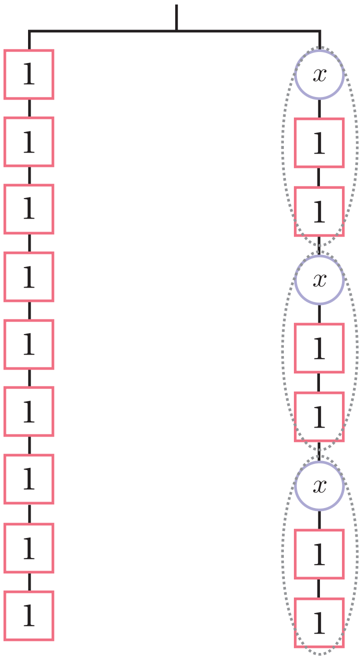
$$\begin{aligned} 3(x + 1) &= 9 \\ 3(x + 1) \div 3 &= 9 \div 3 \\ x + 1 &= 3 \\ x + 1 - 1 &= 3 - 1 \\ x &= 2 \end{aligned}$$

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Refer to the hanger diagram.



a Explain how each part of the equation $9 = 3(x + 2)$ is represented in the hanger diagram.

x ....

9 ....

3 ....

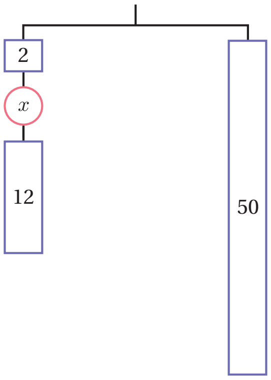
$x + 2$ ....

$3(x + 2)$ ....

$=$ ....

b Solve the equation $9 = 3(x + 2)$.

2. Clare drew the hanger diagram to represent the equation $2(x + 12)=50$. She got an incorrect answer when she solved it.



a What equation does Clare's hanger diagram actually represent?

b Draw the correct hanger diagram for the equation $2(x + 12) = 50$.

c Solve the equation $2(x + 12) = 50$. Show or explain your thinking.

Name: ______________________________ Date: ____________ Period: __________

3. Solve each equation. Show your work.

a $0.5(x + 9) = 32$

b $\frac{1}{3}(y + 8) = 4$

4. Check the box under the value that matches each expression.

Expression	2	−2
$(-2)(-1)$		
$2 \bullet (-1)$		
$0 - (-2)$		
$2 - 0$		
$0 - 2$		
$2 \div (-1)$		
$-2 \div (-1)$		

5. Mentally solve each equation.

a $2x = 10$

b $-3x = 21$

c $\frac{1}{3}x = 6$

d $-\frac{1}{2}x = 7$

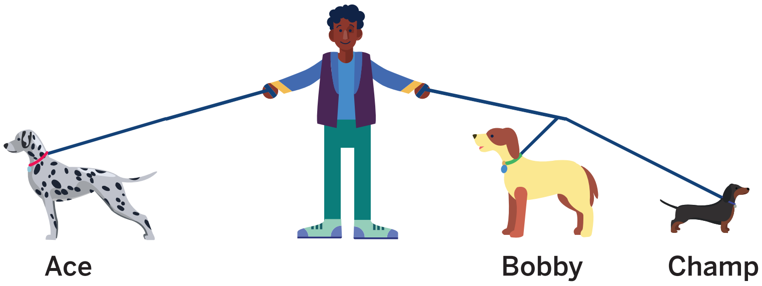
Unit 6 | Lesson 5

Dealing With Negative Numbers

Let's show that doing the same thing to each side of an equation also works for equations with negative numbers.

Warm-up Dogs in Different Directions

Refer to the diagram.
(Assume that the dog walker feels an equal pull from both directions.)



1. Which dog is the strongest? Explain your thinking.

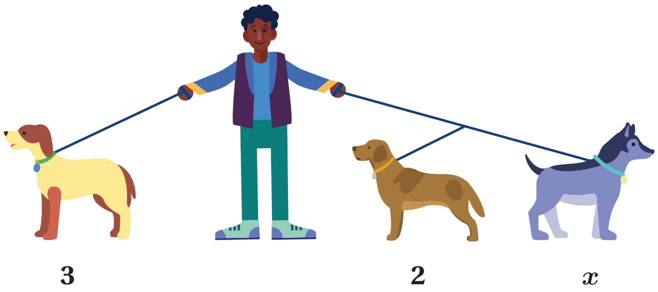
2. If Ace's strength is 10 and Champ's strength is 14, what is Bobby's strength? Explain your thinking.

3. Write an equation that represents the diagram.

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 New Solutions, Old Ways



1. Write an equation that represents the diagram.

2. How else could the dogs be arranged so that the strengths are still balanced? Sketch a diagram of the new arrangement.

3. Write an equation that represents *your* diagram.

4. Determine the value of x for both of the equations you wrote.

Collect and Display: As you describe how your arrangements and equations are similar and different, your teacher will add the language you use to a class display that you can refer to during this unit.

Activity 2 Keeping it True

For each of the three equations shown, complete the following.

- **Analyze each equation.**
- **Place a check next to each equation that is equivalent to the given equation for each problem.**
- **Next, circle the equation that you think represents the best next step for finding the value of x in the original equation.**
- **Lastly, solve the equation for x.**

1. $-x = 10$

☐ $2 \cdot (-x) = 2 \cdot 10$ ☐ $-x + 3 = 10 + 3$ ☐ $-1 \cdot (-x) = -1 \cdot 10$

$x =$ ....

2. $3 - 2x = -5$

☐ $3 - 2x + 4 = -5 + 4$ ☐ $3 - 2x + (-3) = -5 + (-3)$ ☐ $3 - \frac{2x}{2} = -\frac{5}{2}$

$x =$ ....

3. $19 = 3(x - 2)$

☐ $19 + 2 = 3(x - 2) + 2$ ☐ $19 \div 3 = 3(x - 2) \div 3$ ☐ $19 = 3x - 6$

$x =$ ....

STOP

Name: ______________________ Date: ____________ Period: __________

Summary

In today's lesson . . .

You explored how to solve equations with negative numbers. Because negative numbers are just numbers, performing the same operations to each side of an equation involving negative numbers results in an equivalent equation — just as it does with positive numbers. Whenever you perform the same operations to each side of an equation — even if it does not help you solve it — this results in an ***equivalent equation***, which has the same solution.

You can use moves that maintain equality to generate equivalent equations that all have the same solution. Helpful combinations of moves will eventually lead to an equation in which the unknown is by itself on one side of the equal sign — such as $x = 5$. When this happens, you have solved the equation — and every equivalent equation you wrote — in the process.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Match each equation with an equivalent equation.

a $2x - 8 = 16$

b $2x + 8 = -16$

c $-2x + 8 = -16$

d $-2(x + 4) = 16$

.... $2x + 8 - 8 = -16 - 8$

.... $-2x = -24$

.... $2x - 8 + 8 = 16 + 8$

.... $x + 4 = -8$

2. Elena solved the following equation. Her work is shown. She thinks she may have made a mistake. Write a note to Elena telling her whether you think she made a mistake. Explain your thinking.

Elena's work:

$$-7x + 7 = 10$$
$$-7x + 7 - 7 = 10 - 7$$
$$-7x = 3$$
$$-7x \div (-7) = 3 \div (-7)$$
$$x = -\frac{3}{7}$$

3. Solve each equation. Show your thinking.

a $2(x - 5) = -6$

b $-3x + 6 = -8$

Name: .... Date: .... Period: ....

4. Here are some prices customers paid for different items at a farmer's market. Determine the cost for 1 lb of each item.

a $\$5$ for 4 lb of apples

b $\$3.50$ for $\frac{1}{2}$ lb of cheese

c $\$8.25$ for $1\frac{1}{2}$ lb of coffee beans

d $\$6.75$ for $\frac{3}{4}$ lb of fudge

e $\$5.50$ for a $6\frac{1}{4}$ lb pumpkin

5. Here is a set of data showing temperatures. The *range* of a set of data is the distance between its least and greatest values. What is the range of these temperatures?

$9°, -3°, 22°, -5°, 11°, 15°$

6. Evaluate each expression without a calculator.

a $\frac{1}{3} \div 2$

b $7 \div \frac{1}{7}$

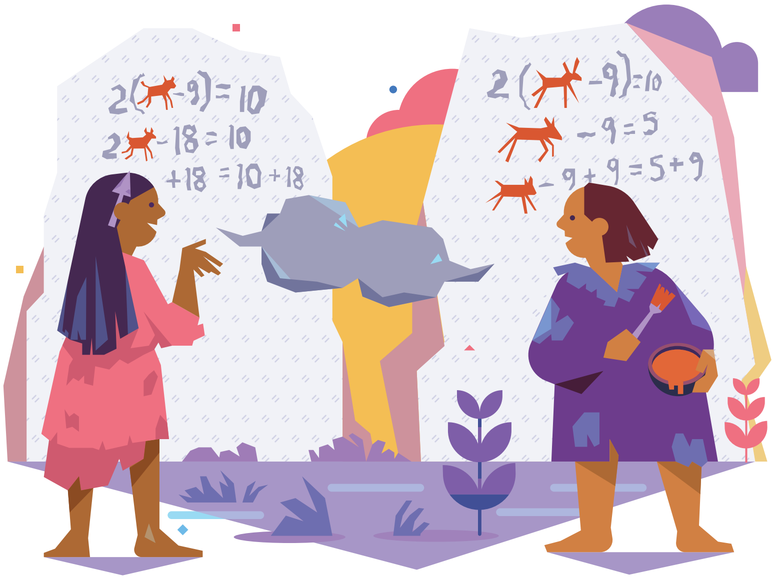
c $-4 \cdot \frac{1}{3}$

d $6 \cdot \frac{2}{3}$

Unit 6 | Lesson 6

Two Ways to Solve One Equation

Let’s think about efficient ways to solve equations with parentheses.



Warm-up Error Analysis

Noah attempted to solve the equation $-2(x - 7) = 32$ and found the solution was $x = -23$. Analyze his work.

$-2(x - 7) = 32$

$-2x - 14 = 32$ Distribute (–2) on the left side.

$-2x - 14 + 14 = 32 + 14$ Add 14 to each side.

$-2x = 46$

$-2x \div (-2) = 46 \div (-2)$ Divide each side by (–2).

$x = -23$

1. Check Noah’s solution by substituting it into the original equation. Show your work.

2. Is Noah correct? If not, explain his error.

3. Solve the equation correctly. Show your work.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Analysis of Work

Elena and Han are each solving the equation $2(x-9)=10$, but they did not finish their work. Their first steps are shown.

Elena's work:	**Han's work:**
$2(x-9)=10$	$2(x-9)=10$
$2x-18=10$	$x-9=5$

1. Which step did Elena do first? What step did Han do first?

2. Complete both methods in their respective boxes.

3. Did you get the same value for x? Why or why not?

4. Which method do you prefer? Explain your thinking.

Reflect: How well did you manage your stress level by staying organized?

Activity 2 Solution Pathways

Try solving each equation by two methods: dividing both sides by the factor in front of parentheses first, or applying the Distributive Property first. Some equations may be more challenging to solve by one method than the other. If this happens, stop the more challenging method and record why you stopped.

	Dividing	Distributive Property
1.	$2\left(x+\frac{5}{4}\right)=3.5$	$2\left(x+\frac{5}{4}\right)=3.5$
2.	$\frac{1}{4}(4+x)=\frac{3}{4}$	$\frac{1}{4}(4+x)=\frac{3}{4}$
3.	$-10(x-1.7)=-3$	$-10(x-1.7)=-3$

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You analyzed equations that could be solved in two specific ways: first applying the Distributive Property, or first dividing by the factor in front of the parentheses. In some cases, it can be more efficient to first apply the Distributive Property, and in other cases, it can be more efficient to first divide by the factor in front of the parentheses.

Because you know different ways to solve equations, you have more tools in your toolbox to help you. You can always stop one method if you feel it is not efficient — or more challenging, and start using the other method.

Reflect:

Name: .... Date: .... Period: ....

1. Kiran and Mai each solved the same equation, but did not show how they arrived at each step. Analyze each student's partial work.

Kiran's work:	**Mai's work:**
$3.3(x - 10) = 66$	$3.3(x - 10) = 66$
$3.3x - 33 = 66$	$x - 10 = 20$
$3.3x = 99$	$x = 30$
$x = 30$	

a How did Kiran begin? Mai?

b Complete the missing steps for each student.

c Which method do you prefer to use for this equation? Why?

2. Solve each equation. Show your thinking.

a $2(x - 3) = 14$

b $-5(x - 1) = 40$

c $\frac{5}{7}(x - 9) = 25$

d $\frac{1}{6}(x + 6) = 11$

Name: .... Date: .... Period: ....

3. Lin and Noah are each solving the equation $7(x + 2) = 91$. Lin begins by using the Distributive Property while Noah begins by dividing each side by 7.

a Show what Lin's and Noah's full solution methods might look like.

b What is the same and what is different about their methods?

4. Complete the magic square so that the sum of each row, each column, and each diagonal in the grid are the same.

0	7	2
	3	

5. Andre wants to buy a backpack. The normal price of the backpack is $\$40$. He notices that a store selling the backpack is having a 30% off sale. What is the sale price of the backpack?

6. Solve each equation. Show your thinking.

a $\frac{1}{3}x + 4 = 3\frac{5}{6}$

b $6(x - 2) = 11$

Unit 6 | Lesson 7

Practice Solving Equations

Let’s practice.

Warm-up Mystery Equation

Using only whole numbers from 1 to 9 at most once, fill in the boxes so that the solution of the equation is as great as possible.

$$\boxed{} + \boxed{}x = \boxed{}$$

Show or explain why your equation gives the greatest possible solution of the equation.

Name: .... Date: .... Period: ....

Activity 1 Equation Chain

This is an equation chain. The solution to each previous equation helps you complete the next one.

- **Solve the equation in Problem 1.**
- **Write the solution for Problem 1 in the box for Problem 2, and solve the new equation.**
- **Continue this process until all four problems are completed.**
- **Your solution to Problem 4 should complete the chain by being one of the numbers from Problem 1.**

1. $4x + 19 = 31$

2. Write your solution from Problem 1 in the box and solve the equation.

$3y - 18 = \boxed{}$

3. Write your solution from Problem 2 in the box and solve the equation.

$3(s + 2) = \boxed{}$

4. Write your solution from Problem 3 in the box and solve the equation.

$-\frac{1}{9}(28 - p) = \boxed{}$

Activity 2 Trading Equations

Plan ahead: How will you communicate that you need help? How will you communicate that you have help to offer?

You will be given a sheet containing some incomplete equations. You and your partner will take turns solving each other's equations.

For each incomplete equation, complete these tasks:

- **Choose a secret number and write it down (so you do not forget it). Do not show it to your partner.**
- **Substitute your secret number for x and evaluate the left side of the equation. Write this value in the empty box on the right side of the equation.**
- **Fill in the final number on the sheet you received from your teacher.**
- **Switch sheets with your partner. Solve your partner's equation and check each other's answers when you are both ready.**

Repeat the process for Equation 2.

Equation 1 secret number: ....

Work Space:

$$\frac{1}{9}(x+7)=\boxed{}$$

Fill this value in the box on the sheet for your partner.

Equation 2 secret number: ....

Work Space:

$$6(x-2)=\boxed{}$$

Fill this value in the box on the sheet for your partner.

STOP

Name: ______________________ Date: __________ Period: ________

Summary

In today's lesson . . .

You applied your understanding of solving equations to fluently solve equations with and without parenthesis.

Equation without parentheses	Equation with parentheses
$3x - 6 = 9$ $3x - 6 + 6 = 9 + 6$ $3x = 15$ $3x \div 3 = 15 \div 5$ $x = 5$	$3(x - 6) = 9$ $3(x - 6) \div 3 = 9 \div 3$ $x - 6 = 3$ $x - 6 + 6 = 3 + 6$ $x = 9$

You checked your solutions by substituting the value for the variable, and evaluating to determine whether the equation is true.

$3x - 6 = 9$ $3(5) - 6 = 9$ $15 - 6 = 9$ $9 = 9$ true	$3(x - 6) = 9$ $3(9 - 6) = 9$ $3(3) = 9$ $9 = 9$ true

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Solve each equation. Show your thinking.

a $31 = 10 + 2y$

b $6.8 = 2z + 2.2$

2. Solve each equation. Show your thinking.

a $2000(x - 0.03) = 6000$

b $-\frac{3}{2}(x + 12) = \frac{15}{2}$

3. Select *all* expressions that represent a correct solution to the equation $6(x + 4) = 20$.

A. $(20 - 4) \div 6$

B. $\frac{1}{6}(20 - 4)$

C. $20 - 6 - 4$

D. $20 \div 6 - 4$

E. $\frac{1}{6}(20 - 24)$

F. $(20 - 24) \div 6$

4. A 200 lb person weighs 33 lb on the Moon.

a By how many pounds did the person's weight decrease?

b By what percent did the person's weight decrease?

Name: .... Date: .... Period: ....

5. What are the missing operations? Complete each equation with +, –, •, or ÷.

a $48 \boxed{} (-8) = -6$

b $-40 \boxed{} 8 = -5$

c $12 \boxed{} (-2) = 14$

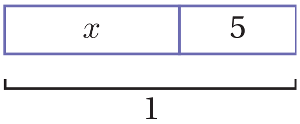
d $18 \boxed{} (-12) = 6$

e $18 \boxed{} (-20) = -2$

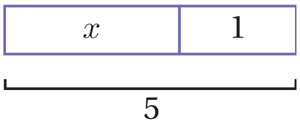
f $22 \boxed{} (-0.5) = -11$

6. Lin had 5 pencils. She gave some away. Now she has 1 pencil left. Which tape diagram matches this story? Select *all* that could apply.

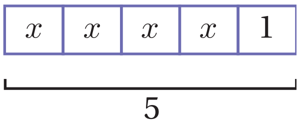
A.



B.



C.



Explain your thinking.

My Notes:

SUB-UNIT

2 Solving Real-World Problems Using Two-Step Equations

Narrative Connections

Who were the VIPs of ancient Egypt?

The scribes and the accountants. Seriously.

In ancient Egypt, if you were a scribe, you were high on the food chain. Almost anything anyone wanted done had to go through you.

Stocking grain? Get a scribe.

Building a bridge? Get a scribe.

Waging war? Get a scribe.

And practically everything we know about ancient Egypt is thanks to their meticulous record-keeping. When only one percent of the population could read or write, scribes had their fingers in everything: business, politics, religion, and engineering.

Knowing how to read and write was great, but if you wanted to go even further, you had to know your math. Advanced scribes were trained to calculate how to build ramps, allocate land, distribute food, and supply armies. Where we might use a variable such as x, an ancient Egyptian may have used the “Aha” — a hieroglyph meaning “heap.”

The ability to do math, such as solving equations to solve real-world problems, was a sign of great skill in a scribe. There are even records of scribes challenging each other with math problems, with all the swagger of a mic drop.

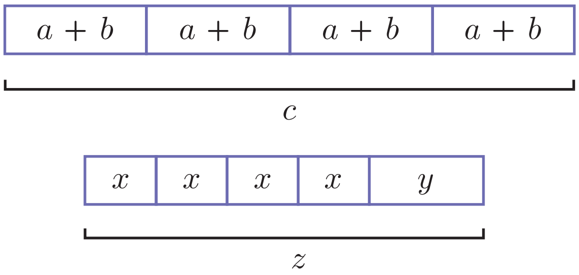
Unit 6 | Lesson 8

Reasoning With Tape Diagrams

Let’s use tape diagrams to write equations for different scenarios.

Warm-up Notice and Wonder

Study the tape diagrams.
What do you notice?
What do you wonder?



1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

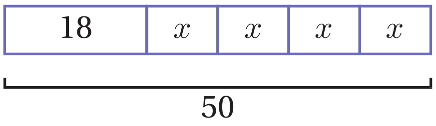
Name: .... Date: .... Period: ....

Activity 1 Every Picture Tells a Story

Here are two stories. Each story has a tape diagram that represents it. For each story:

- **Explain how the diagram represents the story.**
- **Describe the unknown amount in the story.**
- **Write a question about the story the diagram could help answer.**

1. Pashedu ordered 50 limestone blocks for the foundation of a pillar. Among his 5 workers, the first worker was given 18 blocks, while the remaining blocks were divided equally among the other 4 workers.

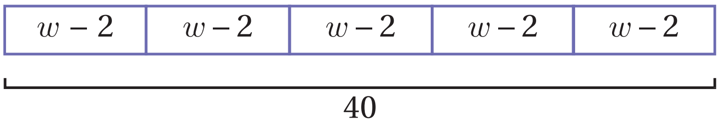


Explanation:

Unknown:

Question:

2. To decorate the pillar, Pashedu distributed a stack of gold foil sheets equally among his 5 workers. But, then he remembered that he also needed some gold foil to decorate the Pharaoh's palace, so he asked for 2 sheets back from each worker. After they gave him these sheets, the workers had a total of 40 sheets left.



Explanation:

Unknown:

Question:

Activity 2 Card Sort: Sorting Tape Diagrams

You will be given a set of tape diagram cards. Sort the tape diagrams into two categories of your choosing. In the following boxes, explain the criteria for each category. Then place each tape diagram in the box under its corresponding category.

Criteria for Category 1:

Tape diagrams:

Criteria for Category 2:

Tape diagrams:

Are you ready for more?

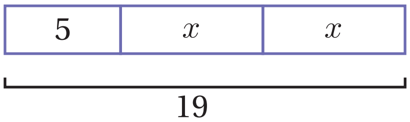
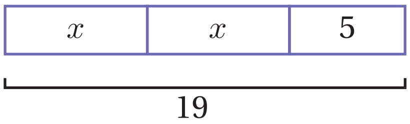
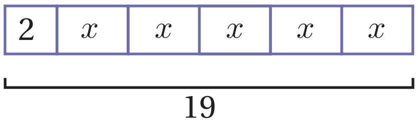
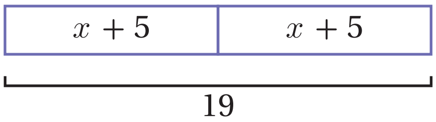
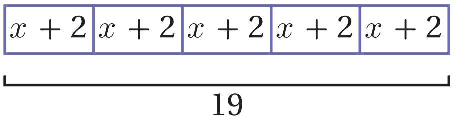
You will be given a set of story cards. Match each story to the tape diagram that it represents. One story may have more than one match.

Name: ________________ Date: ________ Period: ________

Activity 3 Matching Equations and Tape Diagrams

Match each equation with the tape diagram it represents. You may use some equations more than once. Be prepared to explain how each equation represents its diagram.

$2x + 5 = 19$	$5(x + 2) = 19$	$19 = 5 + 2x$
$2(x + 5) = 19$	$2 + 5x = 19$	$(x + 5) \cdot 2 = 19$

Tape diagram	Equation(s)
a 	
b 	
c 	
d 	
e 	

Are you ready for more?

1. Match each of these equations with the tape diagram it represents.

   $2x + 10 = 19$ $\quad$ $19 \div 2 = x + 5$ $\quad$ $19 - 2 = 5x$ $\quad$ $\frac{1}{5} \cdot 19 = x + 2$

2. Write your own equation for each tape diagram.

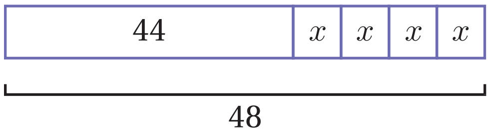
STOP

Summary

In today's lesson . . .

You used tape diagrams and equations to represent stories. Because of the properties of addition and multiplication, more than one equation can represent a tape diagram. When two or more equations can be used to represent the same diagram, the equations are *equivalent*.

A stall at an Egyptian market had 48 olives in a basket. One shopper traded for 44 of them, and the remainder were divided evenly between 4 more shoppers.



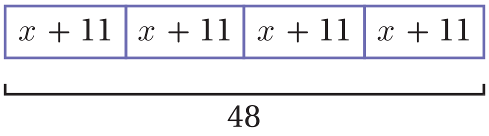
Here are some equations that describe this diagram:

$44 + 4x = 48$

$4x + 44 = 48$

$48 = 4x + 44$

A stall in an Egyptian market has 48 olives in a basket. The olives came from two farms. The olives from the first farm were traded in equal amounts to 4 shoppers. When the olives arrived from the second farm, each of the 4 shoppers traded for 11 additional olives.



Here are some equations that describe this diagram:

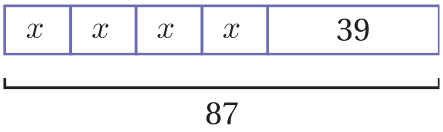
$4(x + 11) = 48$

$(x + 11) \cdot 4 = 48$

Reflect:

Name: .... Date: .... Period: ....

1. Circle *all* the stories this tape diagram can represent.



A. There are 87 children and 39 adults at a show. The seating in the theater is partitioned into 4 equal sections.

B. There are 87 first-graders in after-school care. After 39 students are picked up by their families, the teacher places the remaining students into 4 equal groups for an activity.

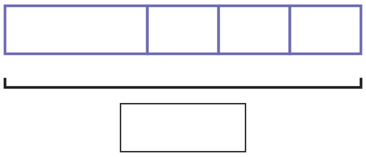
C. Lin buys a pack of 87 pencils. She gives 39 to her teacher and shares the remaining pencils among herself and 3 friends.

D. Andre buys 4 packs of paper clips with 39 paper clips in each pack. Then he gives 87 paper clips to his teacher.

E. Diego's family spends \$87 on 4 tickets to the fair and a dinner that costs \$39.

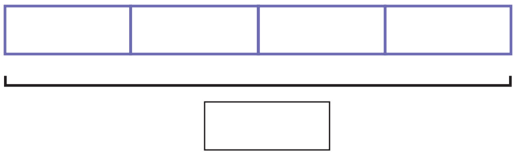
2. Label each part of each tape diagram with either a number, a variable, or an expression to match the story. Then write an equation that represents the diagram and the story.

a Today at doggy daycare, there are 12 dogs. Six of the dogs went outside to play. The rest of the dogs were divided evenly among three inside rooms: one for eating, one for playing, and one for grooming.



Equation:

b Priya is feeding her four cats. Each cat has its own bowl. She puts the same number of scoops of food in each bowl. Then she adds two more scoops to each bowl. Altogether she uses 16 scoops of food.

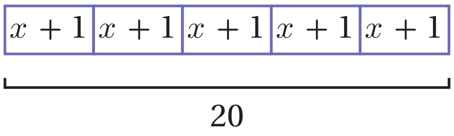
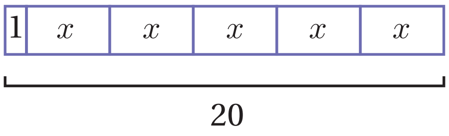


Equation:

Name: .... Date: .... Period: ....

3. The equations $5(x + 1) = 20$ and $5x + 1 = 20$ each represent one of the following tape diagrams. For each tape diagram, complete the following tasks.

- Write each equation under the diagram it represents.
- Write an equivalent equation that could also represent the tape diagram.
- Write two stories that could each be described by each tape diagram.

Tape diagram		
Equation		
Equivalent equation		
Story		

4. Without evaluating each expression, determine which value is the greatest. Explain your thinking.

A. $7\frac{5}{6} - 9\frac{3}{4}$

B. $\left(-7\frac{5}{6}\right) + \left(-9\frac{3}{4}\right)$

C. $\left(7\frac{5}{6}\right) \cdot \left(-9\frac{3}{4}\right)$

D. $\left(-7\frac{5}{6}\right) \div \left(-9\frac{3}{4}\right)$

5. Diego has a total of 30 pieces of clay. He wants to divide it evenly between himself and 5 friends. Write an equation to represent this scenario. Identify what the variable represents in your equation.

Name: .... Date: .... Period: ....

Unit 6 | Lesson 9

Reasoning About Equations and Tape Diagrams (Part 1)

Let’s see how tape diagrams can help us answer questions about unknown amounts in stories.

Warm-up Equation String

Mentally solve each equation. Write each solution, and be prepared to discuss any patterns you notice.

$x + 4 = 10$

$x + 8 = 10$

$12 + x = 10$

$x + 4 = 20$

$x + 8 = 20$

$12 + x = 20$

$x + 4 = 30$

$x + 8 = 30$

$12 + x = 30$

Log in to Amplify Math to complete this lesson online.

Activity 1 Scenarios and Diagrams

Draw a tape diagram to represent each scenario. For some scenarios, you first need to choose a variable to represent the unknown quantity.

Scenario	Tape diagram
1. A builder has 7 equal sets of large stones for building a pyramid. After 9 additional stones are delivered, he has a total of 30 large stones.	
2. A scribe has a scroll of papyrus measuring 30 cubits long (a cubit is about 18 in.). He cuts off 7 cubits, and then cuts the remaining length into 9 equal lengths of x cubits each.	
3. A merchant in the market had 7 identical duck eggs, and a small goose egg weighing 9 debens (a deben is about 13.6 ounces). The total weight of the eggs was 30 debens. The variable is ________ and it represents ________________________________.	
4. Two scribes made a trade. The first scribe offered an equal number of cloves of garlic for each of the 7 eggs he received plus an additional 9 cloves to seal the deal. In total, he traded 30 cloves of garlic for the 7 eggs. The variable is ________ and represents ________________________________.	
5. A baker baked 9 large loaves of bread. He kept 7 for his shop, then divided the remaining loaves into 30 identical slices to sell at the market. The variable is ________ and represents ________________________________.	

Name: .... Date: .... Period: ....

Activity 2 Scenarios, Diagrams, and Equations

Using the first two scenarios from Activity 1, complete the following tasks. Use your diagrams from Activity 1 to help you.

1. A builder has 7 equal sets of large stones for building a pyramid. After 9 additional stones are delivered, he has a total of 30 large stones.

$7x + 9 = 30$ $9x + 7 = 30$ $30x + 7 = 9$

a Which equation represents the scenario? ....

b x represents ....

c Solve the equation.

d Interpret the solution.

2. A scribe has a scroll of papyrus measuring 30 cubits long. He cuts off 7 cubits, and then cuts the remaining length into 9 equal lengths of x cubits each.

$7x + 9 = 30$ $9x + 7 = 30$ $30x + 7 = 9$

a Which equation represents the scenario? ....

b x represents ....

c Solve the equation.

d Interpret the solution.

STOP

Summary

In today's lesson . . .

You used tape diagrams to help you to write equations of the form $px + q = r$. Writing an equation to represent a scenario can help you express how quantities in the scenario are related to each other. Writing an equation can also help you reason about unknown quantities whose values you want to find.

In the next lesson, you will work with equations of the form $p(x + q) = r$.

Reflect:

Name: .... Date: .... Period: ....

1. Consider these two scenarios.

Scenario 1: A family buys 3 tickets to a show. They also pay a \$6 parking fee. They spend a total of \$27 to see the show.

Scenario 2: Diego has 27 oz of juice. He pours equal amounts for each of his 6 friends and has 3 oz left for himself.

Here are two equations: $6x + 3 = 27$ and $3x + 6 = 27$.

For each scenario, complete the following tasks. Use the table to organize your responses.

- **a** Decide which equation represents each scenario. What does x represent in each equation?
- **b** Determine the solution to each equation. Show or explain your thinking.
- **c** What does each solution tell you about its scenario?

	Scenario 1	Scenario 2
a		
b		
c		

2. Draw a tape diagram and write an equation to represent the following scenario. Tyler and 3 of his friends each spend $\$x$ for movie tickets. They also spend \$13 on popcorn. The total cost of the tickets and popcorn is \$39.

Name: .... Date: .... Period: ....

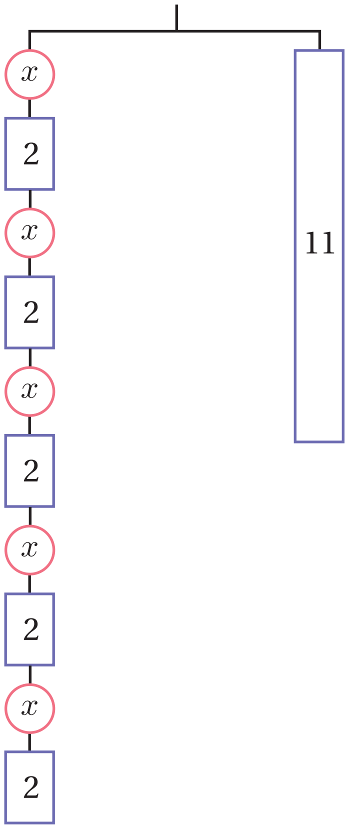
3. Priya designed an app to help students organize their homework time. The app was downloaded 300 times in the first week, and, in each following week, it was downloaded 400 more times. How many more weeks did it take for the app to be downloaded 1,500 times? Write and solve an equation. Explain what your solution represents in the scenario.

4. On their first math exam, 16 students received an A for a grade. On the second math exam, 12 students received an A for a grade. What is the percent change of students receiving an A for a grade? Show or explain your thinking.

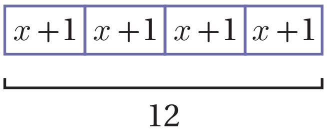
5. Refer to the hanger diagram.

a Write an equation that represents the hanger diagram.

b Solve the equation by reasoning about the equation or the hanger diagram. Show or explain your thinking.



6. Choose *all* of the expressions that match the tape diagram.



A. $4x + 4 = 12$

B. $4(x + 1) = 12$

C. $x + 1 = 12 \cdot 4$

D. $12 - (x + 1) = 3(x + 1)$

E. $x + 4 = 12$

Name: ________________ Date: ________ Period: ________

Unit 6 | Lesson 10

Reasoning About Equations and Tape Diagrams (Part 2)

Let's see how tape diagrams can help us answer questions about unknown amounts in stories.

Warm-up Equation String

Solve each equation and write the answers. Be prepared to discuss any patterns you notice.

$2x + 4 = 10$

$2x + 8 = 10$

$12 + 2x = 10$

$2x + 4 = 20$

$2x + 8 = 20$

$12 + 2x = 20$

$2x + 4 = 30$

$2x + 8 = 30$

$12 + 2x = 30$

Log in to Amplify Math to complete this lesson online.

Activity 1 Connecting Scenarios and Diagrams

Draw a tape diagram to represent each scenario. For some scenarios, you will need to choose a variable to represent an unknown quantity.

Scenario	Tape diagram
1. A Pharoah's tomb has 5 baskets with x pieces of fruit in each. A priest adds 3 more pieces of fruit to each basket. Altogether, the baskets contain 20 pieces of fruit.	
2. Noah draws a model of a pyramid where the face is an equilateral triangle with sides that are 5 in. long. He wants to increase the length of each side by x in., so the triangle is still equilateral, but has a perimeter of 20 in.	
3. An art class charges each student \$3 to attend, plus a fee for supplies. Today, \$20 was collected for the 5 students attending the class. The variable is ________ and represents ____________________	
4. The northern division of the Pharoah's army marched 20 miles. This was 3 times as far as the southern division marched. The southern division marched 5 more miles than the western division. The variable is ________ and represents ____________________	

Name: .... Date: .... Period: ....

Activity 2 More Scenarios, Diagrams, and Equations

Plan ahead: How will you encourage your partner to show self-discipline throughout the task?

Using the scenarios from Activity 1, complete the following.

- **Match each scenario to one of these equations. Use your diagrams from Activity 1 to help you.**

$(x + 3) \bullet 5 = 20$ $\quad$ $3(x + 5) = 20$

- **Find the solution to each equation.**
- **Interpret the solution within the context of the problem.**

1. A Pharoah's tomb has 5 baskets with x pieces of fruit in each. A priest adds 3 more pieces of fruit to each basket. Altogether, the baskets contain 20 pieces of fruit.

a Equation: ....

b x represents ....

c Solve the equation.

d Interpret the solution.

2. Noah draws a model of a pyramid where the face is an equilateral triangle with sides that are 5 in. long. He wants to increase the length of each side by x in., so the triangle is still equilateral, but has a perimeter of 20 in.

a Equation: ....

b x represents ....

c Solve the equation.

d Interpret the solution.

STOP

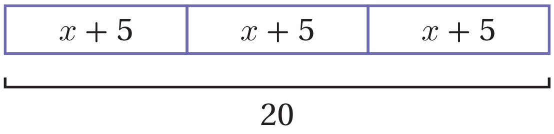
Summary

In today's lesson . . .

You spent time representing, writing, and solving equations of the form $p(x + q) = r$. For example, consider the scenario:

Elena ran 20 miles this week, which was 3 times as far as Clare ran this week. Clare ran 5 more miles this week than she did last week.

If x represents the number of miles Clare ran last week, this scenario can be modeled using a tape diagram:



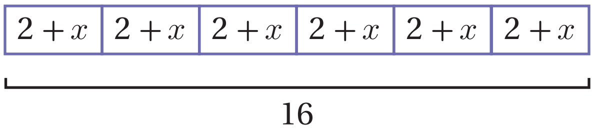
It can also be represented by the equation $3(x + 5) = 20$.

Using a tape diagram helps to make sense of a scenario in order to be able to write an algebraic representation.

Reflect:

Name: ________________ Date: ________ Period: ________

1. Refer to the tape diagram.



Lin claims that the tape diagram could match the following scenario:

Kiran had 2 bags and filled each bag with 6 erasers. Then, he added the same number of erasers to each bag. He used 16 erasers in all.

Do you agree with Lin? Why or why not? Explain your thinking.

2. Match each story with the equation that represents it. Explain your thinking.

Stories

a A family of 6 buys tickets to a show. Each member of the family also spends \$3 on a snack. They spend \$24 in all.

b Diego pours a total of 24 oz of juice for his 3 friends. He first pours an equal amount for each friend, and then adds 6 oz to each.

Equations

________ $3(x+6)=24$

________ $6(x+3)=24$

3. Solve each equation. Show your thinking.

a $45 = 4(x+6)$

b $\frac{2}{5}(x+8)=12$

Name: .... Date: .... Period: ....

4. Determine each product.

a $\frac{2}{3} \cdot \left(-\frac{4}{5}\right)$

b $-\frac{2}{39} \cdot 39$

c $-\frac{5}{7} \cdot \left(\frac{7}{5}\right)$

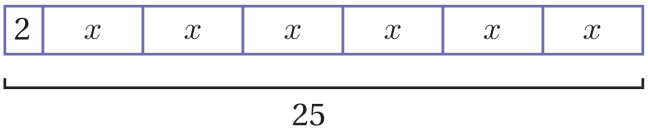
d $\frac{2}{5} \cdot \left(-\frac{3}{4}\right)$

5. Using any whole number from 1 to 9 at most once, fill in each box so that the solution of the equation is as great as possible.

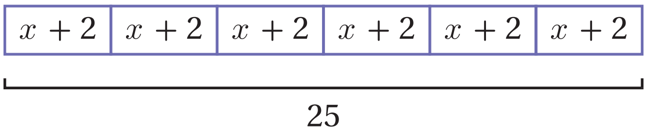
$\square(x + \square) = \square$

6. Complete the following problems about the tape diagrams.

a This diagram can be represented by the equation $25 = 2 + 6x$. Explain where you can see the 6 in the diagram.



b This diagram can be represented by the equation $25 = 6(x + 2)$. Explain where you can see the 6 in the diagram.



Name: ______________________ Date: __________ Period: __________

Unit 6 | Lesson 11

Using Equations to Solve Problems

Let's use equations with and without parentheses to solve problems.

Warm-up Which One Doesn't Belong?

Which equation doesn't belong? Explain your thinking.

A. $4(x + 3) = 9$

B. $4x + 3 = 9$

C. $4 \bullet x + 12 = 9$

D. $9 = 12 + 4x$

Log in to Amplify Math to complete this lesson online.

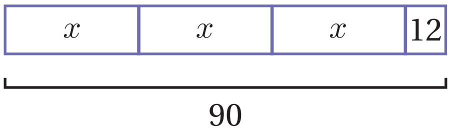
Activity 1 Scenarios, Diagrams, and Equations

Lin is learning about the difference between thermal energy and temperature in science class. She learns if two different amounts of liquids — like a small cup of water and a large bathtub of water — have the same temperature — they will have different amounts of thermal energy. This is because the bathtub has more hot water than the cup, and so it has more thermal energy.

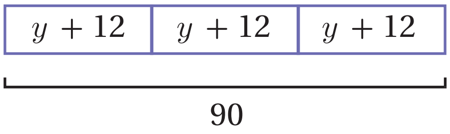
She is conducting experiments with three cups of water that all start out with 12 units of thermal energy.

Use the tape diagrams and scenarios to complete the following problems.

Tape diagram A



Tape diagram B



Scenario 1: In the first experiment, Lin adds three identical immersion heaters to one small cup of water, and the amount of thermal energy in the cup increases to 90 units. How many units of thermal energy did each heater add to the water?

Scenario 2: In the second experiment, Lin heats three small cups of water on three identical hot plates, and then mixes the cups of water all together. The total amount of thermal energy in the mixture is 90 units. How many units of thermal energy did each hot plate add to each small cup of water?

1. Which tape diagram represents each scenario? Explain your thinking.

2. In each tape diagram, what part of the scenario does the variable represent?

Tape diagram A:

Tape diagram B:

Name: ______________________ Date: __________ Period: __________

Activity 1 Scenarios, Diagrams, and Equations (continued)

3. Write an equation that represents each scenario. Use the tape diagrams from the previous page to help with your thinking. Then solve the equation and interpret the solution within the context of the scenario.

Scenario 1

Equation:

Solution:

Interpret the solution:

Scenario 2

Equation:

Solution:

Interpret the solution:

Activity 2 Science Club

Stronger and Clearer: You will create a visual display of one of these problems and participate in a Gallery Tour at the end of this activity. Use any feedback you receive to improve your display.

Read each scenario. Define a variable to represent the unknown, and use it to write an equation that represents the scenario. Then solve your equation to answer the question, and describe what the solution represents in the scenario. Draw a tape diagram to help, if needed.

1. Priya and Elena are members of the Science Club at school. Last week, they observed two families of birds nesting in the woods. The members divided into two equal groups; each group studied one family. Every day after school for 5 days, each group hiked to visit their bird family. Priya's group hiked 2.5 fewer kilometers each day than Elena's group. If Priya's group hiked a total of 19 km last week, how far did Elena's group hike each day?

Variable:

Equation:

Description:

2. Priya and Elena plan a fundraiser for the Science Club. They begin with a balance of $-\$80$, because of expenses. In the first hour of the fundraiser, they collect equal donations of $\$4.50$ from a number of families, bringing their balance to $-\$44$. How many families donated money?

Variable:

Equation:

Description:

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You analyzed two types of equations that can represent different kinds of real-world problems.

$4t - 5 = 27$	$4(t - 5) = 27$
The Science Club ordered the same number of small, medium, large, and extra large t-shirts to sell for a fundraiser. After selling 5 shirts, there were 27 remaining. How many of each size did they order?	Han paid \$27 for 4 identical t-shirts. What was the original price of one t-shirt if each t-shirt had a \$5 discount?

The equation $4t - 5 = 27$ would represent a scenario in which 4 of an unknown amount minus 5 equals 27. The equation $4(t - 5) = 27$ would represent a scenario in which there are 4 groups — but *each group* is equal to an unknown amount minus 5 — with the same total of 27.

Reflect:

Name: .... Date: .... Period: ....

1. Match each equation with a scenario it could represent. Two of the scenarios will match with the same equation.

Equation

- **a** $3(x + 5) = 17$
- **b** $3x + 5 = 17$
- **c** $5(x + 3) = 17$
- **d** $5x + 3 = 17$

Scenario

- .... Jada's teacher fills a travel bag with 5 copies of a textbook. The weight of the bag and books is 17 lb. The empty travel bag weighs 3 lb. How much does each book weigh?
- .... Three identical rectangles are used as scenery in a play. Each rectangle was 5 ft long, and then increased by the same amount. The total length of the 3 rectangles is now 17 ft. By what amount was each rectangle increased?
- .... Elena spends \$17 and buys each of her 5 cousins a \$3 book and a bookmark. How much does each bookmark cost?
- .... Noah packs up bags at the food pantry to deliver to families. He packs 5 bags that weigh a total of 17 lb. Each bag contains 3 lb of groceries and a packet containing health-related information. How much does each packet weigh?
- .... Andre has 5 pens and 3 times as many pencils as Noah. He has 17 pens and pencils altogether. How many pencils does Noah have?

Write and solve an equation to solve each of Problems 2 and 3. Then describe what the solution represents in the scenario. Draw a tape diagram to help, if needed.

2. Kiran works in a bookstore and is stocking a shelf with a new bestseller. He can fit 38 copies of the book on the shelf, with 3 in. left over. If the shelf is 60 in. wide, how wide is each book?

a **Equation:**

b **Description:**

Name: ________________ Date: ________ Period: ________

3. Diego scored 9 points less than Andre in the basketball game. Noah scored twice as many points as Diego. If Noah scored 10 points, how many points did Andre score?

a **Equation:**

b **Description:**

4. In football, the team that has possession of the ball has four chances, called *downs,* to gain at least ten yards. If they do not gain at least ten yards, the other team gets the ball. Select *all* of the sequences of plays that result in the team keeping possession of the ball. (Positive numbers represent a gain of yards and negative numbers represent a loss.)

A. $8, -3, 4, 21$

B. $6, -7, 10, -12$

C. $2, 16, -5, -3$

D. $5, -2, 20, -1$

E. $7, -3, -13, 2$

5. Select *all* of the expressions that represent 75% of x.

A. $0.75x$

B. $75x$

C. $0.50x + 0.25x$

D. $1 - 0.25x$

E. $x - 0.25x$

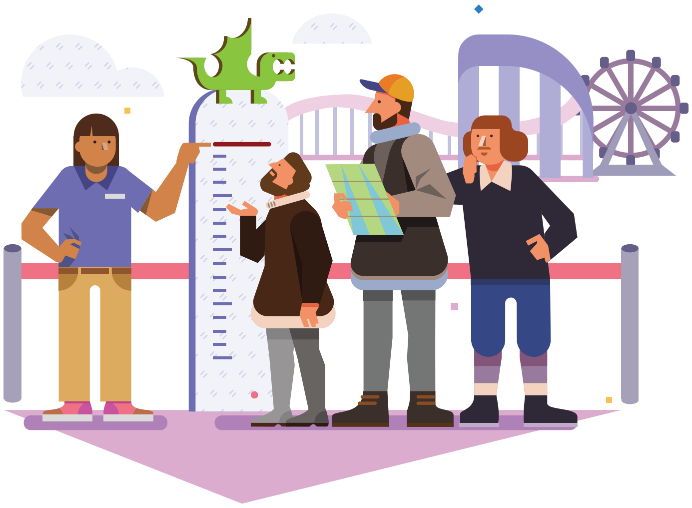
F. $\frac{75}{100}x$

G. $(1 - 0.25)x$

Unit 6 | Lesson 12

Solving Percent Problems in New Ways

Let's use tape diagrams, equations, and reasoning to solve problems with negative numbers and percents.



Warm-up Locating Expressions

1. Draw a line from each expression to its location on the line segment from 0 to x. If an expression does not fit on the line segment, cross it out.

$\frac{20}{100}x$	$x - \frac{20}{100}x$	$(1 - 0.20)x$	$\frac{100-20}{100}x$	$0.80x$	$(100 - 20)x$

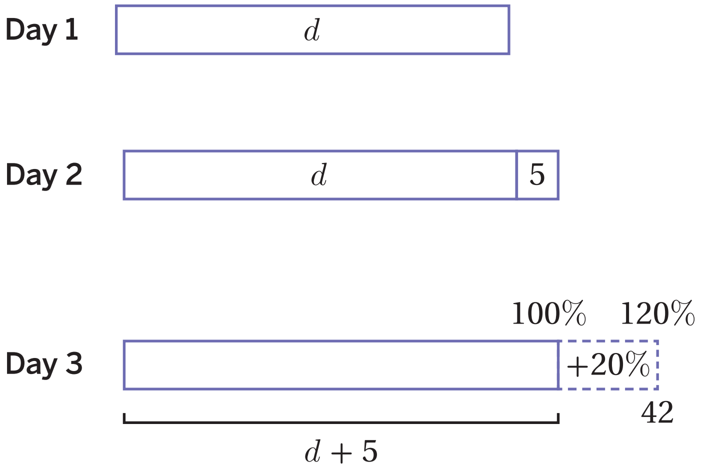
2. An item costs x dollars, and then a 20% discount is applied. Of the given expressions, which expression(s) represent this amount? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Training More Each Day

Mai started a new exercise program. On Day 2, she completed 5 more push-ups than on Day 1. On Day 3, she increased her number of push-ups from Day 2 by 20%, and completed 42. Mai likes to visualize her improvement, so she drew these tape diagrams.



How many push-ups did Mai complete on Day 1? Show or explain your thinking.

Activity 2 Selling Shoes

Solve these problems using any strategy you find helpful.

1. A store is having a sale in which all shoes are discounted by 20%. Diego has a coupon for $\$3$ off the regular price for one pair of shoes. The store first applies this coupon, and then takes 20% off the reduced price. If Diego pays $\$18.40$ for a pair of shoes, what was the original price before the sale and without the coupon?

2. Before the sale, the store had 100 pairs of flip-flops in stock. After selling some, they notice that $\frac{3}{5}$ of the pairs they have left are blue. If the store has 39 pairs of blue pairs left, how many pairs of flip-flops (of any color) have they sold?

Are you ready for more?

A coffee shop offers a special: receive 33% more coffee for free, or receive 33% off the regular price. Which offer is a better deal? Explain your thinking.

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You solved problems in which there was a percent increase or decrease by using what you know about equations and tape diagrams. You can use either method — solving an equation or reasoning using tape diagrams — to solve these types of problems.

Reflect:

Practice

Name: ____________________ Date: ____________ Period: ____________

1. Select *all* the expressions that represent the quantity x increased by 35%.

A. $1.35x$

B. $\frac{35}{100}x$

C. $x + \frac{35}{100}x$

D. $(1 + 0.35)x$

E. $\frac{100 + 35}{100}x$

F. $(100 + 35)x$

2. When a store had sold $\frac{2}{9}$ of the boots that were on display, they brought out another 34 pairs from the stockroom. Then, there were 174 pairs of boots out on display. How many pairs were on display originally? Show your thinking.

3. A store donated 50 pairs of shoes to a homeless shelter. Then they sold 64% of their remaining inventory during a sale. If the store had 288 pairs after the donation and the sale, how many pairs of shoes did they have before donating to the homeless shelter? Show your thinking.

Name: .... Date: .... Period: ....

4. Determine each product.

a $\frac{2}{5} \cdot (-10)$

b $-8 \cdot \left(-\frac{3}{2}\right)$

c $\frac{10}{6} \cdot 0.6$

d $\left(-\frac{100}{37}\right) \cdot (-0.37)$

5. Complete each sentence with the word *discount, deposit,* or *withdrawal.* A word may be used more than once.

a Clare took \$20 out of her bank account.
She made a ....

b Kiran used a coupon when he bought a pair of shoes.
He received a ....

c Priya added \$20 into her bank account.
She made a ....

d Lin paid less than usual for a pack of gum because it was on sale.
She received a ....

6. Determine if each value is a possible solution given the scenario or inequality.

	0	14	15	16	20
An elevator can hold up to 15 people	☐	☐	☐	☐	☐
Han has exactly 15 video games	☐	☐	☐	☐	☐
$x > 15$	☐	☐	☐	☐	☐

My Notes:

SUB-UNIT

3

Inequalities

Narrative Connections

Did a member of the School of Night infiltrate your math class?

More or less . . .

Meet Thomas Harriot.

Harriot was thought to be a member of the School of Night, a secret society from the late 1500s and led by Sir Walter Raleigh. They supposedly studied what was “forbidden knowledge” at the time. (Most scholars do not believe the School of Night ever existed. *But that is exactly the kind of thing you would expect for a secret society*!)

You might recognize Raleigh’s name. He organized voyages from England to Virginia in the 1580s, including establishing the colony on Roanoke Island. Thomas Harriot helped design Raleigh’s ships, and provided considerable navigational expertise. But of all his accomplishments, the one you are most likely to know is the creation of the greater than and less than symbols.

No one knows *how* Harriot came up with these symbols, or if Harriot was truly the original author. One theory suggests that Harriot saw a ⪤ design tattooed on the arm of an indigenous person at Roanoke, which Harriot then separated out to make the > and < symbols we know.

Today, mathematicians use the symbols > and < to describe inequalities. You can use them to show that one value is more or less than another, like how five ships is greater than three ships. But you can also use the same symbols to describe a *need* – like needing more than five ships’ worth of supplies to keep a colony stocked.

Unit 6 | Lesson 13

Reintroducing Inequalities

Let’s work with inequalities.

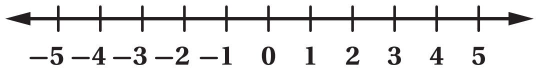
Warm-up Greater Than One

Study the diagram.

