Amplify Math

Grade 8

Volume 2: Units 5–8

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Hello, curious mind!

Welcome to Grade 8. The math you'll see is getting a little more formal . . . but it will be no less interesting.

You see, this year, you'll test some tessellations, make a piece of cardboard come alive, spot which work of art is a forgery, and even see how you can use a triangle to measure the shape of the Universe. And that's just Unit 1!

We promise you, it's all possible. This year, you'll see not only that math makes sense, but it can be fun, too.

Before you dig in, we want you to know two things:

This book is special! It's where you'll record your thoughts, strategies, explanations, and, occasionally, any award-winning doodles that might come to mind.

When you go online, you won't be mindlessly plugging numbers into your device . . . You'll be pushing, pulling, crawling, teleporting, melting . . . , well, let's just say you'll be doing a lot of things, and you'll be teaming up with your classmates as you do.

Let's dig in!

Sincerely,
The Amplify Math Team

Volume 1

Unit 1 Rigid Transformations and Congruence

Unit Narrative:
The Art of Transformation

Shapes are all around us. We find them in art, architecture, and even in animated dancing frogs. In this unit, you will find out what happens when you slide, flip, and turn figures of all shapes and sizes. Plus, you may even create a masterpiece artwork along the way.

How do you make a piece of cardboard come alive?
Pack your geometry toolkits for a transformational journey into the movement of figures.

How can a crack make a piece of art priceless?
Something special happens when you perform rigid transformations on a figure.

What's got 10 billion galaxies and goes great with maple syrup?
Construct a triangle from a straight angle and cut two parallel lines to see what angle relationships you notice.

Unit 2 Dilations and Similarity

The way our brain interprets how objects appear — how big or small they are, how near or far — comes back to dilation. Learn to dilate figures and uncover the magic of this special type of transformation.

Unit Narrative:
More than Meets the Eye

Would you put poison in your eye?
Shrink and stretch objects on and off the plane and study the characteristics of the figures you dilate.

Do you really get what you pay for?
Learn how some companies use dilations to create similar, and slightly smaller, sized packaging, in a process called "shrinkflation."

Unit 3 Linear Relationships

How many cups tall is your teacher? Find out in this unit as you make connections between proportional and linear relationships.

Unit Narrative:
A Straight Change

How fast is a geography teacher?
On your mark, get set, go! Use your understanding of slope to show how a geography teacher shocked the world with her record-setting speed.

How did a coal mine help build America's most famous amusement park?
Use linear relationships to collect as many coins as you can at Honest Carl's Funtime World amusement park.

How did a 16-year-old take down a Chicago Bull?
Create equations from linear relationships and find how a 16-year-old was able to beat Michael Jordan in a game of basketball.

Unit 4 Linear Equations and Systems of Linear Equations

In this unit, you will learn the algebraic skills you need to find a solution for x. But wait, what do you mean there is no solution?

Unit Narrative:
The Path the Mind Takes

Who was the Father of Algebra?
When traders in 9th century Baghdad needed a better system for solving problems, a mathematician developed a new method he called "al-jabr" or algebra.

How is anesthesia like buying live lobsters?
Now that you have practiced solving equations, take a closer look at how you can use linear equations to solve everyday problems.

Volume 2

Unit 5 Functions and Volume

By studying functional relationships in this unit, you will soon be able to explain how height affects the volume of a sphere, calculate how the hare outran the tortoise, and produce your own version of the Happy Birthday song using a graph.

Unit Narrative: Pumping up the Volume on Functions

Who has the better camera: you or your grandparents?

Learn how functions can help you tell stories.

Who invented the waffle cone?

Use your prior knowledge about finding the volume of rectangular prisms to derive formulas for finding the volumes of cylinders, cones, and spheres.

Unit 6 Exponents and Scientific Notation

Unit Narrative:
From Teeny-Tiny to Downright Titanic

Imagine the *smallest* number you can think of. Now imagine the *largest* number you can think of. How can you write these numbers? How can you work with these numbers? In this unit, you'll learn about the power of exponents (pun intended), and how you can use them to work with numbers as small as the mass of a bacterium or as large as the number of atoms in the Universe.

How many carbs are in a game of chess?
You probably already know a thing or two about exponents, but what happens when you multiply or divide expressions with exponents?

Who should we call when we run out of numbers?
You'll work with numbers that are super small and incredibly large. But you won't waste your time writing pesky zeros!

Unit 7 Irrationals and the Pythagorean Theorem

Discover how three squares can prove something radical about triangles that has captivated mathematicians for centuries.

Unit Narrative:
The Mystery of the Pythagoreans

How rational were the Pythagoreans?
Find out if every number can be represented by a fraction.

What do the President of the United States and Albert Einstein have in common?
Uncover a special property of right triangles when you explore one of the nearly 500 proofs of the Pythagorean Theorem.

Unit 8 Associations in Data

Data literacy — being able to tell and interpret stories using data — is one of the most important skills you will ever need. In this unit, you will make sense of data in the world around you, represented in different forms. By the end of the unit, you will put your new data literacy skills to the test by examining the accuracy of newspaper headlines.

Unit Narrative:
Data and the Ozone Layer

Who is the biggest mover and shaker in the Antarctic Ocean?

Explore the ozone hole using scatter plots, while learning about the different kinds of associations data can have.

UNIT 5

Functions and Volume

By studying functional relationships in this unit, you will soon be able to explain how height affects the volume of a sphere, calculate how the hare outran the tortoise, and produce your own version of the *Happy Birthday* song using a graph.

Essential Questions

- What makes a relationship a function?
- How can you compare multiple representations of linear functions to determine which is changing at a faster rate, or which is slower?
- How are the volumes of a cylinder, cone, and sphere related if their dimensions are the same?
- *(By the way, can you create your own music just by manipulating a graph?)*

SUB-UNIT

1 Representing and Interpreting Functions

Narrative: Similar to a function, a camera takes an input — light — and provides an output — a photo.

You'll learn . . .

- what it means for a relationship to be a function.
- how real-world data can be modeled with linear functions.

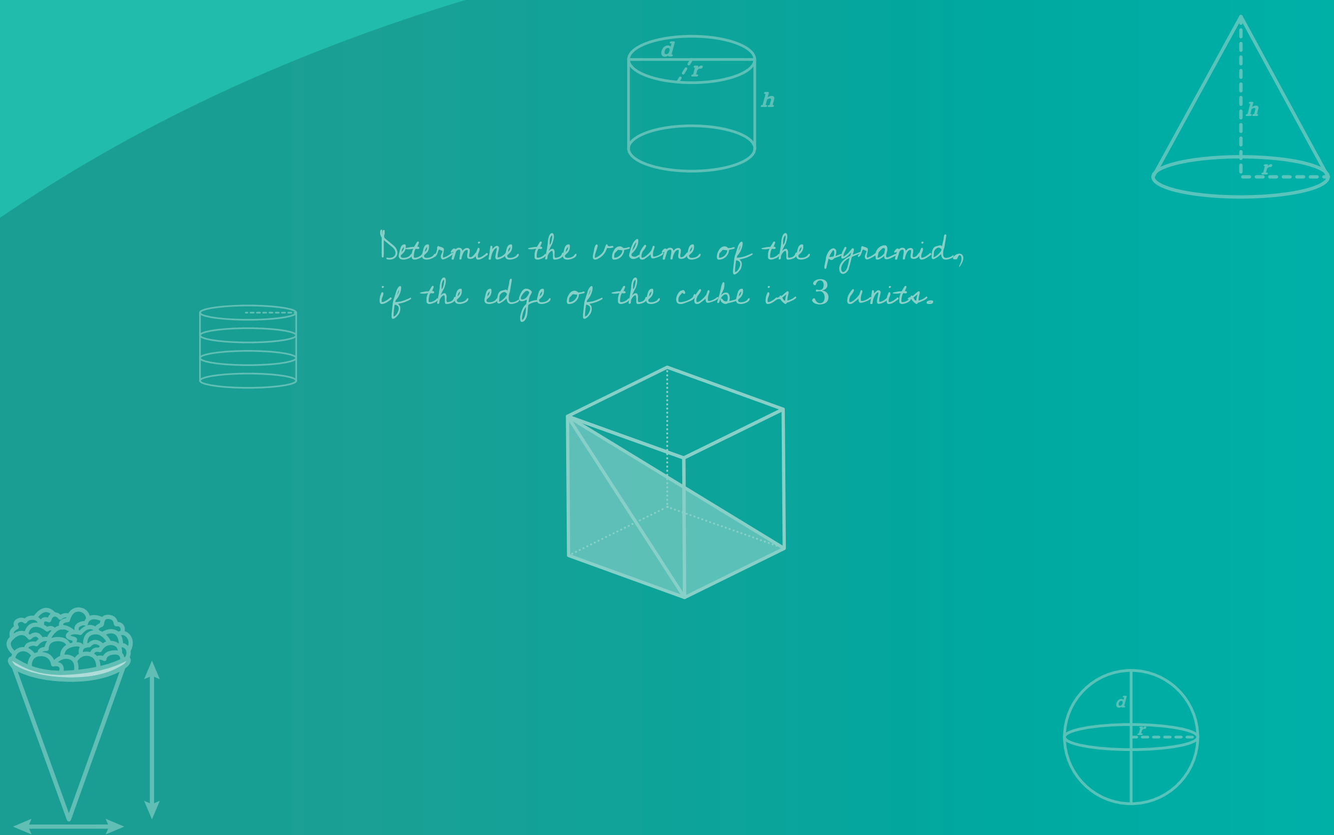
SUB-UNIT

2 Cylinders, Cones, and Spheres

Narrative: Discover how much ice cream a cone can hold.

You'll learn . . .

- how volume can be considered a function.
- how to solve problems involving the volume of a cylinder, cone, or sphere.

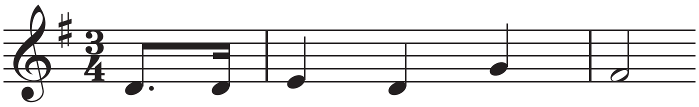


Pick a Pitch

Let's make connections between music and math.

Warm-up Notice and Wonder

You will be given an audio sample of the song, *Happy Birthday*. The audio sample can be represented in written form by the piece of sheet music shown. What do you notice? What do you wonder?



Patty and Mildred J. Hill, "Happy Birthday". Accessed 4/14/21. Riffspot.com

1. I notice . . .

2. I wonder . . .

Co-craft Questions: Share what you notice and wonder with a partner. Together, write 2–3 questions you have about how math might be related to music.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Pitch Perfect

You will use the same audio sample from the Warm-up and will access a graph that shows the pitch of the audio over time.

Hint: *Pitch* is the quality of a sound that makes it possible to judge sounds (notes) as "higher" and "lower." Use this term to help describe what you hear and see.

1. Are there any patterns that you notice between the audio sample and the graph?

2. What do you think the axes labels might be?

3. How many notes are being played at a given time? How can you tell?

4. What similarities do you notice between this graph and the sheet music you saw in the Warm-up?

Activity 2 Produce Your Own Track

Suppose you are a music producer as you access an audio sample that could use some editing. Use the audio sample and digital graph for the following problem.

Adjust the graph to make the audio sample match the original song as closely as possible. After you are finished, describe your process.

Name: ____________ Date: ________ Period: ________

Activity 3 Digital DJ

Some musicians, such as the composer Philip Glass, have used math to help them create music.

You will be given access to several audio samples in the Amps Studio. Use the graph to create music of your own.

1. How would you describe the music you created using the graph?

Featured Mathematician

Philip Glass

"1, 2, 3, 4, 1, 2, 3, 4, 5, 6 . . ." and so begins the opera, *Einstein on the Beach*, written by American composer Philip Glass (1937–present). While the subject of the opera is related to math, perhaps what is most notable about the work is the math behind the music. Glass, who studied math in college, composed the music for the opera using mathematical features, such as repetition, symmetry, and arithmetic patterns. Glass has composed many other pieces of music using similar methods, including an opera called *Kepler*, inspired by the life of Renaissance astronomer Johannes Kepler.

STOP

Unit 5 Functions and Volume

Pumping up the Volume on Functions

When a song is in tune, somehow we just know it — no analysis necessary.

But what makes a song sound "in tune" or "out of tune?" Most of the music we listen to is built using musical scales, which themselves follow mathematical patterns. Over the years, we have become so used to these patterns that we understand them intuitively. When someone hits a wrong note, you can *feel* it without having to break out pencil and paper.

But, even if we don't notice the math when we are listening to music, math can help us describe and even modify music. Take pitch correction, for example. Used in technologies such as Auto-Tune®, pitch correction takes the notes sung by an unprocessed human voice and applies a mathematical rule to it. Each note is then adjusted to fit somewhere along a musical scale. What you end up with is a melody that is perfectly in tune every time!

But it's not just music. Much of what many people find pleasing is driven by hidden mathematical rules. Functions allow us to take what we have and change it into something new. Although we might not always notice these functions, understanding the part they play gives us options for experimenting and seeing what new outcomes might result.

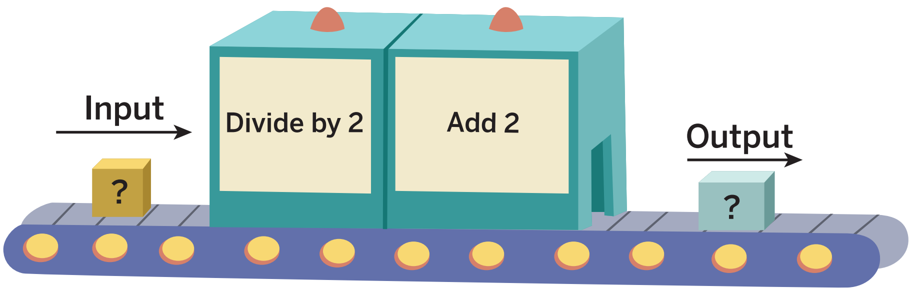
Welcome to Unit 5.

Name: ______ Date: ______ Period: ______

Practice

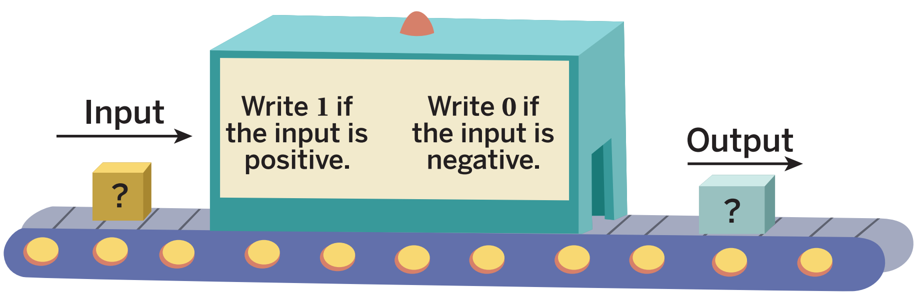
1. Consider the two number machines. For each number machine, calculate the output value for each given input value and record the results in the tables.

a



Input	0	1	2	3	4	5
Output						

b



Input	−15	−10	−5	5	10	15
Output						

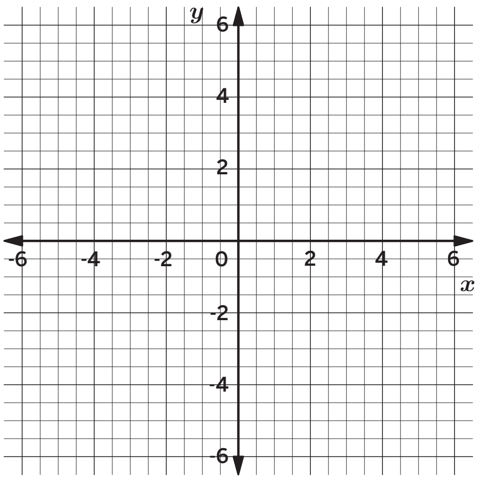
2. Solve the following system of linear equations. Show your thinking.

$$\begin{cases} y = x - 4 \\ y = 6x - 10 \end{cases}$$

Practice

Name: ________ Date: ________ Period: ________

3. Graph a system of linear equations with no solutions. Write an equation for each line you graph.



Line 1:

Line 2:

4. Study these statements carefully.

$12 \div 3 = 4$ because $12 = 4 \bullet 3$.	$6 \div 0 = x$ because $6 = x \bullet 0$.

What value can be used in place of x to create true statements? Explain your thinking.

5. Describe the pattern between x and y shown in the table.

x	y
1	15
2	25
3	35
4	45
5	55

SUB-UNIT

1 Representing and Interpreting Functions

Narrative Connections

Who has the better camera: you or your grandparents?

These days, most people can pop out their phone and snap a selfie. But thirty years ago, things weren't so simple. To capture that perfect moment, you needed an actual, honest-to-goodness analog camera. Digital cameras (like those built into smartphones) and analog cameras (which use film) might produce similar images, but differences in *how they work* affect the kind of image you end up with.

Let's start with an analog camera. When you take a picture, light bounces off what you are looking at. The light then travels through the camera's lens, and hits tiny crystals of silver halide in your film. These crystals undergo a chemical reaction, creating a "latent image," which is later treated with chemicals to produce the photograph.

Digital cameras work differently. Rather than hitting film, the light from your subject hits a tiny grid of millions of photosites. A sensor measures the color and brightness of each photosite and stores that information as a number. This process is called "sampling." These numbers are later recombined to create the image.

There are pros and cons to both methods. Sampling is less precise than film. You end up with an image that is not as sharp, and that does not have the same range of colors. But compared to analog photography, digital photography is more efficient. By turning a picture into numbers, the information can be squeezed down to a tiny file size, making it easier to share.

Whatever your preference is, both analog and digital photography are examples of functions. They take raw information — the light bouncing off your subject — and convert it into something you can keep and carry with you.

Unit 5 | Lesson 2

Introduction to Functions

Let's explore the concept of a function.

Warm-up What Is the Rule?

Your teacher knows two secret rules. You will provide your teacher with an input, and your teacher will tell you the output. Try to guess each rule. Use the table to help organize your thinking.

Input → Rule ? → Output

Rule A:
Tell your teacher a color.

Input	Output

The rule is . . .

Rule B:
Tell your teacher a number.

Input	Output

The rule is . . .

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Guessing My Rule

You will be given a set of cards. Each member in your group will take turns as the rule holder and rule guesser. Place the cards face down and decide who will pick up a card first as the rule holder. The other members in your group will be the rule guessers.

If you are the rule holder . . .	If you are the rule guesser . . .
• Select a card and silently read the rule given on the card. Do not show or read the card to the other group members. • Take turns asking each group member for an input. Provide them with an output by following the rule written on the card. If there is no output, tell them "no output." • When your group members are ready to guess the rule, tell them whether their guess is correct or incorrect. If their guess is incorrect, continue asking for inputs until they guess the rule correctly.	• Your goal is to guess the rule written on the rule holder's card. • Take turns providing an input to the rule holder, and the rule holder will provide you an output. • Record the input-output pairs in the table provided on the next page. • When you have a guess, discuss it with the other rule guessers in your group. Once your group reaches an agreement, tell your guess to the rule holder. The rule holder will tell you whether you are correct. If your guess was incorrect, continue providing inputs until your group guesses the rule correctly.

Repeat the activity, trading roles with your partners.

Are you ready for more?

After completing Rules 1–4, continue the process for Rule 5 and Rule 6.

Rule 5:

Input	Output

The rule is . . .

Rule 6:

Input	Output

The rule is . . .

Activity 1 Guessing My Rule (continued)

If you are the rule guesser, record the input-output pairs in the tables to help organize your thinking.

Rule 1:

Input	Output

The rule is . . .

Rule 2:

Input	Output

The rule is . . .

Rule 3:

Input	Output

The rule is . . .

Rule 4:

Input	Output

The rule is . . .

Name: ________________ Date: ________ Period: ________

Activity 2 Is It a Function?

From music to computer programming, functions are used in many real-world situations to provide rules and structure. Mathematician Ada Lovelace, known as "the first computer programmer," used functions to write programs for an early computing machine. Inputs, outputs, and functions are what make it possible for you to shop online, use apps, and browse websites!

Determine whether the situation for each problem represents a function. Write a statement that describes which column is, or is not, a function of the other.

Here is an example: The table shows the key pressed on a keyboard and the corresponding movement of a video game character.

The video game character's movement is a function of the key pressed.

Key pressed	Movement
Right arrow	Walk right
Left arrow	Walk left
Up arrow	Jump
Down arrow	Squat

1. The button selected on a vending machine and the drink received.

Button selected	Drink received
A	Water
B	Seltzer
C	Juice
D	Water

2. The amount of money spent and the number of items purchased at a store.

Amount of money spent ($)	Number of items purchased
1	2
8	12
7	1
1	3

Activity 2 Is It a Function? (continued)

3. A person's height given in feet and their height given in inches.

Height (ft)	Height (in.)
5	60
4.5	54
6	72
5.4	64.8

4. The musical note that is sung by a singer as time passes.

Time	Musical note
1.25	D
1.5	D
1.75	E
2	D

Featured Mathematician

Ada Lovelace

Augusta Ada Byron, better known as Ada Lovelace, has been recognized as "the first computer programmer" for writing an algorithm for a computing machine in the mid-1800s. Inspired by her mentor, Charles Babbage, Lovelace used the idea of *inputs* and *outputs* to describe how codes could be written and looped on computing machines. Lovelace saw the potential of computing machines and even correctly predicted that they could be used to compose music, produce graphics, and analyze scientific data.

Today, many people around the world celebrate Ada Lovelace Day, held each year on the second Tuesday of October, to recognize the achievements of women in STEM (Science, Technology, Engineering, and Mathematics).

STOP

"Woodcut-style graphic of Ada Lovelace" by Colin Adams, for the Ada Initiative, based on a watercolor portrait by Alfred Edward Chalon. Public Domain via Wikimedia Commons.

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You identified rules that produced different input-output pairs. Suppose you have an input-output rule that, for each allowable input, gives exactly one output. Then you can say the output depends on the input, or the output is a *function* of the input. A ***function*** is a rule that assigns exactly one output to each possible input.

For example, a group of students are timed while sprinting 100 m. The two tables show the distance (meters) and time (seconds) for several students.

Table A

Time (seconds)	Distance (m)
13.8	100
15.9	100
16.3	100
17.1	100

Distance is a function of time because, for each time shown, there is only one possible distance (100 m).

Table B

Distance (m)	Time (seconds)
100	13.8
100	15.9
100	16.3
100	17.1

Time is not a function of distance because for the distance of 100 m, there are many different times shown.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Each table has a set of inputs and outputs. Determine whether each table could represent a function. Explain your thinking.

Table A

Input	Output
4	-2
1	-1
0	0
1	1
4	2

Table B

Input	Output
-2	4
-1	1
0	0
1	1
2	4

2. A group of students is timed while sprinting 100 m. Each student's speed can be calculated by dividing 100 m by their time. Determine whether each statement is *true* or *false*. Explain your thinking.

a. Speed is a function of time.

b. Time is a function of distance.

c. Speed is a function of the number of students racing.

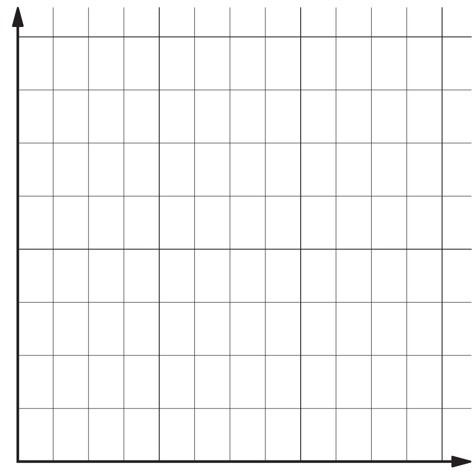
d. Time is a function of speed.

Name: ______________________ Date: ____________ Period: ____________

3. An input-output table is shown. Complete the table so that it represents a function.

Input	Output
-2	0
0	
	10
4	
	0

4. Diego's history teacher writes a test for the class with 30 problems. The test is worth 50 points and has two types of problems: multiple choice problems worth 1 point each, and short answer problems worth 3 points each. How many short answer problems are on the test? Use the graph to help your thinking.



5. Which equations are equivalent to $x + y = 6$? Select *all* that apply.

A. $y = 6 - x$

B. $y = x - 6$

C. $y = 6x$

D. $x = 6 - y$

E. $x = y + 6$

Unit 5 | Lesson 3

Equations for Functions

Let's connect equations and graphs of functions.

Warm-up A Square's Area

Consider the following input-output diagram. Complete the table with the output for each given input.

Input
The side length of a square → **Rule ?** → **Output**
The area of the square

The side length of a square	The area of the square
3	
5	
8	
10	
12	
s	

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Equations and Descriptions

For each description:

- **Write an equation that expresses the output as a function of the input.**
- **Calculate the output when the input is 5.**
- **Determine the independent and dependent variables.**

You may draw an input-output diagram if it helps your thinking.

	Equation	The output when the input is 5	Independent and dependent variable
1. The output y, when you triple the input x and subtract 4.			Independent variable: Dependent variable:
2. The volume of a cube V, given its edge length s.			Independent variable: Dependent variable:
3. The distance in miles d, that you would travel in t hours if you drive at a constant speed of 60 miles per hour.			Independent variable: Dependent variable:
4. The circumference C of a circle with radius r.			Independent variable: Dependent variable:

Activity 2 Apples and Oranges

Jada decides to purchase some apples and oranges at her local farmer's market. Apples cost $\$1$ each, and oranges cost $\$2$ each. The equation $a + 2r = 16$ represents the number of apples a, and the number of oranges r that Jada can purchase for $\$16$.

1. Determine the number of apples Jada can buy if she decides to purchase . . .

a. 2 oranges.

b. 6 oranges.

2. Determine the number of oranges Jada can buy if she decides to purchase . . .

a. 2 apples.

b. 6 apples.

3. Which of the following is true?

A. The number of oranges purchased is a function of the number of apples purchased.

B. The number of apples purchased is a function of the number of oranges purchased.

C. Both A and B

D. Neither A nor B

4. Rewrite the equation so that it gives the number of apples as the dependent variable in terms of the number of oranges as the independent variable.

5. Rewrite the equation so that it gives the number of oranges as the dependent variable in terms of the number of apples as the independent variable.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You discovered that, for some functions, you can describe the relationship between the variables with an equation. Sometimes you can choose, depending on the situation, which variable should be the *independent variable* and which should be the *dependent variable*. The ***independent variable*** represents the input of the function, while the ***dependent variable*** represents the output of a function.

For example, the area of a square A, given its side length s, could be represented by the equation $A = s^2$, for all non-negative numbers. In this example, the input s represents the independent variable, and the output A represents the dependent variable.

Input:
Independent variable
The side length of a square.

→ $\boxed{s^2}$ →

Output:
Dependent variable
The area of the square.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. For each description, write an equation that expresses the output as a function of the input. Then determine the independent and dependent variables.

   a. The perimeter p, of a square with a side length s.

      Equation:

      Independent variable:

      Dependent variable:

   b. The area A, of a circle with radius r.

      Equation:

      Independent variable:

      Dependent variable:

   c. The total cost c, after a sales tax of 7% is applied to the cost of a purchase p.

      Equation:

      Independent variable:

      Dependent variable:

2. Consider the equation $y = \frac{1}{2}x$. Which equation(s) represent the same relationship between x and y? Select *all* that apply.

   A. $x = 2y$

   B. $x = \frac{1}{2}y$

   C. $y = 2x$

   D. $y - \frac{1}{2}x = 0$

   E. $x = \frac{y}{2}$

3. At a local craft fair, Lin purchases necklaces and bracelets. Necklaces cost \$4 each and bracelets cost \$1 each. Lin spends \$20 on these items. Let n represent the number of necklaces Lin buys and b represent the number of bracelets Lin buys.

   a. Write an equation relating the two variables.

   b. Rewrite the equation so that it shows b as the dependent variable in terms of n the independent variable.

   c. Rewrite the equation so that it shows n as the dependent variable in terms of b the independent variable.

Name: ______ Date: ______ Period: ______

Practice

4. Solve each equation. Show your thinking.

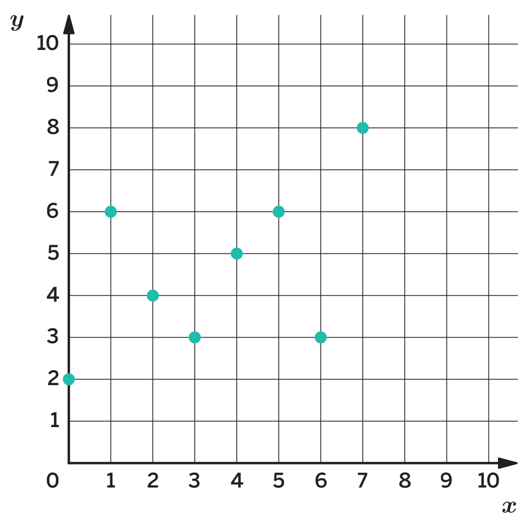
a $4z + 5 = -3z - 8$

b $\frac{1}{2} - \frac{1}{8}q = \frac{q-1}{4}$

5. Consider the graph shown.

a What is the value of y when $x = 2$?

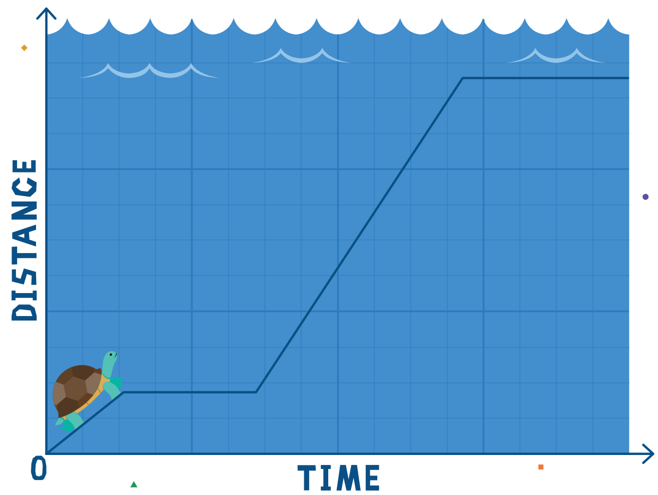
b What is the value of x when $y = 6$?



Unit 5 | Lesson 4

Graphs of Functions (Part 1)

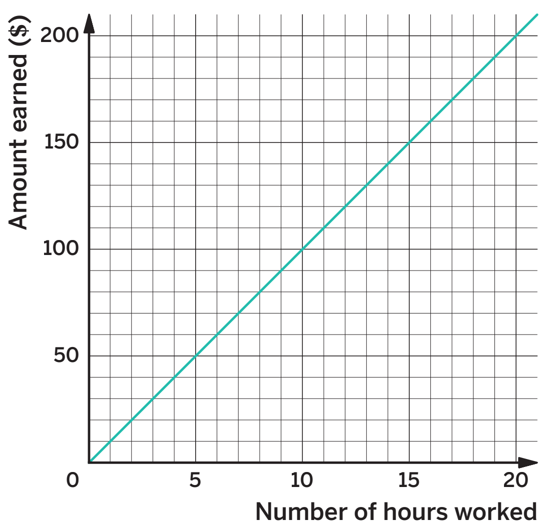
Let's interpret graphs of functions.



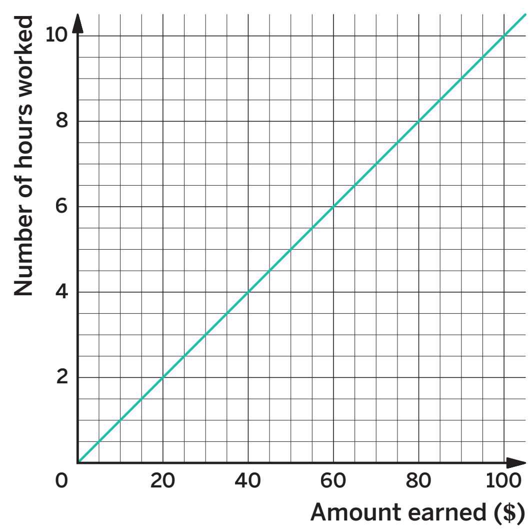
Warm-up Notice and Wonder

Lin earns $10 per hour at her part-time job. The two graphs represent the relationship between Lin's earnings and the number of hours she worked. What do you notice? What do you wonder?

Graph A



Graph B



1. I notice . . .

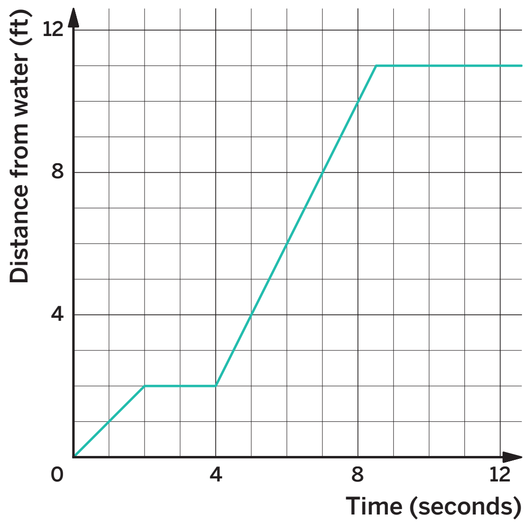
2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Turtle Crossing

The graph shows a turtle's distance from the water, in feet, over time, in seconds.



1. Determine the turtle's distance from the water after:

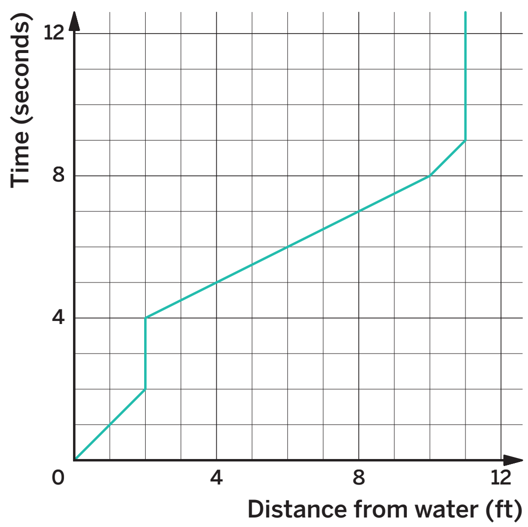
   a. 4 seconds.

   b. 6 seconds.

   c. 8 seconds.

2. For this situation, is the turtle's distance from the water a function of time? If yes, determine the independent and dependent variables and explain why it is a function. If no, explain why it is not a function.

Clare drew the graph shown to represent the turtle's journey.



3. Using Clare's graph, determine the time when the turtle's distance from the water was:

   a. 1 ft.

   b. 8 ft.

   c. 11 ft.

4. For this situation, is the time a function of the turtle's distance from the water?
   If yes, determine the independent and dependent variables and explain why it is a function. If no, explain why it is not a function.

Reflect: How did you keep control over your impulses?

Activity 2 Card Sort: Is It a Function?

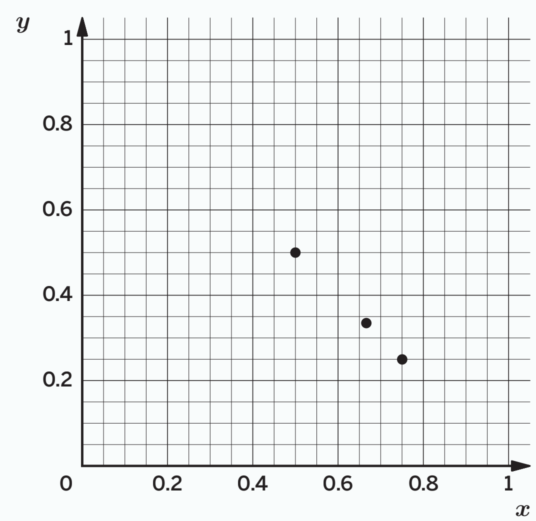
You will be given a set of cards. Each card contains either a graph or a set of ordered pairs. You will sort them into two categories: y *is a function of* x and y *is not a function of* x. Record the card numbers in the table. Be prepared to explain your thinking. Hint: For Card 7 and Card 8, you may plot the points on a graph if it helps your thinking.

y is a function of x	y is *not* a function of x

Are you ready for more?

The inputs to a function are fractions $\frac{a}{b}$ between 0 and 1, where a and b have no common factors, and the output is the fraction $\frac{1}{b}$. For example, given the input $\frac{3}{4}$, the function outputs $\frac{1}{4}$. For the input $\frac{1}{2}$, the function outputs $\frac{1}{2}$. For the input $\frac{2}{3}$, the function outputs $\frac{1}{3}$. These three input-output pairs are shown on the graph.

1. Plot at least 10 more points on the graph of this function.
2. Are most of the points on the graph above or below a height of 0.3?
3. Are most of the points on the graph above or below a height of 0.01?



STOP

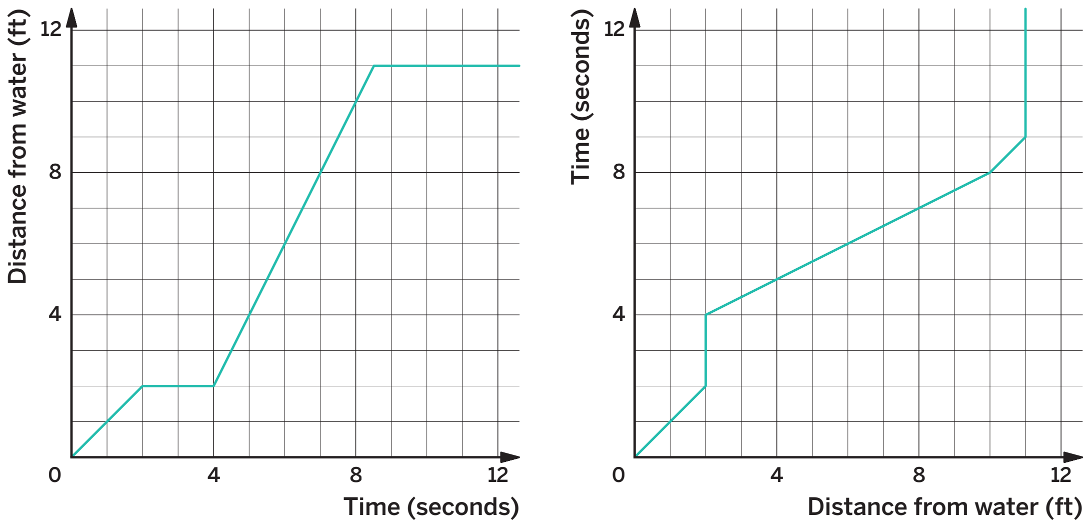
Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You connected different function representations and learned the conventions used to label the graph of a function. The *independent variable* (input) is labeled along the horizontal x-axis and the *dependent variable* (output) is labeled along the vertical y-axis. You also analyzed graphs of functions and non-functions and saw that the graph of a function will not have multiple y-values for the same x-value, because each input will have only one output.

Consider the graphs from Activity 2.



This graph represents a function because, for every second x, there is only one corresponding distance y.

This graph does not represent a function because there are multiple corresponding times y, for 2 ft and for 11 ft.

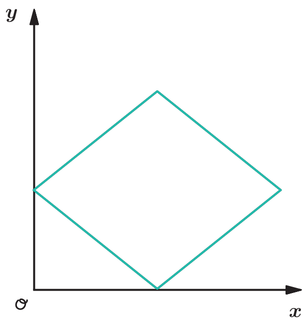
Reflect:

Practice

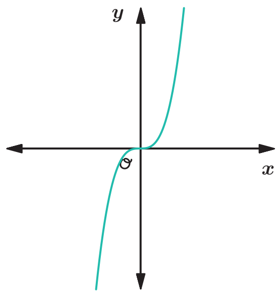
Name: ______________ Date: ______________ Period: ______________

1. Which graph represents y as a function of x?

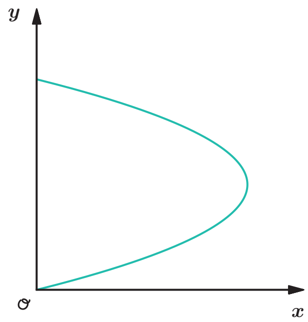
A.



B.

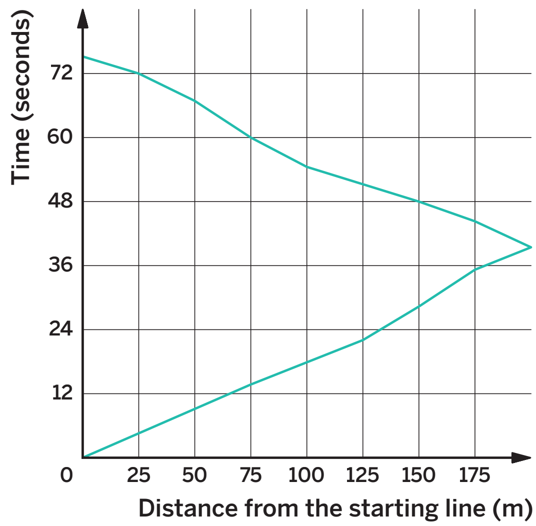


C.

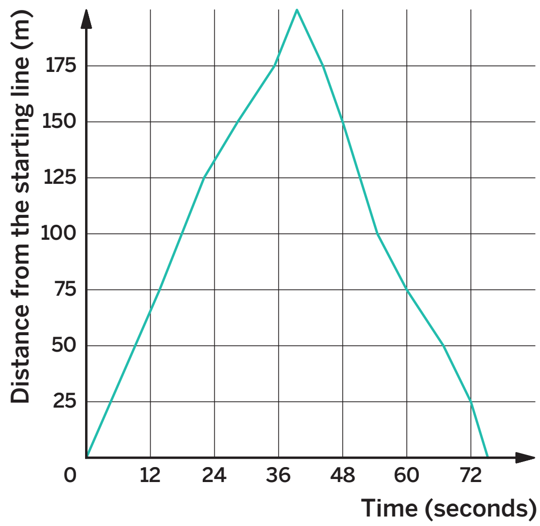


2. Diego runs one lap around a track. Both graphs show the relationship between his time and the distance from the starting point.

Graph T



Graph L



Which graph represents Diego's distance from the starting line as a function of time? Explain your thinking.

A. Graph T

B. Graph L

C. Both graphs

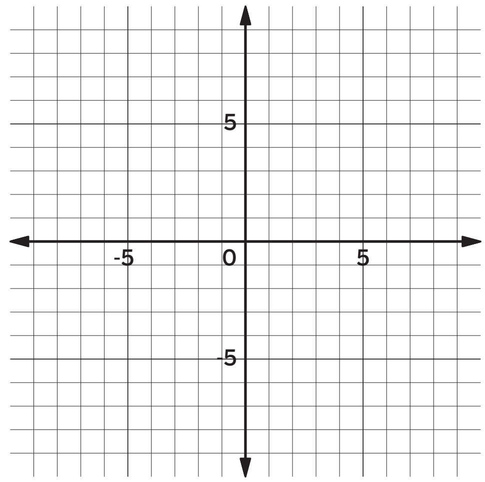
D. Neither graph

Name: .... Date: .... Period: ....

Practice

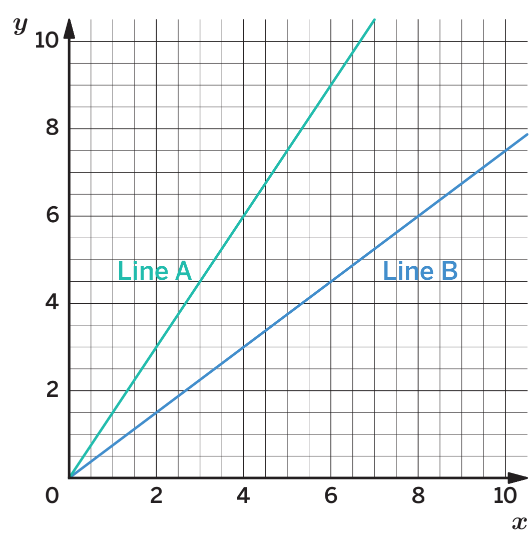
3. Use the equation $2m + 4s = 16$ to complete the table. Plot the points and draw a line using s as the dependent variable and m as the independent variable. Label the axes.

m	0		-2	
s		3		0



4. What value of x makes the expressions $2x + 3$ and $3x - 6$ produce the same value? Show your thinking.

5. Consider the graph shown. Which line has the greater y-value when $x = 6$? Explain your thinking.



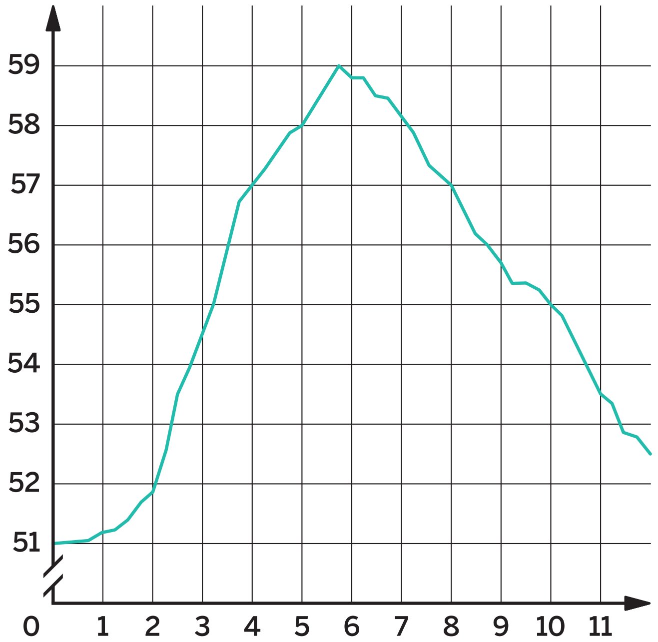
Unit 5 | Lesson 5

Graphs of Functions (Part 2)

Let's interpret graphs of functions.

Warm-up Notice and Wonder

Consider the following graph. What do you notice? What do you wonder?



1. I notice . . .

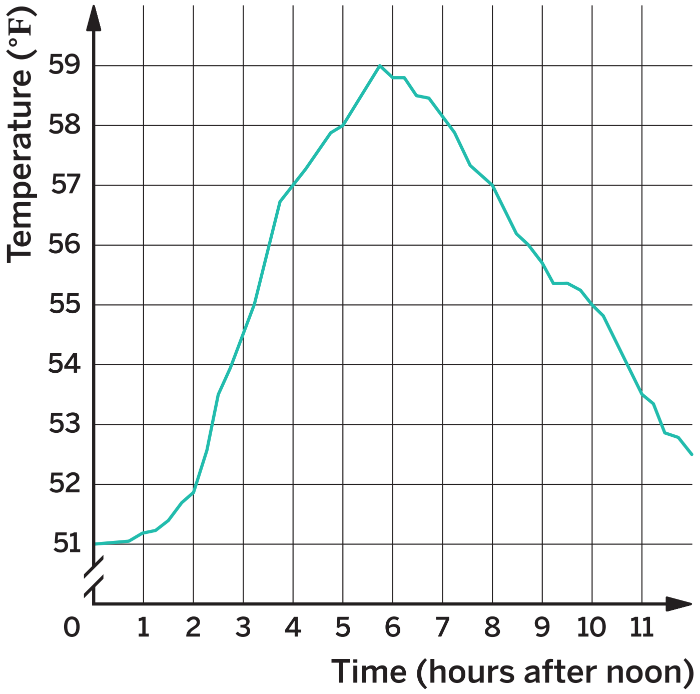
2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Time and Temperature

The graph shows the temperature between noon and midnight for one day in a certain city.



1. Which does the graph show: temperature as a function of time, or time as a function of temperature?

2. Approximately when was the temperature the highest?

3. Was it warmer at 3:00 p.m. or 9:00 p.m.?

4. Determine another time that the temperature was the same as it was at 4:00 p.m. Then estimate the temperature.

5. When the input is 10, what is the output? What does that tell you about the time and temperature?

Activity 2 Highs and Lows

You will be given a graph that shows the temperature of a certain city for 24 hours starting at midnight.

1. Use the graph to respond to the following problems.

   a. What was the high temperature and when did it occur?

   b. What was the low temperature and when did it occur?

   c. Determine a time interval when the temperature was increasing.

   d. Determine a time interval when the temperature was decreasing.

   e. Determine a time interval when the temperature remained the same.

2. With your group, use your responses from Problem 1 and take turns reporting the weather for the city represented by the graph. Consider reporting the information in the role of a meteorologist!

STOP

Name: ______ Date: ______ Period: ______

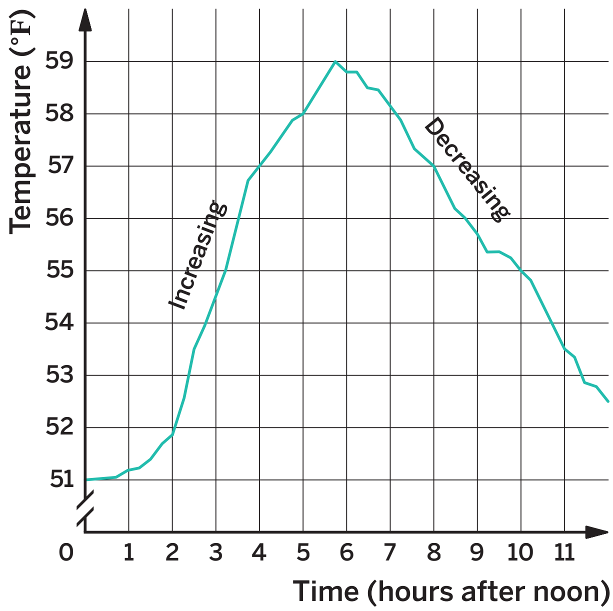
Summary

In today's lesson . . .

You interpreted graphs that represent a function. A graph of a function can tell you what is happening in the context the function represents. The intervals and the overall shape of a graph can be used to interpret the context of the function.

For example, if part of a graph is increasing, this could mean that a value is going up. If part of a graph is decreasing, this could mean that a value is going down.

Determining where a graph is increasing or decreasing is based on reading the graph from left to right.

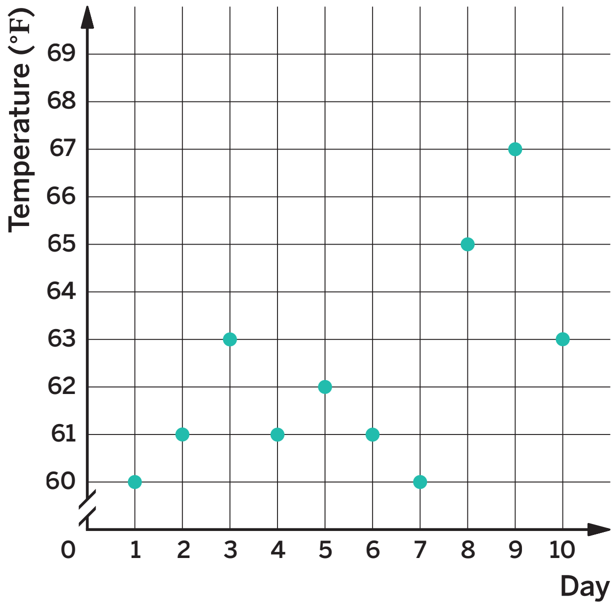


Reflect:

Practice

Name: .... Date: .... Period: ....

1. The graph shows the high temperatures in a city over a 10-day period.

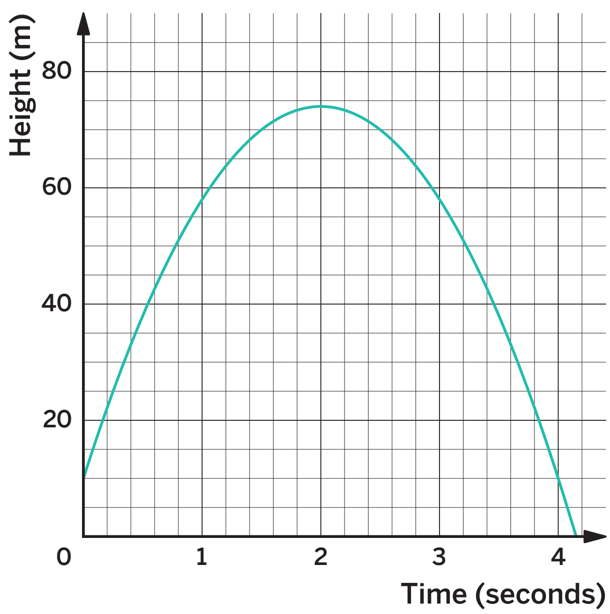


a. What was the high temperature on Day 7?

b. On which day(s) was the high temperature $61°F$?

c. Is the high temperature a function of the day or is the day a function of the high temperature? Explain your thinking. Then determine the independent and dependent variables.

2. The graph represents the height of an object that is launched upwards from a tower and then falls to the ground.



a. How tall is the tower from which the object was launched?

b. Plot the point that represents the greatest height of the object and the time it took the object to reach that height.

c. Determine one time interval when the height of the object was increasing.

d. Determine one time interval when the height of the object was decreasing.

Name: .... Date: .... Period: ....

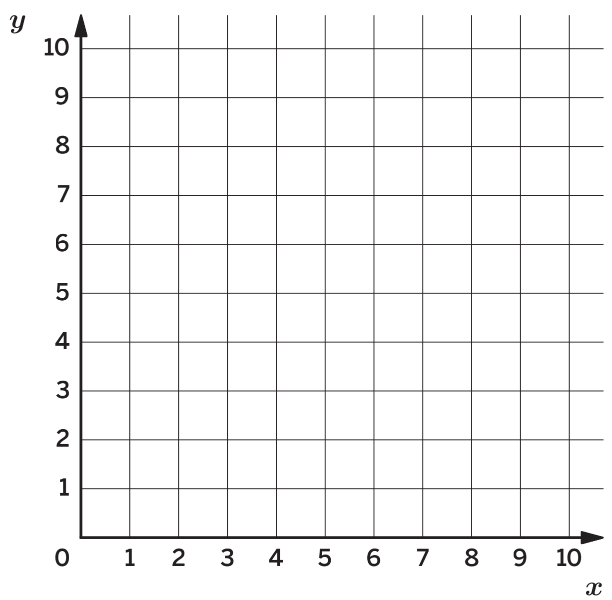
Practice

3. A car is traveling at a speed of either 55 mph or 35 mph, depending on the speed limits, until it reaches its destination 200 miles away. Let x represent the amount of time, in hours, that the car is traveling at 55 mph. Let y represent the amount of time, in hours, that the car is traveling at 35 mph. The equation $55x + 35y = 200$ represents this scenario.

   a. If the car spends 2.5 hours traveling at 35 mph on the trip, how long does it spend traveling at 55 mph? Show or explain your thinking.

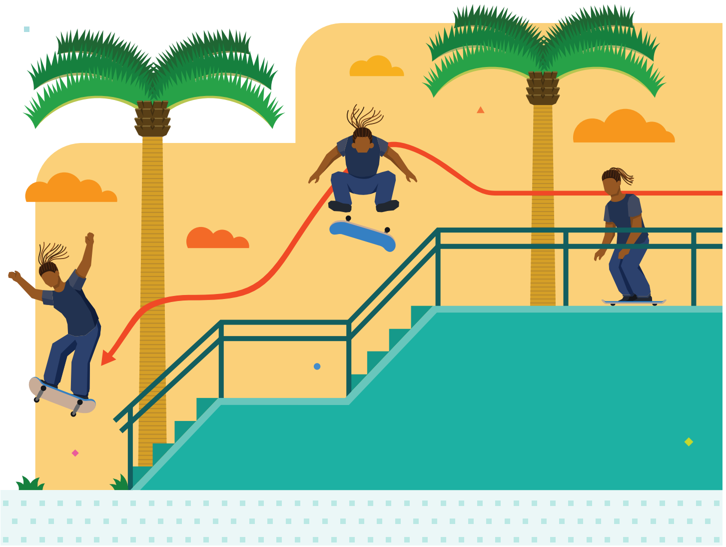
   b. If the car spends no time traveling at 35 mph, how long would the trip take? Show or explain your thinking.

4. Sketch a graph that starts at $(0, 8)$, decreases for the x-values between 0 and 4, and then increases for x-values greater than 4.



Unit 5 | Lesson 6

Graphs of Functions (Part 3)



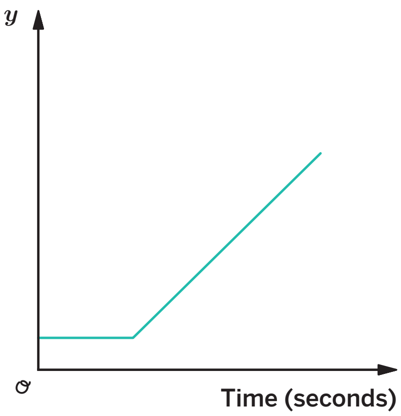
Let's make connections between scenarios and the graphs that represent them.

Warm-up Dog Run

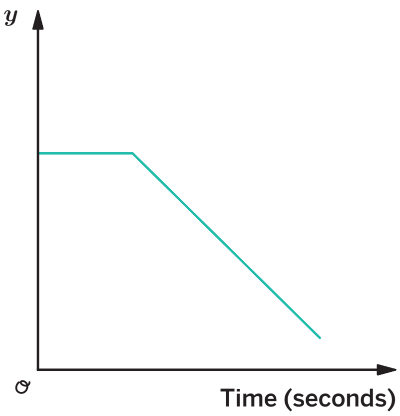
Here are several pictures of a dog at equal intervals of time. Diego and Lin drew different graphs to represent this scenario. They both used time as the independent variable.

Photos courtesy of Ben Simon

Diego's graph



Lin's graph



What do you think each person used for the dependent variable? Explain your reasoning.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Which Graph Is It?

You will be given four graphs.

For each of the following scenarios:

- **Select the graph that best matches the scenario.**
- **Determine the independent and dependent variables.**
- **Label the axes on the graph.**
- **Determine which variable is a function of the other.**
  Be prepared to explain your thinking.

1. The amount of fuel (in gallons) left in a gas tank as a person drives the car a certain distance (in miles).

2. The price (in dollars) of a person's frozen yogurt order based on the weight (in ounces) of the frozen yogurt and toppings.

3. The height (in feet) of a person's shoulders from the ground after time (in seconds) as they go back and forth on a swing.

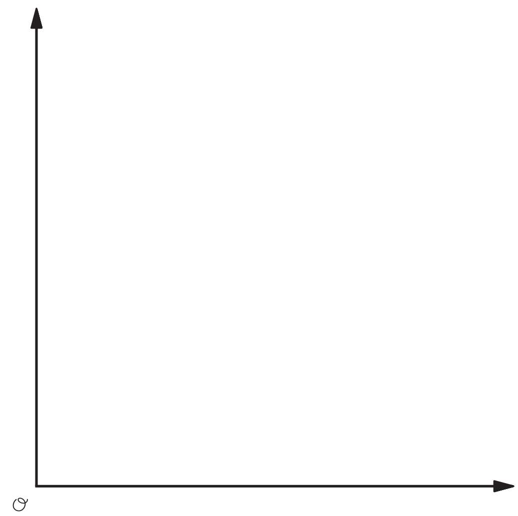
4. The height (in feet) of a basketball from the ground after time (in seconds) as it is shot by a basketball player from the free throw line.

Activity 2 Sketching a Story

Samarria Brevard won the silver medal at the 2017 Minneapolis X Games, where she competed in the Women's Skateboard Street contest. During one of her runs, Brevard executed many skateboarding tricks, including a trick called the Tre Flip. You will be shown a clip of a skateboarder completing the Tre Flip.

1. Sketch a graph representing the distance from the top of the skateboarder's head to the ground, over time. Be sure to label the axes.

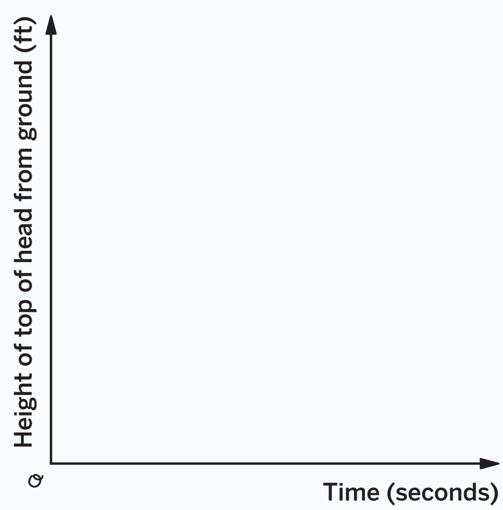
2. Determine which variable is a function of the other. Explain your thinking.



3. Compare your graph with a partner. Does everything make sense? If not, make changes to your work.

Are you ready for more?

In the background of the video there was a ramp and a flight of stairs. What would the graph look like if the person walked down the flight of stairs? Sketch a new graph using the same variables.



STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored graphs of functions that represent a context. For a graph representing a context, there can be multiple representations, so it is important to carefully choose and label variables for the axes. Depending on the independent and dependent variables, distinct graphs can describe different aspects of the same story.

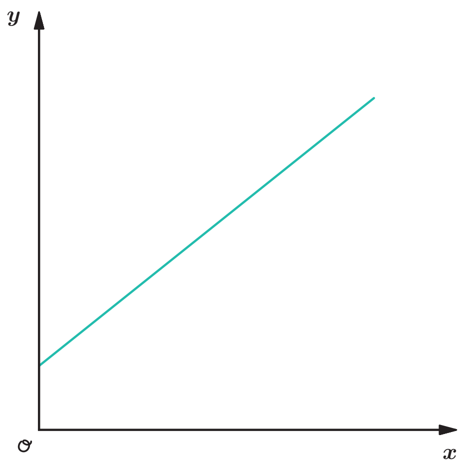
Reflect:

Practice

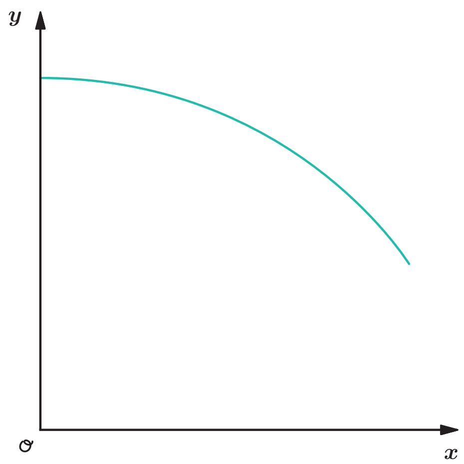
Name: ______ Date: ______ Period: ______

1. For each scenario, determine which graph best represents it. Then identify the possible independent and dependent variables and how you would label the axes. **Hint:** You may select a graph more than once.

Graph 1



Graph 2



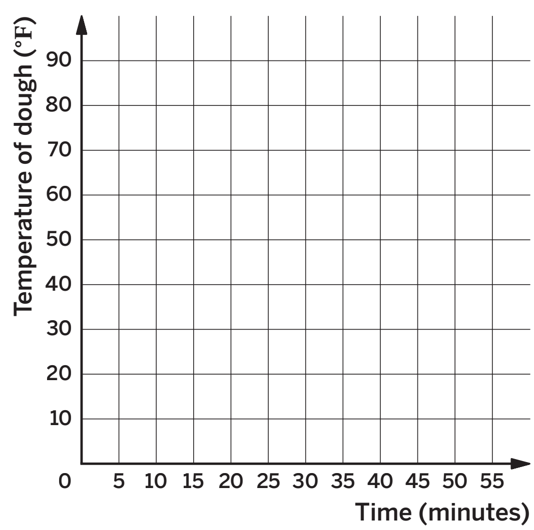
a. A plant grows the same amount every week.

b. The day started very warm, but then it got colder.

c. A carnival has an entry fee of \$5 and tickets for each ride cost \$1.

2. Bard places a batch of homemade pretzels in the refrigerator. The dough takes 15 minutes to cool from 70°F to 40°F. Once it is cool, the dough stays in the refrigerator for another 30 minutes. Bard then places the pretzels into the oven to bake. After 5 minutes in the oven, the temperature of the pretzel dough is 80°F.

Sketch a graph that represents this situation.



Name: .... Date: .... Period: ....

Practice

3. Recall the formula for the area of a circle.

a. Write an equation relating a circle's radius r and area A.

b. Is the area a function of the radius?

c. Is the radius a function of the area?

d. Complete the table. Write your responses in terms of π.

r	3		$\frac{1}{2}$	
A		16π		100π

4. Solve each equation. Show or explain your thinking.

a. $-(-2x+1) = 9 - 14x$

b. $2x + 4(3 - 2x) = \frac{3(2x+2)}{6} + 4$

5. For each relationship shown, determine the output when the input is 2.

a. $y = 3x-7$

b.

x	1	2	3	4	5
y	12	17	22	27	32

Unit 5 | Lesson 7

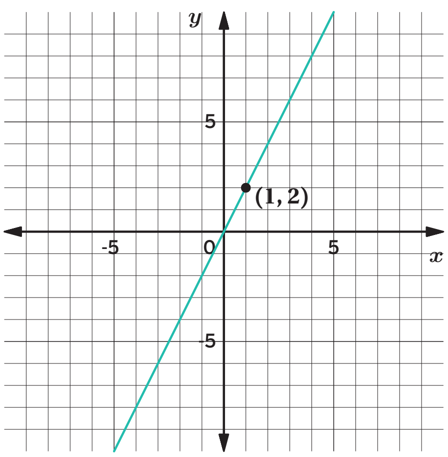
Connecting Representations of Functions

Let's connect tables, equations, graphs, and stories of functions.

Warm-up Three Representations

Refer to the graph, equation, and table shown here.

Graph:



Equation:

$z = 2r$

Table:

p	-2	-1	0	1	2	3
q	-4	-2	0	2	4	6

1. How are they similar?

2. How are they different?

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

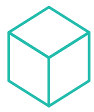
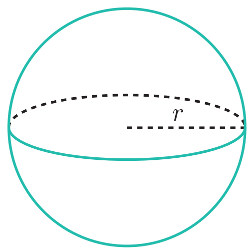
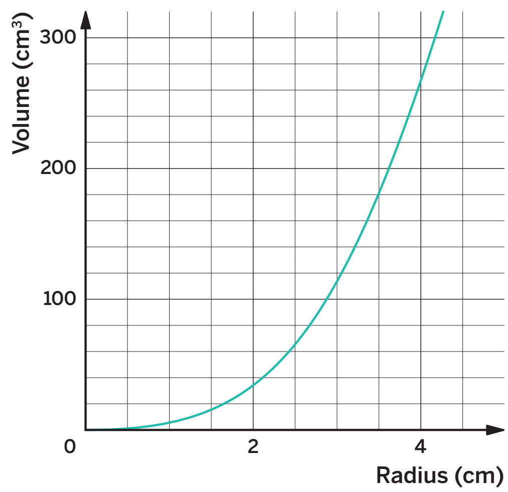
Activity 1 Junior Olympics

Tyler, Elena, and Clare are participating in various sports during the Junior Olympics. You will be given a graph, an equation, and a table representing the number of steps they take over a period of time. With your group, work together to award Tyler, Elena, and Clare a medal for each scenario described. For each scenario, award the gold medal to the person who took the greatest number of steps, and award the bronze medal to the person who took the least number of steps. Be prepared to explain your thinking. An example is shown in the first row.

	Gold medal	Silver medal	Bronze medal
Example: Steps taken in the first 10 minutes.	**Elena:** $s = 130 \cdot 10$ $s = 1300$ 1,300 steps	**Clare:** 1,200 steps	**Tyler:** About 1,000 steps
1. Steps taken in the first 20 minutes.			
2. Steps taken in the first 30 minutes.			
3. Total steps taken.			

Activity 2 Comparing Volumes

Consider the following information about the volume of a cube and the volume of a sphere.

	Volume
Cube 	The volume V of a cube with an edge length of s cm is given by the equation $V = s^3$.
Sphere 	The graph represents the volume of a sphere as a function of its radius (in centimeters). 

1. Is the volume of a cube with an edge length of 3 cm greater than or less than the volume of a sphere with a radius of 3 cm? Explain your thinking.

2. Consider a sphere that has the same volume as a cube with an edge length of 5 cm. Determine the radius of the sphere.

3. Calculate the outputs of the two volume functions when the input is 2.

Compare and Connect:
How does the volume of a cube relate to the volume of a sphere if the cube's side length is equal to the radius of the sphere?

STOP

Name: ______________ Date: ______________ Period: ______________

Summary

In today's lesson . . .

You compared functions represented in a table, graph, and equation. Even though you were looking for the same information, you performed different actions depending on the representation of the function. Each representation gives you the ability to calculate input-output pairs, but each representation has its benefits and drawbacks.

- Graphs require estimation, but can visually provide information, such as the highest point.
- Tables immediately provide output values, but only for limited input values.
- Equations precisely compute outputs for all inputs, but do not provide visual information.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. The equation and the table represent two different functions.

Equation

$b = 4a - 5$

Table

a	-3	0	2	5	10	12
c	-20	7	3	21	19	45

a When $a = -3$, is the value of b or c greater? Explain your thinking.

b When $c = 21$, what is the value of a? What is the value of b for this value of a?

c For what values of a, do you know that the value of c is greater than b? Explain your thinking.

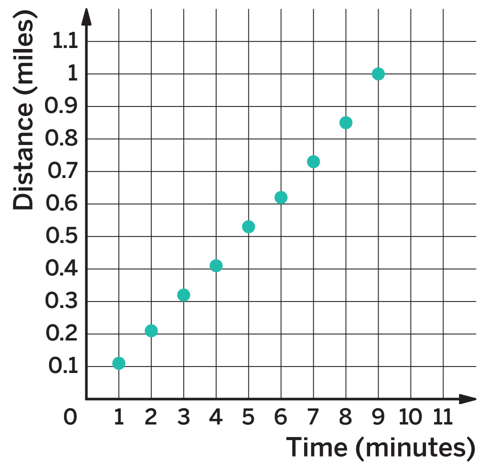
Name: .... Date: .... Period: ....

2. Elena and Lin train for a race. Elena runs her mile at a constant speed of 7.5 mph. Lin's total distance, shown in the table, is recorded every minute.

Time (minutes)	1	2	3	4	5	6	7	8	9
Distance (miles)	0.11	0.21	0.32	0.41	0.53	0.62	0.73	0.85	1

a Who finished their mile first? Explain your thinking.

b The graph represents Lin's progress. On the same graph, draw a line that represents Elena's distance, in miles, and time, in minutes.



3. The solution to a system of equations is $(6, 3)$. Select two equations that could make up the system.

A. $y = -3x + 6$

B. $y = 2x - 9$

C. $y = -5x + 27$

D. $y = 2x - 15$

E. $y = -4x + 27$

4. Determine whether each table could represent a linear relationship. Show or explain your thinking.

a

x	y
1	5
3	6
6	7

b

x	y
1	5
3	6
7	8

Unit 5 | Lesson 8

Comparing Linear Functions

Let's compare linear functions.

Warm-up Saving Money

Jada has $50 in her savings account and saves $7 per week.

1. What are the two variables in this scenario?

2. Which variable is the independent variable and which is the dependent variable? Explain your thinking.

3. Write an equation representing this scenario.

4. Is this relationship a function? Why or why not?

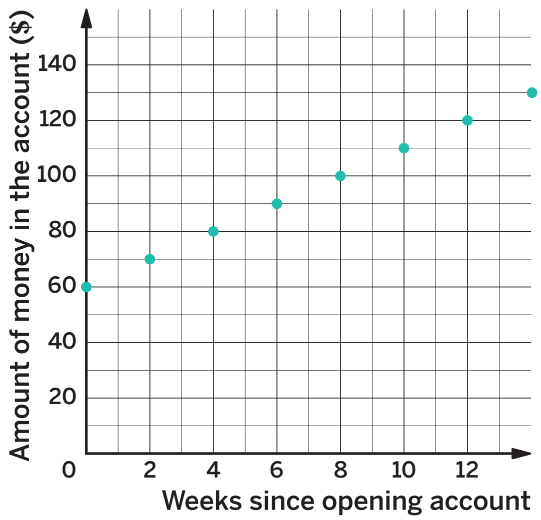
Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Which Is Growing Faster?

