Amplify Math

Grade 6

Volume 1: Units 1–4

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify Math is based on the Illustrative Mathematics (IM) curriculum. IM 6–8 Math™ was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is © 2017–2019 Open Up Resources. Additional adaptations and updates to IM 6–8 Math™ are © 2019 Illustrative Mathematics. IM 9–12 Math™ is © 2019 Illustrative Mathematics. IM 6–8 Math™ and IM 9–12 Math™ are licensed under the Creative Commons Attribution 4.0 International license (CC BY 4.0). Additional modifications contained in Amplify Math are © 2020 Amplify Education, Inc. and its licensors. Amplify is not affiliated with the Illustrative Mathematics organization.

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Dear Teacher,

Thank you for choosing Amplify Math. Our team has worked hard to ensure you have strong, easy-to-follow instructional supports that get all students talking and thinking together about grade-level mat.

You can learn more about the program design in the pages that follow, but we wanted to call out a few things that really drive our point home about the student experience:

Make math social

The student experience is **social and collaborative**. Your students will have plenty of opportunities to talk through their reasoning and work with their peers to come to new understandings. Thanks to our **partnership with Desmos**, you can kick off these social math experiences both offline and while logged in.

Power-ups

Your priority is teaching grade-level math to each and every student in your classroom, every day. To help with that, we've developed **Power-ups** to provide just-in-time support for your students.

Narrative

We kick off each sub-unit with a short, **engaging narrative** about historical or current-day events or phenomena. That way, your students can see the relevance of math in their everyday lives.

Featured Mathematicians

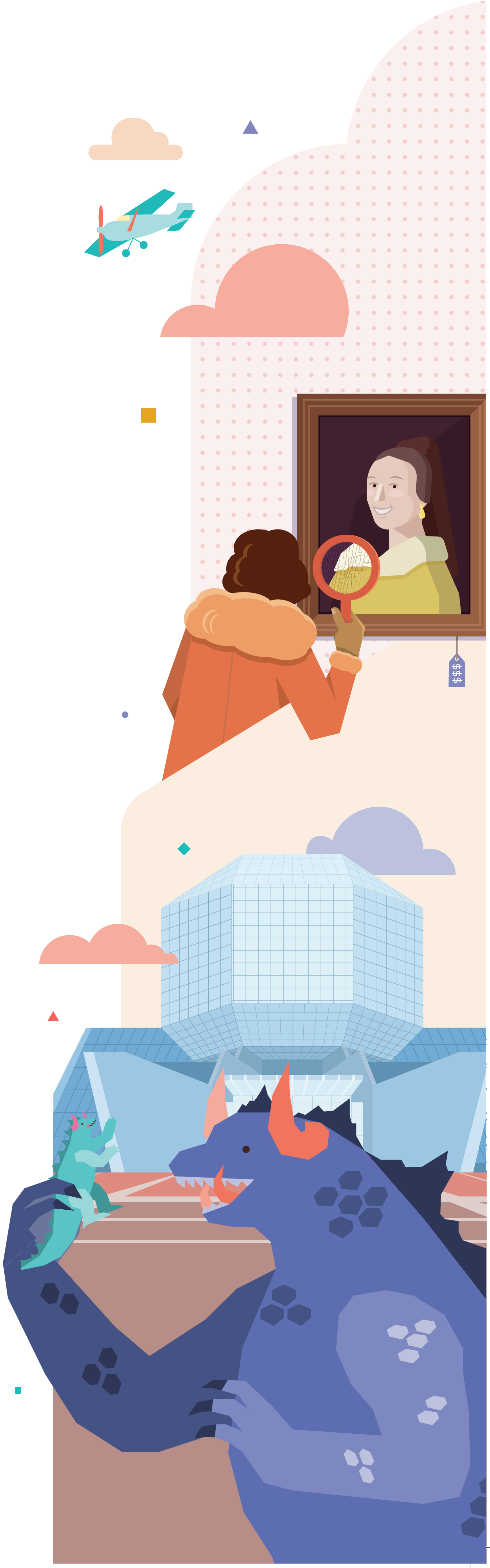
It's important to us that students see themselves in our materials. To that end, we've woven in **the work of innovative mathematical thinkers**. We've also included some of their personal stories, so that students can see themselves mirrored in the living history of mathematics.

Data

We provide plenty of **data to help you drive your instruction** and talk about student performance with your colleagues, as well as with caregivers who may not have had the best experiences with math when they were in school.

We hope you see the quality work of our team of editors and advisors in the program.

Sincerely,
The Amplify Math Team



Acknowledgments

Program Advisors

Amplify gratefully acknowledges the outstanding contributions and work of distinguished program advisors who have been integral to the development of Amplify Math. This product is testimony to their expertise, understanding of student learning needs, and dedication to rigorous and equitable mathematics instruction.

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Amplify gratefully acknowledges the outstanding contributions and work of esteemed members of our Educator Advisory Board. This product exhibits their respected observations and reflections.

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Field Trials

Amplify gratefully acknowledges the time and efforts of educators from the following districts and schools whose participation in field trials provided constructive critiques and resulting improvements. This product reflects their valuable feedback.

Berryessa Union School District, California

Chicago Jesuit Academy, Illinois

Irvine Unified School District, California

Lake Tahoe Unified School District, California

Leadership Learning Academy, Utah

Lusher Charter School, Louisiana

Memphis Grizzlies Preparatory Charter School, Tennessee

Saddleback Valley Unified School District, California

San Juan Unified School District, California

Santa Paula Unified School District, California

Silver Summit Academy, Utah

Streetsboro City Schools, Ohio

West Contra Costa Unified School District, California

Wyoming City Schools, Ohio

Young Women's Leadership School of Brooklyn, New York

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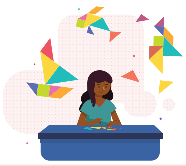
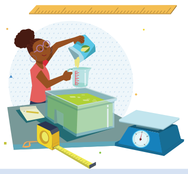
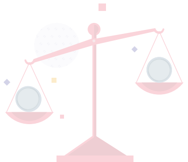
Volume 1





	Unit 1	Unit 2	Unit 3	Unit 4
Grade 6 160 days total	Area and Surface Area 20 Instructional Days 3 Assessment Days 23 days total	Introducing Ratios 20 Instructional Days 2 Assessment Days 22 days total	Rates and Percentages 15 Instructional Days 2 Assessment Days 17 days total	Dividing Fractions 17 Instructional Days 3 Assessment Days 20 days total
Grade 7 153 days total	Scale Drawings 13 Instructional Days 2 Assessment Days 15 days total	Introducing Proportional Relationships 17 Instructional Days 2 Assessment Days 19 days total	Measuring Circles 12 Instructional Days 2 Assessment Days 14 days total	Percentages 13 Instructional Days 2 Assessment Days 15 days total
Grade 8 145 days total	Rigid Transformation and Congruence 18 Instructional Days 3 Assessment Days 21 days total	Dilations and Similarity 12 Instructional Days 2 Assessment Days 14 days total	Linear Relationships 19 Instructional Days 2 Assessment Days 21 days total	Linear Equations and Systems of Linear Equations 17 Instructional Days 2 Assessment Days 19 days total
Algebra 1 157 days total	Linear Equations, Inequalities, and Systems 26 Instructional Days 3 Assessment Days 29 days total	Data Analysis and Statistics 22 Instructional Days 3 Assessment Days 25 days total	Functions and Their Graphs 22 Instructional Days 3 Assessment Days 25 days total	Introducing Exponential Functions 22 Instructional Days 3 Assessment Days 25 days total

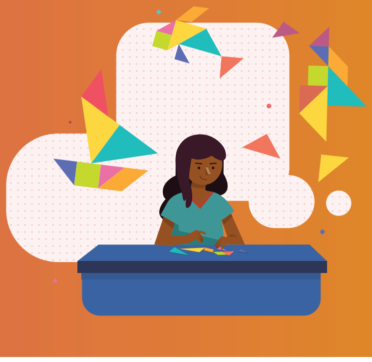
Unit 5	Unit 6	Unit 7	Unit 8
			
Arithmetic in Base Ten 14 Instructional Days 2 Assessment Days 16 days total	Expressions and Equations 19 Instructional Days 2 Assessment Days 21 days total	Rational Numbers 19 Instructional Days 2 Assessment Days 21 days total	Data Sets and Distributions 17 Instructional Days 3 Assessment Days 20 days total
Rational Number Arithmetic 20 Instructional Days 3 Assessment Days 23 days total	Expressions, Equations, and Inequalities 23 Instructional Days 3 Assessment Days 26 days total	Angles, Triangles, and Prisms 18 Instructional Days 3 Assessment Days 21 days total	Probability and Sampling 17 Instructional Days 3 Assessment Days 20 days total
Functions and Volume 21 Instructional Days 3 Assessment Days 24 days total	Exponents and Scientific Notation 15 Instructional Days 2 Assessment Days 17 days total	Irrationals and the Pythagorean Theorem 16 Instructional Days 2 Assessment Days 18 days total	Associations in Data 9 Instructional Days 2 Assessment Days 11 days total
Introducing Quadratic Functions 23 Instructional Days 3 Assessment Days 26 days total	Quadratic Equations 24 Instructional Days 3 Assessment Days 27 days total		

Volume 1

Unit 1 Area and Surface Area

Students extend their elementary understanding of area as compositions and decompositions for covering, shifting from limited experiences with rectangles and unit-square thinking to more general formulas for parallelograms and triangles. They leverage these in working with three-dimensional figures as well, recognizing surface area as a different measure than volume.

Unit Narrative:
A Place for Space



Sub-Unit Narrative:
Can a sum ever really be greater than its parts?
Polygons are shapes whose sides are all line segments, and they can be decomposed and rearranged without changing their area.

Sub-Unit Narrative:
How did a misplaced ruler change the way you shop?
Polyhedra are three-dimensional figures composed of polygon faces. Their surfaces can be decomposed.

Unit 2 Introducing Ratios

Students understand ratios using three of their five senses. They use written and visual representations to learn the language of ratios, and scale up (with multiplication) or down (with division) to calculate equivalent ratios. Ratios are also used for thinking about constant rates or occurrences happening at the same rate.

Unit Narrative:
Sensing a Ratio

Sub-Unit Narrative:
How does an eggplant become a plum?

Ratios represent comparisons between quantities by multiplication or division. First, you must first learn the language of ratios and how quantities "communicate."

Sub-Unit Narrative:
How do you put your music where your mouth is?

Equivalent ratios involve relationships between ratios themselves. They speak to each other through music and rhythm, beats and time.

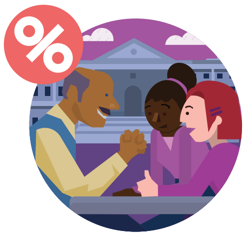
Sub-Unit Narrative:
Who brought Italy to India and back again?

Now it is your turn to choose the information to represent and compare ratios.

Unit 3 Rates and Percentages

Students understand the concept of unit rate in the contexts of constant price and speed, recognizing that equivalent ratios have the same unit rates. They use several visual and algebraic representations of percentages to determine missing percentages, parts, and wholes.

Unit Narrative:
Stand and
Be Counted



Sub-Unit Narrative:
How did student governments come to be?

Rates describe relationships between quantities like price and speed. Unit rates reveal which is a better deal or who is faster.

Sub-Unit Narrative:
What can a corpse teach us about governing?

Percentages are rates per 100. They can compare relationships between parts and wholes, even when two quantities have different total amounts.

Unit 4 Dividing Fractions

Students extend their understanding of partitive and quotitive division from whole numbers to fractions. They use this along with the relationship between multiplication and division to construct models and develop an algorithm for dividing fractions, and they apply it to problems involving lengths, areas, and volumes.

Unit Narrative:
Crossing the
Fractional Divide

Sub-Unit Narrative:
Which item costs between 100 and 1,000 spök-bucks?

Multiplication and division are related, and the relationship between fractions and division can be used to estimate quotients.

Sub-Unit Narrative:
How long is the bolt Samira needs?

To divide fractions, you can use multiplication, common denominators, or an algorithm. Apply these to determine the length of an oddly labeled bolt.

Sub-Unit Narrative:
How can Maya fit Penny in the box?

When you know an area or volume, but not every side length, you will often divide fractions.

Volume 2

Unit 5 Arithmetic in Base Ten

Students synthesize previous learning of place value, properties of operations,and relationships between operations to complete their understanding of both the "whys" and "hows" of the four operations with positive rational numbers. They develop general algorithms for working with whole numbers and decimals, containing any arbitrary number of digits.

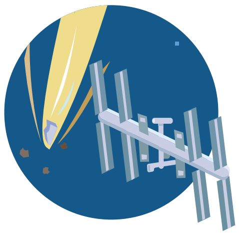
Unit Narrative:
Making Moves
With Decimals

Sub-Unit Narrative:
How did a decimal decide an Olympic race?

Determine the results of high stakes competitions and identify record-setting moments by adding and subtracting decimals, as precisely as you need.

Sub-Unit Narrative:
What happens when you make a small change to a big bridge?

To reproduce something at large or small scales so it looks the same, you need decimals and multiplication.



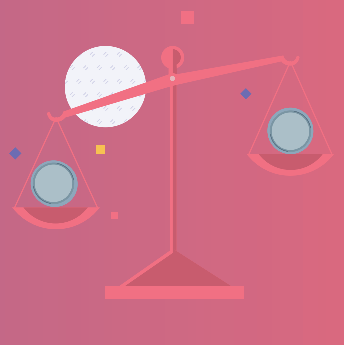
Sub-Unit Narrative:
How do you dodge a piece of space junk?

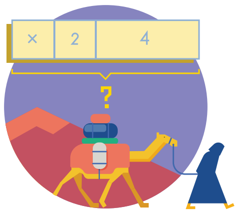
Dividing whole numbers and decimals with many digits is the final set of operations you need to complete your trophy case.

Unit 6 Expressions and Equations

Students discover that the equal sign is more than a prompt, it's also a way to indicate balance — a critical understanding that allows them to move beyond the strictly numeric world and into the realm of algebra.

Unit Narrative:
The Power
of Balance



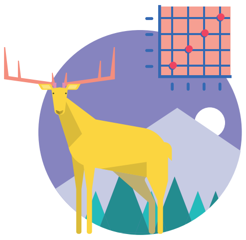


Sub-Unit Narrative:
What's a bag of chips worth in Timbuktu?

Learn about the 14th century African salt trade, as you explore expressions and equations with tape diagrams and hanger diagrams.

Sub-Unit Narrative:
How did a Welshman equalize England's upper crust with its common folk?

Extend the concept of equality as you investigate equivalent expressions, the all-important Distributive Property, and exponents.



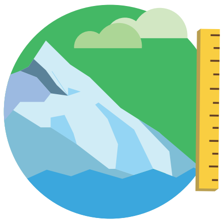
Sub-Unit Narrative:
What's more dangerous: a pack of wolves or a gang of elk?

Balance is everywhere, especially in ecosystems. You'll look at systems that are in and out of balance.

Unit 7 Rational Numbers

Students recognize a need to expand their concept of number to represent both magnitude and direction, extending the number line and coordinate plane to include negative rational numbers. They compare these numbers, as well as their absolute values, and write inequality statements using variables.

Unit Narrative:
Getting Where We're Going



Sub-Unit Narrative:
What's the tallest mountain in the world?
Consider the most extreme locations on Earth as you discover negative numbers, which lend new meaning to positive numbers and zero.

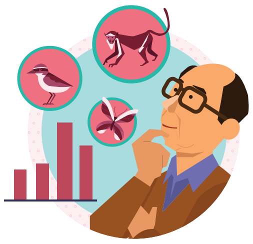
Sub-Unit Narrative:
How do you keep a quantity from wandering off?
A variable represents an unknown quantity. And sometimes it represents many possible values, which can be expressed as an inequality.

Sub-Unit Narrative:
How did Greenland get so big?
Armed with the opposites of positive rational numbers, it's time you expanded your coordinate plane. Welcome to the four quadrants!

Unit 8 Data Sets and Distributions

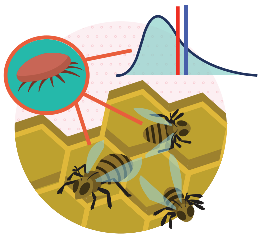
Unit Narrative:
Walk on the Wild Side With Data

In this unit, students learn about populations and study variables associated with a population, focusing on populations of animal species and their respective endangerment classifications. They distinguish numerical and categorical data, relative to survey and statistical questions, and represent and describe the distributions of response data. Students first interpret frequency tables, dot plots, and histograms, before calculating measures of center — mean and median — and measures of variability — mean absolute deviation (MAD), range, and interquartile range (IQR). They then construct box plots in addition to interpreting these measures in context, and relating the shape and features of a distribution to the best choice of measures.



Sub-Unit Narrative:
How do you keep track of a disappearing animal?

When questions have more than one answer, it is helpful to visualize and describe a typical answer. For numbers, you can also identify the center and describe the spread of the numbers.



Sub-Unit Narrative:
What's the buzz on honey bees?

For numerical data, you can summarize an entire data set by a single value representing the center of the distribution. The mean and the median represent two ways you can do this.

Sub-Unit Narrative:
Where have the giant sea cows gone?

For numerical data, you can summarize an entire data set by a single value representing the variability of the distribution. The MAD, range, and IQR represent three ways you can do this.

Get all students talking and thinking about grade-level math

Amplify Math was designed around the idea that core math needs to serve 100% of students in accessing grade-level math every day. To that end, the program delivers:

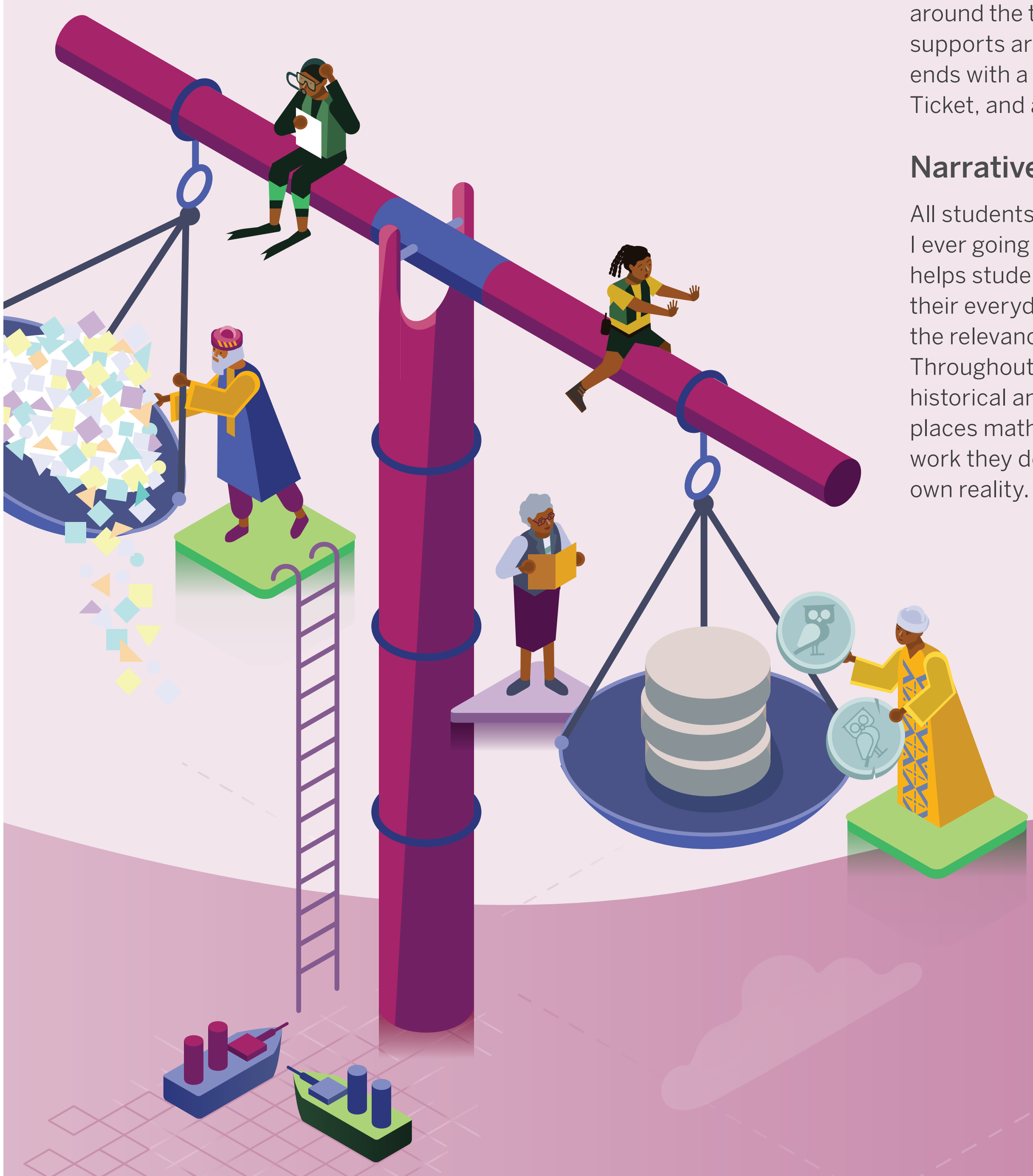
1 Productive discourse made easier to facilitate and more accessible for students

Clean and clear lesson design

The lessons all include straightforward "1, 2, 3 step" guidance for launching and facilitating discussions around the tasks. Thoughtful and specific differentiation supports are included for every activity. Every lesson ends with a summary and reflection moment, an Exit Ticket, and a practice problem set.

Narrative and storytelling

All students ask "Why do I need to know this? When am I ever going to use this in the real world?" Amplify Math helps students make the connections with math and their everyday lives to help them see and appreciate the relevance of the math they're figuring out in class. Throughout the units, students will be introduced to historical and current narratives that show the many places mathematics inhabits in our world and how the work they do in class connects to our history and their own reality.



2 Flexible, social problem-solving experiences online

Social learning experiences online

By partnering with Desmos, we've been able to deliver digital lessons, which we call **Amps**, that get students thinking, talking, revising, and celebrating their ideas. As students work in the interactive slides, new functionality may appear and they will often be asked to justify their actions and thinking. All of this is made visible to the teacher in real time.

Automatically differentiated activities

Our **Power-ups** automatically provide differentiated activities to students who need pre-requisite support, based on performance on past problems and assessments. They're available in this Teacher Edition, too. Phil Daro partnered with us on this feature to ensure we were giving all students, even the ones who might be three years behind in math, but only 15 minutes behind the day's lesson, the chance to experience success in math.

3 Real-time insights, data, and reporting that inform instruction

Teacher orchestration tools

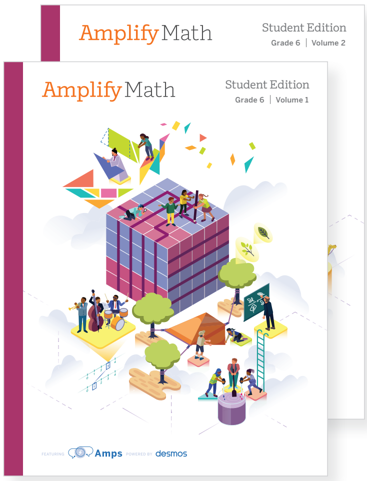
Once a teacher launches an Amp, students will be automatically moved to the lesson of the day and will see the interactive screens. Teachers will have the ability not only to pace the lesson the way they want to, but also to see student work in real time. The orchestration tools offer teachers ways to overlay student work to spot misconceptions and also the ability to spotlight student work anonymously to discuss with the class.

Embedded and standalone assessments

Amplify Math includes both a suite of standalone assessments and embedded assessments that allow teachers and leaders insights into where students are and how they might best be supported. The full reporting suite covers student and class performance based on work done in lessons, Exit Tickets, and practice sets, performance by standards, and performance on Interim assessments.

Amplify Math resources

Student Materials



Student workbooks, 2 volumes

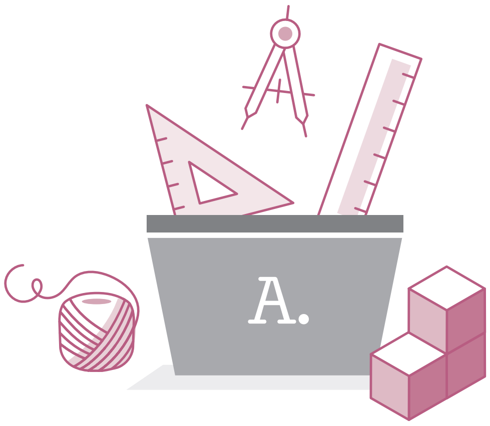
Activity 1: Walking Dogs Like a Pro

Welcome to Pawston University! We've produced some of the nation's finest dog walkers. Your first challenge is to balance some leashes.

You can move the dogs by dragging the points below them. The number below each dog tells you the strength of that dog's pull.

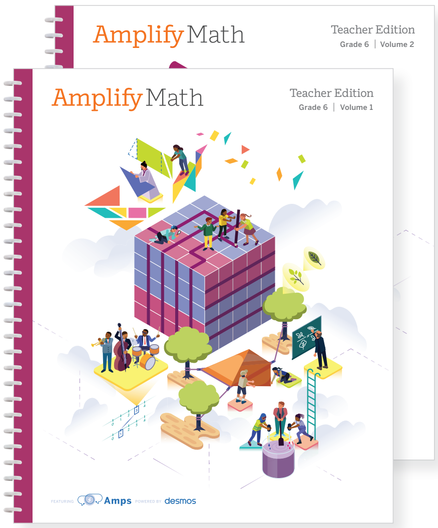
How can you balance all five dogs on two leashes, holding one leash in each hand?

Amps, our exclusive collection of digital lessons powered by desmos



Hands-on manipulatives (optional)

Teacher Materials



Teacher Edition, 2 volumes

Launch: Keeping the B...

Math 7 · Period 3

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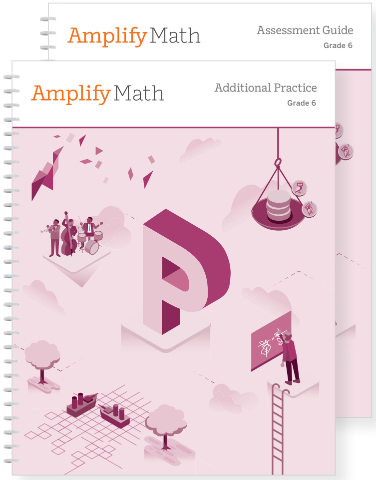
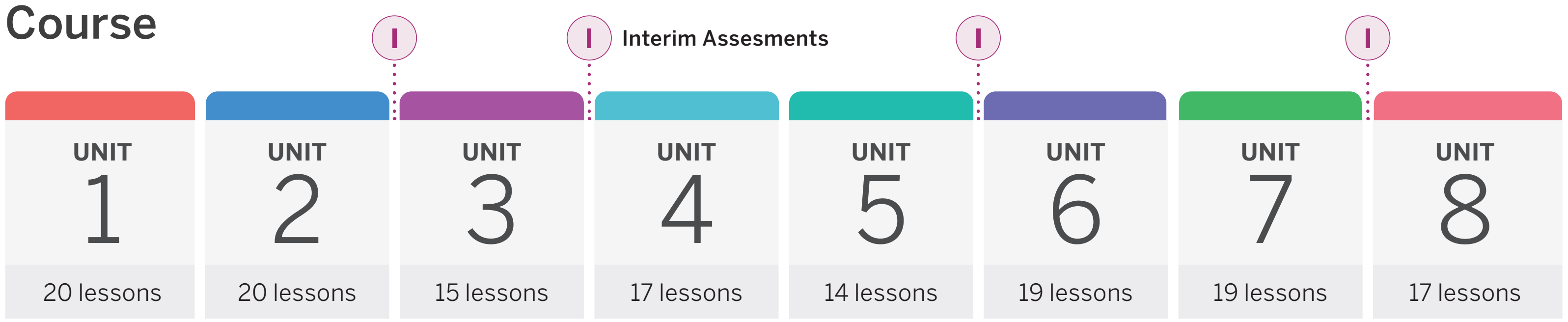
Digital Teacher Edition and classroom monitoring tools



Additional Practice and Assessment Guide blackline masters

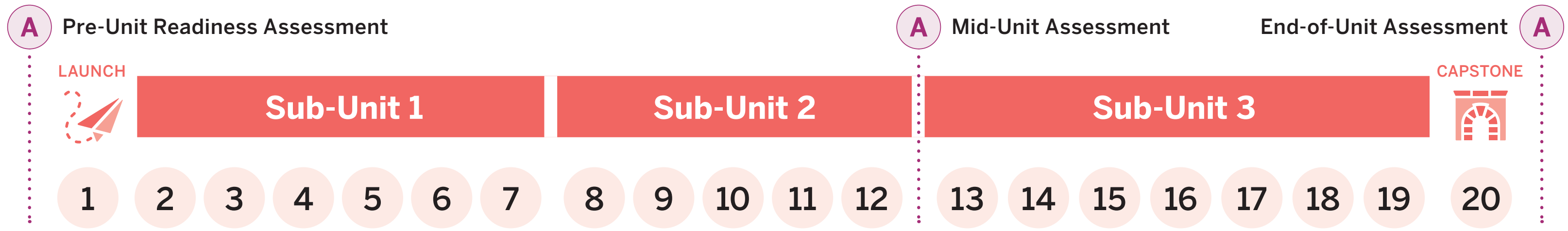
Program architecture

Course



Note: Interim assessments may be administered according to your district/school's timeline; this depiction is just one of many possible administrations.

Unit



Note: The number of sub-units and lessons vary from unit to unit; this depiction shows the general structure of a unit.

Lesson

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket	Practice
5 min	15 min	15 min	5 min	5 min	timing varies
Independent Pairs Small Groups Whole Class	Independent Pairs Small Groups Whole Class	Independent Pairs Small Groups Whole Class	Whole Class	Independent	Independent

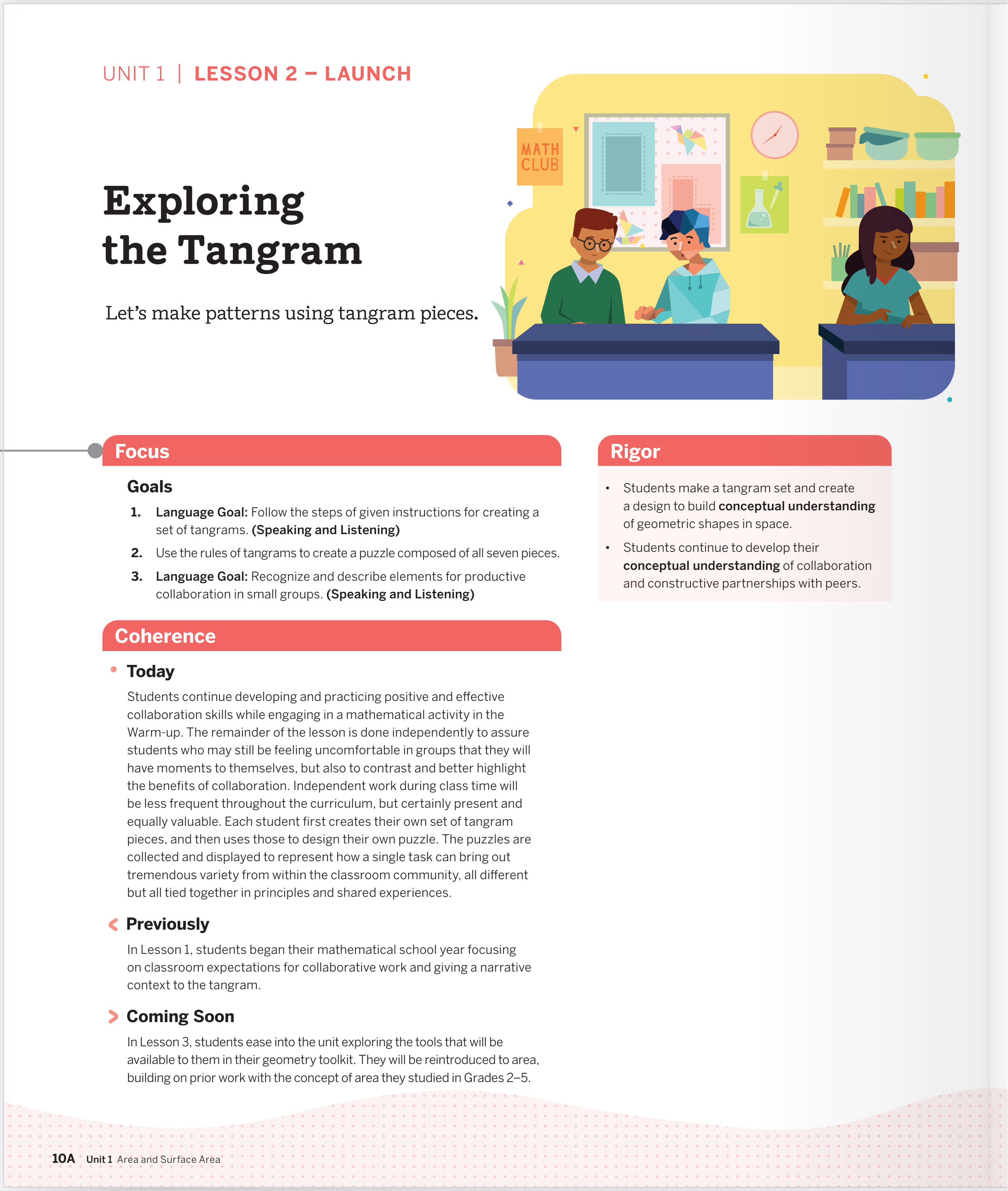
Note: The number of activities vary from lesson to lesson; this depiction shows the general structure of a lesson.

Key:
Independent
Pairs
Small Groups
Whole Class

Navigating This Program

Lesson Brief

Lesson goals, **coherence mapping**, and a breakdown for how **conceptual understanding**, **procedural fluency**, and **application** are addressed are included for each lesson.



UNIT 1 | **LESSON 2 – LAUNCH**

Exploring the Tangram

Let's make patterns using tangram pieces.

Focus

Goals

1. **Language Goal:** Follow the steps of given instructions for creating a set of tangrams. **(Speaking and Listening)**
2. Use the rules of tangrams to create a puzzle composed of all seven pieces.
3. **Language Goal:** Recognize and describe elements for productive collaboration in small groups. **(Speaking and Listening)**

Coherence

Today

Students continue developing and practicing positive and effective collaboration skills while engaging in a mathematical activity in the Warm-up. The remainder of the lesson is done independently to assure students who may still be feeling uncomfortable in groups that they will have moments to themselves, but also to contrast and better highlight the benefits of collaboration. Independent work during class time will be less frequent throughout the curriculum, but certainly present and equally valuable. Each student first creates their own set of tangram pieces, and then uses those to design their own puzzle. The puzzles are collected and displayed to represent how a single task can bring out tremendous variety from within the classroom community, all different but all tied together in principles and shared experiences.

Previously

In Lesson 1, students began their mathematical school year focusing on classroom expectations for collaborative work and giving a narrative context to the tangram.

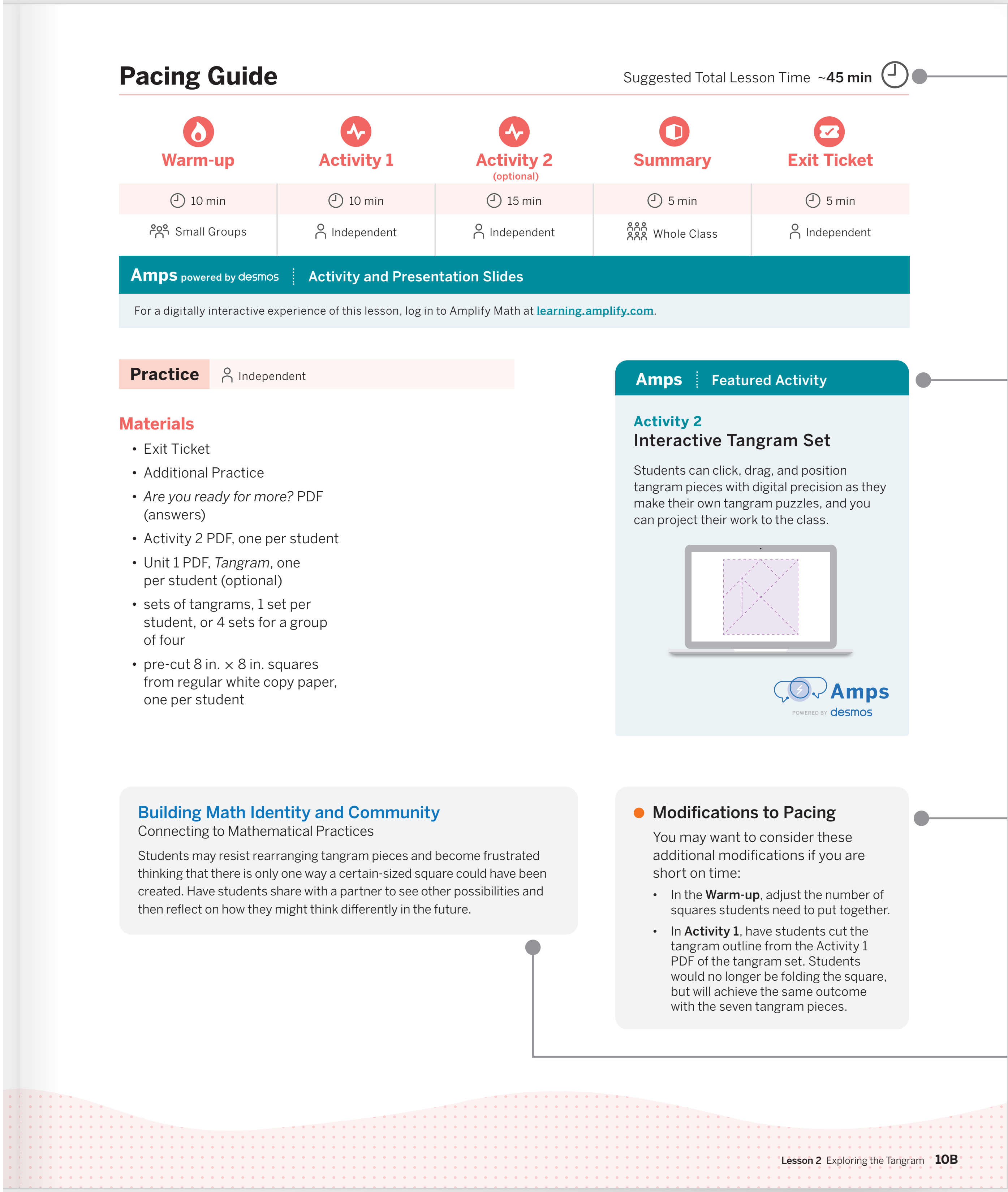
Coming Soon

In Lesson 3, students ease into the unit exploring the tools that will be available to them in their geometry toolkit. They will be reintroduced to area, building on prior work with the concept of area they studied in Grades 2–5.

Rigor

- Students make a tangram set and create a design to build **conceptual understanding** of geometric shapes in space.
- Students continue to develop their **conceptual understanding** of collaboration and constructive partnerships with peers.

10A **Unit 1** Area and Surface Area



Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2 (optional)	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Small Groups	Independent	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- *Are you ready for more?* PDF (answers)
- Activity 2 PDF, one per student
- Unit 1 PDF, *Tangram*, one per student (optional)
- sets of tangrams, 1 set per student, or 4 sets for a group of four
- pre-cut 8 in. × 8 in. squares from regular white copy paper, one per student

Amps | Featured Activity

Activity 2
Interactive Tangram Set

Students can click, drag, and position tangram pieces with digital precision as they make their own tangram puzzles, and you can project their work to the class.

Building Math Identity and Community

Connecting to Mathematical Practices

Students may resist rearranging tangram pieces and become frustrated thinking that there is only one way a certain-sized square could have been created. Have students share with a partner to see other possibilities and then reflect on how they might think differently in the future.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time:

- In the **Warm-up**, adjust the number of squares students need to put together.
- In **Activity 1**, have students cut the tangram outline from the Activity 1 PDF of the tangram set. Students would no longer be folding the square, but will achieve the same outcome with the seven tangram pieces.

Lesson 2 Exploring the Tangram **10B**

Suggested timing for the lesson and each activity is included for quick reference.

The benefits of teaching one or more of the activities **online** are outlined for each lesson.

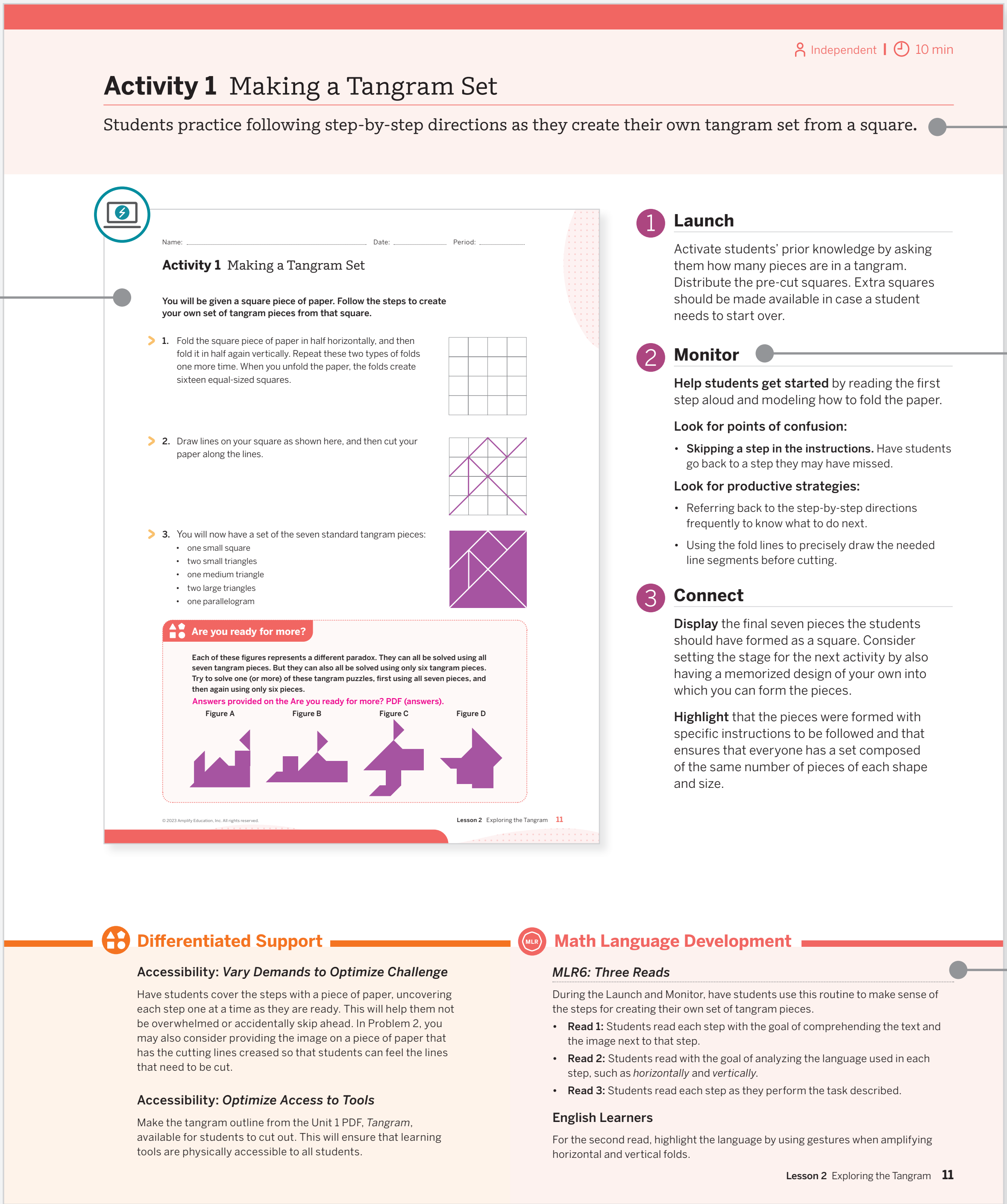
Every lesson pacing guide includes **modification** suggestions.

Building Math Identity and Community supports for teachers are included in the Lesson Brief. Student supports appear online and in the printed Student Edition.

Navigating This Program

Lesson

The **student-facing** content is presented to the left.



Independent | 10 min

Activity 1 Making a Tangram Set

Students practice following step-by-step directions as they create their own tangram set from a square.

Name: Date: Period:

Activity 1 Making a Tangram Set

You will be given a square piece of paper. Follow the steps to create your own set of tangram pieces from that square.

1. Fold the square piece of paper in half horizontally, and then fold it in half again vertically. Repeat these two types of folds one more time. When you unfold the paper, the folds create sixteen equal-sized squares.
2. Draw lines on your square as shown here, and then cut your paper along the lines.
3. You will now have a set of the seven standard tangram pieces:
   - one small square
   - two small triangles
   - one medium triangle
   - two large triangles
   - one parallelogram

Are you ready for more?

Each of these figures represents a different paradox. They can all be solved using all seven tangram pieces. But they can also all be solved using only six tangram pieces. Try to solve one (or more) of these tangram puzzles, first using all seven pieces, and then again using only six pieces.

Answers provided on the Are you ready for more? PDF (answers).

Figure A Figure B Figure C Figure D

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Lesson 2 Exploring the Tangram 11

① Launch

Activate students' prior knowledge by asking them how many pieces are in a tangram. Distribute the pre-cut squares. Extra squares should be made available in case a student needs to start over.

② Monitor

Help students get started by reading the first step aloud and modeling how to fold the paper.

Look for points of confusion:

- **Skipping a step in the instructions.** Have students go back to a step they may have missed.

Look for productive strategies:

- Referring back to the step-by-step directions frequently to know what to do next.
- Using the fold lines to precisely draw the needed line segments before cutting.

③ Connect

Display the final seven pieces the students should have formed as a square. Consider setting the stage for the next activity by also having a memorized design of your own into which you can form the pieces.

Highlight that the pieces were formed with specific instructions to be followed and that ensures that everyone has a set composed of the same number of pieces of each shape and size.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

Have students cover the steps with a piece of paper, uncovering each step one at a time as they are ready. This will help them not be overwhelmed or accidentally skip ahead. In Problem 2, you may also consider providing the image on a piece of paper that has the cutting lines creased so that students can feel the lines that need to be cut.

Accessibility: ***Optimize Access to Tools***

Make the tangram outline from the Unit 1 PDF, *Tangram*, available for students to cut out. This will ensure that learning tools are physically accessible to all students.

Math Language Development

MLR6: Three Reads

During the Launch and Monitor, have students use this routine to make sense of the steps for creating their own set of tangram pieces.

- **Read 1:** Students read each step with the goal of comprehending the text and the image next to that step.
- **Read 2:** Students read with the goal of analyzing the language used in each step, such as *horizontally* and *vertically*.
- **Read 3:** Students read each step as they perform the task described.

English Learners

For the second read, highlight the language by using gestures when amplifying horizontal and vertical folds.

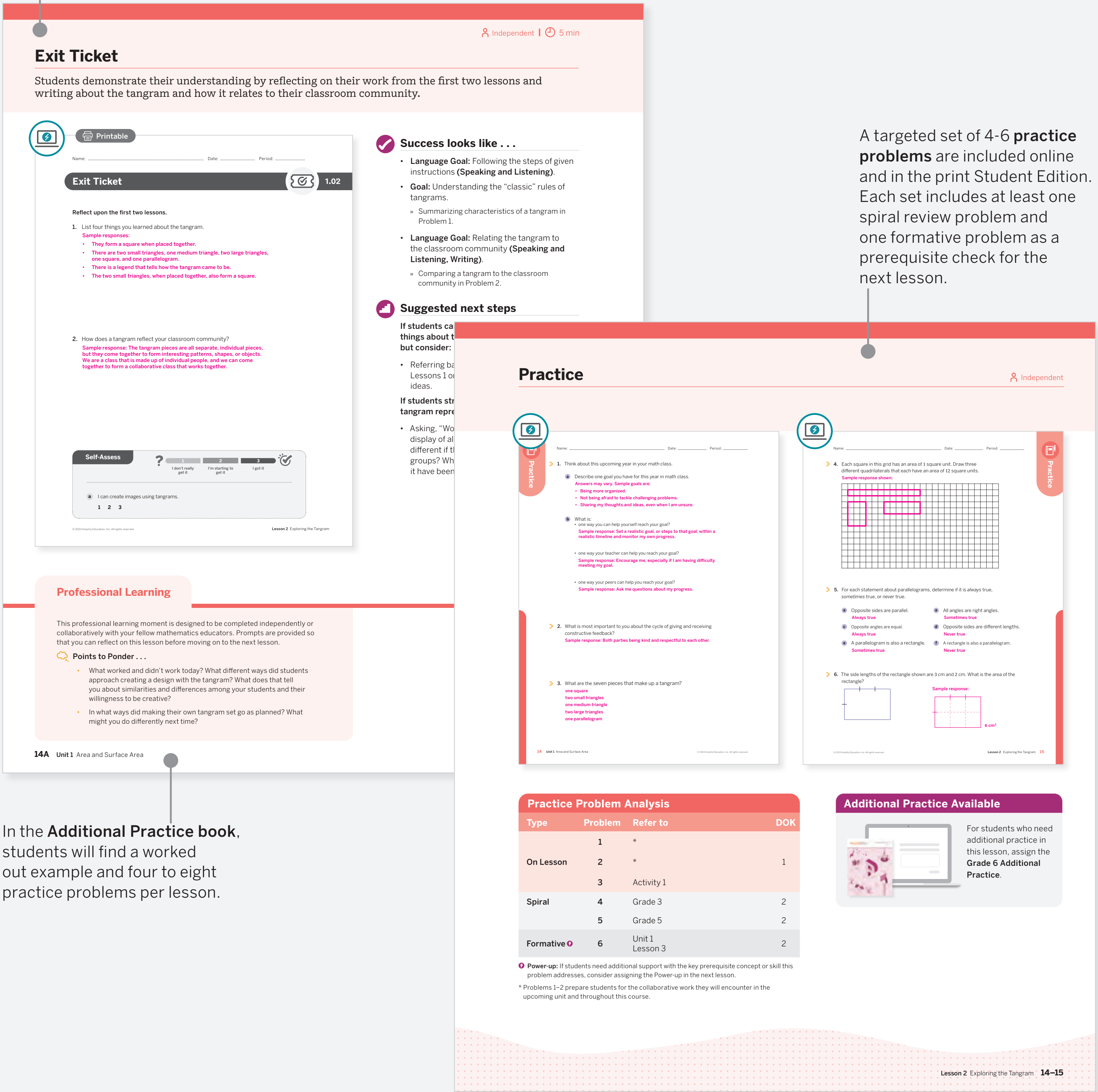
Lesson 2 Exploring the Tangram **11**

A short **description of the activity and its targeted goal** is outlined at the top.

Easy 1-2-3 guidance for teachers shortens the amount of time required to plan. The "look for" prompts are helpful to scan while teaching.

Differentiation supports, including our alternative warm-ups called Power-ups, provide practical guidance for scaffolding or extending the learning for all students. Differentiation supports, including our just-in-time supports called Power-ups, provide practical guidance for scaffolding or extending the learning for all students.

Each lesson ends with an **Exit Ticket** which includes a self assessment for students.



Independent | 5 min

Exit Ticket

Students demonstrate their understanding by reflecting on their work from the first two lessons and writing about the tangram and how it relates to their classroom community.

Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 1.02

Reflect upon the first two lessons.

1. List four things you learned about the tangram.
   Sample responses:
   - **They form a square when placed together.**
   - **There are two small triangles, one medium triangle, two large triangles, one square, and one parallelogram.**
   - **There is a legend that tells how the tangram came to be.**
   - **The two small triangles, when placed together, also form a square.**

2. How does a tangram reflect your classroom community?
   Sample response: The tangram pieces are all separate, individual pieces, but they come together to form interesting patterns, shapes, or objects. We are a class that is made up of individual people, and we can come together to form a collaborative class that works together.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can create images using tangrams.
1 2 3

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Lesson 2 Exploring the Tangram

Success looks like . . .

- **Language Goal:** Following the steps of given instructions **(Speaking and Listening)**.
- **Goal:** Understanding the "classic" rules of tangrams.
  - » Summarizing characteristics of a tangram in Problem 1.
- **Language Goal:** Relating the tangram to the classroom community **(Speaking and Listening, Writing)**.
  - » Comparing a tangram to the classroom community in Problem 2.

Suggested next steps

If students ca
things about t
but consider:

- Referring ba
  Lessons 1 o
  ideas.

If students str
tangram repre

- Asking, "Wo
  display of al
  different if t
  groups? Wh
  it have been

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What different ways did students approach creating a design with the tangram? What does that tell you about similarities and differences among your students and their willingness to be creative?
- In what ways did making their own tangram set go as planned? What might you do differently next time?

14A **Unit 1** Area and Surface Area

Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. Think about this upcoming year in your math class.
   a Describe one goal you have for this year in math class.
   Answers may vary. Sample goals are:
   - **Being more organized.**
   - **Not being afraid to tackle challenging problems.**
   - **Sharing my thoughts and ideas, even when I am unsure.**

   b What is:
   - one way you can help yourself reach your goal?
     Sample response: Set a realistic goal, or steps to that goal, within a realistic timeline and monitor my own progress.
   - one way your teacher can help you reach your goal?
     Sample response: Encourage me, especially if I am having difficulty meeting my goal.
   - one way your peers can help you reach your goal?
     Sample response: Ask me questions about my progress.

2. What is most important to you about the cycle of giving and receiving constructive feedback?
   Sample response: Both parties being kind and respectful to each other.

3. What are the seven pieces that make up a tangram?
   one square
   two small triangles
   one medium triangle
   two large triangles
   one parallelogram

14 **Unit 1** Area and Surface Area
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Name: ______ Date: ______ Period: ______

4. Each square in this grid has an area of 1 square unit. Draw three different quadrilaterals that each have an area of 12 square units.
   Sample response shown:

5. For each statement about parallelograms, determine if it is *always* true, *sometimes* true, or *never* true.
   a Opposite sides are parallel.
   Always true
   a All angles are right angles.
   Sometimes true
   c Opposite angles are equal.
   Always true
   d Opposite sides are different lengths.
   Never true
   e A parallelogram is also a rectangle.
   Sometimes true
   f A rectangle is also a parallelogram.
   Never true

6. The side lengths of the rectangle shown are 3 cm and 2 cm. What is the area of the rectangle?
   Sample response:
   6 cm²

Practice

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Lesson 2 Exploring the Tangram 15

Practice Problem Analysis

Type	Problem	Refer to	DOK
	1	*	
On Lesson	2	*	1
	3	Activity 1	
Spiral	4	Grade 3	2
	5	Grade 5	2
Formative	6	Unit 1 Lesson 3	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

* Problems 1–2 prepare students for the collaborative work they will encounter in the upcoming unit and throughout this course.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Lesson 2 Exploring the Tangram **14–15**

A targeted set of 4-6 **practice problems** are included online and in the print Student Edition. Each set includes at least one spiral review problem and one formative problem as a prerequisite check for the next lesson.

In the **Additional Practice book**, students will find a worked out example and four to eight practice problems per lesson.

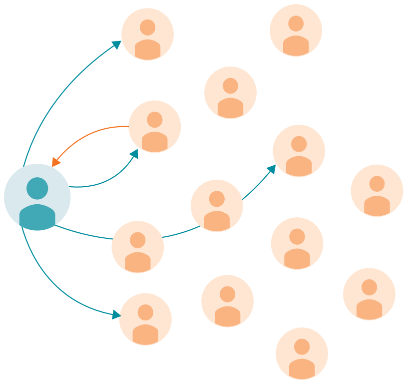
Social, collaborative digital experiences

Digital lessons, when designed the right way, can be powerful in their ability to surface student thinking and spark interesting and productive discussions. To bring our vision of what digital lessons can and should be to life, we've partnered with Desmos to create our complete library of **Amps**—social, collaborative lessons powered by Desmos technology that recast technology from simply mirroring what can be done in a workbook to presenting captivating scenarios where students work together and interact with the mathematics in real time.

Amps
POWERED BY desmos

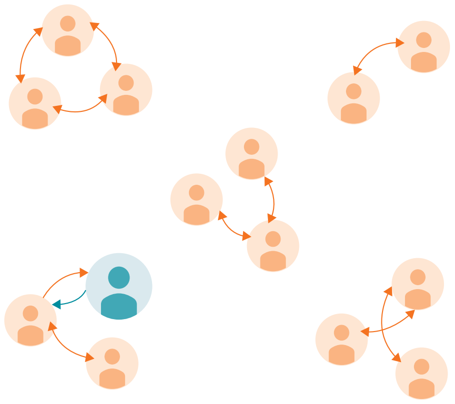
1 Launch

Teachers launch an activity and ensure students understand what's being asked.

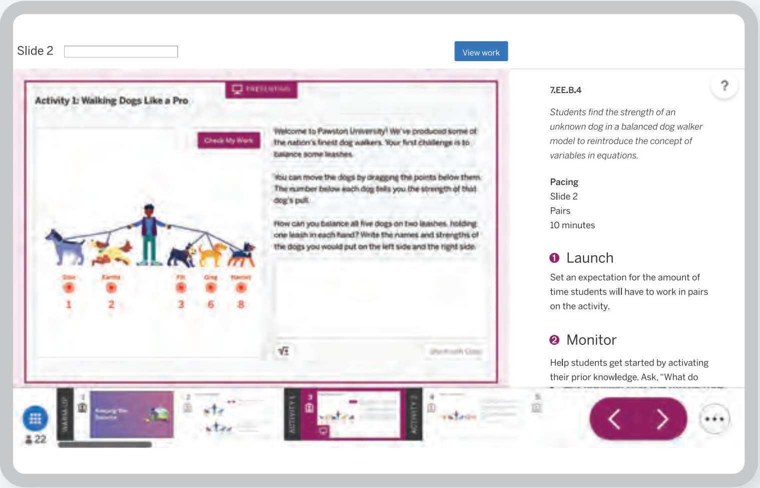


2 Monitor

Students interact with each other to discuss and work out strategies for solving a problem.



Teacher experience



Slide 2

View work

Activity 1: Walking Dogs Like a Pro

Check My Work

Welcome to Pawston University! We've produced some of the nation's finest dog walkers. Your first challenge is to balance some leashes.

You can move the dogs by dragging the points below them. The number below each dog tells you the strength of that dog's pull.

How can you balance all five dogs on two leashes, holding one leash in each hand? Write the names and strengths of the dogs you would put on the left side and the right side.

7.EE.B.4

Students find the strength of an unknown dog in a balanced dog walker model to reintroduce the concept of variables in equations.

Pacing
Slide 2
Pairs
10 minutes

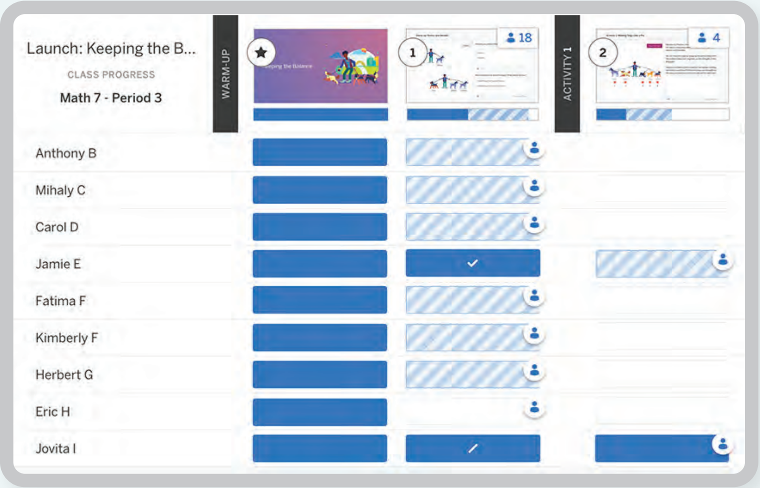
❶ Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

❷ Monitor

Help students get started by activating their prior knowledge. Ask, "What do

When you launch a lesson, you'll have access to **easy-to-skim teacher notes and all of the controls necessary** to manage the lesson.



Launch: Keeping the B...
CLASS PROGRESS
Math 7 - Period 3

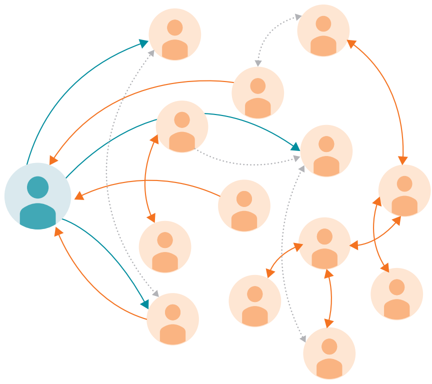
Anthony B
Mihaly C
Carol D
Jamie E
Fatima F
Kimberly F
Herbert G
Eric H
Jovita I

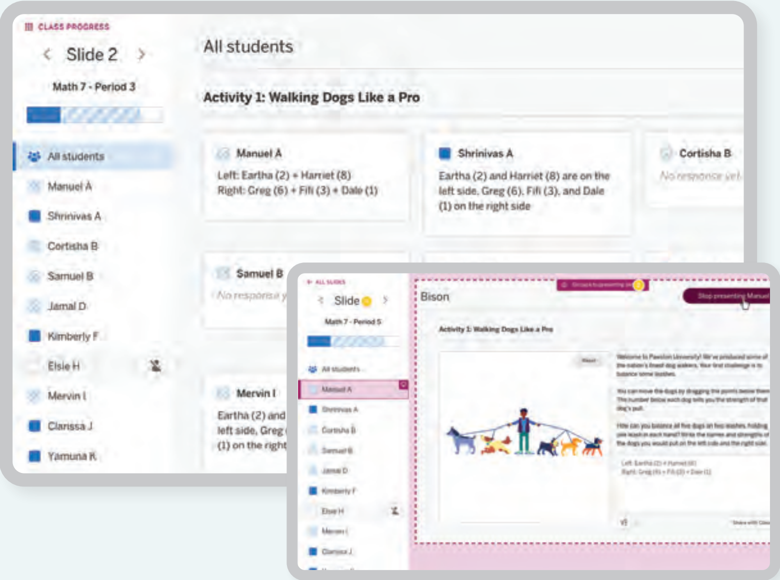
After students have started working you can access the Class Progress screen to **see where students are in the lesson and even control which problems they have access to.**

When you launch an **Amp**, you will be kickstarting small group and whole class discussions where students can see how their thinking can impact a situation and learn how their peers are justifying their actions and decisions.

3 Connect

Students construct viable arguments and critique each other's reasoning, then synthesize with the teacher at the end.





All students

Activity 1: Walking Dogs Like a Pro

Manuel A
Left: Eartha (2) + Harriet (8)
Right: Greg (6) + Fifi (3) + Dale (1)

Shrinivas A
Eartha (2) and Harriet (8) are on the left side. Greg (6), Fifi (3), and Dale (1) on the right side

Cortisha B
No response yet

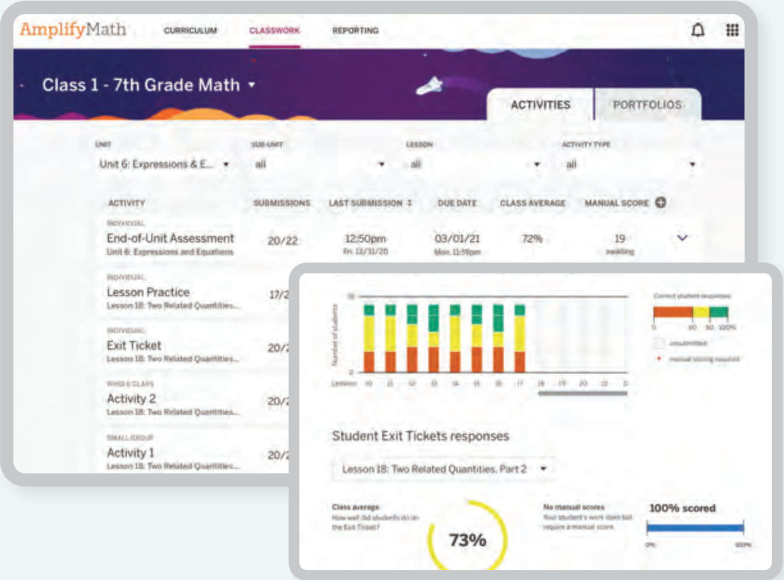
Samuel B
No response yet

Mervin I
Eartha (2) and left side, Greg (1) on the right

All student responses can be viewed easily on the All Students screen. You can often view a composite view of responses and spotlight student work anonymously.

4 Review

After class, teachers can provide feedback on submitted student work and run reports.



UNIT	SUB-UNIT	LESSON	ACTIVITY TYPE
Unit 6: Expressions & E...	all	all	all

ACTIVITY	SUBMISSIONS	LAST SUBMISSION	DUE DATE	CLASS AVERAGE	MANUAL SCORE
INDIVIDUAL End-of-Unit Assessment Unit 6: Expressions and Equations	20/22	12:50pm Fri. 12/31/20	03/01/21 Mon. 11:59pm	72%	19 awaiting
INDIVIDUAL Lesson Practice Lesson 18: Two Related Quantities...	17/2				
INDIVIDUAL Exit Ticket Lesson 18: Two Related Quantities...	20/2				
WHOLE CLASS Activity 2 Lesson 18: Two Related Quantities...	20/2				
SMALL GROUP Activity 1 Lesson 18: Two Related Quantities...	20/2				

Student Exit Tickets responses

Lesson 18: Two Related Quantities, Part 2

After students complete work that's ready for grading, you can head to Classwork to **quickly provide feedback.**

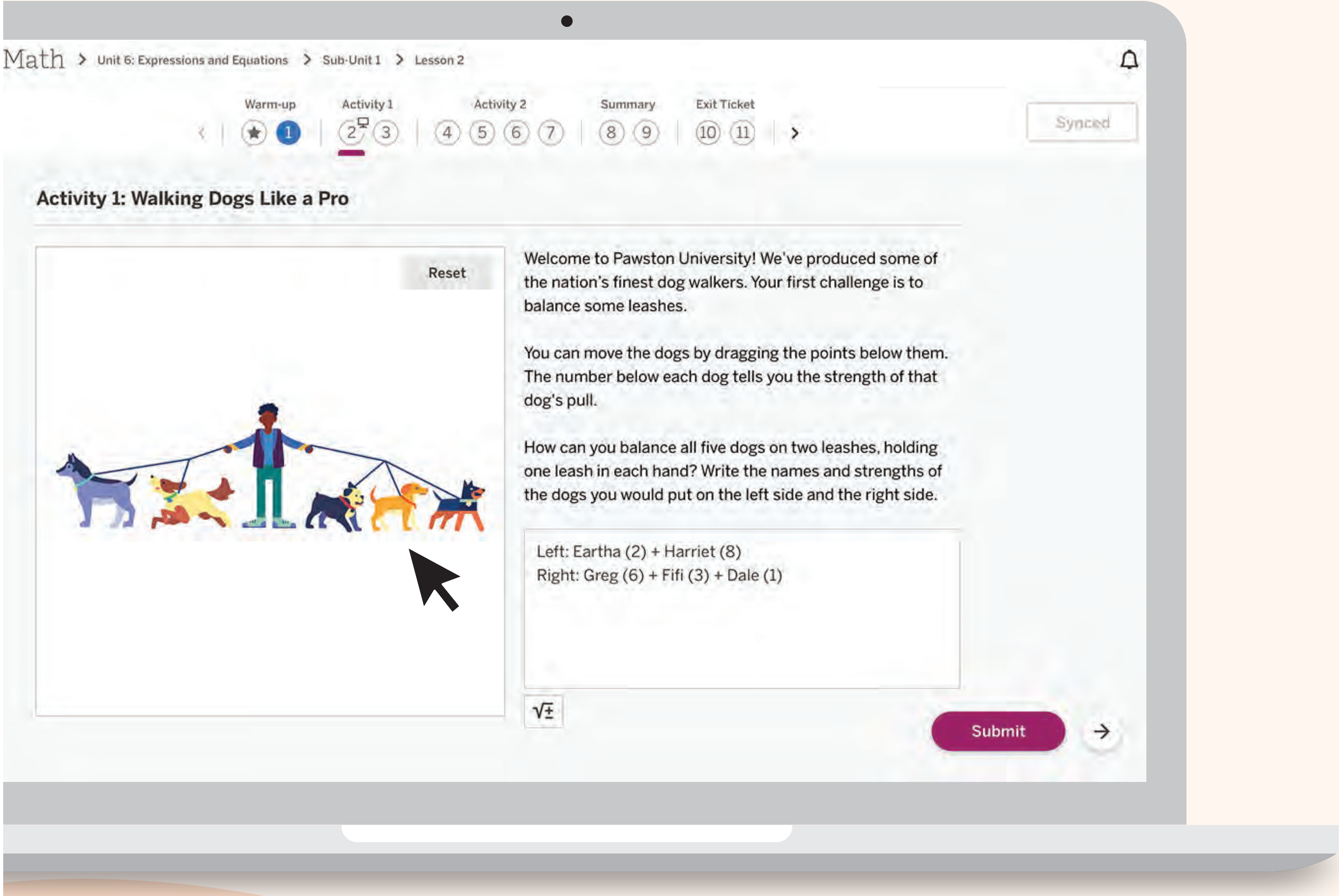
Once students have completed an Exit Ticket, a practice problem set, or an assessment, you can **run reports at the class, student, and standards levels to check in on student progress.**

Connecting everyone in the classroom

The student experience is interactive and responsive. As students manipulate the interactive elements of the Amps, they will be asked to justify their thinking, and often they will get to see how their peers are thinking, too. All along the way, teachers can monitor work in real time.

Student experience

The student experience is intuitive and engaging, offering students **low floors and high ceilings** as they engage with the lesson content.



Activity 1: Walking Dogs Like a Pro

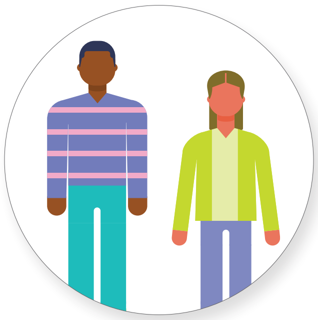
Welcome to Pawston University! We've produced some of the nation's finest dog walkers. Your first challenge is to balance some leashes.

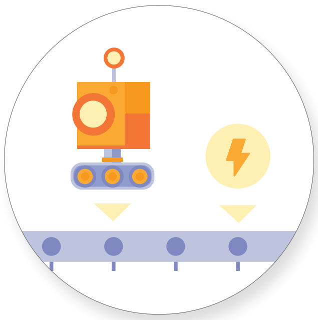
You can move the dogs by dragging the points below them. The number below each dog tells you the strength of that dog's pull.

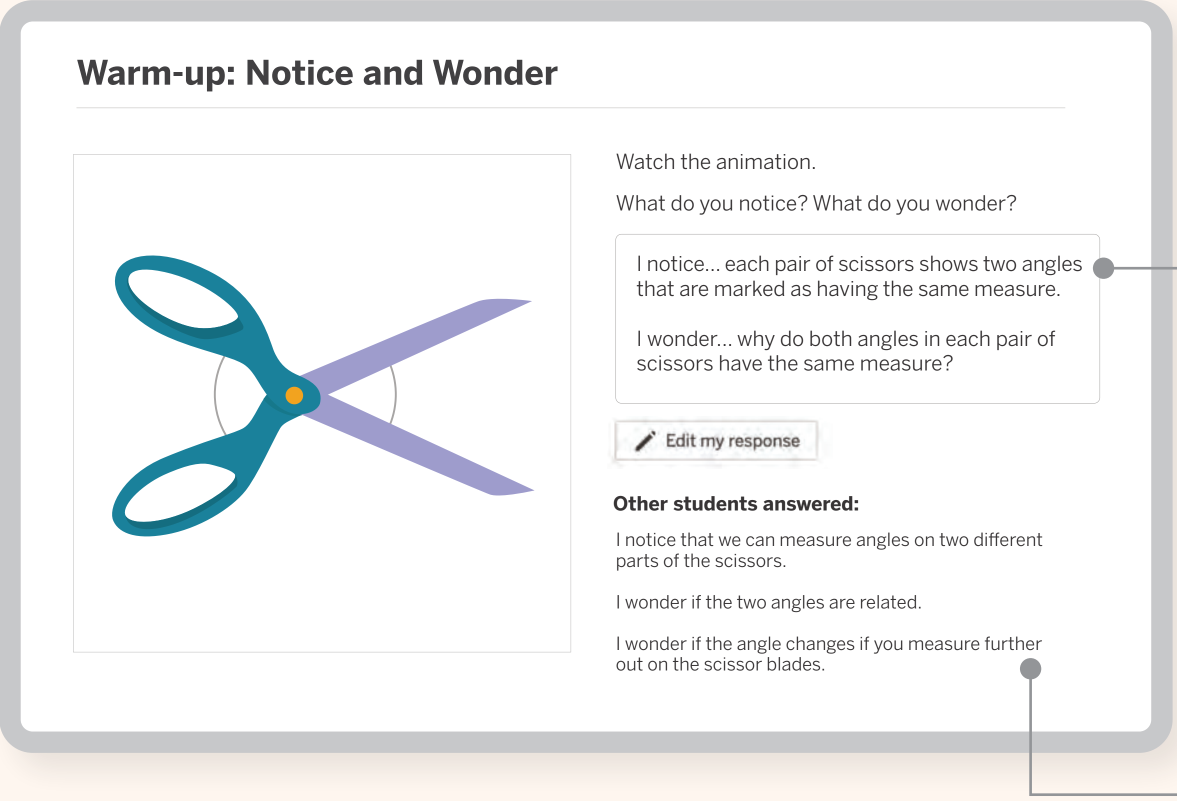
How can you balance all five dogs on two leashes, holding one leash in each hand? Write the names and strengths of the dogs you would put on the left side and the right side.

Left: Eartha (2) + Harriet (8)
Right: Greg (6) + Fifi (3) + Dale (1)

Many of the lessons feature rich, visual moments that students can interact with. Amps are meant to feel intuitive, immersive, and sometimes even playful.







Warm-up: Notice and Wonder

Watch the animation.

What do you notice? What do you wonder?

I notice... each pair of scissors shows two angles that are marked as having the same measure.

I wonder... why do both angles in each pair of scissors have the same measure?

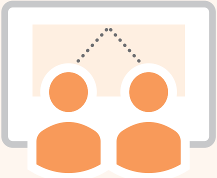
Other students answered:

I notice that we can measure angles on two different parts of the scissors.

I wonder if the two angles are related.

I wonder if the angle changes if you measure further out on the scissor blades.

As students work, the slides change, prompting students to **describe their strategies**. Teachers can see student work in real time and spotlight responses anonymously to support in-class discussion.



When working online, students will sometimes **see their peers' thinking** on their own screens. These connection points are great for sparking discussions ahead of the synthesis moment in the lesson.

Routines in Amplify Math

Routines help you and your students to maintain a sense of familiarity and structure throughout the school year. As the year progresses, routines free up time you would otherwise spend giving new directions for each activity.

Routine	What is it?	Where is it?
Turn and Talk	Turn and Talk can be done anytime, needs little preparation, and only takes a minute or two. Students turn and talk to their partner about another student's thinking, eventually deciding whether they agree or not and why. The students then switch roles. Eventually you will call on a few students to say what they think, or report what their partner said.	Use anytime students are working
Ask Three Before Me	Do you find yourself responding to all of your students' questions? This routine saves you time and empowers your students' agency and voice, while allowing them to view each other as knowledgeable resources. The routine is simple: When a student has a question, let them know they should ask three other students before they ask you.	
Go Find a Good Idea	When students are stuck and productive struggle has stalled, direct them to get up and walk around the room to find a good idea in the written work of other students. Encourage them to ask questions of other students and to explain their work. Then, they should bring a good idea back and continue working with it, citing from whom they got it.	
Notice and Wonder	Students are shown some media or a mathematical representation. They are prompted with "What do you notice? What do you wonder?," and are given a few minutes to think and share with a partner. Then you then ask several students to share what they noticed and wondered, recording responses for all to see. ***Note:*** Notice and Wonder and I Notice/I Wonder are trademarks of NCTM and the Math Forum and used in these materials with permission.	Warm-ups, Activity launches
Math Talks and Strings	Typically, one problem is displayed at a time. Problems are purposefully chosen to elicit different approaches, often in a way that builds from one problem to the next. These problems could be numerical or algebraic in nature. Give students a few moments to quietly think and give you a signal when they have an answer and a strategy, then select some to share different strategies for each problem. Ask questions like "Who thought about it a different way?"	Warm-ups
Which One Doesn't Belong?	Students are asked "Which one doesn't belong?" when presented with several figures, diagrams, graphs, or expressions. The best part is that every answer choice is correct! It's the similarities and differences among the options that are mathematically meaningful here. Prompt students to explain their rationale for deciding which option doesn't belong, and give them opportunities to make their rationale more precise.	Warm-ups
Card Sort	A card sort uses cards or slips of paper that can be manipulated and moved around. Individually, in pairs, or in small groups students put things into categories or groups based on shared characteristics or connections.	Activities
Find and Fix	Students are presented with the work of another hypothetical student whose work contains a flaw or error. They first identify any errors, then show or explain how to correct these errors, and explain why the person may have made the error.	Activities
Group Presentations and Gallery Tours	Instruct students—typically in groups—to create a visual display of their work, such as how they solved a problem with mathematical modeling, invented a new problem, designed a simulation or experiment, or organized and displayed data. In the Gallery Tour version of this routine, student work is captured on a piece of paper, a poster, or on an assigned portion of the board. Students then move around the room to observe, record notes or questions on their own paper, or write on each other's work (posing clarifying questions, giving kudos, or identifying portions they may disagree with). You lead a discussion, allowing students to respond to questions or critiques of their work.	Activities
Info Gap	One partner receives a problem card with a math question that doesn't have enough given information, and the other partner receives a data card with information relevant to the problem card. Students ask each other questions, such as "What information do you need?" and "Why do you need this information?" and are expected to explain what they will do with the information.	Activities

Math language development

All students, including English Learners, benefit from math lessons that are designed with strong math language development supports. Working in close partnership with the **English Learners Success Forum** (ELSF), the Amplify curriculum writers ensured the math language development strategies and supports in the program were clear, useful, and appropriate for all students. This was achieved by infusing the instruction with research-based Math Language Routines (MLRs) and by providing sentence frames where appropriate, both in the teacher language provided for each task and in the differentiation supports section. ELSF has helped review all student-facing content to ensure it's developmentally appropriate and reflects the fact that math is a new language students are mastering.

The math language development story starts at the unit level where teachers will see new vocabulary and a correlation of MLRs to lessons listed in the unit planning materials. In the Unit Study materials, one of the MLRs is highlighted under the Focus on Differentiated support section.

Embedded language development support

- **Course level:** The course design centers the development of communication skills.
- **Unit level:** Teachers will understand how language development progresses throughout the unit.
- **Lesson level:** Each lesson includes definitions of new vocabulary and language goals.
- **Activities:** Math Language Routines support the development of new vocabulary and activities often include additional supports based on language demands.
- **Assessments:** Suggested next steps offer guidance for teachers to support students' mathematical language development, based on their performance on assessment items.

Sentence frames

Sentence frames can give a student a sensible jumping off point for verbalizing or writing out an idea. Amplify Math leans on sentence frames that all serve one or more language functions, including describing a topic, explaining it, justifying thinking, generalizing, critiquing, representing, and interpreting ideas.

Math Language Routines

The Math Language Routines deployed throughout the Teacher Edition:

MLR1: Stronger and Clearer Each Time

MLR2: Collect and Display

MLR3: Critique, Correct, Clarify

MLR4: Information Gap

MLR5: Co-craft Questions

MLR6: Three Reads

MLR7: Compare and Connect

MLR8: Discussion Supports

Some routines adapted from Zwiers, J. (2014). Building academic language: Meeting Common Core Standards across disciplines, grades 5–12 (2nd ed.). San Francisco, CA: Jossey-Bass.

UNIT 1

Area and Surface Area

Students extend their elementary understanding of area as compositions and decompositions for covering, shifting from limited experiences with rectangles and unit-square thinking to more general formulas for parallelograms and triangles. They leverage these in working with three-dimensional figures as well, recognizing surface area as a different measure than volume.

Essential Questions

- What does it mean when you say two shapes have the same area?
- How is surface area different from volume?
- *(By the way, what do you get when you stitch together 12 pentagons and 20 hexagons?)*

Key Shifts in Mathematics

Focus

In this unit . . .

Students extend their reasoning about area to include shapes that are not composed of rectangles, drawing on their prior work of composing and decomposing shapes. This leads to the development of formulas for the areas of parallelograms and triangles. Students also work with three-dimensional solids, drawing nets and determining surface area, and distinguishing surface area from volume.

Coherence

Previously . . .

In Grades K–2, students composed, decomposed, identified, and measured rectilinear figures. Then, in Grades 3–5, they connected tiling unit squares to multiplication and the area of a rectangle, and they solved problems involving rectangular areas, including those with fractional side lengths. Also, in Grade 5, students connected packing unit cubes to multiplication and the volume of a rectangular prism.

Coming soon . . .

In Unit 4 of this grade, students will revisit volume of prisms, now with fractional side lengths. In Grades 7–8, students will continue to compose and decompose two- and three-dimensional shapes in order to derive formulas for calculating surface area and volume, and they will also identify congruent figures using rigid transformations.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Composition and decomposition preserves area and allows known areas to be used to determine unknown areas (Lessons 3–5), particularly of parallelograms (Lessons 6–7) and of triangles (Lesson 9). This also applies to nets and surface area (Lessons 14–16).

Procedural Fluency

The formula for the area of a rectangle can be used to derive similar formulas for the area of any parallelogram (Lesson 8) and any triangle (Lesson 11). Exponents can also be used to simplify formulas for the surface area and volume of cubes (Lessons 18–19).

Application

Area formulas and strategies can be extended to any polygons and polyhedra with known measurements, including those that model real-world shapes and figures (Lessons 10, 12–13, 17, 20).

A Place for Space

SUB-UNIT

1

Lessons 3–13

Area of Special Polygons

Students use general principles of the conservation of ***area*** to decompose two-dimensional shapes in order to determine their areas, eventually identifying critical measurements of ***base*** and ***height*** to derive formulas for the area of any parallelogram, triangle, or any ***polygon*** composed of those shapes.

Narrative: Discover how something can never be greater than the sum of its parts.

SUB-UNIT

2

Lessons 14–19

Nets and Surface Area

Students recognize ***surface area*** as a two-dimensional measure of a three-dimensional figure, and they construct and use ***nets*** representing basic and complex polyhedra to determine surface area. Students also distinguish between surface area and volume, and use ***exponents*** to simplify expressions involving repeated multiplication.

Narrative: From cardboard boxes to suspended tents, areas folded in three dimensions have got you covered!

Launch

Lesson 1–2

The Tangram; Exploring the Tangram

Students learn about the history of the tangram and apply area reasoning to solve a variety of tangram puzzles. These two launch lessons also provide you and your students opportunities to: practice collaborating in a variety of ways that will be used throughout the course and establish some goals for the year.

Capstone

Lesson 20

Designing a Suspended Tent

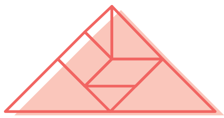
Students collaboratively design a tent that can hang from a tree, and determine how much fabric is needed to make it. They model their design using a net and calculate surface area, while managing real-world assumptions and implications.

Unit at a Glance

Spoiler Alert: Just like for a rectangle, the area of a parallelogram can be determined by a formula, multiplying base by height. And similarly for triangles, by taking half of that product.

Assessment

Launch Lesson



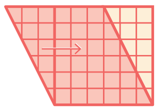
A Pre-Unit Readiness Assessment

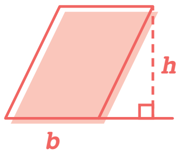
1 The Tangram

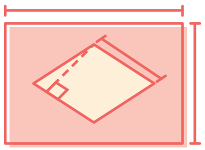
The same seven pieces can be rearranged to compose infinite different designs.

2 Exploring the Tangram

Solve and create more tangram puzzles.







6 Parallelograms

Decompose and rearrange parallelograms to determine area using related rectangles.

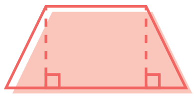
7 Bases and Heights of Parallelograms

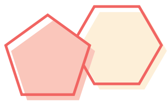
Identify bases and heights of parallelograms and relate their measures to a formula for area.

8 Area of Parallelograms

Practice determining areas of parallelograms in real-world and mathematical settings.

Assessment





12 From Triangles to Trapezoids ●

Decompose trapezoids into parallelograms and triangles to determine area.

13 Polygons ●

Identify polygons and decompose them into parallelograms and triangles to determine area.

A Mid-Unit Assessment

Key Concepts

Lesson 8: The area of any parallelogram is the product of its base and height.
Lesson 11: The area of any triangle is the half the product of its base and height.
Lesson 16: The surface area of a polyhedron is equal to the sum of the areas of its faces.

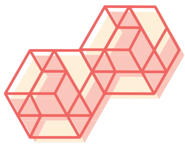
Pacing

20 Lessons: 45 min each
3 Assessments: 45 min each

Full Unit: 23 days
● **Modified Unit:** 14–15 days

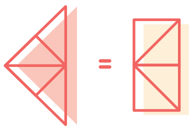
Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

Sub-Unit 1: Areas of Special Polygons



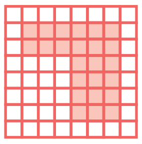
3 Tiling the Plane ●

Compare the relative areas covered by related shapes in a tiling pattern.



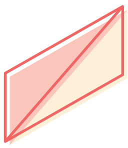
4 Composing and Rearranging to Determine Area

Composition, decomposition, and rearranging preserve area and can be used to determine unknown areas.



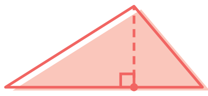
5 Reasoning to Determine Area

Relate composition and decomposition to unit squares and regions with known dimensions.



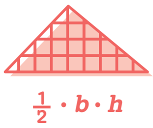
9 From Parallelograms to Triangles

Compose a parallelogram from identical copies of a triangle to determine the area of the triangle.



10 Bases and Heights of Triangles

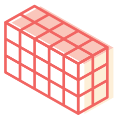
Identify bases and heights of triangles.



11 Formula for the Area of a Triangle

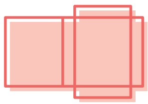
Relate bases and heights of triangles and their measures to corresponding dimensions of parallelograms and to a formula for area.

Sub-Unit 2: Nets and Surface Area



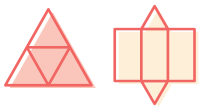
14 What is Surface Area?

Understand surface area of a three-dimensional figure as the sum of the areas of every face.



15 Nets and Surface Area of Rectangular Prisms

Identify and create nets as two-dimensional representations of a three-dimensional rectangular prism that can be used to determine surface area.



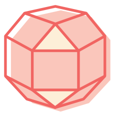
16 Nets and Surface Area of Prisms and Pyramids

Identify, create, and use nets to determine surface area of prisms and pyramids.

Unit at a Glance

Spoiler Alert: Just like for a rectangle, the area of a parallelogram can be determined by a formula, multiplying base by height. And similarly for triangles, by taking half of that product.

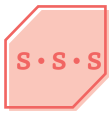
continued



17

Constructing a Rhombicuboctahedron ●

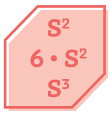
Apply understanding of nets and surface area of simple polyhedra to determine surface area of a more complex polyhedron.



18

Simplifying Expressions for Square and Cubes ●

Identify repeated terms and factors in measures of squares and cubes to simplify expressions for calculating.



19

Simplifying Expressions Even More Using Exponents

Understand exponents as indicating repeated multiplication. Write and evaluate expressions for measures of squares and cubes using exponents.

Key Concepts

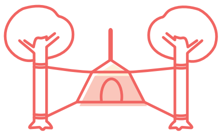
Lesson 8: The area of any parallelogram is the product of its base and height.
Lesson 11: The area of any triangle is half the product of its base and height.
Lesson 16: The surface area of a polyhedron is equal to the sum of the areas of its faces.

Pacing

20 Lessons: 45 min each	**Full Unit:** 23 days
3 Assessments: 45 min each	● **Modified Unit:** 14–15 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

Capstone Lesson



20 Designing a Suspended Tent ●

Apply understanding of area and surface area to design a tent that is a polyhedron, and calculate the amount of fabric needed to produce it.

Assessment

A End-of-Unit Assessment

● Modifications to Pacing

Lesson 3: This lesson may be omitted, but you should consider allowing some extra time for the Warm-up in Lesson 4, and perhaps borrowing some of the questioning and ideas from Activity 1 of Lesson 3 as well.

Lessons 12–13: The focus of Lesson 12 is trapezoids, which is not an explicit expectation for the grade, and so really serves as practice of applying knowledge of parallelograms and triangles. Lesson 13 introduces the term *polygons*, but otherwise also serves as practice of working with parallelograms and triangles. You can consider omitting one or both lessons, but you should probably define polygons at some other point.

Lesson 17: This lesson may be omitted. It allows students to practice determining surface area and using nets for a more complex figure composed of parallelogram and triangle faces, albeit with a super cool, real building (small-scale model kit included).

Lesson 18: Lesson 18 may be omitted as it mainly sets the stage for using exponents in Lesson 19. Note that if you choose to omit Lesson 18, some of the expressions in Activity 1 of Lesson 19 may not be as familiar as they would be if you covered Lesson 18.

Lesson 20: This capstone lesson may be omitted, but as with all capstones, it offers a fun and challenging culminating application of all of the work of the unit, bringing together area and surface area in a real-world design setting.

Unit Supports

Math Language Development

Lesson	New Vocabulary		
4	compose	decompose	
6	parallelogram	quadrilateral	
7	base (of a parallelogram)	height (of a parallelogram)	
10	base (of a triangle)	height (of a triangle)	
13	polygon		
14	edge	face	surface area
14	vertex	volume	
15	net		
16	base (of a prism)	base (of a pyramid)	polyhedron
16	prism	pyramid	
19	cubed	exponent	squared

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
1, 19	**MLR1:** Stronger and Clearer Each Time
4–6, 8–12, 14, 16, 19	**MLR2:** Collect and Display
1, 5, 8, 13, 15, 18, 19	**MLR3:** Critique, Correct, Clarify
3, 17	**MLR5:** Co-craft Questions
2	**MLR6:** Three Reads
2, 4–7, 9, 14, 16, 20	**MLR7:** Compare and Connect
7, 8, 10, 12, 14–18	**MLR8:** Discussion Supports

Materials

Every lesson includes:

Exit Ticket

Additional Practice

Additional required materials include:

Lesson(s)	Materials
1, 2	Sets of tangrams
2	Pre-cut 8 in. × 8 in. squares from white copy paper
3–5, 20	Geometry toolkits
1–4, 6, 9, 10, 12–13, 16, 17, 19, 20	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
6, 11, 17	Straightedges
10	Index cards
13	Transparencies
14, 18	Unit cubes
17	Tape/glue
17	Scissors

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routines
6, 8, 13, 20	Gallery Tour
4, 14, 17	Notice and Wonder
3	Take Turns
4, 5, 10, 12–14	Think-Pair-Share

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 13
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 19

Amps
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Social & Collaborative Digital Moments

Featured Activity

Stained Glass

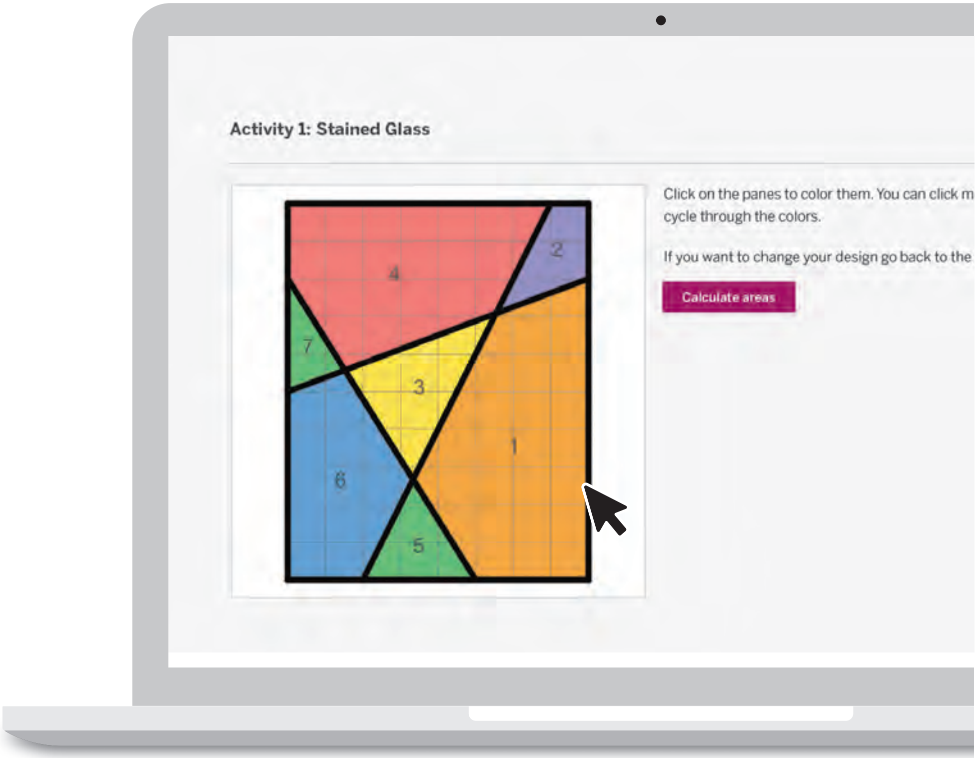
Put on your student hat and work through **Lesson 13, Activity 1:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Creating Your Own Tangram Puzzle **(Lesson 2)**
- Determining the Area of Triangles **(Lesson 9)**
- Constructing a Model of the Library **(Lesson 17)**
- Building Perfect Cubes **(Lesson 18)**



Activity 1: Stained Glass

Click on the panes to color them. You can click m
cycle through the colors.

If you want to change your design go back to the

Unit Study
Professional Learning

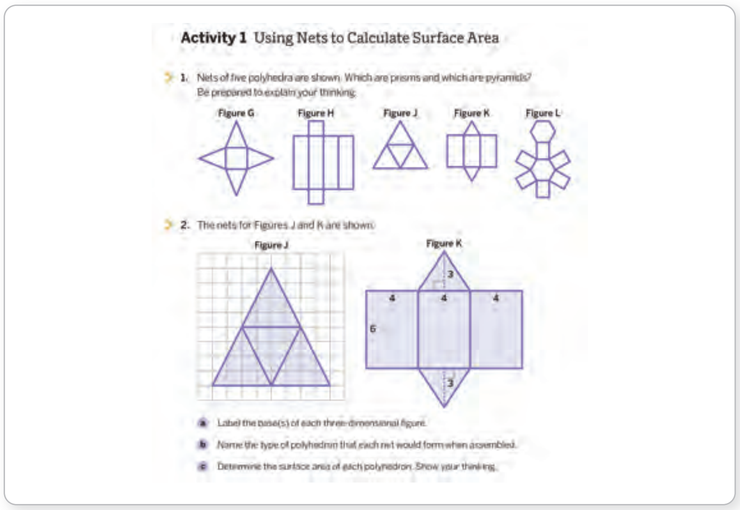
This unit study is designed to be completed independently, or collaboratively, with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 2 introduces the idea of the surface area of three-dimensional shapes of pyramids, prisms, and other polyhedra. Students begin to use nets to help them determine the surface area, and they also calculate the volume of rectangular prisms. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from **Lesson 16, Activity 1:**



Activity 1 Using Nets to Calculate Surface Area

1. Nets of five polyhedra are shown. Which are prisms and which are pyramids? Be prepared to explain your thinking.

2. The nets for Figures J and K are shown.

a. Label the base(s) of each three-dimensional figure.

b. Name the type of polyhedron that each net would form when assembled.

c. Determine the surface area of each polyhedron. Show your thinking.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

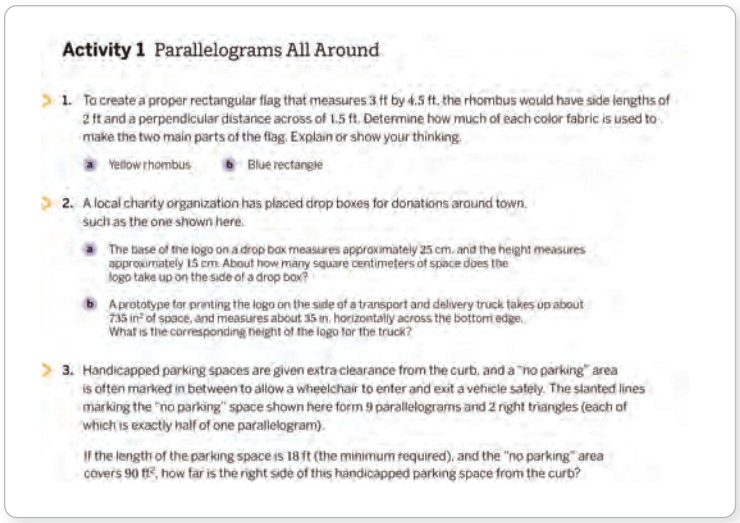
- Unit 1 uses examples and non-examples to guide students into noticing and wondering for themselves about a particular definition or concept. Would you use a similar strategy to guide students into distinguishing between a prism and a pyramid?
- Other than providing grid paper for students to draw the nets of different polyhedra, what other tools or manipulatives (including digital) might be useful?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Gallery Tour

Rehearse . . .

How you'll facilitate the ***Gallery Tour*** instructional routine in **Lesson 8, Activity 1:**



Activity 1 Parallelograms All Around

1. To create a proper rectangular flag that measures 3 ft by 4.5 ft, the rhombus would have side lengths of 2 ft and a perpendicular distance across of 1.5 ft. Determine how much of each color fabric is used to make the two main parts of the flag. Explain or show your thinking.

   a. Yellow rhombus   b. Blue rectangle

2. A local charity organization has placed drop boxes for donations around town, such as the one shown here.

   a. The base of the logo on a drop box measures approximately 25 cm, and the height measures approximately 15 cm. About how many square centimeters of space does the logo take up on the side of a drop box?

   b. A prototype for printing the logo on the side of a transport and delivery truck takes up about 735 in^2 of space, and measures about 35 in. horizontally across the bottom edge. What is the corresponding height of the logo for the truck?

3. Handicapped parking spaces are given extra clearance from the curb, and a "no parking" area is often marked in between to allow a wheelchair to enter and exit a vehicle safely. The slanted lines marking the "no parking" space shown here form 9 parallelograms and 2 right triangles (each of which is exactly half of one parallelogram).

   If the length of the parking space is 18 ft (the minimum required), and the "no parking" area covers 90 ft^2, how far is the right side of this handicapped parking space from the curb?

Points to Ponder . . .

- How will you organize the displays around the classroom – by problem or by group?

This routine . . .

- Helps students organize their work so that others can follow it.
- Allows students to review, analyze, and critique the work of others.
- Can provide an opportunity for students to verbally articulate and present their work and thinking using their own artifacts.
- Has the potential to reduce class time needed for separate presentations or discussion of multiple strategies or results.

Anticipate . . .

- Some groups will only have one problem completed while others have time for all three, and they may have varying amounts of work.
- Students may not know how to observe or comment on the work of others. Consider providing sentence frames or displaying guidance.
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Establish mathematics goals to focus learning.

This effective teaching practice . . .

- Establishes clear goals for both you and your students so that you both know what the lesson is targeting. If you know the target, you know whether or not you hit it.
- Provides a benchmark which will help you to make instructional decisions based on your students' performance.

Points to Ponder . . .

- How will understanding the target goals for each lesson or activity help you when planning how to spend your instructional time?
- How can you use the lesson goals to know if you need to redirect instruction or provide additional support?

Math Language Development

MLR7: Compare and Connect

MLR7 appears in Lessons 2, 4–7, 9, 14, 16, and 20.

- In Lesson 7, as students identify bases and heights of parallelograms, this routine allows them to see and compare a wide variety of different-looking parallelograms that are connected by the same base and height measurements.
- In Lesson 14, as students make connections between the various strategies used to cover the cabinet, this routine allows them to be exposed to multiple solution pathways that result in the same number of sticky notes needed.
- **English Learners:** Multiple strategies are provided to support students' understanding of mathematical language, including using gestures, concrete manipulatives, and technology, and allowing students to speak first in their primary language.

Point to Ponder . . .

- What are some strategies you can use to leverage tasks like these, with multiple representations or solution pathways, to help students build confidence and come to recognize math as a discipline that is not as rigid as they may have previously thought?

Unit Assessments

Use the results of the Pre-Unit Readiness Assessment to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the Mid- and End-of-Unit Assessments, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with determining area and surface area throughout the unit? Do you think your students will generally:
  - hone in on one strategy that works and use it repeatedly even when inefficient?
  - display procedural difficulties when using and applying formulas?

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools, Optimize Access to Technology*

Opportunities to provide visual support, physical manipulatives, or the use of technology appear in Lessons 1–4, 6–10, 12–16, and 18.

- Activate or supply background knowledge about area by providing grids, unit squares, and physical cutouts of shapes to help students think about measuring area in tangible and tactile ways first, before moving to abstractions and calculations.
- Use color and annotations to illustrate student thinking as they compose, decompose, and rearrange shapes to form new shapes with the same area, helping them to identify and keep track of common measurements and areas being preserved.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide whether to use images, concrete physical manipulatives, or technology (through the Amps slides for each activity) to support students' understanding of composing and decomposing shapes to find area?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their self-awareness and social awareness skills.

Points to Ponder . . .

- What are students' strengths, and what do they know about area and relationships between shapes that they can use to identify and make sense of the structure in more complex shapes?
- Are students able to anticipate how their own arguments and explanations may be interpreted or received, and take on the perspectives of others?

The Tangram

Let's discover the tangram.

Focus

Goals

1. **Language Goal:** Recognize and describe elements for productive collaboration with a partner. **(Speaking and Listening, Writing)**
2. Use spatial reasoning to solve tangram puzzles.

Coherence

Today

Students begin their yearlong journey into Grade 6 math by practicing some of the behaviors of successful mathematicians — both as an individual and as a collaborator. For most of the activities, they work together in pairs, communicating and sharing their thinking, to persevere through mathematical problems. The legend of the origin of the tangram puzzle is told, providing a backdrop to composing figures with tangram pieces. Then students explore some paradoxes in which similar yet different shapes are created using the same tangram pieces. This highlights necessary precision in composing and comparing areas of shapes.

Previously

In Grades 2–5, students reasoned with the concept of area first in equipartitioning contexts and then in using unit squares to determine rectilinear areas and relate geometric representations to the arithmetic concept of multiplication.

Coming Soon

In Lesson 2, students will continue exploring the tangram. They will make their own tangram set and create their own unique puzzle using their tangram pieces. Lesson 3 then begins the formal introduction to the unit, reviewing and expanding general principles of area.

Rigor

- Students use the tangram to develop **conceptual understanding** of geometric shapes in space.
- Students develop their **conceptual understanding** of collaboration and constructive partnerships with peers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2 (optional)	Summary	Exit Ticket
10 min	25 min	20 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

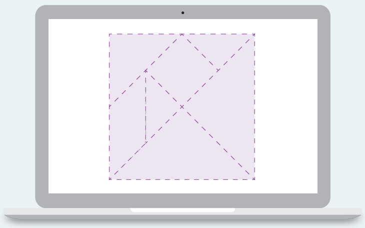
- Exit Ticket
- Additional Practice
- Activity 1 PDF, one set of templates per pair (double-sided)
- Activity 1 PDF, *Are you ready for more?* (double-sided)
- Activity 2 PDF (answers, for display)
- Activity 2 PDF, *Are you ready for more?*, one set per pair (double-sided)
- commercial sets of tangram pieces, one set per pair
- *Tangram* PDF template, one per student (as needed)

Amps | Featured Activity

Activity 1

Interactive Tangram Set

Students can click, drag, and position tangram pieces with digital precision, and you can project their work to the class.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may want to give up right away if the way to solve any of the puzzles is not immediately clear. Model thinking out loud and responding to the thoughts of a partner as examples of how to work well with a partner to determine what can be done together to make progress.

Modifications to Pacing

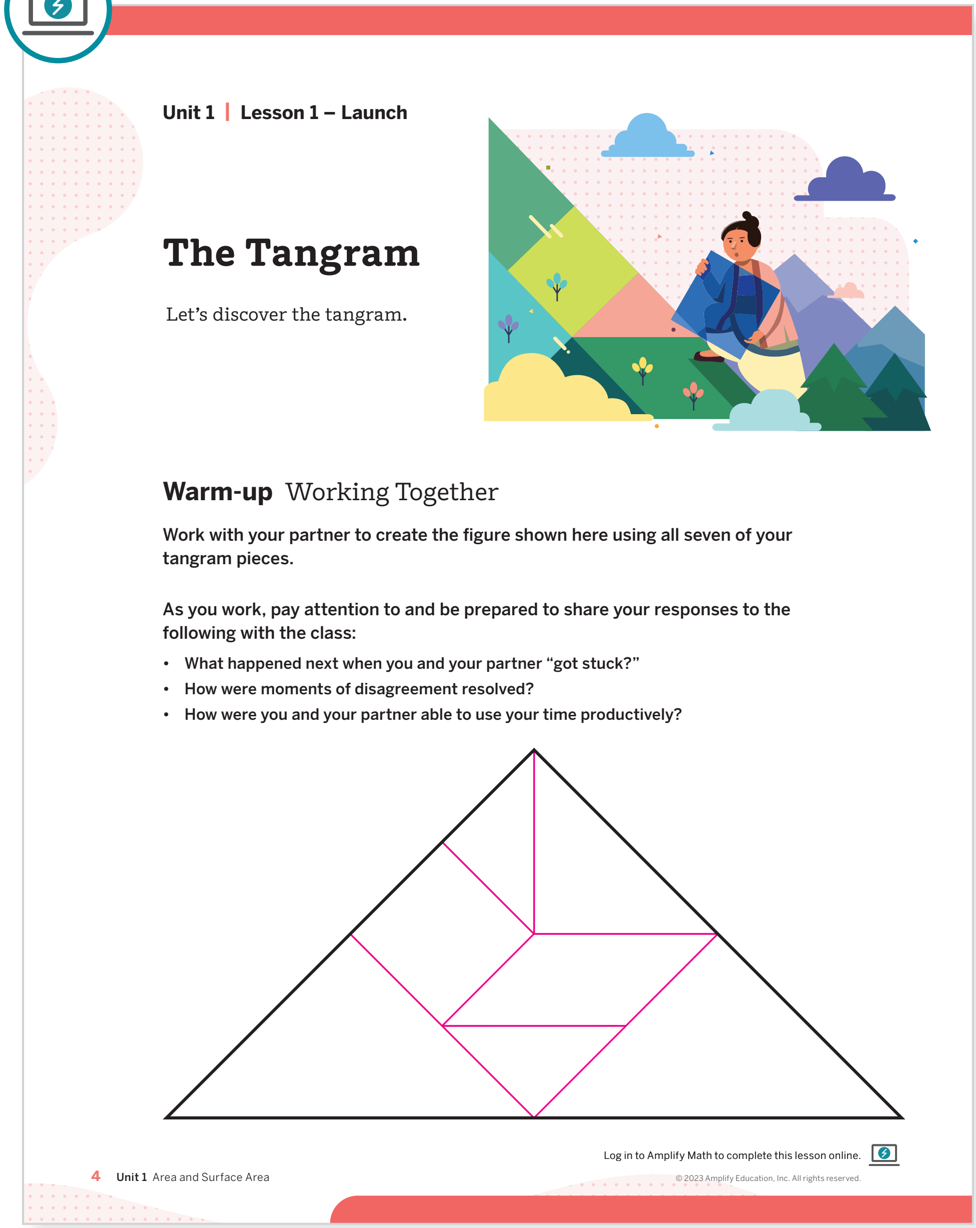
You may want to consider these additional modifications if you are short on time:

- In **Activity 1**, have students only complete a select number of figures to recreate, leaving time for the square to be completed.
- In **Activity 2**, have students only complete Problem 1.

Pairs | 10 min

Warm-up Working Together

Partners work on solving a tangram puzzle, reflecting on how to collaborate effectively and productively.



Unit 1 | **Lesson 1 – Launch**

The Tangram

Let's discover the tangram.

Warm-up Working Together

Work with your partner to create the figure shown here using all seven of your tangram pieces.

As you work, pay attention to and be prepared to share your responses to the following with the class:

- **What happened next when you and your partner "got stuck?"**
- **How were moments of disagreement resolved?**
- **How were you and your partner able to use your time productively?**

Log in to Amplify Math to complete this lesson online.

4 **Unit 1** Area and Surface Area

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① Launch

Read the directions for the Warm-up.
Ask, "What are some characteristics of an effective partnership?"

② Monitor

Help students get started by having them introduce themselves and providing a sentence frame to help students get a conversation going such as, "How do you think you can arrange the pieces to make this shape?"

Look for points of confusion:

- **Getting "stuck" on a certain combination (i.e., thinking a large triangle must make the top vertex).** Encourage students to discuss different configurations or have them identify all the pieces that could fill other areas.

Look for productive strategies:

- Discussing and planning moves first.
- Demonstrating active listening skills, such as facing the speaker and making eye contact.
- Working with different compositions to persevere at solving the puzzles.

③ Connect

Display a two-column chart labeled at the top with the headings "Good collaborators . . . look like: | sound like:". Record students' observations about positive collaboration in the table.

Ask, "Does a successful partnership mean that you have to get a correct solution every time?"

Have pairs of students share what made their partnership successful, focusing on using active listening skills to solve the puzzles.

Highlight that there will be many times throughout the year when positive collaborative work in pairs and in groups will be essential to individual *and* collective mathematical learning.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Create expectations for partner and group work. Collect this information and place it in a personalized checklist for students to refer to during times of collaborative work.

Accessibility: *Guide Processing and Visualization*

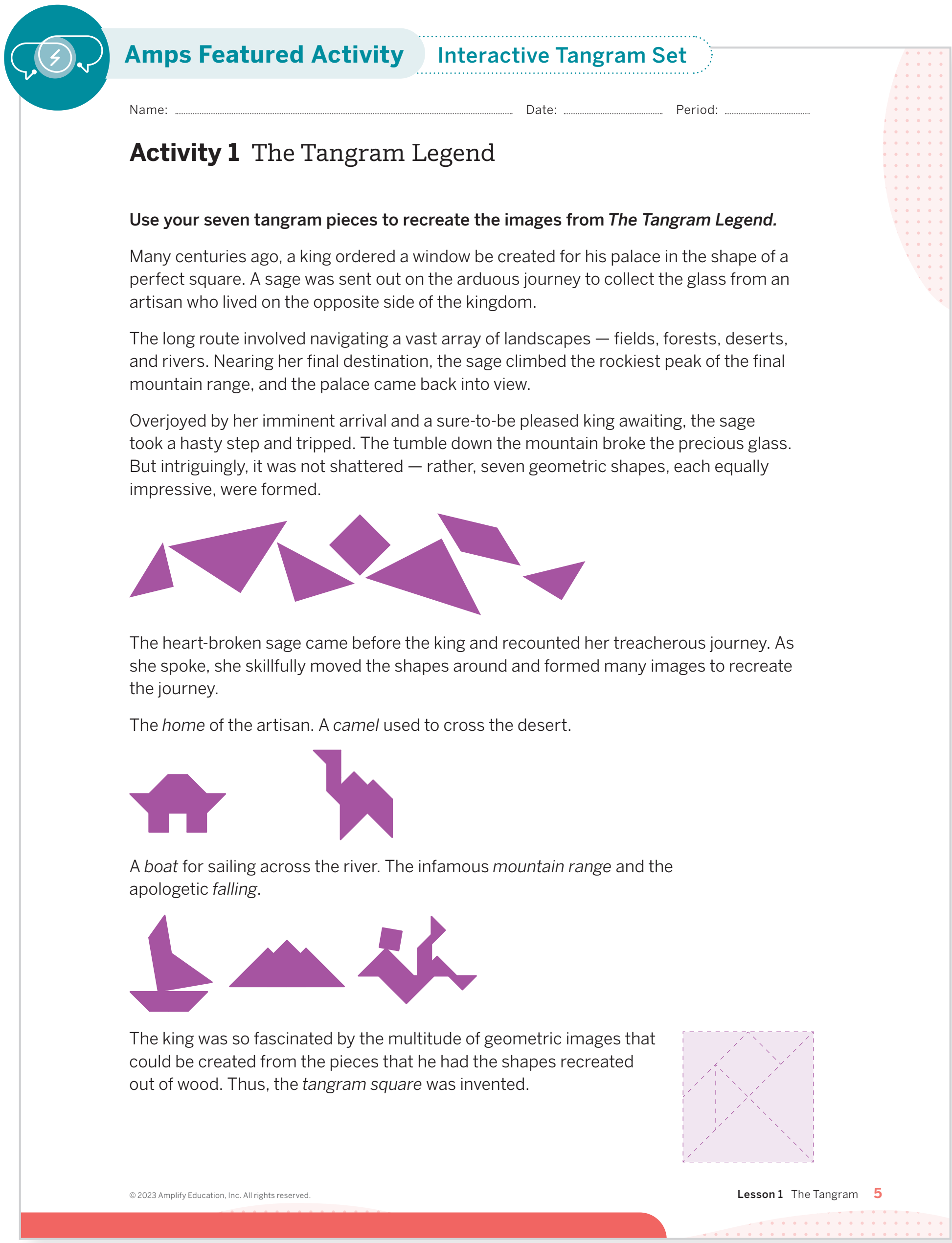
Provide options for students to engage with a replica border of the triangle made out of popsicle/geometry sticks. This will allow them to access different ways of knowing, thinking, and doing by feeling the shape and size of the triangle in order to then construct it using the tangram pieces.

Pairs | 25 min

Activity 1 The Tangram Legend

Partners demonstrate productive collaboration habits as they use templates to recreate the figures in the Tangram Legend.

Amps Featured Activity Interactive Tangram Set



Name: ______ Date: ______ Period: ______

Activity 1 The Tangram Legend

Use your seven tangram pieces to recreate the images from *The Tangram Legend.*

Many centuries ago, a king ordered a window be created for his palace in the shape of a perfect square. A sage was sent out on the arduous journey to collect the glass from an artisan who lived on the opposite side of the kingdom.

The long route involved navigating a vast array of landscapes — fields, forests, deserts, and rivers. Nearing her final destination, the sage climbed the rockiest peak of the final mountain range, and the palace came back into view.

Overjoyed by her imminent arrival and a sure-to-be pleased king awaiting, the sage took a hasty step and tripped. The tumble down the mountain broke the precious glass. But intriguingly, it was not shattered — rather, seven geometric shapes, each equally impressive, were formed.

The heart-broken sage came before the king and recounted her treacherous journey. As she spoke, she skillfully moved the shapes around and formed many images to recreate the journey.

The *home* of the artisan. A *camel* used to cross the desert.

A *boat* for sailing across the river. The infamous *mountain range* and the apologetic *falling*.

The king was so fascinated by the multitude of geometric images that could be created from the pieces that he had the shapes recreated out of wood. Thus, the *tangram square* was invented.

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Lesson 1 The Tangram 5

1 Launch

Read the legend to the whole class and distribute the Activity 1 PDF templates. Tell students that all seven tangram pieces must be used to create each shape. They should solve the puzzle first, and then they can verify their work by using the solutions on the back sides of the templates.

2 Monitor

Help students get started by helping them identify the small, medium, and large triangles in the set and on the puzzle solution for the home of the artisan.

Look for points of confusion:

- **Not working productively.** Remind students about the characteristics of positive working partnerships.

Look for productive strategies:

- Identifying parts of the puzzles that can only be formed by one specific piece, and then narrowing down the likely positions of other pieces.
- Flexibly working with the two small triangles and the square and parallelogram.

3 Connect

Display the legend and the puzzles as needed.

Have pairs of students share which shape(s) they found challenging and why.

Ask, "What motivated you to keep trying even when you hit challenging moments?"

Highlight what students did to work together to stay focused on solving the puzzles. Say, "This is true of working in pairs, groups, or as a whole class." Note how the same seven pieces were rearranged and placed together in several different ways to create very different-looking shapes and figures.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Technology*

Provide copies of the *Tangram* PDF. Students can match each tangram piece to the outlined shape. Creating the square is the most important as it is referenced in Activity 2 and in Lesson 2. Alternatively, have students use the Amps slides, in which they can position tangram pieces that snap to each other, eliminating the chance for gaps or overlaps.

Extension: *Math Enrichment*

Have students complete the puzzles of the sage and king from the *Are you ready for more?* PDF.

Math Language Development

MLR3: Critique, Correct, Clarify

Show the two large triangles forming the top of the sailboat sail. Ask:

Critique: "What if someone suggested using the two triangles to fill this space. Would you agree or disagree?"

Correct, Clarify: "Could you form the figure if the two triangles are used in this way? How would you explain why it has to be the medium triangle at the top of the sail?"

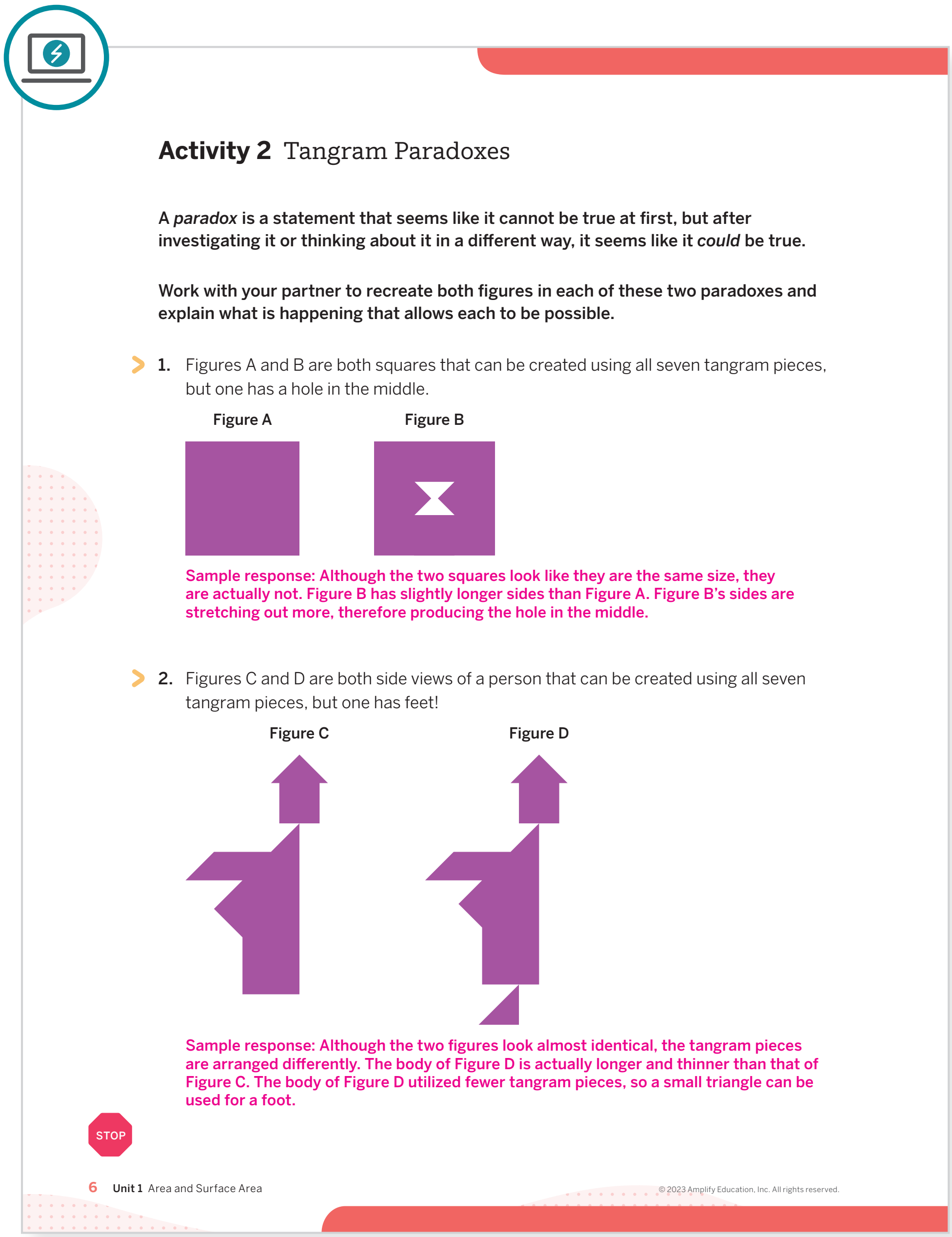
English Learners

Have students physically demonstrate filling the space using the two large triangles and the medium triangle.

Pairs | 20 min

Activity 2 Tangram Paradoxes

Students use their tangram pieces to recreate shapes that *appear* to be identical.



Activity 2 Tangram Paradoxes

A *paradox* is a statement that seems like it cannot be true at first, but after investigating it or thinking about it in a different way, it seems like it *could* be true.

Work with your partner to recreate both figures in each of these two paradoxes and explain what is happening that allows each to be possible.

1. Figures A and B are both squares that can be created using all seven tangram pieces, but one has a hole in the middle.

   Figure A

   Figure B

   Sample response: Although the two squares look like they are the same size, they are actually not. Figure B has slightly longer sides than Figure A. Figure B's sides are stretching out more, therefore producing the hole in the middle.

2. Figures C and D are both side views of a person that can be created using all seven tangram pieces, but one has feet!

   Figure C

   Figure D

   Sample response: Although the two figures look almost identical, the tangram pieces are arranged differently. The body of Figure D is actually longer and thinner than that of Figure C. The body of Figure D utilized fewer tangram pieces, so a small triangle can be used for a foot.

STOP

6 **Unit 1** Area and Surface Area

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1 Launch

Rearrange students into new pairs. Ensure students understand the meaning of the term *paradox*. Note to students that they have already made the first square in Activity 1.

2 Monitor

Help students get started by encouraging them to share ideas to unblock themselves, asking, "How could you get started on this activity?"

Look for points of confusion:

- **Thinking one puzzle can be made using six pieces and the other by just adding the last piece.** Remind students that all seven pieces must be used to form both figures, and their arrangements may actually be quite different.

Look for productive strategies:

- Recognizing the two small triangles can be used as substitutes for squares or parallelograms.
- Noticing that the reason why the sizes of the two figures are slightly different is due to the different composition of tangram pieces.
- Placing the outline of one figure on top of the other to compare.

3 Connect

Display the outlined solutions from the Activity 2 PDF, showing Figures A and B first, and then Figures C and D.

Have pairs of students share what they noticed when recreating and comparing each pair of figures. Note if students say something about placing one on top of the other and they are not matching up. If this does not come up in the share time, make sure to highlight this to students.

Highlight that mathematicians are precise. Students cannot assume things are the same just because they "look" the same, as was the case with the silhouettes .

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide students with a copy of the Activity 2 PDF and have them cut out Figures A and B, or pre-cut the figures for them. By actually manipulating the squares, they may be able to more readily realize that Figure B is slightly larger than Figure A. Alternatively, consider providing the figures already precreated for students to use.

Extension: *Math Enrichment*

Have students work on the eight tangram puzzles from the *Tangram Puzzles* PDF. Answers are provided for each puzzle within the PDF.

Math Language Development

MLR1: Stronger and Clearer Each Time

Partners should individually write their responses to each problem, share their responses, and revise their responses together to clarify their oral and written language.

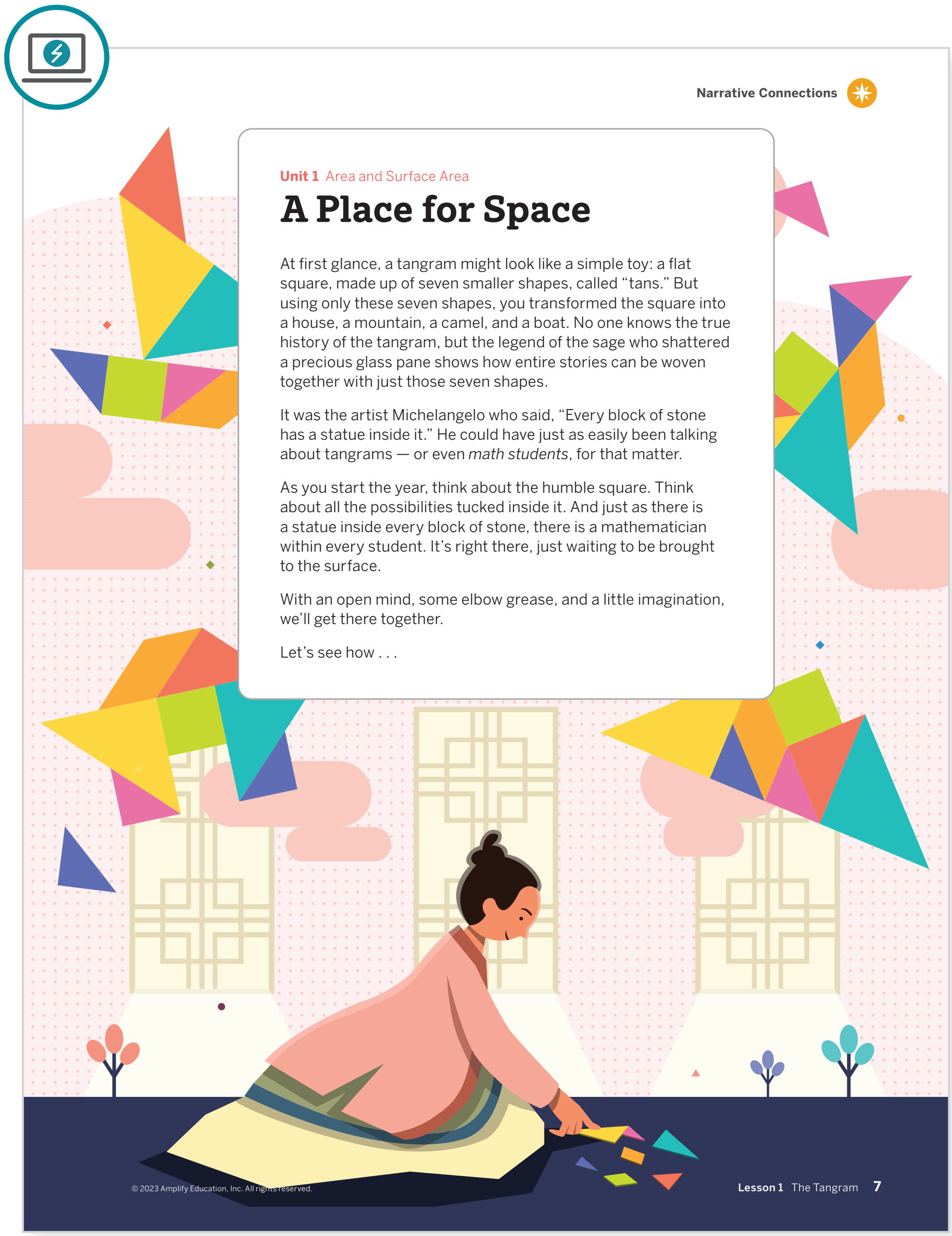
English Learners

Encourage students to write their first response in their primary language. This will help to lower the students' affective filter as they make sense of and explain what is happening with each of the tangram figures.

Whole Class | 5 min

Summary A Place for Space

Review and synthesize how a classroom composed of individuals is like a figure harmoniously composed of tangram pieces.



Narrative Connections

Unit 1 Area and Surface Area

A Place for Space

At first glance, a tangram might look like a simple toy: a flat square, made up of seven smaller shapes, called "tans." But using only these seven shapes, you transformed the square into a house, a mountain, a camel, and a boat. No one knows the true history of the tangram, but the legend of the sage who shattered a precious glass pane shows how entire stories can be woven together with just those seven shapes.

It was the artist Michelangelo who said, "Every block of stone has a statue inside it." He could have just as easily been talking about tangrams — or even *math students*, for that matter.

As you start the year, think about the humble square. Think about all the possibilities tucked inside it. And just as there is a statue inside every block of stone, there is a mathematician within every student. It's right there, just waiting to be brought to the surface.

With an open mind, some elbow grease, and a little imagination, we'll get there together.

Let's see how . . .

Lesson 1 The Tangram 7

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display and read the Summary aloud.

Ask:

- "How is our class like a tangram set?"
- "How is working together a lifelong *math* skill?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

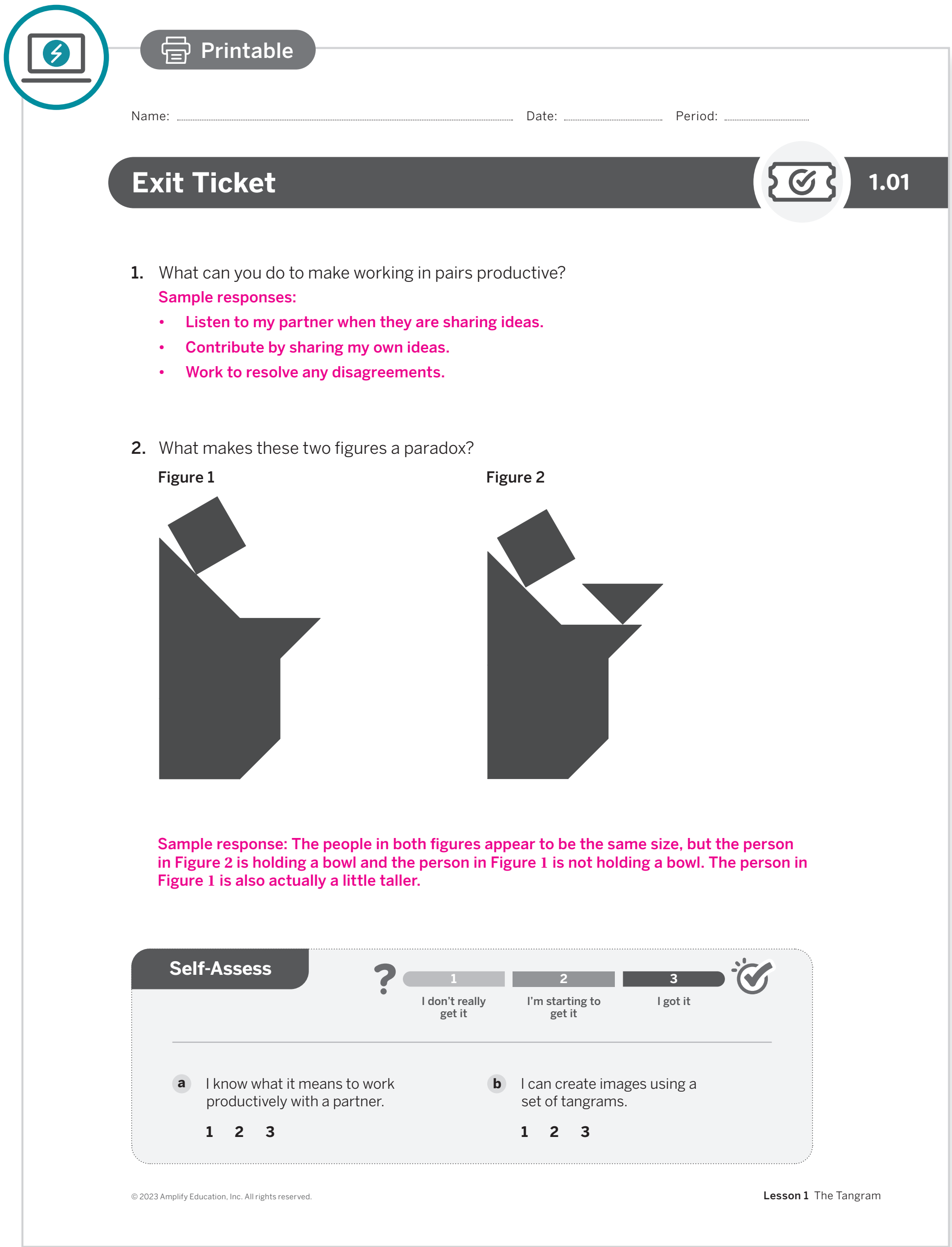
- "What do you think was the author's message in the Tangram Legend?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of successful partnerships by writing how they can be productive collaborators and analyzing a tangram paradox.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.01

1. What can you do to make working in pairs productive?

Sample responses:

- Listen to my partner when they are sharing ideas.
- Contribute by sharing my own ideas.
- Work to resolve any disagreements.

2. What makes these two figures a paradox?

Sample response: The people in both figures appear to be the same size, but the person in Figure 2 is holding a bowl and the person in Figure 1 is not holding a bowl. The person in Figure 1 is also actually a little taller.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I know what it means to work productively with a partner. 1 2 3

b I can create images using a set of tangrams. 1 2 3

Lesson 1 The Tangram

Success looks like . . .

- **Language Goal:** Recognizing and describing elements for productive collaboration with a partner **(Speaking and Listening, Writing)**.
  - » Listing different ways to work effectively with a partner in Problem 1.
- **Goal:** Using spatial reasoning to solve tangram puzzles.
  - » Determining why Figures 1 and 2 are a paradox in Problem 2.

Suggested next steps

If students write about work relationships as only being successful when the correct solution is reached in the end, consider:

- Having a pair of students model respectful speaking and listening behaviors, without reaching the correct answer.

Math Language Development: If students have trouble understanding what a *paradox* is, refer back to Activity 2 and have the student read the opening paragraph and have them look at the second paradox. Ask, "How can those two figures be composed of the same 7 pieces if one has a foot and one does not?" Although the two figures look almost identical, the tangram pieces are arranged differently. The body of Figure 2 is thinner than that of Figure 1. The body of Figure 1 utilized fewer tangram pieces, so a small triangle can be used for a bowl.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The focus of this lesson was on the tangram. What worked and didn't work today? What did you notice about student engagement with the tangram today?
- How did students collaborate today? How will you help students be more aware of their actions and behaviors as collaborators the next time you facilitate this lesson?

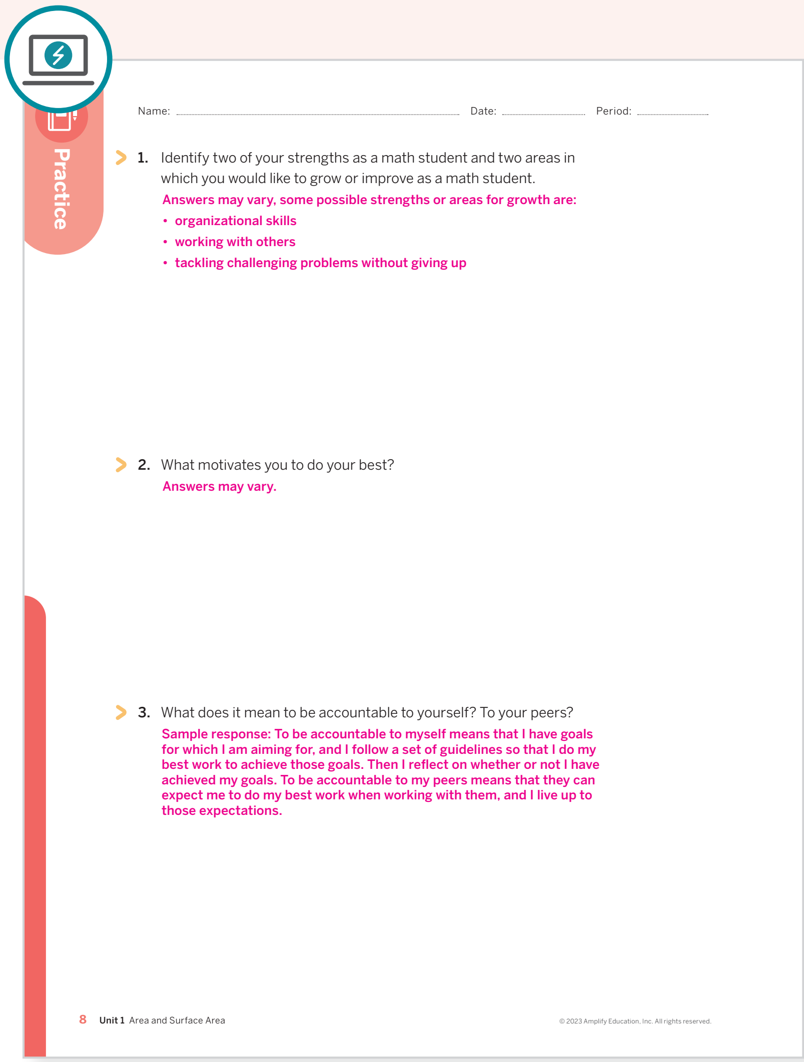
Differentiated Support

Accessibility: ***Optimize Access to Tools, Guide Processing and Visualization***

For Problem 2, provide students with a physical copy of Figures 1 and 2 to use as they respond to this problem.

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Identify two of your strengths as a math student and two areas in which you would like to grow or improve as a math student.
   Answers may vary, some possible strengths or areas for growth are:
   - **organizational skills**
   - **working with others**
   - **tackling challenging problems without giving up**

2. What motivates you to do your best?
   Answers may vary.

3. What does it mean to be accountable to yourself? To your peers?
   Sample response: To be accountable to myself means that I have goals for which I am aiming for, and I follow a set of guidelines so that I do my best work to achieve those goals. Then I reflect on whether or not I have achieved my goals. To be accountable to my peers means that they can expect me to do my best work when working with them, and I live up to those expectations.

8 **Unit 1** Area and Surface Area

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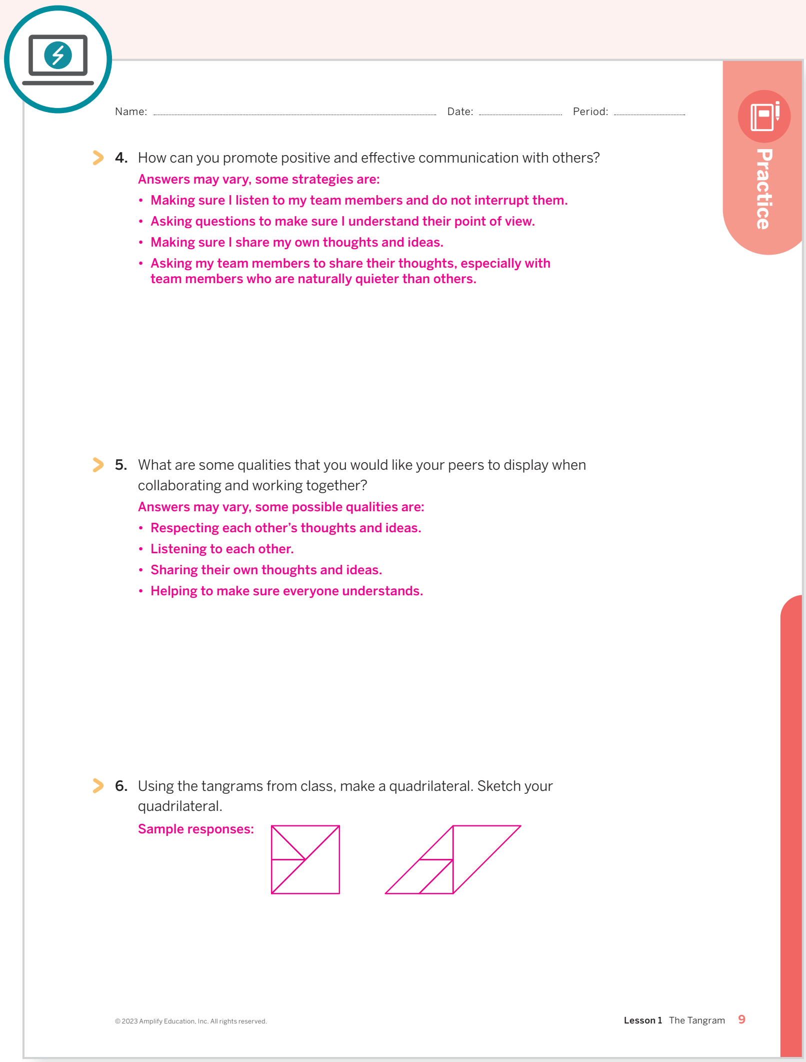

Practice

Name: ______ Date: ______ Period: ______

4. How can you promote positive and effective communication with others?
   Answers may vary, some strategies are:
   - **Making sure I listen to my team members and do not interrupt them.**
   - **Asking questions to make sure I understand their point of view.**
   - **Making sure I share my own thoughts and ideas.**
   - **Asking my team members to share their thoughts, especially with team members who are naturally quieter than others.**

5. What are some qualities that you would like your peers to display when collaborating and working together?
   Answers may vary, some possible qualities are:
   - **Respecting each other's thoughts and ideas.**
   - **Listening to each other.**
   - **Sharing their own thoughts and ideas.**
   - **Helping to make sure everyone understands.**

6. Using the tangrams from class, make a quadrilateral. Sketch your quadrilateral.
   Sample responses:

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Lesson 1 The Tangram 9

Practice Problems 1–6 prepare students for the collaborative work they will encounter in the upcoming unit and throughout this course.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Exploring the Tangram

Let’s make patterns using tangram pieces.

Focus

Goals

1. **Language Goal:** Follow the steps of given instructions for creating a set of tangrams. **(Speaking and Listening)**
2. Use the rules of tangrams to create a puzzle composed of all seven pieces.
3. **Language Goal:** Recognize and describe elements for productive collaboration in small groups. **(Speaking and Listening)**

Coherence

Today

Students continue developing and practicing positive and effective collaboration skills while engaging in a mathematical activity in the Warm-up. The remainder of the lesson is done independently to assure students who may still be feeling uncomfortable in groups that they will have moments to themselves, but also to contrast and better highlight the benefits of collaboration. Independent work during class time will be less frequent throughout the curriculum, but certainly present and equally valuable. Each student first creates their own set of tangram pieces, and then uses those to design their own puzzle. The puzzles are collected and displayed to represent how a single task can bring out tremendous variety from within the classroom community, all different but all tied together in principles and shared experiences.

Previously

In Lesson 1, students began their mathematical school year focusing on classroom expectations for collaborative work and giving a narrative context to the tangram.

Coming Soon

In Lesson 3, students ease into the unit exploring the tools that will be available to them in their geometry toolkit. They will be reintroduced to area, building on prior work with the concept of area they studied in Grades 2–5.

Rigor

- Students make a tangram set and create a design to build **conceptual understanding** of geometric shapes in space.
- Students continue to develop their **conceptual understanding** of collaboration and constructive partnerships with peers.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2 (optional)	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Small Groups	Independent	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

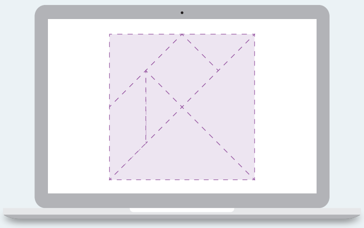
Materials

- Exit Ticket
- Additional Practice
- *Are you ready for more?* PDF (answers)
- Activity 2 PDF, one per student
- Unit 1 PDF, *Tangram*, one per student (optional)
- sets of tangrams, 1 set per student, or 4 sets for a group of four
- pre-cut 8 in. × 8 in. squares from regular white copy paper, one per student

Amps | **Featured Activity**

Activity 2
Interactive Tangram Set

Students can click, drag, and position tangram pieces with digital precision as they make their own tangram puzzles, and you can project their work to the class.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may resist rearranging tangram pieces and become frustrated thinking that there is only one way a certain-sized square could have been created. Have students share with a partner to see other possibilities and then reflect on how they might think differently in the future.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time:

- In the **Warm-up**, adjust the number of squares students need to put together.
- In **Activity 1**, have students cut the tangram outline from the Activity 1 PDF of the tangram set. Students would no longer be folding the square, but will achieve the same outcome with the seven tangram pieces.

Small Groups | 10 min

Warm-up How Many Squares?

Students practice collaborating with others as they work in small groups to create any number of squares out of tangram pieces.

Unit 1 | Lesson 2 – Launch

Exploring the Tangram

Let's make patterns using tangram pieces.

Warm-up How Many Squares?

Using four or more of the pieces from a tangram set, how many different ways can you build a square of any size? Record the tangram pieces used for each square that your group builds, and then sketch the square.

Sample responses:

Tangram pieces used	Square
four pieces: two small triangles, one medium triangle, one large triangle	
four pieces: two small triangles, one large triangle, one parallelogram	
four pieces: two small triangles, one large triangle, one square	
five pieces: two small triangles, one medium triangle, one parallelogram, one square	

Compare and Connect: After completing the Warm-up, share with a partner what is different and what is similar among the ways the squares have been composed.

10 **Unit 1** Area and Surface Area

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1 Launch

Distribute tangram sets or the Unit 1 PDF. Say, "In Lesson 1, you formed a square using all 7 tangram pieces. In this lesson, you will work in groups to try and form squares of different sizes using 4 or more pieces." Model forming squares with 1, 2, and 3 tangram pieces. Give students 5 minutes of work time and remind them of the "Good Collaborators" chart.

2 Monitor

Help students get started by identifying pieces and asking, "What about these pieces (e.g., two small triangles, one medium triangle, one large triangle)? Can you form a square with these?"

Look for points of confusion:

- **Thinking only one possible square configuration can be made.** Encourage students to think of new configurations.

Look for productive strategies:

- Arranging and rearranging using special combinations of pieces to fill the same space e.g., two small triangles to fill the space of the square or medium triangle.

3 Connect

Display a tangram set that can be used to show different square configurations.

Have groups of students share the squares they formed, starting with those who used four pieces, and then those who used five pieces. Focus on how they worked as a group to place them together.

Ask, "Did any group form a square using six pieces?" **Note:** This is not possible.

Highlight how working together and persevering as a group is similar to effectively working in pairs: you need to be respectful, listen to others, and freely share ideas.

Math Language Development

MLR7: Compare and Connect

During the Connect, using the three different configurations of four tangram pieces as examples, ask students to reflect on and linguistically respond to what is different and what is similar among the ways the squares have been composed.

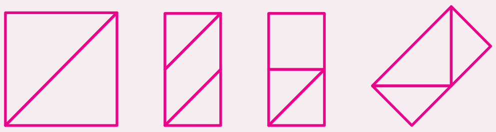
English Learners

Have students first use highlighters or colored pencils to highlight what is similar (one color) and different (another color) among the tangram pieces used.

Power-up

To power up students' ability to create polygons with specific characteristics using tangrams, have students create as many unique (different-sized) rectangles as they can using tangrams.

Sample responses:



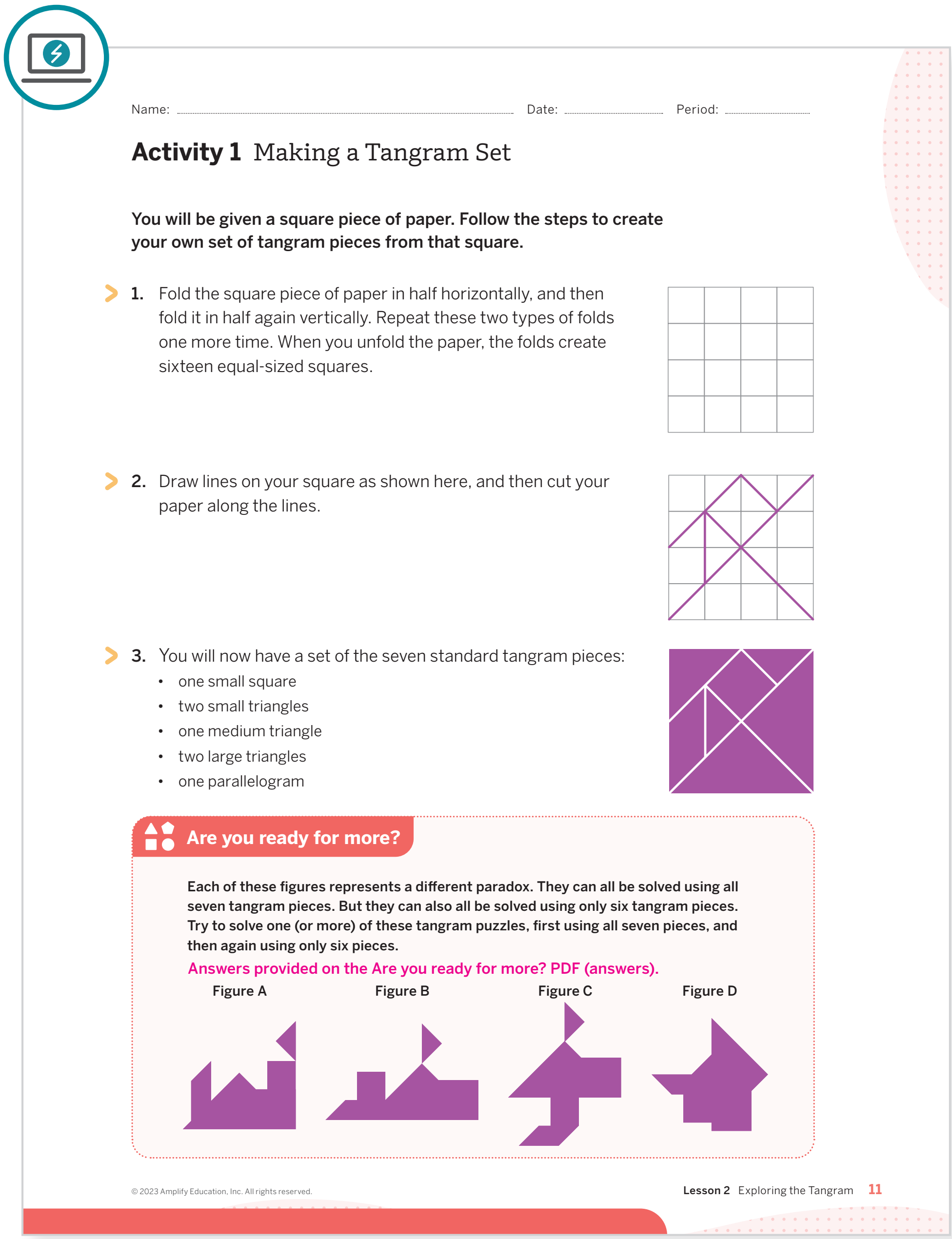
Use: Before the Warm-up.

Informed by: Lesson 1, Practice Problem 6.

Independent | 10 min

Activity 1 Making a Tangram Set

Students practice following step-by-step directions as they create their own tangram set from a square.



Name: ____________ Date: ________ Period: ________

Activity 1 Making a Tangram Set

You will be given a square piece of paper. Follow the steps to create your own set of tangram pieces from that square.

1. Fold the square piece of paper in half horizontally, and then fold it in half again vertically. Repeat these two types of folds one more time. When you unfold the paper, the folds create sixteen equal-sized squares.

2. Draw lines on your square as shown here, and then cut your paper along the lines.

3. You will now have a set of the seven standard tangram pieces:
   - one small square
   - two small triangles
   - one medium triangle
   - two large triangles
   - one parallelogram

Are you ready for more?

Each of these figures represents a different paradox. They can all be solved using all seven tangram pieces. But they can also all be solved using only six tangram pieces. Try to solve one (or more) of these tangram puzzles, first using all seven pieces, and then again using only six pieces.

Answers provided on the Are you ready for more? PDF (answers).

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Lesson 2 Exploring the Tangram 11

1 Launch

Activate students' prior knowledge by asking them how many pieces are in a tangram. Distribute the pre-cut squares. Extra squares should be made available in case a student needs to start over.

2 Monitor

Help students get started by reading the first step aloud and modeling how to fold the paper.

Look for points of confusion:

- **Skipping a step in the instructions.** Have students go back to a step they may have missed.

Look for productive strategies:

- Referring back to the step-by-step directions frequently to know what to do next.
- Using the fold lines to precisely draw the needed line segments before cutting.

3 Connect

Display the final seven pieces the students should have formed as a square. Consider setting the stage for the next activity by also having a memorized design of your own into which you can form the pieces.

Highlight that the pieces were formed with specific instructions to be followed and that ensures that everyone has a set composed of the same number of pieces of each shape and size.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students cover the steps with a piece of paper, uncovering each step one at a time as they are ready. This will help them not be overwhelmed or accidentally skip ahead. In Problem 2, you may also consider providing the image on a piece of paper that has the cutting lines creased so that students can feel the lines that need to be cut.

Accessibility: *Optimize Access to Tools*

Make the tangram outline from the Unit 1 PDF, *Tangram*, available for students to cut out. This will ensure that learning tools are physically accessible to all students.

Math Language Development

MLR6: Three Reads

During the Launch and Monitor, have students use this routine to make sense of the steps for creating their own set of tangram pieces.

- **Read 1:** Students read each step with the goal of comprehending the text and the image next to that step.
- **Read 2:** Students read with the goal of analyzing the language used in each step, such as *horizontally* and *vertically*.
- **Read 3:** Students read each step as they perform the task described.

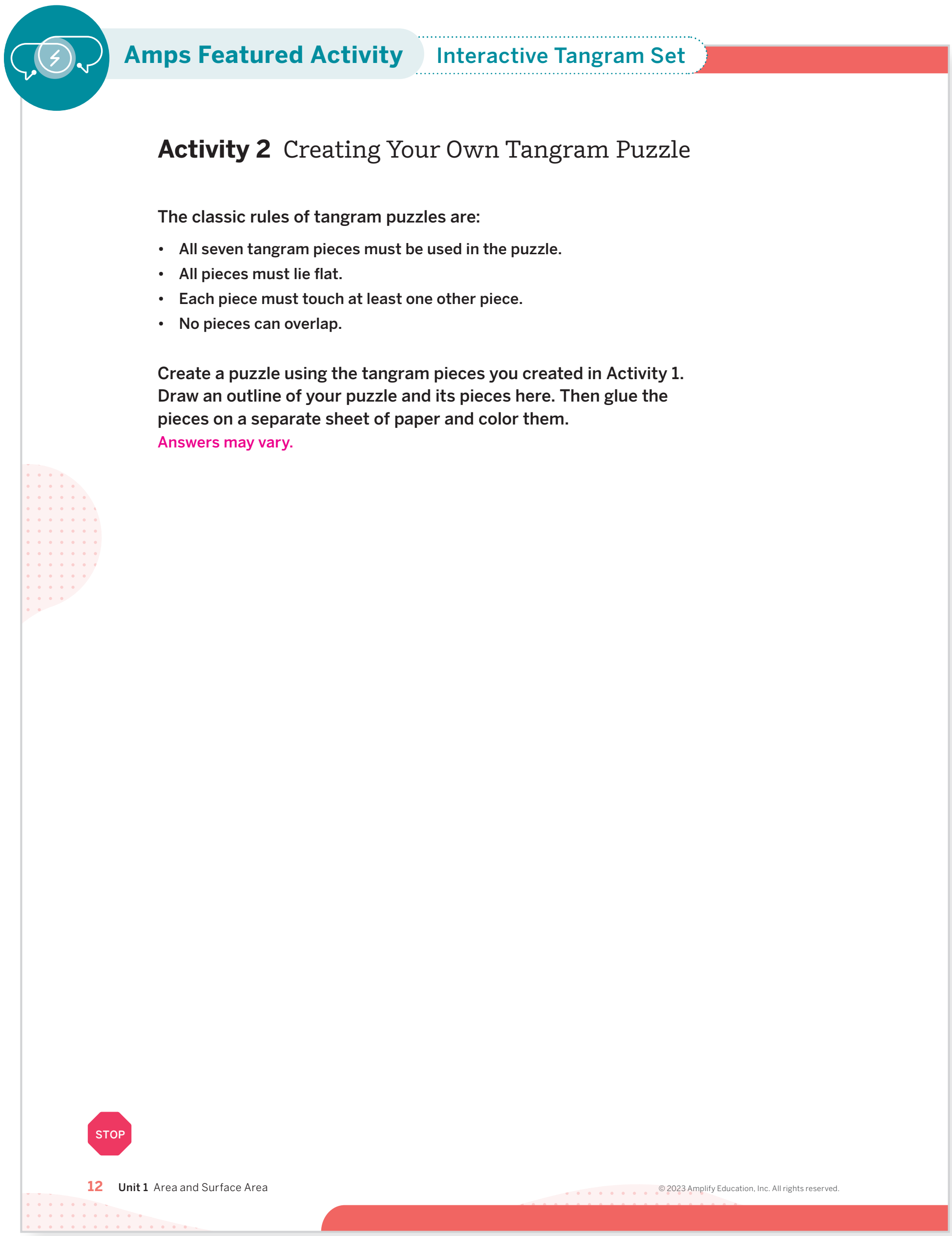
English Learners

For the second read, highlight the language by using gestures when amplifying horizontal and vertical folds.

Independent | 15 min

Activity 2 Creating Your Own Tangram Puzzle

Students create a design with their tangram set. When displayed together in the classroom, these designs represent the individual and collective nature of the class.



Amps Featured Activity Interactive Tangram Set

Activity 2 Creating Your Own Tangram Puzzle

The classic rules of tangram puzzles are:

- All seven tangram pieces must be used in the puzzle.
- All pieces must lie flat.
- Each piece must touch at least one other piece.
- No pieces can overlap.

Create a puzzle using the tangram pieces you created in Activity 1. Draw an outline of your puzzle and its pieces here. Then glue the pieces on a separate sheet of paper and color them.
Answers may vary.

STOP

12 **Unit 1** Area and Surface Area

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1 Launch

Review the directions for the activity, making sure students understand that the best way to trace their design is to outline one piece at a time (i.e., avoid pushing all pieces aside and trying to free draw them).

Note: If you have additional time after students complete their designs, consider allowing 10 or more minutes for them to solve each other's puzzles using the Activity 2 PDF.

2 Monitor

Help students get started by reassuring them that there is no right or wrong design. They should experiment and explore until they like their design.

Look for points of confusion:

- **Thinking their design has to represent an object.** Remind students that their design can be an object or an abstract design.

Look for productive strategies:

- Arranging and rearranging tangram pieces to form desired shapes and parts of their design.
- Tracing one piece at a time as they make the outline.

3 Connect

Display all of the individual puzzle designs.

Have individual students share their inspiration and thinking behind creating and constructing their puzzle designs.

Ask, "What similarities do you notice in the designs of your classmates?"

Highlight how individual student work coming together to form a classroom display is a representation of individual ideas contributing to the classroom community.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

While the goal of this activity is for students to explore and experiment, consider providing a checklist to help students understand the task, plan the task, and ensure that each of the rules of the tangram puzzles are followed.

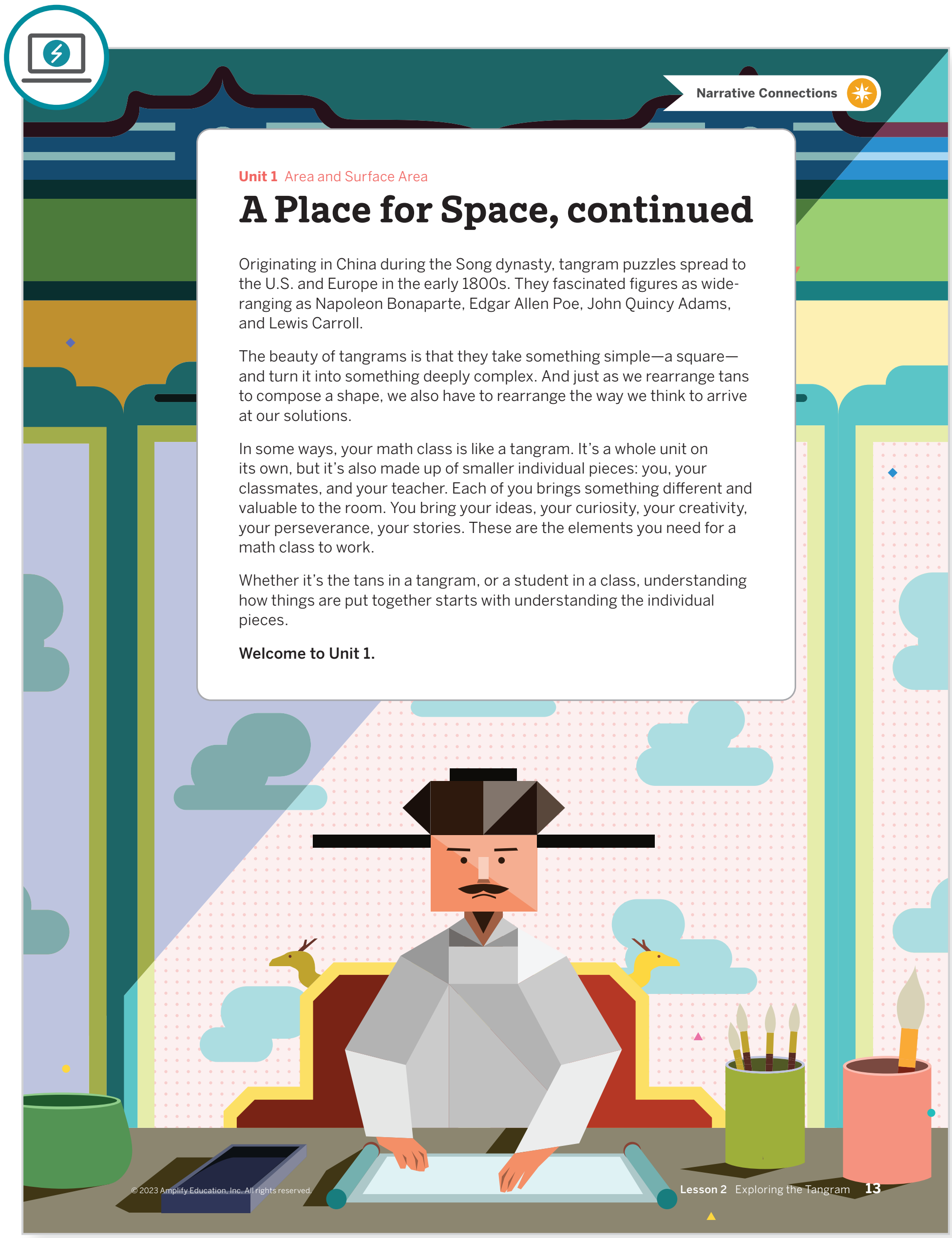
Accessibility: *Optimize Access to Tools*

Provide opportunities for students to engage with the actual tangram puzzle pieces, if available, rather than the paper version. This will allow students to benefit from a greater sense of tactile input.

Whole Class | 5 min

Summary A Place for Space (continued)

Review and synthesize how their work with the tangram reflects the collaboration, creativity, and flexible thinking of a mathematician.



Narrative Connections

Unit 1 Area and Surface Area

A Place for Space, continued

Originating in China during the Song dynasty, tangram puzzles spread to the U.S. and Europe in the early 1800s. They fascinated figures as wide-ranging as Napoleon Bonaparte, Edgar Allen Poe, John Quincy Adams, and Lewis Carroll.

The beauty of tangrams is that they take something simple—a square—and turn it into something deeply complex. And just as we rearrange tans to compose a shape, we also have to rearrange the way we think to arrive at our solutions.

In some ways, your math class is like a tangram. It's a whole unit on its own, but it's also made up of smaller individual pieces: you, your classmates, and your teacher. Each of you brings something different and valuable to the room. You bring your ideas, your curiosity, your creativity, your perseverance, your stories. These are the elements you need for a math class to work.

Whether it's the tans in a tangram, or a student in a class, understanding how things are put together starts with understanding the individual pieces.

Welcome to Unit 1.

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Lesson 2 Exploring the Tangram **13**

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display and read the Summary aloud.

Ask, "What was important to keep in mind as you were creating your puzzle design?" Sample response: It was important to keep in mind that the puzzle should have no gaps or overlaps of the pieces. It was also important to understand that if I am not happy with one design, I can rearrange the pieces and try again by being resilient and flexible.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

- "What did you discover about the tangram today?"

Differentiated Support

Extension: ***Math Around the World***

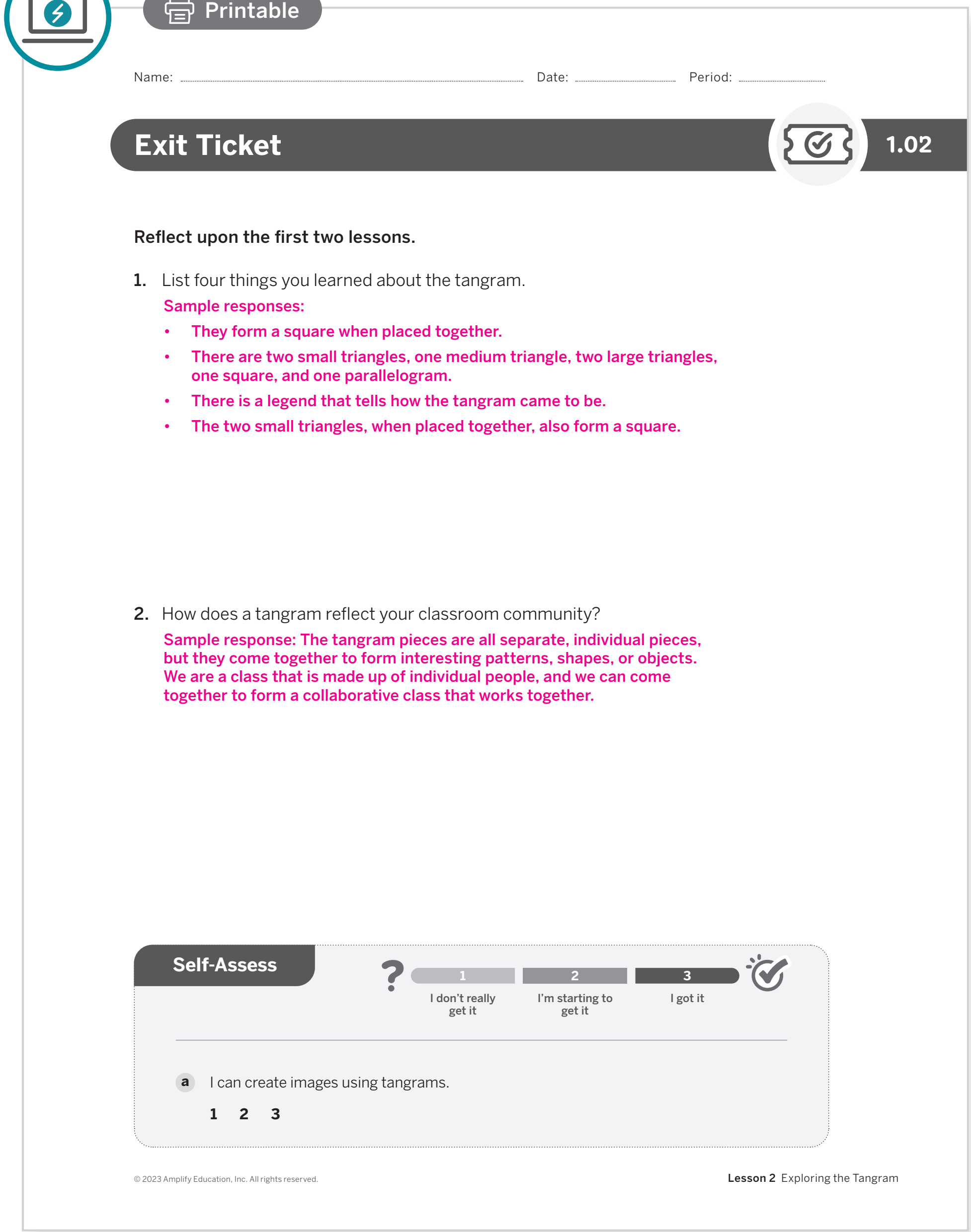
While the exact origin of the tangram puzzle is unknown, many scholars believe that it may have originated in China before the 18th century. The Chinese name for tangram is *qiqiao ban*, which means "7 ingenious pieces." Have students use the internet or another source to research the history of the tangram puzzle and explore other puzzles, such as the Huarong Pass Sliding Block Puzzle and Interlocking Burr Puzzles.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by reflecting on their work from the first two lessons and writing about the tangram and how it relates to their classroom community.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 1.02

Reflect upon the first two lessons.

1. List four things you learned about the tangram.

Sample responses:

- They form a square when placed together.
- There are two small triangles, one medium triangle, two large triangles, one square, and one parallelogram.
- There is a legend that tells how the tangram came to be.
- The two small triangles, when placed together, also form a square.

2. How does a tangram reflect your classroom community?

Sample response: The tangram pieces are all separate, individual pieces, but they come together to form interesting patterns, shapes, or objects. We are a class that is made up of individual people, and we can come together to form a collaborative class that works together.

Self-Assess

?	1	2	3
	I don't really get it	I'm starting to get it	I got it

a. I can create images using tangrams.

1 2 3

Lesson 2 Exploring the Tangram

Success looks like . . .

- **Language Goal:** Following the steps of given instructions **(Speaking and Listening)**.
- **Goal:** Understanding the "classic" rules of tangrams.
  - » Summarizing characteristics of a tangram in Problem 1.
- **Language Goal:** Relating the tangram to the classroom community **(Speaking and Listening, Writing)**.
  - » Comparing a tangram to the classroom community in Problem 2.

Suggested next steps

If students can only come up with one or two things about the tangram, this is acceptable, but consider:

- Referring back to any of the activities from Lessons 1 or 2 to help students generate ideas.

If students struggle to express how the tangram represents the class, consider:

- Asking, "Would you have expected the final display of all of the puzzles to be the same or different if that activity was done in pairs or groups? Why or why not? In what ways might it have been the same or different?"

Professional Learning

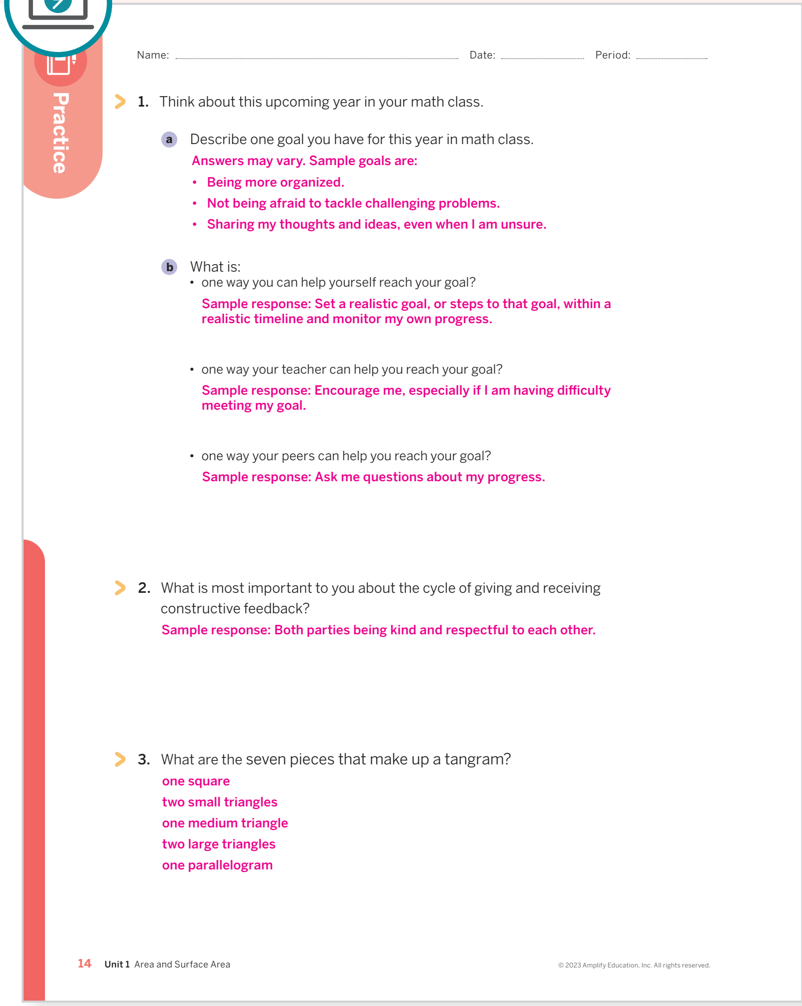
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What different ways did students approach creating a design with the tangram? What does that tell you about similarities and differences among your students and their willingness to be creative?
- In what ways did making their own tangram set go as planned? What might you do differently next time?

Practice

Independent


Practice

Name: ________ Date: ________ Period: ________

1. Think about this upcoming year in your math class.

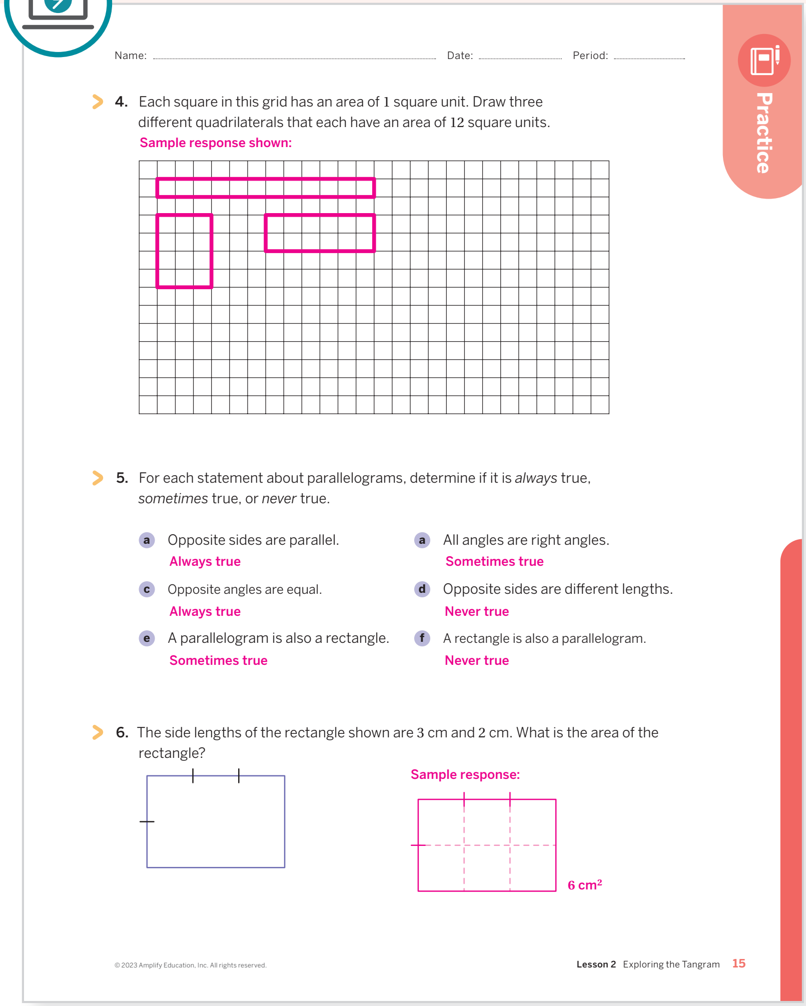
   a. Describe one goal you have for this year in math class.
   Answers may vary. Sample goals are:
   - **Being more organized.**
   - **Not being afraid to tackle challenging problems.**
   - **Sharing my thoughts and ideas, even when I am unsure.**

   b. What is:
   - one way you can help yourself reach your goal?
   Sample response: Set a realistic goal, or steps to that goal, within a realistic timeline and monitor my own progress.
   - one way your teacher can help you reach your goal?
   Sample response: Encourage me, especially if I am having difficulty meeting my goal.
   - one way your peers can help you reach your goal?
   Sample response: Ask me questions about my progress.

2. What is most important to you about the cycle of giving and receiving constructive feedback?
Sample response: Both parties being kind and respectful to each other.

3. What are the seven pieces that make up a tangram?
one square
two small triangles
one medium triangle
two large triangles
one parallelogram

14 **Unit 1** Area and Surface Area


Practice

Name: ________ Date: ________ Period: ________

4. Each square in this grid has an area of 1 square unit. Draw three different quadrilaterals that each have an area of 12 square units.
Sample response shown:

5. For each statement about parallelograms, determine if it is *always* true, *sometimes* true, or *never* true.

   a. Opposite sides are parallel.
   Always true

   a. All angles are right angles.
   Sometimes true

   c. Opposite angles are equal.
   Always true

   d. Opposite sides are different lengths.
   Never true

   e. A parallelogram is also a rectangle.
   Sometimes true

   f. A rectangle is also a parallelogram.
   Never true

6. The side lengths of the rectangle shown are 3 cm and 2 cm. What is the area of the rectangle?

Sample response:

6 cm²

Lesson 2 Exploring the Tangram 15

Practice Problem Analysis

Type	Problem	Refer to	DOK
On Lesson	1	*	1
	2	*	
	3	Activity 1	
Spiral	4	Grade 3	2
	5	Grade 5	2
Formative ⚡	6	Unit 1 Lesson 3	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

* Problems 1–2 prepare students for the collaborative work they will encounter in the upcoming unit and throughout this course.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 1
Area of Special Polygons

Whole Class

In this Sub-Unit, students compose and decompose two-dimensional shapes in order to determine their areas. They identify critical measurements and derive formulas for the area of any parallelogram and any triangle.

SUB-UNIT

1 Area of Special Polygons

Narrative Connections

Can a sum ever *really* be greater than its parts?

Nope. At least, not when it comes to geometry.

Most anything you can take apart, you can also put back together. But putting things together is the hard part!

You got a taste of this challenge with the tangrams you just saw. And if you've ever tried to assemble furniture, you know that things may not end up looking how they're supposed to, even when you use all the pieces.

So, while two different figures can be made up of exactly the same parts and take up the same amount of space, they can *still* look completely different.

When we're talking about two-dimensional figures, the amount of space a shape takes up is called its *area*.

Different shapes have their own relationships between their area and their dimensions — that is, their lengths. And if you know the area of one shape, you can figure out the area of any shape that can be made from it. As you'll see, when it comes to polygons, it always comes back to the humble triangle.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore breaking shapes apart and putting them back together in the following places:

- **Lesson 3, Activity 1:** Tiling the Plane
- **Lesson 9, Activity 1:** Decomposing Parallelograms
- **Lesson 13, Activity 1:** Stained Glass

UNIT 1 | **LESSON 3**

Tiling the Plane

Let's look at tiling patterns and think about area.

Focus

Goals

1. **Language Goal:** Compare areas of the shapes that make up a geometric pattern. **(Speaking and Listening)**
2. **Language Goal:** Comprehend that the term *area* refers to how much of the plane a shape covers. **(Speaking and Listening, Writing)**

Coherence

Today

This lesson begins with students exploring their geometry toolkits and discussing the possible uses for each tool. Students recall what they know about area and discover, or are reminded of, two important ideas:

- If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- A region can be decomposed and rearranged without changing its area.

Students also engage in activities that require them to make sense of problems and persevere in solving them. This opening lesson does not introduce complicated mathematics in order to intentionally leave space for establishing classroom routines and norms, particularly focusing on the expectations for mathematical discourse.

Previously

In Grade 3, students recognized area as an attribute of two-dimensional shapes that is measured by tiling unit squares without gaps or overlaps and is equal to the product of the side lengths (in the case of rectangles with whole number side lengths). This was extended to rectangles with fractional side lengths in Grade 5.

Coming Soon

In Lessons 4 and 5 students will focus on two-dimensional shapes and their areas, leading to a formal definition of *area* that can be leveraged in later lessons.

Rigor

- Students begin to develop their **conceptual understanding** of area by covering a space with no gaps or overlaps.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
15 min	20 min	5 min	5 min
Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, (for display)
- Unit 1 PDF, *Tangram*, one per person (optional)
- *Set of Pattern Blocks* PDF, one per person (optional)
- geometry toolkits: tracing paper, graph paper, colored pencils, scissors, index cards, set of paper pattern blocks (optional), set of paper tangrams (optional)

Math Language Development

Review words

- *area*
- *region*

Amps | **Featured Activity**

Activity 1
Interactive Grid

Students can toggle an isometric grid on or off as they make hypotheses about tiling the plane.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might be tempted to “play” with the tools and ignore how they could be used in math class or only think about less relevant uses. Use interactive modeling to show students that constructive play is allowed (and encouraged) but should be productive in discovering when and how to use tools and resources strategically and appropriately.

Modifications to Pacing

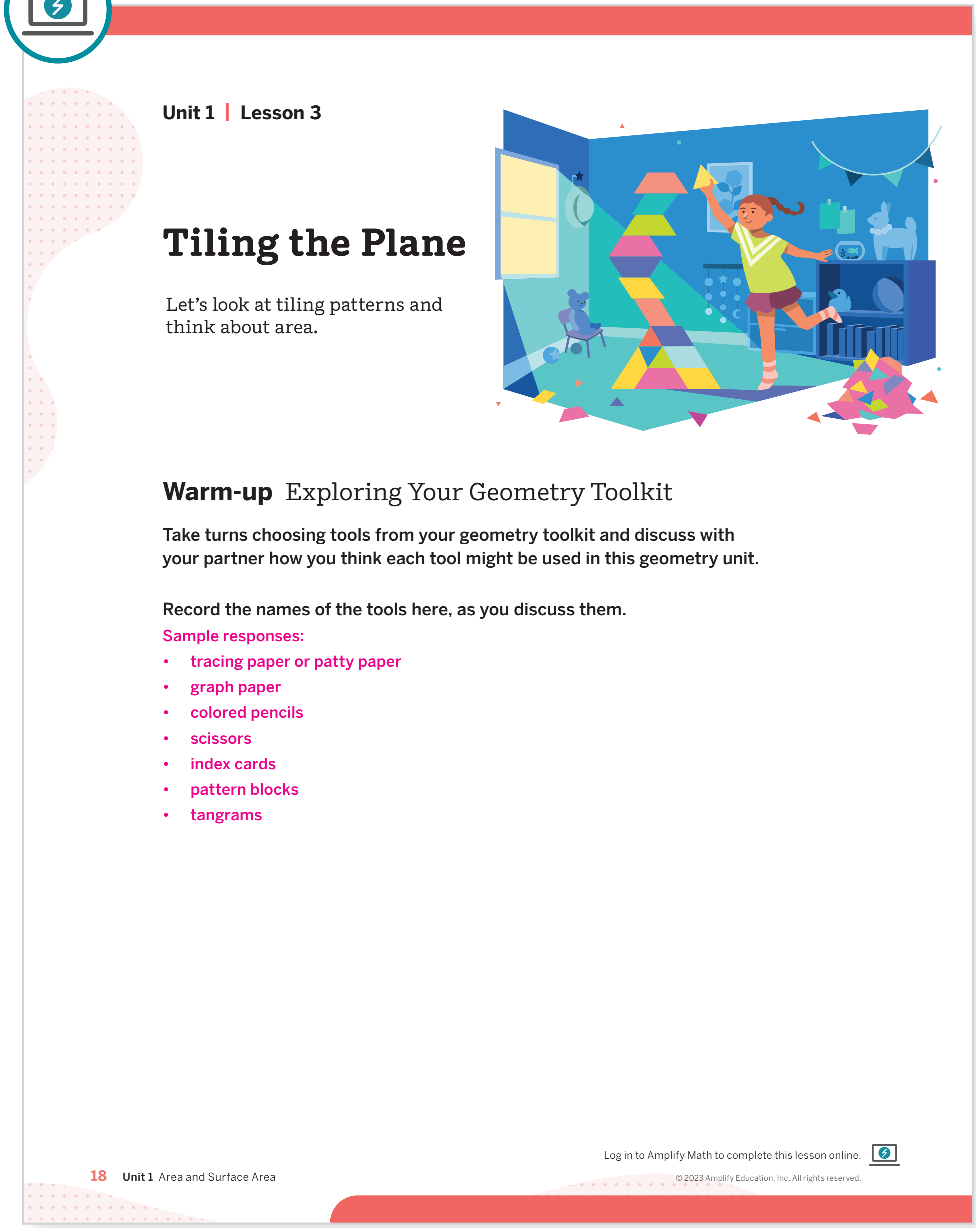
You may want to consider this additional modification if you are short on time.

- In the **Warm-up**, consider assigning each pair only one or two items from their toolkit to discuss.

Pairs | 15 min

Warm-up Exploring Your Geometry Toolkit

Students explore the tools in their geometry toolkits, discussing how each item might be used in a mathematical context.



Unit 1 | Lesson 3

Tiling the Plane

Let's look at tiling patterns and think about area.

Warm-up Exploring Your Geometry Toolkit

Take turns choosing tools from your geometry toolkit and discuss with your partner how you think each tool might be used in this geometry unit.

Record the names of the tools here, as you discuss them.

Sample responses:

- tracing paper or patty paper
- graph paper
- colored pencils
- scissors
- index cards
- pattern blocks
- tangrams

Log in to Amplify Math to complete this lesson online.

18 **Unit 1** Area and Surface Area

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1 Launch

Provide access to geometry toolkits, one per student or one per pair. Review and clarify the instructions with the whole class, and then give pairs time to work collaboratively, using the *Take Turns* routine.

2 Monitor

Help students get started by having them describe each tool first, before thinking about its possible use.

Look for points of confusion:

- **Discussing the tool in a non-math context.** Acknowledge the viability of their claim, but then ask, "How could you use this tool related to mathematics?" Activate prior knowledge by asking how they might have used the tool in math class in previous years.

Look for productive strategies:

- Identifying multiple ways in which a tool could be used.

3 Connect

Display the list of items in the geometry toolkit.

Have pairs of students share one idea for one tool, followed by other pairs' ideas for the same tool. Continue soliciting responses until all the tools have been discussed.

Ask, "How can you show respect for the tools you use? Why is this important?"

Highlight that these tools will always be available by saying, "Your toolkits will always be here, ready to be used whenever you feel that they can be of help."

Math Language Development

MLR5: Co-craft Questions

As students explore each of the tools in their geometry toolkit, use the *Notice and Wonder* routine to elicit student questions. For example, students can ask, "Could tracing paper be used to trace shapes?"

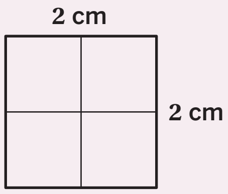
English Learners

Model for students an example of a question based on one of the tools provided in the toolkit. Consider using a *Think Aloud* strategy to demonstrate this. For example, say, "As I examine these paper pattern blocks, I wonder if I could use the scissors to cut these and rearrange them into a different pattern?"

Power-up

To power up students' ability to determine the area of rectangles with whole number side lengths, have students complete:

The side lengths of the square shown are 2 cm and 2 cm. Determine the area of the square.
4 cm^2



Use: Before Activity 1.

Informed by: Performance on Lesson 2, Practice Problem 5 and and Pre-Unit Readiness Assessment, Problem 1.

Pairs | 20 min

Activity 1 Tiling the Plane

Students compare the relative amounts of the plane covered by two similar patterns, reasoning about the conservation of area and composition and decomposition of shapes.

Amps Featured Activity Interactive Grid

Name: ______ Date: ______ Period: ______

Activity 1 Tiling the Plane

Filling spaces with tiles of different shapes and colors can be both decorative and beautiful. It is also very mathematical; mathematicians, such as Laura Escobar, have studied the connections between certain patterns of rhombus-shaped tiles and how letters can be ordered in different ways.

For now, you will be assigned either Pattern A or Pattern B. Determine which shape — rhombus, trapezoid, or triangle — covers more of the plane in your pattern. Be prepared to explain your thinking.

Pattern A

Sample response: The trapezoids cover more of the plane in both Pattern A and Pattern B, which are made up of the same quantity of each shape. In each large hexagon, there are 7 triangles, 4 rhombuses (with a total area equal to the area covered by 8 triangles), and 3 trapezoids (with a total area equal to the area covered by 9 triangles).

Pattern B

Collect and Display: Your teacher will circulate and collect key terms and phrases to add to a class display as your group discusses the patterns. Refer to this display during future discussions.

Featured Mathematician

Laura Escobar

Hailing from Bogotá, Colombia, Laura Escobar is a professor of mathematics at Washington University in St. Louis, Missouri. Her research lies at the intersection of algebra, geometry, and combinatorics (the mathematics of how things are arranged, and how many ways there are to arrange them). In 2016, she was the lead author on a paper that explored connections between special tilings of rhombuses, combinations of letters, and what are known as "Bott-Samelson varieties."

STOP

Lesson 3 Tiling the Plane 19

1 Launch

Arrange students into new pairs. Review and clarify the instructions with the whole class.

2 Monitor

Help students get started by asking, "How many triangles do you think could fit on top of a trapezoid?"

Look for points of confusion:

- **Thinking the shape that has the highest count of appearances is the shape that covers the most area.** Ask, "How could you prove whether your observation is correct?"

Look for productive strategies:

- Relating the area of each shape using a comparable unit — the triangle.
- Recognizing that the pattern being repeated makes up a larger hexagon, so only the shapes within one larger hexagon need to be compared.

3 Connect

Display both Pattern A and Pattern B, and consider having the Activity 1 PDF ready to display as well.

Have pairs of students share their answers and explanations, focusing on decomposing and rearranging figures so that they match when placed on top of each other, thus having the same area.

Highlight that the two patterns are made up of the same shapes, and therefore share the same conclusion about which shape covers more of the plane.

Ask:

- "What are the relationships among the areas of all the different shapes?"
- "Is it possible to compare the area of the rhombuses in Pattern A and the area of the triangles in Pattern B? Why?"

Differentiated Support

Accessibility: *Optimize Access to Tools, Optimize Access to Technology*

Provide students with a copy of the relevant image for the larger hexagon from their pattern, using the Activity 1 PDF. Alternatively, provide physical pattern blocks or have students use the Amps slides for Activity 1 in which they can position pattern blocks that snap to each other, eliminating the chance for gaps or overlaps.

Math Language Development

MLR2: Collect and Display

Collect and display examples of language that students use to describe and compare the different polygons and their areas. Students will revisit this display throughout the unit as they develop their use of mathematical language.

English Learners

Add visual examples to the display where applicable, such as what the phrase *covers the plane* means.