1. Select *all* the values that could represent x so that the dog walker is pulled to the left.

A. 3	**C.** -3	**E.** 5	**G.** 0
B. 0.6	**D.** 1	**F.** 1.05	

2. Plot the solutions from Problem 1 on the number line.



3. Write an inequality to represent *all* the solutions from Problem 2.

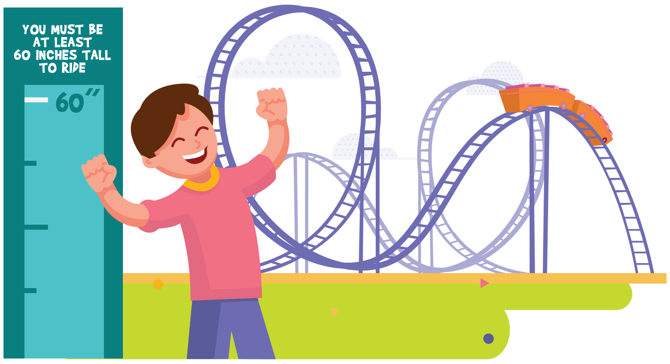
Critique and Correct: Your teacher will provide an incorrect statement about this scenario. Work with a partner to identify the error and correct the statement.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 The Roller Coaster

A sign next to a roller coaster at an amusement park reads, “You must be at least 60 in. tall to ride.” Noah is happy to know that he is tall enough to ride.

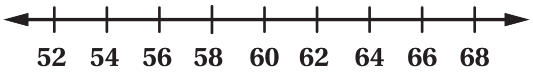


1. Noah is x in. tall. Which of the following can be true: $x > 60$, $x = 60$, or $x < 60$? Explain your thinking.

2. Noah’s friend is 2 in. shorter than Noah. Can you tell if Noah’s friend is tall enough to go on the ride? Explain your thinking.

3. List one possible height for Noah that would mean his friend is tall enough to go on the ride, and another which would mean his friend is too short for the ride.

4. On the number line below, show *all* the possible heights of Noah’s friend.

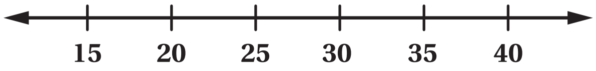


5. Noah’s friend is y in. tall. Use y and one of the symbols $<$, $=$, or $>$ to express his height.

Activity 2 Understanding Inequalities

Clare wants to buy her mother a gift which costs at least $20.
Let x represent the amount of money Clare will need.

1. This scenario is represented by the inequality $x \geq 20$. Graph the solutions to $x \geq 20$ on the number line.



2. If Clare drives to the mall, she must spend $5 on parking. The scenario is now represented by the inequality $x - 5 \geq 20$. Graph the solutions on the number line. How did the solutions change from $x \geq 20$?

3. If Clare's father gives her $5 and drops her off at the mall, the scenario is now represented by $x + 5 \geq 20$. Graph the solutions for the inequality. How did the solutions change from $x \geq 20$?

4. Clare's siblings want to help by dividing the cost of the gift among themselves. Assume her father does not give her $5. There are 5 siblings altogether, so the inequality $5x \geq 20$ can be used to represent this scenario. Graph the solutions for the inequality. Scale your own number line. How did the inequality and the solutions change from $x \geq 20$?

Name: ______ Date: ______ Period: ______

Activity 3 Card Sort: Inequalities

The symbols >, <, ≥, and ≤ help to represent fundamental ideas about comparing amounts, but even these symbols had to come from somewhere. When mathematicians, such as Thomas Harriot or Giuseppe Peano, have brand new mathematical ideas, they create symbols that hold the special meaning of the new idea within them.

You will be given a set of cards. Match each inequality with a solution on the number line. Write the letter of the matches in the table. Have your teacher check your answers when you are finished.

Inequality	Number line
A	
B	
C	
D	

Reflect: Are there other steps that you now know you should have taken during your analysis? Explain.

Explain your strategies for matching.

Featured Mathematician

Giuseppe Peano

How would you explain the concept of numbers without using the words number, math, or even any words related to math? This was a challenge in mathematics until 1889, when Giuseppe Peano wrote a set of axioms that did just that. For Peano's other work, he even had to create new symbols to represent his brand new ideas in mathematical logic. You will likely encounter the symbols ∪, ∩, ∋, ℕ, or ℚ a bit later in your math studies, but you will already know there are some big ideas contained inside of them.

STOP

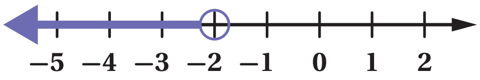
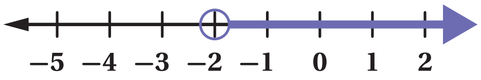
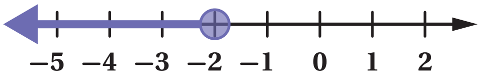
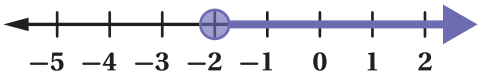
"Giuseppe Peano" Public Domain via Wikimedia Commons

Summary

In today's lesson . . .

You explored inequalities. The ***solutions to an inequality*** are numbers that can replace the variable to make an inequality true. Because inequalities have more than one solution, a number line is used to show *all* the solutions.

Remember, each symbol has its own purpose.

Symbol	Name	Example
<	Less than	$x < -2$ 
>	Greater than	$x > -2$ 
≤	Less than or equal to	$x \leq -2$ 
≥	Greater than or equal to	$x \geq -2$ 

Reflect:

Name: ____________________ Date: __________ Period: __________

1. Express each statement as an inequality, and write two values which will make the inequality true.

a x is less than or equal to 15.

b a is greater than or equal to 0.1.

c 3 is equal to or less than t.

d r is no more than 12.

e p is at least $\frac{3}{4}$.

2. Write an inequality for each scenario.

a A movie ticket costs a minimum of $7.

b For safety reasons, an elevator cannot exceed a $1{,}500$-kg capacity.

c The temperature is higher than $40°$.

d Diego has no more than 3 cousins.

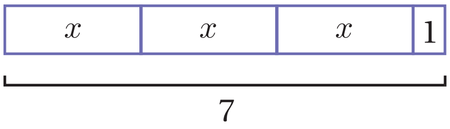
3. Consider the inequality $-3x > 18$.

a List four values for x that would make this inequality true.

b How are the solutions to the inequality $-3x \geq 18$ different from the solutions to $-3x > 18$? Explain your thinking.

Name: ____________ Date: ____________ Period: ____________

4. Select *all* the scenarios that could be represented by the diagram.



A. Andre studies 7 hours this week for his end-of-year exams. He spends 1 hour on English, and an equal number of hours each on math, science, and history.

B. Lin spends \$3 on 7 markers and \$1 on a pen.

C. Diego spends a total of \$1 on 7 stickers and 3 marbles.

D. Noah shares 7 apples with 3 friends. He eats 1 apple and gives each friend an equal number of remaining apples.

E. Elena spends a total of \$7. She buys 3 identical notebooks and spends \$1 on a pen.

5. The table shows the prices for crustless quiche at a certain cafe.

Quiche size	Price (\$)
Small	11.60
Medium	?
Large	16.25

a You have a coupon that makes the price of a large quiche \$13.00. For what percent off was the coupon? Show or explain your thinking.

b Your friend purchased a medium quiche for \$10.31 with a 30%-off coupon. What is the price of a medium quiche without a coupon? Show or explain your thinking.

c Another friend has a 15%-off coupon and \$10. What is the largest quiche they can afford? How much money will be left over? Show or explain your thinking.

6. Determine if $x = -4$ makes each equation true. Show your thinking.

a $x - 9 = -13$

b $-7x = -28$

c $\frac{1}{2}x = -8$

Name: ________________ Date: ________ Period: ________

Unit 6 | Lesson 14

Solving Inequalities

Let’s solve more complicated inequalities.

Warm-up True and False Inequalities

The table shows four inequalities and four possible values for x. Decide whether each value makes each inequality true or false. Complete the table by placing a checkmark for the values that make the inequality true. Be prepared to explain your thinking.

x	0	25	100	-100
$x \leq 25$				
$x + 25 > 25$				
$100 < 4x$				
$10 \geq 35 - x$				

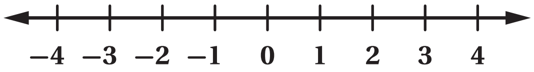
Log in to Amplify Math to complete this lesson online.

Activity 1 Inequalities With Tables (Part 1)

1. Complete the table.

x	-4	-3	-2	-1	0	1	2	3	4
$x - 3$	-7		-5				-1		1

2. Refer to the number line and the values of x in the table from Problem 1.



a Which value of x makes $x - 3 = -2$ true? Mark it on the number line.

b Which values of x make $x - 3$ greater than -2? Mark them in one color on the number line.

c Which values of x make $x - 3$ less than -2? Mark them in another color on the number line.

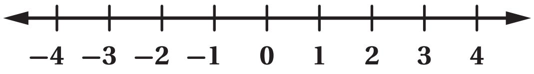
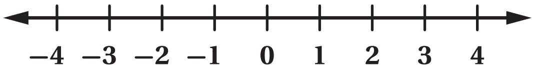
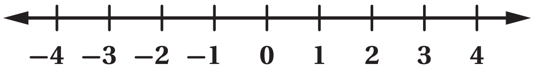
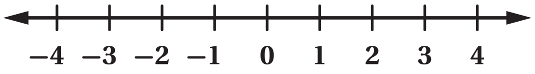
d Find a value of x between -4 and 4 not listed in the table that makes $x - 3$ greater than -2. Plot and label it on the number line. Mark it in the same color you used for part b.

e Find a value of x between -4 and 4 not listed in the table that makes $x - 3$ less than -2. Plot and label it on the number line. Mark it in the same color you used for part c.

Name: ________________ Date: ________ Period: ________

Activity 1 Inequalities With Tables (Part 1) (continued)

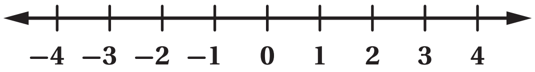
3. Use the number line from Problem 2 to help you think about which values of x will make each of the following inequalities true. Graph the solution to each inequality on the number line, and write an inequality to represent the solution that has x by itself on one side.

Inequality	Graph	Solution
a $x - 3 > -2$		
b $x - 3 \geq -2$		
c $x - 3 < -2$		
d $x - 3 \leq -2$		

Activity 2 Inequalities With Tables (Part 2)

1. Consider the inequality $2x < 6$.

a Predict which values of x will make the inequality $2x < 6$ true, and show them on the number line.



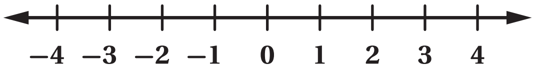
b Complete the table. Compare the values of x in the table with your graph to check your prediction.

x	-4	-3	-2	-1	0	1	2	3	4
$2x$									

c Write an inequality to represent the solutions to the inequality $2x < 6$.

2. Consider the inequality $-2x < 6$.

a Predict which values of x will make the inequality $-2x < 6$ true, and show them on the number line.



b Complete the table. Compare the values of x in the table with your graph to check your prediction.

x	-4	-3	-2	-1	0	1	2	3	4
$-2x$									

c Write an inequality to represent the solutions to $-2x < 6$.

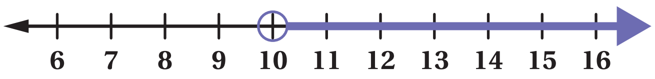
3. How are the solutions to $2x < 6$ different from the solutions to $-2x < 6$?

Name: .... Date: .... Period: ....

Activity 3 Inequality Jeopardy

Each graph is the solution to an inequality. Fill in the boxes with positive or negative numbers to write three inequalities that each have the solution shown. Then trade books with your partner to check each other's work.

1.

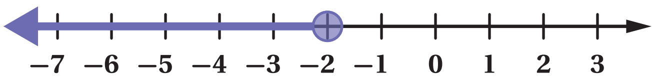


☐ $x >$ ☐

☐ $+ x >$ ☐

☐ $x <$ ☐

2.



☐ $x \leq$ ☐

$x +$ ☐ $\leq$ ☐

☐ $x \geq$ ☐

Reflect: How did checking each other's work deepen your understanding of inequalities?

STOP

Summary

In today's lesson . . .

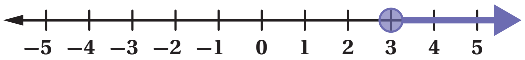
You tested values to determine what values make an inequality true. You used tables to organize your work and to help you write and graph the solutions to inequalities that involve addition, subtraction, or multiplication.

You noticed that in an inequality involving multiplication, the sign of the coefficient affected the direction of the solution. For example, consider the solutions to $3x \geq 9$ and $-3x \geq 9$:

$3x \geq 9$

x	0	1	2	3	4
$3x$	0	3	6	9	12

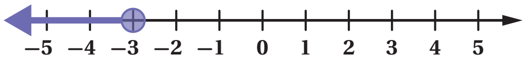
Solution: $x \geq 3$



$-3x \geq 9$

x	0	−1	−2	−3	−4
$-3x$	0	3	6	9	12

Solution: $x \leq -3$



Reflect:

Name: .... Date: .... Period: ....

1. Select *all* values of x that make the inequality $x - 6 \geq -4$ true.

A. 10

B. 1.9

C. -2

D. 7

E. -10

F. 0

G. 2

H. 2.01

2. Diego is solving the inequality $-3x \geq 45$. He solves the equation $-3x = 45$ to determine $x = -15$. What is the solution to the inequality?

A. $x < -15$

B. $x > -15$

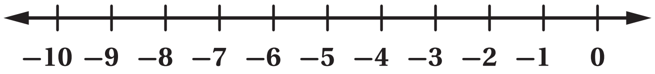
C. $x \leq -15$

D. $x \geq -15$

3. Complete the table to determine the solution to each inequality. Show your solution as a graph on the number line and write an inequality to represent it.

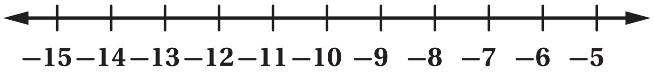
a $-6x > 30$

x	-7	-6	-5	-4	-3	-2	-1	0
$-6x$								



b $\frac{3}{4}x < -\frac{15}{2}$

x	-16	-14	-12	-10	-8	-6	-4	-2
$\frac{3}{4}x$								



Practice

Name: .... Date: .... Period: ....

4. Solve each equation.

a $a \cdot 3 = -30$

b $-9b = 45$

c $-89 \cdot 12 = c$

d $d \cdot 88 = -88000$

5. A backpack normally costs \$25, but is on sale for \$21. What percent is the discount?

6. Select *all* the values of x that make the inequality $-x + 6 \geq 10$ true.

A. -3.9

B. 4

C. -4.01

D. -4

E. 4.01

F. 3.9

G. 0

H. -7

Name: .... Date: .... Period: ....

Unit 6 | Lesson 15

Finding Solutions to Inequalities in Context

Let's solve more complicated inequalities.

Warm-up One Solution or Many Solutions?

1. Solve each equation. Show your thinking.

   a. $-x = 10$

   b. $2x = -20$

2. What do you notice about the solutions in Problem 1?

3. Determine two solutions for each inequality.

   a. $-x > 10$

   b. $2x > -20$

4. What do you notice about the solutions in Problem 3?

Log in to Amplify Math to complete this lesson online.

Activity 1 Earning Money for Soccer Apparel

Han was hired for a summer job selling magazine subscriptions. He will earn $25 per week, plus $3 for every subscription he sells. Han hopes to make enough money this week to buy a new pair of soccer cleats.

1. Let n represent the number of magazine subscriptions Han will sell this week. Write an expression for the amount of money he will make.

2. The most affordable cleats in the store will cost Han $67. Write and solve an equation to determine how many magazine subscriptions he will need to sell to earn $67. Show your thinking.

3. If Han sells 16 subscriptions this week, will he reach his goal and be able to buy the new cleats? Explain your thinking.

4. What are some other numbers of subscriptions Han could sell to reach his goal?

5. Write an inequality expressing how much Han will have to earn to afford at least $67 for the cleats.

6. Write an inequality describing the number of subscriptions Han must sell to reach his goal.

Name: ______________________ Date: __________ Period: __________

Activity 2 Earning More Money for Soccer Apparel

Elena has budgeted $35 from her summer job for new shorts and socks for the upcoming soccer season. She needs 5 pairs of socks and a pair of shorts. The socks cost different amounts in different stores. The shorts she needs cost $19.95.

1. Let x represent the price of one pair of socks. Write an expression for the total cost of the socks and shorts.

2. Write an inequality showing the total cost should be at most $35.

3. Write and solve an equation showing Elena spent exactly $35 on the socks and shorts. What does the solution mean in this scenario?

4. Remember, Elena has $35 to spend on soccer apparel. What are some other sock prices that will keep Elena within her budget?

5. Write an inequality to represent the amount Elena can spend on a single pair of socks.

6. The price of shorts just went up to $22. Should Elena buy more expensive socks or less expensive socks to stay within her $35 budget? Explain your thinking.

STOP

Summary

In today's lesson . . .

You wrote and solved some more complicated inequalities. Writing inequalities is similar to writing equations, but you must also pay attention to the words that compare the two expressions.

The table shows words that are represented by each equality or inequality symbol.

$=$	$<$	$>$	$\leq$	$\geq$
equal	less than	greater than	less than or equal to	greater than or equal to
is	fewer than	more than	at most	at least
the same as	below	above	at a maximum	at a minimum
	lower than	higher than	no more than	no less than
		exceeds	does not exceed	

Reflect:

Name: .... Date: .... Period: ....

1. Jada is scuba diving and starts 1 ft above the water in a boat. She swims down at a rate of 3 ft per minute. The times when Jada is 29 ft below the surface can be represented by the inequality $1 - 3x < -29$, where x represents the number of minutes spent swimming. Does $x < 10$ or $x > 10$ represent the solutions? Explain your thinking.

2. It is currently 0 degrees outside, and the temperature is dropping 4 degrees every hour. The temperature after h hours is $-4h$.

a Explain what the equation $-4h = -14$ represents.

b What value of h makes the equation true?

c Explain what the inequality $-4h < -14$ represents.

d What values of h make the inequality true?

3. Lin budgeted $\$85$ for school supplies. At the first store, she spent $\$47.25$. Later, she found a bargain of $\$7.55$ per pack for her favorite pencils.

a Let p represent how many packs of pencils Lin can buy. Write an expression for her total cost in school supplies.

b Write an equation that shows Lin spending her entire budget on school supplies.

c Solve the equation you wrote in part b. What does the solution mean in context of the scenario?

d Write an inequality showing Lin spending no more than the budgeted $\$85$.

e Write an inequality showing the possible number of packs of pencils Lin can buy and stay within budget.

Name: .... Date: .... Period: ....

4. Which inequality is true when the value of x is -3?

A. $-x - 6 < -3.5$

B. $-x - 6 > 3.5$

C. $-x - 6 > -3.5$

D. $x - 6 > -3.5$

5. One year ago, Clare was 4 ft 6 in. tall. Now, Clare is 4 ft 10 in. tall. What was the percent change of Clare's height in the last year? Show or explain your thinking.

6. Which values of x make the inequality $-2x + 6 < 14$ true? Select *all* that apply.

A. $x = -4$

B. $x = 0$

C. $x = -10$

D. $x = 4$

E. $x = -3.5$

Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 16

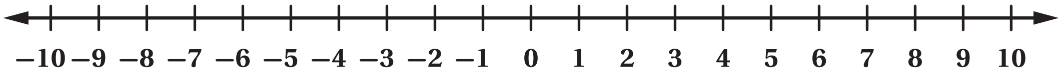
Efficiently Solving Inequalities

Let’s solve more complicated inequalities.

Warm-up Predict the Solution

Consider the inequality $-x \geq -4$.

1. Predict where you think the solutions on the number line will be.



2. Select *all* the values that are solutions to $-x \geq -4$.

- **A.** 3
- **B.** -3
- **C.** 4
- **D.** -4
- **E.** 4.001
- **F.** -4.001

3. Use your solutions from Problem 2 to check your prediction. Was your prediction correct? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Which Side Has the Solutions?

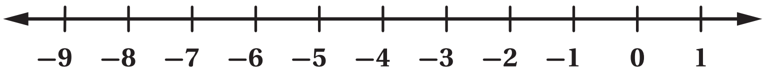
1. Let's investigate the inequality $-4x + 5 \geq 25$.

a Solve the equation $-4x + 5 = 25$, and place an open circle at the solution on the number line below.

b Is the inequality $-4x + 5 \geq 25$ true when:

- x equals the solution to the equation $-4x + 5 = 25$? Explain your thinking.
- x is *greater than* the solution to the equation? Explain your thinking.
- x is *less than* the solution to the equation? Explain your thinking.

c Complete the graph to show the solutions to the inequality $-4x + 5 \geq 25$ on the number line. Then write an inequality to represent the solution.



Solution:

Name: .... Date: .... Period: ....

Activity 1 Which Side Has the Solutions? (continued)

2. Let's investigate the inequality $3(x + 4) < 17.4$.

a Solve the equation $3(x + 4) = 17.4$, and place an open circle at the solution on the number line below.

b Is the inequality $3(x + 4) < 17.4$ true when:

- x equals the solution to the equation $3(x + 4) = 17.4$?
- x is *greater than* the solution to the equation?
- x is *less than* the solution to the equation?

c Complete the graph to show the solutions to the inequality $3(x + 4) < 17.4$ on the number line. Then write an inequality to represent the solution.

Solution:

Activity 2 Solving Inequalities

1. Consider the inequality $\frac{23}{3} < \frac{4}{3}x + 3$.

Solve the related equation and test values less than and greater than the solution. Then graph the solution on the number line and write an inequality to represent the solution.

Solution:

2. Consider the inequality $-3\left(x - \frac{4}{3}\right) \leq 6$.

Solve the related equation and test values less than and greater than the solution. Then graph the solution on the number line and write an inequality to represent the solution.

Solution:

STOP

Name: ____________ Date: ____________ Period: ____________

Summary

In today's lesson . . .

You explored how to use equations to solve more complicated inequalities. You first wrote and solved an equation related to the inequality. Then you tested values greater than and less than the solution, to see which value made the inequality true. Finally, you graphed the solution on a number line and wrote an inequality to represent it.

Your understanding of solving equations helps you to reason about inequalities and their solutions.

> **Reflect:**

Name: .... Date: .... Period: ....

1. Consider the inequality $-4(x - 6) > 16$.

   a Which numbers are solutions to the inequality?

   - **A.** 2
   - **B.** -2
   - **C.** 0
   - **D.** -2.1
   - **E.** 1.9
   - **F.** 2.1
   - **G.** $\frac{2}{3}$

   b Write two more solutions to the inequality.

2. Diego is solving the inequality $100 - 3x \geq -50$. He first solves the equation $100 - 3x = -50$ and obtains $x = 50$. What is the solution to the inequality?

   - **A.** $x < 50$
   - **B.** $x \leq 50$
   - **C.** $x > 50$
   - **D.** $x \geq 50$

3. Solve each inequality. Show your solution as a graph on the number line and write an inequality to represent it.

   a $-10x - 8 \leq -13$

   b $9(x + 15) < -108$

Name: .... Date: .... Period: ....

4. The price of a pair of earrings is $\$22$, but Priya buys them on sale for $\$13.20$.

a How much, in dollars, was the price discounted?

b What was the percent of the discount? Show or explain your thinking.

5. Complete the magic square so that the sums of each row, column, and diagonal in the grid are equal.

1		
	3	-2
		5

6. You are building a tower with 2-in. tall blocks on top of a 10-in. tall base. Your tower will topple when it is taller than 48 in. Which inequality represents the number of blocks you can use to build your tower?

A. $2x - 10 < 48$

B. $2x + 10 < 48$

C. $2x - 10 \leq 48$

D. $2x + 10 \leq 48$

Unit 6 | **Lesson 17**

Interpreting Inequalities

Let's write some inequalities.

Warm-up Mystery Inequalities

Using each of the numbers from 1 to 9 at most once, fill in the boxes so that the solution to the inequality is $x \leq 2$. Repeat as many times as possible to determine different sets of numbers.

$$\square x - \square \leq \square$$

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Matching an Inequality to a Scenario

The Science Club is investigating the effect of a liquid's density on the height of an object floating within that liquid. They place an egg in a 25-cm tall beaker filled with salt water. It floats 5 cm above the bottom. They notice that each time they add a spoonful of salt, the egg rises $\frac{1}{2}$ cm. How many spoonfuls of salt can be added without the egg reaching the top of the cup?

1. Choose the inequality that best matches the scenario.

- **A.** $25x + 5 < \frac{1}{2}$
- **B.** $\frac{1}{2}x + 5 < 25$
- **C.** $\frac{1}{2}x + 25 < 5$
- **D.** $5x + \frac{1}{2} < 25$

2. Explain what each part of the inequality represents.

3. Solve for x, graph the solution, and write an inequality to represent the solution. Show your work.

Solution:

4. Explain what the solution means in terms of the scenario.

Activity 2 Writing an Inequality for a Scenario

Three Reads: Read the introductory information three times.

1. Make sense of the scenario.
2. What mathematical quantities are given?
3. Brainstorm strategies to solve the problem.

The Chemistry Club is experimenting with different mixtures of water and a chemical called sodium polyacrylate to make fake snow.

Each mixture starts with some amount of water, measured in grams. The amount of the chemical used in the mixture is $\frac{1}{7}$ of the amount of water used, plus 9 more grams of the chemical. The chemical is expensive, so there must be less than 50 g of the chemical in any one mixture. How much water can the students use in the experiment?

1. Describe the unknown amount that the variable x will represent.

2. Write an inequality that represents the scenario, graph the solution, and write an inequality to represent the solution.

Solution:

3. Explain what the solution means in terms of the scenario.

STOP

Name: ______________ Date: ________ Period: ________

Summary

In today's lesson . . .

You wrote and solved inequalities to help you solve real-world problems. You can represent and solve many real-world problems with inequalities. Writing an inequality to represent a real-world problem is very similar to writing an equation. The expressions that make up the inequalities are the same as the ones you saw in earlier lessons for equations.

For inequalities, you also have to think about how expressions compare to each other: which is greater, which is less, and which ones might be equal.

Reflect:

Practice

Name: ………………………… Date: ………………… Period: …………………

1. Priya looks at the inequality $12 - x > 5$ and says, "I subtract a number from 12 and want a result that is greater than 5. That means that the solutions should be the values of x that are less than something." Do you agree with Priya? Explain your thinking and include solutions to the inequality in your explanation.

2. The Stock Market Club is planning a field trip. They already have $\$400$, but they need at least $\$1{,}200$ to fund the trip. They have decided that each of their 10 members will be responsible for equally fundraising the remainder of the money. Which inequality and explanation best matches this scenario?

A. $400 + 10x \geq 1200$, where x represents the least amount of money each member will need to raise.

B. $400 + 10x \geq 1200$, where x represents the number of members that will raise the money.

C. $400 + 10x \leq 1200$, where x represents the least amount of money each member will need to raise.

D. $400 + 10x \leq 1200$, where x represents the number of members that will raise the money.

3. After a store sold $\frac{2}{5}$ of the shirts that were on display, they brought out another 30 from the stockroom, because the store manager likes to keep at least 150 shirts on display. The manager wrote the inequality $\frac{3}{5}x + 30 \geq 150$ to describe the situation.

a Explain what $\frac{3}{5}$ means in the inequality.

b Solve the inequality. Explain what the solution means in terms of the scenario.

Solution:

Name: .... Date: .... Period: ....

4. You are at a skateboard shop browsing items. Solve each problem.

a The price tag on a shirt reads $\$12.58$. Sales tax is 7.5% of the price. How much will you pay for the shirt? Show or explain your thinking.

b The shop manager buys a helmet for $\$19.00$ and sells it for $\$31.50$. What percent was the markup? Show or explain your thinking.

c A shop pays workers $\$14.25$ per hour, plus 5.5% commission. If someone works 18 hours and sells $\$250$ worth of merchandise, what is the total amount of their paycheck for this pay period? Show or explain your thinking.

5. Match each scenario with the inequality that could represent it.

Scenario	**Inequality**
a Han got $2 from Clare, but still has less than $20.	.... $2x < 20$
b Mai spent $2 and now has less than $20.	.... $x + 2 < 20$
c If Tyler had twice the amount of money he currently has, he would still have less than $20.	.... $\frac{1}{2}x < 20$
d If Priya had half the money she currently has, she would have less than $20.	.... $x - 2 < 20$

6. Diego is buying juice packs for his party this weekend. Each pack of juice comes with 6 cartons, and he knows he wants enough to serve at least 16 people, so he writes the inequality $6j \geq 16$. After solving, he determines the solution is $j \geq 2\frac{2}{3}$. Is it possible for Diego to purchase $2\frac{2}{3}$ juice packs? How many would you tell him to buy?

Unit 6 | Lesson 18

Modeling With Inequalities

Let's look at solutions to inequalities.

Warm-up Possible Values

The stage manager of the school musical is trying to determine how many sandwiches he can order with the $\$83$ he collected from the cast and crew. Sandwiches cost $\$5.99$ each, so he lets x represent the number of sandwiches he can order and writes the inequality $5.99x \leq 83$. He solves the inequality and rounds the solution to two decimal places, resulting in $x \leq 13.86$.

Which of these are reasonable statements about this scenario? Select *all* that apply.

A. He can order exactly 13.86 sandwiches.

B. He can round up and order 14 sandwiches.

C. He can order 12 sandwiches.

D. He can order 9.5 sandwiches.

E. He can order 2 sandwiches.

F. He can order -4 sandwiches.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Loading an Elevator

A mover is loading an elevator with many identical 48-lb boxes. The mover weighs 185 lb. The elevator can carry at most 2,000 lb.

1. Write an inequality that shows the mover would not overload the elevator on a particular ride.

2. Solve your inequality and graph the solution on a number line.

Solution:

3. The mover asks, "How many boxes can I load on this elevator at a time?" What do you tell them?

Activity 2 Info Gap: Giving Advice

You will be given either a *problem card* or a *data card*.
Do not show or read your card to your partner.

If you are given a *problem card*:	If you are given a *data card*:
1. Silently read your card and think about what information you need to be able to solve the problem.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner *"What specific information do you need?"* and wait for them to ask for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask *"Why do you need that information?"* Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently in the space below.	**4.** Read the *problem card* and solve the problem independently in the space below.
5. Read the *data card* and discuss your reasoning.	**5.** Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. You will be given a new set of cards and repeat the activity, trading roles with your partner.

Problem 1	Problem 2

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You used inequalities to represent and solve some real-world problems. Whenever you write an inequality, it is important to decide what quantity you are representing with a variable. After you make that decision, you can connect the quantities in the scenario to write an expression, and then the whole inequality.

As you solve the inequality, it is important to keep the meaning of each quantity in mind. This helps you decide whether the final answer makes sense in context of the scenario. Some scenarios require only whole number values (number of people, number of buses, etc.) and other scenarios are continuous (length of a rectangle, weight of a package, etc.).

Reflect:

Name: .... Date: .... Period: ....

1. Tyler's teachers are planning a field trip for 175 seventh graders. They are using one van, which carries 14 students, and buses, which carry 30 students each.

a Write an inequality determining the number of buses they could order.

b Solve the inequality and explain what the solution means in context.

2. Write an inequality to represent each real-world problem.

a In the cafeteria, there is one large 10-seat table and many smaller 4-seat tables. There are enough tables to fit at most 210 students. Write an inequality whose solution is the possible number of 4-seat tables in the cafeteria.

b 5 barrels catch rainwater in the schoolyard. 4 barrels are the same size, and the fifth barrel holds 10 liters of water. The Environmental Club is hoping that the 5 barrels can catch at least 210 liters of water to use to water the school's garden. Write an inequality whose solution is the possible size of each of the 4 barrels.

c How are these two problems similar? How are they different?

3. Most shipping companies charge a rate based on the weight of the package and the speed of delivery. Priya purchases $5 worth of packaging supplies and has a 14-oz package to mail. She wants to spend no more than $30 total to mail the package. Write and solve an inequality to determine the cost per ounce Priya can afford.

Name: .... Date: .... Period: ....

4. Solve each equation. Show your thinking.

a $5(n - 4) = -60$

b $-3t + (-8) = 25$

c $7p - 8 = -22$

d $\frac{2}{5}(j + 40) = -40$

5. Select *all* the inequalities that have the same solutions as $x < 4$.

A. $x < 2$

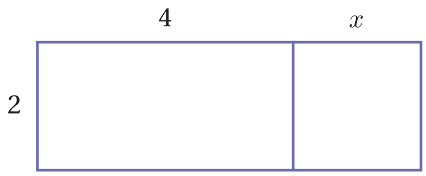
B. $x + 6 < 10$

C. $5x < 20$

D. $x - 2 > 2$

E. $x < 8$

6. Select *all* of the expressions that represent the area of the largest rectangle.



A. $2 \cdot 4 + 2 \cdot x$

B. $8 + x$

C. $8 + 2x$

D. $2(4 + x)$

E. $2 + 4 + x$

F. $(4 + x) + (4 + x)$

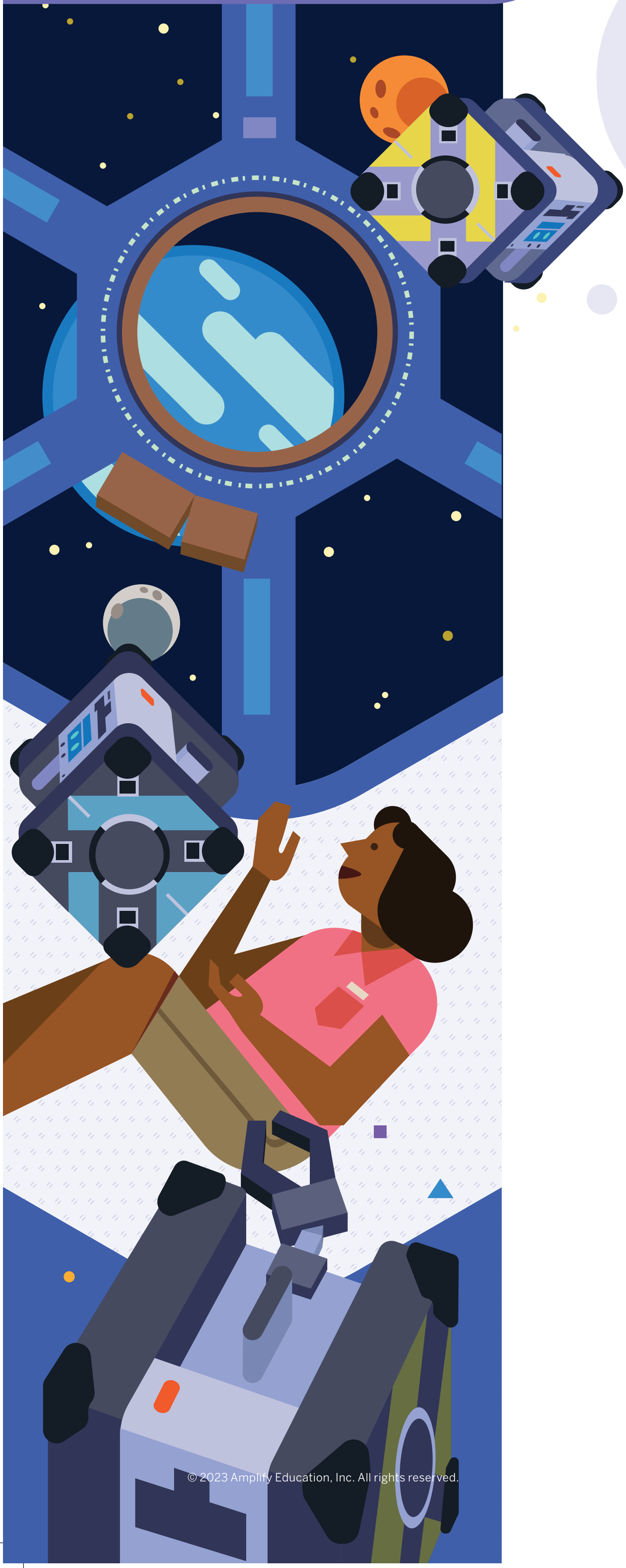
My Notes:

SUB-UNIT

4 Equivalent Expressions

Narrative Connections

Which three blockheads did NASA send into space?



In 2019, NASA launched three robots to the International Space Station. But these were not the clunky bots you would find in a science fiction film. Instead, imagine three cubes, each about a foot wide, and equipped with advanced sensors, touch screens, and a set of electric fans for zipping around.

These were the “Astrobees”: Bumble, Honey, and Queen. NASA hoped these boxy bots would help astronauts with their daily routines. That way the humans onboard would have time to focus on more important things.

The Astrobees were programmed to navigate the space station on their own, dock themselves whenever their batteries ran low, and even put themselves away. They could also track tools, take inventory, help with experiments, and move cargo, making life on the space station easier and more productive.

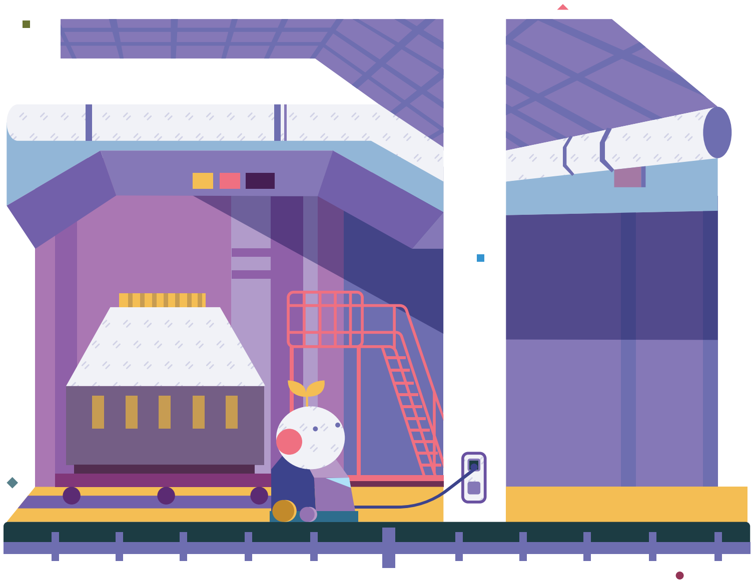
As a species, we humans have often used technology to do things better. And look how far we have come! From spears and clubs, to the steam engine, to personal computers, and now to robots. By finding more efficient ways to get things done, we can be faster, make fewer mistakes, and arrive at more elegant solutions.

As you’ll see, the same goes for solving equations. By writing expressions into equivalent forms, you can see structure (and solutions) where you might not have before.

Unit 6 | Lesson 19

Subtraction in Equivalent Expressions

Let’s find ways to work with subtraction in expressions.



Warm-up Which One Doesn’t Belong?

Circle one of the expressions below and explain why it does not belong with the others.

A. $10 + 3 + (-8)$

B. $10 - 8 + 3$

C. $10 - 3 + 8$

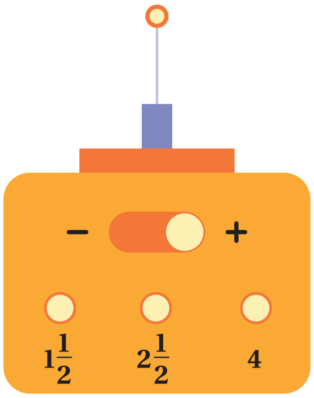
D. $-8 + 3 + 10$

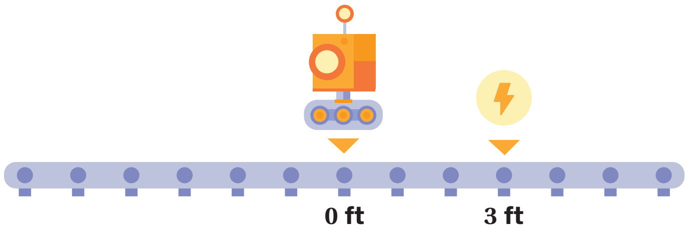
Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Robot Recharge

This robot needs a charge! It only has enough power to make three specific moves, in either direction. One must be $1\frac{1}{2}$ ft, another $2\frac{1}{2}$ ft, and another 4 ft. Find as many different ways as possible to program the robot so that it stops at the power outlet. Write each set of moves as a number sentence.





Number sentence 1: ....

Number sentence 2: ....

Number sentence 3: ....

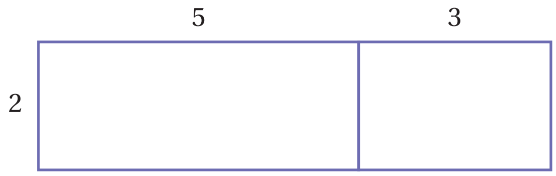
Number sentence 4: ....

Number sentence 5: ....

Number sentence 6: ....

Activity 2 The Distributive Property, Revisited

Write at least three different expressions that represent the area of the largest rectangle shown.



Expression 1:

Expression 2:

Expression 3:

Name: ________ Date: ________ Period: ________

Activity 3 Including Negatives in Area Models

Match each expression with the area model diagram it represents. Be prepared to explain how each expression matches each area model diagram.

$\frac{1}{5}(8y - x - 12)$

$\frac{8}{5}y - \frac{x}{5} + \frac{12}{5}$

$\frac{8}{5}y + \left(-\frac{x}{5}\right) + \frac{12}{5}$

$\frac{8}{5}y - \frac{x}{5} - \frac{12}{5}$

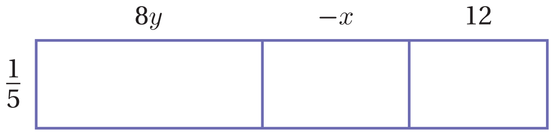
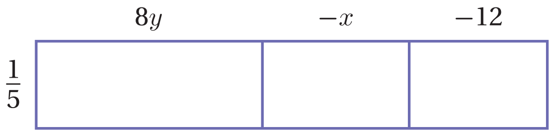
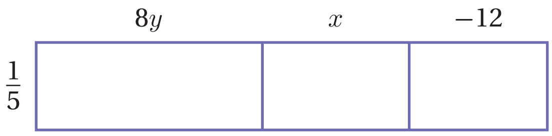
$\frac{1}{5}(8y + x - 12)$

$\frac{x}{5} + \frac{8}{5}y + \left(-\frac{12}{5}\right)$

$\frac{1}{5}(8y - x + 12)$

$\frac{8}{5}y + \frac{x}{5} - \frac{12}{5}$

$\frac{8}{5}y + \left(-\frac{x}{5}\right) + \left(-\frac{12}{5}\right)$

	Area model	Expressions
a		
b		
c		

STOP

Summary

In today's lesson . . .

You explored how to work with subtraction when writing expressions. Working with subtraction and signed numbers can sometimes get tricky. You can apply what you know about the relationship between addition and subtraction — that subtracting a number gives the same result as adding its opposite — to your work with expressions. Then you can make use of the properties of addition that allow you to add and group in any order, which can make calculations simpler.

Reflect:

Name: .... Date: .... Period: ....

1. Tyler analyzed the expression $2.5 + 3 - 4\frac{1}{3}$ and tried to write as many equivalent expressions as he could. He wrote:

$2.5 + 3 + \left(-4\frac{1}{3}\right)$

$-4\frac{1}{3} + 2.5 + 3$

$-4\frac{1}{3} + 3 + 2.5$

Is Tyler missing any expressions? If yes, write the others below. If not, explain how you know.

2. Use the Distributive Property to write an expression that is equivalent to each expression. Consider drawing boxes to help in rewriting each expression and organizing your work.

a $9\left(4x - 3y - \frac{2}{3}\right)$

b $2(-6x + 3y - 1)$

c $\frac{1}{5}(20y - 4x - 13)$

d $8\left(-x - \frac{1}{2}\right)$

e $-8\left(-x - \frac{3}{4}y + \frac{7}{2}\right)$

Practice

Name: .... Date: .... Period: ....

3. Kiran wrote the expression $x - 10$ for the following number puzzle: "Pick a number, add -2, and multiply by 5."

   Lin thinks Kiran made a mistake. How can she convince Kiran he made a mistake? What would be a correct expression for this number puzzle?

4. A store is having a 25%-off sale on all shirts. Show two different ways to calculate the sale price for a shirt that normally costs $\$24$.

5. Use reasoning to complete each problem.

   a. If $(11 + x)$ is positive, but $(4 + x)$ is negative, what is one number that x could be?

   b. If $(-3 + y)$ is positive, but $(-9 + y)$ is negative, what is one number that y could be?

   c. If $(-5 + z)$ is positive, but $(-6 + z)$ is negative, what is one number that z could be?

6. List *all* the common factors for each pair.

   a. 9 and 12

   b. 15 and 60

   c. 17 and 33

   d. $4x$ and $5x$

Name: ____________ Date: ____________ Period: ____________

Unit 6 | Lesson 20

Expanding and Factoring

Let's use the Distributive Property to write expressions in different ways.

Warm-up Number Talk

Mentally determine the value of each expression. Record each solution.

1. $2 + 3 \cdot 4$

2. $(2 + 3) \cdot 4$

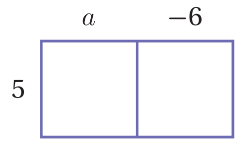
3. $2 - 3 \cdot 4$

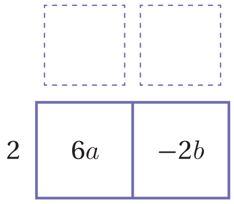
4. $2 - (3 + 4)$

Log in to Amplify Math to complete this lesson online.

Activity 1 Factoring and Expanding With Area Model Diagrams

1. Fill in the boxes to complete each area model.





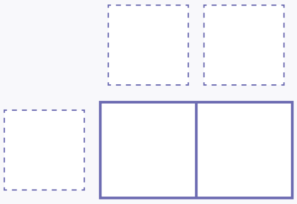
2. Use the diagrams to write an equivalent expression for each expression.

a $5(a - 6)$

b $6a - 2b$

Are you ready for more?

Fill in the boxes to complete the area model. Then write two equivalent expressions to describe the model.



Expression 1:

Expression 2:

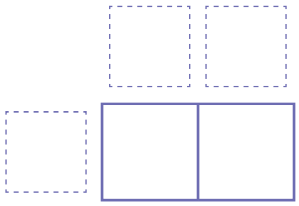
Name: ________ Date: ________ Period: ________

Activity 2 Card Sort: Matching Equivalent Expressions

You will be given a set of expressions. Match each expression to an equivalent expression.

Write each pair of matching expressions and complete the area model diagram to show that they are equivalent.

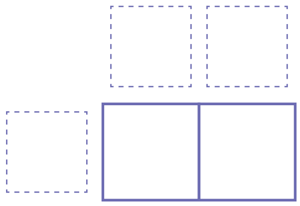
a



Factored expression:

Expanded expression:

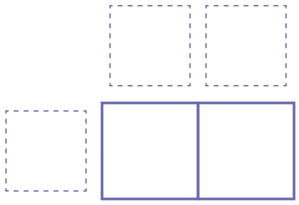
b



Factored expression:

Expanded expression:

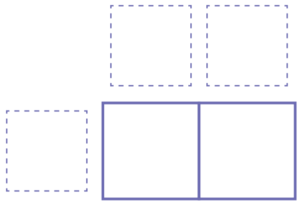
c



Factored expression:

Expanded expression:

d



Factored expression:

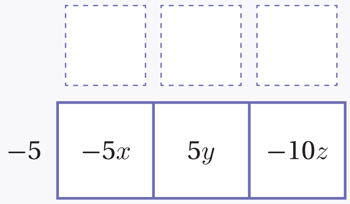
Expanded expression:

Are you ready for more?

Fill in the boxes to complete the area model. Then write two equivalent expressions to describe the model.

Factored expression:

Expanded expression:



Activity 3 Factor and Expand

Complete the table by writing an equivalent expression in each row.

Factored	Expanded
	$12x - 18a$
$-2(4a - 5b)$	
	$10a - 13a$
	$4ab - 4b$
$\frac{2}{3}(-6a - x)$	
$-(x - 2b)$	

Are you ready for more?

Complete the table by writing an equivalent expression in each row.

Factored	Expanded
	$ab - bc - 3bd$
	$20x - 10a - 15$

STOP

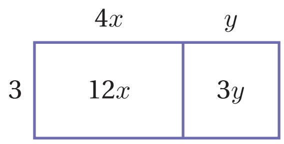
Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw two methods for writing equivalent expressions: ***expanding*** (using the Distributive Property to write a product as a sum or difference) and ***factoring*** (using the Distributive Property to write a sum or difference as a product).

An area model can help to show how both forms relate to each other.

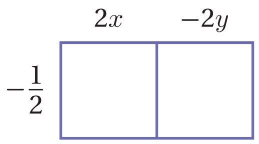
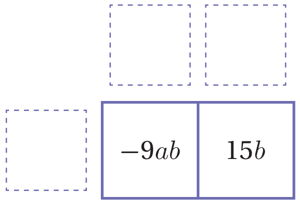


Factored	Expanded
$3(4x + y)$	$12x + 3y$

Reflect:

Name: .... Date: .... Period: ....

1. Complete each area model diagram. Then write two equivalent expressions to describe each diagram.

Area model		
Factored expression		
Expanded expression		

2. Write equivalent expressions for each expression. Draw an area model diagram to help, if needed.

a Expand to write an equivalent expression: $-\frac{1}{4}(-8x + 12y)$

b Factor to write an equivalent expression: $36a - 16$

3. Lin missed math class on the day the class worked on expanding and factoring expressions. Kiran is helping Lin catch up.

a Lin understands that expanding is using the Distributive Property, but she does not understand what factoring is or why it works. How can Kiran explain factoring to Lin?

b Lin asks Kiran how the area model diagrams with boxes help with factoring. What should Kiran tell Lin about the boxes?

Name: .... Date: .... Period: ....

c Lin asks Kiran to help her factor the expression $-4xy - 12x$. How can Kiran use this example to help Lin understand factoring?

4. Elena makes her favorite shade of purple paint by mixing 3 cups of blue paint, $1\frac{1}{2}$ cups of red paint, and $\frac{1}{2}$ of a cup of white paint. Elena has $\frac{2}{3}$ of a cup of white paint.

a How much blue paint and red paint will Elena need to mix with $\frac{2}{3}$ of a cup of white paint to make purple paint? Show or explain your thinking.

b How much purple paint could Elena then make?

5. Select *all* the inequalities that have the same solution as $-4x < 20$.

A. $-x < 5$

B. $4x > -20$

C. $4x < -20$

D. $x < -5$

E. $x > 5$

F. $x > -5$

6. Name the property represented by each set of equivalent expressions.

a $4 + 3 + (-2)$ and $4 + (3 + (-2))$

b $5 + (-6) + 7$ and $5 + 7 + (-6)$

c $5 + 0 = 5$

d $3(2 + 6) = 6 + 18$

Unit 6 | Lesson 21

Combining Like Terms (Part 1)

Let's see how we can tell expressions are equivalent.

Warm-up What Does it Equal?

Evaluate the expression $a + a + a + b + a - c - c + b - a - b - b$, when $a = 3$, $b = -4$, and $c = 2$.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 A's and B's

1. The expression from the Warm-up, $a + a + a + b + a - c - c + b - a - b - b$, is equivalent to $a + a + a + b + a + (-c) + (-c) + b + (-a) + (-b) + (-b)$. How was the second expression rearranged? Why can that be done?

2. The expression $a + a + a + b + a + (-c) + (-c) + b + (-a) + (-b) + (-b)$ is equivalent to $a + a + a + a + (-a) + b + b + (-b) + (-b) + (-c) + (-c)$. How was the second expression rearranged? Why can that be done?

3. The expression $a + a + a + a + (-a) + b + b + (-b) + (-b) + (-c) + (-c)$ is equivalent to $(a + a + a + a + (-a)) + (b + b + (-b) + (-b)) + ((-c) + (-c))$. How was the second expression rearranged? Why can that be done?

4. The expression $(a + a + a + a + (-a)) + (b + b + (-b) + (-b)) + ((-c) + (-c))$ is equivalent to $3a + 0b + (-2c)$. How was the second expression rearranged? Why can that be done?

5. The expression $3a + 0b + (-2c)$ is equivalent to $3a + (-2c)$. How was the second expression rearranged? Why can that be done?

Activity 1 A's and B's (continued)

6. The expression $3a + (-2c)$ is equivalent to $3a - 2c$.
   How was the second expression rearranged? Why can that be done?

7. Evaluate $3a - 2c$ when $a = 3$ and $c = 2$.

8. Why are the final solutions for the Warm-up and Problem 7 the same?
   Which expression is more efficient to evaluate? Why?

Name: ______ Date: ______ Period: ______

Activity 2 Making Sides Equal

Fill in each $\boxed{}$ with an expression, making the left side of the equation equivalent to the right side.

1. $6x + \boxed{} = 10x$

2. $6x - \boxed{} = 2x$

3. $6x - \boxed{} = x$

4. $6x + \boxed{} = 2x$

5. $6x + \boxed{} = 0$

6. $6x + \boxed{} = -10x$

7. $6x - \boxed{} = 10x$

8. $6x - \boxed{} = -2x$

Are you ready for more?