Noah and Elena both open a savings account on the same day. The amount each person has in their account is a function of the time since the account was opened. Each person's account is represented below.

Noah's account:



Elena's account:
$a = 8w + 60$, where w is the number of weeks since opening the account, and a is the total amount, in dollars, in the account.

1. Who started out with more money in their account? Explain your thinking.

2. Who is saving money at a faster rate? Explain your thinking.

3. How much will Noah save over the course of a year if he does not make any withdrawals? How long will it take Elena to save that much? **Hint:** There are 52 weeks in a year.

Activity 2 Is It Charging or Losing Charge?

Plan ahead: What will you need to do to communicate your thinking thoroughly and well?

Four electric vehicle batteries are tested in different conditions. The percent of charge p is measured over a given time period t, in minutes.

Battery A:

The percent of charge p is given by the function $p = 65 + 2t$, where t represents time in minutes.

Battery B:

The percent of charge p starts at 40 and is decreasing at a constant rate of 1.5% per minute.

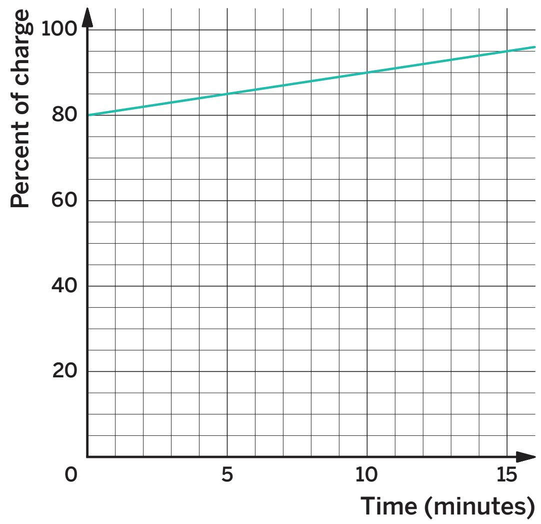
Battery C:

The percent of charge is changing at a constant rate and can be represented by the following table.

Time (minutes)	Percent of charge
1	79
3	72
7	58

Battery D:

The percent of charge can be represented by the following graph.



1. Which of the batteries are being charged? Explain your thinking.

Name: .... Date: .... Period: ....

Activity 2 Is It Charging or Losing Charge? (continued)

2. Which of the batteries are losing charge? Explain your thinking.

3. Which battery started with the greatest percent of charge? Explain your thinking.

4. Which battery has the greatest rate of change? Explain your thinking.

STOP

Summary

In today's lesson . . .

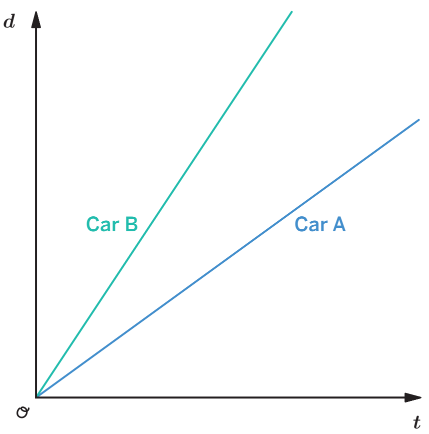
You compared linear functions using different representations. A ***linear function*** is a linear relationship which assigns exactly one output for every possible input.

When you are given more than one linear function — even if they are represented differently — you can determine the slope and y-intercept from each representation and use them to compare the functions.

Reflect:

Name: .... Date: .... Period: ....

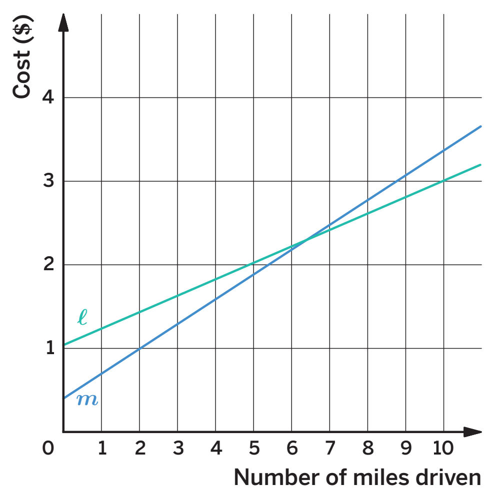
1. Two cars are traveling on the same highway in the same direction. The graphs show the distance d of each car from the starting point as a function of time t. Which car is traveling faster? Explain your thinking.



2. Two car services offer to pick up a customer and take them to their destination. Service A charges a flat fee of $\$0.40$ plus $\$0.30$ for each mile of their trip. Service B charges a flat fee of $\$1.10$ plus c dollars for each mile of their trip.

   a. Match the services to the lines ℓ and m.

   b. For Service B, is the additional charge per mile greater than or less than $\$0.30$ per mile of the trip? Explain your thinking.



3. Kiran and Clare like to race each other home from school. They run at the same speed, but Kiran's house is slightly closer to the school than Clare's house. On a graph, their distance from their house, in meters, is a function of the time from when they begin the race, in seconds.

   a. As you read the graphs from left to right, would you expect the lines to increase or decrease?

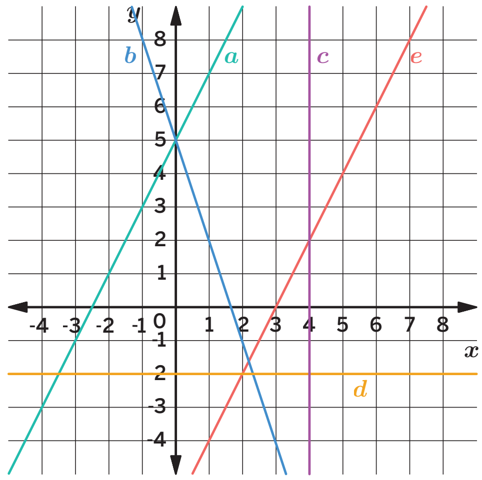
   b. What would you expect to be different about the lines representing Kiran's run and Clare's run? Explain your thinking.

   c. What would you expect to be similar about the lines representing Kiran's run and Clare's run? Explain your thinking.

Practice

Name: ______ Date: ______ Period: ______

4. Write an equation for each line shown in the graph.



5. Consider the incomplete system of equations.

$$\begin{cases} y = -3x + 2.8 \\ \end{cases}$$

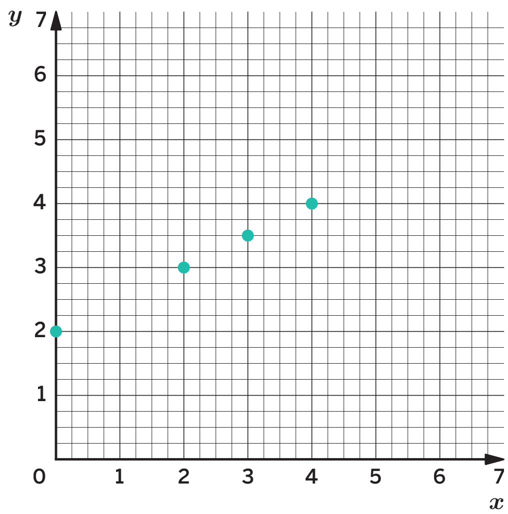
Write a second equation so that the system of linear equations has:

a. Exactly one solution.

b. No solution.

c. Infinitely many solutions.

6. Consider the graph. Use the given points to predict the y-value when $x = 6$.
Show or explain your thinking.

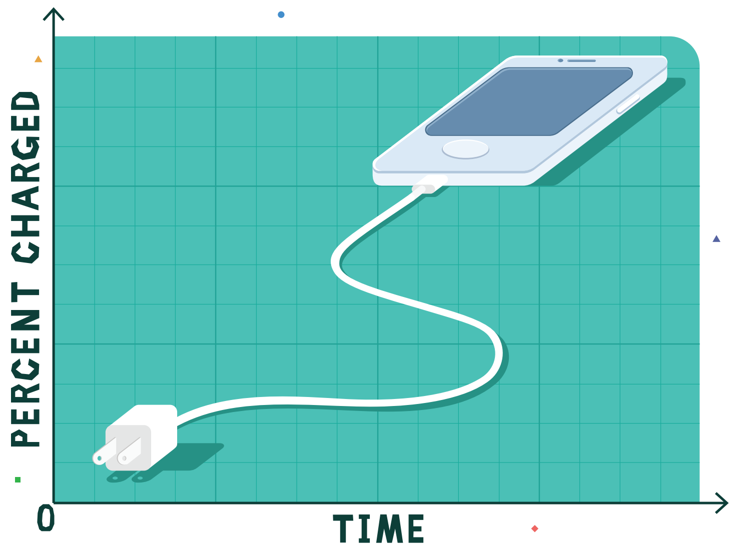


Name: ________________ Date: ________ Period: ________

Unit 5 | Lesson 9

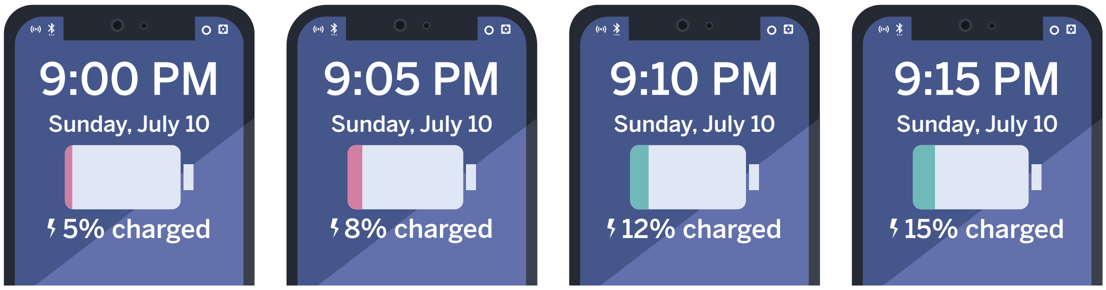
Modeling With Linear Functions

Let's model situations with linear functions.



Warm-up Charge!

The images show the percent a phone is charged over a 15 minute time period.



Estimate the time when the phone will be fully charged. Be prepared to explain your thinking.

Log in to Amplify Math to complete this lesson online.

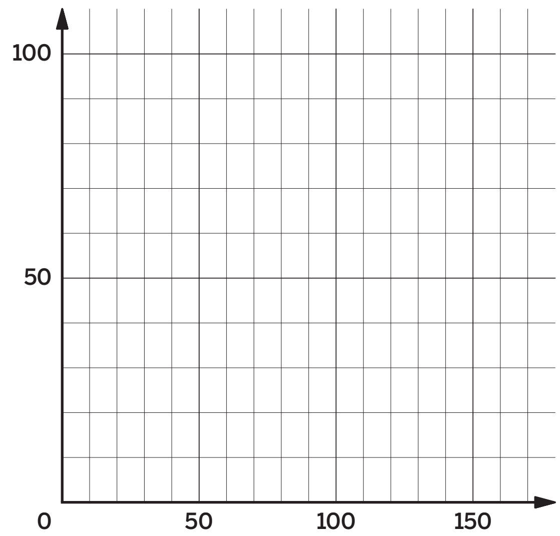
Activity 1 Charging a Phone

Consider the description that corresponds with the images from the Warm-up. At 9 p.m., a phone is charged 5%. After 5 minutes, the phone is charged 8%. After 10 minutes, the phone is charged 12%. After 15 minutes, the phone is charged 15%.

1. Determine the independent variable and the dependent variables in this situation. Which variable is a function of the other?

2. Graph the data that describes the time elapsed since 9 p.m. and the percent the phone is charged. Be sure to label the axes.

3. Does the data appear to be linear? Explain your thinking.

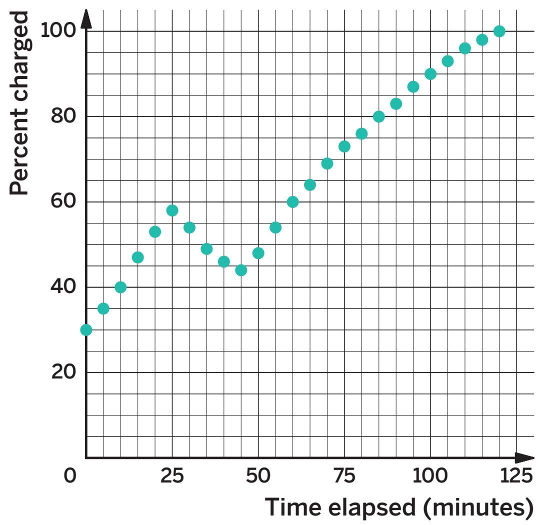


4. When do you think the phone will fully charge to 100%? Show or explain your thinking. How does your response compare with your response from the Warm-up?

Name: ________ Date: ________ Period: ________

Activity 2 Charging a Laptop

Elena charges her laptop. After 25 minutes, she unplugs her laptop to complete her homework. After Elena completes her homework, she charges her laptop until it is fully charged. The graph shows the percent of charge of Elena's laptop over time.



1. Sketch a linear function that models the percent Elena's laptop is charged from 0 to 25 minutes. For what time(s) is the linear function good at predicting the percent charged? For which time(s) is it not as good?

2. Select another time interval to model with a sketch of a linear function. For which time(s) is the linear function good at predicting percent charged? For which time(s) is it not as good?

3. Write an equation for each linear function you created. Be sure to define any variables you use.

4. How do the charging rates compare based on your equations?

STOP

Summary

In today's lesson . . .

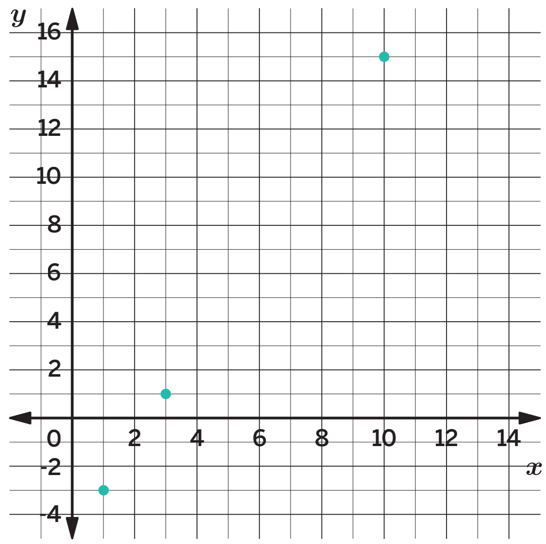
You discovered that, if you are given data for a function, you can sometimes use a line to model the data. You also saw that you can use several linear functions to model data for different time periods. Although a function might not be linear, parts of the data might be modeled by a linear function which you can use to help you make predictions.

Reflect:

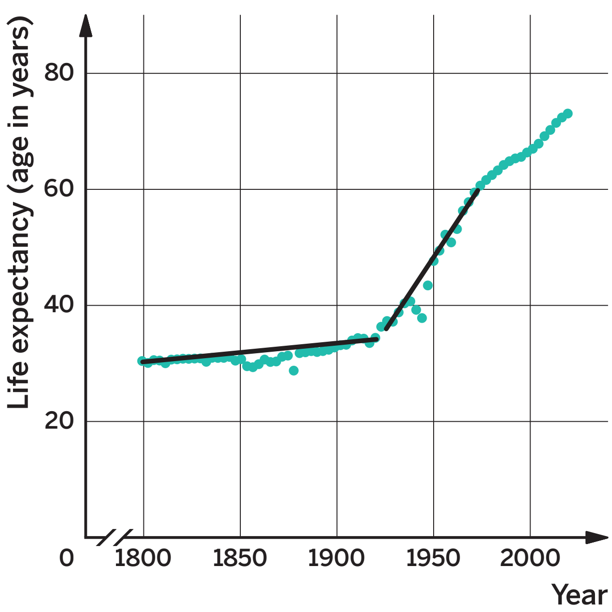
Name: ________________ Date: ________ Period: ________

Practice

1. The graph shows the high temperature, y, (in degrees Celsius) for the number of days after new year, x, for a certain city. Based on this information, is the high temperature in this city a linear function of the number of days after the new year? Explain your thinking.



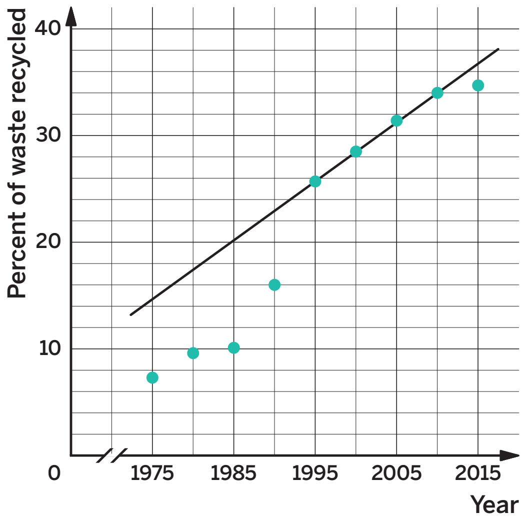
2. This graph shows the human life expectancy over time. Kiran started to model the data with two linear functions. Sketch one more linear function to complete Kiran's work.



Practice

Name: .... Date: .... Period: ....

3. The graph shows the percent of waste produced in the United States that gets recycled over time. A student draws a linear function that models the change from 1975 to 2015. For what years is the model good at making predictions? For which years is it not as good?



4. Solve each equation. Show or explain your thinking.

a $2(3x+2)=2x+28$

b $5y+14=-43-3y$

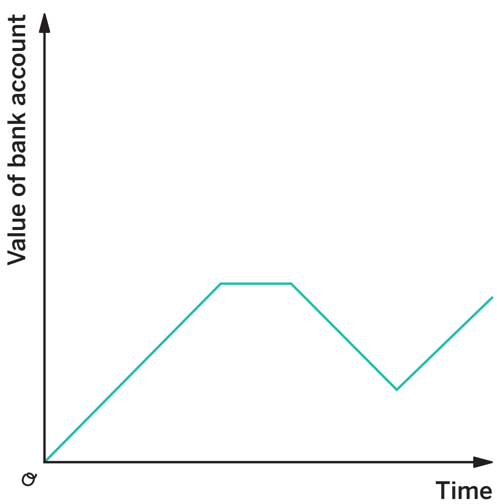
c $4(2a+2)=8(2-3a)$

5. The graph shows the value of a bank account as a function of time.

a Draw a triangle next to any sections of the graph that show the value of the bank account increasing.

b Draw a square next to any sections of the graph that show the value of the bank account decreasing.

c Draw a circle next to any sections of the graph that show the value of the bank account staying the same.

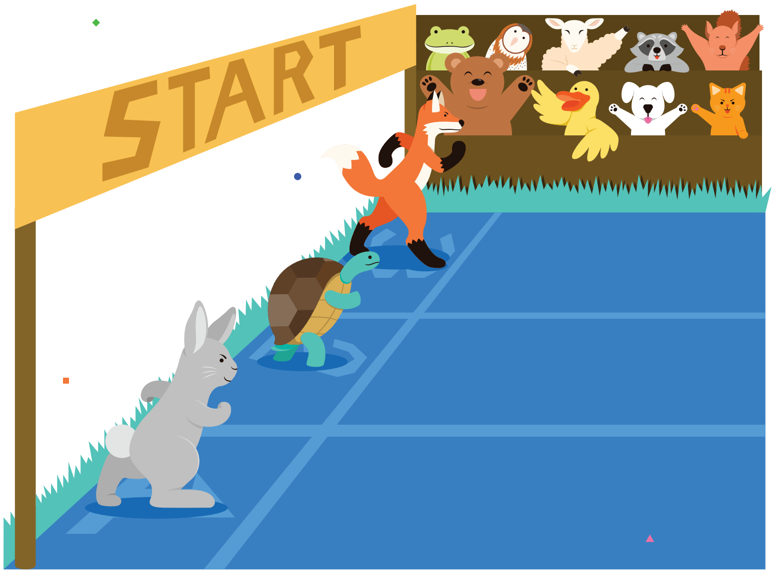


Name: ________ Date: ________ Period: ________

Unit 5 | Lesson 10

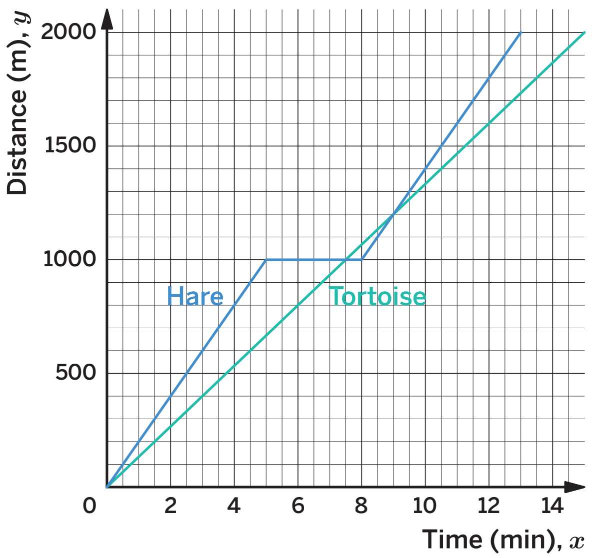
Piecewise Functions

Let's explore functions built from pieces of linear functions.



Warm-up Notice and Wonder

The graph shows a race between a hare and tortoise. What do you notice? What do you wonder?



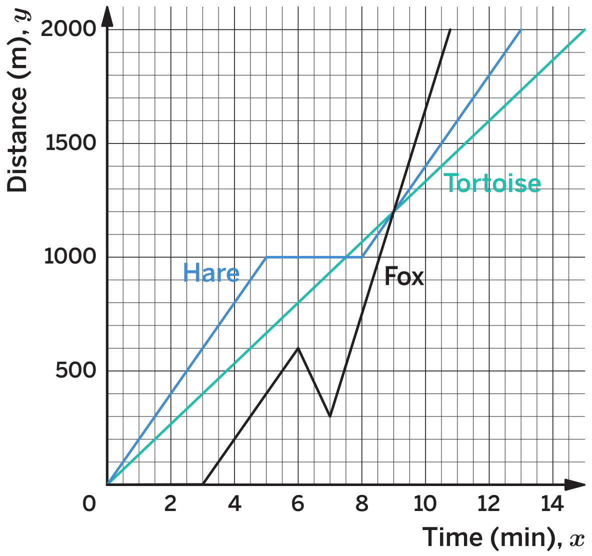
1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 The Tortoise and the Hare . . . and the Fox

The graph from the Warm-up now shows a third animal, a fox, whose distance is graphed as a function of time.



1. Use the graph to write a story about the fox's journey during the race.

2. State whether each statement is *true* or *false*.
   - **a** At 6 minutes, the fox is 200 m behind the tortoise.
   - **b** The fox's distance is always increasing.
   - **c** The hare and the fox are travelling at the same speed from 3 to 5 minutes.
   - **d** When the hare reaches 500 m, the fox is still at the starting line.
   - **e** All three animals are tied at 9 minutes.
   - **f** The tortoise wins the race.

3. Determine the speed of the fox over the following intervals:
   - **a** 0 to 3 minutes.
   - **b** 3 to 6 minutes.
   - **c** 6 to 7 minutes.

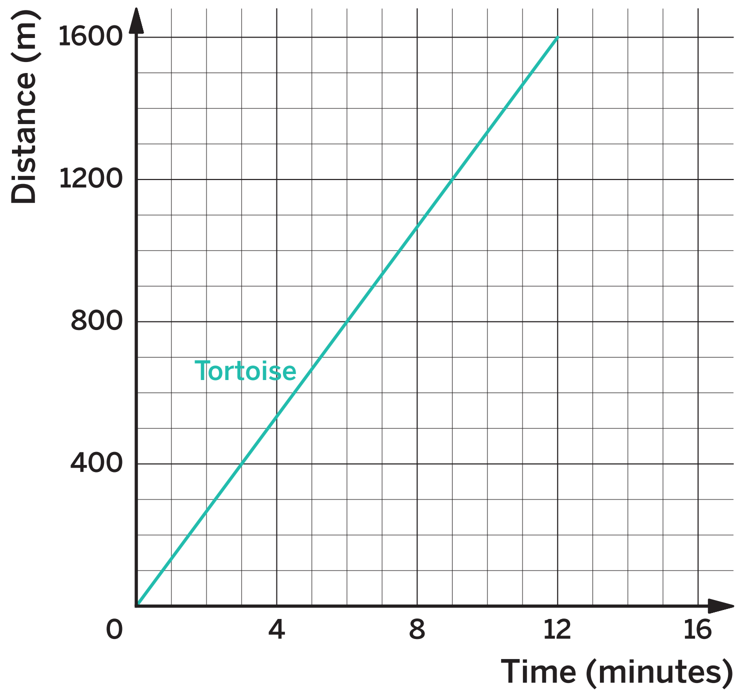
4. Who is traveling the fastest? Explain your thinking

Name: ______ Date: ______ Period: ______

Activity 2 The Tortoise and the Dog

Next, the tortoise races a dog. Draw a graph showing distance as a function of time for the dog that makes *all* of the following statements true.

- The dog gets a head start, but loses the race.
- The dog's distance from the start decreases from 3 to 6 minutes.
- The dog and the tortoise meet at 400 m.
- The dog meets the tortoise three times.
- The dog has a constant speed of 900 m per minute between 6 and 7 minutes.



STOP

Summary

In today's lesson . . .

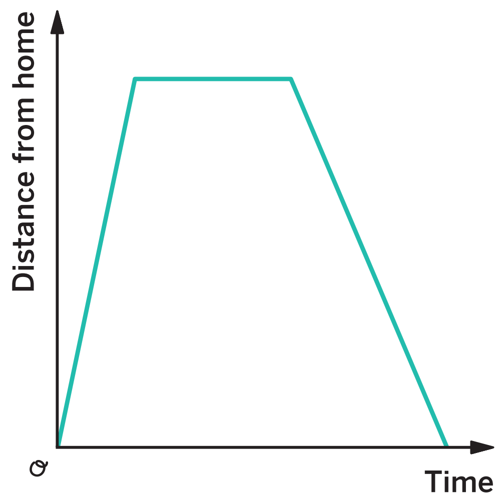
You compared the graphs of linear functions and piecewise functions. A ***piecewise function*** is a function built from pieces of different functions over different intervals. It can be used to model situations in which a quantity changes at a constant rate for a while and then switches to a different constant rate.

Reflect:

Name: ________ Date: ________ Period: ________

Practice

1. The graph shows the distance of a cat from its home as a function of time. Describe what a person watching the cat may be seeing.



2. Elena filled up the tub and gave her dog a bath. Then she let the water in the tub drain.

   a. The graph shows the amount of water in the tub, in gallons, as a function of time, in minutes. Label the axes.

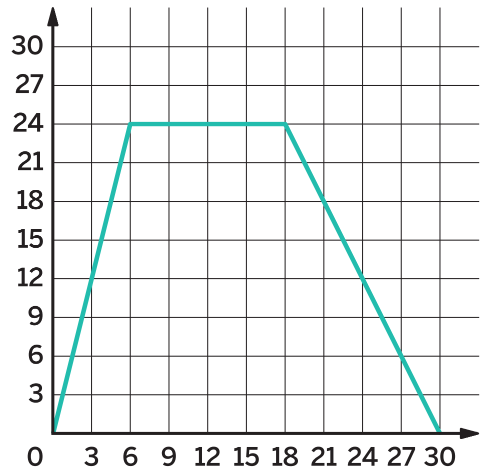
   b. When did she turn off the water faucet?

   c. How much water was in the tub when she bathed her dog?

   d. How long did it take the tub to drain completely?

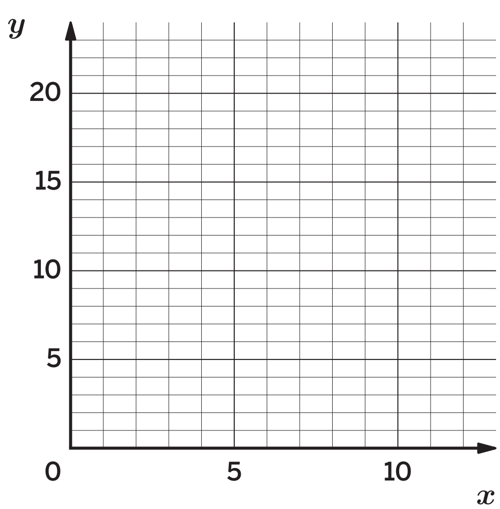
   e. At what rate did the faucet fill the tub?

   f. At what rate did the water drain from the tub?



3. The following statements are descriptions of the linear segments that make up a piecewise function. Draw a graph of the piecewise function described.

   - Starts at the origin.
   - Increasing at a constant rate from $x = 0$ to $x = 3$.
   - Has a slope of 0 from $x = 3$ to $x = 5$.
   - Increasing at its greatest constant rate from $x = 5$ to $x = 9$.
   - Decreasing at a constant rate from $x = 9$ to $x = 10$.



Practice

Name: ____________ Date: ________ Period: ________

4. The expression $-25t + 1250$ represents the volume of liquid in a container after t seconds. The expression $50t + 250$ represents the volume of liquid in another container after t seconds. What does the equation $-25t + 1250 = 50t + 250$ represent in this situation?

5. Which linear function has a greater rate of change? Show or explain your thinking.

Function A:

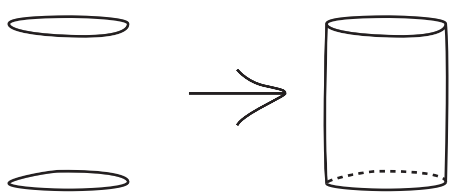
$$y = \frac{1}{6}x + \frac{2}{5}$$

Function B:

x	y
-9	3
1	5
21	9

6. Refer to this method for quickly sketching a cylinder.

- Draw two ovals.
- Connect the edges.
- Consider which parts of your drawing would be hidden behind the cylinder. Draw these as dashed lines.



Practice by sketching at least two cylinders.

SUB-UNIT

2 Cylinders, Cones, and Spheres

Who invented the waffle cone?

Nobody knows for sure! But over the years, many candidates have tried to claim the title.

First was Italo Marchiony, an Italian immigrant from New Jersey. In the late 1800s, Marchiony sold iced treats out of a pushcart. These ices were originally served out of glass cups. But these cups were impractical. They had to constantly be cleaned. Customers also had the nasty habit of walking off with them or breaking them. This inspired Marchiony to invent a more practical container that his customers could eat.

Then there was the case of David Avayou. Avayou was a Turkish immigrant who worked at the St. Louis Fair in 1904. He noticed that fairgoers were avoiding buying ice cream because of the hassle of eating from a plate. Having seen ice cream in paper cones during his travels in France, he set to work creating an edible version for the fair.

There was also the story of Ernest Hamwi. Hamwi was a Syrian immigrant who was also working at the very same St. Louis Fair. Legend has it, Hamwi was selling a Middle Eastern waffle treat, called zalabia, next to an ice cream vendor. When the vendor ran out of plates, Hamwi helped out by rolling the zalabia into a cone. This created a crisp edible container for the vendor's customers.

These are just a few of the people who have claimed to be the waffle cone's inventor. But just because we don't know its origins for certain, doesn't mean we can't appreciate the cone's design. A well-made waffle cone is easy to grab, reduces waste, and keeps your hands clean.

Let's take a closer look at what makes a cone a cone, and how this shape's dimensions relate to its volume through functions.

Unit 5 | Lesson 11

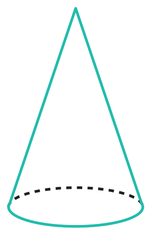
Filling Containers

Let's explore how functions can model the volume of a cylinder.

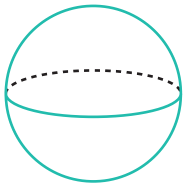
Warm-up Which One Doesn't Belong?

Examine the three-dimensional shapes shown. Which one doesn't belong? Explain your thinking.

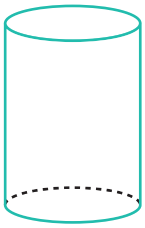
A.



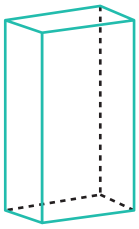
B.



C.



D.



Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 Exploring Height and Volume

You will be given the materials for this activity.

1. To conduct your experiment, follow these steps:

   a. Record the measurement of the radius of your cylinder here: ________.

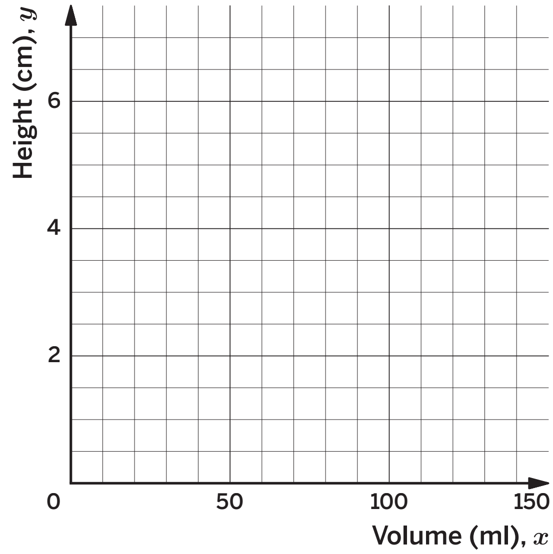
   b. Fill your cylinder with water, and record the amount of water and the height of the water in the cylinder in the table. Repeat the experiment at least five times using different amounts of water.

Volume (ml)	Height (cm)

Activity 1 Exploring Height and Volume (continued)

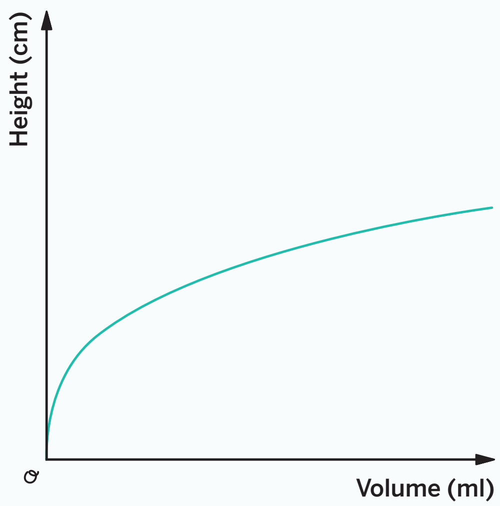
2. Create a graph that shows the height of the water in the cylinder as a function of the volume of water.

3. Choose a point on the graph and explain its meaning in the context of the situation.



Are you ready for more?

The graph shows the height of an unknown container as a function of its volume. What shape could this container have? Explain how you know and draw a possible container.



Name: ________________ Date: ________ Period: ________

Activity 2 Card Sort: What Is the Shape?

You will be provided with a set of cards showing containers and graphs. The graphs show the heights of the containers as a function of their volume.

1. Match the containers with the corresponding piecewise graphs by completing the sentences shown. One of the containers will not have a corresponding graph.

Container ________ corresponds to Graph ________.

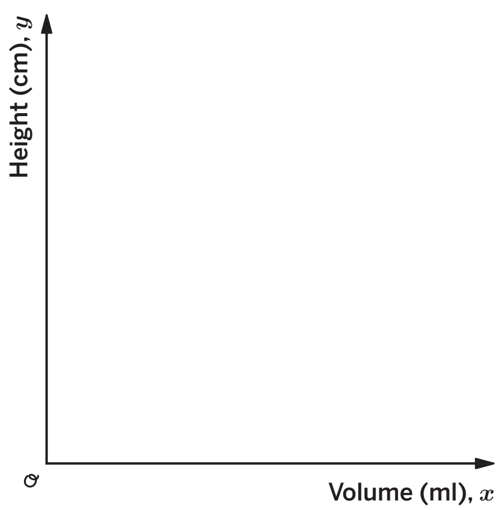
Container ________ corresponds to Graph ________.

Container ________ does not have a corresponding graph.

2. Explain how you matched one of the containers to its graph.

Stronger and Clearer: After you respond to Problem 3, you will meet with 1–2 other pairs of students to give and receive feedback. Use the feedback you receive to revise your response.

3. Sketch a graph for the container that did not have a corresponding graph. Explain your thinking.



STOP

Summary

In today's lesson . . .

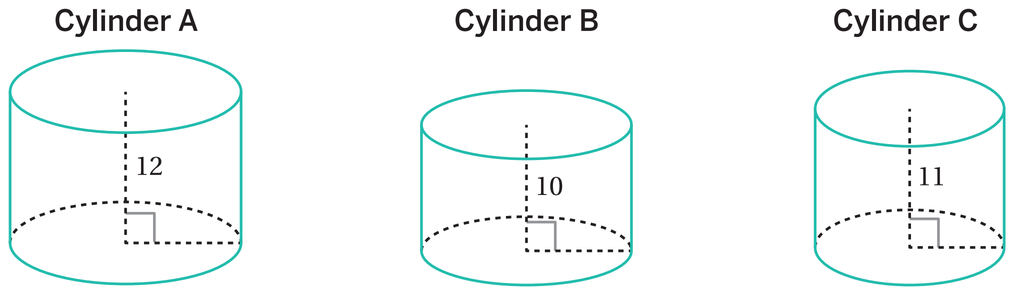
You explored how dimensions of the cylinder are related to each other. When filling different cylinders with water, you noticed that the height of water in a cylinder is a function of its volume. You also saw that the greater the radius, the greater the volume of the cylinder.

Reflect:

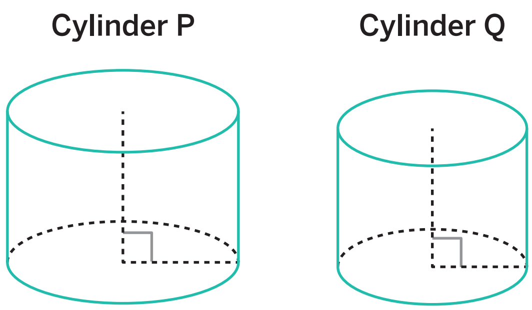
Practice

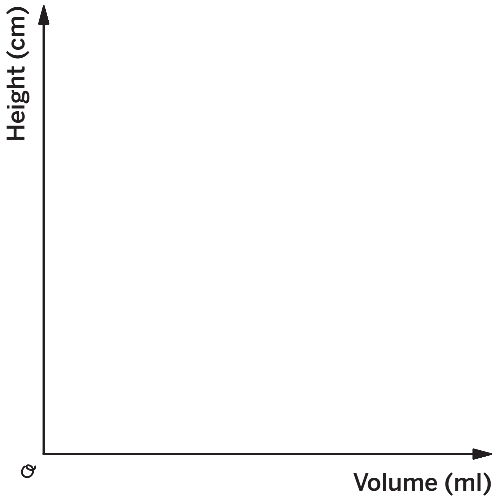
Name: ______ Date: ______ Period: ______

1. Cylinders A, B, and C have the same radius but different heights. Order the cylinders from *least* to *greatest* volume.

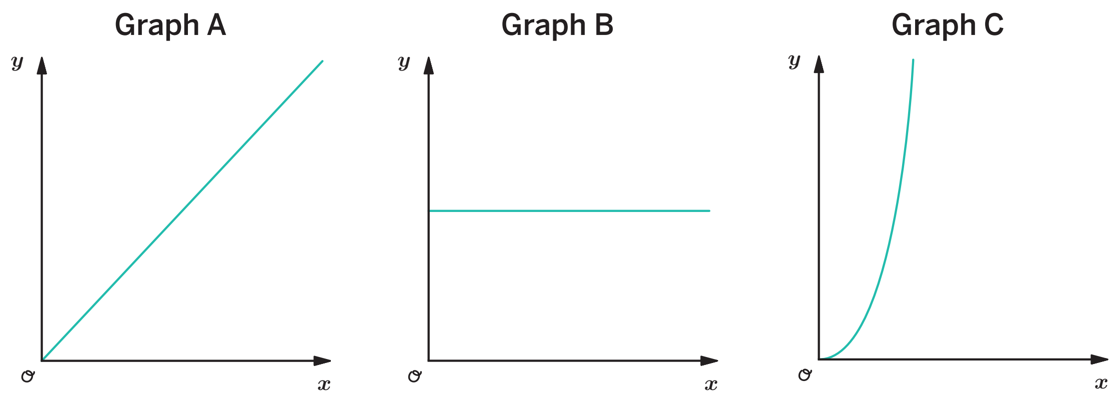


2. Two empty cylinders, P and Q are filled with water. Draw a graph that shows how the heights of the water change as the volume of the water increases in each cylinder. Label your lines P and Q. Explain your thinking.





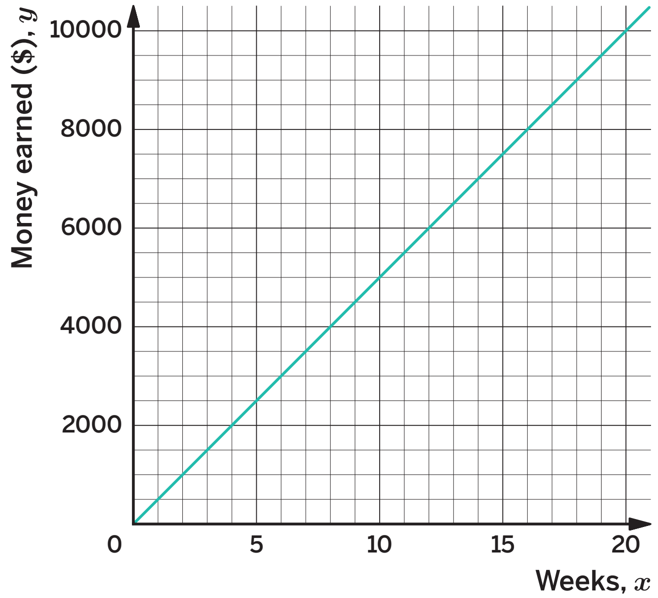
3. Which of the following graphs could represent the volume of water in a cylinder, with a specific radius, as a function of its height? Explain your thinking.



Practice

Name: .... Date: .... Period: ....

4. Mai earns $\$1{,}710$ every 3 weeks by working as a freelance photographer. Jada is also a freelance photographer and her earnings are represented by the graph shown. Who earns more per week? How much more?



5. Select *all* expressions that are equal to the expression $3 \times 3 \times 3 \times 3 \times 3$.

A. 3×5

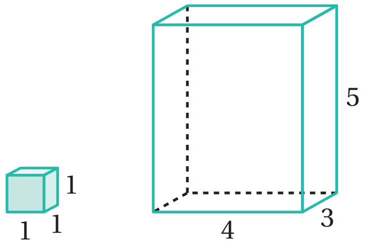
B. 3^5

C. $3^4 \times 3$

D. 5×3

E. 5^3

6. Refer to the unit cube and the rectangular prism shown. How many unit cubes fill the prism completely? Explain your thinking.



Name: ________ Date: ________ Period: ________

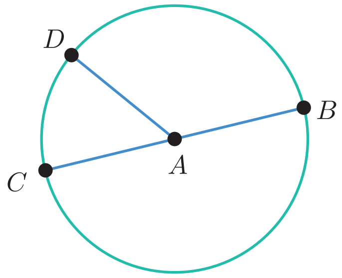
Unit 5 | Lesson 12

The Volume of a Cylinder

Let's explore cylinders and their volumes.

Warm-up A Circle's Dimensions

In the circle, points A, B, C, and D and segments AD and BC are shown.



If the measure of segment AD is 4 units, what is the area of the circle, in square units? Select *all* that apply.

A. 4π

B. 64π

C. 16π

D. $\pi 4^2$

E. approximately 25

F. approximately 50

Log in to Amplify Math to complete this lesson online.

Activity 1 Determining Circular Volumes

For each figure, what is the base area and the exact volume? Record your responses in the table.

Figure A

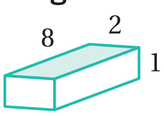


Figure B

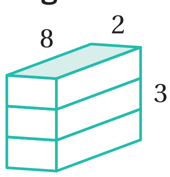


Figure C

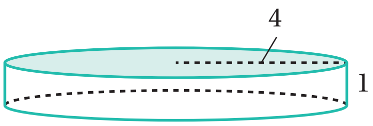


Figure D

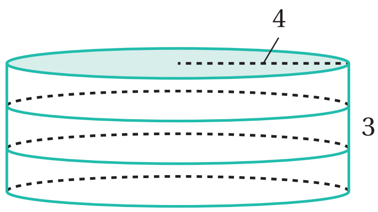
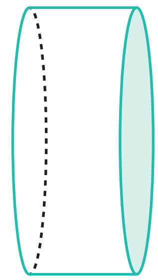


Figure	Area of Base (square units)	Height (units)	Volume (cubic units)
A		1	
B		3	
C		1	
D		3	

Name: ________ Date: ________ Period: ________

Activity 2 Calculating a Cylinder's Volume

1. A cylinder with a height of 4 units and a diameter of 10 units is shown.



   a. Draw and label the height and diameter with their measures.

   b. What is the area of the cylinder's base? Write your response in terms of π.

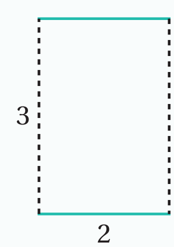
   c. What is the volume of this cylinder? Write your response in terms of π.

2. A silo is a cylindrical container that is used on farms to hold large amounts of goods, such as grain. On a particular farm, a silo has a height of 18 ft and diameter of 6 ft. Determine the approximate amount of cubic feet of grain this silo can hold.

Are you ready for more?

One way to construct a cylinder is to take a rectangle, such as a piece of paper, curl two opposite edges together, and glue them in place.

For the rectangle shown, which has the greater volume – the cylinder created by gluing the two dashed edges together, or the cylinder made by gluing the two solid edges together? Explain your thinking.



STOP

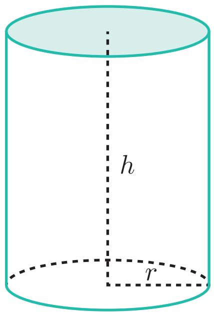
Summary

In today's lesson . . .

You saw how you can determine the volume of a cylinder with radius r and height h using two two mathematical concepts you have previously studied.

- The volume of a rectangular prism is a result of multiplying the area of its base by its height.
- The base of the cylinder is a circle with radius r, so the base area is determined by the expression πr^2.

The base of a cylinder with radius r units has an area of πr^2 square units. If the height is h units, then the volume V, in cubic units, is $V = \pi r^2 h$.

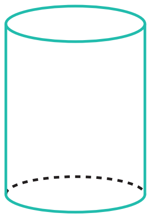


Reflect:

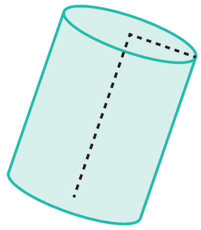
Name: ______________ Date: ______________ Period: ______________

1. For Cylinders A–D, draw the radius and the height. Label each radius with r and each height with h.

Cylinder A

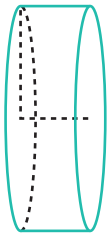


Cylinder B



Cylinder C

Cylinder D



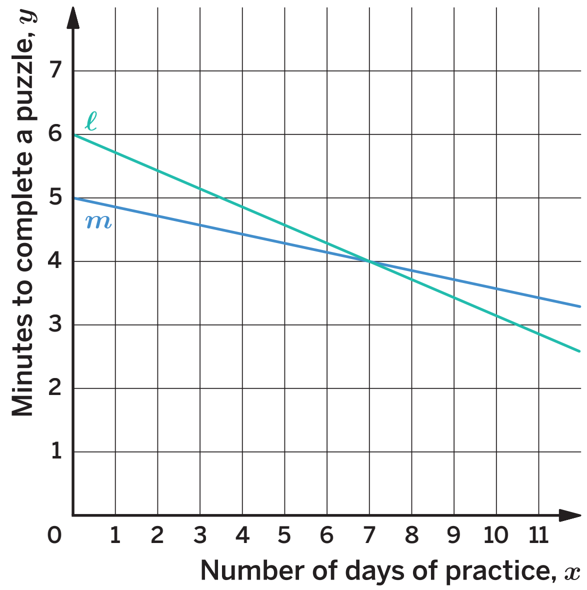
2.
   a. Draw a cylinder. Label the radius 3 units and the height 10 units.
   b. Determine the area of the base. Write your response in terms of π.
   c. Determine the volume of the cylinder. Write your response in terms of π.

3. At a farm, animals are fed bales of hay and buckets of grain. Each bale of hay is in the shape of a rectangular prism. The base has side lengths of 2 ft and 3 ft, and the height is 5 ft. Each bucket of grain is a cylinder with a diameter of 4 ft. The height of the bucket is 5 ft, which is the same height as the bale.
   a. Which is larger in area, the rectangular base of the bale or the circular base of the bucket? Show your thinking.
   b. Which is larger in volume, the bale or the bucket? Show your thinking.

Practice

Name: .... Date: .... Period: ....

4. Two students join a puzzle solving club and they each improve their completion times as they practice. Student A improves their completion times at a faster rate than Student B.



a Match each student with the line that represents their time.

b Which student completed puzzles faster before practicing? Explain your thinking.

5. The expression 6^4 is equal to $1{,}296$. Use this information to evaluate each expression.

a 6^5

b 6^3

6. Complete the table.

Radius	Diameter	Circumference	Area
3 cm			
		16π in.	

Name: .... Date: .... Period: ....

Unit 5 | Lesson 13

Determining Dimensions of Cylinders

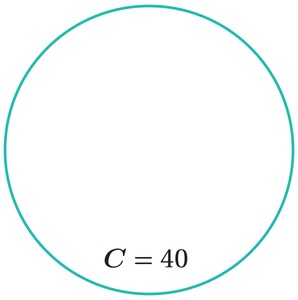
Let's apply the volume formula for a cylinder to determine missing dimensions.

Warm-up Working With Pi

Consider the circle shown.

Shawn claims that the diameter of the circle is 40π and the radius is 20π. Tyler claims that the diameter of the circle is $\frac{40}{\pi}$ and the radius is $\frac{20}{\pi}$.

Who is correct? Explain your thinking.

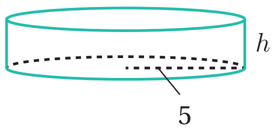


Log in to Amplify Math to complete this lesson online.

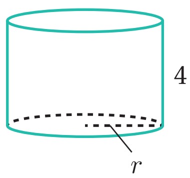
Activity 1 Determining the Unknown Dimension

In Problems 1 and 2, each cylinder has an unknown dimension for you to determine.

1. The cylinder has a radius of 5 units. Its volume is 50π cubic units. What is the height of this cylinder? Show your thinking.



2. The height of the cylinder is 4 cm. Its volume is 36π cm^3. What is the radius of this cylinder? Show your thinking.



Are you ready for more?

Suppose a cylinder has a volume of 36π in^3.

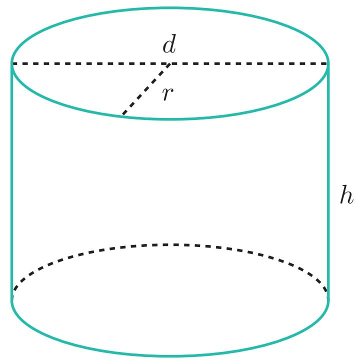
1. Name some different pairs of dimensions for this cylinder.

2. How many different cylinders can you identify that have a volume of 36π in^3?

Name: .... Date: .... Period: ....

Activity 2 What's the Dimension?

Each row of the table has information about a particular cylinder. Determine the missing dimensions and complete the table.



Diameter (units)	Radius (units)	Area of base (square units)	Height (units)	Volume (cubic units)
	3		5	
	4			16π
			11	99π
20				314
			b	$\pi \bullet b \bullet a^2$

STOP

Summary

In today's lesson . . .

You explored how you can determine a missing dimension of a cylinder by using the volume formula. In an earlier lesson, you learned that the volume V of a cylinder with radius r and height h is $V = \pi r^2 h$. You used this formula to solve for that missing dimension.

Volume depends on the radius and height of the cylinder, and, if you know the radius and height, you can determine the cylinder's volume. It is also true that, if you know the volume and one dimension (either radius or height), you can determine the other dimension. You can do so by writing an equation and solving for the missing dimension.

Reflect:

Name: .... Date: .... Period: ....

1. Each row of the table has information about a particular cylinder. Determine the missing dimensions and complete the table.

Diameter (units)	Area of base (square units)	Height (units)	Volume (cubic units)
4		10	
6			63π
	25π	6	

2. A cylinder has a volume of 63π in^3 and a radius of 3 in. What is its height? Show or explain your thinking.

3. A gas company's delivery truck has a cylindrical tank that has a diameter of 14 ft and has a height of 40 ft.

   a. Draw the tank, and label the radius and the height.

   b. How much gas can fit in the tank?

Practice

Name: .... Date: .... Period: ....

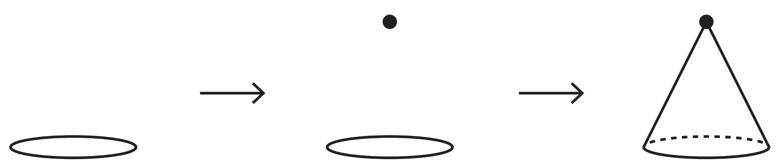
4. A botanist monitors the heights of two plants over time. One plant has an initial height of 8 in. and grows at a rate of 0.5 in. per week. The other plant has a height of 6.25 in. and grows at a rate of 0.75 in. per week.

a Write a system of equations that describes the scenario. Be sure to define your variables.

b When will the plants have the same height? What height will they be? Show or explain your thinking.

5. Refer to this method for quickly sketching a cone.

- Draw an oval.
- Draw a point centered above the oval.
- Connect the edges of the oval to the point.
- Consider which parts of your drawing would be hidden behind the object. Make these parts dashed lines.



Practice by sketching at least two cones.

Name: ______ Date: ______ Period: ______

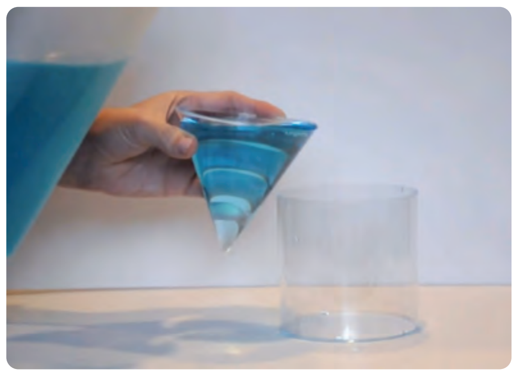
Unit 5 | Lesson 14

The Volume of a Cone

Let's explore cones and their volumes.

Warm-up Notice and Wonder

Refer to the shot about the volumes of a cylinder and a cone from the video, *How Many Cones Does it Take to Fill a Cylinder?*



Illustrative Mathematics

What do you notice? What do you wonder?

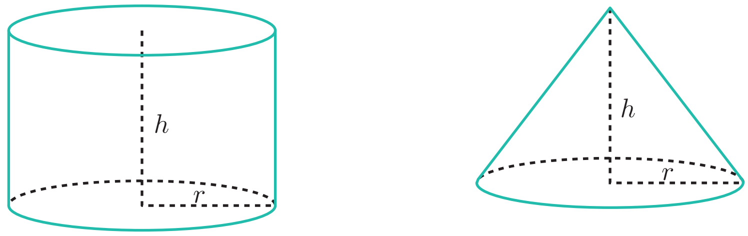
1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 From Cylinders to Cones

The cone and cylinder shown have the same height, and their bases are congruent circles.



1. If the volume of the cylinder is 90 cm^3, what is the volume of the cone? Explain your thinking.

2. If the volume of the cone is 120 cm^3, what is the volume of the cylinder? Explain your thinking.

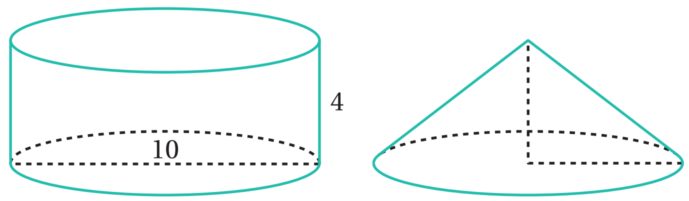
3. If the volume of the cylinder is $V = \pi r^2 h$, what is the volume of the cone? Either write equation for the volume of the cone or explain the relationship between the volumes in words.

Reflect: How were you able to use your prior knowledge to build your self-confidence?

Name: ________ Date: ________ Period: ________

Activity 2 Calculating the Volume of a Cone

1. The cylinder and cone shown have the same height and the same base area.



a Draw and label the radius and height of the cone.

b What is the exact volume of each figure? Write your response in terms of π.

Volume of cylinder: ________ **Volume of cone:** ________

2. A cone-shaped frozen yogurt cup has a radius of 5 cm and a height of 9 cm. How many cubic centimeters of frozen yogurt can the cup hold? Approximate your answer to the nearest hundredth.

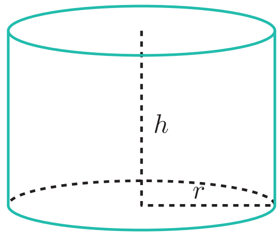
Critique and Correct: Your teacher will display an incorrect response to Problem 1. Work with a partner to critique the response, correct it, and explain your thinking.

STOP

Summary

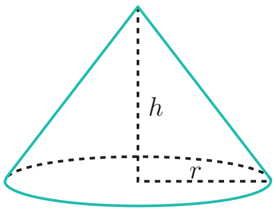
In today's lesson . . .

You saw that, if a cone and a cylinder have the same base and the same height, then the volume of the cone is one third of the volume of the cylinder.



Volume of cylinder:

$V = \pi r^2 h$



Volume of cone:

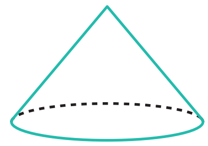
$V = \frac{1}{3}\pi r^2 h$

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. The volume of this cone is 33π cubic units. What is the volume of a cylinder that has the same base area and the same height? Explain your thinking.



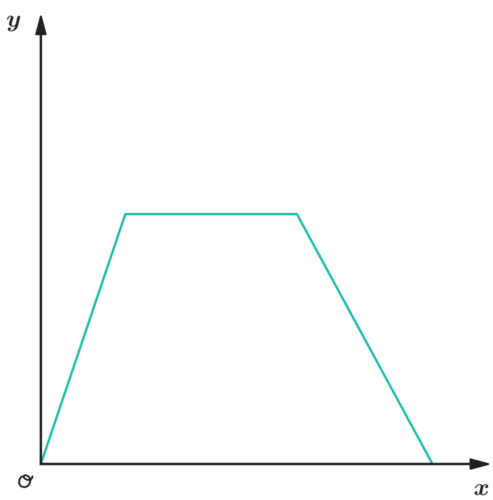
2. A cone-shaped container is used to serve roasted almonds at a hockey game. The container has a diameter of 6 cm and a height of 7 cm.

   a. Draw the cone. Label its height and radius.

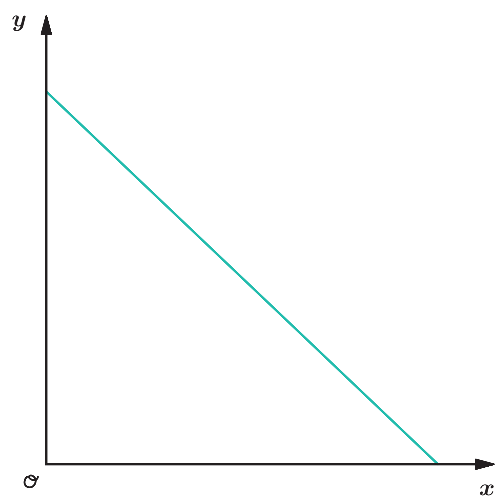
   b. If the container is filled completely with roasted almonds, about how many cubic centimeters can the container hold?

3. In the following graphs, the horizontal axis represents time and the vertical axis represents distance from school. Write a possible story for each graph. Be sure to include what the initial value of each graph represents in your story.

Graph A



Graph B



Graph A story:

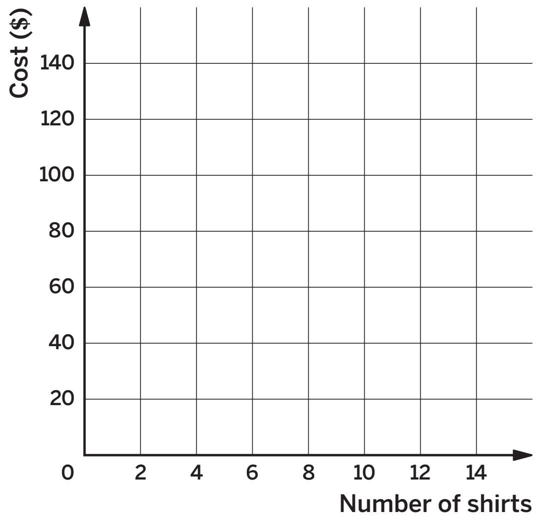
Graph B story:

Practice

Name: .... Date: .... Period: ....

4. Lin wants to get some custom t-shirts printed for her basketball team. If 10 or fewer shirts are ordered, each t-shirt costs $10. If 11 or more shirts are ordered, each costs $9.

a Create a graph that shows the total cost of buying shirts, for 0 through 15 shirts.

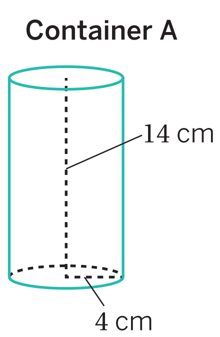


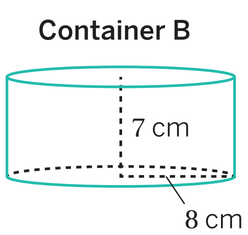
b There are 10 people on the team. Do they save money if they buy an extra shirt? Explain your thinking.

c What is the slope of the graph between 0 and 10? What does it mean in the story?

d What is the slope of the graph between 11 and 15? What does it mean in the story?

5. Two containers of grain are sold at the market. Each container costs the same price, $4. Which is the better buy? Explain your thinking.





Name: ______ Date: ______ Period: ______

Unit 5 | Lesson 15

Determining Dimensions of Cones

Let's apply the volume formula for a cone to determine missing dimensions.

Warm-up Number Talk

For each equation, determine what value, if any, would make it true.

a. $27 = \frac{1}{3}h$

b. $27 = \frac{1}{3}r^2$

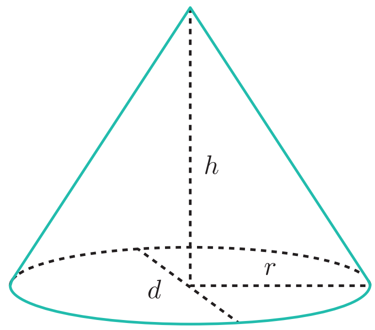
c. $12\pi = \frac{1}{3}\pi a$

d. $12\pi = \frac{1}{3}\pi b^2$

Log in to Amplify Math to complete this lesson online.

Activity 1 Determining Unknown Dimensions

Each row of the table has information about dimensions of a particular cone. Determine the missing values and complete the table.

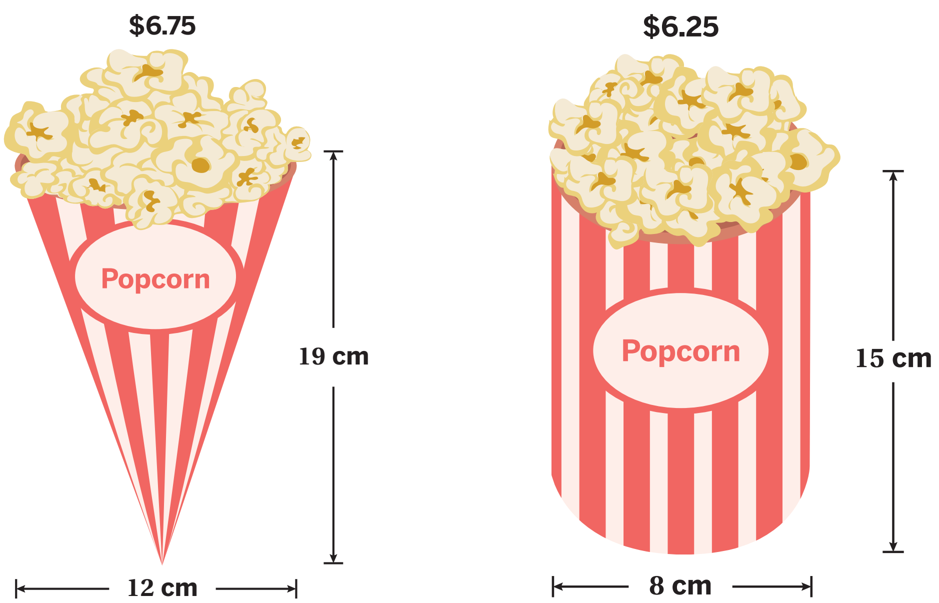


Diameter (units)	Radius (units)	Area of base (square units)	Height (units)	Volume (cubic units)
	4		9	
		144π	$\frac{1}{4}$	
20				200π
			3	3.14
			b	$\frac{1}{3} \cdot \pi \cdot b \cdot a^2$

Name: ______________________ Date: __________ Period: __________

Activity 2 Which Is the Better Deal?

A movie theater offers two containers of popcorn for the prices advertised:



Which container is the better value? Show or explain your thinking.

Are you ready for more?

To make the cone of popcorn have the same volume as the cylinder of popcorn, what must be the height of the cylinder?

STOP

Summary

In today's lesson . . .

As you saw with cylinders, the volume V of a cone is a function of the radius r of the base and the height h. If you know the radius and the height, you can determine the volume, by using the formula for the volume of a cone.

$$V = \frac{1}{3}\pi r^2 h$$

If you know the volume and one of the dimensions, either the radius or height, you can determine the other dimension by writing and solving an equation using the formula for the volume of a cone.

Reflect:

Name: ______________ Date: ______________ Period: ______________

Practice

1. The volume of a cylinder is 175π cubic units. What is the exact volume of a cone that has the same base area and the same height? Explain your thinking.

2. A cone has volume 12π in^3. Its height is 4 in. What is its radius? Show or explain your thinking.

3. Three cones have the same volume of 192π cm^3. Determine the height of each cone.

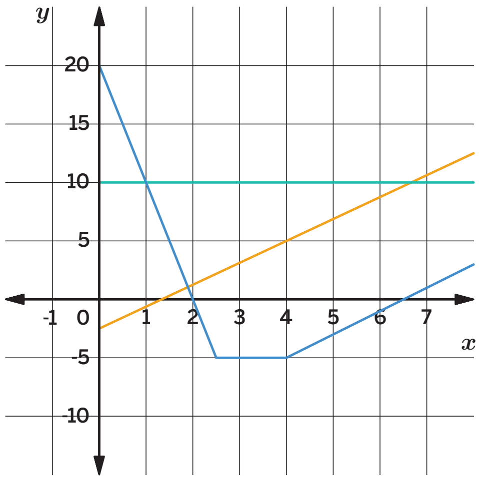
Radius (cm)	2	3	4
Height (cm)			

Practice

Name: .... Date: .... Period: ....

4. Three people are playing near the water. Person A stands on the dock. Person B starts at the top of a pole and ziplines into the water and then climbs out of the water. Person C climbs out of the water and up the zipline pole.

Match the people to the graphs where the horizontal axis represents time in seconds and the vertical axis represents height above the water level in feet. Label the graphs a for Person A, b for Person B, and c for Person C.



5. A room has a ceiling height of 15 ft. An architect wants to include a window that is 6 ft tall. The distance between the floor and the bottom of the window is b ft. The distance between the ceiling and the top of the window is a ft. This relationship can be described by the equation $a = 15 - (b + 6)$.

a Based on the equation, which is the independent variable? Explain your thinking.

b If the architect wants b to be 3, what does this mean? What value of a would work with the given value for b?

c The customer wants the window to have 5 ft of space above it. Is the customer describing variable a or b? What is the value of the other variable?

6. A baseball fits snugly inside a transparent display cube. The length of an edge of the cube is 2.9 in. Is the baseball's volume *greater than, less than,* or *equal to* 2.9^3 in^3? Explain your thinking.

Name: .... Date: .... Period: ....

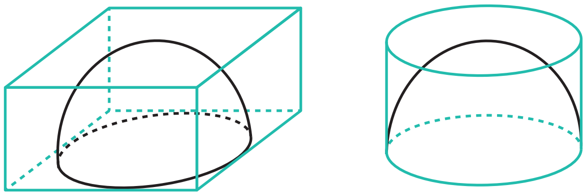
Unit 5 | Lesson 16

Estimating a Hemisphere

Let's estimate the volume of hemispheres using figures we know.

Warm-up Which One Fits Better?

A hemisphere is half of a sphere. The diagram shows the same hemisphere placed snugly inside a square prism and inside a cylinder.

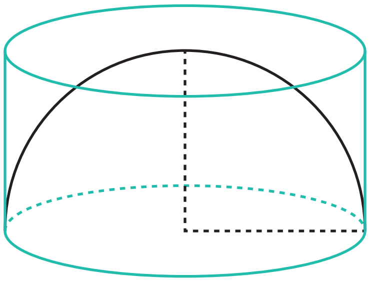


1. If the radius of the hemisphere is 4 cm, what is the height of the square prism and the cylinder? What are the dimensions of the square base? Explain your thinking.

2. Which one — the square prism or the cylinder — is closer in volume to the hemisphere? Explain your thinking.

Activity 1 Estimating Hemispheres (Part 1)

This diagram shows a hemisphere with a radius of 1 unit placed snugly inside a cylinder.



1. What is the radius of the cylinder? What is the height of the cylinder? Label these dimensions on the diagram.

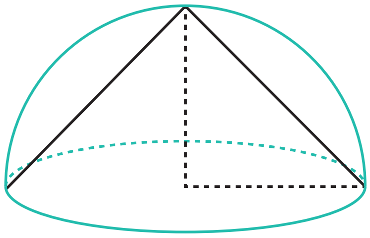
2. Calculate the volume of the cylinder. Write your response in terms of π.

3. Estimate the volume of the hemisphere. Explain your thinking.

Name: ________________ Date: ________ Period: ________

Activity 2 Estimating Hemispheres (Part 2)

This diagram shows a cone placed snugly inside a hemisphere. The hemisphere has a radius of 1 unit.



1. What is the radius of the cone? What is the height of the cone? Label these dimensions on the diagram.

2. What is the volume of the cone? Write your response in terms of π.

3. Estimate the volume of the hemisphere. Explain your thinking.

4. Compare your estimate for the hemisphere with the cone inside it to your estimate of the hemisphere inside the cylinder. Estimate the volume of the hemisphere based on the calculations from Activity 1 and Activity 2.

Activity 3 Estimating Hemispheres (Part 3)

A hemisphere-shaped security mirror fits exactly inside a box with a square base that has an edge length of 12 in. What is a reasonable estimate for the volume of this mirror? Round your responses to the nearest hundredth. Show or explain your thinking.

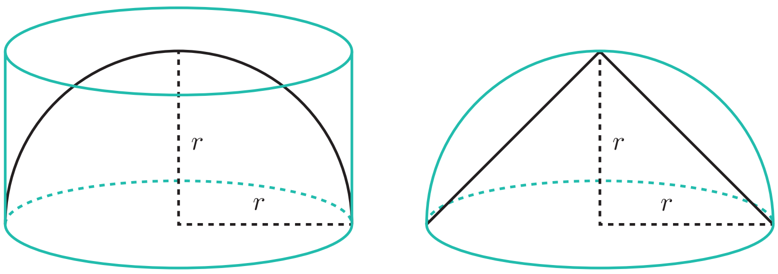
STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You estimated the volume of a ***hemisphere***, which is half of a sphere, by comparing it to other shapes for which you know the volume. You saw that the volume of a hemisphere with a radius of r is less than the volume of a cylinder with a radius and height of r, but greater than a cone with radius and height of r.