Featured Mathematician

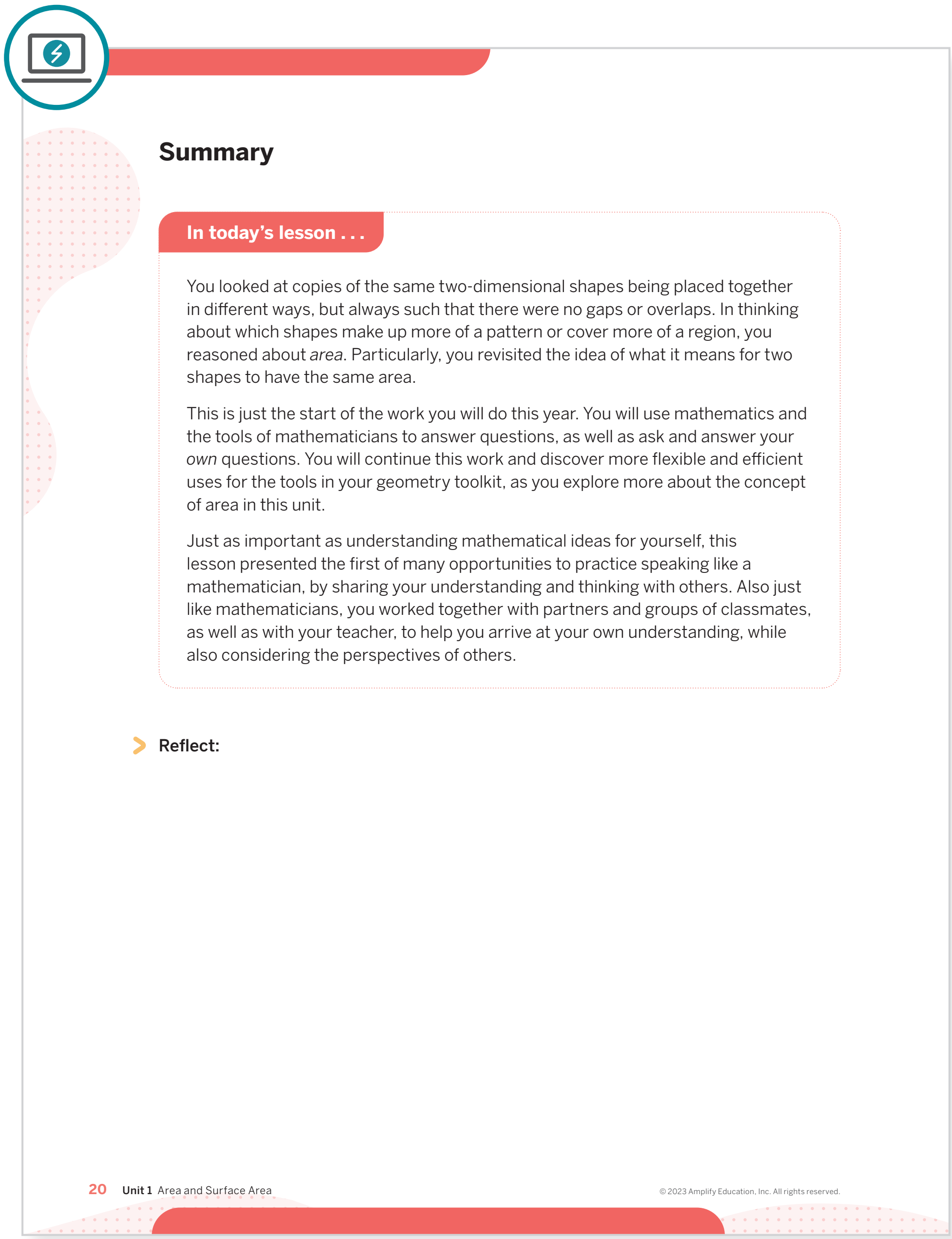
Laura Escobar

Have students read about Laura Escobar, Professor of Mathematics at Washington University in St. Louis, Missouri, and her research with combinatorics and special tilings.

Whole Class | 5 min

Summary

Review and synthesize what it means for two-dimensional shapes to have the same area, and remind students that their geometry toolkits will be used throughout the unit.



Summary

In today's lesson . . .

You looked at copies of the same two-dimensional shapes being placed together in different ways, but always such that there were no gaps or overlaps. In thinking about which shapes make up more of a pattern or cover more of a region, you reasoned about *area*. Particularly, you revisited the idea of what it means for two shapes to have the same area.

This is just the start of the work you will do this year. You will use mathematics and the tools of mathematicians to answer questions, as well as ask and answer your *own* questions. You will continue this work and discover more flexible and efficient uses for the tools in your geometry toolkit, as you explore more about the concept of area in this unit.

Just as important as understanding mathematical ideas for yourself, this lesson presented the first of many opportunities to practice speaking like a mathematician, by sharing your understanding and thinking with others. Also just like mathematicians, you worked together with partners and groups of classmates, as well as with your teacher, to help you arrive at your own understanding, while also considering the perspectives of others.

Reflect:

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Synthesize

Highlight that students have built on their understanding of the concept of area from earlier grades. They held conversations as mathematicians and used tools to help them in their pursuit of understanding mathematical problems. Throughout this unit, students will continue to reason about area in more complex ways, and throughout the year, they will talk about mathematics with their peers and to learn how to select and use tools strategically.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

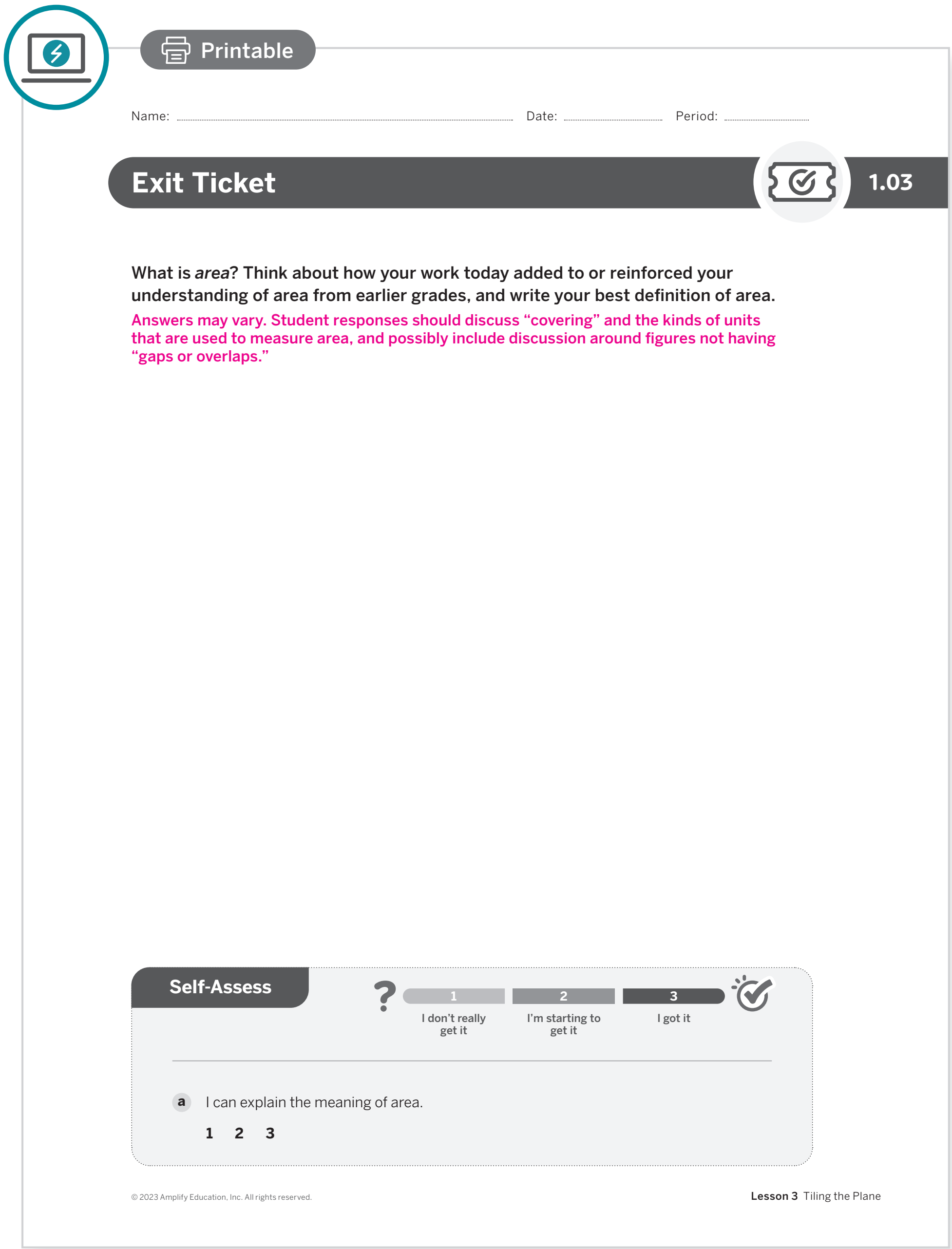
- "What do you remember about area from previous grades?"
- "What tools did you use today to help you determine the area of shapes?"

Independent | 5 min

Exit Ticket

Students demonstrate their working understanding of area by writing their own definition.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 1.03

What is *area*? Think about how your work today added to or reinforced your understanding of area from earlier grades, and write your best definition of area.

Answers may vary. Student responses should discuss "covering" and the kinds of units that are used to measure area, and possibly include discussion around figures not having "gaps or overlaps."

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain the meaning of area.

1 2 3

Lesson 3 Tiling the Plane

Success looks like . . .

- **Language Goal:** Comparing areas of the shapes that make up a geometric pattern **(Speaking and Listening)**.
- **Language Goal:** Comprehending that the term *area* refers to how much of the plane a shape covers **(Speaking and Listening, Writing)**.
  - » Explaining and defining the term *area*.

Suggested next steps

Math Language Development: If students cannot formulate a definition, consider having them write a brief summary of what they did during this lesson that they think was mathematical. Make it clear that they are not expected to be able to write a "dictionary definition" at this point. The most important part of this exercise is to begin practicing writing about mathematics by using their own words and perhaps starting to incorporate words they hear others use that make sense to them.

Professional Learning

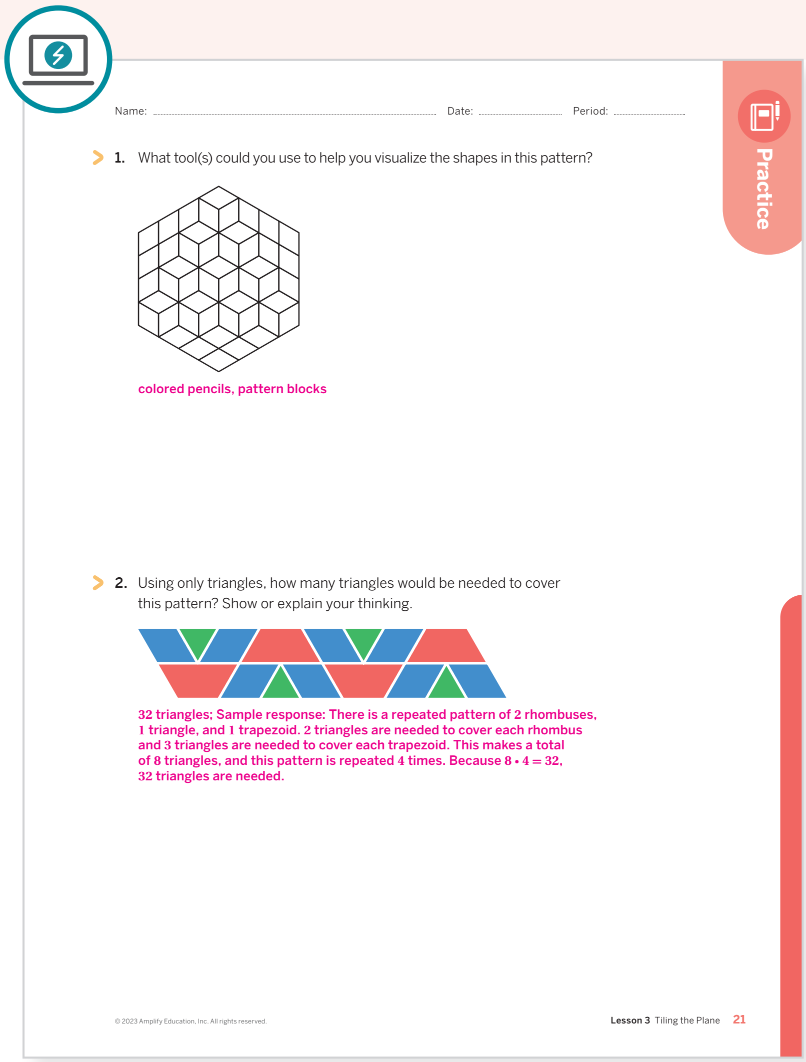
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students focused on the idea of covering a space with no gaps or overlaps. What worked and didn't work today? How did that build on the earlier work students did with recognizing area as an attribute of plane figures?
- In this lesson, students explored their geometry toolkits. What might you change the next time you facilitate this activity?

Practice

Independent



Name: Date: Period:

Practice

1. What tool(s) could you use to help you visualize the shapes in this pattern?

colored pencils, pattern blocks

2. Using only triangles, how many triangles would be needed to cover this pattern? Show or explain your thinking.

32 triangles; Sample response: There is a repeated pattern of 2 rhombuses, 1 triangle, and 1 trapezoid. 2 triangles are needed to cover each rhombus and 3 triangles are needed to cover each trapezoid. This makes a total of 8 triangles, and this pattern is repeated 4 times. Because $8 \cdot 4 = 32$, 32 triangles are needed.

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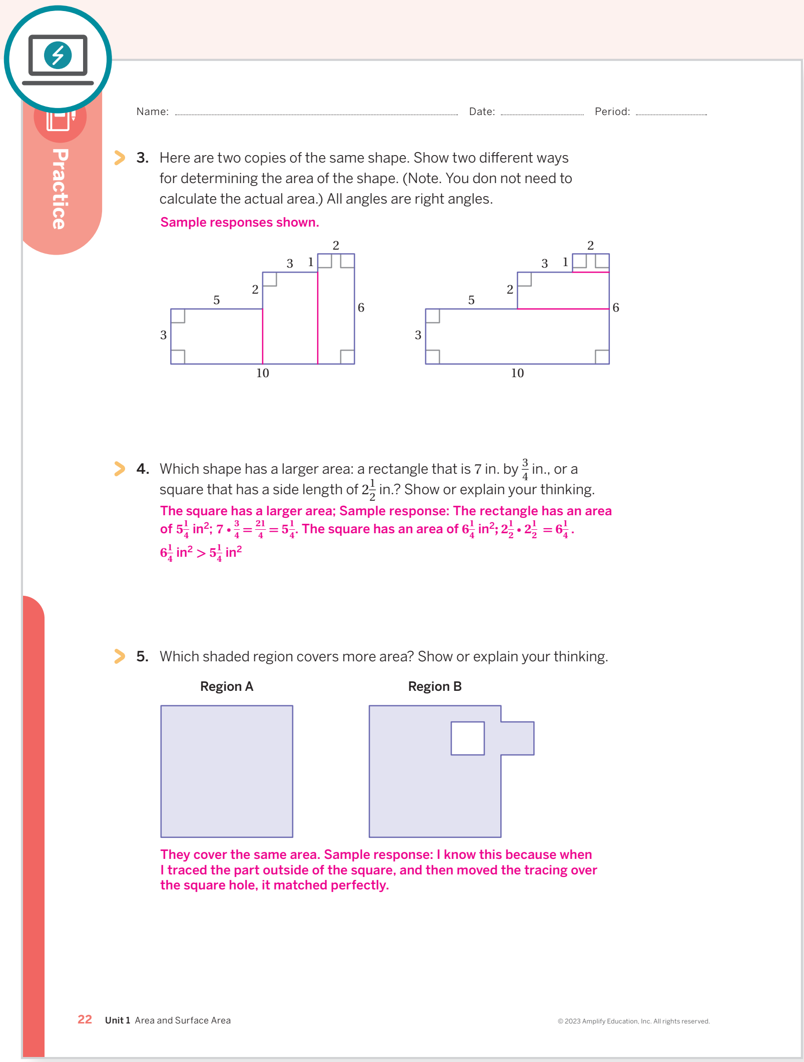
Lesson 3 Tiling the Plane 21



Name: Date: Period:

Practice

3. Here are two copies of the same shape. Show two different ways for determining the area of the shape. (Note. You don not need to calculate the actual area.) All angles are right angles.

Sample responses shown.

4. Which shape has a larger area: a rectangle that is 7 in. by $\frac{3}{4}$ in., or a square that has a side length of $2\frac{1}{2}$ in.? Show or explain your thinking.

The square has a larger area; Sample response: The rectangle has an area of $5\frac{1}{4}$ in^2; $7 \cdot \frac{3}{4} = \frac{21}{4} = 5\frac{1}{4}$. The square has an area of $6\frac{1}{4}$ in^2; $2\frac{1}{2} \cdot 2\frac{1}{2} = 6\frac{1}{4}$. $6\frac{1}{4}$ in^2 $> 5\frac{1}{4}$ in^2

5. Which shaded region covers more area? Show or explain your thinking.

They cover the same area. Sample response: I know this because when I traced the part outside of the square, and then moved the tracing over the square hole, it matched perfectly.

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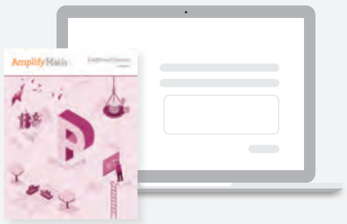
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	2
	2	Activity 1	2
Spiral	3	Grade 4	2
	4	Grade 5	2
Formative ⚡	5	Unit 1 Lesson 4	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 4**

Composing and Rearranging to Determine Area

Let's create shapes and determine their areas.

Focus

Goals

1. **Language Goal:** Calculate the area of a region by decomposing it and rearranging the pieces, and explain the solution method. **(Speaking and Listening, Writing)**
2. **Language Goal:** Recognize and explain that if two figures can be placed one on top of the other so that they match up exactly, they must have the same area. **(Speaking and Listening)**
3. Show that area is additive by composing polygons with a given area.

Coherence

Today

Students begin by comparing the area covered by different-sized squares. They extend their understanding that area is additive to non-rectangular shapes. They compose figures, consisting of triangles and a square, into shapes, using the square as the unit for determining a shape's area. Because students have only one square, they need to use these principles in their reasoning:

- If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- If a figure is decomposed and rearranged to compose another figure, then its area is the same as the area of the original figure.

Previously

In Grade 3, students learned to determine the area of a rectilinear figure by decomposing it into non-overlapping rectangles and adding their areas.

Coming Soon

Students will continue to reason with areas in Lesson 4. Then, in Lessons 6–13, students will apply their understanding of area to parallelograms, triangles, and other polygons.

Rigor

- Students are introduced to composing, decomposing, and rearranging shapes to build their **procedural skills** when determining area.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	25 min	5 min	5 min
Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- geometry toolkits

Math Language Development

New words

- *compose*
- *decompose*

Review words

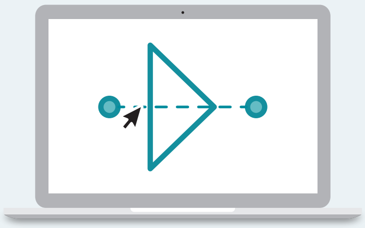
- *area*
- *region*

Amps | Featured Activity

Activity 1

Interactive Shapes

Students can click, drag, and position shapes with digital precision, to see how shapes can be composed and decomposed.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may not recognize connections among the problems in Activity 1. Have students focus on one problem at a time, and as they move from one problem to the next, model how they can ask themselves, "What is similar about this problem and ones I have already worked on? What is different or new about what it is asking me?" Encourage them to apply this reasoning to other problems, making the overall activity more manageable.

Modifications to Pacing

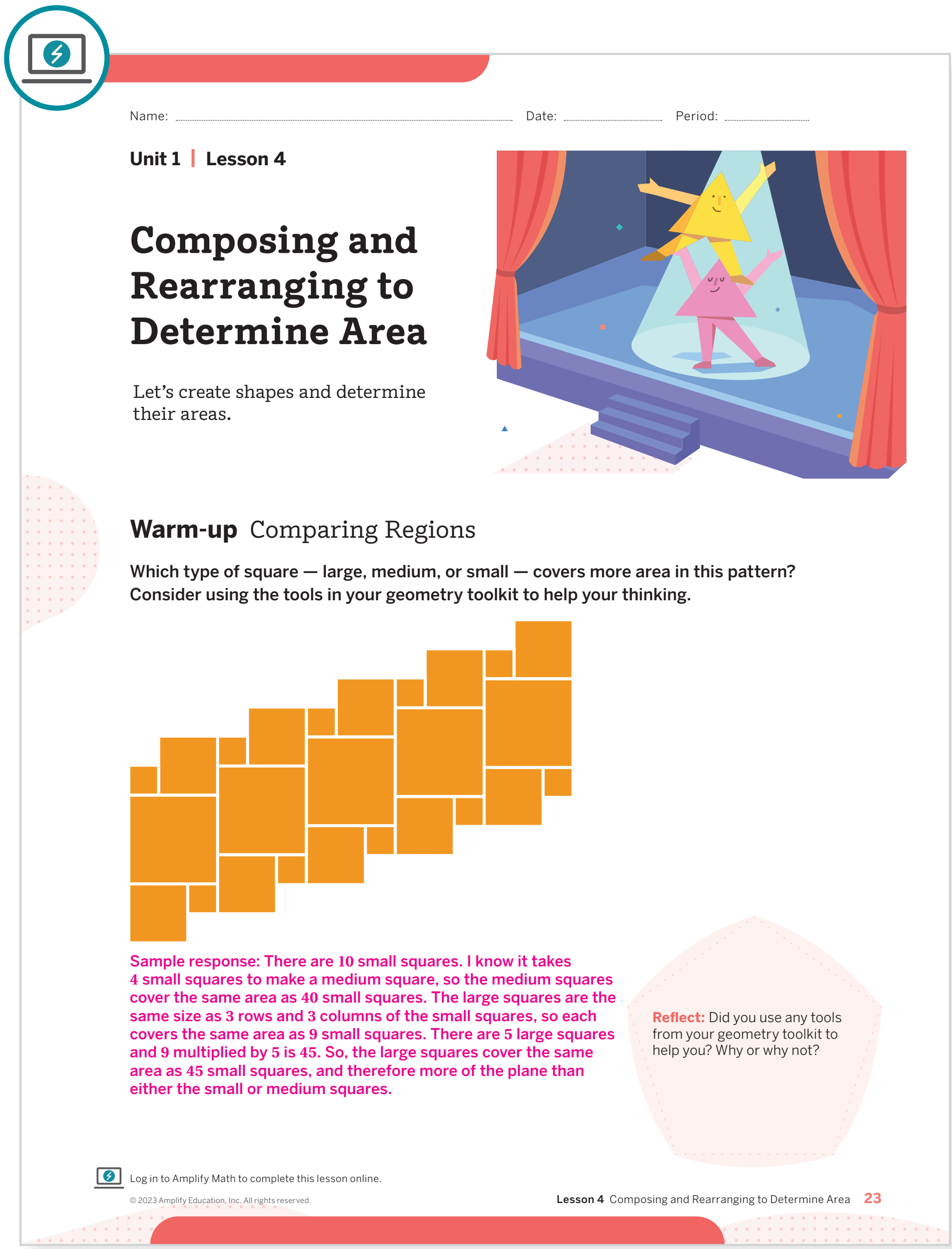
You may want to consider these additional modifications if you are short on time:

- In the **Warm-up**, discuss the problem as a whole class to reduce time spent on the *Think-Pair-Share* routine.
- In **Activity 1,** consider omitting Part 3.

Pairs | 10 min

Warm-up Comparing Regions

Students compare the amounts of the plane covered by three different-sized squares, reasoning about the area covered by each, relative to a common square unit.



Name: ______ Date: ______ Period: ______

Unit 1 | Lesson 4

Composing and Rearranging to Determine Area

Let's create shapes and determine their areas.

Warm-up Comparing Regions

Which type of square — large, medium, or small — covers more area in this pattern? Consider using the tools in your geometry toolkit to help your thinking.

Sample response: There are 10 small squares. I know it takes 4 small squares to make a medium square, so the medium squares cover the same area as 40 small squares. The large squares are the same size as 3 rows and 3 columns of the small squares, so each covers the same area as 9 small squares. There are 5 large squares and 9 multiplied by 5 is 45. So, the large squares cover the same area as 45 small squares, and therefore more of the plane than either the small or medium squares.

Reflect: Did you use any tools from your geometry toolkit to help you? Why or why not?

Log in to Amplify Math to complete this lesson online.

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Lesson 4 Composing and Rearranging to Determine Area 23

1 Launch

Have students use the *Think-Pair-Share* routine as they complete the Warm-up. Provide access to geometry toolkits for the duration of this lesson.

2 Monitor

Help students get started by referring back to Lesson 3, Activity 1. Ask, "How did you know that two triangles covered the same area as one rhombus? Which of these squares can be used in the same way as that triangle?"

Look for points of confusion:

- **Assuming the large square covers more of the plane.** Ask, "How could you prove that your observation is correct?"
- **Drawing "unit" squares randomly.** Ask, "Is there a tool in your geometry toolkit that can help you make the squares uniform without any gaps or overlaps?"

Look for productive strategies:

- Tracing the small square and using this to cover the medium and large squares without gaps or overlaps.
- Using a ruler to measure the small square and recreate it as a unit square in the larger squares.
- Reasoning that a medium square can be composed of four small squares and a large square can be composed of nine small squares.

3 Connect

Display the pattern.

Have pairs of students share their strategies.

Ask:

- "Which square was most helpful to use? Why?"
- "What tools did you use to complete this activity?"

Highlight that a larger shape can be seen as being composed of copies of a smaller shape. The medium and large squares can be composed of different numbers of copies of the small square.

Math Language Development

MLR7: Compare and Connect

As students share their strategies during the Connect, ask them to compare their approach to their partner's approach and make connections between the squares and tools that were most helpful to use. Amplify language around *square units*, *small*, *medium*, and *large squares*, and how these can be seen as composed and decomposed copies of each other.

English Learners

Allow students to describe the tools in their primary language first and then provide them with the corresponding English terms.

Power-up

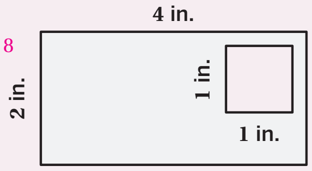
To power up students' ability understanding decomposition and composition of area with rectilinear figures on and off grids, have students complete:

Use the figure to answer each question:

a. What is the area of the large rectangle? 8 in^2; $4 \bullet 2 = 8$

b. What is the area of the inner square? 1 in^2; $1 \bullet 1 = 1$

c. What is the area of the shaded region?
7 in^2; $8 - 1 = 7$



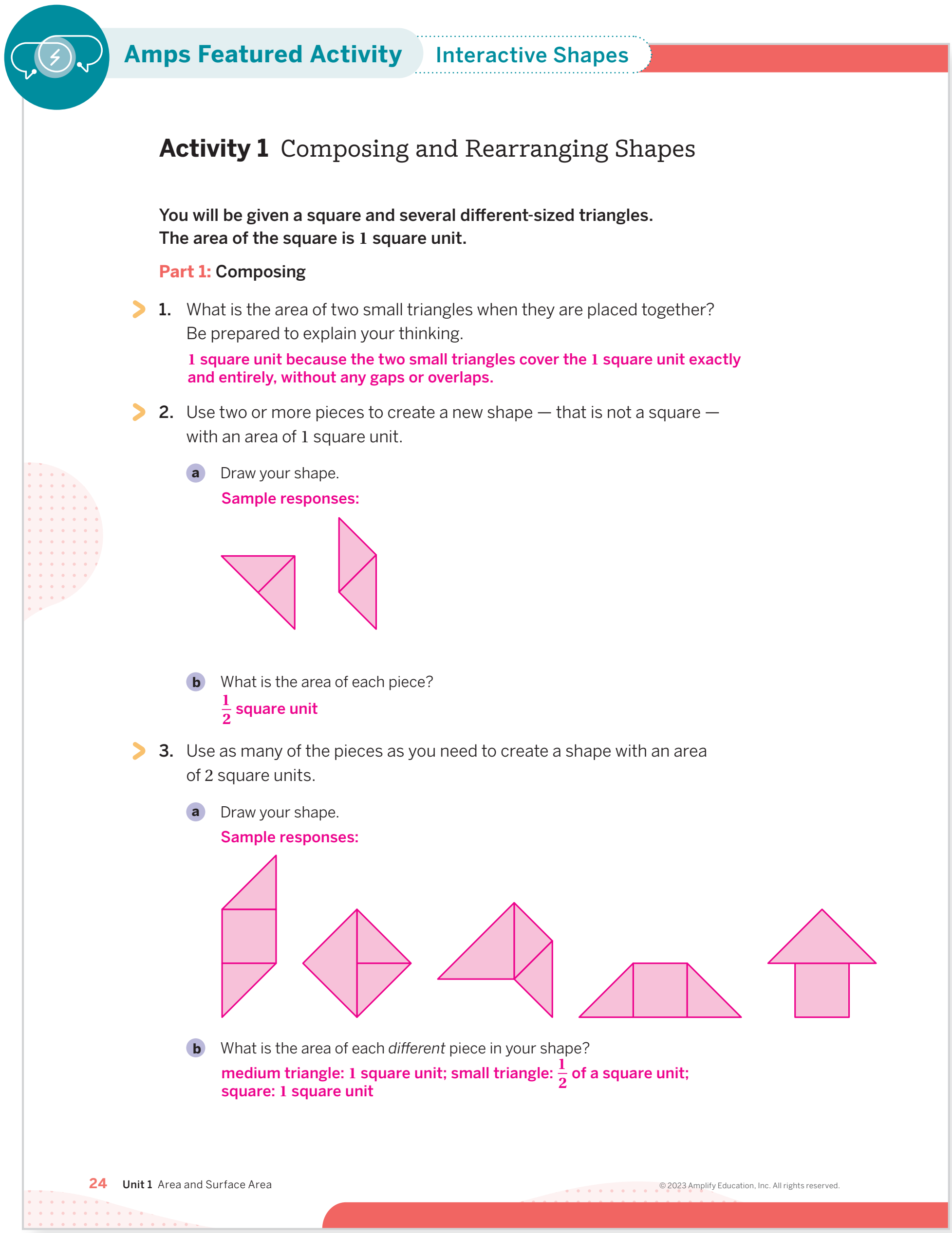
Use: Before the Warm-up.
Informed by: Performance on Lesson 3, Practice Problem 5 and Pre-Unit Readiness Assessment, Problems 2 and 8.

Pairs | 25 min

Activity 1 Composing and Rearranging Shapes

Students use tangram pieces to compose shapes of a given area based on the square piece having an area of 1 square unit.

Amps Featured Activity Interactive Shapes



Activity 1 Composing and Rearranging Shapes

You will be given a square and several different-sized triangles. The area of the square is 1 square unit.

Part 1: Composing

1. What is the area of two small triangles when they are placed together? Be prepared to explain your thinking.
   1 square unit because the two small triangles cover the 1 square unit exactly and entirely, without any gaps or overlaps.
2. Use two or more pieces to create a new shape — that is not a square — with an area of 1 square unit.
   - **a** Draw your shape.
     Sample responses:
   - **b** What is the area of each piece?
     $\frac{1}{2}$ square unit
3. Use as many of the pieces as you need to create a shape with an area of 2 square units.
   - **a** Draw your shape.
     Sample responses:
   - **b** What is the area of each *different* piece in your shape?
     medium triangle: 1 square unit; small triangle: $\frac{1}{2}$ of a square unit; square: 1 square unit

24 **Unit 1** Area and Surface Area

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1 Launch

Distribute a set of tangram pieces to each pair that consists of one square, four small triangles, one medium triangle, and two large triangles.

2 Monitor

Help students get started by asking, "Of which shape do you know the area? How could you compare the area of one triangle to that shape? How could you compare the area of two triangles to that shape?"

Look for points of confusion:

- **Not understanding how a triangular shape can have an area with *square* units.** Draw a picture or use your finger to show how a square could be partitioned into two rectangles. Ask, "What is the area of one rectangle?" Then show how the same square could be partitioned into two triangles. Just like the two rectangles have the same area as one square, so do the two triangles, no matter how they are arranged.
- **Thinking each and every piece has the stated total area or haphazardly guessing the area of each piece.** Ask, "How could you prove that your observation is correct?"

Look for productive strategies:

- Recognizing that all the other shapes can be recreated using only small triangles.
- Recognizing that areas of all the pieces add up to the total area of a created figure.
- Understanding that when pieces match up exactly, they have the same area, even when they are rearranged.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Parts 1 and 2 and, if they have time available, work on Part 3.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can position shapes that snap to each other, eliminating the chance for gaps or overlaps.

Math Language Development

MLR2: Collect and Display

Circulate and listen to the ways students describe composing, decomposing, and rearranging the shapes. Collect and display common phrases you hear students say about each, such as *building*, *breaking apart*, and *moving*. Update the display as needed throughout the remainder of the lesson. Make connections between the phrases students use and the mathematical language developed in the Connect section: *compose*, *decompose*, and *rearrange*.

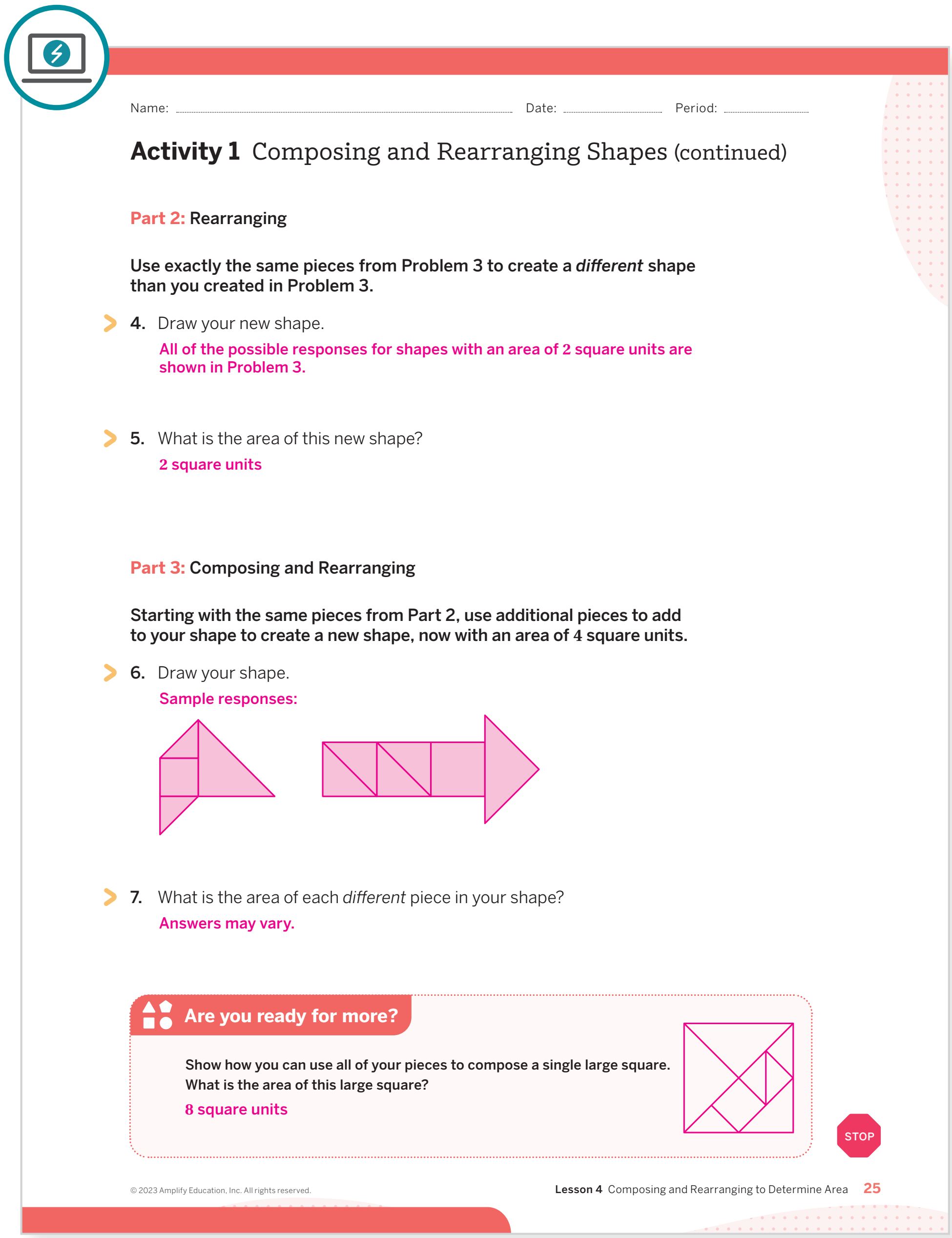
English Learners

Include relevant images or drawings to support students' understanding of the common phrases used.

Pairs | 25 min

Activity 1 Composing and Rearranging Shapes (continued)

Students use tangram pieces to compose shapes of a given area based on the square piece having an area of 1 square unit.



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Activity 1 Composing and Rearranging Shapes (continued)

Part 2: Rearranging

Use exactly the same pieces from Problem 3 to create a *different* shape than you created in Problem 3.

4. Draw your new shape.

All of the possible responses for shapes with an area of 2 square units are shown in Problem 3.

5. What is the area of this new shape?

2 square units

Part 3: Composing and Rearranging

Starting with the same pieces from Part 2, use additional pieces to add to your shape to create a new shape, now with an area of 4 square units.

6. Draw your shape.

Sample responses:

7. What is the area of each *different* piece in your shape?

Answers may vary.

Are you ready for more?

Show how you can use all of your pieces to compose a single large square. What is the area of this large square?

8 square units

STOP

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Lesson 4 Composing and Rearranging to Determine Area 25

3 Connect

Display the tangram pieces from the activity. Consider projecting the digital lesson with movable pieces.

Have pairs of students share the figures that they composed and how they composed them, focusing on how they know each has the required total area, and then how they determined the area of each piece. As students share, restate their moves explicitly by using the terms *compose*, *decompose*, and *rearrange*, and encourage students to adopt this language as well.

Ask, "How is the area of each individual piece related to the total area of a figure?"

Highlight:

- If two shapes can be placed one on top of the other so that they match up exactly, then they have the same area.
- If a shape can be decomposed and rearranged to compose another shape, then the area of the new shape is the same as the area of the original shape.

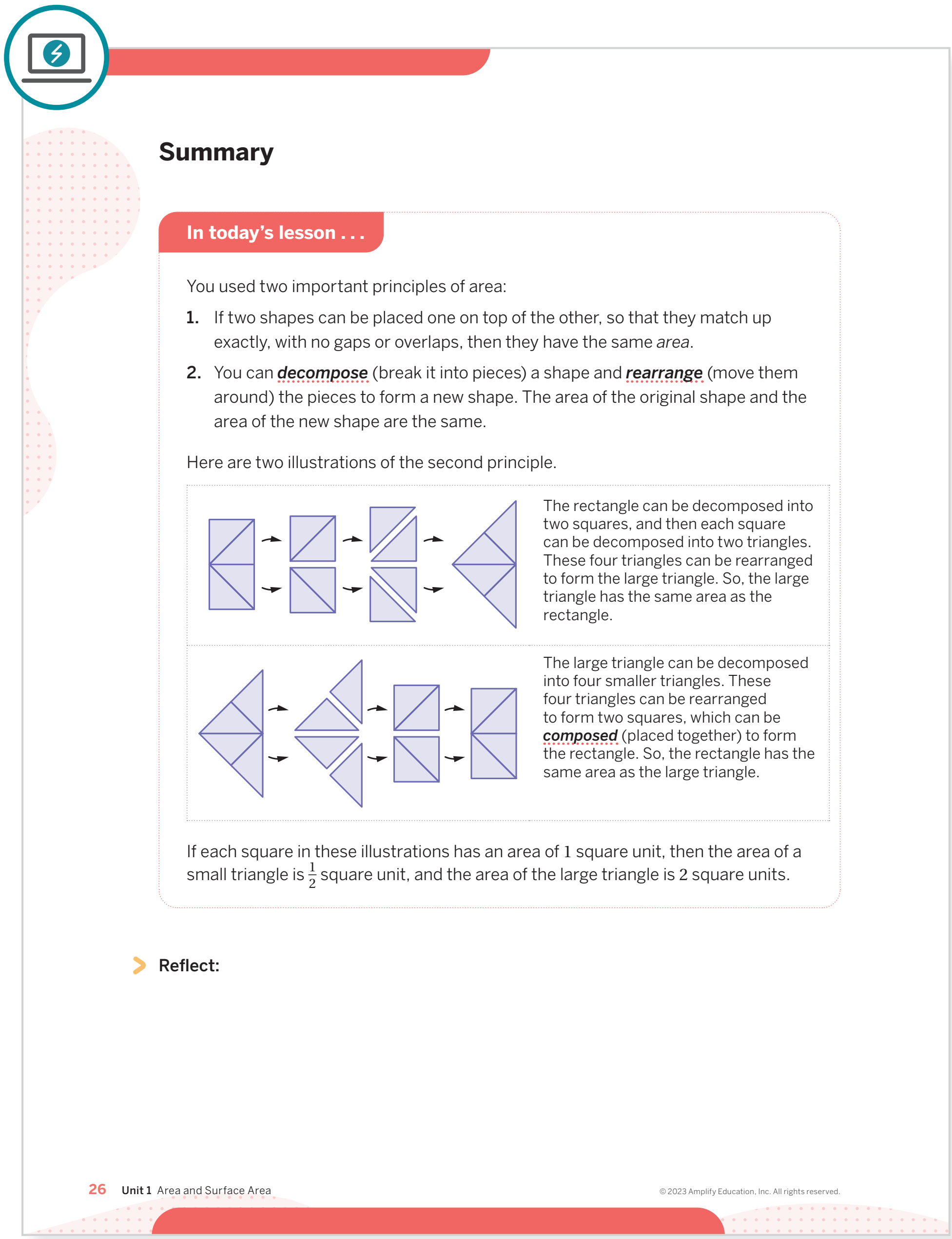
Define:

- ***compose*** as meaning "place together" or "combine." This term is used to describe how more than one smaller shape can be placed together to make a new, larger shape.
- ***decompose*** as meaning "take apart" or "break into pieces." This term is used to describe how a larger shape can be taken apart to make more than one new, smaller shape.

Whole Class | 5 min

Summary

Review and synthesize the different strategies for reasoning about area and what it means for two shapes to have the same area.



Summary

In today's lesson . . .

You used two important principles of area:

1. If two shapes can be placed one on top of the other, so that they match up exactly, with no gaps or overlaps, then they have the same *area*.
2. You can ***decompose*** (break it into pieces) a shape and ***rearrange*** (move them around) the pieces to form a new shape. The area of the original shape and the area of the new shape are the same.

Here are two illustrations of the second principle.

The rectangle can be decomposed into two squares, and then each square can be decomposed into two triangles. These four triangles can be rearranged to form the large triangle. So, the large triangle has the same area as the rectangle.

The large triangle can be decomposed into four smaller triangles. These four triangles can be rearranged to form two squares, which can be ***composed*** (placed together) to form the rectangle. So, the rectangle has the same area as the large triangle.

If each square in these illustrations has an area of 1 square unit, then the area of a small triangle is $\frac{1}{2}$ square unit, and the area of the large triangle is 2 square units.

Reflect:

Synthesize

Display the two illustrations of decomposing and rearranging rectangles and triangles.

Highlight that the two principles introduced in this lesson that can help with reasoning about area are:

1. If two shapes can be placed one on top of the other so that they match up exactly, with no gaps or overlaps, then they have the same area.
2. If a shape is decomposed (broken into pieces) and rearranged (moved around in a different way) to form a new shape, then the area of the original shape and the area of the new shape are the same.

Ask:

- "If you decompose (cut or break apart) a given figure into pieces, how could you determine the area of the figure?"
- "If you compose (place together) a new figure from smaller pieces without overlapping them, how could you determine its area?"

Formalize vocabulary:

- *compose*
- *decompose*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does it mean when you say two shapes have the same area?"

Math Language Development

MLR2: Collect and Display

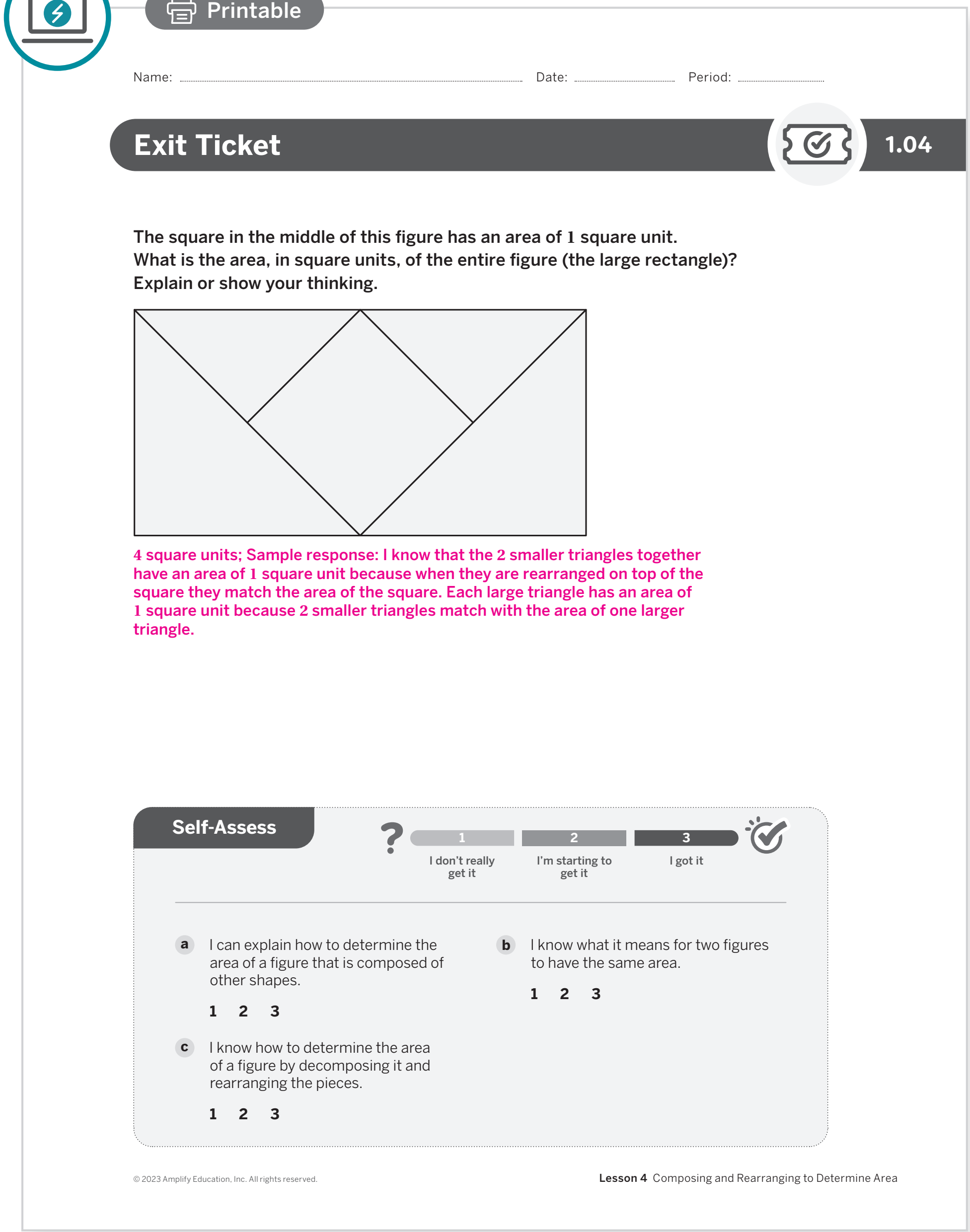
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 3. Ask them to review and reflect on any terms and phrases related to the terms *compose*, *decompose*, or *rearrange* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of determining the area of a figure by composing, decomposing, and rearranging parts of it.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.04

The square in the middle of this figure has an area of 1 square unit. What is the area, in square units, of the entire figure (the large rectangle)? Explain or show your thinking.

4 square units; Sample response: I know that the 2 smaller triangles together have an area of 1 square unit because when they are rearranged on top of the square they match the area of the square. Each large triangle has an area of 1 square unit because 2 smaller triangles match with the area of one larger triangle.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain how to determine the area of a figure that is composed of other shapes. 1 2 3

b. I know what it means for two figures to have the same area. 1 2 3

c. I know how to determine the area of a figure by decomposing it and rearranging the pieces. 1 2 3

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Lesson 4 Composing and Rearranging to Determine Area

Success looks like . . .

- **Language Goal:** Calculating the area of a region by decomposing it and rearranging the pieces, and explaining the solution method **(Speaking and Listening, Writing)**.
  - » Determining the area of the figure given that the area of the square is 1 square unit.
- **Language Goal:** Recognizing and explaining that if two figures can be placed one on top of the other so that they match up exactly, they must have the same area **(Speaking and Listening)**.
  - » Placing the 2 smaller triangles on top of the square to determine that the area of the triangles is 1 square unit.
- **Goal:** Showing that area is additive by composing polygons with a given area.
  - » Determining the area is 4 square units by adding the areas of the smaller triangles with the square.

Suggested next steps

If students have difficulty relating the areas of the different shapes, consider:

- Asking, "What could you use in your geometry toolkit that would help you determine the areas of each shape?" tracing paper
- Referring back to the Summary and asking, "How does this diagram illustrate decomposition and rearranging?"
- Having students create a table to organize their thinking:

1 square =	___ small triangles
2 small triangles =	___ large triangle
1 large triangle =	___ square

Professional Learning

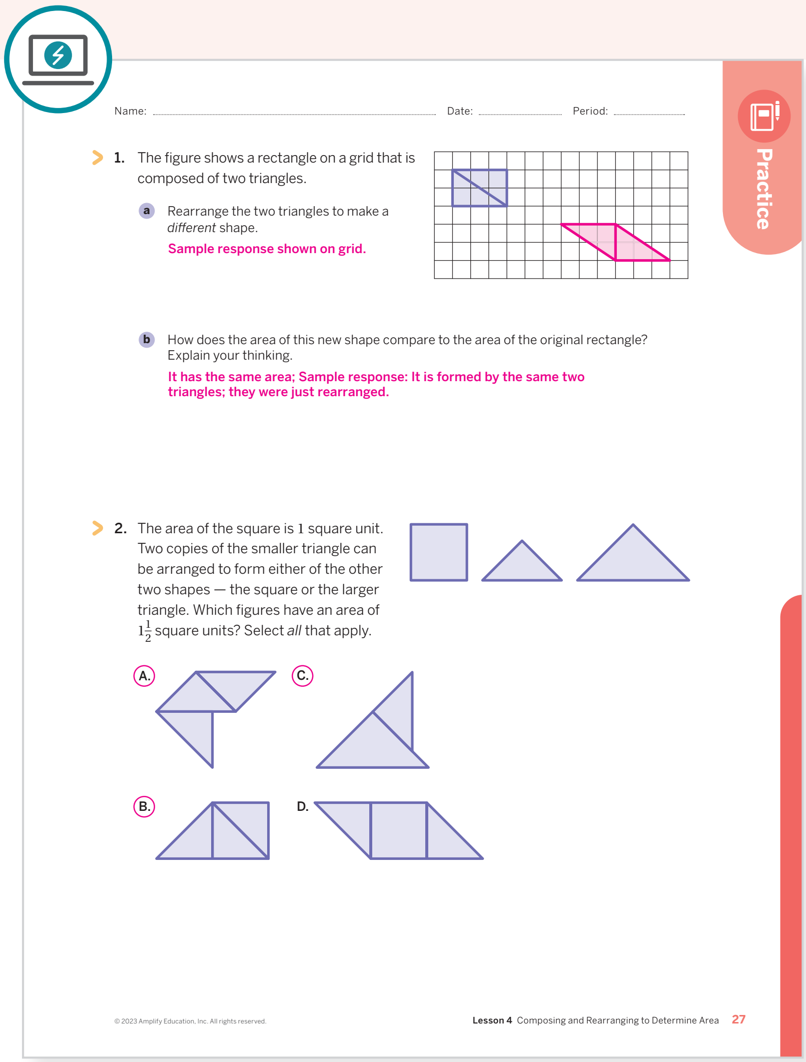
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?
- What could you do the next time you teach this lesson to help student understanding of the terms *compose*, *decompose*, and *rearrange*?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. The figure shows a rectangle on a grid that is composed of two triangles.

   a Rearrange the two triangles to make a *different* shape.

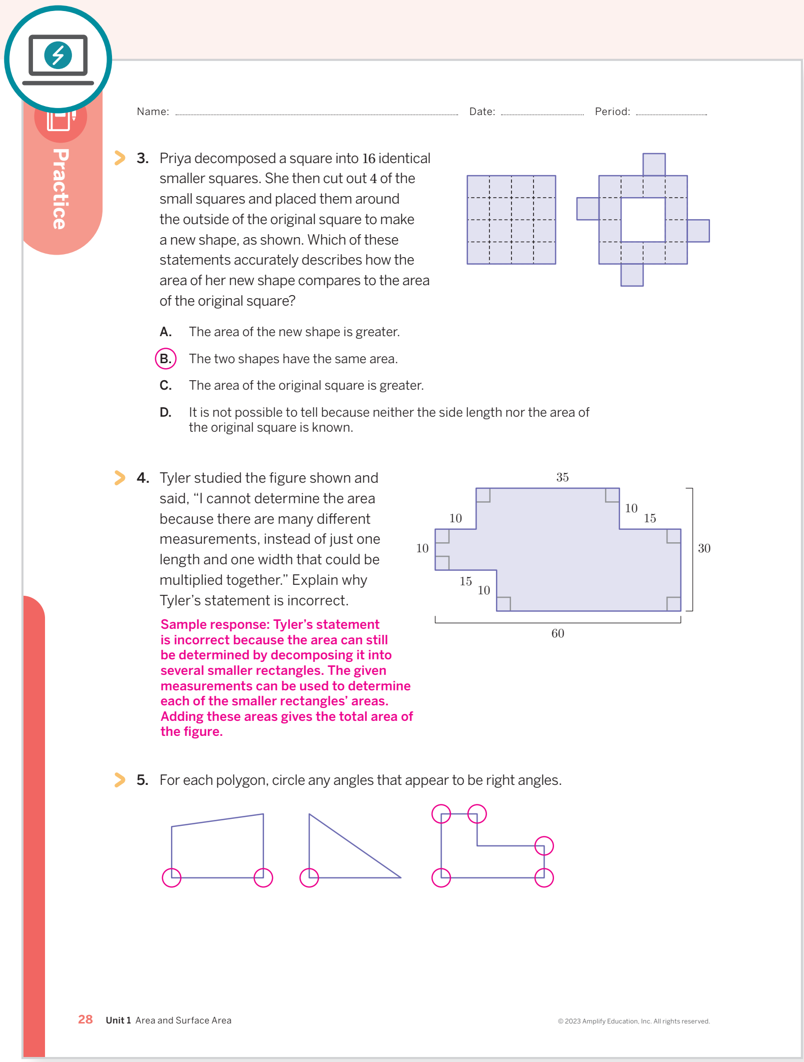
   Sample response shown on grid.

   b How does the area of this new shape compare to the area of the original rectangle? Explain your thinking.

   It has the same area; Sample response: It is formed by the same two triangles; they were just rearranged.

2. The area of the square is 1 square unit. Two copies of the smaller triangle can be arranged to form either of the other two shapes — the square or the larger triangle. Which figures have an area of $1\frac{1}{2}$ square units? Select *all* that apply.

   [✓ A.] [✓ C.] [✓ B.] D.



Name: ______ Date: ______ Period: ______

3. Priya decomposed a square into 16 identical smaller squares. She then cut out 4 of the small squares and placed them around the outside of the original square to make a new shape, as shown. Which of these statements accurately describes how the area of her new shape compares to the area of the original square?

   A. The area of the new shape is greater.

   [✓ B.] The two shapes have the same area.

   C. The area of the original square is greater.

   D. It is not possible to tell because neither the side length nor the area of the original square is known.

4. Tyler studied the figure shown and said, "I cannot determine the area because there are many different measurements, instead of just one length and one width that could be multiplied together." Explain why Tyler's statement is incorrect.

   Sample response: Tyler's statement is incorrect because the area can still be determined by decomposing it into several smaller rectangles. The given measurements can be used to determine each of the smaller rectangles' areas. Adding these areas gives the total area of the figure.

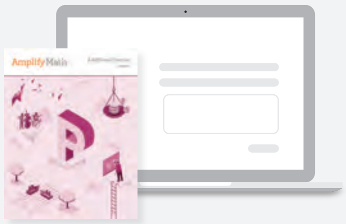
5. For each polygon, circle any angles that appear to be right angles.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Grade 4	2
Formative ⚡	5	Unit 1 Lesson 5	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 5**

Reasoning to Determine Area

Let's use different strategies to determine the area of a shape.

Focus

Goals

1. **Language Goal:** Compare and contrast different strategies for determining the area of a polygon. **(Speaking and Listening)**
2. **Language Goal:** Determine the area of a polygon by decomposing, rearranging, and subtracting or enclosing shapes, and explain the solution method. **(Speaking and Listening, Writing)**
3. Include appropriate units when stating the area of a polygon.

Coherence

Today

Students rethink and revise their definition of area as a class and then build on the principles for reasoning about figures to determine area:

- If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- If a figure is composed of pieces that do not overlap, the sum of the areas of the pieces is the area of the figure.
- If a figure is decomposed, then its area is the sum of the areas of the pieces.

Students can use the following strategies to determine the area of a figure: decompose, decompose and rearrange, subtractive, and enclose. Use of these strategies involves looking for and making use of structure and explaining them involves constructing logical arguments.

Previously

In Grade 3, students decomposed rectilinear figures into non-overlapping rectangles and added the areas of the non-overlapping parts.

Coming Soon

In Lessons 6–13, students will apply what they know about area to parallelograms, triangles, and other polygons.

Rigor

- Students build their **procedural skills** by measuring area with square units.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	20 min	15 min	5 min	5 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *area*
- *compose*
- *decompose*
- *rearrange*

Amps | Featured Activity

Exit Ticket
Real-Time Exit Ticket

Check in real time if your students can decompose or rearrange a figure to find its area, using a digital Exit Ticket that is automatically scored.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel lost when they first encounter shapes that are no longer on grids in Activity 2. Encourage them to look for familiar structures even without the grid. For example, ask them to shift their perspective by viewing a figure as rectangular pieces, just as they did in Activity 1, and imagine (or draw) the measurements given as grid squares.

Modifications to Pacing

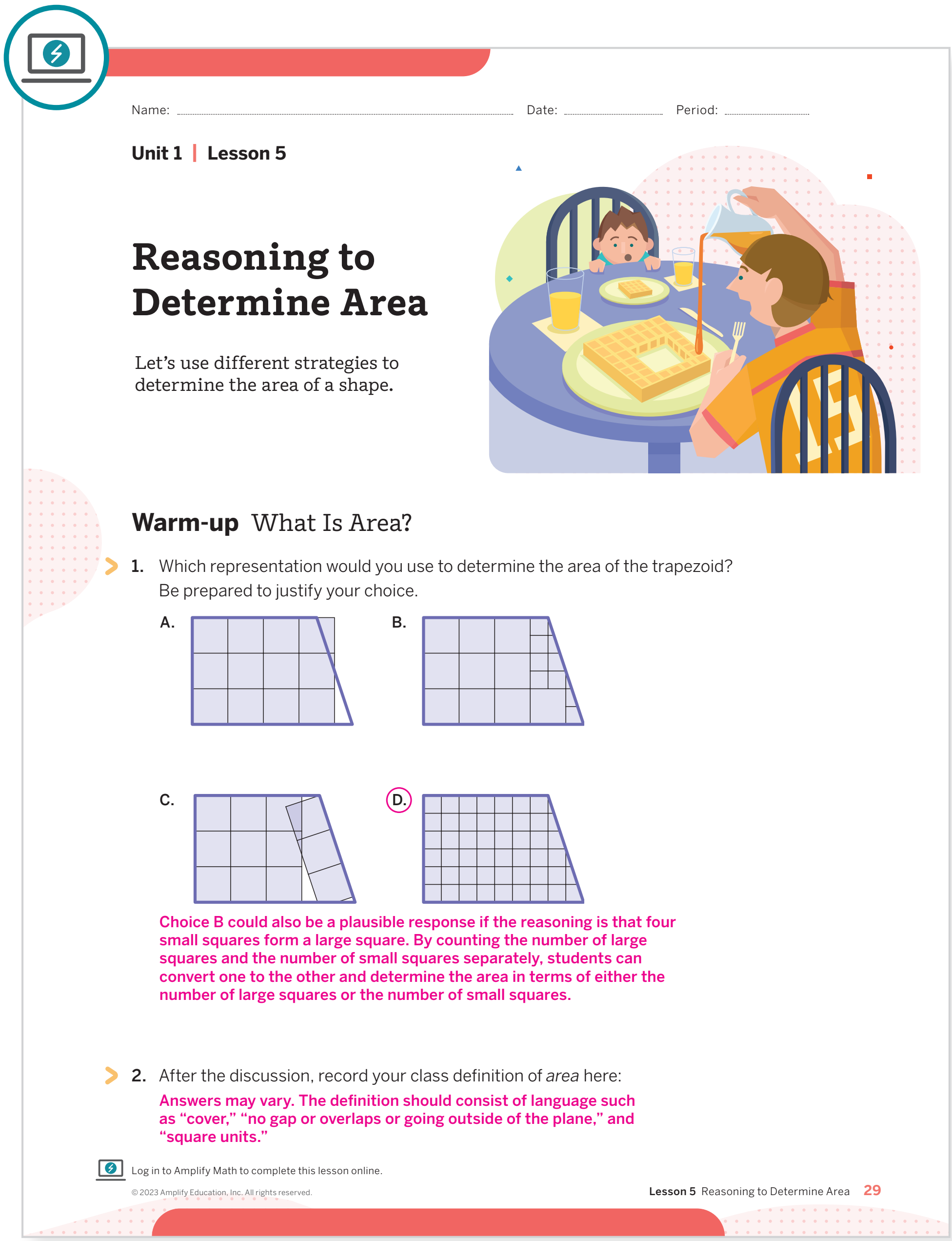
You may want to consider these additional modifications if you are short on time:

- In **Activity 1**, discuss the problems and their solutions as a class.
- In **Activity 2**, omit the *Think-Pair-Share* routine and move straight into the class discussion.

Independent | 10 min

Warm-up What is Area?

Students use and refine their prior knowledge of area to evaluate four ways a region is tiled. A class definition for *area* is co-authored.



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Unit 1 | Lesson 5

Reasoning to Determine Area

Let's use different strategies to determine the area of a shape.

Warm-up What Is Area?

1. Which representation would you use to determine the area of the trapezoid? Be prepared to justify your choice.

A.

B.

C.

[✓ D.]

Choice B could also be a plausible response if the reasoning is that four small squares form a large square. By counting the number of large squares and the number of small squares separately, students can convert one to the other and determine the area in terms of either the number of large squares or the number of small squares.

2. After the discussion, record your class definition of *area* here:

Answers may vary. The definition should consist of language such as "cover," "no gap or overlaps or going outside of the plane," and "square units."

Log in to Amplify Math to complete this lesson online.

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Lesson 5 Reasoning to Determine Area 29

1 Launch

Set an expectation for the amount of time students will have to work on the activity.

2 Monitor

Help students get started by asking, "Is there a representation that you would not use?"

Look for points of confusion:

- **Focusing on trying to calculate the area instead of thinking more generally about what area means.** Say, "You do not need to know the area, but the representation should make it possible to be known."
- **Thinking none of the options could be used because they all contain partial squares.** Remind students that area can be equal to a non-whole number.

Look for productive strategies:

- Ruling out Choices A and C because there are gaps and tiles that extend beyond the area of the trapezoid.
- Recognizing that the large squares in Choice B are decomposed into small squares in Choice D.

3 Connect

Ask, for Choice C first, followed by Choice A, then Choice B, and finally Choice D:

- "Who chose this representation?"
- "What is it about this representation that would be helpful in determining the area? Is there anything that might make it unhelpful?"

Have individual students share their reasoning for or against a representation.

Highlight the language *square units*, *covering a region*, and *no gaps or overlaps*.

Define the term *area* as a class, encouraging the use of terms encountered in Lessons 3 and 4 during discussion, and allowing students to modify and refine the definition before formalizing it for them to record it.

Math Language Development

MLR2: Collect and Display

Circulate and collect examples of language students use to describe and emphasize their understanding of the phrases *square units*, *covering a region*, *area*, *and no gaps and overlaps*. Add these terms and phrases to the class display and update throughout the lesson as needed.

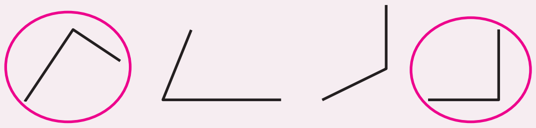
English Learners

Use images and gestures to highlight the different terms and phrases added to the class display.

Power-up

To power up students' ability to identify right angles in polygons, have students complete:

Recall that a *right angle* measures exactly 90°. Identify *all* the right angles.



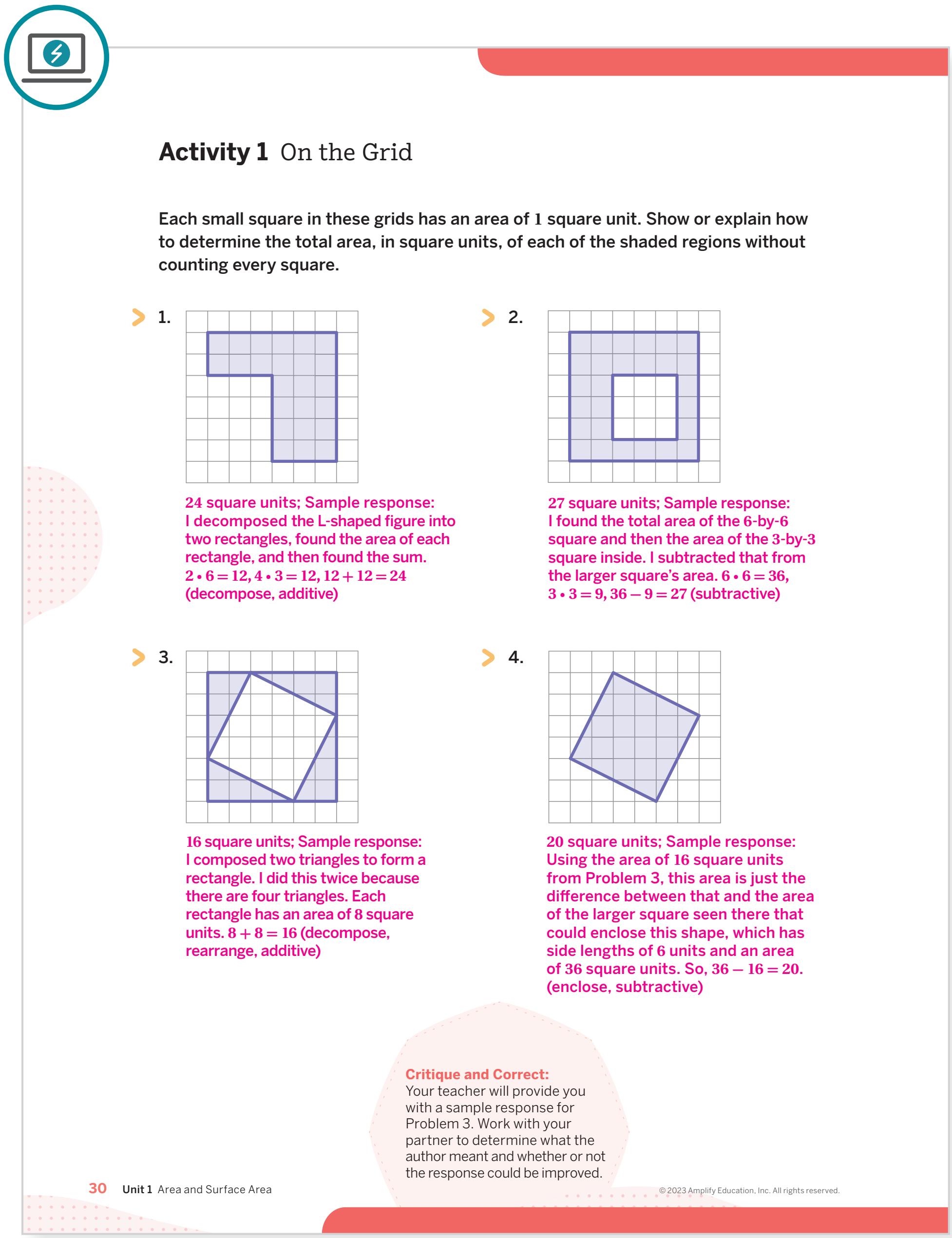
Use: Before Activity 1.

Informed by: Performance on Lesson 4, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 7.

Pairs | 20 min

Activity 1 On the Grid

Students apply a variety of strategies to calculate the areas of different regions formed by rectilinear shapes on a grid.



Activity 1 On the Grid

Each small square in these grids has an area of 1 square unit. Show or explain how to determine the total area, in square units, of each of the shaded regions without counting every square.

1\.

24 square units; Sample response: I decomposed the L-shaped figure into two rectangles, found the area of each rectangle, and then found the sum. $2 \cdot 6 = 12, 4 \cdot 3 = 12, 12 + 12 = 24$ (decompose, additive)

2\.

27 square units; Sample response: I found the total area of the 6-by-6 square and then the area of the 3-by-3 square inside. I subtracted that from the larger square's area. $6 \cdot 6 = 36$, $3 \cdot 3 = 9, 36 - 9 = 27$ (subtractive)

3\.

16 square units; Sample response: I composed two triangles to form a rectangle. I did this twice because there are four triangles. Each rectangle has an area of 8 square units. $8 + 8 = 16$ (decompose, rearrange, additive)

4\.

20 square units; Sample response: Using the area of 16 square units from Problem 3, this area is just the difference between that and the area of the larger square seen there that could enclose this shape, which has side lengths of 6 units and an area of 36 square units. So, $36 - 16 = 20$. (enclose, subtractive)

Critique and Correct: Your teacher will provide you with a sample response for Problem 3. Work with your partner to determine what the author meant and whether or not the response could be improved.

30 **Unit 1** Area and Surface Area

1 Launch

Have students use the *Think-Pair-Share* routine. One partner should complete Problems 1 and 3, while the other partner completes Problems 2 and 4.

2 Monitor

Help students get started with Problem 1 by counting squares or identifying arrays. Confirm that the area is 24 square units, and then have students try to think of a different decomposition strategy.

Look for points of confusion:

- **Counting complete and partial grid squares.** Ask, "Is there a way to decompose and rearrange the shaded pieces so that there are no partial areas?"

Look for productive strategies:

- Problems 1 and 2: Decomposing and adding
- Problem 2: Subtracting
- Problem 3: Rearranging and composing
- Problems 3 and 4: Decomposing and rearranging
- Problem 4: Enclosing and subtracting

3 Connect

Display the four figures.

Have individual students share their strategies for determining the areas of each figure.

Highlight the different strategies, focusing on: decomposing (Problems 1 and 2); decomposing and rearranging (Problems 3 and 4), subtracting (Problem 2), and enclosing and subtracting (Problem 4).

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1, 2, and 3, and, if they have time available, work on Problem 4.

Extension: *Math Enrichment*

Have students rearrange the triangles from Problem 3 so that they fit inside the figure in Problem 4. Have them draw and shade a diagram that represents their work.

Math Language Development

MLR3: Critique, Correct, Clarify

Using Problem 3, provide this draft explanation: "I saw triangles, so I used those to get my answer."

Critique: Pairs should discuss what they think the author meant. Provide sentence frames for scaffolding:

- "I think the author is trying to use the strategy ___ because . . ."
- "The part that is most unclear to me is ___ because . . ."

Correct and Clarify: Have students improve the draft response and explain their improvements.

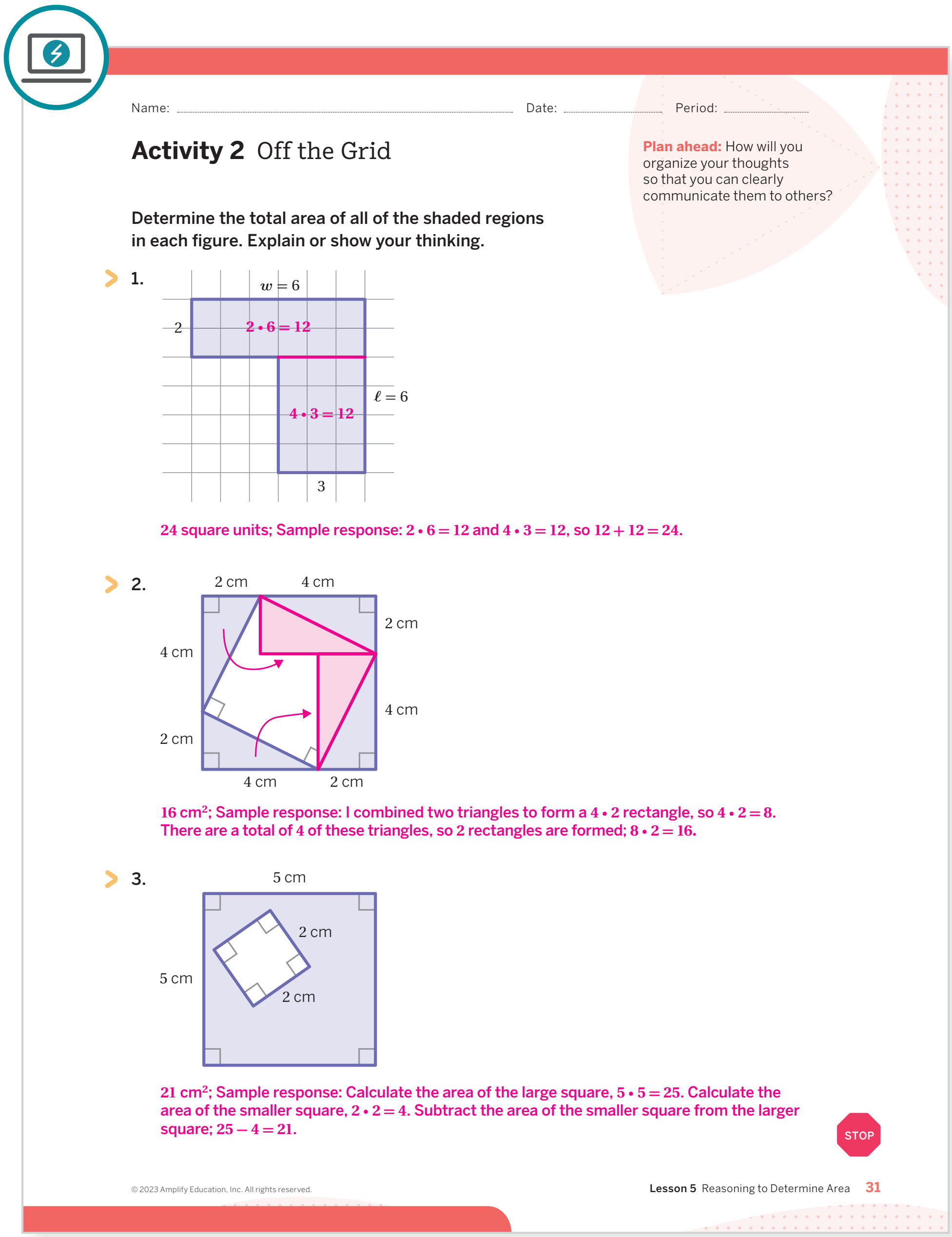
English Learners

Provide a few examples of some improved draft responses for students to reference as they create their own.

Small Groups | 15 min

Activity 2 Off the Grid

Students apply the various strategies for determining area from the previous activity to similar shapes that are no longer represented on a grid.



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Activity 2 Off the Grid

Plan ahead: How will you organize your thoughts so that you can clearly communicate them to others?

Determine the total area of all of the shaded regions in each figure. Explain or show your thinking.

1.

24 square units; Sample response: $2 \cdot 6 = 12$ and $4 \cdot 3 = 12$, so $12 + 12 = 24$.

2.

16 cm^2; Sample response: I combined two triangles to form a $4 \cdot 2$ rectangle, so $4 \cdot 2 = 8$. There are a total of 4 of these triangles, so 2 rectangles are formed; $8 \cdot 2 = 16$.

3.

21 cm^2; Sample response: Calculate the area of the large square, $5 \cdot 5 = 25$. Calculate the area of the smaller square, $2 \cdot 2 = 4$. Subtract the area of the smaller square from the larger square; $25 - 4 = 21$.

STOP

Lesson 5 Reasoning to Determine Area 31

1 Launch

Give students 6 or 7 minutes to work individually, and then arrange students in groups of 4 to compare their responses and discuss their strategies.

2 Monitor

Help students get started by having them match each figure to one from Activity 1, asking, "How did you determine the area when it was on a grid?"

Look for points of confusion:

- **Not recognizing that the triangles can be rearranged in Problem 2.** Refer students back to Problem 3 from Activity 1, and suggest they use tracing paper to help them apply a similar strategy here.
- **Thinking the right angle symbol indicates the size of a square unit in Problem 3.** Remind students that those symbols indicate 90° angles and the unit size or scales of these figures are indicated by given lengths.

Look for productive strategies:

- Noticing that the figures are all similar to the ones from Activity 1, and thus similar strategies can be used: composing, decomposing, rearranging, enclosing, subtracting, or adding.

3 Connect

Display the figures.

Have individual students share the strategies they used to determine the area of each figure, focusing on composing, decomposing, adding, or subtracting.

Ask, "How were your strategies similar or different?"

Highlight that the same strategies for calculating area can be used whether the measurements are indicated by a grid or are given lengths without a grid.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1 and 2, and, if they have time available, work on Problem 3. You may also consider chunking this task into more manageable parts. After students complete Problem 1, check in with groups of students or the entire class. Invite volunteers to share and describe the strategies they used and then ask them to predict a strategy they may find useful for Problem 2.

Extension: *Math Enrichment*

Ask students to compare the figures in Problems 2 and 3 and explain whether they can use the same strategy for both figures.

Math Language Development

MLR7: Compare and Connect

During the Connect, have students create a visual display showing how they made sense of one of the figures. Consider using these prompts to help students compare and connect their strategies:

- "Can you find any connections between the representations?"
- "Why did different strategies for Figure___ lead to the same outcome?"

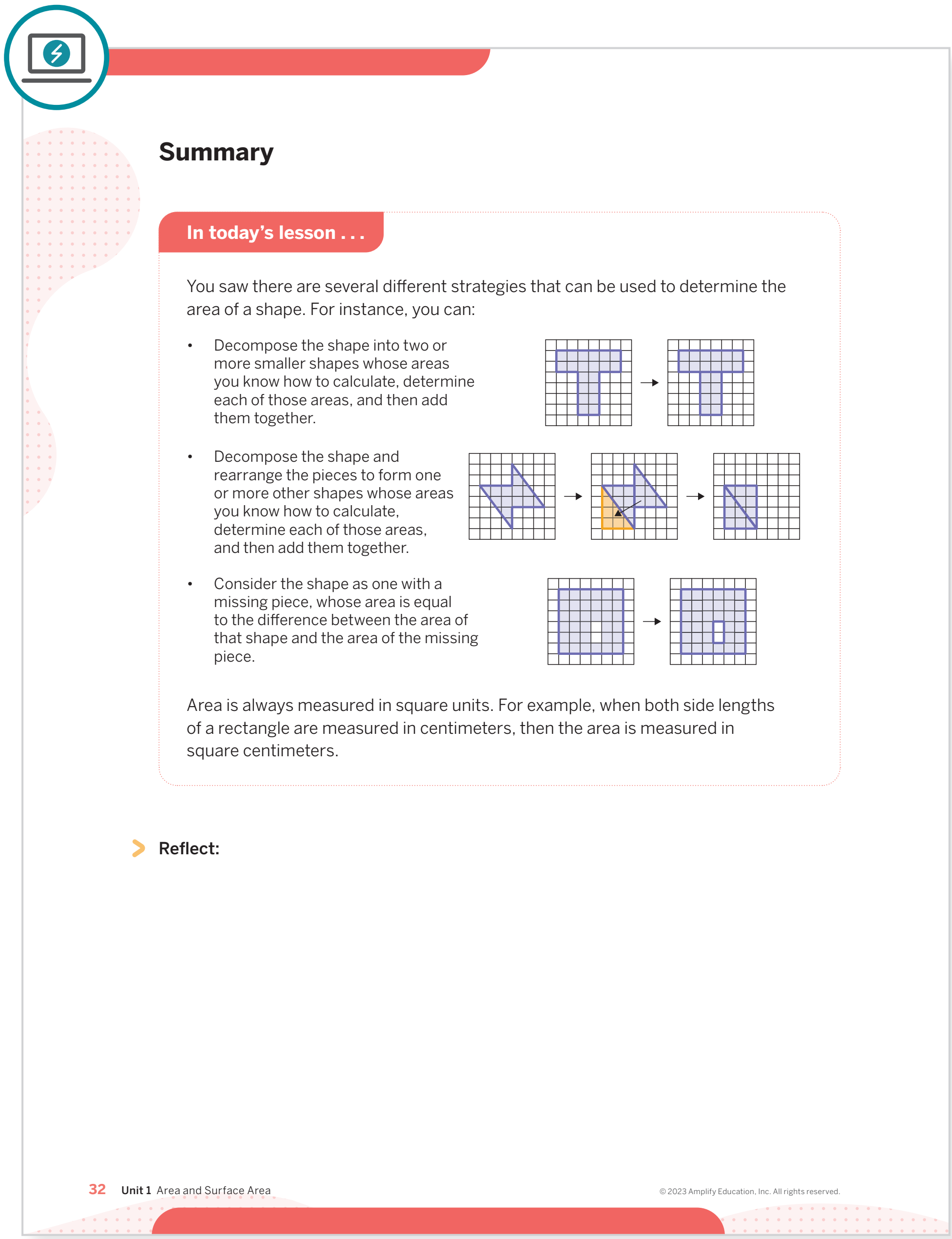
English Learners

Encourage students to revisit the class display/anchor chart of relevant terms and phrases to assist them in using developing mathematical language to compare and connect their strategies.

Whole Class | 5 min

Summary

Review and synthesize the different strategies for determining the areas of figures.



Summary

In today's lesson . . .

You saw there are several different strategies that can be used to determine the area of a shape. For instance, you can:

- Decompose the shape into two or more smaller shapes whose areas you know how to calculate, determine each of those areas, and then add them together.
- Decompose the shape and rearrange the pieces to form one or more other shapes whose areas you know how to calculate, determine each of those areas, and then add them together.
- Consider the shape as one with a missing piece, whose area is equal to the difference between the area of that shape and the area of the missing piece.

Area is always measured in square units. For example, when both side lengths of a rectangle are measured in centimeters, then the area is measured in square centimeters.

Reflect:

32 Unit 1 Area and Surface Area

Synthesize

Display the three examples from the Summary.

Ask students to go back through the activities and find problems in which each strategy was used — one at a time. Tell students that they will have several opportunities to use these strategies in upcoming lessons.

Highlight that the area of a figure can be found both on and off a grid by using one or more of these strategies:

- decomposing it into familiar figures.
- decomposing it and rearranging the pieces to compose familiar figures.
- considering it as a figure with one or more missing pieces, and then subtracting the area(s) of the missing piece(s) from the area of the figure.
- enclosing a figure with a larger square or rectangle and then subtracting the areas of the figures.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- “How did you calculate area today?”
- “Can you only decompose, rearrange, then compose? What about compose, decompose, then rearrange?”

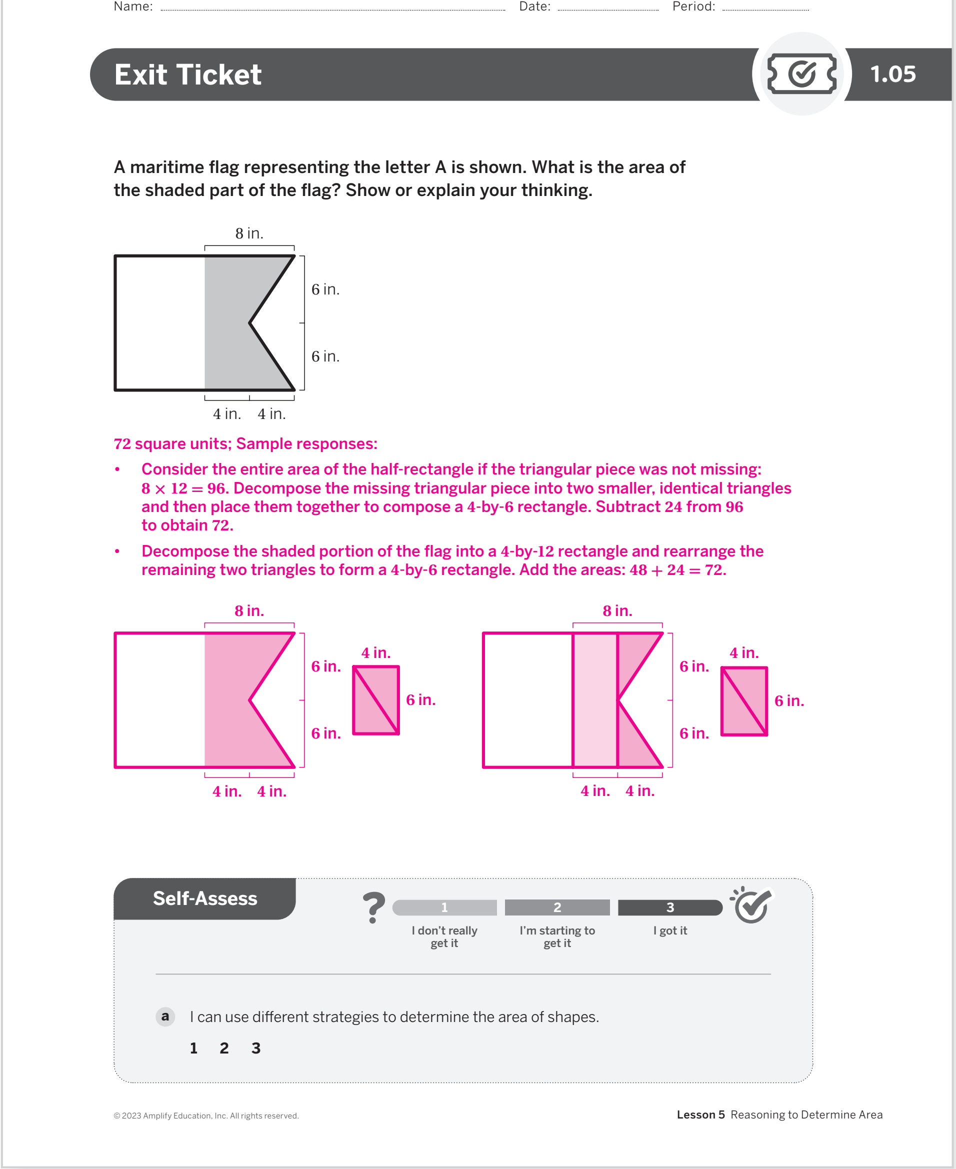
Independent | 5 min

Exit Ticket

Students demonstrate their understanding by decomposing and rearranging to determine the area of a given figure.

Amps Featured Activity Real-Time Exit Ticket

Printable



Name: Date: Period:

Exit Ticket 1.05

A maritime flag representing the letter A is shown. What is the area of the shaded part of the flag? Show or explain your thinking.

72 square units; Sample responses:

- Consider the entire area of the half-rectangle if the triangular piece was not missing: $8 \times 12 = 96$. Decompose the missing triangular piece into two smaller, identical triangles and then place them together to compose a 4-by-6 rectangle. Subtract 24 from 96 to obtain 72.
- Decompose the shaded portion of the flag into a 4-by-12 rectangle and rearrange the remaining two triangles to form a 4-by-6 rectangle. Add the areas: $48 + 24 = 72$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can use different strategies to determine the area of shapes.

1 2 3

Lesson 5 Reasoning to Determine Area

Success looks like . . .

- **Language Goal:** Comparing and contrasting different strategies for determining the area of a polygon **(Speaking and Listening)**.
- **Goal:** Determining the area of a polygon by decomposing, rearranging, and subtracting or enclosing shapes, and explaining the solution method.
  - » Decomposing the flag into rectangles and triangles to calculate its area.
- **Goal:** Including appropriate units (in spoken and written language) when stating the area of a polygon.
  - » Writing the area of the flag in square inches.

Suggested next steps

If students have difficulty visualizing how the triangle can be rearranged to form a rectangle, consider:

- Asking, "What tool in your toolkit could you use to help you here?" Sample response: Tracing paper can be used to trace one half of the triangle. The paper can be moved to trace the other half of the triangle to fit the two together. Or graph paper can be used to recreate the two 4×6 right triangles forming a 4×6 rectangle.

If students have difficulty determining the area (particularly of the triangles created by decomposing the shaded region), consider:

- Referring them to the first two strategies shown in the Summary, and then asking, "How was the first figure decomposed? What does the second diagram indicate you should do to compose the triangles into a shape whose area you know how to determine? Can you do something similar with this maritime flag?"

Professional Learning

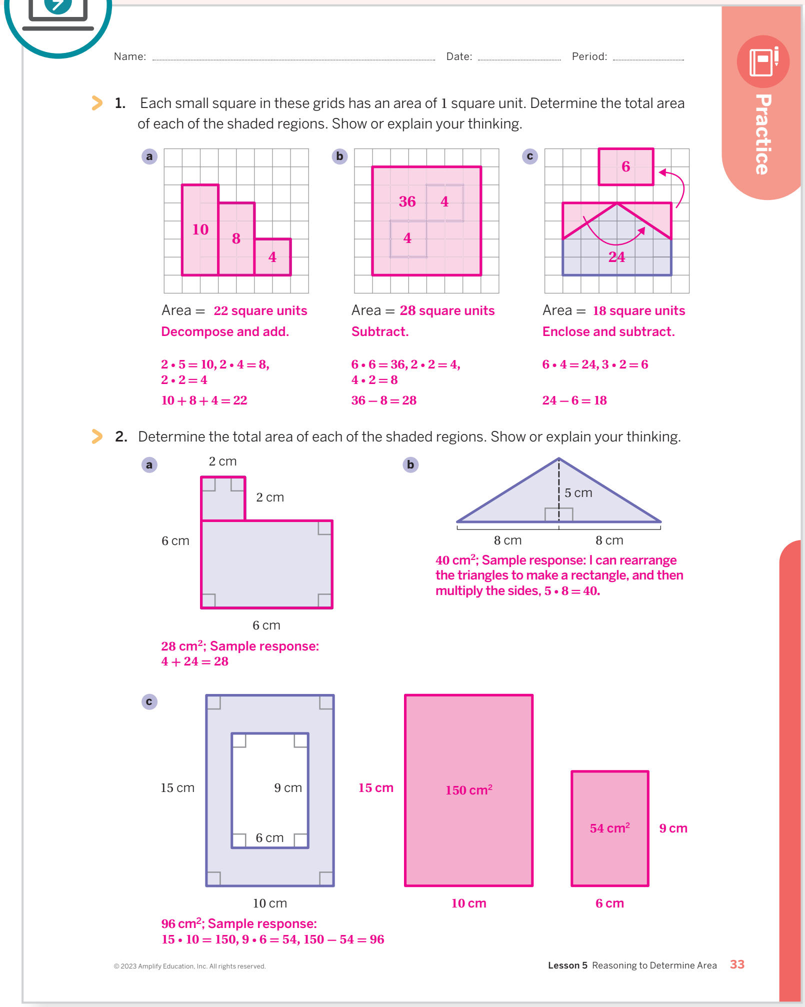
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did you encourage each student to contribute to the class authoring of the definition of area?
- How would you encourage more student participation the next time you teach this lesson?

Practice

Independent



Name: .... Date: .... Period: ....

1. Each small square in these grids has an area of 1 square unit. Determine the total area of each of the shaded regions. Show or explain your thinking.

a Area = **22 square units**
Decompose and add.
$2 \cdot 5 = 10, 2 \cdot 4 = 8,$
$2 \cdot 2 = 4$
$10 + 8 + 4 = 22$

b Area = **28 square units**
Subtract.
$6 \cdot 6 = 36, 2 \cdot 2 = 4,$
$4 \cdot 2 = 8$
$36 - 8 = 28$

c Area = **18 square units**
Enclose and subtract.
$6 \cdot 4 = 24, 3 \cdot 2 = 6$
$24 - 6 = 18$

2. Determine the total area of each of the shaded regions. Show or explain your thinking.

a **28 cm^2; Sample response:**
$4 + 24 = 28$

b **40 cm^2; Sample response: I can rearrange the triangles to make a rectangle, and then multiply the sides, $5 \cdot 8 = 40$.**

c **96 cm^2; Sample response:**
$15 \cdot 10 = 150, 9 \cdot 6 = 54, 150 - 54 = 96$

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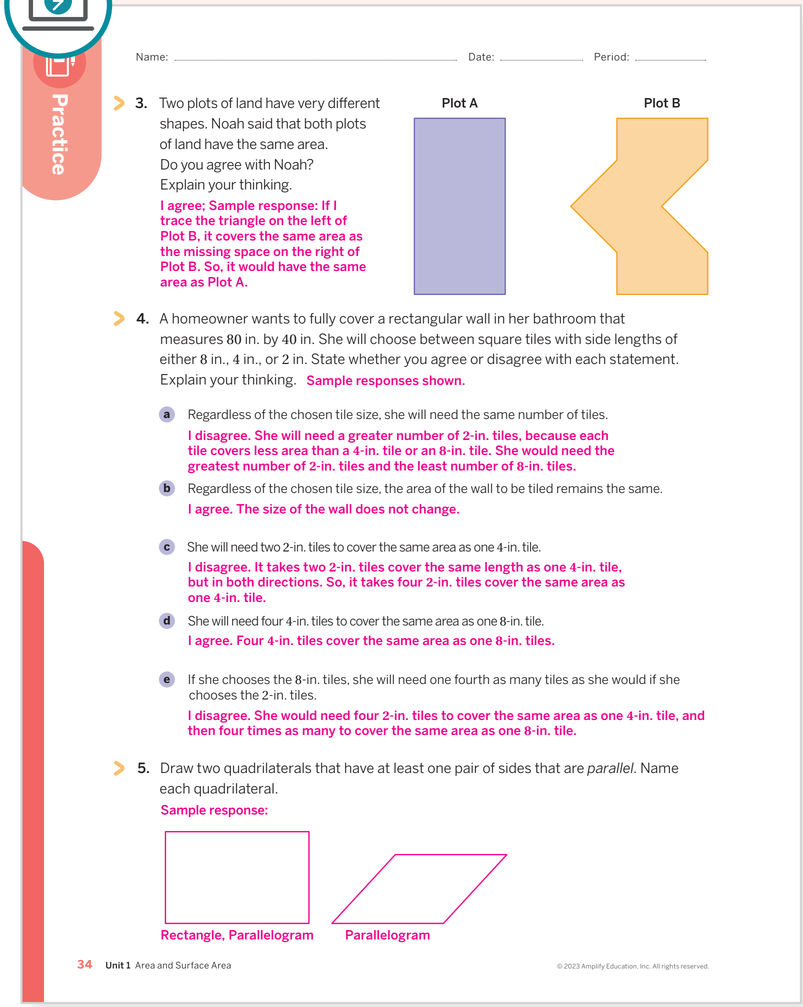
Lesson 5 Reasoning to Determine Area 33



Name: .... Date: .... Period: ....

3. Two plots of land have very different shapes. Noah said that both plots of land have the same area. Do you agree with Noah? Explain your thinking.

I agree; Sample response: If I trace the triangle on the left of Plot B, it covers the same area as the missing space on the right of Plot B. So, it would have the same area as Plot A.

4. A homeowner wants to fully cover a rectangular wall in her bathroom that measures 80 in. by 40 in. She will choose between square tiles with side lengths of either 8 in., 4 in., or 2 in. State whether you agree or disagree with each statement. Explain your thinking. **Sample responses shown.**

a Regardless of the chosen tile size, she will need the same number of tiles.
I disagree. She will need a greater number of 2-in. tiles, because each tile covers less area than a 4-in. tile or an 8-in. tile. She would need the greatest number of 2-in. tiles and the least number of 8-in. tiles.

b Regardless of the chosen tile size, the area of the wall to be tiled remains the same.
I agree. The size of the wall does not change.

c She will need two 2-in. tiles to cover the same area as one 4-in. tile.
I disagree. It takes two 2-in. tiles cover the same length as one 4-in. tile, but in both directions. So, it takes four 2-in. tiles cover the same area as one 4-in. tile.

d She will need four 4-in. tiles to cover the same area as one 8-in. tile.
I agree. Four 4-in. tiles cover the same area as one 8-in. tiles.

e If she chooses the 8-in. tiles, she will need one fourth as many tiles as she would if she chooses the 2-in. tiles.
I disagree. She would need four 2-in. tiles to cover the same area as one 4-in. tile, and then four times as many to cover the same area as one 8-in. tile.

5. Draw two quadrilaterals that have at least one pair of sides that are *parallel*. Name each quadrilateral.

Sample response:

Rectangle, Parallelogram **Parallelogram**

34 **Unit 1** Area and Surface Area

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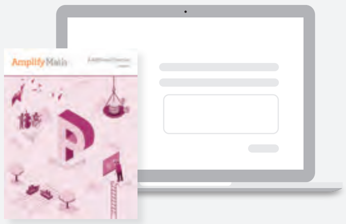
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 3	2
Formative ⚡	5	Unit 1 Lesson 6	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Parallelograms

Let's investigate the features and area of parallelograms.

Focus

Goals

1. **Language Goal:** Compare and contrast different strategies for determining the area of a parallelogram. **(Speaking and Listening)**
2. **Language Goal:** Describe observations about the opposites sides and opposite angles of parallelograms. **(Speaking and Listening, Writing)**
3. **Language Goal:** Explain how to find the area of a parallelogram by rearranging or enclosing it in a rectangle. **(Speaking and Listening, Writing)**

Coherence

Today

Students recall and analyze the defining attributes of parallelograms. They use different strategies to decompose and rearrange parallelograms into rectangles. They apply reasoning and composition strategies from previous lessons to generalize strategies for determining the area of any parallelogram, recognizing:

- A parallelogram has the same area as a related rectangle (a rectangle with the same base and height).
- A parallelogram can be enclosed in a rectangle that is then composed of the parallelogram and two identical right triangles (which form a smaller rectangle). By subtracting the area of this smaller rectangle from the larger rectangle, the area of the parallelogram can be determined.

Previously

In elementary grades, students named and identified attributes of special quadrilaterals. In Lessons 3–5, students saw that area is conserved when polygons are decomposed and rearranged, and they also found the area of shapes composed of polygons with known areas by adding.

Coming Soon

In Lesson 7, students will identify bases and heights of parallelograms, and they will combine the meaning of those terms with the strategies from this and previous lessons to discover the formula for the area of a parallelogram.

Rigor

- Students decompose, rearrange and enclose parallelograms to develop **conceptual understanding** of how to determine the area of a parallelogram.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	5 min	5 min
Pairs	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, Grids, one grid per student
- Activity 2 PDF, Table, one per student
- straightedges

Math Language Development

New words

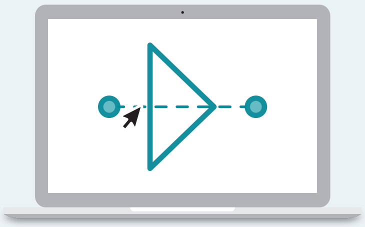
- *parallelogram*
- *quadrilateral*

Amps | **Featured Activity**

Activity 1

Digital Sketch

Students illustrate how they decompose a parallelogram in order to determine its area.



Amps
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Building Math Identity and Community

Connecting to Mathematical Practices

Students might become frustrated when they are asked to show more than one strategy because they already found one that “works.” Explain that not all strategies work for all shapes, due to the shapes’ structures. Therefore, they should seek to learn as many strategies as possible. Encourage students to actively observe or ask questions of their peers in order to build the number of strategies that they have to use.

Modifications to Pacing

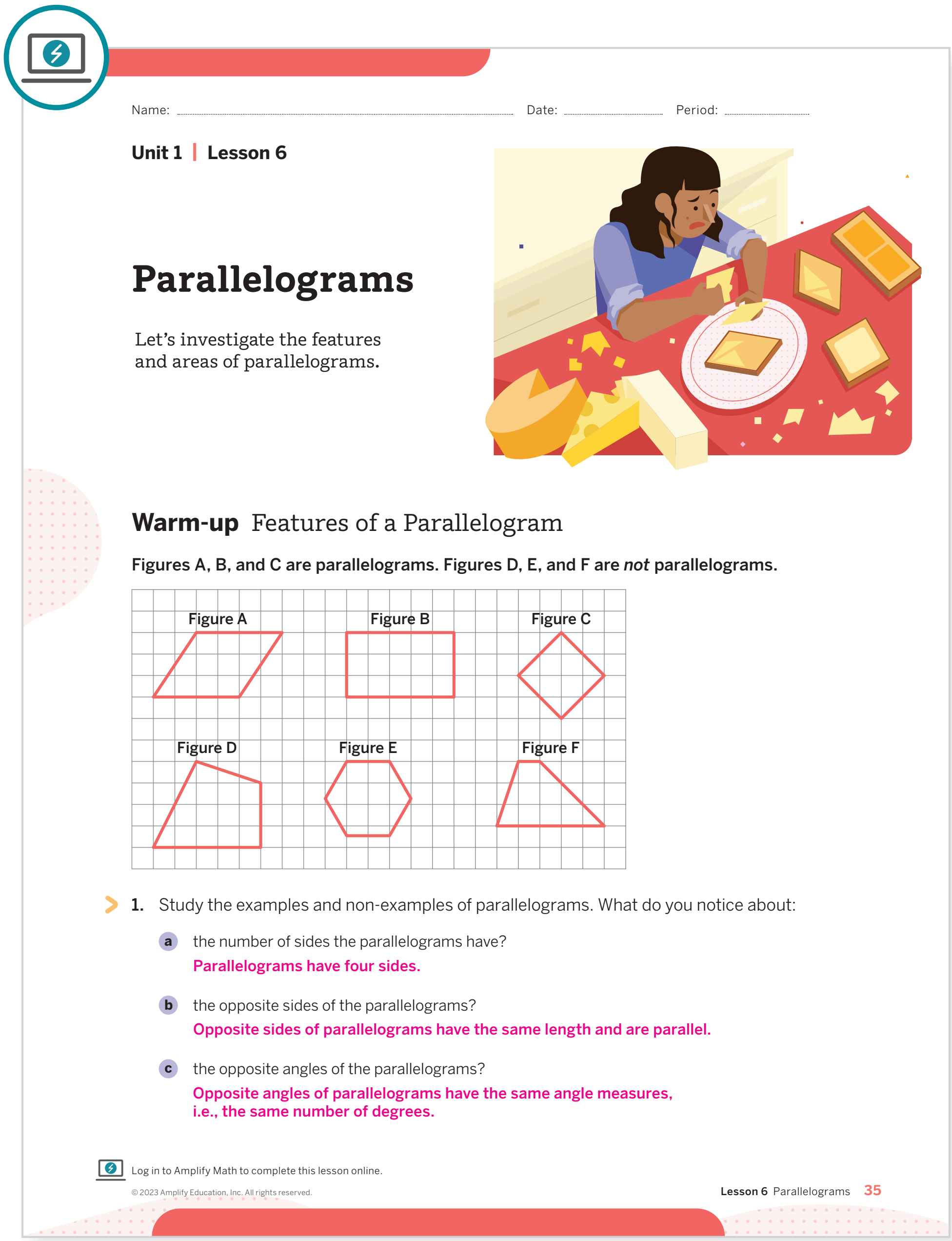
You may want to consider these additional modifications if you are short on time:

- In **Activity 1**, select either Problem 1 or Problem 2 and have students work together to find different strategies for the given shape.
- In **Activity 2**, have students work in pairs, instead of groups of three, to reduce the number of parallelograms for which they need to determine the area.

Pairs | 10 min

Warm-up Features of a Parallelogram

Students identify defining features of parallelograms by comparing and contrasting given examples and non-examples.



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Unit 1 | Lesson 6

Parallelograms

Let's investigate the features and areas of parallelograms.

Warm-up Features of a Parallelogram

Figures A, B, and C are parallelograms. Figures D, E, and F are *not* parallelograms.

1. Study the examples and non-examples of parallelograms. What do you notice about:
   - a the number of sides the parallelograms have?
     Parallelograms have four sides.
   - b the opposite sides of the parallelograms?
     Opposite sides of parallelograms have the same length and are parallel.
   - c the opposite angles of the parallelograms?
     Opposite angles of parallelograms have the same angle measures, i.e., the same number of degrees.

Log in to Amplify Math to complete this lesson online.

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Lesson 6 Parallelograms 35

1 Launch

Give students up to a minute to make individual observations before sharing their responses with a partner and then completing the problem.

2 Monitor

Help students get started by asking, "What is one thing Figures A–C all have in common? Do any of Figures D–F also have that feature? If not, how are they different?"

Look for points of confusion:

- **Not knowing how to determine if lines are parallel.** Show examples of parallel lines and explain how they will never intersect.
- **Generating statements that are true for some, but not all parallelograms.** Ask, "Is your statement true for every parallelogram?"

Look for productive strategies:

- Recognizing that Figures A–C all have four sides, but so do Figures D and F.
- Recognizing opposite sides appear parallel in Figures A–C and also in Figure E (except Figure E has six sides). Noticing that Figure F only has one pair of parallel sides.

3 Connect

Have students share their responses and thinking. Invite other students to agree or disagree and explain their thinking.

Define a ***quadrilateral*** as a two-dimensional shape that has four sides, and a ***parallelogram*** as a type of quadrilateral that has two pairs of parallel sides.

Highlight that all parallelograms are quadrilaterals, but not all quadrilaterals are parallelograms.

Ask, "Why are each of the Figures D, E, and F not parallelograms?"

Math Language Development

MLR2: Collect and Display

As students share their responses during the Connect, collect and display language used to describe features of parallelograms. Amplify phrases, such as "all parallelograms are quadrilaterals, but not all quadrilaterals are parallelograms."

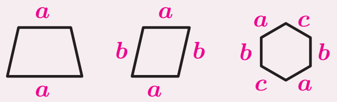
English Learners

When discussing parallel lines, use hand gestures to demonstrate that parallel lines will never intersect.

Power-up

To power up students' ability to identify parallel lines segments, have students complete:

Recall that two lines are *parallel* if they will never intersect, no matter how far they extend. In each shape identify *all* the pairs of parallel sides.



Use: Before the Warm-up.

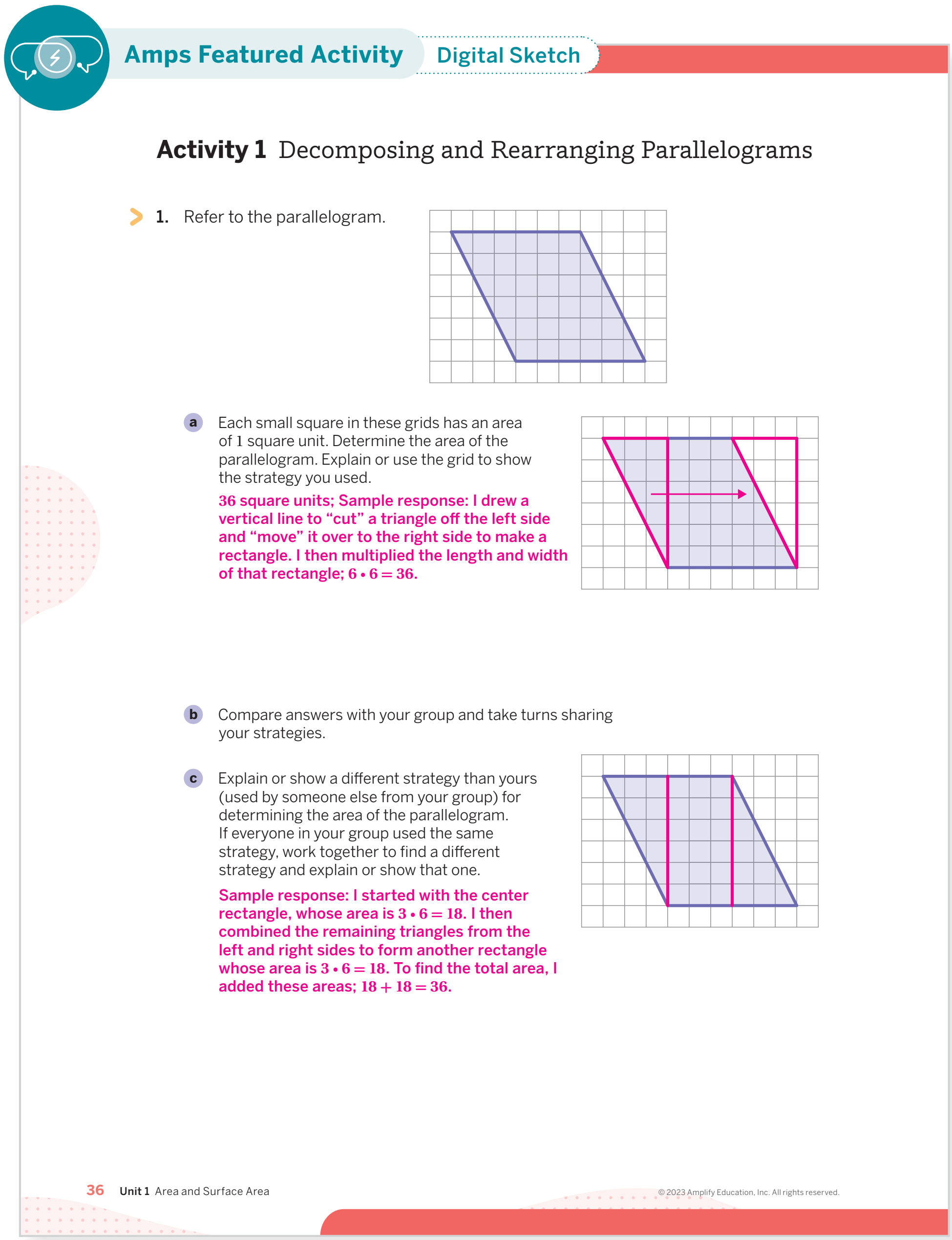
Informed by: Performance on Lesson 5, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4.

Small Groups | 15 min

Activity 1 Decomposing and Rearranging Parallelograms

Students explore different methods of decomposing parallelograms and rearranging the resulting pieces to help determine the area of parallelograms.

Amps Featured Activity Digital Sketch



Activity 1 Decomposing and Rearranging Parallelograms

1. Refer to the parallelogram.

ⓐ Each small square in these grids has an area of 1 square unit. Determine the area of the parallelogram. Explain or use the grid to show the strategy you used.

36 square units; Sample response: I drew a vertical line to "cut" a triangle off the left side and "move" it over to the right side to make a rectangle. I then multiplied the length and width of that rectangle; $6 \cdot 6 = 36$.

ⓑ Compare answers with your group and take turns sharing your strategies.

ⓒ Explain or show a different strategy than yours (used by someone else from your group) for determining the area of the parallelogram. If everyone in your group used the same strategy, work together to find a different strategy and explain or show that one.

Sample response: I started with the center rectangle, whose area is $3 \cdot 6 = 18$. I then combined the remaining triangles from the left and right sides to form another rectangle whose area is $3 \cdot 6 = 18$. To find the total area, I added these areas; $18 + 18 = 36$.

36 **Unit 1** Area and Surface Area

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① Launch

Arrange students in groups of 2–3 to complete Problem 1a individually before discussing their responses with their group. Then have them work together to complete the remainder of Problems 1 and 2. Consider preparing extra copies of the parallelogram images for students who might wish to cut the parallelograms out.

② Monitor

Help students get started by asking, "What are some strategies you used for determining areas in previous lessons that could help you here?"

Look for points of confusion:

- **Relying solely on counting squares to determine area.** Refer back to Lesson 5 and note the different decomposition and rearranging strategies. Ask students to apply one of these strategies.
- **Thinking there is only one way to determine area.** Ask students if they can decompose or rearrange the parallelogram in a different way.

Look for productive strategies:

- Decomposing a parallelogram and rearranging the pieces to create one rectangle with the same base and height. **Note:** These terms have not yet been introduced, and students are not expected to use them formally at this point, or at all.
- Decomposing a parallelogram into a rectangle and two identical triangles that can also be composed to form a rectangle, and then adding the areas of the two rectangles.
- Enclosing a parallelogram within a rectangle, rearranging the non-shaded triangular regions to compose another rectangle, and then subtracting the area of the smaller rectangle from the larger, enclosing rectangle, recognizing the difference is the area of the shaded region that is the parallelogram.
- Recognizing when and how the same strategy can be applied to more than one parallelogram.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide a "slightly tilted" parallelogram that is almost a rectangle, slanted by only 1 or 2 units horizontally. Have students determine the area. Alternatively, display multiple copies of each parallelogram. As students describe their strategies, use color and annotations to scribe their thinking. Label each parallelogram with the strategy described.

Extension: *Math Enrichment*

Provide graph paper to students and have them draw their own parallelogram. Have them determine the area by using and explaining more than one strategy.

Math Language Development

MLR2: Collect and Display

Listen to students talk about different methods to determine the area of the parallelograms. Record important phrases on an anchor chart. During discussions, remind students to borrow language from the display as needed.

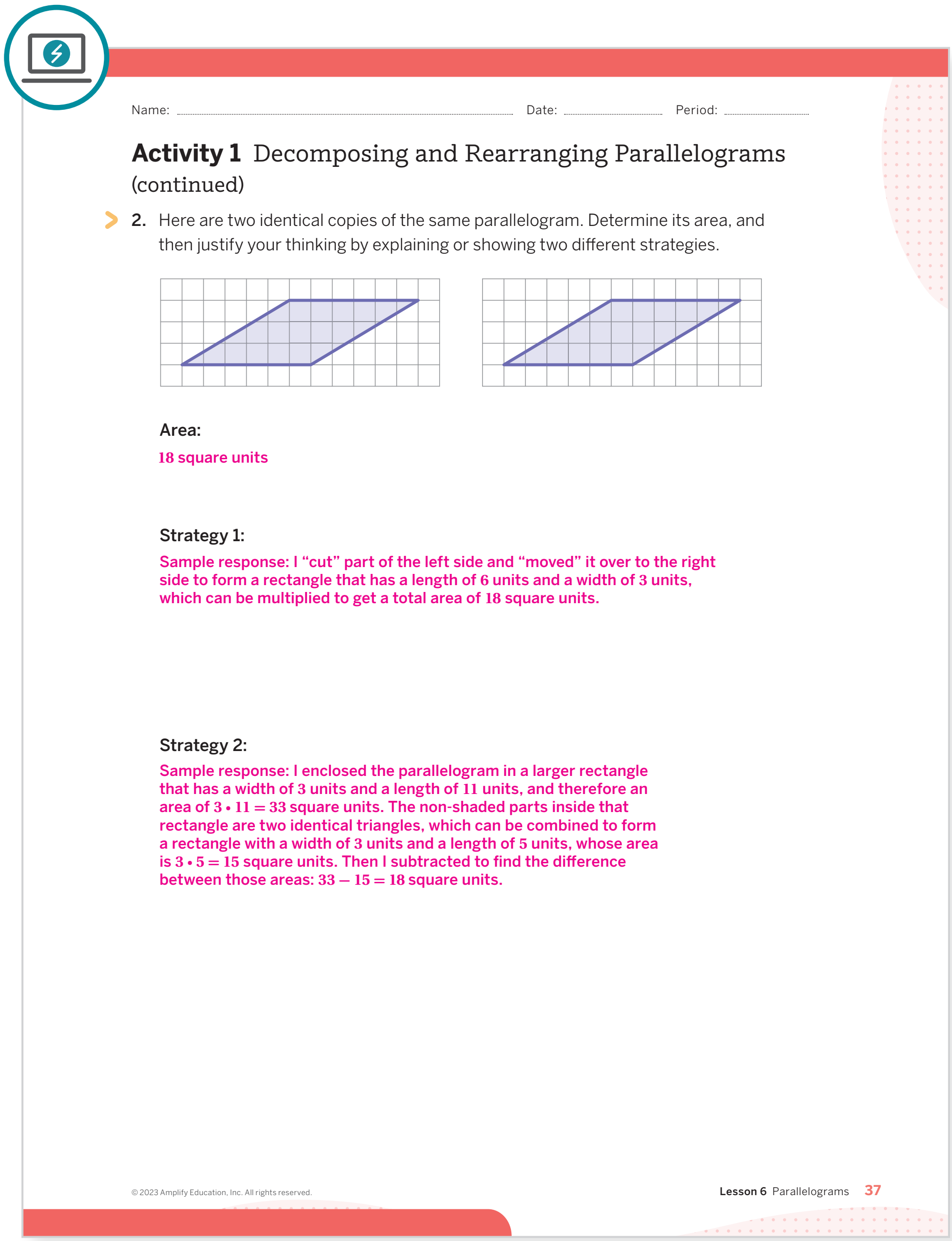
English Learners

Provide multiple cutouts of the parallelograms to students so that they can refer to the physical cutouts during the discussion.

Small Groups | 15 min

Activity 1 Decomposing and Rearranging Parallelograms (continued)

Students explore different methods of decomposing parallelograms and rearranging the resulting pieces to help determine the area of parallelograms.



Name: ______ Date: ______ Period: ______

Activity 1 Decomposing and Rearranging Parallelograms (continued)

2. Here are two identical copies of the same parallelogram. Determine its area, and then justify your thinking by explaining or showing two different strategies.

Area:

18 square units

Strategy 1:

Sample response: I "cut" part of the left side and "moved" it over to the right side to form a rectangle that has a length of 6 units and a width of 3 units, which can be multiplied to get a total area of 18 square units.

Strategy 2:

Sample response: I enclosed the parallelogram in a larger rectangle that has a width of 3 units and a length of 11 units, and therefore an area of $3 \cdot 11 = 33$ square units. The non-shaded parts inside that rectangle are two identical triangles, which can be combined to form a rectangle with a width of 3 units and a length of 5 units, whose area is $3 \cdot 5 = 15$ square units. Then I subtracted to find the difference between those areas: $33 - 15 = 18$ square units.

3 Connect

Display the first parallelogram by itself, and then add the second parallelogram when appropriate, based on the flow of the class discussion. Consider providing space for multiple copies of each to show multiple strategies as they are shared.

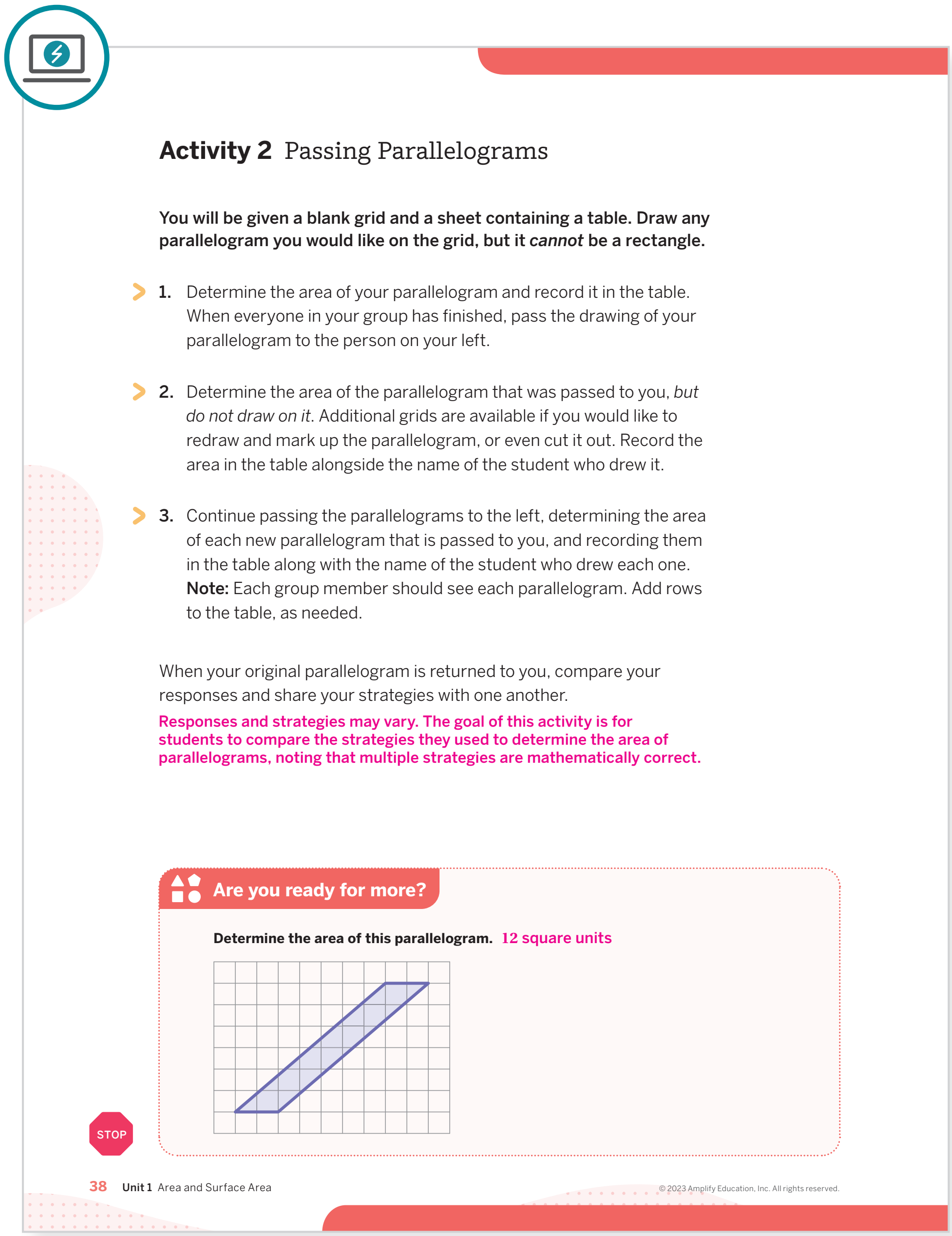
Have students share the various ways they determined the area, starting with decomposition strategies, and then by using a rectangle to enclose the parallelogram. Begin to construct one or more anchor charts that can remain on display and be added to throughout this sub-unit. Consider including, at this point, types of polygons a hierarchy of quadrilaterals with definitions and properties, and also the various decomposition strategies (composing, decomposing, rearranging, enclosing).

Highlight that more than one strategy can be used for each parallelogram, and that each of those strategies could also be used for both parallelograms. Consider introducing any strategies that students did not arrive at on their own. Reinforce the usefulness of rectangles as a familiar shape whose area can be determined.

Small Groups | 10 min

Activity 2 Passing Parallelograms

Students draw their own parallelograms, then determine the area of parallelograms drawn by other members of their groups, and finally discuss results and strategies.



Activity 2 Passing Parallelograms

You will be given a blank grid and a sheet containing a table. Draw any parallelogram you would like on the grid, but it *cannot* be a rectangle.

1. Determine the area of your parallelogram and record it in the table. When everyone in your group has finished, pass the drawing of your parallelogram to the person on your left.
2. Determine the area of the parallelogram that was passed to you, *but do not draw on it*. Additional grids are available if you would like to redraw and mark up the parallelogram, or even cut it out. Record the area in the table alongside the name of the student who drew it.
3. Continue passing the parallelograms to the left, determining the area of each new parallelogram that is passed to you, and recording them in the table along with the name of the student who drew each one. **Note:** Each group member should see each parallelogram. Add rows to the table, as needed.

When your original parallelogram is returned to you, compare your responses and share your strategies with one another.

Responses and strategies may vary. The goal of this activity is for students to compare the strategies they used to determine the area of parallelograms, noting that multiple strategies are mathematically correct.

Are you ready for more?

Determine the area of this parallelogram. 12 square units

STOP

38 **Unit 1** Area and Surface Area

1 Launch

Use the same groups from Activity 1. Provide each student with a straightedge, one pre-cut grid from the Activity 2 PDF (Grids), one copy of the Activity 2 PDF (Table), and one straightedge for drawing parallelograms.

2 Monitor

Help students get started by having them list the attributes of a parallelogram.

Look for points of confusion:

- **Not knowing how to draw a pair of parallel sides that are not horizontal or vertical.** Refer students to parallelograms drawn on grids from previous activities, and remind them that opposite sides of a parallelogram are the same length.

Look for productive strategies:

- Applying previously seen decomposition or enclosing strategies to determine the area.
- Recognizing that some strategies can be used repeatedly, while some figures require different strategies. If any students use area calculations, acknowledge that this will be explored further in upcoming lessons.

3 Connect

Have individual students share their parallelogram and how their group members determined its area. Ensure each group presents once and all strategies are highlighted. You may also consider conducting the ***Gallery Tour*** routine for students to view the different parallelograms drawn.

Ask, "What strategies did you use to determine the area?"

Highlight that decomposing, rearranging, or enclosing strategies can be used for any parallelogram, but depending on the given figure, some may be clearer or more efficient than others.

Differentiated Support

Accessibility: *Optimize Access to Tools*

To assist students in drawing their parallelograms, provide tangrams or pattern blocks that they can trace. Consider also providing additional grids for students to redraw the figures, cut them out, or make additional markings.

Math Language Development

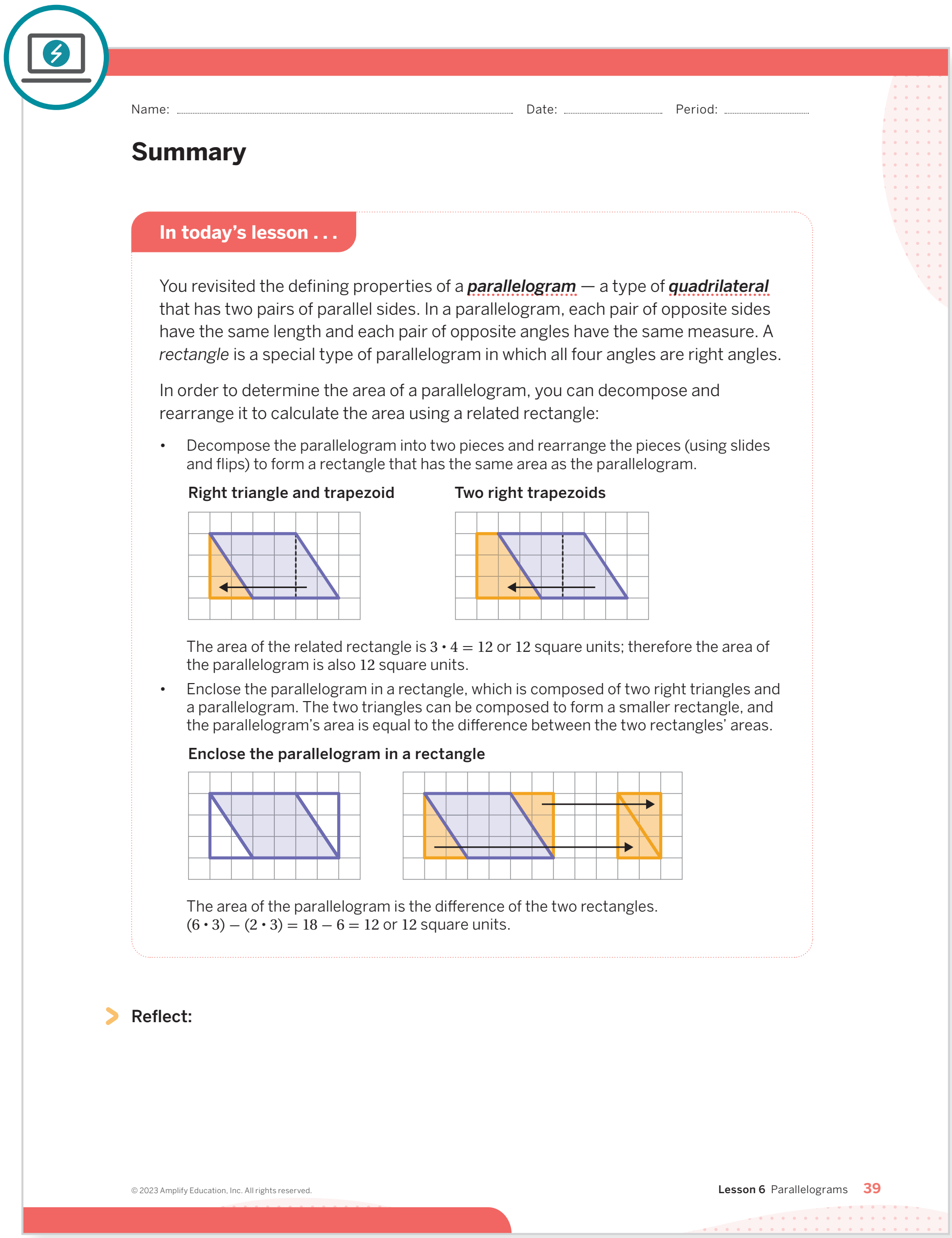
MLR7: Compare and Connect

After students complete Problem 3, have them work with their group members to compare and connect the different solution pathways they took to determine the area of the parallelogram. Encourage students to refer to the anchor chart display with the important words and phrases used to describe finding the area of a parallelogram during their discussions. You might consider using the *Gallery Tour* routine referenced in the Connect for students to compare and connect their strategies.

Whole Class | 5 min

Summary

Review and synthesize the attributes of a parallelogram and the different methods for determining its area.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You revisited the defining properties of a ***parallelogram*** — a type of ***quadrilateral*** that has two pairs of parallel sides. In a parallelogram, each pair of opposite sides have the same length and each pair of opposite angles have the same measure. A *rectangle* is a special type of parallelogram in which all four angles are right angles.

In order to determine the area of a parallelogram, you can decompose and rearrange it to calculate the area using a related rectangle:

- Decompose the parallelogram into two pieces and rearrange the pieces (using slides and flips) to form a rectangle that has the same area as the parallelogram.

Right triangle and trapezoid **Two right trapezoids**

The area of the related rectangle is $3 \cdot 4 = 12$ or 12 square units; therefore the area of the parallelogram is also 12 square units.

- Enclose the parallelogram in a rectangle, which is composed of two right triangles and a parallelogram. The two triangles can be composed to form a smaller rectangle, and the parallelogram's area is equal to the difference between the two rectangles' areas.

Enclose the parallelogram in a rectangle

The area of the parallelogram is the difference of the two rectangles.
$(6 \cdot 3) - (2 \cdot 3) = 18 - 6 = 12$ or 12 square units.

Reflect:

Lesson 6 Parallelograms 39

Synthesize

Display the Summary that shows the different ways to decompose, rearrange, and enclose parallelograms to determine their areas.

Ask:

- "What are the benefits or drawbacks of these different strategies?"
- "Do you think certain strategies are better for certain parallelograms? Why or why not?"
- "Are there any strategies that can be used for *any* parallelogram?"

Have students share how the shape of a parallelogram helps them decide which strategy to use.

Highlight that the various strategies for determining area share common ways of thinking, such as decomposing and rearranging, and also rely on knowing the similar types of information, such as the area of a rectangle. Often more than one strategy can be used, but many times, the same strategy can be used, even when two or more parallelograms look different.

Formalize vocabulary:

- *quadrilateral*
- *parallelogram*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How does knowing how to decompose, rearrange, and enclose help you when analyzing shapes?"
- "Is one strategy more helpful than any others? How so?"

Math Language Development

MLR2: Collect and Display

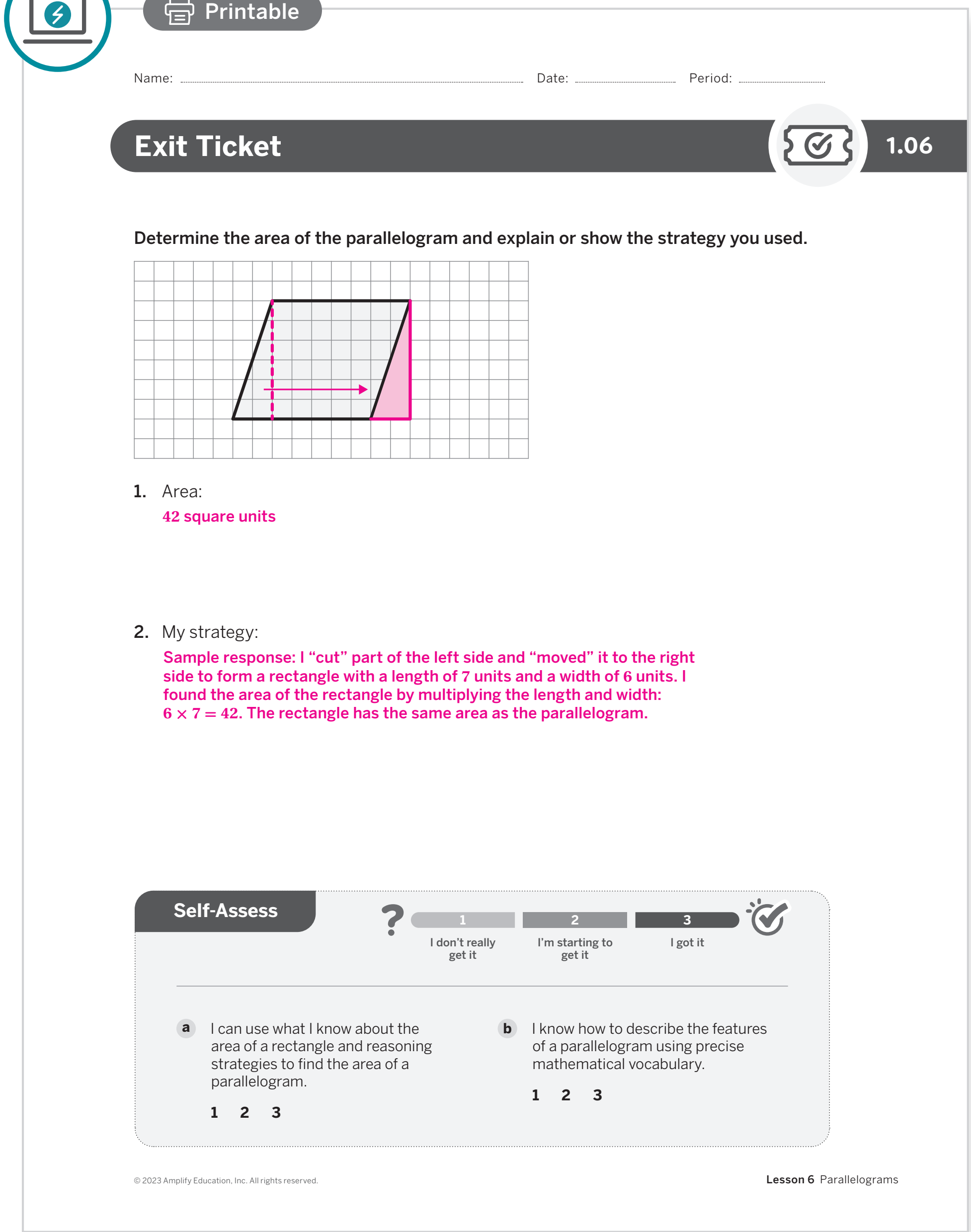
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 3. Ask them to review and reflect on any terms and phrases related to the terms *quadrilaterals* or *parallelograms* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by calculating the area of a parallelogram by decomposing, rearranging, or enclosing.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.06

Determine the area of the parallelogram and explain or show the strategy you used.

1. Area:
   42 square units

2. My strategy:
   Sample response: I "cut" part of the left side and "moved" it to the right side to form a rectangle with a length of 7 units and a width of 6 units. I found the area of the rectangle by multiplying the length and width: $6 \times 7 = 42$. The rectangle has the same area as the parallelogram.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can use what I know about the area of a rectangle and reasoning strategies to find the area of a parallelogram.
   1 2 3

b. I know how to describe the features of a parallelogram using precise mathematical vocabulary.
   1 2 3

Lesson 6 Parallelograms

Success looks like . . .

- **Language Goal:** Comparing and contrasting different strategies for determining the area of a parallelogram **(Speaking and Listening)**.
- **Language Goal:** Describing observations about the opposite sides and opposite angles of parallelograms **(Speaking and Listening, Writing)**.
- **Language Goal:** Explaining how to find the area of a parallelogram by rearranging or enclosing it in a rectangle **(Speaking and Listening, Writing)**.
  - » Explaining how to cut and move a triangle from a parallelogram to create a rectangle in order to determine the area in Problem 2.

Suggested next steps

If students miscalculate the area, consider:

- Providing them with a copy of the shape to cut out and rearrange the pieces on a grid.
- Having them check their work by using a different strategy.
- Reviewing their calculations.

If students have trouble articulating their strategy, consider:

- Referring back to the Summary and going over the different strategies presented.
- Reviewing Activity 1 and asking, "What strategies were used here?"
- Encouraging them to draw pictures to show their thinking rather than explaining in words.

Professional Learning

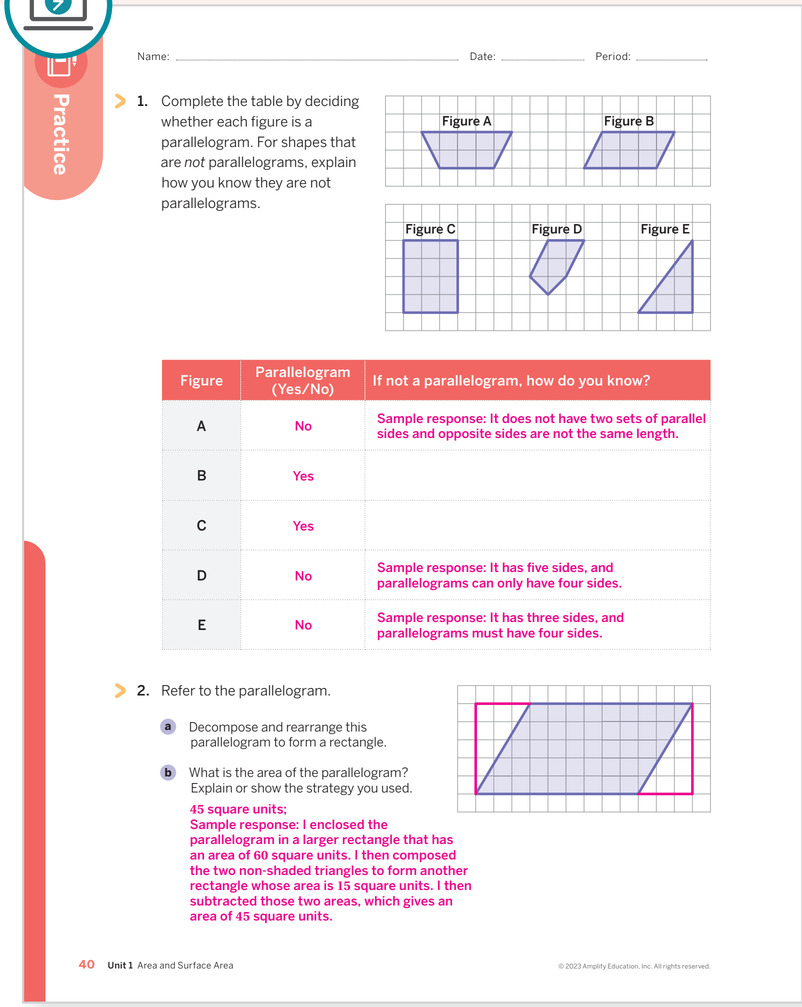
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students worked on the different strategies to determine the area of a parallelogram?
- What was especially satisfying about how the lesson went today? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Complete the table by deciding whether each figure is a parallelogram. For shapes that are *not* parallelograms, explain how you know they are not parallelograms.

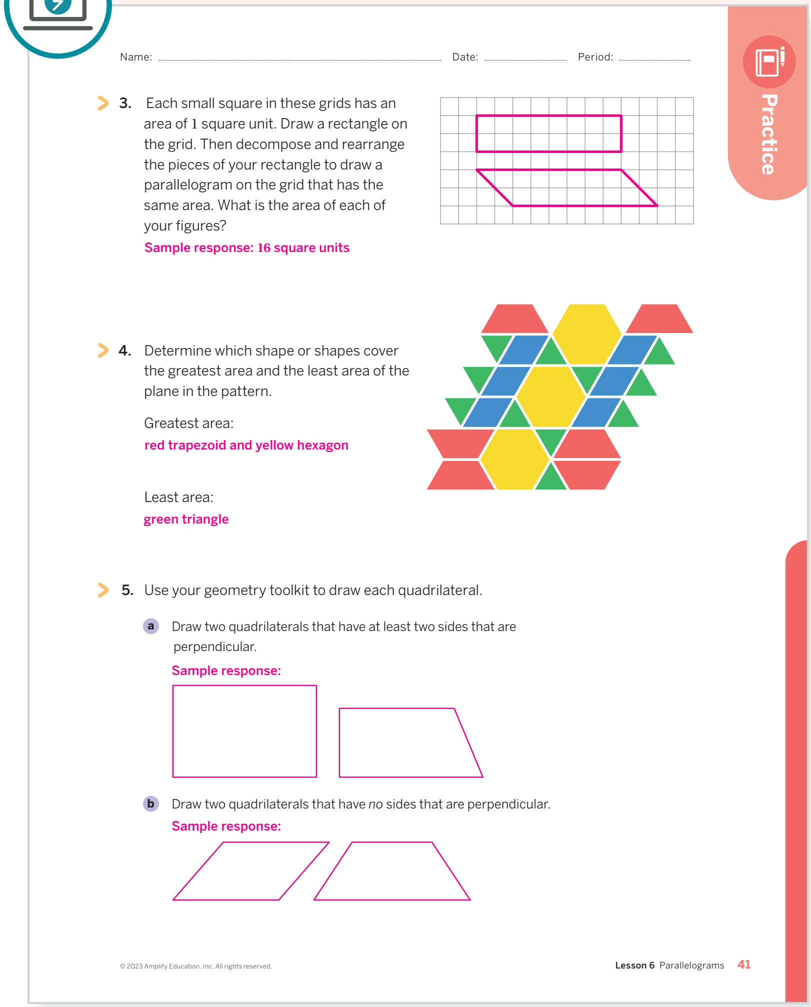
Figure	Parallelogram (Yes/No)	If not a parallelogram, how do you know?
A	No	Sample response: It does not have two sets of parallel sides and opposite sides are not the same length.
B	Yes	
C	Yes	
D	No	Sample response: It has five sides, and parallelograms can only have four sides.
E	No	Sample response: It has three sides, and parallelograms must have four sides.

2. Refer to the parallelogram.

a. Decompose and rearrange this parallelogram to form a rectangle.

b. What is the area of the parallelogram? Explain or show the strategy you used.

45 square units;
Sample response: I enclosed the parallelogram in a larger rectangle that has an area of 60 square units. I then composed the two non-shaded triangles to form another rectangle whose area is 15 square units. I then subtracted those two areas, which gives an area of 45 square units.



Practice

Name: Date: Period:

3. Each small square in these grids has an area of 1 square unit. Draw a rectangle on the grid. Then decompose and rearrange the pieces of your rectangle to draw a parallelogram on the grid that has the same area. What is the area of each of your figures?

Sample response: 16 square units

4. Determine which shape or shapes cover the greatest area and the least area of the plane in the pattern.

Greatest area:

red trapezoid and yellow hexagon

Least area:

green triangle

5. Use your geometry toolkit to draw each quadrilateral.

a. Draw two quadrilaterals that have at least two sides that are perpendicular.

Sample response:

b. Draw two quadrilaterals that have *no* sides that are perpendicular.

Sample response:

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 3	2
Formative ⚡	5	Unit 1 Lesson 7	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 7**

Bases and Heights of Parallelograms

Let's continue to investigate the areas of parallelograms.

Focus

Goals

1. **Language Goal:** Comprehend the terms *base* and *height* to refer to one side of a parallelogram (base) and the perpendicular distance (height) between that side and the opposite side. **(Speaking and Listening, Writing)**
2. Identify a base and the corresponding height for a parallelogram, and understand that there are two different base and height pairs for any parallelogram.
3. **Language Goal:** Generalize a process for determining the area of a parallelogram, using the length of a base and its corresponding height. **(Speaking and Listening, Writing)**

Coherence

Today

Students define the terms *base* and *height* for parallelograms. They recognize and identify different pairs of bases and corresponding heights within the same parallelogram. Then using decomposition strategies to determine area. Students investigate and discover a pattern among the measurements of the base, corresponding height, and area in order to generalize a formula for the area of a parallelogram.

Previously

In Lesson 6, students determined the area of parallelograms by using decomposing, rearranging, and enclosing strategies to create rectangles with known areas.

Coming Soon

Students will apply the formula for the area of a parallelogram to real-world scenarios in Lesson 8, as well as leverage the relationship between parallelograms and triangles to determine the area of triangles in Lessons 9–11.

Rigor

- Students practice identifying the base and height of a parallelogram to build **procedural skills** for determining area.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

New words

- ***base**** *(of a parallelogram)*
- ***height*** *(of a parallelogram)*

Review words

- *quadrilateral*
- *parallelogram*

*Students may confuse the term *base*, which refers to the *base* of a parallelogram, with its meaning in other contexts, such as the first *base* in baseball or the *base* and exponent of a power. Be ready to address these differences.

Amps | **Featured Activity**

Exit Ticket

Real-Time Exit Ticket

Check in real time whether your students can find the values for the base, height, and area of the parallelograms by using a digital Exit Ticket that is automatically scored.

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, some students might feel frustrated in remembering how to correctly identify the base and height pairs of parallelograms and may confuse the two terms. Encourage students to brainstorm solutions to feel more confident, such as writing down the definitions or guidelines for drawing and labeling the base and height of a parallelogram, and rehearsing these strategies aloud.

Modifications to Pacing

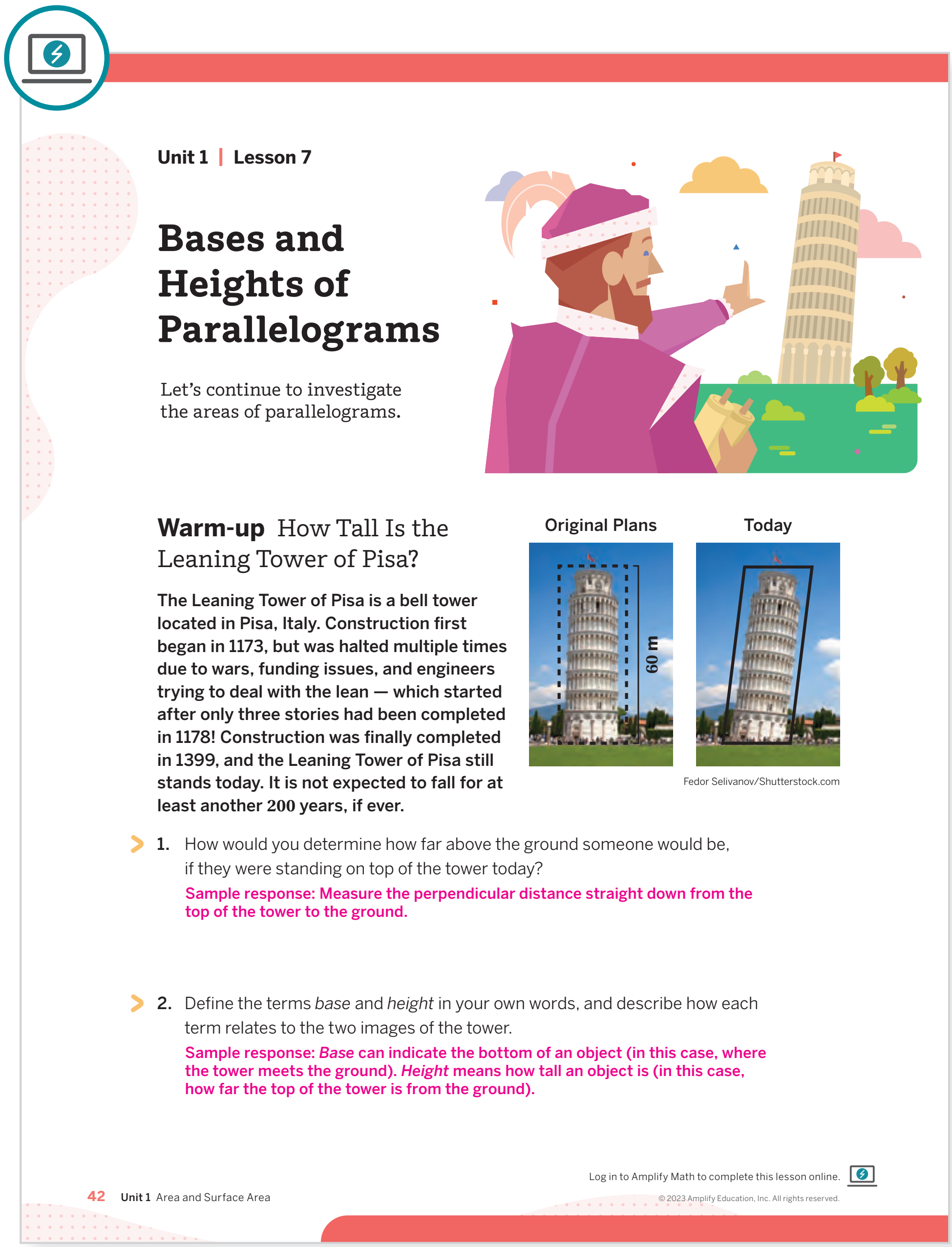
You may want to consider these additional modifications if you are short on time:

- In **Activity 1**, consider discussing Problem 1 as a class, being sure students know the relationship between a base and height. Then have students complete only Problem 3. If you have more time, consider telling students which statements from Problem 2 are false, and have them discuss and share possibly related true statements.
- In **Activity 2**, have students only complete rows of the table for two or three given parallelograms. Alternatively, place students in groups of four and have each student complete one parallelogram and then work together to determine the formula for the area of a parallelogram.

Independent | 10 min

Warm-up How Tall Is the Leaning Tower of Pisa?

Students begin to think about the terms *base* and *height* as they relate to parallelograms.



Unit 1 | Lesson 7

Bases and Heights of Parallelograms

Let's continue to investigate the areas of parallelograms.

Warm-up How Tall Is the Leaning Tower of Pisa?

The Leaning Tower of Pisa is a bell tower located in Pisa, Italy. Construction first began in 1173, but was halted multiple times due to wars, funding issues, and engineers trying to deal with the lean — which started after only three stories had been completed in 1178! Construction was finally completed in 1399, and the Leaning Tower of Pisa still stands today. It is not expected to fall for at least another 200 years, if ever.

Original Plans

Today

60 m

Fedor Selivanov/Shutterstock.com

1. How would you determine how far above the ground someone would be, if they were standing on top of the tower today?
   Sample response: Measure the perpendicular distance straight down from the top of the tower to the ground.

2. Define the terms *base* and *height* in your own words, and describe how each term relates to the two images of the tower.
   Sample response: *Base* can indicate the bottom of an object (in this case, where the tower meets the ground). *Height* means how tall an object is (in this case, how far the top of the tower is from the ground).

42 Unit 1 Area and Surface Area

Log in to Amplify Math to complete this lesson online.

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1 Launch

Activate background knowledge by reading about the Leaning Tower of Pisa, then set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by asking, "If you dropped a penny from the top of the tower, how could you determine how far it fell?"

Look for points of confusion:

- **Thinking the height cannot be determined without measurements.** Reiterate that the question is asking "how" to determine the height, not "what" it is in terms of any measure.
- **Thinking the height is the slanted side of the tower.** Ask, "If you wanted to know the shortest distance from the top of the tower to the ground, would you still measure the slanted side?"
- **Having trouble articulating their own definitions of base and height.** Have students begin with the tower example and use the image to guide their writing.

Look for productive strategies:

- Connecting their thinking in Problem 1 to Problem 2, and recognizing that the height is the distance from the top of the building perpendicular to the ground.

3 Connect

Have students share how they would determine the distance from the top of the tower to the ground, as well as how that relates to their understanding of base and height.

Define the ***base*** of a parallelogram as any chosen side, and a ***height*** of a parallelogram as a segment measuring the shortest distance from the chosen base to the opposite side. A ***height*** will always intersect the base at a right angle. Consider expanding an existing anchor chart or creating a new one.

Ask students to find an example of a parallelogram in the room and to identify its base and height.

Math Language Development

MLR8: Discussion Supports

Utilize a strategic reading strategy to help students make sense of the questions being asked. For example, before reading aloud the background information, direct students to the two questions. Ask students to discuss with their partners what each of the questions is asking them to do. After partners have had a chance to make sense of the questions, read aloud the background information about the Leaning Tower of Pisa.

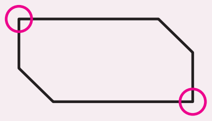
English Learners

When defining *base* and *height* of a parallelogram, use a physical example of a parallelogram to connect to the definitions.

Power-up

To power up students' ability to identify perpendicular line segments in polygons ask, have students complete:

Recall that perpendicular lines meet to form a right angle. Circle the vertices where perpendicular lines meet on the shape.

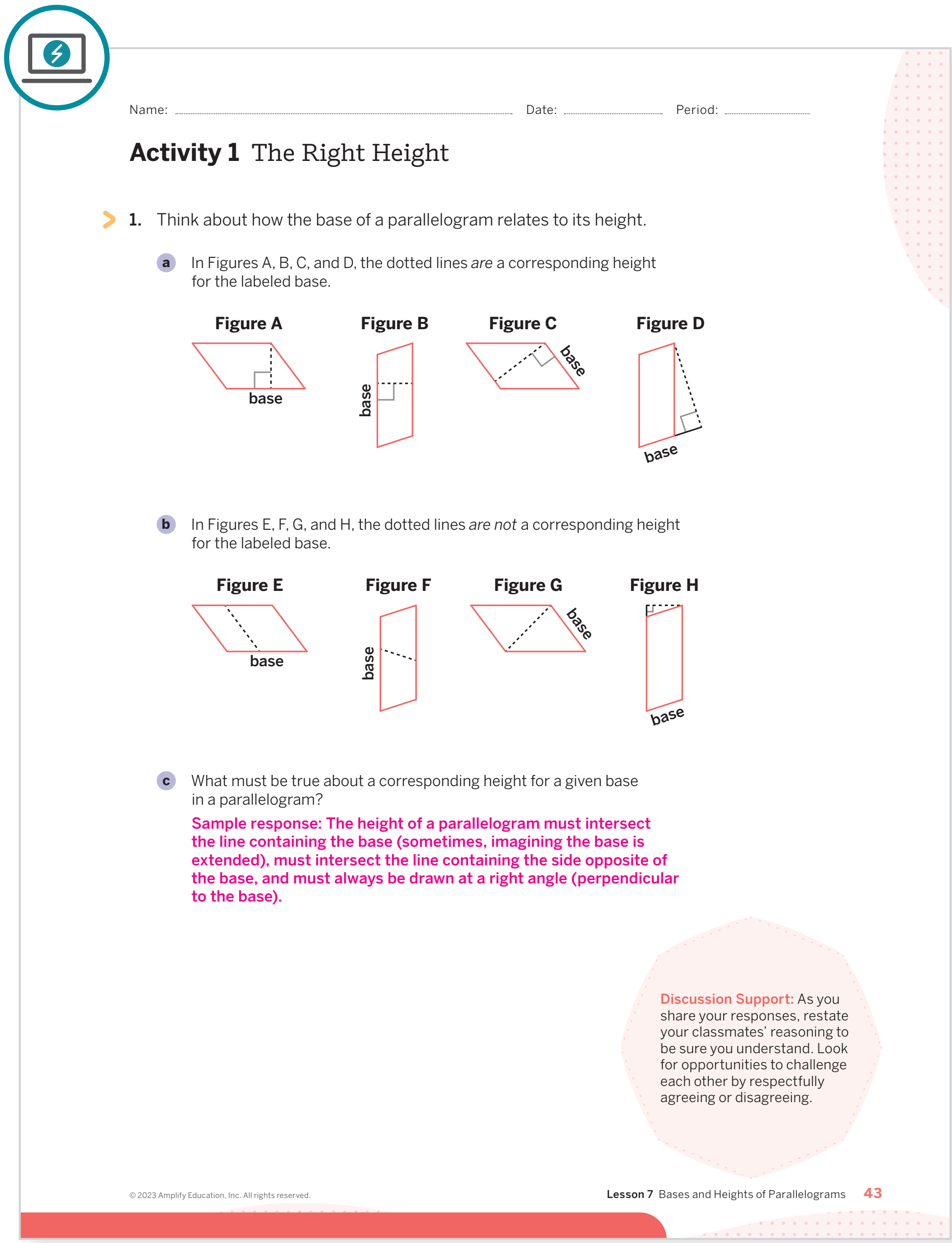


Use: Before the Warm-up.
Informed by: Performance on Lesson 6, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 5.

Pairs | 15 min

Activity 1 The Right Height

Students make generalizations about a parallelogram's base and height by studying examples and non-examples.



Name: ______ Date: ______ Period: ______

Activity 1 The Right Height

1. Think about how the base of a parallelogram relates to its height.

a In Figures A, B, C, and D, the dotted lines *are* a corresponding height for the labeled base.

Figure A | Figure B | Figure C | Figure D

b In Figures E, F, G, and H, the dotted lines *are not* a corresponding height for the labeled base.

Figure E | Figure F | Figure G | Figure H

c What must be true about a corresponding height for a given base in a parallelogram?

Sample response: The height of a parallelogram must intersect the line containing the base (sometimes, imagining the base is extended), must intersect the line containing the side opposite of the base, and must always be drawn at a right angle (perpendicular to the base).

Discussion Support: As you share your responses, restate your classmates' reasoning to be sure you understand. Look for opportunities to challenge each other by respectfully agreeing or disagreeing.

1 Launch

Display the examples and non-examples of bases and heights of parallelograms and have students respond individually to Problem 1, followed by a brief discussion of the properties of a parallelogram. Then have students work with a partner to complete Problems 2 and 3.

2 Monitor

Help students get started by having them compare Figures A and E and then Figures D and H, asking each time, "How are these parallelograms different?"

Look for points of confusion:

- **Not understanding heights drawn outside the shape.** Have students study all four examples and analyze how drawing a height outside the shape is also correct.
- **Not paying attention to right angles.** Remind students what a right angle is and how to use an edge of a paper to identify one, as well as how to look for the right angle symbol.
- **Thinking that the base can only be the "bottom" or a horizontal or vertical side.** Ask, "Why does Figure C show correct base and height pairings, but parallelogram G does not?"
- **Marking the last statement in the table for Problem 2 as false by thinking there can be only one corresponding height drawn from a given base.** Show students that there are few different ways to draw a corresponding height.
- **Thinking that Figure M in Problem 3 is incorrectly drawn.** Have students rotate their paper to see that the height can be vertical or horizontal, as long as it meets the requirements of being perpendicular to the base and of meeting at a vertex of the figure.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students have trouble identifying the difference between the examples and non-examples from Problem 1, explain the rule using a figure from each set. Then see if they can extrapolate that rule by studying some of the other figures. Have them attempt Problem 2 on their own.

Extension: *Math Enrichment*

Using a grid, have students draw their own examples and non-examples of bases and heights of parallelograms and then explain the differences between the two sets of images.

Math Language Development

MLR8: Discussion Supports—Restate It!

During the Connect, as students share responses, have them restate each other's reasoning using the terms *base, height,* and *perpendicular.* Encourage students to challenge each other when they disagree, using prompts such as:

- "I agree because . . ."
- "I disagree because . . ."

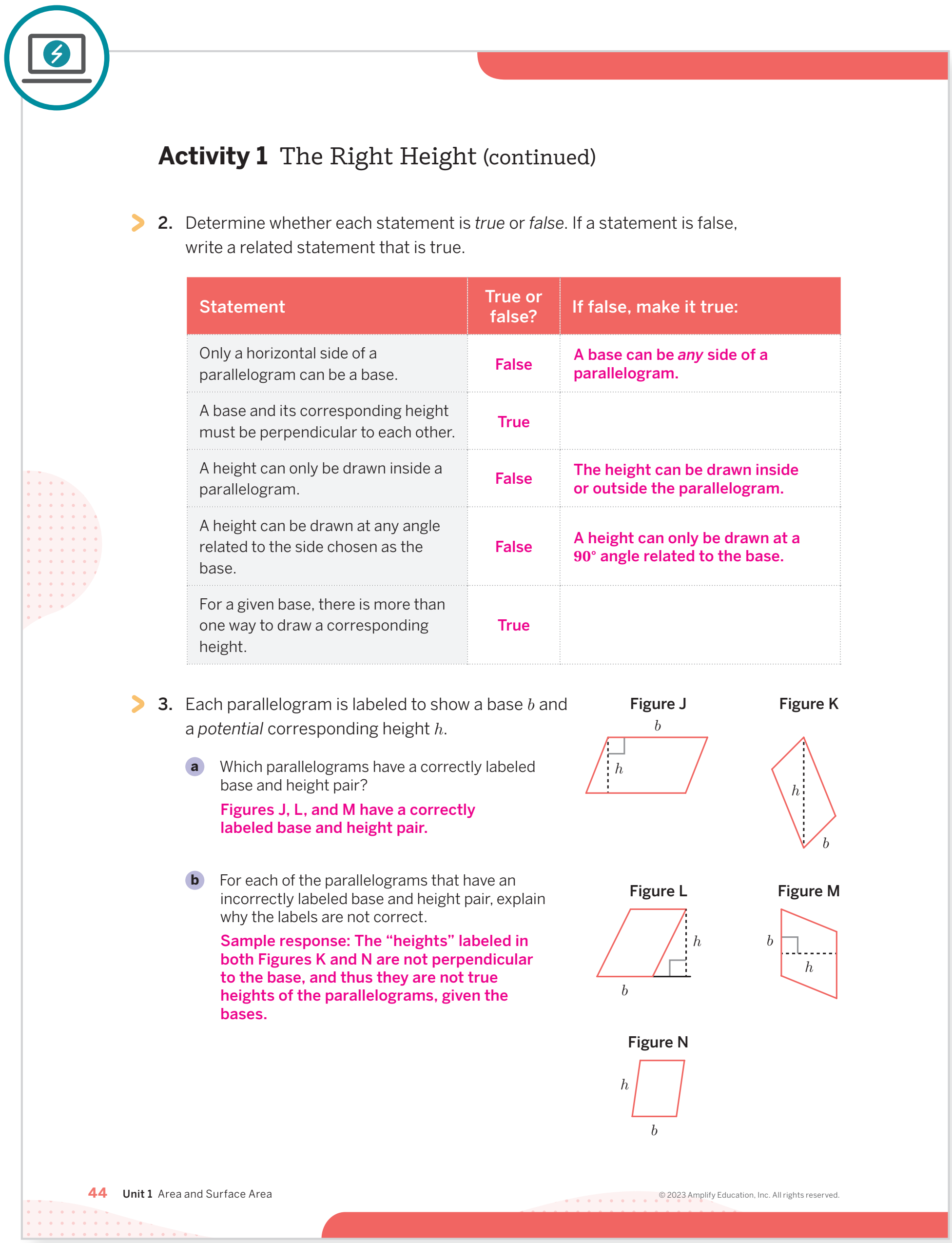
English Learners

When addressing Problem 1c, the term *intersect* might be a new term for many students. Be ready to address what it means for the height and the base of a parallelogram to intersect. Use drawings and gestures to highlight what this intersection looks like.

Pairs | 15 min

Activity 1 The Right Height (continued)

Students make generalizations about a parallelogram's base and height by studying examples and non-examples.



Activity 1 The Right Height (continued)

2. Determine whether each statement is *true* or *false*. If a statement is false, write a related statement that is true.

Statement	True or false?	If false, make it true:
Only a horizontal side of a parallelogram can be a base.	False	A base can be *any* side of a parallelogram.
A base and its corresponding height must be perpendicular to each other.	True	
A height can only be drawn inside a parallelogram.	False	The height can be drawn inside or outside the parallelogram.
A height can be drawn at any angle related to the side chosen as the base.	False	A height can only be drawn at a 90° angle related to the base.
For a given base, there is more than one way to draw a corresponding height.	True	

3. Each parallelogram is labeled to show a base b and a *potential* corresponding height h.

   a. Which parallelograms have a correctly labeled base and height pair?

   Figures J, L, and M have a correctly labeled base and height pair.

   b. For each of the parallelograms that have an incorrectly labeled base and height pair, explain why the labels are not correct.

   Sample response: The "heights" labeled in both Figures K and N are not perpendicular to the base, and thus they are not true heights of the parallelograms, given the bases.

Look for productive strategies:

- Understanding the meanings of the terms *base* and *height* in any given parallelogram.
- Correctly identifying a base and height pair within each parallelogram, applying the "rules" from the other drawings.
- Knowing that there are a few different ways to draw a corresponding height from the base, as long as it is perpendicular to the base and meets at an opposite vertex.

3 Connect

Display the table from Problem 2 and provide the correct responses for the *True or False*? column. Then display Figures J–N from Problem 3. Consider leaving the table visible throughout the discussion, if possible.

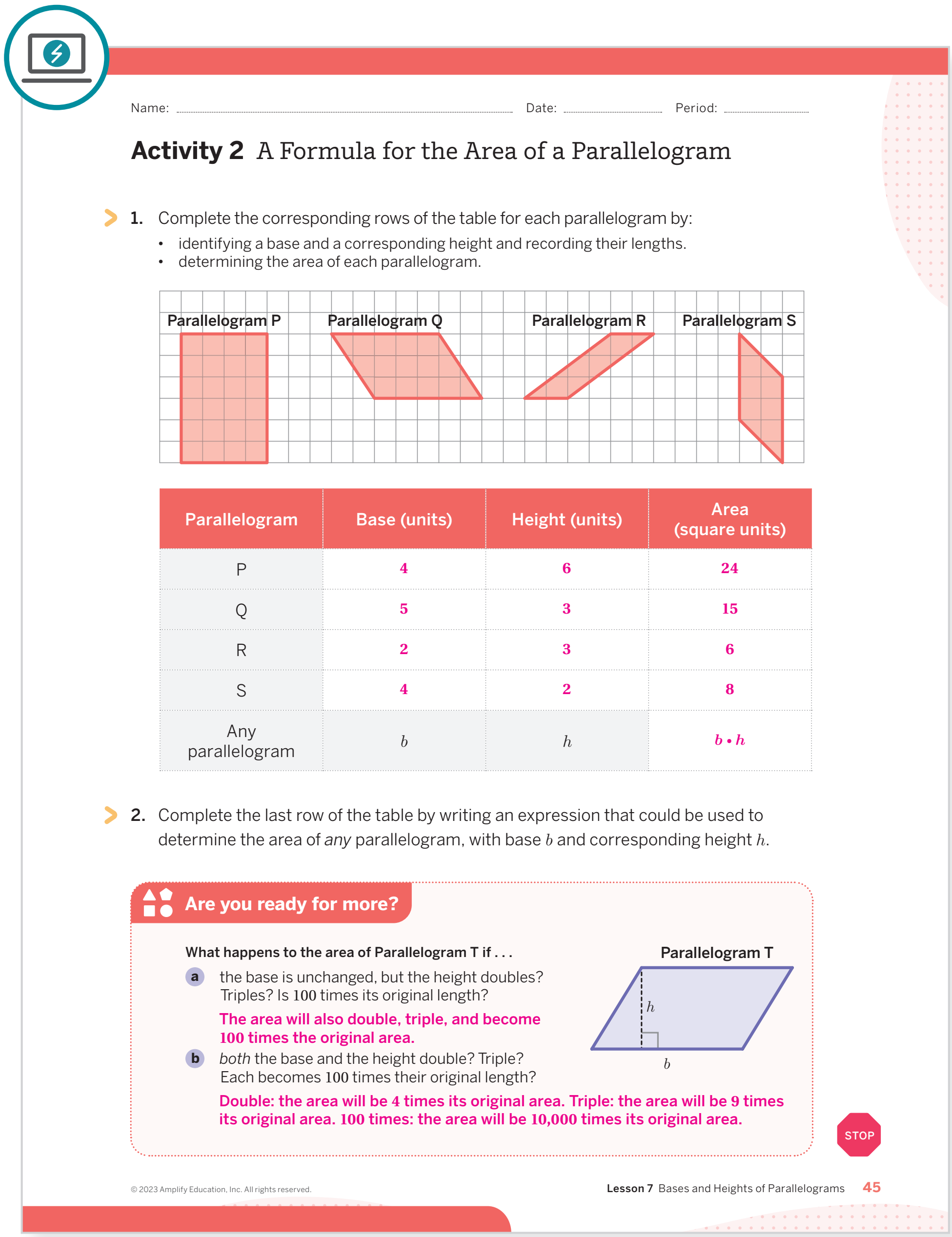
Have individual students share whether they disagree with any responses for Problem 2, and allow other students to help explain why they are correct, leveraging their related statements from the third column when applicable. Ensure all false statements have been corrected before students then share their responses and thinking for Problem 3, referencing the table as needed for discussion.

Highlight that any side of a parallelogram can be a base. To find the corresponding height for a chosen base, draw a segment that joins the base and its opposite side at a right angle. A height can also be drawn outside of the parallelogram by extending the line containing the base, or even also extending the line containing the opposite side to the base. For any parallelogram, many different heights can be drawn — some will be inside and some outside — but they will all have the same length.

Pairs | 10 min

Activity 2 A Formula for the Area of a Parallelogram

Students identify and label bases and heights of parallelograms to determine the area. They generalize their experience to develop a formula for the area of a parallelogram.



Name: .... Date: .... Period: ....

Activity 2 A Formula for the Area of a Parallelogram

1. Complete the corresponding rows of the table for each parallelogram by:
   - identifying a base and a corresponding height and recording their lengths.
   - determining the area of each parallelogram.

Parallelogram	Base (units)	Height (units)	Area (square units)
P	4	6	24
Q	5	3	15
R	2	3	6
S	4	2	8
Any parallelogram	b	h	$b \cdot h$

2. Complete the last row of the table by writing an expression that could be used to determine the area of *any* parallelogram, with base b and corresponding height h.

Are you ready for more?

What happens to the area of Parallelogram T if . . .

a. the base is unchanged, but the height doubles? Triples? Is 100 times its original length?

The area will also double, triple, and become 100 times the original area.

b. *both* the base and the height double? Triple? Each becomes 100 times their original length?

Double: the area will be 4 times its original area. Triple: the area will be 9 times its original area. 100 times: the area will be 10,000 times its original area.

1 Launch

Have students complete the table for Figures P–S independently before sharing and working collaboratively to complete the bottom row of the table for any parallelogram.

2 Monitor

Help students get started by asking, "Where would the base and corresponding height be?" Use the examples from Activity 1 as models.

Look for points of confusion:

- **Thinking that the height can only be inside the parallelogram.** Refer to Figure D from Activity 1.
- **Choosing a diagonal side as the base.** Ask, "Could a different side be the base?" While any side can be a base, the measurement of the diagonal cannot be determined here.

Look for productive strategies:

- Determining the pattern from the table in order to write an expression for the area of a parallelogram.

3 Connect

Display the four parallelograms and the table.

Have students share their strategies for determining the values for each parallelogram and the expression for the area of any parallelogram.

Highlight that because any parallelogram can be decomposed and rearranged to form a rectangle with side lengths equal to the base and height of the parallelogram, the formula for the area A of a parallelogram with base b and height h is $A = b \cdot h$. Review that while any side of a parallelogram can indeed be the base, there are more strategic ones to use here such as the horizontal or vertical sides. Consider adding examples and formulas to an anchor chart.

Ask, "How could Figure S be decomposed and rearranged to form a rectangle with length b and width h?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Parallelograms P and Q. If they have time available, then have them work on the others.

Accessibility: *Demonstrate, Guide Processing and Visualization*

Demonstrate and encourage students to use color coding and/or annotations to highlight the base and height pairs for each parallelogram as they complete the table in Problem 1.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies, ask them to identify similarities and differences for how they identified base and height pairs. Listen for and amplify key findings.

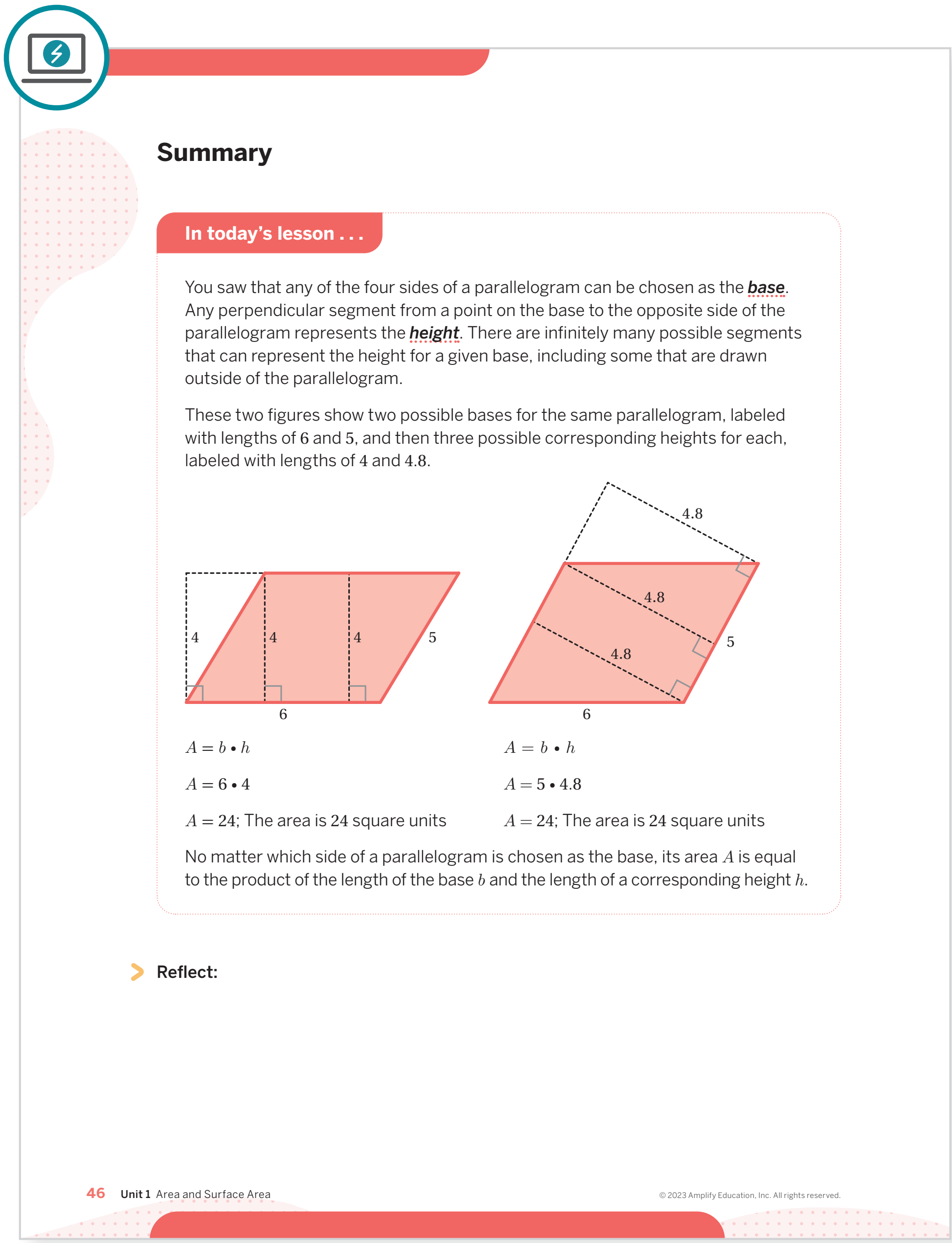
English Learners

Provide parallelogram cutouts to students to cut and rearrange as they make sense of language, such as "any parallelogram can be decomposed and rearranged to form a rectangle with side lengths equal to the base and height of the parallelogram."

Whole Class | 5 min

Summary

Review and synthesize how to identify the base and height of a parallelogram and how to use the measures of each to determine the area of a parallelogram.



Summary

In today's lesson . . .

You saw that any of the four sides of a parallelogram can be chosen as the ***base***. Any perpendicular segment from a point on the base to the opposite side of the parallelogram represents the ***height***. There are infinitely many possible segments that can represent the height for a given base, including some that are drawn outside of the parallelogram.

These two figures show two possible bases for the same parallelogram, labeled with lengths of 6 and 5, and then three possible corresponding heights for each, labeled with lengths of 4 and 4.8.

$A = b \cdot h$	$A = b \cdot h$
$A = 6 \cdot 4$	$A = 5 \cdot 4.8$
$A = 24$; The area is 24 square units	$A = 24$; The area is 24 square units

No matter which side of a parallelogram is chosen as the base, its area A is equal to the product of the length of the base b and the length of a corresponding height h.

Reflect:

Synthesize

Have students share their responses to the following questions.

Ask:

- "How many possible bases are there for a given parallelogram?" Any side can be considered a base.
- "How many different ways can a height be drawn for a chosen base in a given parallelogram? Will they always have the same length?" The height can be drawn in several ways, inside the parallelogram or outside the parallelogram, as long as it is drawn perpendicular to the base. The corresponding heights drawn for any given base will always have the same length.
- "What is the relationship between the lengths of the base and a corresponding height of a parallelogram and its area?" The product of the base and corresponding height is the area of the parallelogram.

Formalize vocabulary:

- ***base of a parallelogram***
- ***height of a parallelogram***

Highlight that there are two possible values for the base in a parallelogram (either side of the parallelogram) and many different ways to draw the height relative to the corresponding base, but the measure of any of those heights will always be the same. Reiterate the formula for the area A of a parallelogram as $A = b \cdot h$, where b represents the length of the base and h represents the length of the corresponding height and • represents multiplication.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What did you discover about the relationship between the base and height of a parallelogram?"
- "How do you determine the base of any parallelogram?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 3. Ask them to review and reflect on any terms and phrases related to the terms *base* or *height* (of a parallelogram) that were added to the display during the lesson.

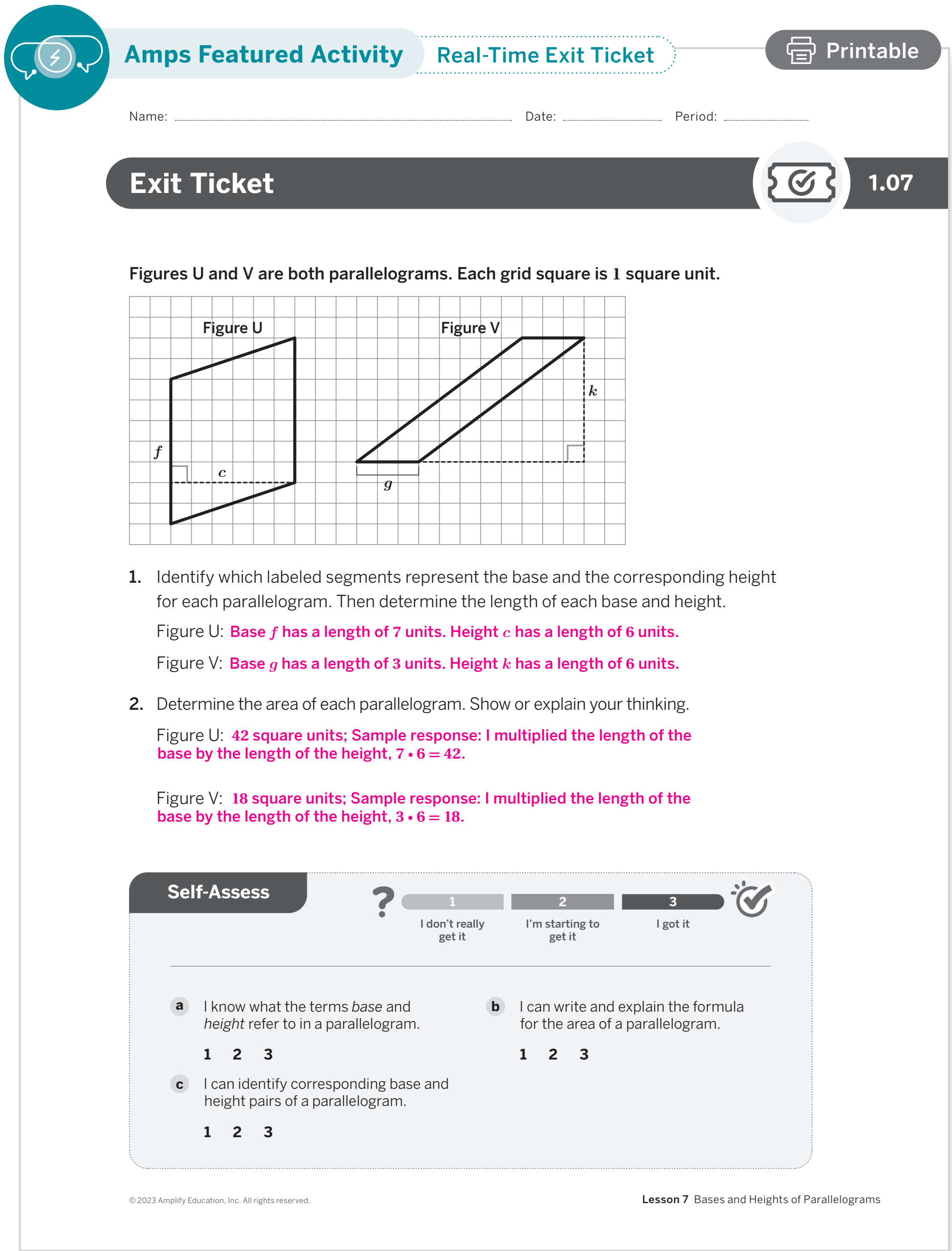
Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying the bases and heights of two parallelograms to determine their areas.

Amps Featured Activity Real-Time Exit Ticket

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.07

Figures U and V are both parallelograms. Each grid square is 1 square unit.

1. Identify which labeled segments represent the base and the corresponding height for each parallelogram. Then determine the length of each base and height.

   Figure U: **Base f has a length of 7 units. Height c has a length of 6 units.**

   Figure V: **Base g has a length of 3 units. Height k has a length of 6 units.**

2. Determine the area of each parallelogram. Show or explain your thinking.

   Figure U: **42 square units; Sample response: I multiplied the length of the base by the length of the height, $7 \cdot 6 = 42$.**

   Figure V: **18 square units; Sample response: I multiplied the length of the base by the length of the height, $3 \cdot 6 = 18$.**

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I know what the terms *base* and *height* refer to in a parallelogram.
1 2 3

b I can write and explain the formula for the area of a parallelogram.
1 2 3

c I can identify corresponding base and height pairs of a parallelogram.
1 2 3

Lesson 7 Bases and Heights of Parallelograms

Success looks like . . .

- **Language Goal:** Comprehending the terms *base* and *height* to refer to one side of a parallelogram (base) and the perpendicular distance (height) between that side and the opposite side **(Speaking and Listening, Writing)**.
  - » Showing comprehension by identifying the base and height of each figure in Problem 1.
- **Goal:** Identifying a base and the corresponding height for a parallelogram, and understanding that there are two different base and height pairs for any parallelogram.
  - » Identifying a base and height for each figure in Problem 1.
- **Language Goal:** Generalizing a process for determining the area of a parallelogram, using the length of a base and its corresponding height **(Speaking and Listening, Writing)**.
  - » Explaining how to determine the area of each parallelogram in Problem 2.

Suggested next steps

If students misidentify the lengths of the bases and heights, consider:

- Modeling how to count the squares in the grid, making sure to clarify how students know where to begin and end using the sides and vertices of the parallelogram.

If students are unable to determine the areas of the parallelograms for Problem 2, consider:

- Referring to the table from Activity 2 and asking, "How were these areas related to the other measures?"
- Suggesting that students use prior strategies of decomposing, rearranging, or enclosing to determine the areas first. Then relate these values to the measures of base and height and the formula for the area.

Professional Learning

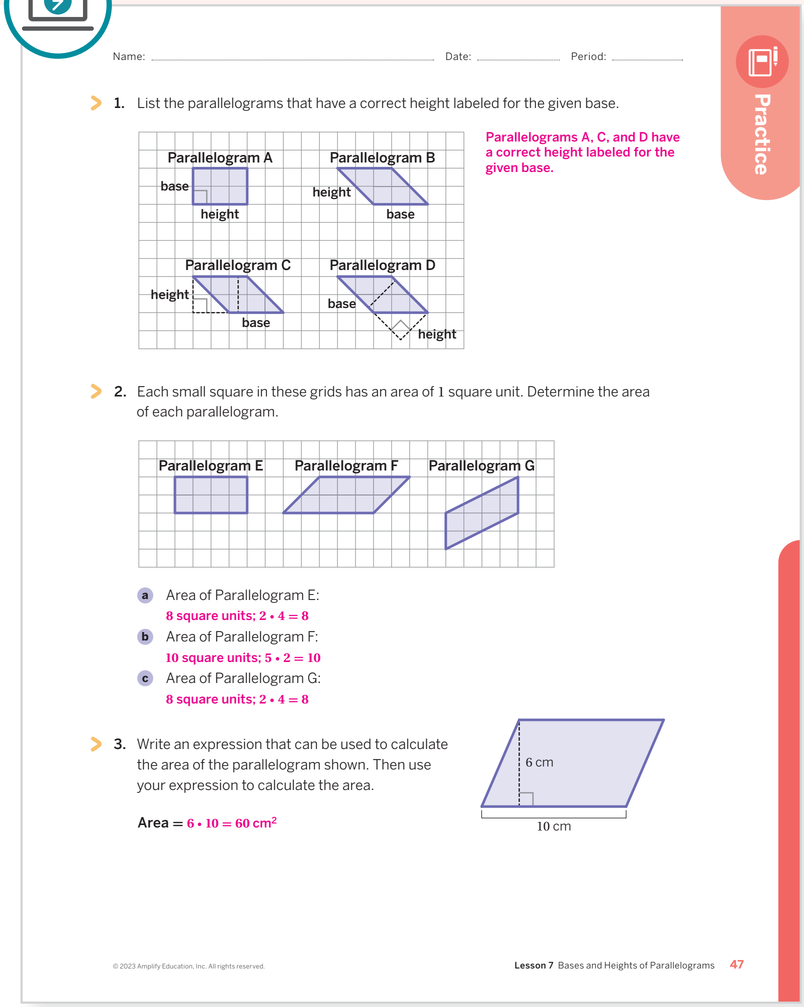
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The focus of this lesson was exploring how to determine the base and area of a parallelogram to think about how it relates to the formula for area of a parallelogram. How did it go?
- In what ways did things happen that you did not expect? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. List the parallelograms that have a correct height labeled for the given base.

Parallelograms A, C, and D have a correct height labeled for the given base.

2. Each small square in these grids has an area of 1 square unit. Determine the area of each parallelogram.

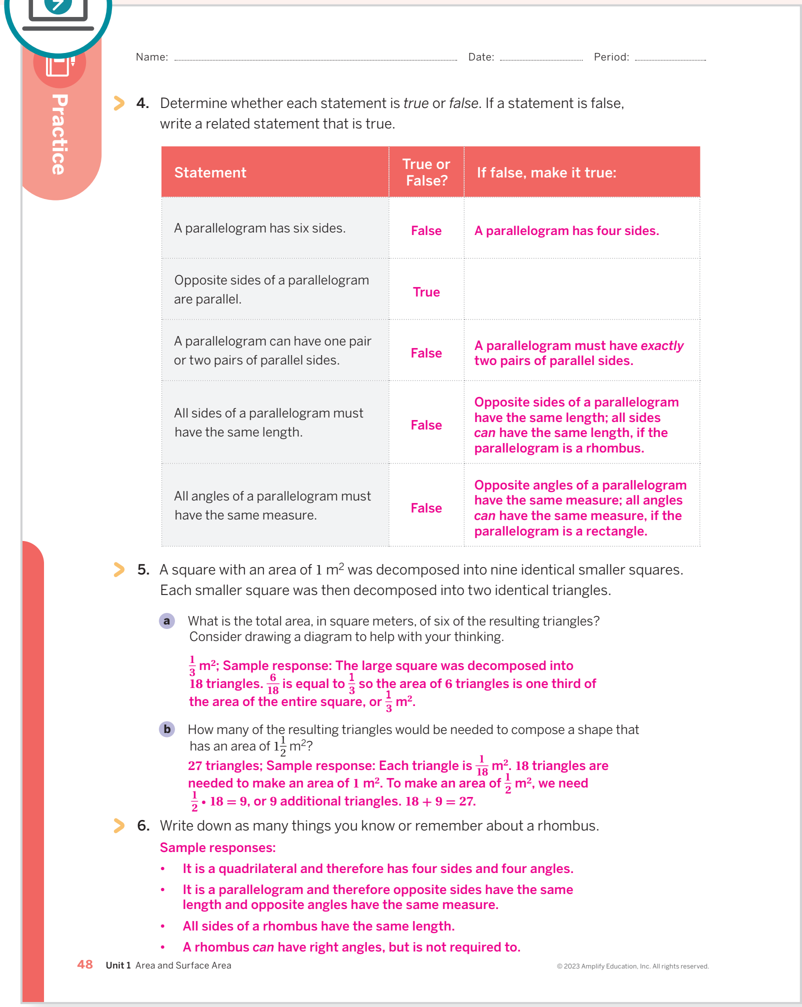
a. Area of Parallelogram E:
8 square units; $2 \cdot 4 = 8$

b. Area of Parallelogram F:
10 square units; $5 \cdot 2 = 10$

c. Area of Parallelogram G:
8 square units; $2 \cdot 4 = 8$

3. Write an expression that can be used to calculate the area of the parallelogram shown. Then use your expression to calculate the area.

Area = **$6 \cdot 10 = 60$ cm^2**



Practice

Name: ______ Date: ______ Period: ______

4. Determine whether each statement is *true* or *false*. If a statement is false, write a related statement that is true.

Statement	True or False?	If false, make it true:
A parallelogram has six sides.	**False**	**A parallelogram has four sides.**
Opposite sides of a parallelogram are parallel.	**True**	
A parallelogram can have one pair or two pairs of parallel sides.	**False**	**A parallelogram must have *exactly* two pairs of parallel sides.**
All sides of a parallelogram must have the same length.	**False**	**Opposite sides of a parallelogram have the same length; all sides *can* have the same length, if the parallelogram is a rhombus.**
All angles of a parallelogram must have the same measure.	**False**	**Opposite angles of a parallelogram have the same measure; all angles *can* have the same measure, if the parallelogram is a rectangle.**

5. A square with an area of 1 m^2 was decomposed into nine identical smaller squares. Each smaller square was then decomposed into two identical triangles.

a. What is the total area, in square meters, of six of the resulting triangles? Consider drawing a diagram to help with your thinking.

$\frac{1}{3}$ m^2; Sample response: The large square was decomposed into 18 triangles. $\frac{6}{18}$ is equal to $\frac{1}{3}$ so the area of 6 triangles is one third of the area of the entire square, or $\frac{1}{3}$ m^2.

b. How many of the resulting triangles would be needed to compose a shape that has an area of $1\frac{1}{2}$ m^2?

27 triangles; Sample response: Each triangle is $\frac{1}{18}$ m^2. 18 triangles are needed to make an area of 1 m^2. To make an area of $\frac{1}{2}$ m^2, we need $\frac{1}{2} \cdot 18 = 9$, or 9 additional triangles. $18 + 9 = 27$.