Replace each box with an expression that makes the left side of the equation equivalent to the right side.

1. $6x - \boxed{} = 6$

2. $6x + \boxed{} = 10$

3. $6x - \boxed{} = 4x - 10$

STOP

Summary

In today's lesson . . .

You explored how some equivalent expressions have fewer terms than other equivalent expressions. There are many ways to write equivalent expressions, some of which may look very different from each other. You have several tools for determining whether two expressions are equivalent.

- Two expressions are *not* equivalent if they have different values when you substitute the same number for the variable.
- If two expressions are equal for many different values of the variable, then the expressions *may be* equivalent. You do *not* know for sure, because it is impossible to compare expressions for all values.

To determine whether two expressions are equivalent, you can use properties of operations to write them with fewer terms. You can also combine ***like terms*** — parts of an expression that have the same variable and can be added together, such as $7x$ and $9x$. If both expressions can be written as the same expression, then they are equivalent.

Reflect:

Name: .... Date: .... Period: ....

1. Consider the expression $x + (-x) + x + x - y - y$.

a Evaluate the expression when $x = 3$ and $y = -2$

b Is it equivalent to the expression $2x + 2y$? Show or explain your thinking.

2. Select *all* of the expressions that are equivalent to the expression $-3x + 9x + 5 - 10$.

A. $-5 + 6x$

B. $-6x + 5$

C. $5 - 6x$

D. $6x - 5$

E. $6x + 5$

F. $-6x - 5$

3. Select *all* of the statements which are true for any value of x.

A. $7x + 2x + 7 = 9x + 7$

B. $7x + 2x - 1 = 9x + 1$

C. $\frac{1}{2}x + 3 - \frac{1}{2}x = 3$

D. $5x - 8 + 6x = -x - 8$

E. $0.4x - 0.2x + 8 = 0.2x - 8$

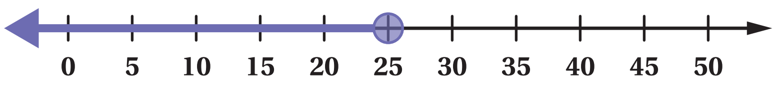
F. $6x - 2x - 4 = 4x - 4$

4. For each scenario or number line, write whether you would represent it with $x < 25$, $x > 25$, $x \leq 25$, or $x \geq 25$. You will use each inequality once.

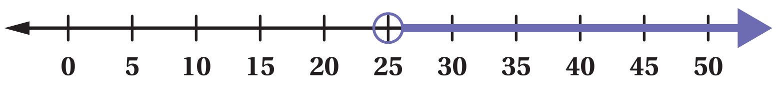
a The library is having a party for any student who read at least 25 books over the summer. Priya read x books and was invited to the party.

b Kiran read x books over the summer, but was not invited to the party.

c



d



Practice

Name: ______ Date: ______ Period: ______

5. Consider the following problem: A bucket is being filled with water from a faucet at a constant rate. When will the bucket be full?

What information would you need to be able to solve the problem?

6. Circle *all* the statements that are true.

A. $4 - 2(3 + 7) = 4 - 2 \bullet 3 - 2 \bullet 7$

B. $4 - 2(3 + 7) = 4 + (-2) \bullet 3 + (-2) \bullet 7$

C. $4 - 2(3 + 7) = 4 - 2 \bullet 3 + 2 \bullet 7$

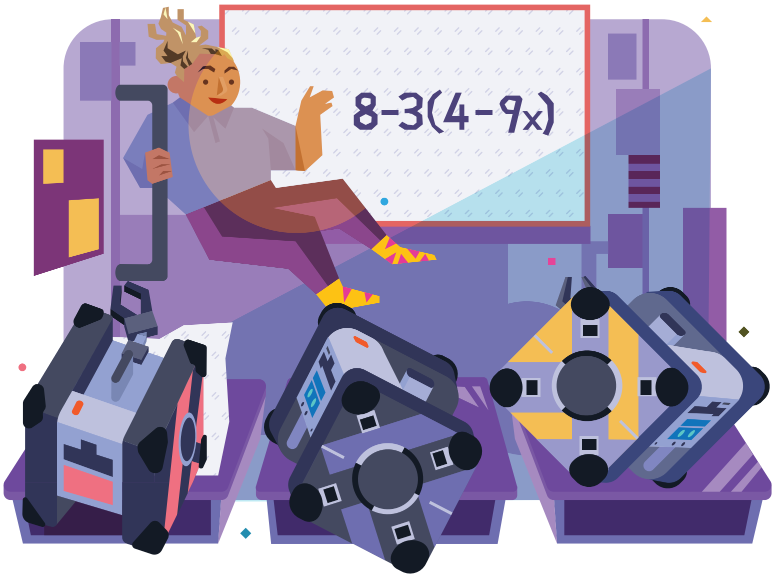
D. $4 - 2(3 + 7) = 4 - (2 \bullet 3 + 2 \bullet 7)$

Name: .... Date: .... Period: ....

Unit 6 | Lesson 22

Combining Like Terms (Part 2)

Let’s see how to write equivalent expressions with parentheses and negative numbers.



Warm-up Make the Equation True

Place parentheses in the equation to make a true equation.

$$44 - 2 \cdot 3 + 7 = 24$$

Log in to Amplify Math to complete this lesson online.

Activity 1 Seeing It Differently

Some students are trying to write an expression with fewer terms that is equivalent to the expression $8 - 3(4 - 9x)$. Their responses are shown. Review each student's response. Then complete the problems.

Noah: "I worked the problem from left to right and ended up with $20 - 45x$."

$8 - 3(4 - 9x)$

$= 5(4 - 9x)$

$= 20 - 45x$

Lin: "I started inside the parentheses and ended up with $23x$."

$8 - 3(4 - 9x)$

$= 8 - 3(-5x)$

$= 8 + 15x$

$= 23x$

Jada: "I used the Distributive Property and ended up with $27x - 4$."

$8 - 3(4 - 9x)$

$= 8 - (12 - 27x)$

$= 8 - 12 - (-27x)$

$= 27x - 4$

Andre: "I also used the Distributive Property, but I ended up with $-4 - 27x$."

$8 - 3(4 - 9x)$

$= 8 - 12 - 27x$

$= -4 - 27x$

1. Do you agree with any of the students? Explain your thinking.

2. For each strategy you disagree with, identify and describe the errors.

Name: ________ Date: ________ Period: ________

Activity 2 Grouping Differently

1. Diego was taking a math quiz. One question asked the students to write the expression $5 + 8x - 9 - 12x$ as an equivalent expression with fewer terms. Complete the task and write the expression $5 + 8x - 9 - 12x$ as an equivalent expression with fewer terms.

2. Diego's teacher told the class there was a typo, and that the expression $5 + 8x - 9 - 12x$ was supposed to have one set of parentheses in it. Diego and his classmates placed parentheses around different parts of the expression and wrote the following expressions.

$5 + (8x - 9) - 12x$

$(5 + 8x) - 9 - 12x$

$5 + 8x - (9 - 12x)$

$5 + (8x - 9 - 12x)$

$5 + 8(x - 9) - 12x$

a Circle the new expressions that are equivalent to the expression $5 + 8x - 9 - 12x$. Explain your thinking.

b Choose one of the new expressions that is *not* equivalent to the expression $5 + 8x - 9 - 12x$, and explain why it is not equivalent.

STOP

Summary

In today's lesson . . .

You furthered your understanding of writing equivalent expressions with fewer terms, which is called *combining like terms*. Combining like terms can be tricky with long expressions, parentheses, and negatives.

You should always follow the order of operations when combining like terms and remember that only like terms can be combined using addition and subtraction. Also, remember to carefully consider the sign of terms when distributing with negative values or subtraction.

Reflect:

Name: .... Date: .... Period: ....

1. Noah and Elena are writing the expression $9x - 2x + 4x$ as an equivalent expression with fewer terms.

- Noah says $9x - 2x + 4x$ is equivalent to $3x$ because the subtraction sign means to subtract everything that comes after $9x$.
- Elena says $9x - 2x + 4x$ is equivalent to $11x$ because the subtraction only applies to $2x$.

Do you agree with either of them? Explain your thinking.

2. Tyler used the Distributive Property to rewrite the expression $9 - 4(5x - 6)$ with fewer terms. His work is shown.

$$\begin{aligned} 9 - 4(5x - 6) &= 9 + (-4)(5x + (-6)) \\ &= 9 + (-20x) + (-6) \\ &= 3 - 20x \end{aligned}$$

a Circle the step where Tyler made an error.

b Explain his error.

c Correct Tyler's work.

3. Write the expression $4 + 2x - \frac{1}{2}(10 - 4x)$ as an equivalent expression with fewer terms. Show your thinking.

Name: .... Date: .... Period: ....

4. The school marching band has a budget of up to $750 to pay for 15 new uniforms and competition fees that total $300. How much can they spend for each uniform?

a Write an inequality to represent this scenario.

b Solve the inequality and describe what the solution represents in the scenario. Show your thinking.

5. A certain shade of blue paint is made by mixing $1\frac{1}{2}$ qt of blue paint with 5 qt of white paint. If you need a total of 16.25 gallons of this shade of blue paint, how much of each color should you mix?

6. How many flags will there be in Figure 10? Show or explain your thinking.

Figure 1

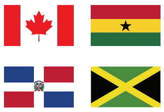


Figure 2

Figure 3

Figure 4

Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 23 – Capstone

Pattern Thinking

Let's write expressions to describe patterns of growth.

Warm-up Predicting the Future

Consider the pattern.

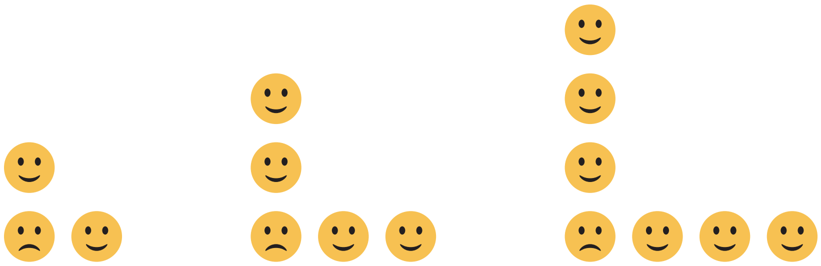


Figure 1 Figure 2 Figure 3

1. Describe how the pattern grows with each step.

2. How many faces will there be in Figure 10? Explain your thinking.

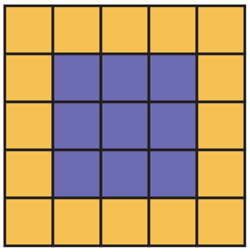
3. How many faces will there be in Figure 1000? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

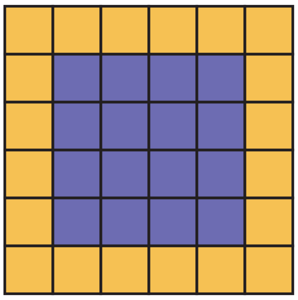
Activity 1 Tiling the Border

Suppose your job is to buy the right number of tiles for the border of different swimming pools.

a The first pool measures 3-by-3. How many tiles are needed for the border?



b The second pool measures 4-by-4. How many tiles are needed for the border?



c How does the number of border tiles relate to the size of the pool? Explain your thinking.

Compare and Connect: Be prepared to convince a friend how you know how many border tiles are needed for a 5-by-5 pool, based on the patterns you discovered.

Name: .... Date: .... Period: ....

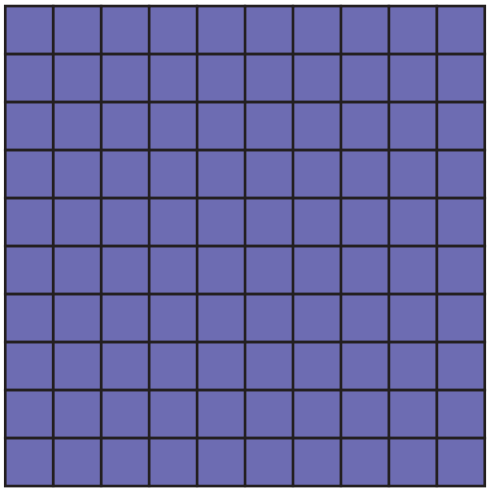
Activity 2 Bigger Borders

Determine the number of tiles you need for the border of each of the following pools.

1. Number of border tiles needed:

   Explain your thinking:

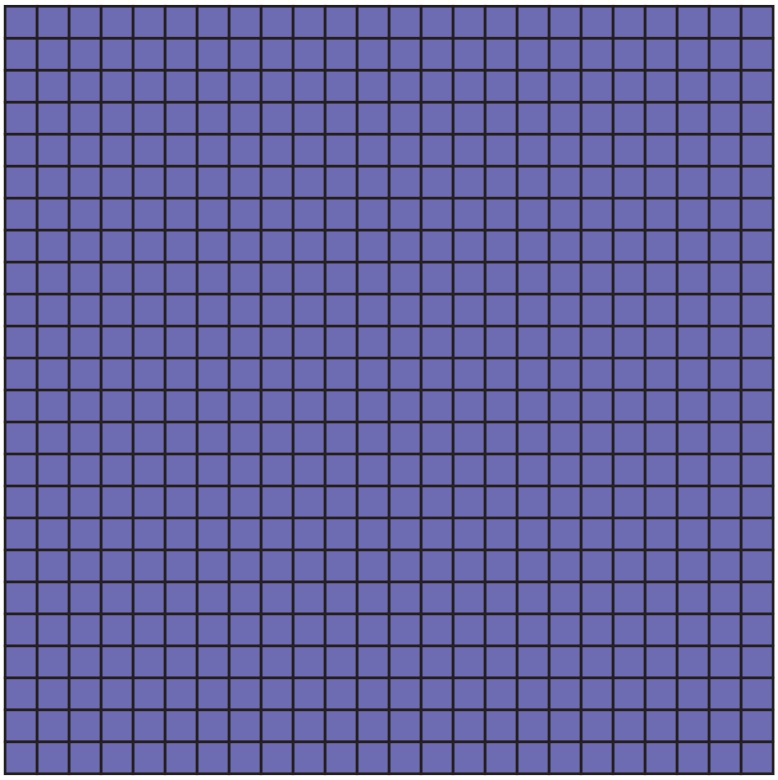
Side length: 10



2. Number of border tiles needed:

   Explain your thinking:

Side length: 24



Activity 3 Booming Business

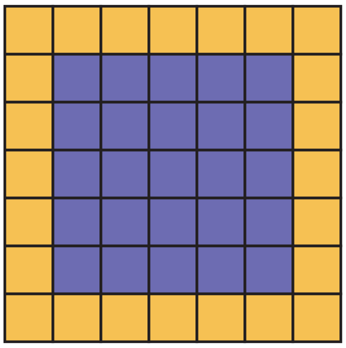
Many customers are interested in having you put borders around their pools. It would be useful to have a rule for determining the number of border tiles for a pool of any size.

Write a rule for a pool of unknown side length n to represent the total number of border tiles b. You will test your rule in the next step and can return to change it, if needed.

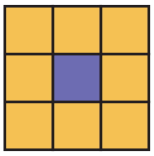
Your rule:

Check that your rule works for both of the following pools.

$n = 5$



$n = 1$



STOP

Unit Summary

The world is an imperfect place. You may not always have the information you want. Maybe the pool is larger than the owner said it would be. Or maybe the tiles you ordered are half the size you thought they were. Life can be full of little curveballs like these, but do not let it get you down.

The first step to solving any problem is knowing how to describe that problem properly. That means not only capturing all its parts, but how those parts relate to each other. There are a few ways to do that: You might use words, for example, or sketch out a diagram. Or you might use symbols. And just as words are composed into a sentence, symbols and numbers are composed into mathematical expressions.

Expressions can be powerful things. When we write them, we write them as statements of fact. Equations, for example, tell us definitively how much a quantity *is*. Meanwhile, inequalities allow us to set thresholds for how much or how little a quantity could be.

But expressions also give us a way to talk about quantities we are unsure of. We can represent these unknowns in our expressions through variables. And once they are in their proper place, it is just a matter of keeping your expression in balance before arriving at your solution.

See you in Unit 7.

Practice

Name: .... Date: .... Period: ....

1. Select *all* the expressions that are equivalent to the expression $0.4x - 2$.

A. $0.4x + (-2)$

B. $0.4(x - 5)$

C. $0.4(x + 5)$

D. $0.4(x - 2)$

E. $2 - 0.4x$

F. $-2 + 0.4x$

2. Consider the pattern of emojis. Determine the total number of emojis in Figure 43. Explain your thinking.

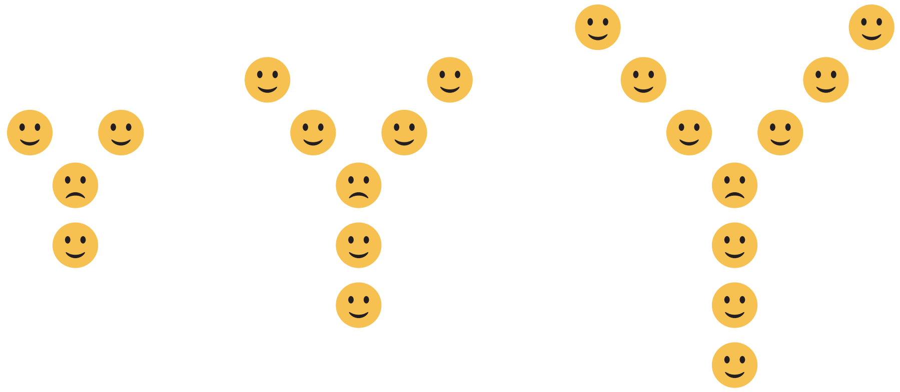


Figure 1 **Figure 2** **Figure 3**

3. Mai claimed that the expression $n + 1 + 2n$ can be used to determine the number of emojis in any step, n. Do you agree with her? Explain your thinking.

Name: .... Date: .... Period: ....

4. Solve each equation. Show your thinking.

a $-\frac{1}{8}d - 4 = -\frac{3}{8}$

b $-\frac{1}{4}m + 5 = 16$

5. Solve each inequality. Show your thinking.

a $10b + (-45) \leq -43$

b $-8(y - 1.25) > 4$

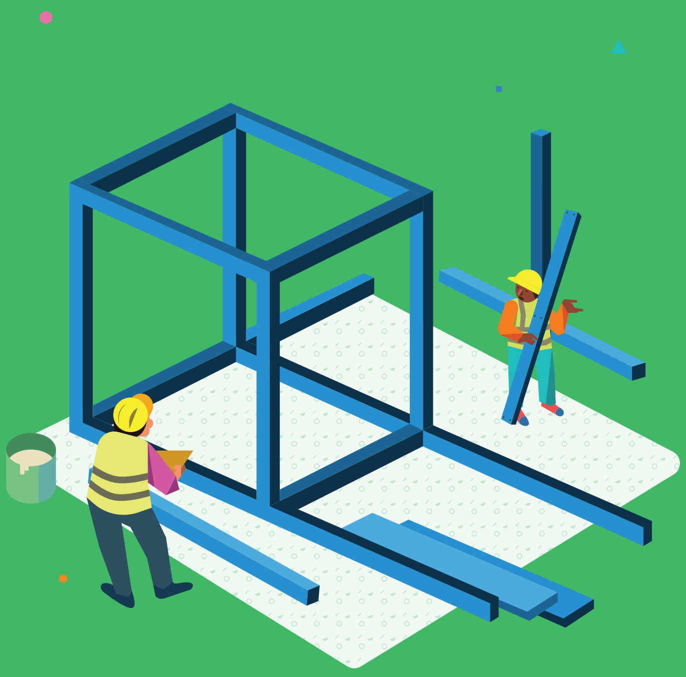
UNIT 7

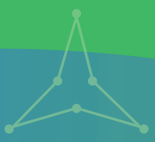
Angles, Triangles, and Prisms

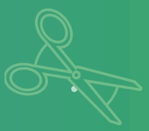
This unit is about the math of what can be seen and what can be held. Get ready to measure, build, and slice your way through an array of geometric figures.

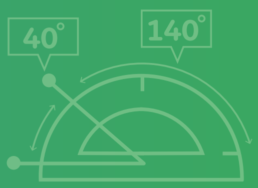
Essential Questions

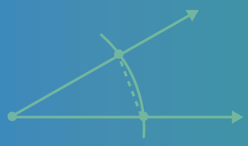
- When do combinations of angles form special angles?
- Given certain segments and angles, how many unique polygons can be made?
- What shapes can be seen when you slice through solid figures?
- *(By the way, why are triangles stronger than squares?)*











SUB-UNIT

1

Angle Relationships

Narrative: Whether you plan to sail across an ocean or not, using some angles to calculate unknown angles can be a powerful tool.

You'll learn . . .

- how complementary and supplementary angles are related.
- about vertical angles.

SUB-UNIT

2

Drawing Polygons With Given Conditions

Narrative: Triangles are an important part of construction. So what are the important parts of triangles?

You'll learn . . .

- to build polygons with specific side lengths and angles.
- about triangles with common measures.

SUB-UNIT

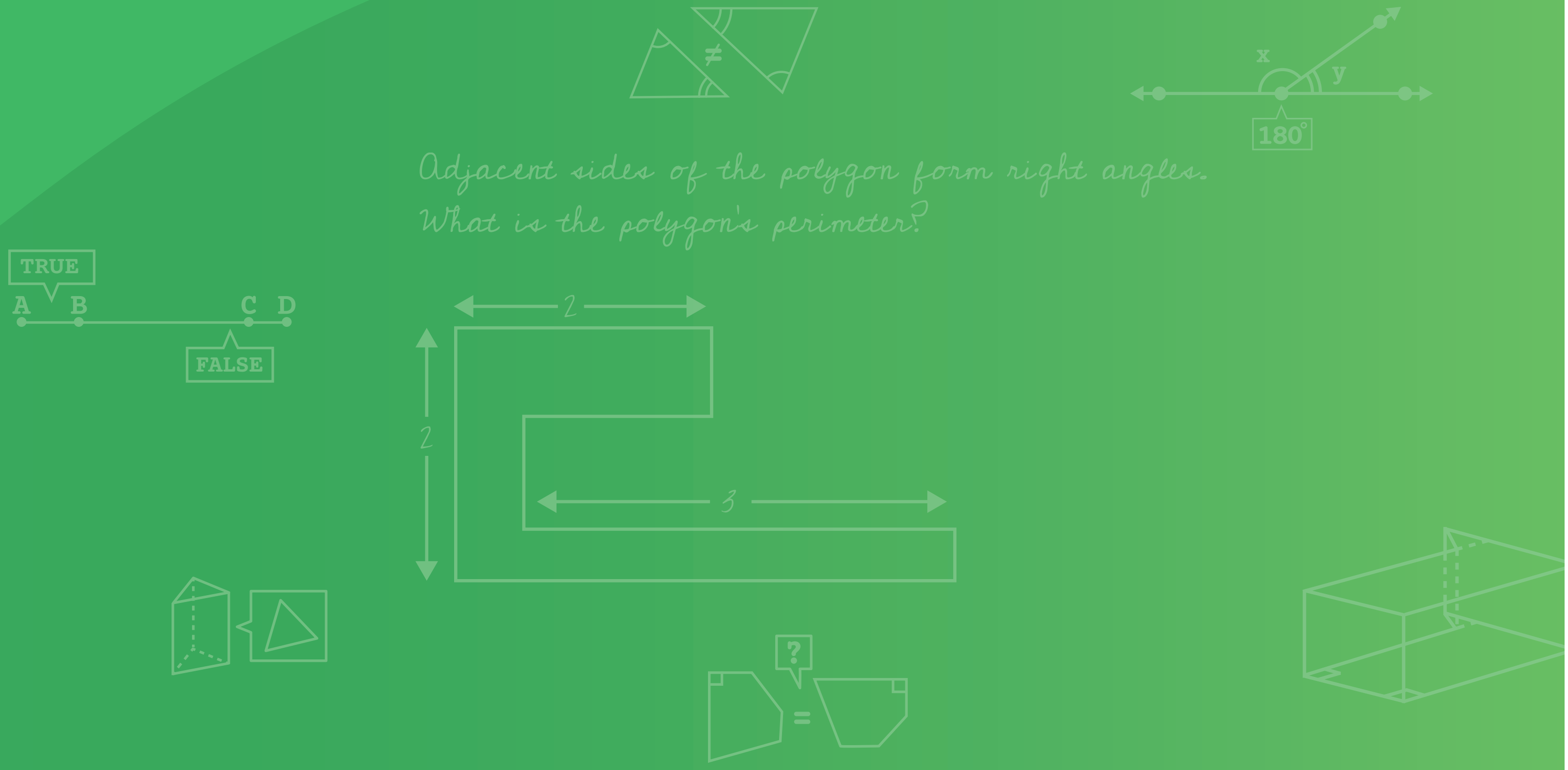
3

Solid Geometry

Narrative: Volume and surface area are important measures of solids. When peering inside, cross sections are another great tool.

You'll learn . . .

- what cross sections look like for different solids.
- strategies for computing volume and surface area.



Unit 7 | Lesson 1 – Launch

Shaping Up

Let's reacquaint ourselves with polygons.

Warm-up Which One Doesn't Belong?

Study the figures. Which one doesn't belong? Explain your thinking.

Figure A

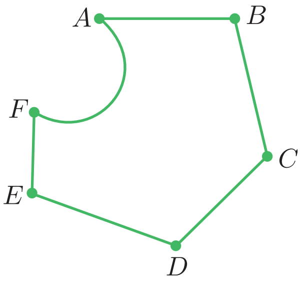


Figure B

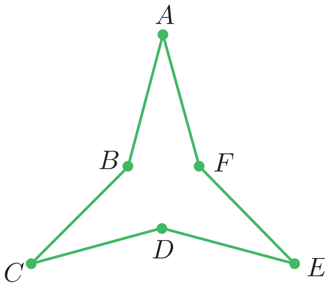


Figure C

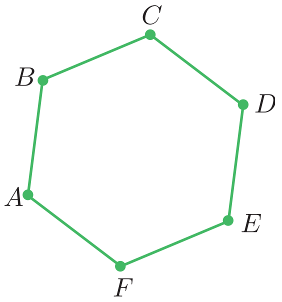
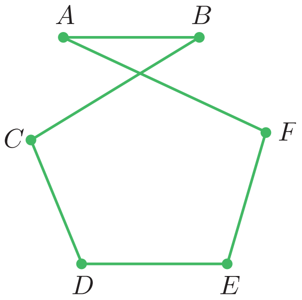


Figure D



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Name: ________________ Date: ________ Period: ________

Activity 1 Team Building

Part 1

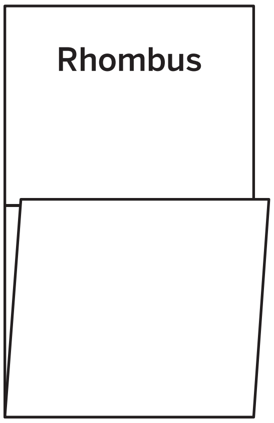
Write what you already know about each of the listed polygons.

Rhombus	Square
Rectangle	**Parallelogram**
Hexagon	**Regular Hexagon**

Activity 1 Team Building (continued)

Part 2

Indigenous peoples around the world have made shapes and patterns by looping strings around their hands for millennia. Mathematicians, including Thomas Storer, have studied these string game sequences to better learn the complex mathematical ideas embedded in each of them. You will be given a set of cards in an envelope and a length of string.



1. With your group, choose one card from the envelope, looking only at the name of the shape.
2. Use the string to build that shape.
3. Be ready to explain to the class how you know your built shape has the specific properties needed to define it.

Featured Mathematician

Thomas Storer

Thomas Storer was a professor of mathematics at the University of Michigan, where he also taught the Ojibwa language. A Navajo, he was one of the first Native Americans to earn a Ph.D. in mathematics. He is known for his studies of string figures — patterns made from a loop of string stretched and woven with one's hands — which he first learned from his grandmother as a child.

STOP

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Unit 7 Angles, Triangles, and Prisms

Journey to the Third Dimension

Hooray for the humble honeybee! These pint-sized pollinators might not look like much, but in the whole animal kingdom, who can match the bee's ingenuity?

They are the industrious and dedicated builders of the natural world. They even work weekends!

And their tool of choice is the hexagon. Every honeycomb is a dazzling pattern of interlocking hexagons, each one a sturdy, stackable cubbyhole for storing honey and housing their young.

Using a brain the size of a sesame seed, the honeybee not only understands the shape of hexagons, but can build them in three dimensions. Like architects and engineers, the honeybee takes an idea that once only lived in its brain and, layer-by-layer, brings it to the world.

Why did the bee choose the hexagon to use? Have humans chosen a shape to use when building? What is important about the shapes we choose to use?

Some shapes hold their form better than others. Some shapes fit together more easily. In this unit, you will explore what makes certain shapes special, build those shapes, and think more deeply about three-dimensional shapes.

Welcome to Unit 7.

Name: .... Date: .... Period: ....

1. Place a check mark in each cell of the table that indicates whether the property listed is a property of each type of polygon.

Property	Square	Rhombus	Rectangle	Parallelogram	Hexagon	Regular hexagon
All side lengths are equal.						
All angle measures are equal.						
There are four sides.						
There are six sides.						
Opposite sides have equal lengths.						
Opposite angles have equal measures.						
All angles are right angles.						

2. Elena claims that a square is just a special type of rectangle. Do you agree or disagree? Explain your thinking.

Name: ________________ Date: ________ Period: ________

3. A train travels at a constant speed for a long distance. Write the two constants of proportionality that represent the relationship between distance traveled and elapsed time. Explain what each constant means.

Time elapsed (hours)	Distance (miles)
1.2	54
3	135
4	180

4. Write as many properties as you can about each shape.

a Square

b Trapezoid

c Rhombus

5. Draw an example of each type of angle.

Acute	Right	Obtuse

My Notes:

SUB-UNIT

1 Angle Relationships

Narrative Connections

Did radio kill the aviation star?

Few pilots are more famous than Amelia Earhart. She was a national hero and a pioneer. She was the first woman to fly solo across the Atlantic, and the breaker of countless distance, speed, and altitude records.

But tragedy struck in 1937, when she attempted to fly around the world.

For the flight, her plane was outfitted with state-of-the-art radio equipment. This would allow her to communicate with nearby ships. Those ships would then use radio signals to get a bearing on her plane.

The problem was that neither Earhart nor her navigator Fred Noonan, had training experience with radios. This led to a breakdown in communication, which cost the two of them their lives while flying over the South Pacific.

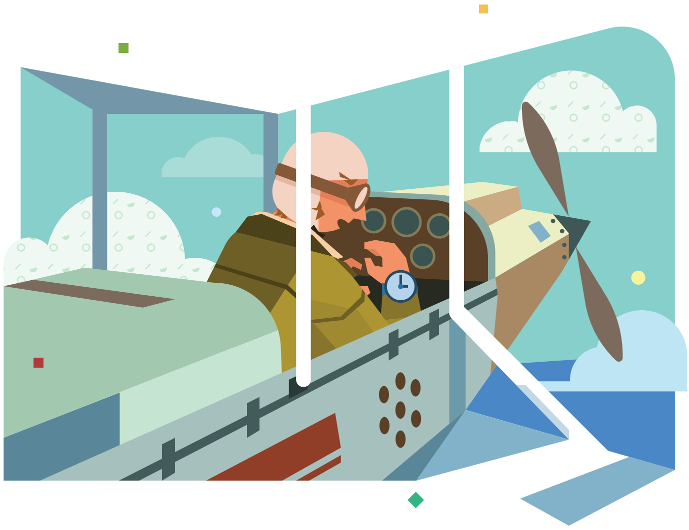
Traditionally, navigators relied on “celestial navigation” to find their way. Using a tool called a sextant, navigators **measured the angle** of a star (usually the Sun) or other celestial body, over the horizon. They would then cross-reference that angle against a nautical chart. With a few calculations they could get a rough idea of their position on a map.

Unfortunately, heavy rain and clouds over the South Pacific made celestial navigation difficult. Had they not put blind trust in their technology, they might have chosen a flight path with clearer skies, where they could have used this tried-and-true method of navigation.

Unit 7 | Lesson 2

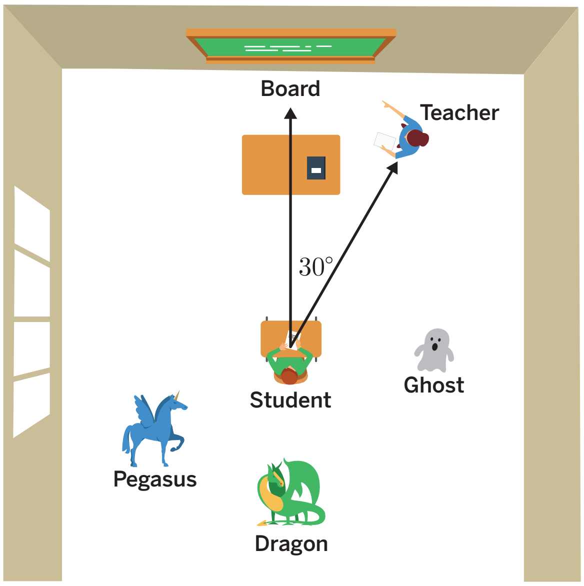
Relationships of Angles

Let's investigate some special angles.



Warm-up 3 O'Clock

While his teacher reads a fiction story aloud, a student visualizes the characters from the story in his mind. Study the diagram that shows some of the characters he is thinking about and their location in the classroom.



1. Which object is at the student's 3 o'clock? What angle is formed by this object, the student, and the board?

2. Which object is at the student's 6 o'clock? What angle is formed by this object, the student, and the board?

3. What angle is formed by the board, the student, and the teacher? What "time" would this be?

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Pattern Block Angles

You will be given a set of pattern blocks.

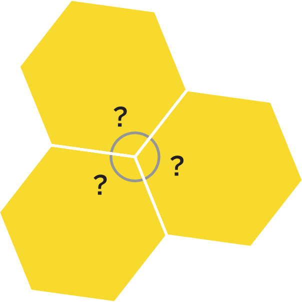
1. Study the different pattern blocks.

   a. Which shapes have only one unique angle measure?

   b. Which shapes have two unique angle measures?

2. If you place three copies of the hexagon together so that one vertex from each hexagon touches the same point, as shown, they fit together without any gaps or overlaps.

   Each of the three labeled angles has the same measure. Determine the measure, in degrees, of one of these angles. Show or explain your thinking.

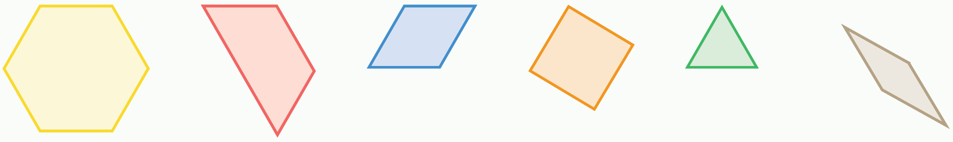


Reflect: How did communication help you and others draw conclusions in the activity?

3. Arrange as many squares as possible so they touch at the same vertex without overlapping. Use this to determine the measure of the angle, in degrees, inside each square pattern block. Does this match what you know about the angles of a square?

Are you ready for more?

Determine the degree measures for as many pattern blocks as you can. Explain your thinking.



Activity 2 More Pattern Block Angles

Use the pattern blocks from Activity 1.

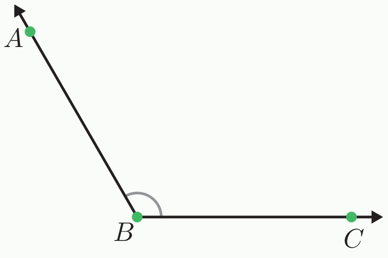
Discussion Support: If you know all the angle measures in a shape have a sum of 180° or 360°, can you determine a single angle measure? Why or why not? Discuss with your partner.

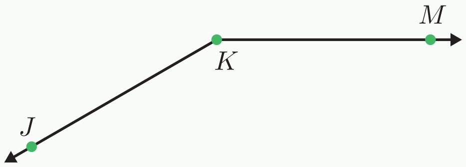
1. Determine which shapes have a sum of 180° for all of its angles. Show your thinking, including a sketch of your pattern block arrangement.

2. Determine which shapes have a sum of 360° for all of its angles. Show your thinking, including a sketch of your pattern block arrangement.

Are you ready for more?

Use the pattern blocks to determine the measures of the following angles. Explain your thinking.





STOP

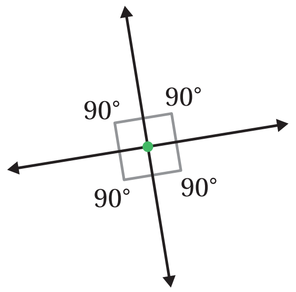
Name: ____________ Date: ____________ Period: ____________

Summary

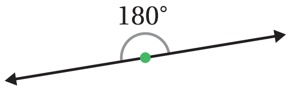
In today's lesson . . .

You composed and decomposed pattern blocks to help you understand the angle measures of these shapes. While doing so, you reviewed the following types of angles:

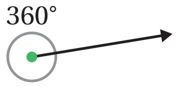
Right angles ($90°$)



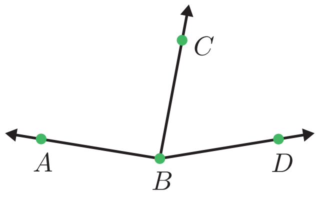
Straight angles ($180°$)



"All the way around" angles ($360°$)



Two angles that share a common ray and common vertex are called ***adjacent angles***. In the diagram, angle ABC is adjacent to angle DBC because both angles share the vertex B and the ray BC.



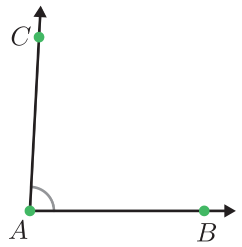
You can use the symbol $\angle$ to represent the word *angle* to save time and space. You can also use the notation $\text{m}\angle$ to represent the *measurement of an angle*.

Reflect:

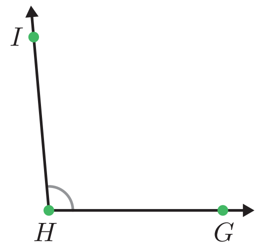
Name: .... Date: .... Period: ....

1. Estimate the measure of each angle.

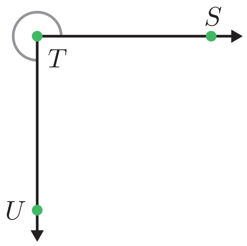
a



b



c



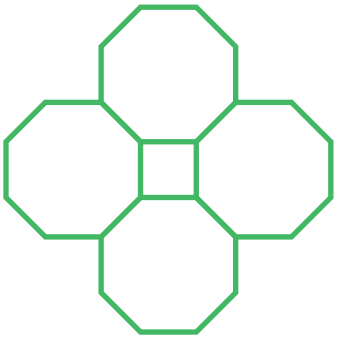
2. An equilateral triangle's angles each have a measure of 60°.

a Can you arrange copies of an equilateral triangle together to form a straight angle? Explain your thinking.

b Can you arrange copies of an equilateral triangle together to form a right angle? Explain your thinking.

c Can you arrange copies of an equilateral triangle together to form a 360° angle? Explain your thinking.

3. In the following pattern, all of the angles inside the octagon have the same measure. The shape in the center is a square. Determine the measure of one of the angles inside one of the octagons. Explain your thinking.



Name: ________________ Date: ________ Period: ________

4. Write an equivalent expression using the Distributive Property.

a $-3(2x - 4)$

b $0.1(-90 + 50a)$

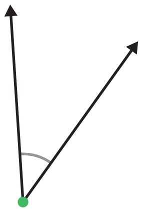
c $-7(-x - 9)$

d $\frac{4}{5}[10y + (-x) + (-15)]$

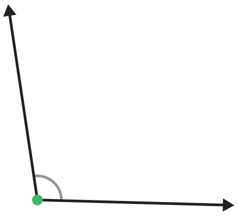
5. Lin's puppy is gaining weight at a rate of 0.125 lb per day. Describe the weight gain in days per pound.

6. Select *all* the angles that have measures less than $90°$. Verify your responses by measuring each angle.

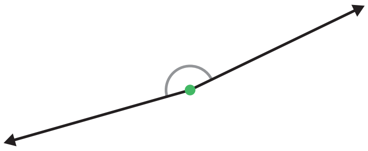
A.



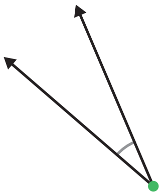
C.



B.



D.



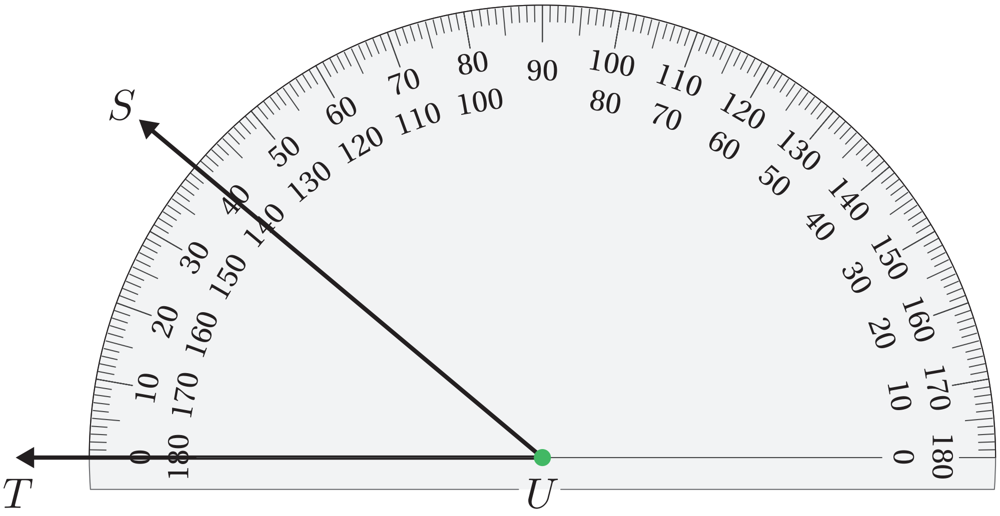
Unit 7 | Lesson 3

Supplementary and Complementary Angles (Part 1)

Let's investigate some special pairs of angles.

Warm-up Measuring Like This or Like That?

Tyler and Priya are both measuring angle TUS.



Priya thinks the angle measures 40°. Tyler thinks the angle measures 140°. Do you agree with either of them? Explain your thinking.

Stronger and Clearer: Meet with a partner to give and receive feedback on your explanation. Revise your explanation, making it stronger and clearer.

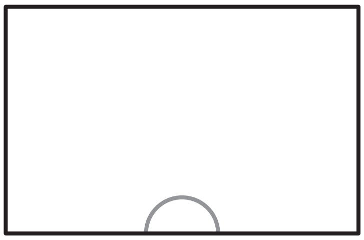
Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Cutting Rectangles One Way

You will be given a sheet of paper.

1. Draw a small half-circle in the middle of one side as shown.



2. Cut a straight line, starting from the center of the half-circle, all the way across the paper to make two separate pieces. Your cut does *not* need to be perpendicular to the side of the paper.

3. On each of these two pieces, measure the angle that is marked by part of the circle. Label the angle measures on the pieces and record them here.

4. Compare angle measures with your group members. What do you notice about the measures of each pair of angles?

Are you ready for more?

Clare measured 70° on one of her pieces. Predict the angle measure of her other piece.

Activity 2 Cutting Rectangles Another Way

You will be given a sheet of paper.

1. Draw a small quarter-circle in one of the corners as shown.

2. Cut a straight line, starting from the corner with the half-circle, all the way across the paper to make two separate pieces. The pieces do *not* have to be the same size.

3. On each of these two pieces, measure the angle that is marked by part of the circle. Label the angle measures on the pieces and record them here.

4. Compare angle measures with your group members. What do you notice about the measures of each pair of angles?

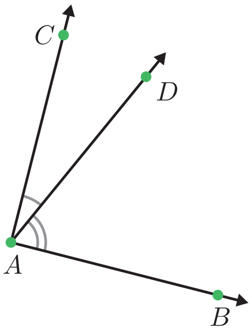
Are you ready for more?

Priya measured 47° on one of her pieces. Predict the angle measure of her other piece.

Name: ____________ Date: ____________ Period: ____________

Activity 3 Are They Complementary or Supplementary?

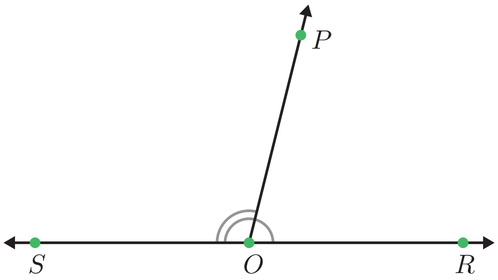
1. Angle CAB is a right angle.



a Use your protractor to measure angle DAC. Label the measurement on the diagram.

b Determine the measurement of angle DAB. Label the measurement on the diagram.

2. Point O is on line RS.



a Use your protractor to measure angle POR. Label the measurement on the diagram.

b Determine the measurement of angle POS. Label the measurement on the diagram.

STOP

Summary

In today's lesson . . .

You discovered two important and special types of angle pairs:

- ***complementary angles*** are two angles whose measures add up to $90°$.
- ***supplementary angles*** are two angles whose measures add up to $180°$.

Reflect:

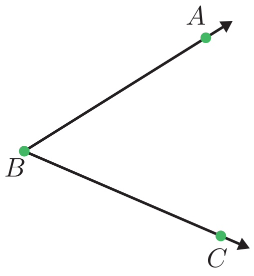
Name: .... Date: .... Period: ....

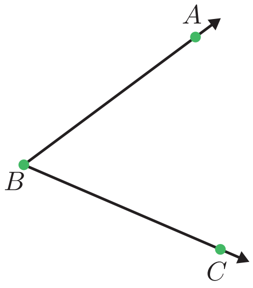
Practice

1. For each diagram, draw an angle that is:

a Complementary to angle ABC.

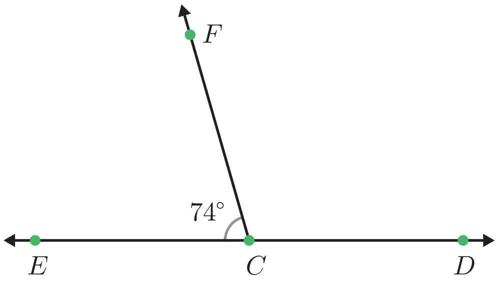
b Supplementary to angle ABC.





2. Determine the missing angle measure in each diagram.

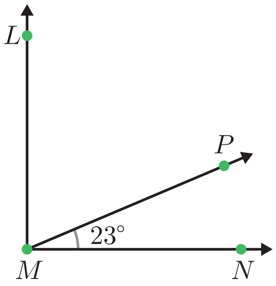
a Point C is on line ED. Determine the measure of $\angle FCD$.



The figure may not be drawn to scale.

$m\angle FCD =$

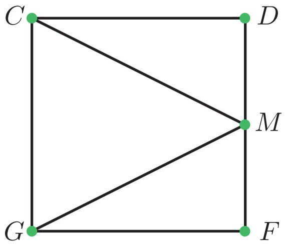
b Angle LMN is a right angle. Determine the measure of $\angle LMP$.



The figure may not be drawn to scale.

$m\angle LMP =$

3. Refer to Square $CDFG$. Name three angles that have a sum of $180°$.



Practice

Name: .... Date: .... Period: ....

4. Here are three equations. For each table, write the equation that represents the same proportional relationship.

$y = 1.5x$ $\quad y = 1.25x$ $\quad y = 4x$

a

x	y
2	8
3	12
4	16
5	20

b

x	y
3	4.5
6	9
7	10.5
10	15

c

x	y
2	$\frac{5}{2}$
4	5
6	$\frac{15}{2}$
12	15

5. Complete the equation with a number that makes the expression on the right side of the equal sign equivalent to the expression on the left side.

$5x - 2.5 + 6x - 3 = \boxed{}(2x - 1)$

6. For each angle listed, write the angle measure of its complementary angle, called its *complement*, and its supplementary angle, called its *supplement*.

Angle	Complement	Supplement
43°		
78°		
22°		

Name: .... Date: .... Period: ....

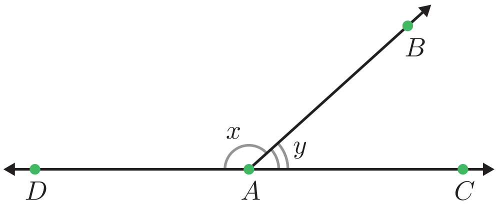
Unit 7 | Lesson 4

Supplementary and Complementary Angles (Part 2)

Let's investigate angles that are not right next to each other.

Warm-up Finding Related Statements

Point A is on the line DC, $m\angle DAB = x$, and $m\angle BAC = y$.



1. What type of angle do $\angle DAB$ and $\angle BAC$ form together?

2. Select *all* the statements which must be true.

A. $x = 180 - y$

B. $x - 180 = y$

C. $360 = 2x + 2y$

D. $x = 90°$ and $y = 90°$

E. $x - y = 90$

3. How do you know these angles are supplementary? Where in the diagram do you see this?

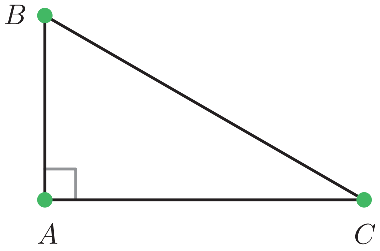
Discussion Support: Share your responses to Problem 3 with another pair of students. What math terms or language did you use in your response?

Log in to Amplify Math to complete this lesson online.

Activity 1 Triangle Angles

1. For each triangle, use a protractor to measure each non-right angle. Then determine the sum of their measures.

a

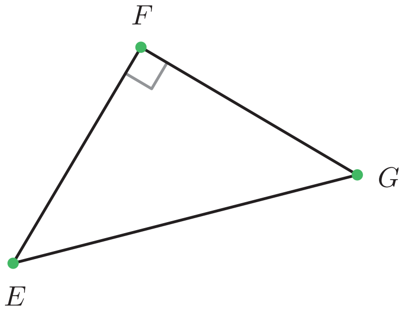


$m\angle B =$ ....

$m\angle C =$ ....

$m\angle B + m\angle C =$ ....

b

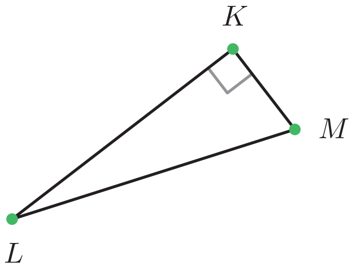


$m\angle E =$ ....

$m\angle G =$ ....

$m\angle E + m\angle G =$ ....

c



$m\angle L =$ ....

$m\angle M =$ ....

$m\angle L + m\angle M =$ ....

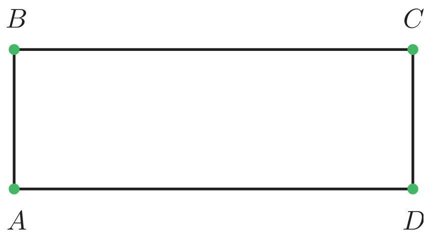
2. What do you notice?

Name: .... Date: .... Period: ....

Activity 2 Parallelogram Angles

1. For each parallelogram, use a protractor to measure each angle.

a



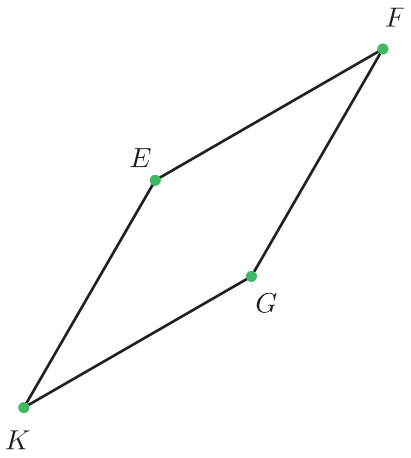
$\mathrm{m}\angle A =$ ....

$\mathrm{m}\angle B =$ ....

$\mathrm{m}\angle C =$ ....

$\mathrm{m}\angle D =$ ....

b



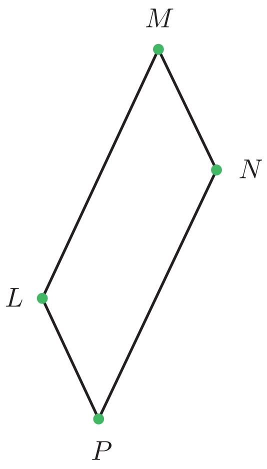
$\mathrm{m}\angle E =$ ....

$\mathrm{m}\angle F =$ ....

$\mathrm{m}\angle G =$ ....

$\mathrm{m}\angle K =$ ....

c



$\mathrm{m}\angle L =$ ....

$\mathrm{m}\angle M =$ ....