For now, you can estimate that the volume of the hemisphere with radius r is about $\frac{2}{3}\pi \cdot r^3$ cubic units.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. Complete the following table for different hemispheres.

Radius	6 cm		$\frac{1000}{3}$ m		9.008 ft	
Diameter		6 cm		$\frac{1000}{3}$ m		9.008 ft

2. A hemisphere fits snugly inside a cylinder with a radius of 4 cm. A cone fits snugly inside the same hemisphere.

a. What is the volume of the cylinder? Round to the nearest hundredth.

b. What is the volume of the cone? Round to the nearest hundredth.

c. Estimate the volume of the hemisphere by calculating the average of the volumes of the cylinder and cone.

3. For each part, select two equations that might make a system of linear equations with the following conditions.

a. The system does not have a solution.

A. $y = 3$

B. $3x = y$

C. $y + 2x = 0$

D. $y = 3x - 4$

b. The only solution the system has is $(0, 0)$.

A. $y = 3$

B. $3x = y$

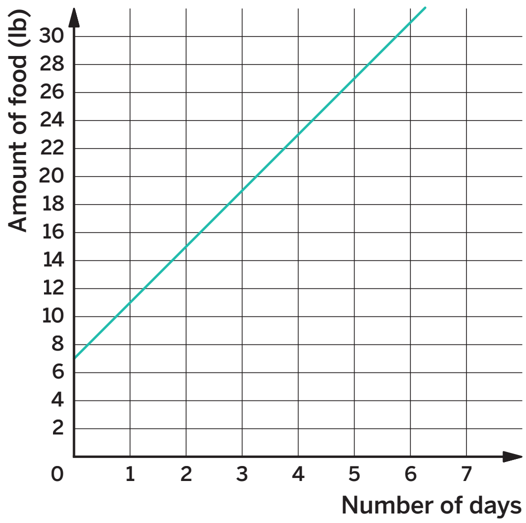
C. $y + 2x = 0$

D. $y = 3x - 4$

Name: ________ Date: ________ Period: ________

Practice

4. Jada and Bard have been collecting food for a local food bank and have modeled their food collection. Jada's collection can be modeled with the equation $p = 7d$, where d represents the number of days collected and p is the amount of food in pounds. Bard's food collection is represented in the graph shown.



a. Who is collecting food at a faster rate? Explain your thinking.

b. Who started out with more food collected? Explain your thinking.

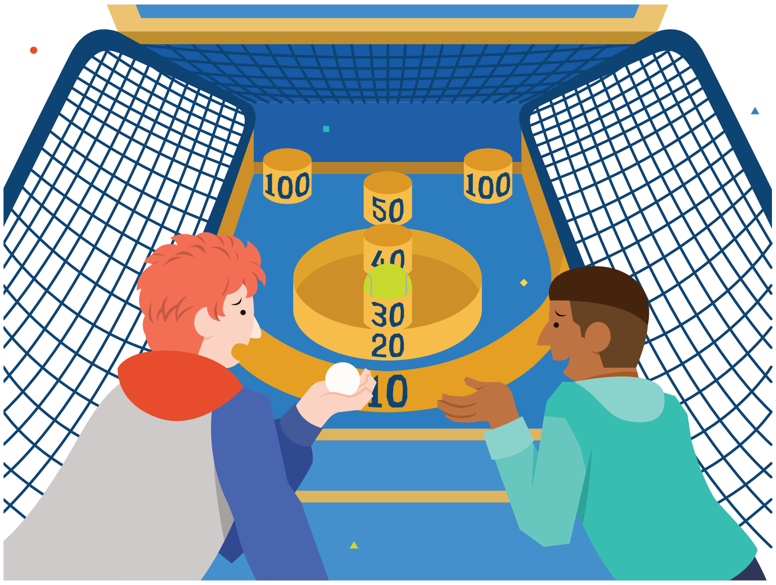
5. Complete the missing cells in the table. The first row has been completed for you.

Exponent	Expanded
3^4	$3 \cdot 3 \cdot 3 \cdot 3$
4^2	
	$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5$
	$-2 \cdot (-2) \cdot (-2)$

Unit 5 | Lesson 17

The Volume of a Sphere

Let's explore spheres and their volumes.



Warm-up Notice and Wonder

Refer to the still shot from the video, *Volume of a Cylinder, Sphere, and Cone*.

What do you notice? What do you wonder?


Illustrative Mathematics

1. I notice . . .

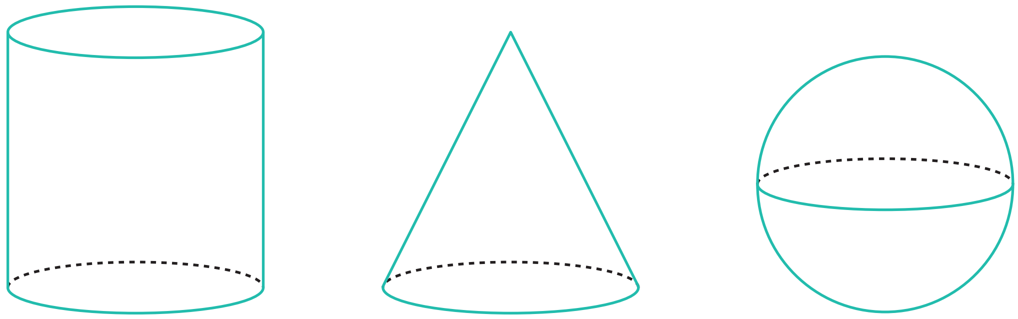
2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: ____________ Period: ____________

Activity 1 A Sphere in a Cylinder (Part 1)

The cylinder, cone, and sphere all have the same radius and height.



1. The radius of the sphere is 5 in. Draw and label the radius and height on each figure.

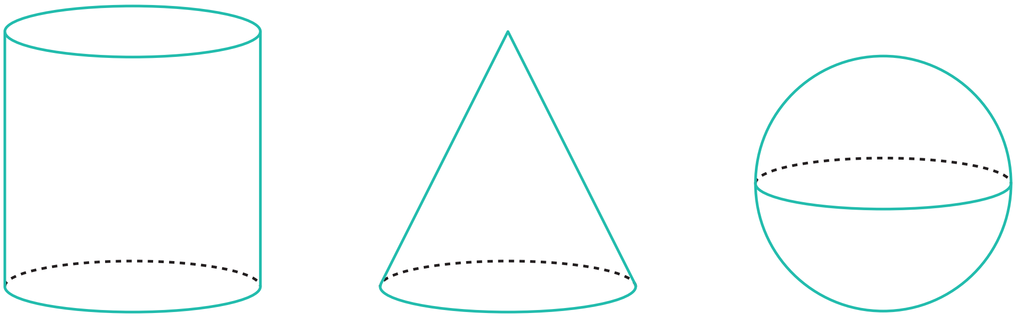
2. What is the volume of the cylinder? Express your responses in terms of π.

3. What is the volume of the cone? Write your response in terms of π.

4. What is the volume of the sphere? Write your response in terms of π and explain your thinking.

Activity 2 A Sphere in a Cylinder (Part 2)

The cylinder, cone, and sphere all have the same radius and height.



1. The radius of the sphere is r units. Draw and label the radius and height on each object in terms of r.

2. What is the volume of the cylinder in terms of r? Show your thinking.

3. What is the volume of the cone in terms of r? Show your thinking.

4. What is the volume of the sphere in terms of r? Show your thinking.

5. The volume of the cone is $\frac{1}{3}$ the volume of the cylinder with the same radius and same height. The volume of the sphere is what fraction of the volume of the cylinder with the same radius and height?

Name: ______ Date: ______ Period: ______

Activity 3 How Are the Volumes Related?

Use the relationship between the volumes of a cylinder, cone, and sphere to solve the following problems. Write your responses in terms of π.

1. A cylinder and a sphere have the same radii and the height of the cylinder is the same as the diameter of the sphere.

a If the cylinder has a volume of 144π cm^3, what is the volume of the sphere?

b If the sphere has a volume of 18π cm^3, what is the volume of the cylinder?

2. A cone and a sphere have the same dimensions.

a If the cone has a volume of 144π cm^3, what is the volume of the sphere?

b If the sphere has a volume of 18π cm^3, what is the volume of the cone?

Historical Moment

Archimedes

The Greek mathematician Archimedes (287–212 BCE) was the first person to discover that the volume of a sphere is $\frac{2}{3}$ the volume of a cylinder with the same dimensions. This was his favorite mathematical discovery, so much that it was engraved on his tombstone.

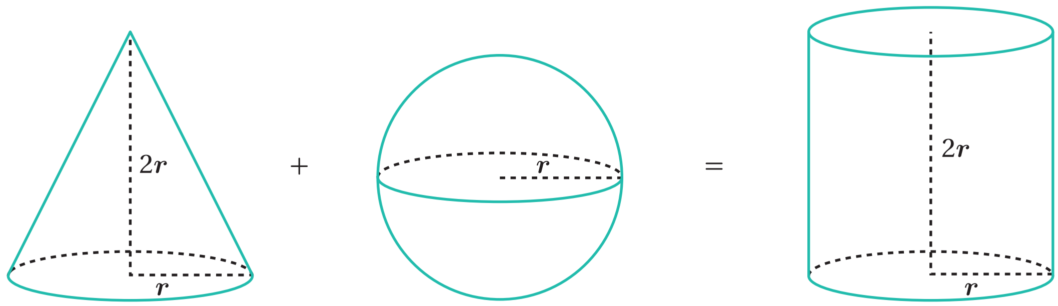
STOP

Summary

In today's lesson . . .

You related the volumes of a sphere, a cone, and a cylinder with the same dimensions. If you filled the cone and sphere with water and poured that water into the cylinder, then the cylinder would be completely filled. This means the volume of a cone and sphere (with the same dimensions) together equals the volume of the cylinder.

In previous lessons, you learned that the volume of a cone with the same height and radius is $\frac{1}{3}$ the volume of the cylinder. Therefore, the volume of the sphere must be $\frac{2}{3}$ of the volume of the cylinder.



Volume of cone + **Volume of sphere** = **Volume of cylinder**

$\left(\frac{1}{3}V \text{ of cylinder}\right)$ $\left(\frac{2}{3}V \text{ of cylinder}\right)$

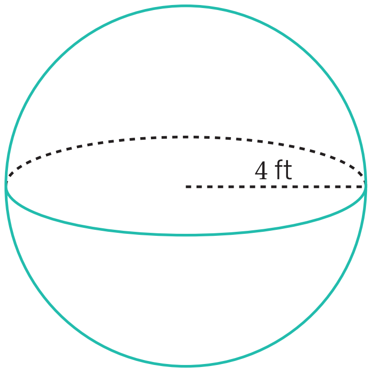
You saw the volume of a cylinder with radius r and height $2r$ is determined by $2\pi r^3$, so the volume of of a sphere is determined by $\frac{2}{3} \cdot 2\pi r^3$, which is equivalent to $\frac{4}{3}\pi r^3$.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Recall that the volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$. What is the volume of the sphere with a radius of 4 ft? Write your response in terms of π.



2. A cube's volume is 512 cubic units.

   a. What is the length of its edge?

   b. If a sphere fits snugly inside this cube, what is the volume of the sphere?

   c. What fraction of the cube is taken up by the sphere? What percent is this? Show or explain your thinking.

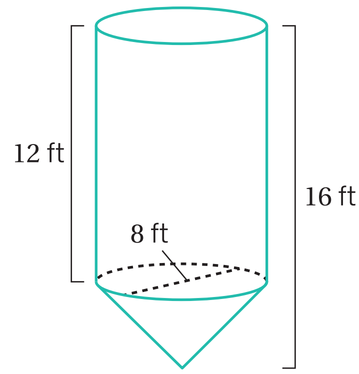
3. Match the description of each sphere to its correct volume.

a. Sphere A: radius of 4 cm	....	288π cm^3
b. Sphere B: diameter of 6 cm	....	$\frac{256}{3}\pi$ cm^3
c. Sphere C: radius of 8 cm	....	36π cm^3
d. Sphere D: radius of 6 cm	....	$\frac{2048}{3}\pi$ cm^3

Practice

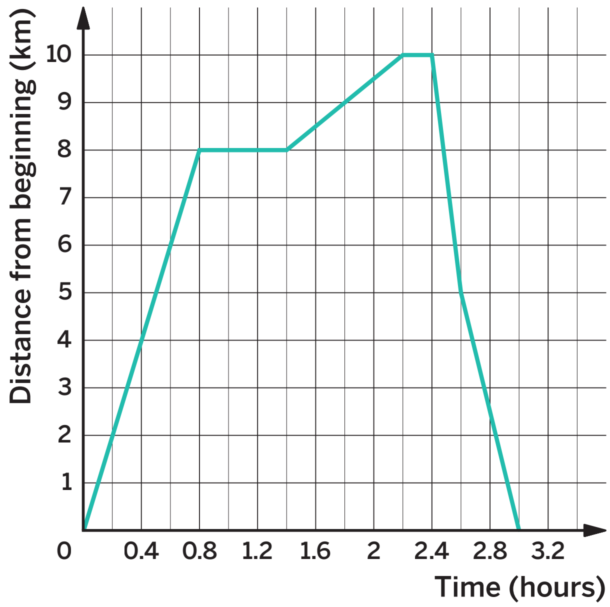
Name: .... Date: .... Period: ....

4. A grain silo has a cone-shaped spout on the bottom in order to regulate the flow of grain out of the silo. The diameter of the silo is 8 ft. The height of the cylindrical part of the silo above the cone spout is 12 ft while the height of the entire silo is 16 ft.



Approximately, how many cubic feet of grain can the entire silo hold? Show or explain your thinking.

5. The graph shows a trip on a bike trail.



a When was the bike rider going the fastest?

b During what times did the rider stop?

c Not including when the bike rider was stopped, when was the bike rider going the slowest?

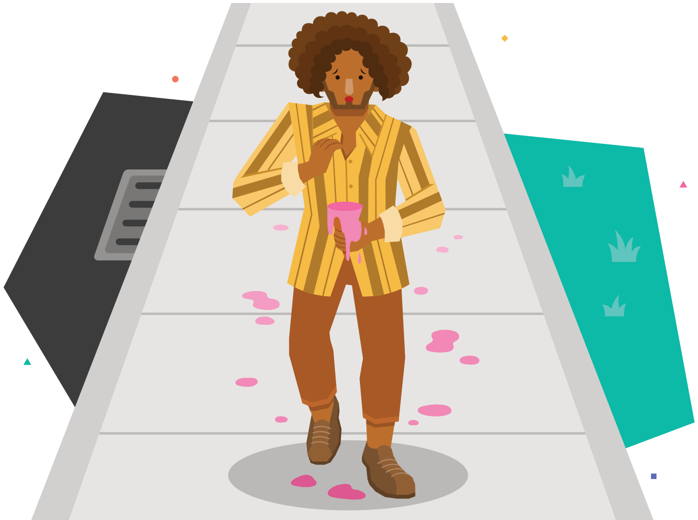
d During what times was the rider going away from the beginning of the trail?

e During what times was the rider going back toward the beginning of the trail?

6. Determine the volume of a sphere with a radius of 8 cm. Write your response in terms of π and approximate the volume by rounding to the nearest hundredth.

Name: ____________ Date: ____________ Period: ____________

Unit 5 | Lesson 18



Cylinders, Cones, and Spheres

Let's determine the volumes of circular solids.

Warm-up Spherical Arguments

Four students each calculated the volume of a sphere with a radius of 9 cm. Each student determined a different answer.

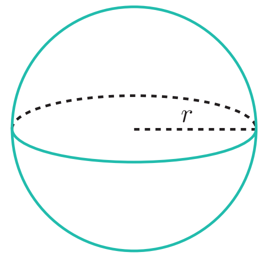
- **Han claimed it is 108 cm^3.**
- **Jada calculated 108π cm^3.**
- **Tyler calculated 972 cm^3.**
- **Mai claimed it is 972π cm^3.**

Do you agree with any of them? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 A Sphere's Radius

Theoretical Physicist Stephen Hawking studied the mass and volumes of one of the great unknowns of our universe — black holes, which are spheres (unless they spin . . .).



This sphere has an unknown dimension r.

1. The volume of this sphere with radius r is 36π cubic units. Shawn wrote the equation: $36\,\pi = \frac{4}{3}\pi \cdot r^3$. Explain how Shawn wrote this equation.

2. Determine the value of r for this sphere. Show or explain your thinking.

Featured Mathematician

Stephen Hawking

Suppose you wanted to determine the volume of the most unusual sphere in the universe, a black hole – a place in space where the pull of gravity is so strong that nothing can escape. There is still much we do not know about black holes, but, thanks to Stephen Hawking, we are much closer to understanding these extraordinary mysteries. Hawking, a British mathematician and theoretical physicist, studied the mass and volumes of black holes and discovered something unexpected: Black holes can "evaporate," which means they lose mass through a phenomenon now called Hawking radiation.

Stephen Hawking' by ²°¹°°. Public Domain via Wikimedia Commons.

Name: ______ Date: ______ Period: ______

Activity 2 Melted Frozen Yogurt

A spherical scoop of frozen yogurt with a 3-in. diameter has melted.

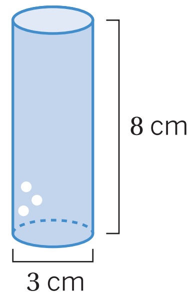
1. How tall must a cone of the same diameter be to hold the melted frozen yogurt?

Discussion Support:
For each problem, will you use the volume formulas or the relationship between the volumes of circular solids? Be prepared to share your strategy with the class.

2. How tall must a cylinder of the same diameter be to completely be filled by the melted frozen yogurt?

Activity 3 The Right Fit

A cylinder with a diameter of 3 cm and a height of 8 cm is filled with water. Determine which figure, if any, could hold all of the water from the cylinder. Explain your thinking.



1. A cone with a height of 8 cm and a diameter of 3 cm

2. A cylinder with a diameter of 6 cm and height of 2 cm

3. A rectangular prism with a length of 3 cm, width of 4 cm, and height of 8 cm

4. A sphere with a radius of 2 cm

STOP

Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You determined the radius of a sphere when given its volume and explored how changes in the dimensions affect the volume of various circular solids.

Volume of a cylinder	Volume of a cone	Volume of a sphere, where $h = 2r$.
$V =$ Area of the base • Height $V = \pi r^2 \cdot h$	$V = \frac{1}{3} \cdot$ Volume of cylinder $V = \frac{1}{3}\pi r^2 \cdot h$	$V = \frac{2}{3} \cdot$ Volume of cylinder $V = \frac{2}{3}\pi r^2 \cdot h$ $V = \frac{2}{3}\pi r^2 \cdot 2r$ $V = \frac{4}{3}\pi r^3$

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Calculate the volume of the following shapes with the given information. For the first three questions, give each response both in terms of π and by using 3.14 to approximate. Make sure to include units.

   a. Cylinder with a height of 6 in. and a diameter of 6 in.

   b. Cone with a height of 6 in. and a radius of 3 in.

   c. Sphere with a diameter of 6 in.

   d. How are these three volumes related?

2. A soccer ball has a diameter of 22 cm. How much air fits inside the ball when it is fully inflated?

Name: .... Date: .... Period: ....

Practice

3. A coin-operated bouncy ball dispenser has a large glass sphere that holds many spherical balls. The large glass sphere has a radius of 9 in. Each bouncy ball has a radius of 1 in. and sits inside the dispenser. If there are 243 bouncy balls in the large glass sphere, what proportion of the large glass sphere's volume is taken up by bouncy balls? Show or explain your thinking.

4. The tables correspond to inputs and outputs. For each table, determine if it could represent a function or could not represent a function. Explain your thinking.

Input	Output
1	0
2	0
3	0
4	0
5	0

Input	Output
0	1
0	2
0	3
0	4
0	5

5. Which change do you think would increase the volume of a cylinder the most — doubling the radius or doubling the height? Explain your thinking.

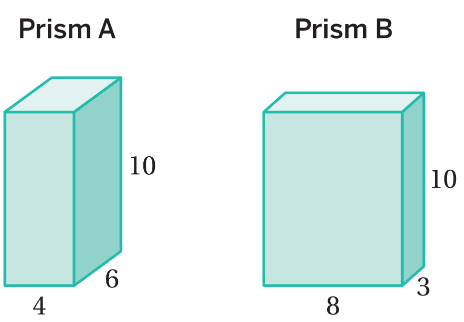
Unit 5 | Lesson 19

Scaling One Dimension

Let's see how changing one dimension changes the volume of a shape.

Warm-up Which One Has a Greater Volume?

Determine which right rectangular prism has the greater volume. Explain your thinking.

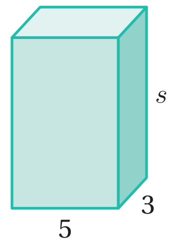


Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

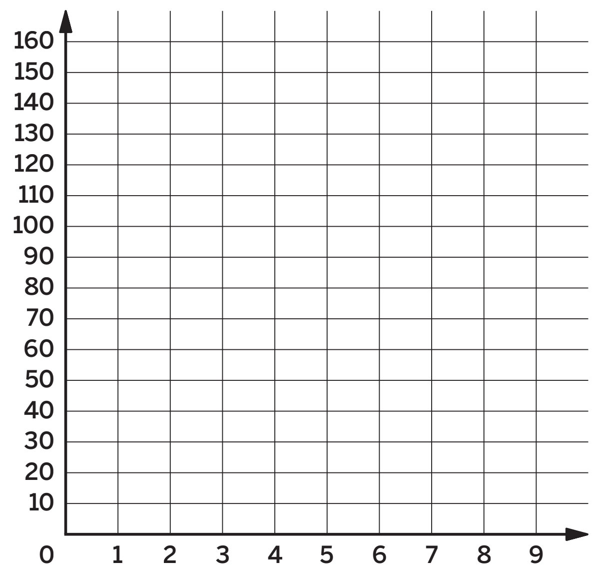
Activity 1 Double the Edge

There are many right rectangular prisms with one edge of length 5 units and another edge of length 3 units. Let s represent the length of the third edge and V represent the volume of these prisms.



1. Write an equation that represents the relationship between s and V.

2. Graph this equation and label the axes.



3. Let's determine what happens to the volume if the third edge length is doubled.

a Choose a side length.

b What is the volume?

c Double the side length from part a.

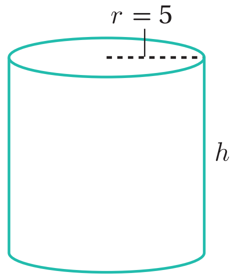
d What is the volume of the prism with the side length chosen from part c.

4. Make a conjecture about what happens to the volume if you double the edge length s? Where do you see this in the graph? Where do you see it in the equation?

Plan ahead: What will you do to analyze the problem in order to form a conjecture?

Activity 2 Halve the Height

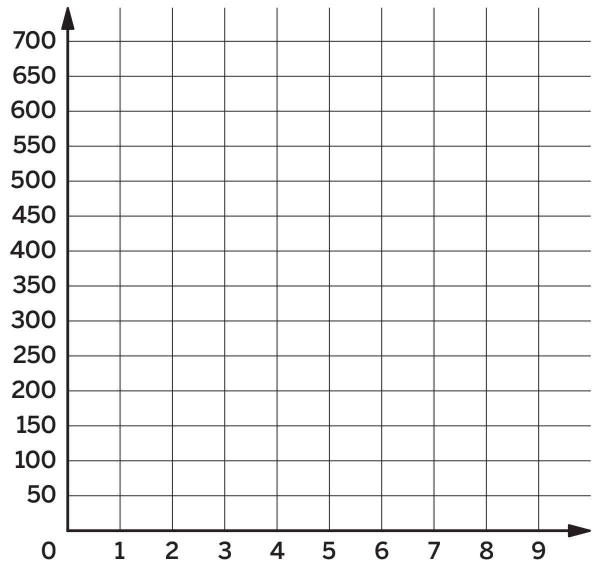
There are many cylinders with a radius of 5 units. Let h represent the height and V represent the volume of each cylinder.



1. Write an equation that represents the relationship between V and h. Use 3.14 as an approximation for π.

2. Graph this equation and label your axes.

3. Make a conjecture about what happens to the volume if you halve the height? Where can you see this in the graph? Where can you see this in the equation?



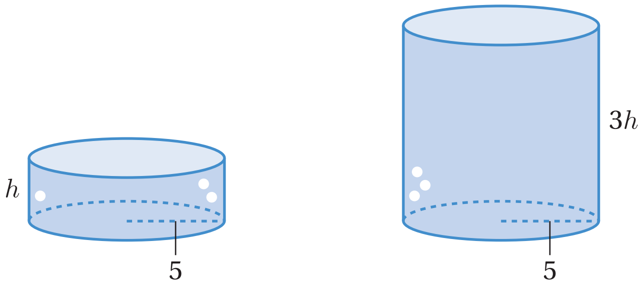
STOP

Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw how changing a single dimension affects the volume. Imagine a cylinder with a radius of 5 cm being filled with water. As the height of the water increases, the volume of water increases proportionally.



$V = \pi \cdot 5^2 \cdot h = 25\pi h$ $V = \pi \cdot 5^2 \cdot 3h = 3(25\pi h)$

In general, when one quantity in a proportional relationship changes by a given factor, the other quantity changes by the same factor.

Remember that proportional relationships are examples of linear relationships, which can also be thought of as functions. So, in this example, V, the volume of water in the cylinder, is a function of the height h of the water.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A cylinder has a volume of 48π cm^3 and a height h. Complete this table for the volumes of cylinders with the same radius but different heights.

Height (cm)	Volume (cm^3)
h	48π
$2h$	
$5h$	
$\frac{h}{2}$	
$\frac{h}{5}$	

2. A cylinder has a radius of 3 cm and a height of 5 cm.

   a. What is the volume of the cylinder? Write your answers in terms of π and approximate your answers to the nearest hundredth.

   b. What is the volume of the cylinder when the height is tripled?

   c. What is the volume of the cylinder when the height is halved?

3. A graduated cylinder that is 24 cm tall can hold 1 liter of water. What is the height of the 500 ml mark? The 250 ml mark? Explain your thinking. **Hint:** 1 liter is equal to 1,000 ml, and 1 liter is equal to 1,000 cm^3.

Practice

Name: ______ Date: ______ Period: ______

4. Evaluate each expression.

a $\left(\frac{1}{2}\right)^2 \cdot 8$

b $10 - 2^3$

c $2 \cdot 4^2$

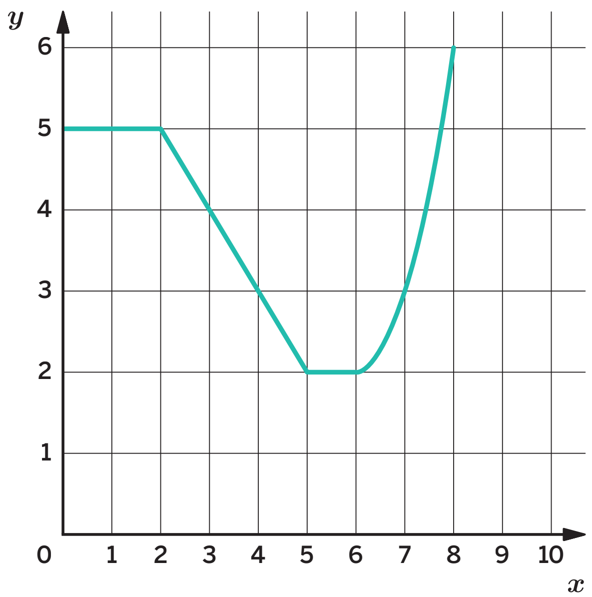
d $3^2 + 2^3$

e $\left(\frac{1}{3}\right)^2 \cdot 3^2$

f $(2 \cdot 3)^2$

5. A frozen yogurt shop offers two cones. The waffle cone holds 12 oz and is 5 in. tall. The regular cone also holds 12 oz, but is 8 in. tall. Which cone has a larger radius? Explain your thinking.

6. Describe each section of the graph using the words *nonlinear, linear, increasing, decreasing, constant*, etc.



Scaling Two Dimensions

Let's change more dimensions of shapes.

Warm-up Tripling Statements

Suppose m, n, a, b, and c all represent positive integers. Consider these two equations:

$m = a + b + c$

$n = abc$

1. Which of these statements are true? Select *all* that apply.
   - **A.** If a is tripled, then m is tripled.
   - **B.** If a, b, and c are all tripled, then m is tripled.
   - **C.** If a is tripled, then n is tripled.
   - **D.** If a, b, and c are all tripled, then n is tripled.
   - **E.** If b and c are tripled, n then is 9 times larger.

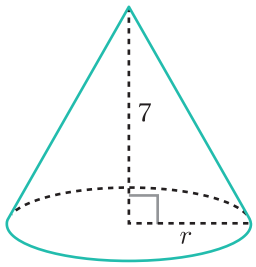
2. Create a true statement of your own about one of the equations.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Playing With Cones

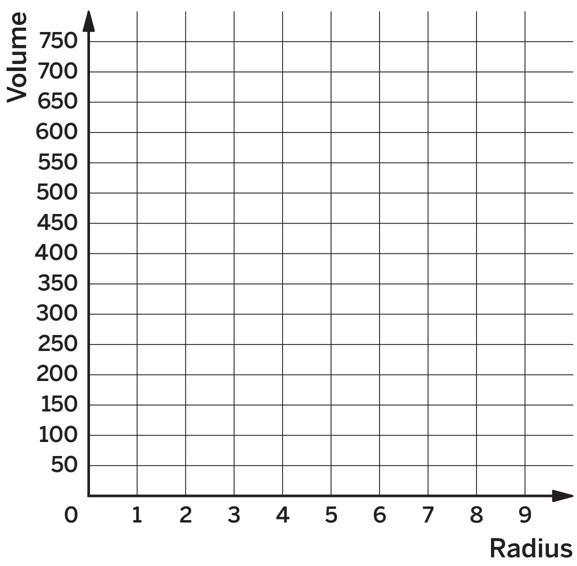
There are many cones with a height of 7 units. Let r represent the radius and V represent the volume of each cone.



1. Write an equation that expresses the relationship between V and r. Use 3.14 as an approximation for π.

2. Complete the table of values and graph the ordered pairs.

Radius (units)	Volume (cubic units)
1	
2	
3	
4	
5	
6	
7	
8	



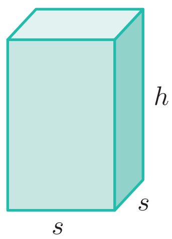
3. What happens to the volume if the radius is tripled? Where do you see this in the graph? Where do you see it algebraically?

Activity 2 Which One Has a Greater Volume?

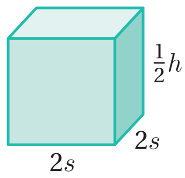
For each set of solids, determine which one has a greater volume. Explain your thinking.

1\.

a

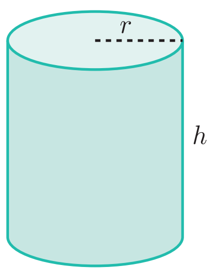


b

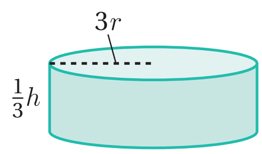


2\.

a



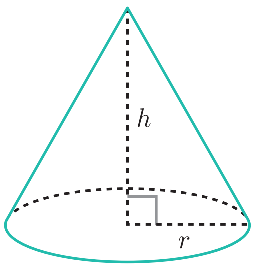
b



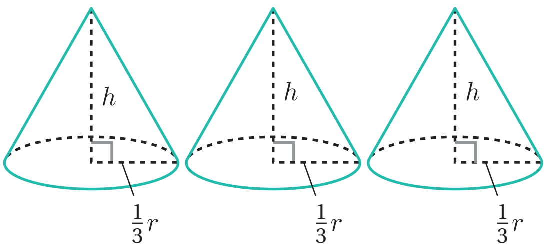
Name: ______ Date: ______ Period: ______

Activity 2 Which One Has a Greater Volume? (continued)

3. a



b



Are you ready for more?

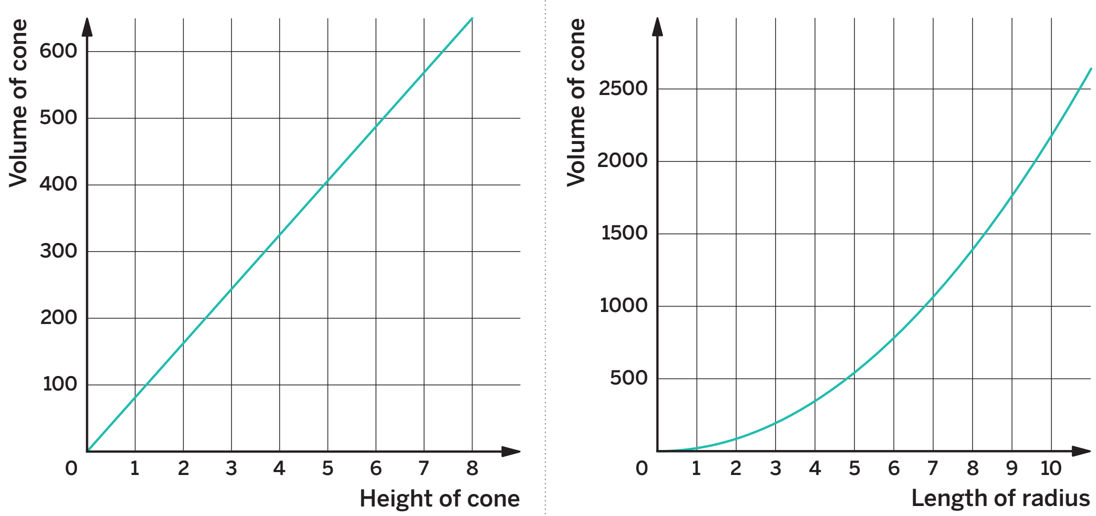
If a sphere's radius is doubled, how does this affect the volume? Explain your thinking.

STOP

Summary

In today's lesson . . .

You saw how changes in two dimensions of a solid figure changes the volume of the solid. In particular, changing the radius or height of a cone results in a change in its volume, $V = \frac{1}{3}\pi r^2 h$. Scaling the height gives a constant change in the volume of the cone. This is represented by the linear function shown in the graph on the left. On the other hand, scaling the radius changes the volume of the cone by a nonconstant amount. This is represented with a nonlinear function, as seen in the graph on the right.



If the height is multiplied by a factor of a, the volume is multiplied by a factor of a.

$$V = \frac{1}{3}\pi r^2 (ah)$$

$$V = a\left(\frac{1}{3}\pi r^2 h\right)$$

If the radius is multiplied by a factor of a, then the volume is multiplied by a factor of a^2.

$$V = \frac{1}{3}\pi (ar)^2 h$$

$$V = a^2\left(\frac{1}{3}\pi r^2 h\right)$$

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. There are many cylinders with a height of 18 m. Let r represent the radius in meters and let V represent the volume in cubic meters of these cylinders.

r(m)	V(m^3)
1	

a. Write an equation that represents the volume V as a function of the radius r.

b. Complete the table, giving three possible examples.

c. If the radius of a cylinder is doubled, does the volume double? Explain your thinking.

d. Is the graph of the volume as a function of the radius linear? Explain how you know.

2. A cone has a radius of 3 units and a height of 4 units.

a. What is the exact volume of this cone?

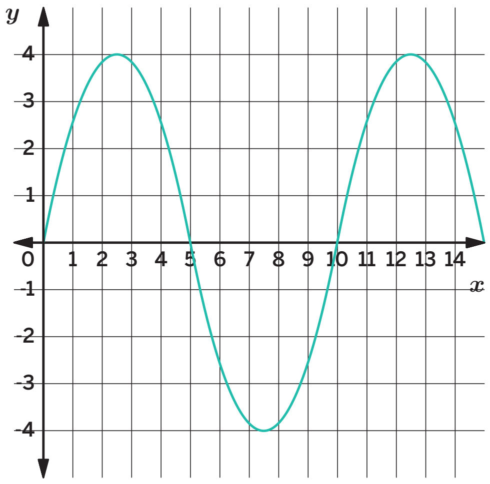
b. The radius of another cone is quadrupled but has the same height. How many times larger is the new cone's volume?

3. A farmer has a water tank for cows in the shape of a cylinder with a radius of 7 ft and a height of 3 ft. The tank comes equipped with a sensor to alert the farmer to fill it when the water falls to 20% capacity. What is the volume of the tank when the farmer is notified?

Practice

Name: ______ Date: ______ Period: ______

4. The table and graph represent two functions.



x	y
1	3
2	-1
3	0
4	4
5	5
6	-1

a For which values of x is the output from the table less than the output from the graph?

b In the graphed function, which values of x give an output of 0?

5. A cylinder and a sphere have the same height.

a If the sphere has a volume of 36π cubic units, what is the height of the cylinder?

b What is a possible volume for the cylinder? Show or explain your thinking.

6. A sphere has a radius of 4.8 cm. If 4 spheres are stacked on top of each other, how tall is the stack? Show or explain your thinking.

Name: ________ Date: ________ Period: ________

Unit 5 | Lesson 21 – Capstone

Packing Spheres

Let's apply our understanding of the volume of spheres to packaging problems.

Warm-up Volume of a Sphere

1. According to the International Tennis Federation, the maximum diameter of a tennis ball is 2.7 in. Determine the volume of a tennis ball.

Log in to Amplify Math to complete this lesson online.

Activity 1 Tennis Balls in a Can

Usually, 3 tennis balls are snuggly packed in a cylinder.

1. Determine the approximate volume of the empty space in the cylinder. Show or explain your thinking.

2. A case (in the shape of a rectangular prism) of tennis balls contains 24 cylinders. If the height of each cylinder is equal to the height of the case, determine the volume of the empty space in the case. Show or explain your thinking.

Are you ready for more?

In Problem 2, did you account for the empty space inside each cylinder? If not, determine the total empty space in the case along with the empty space inside each cylinder. Show or explain your thinking.

Name: ________ Date: ________ Period: ________

Activity 2 Shipping Spheres

Shipping spheres, such as bowling balls, basketballs, baseballs, etc., can be a challenge. Companies think about the amount of material used for the packaging, how much empty space is in the package, and the weight of the package. (For example, bowling balls can weigh as much as 16 lbs each. Can you imagine being a postal carrier with a package of 10 bowling balls to deliver?)

You are going to decide which container to use to ship the spheres you are given. Answer the following problems on a separate sheet of paper.

1. Determine the total volume of the spheres you were given.

2. Sketch an arrangement of how to package the spheres. Include measurements of the dimensions.

3. Determine the volume of your shipping container.

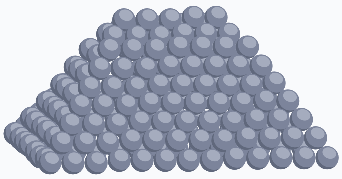
4. What is the amount of empty space in your container?

5. What percent of the container is filled with the spheres? What does this tell you about your container?

Historical Moment

What is the best way to pack spheres?

While sailing from England to North Carolina, Thomas Harriot (1560–1621) was asked to determine the most efficient way to stack cannonballs on the ship. This led Johannes Kepler (1571–1630) to pose his famous Kepler Conjecture, which suggests the greatest percent of a container holding spheres is approximately 74%. More than 200 years later, Gauss (1777–1855) proved this would work for certain arrangements, known as lattices, as seen in the picture. Thomas Hales, in 1998, proved the Kepler Conjecture for any arrangement of spheres, including irregular ones. You thought your homework was tough! This problem took over 400 years to solve!



STOP

Narrative Connections

Unit Summary

From measuring tennis balls packed in a cylinder to recording pitch-perfect tunes, functions are everywhere.

They work by taking an input, applying a rule to it, and getting back one output. Sometimes, this process gives you something completely new, like the way a camera transforms a scene into a photo.

Other times, these functions simply describe the relationships inherent in an object. For example, consider solids like spheres, cylinders, and cones. While we all know what they *look* like, functions tell us more about them. If you know the radius of a sphere, you can use a function to determine its volume. And if you start with the volume, you can probably use another function to determine its radius.

To better see how functions, well, *function*, we can represent their inputs and outputs using graphs, equations, and tables. This allows us to better compare things like rates and speed. We can even tell mathematical stories that have different parts or phases, using piecewise functions.

When you understand how functions work, you'll start to recognize their applications in the world around you. From a tasty treat to jamming tunes, functions have got you covered.

See you in Unit 6.

Practice

Name: ________ Date: ________ Period: ________

1. The table shows radius measurements of various spheres. Complete the table by determining the missing volume of each sphere in terms of π.

Radius (units)	Volume (cubic units)
1	$\frac{4}{3}\pi$
2	
3	
$\frac{1}{2}$	
100	
r	

a. How does the volume of a sphere with radius 2 cm compare to the volume of a sphere with radius 1 cm?

b. How does the volume of a sphere with radius $\frac{1}{2}$ cm compare to the volume of a sphere with radius 1 cm?

2. A sphere has a radius of length r. What happens to the volume of this sphere if its radius is doubled? Show or explain your thinking.

3. The height of a certain cylinder is 6 in. and its volume is 24π in^3. What is the radius of the cylinder?

Practice

Name: .... Date: .... Period: ....

4. A sphere has a radius of 5.1 cm. Approximate the following volumes by using 3.14 for the value of π. Show or explain your thinking.

a Determine the approximate volume of the sphere.

b Determine the approximate volume of a cylinder with the same dimensions as the sphere.

c Determine the volume of a cone with the same dimensions as the sphere.

5. A 6-oz paper cup is shaped like a cone with a diameter of 4 in. How many ounces of water will a cylindrical cup with a diameter of 4 in. hold if it is the same height as the paper cup? Explain your thinking.

My Notes:

UNIT 6

Exponents and Scientific Notation

Imagine the *smallest* number you can think of. Now imagine the *largest* number you can think of. How can you write these numbers? How can you work with these numbers? In this unit, you'll learn about the power of exponents (pun intended), and how you can use them to work with numbers as small as the mass of a bacterium or as large as the number of atoms in the Universe.

Essential Questions

- What happens when expressions containing exponents are multiplied or divided?
- Is there a more efficient way to write really small and really large numbers?
- What strategies can be used when working with very large and very small numbers?
- *(By the way, which weighs more: the Burj Khalifa or all the pennies it cost to build the Burj Khalifa?)*

SUB-UNIT

1 Exponent Rules

Narrative: A chess board can reveal the power of exponents.

You'll learn . . .

- to formulate rules for multiplying and dividing powers.
- what it means for an exponent to be zero or negative.

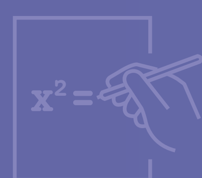
SUB-UNIT

2 Scientific Notation

Narrative: Discover a more efficient way to talk about the distance across the Universe.

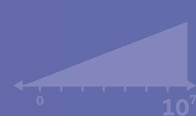
You'll learn . . .

- how a new notation can express quantities with greater ease and efficiency.
- how to compute with very large and small numbers.



Let N be the largest value you can create using four 2s, addition, subtraction, multiplication, division, exponents, and parentheses. What is the last digit of N?

$(10^2)^4$



$3^3 \cdot 5^3 = 15^3$

Unit 6 | Lesson 1 – Launch

Create a Sierpiński Triangle

Let's draw some triangles.

Warm-up Notice and Wonder

Your teacher will show you an animation. What do you notice? What do you wonder?

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Drawing Triangles

Let's create a pattern similar to the one seen in the Warm-up. You will be given a sheet with an equilateral triangle.

1. The midpoints of the sides on the equilateral triangle are marked by dots. Connect the dots, and then shade the middle triangle formed by these dots. Write the number of unshaded triangles in the table for Stage 1.

2. For each unshaded triangle, find the midpoint of each side. Connect the dots, and then shade the middle triangles formed by these dots. Write the number of unshaded triangles in the table for Stage 2.

Stage	Number of unshaded triangles
1	
2	
3	
4	

3. Continue the pattern and complete the table with the number of unshaded triangles for Stages 3 and 4.

4. What patterns do you notice?

5. Do you think the number of unshaded triangles in Stage 10 will be less than or greater than the number of students in your school?

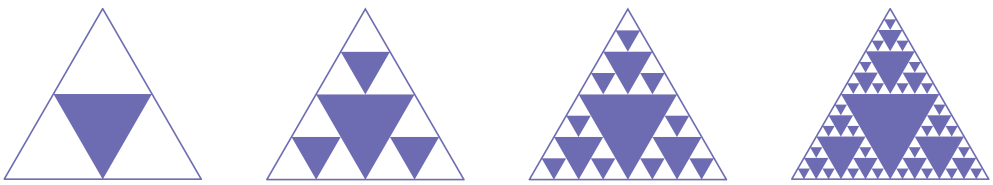
Compare and Connect: How did the patterns you noticed in Problem 4 help you think about the number of unshaded triangles in Stage 10?

Activity 2 Unshaded Area

The triangle pattern you created in the previous activity is called the Sierpiński triangle. In 1915, Waclaw Sierpiński discovered this geometric shape that shows a repeating pattern at different scales.

Study the Sierpiński triangles for the first four stages. The unshaded area of the triangle in each of the first three stages is written as a fraction of the total area, in square units.

Stage 1	**Stage 2**	**Stage 3**	**Stage 4**



$\frac{3}{4}$	$\frac{9}{16}$	$\frac{27}{64}$	....

1. How does the unshaded area change from one stage to the next?

2. Use the patterns to write the unshaded area for Stage 4.

3. Do you think the area for the unshaded triangles in Stage 50 will be less than or greater than the surface area of a grain of salt? Be prepared to explain your thinking.

Featured Mathematician

Wacław Sierpiński

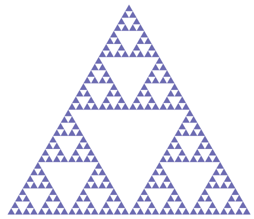
Polish mathematician, Wacław Sierpiński (1882–1969), dedicated his career to the study of set theory, number theory, function theory, and topology. He is perhaps best known for his creation of three fractals that now bear his name: the Sierpiński triangle, the Sierpiński carpet, and the Sierpiński curve. One way to think about a fractal is to imagine zooming in and out on a geometric pattern. If the pattern appears to be the same, you might be looking at a fractal. You just learned about a Sierpiński triangle — research what a Sierpiński carpet and Sierpiński curve look like.

STOP

"Wacław Sierpiński" Courtesy National Digital Archive, Poland.

Unit 6 Exponents and Scientific Notation

From Teeny-Tiny to Downright Titanic



We call this the Sierpiński triangle. Pretty, right? We imagine shapes as things that are settled, static. You wouldn't expect a triangle to grow a pair of legs and hop around the page. But this guy is different! In some ways, it's like a living thing. It moves. It changes. You can see it growing versions of itself, within itself. And those versions grow their own versions. On, and on, and on.

With enough rounds, you'll have a quilt-work of nested triangles. And in each version, those triangles become smaller and smaller — smaller than a pen point, smaller than even the space between atoms.

And yet with each round, the number of triangles keeps increasing — growing into values so colossal, it would cramp the hand, circle the equator, and stagger even the most advanced computer.

These are the numbers that govern how we talk about the vast expanse between planets, or the width of a bacterium, or the chances of getting attacked by an elephant.

Welcome to the Land of Colossal and Infinitesimal Numbers. Your guide? Our old friend, the *exponent*.

Welcome to Unit 6.

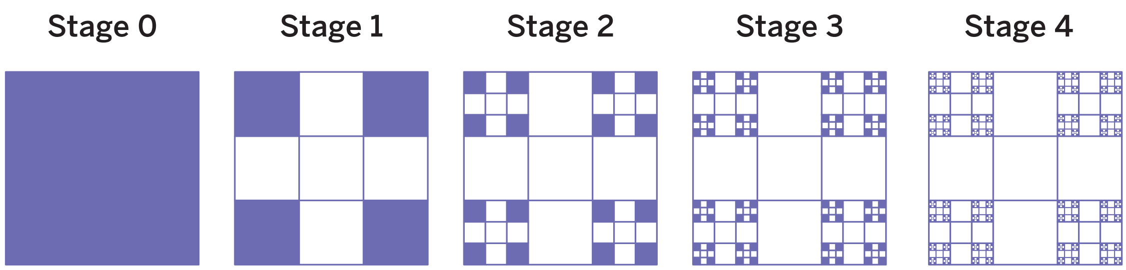
Practice

Name: ______________ Date: ______________ Period: ______________

1. Complete the table by writing the missing single power or expanded form.

Single power	Expanded form
	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
5^4	
$\left(\frac{2}{3}\right)^3$	
	$2\frac{1}{3} \cdot 2\frac{1}{3}$
$\left(\frac{5}{7}\right)^1$	

2. The pattern shown is called the Cantor dust. Study the pattern. The shaded area of each square in Stages 0–3 is written as a fraction of the total area, in square units. Write the fraction that represents the shaded area for Stage 4.



Stage 0	Stage 1	Stage 2	Stage 3	Stage 4
1	$\frac{4}{9}$	$\frac{16}{81}$	$\frac{64}{729}$	______

Name: ______ Date: ______ Period: ______

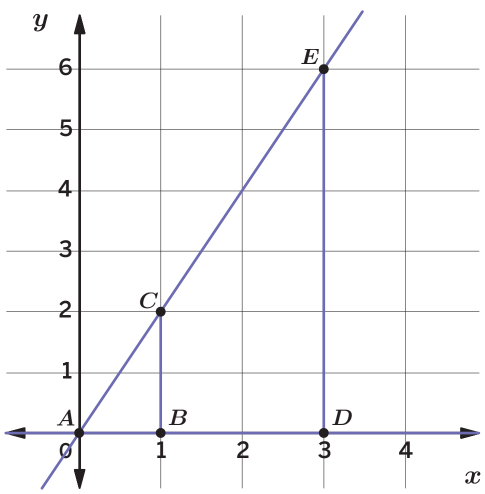
Practice

3. Clare made $\$160$ babysitting last summer. She put the money in a savings account that pays 3% interest per year. If Clare does not touch the money in her account, she can determine the amount she will have the next year by multiplying her current amount by 1.03.

a How much money will Clare have in her account after 1 year?

b How much money will Clare have in her account after 3 years? Explain your thinking.

4. The diagram shows a pair of similar triangles, AED and ACB one contained in the other. Which point represents the center of the dilation mapping the larger triangle onto the smaller one. What scale factor is used?



5. Evaluate each expression.

a $-2 \cdot (-4)$

b $-7 \cdot 2$

c $9 \cdot (-10)$

d $-2 \cdot (-6) \cdot (5)$

e $-8 \cdot (-2) \cdot (-9)$

My Notes:

SUB-UNIT

1 Exponent Rules

Narrative Connections

How many carbs are in a game of chess?

The story goes that there once was a King Shirham of India, whose Grand Vizier, Sissa ben Dahir, invented the game of chess.

The king loved the game so much, he asked ben Dahir to name his reward for having invented it.

The vizier answered, "Your majesty, instead of riches, I ask for wheat. Award me with 2 grain on the first square of the chessboard, 4 grains on the second square, 8 grains on the third, and so on, doubling the number with each square until all 64 squares have been accounted for."

King Shirham agreed. So they started: 2 grain on the first square; 4 grains on the second square; 8 grains on the third, 16 grains on the fourth . . .

What King Shirham didn't realize was that the number of grains was growing a lot faster than he'd previously thought. They were growing ***exponentially***. By the time the King reached the last square, he would've had to place $36{,}893{,}488{,}147{,}419{,}103{,}231$ grains (or just about 2^{65}) — enough to deplete all the wheat in the entire kingdom many times over!

You may remember powers and exponents from prior grades. But what happens when you multiply two powers? Divide two powers? How many times more grains of wheat are on the 30th square than the 12th square? If you recall that exponents represent repeated multiplication, you can answer these questions.

Unit 6 | Lesson 2

Reviewing Exponents

Let's review exponents.

Warm-up How Many Times Greater?

Lin read the story about Sissa ben Dahir. She decided to create the following table to show the number of grains of wheat on the first five squares of the chessboard. Study the table to look for any patterns.

Chessboard square	Grains of wheat	Single power	Expanded form
1	2	2^1	2
2	4	2^2	$2 \cdot 2$
3	8	2^3	$2 \cdot 2 \cdot 2$
4	16	2^4	$2 \cdot 2 \cdot 2 \cdot 2$
5	32	2^5	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

1. How many grains of wheat will be on the 64th square? Write your response as a single power.

2. How many times greater is the number of grains of wheat on square 64 than on square 60? How can the expression — written in expanded form or as a single power — help you determine your response?

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Comparing Expressions

1. Compare each pair of expressions using the symbol >, <, or =.

a. $2^3 \ \boxed{}\ 2^5$

b. $10^8 \ \boxed{}\ 10^2$

c. $\left(\frac{1}{2}\right)^3 \ \boxed{}\ \left(\frac{1}{2}\right)^5$

d. $1^9 \ \boxed{}\ 1^4$

e. $4^3 \ \boxed{}\ 3^4$

f. $6^{10} \ \boxed{}\ 7^{10}$

g. $\left(\frac{1}{2}\right)^3 \ \boxed{}\ \left(\frac{1}{3}\right)^3$

h. $18^1 \ \boxed{}\ 1^{18}$

2. How many times greater is the first expression in the pair than the second expression? Be prepared to explain your thinking.

a. 2^5 is ______ times greater than 2^2.

b. 5^5 is ______ times greater than 5^3.

c. $\left(\frac{1}{2}\right)^1$ is ______ times greater than $\left(\frac{1}{2}\right)^3$.

d. $\left(\frac{1}{3}\right)^2$ is ______ times greater than $\left(\frac{1}{3}\right)^5$.

Are you ready for more?

Write a possible value of x that makes each statement true.

1. $x^5 > x^2$
2. $x^5 < x^2$
3. $x^5 = x^2$

Activity 2 Sorting Expressions

1. Write each expression under its value in the table.

-5^3	4^2	$-4 \cdot (-4)$
3^4	$(-5) \cdot (-5) \cdot (-5)$	$(-3)^4$
$4 \cdot 4$	$(-4)^2$	5^3
$3 \cdot 3 \cdot 3 \cdot 3$	$5 \cdot 5 \cdot 5$	$(-3) \cdot (-3) \cdot (-3) \cdot (-3)$

Expressions equivalent to -125	Expressions equivalent to 125	Expressions equivalent to 16	Expressions equivalent to 81

2. What patterns do you notice?

Name: Date: Period:

Activity 3 Positive or Negative?

Determine whether the value of the expression is positive or negative by placing a checkmark in the appropriate column. Be prepared to explain your thinking.

Expression	Positive	Negative
$(-8)^4$		
23^5		
$(-23)^5$		
$\left(-\frac{1}{5}\right)^7$		
-3^2		

Are you ready for more?

Determine for which values of a and b the expression is positive or negative.

1. a^3

2. $(-b)^4$

STOP

Summary

In today's lesson . . .

You reviewed exponents and compared two expressions that contain exponents to determine which one has a greater value. To compare expressions written as a single power, you can look at the structure of the expression.

When you write an expression like 2^n, 2 represents the base and n represents the exponent. If n is a positive whole number, it tells you how many factors of 2 to multiply to determine the value of the expression.

A negative base with an odd power will result in a negative number. A negative base with an even power will result in a positive number.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. Evaluate each expression.

a. 2^5

b. $\left(\frac{1}{2}\right)^3$

c. $(-4)^3$

d. $(-5)^4$

e. $4 \cdot 4^2$

f. $-2^2 + 2^3$

g. $3 + 3^3$

h. $2^3 \cdot 2^4$

2. Compare each pair of expressions using the symbol >, <, or =.

a. $10^5 \ \square \ 10^8$

b. $7^8 \ \square \ 7^3$

c. $9^2 \ \square \ 17^2$

d. $9^4 \ \square \ 6^4$

e. $24^1 \ \square \ 1^{24}$

f. $\left(\frac{2}{3}\right)^2 \ \square \ \left(\frac{2}{3}\right)^4$

g. $\left(\frac{1}{3}\right)^5 \ \square \ \left(\frac{1}{2}\right)^5$

h. $3^6 \ \square \ 3 \cdot 3^5$

Practice

Name: .... Date: .... Period: ....

3. How many times greater is the first expression in the pair than the second expression?

   a. 2^{10} is .... times greater than 2^6.

   b. $\left(\frac{1}{2}\right)^2$ is .... times greater than $\left(\frac{1}{2}\right)^7$.

4. The equation $y = 5280x$ gives the number of feet y, in x miles. What does the number $5,280$ represent in this relationship?

5. The points $(2, 4)$ and $(6, 7)$ fall on a line. What is the slope of the line?

   A. 1

   B. 2

   C. $\frac{4}{3}$

   D. $\frac{3}{4}$

6. What exponent will make the following equation true?

$$3^{\boxed{}} = 3$$

Name: .... Date: .... Period: ....

Unit 6 | Lesson 3

Multiplying Powers

Let's explore patterns when we multiply powers with the same base.

Warm-up Which One Doesn't Belong?

Which expression does not belong? Explain your thinking.

A. $10 \cdot 10 \cdot 10 \cdot 10$

B. 10^4

C. $10^3 \cdot 10^1$

D. 10,000

Log in to Amplify Math to complete this lesson online.

Activity 1 Card Sort: Multiplying Powers of 10

1. You will be given a set of cards. Match each expression with its expanded form and then as a single power. Record your matched sets in the table.

Expression	Expanded form	Single power
$10^2 \cdot 10^3$		
$10^4 \cdot 10^3$		
$10^3 \cdot 10^3$		
$10^3 \cdot 10^5$		
$10^2 \cdot 10^7$		

2. What patterns do you notice?

Name: ________ Date: ________ Period: ________

Activity 2 Multiplying Powers With Bases Other Than 10

The table shows similar expressions as in Activity 1, but now with bases other than 10.

1. Complete the table to explore patterns among the exponents when multiplying powers with the same base.

Expression	Expanded form	Single power
$2^3 \cdot 2^5$	$(2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$	
$3^7 \cdot 3$		
	$\left(\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}\right) \cdot \left(\frac{1}{5} \cdot \frac{1}{5}\right)$	$\left(\frac{1}{5}\right)^8$
$a^3 \cdot a^4$		

2. What patterns do you notice?

Are you ready for more?

Write whole numbers 0 through 9 as exponents, so that the three expressions are equivalent. Use each number only once. You will not use all of the numbers.

$5^{\square} \cdot 5^{\square}$ $\quad$ $5^{\square} \cdot 5^{\square} \cdot 5^{\square}$ $\quad$ $5^{\square}$

Activity 3 Three Challenges

In each challenge, two expressions are equivalent and one is not. Circle the expression that is *not* equivalent. Then explain to your partner why it is not equivalent. If you disagree, discuss your thinking until you reach an agreement.

1. **Challenge 1:** $3 + 3 + 3 + 3 + 3$ $\qquad$ $3^2 \cdot 3 \cdot 3 \cdot 3$ $\qquad$ 3^5

   How would you change the expression so that it has the same value as the others?

2. **Challenge 2:** $5 \cdot 5 \cdot 5 \cdot 4 \cdot 4 \cdot 4$ $\qquad$ 20^3 $\qquad$ $(5 \cdot 3) \cdot (4 \cdot 3)$

   How would you change the expression so that it has the same value as the others?

3. **Challenge 3:** $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$ $\qquad$ $6^{\frac{1}{2}}$ $\qquad$ $\frac{1}{2^6}$

   How would you change the expression so that it has the same value as the others?

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored patterns among the exponents when multiplying powers that have the same base. In doing so, you developed a rule for multiplying powers with the same base. The rule can be expressed as $a^m \bullet a^n = a^{m+n}$, for $a \neq 0$.

This means that when you multiply powers that have the same base, the product also has the same base and the exponent on the product is the sum of the exponents of the two original powers. In other words, you keep the same base and add the exponents. For example, $4^3 \bullet 4^8 = 4^{(3+8)}$, or 4^{11}.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Write each expression as a single power.

a $10^3 \cdot 10^9$

b $23^5 \cdot 23^6$

c $11^4 \cdot 11^4$

d $\left(\frac{4}{5}\right)^8 \cdot \left(\frac{4}{5}\right)^7$

e $(6.2)^{11} \cdot (6.2)^2$

f $(-8)^2 \cdot (-8)$

g $4^{14} \cdot 4^8 \cdot 4^2$

2. A large rectangular swimming pool is $1{,}000$ ft long, 100 ft wide, and 10 ft deep. The pool is filled to the top with water.

a Write each measurement of the pool as a single power of 10.

Length: **Width:** **Depth:**

b Use your responses from part a to write an expression that represents the volume of the pool in cubic feet.

c How much water does the pool hold? Write your response as a single power of 10.

3. Replace the empty box with a single power of 2 to make each equation true.

a $2^6 \cdot \boxed{} = 2^{14}$

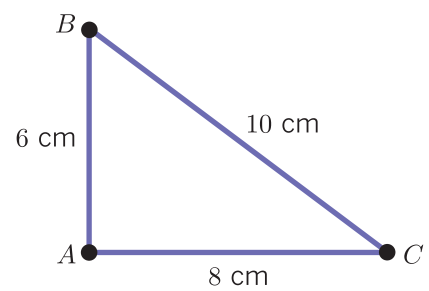
b $\boxed{} \cdot 2^6 = 2^7$

c $2^{12} \cdot \boxed{} \cdot 2 = 2^{15}$

Name: ________ Date: ________ Period: ________

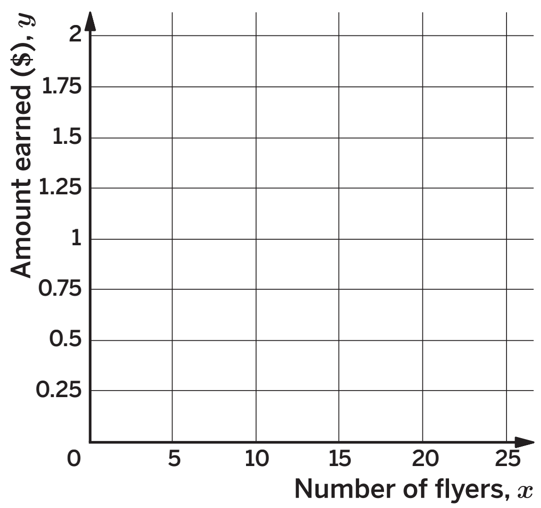
Practice

4. Triangle ABC is shown. Triangle $A'B'C'$, not shown, represents a dilation of Triangle ABC. The length of side $B'C'$ is 5 cm. What are the lengths of sides $A'B'$ and $A'C'$, in centimeters?



5. Elena and Jada distribute flyers for different advertising companies. Elena gets paid 50 cents for every 10 flyers she distributes, and Jada gets paid 75 cents for every 12 flyers she distributes.

a. Graph each relationship representing the total amount y each person earned after distributing x flyers.



b. Who earns more after distributing 20 flyers? Explain your thinking.

6. Evaluate the following expression.

$$\frac{2^3 + 10}{2 + 4^2}$$

Unit 6 | Lesson 4

Dividing Powers

Let's explore patterns with exponents when we divide powers with the same base.

Warm-up Evaluating the Expression

Evaluate the following expression. Show your thinking.

$$\frac{2^5 \cdot 3^3 \cdot 3}{2^2 \cdot 3^4 \cdot 2^3}$$

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Card Sort: Dividing Powers of 10

1. You will be given a set of cards. Match each expression with its expanded form and then as a single power. Record your matched sets in the table.

Expression	Expanded form	Single power
$10^4 \div 10^2$		
$10^7 \div 10^3$		
$10^6 \div 10^3$		
$10^3 \div 10^2$		

2. What patterns do you notice?

Activity 2 Dividing Powers With Bases Other Than 10

The table shows similar expressions as in Activity 1, but now with bases other than 10.

1. Complete the table to explore patterns in the exponents when dividing powers with the same base.

Expression	Expanded form	Single power
$3^7 \div 3^2$		3^5
$(-7)^4 \div (-7)$		
$\left(\frac{2}{3}\right)^3 \div \left(\frac{2}{3}\right)^2$	$\frac{\left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)}{\left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)}$ $= \frac{\left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)}{\left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)} \cdot \left(\frac{2}{3}\right) = 1 \cdot \left(\frac{2}{3}\right)$	
	$\frac{a \cdot a \cdot a \cdot a \cdot a \cdot a}{a \cdot a}$ $= \frac{a \cdot a}{a \cdot a} \cdot a \cdot a \cdot a \cdot a$ $= 1 \cdot a \cdot a \cdot a \cdot a$	a^4

2. What patterns do you notice?

3. Using the patterns you found, write $4^3 \div 4^3$ as a single power, and then evaluate the expression.

Name: ________________ Date: ________ Period: ________

Activity 3 Earning a Medal

Can your math skills earn you a medal? Start from the bronze medal and work your way up to the next medal. What is the highest medal you can earn?

Bronze Medal

Evaluate each expression.

a. $2^0 \cdot 2^5$

b. $\frac{4^2}{4^0}$

c. $\frac{6^4}{6^4}$

Silver Medal

Write each expression as a single power.

a. $\frac{7^9 \cdot 7^2}{7^0 \cdot 7^3}$

b. $\frac{13^8}{13^4} \cdot 13^{12}$

c. $\frac{z^9 \cdot z^2 \cdot z}{z^2 \cdot z^{10}}$

Gold Medal

Write each expression as a single power.

a. $\dfrac{\left(\frac{1}{10}\right)^3 \cdot \left(\frac{1}{10}\right)^2}{\left(\frac{2}{5}+\frac{1}{4}\right)^0}$

b. $\left(\frac{7^{18} \cdot 7^{11} \cdot 7}{7 \cdot 7}\right) \cdot \left(\frac{7^6 \cdot 7^2}{7^7 \cdot 7}\right)$

c. $a^x \cdot \left(\frac{a^y}{a^0}\right)$

STOP

Summary

In today's lesson . . .

You explored patterns among the exponents when dividing powers that have the same base. In doing so, you developed a rule for dividing powers with the same base. The rule can be expressed as $a^m \div a^n = a^{m-n}$, for $a \neq 0$.

This means that when you divide powers with the same base, the quotient also has the same base and the exponent on the quotient is the difference of the exponents of the two original powers. In other words, you keep the same base and subtract the exponents. Be sure to subtract the second exponent from the first. For example, $6^{11} \div 6^4 = 6^{11-4}$, or 6^7.

You also discovered that a nonzero base with an exponent of 0 has a value of 1. This rule can be expressed as $a^0 = 1$, for $a \neq 0$.

Reflect:

Name: ________ Date: ________ Period: ________

Practice

1. Evaluate each expression.

a 10^0

b $\frac{8^3}{8^3}$

c $\frac{2^4}{11^0}$

d $(-9)^0 + 7$

e $\left(\frac{2}{3}\right)^0 + \frac{1}{4}$

f $\frac{3^3}{3^0} \cdot 3$

2. Write each expression as a single power.

a $\frac{10^7}{10^2}$

b $\frac{11^8}{11^2}$

c $\frac{7^3 \cdot 7^4}{7^5}$

d $\left(\frac{8^{12}}{8^7}\right) \cdot 8^4$

e $\frac{10^{13} \cdot 10^5 \cdot 10^0}{10^3 \cdot 10^7}$

f $\frac{a^3 \cdot a^5}{a^2 \cdot a^6}$

3. The Sun is roughly 10^2 times as wide as Earth. The star KW Sagittarii is roughly 10^5 times as wide as Earth. About how many times as wide as the Sun is KW Sagittarii? Explain your thinking.

4. Bananas cost \$1.50 per pound, and guavas cost \$3.00 per pound. Kiran spends \$12 on fruit for a breakfast his family is hosting. Let b represent the number of pounds of bananas Kiran buys and g represent the number of pounds of guavas he buys.

a Write an equation relating the two variables.

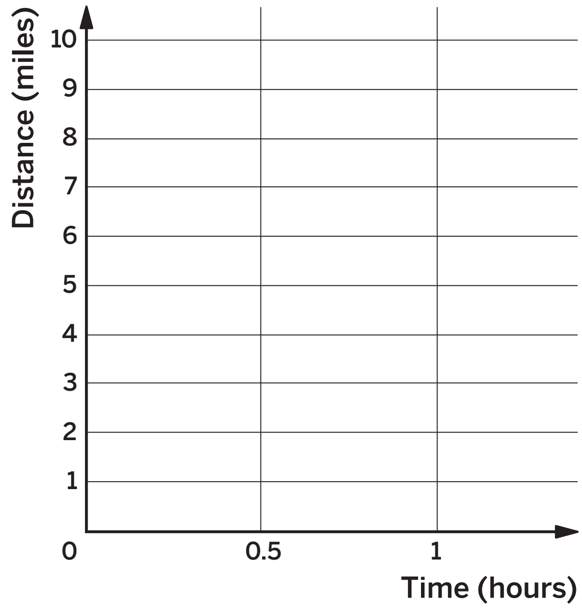
b Rewrite the equation so that g is the independent variable.

c Rewrite the equation so that b is the independent variable.

Practice

Name: ________ Date: ________ Period: ________

5. Lin's mom bikes at a constant speed of 12 mph. Lin walks at a constant speed that is $\frac{1}{3}$ of the speed at which her mom bikes. Sketch a graph of each relationship.



6. Write each expression under an equivalent value in the table.

$-5-7$	$-5+7$	$-5-(-7)$	$5+(-7)$	$5-7$

Equivalent to -2	Equivalent to 2	Equivalent to -12

Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 5

Negative Exponents

Let's see what happens when exponents are negative.

Warm-up Notice and Wonder

Study the table. What do you notice? What do you wonder?

Single power	Value
3^3	27
3^2	9
3^1	3
3^0	1
3^{-1}	$\frac{1}{3}$
3^{-2}	$\frac{1}{9}$
3^{-3}	$\frac{1}{27}$

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Looking at Negative Exponents

1. Complete the table.

Single power	Expanded form	Value
2^4	$2 \cdot 2 \cdot 2 \cdot 2$	16
2^3		8
2^2	$2 \cdot 2$	4
2^1		
	1	1
2^{-1}	$\frac{1}{2}$	$\frac{1}{2}$
2^{-2}	$\frac{1}{2 \cdot 2}$	
2^{-3}		$\frac{1}{8}$

2. How is the exponent related to the expression written in expanded form?

Discussion Support: What math terms can you use in your response to Problem 2? Be ready to share these during the upcoming class discussion.

3. Write an equivalent expression for 2^{-3} with a single, positive exponent.

4. The value of 2^4 is equal to 16. Use this to predict the value of 2^{-4}. Explain your thinking.

Name: ________ Date: ________ Period: ________

Activity 2 Follow the Exponent Rules

With your partner, decide who will complete Problem A, and who will complete Problem B. After each problem, share your response with your partner. Although the problems are different, your responses should be the same. If they are not the same, work together to correct any errors or resolve any disagreements.

Plan ahead: What choices can you make about your behavior to help you and your partner focus on the structures of the expressions?

1. Write each expression in expanded form. Then write the expression with a single, positive exponent.

Problem A

$10^{-2} \cdot 10^{-3}$

Problem B

$10^{-7} \cdot 10^{2}$

2. Recall the rule you developed in Lesson 3 when multiplying powers with the same base, $a^m \cdot a^n = a^{m+n}$. Could the same rule be used when multiplying powers involving negative exponents? Explain your thinking.