6. Write down as many things you know or remember about a rhombus.

Sample responses:

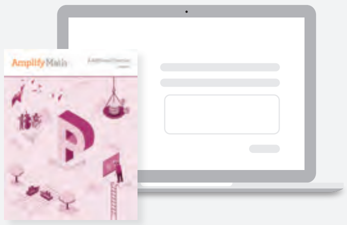
- **It is a quadrilateral and therefore has four sides and four angles.**
- **It is a parallelogram and therefore opposite sides have the same length and opposite angles have the same measure.**
- **All sides of a rhombus have the same length.**
- **A rhombus *can* have right angles, but is not required to.**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 6	2
	5	Unit 1 Lesson 4	2
Formative ⚡	6	Unit 1 Lesson 8	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 8**

Area of Parallelograms

Let's practice determining the area of parallelograms seen in the world.

Focus

Goals

1. **Language Goal:** Apply the formula for area of a parallelogram to find the area, the length of the base, or the height, and explain the solution method. **(Speaking and Listening, Writing)**
2. **Language Goal:** Choose which measurements to use for determining the area of a parallelogram when more than one base or height measurement is given, and explain the choice. **(Speaking and Listening, Writing)**

Coherence

Today

Students practice using decomposition strategies and the formula for the area of parallelograms in several real-world contexts. They identify appropriate measurements for the base, height, and area from given information and determine the missing value. In the optional second activity, students show how parallelograms with the same area can have very different base and height measurements.

Previously

By decomposing, rearranging, and enclosing parallelograms in Lessons 6 and 7, students were able to determine the area of a parallelogram by relating it to a rectangle with the same area. This led to visible patterns and relationships among the measurements, and the formula for the area of a parallelogram was discovered in Lesson 7.

Coming Soon

Students will continue to explore the concept of area with what will ultimately become the most useful shape — the triangle. In Lesson 9, they will first revisit familiar decomposition strategies and relate the area of a triangle to the area of a corresponding parallelogram. And in Lesson 11, the formula for the area of a triangle will be discovered, based on patterns similar to those for parallelograms.

Rigor

- Students use real-world objects to develop **procedural fluency** in identifying the base, height, and area of a parallelogram.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2 (optional)	Summary	Exit Ticket
10 min	25 min	10 min	5 min	5 min
Independent	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

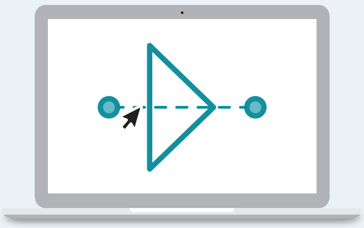
Review words

- *rhombus*
- *quadrilateral*
- *parallelogram*
- *base (of a parallelogram)*
- *height (of a parallelogram)*

Amps | Featured Activity

Activity 2
Interactive Geometry

Students can draw parallelograms by using a grid without needing a straightedge. They can also easily adjust lines without having to erase.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students may become overwhelmed or confused by the shift from mostly well-defined geometric and mathematical problems in earlier lessons to the more open nature of real-world problems in context. Help them remember previous successes and all that they have learned in recent lessons, and encourage them to focus on modeling the given problems so they look more familiar, with the context temporarily stripped away.

Modifications to Pacing

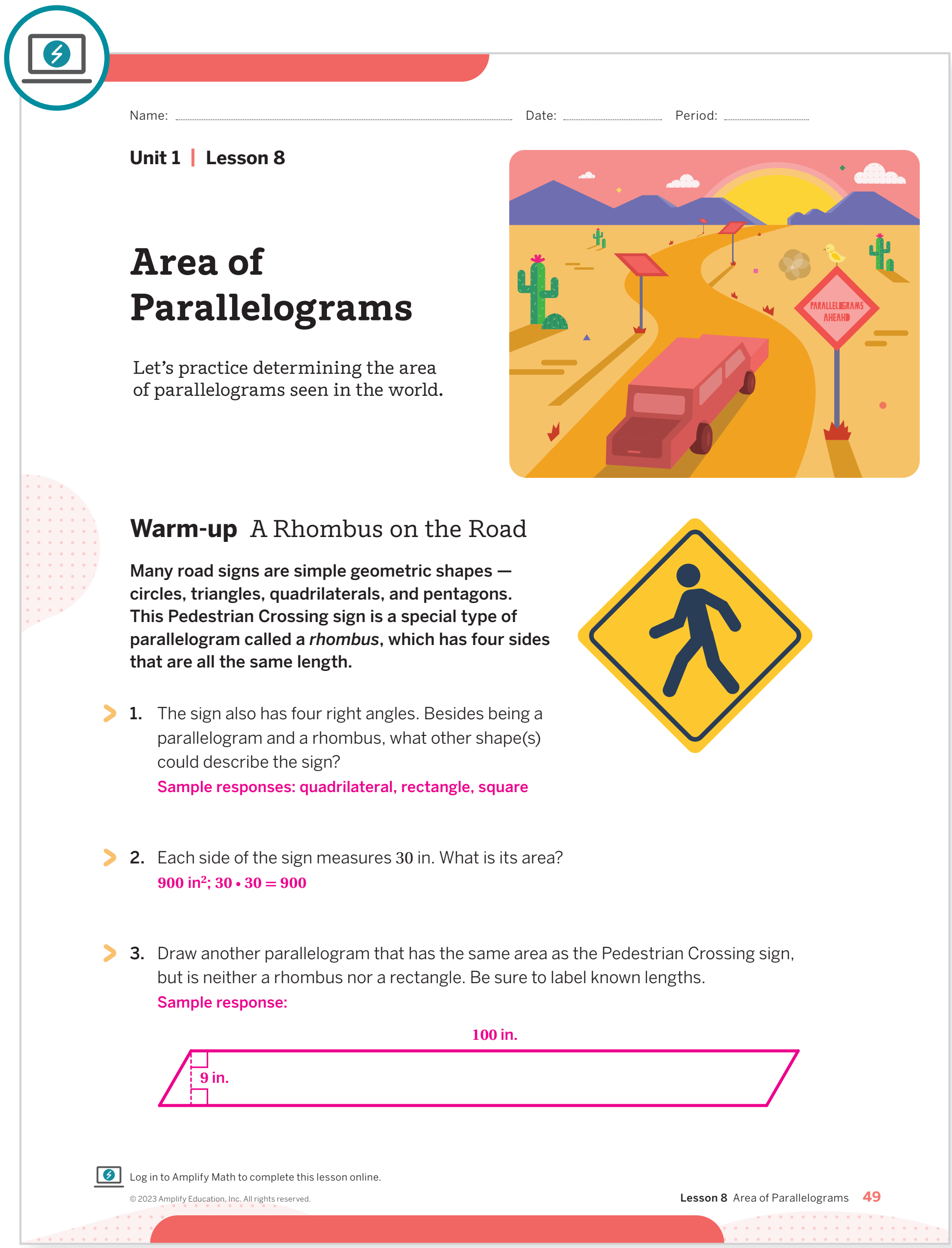
You may want to consider this additional modification if you are short on time:

- In **Activity 1**, have students choose only one of the three problems to complete, instead of two.

Independent | 10 min

Warm-up A Rhombus on the Road

Students review the characteristics of a special parallelogram — the rhombus. They use the formula for the area of a parallelogram to calculate the area of the road sign (rhombus).



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Unit 1 | Lesson 8

Area of Parallelograms

Let's practice determining the area of parallelograms seen in the world.

Warm-up A Rhombus on the Road

Many road signs are simple geometric shapes — circles, triangles, quadrilaterals, and pentagons. This Pedestrian Crossing sign is a special type of parallelogram called a *rhombus*, which has four sides that are all the same length.

1. The sign also has four right angles. Besides being a parallelogram and a rhombus, what other shape(s) could describe the sign?
   Sample responses: quadrilateral, rectangle, square

2. Each side of the sign measures 30 in. What is its area?
   900 in^2; $30 \cdot 30 = 900$

3. Draw another parallelogram that has the same area as the Pedestrian Crossing sign, but is neither a rhombus nor a rectangle. Be sure to label known lengths.
   Sample response:

Log in to Amplify Math to complete this lesson online.

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Lesson 8 Area of Parallelograms 49

1 Launch

Have students share observations about the Pedestrian Crossing sign with the class, before working individually on Problems 1–3.

2 Monitor

Help students get started by having them brainstorm other shapes they know, and see if the road sign matches any of those listed.

Look for points of confusion:

- **Forgetting how to determine the area of a parallelogram.** Refer students back to Lesson 7, Activity 2.
- **Struggling to draw the parallelogram for Problem 3.** Ask, "What does its area need to be? Can you think of two different factors that would create the same area?"

Look for productive strategies:

- Recognizing that the base and height of the road sign have the same length, when applying the formula for the area of a parallelogram.
- Determining base and height lengths for Problem 3 as factor pairs of 900.

3 Connect

Display some of the parallelograms drawn for Problem 3.

Have students share the different examples of parallelograms they drew for Problem 3 and the different strategies they used in drawing and labeling their known lengths (base and height).

Highlight that a *rhombus* is a special type of parallelogram that has four equal sides. A rhombus with four right angles is also called a square. Many quadrilaterals have all of the properties of more than one type of quadrilateral because those types are in a hierarchy.

MLR Math Language Development

MLR3: Critique, Correct, Clarify

After students have drawn their parallelogram that is neither a rhombus nor a rectangle, have students pass their shape to 2–3 partners in their group. Each partner should respond to the following questions:

- ***Critique:*** Does the drawn shape fit the given criteria?
- ***Correct:*** What would need to change for this shape to be a rhombus?
- ***Clarify:*** Why is the shape neither a rhombus nor a rectangle?

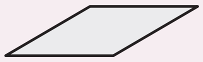
English Learners

Highlight language for students, such as "a rhombus is a special type of parallelogram," and add it to the class anchor chart. Encourage students to refer back to the display during discussions.

Power-up

To power up students' ability to describe characteristics of a rhombus, have students complete:

A rhombus is shown. Recall that other shapes with all the characteristics of a rhombus can also be called rhombuses (like squares!). Mark *all* of the **true** statements about a rhombus.



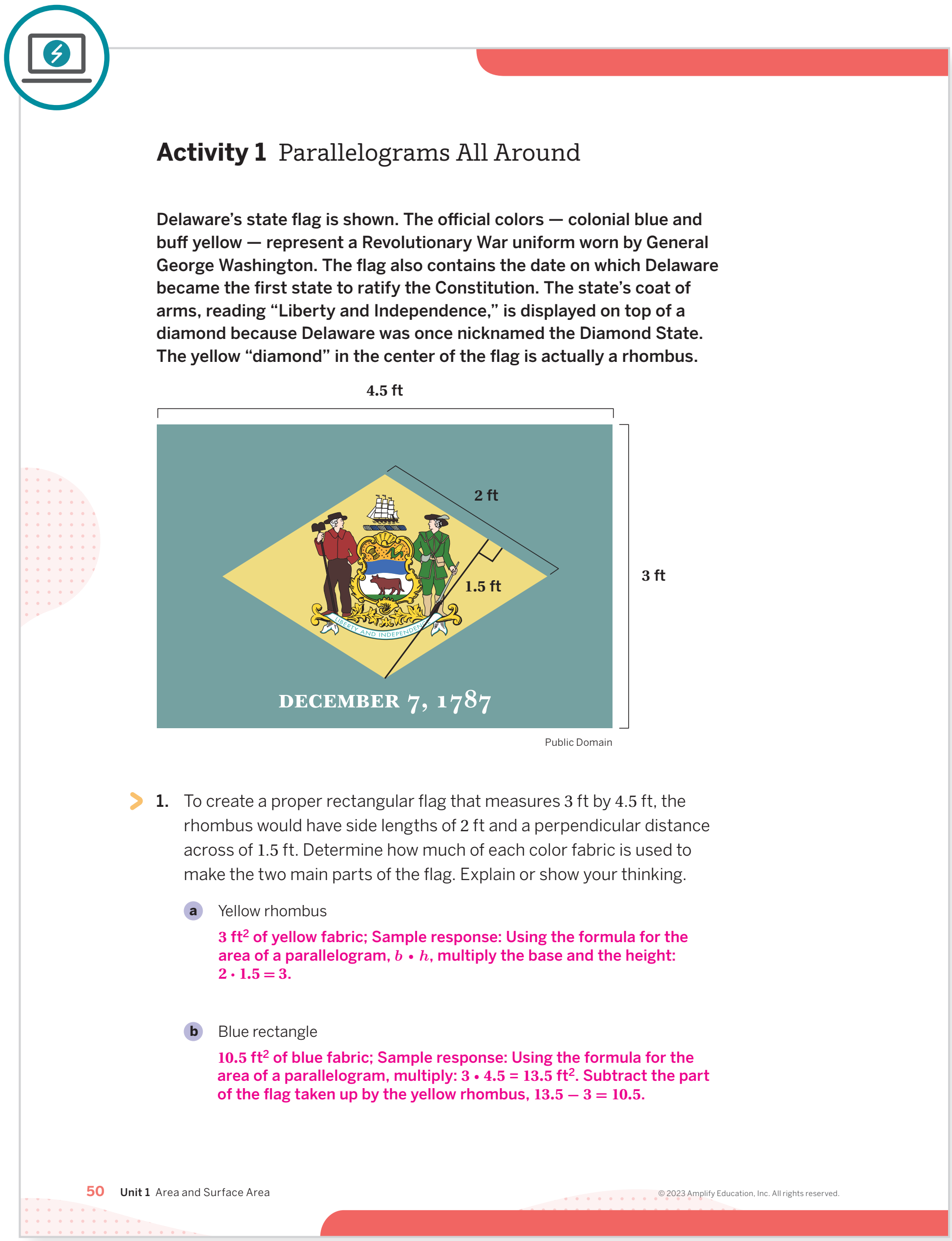
[✓ **A.**] It is a quadrilateral
B. A rhombus never has right angles
[✓ **C.**] All sides have the same length
D. It has two pairs of perpendicular sides
E. Only two sides need to be parallel

Use: Before the Warm-up.
Informed by: Performance on Lesson 7, Practice Problem 6

Pairs | 25 min

Activity 1 Parallelograms All Around

Students calculate the area of a parallelogram in real-world contexts, realizing they need to attend to the given information to know which measure they need to find.



Activity 1 Parallelograms All Around

Delaware's state flag is shown. The official colors — colonial blue and buff yellow — represent a Revolutionary War uniform worn by General George Washington. The flag also contains the date on which Delaware became the first state to ratify the Constitution. The state's coat of arms, reading "Liberty and Independence," is displayed on top of a diamond because Delaware was once nicknamed the Diamond State. The yellow "diamond" in the center of the flag is actually a rhombus.

Public Domain

1. To create a proper rectangular flag that measures 3 ft by 4.5 ft, the rhombus would have side lengths of 2 ft and a perpendicular distance across of 1.5 ft. Determine how much of each color fabric is used to make the two main parts of the flag. Explain or show your thinking.
   - a Yellow rhombus

     3 ft^2 of yellow fabric; Sample response: Using the formula for the area of a parallelogram, $b \cdot h$, multiply the base and the height: $2 \cdot 1.5 = 3$.
   - b Blue rectangle

     10.5 ft^2 of blue fabric; Sample response: Using the formula for the area of a parallelogram, multiply: $3 \cdot 4.5 = 13.5$ ft^2. Subtract the part of the flag taken up by the yellow rhombus, $13.5 - 3 = 10.5$.

50 Unit 1 Area and Surface Area

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1 Launch

Have partners choose two out of the three problems to complete. **Note:** Problem 3 is likely the most challenging. Consider using the *Gallery Tour* routine to display and share student work.

2 Monitor

Help students get started by asking them to identify the shapes and measurements given in the problem.

Look for points of confusion:

- **Not relating the phrase "how much of each color fabric" to area in Problem 1.** Have students draw an outline around the regions they need to consider. Ask, "What type of measurement is that?"
- **Thinking they need 13.5 ft^2 of blue fabric for Problem 1b.** Have students identify the region of the flag to which their calculation corresponds. Ask, "Is that entire region made up of *only* the blue fabric?"
- **Assuming the given values are always multiplied, specifically in Problems 2b and 3.** Ask students to reiterate the area of a parallelogram ($A = b \bullet h$). Have them identify what is given and what is missing, or being asked for, and then how they would use the formula to solve for those values.

Look for productive strategies:

- Using the formula for the area of a parallelogram flexibly to determine base, height, or area (i.e., $A = b \bullet h$, so $A \div b = h$ and $A \div h = b$).
- Attending to the names of attributes of the shape that can be measured (e.g., base, height, area) and using the given values to substitute appropriately into the area formula.

Activity 1 continued ›

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

Have students begin with Problem 1 and ask, "What is the area of the entire flag? What about the yellow rhombus? How can you use those to determine the area of the blue part of the flag?"

Extension: ***Math Enrichment***

Have students study the measurements given in Problem 2 and construct a mathematical argument as to whether the logo on the truck is just a "larger" version of the logo on the mailbox. **Note:** The logos are not proportional.

Math Language Development

MLR2: Collect and Display

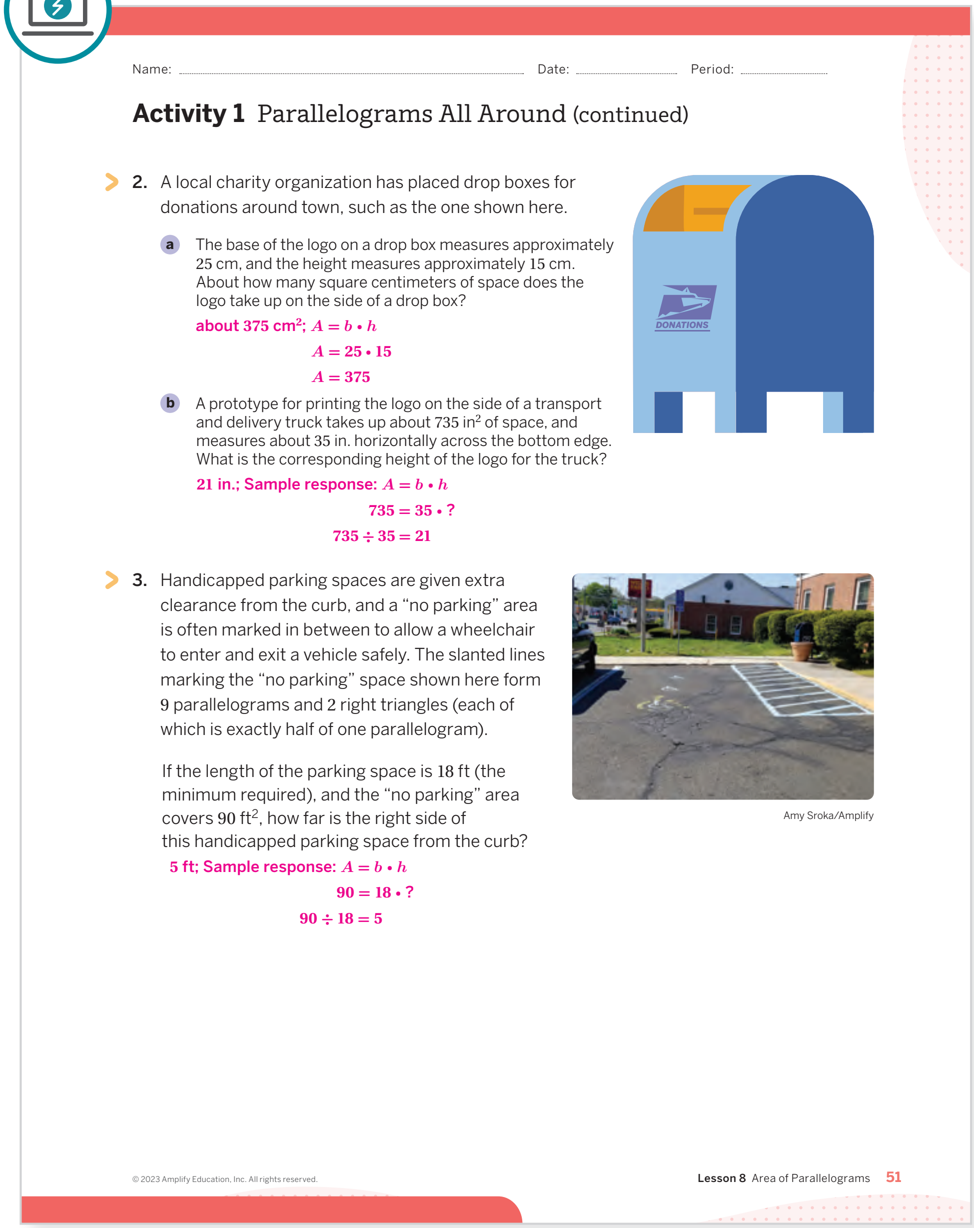
After students participate in the *Gallery Tour* to display and share work, collect and display the posters they created. Leave the posters up and encourage students to refer back to the posters during the Connect discussion.

English Learners

During the Connect discussion, demonstrate the features of each problem by gesturing to the features or annotating them.

Activity 1 Parallelograms All Around (continued)

Students calculate the area of a parallelogram in real-world contexts, realizing they need to attend to the given information to know which measure they need to find.



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Activity 1 Parallelograms All Around (continued)

2. A local charity organization has placed drop boxes for donations around town, such as the one shown here.

   a. The base of the logo on a drop box measures approximately 25 cm, and the height measures approximately 15 cm. About how many square centimeters of space does the logo take up on the side of a drop box?

   about 375 cm^2; $A = b \cdot h$

   $A = 25 \cdot 15$

   $A = 375$

   b. A prototype for printing the logo on the side of a transport and delivery truck takes up about 735 in^2 of space, and measures about 35 in. horizontally across the bottom edge. What is the corresponding height of the logo for the truck?

   21 in.; Sample response: $A = b \cdot h$

   $735 = 35 \cdot ?$

   $735 \div 35 = 21$

3. Handicapped parking spaces are given extra clearance from the curb, and a "no parking" area is often marked in between to allow a wheelchair to enter and exit a vehicle safely. The slanted lines marking the "no parking" space shown here form 9 parallelograms and 2 right triangles (each of which is exactly half of one parallelogram).

   If the length of the parking space is 18 ft (the minimum required), and the "no parking" area covers 90 ft^2, how far is the right side of this handicapped parking space from the curb?

   5 ft; Sample response: $A = b \cdot h$

   $90 = 18 \cdot ?$

   $90 \div 18 = 5$

Amy Sroka/Amplify

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Lesson 8 Area of Parallelograms 51

3 Connect

Have pairs of students share their responses and how they used the formula for the area of a parallelogram to determine the base, height, or area based on which information was given and missing. If you conduct a ***Gallery Tour*** routine, have students record one observation from another group's work to share with the class.

Ask, "What was similar about the work you completed for each of the problems you solved? What was different? What features of the problems are related to these similarities and differences?"

Highlight:

- Area formulas can be used to not only calculate area, but also missing measures (or factors) such as the length of the base or a height of a parallelogram. The measurements that are known and those that need to be determined will inform how to apply the formula. Often the right measures are all known, but sometimes they will have to be determined first as well.
- In the real world, lengths can be *measured* with rulers, tape measures, or even lasers, but areas can generally only be *calculated* or *determined* (or are established as desired targets based on the scenario).

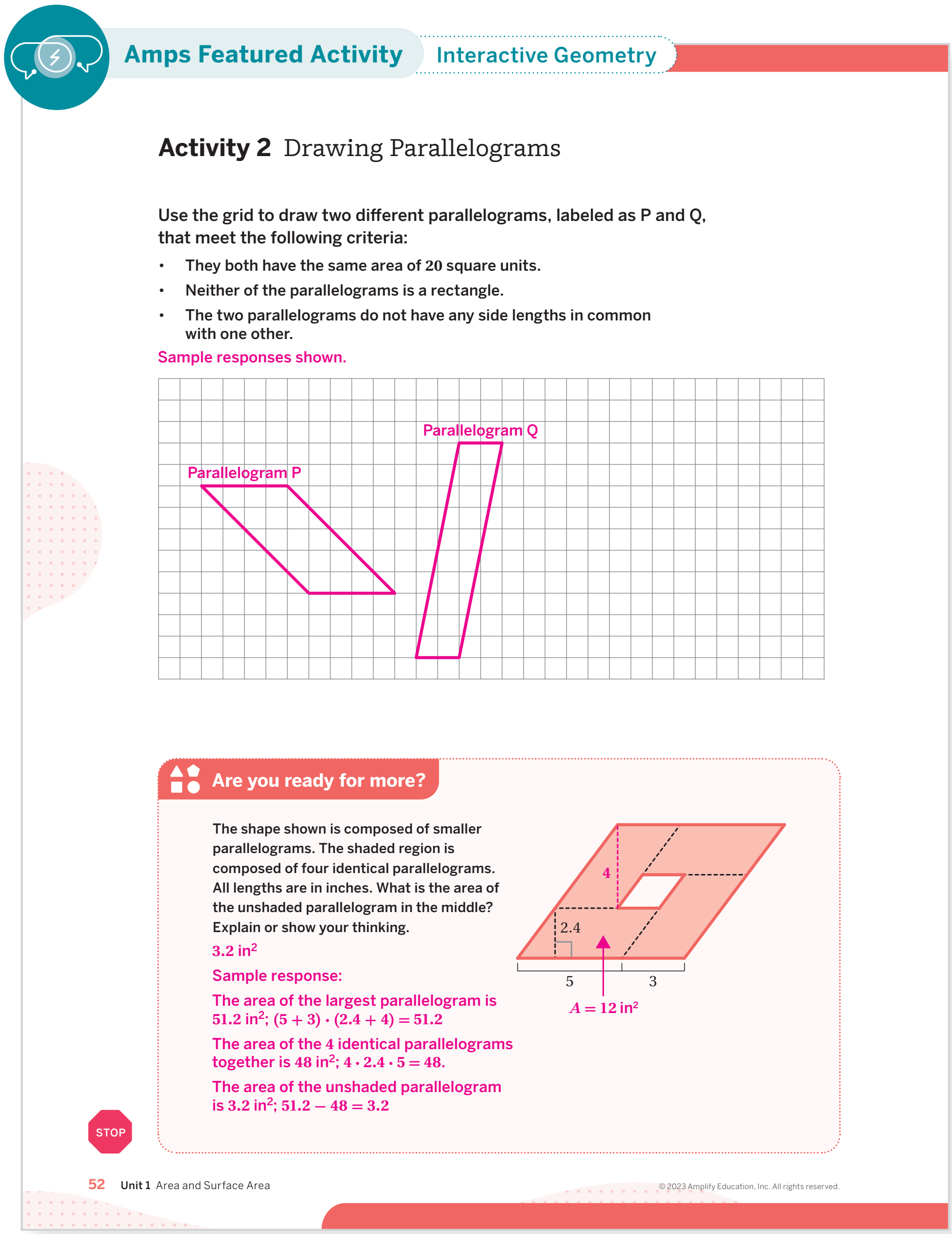
Optional

Independent | 10 min

Activity 2 Drawing Parallelograms

Students draw two different parallelograms with the same given area to understand that two parallelograms can have the same area, yet different base and height lengths.

Amps Featured Activity Interactive Geometry



Activity 2 Drawing Parallelograms

Use the grid to draw two different parallelograms, labeled as P and Q, that meet the following criteria:

- They both have the same area of 20 square units.
- Neither of the parallelograms is a rectangle.
- The two parallelograms do not have any side lengths in common with one other.

Sample responses shown.

Are you ready for more?

The shape shown is composed of smaller parallelograms. The shaded region is composed of four identical parallelograms. All lengths are in inches. What is the area of the unshaded parallelogram in the middle? Explain or show your thinking.

3.2 in^2

Sample response:

The area of the largest parallelogram is 51.2 in^2; $(5 + 3) \cdot (2.4 + 4) = 51.2$

The area of the 4 identical parallelograms together is 48 in^2; $4 \cdot 2.4 \cdot 5 = 48$.

The area of the unshaded parallelogram is 3.2 in^2; $51.2 - 48 = 3.2$

52 Unit 1 Area and Surface Area

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by asking, "If each parallelogram is neither a rectangle nor a rhombus, what do you need to keep in mind when drawing your figures?"

Look for points of confusion:

- **Thinking there is only one way to draw a parallelogram with an area of 20 square units.** Ask students to write down all the factors of 20, and use these factors to help draw their parallelograms.

Look for productive strategies:

- Using factor pairs of 20 to draw a base with the length of one factor along the grid lines, and then using the other factor (the height) to locate the grid line where the opposite side needs to be drawn, translating the segment to ensure the result is *not* a rectangle.

3 Connect

Have individual students share one of the parallelograms they drew and how they used a different base and height pair, but kept the area the same. Ask multiple students to share, each time showing a different shape until all the different examples have been shown.

Highlight that two (or more) parallelograms can have the same area, but can have different measurements of the bases and heights.

Ask, "Would it be possible for a parallelogram to have this same area and have a base with a fractional side length? A height with a fractional side length? Could the parallelogram have *no* sides aligned to the grid? How many parallelograms are possible?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Provide a smaller area, such as 12 square units. Have students list all of the factor pairs for 12 first, and then use those to draw one parallelogram at a time. Have students use the Amps slides for this activity, in which they can easily change lines without having to erase.

Extension: *Math Enrichment*

Have students choose a greater number, with many different factors, such as 54, and have them create as many different parallelograms as possible that have this area (in square units) by using the same directions of the activity.

Math Language Development

MLR8: Discussion Supports — Restate It!

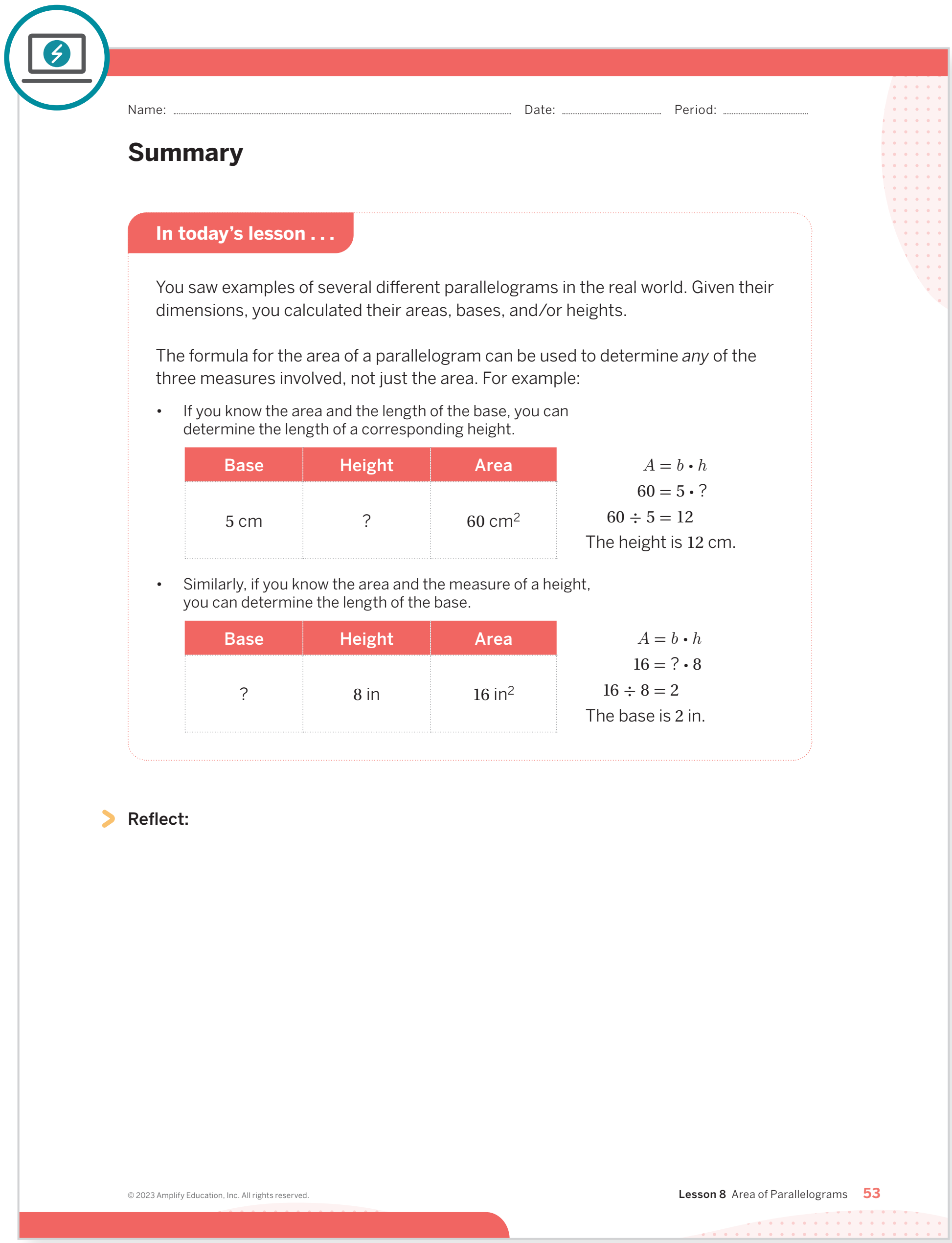
When students explain how they created parallelograms with equal areas, have them restate what they heard from their partner by using developing mathematical language (e.g., *area, product, base,* and *height*). If students are not able to restate, they should ask for clarification.

English Learners

As students restate what they heard from their partner, encourage students to refer back to the *Gallery Tour* posters to help them articulate their developing mathematical language.

Summary

Review and synthesize how to apply the formula for the area of a parallelogram.



Name: __________ Date: __________ Period: __________

Summary

In today's lesson . . .

You saw examples of several different parallelograms in the real world. Given their dimensions, you calculated their areas, bases, and/or heights.

The formula for the area of a parallelogram can be used to determine *any* of the three measures involved, not just the area. For example:

- If you know the area and the length of the base, you can determine the length of a corresponding height.

Base	Height	Area
5 cm	?	60 cm^2

$A = b \cdot h$

$60 = 5 \cdot ?$

$60 \div 5 = 12$

The height is 12 cm.

- Similarly, if you know the area and the measure of a height, you can determine the length of the base.

Base	Height	Area
?	8 in	16 in^2

$A = b \cdot h$

$16 = ? \cdot 8$

$16 \div 8 = 2$

The base is 2 in.

Reflect:

Synthesize

Ask:

- "How do you determine the base and height of any parallelogram?" A base can be any side of the parallelogram and the height is the perpendicular distance to the opposite side.
- "How many possible base and height pairs can there be for any given parallelogram?" two
- "When would you use the formula for the area of a parallelogram to divide two values instead of multiplying them?" When the area is known, but either the base or height is not known.
- "What are some other examples of parallelograms you can think of in the real world? What measure (base, height, or area) do you think is most likely to be unknown in those scenarios? What could each tell you in context or how could each be used?"

Highlight that parallelograms include rectangles and rhombuses and are a common shape in many aspects of buildings and architecture. Most obviously, they represent walls, floors, and ceilings, but they can also be seen in windows, tiles, masonry, and even decorative features. Straight-line geometry has structural appeal to builders, but can also have visual and decorative appeal to humans who are drawn to clean and logically-appearing objects.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

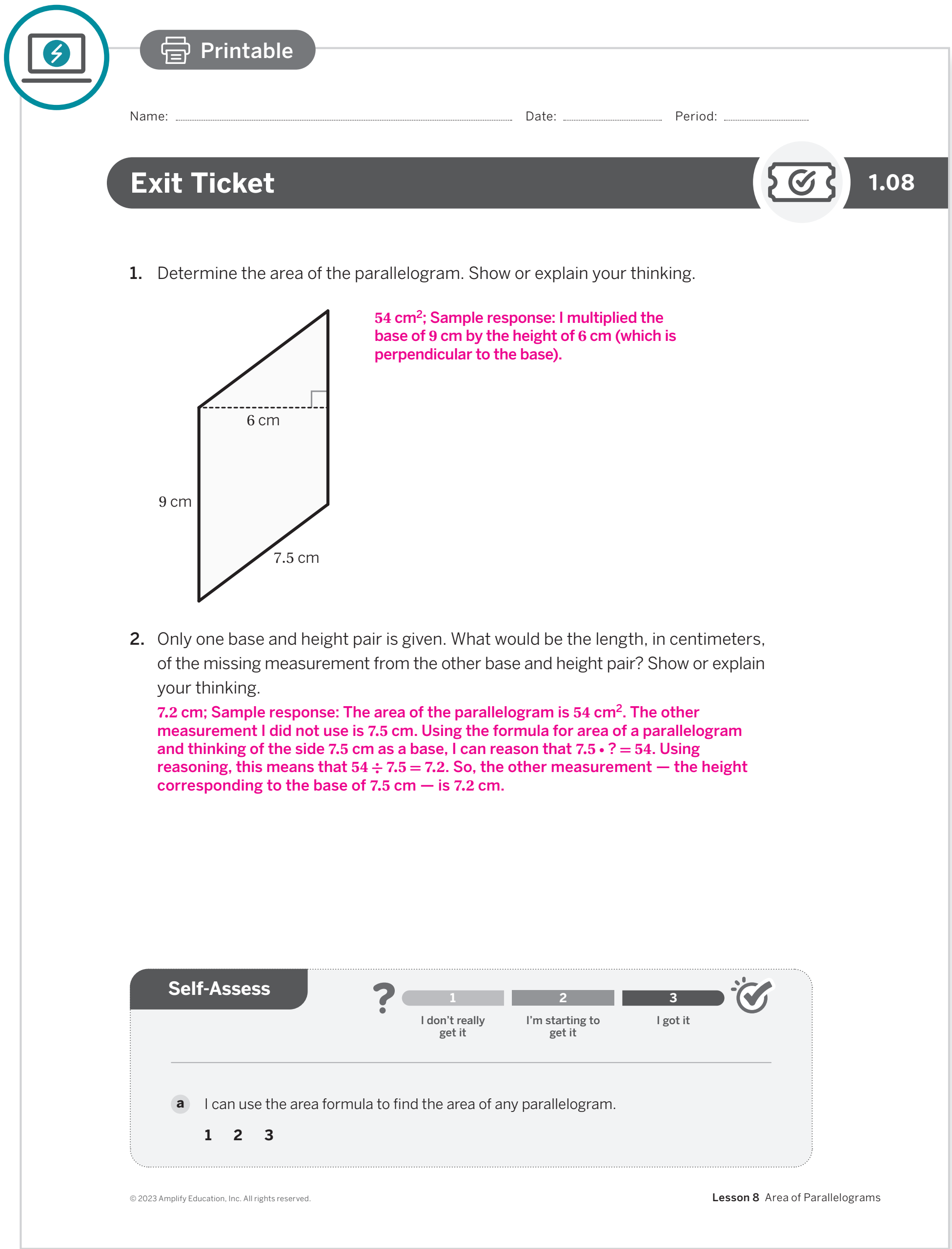
- "How can you use known measurements of base, height, or area of a parallelogram to determine other measurements of base, height, or area?"
- "How many different strategies can you think of for determining the area of a parallelogram? Are some more efficient than others?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how to calculate the area of a parallelogram off a grid by using the formula $A = b \cdot h$.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket

1.08

1. Determine the area of the parallelogram. Show or explain your thinking.

54 cm^2; Sample response: I multiplied the base of 9 cm by the height of 6 cm (which is perpendicular to the base).

2. Only one base and height pair is given. What would be the length, in centimeters, of the missing measurement from the other base and height pair? Show or explain your thinking.

7.2 cm; Sample response: The area of the parallelogram is 54 cm^2. The other measurement I did not use is 7.5 cm. Using the formula for area of a parallelogram and thinking of the side 7.5 cm as a base, I can reason that $7.5 \cdot ? = 54$. Using reasoning, this means that $54 \div 7.5 = 7.2$. So, the other measurement — the height corresponding to the base of 7.5 cm — is 7.2 cm.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can use the area formula to find the area of any parallelogram.

1 2 3

Lesson 8 Area of Parallelograms

Success looks like . . .

- **Language Goal:** Applying the formula for area of a parallelogram to find the area, the length of the base, or the height, and explaining the solution method **(Speaking and Listening, Writing)**.
  - » Multiplying the base and height to determine the area of the parallelogram in Problem 1.
- **Language Goal:** Choosing which measurements to use for determining the area of a parallelogram when more than one base or height measurement is given, and explaining the choice **(Speaking and Listening, Writing)**.
  - » Determining the missing height for the other given base in Problem 2.

Suggested next steps

If students multiply the incorrect values, consider:

- Reviewing examples from Lesson 7 and asking, "What do you know about base and height pairs of parallelograms?" Sample response: The corresponding height must be perpendicular to the base and must meet at the opposite vertex of the figure.
- Reviewing or showing decomposition strategies to determine the area another way.

If students have trouble answering Problem 2, consider:

- Having students draw the missing height and then asking, "What does the area need to be? Can you substitute the two known values into the area formula?"

Professional Learning

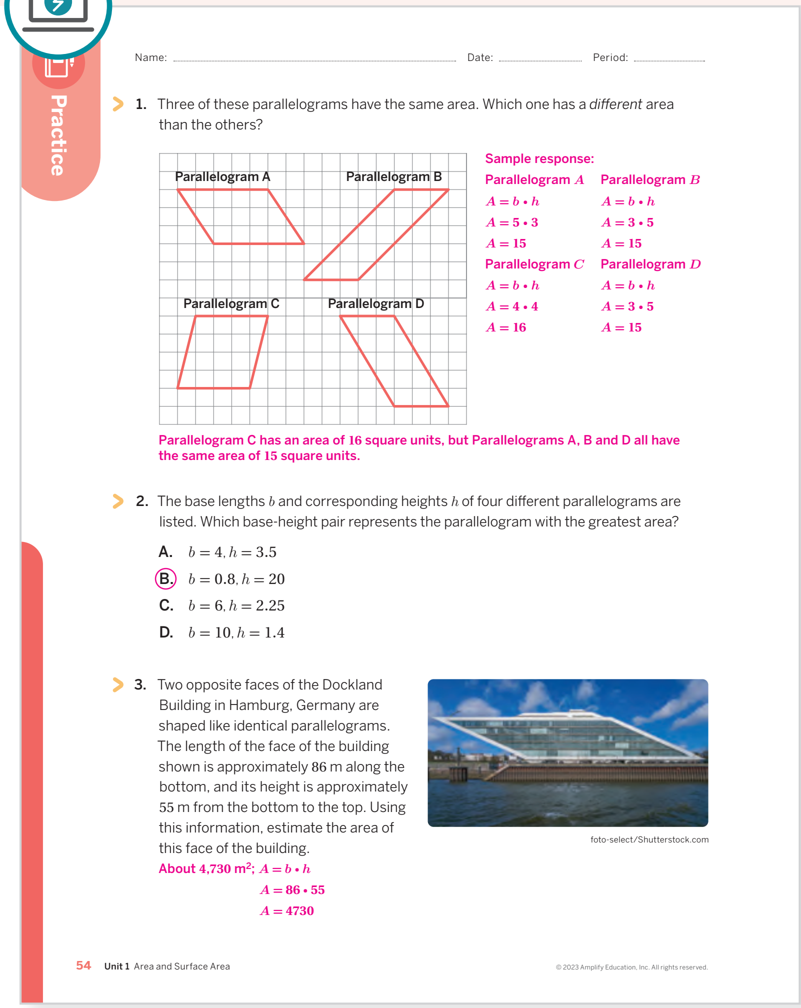
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is?
- What challenges did students encounter as they worked on the activities? Which resources helped them and how might this help you the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Three of these parallelograms have the same area. Which one has a *different* area than the others?

Sample response:

Parallelogram A: $A = b \cdot h$, $A = 5 \cdot 3$, $A = 15$

Parallelogram B: $A = b \cdot h$, $A = 3 \cdot 5$, $A = 15$

Parallelogram C: $A = b \cdot h$, $A = 4 \cdot 4$, $A = 16$

Parallelogram D: $A = b \cdot h$, $A = 3 \cdot 5$, $A = 15$

Parallelogram C has an area of 16 square units, but Parallelograms A, B and D all have the same area of 15 square units.

2. The base lengths b and corresponding heights h of four different parallelograms are listed. Which base-height pair represents the parallelogram with the greatest area?

A. $b = 4, h = 3.5$

[✓ B.] $b = 0.8, h = 20$

C. $b = 6, h = 2.25$

D. $b = 10, h = 1.4$

3. Two opposite faces of the Dockland Building in Hamburg, Germany are shaped like identical parallelograms. The length of the face of the building shown is approximately 86 m along the bottom, and its height is approximately 55 m from the bottom to the top. Using this information, estimate the area of this face of the building.

foto-select/Shutterstock.com

About 4,730 m^2; $A = b \cdot h$

$A = 86 \cdot 55$

$A = 4730$

54 **Unit 1** Area and Surface Area

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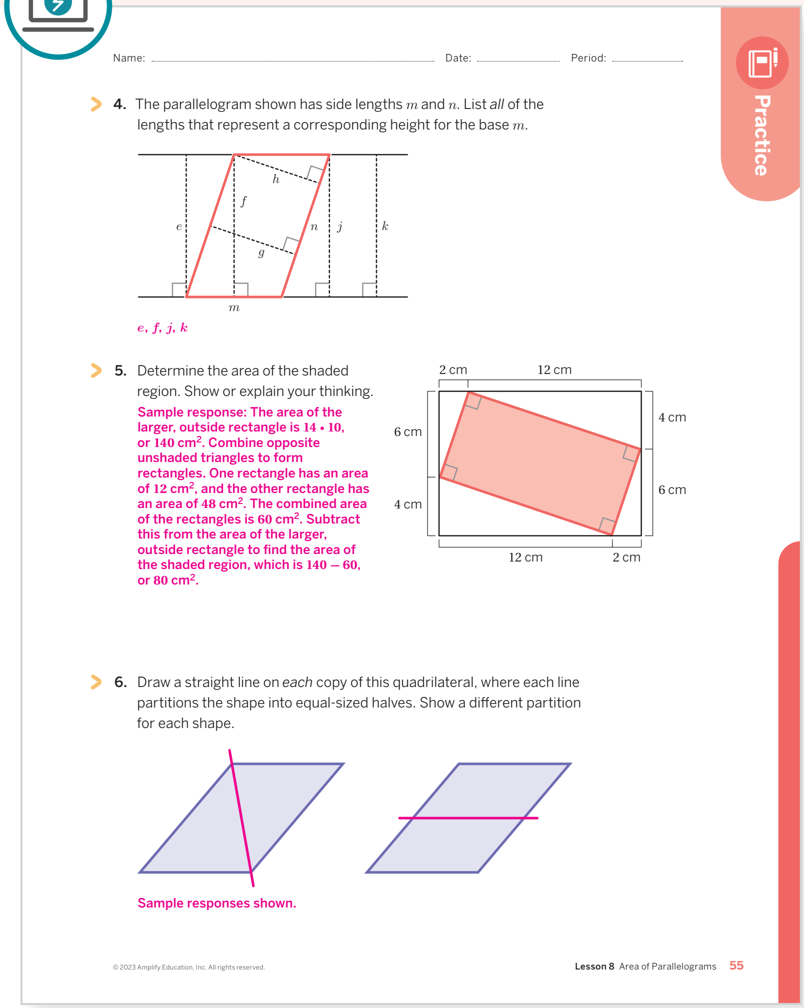

Practice

Name: ______ Date: ______ Period: ______

4. The parallelogram shown has side lengths m and n. List *all* of the lengths that represent a corresponding height for the base m.

e, f, j, k

5. Determine the area of the shaded region. Show or explain your thinking.

Sample response: The area of the larger, outside rectangle is 14 • 10, or 140 cm^2. Combine opposite unshaded triangles to form rectangles. One rectangle has an area of 12 cm^2, and the other rectangle has an area of 48 cm^2. The combined area of the rectangles is 60 cm^2. Subtract this from the area of the larger, outside rectangle to find the area of the shaded region, which is 140 – 60, or 80 cm^2.

6. Draw a straight line on *each* copy of this quadrilateral, where each line partitions the shape into equal-sized halves. Show a different partition for each shape.

Sample responses shown.

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Lesson 8 Area of Parallelograms 55

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 7	2
	5	Unit 1 Lesson 5	2
Formative	6	Unit 1 Lesson 9	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 9**

From Parallelograms to Triangles

Let's use what we know about parallelograms to determine the area of triangles on grids.

Focus

Goals

1. Draw a diagram to show that the area of a triangle is half the area of a related parallelogram.
2. **Language Goal:** Explain strategies for using the base and height of a related parallelogram to determine the area of a triangle. **(Speaking and Listening, Writing)**

Coherence

Today

Students leverage composition of area strategies to relate any given triangle to a related parallelogram formed from two identical copies of the triangle. They use what they know about the area of parallelograms and its area formula to reason that the area of a triangle is exactly half the area of its related parallelogram. Given triangles on grids, students expand on their understand of decomposition, rearranging, and enclosing to discover other methods for determining the area of triangles.

Previously

In Lesson 7, students determined the formula for the area A of a parallelogram as $A = b \cdot h$.

Coming Soon

In Lesson 10, students will continue to explore triangles and their areas, extending their understanding of base and height measures for parallelograms to similar measures for triangles, which will lead to being able to derive a formula for the area of a triangle in Lesson 11.

Rigor

- Students **apply** their understanding of decomposing, rearranging, and enclosing of a parallelogram to a similar procedure of determining the area of a triangle.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

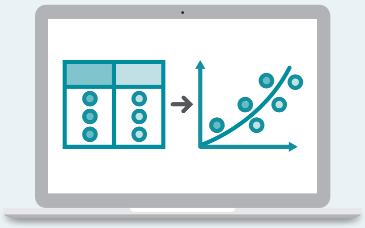
Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, pre-cut triangles, two triangles per student
- Activity 2 PDF, pre-cut grids with triangles (as needed)
- Activity 2 PDF, *Are you ready for more?*, one per pair

Amps | Featured Activity

Activity 1
Using Work From Previous Slides

Students use the parallelograms they formed from two identical triangles in the Warm-up to discover how the area of each triangle is related to the area of its related parallelogram.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

At first, students may feel like they do not have the tools to determine the area of the triangles by extending their study of parallelograms. Ask them to articulate how they can determine the area of a parallelogram and the different ways they can decompose a parallelogram. Reassure them that they have the skills to work through the problems presented by decomposing the parallelograms into two identical triangles.

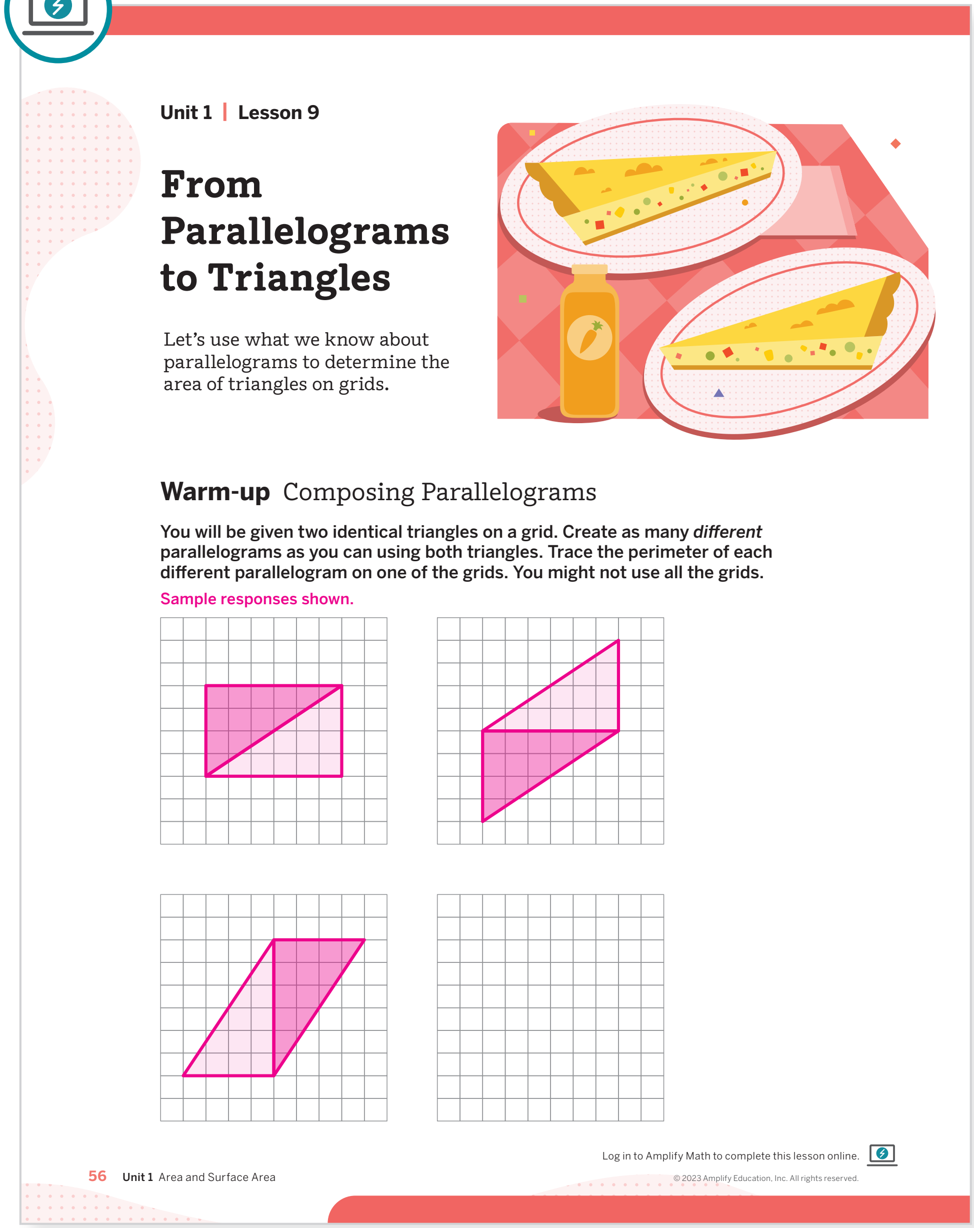
Modifications to Pacing

You may want to consider these additional modifications if you are short on time:

- In **Activity 1**, omit Problem 3.
- In **Activity 2**, have students choose two triangles with which to determine their areas, instead of three.

Warm-up Composing Parallelograms

Students use two copies of the same triangle to form several parallelograms, leading them to understand that a parallelogram can be composed from two identical triangles.



Unit 1 | Lesson 9

From Parallelograms to Triangles

Let's use what we know about parallelograms to determine the area of triangles on grids.

Warm-up Composing Parallelograms

You will be given two identical triangles on a grid. Create as many *different* parallelograms as you can using both triangles. Trace the perimeter of each different parallelogram on one of the grids. You might not use all the grids.

Sample responses shown.

Log in to Amplify Math to complete this lesson online.

56 **Unit 1** Area and Surface Area

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1 Launch

Distribute two pre-cut triangles from the Warm-up PDF to each student.

2 Monitor

Help students get started by having them place any side of one triangle together with any side of the other triangle to see if a parallelogram is formed. Encourage them to try different sides.

Look for points of confusion:

- **Connecting vertices or partial sides.** Ask, "Does that form a parallelogram?" Remind students that an entire side of each triangle must be shared.

Look for productive strategies:

- Recognizing that a parallelogram can be formed only when an entire side of each triangle is shared.

3 Connect

Display the three possible arrangements.

Ask:

- "What do you notice about all of the given parallelograms created?" Each parallelogram is composed of two triangles.
- "Do you think *any* parallelogram can be decomposed into two triangles?" Yes.
- "Could a parallelogram be partitioned into equal halves that are *not* triangles?" Yes, a parallelogram could be partitioned into equal halves that are parallelograms.

Highlight that for any given triangle, one or more parallelograms could be formed using two identical copies of the triangle. Likewise, any given parallelogram can be decomposed into two identical triangles. Make sure all students have all three possible parallelograms drawn on their paper, as these will be needed in Activity 1.

Power-up

To power up students' ability to decompose parallelograms into two triangles, have students complete:

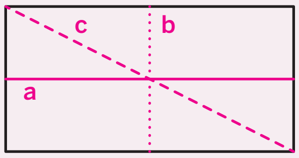
Partition the rectangle into two equal halves by

a. drawing a horizontal line.

b. drawing a vertical line.

c. drawing a diagonal line.

Sample responses shown:



Use: Before the Warm-up.

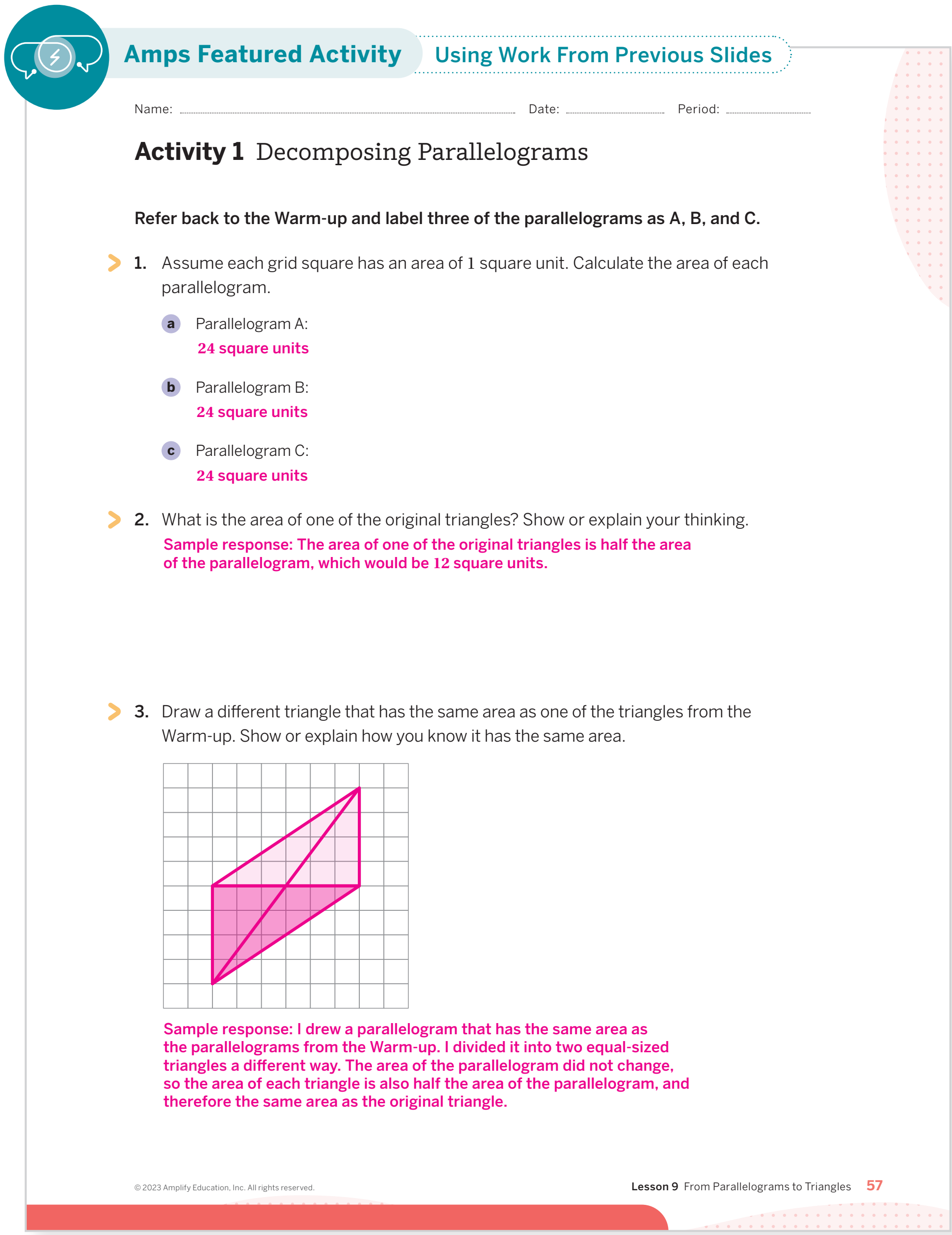
Informed by: Performance on Lesson 8, Practice Problem 6.

Pairs | 10 min

Activity 1 Decomposing Parallelograms

Students calculate the areas of the parallelograms from the Warm-up and begin to consider the relationship between the areas of parallelograms and triangles.

Amps Featured Activity Using Work From Previous Slides



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Activity 1 Decomposing Parallelograms

Refer back to the Warm-up and label three of the parallelograms as A, B, and C.

1. Assume each grid square has an area of 1 square unit. Calculate the area of each parallelogram.
   - a Parallelogram A:
     24 square units
   - b Parallelogram B:
     24 square units
   - c Parallelogram C:
     24 square units
2. What is the area of one of the original triangles? Show or explain your thinking.
   Sample response: The area of one of the original triangles is half the area of the parallelogram, which would be 12 square units.
3. Draw a different triangle that has the same area as one of the triangles from the Warm-up. Show or explain how you know it has the same area.
   Sample response: I drew a parallelogram that has the same area as the parallelograms from the Warm-up. I divided it into two equal-sized triangles a different way. The area of the parallelogram did not change, so the area of each triangle is also half the area of the parallelogram, and therefore the same area as the original triangle.

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Lesson 9 From Parallelograms to Triangles 57

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking how to find the area of a parallelogram. Refer back to Lesson 8 as needed.

Look for points of confusion:

- **Thinking the area cannot be determined without being able to count squares.** Remind students that they can use decomposition and rearranging strategies on the parallelogram to form a rectangle.
- **Determining different areas for each parallelogram.** Make sure students traced the shape outlines correctly and are using the correct area formula.

Look for productive strategies:

- Knowing that the areas of all of the parallelograms must be the same because they are composed of the same two triangles.
- Recognizing that because the triangles are identical, the area of each triangle is equal to exactly half the area of the parallelogram.

3 Connect

Have students share their strategies for Problem 2 and the different triangles created for Problem 3.

Highlight that any parallelogram can be decomposed into two identical triangles and that different ways exist of doing so. All of those triangles will then have an area that is equal to exactly half the area of the parallelogram.

Ask, "How could you draw a different parallelogram with the same area that could be equipartitioned to show even more different triangles with the same area?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on the parallelogram from the Warm-up that is a rectangle, if they have time available, have them work on the remaining ones. **Note:** They will not be able to complete Problem 3 using this rectangle.

Extension: *Math Enrichment*

Challenge students to draw a third and fourth triangle with the same area. Consider offering the hint of using what they know about how to draw parallelograms with the same area.

Math Language Development

MLR7: Compare and Connect

Have students compare their work from Problem 3 with a partner, discussing how they know their triangles have the same area as the triangle from the Warm-up. Highlight language, such as "divided into equal parts" and "the area of each triangle is half the area of the parallelogram."

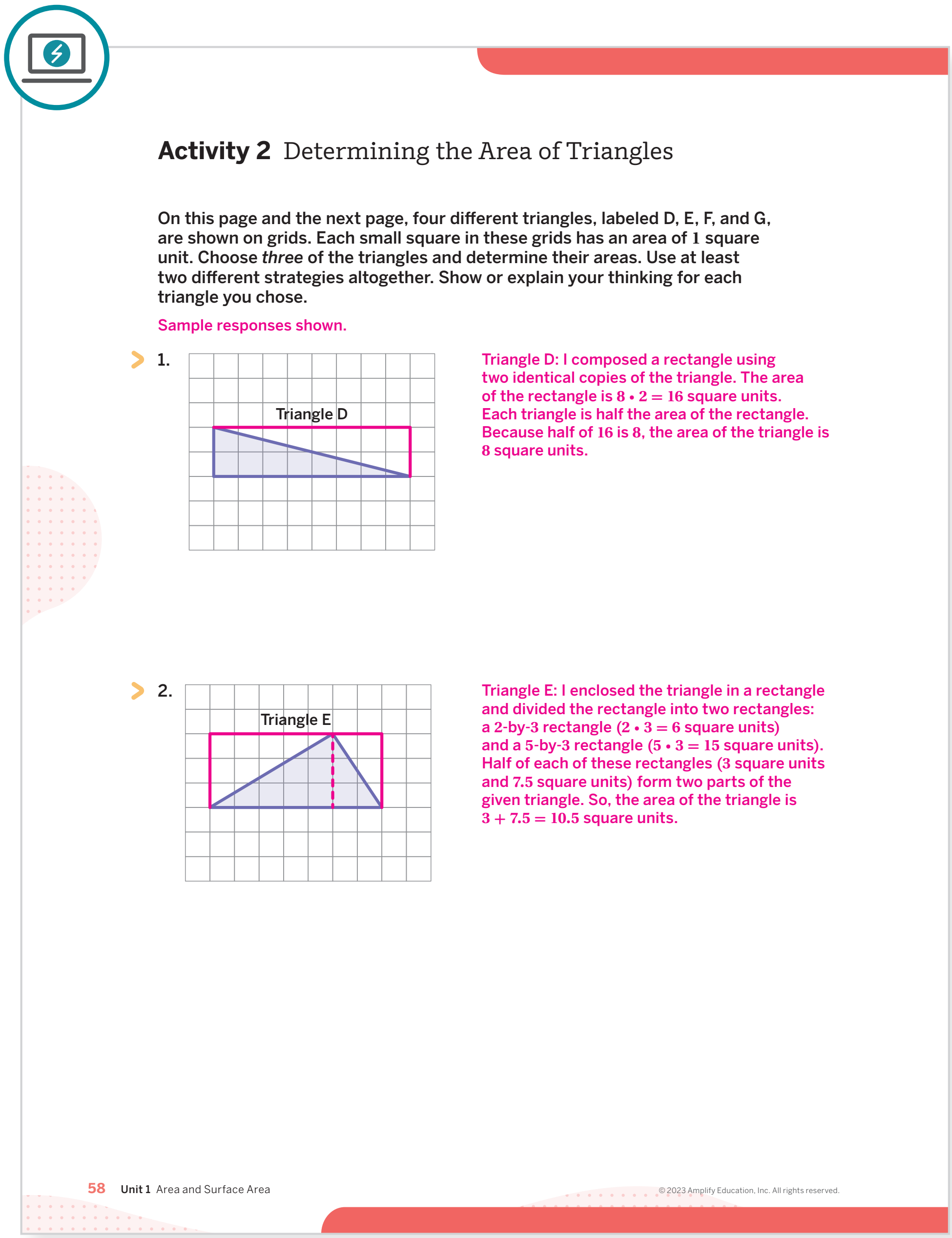
English Learners

The term *equipartitioned* for the question from Connect may be unfamiliar to students. Before asking students how area can be *equipartitioned*, model for students with a physical manipulative, such as a piece of paper, what it looks like to equipartition an object.

Pairs | 20 min

Activity 2 Determining the Area of Triangles

Students explore different strategies for determining the areas of various triangles presented on grids, realizing that multiple strategies can be used.



Activity 2 Determining the Area of Triangles

On this page and the next page, four different triangles, labeled D, E, F, and G, are shown on grids. Each small square in these grids has an area of 1 square unit. Choose *three* of the triangles and determine their areas. Use at least two different strategies altogether. Show or explain your thinking for each triangle you chose.

Sample responses shown.

1.

Triangle D: I composed a rectangle using two identical copies of the triangle. The area of the rectangle is $8 \cdot 2 = 16$ square units. Each triangle is half the area of the rectangle. Because half of 16 is 8, the area of the triangle is 8 square units.

2.

Triangle E: I enclosed the triangle in a rectangle and divided the rectangle into two rectangles: a 2-by-3 rectangle ($2 \cdot 3 = 6$ square units) and a 5-by-3 rectangle ($5 \cdot 3 = 15$ square units). Half of each of these rectangles (3 square units and 7.5 square units) form two parts of the given triangle. So, the area of the triangle is $3 + 7.5 = 10.5$ square units.

58 **Unit 1** Area and Surface Area

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1 Launch

Clarify what is meant by *at least two strategies*, reminding students of the possibilities of decomposing, rearranging, and enclosing, as you think is necessary. Let them know that extra copies of the triangles are available, and distribute the Activity 2 PDF, or pre-cut triangles from it, as needed.

2 Monitor

Help students get started by suggesting they choose a triangle to work with first, such as Triangle D, before assuming a strategy to be used.

Look for points of confusion:

- **Trying to count grid squares and partial squares.** Point out that this could provide a decent estimate, but a precise measure is needed here. Consider suggesting a strategy for them to try.
- **Struggling to draw or visualize an identical triangle.** Provide an identical triangle from the Activity 2 PDF.

Look for productive strategies:

- Choosing different, efficient strategies based on the shape:
  - » for Triangle D, enclosing it in a rectangle and then dividing the area by 2.
  - » for Triangles E or G, decomposing the triangle into two right triangles using a segment from the opposite vertex to a horizontal or vertical side, and recognizing each as half of a rectangle, whose areas can be added together.
- Recognizing that one strategy that would work for all of the triangles is using an identical copy to compose a parallelogram, which has an area equal to exactly twice that of the triangle.

Activity 2 continued ›

Differentiated Support

Accessibility: ***Guide Processing and Visualization***

Consider allowing pairs to form groups of four, or relaxing the requirement of multiple strategies. As students describe how they calculated the area of each triangle, use color and/or annotations to scribe and display their thinking. Label each base and height accordingly.

Extension: ***Math Enrichment***

Have students complete the Activity 2 PDF, *Are you ready for more?,* which asks students to decompose a given triangle and rearrange the pieces to form a rectangle.

Math Language Development

MLR2: Collect and Display

Collect the initial language and representations produced by students when finding the area of a triangle prior to formalizing a formula. Circulate and observe the various strategies they use to find areas. Take pictures of different strategies or sketch them onto a display.

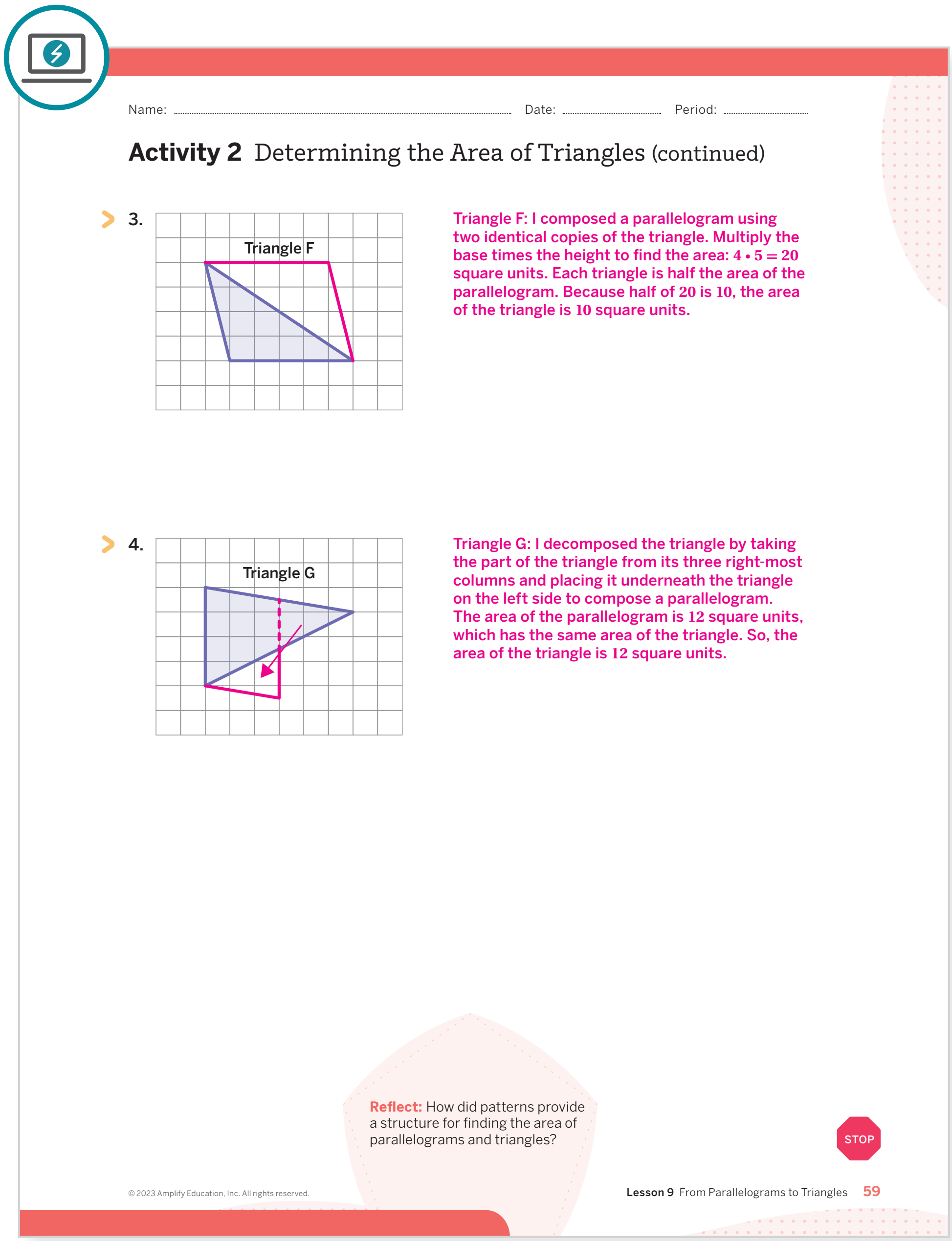
English Learners

Provide cut-out triangles for students to reference as they discuss their strategies for finding the area of a triangle. Emphasize that the area of a triangle will always be half the area of parallelogram and model for students how to use two cut-out triangles to construct a parallelogram.

Pairs | 20 min

Activity 2 Determining the Area of Triangles (continued)

Students explore different strategies for determining the areas of various triangles presented on grids, realizing that multiple strategies can be used.



Name: ____________________ Date: __________ Period: __________

Activity 2 Determining the Area of Triangles (continued)

3\.

Triangle F: I composed a parallelogram using two identical copies of the triangle. Multiply the base times the height to find the area: $4 \cdot 5 = 20$ square units. Each triangle is half the area of the parallelogram. Because half of 20 is 10, the area of the triangle is 10 square units.

4\.

Triangle G: I decomposed the triangle by taking the part of the triangle from its three right-most columns and placing it underneath the triangle on the left side to compose a parallelogram. The area of the parallelogram is 12 square units, which has the same area of the triangle. So, the area of the triangle is 12 square units.

Reflect: How did patterns provide a structure for finding the area of parallelograms and triangles?

STOP

Lesson 9 From Parallelograms to Triangles 59

3 Connect

Display the triangles, focusing on one at a time.

Have individual students share their strategies for determining the areas of the triangles, starting with Triangle D. Ensure that all strategies used for each example are shared before moving to the next. Consider offering and modeling any viable strategies not presented by your students.

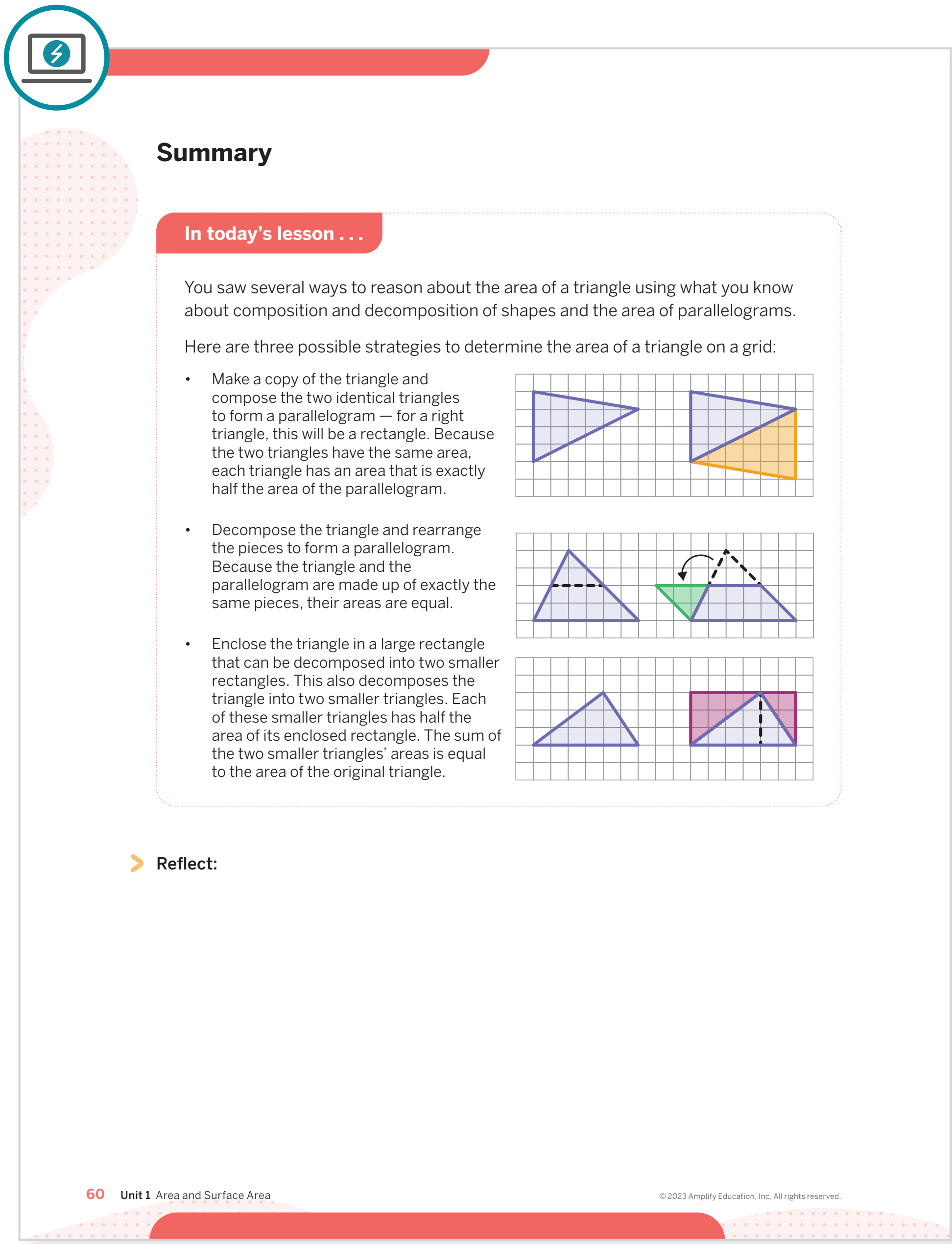
Ask, "Do you think any of the strategies would *not* work for certain triangles? If so, which one(s) and why?" Sample response: Decomposing might not work for Triangle F because I am not sure where to partition it to know for sure that it can be rearranged to make a parallelogram.

Highlight the different ways to determine the area of a triangle. Reiterate that the area of a triangle will always be equal to half the area of a parallelogram that is composed of two copies of the triangle, but depending on its shape other strategies may be just as efficient.

Whole Class | 5 min

Summary

Review and synthesize the different strategies for determining the area of a triangle — making a copy, decomposing, rearranging, or enclosing.



Summary

In today's lesson . . .

You saw several ways to reason about the area of a triangle using what you know about composition and decomposition of shapes and the area of parallelograms.

Here are three possible strategies to determine the area of a triangle on a grid:

- Make a copy of the triangle and compose the two identical triangles to form a parallelogram — for a right triangle, this will be a rectangle. Because the two triangles have the same area, each triangle has an area that is exactly half the area of the parallelogram.
- Decompose the triangle and rearrange the pieces to form a parallelogram. Because the triangle and the parallelogram are made up of exactly the same pieces, their areas are equal.
- Enclose the triangle in a large rectangle that can be decomposed into two smaller rectangles. This also decomposes the triangle into two smaller triangles. Each of these smaller triangles has half the area of its enclosed rectangle. The sum of the two smaller triangles' areas is equal to the area of the original triangle.

Reflect:

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Synthesize

Highlight the different strategies again, focusing on any that may not have been seen or not been shown more than once previously.

Ask, "Do you think any or all of these strategies would work if the triangles were not shown on a grid? Why or why not?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

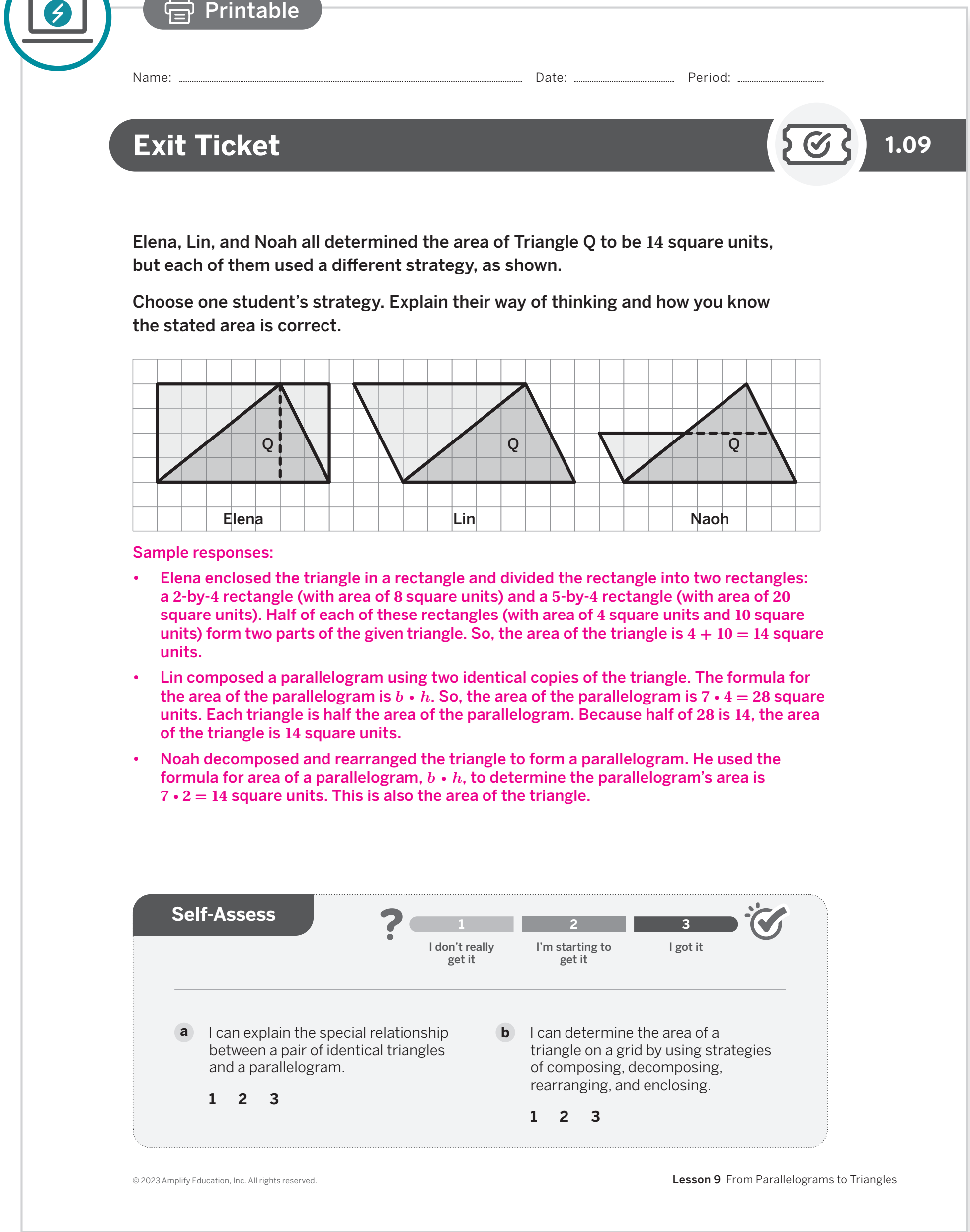
- "Are some of the strategies you saw today more or less efficient for any given triangle?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how the composition, decomposition, and rearranging of areas can be used to determine the area of a triangle.

Printable



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Exit Ticket 1.09

Elena, Lin, and Noah all determined the area of Triangle Q to be 14 square units, but each of them used a different strategy, as shown.

Choose one student's strategy. Explain their way of thinking and how you know the stated area is correct.

Sample responses:

- Elena enclosed the triangle in a rectangle and divided the rectangle into two rectangles: a 2-by-4 rectangle (with area of 8 square units) and a 5-by-4 rectangle (with area of 20 square units). Half of each of these rectangles (with area of 4 square units and 10 square units) form two parts of the given triangle. So, the area of the triangle is $4 + 10 = 14$ square units.
- Lin composed a parallelogram using two identical copies of the triangle. The formula for the area of the parallelogram is $b \cdot h$. So, the area of the parallelogram is $7 \cdot 4 = 28$ square units. Each triangle is half the area of the parallelogram. Because half of 28 is 14, the area of the triangle is 14 square units.
- Noah decomposed and rearranged the triangle to form a parallelogram. He used the formula for area of a parallelogram, $b \cdot h$, to determine the parallelogram's area is $7 \cdot 2 = 14$ square units. This is also the area of the triangle.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain the special relationship between a pair of identical triangles and a parallelogram.

1 2 3

b I can determine the area of a triangle on a grid by using strategies of composing, decomposing, rearranging, and enclosing.

1 2 3

Success looks like . . .

- **Goal:** Drawing a diagram to show that the area of a triangle is half the area of a related parallelogram.
- **Language Goal:** Explaining strategies for using the base and height of a related parallelogram to determine the area of a triangle. **(Speaking and Listening, Writing)**
    - » Explaining why one of the students' strategies is correct for determining the area of Triangle Q.

Suggested next steps

If students struggle explaining any of the strategies, consider:

- Having them look back at their strategy for Triangle E from Activity 2, or referencing an anchor chart that could be useful in determining the area of Triangle E.
- Suggesting they explain Lin's strategy verbally before trying to write anything down.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How was this lesson similar to or different from Lesson 6 on decomposing, rearranging and enclosing parallelograms?
- What routines enabled all students to do the math in today's lesson? What might you change for the next time you teach this lesson?

Math Language Development

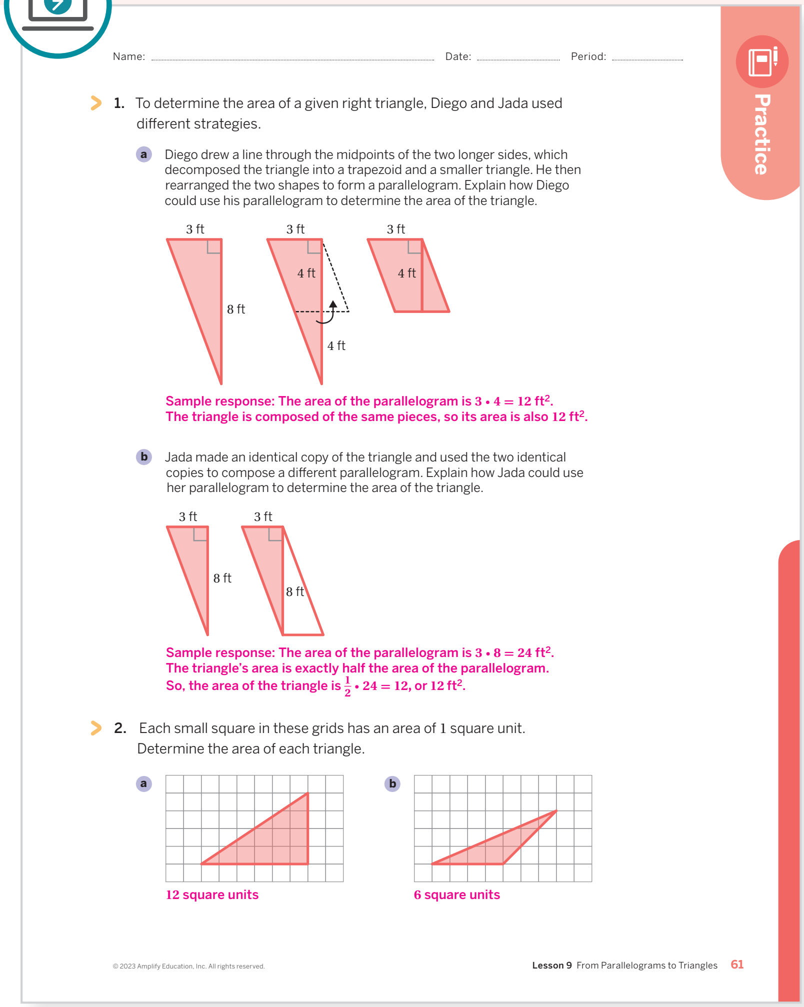
Language Goal: Explaining strategies for using the base and height of a related parallelogram to determine the area of a triangle.

Reflect on students' language development toward this goal.

- How have the language routines used in this lesson helped students explain strategies that can be used to determine the area of a triangle?
- Are there particular routines that have been more helpful? Why or why not?

Practice

Independent



Name: Date: Period:

Practice

1. To determine the area of a given right triangle, Diego and Jada used different strategies.

   a. Diego drew a line through the midpoints of the two longer sides, which decomposed the triangle into a trapezoid and a smaller triangle. He then rearranged the two shapes to form a parallelogram. Explain how Diego could use his parallelogram to determine the area of the triangle.

   Sample response: The area of the parallelogram is $3 \cdot 4 = 12$ ft^2. The triangle is composed of the same pieces, so its area is also 12 ft^2.

   b. Jada made an identical copy of the triangle and used the two identical copies to compose a different parallelogram. Explain how Jada could use her parallelogram to determine the area of the triangle.

   Sample response: The area of the parallelogram is $3 \cdot 8 = 24$ ft^2. The triangle's area is exactly half the area of the parallelogram. So, the area of the triangle is $\frac{1}{2} \cdot 24 = 12$, or 12 ft^2.

2. Each small square in these grids has an area of 1 square unit. Determine the area of each triangle.

   a. **12 square units**

   b. **6 square units**

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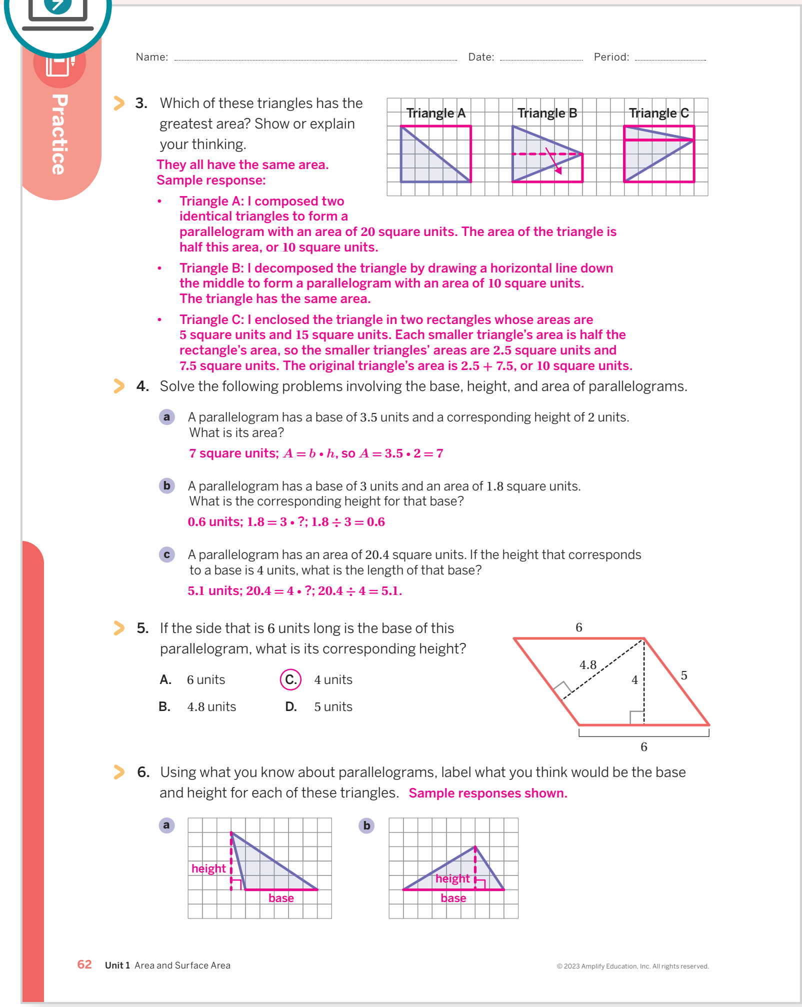
Lesson 9 From Parallelograms to Triangles 61



Name: Date: Period:

Practice

3. Which of these triangles has the greatest area? Show or explain your thinking.

   They all have the same area.
   Sample response:

   - **Triangle A: I composed two identical triangles to form a parallelogram with an area of 20 square units. The area of the triangle is half this area, or 10 square units.**
   - **Triangle B: I decomposed the triangle by drawing a horizontal line down the middle to form a parallelogram with an area of 10 square units. The triangle has the same area.**
   - **Triangle C: I enclosed the triangle in two rectangles whose areas are 5 square units and 15 square units. Each smaller triangle's area is half the rectangle's area, so the smaller triangles' areas are 2.5 square units and 7.5 square units. The original triangle's area is $2.5 + 7.5$, or 10 square units.**

4. Solve the following problems involving the base, height, and area of parallelograms.

   a. A parallelogram has a base of 3.5 units and a corresponding height of 2 units. What is its area?

   7 square units; $A = b \cdot h$, so $A = 3.5 \cdot 2 = 7$

   b. A parallelogram has a base of 3 units and an area of 1.8 square units. What is the corresponding height for that base?

   0.6 units; $1.8 = 3 \cdot ?$; $1.8 \div 3 = 0.6$

   c. A parallelogram has an area of 20.4 square units. If the height that corresponds to a base is 4 units, what is the length of that base?

   5.1 units; $20.4 = 4 \cdot ?$; $20.4 \div 4 = 5.1$.

5. If the side that is 6 units long is the base of this parallelogram, what is its corresponding height?

   A. 6 units
   [✓ C.] 4 units
   B. 4.8 units
   D. 5 units

6. Using what you know about parallelograms, label what you think would be the base and height for each of these triangles. **Sample responses shown.**

   a.

   b.

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 8	2
	5	Unit 1 Lesson 7	2
Formative ⚡	6	Unit 1 Lesson 10	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 10**

Bases and Heights of Triangles

Let's find the bases and heights of triangles.

Focus

Goals

1. **Language Goal:** Recognize that any side of a triangle can be considered its base, and identify a corresponding height. **(Speaking and Listening)**
2. Draw and label a height that corresponds to a given base of a triangle, making sure it is perpendicular to the base and the correct length.

Coherence

Today

Students use examples and non-examples of bases and heights of triangles to analyze and then generalize their properties. They practice both identifying and drawing corresponding heights for given bases of several different triangles. Students also see how an auxiliary line (extending the base or drawing a line through the opposite vertex parallel to the base) can be used to help draw corresponding heights outside the triangle. Note: In these materials, the base refers to the side of the triangle, and its measure is written out as "the length of the base," except in formulas. Similarly, these materials use "a height" to mean a segment that is or can be drawn, and "the height" to mean its measure or length.

Previously

In Lesson 7, students identified bases and corresponding heights for a variety of different parallelograms.

Coming Soon

In Lesson 11, students will build upon their understanding of bases and heights and use patterns in their respective measurements to develop and apply the formula for the area of a triangle.

Rigor

- Students develop **procedural fluency** in determining where to draw the base and height of a triangle.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	5 min	25 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (for display)
- index cards

Math Language Development

New words

- ***base*** (of a triangle)
- ***height*** (of a triangle)

Review words

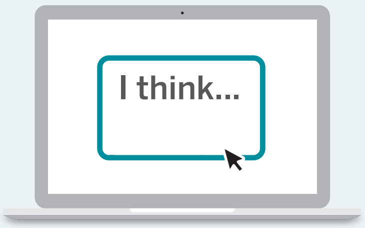
- *vertex**
- *opposite vertex*

*A *vertex* of a two-dimensional shape is a point where two sides intersect. The term *vertex* is defined for three-dimensional figures in Lesson 14 of this Unit. The student glossary contains a definition covering both 2D and 3D.

Amps | Featured Activity

Warm-up
See Student Thinking

Students explain their understanding of the base and height of a triangle and these explanations are available to you digitally, in real time.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may have trouble listening to their partner's ideas about what they think is true and what they think is false in Activity 1. Ask students to fully listen to their partner's comments without interrupting. Then have them restate what their partner shared in their own words and think carefully about how to respond as they provide feedback or critique given statements.

Modifications to Pacing

You may want to consider this additional modification if you are short on time:

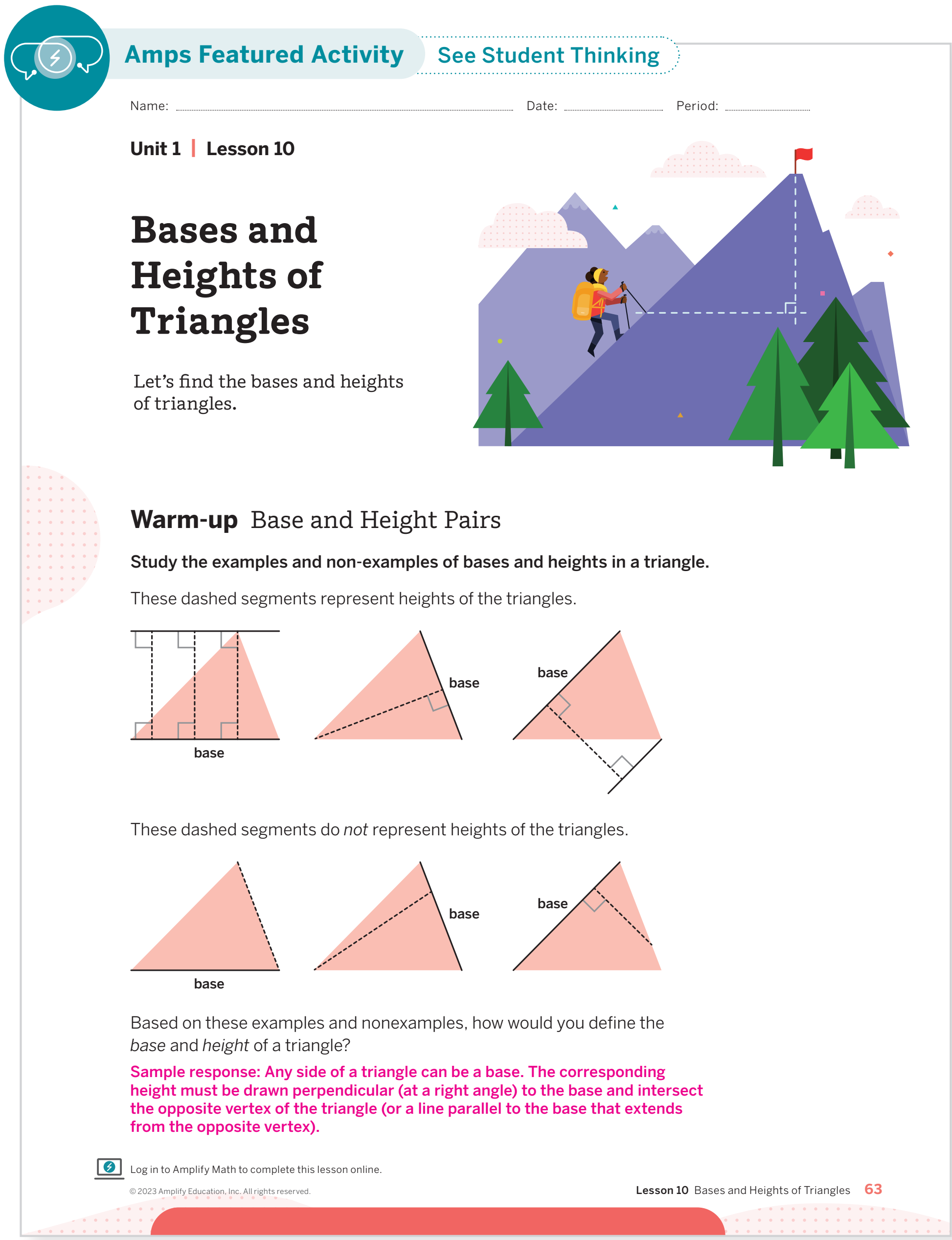
- In **Activity 2**, Problem 1 may be omitted.

Pairs | 5 min

Warm-up Base and Height Pairs

Students study examples and non-examples of bases and heights in different triangles to generalize the properties of bases and heights for triangles.

Amps Featured Activity See Student Thinking



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Unit 1 | Lesson 10

Bases and Heights of Triangles

Let's find the bases and heights of triangles.

Warm-up Base and Height Pairs

Study the examples and non-examples of bases and heights in a triangle.

These dashed segments represent heights of the triangles.

base base base

These dashed segments do *not* represent heights of the triangles.

base base base

Based on these examples and nonexamples, how would you define the *base* and *height* of a triangle?

Sample response: Any side of a triangle can be a base. The corresponding height must be drawn perpendicular (at a right angle) to the base and intersect the opposite vertex of the triangle (or a line parallel to the base that extends from the opposite vertex).

Log in to Amplify Math to complete this lesson online.

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Lesson 10 Bases and Heights of Triangles 63

1 Launch

Conduct the ***Think-Pair-Share*** routine. Review the terms *vertex* and *opposite vertex*, illustrating each using the examples as needed.

2 Monitor

Help students get started by asking, "What is one thing the examples all have in common? Is that also true for all of the non-examples?"

Look for points of confusion:

- **Not recognizing that a height needs to be perpendicular to the base.** Show an example of a parallelogram and highlight its base and height.
- **Assuming a right angle always indicates a height.** Explain that a height must be drawn perpendicular to the base, but it should also correspond to the same length as a segment drawn from the opposite vertex.

Look for productive strategies:

- Recognizing that the base can be any side of the triangle, but many heights are possible as long as they "look a certain way."

3 Connect

Display the examples and non-examples.

Have students share their definitions of *base* and *height*; provide a common language definition for the class before moving on.

Define and consider adding to an anchor chart:

- ***Base*** (of a triangle) as any chosen side of the triangle.
- ***Height*** (of a triangle) as a segment representing the distance between the base and the *opposite vertex*.

Ask:

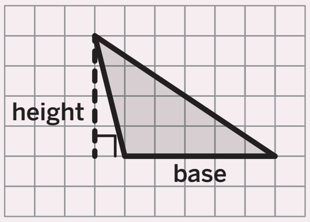
- "What tools could be helpful in drawing a height of a triangle for a given base?"
- "How might the shape of the triangle affect your process or thinking about drawing a height?"

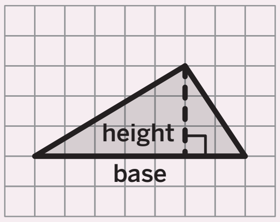
Power-up

To power up students' ability to relate bases and heights of parallelograms and triangles, have students complete:

Consider the diagram showing the heights and bases of two triangles. Select T or F to indicate whether each statement is true or false.

1. The height and base must intersect T or [✓ F]
2. The height of a triangle is always perpendicular to the base [✓ T] or F
3. The height of a triangle is always shorter than its base T or [✓ F]





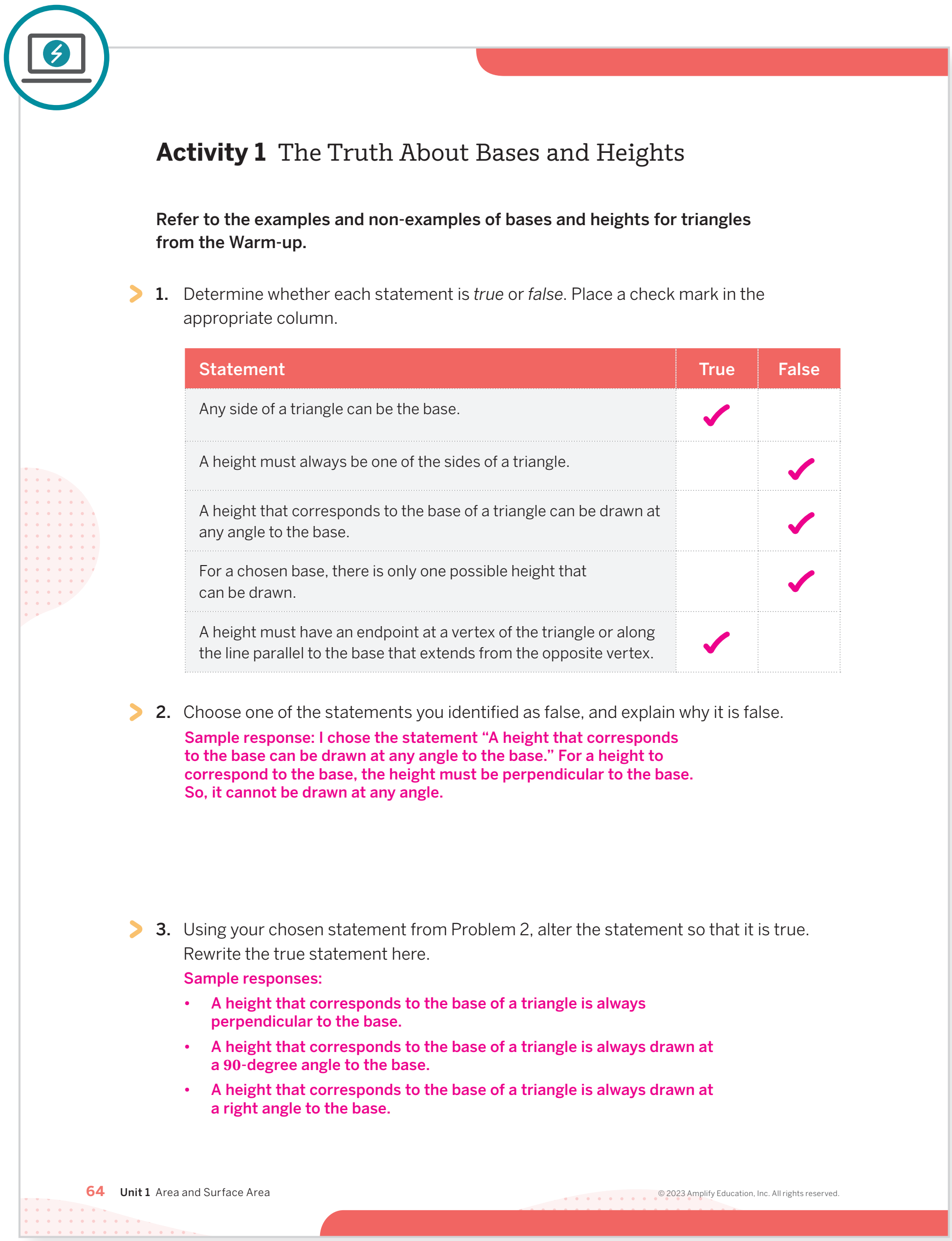
Use: Before the Warm-up.

Performance on Lesson 9, Practice Problem 6.

Pairs | 5 min

Activity 1 The Truth About Bases and Heights

Students determine the validity of statements about the bases and heights of triangles to build understanding about what must be true about base and height pairs of triangles.



Activity 1 The Truth About Bases and Heights

Refer to the examples and non-examples of bases and heights for triangles from the Warm-up.

1. Determine whether each statement is *true* or *false*. Place a check mark in the appropriate column.

Statement	True	False
Any side of a triangle can be the base.	✓	
A height must always be one of the sides of a triangle.		✓
A height that corresponds to the base of a triangle can be drawn at any angle to the base.		✓
For a chosen base, there is only one possible height that can be drawn.		✓
A height must have an endpoint at a vertex of the triangle or along the line parallel to the base that extends from the opposite vertex.	✓	

2. Choose one of the statements you identified as false, and explain why it is false.
Sample response: I chose the statement "A height that corresponds to the base can be drawn at any angle to the base." For a height to correspond to the base, the height must be perpendicular to the base. So, it cannot be drawn at any angle.

3. Using your chosen statement from Problem 2, alter the statement so that it is true. Rewrite the true statement here.
Sample responses:
   - **A height that corresponds to the base of a triangle is always perpendicular to the base.**
   - **A height that corresponds to the base of a triangle is always drawn at a 90-degree angle to the base.**
   - **A height that corresponds to the base of a triangle is always drawn at a right angle to the base.**

1 Launch

Have students refer to the examples and non-examples from the Warm-up.

2 Monitor

Help students get started by asking them if the examples from the Warm-up support or do not support the first statement in the table.

Look for points of confusion:

- **Thinking that the height must always be one side of a triangle.** Refer students back to the examples of heights from the Warm-up.
- **Saying a statement is true because it matches one example.** Remind students that for a statement to be true, it must be true for *all* of the examples.

Look for productive strategies:

- Confirming that each statement identified as true corresponds to every example.
- Identifying at least one non-example that supports their thinking for each statement identified as false.

3 Connect

Display student work showing their responses.

Have pairs of students share their responses to each statement, explaining how they decided whether it was true or false. Encourage the use of examples and non-examples to support their thinking.

Highlight that any side of a triangle can be a base, but a corresponding height must:

- be drawn perpendicular to the base.
- correspond to the distance from the base to the opposite vertex.

A height can also be drawn outside the triangle, represented by any perpendicular segment between the base and the line containing the opposite vertex.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide examples of pre-drawn triangles that illustrate each statement in the table for Problem 1 to help students determine whether the statement is true or false. Have them indicate which pre-drawn triangle(s) helped them make their determination.

Extension: *Math Enrichment*

Have pairs of students write their own statements, one that is true and one that is false, about the base-height pairs of a triangle. Then have them exchange statements with another pair of students. Each pair should determine whether each statement is true or false.

Math Language Development

MLR8: Discussion Supports — Restate It!

As students share responses during the Connect, have them restate each other's reasoning using precise mathematical vocabulary. Encourage them to challenge each other when they disagree, using prompts, such as:

- "I agree because . . ."
- "I disagree because . . ."

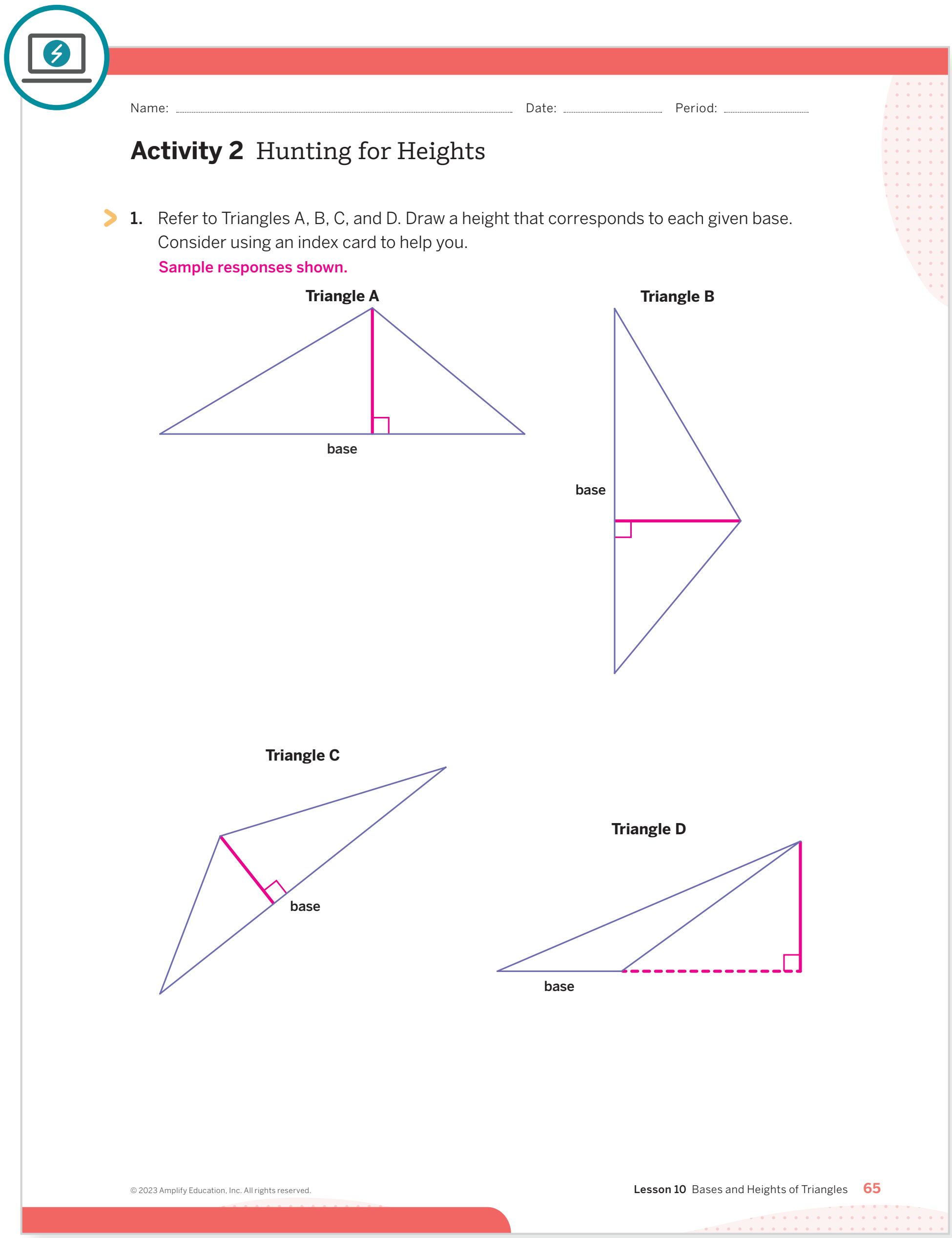
English Learners

Provide a word bank of mathematical vocabulary students can use as they explain whether they agree or disagree, such as *base, height, perpendicular,* and *vertex*.

Pairs | 25 min

Activity 2 Hunting for Heights

Students work with a variety of triangles to develop a concrete strategy for accurately identifying the height of a triangle.



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Activity 2 Hunting for Heights

1. Refer to Triangles A, B, C, and D. Draw a height that corresponds to each given base. Consider using an index card to help you.
Sample responses shown.

1 Launch

Display the Activity 2 PDF for students to read and reference while they work. Consider demonstrating how to use an index card (or similar tool) to draw a corresponding height to the given base for Triangle A. Provide access to index cards or similar tools. Give students time to work with Triangles B–D individually before sharing and completing Problem 2 with a partner.

2 Monitor

Help students get started by having them choose a base for each triangle first, reminding them that they will also need to construct a corresponding height.

Look for points of confusion:

- **Using the index card as a straightedge only and not as a right angle as well.** Remind students that a height needs to be perpendicular to the base, and ask, "How can the index card help you with that?"
- **Struggling to draw a height when it lies is outside the triangle.** Provide an example of a parallelogram (such as those in Lesson 7) where the height lies outside the triangle. Ask, "How could you do something similar for this triangle?"
- **Having difficulty drawing a height when the base is not horizontal.** Have students rotate the paper to make the base "look" horizontal.

Look for productive strategies:

- Ensuring that the height drawn is perpendicular to the base *and* intersects the opposite vertex, even with heights drawn outside the triangle.
- Utilizing the index card to draw accurate heights and right angles.
- Knowing when and how to draw an auxiliary line as needed.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Tools, Demonstrate*

Suggest tools and strategies that students may find useful, such as rotating the page or using an index card to determine perpendicularity. Consider demonstrating how to use these tools and strategies.

Extension: *Math Enrichment*

Have students draw a triangle and label any side as the base. Then using available tools, challenge them to draw three heights, where each height intersects a different vertex. Then have them do the same for each of the other sides as the base, possibly drawing each set of heights in different colors.

Math Language Development

MLR2: Collect and Display

Collect different examples of student drawings of base-height pairs. Display the various examples and ask students to compare the diagrams. Listen for and amplify the mathematical language students use to support their thinking.

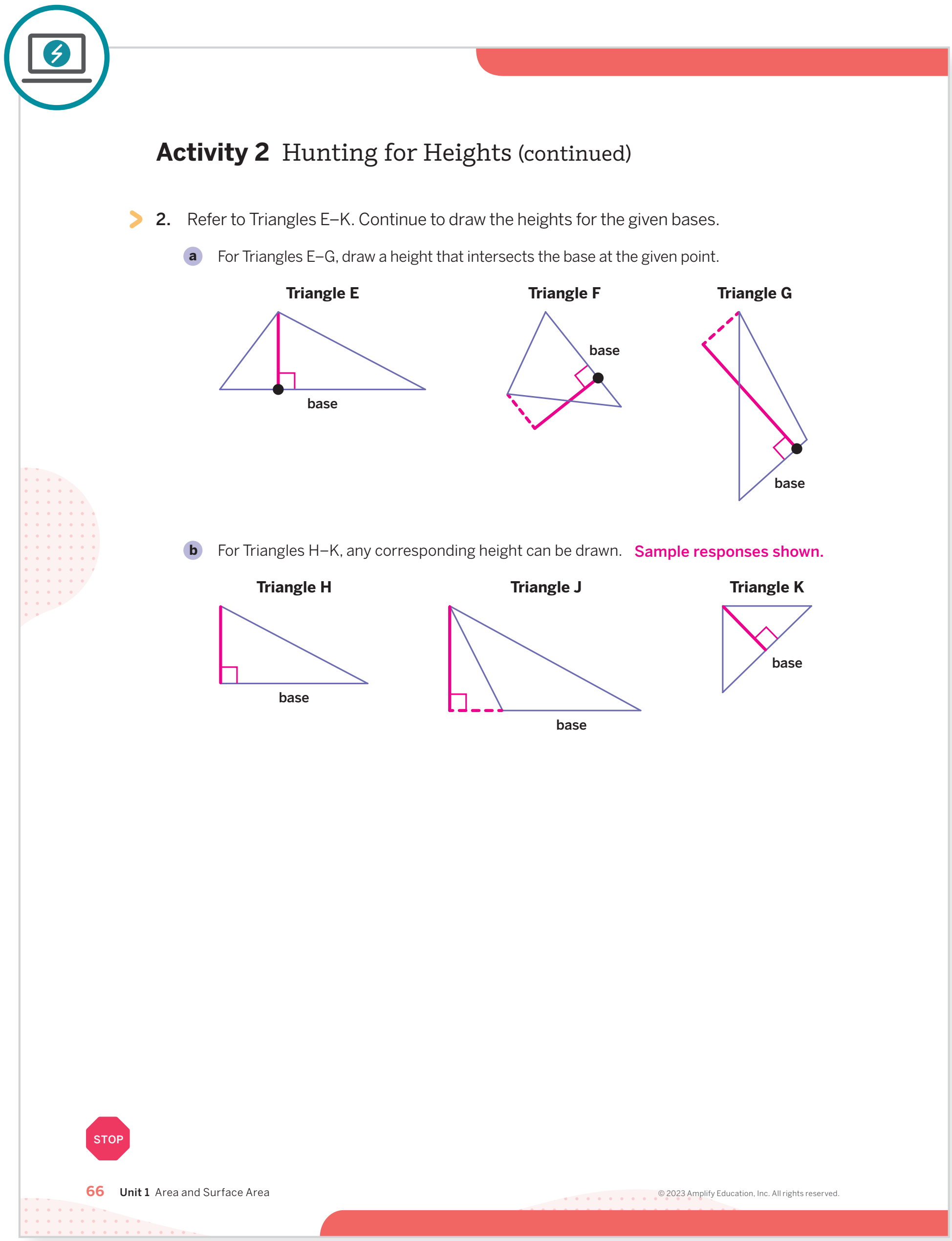
English Learners

Use gestures to support students' understanding of the terms *horizontal, vertical,* and *perpendicular lines*.

Pairs | 25 min

Activity 2 Hunting for Heights (continued)

Students work with a variety of triangles to develop a concrete strategy for accurately identifying the height of a triangle.



Activity 2 Hunting for Heights (continued)

2. Refer to Triangles E–K. Continue to draw the heights for the given bases.

a For Triangles E–G, draw a height that intersects the base at the given point.

b For Triangles H–K, any corresponding height can be drawn. Sample responses shown.

STOP

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3 Connect

Display Triangles B–D first, Triangles E–G and then Triangles H–K, to capture students' work as they share.

Have students share one way to draw a height for each triangle and explain their strategies. Allow other students to offer different possible heights.

Ask:

- "What was different about your strategies for Triangles E–G than the others?" Sample response: There was only one possible way and place to draw the height, and whether it was inside or outside was already determined by the given point.
- "When might you need to draw an auxiliary line, such as extending the base or drawing a line through the opposite vertex?" When the desired height needs to be drawn outside the triangle.

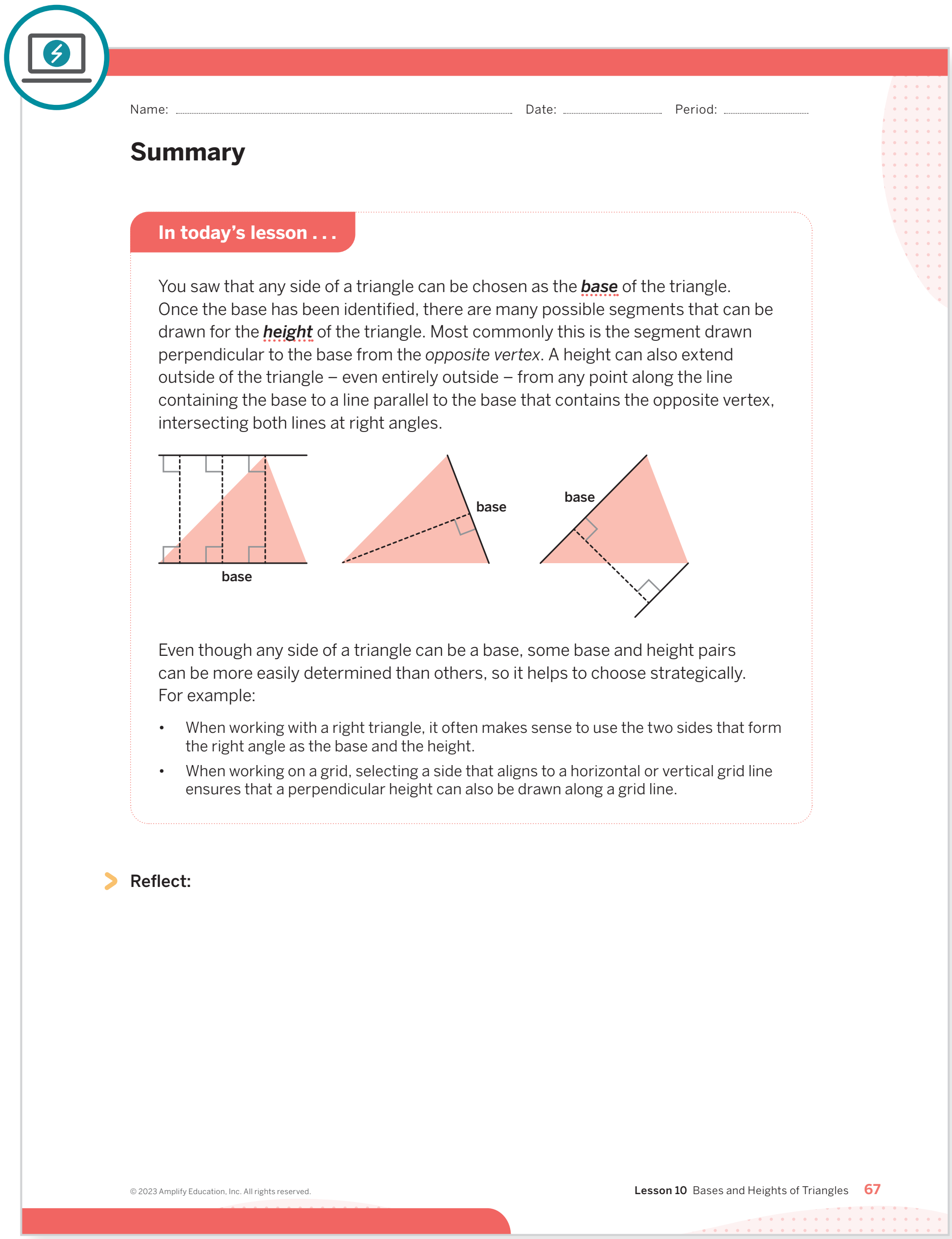
Note: Students may not be familiar with the term *auxiliary line*. This term refers to any line that they can draw on a figure that helps them make sense of the figure or solve problems.

Highlight that for any chosen base there are many possible heights that can be drawn, and students saw how an index card can help them construct a precise segment that is straight and also perpendicular to the base. There is always one and only one height that can be drawn from the opposite vertex, and, for any given point along the base, there is also one and only one possible height, which may require an auxiliary line.

Whole Class | 5 min

Summary

Review and synthesize how to draw a corresponding height of a triangle for any given base.



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Summary

In today's lesson . . .

You saw that any side of a triangle can be chosen as the ***base*** of the triangle. Once the base has been identified, there are many possible segments that can be drawn for the ***height*** of the triangle. Most commonly this is the segment drawn perpendicular to the base from the *opposite vertex*. A height can also extend outside of the triangle – even entirely outside – from any point along the line containing the base to a line parallel to the base that contains the opposite vertex, intersecting both lines at right angles.

Even though any side of a triangle can be a base, some base and height pairs can be more easily determined than others, so it helps to choose strategically. For example:

- When working with a right triangle, it often makes sense to use the two sides that form the right angle as the base and the height.
- When working on a grid, selecting a side that aligns to a horizontal or vertical grid line ensures that a perpendicular height can also be drawn along a grid line.

Reflect:

Synthesize

Display the triangles from the Summary.

Highlight and reiterate as necessary that for every triangle, there are multiple base-height pairs. For many bases, the height drawn from the *opposite vertex* will be inside the triangle, but for others it may be outside the triangle and the base would need to be extended. Yet others still can be drawn by extending a line parallel to the base from the *opposite vertex* and then any perpendicular segment between that line and the base is a height.

Formalize vocabulary:

- ***base*** (of a triangle)
- ***height*** (of a triangle)

Ask:

- "Does it matter which side you choose as the base?" While any side of a triangle can be chosen as the base, some bases are easier to use than others when drawing a corresponding height.
- "What is important to remember about the relationship between a base of a triangle and its corresponding height?" The corresponding height must always be drawn perpendicular to the base and meet at the opposite vertex (or a line parallel to the base containing the opposite vertex).

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do you determine the base of a triangle? its height?"
- "How does working with a right triangle or a triangle on a grid change where you might determine where the base is?"

Math Language Development

MLR2: Collect and Display

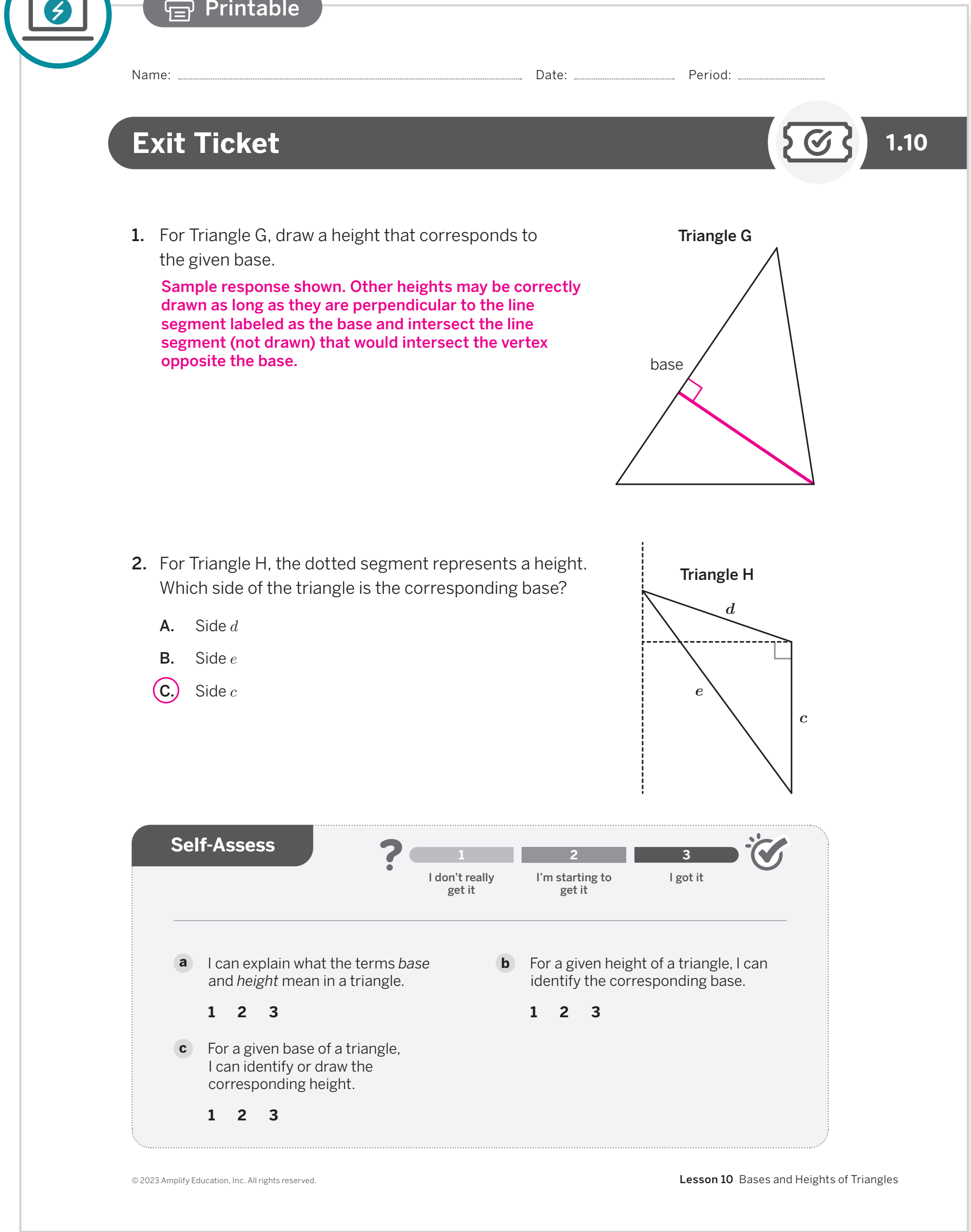
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 3. Ask them to review and reflect on any terms and phrases related to the terms the *base* or *height* of a triangle that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of bases and heights of triangles by identifying a base and drawing its corresponding height.

Printable



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Exit Ticket 1.10

1. For Triangle G, draw a height that corresponds to the given base.

Sample response shown. Other heights may be correctly drawn as long as they are perpendicular to the line segment labeled as the base and intersect the line segment (not drawn) that would intersect the vertex opposite the base.

2. For Triangle H, the dotted segment represents a height. Which side of the triangle is the corresponding base?

A. Side d

B. Side e

[✓ C.] Side c

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can explain what the terms *base* and *height* mean in a triangle. 1 2 3

b. For a given height of a triangle, I can identify the corresponding base. 1 2 3

c. For a given base of a triangle, I can identify or draw the corresponding height. 1 2 3

Lesson 10 Bases and Heights of Triangles

Success looks like . . .

- **Language Goal:** Recognizing that any side of a triangle can be considered its base, and identifying a corresponding height **(Speaking and Listening)**.
    - » Determining where to draw a height in Triangle G for its labeled base in Problem 1.
- **Goal:** Drawing and labeling a height that corresponds to a given base of a triangle, making sure it is perpendicular to the base and the correct length.
    - » Drawing a correct height for Triangle G in Problem 1.

Suggested next steps

For Triangle G, if students struggle with drawing a corresponding height, consider:

- Having them rotate the paper to make the base "look" horizontal.
- Reminding them to use an index card to draw a straight segment from the opposite vertex to the base that is perpendicular to the base.

For Triangle H, if students identify the base as Side d or Side e, consider:

- Referring back to the Warm-up and asking, "Which sides does the height intersect? What has to be true about the base and a corresponding height for a triangle?"

Professional Learning

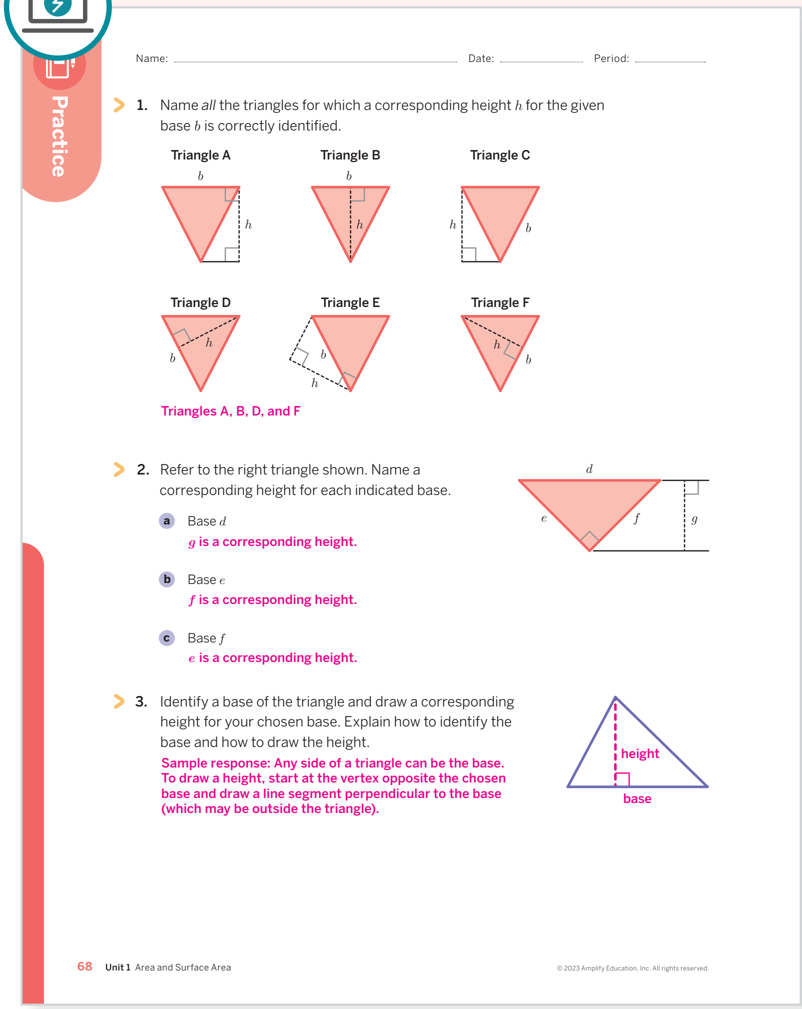
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What trends do you see in participation?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Name *all* the triangles for which a corresponding height h for the given base b is correctly identified.

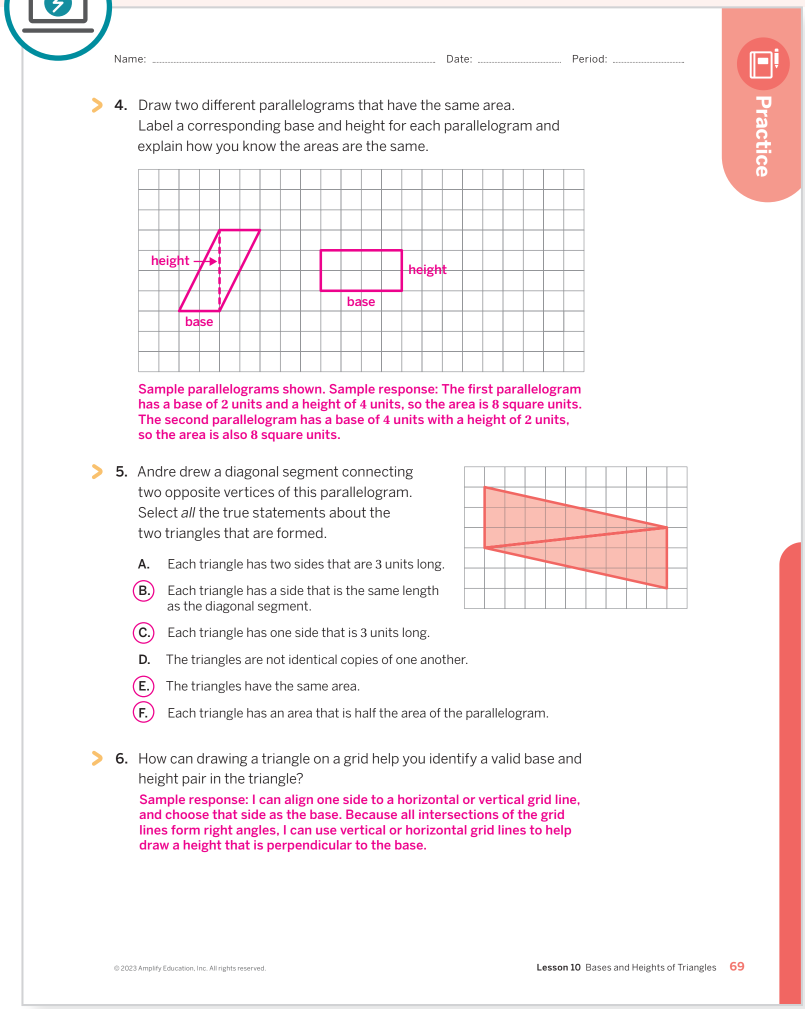
Triangles A, B, D, and F

2. Refer to the right triangle shown. Name a corresponding height for each indicated base.

- a Base d
  g is a corresponding height.
- b Base e
  f is a corresponding height.
- c Base f
  e is a corresponding height.

3. Identify a base of the triangle and draw a corresponding height for your chosen base. Explain how to identify the base and how to draw the height.

Sample response: Any side of a triangle can be the base. To draw a height, start at the vertex opposite the chosen base and draw a line segment perpendicular to the base (which may be outside the triangle).



Name: ________ Date: ________ Period: ________

Practice

4. Draw two different parallelograms that have the same area. Label a corresponding base and height for each parallelogram and explain how you know the areas are the same.

Sample parallelograms shown. Sample response: The first parallelogram has a base of 2 units and a height of 4 units, so the area is 8 square units. The second parallelogram has a base of 4 units with a height of 2 units, so the area is also 8 square units.

5. Andre drew a diagonal segment connecting two opposite vertices of this parallelogram. Select *all* the true statements about the two triangles that are formed.

A. Each triangle has two sides that are 3 units long.
[✓ B.] Each triangle has a side that is the same length as the diagonal segment.
[✓ C.] Each triangle has one side that is 3 units long.
D. The triangles are not identical copies of one another.
[✓ E.] The triangles have the same area.
[✓ F.] Each triangle has an area that is half the area of the parallelogram.

6. How can drawing a triangle on a grid help you identify a valid base and height pair in the triangle?

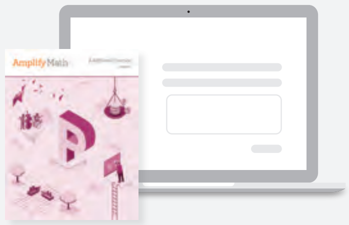
Sample response: I can align one side to a horizontal or vertical grid line, and choose that side as the base. Because all intersections of the grid lines form right angles, I can use vertical or horizontal grid lines to help draw a height that is perpendicular to the base.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 8	2
	5	Unit 1 Lesson 9	2
Formative ⚡	6	Unit 1 Lesson 11	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

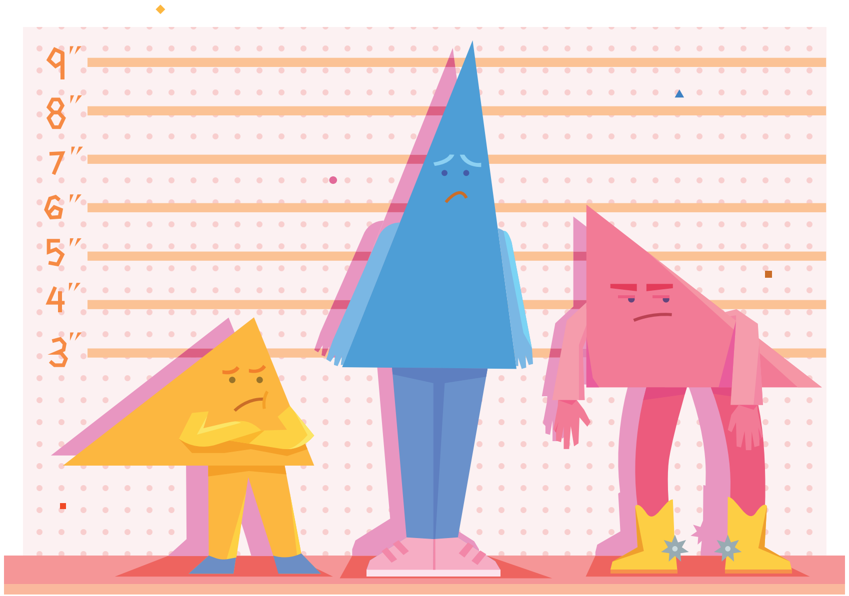


For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 11**

Formula for the Area of a Triangle

Let's write and use a formula to determine the area of any triangle.



Focus

Goals

1. **Language Goal:** Compare, contrast, and critique different strategies for determining the area of a triangle. **(Speaking and Listening)**
2. **Language Goal:** Generalize a process for determining the area of a triangle, and justify why this can be abstracted. **(Speaking and Listening)**
3. **Language Goal:** Evaluate the usefulness of different base-height pairs for determining the area of a given triangle. **(Speaking and Listening)**

Coherence

Today

Students reason about the areas of triangles on a grid by identifying the measures of their bases and heights and relating these to corresponding parallelograms by using composition and decomposition strategies. They realize that some base-height pairs may be more practical or efficient for determining area than others, depending on the shape or orientation of a triangle on a grid. As students determine the areas of several triangles, they identify repeated patterns in both the strategies used and the measures of the triangles to help them write a general formula for the area of a triangle, which can also be applied to triangles not on a grid (given base and height measures).

Previously

In Lessons 9 and 10, students related triangles to corresponding parallelograms and identified bases and heights of triangles. They saw that any side of a triangle can be a base and how the corresponding height can be drawn perpendicular to the base to represent the distance from the opposite vertex.

Coming Soon

In Lesson 12, students will apply both decomposition strategies and the formulas for the area of a parallelogram and a triangle to explore the area of a trapezoid. In Lesson 13, they will use these formulas again to determine areas of polygons in a real-world scenario.

Rigor

- Students use visual models to develop **conceptual understanding** of the formula for the area of a triangle.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	10 min	5 min	10 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- straightedges or index cards

Math Language Development

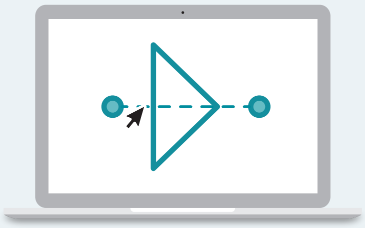
Review words

- *opposite vertex*
- *base* (of a triangle)
- *height* (of a triangle)

Amps | Featured Activity

Warm-up
Interactive Geometry

Students use technology to quickly modify drawings of parallelograms without having to erase.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated if they cannot yet recognize a pattern among bases, heights, and area measurements from the table in Activity 1. Encourage them to persist, consider slowing down, or simplifying the problem by focusing on one row at a time and relating the values to their drawings or strategies for determining area (such as by using a parallelogram). Then they can consider whether the relationship seems to hold for the next triangle, and the next, or they modify and revise their thinking.

Modifications to Pacing

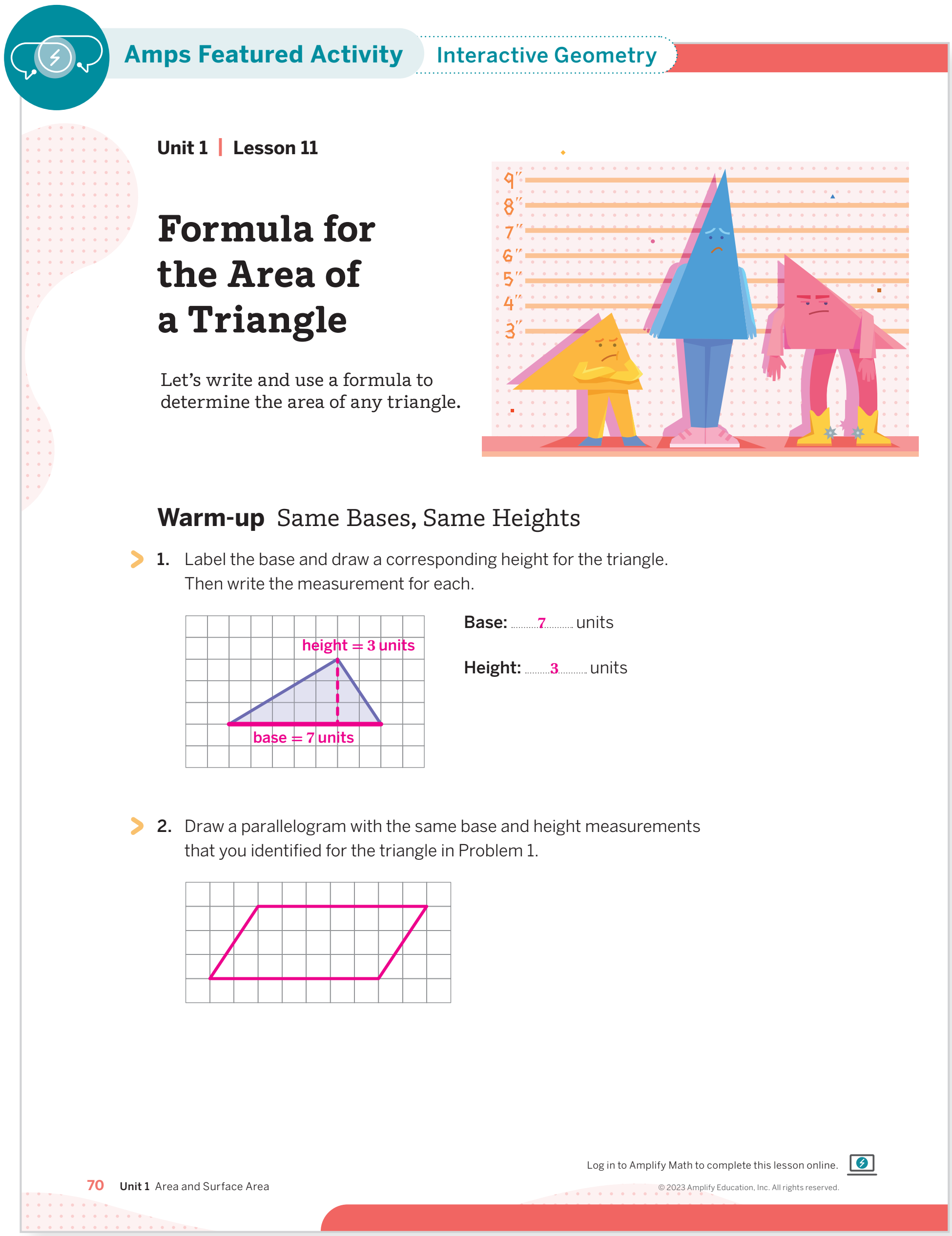
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, assign one triangle to each pair for Problem 2 and crowdsource the values for all when completing the table before attempting Problem 3.
- In **Activity 2**, limit the number of problems that students need to complete to one or two.

Warm-up Same Bases, Same Heights

Students identify the base and height of a triangle and then draw a parallelogram with the same base and height to build the connection between triangles and parallelograms.

Amps Featured Activity Interactive Geometry



Unit 1 | Lesson 11

Formula for the Area of a Triangle

Let's write and use a formula to determine the area of any triangle.

Warm-up Same Bases, Same Heights

1. Label the base and draw a corresponding height for the triangle. Then write the measurement for each.

height = 3 units

base = 7 units

Base: 7 units

Height: 3 units

2. Draw a parallelogram with the same base and height measurements that you identified for the triangle in Problem 1.

70 **Unit 1** Area and Surface Area

Log in to Amplify Math to complete this lesson online.

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1 Launch

Provide students with straightedges or index cards. Remind them that the height may already be indicated as a side of the triangle or it may need to be drawn.

2 Monitor

Help students get started by asking them to explain what the terms *base* and *height* each mean for a triangle. Consider referring to examples from Lesson 10.

Look for points of confusion:

- **Questioning which side of the triangle should be identified as the base.** Remind students that any side can be the base. Ask, "Because you need to be able to determine its length, which side would be best?"
- **Struggling to draw a parallelogram with the same base and height as the triangle.** Have students start by drawing an identical base to that of the triangle. Ask, "What can you say about how the opposite side should look? To have the same height as the triangle, where should it be drawn?"

Look for productive strategies:

- Choosing the horizontal side as the base so its length and the height can both be determined.
- Drawing a parallelogram by using two copies of the triangle from Problem 1.

3 Connect

Have students share their responses to Problem 1 and strategies for drawing a parallelogram in Problem 2.

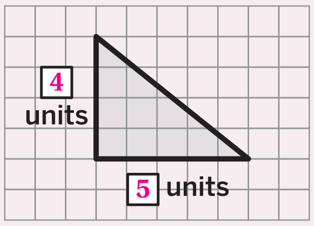
Ask, "Will it always be possible to create a parallelogram with the same base and height measurements as the triangle? How might that be reflected in the expression for the area of any triangle?"

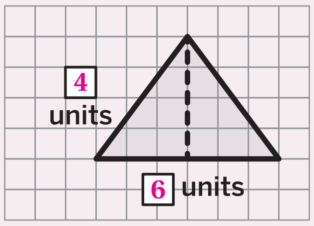
Highlight that there are many different triangles and many different parallelograms with the same base and height, and the formulas will show how these are related.

Power-up

To power up students' ability to determine the base and height of a triangle on a grid have students complete:

Identify the length of the base and the height of each triangle:





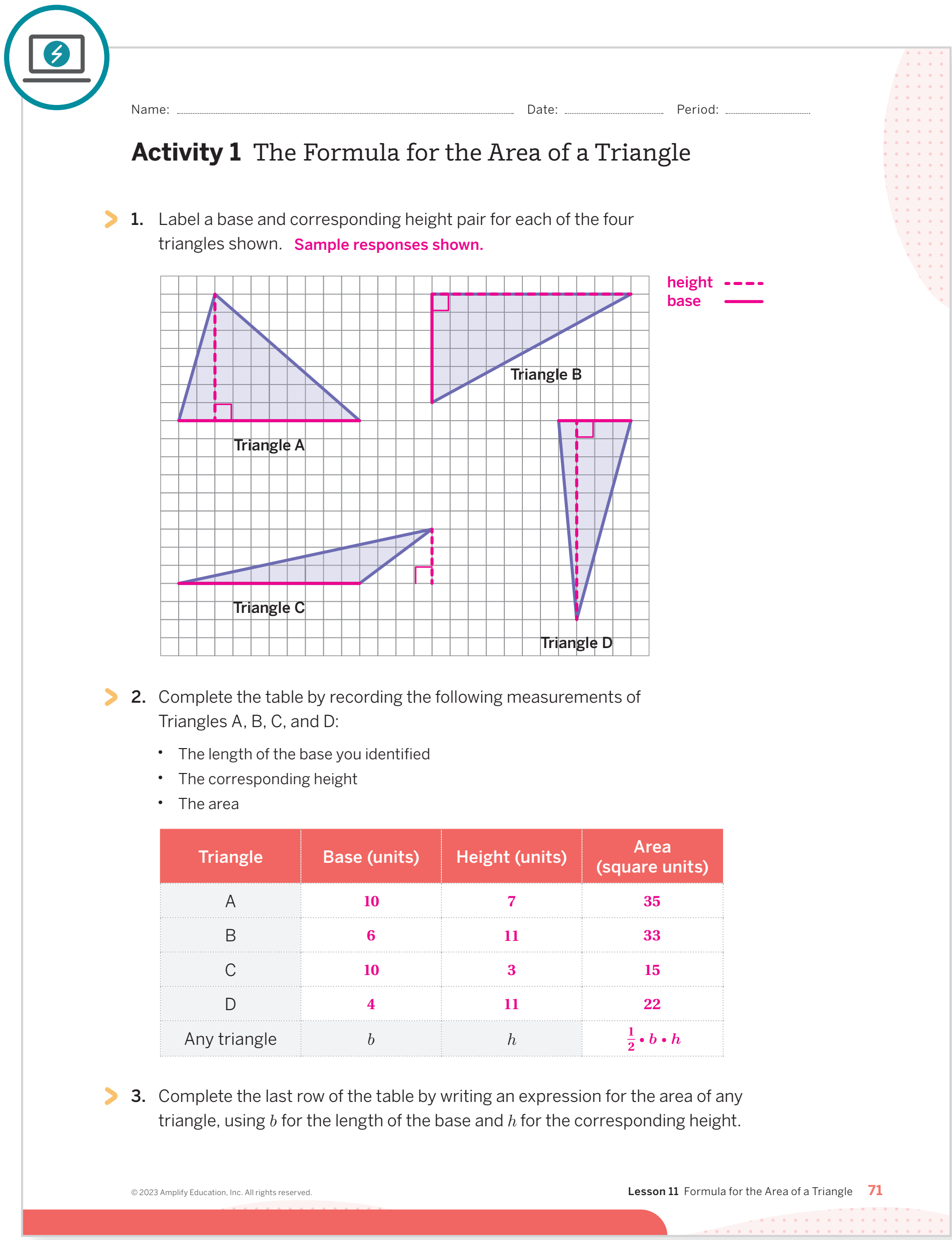
Use: Before the Warm-up.

Informed by: Performance on Lesson 10, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3.

Pairs | 15 min

Activity 1 The Formula for the Area of a Triangle

Students measure the bases and heights of several different triangles to determine their areas and reveal a pattern for generalizing the formula for the area of a triangle.



Name: ____________ Date: ____________ Period: ____________

Activity 1 The Formula for the Area of a Triangle

1. Label a base and corresponding height pair for each of the four triangles shown. **Sample responses shown.**

2. Complete the table by recording the following measurements of Triangles A, B, C, and D:
   - The length of the base you identified
   - The corresponding height
   - The area

Triangle	Base (units)	Height (units)	Area (square units)
A	10	7	35
B	6	11	33
C	10	3	15
D	4	11	22
Any triangle	b	h	$\frac{1}{2} \cdot b \cdot h$

3. Complete the last row of the table by writing an expression for the area of any triangle, using b for the length of the base and h for the corresponding height.

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Lesson 11 Formula for the Area of a Triangle 71

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by having them refer to Triangle A and asking, "Do you know the length of any sides that could be the base?"

Look for points of confusion:

- **Choosing a diagonal base.** Validate that any side can be the base, but only whole-number lengths of horizontal or vertical sides are readily identifiable using the grid.
- **Struggling to determine the height of Triangle C.** Remind students that the height can be drawn outside of the triangle.
- **Not knowing how to arrive at an expression for Problem 3.** Suggest that students label the base and height of one of the triangles with b and h, and then proceed to show how they could determine its area.

Look for productive strategies:

- Using area strategies flexibly, based on the shape of each triangle, to determine areas of related parallelograms.
- Recognizing that the lengths of the base and height are the same as those of a related parallelogram and the area is half the product of the base and height.

3 Connect

Have students share the base-height pairs they identified for the triangles in Problem 1 and how they used those to determine the measures in Problem 2. Have students explain how they determined an expression for Problem 3 using either visual or numerical information to identify a pattern.

Highlight and capture in an anchor chart the connection between a parallelogram composed of two identical triangles and the expressions for the area of a parallelogram, $b \cdot h$, and the area of a triangle, $\frac{1}{2} \cdot b \cdot h$.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students start with Triangle B. Demonstrate how Triangle A could be decomposed into two right triangles, and have students determine the sum of the areas of the two right triangles. Ask them to mimic this strategy for Triangle D.

Extension: *Math Enrichment*

Have students refer to Triangle B. Ask them to draw another right triangle that has twice the area of Triangle B and state the lengths of its base and height. Have them draw another right triangle that has half the area of Triangle B.

Math Language Development

MLR2: Collect and Display

Collect and display language around parallelograms composed of two identical triangles and the corresponding expressions for finding the area of a parallelogram. Add this language to the class anchor chart and encourage students to refer to it during discussions.

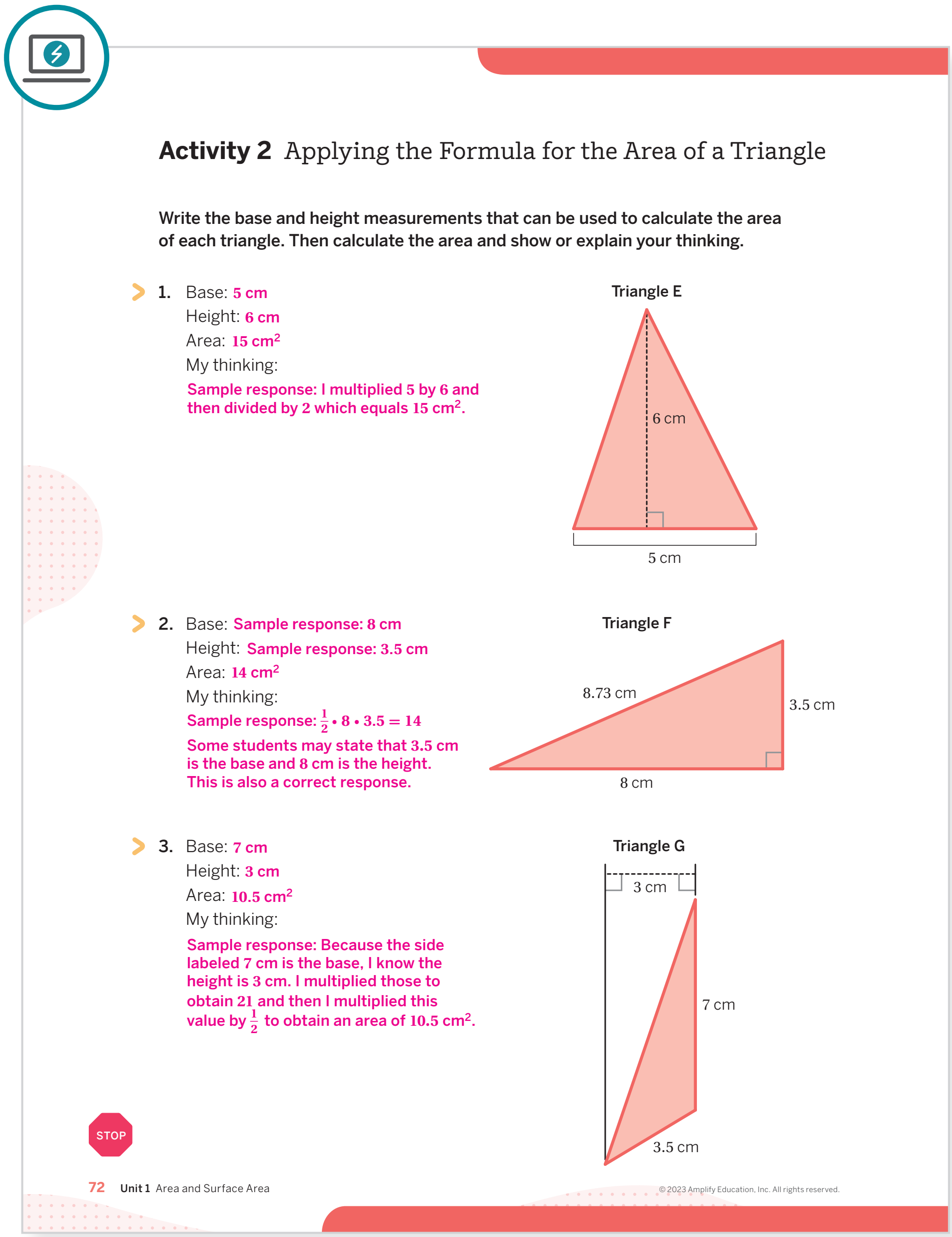
English Learners

Use a visual display to highlight the connections and differences between the variable expressions for the area of a parallelogram and the area of a triangle.

Pairs | 10 min

Activity 2 Applying the Formula for the Area of a Triangle

Students practice using the area formula for triangles, $A = \frac{1}{2} \cdot b \cdot h$, to determine the areas of triangles that are not on a grid.



Activity 2 Applying the Formula for the Area of a Triangle

Write the base and height measurements that can be used to calculate the area of each triangle. Then calculate the area and show or explain your thinking.

1. Base: **5 cm**
Height: **6 cm**
Area: **15 cm^2**
My thinking:
Sample response: I multiplied 5 by 6 and then divided by 2 which equals 15 cm^2.

2. Base: **Sample response: 8 cm**
Height: **Sample response: 3.5 cm**
Area: **14 cm^2**
My thinking:
Sample response: $\frac{1}{2} \cdot 8 \cdot 3.5 = 14$
Some students may state that 3.5 cm is the base and 8 cm is the height. This is also a correct response.

3. Base: **7 cm**
Height: **3 cm**
Area: **10.5 cm^2**
My thinking:
Sample response: Because the side labeled 7 cm is the base, I know the height is 3 cm. I multiplied those to obtain 21 and then I multiplied this value by $\frac{1}{2}$ to obtain an area of 10.5 cm^2.

72 **Unit 1** Area and Surface Area

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking them to first identify the measures for the base and height in each triangle, and then have them record the area formula to proceed.

Look for points of confusion:

- **Not choosing an appropriate base, especially for Triangles F and G.** Ask, "What should a height look like if your chosen side is the base? Do you know the length of this height? Is there another side you could use as a base that would make the height more readily identifiable?"

Look for productive strategies:

- Identifying the most strategic base-height pairs to determine necessary measures.
- Using the formula to determine the area.
- Stating the appropriate units (cm^2).

3 Connect

Have students share how they determined which given measures to use as the base and height for each triangle to calculate its area, noting both possibilities for Triangle F.

Highlight that area can still be calculated when triangles are not on a grid, as long as base and height are known.

Ask:

- "For Triangle E, could you use the 6 cm segment as the base and the 5 cm side as the height? Why or why not?" No, because the base has to be one of the sides of the triangle.
- "Could the area of Triangle E be determined as the sum of the areas of two smaller right triangles? What would you need to know?" Yes, but you would need to know the lengths of each part of the base, which may be 2 cm and 3 cm, but that is not known.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

With the formula for the area of a triangle displayed for students to see, walk them through these steps for one or more triangles:

- Identify a possible height.
- Identify the corresponding base.
- Cross out unnecessary measures (optional).
- Substitute values into the formula for area and evaluate.

Extension: *Math Enrichment*

Using a ruler, have students draw one or more triangles with each side having different measurements (and at least one side having a decimal measurement). Have them identify a base-height pair, measuring again as needed, and then finally calculate the area. Students can also work with a partner, trading triangles with each other to each determine the base, height, and area.

Math Language Development

MLR2: Collect and Display

As students discuss how to identify base-height pairs, record and display common or important phrases, specifically focusing on how they make sense of the base and height of each triangle.

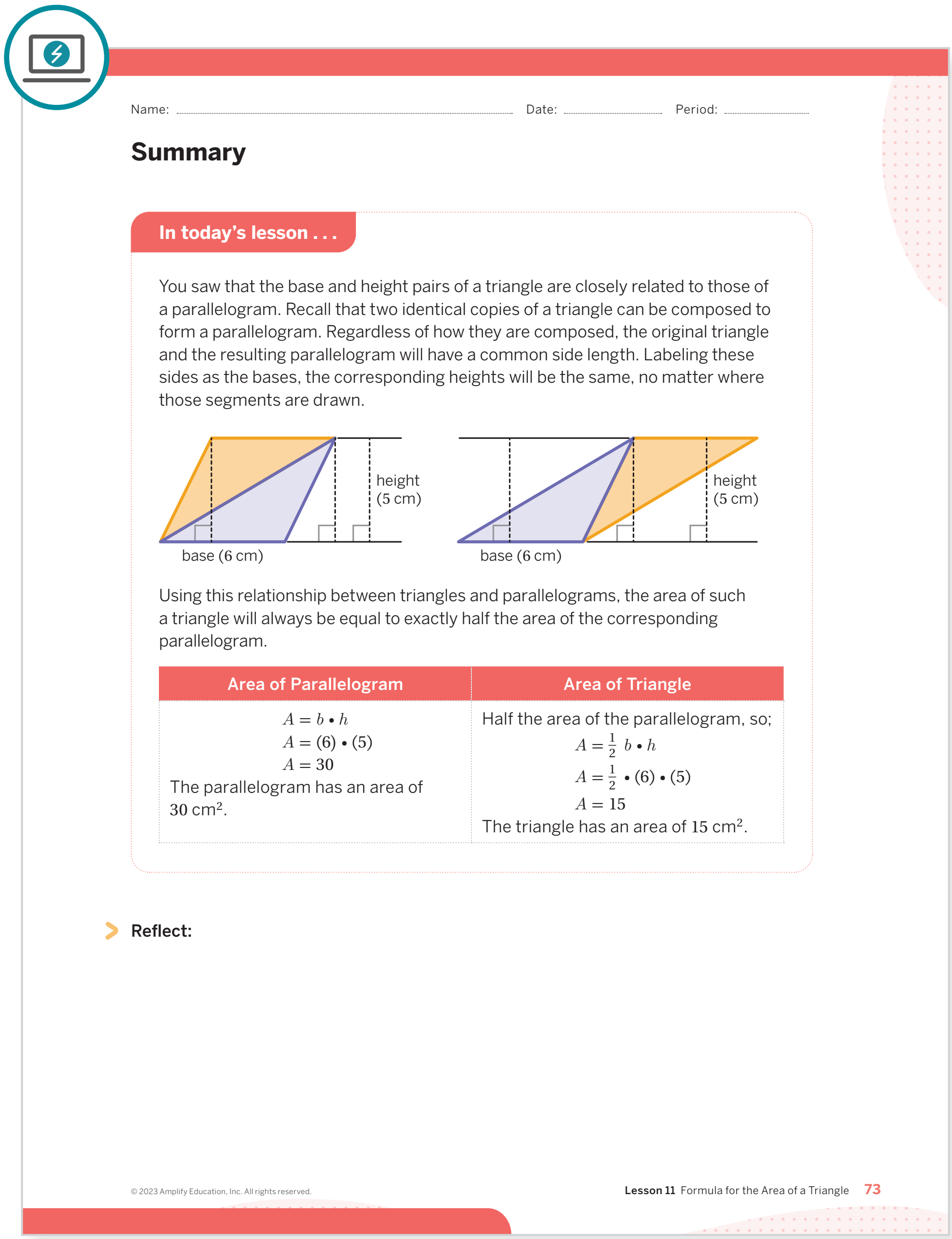
English Learners

Emphasize that *square units* refers to the two-dimensional calculation and not the actual shape with which students are working, in this case, a triangle.

Whole Class | 5 min

Summary

Review and synthesize how the area of a triangle is related to the area of a parallelogram.
Selecting base-height pairs for a triangle impacts determining the area of a triangle.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that the base and height pairs of a triangle are closely related to those of a parallelogram. Recall that two identical copies of a triangle can be composed to form a parallelogram. Regardless of how they are composed, the original triangle and the resulting parallelogram will have a common side length. Labeling these sides as the bases, the corresponding heights will be the same, no matter where those segments are drawn.

Using this relationship between triangles and parallelograms, the area of such a triangle will always be equal to exactly half the area of the corresponding parallelogram.

Area of Parallelogram	Area of Triangle
$A = b \bullet h$ $A = (6) \bullet (5)$ $A = 30$ The parallelogram has an area of 30 cm^2.	Half the area of the parallelogram, so; $A = \frac{1}{2}\ b \bullet h$ $A = \frac{1}{2} \bullet (6) \bullet (5)$ $A = 15$ The triangle has an area of 15 cm^2.

Reflect:

Synthesize

Highlight that similar to parallelograms, the area of triangles can be determined by using base and height measurements. Any side of a triangle can be chosen as the base, but unless students are able to determine necessary lengths (on a grid or using a ruler to physically measure), a base-height pair needs to be chosen where both lengths are known. The measurements of a base-height pair can be used to calculate the area of a triangle, which will always be half the area of a parallelogram with the same base and height measurements, and is represented by the formula $A = \frac{1}{2} \bullet b \bullet h$.

Ask:

- "Is it possible for *both* the base and the height to be sides of the triangle? If so, when?" Yes, when the triangle is a right triangle.
- "Using the formula for the area of a triangle and a drawing, can you show an example of how the sums of the areas of two identical triangles that form a parallelogram is equal to the area of the parallelogram?" **Note:** Students are not necessarily expected to show this algebraically, as $\frac{1}{2} \bullet b \bullet h + \frac{1}{2} \bullet b \bullet h = \left(\frac{1}{2} + \frac{1}{2}\right) \bullet b \bullet h = b \bullet h$, but should be able to qualitatively and quantitatively describe the process and relationships.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

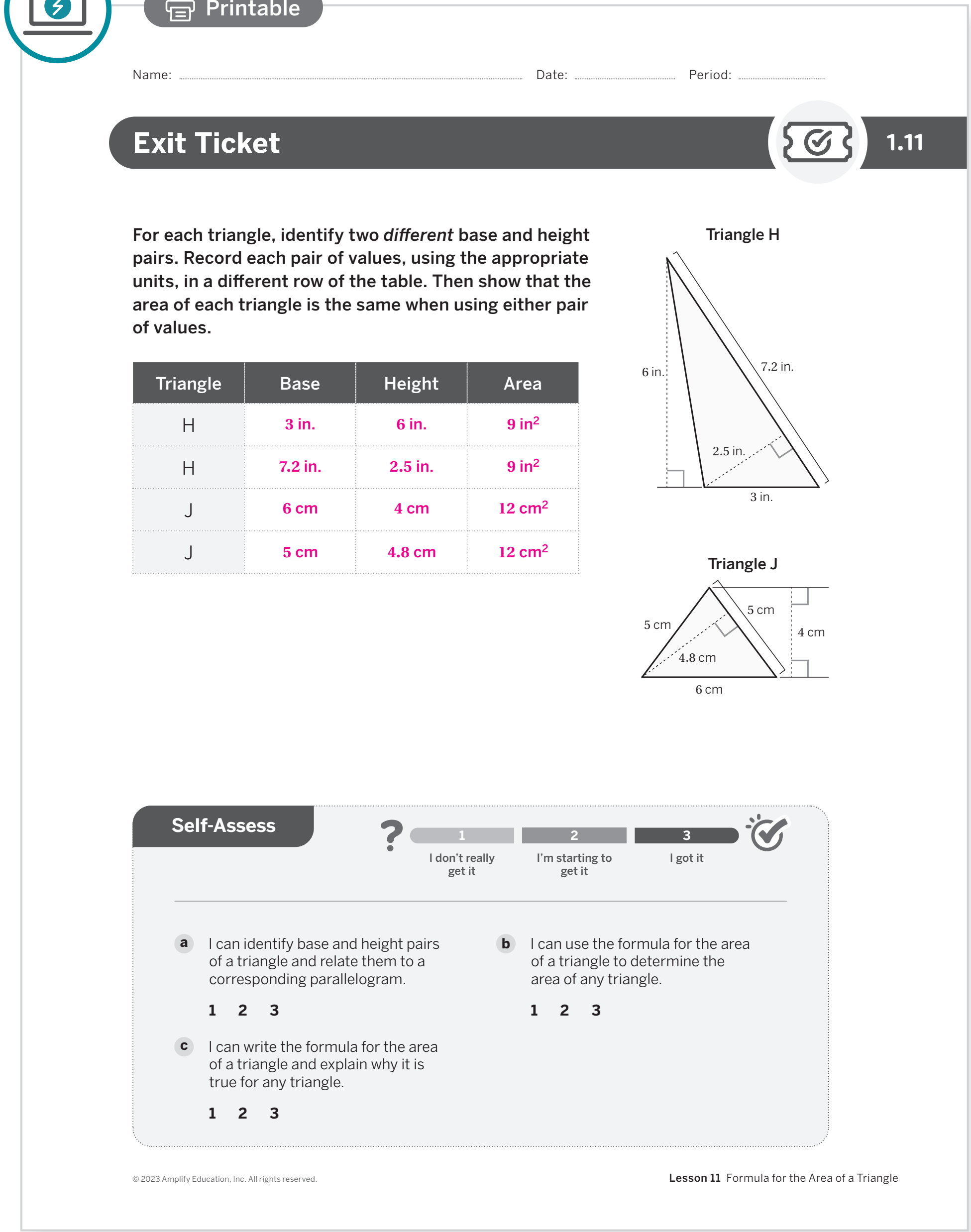
- "How is determining the base and height of a triangle similar or different to determining the base and height of a parallelogram?"
- "What is the formula of a triangle and how is it different/similar to the formula for a parallelogram? Why do you think that is?"

Independent | 10 min

Exit Ticket

Students demonstrate their understanding by strategically choosing base-height pairs of two triangles and then using those measurements to determine their areas.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.11

For each triangle, identify two *different* base and height pairs. Record each pair of values, using the appropriate units, in a different row of the table. Then show that the area of each triangle is the same when using either pair of values.

Triangle	Base	Height	Area
H	3 in.	6 in.	9 in^2
H	7.2 in.	2.5 in.	9 in^2
J	6 cm	4 cm	12 cm^2
J	5 cm	4.8 cm	12 cm^2

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can identify base and height pairs of a triangle and relate them to a corresponding parallelogram.
1 2 3

b I can use the formula for the area of a triangle to determine the area of any triangle.
1 2 3

c I can write the formula for the area of a triangle and explain why it is true for any triangle.
1 2 3

Lesson 11 Formula for the Area of a Triangle

Success looks like . . .

- **Language Goal:** Comparing, contrasting, and critiquing different strategies for determining the area of a triangle **(Speaking and Listening)**.
- **Language Goal:** Generalizing a process for determining the area of a triangle, and justifying why this can be abstracted **(Speaking and Listening)**.
- **Language Goal:** Evaluating the usefulness of different base-height pairs for determining the area of a given triangle **(Speaking and Listening)**.
  - » Identifying two different base-height pairs and determining the area for Triangles H and J.

Suggested next steps

If students have trouble matching the correct base and corresponding height, consider:

- Having students review Activity 2 and compare Triangle H to Triangle G and Triangle J to Triangle E.
- Suggesting students identify one base-height pair first and "eliminate" those values to simplify the options for determining a second pair.

If students are confused why they should calculate the area twice, consider:

- Reminding them there are multiple heights in a given triangle that correspond to the given base; therefore, multiple ways to calculate the area of a triangle. Calculating the area of a triangle by using different strategies should give the same result because the triangle and its area are the same. This serves as a way to check work and calculations.

Professional Learning

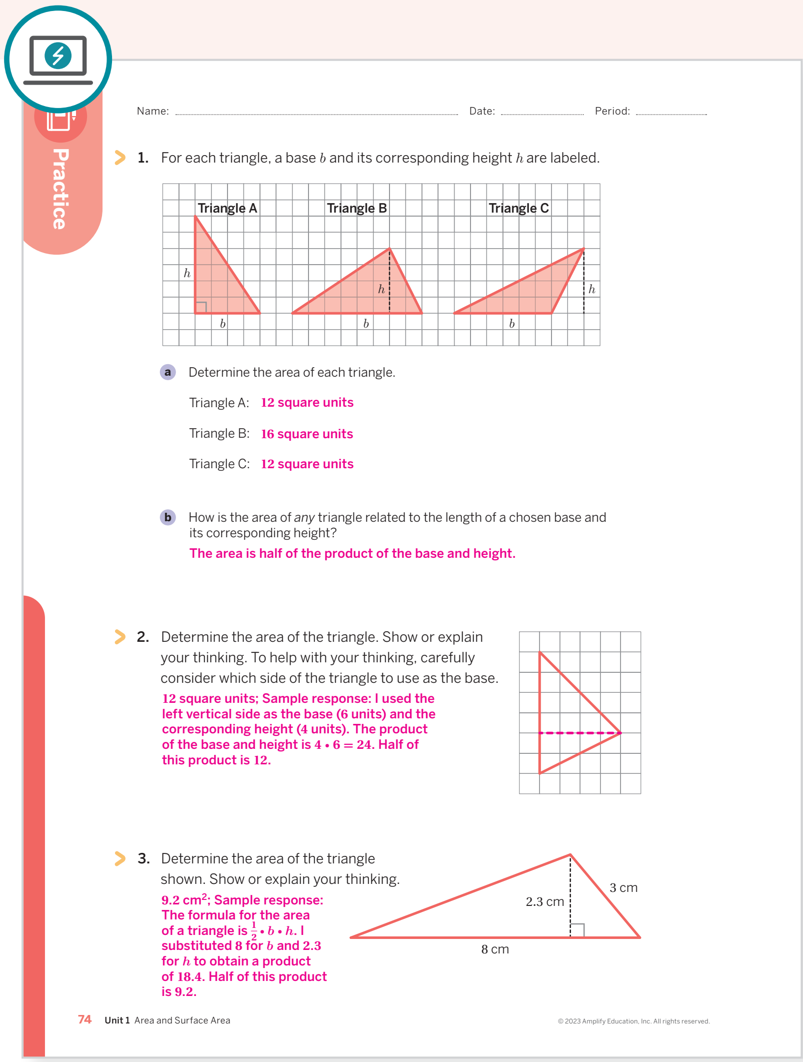
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did students work on their ability to reason about developing a formula for triangles? How are you helping students become aware of how they are progressing in this area?
- What different ways did students approach Activity 1? What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. For each triangle, a base b and its corresponding height h are labeled.

a. Determine the area of each triangle.

Triangle A: **12 square units**

Triangle B: **16 square units**

Triangle C: **12 square units**

b. How is the area of *any* triangle related to the length of a chosen base and its corresponding height?

The area is half of the product of the base and height.

2. Determine the area of the triangle. Show or explain your thinking. To help with your thinking, carefully consider which side of the triangle to use as the base.

12 square units; Sample response: I used the left vertical side as the base (6 units) and the corresponding height (4 units). The product of the base and height is $4 \cdot 6 = 24$. Half of this product is 12.

3. Determine the area of the triangle shown. Show or explain your thinking.

9.2 cm^2; Sample response: The formula for the area of a triangle is $\frac{1}{2} \cdot b \cdot h$. I substituted 8 for b and 2.3 for h to obtain a product of 18.4. Half of this product is 9.2.

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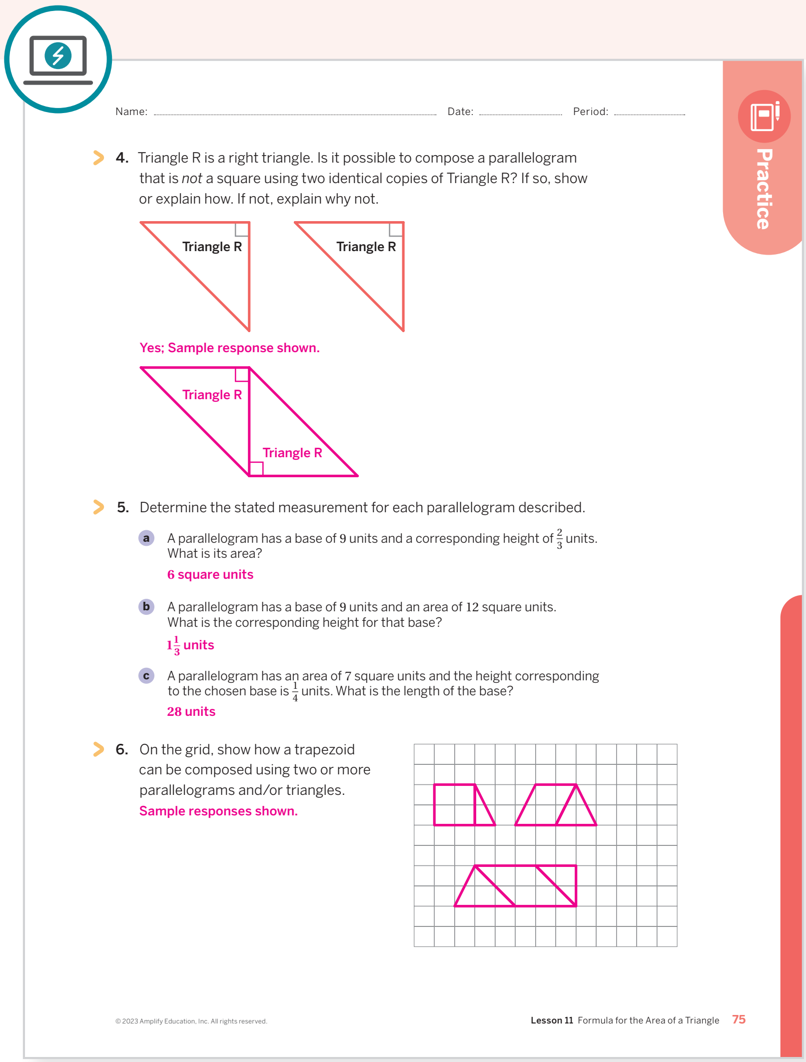

Name: Date: Period:

4. Triangle R is a right triangle. Is it possible to compose a parallelogram that is *not* a square using two identical copies of Triangle R? If so, show or explain how. If not, explain why not.

Yes; Sample response shown.

5. Determine the stated measurement for each parallelogram described.

a. A parallelogram has a base of 9 units and a corresponding height of $\frac{2}{3}$ units. What is its area?

6 square units

b. A parallelogram has a base of 9 units and an area of 12 square units. What is the corresponding height for that base?

$1\frac{1}{3}$ units

c. A parallelogram has an area of 7 square units and the height corresponding to the chosen base is $\frac{1}{4}$ units. What is the length of the base?

28 units

6. On the grid, show how a trapezoid can be composed using two or more parallelograms and/or triangles.

Sample responses shown.

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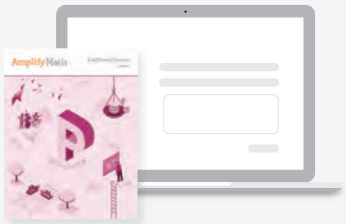
Lesson 11 Formula for the Area of a Triangle 75

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 9	2
	5	Unit 1 Lesson 8	2
Formative ⚡	6	Unit 1 Lesson 12	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

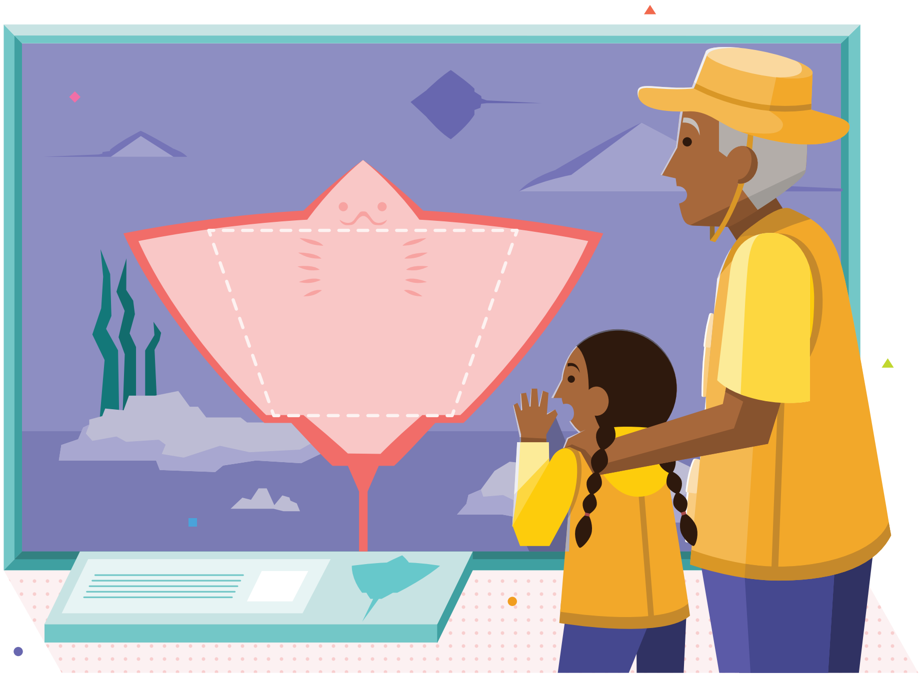


For students that need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 12**

From Triangles to Trapezoids

Let's apply what we know about triangles and parallelograms to trapezoids.



Focus

Goals

1. **Language Goal:** Generalize a process for decomposing a trapezoid into triangles and parallelograms. **(Speaking and Listening)**
2. Use decomposition and known areas of other shapes to determine the area of a trapezoid.

Coherence

Today

Students review the characteristics of the trapezoid, another special quadrilateral, and differentiate between what is a trapezoid and what is not a trapezoid. They explore different ways to decompose trapezoids into triangles and parallelograms, analyzing the features and structure of given shapes to inform the way they choose to decompose a figure. This allows students to see how they can determine the area of a trapezoid by using what they know about bases and heights of triangles and parallelograms. Students then determine the area of several different trapezoids. Some students may be able to write or describe a formula for the area of a trapezoid.

Previously

In Lessons 6–11, students developed and applied common strategies for reasoning about the areas of parallelograms and triangles, recognizing relationships between base and height measurements and area, which led to general formulas for the area of any parallelogram or triangle.

Coming Soon

In Lesson 13, students will continue their exploration of area as they discover that the area of any polygon can be determined by using strategic decompositions and applying area formulas based on given or known measures of a figure.

Rigor

- Students **apply** their understanding of how to determine the area of parallelograms and triangles to think about how to determine the area of a trapezoid.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, *Formula for the Area of a Trapezoid* (for display, optional)

Math Language Development

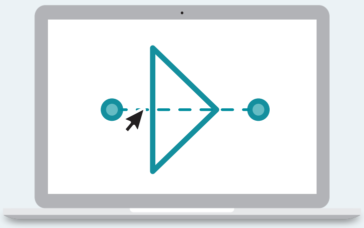
Review word

- *trapezoid*

Amps | Featured Activity

Activity 1
Interactive Geometry

Students can experiment with drawing different partitioning lines to decompose trapezoids and can modify their partitions without having to erase.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students may find it difficult to come up with examples of different types of decompositions of trapezoids because they are only seeing the shapes one particular way or they find the gridlines distracting rather than helpful. Remind them of previous successes with decomposition strategies for parallelograms and triangles, both on and off grids, and have them look back at previous lessons or reflect on their previous work to motivate new and different ways of analyzing the structure of a given figure.

Modifications to Pacing

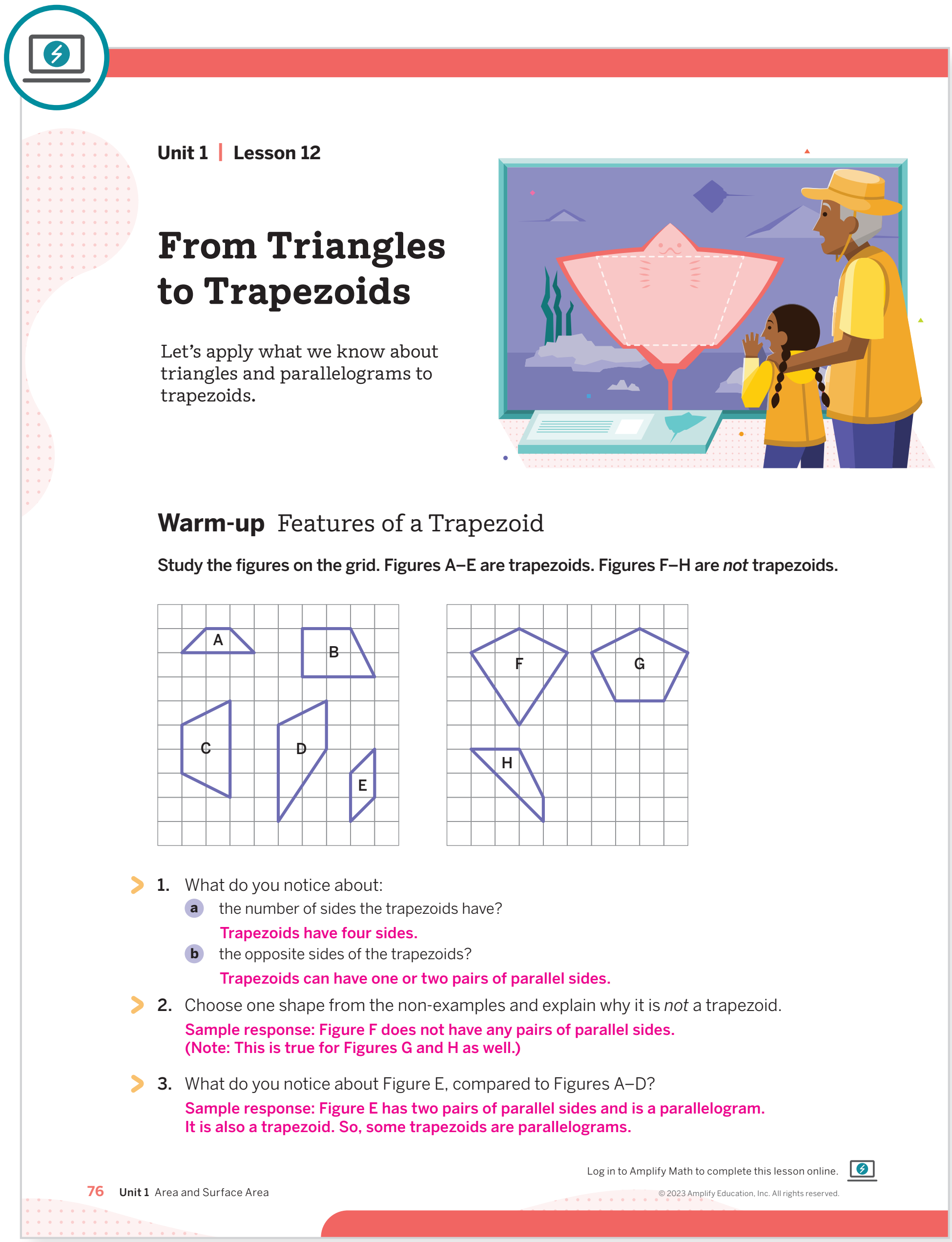
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, assign each pair one trapezoid and have them identify one possible decomposition from the list that can be created. Or consider having them identify all possible decompositions from the list for their assigned shape.
- In **Activity 2**, assign each pair two trapezoids and complete the table by sharing and validating results as a class.

Pairs | 5 min

Warm-up Features of a Trapezoid

Students review the defining features of trapezoids by comparing and contrasting given examples and non-examples.



Unit 1 | Lesson 12

From Triangles to Trapezoids

Let's apply what we know about triangles and parallelograms to trapezoids.

Warm-up Features of a Trapezoid

Study the figures on the grid. Figures A–E are trapezoids. Figures F–H are *not* trapezoids.

1. What do you notice about:
   - a the number of sides the trapezoids have?
     Trapezoids have four sides.
   - b the opposite sides of the trapezoids?
     Trapezoids can have one or two pairs of parallel sides.
2. Choose one shape from the non-examples and explain why it is *not* a trapezoid.
   Sample response: Figure F does not have any pairs of parallel sides. (Note: This is true for Figures G and H as well.)
3. What do you notice about Figure E, compared to Figures A–D?
   Sample response: Figure E has two pairs of parallel sides and is a parallelogram. It is also a trapezoid. So, some trapezoids are parallelograms.

Log in to Amplify Math to complete this lesson online.

76 Unit 1 Area and Surface Area

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1 Launch

Have students use the *Think-Pair-Share* routine for Problem 1, and then complete Problem 2 together.

2 Monitor

Help students get started by asking, "What is one thing that Figures A–E all have in common? Do any of Figures F–H also have that feature? If not, how are they different?"