$\mathrm{m}\angle N =$ ....

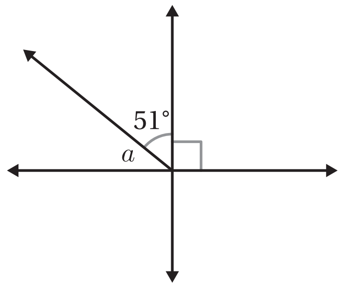
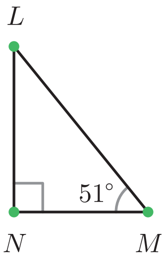
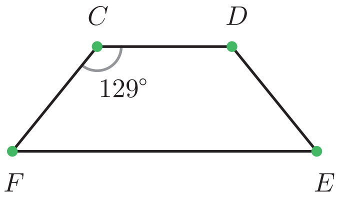
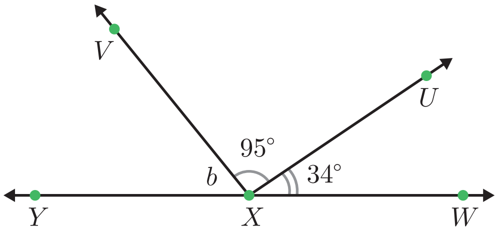
$\mathrm{m}\angle P =$ ....

2. What do you notice?

Activity 3 Partner Problems: Angles

One partner will complete Column A and the other will complete Column B. Complete the problems in your column, then compare responses with your partner. Discuss and resolve any differences.

For each column, write an equation that represents a relationship between the angles in each diagram. Then solve the equation to determine the unknown angle. The figures may not be drawn to scale.

Column A	Column B
1. Determine the value of a. 	In right triangle LMN, angles L and M are complementary. Determine the measure of angle L. 
2. Angle C and angle E are supplementary. Determine the measure of angle E. 	X is on the line WY. Determine the value of b. 

Are you ready for more?

Continue the *Partner Problems* routine with these problems.

Column A

Two angles are complementary. One angle measures 37°. Determine the measure of the other angle.

Column B

Two angles are supplementary. One angle measures 127°. Determine the measure of the other angle.

STOP

Name: ____________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You discovered that the two non-right angles in a right triangle are complementary, and that the adjacent angles of a parallelogram are supplementary. Knowing special angle relationships like these can help you determine unknown angle measures.

- If two angles are *complementary*, then you know the sum of their angle measures must be $90°$.
- If two angles are *supplementary*, then you know the sum of their angle measures must be $180°$.

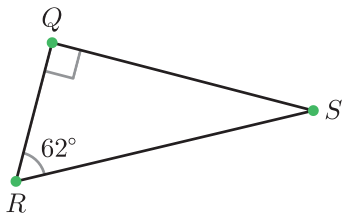
By writing simple equations, you can determine missing angle measures. For example, if you know that two angles are supplementary and one angle measures $56°$, then you can write the equation $56 + x = 180$, and solve the equation to determine that $x = 124$. This means the other angle measure is $124°$.

Reflect:

Name: .... Date: .... Period: ....

1. Angles R and S are complementary. Determine the measure of angle S.

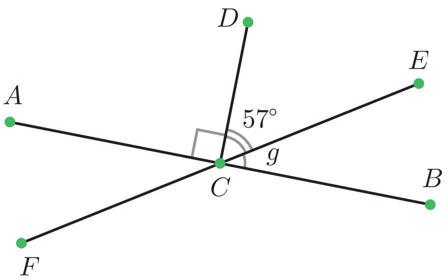
$m\angle S =$ ....



The figure may not be drawn to scale.

2. Segments AB, EF, and CD intersect at point C, and $\angle ACD$ is a right angle. Determine the value of g.

$g =$ ....



The figure may not be drawn to scale.

3. If you know two angles are complementary and are given the measure of one of these angles, can you determine the measure of the other angle? Explain why or why not.

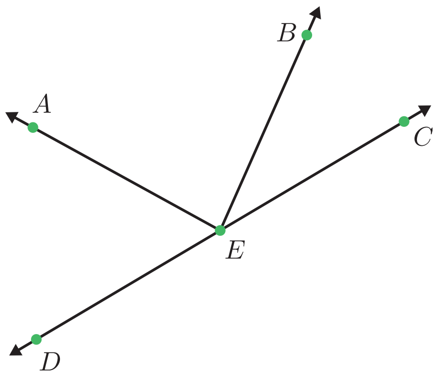
4. Match each expression in the first list with an equivalent expression from the second list.

Expression	**Equivalent expression**
a $5(x + 1) - 2x + 11$	.... $\frac{1}{4}x - 8$
b $2x + 2 + x + 5$	.... $\frac{1}{2}(6x + 14)$
c $-\frac{3}{8}x - 9 + \frac{5}{8}x + 1$	.... $11(9x + 4)$
d $2.06x - 5.53 + 4.98 - 9.02$	.... $3x + 16$
e $99x + 44$	.... $2.06x + (-5.53) + 4.98 + (-9.02)$

Name: .... Date: .... Period: ....

5. A small dog is fed $\frac{3}{4}$ cups of dog food twice a day. Write an equation that gives the total number of cups of food f the dog should be fed over d days. Use the equation to determine how many days a large bag of dog food will last, if it contains 210 cups of food.

6. Name *all* the angles shown in the diagram with measures less than $180°$.



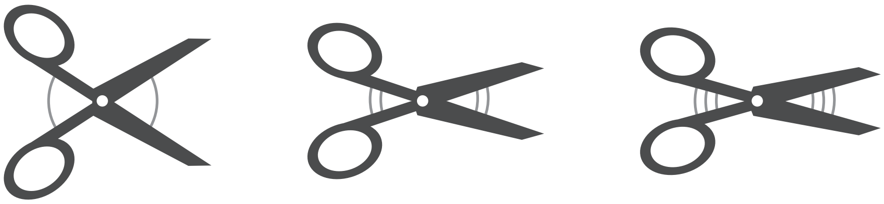
Unit 7 | Lesson 5

Vertical Angles

Let's investigate angles that are across from each other.

Warm-up Notice and Wonder

Refer to the images of scissors. What do you notice. What do you wonder?



1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Vertical Angles

Use the space provided for Problems 1 and 2.

1. Use a ruler to draw two intersecting lines.

2. Use a protractor to measure all four angles whose vertex is located at the intersection. Record the measurements.

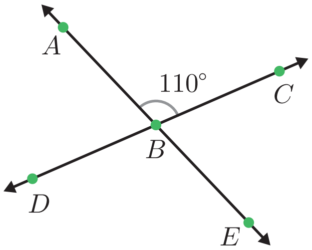
3. Compare your drawing and measurements with others in your group.

4. What do you notice about the relationships between the angle measures at the intersection?

Activity 2 Determine the Missing Angles

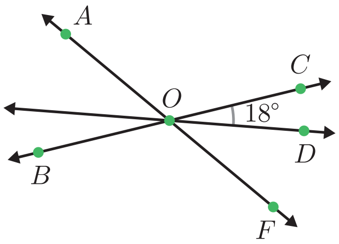
Determine the measures of the unknown angles in each figure. The figures may not be drawn to scale.

1.



$m\angle DBE =$ ....

2.

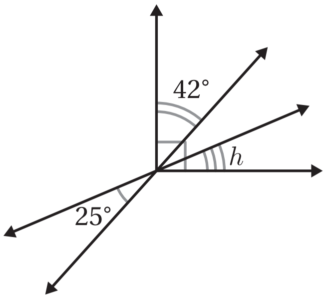


$m\angle COD = 18°$

$m\angle AOB = 63°$

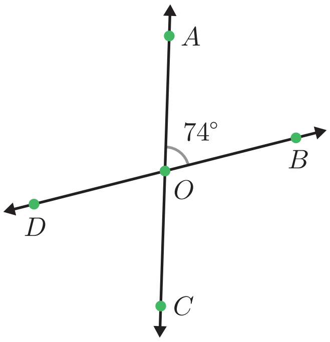
$m\angle DOF =$ ....

3.



$h =$ ....

4.



$m\angle AOD =$ ....

$m\angle DOC =$ ....

$m\angle COB =$ ....

STOP

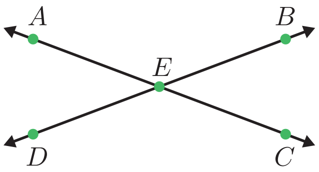
Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You discovered that when two lines intersect, they form two pairs of opposite angles, called ***vertical angles***.

Vertical angles have the same measure.



$\angle AEB$ and $\angle DEC$ are vertical angles, so they have the same measure.

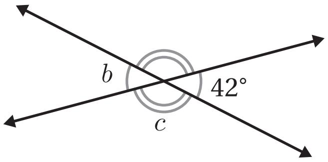
$\angle AED$ and $\angle BEC$ are vertical angles, so they have the same measure.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Determine the values of b and c.

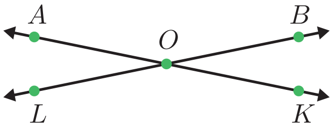


The figure may not be drawn to scale.

$b =$ ....

$c =$ ....

2. Lines AK and LB intersect at point O and $m\angle BOK = 23°$. Determine the measures of angles AOB and LOK.



The figure may not be drawn to scale.

$m\angle AOB =$ ....

$m\angle LOK =$ ....

3. Use what you know about complementary, supplementary, and vertical angles to respond to the following.

a Is it possible for angles to be both vertical and supplementary? If so, can you determine the angle measures? Explain your thinking.

b Is it possible for angles to be both vertical and complementary? If so, can you determine the angle measures? Explain your thinking.

Name: .... Date: .... Period: ....

4. Select *all* the expressions that represent x decreased by 80%.

A. $\frac{20}{100}x$

B. $x - \frac{80}{100}x$

C. $\left(\frac{100-20}{100}\right)x$

D. $0.80x$

E. $(1-0.8)x$

5. Andre is solving the equation $4\left(x+\frac{3}{2}\right)=7$. He says, "I can subtract $\frac{3}{2}$ from each side to get $4x=\frac{11}{2}$. Then divide by 4 to get $x=\frac{11}{8}$."
Kiran says, "I think you made a mistake."

a How can Kiran know for sure that Andre's solution is incorrect?

b Describe Andre's error and explain how to correct his work.

6. Solve each equation.

a $8x - 5.5 = 7.3$

b $2\left(y+\frac{3}{2}\right)=9$

Unit 7 | Lesson 6

Using Equations to Solve for Unknown Angles

Let’s use equations to determine missing angle measures.

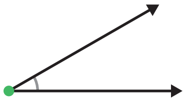
Warm-up It’s All Downhill From Here

In the ancient Roman world, aqueducts, like the Pont du Gard, carried water long distances using only gravity. These structures might look perfectly horizontal, but there is actually a very *slight* difference in elevation — sometimes less than $0.02°$ — that lets water run from higher elevation to lower elevation. Roman engineers measured and built these aqueducts without the use of modern tools!

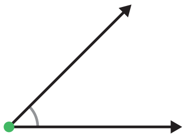
Bertl123/Shutterstock.com

Now it’s your turn. Without using a protractor, estimate the measure of each angle.

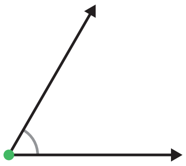
a



b



c



d

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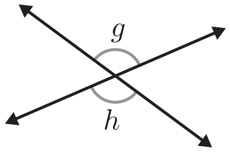
Name: ______ Date: ______ Period: ______

Activity 1 What's the Match?

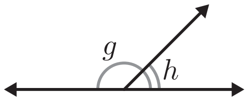
Match each diagram to an equation that represents it. For each match, explain how you know they are a match.

$g + h = 180$	$g = h$	$2h + g = 90$	$g + h + 48 = 180$	$g + h + 35 = 180$

1. 

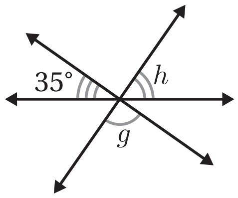
Equation:

Explanation:

2. 

Equation:

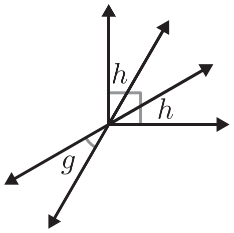
Explanation:

3. 

The figure may not be drawn to scale.

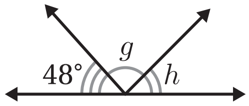
Equation:

Explanation:

4. 

Equation:

Explanation:

5. 

The figure may not be drawn to scale.

Equation:

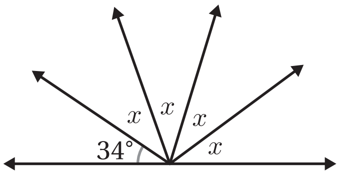
Explanation:

Plan ahead: How can you apply self-discipline to guide your solution process?

Activity 2 What Does It Look Like?

Write an equation that represents a relationship between the angles in each figure. Then solve the equation to determine the unknown angle measure. The figures may not be drawn to scale.

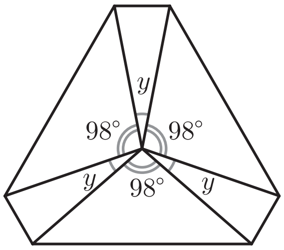
1.



Equation:

Solution:

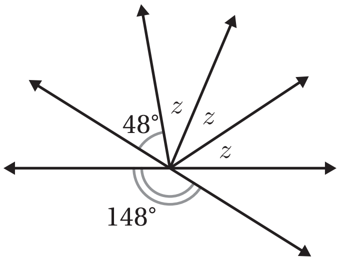
2.



Equation:

Solution:

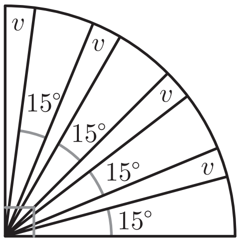
3.



Equation:

Solution:

4.



Equation:

Solution:

Compare and Connect: Your teacher will show you two different solution pathways for Problems 2 and 4. Discuss with a partner which solution is more efficient or which you think is more useful to use and why.

STOP

Name: ______ Date: ______ Period: ______

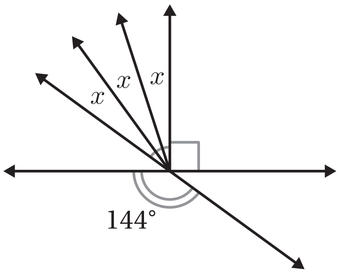
Summary

In today's lesson . . .

You saw that you can write an equation to represent relationships among angles.

Using what you know about the relationships of angles in vertical, complementary, and supplementary pairs can help you write an equation.

Then you can solve the equation to determine an unknown angle measure.



Equation:

$$3x + 90 = 144$$

$$3x + 90 - 90 = 144 - 90$$

$$3x = 54$$

$$x = 18$$

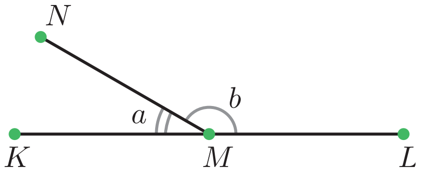
Solution: $x = 18°$

Reflect:

Practice

Name: .... Date: .... Period: ....

1. M is a point on line segment KL. Line segment NM intersects line segment KL.



Select *all* the equations that represent the relationships between the measures of the angles in the diagram.

A. $a = b$

B. $a + b = 90$

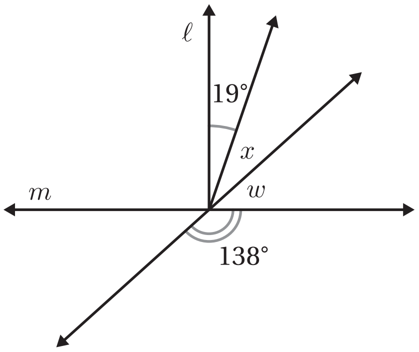
C. $a + b = 180$

D. $b = 90 - a$

E. $180 - a = b$

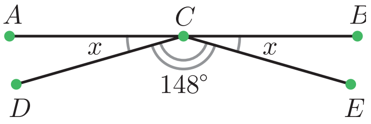
F. $180 = b - a$

2. Ray ℓ is perpendicular to line m. Determine the values of x and w. Show your thinking.



The figure may not be drawn to scale.

3. Segments AB, DC, and EC intersect at point C. The measure of angle DCE is 148°. Determine the value of x.



The figure may not be drawn to scale.

Name: ____________ Date: ________ Period: ________

4. The directors of a dance show expect many students to participate, but do not yet know how many students will actually participate. The directors need 7 students to work on the technical crew. The rest of the students will work on dance routines in groups of 9. The directors need at least 6 full groups working on dance routines.

   a Write and solve an inequality to represent this situation. Then graph the solution on a number line.

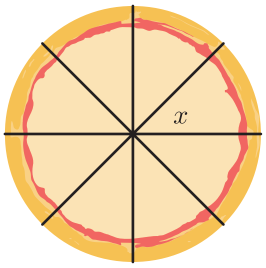
   b Write a sentence to the directors about the number of students they need to participate in the dance show.

5. Solve each equation.

   a $\frac{1}{7}a + \frac{3}{4} = \frac{9}{8}$

   b $\frac{1}{7}\left(x + \frac{3}{4}\right) = \frac{1}{8}$

6. Assuming the vegetable pizza is partitioned equally, determine the value of x. Show or explain your thinking.



Unit 7 | **Lesson 7**

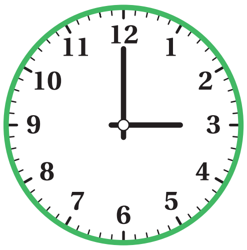
Like Clockwork

Let's apply our understanding of angles and proportional reasoning to the hands on a clock.

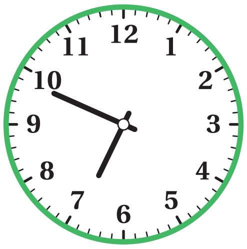
Warm-up Which One Doesn't Belong?

Study the four clocks shown. Which one doesn't belong? Explain your thinking.

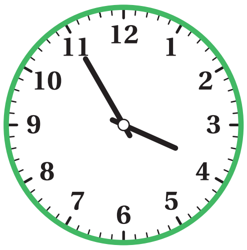
A.



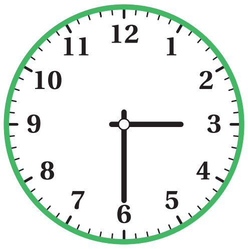
C.



B.



D.



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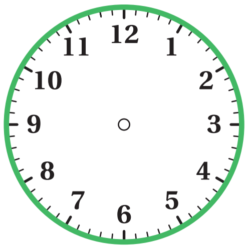
Name: .... Date: .... Period: ....

Activity 1 Time, in Degrees

Think about how the minute and hour hands on a clock turn.
Hint: Sketching the different hands on the clock may help to complete this activity.



1. How many degrees does the minute hand turn in one minute? Explain your thinking.

2. How many degrees does the hour hand turn in one hour? Explain your thinking.

3. How many degrees does the hour hand turn in one minute? Explain your thinking.

Are you ready for more?

Determine a time for which the measure of the angle between the minute hand and the hour hand is about $180°$. Explain your thinking.

Activity 2 Precision Timekeeping

You will be given a set of cards, a protractor, and a ruler to complete this activity.

A time is given on each card. Write an equation that relates the number of hours h and minutes m to the angle formed by the hour hand and 12 o'clock. Then write an equation that relates the number of minutes to the angle formed by the minute hand and 12 o'clock. Then draw the precise location of the hands using your mathematical tools.

Are you ready for more?

What would a clock look like if time was measured using a 10-hour system instead of a 12-hour system? Draw your design here and explain how you came up with it.

STOP

Name: Date: Period:

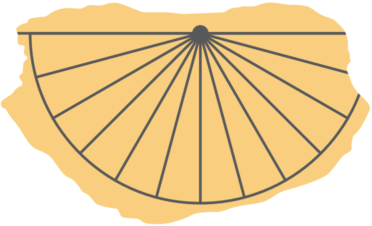
Summary

In today's lesson . . .

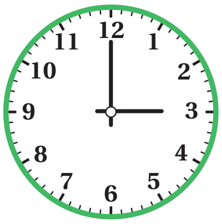
You applied your prior understanding of angles and proportional relationships to clocks, discovering that:

- The minute hand turns $6°$ each minute.
- The hour hand turns $30°$ each hour.
- The hour hand turns $\frac{1}{2}°$ each minute.

The measurement of time and angles have been linked together dating back to some of the earliest civilizations.



Egyptian sundial



Analog clock

The analog clocks still used today are divided into sections that can be partitioned quite nicely into degrees. Because of this, you can explore and reason about the many mathematical relationships between time and angles.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Elena says that a minute hand turns 15° in 15 minutes because it passes 15 marks on the clock. Explain to her why she is incorrect.

2. How many degrees does each hand turn in the given amount of time?

a A minute hand in 20 minutes

b A minute hand in $\frac{3}{4}$ of an hour

c An hour hand in half an hour

d An hour hand in one day

3. Select *all* the equations that represent the relationships between the angles in the diagram.

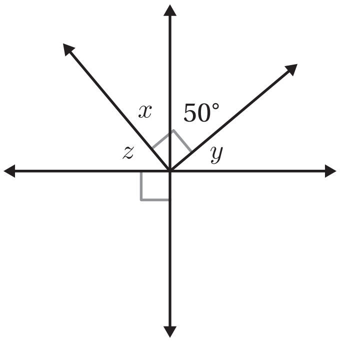
A. $90 = 50 + x$

B. $x + y = 90$

C. $y + z = 90$

D. $180 - y = 50 + x + z$

E. $180 + 50 = x + y + z$



The figure may not be drawn to scale.

Name: .... Date: .... Period: ....

Practice

4. For each inequality, decide whether the solution is represented by $x < 4.5$ or $x > 4.5$.

a $-24 > -6(x - 0.5)$

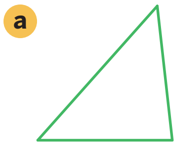
b $-8x + 6 > -30$

c $-2(x + 3.2) < -15.4$

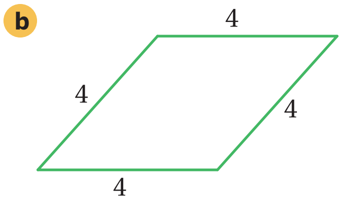
5. Use the word bank to label each figure with *all* the names that apply. Some words may not be used and others may be used more than once.

parallelogram	pentagon	polygon	quadrilateral
rectangle	rhombus	square	triangle

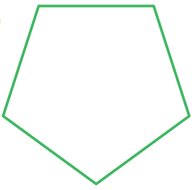
a



b



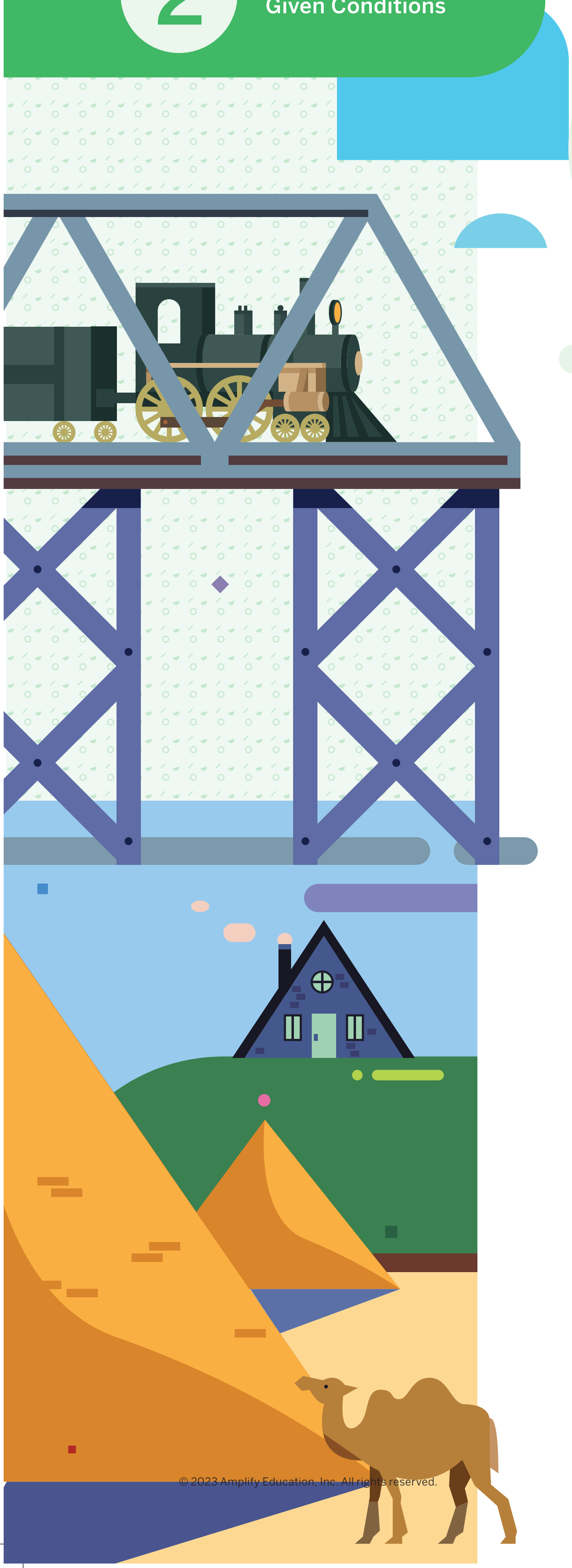
c



My Notes:

SUB-UNIT

2 Drawing Polygons With Given Conditions



Narrative Connections

How did triangles help win a war?

During the Civil War, one advantage the North had over the South was its railroads. While the South had their own railways, theirs was nowhere near as extensive as the North's. The North had over 22,000 miles worth of track. With this network, soldiers and supplies could travel quickly and efficiently to the frontlines.

So where do triangles come in?

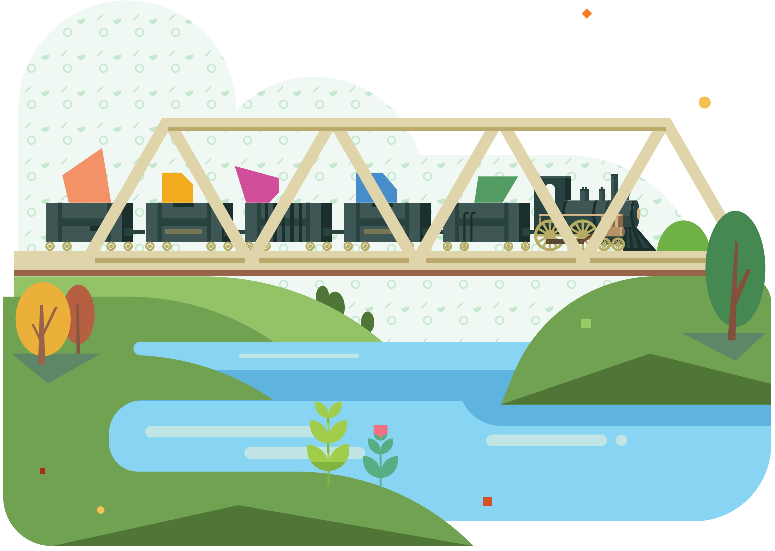
Well, in order for a train to cover these vast distances, it would have to cross many bridges. At the time, most bridges were made of wood. Wood was a strong material, but a far cry from the steel and concrete bridges of today. To make wooden bridges that could support the weight of a locomotive, engineers turned to "trusses". These were structures made of overlapping triangles that made the bridges more stable.

Truss bridges were easy to build and required less material compared to other bridges. Their stability was due to the **rigid** properties of triangles, making these bridges remarkably **strong**. They could easily support the weight of a train carrying a regiment of Union soldiers, without deforming or collapsing.

For centuries, triangles have had a starring role in our architecture. From the simple A-frame house to the Great Pyramids of Giza – if there's a building upright today, chances are that triangles have kept it standing.

Unit 7 | Lesson 8

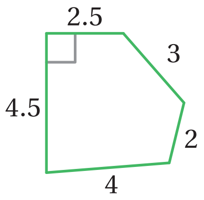
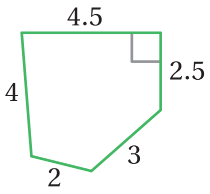
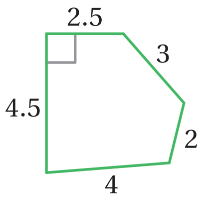
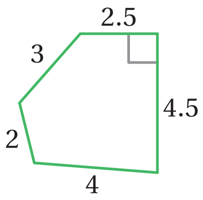
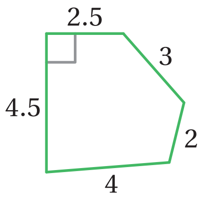
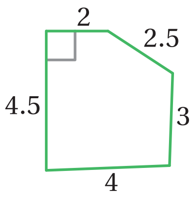
Building Polygons
(Part 1)



Let's build some polygons.

Warm-up Is It Identical?

Compare the polygons in Columns 1 and 2. Decide whether they are identical copies of each other. Explain your thinking.

Column 1	Column 2	Identical? (Yes/No)	Explanation
			
			
			

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 What Can You Build?

You will be given linkage strips of varying lengths and fasteners with which you will use to build polygons. You will also be given sticky notes to use later in the activity. In this activity, you will see if you can build identical shapes given only the side lengths of that shape.

1. Working independently, use the linkage strips to build several polygons, including at least one triangle and one quadrilateral.

2. Select one triangle and one quadrilateral that you have made. Keep them hidden from your partner.

a Write the side lengths of your polygons on the sticky note provided.

b Trade sticky notes with your partner. Each partner should use the lengths on the sticky note to build the polygons.

c Compare the polygons you and your partner built in part b. What do you notice?

Activity 2 How Many Can You Build?

You will use the same linkage strips and fasteners from Activity 1 to build more polygons.

1. Build as many quadrilaterals as you can, with side lengths 4 units, 5 units, 6 units, and 9 units. Sketch each one.

2. Build as many triangles as you can, with side lengths 4 units, 5 units, and 8 units. Sketch each one.

Name: ________________ Date: ________ Period: ________

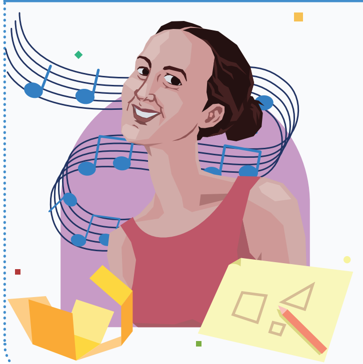
Activity 3 Building a Certain Triangle

Mathematician Vi Hart has created a video library of their “doodles.” In each of these doodles, they draw shapes and patterns with incredible properties, from what are known as “hexaflexagons” to your everyday triangles.

However, some triangles can be more challenging to make (or doodle!). Use the same linkage strips from the previous activities to try to build a triangle with side lengths of 3 units, 4 units, and 9 units.

What do you notice? Explain your thinking.

Featured Mathematician



Vi Hart's

A self-described “mathemusician,” Vi Hart's videos on math doodles have been viewed by millions of people around the world, covering topics from triangles, to fractals, to hexaflexagons, to the mathematics of music. Hart has co-authored research papers in computational geometry and paperfolding, and they have collaborated on educational projects in gaming and virtual reality.

STOP

Summary

In today's lesson . . .

You wrestled with this question: How many different polygons can you make if you need to build a polygon with certain side lengths?

Sometimes, you can make *many* different polygons.

- For example, if you have side lengths of 5 units, 7 units, 11 units, and 14 units, there are many, many quadrilaterals you can make.

Sometimes, there is only *one* polygon that can be made.

- For example, if you are asked to make a triangle with side lengths 3 units, 4 units, and 5 units, you will find that no matter how you arrange the lengths, all of the triangles are identical.

Sometimes, it is *not possible* to make a polygon with certain side lengths.

- For example, can you make a quadrilateral with side lengths of 18 units, 1 unit, 1 unit, and 1 unit? Try it!

You will continue to investigate the polygons that can be made with given measures.

Reflect:

Name: .... Date: .... Period: ....

1. A rectangle has side lengths 6 cm and 3 cm. Can you create a quadrilateral that is not identical to the rectangle using the same four side lengths? If so, sketch it.

2. Determine three side lengths that *cannot* form a triangle. Explain your thinking.

3. How many right angles are needed to form each angle? Show or explain your thinking.

   a 360°

   b 180°

   c 270°

   d a straight angle

   e 330°

Name: .... Date: .... Period: ....

4. Solve each problem. Consider creating a table to help with your thinking.

a You can buy 4 bottles of water from a vending machine for $\$7$. At this rate, how many bottles of water can you buy for $\$28$?

b It costs $\$20$ to buy 5 sandwiches from a vending machine. At this rate, what is the cost for 8 sandwiches?

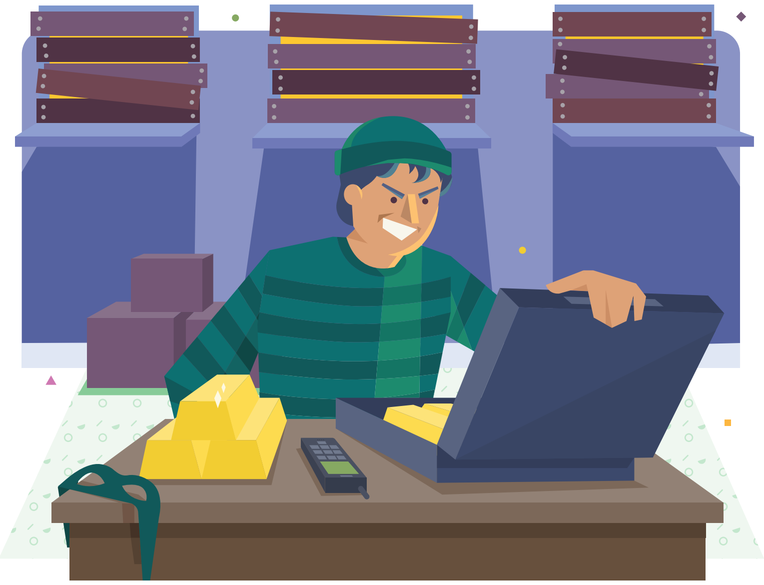
5. Draw a line segment and label its end points A and B. Draw point C somewhere on segment AB. Compare the lengths of segments AC, AB, and BC.

Name: .... Date: .... Period: ....

Unit 7 | Lesson 9

Building Polygons
(Part 2)

Let's build some more triangles.



Warm-up True or False: Length Relationships

In the diagram, points A, B, C, and D lie on the same line segment.

Decide if each of these equations is *true* or *false*. Explain your thinking.

1. $CD + BC = BD$

2. $AB + BD = CD + AD$

3. $AC - AB = AB$

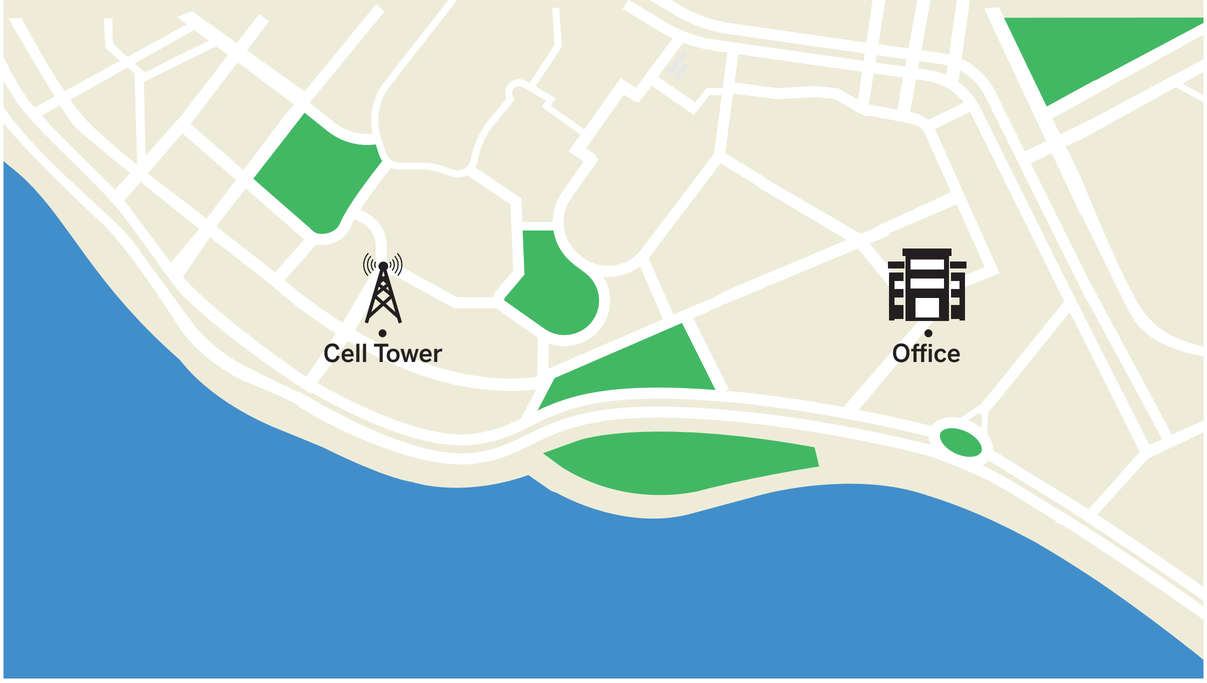
4. $BD - CD = AC - AB$

Log in to Amplify Math to complete this lesson online.

Activity 1 Where Is the Hideout?

Someone just made off with all the gold from the bank's safe! You are the only detective in town working on the case. Here is what you know:

- A cell tower is 5 miles west of your office.
- A phone call by the thief, asking to be picked up at the hideout, just happened at a location 3 miles away from the cell tower.



1. If 1 in. corresponds to 1 mile, use your ruler to label a possible location where the hideout could be. How far is this location from your office?

2. Label some other possible locations for the hideout.

3. What are some places that could *not* be the hideout location? Explain your thinking.

Name: ________ Date: ________ Period: ________

Activity 2 Swinging the Sides Around

You will be given the materials for this activity. You will explore a method for building a triangle that has three specified side lengths. Follow these directions carefully.

1. Draw a 4-in. line segment using the space on the next page, and mark the endpoints A and B.

2. Segment BC is 2 in. long. Use your compass to mark *all* the possible locations for point C.

a What shape have you drawn while determining *all* the possible locations for point C? Why is this the correct shape? Explain your thinking.

b Use your drawing to build two unique triangles, each with a base length of 4 in. and a side length of 2 in. Use a different color for each triangle. Record the side lengths of each of your triangles.

Triangle 1:	**Triangle 2:**

3. Segment AC is 3 in. long. Use your compass to mark *all* the possible locations for point C.

a Using a third color, draw a point where the two circles intersect. Using this third color, draw a triangle with side lengths of 4 in., 2 in., and 3 in.

b What is represented by the points of intersection of the two circles?

Activity 2 Swinging the Sides Around (continued)

Use this space to build your triangle:

STOP

Name: ________________ Date: ________ Period: ________

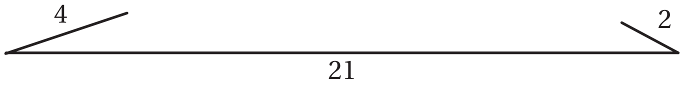
Summary

In today's lesson . . .

You have discovered that sometimes it is *not* possible to build a polygon given a set of side lengths.

For example, if you have one really, really long segment and a bunch of short segments, you may not be able to connect them.

Here is what happens if you try to build a triangle with side lengths 21 units, 4 units, and 2 units:



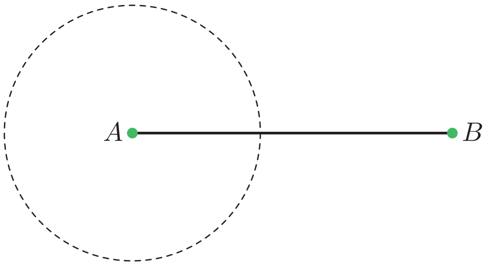
The side lengths of 4 units and 2 units are not long enough to meet at a point, because the side length of 21 units is too long! In general, the longest side length must be *less* than the sum of the other two side lengths. If not, you *cannot* build a triangle!

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. In the diagram, the length of segment AB is 10 units and the radius of the circle centered at A is 4 units. Use this to create two unique triangles, each with side lengths of 10 units and 4 units. Label the sides that have these lengths.



2. Select *all* sets of three side lengths that will form a triangle.

A. 3 in., 4 in., 8 in.

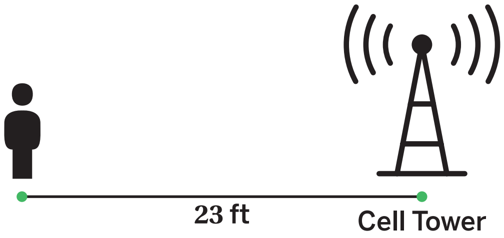
B. 7 in., 6 in., 12 in.

C. 5 in., 11 in., 13 in.

D. 4 in., 6 in., 12 in.

E. 4 in., 6 in., 10 in.

3. Based on signal strength, a person knows that their lost phone is exactly 47 ft from the nearest cell tower. The person is currently standing 23 ft from the same cell tower. What is the closest distance the phone could be to the person? What is the farthest distance their phone could be from them?



Name: .... Date: .... Period: ....

Practice

4. Complete the table so that each row contains the degree measures of two complementary angles.

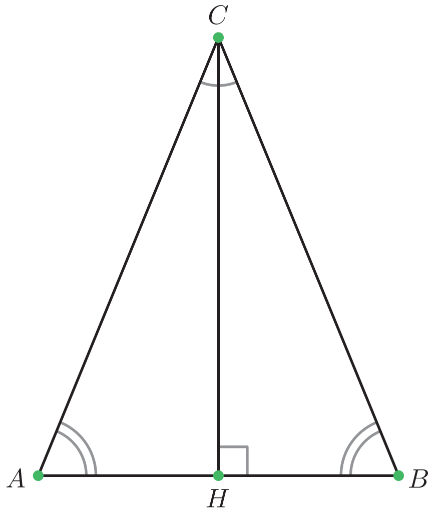
Measure of an angle	Measure of its complement
$80°$	
$25°$	
$54°$	
x	

5. Mai's family is traveling in a car at a constant speed of 65 miles per hour.

a At this speed, how long will it take them to travel 200 miles?

b At this same speed, how far do they travel in 25 minutes?

6. Triangle ABC is shown. List the pairs of angles that have equal measures. Explain your thinking.



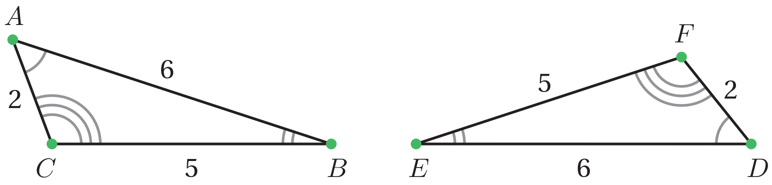
Unit 7 | Lesson 10

Triangles With Three Common Measures

Let's compare triangles that have common side lengths or angle measures.

Warm-up Identical or Not?

Triangle ABC and Triangle DEF have three corresponding angles with the same measure and three corresponding sides of the same length. These triangles are identical copies.



1. Draw two triangles with only one common measurement. Is there enough information to determine that these two triangles are identical copies? Show or explain your thinking.

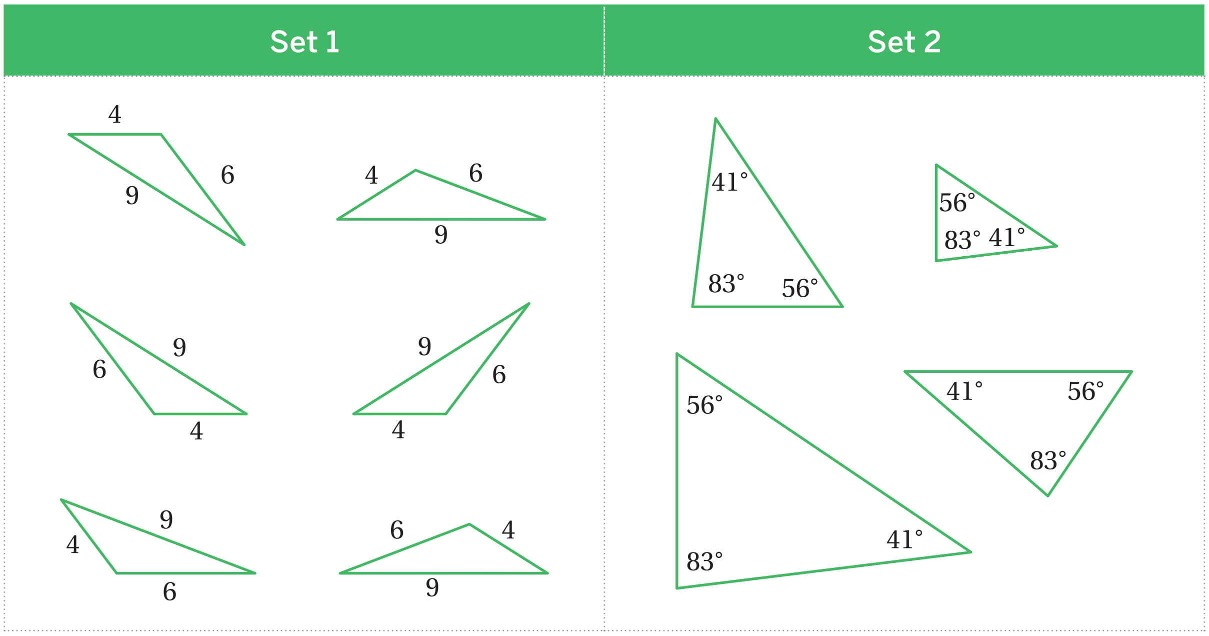
2. Draw two triangles with only *two* common measurements. Is there enough information to determine that these triangles are identical copies? Show or explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Three Sides or Three Angles

Examine each set of triangles. The figures may not be drawn to scale.

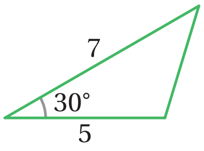


1. What is the same about the triangles in each set?

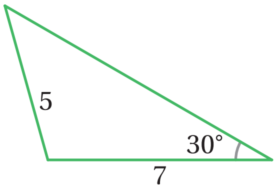
2. What is different between the two sets of triangles?

Activity 2 Two Sides and One Angle

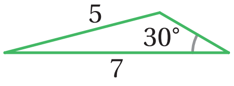
Ever since Thales of Miletus, mathematicians have studied triangles that have side lengths and angles in common. Examine this set of triangles. The figures may not be drawn to scale.



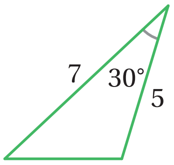
Triangle A



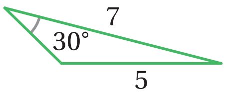
Triangle B



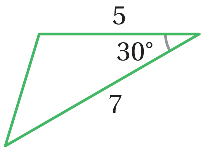
Triangle C



Triangle D



Triangle E



Triangle F

1. What is the same about the triangles in the set?

2. What is different?

3. Which triangles are identical copies of other triangles in the set? Explain or show your thinking.

Collect and Display: What math terms did you use when describing the similarities and differences between the triangles? Add these to your class display.

Featured Mathematician

Thales of Miletus

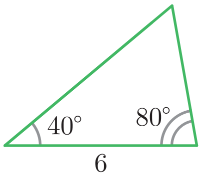
Thales was a mathematician from Ancient Greece, and was born in what is now modern-day Turkey. It is believed he studied under Egyptian scholars. Thales made contributions to scientific reasoning, astronomy, and geometry. He is known for articulating and proving several fundamental theorems about circles and triangles, particularly triangles with side lengths or angles in common.

Wallis, Ernst Bernhard, b. 1842, and Fredrik August Fehr. Illustrerad Verldshistoria. Stockholm: Centraltryckeriets förlag, 1877.

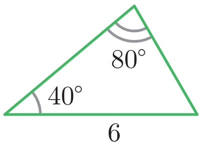
Name: ______________________ Date: __________ Period: __________

Activity 3 One Side and Two Angles

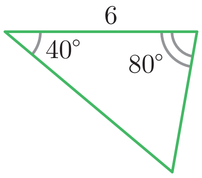
Examine this set of triangles. The figures may not be drawn to scale.



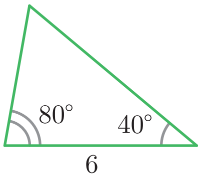
Triangle A



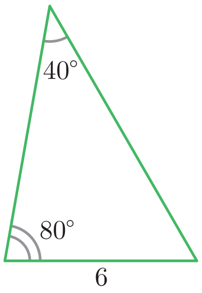
Triangle B



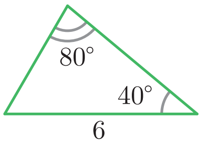
Triangle C



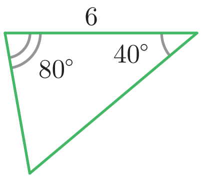
Triangle D



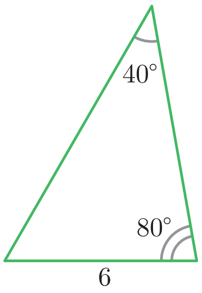
Triangle E



Triangle F



Triangle G



Triangle H

1. What is the same about the triangles in this set?

2. What is different?

3. Which triangles are identical copies of other triangles in the set? Explain or show your thinking.

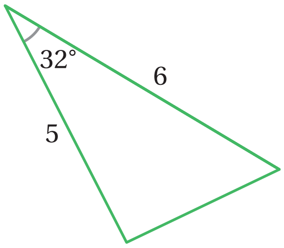
STOP

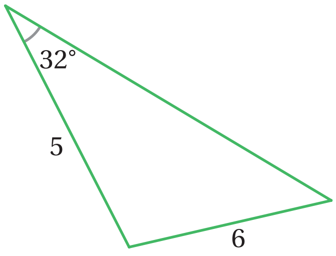
Summary

In today's lesson . . .

You examined sets of triangles with common side lengths or angle measures to determine if any triangles were identical copies of other triangles in the set. To know whether triangles are identical copies of one another, a minimum of three equal measures of corresponding parts are needed. However, it cannot be just *any* three measures.

If you create two triangles with three equal measures, but these measures are *not* next to each other in the same order, it usually means the triangles are different. For example, the two triangles shown are not identical, even though they have three measures in common. The $32°$ angle and the side length of 5 are corresponding parts between the two triangles, but the side length of 6 is not in the same relative position for the two triangles.

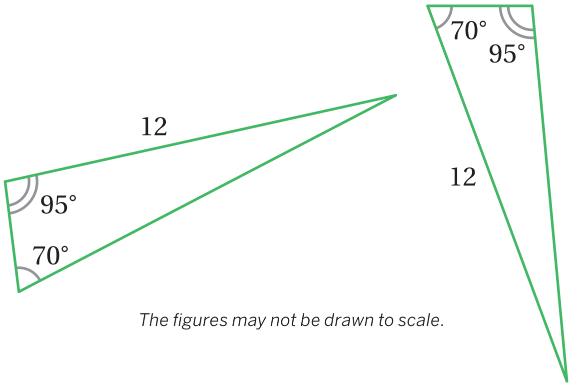




Reflect:

Name: .... Date: .... Period: ....

1. Can you guarantee these two triangles are identical, based solely on the measurements indicated? Explain your thinking.



The figures may not be drawn to scale.

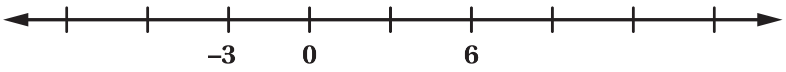
2. Two triangles each have two angle measures of 70° and one angle measure of 40°. Based on these measurements alone, can you guarantee these two triangles are identical? Explain your thinking.

3. Tyler wants to create two triangles, each with side lengths of 8 in. and 10 in. What other information is needed to guarantee that the triangles will be identical? Explain your thinking.

Practice

Name: .... Date: .... Period: ....

4. The markings on the number line are equally spaced. Label the other markings on the number line.



5. Apples cost $\$1.99$ per pound.

a How much do $3\frac{1}{4}$ pounds of apples cost? Round your answer to the nearest cent.

b How much do x pounds of apples cost?

c Clare spent $\$5.17$ on apples. How many pounds of apples did she buy? Round your answer to the nearest tenth of a pound.

6. Draw each angle.

a $45°$ angle

b $123°$ angle

Name: .... Date: .... Period: ....

Unit 7 | Lesson 11

Drawing Triangles (Part 1)

Let's see how many different triangles we can draw with certain measurements.

Warm-up Construct It

1. Draw segment AB with a length of 6 cm. Draw angle BAC so that $m\angle BAC = 56°$ and the length of AC is 3 cm.

2. Connect the ends of the segments to form triangle ABC. What is the side length of the new side?

   $BC =$ ....

3. Measure the other two angles with a protractor. What are their measures?

   $m\angle ACB =$ ....

   $m\angle CBA =$ ....