3. Write each expression in expanded form. Then write the expression with a single, positive exponent.

Problem A

$\frac{10^2}{10^5}$

Problem B

$\frac{10^{-5}}{10^{-2}}$

4. Recall the rule you developed in Lesson 4 when dividing powers with the same base, $\frac{a^m}{a^n} = a^{m-n}$. Could the same rule be used when dividing powers involving negative exponents? Explain your thinking.

Activity 2 Follow the Exponent Rules (continued)

5. Write each expression with a single, positive exponent.

a $10^6 \cdot 10^{-4}$

b $\frac{10^{-3}}{10^5}$

c $10^{-6} \cdot 10^{-4}$

d $\frac{10^3}{10^{-5}}$

e $\frac{10^3}{10^5}$

f $10^{-6} \cdot 10^4$

Are you ready for more?

Write as many different expressions as you can that are equivalent to 10^{-4}.

STOP

Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You explored what happens when expressions contain negative exponents. In doing so, you discovered that negative powers of 2 represent repeated multiplication of $\frac{1}{2}$, and negative powers of 10 represent repeated multiplication of $\frac{1}{10}$.

The rule can be expressed as $a^{-m} = \frac{1}{a^m}$ for $a \neq 0$. For example, $7^{-3} = \frac{1}{7^3}$.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Write each expression with a single *negative* exponent.

a $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

b $\frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7}$

c $\frac{1}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$

d $\frac{1}{x \cdot x}$

2. Write each expression with a single *positive* exponent.

a $10^4 \cdot 10^{-1}$

b $8^{-3} \cdot 8^{-2}$

c $\frac{10^5}{10^7}$

d $\frac{2^3}{2^8}$

3. Which expressions are equivalent to $\frac{1}{10{,}000}$? Select *all* that apply.

A. $10{,}000^{-1}$

B. $-10{,}000$

C. 100^{-2}

D. 10^{-4}

E. $(-10)^2$

4. This problem has two parts. In each part, you will work with the equations shown in Part 1.

Part 1 Match each equation with the scenario it represents.

Equation	Scenario
a $y = 3x$	______ A dump truck is hauling loads of dirt to a construction site. After 20 loads, there are 70 ft^2 of dirt.
b $\frac{1}{2}x = y$	______ I am making a water and salt mixture that has 2 cups of salt for every 6 cups of water.
c $y = 3.5x$	______ A store has a "4 for \$10" sale on hats.
d $y = \frac{5}{2}x$	______ For every 48 cookies I bake, my students receive 24 cookies.

Name: .... Date: .... Period: ....

Practice

Part 2 Explain what the constant of proportionality means in each equation from Part 1.

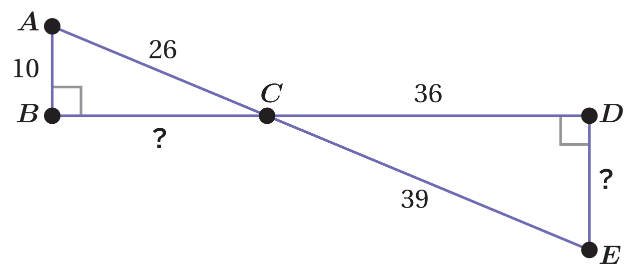
a. $y = 3x$

b. $\frac{1}{2}x = y$

c. $y = 3.5x$

d. $y = \frac{5}{2}x$

5. Refer to the diagram.



a. Explain why Triangle ABC is similar to Triangle EDC.

b. Calculate the missing side lengths.

6. Which expressions are equivalent to 4^3? Select *all* that apply.

A. $4 \cdot 4 \cdot 4$

B. $4 \cdot 3$

C. 3^4

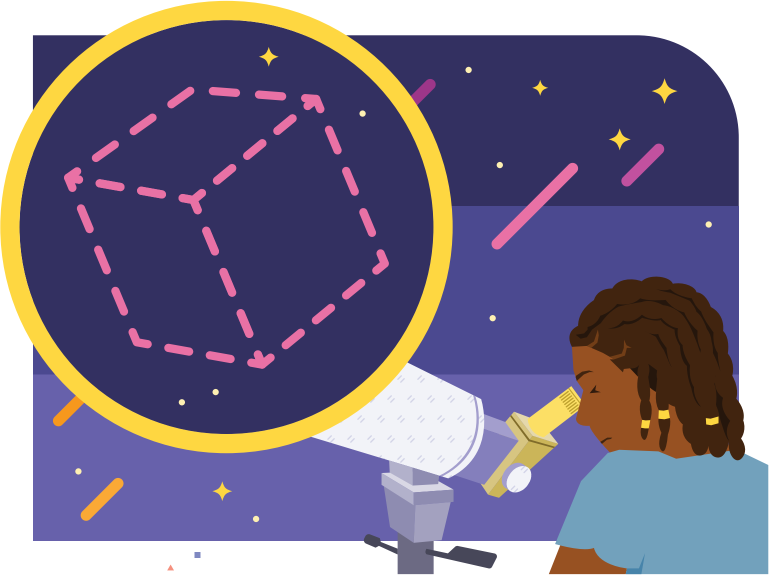
D. $4 \cdot 4^2$

E. $4 + 4 + 4$

Unit 6 | Lesson 6

Powers of Powers

Let's look at powers of powers.



Warm-up A Giant Cube

What is the volume of a giant cube that measures 10,000 km on each side?

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Card Sort: Raising Powers of 10 to Another Power

1. You will be given a set of cards. Match each expression with its expanded form and then as a single power. Record your matched sets in the table.

Expression	Expanded form	Single power
$(10^3)^2$		
$(10^2)^5$		
$(10^3)^4$		
$(10^4)^2$		
$(10^2)^4$		

2. What patterns do you notice?

Activity 2 How Do the Rules Work?

The table shows similar expressions as in Activity 1, but now with bases other than 10.

1. Complete the table to explore patterns when raising a single power to an exponent.

Expression	Expanded form	Single power
$(9^4)^2$		
$(5^2)^3$	$(5 \cdot 5) \cdot (5 \cdot 5) \cdot (5 \cdot 5)$	
	$(5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)$	5^6
	$(0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7) \cdot$ $(0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7) \cdot$ $(0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7) \cdot$ $(0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7)$	
$(a^3)^4$		a^{12}

2. What patterns do you notice?

Name: ______ Date: ______ Period: ______

Activity 3 Making a Match

1. Match each expression in Column A with an equivalent expression from Column B.
Note: Each expression in Column B corresponds to two expressions from Column A.

Column A	Column B
a $(10^2)^3$	______ $\frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)}$
b $(10^{-2})^3$	______ $\left(\frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10}\right)$
c $(10^2)^{-3}$	______ $\frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}}$
d $(10^{-2})^{-3}$	______ $(10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10)$

2. Write $(10^2)^{-3}$ as a single power. Be prepared to explain your thinking.

Are you ready for more?

Use all of the digits 0–9 as exponents to write as many expressions as you can that are equivalent to 10^{24}. Use each number only once.

You may use:

- multiplication of powers of 10
- division of powers of 10
- powers of powers of 10

STOP

Summary

In today's lesson . . .

You explored patterns among the exponents when raising a power to another power. In doing so, you developed a rule for raising a single power to an exponent. The rule can be expressed as $(a^m)^n = a^{m \cdot n}$, for $a \neq 0$.

This means that when you raise a single power to another power, the result is a single power whose exponent is the product of the exponents in the original expression. In other words, you keep the same base and multiply the exponents. For example, $(3^4)^2 = 3^{4 \cdot 2}$, or 3^8.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Write each expression as a single power.

a. $\left(10^{7}\right)^{2}$

b. $\left(3^{5}\right)^{2}$

c. $\left(0.2^{2}\right)^{9}$

d. $\left(9^{-4}\right)^{-2}$

e. $\left(-a^{3}\right)^{2}$

f. $\left(8^{0}\right)^{5}$

2. Replace each box with an expression, written as a single power, that makes each equation true.

a. $\left(\boxed{}\right)^{4}=a^{20}$

b. $\left(\boxed{}\right)^{2}=x^{14}$

c. $\left(\boxed{}\right)^{-4}=t^{8}$

d. $\left(\boxed{}\right)^{n}=c^{2n}$

3. You have $1{,}000{,}000$ number cubes, each measuring 1 in. on a side.

a. If you stacked the cubes on top of one another to make an enormous tower, how high would they reach? Explain your thinking.

b. If you arranged the cubes on the floor to make a square, would the square fit in your classroom? What would its dimensions be? Explain your thinking.

c. If you layered the cubes in rows and columns to make one big cube, what would the dimensions of the big cube be? Explain your thinking.

Practice

4. A rumor spreads at Stifel Middle School. In the first hour, three students hear the rumor. In each additional hour, the number of new students who hear the rumor triples.

   a. How many new students hear the rumor in the second hour?

   b. How many new students hear the rumor in the third hour?

   c. What expression represents the number of new students hearing the rumor in 24 hours?

   d. Why might using exponential notation be preferable to represent the number of new students hearing the rumor for different numbers of hours?

5. Elena noticed that nine years ago, her cousin Katie was twice as old as Elena was then. Then, Elena said, "In four years, I'll be as old as Katie is now!" If Elena is currently e years old and Katie is k years old, which system of equations matches this scenario?

   A. $\begin{cases} k - 9 = 2e \\ e + 4 = k \end{cases}$

   B. $\begin{cases} 2k = e - 9 \\ e = k + 4 \end{cases}$

   C. $\begin{cases} k = 2e - 9 \\ e + 4 = k + 4 \end{cases}$

   D. $\begin{cases} k - 9 = 2(e - 9) \\ e + 4 = k \end{cases}$

6. Which expression *cannot* be written as a single power, using the rule $a^m \bullet a^n = a^{m+n}$?

   A. $5^3 \bullet 5^5$

   B. $4^2 \bullet 4^4$

   C. $3^4 \bullet 4^3$

Name: ______________ Date: ______________ Period: ______________

Unit 6 | Lesson 7

Different Bases, Same Exponent

Let's multiply expressions that have different bases, yet the same exponent.

Warm-up Evaluating Expressions

Evaluate each expression.

1. $2^3 \cdot 2^2$

2. $10^3 \cdot 10^4$

3. $10^2 \cdot 2^2$

Log in to Amplify Math to complete this lesson online.

Activity 1 Power of Products

1. Complete the table to explore patterns when multiplying powers with *different* bases and the *same* exponent. You may skip a single cell in the table, but if you do, be prepared to explain why you skipped it.

Expression	Expanded form	Single power
$3^3 \cdot 4^3$	$(3 \cdot 3 \cdot 3) \cdot (4 \cdot 4 \cdot 4) = (3 \cdot 4)(3 \cdot 4)(3 \cdot 4)$ $= 12 \cdot 12 \cdot 12$	
$2^4 \cdot 3^4$		
		15^3
		30^4
$2^5 \cdot x^5$		
$7^3 \cdot 2^3 \cdot 5^3$		

2. What patterns do you notice?

Reflect: What did you do to remain calm as you reasoned quantitatively about powers with different bases?

Name: ______________ Date: ______________ Period: ______________

Activity 2 True or False?

Determine if each equation is true or false by placing a mark in each box. If the equation is false, change one number in the equation to make it true, and write your altered equation on the line.

	True	False	Altered equation
1. $4^3 \cdot 6^3 = 24^3$	☐	☐	______________
2. $5^7 \cdot 8^7 = 40^{14}$	☐	☐	______________
3. $2^4 \cdot \left(\frac{1}{3}\right)^4 = \frac{2^4}{3}$	☐	☐	______________
4. $(9 \cdot 6)^5 = 9^5 \cdot 6^5$	☐	☐	______________
5. $\left(\frac{1}{5}\right)^3 = \frac{1^3}{5^3}$	☐	☐	______________
6. $10^4 \cdot 1^4 \cdot 8^4 = 80^{12}$	☐	☐	______________
7. $a^7 \cdot b^7 = (ab)^{14}$	☐	☐	______________
8. $(6^7 \cdot 3^7) \cdot (2^7 \cdot 4^7) = 144^7$	☐	☐	______________

Are you ready for more?

Use whole numbers 1–10 to replace the boxes so that the equation is true. Use each number only once.

$$(6^{\square})^{\square} = \frac{6^{\square}}{\square^6 \cdot \square^6}$$

STOP

Summary

In today's lesson . . .

You explored patterns among the exponents when multiplying powers that have *different* bases, but the *same* exponent. In doing so, you developed a rule for multiplying powers that have a different base and same exponent. The rule can be expressed as $a^m \cdot b^m = (a \cdot b)^m$, for $a \neq 0$ and $b \neq 0$. For example, $5^3 \cdot 2^3 = (5 \cdot 2)^3$, or 10^3.

You also explored patterns when dividing powers with *different* bases, but the same exponent. In doing so, you developed the rule $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$, for $a \neq 0$ and $b \neq 0$. For example, $\frac{8^3}{2^3} = \left(\frac{8}{2}\right)^3$, or 4^3.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Which equations are true? Select *all* that apply.

A. $2^8 \cdot 2^9 = 2^{17}$

B. $8^2 \cdot 9^2 = 72^2$

C. $8^2 \cdot 9^2 = 72^4$

D. $2^8 \cdot 2^9 = 4^{17}$

E. $(8 \cdot 9)^2 = 8^2 \cdot 9^2$

2. Replace the boxes with values that make the equation true.

$(3 \cdot 5)^4 \cdot (2 \cdot 3)^5 = 2^{\square} \cdot 3^{\square} \cdot 5^{\square}$

3. Match an expression from Column A with its equivalent expression in Column B.

Column A	Column B
a $(5 \cdot 5 \cdot 5 \cdot 5) \div (3 \cdot 3 \cdot 3 \cdot 3)$	______ $15 \cdot 15$
b $(5 \cdot 5) \cdot (3 \cdot 3)$	______ 48^4
c $(1 \cdot 8)(1 \cdot 8)(1 \cdot 8)(1 \cdot 8)$	______ 14^4
d $(6 \cdot 6 \cdot 6 \cdot 6) \div (2 \cdot 2 \cdot 2 \cdot 2)$	______ $8 \cdot 8 \cdot 8 \cdot 8$
e $6^4 \cdot 8^4$	______ $\left(\frac{5}{3}\right)^4$
f $(7 \cdot 7 \cdot 7 \cdot 7) \cdot (2 \cdot 2 \cdot 2 \cdot 2)$	______ $(6 \div 2)(6 \div 2)(6 \div 2)(6 \div 2)$

4. A cylinder has a radius of 4 cm and a height of 5 cm.

a What is the exact volume of the cylinder?

b What is the exact volume of the cylinder when the radius is tripled?

c What is the exact volume of the cylinder when the radius is halved?

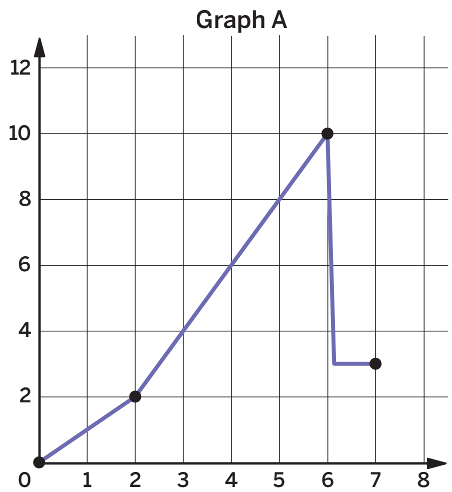
Practice

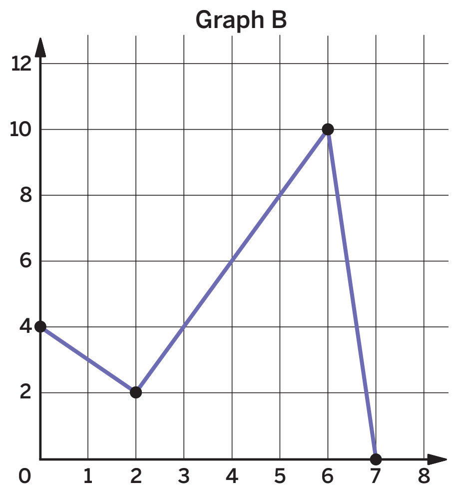
Name: ______ Date: ______ Period: ______

5. Andre sets up a rain gauge to measure rainfall in his backyard. On Tuesday, it rains on and off all day.

- He starts at 10 a.m. with an empty gauge, when it starts to rain.
- Two hours later, he checks the gauge and there are 2 cm of water in it.
- It starts raining even harder, and then at 4 p.m., the rain stops. Andre checks the rain gauge at 4 p.m., and there are 10 cm of water in it.
- While checking the gauge, he accidentally knocks it over and spills most of the water, leaving only 3 cm of water in the gauge.
- When he checks for the last time at 5 p.m., there is no change in the amount of water in the gauge.

a Which of the two graphs could represent Andre's story? Explain your thinking.





b Label the axes of the correct graph with appropriate units.

c Use the graph to determine how much total rain fell on Tuesday.

6. Circle *all* the expressions that *cannot* be rewritten as a single power of 4.

$4^5 \cdot 6^3$ $4^5 \cdot 4^3$ $\frac{4^{10}}{4^2}$ $4^4 \cdot 5^4$ $(4^2)^{-11}$

Name: ____________ Date: ____________ Period: ____________

Unit 6 | Lesson 8

Practice With Rational Bases

Let’s practice with exponents.

Warm-up Ordering Expressions

Here is a list of expressions.

Write the expressions in order from least value to greatest value.

$\frac{2^3}{2^4}$	2^0	$(2^3)^6$	$2^4 \cdot 2^{-2}$	$\left(\frac{1}{2}\right)^3$

	Least value
	Greatest value

Log in to Amplify Math to complete this lesson online.

Activity 1 Partner Problems

With your partner, decide who will complete Column A and who will complete Column B. After each row, share your responses with your partner. Although the problems in each row are different, your responses should be the same. If they are not the same, work together to correct any errors or resolve any disagreements.

1. Write the value of each expression as a single power, with a *non-negative* exponent.

Column A	Column B
$7^3 \bullet 7^5$	$\frac{7^{10}}{7^2}$
$\frac{3^{27}}{3^5}$	$\left(\left(\frac{1}{3}\right)^2\right)^{-11}$
$(12^0)^3$	$3^0 \bullet 4^0$
$4^2 \bullet 3^2$	$12^{-12} \bullet 12^{14}$
$\left(\frac{1}{2}\right)^4 \bullet 2^{-5}$	$(2^3)^{-3}$

2. Replace each box with a value that makes each equation true.

Column A	Column B
$3^3 \bullet \square^3 = 30^3$	$(\square^2 \bullet 9^2)^3 = 90^6$
$2^6 \bullet 2^{\square} = 2^4$	$3^{\square} \bullet 3^7 = 3^5$
$2^2 \bullet 2^{\square} = (2^4)^5$	$9^{\square} \bullet 9^{-2} = (9^8)^2$
$\frac{7^{20} \bullet 4^{20}}{28^{\square}} = 28^{17}$	$\frac{x^2 \bullet x^{\square}}{x^8} = \frac{1}{x^3}$

Name: ____________ Date: ____________ Period: ____________

Activity 2 Covering All Your Bases

How to play: When the time starts, you and your partner will write *one* expression for *each* rule whose value equals the target number your teacher tells you.

- **Your team earns 1 point for every unique expression you write. If another group writes the same expression, no one earns a point.**
- **If an expression uses negative exponents, your team earns 2 points.**
- **You can challenge the other team's expression if you think that its value is not equal to the target number.**

	Rule 1 Multiplying powers with the same base, yet *different* exponents $a^m \cdot a^n = a^{m+n}$	Rule 2 Dividing powers with the same base, yet *different* exponents $\frac{a^m}{a^n} = a^{m-n}$	Rule 3 Multiplying powers with *different* bases, yet the same exponent $a^m \cdot b^m = (a \cdot b)^m$
Round 1			
Round 2			
Round 3			
Round 4			
Round 5			

STOP

Summary

In today's lesson . . .

You practiced working with expressions with exponents. In previous lessons, you determined rules to more easily keep track of repeated factors when using exponents. You also extended these rules to make sense of negative exponents for nonzero bases, as well as defined a number to the power of 0.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. Rewrite each expression with a single, positive exponent.

a $\frac{7^6}{7^2}$

b $(11^4)^5$

c $4^2 \cdot 4^6$

d $(-5)^3 \cdot (-5)^0$

e $7^8 \cdot 2^8$

f $\frac{3^{-10}}{3^5}$

2. Replace each box with a value that makes the equation true.

a $2^{\square} \cdot 2^{10} = 2^{60}$

b $(\square \cdot 6^3) = 24^3$

c $\frac{(9^3)^{\square}}{(9^2)^9}$

3. For each list, circle the expression that is *not* equivalent to the others.

List A

$(5^3)^{-3}$	$\frac{5^{-6}}{5^3}$	$(5^3)^{-2}$	$5^{-4} \cdot 5^{-5}$	$\frac{1}{5^9}$

List B

1	10^0	$\frac{8^2}{8^2}$	$8^3 \cdot 8^{-3}$	0

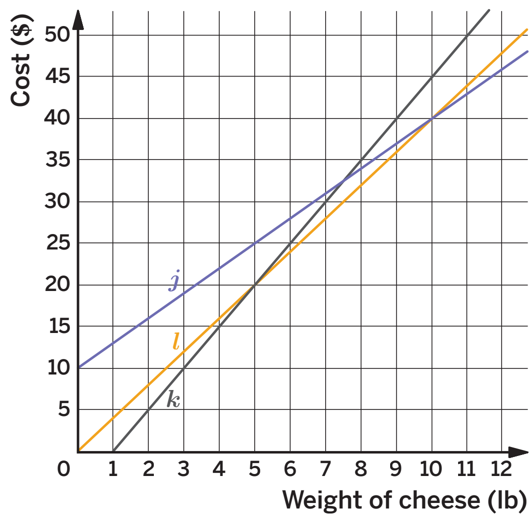
Practice

Name: ____________ Date: ____________ Period: ____________

4. The cost of cheese at three stores is a function of the weight of the cheese. The cheese is not prepackaged, so a customer can buy any amount of cheese.

- Store A sells the cheese for $\$a$ per pound.
- Store B sells the cheese for $\$b$ per pound, and a customer has a coupon for $5 off the total purchase at the store.
- Store C is an online store, selling the same cheese at $\$c$ per pound, but with a $10 delivery fee.

The graph shows the cost functions for Stores A, B, and C.



a Match Stores A, B, and C with each function j, k, and l.

Store A:

Store B:

Store C:

b How much does each store charge for the cheese per pound?

Store A:

Store B:

Store C:

c If a customer wants to buy 6 lb of cheese for a party, which store has the lowest price?

d How many pounds would a customer need to order so that Store C is the best option?

5. Evaluate each expression.

a $256 \cdot 10^1$

b $25.6 \cdot 10^2$

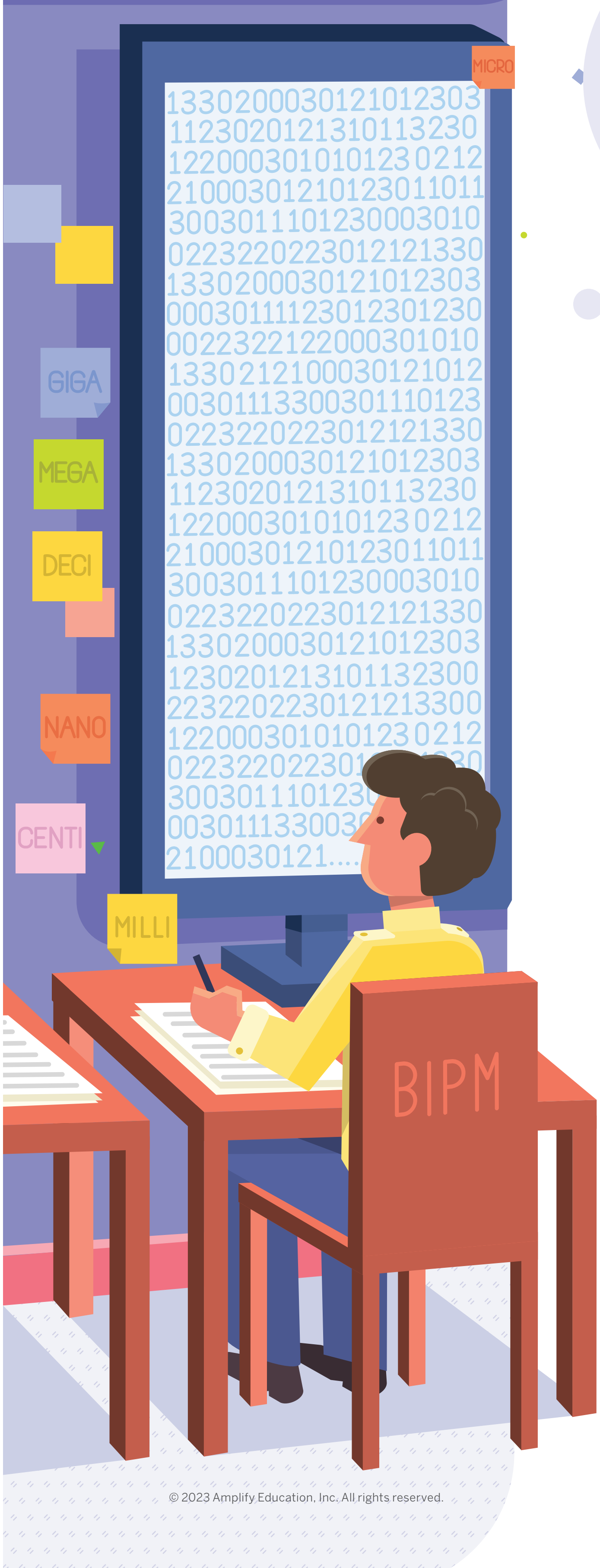
c $2.56 \cdot 10^3$

d $0.256 \cdot 10^4$

SUB-UNIT

2 Scientific Notation

Narrative Connections



Who should we call when we run out of numbers?

While we can't *really* run out of numbers, we can run out of *names* for numbers.

For that, we turn to the International Bureau of Weights and Measures. Founded in 1875, the group's mission has been to provide an international system of measurement. That way, a centimeter measured in Oklahoma is just as long as a centimeter measured in Okinawa.

Part of their work is coming up with the prefixes we use in metric measurement (like the ***kilo-*** in kilogram, or the ***giga-*** in gigabyte). It's not as easy as it looks. They look at a prefix's meaning, how easy it is to pronounce, and what symbols they can use to represent them.

So far, the smallest and largest prefixes the committee has recognized are the yocto- and yotta-. A neutrino, a particle smaller than an atom, is about 1 yoctometer, or 0.000000000000000000000001 meters, wide. Meanwhile, the distance across the whole observable Universe is about 880 yottameters, or 880,000,000,000,000,000,000,000,000 meters.

Between these names and all these zeroes, it's enough to make anyone's head spin. If only there were *another way* to represent these numbers . . .

Unit 6 | Lesson 9

Representing Large Numbers on the Number Line

Let's visualize large numbers on the number line using powers of 10.

Warm-up Different Ways to Write Large Numbers

1. Several expressions and values are shown. For each word, write its corresponding expression and value.

Expressions: 10^{12} 10^2 10^9 10^3 10^6

Values: 1,000 1,000,000,000 100 1,000,000 1,000,000,000,000

a Hundred

b Thousand

c Million

d Billion

e Trillion

2. For each number, think of a real-world example of something that can be described by that number. Record your responses here.

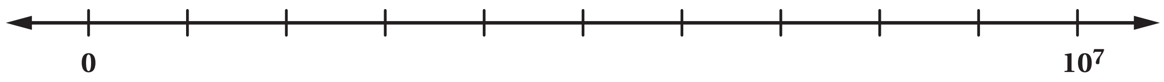
Log in to Amplify Math to complete this lesson online.

Name: ____________ Date: ________ Period: ________

Activity 1 Labeling a Number Line

Very large and very small numbers show up all the time when it comes to computers. While today's devices can perform billions of operations every second, computer scientist Sophie Wilson was among the pioneers who helped engineer these greater speeds. In the 1980s, she designed processors that could reach speeds of up to 10 MHz, or 10^7 operations per second.

1. Label the tick marks on the number line.



2. Trade number lines with a partner and check each other's work. If your labels are not the same, convince your partner that you are correct, or explain why you believe they are incorrect.

Featured Mathematician

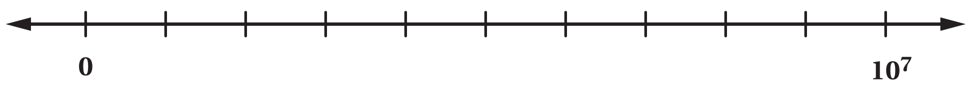
Sophie Wilson

Born in Leeds, England in 1957, Sophie Wilson is an English computer scientist. She designed her first microcomputer while studying at the University of Cambridge, and went on to lead the development of the BBC BASIC programming language. In the 1980s, Wilson helped design processor architectures that are commonly used in today's phones. In 2019, she was named a Commander of the British Empire for her contributions to computing.

Activity 2 Comparing Large Numbers Using a Number Line

1. Plot and label each value on the number line.

4,500,000	9,100,000	$6 \cdot 10^6$
$2 \cdot 10^6$	$7.5 \cdot 106$	$0.3 \cdot 10^7$



2. Which is greater, $2 \cdot 10^6$ or 4,500,000? Estimate how many times greater.

Name: ______ Date: ______ Period: ______

Activity 3 The Speeds of Light

The table shows an approximation of how fast light waves can travel through different materials.

Material	Speed of light (meters per second)
Space	300,000,000
Water	$2.25 \cdot 10^8$
Olive oil	200,000,000
Ice	$2.3 \cdot 10^8$
Diamond	$124 \cdot 10^6$

Order the speeds of light passing through each material from fastest to slowest. Be prepared to explain your thinking.

Material	Speed of light (meters per second)	
		Fastest
		Slowest

STOP

Summary

In today's lesson . . .

You investigated how you can compare large quantities by plotting them on the same number line. You can also compare large quantities by writing the numbers in the same form.

Suppose you want to compare $7{,}400{,}000{,}000$ and $8.9 \cdot 10^9$.

- You can write both numbers in standard form and compare the non-zero digits in the same place-value positions, $7 < 8$.
  $7{,}400{,}000{,}000$ and $8{,}900{,}000{,}000$
- You can also write both as numbers multiplied by the same power of 10 and compare the first factors. The exponents on the power of 10 are the same, and $7.4 < 8.9$.
  $7.4 \cdot 10^9$ and $8.9 \cdot 10^9$

When you compare the numbers using the same form, you see that $7{,}400{,}000{,}000$ is less than $8.9 \cdot 10^9$.
$7{,}400{,}000{,}000 < 8.9 \cdot 10^9$

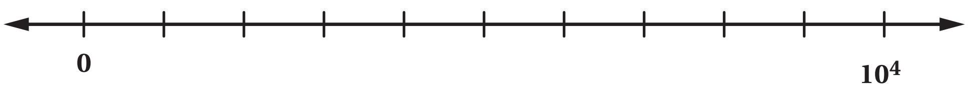
Reflect:

Name: ____________ Date: ____________ Period: ____________

1. Show three different ways to write the number $437{,}000$ as a multiple of a power of 10. For example, one way is writing $437{,}000$ as $4370 \cdot 10^2$.

____________ ____________ ____________

2. Refer to the number line.



a. Label the tick marks on the number line.

b. Plot and label the values $2{,}500$ and $4 \cdot 10^3$ on the number line.

c. Which is less, $2{,}500$ or $4 \cdot 10^3$? Use estimation to compare the values.

3. Each statement contains a quantity. Rewrite each quantity as a multiple of a power of 10.

a. There are about 37 trillion cells, on average, in the human body.

b. The Milky Way contains about 300 billion stars.

Practice

Name: ________ Date: ________ Period: ________

4. A cone has volume V, radius r, and a height of 12 cm.

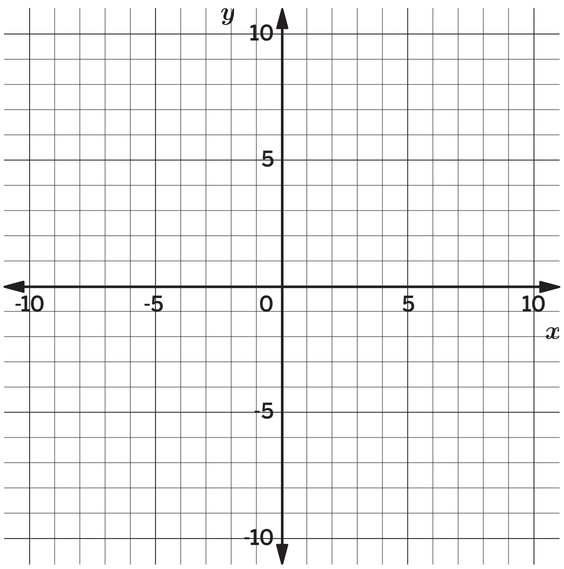
a Write an expression for the cone's volume.

b Another cone has the same height and three times the radius of the original cone. How does the volume of this new cone compare to the volume of the original cone? Show or explain your thinking.

5. Refer to the coordinate plane.

a Graph the line that pases through the point $(-6,\ 1)$ with a slope of $-\frac{2}{3}$.

b What is the equation of the line?



6. Order the numbers 0.00023, 0.0023, and 0.002 from least to greatest.

Least **Greatest**

Name: ____________ Date: ________ Period: ________

Unit 6 | Lesson 10

Representing Small Numbers on the Number Line

Let's visualize small numbers on the number line using powers of 10.

Warm-up Different Ways of Writing Small Numbers

Several expressions and values are shown. For each word, write its corresponding expression and value.

Expressions:	10^{-1}	10^{-3}	10^{-2}	10^{-6}
Values:	0.000001	0.01	0.1	0.001

a One hundredth

b One millionth

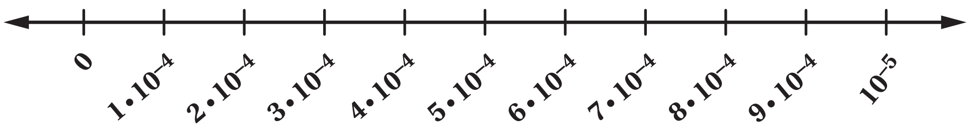
c One thousandth

d One tenth

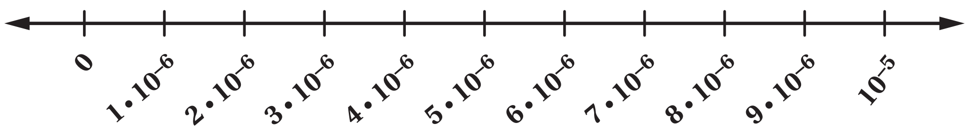
Activity 1 Comparing Small Numbers Using a Number Line

Kiran and Andre both label a number line using negative exponents.

Kiran's number line:



Andre's number line:



1. Whose number line is correct? Explain your thinking.

Critique and Correct: Your teacher will show you incorrect reasoning for one of these number lines. How can you convince either Kiran or Andre as to how they should label the increments?

2. Plot and label each number on the correct number line.

a $4.7 \cdot 10^{-6}$

b 0.0000075

c 0.0000012

Name: ______________________ Date: __________ Period: __________

Activity 2 Deadly Animals

The table shows the estimated percentage of global human deaths caused by different animals in the year 2016.

Animal	Deaths (% of global human deaths)
Dog	$23 \cdot 10^{-3}$
Mosquito	$14 \cdot 10^{-1}$
Shark	0.0000072
Freshwater snail	$0.18 \cdot 10^{-1}$
Elephant	0.00018

Use the data to order the animals from most deadly to least deadly. Be prepared to explain your thinking.

Animal	
	Most deadly
	Least deadly

STOP

Summary

In today's lesson . . .

You investigated how you can compare small quantities by plotting them on the same number line. You can also compare small quantities by writing the numbers in the same form.

Suppose you want to compare $53 \cdot 10^{-4}$ and 0.0034.

- You can write both numbers in standard form and compare the non-zero digits in the same place-value positions.
  0.0053 and 0.0034
  $5 > 3$
- You can also write both as numbers multiplied by the same power of 10 and compare the first factors.
  $53 \cdot 10^{-4}$ and $34 \cdot 10^{-4}$
  $53 > 34$

When you compare the numbers using the same form, you can see that 0.0053 is greater than 0.0034.
$53 \cdot 10^{-4} > 0.0034$

Reflect:

Name: .... Date: .... Period: ....

1. Which expressions are equivalent to $4 \cdot 10^{-3}$? Select *all* that apply.

   A. $4 \cdot \left(\frac{1}{10}\right) \cdot \left(\frac{1}{10}\right) \cdot \left(\frac{1}{10}\right)$

   B. $4 \cdot (-10) \cdot (-10) \cdot (-10)$

   C. $4 \cdot (0.001)$

   D. $4 \cdot (0.0001)$

   E. 0.0004

   F. 0.004

2. Write each value as a multiple of a power of 10.

   a. 0.04

   b. 0.072

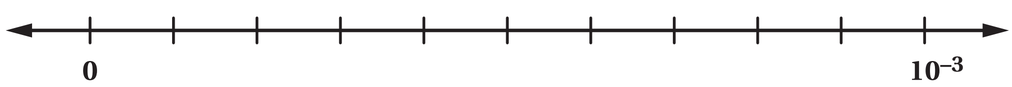
   c. 0.0000325

   d. three thousandths

   e. 23 hundredths

   f. 729 thousandths

3. Refer to the number line.



   a. Label the tick marks on the number line.

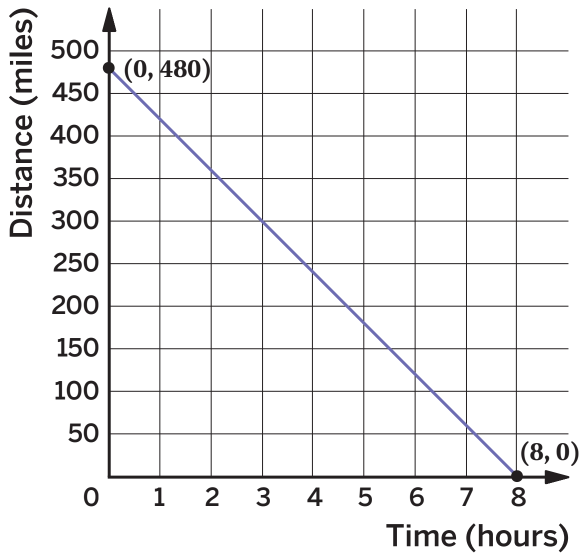
   b. Plot and label the values $2.1 \cdot 10^{-4}$ and 0.00082 on the number line.

   c. Estimate how many times greater 0.00082 is than $2.1 \cdot 10^{-4}$.

Practice

Name: ________ Date: ________ Period: ________

4. A family sets out on a road trip to visit their cousins. They travel at a steady rate. The graph shows the distance remaining to their cousins' house for each hour of the trip.



a. How fast are they traveling? Explain your thinking.

b. Is the slope positive or negative? Explain how you know and why that fits the situation.

c. How far is the trip and how long did it take? Explain your thinking.

5. What information would you need to answer the following questions?

a. How many meter sticks does it take to equal the *mass* of the Moon?

b. If all these meter sticks were laid end-to-end, would they reach the Moon?

Name: .... Date: .... Period: ....

Unit 6 | Lesson 11

Applications of Arithmetic With Powers of 10

Let's use powers of 10 to help us make calculations with large numbers.

Warm-up What Information Do You Need?

The Burj Khalifa, located in Dubai, is the tallest building in the world. It was very expensive to build.

Consider this question: *Which is larger, the mass of the Burj Khalifa or the mass of all the pennies (U.S. currency) that it cost to build the Burj Khalifa?*

What information would you need to answer this question?

M7KK/Shutterstock.com

Log in to Amplify Math to complete this lesson online.

Activity 1 How Many Pennies?

You will be given a calculator and a sheet with information about one penny and information about the Burj Khalifa.

Which has a larger mass, the Burj Khalifa or all of the pennies (U.S. currency) it costs to build the Burj Khalifa? You may want to approximate your calculations as you work to solve this problem.

Name: .... Date: .... Period: ....

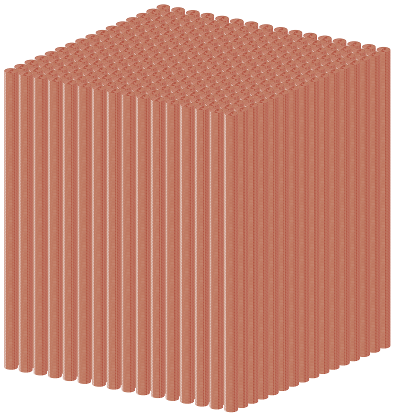
Activity 2 Even More Pennies

You will be given a sheet with information about the Empire State Building, located in New York City.

1. **a** How many pennies are needed to build a stack with a height of 1 in.?

b By placing pennies side-by-side, how many are needed to create a straight line with a length of 1 ft?

2. By placing pennies side-by-side and stacked on top of each other, how many are needed to create a cube with dimensions 1 ft by 1 ft by 1 ft?



3. Consider this question: *How many pennies would it take to fill the Empire State Building, in New York City?*

a What information do you need to answer this question? Record any relevant information you already know and the new information now provided to you.

b Use the information you know to answer the question. You may approximate your answer.

Are you ready for more?

Approximately how many Empire State Buildings could be filled completely, if you have one quadrillion pennies? A quadrillion is 10^{15}.

STOP

Summary

In today's lesson . . .

You worked with powers of 10 to determine how many pennies are needed to fill the Empire State Building and the Burj Khalifa. Powers of 10 can be helpful for making calculations with large or small numbers.

In general, when you want to estimate calculations with very large or small quantities, estimating with powers of 10 and using exponent rules can help simplify the process.

However, if you wanted to find the exact quotient of $2,203,799,778,107$ divided by $318,586,495$, using powers of 10 would not simplify the calculation.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Which is greater, the number of stars in the Universe or the number of grains of sand on Earth?

Some useful information:

- 1 m^3 of sand contains about $8 \cdot 10^9$ grains of sand.
- There are about $1 \cdot 10^{11}$ stars in a galaxy.
- There are about $7.5 \cdot 10^9$ m^3 of sand on Earth.
- The Universe contains about $1 \cdot 10^{10}$ galaxies.

2. The Burj Khalifa in Dubai is the tallest structure in the world at approximately $8.3 \cdot 10^2$ m tall. The Big Ben clock tower in London is 96 m tall. About how many times shorter is the height of Big Ben than the Burj Khalifa? Round the heights first to help you find an approximate answer.

3. Which is greater, the number of meters across the Milky Way, or the total number of cells in all the living humans on Earth? Show or explain your thinking.

Some useful information:

- The Milky Way is about $100,000$ light years across.
- There are about 37 trillion cells in a human body.
- One light year is about 10^{16} m.
- The world population is about 7 billion.

Practice

Name: .... Date: .... Period: ....

4. A cylinder has a volume of 36π cubic cm and height h. Complete the table for the volume of other cylinders with the same radius, but different heights.

Height (cm)	Volume (cm^3)
h	36π
$2h$	
$5h$	
$\frac{h}{2}$	
$\frac{h}{5}$	

5. Determine the value of each expression mentally.

a. $123 \cdot 10000$

b. $3.4 \cdot 1000$

c. $0.42 \cdot 10^3$

d. $7.3 \cdot 0.001$

e. $4 \cdot 10^{-4}$

Name: ______ Date: ______ Period: ______

Unit 6 | Lesson 12

Definition of Scientific Notation

Let's use scientific notation to describe large and small numbers.

Warm-up Ordering Numbers

1. Refer to the list of expressions. This is List 1.

$5 \cdot 10^5$	$4{,}000{,}000$	$75 \cdot 10^5$	$0.6 \cdot 10^7$

Write these expressions in order from least to greatest.

______ ______ ______ ______

Least **Greatest**

2. Refer to the list of expressions shown. This is List 2.

$6 \cdot 10^6$	$7.5 \cdot 10^6$	$4 \cdot 10^6$	$5 \cdot 10^5$

Write these expressions in order from least to greatest.

______ ______ ______ ______

Least **Greatest**

3. Which list required fewer steps to order? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Card Sort: Identifying Scientific Notation

1. You will be given a set of cards. Sort the cards into two groups, expressions written in scientific notation and expressions not written in scientific notation. In the following table, record which cards you sorted into each group.

Scientific notation	Not scientific notation

2. Certain calculators will use "E" notation as a way to write scientific notation. For example, 6.02×10^{23} is represented as $6.02\text{E}23$. Given that these two expressions are equivalent, what do you think "E23" represents in the second expression?

Are you ready for more?

Can you think of information in the real world that might be easier to work with, if the numbers are written in scientific notation? List as many as you can.

Name: .... Date: .... Period: ....

Activity 2 Writing Scientific Notation

Ever wonder how many grains of sand it would take to fill up the Universe? Well, Archimedes did. Back in 215 BCE., he tried to determine it, but the Ancient Greeks did not have a number system like we do today. Instead, letters represented individual numbers. Which meant in order to solve this pressing question, Archimedes had to invent a way to count extremely large numbers.

What he ended up with was the earliest form of scientific notation. By using a letter (M) that represented $10{,}000$, he could describe $10{,}000 \cdot 10{,}000$ (MM), or as we would call it, 10^8. Then he found a way to talk about ten-thousand of ten-thousand of those! And now it is your turn to use powers of ten to describe the cosmos!

1. Mercury orbits the sun at a distance of $36{,}000{,}000$ miles. Why might it be beneficial to write this distance in scientific notation?

2. The following table shows each planet's distance from the Sun. Write each distance in scientific notation.

Planet	Distance from the Sun (miles)	Scientific notation
Mercury	$36{,}000{,}000$	
Venus	$67{,}000{,}000$	
Earth	$92{,}960{,}000$	
Mars	1417×10^5	

Are you ready for more?

Han and Jada were determining which was the correct way to write Jupiter's distance from the Sun, 480 million miles, in scientific notation. Han wrote 48×10^7 miles. Jada wrote 4.8×10^8 miles. Who is correct? Explain your thinking.

Activity 3 Writing Small Numbers in Scientific Notation

You can also use scientific notation to represent small numbers. *Nanoarchaeum equitans* is a single-celled organism found in some naturally occurring pools of boiling water, in places like Iceland or Yellowstone National Park in the United States. Because it is an organism made up of only a single cell, *Nanoarchaeum equitans* is very small — measuring only 400 nanometers in diameter, or 0.0000004 m.

1. The table shows the diameters of three of the smallest microorganisms on Earth. Write each number in scientific notation.

Microorganisms	Diameter (m)	Scientific notation
Nanoarchaeum equitans	0.0000004	
Pelagibacter ubique	0.00000012	
Prasinophyte algae	800×10^{-10}	

Are you ready for more?

Diego analyzed a new microorganism he discovered. With his microscope, he measured the diameter to be 0.000042 cm. He was asked to write this number in meters, so he rewrote it as 4.2×10^{-8} m.

Did he correctly rewrite the value in scientific notation? Explain your thinking.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You used powers of 10 to write large and small numbers in a more efficient way. There are many ways to express a number using a power of 10. One specific way to write a number using a power of 10 is called ***scientific notation***. When a number is written in scientific notation, the first factor is a number greater than or equal to one, but less than ten. The second factor is an integer power of ten.

For example, the number $23{,}000$ can be written as 2.3×10^4, and the number 0.00023 can be written as 2.3×10^{-4}.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. State whether the following is *true* or *false*. Explain your thinking.
52.4 million written in scientific notation is 52.4×10^6.

2. Complete the table.

Number	Scientific notation
$14,700$	
0.00083	
	7.6×10^8
	3.8×10^{-2}
0.38	
3.8	
$3,800,000,000,000$	
	9×10^{-10}

3. Evaluate each expression by first expanding each value inside the parentheses and then simplifying. Write the final result in scientific notation.

a. $(2 \times 10^3) + (6 \times 10^3)$

b. $2 \times (4.1 \times 10^2)$

c. $(1.5 \times 10^3) \times 3$

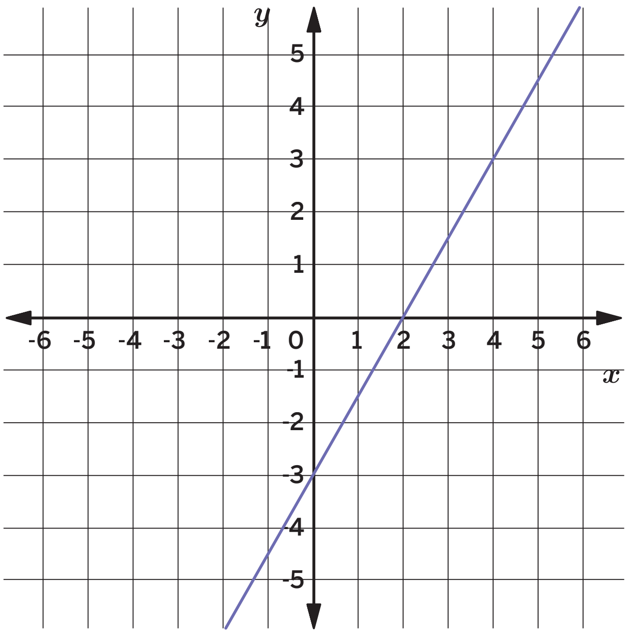
d. $(3 \times 10^3)^2$

e. $(9 \times 10^2) \times (3 \times 10^6)$

Name: ______ Date: ______ Period: ______

Practice

4. Here is a graph for one equation in a system of two equations.



a Write a second equation for the system, so that it has *infinitely many solutions*.

b Write a second equation, whose graph passes through $(0, 2)$, so that the system has *no solution*.

c Write a second equation, whose graph passes through $(2, 2)$, so that the system has *one solution* located at $(4, 3)$. Show your thinking.

5. Solve each equation. Show or explain your thinking.

a $2(3 - 2c) = 30$

b $3x - 2 = 7 - 6x$

6. Approximate each quotient by rounding each dividend and divisor first. Then write each approximation in scientific notation. The first row has been completed for you.

Expression	Related expression	Approximate quotient	Scientific notation
$6132 \div 41$	$6000 \div 40$	150	1.5×10^2
$9031245 \div 143$			
$0.0000304 \div 0.021$			

Unit 6 | Lesson 13

Multiplying, Dividing, and Estimating With Scientific Notation

Let's solve problems by multiplying and dividing numbers in scientific notation.

Warm-up Rewriting Powers of 10

1. Which of the following expressions are equivalent to 5×10^5? Select *all* that apply and be prepared to explain your thinking.

   A. 50×10^4

   B. 50^5

   C. 0.05×10^6

   D. 500×10^3

   E. -5×10^{-5}

2. Replace the box in each equation so that the equation is true.

   a. $6 \times 10^3 = \square \times 10^4$

   b. $4.2 \times 10^6 = 420 \times \square$

   c. $3.1 \times 10^{-2} = \square \times 10^{-3}$

   d. $5 \times 10^{-3} = 0.5 \times \square$

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Multiplying and Dividing With Scientific Notation

1. Consider the expressions $(4 \times 10^5) \times (4 \times 10^4)$ and 16×10^9.

a Evaluate each expression.

$(4 \times 10^5) \times (4 \times 10^4)$

16×10^9

b Compare the values you found in part a. What do you notice?

2. Consider the expressions $\frac{7 \times 10^6}{2 \times 10^2}$ and 3.5×10^4.

a Evaluate each expression.

$\frac{7 \times 10^6}{2 \times 10^2}$

3.5×10^4

b Compare the values you found in part a. What do you notice?

Activity 2 Biomass

Use the table to complete the following problems about different creatures on Earth. Write your responses using scientific notation. Be prepared to explain your thinking.

Creature	Approximate number of individuals on Earth	Typical mass of one individual (kg)
Humans	7.5×10^{9}	6.2×10^{1}
Cows	1.3×10^{9}	4×10^{2}
Sheep	1.75×10^{9}	6×10^{1}
Chickens	2.4×10^{10}	2×10^{0}
Ants	5×10^{16}	3×10^{-6}
Blue whales	4.7×10^{3}	1.9×10^{5}
Antarctic krill	7.8×10^{14}	4.86×10^{-4}
Zooplankton	1×10^{20}	5×10^{-8}
Bacteria	5×10^{30}	1×10^{-12}

1. Identify the least and most numerous creatures on Earth.

a Which creature is the least numerous?

b Which creature is the most numerous?

2. According to the values, approximately how many ants have the same mass as one human?

Name: ________ Date: ________ Period: ________

Activity 2 Biomass (continued)

3. Clare and Diego were trying to determine how many times more massive one ant is than one zooplankton. Review their work.

Clare's strategy:

$$\frac{3 \times 10^{-6}}{5 \times 10^{-8}} = \frac{30 \times 10^{-7}}{5 \times 10^{-8}} = 6 \times 10^{1}$$

Diego's strategy:

$$\frac{3 \times 10^{-6}}{5 \times 10^{-8}} = 0.6 \times 10^{2} = 6 \times 10^{1}$$

What do you notice about the strategy they each used?

4. One blue whale has the same mass of approximately how many humans?

5. There are approximately 57,790,200 horses and 308,000 rabbits (including hares) on the planet. To determine how many times more horses there are than rabbits and hares, Kiran says it will be more efficient to estimate using scientific notation. Tyler says it will be more efficient to estimate using the values given in standard form. Do you agree with Kiran or Tyler? Explain your thinking.

Are you ready for more?

Which has more mass — all the humans or all the bacteria — on Earth? Show or explain your thinking.

STOP

Summary

In today's lesson . . .

You solved problems about the animals on Earth by multiplying and dividing numbers written in scientific notation.

Multiplying numbers in scientific notation is an extension of multiplying decimals. To multiply two numbers in scientific notation, start by multiplying the first factors of each number using the commutative property. Then multiply the powers of 10, using what you have learned about exponents.

- For example, $(a \times 10^m) \times (b \times 10^n) = ab \times 10^{(m+n)}$.

To divide numbers in scientific notation, it can be helpful to first write the expression as a fraction. Divide the first factor in the numerator by the first factor in the denominator, and then divide the powers of 10 using what you have learned about exponents.

- For example, $\frac{a \times 10^m}{b \times 10^n} = \frac{a}{b} \times 10^{m-n}$.

Comparing very large or very small numbers by estimation is often more efficient with scientific notation. In some cases, it may be helpful to rewrite one quantity using a different power of 10 so that the powers of 10 on the two quantities are the same.

- For example, if you want to compare 4×10^5 and 8×10^4, you could rewrite 4×10^5 as 40×10^4.

  $\frac{40 \times 10^4}{8 \times 10^4} = 5$

  So, 4×10^5 is 5 times greater than 8×10^4.

Reflect:

Name: ________ Date: ________ Period: ________

Practice

1. Evaluate each expression. Write the result in scientific notation.

   a. $(1.5 \times 10^2) \times (5 \times 10^{10})$

   b. $\frac{6 \times 10^{-8}}{3 \times 10^{-3}}$

   c. $(5 \times 10^8) \times (4 \times 10^3)$

   d. $(7.2 \times 10^3) \div (1.2 \times 10)^5$

2. On planet Zerg, there are two alien species, zorks and zonks. One zork has a mass of 7.3×10^7 kg and one zonk has a mass of 2.1×10^3 kg. About how many times less in mass is one zonk than one zork?

3. How many buckets would it take to hold all the water in the world's oceans? Write your response in scientific notation.

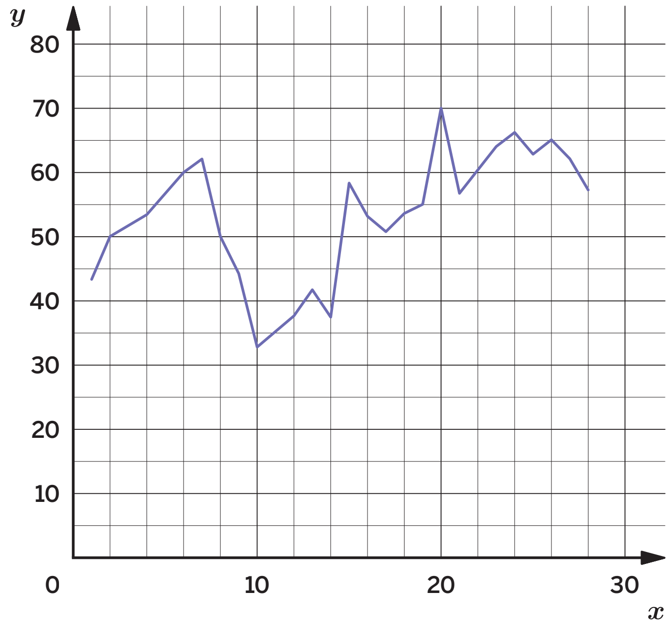
Some useful information:

- The world's oceans hold about 1.4×10^9 km^3 of water.
- A typical bucket holds about $20{,}000$ cm^3 of water.
- There are 10^{15} cm^3 in a cubic kilometer.

Practice

Name: ______________ Date: ______________ Period: ______________

4. The graph represents the closing price per share of stock for a company each day for 28 days.



a. What variable is represented on the horizontal axis?

b. In the first week, was the stock price generally increasing or decreasing?

c. During which time period did the closing price of the stock decrease for at least three days in a row?

5. Write an equation for the line that passes through $(-8.5, 11)$ and $(5, -2.5)$. Show your thinking.

6. Determine each sum or difference.

a. $1.35 + 0.25$

b. $1.35 + 0.025$

c. $1.35 - 0.25$

d. $1.35 - 0.025$

Name: ______________________ Date: __________ Period: __________

Unit 6 | Lesson 14

Adding and Subtracting With Scientific Notation

Let's solve problems by adding and subtracting numbers in scientific notation.

Warm-up Notice and Wonder

Diego and Clare were asked to evaluate the expression $(2 \times 10^3) + (6.5 \times 10^2)$. Consider the following strategies each student used.

Diego's strategy:	Clare's strategy:
$(2 \times 10^3) + (6.5 \times 10^2) = 2000 + 650$ $= 2{,}650$	$(2 \times 10^3) + (6.5 \times 10^2) = (2 \times 10^3) + (0.65 \times 10^3)$ $= 2.65 \times 10^3$

What do you notice? What do you wonder?

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Adding and Subtracting With Scientific Notation

1. Which equation is true? Explain your thinking.

A. $(2.3 \times 10^2) + (3.6 \times 10^2) = 5.9 \times 10^2$

B. $(2.3 \times 10^2) + (3.6 \times 10^2) = 5.9 \times 10^4$

2. Which equation is true? Explain your thinking.

A. $(4.1 \times 10^3) + (5 \times 10^2) = 9.1 \times 10^3$

B. $(4.1 \times 10^3) + (5 \times 10^2) = 4.6 \times 10^3$

Name: ______ Date: ______ Period: ______

Activity 2 A Celestial Dance

Study the table, which shows the diameter of some celestial objects in our solar system as well as each object's distance from the Sun.

Object	Diameter (km)	Distance from the Sun (km)
Sun	1.392×10^6	0×10^0
Mercury	4.878×10^3	5.79×10^7
Venus	1.21×10^4	1.08×10^8
Earth	1.28×10^4	1.47×10^8
Mars	6.785×10^3	2.28×10^8
Jupiter	1.428×10^5	7.79×10^8

Which of these distances is greater? Explain your thinking.

A. The combined distances of each of Mercury, Venus, Earth, and Mars from the Sun.

B. The distance from Jupiter to the Sun.

Stronger and Clearer: You will meet with other pairs of students to give and receive feedback on your explanations. Use this feedback to refine and improve your response.

Are you ready for more?

Suppose the planets listed in the table were placed side-by-side, *except* the Sun. About how much wider is the Sun than these planets placed side-by-side?

Activity 3 Biomass, Revisited

Use this table from Lesson 13 to solve some new problems about different creatures on Earth.

Creature	Approximate number of individuals	Typical mass of one individual (kg)
Humans	7.5×10^9	6.2×10^1
Cows	1.3×10^9	4×10^2
Sheep	1.75×10^9	6×10^1
Chickens	2.4×10^{10}	2×10^0
Ants	5×10^{16}	3×10^{-6}
Blue whales	4.7×10^3	1.9×10^5
Antarctic krill	7.8×10^{14}	4.86×10^{-4}
Zooplankton	1×10^{20}	5×10^{-8}
Bacteria	5×10^{30}	1×10^{-12}

1. A farmer is planning to transport one cow, two sheep, and three chickens to a different farm. The farmer will also transport 100,000 ants, which support healthy soil. What is the total mass of *all* the animals and ants that will be transported?

2. Which is greater, the number of bacteria or the total number of *all* the other animals in the table? Explain your thinking.

Are you ready for more?

Lin, Mai, and Noah are planning a trip to go swimming with blue whales. How much greater is the mass of one blue whale than Lin, Mai, and Noah altogether? Assume each person has the same mass as one human in the table.

STOP

Name: ______________________ Date: ____________ Period: ____________

Summary

In today's lesson . . .

You added and subtracted numbers written in scientific notation to solve problems about the planets in our solar system, and about different creatures on Earth. From your prior experience working with decimals, it is important to pay attention to place value when adding and subtracting decimals. The same is true when adding and subtracting numbers written in scientific notation.

For example, in the expression $(3.4 \times 10^5) + (2.1 \times 10^6)$, it may appear that you can add the numbers 3.4 and 2.1, but those digits are actually *not* in the same place-value position because the exponents on the power of tens are different. If you rewrite one of the numbers so that the power of 10 is the same, then you can add the digits.

$3.4 \times 10^5 = 3.4 \times 10^5$

$2.1 \times 10^6 = 21 \times 10^5$

Now that the power of 10 is the same, you can add 3.4 and 21.
The sum is 24.4×10^5, or 2.44×10^6.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. State whether each statement is true or false. Show or explain your thinking.

a $(3 \times 10^2) + (4 \times 10^3) = 7 \times 10^5$

b $(8 \times 10^2) - (5.1 \times 10^3) = 2.9 \times 10^2$

c $(7 \times 10^{-4}) + (9 \times 10^{-3}) = 9.7 \times 10^{-3}$

2. Evaluate each expression. Write the result in scientific notation.

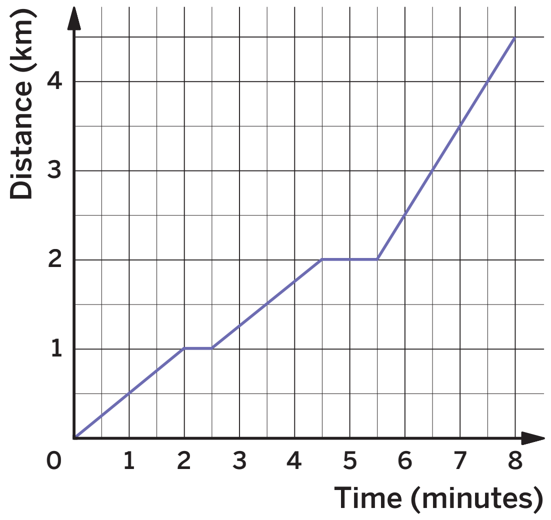
a $(5.3 \times 10^4) + (4.7 \times 10^4)$

b $(3.7 \times 10^6) - (3.3 \times 10^6)$

c $(4.8 \times 10^{-3}) + (6.3 \times 10^{-3})$

d $(6.6 \times 10^{-5}) - (6.1 \times 10^{-5})$

3. Refer to the graph.

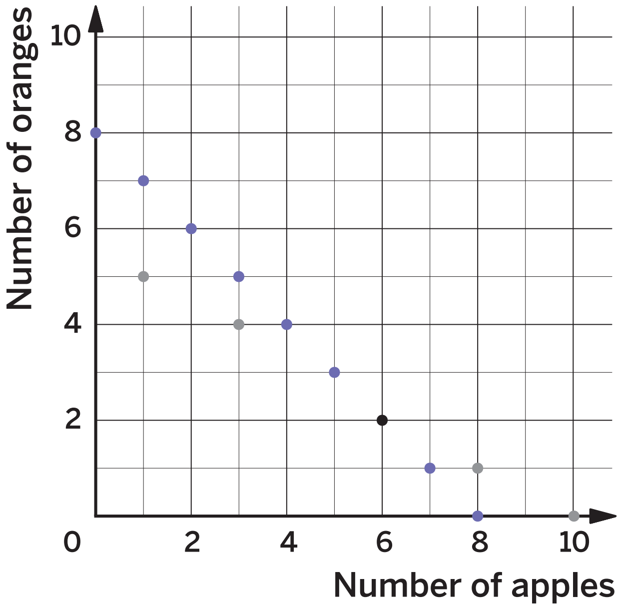


a Write a real-world scenario that describes what is happening in the graph.

b What happens at 5 minutes?

c What does the slope of the line between 6 and 8 minutes mean?

4. Apples cost $1 each. Oranges cost $2 each. You have $10 and want to buy 8 pieces of fruit. One graph shows the combinations of apples and oranges that cost a total of $10. The other graph shows the combinations of apples and oranges that have a total of 8 pieces of fruit.



a. Name one combination of 8 fruits shown on the graph whose total cost is *not* $10.

b. Name one combination of fruits shown on the graph whose total cost is $10.

c. How many apples and oranges will you need in order to have 8 fruits that cost a total of $10?

5. How many centimeters are in 8.7 km? Remember there are 100 cm in 1 m, and 1,000 m in 1 km. Write your final answer in scientific notation.

Unit 6 | Lesson 15 – Capstone

Is a Smartphone Smart Enough to Go to the Moon?

Let's answer some big questions about even larger numbers!

Warm-up Measuring Computer Power

Computing power has changed significantly over the past several decades. You will take a closer look at the history of computing power, going back to when NASA first launched the Apollo program to land a man on the Moon.

First, let's become familiar with some ways to measure computing power. Computing power can be measured by looking at storage, processing speed, and memory.

For reference:

Storage and memory processing are each measured in bytes.

Processor speed is measured in hertz.

Kilo means 1,000.

Mega means 1,000,000.

Giga means 1,000,000,000

Tera means 1,000,000,000,000.

1. Which is greater, 1 kilobyte or 1 megabyte?

2. How many gigabytes are equivalent to 1 terabyte?

3. How many kilohertz are in 5.2 terahertz?

Log in to Amplify Math to complete this lesson online.

Name: ____________ Date: ____________ Period: ____________

Activity 1 Old Hardware, New Hardware

In 1966, the Apollo Guidance Computer was developed to make the calculations that would put humans on the Moon.

You will be given information for different devices from 1966 to 2020. Choose one device and compare that device with the Apollo Guidance Computer. Consider using scientific notation to help with calculations.

1. Which device can store more information? About how many times more information?

2. Which device has a faster processor? About how many times faster?

Are you ready for more?

As you just saw, computing power has increased substantially in the modern era. Recently, a supercomputer was built by a lab in Tennessee that has a storage capacity of 200 petabytes and can perform 200 quadrillion calculations per second, making it one of the most powerful computers in the world! For your reference, 1 petabyte is equal to 10^{15} bytes, and 1 quadrillion is equal to 1 thousand trillion.

1. How many terabytes are equivalent to 200 petabytes?

2. What is 200 quadrillion written in scientific notation?

Activity 2 Counting to a Million and Beyond

Thanks to modern computing, NASA and other space programs are able to explore places far beyond the Moon. This means we can think more deeply about the vast stretches of the Universe.

The size of the Universe is thought to be 93 billion light years, meaning it would take 93 billion years for a ray of light to cross the entire Universe. That's pretty big! To put it another way, astronomers estimate the number of atoms in the Universe to be anywhere from 10^{78} to 10^{82}. That's a lot of atoms!

But that's a drop in the bucket compared to the number *googol*, which is 10^{100}. And if you think a googol is big, a googolplex is 10^{googol} or $10^{10^{100}}$. Is it even possible to count to a googolplex? How long would it take you to write out the whole number?

In 1977, mathematician Ronald Graham proposed a number larger than a googolplex, setting a record at the time for the largest number ever used in a mathematical proof.

Maybe let's start with something a little smaller . . .

1. Jeremy Harper set a record for counting aloud from one to one million. He started on June 18th, 2007, and counted for 16 hours each day, every day, until he reached one million. He was able to count, on average, about 12 numbers per minute. About how many days, counting 16 hours each day, would you estimate it took him to reach 1 million?

2. Suppose Jeremy Harper decided to start counting to one trillion. If he counted for 16 hours each day at the same rate as before, about how many days would it take him to count to one trillion?

Name: ______ Date: ______ Period: ______

Activity 2 Counting to a Million and Beyond (continued)

3. What about counting to a googol?
It would take a *really* long time to count to a googol.
Consider these questions instead.

a Write a googol as a number. How much space do you need?

b Order this list of items from least to greatest based on what you know. Write the numbers 1–8 next to each item, where 1 represents the least number and 8 represents the greatest number.

______ The population of Earth.
______ The number of stars in the Universe.
______ The number of bacteria on Earth.
______ The number of atoms in the Universe.
______ A googol.
______ The number of seconds in an 8-hour school day.
______ The population of the United States.
______ The number of grains of sand on Earth.

Featured Mathematician

Ronald Graham

Imagine discovering a number so large, it gets named after you. That's exactly what mathematician Ronald Graham (1935–2020) did when he proposed the solution to a problem in the mathematical field called Ramsey Theory. At the time, "Graham's number" was the largest number ever used in a mathematical proof, so large that the observable Universe is far too small to contain a digital representation. Over the course of Graham's career, he wrote six books and authored or co-authored hundreds of papers. He worked at IBM's Bell Labs, a research and scientific development company that has won nine Nobel Prizes for the invention of fundamental technologies that power our world today, such as lasers, photovoltaic cells, and computer programming languages.

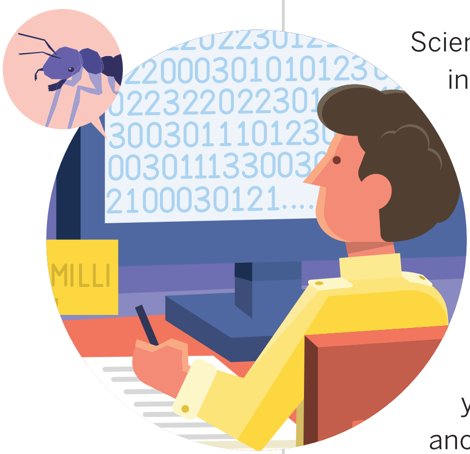
STOP

"Ronald Graham Juggling" by Peter Vidor, courtesy of Wikimedia Commons

Unit Summary

Scientists estimate there are more than 10 million, million, million individual insects alive on the planet right now. Meanwhile, the average human being houses $39{,}000{,}000{,}000{,}000$ bacteria in their gut. And with every shuffle of a deck of 52 cards, you are ordering the cards into one of approximately 80 unvigintillion (that's an 8 followed by 67 zeros!) possible permutations.

Whether we notice it or not, we are surrounded by numbers that are both astronomically large and infinitesimally small. From the number of atoms in the Universe to the weight of a microorganism, it is important to find a way to handle these numbers. And for that, there are no better tools than exponents and scientific notation.



Scientific notation allows us to rewrite incredibly large or incredibly small numbers in a way that is friendly to the eye. Using powers of 10, these numbers can be compared much more efficiently. So rather than reach for the dictionary, we can write 80 unvigintillion as 8×10^{67}, and $39{,}000{,}000{,}000{,}000$ as 3.9×10^{13}.

And when numbers share a common base, multiplying and dividing become a snap! When you are multiplying, just add the exponents. And when you are dividing, subtract. When one power is raised to another, you can multiply the exponents to find the new resulting exponent.

With an assist from exponents and scientific notation, you can tackle any number, great or small.

See you in Unit 7.