Look for points of confusion:

- **Generating statements that are true for some, but not all trapezoids.** Remind students their statements must be true for *all* trapezoids. Ask, "Which example is different? How can you revise your statement?"
- **Struggling to understand why H is an example.** Ask students to describe what Figures A–D all have in common (*exactly one* pair of parallel sides). Then ask them to describe what Figures A–E all have in common (*at least one* pair of parallel sides).

Look for productive strategies:

- Recognizing and explaining the characteristics of trapezoids using the examples and non-examples.
- Referencing an attribute from Problem 1 to identify the rationale for a non-example in Problem 2.

3 Connect

Have students share their responses, citing both examples and non-examples. Invite other students to agree or disagree and explain their thinking.

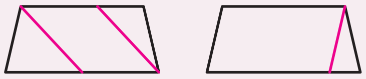
Highlight that a ***trapezoid*** is a special type of quadrilateral that has *at least one* pair of parallel sides, called the *bases* of the trapezoid. The *height* is the distance between the two bases.

Ask, "How could you adjust one vertex of Figure G so that it is a trapezoid?"

Power-up

To power up students' ability to relate composition and decomposition of shapes, have students complete:

Recall that a trapezoid can be decomposed into other familiar shapes. Draw lines to decompose the trapezoids shown into triangles and parallelograms, each in a different way.



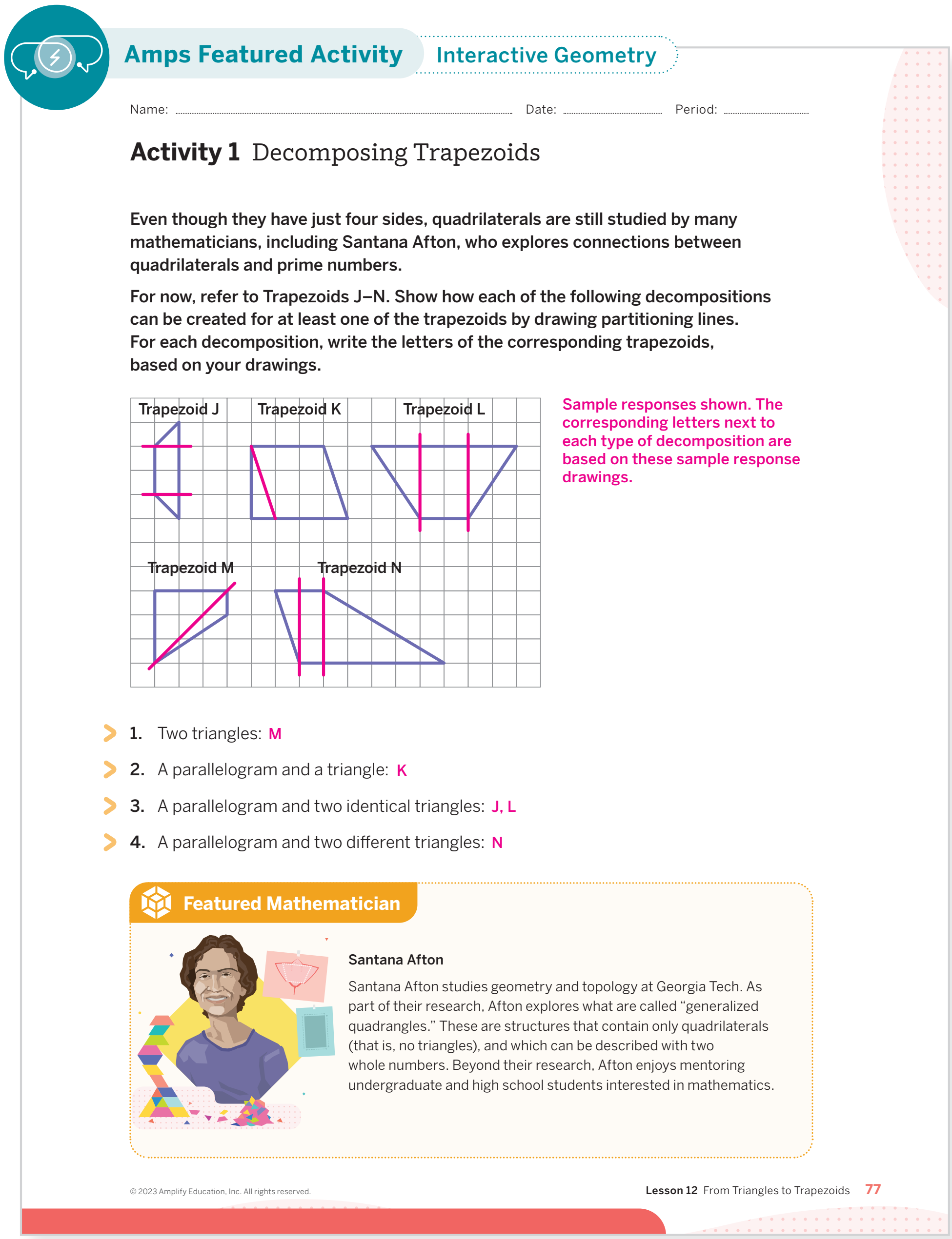
Use: Before Activity 1.

Informed by: Performance on Lesson 11, Practice Problem 6.

Activity 1 Decomposing Trapezoids

Students explore different ways to decompose trapezoids into parallelograms and triangles.

Amps Featured Activity Interactive Geometry



Name: Date: Period:

Activity 1 Decomposing Trapezoids

Even though they have just four sides, quadrilaterals are still studied by many mathematicians, including Santana Afton, who explores connections between quadrilaterals and prime numbers.

For now, refer to Trapezoids J–N. Show how each of the following decompositions can be created for at least one of the trapezoids by drawing partitioning lines. For each decomposition, write the letters of the corresponding trapezoids, based on your drawings.

Trapezoid J
Trapezoid K
Trapezoid L
Trapezoid M
Trapezoid N

Sample responses shown. The corresponding letters next to each type of decomposition are based on these sample response drawings.

1. Two triangles: M
2. A parallelogram and a triangle: K
3. A parallelogram and two identical triangles: J, L
4. A parallelogram and two different triangles: N

Featured Mathematician

Santana Afton

Santana Afton studies geometry and topology at Georgia Tech. As part of their research, Afton explores what are called "generalized quadrangles." These are structures that contain only quadrilaterals (that is, no triangles), and which can be described with two whole numbers. Beyond their research, Afton enjoys mentoring undergraduate and high school students interested in mathematics.

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Lesson 12 From Triangles to Trapezoids 77

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What would happen if you decomposed one of the trapezoids by partitioning it with a segment connecting two vertices?"

Look for points of confusion:

- **Struggling to determine how to partition the trapezoids.** Suggest that they focus on existing sides and first try to "see" a familiar smaller shape within. Ask, "Can you draw one segment from a vertex to another side to create a triangle or parallelogram?"

Look for productive strategies:

- Recognizing that two triangles can be formed by connecting opposite vertices in any trapezoid.
- Using grid lines as partitioning lines that reveal decompositions into triangles and parallelograms.
- Recognizing that there are multiple ways to decompose a given trapezoid into parallelograms and triangles and the shape and orientation of the trapezoid helps to reveal some possibilities.

3 Connect

Ask:

- "Is there always more than one way to decompose a given trapezoid?" Yes.
- "Why do you think all of the given decompositions for this activity included only triangles and parallelograms?" The areas of triangles and parallelograms are readily able to be determined.

Highlight that not all of the given decompositions in this activity are possible, or the most efficient, for a given trapezoid. However, every type of trapezoid can be decomposed into two or more triangles, parallelograms, or both.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them work with Trapezoids J, K, and L. Alternatively, have them choose one type of decomposition at a time and work through all of the shapes trying to apply it (It is okay if they do not have a trapezoid matched to each decomposition).

Math Language Development

MLR2: Collect and Display

Listen to students talking about how and where to draw partitioning lines. Record and display common or useful phrases, as well as examples of their drawings. Encourage students to borrow language from the display as needed.

Featured Mathematician

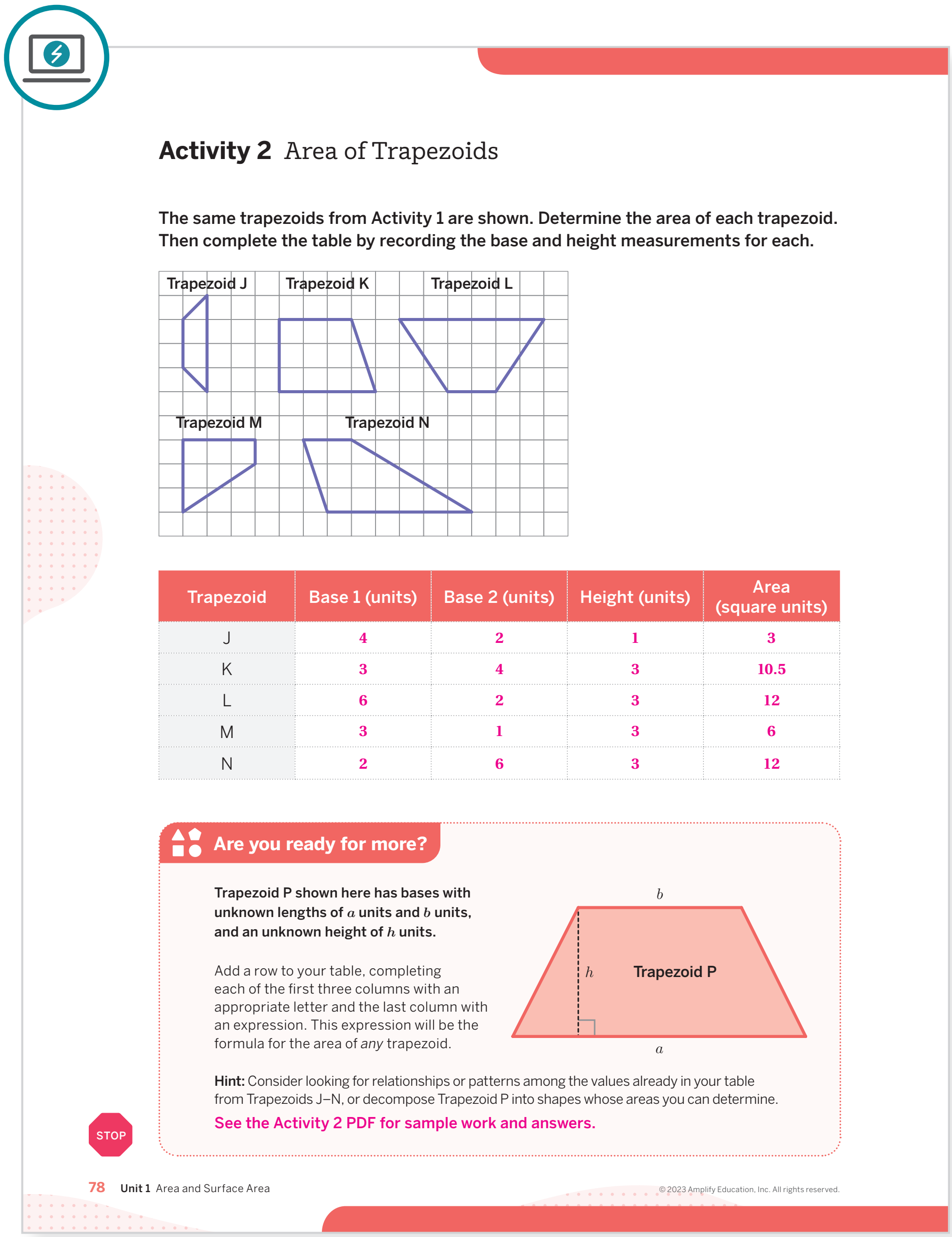
Santana Afton

Have students read about Featured Mathematician Santana Afton, who studies what are called "generalized quadrangles," an advanced mathematical concept that is closely related to the quadrilaterals students explore in this unit.

Pairs | 20 min

Activity 2 Area of Trapezoids

Students apply the decomposition strategies from Activity 1 to determine the areas of several trapezoids. Recording relevant measures in a table allows patterns to be seen.



Activity 2 Area of Trapezoids

The same trapezoids from Activity 1 are shown. Determine the area of each trapezoid. Then complete the table by recording the base and height measurements for each.

Trapezoid	Base 1 (units)	Base 2 (units)	Height (units)	Area (square units)
J	4	2	1	3
K	3	4	3	10.5
L	6	2	3	12
M	3	1	3	6
N	2	6	3	12

Are you ready for more?

Trapezoid P shown here has bases with unknown lengths of a units and b units, and an unknown height of h units.

Add a row to your table, completing each of the first three columns with an appropriate letter and the last column with an expression. This expression will be the formula for the area of *any* trapezoid.

Hint: Consider looking for relationships or patterns among the values already in your table from Trapezoids J–N, or decompose Trapezoid P into shapes whose areas you can determine.

See the Activity 2 PDF for sample work and answers.

78 **Unit 1** Area and Surface Area

1 Launch

Remind students how to identify the two bases and height of a trapezoid, from the Warm-up.

2 Monitor

Help students get started by having students look back at their decomposition of Triangle J from Activity 1 and asking, "Can you determine the areas of those smaller shapes? If not, could you decompose it differently?"

Look for points of confusion:

- **Struggling to identify the two bases in each trapezoid.** Ask them to use the grid lines to identify sides that are parallel, and then say, "Either side can be base 1 or base 2."
- **Thinking the area cannot be determined because their previous decomposition is missing critical measures.** Suggest students use the grid to identify as many measurements as they can, and if needed, they can try a different decomposition.

Look for productive strategies:

- Recognizing that the lengths of the two bases and the height are shared measures with many of the decomposed triangles and parallelograms.
- Determining the base and height measures appropriately, calculating the area of the trapezoids, and perhaps discovering the formula.

3 Connect

Display the trapezoids and the blank table.

Have students share their values for each trapezoid and how they determined the area.

Highlight that the area of any trapezoid can be determined using decomposition strategies, as long as the base and height are known. Just like for parallelograms and triangles, there is also a connection between the lengths of the bases and height of a trapezoid that can be represented by a formula for area. You may go into further discussion by using the Activity 2 PDF, *Formula for the Area of a Trapezoid*.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them work with Trapezoids J, K, and L. If they did not have decompositions in Activity 1 that relied solely on grid lines, help them to establish these as good starting points for seeing the areas that need to be determined.

Accessibility: *Guide Processing and Visualization*

Have students use color and annotations to label the bases and heights of the trapezoids.

Extension: *Math Enrichment*

Provide students with copies of the Activity 2 PDF, *Formula for the Area of a Trapezoid*. Have them complete the activity, which leads them to the formula for the area of a trapezoid. **Note:** The *Are you Ready for More?* activity in the Student Edition also leads students to derive this formula.

Math Language Development

MLR8: Discussion Supports–Restate It!

As students share their strategies during the Connect, have them restate each others' reasoning using the terms *base* and *height*. Encourage students to challenge each other when they disagree, using prompts such as "I agree because . . . " or "I disagree because . . ."

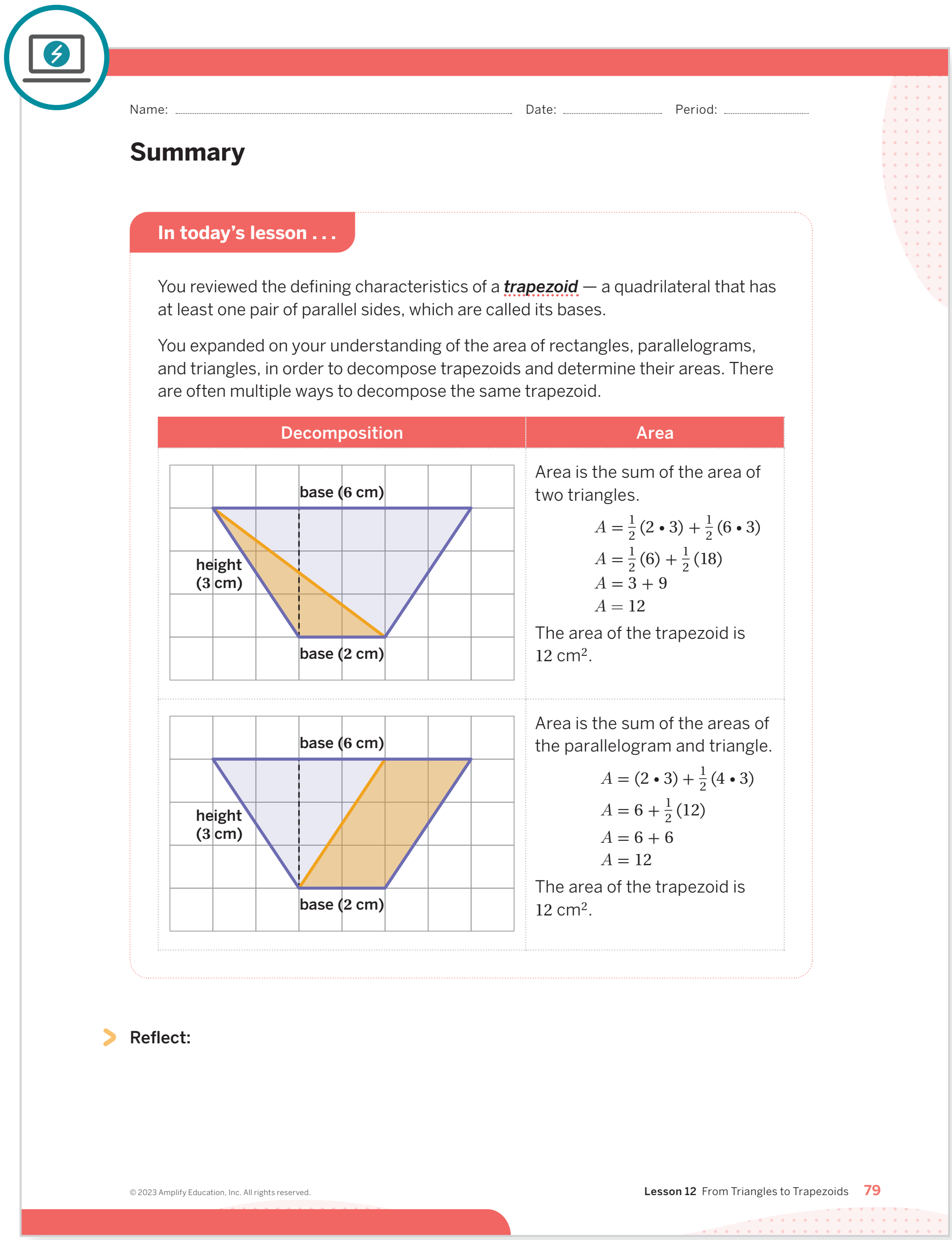
English Learners

Provide a word bank of vocabulary students can use, such as *decompose, base, height, parallel,* etc.

Whole Class | 5 min

Summary

Review and synthesize how a trapezoid can be decomposed into parallelograms and triangles to determine its area.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You reviewed the defining characteristics of a ***trapezoid*** — a quadrilateral that has at least one pair of parallel sides, which are called its bases.

You expanded on your understanding of the area of rectangles, parallelograms, and triangles, in order to decompose trapezoids and determine their areas. There are often multiple ways to decompose the same trapezoid.

Decomposition	Area
base (6 cm) height (3 cm) base (2 cm)	Area is the sum of the area of two triangles. $A = \frac{1}{2}(2 \bullet 3) + \frac{1}{2}(6 \bullet 3)$ $A = \frac{1}{2}(6) + \frac{1}{2}(18)$ $A = 3 + 9$ $A = 12$ The area of the trapezoid is 12 cm².
base (6 cm) height (3 cm) base (2 cm)	Area is the sum of the areas of the parallelogram and triangle. $A = (2 \bullet 3) + \frac{1}{2}(4 \bullet 3)$ $A = 6 + \frac{1}{2}(12)$ $A = 6 + 6$ $A = 12$ The area of the trapezoid is 12 cm².

Reflect:

Synthesize

Highlight that the area of a trapezoid can be decomposed into shapes whose areas can be determined (namely, parallelograms and triangles) and the sum of those areas is equal to the area of a trapezoid. There are many ways to decompose the same trapezoid, and depending on given information some may be more efficient, but the area will always be the same.

Ask:

- "How would you determine the area of a trapezoid *not* drawn on a grid? What information would need to be given or known?" I would still decompose it into triangles and parallelograms, but I would need to be able to determine their areas, so the given measurements might influence how I need to decompose the trapezoid.
- "How could this same thinking and use of decomposition strategies be applied to other polygons?" I could try to draw line segments from the vertices and sides of a polygon to decompose it into triangles and parallelograms, because if I can determine those areas, then the area of the polygon is the sum of all of those individual areas.

Have students share their ideas and responses to these questions, allowing them to draw figures to support their thinking, as time allows.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How does knowing how to determine the area of a parallelogram and triangle help you in determining the area of a trapezoid?"
- "Are there certain strategies that are more or less efficient when determining the area of a trapezoid?"

Differentiated Support

Extension: *Math Around the World, Interdisciplinary Connections*

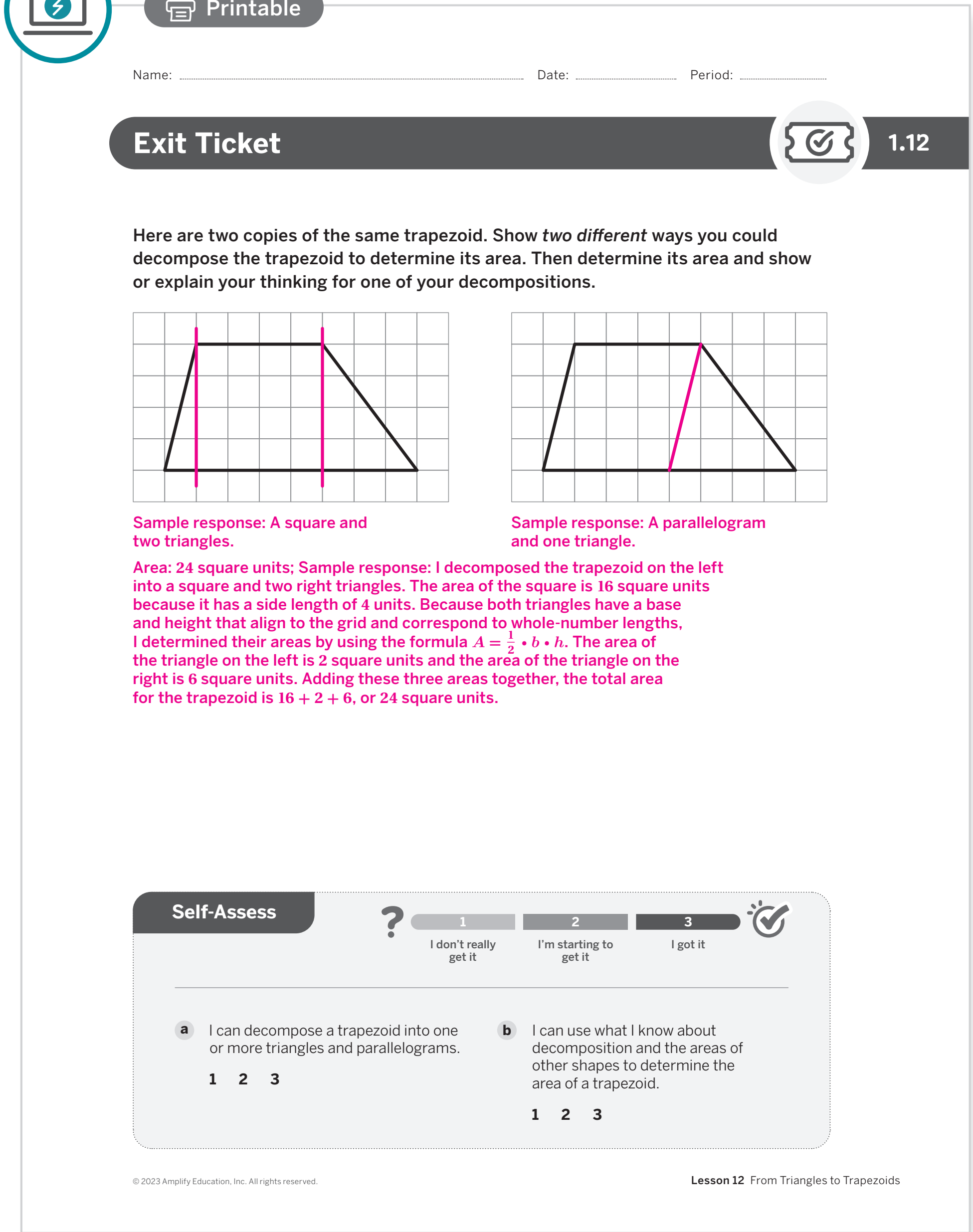
Preview the online article "Signs of Modern Astronomy Seen in Ancient Babylon" from the New York Times, January 28, 2016, that describes how the ancient Babylonian mathematicians used the area of a trapezoid to track the path of Jupiter. Read the article with your students and facilitate a class discussion about how ancient Babylonian mathematicians used advanced mathematics, today known as precalculus, to track the planet. Emphasize that, until this was discovered, this type of mathematical knowledge was only credited to have been used by Europeans almost 15 centuries later. **(History, Science)**

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how the area of a trapezoid will be the same, regardless of how it is decomposed.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.12

Here are two copies of the same trapezoid. Show *two different* ways you could decompose the trapezoid to determine its area. Then determine its area and show or explain your thinking for one of your decompositions.

Sample response: A square and two triangles.

Sample response: A parallelogram and one triangle.

Area: 24 square units; Sample response: I decomposed the trapezoid on the left into a square and two right triangles. The area of the square is 16 square units because it has a side length of 4 units. Because both triangles have a base and height that align to the grid and correspond to whole-number lengths, I determined their areas by using the formula $A = \frac{1}{2} \cdot b \cdot h$. The area of the triangle on the left is 2 square units and the area of the triangle on the right is 6 square units. Adding these three areas together, the total area for the trapezoid is $16 + 2 + 6$, or 24 square units.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can decompose a trapezoid into one or more triangles and parallelograms.

1 2 3

b I can use what I know about decomposition and the areas of other shapes to determine the area of a trapezoid.

1 2 3

Lesson 12 From Triangles to Trapezoids

Success looks like . . .

- **Language Goal:** Generalizing a process for decomposing a trapezoid into triangles and parallelograms. **(Speaking and Listening)**
    - » Decomposing the trapezoid in two different ways to determine its area.
- **Goal:** Using decomposition and known areas of other shapes to determine the area of a trapezoid.
    - » Explaining how one of their chosen decompositions can be used to determine the area of the trapezoid.

Suggested next steps

If students have difficulty decomposing the trapezoid two different ways, consider:

- Reviewing the different possible decomposition statements from Activity 1 and focusing on Trapezoid L as a similar but slightly different trapezoid.

If students are unable to determine the area of the trapezoid, consider:

- Reviewing Activity 2, and reminding them that they know how to determine the area of parallelograms and triangles.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How was Activity 1 similar to or different from decomposing parallelograms or triangles in earlier lessons?
- Have you changed any ideas you used to have about determining the area of a trapezoid as a result of today's lesson? What might you change for the next time you teach this lesson?

MLR Math Language Development

Language Goal: Generalizing a process for decomposing a trapezoid into triangles and parallelograms.

Reflect on students' language development toward this goal.

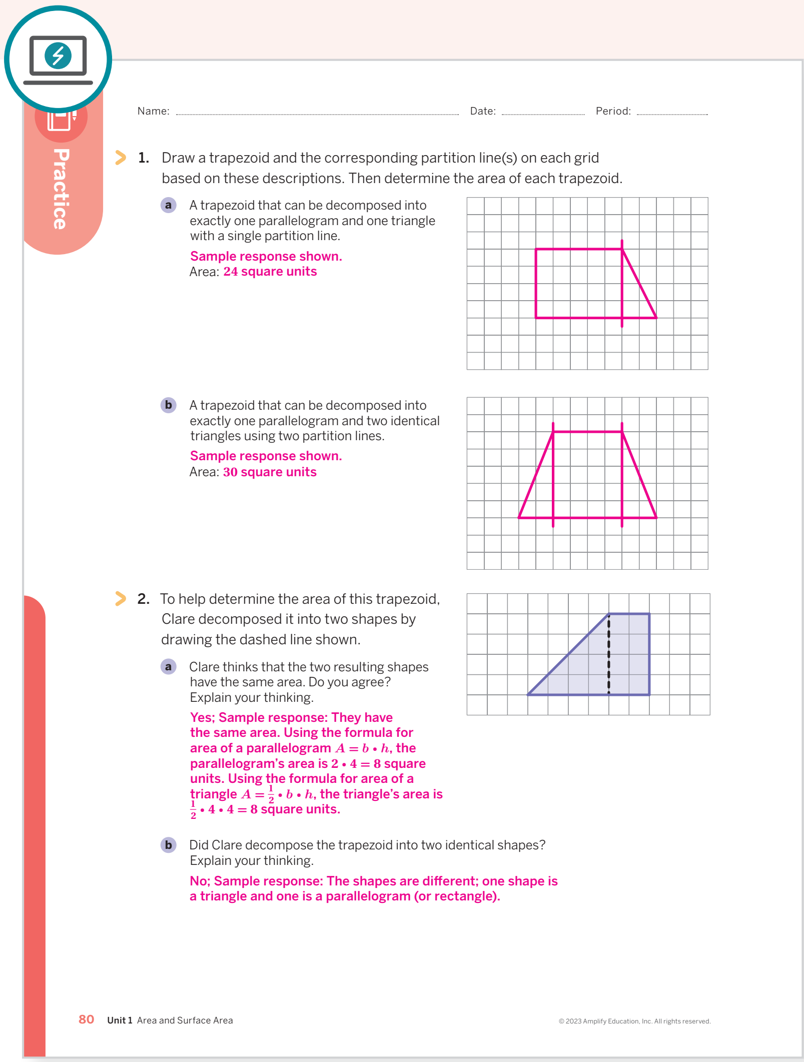
- Earlier in this unit, how did students begin to describe how they can decompose figures into known shapes?
- What is an example of a developing description and how can you help students be more precise in their descriptions?

Sample descriptions:

Emerging	Expanding
Use triangles and squares.	Decompose the trapezoid into a square and two right triangles.

Practice

Independent



Practice

Name: Date: Period:

1. Draw a trapezoid and the corresponding partition line(s) on each grid based on these descriptions. Then determine the area of each trapezoid.
   - **a** A trapezoid that can be decomposed into exactly one parallelogram and one triangle with a single partition line.
     Sample response shown.
     Area: **24 square units**
   - **b** A trapezoid that can be decomposed into exactly one parallelogram and two identical triangles using two partition lines.
     Sample response shown.
     Area: **30 square units**
2. To help determine the area of this trapezoid, Clare decomposed it into two shapes by drawing the dashed line shown.
   - **a** Clare thinks that the two resulting shapes have the same area. Do you agree? Explain your thinking.
     Yes; Sample response: They have the same area. Using the formula for area of a parallelogram $A = b \cdot h$, the parallelogram's area is $2 \cdot 4 = 8$ square units. Using the formula for area of a triangle $A = \frac{1}{2} \cdot b \cdot h$, the triangle's area is $\frac{1}{2} \cdot 4 \cdot 4 = 8$ square units.
   - **b** Did Clare decompose the trapezoid into two identical shapes? Explain your thinking.
     No; Sample response: The shapes are different; one shape is a triangle and one is a parallelogram (or rectangle).

80 **Unit 1** Area and Surface Area

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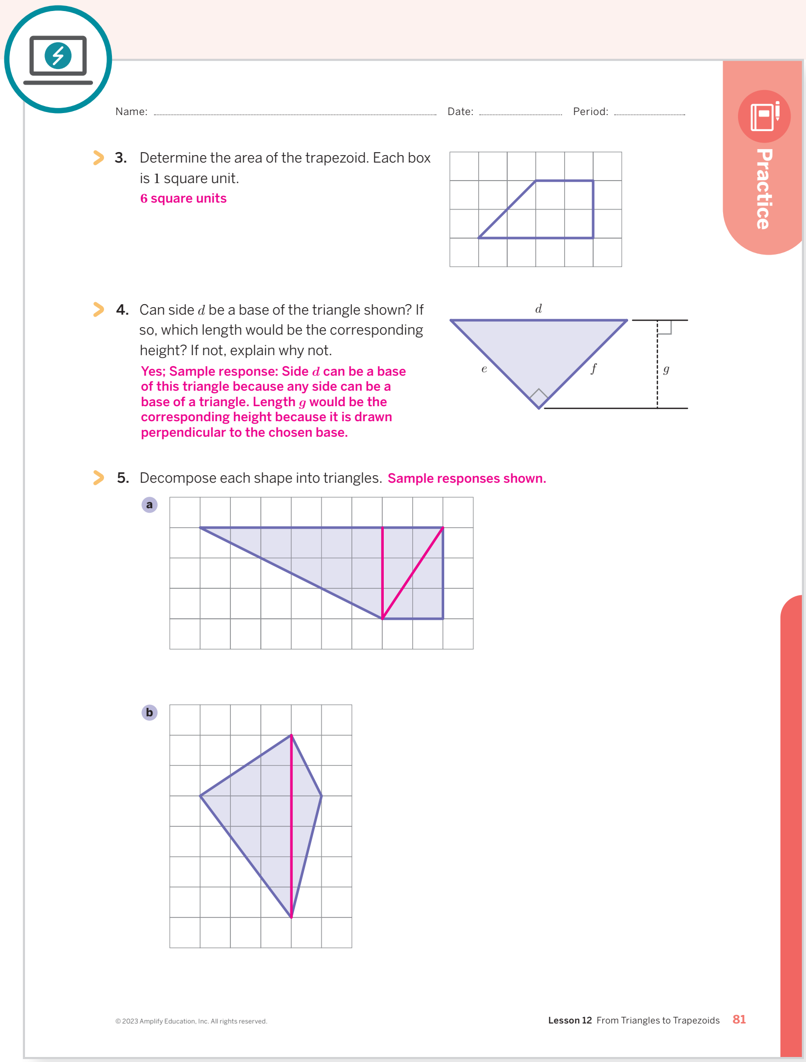

Practice

Name: Date: Period:

3. Determine the area of the trapezoid. Each box is 1 square unit.
   6 square units
4. Can side d be a base of the triangle shown? If so, which length would be the corresponding height? If not, explain why not.
   Yes; Sample response: Side d can be a base of this triangle because any side can be a base of a triangle. Length g would be the corresponding height because it is drawn perpendicular to the chosen base.
5. Decompose each shape into triangles. **Sample responses shown.**
   - **a**
   - **b**

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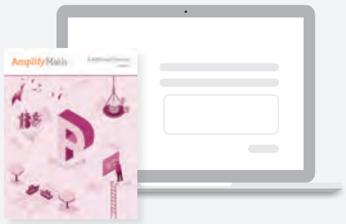
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activities 1 and 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 10	2
Formative	5	Unit 1 Lesson 13	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

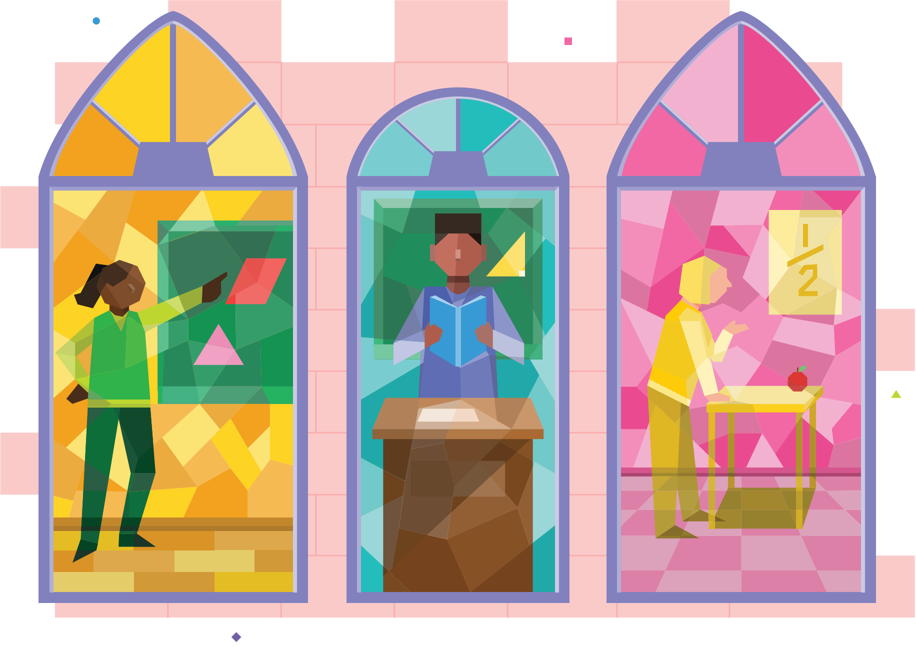


For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 13**

Polygons

Let's investigate polygons and their areas.



Focus

Goals

1. **Language Goal:** Compare and contrast different strategies for determining the area of a polygon. **(Speaking and Listening)**
2. **Language Goal:** Describe the defining characteristics of polygons. **(Speaking and Listening, Writing)**
3. Determine the area of a polygon, by decomposing it into rectangles and triangles, and present the solution method (using multiple representations).

Coherence

Today

Students co-construct a definition for the term *polygon* based on generalizations of similar characteristics observed in given examples and non-examples. They combine their understanding of polygons and strategies for calculating areas of special polygons to design a "stained glass" mosaic. Students choose to work with either centimeters or inches to calculate the areas.

Previously

In Lessons 6–11, students determined the formulas for the area of parallelograms and triangles. In Lesson 12, they applied both decomposition strategies and area formulas to determine the area of trapezoids.

Coming Soon

In Lessons 14–19, students will shift from two-dimensional polygons to three-dimensional polyhedra, focusing on surface area and nets. The strategies and formulas for area from the first half of the unit will be used as they calculate the areas of the polygon faces of the polyhedra.

Rigor

- Students strengthen their **fluency** in determining area by creating and decomposing polygons.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	25 min	5 min	5 min
Independent	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-Up PDF (for display)
- Activity 1 PDF, one per student
- transparencies, one per student

Math Language Development

New word

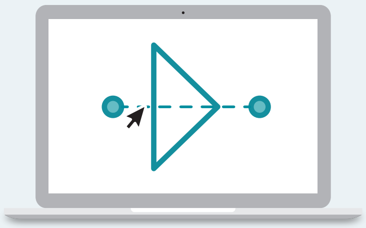
- *polygon*

Amps | Featured Activity

Activity 1

Interactive Geometry

Students create a stained glass design and use digital tools to measure the area of the polygons within it. You will be able to see their reasoning in real-time, to interact, or intervene when necessary.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students may be worried about the openness of creating a design in Activity 1 because limited guidelines for their design were given. Encourage students to use the examples provided and have them start with drawing a single line on their paper. Then have them draw another line to intersect it and point out the polygons that were formed as a result. They can continue to "grow" their design by adding more lines until they have formed an appropriate number of polygons.

Modifications to Pacing

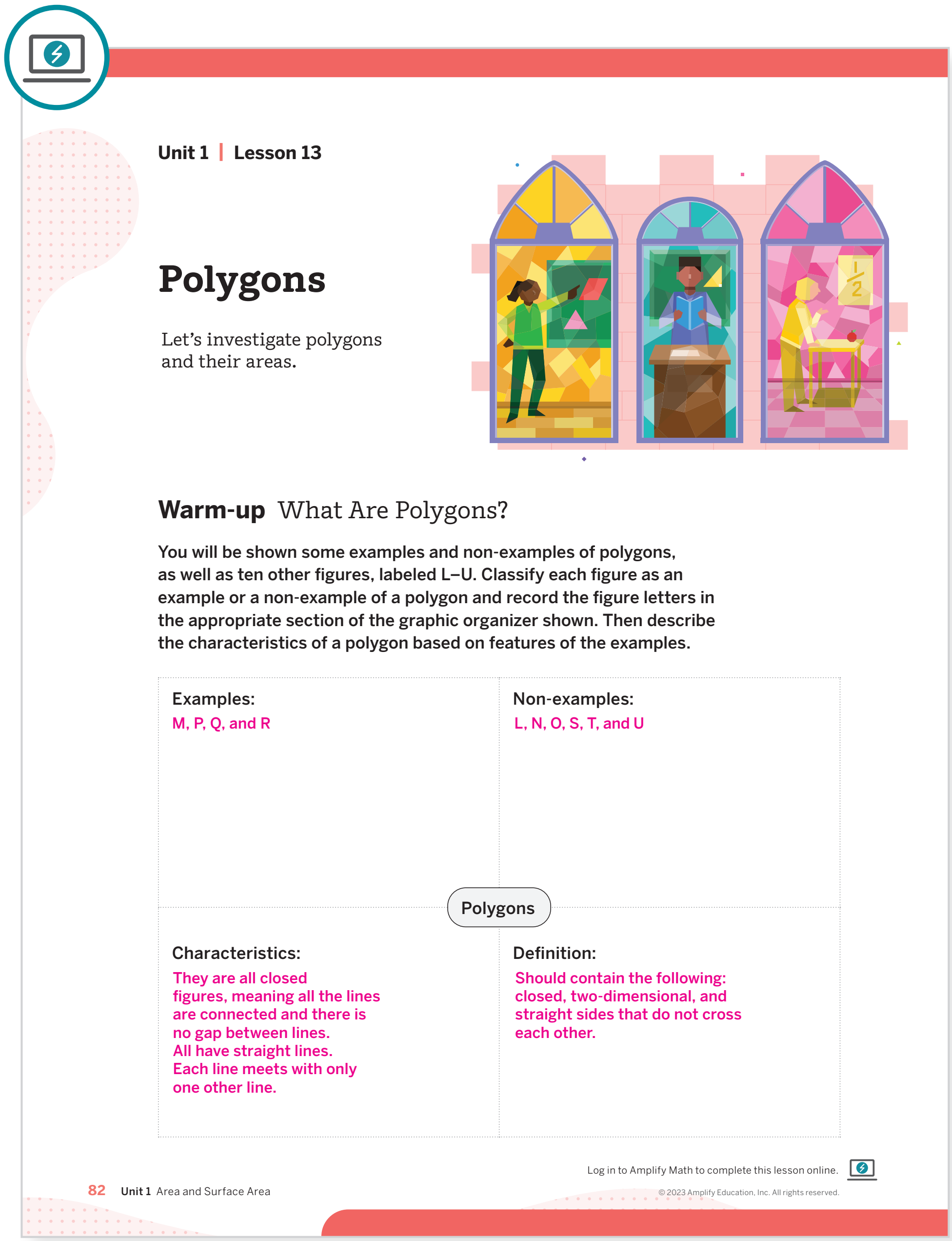
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, the graphic organizer can be completed as a class instead of using the ***Think-Pair-Share*** routine.
- In **Activity 1**, reduce the number of polygons for which students are asked to determine the area. For example, have students calculate the areas of 1 or 2 triangles or parallelograms and 1 or 2 other types of polygons.

Independent | 10 min

Warm-up What Are Polygons?

Students determine commonalities among examples and non-examples to define the term *polygon*.



Unit 1 | Lesson 13

Polygons

Let's investigate polygons and their areas.

Warm-up What Are Polygons?

You will be shown some examples and non-examples of polygons, as well as ten other figures, labeled L–U. Classify each figure as an example or a non-example of a polygon and record the figure letters in the appropriate section of the graphic organizer shown. Then describe the characteristics of a polygon based on features of the examples.

Examples:	Non-examples:
M, P, Q, and R	L, N, O, S, T, and U

Polygons

Characteristics:	Definition:
They are all closed figures, meaning all the lines are connected and there is no gap between lines. All have straight lines. Each line meets with only one other line.	Should contain the following: closed, two-dimensional, and straight sides that do not cross each other.

Log in to Amplify Math to complete this lesson online.

82 Unit 1 Area and Surface Area

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1 Launch

Display the Warm-up PDF. Have students use the *Think-Pair-Share* routine for the first three sections of the graphic organizer. Explain that the definition will be written as a class during the discussion.

2 Monitor

Help students get started by asking, "Which figure can you say is a non-example right away? Why?"

Look for points of confusion:

- **Thinking closed composite figures (Figures N, T, and U) are polygons.** Ask, "Is there an example or non-example that is composed of multiple triangles or quadrilaterals in a similar way?"
- **Not being able to generalize characteristics.** Using Figure Q, ask, "What characteristic makes this not a polygon? What must be a characteristic of a polygon, based on this figure not being a polygon?"

Look for productive strategies:

- Recognizing characteristics of polygons and characteristics of non-examples.

3 Connect

Display the completed graphic organizer.

Have individual students share the common characteristics that they noticed about polygons.

Highlight the important characteristics of polygons:

- They are composed of straight line segments.
- Line segments are connected only at their endpoints.
- The line segments never intersect each other except at their endpoints.
- They are two-dimensional figures.

Define the term ***polygon*** as a class using the characteristics.

Ask, "The word *polygon* has both Greek and Latin roots. *Poly* means many and *gon* refers to angles. Do you think "many angles" captures what a polygon is?"

Math Language Development

MLR3: Critique, Correct, Clarify

Place Figure F in the Examples section of the graphic organizer. Ask students to agree or disagree with the placement, correct the placement if necessary, and explain why it should be placed there.

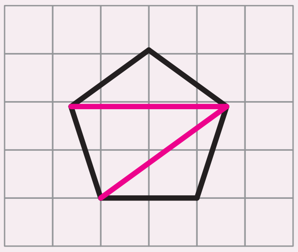
English Learners

Include drawings in the graphic organizer that display examples and nonexamples of polygons. Consider including visual displays to highlight some of the terms in the Characteristics section of the graphic organizer. For example, students might struggle with terms such as *endpoints, closed figures,* and *two-dimensional.*

Power-up

To power up students' ability to decompose a polygon into triangular parts, have students complete:

By adding exactly 2 lines, decompose the shape into 3 triangles.



Sample response shown

Use: Before the Warm-up.

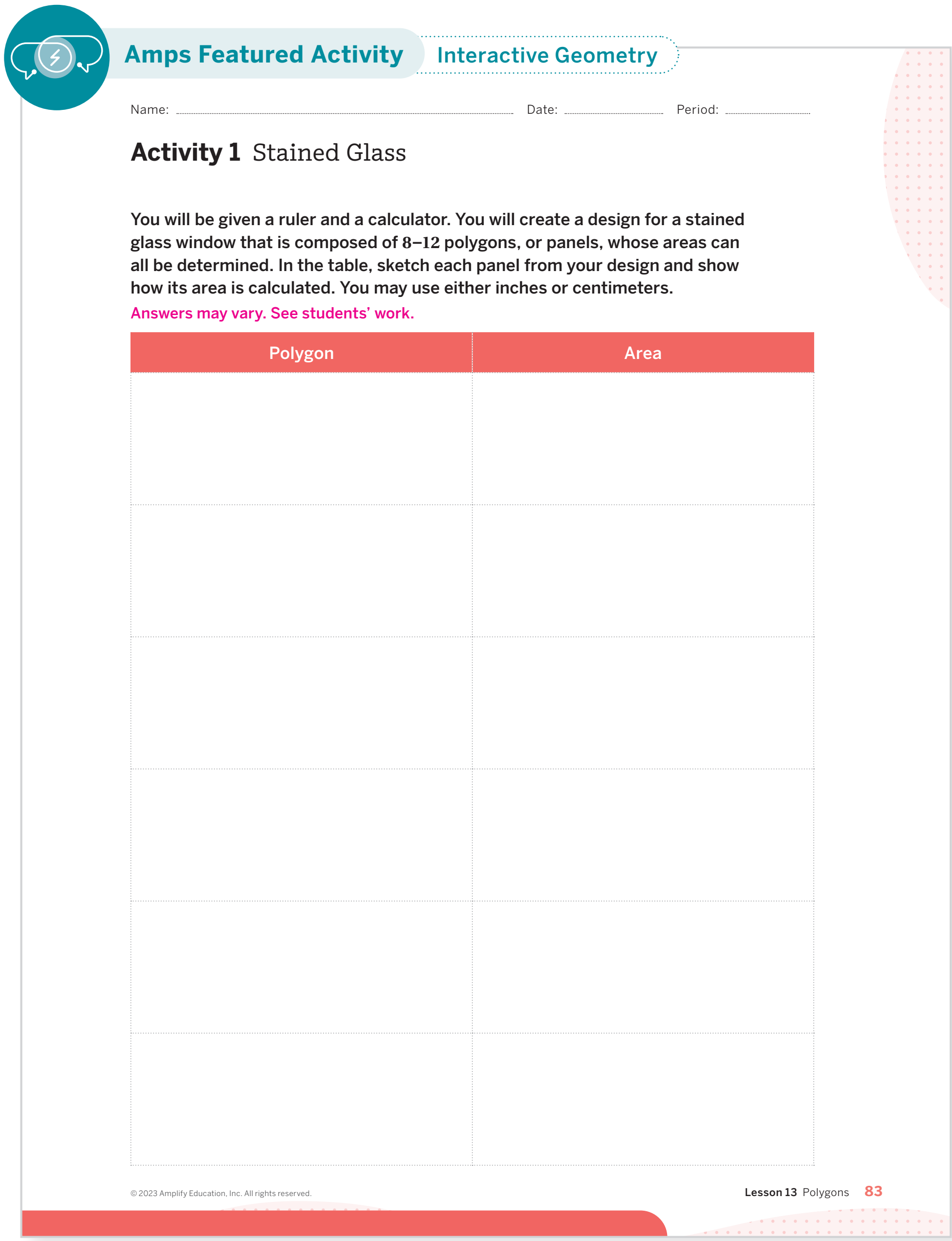
Informed by: Performance on Lesson 12, Practice Problem 5.

Pairs | 25 min

Activity 1 Stained Glass

Students design a piece of stained glass composed of polygons. They calculate the area of each polygon by decomposing and measuring dimensions.

Amps Featured Activity Interactive Geometry



Name: ______________________ Date: __________ Period: __________

Activity 1 Stained Glass

You will be given a ruler and a calculator. You will create a design for a stained glass window that is composed of 8–12 polygons, or panels, whose areas can all be determined. In the table, sketch each panel from your design and show how its area is calculated. You may use either inches or centimeters.

Answers may vary. See students' work.

Polygon	Area

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Lesson 13 Polygons 83

1 Launch

Activate students' background knowledge by asking, "Have you ever seen stained glass on a building? What did it look like?" Refer to the Activity 1 PDF for modeling instructions. These should be reviewed before students begin. Distribute one transparency to each student to help them design their piece of stained glass.

2 Monitor

Help students get started by demonstrating how the lines were made in the example from the Activity 1 PDF.

Look for points of confusion:

- **Thinking that they should make only one repeated shape.** Refer back to either the modeled example or the additional examples and note the different polygons represented to encourage students to use a variety of polygons.
- **Having difficulty determining the base and height.** Refer students back to Lesson 10 by asking, "What tool did you use to help you find the height of triangles?" Once students have identified the height ask, "So, where would the base be then?"

Look for productive strategies:

- Using tools and previous strategies for determining known areas flexibly.
- Applying their understanding of base and height to the triangles.
- Choosing appropriate tools and units of measure for calculating areas.
- Recognizing that any polygon can be decomposed into triangles.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display the example from the Activity 1 PDF for students to reference as a model throughout the activity.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, reduce the number of areas they should determine. For example, have them determine the area of one triangle and one other polygon (perhaps with only 4 or 5 sides).

Pairs | 25 min

Activity 1 Stained Glass (continued)

Students design a piece of stained glass composed of polygons. They calculate the area of each polygon by decomposing and measuring dimensions.

Activity 1 Stained Glass (continued)

Polygon	Area

Are you ready for more?

Determine the sum of the areas of your polygons. The total area of your polygons should equal the area of the piece of paper. Check your work to see if this holds true and explain why it is true.

Note: U.S. Letter size paper has dimensions of $8\frac{1}{2}$ in. by 11 in.

Area in inches: approximately 93.5 in^2
or
Area in centimeters: approximately 603 cm^2

STOP

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3 Connect

Display student work and conduct the ***Gallery Tour*** routine to display student work.

Have individual students share:

- How they created their design and polygons.
- How they determined the areas of their polygons, focusing on how they decomposed shapes with four or more sides.
- Why they chose to measure their shapes using inches or centimeters.

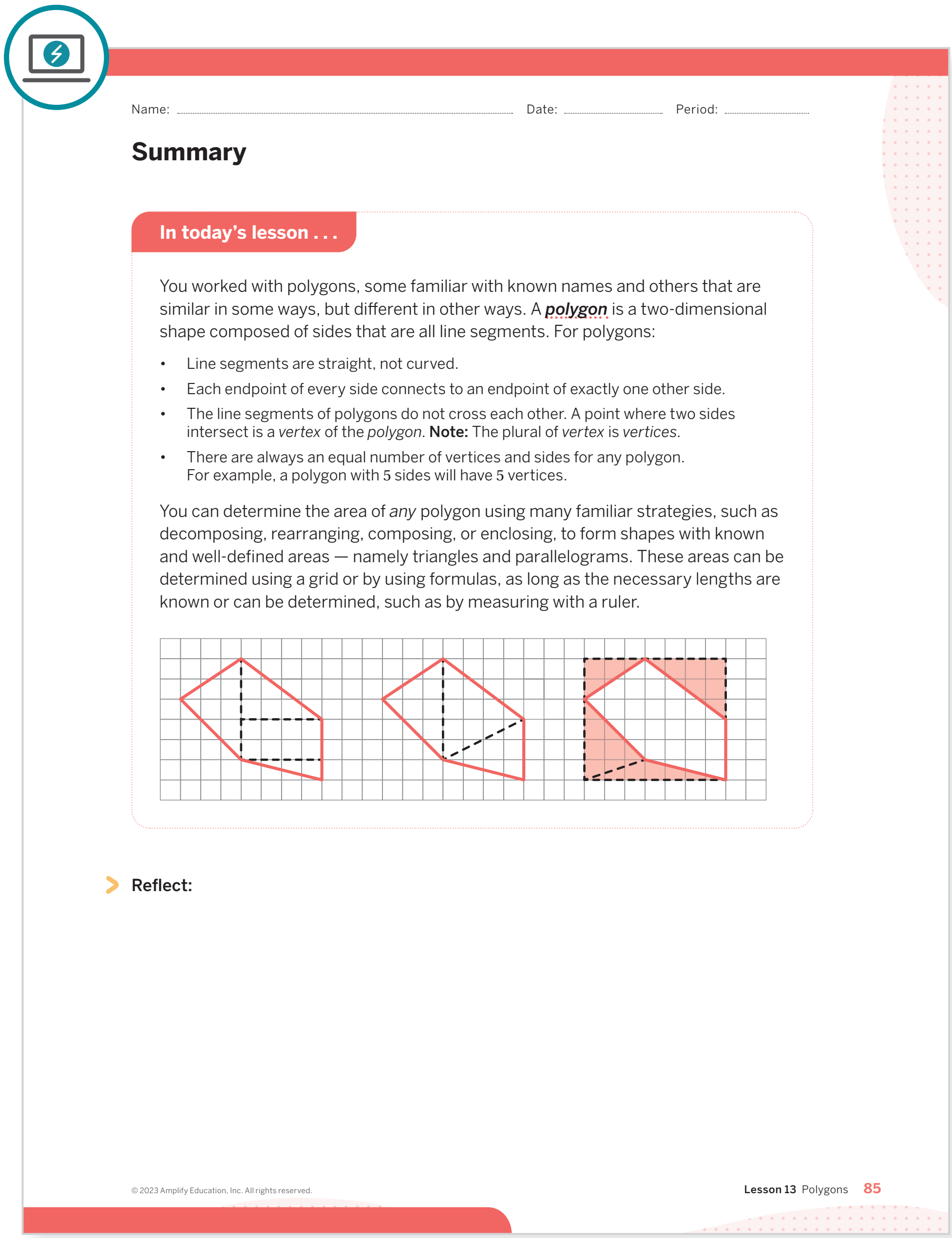
Highlight that any region that is itself a polygon can be decomposed into polygons many different ways. And any individual polygon can be decomposed by using only triangles. The area of any triangle can always be calculated if its base and height can be measured or known.

Ask, “What should the total area of the polygons in your design equal?” The area of the piece of paper.

Whole Class | 5 min

Summary

Review and synthesize what a polygon is and the strategies that can be used to determine the area of any polygon.



Name: Date: Period:

Summary

In today's lesson . . .

You worked with polygons, some familiar with known names and others that are similar in some ways, but different in other ways. A ***polygon*** is a two-dimensional shape composed of sides that are all line segments. For polygons:

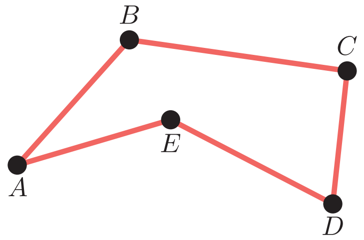
- Line segments are straight, not curved.
- Each endpoint of every side connects to an endpoint of exactly one other side.
- The line segments of polygons do not cross each other. A point where two sides intersect is a *vertex* of the *polygon*. **Note:** The plural of *vertex* is *vertices*.
- There are always an equal number of vertices and sides for any polygon. For example, a polygon with 5 sides will have 5 vertices.

You can determine the area of *any* polygon using many familiar strategies, such as decomposing, rearranging, composing, or enclosing, to form shapes with known and well-defined areas — namely triangles and parallelograms. These areas can be determined using a grid or by using formulas, as long as the necessary lengths are known or can be determined, such as by measuring with a ruler.

Reflect:

Synthesize

Display Polygon $ABCDE$ as shown here by drawing it for the class to see.



Ask:

- "How do you know this figure is a polygon?"
- "What does it mean to determine the area of this polygon?"
- "How can you determine the area of this polygon?"

Formalize vocabulary: ***polygon***

Highlight that to determine the area of any polygon, it can be decomposed into triangles. The area of the triangles can be calculated and added to determine the area of the polygon.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What are some characteristics of the polygons you made in your stained glass design?"

Math Language Development

MLR2: Collect and Display

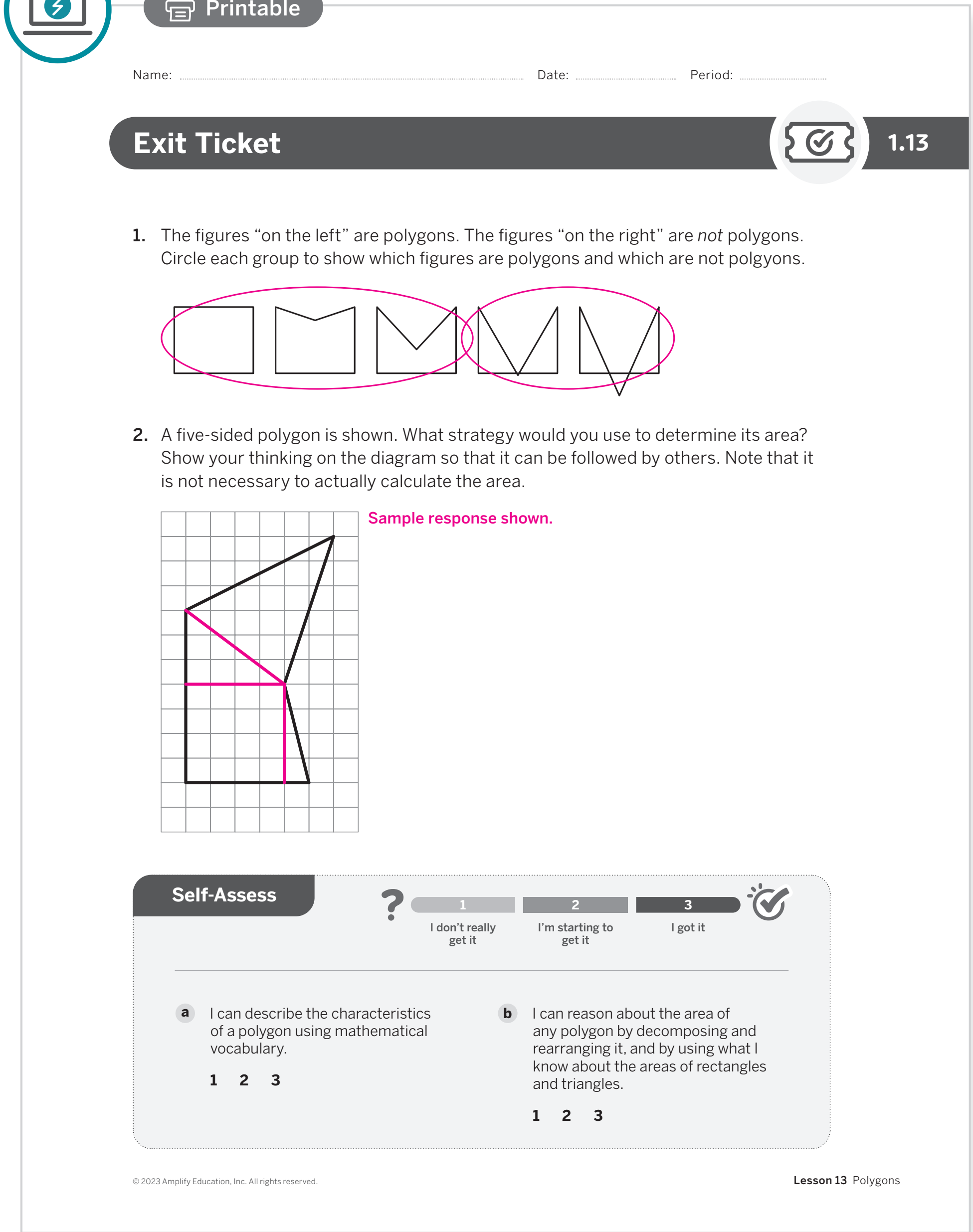
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 3. Ask them to review and reflect on any terms and phrases related to the term *polygon* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of determining the area of a polygon by first critiquing a given strategy, and then by showing how to decompose a five-sided polygon.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.13

1. The figures "on the left" are polygons. The figures "on the right" are *not* polygons. Circle each group to show which figures are polygons and which are not polgyons.

2. A five-sided polygon is shown. What strategy would you use to determine its area? Show your thinking on the diagram so that it can be followed by others. Note that it is not necessary to actually calculate the area.

Sample response shown.

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can describe the characteristics of a polygon using mathematical vocabulary.

1 2 3

b. I can reason about the area of any polygon by decomposing and rearranging it, and by using what I know about the areas of rectangles and triangles.

1 2 3

Lesson 13 Polygons

Success looks like . . .

- **Language Goal:** Comparing and contrasting different strategies for determining the area of a polygon **(Speaking and Listening)**.
- **Language Goal:** Describing the defining characteristics of polygons **(Speaking and Listening, Writing)**.
  - » Selecting figures as polygons by checking for characteristics of polygons in Problem 1.
- **Goal:** Determining the area of a polygon, by decomposing it into rectangles and triangles, and presenting the solution method (using multiple representations).
  - » Drawing a diagram that shows how to determine the area of a five-sided polygon in Problem 2.

Suggested next steps

If students identify any of the last two figures as polygons in Problem 1, consider:

- Referring back to the Warm-up and reviewing the characteristics of a polygon.

If students cannot decompose the polygon in Problem 2 into shapes that would allow its area to be determined, consider:

- Referring to the Summary and reviewing which shapes polygons can be decomposed into (parallelograms and triangles) and ask, "Where could you draw a line segment to form a triangle or a parallelogram?"

Professional Learning

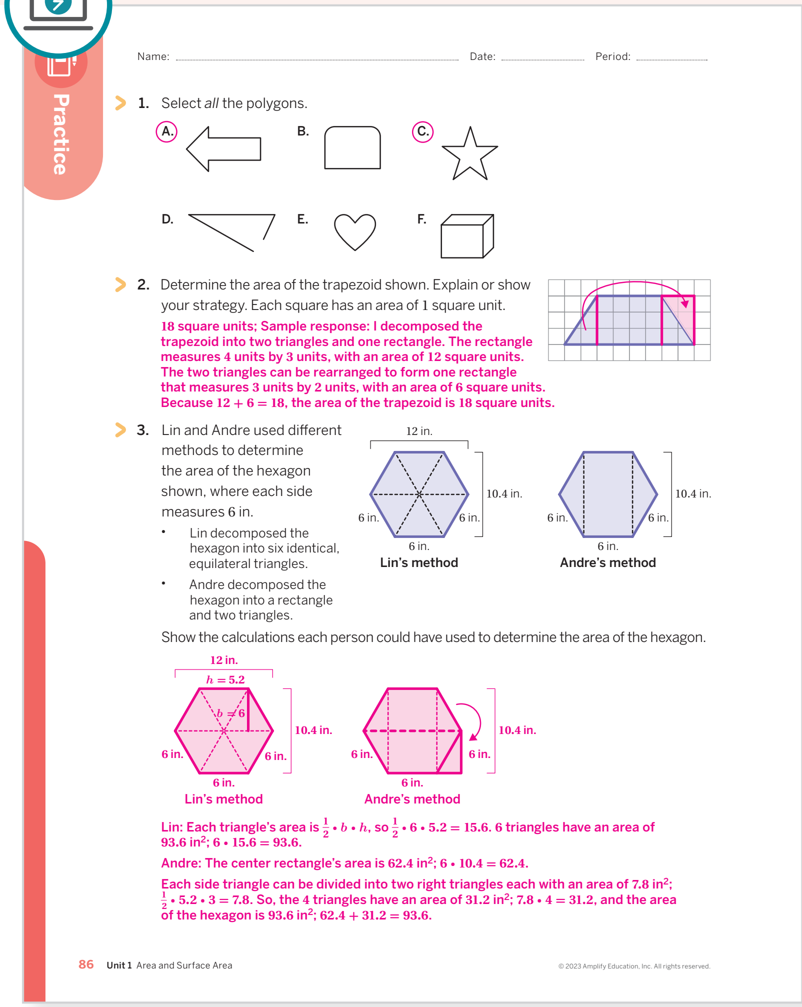
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did working with polygons connect to the learning of the unit?
- What materials could be helpful for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. Select *all* the polygons.

[✓ A.] B. [✓ C.] D. E. F.

2. Determine the area of the trapezoid shown. Explain or show your strategy. Each square has an area of 1 square unit.

18 square units; Sample response: I decomposed the trapezoid into two triangles and one rectangle. The rectangle measures 4 units by 3 units, with an area of 12 square units. The two triangles can be rearranged to form one rectangle that measures 3 units by 2 units, with an area of 6 square units. Because $12 + 6 = 18$, the area of the trapezoid is 18 square units.

3. Lin and Andre used different methods to determine the area of the hexagon shown, where each side measures 6 in.

- Lin decomposed the hexagon into six identical, equilateral triangles.
- Andre decomposed the hexagon into a rectangle and two triangles.

Show the calculations each person could have used to determine the area of the hexagon.

Lin: Each triangle's area is $\frac{1}{2} \cdot b \cdot h$, so $\frac{1}{2} \cdot 6 \cdot 5.2 = 15.6$. 6 triangles have an area of 93.6 in^2; $6 \cdot 15.6 = 93.6$.

Andre: The center rectangle's area is 62.4 in^2; $6 \cdot 10.4 = 62.4$.

Each side triangle can be divided into two right triangles each with an area of 7.8 in^2; $\frac{1}{2} \cdot 5.2 \cdot 3 = 7.8$. So, the 4 triangles have an area of 31.2 in^2; $7.8 \cdot 4 = 31.2$, and the area of the hexagon is 93.6 in^2; $62.4 + 31.2 = 93.6$.

86 **Unit 1** Area and Surface Area

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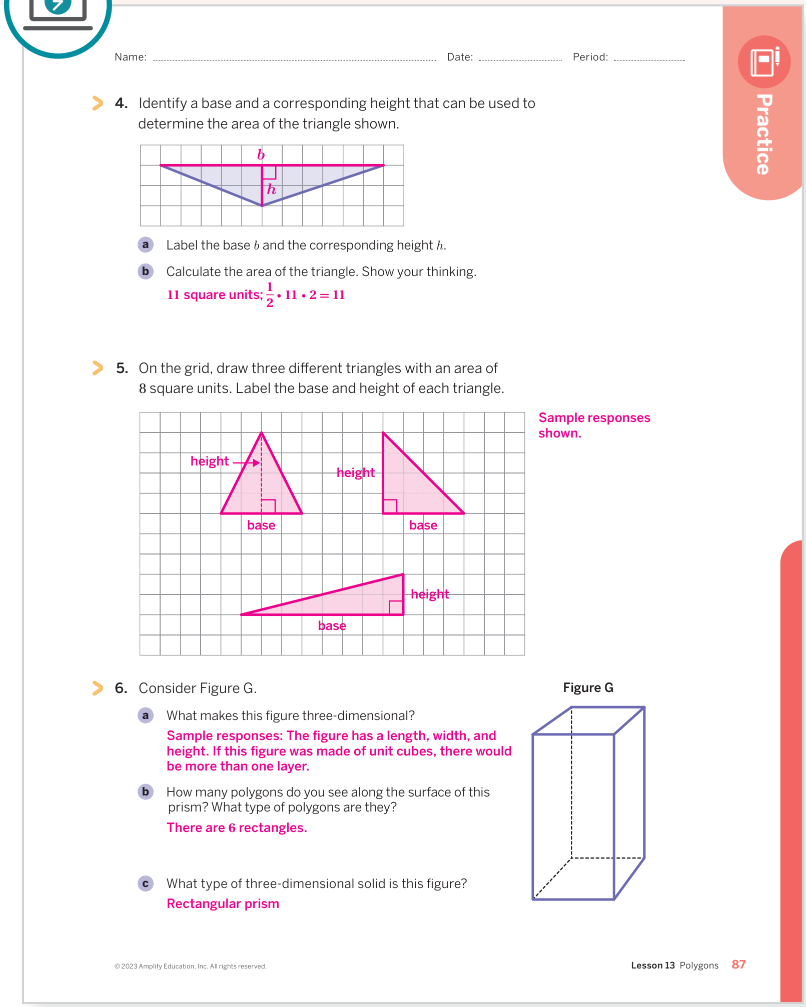

Name: ______ Date: ______ Period: ______

4. Identify a base and a corresponding height that can be used to determine the area of the triangle shown.

a Label the base b and the corresponding height h.

b Calculate the area of the triangle. Show your thinking.

11 square units; $\frac{1}{2} \cdot 11 \cdot 2 = 11$

5. On the grid, draw three different triangles with an area of 8 square units. Label the base and height of each triangle.

Sample responses shown.

6. Consider Figure G.

Figure G

a What makes this figure three-dimensional?

Sample responses: The figure has a length, width, and height. If this figure was made of unit cubes, there would be more than one layer.

b How many polygons do you see along the surface of this prism? What type of polygons are they?

There are 6 rectangles.

c What type of three-dimensional solid is this figure?

Rectangular prism

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Lesson 13 Polygons 87

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	1
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 9	2
	5	Unit 1 Lesson 12	2
Formative ⚡	6	Unit 1 Lesson 14	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

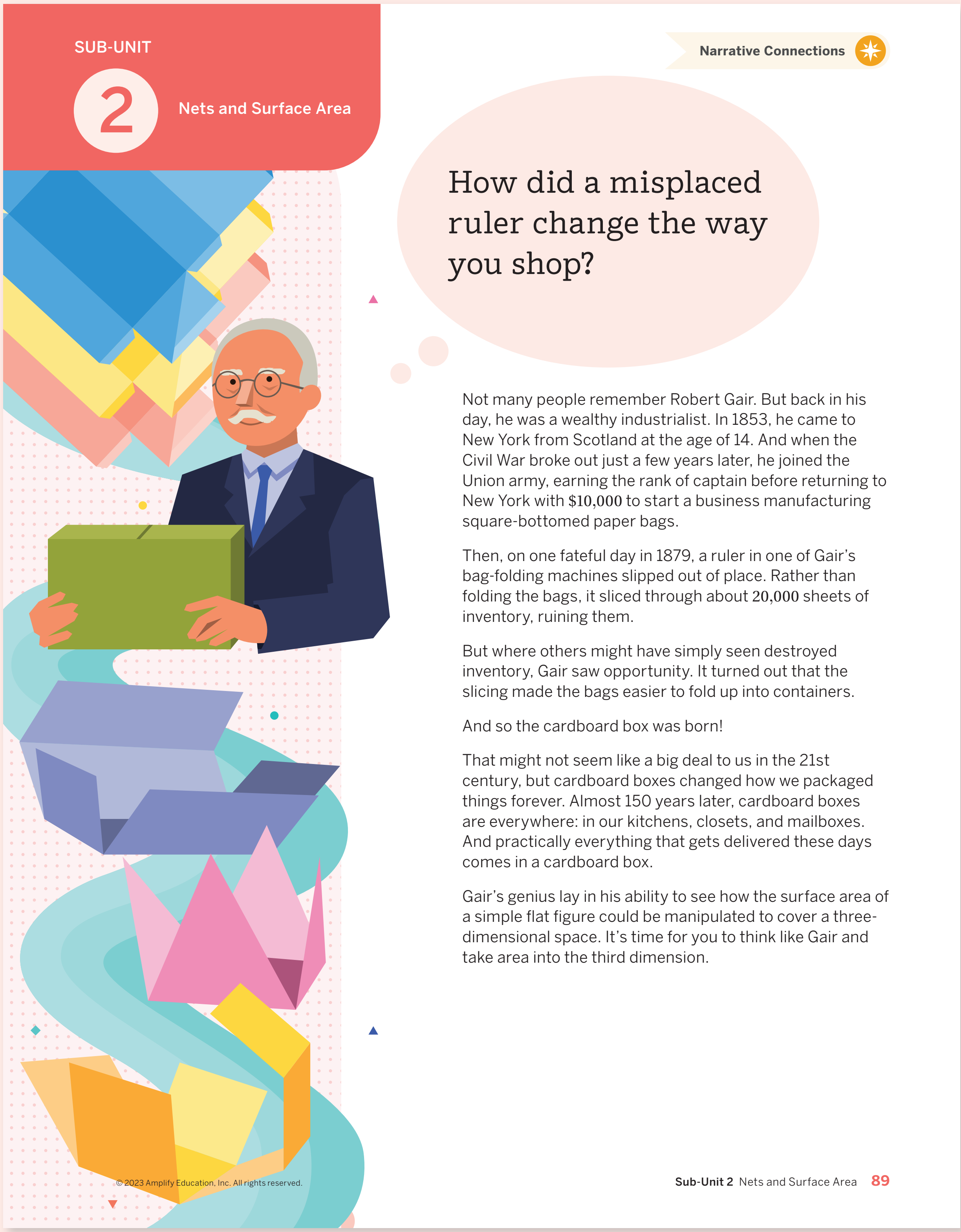
For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 2

Nets and Surface Area

Whole Class

In this Sub-Unit, students construct nets of three-dimensional figures in order to calculate surface area. They also distinguish surface area from volume and use exponents to simplify expressions and formulas involving repeated multiplication.



SUB-UNIT

2 Nets and Surface Area

Narrative Connections

How did a misplaced ruler change the way you shop?

Not many people remember Robert Gair. But back in his day, he was a wealthy industrialist. In 1853, he came to New York from Scotland at the age of 14. And when the Civil War broke out just a few years later, he joined the Union army, earning the rank of captain before returning to New York with $10,000 to start a business manufacturing square-bottomed paper bags.

Then, on one fateful day in 1879, a ruler in one of Gair's bag-folding machines slipped out of place. Rather than folding the bags, it sliced through about 20,000 sheets of inventory, ruining them.

But where others might have simply seen destroyed inventory, Gair saw opportunity. It turned out that the slicing made the bags easier to fold up into containers.

And so the cardboard box was born!

That might not seem like a big deal to us in the 21st century, but cardboard boxes changed how we packaged things forever. Almost 150 years later, cardboard boxes are everywhere: in our kitchens, closets, and mailboxes. And practically everything that gets delivered these days comes in a cardboard box.

Gair's genius lay in his ability to see how the surface area of a simple flat figure could be manipulated to cover a three-dimensional space. It's time for you to think like Gair and take area into the third dimension.

Sub-Unit 2 Nets and Surface Area 89

Narrative Connections

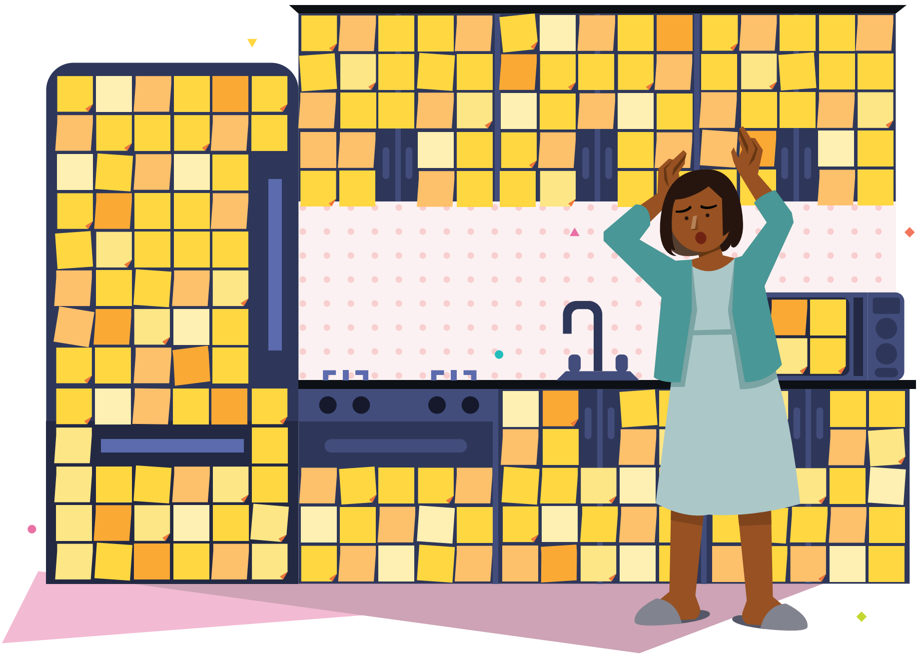
Read the narrative aloud as a class or have students read it individually. Students continue to explore the surfaces of solids (like that of Gair's cardboard box) in the following places:

- **Lesson 14, Activity 1:** Covering the Cabinet
- **Lesson 17, Activity 1:** Constructing a Model of the Library
- **Lesson 20, Activity 1:** Suspended Tent Design

UNIT 1 | **LESSON 14**

What Is Surface Area?

Let's cover the surfaces of some three-dimensional objects.



Focus

Goals

1. **Language Goal:** Comprehend and use the terms face, edge, and vertex to describe rectangular prisms. **(Speaking and Listening, Writing)**
2. **Language Goal:** Comprehend that the term surface area refers to how many square units it takes to cover all the faces of a three-dimensional object. **(Speaking and Listening, Writing)**
3. **Language Goal:** Calculate the surface area of a rectangular prism, and explain the solution method. **(Speaking and Listening, Writing)**
4. **Language Goal:** Comprehend that surface area and volume are two different attributes of three-dimensional objects and are measured in different units. **(Speaking and Listening)**

Coherence

Today

Students extend their work with the area of two-dimensional shapes to understand surface area as a measure of three-dimensional solids. Students begin their exploration with a concrete example of a rectangular prism, using tiling and the area formula for a rectangle to determine the total number of sticky notes it takes to cover a filing cabinet. Students then build rectangular prisms and attend to precise vocabulary as they use their models to distinguish between surface area and volume.

Previously

In Grades 3–5, students calculated the area of rectangles and the volume of rectangular prisms similarly, by tiling with unit squares (area) or packing with unit cubes (volume). They derived the formulas $A = \ell \cdot w$ and $V = \ell \cdot w \cdot h$.

Coming Soon

In Lesson 15, students will explore the relationship between a rectangular prism and its two-dimensional net, interpreting and drawing nets to calculate the surface area of boxes.

Rigor

- Students use the context of covering a cabinet with sticky-notes to develop **conceptual understanding** of the surface area of rectangular prisms.
- Students strengthen their **procedural fluency** in determining the volume of rectangular prisms.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- unit cubes, 12 per student

Math Language Development

New words

- **_face_***
- **_edge_**
- **_surface area_**
- **_vertex_**
- **_volume_****

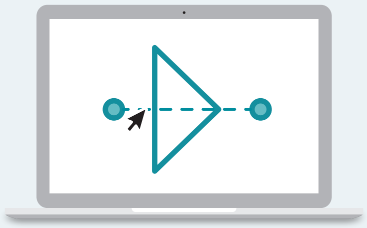
*Students may confuse the term *face*, which refers to the *face* of a three-dimensional shape, with the term *face* as it relates to human anatomy. Be ready to explain how the terms differ.

**Students may confuse the term *volume*, which refers to the *volume* of a three-dimensional shape, with the *volume* of a music player. Be ready to explain how the terms differ.

Amps | Featured Activity

Activity 2
Interactive Rectangular Prisms

Students build rectangular prisms using virtual unit cubes. They can rotate their prism to obtain a 360° view. Tri-colored faces help students recognize patterns and keep track of their work.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may find that the open nature of **Activity 1** makes it particularly challenging to evaluate progress or change course if necessary. Model how to systematically evaluate progress, focusing on recognizing small successes, gauging effort, and looking for efficiencies in repeated processes by using missteps as motivation to devise a new plan.

Modifications to Pacing

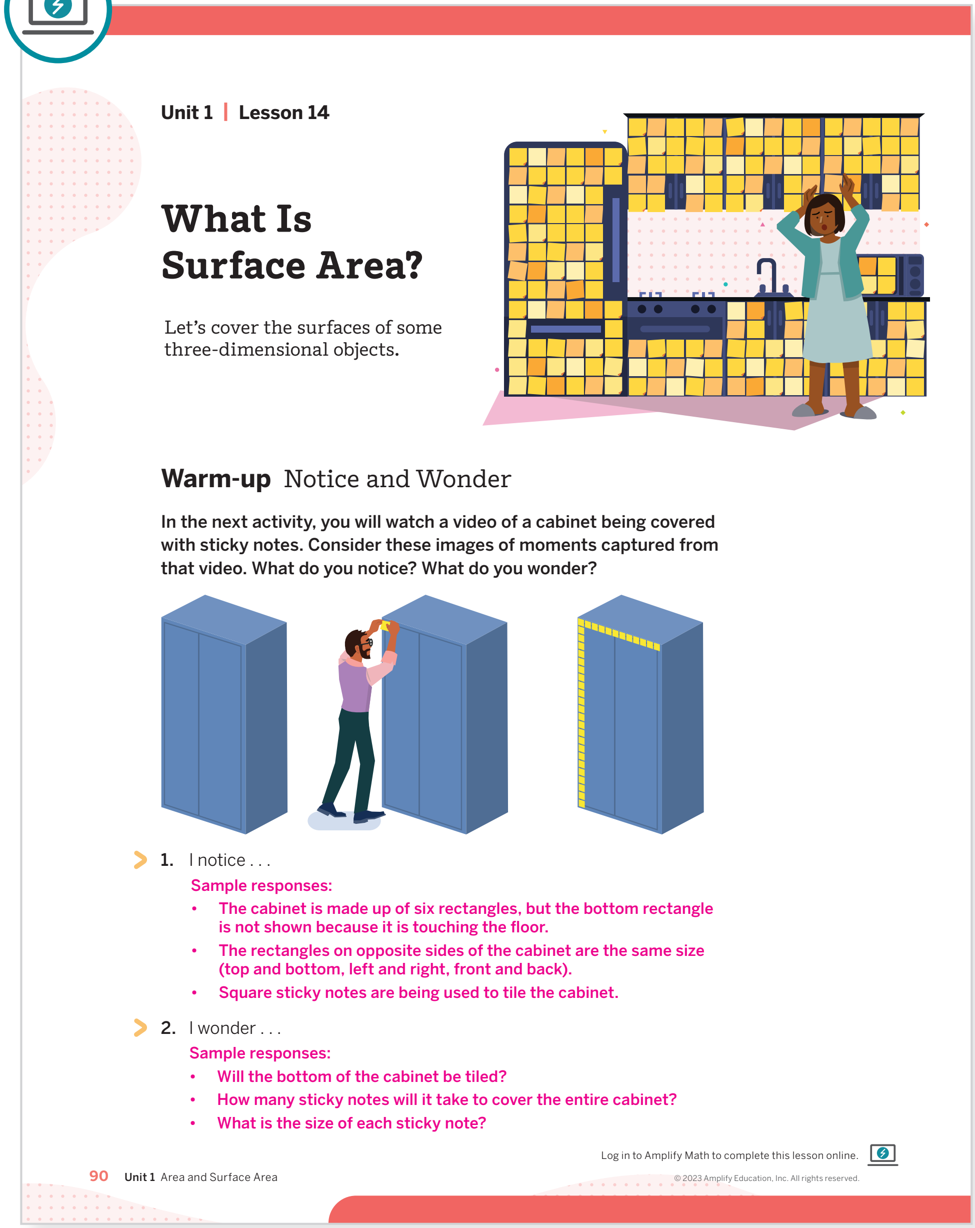
You may want to consider this additional modification if you are short on time.

- The **Warm-up** and **Activity 1**, Part 1 may be omitted. Begin with Clip A of the video and Part 2 of **Activity 1**.

Independent | 10 min

Warm-up Notice and Wonder

Students reason about a series of images depicting a cabinet being covered with sticky notes, preparing them to investigate its surface area in Activity 1.



Unit 1 | Lesson 14

What Is Surface Area?

Let's cover the surfaces of some three-dimensional objects.

Warm-up Notice and Wonder

In the next activity, you will watch a video of a cabinet being covered with sticky notes. Consider these images of moments captured from that video. What do you notice? What do you wonder?

1. I notice . . .

   Sample responses:
   - The cabinet is made up of six rectangles, but the bottom rectangle is not shown because it is touching the floor.
   - The rectangles on opposite sides of the cabinet are the same size (top and bottom, left and right, front and back).
   - Square sticky notes are being used to tile the cabinet.

2. I wonder . . .

   Sample responses:
   - Will the bottom of the cabinet be tiled?
   - How many sticky notes will it take to cover the entire cabinet?
   - What is the size of each sticky note?

Log in to Amplify Math to complete this lesson online.

90 Unit 1 Area and Surface Area

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1 Launch

Conduct the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by asking, "What is happening to the cabinet?"

Look for points of confusion:

- **Considering only the visible sides of the cabinet.** Refer to a real cabinet in your classroom, and ask, "How many sides are there? What shape(s) are the sides?"

Look for productive strategies:

- Recognizing the cabinet as a 3D rectangular prism with six rectangular faces, and the square sticky notes as "unit squares" being used to tile the cabinet.
- Multiplying to calculate the area of each rectangular face.

3 Connect

Have students share their answers, focusing on the six rectangular faces and how the sticky notes tile one row of one side without gaps or overlap.

Display an anchor chart for 3D solids and add new terms (this will be added to over several lessons).

Define a ***face*** of a three-dimensional solid as any of the two-dimensional shapes joined to make its outer surface. Two faces meet at an ***edge***, and two or more edges intersect at a ***vertex***.

Highlight that the cabinet is a rectangular prism with six flat, rectangular faces. Each face has an identical parallel face, and some faces may not be visible (like the bottom of the cabinet).

Ask students to estimate how many sticky notes it would take to cover the cabinet's faces, excluding the bottom. Poll the class and record student responses.

Math Language Development

MLR2: Collect and Display

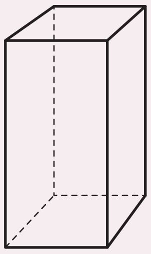
Display the class anchor chart and add new terms for three-dimensional solids, such as *face*, *edge*, and *vertex*. Encourage students to refer to this anchor chart during their class discussions.

English Learners

Include visual examples that illustrate each term. Consider also using physical models and gestures pointing to how these terms represent features of the solids, before adding the terms and visual examples to the class anchor chart.

Power-up

To power up students' ability to name and describe a rectangular prism, have students complete:



Identify *all* of the true statements about rectangular prisms:

[✓ **A.**] There are 6 rectangular faces.

B. There are 8 total faces.

[✓ **C.**] Faces that are parallel to one another are the same.

[✓ **D.**] There are 12 edges.

E. There are 9 edges.

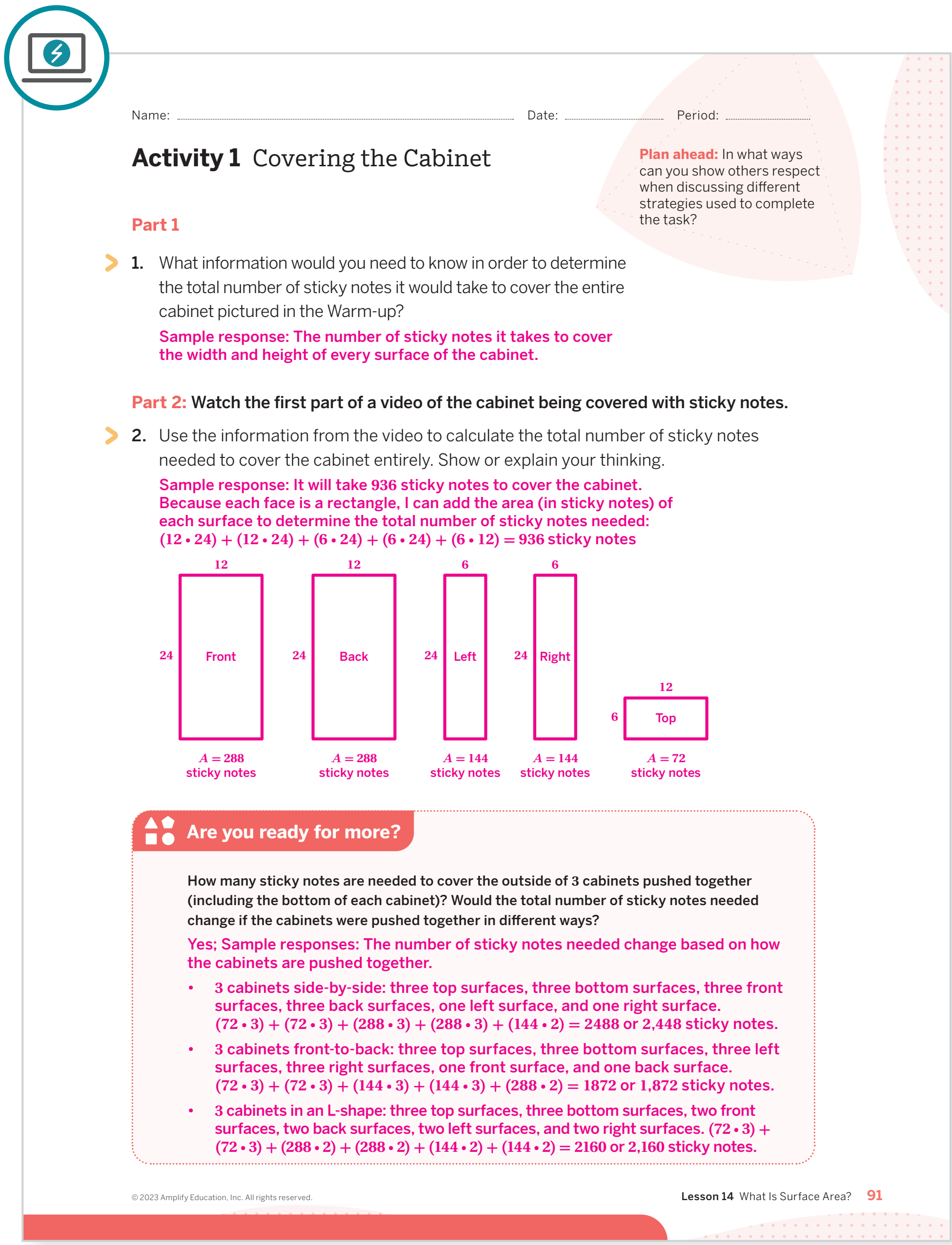
Use: Before the Warm-up.

Informed by: Performance on Lesson 13, Practice Problem 6.

Pairs | 30 min

Activity 1 Covering the Cabinet

Students watch a two-part video, identifying necessary information and devising strategies to determine the total number of sticky notes needed to cover the cabinet.



Name: .... Date: .... Period: ....

Activity 1 Covering the Cabinet

Plan ahead: In what ways can you show others respect when discussing different strategies used to complete the task?

Part 1

1. What information would you need to know in order to determine the total number of sticky notes it would take to cover the entire cabinet pictured in the Warm-up?

Sample response: The number of sticky notes it takes to cover the width and height of every surface of the cabinet.

Part 2: Watch the first part of a video of the cabinet being covered with sticky notes.

2. Use the information from the video to calculate the total number of sticky notes needed to cover the cabinet entirely. Show or explain your thinking.

Sample response: It will take 936 sticky notes to cover the cabinet. Because each face is a rectangle, I can add the area (in sticky notes) of each surface to determine the total number of sticky notes needed:
$(12 \cdot 24) + (12 \cdot 24) + (6 \cdot 24) + (6 \cdot 24) + (6 \cdot 12) = 936$ sticky notes

Are you ready for more?

How many sticky notes are needed to cover the outside of 3 cabinets pushed together (including the bottom of each cabinet)? Would the total number of sticky notes needed change if the cabinets were pushed together in different ways?

Yes; Sample responses: The number of sticky notes needed change based on how the cabinets are pushed together.

- **3 cabinets side-by-side: three top surfaces, three bottom surfaces, three front surfaces, three back surfaces, one left surface, and one right surface. $(72 \cdot 3) + (72 \cdot 3) + (288 \cdot 3) + (288 \cdot 3) + (144 \cdot 2) = 2488$ or 2,448 sticky notes.**
- **3 cabinets front-to-back: three top surfaces, three bottom surfaces, three left surfaces, three right surfaces, one front surface, and one back surface. $(72 \cdot 3) + (72 \cdot 3) + (144 \cdot 3) + (144 \cdot 3) + (288 \cdot 2) = 1872$ or 1,872 sticky notes.**
- **3 cabinets in an L-shape: three top surfaces, three bottom surfaces, two front surfaces, two back surfaces, two left surfaces, and two right surfaces. $(72 \cdot 3) + (72 \cdot 3) + (288 \cdot 2) + (288 \cdot 2) + (144 \cdot 2) + (144 \cdot 2) = 2160$ or 2,160 sticky notes.**

1 Launch

Give pairs 1 minute to complete Part 1, and then have pairs share their responses with the class. Display Clip A from the Activity 1 Amps slides before students complete Part 2.

2 Monitor

Help students get started by asking how many sticky notes are needed to cover the front face.

Look for points of confusion:

- **Thinking all sides (or all but the top) are identical.** Have students compare the dimensions of the faces of another rectangular prism in the room.
- **Calculating $\ell \cdot w \cdot h$.** Remind students that this determines *volume* (the space inside the cabinet). Ask, "How is filling an object different from covering it?"

Look for productive strategies:

- Calculating and then adding the areas of each face.
- Using congruent faces to reduce calculations.

3 Connect

Have students share their calculations and strategies.

Display Clip B from the Activity 1 Amps slides.

Highlight that the total number of sticky notes it takes to cover the entire cabinet, including the bottom, is the *surface area*. However, the bottom face was excluded for the cabinet because tiling it was unnecessary.

Define the term ***surface area*** as the number of unit squares it takes to cover all of the faces of an object without gaps or overlaps. Add the term to the anchor chart.

Ask, "Would the surface area change if you used larger or smaller sticky notes? How?" The larger the sticky notes, the fewer that is needed.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with pre-drawn faces of the cabinet, or have them draw the faces. Ask students to label the dimensions of each face. Consider demonstrating how to do this for one of the faces. Then ask them how many sticky notes it would take to cover each face of the cabinet.

Accessibility: *Optimize Access to Tools, Guide Processing and Visualization*

Consider providing manipulatives that could represent the cabinet, such as cereal boxes, and small sticky notes. Have students experiment with the sticky notes to cover the surface of the cereal box instead of completing Problem 2.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies, ask them to make connections between the various strategies. Ask them why certain methods will result in the same number of sticky notes needed. For example, highlight how the front and back of the cabinet are each covered by the same number of sticky notes.

English Learners

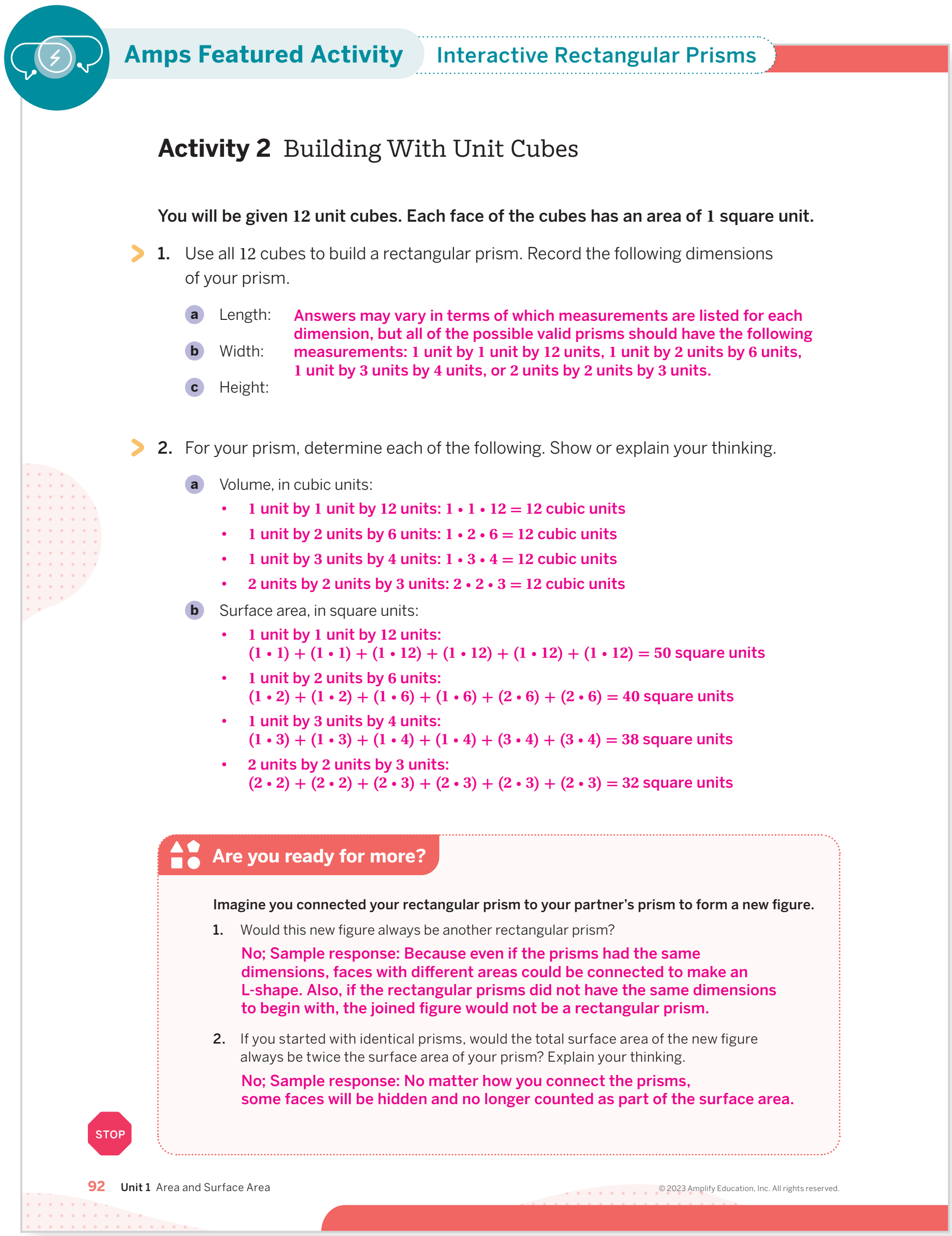
Annotate the diagrams in Problem 2 with the terms *front*, *back*, *left*, *right*, and *top*.

Pairs | 30 min

Activity 2 Building With Unit Cubes

Students build a rectangular prism with unit cubes, distinguishing between volume and surface area. They apply strategies for calculating the area of polygons to surface area.

Amps Featured Activity Interactive Rectangular Prisms



Activity 2 Building With Unit Cubes

You will be given 12 unit cubes. Each face of the cubes has an area of 1 square unit.

1. Use all 12 cubes to build a rectangular prism. Record the following dimensions of your prism.
   - a Length:
   - b Width:
   - c Height:

   Answers may vary in terms of which measurements are listed for each dimension, but all of the possible valid prisms should have the following measurements: 1 unit by 1 unit by 12 units, 1 unit by 2 units by 6 units, 1 unit by 3 units by 4 units, or 2 units by 2 units by 3 units.

2. For your prism, determine each of the following. Show or explain your thinking.
   - a Volume, in cubic units:
     - **1 unit by 1 unit by 12 units: $1 \cdot 1 \cdot 12 = 12$ cubic units**
     - **1 unit by 2 units by 6 units: $1 \cdot 2 \cdot 6 = 12$ cubic units**
     - **1 unit by 3 units by 4 units: $1 \cdot 3 \cdot 4 = 12$ cubic units**
     - **2 units by 2 units by 3 units: $2 \cdot 2 \cdot 3 = 12$ cubic units**
   - b Surface area, in square units:
     - **1 unit by 1 unit by 12 units:** $(1 \cdot 1) + (1 \cdot 1) + (1 \cdot 12) + (1 \cdot 12) + (1 \cdot 12) + (1 \cdot 12) = 50$ **square units**
     - **1 unit by 2 units by 6 units:** $(1 \cdot 2) + (1 \cdot 2) + (1 \cdot 6) + (1 \cdot 6) + (2 \cdot 6) + (2 \cdot 6) = 40$ **square units**
     - **1 unit by 3 units by 4 units:** $(1 \cdot 3) + (1 \cdot 3) + (1 \cdot 4) + (1 \cdot 4) + (3 \cdot 4) + (3 \cdot 4) = 38$ **square units**
     - **2 units by 2 units by 3 units:** $(2 \cdot 2) + (2 \cdot 2) + (2 \cdot 3) + (2 \cdot 3) + (2 \cdot 3) + (2 \cdot 3) = 32$ **square units**

Are you ready for more?

Imagine you connected your rectangular prism to your partner's prism to form a new figure.

1. Would this new figure always be another rectangular prism?

   No; Sample response: Because even if the prisms had the same dimensions, faces with different areas could be connected to make an L-shape. Also, if the rectangular prisms did not have the same dimensions to begin with, the joined figure would not be a rectangular prism.

2. If you started with identical prisms, would the total surface area of the new figure always be twice the surface area of your prism? Explain your thinking.

   No; Sample response: No matter how you connect the prisms, some faces will be hidden and no longer counted as part of the surface area.

STOP

92 **Unit 1** Area and Surface Area

1 Launch

Give each student 12 unit cubes and conduct the ***Think-Pair-Share*** routine. Consider pairing students with different designs or strategies.

2 Monitor

Help students get started by activating prior knowledge. Ask, "What should a rectangular prism look like?"

Look for points of confusion:

- **Using volume strategies for surface area.** Remind students that surface area measures space *around the outside faces*, and volume measures space *inside*.
- **Ignoring the bottom when calculating the surface area.** Ask, "What if you turned your prism on a different side? Shouldn't the surface area be the same?"

Look for productive strategies:

- Using $\ell \cdot w \cdot h$ for volume, and recognizing the result is the same as counting unit cubes.
- Calculating and adding the areas of each face, recognizing the result is the same as counting only visible unit cube faces on all six faces of the prism.

3 Connect

Have pairs of students share how they applied strategies for calculating the area of rectangles to determine the surface area. Encourage students to use vocabulary, such as *face*, *surface area*, *edge*, *vertex*, and *square units*.

Highlight that for a 3D solid, *volume* is a 3D measure of how much can be packed in the solid (space inside), while surface area is a 2D measure of how much it takes to cover the solid (space around the outside). This is similar to the 2D relationship between area and perimeter.

Ask, "Why did every prism have the same volume, but some had different surface areas?" All use 12 unit cubes, but a different number of unit cube faces are visible.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

For students who need more processing time, have them complete the activity using 6 unit cubes, instead of 12 unit cubes.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they build rectangular prisms using virtual unit cubes. They can rotate their prisms and tri-colored faces help them keep track of their work.

Math Language Development

MLR8: Discussion Supports— Restate It!

During the Connect, have students use their developing mathematical language (e.g., *faces*, *surface area*, *square units*), to restate the strategies presented.

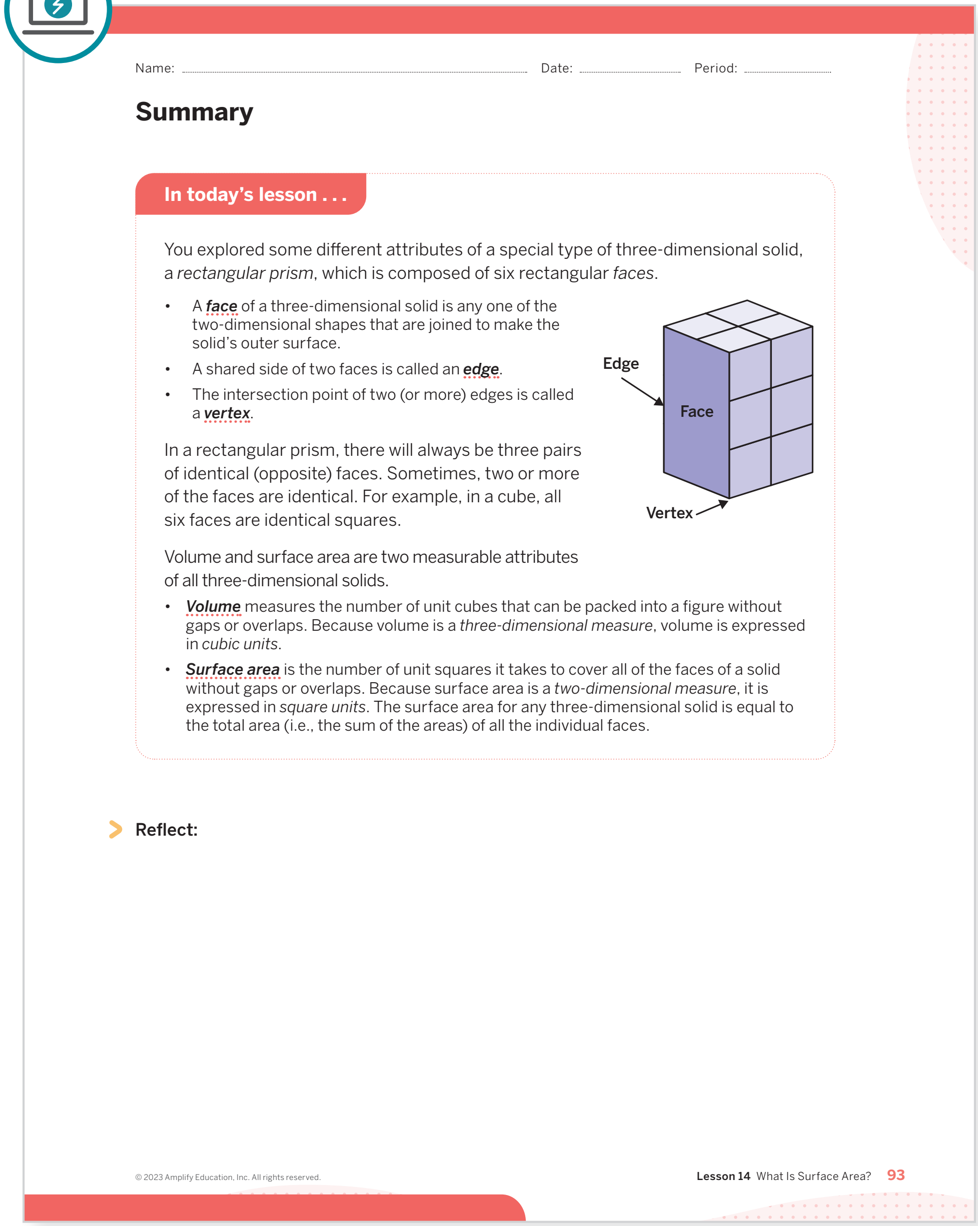
English Learners

Utilizing strategic partners and a ***Think-Pair-Write-Share***, encourage students to use their primary language to present their strategies. Then have students write the restatement of their partner's idea before orally restating in English.

Whole Class | 5 min

Summary

Review and synthesize how to calculate the surface area of a rectangular prism, and how to distinguish it from the area of polygons and volume of rectangular prisms.



Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You explored some different attributes of a special type of three-dimensional solid, a *rectangular prism*, which is composed of six rectangular *faces*.

- A ***face*** of a three-dimensional solid is any one of the two-dimensional shapes that are joined to make the solid's outer surface.
- A shared side of two faces is called an ***edge***.
- The intersection point of two (or more) edges is called a ***vertex***.

Edge

Face

Vertex

In a rectangular prism, there will always be three pairs of identical (opposite) faces. Sometimes, two or more of the faces are identical. For example, in a cube, all six faces are identical squares.

Volume and surface area are two measurable attributes of all three-dimensional solids.

- ***Volume*** measures the number of unit cubes that can be packed into a figure without gaps or overlaps. Because volume is a *three-dimensional measure*, volume is expressed in *cubic units*.
- ***Surface area*** is the number of unit squares it takes to cover all of the faces of a solid without gaps or overlaps. Because surface area is a *two-dimensional measure*, it is expressed in *square units*. The surface area for any three-dimensional solid is equal to the total area (i.e., the sum of the areas) of all the individual faces.

Reflect:

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Lesson 14 What Is Surface Area? 93

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 3. Ask them to review and reflect on any terms and phrases related to the terms *face*, *surface area*, *edge*, *vertex*, or *volume* that were added to the display during the lesson.

Synthesize

Ask:

- "How could you calculate the surface area of a rectangular prism when you can only see three faces?" Because a rectangular prism has three sets of identical, parallel sides, you can calculate the area of the three visible faces, find the sum, and then double this sum to find the surface area.
- "How are calculating surface area and calculating area alike? How are they different?" They both involve finding the number of unit squares that cover a two-dimensional region entirely without gaps and overlaps. Calculating area involves a single face (two-dimensional figure). Calculating surface area involves finding the sum of the areas of multiple faces of a three-dimensional figure.
- "How are volume and surface area similar? How are they different?" Volume and surface area both refer to measures of three-dimensional figures. Volume measures the number of three-dimensional unit cubes that compose or fill a prism. It is measured in cubic units. Surface area measures the total area of each two-dimensional face of the prism. It is measured in square units.

Highlight that the edge lengths of the faces are the critical measures to know when calculating surface area and volume. Depending on what students are calculating, the edge lengths will be used differently. Therefore, it is important to know exactly what is being determined.

Formalize vocabulary:

- ***face***
- ***surface area***
- ***edge***
- ***vertex***
- ***volume***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

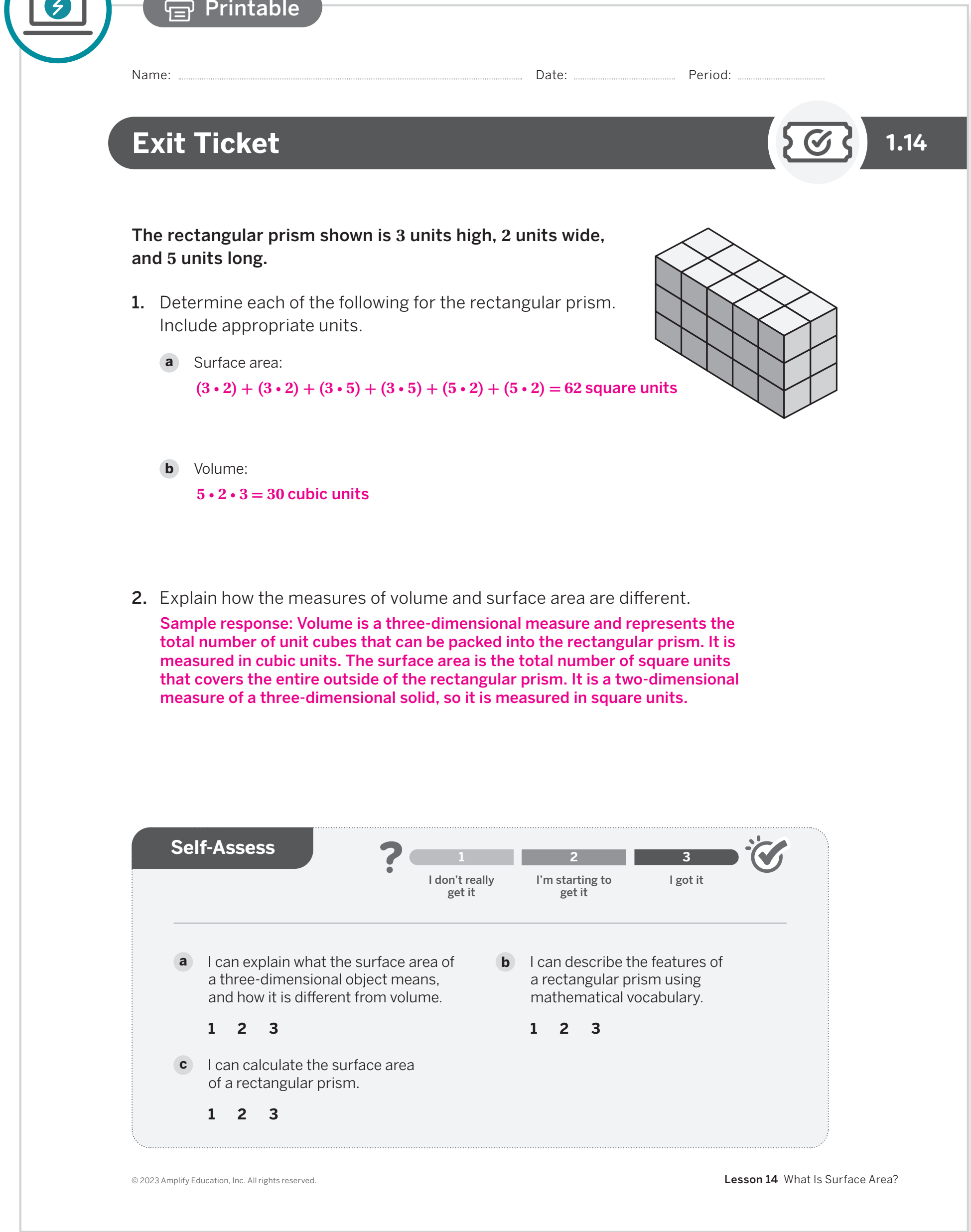
- "What is the most important thing to remember about surface area?"
- "How is surface area different from volume?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by calculating the surface area of a rectangular prism composed of unit cubes.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 1.14

The rectangular prism shown is 3 units high, 2 units wide, and 5 units long.

1. Determine each of the following for the rectangular prism. Include appropriate units.

   a. Surface area:

   (3 • 2) + (3 • 2) + (3 • 5) + (3 • 5) + (5 • 2) + (5 • 2) = 62 square units

   b. Volume:

   5 • 2 • 3 = 30 cubic units

2. Explain how the measures of volume and surface area are different.

   Sample response: Volume is a three-dimensional measure and represents the total number of unit cubes that can be packed into the rectangular prism. It is measured in cubic units. The surface area is the total number of square units that covers the entire outside of the rectangular prism. It is a two-dimensional measure of a three-dimensional solid, so it is measured in square units.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can explain what the surface area of a three-dimensional object means, and how it is different from volume.

1 2 3

b. I can describe the features of a rectangular prism using mathematical vocabulary.

1 2 3

c. I can calculate the surface area of a rectangular prism.

1 2 3

Lesson 14 What Is Surface Area?

Success looks like . . .

- **Language Goal:** Comprehending and using the terms *face*, *edge*, and *vertex* to describe rectangular prisms **(Speaking and Listening, Writing)**.
- **Language Goal:** Comprehending that the term *surface area* refers to how many square units it takes to cover all the faces of a three-dimensional object **(Speaking and Listening, Writing)**.
- **Language Goal:** Calculating the surface area of a rectangular prism, and explaining the solution method **(Speaking and Listening, Writing)**.
  - » Determining the surface area of the rectangular prism by using the square units on the faces in Problem 1.
- **Language Goal:** Comprehending that surface area and volume are two different attributes of three-dimensional objects and are measured in different units **(Speaking and Listening)**.
  - » Explaining the differences between volume and surface area in Problem 2.

Suggested next steps

If students incorrectly calculate the surface area because they:

- Multiply the length, width, and height, consider reviewing Activity 2, and asking, "Why did all of the prisms have the same volume, but some prisms had different surface areas? What is the difference between volume and surface area?"
- Only use the three visible faces, consider referring them to the prism in the *Summary* and asking, "How many faces are in a rectangular prism? What do you know about those faces? How can you use this information to calculate the surface area of a rectangular prism?"

Professional Learning

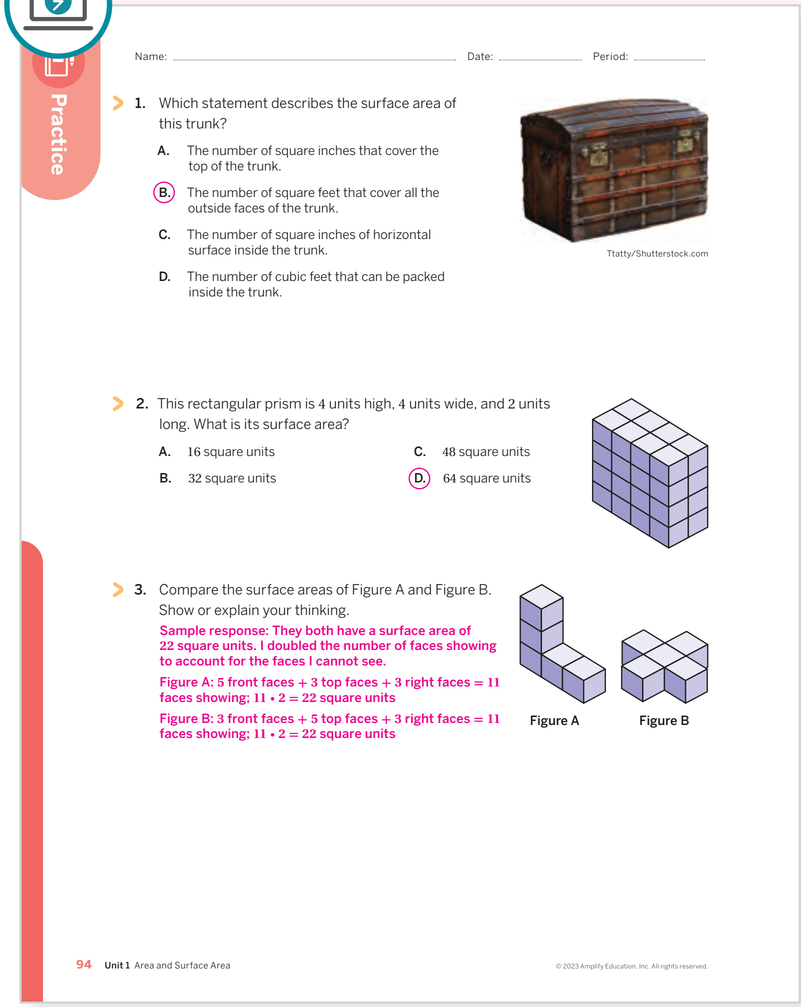
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Knowing where students need to be by the end of this unit, how did students' work in Activity 2 influence that future goal?
- What did the exploratory nature of Activity 1 reveal about your students as learners? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Which statement describes the surface area of this trunk?
   - A. The number of square inches that cover the top of the trunk.
   - [✓ B.] The number of square feet that cover all the outside faces of the trunk.
   - C. The number of square inches of horizontal surface inside the trunk.
   - D. The number of cubic feet that can be packed inside the trunk.

Ttatty/Shutterstock.com

2. This rectangular prism is 4 units high, 4 units wide, and 2 units long. What is its surface area?
   - A. 16 square units
   - B. 32 square units
   - C. 48 square units
   - [✓ D.] 64 square units

3. Compare the surface areas of Figure A and Figure B. Show or explain your thinking.

   Sample response: They both have a surface area of 22 square units. I doubled the number of faces showing to account for the faces I cannot see.

   Figure A: 5 front faces + 3 top faces + 3 right faces = 11 faces showing; $11 \cdot 2 = 22$ square units

   Figure B: 3 front faces + 5 top faces + 3 right faces = 11 faces showing; $11 \cdot 2 = 22$ square units

Figure A

Figure B

94 **Unit 1** Area and Surface Area

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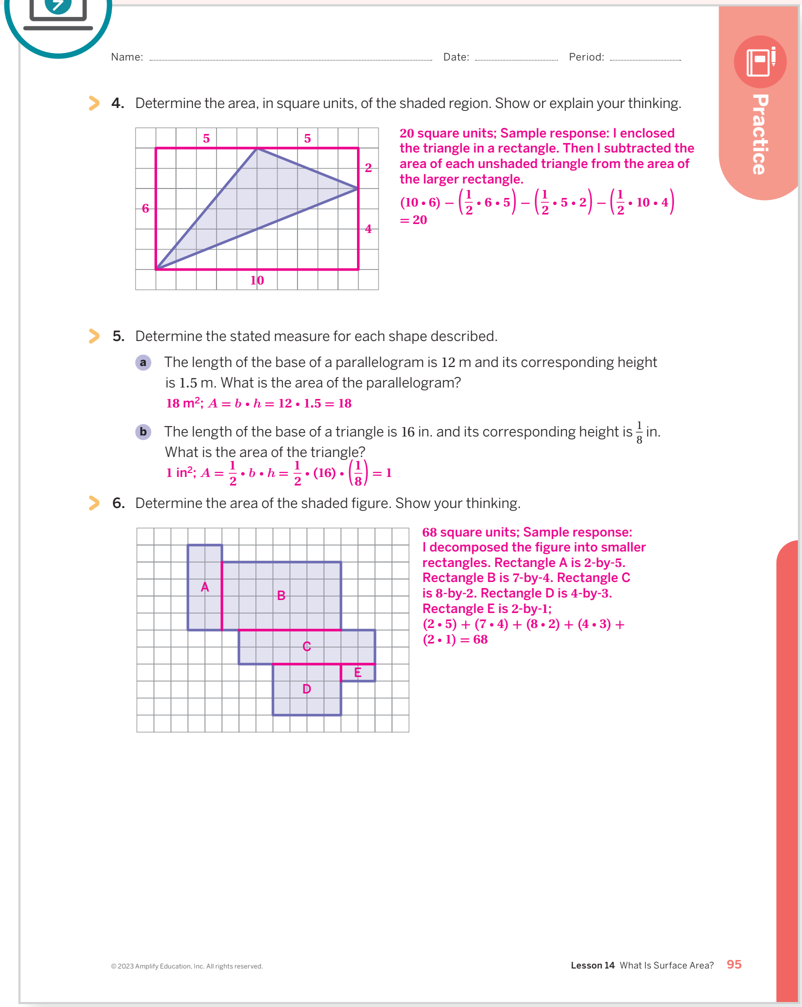

Practice

Name: Date: Period:

4. Determine the area, in square units, of the shaded region. Show or explain your thinking.

20 square units; Sample response: I enclosed the triangle in a rectangle. Then I subtracted the area of each unshaded triangle from the area of the larger rectangle.

$(10 \cdot 6) - \left(\frac{1}{2} \cdot 6 \cdot 5\right) - \left(\frac{1}{2} \cdot 5 \cdot 2\right) - \left(\frac{1}{2} \cdot 10 \cdot 4\right) = 20$

5. Determine the stated measure for each shape described.
   - a. The length of the base of a parallelogram is 12 m and its corresponding height is 1.5 m. What is the area of the parallelogram?

     18 m^2; $A = b \cdot h = 12 \cdot 1.5 = 18$
   - b. The length of the base of a triangle is 16 in. and its corresponding height is $\frac{1}{8}$ in. What is the area of the triangle?

     1 in^2; $A = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot (16) \cdot \left(\frac{1}{8}\right) = 1$

6. Determine the area of the shaded figure. Show your thinking.

68 square units; Sample response: I decomposed the figure into smaller rectangles. Rectangle A is 2-by-5. Rectangle B is 7-by-4. Rectangle C is 8-by-2. Rectangle D is 4-by-3. Rectangle E is 2-by-1;
$(2 \cdot 5) + (7 \cdot 4) + (8 \cdot 2) + (4 \cdot 3) + (2 \cdot 1) = 68$

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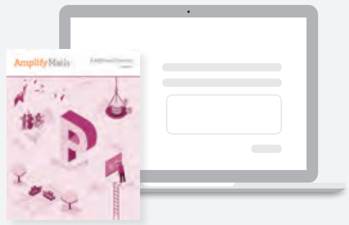
Lesson 14 What Is Surface Area? 95

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 10	2
	5	Unit 1 Lesson 11	2
Formative ⚡	6	Unit 1 Lesson 15	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 15**

Nets and Surface Area of Rectangular Prisms

Let's use nets to calculate the surface area of rectangular prisms.

Focus

Goals

1. **Language Goal:** Understand that the term *net* refers to a two-dimensional figure that can be assembled to form a three-dimensional solid. **(Speaking and Listening, Writing)**
2. **Language Goal:** Use a net with or without gridlines to calculate the surface area of a rectangular prism, and explain the method used. **(Writing)**
3. Draw a net for a given rectangular prism.

Coherence

Today

Students further develop their capacity to reason about surface area as they explore the relationship between a rectangular prism and its two-dimensional net. Using a concrete demonstration of a rectangular prism "unfolding," students label each face in the unfolded net. They then apply their work with area of two-dimensional shapes to calculate the prism's surface area. In the context of comparing the size of boxes, students mentally unfold three-dimensional shapes, draw nets, and use nets to calculate surface area. They continue to compare and contrast surface area and volume as distinct measures of a three-dimensional solid.

Previously

In Lesson 14, students defined surface area as a measure of three-dimensional solids and explored how it is distinct from volume. They used tiling of unit squares and the formula for the area of a rectangle to calculate the surface area.

Coming Soon

In Lesson 16, students will define and classify polyhedra as prisms or pyramids. Extending previous work with rectangular prisms, students will interpret and draw nets to calculate the surface area of prisms and pyramids.

Rigor

- Students build their **conceptual understanding** of two-dimensional nets of three-dimensional rectangular prisms.
- Students develop **procedural fluency** with using nets to determine the surface area of rectangular prisms.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

New word

- *net*

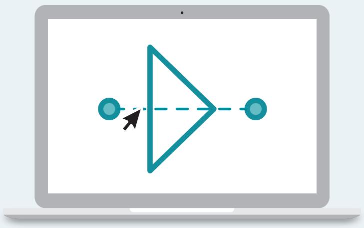
Review words

- *edge*
- *face*
- *surface area*
- *vertex*
- *volume*

Amps | Featured Activity

Warm-up
Interactive "Unfolding" of Rectangular Prisms

Students can "unfold" and "refold" a rectangular prism to recognize the relationship between the three-dimensional solid and its two-dimensional net.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated when a pattern or structure to calculate surface area is not immediately apparent. Encourage students to make sense of problems by connecting to their previous work with the surface area of the cabinet in Lesson 14. Ask, "How did you calculate the surface area of the cabinet? What drawings were helpful? How can you use that same thinking here?"

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

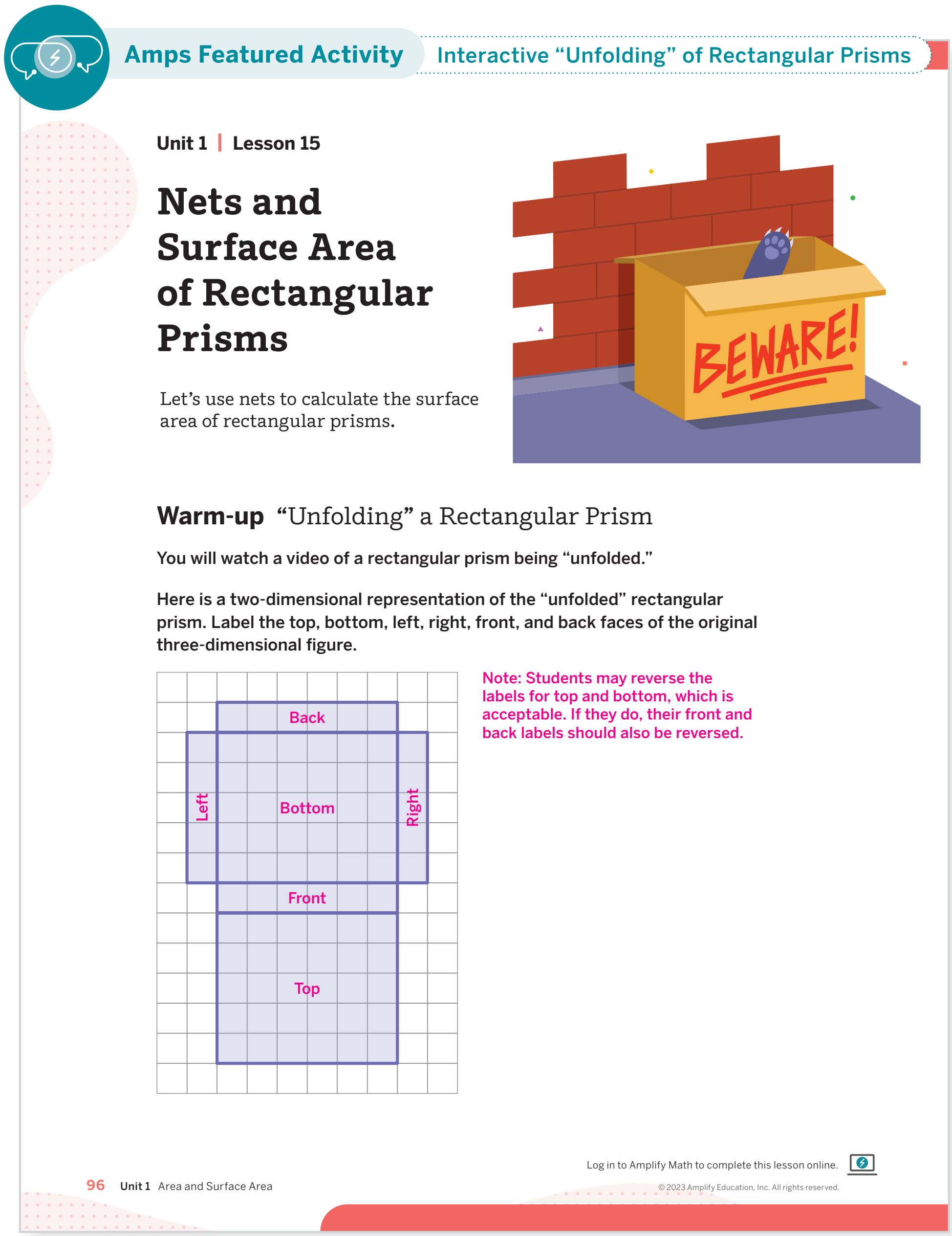
- Combine the **Warm-up** and **Activity 1** into one activity. You may also consider omitting Activity 1, Problem 2.
- In **Activity 2,** Problem 3 may be omitted.

Independent | 5 min

Warm-up "Unfolding" a Rectangular Prism

Students watch a rectangular prism being "unfolded," labeling the parts of the 3D figure on its 2D representation, which prepares them to work with nets in the upcoming activities.

Amps Featured Activity Interactive "Unfolding" of Rectangular Prisms



Unit 1 | Lesson 15

Nets and Surface Area of Rectangular Prisms

Let's use nets to calculate the surface area of rectangular prisms.

Warm-up "Unfolding" a Rectangular Prism

You will watch a video of a rectangular prism being "unfolded."

Here is a two-dimensional representation of the "unfolded" rectangular prism. Label the top, bottom, left, right, front, and back faces of the original three-dimensional figure.

Note: Students may reverse the labels for top and bottom, which is acceptable. If they do, their front and back labels should also be reversed.

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Log in to Amplify Math to complete this lesson online.

1 Launch

Display the animation, Unfolding a Rectangular Prism, from the Warm Up Amps slides, before having students work independently on the Warm-up.

2 Monitor

Help students get started by having them rewatch the video and track the movement of the top and bottom faces.

Look for points of confusion:

- **Labeling the faces based on their placement in the 2D representation (e.g., labeling the back as the top and the top as the bottom).** Ask, "In the 3D prism, which faces share an edge?" Have students identify one face to label and use the known shared edges to continue labeling.

Look for productive strategies:

- Using identical rectangles to label pairs of faces.
- Using shared edges in the 3D prism to identify and label faces in its 2D representation.

3 Connect

Have students share how they used identical rectangles and shared edges to label faces.

Define a ***net*** as a two-dimensional representation of a three-dimensional solid that shows all of its faces. Add this term and a diagram to the anchor chart.

Highlight that the net of a rectangular prism shows all 6 faces, all 12 edges, and all 8 vertices. Most of the faces that share an edge in the solid will also share an edge in the net. The net can be cut out and folded along the edges to construct the prism.

Ask, "If the bottom face was relabeled as the top, how would the other labels change? Would the rebuilt 3D prism look the same?" The front and back faces would need relabeled, but the prism looks the same.

Power-up

To power up students' ability to decompose a figure on a grid to determine its area, have students complete:

Use the following steps to determine the area of the figure:

a. Add one line to the figure to separate it into two rectangles.

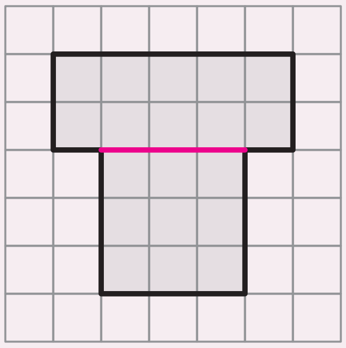
b. Determine the area of each rectangle.

10 units^2; $5 \cdot 2 = 10$ and

9 units^2; $3 \cdot 3 = 9$

c. Determine the total area.

19 units^2; $10 + 9 = 19$



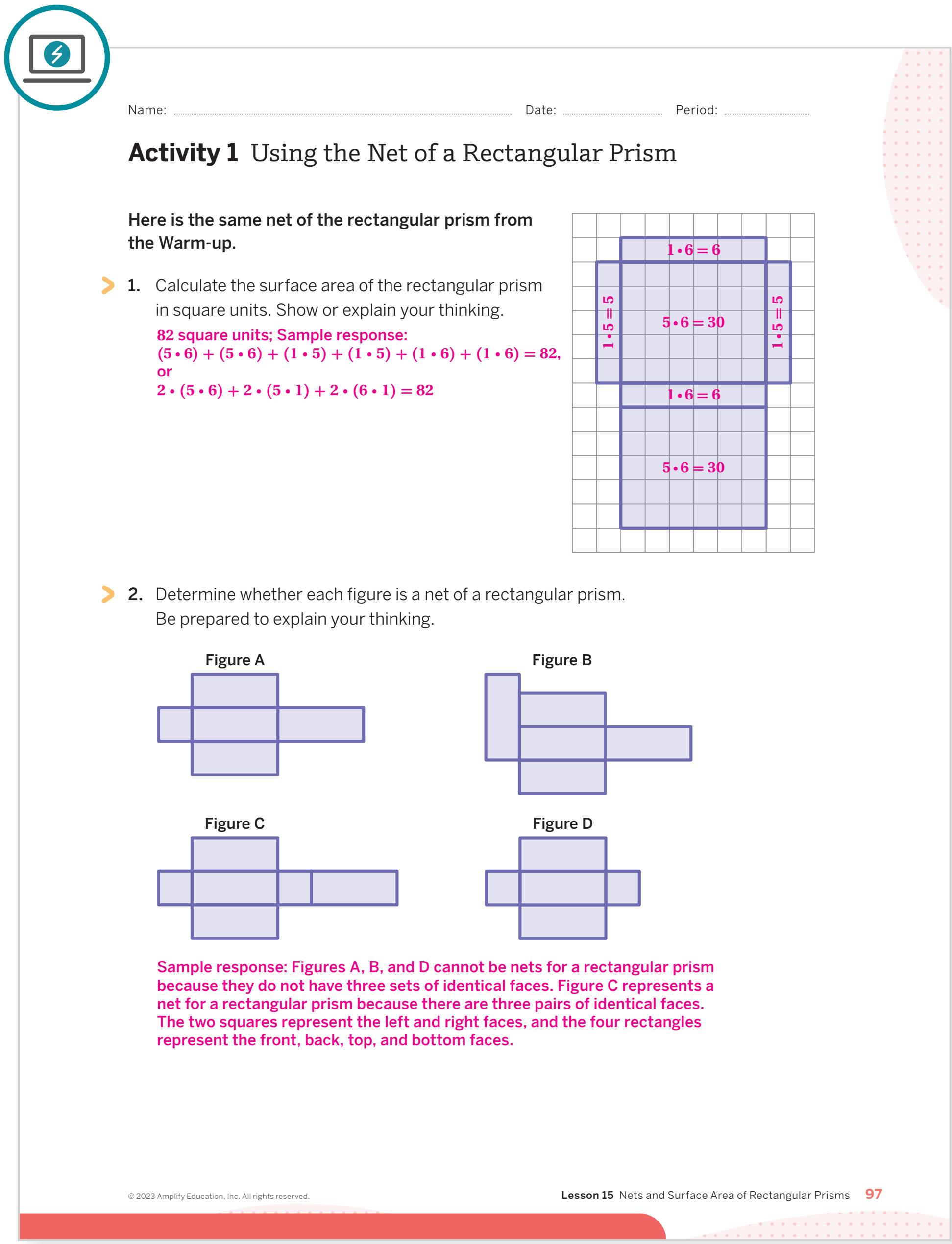
Use: Before the Warm-up.

Informed by: Performance on Lesson 14, Practice Problem 6.

Pairs | 10 min

Activity 1 Using the Net of a Rectangular Prism

Students calculate the surface area of a rectangular prism using its net drawn on a grid to illustrate how a net can be used to determine the surface area.



Name: Date: Period:

Activity 1 Using the Net of a Rectangular Prism

Here is the same net of the rectangular prism from the Warm-up.

1. Calculate the surface area of the rectangular prism in square units. Show or explain your thinking.

   82 square units; Sample response:
   $(5 \cdot 6) + (5 \cdot 6) + (1 \cdot 5) + (1 \cdot 5) + (1 \cdot 6) + (1 \cdot 6) = 82$,
   or
   $2 \cdot (5 \cdot 6) + 2 \cdot (5 \cdot 1) + 2 \cdot (6 \cdot 1) = 82$

2. Determine whether each figure is a net of a rectangular prism. Be prepared to explain your thinking.

   Sample response: Figures A, B, and D cannot be nets for a rectangular prism because they do not have three sets of identical faces. Figure C represents a net for a rectangular prism because there are three pairs of identical faces. The two squares represent the left and right faces, and the four rectangles represent the front, back, top, and bottom faces.

Lesson 15 Nets and Surface Area of Rectangular Prisms 97

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by having them identify the shape of each face. Ask, "How do you determine the area of a rectangle?"

Look for points of confusion:

- **Missing or double-counting the area of a face in Problem 1.** Ask, "How can you record your calculations to ensure you used the correct measurements and accounted for all of the faces?"
- **Not verifying the number of faces shown in Problem 2.** Remind students that a net shows all faces of a rectangular prism.
- **Not attending to the number of faces of each size in Problem 2.** Remind students that the net of a rectangular prism has three sets of identical and parallel rectangles.

Look for productive strategies:

- Calculating and then adding the area of each face.
- Grouping rectangles together and determining the area of the composite shape, e.g., identifying identical faces or common edge lengths.
- Recognizing that a rectangular prism's net should have three sets of identical rectangles in Problem 2.

3 Connect

Have students share, first, how their different strategies for Problem 1 demonstrate that surface area is the total area of all faces, followed by how they can determine whether a given 2D representation is in fact a net of a rectangular prism.

Ask, "How do nets show that surface area is a 2D measure of a 3D solid?"

Highlight the benefits of using nets to calculate surface area systematically.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Optimize Access to Tools*

If students need more processing time, have them focus on completing Problem 1. Then if they have time available, work on Problem 2. You may also consider providing students with concrete manipulatives, such as a pre-assembled rectangular prism and a pre-cut net.

Extension: *Math Enrichment*

Challenge students to draw correct nets for Figures A, B, and D in Problem 2.

Math Language Development

MLR3: Critique, Correct, Clarify

Prior to students completing Problem 2, provide them with an incorrect statement, such as "Figure A represents the net of a rectangular prism because the faces are all rectangles."

Critique: Have students determine whether they agree or disagree.

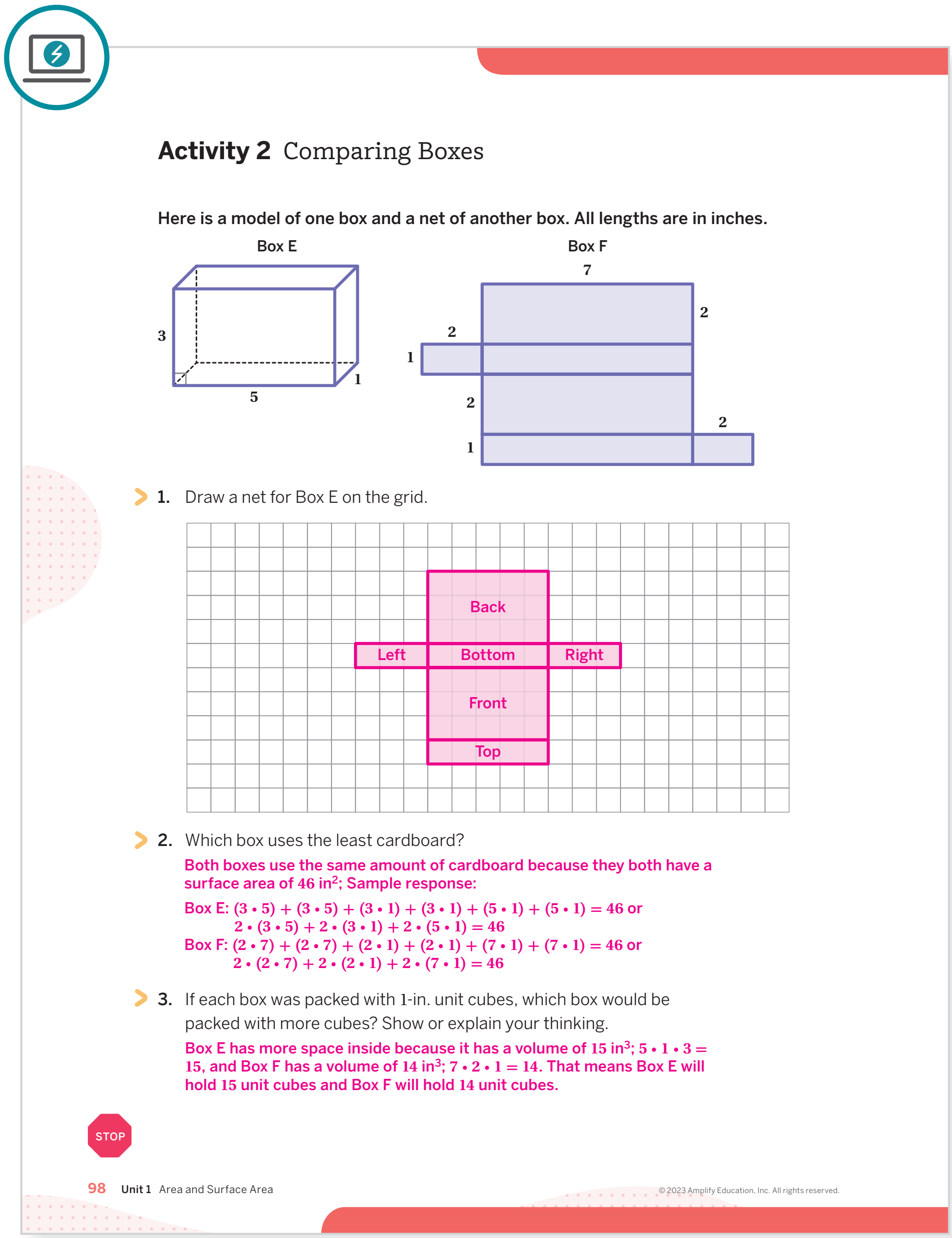
Correct: Have students write a first draft of a corrected statement.

Clarify: Have students work with a partner to review and revise their statements using correct mathematical language. One example of a correct statement is "Figure A does not represent the net of a rectangular prism because the net does not have three sets of identical faces."

Pairs | 20 min

Activity 2 Comparing Boxes

Students use both solids and nets, drawn on or off grids, to compare surface areas and volumes of two rectangular prisms.



Activity 2 Comparing Boxes

Here is a model of one box and a net of another box. All lengths are in inches.

1. Draw a net for Box E on the grid.

2. Which box uses the least cardboard?

   Both boxes use the same amount of cardboard because they both have a surface area of 46 in^2; Sample response:

   Box E: $(3 \cdot 5) + (3 \cdot 5) + (3 \cdot 1) + (3 \cdot 1) + (5 \cdot 1) + (5 \cdot 1) = 46$ or $2 \cdot (3 \cdot 5) + 2 \cdot (3 \cdot 1) + 2 \cdot (5 \cdot 1) = 46$

   Box F: $(2 \cdot 7) + (2 \cdot 7) + (2 \cdot 1) + (2 \cdot 1) + (7 \cdot 1) + (7 \cdot 1) = 46$ or $2 \cdot (2 \cdot 7) + 2 \cdot (2 \cdot 1) + 2 \cdot (7 \cdot 1) = 46$

3. If each box was packed with 1-in. unit cubes, which box would be packed with more cubes? Show or explain your thinking.

   Box E has more space inside because it has a volume of 15 in^3; $5 \cdot 1 \cdot 3 = 15$, and Box F has a volume of 14 in^3; $7 \cdot 2 \cdot 1 = 14$. That means Box E will hold 15 unit cubes and Box F will hold 14 unit cubes.

STOP

98 **Unit 1** Area and Surface Area

1 Launch

Explain that most real boxes would not unfold into a net of a prism because they contain overlapping flaps.

2 Monitor

Help students get started by having them draw each face for Box E in isolation, and then have label the edge lengths.

Look for points of confusion:

- **Drawing the net incorrectly in Problem 1.** Ask, "Which faces share an edge in Box E? How can your net show this?"
- **Misidentifying the measure required to find the amount of cardboard needed in Problem 2.** Remind students that surface area is the space around the outside and volume is the space inside.
- **Struggling to calculate the volume from a net in Problem 3.** Have students sketch the prism for the net and label the edge lengths.

Look for productive strategies:

- Using sides of equal length to draw a net for Box E, and drawing a 3D solid for Box F.
- Labeling edges and correctly and using them to determine the areas of rectangles.
- Recognizing that Problem 2 refers to surface area and Problem 3 refers to volume.

3 Connect

Have pairs of students share different nets, explaining how they represent the same rectangular prism. Then have students share how they knew when to calculate surface area versus volume, followed by how they used nets and drawings to complete the calculations.

Highlight that images of both solids and nets that include measurements can be used to calculate surface area and volume, but nets are more helpful when calculating surface area.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

If students need more processing time, have them focus on completing Problems 1 and 2 by comparing and documenting the differences in area for each corresponding face in the prisms. Consider providing graph paper for students to draw a scaled net of Box E and to redraw Box F for a more direct comparison.

Extension: *Math Enrichment*

Challenge students to draw a net for a box that would have the same volume as either Box E or Box F, but a different surface area.

Math Language Development

MLR8: Discussion Supports

During the Connect, have students write a response to the prompt, "If two prisms have the same surface area, their volume will always/sometimes/never be the same because . . ." As they share, point out how the use of examples and counterexamples help justify their reasoning.

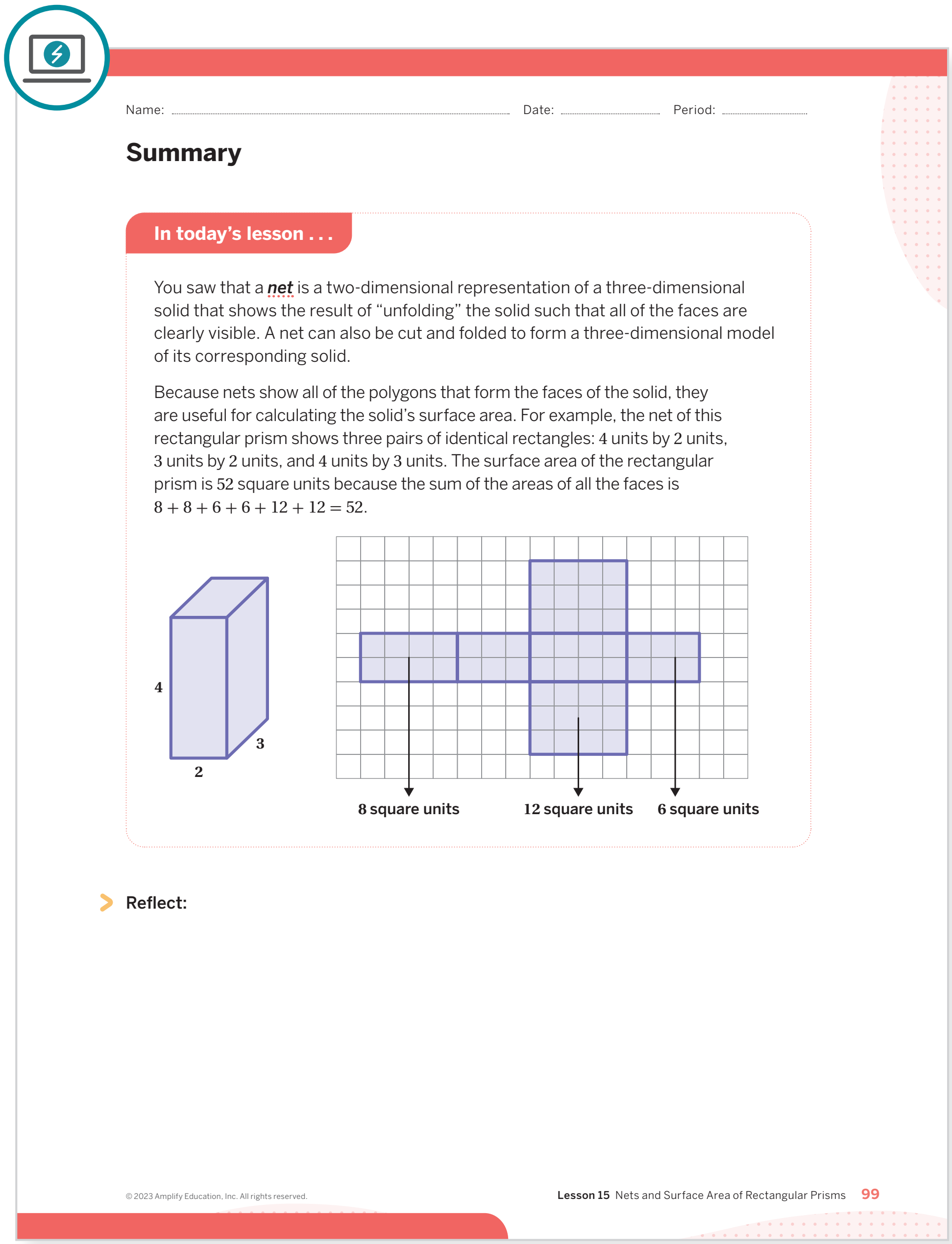
English Learners

Encourage students to include drawings in the examples and counterexamples.

Whole Class | 5 min

Summary

Review and synthesize how to interpret and use nets to calculate the surface area of a rectangular prism.



Name: Date: Period:

Summary

In today's lesson . . .

You saw that a ***net*** is a two-dimensional representation of a three-dimensional solid that shows the result of "unfolding" the solid such that all of the faces are clearly visible. A net can also be cut and folded to form a three-dimensional model of its corresponding solid.

Because nets show all of the polygons that form the faces of the solid, they are useful for calculating the solid's surface area. For example, the net of this rectangular prism shows three pairs of identical rectangles: 4 units by 2 units, 3 units by 2 units, and 4 units by 3 units. The surface area of the rectangular prism is 52 square units because the sum of the areas of all the faces is $8 + 8 + 6 + 6 + 12 + 12 = 52$.

Reflect:

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Lesson 15 Nets and Surface Area of Rectangular Prisms 99

Synthesize

Ask:

- "Are there ways to simplify the calculations used to determine surface area? Or is it more beneficial to find the area of each rectangle one at a time?" When there are identical polygons, I can find the area of one polygon and multiply it by the number of identical polygons in the net. I can also combine polygons and find the area of the combined region (e.g., a group of rectangles with the same side length).
- "How is calculating surface area using a net different from calculating surface area by looking at an image of a rectangular prism?" A net allows me to see all the faces of a rectangular prism at once. When working from a picture or drawing, I need to visualize the hidden faces.
- "When using a net, how do you keep track of calculations and make sure all faces are accounted for?" I can label all the faces and the calculations for each.

Formalize vocabulary: *net*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How does a two-dimensional net reflect a three-dimensional rectangular prism?"
- "How did you calculate surface area today?"

Math Language Development

MLR2: Collect and Display

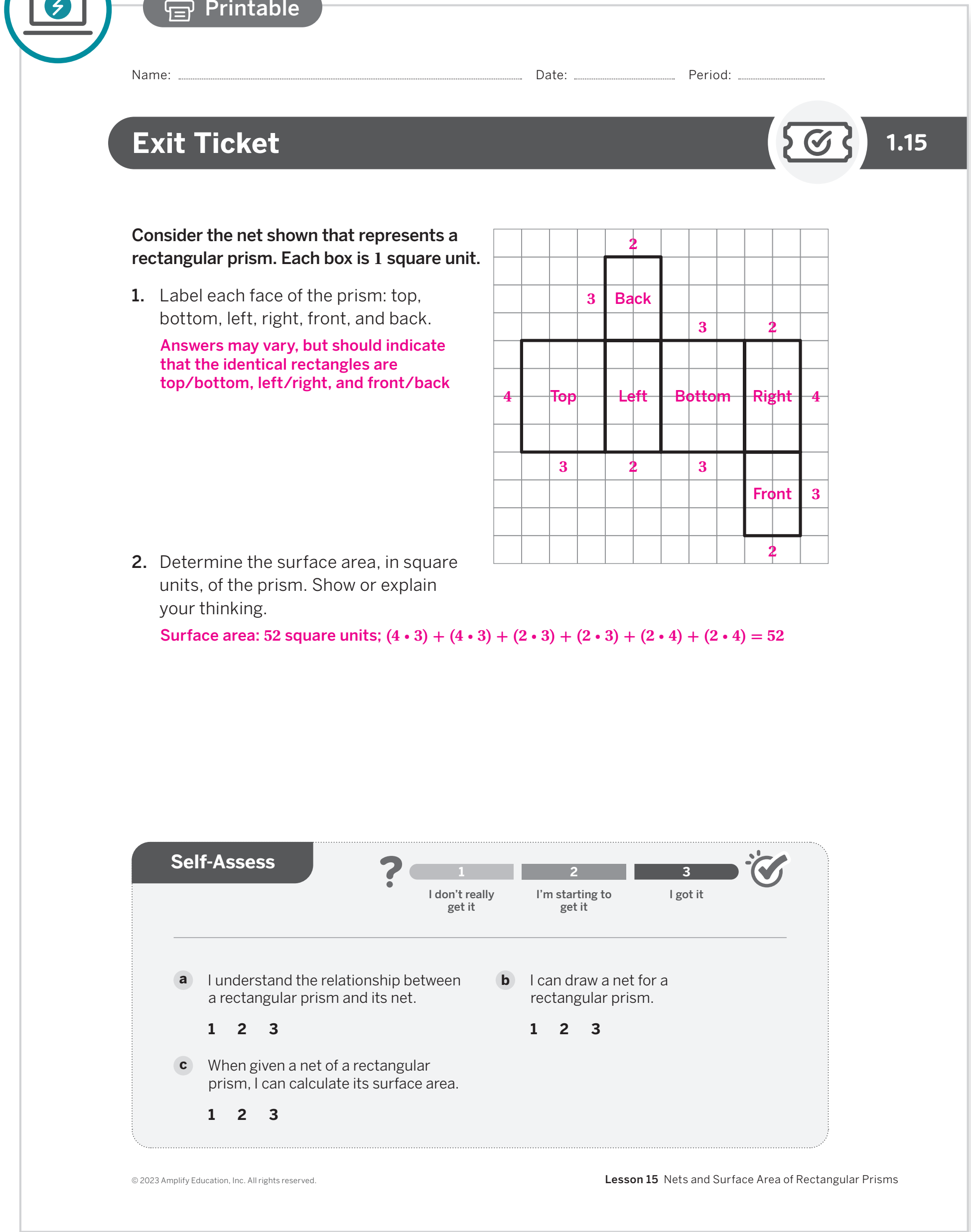
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 3. Ask them to review and reflect on any terms and phrases related to the term *net* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by labeling the faces of a rectangular prism's net and using the net to calculate the prism's surface area.

Printable



Name: Date: Period:

Exit Ticket 1.15

Consider the net shown that represents a rectangular prism. Each box is 1 square unit.

1. Label each face of the prism: top, bottom, left, right, front, and back.
   Answers may vary, but should indicate that the identical rectangles are top/bottom, left/right, and front/back

2. Determine the surface area, in square units, of the prism. Show or explain your thinking.
   Surface area: 52 square units; (4 • 3) + (4 • 3) + (2 • 3) + (2 • 3) + (2 • 4) + (2 • 4) = 52

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I understand the relationship between a rectangular prism and its net.
1 2 3

b I can draw a net for a rectangular prism.
1 2 3

c When given a net of a rectangular prism, I can calculate its surface area.
1 2 3

Lesson 15 Nets and Surface Area of Rectangular Prisms

Success looks like . . .

- **Language Goal:** Understanding that the term *net* refers to a two-dimensional figure that can be assembled to form a three-dimensional solid **(Speaking and Listening, Writing)**.
  - » Labeling a net to show how it represents a rectangular prism in Problem 1.
- **Language Goal:** Using a net (on or off a grid) to calculate the surface area of a rectangular prism, and explain the method used **(Writing)**
  - » Determining the surface area of a rectangular prism using its net in Problem 2.
- **Goal:** Drawing a net for a given rectangular prism.

Suggested next steps

If students mislabel the net, consider:

- Reviewing Activity 2, Problem 1. Have students label the bottom face, and ask, "How can you use shared edges to label each face?"

If students calculate surface area incorrectly (e.g., miss a face or double count faces) or inefficiently (e.g., counting every individual unit square), consider:

- Reviewing Activity 1 and asking, "How did you make sure you counted, and did not double count, each face? How did you calculate the surface area of this rectangular prism? Were there ways to simplify those calculations?"

Professional Learning

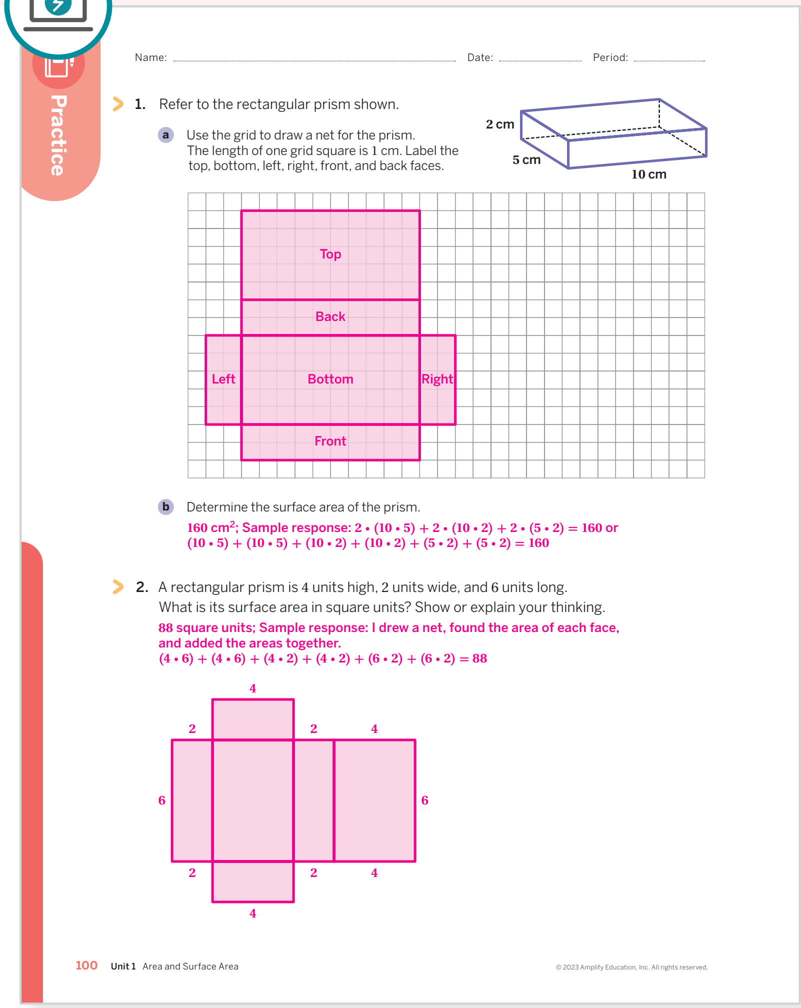
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students determined the surface area of a rectangular prism. How did that build on the earlier work students did with area?
- What was especially satisfying about students' work in Activity 2? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Refer to the rectangular prism shown.

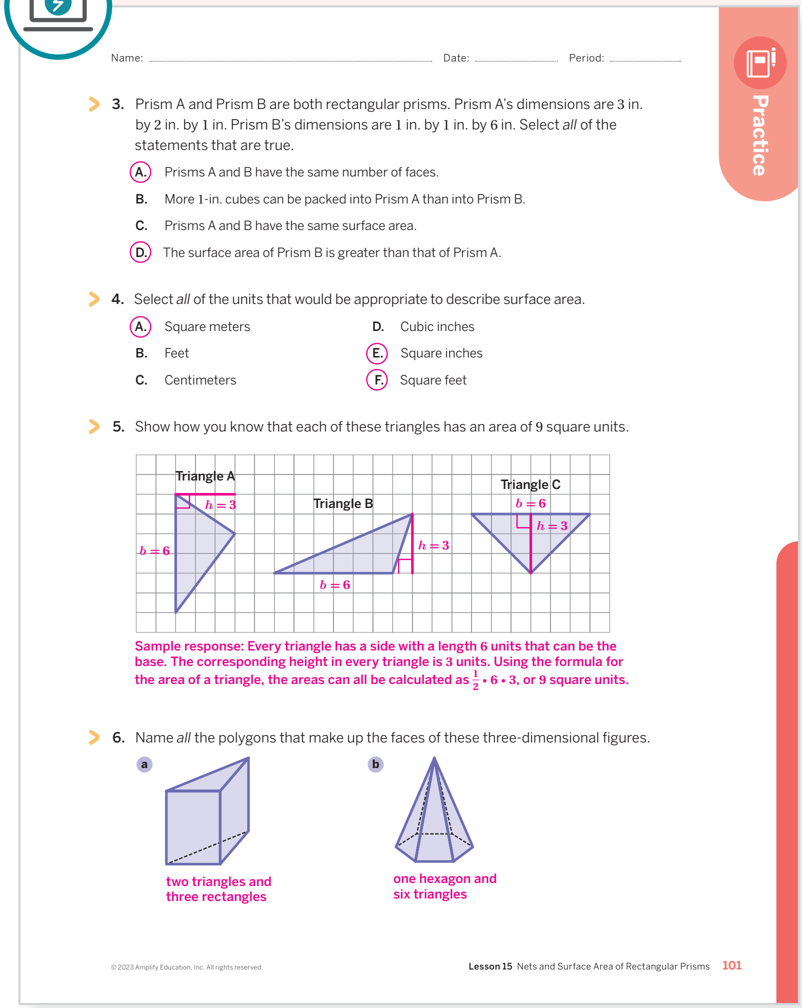
   a. Use the grid to draw a net for the prism. The length of one grid square is 1 cm. Label the top, bottom, left, right, front, and back faces.

   b. Determine the surface area of the prism.

   160 cm^2; Sample response: $2 \cdot (10 \cdot 5) + 2 \cdot (10 \cdot 2) + 2 \cdot (5 \cdot 2) = 160$ or $(10 \cdot 5) + (10 \cdot 5) + (10 \cdot 2) + (10 \cdot 2) + (5 \cdot 2) + (5 \cdot 2) = 160$

2. A rectangular prism is 4 units high, 2 units wide, and 6 units long. What is its surface area in square units? Show or explain your thinking.

   88 square units; Sample response: I drew a net, found the area of each face, and added the areas together.
   $(4 \cdot 6) + (4 \cdot 6) + (4 \cdot 2) + (4 \cdot 2) + (6 \cdot 2) + (6 \cdot 2) = 88$



Practice

Name: ______ Date: ______ Period: ______

3. Prism A and Prism B are both rectangular prisms. Prism A's dimensions are 3 in. by 2 in. by 1 in. Prism B's dimensions are 1 in. by 1 in. by 6 in. Select *all* of the statements that are true.

   [✓ A.] Prisms A and B have the same number of faces.
   B. More 1-in. cubes can be packed into Prism A than into Prism B.
   C. Prisms A and B have the same surface area.
   [✓ D.] The surface area of Prism B is greater than that of Prism A.

4. Select *all* of the units that would be appropriate to describe surface area.

   [✓ A.] Square meters
   B. Feet
   C. Centimeters
   D. Cubic inches
   [✓ E.] Square inches
   [✓ F.] Square feet

5. Show how you know that each of these triangles has an area of 9 square units.

   Sample response: Every triangle has a side with a length 6 units that can be the base. The corresponding height in every triangle is 3 units. Using the formula for the area of a triangle, the areas can all be calculated as $\frac{1}{2} \cdot 6 \cdot 3$, or 9 square units.

6. Name *all* the polygons that make up the faces of these three-dimensional figures.

   a. two triangles and three rectangles

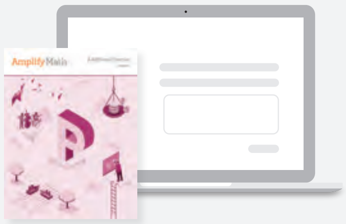
   b. one hexagon and six triangles

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 14	1
	5	Unit 1 Lesson 9	2
Formative ⚡	6	Unit 1 Lesson 16	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

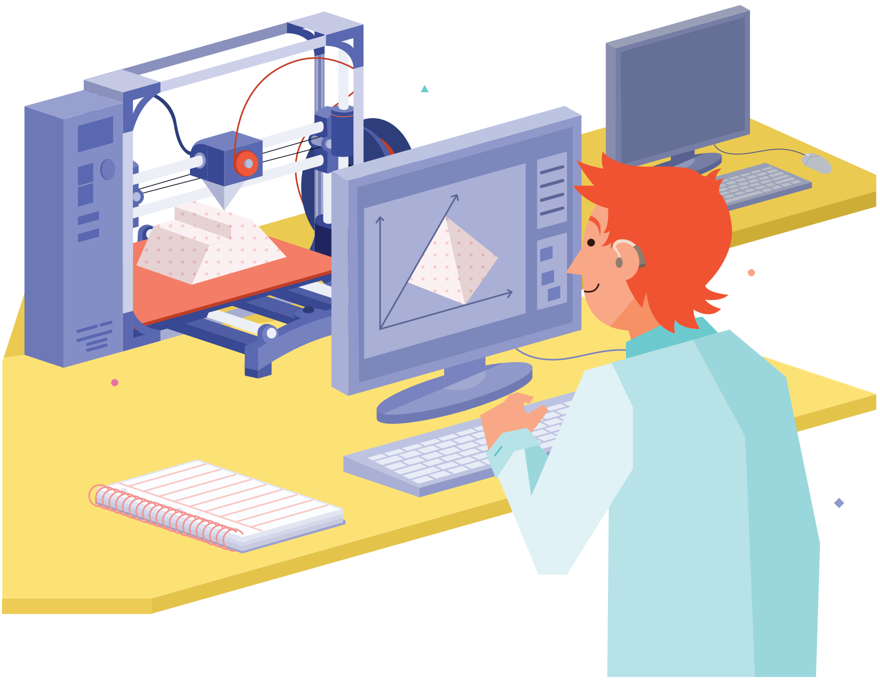


For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 16**

Nets and Surface Area of Prisms and Pyramids

Let's use nets to calculate the surface areas of other polyhedra.



Focus

Goals

1. **Language Goal:** Describe the features of a polyhedron, and compare and contrast features of prisms and pyramids, using the terms *face*, *edge*, *vertex*, and *base*. **(Speaking and Listening, Writing)**
2. **Language Goal:** Visualize and identify the polyhedron that can be assembled from a given net. **(Speaking and Listening)**
3. **Language Goal:** Use a net, with or without gridlines, to calculate the surface area of a prism or pyramid, and explain the method used. **(Writing)**
4. Identify and draw a net for a given prism or pyramid.

Coherence

Today

Students extend previous work with rectangular prisms and their nets, using precise vocabulary to analyze and define the features of polyhedra, prisms, and pyramids. They visualize the polyhedron that could be assembled from a given net, and they use the net to calculate its surface area. Students then work backward, visualizing unfolding a polyhedron, drawing a net, and using the net to calculate its surface area. As they compare and evaluate their drawings and strategies with those of their peers, students recognize that although there are multiple nets for a given solid, the total surface area remains the same.

Previously

In Lessons 14 and 15, students determined the surface area of rectangular prisms when given a net, drawing, or 3D model.

Coming Soon

In Lesson 17, students will calculate the surface area of a polyhedron with 26 square and triangular faces and use its net to construct the 3D solid.

Rigor

- Students build **conceptual understanding** of the nets and surface area of polyhedra.
- Students develop **procedural fluency** with using nets to determine the surface area of polyhedra.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, pre-cut cards, one set per student
- Warm-up PDF, *Defining Polyhedra* (for display)
- Activity 2 PDF, pre-cut cards, one per student
- Activity 2 PDF (answers)

Math Language Development

New words

- ***base*** (of a prism)
- ***base*** (of a pyramid)
- ***polyhedron***
- ***prism***
- ***pyramid***

Review words

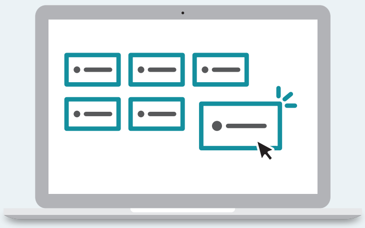
- *edge*
- *face*
- *net*
- *surface area*
- *vertex*

Amps | **Featured Activity**

Warm-up

Digital Sorting

Students group three-dimensional solids by selecting them on screen.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might be intimidated by the number of calculations required to find surface area. Ask how they will motivate themselves to be persistent and not give up. Encourage them to look for a pattern or structure that can give them confidence in their abilities. If students struggle, point out that finding surface area using nets is similar to finding the area of composite figures.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

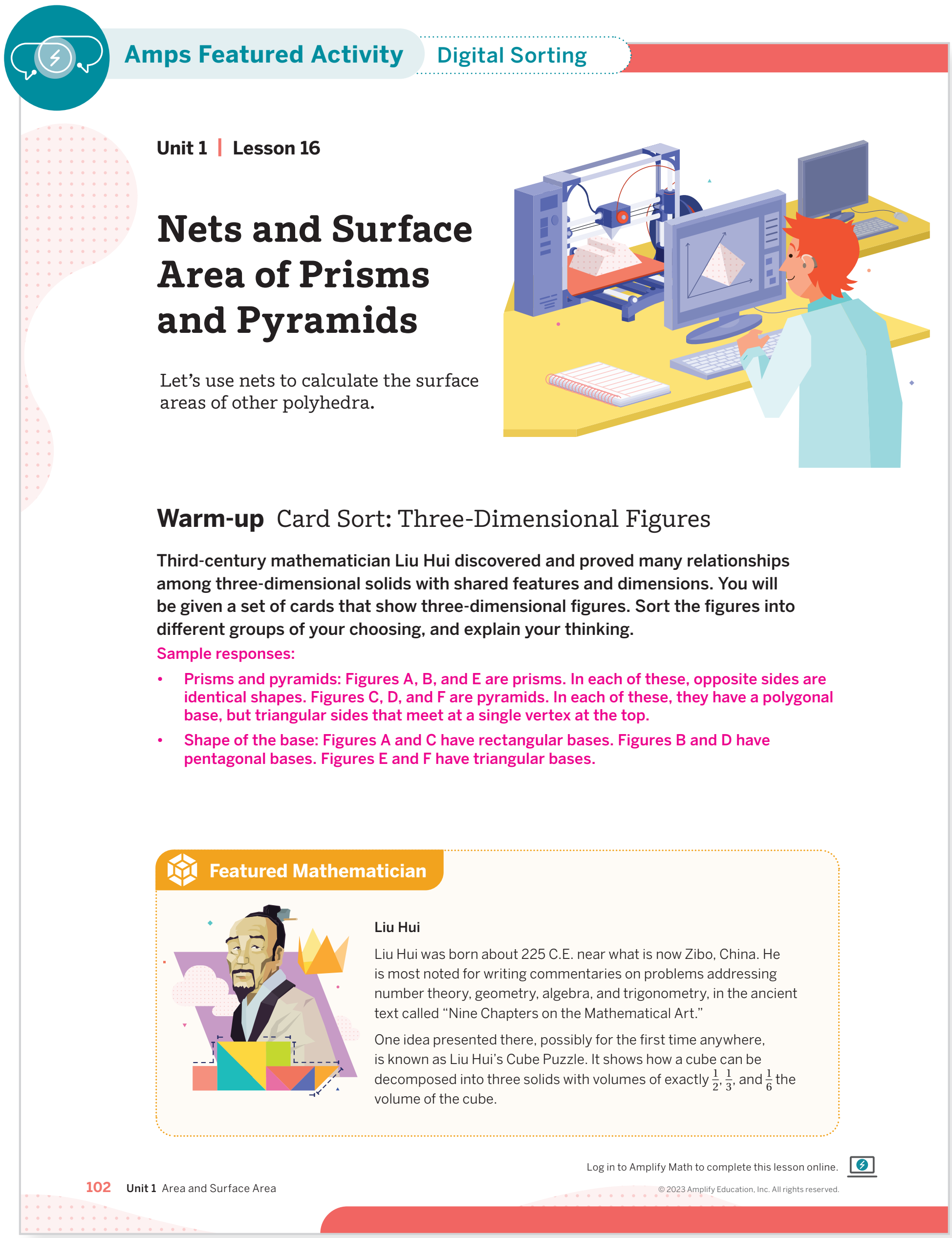
- In the **Warm-up**, display the *Defining Polyhedra* PDF and define *polyhedron*. Then show the Card Sort PDF, focusing on Figures B and C to define the terms *prism* and *pyramid* using the related terms *base*, *vertex*, and *face*. Give students 1 minute to consider whether the others figures are polyhedra.
- In **Activity 1**, Problem 1 may be omitted.
- **Activity 2** may be omitted.

Independent | 10 min

Warm-up Card Sort: Three-Dimensional Figures

Students sort cards to group three-dimensional figures by reasoning about their attributes. They use these groupings to define and classify *polyhedra*, *prisms*, and *pyramids*.

Amps Featured Activity Digital Sorting



Unit 1 | Lesson 16

Nets and Surface Area of Prisms and Pyramids

Let's use nets to calculate the surface areas of other polyhedra.

Warm-up Card Sort: Three-Dimensional Figures

Third-century mathematician Liu Hui discovered and proved many relationships among three-dimensional solids with shared features and dimensions. You will be given a set of cards that show three-dimensional figures. Sort the figures into different groups of your choosing, and explain your thinking.

Sample responses:

- Prisms and pyramids: Figures A, B, and E are prisms. In each of these, opposite sides are identical shapes. Figures C, D, and F are pyramids. In each of these, they have a polygonal base, but triangular sides that meet at a single vertex at the top.
- Shape of the base: Figures A and C have rectangular bases. Figures B and D have pentagonal bases. Figures E and F have triangular bases.

Featured Mathematician

Liu Hui

Liu Hui was born about 225 C.E. near what is now Zibo, China. He is most noted for writing commentaries on problems addressing number theory, geometry, algebra, and trigonometry, in the ancient text called "Nine Chapters on the Mathematical Art."

One idea presented there, possibly for the first time anywhere, is known as Liu Hui's Cube Puzzle. It shows how a cube can be decomposed into three solids with volumes of exactly $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$ the volume of the cube.

Log in to Amplify Math to complete this lesson online.

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1 Launch

Distribute the cards from the Warm-up PDF. Explain that dotted lines are used to show edges of "hidden" faces of a 3D figure in 2D space.

2 Monitor

Help students get started by having them name the different polygons that make up the faces of each solid.

Look for productive strategies:

- Sorting cards into groups based on:
  - » the total number of faces.
  - » solids with at least one triangular face.
  - » the shape of the bottom (base).
  - » the shape of the top (e.g., "flat" or "pointy").
  - » the solid being a prism or a pyramid.

3 Connect

Have students share different grouping methods. Encourage students to use vocabulary, such as *edge*, *vertex*, and *face*.

Define and add to the anchor chart:

- A ***prism*** as a 3D figure with two parallel, identical *faces* called ***bases*** that are connected by a set of rectangles.
- A ***pyramid*** as a 3D figure with one base and a set of triangular faces that meet at a singular vertex.

Highlight that Figures A, B, and E are prisms, and Figures C, D, and F are pyramids. They are named for the shape of their base (Figure B: pentagonal prism, Figure D: pentagonal pyramid).

Display the Warm-up PDF *Defining Polyhedra*. Explain that prisms and pyramids are types of *polyhedra* (singular: *polyhedron*). Have students examine the examples and non-examples.

Ask, "What are the defining attributes of a polyhedron?" It is a closed 3D solid with flat faces that are polygons.

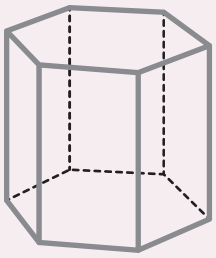
Power-up

To power up students' ability to name the polygons on the faces of a solid, have students complete:

Recall that the *face* of a solid is a two-dimensional shape on its surface.

Identify all of the true statements about the faces of this solid:

A. There are 6 rectangular faces.
B. There are 6 total faces.
C. There are 2 octagonal faces.
D. There are 2 hexagonal faces.



Use: Before Activity 1.

Informed by: Performance on Lesson 15, Practice Problem 6.

Featured Mathematician

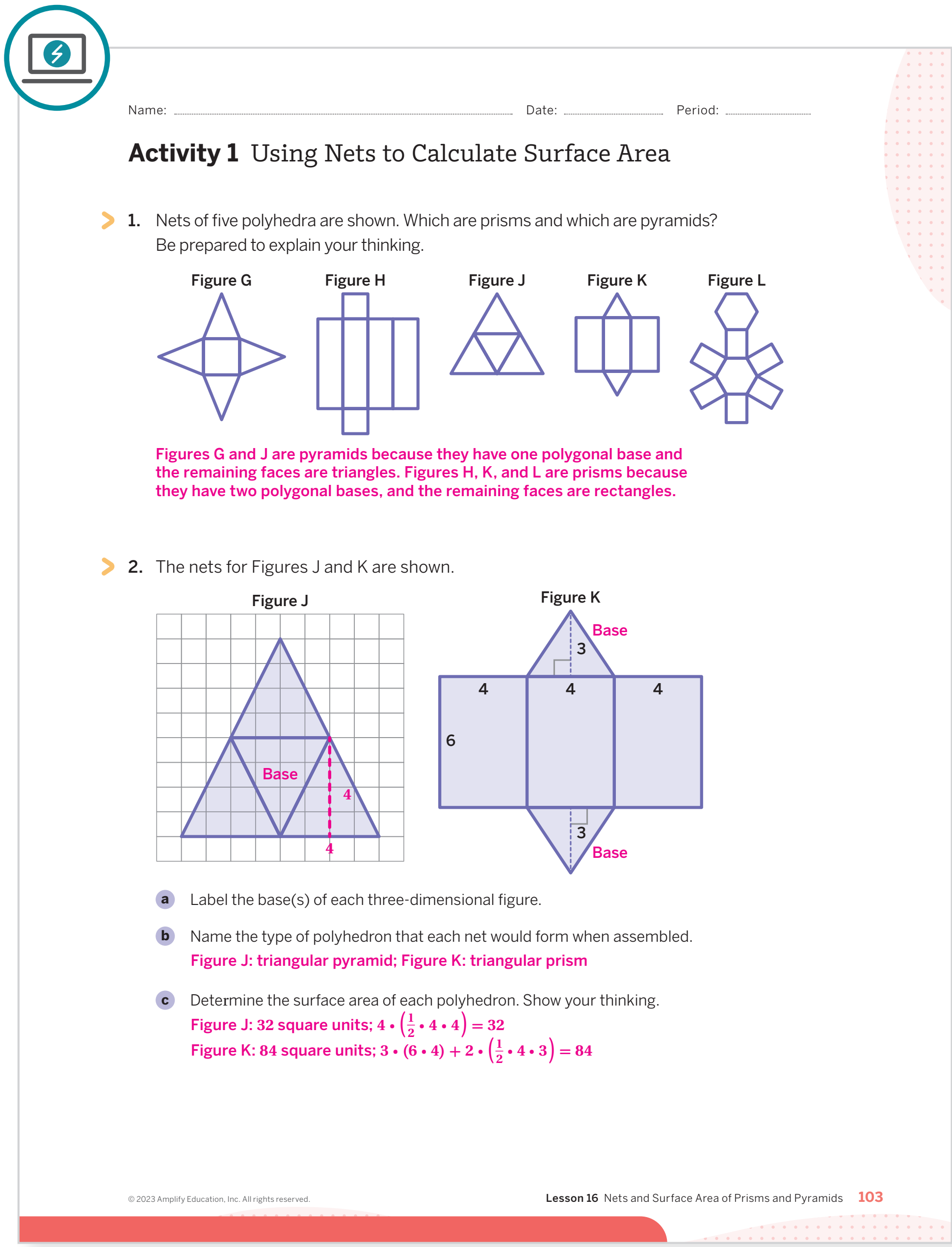
Liu Hui

Have students read about one of the most famous mathematicians of ancient China, Lui Hui, whose ideas are preserved in the Chinese book, "Nine Chapters on the Mathematical Art." One discovery is referred to as "Liu Hui's Cube Puzzle." The puzzle shows how a cube can be dissected into three solids that each have volumes of exactly one half, one third, and one sixth of the cube's volume.

Pairs | 30 min

Activity 1 Using Nets to Calculate Surface Area

Students use nets to classify polyhedra as prisms or pyramids and to calculate surface area, leading them to understand that the nets of prisms and pyramids are different.



Name: Date: Period:

Activity 1 Using Nets to Calculate Surface Area

1. Nets of five polyhedra are shown. Which are prisms and which are pyramids? Be prepared to explain your thinking.

Figures G and J are pyramids because they have one polygonal base and the remaining faces are triangles. Figures H, K, and L are prisms because they have two polygonal bases, and the remaining faces are rectangles.

2. The nets for Figures J and K are shown.

a. Label the base(s) of each three-dimensional figure.

b. Name the type of polyhedron that each net would form when assembled.
Figure J: triangular pyramid; Figure K: triangular prism

c. Determine the surface area of each polyhedron. Show your thinking.
Figure J: 32 square units; $4 \cdot \left(\frac{1}{2} \cdot 4 \cdot 4\right) = 32$
Figure K: 84 square units; $3 \cdot (6 \cdot 4) + 2 \cdot \left(\frac{1}{2} \cdot 4 \cdot 3\right) = 84$

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Lesson 16 Nets and Surface Area of Prisms and Pyramids 103

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by having them identify the polygonal faces in each net.

Look for points of confusion:

- **Struggling to distinguish between Figures G, J, and K.** Refer to Figures E and F in the Warm-up. Ask, "How do a prism and a pyramid that both contain triangular faces differ?"
- **Thinking the bases are rectangles in Figure K of Problem 2a.** Ask, "If those are the bases, then what polygons are the other faces? Is that right?"
- **Miscalculating the surface area in Problem 2c.** Have students label each edge length in the nets and write the areas on top of each face.

Look for productive strategies:

- Using the number of bases and the shapes of both the bases and the faces to name each polyhedron.
- Calculating and then adding the area of each face.
- Grouping polygons together and determining the area of the composite shape, such as identifying identical faces or polygons with common edge lengths.

3 Connect

Have students share what the nets of the prisms have in common and what the nets of the pyramids have in common. Then have students share ways of simplifying calculations by grouping faces.

Highlight the main distinctions between prisms and pyramids: (1) the number of bases (2 or 1), and (2) the shape of the other faces (rectangles or triangles).

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide copies of Figures J and K for students to cut out and manipulate to help them connect the nets given to their actual solids.

Extension: *Interdisciplinary Connections*

Tell students that about 5.8 million square miles on Earth is covered with glacial ice. The total land area of Earth is about 57,393,000 square miles. Ask them to compare, in their own words, the area of Earth covered by glaciers to Earth's land area. **(Science)** The land area is about 10 times greater than the glacier area.

Math Language Development

MLR8: Discussion Supports

During the Connect, use a *Think-Pair-Share* strategy to have students share ways of simplifying calculations by grouping faces. Encourage students to focus on listening and paraphrasing their partners strategy before sharing their own.

English Learners

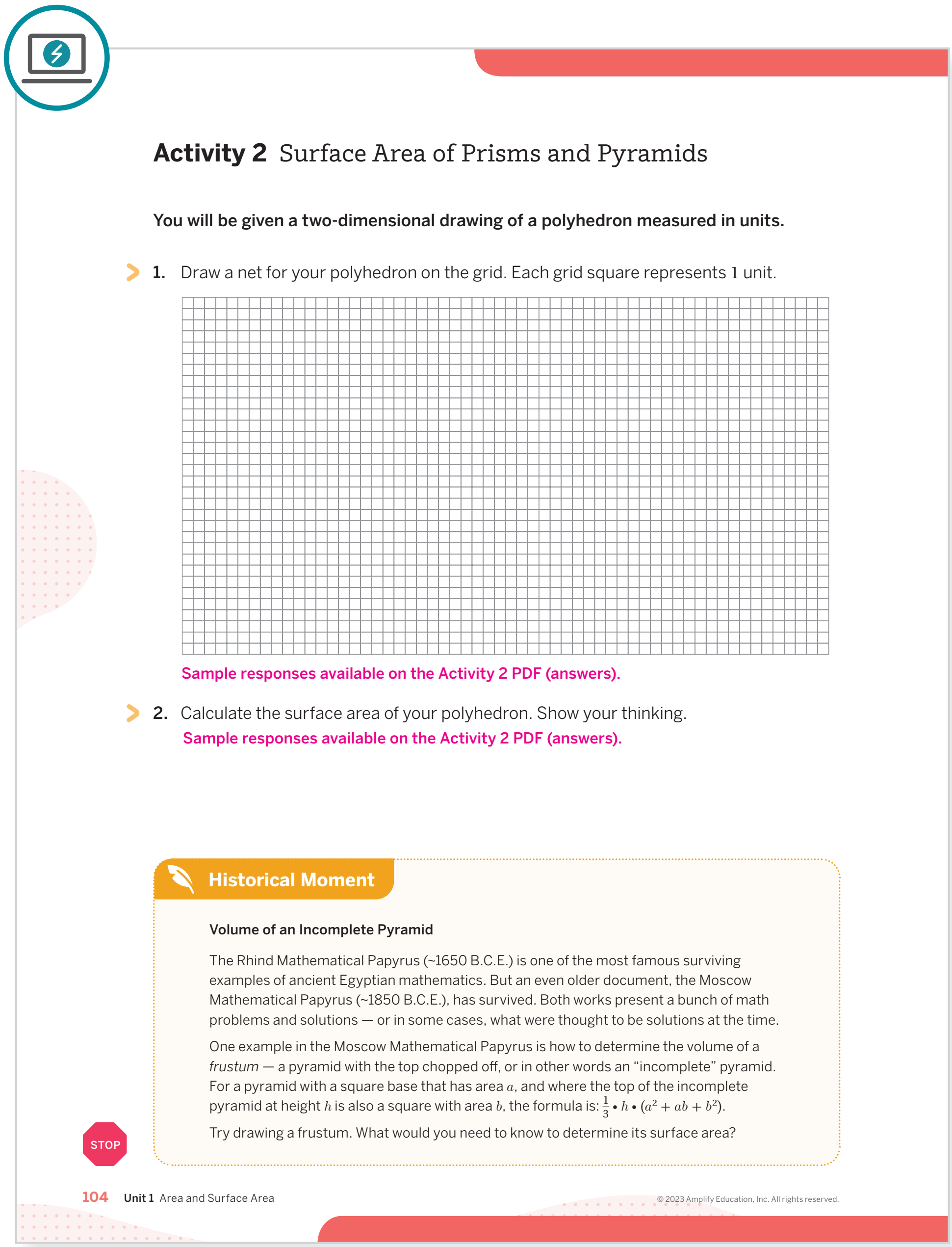
Display sentence frames to support students as they explain their strategies. For example:

- "First, I ___ because . . ."
- "I noticed ___, so I . . . "

Pairs | 15 min

Activity 2 Surface Area of Prisms and Pyramids

Students will draw a net and determine the surface area of a given prism or pyramid, strengthening their understanding of how nets can be used to determine surface area.



Activity 2 Surface Area of Prisms and Pyramids

You will be given a two-dimensional drawing of a polyhedron measured in units.

1. Draw a net for your polyhedron on the grid. Each grid square represents 1 unit.

Sample responses available on the Activity 2 PDF (answers).

2. Calculate the surface area of your polyhedron. Show your thinking.
Sample responses available on the Activity 2 PDF (answers).

Historical Moment

Volume of an Incomplete Pyramid

The Rhind Mathematical Papyrus (~1650 B.C.E.) is one of the most famous surviving examples of ancient Egyptian mathematics. But an even older document, the Moscow Mathematical Papyrus (~1850 B.C.E.), has survived. Both works present a bunch of math problems and solutions — or in some cases, what were thought to be solutions at the time.

One example in the Moscow Mathematical Papyrus is how to determine the volume of a *frustum* — a pyramid with the top chopped off, or in other words an "incomplete" pyramid. For a pyramid with a square base that has area a, and where the top of the incomplete pyramid at height h is also a square with area b, the formula is: $\frac{1}{3} \bullet h \bullet (a^2 + ab + b^2)$.

Try drawing a frustum. What would you need to know to determine its surface area?

STOP

104 Unit 1 Area and Surface Area

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1 Launch

Distribute the cards from the Activity 2 PDF. For Card 3, clarify that each edge with the double line notation has an equal length. Have students work independently for 5–7 minutes before comparing nets and calculations with their partner. Consider pairing students with the same card, but different nets and/or strategies.

2 Monitor

Help students get started by having them identify the polygons that make up the faces of their polyhedron.

Look for points of confusion:

- **Arranging polygons incorrectly on the net.** Have students label one face of the drawing and use known shared edges to continue arranging faces in the net.
- **Mislabeling edge lengths.** Ask, "Is there a parallel and identical edge length labeled in the drawing?"