4. Compare triangles with your partner. Did you construct identical copies?

Log in to Amplify Math to complete this lesson online.

Activity 1 Can You Draw It?

Priya drew a triangle with two angles measuring 75° and 45°, and one side measuring 5 cm.

1. Construct Priya's triangle.

2. Compare your triangle with your partner's triangle. Are they identical copies? How do you know?

3. To make sure everyone in your class has an identical copy of Priya's triangle, what should the directions have been?

Name: ………………………… Date: ……………… Period: ………………

Activity 2 How Many Can You Draw?

1. Draw as many different triangles as you can with each of the these sets of measurements:

   a. **Set A**: Two angles each measure 60° and one side measures 4 cm.

   b. **Set B**: Two angles each measure 90° and one side measures 4 cm.

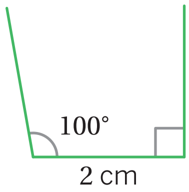
   c. **Set C**: One angle measures 60°, one angle measures 90°, and one side measures 4 cm.

2. Which sets of measurements determine one unique triangle? Explain your thinking.

STOP

Summary

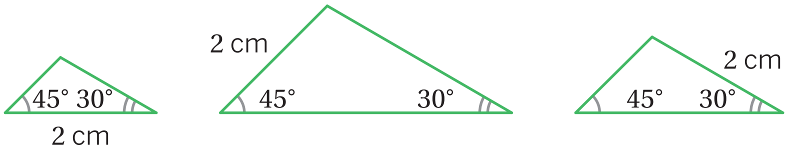
In today's lesson . . .



You explored how many different triangles you could draw, given certain measurements.

Sometimes, you are given the measures of two different angles and a side length, and it is impossible to draw a triangle. For example, it is impossible to draw a triangle with two angle measures of 100° and 90°, and a side length of 2 cm.

Sometimes, you are given the measures of two different angles and a side length, and you can draw multiple triangles. For example, there are three triangles with two angle measures of 45° and 30°, and a side length of 2 cm.



Sometimes, you are given two different angle measures and a side length, and you can draw a *unique* triangle. You need to know where the side length is located (either between the known angles or across from one of the two known angles) to draw the unique triangle. For example, there is only one unique triangle that can be drawn with two angle measures of 45° and 30°, and a side length of 2 cm across from the 30° angle.

Reflect:

Name: .... Date: .... Period: ....

1. A triangle has an angle measuring $90°$, an angle measuring $20°$, and a side measuring 6 cm long. The 6-cm side is between the two known angles.

a Draw and label this triangle.

b How many unique triangles can you draw that satisfy the given criteria?

2. Which of these three triangles is impossible to draw? Use a protractor to draw the triangles that are possible.

a A triangle with one angle measuring $20°$ and another angle measuring $45°$

b A triangle with one angle measuring $120°$ and another angle measuring $50°$

c A triangle with one angle measuring $90°$ and another angle measuring $100°$

Practice

Name: .... Date: .... Period: ....

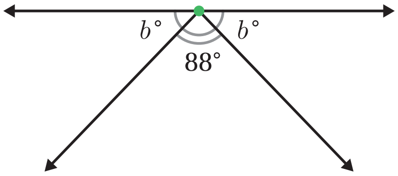
3. Which equation represents the relationship between the angles in the figure?

A. $88 + b = 90$

B. $88 + b = 180$

C. $2b + 88 = 90$

D. $2b + 88 = 180$



The figure may not be drawn to scale.

4. Determine a value of x that satisfies each of the following criteria.

a $-x$ is less than $2x$

b $-x$ is greater than $2x$

5. One of the particles that make up atoms is called an electron. An electron has a charge of -1. Another particle is a proton, which has a charge of $+1$.

The overall charge of an atom is the sum of the charges of the electrons and the protons. Complete the table to determine the overall charge for the atoms that are listed.

	Charge from electrons	Charge from protons	Overall charge
Carbon	-6	$+6$	0
Aluminum cation	-10	$+13$	
Phosphide	-18	$+15$	
Iodide	-54	$+53$	
Tin	-50	$+50$	

6. Using a compass, draw a circle with the given dimension.

a 2 cm radius

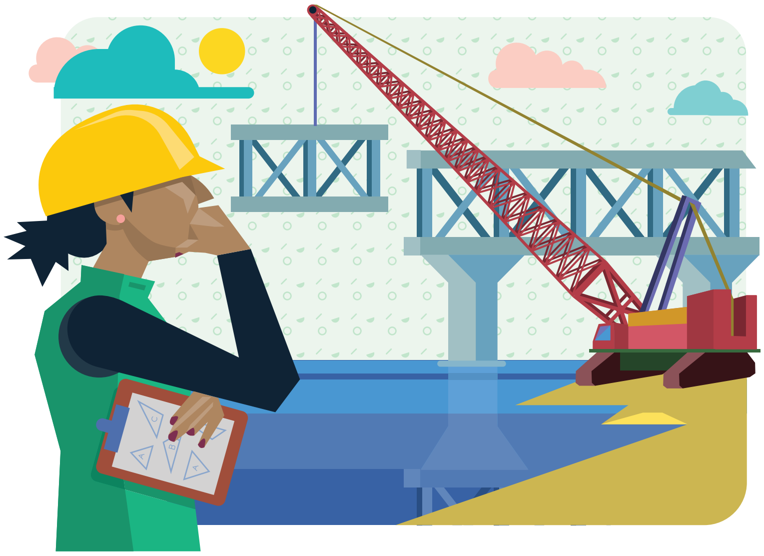
b 2 cm diameter

Name: ____________________ Date: ____________ Period: ____________

Unit 7 | Lesson 12

Drawing Triangles (Part 2)

Let's draw some more triangles.



Warm-up Using a Compass to Estimate Length

1. Draw a 40° angle.

2. Set your compass to a radius of 5 cm. Place the point of the compass at the vertex of your angle. Draw an arc intersecting the two rays to make sure both sides of your angle have a length of 5 cm.

3. Connect the endpoints of the sides to form a triangle. Without measuring, predict the length of the constructed side.

4. How can you use a compass to check if the side length is less than or greater than 5 cm?

Log in to Amplify Math to complete this lesson online.

Activity 1 Can You Draw It?

Elena drew a triangle with side lengths of 3 cm and 4 cm, and an angle measuring $100°$.

1. Construct Elena's triangle.

2. Compare your triangle to your partner's triangle. Are they identical copies? How do you know?

3. To make sure everyone in your class has an identical copy of Elena's triangle, what should the directions have been?

Are you ready for more?

Can you draw a triangle with side lengths of 3 cm and 4 cm, and an angle measuring $100°$ across from the 3-cm side? If yes, draw an example. If not, explain why.

Name: ________________ Date: ________ Period: ______

Activity 2 How Many Can You Draw?

1. Draw as many different triangles as you can that satisfy each set of criteria.

   a. **Set A:** One angle measures $50°$, one measures $60°$, and one measures $70°$.

   b. **Set B:** One angle measures $50°$, one measures $60°$, and one measures $100°$.

Activity 2 How Many Can You Draw? (continued)

c **Set C:** One angle measures 40°, one side measures 4 cm, and one side measures 5 cm.

d **Set D:** Two sides measure 6 cm and one angle measures 100°.

Reflect: How did you motivate yourself to do your best and complete the activity?

2. Which sets of measurements determine one unique triangle? Explain your thinking.

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You drew triangles to satisfy given criteria about side lengths and angle measures. You already know that a triangle has six measures: three side lengths and three angle measures. Suppose you are given three of these six measures and are asked to create a triangle with those measures.

Sometimes, there is *no triangle* that can be created. For example:

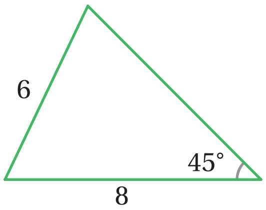
- No triangle can be made with side lengths of 1 unit, 2 units, and 5 units.
- No triangle can be made with all three angles measuring 100°.

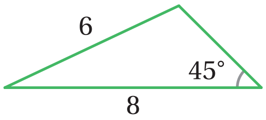
Sometimes, *only one triangle* can be created. This means that all of the triangles created are identical copies of one another. For example:

- If you know the three side lengths will produce a triangle, then the corresponding angles between triangles with these side lengths will always have the same measures. This creates a unique triangle.
- If an angle measuring 45° is placed between side lengths of 6 units and 8 units, only one unique triangle can be created.

Sometimes, *two or more different triangles* can be created. For example:

- If you know two side lengths are 6 units and 8 units and one angle measures 45°, then two triangles can be created. In both triangles, the 45° angle is across from the side length of 6 units.





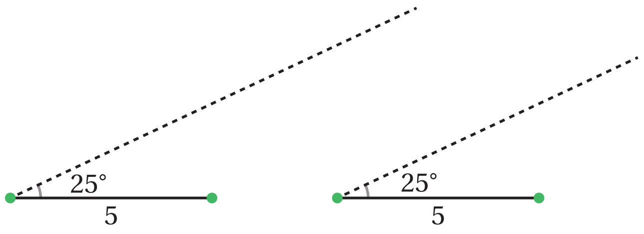
Reflect:

Practice

Name: .... Date: .... Period: ....

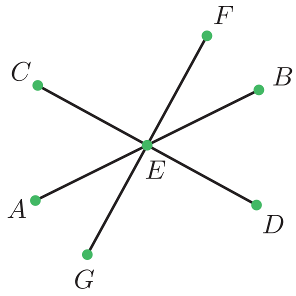
1. A triangle has sides of length 7 cm, 4 cm, and 5 cm. How many unique triangles can be drawn to satisfy the criteria? Explain your thinking.

2. A triangle has one side 5 units long and an adjacent angle measuring $25°$. The other angles in the triangle measure $90°$ and $65°$. Complete the two diagrams to create two *different* triangles with these measurements.



3. Is it possible to create a triangle with angles measuring $90°$, $30°$, and $100°$? If so, draw an example. If not, explain your thinking.

4. Segments CD, AB, and FG intersect at point E. Angle FEC is a right angle. Identify any pairs of angles that are complementary.

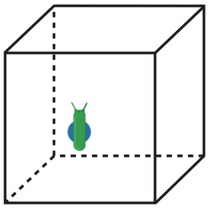


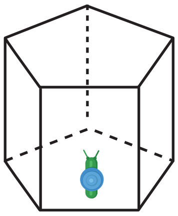
Name: ______________________ Date: __________ Period: __________

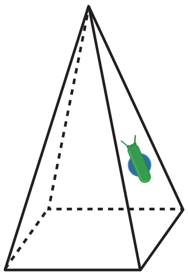
5. Match each equation with a step that will help solve the equation for x.

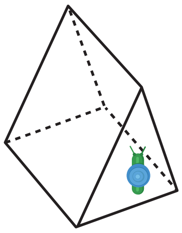
Equation	Step
a $3x = -4$	______ Add $\frac{1}{3}$ to each side.
b $-4.5 = x - 3$	______ Add $-\frac{1}{3}$ to each side.
c $3 = -\frac{x}{3}$	______ Add 3 to each side.
d $\frac{1}{3} = -3x$	______ Add -3 to each side.
e $x - \frac{1}{3} = 0.4$	______ Divide each side by $\frac{1}{3}$.
f $3 + x = 8$	______ Divide each side by $-\frac{1}{3}$.
g $\frac{x}{3} = 15$	______ Divide each side by 3.
h $7 = \frac{1}{3} + x$	______ Divide each side by -3.

6. There is a snail crawling on the outside of each of the solids below. When seen from above, you will see the shell as a blue circle over the green snail. When seen from below, you will see the green snail from antennae to tail. Shade the face of the solid on which the snail is crawling.









My Notes:

SUB-UNIT

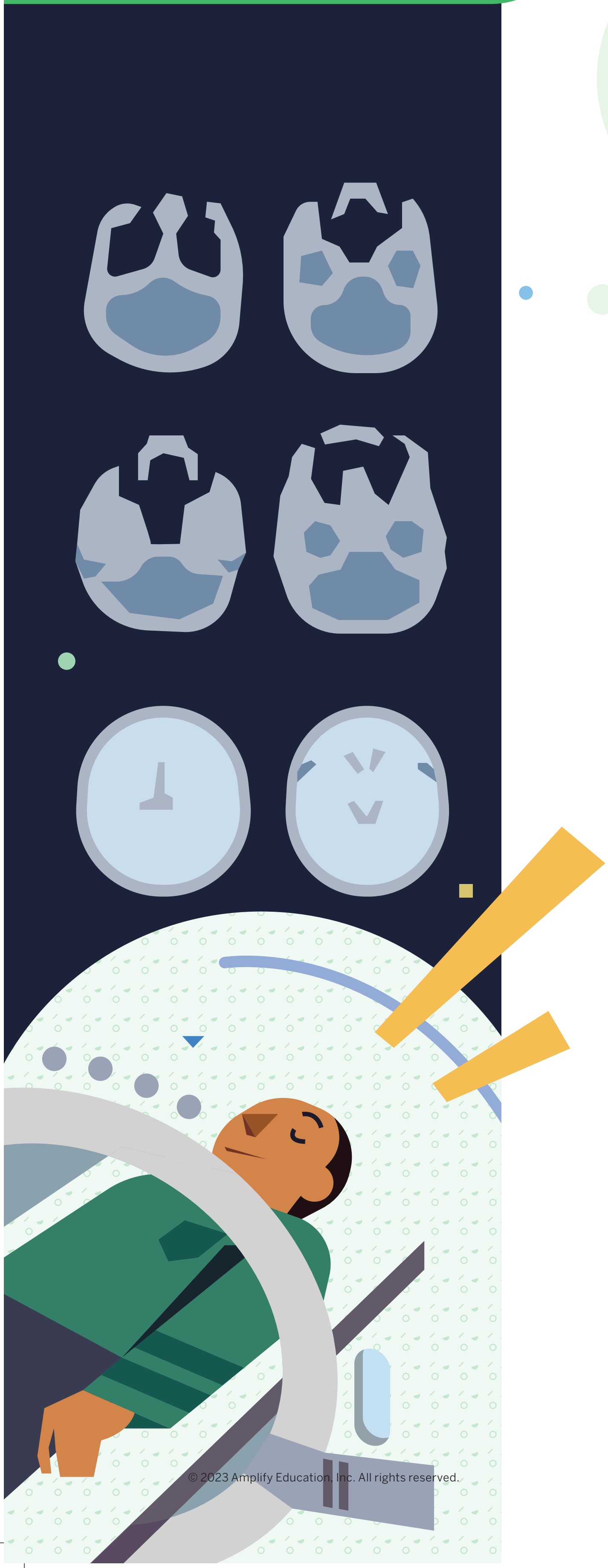
3

Solid Geometry

Narrative Connections

This machine will slice, but will it dice?

Thankfully, no. At least not in the case of our internal organs.



In years past, the only way to see inside of our bodies was to cut them open. Though surgeries have been performed for thousands of years, it was a fairly violent affair. Anaesthesia was developed in the late 1800s to reduce the pain, but infection and death occurred regularly.

In 1972, Godfrey Hounsfield built a machine capable of slicing the human body into **cross sections**, except these slices were *images*. The body didn't have to be cut open to see inside. This process is called a computed tomography scan, or CT scan. It works by taking a series of images of very thin slices of the body. The images are then reassembled next to each other, giving doctors a three-dimensional view of what's inside.

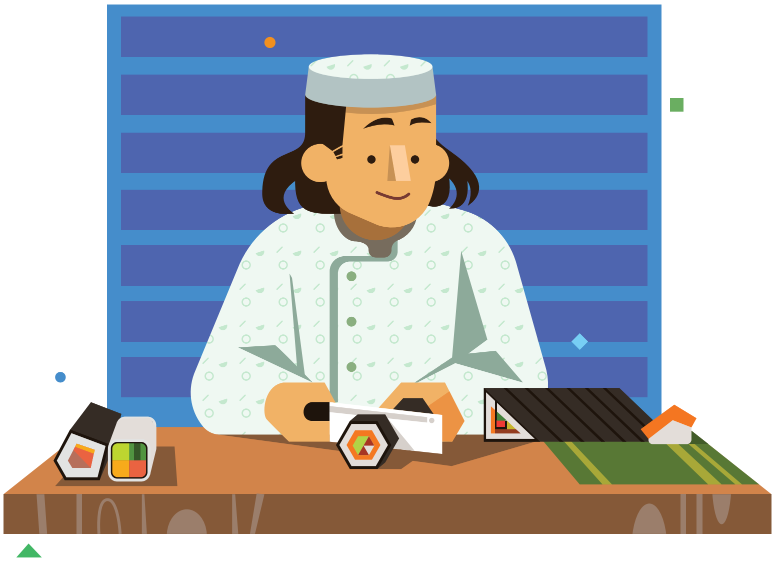
Today, CT scans are used to find everything from tiny fractures to brain tumors, all while keeping our insides intact.

The usefulness of cross sections doesn't end with medical technology, though. We can use these two-dimensional slices to find volumes of three-dimensional figures beyond the rectangular prisms you already know.

Unit 7 | Lesson 13

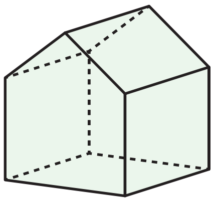
Slicing Solids

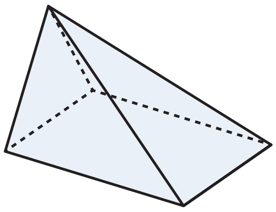
Let’s see what shapes you get when you slice a three-dimensional solid.

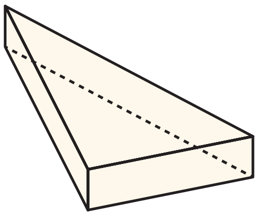


Warm-up Prisms and Pyramids

Describe each figure as precisely as you can.

1. 

2. 

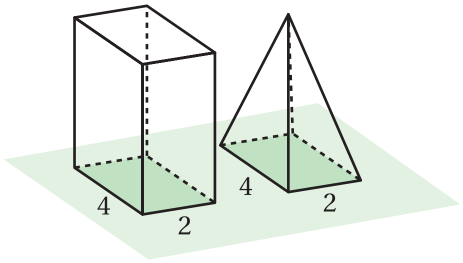
3. 

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 What is the Cross Section?

Here is a rectangular prism and a pyramid with the same base and same height. Watch the animation to see what happens as the plane moves through the solids.



1. If you slice each solid parallel to its base halfway up, what shape of cross section would you get for each? What would be the same about the cross sections? What would be different?

2. If you slice each solid parallel to its base near the top, what shape of cross section would you get for each? What would be the same about the cross sections? What would be different?

Activity 2 Card Sort: Sorting Cross Sections

You will be given a set of cards. Sort the images into groups that make sense to you. Record your sorted groups in the table, along with an explanation of why those cards belong in the same group. You may or may not need all of the rows.

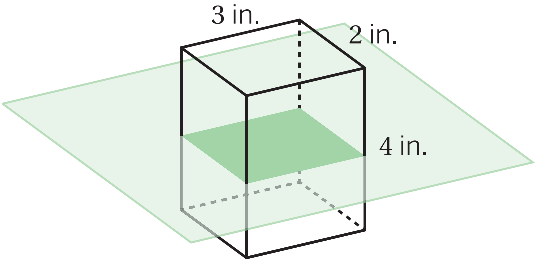
	Cards in this group	Explanation
Group 1		
Group 2		
Group 3		
Group 4		

Name: ________________ Date: ________ Period: ________

Activity 3 Drawing Cross Sections

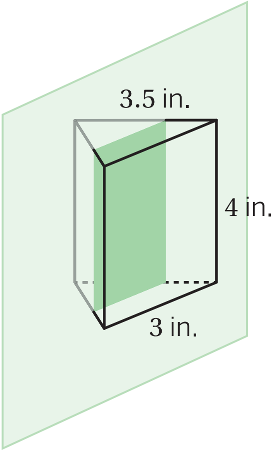
Draw each cross section as if you were looking straight at it. Consider the dimensions of the solid and use them to estimate the dimensions of the cross section. The figures may not be drawn to scale.

1.



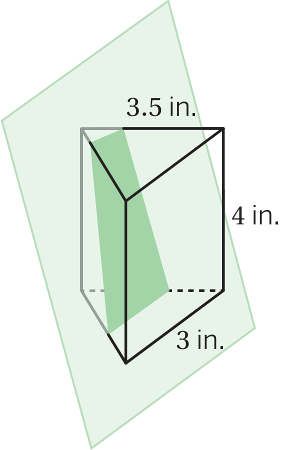
Drawing of cross section:
(Including estimated dimensions.)

2.



Drawing of cross section:
(Including estimated dimensions.)

3.



Drawing of cross section:
(Including estimated dimensions.)

STOP

Summary

In today's lesson . . .

You saw that when you slice a three-dimensional solid, you expose new faces that are two-dimensional. The two-dimensional face is called a ***cross section***. For example, if you slice a rectangular pyramid parallel to the base, the cross section is a rectangle that is smaller than the base.

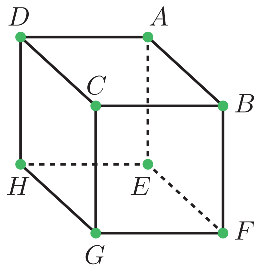
The two-dimensional surface used to slice the figure is called a ***plane***.

Many different cross sections are possible when slicing the same three-dimensional solid. It takes practice visualizing the cross sections of a three-dimensional solid for different slices.

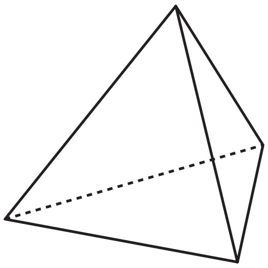
Reflect:

Name: .... Date: .... Period: ....

1. A cube is cut into two pieces by a single, vertical slice that passes through points A and C, perpendicular to base $EFGH$. What shape is the cross section? Which other vertices does the slice pass through?



2. Describe how to slice the three-dimensional figure to result in a triangular cross section. Then describe how to slide the figure to result in a trapezoidal cross section.



3. Each row contains the degree measures of two supplementary angles. Complete the table.

Measure of an angle	Measure of its supplement
$80°$	$100°$
$25°$	
$119°$	
x	

Name: ______________________ Date: ____________ Period: ____________

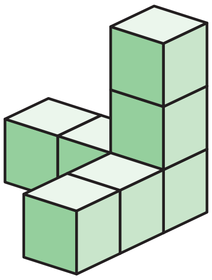
4. Two months ago, the price of a cell phone was c.

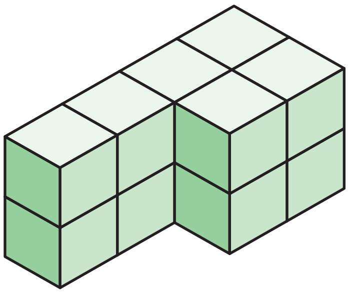
a Last month, the price of the phone increased by 10%. Write an expression for the price of the phone last month.

b This month, the price of the phone decreased by 10%. Write an expression for the price of the phone this month.

c Is the price of the phone this month the same as it was two months ago? Explain your thinking.

5. How many cubes are needed to build each three-dimensional figure? Explain your thinking.





6. Determine the height of a rectangular prism whose volume is 70 in.^{3}, width is $3\frac{1}{2}$ in. and length is $6\frac{1}{4}$ in. Show your thinking.

Name: ________________ Date: ________ Period: ________

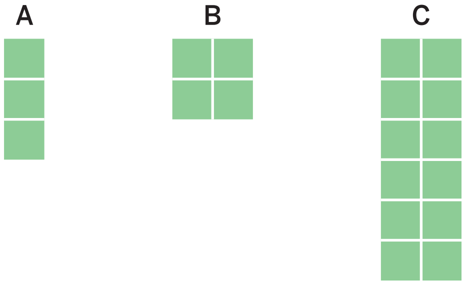
Unit 7 | Lesson 14

Volume of Right Prisms

Let's look at volumes of prisms.

Warm-up Three Prisms With the Same Volume

Rectangles A, B, and C represent bases of three prisms.



1. If each prism has the same height, which one will have the greatest volume? The least? Explain your thinking.

2. If each prism has the same volume, which one will have the tallest height? The shortest? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Determining Volume With Cubes

You will be given a sheet of paper with a figure on it and some snap cubes.

1. Using the face of a snap cube as your unit of area, what is the area of the figure? Explain or show your thinking.

2. Use snap cubes to build the figure. What is the volume of your figure?

3. Add another layer of cubes to the top of the figure. Describe this new figure.

4. What is the volume of your figure? Show or explain your thinking.

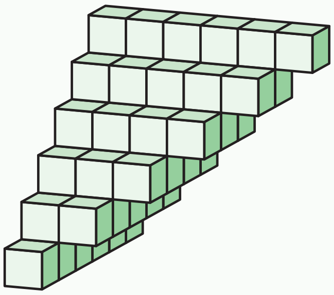
5. Right now, your figure has a height of 2 units. What would the volume be if it had a height of:

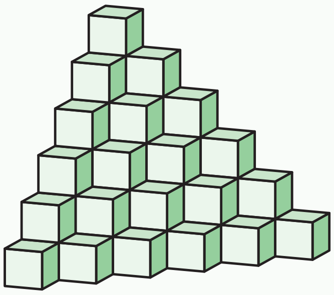
   a 5 units

   b 8.5 units

Are you ready for more?

Which three-dimensional figure has a greater volume? All of the cubes are the same size.





Name: ________________ Date: ________ Period: ________

Activity 2 Can You Determine the Volume?

You will be given a set of three-dimensional figures.

1. First, determine whether each three-dimensional figure is a prism. Record your response in the table.

2. For each prism, record the area of the base, measure the height, and calculate the volume. Record all measurements in the table.

	Is it a prism?	Area of base (cm^2)	Height (cm)	Volume (cm^3)
Figure A				
Figure B				
Figure C				
Figure D				

Are you ready for more?

Imagine a large, solid cube made out of 64 white snap cubes. Someone spray paints all 6 faces of the large cube blue. After the paint dries, they disassemble the large cube into a pile of 64 snap cubes.

1. How many of those 64 snap cubes have exactly 2 faces that are blue?

2. What are the other possible numbers of blue faces the cubes can have? How many of each are there?

3. Try this problem again with some larger cubes made up of more than 64 snap cubes. What patterns do you notice?

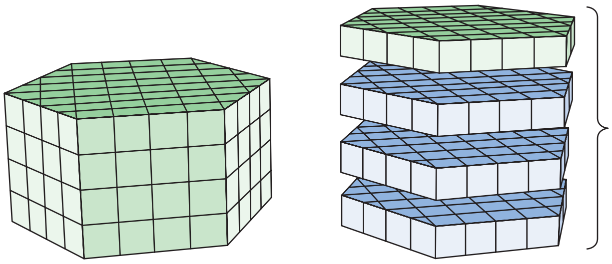
STOP

Summary

In today's lesson . . .

You explored how to determine the volume of cubes and other prisms. Any cross section of a prism that is parallel to the base will be identical to the base. This means you can slice a prism by its layers to help determine its volume.

area of base $= 32$ square units



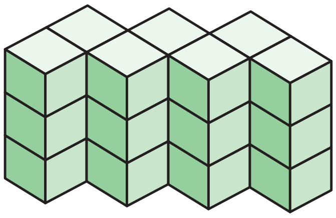
$V = Bh$
$V =$ (area of base • height)
$V = 32 \cdot 4$
$V = 128$ cubic units

This works with any prism!

Reflect:

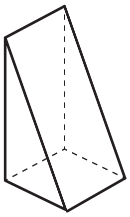
Name: ________________ Date: ________ Period: ________

1. Determine the volume of the solid. Each cube has a volume of 8 in^3.

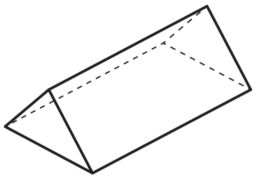


2. Circle the prisms. For each prism, shade one of its bases.

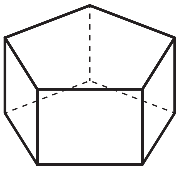
A.



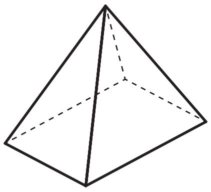
D.



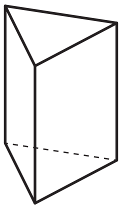
B.



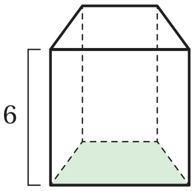
E.

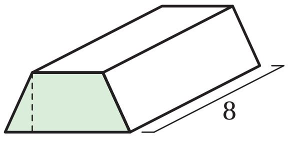


C.



3. The volume of each of these trapezoidal prisms is 24 cubic units. Their heights are 6 units and 8 units, as labeled. What is the area of the trapezoidal base of each prism?





Name: .... Date: .... Period: ....

4. Use what you know about complementary and supplementary angles to solve each problem.

a Two angles are complementary. One angle has a measure of 19°. What is the measure of the other angle?

b Two angles are supplementary. One angle has a measure that is 19°. What is the measure of the other angle?

5. Match each expression in Column A with an equivalent expression from Column B.

Column A	**Column B**
a $7(x+2)-x+3$	.... $\frac{1}{5}x-10$
b $6x+3+4x+5$	.... $6x+17$
c $-\frac{2}{5}x-7+\frac{3}{5}x-3$	.... $2(5x+4)$
d $8x-5+4-9$	.... $12(2x+3)$
e $24x+36$	.... $8x+(-5)+4+(-9)$

6. Determine the area of each figure.

a A triangle with a base of 3 in. and a height of 12 in.

b A trapezoid with bases of lengths 3 ft and 7 ft and height of 4 ft.

Name: ________ Date: ________ Period: ________

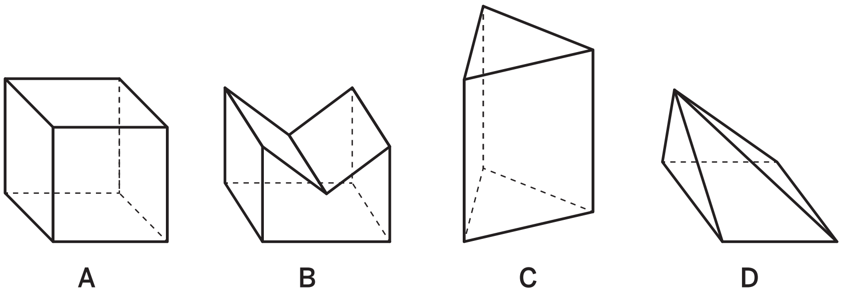
Unit 7 | Lesson 15

Decomposing Bases for Area

Let's look at the bases of different solids.

Warm-up Are These Prisms?

Study these figures.



1. Which of these figures are prisms? Explain your thinking.

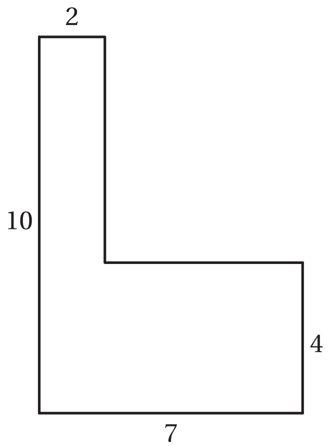
2. For each prism, think about what the base looks like.
   - **a** Shade one of the bases for each prism.
   - **b** Draw a cross section of each prism that is parallel to the base.

Log in to Amplify Math to complete this lesson online.

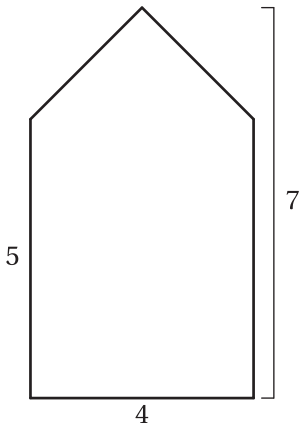
Activity 1 Determine the Area

For each figure, determine the area. Show and organize your work so that it can be understood by others.

1.

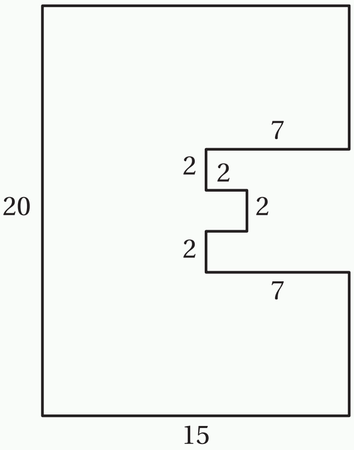


2.



Are you ready for more?

Determine the area of the figure.



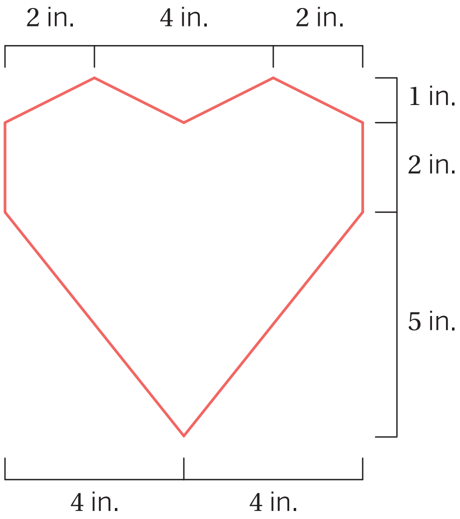
Name: Date: Period:

Activity 2 A Heart-Shaped Box

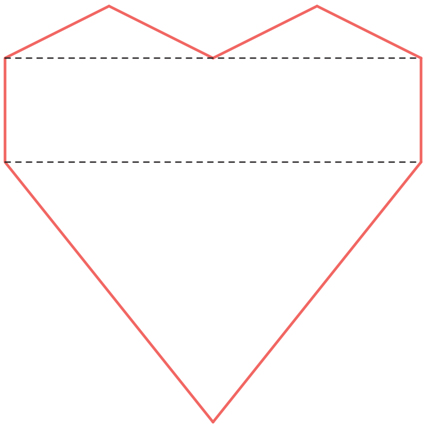
Plan ahead: What will you do to understand the approaches of others in this activity?

A box is a prism with a heart-shaped base and a height of 2 in. The drawing shows the measurements of the base.

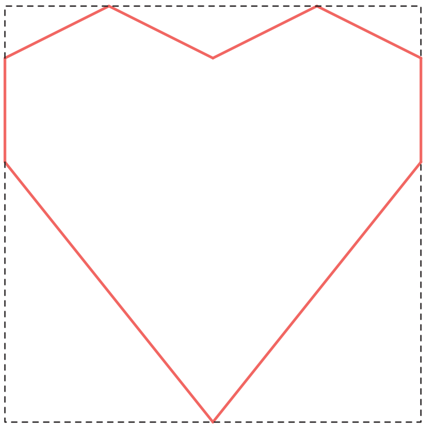
To calculate the volume of the box, two students have drawn line segments showing how they plan on first determining the area of the heart-shaped base.



Jada's Plan



Diego's Plan



1. For each student's plan, describe the shapes for which they need to determine the area, and the operations needed to calculate the total area.

2. Of the two plans, select one and have your partner select the other. Circle the plan you selected. You will use your selected plan for the rest of this activity.

Activity 2 A Heart-Shaped Box (continued)

3. Using the quadrilaterals and triangles drawn in your selected plan, determine the area of the base. Show and organize your work so that it can be understood by others.

4. Trade with your partner and check each other's work. If you disagree, work to reach an agreement.

5. Return your partner's work. Calculate the volume of the box.

STOP

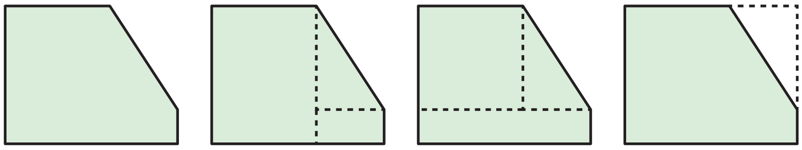
Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You saw that prisms can have bases of any polygonal shape. If the base is not a triangle or rectangle, determining the area does not have to be challenging. You can decompose any polygonal base into rectangles and triangles to determine its area. There are many ways to decompose a polygon.

Here are a few examples:



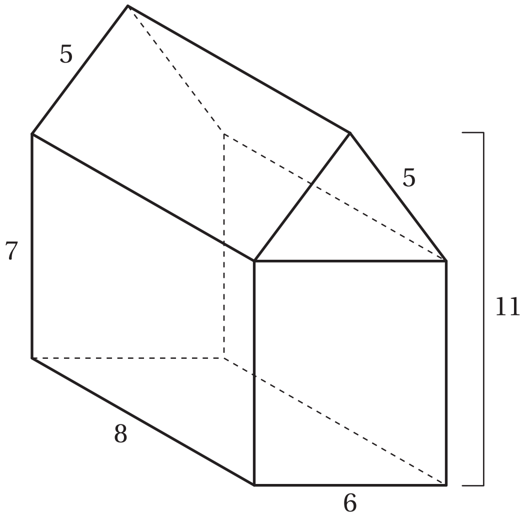
Sometimes it is easier to enclose a polygon in a rectangle, and then subtract the area that is not part of the polygon.

Reflect:

Practice

Name: ____________________ Date: __________ Period: __________

1. A house-shaped prism is created by attaching a triangular prism on the top of a rectangular prism.



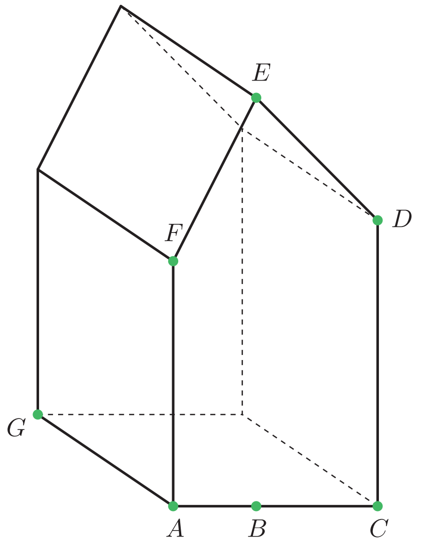
a Draw the base of this prism and label its dimensions.

b What is the area of the base in square units? Explain or show your thinking.

c What is the volume of the prism in cubic units? Explain or show your thinking.

2. You find a crystal in the shape of the prism shown. Determine the volume of the crystal. Point B is directly underneath point E, and the following lengths are known:

- from A to B: 2 mm
- from B to C: 3 mm
- from A to F: 6 mm
- from B to E: 10 mm
- from C to D: 7 mm
- from A to G: 4 mm



Name: ______________________ Date: __________ Period: __________

3. Select *all* the equations that represent a relationship between angles in the figure.

A. $90 - 30 = b$

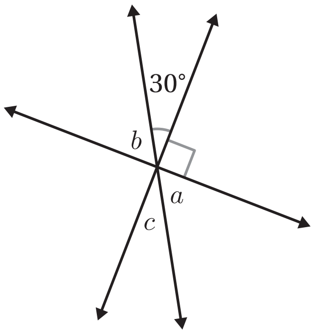
B. $a + c + 30 + b = 180$

C. $a + c = 30$

D. $30 + b = a + c$

E. $a = 30$

F. $90 + a + c = 180$

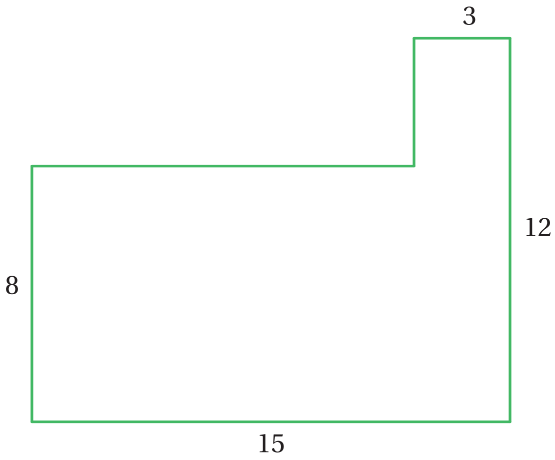


The figure may not be drawn to scale.

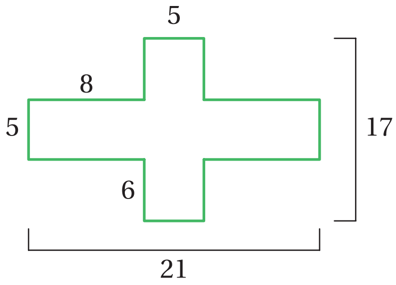
4. A mixture of punch contains 1 qt of lemonade, 2 cups of grape juice, 4 tbsp of honey, and $\frac{1}{2}$ gallons of sparkling water. Determine the percentage of the punch mixture composed of each ingredient. Round to the nearest tenth of a percent. (**Hint**: 1 cups = 16 tbsp.)

5. Determine the area of each figure.

a



b



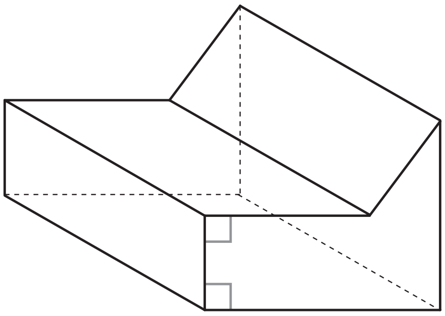
Unit 7 | Lesson 16

Surface Area of Right Prisms

Let's look at the surface area of prisms.

Warm-up Multifaceted Objects

Here is a prism.



1. What are some attributes of the prism you could measure?

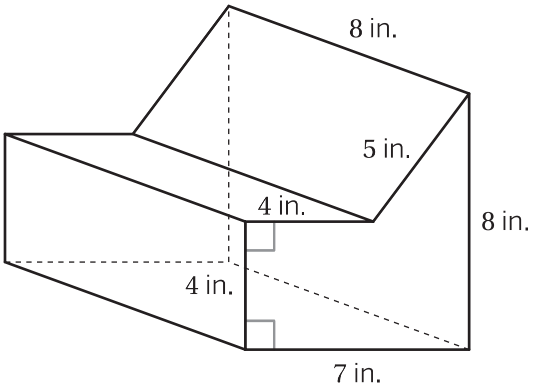
2. What units would you use for these measurements?

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 So Many Faces

Refer to the prism from the Warm-up, now with the dimensions labeled.



Two students are trying to calculate the surface area of this prism.

- Noah says, “This is going to be a lot of work. We need to find the area of 7 different faces and then add them up.”
- Andre says, “It’s not so bad. The 2 bases are the same and the other faces, the 5 rectangles, have the same height.”

1. Do you agree with Noah or Andre? Explain your thinking.

2. Use Noah’s method to determine the surface area. Draw each face and determine its area. Show and organize your work so that it can be understood by others.

Activity 1 So Many Faces (continued)

3. Use Andre's method to determine the surface area.

a Draw one of the bases and determine the area of the base.

b To determine the total area of the rectangular faces Andre wrote $8 \cdot 7 + 8 \cdot 4 + 8 \cdot 4 + 8 \cdot 5 + 8 \cdot 8 = 8(7 + 4 + 4 + 5 + 8)$.

What does each part of the expression on the left side of the equal sign represent? The right side? What did Andre do to determine the expression on the right side?

c Determine the total area of the rectangular faces.

d Determine the surface area.

4. Will Noah's method always work for determining the surface area of any prism? Andre's method? Be prepared to explain your thinking.

5. Which method do you think is the most efficient? Why?

Name: .... Date: .... Period: ....

Activity 2 Determining the Surface Area

Select one of the three following prisms and determine its surface area. Show and organize your work so that it can be understood by others.

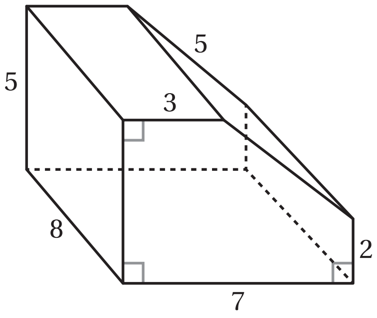


Figure 1

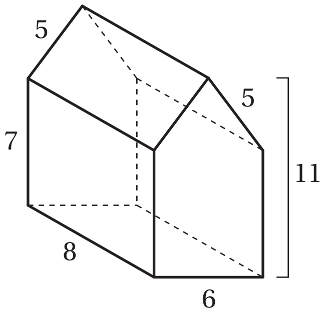


Figure 2

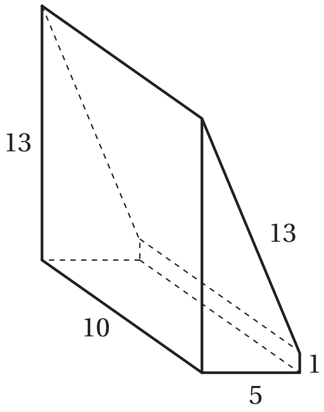


Figure 3

Find someone who selected the same prism that you did. Compare solutions. If you disagree, convince your partner why you are correct, or why you believe they are incorrect.

STOP

Summary

In today's lesson . . .

You explored different ways to determine the surface area of prisms. ***Surface area*** is the number of square units covering *all* the faces of a polyhedron without any gaps or overlaps. To determine the surface area of a prism, you can determine the area of *all* the faces and then add them together.

Sometimes, you can simplify your work by drawing the net of the prism. By doing so, you might recognize the shapes and sizes of the faces and bases of the prism. If this helps, determine the area of each shape and then add them together.

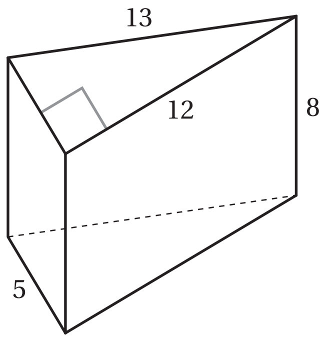
Another way to make work more efficient is to recognize a prism has three parts: two identical bases and one larger rectangle connecting the bases. Think about cutting the prism and unfolding the sides to create this rectangle. The height of the rectangle is the height of the prism and the length of the larger rectangle is found by calculating the perimeter of the base. To determine the total surface area, determine the area of the larger rectangle and add it to the total area of the two bases.

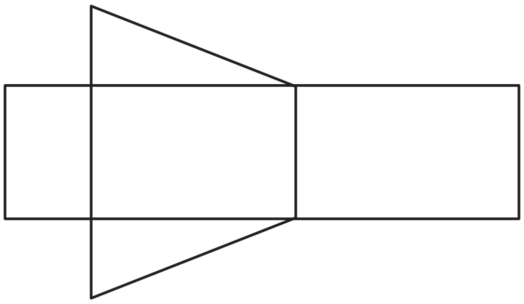
Reflect:

Name: .... Date: .... Period: ....

Practice

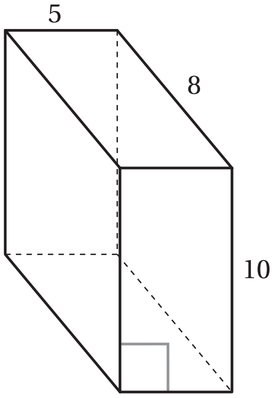
1. Here is a prism and its net. Mark the dimensions on the net and then determine the surface area. Show and organize your work so that it can be understood by others.



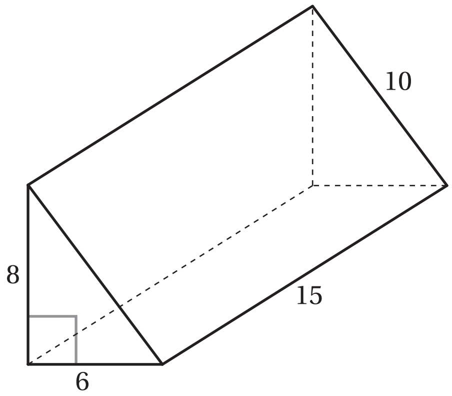


2. The edge lengths of each prism are given in units. Determine the surface area of each prism in square units. Show and organize your work so that it can be understood by others.

a

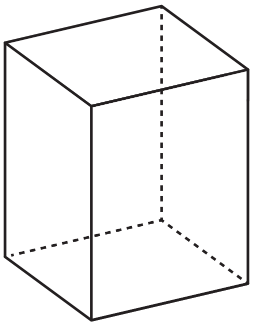


b



Name: .... Date: .... Period: ....

3. Priya says, "No matter which way you slice a rectangular prism, the cross section will be a rectangle." Mai says, "I'm not so sure." Describe a slice Mai might be thinking of. Draw the slice and what the cross section will look like.



4. For each expression, rewrite it using fewer terms.

   a $12m - 4m$

   b $12m - 5k + m$

   c $9m + k - (3m - 2k)$

5. Knowing that 40% of 625 is 250, and 4% of 625 is 25, determine each of the following.

   a What is 44% of 625?

   b What is 4.4% of 625?

   c What is 0.44% of 625?

6. A cube has dimensions of 3 in. Determine the surface area and volume of the cube.

Name: ______________________ Date: __________ Period: __________

Unit 7 | **Lesson 17**

Distinguishing Surface Area and Volume

Let's work with surface area and volume in real-world situations.

Warm-up The Science Fair

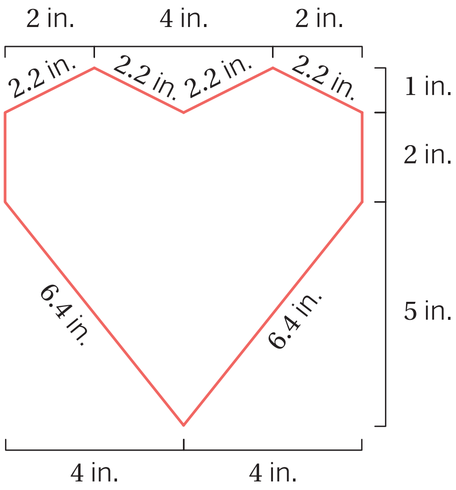
Mai's science teacher told her that when there is a greater amount of ice touching the water in a glass, the ice melts faster. Mai wants to test this statement, so she designs her science fair project to determine if crushed ice or ice cubes will melt faster in a drink. She begins with two cups of warm water. In one cup, she puts one cube of ice. In a second cup, she puts crushed ice that has the same volume as the cube of ice.

What is your hypothesis? Will the ice cube or crushed ice melt faster, or will they melt at the same rate? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 A Heart-Shaped Box, Revisited

Previously, you calculated the volume of this heart-shaped box. Here is a copy of the same image with the slanted side lengths now included. The depth of the box is 2 in. How much cardboard is needed to create the box?



Name: .... Date: .... Period: ....

Activity 2 Surface Area or Volume?

Plan ahead: How can you prepare yourself for both frustration and success during this activity?

1. Read each question. Decide whether it would be more reasonable to consider the surface area or the volume of the figure when answering each question. Place a check mark in the appropriate column.

Question	Surface area	Volume
How much wood is needed to make triangular-shaped stacking blocks?		
How long would it take to fill a rectangular swimming pool?		
How long would it take to paint the outside of a barn?		
How many yards of fabric are needed to sew a pillowcase?		
How long would it take to form a rectangular foundation for a new building by digging out the dirt?		
How much wood is needed to build a birdhouse?		

2. Select two of the above scenarios. Describe what additional information you would need to be able to answer the question.

Activity 3 Sharing Quiche

An egg white and spinach quiche is in the shape of a square prism. The square base measures 20 cm on each side, and the quiche is 5 cm tall. It has crust on the sides and the bottom. A toothpick is placed on the top at the exact center of the square.

1. Draw a figure to represent the quiche and label its dimensions. Then draw the base of the quiche and label its dimensions.

a Draw the quiche:

b Draw the base of the quiche:

2. Calculate the size (volume) of the quiche.

3. Calculate the amount of crust that is on the quiche.

4. Determine a way to cut the quiche into four equal portions, so that all four portions have the same amount of quiche and crust.

5. Determine another way to cut the quiche into four equal portions.

Are you ready for more?

Determine a way to cut the quiche into five equal portions. Draw how you would cut the quiche in the space provided.

STOP

Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You analyzed different real-world situations involving surface area or volume. Sometimes you need to determine the surface area of a prism, and sometimes you need to determine the volume. Here are some things to note:

Surface Area

- Measures how much material is needed to cover a surface.
- Measures how much of an object needs to be painted.
- Is measured in square units, such as in^2 or m^2.

Volume

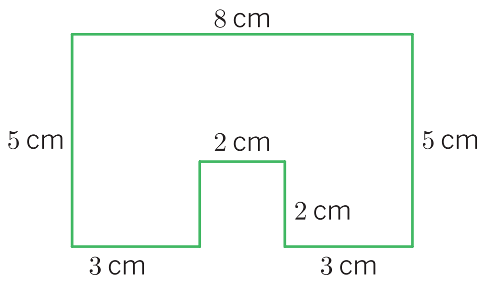
- Measures how much liquid a container can hold.
- Measures how much material is required to build a solid object.
- Is measured in cubic units, such as in^3 or m^3.

Reflect:

Practice

Name: ________________ Date: ________ Period: ________

1. Here is the base of a prism.



a. If the height of the prism is 5 cm, what is the surface area? The volume?

b. If the height of the prism is 10 cm, what is the surface area? The volume?

c. If the height of the prism doubles, what is the percent increase in the surface area? The volume?