Name: Date: Period:

Practice

1. How many times greater is 200 billion than 5 thousand? Write your response in scientific notation.

2. Evaluate each expression and write the result in scientific notation.

a $(4.3 \times 10^2) + (2.1 \times 10^3)$

b $(1 \times 10^{-3}) - (9 \times 10^{-4})$

c $(3 \times 10^3) \times (4 \times 10^5)$

d $\frac{4 \times 10^{-3}}{5 \times 10^{-6}}$

3. Andre and Han are measuring microorganisms in a lab. Andre measures a microorganism that is 4.2×10^{-2} cm in diameter. Han measures a microorganism that is 1.9×10^{-5} cm in diameter. About how many times greater is the diameter of Andre's microorganism than Han's?

4. Jada is making a scale model of the solar system. The distance from Earth to the Moon is about 2.389×10^5 miles. The distance from Earth to the Sun is about 9.296×10^7 miles. She decides to place Earth on one corner to her dresser and the Moon on another corner, about a foot away from each other. Where should she place the Sun? Show or explain your thinking.

A. On a windowsill in the same room.

B. In her kitchen, which is down the hallway.

C. Half of a city block away.

Practice

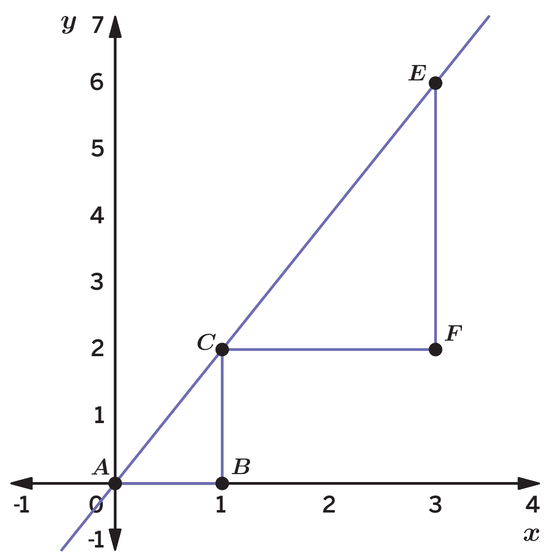
Name: .... Date: .... Period: ....

5. Solve each equation and check your solution. Show your thinking.

a $-2(3x - 4) = 4(x + 3) + 6$

b $-\frac{1}{2}(z + 4) - 6 = -2z + 8$

6. Explain why Triangle ABC is similar to Triangle CFE.



My Notes:

UNIT 7

Irrationals and the Pythagorean Theorem

Discover how three squares can prove something radical about triangles that has captivated mathematicians for centuries.

Essential Questions

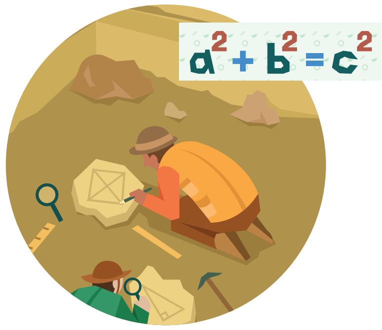
- What is the difference between a rational number and an irrational number?
- How can you estimate the square root of a number? And what does it represent?
- Is it true that $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ for *all* right triangles? If so, can you prove it?
- *(By the way, what is the longest cut you can make in a sandwich?)*

SUB-UNIT

1 Rational and Irrational Numbers

Narrative: Beliefs were challenged with the idea that not every number can be expressed as a ratio.

You'll learn . . .
- about numbers that are *not* rational.
- how to approximate these numbers on the number line.



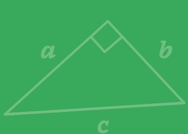
SUB-UNIT

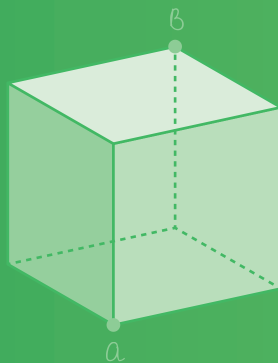
2 The Pythagorean Theorem

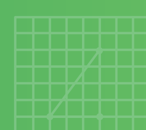
Narrative: Discover and use "the most proven theorem of all time".

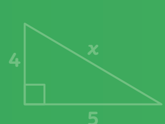
You'll learn . . .
- about the special relationship among the side lengths of a right triangle.
- how to use this relationship to solve real-world and mathematical problems.

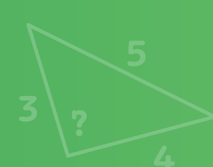
Let ℓ be the length of the shortest path from point A to point B across the surface of a cube with the side length 1. Find ℓ^4.











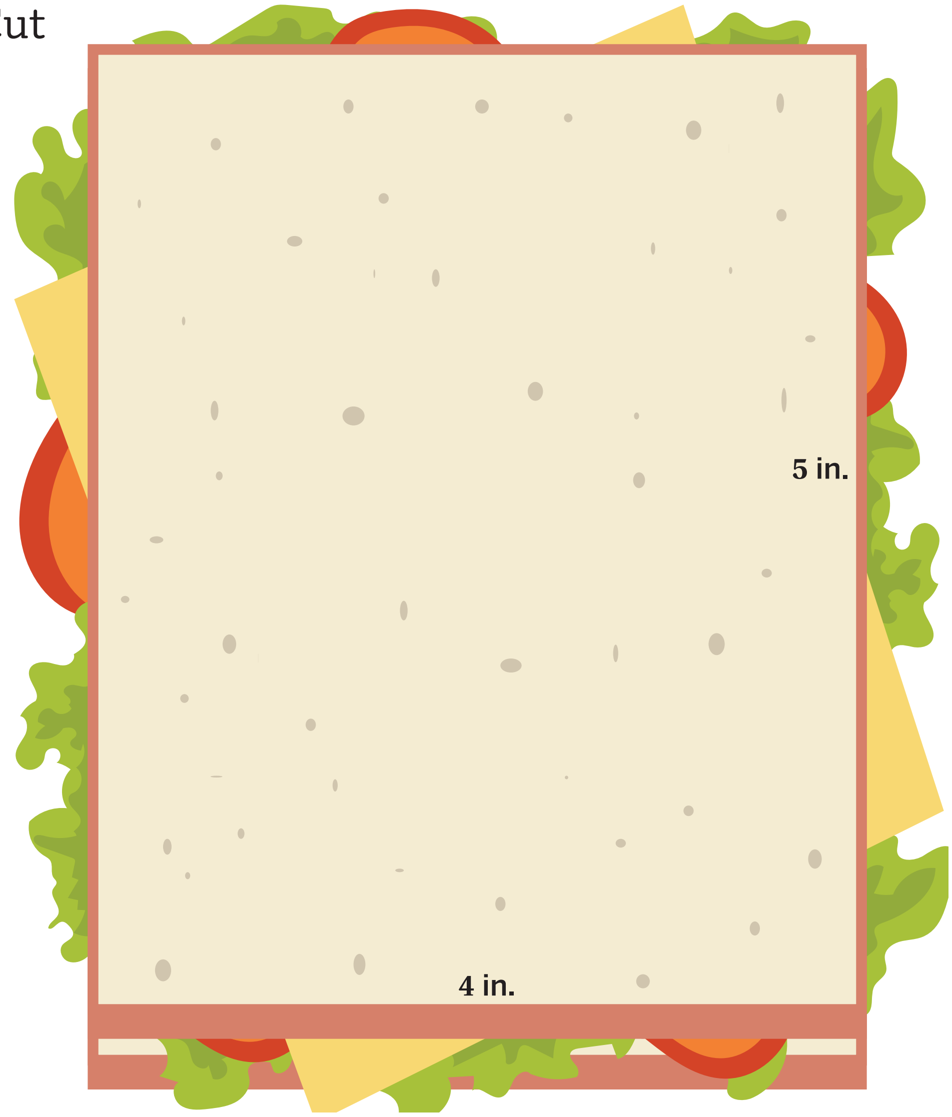
Unit 7 | Lesson 1 – Launch

Sliced Bread

Let's cut some sandwiches.

Warm-up Make a Cut

Draw a line to represent cutting the sandwich into two parts. Label the length of the cut and the sides of each new part.



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Name: ______________________ Date: __________ Period: __________

Activity 1 The Longest Cut

You will be provided with a string and several images of sandwiches.

What is the longest, straight cut you can make on each sandwich? Estimate and then label the length of the cut. Record the measures of the sides and the cut for each sandwich.

	Side (in.)	Side (in.)	Length of cut (in.)
Sandwich A			
Sandwich B			
Sandwich C			
Sandwich D			

Activity 2 The Longest String

You will be provided with string, a ruler, and a box.

1. Sketch your box. Label the length, width, and height of the box.

2. What is the longest straight length of string you can create that fits completely inside the box? Sketch the measure of the string length you found inside the box and label its length.

STOP

Unit 7 Irrationals and the Pythagorean Theorem

The Mystery of the Pythagoreans

What makes a perfect sandwich? Fresh ingredients? Toasty bread?

No! It all comes down to presentation. With one diagonal cut, a boring square is transformed into two perfect right triangles of sandwichy goodness.

Diagonals might not seem like a big deal. We use them everyday — taking a shortcut across a yard, moving a bishop along a chessboard, or folding down a corner for the perfect paper airplane. But would you believe that behind these simple, everyday occurrences, there lies a dark and twisted story, as bewildering as any mystery thriller?

It's a story centuries in the making involving cults, mystics, mysterious deaths, and baffling clues in notebook margins. So, let's roll up our sleeves, get off the grid, and peer into the infinite with the Pythagorean Theorem and irrational numbers.

Welcome to Unit 7.

Practice

Name: ______ Date: ______ Period: ______

1. A kite flies from a string 150 ft above the person holding it, and trails 75 ft behind them as they run. Andre estimates the length of the string is 150 ft. Do you think his estimate is reasonable? Show or explain your thinking.

2. A cat is stuck in a tree 40 ft above the ground. The fire department is called to rescue the cat with their ladder, but must park behind a fence, which is 35 ft from the tree. The firefighter sets up an extension ladder to reach the cat. To what length would the ladder reasonably need to extend? Explain your thinking.

3. Select *all* the expressions equivalent to 3^8.

A. $(3^2)^4$

B. 8^3

C. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

D. $(3^4)^2$

E. $\frac{3^6}{3^{-2}}$

F. $3^6 \cdot 10^2$

Name: .... Date: .... Period: ....

Practice

4. Noah is asked to evaluate $5.4 \times 10^5 + 2.3 \times 10^4$ and give the final result in scientific notation. Noah says, "I can rewrite 5.4×10^5 as 54×10^4. Then I can add the numbers: $54 \times 10^4 + 2.3 \times 10^4 = 56.3 \times 10^4$."

Do you agree with Noah's solution to the problem? Explain your thinking.

5. The high school is hosting an event for seniors, but it will also allow some juniors to attend. The principal approved the event for 200 students and decided the number of juniors should be 25% of the number of seniors. How many juniors are allowed to attend? Show or explain your thinking. **Hint:** Try writing two equations that each represent the number of juniors and seniors at the event.

6. Order these numbers from least to greatest.

4.5	4	5.4	4.57	4.055

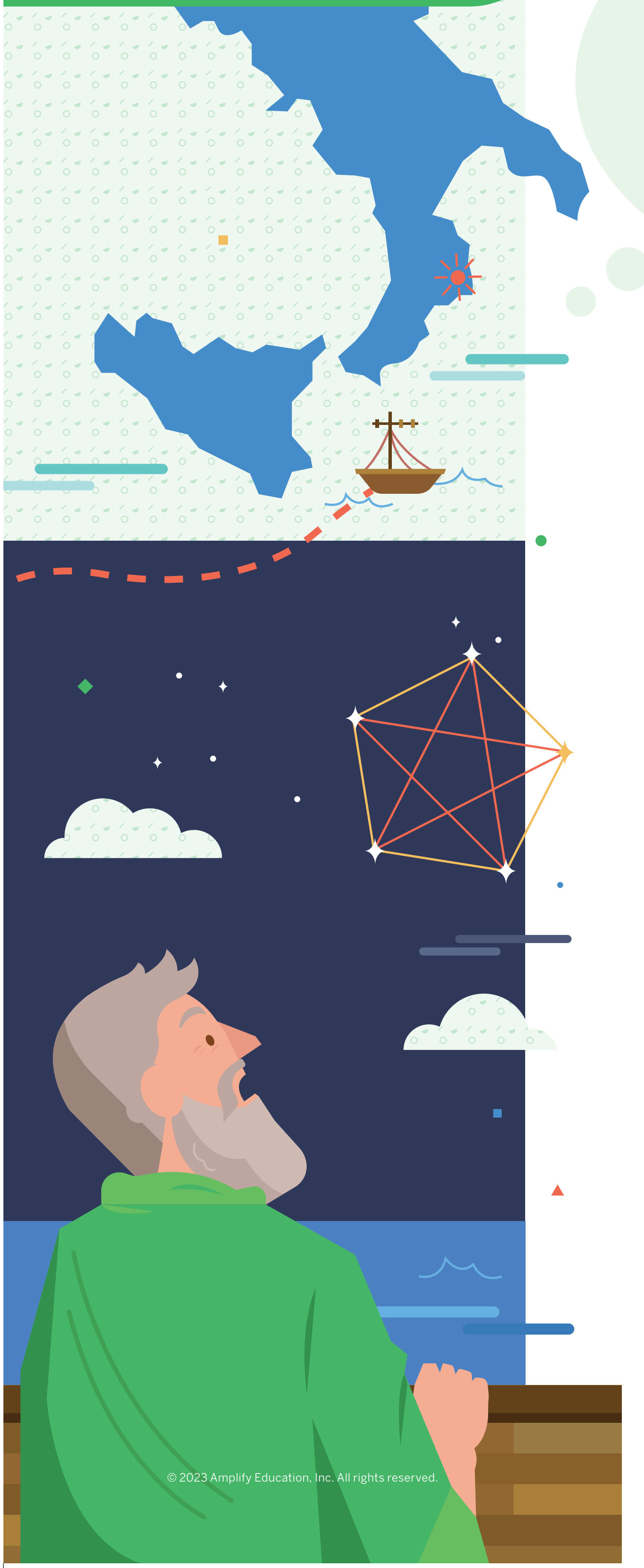
Least **Greatest**

My Notes:

SUB-UNIT

1 Rational and Irrational Numbers

Narrative Connections



How rational were the Pythagoreans?

Around 530 BCE, a stranger had arrived in the city of Croton. He had left from his home on the island of Samos. Years before, Samos had been seized by the tyrant Polycrates, whose constant warring brought instability to the island. But now the man from Samos had arrived at his new home. And so, in the 6th century BCE, on the shores of southern Italy, Pythagoras looked to make his mark.

It was in Croton that Pythagoras would start a movement called "Pythagoreanism." For some, it was a philosophy; for others, a mystical cult. Their most deeply held belief was that whole numbers made up the very fabric of the universe.

They believed that all numbers could be described as a ratio of two whole numbers, such as the fraction $\frac{1}{2}$.

But what happened when this belief was challenged?

The legend goes that there was a Pythagorean named Hippasus. While traveling by ship, he saw a pentagon formed by stars in the sky. Within that pentagon, he drew a five-pointed star. A loyal Pythagorean, he was trying to describe the pentagon's side length as a ratio of two parts of the star. To his horror, he discovered it was impossible. These lengths shared no whole unit in common. When he told his fellow Pythagoreans, they hauled him from the ship and drowned him for heresy.

We know now that this number is called an *irrational number*. For the Pythagoreans, it defied belief and challenged everything they stood for.

Unit 7 | Lesson 2

The Square Root

Let's learn about square roots.

Warm-up Algebra Talk

Determine a solution for each equation.

1. $x^2 = 49$

2. $x^2 = 1$

3. $x^2 = 16$

4. $x^2 = \frac{9}{100}$

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Determining the Value of x

You and your partner will take turns guessing the value of x for the equation $x^2 = 2$.

1. Decide which student will guess first. The other student will use a calculator to determine the value of x^2. Record your results in the table. You and your partner will have a total of 10 guesses. At the end of the 10 guesses, determine which student was the closest to guessing the value of x.

$x^2 = 2$

	x	x^2
Partner A		
Partner B		
Partner A		
Partner B		
Partner A		
Partner B		
Partner A		
Partner B		
Partner A		
Partner B		
Partner A		
Partner B		

2. Do you think there is a value of x so that $x^2 = 2$? Explain your thinking.

Activity 2 Square Roots

Complete the table by writing two solutions for each equation, first using square root notation, and then without it.

	With $\sqrt{}$	Without $\sqrt{}$
$x^2 = 36$	$x = \sqrt{36}$	$x = 6$
$x^2 = 64$		
$x^2 = 144$		
$x^2 = 81$		
$x^2 = \frac{1}{36}$		
$x^2 = 0.01$		
$x^2 = \frac{4}{25}$		

Are you ready for more?

Solve each equation.

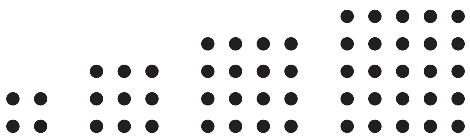
1. $x^2 + 1 = 2$
2. $x^2 + 1 = 1$
3. $x^2 + 1 = 0$

Name: ______________________ Date: __________ Period: __________

Activity 3 Perfect Squares

Pythagoras of Samos was an ancient Greek philosopher who lived from 570 BCE to 495 BCE. Pythagoras and his followers, the Pythagoreans, studied religion, philosophy, and numbers. The Pythagoreans believed that numbers defined the universe and often represented numbers graphically. For example, the Pythagoreans arranged dots into patterns, to help them understand relationships between numbers.

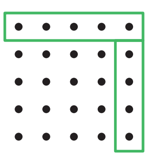
Here are some representations of ***perfect squares*** using dots.



4 9 16 25

1. List all of the perfect squares that are less than 200.

Geometric arrangements of dots can help you look for patterns between perfect squares. For example, 16 dots can be arranged into 4 rows, each row with 4 dots. When 9 more dots are added to the square, you get a larger square with a total of 25 dots. This arrangement shows that some perfect squares can be written as a sum of two other perfect squares.



$25 = 16 + 9$

2. Determine another example where a perfect square can be represented as a sum of two perfect squares.

STOP

Summary

In today's lesson . . .

You determined a solution to equations, such as $x^2 = p$, using ***square root*** notation.

For example, $x = \sqrt{100}$, or 10, is a solution for the equation $x^2 = 100$, so $\left(\sqrt{100}\right)^2 = 100$.

$x = \sqrt{2}$ is a solution for the equation $x^2 = 2$, so $\left(\sqrt{2}\right)^2 = 2$.

You discovered that a ***perfect square*** is a number that is the square of an integer. For example, 9 is a perfect square because $3^2 = 9$, but 8 is not a perfect square because there is no square of an integer that equals 8.

Reflect:

Name: ________________ Date: ________ Period: ________

Practice

1. Which value is a solution to the equation $x^2 = 14$?

A. $x = 7$

B. $x = \sqrt{14}$

C. $x = \sqrt{7}$

D. $x = 196$

2. Determine the solution to each equation. If possible, write your response without using square root notation.

a $x^2 = 81$

b $x^2 = 10$

c $\sqrt{x} = 12$

d $\sqrt{x} = 9$

3. Write an equivalent value *without* using a square root symbol.

a $\sqrt{1}$

b $\sqrt{144}$

c $\sqrt{400}$

d $\sqrt{0}$

e $\sqrt{\frac{4}{49}}$

f $\sqrt{0.09}$

g $\sqrt{0.0025}$

h $\sqrt{\frac{1}{100}}$

4. Select the expression that is equivalent to $(3.1 \times 10^4) \times (2 \times 10^6)$.

A. 5.1×10^{10}

B. 5.1×10^{24}

C. 6.2×10^{10}

D. 6.2×10^{24}

Practice

Name: .... Date: .... Period: ....

5. The table shows the approximate area of four countries.

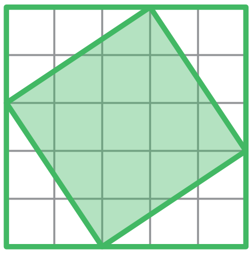
Country	Area (km^2)
Russia	1.71×10^7
Canada	9.98×10^6
Brazil	8.52×10^6
India	3.29×10^6

a. How much larger is Russia than Canada? Write the result in scientific notation. Show your thinking.

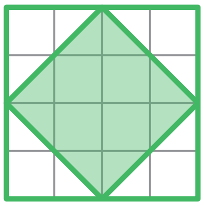
b. Which is greater — the combined area of Brazil and India or the area of Russia? Show your thinking.

6. Determine the area of each shaded square. Show or explain your thinking.

a.



b.



Name: ______ Date: ______ Period: ______

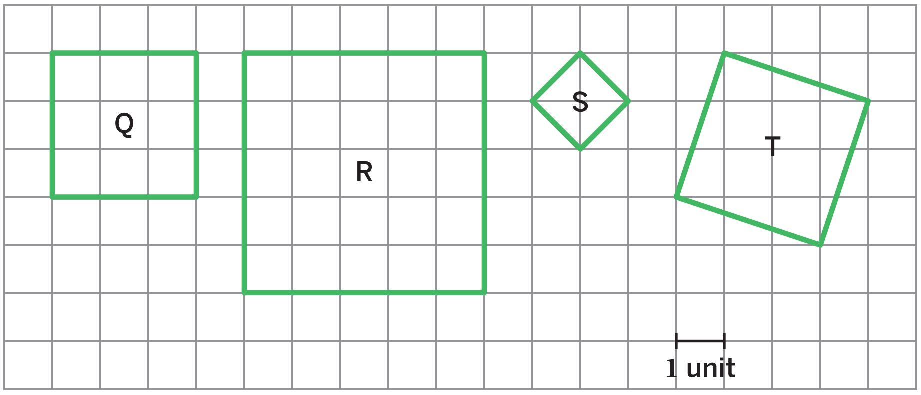
Unit 7 | Lesson 3

The Areas of Squares and Their Side Lengths

Let's investigate the squares and their side lengths.

Warm-up Areas and Side Lengths

Consider the following squares.



1. Determine the area, in square units, of each square.

Square Q: **Square R:**

Square S: **Square T:**

2. Estimate the side length, in units, of each square.

Square Q: **Square R:**

Square S: **Square T:**

Log in to Amplify Math to complete this lesson online.

Activity 1 Comparing Squares

You will be given a sheet with six squares.

1. Label each square with its area and *exact* side length.

2. Which square(s) have a side length that is greater than 2 units, but less than 3 units? Explain your thinking.

3. Which square(s) have a side length that is greater than 4 units, but less than 5 units? Explain your thinking.

4. Which square(s) have a side length that is greater than 6 units, but less than 7 units? Explain your thinking.

Name: ________ Date: ________ Period: ________

Activity 2 Sorting Square Roots

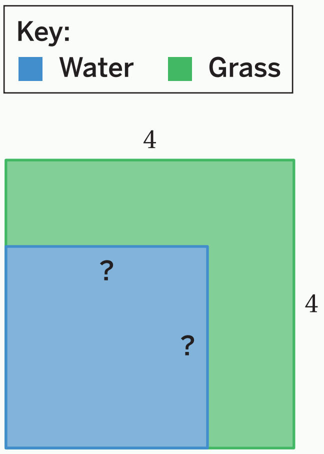
Write each square root under the corresponding column in the table according to its value. It is possible that you may not write all of the square roots.

$\sqrt{6}$ $\sqrt{7}$ $\sqrt{10}$ $\sqrt{15}$ $\sqrt{3}$ $\sqrt{40}$ $\sqrt{2}$

Between 1 and 2	Between 2 and 3	Between 3 and 4

Are you ready for more?

A city has a park enclosed by a fence in the shape of a square with side lengths of 4 m. The city would like to build a pool by making a smaller square filled with water, as shown in the figure. What should the side length of the smaller square be so that half of the area is grass and half is water?



STOP

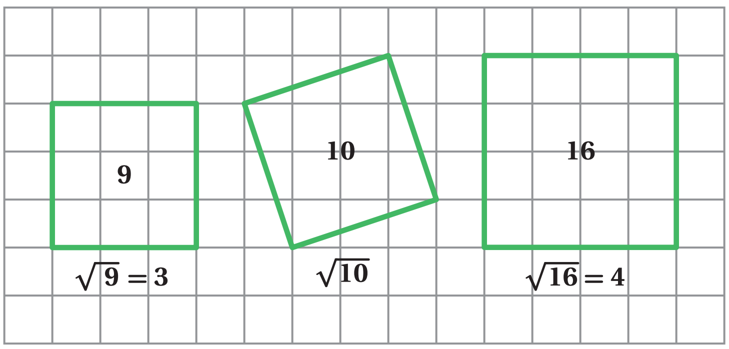
Summary

In today's lesson . . .

You determined the side length of a square given the square's area. The side length of a square is equal to the *square root* of its area.

You also approximated the value of square roots by observing the whole numbers around it, and remembering the relationship between square roots and squares.

For example, the area of a square with a side length of 3 units is 9 square units and the area of a square with a side length of 4 is 16 square units and 10 is between 9 and 16. Therefore, $\sqrt{10}$ is between 3 and 4.



Reflect:

Name: ______________ Date: ______________ Period: ______________

Practice

1. Determine the side length of a square if the area of the square is . . .

   a. 81 in^2

   b. $\frac{4}{25}$ cm^2

   c. 0.49 square units

   d. m^2 square units

2. Compare each pair of expressions using the symbol <, >, or =.

   a. $\sqrt{3}\ \square\ \sqrt{2}$

   b. $\sqrt{99}\ \square\ 98$

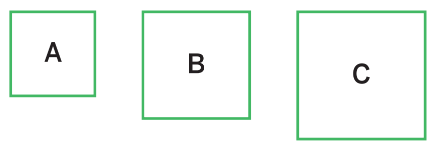
   c. $\sqrt{3}\ \square\ \sqrt{3.7}$

   d. $\sqrt{8}\ \square\ 54$

   e. $\sqrt{121}\ \square\ 11$

   f. $\sqrt{\frac{2}{3}}\ \square\ \sqrt{\frac{2}{5}}$

3. The side lengths of each square shown are $\sqrt{26}$, 4.2, and $\sqrt{11}$. Match each square with its side length. Explain your thinking.



Name: .... Date: .... Period: ....

4. Select *all* the expressions that are equivalent to 10^{-6}.

A. $\frac{1}{1{,}000{,}000}$

B. $-\frac{1}{1{,}000{,}000}$

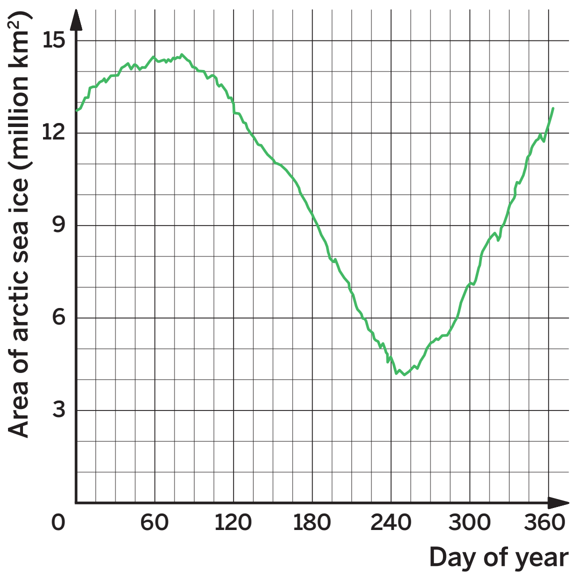
C. $\frac{1}{10^6}$

D. $10^8 \cdot 10^{-2}$

E. $\left(\frac{1}{10}\right)^6$

F. $\frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$

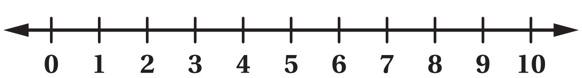
5. The graph represents the area of arctic sea ice, in square kilometers, as a function of each day of the year in 2016.



a. Give an approximate interval of days when the area of arctic sea ice was decreasing.

b. On which days was the area of arctic sea ice 12 million square kilometers?

6. Plot the values for each number: $\sqrt{25}$, $\sqrt{81}$, $\sqrt{9}$.

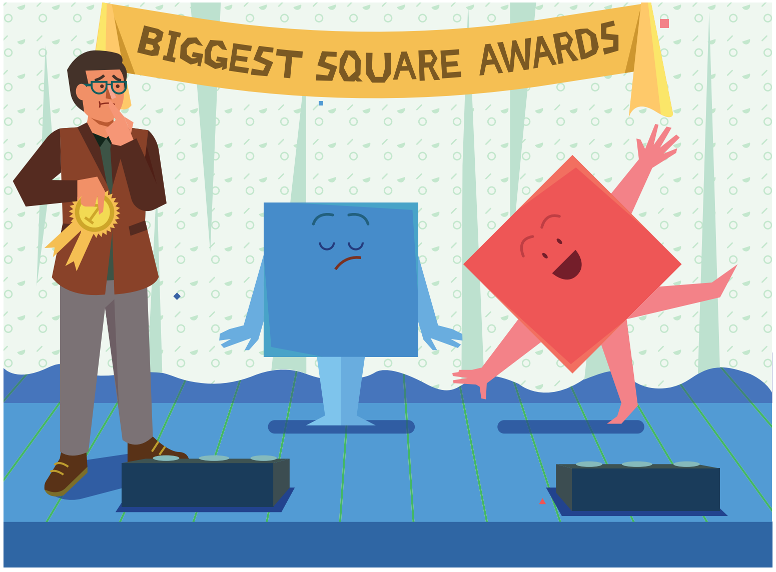


Name: ______ Date: ______ Period: ______

Unit 7 | Lesson 4

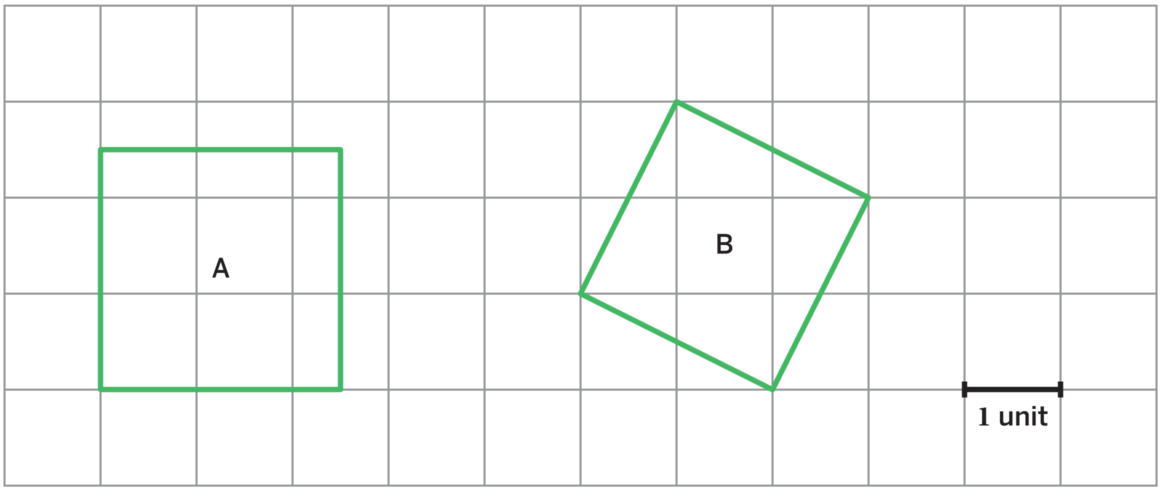
Estimating Square Roots

Let's approximate square roots.



Warm-up Comparing Two Squares

Consider the two squares shown.



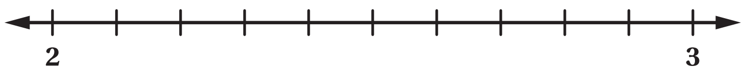
1. Which square has a greater area? Show or explain your thinking.

2. Which square has a greater side length? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Estimating Square Roots

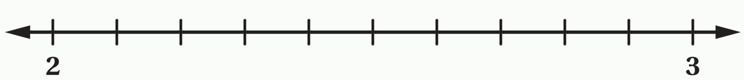
1. Estimate the value of $\sqrt{5}$ and plot it on the number line.



2. Compare your placement of $\sqrt{5}$ with a partner. Who had a more precise estimate? Explain your thinking.

Are you ready for more?

Estimate the value of $\sqrt{7}$ and plot it on the number line. Approximate the value of $\sqrt{7}$ to the nearest thousandth.



Name: .... Date: .... Period: ....

Activity 2 Ordering Square Roots on a Number Line

Your group will be given a card with five numbers.

Create a scale for your number line and then plot and label the approximate location for each number on the number line.

Are you ready for more?

Write an equivalent expression for each of the following.

1. $\sqrt{2^4}$
2. $\sqrt{2^8}$
3. $\sqrt{2^{100}}$

STOP

Summary

In today's lesson . . .

You explored how to make more precise estimates of square roots. For example, $\sqrt{75}$ is between 8 and 9 because $8^2 = 64$ and $9^2 = 81$.

You might start by checking whether $\sqrt{75}$ is less than or greater than 8.5 by calculating 8.5^2. Because $8.5^2 = 72.25$, you can conclude that $\sqrt{75} > 8.5$.

You can then test a number greater than 8.5, such as 8.7, for a more precise estimate. $8.7^2 = 75.69$, so you can conclude that $\sqrt{75} < 8.7$.

Now that you know that $\sqrt{75}$ is greater than 8.5, but less than 8.7, you can test a number between those two numbers, such as 8.6. Because $8.6^2 = 73.96$, you can conclude that $\sqrt{75}$ is between 8.6 and 8.7.

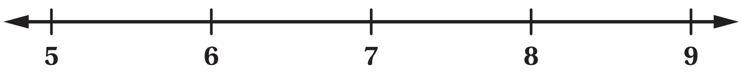
Reflect:

Practice

Name: ________________ Date: ________ Period: ________

1. Plot and label the approximate value of each number on the number line.

6.8	$\sqrt{76}$	$\sqrt{40}$	$\sqrt{64}$	7.5



2. Determine whether each inequality is *true* or *false*.

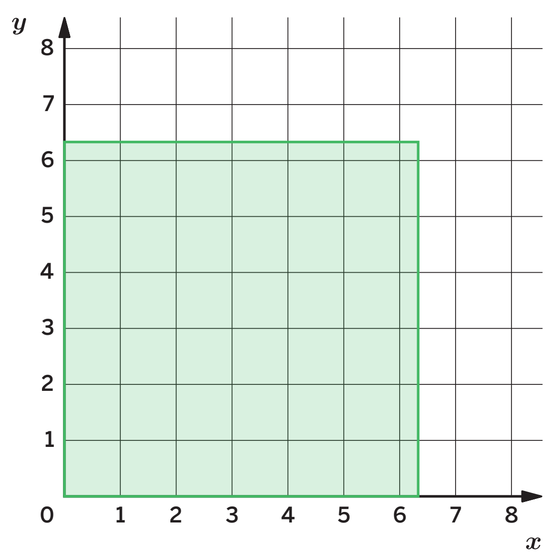
a) $\sqrt{34} > 6$

b) $\sqrt{15} > 3$

c) $\sqrt{3} > 1.7$

d) $\sqrt{2} < 1.4$

3. The shaded square has an area of 40 square units. Use the image to estimate the value of $\sqrt{40}$. Explain your thinking.



Practice

Name: ______ Date: ______ Period: ______

4. Write each expression as a single power.

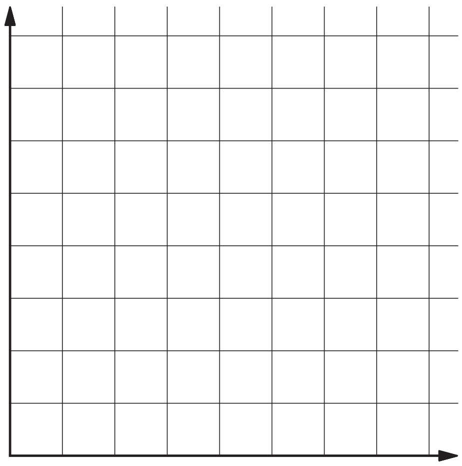
a $(10^2)^{-3}$

b $(3^{-3})^2$

c $3^{-5} \cdot 4^{-5}$

d $2^5 \cdot 3^{-5}$

5. Andre is ordering ribbon to make decorations for a school event. He needs a total of exactly 30 m of blue and green ribbon. Andre needs 50% more blue ribbon than green ribbon for the basic design, plus an extra 5 m of blue ribbon for accents. How much of each color of ribbon does Andre need to order? Use the coordinate grid to help your thinking.



6. Which expression is equivalent to 4^3?

A. $4 \cdot 4 \cdot 4$

B. $4 \cdot 3$

C. $3 \cdot 3 \cdot 3 \cdot 3$

D. $3 + 3 + 3 + 3$

E. $4 + 4 + 4$

Name: ________ Date: ________ Period: ________

Unit 7 | Lesson 5

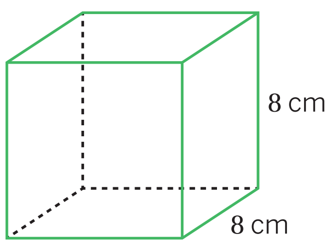
The Cube Root

Let's learn about cube roots.

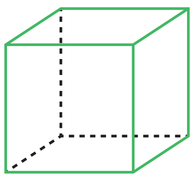
Warm-up Volume and Edge Length

A local market sells reusable, cube-shaped containers for food storage. Using a base and exponent, write an equation that represents the volume of each container. Then determine the missing number.

a $V = ?$



b $V = 27 \text{ cm}^3$



Log in to Amplify Math to complete this lesson online.

Activity 1 Determining the Values of x and x^3

1. The table shows the measurements of different cube-shaped storage containers where x represents the edge length and x^3 represents the volume. Complete the table with the missing measurements.

x (in.)	x^3 (in^3)
2	
7	
	64
	125

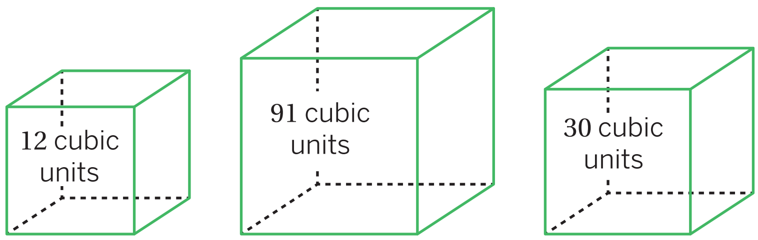
2. The equation $x^3 = 100$ represents a box with a volume of 100 in^3 and an edge length x. Can you determine the *exact* side length? Record your thinking in the table.

x	x^3

Name: ____________ Date: ____________ Period: ____________

Activity 2 Cube Roots

Consider the following cubes, each labeled with its volume.



1. Label each cube with its exact edge length.

2. Plot and label the approximate location for each edge length from Problem 1 on the number line.

3. Choose one edge length and explain how you determined where to plot the approximate location on the number line.

Reflect: How did you communicate clearly? How could you improve your communication?

STOP

Summary

In today's lesson . . .

You discovered that you could represent the solution to equations of the form $x^3 = p$ using ***cube root*** notation. For example, the solution to the equation $x^3 = 100$ could be represented as $x = \sqrt[3]{100}$.

You also discovered that you can use the *cube root* symbol when describing the edge length of a cube given the cube's volume.

A ***perfect cube*** is a number that is the cube of an integer. For example, 8 is a perfect cube because $2 \cdot 2 \cdot 2 = 8$, but 100 is not a perfect cube because there is no cube of an integer that equals 100.

You can approximate the values of cube roots by observing the whole numbers around it and remembering the relationship between cube and cube roots. For example, $\sqrt[3]{20}$ is between 2 and 3 because $2^3 = 8$ and $3^3 = 27$ and 20 is between 8 and 27.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Complete the table by writing the exact values. If possible, write your response *without* using cube root notation.

x	x^3
2	8
10	
	27
$\sqrt[3]{11}$	
	23
a	
	n

2. Write an equivalent value *without* using a cube root symbol.

a $\sqrt[3]{1}$

b $\sqrt[3]{216}$

c $\sqrt[3]{8000}$

d $\sqrt[3]{\frac{1}{64}}$

e $\sqrt[3]{\frac{27}{125}}$

f $\sqrt[3]{0.027}$

3. Compare each pair of expressions using the symbol <, >, or =.

a $\sqrt[3]{23}$ ☐ $\sqrt[3]{12}$

b $\sqrt[3]{54}$ ☐ 4

c $\sqrt[3]{2}$ ☐ 2

d $\sqrt{16}$ ☐ $\sqrt[3]{64}$

Practice

Name: ______ Date: ______ Period: ______

4. Write each expression as a single power of 10.

a. $10^5 \cdot 10^0$

b. $\frac{10^9}{10^0} =$

5. Determine the two consecutive integers between which each square root is located.

a. $\sqrt{10}$

b. $\sqrt{54}$

c. $\sqrt{18}$

d. $\sqrt{99}$

e. $\sqrt{41}$

6. Evaluate each expression. Write your response as a mixed number.

a. $\left(\frac{3}{2}\right)^2$

b. $\left(\frac{5}{3}\right)^2$

c. $\left(\frac{7}{6}\right)^2$

Name: ______ Date: ______ Period: ______

Unit 7 | Lesson 6

Rational and Irrational Numbers

Let's learn about rational and irrational numbers.

Warm-up Number Talk

Write each number as a fraction.

1. 0.5

2. 1.25

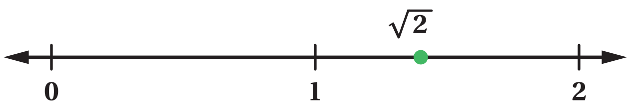
3. 3

4. -0.01

Activity 1 Ratio of Integers

Pythagoras and his followers were consumed by numbers and claimed that *all* numbers could be expressed as the ratio of integers. Let's explore whether this claim is true for $\sqrt{2}$.

Determine a ratio of integers for x that will make the equation $x^2 = 2$ true. Start by trying the fractions given in the table. Use the number line to help your thinking.



x	x^2
$\frac{3}{2}$	
$\frac{4}{3}$	
$\frac{8}{5}$	

Name: ______ Date: ______ Period: ______

Activity 2 Is It Irrational?

Greek mathematician and Pythagorean philosopher, Hippasus of Metapontum, is credited to discovering that $\sqrt{2}$ is an *irrational number*. Hippasus' discovery of irrational numbers shocked Pythagoras because it went against his idea that all numbers could be expressed as the ratio of integers. Let's explore whether there are other irrational numbers.

1. Mai claims that any number written with a square root is an irrational number. Is Mai correct? Explain your thinking.

2. Diego claims that any number written with a cube root is an irrational number. Is Diego correct? Explain your thinking.

3. Provide an example of an irrational number. Explain how you know the number is irrational.

Reflect: Were you able to critique without being critical? How did you show respect for others during this activity?

Activity 3 Is It Rational or Irrational?

1. Determine whether each number is *rational* or *irrational*. If the number is rational, write *R*. If the number is irrational, write *I*.

5	-2.051	$\sqrt{10}$	$\sqrt{16}$
$\sqrt{13}$	$\sqrt[3]{11}$	$2\frac{1}{4}$	3.1
$\sqrt{1}$	$\sqrt[3]{8}$	$\frac{8}{2}$	$-\sqrt{2}$

2. Choose one rational number and one irrational number from Problem 1 and explain how you know the number is rational or irrational.

Critique and Correct: Determine the error in the following statement and explain how you would correct it. "$\sqrt{10}$ is rational because 10 can be written as the fraction $\frac{10}{1}$."

Are you ready for more?

Determine if the expression $1 + \sqrt{2}$ will result in a rational or irrational number. Explain your thinking.

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You explored rational and irrational numbers. A ***rational number*** is a number that can be expressed as a fraction, in other words, as a ratio of two integers. An ***irrational number*** cannot be expressed as a fraction.

Examples of rational numbers

$\frac{7}{4}$ $\quad 0 \quad 0.2$

$-\frac{1}{3} \quad \sqrt{9} \quad \sqrt[3]{27}$

Examples of irrational numbers

$\sqrt{2} \quad \sqrt[3]{17} \quad \pi$

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Write each number as a fraction. If it is impossible, write *irrational number*.

   a. 0.01

   b. $-\sqrt{144}$

   c. $\sqrt{37}$

   d. $\sqrt[3]{125}$

   e. $\sqrt[3]{12}$

2. Noah says that the solution to equation $x^3 = 120$ is rational. Do you agree or disagree with Noah? Explain your thinking.

3. Determine if each statement is true or false. Explain your thinking.

   a. The sum of two positive whole numbers is always a rational whole number.

   b. The sum of two positive rational numbers, that are not whole numbers, is never a whole number.

   c. The sum of two rational numbers is always a rational number.

Name: ________ Date: ________ Period: ________

Practice

4. Determine which expression is larger, and then estimate how many times larger.

a 0.37×10^6 and 700×10^4

b $500{,}000$ and 2.3×10^8

5. Han makes a trip to Mexico. He exchanges some dollars for pesos at a rate of 20 pesos per dollar. While in Mexico, he spends $9{,}000$ pesos. When he returns, he exchanges his pesos for dollars at the same exchange rate. He receives $\frac{1}{10}$ the amount he started with. Determine how many dollars Han exchanged for pesos at the beginning of his trip and explain your thinking.

6. Use long division to write $\frac{3}{16}$ as a decimal. Show your thinking.

Unit 7 | Lesson 7

Decimal Representations of Rational Numbers

Let’s learn more about how rational numbers can be represented.

Warm-up Number Talk

Write each fraction as a decimal.

1. $\frac{99}{100}$

2. $-\frac{8}{10}$

3. $\frac{7}{5}$

4. $\frac{5}{2}$

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Name: ______ Date: ______ Period: ______

Activity 1 Writing Fractions as Decimals

1. With your group, decide who will complete part a, who will complete part b, and who will complete part c. Use long division to write each fraction as a decimal.

   a. $\frac{3}{8}$

   b. $\frac{3}{11}$

   c. $\frac{98}{6}$

2. Compare your responses with your group. What do you notice? What do you wonder?

   a. I notice . . .

   b. I wonder . . .

Activity 2 Bar Notation

Shawn, Mai, and Andre each write the number $30.212212212212\ldots$ using bar notation. Each students' response is shown.

Shawn	Mai	Andre
$30.2\overline{12}$	$30.\overline{212}$	$30.2\overline{122}$

Which student(s) are correct? Explain your thinking.

Historical Moment

Different Notations

There are different methods that countries use to represent repeating decimals.

For example:

- The United States, Switzerland, and Slovakia place a horizontal line, called a *vinculum*, above repeating digits. The decimal $0.0123123\ldots$ is written as $0.0\overline{123}$.
- The United Kingdom, Australia, South Korea place dots above the outermost repeating digits. The decimal $0.0123123\ldots$ is written as $0.0\dot{1}2\dot{3}$.
- Vietnam and Russia place parentheses around repeating digits. The decimal $0.0123123\ldots$ is written as $0.0(123)$.
- Spain and some Latin American countries use an arc notation over repeating digits. The decimal $0.0123123\ldots$ is written as $0.0\overset{\frown}{123}$.

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Activity 3 Is It Terminating or Repeating?

A *unit fraction* is a fraction where the numerator is 1 and the denominator is a positive integer.

1. Write each unit fraction as a decimal, and then place a check mark in the appropriate *Terminating* or *Repeating* column for each fraction. The first two rows are completed for you.

Unit Fraction	Decimal representation	Terminating	Repeating
$\frac{1}{2}$	0.5	✔	
$\frac{1}{3}$	$0.\overline{3}$		✔
$\frac{1}{4}$			
$\frac{1}{5}$			
$\frac{1}{6}$			
$\frac{1}{7}$			
$\frac{1}{8}$			
$\frac{1}{9}$			
$\frac{1}{10}$			

2. How can you predict whether a unit fraction will terminate or repeat? Check your prediction by testing other unit fractions.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You wrote rational numbers using decimal expansion. Some rational numbers can be written as ***terminating decimals***, while other rational numbers can be written as ***repeating decimals***.

To avoid writing the repeating part of a decimal over and over, you can use ***bar notation***, which shows a line over the part of the decimal that repeats.

For example:

$0.7434343\ldots = 0.7\overline{43}$

You also predicted whether the decimal representation of a unit fraction will terminate or repeat: If the factors of the denominator consist of only 2s or 5s, then the decimal representation will terminate. Otherwise, it will repeat.

For example:

- The fraction $\frac{1}{40}$ *terminates* because the denominator, 40, only consists of products of 2 and 5, $40 = 2 \cdot 2 \cdot 2 \cdot 5$.
- The fraction $\frac{1}{12}$ *repeats* because the denominator, 12, includes at least one factor other than 2 or 5, $12 = 2 \cdot 2 \cdot 3$.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Han and Jada discuss how to write $\frac{17}{20}$ as a decimal. Han says he can use long division to divide 17 by 20 to get the decimal. Jada says she can write an equivalent fraction with a denominator of 100 by multiplying by $\frac{5}{5}$, and then writing the number of hundredths as a decimal.

a. Do both strategies work?

b. Which strategy do you prefer? Explain your thinking.

c. Write $\frac{17}{20}$ as a decimal. Explain or show your thinking.

2. Match each repeating decimal with its equivalent expression in bar notation.

Repeating decimal	Bar notation
a. $0.555\ldots$	.... $0.\overline{050}$
b. $0.0555\ldots$	.... $0.0\overline{5}$
c. $0.050505\ldots$	.... $0.\overline{05}$
d. $0.050050050\ldots$	.... $0.\overline{5}$

3. Determine whether the decimal representation of each fraction *terminates* or repeats. Explain your thinking.

a. $\frac{1}{13}$

b. $\frac{1}{16}$

c. $\frac{1}{25}$

d. $\frac{1}{18}$

Practice

Name: .... Date: .... Period: ....

4. Graphite is made up of layers of graphene. Each layer of graphene is about 200 picometers, or 200×10^{-12} m, thick. How many layers of graphene are there in a 1.6-mm thick piece of graphite? Show or explain your thinking. Express your answer in scientific notation.

5. Select *all* of the irrational numbers.

A. $\frac{2}{3}$

B. $-\frac{123}{45}$

C. $\sqrt{14}$

D. $\sqrt{64}$

E. $\sqrt{\frac{9}{1}}$

F. $-\sqrt{99}$

G. $-\sqrt{100}$

6. Write each decimal as a fraction.

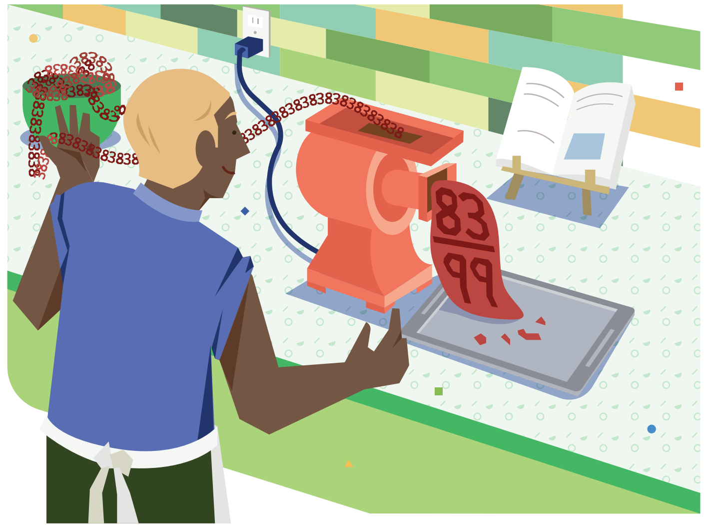
a 0.4

b 1.85

Name: ________ Date: ________ Period: ________

Unit 7 | Lesson 8

Converting Repeating Decimals Into Fractions



Let's convert repeating decimals into fractions.

Warm-up Working With Decimals

Without using a calculator, evaluate each expression.

1. $100.321 - 99.321$

2. $3.\overline{45} \cdot 100$

3. $18.\overline{33} - 1.3\overline{3}$

4. $18.\overline{33} - 1.8\overline{3}$

Compare and Connect:
If you subtract the repeating part of a decimal, what happens? Be ready to share your thoughts during the class discussion.

Log in to Amplify Math to complete this lesson online.

Activity 1 It Just Keeps Going

Do you remember the Pythagoreans and their claim that any number can be expressed as a fraction? Let's check whether this claim is true for repeating decimals.

The following algorithm is one way to turn repeating decimals into fractions.

1. Follow the example for $0.\overline{83}$ to determine how to write $0.\overline{2}$ as a fraction.

Step	$0.\overline{83}$	$0.\overline{2}$
Write a few iterations of the repeating decimal.	$0.83838383\ldots$	
Let $x =$ the original number with extra iterations shown. Multiply by factors of 10.	$x = 0.838383\ldots$ $10x = 8.383838\ldots$ $100x = 83.838383\ldots$ $1000x = 838.383838\ldots$	

2. What do you notice happens as the number is multiplied by factors of 10?

3. Refer to Problems 3 and 4 from the Warm-up. What operation can help eliminate the repeating decimals?

4. Complete the algorithm to rewrite a repeating decimal as a fraction.

Step	$0.\overline{83}$	$0.\overline{2}$
Select two equations which will subtract the decimal values to 0. Be careful!	$x = 0.838383\ldots$ $100x = 83.838383\ldots$	
Subtract the equations and solve the resulting equation. Simplify the fraction, if possible.	$100x = 83.838383\ldots$ $-\quad x = -\ 0.838383\ldots$ $99x = 83$ $x = \frac{83}{99}$	

Name: ______________________ Date: __________ Period: __________

Activity 2 Now You Try

Plan ahead: If you were going to make a mistake in this process, what would it be? What reminder can you give yourself so that you don't make that mistake?

Rewrite each decimal as a fraction. Show your thinking.

1. $0.\overline{8}$

2. $0.\overline{81}$

3. $0.8\overline{1}$

4. $0.\overline{181}$

Are you ready for more?

What do you notice about the repeating decimal patterns in Problems 1, 2, and 4 and their corresponding fractions? Use the pattern you notice to write $0.\overline{1234}$ as a fraction.

STOP

Summary

In today's lesson . . .

You explored how to write repeating decimals as fractions. One algorithm to do this involves multiplying equations by factors of 10 until the repeating decimals can subtract to 0. Once the repetition is removed, the resulting equation can be solved and left in fraction form.

For example, $0.\overline{57} = 0.575757575\ldots$

$$\begin{aligned} x &= 0.575757\ldots \\ 10x &= 5.757575\ldots \\ 100x &= 57.575757\ldots \\ 1000x &= 575.757575\ldots \end{aligned}$$

$$\begin{aligned} 100x &= 57.575757\ldots \\ -\ x &= -\ 0.575757\ldots \\ \hline 99x &= 57 \\ x &= \frac{57}{99} \\ 0.\overline{57} &= \frac{19}{33} \end{aligned}$$

If a decimal expansion of a number is a repeating or terminating decimal, the number is rational. If the digits in the decimal expansion do not repeat (non-repeating) and do not terminate (non-terminating), the number is irrational.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Write 0.7 and $0.\overline{7}$ as fractions.

2. Complete the table.

Fraction	Decimal expansion
$\frac{17}{5}$	
	-1.23
	$0.1\overline{3}$
$-\frac{7}{3}$	
	$-0.\overline{03}$

3. Write each rational number as a fraction. If it is not rational, write *irrational*.

a. 0.2

b. 0.12

c. 1.2

d. $-\sqrt{100}$

e. $\sqrt{1.21}$

f. $\sqrt[3]{0.125}$

Practice

Name: ________ Date: ________ Period: ________

4. Evaluate each expression.

a $\left(\frac{1}{2}\right)^3$

b $\left(\frac{1}{2}\right)^{-3}$

5. Compare each pair of expressions using the symbol <, >, or =.

a $\sqrt{4} \ \square \ \sqrt[3]{8}$

b $\sqrt[3]{45} \ \square \ 4$

c $\sqrt{1.44} \ \square \ \sqrt[3]{0.125}$

d $\sqrt{36.1} \ \square \ 6$

e $\sqrt[3]{64} \ \square \ \sqrt{16}$

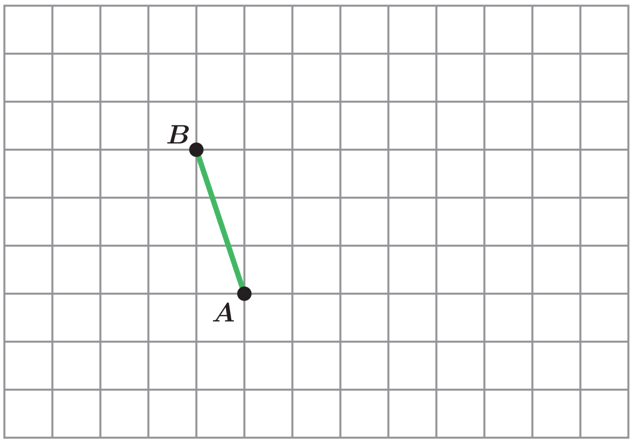
f $\sqrt[3]{\frac{1}{8}} \ \square \ \frac{1}{4}$

6. Consider segment AB.

a Rotate segment AB 90° counterclockwise about point B and label the image CB.

b Rotate segment BC 90° counterclockwise about point C and label the image DC.

c Rotate segment CD 90° counterclockwise about point D. What do you notice?



SUB-UNIT 2

The Pythagorean Theorem

What do President Garfield and Albert Einstein have in common?

There is evidence many cultures have grappled with the Pythagorean Theorem well before Pythagoras. The earliest record came from the ancient Mesopotamians. In 1945, archaeologists unearthed tablets that showed calculations for the lengths of right triangles a thousand years before Pythagoras was even born.

In around the 8th century BCE, the theorem appeared in India in the Baudhāyana Śulbasûtra. The text described rules for calculating a diagonal produced by a stretched rope using its vertical and horizontal lengths. The theorem appeared again in China, around 700 BCE in the Bi Suan Jing.

In the centuries since, the Pythagorean Theorem has not exactly fallen out of style. Even to this day, math enthusiasts are still searching for new proofs. In 1928, math teacher Elisha Loomis published *The Pythagorean Proposition*. His book collected 370 different methods for proving the theorem. Even a young Albert Einstein and President James A. Garfield have authored their own proofs.

In 1995, *The Guinness Book of World Records* declared the Pythagorean Theorem "the most proven theorem of all time." Today, there are nearly 500 different proofs for this humble theorem.

But why?

Well, for one, the theorem is practical. It helps us calculate distances. This makes navigation a breeze. The theorem is also useful for engineers and architects. Knowing the lengths for a right triangle guarantees your constructions are built at right angles. But there's another reason beyond the practical. There's also the simple pleasure in discovering a proof. A well-crafted proof can be a delight — one of those aha moments. Math is not just about solving problems. It's also about training the mind to find solutions that are creative and elegant.

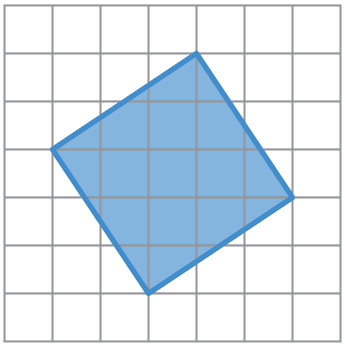
Unit 7 | Lesson 9

Observing the Pythagorean Theorem

Let's determine the side lengths of triangles.

Warm-up Tilted Square

Consider the square shown.



What is the square's side length measure? Explain your thinking.

Co-craft Questions: Work with your partner to write 1–2 other questions you could ask about this square.

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Name: .... Date: .... Period: ....

Activity 1 Recording Triangle Side Lengths

Each member in your group will be provided with a triangle for this activity.

1. Record the side lengths for your triangle here.

	Leg	Leg	Hypotenuse
Triangle			

2. Calculate and record the squared side lengths.

	Leg^2	Leg^2	Hypotenuse^2
Triangle			

Activity 1 Recording Triangle Side Lengths (continued)

3. Record the results from your group members' squared side lengths.

	Leg^2	Leg^2	Hypotenuse^2
Triangle D			
Triangle E			
Triangle F			
Triangle P			
Triangle Q			

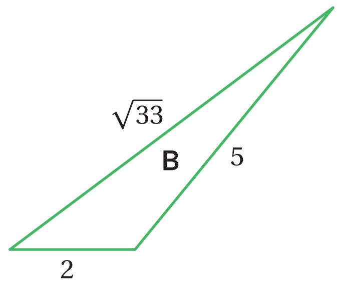
4. What patterns do you notice?

Name: ______ Date: ______ Period: ______

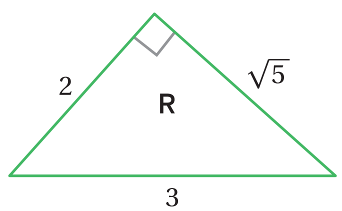
Activity 2 Testing the Theorem

Consider the following statements.

1. Elena claims the Pythagorean Theorem is true for any triangle. Is she correct? Explain your thinking using Triangle B as an example.



2. Kiran claims that the Pythagorean Theorem is only true for *some* right triangles. For example, for Triangle R with sides 2, 3, and $\sqrt{5}$, he determines $2^2 + 3^2 = 4 + 9 = 13$, which is not equal to $(\sqrt{5})^2$. Is he correct? Explain your thinking.

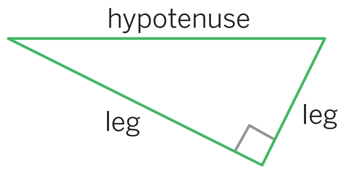


STOP

Summary

In today's lesson . . .

You studied special properties of *right triangles*. In a right triangle, the side opposite the right angle is called the ***hypotenuse***, and the two other sides are called its ***legs***.



The ***Pythagorean Theorem*** states that the sum of squares of the legs of a right triangle is equal to the square of the hypotenuse: $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$. Sometimes this can be presented instead by $a^2 + b^2 = c^2$, where a and b represent the length of the legs and c represents the length of the hypotenuse.

Reflect:

Name: ____________________ Date: __________ Period: __________

1. The length of the three sides are given for several right triangles. Show that the Pythagorean Theorem is true for each of these triangles.

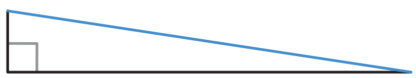
   a. $5, 12$, and 13

   b. $1, 7$, and $\sqrt{50}$

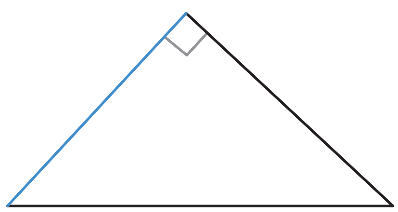
   c. $\sqrt{18}, \sqrt{18}$, and 6

   d. $\sqrt{11}, \sqrt{7}$, and $\sqrt{18}$

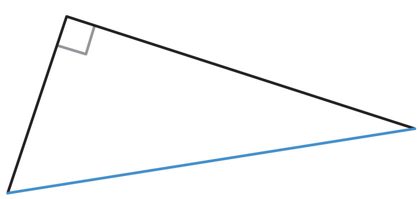
2. Is the highlighted side of each triangle a hypotenuse? Circle *yes* or *no*. Explain your thinking.

   a. 

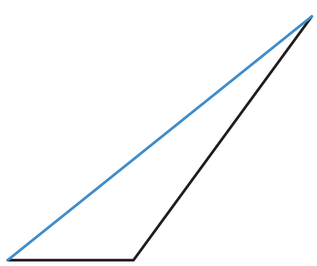
   Yes No

   b. 

   Yes No

   c. 

   Yes No

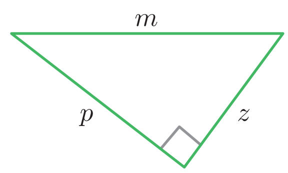
   d. 

   Yes No

Practice

Name: .... Date: .... Period: ....

3. The variables m, p, and z represent the lengths of the three sides of this right triangle. Select *all* the equations that represent the relationship between m, p, and z.



A. $m^2 + p^2 = z^2$

B. $m^2 = p^2 + z^2$

C. $m^2 = z^2 - p^2$

D. $z^2 + p^2 = m^2$

E. $p^2 = m^2 - z^2$

4. The points $(12, 23)$ and $(14, 45)$ lie on a line. What is the slope of the line?

5. Determine a positive solution for each equation. If the solution is irrational, write the solution using square root notation.

a $c^2 = 121$

b $t^2 = 93$

c $u^2 = 0.001$

d $k^2 = \frac{4}{25}$

e $r^2 = 3^2 + 4^2$

f $s^2 = (\sqrt{19})^2 + (\sqrt{17})^2$

6. What is the relationship between the area of the shaded section of Figure A and the area of the shaded section of Figure B? Explain your thinking.

Figure A

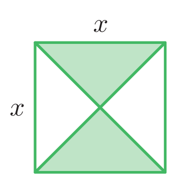
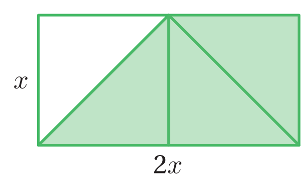


Figure B



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Unit 7 | Lesson 10

Proving the Pythagorean Theorem

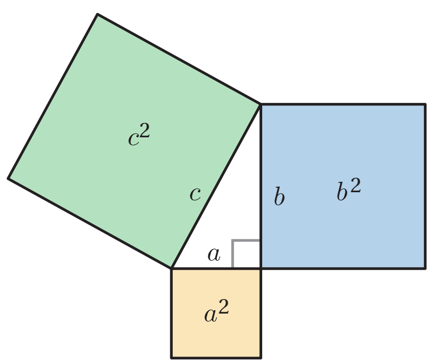
Let's prove the Pythagorean Theorem.

Warm-up Notice and Wonder

The name of the Pythagorean Theorem comes from a mathematician named Pythagoras, but this property of right triangles was also discovered independently by mathematicians in other ancient cultures, including Babylon, India, and China. In fact, there are nearly 500 well-known ways to prove the Pythagorean Theorem (including one by an American president!) In this lesson, you will explore one of these proofs.

Consider the drawing.

What do you notice? What do you wonder?



1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Arranging Shapes

You will be provided with shapes that you will cut out.

1. Decide who will be Partner A and who will be Partner B. Then cut out and arrange your shapes to completely cover your square.

Partner A	**Partner B**

2. Compare your results with your partner. What do you notice?

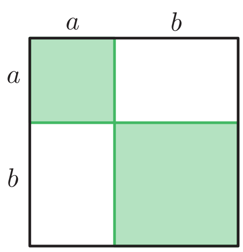
Reflect: How did working with a partner help you succeed in this activity? How did you each use your strengths?

Name: ________________ Date: ________ Period: ________

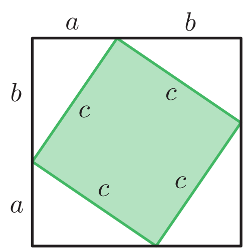
Activity 2 Any Right Triangle

One of the oldest known proofs of the Pythagorean Theorem — long before Pythagoras was even born — is credited to the Chinese mathematician Shang Gao. His proof has some similarities to the proof you are exploring in this lesson. Consider the following figures.

Square G



Square H



1. Write an expression for the shaded areas for each figure in terms of a, b, and c.

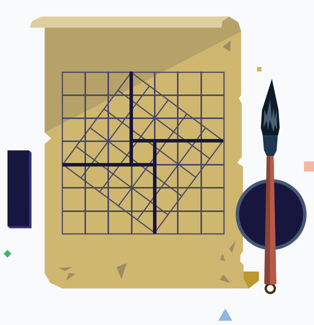
   Square G shaded area:

   Square H shaded area:

2. Write an equation relating the two areas.

3. Does your equation in Problem 2 work for any right triangle? Explain your thinking.

Featured Mathematician



Shang Gao

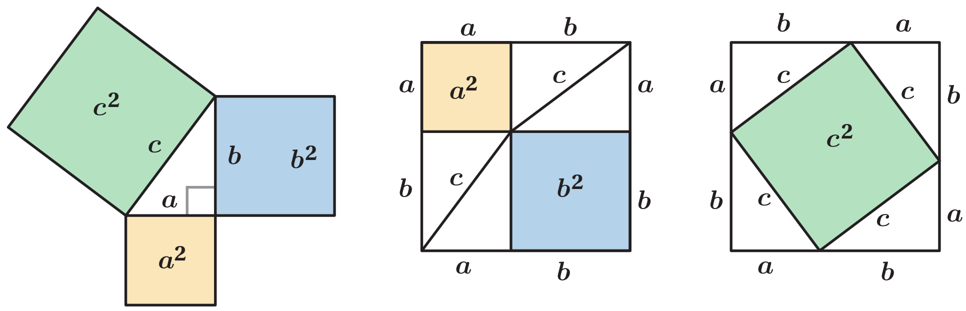
While in much of the world, $a^2 + b^2 + c^2$, is known as the Pythagorean Theorem, to some in China, it is instead known as the "Shang Gao Theorem" (商高定理) or the "Gougu Rule." Why? There is an ancient Chinese text that makes mention of what we now know are Pythagorean triples and right triangles. The text, the *Zhoubi Suanjing*, dates back to the *Zhou Dynasty* (ca. 100 BCE), long before Pythagoras was alive. The Zhoubi Suanjing is a collection of mathematical problems explored and completed by astronomer and mathematician, Shang Gao.

STOP

Summary

In today's lesson . . .

You explored a proof for the Pythagorean Theorem. By rearranging right triangles into squares, you saw why $a^2 + b^2$ as represented by areas of two squares, built on the legs is equal to c^2, as represented by the area of a square built on the hypotenuse.

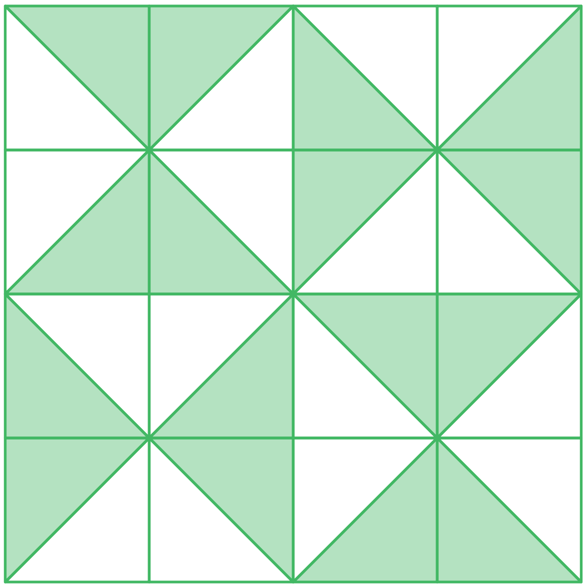


Reflect:

Practice

Name: .... Date: .... Period: ....

1. Study this mosaic.



Identify a right triangle on this mozaic. Next, look for squares that are built on each side.

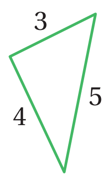
What do you notice? What does this prove?