Look for points of confusion:

- Using known shared edges in the drawing to arrange the polygons in the net, and recognizing that this may result in multiple nets for the same solid.
- Calculating surface area by adding the area of each face or grouping polygons together and calculating the area of the composite shape.

3 Connect

Have pairs of students share how their nets or strategies differed from each other or other partner pairs, and how they evaluated whether the work is correct.

Ask, "How did you arrange the polygons so that, if folded, the net would create the polyhedron in the drawing?" Imagined the solid "unfolding" from top; used shared edges.

Highlight that when given a 2D drawing of a polyhedron, drawing its net is a helpful way to show and label every face.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them work with Card 1 or Card 4. Consider demonstrating how to draw the net for a sample prism and invite students to engage in the process by offering suggested directions as you demonstrate.

Math Language Development

MLR7: Compare and Connect

As students complete Problem 2, ask them how they can display their surface area calculations so that another group can interpret them. When pairs of students join, have them quietly read and interpret each other's work before discussing and comparing.

English Learners

Encourage students to use hand gestures or color coding when interpreting each other's work and comparing nets and strategies.

Historical Moment

Volume of an Incomplete Pyramid

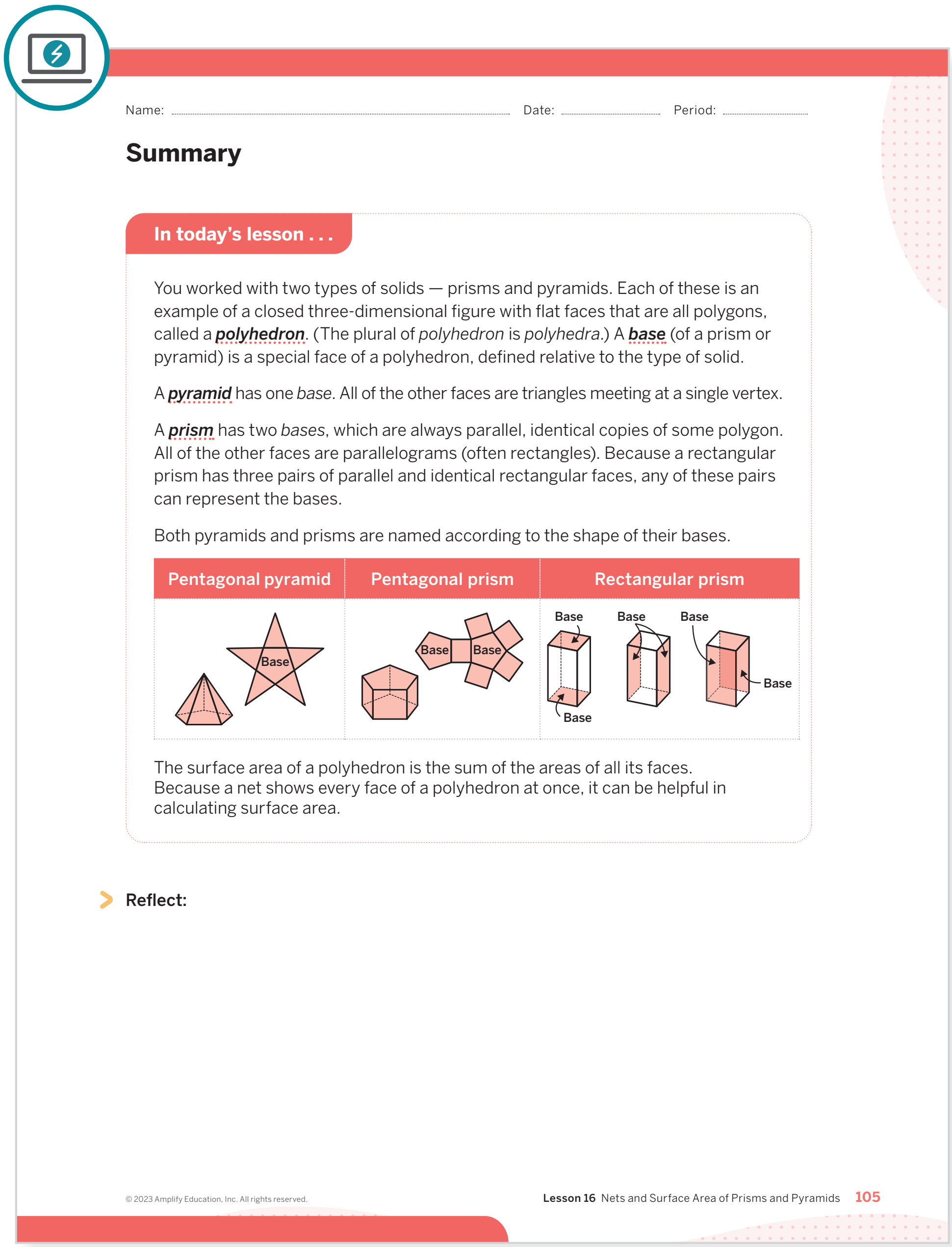
Have students read about the Moscow Mathematical Papyrus, an ancient document that showed how to find the volume of a frustum of a pyramid. Frustums of cones also exist, where the top part is "chopped off". Some examples of frustums in real life are:

- Frustums of pyramids: El Castillo (Chichen Itza) in Yucatán, Mexico
- Frustums of cones: buckets, some drinking glasses

Whole Class | 5 min

Summary

Review and synthesize the key similarities and differences between prisms and pyramids, and how to interpret and use their nets to calculate surface area.



Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You worked with two types of solids — prisms and pyramids. Each of these is an example of a closed three-dimensional figure with flat faces that are all polygons, called a ***polyhedron***. (The plural of *polyhedron* is *polyhedra*.) A ***base*** (of a prism or pyramid) is a special face of a polyhedron, defined relative to the type of solid.

A ***pyramid*** has one *base*. All of the other faces are triangles meeting at a single vertex.

A ***prism*** has two *bases*, which are always parallel, identical copies of some polygon. All of the other faces are parallelograms (often rectangles). Because a rectangular prism has three pairs of parallel and identical rectangular faces, any of these pairs can represent the bases.

Both pyramids and prisms are named according to the shape of their bases.

Pentagonal pyramid	Pentagonal prism	Rectangular prism

The surface area of a polyhedron is the sum of the areas of all its faces. Because a net shows every face of a polyhedron at once, it can be helpful in calculating surface area.

Reflect:

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 3. Ask them to review and reflect on any terms and phrases related to the terms *base* (of a prism or pyramid), *polyhedron, prism*, or *pyramid* that were added to the display during the lesson.

Synthesize

Display the images of three types of polyhedra from the Summary.

Have students share the key differences between a prism and a pyramid, and explain why both are types of polyhedra.

Highlight that all prisms have *one* pair of identical, parallel faces called bases, but rectangular prisms are a special type of prism that actually have *three* pairs of parallel and identical rectangular faces, which means that any pair of those could be the bases.

Ask, "How could you determine the surface area of a prism or pyramid from an image showing the 3D solid without drawing a net?" Label each face in the drawing (e.g., using letters), and make an organized list of each face's edge lengths and area. I can simplify calculations by listing identical faces once, and multiplying the area of one face by the number of identical faces (e.g., in a square pyramid, calculate the area of one triangle face, and multiply the area by 4).

Formalize vocabulary:

- ***base*** (of a prism or pyramid)
- ***polyhedron***
- ***prism***
- ***pyramid***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

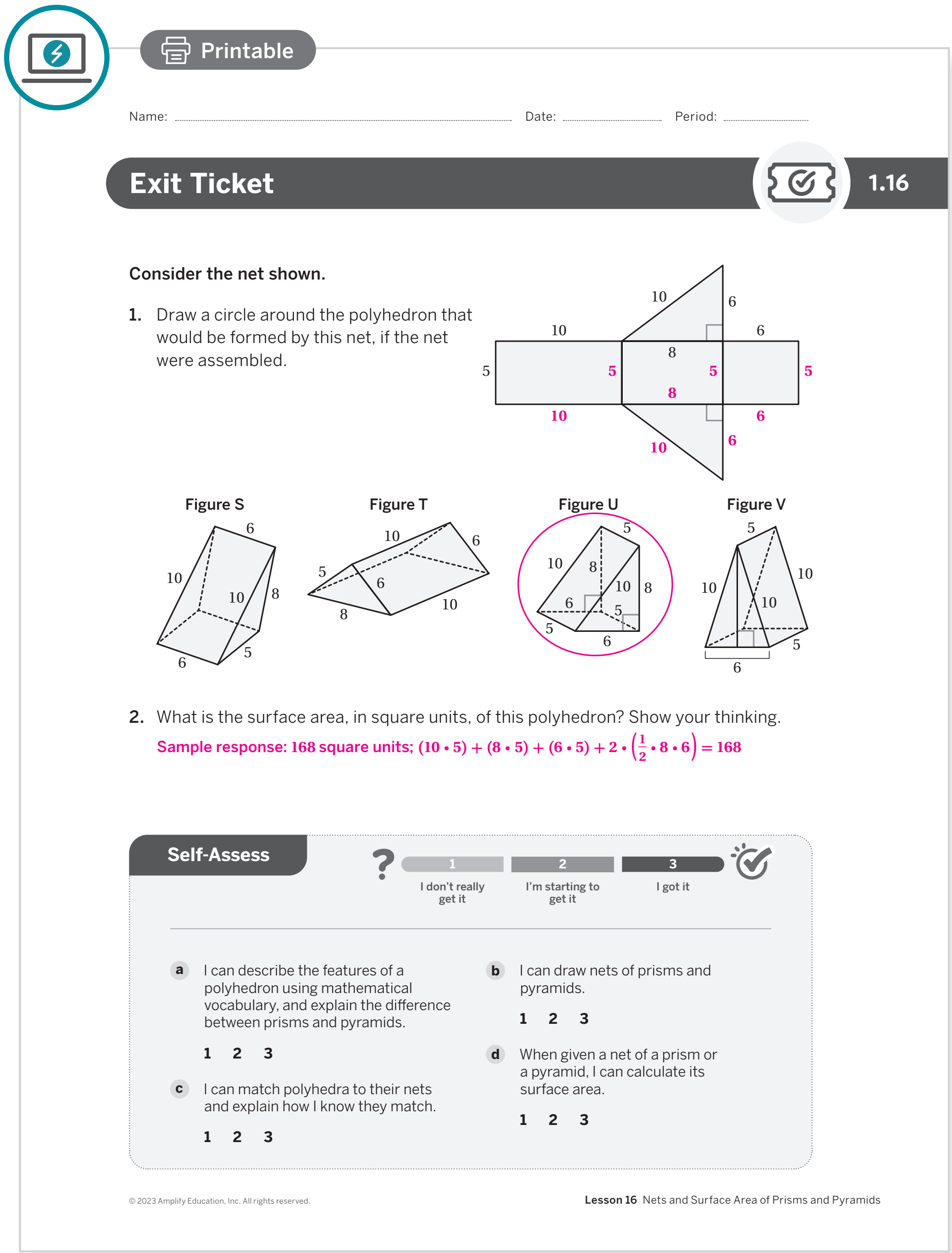
- "How can you determine if a figure is a prism or a pyramid?"
- "How are polyhedra similar to and different from polygons?"
- "How did you calculate surface area today? How was this similar to or different from determining the surface area of a rectangular prism?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by matching the net of a triangular prism to the corresponding drawing, and using the net to calculate the surface area.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.16

Consider the net shown.

1. Draw a circle around the polyhedron that would be formed by this net, if the net were assembled.

2. What is the surface area, in square units, of this polyhedron? Show your thinking.

Sample response: 168 square units; $(10 \cdot 5) + (8 \cdot 5) + (6 \cdot 5) + 2 \cdot \left(\frac{1}{2} \cdot 8 \cdot 6\right) = 168$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can describe the features of a polyhedron using mathematical vocabulary, and explain the difference between prisms and pyramids.

1 2 3

c I can match polyhedra to their nets and explain how I know they match.

1 2 3

b I can draw nets of prisms and pyramids.

1 2 3

d When given a net of a prism or a pyramid, I can calculate its surface area.

1 2 3

Lesson 16 Nets and Surface Area of Prisms and Pyramids

Success looks like . . .

- **Language Goal:** Describing the features of a polyhedron, and comparing and contrasting features of prisms and pyramids, using the terms *face*, *edge*, *vertex*, and *base* **(Speaking and Listening, Writing)**.
- **Language Goal:** Visualizing and identifying the polyhedron that can be assembled from a given net **(Speaking and Listening)**.
  - » Identifying the polyhedron formed from a given net in Problem 1.
- **Language Goal:** Using a net, with or without gridlines, to calculate the surface area of a prism or pyramid, and explain the method used **(Writing)**.
  - » Determining the surface area of a polyhedron using its net in Problem 2.
- **Goal:** Identifying and drawing a net for a given prism or pyramid.

Suggested next steps

If students select the incorrect figure for Problem 1, because they chose:

- Figure T or V; consider asking, "What type of triangle does the net have? Is the triangle in Figure T or Figure V a right triangle? How do you know?"
- Figure S; consider asking, "What is the common edge length for the three rectangular faces in the net? Does this figure show the edge length 5 as a shared edge length?"

If students miscalculate the surface area in Problem 2, consider:

- Reviewing how to calculate surface area by using a net not on a grid, as in Activity 1, Problem 2.

Professional Learning

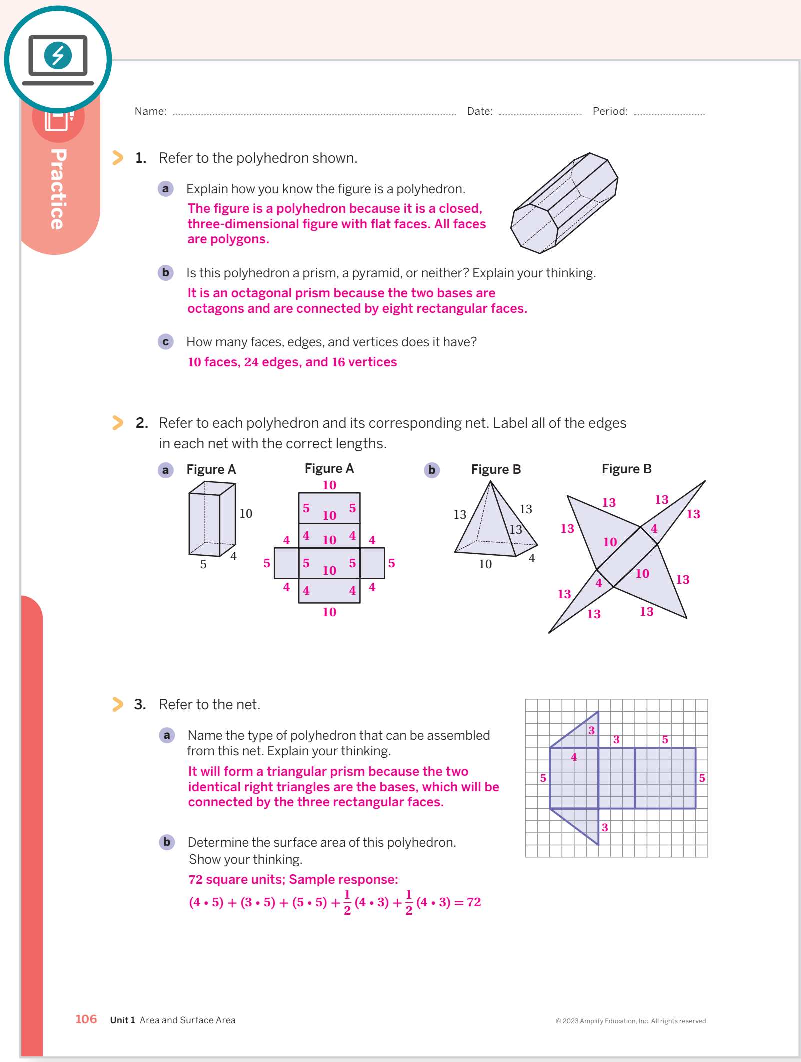
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In earlier lessons, students determined the area of polygons. How did that support students as they determined the surface area of polyhedra?
- What did students find frustrating about Activity 2? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Refer to the polyhedron shown.
   - a Explain how you know the figure is a polyhedron.
     The figure is a polyhedron because it is a closed, three-dimensional figure with flat faces. All faces are polygons.
   - b Is this polyhedron a prism, a pyramid, or neither? Explain your thinking.
     It is an octagonal prism because the two bases are octagons and are connected by eight rectangular faces.
   - c How many faces, edges, and vertices does it have?
     10 faces, 24 edges, and 16 vertices

2. Refer to each polyhedron and its corresponding net. Label all of the edges in each net with the correct lengths.

3. Refer to the net.
   - a Name the type of polyhedron that can be assembled from this net. Explain your thinking.
     It will form a triangular prism because the two identical right triangles are the bases, which will be connected by the three rectangular faces.
   - b Determine the surface area of this polyhedron. Show your thinking.
     72 square units; Sample response:
     $(4 \cdot 5) + (3 \cdot 5) + (5 \cdot 5) + \frac{1}{2}(4 \cdot 3) + \frac{1}{2}(4 \cdot 3) = 72$

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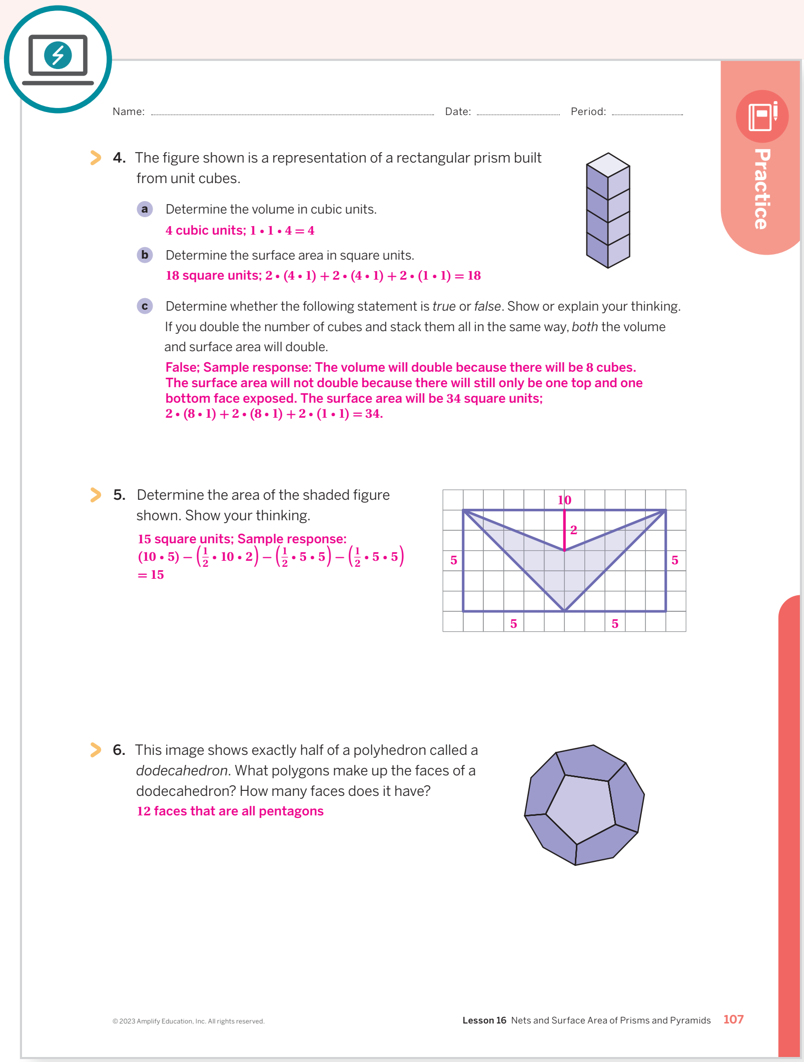

Practice

Name: Date: Period:

4. The figure shown is a representation of a rectangular prism built from unit cubes.
   - a Determine the volume in cubic units.
     4 cubic units; $1 \cdot 1 \cdot 4 = 4$
   - b Determine the surface area in square units.
     18 square units; $2 \cdot (4 \cdot 1) + 2 \cdot (4 \cdot 1) + 2 \cdot (1 \cdot 1) = 18$
   - c Determine whether the following statement is *true* or *false*. Show or explain your thinking.
     If you double the number of cubes and stack them all in the same way, *both* the volume and surface area will double.
     False; Sample response: The volume will double because there will be 8 cubes. The surface area will not double because there will still only be one top and one bottom face exposed. The surface area will be 34 square units; $2 \cdot (8 \cdot 1) + 2 \cdot (8 \cdot 1) + 2 \cdot (1 \cdot 1) = 34$.

5. Determine the area of the shaded figure shown. Show your thinking.
   15 square units; Sample response:
   $(10 \cdot 5) - \left(\frac{1}{2} \cdot 10 \cdot 2\right) - \left(\frac{1}{2} \cdot 5 \cdot 5\right) - \left(\frac{1}{2} \cdot 5 \cdot 5\right) = 15$

6. This image shows exactly half of a polyhedron called a *dodecahedron*. What polygons make up the faces of a dodecahedron? How many faces does it have?
   12 faces that are all pentagons

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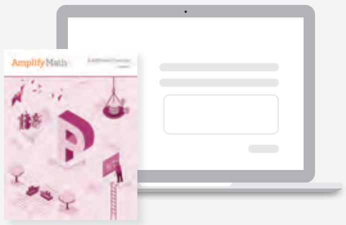
Lesson 16 Nets and Surface Area of Prisms and Pyramids 107

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 14	2
	5	Unit 1 Lesson 5	2
Formative ⚡	6	Unit 1 Lesson 17	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 17**

Constructing a Rhombicuboctahedron

Let’s use nets to construct a rhombicuboctahedron.

Focus

Goals

1. Calculate the surface area of any polyhedron composed of rectangles and triangles.
2. Assemble a polyhedron using a net.

Coherence

Today

Students apply their previous work with area of polygons and surface area of polyhedra to calculate the surface area of the National Library of Belarus, a rhombicuboctahedron composed of 18 square and 8 triangular faces. They then build a scaled model of the Library by using a given net.

Previously

In Lessons 13–16, students extended previous work with two-dimensional shapes to explore and define surface area for three-dimensional solids. Using nets and other images, they devised strategies to calculate the surface area of prisms and pyramids — two special types of polyhedra with familiar polygons as faces.

Coming Soon

In Lesson 18, students will connect two interpretations of the terms *perfect square* and *perfect cube*, as representing both a number and a geometric shape, each with a related set of special characteristics. They will write expressions with variables to represent several measures of a cube: area of a face, surface area, and volume.

Rigor

- Students **apply** their understanding of nets and surface area of polyhedra to determine the surface area and build a model of the National Library of Belarus.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
5 min	30 min	5 min	5 min
Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per pair
- scissors
- tape or glue
- straightedges

Math Language Development

Review words

- *base*
- *edge*
- *face*
- *net*
- *polyhedron*
- *surface area*
- *vertex*

Amps | Featured Activity

Activity 1
See Student Thinking

Students are asked to show and explain their thinking behind their surface area calculations, and these explanations are available to you digitally, in real time.

Building Math Identity and Community

Connecting to Mathematical Practices

Students may struggle to write an expression for the surface area of the library because they do not know how to connect their previous work with surface area to this significantly more complex figure. Ask students what strategies they used in previous lessons to calculate the surface area of prisms and pyramids. Explain that a rhombicuboctahedron is also a polyhedron with only two types of faces, and ask them the two previous types of polyhedra to which this figure is most similar. Encourage them to try familiar strategies.

Modifications to Pacing

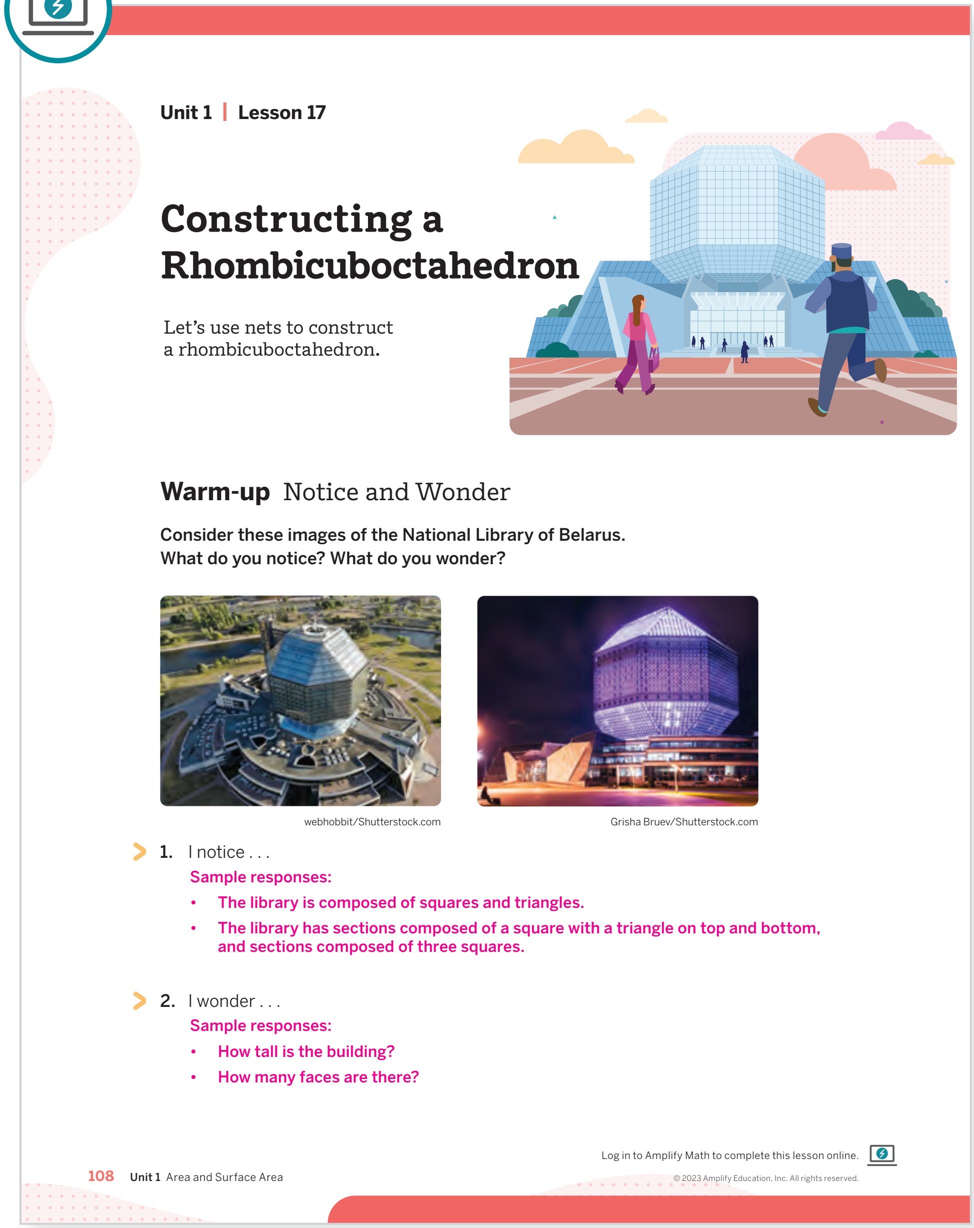
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, provide students with calculators for Problem 1 or let them simply write down expressions without evaluating. You can also provide pre-cut nets for Problem 2; or you may choose to omit this activity altogether or assign it to students to complete during a later time.
- Treat the **Warm-up** as a launch for **Activity 1**, still conducting the ***Notice and Wonder*** routine, but collecting verbal responses as a whole class.

Independent | 5 min

Warm-up Notice and Wonder

Students apply previous understandings of two-dimensional and three-dimensional figures to reason about images of the National Library of Belarus, a 26-sided polyhedron.



Unit 1 | **Lesson 17**

Constructing a Rhombicuboctahedron

Let's use nets to construct a rhombicuboctahedron.

Warm-up Notice and Wonder

Consider these images of the National Library of Belarus. What do you notice? What do you wonder?

webhobbit/Shutterstock.com

Grisha Bruev/Shutterstock.com

1. I notice . . .

Sample responses:

- **The library is composed of squares and triangles.**
- **The library has sections composed of a square with a triangle on top and bottom, and sections composed of three squares.**

2. I wonder . . .

Sample responses:

- **How tall is the building?**
- **How many faces are there?**

Log in to Amplify Math to complete this lesson online.

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1 Launch

Conduct the ***Notice and Wonder*** routine using the two images.

2 Monitor

Help students get started by asking, "Do you notice any patterns on the building?"

Look for points of confusion:

- **Identifying the Library as a sphere.** Ask, "How are the windows arranged? What do you notice about the top of the library? Can it be a sphere if the top is flat, or if some of the faces are flat?"

Look for productive strategies:

- Recognizing that the library is a polyhedron composed of square (or rectangular) and triangular faces arranged in alternating "columns" from top to bottom in this pattern: square-square-square and triangle-square-triangle.

3 Connect

Have students share their responses, focusing on how the library is a polyhedron composed of square and triangular faces, arranged in alternating columns of square-square-square and triangle-square-triangle.

Display the rotating rhombicuboctahedron animation.

Highlight that the National Library of Belarus is a *rhombicuboctahedron*, a polyhedron with 18 square and 8 triangular faces. Each "column" of faces has an identical, opposite column of faces that may not be seen; there is also a square face on top and a square face on the bottom.

Ask, "Why is a rhombicuboctahedron a polyhedron?" It is a closed 3D solid with flat faces that are all polygons.

Math Language Development

MLR5: Co-craft Questions

Use this routine for Problem 2 as students consider their "I wonder" statements. Encourage them to generate mathematical questions about the National Library of Belarus. Have pairs of students compare and share their questions and facilitate a brief class discussion about the questions they generated.

English Learners

Pair students together that have the same primary language. Allow them to generate their questions in their primary language first and then write their questions in English before sharing.

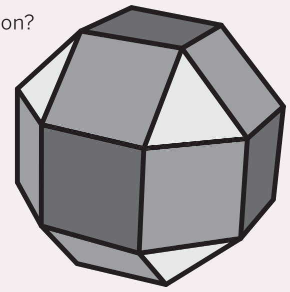
Power-up

To power up students' ability to identify and count the faces of a polyhedron, have students complete:

Use the polyhedron to answer each question:

a. What polygons make up the faces of this polyhedron?
Triangles and squares

b. If each face had an identical "partner" that you cannot see, how many faces are there?
26; 8 triangles and 18 squares



Use: Before the Warm-up.

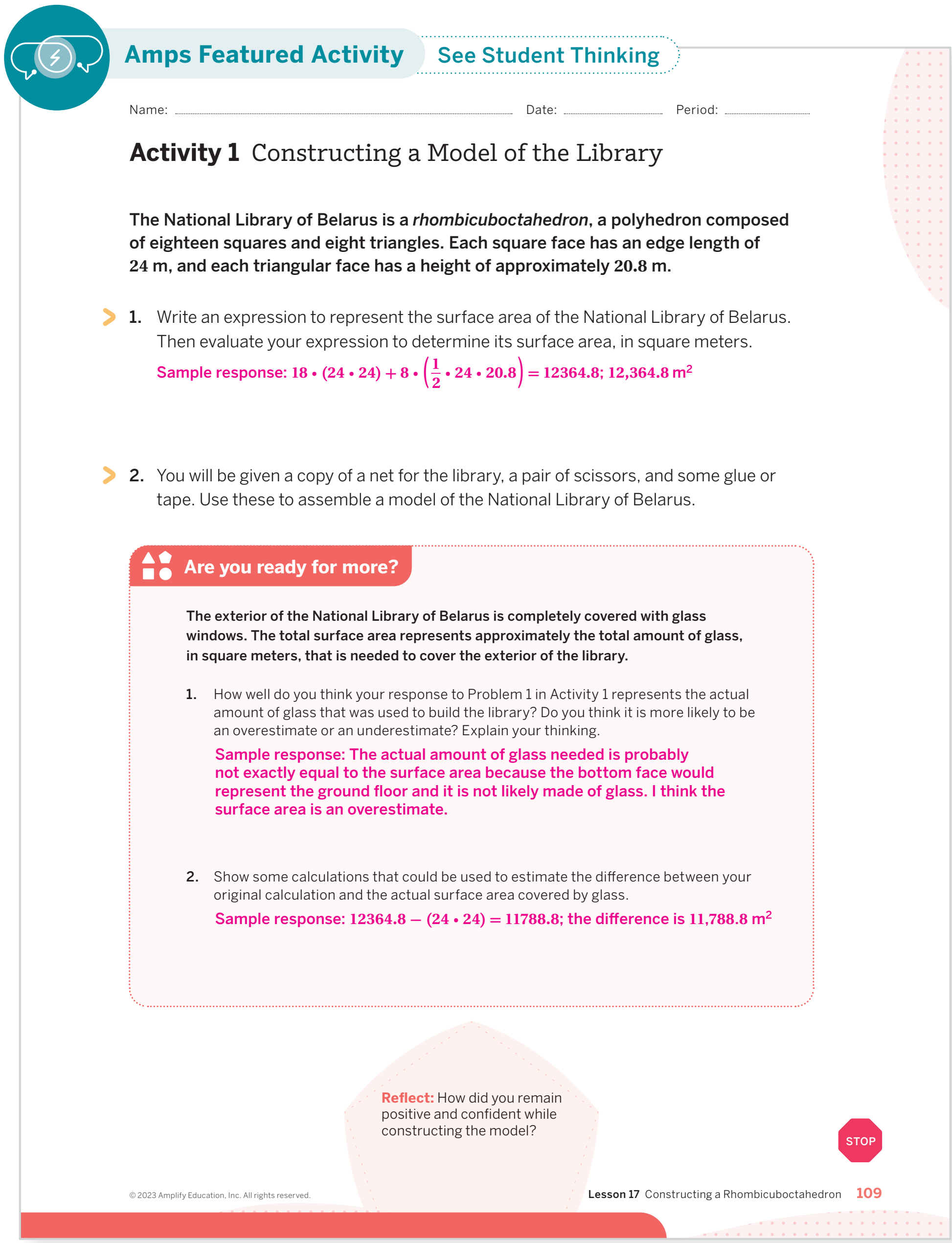
Informed by: Performance on Lesson 16, Practice Problem 6.

Pairs | 30 min

Activity 1 Constructing a Model of the Library

Students calculate the surface area of the National Library of Belarus, realizing that even the surface area of a complex solid is just the sum of the areas of all its surfaces.

Amps Featured Activity See Student Thinking



Name: Date: Period:

Activity 1 Constructing a Model of the Library

The National Library of Belarus is a *rhombicuboctahedron*, a polyhedron composed of eighteen squares and eight triangles. Each square face has an edge length of 24 m, and each triangular face has a height of approximately 20.8 m.

1. Write an expression to represent the surface area of the National Library of Belarus. Then evaluate your expression to determine its surface area, in square meters.
   Sample response: $18 \cdot (24 \cdot 24) + 8 \cdot \left(\frac{1}{2} \cdot 24 \cdot 20.8\right) = 12364.8$; 12,364.8 m^2

2. You will be given a copy of a net for the library, a pair of scissors, and some glue or tape. Use these to assemble a model of the National Library of Belarus.

Are you ready for more?

The exterior of the National Library of Belarus is completely covered with glass windows. The total surface area represents approximately the total amount of glass, in square meters, that is needed to cover the exterior of the library.

1. How well do you think your response to Problem 1 in Activity 1 represents the actual amount of glass that was used to build the library? Do you think it is more likely to be an overestimate or an underestimate? Explain your thinking.
   Sample response: The actual amount of glass needed is probably not exactly equal to the surface area because the bottom face would represent the ground floor and it is not likely made of glass. I think the surface area is an overestimate.

2. Show some calculations that could be used to estimate the difference between your original calculation and the actual surface area covered by glass.
   Sample response: $12364.8 - (24 \cdot 24) = 11788.8$; the difference is 11,788.8 m^2

Reflect: How did you remain positive and confident while constructing the model?

STOP

1 Launch

For Problem 2, provide pairs with one copy of the Activity 1 PDF, scissors, and tape or glue. Explain that each partner will construct part (close to half) of the solid before they connect the two sections. Consider demonstrating how to use the flaps on the nets to accommodate gluing or taping. Explain that the flaps are *not* faces.

2 Monitor

Help students get started by having them draw and label one of each type of polygonal face.

Look for points of confusion:

- **Struggling to write an expression for surface area.** Ask, "What does surface area measure? How have you calculated surface area of other polyhedra? How can you represent those steps in an expression?"
- **Connecting the two sections of the net incorrectly.** Remind students that the top and bottom rows should alternate between triangles and squares. Have them flip one section of the net to confirm.

Look for productive strategies:

- Calculating surface area by grouping polygons together and determining the area of the composite shape, e.g., identifying identical faces or polygons with common edge lengths.

3 Connect

Display students' completed models.

Have pairs of students share their strategies, focusing on how their expressions represent simplified calculations.

Highlight that the real building is almost 200 times larger than models. Explain that the same general strategies used to calculate the surface area of a prism or pyramid can be used for any polyhedron composed of polygons whose areas can be determined.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students assemble the net of the National Library of Belarus in Problem 2, demonstrate how to do so and have them use the copy of the net to write the expression in Problem 1. Consider demonstrating how to begin writing the expression in Problem 1.

Math Language Development

MLR8: Discussion Supports

Encourage students to use sentence frames to explain their thinking during the Connect. For example:

- "This reminds me of ___ because . . ."
- "First I ___ because. . . "
- "This method works/doesn't work because . . ."

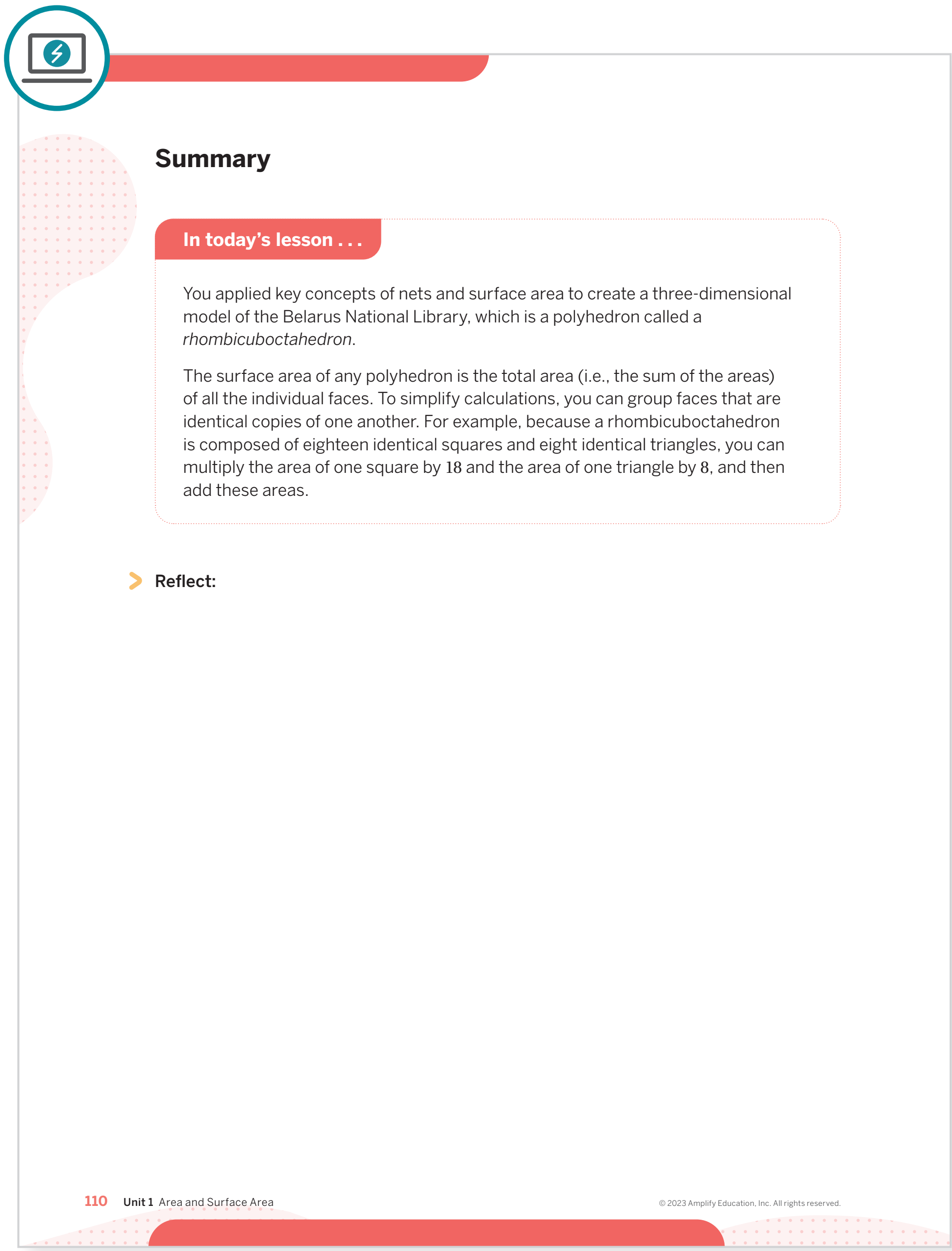
English Learners

Pair students who speak the same primary language. Allow them to share their strategies with each other, using their primary language first, and then have them share in English.

Whole Class | 5 min

Summary

Review and synthesize how to calculate the area of any polyhedron composed of rectangular and triangular faces.



Summary

In today's lesson . . .

You applied key concepts of nets and surface area to create a three-dimensional model of the Belarus National Library, which is a polyhedron called a *rhombicuboctahedron*.

The surface area of any polyhedron is the total area (i.e., the sum of the areas) of all the individual faces. To simplify calculations, you can group faces that are identical copies of one another. For example, because a rhombicuboctahedron is composed of eighteen identical squares and eight identical triangles, you can multiply the area of one square by 18 and the area of one triangle by 8, and then add these areas.

Reflect:

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Synthesize

Ask:

- "How can you calculate the surface area of *any* polyhedron composed of rectangles and triangles?" I can add the areas of each rectangle and triangle, determine the area of each unique shape, and multiply by the number of identical faces, or I can group polygons by using common edge lengths and I can determine the areas of composite shapes.
- "Would any of these strategies need to change if the polyhedron was composed of polygons other than rectangles or triangles (e.g., pentagonal faces)? If so, how?" The method to determine the area of each face may change, but the strategies to calculate the surface area would not change because it's still about determining the total area of all faces.

Highlight that, no matter how complex a three-dimensional figure looks, if students can decompose it into its individual faces, then they can determine its surface area. For any polyhedron, a net can be drawn and the same strategies to calculate surface area can be used, because surface area represents the sum of the areas of every face.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

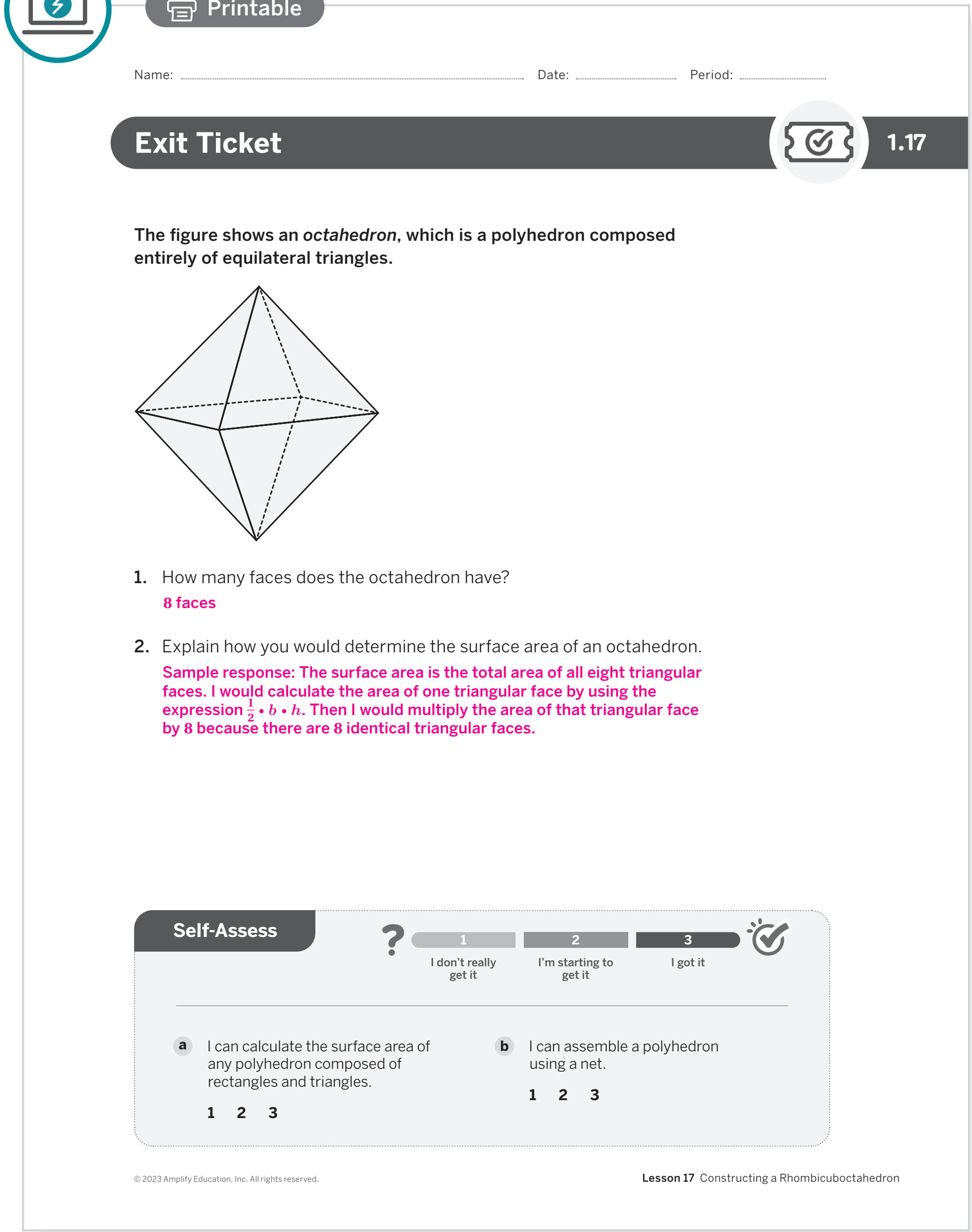
- "How did your previous work with nets and surface area help you today?"
- "By the way, what do you get when you stitch together 12 pentagons and 20 hexagons?" A truncated icosahedron or a soccer ball!

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by explaining how to calculate the area of a polyhedron composed of eight triangles.

Printable



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Exit Ticket 1.17

The figure shows an *octahedron*, which is a polyhedron composed entirely of equilateral triangles.

1. How many faces does the octahedron have?
   8 faces

2. Explain how you would determine the surface area of an octahedron.
   Sample response: The surface area is the total area of all eight triangular faces. I would calculate the area of one triangular face by using the expression $\frac{1}{2} \cdot b \cdot h$. Then I would multiply the area of that triangular face by 8 because there are 8 identical triangular faces.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a I can calculate the surface area of any polyhedron composed of rectangles and triangles.
1 2 3

b I can assemble a polyhedron using a net.
1 2 3

Lesson 17 Constructing a Rhombicuboctahedron

Success looks like . . .

- **Goal:** Calculating the surface area of any polyhedron composed of rectangles and triangles.
  - » Explaining how to calculate the surface area of the polyhedron in Problem 2.
- **Goal:** Assembling a polyhedron using a net.

Suggested next steps

If students misidentify the number of faces in Problem 1 because they:

- Misinterpret the dotted lines; consider reviewing the images in the Warm-up from Lesson 16 and asking, "What did the dotted lines represent in these polyhedra? How does this help you count the number of faces in the octahedron?"
- Count the rectangle in the center of the octahedron; consider reminding them that faces are on the outside of the solid, and surface area measures the total space on the *outside* of a polyhedron.

If students' explanations for Problem 2 are incomplete or incorrect, consider:

- Referring them to Activity 1, and ask, "What were some strategies you used to calculate the surface area of the National Library of Belarus? How can those same strategies help you here?"

Professional Learning

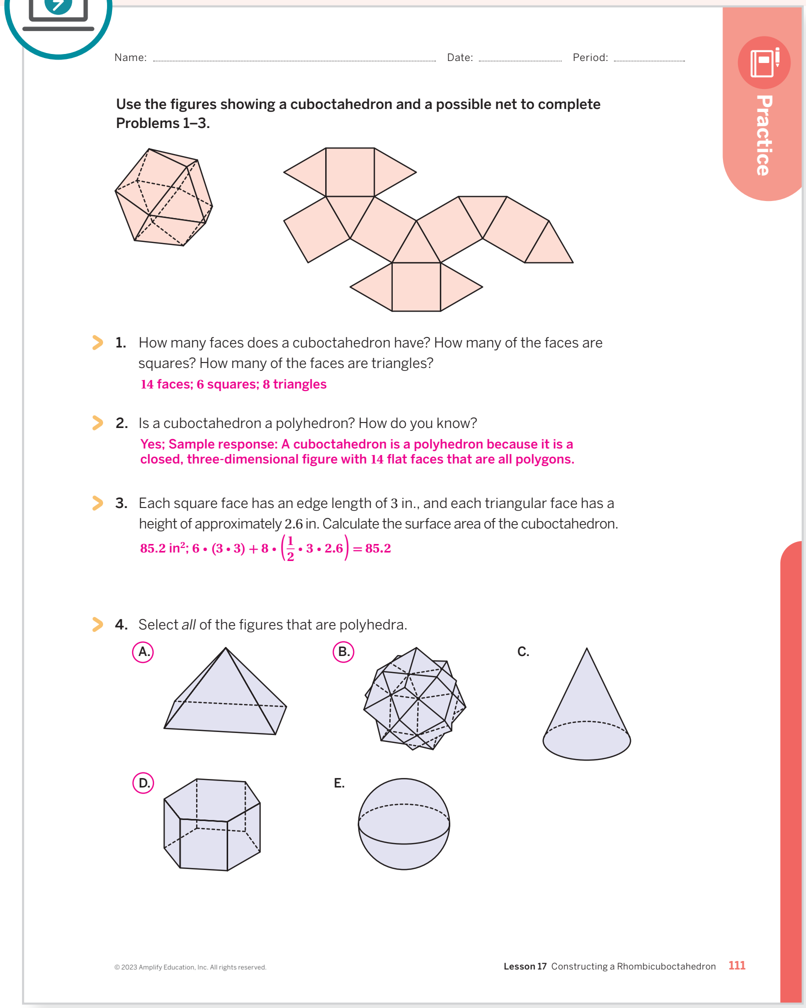
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How was Activity 1 similar to or different from previous work with nets and surface area of polyhedra?
- In what ways have your students gotten better at looking for and using structure to approach complex tasks? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

Practice

Use the figures showing a cuboctahedron and a possible net to complete Problems 1–3.

1. How many faces does a cuboctahedron have? How many of the faces are squares? How many of the faces are triangles?
   14 faces; 6 squares; 8 triangles

2. Is a cuboctahedron a polyhedron? How do you know?
   Yes; Sample response: A cuboctahedron is a polyhedron because it is a closed, three-dimensional figure with 14 flat faces that are all polygons.

3. Each square face has an edge length of 3 in., and each triangular face has a height of approximately 2.6 in. Calculate the surface area of the cuboctahedron.
   85.2 in^2; $6 \cdot (3 \cdot 3) + 8 \cdot \left(\frac{1}{2} \cdot 3 \cdot 2.6\right) = 85.2$

4. Select *all* of the figures that are polyhedra.
   [✓ A.] [✓ B.] C.
   [✓ D.] E.

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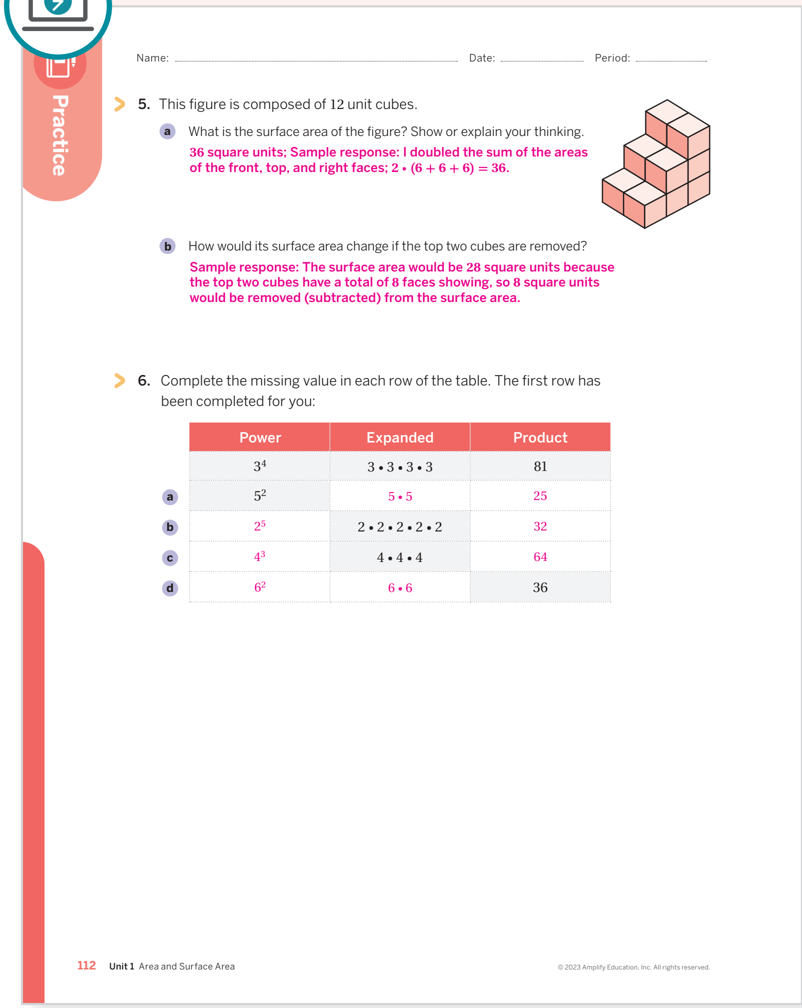
Lesson 17 Constructing a Rhombicuboctahedron 111



Name: ________ Date: ________ Period: ________

Practice

5. This figure is composed of 12 unit cubes.
   - **a** What is the surface area of the figure? Show or explain your thinking.
     36 square units; Sample response: I doubled the sum of the areas of the front, top, and right faces; $2 \cdot (6 + 6 + 6) = 36$.
   - **b** How would its surface area change if the top two cubes are removed?
     Sample response: The surface area would be 28 square units because the top two cubes have a total of 8 faces showing, so 8 square units would be removed (subtracted) from the surface area.

6. Complete the missing value in each row of the table. The first row has been completed for you:

	Power	Expanded	Product
	3^4	$3 \cdot 3 \cdot 3 \cdot 3$	81
a	5^2	$5 \cdot 5$	25
b	2^5	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	32
c	4^3	$4 \cdot 4 \cdot 4$	64
d	6^2	$6 \cdot 6$	36

112 **Unit 1** Area and Surface Area

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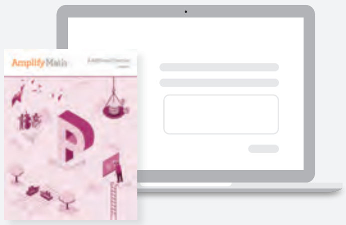
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 16	1
	5	Unit 1 Lesson 13	2
Formative ⚡	6	Unit 1 Lesson 18	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 18**

Simplifying Expressions for Squares and Cubes

Let's write expressions for the attributes of squares and cubes.

Focus

Goals

1. **Language Goal:** Generalize a process for determining the volume of a cube, and justify why this can be abstracted as $s \cdot s \cdot s$. **(Speaking and Listening)**
2. **Language Goal:** Generalize a process for determining the surface area of a cube, and justify why this can be abstracted as $6 \cdot s \cdot s$. **(Speaking and Listening)**
3. **Language Goal:** Include appropriate units when reporting lengths, areas, and volumes (e.g., cm, cm^2, and cm^3). **(Speaking and Listening, Writing)**

Coherence

Today

Students generalize processes for calculating measures of squares and cubes. As they analyze the area of squares and volume of cubes with whole-number side lengths, they solidify understanding of the numbers called "perfect squares" and "perfect cubes." Students apply this reasoning to a cube with side length s, and they write general expressions to calculate the area of a face, $s \cdot s$, volume, $s \cdot s \cdot s$, and surface area, $6 \cdot s \cdot s$. Having seen the geometric motivation to distinguish between square and cubic units, students attend to precise labeling of units.

Previously

In Lessons 13–17, students explored surface area as a two-dimensional measure of a three-dimensional solid. They devised strategies to calculate the surface areas of polyhedra composed of rectangular and triangular faces.

Coming Soon

In Lesson 19, students will use exponents of 2 and 3 to write expressions for measures of squares and cubes.

Rigor

- Students use geometric models of squares and cubes to build their **conceptual understanding** of the numbers called "perfect squares" and "perfect cubes."
- Students **apply** their understanding of area, volume, and surface area to write general expressions for these attributes.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- 32 unit cubes per pair

Math Language Development

Review words

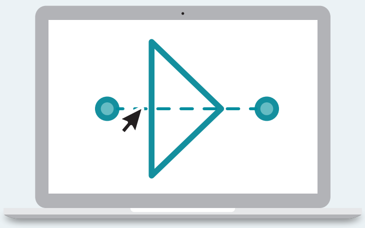
- *face*
- *net*
- *polyhedron*
- *surface area*
- *volume*

Amps | Featured Activity

Activity 2

Drawing Nets

Students draw precise nets for cubes to help them think about surface area.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may find it challenging to write generalized expressions with variables for the measures of the attributes of a cube. Encourage them to consider their work in previous activities or lessons, and add new examples for them to consider. For example, have them write expressions for each attribute of a cube when the side length is 1, 2 or 3 units. Ask, "What actions are being repeated in each calculation, even when you change the side length?"

Modifications to Pacing

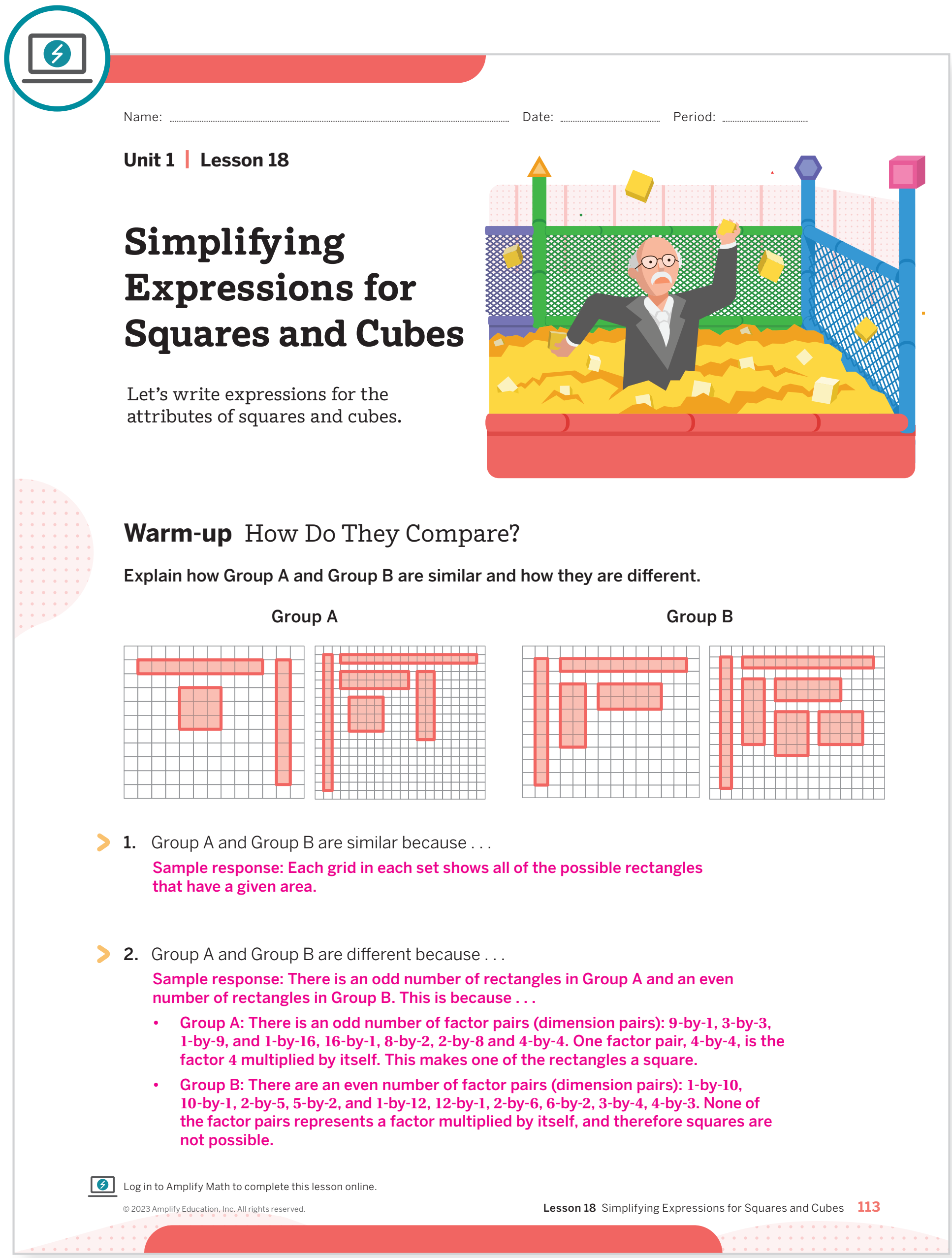
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have students build a cube by using 27 unit cubes and complete Problems 1–3. You may also consider providing them with pre-built cubes composed of 27 unit cubes.
- In **Activity 2**, Problem 1 may be omitted, or you may consider providing them with a pre-drawn net they can use to complete Problem 2.

Independent | 5 min

Warm-up How Do They Compare?

Students discover why some numbers are perfect squares by comparing and contrasting the geometric representation of examples and non-examples.



Name: Date: Period:

Unit 1 | Lesson 18

Simplifying Expressions for Squares and Cubes

Let's write expressions for the attributes of squares and cubes.

Warm-up How Do They Compare?

Explain how Group A and Group B are similar and how they are different.

1. Group A and Group B are similar because . . .

   Sample response: Each grid in each set shows all of the possible rectangles that have a given area.

2. Group A and Group B are different because . . .

   Sample response: There is an odd number of rectangles in Group A and an even number of rectangles in Group B. This is because . . .

   - **Group A: There is an odd number of factor pairs (dimension pairs): 9-by-1, 3-by-3, 1-by-9, and 1-by-16, 16-by-1, 8-by-2, 2-by-8 and 4-by-4. One factor pair, 4-by-4, is the factor 4 multiplied by itself. This makes one of the rectangles a square.**
   - **Group B: There are an even number of factor pairs (dimension pairs): 1-by-10, 10-by-1, 2-by-5, 5-by-2, and 1-by-12, 12-by-1, 2-by-6, 6-by-2, 3-by-4, 4-by-3. None of the factor pairs represents a factor multiplied by itself, and therefore squares are not possible.**

Log in to Amplify Math to complete this lesson online.

Lesson 18 Simplifying Expressions for Squares and Cubes 113

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by having them label the side lengths and area of each rectangle. Have them look for patterns within each grid, before comparing across grids.

Look for points of confusion:

- **Not recognizing the rectangles as geometric representations of factor pairs.** Have students write the equations for the area of each rectangle and relate these to the side lengths.
- **Overlooking that Group A contains squares.** Ask, "What types of polygons are each of these shapes? Could any have a different name? Why do you think that is possible in Group A, but not in group B?"

Look for productive strategies:

- Recognizing that the rectangles on each grid have the same area.
- Understanding that Group A has an odd number of area models and B has an even number.
- Recognizing that Group A has square area models, because the side lengths (factor pairs) are equal.

3 Connect

Have students share their responses, focusing on how side lengths affect the shape of an area model and how this relates to factor pairs.

Highlight that when a number has a factor pair in which a factor is multiplied by itself, the area model will be a square. These types of numbers are called ***perfect squares*** (e.g., $3 \cdot 3 = 9$, so 9 is a perfect square).

Ask, "What is the side length of a square that has an area of 64 cm^2? 8 cm What other numbers are perfect squares?" 1, 4, 25, etc.

Power-up

To power up students' ability to evaluate an expression where a number is raised to a power, have students complete:

Recall that an exponent represents repeated multiplication. For example $10^2 = 10 \cdot 10$. Choose *all* the correct equations.

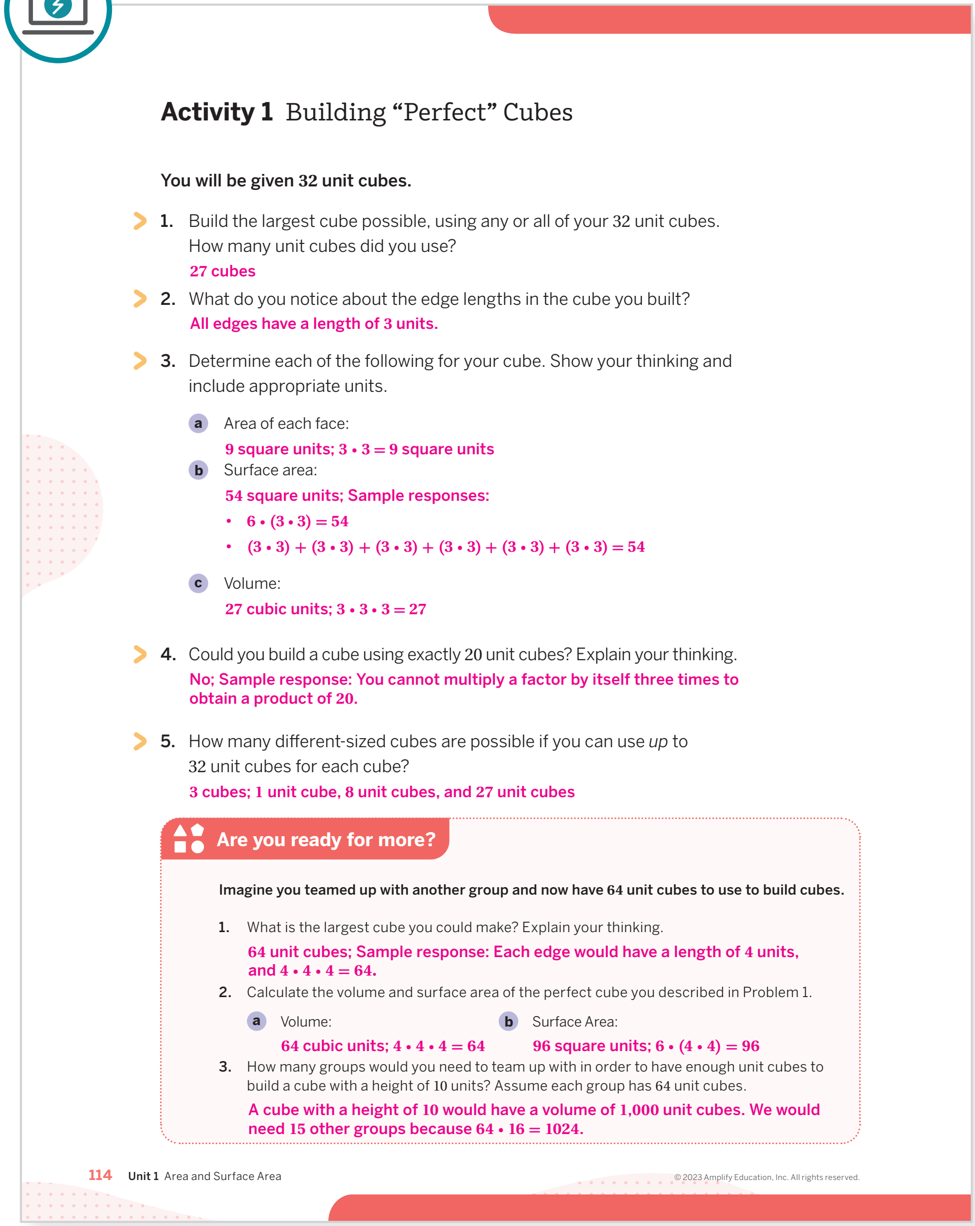
[✓ **A.**] $10 + 10 + 10 + 10 = 10 \cdot 4$
B. $10 + 3 = 10^3$
C. $10 \cdot 5 = 10^5$
[✓ **D.**] $10^3 = 10 \cdot 10 \cdot 10$
E. $10 + 10 = 10^2$

Use: Before Activity 1.
Informed by: Performance on Lesson 17, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 6.

Pairs | 15 min

Activity 1 Building “Perfect” Cubes

Students discover numbers that are perfect cubes by building a cube and determining the area of each face, its surface area, and its volume.



Activity 1 Building “Perfect” Cubes

You will be given 32 unit cubes.

1. Build the largest cube possible, using any or all of your 32 unit cubes. How many unit cubes did you use?
   27 cubes
2. What do you notice about the edge lengths in the cube you built?
   All edges have a length of 3 units.
3. Determine each of the following for your cube. Show your thinking and include appropriate units.
   - a Area of each face:
     9 square units; $3 \cdot 3 = 9$ square units
   - b Surface area:
     54 square units; Sample responses:
     - $6 \cdot (3 \cdot 3) = 54$
     - $(3 \cdot 3) + (3 \cdot 3) + (3 \cdot 3) + (3 \cdot 3) + (3 \cdot 3) + (3 \cdot 3) = 54$
   - c Volume:
     27 cubic units; $3 \cdot 3 \cdot 3 = 27$
4. Could you build a cube using exactly 20 unit cubes? Explain your thinking.
   No; Sample response: You cannot multiply a factor by itself three times to obtain a product of 20.
5. How many different-sized cubes are possible if you can use *up* to 32 unit cubes for each cube?
   3 cubes; 1 unit cube, 8 unit cubes, and 27 unit cubes

Are you ready for more?

Imagine you teamed up with another group and now have 64 unit cubes to use to build cubes.

1. What is the largest cube you could make? Explain your thinking.
   64 unit cubes; Sample response: Each edge would have a length of 4 units, and $4 \cdot 4 \cdot 4 = 64$.
2. Calculate the volume and surface area of the perfect cube you described in Problem 1.
   - a Volume:
     64 cubic units; $4 \cdot 4 \cdot 4 = 64$
   - b Surface Area:
     96 square units; $6 \cdot (4 \cdot 4) = 96$
3. How many groups would you need to team up with in order to have enough unit cubes to build a cube with a height of 10 units? Assume each group has 64 unit cubes.
   A cube with a height of 10 would have a volume of 1,000 unit cubes. We would need 15 other groups because $64 \cdot 16 = 1024$.

1 Launch

Give each pair 32 unit cubes.

2 Monitor

Help students get started by activating prior knowledge. Ask, “What does a cube look like?”

Look for points of confusion:

- **Building a rectangular prism, but not a cube.** Remind students that a cube has six identical faces. Suggest students try to build a smaller cube first.
- **Not using appropriate units.** Remind students how the units reflect that length, area, surface area, and volume are measures of different attributes.
- **Multiplying the edge length by 2 for area and by 3 for volume.** Have students count the unit cube faces (area) or unit cubes (volume) to check calculations.

Look for productive strategies:

- Recognizing the relationship between a cube’s identical edge lengths and repeated multiplication, and using this to determine if a cube could be built using a given number of unit cubes.
- Using appropriate units.

3 Connect

Have pairs of students share how and why they used repeated multiplication for each attribute, and how this differs from multiplying by 2 or 3. Then, have them share how they used this understanding to solve Problems 4 and 5. Encourage students to use appropriate units in their explanations.

Highlight that when a single factor is multiplied by itself three times, like when calculating the volume of a cube, the product is a ***perfect cube*** (e.g., $3 \cdot 3 \cdot 3 = 27$, so 27 is a perfect cube).

Ask, “Why were you unable to use all 32 unit cubes to build a cube?” No factor multiplied by itself three times is equal to 32.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

For students who need more processing time, have them focus on completing Problems 1—3 and consider providing them with fewer unit cubes (at least 8 and no more than 18). Allow students to orally explain how to build a cube to their partner, without actually building it. Their partner can build the cube, based on the oral instructions provided.

Math Language Development

MLR8: Discussion Supports— Press for Details

During the Connect, encourage students to ask peers to elaborate on their ideas. Provide sentence frames for them to use, such as:

- “How do you know . . .?”
- “Tell me more about . . .”
- “I agree/disagree because . . .”

English Learners

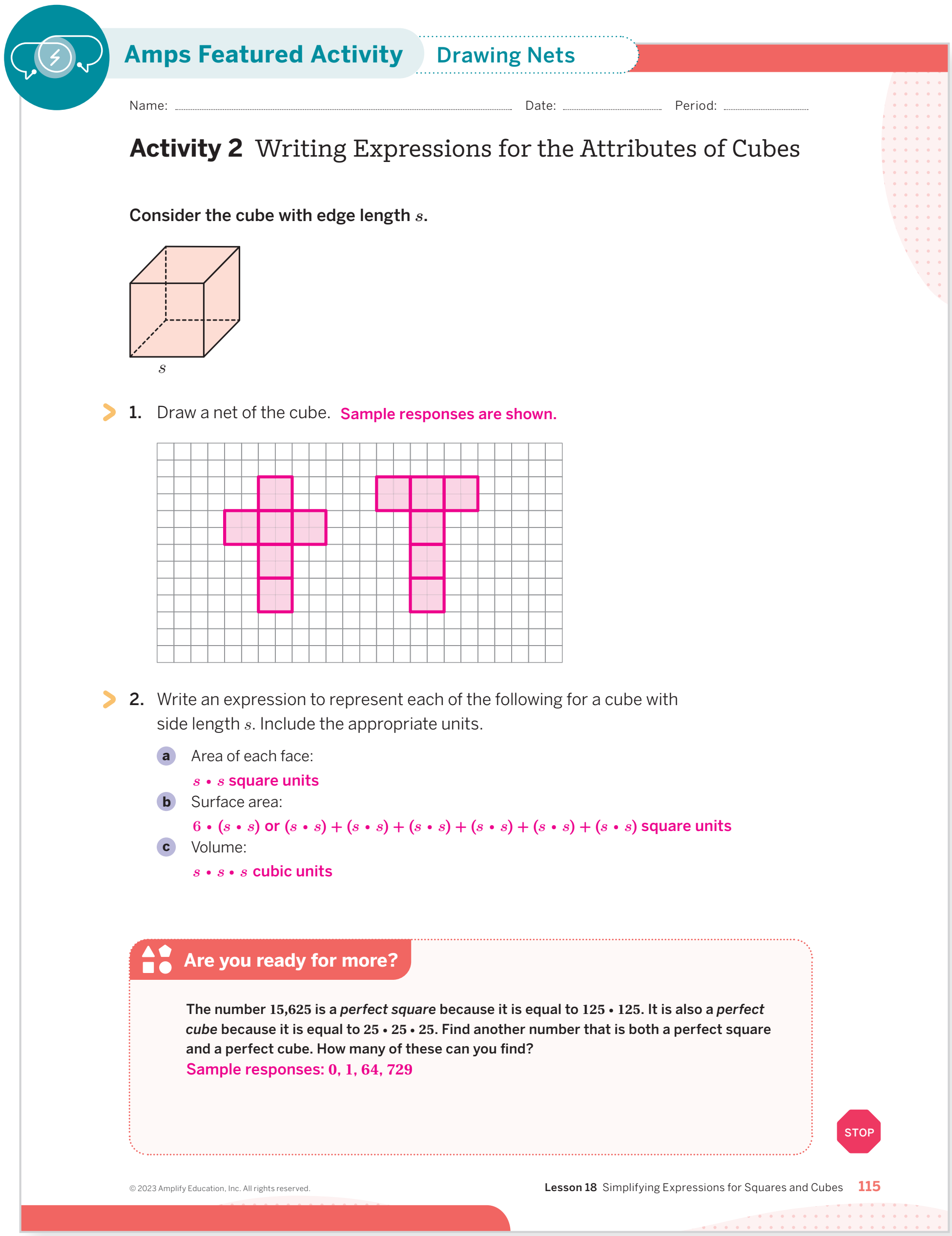
Consider pairing students who speak the same primary language. Allow them to converse in their primary language first, and then have them write their responses in English.

Pairs | 15 min

Activity 2 Writing Expressions for the Attributes of Cubes

Students draw a net of a cube with side length s, and use it to write a general expression for the attributes of any cube.

Amps Featured Activity Drawing Nets



Name: ______ Date: ______ Period: ______

Activity 2 Writing Expressions for the Attributes of Cubes

Consider the cube with edge length s.

1. Draw a net of the cube. **Sample responses are shown.**

2. Write an expression to represent each of the following for a cube with side length s. Include the appropriate units.
   - a Area of each face:
     $s \cdot s$ **square units**
   - b Surface area:
     $6 \cdot (s \cdot s)$ **or** $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$ **square units**
   - c Volume:
     $s \cdot s \cdot s$ **cubic units**

Are you ready for more?

The number 15,625 is a *perfect square* because it is equal to $125 \cdot 125$. It is also a *perfect cube* because it is equal to $25 \cdot 25 \cdot 25$. Find another number that is both a perfect square and a perfect cube. How many of these can you find?
Sample responses: 0, 1, 64, 729

STOP

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Lesson 18 Simplifying Expressions for Squares and Cubes 115

1 Launch

Tell students that they will write expressions using the variable s to represent the edge length.

2 Monitor

Help students get started by identifying the polygons that make up the faces of a cube.

Look for points of confusion:

- **Drawing an incorrect net.** Provide a 3D model of a cube. Ask, "What polygon is represented by all of the faces of a cube?" Have them use shared edges to draw their net.
- **Writing numerical expressions using implied side lengths from their net instead of s.** Remind students the expressions should work for any side length, not just the one in their net.

Look for productive strategies:

- Expressing surface area as the sum of products $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$, or combining like terms to get $6 \cdot (s \cdot s)$.

3 Connect

Have students share nets with different dimensions and explain why all can be correct. Record their expressions as they share, focusing on how and why they used repeated multiplication.

Display if not previously provided, both $6 \cdot (s \cdot s)$ and $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$.

Ask, "How do both expressions represent the surface area of a cube?" Both show 6 groups of the area of each face.

Highlight that general expressions like these help simplify calculations because students can substitute any edge length for the variable and calculate efficiently.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Optimize Access to Tools*

If students need more processing time, have them focus on completing Problem 2. Provide concrete or virtual manipulatives to build several small cubes. Have them write an expression for each attribute in Problem 2 and ask, "What steps or operations did you repeat across all examples?"

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity in which they build rectangular prisms using virtual unit cubes. They can rotate their prisms and tri-colored faces help them keep track of their work.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display the incorrect surface area expression $s + s + s + s + s + s$. As students critique it, listen for those who note that the areas of the six faces, $(s \cdot s)$, must be repeatedly added, not the lengths of the six edges. Have them correct the expression, and clarify their thinking.

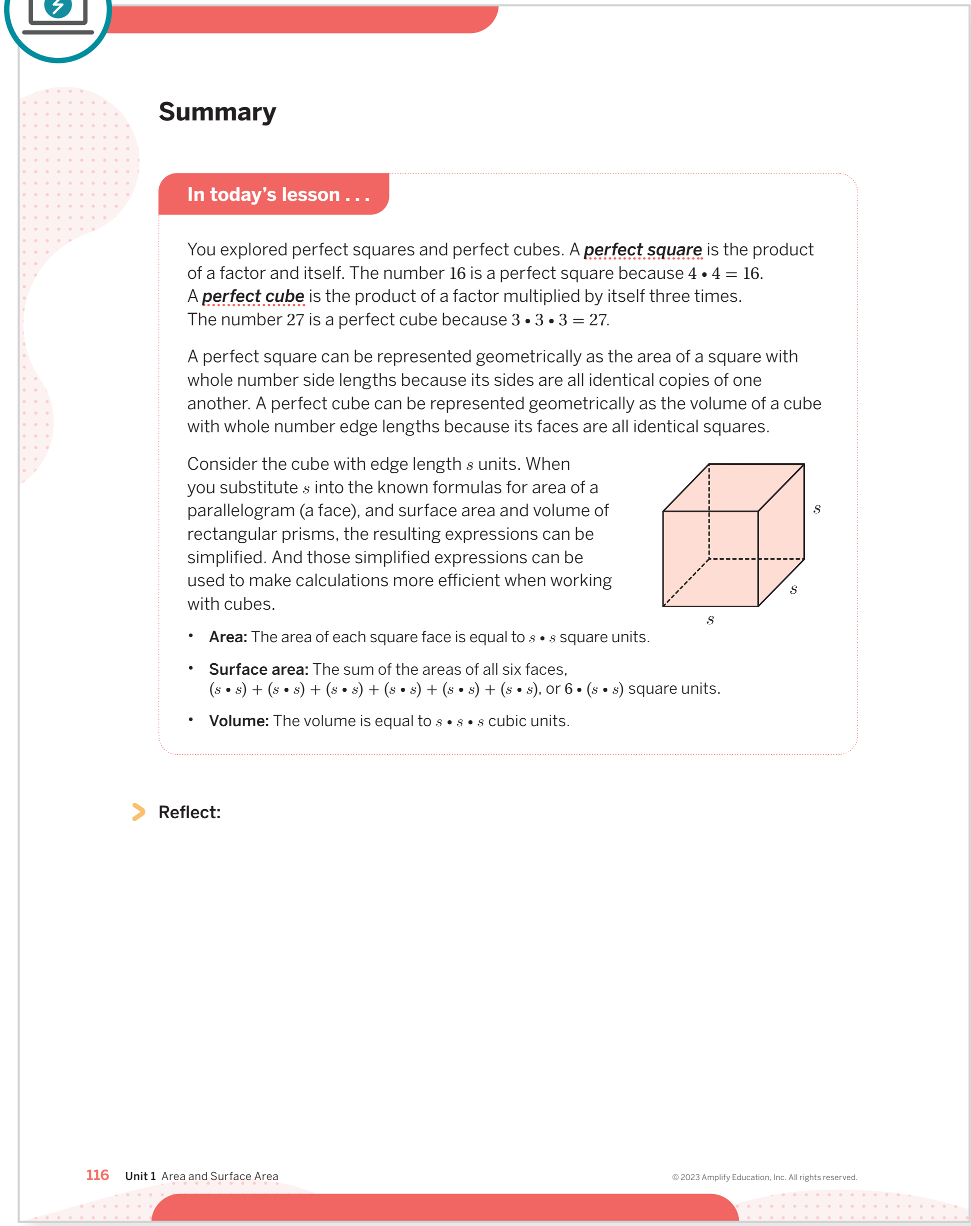
English Learners

Display the expressions $6 \cdot (s \cdot s)$ and $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$. Use the nets of the cube to highlight that general expressions allow students to substitute any edge length for the variable in order to calculate efficiently.

Whole Class | 5 min

Summary

Review and synthesize how the generalized expressions for a cube's attributes relate to each other and help to simplify calculations.



Summary

In today's lesson . . .

You explored perfect squares and perfect cubes. A ***perfect square*** is the product of a factor and itself. The number 16 is a perfect square because $4 \cdot 4 = 16$. A ***perfect cube*** is the product of a factor multiplied by itself three times. The number 27 is a perfect cube because $3 \cdot 3 \cdot 3 = 27$.

A perfect square can be represented geometrically as the area of a square with whole number side lengths because its sides are all identical copies of one another. A perfect cube can be represented geometrically as the volume of a cube with whole number edge lengths because its faces are all identical squares.

Consider the cube with edge length s units. When you substitute s into the known formulas for area of a parallelogram (a face), and surface area and volume of rectangular prisms, the resulting expressions can be simplified. And those simplified expressions can be used to make calculations more efficient when working with cubes.

- **Area:** The area of each square face is equal to $s \cdot s$ square units.
- **Surface area:** The sum of the areas of all six faces, $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$, or $6 \cdot (s \cdot s)$ square units.
- **Volume:** The volume is equal to $s \cdot s \cdot s$ cubic units.

Reflect:

116 Unit 1 Area and Surface Area

Synthesize

Display all the expressions from Activity 2.

Highlight that a *perfect square* is the product of a whole number factor multiplied by itself, and a *perfect cube* is the product of a whole number factor multiplied by itself three times. These names relate to their geometric representations: the square and the cube.

Ask:

- "Which of these expressions could you use to determine whether a number is a perfect square? A perfect cube? How would you use them?" Sample responses:
  - Because a cube's face is a square and its area is calculated by multiplying identical side lengths, or factors, I can use the expression $s \cdot s$ to determine if a number is a perfect square. I can ask myself, "Is this number the product of two of the same factors?"
  - Because the volume of a cube is calculated by multiplying three edge lengths, I can use the formula $s \cdot s \cdot s$ to determine if a number is a perfect cube. I can ask myself, "Is this number a product of three of the same factors?"
- "How are the expressions for the area of a cube's face and its total surface area related? How does this represent the relationship between area and surface area?" Sample response: The faces of a cube are squares with identical edge lengths. The area of each face is calculated by multiplying the side lengths, or $s \cdot s$. Surface area is the total area of each face in a polyhedron. Because a cube has six identical square faces, I can repeatedly add the area of each face six times, or I can multiply the area of one face $(s \cdot s)$ six times.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

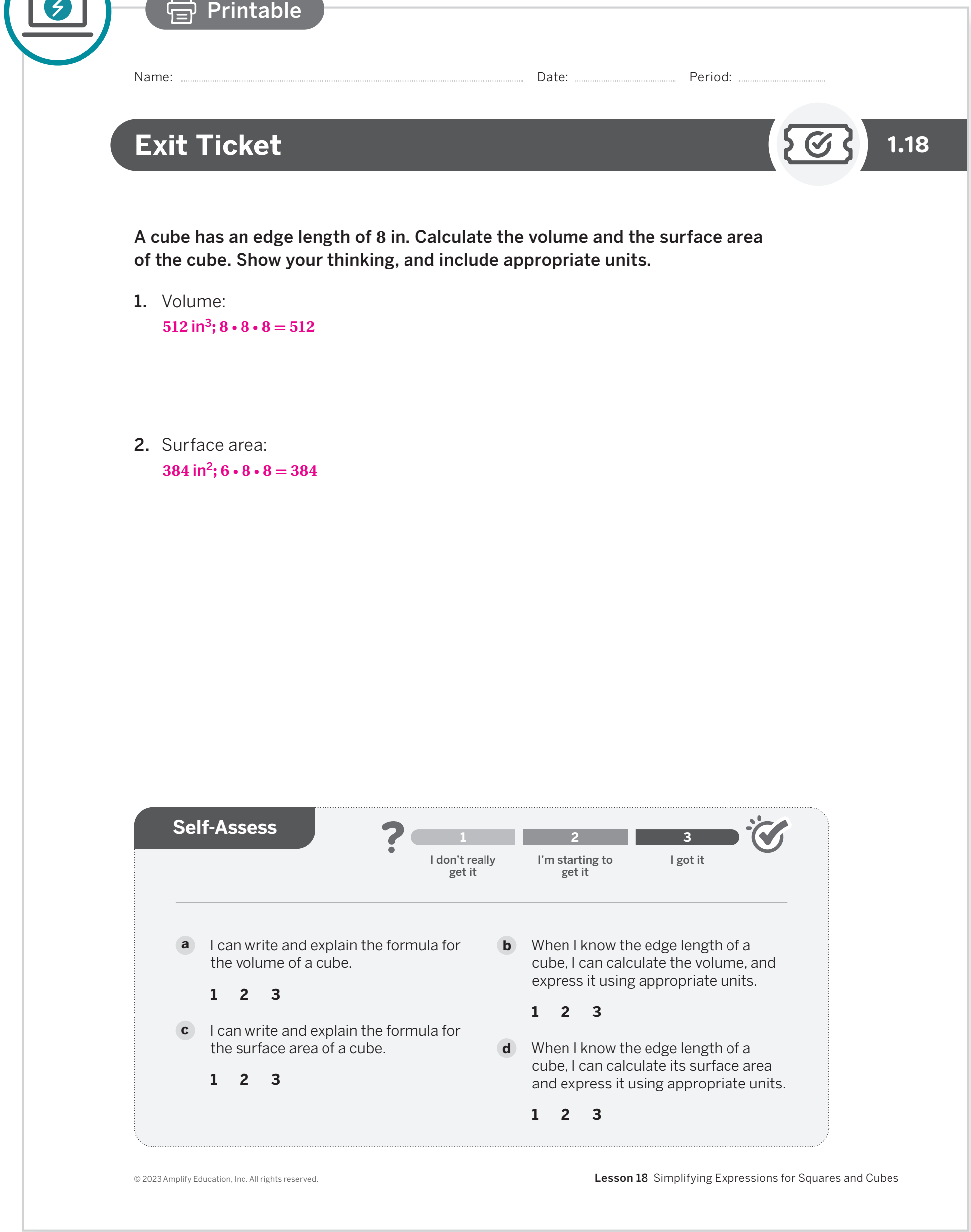
- "What does it mean when you say a number is a perfect square or a perfect cube?"
- "How did your work today build on your previous understanding of area, volume, and surface area?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by calculating the volume and surface area of a cube and labeling each using correct units.

Printable



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Exit Ticket 1.18

A cube has an edge length of 8 in. Calculate the volume and the surface area of the cube. Show your thinking, and include appropriate units.

1. Volume:
   $512 \text{ in}^3; 8 \cdot 8 \cdot 8 = 512$

2. Surface area:
   $384 \text{ in}^2; 6 \cdot 8 \cdot 8 = 384$

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can write and explain the formula for the volume of a cube.
1 2 3

b When I know the edge length of a cube, I can calculate the volume, and express it using appropriate units.
1 2 3

c I can write and explain the formula for the surface area of a cube.
1 2 3

d When I know the edge length of a cube, I can calculate its surface area and express it using appropriate units.
1 2 3

Lesson 18 Simplifying Expressions for Squares and Cubes

Success looks like . . .

- **Language Goal:** Generalizing a process for determining the volume of a cube, and justify why this can be abstracted as $s \cdot s \cdot s$ **(Speaking and Listening)**.
  - » Writing the expression for the volume of a cube in Problem 1.
- **Language Goal:** Generalizing a process for determining the surface area of a cube, and justify why this can be abstracted as $6 \cdot s \cdot s$ **(Speaking and Listening)**.
  - » Writing the expression for the surface area of a cube in Problem 2.
- **Language Goal:** Including appropriate units when reporting lengths, areas, and volumes, e.g., cm, square cm, and cubic cm **(Speaking and Listening, Writing)**.
  - » Writing the volume and surface area of a cube with the appropriate units in Problems 1 and 2.

Suggested next steps

If students do not use an expression, or use the wrong expression to calculate volume or surface area, consider:

- Reviewing Activity 2, Problem 2 and asking, "Which general expression can you use to calculate the volume/surface area? How can you use the expression?"

If students use incorrect units, consider:

- Referring to Activity 1, Problems 2 and 3, and asking, "When did you use units, square units, and cubic units? How does that relate to one-, two-, and three-dimensional measures?"

Professional Learning

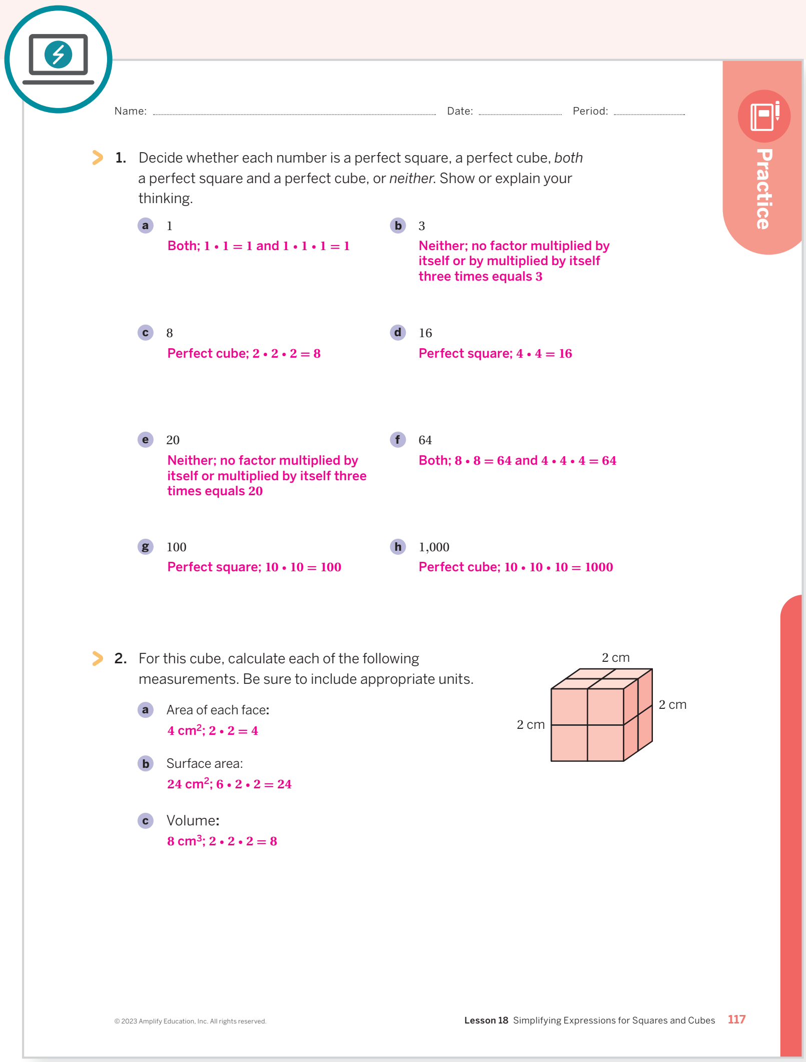
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students generalized expressions for the attributes of squares and cubes. How did that build on the earlier work students did with the area of polygons and surface area of polyhedra?
- Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is? What might you change for the next time you teach this lesson?

Practice

Independent



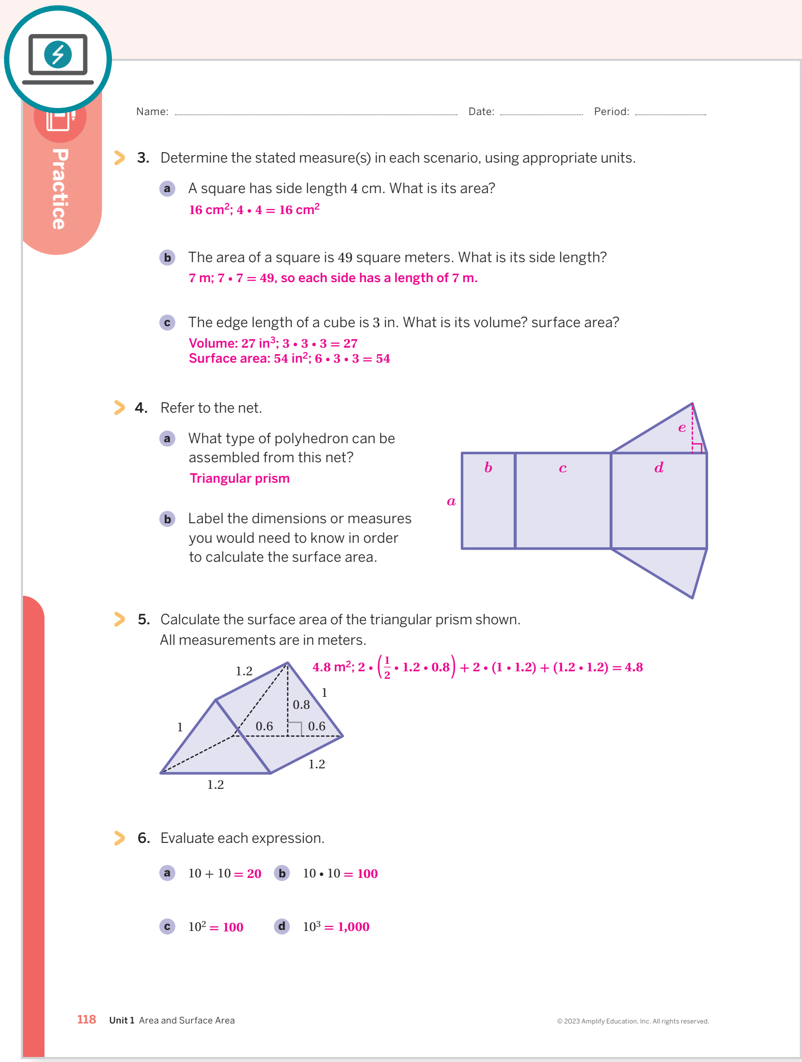
Name: Date: Period:

Practice

1. Decide whether each number is a perfect square, a perfect cube, *both* a perfect square and a perfect cube, or *neither*. Show or explain your thinking.
   - a 1
     Both; 1 • 1 = 1 and 1 • 1 • 1 = 1
   - b 3
     Neither; no factor multiplied by itself or by multiplied by itself three times equals 3
   - c 8
     Perfect cube; 2 • 2 • 2 = 8
   - d 16
     Perfect square; 4 • 4 = 16
   - e 20
     Neither; no factor multiplied by itself or multiplied by itself three times equals 20
   - f 64
     Both; 8 • 8 = 64 and 4 • 4 • 4 = 64
   - g 100
     Perfect square; 10 • 10 = 100
   - h 1,000
     Perfect cube; 10 • 10 • 10 = 1000

2. For this cube, calculate each of the following measurements. Be sure to include appropriate units.

   - a Area of each face:
     4 cm²; 2 • 2 = 4
   - b Surface area:
     24 cm²; 6 • 2 • 2 = 24
   - c Volume:
     8 cm³; 2 • 2 • 2 = 8



Name: Date: Period:

Practice

3. Determine the stated measure(s) in each scenario, using appropriate units.
   - a A square has side length 4 cm. What is its area?
     16 cm²; 4 • 4 = 16 cm²
   - b The area of a square is 49 square meters. What is its side length?
     7 m; 7 • 7 = 49, so each side has a length of 7 m.
   - c The edge length of a cube is 3 in. What is its volume? surface area?
     Volume: 27 in³; 3 • 3 • 3 = 27
     Surface area: 54 in²; 6 • 3 • 3 = 54

4. Refer to the net.
   - a What type of polyhedron can be assembled from this net?
     Triangular prism
   - b Label the dimensions or measures you would need to know in order to calculate the surface area.

5. Calculate the surface area of the triangular prism shown. All measurements are in meters.

$4.8\ \text{m}^2; 2 \cdot \left(\frac{1}{2} \cdot 1.2 \cdot 0.8\right) + 2 \cdot (1 \cdot 1.2) + (1.2 \cdot 1.2) = 4.8$

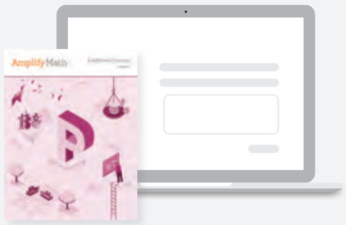
6. Evaluate each expression.
   - a $10 + 10 =$ **20**
   - b $10 \cdot 10 =$ **100**
   - c $10^2 =$ **100**
   - d $10^3 =$ **1,000**

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 15	2
	5	Unit 1 Lesson 15	2
Formative ⚡	6	Unit 1 Lesson 19	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

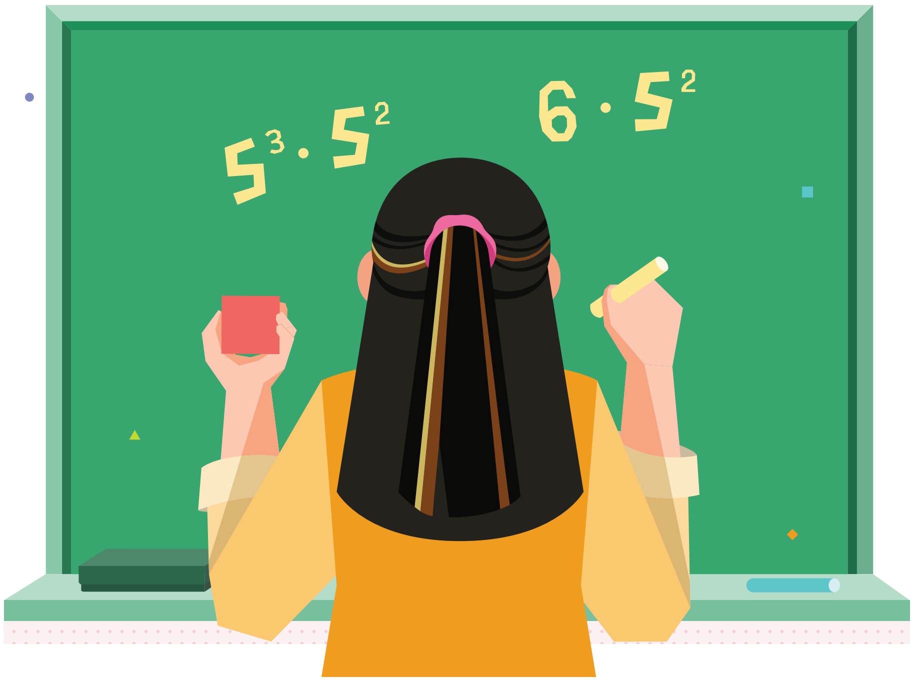


For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 19**

Simplifying Expressions Even More Using Exponents

Let's write expressions with exponents to represent the volume and surface area of cubes.



Focus

Goals

1. **Language Goal:** Generalize processes for determining the surface area and volume of a cube, and justify why these can be abstracted as $6 \cdot s^2$ and s^3, respectively. **(Speaking and Listening)**
2. **Language Goal:** Interpret and write expressions with or without exponents to represent the attributes of a cube. **(Speaking and Listening, Writing)**
3. **Language Goal:** Include appropriate units when reporting lengths, areas, and volumes (e.g., cm, cm^2, cm^3). **(Speaking and Listening, Writing)**

Coherence

Today

Students use exponents to write expressions and the related appropriate units for measures of a cube. They recognize that exponents represent repeated multiplication, and they extend this reasoning to the multiplication of lengths in squares and cubes. Students apply these understandings to sort and match expressions and related units for measures of a cube with side length s. Finally, they evaluate statements about a cube's various attributes and use precise language and units to express lengths, areas and surface areas, and volumes.

Previously

In Grade 5, students used exponents to represent powers of 10. In Lesson 18, students wrote general expressions for the attributes of a cube with side length s as repeated multiplication and repeated addition.

Coming Soon

In Lesson 20, students will apply their understanding of area and surface area to design a suspended tent. They will consider the impact of design decisions on the surface area of their tents.

Rigor

- Students build their **conceptual understanding** of exponents as a way to represent repeated multiplication.
- Students **apply** their understanding of area, volume, and surface area to write expressions with exponents for these attributes.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set of cards per pair

Math Language Development

New words

- ***exponent***
- ***squared***
- ***cubed***

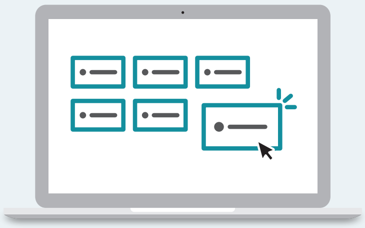
Review words

- *face*
- *surface area*
- *volume*

Amps | Featured Activity

Activity 1
Digital Card Sort

Students group expressions and units that represent the attributes of a cube by dragging and connecting them on screen.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel lost as they sort cards that show exponential notation because a relationship between repeated multiplication and exponents is not immediately apparent. Encourage students to begin by sorting cards that show familiar expressions from the previous lesson. Ask, "How can you connect what you saw about exponents in the Warm-up to these expressions from the previous lesson?"

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, eliminate the sorting exercise and show students the completed groups with the expressions listed in order of the number of terms (greatest to least). Ask students to explain why each card belongs in its group. Then have them discuss the relationships among the expressions in each group, focusing on the relationships between repeated addition and multiplication and repeated multiplication and exponents. You may also consider eliminating Problem 2.
- **Activity 1** and **Activity 2** may both be completed by students individually rather than in pairs.

Warm-up Which Is Greater?

Students recall powers of 10 from Grade 5, using the structure of operations to compare numerical expressions, which prepares them for upcoming work with exponents.

Name: ______ Date: ______ Period: ______

Unit 1 | Lesson 19

Simplifying Expressions Even More Using Exponents

Let's write expressions with exponents to represent the volume and surface area of cubes.

Warm-up Which Is Greater?

Without calculating the value of each expression, use what you know about operations to determine which expression in each pair represents the greater value. Be prepared to explain your thinking.

1. **Expression A:** $10 \cdot 3$ **Expression B:** 10^3

   Sample response: Expression B is greater because it is equivalent to $10 \cdot 10 \cdot 10$, which is greater than $10 \cdot 3$.

2. **Expression A:** 10^2 **Expression B:** $9 \cdot 9$

   Sample response: Expression A is greater because it is equivalent to $10 \cdot 10$, which is greater than $9 \cdot 9$.

3. **Expression A:** $10 + 10 + 10 + 10 + 10 + 10$ **Expression B:** $5 \cdot 10$

   Sample response: Expression A is greater because it is equivalent to $6 \cdot 10$, which represents 6 groups of 10. This is greater than $5 \cdot 10$, or 5 groups of 10.

Log in to Amplify Math to complete this lesson online.

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Lesson 19 Simplifying Expressions Even More Using Exponents 119

1 Launch

Review the directions, emphasizing that students should compare without calculating.

2 Monitor

Help students get started by activating prior knowledge. Ask, "What is another way to write repeated addition?"

Look for points of confusion:

- **Misinterpreting exponents as factors.** Remind them about the meaning of exponents in powers of 10 they learned in Grade 5.
- **Ignoring the relationship between repeated addition and multiplication.** Ask, "How many equal groups of 10 are added or multiplied in each expression?"

Look for productive strategies:

- Using the relationships between repeated addition and multiplication, and between repeated multiplication and exponents, to relate pairs of expressions.

3 Connect

Have students share their responses, focusing on how they used the relationships among operations to compare expressions.

Define an ***exponent*** as the number of times a factor is multiplied by itself. A number raised to the "second power" is read as ***squared***; and a number raised to the "third power" is read as ***cubed***.

Highlight that exponents are used to write expressions involving repeated multiplication more efficiently. The terms "squared" and "cubed" are related to multiplying the side lengths of a square to determine its area and multiplying the edge lengths of a cube to determine its volume.

Math Language Development

MLR2: Collect and Display

As students share their responses, collect and display language used to describe the relationships among operations to compare expressions. Amplify and display terms, such as *second power* and *squared*, *third power* and *cubed*, *exponents*, and *repeated multiplication*. Encourage students to refer back to the display during discussions.

Power-up

To power up students' ability to evaluate powers of 10, have students complete:

Write each expression as a product and then evaluate it.

a. $10^2 = 10 \cdot 10 = 100$
b. $10^3 = 10 \cdot 10 \cdot 10 = 1000$
c. $10^5 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 100000$

Use: Before the Warm-up.

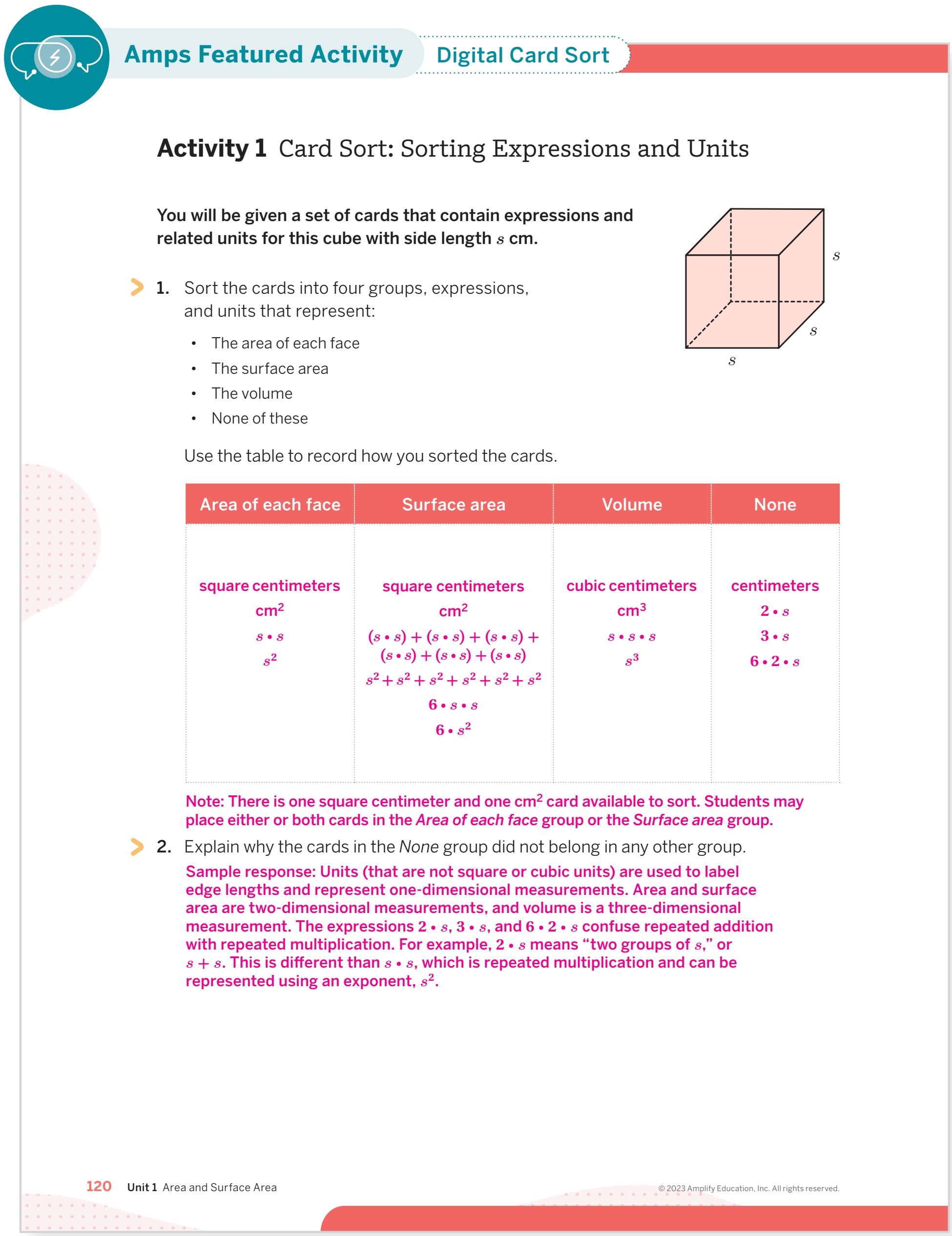
Informed by: Performance on Lesson 18, Practice Problem 6.

Pairs | 15 min

Activity 1 Card Sort: Sorting Expressions and Units

Students sort expressions and related units for the attributes of a cube, identifying connections between repeated multiplication and exponents.

Amps Featured Activity Digital Card Sort



Activity 1 Card Sort: Sorting Expressions and Units

You will be given a set of cards that contain expressions and related units for this cube with side length s cm.

1. Sort the cards into four groups, expressions, and units that represent:
   - The area of each face
   - The surface area
   - The volume
   - None of these

   Use the table to record how you sorted the cards.

Area of each face	Surface area	Volume	None
square centimeters cm^2 $s \cdot s$ s^2	square centimeters cm^2 $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$ $s^2 + s^2 + s^2 + s^2 + s^2 + s^2$ $6 \cdot s \cdot s$ $6 \cdot s^2$	cubic centimeters cm^3 $s \cdot s \cdot s$ s^3	centimeters $2 \cdot s$ $3 \cdot s$ $6 \cdot 2 \cdot s$

Note: There is one square centimeter and one cm^2 card available to sort. Students may place either or both cards in the *Area of each face* group or the *Surface area* group.

2. Explain why the cards in the *None* group did not belong in any other group.

   Sample response: Units (that are not square or cubic units) are used to label edge lengths and represent one-dimensional measurements. Area and surface area are two-dimensional measurements, and volume is a three-dimensional measurement. The expressions $2 \cdot s$, $3 \cdot s$, and $6 \cdot 2 \cdot s$ confuse repeated addition with repeated multiplication. For example, $2 \cdot s$ means "two groups of s," or $s + s$. This is different than $s \cdot s$, which is repeated multiplication and can be represented using an exponent, s^2.

120 **Unit 1** Area and Surface Area

1 Launch

Distribute the cards from the Activity 1 PDF.

2 Monitor

Help students get started by asking, "What do you know about a cube with side length s?"

Look for points of confusion:

- **Misunderstanding how to sort cards that contain exponents.** Review the Warm-up and ask, "What is another way to write repeated multiplication?"
- **Confusing multiplication with exponents (e.g., replacing s^2 with $2 \cdot s$).** Refer to Problem 1 from the Warm-up. Ask, "What is the difference between these two expressions? How does that relate to this cube?"

Look for productive strategies:

- Using the relationships between repeated addition and multiplication, and repeated multiplication and exponents, to sort the expressions and units.

3 Connect

Display the correct groupings, with expressions ordered by number of terms (greatest to least).

Have pairs of students share their reasoning and why the "square centimeters" and cm^2 cards are not in the same group. Have them share responses to Problem 2, focusing on distinguishing multiplication from exponents.

Highlight that because length is a one-dimensional measure reported in units, exponents do not apply. Area, surface area, and volume are two- and three-dimensional measures, so exponents can be used in their expressions and related units.

Ask, "How does each expression relate to the others in the same group?" Exponents show repeated multiplication; multiplication shows repeated addition.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problem 1. They should only work on Problem 2 if they have time available. You may also consider limiting the number of cards students need to sort. For example, consider eliminating one or two of the expressions for the surface area category.

Extension: *Math Enrichment*

Have students list other possible points of confusion that could be added to the *None* group and explain their thinking (e.g., $s^2 \cdot s^2 \cdot s^2 \ldots$ or $(s \cdot s) \cdot (s \cdot s) \cdot (s \cdot s) \ldots$).

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, place the $2 \cdot s$ card in the *Area* group, and explain it belongs there because it means to multiply 2 sides, each with with length s. Have students identify the error and critique the reasoning. Listen for students who clarify that $2 \cdot s$ means 2 groups of s and explain how this differs from $s \cdot s$, or s^2.

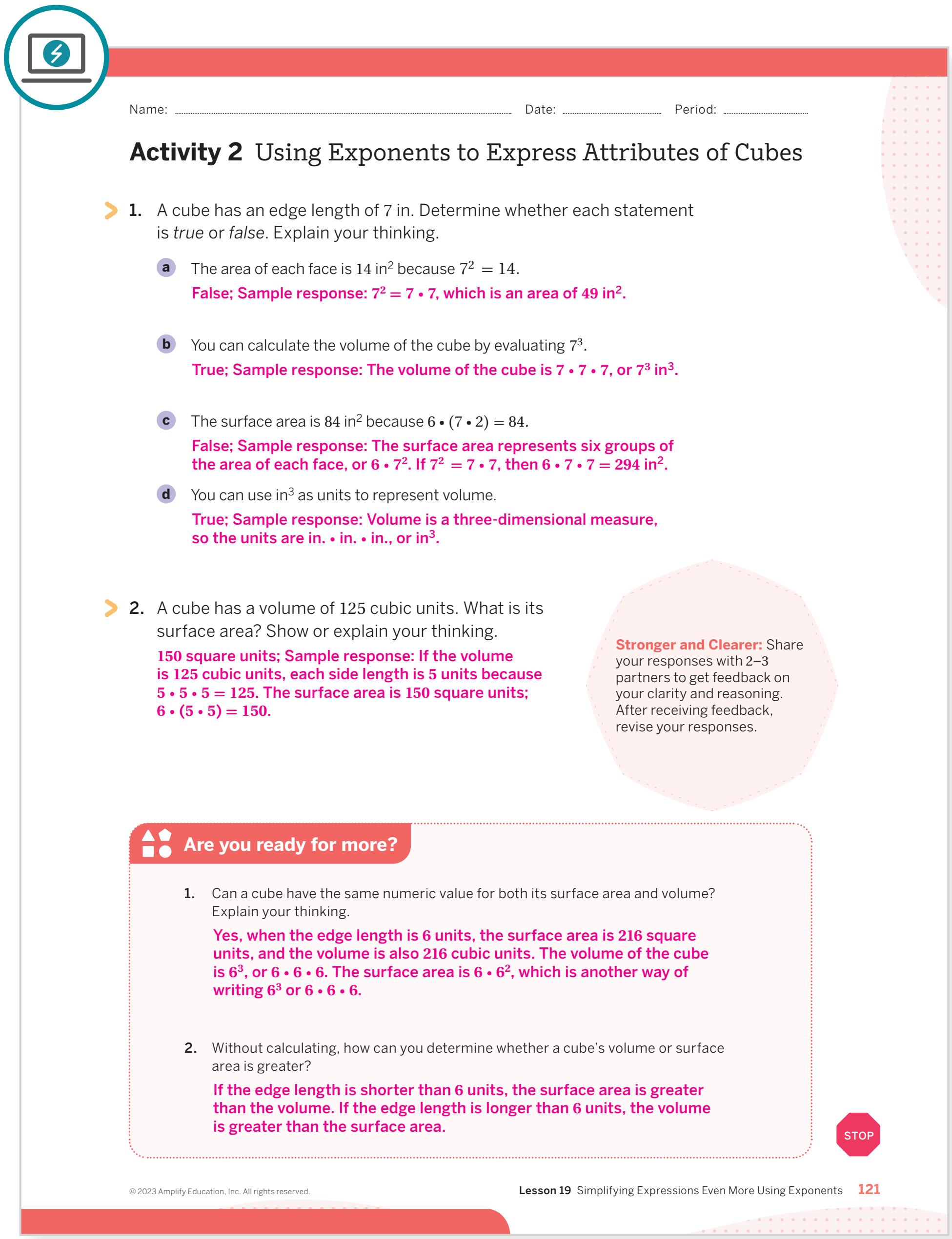
English Learners

Use gestures or images as you state the incorrect statement. For example, point to 2 sides of one face and then point to the expression s that represents the length of the face.

Pairs | 15 min

Activity 2 Using Exponents to Express Attributes of Cubes

Students evaluate whether statements about a cube's attributes are true or false, applying their understanding of exponents and the generalized expressions for a cube's attributes.



Name: ________ Date: ________ Period: ________

Activity 2 Using Exponents to Express Attributes of Cubes

1. A cube has an edge length of 7 in. Determine whether each statement is *true* or *false*. Explain your thinking.

   a. The area of each face is 14 in^{2} because $7^2 = 14$.
   False; Sample response: $7^2 = 7 \bullet 7$, which is an area of 49 in^{2}.

   b. You can calculate the volume of the cube by evaluating 7^3.
   True; Sample response: The volume of the cube is $7 \bullet 7 \bullet 7$, or 7^3 in^{3}.

   c. The surface area is 84 in^{2} because $6 \bullet (7 \bullet 2) = 84$.
   False; Sample response: The surface area represents six groups of the area of each face, or $6 \bullet 7^2$. If $7^2 = 7 \bullet 7$, then $6 \bullet 7 \bullet 7 = 294$ in^{2}.

   d. You can use in^{3} as units to represent volume.
   True; Sample response: Volume is a three-dimensional measure, so the units are in. • in. • in., or in^{3}.

2. A cube has a volume of 125 cubic units. What is its surface area? Show or explain your thinking.
   150 square units; Sample response: If the volume is 125 cubic units, each side length is 5 units because $5 \bullet 5 \bullet 5 = 125$. The surface area is 150 square units; $6 \bullet (5 \bullet 5) = 150$.

Stronger and Clearer: Share your responses with 2–3 partners to get feedback on your clarity and reasoning. After receiving feedback, revise your responses.

Are you ready for more?

1. Can a cube have the same numeric value for both its surface area and volume? Explain your thinking.
   Yes, when the edge length is 6 units, the surface area is 216 square units, and the volume is also 216 cubic units. The volume of the cube is 6^3, or $6 \bullet 6 \bullet 6$. The surface area is $6 \bullet 6^2$, which is another way of writing 6^3 or $6 \bullet 6 \bullet 6$.

2. Without calculating, how can you determine whether a cube's volume or surface area is greater?
   If the edge length is shorter than 6 units, the surface area is greater than the volume. If the edge length is longer than 6 units, the volume is greater than the surface area.

STOP

1 Launch

Set an expectation for the amount of time students will have to work on the activity.

2 Monitor

Help students get started by asking, "What is true about a cube with an edge length of 7 in.?"

Look for points of confusion:

- **Misinterpreting exponents as factors.** Refer to Problem 1 from the Warm-up, and ask, "How are multiplication and exponent notation different?"
- **Using incorrect expressions to evaluate.** Refer to Activity 1 and ask, "Which expressions represent the attribute you are evaluating?"
- **Misapplying units.** Refer to Activity 1 and ask, "How do the units show that an attribute or measure is one-, two-, or three-dimensional?"

Look for productive strategies:

- Recognizing exponents as repeated multiplication to correctly write and evaluate expressions.
- Using a cube's generalized expressions from Activity 1 to substitute values and evaluate.

3 Connect

Have students share their responses, focusing on how they used generalized expressions and interpreted exponents as repeated multiplication. Encourage them to use precise vocabulary, such as *exponent*, *squared*, and *cubed*.

Highlight that a cube's edge length is critical for calculating its other measures. If the edge length is not known, but higher dimensional measures are, then understanding exponents allows students to determine the edge length.

Ask, "Given the area of a cube's face, how can you determine its edge length?" Sample response: I can ask myself what factor multiplied by itself results in that area.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problem 1. Then, if they have time available, work on Problem 2.

Math Language Development

MLR1: Stronger and Clearer Each Time

Have students share their responses to Problems 1 and 2 with 2 other partners, asking questions for clarity and reasoning. Have them write a second draft that reflects shared ideas and refinement of their initial thoughts.

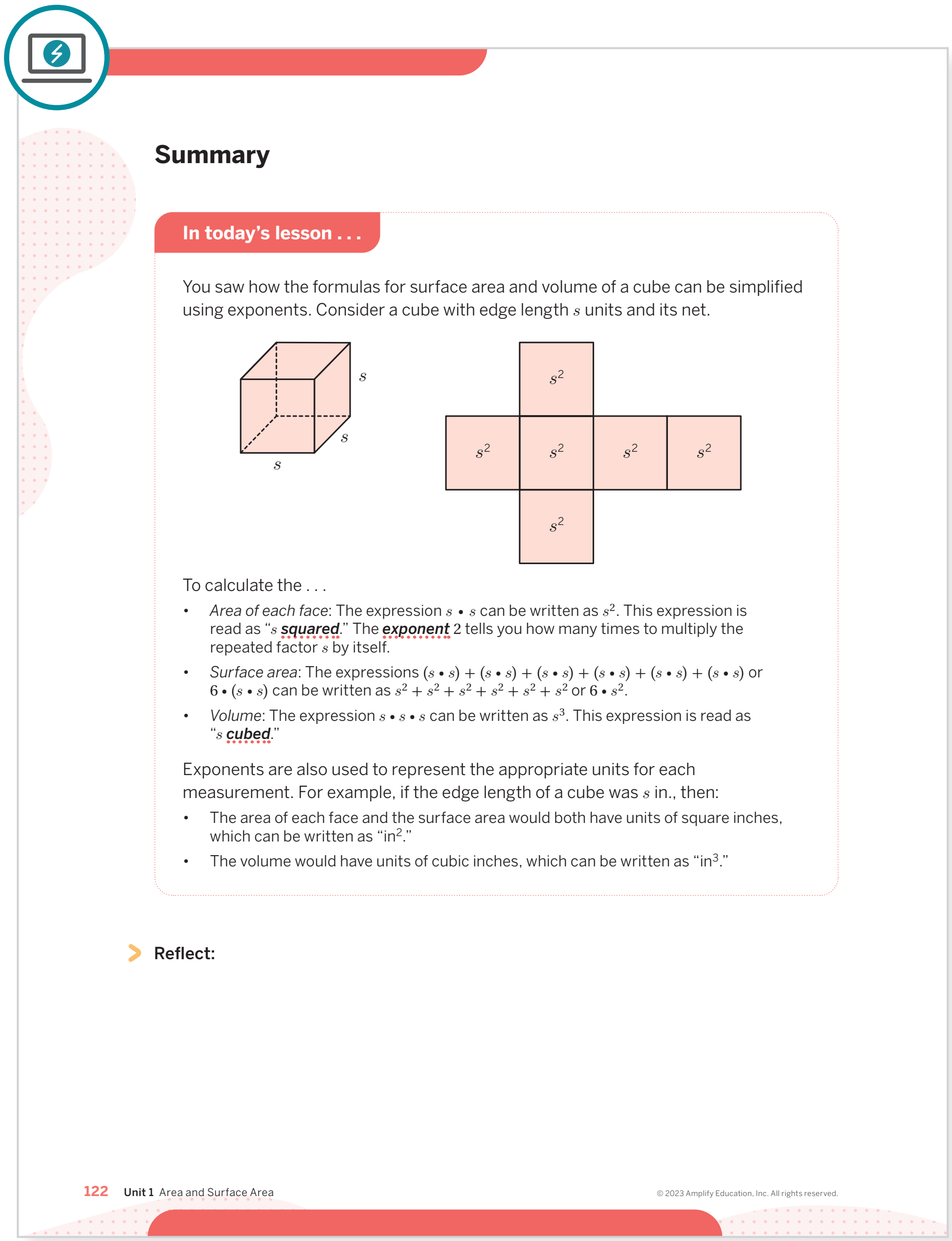
English Learners

Allow students to write their first draft in their primary language.

Whole Class | 5 min

Summary

Review and synthesize the expressions for the attributes of a cube, and how exponents represent repeated multiplication.



Summary

In today's lesson . . .

You saw how the formulas for surface area and volume of a cube can be simplified using exponents. Consider a cube with edge length s units and its net.

To calculate the . . .

- *Area of each face*: The expression $s \cdot s$ can be written as s^2. This expression is read as "s ***squared***." The ***exponent*** 2 tells you how many times to multiply the repeated factor s by itself.
- *Surface area*: The expressions $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$ or $6 \cdot (s \cdot s)$ can be written as $s^2 + s^2 + s^2 + s^2 + s^2 + s^2$ or $6 \cdot s^2$.
- *Volume*: The expression $s \cdot s \cdot s$ can be written as s^3. This expression is read as "s ***cubed***."

Exponents are also used to represent the appropriate units for each measurement. For example, if the edge length of a cube was s in., then:

- The area of each face and the surface area would both have units of square inches, which can be written as "in^2."
- The volume would have units of cubic inches, which can be written as "in^3."

Reflect:

Synthesize

Display the groupings from Activity 1.

Ask, "If a cube has a surface area of 54 in^2, what is its edge length?" 3 in. ; Sample response: $3 \cdot 3 = 9$ (or $3^2 = 9$) and $6 \cdot 9 = 54$. Or $54 \div 6 = 9$, and $3 \cdot 3 = 9$ (or $3^2 = 9$).

Highlight that general expressions with or without exponents can help students determine edge lengths, area, surface area, and volume for any cube.

Formalize vocabulary:

- ***exponent***
- ***squared***
- ***cubed***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is surface area different from volume?"

Math Language Development

MLR2: Collect and Display

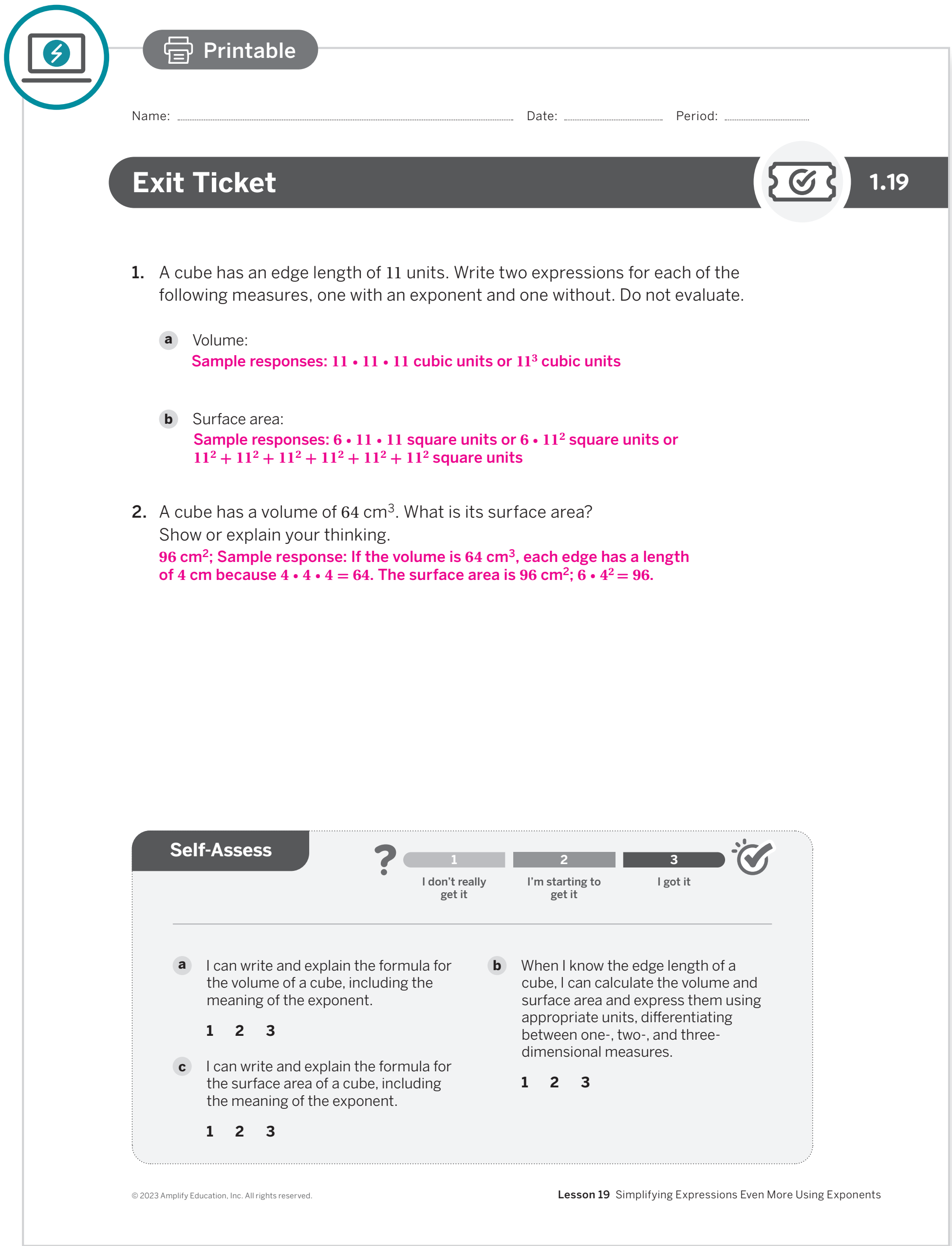
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 3. Ask them to review and reflect on any terms and phrases related to the terms *exponent*, *squared*, or *cubed* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing expressions for the surface area and volume of a cube, and explaining how to find a cube's surface area given the volume.

Printable



Name: Date: Period:

Exit Ticket 1.19

1. A cube has an edge length of 11 units. Write two expressions for each of the following measures, one with an exponent and one without. Do not evaluate.

   a. Volume:
   Sample responses: $11 \cdot 11 \cdot 11$ cubic units or 11^3 cubic units

   b. Surface area:
   Sample responses: $6 \cdot 11 \cdot 11$ square units or $6 \cdot 11^2$ square units or $11^2 + 11^2 + 11^2 + 11^2 + 11^2 + 11^2$ square units

2. A cube has a volume of 64 cm^3. What is its surface area? Show or explain your thinking.
   96 cm^2; Sample response: If the volume is 64 cm^3, each edge has a length of 4 cm because $4 \cdot 4 \cdot 4 = 64$. The surface area is 96 cm^2; $6 \cdot 4^2 = 96$.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a. I can write and explain the formula for the volume of a cube, including the meaning of the exponent.
1 2 3

b. When I know the edge length of a cube, I can calculate the volume and surface area and express them using appropriate units, differentiating between one-, two-, and three-dimensional measures.
1 2 3

c. I can write and explain the formula for the surface area of a cube, including the meaning of the exponent.
1 2 3

Success looks like . . .

- **Language Goal:** Generalizing processes for determining the surface area and volume of a cube, and justify why these can be abstracted as $6 \cdot s^2$ and s^3 respectively. **(Speaking and Listening)**
  - » Writing expressions for the volume and surface area of a cube in Problems 1 and 2.
- **Language Goal:** Interpreting and writing expressions with or without exponents to represent the attributes of a cube. **(Speaking and Listening, Writing)**
  - » Writing expressions for the volume and surface area of a cube in Problems 1 and 2.
- **Language Goal:** Including appropriate units when reporting lengths, areas, and volumes, e.g., cm, cm^2, and cm^3. **(Speaking and Listening, Writing)**
  - » Using cm^2 as units for surface area in Problem 2.

Suggested next steps

If students write expressions that misuse the generalized formulas or exponents in Problem 1, consider:

- Reviewing the groupings in Activity 1, and asking, "How can you use the expressions in these groups to help you write expressions with exponents? Without exponents?"

If students incorrectly solve Problem 2, consider:

- Reviewing strategies used to solve Activity 2, Problem 2, and remind students that the edge length is a critical measure.

Professional Learning

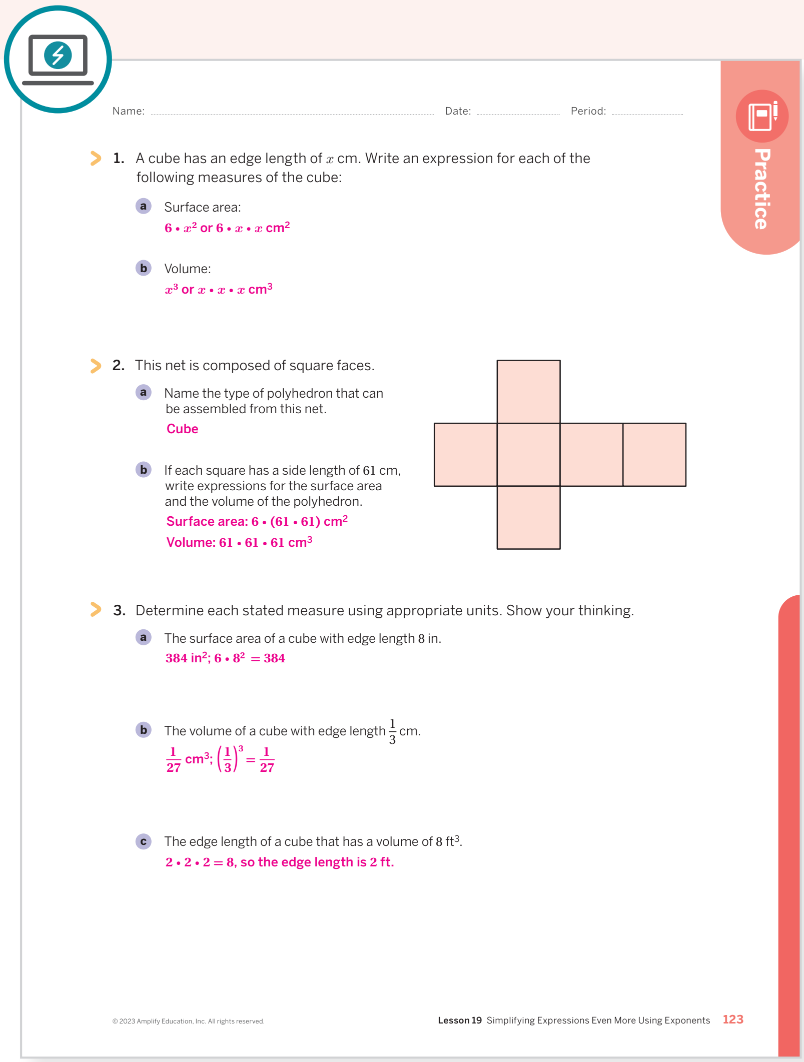
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The instructional goal for this lesson was for students to interpret and write expressions to represent the attributes of a cube. How well did students accomplish this? What did you specifically do to help students accomplish it?
- Which groups of students did and didn't have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. A cube has an edge length of x cm. Write an expression for each of the following measures of the cube:

   a. Surface area:

   $6 \cdot x^2$ or $6 \cdot x \cdot x$ cm^2

   b. Volume:

   x^3 or $x \cdot x \cdot x$ cm^3

2. This net is composed of square faces.

   a. Name the type of polyhedron that can be assembled from this net.

   Cube

   b. If each square has a side length of 61 cm, write expressions for the surface area and the volume of the polyhedron.

   Surface area: $6 \cdot (61 \cdot 61)$ cm^2

   Volume: $61 \cdot 61 \cdot 61$ cm^3

3. Determine each stated measure using appropriate units. Show your thinking.

   a. The surface area of a cube with edge length 8 in.

   384 in^2; $6 \cdot 8^2 = 384$

   b. The volume of a cube with edge length $\frac{1}{3}$ cm.

   $\frac{1}{27}$ cm^3; $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

   c. The edge length of a cube that has a volume of 8 ft^3.

   $2 \cdot 2 \cdot 2 = 8$, so the edge length is 2 ft.

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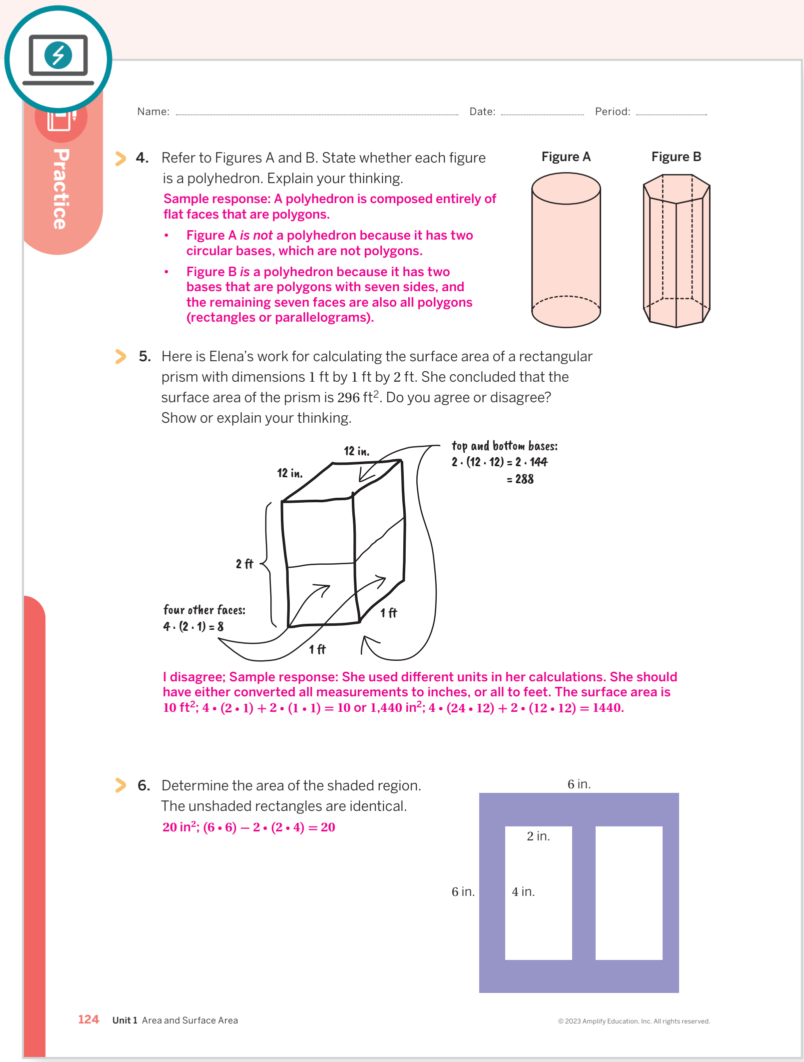
Lesson 19 Simplifying Expressions Even More Using Exponents 123



Name: Date: Period:

4. Refer to Figures A and B. State whether each figure is a polyhedron. Explain your thinking.

   Sample response: A polyhedron is composed entirely of flat faces that are polygons.

   - Figure A *is not* a polyhedron because it has two circular bases, which are not polygons.
   - Figure B *is* a polyhedron because it has two bases that are polygons with seven sides, and the remaining seven faces are also all polygons (rectangles or parallelograms).

5. Here is Elena's work for calculating the surface area of a rectangular prism with dimensions 1 ft by 1 ft by 2 ft. She concluded that the surface area of the prism is 296 ft^2. Do you agree or disagree? Show or explain your thinking.

   top and bottom bases:
   $2 \cdot (12 \cdot 12) = 2 \cdot 144$
   $= 288$

   four other faces:
   $4 \cdot (2 \cdot 1) = 8$

   I disagree; Sample response: She used different units in her calculations. She should have either converted all measurements to inches, or all to feet. The surface area is 10 ft^2; $4 \cdot (2 \cdot 1) + 2 \cdot (1 \cdot 1) = 10$ or 1,440 in^2; $4 \cdot (24 \cdot 12) + 2 \cdot (12 \cdot 12) = 1440$.

6. Determine the area of the shaded region. The unshaded rectangles are identical.

   20 in^2; $(6 \cdot 6) - 2 \cdot (2 \cdot 4) = 20$

124 **Unit 1** Area and Surface Area

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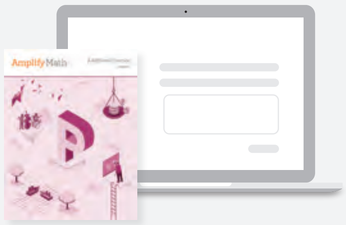
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 16	2
	5	Unit 1 Lesson 14	2
Formative ⚡	6	Unit 1 Lesson 20	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 1 | **LESSON 20 – CAPSTONE**

Designing a Suspended Tent

Let's design a tent that can hang from trees.

Focus

Goals

1. **Language Goal:** Apply understanding of surface area to estimate the amount of fabric needed to manufacture a tent, and explain the estimation strategy. **(Speaking and Listening, Writing)**
2. **Language Goal:** Interpret given information about tents and sleeping bags using multiple representations. **(Speaking and Listening)**
3. **Language Goal:** Compare and contrast different tent designs mathematically. **(Speaking and Listening)**

Coherence

Today

In this capstone lesson, students collaboratively design a tent that can hang from a tree, and they determine how much fabric is needed to make it. Students model their design using area and surface area, while also navigating assumptions and real-world implications in order to plan their path to a solution. To calculate the surface area of the tent, students consider the structure of its two- and three-dimensional attributes before applying methods of grouping shapes and terms for simplifying calculations in formulas. They present their designs and justify their mathematical reasoning.

Previously

In Lessons 1 and 2, students developed the collaborative skills of mathematicians. In Lessons 3–13, they developed strategies to determine the area of two-dimensional shapes, which were leveraged in developing strategies to determine the surface area of three-dimensional figures in Lessons 14–19.

Coming Soon

In Grade 7, students will apply and extend their work with area, surface area, and volume to determine the area of circles, and volume and surface area of solids composed of triangles, quadrilaterals, and other polygons.

Rigor

- Students **apply** their understanding of area and volume to calculating the amount of material(s) needed for a suspended tent.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
5 min	30 min	5 min	5 min
Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per group
- geometry toolkits

Math Language Development

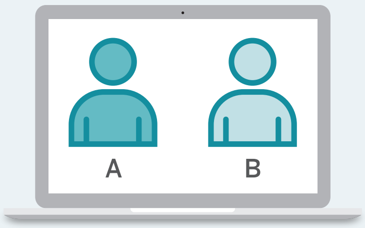
Review words

- *exponent*
- *squared*
- *cubed*
- *face*
- *surface area*
- *volume*

Amps | Featured Activity

Activity 1
Digital Collaboration

Students work together to create a suspended tent. This capstone activity sets the expectations for working collaboratively on a digital platform.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated when trying to design a 3D figure in a 2D space because they get too caught up in the more superficial details of their design. Help students recognize when they should pause to check their progress and results, and model productive techniques for monitoring the given information, their current decisions, how their models reflect those, and whether any reconsiderations of process, design, or models are needed.

Modifications to Pacing

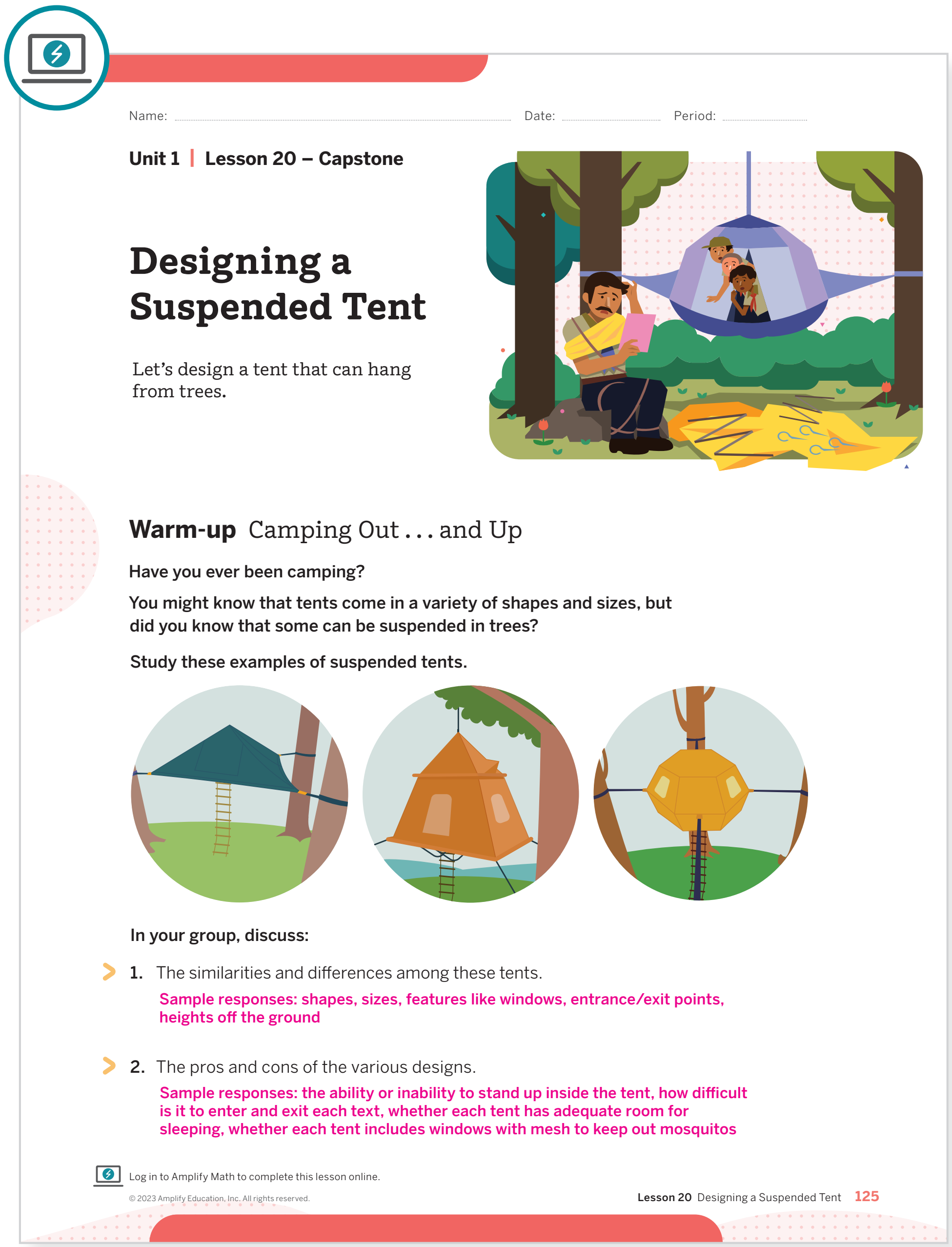
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** can be treated as a part of the Launch for Activity 1. The discussion of pros and cons of designs can be conducted as a whole class, reducing partner work time.
- In **Activity 1**, have groups choose a design closely related to one of the three provided examples.

Small Groups | 5 min

Warm-up Camping Out . . . and Up

Students compare and critique three designs of suspended tents.



Name: Date: Period:

Unit 1 | **Lesson 20 – Capstone**

Designing a Suspended Tent

Let's design a tent that can hang from trees.

Warm-up Camping Out . . . and Up

Have you ever been camping?

You might know that tents come in a variety of shapes and sizes, but did you know that some can be suspended in trees?

Study these examples of suspended tents.

In your group, discuss:

1. The similarities and differences among these tents.

   Sample responses: shapes, sizes, features like windows, entrance/exit points, heights off the ground

2. The pros and cons of the various designs.

   Sample responses: the ability or inability to stand up inside the tent, how difficult is it to enter and exit each text, whether each tent has adequate room for sleeping, whether each tent includes windows with mesh to keep out mosquitos

Log in to Amplify Math to complete this lesson online.

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Lesson 20 Designing a Suspended Tent 125

1 Launch

Explain *pros* are positive attributes or benefits relative to an intended purpose or goal, and explain *cons* are negative attributes or downsides that might be counterproductive to an intended purpose or goal.

2 Monitor

Help students get started by asking, "Would you want to sleep in one of these tents more than the others? Why that one?"

Look for points of confusion:

- **Not understanding what "suspended" means.** Ask students how these tents differ from tents that lay flat on the ground.
- **Noticing "less important" similarities or differences, such as the color of the tent.** "What are some shapes that you see that make up these tents?"

Look for productive strategies:

- Comparing how people would be able to enter/exit, move around, and sleep in each tent.
- Wondering about the windows in some of the tents and how that would affect the interior space.
- Comparing how people would lay down so they had enough space.
- Discussing the shape of the bottom of the tent.

3 Connect

Display the images of the suspended tents for students to refer to when sharing.

Have groups of students alternate sharing something related to either the first or second discussion point.

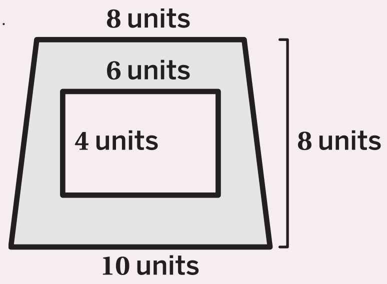
Highlight the points that students shared that related to the comfort of moving inside the tent, such as ease of entering, adequate space for sleeping, possibly mesh to keep bugs out, and air circulating, etc..

Power-up

To power up students' ability to determining the shaded area of a figure have them complete:

Determine the shaded area of the given figure.

120



Use: Before Activity 1

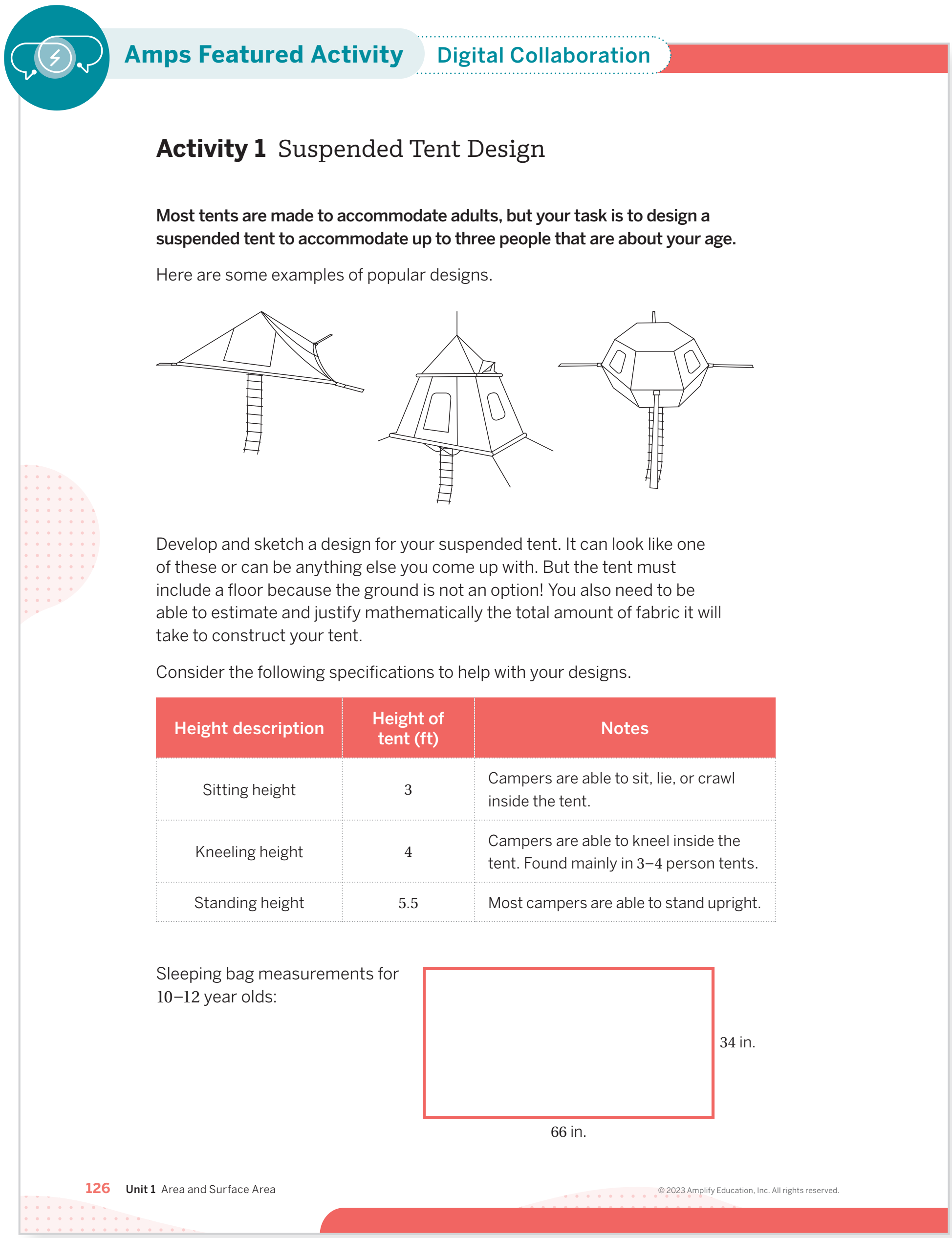
Informed by: Performance on Lesson 19, Practice Problem 6

Small Groups | 30 min

Activity 1 Suspended Tent Design

Students work collaboratively to design a suspended tent and calculate its surface area.

Amps Featured Activity Digital Collaboration



Activity 1 Suspended Tent Design

Most tents are made to accommodate adults, but your task is to design a suspended tent to accommodate up to three people that are about your age.

Here are some examples of popular designs.

Develop and sketch a design for your suspended tent. It can look like one of these or can be anything else you come up with. But the tent must include a floor because the ground is not an option! You also need to be able to estimate and justify mathematically the total amount of fabric it will take to construct your tent.

Consider the following specifications to help with your designs.

Height description	Height of tent (ft)	Notes
Sitting height	3	Campers are able to sit, lie, or crawl inside the tent.
Kneeling height	4	Campers are able to kneel inside the tent. Found mainly in 3–4 person tents.
Standing height	5.5	Most campers are able to stand upright.

Sleeping bag measurements for 10–12 year olds:

34 in.

66 in.

126 Unit 1 Area and Surface Area

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1 Launch

Have students read the directions aloud. Explain that their tent needs to be waterproof. Distribute one copy of the Activity 1 PDF to each group. Encourage students to come up with their own designs, if they want to, or they can use one of the three provided examples as inspiration for a similar, but different design.

2 Monitor

Help students get started by asking, “Is there a tent here that you would be interested in using as a starting point for your design?”

Look for points of confusion:

- **Incorrectly drawing the net.** Refer back to the nets from Lessons 15 and 16 to help connect the 3D shape of the tent to a net.
- **Thinking that they cannot have any windows or doors if the tent is to be waterproof.** Refer back to the second tent in the Warm-up and ask, “How did they incorporate the window and door here?”
- **Not knowing how to partition the tent to determine the surface area.**
    - Ask, “What 2D shapes make up your tent’s sides?”

Look for productive strategies:

- Sharing and exchanging ideas equitably.
- Drawing a net of the tent to determine its surface area.
- Decomposing the tent into individual polygons and grouping identical polygons to simplify calculations.
- Using tools from their geometry toolkits to help design the tent and calculate its surface area.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have groups choose a design from one of the three provided examples. Provide a project checklist that chunks the various steps of the project into a set of manageable tasks.

Math Language Development

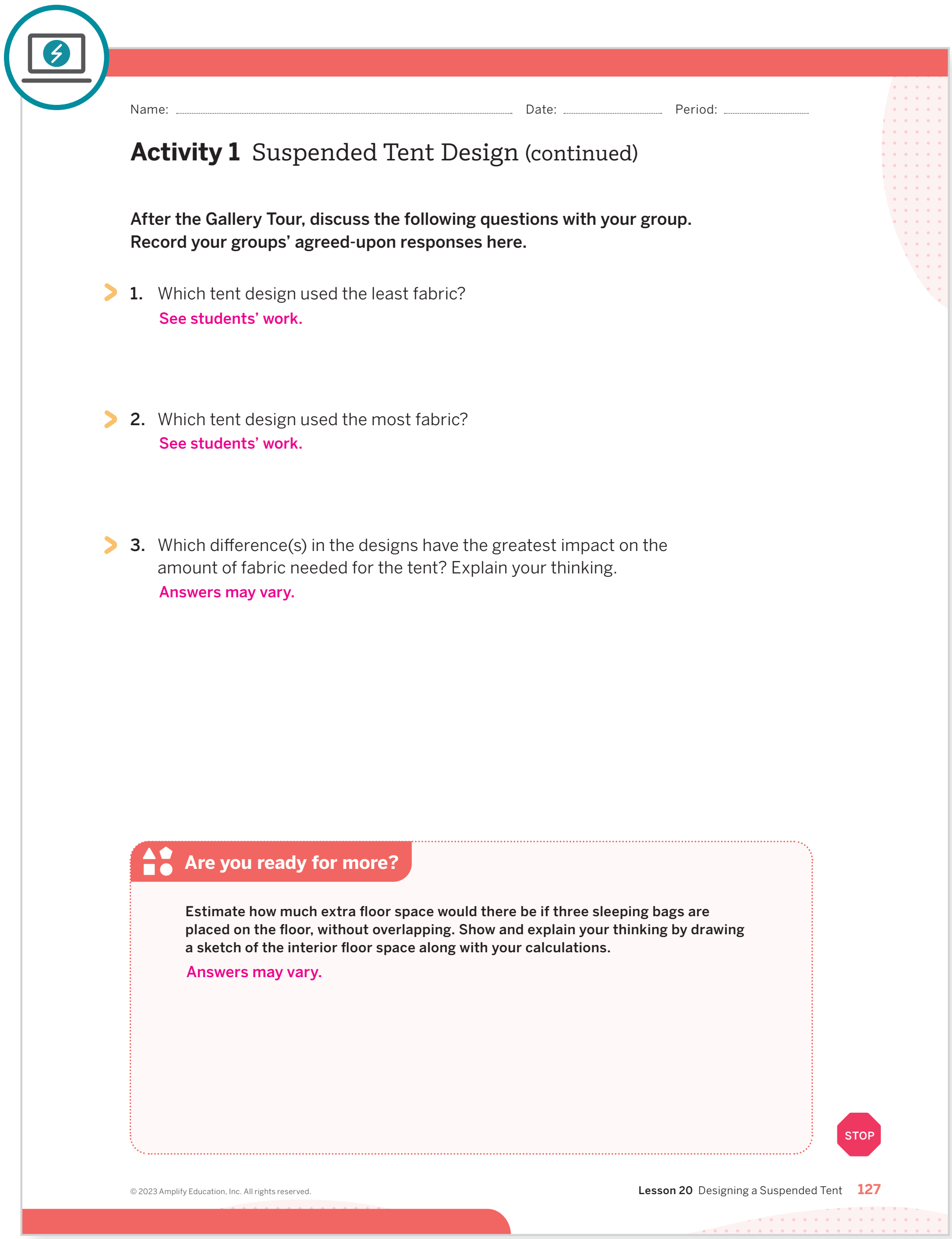
MLR7: Compare and Connect

Help students consider the audience when preparing their work for the *Gallery Tour*. Record ideas for details that could be included in their tent sketch. Examples include: the clarity of any drawn diagrams, written notes or details to clarify diagrams, use of specific vocabulary or phrases, or use of color or arrows to show connections between representations.

Small Groups | 30 min

Activity 1 Suspended Tent Design (continued)

Students work collaboratively to design a suspended tent and calculate its surface area.



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Activity 1 Suspended Tent Design (continued)

After the Gallery Tour, discuss the following questions with your group. Record your groups' agreed-upon responses here.

1. Which tent design used the least fabric?
   See students' work.

2. Which tent design used the most fabric?
   See students' work.

3. Which difference(s) in the designs have the greatest impact on the amount of fabric needed for the tent? Explain your thinking.
   Answers may vary.

Are you ready for more?

Estimate how much extra floor space would there be if three sleeping bags are placed on the floor, without overlapping. Show and explain your thinking by drawing a sketch of the interior floor space along with your calculations.

Answers may vary.

STOP

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Lesson 20 Designing a Suspended Tent 127

3 Connect

Display the group tent designs and conduct the ***Gallery Tour*** routine.

Have groups of students share and justify their responses to the post-***Gallery Tour*** questions that follow.

Ask:

- "What design choices led to using less fabric?" Sample responses: having the floor area just large enough for three sleeping bags; sitting height vs. kneeling and standing heights.
- "What design choices led to using more fabric?" Sample responses: having a taller height; providing more space between sleeping bags; incorporating windows that needed additional mesh fabric.
- "What are some ways that tents designed to accommodate the same number of people could use very different amounts of fabric?" Sample response: the differences in height and interior space.
- "What kinds of details helped you understand another group's tent design and how much fabric they needed?" Sample responses: showing the net of the tent with measurements corresponding to each area; showing identical shapes being grouped to find area multiplicatively; color coding identical sides; diagrams showing how area was calculated with shapes such as triangles and trapezoids; presenting a diagram with any features such as entrance/exit ways or windows.

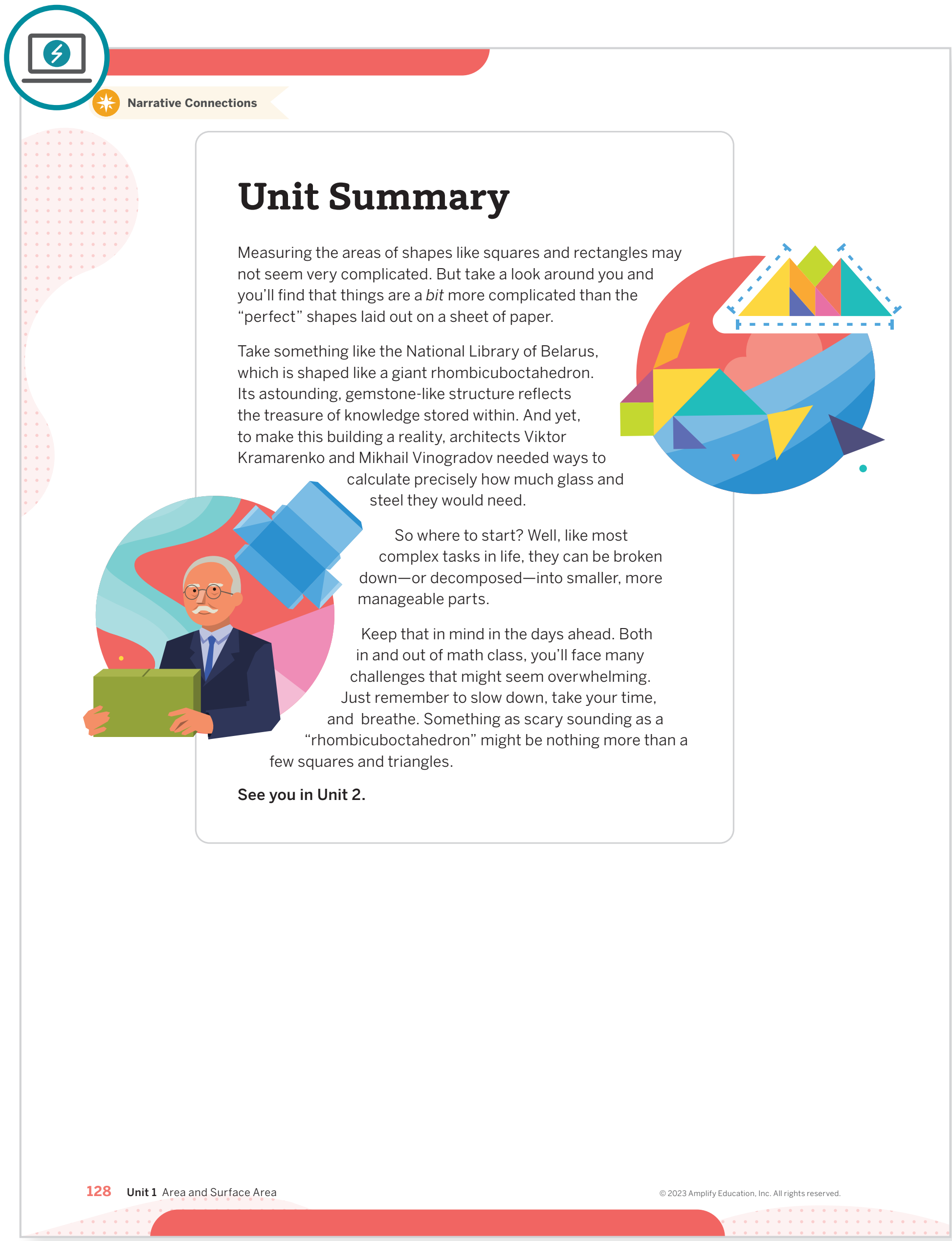
Highlight that students designed an item that could be useful in the real world. Tent designers and manufacturers go through a very similar process that students experienced today.

Ask, "If you were to market your suspended tent, what would you want your buying audience to know about it? In other words, what are the "pros" of your tent design?" Sample responses: how much space there is inside the tent for the three sleeping bags; if one can stand, kneel, or crawl; the added benefit of any features such as windows.

Whole Class | 5 min

Unit Summary

Review and synthesize how the tent design process required students to use key concepts from this unit including area and surface area, and also collaborative work behaviors.



Narrative Connections

Unit Summary

Measuring the areas of shapes like squares and rectangles may not seem very complicated. But take a look around you and you'll find that things are a *bit* more complicated than the "perfect" shapes laid out on a sheet of paper.

Take something like the National Library of Belarus, which is shaped like a giant rhombicuboctahedron. Its astounding, gemstone-like structure reflects the treasure of knowledge stored within. And yet, to make this building a reality, architects Viktor Kramarenko and Mikhail Vinogradov needed ways to calculate precisely how much glass and steel they would need.

So where to start? Well, like most complex tasks in life, they can be broken down—or decomposed—into smaller, more manageable parts.

Keep that in mind in the days ahead. Both in and out of math class, you'll face many challenges that might seem overwhelming. Just remember to slow down, take your time, and breathe. Something as scary sounding as a "rhombicuboctahedron" might be nothing more than a few squares and triangles.

See you in Unit 2.

128 **Unit 1** Area and Surface Area

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Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the tent designs.

Have students share what they enjoyed about working with their group for this activity, and how each member contributed something unique.

Ask, "When calculating the surface area of your tent, what skills and strategies from this unit did you find most useful?"

Highlight the progression over the course of the unit, starting from determining the area of 2D shapes to the surface area of 3D figures, and the connections between them.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

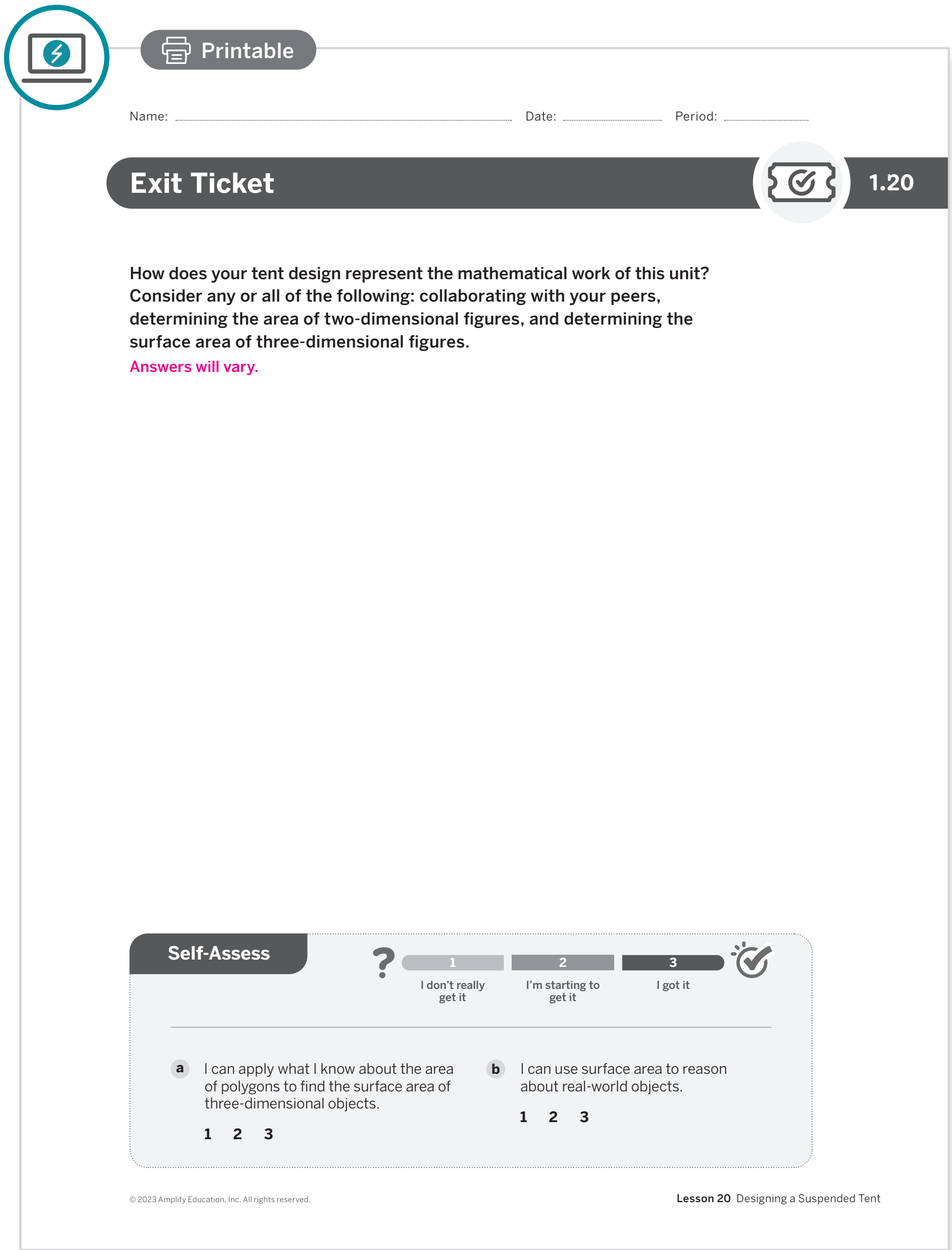
- Did anything surprise you while reading the narratives of this unit?
- Is there anything you would like to learn more about? What are some steps you can take to learn more?

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how the work of the unit is reflected in the capstone activity.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.20

How does your tent design represent the mathematical work of this unit? Consider any or all of the following: collaborating with your peers, determining the area of two-dimensional figures, and determining the surface area of three-dimensional figures.

Answers will vary.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a. I can apply what I know about the area of polygons to find the surface area of three-dimensional objects.

1 2 3

b. I can use surface area to reason about real-world objects.

1 2 3

Lesson 20 Designing a Suspended Tent

Success looks like . . .

- **Language Goal:** Applying understanding of surface area to estimate the amount of fabric needed to manufacture a tent, and explain the estimation strategy. **(Speaking and Listening, Writing)**
  - » Determining the surface area of the tent.
- **Language Goal:** Interpreting given information about tents and sleeping bags using multiple representations. **(Speaking and Listening)**
- **Language Goal:** Comparing and contrasting different tent designs mathematically. **(Speaking and Listening)**
  - » Comparing tent designs with peers.

Suggested next steps

If students write about less important things, such as color or prints on the fabric, consider:

- Referring them to the anchor charts from the unit and ask how those concepts were used throughout the tent design process.

Professional Learning

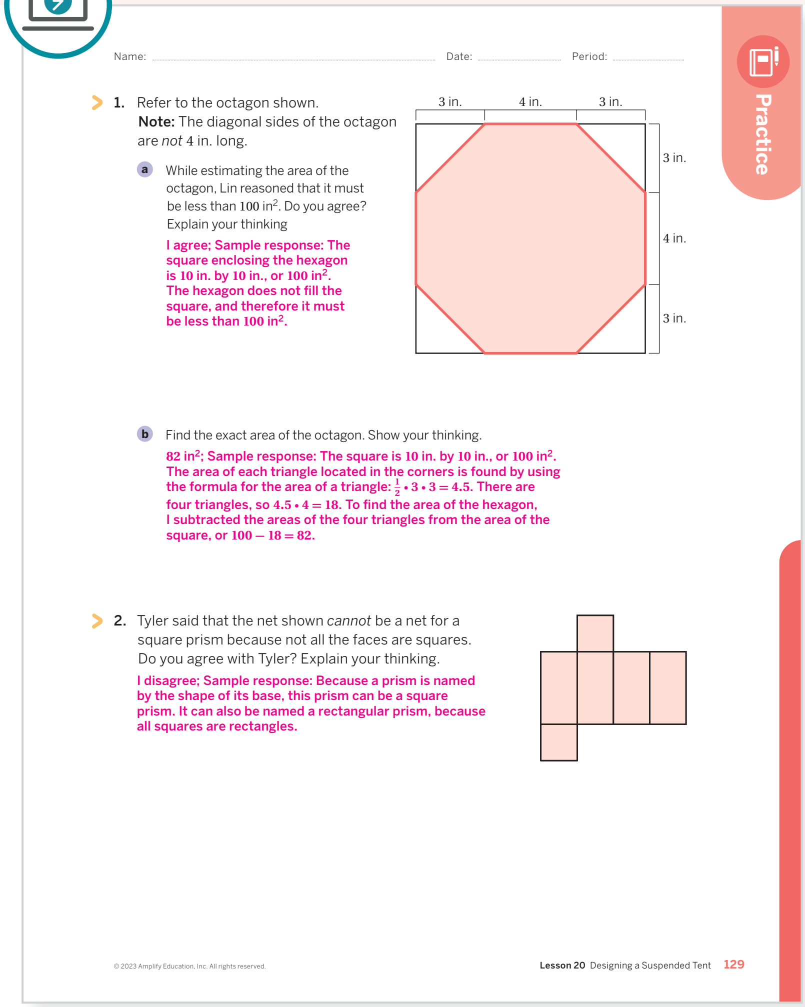
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? When you compare and contrast today's work with work students did earlier in this unit, what similarities and differences do you see?
- Assess your students' collaboration. How has it developed? What can you do to facilitate improving your students' collaboration skills?

Practice

Independent



Name: Date: Period:

1. Refer to the octagon shown.
   Note: The diagonal sides of the octagon are *not* 4 in. long.

   a. While estimating the area of the octagon, Lin reasoned that it must be less than 100 in^2. Do you agree? Explain your thinking

   I agree; Sample response: The square enclosing the hexagon is 10 in. by 10 in., or 100 in^2. The hexagon does not fill the square, and therefore it must be less than 100 in^2.

   b. Find the exact area of the octagon. Show your thinking.

   82 in^2; Sample response: The square is 10 in. by 10 in., or 100 in^2. The area of each triangle located in the corners is found by using the formula for the area of a triangle: $\frac{1}{2} \cdot 3 \cdot 3 = 4.5$. There are four triangles, so $4.5 \cdot 4 = 18$. To find the area of the hexagon, I subtracted the areas of the four triangles from the area of the square, or $100 - 18 = 82$.

2. Tyler said that the net shown *cannot* be a net for a square prism because not all the faces are squares. Do you agree with Tyler? Explain your thinking.

   I disagree; Sample response: Because a prism is named by the shape of its base, this prism can be a square prism. It can also be named a rectangular prism, because all squares are rectangles.

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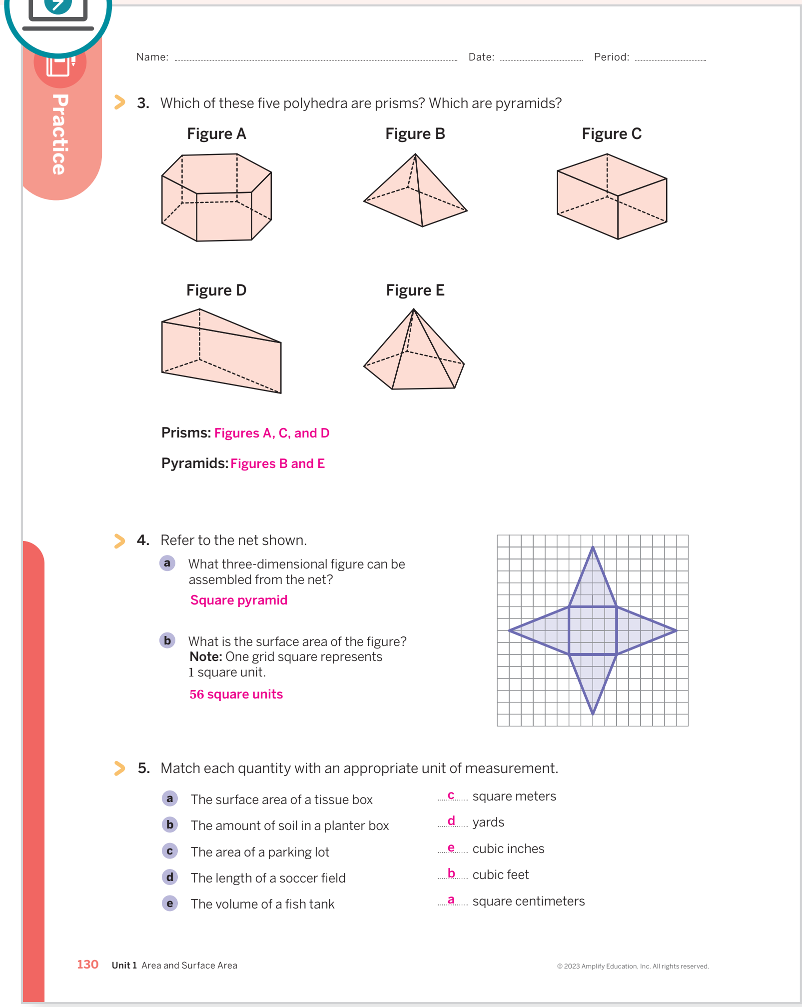
Lesson 20 Designing a Suspended Tent 129



Name: Date: Period:

3. Which of these five polyhedra are prisms? Which are pyramids?

   Prisms: **Figures A, C, and D**

   Pyramids: **Figures B and E**

4. Refer to the net shown.

   a. What three-dimensional figure can be assembled from the net?

   Square pyramid

   b. What is the surface area of the figure?
   Note: One grid square represents 1 square unit.

   56 square units

5. Match each quantity with an appropriate unit of measurement.

a	The surface area of a tissue box	c	square meters
b	The amount of soil in a planter box	d	yards
c	The area of a parking lot	e	cubic inches
d	The length of a soccer field	b	cubic feet
e	The volume of a fish tank	a	square centimeters

130 **Unit 1** Area and Surface Area

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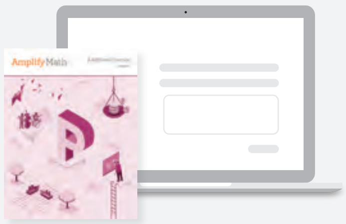
Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 1 Lesson 5	2
	2	Unit 1 Lesson 15	2
	3	Unit 1 Lesson 16	2
	4	Unit 1 Lesson 16	2
	5	Unit 1 Lesson 19	1

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

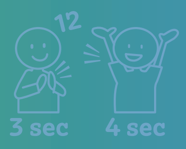
UNIT 2

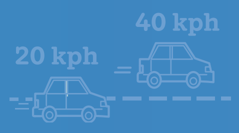
Introducing Ratios

Students understand the concept of ratios in the context of three of their five senses. They use written and visual representations to learn the language of ratios. Students determine the relationship between numbers by scaling up (multiplication) or scaling down (division) to calculate equivalent ratios. Ratios are also used as a way to think about constant rates or things happening at the same rate.

Essential Questions

- What does a ratio say about the relationship between quantities?
- How can ratios reflect fairness?
- How can ratios help you estimate solutions to seemingly impossible real-world problems?
- *(By the way, is it possible to have too much cowbell?)*





Key Shifts in Mathematics

Focus

In this unit . . .

Students learn that a *ratio* is an association between two quantities by multiplication or division. Students first encounter equivalent ratios in thinking about multiple batches of a variety of recipes. Building on these experiences, students analyze situations involving both discrete and continuous quantities using double number lines, ratio tables, and tape diagrams. They then use these tools to compare ratios and determine missing values.

Coherence

Previously . . .

Starting in Grade 3, students worked with relationships that can be expressed in terms of ratios and rates (e.g., conversions between measurements in inches and in yards), but they did not use these terms. In Grade 4, students studied multiplicative comparison. In Grade 5, they began to interpret multiplication as scaling, preparing them to think about simultaneously scaling two quantities by the same factor. They learned what it means to divide one whole number by another, so they are well equipped to consider the quotients $\frac{a}{b}$ and $\frac{b}{a}$ associated with a ratio $a : b$ for non-zero whole numbers a and b.

Unit 1 began to set the mathematical community tone for the year. The importance of effective collaboration and perseverance is essential to students' experience as mathematicians.

Coming soon . . .

Unit 3 continues to explore ratios, with a focus on unit rate, just as the major use of part-part-whole ratios occurs with certain kinds of percentage problems.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Ratio language provides the foundation for developing the concept of ratios as a relationship between two quantities (Lessons 2–5). This concept builds as students look for the relationship between two ratios and equivalent ratios (Lessons 6–7, 11–14).

Procedural Fluency

Ample practice scaling ratios up and down prepares students for fluently determining equivalent ratios (Lessons 4–6). Ratio tables and double number lines support procedural fluency (Lessons 7, 11–13). Lessons on common factors and common multiples (Lessons 9–10) provide opportunities to fluently and flexibly identify relationships between numbers.

Application

Ratios and equivalent ratios are extended to real-world scenarios, including rate problems (Lessons 14, 16–19).

Sensing a Ratio

SUB-UNIT

1

Lessons 2–5

What are Ratios?

Students are introduced to the language of ***ratios***. They are given multiple opportunities to use this language as they move into abstractly representing ***ratio relationships***. Students work with scaling ratios up and down as they also create the foundation for equivalent ratios. Beware . . . the making of oobleck is an option your students will be sure to enjoy, but it could get messy.

Narrative: Whether it's colors or tiles, having the right amount of each part is the key.

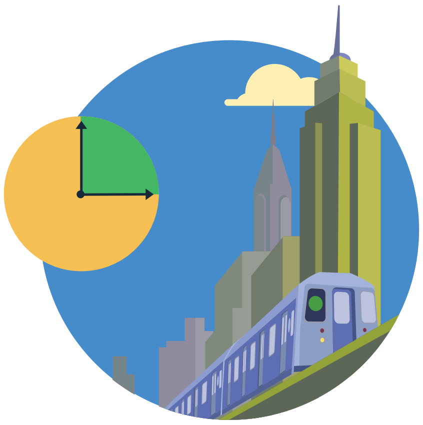
SUB-UNIT

2

Lessons 6–13

Equivalent Ratios

Equivalent ratios are formally defined early in this Sub-Unit. Equivalent ratios are explored by using representations such as tables and double number lines. Mid-way through the Sub-Unit, students focus shifts to identifying and using the ***common factor***, ***greatest common factor***, ***common multiple***, and ***least common multiple***. Students take this new understanding of common factors and multiples and apply it to more efficiently navigate ratio tables.



Narrative: Ratios can help you keep a rhythm and balance the sounds of music.

Launch

Lesson 1

Fermi Problems

Students tackle problems that seem impossible to solve, but discover, with the help of their peers, that it can be done! This lesson continues the sense of teamwork and perseverance begun in Unit 1, as well as touching on the math of the unit which involves finding base numbers by multiplying or dividing, scaling up or down, to solve a problem.

SUB-UNIT

3

Lessons 14–19

Solving Ratio Problems

This Sub-Unit puts ratios into real-world application problems, involving everything from hot chocolate to hot chilies. Students compare the heat of a variety of chilies — but don't worry, unlike the oobleck option earlier, your students will not be testing chilies! As students compare ratios, they study and determine the information needed to solve these problems.

Narrative: Every good cook knows that ratios are an important ingredient of any recipe.

Capstone

Lesson 20

More Fermi Problems

Students once again tackle Fermi problems, but now they are armed with oobleck and chilies, ratio knowledge and information gathering, and analytical skills.

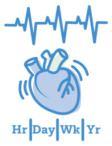
Unit at a Glance

Spoiler Alert: What's good for the goose is good for the gander. If you multiply one part of a ratio by a number, you have to multiply the other part by the same number. If you divide one part of a ratio by a number, you have to divide the other part by the same number.

Assessment

A Pre-Unit Readiness Assessment

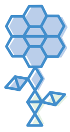
Launch Lesson



1 Fermi Problems

Reason with questions that seem impossible to answer.

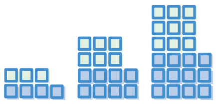
Sub-Unit 1: What are Ratios?



2 Introducing Ratios and Ratio Language ●

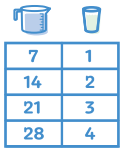
Sentence structures for describing ratio relationships are introduced.

Sub-Unit 2: Equivalent Ratios



6 Defining Equivalent Ratios

The term *equivalent ratio* is formalized.



7	1
14	2
21	3
28	4

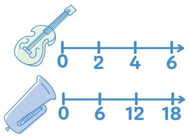
7 Representing Equivalent Ratios With Tables

Ratio tables help visualize and organize a set of scaled-up equivalent ratios.

$\div 12 \cdot 17$

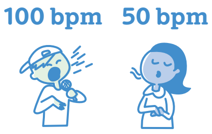
8 Reasoning With Multiplication and Division (optional) ●

Sequences of multiplication and division provide practice getting from one number to another.



12 Tables and Double Number Line Diagrams

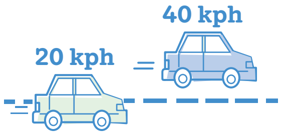
Relate representations of equivalent ratios on tables and double number lines.



13 Tempo and Double Number Lines

Practice creating double number lines to represent equivalent ratios.

Sub-Unit 3: Solving Ratio Problems



14 Solving Equivalent Ratio Problems

Solve for missing values in equivalent ratios.

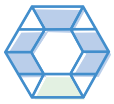
Key Concepts

Lesson 3: Ratios can be represented by abstract diagrams.
Lesson 6: Equivalent ratios are found by multiplying or dividing the parts by the same number.
Lesson 14: Equivalent ratios can be used to solve for a missing value for one quantity.

Pacing

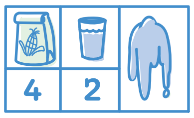
20 Lessons: 45 min each
2 Assessments: 45 min each
Full Unit: 22 days
● **Modified Unit:** 18 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



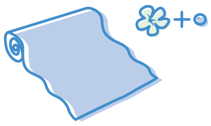
3 Representing Ratios With Diagrams

Ratio language is translated into abstracted diagrams.



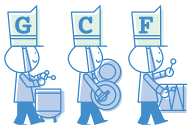
4 A Recipe for Purple Oobleck

Ratios that are equivalent are introduced by scaling up, or multiplying two amounts by the same number.



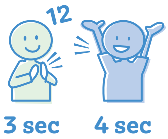
5 Kapa Dyes

Ratios that are equivalent are introduced by scaling down, or dividing two amounts by the same number.



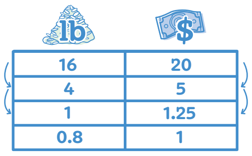
9 Common Factors

Factors of two numbers are explored in real-world contexts, including the identification of greatest common factors.



10 Common Multiples

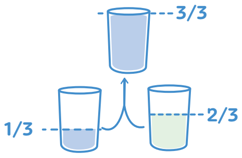
Patterns finding common multiples and least common multiple are explored in mathematical and real-world contexts.



lb	$
16	20
4	5
1	1.25
0.8	1

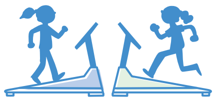
11 Navigating a Table of Equivalent Ratios

Determine equivalent ratios by using a ratio table.



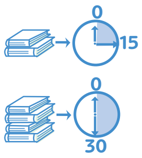
15 Part-Part-Whole Ratios

Justify starting with the part or the whole to solve ratio problems.



16 Comparing Ratios

Strategies to compare two ratios are explored.



17 More Comparing and Solving ●

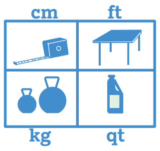
Equivalent ratios are developed and compared when total parts are considered.

Unit at a Glance

Spoiler Alert: What's good for the goose is good for the gander. If you multiply one part of a ratio by a number, you have to multiply the other part by the same number. If you divide one part of a ratio by a number, you have to divide the other part by the same number.

continued

Capstone Lesson



18

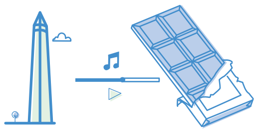
Measuring With Different-Sized Units ●

Concepts of converting standard units of length, weight, and volume are reviewed.

19

Converting Units

See how ratios connect to the process for converting units.



20

More Fermi Problems ●

Apply ratio reasoning to Fermi problems that seem impossible to solve.

Key Concepts

Lesson 3: Ratios can be represented by abstract diagrams.
Lesson 6: Equivalent ratios are found by multiplying or dividing the parts by the same number.
Lesson 14: Equivalent ratios can be used to solve for a missing value for one quantity.

Pacing

20 Lessons: 45 min each
2 Assessments: 45 min each
Full Unit: 22 days
● **Modified Unit:** 18 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

Assessment

A **End-of-Unit Assessment**

● Modifications to Pacing

Lessons 2: Lesson 2's focus on language could be absorbed into Lesson 3, connecting it to the diagrams evaluated and created in Lesson 3. However, students really do love the "Two Truths and a Lie" activity, so it would be better that you don't lie about there not being a Lesson 2.

Lesson 8: This optional lesson can be omitted.

Lessons 17–18: Lesson 17 may be omitted as it is a further extension of material explored in Lesson 16. However, you may want to look at the strategies highlighted in Lesson 17 and bring them into the Lesson 16 fold. Given that Lesson 18 is more of a review to prepare students for Lesson 19, it may be omitted as well.