2. Select *all* the situations where knowing the volume of an object would be more useful than knowing its surface area.

A. Determining the amount of paint needed to paint a barn.

B. Determining the monetary value of a piece of gold jewelry.

C. Filling an aquarium with buckets of water.

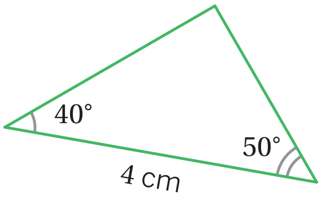
D. Deciding how much wrapping paper a gift will need.

E. Packing a box with watermelons for shipping.

F. Charging a company for ad space on your race car.

G. Measuring the amount of gasoline left in the tank of a tractor.

3. Han draws a triangle with a 50° angle, a 40° angle, and a side length of 4 cm, as shown. Can you draw a different triangle with these same criteria? Explain your thinking.

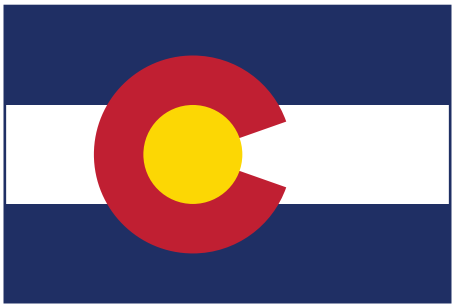


Name: .... Date: .... Period: ....

Practice

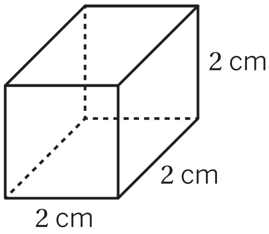
4. Angle H is half the measure of angle J. Angle J is one fourth the measure of angle K. Angle K has measure of $240°$. What is the measure of angle H?

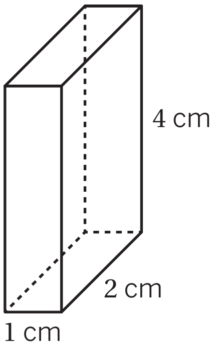
5. The Colorado state flag consists of three horizontal stripes of equal height. The side lengths of the rectangular flag are in the ratio $2 : 3$. The diameter of the gold circle inside the letter C is equal to the height of the center stripe. What percentage of the flag is gold?



Creative Commons

6. The solids below have the same volume. Determine if their surface areas are also the same. Explain or show your thinking.





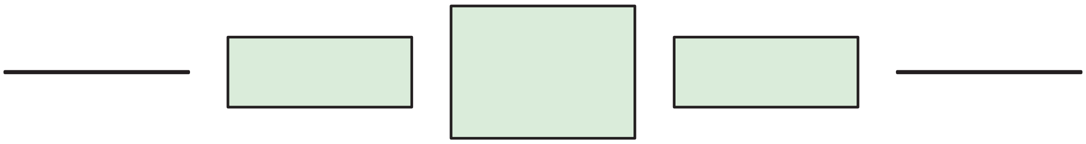
Unit 7 | Lesson 18 – Capstone

Applying Volume and Surface Area

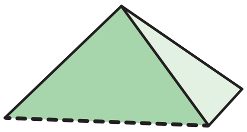
Let's explore applications of volume and surface area.

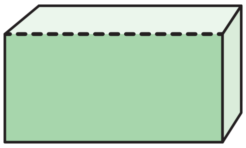
Warm-up A Mysterious Figure

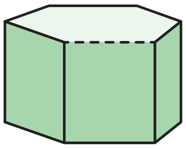
A solid has been cut by five parallel planes. The following set shows the obtained cross sections.



Which of the following figures could be the solid? (The dashed line segment indicates the location of the first slice). Explain your thinking.







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Name: .... Date: .... Period: ....

Activity 1 Office Building

The National Museum of African American History and Culture Cvandyke/Shutterstock.com

They say that architecture is the relationship between form and space, such as the shape of a building and the volume it contains. How these two elements play off of each other affect how people experience different types of buildings.

Air, light, space, proximity, distance, and beauty all play a role in what is designed and built.

At the center of decisions is the trade-off between volume and area.

Imagine you are an architect, in charge of designing a new office building.

Using your cubes, build a model of a building that meets the following specifications. Each cube counts as one office, each face of a cube on the bottom base counts as a unit of land, and every face of a cube on the side of the building will be a window. It is important that you keep the costs as low as possible.

Building specifications	Building costs
• The shape must be a rectangular prism.	• Each office costs $\$10,000$.
• There should be 72 office units.	• Each square unit of land costs $\$5,000$.
• All exterior faces are glass windows.	• Each square unit of windows costs $\$1,000$.

1. Design a building and determine its dimensions, volume, surface area, and cost.

Activity 1 Office Building (continued)

2. Design a second building and determine its dimensions, volume, surface area, and cost.

3. How did the costs change? Why does one building have a lower cost than another?

STOP

Unit Summary

Like the humble honeybee, we humans love to make things. Everything from the pencils in your bookbag to your school is the result of someone's careful thought and planning.

Many inventions begin as a sketch on a flat piece of paper, where different shapes are good for different uses. A triangle is a **rigid** shape, making it great for supporting weight. That's why we use triangles in structures like bridges and roofs. Meanwhile, hexagons, like those in honeycombs, are great for stacking and saving space.

What makes these shapes unique has everything to do with their sides, their angles, and their relationships to each other.

And as we lift these shapes into three-dimensional space, we can see how these shapes form prisms. Prisms take up space, or volume, and also have surface area. These properties determine how spacious a room is, or how impressive a monument looks.

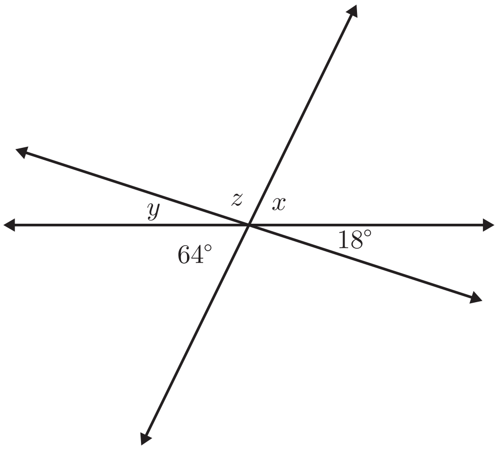
By understanding the properties of triangles, other polygons, and prisms, we can craft structures that dazzle the eye, and stand the test of time. Just like the honeybee.

See you in Unit 8.

Practice

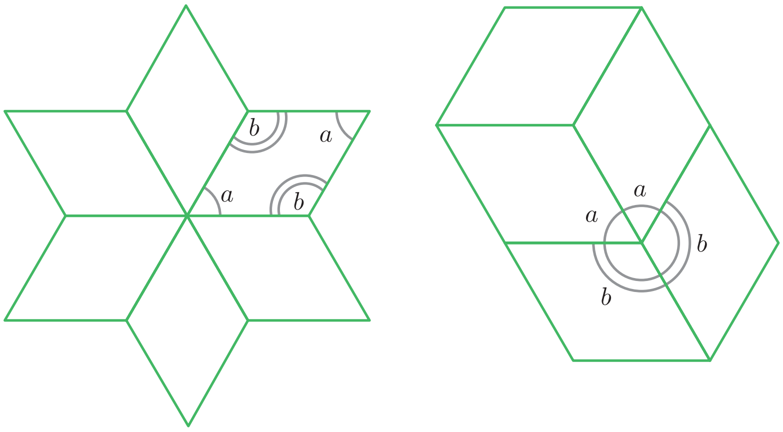
Name: ______________________ Date: __________ Period: __________

1. Determine the values of x, y, and z.



The figure may not be drawn to scale.

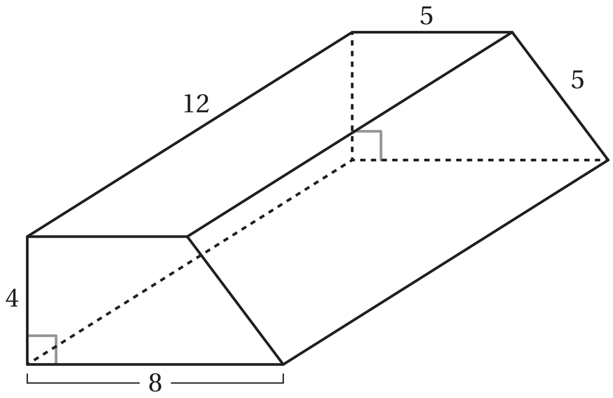
2. Here are two patterns using identical rhombuses. Without using a protractor, determine the values of a and b. Show or explain your thinking.



3. Can you draw a triangle with side lengths of 4 cm, 3 cm, and 10 cm? If so, draw one. If not, explain why.

Name: .... Date: .... Period: ....

4. Refer to the trapezoidal prism.



a Shade a base of the prism.

b Determine the area of the base you shaded.

c Determine the volume of the prism.

d Determine the surface area of the prism.

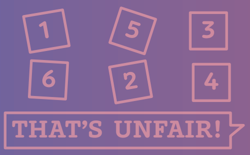
UNIT 8

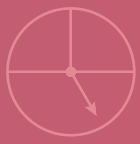
Probability and Sampling

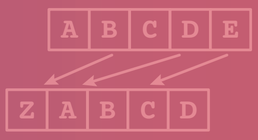
It is impossible to see into the future, but that should not stop us from trying, should it? Making predictions — taking limited information and making our best guess about what will happen — is all about knowing what is possible, what is impossible, and what is likely.

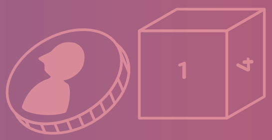
Essential Questions

- When faced with more than one possibility, how can you determine which is more likely to happen?
- Our world is complicated — how can we simulate parts of it to make better predictions?
- When is a sample *not* representative of a population?
- *(By the way, how do you crack a Caesar cipher-encoded message?)*









SUB-UNIT

1 Probabilities of Single-Step Events

Narrative: The women of Bletchley Park use probability to decode enemy messages during World War II.

You'll learn . . .

- how to describe the chance that a single-step event will occur.
- how probability is connected to ratios.

SUB-UNIT

2 Probabilities of Multi-Step Events

Narrative: Discover how to determine the chances of drawing both Blazing Shoal and Dragonstorm.

You'll learn . . .

- how to organize the sample space of a multi-step event.
- how simulating an event can help you estimate its probability.

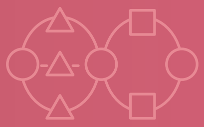
SUB-UNIT

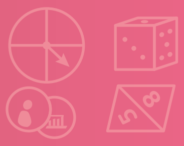
3 Sampling

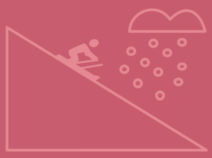
Narrative: Use sampling and statistics to answer the questions that interest *you*.

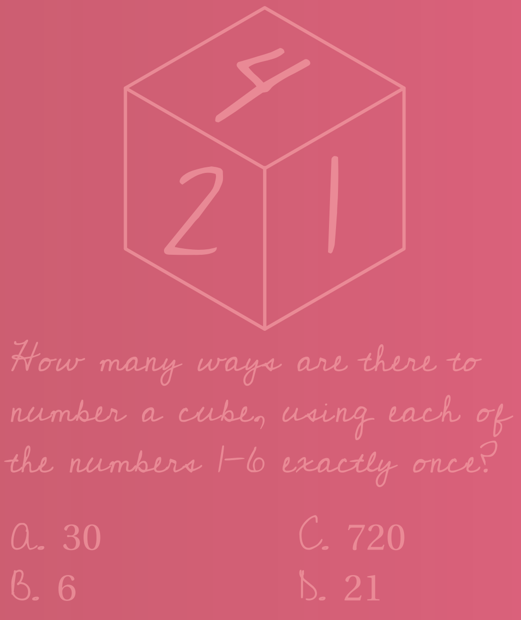
You'll learn . . .

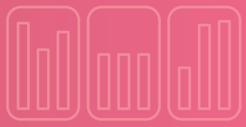
- what it means for a sample to be representative of a population.
- about the importance of random sampling and reducing bias.











The Invention of Fairness

Let's figure out how to make complex games fair.

Warm-up Fair or Unfair?

What makes a game "unfair?" Describe a game that you think is unfair. Then describe a game that you think is fair. Explain your thinking.

Collect and Display:
As you describe what makes a game "fair," your teacher will add the language you use to a class display that you can refer to during this unit.

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ______

Activity 1 Game Time

You will be given two number cubes to play the game.

Round 1 rules:

1. Each player rolls two number cubes 10 times, and then gives them to the next player. Players may win points on any roll — not just their own. Track the results in the table.

	Wins if . . .	Reward (points)	Win tally	Total points
Player A	The sum of the number cubes is 4.	1		
Player B	The sum of the number cubes is 7.	1		
Player C	The sum of the number cubes is 12.	1		

2. Does this game seem fair? Explain your thinking.

3. If you had the choice, which player would you choose to be the next time the game is played? Explain your thinking.

Round 2 rules:

4. Discuss with your group what reward each player should get to make the game more fair. Record what you decide in the table. Then play as you did before. Track the results in the table.

	Wins if . . .	Reward (points)	Win tally	Total points
Player A	The sum of the number cubes is 4.			
Player B	The sum of the number cubes is 7.			
Player C	The sum of the number cubes is 12.			

Activity 1 Game Time (continued)

5. Are the new rules more fair? How do you know? Explain your thinking.

6. List all of the possible sums you can get from rolling two number cubes.

7. What sum(s) do you think will be most likely when rolling two number cubes? Least likely? Explain your thinking.

Are you ready for more?

Design and play a new game using two number cubes. After playing, decide whether the game is fair or not. Explain your thinking.

	Wins if . . .	Reward (points)	Win tally	Total points
Player A				
Player B				
Player C				

STOP

Reflect: Why is fairness important? How do you feel when something is not fair?

Unit 8 Probability and Sampling

Winning Chance

Imagine a game with two players, where the first to win 10 rounds gets a big cash prize. Now imagine you are in the middle of this game, winning and losing rounds here and there. But before anyone can claim that fateful 10th round, the game is forced to stop!

How much prize money should you get?

This is the problem that plagued the mathematician Blaise Pascal in the summer of 1654. He was 31, a year away from publishing his famous treatise on arithmetic triangles.

Pascal wrote to the esteemed Pierre de Fermat for help. Fermat, a lawyer, was already well revered in mathematical circles. He had produced ground-breaking work in number theory and analytic geometry. Over several months, the two corresponded by mail — Pascal in Paris, Fermat from his home in southern France. If you read their letters, it's clear how much the men admired each other. Over the course of working together, and seeing how the other thought, they formed a deep and profound friendship.

Together, they recognized that the fairest way to split the prize was based on the probability of each player winning the game.

But what was the likelihood that a young Pascal would reach out to the older and more esteemed Fermat for help?

And what was the likelihood that Fermat would agree?

Thanks to their friendship, we have better ways to describe moments of uncertainty. From drawing a card to predicting the weather, we can use probability as a check on our intuition, and to help us make more informed decisions.

Welcome to Unit 8.

Practice

Name: ________ Date: ________ Period: ________

1. A game is played with two tetrahedral (4-sided) dice. The dice are in the shape of a pyramid with the numbers 1, 2, 3, and 4 written on each triangular side. After the dice are rolled, the numbers on the sides facing down are added together.

a What are all of the possible sums for rolling two tetrahedral dice?

b What sum do you think will be most likely to get when rolling two tetrahedral dice? Least likely? Explain your thinking.

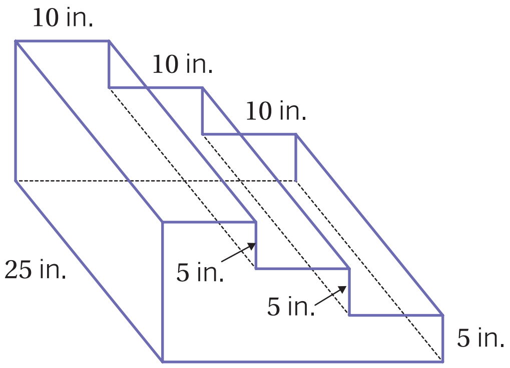
2. Complete the table to create the rules for a fair game using two tetrahedral (4-sided) dice.

a

	Wins if . . .	Reward (points)
Player A		
Player B		

b Explain why you think your game is fair.

3. The small staircase shown is built so that the horizontal piece of each step measures 10 in. long and 25 in. wide. Each step is 5 in. higher than the previous step. What is the surface area of this staircase? Include the bottom face. Show or explain your thinking.



Name: .... Date: .... Period: ....

4. Solve each equation. Show your thinking.

a $\frac{2}{3}b + 3 = -10$

b $3 - \frac{k}{2} = -\frac{5}{2}$

c $-\frac{5}{9} = \frac{2}{3}c + \frac{1}{3}$

d $-\frac{2}{3}(4 + j) = -\frac{5}{6}$

5. Use the same number line for both problems.

a Construct a number line for 0 to 1.25 marking every 0.25 with a dash.

b Plot and label the following values on your number line:

My Notes:

SUB-UNIT

1 Probabilities of Single-step Events

Narrative Connections

How did the women of Bletchley Park save the free world?

Before World War II, Bletchley Park was a quaint country estate outside of London. But by 1938, it had become the headquarters for a crack team of British codebreakers. This team, composed mostly of women, was tasked with decoding messages intercepted from the Nazis and their allies.

But how does code breaking work? The key is knowing the language a code is written in. For example, in English, 'e' is the most common letter. So if you were decoding a secret message written in English, chances are good that the symbol that occurred most would represent the letter 'e'. Once you figure out one letter, it becomes easier to deduce the rest.

That is exactly what the codebreakers did. By looking at a message's length and the frequency of different letters, codebreakers such as Mavis Batey, Jane Fawcett, and Joan Clarke decoded enemy messages back into their original language.

For years, their contribution remained a secret. But in 2009 the British government finally acknowledged the teams' work. Thanks to their efforts, the Allies gained valuable information. They learned about enemy troop movements, attack plans, and even spy activity. Historians estimate that the Bletchley codebreakers shortened the war by about three years, saving countless lives.

Unit 8 | Lesson 2

Chance Experiments

Let's investigate experiments of chance.

Warm-up Which Is More Likely?

Which event is more likely to happen? Explain your thinking.

A. When reaching into a dark closet to randomly select a shoe from a pile of 10 pairs of shoes, you pull out a left shoe.

B. While listening to a playlist with 20 songs in shuffle mode, your favorite song plays first.

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: ____________ Period: ____________

Activity 1 How Likely Is It?

1. Think about the likelihood of each event. Then describe the likelihood of each event using one of these terms: *impossible*, *possible*, or *certain*.

   a You will win the grand prize in a raffle if you purchase 2 out of 10,000 total tickets.

   b When randomly selecting a letter from the word *MATH*, you select the letter *M*.

   c No one will be late to class next week.

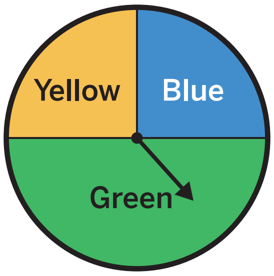
   d You will guess the correct answer on a multiple-choice question without reading the problem.

   e A unicorn will trot into your classroom this period.

   f The Earth will complete one rotation in the next 24 hours.

   g When spinning this spinner, it will land on *Green*.

   h When spinning the spinner in part g, it will land on *Red*.



2. Discuss your responses to Problem 1 with your partner. If they are not the same, convince your partner that you are correct, or why you believe they are incorrect.

Are you ready for more?

Describe three more events: one which is impossible, one which is possible, and one which is certain.

Activity 2 Card Sort: Likelihood

You will be given cards with descriptions of events on them.

1. Order the events from *most likely* to *least likely*. Record the card letters in the table.

	Most likely
	Least likely

After ordering the first set of cards, pause here and wait for further instructions. Then you will be given additional cards.

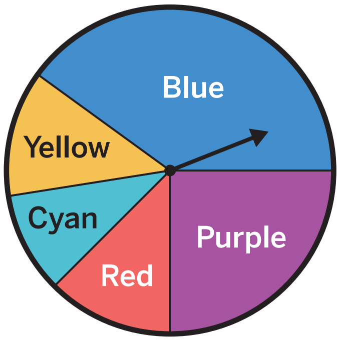
2. Add the additional cards to the first set. Reorder all of the cards from *most likely* to *least likely* and record the card letters in the table.

	Most likely
	Least likely

Name: .... Date: .... Period: ....

Activity 3 Spin to Win

Refer to the spinner shown.



1. What are the possible colors on which this spinner can land, once it is spun?

2. From these colors:

- **a** On which color is the spinner *most likely* to land?
- **b** On which color is the spinner *least likely* to land?

3. You will be given a different spinner.

- **a** If you were to spin the spinner 10 times, predict how many times the spinner would land on each color.
- **b** Spin the spinner 10 times. Track your results in the table. Make at least one observation about the results.

Spin	1	2	3	4	5	6	7	8	9	10
Result										

STOP

Summary

In today's lesson . . .

You explored events of chance and how likely they are to occur. A ***chance experiment*** is something that happens in which the outcome is unknown. For example, if you toss a coin, you do not know whether the result will be heads facing up or tails facing up until the coin lands.

An ***outcome*** is any one of the possible results that can happen when you perform a chance experiment. For example, when you toss a coin, one possible outcome is that the coin will land heads facing up. An ***event*** is a set of one or more outcomes that are favorable, or desirable.

You can describe the likelihood of events using these phrases:

- *impossible*
- *unlikely*
- *equally likely as not*
- *likely*
- *certain*

Reflect:

Name: .... Date: .... Period: ....

1. Determine whether each event is *impossible*, *unlikely*, *equally likely as not*, *likely*, or *certain*.

   a Selecting a red marble from a jar containing 7 red marbles, 2 blue marbles, and 3 green marbles.

   b Randomly selecting a green card from a deck of cards containing 2 purple cards, 4 orange cards, 3 yellow cards, and 1 blue card.

   c Rolling a number less than 4 on a number cube.

   d A spinner has 5 equal-sized sections labeled 1 through 5. You spin the spinner and it lands on a number greater than 4.

2. Kiran will randomly select a letter from the word *DEBATE*. Bard will randomly select a letter from the word *MEANING*. Who is more likely to select the letter *A*? Explain your thinking.

3. There are 4 prime numbers between 1 and 10. There are 25 prime numbers between 1 and 100. Which of these events is more likely? Explain your thinking.

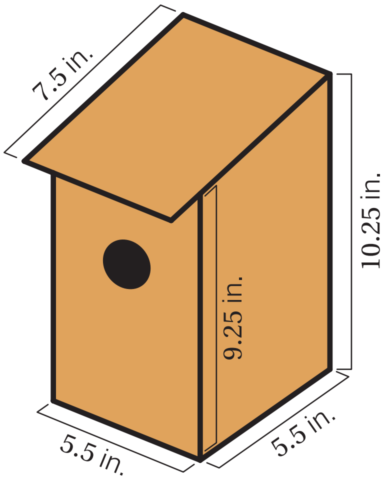
   A. A computer generates a random number between 1 and 10 that is prime.

   B. A computer generates a random number between 1 and 100 that is prime.

Name: .... Date: .... Period: ....

4. To make a specialty pizza, you need $4\frac{3}{4}$ cups of cheese, $\frac{3}{4}$ cup of olives, and $2\frac{1}{4}$ cups of sausage. How much of each ingredient is needed to make 10 specialty pizzas? Explain or show your thinking.

5. Elena is planning to build a birdhouse. The diagram shows the type of birdhouse she wants to build. About how many square inches of wood does she need to build this birdhouse? Show your thinking.



6. Shawn's goal was to read for 30 min every day. For each day of the week determine what percent of the goal Shawn achieved. The first day is done for you.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Minutes	30	24	33	15	45	28.5
Percent of Goal	100%					

Name: ________________ Date: ________ Period: ________

Unit 8 | **Lesson 3**

What Are Probabilities?

Let's find out what's possible.

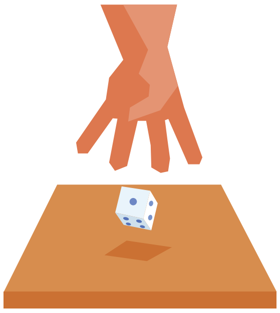
Warm-up Would You Rather?

Would you rather choose to play Game 1 or Game 2? Explain your thinking.

Game 1

Toss a coin. You win the game if the coin lands heads facing up.

Game 2



Roll a number cube. You win the game if you roll a 1.

Log in to Amplify Math to complete this lesson online.

Activity 1 What's Possible?

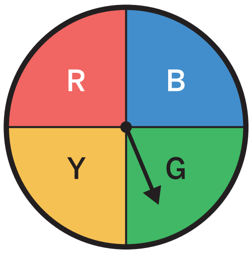
1. For each situation, list the ***sample space*** and write the number of outcomes that are in the sample space.

a Han rolls a number cube once.

Sample space:

b Clare spins the spinner shown once.

Sample space:



c Kiran selects a letter from the word *MATH*.

Sample space:

d Mai selects a letter from the alphabet.

Sample space:

e Noah selects a card from a stack of cards numbered 1 through 6.

Sample space:

2. Compare the likelihood of each event. Explain your thinking.

a Clare spins the spinner from Problem 1b. Is it more likely for the spinner to land on R or B?

b Refer to Problem 1c and 1d. Who is more likely to select the letter *T*: Kiran or Mai?

c Refer to Problem 1a and 1e. Which event is more likely to happen: Han rolling a 2 or Noah selecting a number divisible by 2?

Name: .... Date: .... Period: ....

Activity 2 What's in the Bag?

Your teacher will give your group a bag of paper slips with a letter printed on each slip.

- **Without looking in the bag, make a guess as to what letter might be printed on one of the slips. Record your guess in the table.**
- **Without looking in the bag, take out one of the slips and show it to the group.**
- **Everyone in the group records what is printed on the slip.**
- **Replace the slip back in the bag. Shake the bag and pass it to the next person.**

	Guess	Actual
Person 1		
Person 2		
Person 3		
Person 4		

Repeat these steps until everyone in your group has had a turn.

1. After everyone has taken a turn, can you be certain whether you have seen all of the letters that are printed on all of the slips? Explain your thinking.

2. Is it possible to know the ***probability*** of selecting a slip of paper with a particular letter printed on it? Explain your thinking.

3. Take out all of the slips from the bag and study them. Are all the possible outcomes — selecting a slip with a particular letter — equally likely? Explain your thinking.

4. Based on what is in your bag, determine the probability that you would select a slip with a vowel printed on it.

STOP

Summary

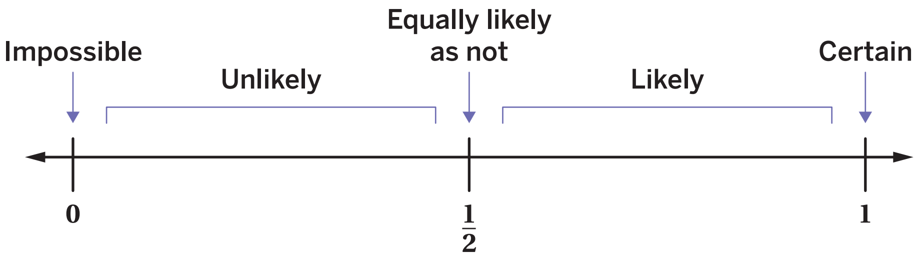
In today's lesson . . .

You listed all of the possible outcomes for a chance experiment, which is known as the ***sample space*** of the experiment. If outcomes are equally likely, identifying the sample space can help you determine the probability of an event occurring.

When all of the outcomes are equally likely, the ***probability*** of an event is the ratio of the number of favorable outcomes to the total possible number of outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total possible number of outcomes}}$$

Probabilities are expressed using numbers from 0 to 1, where 0 represents an event that is impossible and 1 represents an event that is certain. Often, probabilities are expressed as ratios, fractions, or percentages.



Reflect:

Name: .... Date: .... Period: ....

1. List the sample space for each chance experiment.
   - **a** Tossing a coin.
   - **b** Randomly selecting a season of the year.
   - **c** Randomly selecting a day of the week.

2. A computer randomly selects a letter from the English alphabet.
   - **a** How many different outcomes are in the sample space?
   - **b** What is the probability the computer selects the first letter of your first name?

3. What is the probability of randomly selecting a month of the year that starts with the letter *J*? Explain your thinking.

Practice

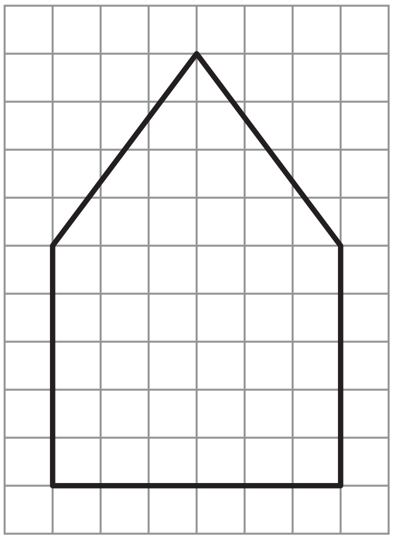
Name: .... Date: .... Period: ....

4. Let E represent an object's weight on Earth and M represent that same object's weight on the Moon. The equation $M = \frac{1}{6}E$ represents the relationship between these quantities.

a What does the value $\frac{1}{6}$ represent in this situation?

b Give an example of what a person might weigh on Earth and the Moon.

5. A bird feeder is in the shape of a pentagonal prism. The diagram of the base is shown. Suppose each small square on the grid has a side length of 1 in. The distance between the two bases is 8 in. What is the volume of the bird feeder? Show your thinking.



6. A coach can choose only one player to take a penalty shot in a soccer game. The coach must decide between Tyler, who scored on 2 out of his previous 3 penalty shots, and Bard, who scored on 15 of Bard's previous 25 penalty shots. Would you advise the coach to choose Tyler or Bard? Explain your thinking.

Name: .... Date: .... Period: ....

Unit 8 | Lesson 4

Estimating Probabilities Through Repeated Experiments

Let's do some experimenting.

Warm-up Decimals on the Number Line

1. Locate and label these numbers on the number line.

0.5   0.75   0.33   0.25

2. Choose one of these numbers. Describe an event for which the number represents the probability of that event occurring.

Log in to Amplify Math to complete this lesson online.

Activity 1 In the Long Run

Let's test how often a number cube will roll a 5 or a 6 during an experiment. You will be given a number cube to use starting with Problem 3.

1. List the sample space for rolling a number cube.

2. The number cube is rolled once. What is the probability of rolling a 5 or a 6? What does this mean?

3. With a partner, roll the number cube 10 times. One partner should roll while the other records the results. Trade roles and repeat the experiment for Round 2.

Round	Record the results. What number is rolled each time?									
1										
2										

4. Determine the ratio of the number of 5s or 6s rolled to the total number of rolls for each round.

a Round 1

b Round 2

5. Determine the ratio of the number of 5s or 6s rolled to the total number of rolls combined for each round.

a Round 1

b Round 2

6. After 20 rolls, how close was the result of your experiment to the expected probability from Problem 2?

Pause and wait for further instructions while your teacher collects the class's data.

7. Pool the class results. What was the ratio of the total number of 5s or 6s rolled to the total number of rolls for your entire class? How close is this ratio to the expected probability from Problem 2?

Name: .... Date: .... Period: ....

Activity 2 Due for a Win

Let's think about whether certain outcomes are surprising.

1. For each situation, do you think the outcome is surprising or not surprising? Explain your thinking.
   - **a** You toss a coin once, and it lands heads facing up.
   - **b** You toss a coin twice, and it lands heads facing up both times.
   - **c** You toss a coin 100 times, and it lands heads up all 100 times.

2. If you toss a coin 100 times, how many times would you expect the coin to land heads facing up? Explain your thinking.

3. If you toss a coin 100 times, what are some other results that would *not* be surprising?

4. You tossed a coin 3 times, and it has landed heads facing up exactly once. The ratio of heads to the number of tosses is currently $\frac{1}{3}$. If you toss the coin one more time, will it land heads facing up to make the ratio $\frac{2}{4}$? Explain your thinking.

Are you ready for more?

You toss a coin 100 times, and it lands heads facing up all 100 times. Then you toss it once more, and it lands heads facing up again! How surprising is this to you?

STOP

Summary

In today's lesson . . .

You estimated probabilities of events, based on what you saw happen during an experiment and thought about whether those events were surprising. A probability for an event represents a ratio of the number of times the event is expected to occur in the long run. For example, the probability of a tossed coin landing heads facing up is $\frac{1}{2}$. This means that if the coin is tossed many times, it is expected to land heads facing up about half of the time.

Even though the probability tells you what you should expect if you toss a coin many times, that does not mean the coin is more likely to land heads facing up if it just landed tails facing up three times in a row. Each toss is an independent event, and the chances of landing heads facing up are the same each time the coin was tossed, regardless of the outcomes for prior tosses.

Reflect:

Name: ______________________ Date: __________ Period: __________

1. The Student Government is surveying students from two different grades about what they typically do after dismissal. The tables summarize the survey responses. Suppose you randomly selected one student in the school. What is the probability that their typical activity would be practicing a sport? Show your thinking.

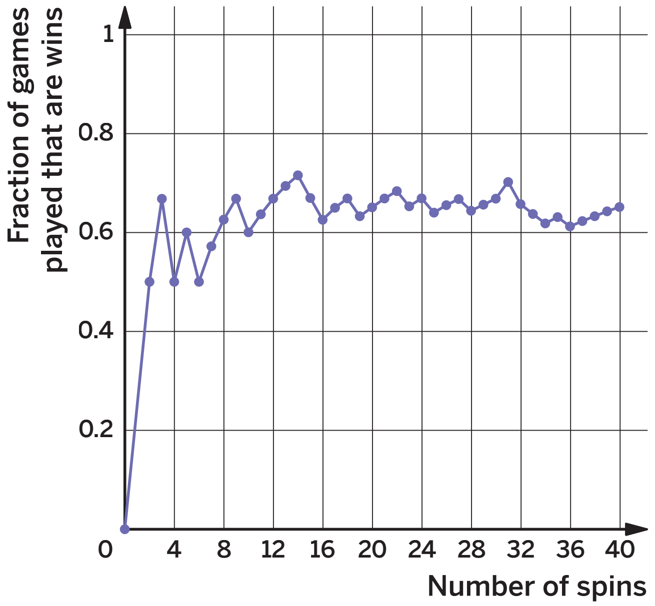
Grade 7

Activity	Number of students
Practice a sport	21
Play video games	9
Start homework	10
Watch a younger sibling	4
Hang out with friends	16
Other	5
Total	**65**

Grade 8

Activity	Number of students
Practice a sport	6
Play video games	4
Start homework	3
Watch a younger sibling	4
Hang out with friends	10
Other	3
Total	**30**

2. A spinner is spun 40 times in a game. The graph shows the fraction of spins that were considered wins. Estimate the probability of a winning spin in this game, based on the graph.



Name: ______________________ Date: ____________ Period: ____________

3. Lin wants to know whether tossing a quarter has a probability of landing heads facing up of $\frac{1}{2}$, so she tosses a quarter 10 times. It lands heads facing up 3 times and tails facing up 7 times. Does this prove that the probability is *not* $\frac{1}{2}$? Explain your thinking.

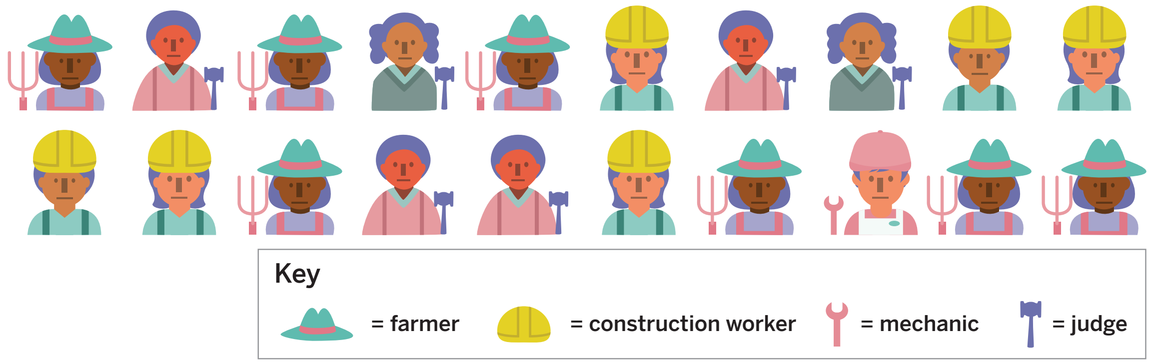
4. Solve each inequality. Show your thinking.

a $12 < 14 - 3x$

b $\frac{1}{3}x - 5 \leq 10$

5. Which event is more likely: rolling a number cube and rolling an even number, or tossing a coin and having it land heads facing up? Explain your thinking.

6. In this group of people, what percentage are dressed as construction workers? Explain your thinking.



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Name: ________ Date: ________ Period: ________

Unit 8 | Lesson 5

Code Breaking
(Part 1)

Let's use probability to decode encrypted messages.

Warm-up Comparing Chance Experiments

What is similar about these two experiments? What is different? Explain your thinking.

Experiment 1: Randomly selecting a letter from the word *ALABAMA*.

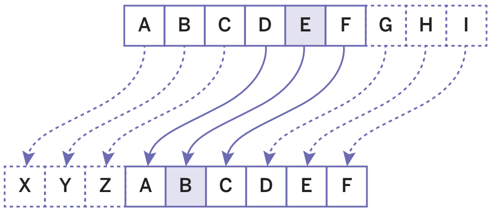
Experiment 2: Randomly selecting a letter from the word *LAMB*.

Log in to Amplify Math to complete this lesson online.

Activity 1 Caesar Cipher

Julius Caesar had good reason to be suspicious — he would eventually be assassinated by some of his closest friends. So, it was understandable why the Roman emperor developed one of the earliest secret codes, encrypting important messages that would be sent across his empire.

The Caesar cipher shifts all the letters in the alphabet by some amount. If the cipher shifted the letters by three places to the left, you could spell *CAFE* as *ZXCB* (with *Z* representing the letter *C*, *X* representing the letter *A*, and so on).



Where many of Caesar's enemies were illiterate and thus incapable of cracking this code, modern codebreakers have found a tool — using probability — to crack the code.

Consider this encrypted message:

Pm ol ohk hufaopun jvumpkluaphs av zhf, ol dyval pa pu jpwoly, aoha pz, if zv johunpun aol vykly vm aol slaalyz vm aol hswohila, aoha uva h dvyk jvbsk il thkl vba.

1. Choose a few letters from the encrypted message to analyze. Determine the number of times each letter you chose occurs in the message. Then determine the relative frequency for each letter. Use the table to collect and organize your data.

Letter	Number of occurrences	Relative frequency $\frac{\text{number of times the letter occurs}}{\text{number of letters in the text}}$

2. Do you have any guesses about which letters in the coded message represent the actual letters in the English alphabet? Discuss with a partner and write your thoughts.

Name: Date: Period:

Activity 2 Crack the Code

The Caesar cipher is an example of a "substitution cipher," in which each letter is substituted with another. The Nazi ciphers of World War II were more complex, and it took mathematicians, such as Alan Turing, and early computers to break these codes.

Next, you will be given a Caesar Cipher Decoder and a table showing the approximate frequency of letters used in the English language.

1. Compare the frequencies you observed in the coded message in Activity 1 to the frequencies in the table that represents typical English usage. Do any frequencies seem to match? Explain your thinking.

2. Use the decoder to shift the alphabet according to the matched frequencies. By how many letters do you think the encrypted message is shifted?

Encrypted message:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Typical English usage:

Activity 2 Crack the Code (continued)

3. Decode the following message.

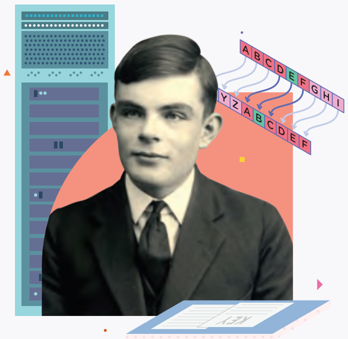
Pm ol ohk hufaopun jvumpkluaphs av

zhf, ol dyval pa pu jpwoly, aoha pz, if zv

johunpun aol vykly vm aol slaalyz vm aol

hswohila, aoha uva h dvyk jvbsk il thkl vba.

Featured Mathematician



Alan Turing

Alan Turing was an English mathematician, computer scientist, and pioneer in artificial intelligence. His formulation of what are today called "Turing machines" explored the limits of what computers could do – decades before the invention of the personal computer. During World War II, Turing worked at Bletchley Park as part of Britain's codebreaking efforts of German ciphers. Turing's career ended prematurely, when he was prosecuted by the British government in 1952 for being a homosexual. In 2013, Turing was posthumously pardoned by the Queen of England, and he now appears on the Bank of England 50 pound note.

STOP

Heritage Images/Getty Images

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You analyzed the frequency of letters in a coded message and matched them to the frequencies of letters used in the English language. You saw that the outcomes in the sample space of letters of the alphabet were not equally likely; some letters are more frequently used than others.

The ***relative frequency*** is the ratio of the number of times an outcome occurs in a set of data, or a set of possible outcomes. For example, in typical English usage, the relative frequency of the letter *E* is about 11.2%. This means that for every 100 letters used in the English language, about 11 of them are expected to be the letter *E*. The relative frequency is expressed as a fraction, a decimal, or a percentage. This type of probability analysis is used frequently in scientific research.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

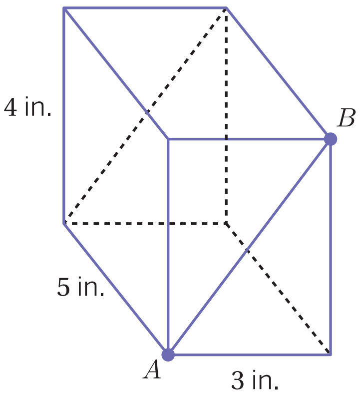
1. Write a sentence in which the relative frequency of the letter *E* (or *e*) is exactly 10%.

2. Andre conducts an experiment in which he selects one block out of a bag, records its color, and then places the block back in the bag. He repeats this experiment 60 times and notes that 43 of the blocks he selected were green.

a What could Andre estimate for the probability of selecting a green block from this bag?

b Mai looks in the bag and sees that there are 6 blocks in the bag. Should Andre change his estimate based on this information? If so, what should the new estimate be? If not, explain your thinking.

3. A rectangular prism is cut along a diagonal on each face to create two triangular prisms. The distance between points A and B is 5 in.



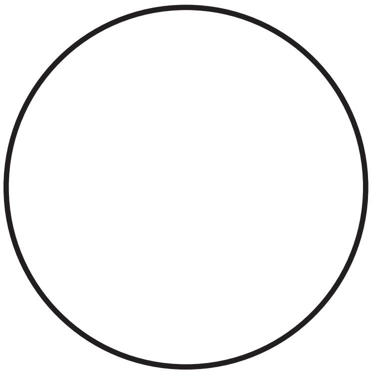
a What is the surface area of the original rectangular prism?

b What is the total surface area of the two triangular prisms combined?

Name: .... Date: .... Period: ....

4. Use the circle to design and label a spinner, with as many sections as needed, so that all of the following criteria are met:

- The chance of landing on the section labeled *A* is equally likely as not.
- The chance of landing on the section labeled *B* is unlikely.
- The chance of landing on the section labeled *C* is less than landing on *A* but greater than landing on *B*.



5. The names of four months are written in code. Can you determine which months they are? Explain your thinking.

MXOB　DXJXVW　MXQH　PDB

Unit 8 | Lesson 6

Code Breaking
(Part 2)

Let’s use probability to decode encrypted messages.

Warm-up What’s the Shift?

Use the letter frequency analysis and your pattern-recognition skills to reason about how the alphabet has been shifted using a Caesar cipher.

Typical English usage (%):

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
8	1	2	4	11	2	2	6	8	0	1	4	2	7	8	2	0	8	6	9	3	1	3	0	2	0

Mystery coded text (%):

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	2	4	0	4	0	8	2	4	3	11	2	2	6	8	0	2	4	4	7	8	2	5	4	9	4

What is the shift? Explain your thinking.

Name: .... Date: .... Period: ....

Activity 1 Send a Secret Message

You will need your Caesar Cipher Decoder for this activity. You will use the message you previously prepared to encode it, and your teacher will give your coded message to another classmate.

1. Write the message you wish to encode. Place each word in its own cell in the table. Try to keep the message to no more than 20 words.

2. Using your decoder, select a shift for your cipher text and rewrite your message in code, again placing each coded word in its own cell in the table.

Original word							
Coded word							

Original word							
Coded word							

Original word							
Coded word							

3. Rewrite your coded message on a separate sheet of paper using your neatest handwriting.

Activity 2 Decoding the Secret Message

You will receive a secret message from another classmate. Use a frequency analysis to determine the shift of the Caesar cipher and decode the message. You should only need to analyze a few letters to determine the shift.

1. Select a few letters to analyze. Use the table to organize your work. You may add rows to the table, if needed.

Letter	Number of occurrences	Relative frequency $\frac{\text{number of times the letter occurs}}{\text{total number of letters in the text}}$

2. Use this table of typical English letter frequencies and the blank table for your analysis.

Typical English usage (%):

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
8	1	2	4	11	2	2	6	8	0	1	4	2	7	8	2	0	8	6	9	3	1	3	0	2	0

Blank table for your analysis (%):

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

3. What is the shift of the letter in the coded message? Use your decoder to decode the message on the same paper that has the message.

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You performed a sophisticated analysis of a ciphered text using your understanding of relative frequencies and by reasoning about patterns that exist in the English language. Alan Turing, and perhaps even Julius Caesar himself, would be very proud of the work you did in this lesson.

In Activity 2, you selected a sample of letters to analyze. As you continue through this unit, you will see just how useful sampling can be to help you make accurate predictions.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Analyze this short excerpt in Spanish. Compare the letter frequencies of some letters to those of typical English usage. What do you notice?

> **Nunca serás capaz de cruzar el océano hasta que pierdas de vista la costa.**

Typical English usage (%):

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
8	1	2	4	11	2	2	6	8	0	1	4	2	7	8	2	0	8	6	9	3	1	3	0	2	0

2. Suppose you are recording how many times a coin lands heads facing up. How many tosses of a coin would you expect to perform in order to have the coin land heads facing up 15 times?

3. A textbook has 428 pages numbered in order, starting with 1. Suppose that without looking, you randomly turn to one page in the book.

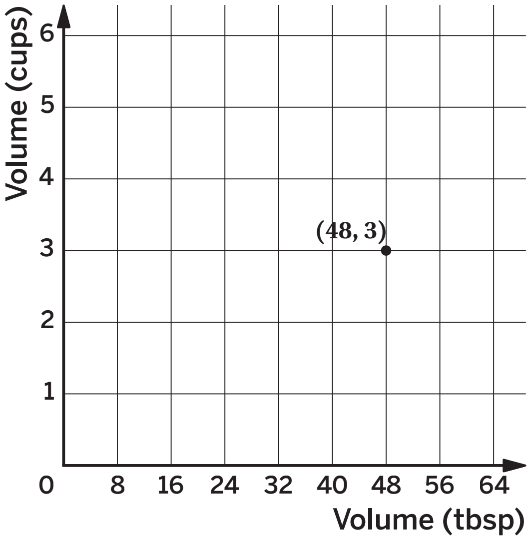
a What is the sample space for this experiment?

b What is the probability that you turn to page 45?

c What is the probability that you turn to an even numbered page?

d If you repeat this experiment 50 times, about how many times do you expect to turn to an even numbered page?

4. There is a proportional relationship between volume measured in cups and the same volume measured in tablespoons. As shown on the graph, 48 tbsp is equivalent to 3 cups.



a Plot and label two more points that represent this relationship.

b Draw a line that passes through these points and represents this proportional relationship.

c For which value of y is the point $(1, y)$ on the line you drew in part b?

d What is the constant of proportionality for this relationship? What does it mean within the context of this scenario?

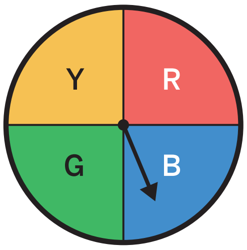
e Write an equation representing this relationship. Use c for cups and T for tablespoons.

5. List all the possible outcomes for each event.

a Tossing a coin.

b Rolling a number cube.

c Spinning the spinner shown.



My Notes:

SUB-UNIT

2 Probabilities of Multi-step Events

Narrative Connections

How did a Blazing Shoal bring the Philadelphia Convention Center to its feet?

It was a warm September day in the city of brotherly love. But inside the Philadelphia Convention Center, it was an intense competition. That day in 2011, players from all over the world had gathered to compete in a tournament for a popular collectible fantasy card game. Here, players assumed the role of powerful wizards, drawing cards from custom decks that allowed them to summon monsters and cast spells to score points against their opponents.

Facing off were Sam Black and Josh Leyton-Utter. That year, Black had built a deck never seen before in tournament play. How it worked is, well, complicated — but it boiled down to drawing the right cards in the right order.

At the heart of his strategy was to use a card called “Blazing Shoal,” which could end the game in a single turn. But for this to work, Black first had to use another card called “Dragonstorm.” This strategy was so powerful, it would later be banned from tournament play. But it was also risky. From a deck of 75 cards, Black had to draw *both* Blazing Shoal and Dragonstorm.

In the final round, Black was a turn away from defeat. But with Dragonstorm already in his hand, all he needed was Blazing Shoal in order to win. The crowd was tense as Black reshuffled his deck. With one last draw, Black slammed down the card without even looking.

A roar went through the convention center. He looked down and saw that he had failed. With a grin, Black shook his opponent’s hand in defeat.

Drawing *both* Blazing Shoal and Dragonstorm is an example of a *multi-step* event, because more than one event has to occur. You can apply probability concepts to multi-step events to help you win at games of chance.

Unit 8 | Lesson 7

Keeping Track of All Possible Outcomes

Let's explore sample spaces for experiments with multiple events.

Warm-up Questeros

Your character is about to embark on an epic journey in the land of Questeros.

- **You begin at Level 1 and must choose a path.**
- **The first choice is which skill you will learn before reaching Level 2: navigation, agriculture, or warfare.**
- **As you continue, you must make a difficult decision for how to get to Level 3: journey through the Dark Forest or cross the Creature Crossing.**
- **You cannot move backwards on any path.**

1. Which path will you choose? Why?

2. What are all the possible paths that may be chosen?

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Lists, Tables, and Tree Diagrams

Consider this experiment: A person tosses a coin, then rolls a number cube.

Elena, Kiran, and Priya each use a different method for determining the sample space for this experiment.

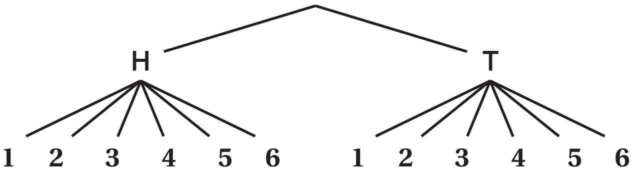
Organized list: Elena carefully creates an organized list of all the options.

heads and 1, heads and 2, heads and 3, heads and 4, heads and 5, heads and 6, tails and 1, tails and 2, tails and 3, tails and 4, tails and 5, tails and 6

Table: Kiran creates a table.

	1	2	3	4	5	6
Heads	H1	H2	H3	H4	H5	H6
Tails	T1	T2	T3	T4	T5	T6

Tree diagram: Priya draws a tree diagram with branches in which each pathway represents a different outcome.



Compare the three methods.

1. What is the same about each method?

2. What is different about the methods?

3. Why does each method show all the different outcomes without repeating any?

4. Which method would you choose to show the sample space? Why?

Activity 2 Multi-step Events

Plan ahead: How will you organize your sample spaces? How will you keep yourself organized?

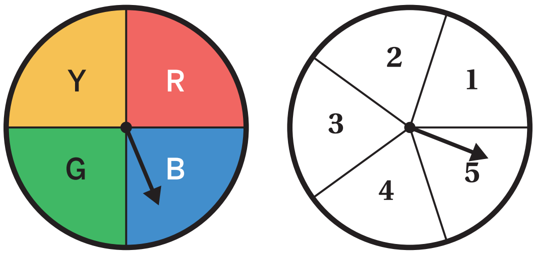
Select two of the following four experiments (A, B, C, D). For each experiment you select, complete the following tasks.

- **Use any method to determine the sample space. Make sure you list all of the possible outcomes without repeating any outcome.**
- **Determine the total number of outcomes for your chosen experiments.**

Experiment A: Toss a dime, then toss a nickel, and then toss a penny. Record whether each lands heads facing up or tails facing up.

Experiment B: Han's closet has a blue shirt, a gray shirt, a white shirt, blue pants, khaki pants, and black pants. He will randomly select one shirt and one pair of pants to wear for the day.