Practice

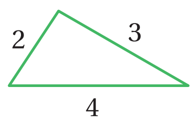
Name: ________ Date: ________ Period: ________

2. For which of the following triangles is the Pythagorean Theorem true? Explain your thinking.

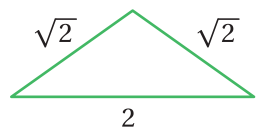
Triangle A



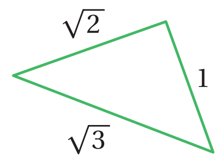
Triangle B



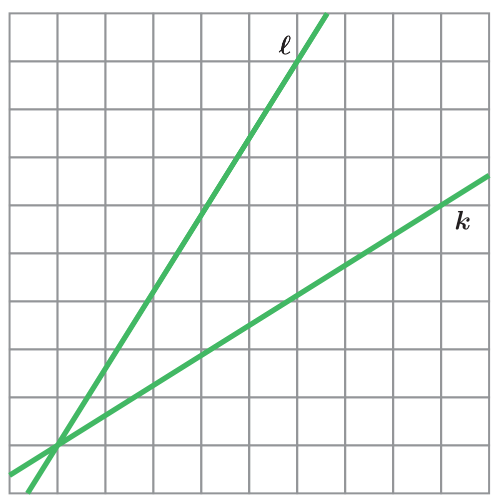
Triangle C



Triangle D



3. Which line has a slope of 0.625, and which line has a slope of 1.6? Explain why the slopes of these lines are 0.625 and 1.6.



4. If $y = \sqrt{7}$, solve the following equation for x. Show your thinking.

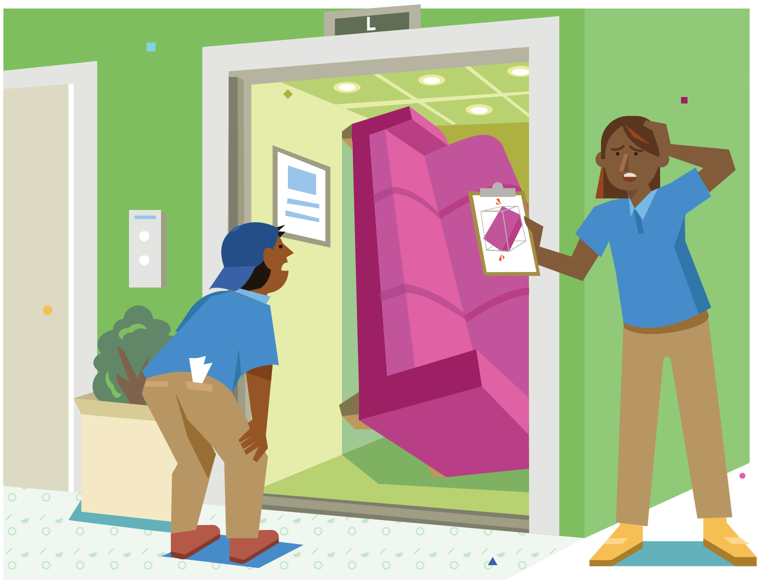
$x^2 + y^2 = 23$

Name: ______________________ Date: __________ Period: __________

Unit 7 | Lesson 11

Determining Unknown Side Lengths

Let's determine missing side lengths of right triangles.



Warm-up Which One Doesn't Belong?

Which equation doesn't belong? Explain your thinking.

A. $3^2 + s^2 = 5^2$

B. $s^2 = 5^2 - 3^2$

C. $3^2 + 5^2 = s^2$

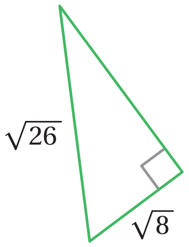
D. $3^2 + 4^2 = 5^2$

Log in to Amplify Math to complete this lesson online.

Activity 1 Determine the Unknown Length

1. Determine the length of the hypotenuse. Show your thinking.

2. Determine the missing side length. Show your thinking.

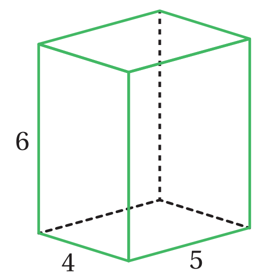


3. A right triangle has two side lengths of 10 and 15 units. What could be the length of the third side? Show your thinking.

Name: ______ Date: ______ Period: ______

Activity 2 Internal Diagonal

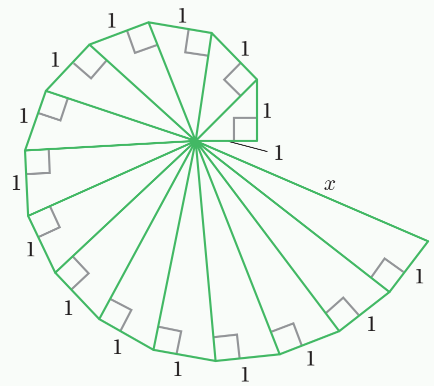
Determine the exact length of the diagonal in the rectangular prism shown. Show or explain your thinking.



Are you ready for more?

The spiral in the figure is made by starting with a right triangle with both legs measuring one unit each. Then a second right triangle is built with one leg measuring one unit, and the other leg being the hypotenuse of the first triangle. A third right triangle is built on the second triangle's hypotenuse, again with the other leg measuring one unit, and so on.

Find the length, x, of the hypotenuse of the last triangle constructed in the figure.



STOP

Summary

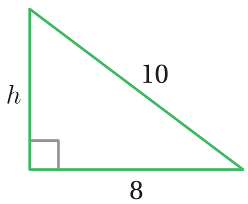
In today's lesson . . .

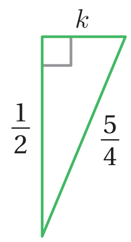
You saw examples where the lengths of two legs of a right triangle are known and can be used to determine the length of the hypotenuse. The Pythagorean Theorem can also be used if the length of the hypotenuse and one leg is known, and you want to determine the length of the other leg. In each instance, use the equation $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ or $a^2 + b^2 = c^2$, and then solve for the unknown quantity.

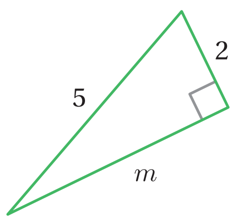
Reflect:

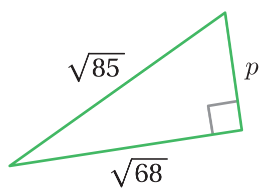
Name: ____________ Date: ____________ Period: ____________

1. Determine the exact value of the variable that represents a side length of each right triangle.

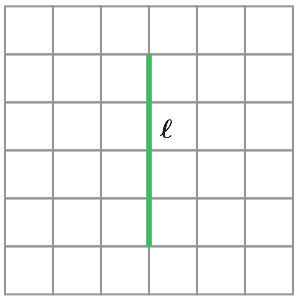
a 

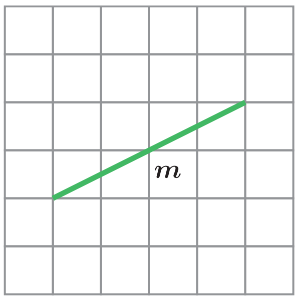
b 

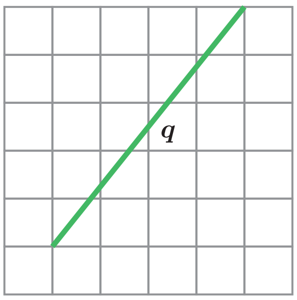
c 

d 

2. What is the exact length of each line segment if each grid square is 1 square unit?

a 

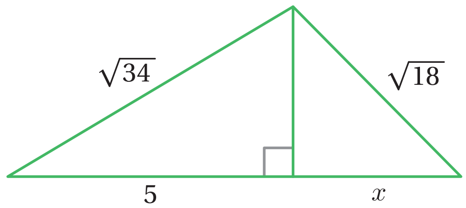
b 

c 

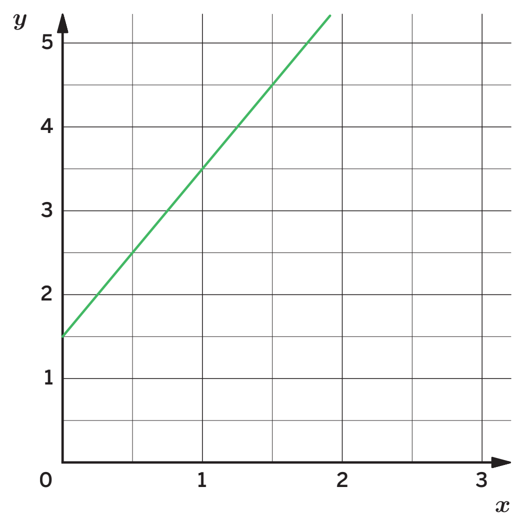
Practice

Name: ____________ Date: ____________ Period: ____________

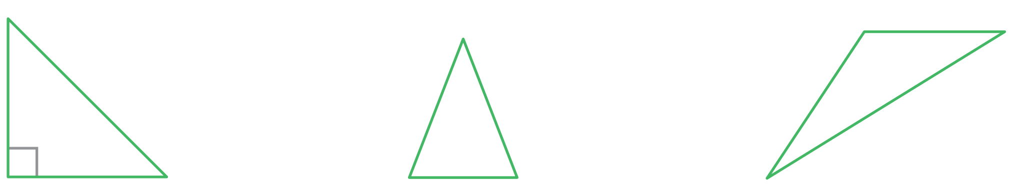
3. Determine the value of x in the figure shown. Show your thinking.



4. Write an equation that represents the graph of the line shown. Show or explain your thinking.



5. Determine whether each triangle is *obtuse*, *right* or *acute*.



Name: ______ Date: ______ Period: ______

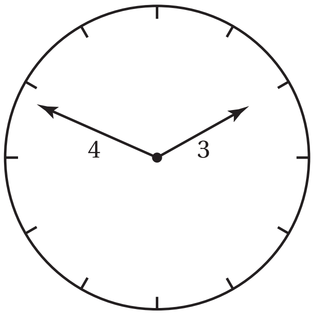
Unit 7 | Lesson 12

Converse of the Pythagorean Theorem

Let's determine whether a triangle is a right triangle.

Warm-up Clock Hands

Consider the tips of the hands of an analog clock that has an hour hand that is 3 cm long and a minute hand that is 4 cm long.



Over the course of a day:

1. What is the farthest distance apart the two tips will be?

2. What is the closest distance the two tips will be?

3. Are the two tips ever exactly 5 cm apart? Show or explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Making Triangles

1. Using side lengths of 5 cm and 12 cm, draw three triangles such that:
   - The longest side is less than 13 cm.
   - The longest side is equal to 13 cm.
   - The longest side is greater than 13 cm.

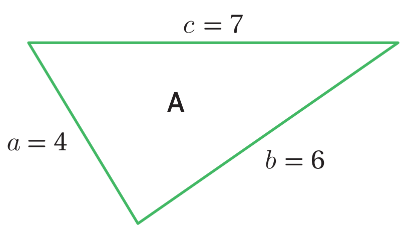
2. Share the triangles you drew with your group. What do you notice about the triangles in your group?

Name: ____________ Date: ________ Period: ________

Activity 2 Is This a Right Triangle?

Long before the time of Pythagoras, ancient Egyptians used the structure of right triangles in the form of knotted cords to aid in construction of buildings, including the pyramids. Today, land surveyors create right triangles with laser meters to aid in construction projects. Builders must be able to determine whether a triangle is a right triangle; otherwise, their buildings might collapse.

Now, it is your turn to try. Is Triangle A a right triangle? If it is, explain how you know. If not, change one of the values to change it into a right triangle. Explain your thinking.



Activity 3 Acute, Right, or Obtuse

The side lengths of three triangles are given. Decide whether each set of measures would form an *acute*, *right* or *obtuse* triangle. Show or explain your thinking.

1. $6, 8, 12$

2. $2, 3, \sqrt{11}$

3. $9, 12, 15$

Are you ready for more?

Consider a right triangle with side lengths of 3, 4, and 5 units. Suppose you were to dilate the triangle about any vertex by a scale factor of 2. Will it still be a right triangle?

STOP

Name: Date: Period:

Summary

In today's lesson . . .

You saw that if a triangle has side lengths a, b, and c, with c being the longest of the three, then the converse of the Pythagorean Theorem tells you that you must have a right triangle.

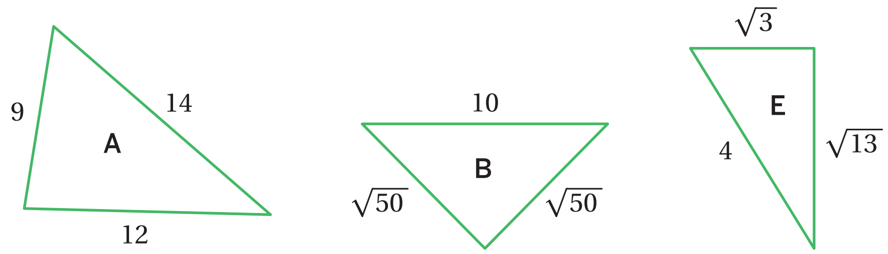
If the sides of a triangle *do not* make the equation $a^2 + b^2 = c^2$ true, then you know it is *not* a right triangle. In this case, if the longest side of the triangle is greater than c, the triangle must be obtuse. If the longest side is less than c, it must be an acute triangle.

Reflect:

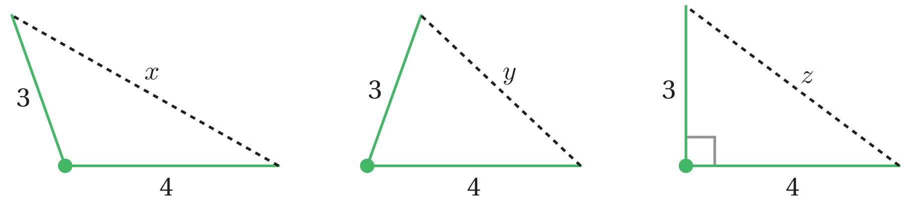
Practice

Name: .... Date: .... Period: ....

1. Which of these triangles are right triangles? Explain how you know.



2. Here are three triangles with side length measures of 3 and 4 units, and the third side of unknown length.

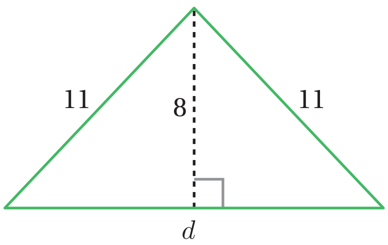


Order the numbers 1, 5, 7, x, y, and z from least to greatest. Put an equal sign between any that are equal. Explain your thinking.

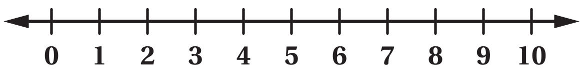
Practice

Name: ________ Date: ________ Period: ________

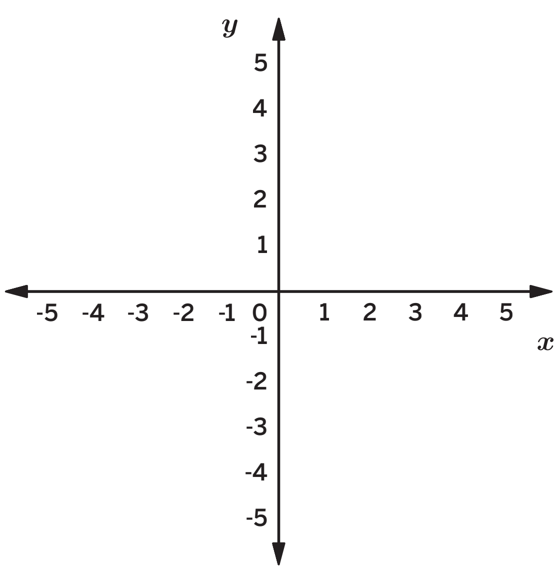
3. Determine the measure of d. Show or explain your thinking.



4. Plot and label $\sqrt{27}$ and $\sqrt[3]{27}$ on the number line. Explain your thinking.



5. Using the letters a–e, order the following distances from least to greatest. Explain your thinking. If needed, use the coordinate plane to help with your thinking.



a Distance between $(-2, 0)$ and $(-2, 2)$

b Distance between $(2, 2)$ and $(2, -3)$

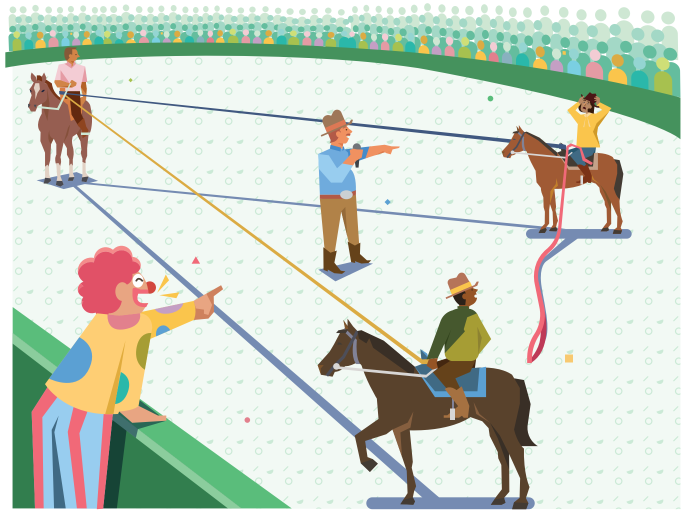
c Distance between $(4, 0)$ and $(-2, 0)$

d Distance between $(-5, -3)$ and $(2, -3)$

e Distance between $(-2, 2)$ and $(2, 2)$

Unit 7 | Lesson 13

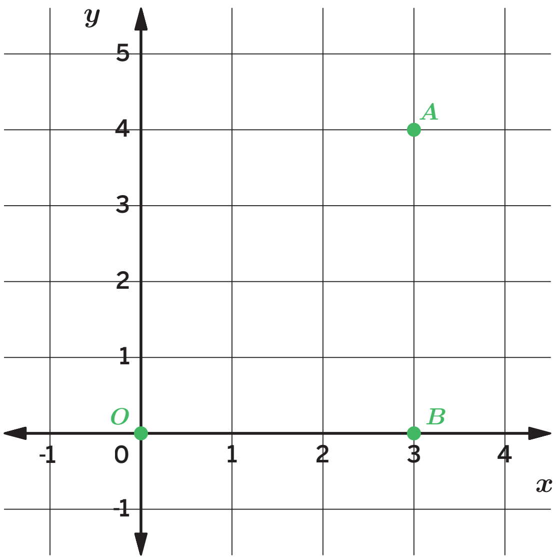
Distances on the Coordinate Plane (Part 1)



Let's determine the distance between two points on the coordinate plane.

Warm-up Distance to the Origin

Refer to the graph.



1. Connect the points O and B and determine the distance between them.

2. Connect the points A and B and determine distance between them.

3. Connect the points O and A. How can you use the information from Problems 1 and 2 to find the distance between points O and A?

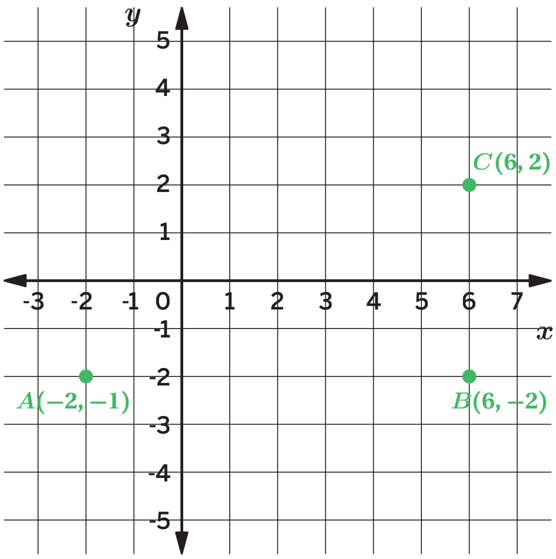
4. Determine the distance between points O and A. Show your thinking.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Distance Between Any Two Points

Study the points on the coordinate plane.



1. What is the distance between points A and B? Explain your thinking.

2. What is the distance between points C and B? Explain your thinking.

3. What do you notice about the points C and A? Can you determine the distance between them the same way you did in Problems 1 and 2?

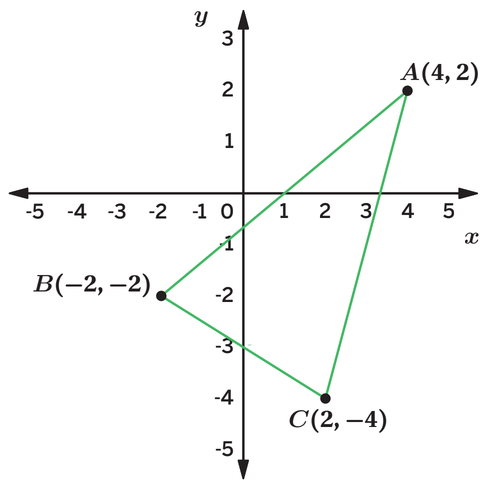
4. What type of triangle is Triangle ABC? Explain your thinking.

5. Use what you know about the Pythagorean Theorem to determine the exact distance between the points C and A.

6. How can you use the coordinates of points C and A to determine the distance between the points C and A?

Activity 2 Determining a Perimeter

Let's determine the perimeter of Triangle ABC.



1. Determine the distance between A and B and round to the nearest tenth.

2. Determine the distance between B and C and round to the nearest tenth.

3. Determine the distance between A and C. Round to the nearest tenth.

4. What is the approximate perimeter of Triangle ABC?

STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You used the Pythagorean Theorem to determine the distance between two points on a coordinate plane. To determine the length of a diagonal line segment, draw the horizontal and vertical legs forming a right triangle. Then use the Pythagorean Theorem to determine the length of the hypotenuse, which will be the distance between the two points.

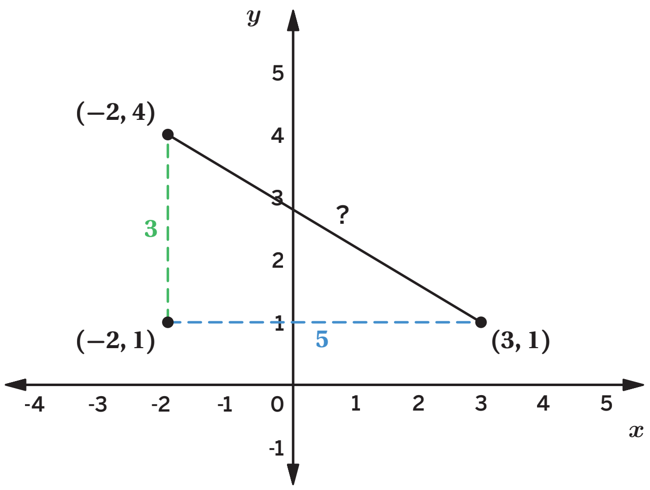
$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$

$$3^2 + 5^2 = x^2$$

$$9 + 25 = x^2$$

$$34 = x^2$$

$$\sqrt{34} = x$$

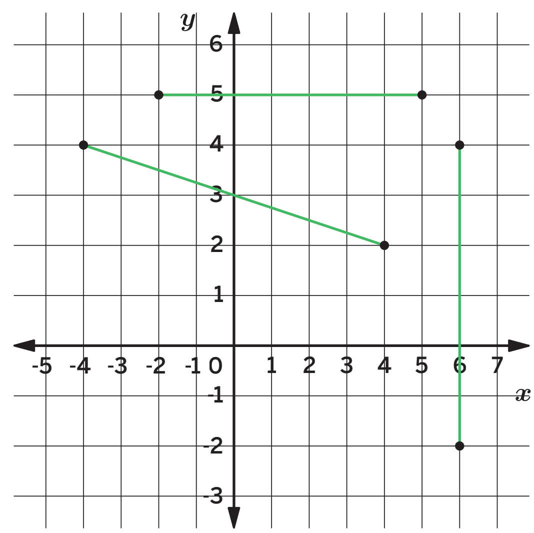


Reflect:

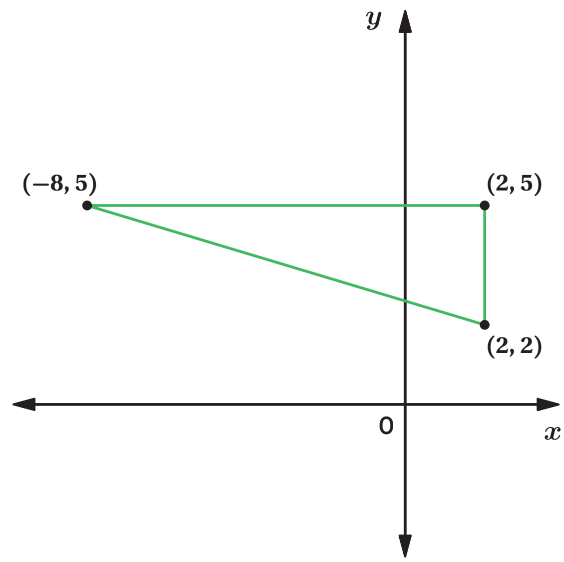
Practice

Name: ______ Date: ______ Period: ______

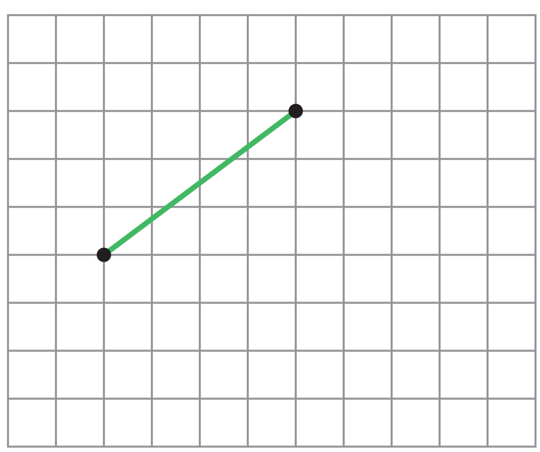
1. Determine the exact length of each line segment. Show your thinking.



2. The right triangle is drawn on the coordinate plane, and the coordinates of its vertices are labeled. Determine the exact length of each side. Show your thinking.



3. Here is a segment with a certain length. Without measuring, draw 3 segments which have the same length and whose vertices are at the nodes of the grid.



Name: ______ Date: ______ Period: ______

Practice

4. Select *all* the sets that could represent the three side lengths of a right triangle. Show your thinking.

A. $8, 7, 15$

D. $\sqrt{5}, \sqrt{11}, 16$

B. $\sqrt{8}, 11, \sqrt{129}$

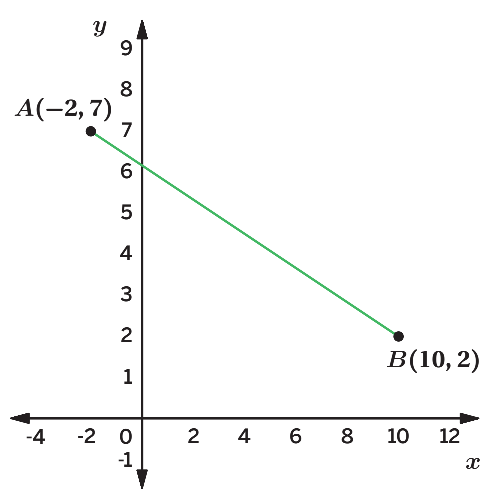
E. $\sqrt{1}, 2, \sqrt{3}$

C. $4, 10, \sqrt{84}$

F. $4, \sqrt{9}, \sqrt{13}$

5. Noah and Han are preparing for a jump rope contest. Noah can jump 40 times in 0.5 minutes. Han can jump y times in x minutes, where $y = 78x$. If they both jump for 2 minutes, who jumps more times? How many more?

6. Determine the distance between point $A(-2, 7)$ and point $B(10, 2)$. Show your thinking.



Distances on the Coordinate Plane (Part 2)

Let's determine more distances between two points.

Warm-up What's the Length?

Order the following pairs of points from closest to farthest apart. Be prepared to explain your thinking.

A. $(2, 4)$ and $(2, 10)$

B. $(-3, 6)$ and $(5, 6)$

C. $(7, 0)$ and $(7, -9)$

D. $(1, -10)$ and $(-4, -10)$

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 What's the Shape?

You will be given a set of cards showing different points. Divide them equally among your group members, but be sure to distribute points C, E, H, and K among different group members.

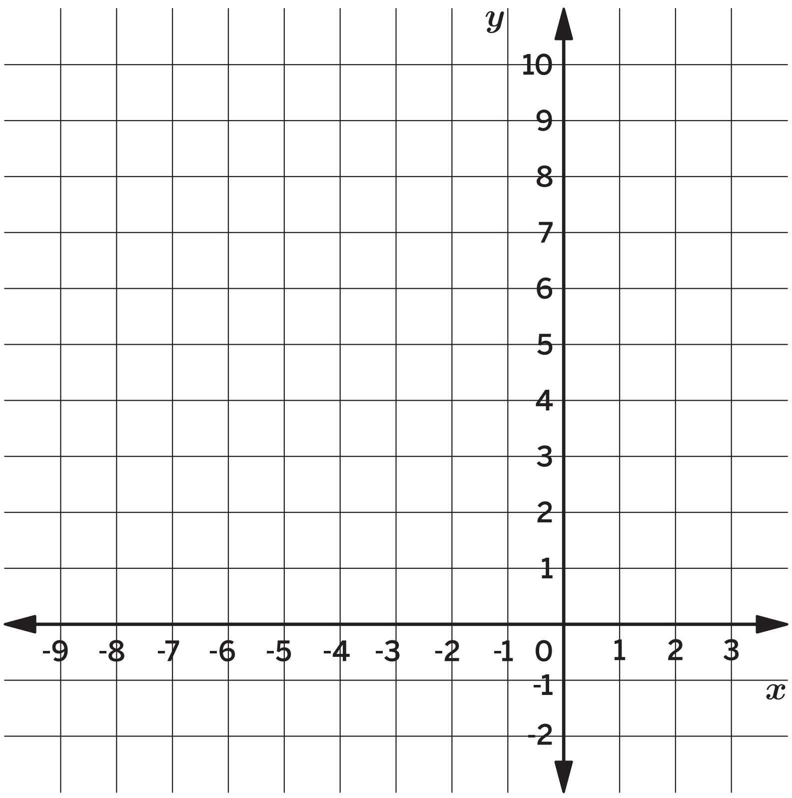
1. For each of your points, determine the distance from your point to point $P(-3, 4)$. Show your thinking.

Point	Distance to point P (units)

2. Compare your points and distances to your group members' points and distances. What do you notice?

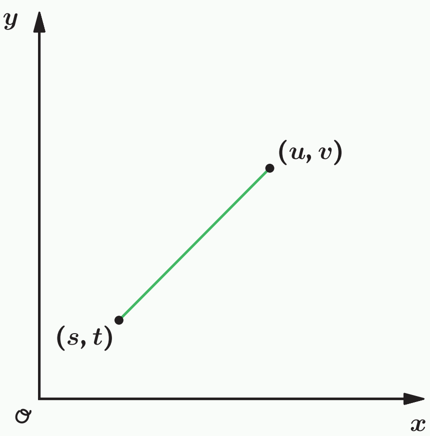
Activity 1 What's the Shape? (continued)

3. Predict the shape the points make. Then plot the points on the coordinate plane to check your prediction.



Are you ready for more?

The points (s, t) and (u, v) are plotted on the coordinate plane. Write an equation that represents the distance d between the points.



STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that you can determine the distance between any two points without plotting them.

For example, determine the distance between points $A(-2, 4)$ and $B(3, 1)$. Think of the distance between A and B, or the length of segment AB, as the hypotenuse of a right triangle. The lengths of the legs can be deduced from the coordinates of the points.

The length of the vertical leg is 3 units, because $|4 - 1| = 3$.

The length of the horizontal leg is 5 units, because $|-2 - 3| = 5$.

Now, apply the Pythagorean Theorem to determine the length of the hypotenuse.

$$\begin{aligned} \text{leg}^2 + \text{leg}^2 &= \text{hypotenuse}^2 \\ 3^2 + 5^2 &= x^2 \\ 9 + 25 &= x^2 \\ 34 &= x^2 \\ \sqrt{34} &= x \end{aligned}$$

The distance between points A and B is exactly $\sqrt{34}$ units.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. Find the distance between each pair of points. To help with your thinking, try plotting the points on graph paper.

   a. $(0, -11)$ and $(0, 2)$

   b. $(0, 0)$ and $(-3, -4)$

2. Which set of points has a greater distance between them? Show your thinking.

   Set A: $(-3, 1)$ and $(5, 7)$

   Set B: $(-2, 4)$ and $(3, 11)$

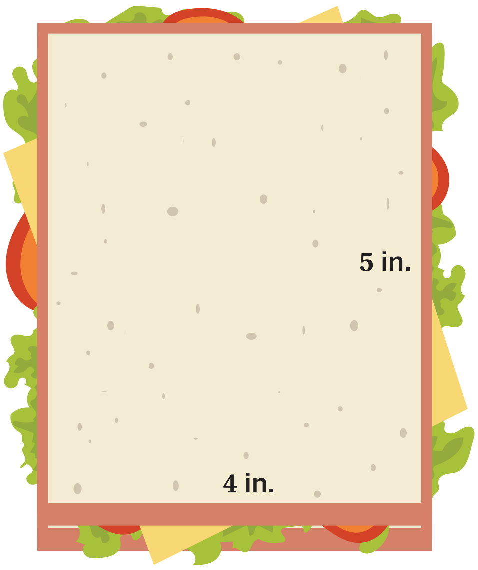
3. Point M has coordinates $(1, 2)$ and is 5 units from point N which has an x-coordinate of 4. What is the y-coordinate of point N? Show or explain your thinking.

Name: ________________ Date: ________ Period: ________

4. A line contains the point $(3, 5)$. If the line has a negative slope, which of these points could also be on the line? Explain your thinking.

   A. $(2, 0)$ B. $(4, 7)$ C. $(5, 4)$ D. $(6, 5)$

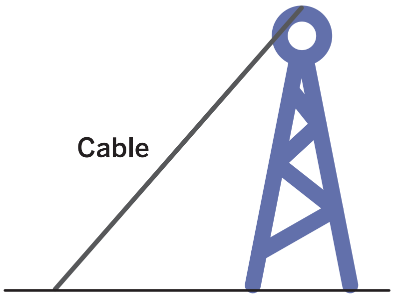
5. Do you remember the sandwiches from Lesson 1? You now have the tools to determine the exact measure of the longest cut. Draw and label the measure of the longest cut. Explain your thinking.



The figure may not be drawn to scale.

6. Noah works for a cell tower company. A cable is being placed on level ground to support a tower. One end of the cable should be connected to the top of the tower and the other end to the ground.

   Can this problem be solved using the Pythagorean Theorem? If so, what information would Noah need to know to determine how far away the cable can connect to the ground? Explain your thinking.

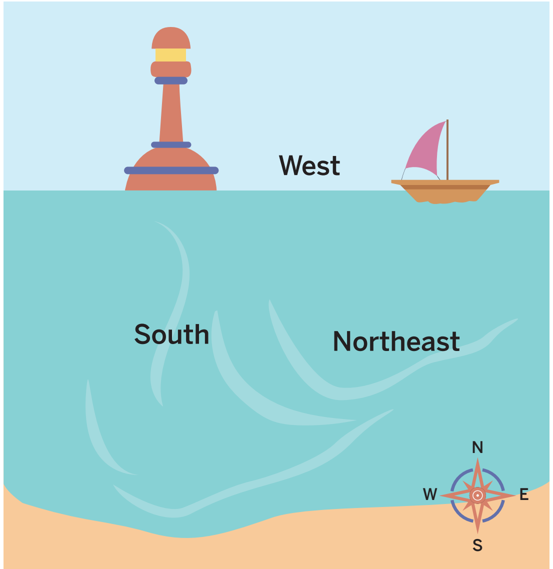


Applications of the Pythagorean Theorem

Let's solve problems using the Pythagorean Theorem.

Warm-up Charting a Path

Tyler drives his motorboat from a dock directly west for some time until he reaches a buoy and then directly south for some time before stopping in a bay for lunch. If Tyler needs to return to the dock in the fastest possible time, which direction should he travel in, assuming there is nothing obstructing his path? Show or explain your thinking.



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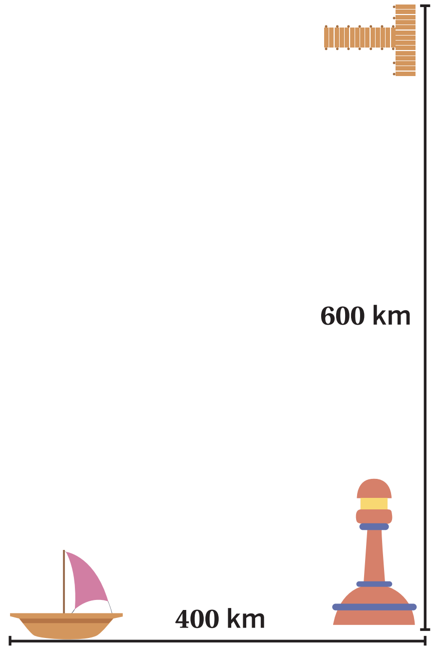
Name: .... Date: .... Period: ....

Activity 1 Navigating the Seas

The idea that right triangle lengths have a fixed relationship to each other has a long history of helping people make sense of the space around them. In the Age of Discovery, explorers looked up at the stars and used triangles and angles to help them navigate the sea. What about the blinking dot that tells you your exact location on a digital map? Turns out the technology behind it, the Global Positioning System, has its origins in the Pythagorean Theorem as well, thanks to pioneering mathematicians such as Gladys West, one of the founders of GPS.

Jada and Mai are taking a multi-day sailing trip. They calculate their boat's position as shown on the map. If the boat travels to the dock taking the shortest path possible at a speed of 20 km per hour, when will the boat arrive at the loading dock?



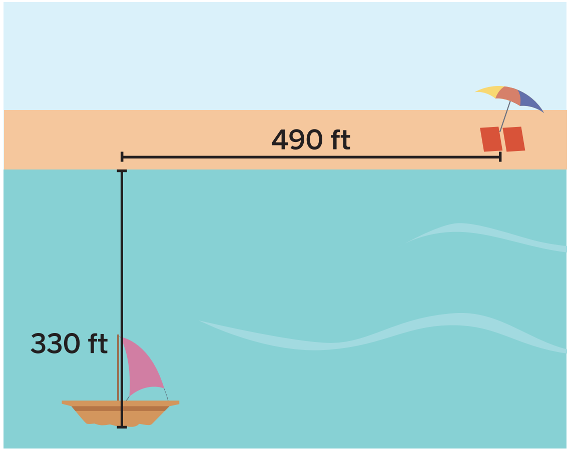
Featured Mathematician

While the Pythagorean Theorem can be used to help find an exact location in 2D space, more advanced mathematics is needed for 3D location. The Global Positioning System (also known as GPS) is a network of satellites that work together using mathematics and technology. GPS was built on the structure found in triangles, circles, and spheres. All of these elements work in concert to determine a precise location on earth with great accuracy. These advances were made possible in large part because of the work of mathematician Gladys West. In the 1960s and 1970s, West worked for what was then known as the Naval Proving Ground, where she collected pivotal data about Earth's precise shape. (It may surprise you that we do not live on a perfect sphere.) This data served as a backbone for the newly developed GPS system.

"Gladys West" by Adrian Cadiz, Courtesy United States Air Force

Activity 2 Fastest Route

Jada and Mai want to jump off their boat anchored in a lake and swim back to their towels and umbrella set up on the beach. They decide to race to the umbrella.



1. Jada and Mai decide to take separate routes. They can each swim 3 ft per second. Their speed on the sand is 5 ft per second. Mai decides to swim directly to the umbrella and Jada decides to swim directly to shore and then run to the umbrella. Who will reach the umbrella first?

2. Is there a path the person who finished second could have taken to reach the umbrella first? Sketch your path and explain your thinking.

Stronger and Clearer: Share your responses to Problems 1 and 2 with another pair of students to give and receive feedback. Use the feedback you receive to improve your responses.

STOP

Name: ________________ Date: ________ Period: ________

Summary

In today's lesson . . .

You applied the Pythagorean Theorem to real-world scenarios. The Pythagorean Theorem can be used to solve any problem that can be modeled with a right triangle, where the lengths of two sides are known and the length of the other side needs to be determined.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Consider a boat that has traveled 6 miles directly south and then 8 miles directly east. The boat traveled at an average speed of 5 miles per hour.

   a. What is the shortest distance the boat can travel directly back to its original location? Show or explain your work.

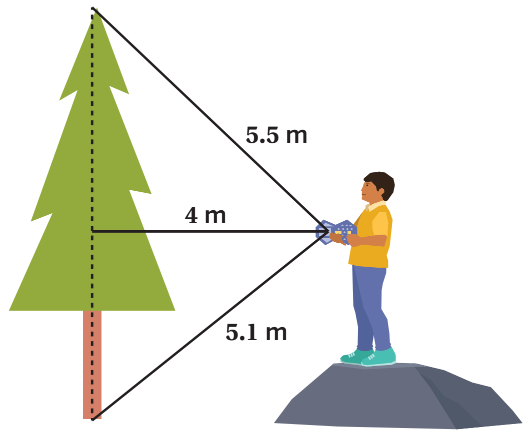
   b. How long will it take for the boat to make its entire journey?

2. At a restaurant, a trash can's opening is rectangular and measures 7 in. by 9 in. The restaurant serves food on trays that measure 11 in. by 15 in. Jada says it is impossible for the tray to accidentally fall through the trash can opening because the shortest side of the tray is longer than either edge of the opening. Do you agree or disagree with Jada's explanation? Explain your thinking.

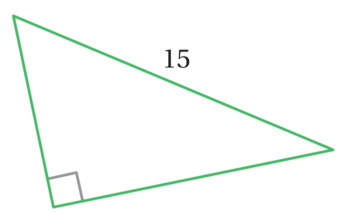
Practice

Name: ________ Date: ________ Period: ________

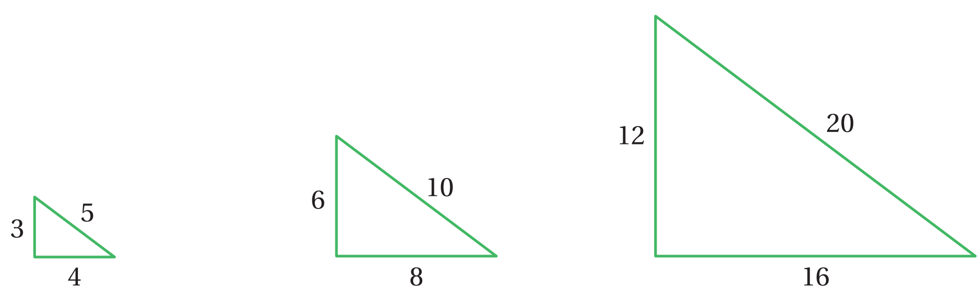
3. A laser range finder is a tool used to measure distances. By pointing the laser at an object, the tool can tell you the distance from the tool to the object along the line you are pointing. Han measures a tree from a hill. What is the height of the tree? Label the measurement on the diagram.



4. A right triangle has a hypotenuse of 15 cm. What are possible lengths for the two legs of the triangle? Explain your reasoning.



5. Which of these triangles are right triangles? Explain how you know.



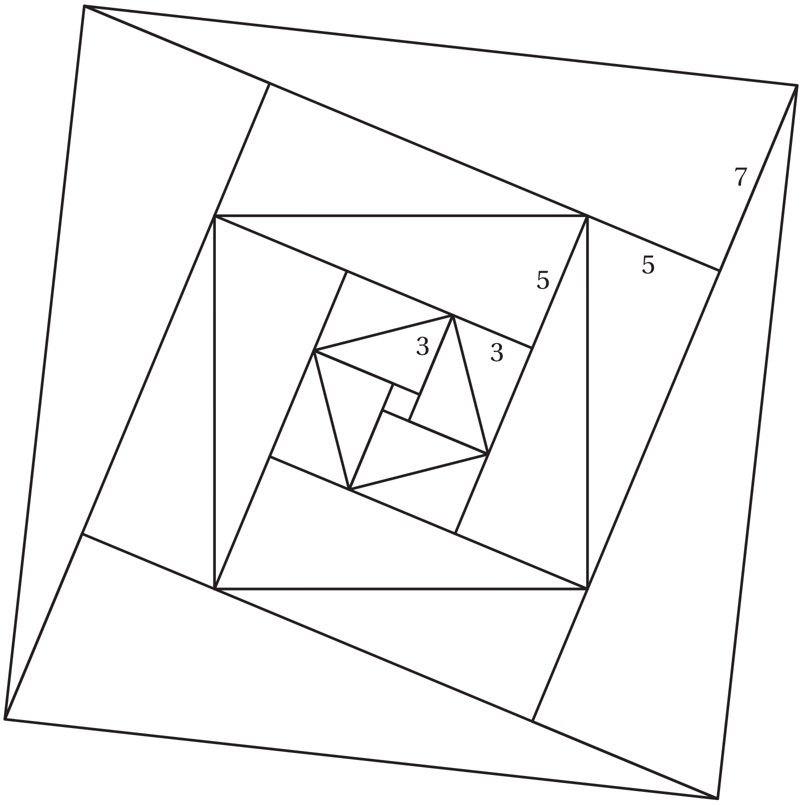
Triangle A **Triangle B** **Triangle C**

Pythagorean Triples

Let's try to recognize patterns that will make Pythagorean triples.

Warm-up Spiraling Squares

Consider the figure shown. In the center is a square with a side length of 1 unit. It is surrounded by additional squares. Determine as many side lengths as you can. Write the lengths on the figure.



Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 Looking for a Pattern

A *Pythagorean triple* is a set of three positive integers a, b, and c, such that $a^2 + b^2 = c^2$.

The side lengths of triangles from Warm-up are all examples of Pythagorean triples.

Leg	Leg	Hypotenuse	Pythagorean Theorem
3	4	5	$3^2 + 4^2 = 5^2$
5	12	13	$5^2 + 12^2 = 13^2$
7	24	25	$7^2 + 24^2 = 25^2$

1. What pattern(s) do you see among these Pythagorean triples?

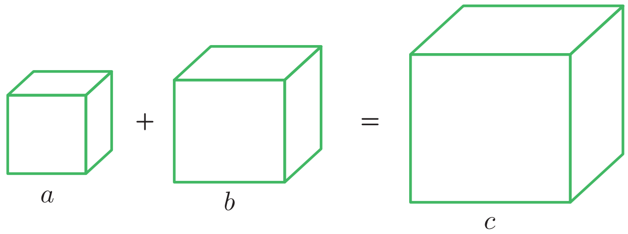
2. Use the pattern you found in Problem 1 to write another Pythagorean triple.

3. Use $a^2 + b^2 = c^2$ to check whether your solution in Problem 2 is a Pythagorean triple. If needed, correct any errors and try again.

Activity 2 Fermat's Last Theorem

Throughout this unit, you worked with the Pythagorean Theorem, which states the sum of the squares of the legs of a right triangle equals the square of the hypotenuse, or more commonly represented as $a^2 + b^2 = c^2$. There are many integer values that make that statement true; refer to Activity 1 if you need a reminder of a few possibilities.

Pythagorean triples have long been a popular area of research. For example, mathematician Jennifer Balakrishnan studies these and integer solutions for more complex equations. Meanwhile, 17th century mathematician Pierre de Fermat tried to find integer solutions for equations with powers greater than 2. For example, try to find three positive integers a, b, and c such that $a^3 + b^3 = c^3$.



1. Have each group member choose different positive integers for a and b. Determine what value of c will make the equation $a^3 + b^3 = c^3$ true. Is it a positive integer?

2. Have each group member choose another set of different positive integers for a and b. Determine what value of c will make the equation true. Is it a positive integer?

3. Compare your answers with your group members. Did anyone get a positive integer for c? What conclusions, if any, can you make about the values that make the statement $a^3 + b^3 = c^3$ true?

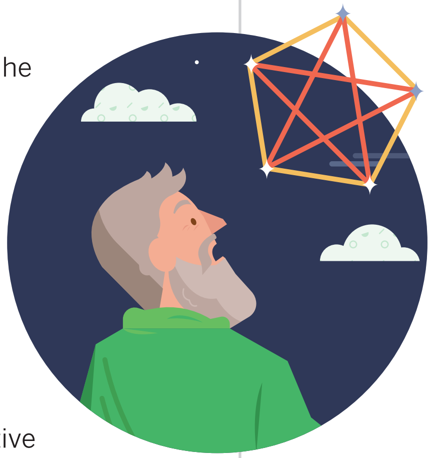
Featured Mathematician

Jennifer Balakrishnan

Originally from Guam, Balakrishnan is a mathematics professor at Boston University. In 2017, she and her collaborators proved there are exactly seven rational solutions to what is known as the "cursed curve": $y^4 + 5x^4 - 6x^2y^2 + 6x^3z + 26x^2yz + 10xy^2z - 10y^3z - 32x^2z^2 - 40xyz^2 + 24y^2z^2 + 32xz^3 - 16yz^3$. Balakrishnan is also a fan of problems related to Pythagorean triples. Among her favorites: Can you find a right triangle and an isosceles triangle whose side lengths are all integers, and which have the same perimeter and the same area as each other?

STOP

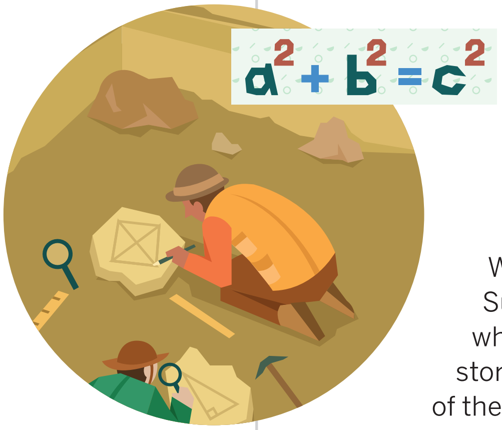
Unit Summary



When mathematician Pierre de Fermat died in 1665, he left behind a copy of Diophantus's *Arithmetica*. This text covered many mathematical topics. In one section on the Pythagorean Theorem, Fermat had written something strange in the margin.

Fermat knew that there were many integer solutions to the equation in the Pythagorean Theorem, $a^2 + b^2 = c^2$. But Fermat claimed that $a^3 + b^3 = c^3$ had no positive integer solutions, nor did $a^4 + b^4 = c^4$. In fact, there was no exponent greater than 2 in which any such equation had positive integer solutions.

But how did he know? Fermat wrote: "I have discovered a truly marvelous proof of this, which this margin is too narrow to contain." And so, one mathematician's decision not to show his work launched one of the greatest mathematical mysteries of the world: Fermat's Last Theorem.



This was a tantalizing puzzle. It wasn't enough to simply test it with a few integers, or even a few million integers. The "theorem" had to be proven. For over three centuries, different mathematicians tried. It wasn't until 1995 when Professor Andrew Wiles finally proved it to be true.

Wiles' proof is too complicated to get into here. Suffice it to say, it involved mathematics beyond what anyone understood in Fermat's time. But the story of Fermat's theorem is just another example of the Pythagorean Theorem's effect on the world. For ages, the theorem has helped make sense of the space around us, from navigation to construction. But beyond its notoriety and countless proofs, the theorem is incredibly useful in how it links the disciplines of algebra and geometry.

See you in Unit 8.

Practice

Name: .... Date: .... Period: ....

1. Determine the exact positive solution to each equation.

a. $x^2 = 90$

b. $w^2 = 36$

c. $z^2 = 0$

d. $y^3 = 1$

e. $p^3 = 90$

f. $h^3 = 64$

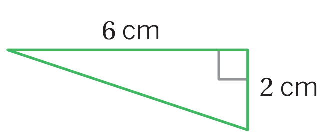
2. Determine the distance between each pair of points. Show your thinking. Consider plotting the points on graph paper.

a. $(5, 0)$ and $(-4, 0)$

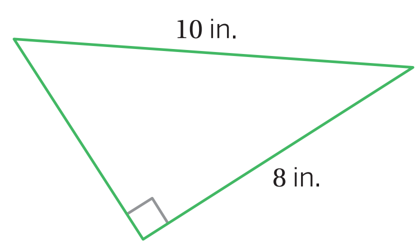
b. $(-21, -29)$ and $(0, 0)$

3. Determine the lengths of the unlabeled sides. Show your thinking.

a.



b.



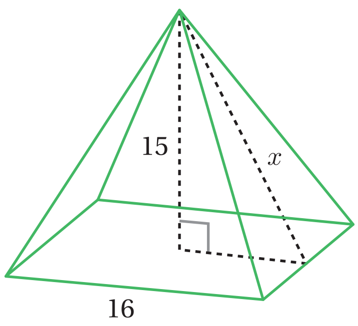
Name: ______________________ Date: __________ Period: __________

Practice

4. How are the numbers 0.444 and $0.\overline{4}$ similar? How are they different?

5. A standard city block in Manhattan is a rectangle measuring 80 m by 270 m. A resident wants to get from one corner of a block to the opposite corner of a block that contains a park. She wonders about the difference between cutting across the diagonal through the park compared to going around the park, along the streets. How much shorter would her walk be going through the park? Round to the nearest meter.

6. Consider this right square pyramid.



a. What is the measurement of the slant height x of the triangular face of the pyramid? To help with your thinking, use a cross section of the pyramid.

b. What is the surface area of the pyramid? Explain your thinking.

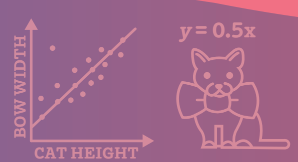
UNIT 8

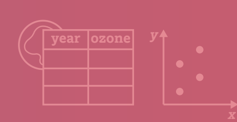
Associations in Data

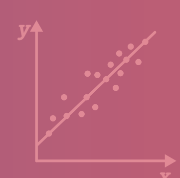
Data literacy — being able to tell and interpret stories using data — is one of the most important skills you will ever need. In this unit, you will make sense of data in the world around you, represented in different forms. By the end of the unit, you will put your new data literacy skills to the test by examining the accuracy of newspaper headlines.

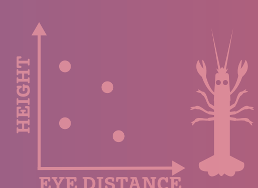
Essential Questions

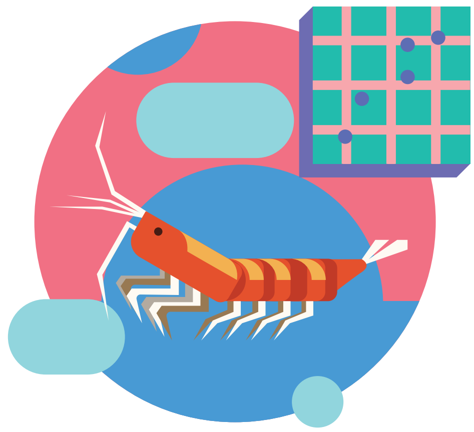
- What is a scatter plot? And what can it tell you?
- How can you model data in a scatter plot? And what does that model tell you?
- What associations can you find, if any, in bivariate data?
- *(By the way, how can you use data to check the accuracy of news headlines?)*











SUB-UNIT

1 Associations in Data

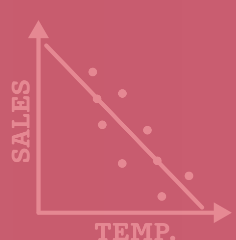
Narrative: Understanding statistics can help us study and preserve the balance of ecosystems.

You'll learn . . .

- how to create and interpret various representations of bivariate data.
- how linear models can be used to fit data and make predictions.

Solve for x in the following two-way table.

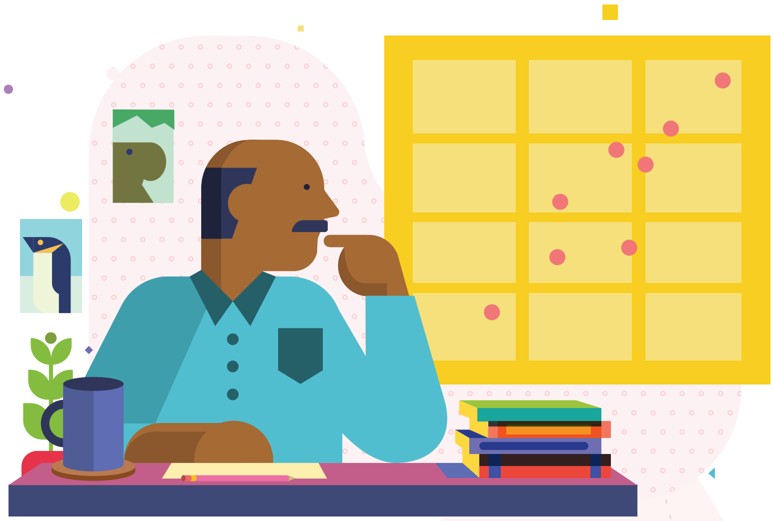
			Total
	x		7
		$\frac{1}{2}x$	
Total	9		16



Unit 8 | Lesson 1 – Launch

Creating a Scatter Plot

Let's find ways to show patterns in data.

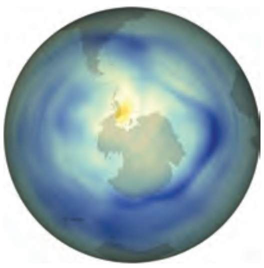


Warm-up Notice and Wonder

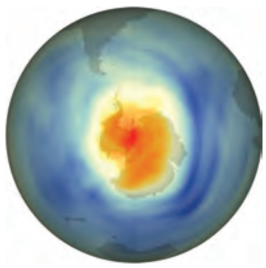
The images show the hole in the ozone layer over Antarctica. The size and shape of the hole are monitored yearly. What do you notice? What do you wonder?

Plan ahead: What will you say to yourself to manage your stress during the activity?

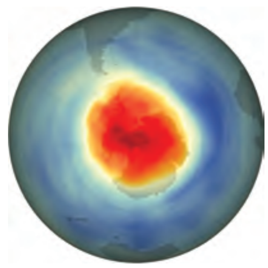
October 16, 1980



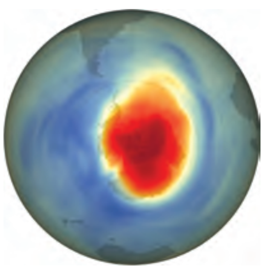
October 3, 1984



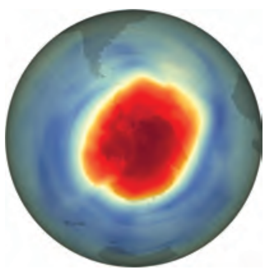
October 7, 1989



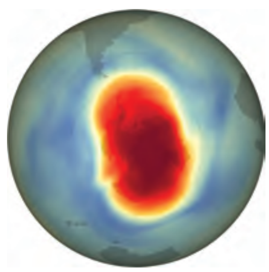
October 11, 1992



October 6, 1998



October 6, 2006



"World of Change: Antarctic Ozone Hole", NASA Earth Observatory,

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Data Tables

NASA records the area of the ozone hole between September and October every year. The Australian government records the number of new cases of melanoma, a type of skin cancer, in Australia.

The following tables show the data they collected between 1982 and 2008.

Year	Ozone hole area (million km^2)
1982	5
1986	11
1988	10
1991	19
1997	22
2001	25
2005	24
2006	27
2007	22
2008	25

Year	Number of skin cancer cases in Australia
1982	3,541
1986	4,712
1988	6,013
1991	5,970
1997	8,444
2001	9,000
2005	10,832
2006	10,427
2007	10,450
2008	11,135

1. Use the tables to estimate the missing values for the ozone hole area and number of skin cancer cases.

a In 1983, the ozone hole area was about ______ million km^2, and the number of skin cancer cases was about ______.

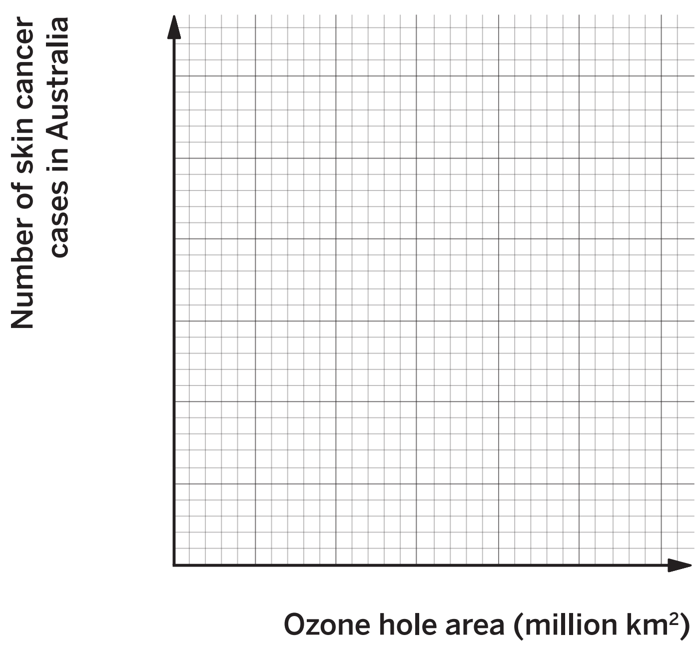
b In 2000, the ozone hole area was about ______ million km^2 and the number of skin cancer cases was about ______.

2. How can you organize the measurements from the tables so that any patterns are easier to see?

Activity 2 Creating a Scatter Plot

On the following graph, create a scale for the data. Then for each year represented in the table, plot a point to represent both the ozone hole area and the number of skin cancer cases.

Year	Ozone hole area (million km^2)	Number of skin cancer cases in Australia
1982	5	3,541
1986	11	4,712
1988	10	6,013
1991	19	5,970
1997	22	8,444
2001	25	9,000
2005	24	10,832
2006	27	10,427
2007	22	10,450
2008	25	11,135



Collect and Display:
As your class discusses the new type of graph introduced in this activity, your teacher will add the language you use to a class display that you can refer to during this unit.

Name: ____________ Date: ____________ Period: ____________

Activity 3 Interpreting a Scatter Plot

Use the table and scatter plot from Activity 2 to complete these problems.

1. Describe how the scatter plot represents the data in the table.

2. What patterns, if any, do you see in the data when it is organized as a scatter plot?

3. How is reading data from a table *similar* to reading data in a scatter plot?

4. How is reading data from a table *different* from reading data in a scatter plot?

STOP

Narrative Connections

Unit 8 Associations in Data

Data and the Ozone Layer

It's easy to run your eyes down a table of numbers or a bunch of graphs and not feel very much. But numbers give us direction. They can tell us stories about where we are, and where we still need to go.

Here, we'll be looking at one such story — a story, told through data, about how our actions matter.

It is a story with billions of characters — heroes and villains, winners and losers. It's a race against time, and in the balance, the fate of an entire planet.

This is the story of our ozone layer. This dense shield of gas in the upper reaches of the stratosphere that protects us from the sun's radiation. It is a story of how that shield weakened and cracked, and what it took to put it back together.

In these next lessons, we'll tell this story through representations like scatter plots. These graphs show the values of two different variables on a coordinate plane. We'll use them to see how the different pieces of that story fit together, and what we had to do to pull our planet from the brink.

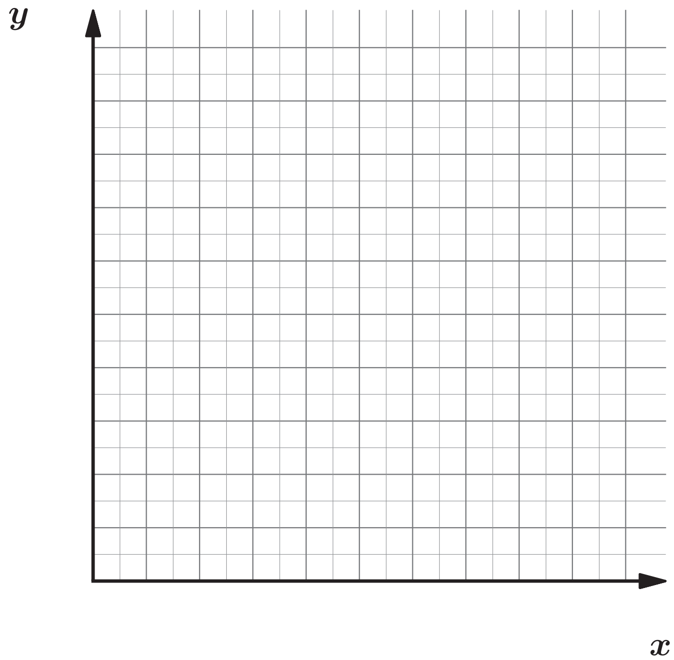
Welcome to Unit 8.

Practice

Name: ____________ Date: ____________ Period: ____________

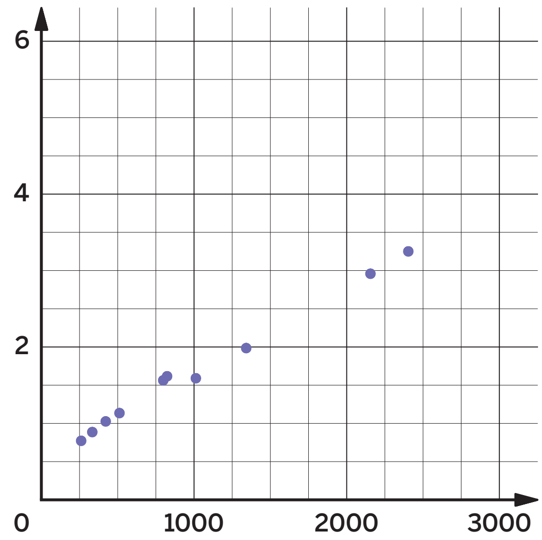
1. Use the table to create a scale for the graph. Then create a scatter plot from the data.

x	y
2	3
5	5
4	2
8	3
6	3
3	5
1	6
4	4



2. When do you think it is better to use a table to represent data? When do you think it is better to use a scatter plot?

3. The graph shows the relationship between flight time, in hours, and flight distance, in kilometers, for 10 flights. Label the axes with this information.



Practice

Name: .... Date: .... Period: ....

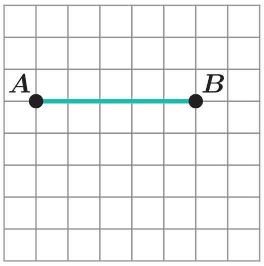
4. Write each decimal as a fraction.

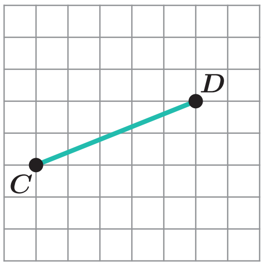
a $0.\overline{45}$

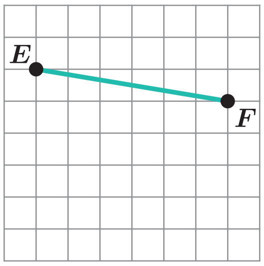
b $0.\overline{8}$

c $0.\overline{6}$

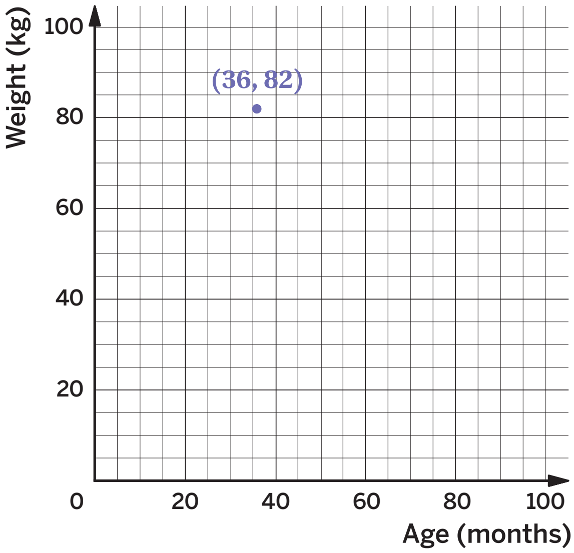
5. What is the exact length of each line segment shown? Each grid square represents 1 square unit. Explain your thinking.

a 

b 

c 

6. The age and weight of a giant panda are recorded on the graph.



a What does the point on the graph tell you about the panda?

b Plot a point that represents a giant panda that is 24 months old and weighs 50 kg.

SUB-UNIT

1 Associations in Data

Narrative Connections

Who is the biggest mover and shaker in the Antarctic Ocean?

The humble krill!

They may be just a few inches long, but these pint-sized crustaceans play a gigantic role in the ecosystem.

Before we get into why they're so important, let's talk about another life form: phytoplankton — tiny, single-celled plants that float along the ocean's surface. Like all plants, they absorb sunlight and convert it to energy. They're also really good at reproducing. While you would need a microscope to see an individual phytoplankton, phytoplankton "blooms" can get so large that they can be seen from space.

Their ability to convert sunlight into energy and reproduce quickly should make them an ideal food source for many animals. The problem is that they're too small for most animals to eat. And that's where the krill comes in.

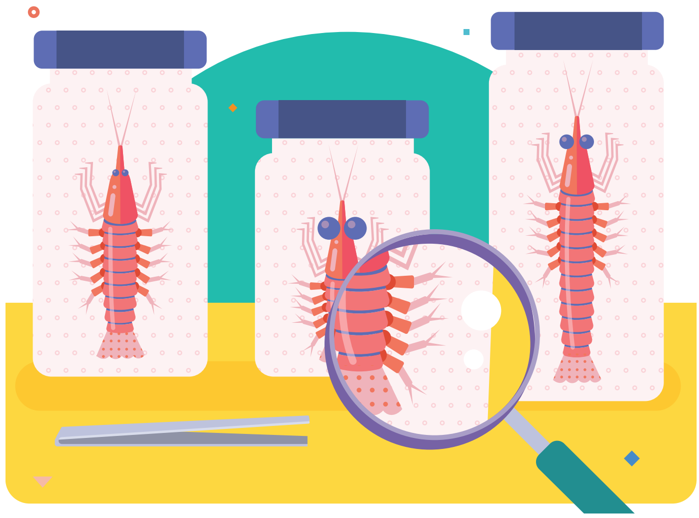
Krill are small enough to eat plankton, but big enough to make a worthwhile meal for fish, whales, seals, squid, penguins, and birds. In fact, they are *the* main source of food in the Antarctic Ocean, making them a vital part of the food chain. Krill are so important that they are considered a "keystone species," providing support and balance to the entire Antarctic ecosystem.

But what happens when something comes along and disrupts that balance? Mathematicians and scientists around the world use data and statistics to study ecosystems. They look for patterns and any associations in the data which can then be used to make predictions as well as generate ideas for ways to restore balance.

Unit 8 | Lesson 2

Interpreting Points on a Scatter Plot

Let's investigate points on a scatter plot.

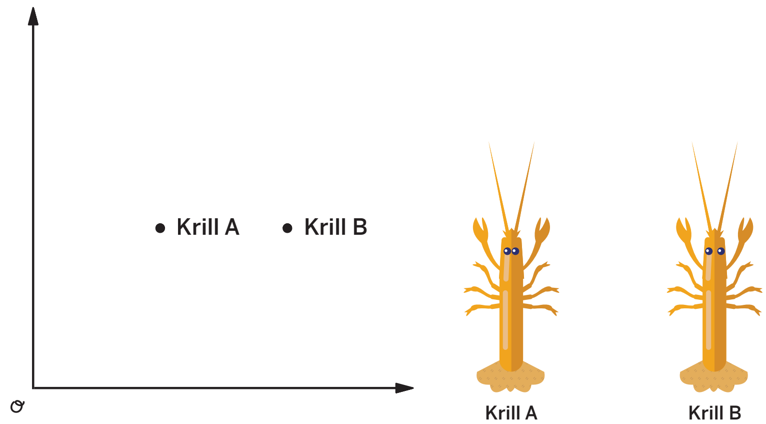


Warm-up Notice and Wonder

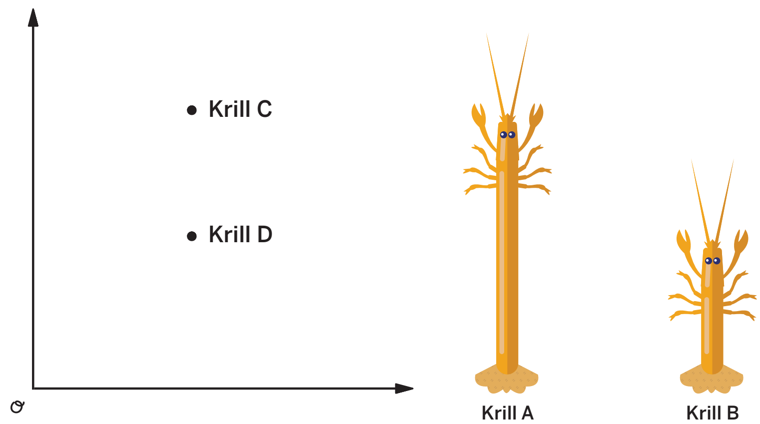
The Oceanological Institute is studying the effects of harmful UV radiation on different krill populations. Here are several specimens they have captured.