Lessons 20: This capstone lesson may be omitted. However, not only will the fun application be taken away, but also the possibility for students to feel as though they have reached mathematical nirvana by solving a Fermi problem!

Unit Supports

Math Language Development

Lesson	New Vocabulary
2	ratio relationship
4	equivalent ratio
6	equivalent ratios
7	ratio table
9	common factor greatest common factor
10	common multiple least common multiple
11	per

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Language Routines
2, 6, 10, 12, 13, 20	**MLR1:** Stronger and Clearer Each Time
2, 3, 7, 9–11, 15, 19	**MLR2:** Collect and Display
5, 17, 18, 19	**MLR3:** Critique, Correct, Clarify
14	**MLR4:** Information Gap
1	**MLR5:** Co-craft Questions
9, 15, 16, 17	**MLR6**: Three Reads
2–7, 9, 12, 13, 15, 19	**MLR7:** Compare and Connect
1, 3–5, 7, 8, 11, 13, 16, 18, 20	**MLR8:** Discussion Supports

Materials

Every lesson includes:

Exit Ticket Additional Practice

Lesson(s)	Additional required materials
1, 19, 20	calculators
1, 3	colored Pencils
4, 5	counters
3	envelopes
18	four 1-liter bottles four 1-quart bottles one 1-gallon jug rulers and scales select objects to be measured (textbook, stapler, etc.)
1	markers
20	materials for creating a visual display computers
2, 3	pattern blocks
1–6, 9, 10, 13–16, 19, 20	PDFs are required for these lessons. Refer to each lesson to see the required PDFs.
4	Optional supplies: food coloring (red/blue) measuring cup cornstarch 3–4 bowls clear cups graph paper snap cubes

Instructional Routines

Activities throughout this unit include these routines:

Lesson(s)	Instructional Routines
1	Carousel
12, 20	Gallery Tour
14	Info Gap
2	Mix and Mingle, Two Truths and a Lie
16	Notice and Wonder
4, 5, 11, 12	Number Talk
3, 18	Take Turns
1, 7, 11, 13, 19	Think-Pair-Share
7	Turn and Talk

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 20

Amps

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Social & Collaborative Digital Moments

Featured Activity

Faster and Slower Tempos

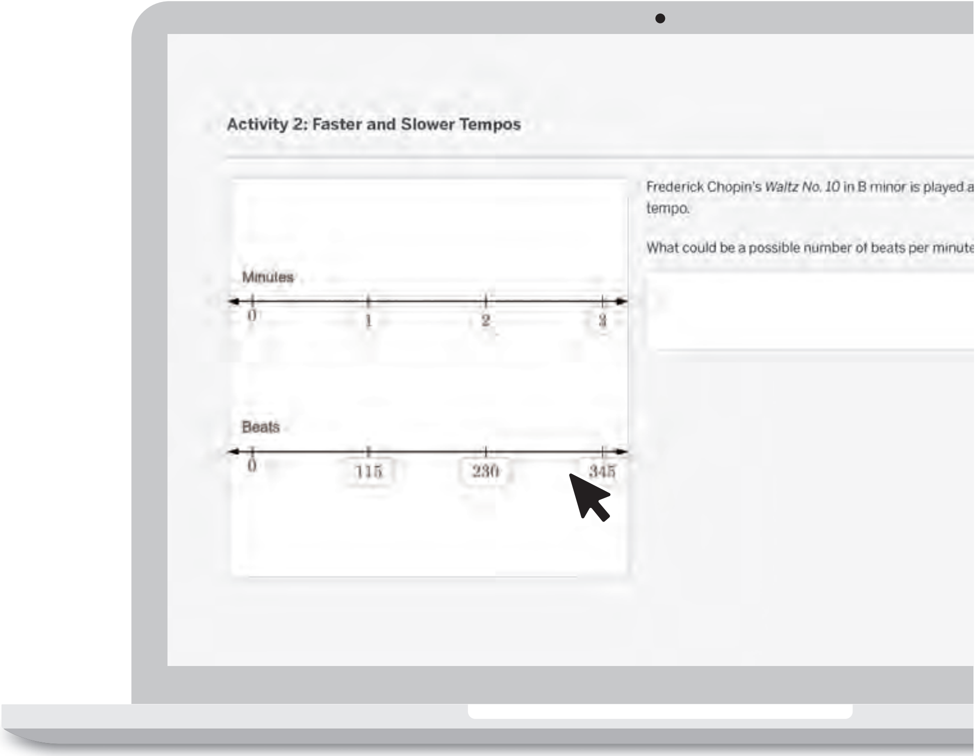
Put on your student hat and work through **Lesson 13, Activity 2:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Kapa Dyes **(Lesson 5)**
- What Are Equivalent Ratios? **(Lesson 6)**
- Comparing Chilli Peppers **(Lesson 16)**
- Bliss Point **(Lesson 17)**



Activity 2: Faster and Slower Tempos

Frederick Chopin's *Waltz No. 10* in B minor is played a

tempo.

What could be a possible number of beats per minute

Unit Study
Professional Learning

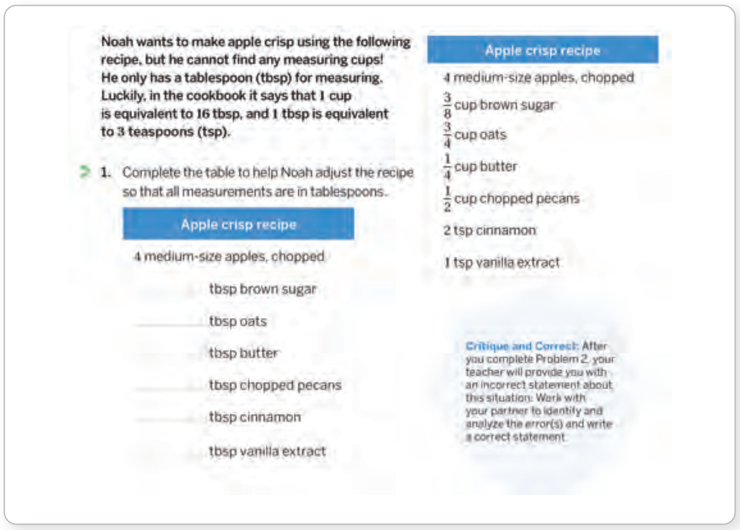
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 3 introduces students to solving problems using ratios. They work with equivalent ratios and unit ratios. Students learn to solve problems by finding the greatest common factor or the least common multiple. They can compare ratios and understand that ratios can be combined or added to solve problems. The unit wraps up with students learning to convert units in real-world applications. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from **Lesson 19, Activity 1:**



Noah wants to make apple crisp using the following recipe, but he cannot find any measuring cups! He only has a tablespoon (tbsp) for measuring. Luckily, in the cookbook it says that 1 cup is equivalent to 16 tbsp, and 1 tbsp is equivalent to 3 teaspoons (tsp).

1. Complete the table to help Noah adjust the recipe so that all measurements are in tablespoons.

Apple crisp recipe

4 medium-size apples, chopped

_____ tbsp brown sugar

_____ tbsp oats

_____ tbsp butter

_____ tbsp chopped pecans

_____ tbsp cinnamon

_____ tbsp vanilla extract

Apple crisp recipe

4 medium-size apples, chopped

$\frac{3}{8}$ cup brown sugar

$\frac{3}{4}$ cup oats

$\frac{1}{4}$ cup butter

$\frac{1}{2}$ cup chopped pecans

2 tsp cinnamon

1 tsp vanilla extract

Critique and Correct: After you complete Problem 2, your teacher will provide you with an incorrect statement about this situation. Work with your partner to identify and analyze the error(s) and write a correct statement.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

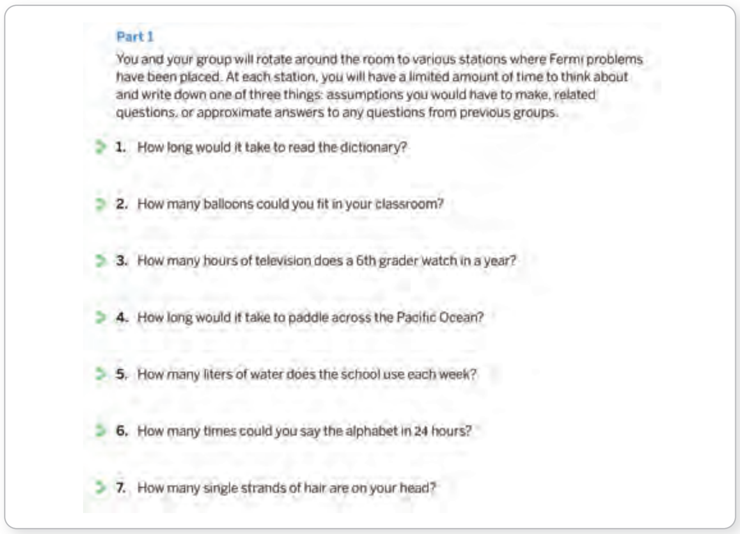
- What was it like to engage in this problem as a learner?
- What approaches might your students take?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Carousel Routine

Rehearse . . .

How you'll facilitate the ***Carousel*** instructional routine in **Lesson 1, Activity 1:**



Part 1

You and your group will rotate around the room to various stations where Fermi problems have been placed. At each station, you will have a limited amount of time to think about and write down one of three things: assumptions you would have to make, related questions, or approximate answers to any questions from previous groups.

1. How long would it take to read the dictionary?
2. How many balloons could you fit in your classroom?
3. How many hours of television does a 6th grader watch in a year?
4. How long would it take to paddle across the Pacific Ocean?
5. How many liters of water does the school use each week?
6. How many times could you say the alphabet in 24 hours?
7. How many single strands of hair are on your head?

Points to Ponder . . .

- How will you balance too little time at a station with too much time at a station?

This routine . . .

- Gets students up and moving.
- Provides variety given that each rotation has a different focus.
- Allows students to see *all* of the problems instead of just one.
- Builds collective information to be reviewed, analyzed, and used to solve a problem.

Anticipate . . .

- A wide variety of responses, both relevant and not.
- Timing to feel too fast or too slow at times. Consider projecting a stopwatch so students are aware of the time.
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Support productive struggle in learning mathematics.

This effective teaching practice . . .

- Provides students with the opportunity to wrestle with mathematical concepts and relationships before you intervene, which builds student confidence and perseverance.
- Allows you greater opportunities to monitor student progress, probe for student understanding, and offer differentiation support.

Points to Ponder . . .

- How comfortable are you with allowing students the time to wrestle with mathematical ideas, before you intervene?
- When is the right moment to intervene? What can you look for as you monitor student work and student conversations to know when they are engaging in productive struggle or unproductive struggle?

Math Language Development

MLR2: Collect and Display

MLR2 appears in Lessons 2, 3, 7, 9–11, 15, 19.

- In Lesson 11, as students share their responses, you can highlight and collect terms and phrases they use to describe ratios, such as *per* and for *each*.
- Throughout the unit, as students formalize the new vocabulary they are learning, have them refer to the class display to continually review and reflect on new terms that are added.
- **English Learners:** Add diagrams or illustrations to the class display so that students can visualize the terms or phrases.

Point to Ponder . . .

- How will you encourage or guide students toward using their developing ratio language to describe ratio relationships in this unit?

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Opportunities to vary the demands of a task or activity appear in Lessons 1, 3, 4, 6–13, 15–20.

- Chunking a complex task into smaller, more manageable parts allows students to focus on one part at a time.
- In Lesson 18, Activity 2, instead of having students measure, provide sample measurements of the objects, because the activity goal is to notice that it takes more centimeters than inches to measure the length of an object, not to perform the actual act of measuring.
- Some students may benefit from more processing time. When restricting or altering tasks, consider allowing students to choose which problem(s) to complete. Students are often more engaged when they have a choice.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to vary the demands of a particular task or activity? What clues will you gather from your students?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **End-of-Unit Assessment**, noting the concepts and skills assessed.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with solving one-step equations with a variable throughout the unit? Do you think your students will generally:
  - miss the underlying concept of balance and mathematical equality?
  - simply struggle with the concept of variables and unknowns?
  - be fully ready to solve procedurally and efficiently, but misapply the properties of equality?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

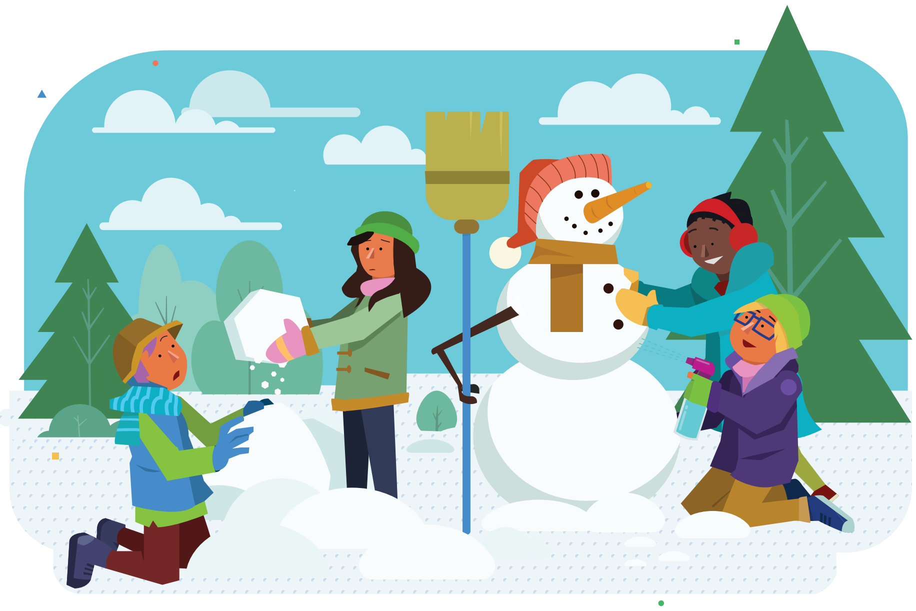
In this unit, pay particular attention to supporting students in building their self-awareness and self-management skills.

Points to Ponder . . .

- What are their strengths and what do they know about numerical reasoning that they can build upon and use to begin reasoning algebraically?
- Are students able to exercise patience and persist in order to understand the underlying conceptual structure of one-variable equations, rather than asking for or jumping to a procedural shortcut?

Fermi Problems

Let's explore Fermi problems.



Focus

Goals

1. **Language Goal:** Ask questions that can help gather more information that is useful for solving real-world problems. **(Speaking and Listening, Writing)**
2. Identify information that is relevant and useful for solving real-world problems.

Coherence

Today

Students reason with information related to an unfamiliar, Fermi-type problem. They must take a question that may at first seem impossible to answer and make assumptions and approximations to simplify the problem so that a reasonable answer can be determined, which requires sense-making and perseverance. Students collaborate to understand what the problem is asking by breaking down larger questions into manageable sub-questions. They make assumptions, plan an approach, and reason with the mathematics and the information they know. As a group, they draw models and diagrams to illustrate the thinking behind their process for solving the problem.

Previously

In Grades 4 and 5, students worked with converting measurements of time, length, and volume. In Unit 1 of this grade, students calculated the volume of right prisms.

Coming Soon

In Lessons 2–3, students will build a basis for understanding ratio relationships and ratio language through encounters with examples of ratio relationships in a variety of contexts, such as mixing colors and recipes.

Rigor

- Students **apply** prior skills and knowledge for solving word problems to Fermi problems.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	25 min	5 min	5 min
Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

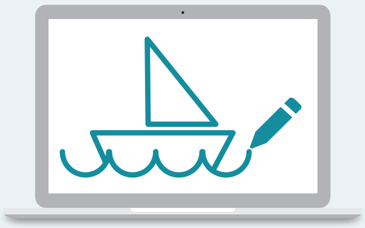
- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Carousel Routine: Teacher Instructions*
- Activity 1, Part 1 PDF, one per Fermi problem or one per group
- Activity 1, Part 2 PDF, one per Fermi problem or one per group
- Activity 1, Part 2 PDF, *Four-Square Graphic Organizer* (as needed)
- calculators
- markers
- colored pencils

Amps | **Featured Activity**

Activity 1

Digital Poster

Students create their posters digitally to share in small groups, but they can also be visible for everyone.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students may want to impulsively write anything down just to get something down. However, in order to make sense of each problem, they should think thoughtfully and ask meaningful questions that will help them build toward a solution. Encourage students to think through their responses before writing anything down. Remind students that reasonableness requires thoughtful attention to the problem and to the information needed to solve it.

Modifications to Pacing

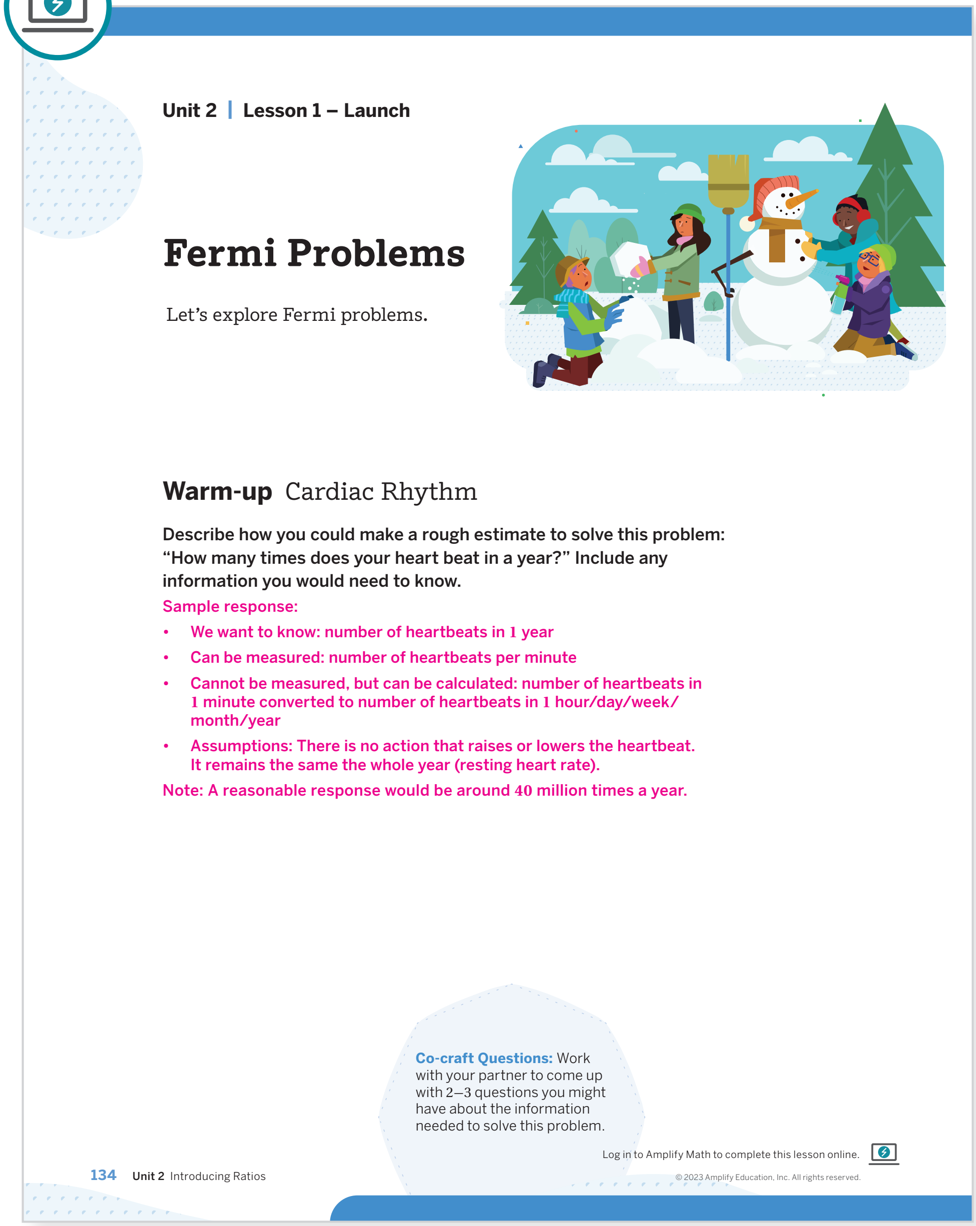
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, the number of Fermi problems and rotations can be reduced.

Pairs | 10 min

Warm-up Cardiac Rhythm

Students determine what relevant information needs to be identified and how it can be used or pieced together mathematically to solve a problem.



Unit 2 | **Lesson 1 – Launch**

Fermi Problems

Let's explore Fermi problems.

Warm-up Cardiac Rhythm

Describe how you could make a rough estimate to solve this problem: "How many times does your heart beat in a year?" Include any information you would need to know.

Sample response:

- We want to know: number of heartbeats in 1 year
- Can be measured: number of heartbeats per minute
- Cannot be measured, but can be calculated: number of heartbeats in 1 minute converted to number of heartbeats in 1 hour/day/week/month/year
- Assumptions: There is no action that raises or lowers the heartbeat. It remains the same the whole year (resting heart rate).

Note: A reasonable response would be around 40 million times a year.

Co-craft Questions: Work with your partner to come up with 2–3 questions you might have about the information needed to solve this problem.

Log in to Amplify Math to complete this lesson online.

134 **Unit 2** Introducing Ratios

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1 Launch

Conduct the ***Think-Pair-Share*** routine.

2 Monitor

Help students get started by having them work with a partner to brainstorm ideas instead of having independent think time.

Look for points of confusion:

- **Thinking they need to solve the problem.** Remind students that they write about the steps to solve, not actually find the solution.
- **Not breaking down the unit of one year.** Ask, "Would it be reasonable to count every time your heart beats in one year, or could you break that down into a smaller unit of time?"

Look for productive strategies:

- Assuming this is a resting heartbeat.
- Estimating a solution first and then breaking the problem into smaller parts.

3 Connect

Display the following categories:

- Information you know
- Information you can measure
- Information that cannot be measured directly but can be calculated
- Assumptions

Have pairs of students share a piece of information or statement relevant to solving the problem, why it is important, and how it would be used.

Highlight ways information can be communicated more precisely. For example, revise "X beats in a minute," as "For every minute, there are X heartbeats."

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students count the number of heartbeats in one minute by counting their pulse. Have them record this value and return to it during the Connect as you highlight the phrases "X beats in a minute" and "For every minute, there are X heartbeats."

Math Language Development

MLR5: Co-craft Questions

Have pairs of students work together to co-craft questions they may have about the question given in the Warm-up, or co-craft information they might know or want to know in order to help answer the question.

English Learners

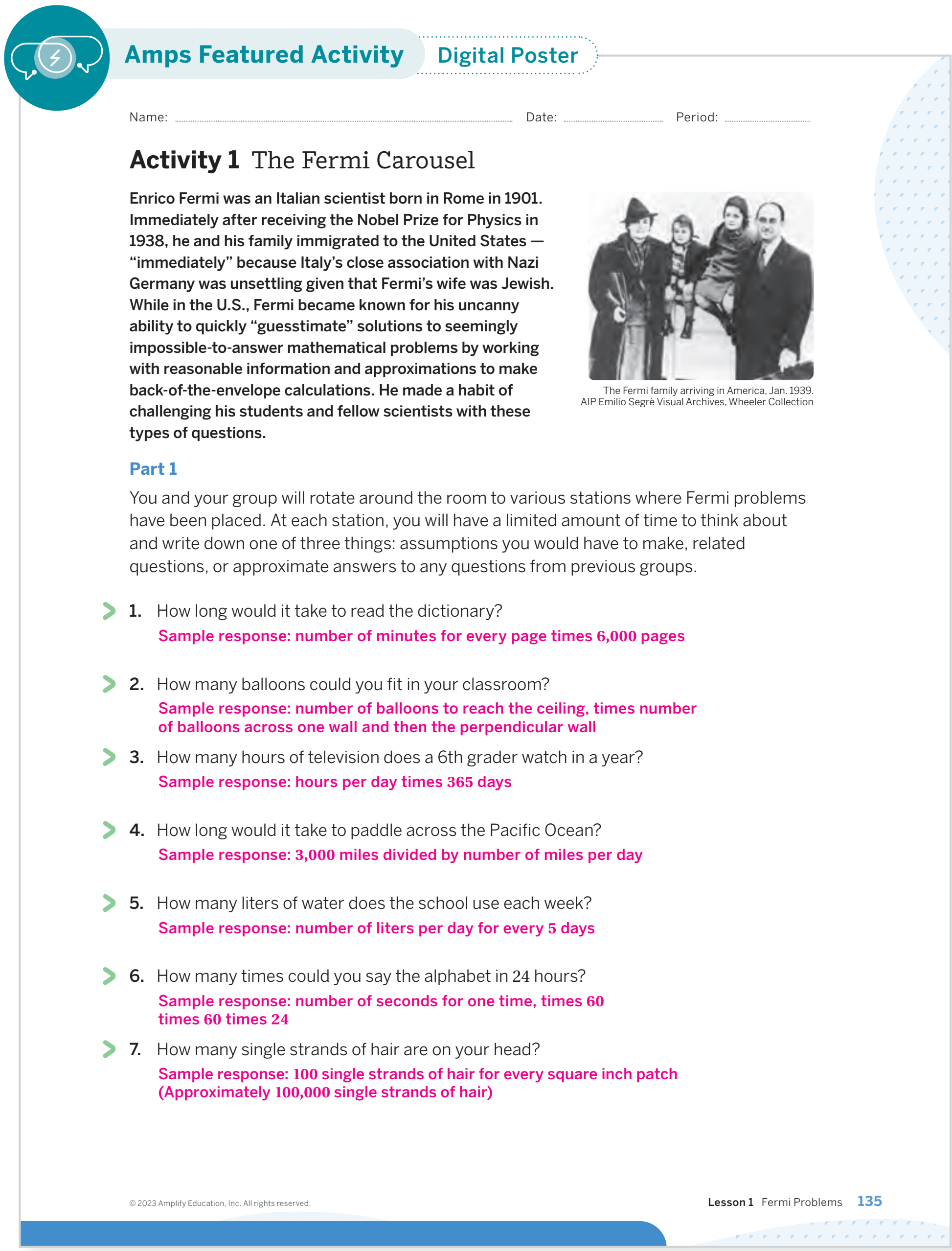
Model for students how to craft 1–2 questions, such as using a think-aloud: "I can find out how many times my heart beats in 1 minute. How can I use that to find out how many times my heart beats in 1 year?"

Small Groups | 25 min

Activity 1 The Fermi Carousel

Students rotate to read several Fermi problems and cycle through writing relevant assumptions, questions, and answers. Each group then works to solve one problem.

Amps Featured Activity Digital Poster



Name: ______ Date: ______ Period: ______

Activity 1 The Fermi Carousel

Enrico Fermi was an Italian scientist born in Rome in 1901. Immediately after receiving the Nobel Prize for Physics in 1938, he and his family immigrated to the United States — "immediately" because Italy's close association with Nazi Germany was unsettling given that Fermi's wife was Jewish. While in the U.S., Fermi became known for his uncanny ability to quickly "guesstimate" solutions to seemingly impossible-to-answer mathematical problems by working with reasonable information and approximations to make back-of-the-envelope calculations. He made a habit of challenging his students and fellow scientists with these types of questions.

The Fermi family arriving in America, Jan. 1939.
AIP Emilio Segrè Visual Archives, Wheeler Collection

Part 1

You and your group will rotate around the room to various stations where Fermi problems have been placed. At each station, you will have a limited amount of time to think about and write down one of three things: assumptions you would have to make, related questions, or approximate answers to any questions from previous groups.

1. How long would it take to read the dictionary?
   Sample response: number of minutes for every page times 6,000 pages
2. How many balloons could you fit in your classroom?
   Sample response: number of balloons to reach the ceiling, times number of balloons across one wall and then the perpendicular wall
3. How many hours of television does a 6th grader watch in a year?
   Sample response: hours per day times 365 days
4. How long would it take to paddle across the Pacific Ocean?
   Sample response: 3,000 miles divided by number of miles per day
5. How many liters of water does the school use each week?
   Sample response: number of liters per day for every 5 days
6. How many times could you say the alphabet in 24 hours?
   Sample response: number of seconds for one time, times 60 times 60 times 24
7. How many single strands of hair are on your head?
   Sample response: 100 single strands of hair for every square inch patch (Approximately 100,000 single strands of hair)

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Lesson 1 Fermi Problems 135

1 Launch

Read the opening paragraph aloud, and then explain the ***Carousel*** routine that groups will be using for Part 1. Place the printed PDFs of each problem around the room, and make calculators available for Part 2.

2 Monitor

Help students get started by referring back to the four categories from the Warm-up. Say, "Try using these as guiding questions to help organize your thoughts."

Look for points of confusion:

- **Thinking that there is a right or wrong solution to the problem.** Remind students that this activity is more about the process of solving rather than getting a "right answer."
- **Not considering the reasonableness of answers given.** Ask, "What would be a guess that is way too low? Why? What would be a guess that is too high? Why? What, then, is a reasonable estimate?"
- **Showing thinking with numbers only in Part 2.** Encourage students to create an illustration or diagram showing their process along with their numbers.

Look for productive strategies:

- Making sense of the problem by breaking it down into smaller parts or questions, and determining what information is necessary to determine a solution.
- Formulating reasonable estimates based on understanding and on estimates of values that are too high or too low.
- Effectively modeling and communicating how they interpreted the strategy for solving the problem in a visual form, such as a diagram.

Activity 1 continued >

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Modify the Fermi problems by changing its unit. For example, modify Problem 3 to be "How many hours of television does a 6th grader watch in a week?", instead of a year. You may also consider providing students with a four-square graphic organizer with the categories from the Warm-up to help them organize their thinking and communicate with their partners during Part 2 of the activity.

- I know . . .
- I can measure . . .
- I can calculate . . .
- I can assume . . .

Math Language Development

MLR8: Discussion Supports—Press for Reasoning

Use this routine to support student understanding of Fermi problems. Present nonexamples such as,

- "How many students are in our classroom right now?"
- "How many desks are in our classroom?"

Ask pairs of students if these questions require them to estimate. As they discuss, highlight the reasoning they used to break down the Fermi problems in this activity to approximate the solutions.

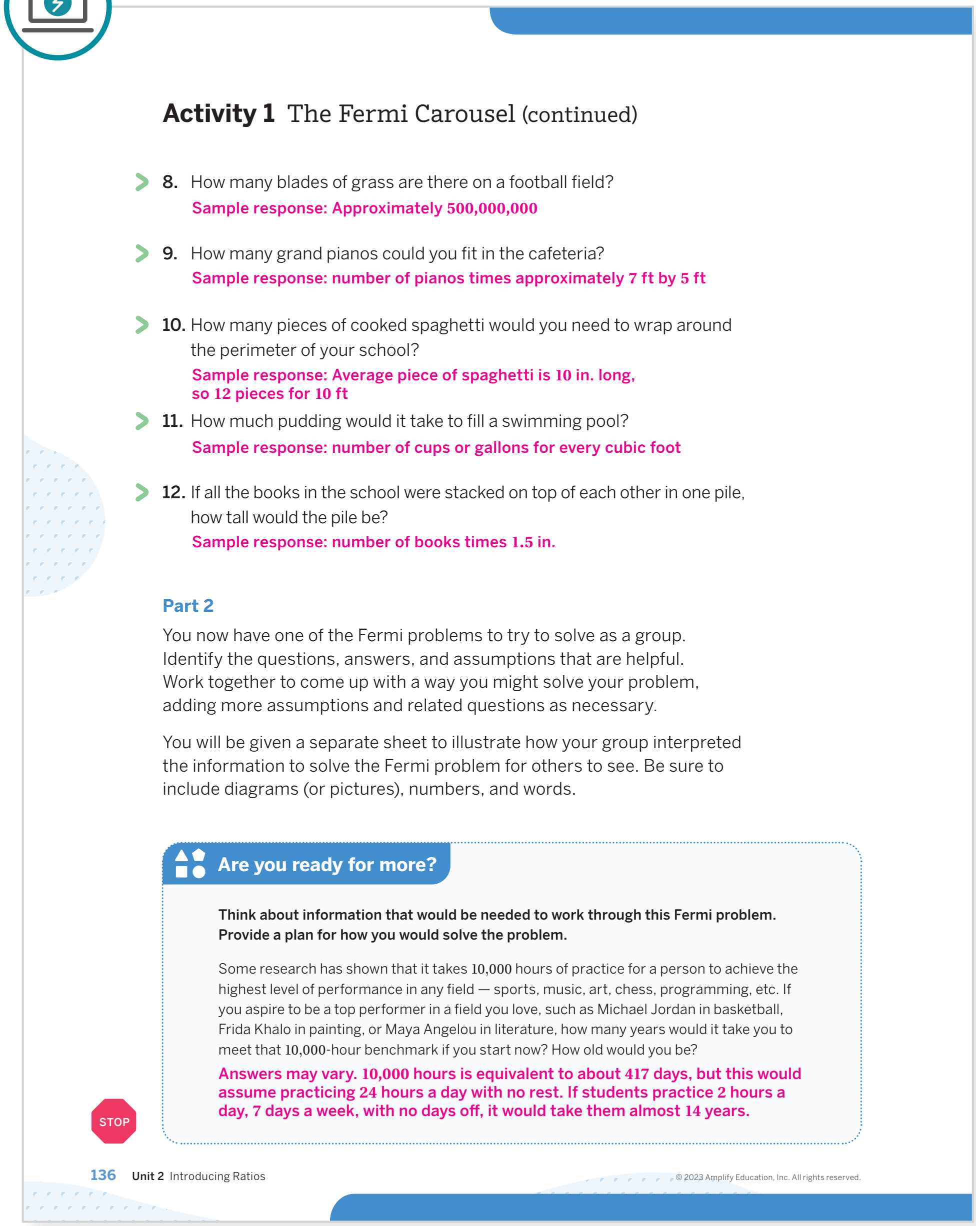
English Learners

Provide examples of some of the terms used in each Fermi problem. For example, in Problem 1, be sure students understand what a *dictionary* is by providing an example of a dictionary.

Small Groups | 25 min

Activity 1 The Fermi Carousel (continued)

Students rotate to read several Fermi problems and cycle through writing relevant assumptions, questions, and answers. Each group then works to solve one problem.



Activity 1 The Fermi Carousel (continued)

8. How many blades of grass are there on a football field?
   Sample response: Approximately 500,000,000
9. How many grand pianos could you fit in the cafeteria?
   Sample response: number of pianos times approximately 7 ft by 5 ft
10. How many pieces of cooked spaghetti would you need to wrap around the perimeter of your school?
    Sample response: Average piece of spaghetti is 10 in. long, so 12 pieces for 10 ft
11. How much pudding would it take to fill a swimming pool?
    Sample response: number of cups or gallons for every cubic foot
12. If all the books in the school were stacked on top of each other in one pile, how tall would the pile be?
    Sample response: number of books times 1.5 in.

Part 2

You now have one of the Fermi problems to try to solve as a group. Identify the questions, answers, and assumptions that are helpful. Work together to come up with a way you might solve your problem, adding more assumptions and related questions as necessary.

You will be given a separate sheet to illustrate how your group interpreted the information to solve the Fermi problem for others to see. Be sure to include diagrams (or pictures), numbers, and words.

Are you ready for more?

Think about information that would be needed to work through this Fermi problem. Provide a plan for how you would solve the problem.

Some research has shown that it takes 10,000 hours of practice for a person to achieve the highest level of performance in any field — sports, music, art, chess, programming, etc. If you aspire to be a top performer in a field you love, such as Michael Jordan in basketball, Frida Khalo in painting, or Maya Angelou in literature, how many years would it take you to meet that 10,000-hour benchmark if you start now? How old would you be?

Answers may vary. 10,000 hours is equivalent to about 417 days, but this would assume practicing 24 hours a day with no rest. If students practice 2 hours a day, 7 days a week, with no days off, it would take them almost 14 years.

STOP

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3 Connect

Display the completed Activity 1, Part 1 PDF pages and group illustrations for three different Fermi problems in different areas of the classroom (e.g., each of the four corners of the classroom).

Have groups of students share their illustrations within these smaller groups, focusing on effectively communicating their ideas with appropriate language.

Ask:

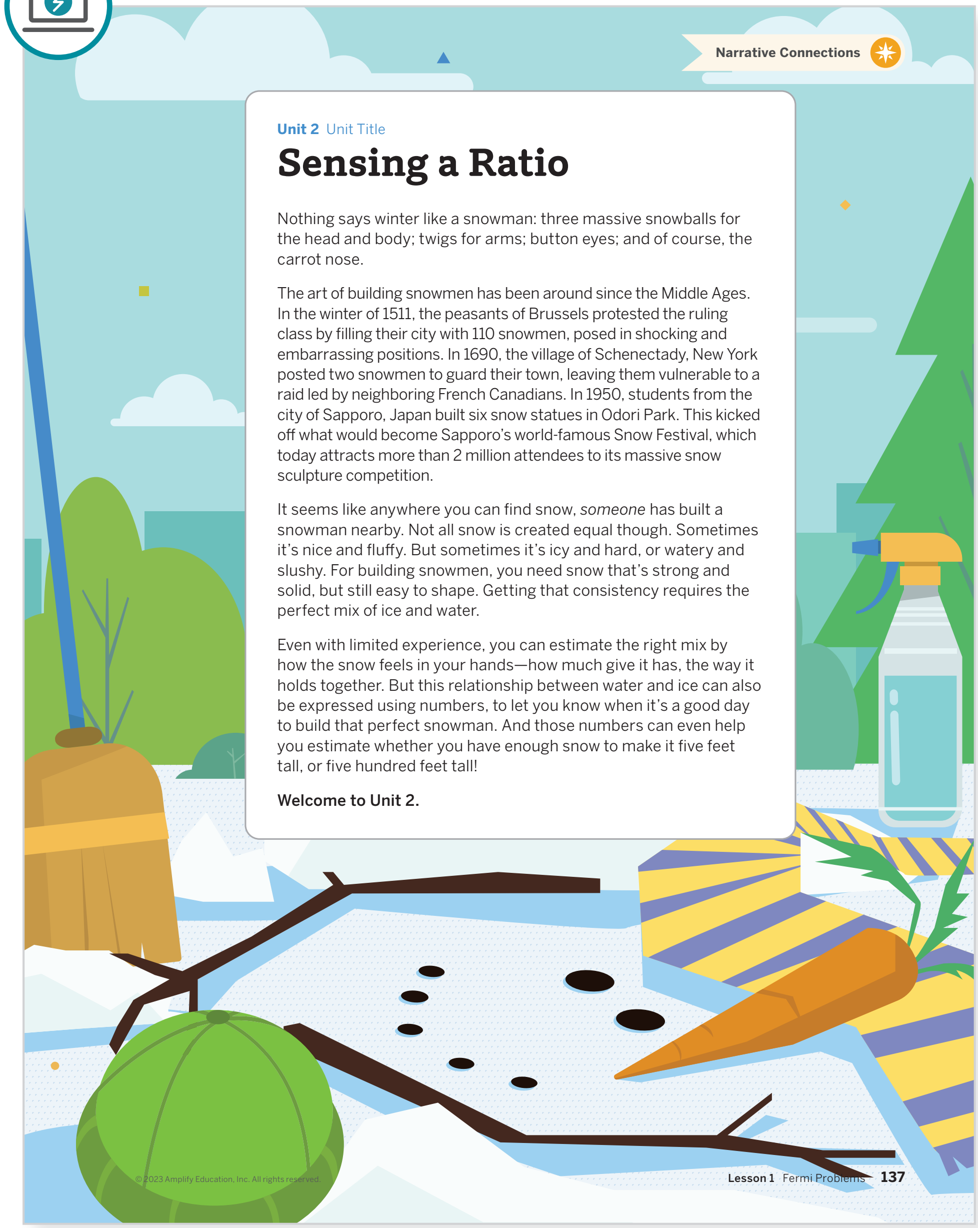
- "How did you determine whether a piece of information was relevant for helping you determine an answer to the problem?" I thought about if the number was reasonable.
- "What do the illustrations or solution strategies have in common in all the Fermi problems?" In most — if not every problem — multiplication was used, every problem had at least two quantities, each problem took multiple steps to solve, and each required some assumptions and estimates.

Highlight the use of language to compare quantities, revoicing with the phrase *for every* when appropriate.

Whole Class | 5 min

Summary Sensing a Ratio

Review and synthesize how students engaged with Fermi problems by identifying and gathering necessary information not given to them, and then estimating or calculating.



Unit 2 Unit Title

Sensing a Ratio

Nothing says winter like a snowman: three massive snowballs for the head and body; twigs for arms; button eyes; and of course, the carrot nose.

The art of building snowmen has been around since the Middle Ages. In the winter of 1511, the peasants of Brussels protested the ruling class by filling their city with 110 snowmen, posed in shocking and embarrassing positions. In 1690, the village of Schenectady, New York posted two snowmen to guard their town, leaving them vulnerable to a raid led by neighboring French Canadians. In 1950, students from the city of Sapporo, Japan built six snow statues in Odori Park. This kicked off what would become Sapporo's world-famous Snow Festival, which today attracts more than 2 million attendees to its massive snow sculpture competition.

It seems like anywhere you can find snow, *someone* has built a snowman nearby. Not all snow is created equal though. Sometimes it's nice and fluffy. But sometimes it's icy and hard, or watery and slushy. For building snowmen, you need snow that's strong and solid, but still easy to shape. Getting that consistency requires the perfect mix of ice and water.

Even with limited experience, you can estimate the right mix by how the snow feels in your hands—how much give it has, the way it holds together. But this relationship between water and ice can also be expressed using numbers, to let you know when it's a good day to build that perfect snowman. And those numbers can even help you estimate whether you have enough snow to make it five feet tall, or five hundred feet tall!

Welcome to Unit 2.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display and read the Summary.

Ask:

- "What Fermi problems could you make after reading this Summary?" (Select one or two responses to be used for the subsequent questions.)

  Sample responses:
  - How many snowflakes are in one snowball?
  - How many snowflakes make up a snowperson?
- "What information do you need to solve the problems?"
  - How big is each snowball?
  - How tall is the snowperson?
- "What do you think they mean by 'the perfect mix of ice and water'?" Sample response: The balance between ice and water needs to be right.
- "How many batches of this 'perfect mix of ice and water' do you need to make a snowperson?" Sample response: Typically three
- "Do the batches need to be the same amount? Do they need the same consistency?" Sample response: No, each one would get a little bit smaller as you go up. Yes the consistency would still have to be the same, just in smaller amounts.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

- "How did you feel when tackling these seemingly impossible problems today?"
- "What resources did you use to help you with your Fermi problem?"

Fostering Diverse Thinking

Recognizing the Contributions of Immigrants

Enrico Fermi wasn't just known for his Fermi problems. His research in radioactivity led to the discovery of nuclear fission, for which he was awarded the Nobel Prize in 1938. (Nuclear fission releases a significant amount of energy and is used by nuclear power plants to generate heat and power electrical generators. About 20% of electricity in the U.S. is generated by nuclear power plants!) Around this same time, the political situation in Italy worsened. Mussolini and his followers consolidated power through a series of laws that transformed the country into a one-party dictatorship and began to ally with Nazi Germany. Fermi's wife was Jewish and they decided to leave Italy. Immediately after the 1938 Nobel Prize ceremony, Fermi's family fled to the U.S., where his research and contributions to physics continued.

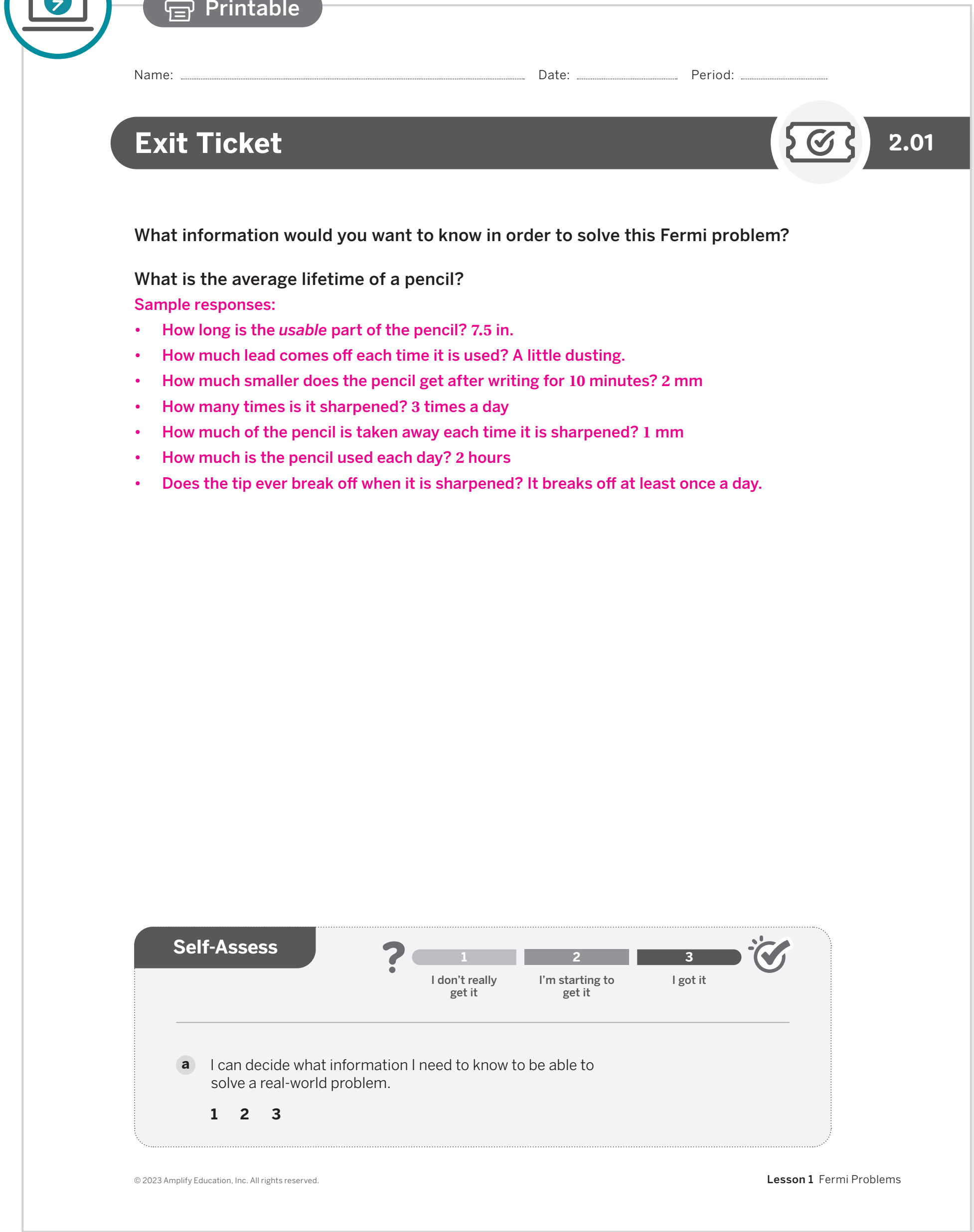
Have students research the contributions of different immigrants to the U.S., such as Kurt Gödel (mathematics), Martina Navratilova (tennis), I.M. Pei (architecture), Kahlil Gibran (author), Liz Claiborne (fashion), Madeleine Albright (diplomacy), Yo-Yo Ma (music), Amar Bose (engineering), and Subrahmanyan Chandrasekhar (physics). Have students choose one person to study and ask them to prepare a brief write-up or presentation about their chosen person's life and contributions to the U.S. or the world. Ask students to read the following quote, by former U.S. President, John F. Kennedy, whose grandparents were immigrants from Ireland: "Every American who has ever lived, with the exception of one group, was either an immigrant himself or a descendant of immigrants."

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying information that would be of use to solve a Fermi problem.

Printable



Name: Date: Period:

Exit Ticket 2.01

What information would you want to know in order to solve this Fermi problem?

What is the average lifetime of a pencil?

Sample responses:

- How long is the *usable* part of the pencil? 7.5 in.
- How much lead comes off each time it is used? A little dusting.
- How much smaller does the pencil get after writing for 10 minutes? 2 mm
- How many times is it sharpened? 3 times a day
- How much of the pencil is taken away each time it is sharpened? 1 mm
- How much is the pencil used each day? 2 hours
- Does the tip ever break off when it is sharpened? It breaks off at least once a day.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can decide what information I need to know to be able to solve a real-world problem.

1 2 3

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Lesson 1 Fermi Problems

Success looks like . . .

- **Language Goal:** Asking questions that can help gather information that is useful for solving real-world problems. **(Speaking and Listening, Writing)**
  - » Writing questions that can be used to help determine the average lifetime of a pencil.
- **Goal:** Identifying information that is relevant and useful for solving real-world problems.
  - » Identifying information that is relevant to determining the average lifetime of a pencil.

Suggested next steps

If students phrase their information in a form of a statement, consider:

- Having students reread the directions.
- Reminding them that you need a question to get an answer.

If students struggle to write questions related to the use of a pencil, consider asking:

- "What happens every time you use a pencil?"
- "Have you ever seen those tiny pencils? How do you think they get that small?"

If students record unreasonable responses, consider:

- Asking how they thought of those responses and if they can justify them.

Professional Learning

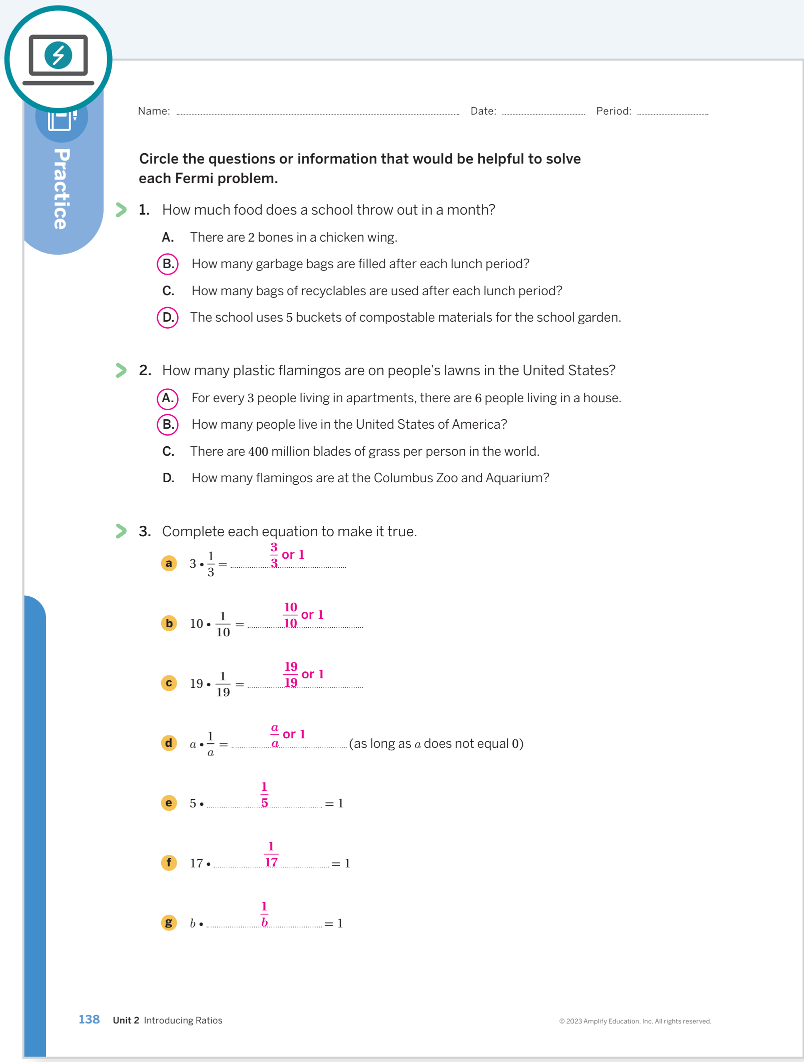
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What different ways did students approach the problems today? What does that tell you about similarities and differences among your students?
- What resources did students use as they worked on their Fermi problem? Which resources were especially helpful? What resources would you want to have available the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Circle the questions or information that would be helpful to solve each Fermi problem.

1. How much food does a school throw out in a month?
   - A. There are 2 bones in a chicken wing.
   - [✓ B.] How many garbage bags are filled after each lunch period?
   - C. How many bags of recyclables are used after each lunch period?
   - [✓ D.] The school uses 5 buckets of compostable materials for the school garden.

2. How many plastic flamingos are on people's lawns in the United States?
   - [✓ A.] For every 3 people living in apartments, there are 6 people living in a house.
   - [✓ B.] How many people live in the United States of America?
   - C. There are 400 million blades of grass per person in the world.
   - D. How many flamingos are at the Columbus Zoo and Aquarium?

3. Complete each equation to make it true.

a $3 \cdot \frac{1}{3} =$ $\frac{3}{3}$ or 1

b $10 \cdot \frac{1}{10} =$ $\frac{10}{10}$ or 1

c $19 \cdot \frac{1}{19} =$ $\frac{19}{19}$ or 1

d $a \cdot \frac{1}{a} =$ $\frac{a}{a}$ or 1 (as long as a does not equal 0)

e $5 \cdot$ $\frac{1}{5}$ $= 1$

f $17 \cdot$ $\frac{1}{17}$ $= 1$

g $b \cdot$ $\frac{1}{b}$ $= 1$

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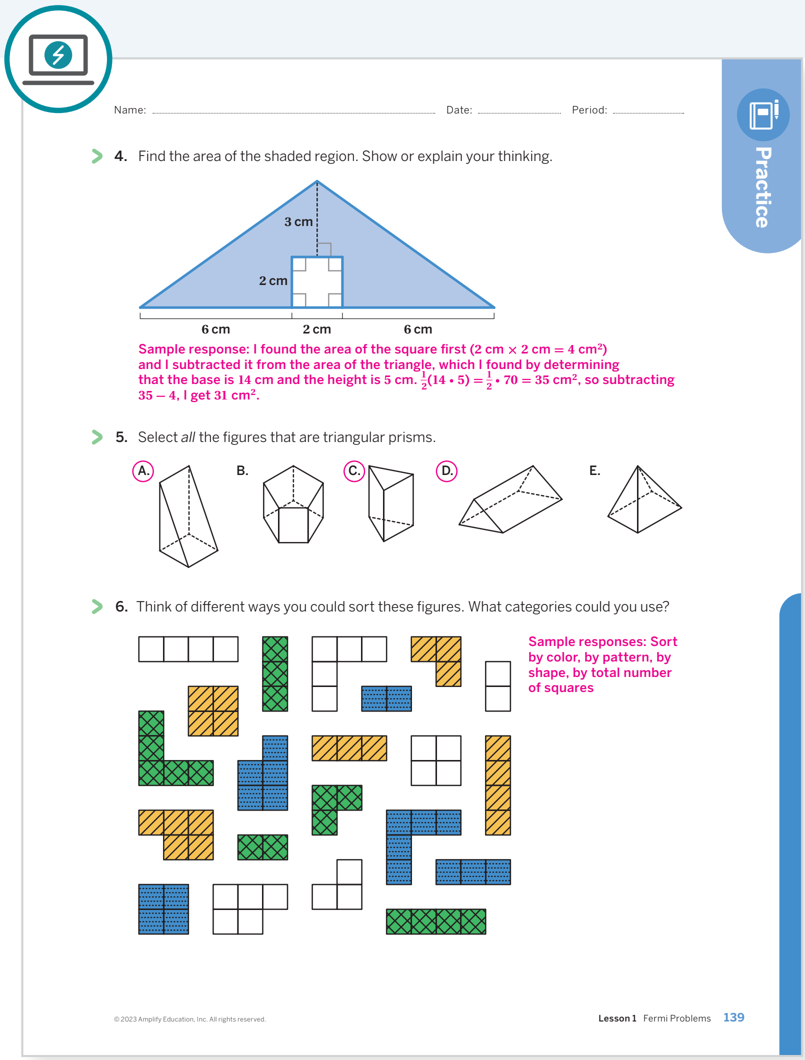
Practice



Name: Date: Period:

4. Find the area of the shaded region. Show or explain your thinking.

Sample response: I found the area of the square first (2 cm × 2 cm = 4 cm^2) and I subtracted it from the area of the triangle, which I found by determining that the base is 14 cm and the height is 5 cm. $\frac{1}{2}(14 \cdot 5) = \frac{1}{2} \cdot 70 = 35$ cm^2, so subtracting $35 - 4$, I get 31 cm^2.

5. Select *all* the figures that are triangular prisms.

[✓ A.] B. [✓ C.] [✓ D.] E.

6. Think of different ways you could sort these figures. What categories could you use?

Sample responses: Sort by color, by pattern, by shape, by total number of squares

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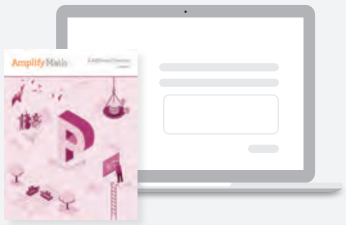
Practice

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
Spiral	3	Grade 5	2
	4	Unit 1 Lesson 11	1
	5	Unit 1 Lesson 16	2
Formative ⚡	6	Unit 2 Lesson 2	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 1

What are Ratios?

Whole Class

In this Sub-Unit, students are introduced to the language of ratios and their representations, and they develop an informal understanding of equivalence by scaling ratios up and down.

SUB-UNIT

1 What Are Ratios?

Narrative Connections

How does an eggplant become a plum?

Few things are more important to painters than color.

“Color in painting,” according to Vincent Van Gogh, “is like enthusiasm in life.” In her diaries, Mexican painter Frida Kahlo described herself as “CHROMOPHORE—the one who gives color.” Meanwhile French impressionist Claude Monet described color as his “day-long obsession, joy and torment.” It is through color that artists give their paintings a sense of life and motion, enabling them—in the words of Georgia O’Keefe—to “say things . . . [they] couldn’t say any other way.”

In painting, there are three primary colors: red, yellow, and blue. They’re called “primary” because they can’t be mixed from the *other* colors. They can, however, be combined to create a wide range of other colors. Green, purple, orange, and the shades in between are made from these primary colors. With the right combinations, artists can create light and shadow, or give an image depth. They can draw attention to certain areas of the canvas, or evoke a particular emotion within us.

One of the greatest innovations in color came not from an artist, but from the mathematician Isaac Newton. Newton bent a sunbeam through a prism, creating a rainbow spectrum. He arranged the resulting colors onto a wheel. Over time, this color wheel would evolve into a useful tool to help artists choose meaningful color combinations.

Painters must use these combinations of primary colors in just the right amounts. The wrong combination could mean the difference between blush and cerise, or eggplant and plum! To get the exact right shade, we need a way to express the relationship between the amounts of different pigments.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore ratios within rhythm and music in the following places:

- **Lesson 6, Activity 1:** Clapping a Rhythm
- **Lesson 12, Activity 1:** A Larger Orchestra
- **Lesson 13, Activity 1:** Song Tempos

UNIT 2 | **LESSON 2**

Introducing Ratios and Ratio Language

Let's use visuals to describe how quantities relate to each other.

Focus

Goals

1. **Language Goal:** Comprehend the word *ratio* and the notation $a : b$ to refer to an association between quantities. **(Speaking and Listening, Writing)**
2. **Language Goal:** Describe associations between quantities using the language "For every a of these, there are b of those." and "The ratio of these to those is $a : b$ (or a to b)." **(Speaking and Listening, Writing)**

Coherence

Today

Students use pattern block designs to make sense as they begin to use ratio language. Three sentence structures for describing ratio relationships are introduced: "For every a of these, there are b of those;" "The ratio of these to those is a to b;" and "The ratio of these to those is $a : b$." Although the term *ratio* is not defined until later, expressing associations between quantities in a context still requires students to use ratio language precisely. They also create their own designs by using groups of pattern blocks and relate those to the language of *for each* or *for every* in a concrete way.

Previously

Students identified and generated number and shape patterns in elementary grades. In Lesson 1 of this unit, they worked with Fermi problems to identify and to explain different ways that two quantities can be related to one another.

Coming Soon

In Lesson 3, students will draw diagrams to represent and to interpret ratios abstractly.

Rigor

- Students build **conceptual understanding** of ratios by describing ratio relationships in words.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	15 min	10 min	5 min	5 min
Independent	Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, one per group
- Activity 3 PDF (answers)
- pattern blocks

Math Language Development

New word

- *ratio relationship**

*This is a *working* definition of *ratio* that describes a specific type of association between numbers. The term *ratio* will be formally defined in Lesson 4.

Amps | Featured Activity

Activities 1 and 2
Pattern Blocks

Students analyze displays of pattern blocks to represent their ratio thinking, which can be checked in real time.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practice

Students may not take special care to consistently use the precise wording or notations. To help students self-regulate their thoughts and behaviors, create a bank of sentence frames that students can use when having mathematical conversations about ratios. Frames should include the sentence structure and language from this lesson: "For every a of these, there are b of those;" "The ratio of these to those is a to b;" and "The ratio of these to those is $a : b$."

● Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, limit students to two or three sentences for Problem 2, and then have a whole-class discussion to share different sentences. Problem 3 can be incorporated into discussion.
- **Activity 3** is optional.

Independent | 5 min

Warm-up Categorizing Flags of the World

Students sort flags into categories using precise, descriptive language to compare different groupings.

Unit 2 | Lesson 2

Introducing Ratios and Ratio Language

Let's use visuals to describe how quantities relate to each other.

Warm-up Categorizing Flags of the World

Think about how you could group these flags into two or more categories. Be prepared to explain your thinking.

Bosnia and Herzegovina
Cambodia
Italy
Kiribati
United Arab Emirates
Bhutan
Mali
Belgium
Jordan
Papua New Guinea
Uruguay
Sri Lanka
Uzbekistan
Namibia
Romania
Honduras

Public Domain

Sample responses: Sort by color, by shape, by stars/no stars, by whether there are horizontal/vertical/diagonal stripes or lines, by whether there are triangles/no triangles, by continent.

Log in to Amplify Math to complete this lesson online.

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1 Launch

Display the image of the flags and review the directions.

2 Monitor

Help students get started by asking, "What is a feature of one flag that jumps out at you? Are there other flags that have that same feature that could make a group?"

Look for productive strategies:

- Wondering if the flags could be sorted by continent or using the first letters of the names of the countries.
- Using a variety of descriptive language relating to colors, sizes, orientations, shapes, and details of the flags.
- Thinking about ways to assign numbers or values to describe categories, such as total number of colors, relative numbers of stripes of different colors, or estimating the relative area of each color on a flag.

3 Connect

Have individual students share how they grouped certain flags, focusing on categorizing the flags by stripes or overall orientations of vertical, horizontal, and diagonal — it can be said that there are 5 vertical, 7 horizontal, and 4 diagonal flags. Revoice student responses using *for every* as they make observations. For example, say, "There are 5 vertical flags for every 7 horizontal flags."

Highlight that in order to categorize objects, students need to use clear and descriptive language to accurately communicate their thinking. In this case, students used their sense of sight to reason about similarities and differences in flags.

Power-up

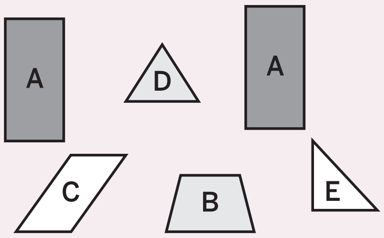
To power up students' ability to identify attributes to categorize, have students complete:

Determine the number of groups would be created if the figures were categorized by each characteristic:

a. Color: 3

b. Shape: 2 or 4

c. Letter: 5



Use: Before the Warm-up.

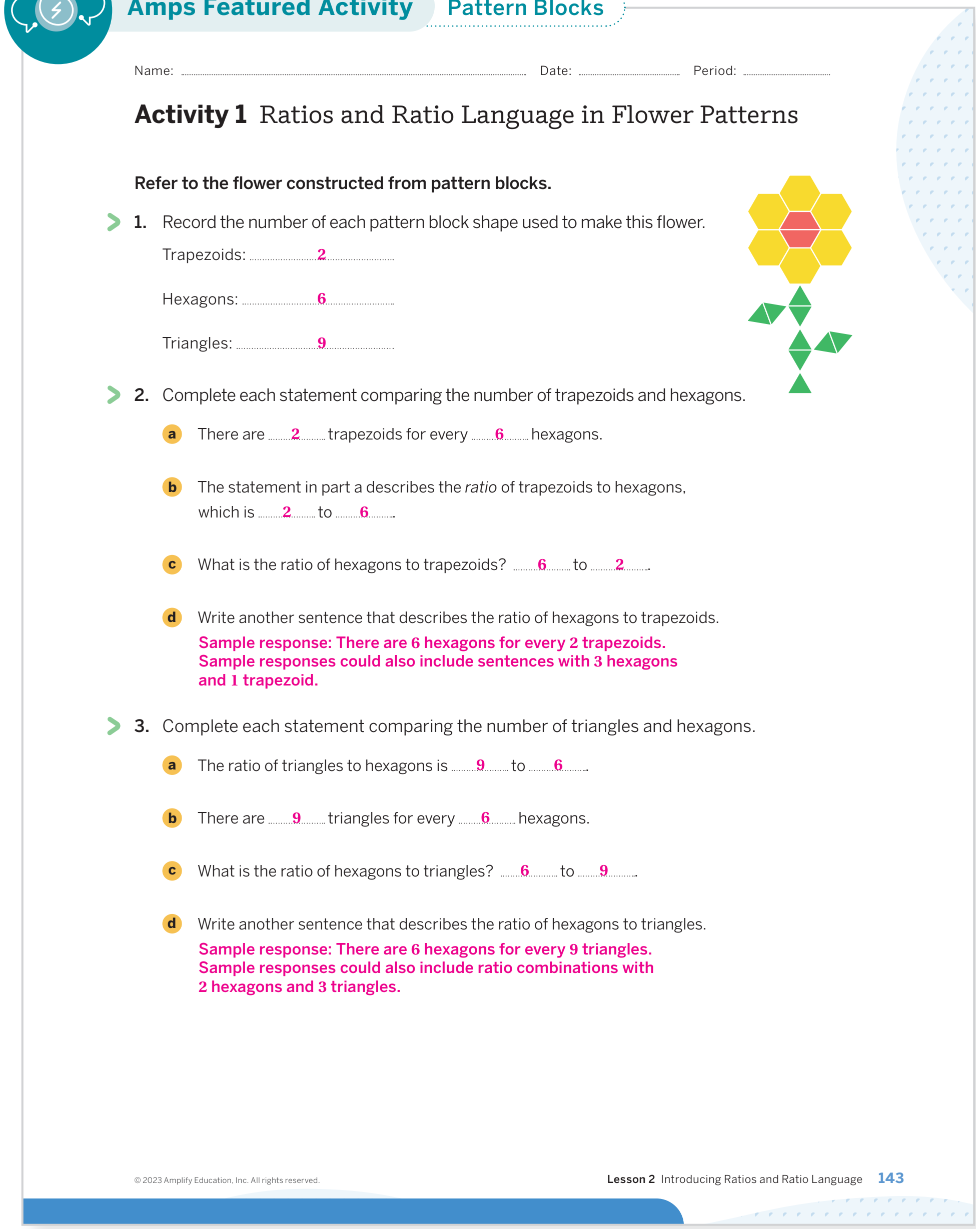
Informed by: Performance on Lesson 1, Practice Problem 6

Independent | 15 min

Activity 1 Ratios and Ratio Language in Flower Patterns

Students are introduced to ratio language and notation by working with sentences that describe the relative amounts of pattern blocks in a design.

Amps Featured Activity Pattern Blocks



Name: Date: Period:

Activity 1 Ratios and Ratio Language in Flower Patterns

Refer to the flower constructed from pattern blocks.

1. Record the number of each pattern block shape used to make this flower.

Trapezoids: 2

Hexagons: 6

Triangles: 9

2. Complete each statement comparing the number of trapezoids and hexagons.

a. There are 2 trapezoids for every 6 hexagons.

b. The statement in part a describes the *ratio* of trapezoids to hexagons, which is 2 to 6.

c. What is the ratio of hexagons to trapezoids? 6 to 2

d. Write another sentence that describes the ratio of hexagons to trapezoids.

Sample response: There are 6 hexagons for every 2 trapezoids. Sample responses could also include sentences with 3 hexagons and 1 trapezoid.

3. Complete each statement comparing the number of triangles and hexagons.

a. The ratio of triangles to hexagons is 9 to 6.

b. There are 9 triangles for every 6 hexagons.

c. What is the ratio of hexagons to triangles? 6 to 9.

d. Write another sentence that describes the ratio of hexagons to triangles.

Sample response: There are 6 hexagons for every 9 triangles. Sample responses could also include ratio combinations with 2 hexagons and 3 triangles.

1 Launch

Have students complete Problem 1. Complete Problems 2a and 2b together during a class discussion. Then have students complete Problem 3.

2 Monitor

Help students get started by referring back to Problem 2a and having students substitute *to* with *for every* in Problem 2c.

Look for points of confusion:

- **Not attending to the order of how the ratio is set up.** Refer to Problem 2b and ask, "Why is this written as '2 to 6' and not '6 to 2'?"

Look for productive strategies:

- Using ratio language, such as "for every" or "for each."
- Noticing that there are three times as many hexagons as trapezoids.

3 Connect

Have individual students share the sentences they wrote in Problems 2d and 3d, focusing on the consistent use of ratio language.

Define a ***ratio relationship*** as a specific type of relationship between two or more quantities. (Explain that the term *ratio* will be defined in a later lesson.)

Highlight that in addition to ratio sentences, another way is to use colon notation, writing the ratio of "a to b" as $a : b$. For Problem 2, a sentence could read, "The ratio of hexagons to trapezoids is $6 : 2$." Have students add this sentence to their page.

Ask, "Can you reverse the numbers in the ratio sentence?" Not when the sentence dictates "these" to "those."

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to pattern blocks, or copies of paper pattern blocks, for students to use and manipulate during this activity.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can click, drag, and position pattern blocks to help represent their thinking about ratios.

Math Language Development

MLR1: Stronger and Clearer Each Time

For both Problems 2d and 3d, have students first write sentences individually and then work with a partner to clarify their thinking and ideas through conversation. Ultimately, they should revise their writing based on language clarifications.

English Learners

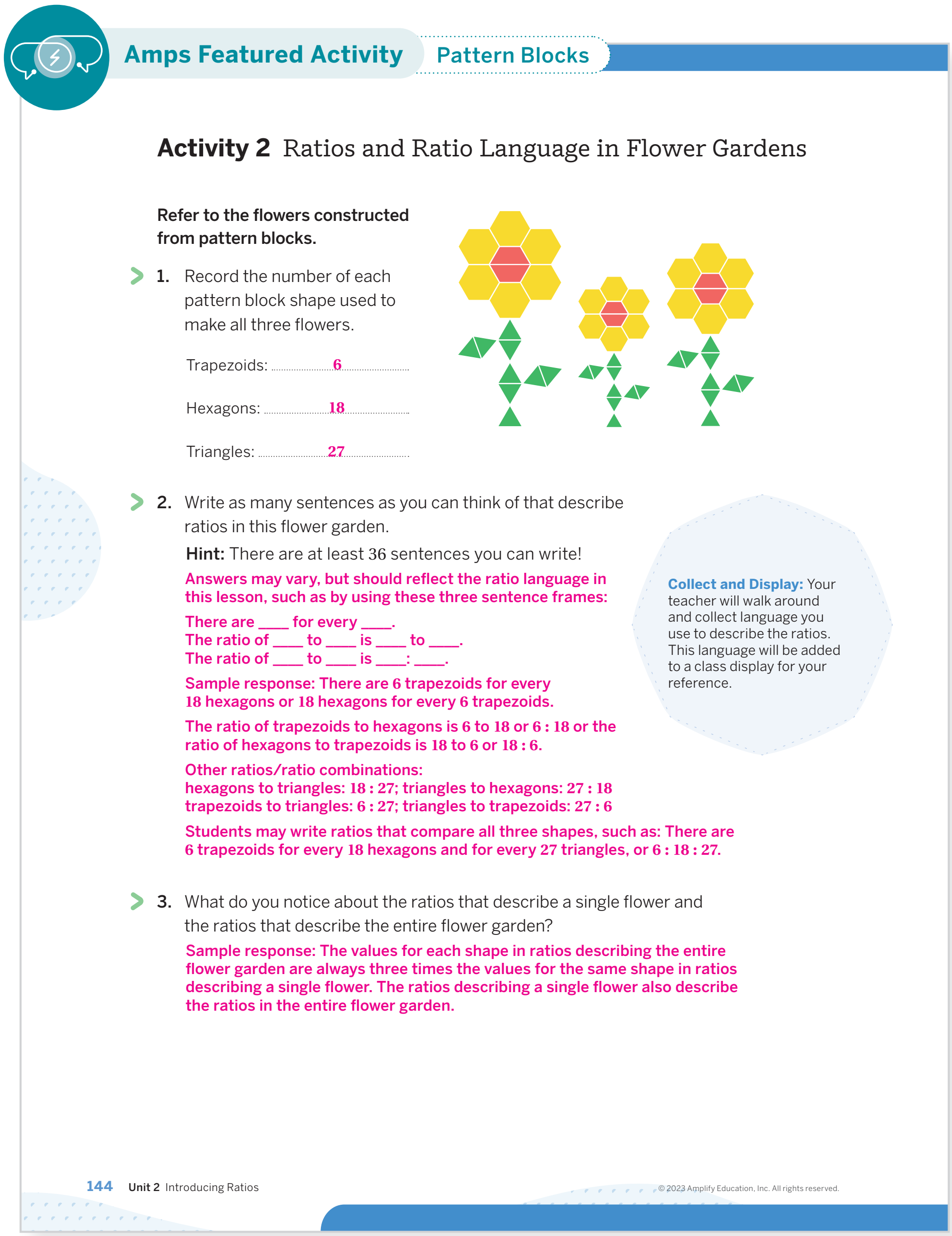
Encourage students to write their first sentences in their primary language before working with a partner to clarify their thinking. Their revision should be written in English.

Pairs | 15 min

Activity 2 Ratios and Ratio Language in Flower Gardens

Students write ratio sentences to describe a garden of flowers to reinforce the many ways to write ratios for the same situation.

Amps Featured Activity Pattern Blocks



Activity 2 Ratios and Ratio Language in Flower Gardens

Refer to the flowers constructed from pattern blocks.

1. Record the number of each pattern block shape used to make all three flowers.

Trapezoids: 6

Hexagons: 18

Triangles: 27

2. Write as many sentences as you can think of that describe ratios in this flower garden.

Hint: There are at least 36 sentences you can write!

Answers may vary, but should reflect the ratio language in this lesson, such as by using these three sentence frames:

There are ____ for every ____.
The ratio of ____ to ____ is ____ to ____.
The ratio of ____ to ____ is ____: ____.

Sample response: There are 6 trapezoids for every 18 hexagons or 18 hexagons for every 6 trapezoids.

The ratio of trapezoids to hexagons is 6 to 18 or 6 : 18 or the ratio of hexagons to trapezoids is 18 to 6 or 18 : 6.

Other ratios/ratio combinations:
hexagons to triangles: 18 : 27; triangles to hexagons: 27 : 18
trapezoids to triangles: 6 : 27; triangles to trapezoids: 27 : 6

Students may write ratios that compare all three shapes, such as: There are 6 trapezoids for every 18 hexagons and for every 27 triangles, or 6 : 18 : 27.

Collect and Display: Your teacher will walk around and collect language you use to describe the ratios. This language will be added to a class display for your reference.

3. What do you notice about the ratios that describe a single flower and the ratios that describe the entire flower garden?

Sample response: The values for each shape in ratios describing the entire flower garden are always three times the values for the same shape in ratios describing a single flower. The ratios describing a single flower also describe the ratios in the entire flower garden.

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1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by referring back to Activity 1 and asking, "Which sentence frames was repeated for both Problems 2 and 3?"

Look for points of confusion:

- **Writing incomplete ratio sentences with only quantities.** Remind students that a ratio sentence must include the numerical amounts and to which quantities they correspond.
- **Commenting on the size of the flowers in Problem 3.** Ask, "Does the size of the flowers change the number of blocks used?"

Look for productive strategies:

- Flexibly writing multiple statements for the same ratio.
- Noticing a multiplicative relationship between the amounts of two of the shapes, such as, "There are 3 times as many hexagons as trapezoids."

3 Connect

Have pairs of students share their sentences for Problem 2, focusing on using the colon notation.

Highlight three ways the same ratio can be described:

- For every a of "this," there is b of "that."
- The ratio of "this" to "that" is a to b.
- The ratio of "this" to "that" is $a : b$.

Ask:

- "Can you reverse the order in the sentences?" Yes, as long as the written part of the ratio matches the numerical ratio.
- "Why is it not possible to reverse the order in Problems 2d and 3d in Activity 1?" They stated "hexagons to trapezoids" and "hexagons to triangles."

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to pattern blocks, or copies of paper pattern blocks, for students to use and manipulate during this activity.

Extension: *Math Enrichment*

If students have not considered that they can write ratio sentences that compare all three shapes, ask them to do so now. Consider providing them with a sample sentence frame, such as "For every ___ trapezoids, there are ___ hexagons and ___ triangles."

Math Language Development

MLR2: Collect and Display

Circulate and listen to students talk as they work with their partners on the activity. Display these sentence frames they can use.

- There are ___ for every ___.
- The ratio of ___ to ___ is ___ to ___.
- The ratio of ___ to ___ is ___ : ___.

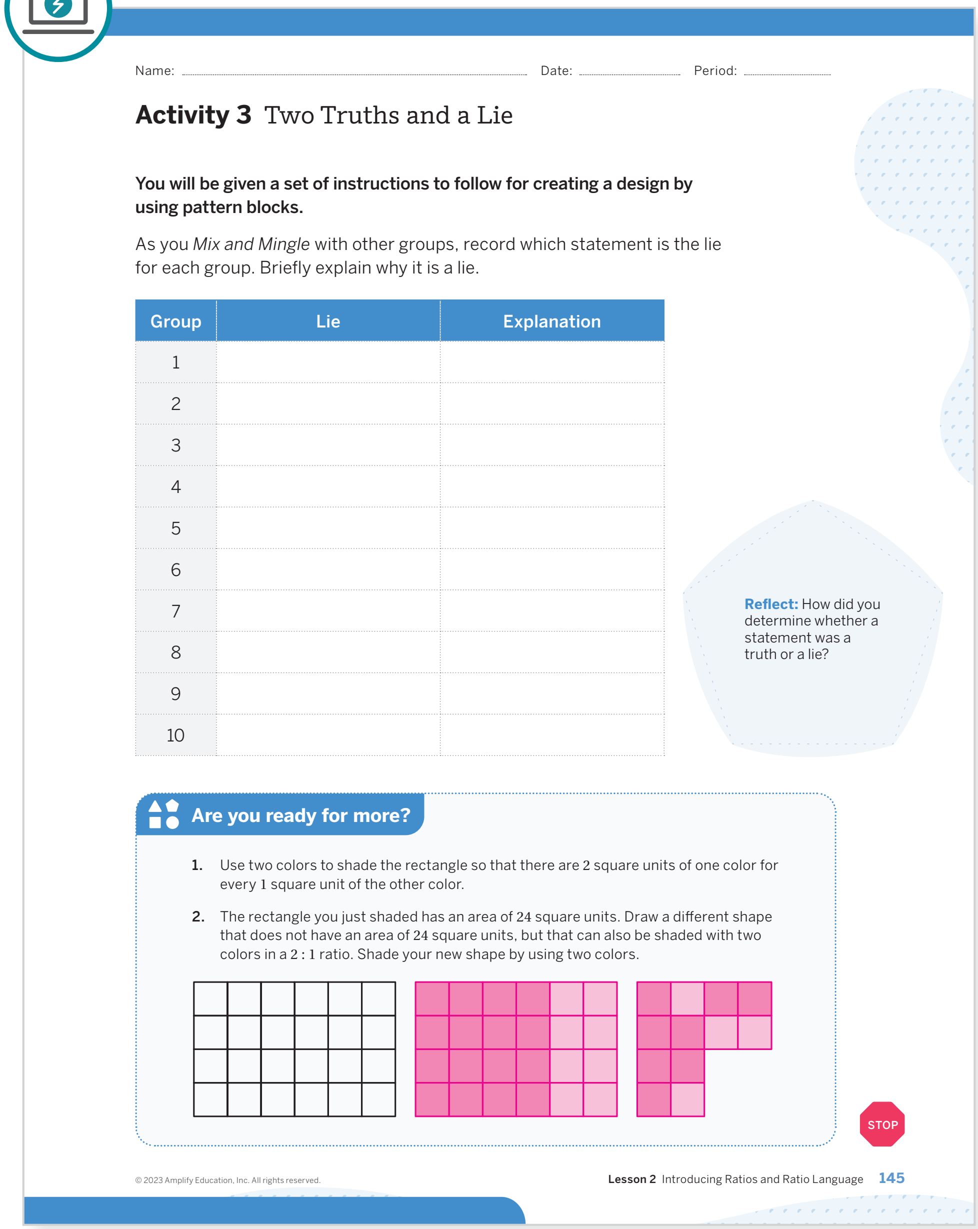
Ask students to think about how these sentence frames communicate ideas of ratio more precisely. Start a class display of mathematical words and phrases related to ratios and encourage students to refer to the display during future discussions in this unit.

Optional

Small Groups | 10 min

Activity 3 Two Truths and a Lie

Students create a pattern block design and describe it using ratio language, but one of the sentences is incorrect. They then circulate to identify the lie presented by others.



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Activity 3 Two Truths and a Lie

You will be given a set of instructions to follow for creating a design by using pattern blocks.

As you *Mix and Mingle* with other groups, record which statement is the lie for each group. Briefly explain why it is a lie.

Group	Lie	Explanation
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Reflect: How did you determine whether a statement was a truth or a lie?

Are you ready for more?

1. Use two colors to shade the rectangle so that there are 2 square units of one color for every 1 square unit of the other color.
2. The rectangle you just shaded has an area of 24 square units. Draw a different shape that does not have an area of 24 square units, but that can also be shaded with two colors in a 2 : 1 ratio. Shade your new shape by using two colors.

STOP

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Lesson 2 Introducing Ratios and Ratio Language 145

1 Launch

Assign each group a number, and distribute the Activity 3 PDF. Give students 4 minutes to make their design, 2 minutes to write their sentences, and 5 minutes to conduct the ***Mix and Mingle*** routine. Explain that students may not have a chance to view every group's designs.

2 Monitor

Help students get started by encouraging groups to work together to create a design. Note that designs do not have to represent an actual object.

Look for points of confusion:

- **Writing incomplete ratio sentences with only quantities.** Remind students that a ratio sentence must include both the numerical amounts and to which quantities they correspond.
- **Identifying incorrect false statements.** Ask, "Why do you think this is the lie?"

Look for productive strategies:

- Carefully crafting ratio sentences that accurately represent the design.
- Analyzing each statement and identifying what made the lie a false statement based on the structure of the statement and its representation of the design.

3 Connect

Display the ratio sentence frames captured in Activity 2.

Have pairs of students share a lie that they identified from another pair, and how they knew. Continue until all groups' lies have been discussed.

Highlight the various ways that students chose to write their lies. **Note:** This activity can also be used for students who are struggling with using ratio language.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide a pre-created design using pattern blocks and have students write two true statements and one false statement. Alternatively, provide pre-created sets of true and false statements for students to choose from.

Math Language Development

MLR7: Compare and Connect

Use this routine to help students consider their audience when preparing to display their work. Display the list of items that should be included on the pattern block design and ask students, "What kinds of details could you include in your design to help someone understand the ratios you used?"

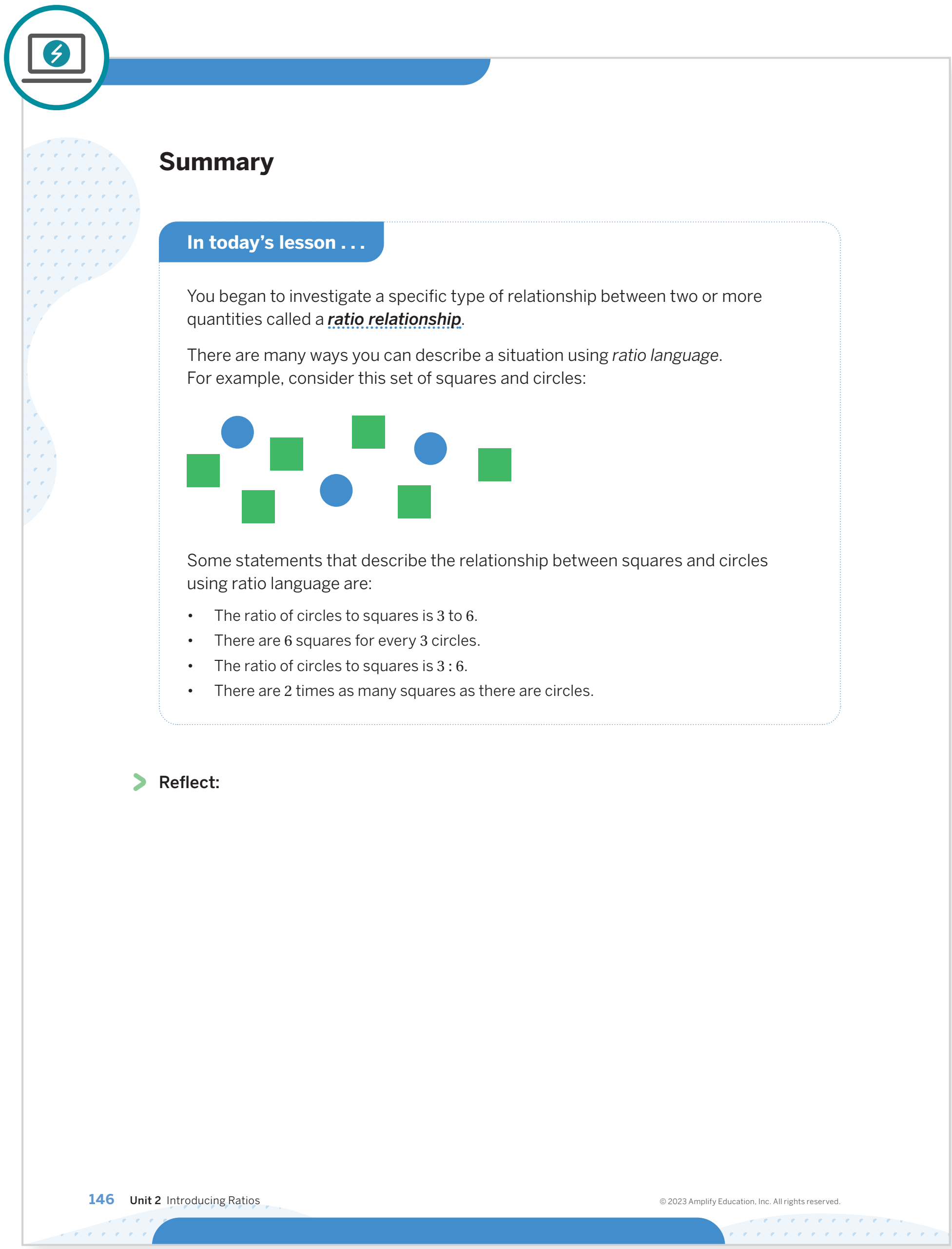
English Learners

Allow students time to formulate what they will share during the ***Mix and Mingle***.

Whole Class | 5 min

Summary

Review and synthesize the different ways ratio language and notation can be used to describe an association between two or more quantities.



Summary

In today's lesson . . .

You began to investigate a specific type of relationship between two or more quantities called a ***ratio relationship***.

There are many ways you can describe a situation using *ratio language*. For example, consider this set of squares and circles:

Some statements that describe the relationship between squares and circles using ratio language are:

- The ratio of circles to squares is 3 to 6.
- There are 6 squares for every 3 circles.
- The ratio of circles to squares is 3 : 6.
- There are 2 times as many squares as there are circles.

Reflect:

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Synthesize

Display the 6 squares and 3 circles from the Summary page. These can be quickly drawn on the board or have students refer to their books.

Highlight the three ratio sentences repeated in this lesson using the ratio of squares and circles:

- The ratio of squares to circles is 6 : 3, or circles to squares is 3 to 6.
- There are 6 squares for every 3 circles or 3 circles for every 6 squares.
- The ratio of squares to circles is 6 : 3 or circles to squares is 3 : 6.

Formalize vocabulary: *ratio relationship*

Ask,

- "What are some words, phrases, or symbols that are used to write a ratio?" *for every*, *to*, and the : notation
- "What must you pay attention to when writing a ratio?" The order of the quantities in the ratio. (This point can be made by referring back to the three sentences for squares to circles and reversing them to circles to squares.)

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does a ratio say about the relationship between quantities?"

Math Language Development

MLR2: Collect and Display

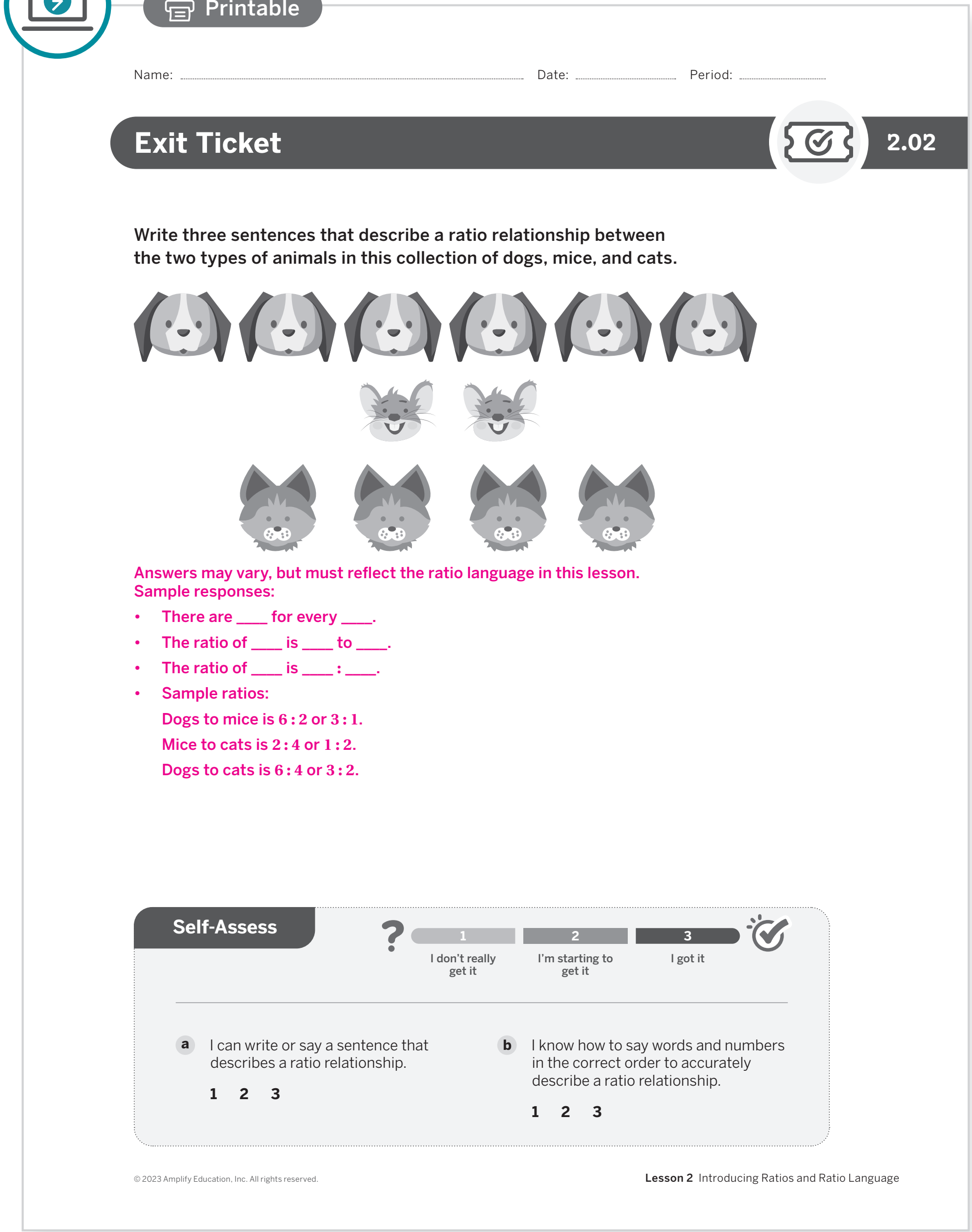
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this lesson. Ask them to review and reflect on any terms and phrases related to the term *ratio relationship* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of ratio language by writing three sentences to describe a scenario involving two quantities.

Printable



Name: ____ Date: ____ Period: ____

Exit Ticket 2.02

Write three sentences that describe a ratio relationship between the two types of animals in this collection of dogs, mice, and cats.

Answers may vary, but must reflect the ratio language in this lesson. Sample responses:

- There are ____ for every ____.
- The ratio of ____ is ____ to ____.
- The ratio of ____ is ____ : ____.
- Sample ratios:
  Dogs to mice is 6 : 2 or 3 : 1.
  Mice to cats is 2 : 4 or 1 : 2.
  Dogs to cats is 6 : 4 or 3 : 2.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can write or say a sentence that describes a ratio relationship.
1 2 3

b I know how to say words and numbers in the correct order to accurately describe a ratio relationship.
1 2 3

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Lesson 2 Introducing Ratios and Ratio Language

Success looks like . . .

- **Language Goal:** Comprehending the word *ratio* and the notation $a : b$ to refer to an association between quantities. **(Speaking and Listening, Writing)**
  - » Writing ratios in the form $a : b$ to represent the relationship between two groups of animals.
- **Language Goal:** Describing associations between quantities using the language "For every a of these, there are b of those" and "The ratio of these to those is $a : b$ (or a to b)." **(Speaking and Listening, Writing)**
  - » Using ratio language to represent the relationship between two groups of animals.

Suggested next steps

If students confuse the order of the ratios with the sentence structure, consider:

- Having students write the corresponding number above the animal and how that corresponds to the order of the ratio.

If students incorrectly reduce ratios, consider:

- Having them circle the groups as shown in the summary.

Professional Learning

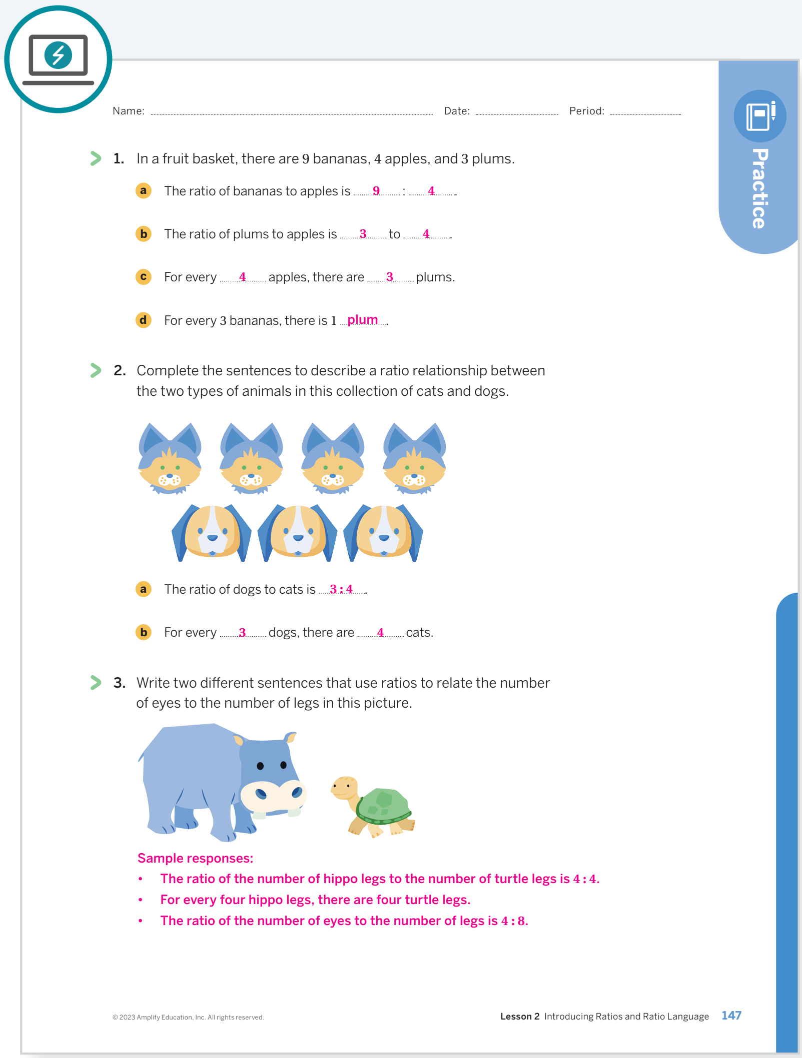
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did students use mathematical language today? How are you helping students become aware of how they are progressing in this area?
- Knowing where students need to be by the end of this unit, how did Activity 3 influence that future goal? What might you change the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. In a fruit basket, there are 9 bananas, 4 apples, and 3 plums.
   - a The ratio of bananas to apples is 9 : 4.
   - b The ratio of plums to apples is 3 to 4.
   - c For every 4 apples, there are 3 plums.
   - d For every 3 bananas, there is 1 plum.

2. Complete the sentences to describe a ratio relationship between the two types of animals in this collection of cats and dogs.
   - a The ratio of dogs to cats is 3 : 4.
   - b For every 3 dogs, there are 4 cats.

3. Write two different sentences that use ratios to relate the number of eyes to the number of legs in this picture.

   Sample responses:
   - **The ratio of the number of hippo legs to the number of turtle legs is 4 : 4.**
   - **For every four hippo legs, there are four turtle legs.**
   - **The ratio of the number of eyes to the number of legs is 4 : 8.**

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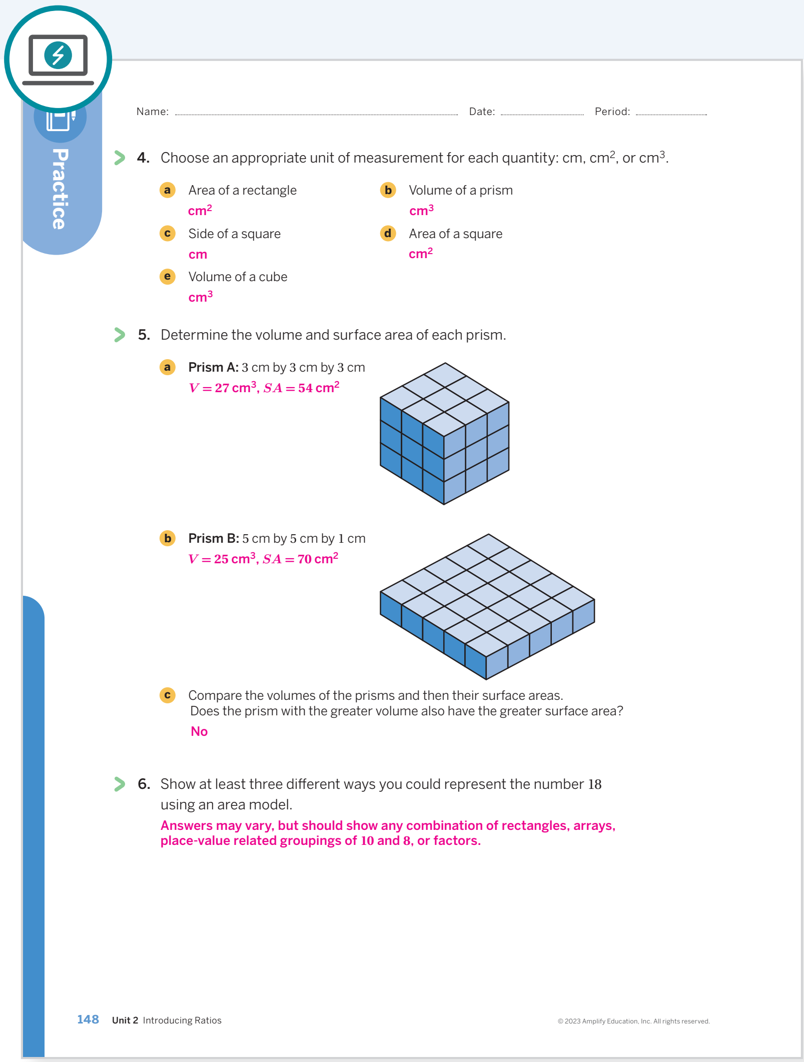
Lesson 2 Introducing Ratios and Ratio Language 147



Practice

Name: Date: Period:

4. Choose an appropriate unit of measurement for each quantity: cm, cm^2, or cm^3.
   - a Area of a rectangle
     cm^2
   - b Volume of a prism
     cm^3
   - c Side of a square
     cm
   - d Area of a square
     cm^2
   - e Volume of a cube
     cm^3

5. Determine the volume and surface area of each prism.
   - a **Prism A:** 3 cm by 3 cm by 3 cm
     $V = 27\text{ cm}^3$, $SA = 54\text{ cm}^2$
   - b **Prism B:** 5 cm by 5 cm by 1 cm
     $V = 25\text{ cm}^3$, $SA = 70\text{ cm}^2$
   - c Compare the volumes of the prisms and then their surface areas. Does the prism with the greater volume also have the greater surface area?
     No

6. Show at least three different ways you could represent the number 18 using an area model.
   Answers may vary, but should show any combination of rectangles, arrays, place-value related groupings of 10 and 8, or factors.

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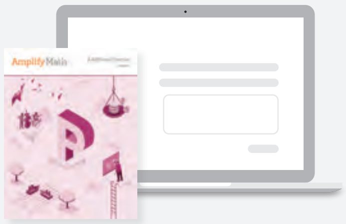
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 15	2
	5	Unit 1 Lesson 18	2
Formative	6	Unit 2 Lesson 3	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 2 | **LESSON 3**

Representing Ratios With Diagrams

Let’s use diagrams to represent ratios.

Focus

Goals

1. Coordinate discrete diagrams and multiple written sentences describing the same ratios.
2. Draw and label discrete diagrams to represent situations involving ratios.
3. **Language Goal:** Practice reading and writing sentences describing ratios, e.g., “The ratio of these to those is $a : b$. The ratio of these to those is a to b. For every a of these, there are b of those.” **(Speaking and Listening, Writing)**

Coherence

Today

Students use diagrams to represent situations involving ratio relationships and continue to develop ratio language. Examples of very simple diagrams with discrete objects help guide students toward more abstract representations while still relying on visual or spatial cues to support reasoning. These diagrams also help students see associations between two or more quantities in different ways. Both the visual and verbal descriptions of ratios demand careful interpretation and use of language. **Note:** The term *equivalent ratio* is not defined until Lesson 6, but students build toward that idea using the contextual notions of “same color” and “same taste.” Students should begin connecting different ways to write a ratio for the same relationship with different numbers.

Previously

Students used pattern blocks to learn about ratio relationships and ratio language in Lesson 2, with a focus on precision of language.

Coming Soon

Lessons 4 and 5 continue to explore equivalent ratios, first with larger batches of recipes and then with smaller batches.

Rigor

- Students continue to build **conceptual understanding** of ratios by creating visual representations of ratios.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, one per group
- Activity 2 PDF (answers)
- colored pencils
- envelopes for the Activity 2 PDF answer keys
- pattern blocks

Math Language Development

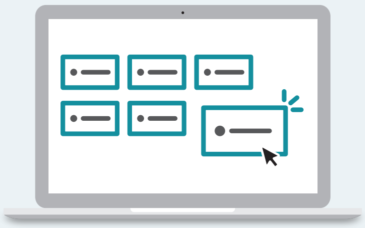
Review word

- *ratio relationship*

Amps | Featured Activity

Activity 2
Digital Card Sort

Students match digital diagrams and sentence cards.



Building Math Identity and Community

Connecting to Mathematical Practice

Students may have trouble working with a peer, which reduces the effectiveness of being able to make sense of mathematical problems and solving them. Remind students of the purpose of working in pairs and how to be a good contributor *and* listener. Ask, “We talked about how to be a good partner. What are some of the ways you can be a good partner?”

Modifications to Pacing

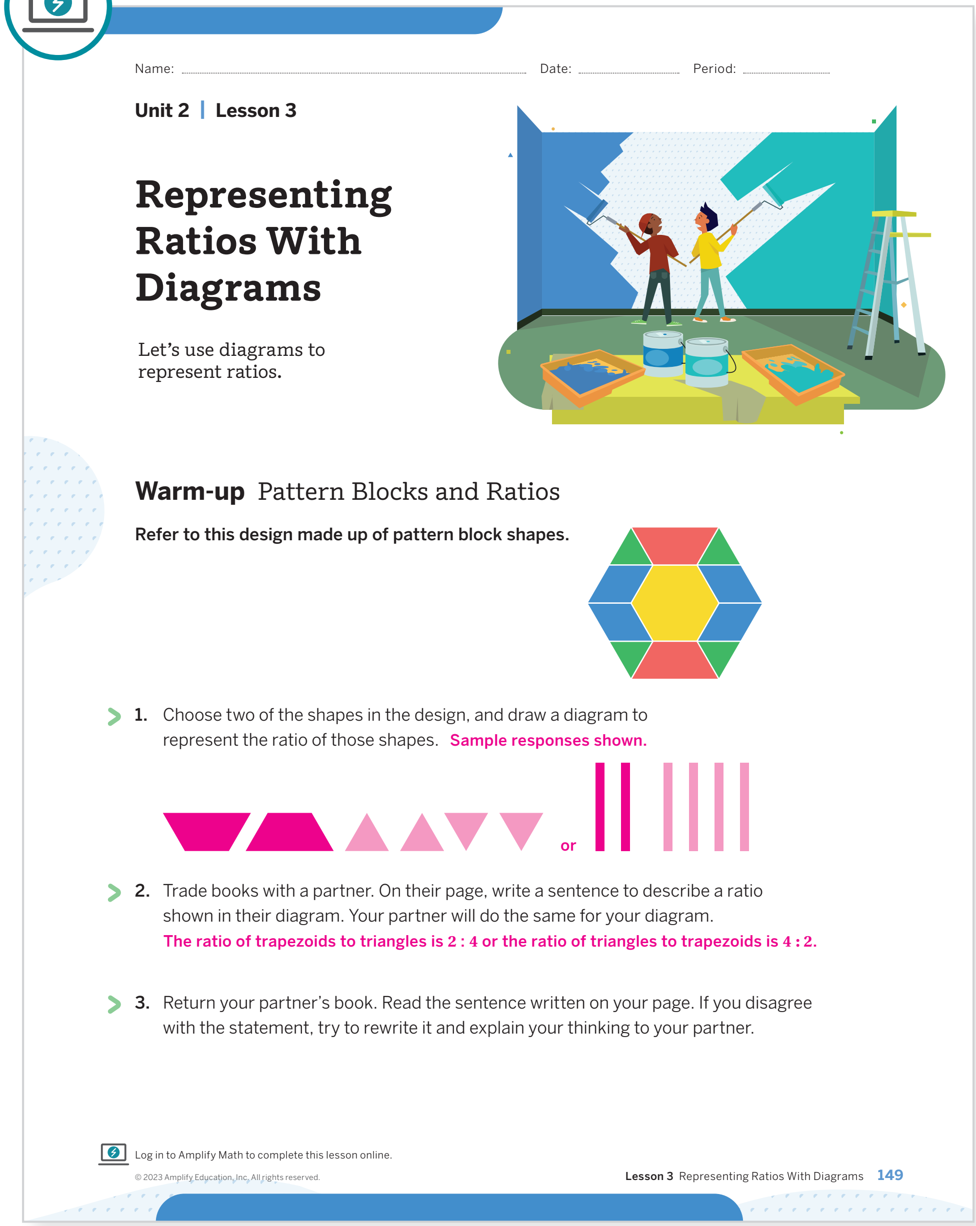
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, ask, “What connections can you make between these expressions?” Have a group discussion instead of allowing individual think-time for each problem.
- In **Activity 1**, Problem 2 can be done and discussed as a whole class.
- In **Activity 2**, Problems 3 and 4 may be omitted or discussed as a whole class.

Pairs | 5 min

Warm-up Pattern Blocks and Ratios

Students interpret ratio information from a pattern block design and represent ratios with diagrams and in sentences.



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Unit 2 | Lesson 3

Representing Ratios With Diagrams

Let's use diagrams to represent ratios.

Warm-up Pattern Blocks and Ratios

Refer to this design made up of pattern block shapes.

1. Choose two of the shapes in the design, and draw a diagram to represent the ratio of those shapes. **Sample responses shown.**

2. Trade books with a partner. On their page, write a sentence to describe a ratio shown in their diagram. Your partner will do the same for your diagram.
The ratio of trapezoids to triangles is 2 : 4 or the ratio of triangles to trapezoids is 4 : 2.

3. Return your partner's book. Read the sentence written on your page. If you disagree with the statement, try to rewrite it and explain your thinking to your partner.

Log in to Amplify Math to complete this lesson online.

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Lesson 3 Representing Ratios With Diagrams 149

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by reviewing the meaning of *diagram*. For example, to represent two green triangles, you might draw two green triangles on the board, or two triangles labeled "G."

Look for points of confusion:

- **Not including all quantities or units in ratio sentences.** Remind students that a ratio sentence must include both the amounts and the units.
- **Worrying about how to draw the shapes.** Emphasize that a diagram represents the number and type of objects and does not need to be an identical representation.
- **Basing ratios on area rather than just counts.** Remind students that they are comparing the number of block shapes, not area.

Look for productive strategies:

- Understanding that the order of the ratio must match in both quantity and units in the sentence.
- Drawing simplified representations of the shapes, such as red and green lines.
- Recognizing ratios that can be simplified to equivalent forms.

3 Connect

Have pairs of students share how they were able to interpret one another's diagrams accurately — or if not, how they resolved their differences.

Ask, "How could there be two (or more) correct sentences representing one diagram?"

Highlight that it is important to use precise ratio language, which can be interpreted from a diagram that it is representing. However, the diagrams do not have to be complex.

Math Language Development

MLR7: Compare and Connect

To support students' sense making for Problems 2 and 3, have partners compare ratio sentences. Ask students to determine if they agree or disagree with their partner's ratio sentence using their developing ratio language.

English Learners

Encourage students to refer to the class display to support their use of ratio language.

Power-up

To power up students' ability to determine factors in multiplicative comparisons have students complete:

Determine the unknown value in each problem. Use an area model to show your thinking for at least one problem.

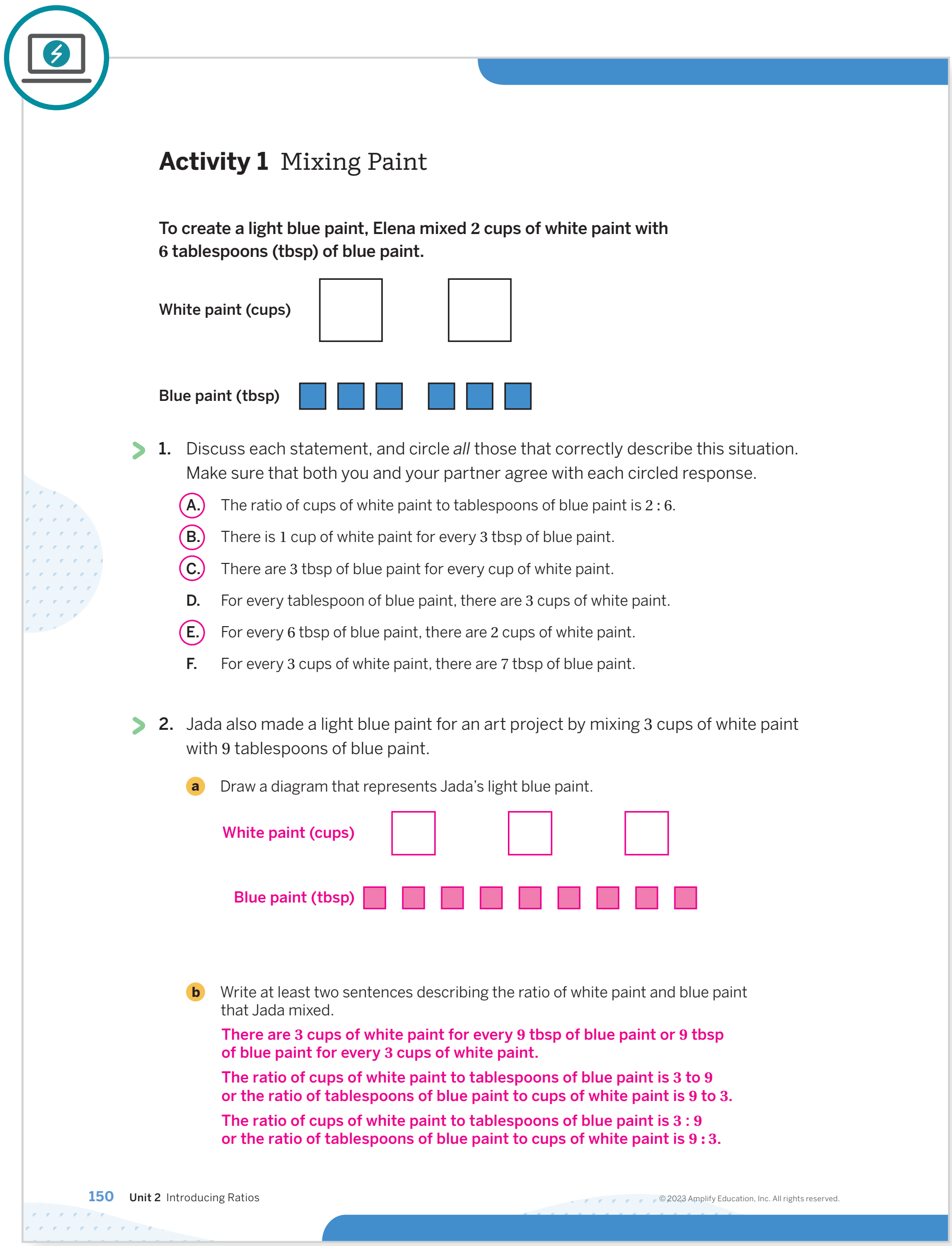
1. 28 is 7 times what number? 4
2. 32 is 8 times what number? 4
3. 4,000 is 4 times what number? 1,000

Use: Before the Warm-up.
Informed by: Performance on Lesson 2, Practice Problem 6 and and Pre-Unit Readiness Assessment, Problem 2.

Pairs | 15 min

Activity 1 Mixing Paint

Students use ratio language to make connections between a diagram and the ratio it represents.



Activity 1 Mixing Paint

To create a light blue paint, Elena mixed 2 cups of white paint with 6 tablespoons (tbsp) of blue paint.

1. Discuss each statement, and circle *all* those that correctly describe this situation. Make sure that both you and your partner agree with each circled response.
   - [✓ A.] The ratio of cups of white paint to tablespoons of blue paint is 2 : 6.
   - [✓ B.] There is 1 cup of white paint for every 3 tbsp of blue paint.
   - [✓ C.] There are 3 tbsp of blue paint for every cup of white paint.
   - D. For every tablespoon of blue paint, there are 3 cups of white paint.
   - [✓ E.] For every 6 tbsp of blue paint, there are 2 cups of white paint.
   - F. For every 3 cups of white paint, there are 7 tbsp of blue paint.

2. Jada also made a light blue paint for an art project by mixing 3 cups of white paint with 9 tablespoons of blue paint.
   - a. Draw a diagram that represents Jada's light blue paint.

   - b. Write at least two sentences describing the ratio of white paint and blue paint that Jada mixed.

There are 3 cups of white paint for every 9 tbsp of blue paint or 9 tbsp of blue paint for every 3 cups of white paint.

The ratio of cups of white paint to tablespoons of blue paint is 3 to 9 or the ratio of tablespoons of blue paint to cups of white paint is 9 to 3.

The ratio of cups of white paint to tablespoons of blue paint is 3 : 9 or the ratio of tablespoons of blue paint to cups of white paint is 9 : 3.

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1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help pairs get started by suggesting they discuss the diagram before looking at the statements. Ask, "What do you notice about the diagram?"

Look for points of confusion:

- **Drawing realistic cups and tablespoons.** Ask, "What is a more efficient way to make the diagram?"
- **Not connecting the numbers 1 and 3 to the diagram.** Ask, "Do you see a repeating group?"

Look for productive strategies:

- Analyzing each statement in Problem 1 to identify correct ratio descriptions.
- Using ratio language correctly to describe and label the diagrams.

3 Connect

Display Elena's diagram and the correct statements.

Ask:

- "Would statement A still be correct if it said, 'The ratio of tablespoons of blue paint to cups of white paint is 2 : 6.'?"
- "Does the order of the ratio matter?" It depends on how it is labeled or how the statement is written.
- "Why is statement F not correct?"

Have pairs of students share their diagrams for Jada's mixture and their responses to Problem 2b.

Highlight that both a diagram and sentences help to make sense of the ratio relationship in a given scenario.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Inform students that there are two false statements and have them determine which statements are the false ones.

Extension: *Math Enrichment*

Have students complete the following problem:
Is Jada's light blue paint the same shade as the light blue paint made by Elena? How do you know? Yes; Sample response: There is still one cup of white paint for every 3 tbsp of blue paint. Jada's mixture just adds another group of white and blue amounts.

Math Language Development

MLR2: Collect and Display

Circulate and listen to students talk with their partner as they complete the activity. Display important words and phrases (e.g., *for every, the ratio of*). Refer back to this list and ask students to clarify the meanings of these phrases. Ask them to think about how these words and phrases help communicate ratio relationships.

English Learners

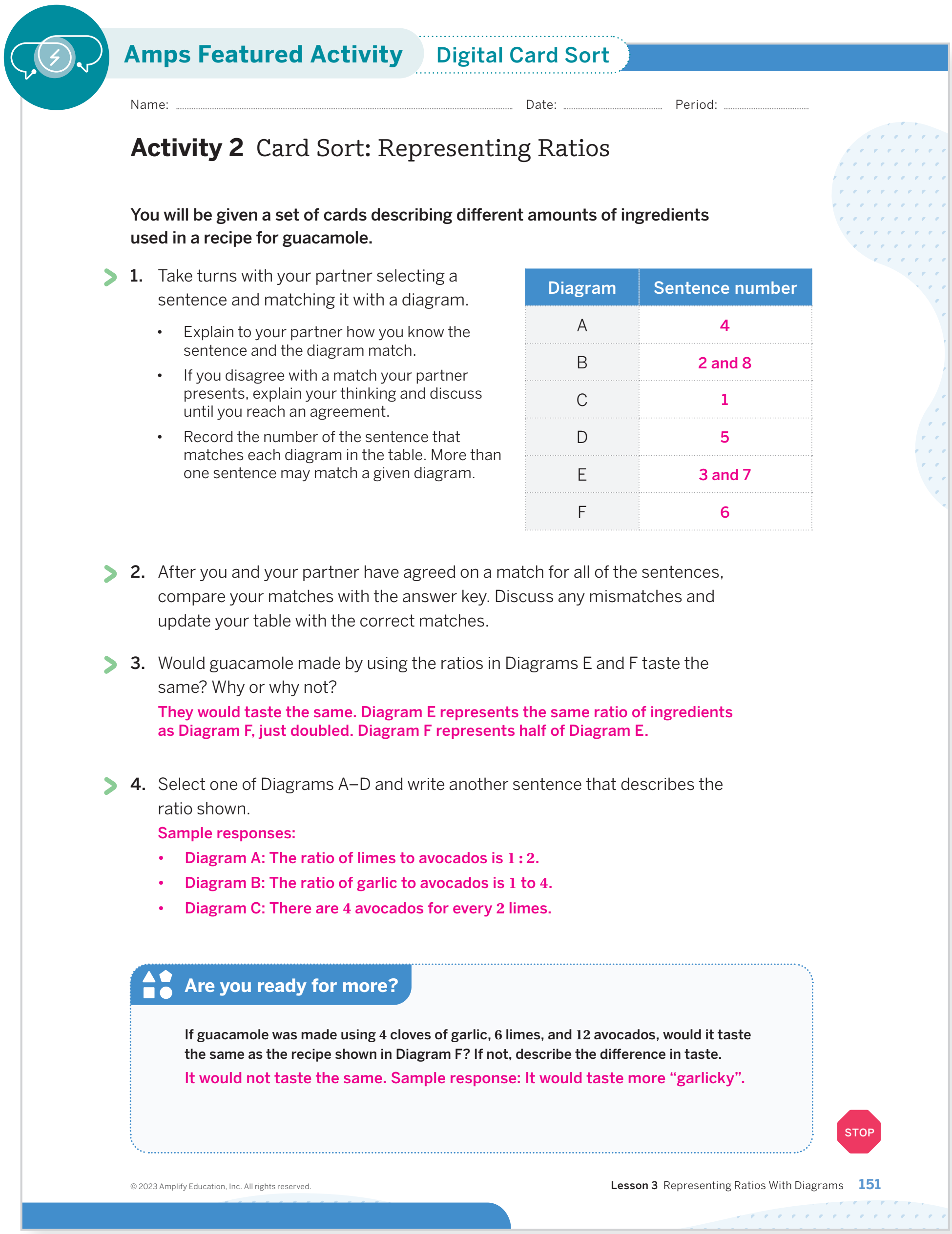
Include related diagrams or examples on the display to represent the ratio words and phrases.

Pairs | 15 min

Activity 2 Card Sort: Representing Ratios

Students further develop their ability to describe ratio situations precisely by attending carefully to the quantities, their units, and their order in ratio sentences.

Amps Featured Activity Digital Card Sort



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Activity 2 Card Sort: Representing Ratios

You will be given a set of cards describing different amounts of ingredients used in a recipe for guacamole.

1. Take turns with your partner selecting a sentence and matching it with a diagram.
   - Explain to your partner how you know the sentence and the diagram match.
   - If you disagree with a match your partner presents, explain your thinking and discuss until you reach an agreement.
   - Record the number of the sentence that matches each diagram in the table. More than one sentence may match a given diagram.

Diagram	Sentence number
A	4
B	2 and 8
C	1
D	5
E	3 and 7
F	6

2. After you and your partner have agreed on a match for all of the sentences, compare your matches with the answer key. Discuss any mismatches and update your table with the correct matches.

3. Would guacamole made by using the ratios in Diagrams E and F taste the same? Why or why not?

   They would taste the same. Diagram E represents the same ratio of ingredients as Diagram F, just doubled. Diagram F represents half of Diagram E.

4. Select one of Diagrams A–D and write another sentence that describes the ratio shown.

   Sample responses:
   - Diagram A: The ratio of limes to avocados is 1 : 2.
   - Diagram B: The ratio of garlic to avocados is 1 to 4.
   - Diagram C: There are 4 avocados for every 2 limes.

Are you ready for more?

If guacamole was made using 4 cloves of garlic, 6 limes, and 12 avocados, would it taste the same as the recipe shown in Diagram F? If not, describe the difference in taste.

It would not taste the same. Sample response: It would taste more "garlicky".

STOP

Lesson 3 Representing Ratios With Diagrams 151

1 Launch

Provide pairs of students with a set of the Guacamole cards from the Activity 2 PDF and review the ***Take Turns*** routine. Point out where the answer keys will be for Problem 2.

2 Monitor

Help students get started by having partners choose one sentence and find its matching diagram.

Look for points of confusion:

- **Thinking the shapes in the diagram need to be drawn in the same order as the ingredients appear in the sentence.** Explain that the diagram shows the ingredients and the important thing is that the number in the diagram matches the number in the sentence.

Look for productive strategies:

- Eliminating matches that cannot be possible.
- Identifying that Diagram E/Sentence 6 is half of the same recipe in Diagram D/Sentence 3.

3 Connect

Have pairs of students share their responses for Problem 3, focusing on the doubling relationship, and then their sentences for Problem 4, including Diagrams A–D.

Ask:

- "Which matches required more think time? Why? Explain why."
- "What would happen if you used an incorrect ratio of ingredients when making this guacamole?"

Highlight that describing ratios requires precise language, and paying attention to the quantities, their units, and their order in the ratio statements.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate how Diagram A matches with Sentence 4 by highlighting the words "1 lime" and the image of the lime in one color, and the words "2 avocados" and the image of the two avocados in another color.

Extension: *Math Enrichment*

Ask students to draw diagrams that represent the following sentence: There is $\frac{1}{2}$ of a lime for every $\frac{1}{3}$ avocado. Challenge students to draw more than one diagram that could represent each sentence. For example, for the first sentence, they could draw 1 lime and $\frac{2}{3}$ avocado, or 3 limes and 2 avocados.

Math Language Development

MLR8: Discussion Supports—Revoicing

To demonstrate mathematical language and to help students with more clearly communicating their reasoning, revoice their ideas and press for details in explanations. For example, if a student says that they matched Diagram D with Sentence 5, ask, "What did you see in Diagram D that matched with the words of Sentence 5?"

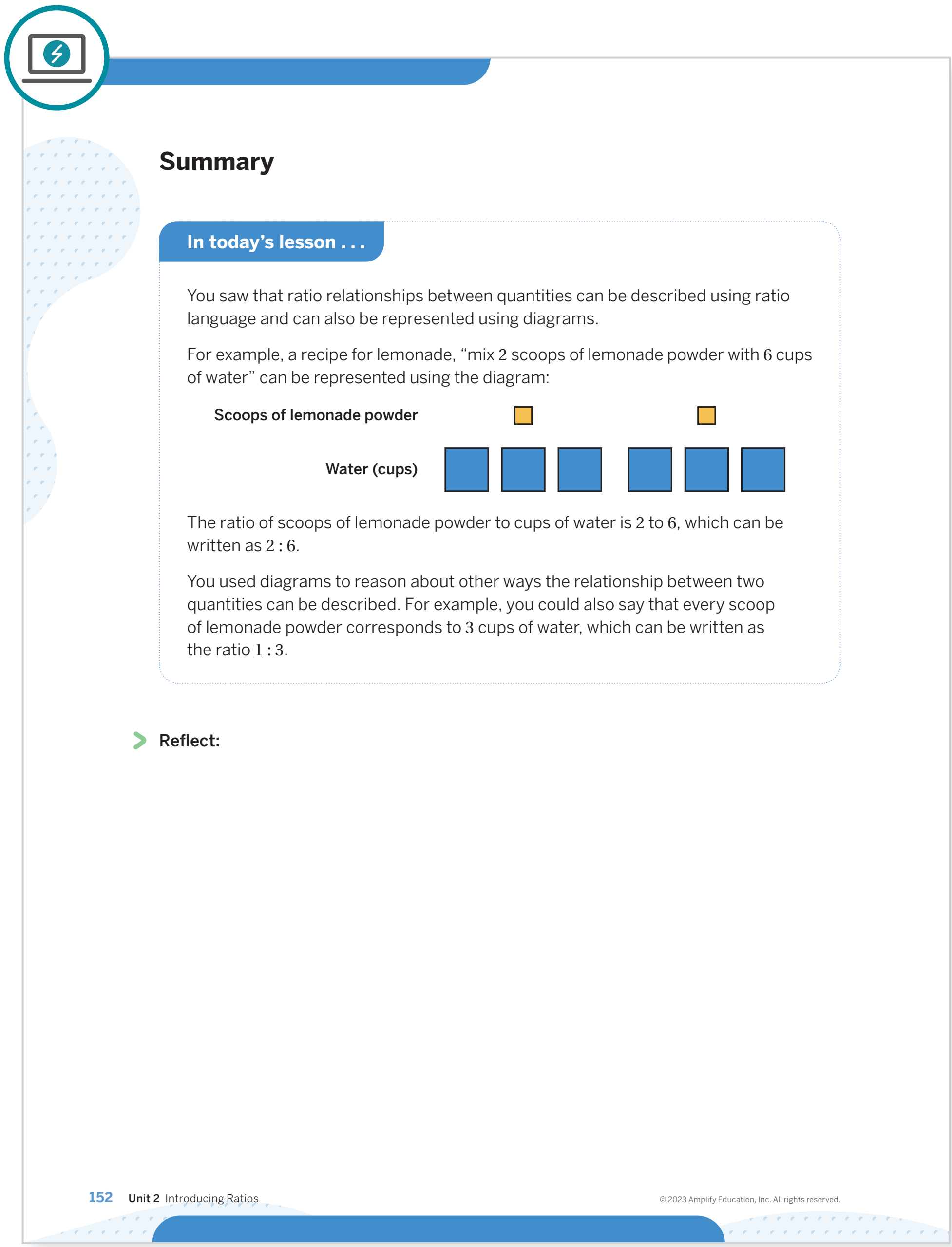
English Learners

Provide sentence frames as students explain their matches, such as "Diagram ____ matches with Sentence ____ because . . ."

Whole Class | 5 min

Summary

Review and synthesize how diagrams can be used to represent and interpret ratio relationships.



Summary

In today's lesson . . .

You saw that ratio relationships between quantities can be described using ratio language and can also be represented using diagrams.

For example, a recipe for lemonade, "mix 2 scoops of lemonade powder with 6 cups of water" can be represented using the diagram:

Scoops of lemonade powder 🟨 🟨

Water (cups) 🟦 🟦 🟦 🟦 🟦 🟦

The ratio of scoops of lemonade powder to cups of water is 2 to 6, which can be written as 2 : 6.

You used diagrams to reason about other ways the relationship between two quantities can be described. For example, you could also say that every scoop of lemonade powder corresponds to 3 cups of water, which can be written as the ratio 1 : 3.

Reflect:

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Synthesize

Display the diagram from the Summary page or have students refer to them in their books.

Ask:

- "What are some good things to remember when you draw a diagram of a ratio?" You need necessary information, including labels. You could include shapes, color-coded boxes, or initials to represent each object within the set. It is helpful to organize the types of items in rows.
- "How can a diagram help you make sense of a situation involving a ratio?" It is easier to write ratio statements when you have the visual representation to refer to. Also, you might be able to see other ways the objects can be grouped.

Highlight that ratio sentences can be represented visually in diagrams, with appropriate labels or units. By doing so, the diagrams may help students see how ratios can be grouped, such as seeing the same relationship as 2 : 6 and 1 : 3 representing the same set of objects.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What did it mean to represent a ratio in this lesson?"
- "How did you represent ratios in the scenarios presented in this lesson?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of diagrams representing ratios by both drawing diagrams and writing corresponding ratio sentences.

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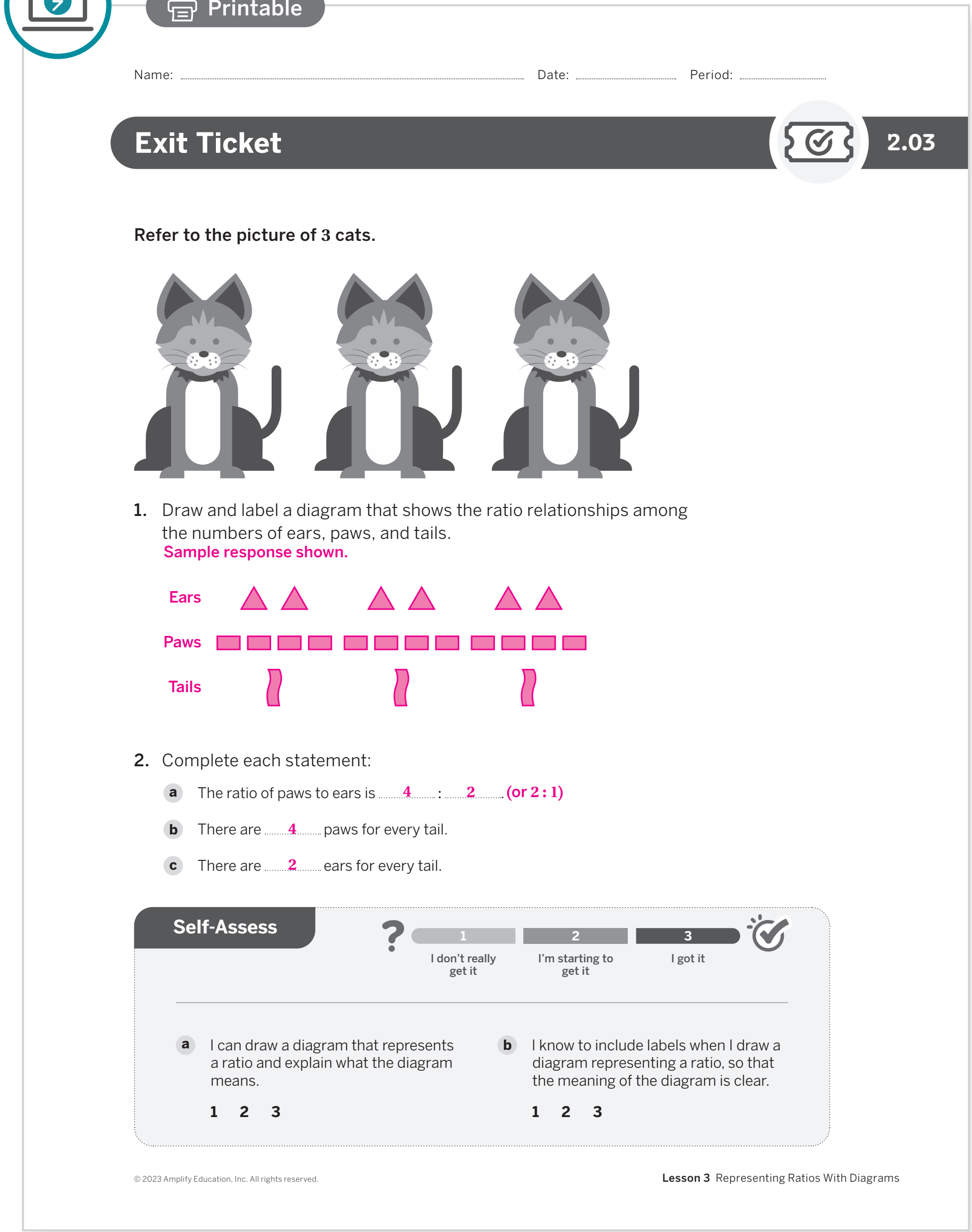


Name: ____________ Date: ________ Period: ________

Exit Ticket 2.03

Refer to the picture of 3 cats.

1. Draw and label a diagram that shows the ratio relationships among the numbers of ears, paws, and tails.
Sample response shown.

Ears

Paws

Tails

2. Complete each statement:

   a. The ratio of paws to ears is 4 : 2 (or 2 : 1)

   b. There are 4 paws for every tail.

   c. There are 2 ears for every tail.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can draw a diagram that represents a ratio and explain what the diagram means.

1 2 3

b. I know to include labels when I draw a diagram representing a ratio, so that the meaning of the diagram is clear.

1 2 3

Lesson 3 Representing Ratios With Diagrams

Success looks like . . .

- **Goal:** Coordinating discrete diagrams and multiple written sentences describing the same ratios.
  - » Completing the statements correctly in Problem 2.
- **Goal:** Drawing and labeling discrete diagrams to represent situations involving ratios.
  - » Drawing and labeling a correct diagram in Problem 1.
- **Language Goal:** Practicing reading and writing sentences describing ratios, e.g., "The ratio of these to those is $a : b$. The ratio of these to those is a to b. For every a of these, there are b of those." **(Speaking and Listening, Writing)**

Suggested next steps

If students lose time because they are trying to realistically draw the different components of the ratio, consider:

- Asking, "Is there a more efficient way you can draw this diagram?"

If students confuse the order of the quantities in Problem 2a, consider:

- Asking, "What comes first in the written sentence?" paws "So, what needs to come first in the numerical ratio?" 4

If students say the ratio is 2 paws to 2 ears in Problem 2a, consider:

- Having students think of the ratio in terms of "for every."

If students have trouble with Problem 2, consider:

- Having students circle the components in the problem.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is?
- What might you change the next time you teach this lesson?

Math Language Development

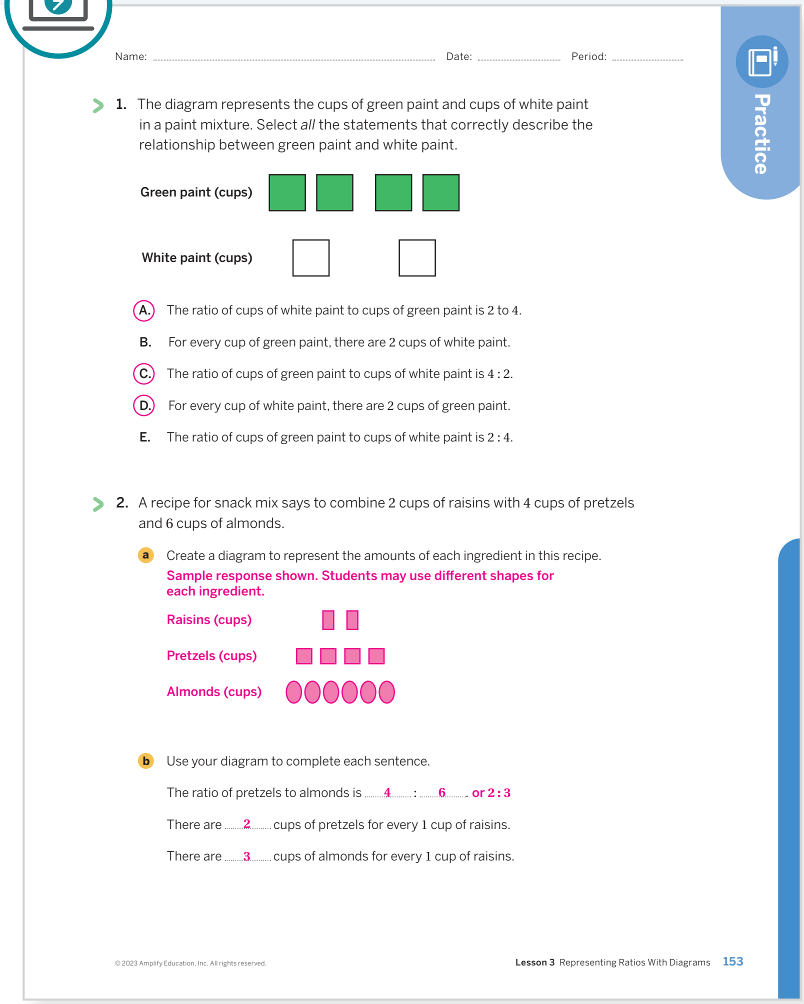
Language Goal: Practicing reading and writing sentences describing ratios.

Reflect on students' language development toward this goal.

- How have students progressed in reading and writing statements that describe ratio relationships from Lesson 2 to Lesson 3?
- Do students understand the meanings of the phrases *for every and for each* as they describe ratios? How has providing them with sentence frames helped them develop this language?

Practice

Independent



Name: Date: Period:

Practice

1. The diagram represents the cups of green paint and cups of white paint in a paint mixture. Select *all* the statements that correctly describe the relationship between green paint and white paint.

Green paint (cups) 🟩 🟩 🟩 🟩

White paint (cups) ☐ ☐

[✓ A.] The ratio of cups of white paint to cups of green paint is 2 to 4.

B. For every cup of green paint, there are 2 cups of white paint.

[✓ C.] The ratio of cups of green paint to cups of white paint is 4 : 2.

[✓ D.] For every cup of white paint, there are 2 cups of green paint.

E. The ratio of cups of green paint to cups of white paint is 2 : 4.

2. A recipe for snack mix says to combine 2 cups of raisins with 4 cups of pretzels and 6 cups of almonds.

a. Create a diagram to represent the amounts of each ingredient in this recipe.

Sample response shown. Students may use different shapes for each ingredient.

Raisins (cups) ▯ ▯

Pretzels (cups) ■ ■ ■ ■

Almonds (cups) ● ● ● ● ● ●

b. Use your diagram to complete each sentence.

The ratio of pretzels to almonds is **4** : **6**. **or 2 : 3**

There are **2** cups of pretzels for every 1 cup of raisins.

There are **3** cups of almonds for every 1 cup of raisins.

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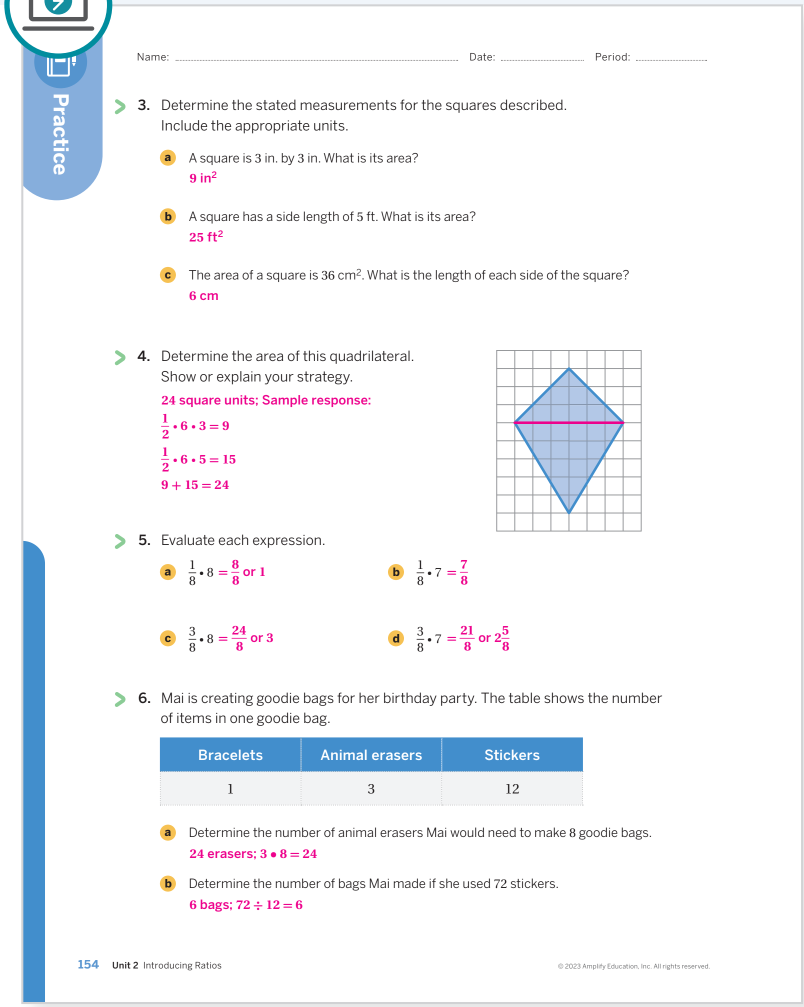
Lesson 3 Representing Ratios With Diagrams 153



Name: Date: Period:

Practice

3. Determine the stated measurements for the squares described. Include the appropriate units.

a. A square is 3 in. by 3 in. What is its area?

9 in^2

b. A square has a side length of 5 ft. What is its area?

25 ft^2

c. The area of a square is 36 cm^2. What is the length of each side of the square?

6 cm

4. Determine the area of this quadrilateral. Show or explain your strategy.

24 square units; Sample response:

$\frac{1}{2} \bullet 6 \bullet 3 = 9$

$\frac{1}{2} \bullet 6 \bullet 5 = 15$

$9 + 15 = 24$

5. Evaluate each expression.

a. $\frac{1}{8} \bullet 8 = \frac{8}{8}$ **or 1**

b. $\frac{1}{8} \bullet 7 = \frac{7}{8}$

c. $\frac{3}{8} \bullet 8 = \frac{24}{8}$ **or 3**

d. $\frac{3}{8} \bullet 7 = \frac{21}{8}$ **or** $2\frac{5}{8}$

6. Mai is creating goodie bags for her birthday party. The table shows the number of items in one goodie bag.

Bracelets	Animal erasers	Stickers
1	3	12

a. Determine the number of animal erasers Mai would need to make 8 goodie bags.

24 erasers; $3 \bullet 8 = 24$

b. Determine the number of bags Mai made if she used 72 stickers.

6 bags; $72 \div 12 = 6$

154 **Unit 2** Introducing Ratios

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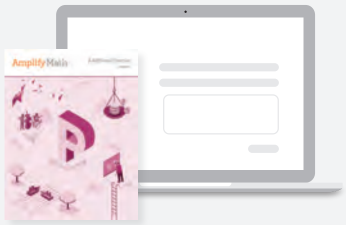
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
Spiral	3	Unit 1 Lesson 18	1
	4	Unit 1 Lesson 13	2
	5	Unit 2 Lesson 2	1
Formative ⚡	6	Unit 2 Lesson 4	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 2 | **LESSON 4**

A Recipe for Purple Oobleck

Let's explore ratios in recipes.

Focus

Goals

1. Draw and label a discrete diagram with groupings to represent multiple batches of a recipe.
2. Understand that doubling or tripling a recipe involves multiplying the amount of each ingredient by the same number, yielding something that feels or looks the same.
3. **Language Goal:** Explain ratios that are equivalent in terms of different-sized batches of the same recipe have the same consistency or color. **(Speaking and Listening, Writing)**

Coherence

Today

This is the first of two lessons that develop the idea of equivalent ratios informally through familiar contexts and physical experiences. The key understanding is that you can change the amount of something and the result can still be "the same" in some meaningful way. In this lesson the focus is on *making more*, such as scaling a recipe *up* to make multiple batches or larger batches. If the ingredients are in the same ratio, then the resulting mixtures "feel or "look" the same. Students recognize that this requires multiplying the amounts of each ingredient by the same factor, e.g., doubling a recipe means multiplying the amount of each ingredient by 2. They continue to use discrete diagrams as a tool to represent ratio situations.

Previously

In Lessons 2 and 3, students gained an understanding of ratio relationships and the language and notation used to represent ratios.

Coming Soon

In Lesson 5, students will continue exploring equivalent ratios informally, focusing on making less, or scaling recipes down.

Rigor

- Students begin to build **conceptual understanding** of equivalent ratios through the scaling up of recipes.
- Students strengthen their **fluency** in using ratio language and notation.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (instructions)
- Activity 2 PDF (instructions)
- counters

For oobleck demonstration (optional):

- cornstarch
- water
- measuring cup
- 3–4 bowls

For purple coloring demonstration (optional):

- red food coloring
- blue food coloring
- clear cups

Math Language Development

New words

- *ratio*
- *equivalent**

*The term *equivalent* is introduced for the context of ratios, and students construct a *working* definition, but the term *equivalent ratios* will not be formally defined until Lesson 6.

Amps | Featured Activity

Activity 2

Mixing Colors

Students can add and remove amounts of red and blue to make equivalent ratios.

Building Math Identity and Community

Connecting to Mathematical Practice

At first, students may feel lost when the pattern of increasing a ratio by multiplication is not immediately apparent. Encourage students to persist as they look for structure. For example, ask them to shift their perspective by viewing doubling as multiplying by 2 and tripling as multiplying by 3. The use of actual ingredients in the recipes helps by demonstrating the concept of increasing ratios.

Modifications to Pacing

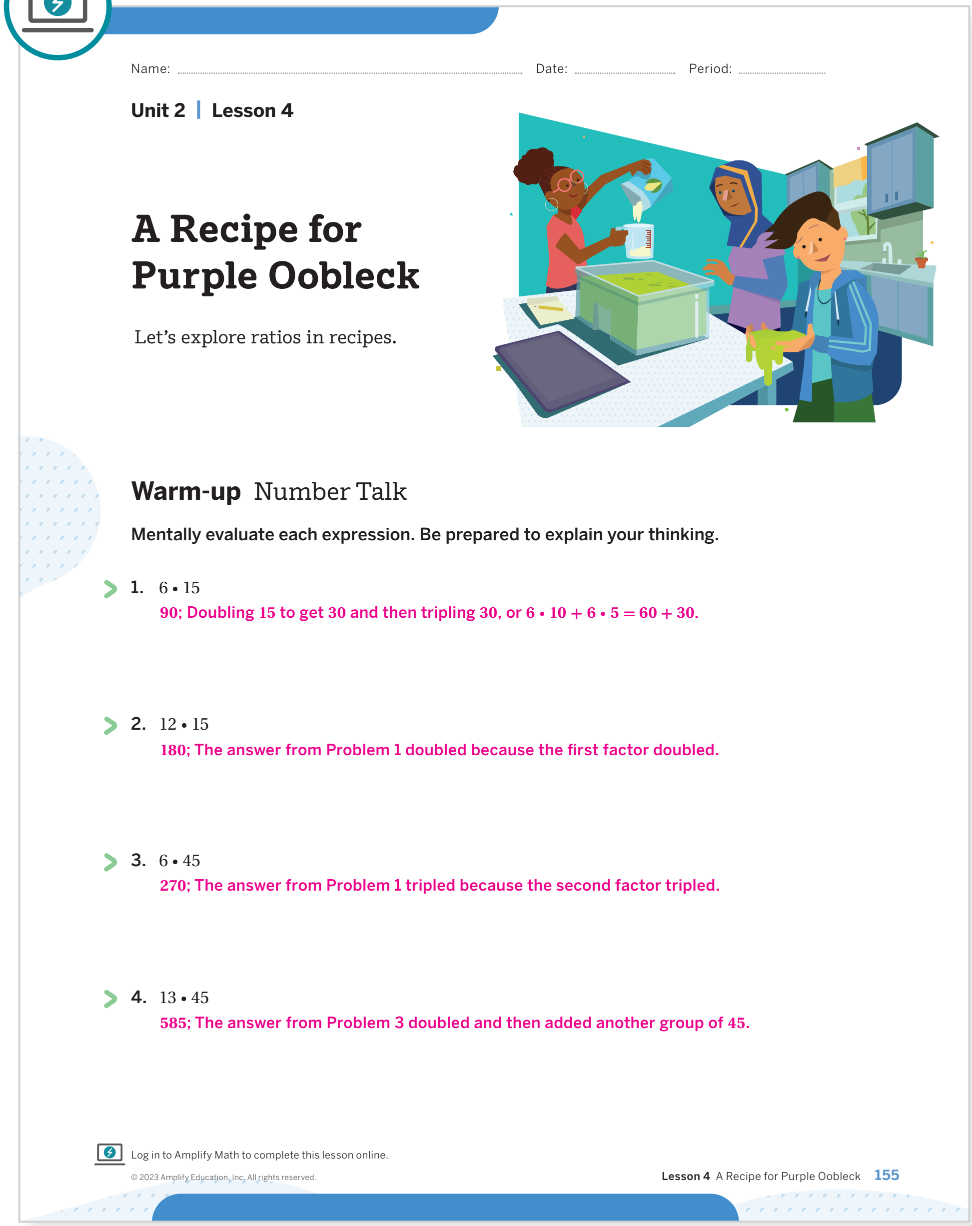
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, Problems 1 and 3 may be omitted.

Independent | 5 min

Warm-up Number Talk

Students use the structure of base ten numbers and the properties of operations to evaluate related products.



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Unit 2 | Lesson 4

A Recipe for Purple Oobleck

Let's explore ratios in recipes.

Warm-up Number Talk

Mentally evaluate each expression. Be prepared to explain your thinking.

1. $6 \cdot 15$
   90; Doubling 15 to get 30 and then tripling 30, or $6 \cdot 10 + 6 \cdot 5 = 60 + 30$.
2. $12 \cdot 15$
   180; The answer from Problem 1 doubled because the first factor doubled.
3. $6 \cdot 45$
   270; The answer from Problem 1 tripled because the second factor tripled.
4. $13 \cdot 45$
   585; The answer from Problem 3 doubled and then added another group of 45.

Log in to Amplify Math to complete this lesson online.

1 Launch

Display one expression at a time. Give students approximately 15 seconds to evaluate each and then share their responses and strategies, before displaying the next expression. Consider gathering only two or three different strategies per problem. The focus of this string of problems is for students to see how adjusting a factor impacts the product, and how this insight can be used to reason about other problems. Encourage terms such as *doubled*, *tripled*, *two times*, and *three times*.

2 Monitor

Help students get started by asking, "What would you get if you double the 15? How could you use that?"

Look for points of confusion:

- **Trying to evaluate each expression as a brand new exercise.** Ask, "Is there a way you could use the factors or product from a previous expression to help you evaluate this one?"

Look for productive strategies:

- Using the expanded form of 15 $(10 + 5)$ to evaluate products with a factor of 15 as being the sum of the other factor times 10 and then half of that again.
- Recognizing $45 = 3 \cdot 15$ and other relationships among factors, such as $2 \cdot 6 = 12$ and $2 \cdot 6 + 1 = 13$.
- Referring to and connecting each expression to a previous expression or mental strategy.

3 Connect

Ask, "How did the factors in the problem impact your thinking and choice of strategy?"

Highlight that, individually, these seem like separate problems, but together there are connections between factors. In this case, factors and multiples of 6 and 15 could be used to help you mentally evaluate related expressions.

Math Language Development

MLR8: Discussion Supports

Display sentence frames to support students when they explain their strategies. For example, use these sentence frames:

- "First, I ___ because . . ."
- "I noticed ___, so I . . ."

Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Power-up

To power up students' ability to reason with multiplication as scaling, have students complete:

A recipe calls for 2 cups of sliced apples to make one batch of muffins. How many cups are needed for

a. 2 batches? 4 cups
b. 3 batches? 6 cups
c. 4 batches? 8 cups
d. 10 batches? 20 cups
e. 100 batches? 200 cups

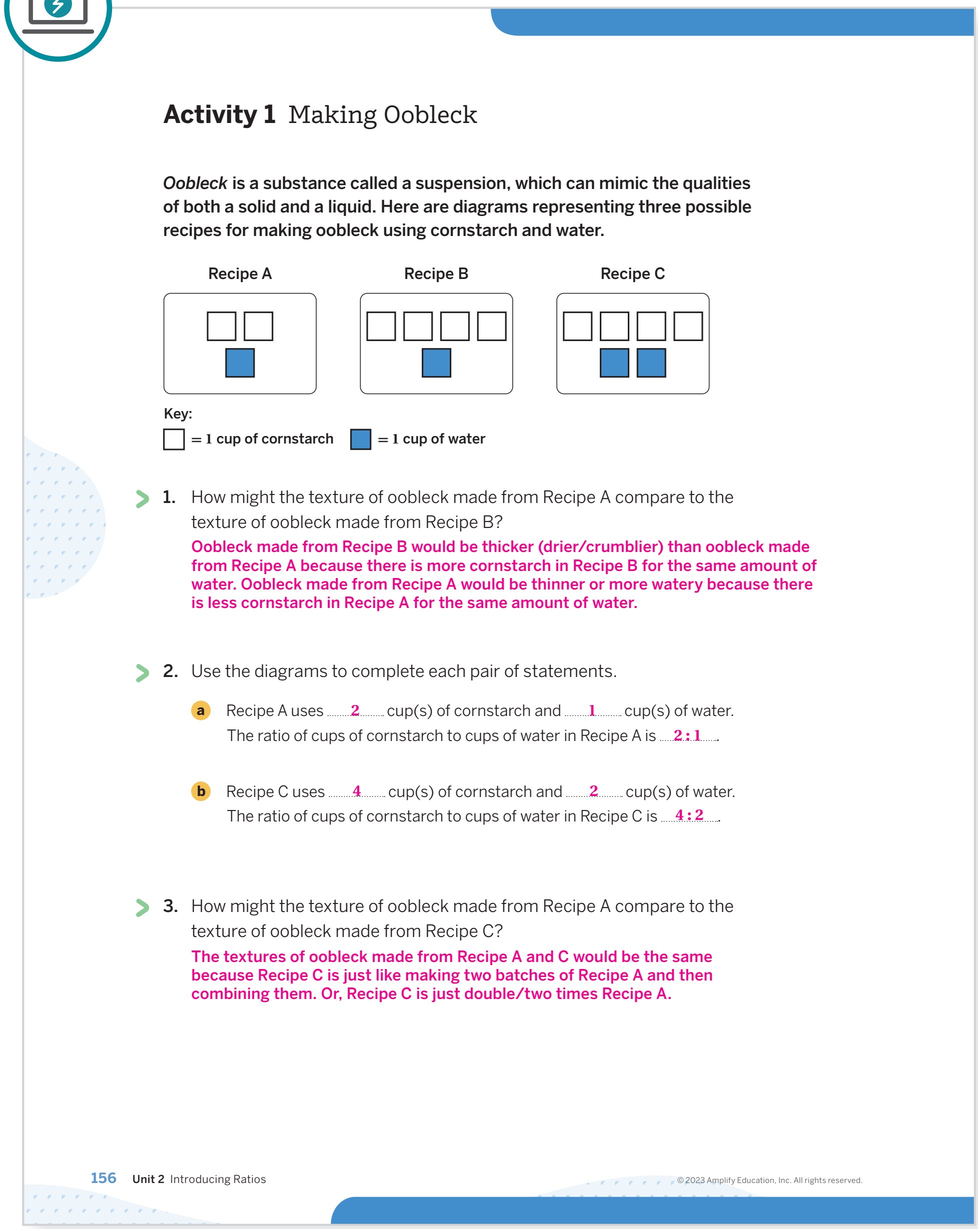
Use: Before Activity 1.

Informed by: Performance on Lesson 3, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 5.

Pairs | 15 min

Activity 1 Making Oobleck

Students work with ratio diagrams to represent and compare recipes for varying quantities and consistencies of oobleck. The term *ratio* is formally defined.



Activity 1 Making Oobleck

Oobleck is a substance called a suspension, which can mimic the qualities of both a solid and a liquid. Here are diagrams representing three possible recipes for making oobleck using cornstarch and water.

Key:
☐ = 1 cup of cornstarch 🟦 = 1 cup of water

1. How might the texture of oobleck made from Recipe A compare to the texture of oobleck made from Recipe B?
   Oobleck made from Recipe B would be thicker (drier/crumblier) than oobleck made from Recipe A because there is more cornstarch in Recipe B for the same amount of water. Oobleck made from Recipe A would be thinner or more watery because there is less cornstarch in Recipe A for the same amount of water.

2. Use the diagrams to complete each pair of statements.
   - a Recipe A uses 2 cup(s) of cornstarch and 1 cup(s) of water.
     The ratio of cups of cornstarch to cups of water in Recipe A is 2 : 1.
   - b Recipe C uses 4 cup(s) of cornstarch and 2 cup(s) of water.
     The ratio of cups of cornstarch to cups of water in Recipe C is 4 : 2.

3. How might the texture of oobleck made from Recipe A compare to the texture of oobleck made from Recipe C?
   The textures of oobleck made from Recipe A and C would be the same because Recipe C is just like making two batches of Recipe A and then combining them. Or, Recipe C is just double/two times Recipe A.

156 Unit 2 Introducing Ratios

1 Launch

Make counters available to support student thinking. **Note:** Including a demonstration is recommended, if possible; a suggested sequence is available in the Activity 1 PDF. (The actual recipe for oobleck has a ratio of cornstarch to water of 2 : 1.)

2 Monitor

Help students get started by asking, “What is the same in all three diagrams? What is different?”

Look for points of confusion:

- Thinking Recipes A and C do not feel the same. Ask, “What if you made two separate batches of mixture A and then combined them? Would the new batch feel the same?” Combining two of the same makes a larger amount of the same thing.
- Not connecting the ratios in Problem 5 to doubling and tripling. Have students write the three ratios in a vertical-list format and ask, “What do you notice is happening to each part of the ratios as you look down your list?”
- Not understanding or envisioning “same texture.” Suggest they think about color, or change the cornstarch to sugar and think about “sweetness.”

Look for productive strategies:

- Recognizing that doubling and tripling the batch, or using 4 : 2 and 6 : 3 ratios, will have the same consistency as a single batch of 2 : 1 because the process is making that same recipe two or three times and then combining the batches.
- Recognizing that the number of cups of cornstarch is always 2 times the number of cups of water in the recipes that make oobleck with the same texture.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Problems 1–3, relying heavily on the diagrams and making sense of the ratio language, and what it means to have the “same texture.” If there is time available, have them complete Problems 4 and 5.

Accessibility: *Clarify Language and Symbols*

Provide some words students could use to describe the textures of each recipe, such as *thick*, *dry*, *crumbly*, *flaky*, and *watery*.

Math Language Development

MLR7: Compare and Connect

Ask groups of students to share their thinking for Problem 5, focus on comparing recipes A, C, and E and make the connection that the quantities of ingredients are *multiples* of each other.

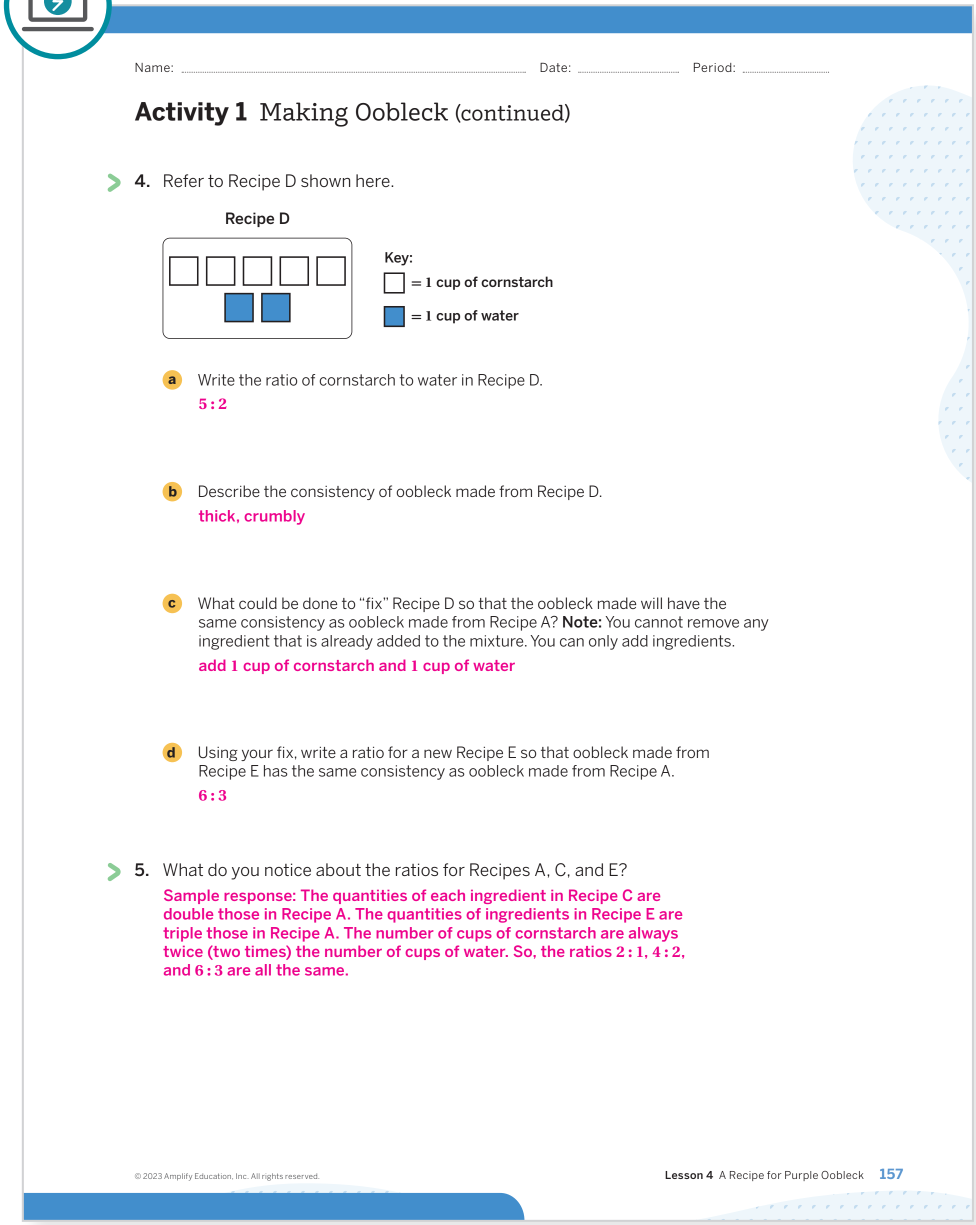
English Learners

Reinforce language that indicates multiples, such as *twice*, *double*, *triple*, *two times as many*, *three times as many*, etc.

Pairs | 15 min

Activity 1 Making Oobleck (continued)

Students work with ratio diagrams to represent and compare recipes for varying quantities and consistencies of oobleck. The term *ratio* is formally defined.



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Activity 1 Making Oobleck (continued)

4. Refer to Recipe D shown here.

Recipe D

☐☐☐☐☐
■■

Key:
☐ = 1 cup of cornstarch
■ = 1 cup of water

a. Write the ratio of cornstarch to water in Recipe D.

$5 : 2$

b. Describe the consistency of oobleck made from Recipe D.

thick, crumbly

c. What could be done to "fix" Recipe D so that the oobleck made will have the same consistency as oobleck made from Recipe A? **Note:** You cannot remove any ingredient that is already added to the mixture. You can only add ingredients.

add 1 cup of cornstarch and 1 cup of water

d. Using your fix, write a ratio for a new Recipe E so that oobleck made from Recipe E has the same consistency as oobleck made from Recipe A.

$6 : 3$

5. What do you notice about the ratios for Recipes A, C, and E?

Sample response: The quantities of each ingredient in Recipe C are double those in Recipe A. The quantities of ingredients in Recipe E are triple those in Recipe A. The number of cups of cornstarch are always twice (two times) the number of cups of water. So, the ratios $2 : 1$, $4 : 2$, and $6 : 3$ are all the same.

3 Connect

Display all the recipe diagrams, beginning with A, B, and C and the responses to Problems 1–3. Then add Recipe D for students to reference as they share their thinking for Problems 4 and 5.

Have pairs of students share their responses and thinking for Problems 4 and 5. Be sure to include and focus on explanations that suggest the ratios for Recipes A, C, and E are the same because either the two quantities are always the same multiple of each other, or each ratio is a multiple of another.

Define a ***ratio*** as a comparison of two quantities, such that for every a units of one quantity, there are b units of another quantity.

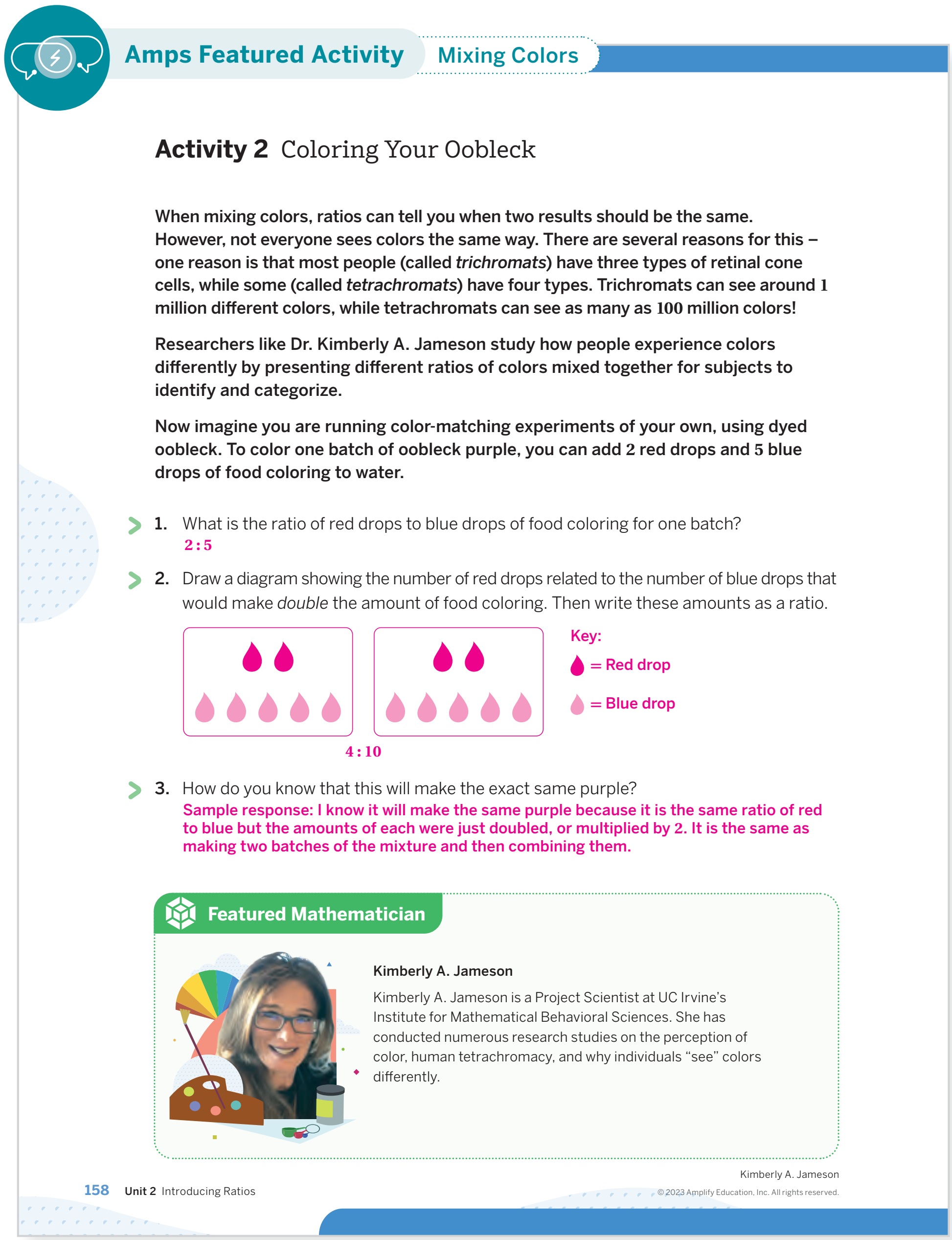
Highlight that the ratio $2 : 1$ of cornstarch to water represents a comparison of the amount of cornstarch relative to the amount of water in making oobleck. There is 2 times as much cornstarch as water. That means that the ratio $4 : 2$ is the *same ratio* as $2 : 1$ because there is still 2 times as much cornstarch.

Another way students can think about two ratios with different values for each quantity as still being the same is to think of the amount of each ingredient being doubled (multiplied by 2), tripled (multiplied by 3), and so on. So, the ratio $6 : 3$ is the same ratio as $2 : 1$ as well, because there is still 2 times as much cornstarch as water.

Small Groups | 15 min

Activity 2 Coloring Your Oobleck

Students describe how mixtures of different amounts of food coloring can make the same color (same ratio, or equivalent) or different colors (different ratios).



Amps Featured Activity Mixing Colors

Activity 2 Coloring Your Oobleck

When mixing colors, ratios can tell you when two results should be the same. However, not everyone sees colors the same way. There are several reasons for this – one reason is that most people (called *trichromats*) have three types of retinal cone cells, while some (called *tetrachromats*) have four types. Trichromats can see around 1 million different colors, while tetrachromats can see as many as 100 million colors!

Researchers like Dr. Kimberly A. Jameson study how people experience colors differently by presenting different ratios of colors mixed together for subjects to identify and categorize.

Now imagine you are running color-matching experiments of your own, using dyed oobleck. To color one batch of oobleck purple, you can add 2 red drops and 5 blue drops of food coloring to water.

1. What is the ratio of red drops to blue drops of food coloring for one batch?
   2 : 5
2. Draw a diagram showing the number of red drops related to the number of blue drops that would make *double* the amount of food coloring. Then write these amounts as a ratio.

   Key:
   = Red drop
   = Blue drop

   4 : 10
3. How do you know that this will make the exact same purple?
   Sample response: I know it will make the same purple because it is the same ratio of red to blue but the amounts of each were just doubled, or multiplied by 2. It is the same as making two batches of the mixture and then combining them.

Featured Mathematician

Kimberly A. Jameson

Kimberly A. Jameson is a Project Scientist at UC Irvine's Institute for Mathematical Behavioral Sciences. She has conducted numerous research studies on the perception of color, human tetrachromacy, and why individuals "see" colors differently.

Kimberly A. Jameson

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① Launch

Make counters available to support student thinking. **Note:** Including an experiment or demonstration using food coloring or paint is recommended, if possible; two suggested options are available in the Activity 2 PDF.

② Monitor

Help students get started by asking, "What does it mean to double something? What operation can you use to show the doubling of something?"

Look for points of confusion:

- **Explaining their thinking in Problem 4.** Suggest students draw a diagram first.
- **Applying the idea of multiplying for equivalent ratios in Problems 6 and 7.** Refer back to Problems 2 and 4 and ask, "How did you obtain the ratios 4 : 10 and 6 : 15? So now, try a new number by which to multiply each of the quantities in the ratio 2 : 5."

Look for productive strategies:

- Understanding that *equivalent* in this context means the same shade of purple.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can add and remove amounts of red and blue food coloring to see the varying shades of purple.

Math Language Development

MLR8: Discussion Supports

To support students in justifying their reasoning, display sentence frames such as:

- "I noticed ___, so I . . ."
- "In my diagram, ___ represents . . ."
- "To find a ratio that has the same consistency, but greater amounts, first I ___ because . . ."

As students share, highlight words and phrases that indicate *equivalent ratios*, such as *the same consistency*.

Featured Mathematician

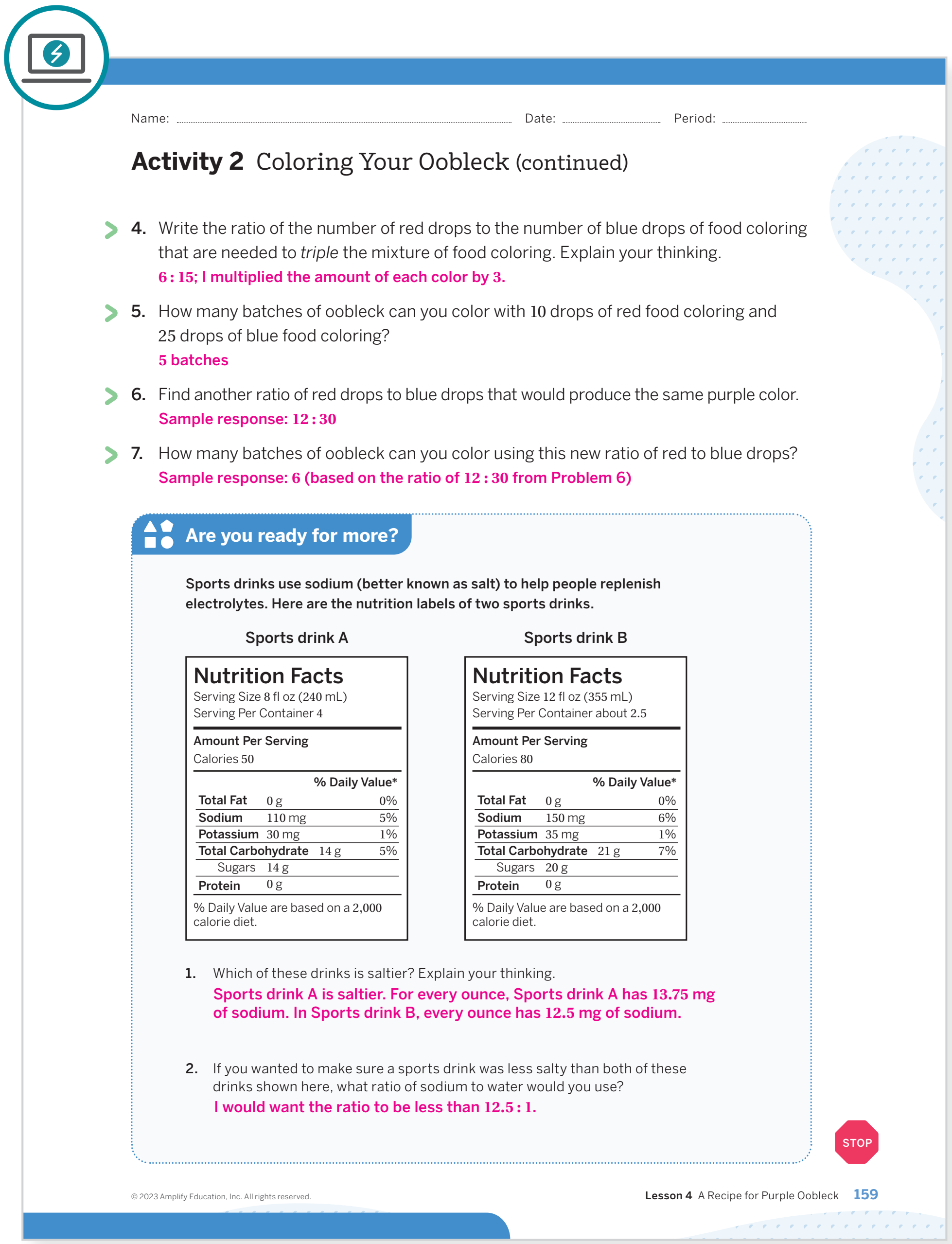
Kimberly A. Jameson

Have students read about Kimberly A. Jameson, Project Scientist at UC Irvine's Institute for Mathematical Behavioral Sciences, and her work with tetrachromacy.

Small Groups | 15 min

Activity 2 Coloring Your Oobleck (continued)

Students describe how mixtures of different amounts of food coloring can make the same color (same ratio, or equivalent) or different colors (different ratios).



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Activity 2 Coloring Your Oobleck (continued)

4. Write the ratio of the number of red drops to the number of blue drops of food coloring that are needed to *triple* the mixture of food coloring. Explain your thinking.
   6 : 15; I multiplied the amount of each color by 3.
5. How many batches of oobleck can you color with 10 drops of red food coloring and 25 drops of blue food coloring?
   5 batches
6. Find another ratio of red drops to blue drops that would produce the same purple color.
   Sample response: 12 : 30
7. How many batches of oobleck can you color using this new ratio of red to blue drops?
   Sample response: 6 (based on the ratio of 12 : 30 from Problem 6)

Are you ready for more?

Sports drinks use sodium (better known as salt) to help people replenish electrolytes. Here are the nutrition labels of two sports drinks.

Sports drink A

Nutrition Facts
Serving Size 8 fl oz (240 mL)
Serving Per Container 4

Amount Per Serving
Calories 50

		% Daily Value*
Total Fat	0 g	0%
Sodium	110 mg	5%
Potassium	30 mg	1%
Total Carbohydrate	14 g	5%
Sugars	14 g	
Protein	0 g	

% Daily Value are based on a 2,000 calorie diet.

Sports drink B

Nutrition Facts
Serving Size 12 fl oz (355 mL)
Serving Per Container about 2.5

Amount Per Serving
Calories 80

		% Daily Value*
Total Fat	0 g	0%
Sodium	150 mg	6%
Potassium	35 mg	1%
Total Carbohydrate	21 g	7%
Sugars	20 g	
Protein	0 g	

% Daily Value are based on a 2,000 calorie diet.

1. Which of these drinks is saltier? Explain your thinking.
   Sports drink A is saltier. For every ounce, Sports drink A has 13.75 mg of sodium. In Sports drink B, every ounce has 12.5 mg of sodium.
2. If you wanted to make sure a sports drink was less salty than both of these drinks shown here, what ratio of sodium to water would you use?
   I would want the ratio to be less than 12.5 : 1.

STOP

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Lesson 4 A Recipe for Purple Oobleck 159

3 Connect

Have groups of students share their responses to Problems 5, 6, and 7, focusing on how the ratio is related to that of one batch.

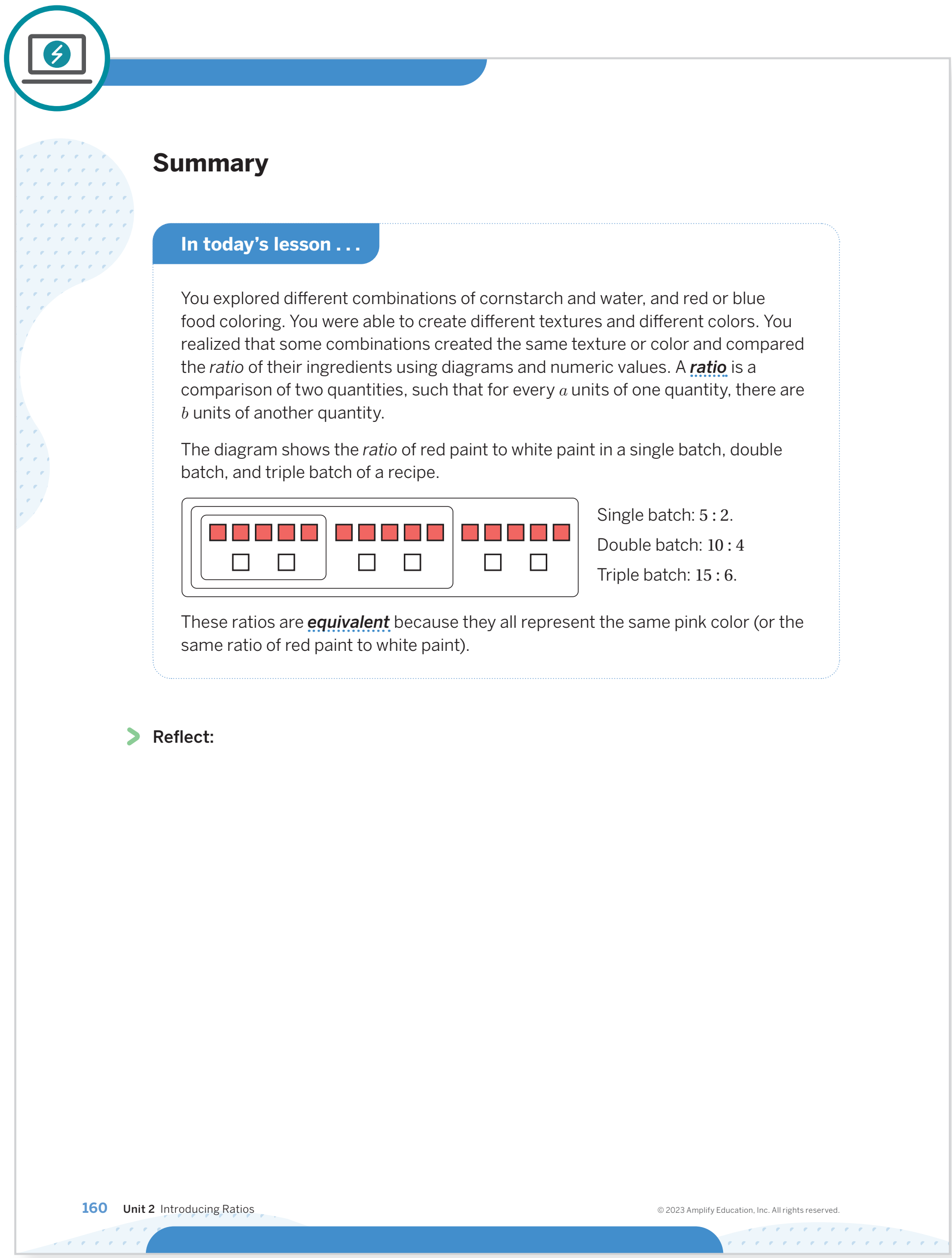
Define *equivalent* (in the context of ratios) as simply meaning "the same." (This will act as a *working definition* for *equivalent ratios*, formalized in Lesson 6.)

Highlight that whether the ratio of cups of red to blue is 2 : 5, 4 : 10, or 6 : 15, the recipes would make the same color of oobleck. Explain that students can say these ratios are *equivalent* because, even though there are different amounts, the relationship between red and blue is the same. There is exactly 2.5 times as much blue as there is red.

Whole Class | 5 min

Summary

Review and synthesize what it means to have more of something while one attribute remains the same, such as doubling and tripling mixtures, and how this relates to ratios.



Summary

In today's lesson . . .

You explored different combinations of cornstarch and water, and red or blue food coloring. You were able to create different textures and different colors. You realized that some combinations created the same texture or color and compared the *ratio* of their ingredients using diagrams and numeric values. A ***ratio*** is a comparison of two quantities, such that for every a units of one quantity, there are b units of another quantity.

The diagram shows the *ratio* of red paint to white paint in a single batch, double batch, and triple batch of a recipe.

Single batch: $5:2$.
Double batch: $10:4$
Triple batch: $15:6$.

These ratios are ***equivalent*** because they all represent the same pink color (or the same ratio of red paint to white paint).

Reflect:

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the terms *ratio* and *equivalent* that were added to the display during the lesson.

Synthesize

Highlight these four main ideas to conclude the lesson:

- To double or triple a recipe, you need to double or triple the amount of each ingredient.
- Making more of a recipe results in a substance that feels the same and looks the same as the original recipe.
- A ratio that represents a recipe for one batch is *equivalent* to a ratio that represents multiple batches of the same recipe.
- Any two ratios that are *equivalent* share the same relationship between the values of the two quantities, and this relationship can always be described by multiplication or division.

Formalize vocabulary:

- ***ratio***
- ***equivalent***

Ask:

- "When doubling a recipe, how does the amount of each individual ingredient change?" Each ingredient is doubled. The new ratio of ingredients is called an equivalent ratio.
- "When tripling a recipe, how does the amount of each individual ingredient change?" Each ingredient is tripled. The new ratio of ingredients is called an equivalent ratio.
- "How do different numbers of batches of the same recipe feel or look? Why?" Sample response: They feel or look exactly the same because there is always the same relative amounts of each quantity, even when the actual amounts are greater.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

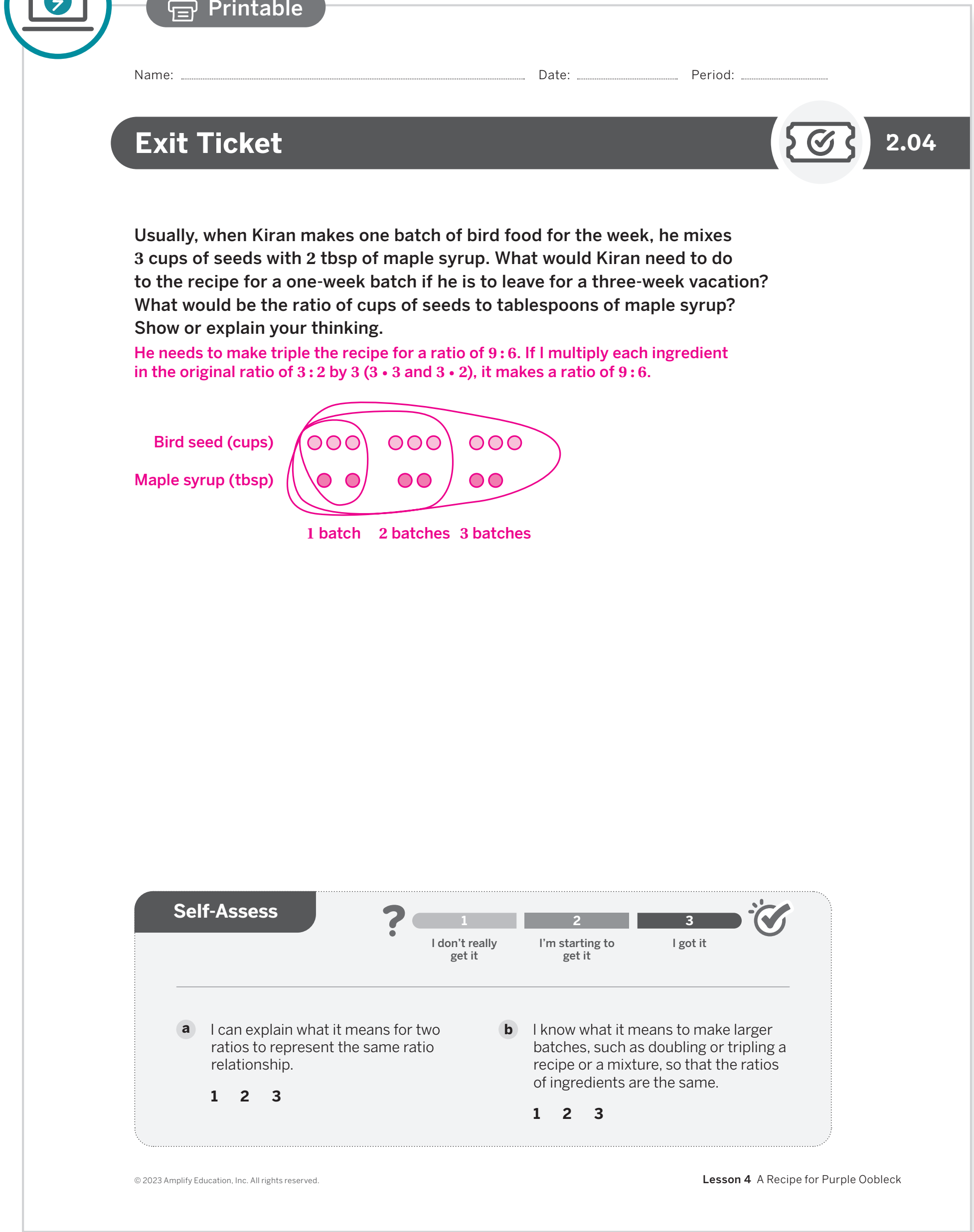
- "What strategies or tools did you find helpful today when finding all of the possible outcomes of the Oobleck experiment? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding that tripling a batch of bird food means tripling each ingredient represented in the ratio of the recipe.

Printable



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Exit Ticket 2.04

Usually, when Kiran makes one batch of bird food for the week, he mixes 3 cups of seeds with 2 tbsp of maple syrup. What would Kiran need to do to the recipe for a one-week batch if he is to leave for a three-week vacation? What would be the ratio of cups of seeds to tablespoons of maple syrup? Show or explain your thinking.

He needs to make triple the recipe for a ratio of 9 : 6. If I multiply each ingredient in the original ratio of 3 : 2 by 3 (3 • 3 and 3 • 2), it makes a ratio of 9 : 6.

Bird seed (cups)
Maple syrup (tbsp)
1 batch
2 batches
3 batches

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can explain what it means for two ratios to represent the same ratio relationship.
1 2 3

b I know what it means to make larger batches, such as doubling or tripling a recipe or a mixture, so that the ratios of ingredients are the same.
1 2 3

Lesson 4 A Recipe for Purple Oobleck

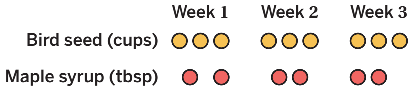
Success looks like . . .

- **Goal:** Drawing and labeling a discrete diagram with groupings (e.g., circled, divided, spaced apart) to represent multiple batches of a recipe.
  - » Drawing and labeling a correct diagram and using circles to represent multiple batches.
- **Goal:** Understanding that doubling or tripling a recipe involves multiplying the amount of each ingredient by the same number, yielding something that feels or looks the same.
- **Language Goal:** Explaining ratios that are equivalent in terms of different-sized batches of the same recipe have the same consistency or color. **(Speaking and Listening, Writing)**

Suggested next steps

If students use repeated addition to solve, consider:

- Reviewing multiplicative strategies from Activity 2:
  - » If students do not label their diagram, Ask, "How do I know which is seeds and which is maple syrup?"
  - » If students did not connect "three weeks" to tripling the ratio, consider making a diagram that shows the amounts needed by weeks (instead of batches).



If students reverse the order of the ratio, consider:

- Referring to Lesson 2 and having them write the ratio as a complete sentence using one of the sentence structures:
  - » There are ___ for every ___.
  - » The ratio of ___ to ___ is ___ to ___.
  - » The ratio of ___ to ___ is ___ : ___.