Experiment C: Spin a color and then spin a number.



Experiment D: Spin the hour hand on an analog clock, and then choose a.m. or p.m.

For each experiment you selected, determine the number of outcomes for each event. Then study the relationship between the number of outcomes for each event and the total number of outcomes in the sample space. What do you notice?

Name: ______________________ Date: __________ Period: ________

Activity 3 How Many Options?

It is estimated that the average adult makes about 35,000 decisions per day! Let's look at a few decisions that someone might make throughout the day.

1. Elena's closet contains 15 shirts, 5 pair of pants, and 3 pairs of shoes. How many different outfits are possible if it consists of one shirt, one pair of pants, and one pair of shoes? Show or explain your thinking.

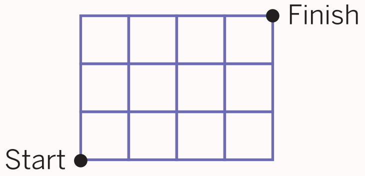
2. Elena's school cafeteria offers the items shown for lunch. How many different meals are possible if it consists of one item from each category? Show or explain your thinking.

Main	Side	Beverage	Dessert
sandwich pasta veggie pizza black bean burger	salad soup baked potato carrots pretzels veggie crisps	water milk seltzer orange juice apple juice	zucchini bread pudding fruit

3. Elena registers as a new user for an online game, where she is asked to create a five letter password. How many passwords are possible if the letters are not case sensitive and can be repeated? Show or explain your thinking.

Are you ready for more?

Suppose a mail carrier delivers mail using the map shown, where each grid square represents a block. How many different routes can the mail carrier travel, from start to finish, if she does not backtrack and only travels north and east?



STOP

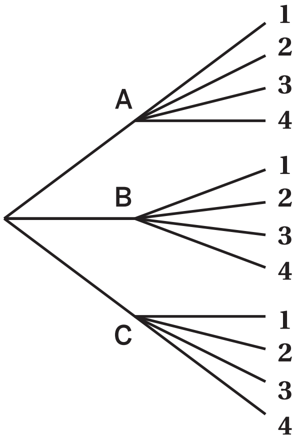
Summary

In today's lesson . . .

You explored how to determine the sample space for an experiment with multiple events. An event that consists of more than one event is called a ***multi-step event***. Up until this lesson, you studied single-step events, which just include one event.

Suppose a multi-step event consists of choosing a letter from A, B, or C, and then choosing a number from 1, 2, 3, or 4. Sometimes, it is helpful to use a systematic way to count the number of outcomes which are possible. You can use tree diagrams, tables, and organized lists to count the possible outcomes of a multi-step event.

- With a ***tree diagram***, each branch represents an outcome and the end of branches can be counted to determine the total number of possible outcomes.
  In this example, there are 3 events followed by 4 events, giving a total of $3 \cdot 4$, or 12 outcomes.
- A table also can represent the possible outcomes.
  This display also shows a total of $3 \cdot 4$, or 12 outcomes.



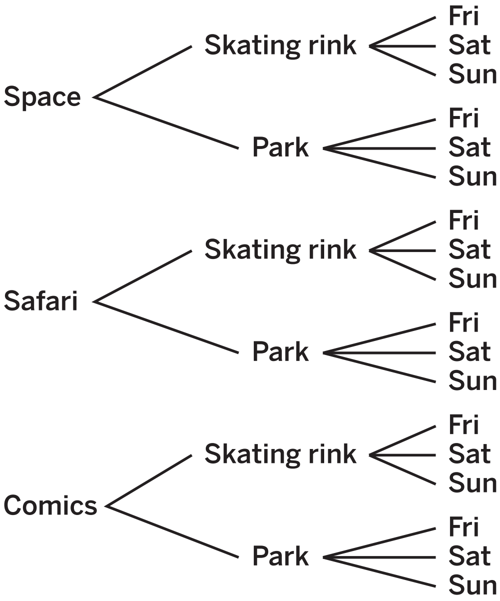
	1	2	3	4
A	A1	A2	A3	A4
B	B1	B2	B3	B4
C	C1	C2	C3	C4

- An organized list for these two events also shows the same number of total possible outcomes.
  A1, A2, A3, A4, B1, B2, B3, B4, C1, C2, C3, C4

Reflect:

Name: .... Date: .... Period: ....

1. Noah is planning his birthday party. Here is a tree diagram showing all of the possible themes, locations, and days of the week Noah is considering.



a How many themes is Noah considering?

b How many locations is Noah considering?

c How many days of the week is Noah considering?

d One possibility is a party with a space theme at the skating rink on Sunday. Write two other possible parties Noah is considering.

c How many different possible outcomes are in the sample space?

2. For each event, use any method to write the sample space. Then determine the number of outcomes.

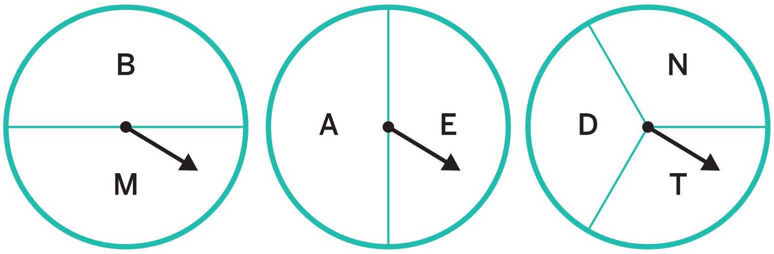
a Lin selects one type of lettuce and one kind of dressing to make a salad.

Lettuce types: iceberg, romaine

Dressings: ranch, Italian, French

b Diego chooses rock, paper, or scissors, and Jada chooses rock, paper, or scissors.

c These three spinners are each spun once.



Practice

Name: .... Date: .... Period: ....

3. For each event, write the sample space and determine the number of outcomes.

a Roll a number cube, and then toss a quarter.

b Select a month, and then select the year 2022 or 2025.

4. A pentagonal prism has a volume of 162 in^3 and a height of 6 in. What is the area of the pentagonal base? Show your thinking.

5. Use the Distributive Property to rewrite each expression.

a $4(x + 2y - 3)$

b $-3(2x - 7)$

c $\frac{1}{3}(6a + 9b)$

6. Use any method to list the sample space for rolling two number cubes. How many possible outcomes are there?

Name: .... Date: .... Period: ....

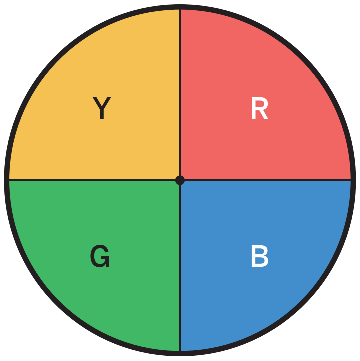
Unit 8 | Lesson 8

Experiments With Multi-step Events

Let's look at probabilities of experiments with multi-step events.

Warm-up Spinner

The spinner shown is divided into equal sections.



1. What is the probability of landing on section Y?

2. What is the probability of landing on section R?

3. What is the probability of landing on section Y or R ?

4. What is the probability of *not* landing on section B?

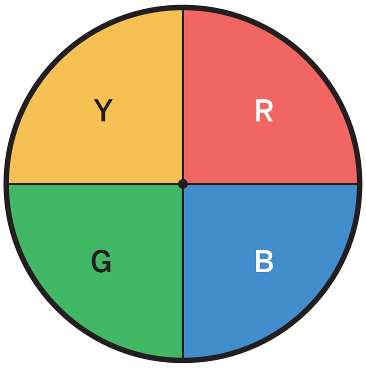
5. List the sample space for *two* spins of the spinner.

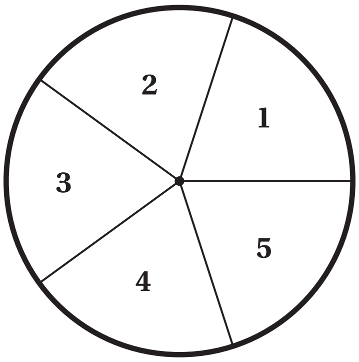
Log in to Amplify Math to complete this lesson online.

Activity 1 Spinning a Color and a Number

In the previous lesson, you wrote the sample space for spinning each of these spinners once. Let's see what happens if we create a multi-step event and spin both of them at the same time.

Co-craft Questions:
Work with your partner to write 2–3 questions you could ask about spinning both of these spinners at the same time.





What is the probability of spinning:

1. G and 3? Explain your thinking.

   P(G and 3) =

2. B and any odd number? Explain your thinking.

   P(B and odd number) =

3. Any section on the first spinner other than R and any number other than 2? Explain your thinking.

   P(*not* R and *not* 2) =

Name: ________________ Date: ________ Period: ________

Activity 2 Cubes and Coins

In the previous lesson, you analyzed an organized list, a table, and a tree diagram showing the sample space for tossing a coin and rolling a number cube.

Your teacher will assign you one of these methods to use to complete the following problems. Be prepared to explain your thinking.

1. What is the probability of the coin landing tails facing up and rolling a 6? Explain your thinking.

$P(\text{tails and } 6) =$

2. What is the probability of the coin landing heads facing up and rolling an odd number? Explain your thinking.

$P(\text{heads and odd number}) =$

Are you ready for more?

Jada tosses three quarters. What is the probability all three will land showing the same side facing up?

Activity 3 Two Cubes

Suppose you roll two number cubes. What is the probability of rolling each of the following?

1. *Both* cubes showing the same number

2. *Exactly* one cube showing an even number

3. *At least* one cube showing an even number

4. A sum of 8

5. A sum of 13

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You explored how to determine probabilities for multi-step events. Writing the sample space using an organized list, a table, or a tree diagram can help you determine the total number of possible outcomes. Another way to find the total number of possible outcomes is by multiplying the number of outcomes for each event. For example, if a multi-step event consists of two events, and the first event has 3 outcomes and the second event has 5 outcomes, then there are 15 total possible outcomes, because $3 \cdot 5 = 15$.

In general, if the outcomes in an experiment are equally likely, then the probability of an event is the ratio of the number of favorable outcomes to the total number of possible outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total possible outcomes}}$$

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A vending machine has 5 colors of gumballs (white, red, green, blue, and yellow). A second machine has 4 different animal-shaped rubber bands (lion, elephant, horse, and alligator). Each machine randomly dispenses one item for every purchase. If you buy one item from each machine, what is the probability of getting a yellow gumball and a lion band? Explain your thinking.

2. The numbers 1 through 10 are each written on a card and placed into a bag. The numbers 5 through 14 are each written on a card and placed into another bag. You will randomly select one card from each bag. What is the probability that you will select the same number from each bag? Explain your thinking.

3. When rolling three number cubes, the probability of rolling three numbers that are the same is $\frac{6}{216}$. What is the probability the three numbers are *not* all the same? Explain your thinking.

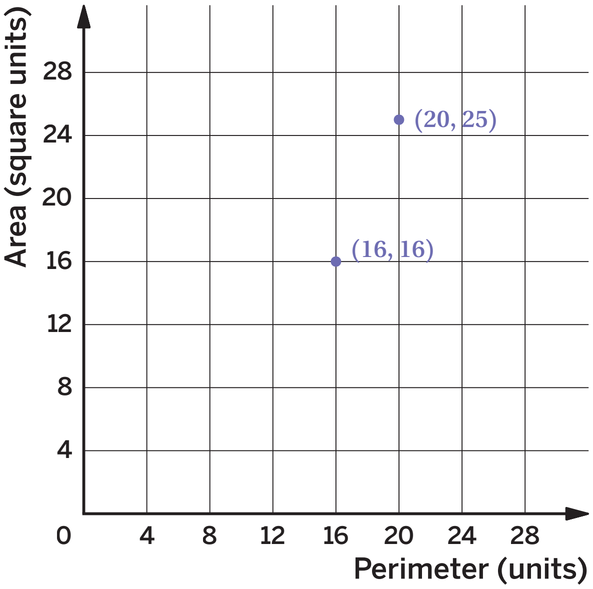
4. Ridgemont High's individual record for all-time points scored in a basketball season is $1{,}022$ points. Bard currently has scored 940 all-time points. Write and solve an inequality to determine the amount of two-point shots Bard must make to have the record. Show your thinking. Describe the solution within the context of the problem.

Name: ______ Date: ______ Period: ______

5. The graph of the relationship between the perimeter of a square and its area is shown. Two points are already plotted.

a Plot and label two more points that represent the relationship.

b Is there a proportional relationship between the area and perimeter of a square? Explain your thinking.



6. The weather forecast says it may snow every day this week. List or show the sample space for *snow* or *no snow* for four consecutive days. What is the number of total possible outcomes?

Unit 8 | Lesson 9

Simulating Multi-step Events

Let's simulate multi-step events.

Warm-up Graphington Slopes (Part 1)

Graphington Slopes is a ski business. To make money over spring break, it needs to snow at least 4 out of the 10 days of spring break. The weather forecast indicates a $\frac{1}{3}$ chance it will snow on each day during spring break.

1. Describe how a spinner could be used to model an experiment to determine the probability of snow on the first day of spring break.

2. Describe how a number cube could be used to model the probability of snow on the first day of spring break.

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Graphington Slopes (Part 2)

Recall the ski business, Graphington Slopes, from the Warm-up. To make money over spring break, it needs to snow at least 4 out of the 10 days of spring break. The weather forecast indicates a $\frac{1}{3}$ chance it will snow on each day during spring break.

1. How could a simulation be used to determine whether Graphington Slopes will make money?

2. Run your simulation for ten days to see if Graphington Slopes will make money over spring break. Record your results in the first row (Simulation 1) of the table.

Simulation	Did it snow? (✓ or ✗)									
1										
2										
3										
4										
5										

3. Complete the simulation four more times and record your results in the table (Simulations 2–5).

4. For each simulation, determine the frequency of days with snow and whether or not Graphington Slopes made money. Record your responses in the table.

Simulation	Frequency of days with snow	Did they make money? (Yes or No)
1		
2		
3		
4		
5		

Activity 1 Graphington Slopes (Part 2) (continued)

5. Based on your simulation results, estimate the probability that Graphington Slopes makes money over spring break.

$$P(\text{Graphing Slopes makes money}) = \frac{\text{number of simulations resulting in “yes”}}{\text{number of completed simulations}}$$

$$= \frac{\square}{\square}$$

Pause and wait for further directions while your teacher collects the class's data.

6. Based on the class simulation results, estimate the probability that Graphington Slopes makes money over spring break.

Name: ________________ Date: ________ Period: ________

Activity 2 Simulation Nation

Consider Simulation A, B, C, and D. For Problems 1–4, which simulation could be used to determine the indicated probability?

Simulation A

- Toss a number cube 2 times and record the outcomes.
- Repeat this process as many times as needed.
- Find the ratio of the simulated results in which a 1 or 2 appeared both times.

Simulation B

- Create a spinner with four equal-sized sections labeled 1, 2, 3, and 4. Spin the spinner 5 times and record the outcomes.
- Repeat this process as many times as needed.
- Find the ratio of the simulated results in which a 4 appears three or more times.

Simulation C

- Toss a fair coin 4 times and record the outcomes.
- Repeat this process as many times as needed.
- Find the ratio of the simulated results in which the coin lands heads facing up exactly 3 times.

Simulation D

- Place 8 blue chips and 2 red chips in a bag. Shake the bag, randomly select a chip, record its color, and then place it back in the bag.
- Repeat this 4 more times to obtain one outcome. Then repeat this process as many times as needed.
- Find the ratio of the simulated results in which exactly 4 blue chips are selected.

1. In a small lake, 25% of the fish are female. Suppose you catch a fish, record whether it is male or female, and toss the fish back into the lake. If you repeat this process 5 times, what is the probability at least 3 of the 5 fish you catch are female?

2. Elena makes about 80% of her free throws. Based on this, what is the probability she will make exactly 4 out of the 5 free throws in her next basketball game?

3. On a game show, a contestant will randomly select one of three doors. There are two rounds. In each round, one of the three doors contains a prize. In Round 1, the prize is a vacation. In Round 2, the prize is a new car. What is the probability of winning a vacation and a car?

4. Bard's choir is singing in 4 concerts. Bard and one other classmate both learned the solo. Before each concert, the choir director will randomly select Bard or the other classmate to sing the solo. What is the probability Bard will be selected to sing the solo in exactly 3 of the 4 concerts?

STOP

Summary

In today's lesson . . .

You saw the more complex an experiment is, the more challenging it can be to estimate the probability of a particular event. Well-designed simulations are ways to estimate a probability in a complex experiment, especially when it would be challenging or impossible to determine the probability from reasoning alone.

To design a good ***simulation*** — an experiment to model a real-world event — you need to know or be able to determine the probability of the individual events you wish to find. These probabilities can help you design the simulation. For example, if an event has the probability of $\frac{1}{2}$, you can use a coin toss to simulate the experiment. You can also use a number cube, in which rolling three out of the six possible outcomes is favorable.

As the number of trials of the simulation increases, the experimental (observed) probability should approach the theoretical (expected) probability.

Reflect:

Name: .... Date: .... Period: ....

1. Based on prior orders, a customer notices that when ordering takeout food from a particular restaurant, napkins were not included in the bag 50% of the time. Design a simulation you can use to estimate the probability of napkins not being included on every one of your next three takeout orders.

2. Priya's cat is pregnant with a litter of 5 kittens. Each kitten has a 30% chance of being chocolate brown. Priya wants to know the probability that at least two of the kittens will be chocolate brown. To estimate this probability, Priya designed and conducted the following simulation. The table shows her results. The letter W represents selecting a white cube and the letter G represents selecting a green cube.

Trial	Outcome
1	GGGGG
2	GGGWG
3	WGWGW
4	GWGGG
5	GGGWG
6	WWGGG
7	GWGGG
8	GGWGW
9	WWWGG
10	GGGGW
11	WGGWG
12	GGGWG

Simulation

- Place 3 white cubes and 7 green cubes in a bag.
- Randomly select a cube, record its color, and place it back in the bag.
- Repeat this 5 times to simulate one trial representing a litter of 5 kittens. Perform 12 trials.

a How many successful trials were there? Describe how you determined if a trial was a success.

b Based on this simulation, estimate the probability that *exactly* two kittens will be chocolate brown.

c Based on this simulation, estimate the probability that *at least* two kittens will be chocolate brown.

d Write and solve another probability problem Priya could estimate using this simulation.

e How could Priya increase the accuracy of her predictions?

Name: .... Date: .... Period: ....

3. Based on prior games played, a team's record shows they have won 75% of the games they have played so far this season. They will play 3 games this week. To estimate the probability that the team will win at least two of these games, Clare designs the simulation shown in the table. She obtains a result of "win," "win," "lose."

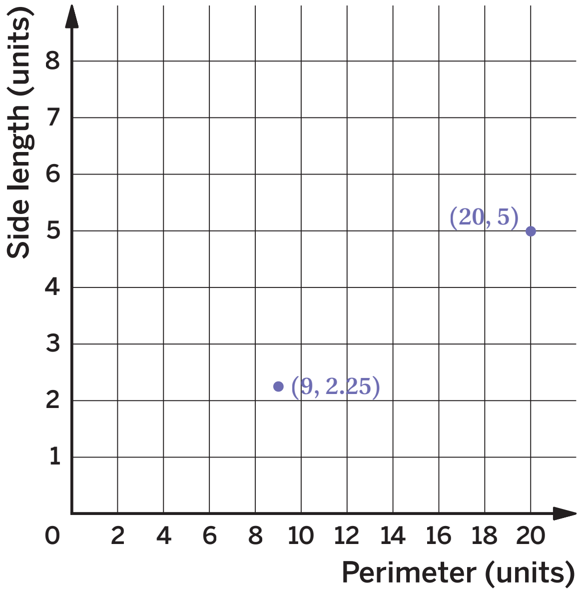
What is the estimated probability that the team will win at least two games? Explain whether Clare can trust this estimate.

Simulation
• Place 4 cards in a bag, 3 of which are labeled "win" and 1 of which is labeled "lose." • Randomly select a card, record the result, and place the card back in the bag. This represents one trial of playing a game. • Perform three trials to simulate playing three games.

4. The graph of the relationship between the perimeter of a square and its side length are shown. Two points are plotted.

a Plot and label two more points that represent the relationship.

b Is there a proportional relationship between the perimeter of a square and its side length? Explain your thinking.



5. Design a simulation of an experiment in which the favorable outcome has a probability of 0.25.

Name: .... Date: .... Period: ....

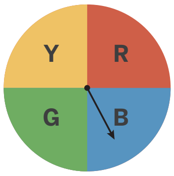
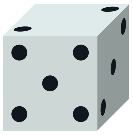
Unit 8 | Lesson 10

Designing Simulations

Let's simulate some
real-world experiments.

Warm-up Breeding Mice (Part 1)

A scientist is studying the genes that determine the color of a mouse's fur. When two mice with brown fur breed, there is a 25% chance that each baby will have gray fur. Choose one of the following tools or describe your own. Describe how you would use the tool to simulate this experiment.

Spinner	Number cube	Two coins	Bag containing 3 white cubes and 1 blue cube	Octahedral (8-sided) die
				

Other tool: ....

Log in to Amplify Math to complete this lesson online.

Activity 1 Breeding Mice (Part 2)

Plan ahead: What are some ways you can manage your stress throughout the activity?

You will be given materials to perform a simulation. Refer to the scenario from the Warm-up. When two mice with brown fur breed, there is a 25% chance that each baby will have gray fur. For the scientist's experiment to continue, at least 2 out of 5 baby mice born need to have gray fur.

1. What tool(s) are you using to simulate this experiment? List the sample space and circle the outcome representing the mouse having gray fur.

2. How many trials will need to be completed to represent one simulation? Explain your thinking.

3. How do you know whether the scientist's experiment can continue?

4. Perform five simulations and record your results in the table. Let the letter G represent a mouse born with gray fur and let X represent a mouse born without gray fur.

Simulation	Results (G or X)					Can the experiment continue? (Yes or No)
1						
2						
3						
4						
5						

Name: Date: Period:

Activity 1 Breeding Mice (Part 2) (continued)

5. Based on your simulation results, what is the probability of the scientist being able to continue the experiment? Explain your thinking.

Pause here and wait for teacher directions.

6. Based on the class simulation results, estimate the probability of the scientist continuing the experiment.

7. How could your class improve the estimated probability?

Are you ready for more?

You will be given materials and a recording sheet. Design and perform a simulation for one of the following scenarios. Use the simulation to complete the problems.

1. Based on the prior year's data, about 20% of animals brought into a small animal emergency hospital need to stay overnight. The hospital can only accommodate two animals per night. On a particular day, five animals were brought into the hospital. What is the probability that *at least* three of the animals may need to stay overnight?

2. A man has 5 grandchildren, 4 girls, and 1 boy. He thinks this is unusual. If the probability that any child born will be a girl is $\frac{1}{2}$, what is the probability that a person who has 5 grandchildren will have exactly 4 granddaughters?

3. In a basketball tournament, two teams are playing a best-of-seven series. In head-to-head matches, Elena's favorite team wins 3 out of 5 matches against Kiran's favorite team. Based on these results, what is the probability that Elena's favorite team will win the tournament?

STOP

Summary

In today's lesson . . .

You designed a simulation to model a real-world, multi-step event. You can use simulations to estimate the probability of an event. The more simulations performed, the closer the estimated probability should be to the expected probability.

Many real-world events are complicated to reproduce multiple times. So, scientists, computer programmers, financial analysts, sports analysts, environmental scientists, and others create simulations to model the outcomes. Using computer software, they are able to perform thousands of simulations to answer questions about everyday phenomena!

Reflect:

Name: .... Date: .... Period: ....

1. About 40% of people in the United States have brown eyes. Jada and Elena want to estimate the probability that at least one person in a randomly selected group of 4 people have brown eyes. They design the following simulation.

Simulation

- Place 10 slips of paper in a bag. Four slips are labeled "brown eyes". The remaining six slips have nothing written on them.
- Randomly select one slip, record its result, and place it back in the bag.
- Repeat this 4 times to represent the group of 4 people. This represents one trial. Perform 15 trials.

a Jada says they could improve the accuracy of their predictions by using 100 slips of paper and writing "brown eyes" on 40% of them. Do you agree with Jada? Explain your thinking.

b Elena says they could improve the accuracy of their predictions by conducting 30 trials instead of 15. Do you agree? Explain your thinking.

c Describe another method of simulating this experiment.

2. Priya is programming a video game. When a player reaches a certain level, they earn a type of power-up. The computer can run a program to randomly select an integer between 1 and 100. She wants a super power-up to be rewarded 15% of the time. Explain how Priya could use the computer to simulate the player receiving the super power-up at least 2 out of 3 times.

Name: .... Date: .... Period: ....

3. A bag contains 3 yellow cubes, 2 red cubes, 1 green cube, and 3 blue cubes. All of the cubes are the exact same size. What is the probability of randomly selecting a red cube?

4. Match each expression in the first list with an equivalent expression from the second list.

a $(8x + 6y) + (2x + 4y)$

b $(8x + 6y) - (2x + 4y)$

c $(8x + 6y) - (2x - 4y)$

d $8x - 6y - 2x + 4y$

e $8x - 6y + 2x - 4y$

f $8x - (-6y - 2x + 4y)$

.... $6\left(x - \frac{1}{3}y\right)$

.... $10(x + y)$

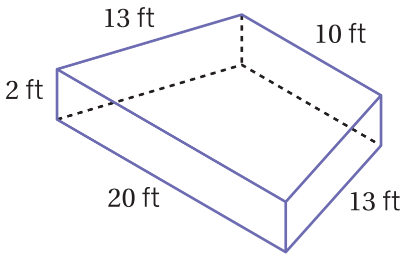
.... $10(x - y)$

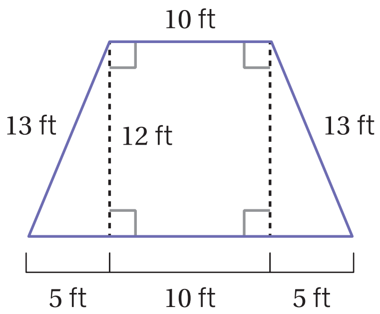
.... $8x + 6y + 2x - 4y$

.... $8x + 6y - 2x + 4y$

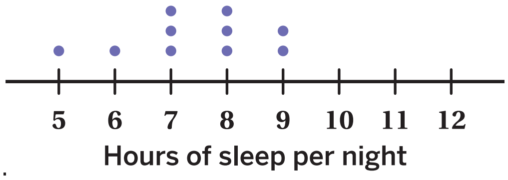
.... $8x - 2x + 6y - 4y$

5. The figure on the left is a trapezoidal prism. The figure on the right represents its base. Determine the volume of the prism. Organize your work so it can be followed by others.





6. Bard recorded the number of hours slept per night over 10 nights: $5, 6, 7, 7, 7, 8, 8, 8, 9, 9$. The dot plot shows the results.



a Determine the mean number of hours Bard slept.

b Calculate the mean absolute deviation (MAD) for this data.

SUB-UNIT

3

Sampling

Narrative Connections

What’s on your mind?

Forget about school for a second. Let’s talk about you.

What are the things *you* enjoy? What are you passionate about?

Everyone has something that drives them. Maybe it is the environment. Maybe it is community organizing. Maybe it is competitive chicken grooming. As you think about what you want to do when you grow up, do not lose track of the things you care about.

What starts as an interest can turn into a hobby, then a passion, and even a full-fledged career!

The trick is to stay curious about what you love. In fact, that’s what researchers do. They look at the things that interest them, and turn these things into research questions.

Is the new traffic light on Main Street keeping people safe?

Does owning a pet help you live longer?

Do video games help with problem-solving?

With the right questions, you can dig deep into the way people think, open doors to new knowledge, and even change the way people behave.

In these next few lessons, it will be up to you to develop your own research question. And whatever that question may be, the key to getting accurate answers will come down to finding the right group of people to ask.

Unit 8 | Lesson 11

Comparing Two Populations

Let's compare two populations of data.

Warm-up Poll the Class

1. Your school wants to organize one new sports team and can choose from the following sports. Which sport would you choose?

Alpine skiing

Jag_cz/Shutterstock.com

Gymnastics

Alex Kravtsov/
Shutterstock.com

Rowing

Ivan Smuk/Shutterstock.com

Volleyball

Eugene Onischenko/
Shutterstock.com

2. Suggest a way for the school to collect the data to decide which sport will get a new team. Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: ____________________ Date: __________ Period: __________

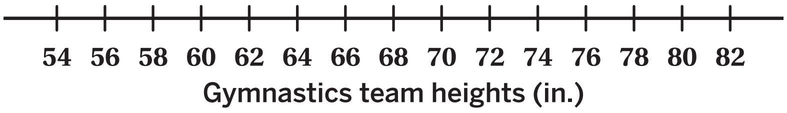
Activity 1 Team Heights

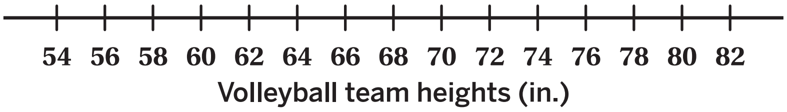
Let's compare the heights of the Olympic gymnastics team members and the Olympic volleyball team members.

Gymnastics team heights (in.)
56 60 62 62 62 63 63 64 68 71

Volleyball team heights (in.)
70 75 76 77 78 79 80 80 81 82

1. Construct two dot plots to show the heights of each team.





2. Which team has a taller population? Explain your thinking.

3. Determine the mean height for each team.

a Mean height of the gymnastics team members:

b Mean height of the volleyball team members:

4. Compare the mean heights of the two teams.

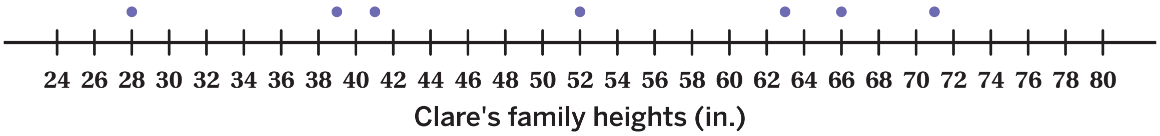
Activity 2 Family Heights

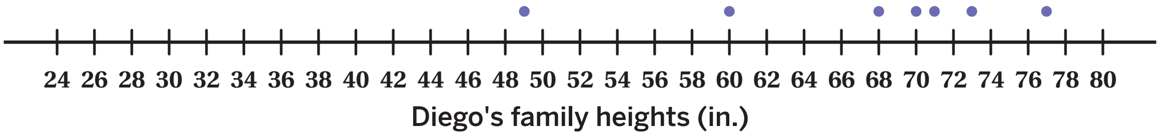
Clare and Diego are curious to know which family has taller members. They each ask their family members for their heights and decide to compare the data they collected.

Clare's family heights (in.)
28 39 41 52 63 66 71

Diego's family heights (in.)
49 60 68 70 71 73 77

The dot plots show the heights of Clare's and Diego's families.





1. Which family has taller members? Explain your thinking.

2. Determine the mean height for each family.

- **a** Mean height of Clare's family:
- **b** Mean height of Diego's family:

3. Compare the mean heights of the two families.

Name: ______________________ Date: __________ Period: __________

Activity 3 Comparing Two Populations Using the MAD

Decide, among your group, who will analyze each data set. Circle your data set. You will be given a recording sheet for your data.

Gymnastics team heights (in.)

56 60 62 62 62 63 63 64 68 71

Volleyball team heights (in.)

70 75 76 77 78 79 80 80 81 82

Clare's family heights (in.)

28 39 41 52 63 66 71

Diego's family heights (in.)

49 60 68 70 71 73 77

1. Use your data set and the recording sheet to complete the following tasks:

- List your data in the data column. You may have unused boxes.
- Record the mean. Remember, these means were determined in Activities 1 and 2.
- Subtract the mean from each data value and complete Column 3.
- In Column 4, determine the absolute value for each entry from Column 3.
- Calculate the sum of the absolute values and calculate the mean of the absolute values. This is the *mean absolute deviation (MAD)* for your data set.
- Record the MAD for your data set here.

	Gymnastics team's heights	Volleyball team's heights	Clare's family heights	Diego's family heights
MAD				

2. Share responses with your group members until the group has determined each of the four team's MADs. Record each MAD.

3. Compare the MADs for the Olympic team members with the MADs for the two families. What do you notice?

Compare and Connect: How do the numerical values for the MADs compare to the visual displays of the dot plots from Activity 1?

STOP

Summary

In today's lesson . . .

You decided whether two sets of data were very different from each other. Comparing two individuals or objects is fairly straightforward. The question, "Which dog is taller?" can be answered by measuring the heights of two dogs and comparing them directly. Comparing two populations or two data sets requires some additional analysis.

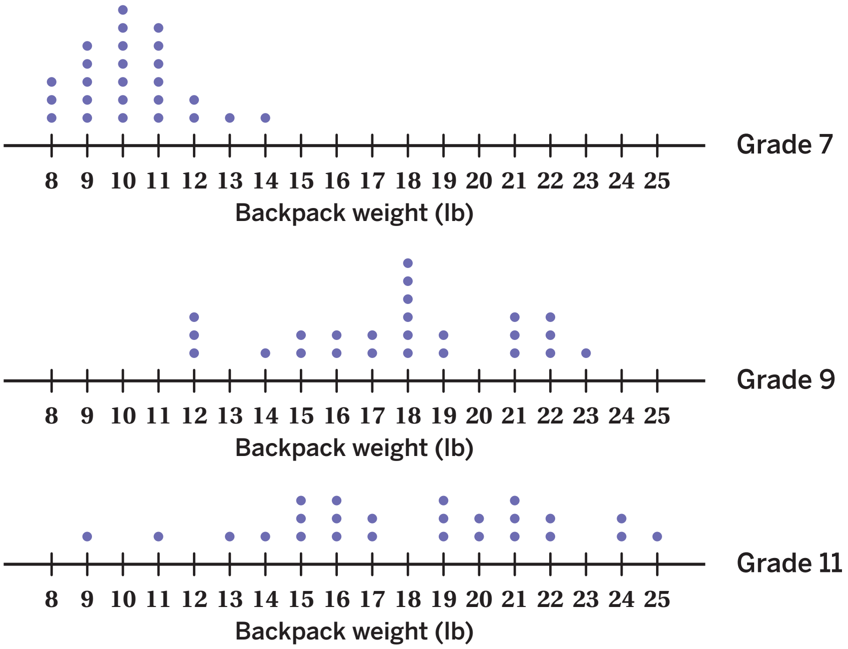
Generally speaking, two data sets are said to be very different from each other if the difference in their means is *more than twice* the greater mean absolute deviation.

If the difference in means is greater than 2 • (the greater MAD), then the data sets are very different.

Reflect:

Name: ________________ Date: ________ Period: ________

1. The dot plots show the weights of backpacks for several students in three different grades. Compare the weights of the backpacks. What do you notice?



2. A school's art club held a fundraiser to raise money for art supplies. The table shows the number of t-shirts sold each week during each season.

Fall

20	26	25	24	29
20	19	19	24	24

MAD $= 2.8$

Spring

19	27	29	21	25
22	26	21	25	25

MAD $= 2.6$

a. Determine the mean number of t-shirts sold in the fall and in the spring.

b. Express the difference in means as a multiple of the larger MAD.

c. Based on this data, do you think that sales are generally higher in the spring than in the fall? Explain your thinking.

Name: .... Date: .... Period: ....

3. Consider the word *PINEAPPLE*. Each letter in the word will be written on a slip of paper and placed in a bag. One slip of paper will be randomly selected from the bag.

a What is the sample space?

b Which is more likely: selecting the letter *E* or the letter *P*? Explain your thinking.

4. A bookstore marked down the price for all the books in a certain series by 15%.

a How much is the discount on a book in the series that normally costs $\$18.00$? Show your thinking.

b After the discount, how much would the book cost? Show your thinking.

5. Angles C and D are complementary. The ratio of the measure of Angle C to the measure of Angle D is $2 : 3$. Determine the measure of each angle. Show or explain your thinking.

6. Suppose you wanted to collect data about the number of students in your school who play an instrument.

a How would you begin to collect this data?

b Would asking 40 students in Grade 7 whether they plan an instrument be sufficient to answer this question? Explain your thinking.

Name: .... Date: .... Period: ....

Unit 8 | Lesson 12

Larger Populations

Let's compare larger populations of data.

Warm-up Siblings and Pets

Consider this statistical question:

Do families with only one child have more pets than families with more than one child?

1. If you needed to respond to this question by the end of class today, how would you gather data?

2. If you could come back tomorrow with your response to this question, how would you gather data?

3. If someone else in the class came back tomorrow with a response different than yours, what could this mean? How would you determine which response better represented the actual answer to this question?

Log in to Amplify Math to complete this lesson online.

Activity 1 Card Sort: Population or Sample?

Next, you will explore examples of samples and populations. One place you might encounter these terms is in polls, which are samples of public opinion. Statisticians, such as Courtney Kennedy of the Pew Research Center, frequently work with samples and populations, and must understand how (and why) they are similar or different.

In this activity, you will be given a set of cards. Decide which card identifies a population and which card identifies a sample. Match each scenario with the population and the sample. Record your matches in the table.

Scenario	Population	Sample
Jada noticed a picture of her teacher's pet cat and dog on the teacher's desk. Jada wondered how many teachers at her school have pets.		
Bard was eating falafel patties at lunch and offered to share some with Priya. When Priya reached in, she pulled out two falafel patties that were stuck together. Bard and Priya wondered how often falafel patties get stuck together.		
Mai was curious about the average length of popular songs from a playlist she listened to for one week on her music-streaming app.		
Kiran wondered which movie-streaming service, Webflicks or Whooloo, is more popular.		

Featured Mathematician

Courtney Kennedy

Courtney Kennedy is the director of survey research at Pew Research Center, a nonpartisan think tank based in Washington, D.C. Pew Research Center conducts countless public opinion polls every year, often by calling randomly selected phone numbers. In her work, Kennedy is responsible for the research and methodology behind the surveys.

"Courtney Kennedy." Pew Research Center, Washington, D.C.

Name: ______________________ Date: __________ Period: __________

Activity 2 John Jacob Jingleheimer Schmidt

Consider these statistical questions:

In general, do the students at your school have more letters in their first name or last name? How many more letters?

1. How many letters are in your first name? In your last name?

2. Does the number of letters in your own first and last names give you enough information to draw conclusions about students' names in your entire school? Explain your thinking.

Pause here while your class shares data.

3. Calculate the mean number of letters for the first names and last names of the students in your class. Then calculate the mean absolute deviation (MAD) of each data set. Record the results in the table.

	Mean	MAD
The first names of the students in your class.		
The last names of the students in your class.		

4. How similar or different are the two data sets? Calculate the ratio of the difference of the means to the larger MAD to support your thinking.

5. After analyzing the data for your class, do you have enough information to draw conclusions about students' names for your entire school? Explain your thinking.

STOP

Summary

In today's lesson . . .

You saw that to answer a question about a population of data, it is sometimes unreasonable to collect data from the *entire* population. Instead, data is often collected from a sample of the population.

A ***population*** is a set of people or objects that you want to study.
A ***sample*** is a part of the population.

Here are some examples of populations and samples.

Population	Sample
All of the people in the world.	The leaders of each country in the world.
All Grade 7 students in your school.	The Grade 7 students in your school who are in band.
All apples grown in the U.S.	The apples in your school cafeteria.

Reflect:

Name: ______ Date: ______ Period: ______

1. For each sample, describe two possible populations to which it could belong.

Sample	Possible population 1	Possible population 2
The prices for apples at two stores near your house.		
The daily high temperatures for the capital cities of all 50 U.S. states over the past year.		

2. Identify the population and a possible sample for the following statistical question. *What is the median salary for teachers in North America?*

The last lesson of this unit is a capstone project. In this project, you will:

1. **Pose a statistical question.**
2. **Create a survey.**
3. **Decide on a sample.**
4. **Collect and analyze data from the sample.**
5. **Present your findings.**

In the remaining lessons in this unit, there are problems marked *Capstone project helper*. These problems will help you prepare for the project.

3. *Capstone project helper*. Write a statistical question you are interested in studying. Be sure it meets the criteria shown in the table. An example statistical question is: *Do students in my school who have purchased a new phone within the last* 3 *months tend to have more apps on their phone?*

Criteria for Statistical Questions
• Can it be answered by collecting data? • Will there be some variability in the data? • Is it possible to actually collect this data?

Practice

Name: .... Date: .... Period: ....

4. Six coins are tossed. Determine the probability of each event. Show or explain your thinking.

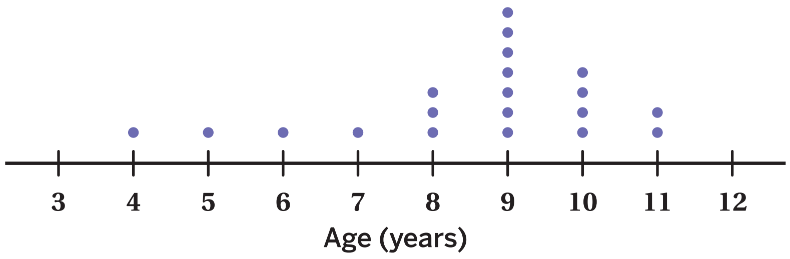
a *All* of the coins land tails facing up.

b *At least* 1 landing heads facing up.

5. A school is selling candles for a fundraiser. The school keeps 40% of the total sales, and they pay the rest to the candle company. The table shows the price and number sold of each candle size. How much money will the school pay to the candle company? Show or explain your thinking.

Candle Size	Price of candle ($)	Number of candles sold
Small	11	68
Medium	18	45
Large	25	21

6. Describe what the terms *shape*, *center*, and *spread* mean in your own words. Use the following dot plot, which shows the ages of the first 20 people surveyed at a movie theater, as an example in your explanation.



Name: .... Date: .... Period: ....

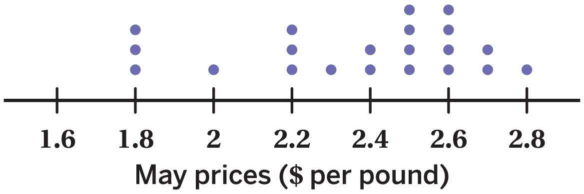
Unit 8 | Lesson 13

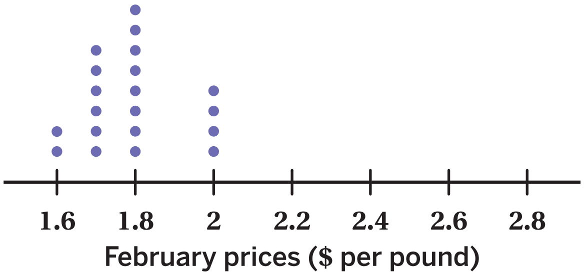
What Makes a Good Sample?

Let's see what makes a good sample.

Warm-up Describing Distributions

The following dot plots show the prices of catfish recorded during two months at a fish market.





1. Describe each distribution by mentioning the shape, center, and spread.

2. Compare the distributions.

Log in to Amplify Math to complete this lesson online.

Activity 1 Fish Market

A saltwater fisherman caught and sold 10 different fish. The mean selling price was $379 per fish.

1. The first two fish she sold were sold for $50 and $410. Are the prices of these two fish a good representation of the 10 fish? Explain your thinking.

2. The fisherman sold three whole tuna fish for $250, $400, and $1,200. Are the prices of these three fish a good representation of the 10 fish? Explain your thinking.

3. The fisherman sold three groupers for $410, $350, and $375. Are the prices of these groupers a good representation of the 10 fish? Explain your thinking.

4. The table shows the selling prices for all 10 fish. Now that you have seen the entire population, which sample from Problems 1–3 is a better representation of the 10 fish? Explain your thinking.

Prices of 10 fish ($)

50	200	250	275	280
350	375	400	410	1,200

Reflect: How did showing respect for others guide your behavior during the activity?

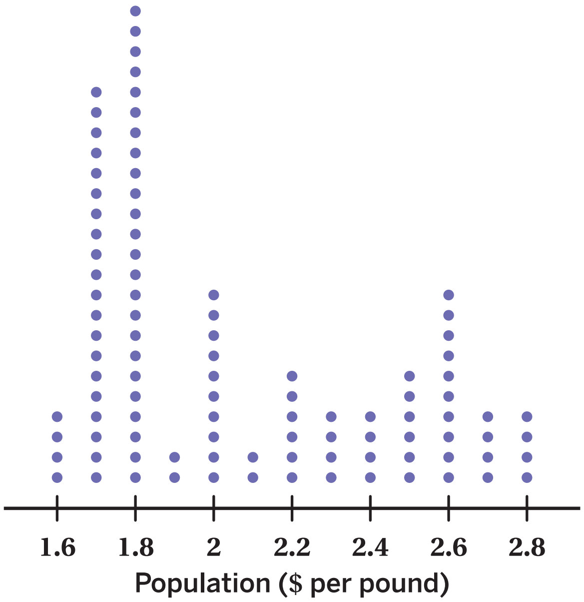
Are you ready for more?

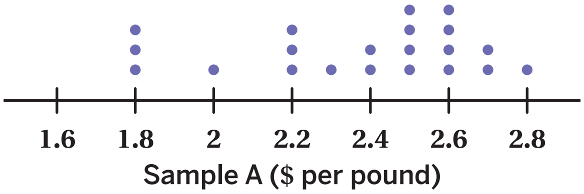
If the fisherman decided to sell all her fish for $379 each, which sample from Problems 1–3 would give the buyer the best deal? Explain your thinking.

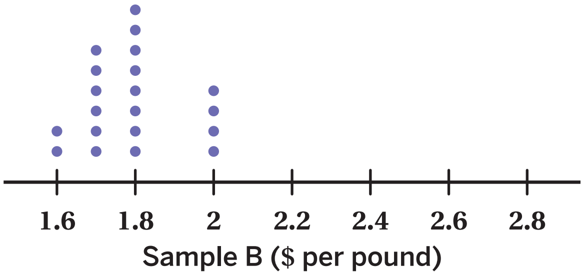
Name: .... Date: .... Period: ....

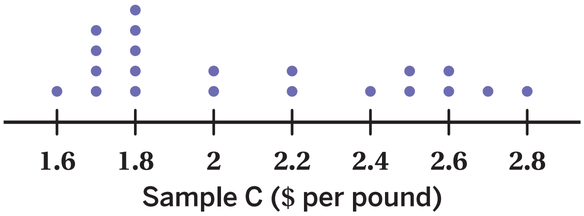
Activity 2 Sampling the Fish Market (Part 1)

The price per pound of catfish at a market was recorded for 100 weeks. Consider the dot plots showing the population and three different samples from the population. If the goal is to have the sample represent the population, which of the samples would work best? Which would not work as well? Explain your thinking.



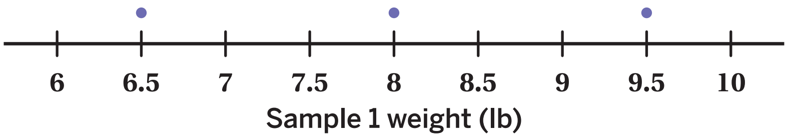


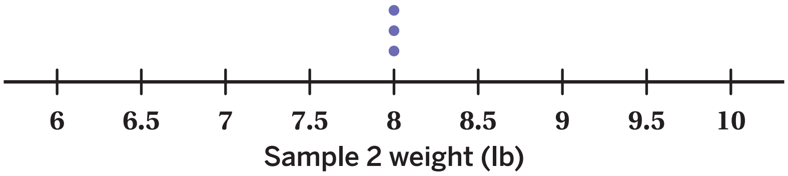


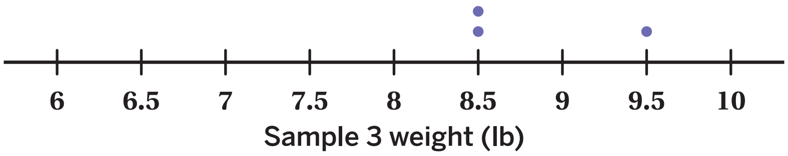


Activity 3 Sampling the Fish Market (Part 2)

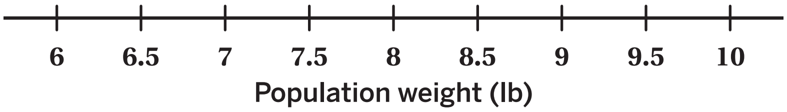
Salmon is a common type of fish to purchase in a market. To ensure quality, weight measurements were taken from three different samples of salmon. Study the samples.







Use the samples to draw a dot plot of what the population data might look like, displaying the weight of at least 12 salmon.



STOP

Name: ______________________ Date: __________ Period: ________

Summary

In today's lesson . . .

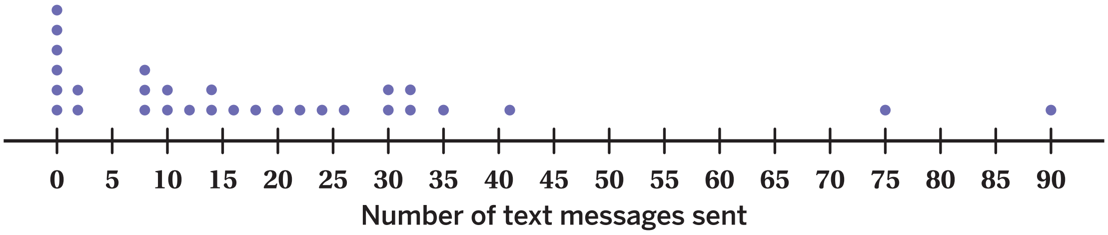
You explored samples of populations and asked yourself "What makes a good sample?" Samples are used when the population is too large to survey or measure each individual or object. You saw that samples are not always good representations of the population. A ***representative sample*** of a population has a distribution which closely resembles the distribution of the population with respect to its shape, center, and spread. Representative samples are useful in making statistical inferences about the whole population.

Reflect:

Name: ________________ Date: ________ Period: ________

1. Andre's school held a canned food drive. Suppose 45% of all the students at his school brought in a can of food. Andre selects a representative sample of 25 students from the school and determines the sample's percentage of students who brought food. He expects the percentage of students bringing food for this sample to be 45%. Do you agree? Explain your thinking.

2. Kiran wants to know how often students at his school send text messages. The following sample consists of 30 students and is representative of the population.



a What do the six dots above 0 represent?

b Because this sample is representative of the population, describe what you think a dot plot for the entire population might look like?

3. *Capstone project helper.* Design a short survey, 1–3 questions, that will help you answer the statistical question you created in the previous lesson. Be sure your survey meets the criteria shown in the table. Example survey questions are: *Did you get a new cell phone within the last 3 months? How many apps do you have on your phone?*

Survey criteria

- Include a question about what makes the population special.
- Include a question about what you want to answer about the population under study.

Name: .... Date: .... Period: ....

4. A chef is making a large pot of chicken soup and needs to taste it to make sure the amount of seasoning is correct. Why would it be a better idea for the chef to taste a sample of the soup instead of tasting the entire population of the soup?

5. How many different outcomes are in each sample space? Show or explain your thinking. You do not need to list the actual outcomes in the sample space.

a A letter of the English alphabet followed by a digit ranging from 0 to 9.

b A baseball team's cap design is selected from 3 different colors, 2 different clasps, and 4 different placements for the team logo. A decision is also made to include or not include reflective piping.

c A locker combination, such as $7–23–11$, uses three numbers. Each number ranges from 1 to 40. Numbers can be used more than once, such as in the locker combination $7–23–7$.

6. List three ways to select a sample of Grade 7 students at your school.

Unit 8 | Lesson 14

Sampling in a Fair Way

Let's explore ways to obtain representative samples.

Warm-up Comparing Methods for Selecting Samples

Lin is running for president of the seventh grade. To predict her chances of winning, she thinks of the following methods she can use to survey a sample of the students who will vote in the election.

Method 1	Method 2	Method 3
Survey everyone on her basketball team.	Survey every third student waiting in the lunch line.	Survey the first 15 students to arrive at school one morning.

1. What are the benefits of each method?

   Method 1:

   Method 2:

   Method 3:

2. Who might each method exclude?

   Method 1:

   Method 2:

   Method 3:

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 That's the First Straw!

Students from your class will select cut straws from a paper bag, and use a centimeter ruler to measure the lengths of the straws selected.

1. As each straw is selected and measured, record its length, in centimeters, in the table.

Length of straws (cm)

	Straw 1	Straw 2	Straw 3	Straw 4	Straw 5
Sample 1					
Sample 2					

2. Calculate the mean length of all the straws based on:

a The mean of the first sample.

b The mean of the second sample.

3. Were the means of the samples the same? Did the mean length of all the straws in the bag change in between selecting the two samples? Explain your thinking.

4. The actual mean length of all the straws in the bag is 2.46 cm. How do the sample means compare to the actual mean length? Explain why this may have happened.

5. Suppose you repeat this experiment again, yet this time you select a larger sample — such as 10 or 20 straws — instead of just 5 straws. Would your sample's mean be more accurate? Explain your thinking.

Activity 2 That's the Last Straw!