Study Krills A–D and both graphs. What do you notice? What do you wonder?

1. I notice . . .



2. I wonder . . .



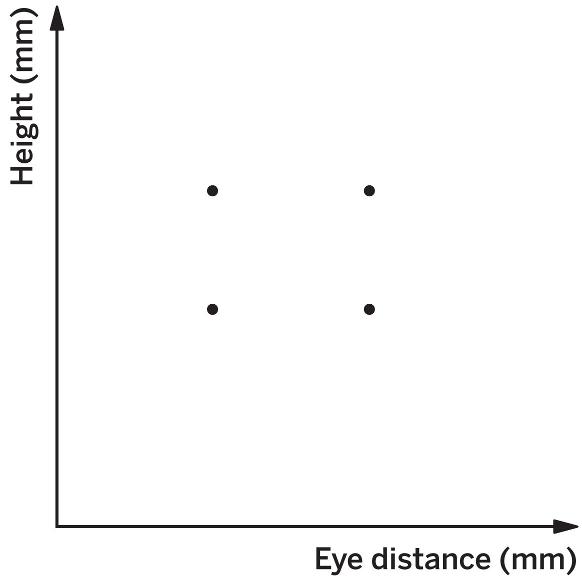
3. Label the axes for both graphs.

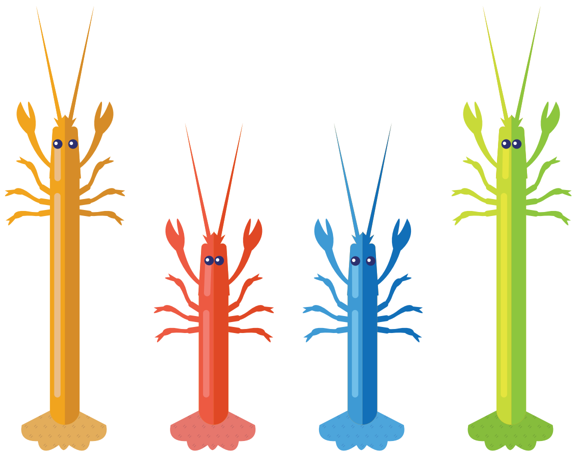
Log in to Amplify Math to complete this lesson online.

Name: ____________ Date: ____________ Period: ____________

Activity 1 Matching Krill

Here are four different species of krill currently being studied by marine scientists at the Oceanological Institute. Label each point on the graph with its corresponding krill color, based on its features. Explain your thinking.





Activity 2 Adding a Point

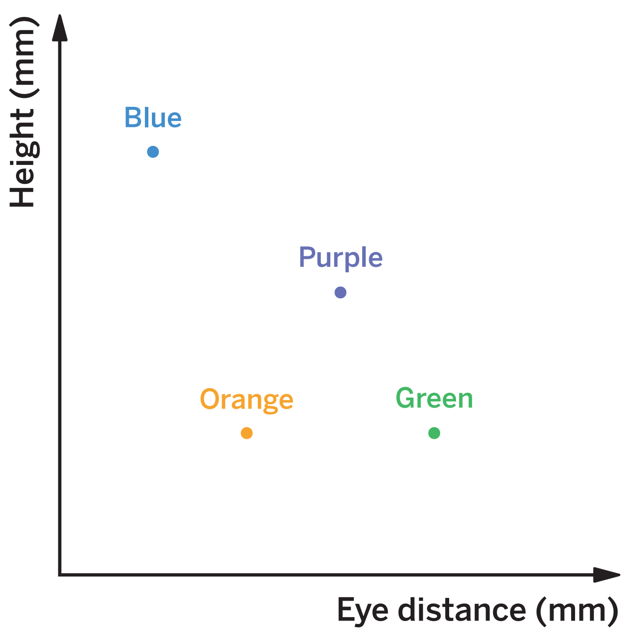
Plan ahead: What will you say to yourself to manage your stress during the activity?

The table shows the heights and eye distances for five different krill.

Name	Eye distance (mm)	Height (mm)
Blue	2	30
Orange	4	10
Green	8	10
Purple	6	20
Red	8	20

1. Add a point to the graph that represents the red krill.

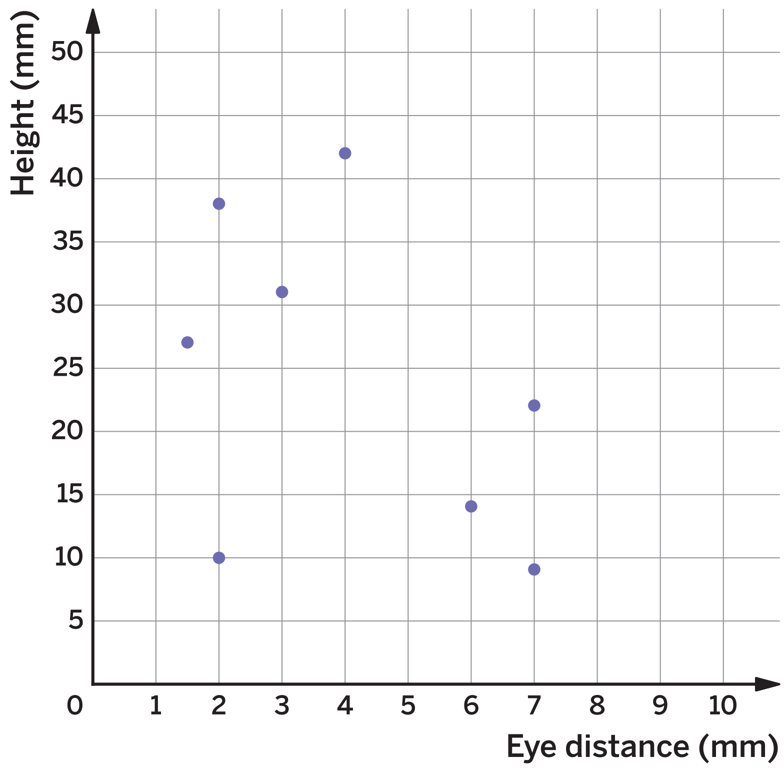
2. Explain how you decided where to place the point.



Name: ________ Date: ________ Period: ________

Activity 3 What's the Point?

The graph shows the heights and eye distances for eight different krill.



1. What are the coordinates of the krill with the shortest eye distance? What does this tell you about the eye distance and height of the krill?

2. Add a point to the graph that represents a krill that makes the following statement true: The red krill is taller than the other eight krill, and its eye distance is 5 mm. How did you decide where to put your point?

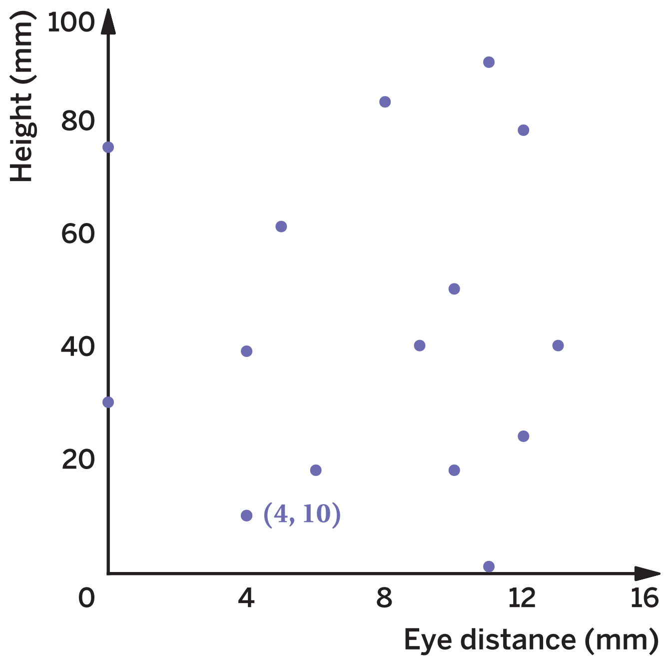
STOP

Summary

In today's lesson . . .

You investigated the coordinates of points on a scatter plot. A ***scatter plot*** is a graph that shows the values of two variables on a coordinate plane. It allows you to investigate connections between the two variables.

A point in a scatter plot represents the measures for an individual data value in a population of data. The axes labels tell you how to interpret the coordinates of each point. In this example, the point $(4, 10)$ represents a krill with an eye distance of 4 mm and a height of 10 mm.

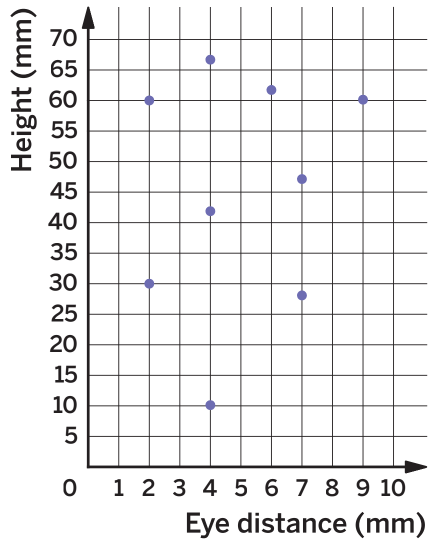


Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. Circle the point for the krill with the shortest eye distance. What are the height(s) and eye distance(s) of the krill?

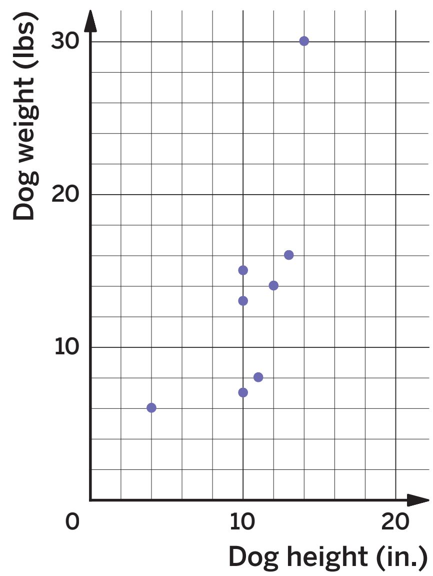


2. The scatter plot shows the heights and weights of 8 different dogs.

   a. How much does the tallest dog weigh?

   b. What are the height and weight of the shortest dog?

   c. Plot a point on the graph that represents a dog that is shorter than 8 in. and has a weight of 10 lb.



3. Here is a table that compares the average points per game to the average free throw attempts per game for a few players on a basketball team during a tournament. What does the ordered pair $(2.1, 18.6)$ represent?

Player	Free throw attempts	Points
Player A	5.5	28.3
Player B	2.1	18.6
Player C	4.1	13.7
Player D	1.6	10.6
Player E	3.1	10.4

Name: .... Date: .... Period: ....

4. Determine whether each number is *rational* or *irrational*.

a $\sqrt{27}$

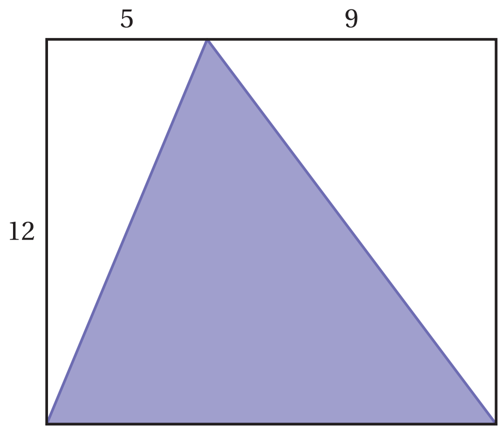
b 2.654

c $\sqrt{54}$

d -95

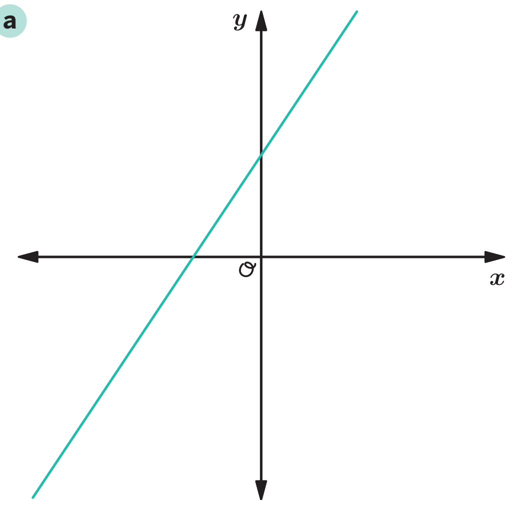
e $\sqrt{144}$

5. Refer to the 12-by-14 rectangle partitioned into triangles.
Is the shaded triangle a right triangle? Explain your thinking.

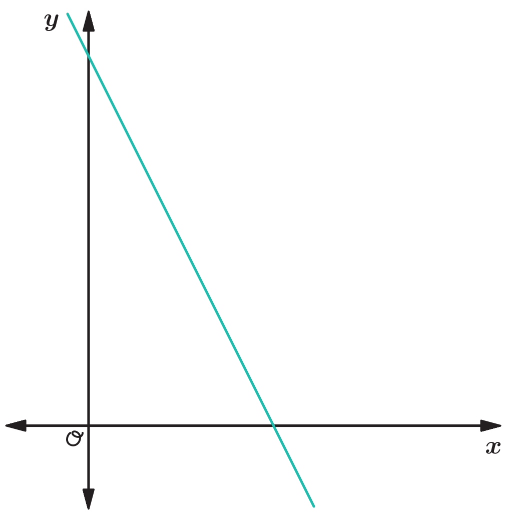


6. Identify whether the slope of each line is *positive* or *negative*.

a



b



Name: ________ Date: ________ Period: ________

Unit 8 | Lesson 3

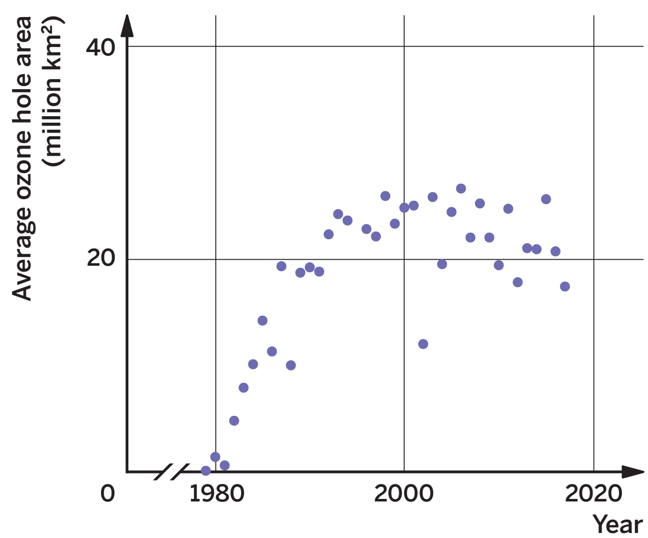
Observing Patterns in Scatter Plots

Let's look for patterns in scatter plots.

Warm-up Notice and Wonder

Study the graph. What do you notice? What do you wonder?

Antarctic ozone hole area



1. I notice . . .

2. I wonder . . .

Activity 1 Card Sort: Associations in Scatter Plots (Part 1)

You will be given a set of cards. Sort the cards into two categories: *linear association* and *nonlinear association*. Compare with your partner and discuss the strategies you used to sort each card.

Linear association	Nonlinear association

Name: ________ Date: ________ Period: ________

Activity 2 Card Sort: Associations in Scatter Plots (Part 2)

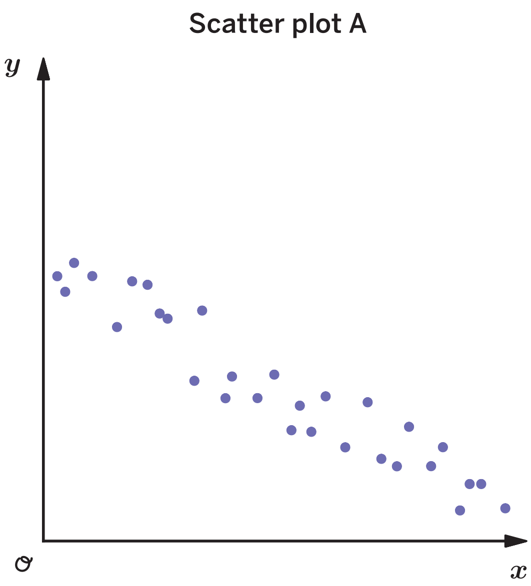
Now, sort the cards from Activity 1 into three categories: *positive association*, *negative association*, and *neither positive nor negative association*. Compare with your partner and discuss the strategies you used to sort each card.

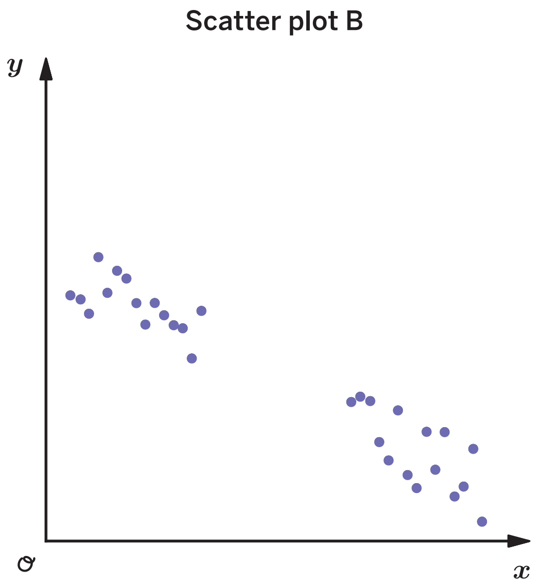
Positive association	Negative association	Neither positive nor negative association

Explain how you determined which cards were sorted as having "neither positive nor negative association."

Activity 3 Spot the Difference

Compare the scatter plots.





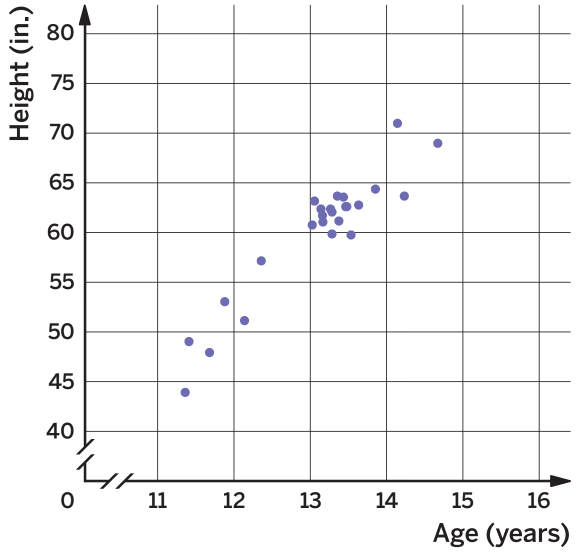
1. What similarities do you see?

2. What differences do you see?

Name: ________ Date: ________ Period: ________

Activity 4 Identifying Clusters

1. The scatter plot shows the ages and heights of several students in Jada's school.



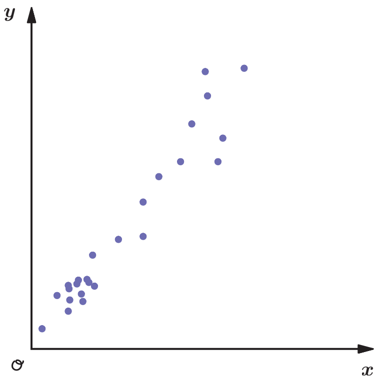
a. Circle the cluster in the data.

b. What does the cluster represent?

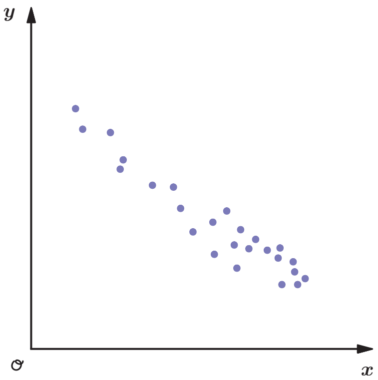
c. What might cause this cluster?

2. Circle any clusters you see in the scatter plots.

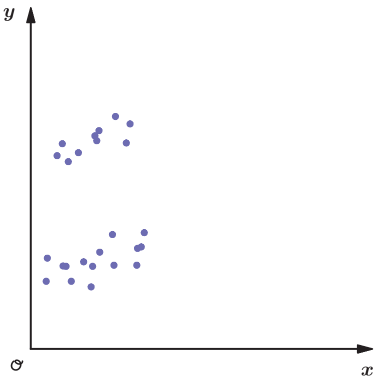
a.



b.



c.



STOP

Summary

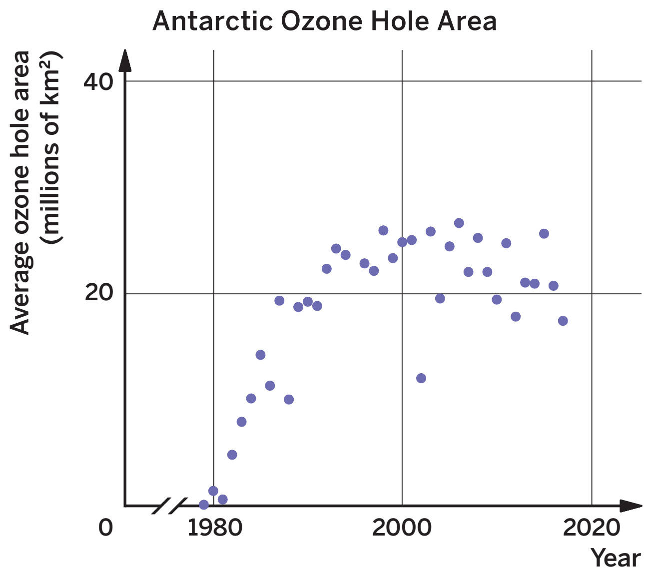
In today's lesson . . .

You observed patterns in scatter plots. If a straight line can model the data, the data have a ***linear association***. If a straight line cannot model the data, the data have a ***nonlinear association***.

Both linear and nonlinear associations can have positive or negative associations. A ***positive association*** means that when one variable increases, the other also increases. A ***negative association*** means that when one variable increases, the other decreases.

A ***cluster*** represents data values that are grouped closely together.

This scatter plot shows a nonlinear association.



Reflect:

Name: .... Date: .... Period: ....

Practice

1. Select *all* the following terms that describe the association in the scatter plot.

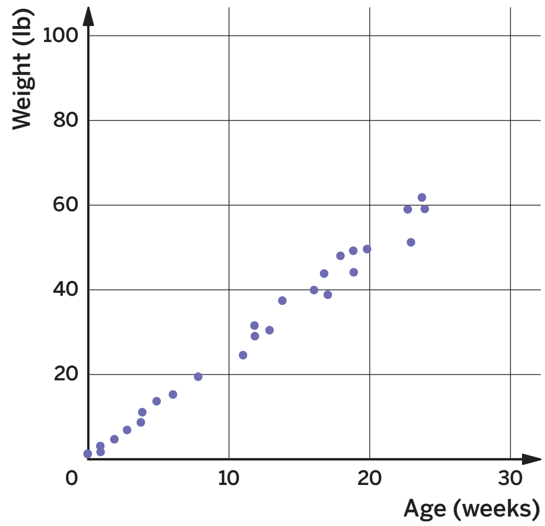
   A. linear association

   B. nonlinear association

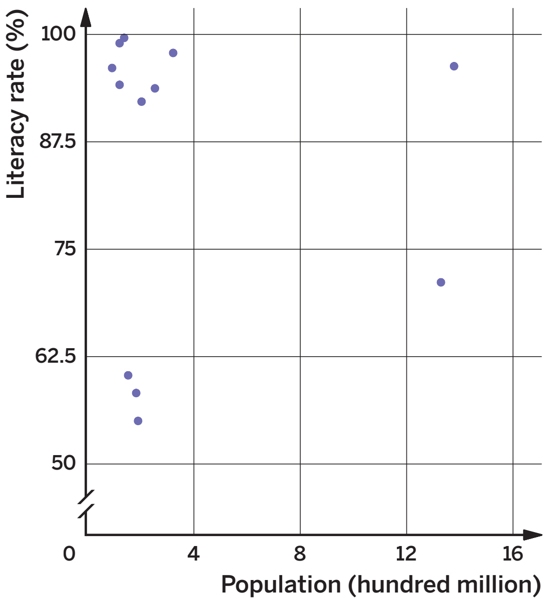
   C. positive association

   D. negative association

   E. no association



2. Circle any clusters in the data.

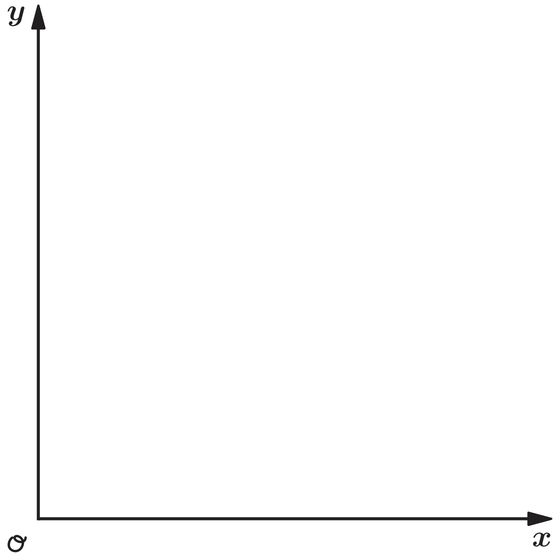


Explain what the clusters mean in context.

Practice

Name: ______________________ Date: ____________ Period: ____________

3. Create a scatter plot that has a positive linear association with clustering.

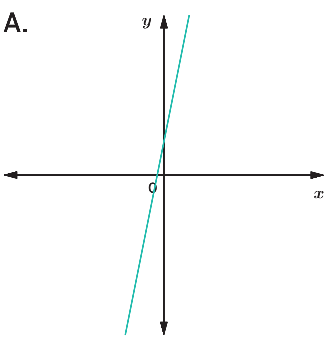


4. Write each expression as a single power.

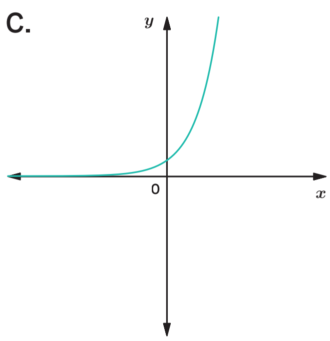
a $\frac{9^7}{9^3}$ **b** $(5^4)^3$ **c** $3^2 \cdot 3^{12}$ **d** $11^4 \cdot 5^4$

5. Circle *all* the graphs that represent a linear function.

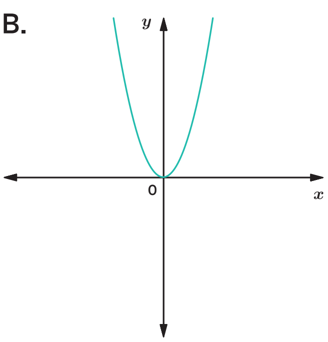
A.



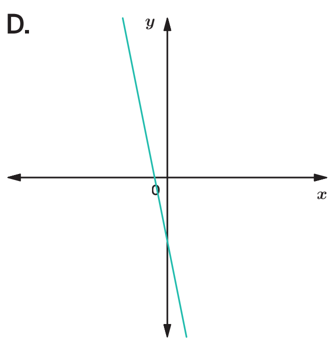
C.



B.



D.



Name: ________ Date: ________ Period: ________

Unit 8 | Lesson 4

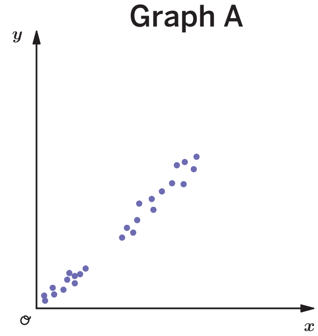
Fitting a Line to Data

Let's draw a line to fit data.

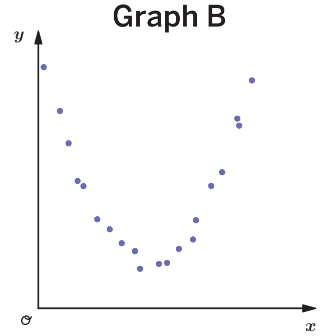
Warm-up Which One Doesn't Belong?

Study the following graphs. Which graph doesn't belong? Explain your thinking.

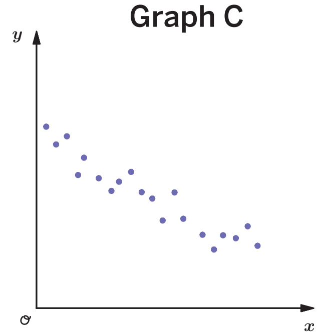
Graph A



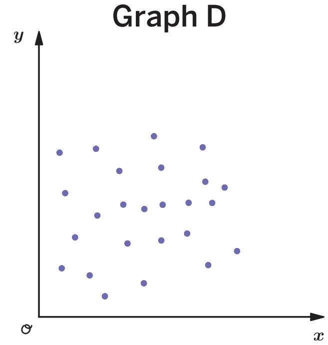
Graph B



Graph C



Graph D

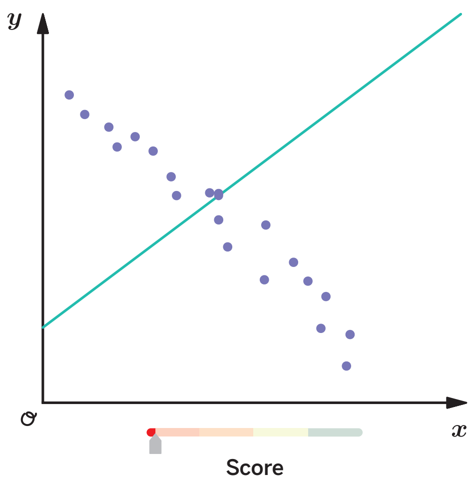


Log in to Amplify Math to complete this lesson online.

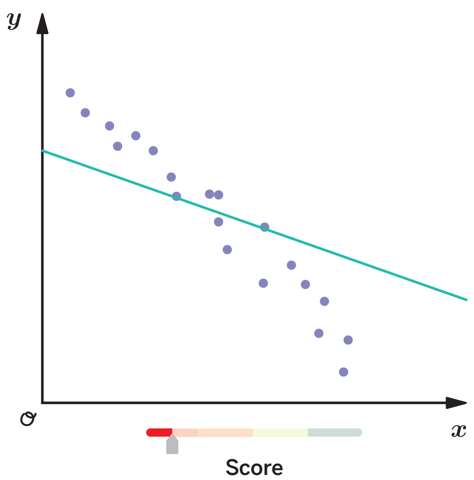
Activity 1 Survival of the Fittest

Each graph shows the same set of data. Study these graphs to determine what the score meter is measuring, and how a rating is determined. For each meter, scores closer to the left represent lower scores and scores closer to the right represent higher scores.

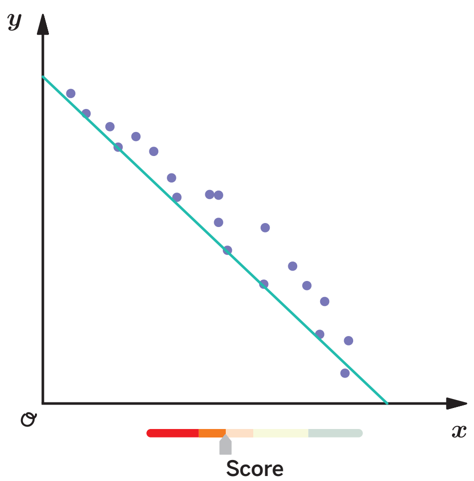
Graph A



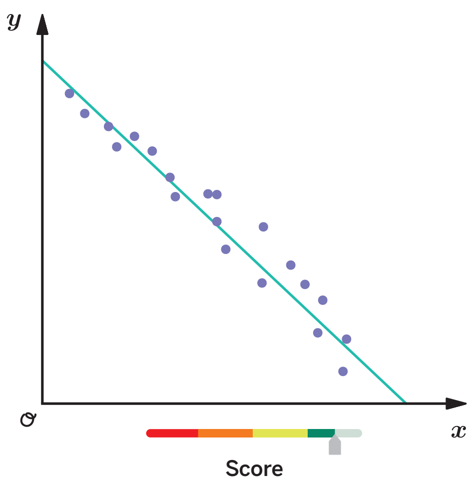
Graph B



Graph C



Graph D



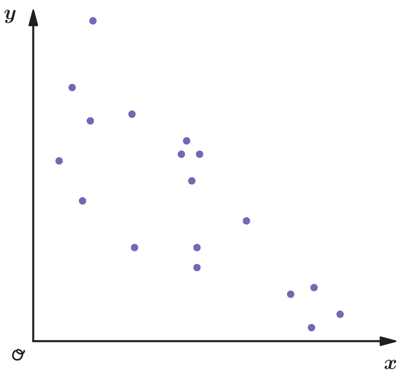
1. Describe how the line affects the score meter. Why is the score low for Graph A, Graph B, and Graph C?

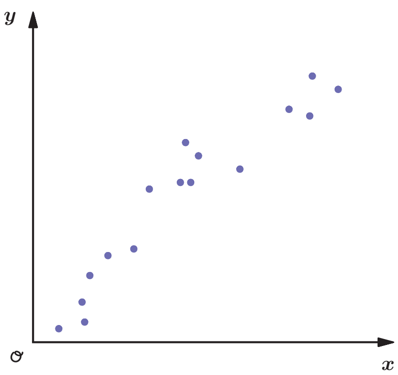
2. How can you earn a high score?

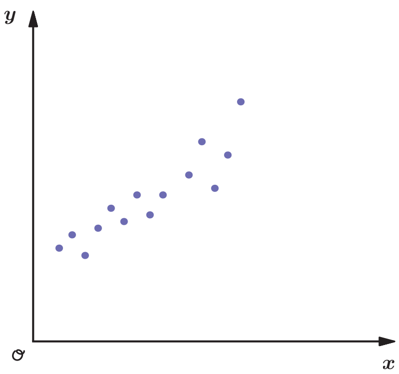
Name: ________ Date: ________ Period: ________

Activity 2 Drawing a Line of Fit

State whether each scatter plot shows *positive* or *negative* association. Then use the strategies you learned from Activity 1 to draw a good line of fit. Compare your lines of fit with a partner.

1. 

2. 

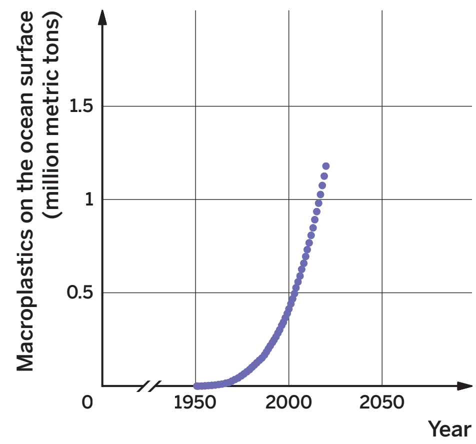
3. 

Activity 3 Is It a Good Fit?

Discarded plastic has done great harm to the environment, contaminating groundwater, injuring marine life, and even causing changes in human health.

The following scatter plot shows the amount of macroplastics on the ocean surface from 1950 to 2020.

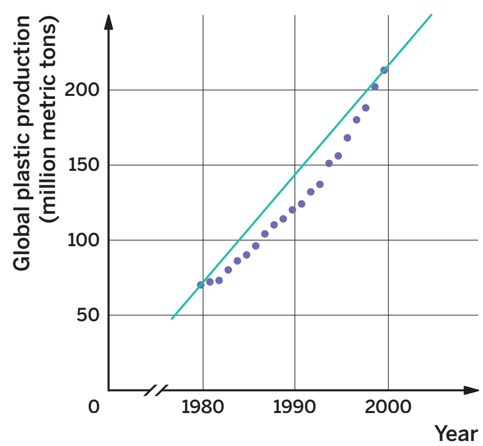
1. Do you think a line would be a good fit for the data? Explain your thinking.



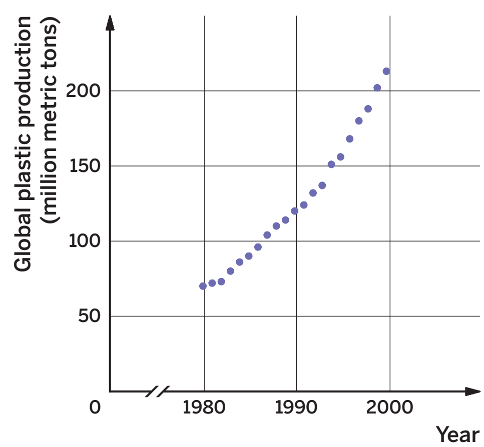
2. This scatter plot shows global plastic production between 1980 and 2000.

a Priya thinks this line is a good fit for the data because it passes through the first and last point.

Explain why Priya is incorrect.



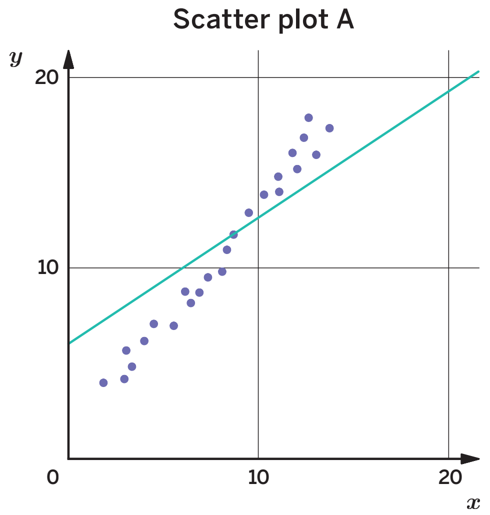
b Draw your own line of fit for the data. Does the scatter plot have a positive or negative association? What does this association tell you about global plastic production?

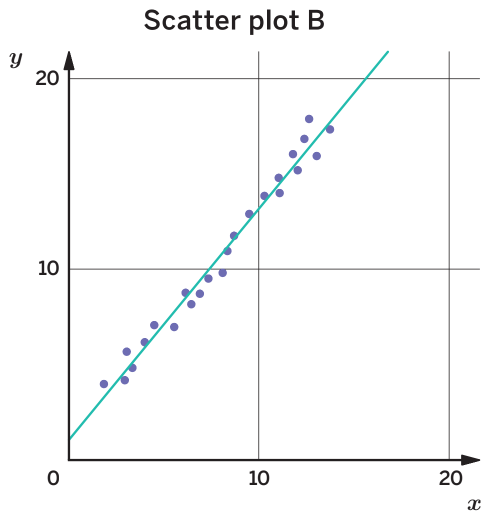


Name: .... Date: .... Period: ....

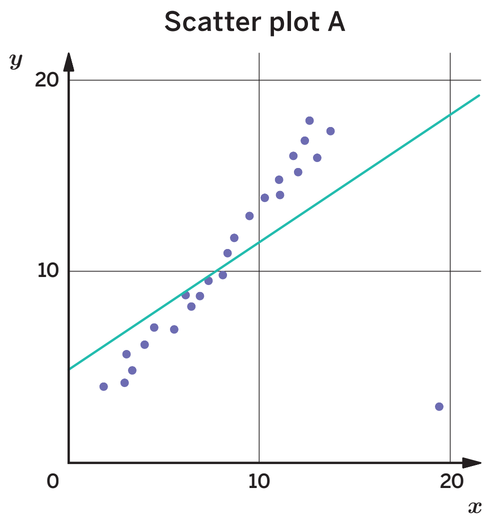
Activity 4 Making a Choice

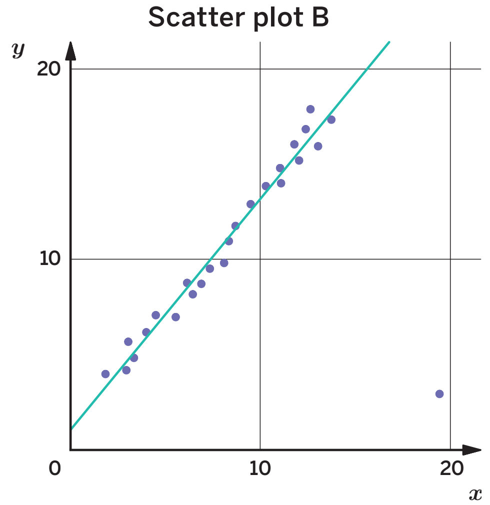
1. The two scatter plots have the same data points, but different lines. Which line do you think is a better fit? Explain your thinking.





2. Here are the same scatter plots from Problem 1, but one data point has been added that is far away from the rest of the data. Does the additional point change your thinking about the line of fit?





STOP

Summary

In today's lesson . . .

You investigated how to draw a line that fits a set of data. When data has a linear association, you can draw a straight line to model the data. A good line of fit follows the trend of the data, and has a balance of points above and below the line. The line may pass through some, all, or none of the points.

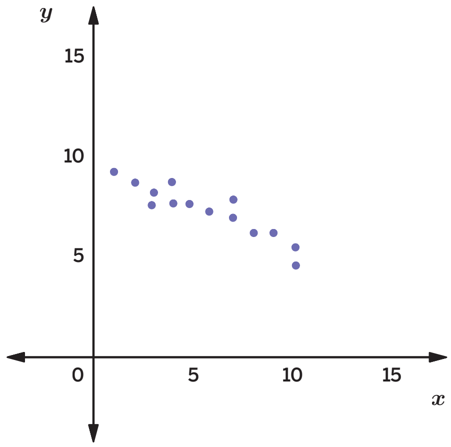
Reflect:

Name: ______________________ Date: __________ Period: __________

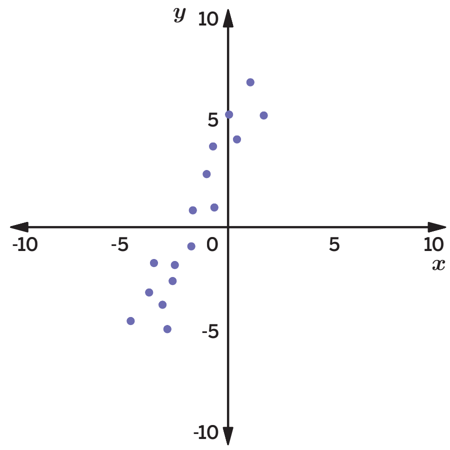
Practice

1. For each graph, draw a line that models the data.

Scatter plot A

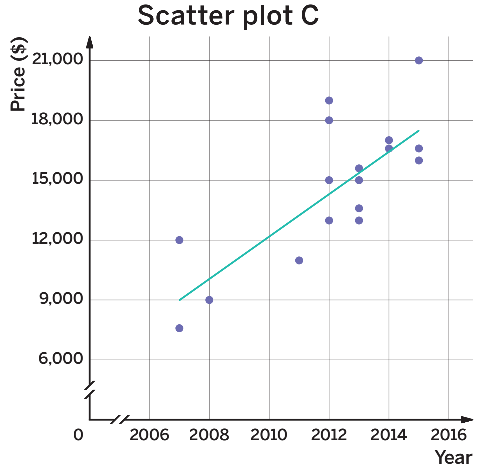


Scatter plot B

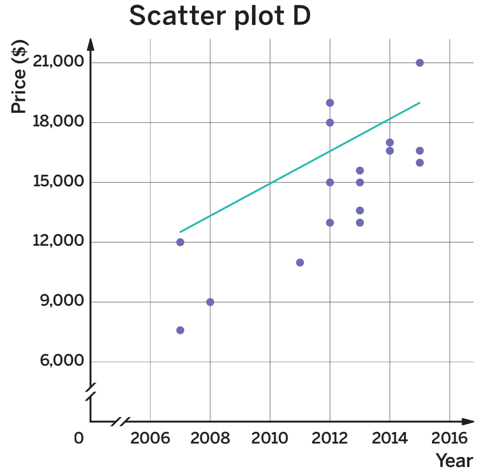


2. The two scatter plots have the same data, but different lines. Which line do you think is a better fit? Explain your thinking.

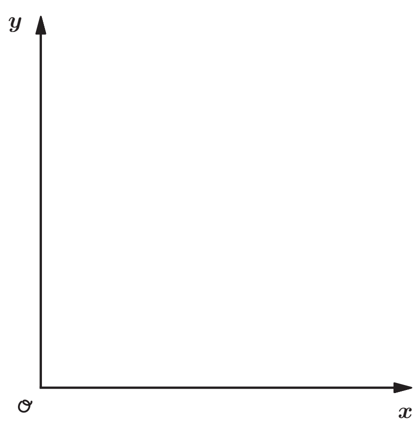
Scatter plot C



Scatter plot D



3. Create a scatter plot that has a negative linear association with clustering. Then draw a line of fit for your scatter plot.



Practice

Name: ________________ Date: ________ Period: ________

4. In each problem, a and b represent the lengths of the legs of a right triangle, and c represents the length of its hypotenuse. Determine the missing length, given the other two lengths. Show or explain your thinking.

a $a = 3, b = 4, c = ?$

b $a = ?, b = 15, c = 17$

5. Determine between which two consecutive integers each square root lies. Explain your thinking.

a $\sqrt{13}$

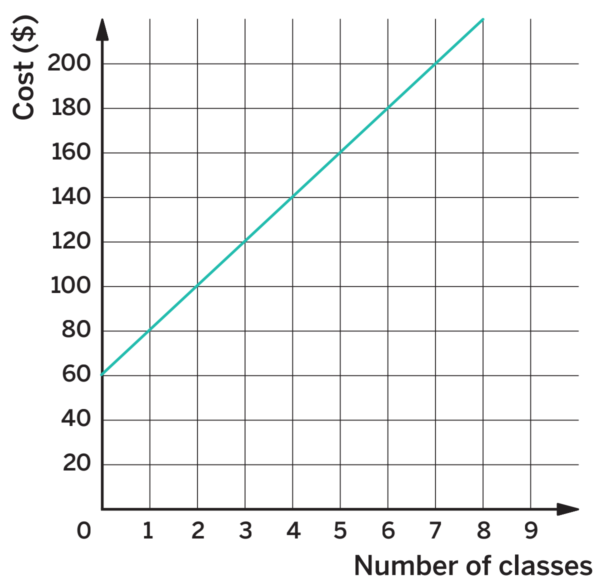
b $\sqrt{63}$

c $\sqrt{14}$

d $\sqrt{115}$

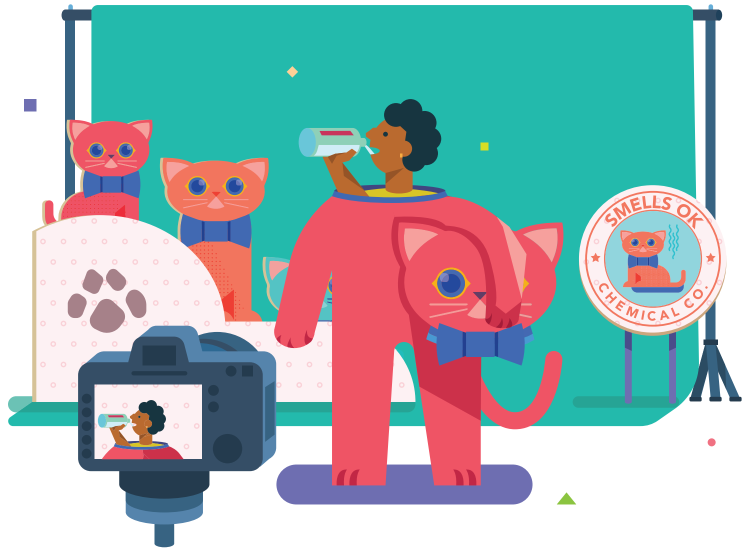
e $\sqrt{26}$

6. Customers at a gym pay a membership fee to join and then an additional fee for each class they attend. The gym manager uses this graph to represent the cost. What is the cost of 6 classes?



Name: ______ Date: ______ Period: ______

Unit 8 | Lesson 5



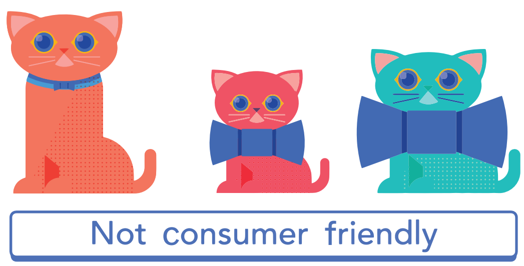
Using a Linear Model

Let's identify potential outliers and use a linear model to predict values.

Warm-up What Makes a Good Logo?

The Smells OK Chemical Company is testing a logo for its new litter box deodorizer. The mascot, Chemy Cat, is currently undergoing a redesign so that it's more consumer friendly.

While the designers were allowed to adjust the mascot however they liked, some designs are consumer friendly, but others are not.



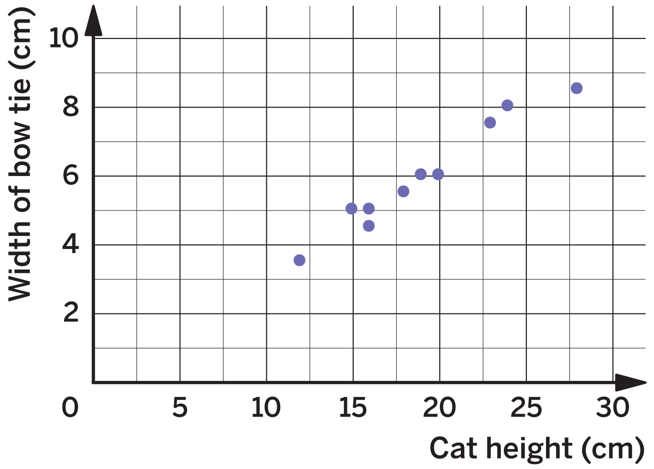
1. What do you think makes a cat "consumer friendly"?

2. What do you think makes a cat "not consumer friendly"?

Log in to Amplify Math to complete this lesson online.

Activity 1 Measuring Chemy Cat

The design team at Smells OK Chemical Company created and reviewed 10 different logos, and then plotted the measurements on the graph. You will be given four more Chemy Cat logos that the designers would like you to review.



1. Use your ruler to measure the cat's height and bow-tie width in centimeters. Add your group's data (*cat height, width of bow tie*) to the graph. Then label each new point with the letter of the corresponding cat.

2. Describe any patterns you see in the data.

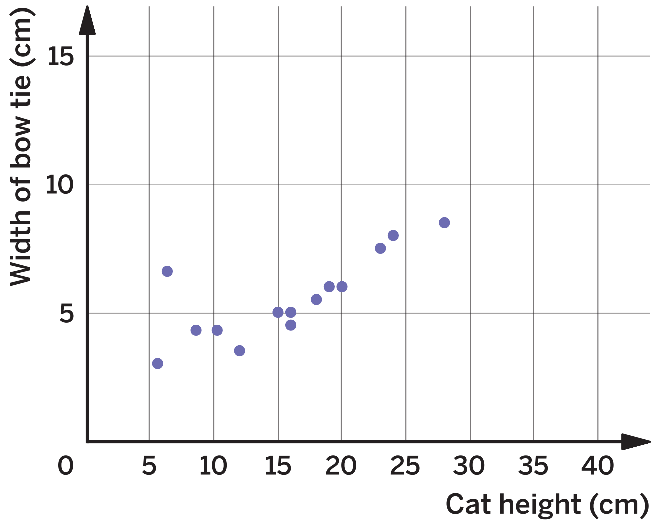
3. What did you notice about the point that represents Cat C? How does its height and bow-tie width compare to the other cat designs?

4. Add a point to the scatter plot that represents a cat that is "*not* consumer friendly". How does its height and bow-tie width compare to those of a "consumer friendly" cat?

Name: ________________ Date: ________ Period: ________

Activity 2 Adding a Line

Precise methods for fitting lines to data were developed several hundred years ago by mathematicians including Adrien-Marie Legendre. For now, you will continue to draw and judge these lines by eye.



1. Predict the bow-tie width for a cat that is 35 cm tall.

2. Draw a line of fit to model the data.

3. How can drawing a line that models the data help you to make a prediction?

Featured Mathematician

Adrien-Marie Legendre

A French mathematician, Legendre was born in 1752. He made numerous contributions to statistics — including fitting lines and curves to data — as well as number theory and physics. While he received numerous honors, and even has a crater on the moon named after him, no one knows quite what he looked like. His only surviving portrait is a caricature!

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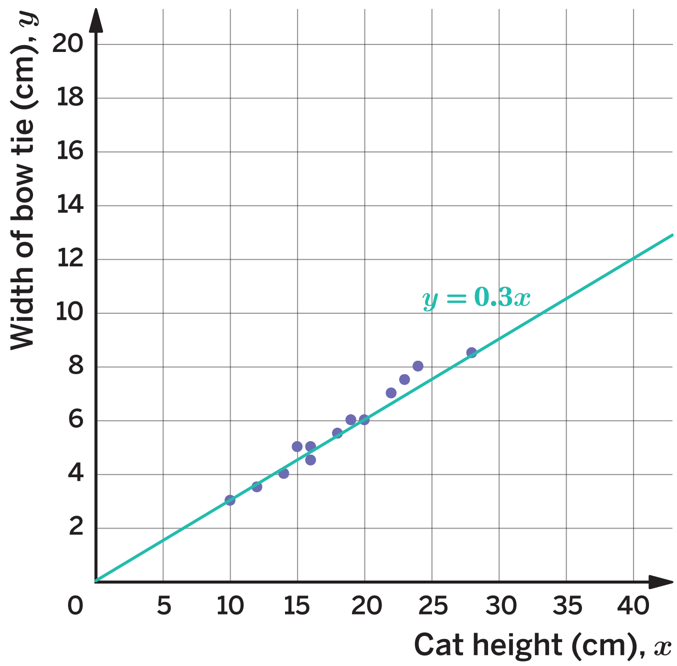
Activity 3 Using a Linear Model to Predict Data

A student from Ms. Sutula's class drew a linear model represented by the equation $y = 0.3x$.



1. Use the linear model to determine the bow-tie width for cats with the following heights.

   a Cat height: 32 cm

   b Cat height: 40 cm

2. Use the linear model to determine the height for a cat with the following bow-tie width.

   a Bow-tie width: 10 cm

   b Bow-tie width: 60 cm

3. A cat logo design is 40 cm tall with a bow-tie width of 8 cm. What height does the linear model predict for a cat with a bow-tie width of 8 cm? How does this compare to the actual height of the cat design?

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You used a linear model on a scatter plot to predict values. A ***linear model*** helps you to see trends in data more clearly. You can use a linear model to make a prediction by matching the corresponding values of x and y on the line, or by substituting values into the equation.

When a data point is above the line of fit, it means that the actual value is greater than the predicted value. When the data point is below the line, it means the actual value is less than the predicted value.

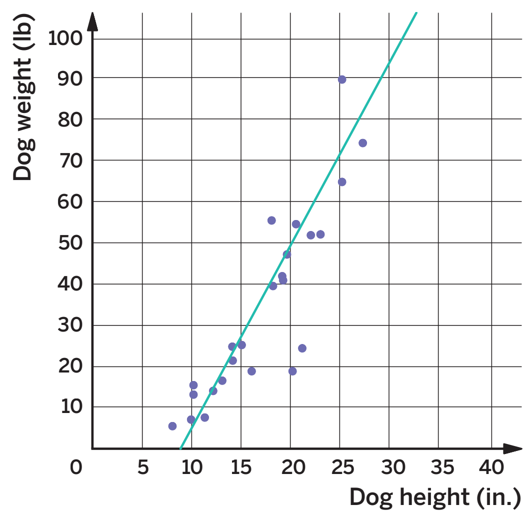
You can identify an ***outlier*** by looking for points that are far away from their predicted values.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

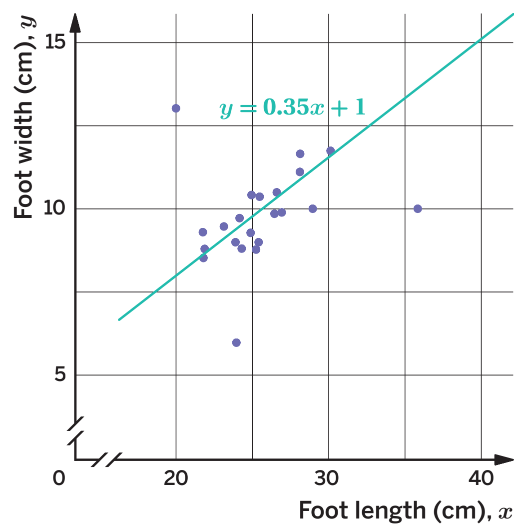
1. The scatter plot shows the height and weight of 25 dogs, along with a linear model to represent the relationship between a dog's height and its weight.



a. Predict a dog's weight if its height is 16 in.

b. Predict a dog's height if its weight is 61 lb.

2. The scatter plot shows several foot lengths and widths, along with a linear model represented by the equation $y = 0.35x + 1$.



a. Use the linear model's equation to predict the width of a foot that is 50 cm long.

b. Does the scatter plot appear to have any outliers? If so, circle them and describe what they represent about foot length and foot width.

3. In your own words, what does an outlier represent?

Name: .... Date: .... Period: ....

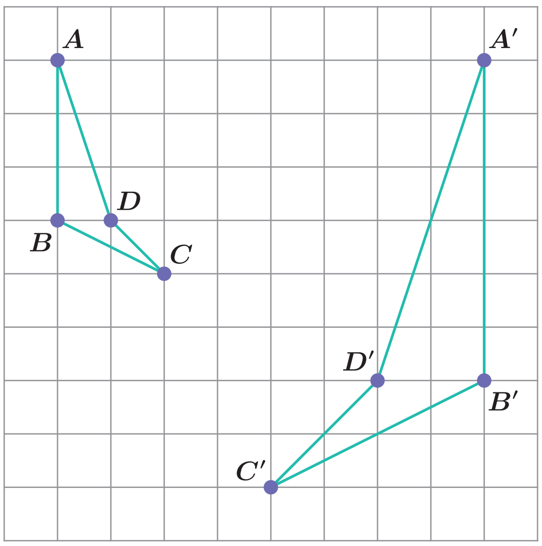
Practice

4. Solve each system of equations. Write the solution as an ordered pair. Show your thinking.

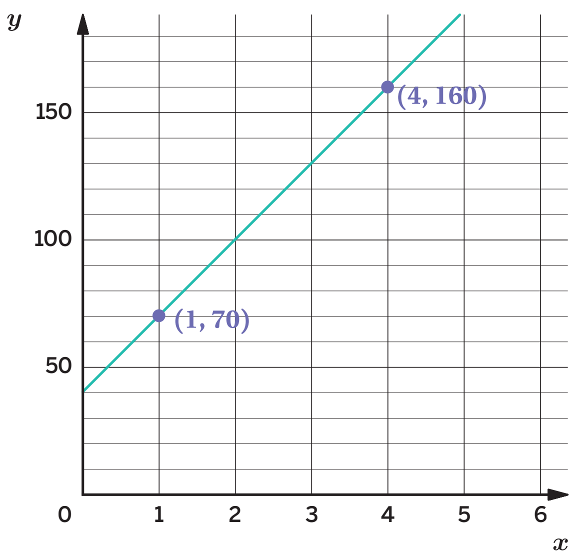
a $\begin{cases} y = -3x + 13 \\ y = -2x + 1 \end{cases}$

b $\begin{cases} y = x + 1 \\ y = -x + 5 \end{cases}$

5. Describe a sequence of transformations that maps Polygon $ABCD$ onto Polygon $A'B'C'D'$.



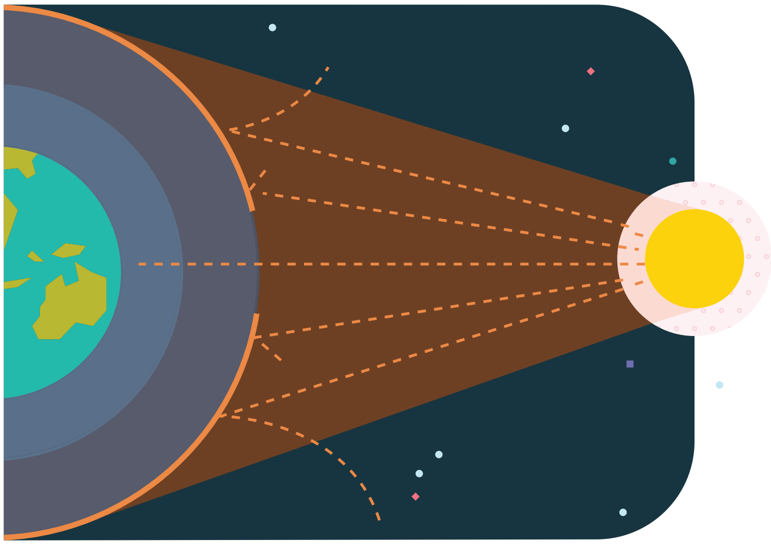
6. Determine the slope and and y-intercept of the line shown. Use the slope and y-intercept to write the equation fo the line. Show or explain your thinking.



Unit 8 | Lesson 6

Interpreting Slope and y-intercept

Let's see what the slope and y-intercept represent in context.

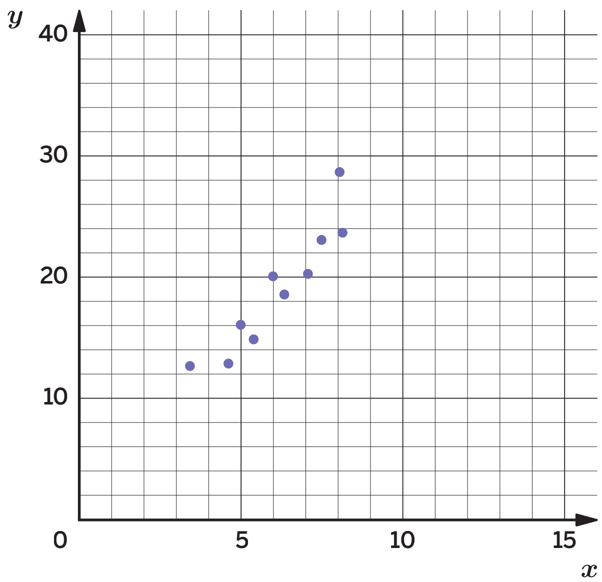


Warm-up Creating a Linear Model

Refer to the scatter plot shown.

1. Draw a line to model the data.

2. Write an equation for your line.

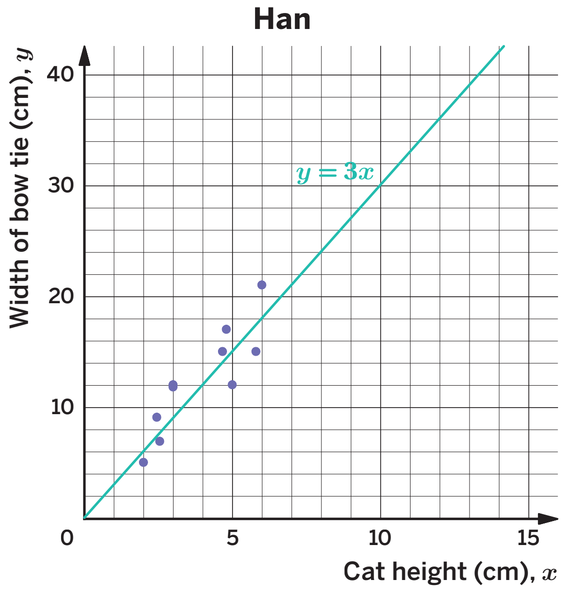


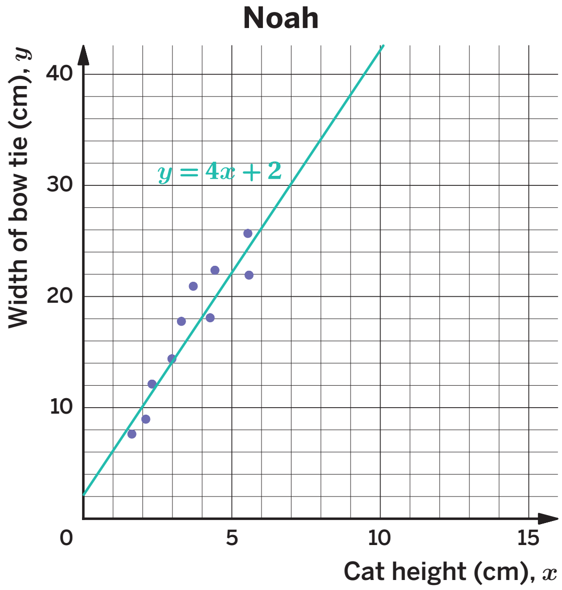
Log in to Amplify Math to complete this lesson online.

Name: ____________ Date: ____________ Period: ____________

Activity 1 Comparing Linear Models

The scatter plots and linear models show two cat logo designs created by Han and Noah.





1. What are the slopes of the linear models? What do the slopes represent in terms of cat height and bow-tie width?

   Han: **Noah:**

2. Based on the slopes, how would you describe the associations between bow-tie width and cat height?

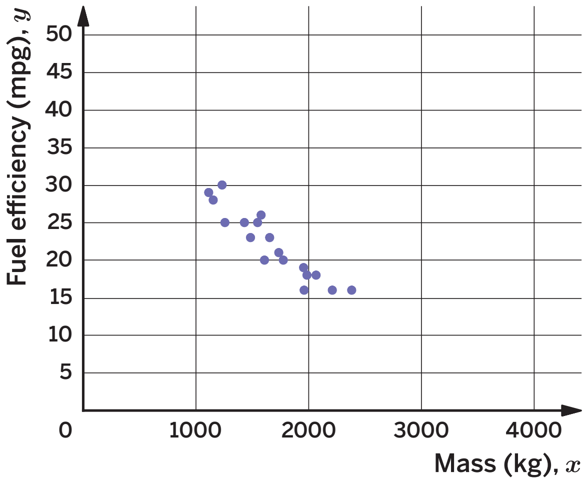
   Han: **Noah:**

3. What are the y-intercepts of the linear models? What do the y-intercepts represent in terms of cat height and bow-tie width?

   Han: **Noah:**

Activity 2 Interpreting a Negative Slope

The scatter plot shows the mass, in kilograms, and fuel efficiency, in miles per gallon, of 20 new cars.



1. What happens to the fuel efficiency as the mass increases? Describe any associations in the data.

2. Draw a line that models the data.

3. Write an equation for your line.

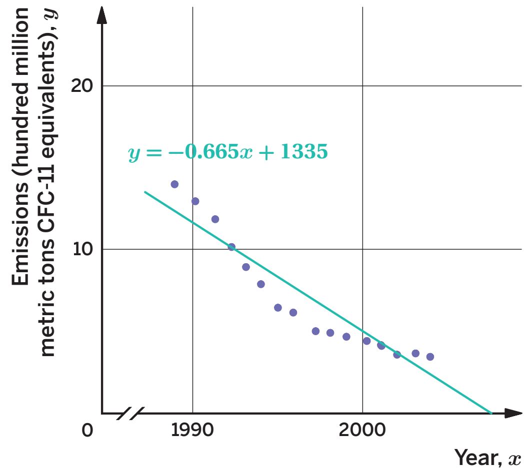
4. What is the value of the slope, and what does it mean in this context?

Name: ________ Date: ________ Period: ________

Activity 3 Two Truths and a Lie

Everyday human activities, such as using an older model refrigerator and driving a car, can emit harmful substances and affect the Earth. For each problem, use the linear model to help you identify the false statement.

1. Chlorofluorocarbons (CFCs), once used in almost all refrigeration and air conditioning, can be harmful to the ozone layer. The scatter plot shows emissions of the chemical CFC-11, with a linear model represented by equation $y = -0.665x + 1335$, where x represents the year and y represents the emissions measured in hundred million metric tons of CFC-11 equivalents. Of the following three statements, choose the one that is *false*.

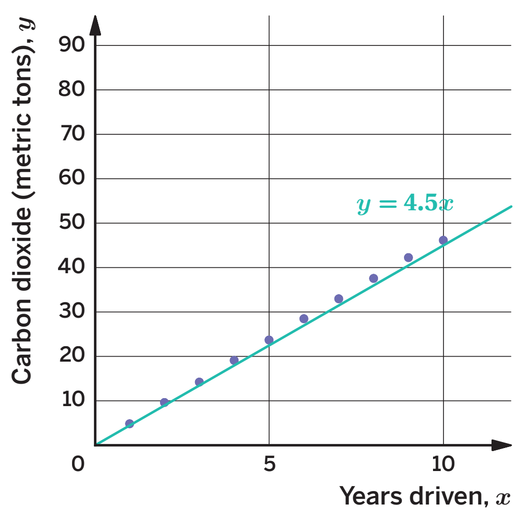


   A. For every 1 year increase, the emission is predicted to decrease by 0.665 hundred million metric tons.

   B. The slope of the linear model is $1{,}335$.

   C. There is a negative association between the years and amount of emissions.

2. This scatter plot shows the average carbon dioxide emissions of a typical passenger vehicle, with a linear model represented by the equation $y = 4.5x$, where x represents the number of years driven and y represents the average emissions of carbon dioxide in metric tons. Of the following three statements, choose the one that is *false*.



   A. A typical passenger vehicle emits about 4.5 metric tons of carbon dioxide each year.

   B. The linear model predicts a typical passenger vehicle will emit 50 metric tons of carbon dioxide in 5 years.

   C. There is a positive association between years driven and the amount of carbon dioxide emitted by a vehicle.

STOP

Summary

In today's lesson . . .

You investigated what the slope and y-intercept mean within the context of a real-world scenario.

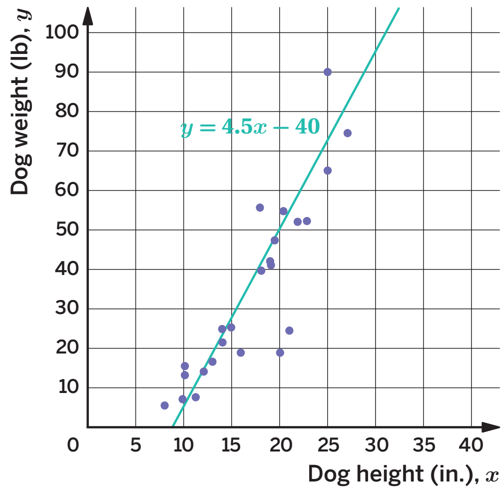
- If a scatter plot has a *positive linear association*, the slope of its linear model will be *positive*. For example, if the slope is 4, then for every increase of 1 unit of the independent variable, the model predicts that the dependent variable will increase by 4 units.
- If the scatter plot has a *negative linear association*, the slope of its linear model will be *negative*. For example, if the slope is -4, then for every increase of 1 unit of the independent variable, the model predicts the dependent variable will decrease by 4 units.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. The scatter plot shows the height and weight of 25 dogs, along with a linear model represented by the equation $y = 4.5x - 40$.