Professional Learning

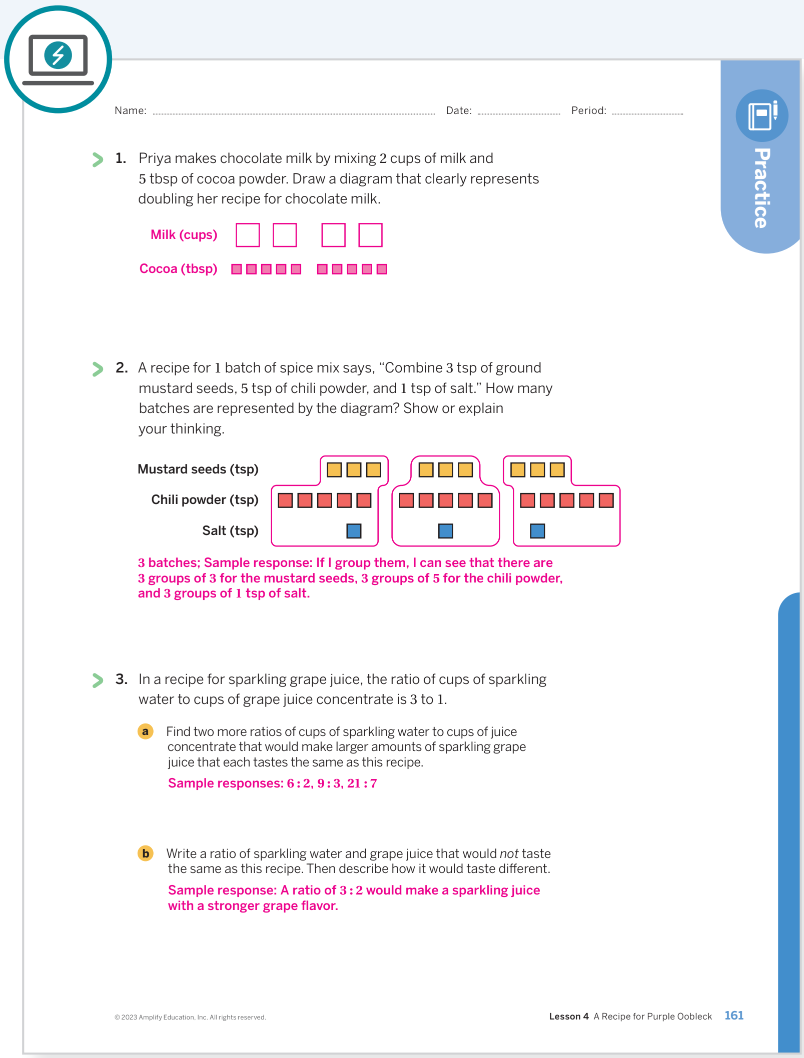
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which teacher actions made understanding that abstract representations are more efficient clear to your students?
- If you made oobleck today in class, how did it go? What might you change the next time you teach this lesson? If you did not make the oobleck today, what added benefit could it provide the next time you teach this lesson?

Practice

Independent


Practice

Name: Date: Period:

1. Priya makes chocolate milk by mixing 2 cups of milk and 5 tbsp of cocoa powder. Draw a diagram that clearly represents doubling her recipe for chocolate milk.

2. A recipe for 1 batch of spice mix says, “Combine 3 tsp of ground mustard seeds, 5 tsp of chili powder, and 1 tsp of salt.” How many batches are represented by the diagram? Show or explain your thinking.

3 batches; Sample response: If I group them, I can see that there are 3 groups of 3 for the mustard seeds, 3 groups of 5 for the chili powder, and 3 groups of 1 tsp of salt.

3. In a recipe for sparkling grape juice, the ratio of cups of sparkling water to cups of grape juice concentrate is 3 to 1.

   a. Find two more ratios of cups of sparkling water to cups of juice concentrate that would make larger amounts of sparkling grape juice that each tastes the same as this recipe.

   Sample responses: 6 : 2, 9 : 3, 21 : 7

   b. Write a ratio of sparkling water and grape juice that would *not* taste the same as this recipe. Then describe how it would taste different.

   Sample response: A ratio of 3 : 2 would make a sparkling juice with a stronger grape flavor.

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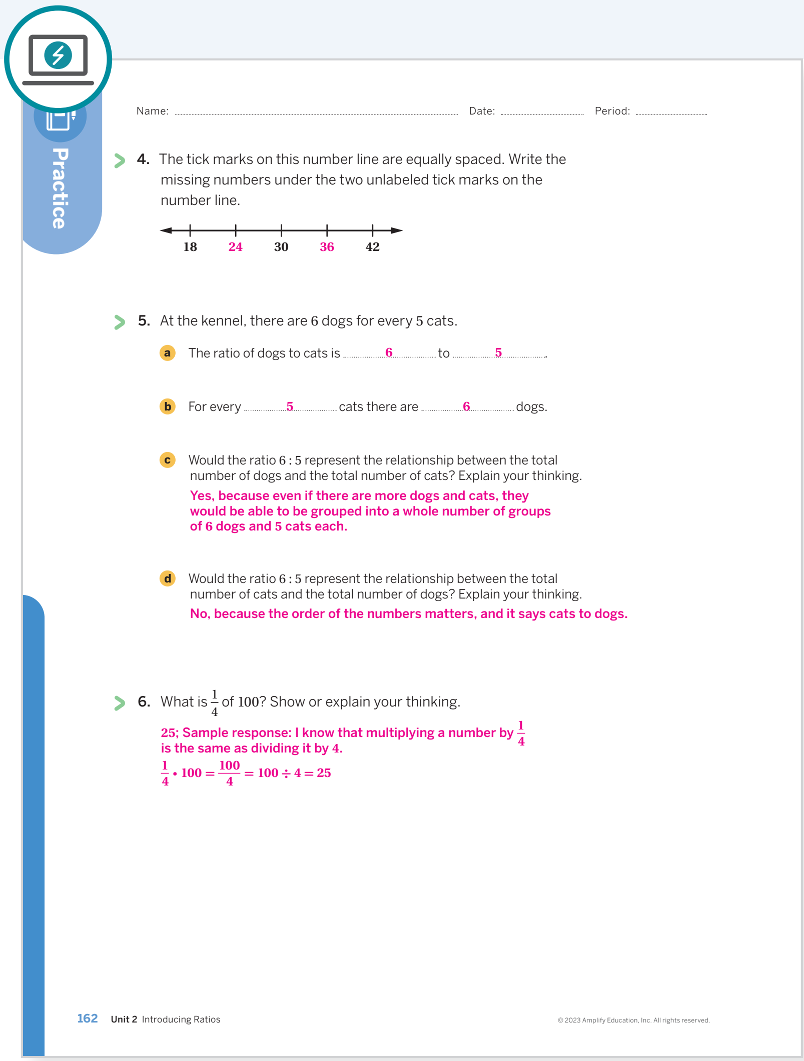
Lesson 4 A Recipe for Purple Oobleck 161


Practice

Name: Date: Period:

4. The tick marks on this number line are equally spaced. Write the missing numbers under the two unlabeled tick marks on the number line.

5. At the kennel, there are 6 dogs for every 5 cats.

   a. The ratio of dogs to cats is **6** to **5**.

   b. For every **5** cats there are **6** dogs.

   c. Would the ratio 6 : 5 represent the relationship between the total number of dogs and the total number of cats? Explain your thinking.

   Yes, because even if there are more dogs and cats, they would be able to be grouped into a whole number of groups of 6 dogs and 5 cats each.

   d. Would the ratio 6 : 5 represent the relationship between the total number of cats and the total number of dogs? Explain your thinking.

   No, because the order of the numbers matters, and it says cats to dogs.

6. What is $\frac{1}{4}$ of 100? Show or explain your thinking.

25; Sample response: I know that multiplying a number by $\frac{1}{4}$ is the same as dividing it by 4.

$\frac{1}{4} \cdot 100 = \frac{100}{4} = 100 \div 4 = 25$

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Grade 4	1
	5	Unit 2 Lesson 3	1
Formative ⚡	6	Unit 2 Lesson 5	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 2 | **LESSON 5**

Kapa Dyes

Let's see how mixing colors relates to ratios.

Focus

Goals

1. Draw and label a discrete diagram with groups to represent fewer batches of a color mixture.
2. Understand that halving or making smaller batches of a color mixture involves dividing the amount of each ingredient by the same number, yielding something that looks the same.
3. **Language Goal:** Explain ratios that are equivalent in terms of the amounts of each color in a mixture being divided by the same number to create another mixture that is the same color. **(Speaking and Listening, Writing)**

Coherence

Today

This is the second of two lessons that help students make sense of equivalent ratios through familiar contexts and physical experiences. In this lesson, the focus is on *making less*, such as scaling a recipe for a color mixture *down* to create fewer or smaller batches of dye. If the ingredients are in the same ratio, then the resulting mixtures "look" the same. Students recognize that this requires dividing the amounts of each ingredient by the same factor (e.g., halving a recipe means dividing the amount of each ingredient by 2), or coordinating division and multiplication to determine other ratios that are equivalent. Students continue to use discrete diagrams as a tool to represent ratio situations.

Previously

In Lesson 4, students defined the term *ratio* formally and began scaling up recipes by using multiplication.

Coming Soon

The next Sub-Unit will formally address equivalent ratios.

Rigor

- Students continue to build **conceptual understanding** of equivalent ratios through scaling down of recipes.
- Students continue to strengthen their **fluency** in using ratio language and notation.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Kapa Dye Color Wheel* (for display)
- counters

For the demonstrations in Activities 1–2 (optional):

- red, blue, and yellow food coloring
- water
- clear cups

Math Language Development

Review words

- *ratio*
- *equivalent*

Amps | Featured Activity

Activity 2
Mixing Colors

Students can add and remove amounts of two different colors make equivalent ratios.

Building Math Identity and Community

Connecting to Mathematical Practice

At first, students may feel lost when the pattern of decreasing a ratio by dividing is not immediately apparent. Remind students that they have drawn diagrams and seen how a large group can be divided into equal smaller groups. They also worked with multiplication and division as "opposite" operations. Consider providing manipulatives so students can recreate the groups and physically sort them to scale down.

Modifications to Pacing

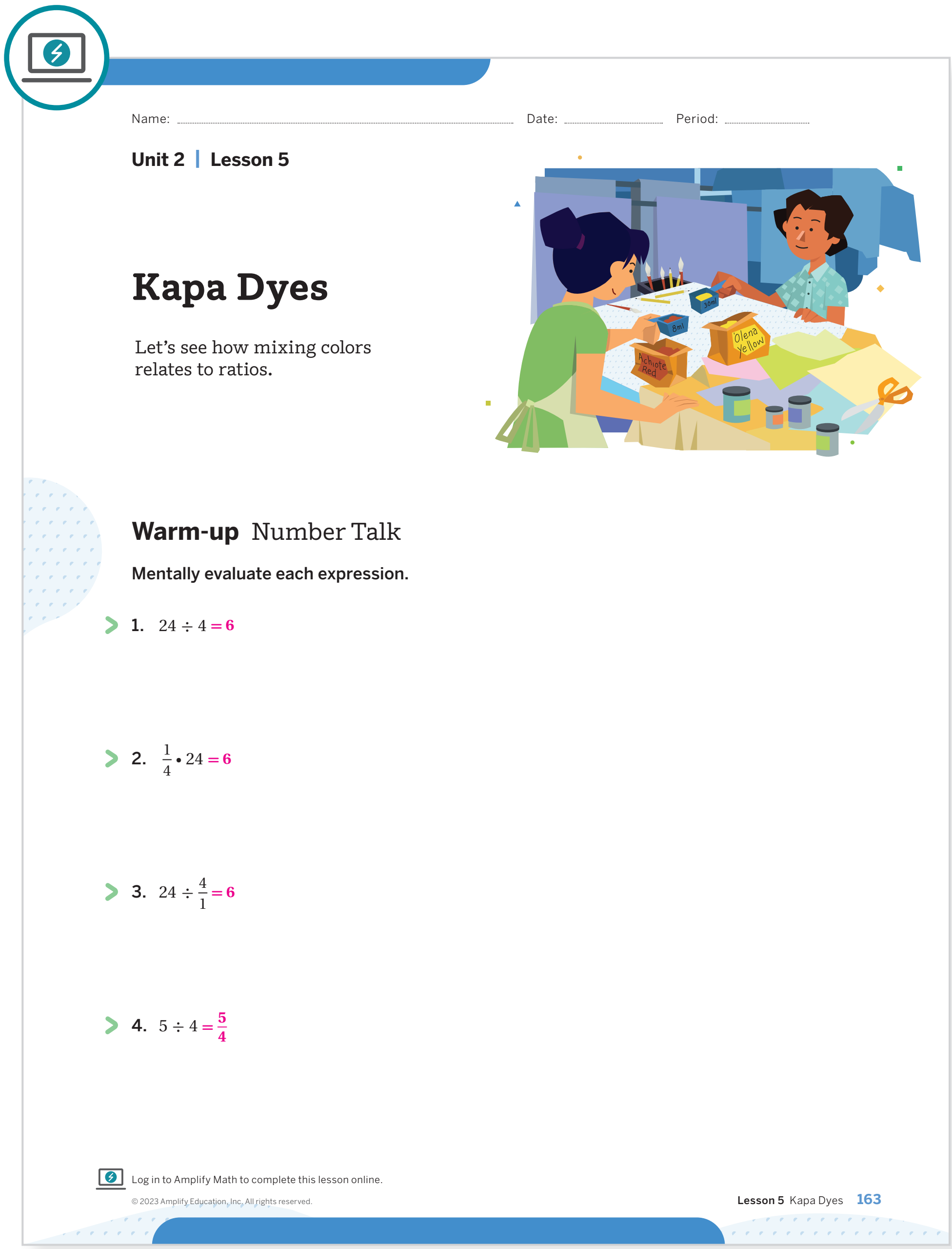
You may want to consider this additional modification if you are short on time.

- **Activity 2** may be completed as a whole class, instead of in small groups, cutting down the time needed for the social component.

Independent | 5 min

Warm-up Number Talk

Students mentally evaluate expressions to recall that dividing by a number is the same as multiplying by its reciprocal.



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Unit 2 | Lesson 5

Kapa Dyes

Let's see how mixing colors relates to ratios.

Warm-up Number Talk

Mentally evaluate each expression.

1. $24 \div 4 = 6$
2. $\frac{1}{4} \cdot 24 = 6$
3. $24 \div \frac{4}{1} = 6$
4. $5 \div 4 = \frac{5}{4}$

Log in to Amplify Math to complete this lesson online.

Lesson 5 Kapa Dyes 163

1 Launch

Display one expression at a time and have students share their mental strategy for solving. The first three problems should be given 1 minute of discussion time, allowing for more time for the fourth problem and its discussion.

2 Monitor

Help students get started by asking, "Is there a fact family for 24 and 4?"

Look for points of confusion:

- **Using a division algorithm for Problem 4.** Ask, "How is this related to the first three problems?"

Look for productive strategies:

- Connecting the multiplication and division expressions in the first three problems.
- Recognizing that Problem 4 can also be thought of as the expression $5 \cdot \frac{1}{4}$.

3 Connect

Ask:

- "What do you notice about the first three problems, other than their solutions are the same?" Problem 2 is the same as dividing by 4. Problem 3 is the same as dividing by 4 in Problem 1, but can also be seen as multiplying by the reciprocal, $\frac{1}{4}$, as in Problem 2.
- "Do you notice the same thing if we divide 5 by 4? Why?" This could be thought of as 5 times $\frac{1}{4}$, based on what I know from Problem 3; or also $\frac{5}{4}$ or $1\frac{1}{4}$.

Highlight that Problems 1–3 all have the same result of 6 because of the relationship between multiplication and division — multiplying by $\frac{1}{4}$ is the same as dividing by 4.

Math Language Development

MLR7: Compare and Connect

Have groups of students compare their strategies to mentally evaluate the expressions and make connections between the differing approaches. Encourage students to make connections between Problem 1 and 2 by asking, "How are Problems 1 and 2 related?"

Power-up

To power up students' ability to connect fractions, multiplication, and division, have students complete:

Determine which of the following expressions are equivalent to $\frac{1}{4}$ of 100. Select *all* that apply.

[✓ A.] $100 \cdot \frac{1}{4}$
B. $100 \cdot 4$
[✓ C.] $100 \div 4$
D. $100 \div \frac{1}{4}$

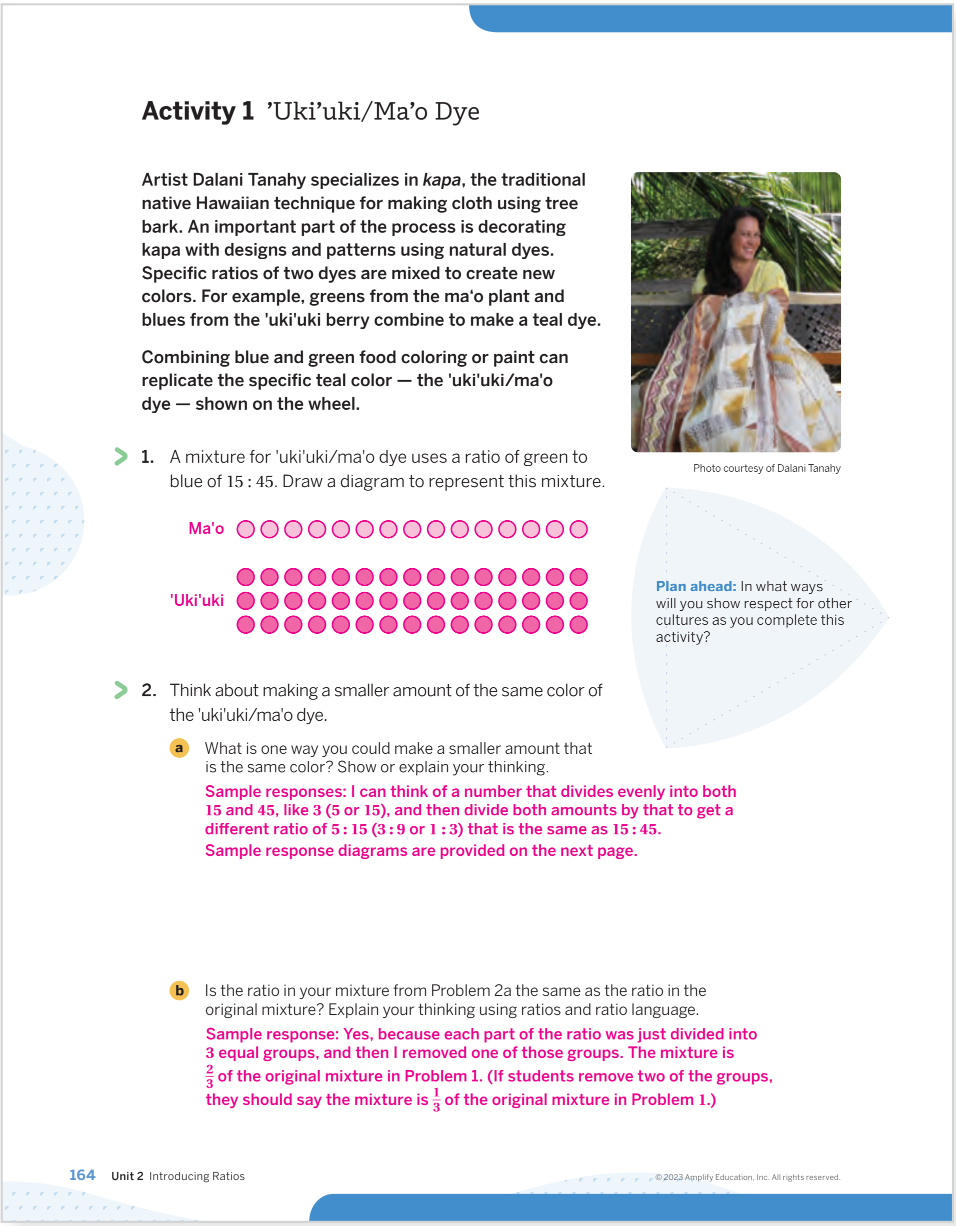
Use: Before the Warm-up.

Informed by: Performance on Lesson 2, Practice Problem 6.

Small Groups | 15 min

Activity 1 'Uki'uki/Ma'o Dye

Students work with dividing batches of dye into equal groups to build their understanding of equivalent ratios where the value of each quantity is less than a given value.



Activity 1 'Uki'uki/Ma'o Dye

Artist Dalani Tanahy specializes in *kapa*, the traditional native Hawaiian technique for making cloth using tree bark. An important part of the process is decorating kapa with designs and patterns using natural dyes. Specific ratios of two dyes are mixed to create new colors. For example, greens from the ma'o plant and blues from the 'uki'uki berry combine to make a teal dye.

Combining blue and green food coloring or paint can replicate the specific teal color — the 'uki'uki/ma'o dye — shown on the wheel.

Photo courtesy of Dalani Tanahy

1. A mixture for 'uki'uki/ma'o dye uses a ratio of green to blue of 15 : 45. Draw a diagram to represent this mixture.

Plan ahead: In what ways will you show respect for other cultures as you complete this activity?

2. Think about making a smaller amount of the same color of the 'uki'uki/ma'o dye.

   a. What is one way you could make a smaller amount that is the same color? Show or explain your thinking.

   Sample responses: I can think of a number that divides evenly into both 15 and 45, like 3 (5 or 15), and then divide both amounts by that to get a different ratio of 5 : 15 (3 : 9 or 1 : 3) that is the same as 15 : 45. Sample response diagrams are provided on the next page.

   b. Is the ratio in your mixture from Problem 2a the same as the ratio in the original mixture? Explain your thinking using ratios and ratio language.

   Sample response: Yes, because each part of the ratio was just divided into 3 equal groups, and then I removed one of those groups. The mixture is $\frac{2}{3}$ of the original mixture in Problem 1. (If students remove two of the groups, they should say the mixture is $\frac{1}{3}$ of the original mixture in Problem 1.)

164 **Unit 2** Introducing Ratios

1 Launch

Read aloud the opening paragraphs and display the Activity 1 PDF, *Kapa Dye Color Wheel*.

Note: Including an experiment or demonstration by using food coloring or paint is recommended.

2 Monitor

Help students get started by asking, "What jumps out at you when you see the numbers 15 and 45?"

Look for points of confusion:

- **Subtracting instead of dividing to make smaller amounts.** Explain that while subtraction and division are related because they reduce amounts, division makes equal groups.
- **Not labeling their diagram clearly.** Ask, "How could you show exactly what the diagram is representing?"

Look for productive strategies:

- Understanding that determining lesser amounts in the same ratio means dividing both quantities into the same number of equal groups.
- Using multiplication to check solutions, and connecting this to the doubling and tripling of ratios in Lesson 4.
- Connecting the original ratio and its equivalent ratios to strategies from the Warm-up.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Tools, Optimize Access to Technology*

Provide access to counters for students to physically manipulate as they partition the original ratio into equivalent ratios with smaller quantities. Alternatively, have students use the Amps slides for this activity, in which they can add and remove amounts of two different colors to see the resulting hues of a third color.

Extension: *Math Enrichment*

Have students think about ratios that could make a bluer, more ʻukiʻuki dye, and draw a diagram representing the ratio. Sample responses: 10 : 15 or 5 : 10 Then have them explain how they know this will result in a bluer hue.

Math Language Development

MLR8: Discussion Supports—Press for Reasoning

Use a ***Jigsaw*** routine and have members from each group ask questions of other groups. Provide question prompts such as:

- "How can you be sure this will make the same color?"
- "To make a smaller amount with the same color, can you explain what I should do?"

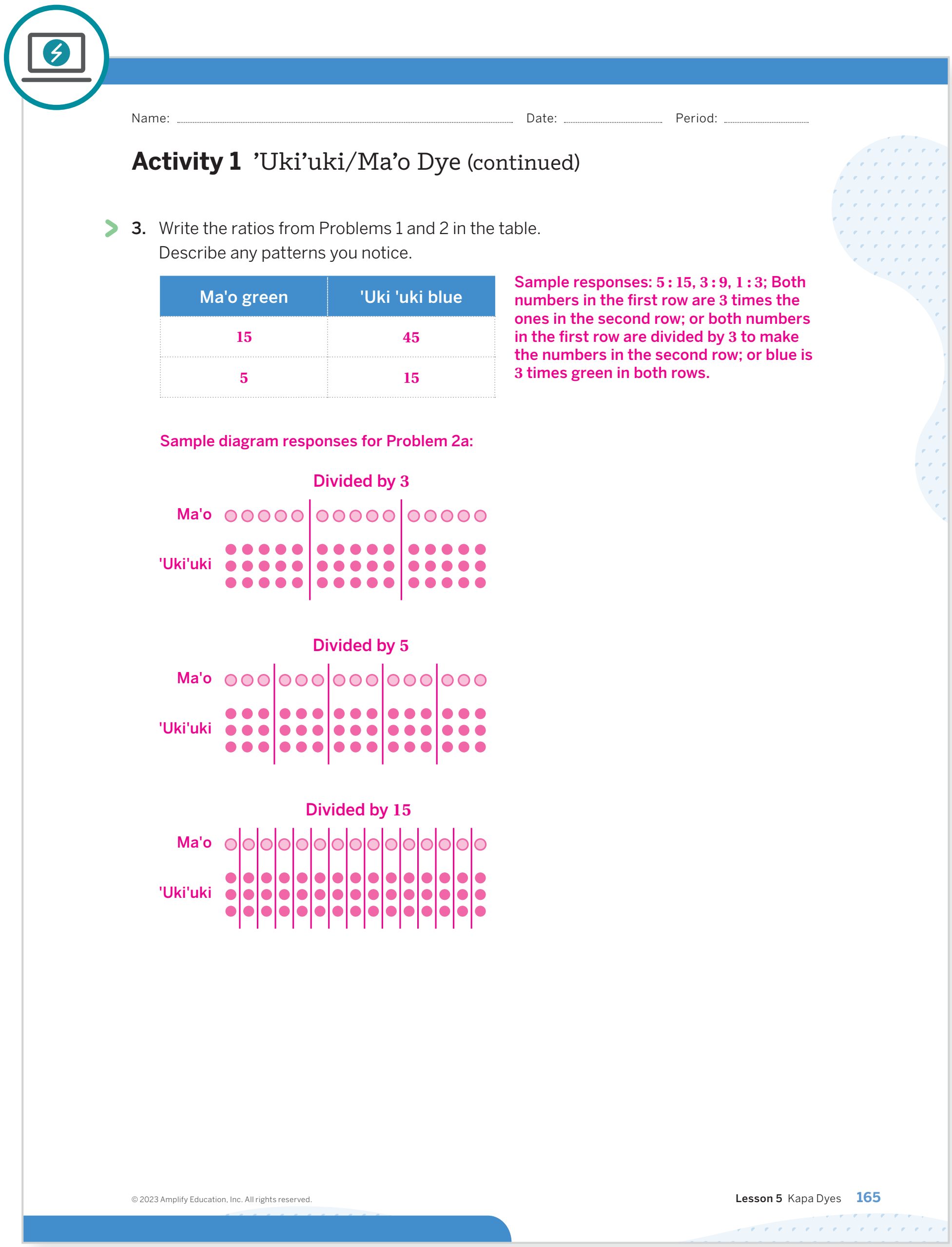
English Learners

Encourage students to use diagrams as they reason through the provided questions.

Small Groups | 15 min

Activity 1 ’Uki’uki/Ma’o Dye (continued)

Students work with dividing batches of dye into equal groups to build their understanding of equivalent ratios where the value of each quantity is less than a given value.



Name: ________ Date: ________ Period: ________

Activity 1 ’Uki’uki/Ma’o Dye (continued)

3. Write the ratios from Problems 1 and 2 in the table.
Describe any patterns you notice.

Ma'o green	'Uki 'uki blue
15	45
5	15

Sample responses: 5 : 15, 3 : 9, 1 : 3; Both numbers in the first row are 3 times the ones in the second row; or both numbers in the first row are divided by 3 to make the numbers in the second row; or blue is 3 times green in both rows.

Sample diagram responses for Problem 2a:

Divided by 3

Ma'o

'Uki'uki

Divided by 5

Ma'o

'Uki'uki

Divided by 15

Ma'o

'Uki'uki

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Lesson 5 Kapa Dyes 165

3 Connect

Display a diagram for Problem 1, and then a blank table for Problem 3. These can be drawn or projected for students as they explain/model their thinking.

Have groups of students share their responses, focusing on Problem 2.

Highlight that the original ratio of 15 : 45 can be divided by 15, 5, or 3 to make smaller equal groups using the diagram. This results in colors that are the same, and ratios that are equivalent.

Ask, “How did you know you could use 3, 5, and 15 to divide the 15 : 45 ratio?” 15 and 45 are both divisible by those numbers.

Differentiated Support

Extension: *Math Around the World*

Display or provide copies of the Activity 1 PDF, *Kapa Dye Color Wheel*. Ask students to respond to the following questions:

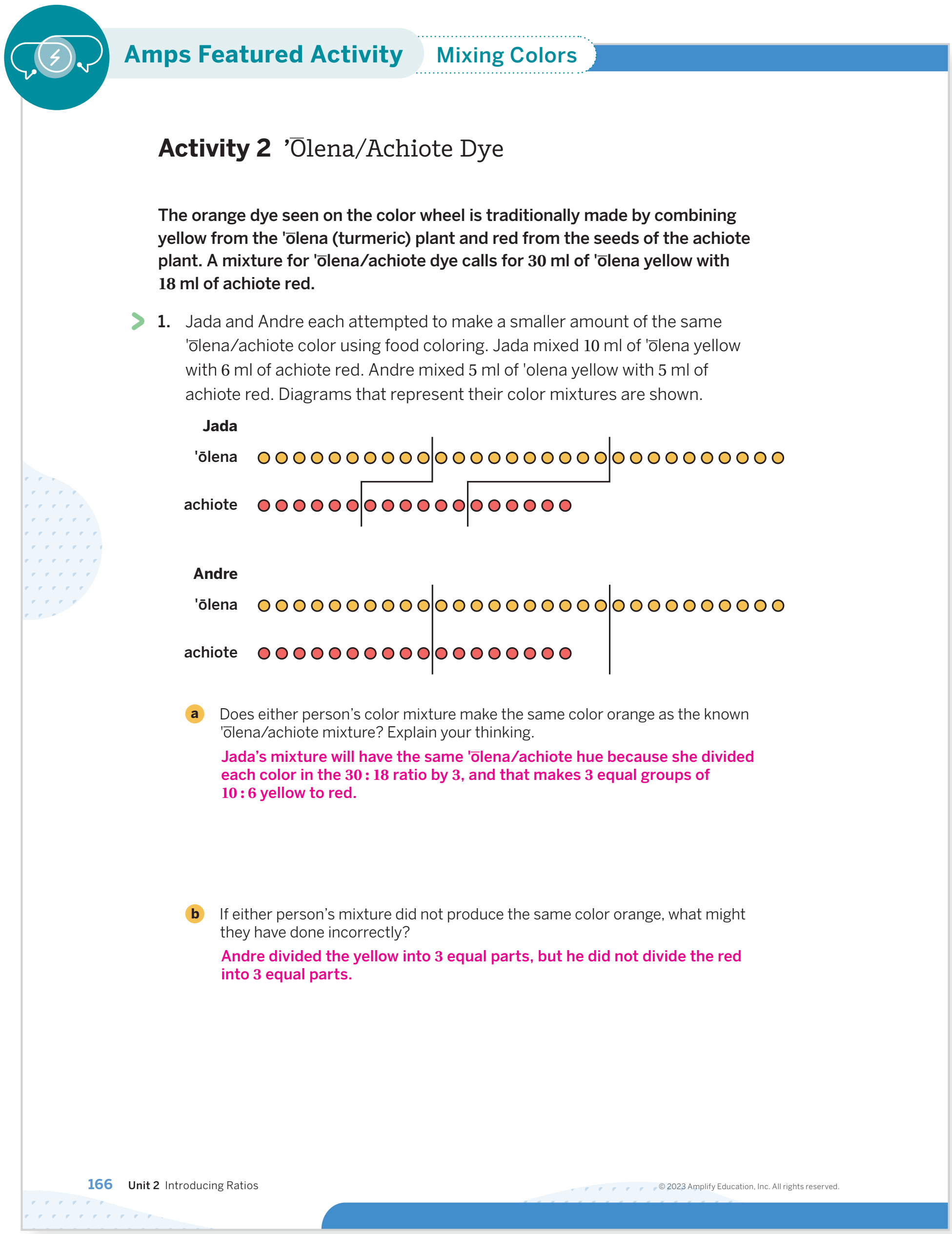
- A mixture for 'ōlena/achiote dye uses a ratio of 'ōlena to achiote of 4 : 5. Describe the ratio of 'ōlena to achiote for a mixture that has a similar color, but has more achiote than 'ōlena. Sample response: A ratio of 'ōlena to achiote of 4 : 6, or 2 : 3.
- A mixture for ma'o/'ōlena dye uses a ratio of ma'o to 'ōlena of 2 : 3. Describe the ratio of ma'o to 'ōlena for a mixture that has a similar color, but is more green. Sample response: A ratio of ma'o to 'ōlena of 3 : 3, or 1 : 1.
- A mixture for uki'uki/ma'o dye uses a ratio of ma'o to uki'uki of 5 : 6. Describe the ratio of ma'o to uki'uki for a mixture that has a similar color, but is more blue. Sample response: A ratio of ma'o to uki'uki of 5 : 7.

Small Groups | 15 min

Activity 2 ’Ōlena/Achiote Dye

Students continue working with color mixing to further their understanding of equivalent ratios with lesser values.

Amps Featured Activity Mixing Colors



Activity 2 ’Ōlena/Achiote Dye

The orange dye seen on the color wheel is traditionally made by combining yellow from the 'ōlena (turmeric) plant and red from the seeds of the achiote plant. A mixture for 'ōlena/achiote dye calls for 30 ml of 'ōlena yellow with 18 ml of achiote red.

1. Jada and Andre each attempted to make a smaller amount of the same 'ōlena/achiote color using food coloring. Jada mixed 10 ml of 'ōlena yellow with 6 ml of achiote red. Andre mixed 5 ml of 'olena yellow with 5 ml of achiote red. Diagrams that represent their color mixtures are shown.

a. Does either person’s color mixture make the same color orange as the known 'ōlena/achiote mixture? Explain your thinking.

Jada’s mixture will have the same 'ōlena/achiote hue because she divided each color in the 30 : 18 ratio by 3, and that makes 3 equal groups of 10 : 6 yellow to red.

b. If either person’s mixture did not produce the same color orange, what might they have done incorrectly?

Andre divided the yellow into 3 equal parts, but he did not divide the red into 3 equal parts.

166 **Unit 2** Introducing Ratios

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1 Launch

Display the diagrams of Jada and Andre. Give students a minute to look at and think about what each diagram represents before working on Problem 1.

2 Monitor

Help students get started by asking, “What do you notice about these two diagrams? Did each person group both of the colors the same way?”

Look for points of confusion:

- **Not dividing each part by the same amount.** Have students model their ratio by using counters and have them analyze the model. Ask, “Is each color divided by the same number? Are equal groups formed of the red? Of the yellow?”
- **Thinking neither diagram represents a ratio that would result in the same color orange.** Refer back to Activity 1. Ask, “How did you know that the same color dye would be made?” Then ask, “Does either diagram show both colors being divided the same way?”

Look for productive strategies:

- Justifying their responses by using appropriate ratio language.
- Generalizing that equivalent ratios with lesser values can be represented by smaller equal groups of each color.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Tools*

Allow students access to virtual or concrete manipulatives, such as counters, cubes, or printed representations, to help make sense of Jada’s and Andre’s diagrams.

Extension: *Math Enrichment*

Have students complete the following problem:
An orange paint was made by using different amounts of yellow and red. Some yellow was added, and then the same amount of red was added. The result was the same color orange as 30 : 18. Explain how this is possible. The first orange was not the same (e.g., 23 : 11), so adding the same amount of each (e.g., 7 ml) changed the color.

Math Language Development

MLR3: Critique, Correct, Clarify

Before students work on Problem 2, present a flawed response. Ask students to identify the error, critique the reasoning, and write a correct explanation. Listen for and amplify the language students use to justify the ratios are equivalent.

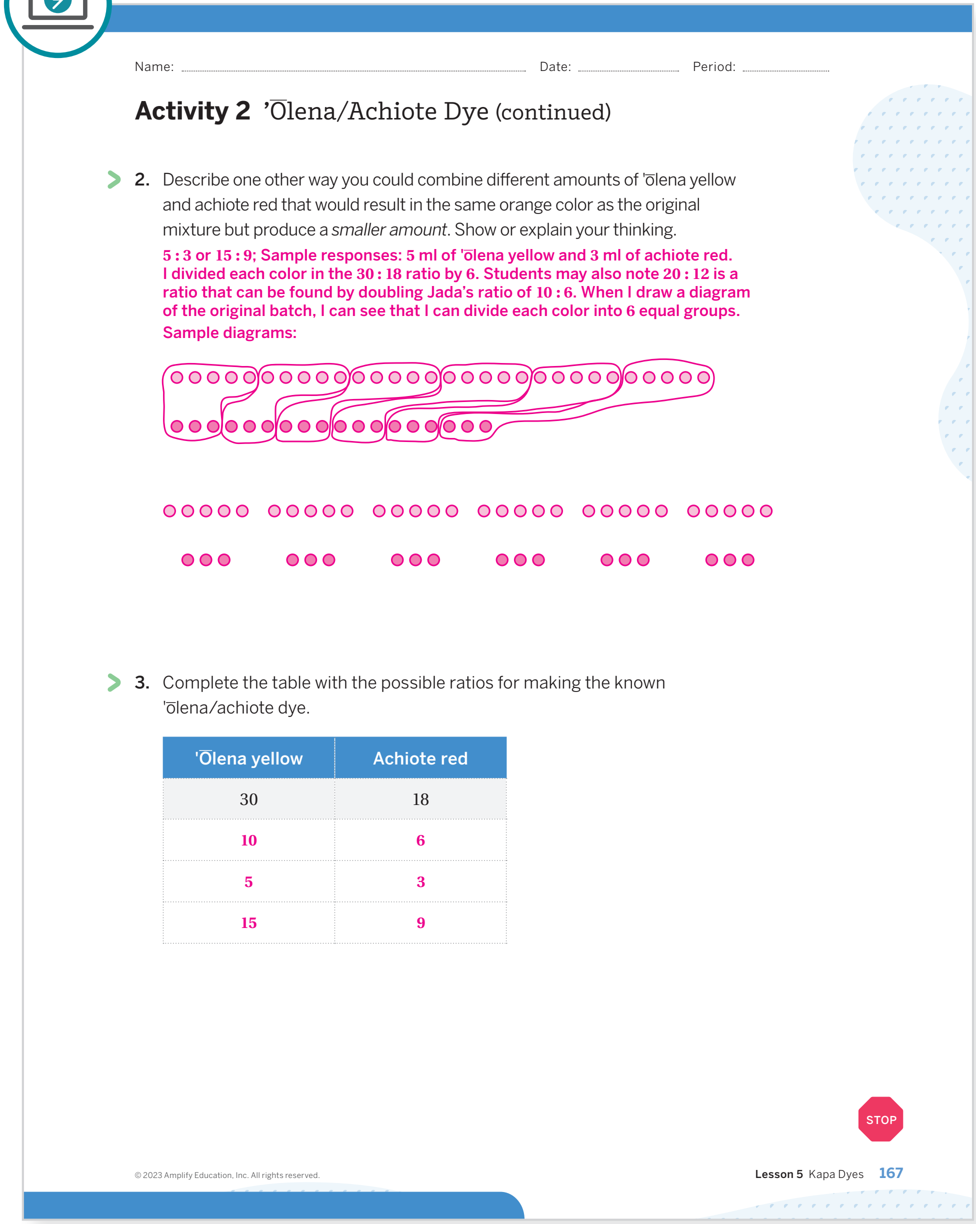
English Learners

Encourage students to use counters to model the language used for justifying that the ratios are equivalent.

Small Groups | 15 min

Activity 2 ʼŌlena/Achiote Dye (continued)

Students continue working with color mixing to further their understanding of equivalent ratios with lesser values.



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Activity 2 ʼŌlena/Achiote Dye (continued)

2. Describe one other way you could combine different amounts of ʻōlena yellow and achiote red that would result in the same orange color as the original mixture but produce a *smaller amount*. Show or explain your thinking.

   5 : 3 or 15 : 9; Sample responses: 5 ml of ʻōlena yellow and 3 ml of achiote red. I divided each color in the 30 : 18 ratio by 6. Students may also note 20 : 12 is a ratio that can be found by doubling Jada's ratio of 10 : 6. When I draw a diagram of the original batch, I can see that I can divide each color into 6 equal groups.

   Sample diagrams:

3. Complete the table with the possible ratios for making the known ʻōlena/achiote dye.

ʻŌlena yellow	Achiote red
30	18
10	**6**
5	**3**
15	**9**

STOP

3 Connect

Display the diagrams from Problem 1 and the blank table from Problem 3. These can be drawn or projected for students as they explain/model their thinking.

Have groups share their responses for Problem 1 and how they determined equivalent ratios for Problem 2. Focus on different ways equal groupings were shown.

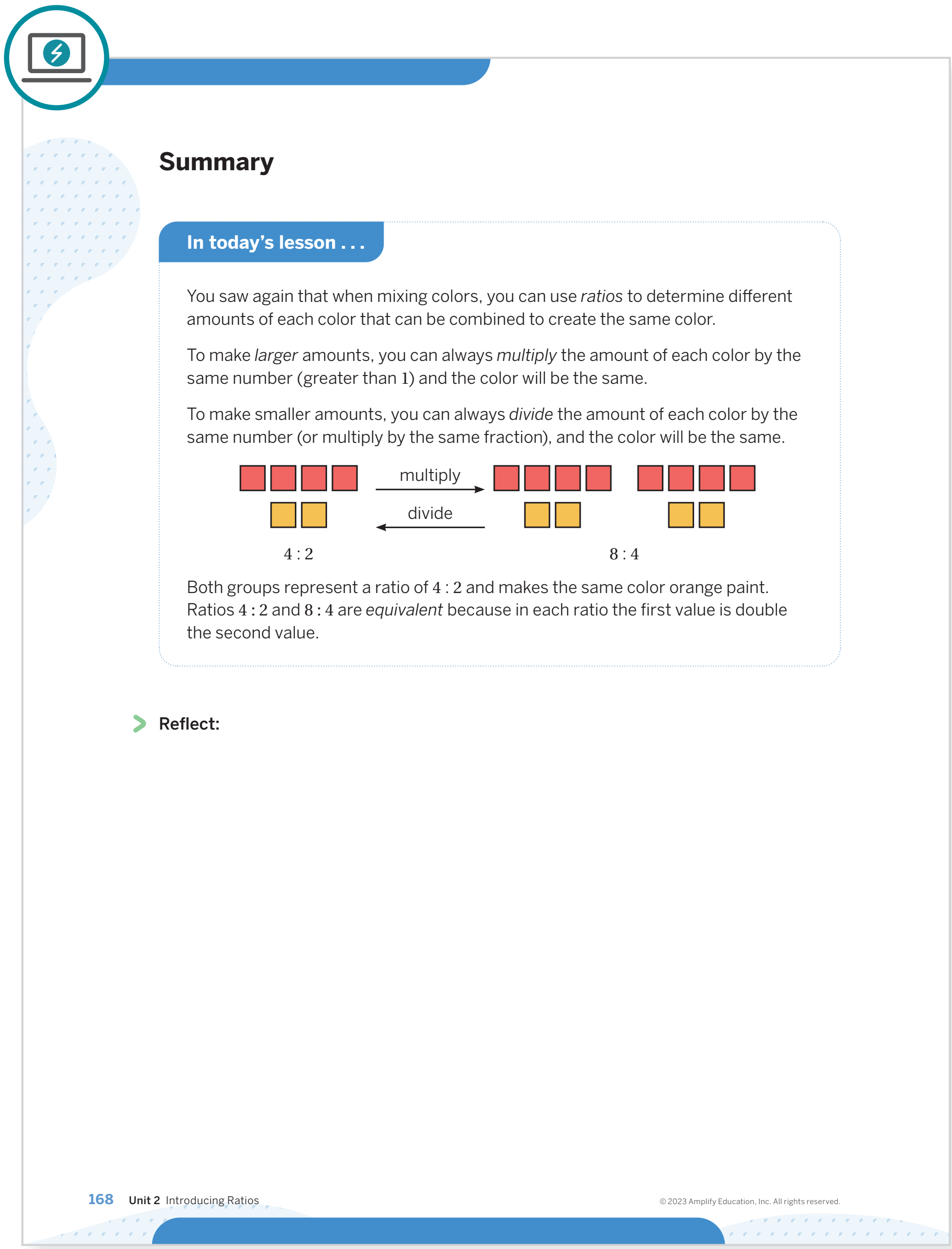
Highlight that when making equivalent ratios with lesser values, students must divide each quantity in the ratio by the *same* number.

Ask, “How would Jada know that dividing the ratio by 3 would make the same ʻōlena/achiote color? Why not 2 or 5? Why 3?” Jada knew that 30 and 9 are both divisible by 3.

Whole Class | 5 min

Summary

Review and synthesize how diagrams and coordinated division or equipartitioning can be used to determine and to verify equivalent ratios with lesser values.



Summary

In today's lesson . . .

You saw again that when mixing colors, you can use *ratios* to determine different amounts of each color that can be combined to create the same color.

To make *larger* amounts, you can always *multiply* the amount of each color by the same number (greater than 1) and the color will be the same.

To make smaller amounts, you can always *divide* the amount of each color by the same number (or multiply by the same fraction), and the color will be the same.

Both groups represent a ratio of 4 : 2 and makes the same color orange paint. Ratios 4 : 2 and 8 : 4 are *equivalent* because in each ratio the first value is double the second value.

Reflect:

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Synthesize

Display the diagram from the Summary.

Ask:

- "Would 2 : 1 be another equivalent ratio to 4 : 2?"
- "How do you know?"
- "Where do you see the ratio 2 : 1 in either of the diagrams?"
- "How would you model it on the diagram?" Models should include lines, circles, or separating by grouping.

Have students share their ideas about how 2 : 1 is also equivalent, focusing on how they use precise language to arrive at their conclusion.

Highlight that to create more batches with lesser amounts of the same color recipe, students must divide the amount of each ingredient by the same number. Say, "Similarly to what you saw in Lesson 4, you can think of equivalent ratios as representing different numbers of batches of the same recipe, but instead of multiples, (doubling, tripling, etc.) they are divided into smaller groups."

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

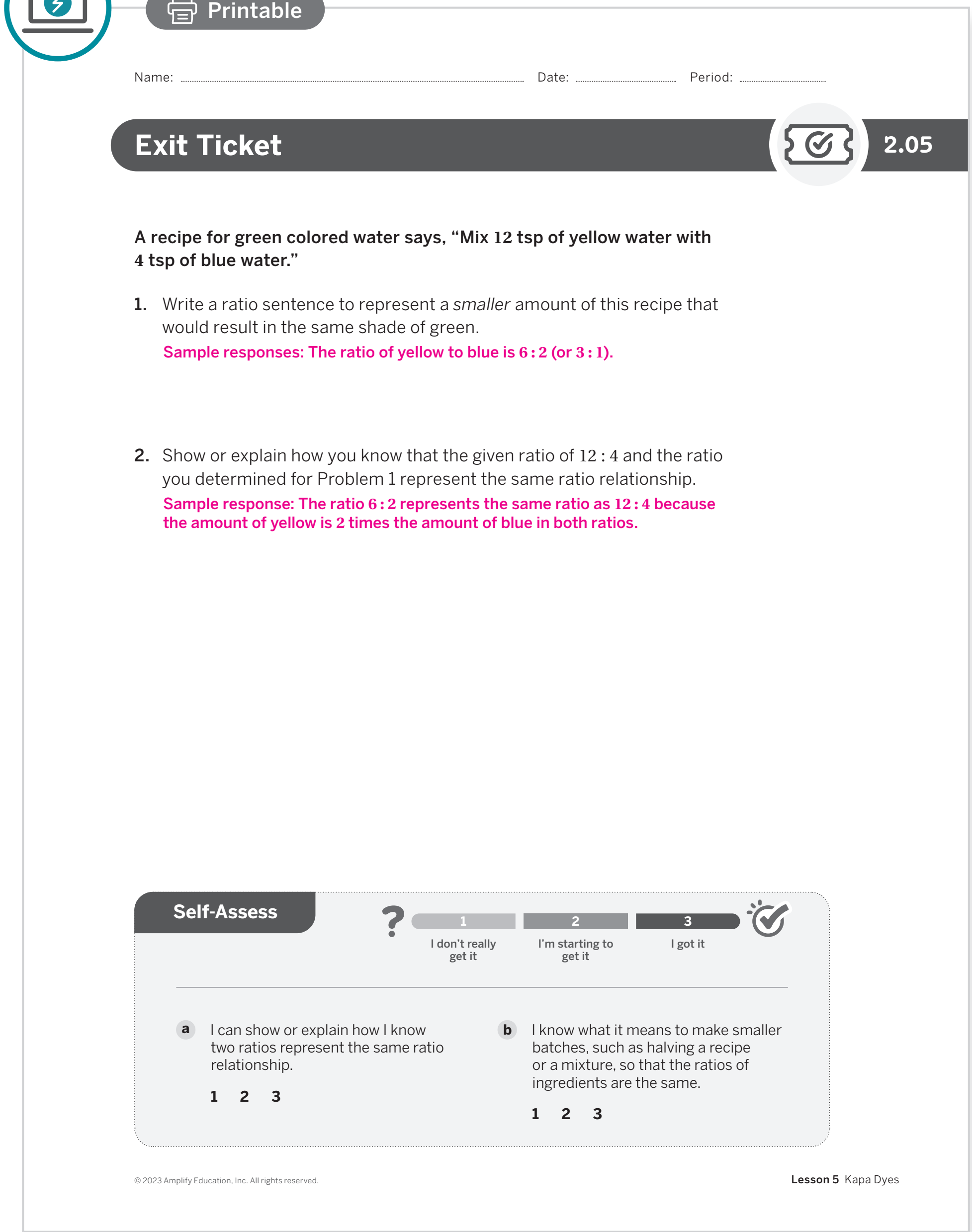
- "What did you find interesting about the ancient Hawaiian use of kapa?"
- "When did you realize you had a ratio that was equivalent to another? How did you know the ratios were equivalent?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding dividing ratios into smaller, equivalent ratios.

Printable



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Exit Ticket 2.05

A recipe for green colored water says, "Mix 12 tsp of yellow water with 4 tsp of blue water."

1. Write a ratio sentence to represent a *smaller* amount of this recipe that would result in the same shade of green.
   Sample responses: The ratio of yellow to blue is 6 : 2 (or 3 : 1).

2. Show or explain how you know that the given ratio of 12 : 4 and the ratio you determined for Problem 1 represent the same ratio relationship.
   Sample response: The ratio 6 : 2 represents the same ratio as 12 : 4 because the amount of yellow is 2 times the amount of blue in both ratios.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

- a I can show or explain how I know two ratios represent the same ratio relationship.
  1 2 3
- b I know what it means to make smaller batches, such as halving a recipe or a mixture, so that the ratios of ingredients are the same.
  1 2 3

Lesson 5 Kapa Dyes

Success looks like . . .

- **Goal:** Drawing and labeling a discrete diagram with groups to represent fewer batches of a color mixture.
- **Goal:** Understanding that halving or making smaller batches of a color mixture involves dividing the amount of each ingredient by the same number, yielding something that looks the same.
- **Language Goal:** Explaining ratios that are equivalent in terms of the amounts of each color in a mixture being divided by the same number to create another mixture that is the same color. **(Speaking and Listening, Writing)**
  - » Explaining how 12 : 4 is equivalent to another ratio in Problem 2.

Suggested next steps

If students do not use precise language in the ratio sentence, consider:

- Referring back to Lesson 2, Activity 1 and say, "Tell me one of the three ways you can write a ratio sentence."

If students cannot think of a smaller ratio, consider:

- » Suggesting that students draw a diagram to represent the 12 : 4 ratio and divide the diagram into equal groups.
- » Referring back to Activity 1, Problem 2 and asking, "How did you show dividing the 15 : 45 into equal groups?"

If students incorrectly divide the original ratio because they are dividing the amount of each ingredient by a different number, consider:

- Referring back to Andre's diagram in Activity 2 and asking, "What did Andre do that led to an incorrect equivalent ratio?"

Professional Learning

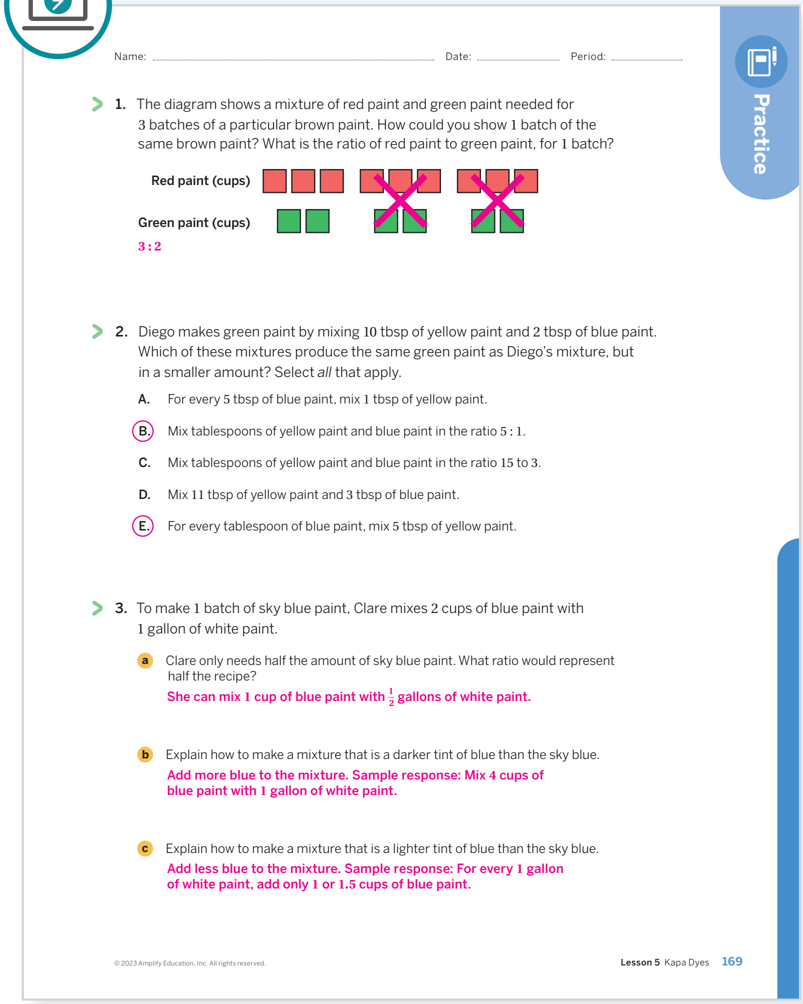
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did students find frustrating about scaling down? What helped them work through this frustration?
- What surprised you as your students compared ways of scaling down ratios? What might you change the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. The diagram shows a mixture of red paint and green paint needed for 3 batches of a particular brown paint. How could you show 1 batch of the same brown paint? What is the ratio of red paint to green paint, for 1 batch?

Red paint (cups)

Green paint (cups)

3 : 2

2. Diego makes green paint by mixing 10 tbsp of yellow paint and 2 tbsp of blue paint. Which of these mixtures produce the same green paint as Diego's mixture, but in a smaller amount? Select *all* that apply.

A. For every 5 tbsp of blue paint, mix 1 tbsp of yellow paint.

[✓ B.] Mix tablespoons of yellow paint and blue paint in the ratio 5 : 1.

C. Mix tablespoons of yellow paint and blue paint in the ratio 15 to 3.

D. Mix 11 tbsp of yellow paint and 3 tbsp of blue paint.

[✓ E.] For every tablespoon of blue paint, mix 5 tbsp of yellow paint.

3. To make 1 batch of sky blue paint, Clare mixes 2 cups of blue paint with 1 gallon of white paint.

a. Clare only needs half the amount of sky blue paint. What ratio would represent half the recipe?

She can mix 1 cup of blue paint with $\frac{1}{2}$ gallons of white paint.

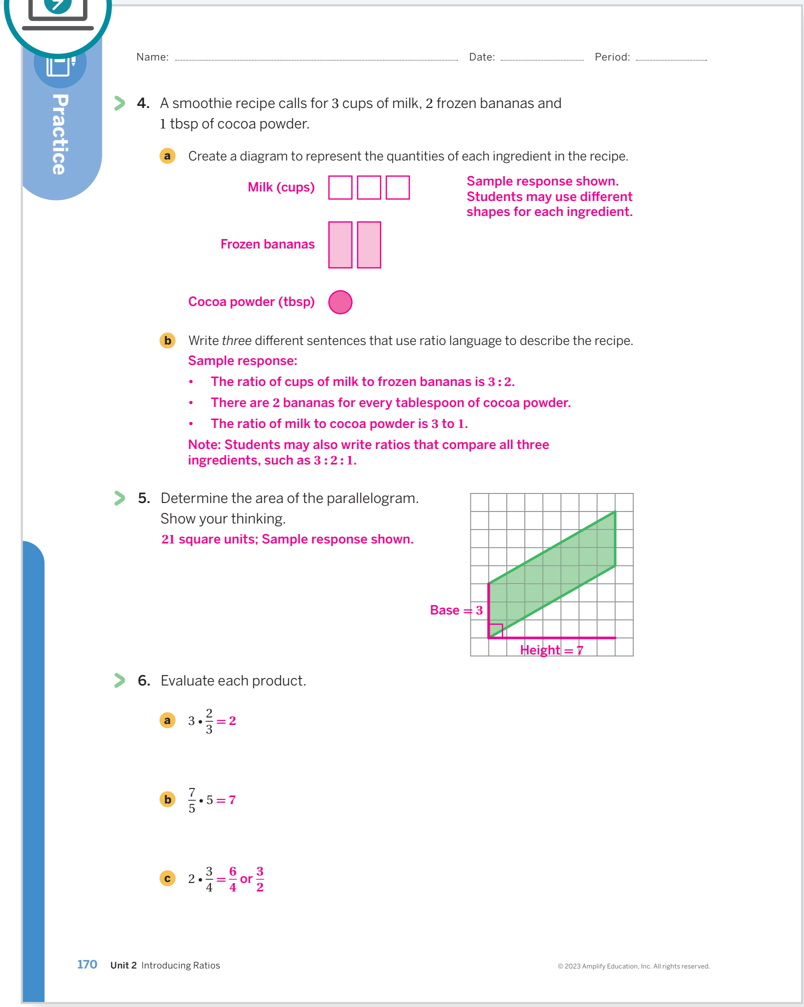
b. Explain how to make a mixture that is a darker tint of blue than the sky blue.

Add more blue to the mixture. Sample response: Mix 4 cups of blue paint with 1 gallon of white paint.

c. Explain how to make a mixture that is a lighter tint of blue than the sky blue.

Add less blue to the mixture. Sample response: For every 1 gallon of white paint, add only 1 or 1.5 cups of blue paint.

Lesson 5 Kapa Dyes 169



Name: ______ Date: ______ Period: ______

Practice

4. A smoothie recipe calls for 3 cups of milk, 2 frozen bananas and 1 tbsp of cocoa powder.

a. Create a diagram to represent the quantities of each ingredient in the recipe.

Milk (cups)

Frozen bananas

Cocoa powder (tbsp)

Sample response shown. Students may use different shapes for each ingredient.

b. Write *three* different sentences that use ratio language to describe the recipe.

Sample response:

- **The ratio of cups of milk to frozen bananas is 3 : 2.**
- **There are 2 bananas for every tablespoon of cocoa powder.**
- **The ratio of milk to cocoa powder is 3 to 1.**

Note: Students may also write ratios that compare all three ingredients, such as 3 : 2 : 1.

5. Determine the area of the parallelogram. Show your thinking.

21 square units; Sample response shown.

Base = 3

Height = 7

6. Evaluate each product.

a. $3 \cdot \frac{2}{3} = 2$

b. $\frac{7}{5} \cdot 5 = 7$

c. $2 \cdot \frac{3}{4} = \frac{6}{4}$ or $\frac{3}{2}$

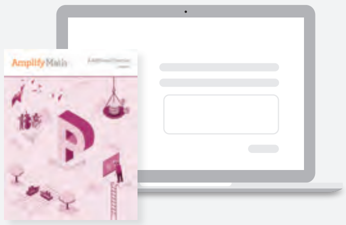
170 **Unit 2** Introducing Ratios

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 3	2
	5	Unit 1 Lesson 6	1
Formative ⚡	6	Unit 2 Lesson 6	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 2

Equivalent Ratios

Whole Class

In this Sub-Unit, students utilize greatest common factors, least common multiples, and other strategies to complete tables of equivalent ratios, and also represent them using double number lines and coordinate graphs.

SUB-UNIT

2 Equivalent Ratios

Narrative Connections

How do you put your music where your mouth is?

Antoinette Clinton was just 20 years old when she took the stage in Leipzig, Germany. Better known by her stage name, Butterscotch, she was born in Sacramento, California to a musical family. Her mother was a piano teacher. Her siblings played trumpet, cello, clarinet, and trombone. But tonight was the night of the first Beatbox Battle World Championship. She had come to showcase a different musical instrument: herself!

Beatboxing has long been a core element of hip-hop. Pioneered by artists like Doug E. Fresh, Biz Markie, and Darrell "Buffy" Robinson, performers use their mouth, throat, and nose to imitate a drum kit. MC's would then rap over their beats.

More than 20 years later, beatboxing re-emerged as an international phenomenon. In 2005, Butterscotch was crowned the first Individual Female Beatbox Battle World Champion. Two years later, she beat out 18 men to become the West Coast beatboxing champion.

To be a champion beatboxer, you need a strong sense of timing. An artist needs to know the length of each of their "hits", as well as how many "hits" they can fit into a measure of music. Ratios give performers a way to conceptualize and map those hits so that they never miss a beat.

Sub-Unit 2 Equivalent Ratios 171

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore ratios within rhythm and music in the following places:

- **Lesson 6, Activity 1:** Clapping a Rhythm
- **Lesson 12, Activity 1:** A Larger Orchestra
- **Lesson 13, Activity 1:** Song Tempos

Fostering Diverse Thinking

Play part of a Butterscotch performance for your class. Butterscotch describes her mission as "empowering and elevating people through music and compassion." Ask:

- Where do you hear ratios in Butterscotch's beatboxing?
- How do you think artists can use their music to help make a difference in society?

UNIT 2 | **LESSON 6**

Defining Equivalent Ratios

Let's investigate equivalent ratios.

Focus

Goals

1. **Language Goal:** Generate equivalent ratios and justify that they are equivalent. **(Reading and Writing)**
2. **Language Goal:** Present a definition of equivalent ratios, including examples and non-examples. **(Reading and Writing)**

Coherence

Today

Students formalize their understanding of equivalent ratios, moving away from concrete representations to more abstract thinking, both in the context of music and in working simply with ratios of numbers. They understand and articulate the relationship "between" all ratios that are equivalent to $a : b$ as those ratios that can be generated by multiplying or dividing both a and b by the same number. Students also use a ratio box to represent equivalent ratios and to help generate equivalent ratios, noticing that there is also a constant multiplicative relationship (related by the same factor) "within" the values for both quantities in a set of equivalent ratios.

Previously

In Lessons 4 and 5, students worked with equivalent ratios informally as they determined how to make larger and smaller batches of recipes for oobleck and color mixtures that would preserve an attribute of the results, such as the texture or the hue of the color.

Coming Soon

In Lesson 7, students extend the ratio box representation to create tables of equivalent ratios, which can be used to organize information and as a flexible tool for solving problems involving ratios.

Rigor

- Students build **conceptual understanding** of equivalent ratios.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Independent	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (for display)
- Activity 2 PDF, *Ratio Boxes* (for display)

Math Language Development

New word

- ***equivalent ratios***

Review word

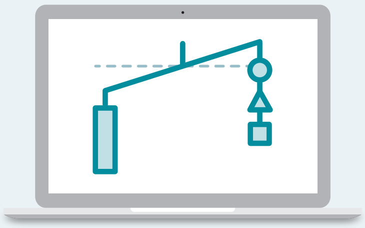
- *equivalent*

Amps | **Featured Activity**

Activity 2

Interactive Ratio Boxes

Students will use interactive ratio boxes to determine whether ratios are equivalent.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may become frustrated if they are unable to communicate a clear and precise description of the information requested in Activity 2. Encourage their partner to ask the student clarifying questions or to reword their request. Their partner can then repeat the request in their own words to make sure that both students have the same understanding of the request.

Modifications to Pacing

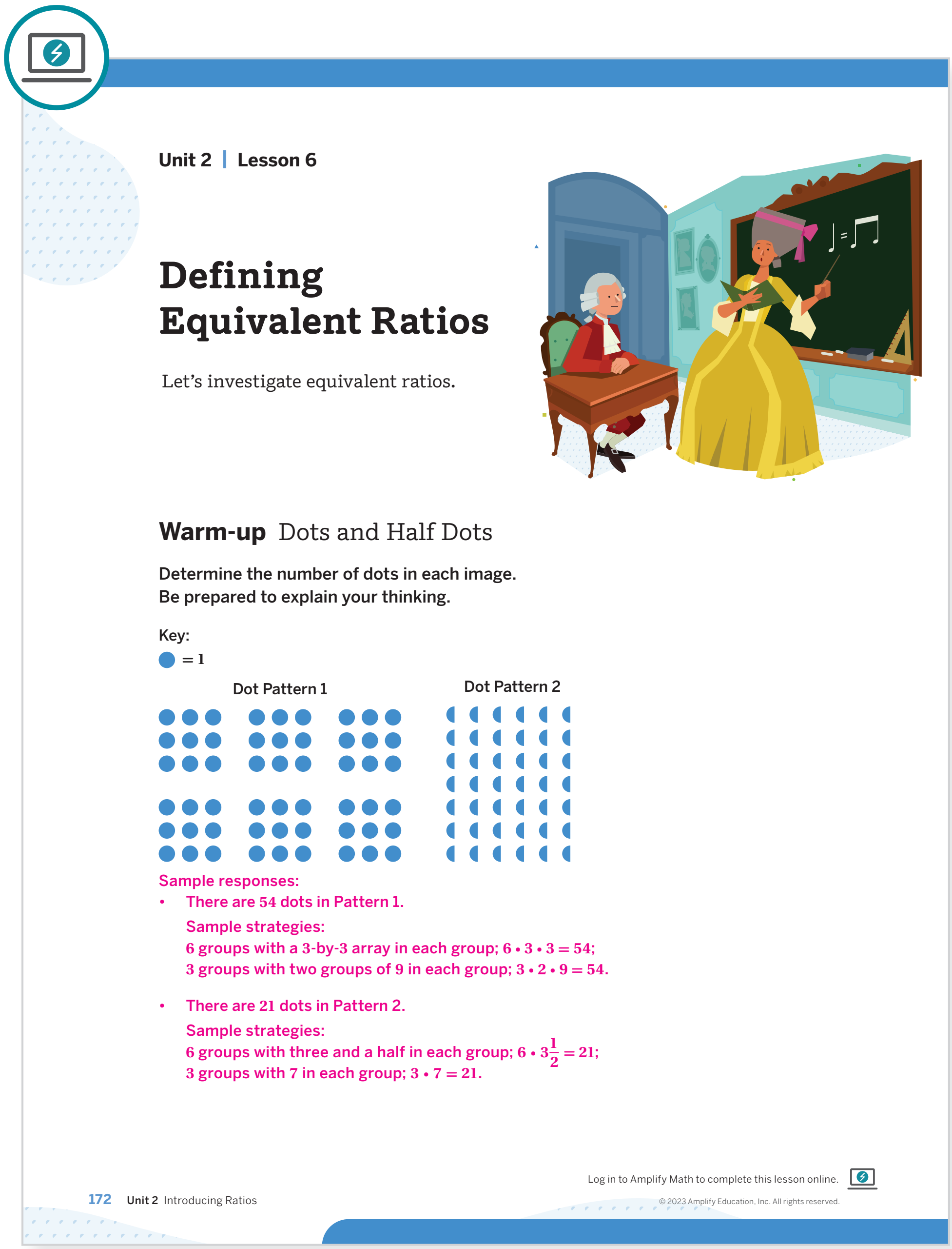
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 2 may be omitted.
- In **Activity 1**, you could skip the participatory demonstration and answer Problems 1 and 2 as a class.

Independent | 10 min

Warm-up Dots and Half Dots

Students visualize and articulate different ways to mentally calculate totals in arrays of dots and half-dots using equal groups, which are informal equivalent ratios.



Unit 2 | **Lesson 6**

Defining Equivalent Ratios

Let's investigate equivalent ratios.

Warm-up Dots and Half Dots

Determine the number of dots in each image. Be prepared to explain your thinking.

Key:

● $= 1$

Dot Pattern 1

Dot Pattern 2

Sample responses:

- **There are 54 dots in Pattern 1.**

  Sample strategies:

  6 groups with a 3-by-3 array in each group; $6 \cdot 3 \cdot 3 = 54$;
  3 groups with two groups of 9 in each group; $3 \cdot 2 \cdot 9 = 54$.

- **There are 21 dots in Pattern 2.**

  Sample strategies:

  6 groups with three and a half in each group; $6 \cdot 3\frac{1}{2} = 21$;
  3 groups with 7 in each group; $3 \cdot 7 = 21$.

Log in to Amplify Math to complete this lesson online.

172 **Unit 2** Introducing Ratios

1 Launch

Say, "I will show you an image for a few seconds and you need to determine mentally how many total dots are in the pattern." Display the first image from the Warm-up PDF for 3 seconds and then hide it. Give students several more seconds to think before displaying the same image for 3 seconds again. Repeat for the second image.

2 Monitor

Help students get started by suggesting they look at how the dots are arranged, trying to find groups to determine a total without counting.

Look for points of confusion:

- **Thinking that they need to count all of the dots.** Remind students that they will not have time to count and should look for a pattern.

Look for productive strategies:

- Noticing that there are 6 groups of 9 in the first pattern.
- Noticing that there are 6 groups of $3\frac{1}{2}$ in the second pattern.
- Identifying ways of grouping pairs of half-dots.

3 Connect

Display each image again and discuss.

Have individual students share the total number of dots they calculated in each image and how they saw the dots as a pattern to determine a total.

Highlight that several expressions can be used to describe the groupings of dots in each of the two problems, and note how those are similar. For example, the first pattern could be described as $6 \cdot 3 \cdot 3 = 54$ or $3 \cdot 2 \cdot 9 = 54$.

Ask:

- "How do these patterns and your expressions represent the properties of multiplication?"
- "How do your groups of the dots connect to what you have learned about ratios?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

To support working memory, show the image in the Warm-up for a longer period of time, or show the image for multiple, shorter periods of time.

Power-up

To power up students' ability to multiply whole numbers and fractions, have students complete:

Recall that any whole number can be rewritten as a fraction with a denominator of 1. For example, $3 = \frac{3}{1}$. Rewrite the expression as the product of two fractions, then evaluate.

$$4 \cdot \frac{3}{8} = \frac{4}{1} \cdot \frac{3}{8}$$
$$= \frac{12}{8}$$
$$= \frac{3}{2} \text{ or } = 1\frac{1}{2}$$

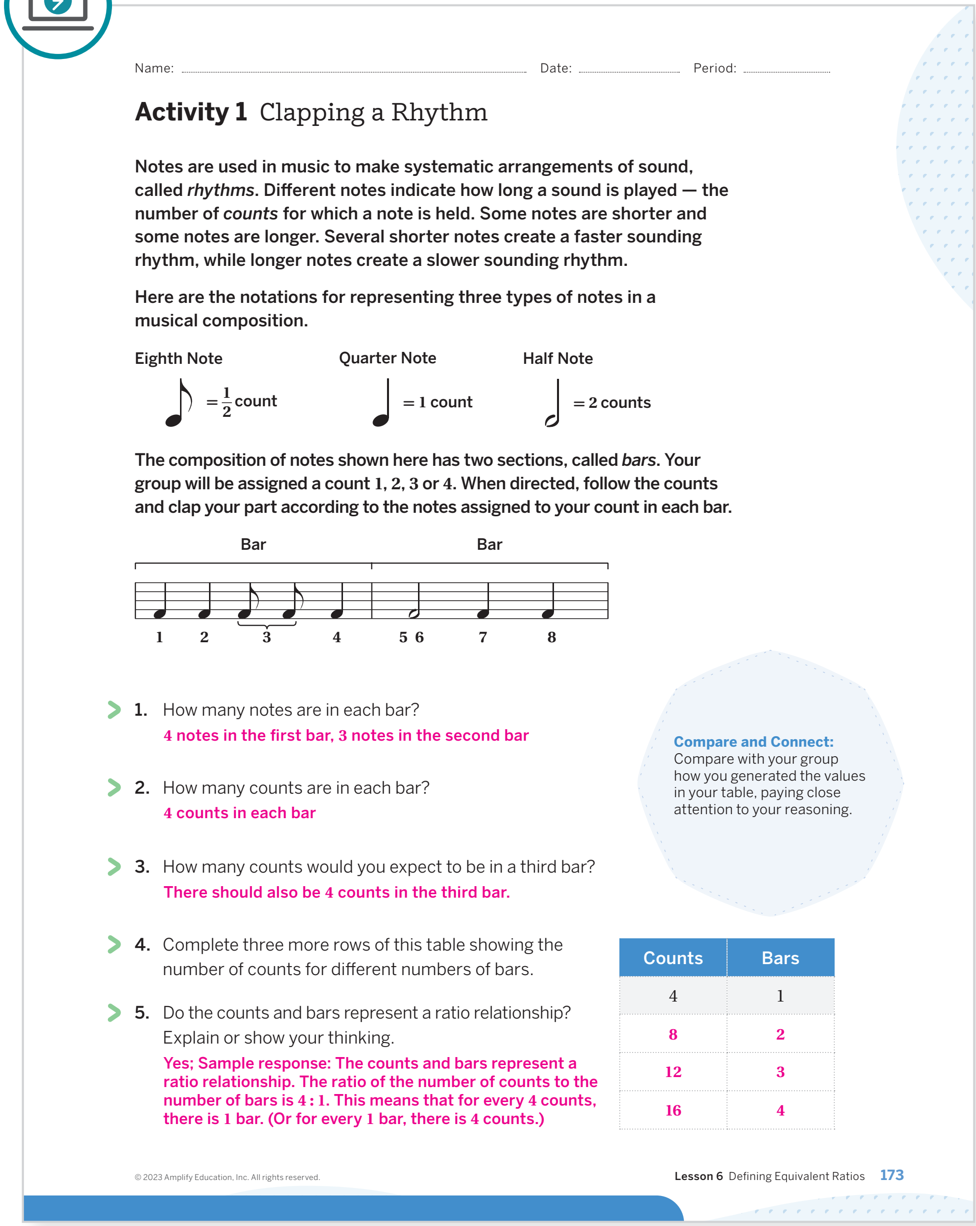
Use: Before the Warm-up.

Informed by: Performance on Lesson 5, Practice Problem 6.

Small Groups | 10 min

Activity 1 Clapping a Rhythm

Students clap a rhythm and represent counts and bars in music as ratio relationships.



Name: ______ Date: ______ Period: ______

Activity 1 Clapping a Rhythm

Notes are used in music to make systematic arrangements of sound, called *rhythms*. Different notes indicate how long a sound is played — the number of *counts* for which a note is held. Some notes are shorter and some notes are longer. Several shorter notes create a faster sounding rhythm, while longer notes create a slower sounding rhythm.

Here are the notations for representing three types of notes in a musical composition.

Eighth Note $= \frac{1}{2}$ count

Quarter Note $= 1$ count

Half Note $= 2$ counts

The composition of notes shown here has two sections, called *bars*. Your group will be assigned a count $1, 2, 3$ or 4. When directed, follow the counts and clap your part according to the notes assigned to your count in each bar.

1. How many notes are in each bar?
   4 notes in the first bar, 3 notes in the second bar

2. How many counts are in each bar?
   4 counts in each bar

3. How many counts would you expect to be in a third bar?
   There should also be 4 counts in the third bar.

4. Complete three more rows of this table showing the number of counts for different numbers of bars.

5. Do the counts and bars represent a ratio relationship? Explain or show your thinking.
   Yes; Sample response: The counts and bars represent a ratio relationship. The ratio of the number of counts to the number of bars is $4:1$. This means that for every 4 counts, there is 1 bar. (Or for every 1 bar, there is 4 counts.)

Compare and Connect: Compare with your group how you generated the values in your table, paying close attention to your reasoning.

Counts	Bars
4	1
8	2
12	3
16	4

1 Launch

Say, "I'm going to count to 4. For eighth notes, you clap twice on each count. Let's try it. 1, 2, 3, 4. 1, 2, 3, 4. For quarter notes, you clap on each count. Let's try it. 1, 2, 3, 4. 1, 2, 3, 4. Now let's do half notes. When will you clap? Let's try it. 1, 2, 3, 4."

2 Monitor

Help students get started by reminding them that an eighth note gets 2 claps in 1 count, a quarter note gets 1 clap in 1 count, and a half note gets 1 clap over 2 counts.

Look for points of confusion:

- **Thinking that there are the same number of notes in each bar because they have 4 counts.** Remind students that there are the same number of counts for each bar but not necessarily the same number of notes.

Look for productive strategies:

- Noticing that there is a pattern in the table for the counts and bars.
- Noticing that the counts are in multiples of 4, or adding 4 every row.
- Noticing that the ratio of counts to bars is $4:1$.

3 Connect

Display the completed table.

Ask, "What pattern do you notice in the counts and the bars?"

Have groups of students share how they generated their values in the table. Have students share what they notice about the patterns and the ratios by using the table.

Highlight that there is a common ratio of $4:1$ between the counts and the bars. This means that the number of counts is always equal to four times the number of bars.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate how to clap for eighth notes, quarter notes, and half notes so that students can see and hear an example for each. Then have students clap. Consider clapping with them so they can follow your lead.

Extension: *Math Enrichment*

Have students complete the following problem:
What would the rows in the table look like for 2 counts? 1 count? Explain your thinking. For every 4 counts, there is 1 bar. So, for every 2 counts, there would be $\frac{1}{2}$ bars. For every 1 count, there would be $\frac{1}{4}$ bars.

Math Language Development

MLR7: Compare and Connect

Have groups share and compare how they generated the values in the table and then discuss what patterns they notice. Encourage students to explain how they each approached completing the table. Ask, "What connections can you make between your approach and your group members' approaches?"

English Learners

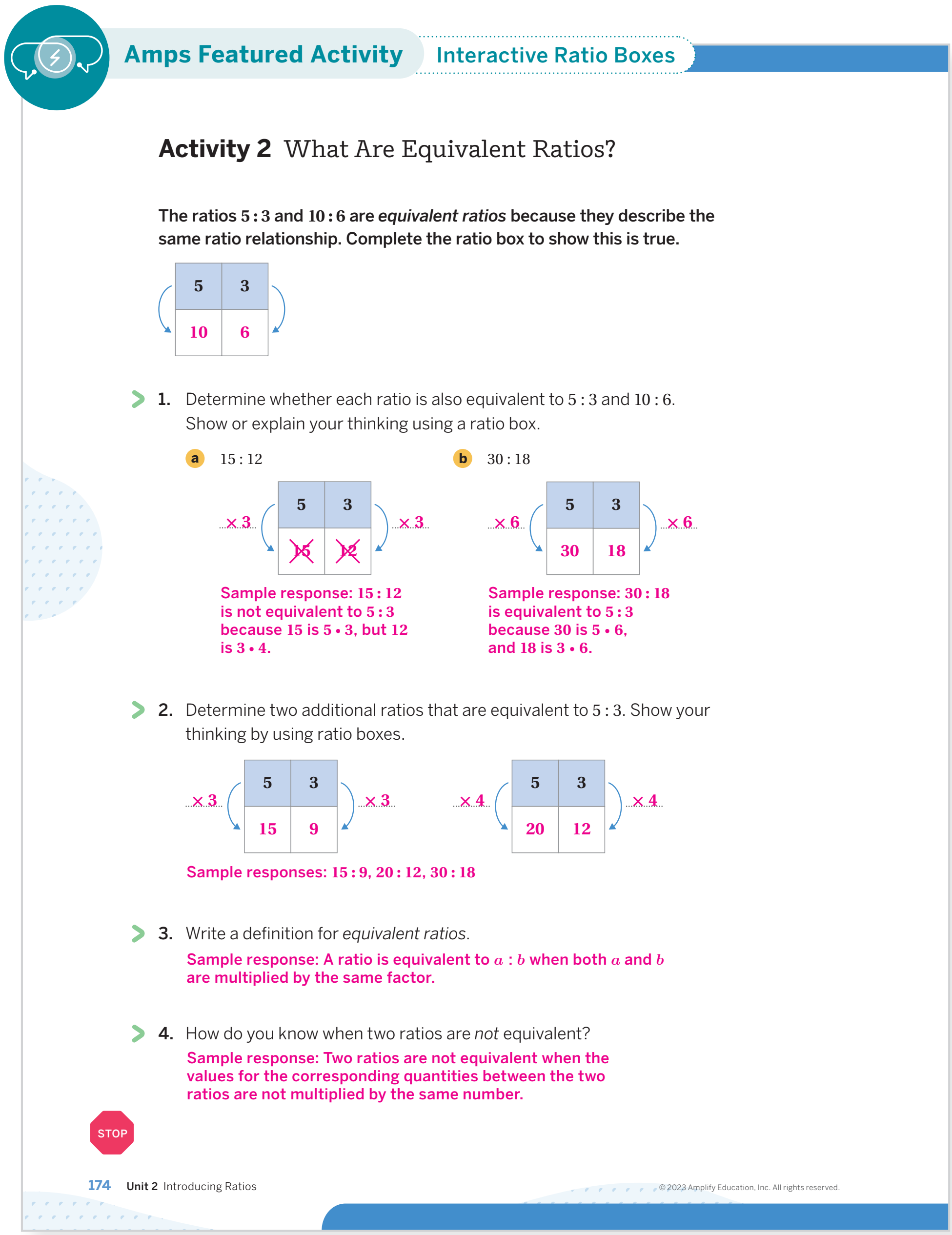
Annotate a copy of the completed table illustrating the different strategies used.

Pairs | 15 min

Activity 2 What Are Equivalent Ratios?

Students use an example of equivalent ratios to identify how corresponding values are related by multiplication or division, and they generate equivalent ratios.

Amps Featured Activity Interactive Ratio Boxes



Activity 2 What Are Equivalent Ratios?

The ratios 5 : 3 and 10 : 6 are *equivalent ratios* because they describe the same ratio relationship. Complete the ratio box to show this is true.

5	3
10	6

1. Determine whether each ratio is also equivalent to 5 : 3 and 10 : 6. Show or explain your thinking using a ratio box.

   a 15 : 12

   × 3 × 3

5	3
~~15~~	~~12~~

   Sample response: 15 : 12 is not equivalent to 5 : 3 because 15 is 5 • 3, but 12 is 3 • 4.

   b 30 : 18

   × 6 × 6

5	3
30	18

   Sample response: 30 : 18 is equivalent to 5 : 3 because 30 is 5 • 6, and 18 is 3 • 6.

2. Determine two additional ratios that are equivalent to 5 : 3. Show your thinking by using ratio boxes.

   × 3 × 3

5	3
15	9

   × 4 × 4

5	3
20	12

   Sample responses: 15 : 9, 20 : 12, 30 : 18

3. Write a definition for *equivalent ratios*.

   Sample response: A ratio is equivalent to $a : b$ when both a and b are multiplied by the same factor.

4. How do you know when two ratios are *not* equivalent?

   Sample response: Two ratios are not equivalent when the values for the corresponding quantities between the two ratios are not multiplied by the same number.

STOP

174 Unit 2 Introducing Ratios

1 Launch

Display the ratio box from the Activity 2 PDF and have students follow along as you fill in the values. Then have pairs of students complete Problems 1–4.

2 Monitor

Help students get started by asking, "Where should you write the 15 and the 12? What operation can you write along each downward arrow?"

Look for points of confusion:

- **Not recognizing they need to multiply both parts of the ratio by the same value.** Have students use the ratio box to multiply both values on the top by the same value.

Look for productive strategies:

- Recognizing that the ratio of 30 : 18 is equivalent to 5 : 3 because students can multiply by 6, but 15 : 12 is not equivalent to 5 : 3.
- Multiplying both values in the ratio 5 : 3 by the same number, such as 3, 4, or 5, to get equivalent ratios.

3 Connect

Display the completed ratio boxes.

Have groups of students share how they completed their ratios boxes and any patterns they notice. Then have students share their definition for equivalent ratios and their response to Problem 4.

Ask, "If we drew arrows going across the box, could you write an operation that connects the values that way?"

Highlight that ratio boxes can be used to perform coordinated multiplication or division between the rows, to generate or identify equivalent ratios.

Define ***equivalent ratios*** as any two ratios where the values for one quantity in each ratio can be multiplied or divided by the same number to generate the values for the other quantity.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Chunk this task into smaller, more manageable parts. Have students complete Problem 1a, and discuss before having them complete Problem 1b. Pause for discussion before moving on to Problem 2.

Extension: *Math Enrichment*

Have students complete the following problem:
Can you complete a ratio box for 5 : 3 so that the number 1 is in one of the cells? Explain your thinking. Sample response: If I multiply 3 by $\frac{1}{3}$, that gives the number 1. Then I can multiply 5 by $\frac{1}{3}$, which gives the number $\frac{5}{3}$.

Math Language Development

MLR1: Stronger and Clearer Each Time

For Problems 3 and 4, have students create a first draft and then work with a partner to share and refine their response through conversation. While meeting, listeners should ask questions such as, "What did you mean by . . . ?" Have students write a second draft of their response that reflects ideas from their partners.

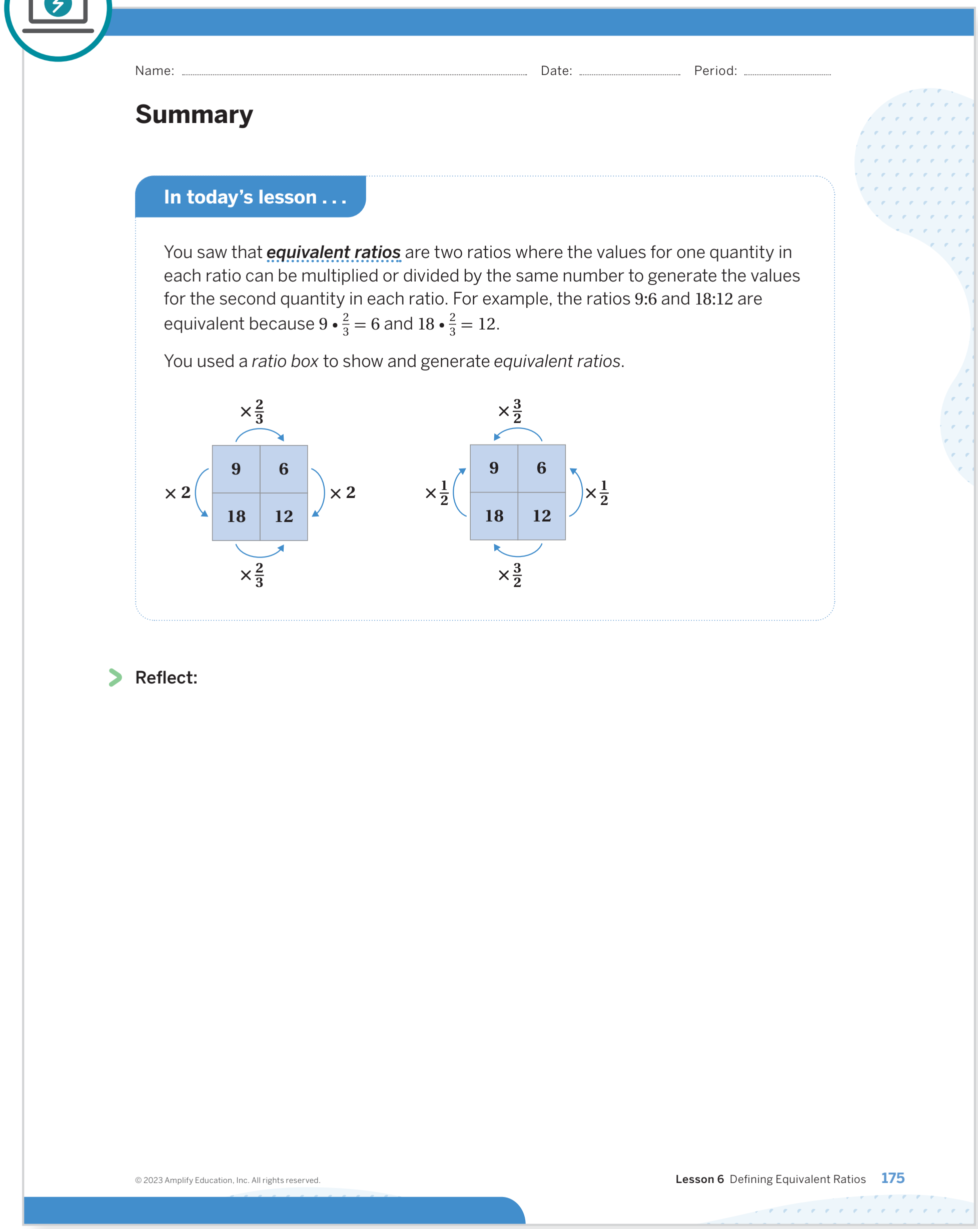
English Learners

Consider allowing students to write their first draft in the primary language before writing a second draft in English.

Whole Class | 5 min

Summary

Review and synthesize the meaning of equivalent ratios and how a ratio box can be used to verify or generate equivalent ratios to a given ratio.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that ***equivalent ratios*** are two ratios where the values for one quantity in each ratio can be multiplied or divided by the same number to generate the values for the second quantity in each ratio. For example, the ratios 9:6 and 18:12 are equivalent because $9 \bullet \frac{2}{3} = 6$ and $18 \bullet \frac{2}{3} = 12$.

You used a *ratio box* to show and generate *equivalent ratios*.

Reflect:

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Lesson 6 Defining Equivalent Ratios 175

Synthesize

Display the ratio 3 : 4.

Have students share how they would determine an equivalent ratio.

Highlight that students can multiply both 3 and 4 by the same value, such as 2 or 200. They can then divide the new ratio by that same value, or multiply by its reciprocal, to arrive at the original ratio of 3 : 4.

Formalize vocabulary: *equivalent ratios*

Ask, "If you wanted to make a larger amount of a food recipe, how would you ensure that the result would taste the same?" The amount of each ingredient in the recipe must be multiplied by the same value.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does a ratio say about the relationship between quantities?"

Math Language Development

MLR2: Collect and Display

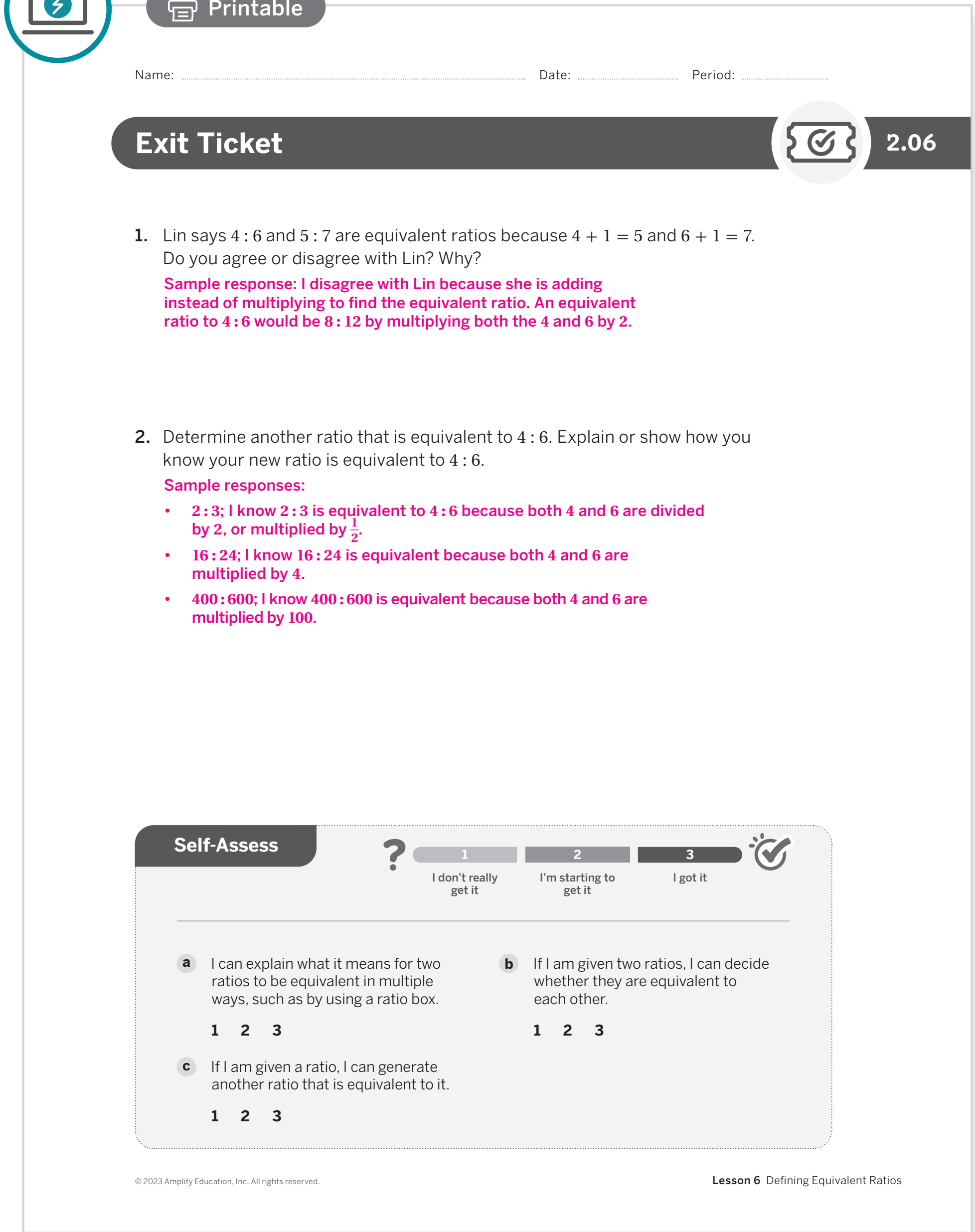
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the term *equivalent ratios* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of equivalent ratios by writing an equivalent ratio of a given example.

Printable



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Exit Ticket 2.06

1. Lin says 4 : 6 and 5 : 7 are equivalent ratios because 4 + 1 = 5 and 6 + 1 = 7. Do you agree or disagree with Lin? Why?

   Sample response: I disagree with Lin because she is adding instead of multiplying to find the equivalent ratio. An equivalent ratio to 4 : 6 would be 8 : 12 by multiplying both the 4 and 6 by 2.

2. Determine another ratio that is equivalent to 4 : 6. Explain or show how you know your new ratio is equivalent to 4 : 6.

   Sample responses:
   - **2 : 3; I know 2 : 3 is equivalent to 4 : 6 because both 4 and 6 are divided by 2, or multiplied by $\frac{1}{2}$.**
   - **16 : 24; I know 16 : 24 is equivalent because both 4 and 6 are multiplied by 4.**
   - **400 : 600; I know 400 : 600 is equivalent because both 4 and 6 are multiplied by 100.**

Self-Assess

a) I can explain what it means for two ratios to be equivalent in multiple ways, such as by using a ratio box.
1 2 3

b) If I am given two ratios, I can decide whether they are equivalent to each other.
1 2 3

c) If I am given a ratio, I can generate another ratio that is equivalent to it.
1 2 3

Lesson 6 Defining Equivalent Ratios

Success looks like . . .

- **Language Goal:** Generating equivalent ratios and justifying that they are equivalent. **(Reading and Writing)**
  - » Creating an equivalent ratio to a given ratio and explaining why the two ratios are equivalent in Problem 2.
- **Language Goal:** Presenting a definition of equivalent ratios, including examples and non-examples. **(Reading and Writing)**
  - » Explaining why Lin's ratios are not equivalent in Problem 1.

Suggested next steps

If students think they can add the same number to both quantities to make equivalent ratios, consider:

- Reviewing the ratio box in Activity 2.
- Assigning Practice Problem 1,
- Asking, "How would you solve this instead of adding both parts of the ratio?"

If students use addition to solve Problem 2, consider:

- Assigning Practice Problem 1.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The instructional goal for this lesson was to generate equivalent ratios and justify that they are equivalent. How well did students accomplish this? What did you specifically do to help students accomplish it?
- How did students communicate clear and precise descriptions of equivalent ratios today? How are you helping students become aware of how they are progressing in this area? What might you change the next time you teach this lesson?

Math Language Development

Language Goal: Generating equivalent ratios and justifying that they are equivalent.

Reflect on students' language development toward this goal.

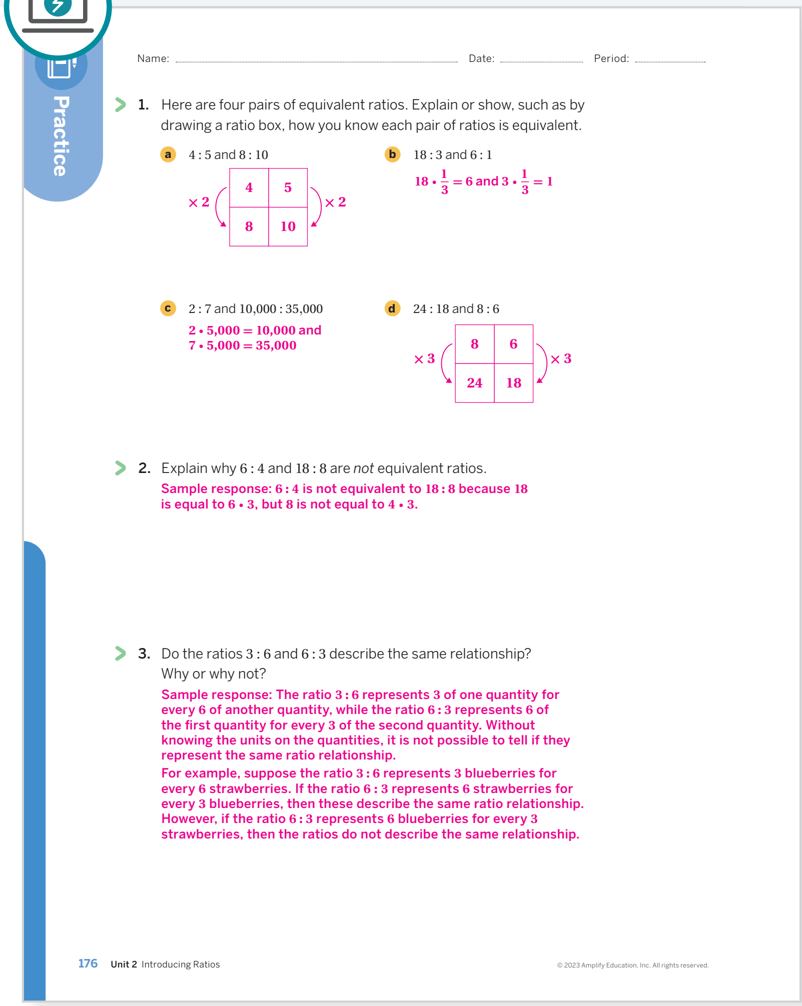
- In what ways did students use their developing math language to justify their response to Problem 1 on the Exit Ticket?
- What support do they still need in order to be more precise in their justifications?

Sample descriptions:

Emerging	Expanding
I disagree because Lin added.	I disagree because only multiplying the same number by each quantity produces equivalent ratios, not adding.

Practice

Independent



Practice

Name: Date: Period:

1. Here are four pairs of equivalent ratios. Explain or show, such as by drawing a ratio box, how you know each pair of ratios is equivalent.

a 4 : 5 and 8 : 10

4	5
8	10

b 18 : 3 and 6 : 1

$18 \cdot \frac{1}{3} = 6$ and $3 \cdot \frac{1}{3} = 1$

c 2 : 7 and 10,000 : 35,000

$2 \cdot 5{,}000 = 10{,}000$ and
$7 \cdot 5{,}000 = 35{,}000$

d 24 : 18 and 8 : 6

8	6
24	18

2. Explain why 6 : 4 and 18 : 8 are *not* equivalent ratios.

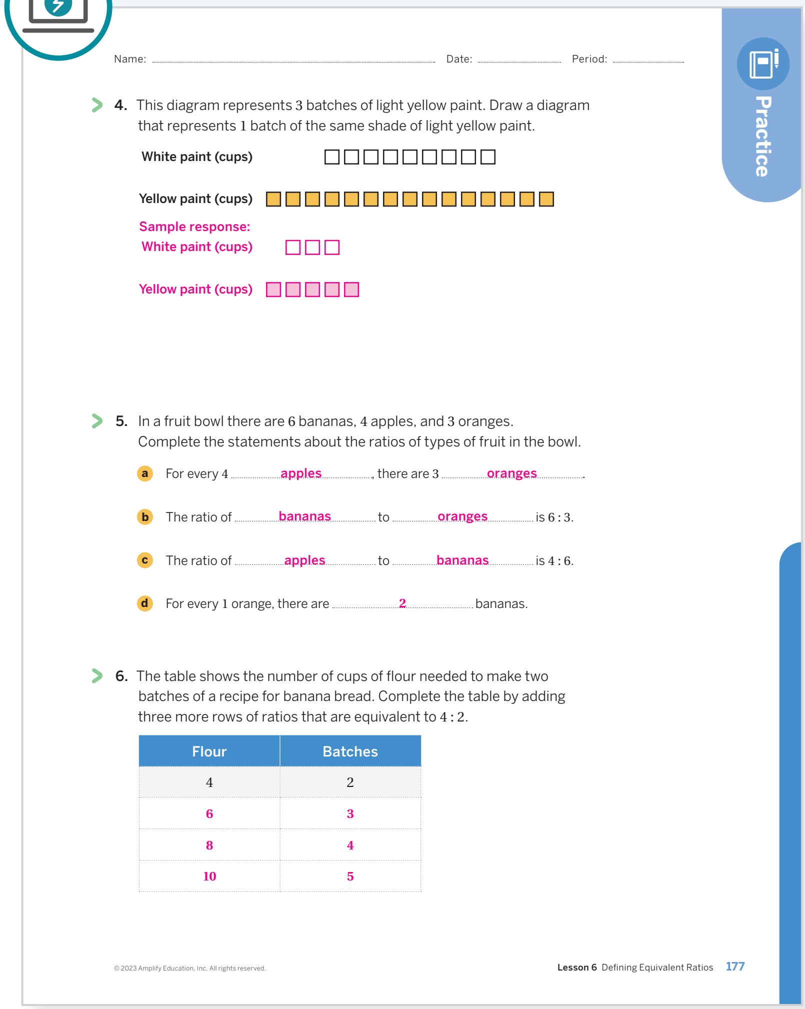
Sample response: 6 : 4 is not equivalent to 18 : 8 because 18 is equal to $6 \cdot 3$, but 8 is not equal to $4 \cdot 3$.

3. Do the ratios 3 : 6 and 6 : 3 describe the same relationship? Why or why not?

Sample response: The ratio 3 : 6 represents 3 of one quantity for every 6 of another quantity, while the ratio 6 : 3 represents 6 of the first quantity for every 3 of the second quantity. Without knowing the units on the quantities, it is not possible to tell if they represent the same ratio relationship.

For example, suppose the ratio 3 : 6 represents 3 blueberries for every 6 strawberries. If the ratio 6 : 3 represents 6 strawberries for every 3 blueberries, then these describe the same ratio relationship. However, if the ratio 6 : 3 represents 6 blueberries for every 3 strawberries, then the ratios do not describe the same relationship.

176 **Unit 2** Introducing Ratios



Practice

Name: Date: Period:

4. This diagram represents 3 batches of light yellow paint. Draw a diagram that represents 1 batch of the same shade of light yellow paint.

White paint (cups) ☐☐☐☐☐☐☐☐☐

Yellow paint (cups) ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐

Sample response:

White paint (cups) ☐☐☐

Yellow paint (cups) ☐☐☐☐☐

5. In a fruit bowl there are 6 bananas, 4 apples, and 3 oranges. Complete the statements about the ratios of types of fruit in the bowl.

a For every 4 apples, there are 3 oranges.

b The ratio of bananas to oranges is 6 : 3.

c The ratio of apples to bananas is 4 : 6.

d For every 1 orange, there are 2 bananas.

6. The table shows the number of cups of flour needed to make two batches of a recipe for banana bread. Complete the table by adding three more rows of ratios that are equivalent to 4 : 2.

Flour	Batches
4	2
6	3
8	4
10	5

Lesson 6 Defining Equivalent Ratios 177

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	3
	3	Activity 2	3
Spiral	4	Unit 2 Lesson 4	2
	5	Unit 2 Lesson 1	1
Formative ⚡	6	Unit 6 Lesson 7	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Representing Equivalent Ratios With Tables

Let's use tables to represent equivalent ratios.

Focus

Goals

1. **Language Goal:** Comprehend the words *row* and *column* as they are used to describe a table of equivalent ratios. **(Speaking and Listening, Writing)**
2. **Language Goal:** Explain how to generate new rows in a table of equivalent ratios. **(Speaking and Listening, Writing)**
3. **Language Goal:** Interpret a table of equivalent ratios that represents different-sized batches of a recipe. **(Writing)**

Coherence

Today

Students use a ratio table to organize a set of equivalent ratios. They generate equivalent ratios using a common multiplier for the values in each row in a ratio table. Students use a common multiplier that can be determined between the values in a given row. That multiplier can be used to multiply any chosen value for one quantity to get the corresponding value for the other quantity.

Previously

In Lesson 6, students worked with equivalent ratios in the context of music and in working simply with ratios of numbers.

Coming Soon

In Lessons 8–10, students will develop skills to more efficiently navigate ratio tables, focusing on factors and multiples.

Rigor

- Students are introduced to ratio tables to build **procedural skills** for determining equivalent ratios.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (answers)

Math Language Development

New word

- *ratio table*

Review words

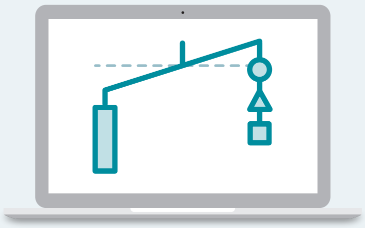
- *equivalent ratios*
- *equivalent*

Amps | Featured Activity

Activity 1

Jazz Rhythm and Horn Sections

Students get a little music education with this activity. They compare ratios of a jazz orchestra to the Marsalis family.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may experience an increase in their stress levels during Activities 1 and 2 as they try to discern the structure of tables with ratios. Encourage them by acknowledging the challenge set forth. Remind them to seek out support from other sources, such as other students or you, as a general guideline when they need help regulating their emotions.

Modifications to Pacing

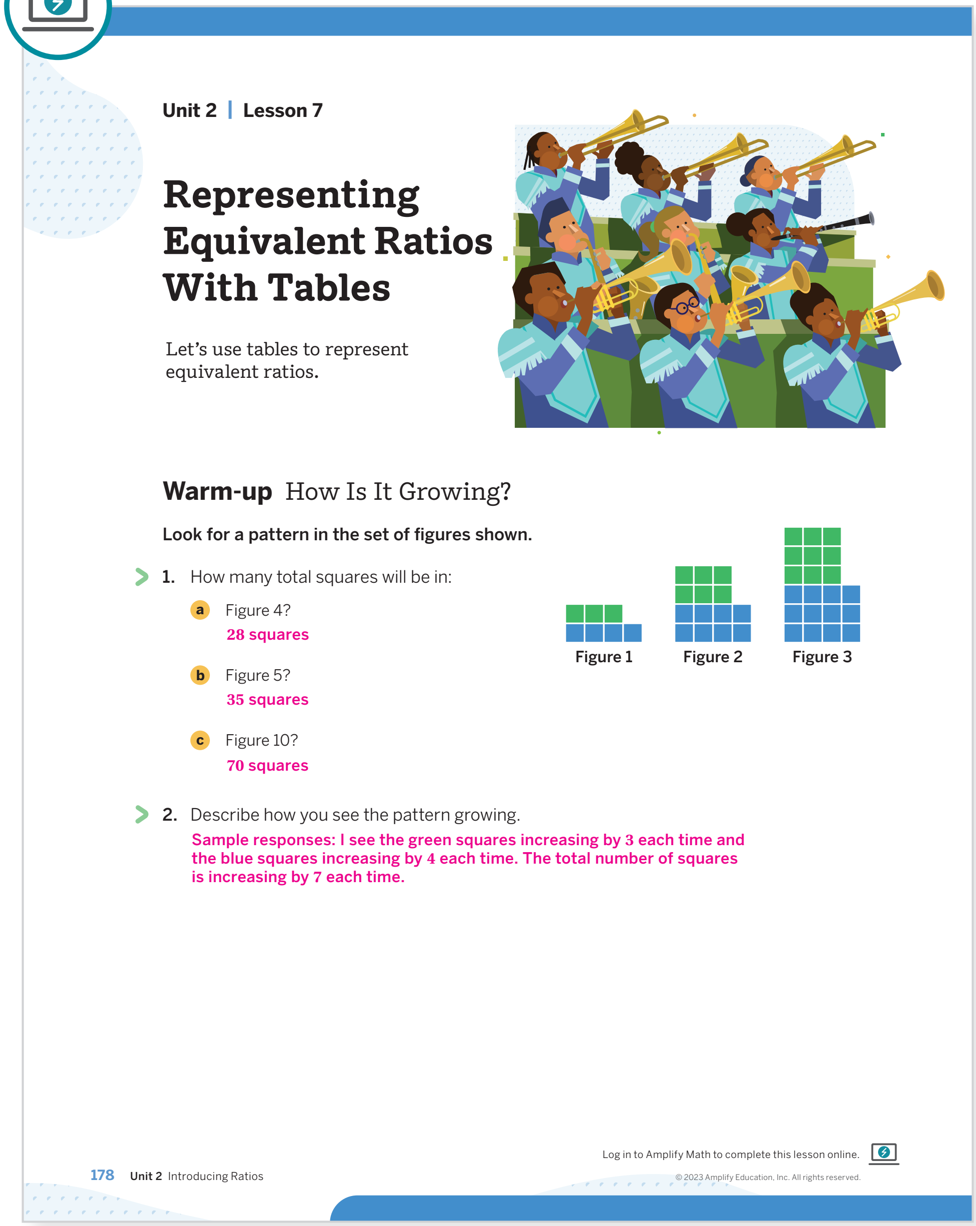
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problems 1b and 1c may be omitted.
- In **Activity 1**, Problems 1 and 2 may be omitted and students could be asked to only add one or two rows to the table.
- In **Activity 2**, students could be asked to determine only one "larger" and one "smaller" ratio.

Pairs | 10 min

Warm-up How Is It Growing?

Students determine the number of squares in future figures of a growing pattern of squares, noting how each color and the total are increasing, but the ratio stays the same.



Unit 2 | Lesson 7

Representing Equivalent Ratios With Tables

Let's use tables to represent equivalent ratios.

Warm-up How Is It Growing?

Look for a pattern in the set of figures shown.

1. How many total squares will be in:
   - a Figure 4?
     28 squares
   - b Figure 5?
     35 squares
   - c Figure 10?
     70 squares

Figure 1 Figure 2 Figure 3

2. Describe how you see the pattern growing.
   Sample responses: I see the green squares increasing by 3 each time and the blue squares increasing by 4 each time. The total number of squares is increasing by 7 each time.

Log in to Amplify Math to complete this lesson online.

178 **Unit 2** Introducing Ratios

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1 Launch

Display the three figures from the pattern of growing squares. Have students answer Problems 1 and 2, and tell them to give a signal when they have responses and a strategy. Then have students conduct the *Turn and Talk* routine.

2 Monitor

Help students get started by asking, "What pattern do you see with the blue squares? The green squares?"

Look for points of confusion:

- **Thinking that the total number of squares is doubling every time.** Have students write down how many of each color are being added for each image to find the pattern.

Look for productive strategies:

- Multiplying the number of each color square by the number of the figure in the sequence to get the totals.
- Writing expressions for the patterns, such as $(3 \cdot 1) + (4 \cdot 1)$, $(3 \cdot 2) + (4 \cdot 2)$.

3 Connect

Display the Warm-up PDF (answers).

Have pairs of students share their strategies and reasoning for determining each total.

Ask, "How does the number of the figure in the sequence relate to the total number of squares? To the counts of each color?"

Highlight that the total squares in each figure is equal to the number of squares in the first figure (7) times the number of the figure in the sequence. Also, each color is growing by multiples of the amounts in the first figure, and those counts together represent equivalent ratios.

Math Language Development

MLR8: Discussion Supports — Press for Reasoning

Encourage students to create a representation, such as a table, to track the pattern, and ask, "How do you see each colored set of squares growing? How does the growth of each colored set affect the total growth of the figures?"

English Learners

Provide sentence frames for students to use, such as:

- The green squares are growing by _____ each time.
- The blue squares are growing by _____ each time.

Power-up

To power up students' ability to understand ratio tables, have students complete:

Determine which of the following rows could be added to the ratio table to keep the ratio of raisins to peanuts equivalent. Select *all* that apply:

[✓ A.]	4	6
B.	9	6
[✓ C.]	1	$1\frac{1}{2}$
[✓ D.]	10	15

Raisins	Peanuts
2	3
8	12

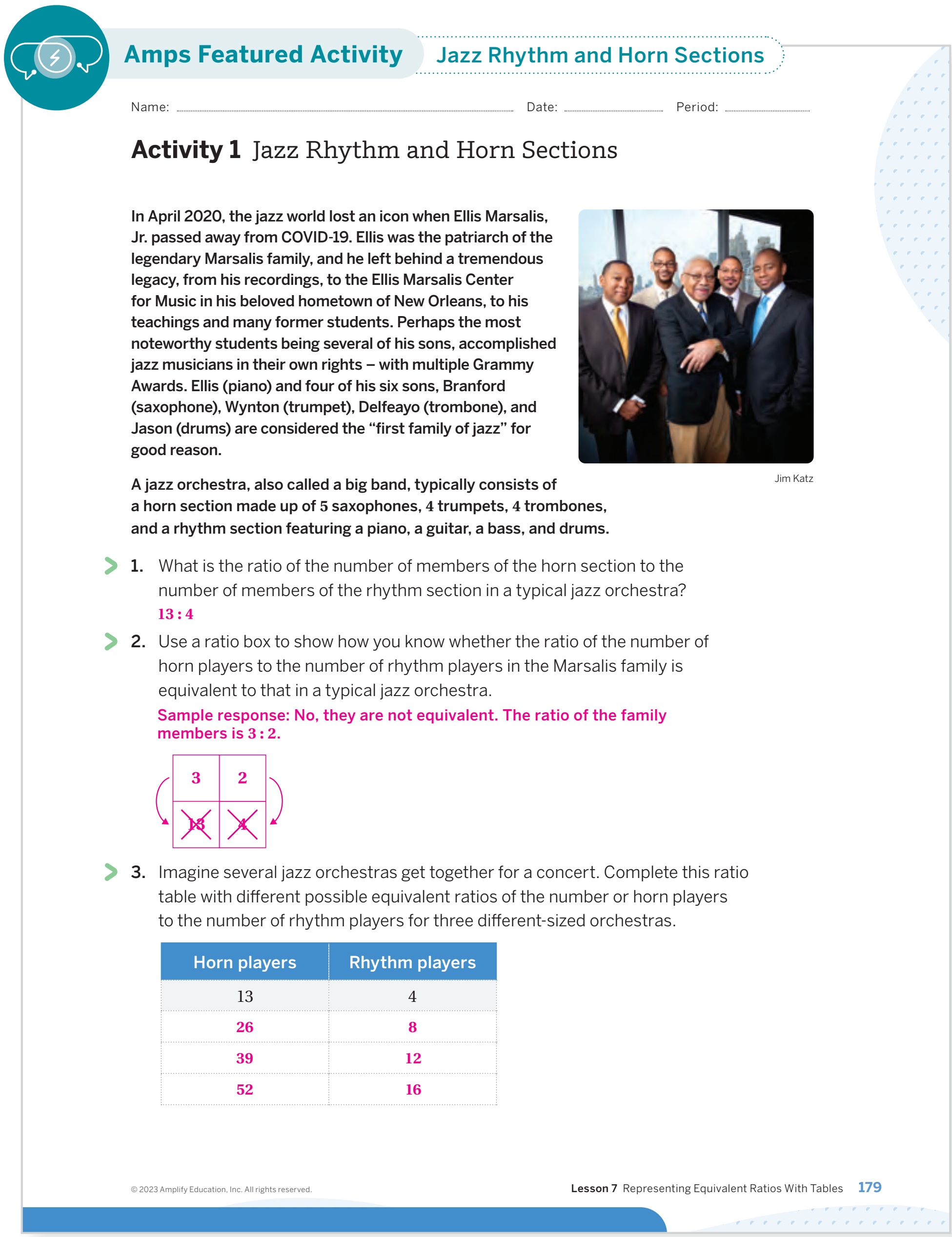
Use: Before Activity 1.

Informed by: Performance on Lesson 6, Practice Problem 6.

Pairs | 15 min

Activity 1 Jazz Rhythm and Horn Sections

Students extend the 2 × 2 ratio box to a ratio table relating several equivalent ratios of rhythm players and of horn players in jazz ensembles.



Amps Featured Activity Jazz Rhythm and Horn Sections

Name: ______ Date: ______ Period: ______

Activity 1 Jazz Rhythm and Horn Sections

In April 2020, the jazz world lost an icon when Ellis Marsalis, Jr. passed away from COVID-19. Ellis was the patriarch of the legendary Marsalis family, and he left behind a tremendous legacy, from his recordings, to the Ellis Marsalis Center for Music in his beloved hometown of New Orleans, to his teachings and many former students. Perhaps the most noteworthy students being several of his sons, accomplished jazz musicians in their own rights – with multiple Grammy Awards. Ellis (piano) and four of his six sons, Branford (saxophone), Wynton (trumpet), Delfeayo (trombone), and Jason (drums) are considered the "first family of jazz" for good reason.

Jim Katz

A jazz orchestra, also called a big band, typically consists of a horn section made up of 5 saxophones, 4 trumpets, 4 trombones, and a rhythm section featuring a piano, a guitar, a bass, and drums.

1. What is the ratio of the number of members of the horn section to the number of members of the rhythm section in a typical jazz orchestra?
   13 : 4

2. Use a ratio box to show how you know whether the ratio of the number of horn players to the number of rhythm players in the Marsalis family is equivalent to that in a typical jazz orchestra.
   Sample response: No, they are not equivalent. The ratio of the family members is 3 : 2.

3	2
~~13~~	~~4~~

3. Imagine several jazz orchestras get together for a concert. Complete this ratio table with different possible equivalent ratios of the number or horn players to the number of rhythm players for three different-sized orchestras.

Horn players	Rhythm players
13	4
26	8
39	12
52	16

Lesson 7 Representing Equivalent Ratios With Tables 179

1 Launch

Keep students in the same pairs. Have students use the *Think-Pair-Share* routine as they complete Problems 1 and 2. Allow students a few minutes of think time before discussing with a partner. Students should complete Problem 3 together.

2 Monitor

Help students get started by asking them to add up the total number of horn players and the total number of rhythm players. **Note:** One person plays all the drums.

Look for points of confusion:

- **Thinking that the ratio of the Marsalis family and a typical jazz orchestra is the same.** Remind students to find the ratio of the family out of the first paragraph.

Look for productive strategies:

- Multiplying both 13 and 4 systematically by different whole numbers, such as 2, 3, and 4.
- Using the equivalent ratios 3 : 2 and 6 : 4 to see that 3 : 2 is not the same ratio as 13 : 4.

3 Connect

Display a blank ratio table.

Have pairs of students share how they were able to determine the equivalent ratios in the table. Fill in the blank table with students' responses.

Highlight that generating equivalent ratios can be done using a common multiplier for the values in each column to add rows of values to a ratio table. Similarly, a common multiplier can be determined between the values in a given row and that multiplier can be used to multiply any chosen value for one quantity to get the corresponding value for the other quantity.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Read the introduction aloud to students as they follow along. Ask students whether they have heard of the phrase *big band* or whether any of them play a musical instrument.

Extension: *Math Enrichment*

Have students determine the number of each type of instrument there would be if 15 jazz orchestras got together. 75 saxophones, 60 trumpets, 60 trombones, 15 pianos, 15 guitars, 15 bass, and 15 sets of drums.

Math Language Development

MLR7: Compare and Connect

Ask students to describe to a partner how multiplication appears in the table in Problem 3, and then invite listeners to restate or revoice what they heard back to their partner by using mathematical language. After students have a chance to share with a partner, select a few to share their thinking with the class.

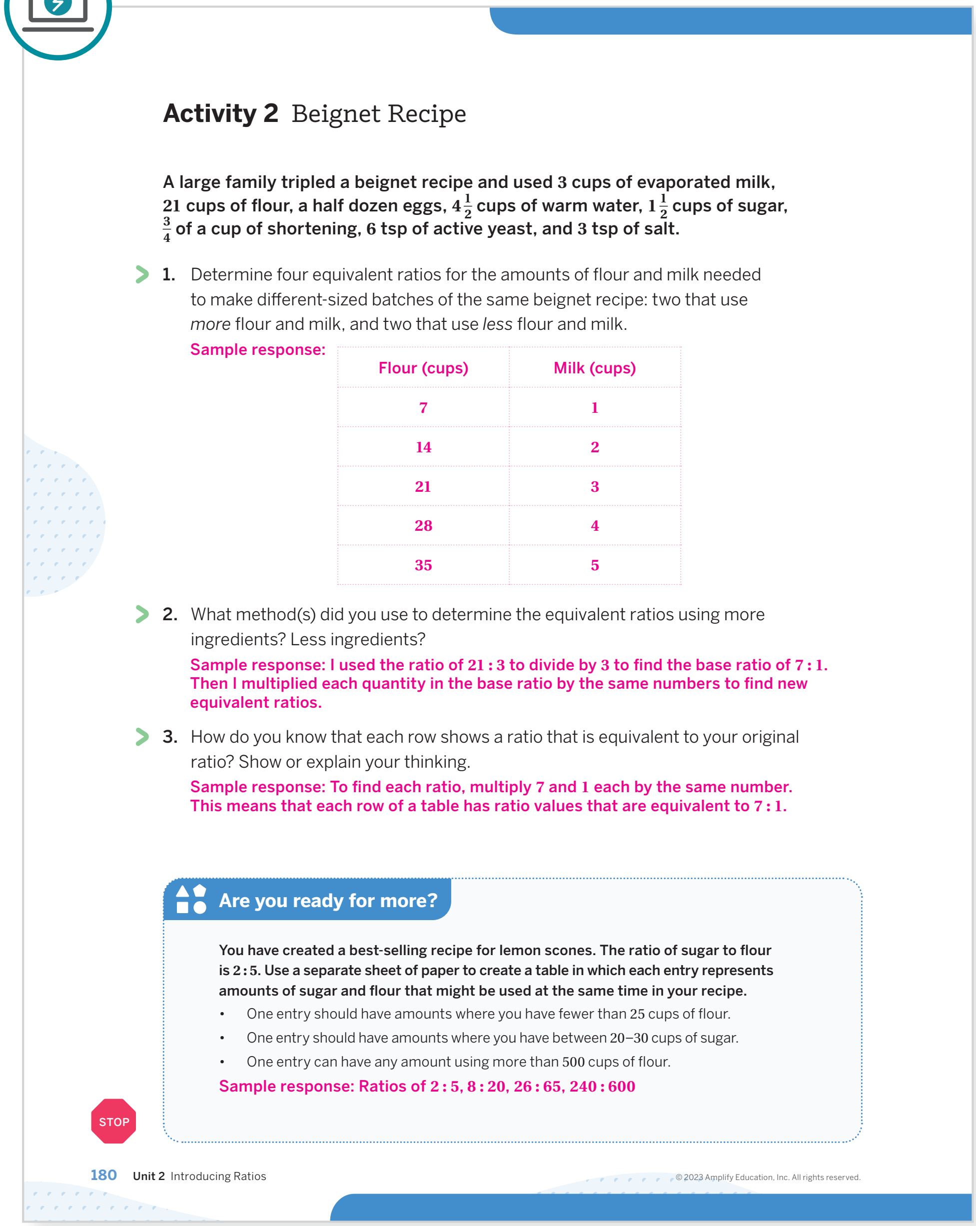
English Learners

Annotate how multiplication appears in the table by drawing arrows from the first row to each of the other rows.

Pairs | 10 min

Activity 2 Beignet Recipe

Students use the amounts of two ingredients in a given recipe to determine equivalent ratios with both greater and lesser values by using a table.



Activity 2 Beignet Recipe

A large family tripled a beignet recipe and used 3 cups of evaporated milk, 21 cups of flour, a half dozen eggs, $4\frac{1}{2}$ cups of warm water, $1\frac{1}{2}$ cups of sugar, $\frac{3}{4}$ of a cup of shortening, 6 tsp of active yeast, and 3 tsp of salt.

1. Determine four equivalent ratios for the amounts of flour and milk needed to make different-sized batches of the same beignet recipe: two that use *more* flour and milk, and two that use *less* flour and milk.

   Sample response:

Flour (cups)	Milk (cups)
7	1
14	2
21	3
28	4
35	5

2. What method(s) did you use to determine the equivalent ratios using more ingredients? Less ingredients?

   Sample response: I used the ratio of 21 : 3 to divide by 3 to find the base ratio of 7 : 1. Then I multiplied each quantity in the base ratio by the same numbers to find new equivalent ratios.

3. How do you know that each row shows a ratio that is equivalent to your original ratio? Show or explain your thinking.

   Sample response: To find each ratio, multiply 7 and 1 each by the same number. This means that each row of a table has ratio values that are equivalent to 7 : 1.

Are you ready for more?

You have created a best-selling recipe for lemon scones. The ratio of sugar to flour is 2 : 5. Use a separate sheet of paper to create a table in which each entry represents amounts of sugar and flour that might be used at the same time in your recipe.

- One entry should have amounts where you have fewer than 25 cups of flour.
- One entry should have amounts where you have between 20–30 cups of sugar.
- One entry can have any amount using more than 500 cups of flour.

Sample response: Ratios of 2 : 5, 8 : 20, 26 : 65, 240 : 600

STOP

180 Unit 2 Introducing Ratios

1 Launch

Keep students in pairs. Have students use the *Think-Pair-Share* routine for Problem 1, and then work together with their partner on Problems 2–3.

2 Monitor

Help students get started by asking, "Because the recipe was tripled (3 batches), how could you find the amounts for the original recipe (1 batch)?"

Look for points of confusion:

- **Thinking that 7 : 1 is the only possible "smaller" ratio because 21 and 3 can only be divided evenly by 3.** Ask, "How many batches does 7 : 1 make? What about 2 batches?"

Look for productive strategies:

- Flexibly multiplying or dividing both values in one row by the same factor to generate new rows.
- Determining and using the ratio of 7 : 1 to generate greater and lesser equivalent ratios.
- Recognizing the number of cups of flour is always 7 times the number of cups of milk.

3 Connect

Display a blank ratio table.

Have pairs of students share how they determined equivalent ratios for the table, focusing on different strategies for using more and less ingredients.

Define a ***ratio table*** as a table of values organized in columns and rows that contains equivalent ratios.

Highlight that not every equivalent ratio comes from whole-number factors, but any row can be used to generate new rows.

Ask, "What might be a result of adjusting ingredients not using equivalent ratios?"

Differentiated Support

Accessibility: *Activate Background Knowledge*

Explain that a *beignet* is a square piece of dough that is fried and covered with powdered sugar, somewhat similar to a donut. Consider showing an image of a beignet to help students visualize one.

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Have students annotate or highlight the quantities of flour and milk in the introduction. Demonstrate to students how to determine one equivalent ratio that uses more flour and milk and start a table they can use to determine the remaining three equivalent ratios.

Math Language Development

MLR2: Collect and Display

Collect and display words, phrases, and diagrams that highlight ratio language, such as *equivalent ratios* and *ratio tables*. Focus on words students use to describe scaling up or down, and the words used to verify that the recipe amounts are the same. Encourage students to refer back to the display during future discussions.

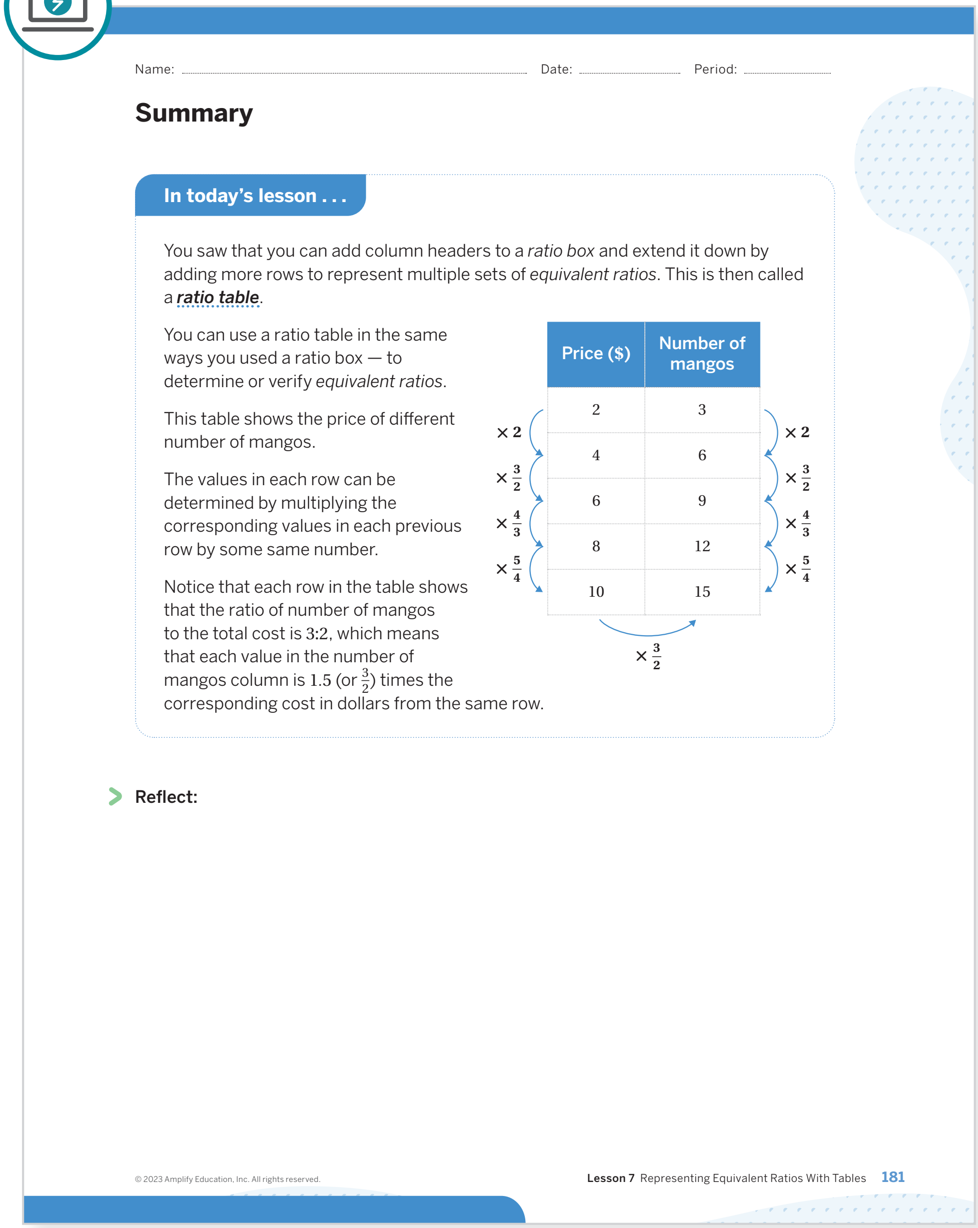
English Learners

As you add to the display, explicitly make connections between the words and phrases and how they connect to the ratio table.

Whole Class | 5 min

Summary

Review and synthesize how a ratio table can be used to organize and determine a set of equivalent ratios.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that you can add column headers to a *ratio box* and extend it down by adding more rows to represent multiple sets of *equivalent ratios*. This is then called a ***ratio table***.

You can use a ratio table in the same ways you used a ratio box — to determine or verify *equivalent ratios*.

This table shows the price of different number of mangos.

The values in each row can be determined by multiplying the corresponding values in each previous row by some same number.

Notice that each row in the table shows that the ratio of number of mangos to the total cost is 3:2, which means that each value in the number of mangos column is 1.5 (or $\frac{3}{2}$) times the corresponding cost in dollars from the same row.

	Price ($)	Number of mangos	
	2	3	
$\times 2$	4	6	$\times 2$
$\times \frac{3}{2}$	6	9	$\times \frac{3}{2}$
$\times \frac{4}{3}$	8	12	$\times \frac{4}{3}$
$\times \frac{5}{4}$	10	15	$\times \frac{5}{4}$

$\times \frac{3}{2}$

Reflect:

Synthesize

Display a ratio table with one complete row containing a ratio that can be simplified, such as 20 : 12.

Ask,

- "How could you determine an equivalent ratio with greater values? Give an example."
- "How could you determine an equivalent ratio with lesser values? Give an example."

Have students share their responses, adding rows to the table to record them and draw arrows to show the common divisors, or multipliers, used.

Formalize vocabulary: *ratio table*

Highlight that any two rows in a ratio table can be related by coordinated multiplication or division, even when the values are not whole-number factors or multiples of one another.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How are multiplication and division used in ratio tables?"

Math Language Development

MLR2: Collect and Display

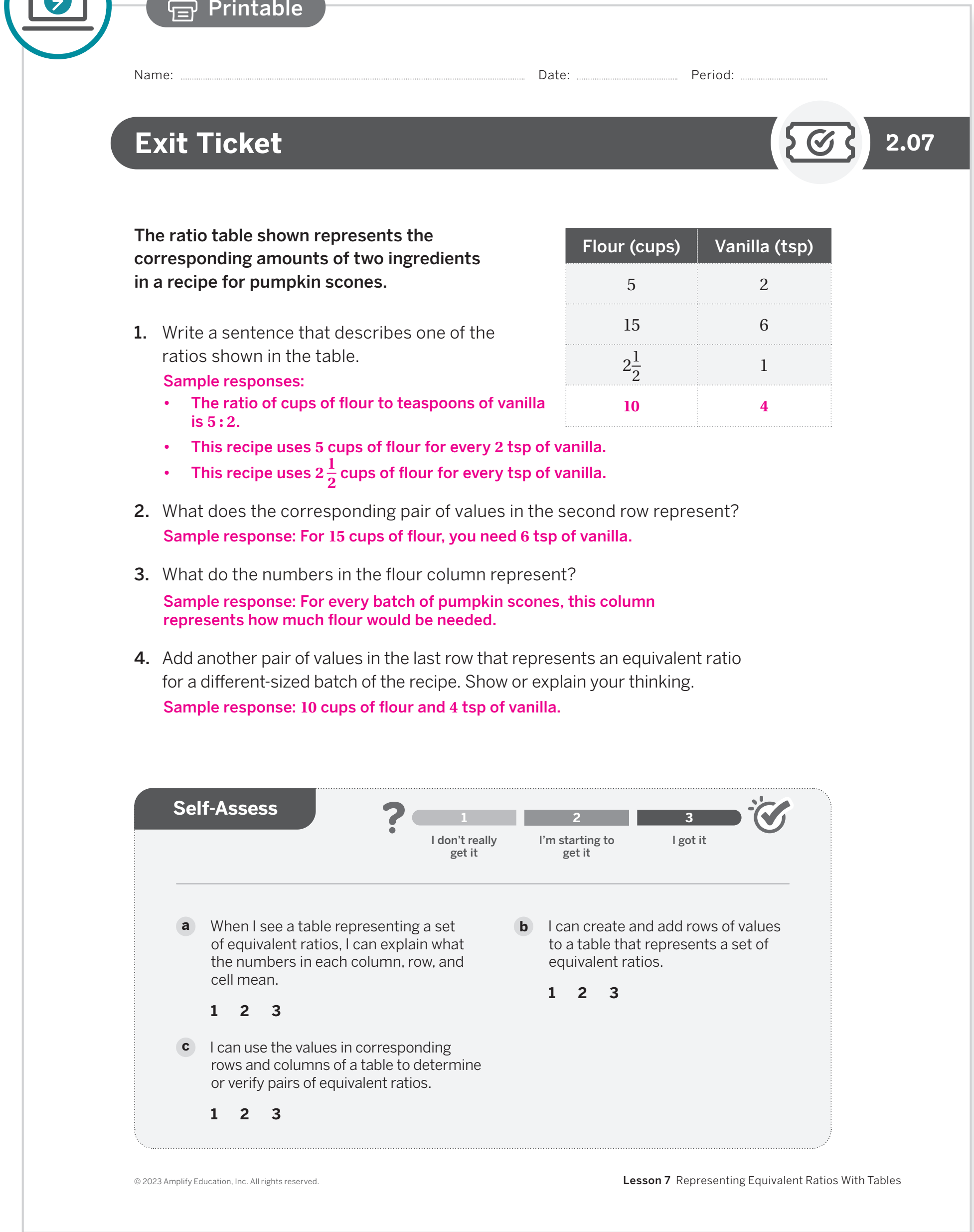
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the term *ratio table* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how to determine equivalent ratios by using a ratio table.

Printable



Name: ________________ Date: ________ Period: ________

Exit Ticket 2.07

The ratio table shown represents the corresponding amounts of two ingredients in a recipe for pumpkin scones.

Flour (cups)	Vanilla (tsp)
5	2
15	6
$2\frac{1}{2}$	1
10	4

1. Write a sentence that describes one of the ratios shown in the table.
   Sample responses:
   - The ratio of cups of flour to teaspoons of vanilla is 5 : 2.
   - This recipe uses 5 cups of flour for every 2 tsp of vanilla.
   - This recipe uses $2\frac{1}{2}$ cups of flour for every tsp of vanilla.
2. What does the corresponding pair of values in the second row represent?
   Sample response: For 15 cups of flour, you need 6 tsp of vanilla.
3. What do the numbers in the flour column represent?
   Sample response: For every batch of pumpkin scones, this column represents how much flour would be needed.
4. Add another pair of values in the last row that represents an equivalent ratio for a different-sized batch of the recipe. Show or explain your thinking.
   Sample response: 10 cups of flour and 4 tsp of vanilla.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. When I see a table representing a set of equivalent ratios, I can explain what the numbers in each column, row, and cell mean.
1 2 3

b. I can create and add rows of values to a table that represents a set of equivalent ratios.
1 2 3

c. I can use the values in corresponding rows and columns of a table to determine or verify pairs of equivalent ratios.
1 2 3

Lesson 7 Representing Equivalent Ratios With Tables

Success looks like . . .

- **Language Goal:** Comprehending the words *row* and *column* as they are used to describe a table of equivalent ratios. **(Speaking and Listening, Writing)**
  - » Explaining how the second row and the first column each represent a ratio in Problems 2 and 3.
- **Language Goal:** Explaining how to generate new rows in a table of equivalent ratios. **(Speaking and Listening, Writing)**
  - » Generating a new row for a different sized batch of the recipe in Problem 4.
- **Language Goal:** Interpreting a table of equivalent ratios that represents different-sized batches of a recipe. **(Writing)**
  - » Describing ratios for different-sized batches of pumpkin scones in Problem 1.

Suggested next steps

If students cannot describe the ratio represented in the table, consider:

- Reviewing the table in Activity 1.
- Assigning Practice Problem 1.
- Asking, "How many cups of flour are there for every 1 tsp of vanilla?"

If students struggle with adding another pair of values to the ratio table, consider:

- Reviewing the table in Activity 2.
- Assigning Practice Problem 1.
- Asking, "How are the ratios 5 : 2 and 15 : 6 related? Could you use similar thinking with either of those ratios to generate another equivalent ratio that is not listed?"

Professional Learning

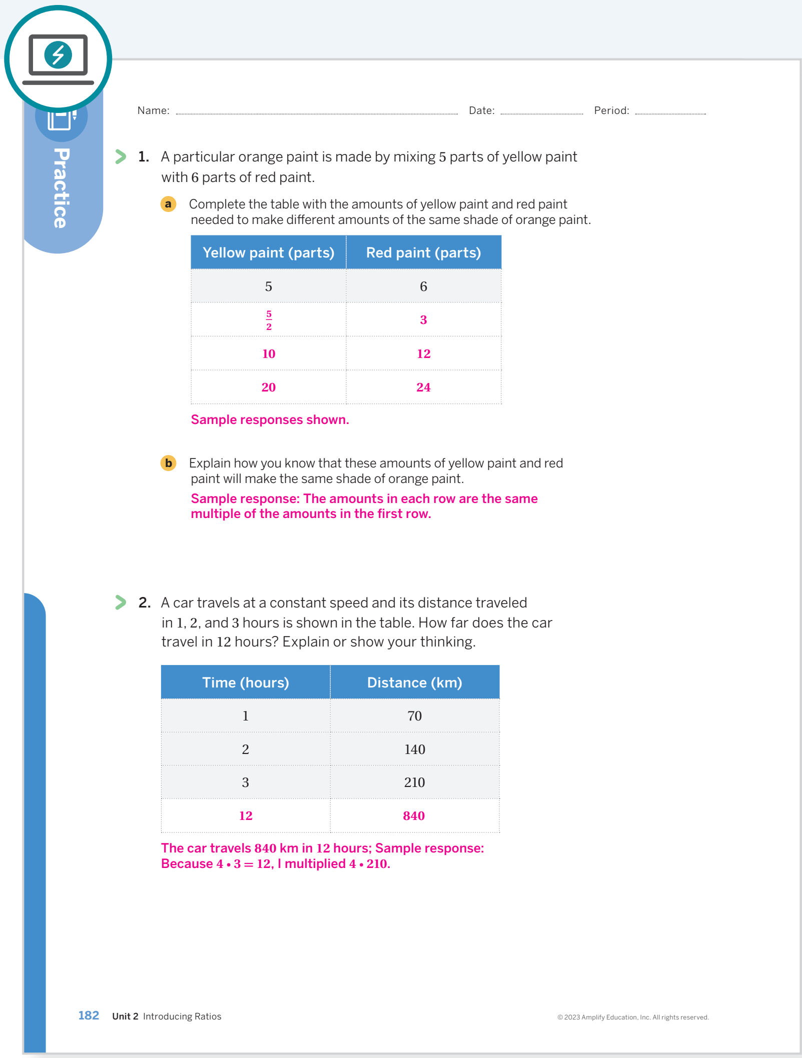
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How was using tables with ratios similar to or different from equivalent ratio tables that you used in the last lesson?
- What different ways did students approach tables with ratios? What does that tell you about similarities and differences among your students? What might you change the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. A particular orange paint is made by mixing 5 parts of yellow paint with 6 parts of red paint.
   - **a** Complete the table with the amounts of yellow paint and red paint needed to make different amounts of the same shade of orange paint.

Yellow paint (parts)	Red paint (parts)
5	6
$\frac{5}{2}$	3
10	12
20	24

Sample responses shown.

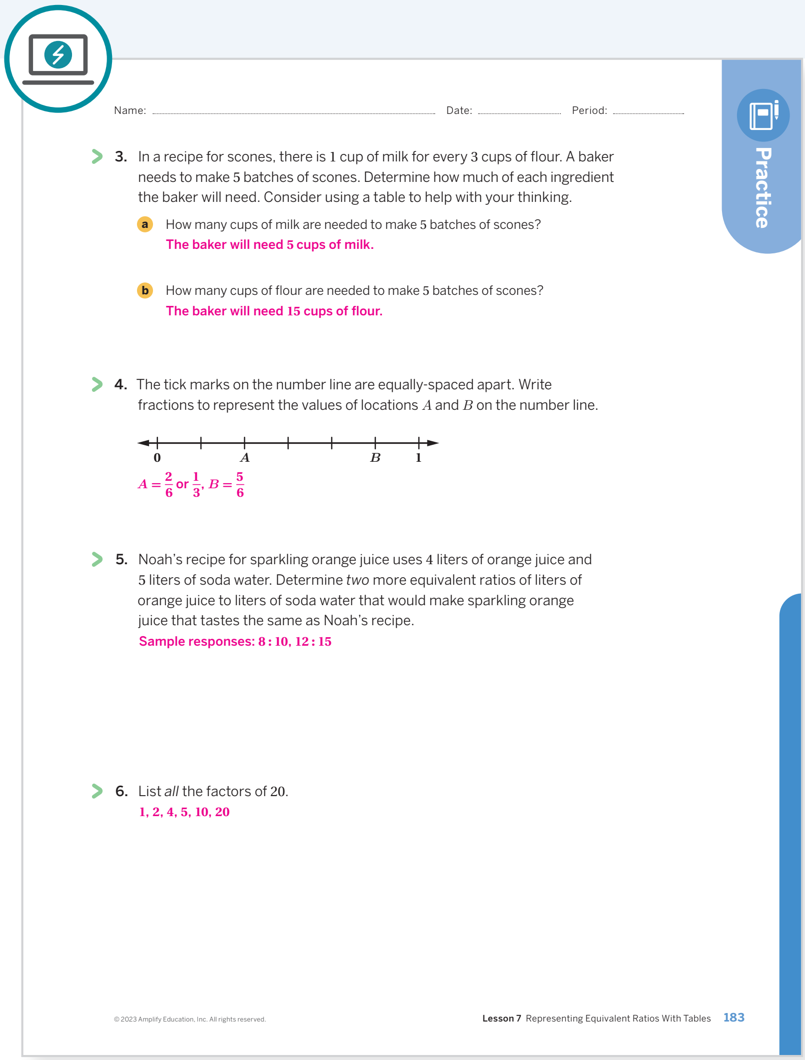
   - **b** Explain how you know that these amounts of yellow paint and red paint will make the same shade of orange paint.
     Sample response: The amounts in each row are the same multiple of the amounts in the first row.

2. A car travels at a constant speed and its distance traveled in 1, 2, and 3 hours is shown in the table. How far does the car travel in 12 hours? Explain or show your thinking.

Time (hours)	Distance (km)
1	70
2	140
3	210
12	840

The car travels 840 km in 12 hours; Sample response: Because $4 \cdot 3 = 12$, I multiplied $4 \cdot 210$.

182 **Unit 2** Introducing Ratios



Practice

Name: Date: Period:

3. In a recipe for scones, there is 1 cup of milk for every 3 cups of flour. A baker needs to make 5 batches of scones. Determine how much of each ingredient the baker will need. Consider using a table to help with your thinking.
   - **a** How many cups of milk are needed to make 5 batches of scones?
     The baker will need 5 cups of milk.
   - **b** How many cups of flour are needed to make 5 batches of scones?
     The baker will need 15 cups of flour.

4. The tick marks on the number line are equally-spaced apart. Write fractions to represent the values of locations A and B on the number line.

$A = \frac{2}{6}$ or $\frac{1}{3}$, $B = \frac{5}{6}$

5. Noah's recipe for sparkling orange juice uses 4 liters of orange juice and 5 liters of soda water. Determine *two* more equivalent ratios of liters of orange juice to liters of soda water that would make sparkling orange juice that tastes the same as Noah's recipe.
   Sample responses: 8 : 10, 12 : 15

6. List *all* the factors of 20.
   1, 2, 4, 5, 10, 20

Lesson 7 Representing Equivalent Ratios With Tables 183

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	3
Spiral	4	Grade 4	1
	5	Unit 2 Lesson 3	2
Formative ⚡	6	Unit 2 Lesson 8	3

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Optional

UNIT 2 | **LESSON 8**

Reasoning With Multiplication and Division

Let's use multiplication and division to go from a given number to any other number.

Focus

Goals

1. Identify sequences of multiplication and division to get from one number to another.
2. Identify a single factor to get from one number to another.

Coherence

Today

In this optional lesson, students will apply what they already know about multiples and factors, as well as the relationship between multiplication and division, to write out expressions showing sequences of operations that can be used to get from one number to another. They reason first with sets of related products and quotients by using the associative and commutative properties. Then students will coordinate multiplication and division to go from a lesser value to a greater value, or a greater value to a lesser value, in two steps. Lastly, students recognize this process can always be simplified to one step by using the interpretation of fractions as division.

Previously

In Grade 4, students recognized factors and multiples of whole numbers within 100 and identified factor pairs. Then in Grade 5, they wrote expressions for multi-step numerical calculations, and they interpreted multiplication as scaling and fractions as division.

Coming Soon

In Lessons 9–10, students will identify common factors and multiples and also the greatest common factor and least common multiple of two whole numbers. Then, in Lesson 11, they will apply these concepts and properties of operations in order to efficiently determine equivalent ratios.

Rigor

- Students exercise **fluency** with division of whole numbers and multiplication of whole numbers and fractions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
15 min	10 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *product**
- *quotient*

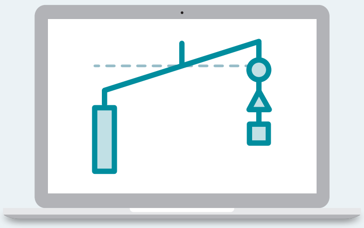
*Students may confuse the term *product* with the everyday meaning of the term, such as the *product* of a company. Be ready to address the differences between these terms.

Amps | Featured Activity

Activity 1

Interactive Expression Builder

Students can input factors and divisors to an expression involving both multiplication and division and then see immediate validation of the result.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel disorganized and lost if they cannot determine one operation because they are not always recording their two multiplication and division operations in the same order in Activity 2. Have students identify an example they were able to complete, or suggest they create two columns within the *Two operations* column: *Multiplication* and *Division*.

Modifications to Pacing

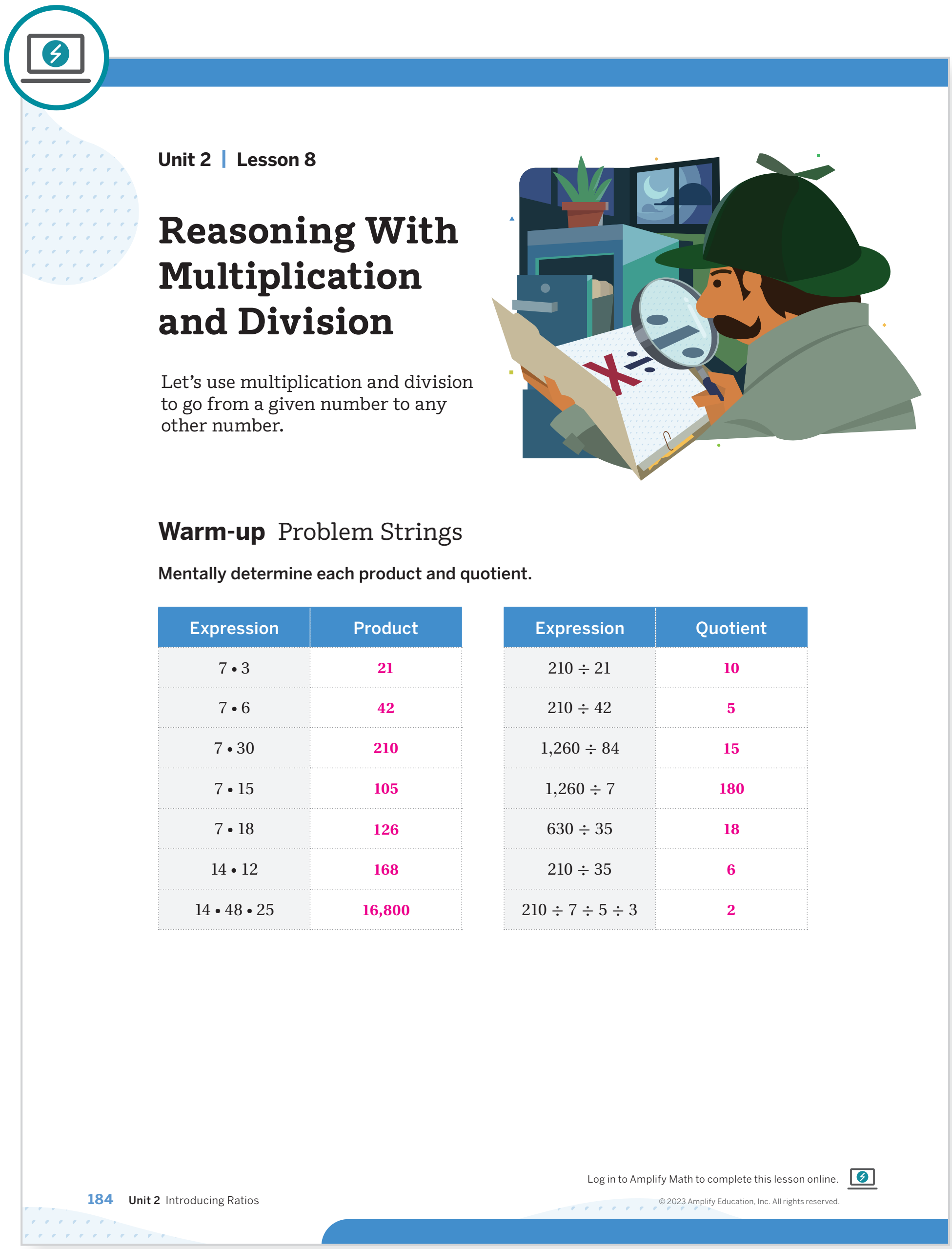
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, have students evaluate only the top four expressions in each column.
- **Activity 1** may be omitted.

Pairs | 15 min

Warm-up Problem Strings

Students mentally evaluate sequences of related products and related quotients, leveraging the commutative and associative properties.



Unit 2 | **Lesson 8**

Reasoning With Multiplication and Division

Let's use multiplication and division to go from a given number to any other number.

Warm-up Problem Strings

Mentally determine each product and quotient.

Expression	Product
7 • 3	21
7 • 6	42
7 • 30	210
7 • 15	105
7 • 18	126
14 • 12	168
14 • 48 • 25	16,800

Expression	Quotient
210 ÷ 21	10
210 ÷ 42	5
1,260 ÷ 84	15
1,260 ÷ 7	180
630 ÷ 35	18
210 ÷ 35	6
210 ÷ 7 ÷ 5 ÷ 3	2

Log in to Amplify Math to complete this lesson online.

184 **Unit 2** Introducing Ratios

1 Launch

Tell students they will have 10 minutes to determine as many of the products and quotients as they can. Emphasize that they should first attempt to not write out any calculations, but also that determining more correct responses is more important than working through all of the expressions.

2 Monitor

Help students get started by noting that all of the multiplication expressions are related and all of the division expressions are related. Have students consider how each product/quotient relates to another.

Look for points of confusion:

- **Thinking about each expression as an isolated problem.** Allow students to rewrite expressions, "breaking" a factor into an operation itself, to help relate the expressions.

Look for productive strategies:

- Noticing every product includes a factor of 7 or 14.
- Noticing every quotient includes a dividend that is a multiple of 210 and a divisor that is a multiple of 7.
- Using factors, multiples, and properties of operations to evaluate related products and quotients.

3 Connect

Display the expressions, starting with the products and followed by the quotients.

Have students share their responses and their strategies for evaluating each product and quotient, focusing on those who used relationships between factors and previous results.

Highlight that multiplication is commutative and associative, and that multiplication and division can be performed in any order, which can be used to evaluate related expressions.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a rectangular area model or array (or provide counters) for the product 7 • 3 and ask them to show how it would change for 7 • 6. Have them draw or explain similar reasoning for the next three expressions as well. If time permits, have them conduct a similar exercise with the quotient 210 ÷ 21, and any other quotients they can relate to that, and show or explain how their model would change

Power-up

To power up students' ability to determine the factors of a value, have students complete:

Recall that a *factor* is a number that divides evenly into a given whole number. Circle all of the factors of 12

[✓ 1] [✓ 2] [✓ 3] [✓ 4] 5 [✓ 6]
7 8 9 10 11 [✓ 12]

Use: Before the Warm-up.

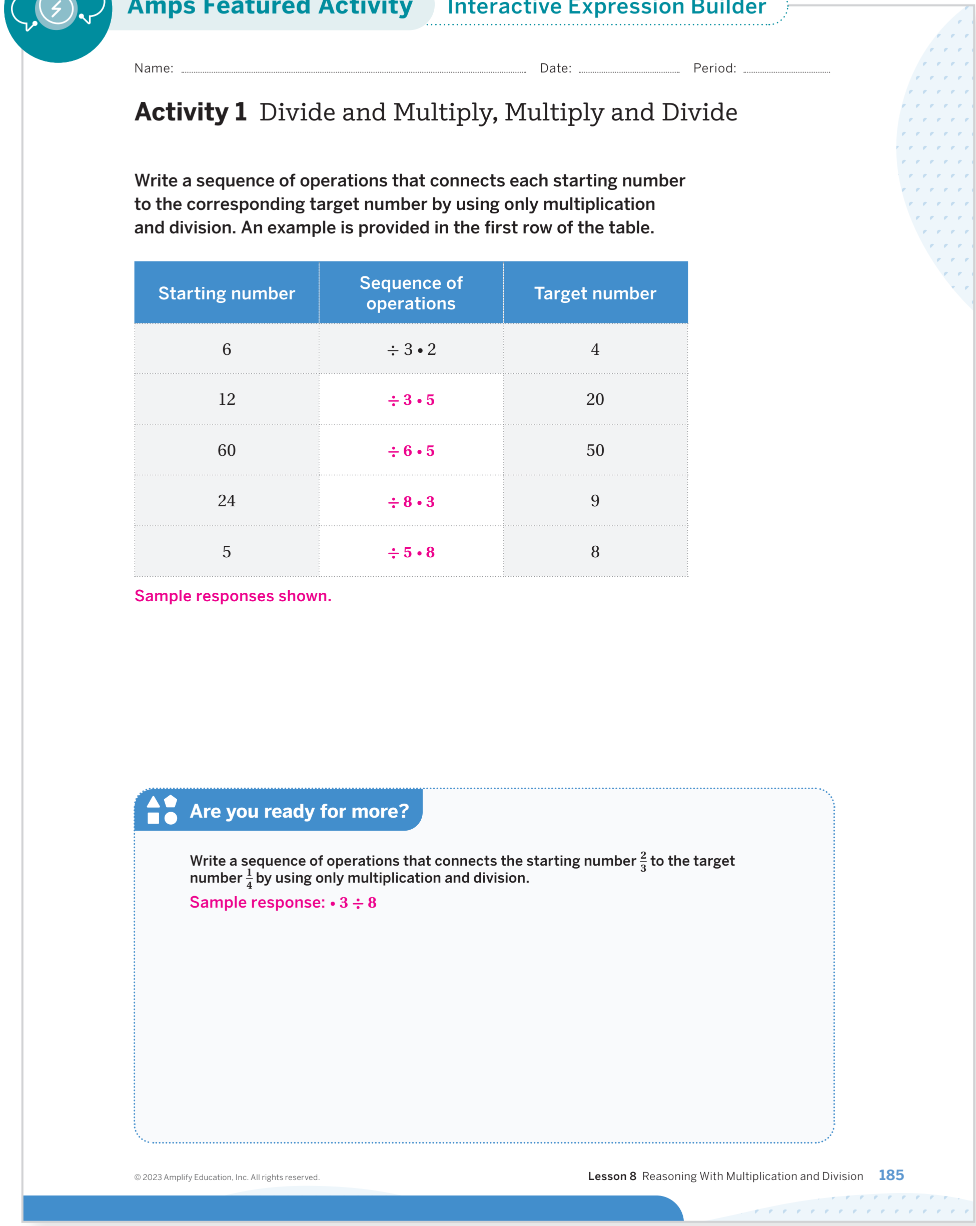
Informed by: Performance on Lesson 7, Practice Problem 6.

Pairs | 10 min

Activity 1 Divide and Multiply, Multiply and Divide

Students determine and write expressions containing one multiplication and one division operation to connect two given numbers.

Amps Featured Activity Interactive Expression Builder



Name: Date: Period:

Activity 1 Divide and Multiply, Multiply and Divide

Write a sequence of operations that connects each starting number to the corresponding target number by using only multiplication and division. An example is provided in the first row of the table.

Starting number	Sequence of operations	Target number
6	÷ 3 • 2	4
12	**÷ 3 • 5**	20
60	**÷ 6 • 5**	50
24	**÷ 8 • 3**	9
5	**÷ 5 • 8**	8

Sample responses shown.

Are you ready for more?

Write a sequence of operations that connects the starting number $\frac{2}{3}$ to the target number $\frac{1}{4}$ by using only multiplication and division.

Sample response: • 3 ÷ 8

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Lesson 8 Reasoning With Multiplication and Division 185

1 Launch

Set an expectation for the amount of time that pairs will have to work on the activity.

2 Monitor

Help students get started by asking, "Why do you think the example would start with ÷ 3? How could you do something similar starting from 12 and knowing you want to get to 20?"

Look for points of confusion:

- **Thinking connections are impossible because none of the starting numbers is a factor of its corresponding target number.** Note that this is also true for 6 and 4. Ask, "What does that tell you about the operations you need to use?"

Look for productive strategies:

- Recognizing that to go from a lesser to a greater value, the multiplier must be greater than the divisor, and vice versa.
- Using factors and multiples to determine a correct sequence of operations.
- Recognizing that you can always divide by the starting number and multiply by the target number.
- Using relationships between the starting numbers or target numbers in multiple rows to adjust factors and divisors.

3 Connect

Display a table to show each pair of numbers.

Have pairs of students share their steps (operations) and explain their thinking for one pair of numbers at a time. Allow multiple pairs of students to share different sets of operations.

Highlight that even when two numbers are not multiples, they can be related by multiplication and division. Knowing their factors is helpful, but it always works to divide by the starting number and multiply by the target number.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can input factors and divisors and then see immediate validation of the result.

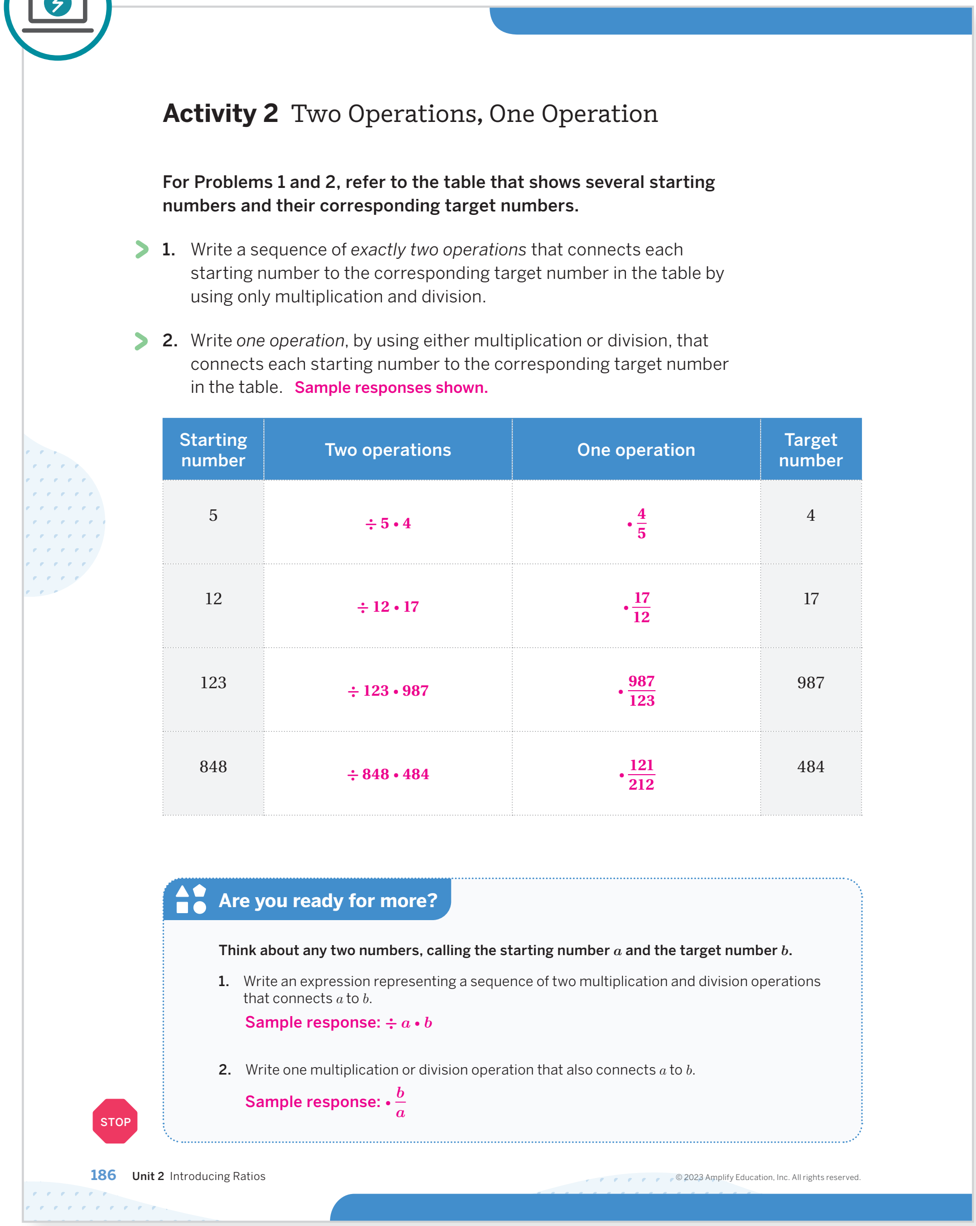
Accessibility: *Guide Processing and Visualization*

Walk through the example that was already provided in the table. Demonstrate how the sequence of operations results in the target number of 4 and ask, "If you divided 6 by another number, such as 2, could you still get to the target number (by only using whole numbers)? What if you divided 6 by 6?"

Pairs | 10 min

Activity 2 Two Operations, One Operation

Students determine and write expressions containing one multiplication or division operation to connect two given numbers, and generalize this as fraction multiplication.



Activity 2 Two Operations, One Operation

For Problems 1 and 2, refer to the table that shows several starting numbers and their corresponding target numbers.

1. Write a sequence of *exactly two operations* that connects each starting number to the corresponding target number in the table by using only multiplication and division.
2. Write *one operation*, by using either multiplication or division, that connects each starting number to the corresponding target number in the table. **Sample responses shown.**

Starting number	Two operations	One operation	Target number
5	$\div 5 \cdot 4$	$\cdot \frac{4}{5}$	4
12	$\div 12 \cdot 17$	$\cdot \frac{17}{12}$	17
123	$\div 123 \cdot 987$	$\cdot \frac{987}{123}$	987
848	$\div 848 \cdot 484$	$\cdot \frac{121}{212}$	484

Are you ready for more?

Think about any two numbers, calling the starting number a and the target number b.

1. Write an expression representing a sequence of two multiplication and division operations that connects a to b.
   Sample response: $\div a \cdot b$
2. Write one multiplication or division operation that also connects a to b.
   Sample response: $\cdot \frac{b}{a}$

STOP

186 **Unit 2** Introducing Ratios

1 Launch

Set an expectation for the amount of time pairs will have to work on the activity.

2 Monitor

Help students get started by asking, "What is another way you can represent division?" I can represent division as a fraction.

Look for points of confusion:

- **Thinking to go from a greater to a lesser value you must divide.** Ask, "What is another way to think about division by a fraction?"
- **Thinking one operation is impossible because the values are not multiples.** Have students write the two operations, and have them order the multiplication first. Then ask, "Is there another way to represent the division part?"

Look for productive strategies:

- Recognizing that division can be represented as a fraction, and every pair of multiplication and division operations can be represented as fraction multiplication.
- Simplifying fractions when possible, but also recognizing that does not change the results.

3 Connect

Display a table to show each pair of numbers.

Have pairs of students share their steps (operations) and explain their thinking for one pair of numbers at a time.

Highlight that since students can always get from one number to another by using one multiplication operation and one division operation, and since division can be represented as a fraction, students can also always write a single multiplication operation (by a fraction) to go between those numbers. Consider relating cases where the operators are factors or multiples to equivalent fractions.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them work with the following start and target numbers instead of the ones given in the table.

- Start: 1, Target: 2
- Start: 2, Target: 4
- Start: 3, Target: 2

Extension: *Math Enrichment*

Ask students to explain why the operation indicated in the *One Operation* column is multiplication and not division. Then ask them if they can write an expression using one operation for the first row in the table where the operation is division, not multiplication. $\div \frac{5}{4}$

Math Language Development

MLR8: Discussion Supports — Press for Details

Facilitate a class discussion highlighting connections between two operations and one operation. Ask, "How are the *Two operations* and *One operation* columns related? What patterns do you see?"

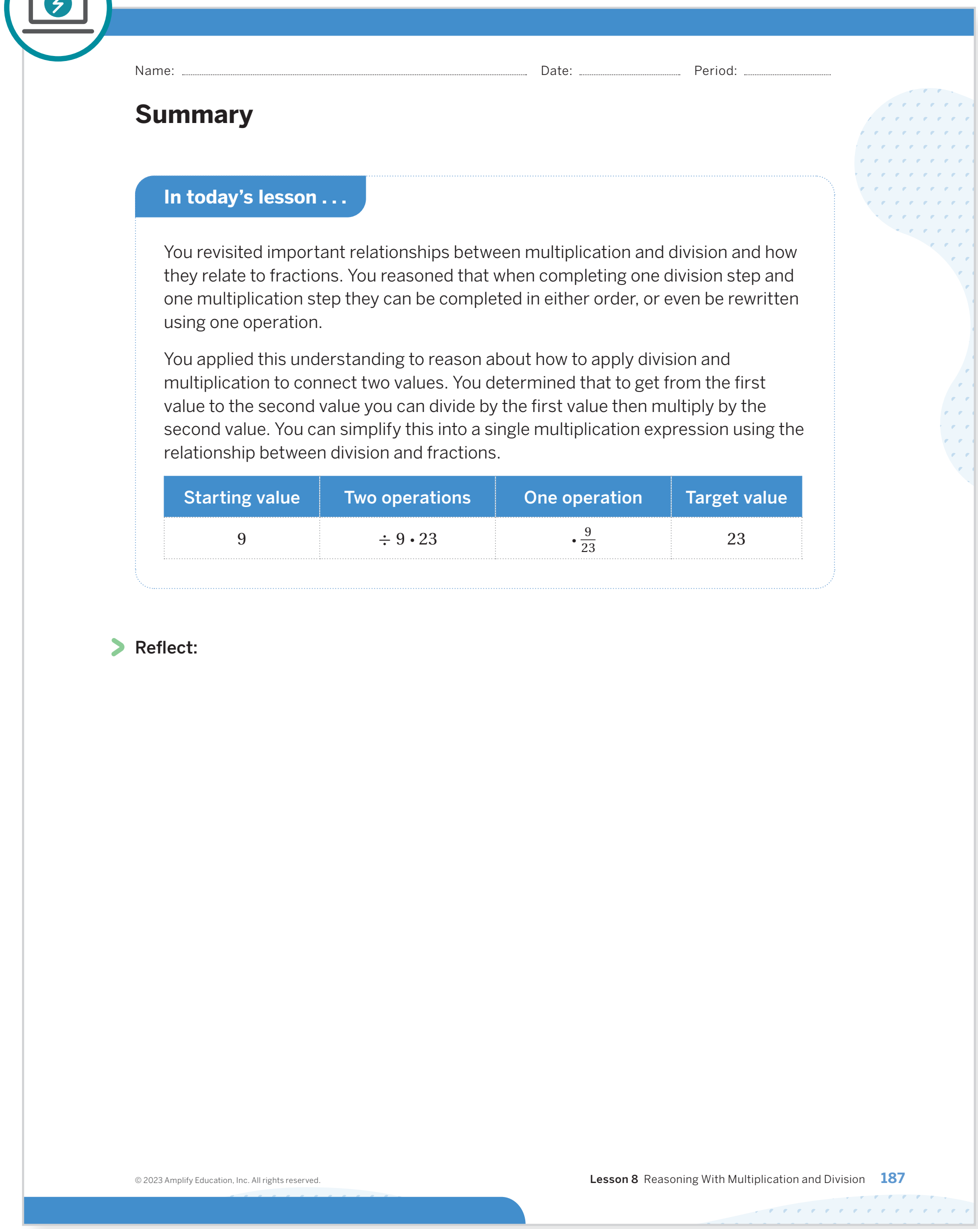
English Learners

As students describe the patterns they see, annotate the *Two operations* and *One operation* columns, using one color to illustrate the relationship between division and the denominator of the fraction and another color to illustrate the multiplication and the numerator of the fraction.

Whole Class | 5 min

Summary

Review and synthesize how multiplication and division can be used to connect any two whole numbers.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You revisited important relationships between multiplication and division and how they relate to fractions. You reasoned that when completing one division step and one multiplication step they can be completed in either order, or even be rewritten using one operation.

You applied this understanding to reason about how to apply division and multiplication to connect two values. You determined that to get from the first value to the second value you can divide by the first value then multiply by the second value. You can simplify this into a single multiplication expression using the relationship between division and fractions.

Starting value	Two operations	One operation	Target value
9	$\div 9 \cdot 23$	$\cdot \frac{9}{23}$	23

Reflect:

Synthesize

Display the numbers 9 and 23.

Ask,

- "How could you go from 9 to 23 by using only multiplication and division?"
- "How could you go from 23 to 9 by using only multiplication and division?"
- "How could you represent each of those by using only one multiplication or division operation?"
- "How are those fractions related?"
- (optional) "How might any of this be related to working with ratios and equivalent ratios?"

Have students share responses to the questions, one at a time, and capture two sets of work for all to see.

Highlight that when going from a lesser value to a greater value, the divisor will always be greater than the corresponding multiplication factor. This also makes sense because, for the related single fraction multiplication, the numerator will be greater than the denominator. This means students are multiplying by a value greater than 1. And the opposite, going from a greater value to a lesser value, is also always true. Students will be working more with factors and multiples in the next two lessons, and all of this thinking will prove useful when students get back to working with ratios and equivalent ratios.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can fact families help you in writing multiplication and division expressions?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of relating two given whole numbers using multiplication, division, and fractions.

Printable

Name: ______________________ Date: __________ Period: __________

Exit Ticket

2.08

Consider the starting number 30 and the target number 18.

1. Write *two different expressions* that each shows a sequence of exactly *two* multiplication or division operations to connect those numbers.
   Sample response: $30 \div 15 \cdot 9$; $30 \div 5 \cdot 3$

2. Write *two different expressions* that each shows exactly *one* multiplication or division operation to connect those numbers.
   Sample response: $30 \cdot \frac{9}{15}$; $30 \cdot \frac{3}{5}$

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a Given a multiplication or division expression, I can write a related expression to evaluate the same product or quotient.
1 2 3

b Given any two whole numbers, I can write a sequence of multiplication and division operations that relates one value to the other.
1 2 3

c Given any two whole numbers, I can write a single multiplication or division operation that relates one value to the other.
1 2 3

Lesson 8 Reasoning With Multiplication and Division

Success looks like . . .

- **Goal:** Identifying sequences of multiplication and division to get from one number to another.
    - » Writing two expressions using multiplication and division to show how the number 30 connects to 18 in Problem 1.
- **Goal:** Identifying a single factor to get from one number to another.
    - » Writing an expression with either multiplication or division to connect the number 30 to the number 18 in Problem 2.

Suggested next steps

If students cannot write one expression by using two operations (Problem 1), consider:

- Having them refer back to the Two operations column in their table from Activity 2.

If students cannot write two expressions by using two operations (Problem 1), consider:

- Using their first expression and asking, "What if you doubled what you were multiplying by there?"

If students cannot write corresponding multiplication operations with fractions (Problem 2), consider:

- Having them refer back to the One operation column in their table from Activity 2.

Professional Learning

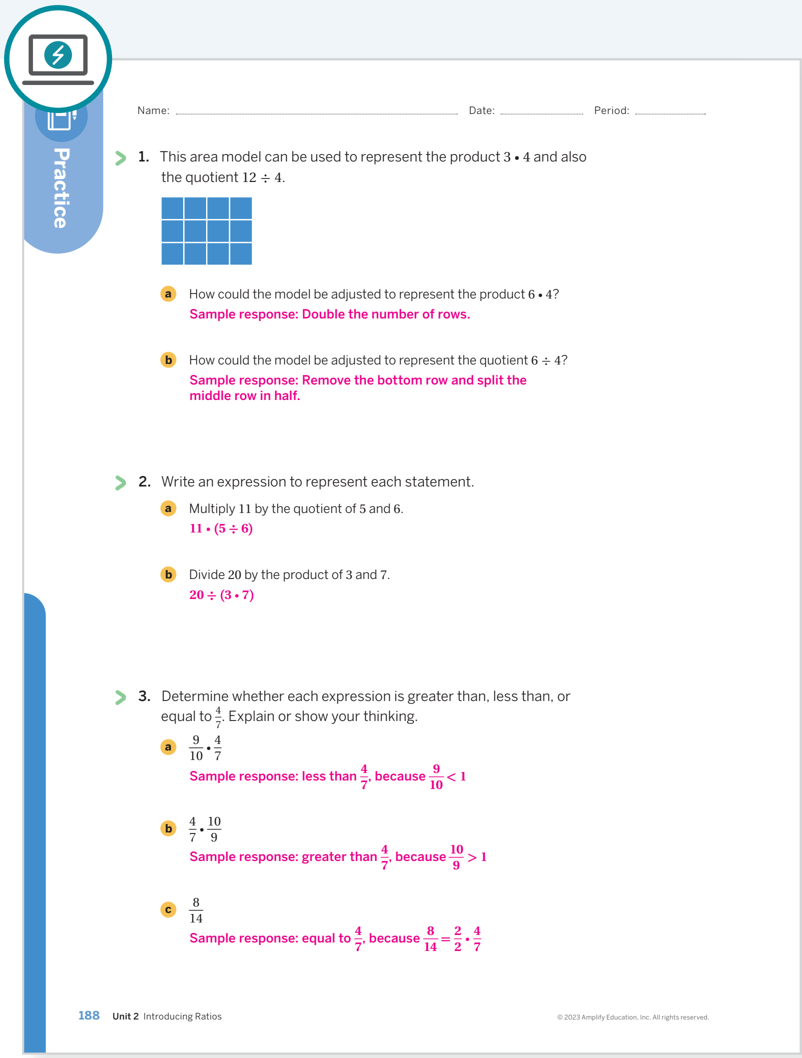
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In earlier lessons, students worked with equivalent ratios. How did that support multiplication and division?
- What trends do you see in participation? What might you change the next time you teach this lesson?

Practice

Independent


Practice

Name: Date: Period:

1. This area model can be used to represent the product 3 • 4 and also the quotient 12 ÷ 4.

   a. How could the model be adjusted to represent the product 6 • 4?
   Sample response: Double the number of rows.

   b. How could the model be adjusted to represent the quotient 6 ÷ 4?
   Sample response: Remove the bottom row and split the middle row in half.

2. Write an expression to represent each statement.

   a. Multiply 11 by the quotient of 5 and 6.
   $11 \bullet (5 \div 6)$

   b. Divide 20 by the product of 3 and 7.
   $20 \div (3 \bullet 7)$

3. Determine whether each expression is greater than, less than, or equal to $\frac{4}{7}$. Explain or show your thinking.

   a. $\frac{9}{10} \bullet \frac{4}{7}$
   Sample response: less than $\frac{4}{7}$, because $\frac{9}{10} < 1$

   b. $\frac{4}{7} \bullet \frac{10}{9}$
   Sample response: greater than $\frac{4}{7}$, because $\frac{10}{9} > 1$

   c. $\frac{8}{14}$
   Sample response: equal to $\frac{4}{7}$, because $\frac{8}{14} = \frac{2}{2} \bullet \frac{4}{7}$

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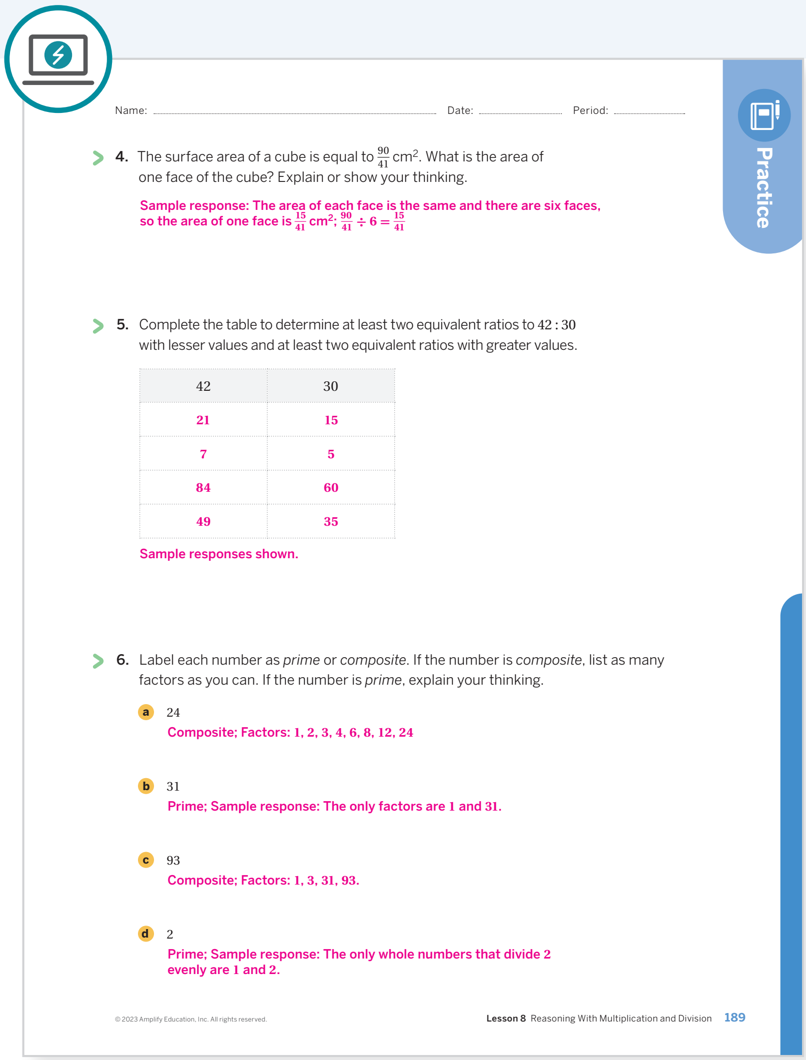
Practice

Name: Date: Period:

4. The surface area of a cube is equal to $\frac{90}{41}$ cm^2. What is the area of one face of the cube? Explain or show your thinking.
   Sample response: The area of each face is the same and there are six faces, so the area of one face is $\frac{15}{41}$ cm^2; $\frac{90}{41} \div 6 = \frac{15}{41}$

5. Complete the table to determine at least two equivalent ratios to 42 : 30 with lesser values and at least two equivalent ratios with greater values.

42	30
21	**15**
7	**5**
84	**60**
49	**35**

   Sample responses shown.

6. Label each number as *prime* or *composite*. If the number is *composite*, list as many factors as you can. If the number is *prime*, explain your thinking.

   a. 24
   Composite; Factors: 1, 2, 3, 4, 6, 8, 12, 24

   b. 31
   Prime; Sample response: The only factors are 1 and 31.

   c. 93
   Composite; Factors: 1, 3, 31, 93.

   d. 2
   Prime; Sample response: The only whole numbers that divide 2 evenly are 1 and 2.

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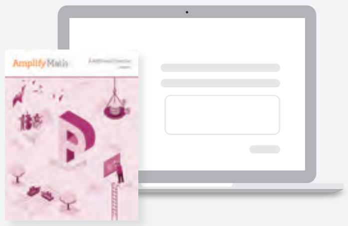
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	2
	2	Activity 1	1
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 18	3
	5	Unit 2 Lesson 7	2
Formative ⚡	6	Unit 2 Lesson 9	1, 2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 2 | **LESSON 9**

Common Factors

Let's use factors to solve problems.

Focus

Goals

1. **Language Goal:** Comprehend the terms *factor*, *common factor*, and *greatest common factor*. **(Speaking and Listening, Writing)**
2. **Language Goal:** Explain how to determine the greatest common factor of two whole numbers less than 100. **(Speaking and Listening, Writing)**
3. **Language Goal:** List the factors of a number and identify common factors for two numbers in a real-world scenario. **(Writing)**

Coherence

Today

Students identify common factors of two whole numbers in both mathematical and real-world contexts, such as forming equal groups of percussionists. They recognize the greatest common factor for two whole numbers as the common factor whose value is the greatest. Students attend to the meanings of these terms in working with both mathematical problems and a geometric context of covering an area using same-size squares.

Previously

In Lesson 7, students learned to organize a set of equivalent ratios in a table. If students completed optional Lesson 8, they also explored different ways of relating two whole numbers with multiplication, division, or both.

Coming Soon

In Lesson 10, students will learn about multiples and the least common multiple. As students continue to work with ratio problems in the rest of this unit, they will be able to apply common factors to more efficiently navigate a set of equivalent ratios and to determine missing values. In Unit 6, students will relate common factors to the Distributive Property in order to write equivalent expressions.

Rigor

- Students develop **conceptual understanding** of what the terms *common factor* and *greatest common factor* mean and how they relate to each other.
- Students **apply** common factors in real-world and mathematical contexts.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- graph paper (optional)
- snap cubes (optional)

Math Language Development

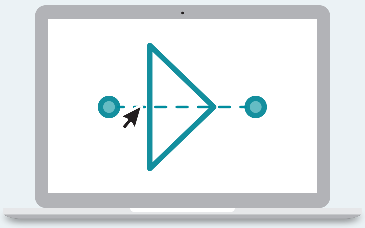
New words

- *common factor*
- *greatest common factor*

Amps | **Featured Activity**

Activity 2
Square Tiling

Students use common factors to determine dimensions for square pieces of paper that will cover a bulletin board area with no gaps or overlaps.



Building Math Identity and Community

Connecting to Mathematical Practices

When describing how the new term *greatest common factor* relates to the bulletin board in Activity 2, students may be frustrated that others do not understand what they are trying to say. To promote clear communication, encourage partners to revoice what the other is saying in their own words so that both students have the same understanding and so that students can revise what and how they are communicating for better clarity.

Modifications to Pacing

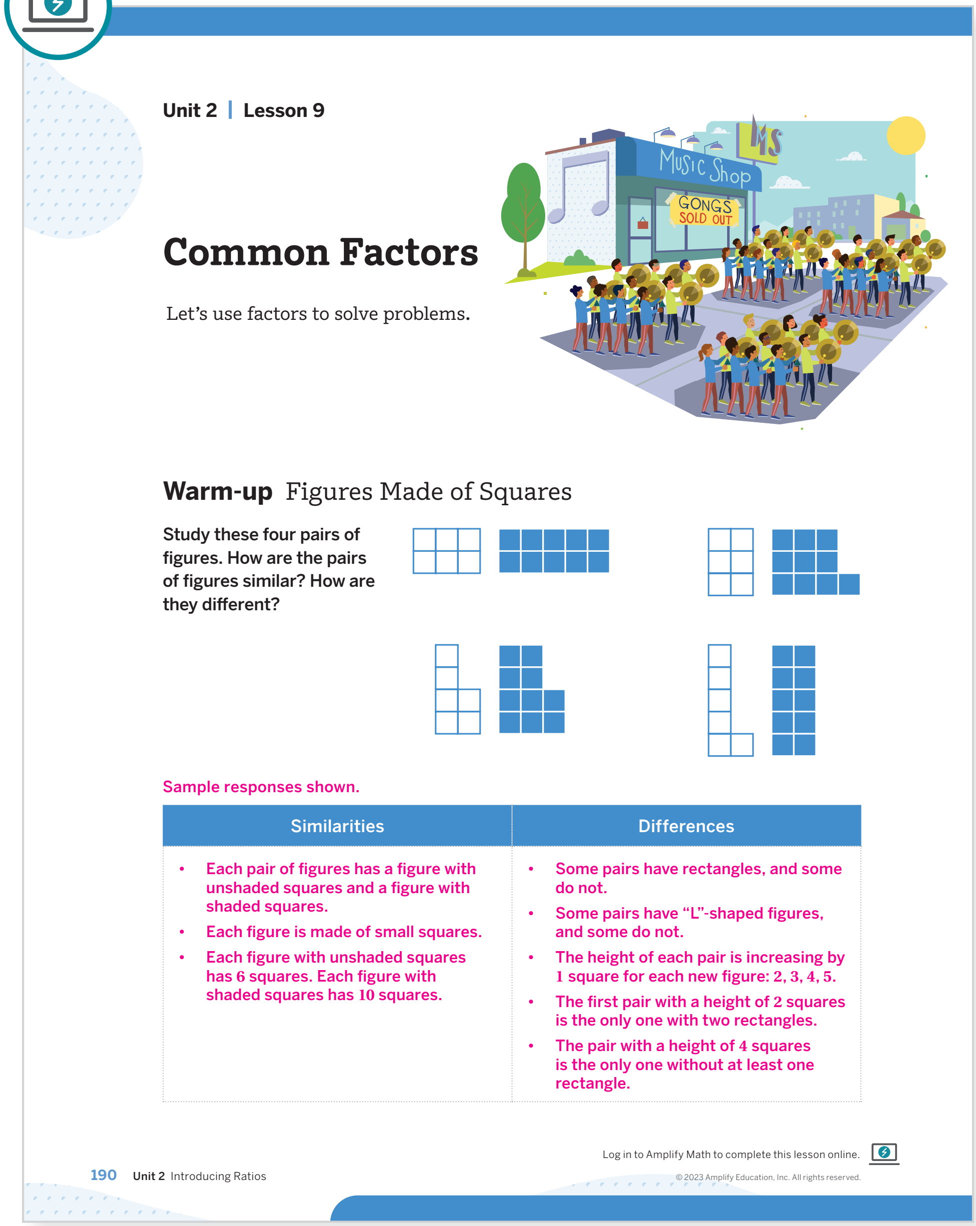
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have students only complete Problems 1 and 2, and if time allows, discuss Problem 3 as a whole class.
- In **Activity 2**, present the definition of *greatest common factor* for students in Problem 1 and then have students only complete Problems 2 and 3.

Independent | 5 min

Warm-up Figures Made of Squares

Students identify the similarities and differences in groups of 6 and 10 squares that do and do not form rectangles, as a basis for thinking about factors and common factors.



Unit 2 | Lesson 9

Common Factors

Let's use factors to solve problems.

Warm-up Figures Made of Squares

Study these four pairs of figures. How are the pairs of figures similar? How are they different?

Sample responses shown.