Imagine arranging the 35 straws from Activity 1 in order from shortest to longest, and then assigning each straw a number from 1 to 35. For each method shown, decide whether it would be an *unbiased* — or fair — way to select a sample of 5 straws. Explain your thinking.

1. Select the straws numbered 1 through 5.

2. Write the numbers 1 through 35 on slips of paper of equal size. Place the slips into a bag. Without looking, select five slips from the bag. Select the straws that correspond to the numbered results for the sample.

3. Write the numbers 1 through 35 on slips of paper of equal size. Place the slips into a bag. Select one slip from the bag. Select the first straw based on this number. Use the next 4 numbers in order to complete the sample. For example, if you select 17, then you would select straws 18, 19, 20, and 21 for the sample.

4. Create a spinner with 35 equal-sized sections numbered 1 through 35. Spin the spinner 5 times. Select the straws that correspond to the numbered results for the sample.

Are you ready for more?

Describe another unbiased sampling method for selecting 5 straws.

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that some samples from a population can be *biased*, or unfair, which means the sample is not representative of the population as a whole.

- For example, if you select the first 5 students who walk into the classroom, this will not give you an unbiased sample of all of the students in the class. It will be biased against students who are typically late.

A ***random sample*** is a sample of a population which has an equal chance of being selected as every other sample of the same size.

- For example, to obtain a random sample of all of the students in a class, you can write each student's name on a slip of paper, place the slips into a bag, and without looking, select a sample of 5 slips from the bag.

It is not always possible to select a sample at random.

- For example, if you want to know the average length of wild salmon, it is not possible to identify each fish, select a few at random from the list, and then capture and measure those exact fish.

When a sample cannot be selected at random, it is important to try to reduce bias as much as possible when determining how to select the sample.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Select *all* the reasons why random samples are preferred over other methods of collecting a sample.

   A. A random sample is the only way to determine how many people or objects you want in the sample.

   B. A random sample is always the most convenient way to select a sample from a population.

   C. A random sample is likely to give you a representative sample of the population.

   D. A random sample is a fair way to select a sample, because each person in the population has an equal chance of being selected.

   E. If you use a random sample, the sample mean will always be the same as the population mean.

2. Jada wants to create a representative sample of 6 whole numbers between 1 and 100. She uses a computer's random number generator to produce these numbers: 1, 2, 3, 4, 5, and 6. Should she use these numbers or have the computer generate a new set of numbers? Explain your thinking.

3. *Capstone project helper.* Collect data to answer your statistical question by giving the survey you created in the previous lesson to a representative sample. Example method: *I will survey every 10th student who arrives at school in the morning.* Describe your method here:

Important items to consider

- A representative sample that is randomly selected.
- The process you will use to select your sample.
- A realistic sample size.

Name: .... Date: .... Period: ....

4. At an office party, 100 people are divided into 5 groups with 20 people in each group. One person's name is randomly chosen, and everyone in their group wins a prize. Noah simulates this event by writing 100 different names on slips of papers, placing them in a bag, and then selecting one slip of paper without looking. Explain a way to simulate this event using fewer than 100 slips of paper.

5. Data collected from a survey of American teenagers, ages 13 to 17, was used to estimate that 29% of teens believe in ghosts. This estimate was based on data from 510 American teenagers. Which of the following represents the population that this survey was studying?

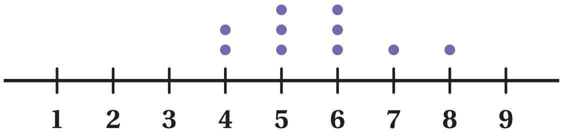
A. All American teenagers.

B. The 510 teens that were surveyed.

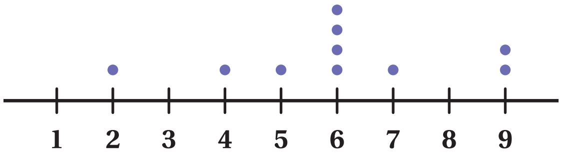
C. All American teens who are between the ages of 13 and 17.

D. The 29% of the teens surveyed who said they believe in ghosts.

6. Suppose the average shoe size of a Grade 7 student in a school is 5.5. Each dot plot shows the shoe size for a sample of ten Grade 7 students. Which sample is more representative of the population of Grade 7 students at the school? Explain your thinking.



Sample A shoe sizes



Sample B shoe sizes

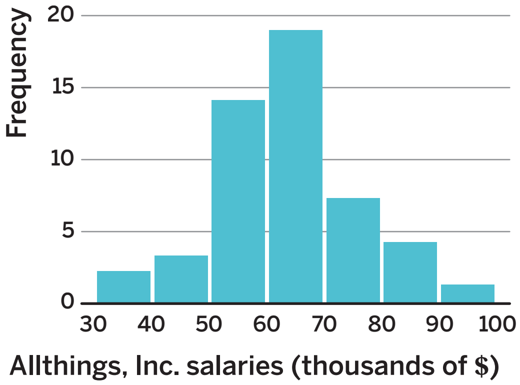
Unit 8 | Lesson 15

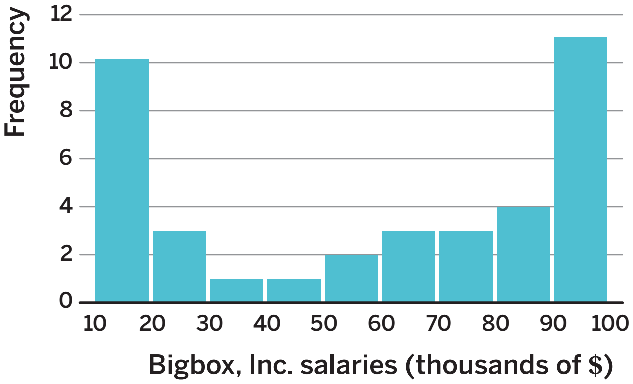
Estimating Population Measures of Center

Let's think about when the mean is an appropriate measure of center, and when it is not.

Warm-up Would You Rather?

Suppose you have been offered jobs by two different companies. The work is the same, and you do not know how much money you will be paid yet. The histograms show a random sample of the salaries of employees at each company. Would you rather work at Allthings, Inc. or Bigbox, Inc.? Explain your thinking.





Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Three Different Shows

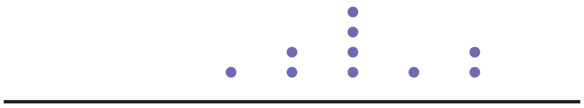
The streaming-media company, Webflicks, tracks all kinds of data about the people who watch their shows. The table shows the ages of a sample of 10 viewers for three different shows.

Sample	Ages of viewers (years)	Mean	MAD
Show 1	6, 6, 5, 4, 8, 5, 7, 8, 6, 6		
Show 2	15, 1, 12, 13, 12, 10, 12, 11, 10, 8		
Show 3	43, 60, 50, 36, 58, 50, 73, 59, 63, 51		

1. Calculate the mean and the MAD for each show and complete the table.

2. These dot plots display the data and the titles for the three shows, but are missing their scales. Match each dot plot with a show. Explain your thinking.

a **Show** ....



Learning to Read

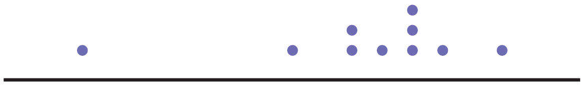
Explanation:

b **Show** ....

Cooking for Health

Explanation:

c **Show** ....



Science Experiments at Home!

Explanation:

Activity 2 Making a Recommendation

Let's continue to analyze the data for two more shows on Webflicks. The table shows the mean and MAD for these shows. The measurements are in years representing the ages of viewers.

Sample	Mean	MAD	Ages of viewers
Show 4	17.1	6.6	
Show 5	13.5	6.6	

1. Based on the mean and the MAD, which show would you recommend for a 13-year old? Explain your thinking.

2. You will be given the ages of viewers in the samples for each show. Record them in the table. Based on these ages, would you change your recommendation in Problem 1? Explain your thinking.

3. Study the means, the MADs, and the ages of viewers for Shows 4 and 5, compared to Shows 1 and 2 in Activity 1.

a What do you notice about the MADs for Shows 4 and 5 compared to Shows 1 and 2?

b What do you notice about the means and the ages of viewers for Shows 4 and 5, compared to Shows 1 and 2?

Are you ready for more?

Choose either Show 4 or Show 5. Write a different set of ages that gives approximately the same mean and MAD.

STOP

Name: ________________ Date: ________ Period: ________

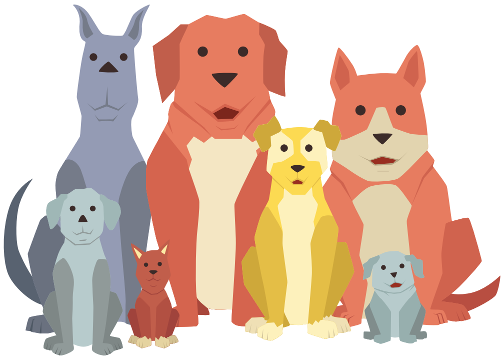
Summary

In today's lesson . . .

You discovered that when the mean absolute deviation (MAD) is lower, you are more likely to be able to predict that the mean of a sample is close to the population mean. Because the MAD is a measure of how spread out a set of data is, a lower MAD indicates that the data are closer together and near the mean.

For example, you can expect greater variability in the weights of dogs at a dog park than at a Corgi meetup.

Dogs at a dog park



Mean weight: 12.8 kg
MAD: 2.3 kg

Corgi meetup

Mean weight: 10.1 kg
MAD: 0.8 kg

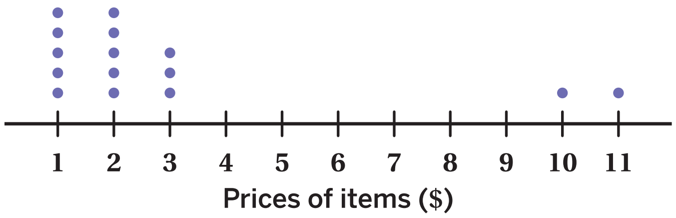
The lower MAD indicates there is less variability in the weights of the Corgis. This is because there is less variability in one type of dog. The mean weight of the Corgi meetup sample is likely to be close to the mean weight of all Corgis.

In general, a sample from a population with less variability is more likely to have a mean that is close to the population mean.

Reflect:

Name: .... Date: .... Period: ....

1. A random sample of 15 items were selected at a grocery store. The dot plot shows their prices. Is the mean of this sample likely to be representative of the population mean? Explain your thinking.



2. The teacher in Han's and Lin's class says that the students in class never bring a pen or a pencil. Han and Lin design a survey to ask, "*How many pens and pencils did you bring to class today?*" Out of 25 students, they each select a random sample of 5 students. The sample mean for Han's survey is 2.2. The sample mean for Lin's survey is 1.88.

a Does it surprise you that the two sample means are different from each other? Why or why not?

b If Han and Lin each determined the population means, would they be different from each other? Explain your thinking.

3. Clare and Priya each selected a random sample of 25 students at their school and designed a single-question survey. Their survey question and results are shown in the table.

Clare's survey	Priya's survey
How many hours do you spend doing homework each night?	*How many hours do you spend watching TV each night?*
Sample mean: 1.2 hours **Sample MAD:** 0.6 hours	**Sample mean:** 2 hours **Sample MAD:** 1.3 hours

Clare estimates the students at her school spend an average of 1.2 hours each night doing homework. Priya estimates the students at her school spend an average of 2 hours each night watching TV. Which estimate is likely to be closer to the actual mean for all the students at their school? Explain your thinking.

Name: ________________ Date: ________ Period: ________

4. *Capstone project helper.* Organize the data you collected in the previous lesson for your survey. Use a separate sheet of paper to display your collected data in a table or chart.

Important items to consider

- Your visual representation could be a dot plot, histogram, or table.
- Choose a display that you think best shows your collected data.

5. A high school plans to take all of its students to see a documentary on climate change at a large movie theater. The school has $1{,}325$ students. Each screen has enough seats for 250 students. How many screens are needed? Write and solve an inequality and explain what the solution means in context.

6. Han and Clare want to know how students in their classes travel to school.

a In Han's class, 3 out of 17 students take the bus to get to school. What fraction of the students take the bus?

b In Clare's class, $\frac{2}{5}$ of the students walk to school. If there are 35 students in Clare's class, how many students walk to school?

Unit 8 | Lesson 16

Estimating Population Proportions

Let's use samples to estimate population proportions.

Warm-up Getting to School

Your class will answer this question: ***How many minutes does it take you to travel to school?***

1. Record your classmates' responses in the table.

<table>
<tr><th colspan="10">Time to travel to school (minutes)</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
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</table>

2. What fraction of the students in your class reported that it takes:

   a Exactly 5 minutes to travel to school?

   b More than 25 minutes to travel to school?

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Travel Times

Suppose a group of concerned students met with the school's administration about the length of students' travel times in the morning. The students claimed the travel times are too long to start school at the designated time. The administration asked the group to survey a random sample of students how long it takes them to travel to school.

You will be given a set of cards. Each card shows the number of minutes it takes for one student to travel to school.

1. Work with your partner to select a random sample of 20 students' travel times, and record the travel times in the table.

Time to travel to school (minutes)									

2. What fraction of your sample has a travel time of less than 15 minutes?

3. Based on these results, estimate the number of all Grade 7 students at your school who have a travel time of less than 15 minutes.

4. Suppose another group in your class comes up with a different estimate for Problem 3.

- **a** What is another estimate that would be reasonable?
- **b** What is an estimate you would consider unreasonable?

Activity 2 A New Comic Book Hero

A survey asked a randomly selected group of 20 people who read *The Adventures of Super Sam* this question: *What superpower do you think a new superhero should have*? The table shows the survey results.

Response	Superpower	Response	Superpower
1	fly	11	freeze
2	freeze	12	freeze
3	freeze	13	fly
4	fly	14	invisibility
5	fly	15	freeze
6	freeze	16	fly
7	fly	17	freeze
8	super strength	18	fly
9	freeze	19	super strength
10	fly	20	freeze

1. What fraction of this sample thinks a new superhero should be able to fly?

2. If there are 2,024 dedicated readers of *The Adventures of Super Sam*, estimate the number of readers who would want the new superhero to be able to fly. Explain or show your thinking.

3. Based on the data from the survey, which superpower would you recommend the new superhero to have? Explain your thinking.

Critique and Correct: Your teacher will display an incorrect statement about this survey. Work with a partner to identify the error and correct the statement.

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You estimated the ***proportion*** of a population based on a sample.

For example, suppose you want to know how many of the 60 teachers at your school play video games in their spare time. You survey a randomly selected group of 20 teachers and record that 7 of them play video games in their spare time. You could make a prediction that $\frac{7}{20}$ of all the teachers in your school play video games in their spare time, as long as your sample data was collected without bias. Because $60 \cdot \frac{7}{20} = 21$, you can predict that about 21 teachers in your school play video games in their spare time.

Reflect:

Practice

Name: .... Date: .... Period: ....

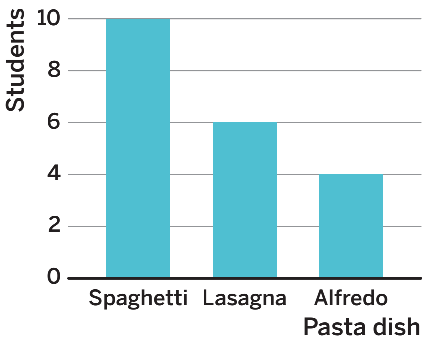
1. Tyler wonders what fraction of students at his school would dye their hair blue if they were permitted to do so. He surveyed a random sample of 10 students at his school, and 2 of them said they would. Kiran did not think Tyler's estimate was very accurate, so he surveyed a random sample of 100 students, and 17 of them said they would dye their hair blue.

a Based on Tyler's sample, estimate what fraction of the students would dye their hair blue.

b Based on Kiran's sample, estimate what fraction of the students would dye their hair blue.

c Whose estimate is likely more accurate? Explain your thinking.

2. There are 917 students at Han's school. He surveys a randomly selected group of students as to their favorite pasta dish served in the cafeteria. The bar graph shows the results. Based on these results, estimate the total number of the students in the school who would choose lasagna as their favorite pasta dish. Explain or show your thinking.



3. Elena wants to know the proportion of pet owners in her town that have cats. Describe a method she could use to estimate an answer to her question.

Name: ______________________ Date: __________ Period: __________

4. The science teacher and the Spanish teacher both give daily homework. The table shows the mean and MAD number of homework problems for a random sample of days. Which sample mean is more likely to represent the population mean for each class? Explain your thinking.

Science homework	Spanish homework
Sample mean: 5 problems	**Sample mean:** 10 problems
Sample MAD: 4 problems	**Sample MAD:** 1 problems

5. *Capstone project helper.* Write an elevator pitch for your statistical study. An *elevator pitch* is a short description of your study (no more than 30 words). The term *elevator pitch* comes from the idea that you might only have a short elevator ride to tell someone about why the work you are doing is special and valuable.

6. *Capstone project helper.* Analyze the center, spread, and shape of the data you have collected using the display you constructed in the previous lesson. Use these values to write an answer to your statistical question. Be sure to address the criteria shown in the table.

Criteria for your analysis
• Use at least one measure of center. • Use at least one measure of variability. • Describe the shape of the distribution. • Your answer to your statistical question should be at least 5 sentences.

Unit 8 | **Lesson 17 – Capstone**

Presentation of Findings

Let's share some new ideas, based on what we've learned during this unit.

Warm-up Title Your Presentation

Over the last few weeks, you worked on collecting and analyzing data related to a statistical question as you completed the *Capstone project helper* problems in the previous lessons. Today, you will put the finishing touches on your presentation and share your findings with your classmates.

Think of a title for your presentation. The table shows some important things to consider when selecting a title for a statistical study.

Important items to consider
• Informative (What is the study mostly about?) • Formal (Avoids contractions and other informal language) • Catchy (Helps to grab the reader's interest)

1. Brainstorm some titles for your study.

2. After discussing with a classmate, select the title you will use. Write it on your presentation and record it here.

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Gallery Tour, With Feedback

You will be given some sticky notes to use during the Gallery Tour.

During the Gallery Tour, use your sticky notes to leave feedback on your classmates' presentations. Each sticky note should include two comments:

+ One aspect you appreciated about the presentation.

? One aspect about which you are still curious.

Here is an example:

Sample Sticky Note

+ Using a dot plot made it straightforward to see the shape of the data that was collected.

? I'm curious about why people with newer phones have fewer apps.

Activity 2 Time to Reflect

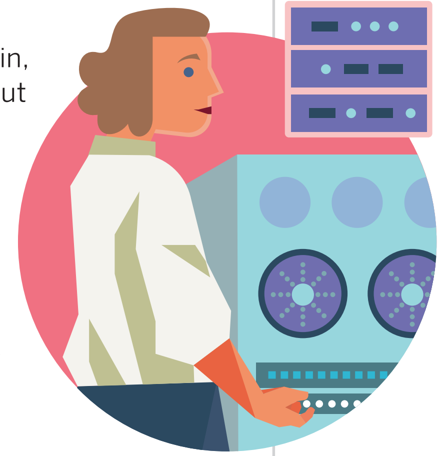
Read through the sticky notes for your presentation.

1. If you were to conduct your same study again, would you change anything? Explain your thinking. If there is nothing you would change, defend your reasoning.

2. What other statistical questions interest you? Include any questions that you may not even have the ability to study at this moment. In a few years, you may be able to get someone to pay you to research your question!

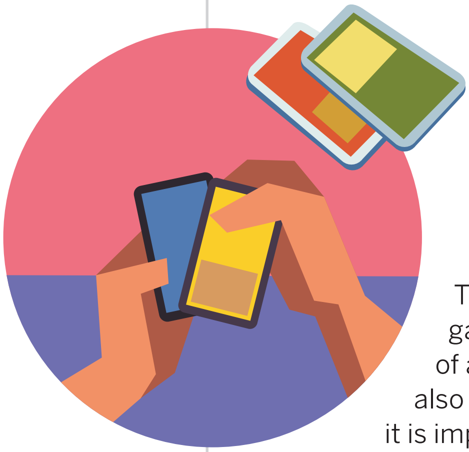
STOP

Unit Summary



Whether it's the roll of the dice or the flip of a coin, there is no way to know what the future holds. But just because we can't predict the future doesn't mean we are helpless against it.

For centuries, people have used their intuition to make educated guesses. These guesses help hedge our bets against uncertainty. It wasn't until Pascal and Fermat's groundbreaking work that we finally had a system for expressing that uncertainty mathematically.



We can express the likelihood of an event's outcome as a ratio. The first number is how many ways something can happen. The second is the total number of ways anything can happen.

These ratios allow us to make smart moves in games, crack secret codes, and manage the risk of a venture we might undertake. But probability also tells us something about large numbers. When it is impractical to collect information from an entire population, probability allows us to make inferences through a smaller sample.

Unlike horoscopes and magic eight-balls, probability makes no promises about the future. Instead, it gives us a way to understand our risk, so that we can make a well-informed choice in the face of uncertainty.

See you next year.

Practice

Name: .... Date: .... Period: ....

1. Noah will randomly select one letter from the word *FLUTE*. Lin will randomly select one letter from the word *CLARINET*. Which person is more likely to select the letter *E*? Explain your thinking.

2. A computer program simulates tossing a coin 100 times. This is considered one trial. The computer program conducts five trials, and then counts the longest string of the coin landing heads facing up in a row. The table shows the results. Based on these results, estimate the probability of landing heads facing up at least 15 times in a row when a coin is tossed 100 times.

Trial	Most number of heads facing up, in a row
1	8
2	6
3	5
4	11
5	13

3. Using each of the numbers 1 to 9 only once, complete the boxes to make the statement true.

The probability of flipping ☐ coin(s) and getting exactly ☐ tail(s) is $\frac{\square}{\square}$.

Name: .... Date: .... Period: ....

4. Write an equation, using at least two different operations, for which the value of x is 5.

5. An item at a store is discounted by 15%. Select *all* the equations representing the discounted price p for an item that originally costs c.

A. $p = 0.15c$

B. $p = 0.85c$

C. $p = 1 - 0.15c$

D. $p = c \div 0.15$

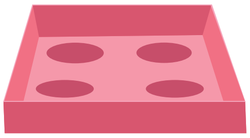
E. $p = c - 0.15c$

6. Andre wants to buy a small box of muffins from Pi Bakery. The bakery offers the muffin choices shown. How many *unique* combinations are possible if the bakery offers these 3 muffins and the small box fits 4 muffins in the box? Explain your thinking.

Lemon

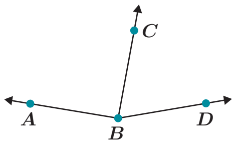
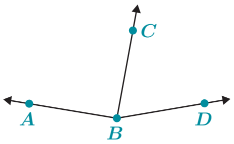
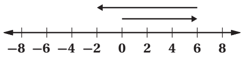
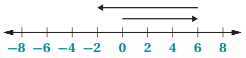
Blueberry

Banana Nut



Glossary/Glosario

A

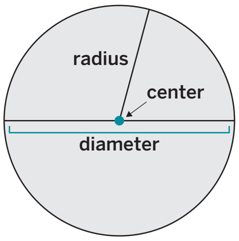
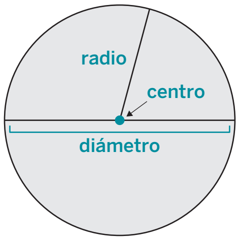
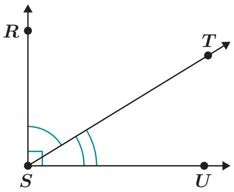
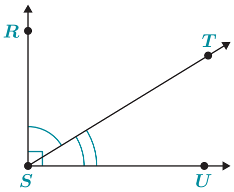
English	Español
absolute value The value that represents the distance between a number and zero. For example, because the distance between -3 and 0 is 3, the absolute value of -3 is 3, or $\|-3\| = 3$.	**valor absoluto** Valor que representa la distancia entre un número y cero. Por ejemplo, dado que la distancia entre -3 y 0 es 3, el valor absoluto de -3 es 3, o $\|-3\| = 3$.
Addition Property of Equality A property stating that, if $a = b$, then $a + c = b + c$.	**Propiedad de igualdad en la suma** Propiedad que establece que si $a = b$, entonces $a + c = b + c$.
additive inverse The additive inverse of a number a is the number that, when added to a, gives a sum of zero. It is the number's opposite.	**inverso aditivo** El inverso aditivo de un número a es el número que, cuando se suma a a, resulta en cero. Es el opuesto del número.
adjacent angles Angles that share a common side and vertex. For example, $\angle ABC$ and $\angle CBD$ are adjacent angles. 	**ángulos adyacentes** Ángulos que comparten un lado y un vértice. Por ejemplo, $\angle ABC$ y $\angle CBD$ son ángulos adyacentes. 
area The number of unit squares needed to fill a two-dimensional figure without gaps or overlaps.	**área** Número de unidades cuadradadas necesario para llenar una figura bidimensional sin dejar espacios vacíos ni superposiciones.
arrow diagram A model used in combination with a number line to show positive and negative numbers and operations on them. 	**diagrama de flechas** Modelo que se utiliza en combinación con una línea numérica para mostrar números positivos y negativos, y operaciones sobre estos. 
Associative Property of Addition A property stating that how addends are grouped does not change the result. For example, $(a + b) + c = a + (b + c)$.	**Propiedad asociativa de la suma** Propiedad que establece que la forma en que se agrupan los sumandos en una suma no cambia el resultado. Por ejemplo, $(a + b) + c = a + (b + c)$.
Associative Property of Multiplication A property stating that how factors are grouped in multiplication does not change the product. For example, $(a \bullet b) \bullet c = a \bullet (b \bullet c)$.	**Propiedad asociativa de la multiplicación** Propiedad que establece que la forma en que se agrupan los factores en una multiplicación no cambia el producto. Por ejemplo, $(a \bullet b) \bullet c = a \bullet (b \bullet c)$.

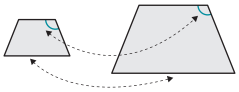
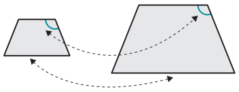
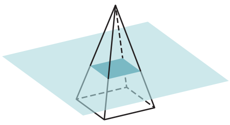
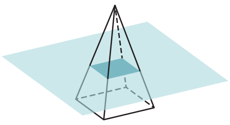
B

English	Español
balance The amount that represents the difference between positive and negative amounts of money in an account.	**balance** Cantidad que representa la diferencia entre cantidades positivas y negativas de dinero en una cuenta bancaria.
bar notation Notation that indicates the repeated part of a repeating decimal. For example, $0.\overline{6} = 0.66666\ldots$.	**notación de barras** Notación que indica la parte repetida de un número decimal periódico. Por ejemplo, $0.\overline{6} = 0.66666\ldots$.
base (of a prism) Either of the two identical, parallel faces of a prism that are connected by a set of rectangular faces.	**base (de un prisma)** Cualquiera de las dos caras idénticas y paralelas de un prisma que están conectadas por un conjunto de caras rectangulares.
base (of a pyramid) The face of a pyramid that is opposite from the vertex, where all the triangular faces connect.	**base (de una pirámide)** La cara de una pirámide que se opone al vértice, donde todas las caras triangulares se conectan.

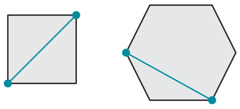
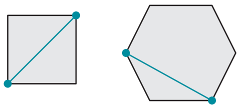
Glossary/Glosario

C

English	Español
center of a circle The point that is the same distance from all points on the circle.	**centro de un círculo** Punto que está a la misma distancia de todos los puntos del círculo.
certain A certain event is an event that is sure to happen. (The probability of the event happening is 1.)	**seguro** Un evento seguro es un evento que ocurrirá con certeza. (La probablidad de que el evento ocurra es 1.)
chance experiment An experiment that can be performed multiple times, in which the outcome may be different each time.	**experimento aleatorio** Experimento que puede ser llevado a cabo muchas veces, en cada una de las cuales el resultado será diferente.
circle A shape that is made up of all of the points that are the same distance from a given point. 	**círculo** Forma compuesta de todos los puntos que están a la misma distancia de un punto dado. 
circumference The distance around a circle.	**circunferencia** Distancia alrededor de un círculo.
coefficient A number that is multiplied by a variable, typically written in front of or "next to" the variable, often without a multiplication symbol.	**coeficiente** Número por el cual una variable es multiplicada, escrito comúnmente frente o junto a la variable.
commission A fee paid for services, usually as a percentage of the total cost.	**comisión** Pago realizado a cambio de algún servicio, usualmente como porcentaje del costo total.
common factor A number that divides evenly into each of two or more given numbers.	**factor común** Número que divide en partes iguales cada número de entre dos o más números dados.
commutative property Changing the order in which numbers are either added or multiplied does not change the value of the sum or product.	**propiedad conmutativa** Cambiar el orden de los operandos en una suma o multiplicación no cambia el valor final de la suma o el producto.
complementary angles Two angles whose measures add up to 90 degrees. For example, $\angle RST$ and $\angle TSU$ are complementary angles. 	**ángulos complementarios** Dos ángulos cuyas medidas suman 90 grados. Por ejemplo, $\angle RST$ y $\angle TSU$ son ángulos complementarios. 
constant of proportionality The number in a proportional relationship by which the value of one quantity is multiplied to get the value of the other quantity.	**constante de proporcionalidad** En una relación proporcional, el número por el cual el valor de una cantidad es multiplicado para obtener el valor de otra cantidad.
coordinate plane A two-dimensional plane that represents all the ordered pairs (x, y), where x and y can both represent values that are positive, negative, or zero.	**plano de coordenadas** Plano bidimensional que representa todos los pares ordenados (x, y), donde tanto x como y pueden representar valores positivos, negativos o cero.

English	Español
corresponding parts Parts of two scaled copies that match up, or "correspond" with each other. These corresponding parts could be points, segments, angles, or lengths. 	**partes correspondientes** Partes de dos copias a escala que coinciden, o "se corresponden" entre sí. Estas partes correspondientes pueden ser puntos, segmentos, ángulos o longitudes. 
cross section A cross section is the new face seen when slicing through a three-dimensional figure. For example, a rectangular pyramid that is sliced parallel to the base has a smaller rectangle as the cross section. 	**corte transversal** Un corte transversal es la nueva cara que aparece cuando una figura tridimensional es rebanada. Por ejemplo, una pirámide rectangular que es rebanada en forma paralela a la base tiene un rectángulo más pequeño como corte transversal. 

D

debt Amount of money that has been borrowed and owed to the person or bank from which it was borrowed.	**deuda** Cantidad de dinero que ha sido pedida prestada y se le debe a la persona o al banco que la prestó.
deposit Money put into an account.	**depósito** Dinero colocado en una cuenta.
diagonal A line segment connecting two vertices on different sides of a polygon. The *diagonal* of a square connects opposite vertices. 	**diagonal** Segmento de una línea que conecta dos vértices en lados diferentes de un polígono. La *diagonal* de un cuadrado conecta vértices opuestos. 
diameter The distance across a circle through its center. The line segment with endpoints on the circle, that passes through its center. (See also *circle*.)	**diámetro** Distancia a través de un círculo que atraviesa su centro. Segmento de línea cuyos extremos limitan con el círculo y que pasa por su centro. (Ver también *círculo*.)
discount A reduction in the price of an item, typically due to a sale.	**descuento** Reducción del precio de un artículo, usualmente debido a una venta de rebaja.
Distributive Property A property that states the product of a number and a sum of numbers is equal to the sum of two products: $a\,(b + c) = ab + ac$.	**Propiedad distributiva** Propiedad que establece que el producto de un número y una suma de números es igual a la suma de dos productos: $a\,(b + c) = ab + ac$.

E

equally likely as not An event that has equal chances of occurring and not occurring. (The probability of the event happening is exactly $\frac{1}{2}$.)	**tan probable como improbable** Evento que tiene las mismas posibilidades de ocurrir que de no ocurrir. (La probabilidad de que ocurra es exactamente $\frac{1}{2}$.)
equation Two expressions with an equal sign between them. When the two expressions are equal, the equation is true. An equation can also be false, when the values of the two expressions are not equal.	**ecuación** Dos expresiones con un signo igual entre sí. Cuando las dos expresiones son iguales, la ecuación es verdadera. Una ecuación también puede ser falsa, cuando los valores de las dos expresiones no son iguales.

Glossary/Glosario

English	Español
equivalent equations Equations that have the same solution.	**ecuaciones equivalentes** Ecuaciones que tienen la misma solución.
equivalent expressions Two expressions whose values are equal when the same value is substituted into the variable for each expression.	**expresiones equivalentes** Dos expresiones cuyos valores son iguales cuando se sustituye el mismo valor en la variable de cada expresión.
equivalent ratios Any two ratios in which the values for one quantity in each ratio can be multiplied or divided by the same number to get the values for the other quantity in each ratio.	**razones equivalentes** Dos razones entre las cuales los valores de una cantidad en cada razón pueden ser multiplicados o divididos por el mismo número para obtener así los valores de la otra cantidad en cada razón.
equivalent scales Different scales (relating scaled and actual measurements) that have the same scale factor.	**escalas equivalentes** Diferentes escalas (que relacionan medidas a escala y reales) que tienen el mismo factor de escala.
error interval A range of values above and below an exact value, expressed as a percentage.	**intervalo de error** Rango de valores por sobre y por debajo de un valor exacto, expresado como porcentaje.
event A set of one or more outcomes in a chance experiment.	**evento** Conjunto de uno o más resultados de un experimento aleatorio.
expand To expand an expression means to use the Distributive Property to rewrite a product as a sum. The new expression is equivalent to the original expression.	**expandir** Expandir una expresión significa usar la Propiedad distributiva para volver a escribir un producto como una suma. La nueva expresión es equivalente a la expresión original.

F

factor To factor an expression means to use the Distributive Property to rewrite a sum as a product. The new expression is equivalent to the original expression.	**factorizar** Factorizar una expresión significa usar la Propiedad distributiva para volver a escribir una suma como un producto. La nueva expresión es equivalente a la expresión original.

G

gratuity See the definition for *tip*.	**gratificación** Ver *propina*.
greater than or equal to $x \geq a$, x is greater than a or x is equal to a.	**mayor o igual a** $x \geq a$, x es mayor que a o x es igual a a.

H

hanger diagram A model in which quantities are represented as weights attached to either side of a hanger. When the hanger is balanced, the sum of the quantities on either side must be equal.	**diagrama de colgador** Modelo en el cual ciertas cantidades son representadas como pesos sujetos a cada lado de un colgador. Cuando el colgador está en equilibrio, la suma de las cantidades en cualquiera de los lados debe ser igual.

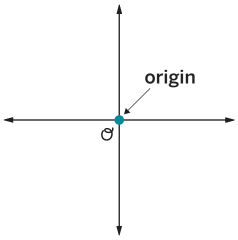
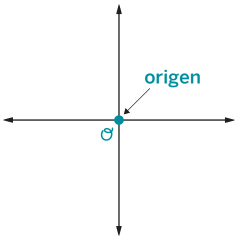
English	Español
I	
impossible An impossible event is an event that has no chance of occurring. The probability of the event happening is 0.	**imposible** Un evento imposible es un evento que no tiene posibilidad de que ocurra. La probabilidad de que ocurra es 0.
inequality A statement relating two numbers or expressions that are not equal. The phrases *less than*, *less than or equal to*, *greater than*, and *greater than* or *equal to* describe inequalities.	**desigualdad** Enunciado que relaciona dos números o expresiones que no son iguales. Las expresiones “menor que”, “menor o igual a”, “mayor que” o “mayor o igual a” describen desigualdades.
integers Whole numbers and their opposites.	**enteros** Números completos y sus opuestos.
inverse operations Operations that “undo” each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.	**operaciones inversas** Operaciones que se cancelan entre sí. La suma y la resta son operaciones inversas. La multiplicación y la división son operaciones inversas.
L	
less than or equal to $x \leq a$, x is less than a or x is equal to a.	**menor o igual a** $x \leq a$, x es menor que a o x es igual a a.
like terms Terms in an expression that have the same variables and can be combined, such as $7x$ and $9x$.	**términos semejantes** Partes de una expresión que tiene la misma variable y que pueden ser sumadas, tales como $7x$ and $9x$.
likely A likely event is an event that has a greater chance of occurring than not occurring. (The probability of happening is more than $\frac{1}{2}$.)	**probable** Un evento probable es un evento que tiene más posibilidad de ocurrir que de no ocurrir. (La probabilidad de que ocurra es mayor que $\frac{1}{2}$.)
long division A method that shows the steps for dividing base ten whole numbers and decimals, dividing one digit at a time, from left to right. $\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\;4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$	**división larga** Método que muestra los pasos necesarios para dividir números enteros en base diez y decimales, por medio de la división de un dígito a la vez, de izquierda a derecha. $\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\;4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$
M	
magnitude The absolute value of a number, or the distance of a number from 0 on the number line.	**magnitud** Valor absoluto de un número, o distancia de un número con respecto a 0 en la línea numérica.
markdown An amount, expressed as a percentage, subtracted from the cost of an item.	**descuento** Monto, expresado como porcentaje, que se resta al costo de un producto.
markup An amount, expressed as a percentage, added to the cost of an item.	**sobreprecio** Monto, expresado como porcentaje, que se agrega al costo de un producto.
multi-step event When an experiment consists of two or more events, it is called a multi-step event.	**evento de varios pasos** Cuando un experimento consiste en dos o más eventos, es llamado un evento de varios pasos.
multiplicative inverse Another name for the reciprocal of a number; The multiplicative inverse of a number a is the number that, when multiplied by a, gives a product of 1.	**inverso multiplicativo** Otro nombre para el recíproco de un número. El inverso multiplicativo de un número a es el número que, cuando se multiplica por a, tiene como producto 1.

Glossary/Glosario

N

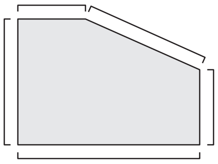
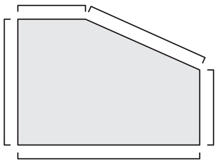
English	Español
negative numbers Numbers whose values are less than zero.	**números negativos** Números cuyos valores son menores que cero.
nonproportional relationship A relationship between two quantities in which the corresponding values do not have a constant ratio. (In other words, a relationship that is *not* a proportional relationship.)	**relación no proporcional** Relación entre dos cantidades, en la cual los valores correspondientes no tienen una razón constante. (En otras palabras, una relación que *no* es una relación proporcional.)

O

English	Español
opposites Two numbers that are the same distance from 0, but are on different sides of the number line.	**opuestos** Dos números que están a la misma distancia de 0, pero que están en lados diferentes de la línea numérica.
order of operations When an expression has multiple operations, they are applied in a consistent order (the "order of operations") so that the expression is evaluated the same way by everyone.	**orden de las operaciones** Cuando una expresión contiene múltiples operaciones, estas se aplican en cierto orden consistente (el "orden de las operaciones") de forma que la expresión sea evaluada de la misma manera por todas las personas.
ordered pair Two values, written as (x, y), that represent a point on the coordinate plane.	**par ordenado** Dos valores, escritos como (x, y), que representan un punto en el plano de coordenadas.
origin The point represented by the ordered pair $(0, 0)$ on the coordinate plane. The *origin* is where the x- and y-axes intersect.  	**origen** Punto representado por el par ordenado $(0, 0)$ en el plano de coordenadas. El *origen* es donde los ejes x y y se intersecan. 
outcome One of the possible results that can happen when an experiment is performed. For example, the possible outcomes of tossing a coin are heads and tails.	**resultado** El resultado de un experimento aleatorio es una de las cosas que pueden ocurrir cuando se realiza el experimento. Por ejemplo, los posibles resultados de tirar una moneda al aire son cara o cruz.

P

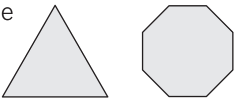
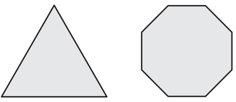
English	Español
percent change How much a quantity changed (increased or decreased), expressed as a percentage of the original amount.	**cambio porcentual** Cuánto ha cambiado una cantidad (aumentado o disminuido), expresado en un porcentaje del monto original.
percent decrease The amount a value has gone down, expressed as a percentage of the original amount.	**disminución porcentual** Cantidad en que un valor ha disminuido, expresada como porcentaje del monto original.
percent error The difference between approximate and exact values, as a percentage of the exact value.	**error porcentual** Diferencia entre valores aproximados y valores exactos, expresada como porcentaje del valor exacto.

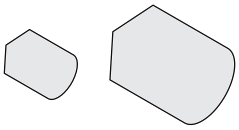
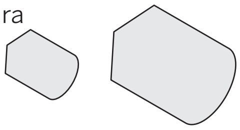
English	Español
percent increase The amount a value has gone up, expressed as a percentage of the original amount.	**aumento porcentual** Monto en que un valor ha incrementado, expresado como porcentaje del monto original.
percentage A rate per 100. (A specific *percentage* is also called a *percent*, such as "70 percent.")	**porcentaje** Tasa por cada 100. (Un *porcentaje* específico también es llamado *por ciento*, como por ejemplo "70 por ciento.")
perimeter The total distance around the sides of a two-dimensional figure. 	**perímetro** Distancia total alrededor de los lados de una forma bidimensional. 
pi, or π The ratio between the circumference and the diameter of a circle.	**pi, o π** Razón entre la circunferencia y el diámetro de un círculo.
polygon A closed, two-dimensional shape with straight sides that do not cross each other.	**porcentaje** Tasa por cada 100. (Un porcentaje específico también es llamado "por ciento", como por ejemplo "70 por ciento".)
population A set of people or objects that are to be studied. For example, if the heights of people on different sports teams are studied, the population would be all the people on the teams.	**población** Una población es un conjunto de personas o cosas por estudiar. Por ejemplo, si se estudia la altura de las personas en diferentes equipos deportivos, la población constaría de todas las personas que conforman los equipos.
population proportion A number in statistics, between 0 and 1 that represents the fraction of the data that fits into the desired category.	**proporción de la población** En estadística, número entre 0 y 1 que representa la fracción de los datos que cabe en la categoría deseada.
positive numbers Numbers whose values are greater than zero.	**números positivos** Números cuyos valores son mayores que cero.
prism A three-dimensional figure with two parallel, identical faces (called *bases*) that are connected by a set of rectangular faces.	**prisma** Forma tridimensional con dos caras iguales y paralelas (llamadas *bases*) que se conectan entre sí a través de un conjunto de caras rectangulares.
probability The ratio of the number of favorable outcomes to the total possible number of outcomes. A probability of 1 means the event will always happen. A probability of 0 means the event will never happen.	**probabilidad** La razón entre el número de resultados favorables y el número total posible de resultados. Una probabilidad de 1 significa que el evento siempre ocurrirá. Una probabilidad de 0 significa que el evento nunca va a ocurrir.
profit The amount of money earned, minus expenses.	**ganancia** Monto del dinero obtenido, menos los gastos.
properties of equality Rules that apply to all equations. These include properties of addition, subtraction, multiplication, and division, which state that, if an equation is true, then applying the same operation to both sides will give a new equation that is also true.	**propiedades de igualdad** Reglas que son aplicables a todas las ecuaciones. Incluyen las propiedades de suma, resta, multiplicación y división, las cuales señalan que si una ecuación es verdadera, al aplicar la misma operación a ambos lados se obtendrá una nueva ecuación que también es verdadera.

Glossary/Glosario

English	Español
proportional relationship A relationship in which the values for one quantity are each multiplied by the same number (the *constant of proprtionality*) to get the values for the other quantity.	**relación proporcional** Relación en la que los valores de una cantidad se multiplican cada uno por el mismo número (la *constante de proporcionalidad*) para encontrar los valores de la otra cantidad.
pyramid A three-dimensional figure with one base and a set of triangular faces that meet at a singular vertex.	**pirámide** Forma tridimensional con una base y un conjunto de caras triangulares que se intersecan en un solo vértice.

R

English	Español
radius A line segment that connects the center of a circle with a point on the circle. The term *radius* can also refer to the length of this segment. (See also *circle*.)	**radio** Segmento de una línea que conecta el centro de un círculo con un punto del círculo. *Radio* también puede referirse a la longitud de este segmento. (Ver también *círculo*.)
random sample A sample that has an equal chance of being selected from the population as any sample of the same size	**muestra al azar** Muestra que tiene la misma posibilidad de ser seleccionada de entre la población que cualquier otra muestra del mismo tamaño.
rate A comparison of how two quantities change together.	**tasa** Comparación de cuánto cambian dos cantidades en conjunto.
ratio A comparison of two quantities by multiplication or division.	**razón** Comparación de dos cantidades a través de la multiplicación o la división.
rational numbers The set of all numbers, positive and negative, that can be written as fractions. For example, any whole number is a rational number.	**números racionales** Conjunto de todos los números positivos y negativos que pueden ser escritos como fracciones. Por ejemplo, todo número entero es un número racional.
reciprocal Two numbers whose product is 1 are *reciprocals* of each other. (For example, $\frac{3}{5}$ and $\frac{5}{3}$ are reciprocals.)	**recíproco/a** Dos números cuyo producto es 1 son *recíprocos* entre sí. (Por ejemplo, $\frac{3}{5}$ y $\frac{5}{3}$ son recíprocos.)
regular polygon A polygon whose sides all have the same length and whose angles all have the same measure. 	**polígono regular** Polígono cuyos lados tienen todos la misma longitud y cuyos ángulos tienen todos la misma medida. 
relative frequency The relative frequency is the ratio of the number of times an outcome occurs in a set of data. The relative frequency can be written as a fraction, a decimal, or a percentage.	**frecuencia relativa** La frecuencia relativa es la razón del número de veces que ocurre un resultado en un conjunto de datos. Se puede escribir como una fracción, un decimal o un porcentaje.
repeating decimal A decimal in which there is a sequence of non-zero digits that repeat indefinitely.	**número decimal periódico** Decimal que tiene una secuencia de dígitos distintos de cero que se repite de manera indefinida.
representative sample A sample is representative of a population if its distribution resembles the population's distribution in center, shape, and spread.	**muestra representativa** Una muestra es representativa de una población si su distribución asemeja la distribución de la población en centro, forma y extensión.

English	Español
retail price The price a store typically charges for an item	**precio de venta al público** Precio que una tienda comercial usualmente cobra por un producto.
right angle An angle whose measure is 90 degrees.	**ángulo recto** Ángulo cuya medida es de 90 grados.
S	
sales tax An additional cost, as a rate to the cost of certain goods and services, applied by the government.	**impuesto de venta** Costo adicional, como una tasa del costo de ciertos bienes y servicios, aplicado por el gobierno.
sample Part of a population. For example, a population could be all the seventh graders at one school. One sample of that population is all the seventh graders who are in band.	**interés simple** Monto de dinero que se agrega a un monto original, usualmente pagado al titular o a la titular de una cuenta bancaria de ahorros.
sample space A list of every possible outcome for a chance experiment.	**espacio de muestra** Lista de cada resultado posible de un experimento aleatorio.
scale A ratio, sometimes shown as a segment, that indicates how the measurements in a scale drawing represent the actual measurements of the object shown.	**escala** Razón, a veces mostrada como segmento, que indica de qué forma las medidas de un dibujo a escala representan las verdaderas medidas del objeto mostrado.
scale drawing A drawing that represents an actual place, object, or person. All of the measurements in the scale drawing correspond to the measurements of the actual object by the same scale.	**dibujo a escala** Dibujo que representa un lugar, objeto o persona real. Todas las medidas en el dibujo a escala corresponden en la misma escala a las medidas del objeto real.
scale factor The value that side lengths are multiplied by to produce a certain scaled copy.	**factor de escala** Valor por el cual las longitudes de cada lado se multiplican para producir cierta copia a escala.
scaled copy A copy of a figure where every length in the original figure is multiplied by the same value to produce corresponding lengths in the copy. 	**copia a escala** Copia de una figura donde cada longitud de la figura original es multiplicada por el mismo valor, para producir longitudes correspondientes en la copia. 
simple interest An amount of money that is added on to an original amount, usually paid to the holder of a bank savings account.	**muestra** Una muestra es una parte de la población. Por ejemplo, una población podría ser todos/as los/as estudiantes de séptimo grado en una escuela. Una muestra de esa población son todos/as los/as estudiantes de séptimo grado que están en una banda.
simulation An experiment that is used to estimate the probability of a real-world event.	**simulación** Un experimento que es utilizado para estimar la probabilidad de un evento en el mundo real.
solution to an equation A value that will make an equation true when substituted into the equation.	**solución a una ecuación** Número que puede sustituir una variable para volver verdadera una ecuación.
solution to an inequality A value that will make an inequality a true statement when substituted into the inequality.	**solución a una desigualdad** Cualquier número que puede sustituir una variable para volver verdadera una desigualdad.
straight angle An angle whose measure is 180 degrees. For example, $\angle EFH$ is a straight angle. 	**ángulo llano** Ángulo cuya medida es de 180 grados. Por ejemplo, $\angle EFH$ es un ángulo llano. 

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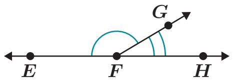
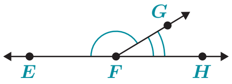
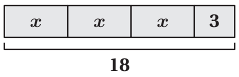
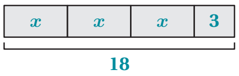
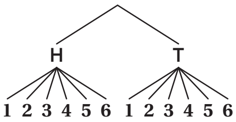
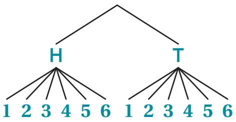
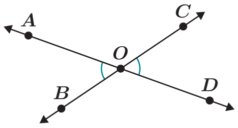
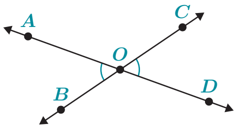
English	Español
supplementary angles Two angles whose measures add up to 180 degrees. For example, $\angle EFG$ and $\angle GFH$ are supplementary angles. 	**ángulos suplementarios** Dos ángulos cuyas medidas suman 180 grados. Por ejemplo, $\angle EFG$ y $\angle GFH$ son ángulos suplementarios. 
surface area The number of unit squares needed to cover all of the faces of a three-dimensional figure without gaps or overlaps.	**área de superficie** Número de unidades cuadradas necesarias para cubrir todas las caras de una figura tridimensional sin dejar espacios vacíos ni superposiciones.
T	
tape diagram A model in which quantities are represented as lengths (of tape) placed end-to-end, and which can be used to show addition, subtraction, multiplication, and division. 	**diagrama de cinta** Modelo en el cual las cantidades están representadas como longitudes (de cinta) colocadas de forma continua, y que pueden ser usadas para mostrar suma, resta, multiplicación y división. 
term A term is a part of an expression. It can be a number, a variable, or a product of a number and a variable.	**término** Un término es una parte de una expresión. Puede ser un número individual, una variable o el producto de un número y una variable.
terminating decimal A decimal that ends at a specific place value.	**decimal exacto** Un decimal que termina en un valor posicional específico.
tip An amount given to a server at a restaurant (or other service provider) that is calculated as a percentage of the bill.	**propina** Cantidad dada a un mesero o mesera en un restaurante (o a una persona que presta cualquier otro servicio) que se calcula como porcentaje de la cuenta.
tree diagram A diagram that represent all the possible outcomes in an experiment. 	**diagrama de árbol** Diagrama que representa todos los resultados posibles. 
U	
unit rate How much one quantity changes when the other changes by 1.	**tasa unitaria** Cuánto cambia cierta cantidad cuando la otra cambia por un factor de 1.
unlikely An unlikely event is an event that has small chance of occurring. (The probablity of the event happening is less than $\frac{1}{2}$.)	**improbable** Un evento improbable es un evento que tiene pocas posibilidades de ocurrir. (La probabilidad de que ocurra es menor que $\frac{1}{2}$.)

English	Español
V	
variable A letter that represents an unknown number in an expression or equation.	**variable** Letra que representa un número desconocido en una expresión o ecuación.
velocity A quantity that represents the speed and the direction of motion. In general, speed, like distance, is always positive, but velocity can be either positive or negative.	**velocidad** Cantidad que representa la rapidez y la dirección de un movimiento. En general, la rapidez, como la distancia, es siempre positiva, pero la velocidad puede ser tanto positiva como negativa.
vertical angles Opposite angles that share the same vertex. They are formed by a pair of intersecting lines. Their angle measures are equal. For example, $\angle AOB$ and $\angle COD$ are vertical angles. 	**ángulos verticales** Ángulos opuestos que comparten el mismo vértice. Están compuestos de un par de líneas que se intersecan. Sus medidas de ángulo son iguales. Por ejemplo, $\angle AOB$ y $\angle COD$ son ángulos verticales. 
volume The number of unit cubes needed to fill a three-dimensional figure without gaps or overlaps.	**volumen** Número de unidades cúbicas necesario para llenar una figura tridimensional sin dejar espacios vacíos ni superposiciones.
W	
withdrawal Money taken out of an account.	**retiro** Dinero que es extraído de una cuenta.

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