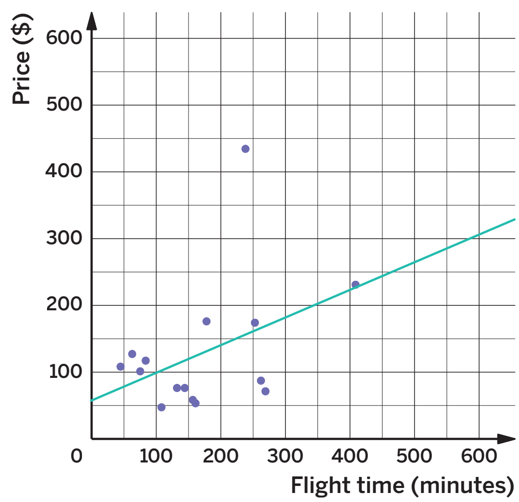


a What is the slope of the linear model?

b What information does the slope represent in terms of the dog's weight and height?

c Is the trend of the data positive or negative? Explain your thinking.

2. Nonstop, one-way flight times from Chicago's O'Hare Airport, as well as prices of a one-way ticket, are shown in the scatter plot.



a Circle any data points that appear to be outliers.

b Estimate the difference between any potential outliers and their predicted values.

3. Select *all* the relationships that demonstrate a negative association between variables.

A. The price of apples and the amount of apples you can buy for $\$10$.

B. The speed of a car and the distance the car has traveled.

C. The number of people in a check-out line and how long you have to wait to check out.

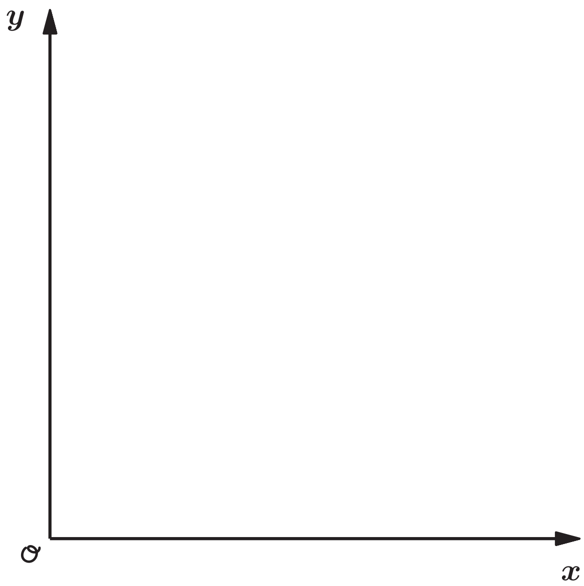
D. The time you spend running on a treadmill and the amount of calories you have burned.

E. The speed you have walked and the amount of time it takes to walk a certain distance.

Practice

Name: .... Date: .... Period: ....

4. Create a scatter plot with a positive, linear association that also has an outlier.



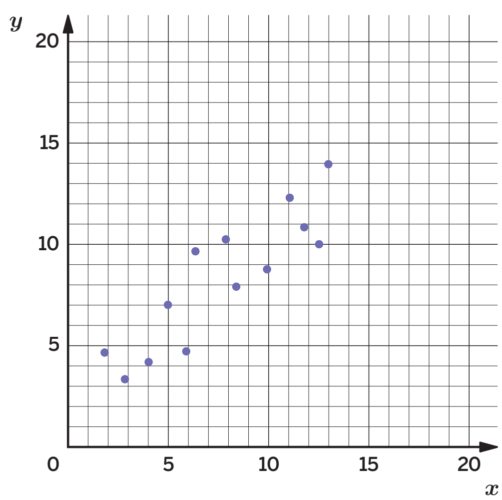
5. Write each expression as a single power of 10.

a $\dfrac{10^5 \cdot 10^8}{10^2}$

b $\dfrac{10^{18}}{10^7} \cdot 10^3$

c $\left(\dfrac{10^7}{10^3}\right)^6$

6. Refer to the scatter plot.



a Draw a line to model the data.

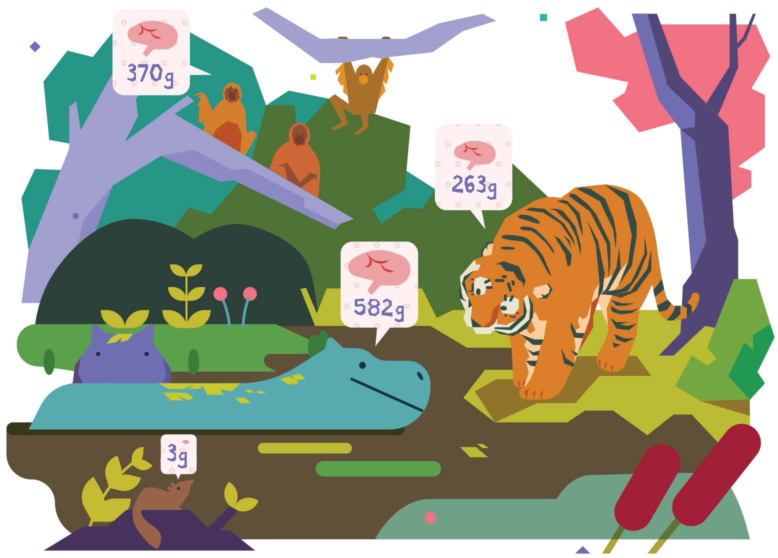
b Write an equation for your line.

c Use your linear model to predict the value of y when $x = 15$.

Name: ________ Date: ________ Period: ________

Unit 8 | Lesson 7

Analyzing Bivariate Data

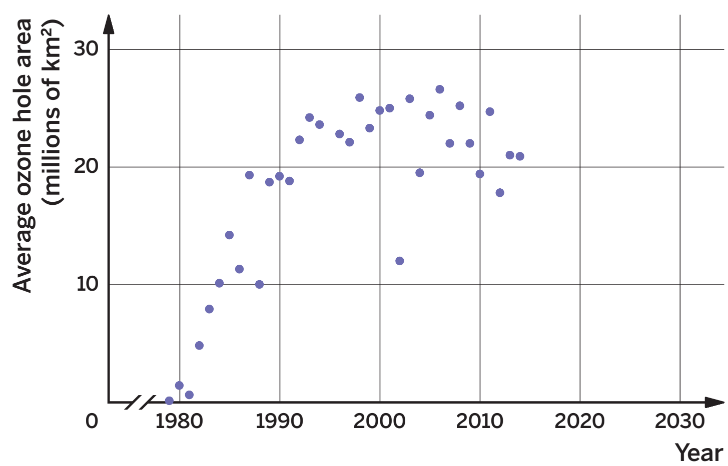


Let’s analyze data like a pro.

Warm-up Making a Prediction

In 1987, several countries agreed to take action and protect the ozone layer. The goal of their international treaty, called the Montreal Protocol, was to limit and eliminate human-made substances that harm the Earth.

The scatter plot shows the average size of the ozone hole over Antarctica over several years.



Make a prediction for the area of the ozone hole in the year 2030. Explain your thinking.

Activity 1 Animal Brains

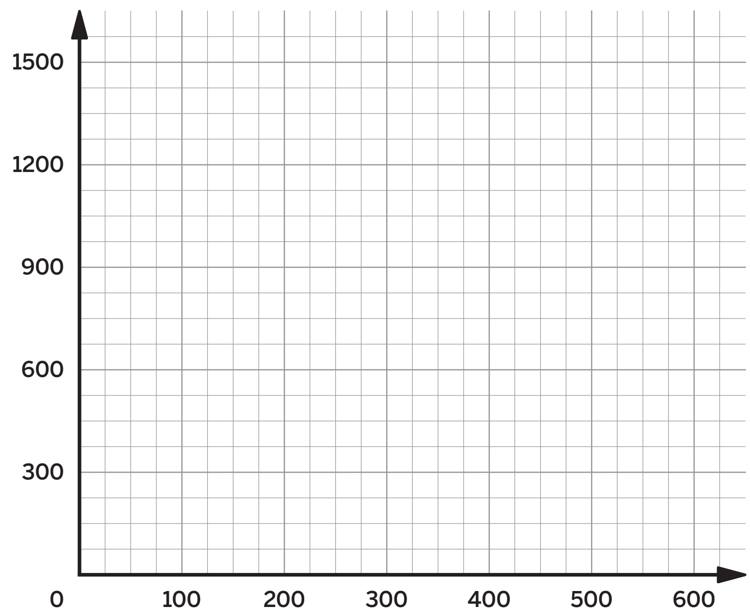
The table shows the data of body weight and brain weight for several animals. Study the table. You will refer to this table as you continue the activity on the next page.

Animal	Body weight (kg)	Brain weight (g)
Giraffe	529	680
Tiger	157	264
Goat	28	115
Cow	465	423
Grey Wolf	36	120
Potar Monkey	10	115
Cat	3	26
Rhesus Monkey	7	179
Sheep	56	175
Lion	159	240
Dog	10	
Pig	192	
Horse	521	

Name: .... Date: .... Period: ....

Activity 1 Animal Brains (continued)

1. Label the axes on the graph, and plot the points for the first ten animals.



2. Based on the scatter plot, choose the types of association you see between brain weight and body weight. Select *all* that apply.

A. Positive association

B. Negative association

C. No association

D. Linear association

E. Nonlinear association

3. Use the patterns in the scatter plot to help you plot three more points on the graph for the dog, pig, and horse. Write your predictions in the table on the previous page.

Activity 2 Drawing and Using a Linear Model

Use the graph from Activity 1 to complete these problems.

1. Draw a linear model for the data.

2. Use the linear model to predict the brain weight of a gorilla, a jaguar, and a human. Plot your points on the scatter plot, and then write your predictions in the table.

Animal	Body weight (kg)	Predicted brain weight (g)
Gorilla	207	
Jaguar	100	
Human	62	

Name: ______________________ Date: __________ Period: __________

Activity 3 What Does It Represent?

Use the graph from Activity 1 to complete the following problems. Write your predicted brain weights in the following table. Your teacher will reveal to you the actual brain weights for these animals.

Animal	Predicted brain weight (g)	Actual brain weight (g)
Gorilla		
Jaguar		
Human		

1. Plot the actual brain weights.

2. What do you notice about the human body weight and brain weight? How did your prediction compare to the actual weights?

3. Write an equation for your linear model.

4. What does your linear model's slope represent in this context?

5. What does your linear model's y-intercept represent in this context?

6. Choose one of the animals in the table and show how to determine the predicted brain weight using both the line of fit and the equation.

STOP

Summary

In today's lesson . . .

You used everything you have studied in the unit so far to analyze and interpret data in context. You plotted points on a scatter plot, identified potential outliers, and used a linear model to make predictions.

You compared actual and predicted values of body weights and brain weights for different animals and saw a positive, linear association. When you created and used a linear model, it helped you make more accurate predictions.

Reflect:

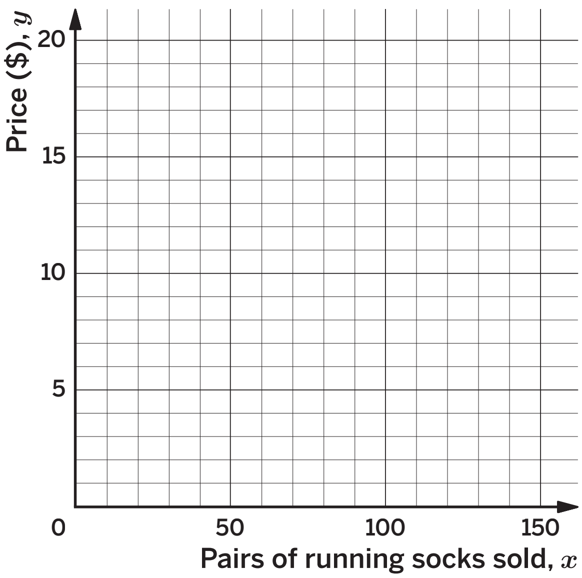
Practice

Name: ________ Date: ________ Period: ________

Have you ever wondered why different stores sell the same product at different prices? Stores base their retail prices on several factors, such as store location, consumer demand, and operational costs. In Problems 1–3, consider a pair of running socks sold at different stores across the country.

1. Refer to the table that shows the number of running socks sold, in pairs, and their price at 9 different stores. Plot the points to create a scatter plot of the data.

Pairs of running socks sold	Price ($)
53	11.25
60	10.50
30	12.10
81	8.45
70	9.25
80	9.75
120	7.25
37	12
130	9.99



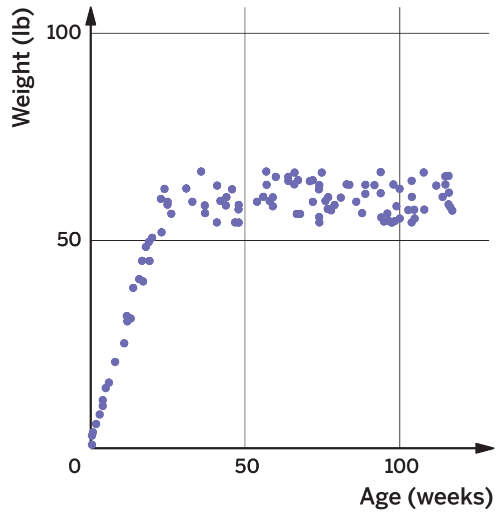
2. Draw a line to model the data. Then write an equation for your line.

3. Use your line or equation to predict the price if 100 pairs of running socks are sold. Explain your thinking.

Practice

Name: .... Date: .... Period: ....

4. Which of the following describes the association in the scatter plot? Select *all* that apply.



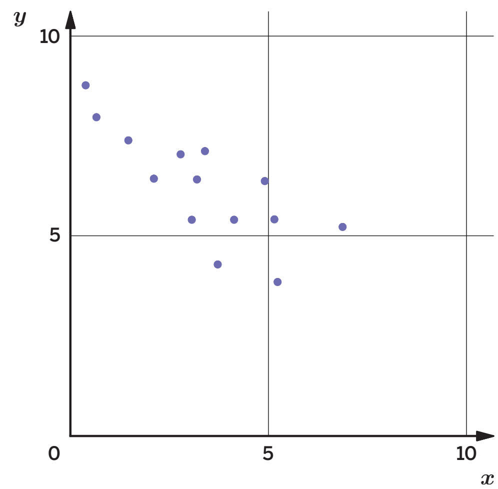
A. Linear association

B. Nonlinear association

C. Positive association

D. Negative association

5. Lin wants to draw a line to model the data. What type of slope would best fit this line?



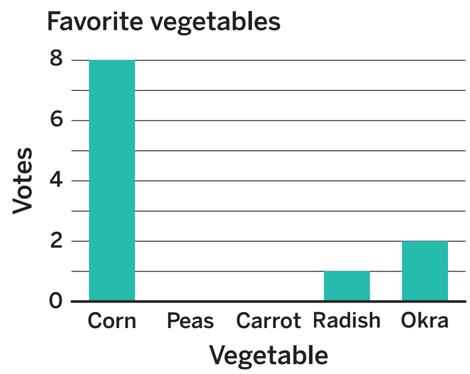
A. Positive slope

B. Negative slope

C. Zero slope

6. Complete the missing sections in the table as well as the missing bars in the bar graph so that they are displaying the same data.

Vegetable	Corn	Peas	Carrot	Radish	Okra
Number of votes	8	6	3		



Name: ______________________ Date: __________ Period: __________

Unit 8 | Lesson 8

Looking for Associations

Let's look for associations in data.

Warm-up Like Riding a Bike

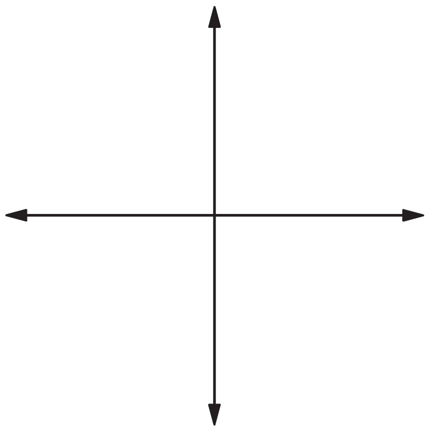
To determine a solution to the ozone crisis, individuals, companies, and governments had to make some difficult changes in their behavior. Similarly, as many communities around the world seek to reduce carbon emissions from automobiles, some individuals are deciding to ride bicycles to work instead of driving.

Consider these questions: How easy or difficult do you find riding a bicycle? Do you ride a bicycle often or rarely?

1. Plot a point on the scatter plot to identify your responses to these questions.

2. Plot a point to represent your partner's response on the same scatter plot.

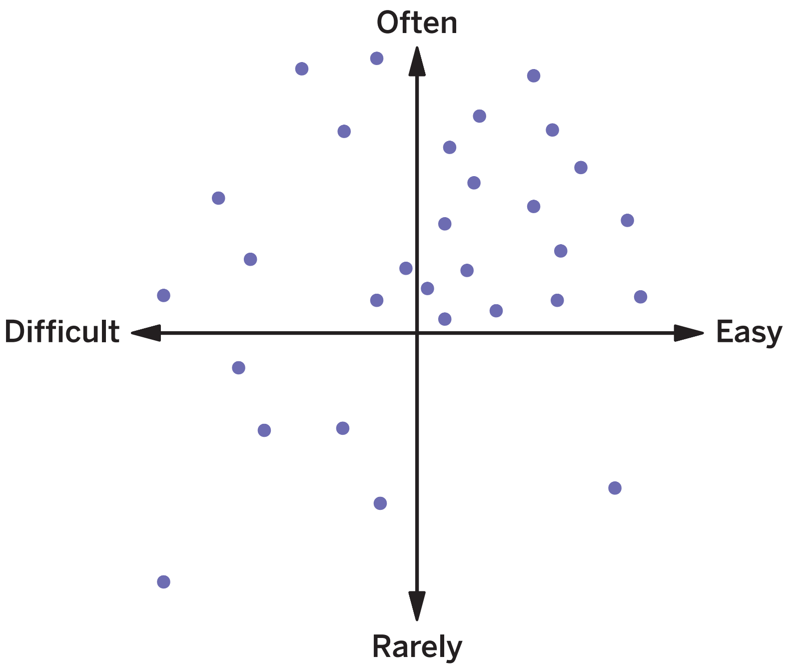
3. Describe how the scatter plot represents your responses.



Log in to Amplify Math to complete this lesson online.

Activity 1 Two-way Tables

Students in Mr. Diaz's class also responded to the same two questions about riding a bicycle. The students' responses are represented by the scatter plot shown.



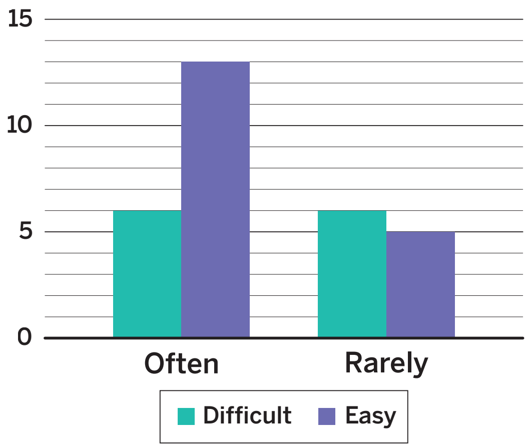
Complete the two-way table to match the data from the scatter plot. The total number of students, 30, has already been completed for you.

	Difficult	Easy	Total
Often			
Rarely			
Total			30

Name: ________ Date: ________ Period: ________

Activity 2 Bar Graphs

A different eighth-grade class collected data on the same two questions about riding a bicycle. This time, a student displayed the data using a different visual representation: a double bar graph. Complete the two-way table using the data from the double bar graph. The total number of students, 30, has already completed for you.



	Difficult	Easy	Total
Often			
Rarely			
Total			30

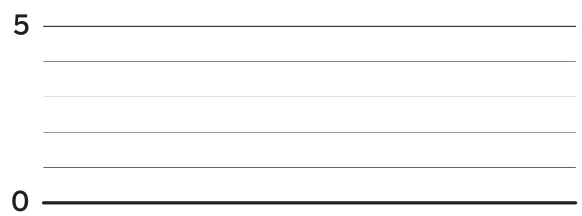
Are you ready for more?

Han analyzes the data above and says, "Students who find it easy to ride a bike are more likely to ride their bike often." Is he correct? Justify your response based on the double bar graph or two-way table.

Activity 3 Other Forms of Transportation

The two-way table represents how students in an eighth-grade classroom feel about riding a scooter. Use the two-way table to create a double bar graph that represents the data. Be sure to label the axes and include a key.

	Difficult	Easy	Total
Often	4	3	7
Rarely	2	1	3
Total	6	4	10



Are you ready for more?

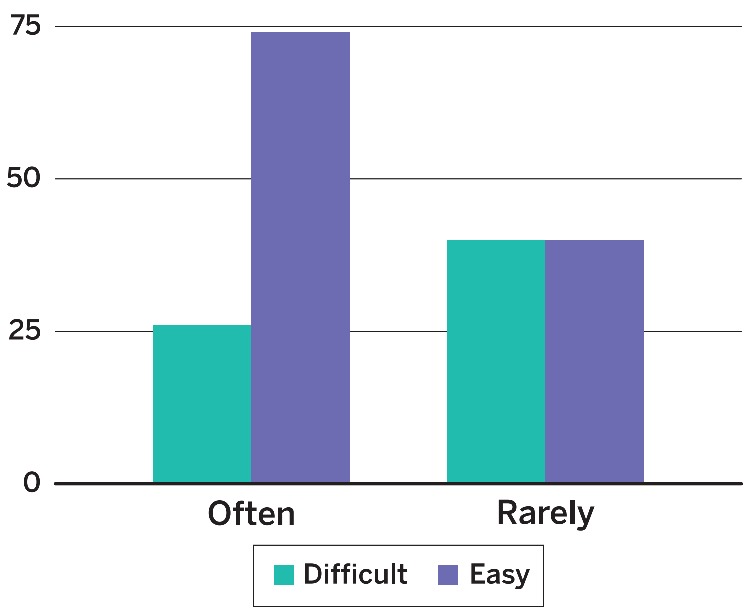
Jada is preparing a presentation for her class to discuss the trends in the data about riding a scooter. Which representation would you recommend she use: a double bar graph, a scatter plot, or a two-way table? Explain your thinking.

Name: ______________________ Date: __________ Period: __________

Activity 3 Other Forms of Transportation (continued)

Similar to riding a bicycle or a scooter, using public transportation, instead of driving a car, is another choice that can reduce carbon emissions. The double bar graph and two-way table represent how one school's eighth graders feel about using public transportation.

	Difficult	Easy	Total
Often	26	74	100
Rarely	40	40	80
Total	66	114	180



Co-craft Questions: Work with your partner to write 2–3 mathematical questions you could ask about the two-way table and double bar graph, before completing Problems 1 and 2.

1. What conclusions, if any, can you draw from the data set?

2. What are the advantages of each representation: two-way table and double bar graph? Explain your thinking.

STOP

Summary

In today's lesson . . .

You used scatter plots and ***two-way tables*** to explore associations in data. You can collect data by counting things in various categories, such as the number of students who find riding a bicycle difficult and the number of students who ride a bicycle often. Two-way tables, scatter plots, and double bar graphs can all be used to represent data. Each representation has advantages and disadvantages. You can use these representations to investigate possible connections between variables.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. The two-way table shows the number of students in a middle school who have a cell phone. What does the number 13 represent in the table?

	Has a cell phone	Does not have a cell phone	Total
10 or 11 years old	30	41	71
14 or 15 years old	65	13	78
Total	95	54	149

A. 13 students, who are 10 or 11 years old, do not have a cell phone.

B. 13 students, who are 14 or 15 years old, do not have a cell phone.

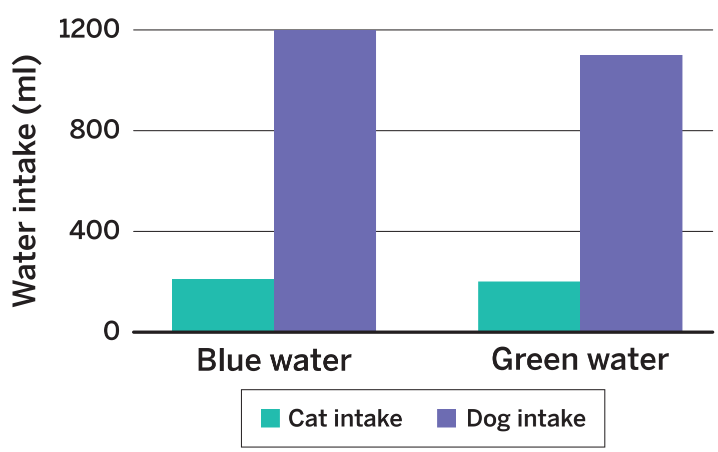
C. 13 students, who are 10 or 11 years old, have a cell phone.

D. 13 students, who are 14 or 15 years old, have a cell phone.

2. A scientist wants to know whether the color of water affects how much animals drink. The average amount of water each animal drank was recorded in milliliters for one week and then graphed using the bar graph shown.

Is there evidence to suggest an association between the color of the water and the type of animal?

	Cat intake (ml)	Dog intake (ml)	Total
Blue water	210	1,200	1,410
Green water	200	1,100	1,300
Total	410	2,300	2,710



Practice

Name: .... Date: .... Period: ....

3. A farmer brings his produce to the farmer's market and records whether people buy lettuce, apples, both, or something else. Complete the two-way table with the missing values. Then create a double bar graph to represent the data.

	Bought apples	Did not buy apples	Total
Bought lettuce	14		72
Did not buy lettuce		29	
Total	22		109

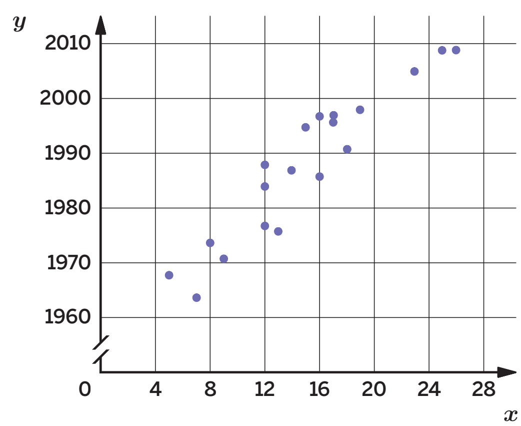
4. For the scatter plot shown, what is a reasonable slope of a linear model that fits the data?

A. -2.5

B. -1

C. 1

D. 2.5



5. The table shows the number of students in an eighth-grade class who like or do not like playing video games. What percent of the class likes playing video games?

	Likes	Does not like	Total
Number of students	15	10	25

A. 15%

B. 60%

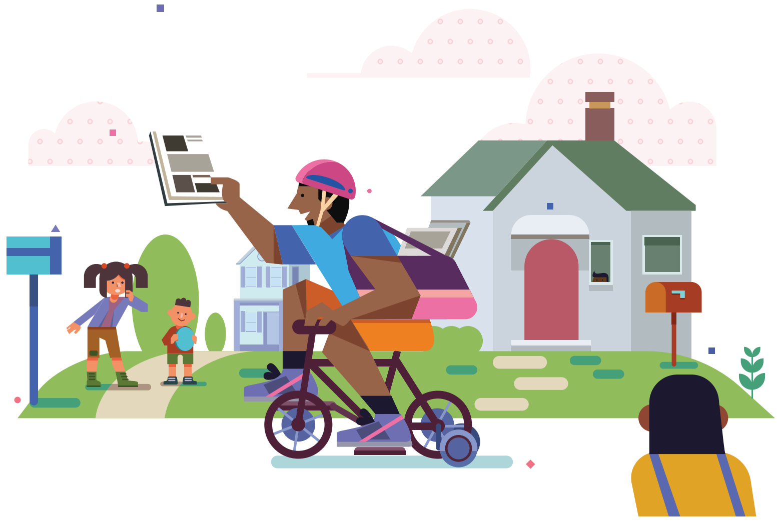
C. 75%

D. 40%

Name: .... Date: .... Period: ....

Unit 8 | Lesson 9 – Capstone

Using Data Displays to Find Associations



Let's use data displays to find associations.

Warm-up Headline News

The newspaper headline, "Adults are more likely to like riding a bicycle than kids," is based upon the table. Study both the headline and the table.

	Likes riding a bicycle	Does not like riding a bicycle
Kids	30	10
Adults	40	60

Based on the table, do you think the headline is accurate? Explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Relative Frequencies

Many statisticians, such as Professor Kimberly Sellers, analyze data that arises from counting (meaning the data takes on whole number values), finding associations and other patterns.

	Likes riding a bicycle	Does not like riding a bicycle
Kids	30	10
Adults	40	60

1. Based on the table, what percent of kids like riding a bicycle?

2. Based on the table, what percent of adults like riding a bicycle?

3. Based on the table, approximately what percent of people who *do not like riding a bicycle* are adults?

4. Revisit your response about the headline in the Warm-up. Do the relative frequencies you just calculated support your response?

5. How else might you represent the data to better identify associations?

Featured Mathematician

Kimberly Sellers

Kimberly Sellers is a Professor of Mathematic and Statistics at Georgetown University in Washington D.C., where she studies methods for analyzing count data, as well as mathematical techniques for automatically aligning images and finding features within them. She is a principal researcher at the U.S. Census Bureau, and is working to increase gender and racial diversity in mathematics and statistics.

Name: .... Date: .... Period: ....

Activity 2 Segmented Bar Graphs

As part of a survey, a class of eighth-grade students were asked whether they ride a bicycle and whether they use a reusable water bottle.

1. Some data from the survey is represented in the following tables. Complete the two-way and relative frequency tables so that they represent the same data.

Two-way table

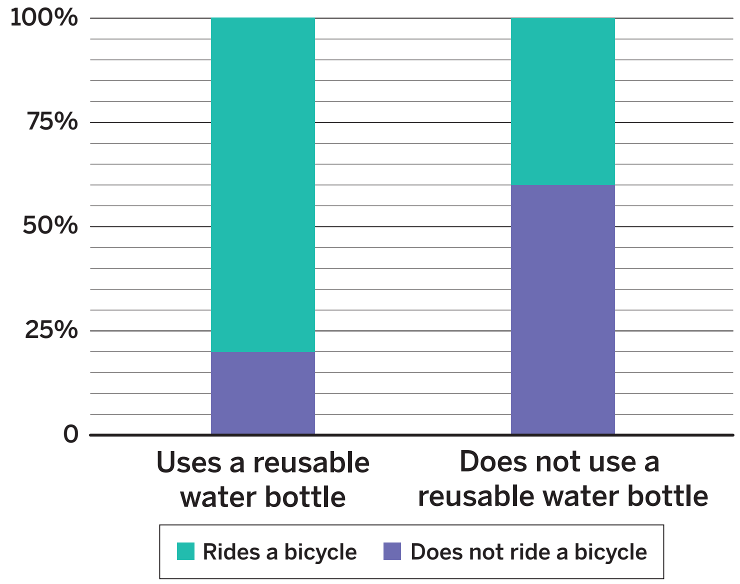
	Rides a bicycle	Does not ride a bicycle	Total
Uses a reusable water bottle			15
Does not use a reusable water bottle	4	6	10

Relative frequency table

	Rides a bicycle	Does not ride a bicycle	Total
Uses a reusable water bottle	80%		100%
Does not use a reusable water bottle			100%

Activity 2 Segmented Bar Graphs (continued)

2. The segmented bar graph shows the percentages from the relative frequency table. In your own words, how does the segmented bar graph represent the data?



3. Based on the different displays of data, do you think there is an association between using a reusable water bottle and riding a bicycle? Explain your thinking.

Name: .... Date: .... Period: ....

Activity 3 Frequency Tables and Segmented Bar Graphs

1. Complete the frequency table to represent the data from Activity 2, showing relative frequencies by column. Round to the nearest percent.

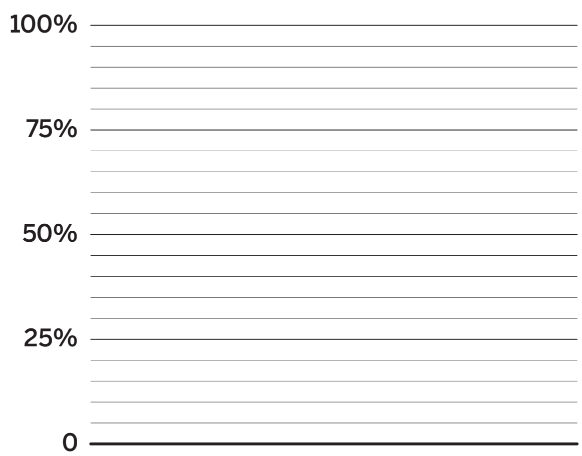
Two-way table

	Rides a bicycle	Does not ride a bicycle
Uses a reusable water bottle	12	3
Does not use a reusable water bottle	4	6
Total	16	9

Relative frequency table

	Rides a bicycle	Does not ride a bicycle
Uses a reusable water bottle	75%	
Does not use a reusable water bottle		
Total		

2. Using the values in the relative frequency table, create a segmented bar graph for each column of the table. Be sure to include labels and a key.

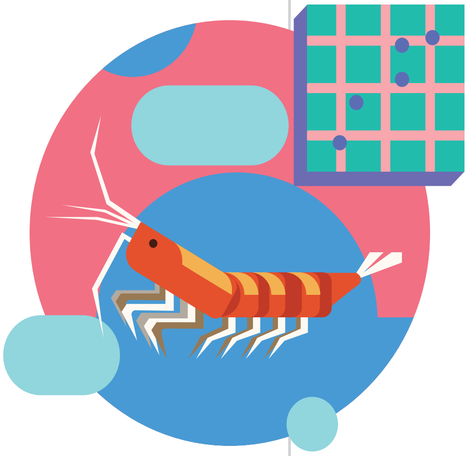


3. Compare your segmented bar graph to the segmented bar graph from Activity 2. What do you notice?

Reflect: How could you improve your performance in the activity? What would you change about what you did?

STOP

Unit Summary



In the late 1970s, the hole in the ozone layer was one of the biggest threats to our planet. Chemicals released into the atmosphere were stripping it away, letting in harmful ultraviolet radiation from the sun. This bombardment of UV radiation increased the incidence of deadly diseases and caused dramatic damage to the world's ecosystems.

If left unchecked, it could have spelled doom for our planet.

But in the end, our world wasn't saved by costumed heroes. It was protected by scientists and government officials, coming together under a spirit of cooperation.

In 1987, members of the United Nations signed the Montreal Protocol. Member nations pledged to phase out the use of ozone-depleting chemicals. Over the next few decades, the ozone layer began to stabilize. In some areas, the hole even closed.

This reversal wouldn't have been possible without passionate investigators. who pored over the global data. Using visualizations like scatter plots and two-way tables, they found connections between the data and created linear (and other) models that could predict change.

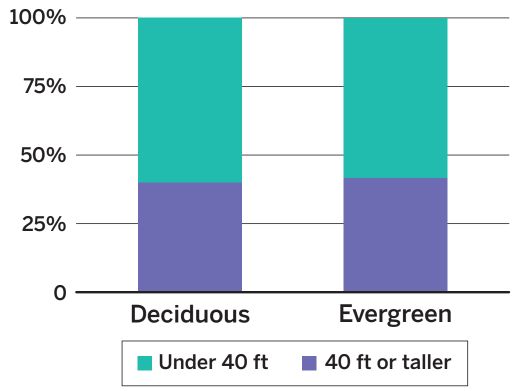
Through careful study of the data, these heroes saw the story in the numbers and rallied the world into making a difference.

See you next year.

Name: .... Date: .... Period: ....

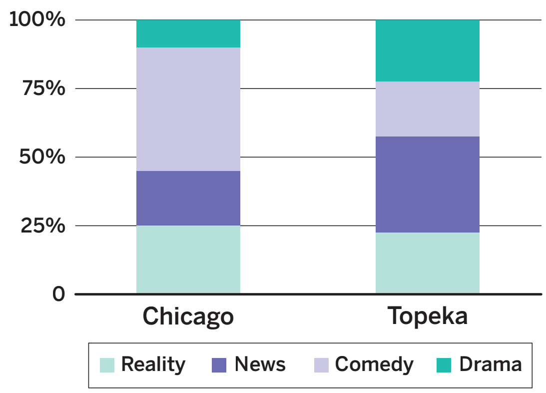
Practice

1. An ecologist is studying a forest with a mixture of tree types. Because the average tree height in the area is 40 ft, she measures the height of the tree against that. She also records the type of tree. The results are shown in the table and in the segmented bar graph. Is there evidence of an association between tree height and tree type? Explain your reasoning.



	Under 40 ft	40 ft or taller	Total
Deciduous	45	30	75
Evergreen	14	10	24
Total	59	40	99

2. Workers at an advertising agency are interested in people's TV-viewing habits. They survey people in two cities to try to determine patterns in the types of shows they watch. The results are recorded in a table and shown in a segmented bar graph. Is there evidence of different viewing habits? If so, explain.



	Reality	News	Comedy	Drama
Chicago	50	40	90	20
Topeka	45	70	40	45

Name: .... Date: .... Period: ....

3. A scientist is interested in whether two species of butterfly like certain types of local flowers. The scientist captures butterflies in two zones with different flower types and records the number caught. Do these data show an association between butterfly type and zone? Explain your reasoning.

	Zone 1	Zone 2
Eastern tiger swallowtail	16	34
Monarch	24	46

4. Create a segmented bar graph to show the relative frequencies of how likely people who meditated and did not meditate were to be calm or agitated. Be sure to label your graph and provide a key.

	Meditated	Did not meditate	Total
Calm	48	5	53
Agitated	23	21	44
Total	71	26	97

5. Students in a class were asked whether they play a sport and whether they play a musical instrument. Some of the results from the survey are shown in the two-way table. Complete the table, assuming all students responded to both questions.

	Plays instrument	Does not play instrument	Total
Plays sport	5		16
Does not play sport			
Total		15	25

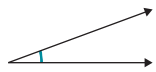
Glossary/Glosario

English

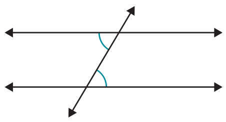
A

absolute value The value that represents the distance between a number and zero. For example, because the distance between -3 and 0 is 3, the absolute value of -3 is 3, or $|-3| = 3$.

acute angle An angle whose measure is less than 90 degrees.



alternate interior angles Alternate interior angles are created when a pair of parallel lines are intersected by a transversal. These angles lie inside the parallel lines and on opposite (alternate) sides of the transversal.

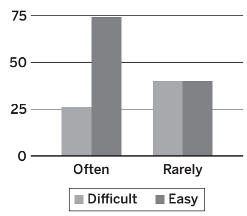


angle of rotation See the definition for *rotation*.

area The number of unit squares needed to fill a two-dimensional shape without gaps or overlaps.

B

bar graph A graph that presents data using rectangular bars that have heights proportional to the values that they represent.



bar notation Notation that indicates the repeated part of a repeating decimal. For example, $0.\overline{6} = 0.66666\ldots$.

base The number that is raised to a power. The power indicates the number of times the base is multiplied by itself.

C

center of dilation See the definition for *dilation*.

center of rotation See the definition for *rotation*.

circle A shape that is made up of all of the points that are the same distance from a given point.

circumference The distance around a circle.

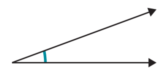
clockwise A rotation in the same direction as the way hands on a clock move is called a *clockwise* rotation.

Español

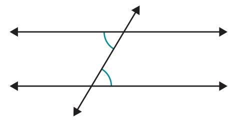
A

valor absoluto Valor que representa la distancia entre un número y cero. Por ejemplo, dado que la distancia entre -3 y 0 es 3, el valor absoluto de -3 es 3, o $|-3| = 3$.

ángulo agudo Ángulo cuya medida es menor que 90 grados.



ángulos interiores alternos Se crean ángulos interiores alternos cuando un par de líneas paralelas son intersecadas por una transversal. Estos ángulos están dentro de las líneas paralelas y en lados opuestos (alternos) de la transversal.

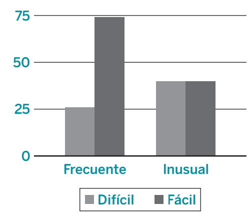


ángulo de rotación Ver *rotación*.

área Número de unidades cuadradas necesario para llenar una forma bidimensional sin dejar espacios vacíos ni superposiciones.

B

gráfica de barras Gráfica que presenta datos por medio de barras con alturas proporcionales a los valores que representan.



notación de barras Notación que indica la parte repetida de un número decimal periódico. Por ejemplo, $0.\overline{6} = 0.66666\ldots$.

base Número que se eleva a una potencia. La potencia indica el número de veces que la base debe multiplicarse por sí mismo.

C

centro de dilatación Ver *dilatación*.

centro de rotación Ver *rotación*.

círculo Forma constituida por todos los puntos que están a la misma distancia de un punto dado.

circunferencia Distancia alrededor de un círculo.

en el sentido de las agujas del reloj Una rotación en la misma dirección en que se mueven las agujas de un reloj es llamada una rotación *en el sentido de las agujas del reloj*.

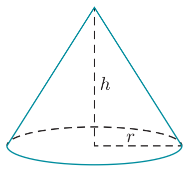
Glossary/Glosario

English

cluster A cluster represents data values that are grouped closely together.

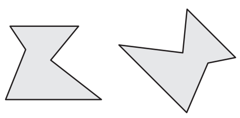
coefficient A constant by which a variable is multiplied, written in front of the variable. For example, in the expression $3x + 2y$, 3 is the coefficient of x.

cone A three-dimensional solid that consists of a circular base connected by a curved surface to a single point.



congruent Two figures are "congruent" to each other if one figure can be mapped onto the other by a sequence of rigid transformations.

congruent Two figures are *congruent* to each other if one figure can be mapped onto the other by a sequence of rigid transformations.



constant A value that does not change, meaning it is not a variable.

constant of proportionality The number in a proportional relationship that the value of one quantity is multiplied by to get the value of the other quantity.

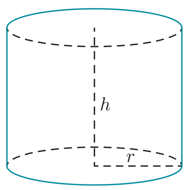
coordinate plane A two-dimensional plane that represents all the ordered pairs (x, y), where x and y can both represent on values that are positive, negative, or zero.

corresponding parts Parts of two scaled copies that match up, or "correspond" with each other. These corresponding parts could be points, segments, angles, or lengths.

counterclockwise A rotation in the opposite direction as the way hands on a clock move is called a *counterclockwise* rotation.

cube root The cube root of a positive number p is a positive solution to equations of the form $x^3 = p$. Write the cube root of p as $\sqrt[3]{p}$.

cylinder A three-dimensional solid that consists of two parallel, circular bases joined by a curved surface.

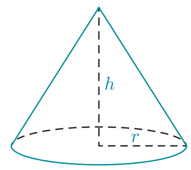


Español

agrupación Una agrupación representa valores de datos que se agrupan de manera cercana entre ellos.

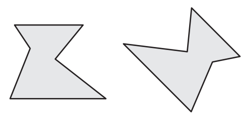
coeficiente Constante por la cual una variable es multiplicada, escrita frente a la variable. Por ejemplo, en la expresión $3x + 2y$, 3 es el coeficiente de x.

cono Sólido tridimensional compuesto de una base circular conectada a un solo punto por medio de una superficie curva.



congruente Dos figuras son "congruentes" si una de las figuras puede mapearse con la otra mediante una secuencia de transformaciones rígidas.

congruente Dos figuras son *congruentes* entre sí, si una figura puede adquirir la forma de la otra figura mediante una secuencia de transformaciones rígidas.



constante Valor que no cambia, lo que significa que no es una variable.

constante de proporcionalidad En una relación proporcional, el número por el cual el valor de una cantidad es multiplicado para obtener el valor de otra cantidad.

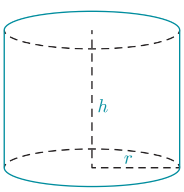
plano de coordenadas Plano bidimensional que representa todos los pares ordenados (x, y), donde tanto x como y pueden representar valores positivos, negativos o cero.

partes correspondientes Partes de dos copias a escala que coinciden, o "se corresponden", entre sí. Estas partes correspondientes pueden ser puntos, segmentos, ángulos o longitudes.

en el sentido contrario a las agujas del reloj Una rotación en la dirección opuesta a la forma en que las agujas de un reloj se mueven es llamada una rotación *en el sentido contrario a las agujas del reloj*.

raíz cúbica La raíz cúbica de un número positivo p es una solución positiva a las ecuaciones de la forma $x^3 = p$. Escribimos la raíz cúbica p como $\sqrt[3]{p}$.

cilindro Sólido tridimensional compuesto por dos bases paralelas y circulares unidas por una superficie curva.



D

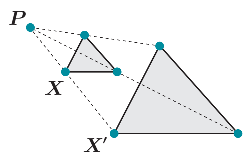
English

dependent variable The dependent variable represents the output of a function.

diagonal A line segment connecting two vertices on different sides of a polygon or polyhedra.

diameter The distance across a circle through its center. The line segment with endpoints on the circle, that passes through its center.

dilation A transformation defined by a fixed point P (called the *center of dilation*) and a scale factor k. The dilation moves each point X to a point X' along ray PX, such that its distance from P changes by the scale factor.



Distributive Property A property relating addition and multiplication: $a(b + c) = ab + ac$.

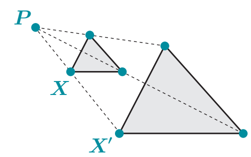
Español

variable dependiente La variable dependiente representa el resultado, o salida, de una función.

diagonal Segmento de línea que conecta dos vértices que están en lados diferentes de un polígono o de un poliedro.

diámetro Distancia que atraviesa un círculo por su centro. El segmento de línea cuyos extremos se ubican en el círculo y que pasa a través de su centro.

dilatación Transformación definida por un punto fijo P (llamado *centro de dilatación*) y un factor de escala k. La dilatación mueve cada punto X a un punto X' a lo largo del rayo PX, de manera tal que su distancia con respecto a P es cambiada por el factor de escala.



Propiedad distributiva Propiedad que relaciona la suma con la multiplicación: $a(b + c) = ab + ac$.

E

English

equation A mathematical statement that two expressions are equal.

equivalent If two mathematical objects (especially fractions, ratios, or expressions) are equal in any form, then they are equivalent.

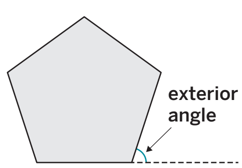
equivalent equations Equations that have the same solution or solutions.

equivalent expressions Two expressions whose values are equal when the same value is substituted into the variable for each expression.

exponent The number of times a factor is multiplied by itself.

expression A quantity that can include constants, variables, and operations.

exterior angle An angle between a side of a polygon and an extended adjacent side.



Español

ecuación Declaración matemática de que dos expresiones son iguales.

equivalente Si dos objetos matemáticos (especialmente fracciones, razones o expresiones) son iguales de cualquier manera, entonces son equivalentes.

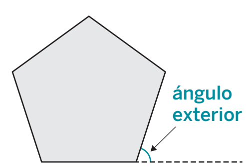
ecuaciones equivalentes Ecuaciones que tienen la misma solución o soluciones.

expresiones equivalentes Dos expresiones cuyos valores son iguales cuando se sustituye el mismo valor en la variable de cada expresión.

exponente Número de veces que un factor es multiplicado por sí mismo.

expresión Cantidad que puede incluir constantes, variables y operaciones.

ángulo exterior Ángulo que se encuentra entre un lado de un polígono y un lado extendido adyacente.



Glossary/Glosario

F

English

function A function is a rule that assigns exactly one output to each possible input.

Español

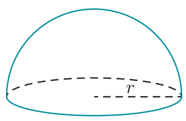
función Una función es una regla que asigna exactamente un resultado, o salida, a cada posible entrada.

H

English

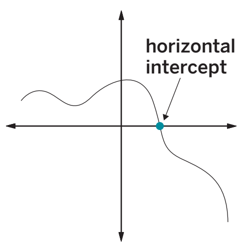
hanger diagram A model in which quantities are represented as weights attached to either side of a hanger. When the hanger is balanced, the sum of the quantities on either side must be equal.

hemisphere Half of a sphere.

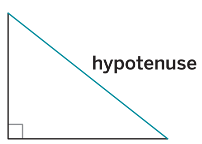


horizontal Running straight from left to right (or right to left).

horizontal intercept A point where a graph intersects the horizontal axis. Also known as the x-intercept, it is the value of x when y is 0.



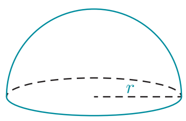
hypotenuse In a right triangle, the side opposite the right angle is called the hypotenuse.



Español

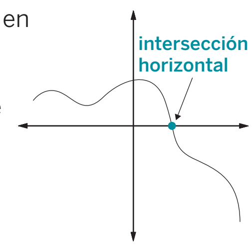
diagrama de colgador Modelo en el cual ciertas cantidades son representadas como pesos sujetos a cada lado de un colgador. Cuando el colgador está en equilibrio, la suma de las cantidades en cualquiera de los lados debe ser igual.

hemisferio La mitad de una esfera.

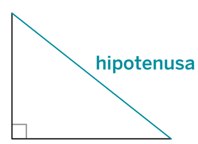


horizontal Que corre en línea recta de izquierda a derecha (o de derecha a izquierda).

intersección horizontal Punto en que una gráfica se interseca con el eje horizontal. Conocida también como intersección x, se trata del valor de x, cuando y es 0.



hipotenusa En un triangulo rectángulo, el lado opuesto al ángulo recto se llama la hipotenusa.



I

English

image A new figure that is created from an original figure (called the *preimage*) by a transformation.

independent variable The independent variable represents the input of a function.

initial value The starting amount in a context.

input The independent variable of a function.

integers Whole numbers and their opposites. For example, -4, 0, and 15 are whole numbers.

interior angle An angle between two adjacent sides of a polygon.

irrational number A number that is not rational. That is, an irrational number cannot be written as a fraction.

Español

imagen Nueva figura que se crea a partir de una figura original (llamada la *preimagen*) por medio de una transformación.

variable independiente La variable independiente representa la entrada de una función.

valor inicial Monto inicial en un contexto.

entrada La variable independiente de una función.

enteros Números completos y sus opuestos. Por ejemplo, -4, 0 y 15 son números enteros.

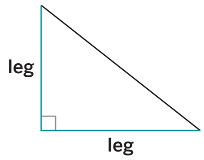
ángulo interior Ángulo que se encuentra entre dos lados adyacentes de un polígono.

número irracional Número que no es racional. Es decir, un número irracional no puede ser escrito como fracción.

English

L

legs The two sides of a right triangle that form the right angle.



like terms Parts of an expression that have the same variables and exponents. *Like terms* can be added or subtracted into a single term.

line of reflection See the definition for *reflection*.

linear association If a straight line can model the data, the data have a linear association.

linear function A linear relationship which assigns exactly one output to each possible input.

linear model A linear equation that models a relationship between two quantities.

linear relationship A relationship between two quantities in which there is a constant rate of change. When one quantity increases by a certain amount, the other quantity increases or decreases by a proportional amount.

long division A way to show the steps for dividing base ten whole numbers and decimals, dividing one digit at a time, from left to right.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{-2\;4} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

N

negative association A negative association is a relationship between two quantities where one tends to decrease as the other increases.

nonlinear association If a straight line cannot model the data, the data have a nonlinear association.

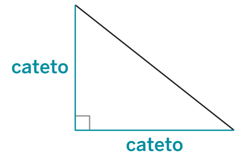
nonlinear function A function that does not have a constant rate of change. Its graph is not a straight line.

nonproportional relationship A relationship between two quantities in which the corresponding values do not have a constant ratio. (In other words, a relationship that is not a proportional relationship.)

Español

L

catetos Los dos lados de un triángulo rectángulo que componen el ángulo recto.



términos similares Partes de una expresión que tienen las mismas variables y exponentes. Los *términos similares* pueden ser reducidos a un solo término mediante su suma o resta.

línea de reflexión Ver *reflexión*.

asociación lineal Si una línea recta puede modelar los datos, los datos tienen una asociación lineal.

función lineal Relación lineal que asigna exactamente un resultado, o salida, a cada entrada posible.

modelo lineal Ecuación lineal que modela una relación entre dos cantidades.

relación lineal Relación entre dos cantidades en la cual existe una tasa de cambio constante. Cuando una cantidad aumenta un cierto monto, la otra cantidad aumenta o disminuye en un monto proporcional.

división larga Forma de mostrar los pasos necesarios para dividir números enteros en base diez y decimales, por medio de la división de un dígito a la vez, de izquierda a derecha.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{-2\;4} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

N

asociación negativa Una asociación negativa es una relación entre dos cantidades, en la cual una tiende a disminuir a medida que la otra aumenta.

asociación no lineal Si una línea recta no puede modelar los datos, los datos tienen una asociación no lineal.

función no lineal Función que no tiene un índice constante de cambio. Su gráfica no es una línea recta.

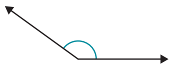
relación no proporcional Relación entre dos cantidades, en la cual los valores correspondientes no tienen una razón constante. (En otras palabras, una relación que no es una relación proporcional.)

Glossary/Glosario

O

English

obtuse angle An angle that measures more than 90 degrees.



order of operations When an expression has multiple operations, they are applied in a consistent order (the *order of operations*) so that the expression is evaluated the same way by everyone.

ordered pair Two values x and y, written as (x, y), that represent a point on the coordinate plane.

orientation The arrangement of the vertices of a figure before and after a transformation. A figure's orientation changes when it is reflected across a line.

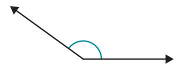
origin The point represented by the ordered pair $(0, 0)$ on the coordinate plane. The *origin* is where the x- and y-axes intersect.

outlier Outliers are points that are far away from their predicted values.

output The dependent variable of a function.

Español

ángulo obtuso Ángulo que mide más de 90 grados.



orden de las operaciones Cuando una expresión tiene múltiples operaciones, estas se aplican en cierto orden consistente (el *orden de las operaciones*) de manera que la expresión sea evaluada de la misma manera por todas las personas.

par ordenado Dos valores x y y, escritos como (x, y), que representan un punto en el plano de coordenadas.

orientación El arreglo de los vertices de una figura antes y después de una transformación. La orientación de una figura cambia cuando esta es reflejada con respecto de una línea.

origen Punto representado por el par ordenado $(0, 0)$ en el plano de coordenadas. El *origen* es donde los ejes x y y se intersecan.

valor atípico Los valores atípicos son puntos que están muy lejos de sus valores predichos.

resultado o salida Variable dependiente de una función.

P

English

perfect cube A number that is the cube of an integer. For example, 8 is a perfect cube because $2^3 = 8$.

perfect square A number that is the square of an integer. For example, 16 is a perfect square because $4^2 = 16$.

pi The ratio of the circumference of a circle to its diameter. It is usually represented by π.

piecewise function A function that is defined by two or more equations. Each equation is valid for some interval.

polygon A closed, two-dimensional shape with straight sides that do not cross each other.

Español

cubo perfecto Número que es el cubo de un número entero. Por ejemplo, 8 es un cubo perfecto porque $2^3 = 8$.

cuadrado perfecto Número que es el cuadrado de un número entero. Por ejemplo, 16 es un cuadrado perfecto porque $4^2 = 16$.

pi Razón entre la circunferencia y el diámetro de un círculo. Usualmente se representa como π.

función por partes Función definida por dos o más ecuaciones. Cada ecuación es válida para alguno de los intervalos.

polígono Forma cerrada y bidimensional de lados rectos que no se entrecruzan.

English	Español
positive association A positive association is a relationship between two quantities where one tends to increase as the other increases.	**asociación positiva** Una asociación positiva es una relación entre dos cantidades, en la cual una tiende a aumentar a medida que la otra disminuye.
preimage See the definition of *image*.	**preimagen** Ver *imagen*.
prime notation A labeling notation that uses a tick mark. *Prime notation* is typically applied to an image, to tell it apart from its preimage.	**notación prima** Notación para etiquetar que usa un signo de prima. Una *notación prima* usualmente se aplica a una imagen, para distinguirla de su preimagen.
Properties of Equality Rules that apply to all equations. These include properties of addition, subtraction, multiplication, and division, which state that if an equation is true, then applying the same operation to both sides will give a new equation that is also true.	**Propiedades de igualdad** Reglas que son aplicables a todas las ecuaciones. Incluyen las propiedades de suma, resta, multiplicación y división, las cuales señalan que si una ecuación es verdadera, al aplicar la misma operación a ambos lados se obtendrá una nueva ecuación que también es verdadera.
proportional relationship A relationship in which the values for one quantity are each multiplied by the same number (the *constant of proprtionality*) to get the values for the other quantity.	**relación proporcional** Relación en la que los valores de una cantidad se multiplican cada uno por el mismo número (la *constante de proporcionalidad*) para obtener los valores de la otra cantidad.
Pythagorean Theorem The Pythagorean Theorem states that, for any right triangle, leg^2 + leg^2 = hypotenuse^2. Sometimes this can be presented as $a^2 + b^2 = c^2$, where a and b represent the length of the legs and c represents the length of the hypotenuse.	**Teorema de Pitágoras** El Teorema de Pitágoras establece que para todo triángulo rectángulo: cateto^2 + cateto^2 = hipotenusa^2. A veces puede ser también presentado como $a^2 + b^2 = c^2$, donde a y b representan las longitudes de los catetos y c representa la longitud de la hipotenusa.
Pythagorean triple Three positive integers a, b, and c, such that $a^2 + b^2 = c^2$.	**Triplete pitagórico** Tres enteros positivos a, b y c, tales como $a^2 + b^2 = c^2$.

Q

English	Español
quadrilateral A polygon with exactly four sides.	**cuadrilátero** Polígono de exactamente cuatro lados.

Glossary/Glosario

English

R

radius A line segment that connects the center of a circle with a point on the circle. The term can also refer to the length of this segment.

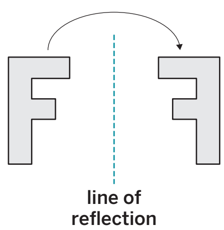
rate of change The amount one quantity (often y) changes when the value of another quantity (often x) increases by 1. The *rate of change* in a linear relationship is also the slope of its graph.

ratio A comparison of two quantities by multiplication or division.

rational numbers The set of all the numbers that can be written as positive or negative fractions.

rectangular prism A polyhedron with two congruent and parallel bases, whose faces are all rectangles.

reflection A transformation that flips each point on a preimage across a *line of reflection* to a point on the opposite side of the line.

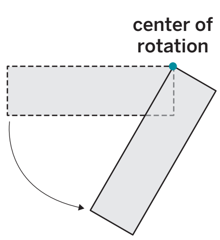


relative frequency The relative frequency is the ratio of the number of times an outcome occurs in a set of data. It can be written as a fraction, a decimal, or a percentage.

repeating decimal A decimal in which there is a sequence of non-zero digits that repeat indefinitely.

rigid transformation A move that does not change any of the measurements of a figure. Translations, rotations, and reflections are all examples of *rigid transformations* (as well as any sequence of these).

rotation A transformation that turns a figure a certain angle (called the *angle of rotation*) about a point (called the *center of rotation*).



Español

radio Segmento de línea que conecta el centro de un círculo con cualquier punto del círculo. El término puede también referirse a la longitud de este segmento.

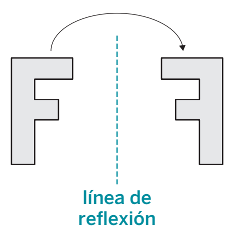
tasa de cambio Monto en que una cantidad (usualmente y) cambia cuando el valor de otra cantidad (usualmente x) aumenta en un factor de 1. La *tasa de cambio* en una relación lineal es también la pendiente de su gráfica.

razón Comparación de dos cantidades a través de una multiplicación o una división.

números racionales Conjunto de todos los números que pueden ser escritos como fracciones positivas o negativas.

prisma rectangular Poliedro con dos bases congruentes y paralelas, cuyas caras son todas rectángulos.

reflexión Transformación que hace girar cada punto de una preimagen a lo largo de una *línea de reflexión* hacia un punto en el lado opuesto de la línea.

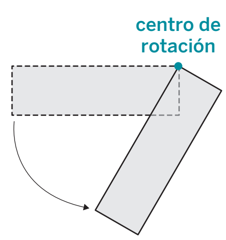


frecuencia relativa La frecuencia relativa es la razón del número de veces que ocurre un resultado en un conjunto de datos. Se puede escribir como una fracción, un decimal o un porcentaje.

número decimal periódico Decimal que tiene una secuencia de dígitos diferentes de cero que se repite de manera indefinida.

transformación rígida Movimiento que no cambia medida alguna de una figura. Traslaciones, rotaciones y reflexiones son ejemplos de *transformaciones rígidas* (como también cualquier secuencia de estas transformaciones).

rotación Transformación que hace girar una figura en cierto ángulo (llamado *ángulo de rotación*) alrededor de un punto (llamado *centro de rotación*).



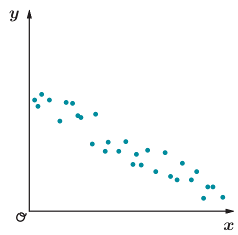
English | Español

S

scale factor The value that side lengths are multiplied by to produce a certain scaled copy.

scaled copy A copy of a figure where every length in the original figure is multiplied by the same value to produce corresponding lengths in the copy.

scatter plot A scatter plot is a graph that shows the values of two variables on a coordinate plane. It allows us to investigate connections between the two variables.

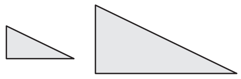


scientific notation A way of writing very large or very small numbers. When a number is written in scientific notation, the first factor is a number greater than or equal to one, but less than ten. The second factor is an integer power of ten. For example, $23000 = 2.3 \times 10^4$ and $0.00023 = 2.3 \times 10^{-4}$.

segmented bar graph A segmented bar graph compares two categories within a data set. The whole bar represents all the data within one category. Then, each bar is separated into parts (segments) that show the percentage of each part in the second category.

sequence of transformations Two or more transformations that are performed in a particular order.

similar Two figures are *similar* if they can be mapped onto each other by a sequence of transformations, including dilations.

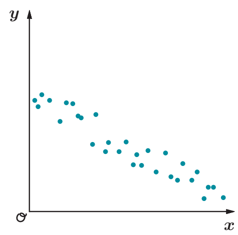


slope The numerical value that represents the ratio of the vertical side length to the horizontal side length in a slope triangle. The rate of change in a linear relationship is also the slope of its graph.

factor de escala Valor por el cual las longitudes de cada lado son multiplicadas para producir una cierta copia a escala.

copia a escala Copia de una figura donde cada longitud de la figura original es multiplicada por el mismo valor, para producir longitudes correspondientes en la copia.

diagrama de dispersión Un diagrama de dispersión es una gráfica que muestra los valores de dos variables en un plano de coordenadas. Nos ayuda a investigar relaciones entre las dos variables.

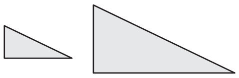


notación científica Manera de escribir números muy grandes o números muy pequeños. Cuando un número es escrito en notación científica, el primer factor es un número mayor o igual a uno, pero menor que diez. El segundo factor es un número entero que es potencia de diez. Por ejemplo, $23000 = 2.3 \times 10^4$ y $0.00023 = 2.3 \times 10^{-4}$.

gráfica de barras segmentada Una gráfica de barras segmentada compara dos categorías dentro de una serie de datos. La barra completa representa la totalidad de los datos dentro de una categoría. Entonces, cada barra es separada en partes (llamadas segmentos) que muestran el porcentaje de cada parte en la segunda categoría.

secuencia de transformaciones Dos o más transformaciones que se llevan a cabo en un orden particular.

similar Dos figuras son *similares* si pueden ser imagen la una de la otra, mediante una secuencia de transformaciones que incluyen las dilataciones.



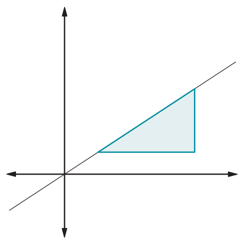
pendiente El valor numérico que representa la razón entre la longitud del lado vertical y la longitud del lado horizontal en un triángulo de pendiente. Dada una línea, todo triángulo de pendiente tiene la misma pendiente.

Glossary/Glosario

English	Español

English

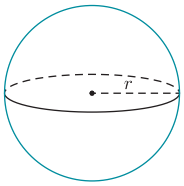
slope triangle A right triangle whose longest side is part of a line, and whose other sides are horizontal and vertical. *Slope triangles* can be used to calculate the slope of a line.



solution A value that makes an equation true.

solution to a system of equations An ordered pair that makes every equation in a system of equations true.

sphere A three-dimensional figure that consists of the set of points, in space, that are the same distance from a given point called the center.



square root The square root of a positive number p is a positive solution to equations of the form $x^2 = p$. Write the square root of p as $\sqrt{p}$.

straight angle An angle that forms a straight line. A straight angle measures 180 degrees.

substitution Replacing an expression with another expression that is known to be equal.

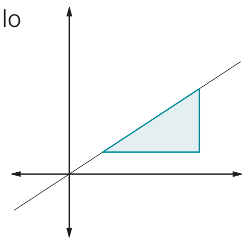
supplementary angles Two angles whose measures add up to 180 degrees.

symmetry When a figure can be transformed in a certain way so that it returns to its original position, it is said to have *symmetry*, or be *symmetric*.

system of equations A set of two equations with two variables. (In a later course, you will see systems with more than two equations and variables.)

Español

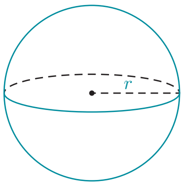
triángulo de pendiente Triángulo rectángulo cuyo lado más largo es parte de una línea, y cuyos otros lados son horizontales y verticales. Los *triángulos de pendiente* pueden ser usados para calcular la pendiente de una línea.



solución Valor que hace verdadera a una ecuación.

solución al sistema de ecuaciones Par ordenado que hace verdadera cada ecuación de un sistema de ecuaciones.

esfera Figura tridimensional que consiste en una serie de puntos en el espacio que están a la misma distancia de un punto específico, llamado centro.



raíz cuadrada La raíz cuadrada de un número positivo p es una solución positiva a las ecuaciones de la forma $x^2 = p$. Escribimos la raíz cuadrada de p como $\sqrt{p}$.

ángulo llano Ángulo que forma una línea recta. Un ángulo llano mide 180 grados.

sustitución Reemplazo de una expresión por otra expresión que se sabe es equivalente.

ángulos suplementarios Dos ángulos cuyas medidas suman 180 grados.

simetría Cuando una figura puede ser transformada de manera tal que regrese a su posición original, se dice que tiene *simetría* o que es *simétrica*.

sistema de ecuaciones Conjunto de dos ecuaciones con dos variables. (En un curso posterior verán sistemas con más de dos ecuaciones y variables.)

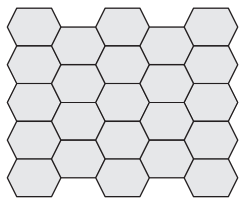
English

T

term An expression with constants or variables that are multiplied or divided.

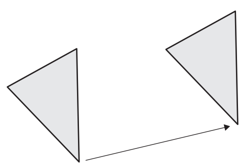
terminating decimal A decimal that ends in 0s.

tessellation A pattern made of repeating shapes that completely covers a plane, without any gaps or overlaps.

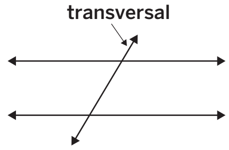


transformation A rule for moving or changing figures on the plane. Transformations include translations, reflections, and rotations.

translation A transformation that slides a figure without turning it. In a *translation*, each point of the figure moves the same distance in the same direction.



transversal A line that intersects two or more other lines.



Triangle Sum Theorem A theorem that states the sum of of the three interior angles of any triangle is 180 degrees.

two-way table A two-way table provides a way to compare two categorical variables. It shows one of the variables across the top and the other down one side. Each entry in the table is the frequency or relative frequency of the category shown by the column and row headings.

U

unit rate How much one quantity changes when the other changes by 1.

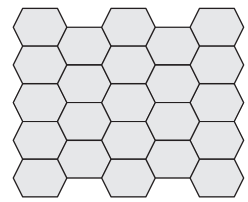
Español

T

término Expresión con constantes o variables que son multiplicadas o divididas.

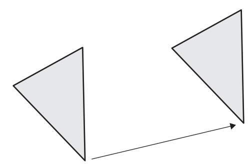
decimal exacto Un decimal que termina en ceros.

teselado Patrón compuesto por formas repetidas que cubren por completo un plano, sin dejar espacios vacíos ni superposiciones.

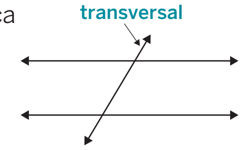


transformación Regla que se aplica al movimiento o al cambio de figuras en el plano. Traslaciones, rotaciones y reflexiones son ejemplos de transformaciones.

traslación Transformación que desliza una figura sin hacerla girar. En una *traslación* cada punto de la figura se mueve la misma distancia en la misma dirección.



transversal Línea que se interseca con dos o más líneas distintas.



Teorema de la suma del triángulo Teorema que afirma que la suma de los tres ángulos interiores de cualquier triángulo es 180 grados.

tabla de dos entradas Una tabla de dos entradas provee una forma de comparar dos variables categóricas. Muestra una de las variables de forma horizontal y la otra de forma vertical. Cada entrada en la tabla es la frecuencia o frecuencia relativa de la categoría mostrada en los encabezados de la columna y la fila.

U

tasa unitaria Cuánto cambia cierta cantidad cuando la otra cambia por un factor de 1.

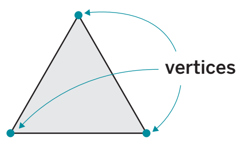
Glossary/Glosario

English

V

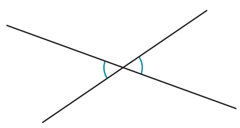
variable A quantity that can take on different values, or that has a single unknown value. Variables are typically represented using letters.

vertex A point where two sides of a two-dimensional shape or two or more edges of a three-dimensional figure intersect. (The plural of *vertex* is *vertices*.)

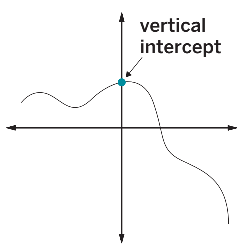


vertical Running straight up or down.

vertical angles Opposite angles that share the same vertex, formed by two intersecting lines. Vertical angles have equal measures.



vertical intercept A point where a graph intersects the vertical axis. Also known as the y-intercept, it is the value of y when x is 0.



volume The number of unit cubes needed to fill a three-dimensional figure without gaps or overlaps.

X

x-intercept See the definition for *horizontal intercept*.

Y

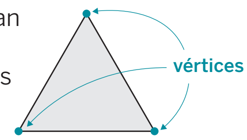
y-intercept See the definition for *vertical intercept*.

Español

V

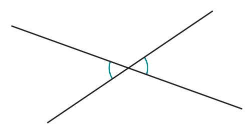
variable Cantidad que puede asumir diferentes valores o que tiene un solo valor desconocido. Las variables usualmente son representadas por letras.

vértice Punto donde se intersecan dos lados de una forma bidimensional, o dos o más aristas de una figura tridimensional.

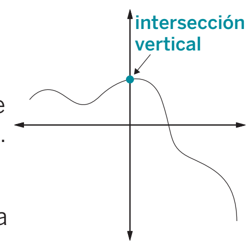


vertical Que corre en línea recta hacia arriba o hacia abajo.

ángulos verticales Ángulos opuestos que comparten el mismo vértice, conformado por dos líneas que se intersecan. Los ángulos verticales tienen las mismas medidas.



intersección vertical Punto en que una gráfica se interseca con el eje vertical. También conocida como intersección y, se trata del valor de y cuando x es 0.



volumen Número de unidades cúbicas necesario para llenar una figura tridimensional sin dejar espacios vacíos ni superposiciones.

X

intersección x Ver *intersección horizontal*.

Y

intersección y Ver *intersección vertical*.

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