Similarities	Differences
Each pair of figures has a figure with unshaded squares and a figure with shaded squares. Each figure is made of small squares. Each figure with unshaded squares has 6 squares. Each figure with shaded squares has 10 squares.	Some pairs have rectangles, and some do not. Some pairs have "L"-shaped figures, and some do not. The height of each pair is increasing by 1 square for each new figure: 2, 3, 4, 5. The first pair with a height of 2 squares is the only one with two rectangles. The pair with a height of 4 squares is the only one without at least one rectangle.

Log in to Amplify Math to complete this lesson online.

190 Unit 2 Introducing Ratios

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1 Launch

Display the four pairs of images as students answer the questions and throughout the discussion.

2 Monitor

Help students get started by asking students to concentrate on one pair of images at a time.

Look for points of confusion:

- **Thinking that the shapes are doubling each time.** Have students look carefully at the shapes and the number of squares so they can realize that the shapes are not doubling in size.

Look for productive strategies:

- Noticing that every unshaded shape contains 6 squares and every blue shape contains 10 squares.
- Observing that, when going clockwise from the upper left, the heights of each pair are increasing by 1 every time.
- Noticing that the top left are both rectangles made up of numbers of squares that are divisible by 2, and only the bottom left has no rectangles.

3 Connect

Have individual students share the similarities and differences they noticed in the sets of shapes. Record and display their responses beside the images.

Ask,

- "If 2 and 3 are both factors of 6, how is this reflected in the diagrams?"
- "If 2 is a factor of both 6 and 10, how is this reflected in the diagrams?"
- "What do the diagrams show you about whether 4 is a factor of 6 or 10?"

Highlight that, if students focus on the height in each pair of images, it represents a factor of the total squares of each color because the shape is a rectangle.

Math Language Development

MLR7: Compare and Connect

Ask students to compare the similarities and differences they notice among the pairs of figures. Highlight the various attributes that students notice.

English Learners

As you highlight connections regarding the rows and columns of the figures, use gestures, such as using your arms to make an "L" shape to help students make sense of the connections.

Power-up

To power up students' ability to identify factors of a given number, have students complete:

Recall that a *factor* is a number that divides evenly into a given whole number. Determine if 9 is a factor of each value.

a. 18 yes

b. 33 no

c. 36 yes

d. 96 no

e. 108 yes

f. 3 no

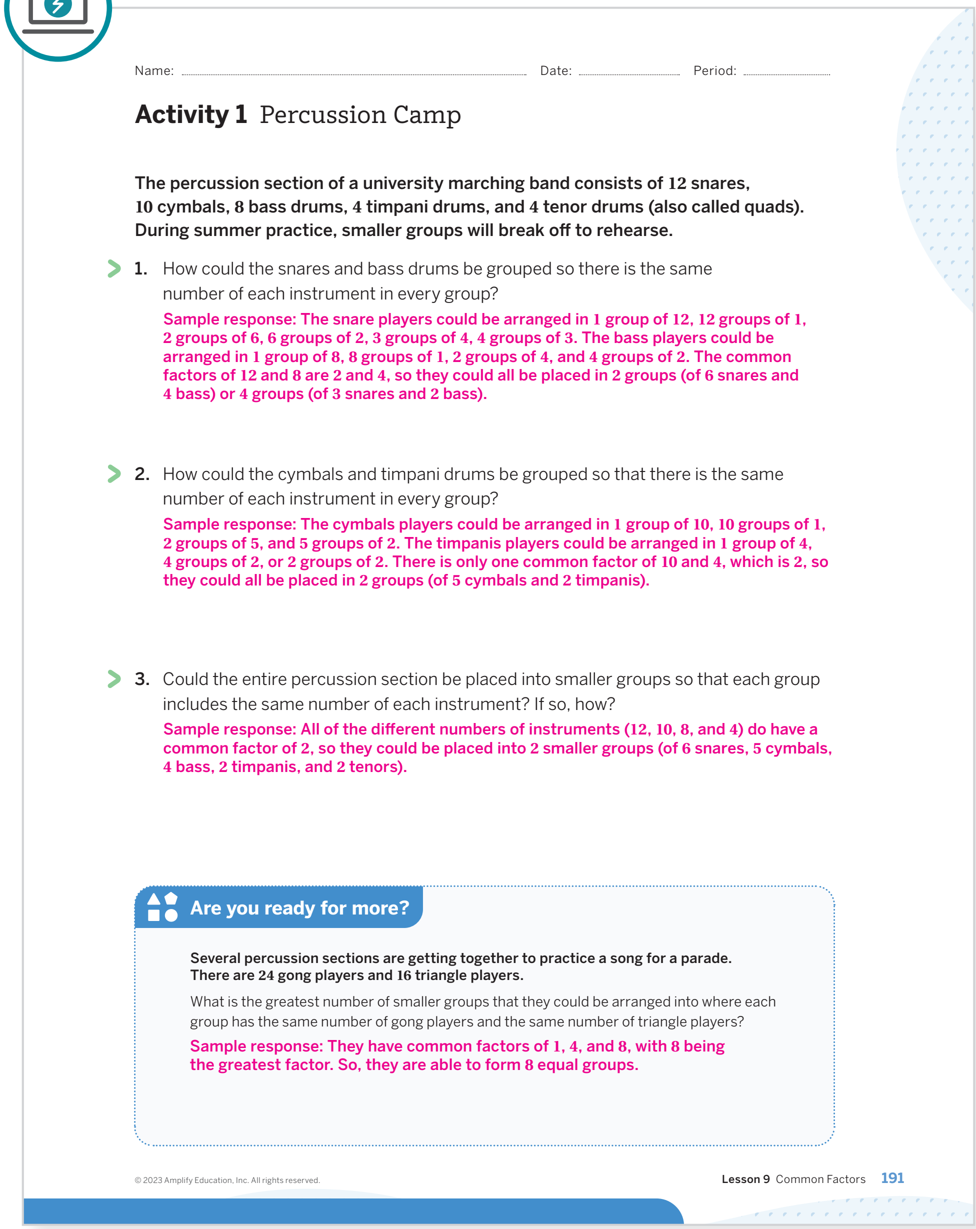
Use: Before the Warm-up.

Informed by: Performance on Lesson 8, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 6.

Pairs | 15 min

Activity 1 Percussion Camp

Students determine the common factors of different numbers of percussion instruments by forming equal groups, which also builds a foundation for the greatest common factor.



Name: ______ Date: ______ Period: ______

Activity 1 Percussion Camp

The percussion section of a university marching band consists of 12 snares, 10 cymbals, 8 bass drums, 4 timpani drums, and 4 tenor drums (also called quads). During summer practice, smaller groups will break off to rehearse.

1. How could the snares and bass drums be grouped so there is the same number of each instrument in every group?

   Sample response: The snare players could be arranged in 1 group of 12, 12 groups of 1, 2 groups of 6, 6 groups of 2, 3 groups of 4, 4 groups of 3. The bass players could be arranged in 1 group of 8, 8 groups of 1, 2 groups of 4, and 4 groups of 2. The common factors of 12 and 8 are 2 and 4, so they could all be placed in 2 groups (of 6 snares and 4 bass) or 4 groups (of 3 snares and 2 bass).

2. How could the cymbals and timpani drums be grouped so that there is the same number of each instrument in every group?

   Sample response: The cymbals players could be arranged in 1 group of 10, 10 groups of 1, 2 groups of 5, and 5 groups of 2. The timpanis players could be arranged in 1 group of 4, 4 groups of 2, or 2 groups of 2. There is only one common factor of 10 and 4, which is 2, so they could all be placed in 2 groups (of 5 cymbals and 2 timpanis).

3. Could the entire percussion section be placed into smaller groups so that each group includes the same number of each instrument? If so, how?

   Sample response: All of the different numbers of instruments (12, 10, 8, and 4) do have a common factor of 2, so they could be placed into 2 smaller groups (of 6 snares, 5 cymbals, 4 bass, 2 timpanis, and 2 tenors).

Are you ready for more?

Several percussion sections are getting together to practice a song for a parade. There are 24 gong players and 16 triangle players.

What is the greatest number of smaller groups that they could be arranged into where each group has the same number of gong players and the same number of triangle players?

Sample response: They have common factors of 1, 4, and 8, with 8 being the greatest factor. So, they are able to form 8 equal groups.

1 Launch

Have students read the introductory paragraph and the three problems independently and allow some think time (without writing) before they work on the problems with their partner.

2 Monitor

Help students get started by asking, "How many snare drums and bass drums are there?"

Look for points of confusion:

- **Not knowing what a factor represents.** Guide students in writing the factor pairs for one number.
- **Not determining all combinations of factor pairs for each number.** Refer to the Warm-up and have students draw a diagram, or use snap cubes to help them determine more possible combinations.

Look for productive strategies:

- Determining the factor pairs for the pairs of numbers in Problems 1 and 2 and then looking for a common factor.
- Concluding that the entire band camp group can only be arranged into 2 smaller groups because they already knew 2 was a common factor of every instrument other than tenors, and 2 is a common factor of 4.

3 Connect

Have pairs of students share their responses for Problems 1–2 and their explanation from Problem 3 with the class.

Define a ***common factor*** of two or more numbers as a number that divides evenly into each number. Add this term and its related information to your classroom anchor chart.

Highlight that in this context, the possible numbers of equal groups can be represented by the common factors. Although there were other common factors in Problem 1, there is only one common factor of 2 for Problem 2 and for every instrument in Problem 3.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1 and 2, and only work on Problem 3 as time allows. Consider having students draw rectangles using factor pairs.

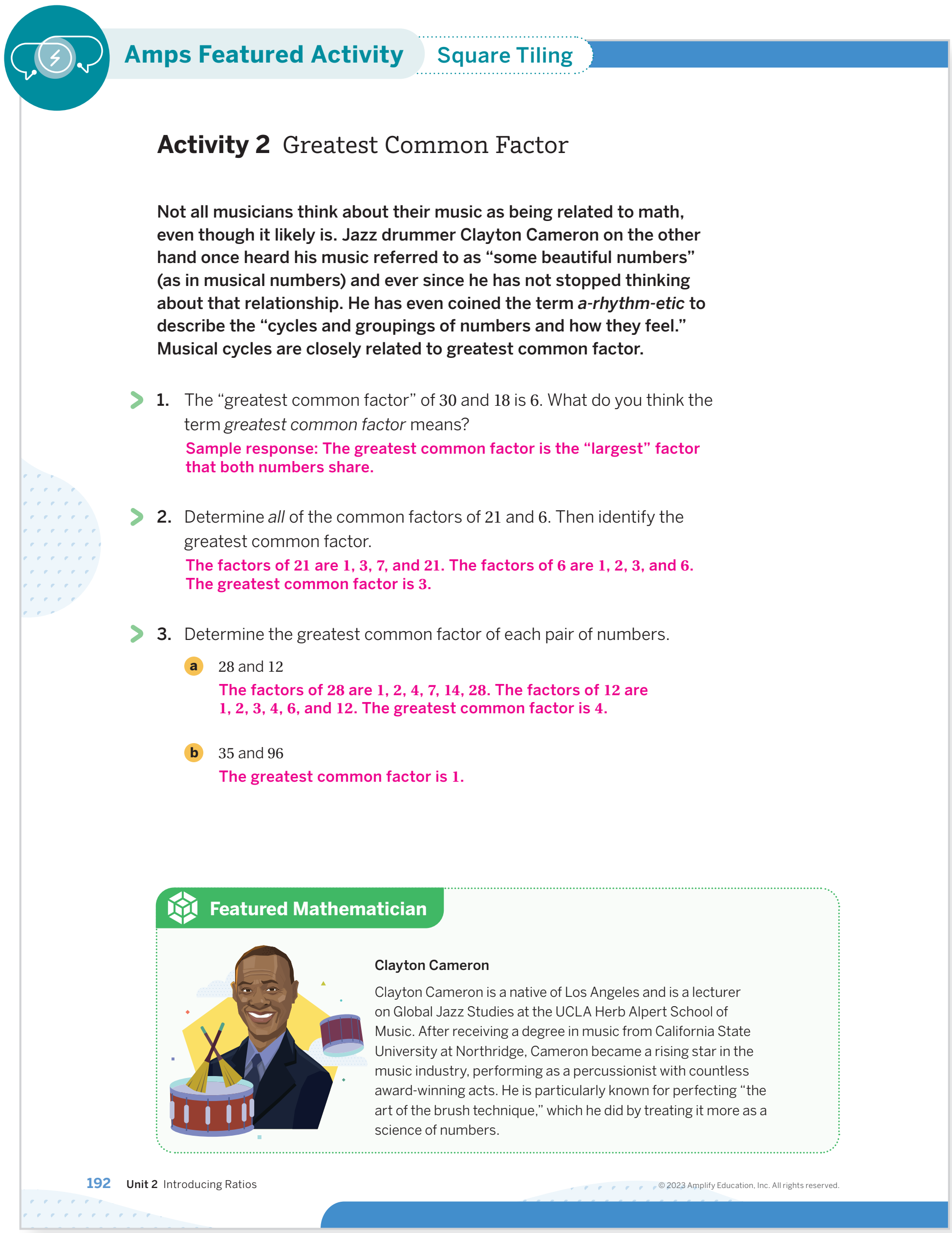
Accessibility: *Optimize Access to Tools*

Provide students with access to virtual or concrete manipulatives, such as snap cubes, counters, or small circles or squares of paper, to represent the instruments.

Pairs | 15 min

Activity 2 Greatest Common Factor

Students extend the concept of common factors to the greatest common factor, which can be used to maximize groups or dimensions.



Amps Featured Activity Square Tiling

Activity 2 Greatest Common Factor

Not all musicians think about their music as being related to math, even though it likely is. Jazz drummer Clayton Cameron on the other hand once heard his music referred to as "some beautiful numbers" (as in musical numbers) and ever since he has not stopped thinking about that relationship. He has even coined the term *a-rhythm-etic* to describe the "cycles and groupings of numbers and how they feel." Musical cycles are closely related to greatest common factor.

1. The "greatest common factor" of 30 and 18 is 6. What do you think the term *greatest common factor* means?
   Sample response: The greatest common factor is the "largest" factor that both numbers share.
2. Determine *all* of the common factors of 21 and 6. Then identify the greatest common factor.
   The factors of 21 are 1, 3, 7, and 21. The factors of 6 are 1, 2, 3, and 6. The greatest common factor is 3.
3. Determine the greatest common factor of each pair of numbers.
   - a 28 and 12
     The factors of 28 are 1, 2, 4, 7, 14, 28. The factors of 12 are 1, 2, 3, 4, 6, and 12. The greatest common factor is 4.
   - b 35 and 96
     The greatest common factor is 1.

Featured Mathematician

Clayton Cameron

Clayton Cameron is a native of Los Angeles and is a lecturer on Global Jazz Studies at the UCLA Herb Alpert School of Music. After receiving a degree in music from California State University at Northridge, Cameron became a rising star in the music industry, performing as a percussionist with countless award-winning acts. He is particularly known for perfecting "the art of the brush technique," which he did by treating it more as a science of numbers.

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1 Launch

Keep students in pairs and ask them to discuss their responses to Problem 1. Choose a few groups to share their thinking, and ensure the class has been presented with an adequate working definition before moving on to Problems 2–4.

2 Monitor

Help students get started by asking, "What are two numbers that can be multiplied together to make a product of 6? What about 21?"

Look for points of confusion:

- **Including some but not all factors of a number.** Have students list out all factor pairs of each number and then put their unique factors in order.

Look for productive strategies:

- Listing factors of each number in order (least to greatest).
- Recognizing for Problem 4, they need to determine the greatest common factor of 12 and 27.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to graph paper for students to draw rectangles with a certain area.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see whether the square pieces of paper — whose dimensions were determined by using common factors — actually cover the area of a bulletin board, with no gaps or overlaps.

Math Language Development

MLR2: Collect and Display

Collect and display the language students use to describe the *greatest common factor* in Problem 1. Listen for students using *common factor* versus *greatest common factor*. Highlight the difference between the two phrases.

English Learners

Focus on first identifying the common factors of two numbers, before discussing the greatest common factor between the two numbers.

Featured Mathematician

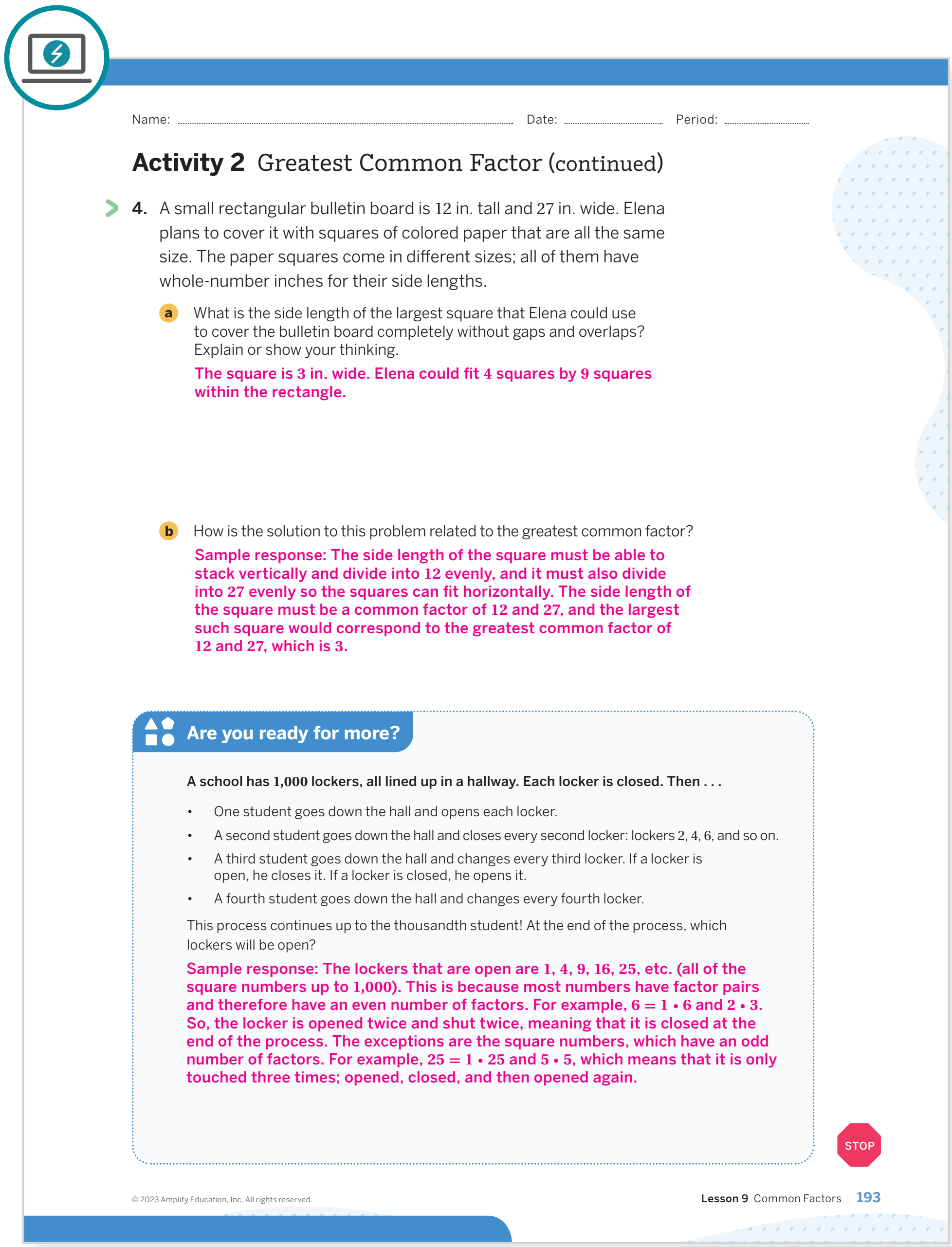
Clayton Cameron

Have students read about Clayton Cameron, jazz musician and lecturer at the UCLA Herb Alpert School of Music, and how he thinks about music as a science of numbers.

Pairs | 15 min

Activity 2 Greatest Common Factor (continued)

Students extend the concept of common factors to the greatest common factor, which can be used to maximize groups or dimensions.



Name: ………… Date: ………… Period: …………

Activity 2 Greatest Common Factor (continued)

4. A small rectangular bulletin board is 12 in. tall and 27 in. wide. Elena plans to cover it with squares of colored paper that are all the same size. The paper squares come in different sizes; all of them have whole-number inches for their side lengths.

   a. What is the side length of the largest square that Elena could use to cover the bulletin board completely without gaps and overlaps? Explain or show your thinking.

   The square is 3 in. wide. Elena could fit 4 squares by 9 squares within the rectangle.

   b. How is the solution to this problem related to the greatest common factor?

   Sample response: The side length of the square must be able to stack vertically and divide into 12 evenly, and it must also divide into 27 evenly so the squares can fit horizontally. The side length of the square must be a common factor of 12 and 27, and the largest such square would correspond to the greatest common factor of 12 and 27, which is 3.

Are you ready for more?

A school has 1,000 lockers, all lined up in a hallway. Each locker is closed. Then . . .

- One student goes down the hall and opens each locker.
- A second student goes down the hall and closes every second locker: lockers 2, 4, 6, and so on.
- A third student goes down the hall and changes every third locker. If a locker is open, he closes it. If a locker is closed, he opens it.
- A fourth student goes down the hall and changes every fourth locker.

This process continues up to the thousandth student! At the end of the process, which lockers will be open?

Sample response: The lockers that are open are 1, 4, 9, 16, 25, etc. (all of the square numbers up to 1,000). This is because most numbers have factor pairs and therefore have an even number of factors. For example, $6 = 1 \cdot 6$ and $2 \cdot 3$. So, the locker is opened twice and shut twice, meaning that it is closed at the end of the process. The exceptions are the square numbers, which have an odd number of factors. For example, $25 = 1 \cdot 25$ and $5 \cdot 5$, which means that it is only touched three times; opened, closed, and then opened again.

STOP

3 Connect

Display the two given numbers for each problem, one problem at a time.

Have pairs of students share their responses.

Define the ***greatest common factor*** (often abbreviated as GCF) of two or more given whole numbers as the common factor of all of the numbers whose value is the greatest. Add this term to your classroom anchor chart.

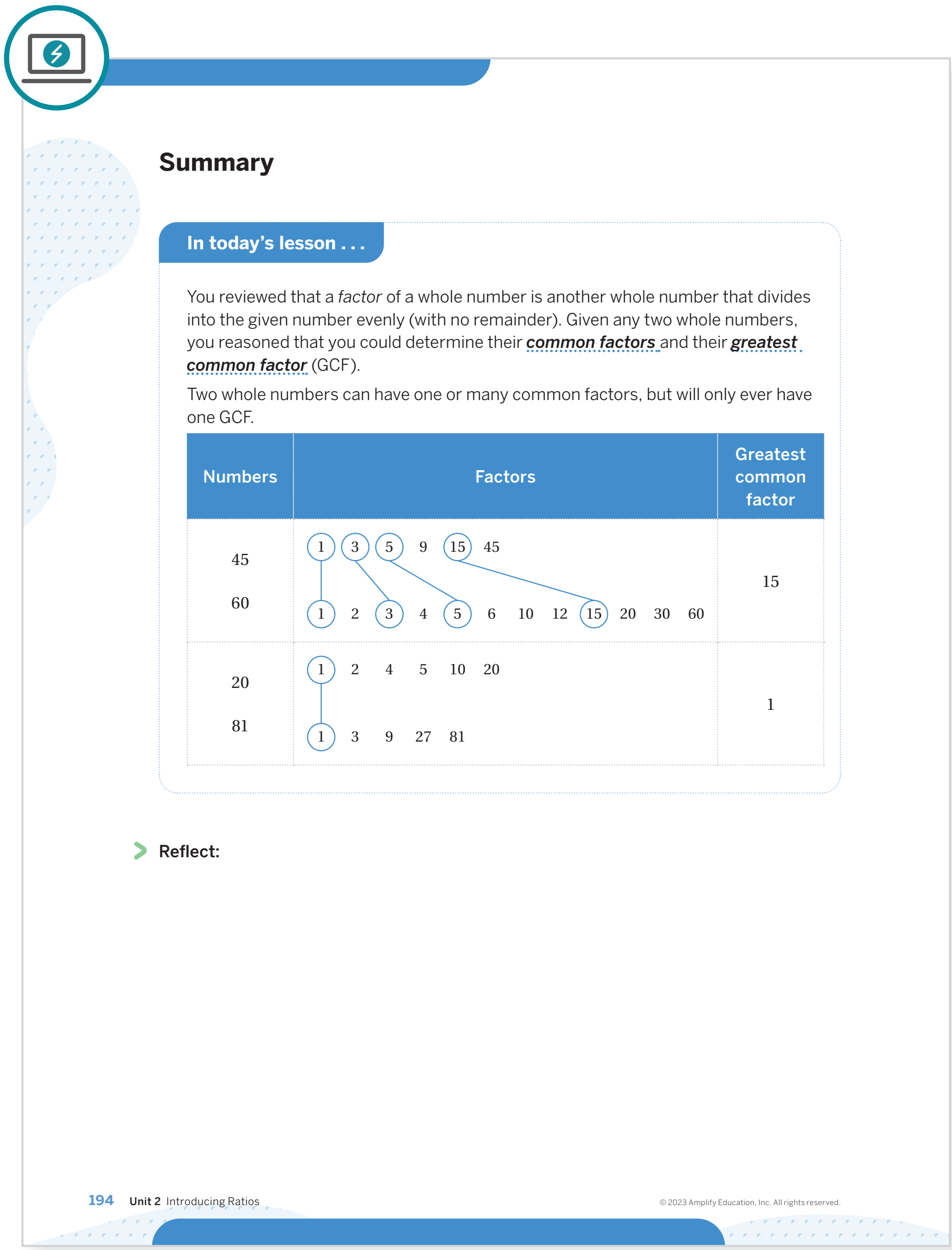
Highlight that every pair of whole numbers has a greatest common factor. Students should list all factors of both numbers in order to determine the GCF. If one number is a factor of the other, then that number is the GCF; or if they share no other common factors, then the GCF is 1.

Ask, "How would your response change if the bulletin board was 18 in. tall and 63 in. wide instead?" Squares of colored paper could have 9-in. sides.

Whole Class | 5 min

Summary

Review and synthesize the meanings of the terms *common factor* and *greatest common factor* for two numbers and how those relate to determining possible equal groups.



Summary

In today's lesson . . .

You reviewed that a *factor* of a whole number is another whole number that divides into the given number evenly (with no remainder). Given any two whole numbers, you reasoned that you could determine their ***common factors*** and their ***greatest common factor*** (GCF).

Two whole numbers can have one or many common factors, but will only ever have one GCF.

Numbers	Factors	Greatest common factor
45 60	① ③ ⑤ 9 ⑮ 45 ① 2 ③ 4 ⑤ 6 10 12 ⑮ 20 30 60	15
20 81	① 2 4 5 10 20 ① 3 9 27 81	1

Reflect:

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Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the terms *common factor* and *greatest common factor* that were added to the display during the lesson.

Synthesize

Display a blank version of the table from the Summary with the numbers 45 and 60.

Have students share factors for each of the two numbers until all have been captured, one number at a time. Record these first in the "factors" column of the table. Then have students identify common factors and circle them in the table. Lastly, have students identify the GCF as the greatest factor circled.

Highlight that two numbers can have more than one common factor but only one greatest common factor.

Formalize vocabulary:

- *common factor*
- *greatest common factor*

Ask,

- "What are some scenarios when it is helpful to use the greatest common factor?" When forming the largest amount of equal mixed groups with no items left over, or when determining the largest side length of a square that can be used to tile a rectangle.
- "Describe a process for how you can determine the greatest common factor of two whole numbers." List the factors of each number, circle the ones that are the same, and then find the largest number that is the same.
- "How would your process change if you were determining the greatest common factor of three whole numbers?" I would list their factors as well and only be looking for ones that are common across all three lists, and then identify the one with the greatest value.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

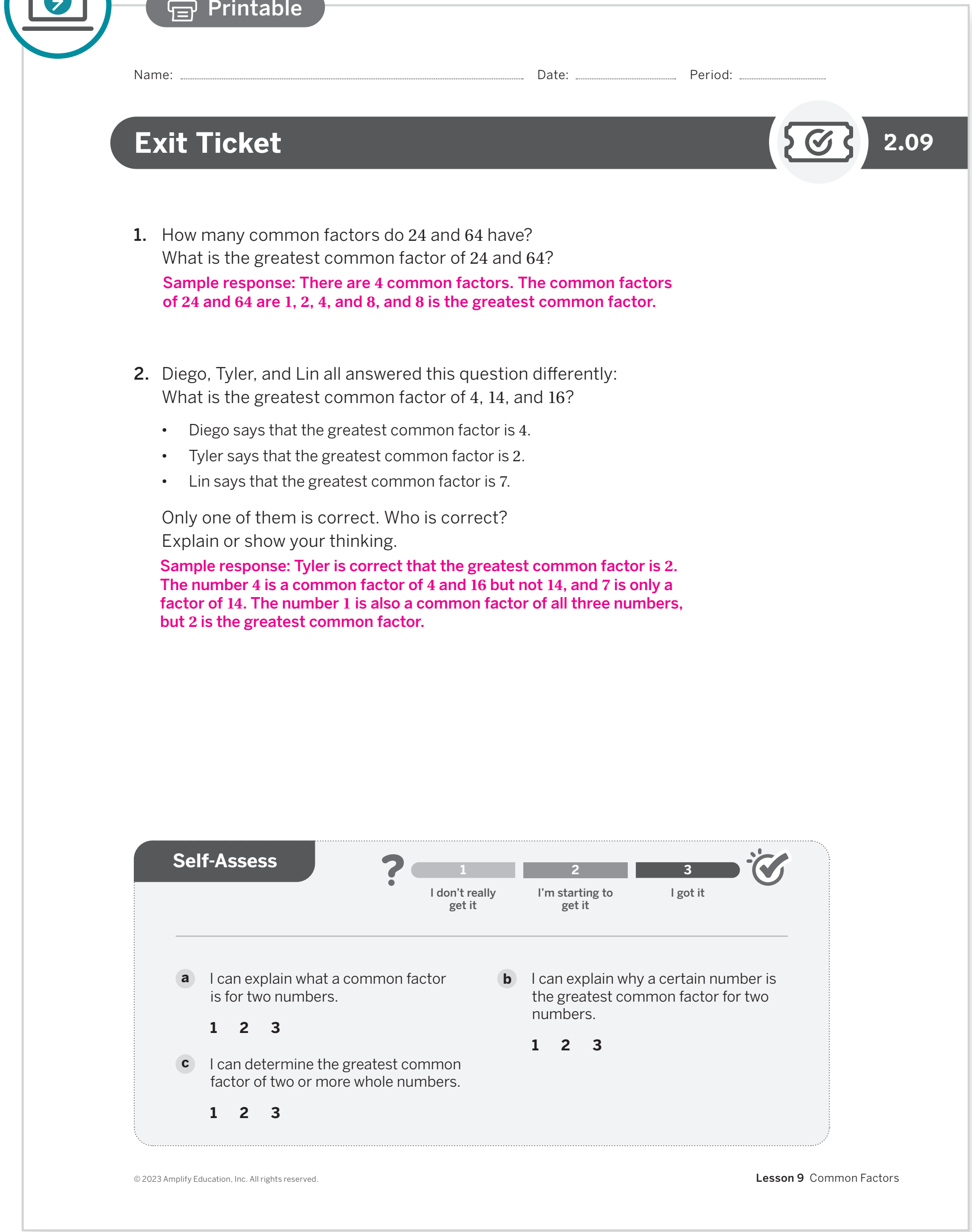
- "How do divisibility rules help you think about common factors?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how to determine the greatest common factor of two whole numbers.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 2.09

1. How many common factors do 24 and 64 have?
   What is the greatest common factor of 24 and 64?
   Sample response: There are 4 common factors. The common factors of 24 and 64 are 1, 2, 4, and 8, and 8 is the greatest common factor.

2. Diego, Tyler, and Lin all answered this question differently:
   What is the greatest common factor of 4, 14, and 16?
   - Diego says that the greatest common factor is 4.
   - Tyler says that the greatest common factor is 2.
   - Lin says that the greatest common factor is 7.

   Only one of them is correct. Who is correct?
   Explain or show your thinking.
   Sample response: Tyler is correct that the greatest common factor is 2. The number 4 is a common factor of 4 and 16 but not 14, and 7 is only a factor of 14. The number 1 is also a common factor of all three numbers, but 2 is the greatest common factor.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can explain what a common factor is for two numbers.
1 2 3

b I can explain why a certain number is the greatest common factor for two numbers.
1 2 3

c I can determine the greatest common factor of two or more whole numbers.
1 2 3

Lesson 9 Common Factors

Success looks like . . .

- **Language Goal:** Comprehending the terms *factor*, *common factor*, and *greatest common factor*. **(Speaking and Listening, Writing)**
- **Language Goal:** Explaining how to determine the greatest common factor of two whole numbers less than 100. **(Speaking and Listening, Writing)**
- **Language Goal:** Listing the factors of a number and identifying common factors for two numbers in a real-world situation. **(Writing)**
  - » Listed the common factors for 24 and 64 in Problem 1.

Suggested next steps

If students cannot identify all of the common factors for Problem 1, consider:

- Reviewing how they found common factors in Activity 1.
- Asking, "What are all the factor pairs of 24? Of 64?"
- Assigning Practice Problem 2.

If students cannot determine the greatest common factor of 4, 14 and 16 in Problem 2, consider:

- Reviewing one or more problems from Activity 2.

Professional Learning

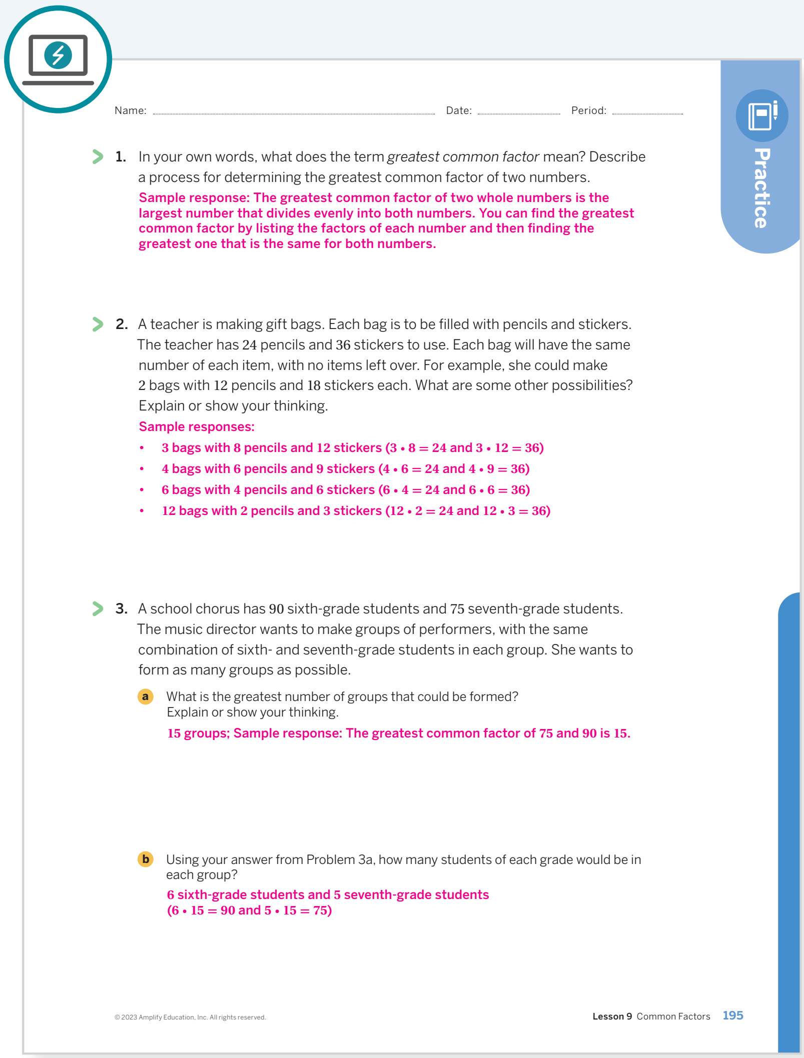
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students determine common factors and the greatest common factor. How will that support work with the least common denominator?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

Practice

1. In your own words, what does the term *greatest common factor* mean? Describe a process for determining the greatest common factor of two numbers.
Sample response: The greatest common factor of two whole numbers is the largest number that divides evenly into both numbers. You can find the greatest common factor by listing the factors of each number and then finding the greatest one that is the same for both numbers.

2. A teacher is making gift bags. Each bag is to be filled with pencils and stickers. The teacher has 24 pencils and 36 stickers to use. Each bag will have the same number of each item, with no items left over. For example, she could make 2 bags with 12 pencils and 18 stickers each. What are some other possibilities? Explain or show your thinking.
Sample responses:
- **3 bags with 8 pencils and 12 stickers ($3 \cdot 8 = 24$ and $3 \cdot 12 = 36$)**
- **4 bags with 6 pencils and 9 stickers ($4 \cdot 6 = 24$ and $4 \cdot 9 = 36$)**
- **6 bags with 4 pencils and 6 stickers ($6 \cdot 4 = 24$ and $6 \cdot 6 = 36$)**
- **12 bags with 2 pencils and 3 stickers ($12 \cdot 2 = 24$ and $12 \cdot 3 = 36$)**

3. A school chorus has 90 sixth-grade students and 75 seventh-grade students. The music director wants to make groups of performers, with the same combination of sixth- and seventh-grade students in each group. She wants to form as many groups as possible.
   a. What is the greatest number of groups that could be formed? Explain or show your thinking.
   15 groups; Sample response: The greatest common factor of 75 and 90 is 15.
   b. Using your answer from Problem 3a, how many students of each grade would be in each group?
   6 sixth-grade students and 5 seventh-grade students ($6 \cdot 15 = 90$ and $5 \cdot 15 = 75$)

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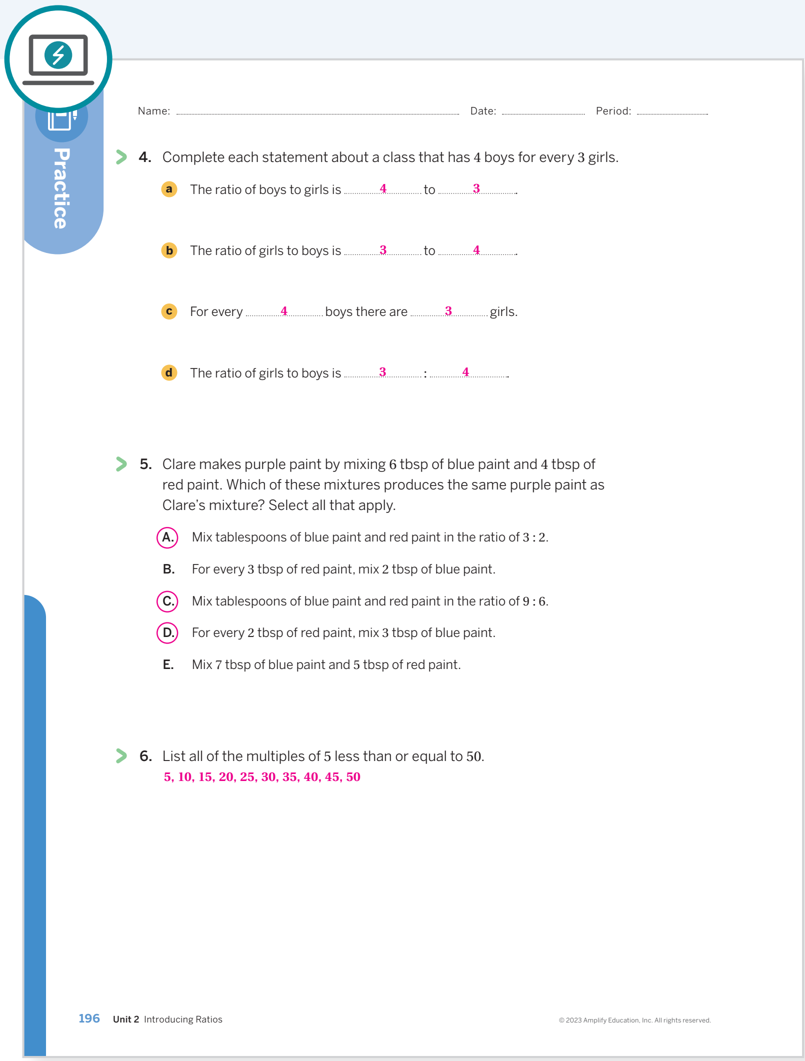
Lesson 9 Common Factors 195



Name: ________ Date: ________ Period: ________

Practice

4. Complete each statement about a class that has 4 boys for every 3 girls.
   a. The ratio of boys to girls is **4** to **3**.
   b. The ratio of girls to boys is **3** to **4**.
   c. For every **4** boys there are **3** girls.
   d. The ratio of girls to boys is **3** : **4**.

5. Clare makes purple paint by mixing 6 tbsp of blue paint and 4 tbsp of red paint. Which of these mixtures produces the same purple paint as Clare's mixture? Select all that apply.
   [✓ A.] Mix tablespoons of blue paint and red paint in the ratio of 3 : 2.
   B. For every 3 tbsp of red paint, mix 2 tbsp of blue paint.
   [✓ C.] Mix tablespoons of blue paint and red paint in the ratio of 9 : 6.
   [✓ D.] For every 2 tbsp of red paint, mix 3 tbsp of blue paint.
   E. Mix 7 tbsp of blue paint and 5 tbsp of red paint.

6. List all of the multiples of 5 less than or equal to 50.
5, 10, 15, 20, 25, 30, 35, 40, 45, 50

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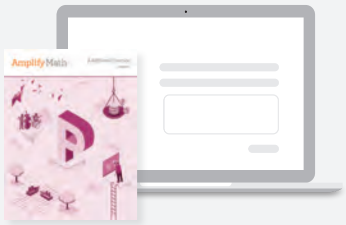
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 1	3
Spiral	4	Unit 2 Lesson 1	1
	5	Unit 2 Lesson 5	2
Formative ⚡	6	Unit 2 Lesson 10	3

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 2 | **LESSON 10**

Common Multiples

Let's use multiples to solve problems.

Focus

Goals

1. **Language Goal:** Comprehend the terms *multiple*, *common multiple*, and *least common multiple*. **(Speaking and Listening, Writing)**
2. **Language Goal:** Explain how to calculate the least common multiple of two whole numbers. **(Speaking and Listening, Writing)**
3. **Language Goal:** List the multiples of a number and identify common multiples for two numbers in a real-world situation. **(Writing)**

Coherence

Today

Students identify common multiples of two whole numbers in both mathematical and real-world contexts, such as identifying when two sounds in a rhythmic pattern will occur on the same beats. They recognize patterns in both multiples and common multiples. And they recognize the least common multiple for two whole numbers as the common multiple whose value is the least. Students attend to the meanings of these terms when working with both mathematical and real-world problems.

Previously

In Lesson 9, students determined common factors and the greatest common factor of two whole numbers and applied this in mathematical and real-world problems.

Coming Soon

In Lesson 11, students will shift back to their exploration of equivalent ratios by using common multiples and common factors to more efficiently navigate a table of equivalent ratios. They will begin to target missing values by determining both ratios containing a 1 for a given ratio.

Rigor

- Students build **conceptual understanding** of common multiples and least common multiples of two numbers.
- Students **apply** common multiples in real-world and mathematical contexts.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Whole Class	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (instructions)
- metronome

Math Language Development

New words

- *common multiple*
- *least common multiple*

Amps | Featured Activity

Activity 2

Interactive Least Common Multiple

Students will enter multiples and receive real-time feedback on accuracy and completeness.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may have difficulty with describing the rhythms in the Warm-up and Activity 1. Lead a discussion on barriers that students may encounter relative to this context and to the mathematics it represents. Have them think about and discuss ways in which they could overcome these obstacles to describe the patterns. Have students use other methods, such as drawing or writing, to express the regularity of the patterns of beats that they hear.

Modifications to Pacing

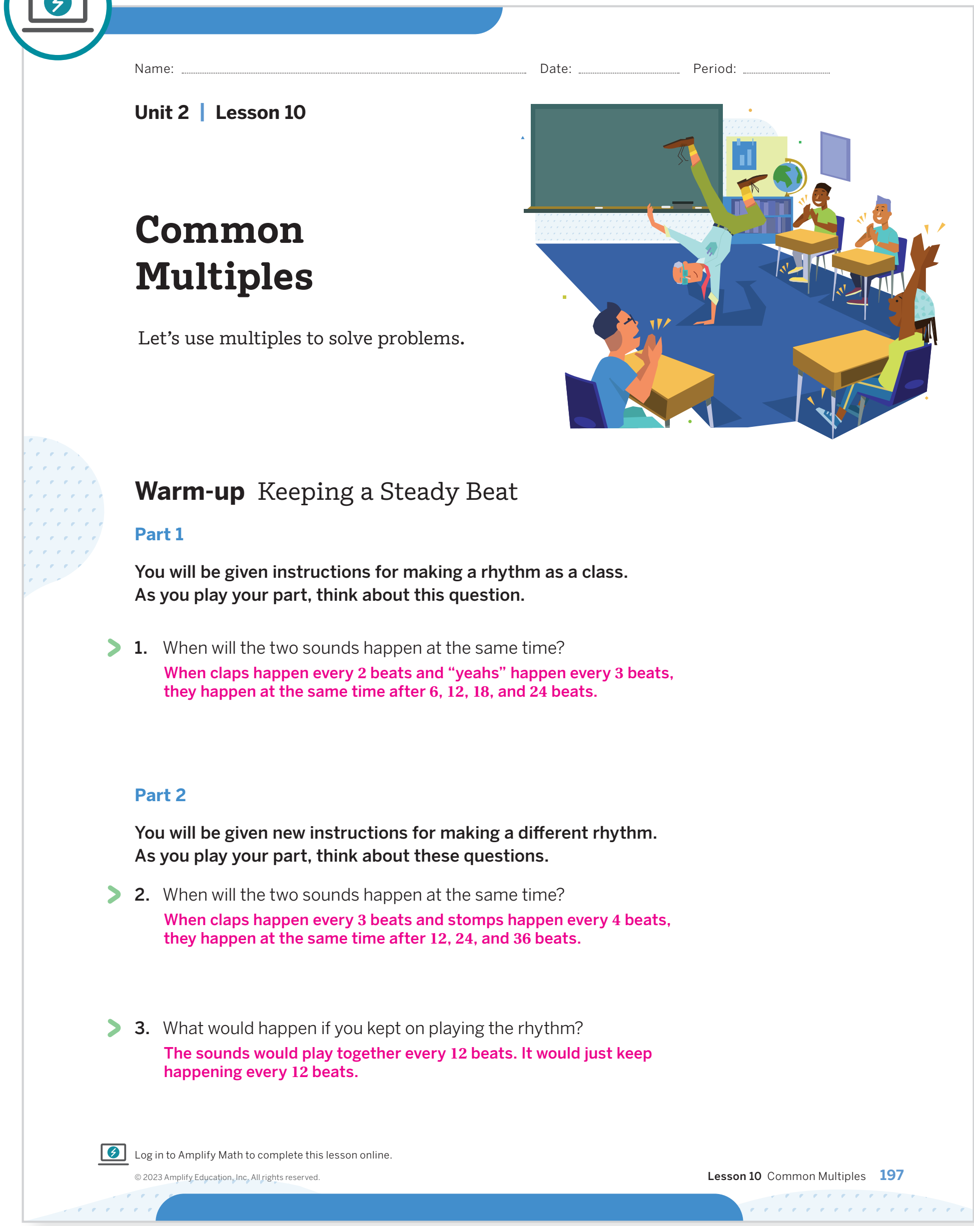
You may want to consider this additional modification if you are short on time.

- You may choose to omit either the **Warm-up** or **Activity 1**, however, be sure to define the term *common multiples*.

Whole Class | 5 min

Warm-up Keeping a Steady Beat

Students create a rhythm as a class to begin thinking about multiples and events (sounds) that occur at the same time (beats), preparing them to determine common multiples.



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Unit 2 | Lesson 10

Common Multiples

Let's use multiples to solve problems.

Warm-up Keeping a Steady Beat

Part 1

You will be given instructions for making a rhythm as a class. As you play your part, think about this question.

1. When will the two sounds happen at the same time?
   When claps happen every 2 beats and "yeahs" happen every 3 beats, they happen at the same time after 6, 12, 18, and 24 beats.

Part 2

You will be given new instructions for making a different rhythm. As you play your part, think about these questions.

2. When will the two sounds happen at the same time?
   When claps happen every 3 beats and stomps happen every 4 beats, they happen at the same time after 12, 24, and 36 beats.

3. What would happen if you kept on playing the rhythm?
   The sounds would play together every 12 beats. It would just keep happening every 12 beats.

Log in to Amplify Math to complete this lesson online.

Lesson 10 Common Multiples 197

1 Launch

Review the Warm-up PDF (instructions). After the first rhythm, allow students to answer the first problem and share their responses. After the second rhythm, allow students time to complete the next two problems.

2 Monitor

Help students get started by doing short demonstrations as you count, with the metronome, 1, 2, 3, but with only one group participating at a time.

Look for points of confusion:

- **Not being able to multitask.** Have these students listen as the rest of the class puts the rhythm out, instead of making any sounds themselves.

Look for productive strategies:

- Recognizing that the beats will occur at the same time at 6, 12, 18, and 24 beats for Part 1 and at 12, 24, and 36 beats for Part 2.
- Recognizing multiples of 6 as multiples of both 2 and 3.

3 Connect

Define a ***common multiple*** for two or more whole numbers as a number that is a multiple of all of the numbers. You may add this to your classroom anchor chart.

Have students share their responses to Problem 3 in Part 2. Discuss that the common multiples will keep on going if the beat keeps going.

Ask, "How do the rhythms relate to common factors or common multiples?"

Highlight that all multiples of a number can be determined by multiplying it by 2, 3, 4, and so on. To determine common multiples for two numbers, look for values that show up on those lists of multiples for both numbers.

Power-up

To power up students' ability to determine multiples of a given value, have students complete:

Recall that a *multiple* is a number that is the product of a given number and a whole number. Determine if each number is a multiple of 4.

a. 2 no **b.** 4 yes

c. 18 no **d.** 36 yes

e. 108 yes **f.** 1 no

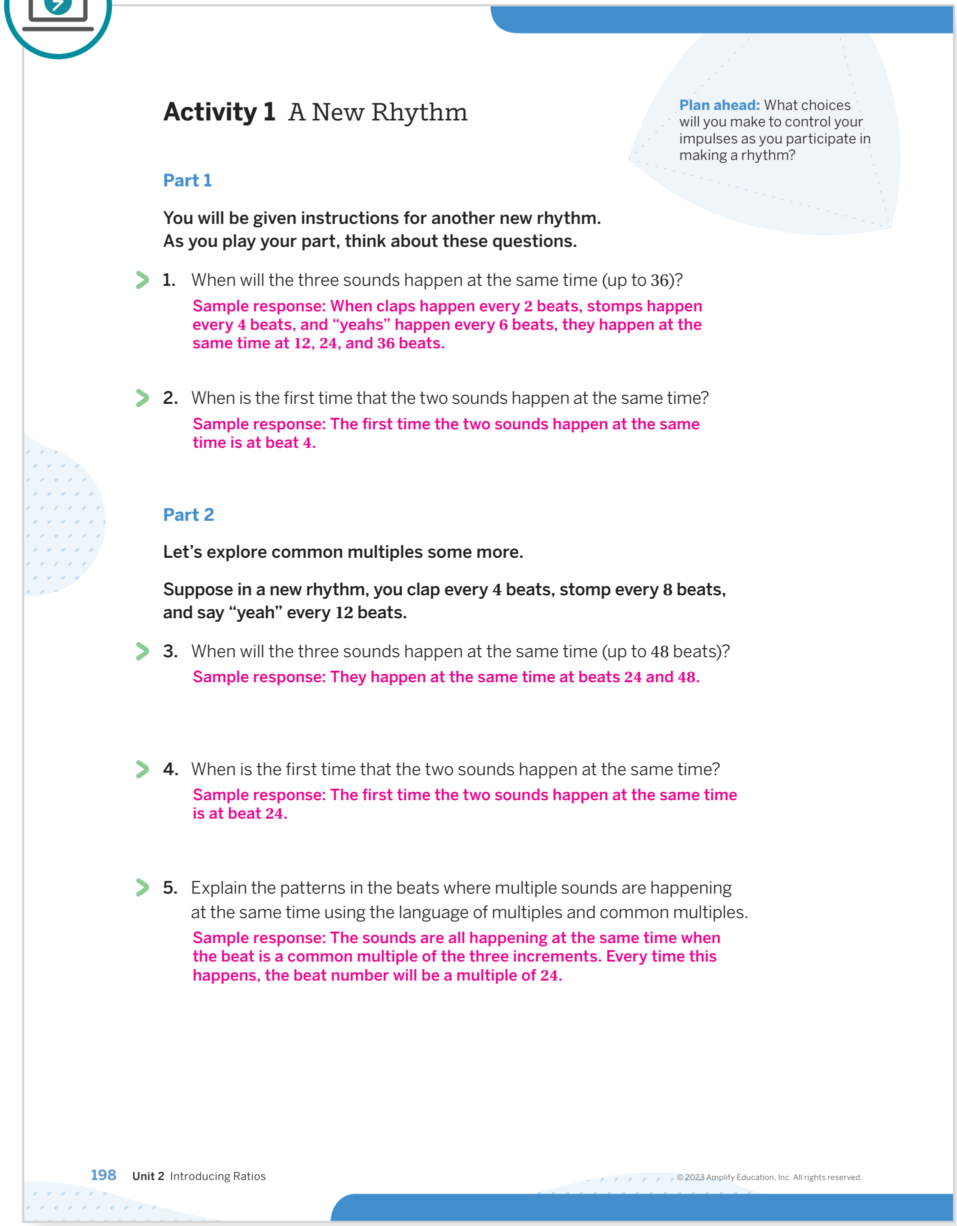
Use: Before the Warm-up.

Informed by: Performance on Lesson 9, Practice Problem 6

Pairs | 15 min

Activity 1 A New Rhythm

Students create more rhythms as a class to determine common multiples of the beats at which different sounds occur at the same time.



Activity 1 A New Rhythm

Plan ahead: What choices will you make to control your impulses as you participate in making a rhythm?

Part 1

You will be given instructions for another new rhythm. As you play your part, think about these questions.

1. When will the three sounds happen at the same time (up to 36)?
   Sample response: When claps happen every 2 beats, stomps happen every 4 beats, and "yeahs" happen every 6 beats, they happen at the same time at 12, 24, and 36 beats.
2. When is the first time that the two sounds happen at the same time?
   Sample response: The first time the two sounds happen at the same time is at beat 4.

Part 2

Let's explore common multiples some more.

Suppose in a new rhythm, you clap every 4 beats, stomp every 8 beats, and say "yeah" every 12 beats.

3. When will the three sounds happen at the same time (up to 48 beats)?
   Sample response: They happen at the same time at beats 24 and 48.
4. When is the first time that the two sounds happen at the same time?
   Sample response: The first time the two sounds happen at the same time is at beat 24.
5. Explain the patterns in the beats where multiple sounds are happening at the same time using the language of multiples and common multiples.
   Sample response: The sounds are all happening at the same time when the beat is a common multiple of the three increments. Every time this happens, the beat number will be a multiple of 24.

198 **Unit 2** Introducing Ratios

1 Launch

Use a metronome to count off beats. Assign some students to clap every 2 beats, some students to stomp every 4 beats, and some students to say "yeah" every 6 beats. Have pairs complete Part 1 before moving on to Part 2 as a class.

2 Monitor

Help students get started by doing short demonstrations as you count with the metronome, 1, 2, 3, 4, 5, 6, but with only one group participating at a time.

Look for points of confusion:

- **Struggling to hear all three sounds.** Have students make a list of the multiples of all three numbers: 2, 4, 6, to represent what they are hearing.

Look for productive strategies:

- Listing out the multiples of 2, 4, and 6 and determining 12 is the first multiple they have in common (Problem 1).
- Listing out the multiples of 4, 8, and 12 and determining that they have multiples of 24 in common (Problem 4).

3 Connect

Display the numbers 2, 4, and 6 for Part 1 and then followed by 4, 8, and 12 for Part 2.

Have pairs share their responses and how they used multiples and common multiples.

Highlight that you can determine common multiples for three or more numbers in the same way as for two numbers, by listing all of the multiples of each. In every case, the list is technically infinite, but the list of common multiples also follows a pattern.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization, Optimize Access to Tools*

Allow students to listen to the class sound out the beat instead of requiring them to physically participate. Provide access to colored pencils or highlighters for students to use to annotate common multiples and the least common multiple.

Extension: *Math Enrichment*

Have students determine the common multiples of 4, 8, and 16, up to 64. Then ask, "What patterns do you notice?" Sample response: All of the common multiples are the multiples of 64 because 4 and 8 are both factors of 64.

Math Language Development

MLR1: Stronger and Clearer Each Time

To support students in explaining their thinking for Problem 5, have them write a first draft response and then share with a partner. Partners should read the draft and ask clarifying questions to help make sense of the writing. After asking questions and discussing the draft, students should revise their writing and create a second draft, based on the feedback from their partner.

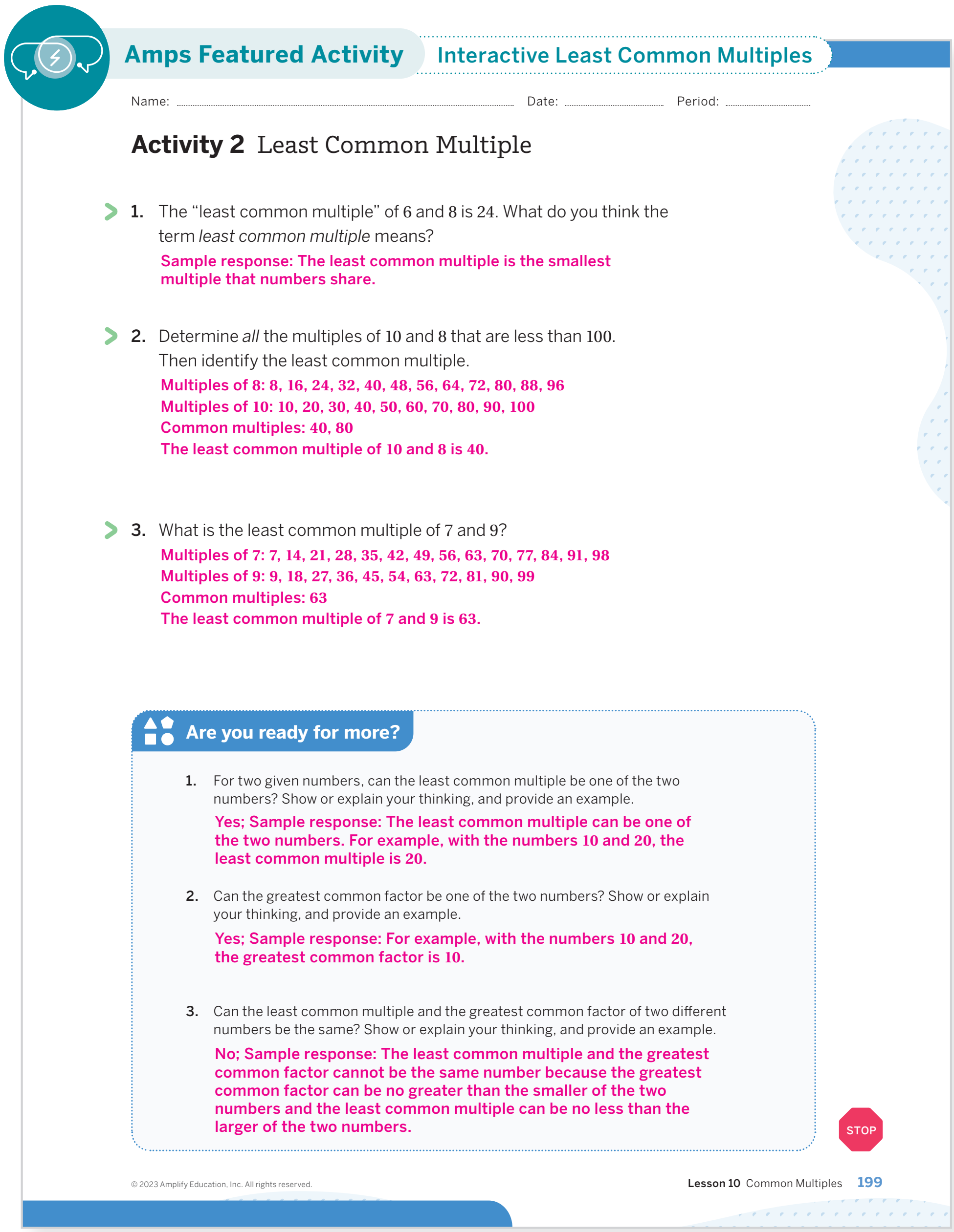
English Learners

Allow students' first draft to be written in their primary language. Their revised draft should be translated in English, with feedback and support from a strategic partner.

Pairs | 15 min

Activity 2 Least Common Multiple

Students clarify the process of finding common multiples to identify the least common multiple.



Amps Featured Activity Interactive Least Common Multiples

Name: ____________ Date: ________ Period: ________

Activity 2 Least Common Multiple

1. The "least common multiple" of 6 and 8 is 24. What do you think the term *least common multiple* means?

   Sample response: The least common multiple is the smallest multiple that numbers share.

2. Determine *all* the multiples of 10 and 8 that are less than 100. Then identify the least common multiple.

   Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96
   Multiples of 10: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100
   Common multiples: 40, 80
   The least common multiple of 10 and 8 is 40.

3. What is the least common multiple of 7 and 9?

   Multiples of 7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98
   Multiples of 9: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99
   Common multiples: 63
   The least common multiple of 7 and 9 is 63.

Are you ready for more?

1. For two given numbers, can the least common multiple be one of the two numbers? Show or explain your thinking, and provide an example.

   Yes; Sample response: The least common multiple can be one of the two numbers. For example, with the numbers 10 and 20, the least common multiple is 20.

2. Can the greatest common factor be one of the two numbers? Show or explain your thinking, and provide an example.

   Yes; Sample response: For example, with the numbers 10 and 20, the greatest common factor is 10.

3. Can the least common multiple and the greatest common factor of two different numbers be the same? Show or explain your thinking, and provide an example.

   No; Sample response: The least common multiple and the greatest common factor cannot be the same number because the greatest common factor can be no greater than the smaller of the two numbers and the least common multiple can be no less than the larger of the two numbers.

STOP

1 Launch

Keep students in pairs and ask them to discuss what the term *least common multiple* means after finishing Problem 1. Choose a few groups to share their thinking before moving on to Problems 2–3.

2 Monitor

Help students get started by having them list the first several multiples for each of the numbers, 6 and 8.

Look for points of confusion:

- **Not being able to list all of the multiples of 6 and 8.** Have students use a multiplication table to list out the multiples of 6 and 8.
- **Not knowing how to find the LCM.** Have students write out all the multiples for each number until there is at least one match.

Look for productive strategies:

- Listing the multiples of both numbers in order.
- Determining the least common multiple of both numbers.

3 Connect

Define the ***least common multiple*** (often abbreviated as LCM) as the common multiple of two or more given whole numbers whose value is the *least*. You may add this term to your classroom anchor chart.

Display Problems 2 and 3.

Have pairs of students share their responses to each problem.

Highlight that, although there may be many multiples of two numbers, there is only one least common multiple. There are also special cases where the least common multiple is the product of the two numbers, such as 7 and 9, in Problem 3.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization, Optimize Access to Tools*

Provide access to colored pencils or highlighters for students to use to annotate common multiples and the least common multiple.

Extension: *Math Enrichment*

Have students determine the least common multiple of all 12 numbers from 1 to 12. 27,720

Math Language Development

MLR2: Collect and Display

While pairs are working, circulate and listen to students talk about the meaning of the phrase *least common multiple*. Write down phrases and representations they use to help determine the least common multiple. Record these on a visual display, as this will help students use mathematical language as they represent least common multiples.

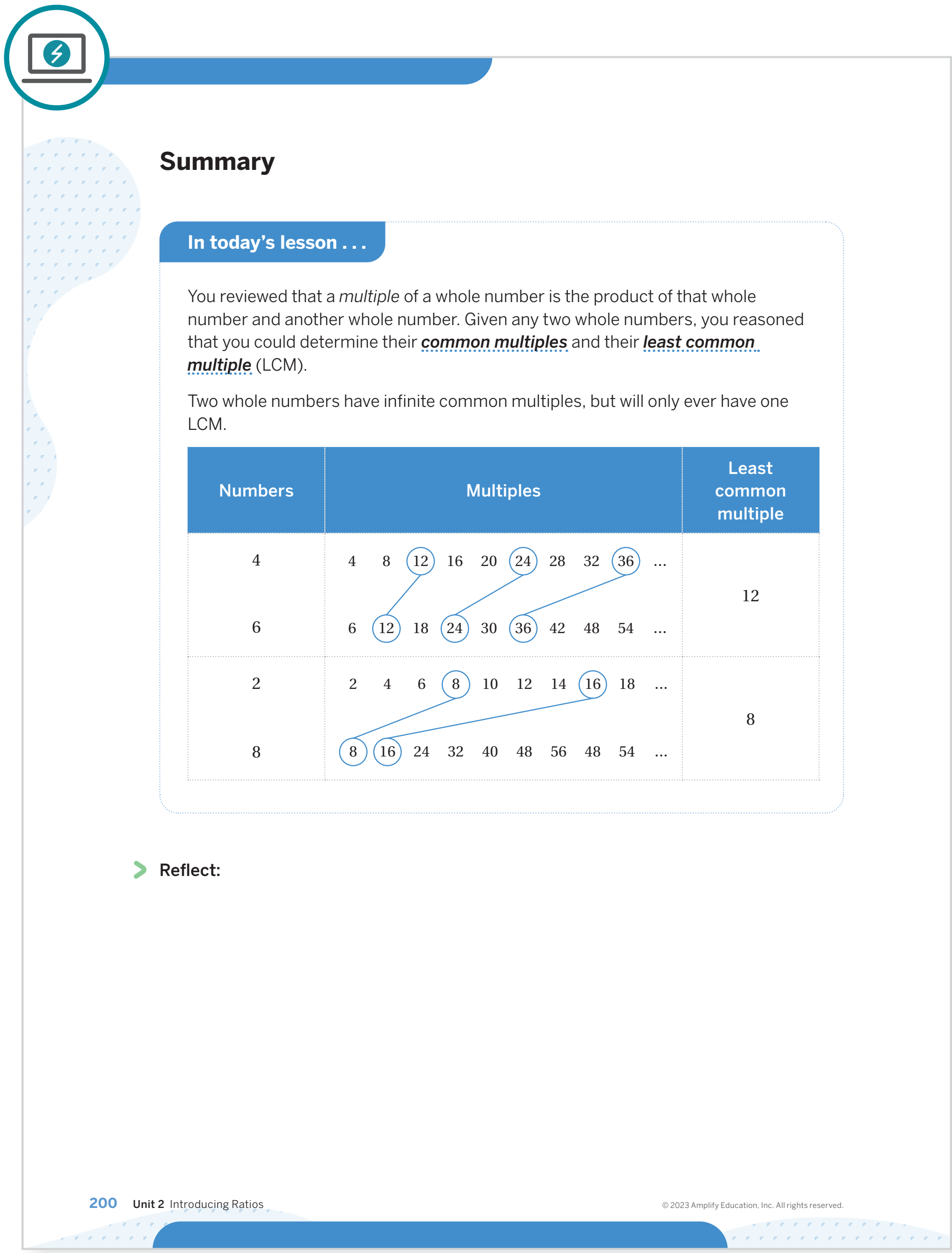
English Learners

Strengthen students' understanding of the terms least and greatest by displaying 4 or 5 numbers, in order, and then annotating the least and greatest numbers.

Whole Class | 5 min

Summary

Review and synthesize common multiples and the least common multiple for two numbers and how those can be used to determine when things happen at the same time.



Summary

In today's lesson . . .

You reviewed that a *multiple* of a whole number is the product of that whole number and another whole number. Given any two whole numbers, you reasoned that you could determine their ***common multiples*** and their ***least common multiple*** (LCM).

Two whole numbers have infinite common multiples, but will only ever have one LCM.

Numbers	Multiples	Least common multiple
4	4 8 12 16 20 24 28 32 36 ...	12
6	6 12 18 24 30 36 42 48 54 ...	
2	2 4 6 8 10 12 14 16 18 ...	8
8	8 16 24 32 40 48 56 48 54 ...	

Reflect:

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Synthesize

Display the numbers 4 and 6.

Have students share how they know what the least common multiple of 4 and 6 is.

Highlight that although many numbers will share many multiples, there is only one least common multiple.

Formalize **vocabulary:**

- ***common multiple***
- ***least common multiple***

Ask:

- "What are some situations when finding the least common multiple is helpful?" It is helpful when forming the smallest number of equal groups, or when two events first happen at the same time.
- "Explain what *least common multiple* means." It is the smallest multiple that numbers share.
- "How can you determine the least common multiple?" List multiples of each number until I find the first one that is common to both lists.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you find the least common multiple of any two given numbers?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the terms *common multiple* and *least common multiple* that were added to the display during the lesson.

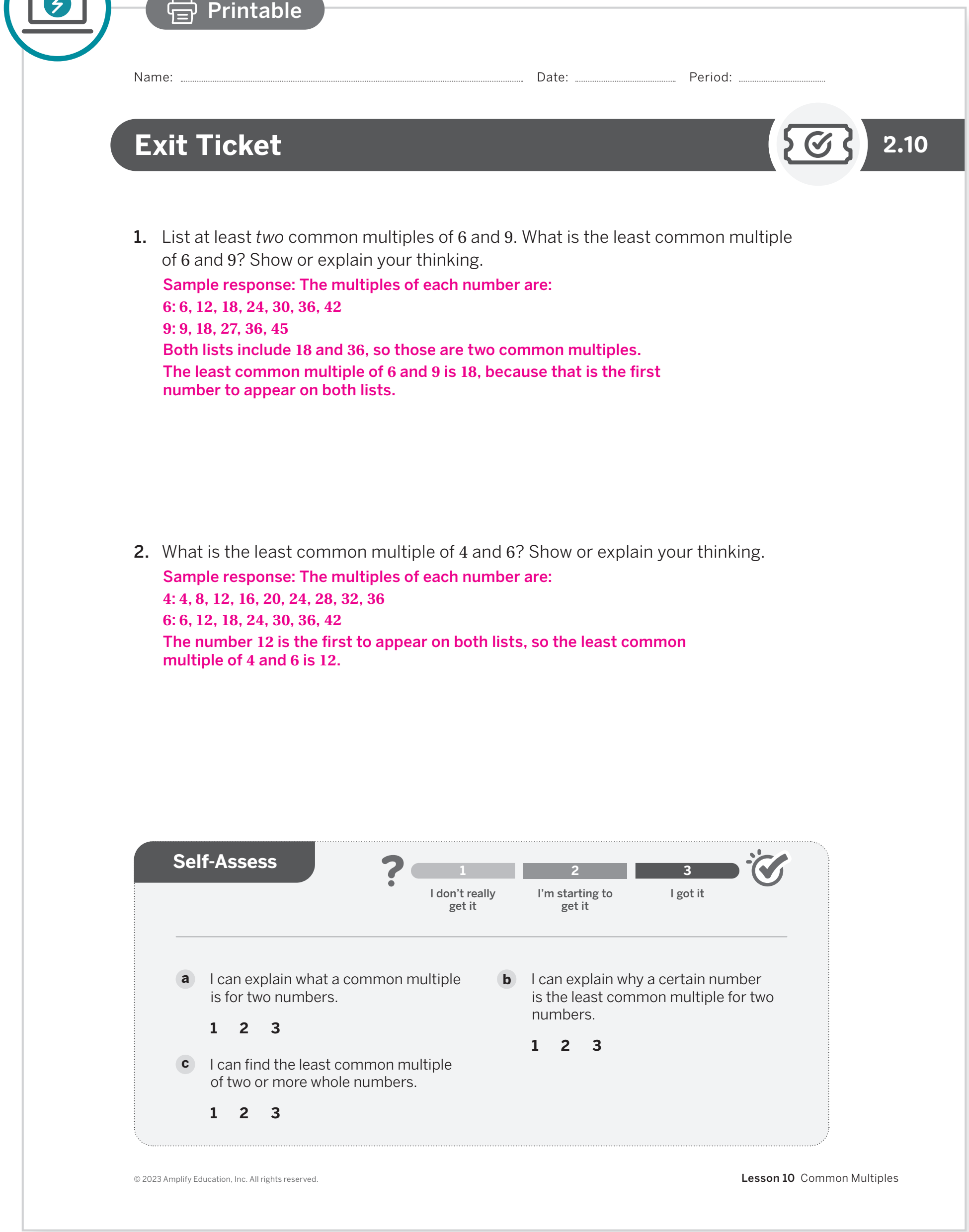
Some students may confuse *greatest common factor* and *least common multiple* and use language such as *least common factor* or *greatest common multiple*. Be sure students understand that multiples can go on forever, so there will be no greatest common multiple. Similarly, the least common factor for any two numbers will always be 1.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of least common multiples by determining the least common multiple of the numbers 6 and 9 and of the numbers 4 and 6.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 2.10

1. List at least *two* common multiples of 6 and 9. What is the least common multiple of 6 and 9? Show or explain your thinking.

Sample response: The multiples of each number are:
6: 6, 12, 18, 24, 30, 36, 42
9: 9, 18, 27, 36, 45
Both lists include 18 and 36, so those are two common multiples.
The least common multiple of 6 and 9 is 18, because that is the first number to appear on both lists.

2. What is the least common multiple of 4 and 6? Show or explain your thinking.

Sample response: The multiples of each number are:
4: 4, 8, 12, 16, 20, 24, 28, 32, 36
6: 6, 12, 18, 24, 30, 36, 42
The number 12 is the first to appear on both lists, so the least common multiple of 4 and 6 is 12.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain what a common multiple is for two numbers.
1 2 3

b I can explain why a certain number is the least common multiple for two numbers.
1 2 3

c I can find the least common multiple of two or more whole numbers.
1 2 3

Lesson 10 Common Multiples

Success looks like . . .

- **Language Goal:** Comprehending the terms *multiple*, *common multiple*, and *least common multiple*. **(Speaking and Listening, Writing)**
  - » Understanding the terms while solving Problems 1 and 2.
- **Language Goal:** Explaining how to calculate the least common multiple of two whole numbers. **(Speaking and Listening, Writing)**
  - » Determine the least common multiple and explaining how to determine it in Problems 1 and 2.
- **Language Goal:** Listing the multiples of a number and identifying common multiples for two numbers in a real-world situation. **(Writing)**
  - » Determining common multiples for the numbers 6 and 9 in Problem 1.

Suggested next steps

If students cannot determine the least common multiple of 6 and 9 in Problem 1, consider:

- Reviewing least common multiples in Activity 2.
- Assigning Practice Problem 2.
- Asking, "What are the multiples of 6? What are the multiples of 9? What are the common multiples?"

If students cannot determine the least common multiple of 4 and 6 in Problem 2, consider:

- Reviewing least common multiples in Activity 2.
- Assigning Practice Problem 2.
- Asking, "What are the multiples of 4? What are the multiples of 6? What are the common multiples?"

Professional Learning

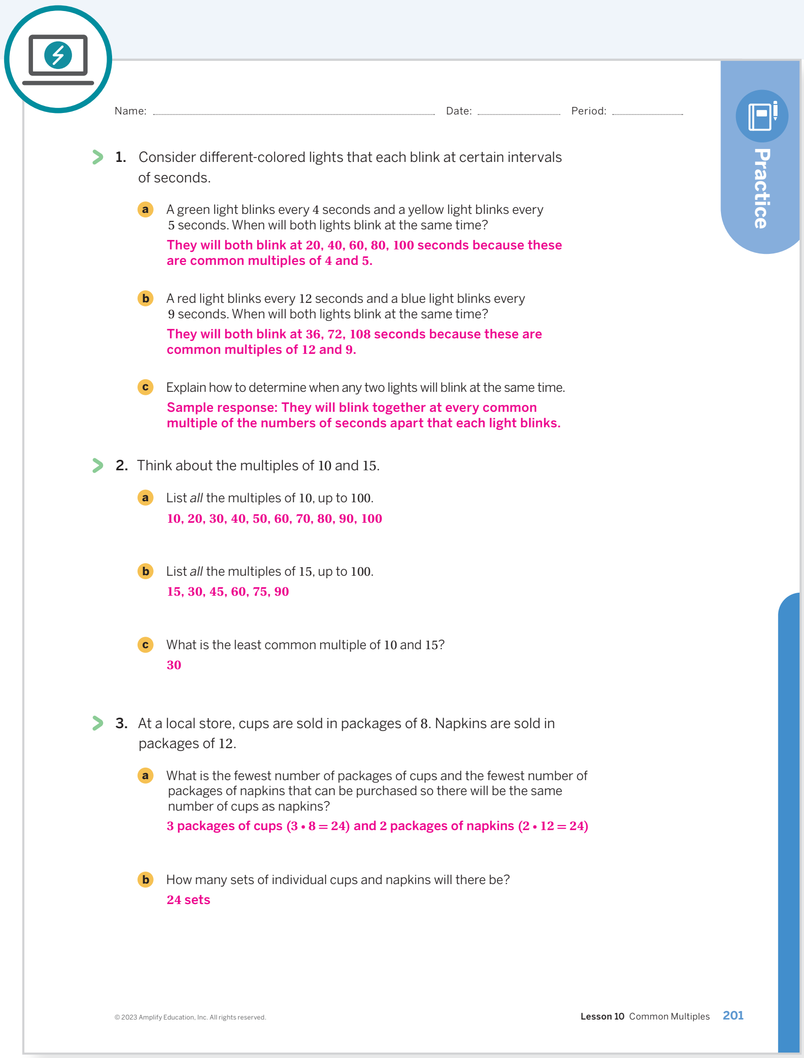
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did determining least common multiples reveal about your students as learners?
- What did students find frustrating about Activity 2? What helped them work through this frustration? What might you change the next time you teach this lesson?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Consider different-colored lights that each blink at certain intervals of seconds.
   - a. A green light blinks every 4 seconds and a yellow light blinks every 5 seconds. When will both lights blink at the same time?
     They will both blink at 20, 40, 60, 80, 100 seconds because these are common multiples of 4 and 5.
   - b. A red light blinks every 12 seconds and a blue light blinks every 9 seconds. When will both lights blink at the same time?
     They will both blink at 36, 72, 108 seconds because these are common multiples of 12 and 9.
   - c. Explain how to determine when any two lights will blink at the same time.
     Sample response: They will blink together at every common multiple of the numbers of seconds apart that each light blinks.

2. Think about the multiples of 10 and 15.
   - a. List *all* the multiples of 10, up to 100.
     10, 20, 30, 40, 50, 60, 70, 80, 90, 100
   - b. List *all* the multiples of 15, up to 100.
     15, 30, 45, 60, 75, 90
   - c. What is the least common multiple of 10 and 15?
     30

3. At a local store, cups are sold in packages of 8. Napkins are sold in packages of 12.
   - a. What is the fewest number of packages of cups and the fewest number of packages of napkins that can be purchased so there will be the same number of cups as napkins?
     3 packages of cups ($3 \cdot 8 = 24$) and 2 packages of napkins ($2 \cdot 12 = 24$)
   - b. How many sets of individual cups and napkins will there be?
     24 sets

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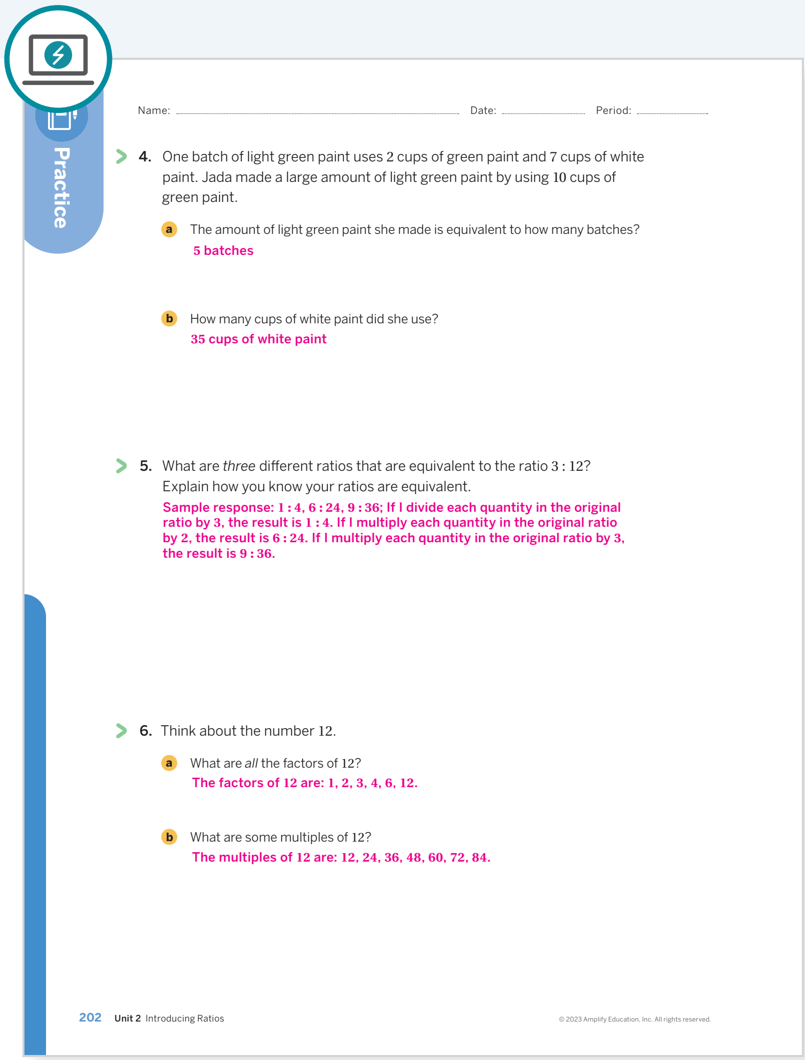
Lesson 10 Common Multiples 201



Practice

Name: ________ Date: ________ Period: ________

4. One batch of light green paint uses 2 cups of green paint and 7 cups of white paint. Jada made a large amount of light green paint by using 10 cups of green paint.
   - a. The amount of light green paint she made is equivalent to how many batches?
     5 batches
   - b. How many cups of white paint did she use?
     35 cups of white paint

5. What are *three* different ratios that are equivalent to the ratio 3 : 12? Explain how you know your ratios are equivalent.
   Sample response: 1 : 4, 6 : 24, 9 : 36; If I divide each quantity in the original ratio by 3, the result is 1 : 4. If I multiply each quantity in the original ratio by 2, the result is 6 : 24. If I multiply each quantity in the original ratio by 3, the result is 9 : 36.

6. Think about the number 12.
   - a. What are *all* the factors of 12?
     The factors of 12 are: 1, 2, 3, 4, 6, 12.
   - b. What are some multiples of 12?
     The multiples of 12 are: 12, 24, 36, 48, 60, 72, 84.

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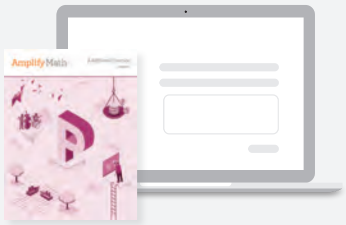
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	3
	3	Activity 2	3
Spiral	4	Unit 2 Lesson 4	2
	5	Unit 2 Lesson 5	2
Formative ⚡	6	Unit 2 Lesson 11	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 2 | **LESSON 11**

Navigating a Table of Equivalent Ratios

Let's use a table of equivalent ratios.

Focus

Goals

1. **Language Goal:** Comprehend and use the word *per* to mean *for each*. **(Speaking and Listening, Writing)**
2. Comprehend that every ratio has two equivalent ratios containing a 1, and determine and represent each using a table with a 1 in the appropriate column.
3. Choose multipliers strategically while solving problems involving specific or multiple equivalent ratios.
4. **Language Goal:** Describe how a table of equivalent ratios was used to solve a problem about related quantities. **(Speaking and Listening, Writing)**

Coherence

Today

Students see how tables can accommodate different ways of reasoning about equivalent ratios. They begin to target specific equivalent ratios by using strategically chosen multipliers, such as using the greatest common factor to determine the equivalent ratio that has the *smallest* pair of whole number values, or the divisor needed to determine an equivalent ratio containing a 1 (representing an amount *per* 1). Students recognize that every ratio has two equivalent ratios containing a 1 and can determine both, or the one that is most useful for determining other equivalent ratios.

Previously

In Lessons 9–10, students determined the greatest common factor and least common multiple of two or more whole numbers, which can be used in determining equivalent ratios and navigating ratio tables.

Coming Soon

In Lesson 12, students explore double number line diagrams as another representational tool for equivalent ratios.

Rigor

- Students continue to build **fluency** with equivalent ratios.
- Students **apply** equivalent ratios to determine practical and meaningful values in real-world contexts.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

New word

- *per*

Review word

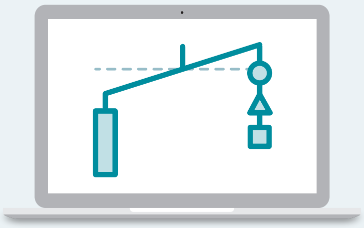
- *equivalent ratios*

Amps | Featured Activity

Activity 2

Interactive Tables

Students will use a pizza crust recipe to determine equivalent ratios by using a dynamic table.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may lose motivation or focus if they do not immediately see how to make use of the structure of the equivalent ratio table when it is given a context in Activity 2. Help them practice taking control of these impulses by suggesting they use their peers as a resource and by asking them who they think might be able to help them with this Activity.

Modifications to Pacing

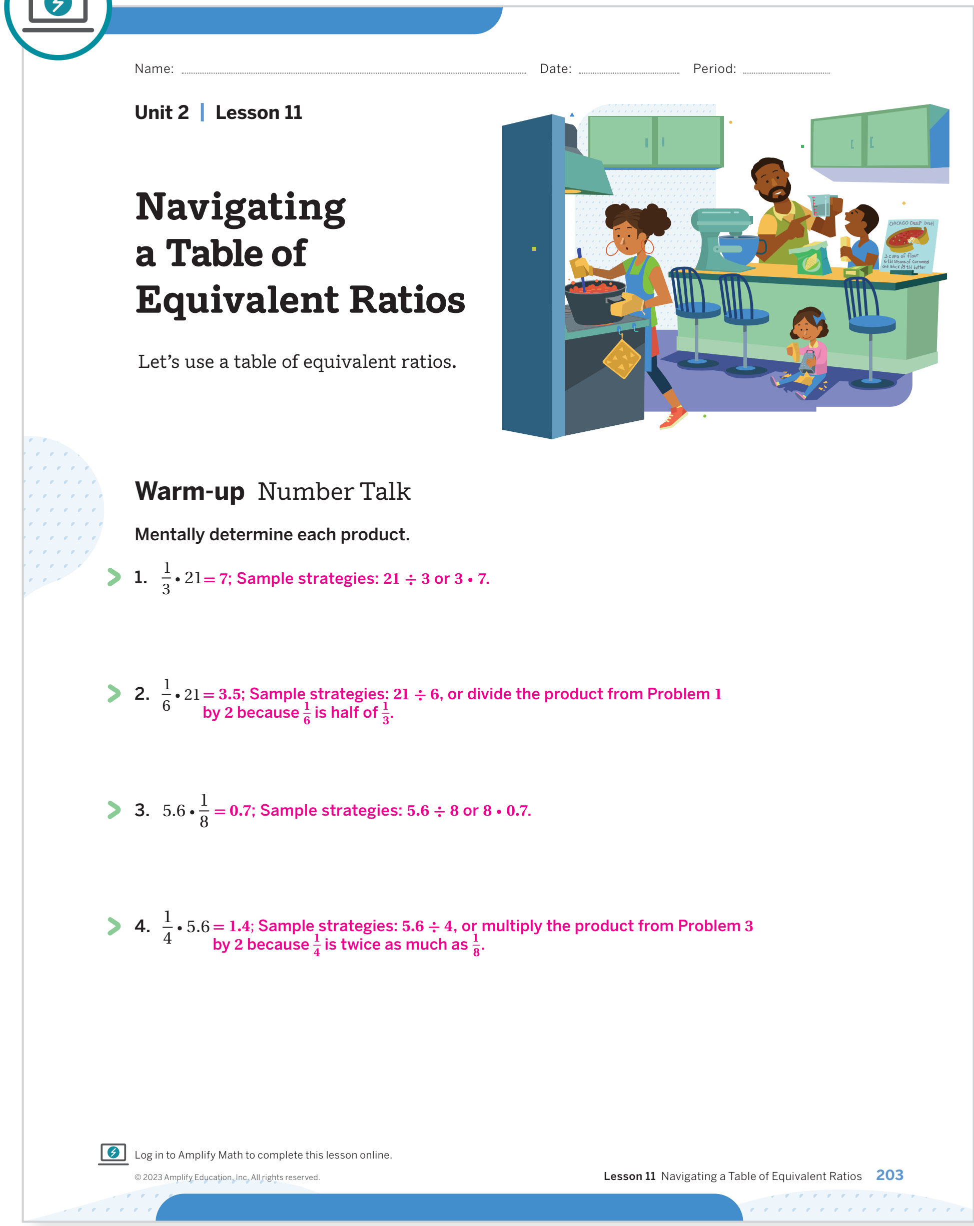
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, Problems 1 and 2 may be omitted.

Independent | 10 min

Warm-up Number Talk

Students review the meaning of fractions and the properties of operations as they determine the products with fractions and decimals.



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Unit 2 | Lesson 11

Navigating a Table of Equivalent Ratios

Let's use a table of equivalent ratios.

Warm-up Number Talk

Mentally determine each product.

1. $\frac{1}{3} \cdot 21 = 7$; Sample strategies: $21 \div 3$ or $3 \cdot 7$.

2. $\frac{1}{6} \cdot 21 = 3.5$; Sample strategies: $21 \div 6$, or divide the product from Problem 1 by 2 because $\frac{1}{6}$ is half of $\frac{1}{3}$.

3. $5.6 \cdot \frac{1}{8} = 0.7$; Sample strategies: $5.6 \div 8$ or $8 \cdot 0.7$.

4. $\frac{1}{4} \cdot 5.6 = 1.4$; Sample strategies: $5.6 \div 4$, or multiply the product from Problem 3 by 2 because $\frac{1}{4}$ is twice as much as $\frac{1}{8}$.

Log in to Amplify Math to complete this lesson online.

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Lesson 11 Navigating a Table of Equivalent Ratios 203

1 Launch

Have students conduct the ***Number Talk*** routine. Display one expression at a time. Tell students to give a signal when they have an answer and strategy.

2 Monitor

Help students get started by having them read the product by using words, for example "one-third of twenty-one." Ask, "How would you think about grouping twenty-one things into thirds?"

Look for points of confusion:

- **Dividing the whole number by the fraction.** Ask, "Is dividing by 2 the same as multiplying by 2? What other operation is the same as multiplying by a unit fraction?" Dividing by the denominator.

Look for productive strategies:

- Using the product from Problem 1 to determine the product of Problem 2; and relating the products in Problems 3 and 4 similarly.

3 Connect

Display all four expressions at once.

Have individual students share their strategies for evaluating each expression.

Highlight that multiplication by a unit fraction is equivalent to division by the denominator (a whole number, or the reciprocal of the unit fraction).

Ask:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone solve the problem the same way but would explain it differently?"
- "Did any previous problem help you in a later problem?"

Math Language Development

MLR8: Discussion Supports— Press for Reasoning

To support students in sharing the reasoning they used to mentally determine each product, ask "How do the denominators of the unit fraction help you?" As students share, encourage them to restate and revoice their classmates' reasoning before adding on to the discussion.

English Learners

Provide wait time for students and encourage them to use the wait time to formulate their response before sharing with the class.

Power-up

To power up students' ability to determine factors or multiples of a number, have students complete:

Determine if each value is a *factor* or *multiple* of 12. If it is both, write "both".

a. 1 factor
b. 24 multiple
c. 12 both
d. 36 multiple
e. 6 factor

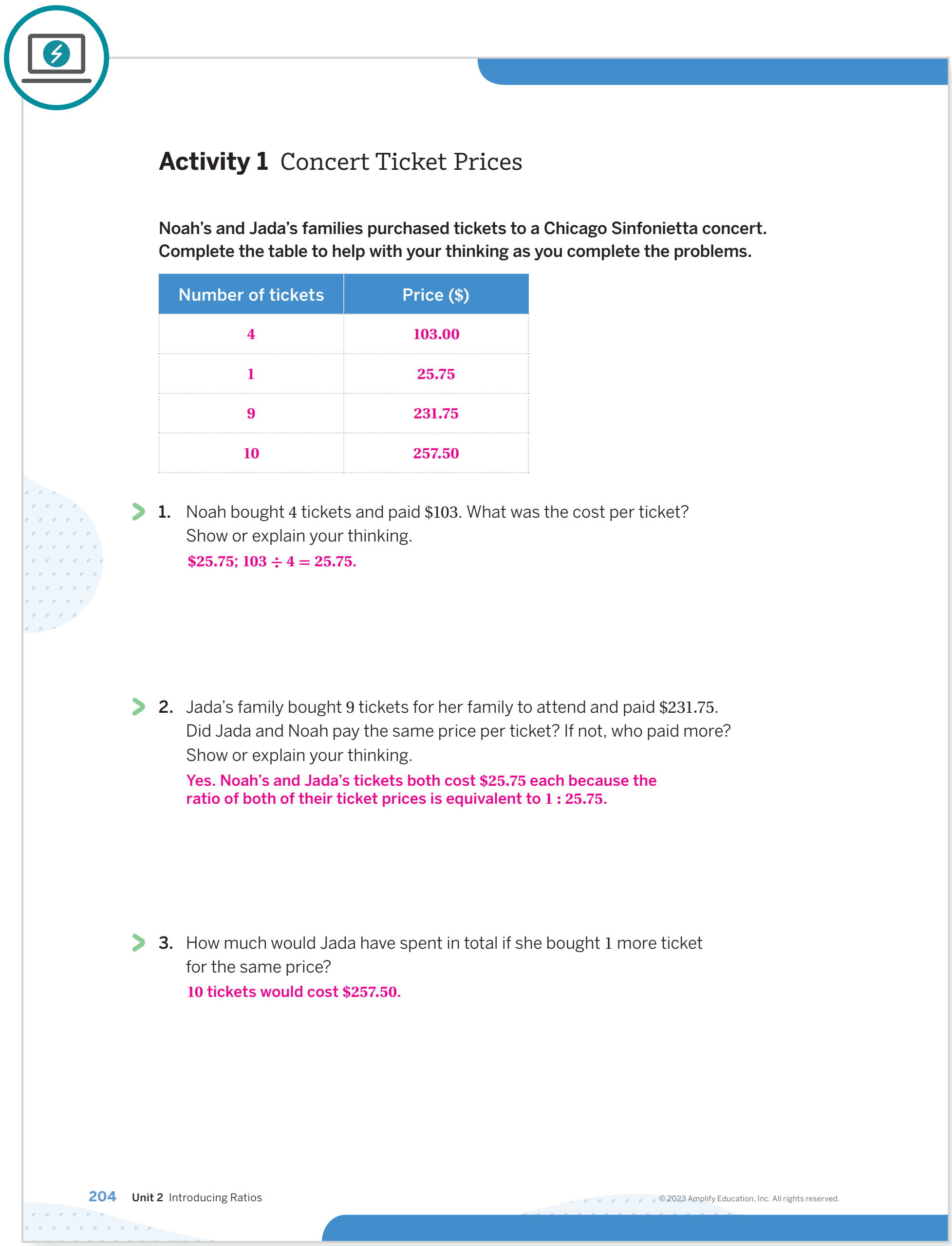
Use: Before the Warm-up.

Informed by: Performance on Lesson 10, Practice Problem 6

Pairs | 10 min

Activity 1 Concert Ticket Prices

Students use a table to compare ratios of concert ticket to prices. This is a first step towards more general missing value and comparison problems in Lessons 14–17.



Activity 1 Concert Ticket Prices

Noah's and Jada's families purchased tickets to a Chicago Sinfonietta concert. Complete the table to help with your thinking as you complete the problems.

Number of tickets	Price ($)
4	103.00
1	25.75
9	231.75
10	257.50

1. Noah bought 4 tickets and paid $103. What was the cost per ticket? Show or explain your thinking.
   $25.75; 103 ÷ 4 = 25.75.

2. Jada's family bought 9 tickets for her family to attend and paid $231.75. Did Jada and Noah pay the same price per ticket? If not, who paid more? Show or explain your thinking.
   Yes. Noah's and Jada's tickets both cost $25.75 each because the ratio of both of their ticket prices is equivalent to 1 : 25.75.

3. How much would Jada have spent in total if she bought 1 more ticket for the same price?
   10 tickets would cost $257.50.

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① Launch

Have students use the ***Think-Pair-Share*** routine to answer the problems.

② Monitor

Help students get started by asking, "How could you represent the price of 1 ticket as a ratio?"

Look for points of confusion:

- **Not using ratios to determine each ticket price.** Have students divide the total cost by the number of tickets to determine individual ticket prices.

Look for productive strategies:

- Determining the ratio of 1 : 25.75, and recognizing it is the same for both Noah and Jada.
- Using that ratio to multiply 25.75 • 10 (Problem 3).
- Using that to add 231.75 + 25.75 (Problem 3).

③ Connect

Display a blank table for tickets and prices.

Have pairs of students share their responses and strategies, adding each ratio used to the table and showing the multipliers between the rows.

Ask, "Does the cost of each ticket change as the number of tickets increases? Why or why not?"

Define ***per*** as "for each."
Note: *Per* can be interpreted as "for each one," or "for every." The latter is more of a focus in Unit 3 with rate contexts.

Highlight that there is an equivalent ratio of 1 : 25.75 for both Noah and Jada, which means they both paid $25.75 for 1 ticket, or *per* ticket. Once this ratio is determined for 1 ticket, it can be used to generate equivalent ratios for any number of tickets, which then also tells students how much those tickets cost.

Fostering Diverse Thinking

Exploring the Chicago Sinfonietta

Have students research the Chicago Sinfonietta, whose concert ticket prices they mathematically analyzed in the activity. The Chicago Sinfonietta is a professional orchestra dedicated to helping change the face of classical music by modeling and valuing diversity. Have students read about the orchestra's mission and history.

Math Language Development

MLR2: Collect and Display

As students share their responses, highlight and collect ratio language, specifically *per*, *for every*, and *for each*.

English Learners

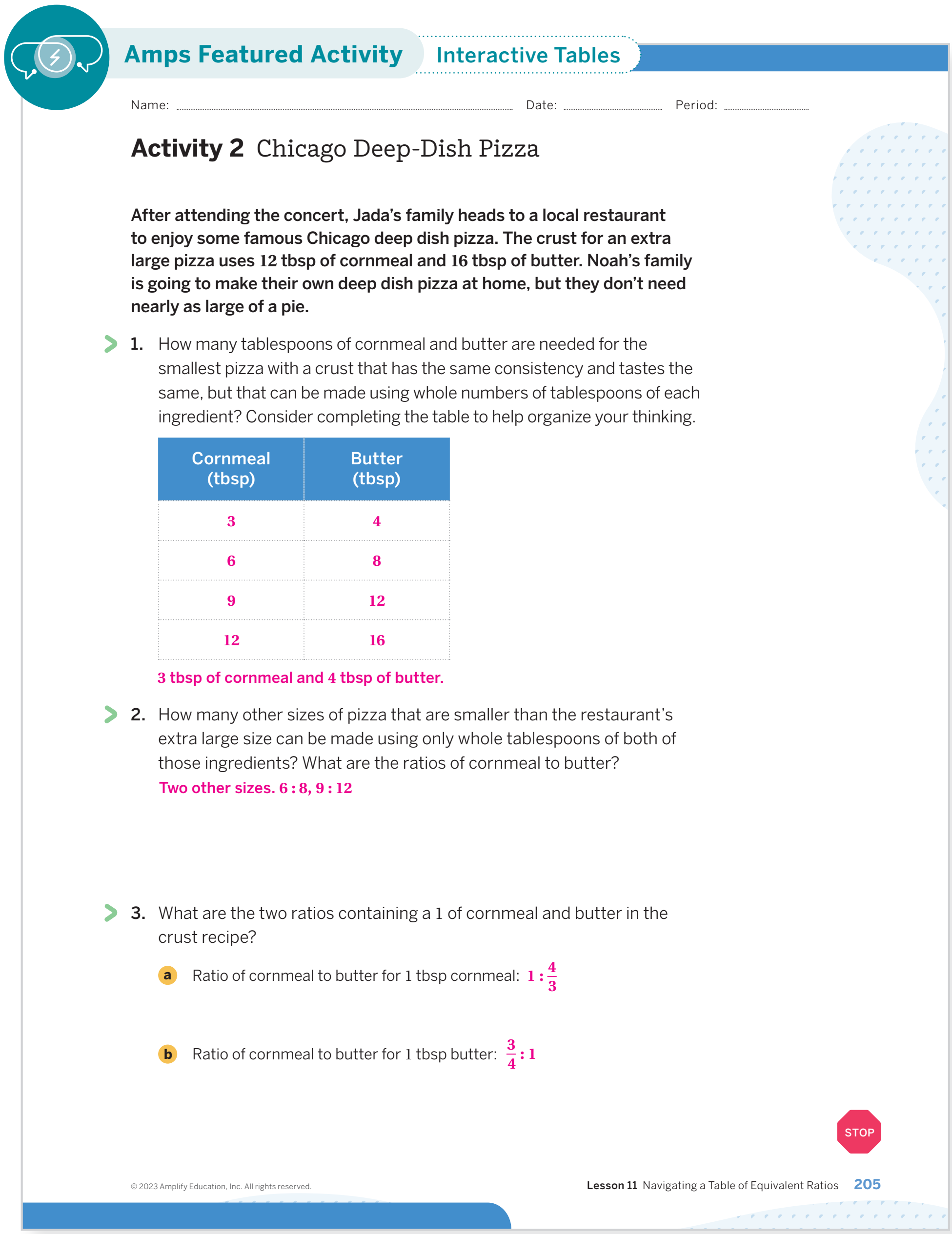
English learners, and even fluent English speakers, may use the phrases *for every* and *for each* interchangeably. There are some nuances between them. *For every* is usually reserved when there are more than two items: for every student, for every book in the library, etc. *For each* can be used regardless of the quantity of items. Allow students to use either phrase; however, you may want to consistently use *for each* to avoid any confusion.

Pairs | 15 min

Activity 2 Chicago Deep-Dish Pizza

Students determine special equivalent ratios, including both ratios containing a 1, for a given ratio in context. Like Activity 1, this continues to build toward problems with a missing value.

Amps Featured Activity Interactive Tables



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Activity 2 Chicago Deep-Dish Pizza

After attending the concert, Jada's family heads to a local restaurant to enjoy some famous Chicago deep dish pizza. The crust for an extra large pizza uses 12 tbsp of cornmeal and 16 tbsp of butter. Noah's family is going to make their own deep dish pizza at home, but they don't need nearly as large of a pie.

1. How many tablespoons of cornmeal and butter are needed for the smallest pizza with a crust that has the same consistency and tastes the same, but that can be made using whole numbers of tablespoons of each ingredient? Consider completing the table to help organize your thinking.

Cornmeal (tbsp)	Butter (tbsp)
3	4
6	8
9	12
12	16

3 tbsp of cornmeal and 4 tbsp of butter.

2. How many other sizes of pizza that are smaller than the restaurant's extra large size can be made using only whole tablespoons of both of those ingredients? What are the ratios of cornmeal to butter?

Two other sizes. $6 : 8$, $9 : 12$

3. What are the two ratios containing a 1 of cornmeal and butter in the crust recipe?

a. Ratio of cornmeal to butter for 1 tbsp cornmeal: $1 : \frac{4}{3}$

b. Ratio of cornmeal to butter for 1 tbsp butter: $\frac{3}{4} : 1$

STOP

1 Launch

Have students use the *Think-Pair-Share* routine to complete Problem 1. Pause for a brief discussion of the "smallest" equivalent ratio, and how they know, before pairs work on Problems 2–3.

2 Monitor

Help students get started by asking, "To determine a "smaller" equivalent ratio, should you multiply or divide?"

Look for points of confusion:

- **Thinking that all smaller pizzas come from dividing 12 and 16 only.** Remind students that if two ratios are equivalent, then any ratio equivalent to one is also equivalent to the other.

Look for productive strategies:

- Determining an equivalent ratio with lesser values for Problem 1, but not the "smallest" ratio with whole numbers, or determining a ratio with fractional amounts.
- Determining other equivalent ratios by flexibly dividing and multiplying (e.g., $12 : 16$ to $3 : 4$).
- Determining the two ratios containing a 1 of $1 : \frac{4}{3}$ and $\frac{3}{4} : 1$.

3 Connect

Display a blank table for cornmeal and butter.

Have pairs of students share how they determined each specified equivalent ratio (Problems 1 and 3) and the other ratio for possible smaller-sized pizzas (Problem 2).

Highlight that specific, well-planned operations can be used to determine specific equivalent ratios, such as the "smallest" whole-number pair. Every ratio has two equivalent ratios, where each quantity is equal to 1, which each describe exact amount of one quantity "per 1" of the other quantity.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can determine equivalent ratios for the pizza crust recipe using a dynamic table.

Extension: *Math Enrichment*

Have students complete this problem as an extension to Problem 3: If you had 15 tbsp of cornmeal and wanted to know how much butter you need, which ratio would you choose? How much butter do you need? $1 : \frac{4}{3}$ gives the ratio of cornmeal to butter for 1 tbsp of cornmeal; 20 tbsp of butter

Math Language Development

MLR8: Discussion Supports— Revoicing

As students share during the Connect, have them revoice their classmates' strategies using their own developing math language. Ask the speakers if their peers accurately restated their thinking. Call students' attention to any language and connections between the original strategy and the revoiced strategy.

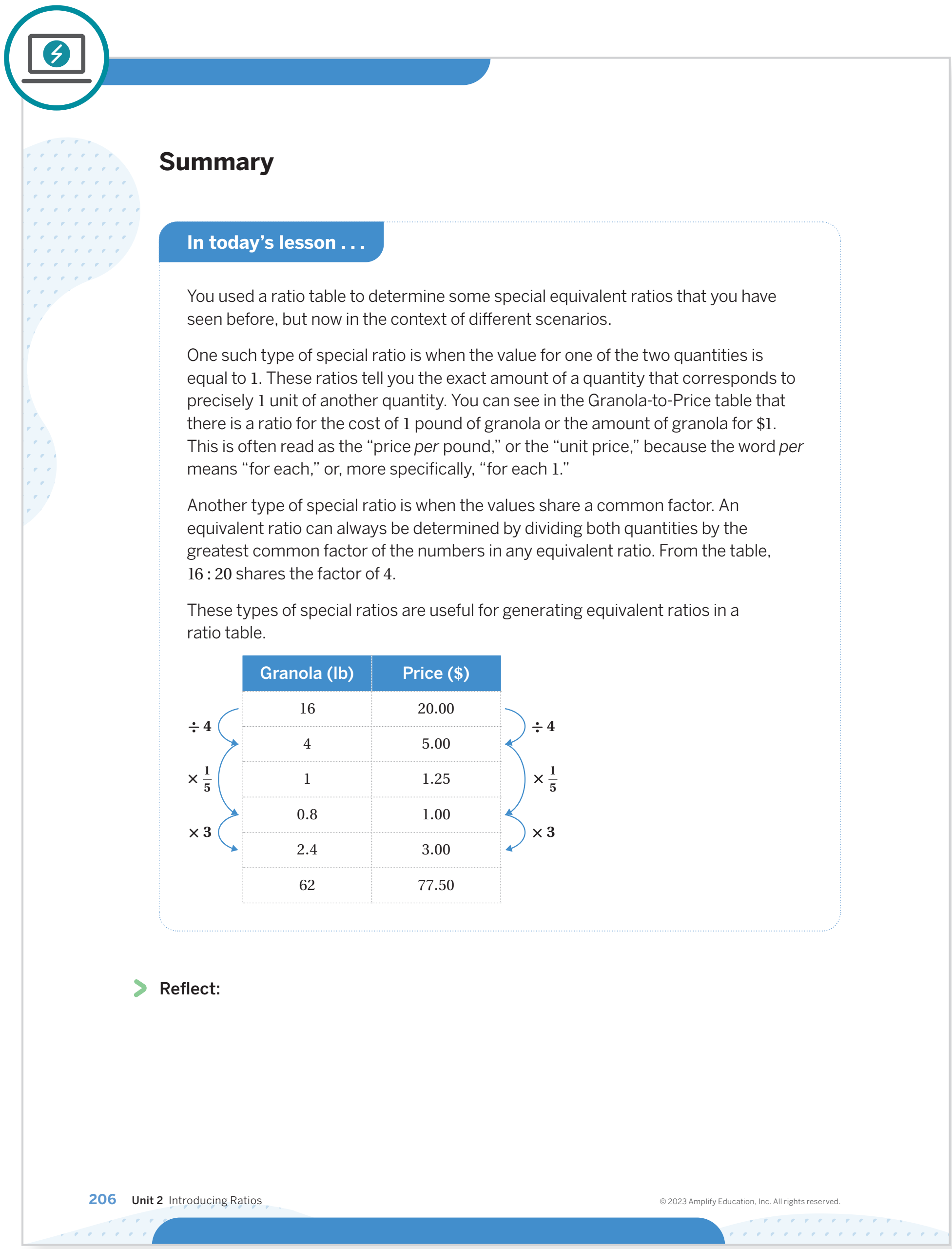
English Learners

During the discussion, highlight language used that has also been added to the class display.

Whole Class | 5 min

Summary

Review and synthesize how not all equivalent ratios can be determined by multiplying or dividing by a whole number, but multiple operations and ratios can be used.



Summary

In today's lesson . . .

You used a ratio table to determine some special equivalent ratios that you have seen before, but now in the context of different scenarios.

One such type of special ratio is when the value for one of the two quantities is equal to 1. These ratios tell you the exact amount of a quantity that corresponds to precisely 1 unit of another quantity. You can see in the Granola-to-Price table that there is a ratio for the cost of 1 pound of granola or the amount of granola for $\$1$. This is often read as the "price *per* pound," or the "unit price," because the word *per* means "for each," or, more specifically, "for each 1."

Another type of special ratio is when the values share a common factor. An equivalent ratio can always be determined by dividing both quantities by the greatest common factor of the numbers in any equivalent ratio. From the table, $16 : 20$ shares the factor of 4.

These types of special ratios are useful for generating equivalent ratios in a ratio table.

Granola (lb)	Price ($)
16	20.00
4	5.00
1	1.25
0.8	1.00
2.4	3.00
62	77.50

Reflect:

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Synthesize

Display the ratio $16 : 20$ or the table from the Summary page.

Ask: (adjusting the language as necessary)

- "What is the smallest whole number pair? How do you know?" $4 : 5$; because the others are decimals.
- "How could you get from the original ratio to each ratio containing a 1?" Dividing by 4 or 5 will give each ratio containing a 1.

Have students share how they would use the table shown in the Summary.

Highlight that, in order to find the base ratio, students will divide by the GCF of the two numbers in the ratio, whereas, the ratio containing a 1 is found by dividing down until one of the ratio values is a 1.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

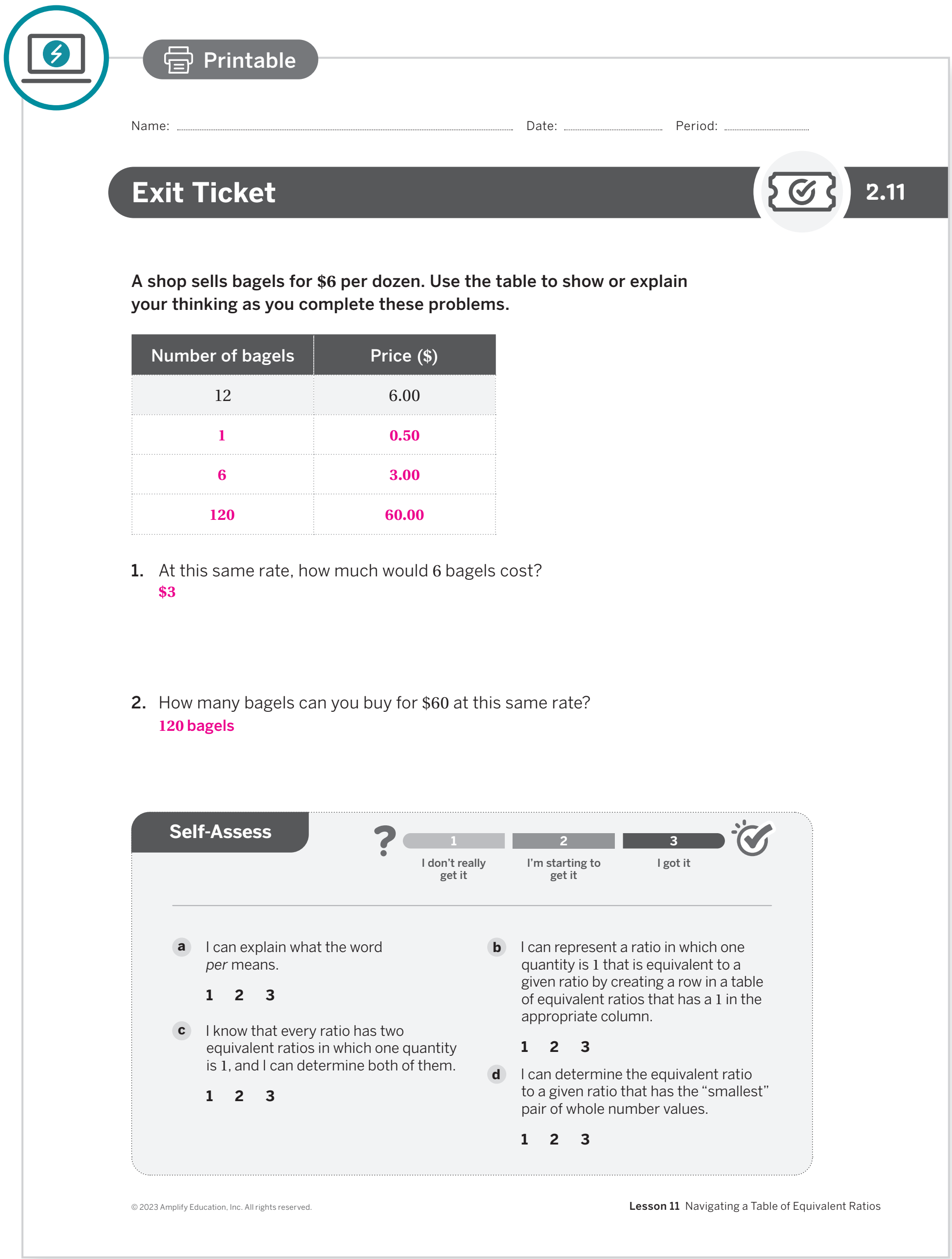
- "How can ratios reflect fairness?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of ratios containing a 1 by using the context of bagel sales.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 2.11

A shop sells bagels for $6 per dozen. Use the table to show or explain your thinking as you complete these problems.

Number of bagels	Price ($)
12	6.00
1	**0.50**
6	**3.00**
120	**60.00**

1. At this same rate, how much would 6 bagels cost?
   $3

2. How many bagels can you buy for $60 at this same rate?
   120 bagels

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain what the word *per* means.
1 2 3

b. I can represent a ratio in which one quantity is 1 that is equivalent to a given ratio by creating a row in a table of equivalent ratios that has a 1 in the appropriate column.
1 2 3

c. I know that every ratio has two equivalent ratios in which one quantity is 1, and I can determine both of them.
1 2 3

d. I can determine the equivalent ratio to a given ratio that has the "smallest" pair of whole number values.
1 2 3

Lesson 11 Navigating a Table of Equivalent Ratios

Success looks like . . .

- **Language Goal:** Comprehending and using the word *per* to mean "for each." **(Speaking and Listening, Writing)**
  - » Understanding that per dozen means for each set of 12 in Problem 1.
- **Goal:** Comprehending that every ratio has two equivalent ratios containing a 1, and determining and representing each using a table with a 1 in the appropriate column.
- **Goal:** Choosing multipliers strategically while solving problems involving specific or multiple equivalent ratios.
  - » Multiplying 12 and 6.00 by appropriate numbers in Problems 1 and 2.
- **Language Goal:** Describing how a table of equivalent ratios was used to solve a problem about related quantities. **(Speaking and Listening, Writing)**
  - » Determining equivalent ratios in the table for Problems 1 and 2.

Suggested next steps

If students multiply the given ratio times 6 in Problem 1, consider:

- Reviewing Activity 1, Problem 1.
- Assigning Practice Problem 1.
- Asking, "How could you find the price of 1 bagel?"

Professional Learning

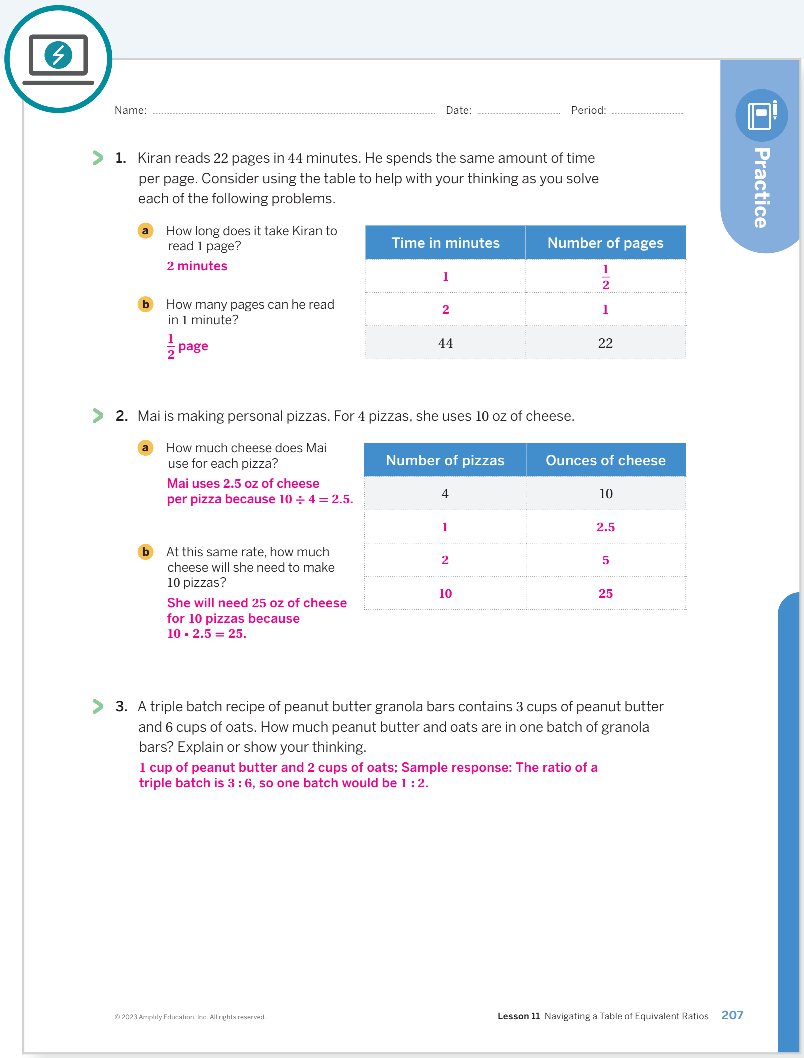
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? When you compare and contrast today's work with work students did earlier this year on equivalent ratios, what similarities and differences do you see?
- Which groups of students did and didn't have their ideas seen and heard today? What might you change the next time you teach this lesson?

Practice

Independent



Practice

Name: .... Date: .... Period: ....

1. Kiran reads 22 pages in 44 minutes. He spends the same amount of time per page. Consider using the table to help with your thinking as you solve each of the following problems.

   a. How long does it take Kiran to read 1 page?
   2 minutes

   b. How many pages can he read in 1 minute?
   $\frac{1}{2}$ page

Time in minutes	Number of pages
1	$\frac{1}{2}$
2	1
44	22

2. Mai is making personal pizzas. For 4 pizzas, she uses 10 oz of cheese.

   a. How much cheese does Mai use for each pizza?
   Mai uses 2.5 oz of cheese per pizza because $10 \div 4 = 2.5$.

   b. At this same rate, how much cheese will she need to make 10 pizzas?
   She will need 25 oz of cheese for 10 pizzas because $10 \cdot 2.5 = 25$.

Number of pizzas	Ounces of cheese
4	10
1	2.5
2	5
10	25

3. A triple batch recipe of peanut butter granola bars contains 3 cups of peanut butter and 6 cups of oats. How much peanut butter and oats are in one batch of granola bars? Explain or show your thinking.
   1 cup of peanut butter and 2 cups of oats; Sample response: The ratio of a triple batch is 3 : 6, so one batch would be 1 : 2.

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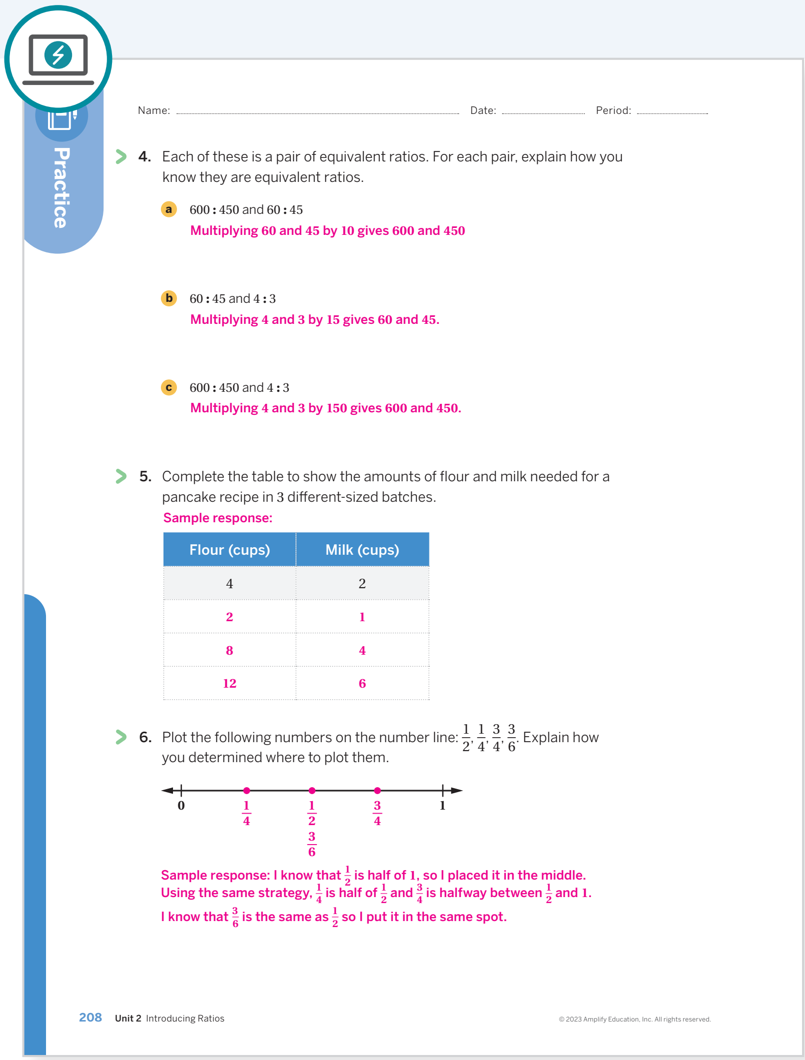

Practice

Name: .... Date: .... Period: ....

4. Each of these is a pair of equivalent ratios. For each pair, explain how you know they are equivalent ratios.

   a. 600 : 450 and 60 : 45
   Multiplying 60 and 45 by 10 gives 600 and 450

   b. 60 : 45 and 4 : 3
   Multiplying 4 and 3 by 15 gives 60 and 45.

   c. 600 : 450 and 4 : 3
   Multiplying 4 and 3 by 150 gives 600 and 450.

5. Complete the table to show the amounts of flour and milk needed for a pancake recipe in 3 different-sized batches.
   Sample response:

Flour (cups)	Milk (cups)
4	2
2	1
8	4
12	6

6. Plot the following numbers on the number line: $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{3}{6}$. Explain how you determined where to plot them.

Sample response: I know that $\frac{1}{2}$ is half of 1, so I placed it in the middle. Using the same strategy, $\frac{1}{4}$ is half of $\frac{1}{2}$ and $\frac{3}{4}$ is halfway between $\frac{1}{2}$ and 1. I know that $\frac{3}{6}$ is the same as $\frac{1}{2}$ so I put it in the same spot.

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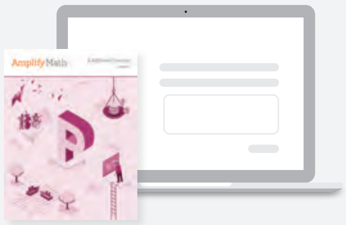
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 6	2
	5	Unit 2 Lesson 7	2
Formative ⚡	6	Unit 2 Lesson 12	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 2 | **LESSON 12**

Tables and Double Number Line Diagrams

Let's use double number lines to represent equivalent ratios.

Focus

Goals

1. **Language Goal:** Explain how to use a double number line diagram to determine equivalent ratios. **(Speaking and Listening)**
2. Label and interpret a double number line diagram that represents a familiar context.
3. **Language Goal:** Compare and contrast double number line diagrams and tables representing the same situation. **(Speaking and Listening)**

Coherence

Today

Students explore double number line diagrams, a useful and efficient tool for reasoning about equivalent ratios. They reason about how to best represent equivalent ratio data by using both double number lines and tables. Students also compare and contrast double number lines with tables and identify when a table or a double number line might be preferable, such as noting that a double number line dictates the ordering of the values on the line, but that pairs of values in a table can be listed in any order.

Previously

In earlier grades, students used number lines to identify arithmetic patterns and record measurement equivalents. In Lessons 6, 7, and 11 of this unit, students worked with equivalent ratios and tables.

Coming Soon

In Lesson 13, students will apply equivalent ratios to convert measurements in recipes by using double number line diagrams and tables to help with their thinking and represent their results.

Rigor

- Students develop their **procedural fluency** for creating equivalent ratios through tables and double number lines.
- Students **apply** equivalent ratios within the context of real-world scenarios.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

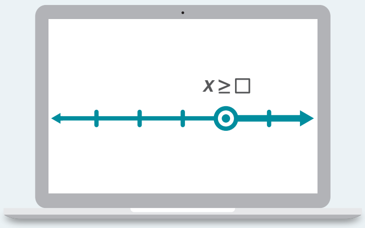
Materials

- Exit Ticket
- Additional Practice

Amps | Featured Activity

Activity 1
Digital Number Lines

Students can freely manipulate double number lines to plot and to represent equivalent ratios, without having to manually draw the lines, and to create appropriate, evenly-spaced intervals.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may develop a preference for using only a double number line or only a table in Activity 2 and may not understand that other methods will help develop their abstract reasoning skills. Help students practice taking control of their own impulses by asking them to think of a time when they learned how to do something in a different way and that ended up being the way they liked more. Reinforce the importance of being open to multiple ways of solving problems.

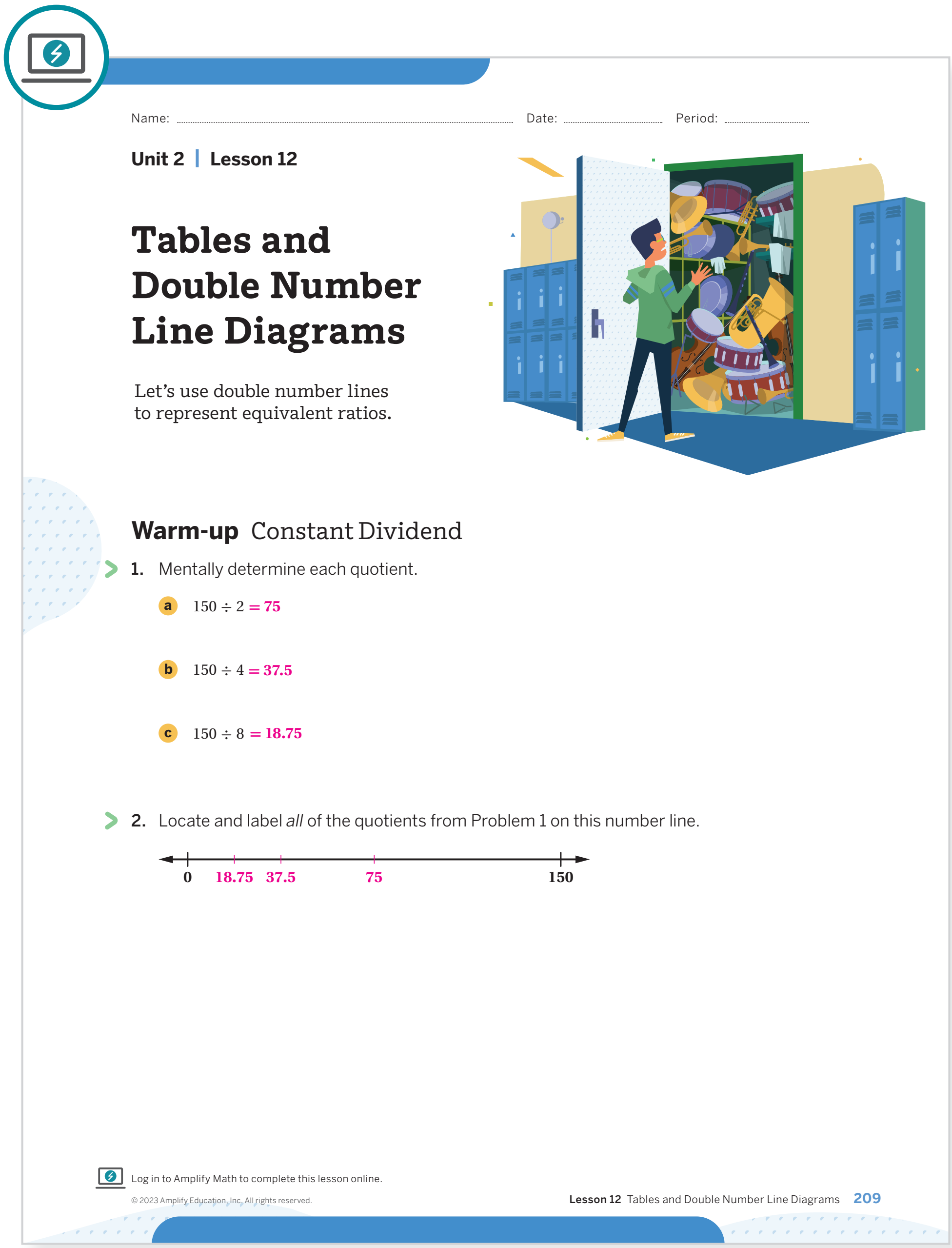
Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, have students choose to complete either Problem 1 or Problem 2.

Warm-up Constant Dividend

Students think about what happens to a quotient when the divisor is doubled, and they represent the quotients on a number line as they prepare to use double number lines.



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Unit 2 | Lesson 12

Tables and Double Number Line Diagrams

Let's use double number lines to represent equivalent ratios.

Warm-up Constant Dividend

1. Mentally determine each quotient.
   - a 150 ÷ 2 = 75
   - b 150 ÷ 4 = 37.5
   - c 150 ÷ 8 = 18.75
2. Locate and label *all* of the quotients from Problem 1 on this number line.

0 18.75 37.5 75 150

Log in to Amplify Math to complete this lesson online.

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1 Launch

Use the ***Number Talk*** routine for Problem 1 and then give students time to plot the quotients on the number line before discussing Problem 2.

2 Monitor

Help students get started by asking how 150 could be broken up to divide it more easily.

Look for points of confusion:

- **Not using the pattern in subsequent divisors.** Have students focus on the relationship between 2 and 4. Ask, "How many times larger is 4 than 2? How would that affect the quotient?"
- **Having trouble locating tick marks precisely on the number line.** Ask, "Where would you place a tick mark halfway between 0 and 150? What would the value be? What about halfway between there and 0 again?" Encourage students to follow this pattern rather than marking every 1.

Look for productive strategies:

- Relating the pattern in the dividends (each is 2 times larger than the previous) to a pattern in the quotients (each is 2 times smaller, or half).

3 Connect

Display the blank number line.

Have students share how they plotted the quotients on the number line.

Ask:

- "Is there more than one way to determine the quotients?"
- "What is the scale between each tick mark? What other labels might go on any other ticks that weren't used?" Scale is 18.75

Highlight that when students divide by a greater value, the quotient is less, and vice versa. As one increases by a factor, the other decreases by the same factor.

Power-up

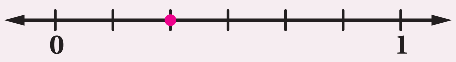
To power up students' ability to identify equivalent fractions, have students complete:

1. Plot each value on the given number line:

a. $\frac{1}{3}$

b. $\frac{1}{4}$

c. $\frac{2}{6}$



2. Which two fractions from Problem 1 are equivalent? $\frac{1}{3}$ and $\frac{2}{6}$.

Use: Before Activity 1.

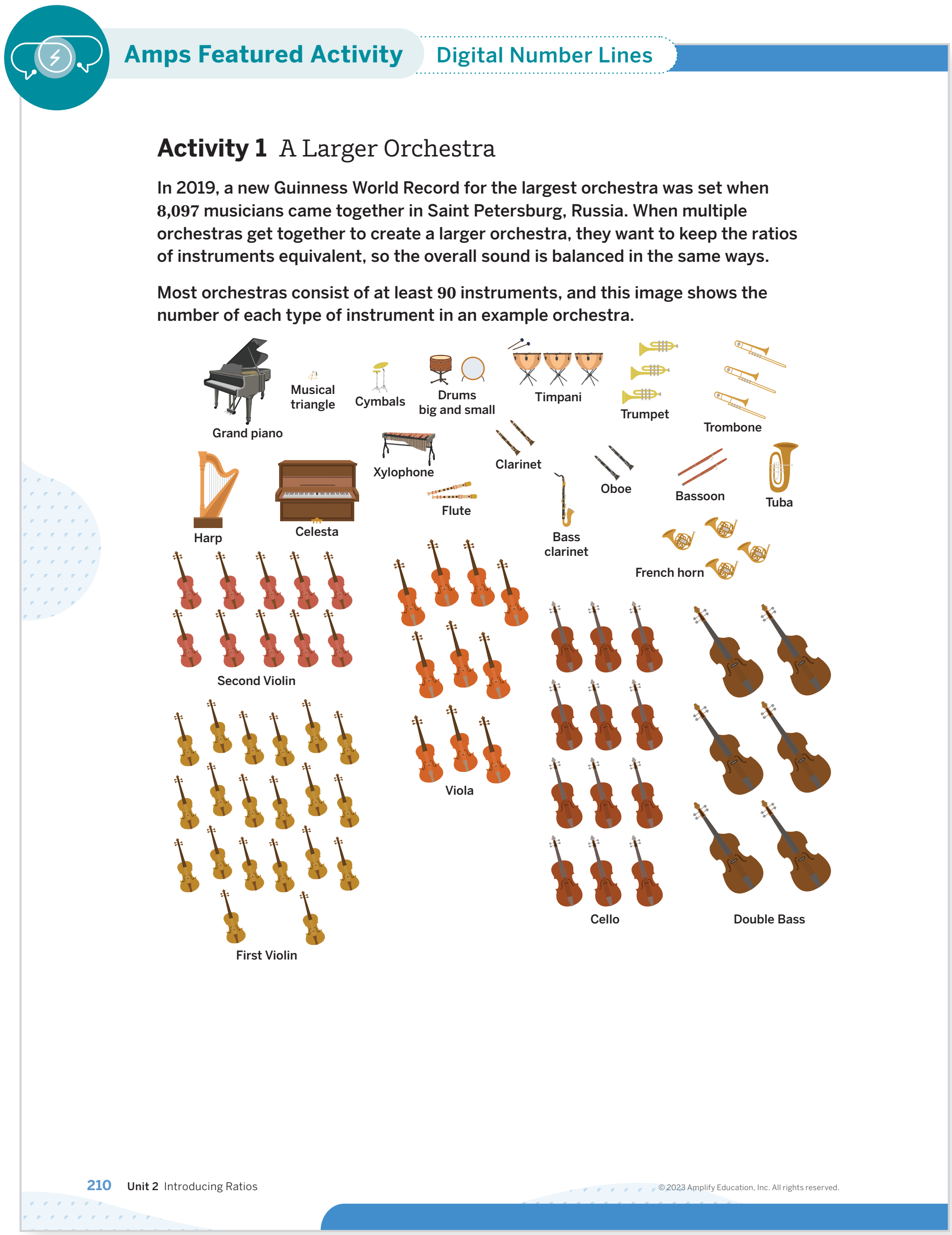
Informed by: Performance on Lesson 11, Practice Problem 6, and Pre-Unit Readiness Assessment, Problem 1.

Pairs | 15 min

Activity 1 A Larger Orchestra

Students consider the ratios of instruments in an orchestra and create double number lines to represent equivalent ratios for more instruments.

Amps Featured Activity Digital Number Lines



Activity 1 A Larger Orchestra

In 2019, a new Guinness World Record for the largest orchestra was set when 8,097 musicians came together in Saint Petersburg, Russia. When multiple orchestras get together to create a larger orchestra, they want to keep the ratios of instruments equivalent, so the overall sound is balanced in the same ways.

Most orchestras consist of at least 90 instruments, and this image shows the number of each type of instrument in an example orchestra.

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1 Launch

Consider showing students an online video of the 2019 Guinness World Record for the largest orchestra ever created.

Have students read the directions, ensuring they understand the reasoning behind keeping the ratios of instruments the same when putting together an orchestra so it will "sound the same." Consider asking students for a sample ratio of instruments from the diagram.

2 Monitor

Help students get started by asking them to count the number of tubas and trombones in the picture and write the figures in the correct boxes.

Look for points of confusion:

- **Struggling to complete the labels on the double number line.** Have students label the first ticks based on the image of a single orchestra. Ask, "If there were two orchestras, how many tubas and trombones would you need?"
- **Having trouble explaining why the ratios are equivalent.** Point to the first pair of numbers and ask, "How could you write that in ratio form? What about the second pair of numbers? The third? What are you doing each time to get the equivalent ratio?" Multiplying by a common factor.

Have students make a table showing the ratios.

Look for productive strategies:

- Knowing how to represent and to interpret larger ratios of instruments using the double number line diagrams.
- Incorporating understanding of factors and multiples to determine equivalent ratios.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can freely manipulate double number lines to plot equivalent ratios. This alleviates the physical requirement of manually drawing the lines and having to create appropriate, evenly-spaced intervals.

Accessibility: *Guide Processing and Visualization*

Provide multiple copies of the orchestra image and have students circle the tubas and trombones *in each copy*. This will provide a visual aid so students can see how the quantities of each set of instruments are multiplied with each additional orchestra.

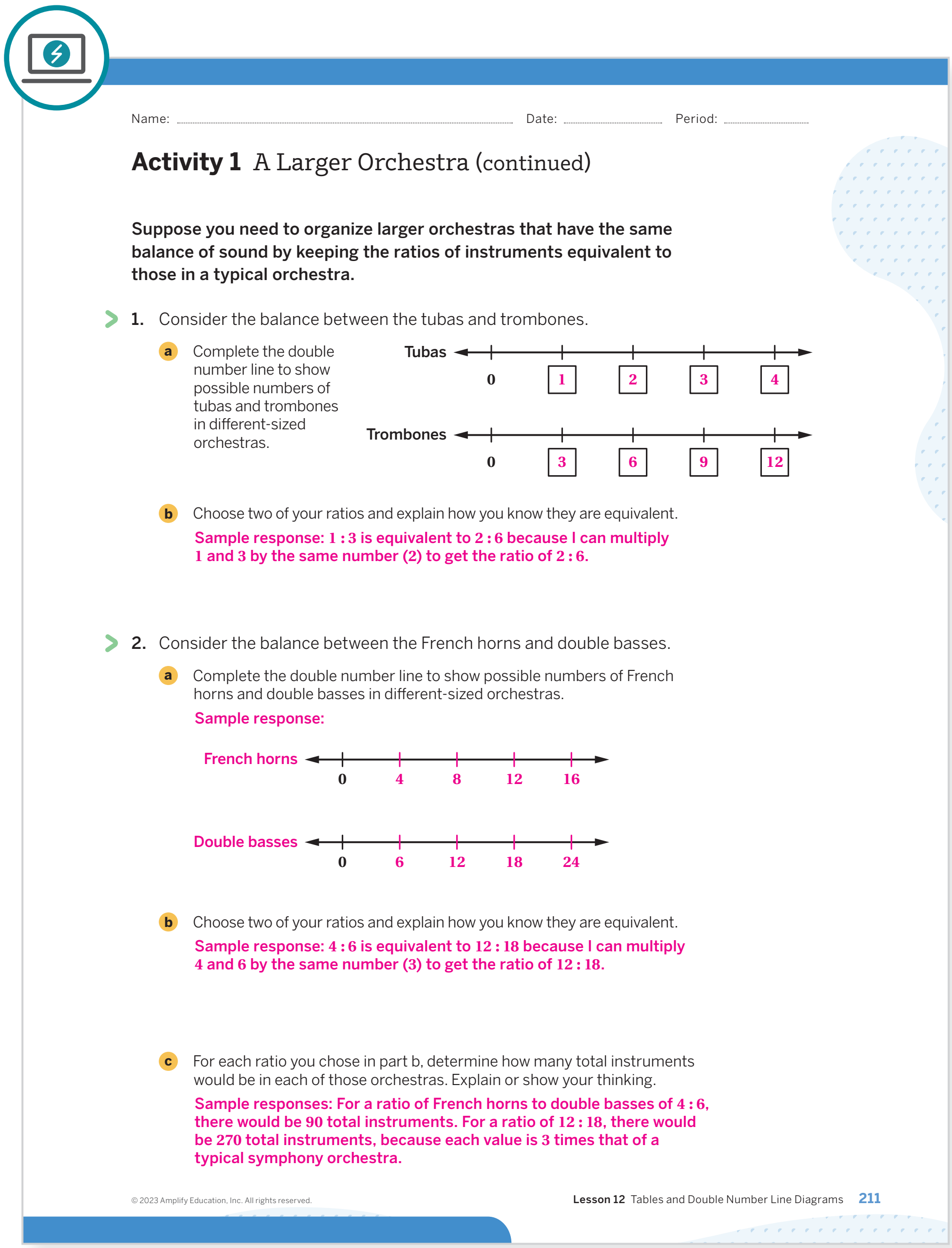
Math Language Development

MLR1: Stronger and Clearer Each Time

Prepare students for the whole-class discussion during the Connect by providing them with opportunities to clarify their reasoning through conversation with their partners. Display prompts for feedback such as, "Can you explain how you used your double number line diagram?"

Activity 1 A Larger Orchestra (continued)

Students consider the ratios of instruments in an orchestra and create double number lines to represent equivalent ratios for more instruments.



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Activity 1 A Larger Orchestra (continued)

Suppose you need to organize larger orchestras that have the same balance of sound by keeping the ratios of instruments equivalent to those in a typical orchestra.

1. Consider the balance between the tubas and trombones.

   a. Complete the double number line to show possible numbers of tubas and trombones in different-sized orchestras.

Tubas	0	1	2	3	4
Trombones	0	3	6	9	12

   b. Choose two of your ratios and explain how you know they are equivalent.
   Sample response: 1 : 3 is equivalent to 2 : 6 because I can multiply 1 and 3 by the same number (2) to get the ratio of 2 : 6.

2. Consider the balance between the French horns and double basses.

   a. Complete the double number line to show possible numbers of French horns and double basses in different-sized orchestras.
   Sample response:

French horns	0	4	8	12	16
Double basses	0	6	12	18	24

   b. Choose two of your ratios and explain how you know they are equivalent.
   Sample response: 4 : 6 is equivalent to 12 : 18 because I can multiply 4 and 6 by the same number (3) to get the ratio of 12 : 18.

   c. For each ratio you chose in part b, determine how many total instruments would be in each of those orchestras. Explain or show your thinking.
   Sample responses: For a ratio of French horns to double basses of 4 : 6, there would be 90 total instruments. For a ratio of 12 : 18, there would be 270 total instruments, because each value is 3 times that of a typical symphony orchestra.

3 Connect

Have students share their strategies of how they labeled and plotted the numbers on the double number lines, and then how they knew the ratios were equivalent.

Display the blank double number line diagram from Problem 2.

Highlight that every double number line will have pairs of associated quantities (consider circling the pairs to show this). Each one of those pairs represents an equivalent ratio to the others and students can use double number line diagrams to determine larger or smaller quantities of a group, a batch or a set. **Note:** The scales of these number lines are not the same, and they were chosen based on the values in the given ratios, so that equivalent ratios are aligned vertically. When that is the case, the ticks and labels themselves align to show and represent the equivalent ratios.

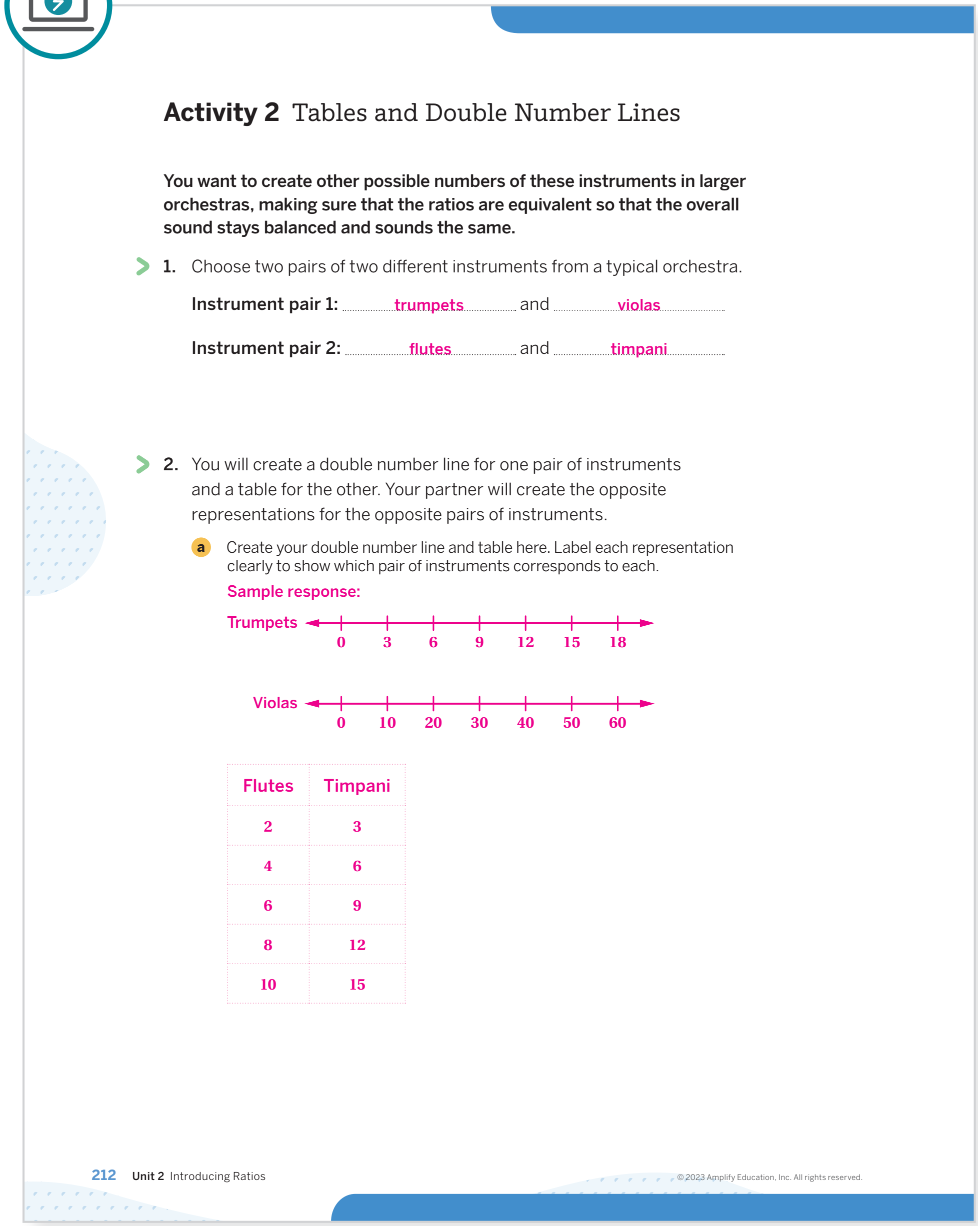
Ask:

- "How did you know how to fill in each number of the number line diagram?" Skip counting, multiplying
- "Describe in your own words what a double number line diagram is and how it can be used." A pair of parallel number lines to represent equivalent ratios and see fewer or larger batches.
- "What might be some benefits of using double number lines, instead of diagrams, such as using the orchestra image, or using a number of squares to represent how many there are of each instrument?" I can use them to show many more batches; they are quicker to draw.
- For Problem 2, "What would the diagram look like if you spaced the intervals on each number line so that the labels represent distance from 0 with the same scale?" Intervals for French horns increase by 4 and contrabasses increase by 6, so the ticks on the contrabasses number line would be farther apart.

Pairs | 15 min

Activity 2 Tables and Double Number Lines

Students compare and contrast representing equivalent ratios by using both double number lines and tables.



Activity 2 Tables and Double Number Lines

You want to create other possible numbers of these instruments in larger orchestras, making sure that the ratios are equivalent so that the overall sound stays balanced and sounds the same.

1. Choose two pairs of two different instruments from a typical orchestra.

Instrument pair 1: trumpets and violas

Instrument pair 2: flutes and timpani

2. You will create a double number line for one pair of instruments and a table for the other. Your partner will create the opposite representations for the opposite pairs of instruments.

a Create your double number line and table here. Label each representation clearly to show which pair of instruments corresponds to each.

Sample response:

Trumpets 0 3 6 9 12 15 18

Violas 0 10 20 30 40 50 60

Flutes	Timpani
2	3
4	6
6	9
8	12
10	15

1 Launch

Keep students in the same pairs. Explain that they will continue to create double number lines showing ratios of instruments in the orchestra, but, this time, they will also represent the information in a table. As they work, they should think about the pros and cons of each representation. The ***Gallery Tour*** routine will be used at the end of the activity.

2 Monitor

Help students get started by having them circle two pairs of instruments they want to compare (crossing out the ones already used in Activity 1) and write down the ratio and the names from the orchestra image.

Look for points of confusion:

- **Not knowing how to represent the data using a table.** Ask, "How many columns do you think there should be? What are you comparing? What should each column be titled? What would go in the first row and each row after?"
- **Struggling to identify differences between the table and the double number line other than appearance.** Ask, "What can you do with a double number line that you cannot do with a table?"

Look for productive strategies:

- Correctly representing the ratios for the two pairs of instruments on a double number line and in a table.
- Using values in a table to flexibly generate equivalent ratios, such as by using repeated doubling or coordinated addition of the values in two rows.
- Being able to reason how a table and number line diagram are different and which might be more useful in different situations.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Provide blank tables and double number lines for students to use. Consider providing the labels *Trumpets*, *Violas*, *Flutes*, and *Timpani* for each representation. You may wish to demonstrate how to label or complete two equivalent ratios and have students complete the rest.

Extension: *Math Enrichment*

Have students complete the following problem: Can a double number line diagram use the same scale on both number lines and have equivalent ratios align vertically? Yes, for a ratio of 1 : 1.

Math Language Development

MLR7: Compare and Connect

Keep students' work displayed from the ***Gallery Tour*** for reference. Highlight productive strategies and displays created, such as double number lines and tables. Annotate an example of a double number line with a phrase, such as "must be in numerical order" or "ratios are aligned vertically." Encourage students to refer back to the various displays to support their use of mathematical language.

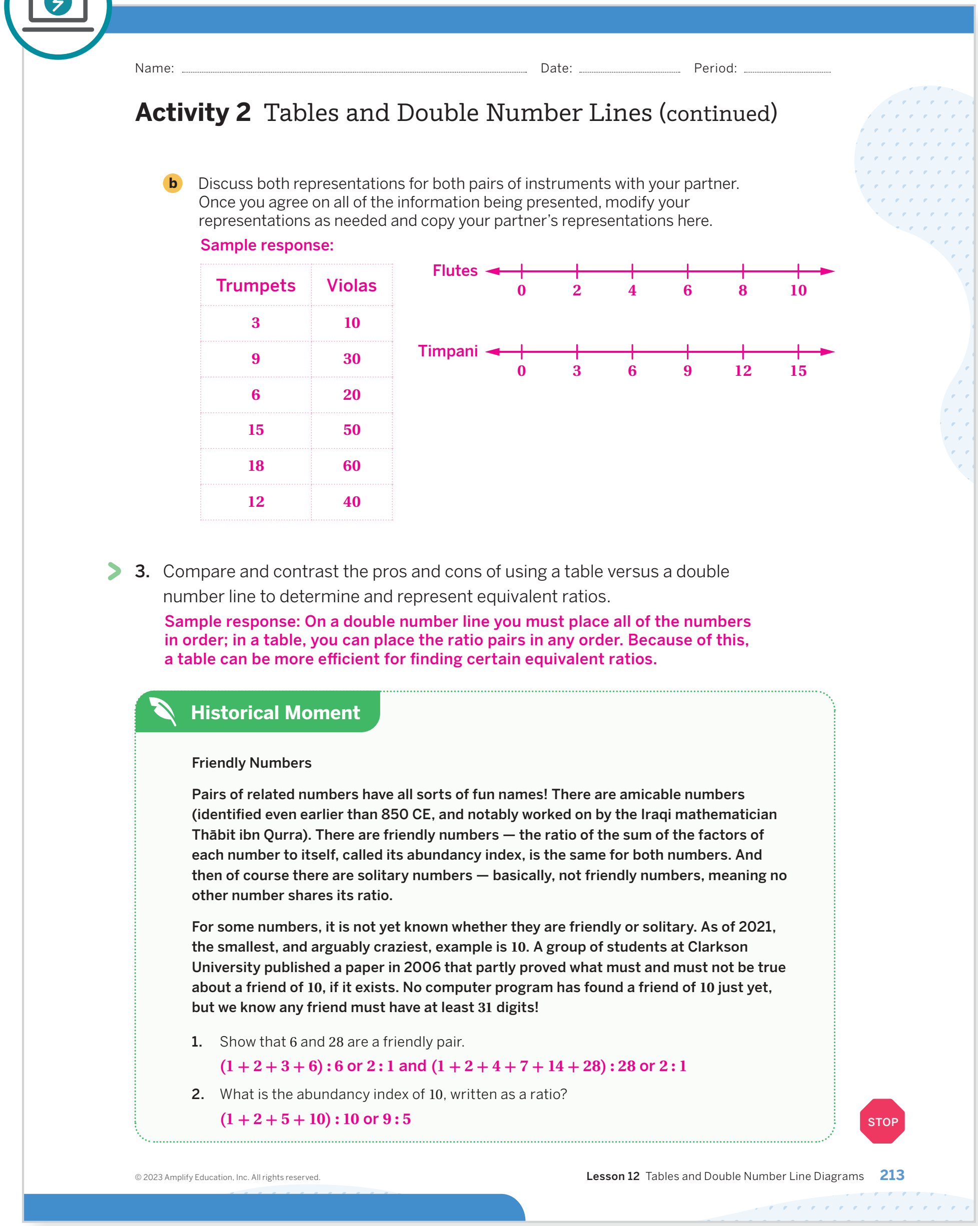
English Learners

Use hand gestures to illustrate what it means for ratios to be aligned vertically.

Pairs | 15 min

Activity 2 Tables and Double Number Lines (continued)

Students compare and contrast representing equivalent ratios by using both double number lines and tables.



Name: Date: Period:

Activity 2 Tables and Double Number Lines (continued)

b Discuss both representations for both pairs of instruments with your partner. Once you agree on all of the information being presented, modify your representations as needed and copy your partner's representations here.

Sample response:

Trumpets	Violas
3	10
9	30
6	20
15	50
18	60
12	40

Flutes: 0, 2, 4, 6, 8, 10

Timpani: 0, 3, 6, 9, 12, 15

3. Compare and contrast the pros and cons of using a table versus a double number line to determine and represent equivalent ratios.

Sample response: On a double number line you must place all of the numbers in order; in a table, you can place the ratio pairs in any order. Because of this, a table can be more efficient for finding certain equivalent ratios.

Historical Moment

Friendly Numbers

Pairs of related numbers have all sorts of fun names! There are amicable numbers (identified even earlier than 850 CE, and notably worked on by the Iraqi mathematician Thābit ibn Qurra). There are friendly numbers — the ratio of the sum of the factors of each number to itself, called its abundancy index, is the same for both numbers. And then of course there are solitary numbers — basically, not friendly numbers, meaning no other number shares its ratio.

For some numbers, it is not yet known whether they are friendly or solitary. As of 2021, the smallest, and arguably craziest, example is 10. A group of students at Clarkson University published a paper in 2006 that partly proved what must and must not be true about a friend of 10, if it exists. No computer program has found a friend of 10 just yet, but we know any friend must have at least 31 digits!

1. Show that 6 and 28 are a friendly pair.
   $(1+2+3+6):6$ or $2:1$ and $(1+2+4+7+14+28):28$ or $2:1$
2. What is the abundancy index of 10, written as a ratio?
   $(1+2+5+10):10$ or $9:5$

STOP

Lesson 12 Tables and Double Number Line Diagrams 213

3 Connect

Display students' work for a ***Gallery Tour***.

Highlight that the numbers on double number lines must be in order and relative distances matter, whereas in a table they do not. On double number lines, ratios are aligned vertically or connected by segments, and in a table, they appear in the same row. If students need both "smaller" values and really "large" values, then double number lines may not be very efficient.

Historical Moment

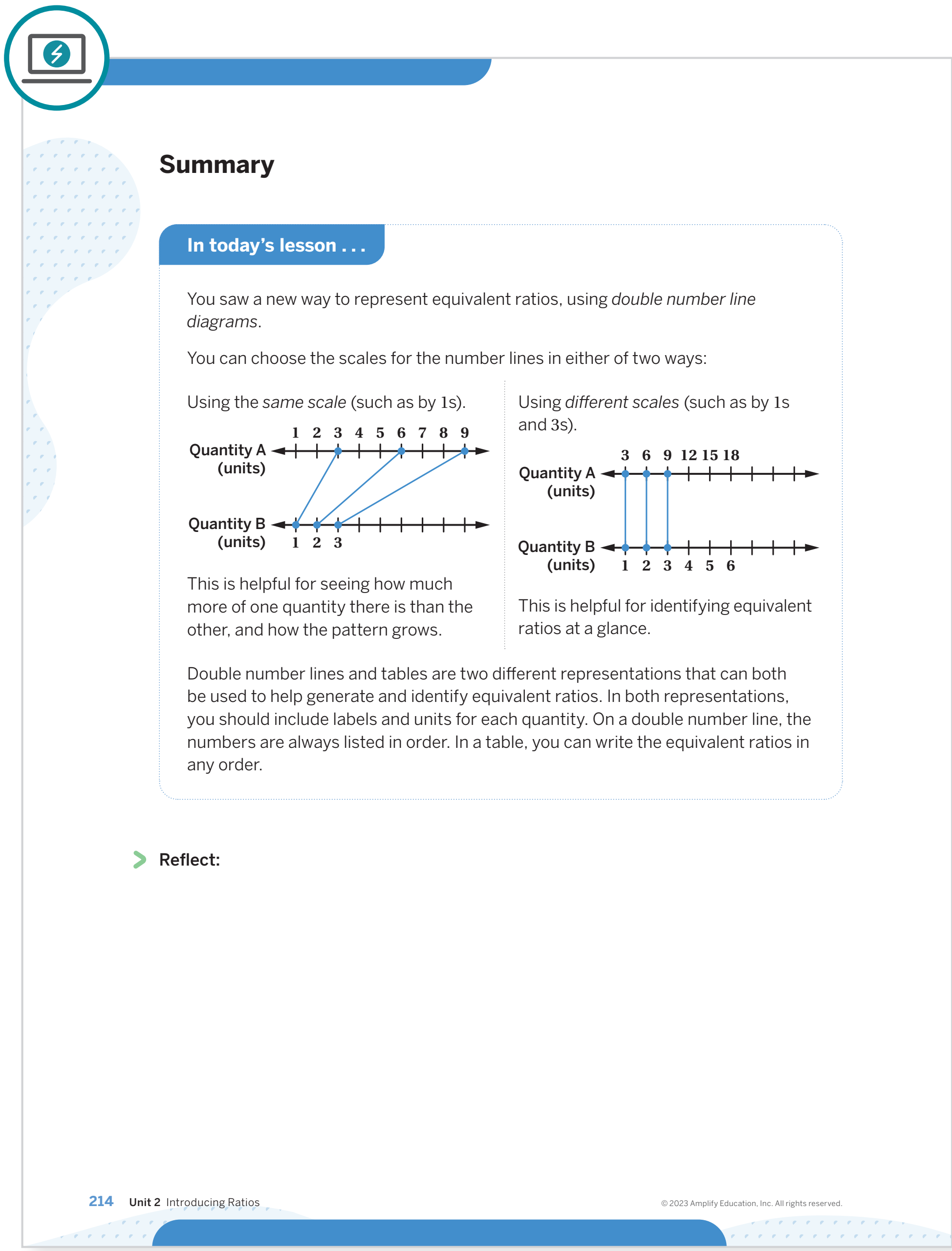
Friendly Numbers

Have students read about some different relationships between pairs of numbers, which date back centuries but also represent an area of mathematics that is ongoing, including problems that are still unsolved or unproven. Number theory topics such as these also represent both extensions of basic number sense, as well as a lens into the wonder and beauty of mathematics and numbers.

Whole Class | 5 min

Summary

Review and synthesize the ways in which tables and number lines are the same and different and how each can be used to represent and generate equivalent ratios.



Summary

In today's lesson . . .

You saw a new way to represent equivalent ratios, using *double number line diagrams*.

You can choose the scales for the number lines in either of two ways:

Using the *same scale* (such as by 1s).

This is helpful for seeing how much more of one quantity there is than the other, and how the pattern grows.

Using *different scales* (such as by 1s and 3s).

This is helpful for identifying equivalent ratios at a glance.

Double number lines and tables are two different representations that can both be used to help generate and identify equivalent ratios. In both representations, you should include labels and units for each quantity. On a double number line, the numbers are always listed in order. In a table, you can write the equivalent ratios in any order.

Reflect:

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Synthesize

Display the same-scale and different-scale double number lines.

Highlight that double number lines can be represented by using the same scale or different scales. Both double number lines and tables are different ways of showing equivalent ratios; sometimes tables are more efficient. Consider choosing one of the double number lines to create a corresponding table ahead of time or do it together if time allows.

Ask:

- "Why is it important to include descriptive labels and units on tables and double number lines?" So you know what and how much each item and measure is.
- "How are double number lines and tables similar? How are they different?" On a double number line, the numbers on each line are always listed in order. In a table, you can write the equivalent ratios in any order.
- "What is the difference between using a same-scale and different-scale double number line?" A different-scale double number line will have equivalent ratios aligned vertically, whereas a same-scale double number line will not have vertical alignment, but diagonal segments can be drawn to see how much more of one quantity there is than the other, and how the pattern grows.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

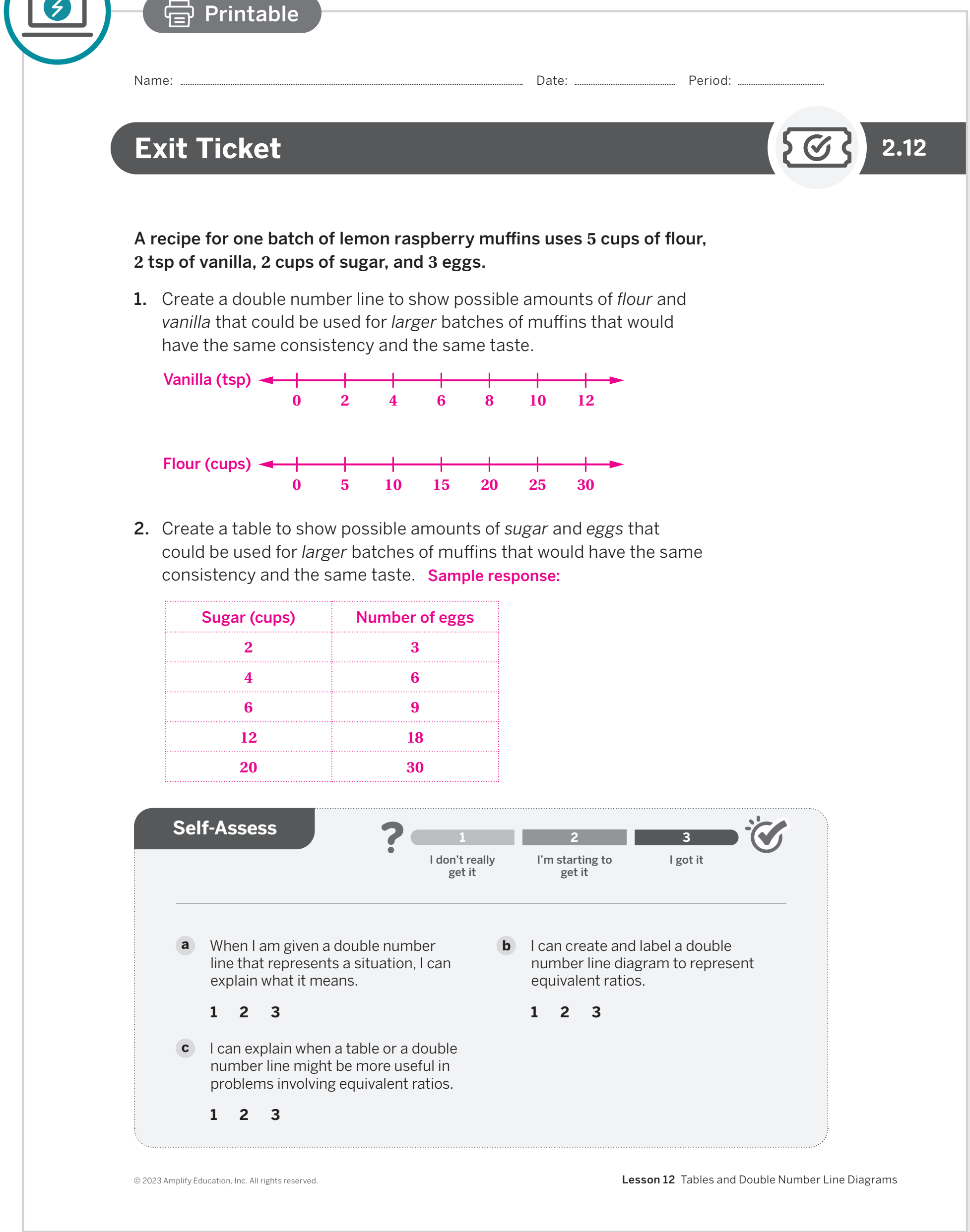
- "Compare/contrast using double number lines to represent equivalent ratios with using a table. Is one more efficient than another? Explain."
- "When might someone use a same-scale double number line versus a different-scale number line?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how to represent equivalent ratios using both a double number line and a table.

Printable



Name: Date: Period:

Exit Ticket 2.12

A recipe for one batch of lemon raspberry muffins uses 5 cups of flour, 2 tsp of vanilla, 2 cups of sugar, and 3 eggs.

1. Create a double number line to show possible amounts of *flour* and *vanilla* that could be used for *larger* batches of muffins that would have the same consistency and the same taste.

Vanilla (tsp) 0 2 4 6 8 10 12

Flour (cups) 0 5 10 15 20 25 30

2. Create a table to show possible amounts of *sugar* and *eggs* that could be used for *larger* batches of muffins that would have the same consistency and the same taste. Sample response:

Sugar (cups)	Number of eggs
2	3
4	6
6	9
12	18
20	30

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a When I am given a double number line that represents a situation, I can explain what it means.

1 2 3

b I can create and label a double number line diagram to represent equivalent ratios.

1 2 3

c I can explain when a table or a double number line might be more useful in problems involving equivalent ratios.

1 2 3

Success looks like . . .

- **Language Goal:** Explaining how to use a double number line diagram to find equivalent ratios. **(Speaking and Listening)**
- **Goal:** Labeling and interpreting a double number line diagram that represents a familiar context.
  - » Creating a double number line to show amounts of ingredients for different-sized batches of muffins in Problem 1.
- **Language Goal:** Comparing and contrasting double number line diagrams and tables representing the same situation. **(Speaking and Listening)**

Suggested next steps

If students have difficulty completing the double number line for Problem 1, consider:

- Asking, "How much vanilla and flour would you need for one batch? What about two batches?"
- Reviewing the double number lines from Activity 1.
- Assigning Practice Problem 2.

If students aren't sure how to start or complete the table, consider:

- Asking, "What is being compared here? What should the headings be in the table? What would the numbers be for the first row — for one batch only? How could you use those numbers to determine larger batches?"
- Reviewing Problem 2 from Activity 2.

Professional Learning

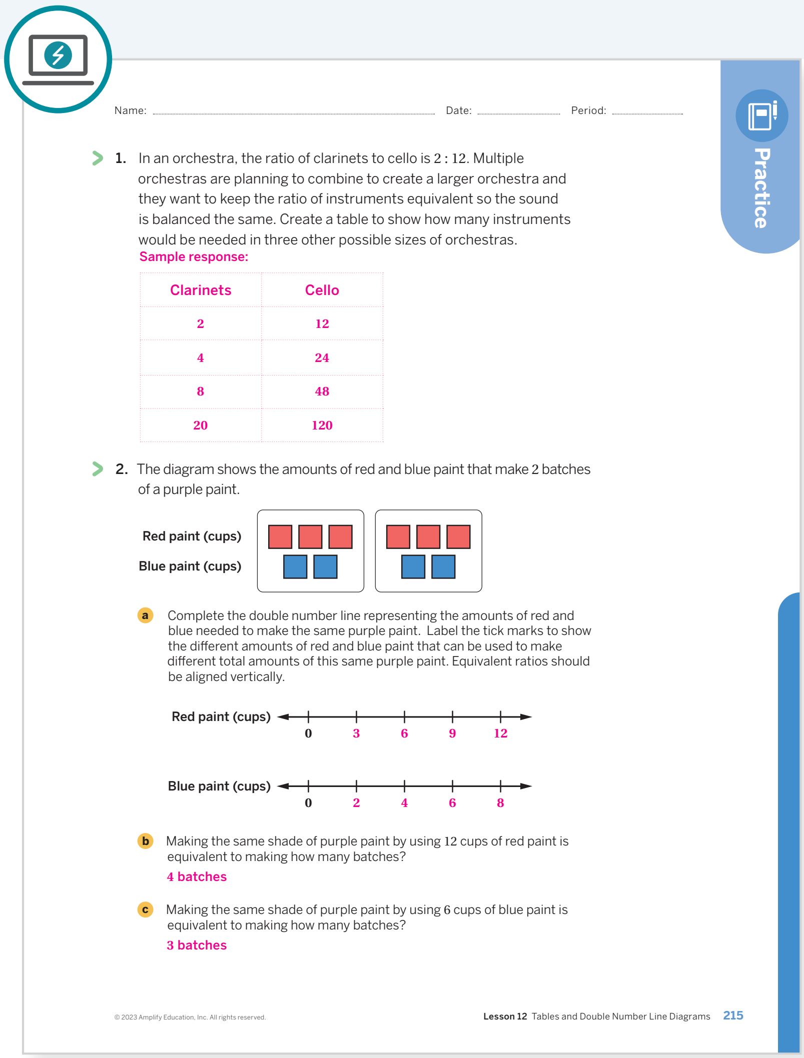
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did the ***Gallery Tour*** routine in Activity 2 impact your students' understanding of the concept?
- Which groups of students did and didn't have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Practice

Independent



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Practice

1. In an orchestra, the ratio of clarinets to cello is 2 : 12. Multiple orchestras are planning to combine to create a larger orchestra and they want to keep the ratio of instruments equivalent so the sound is balanced the same. Create a table to show how many instruments would be needed in three other possible sizes of orchestras.

Sample response:

Clarinets	Cello
2	12
4	24
8	48
20	120

2. The diagram shows the amounts of red and blue paint that make 2 batches of a purple paint.

Red paint (cups)
Blue paint (cups)

a. Complete the double number line representing the amounts of red and blue needed to make the same purple paint. Label the tick marks to show the different amounts of red and blue paint that can be used to make different total amounts of this same purple paint. Equivalent ratios should be aligned vertically.

Red paint (cups)	0	3	6	9	12
Blue paint (cups)	0	2	4	6	8

b. Making the same shade of purple paint by using 12 cups of red paint is equivalent to making how many batches?

4 batches

c. Making the same shade of purple paint by using 6 cups of blue paint is equivalent to making how many batches?

3 batches

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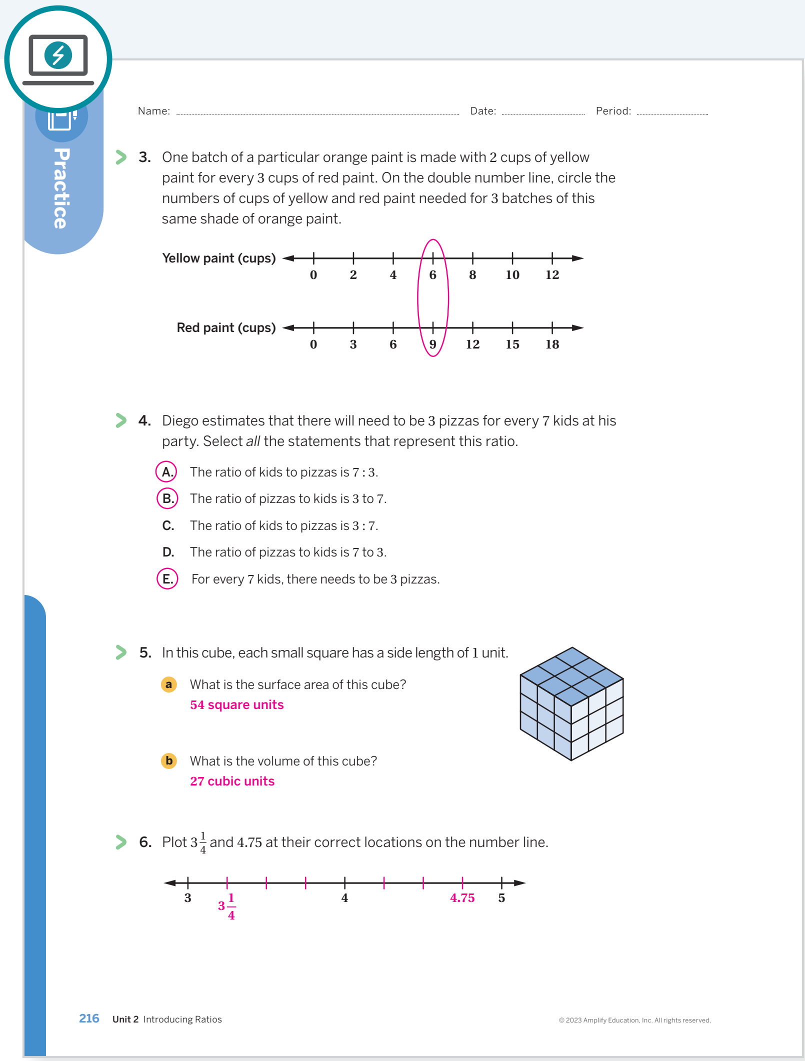

Name: ________ Date: ________ Period: ________

Practice

3. One batch of a particular orange paint is made with 2 cups of yellow paint for every 3 cups of red paint. On the double number line, circle the numbers of cups of yellow and red paint needed for 3 batches of this same shade of orange paint.

Yellow paint (cups)	0	2	4	6	8	10	12
Red paint (cups)	0	3	6	9	12	15	18

4. Diego estimates that there will need to be 3 pizzas for every 7 kids at his party. Select *all* the statements that represent this ratio.

[✓ A.] The ratio of kids to pizzas is 7 : 3.
[✓ B.] The ratio of pizzas to kids is 3 to 7.
C. The ratio of kids to pizzas is 3 : 7.
D. The ratio of pizzas to kids is 7 to 3.
[✓ E.] For every 7 kids, there needs to be 3 pizzas.

5. In this cube, each small square has a side length of 1 unit.

a. What is the surface area of this cube?

54 square units

b. What is the volume of this cube?

27 cubic units

6. Plot $3\frac{1}{4}$ and 4.75 at their correct locations on the number line.

3, $3\frac{1}{4}$, 4, 4.75, 5

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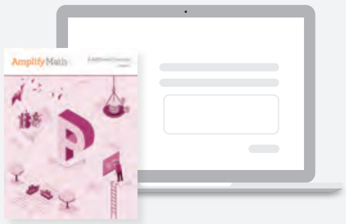
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 2 Lesson 2	2
	5	Unit 1 Lesson 14	2
Formative ⚡	6	Unit 2 Lesson 13	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 2 | **LESSON 13**

Tempo and Double Number Lines

Let’s look at song tempos and draw double number line diagrams.

Focus

Goals

1. Draw and label a double number line diagram from scratch, with parallel lines and equally-spaced tick marks.
2. Use double number line diagrams to find a wider range of equivalent ratios.

Rigor

- Students strengthen their **procedural fluency** using double number lines to show equivalent fractions.
- In thinking about song tempos, students **apply** their equivalent ratios on double number lines.

Coherence

Today

Students create their own double number line diagrams to support their reasoning and represent equivalent ratios in the context of beats per minute of songs and their corresponding tempo markings. They recognize the importance of using parallel lines, equally-spaced tick marks, and descriptive labels.

Previously

Students were introduced to using double number line diagrams as well as using a table to show equivalent ratios. They discussed the pros and cons of each method.

Coming Soon

In later activities and lessons, students make their own strategic choice of an appropriate representation to support their reasoning.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Song Tempos, Problem 3c* (for display)
- Activity 1 PDF, *Song Tempos, Problem 3c* (answer)
- Activity 2 PDF, *Song List With Tempo Markings* (for display)

Math Language Development

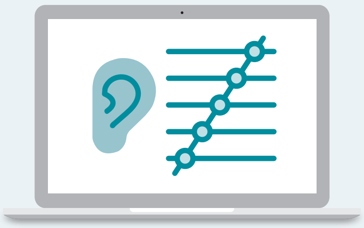
Review word

- *per*

Amps | Featured Activity

Activity 1
Hear the Math

Students listen to different musical tempos and experiment with adjusting equivalent ratios to alter the tempo.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students may not see the rhythm structure when they have to shift from representing patterns of beats per minute to beats per 30 seconds on a double number line. Encourage them to persist in looking for structure in both the numbers and in the physical representation. Have them make a table first, if necessary, to help them see the pattern.

Modifications to Pacing

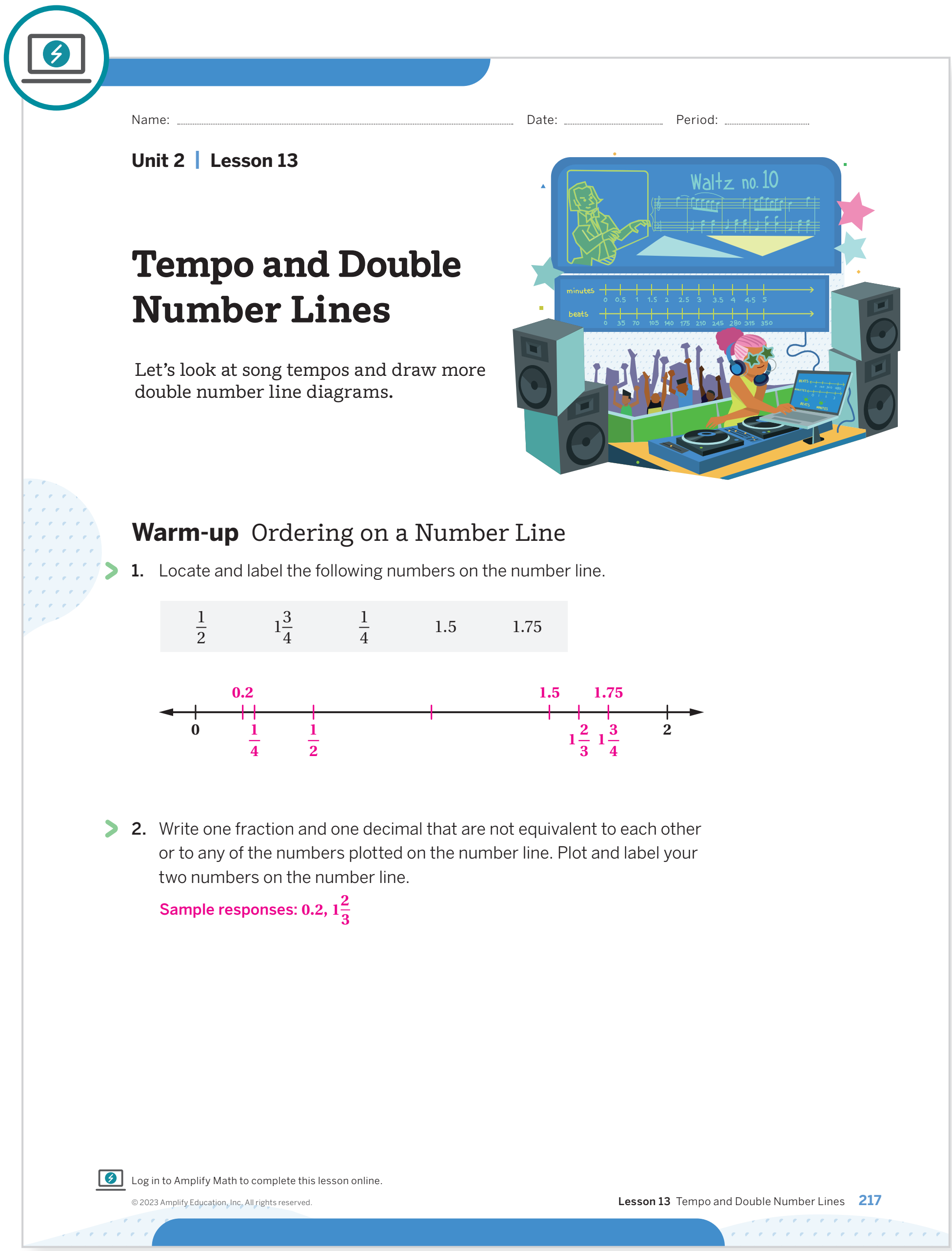
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, skip Problem 1 and have students only complete two of the four parts from Problem 2, in addition to doing Problem 3.
- In **Activity 3**, Problem 1 may be omitted, and after students complete Problem 2, provide a tempo for Problem 1a of 115 bpm, allowing them to complete Problem 3.

Pairs | 5 min

Warm-up Ordering on a Number Line

Students partition a number line and locate fraction and decimal equivalents in preparation for working with double number lines.



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Unit 2 | Lesson 13

Tempo and Double Number Lines

Let's look at song tempos and draw more double number line diagrams.

Warm-up Ordering on a Number Line

1. Locate and label the following numbers on the number line.

$\frac{1}{2}$	$1\frac{3}{4}$	$\frac{1}{4}$	1.5	1.75

2. Write one fraction and one decimal that are not equivalent to each other or to any of the numbers plotted on the number line. Plot and label your two numbers on the number line.

Sample responses: 0.2, $1\frac{2}{3}$

Log in to Amplify Math to complete this lesson online.

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1 Launch

Use the *Think-Pair-Share* routine to have students work through the problems and then explain their responses to their partner.

2 Monitor

Help students get started by asking, "Do you know how to plot any of these numbers? Where would 1 be plotted?"

Look for points of confusion:

- **Plotting $\frac{1}{2}$ in the middle of the number line.** Have them look at the ends of the number line and ask, "What belongs in the middle?"
- **Labeling tick marks in numerical order, but without proper spacing.** Encourage students to partition the number line by using repeated halving down to quarters. Note that not every tick mark will correspond to a given value.

Look for productive strategies:

- Recognizing they need tick marks for fourths and spacing tick marks evenly on the number line.
- Knowing how to plot other fractions and decimals (Problem 2) that might not be halves or quarters.

3 Connect

Display a blank version of the number line.

Have pairs of students share how they partitioned the number line to plot each number from Problem 1. Have them state their different chosen numbers from Problem 2 and how they added tick marks.

Highlight that a number line can be partitioned into different intervals, such as halves or fourths of a given endpoint number. Students can always create smaller intervals from larger ones by partitioning between each tick mark again. This works with fractions and whole numbers.

Math Language Development

MLR7: Compare and Connect

As students compare and share how they partitioned the number line, revoice their language and connect it to the terms *locate*, *label*, and *tick mark*.

English Learners

Support students' understanding of the terms *locate* and *label* by pointing to the number's position on the number line and saying "locate". Then as you write the number, say "label."

Power-up

To power up students' ability to partition number lines in order to plot fraction and decimal values, have students complete:

1. Partition the number line into 5 equal sections:

2. Mark the value $\frac{4}{5}$ on the number line from Problem 1.

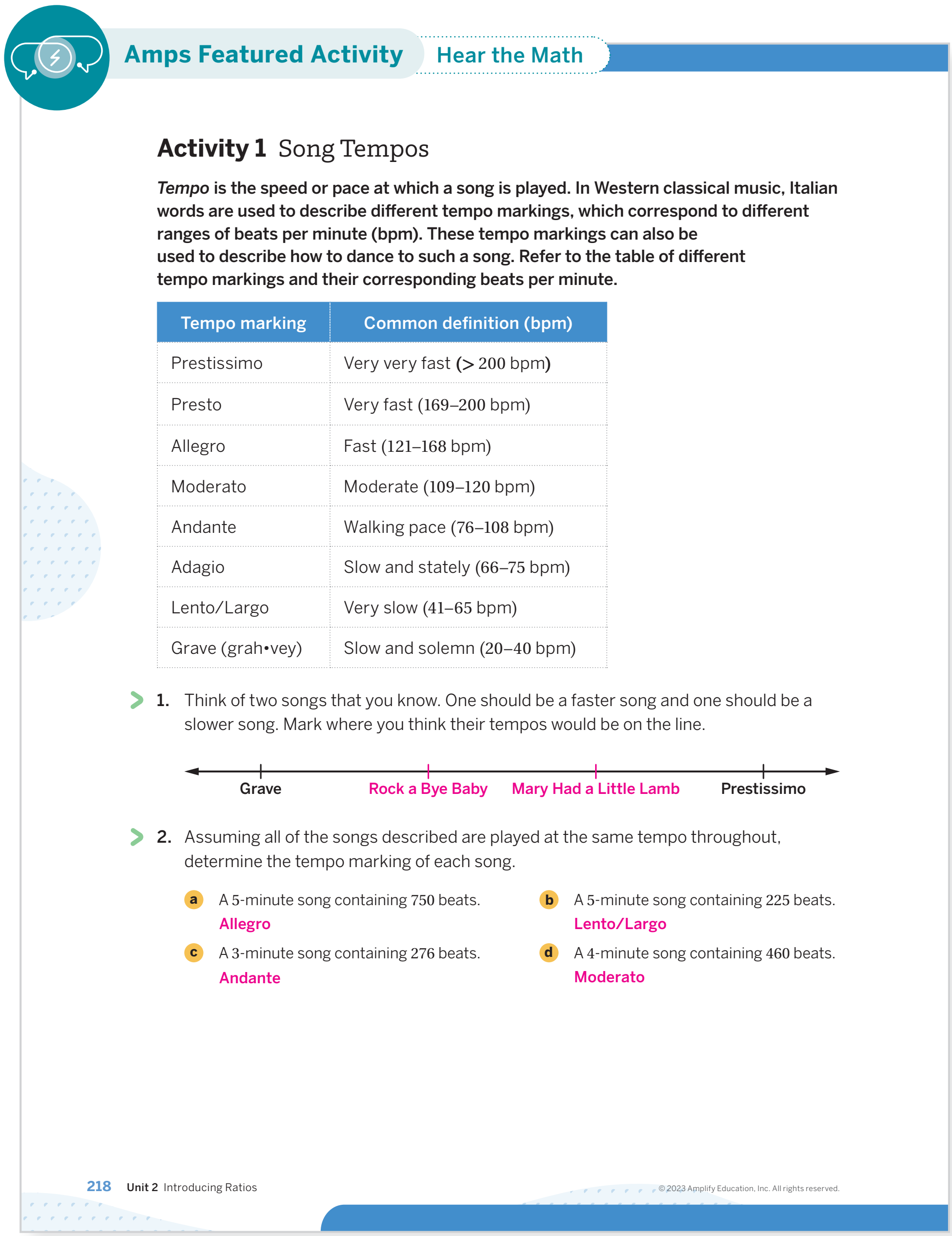
Use: Before the Warm-up

Informed by: Performance on Lesson 12, Practice Problem 6, and Pre-Unit Readiness Assessment, Problems 3 and 4.

Pairs | 15 min

Activity 1 Song Tempos

Students use equivalent ratios to determine the beats per minute of different songs, and create a double number line to represent the beats per minute of a chosen tempo.



Amps Featured Activity Hear the Math

Activity 1 Song Tempos

***Tempo* is the speed or pace at which a song is played. In Western classical music, Italian words are used to describe different tempo markings, which correspond to different ranges of beats per minute (bpm). These tempo markings can also be used to describe how to dance to such a song. Refer to the table of different tempo markings and their corresponding beats per minute.**

Tempo marking	Common definition (bpm)
Prestissimo	Very very fast (> 200 bpm)
Presto	Very fast (169–200 bpm)
Allegro	Fast (121–168 bpm)
Moderato	Moderate (109–120 bpm)
Andante	Walking pace (76–108 bpm)
Adagio	Slow and stately (66–75 bpm)
Lento/Largo	Very slow (41–65 bpm)
Grave (grah•vey)	Slow and solemn (20–40 bpm)

1. Think of two songs that you know. One should be a faster song and one should be a slower song. Mark where you think their tempos would be on the line.

2. Assuming all of the songs described are played at the same tempo throughout, determine the tempo marking of each song.

a. A 5-minute song containing 750 beats.
Allegro

b. A 5-minute song containing 225 beats.
Lento/Largo

c. A 3-minute song containing 276 beats.
Andante

d. A 4-minute song containing 460 beats.
Moderato

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1 Launch

Keep students in pairs. Have them read the paragraph about tempo and study the chart of tempo markings, pausing to answer questions.

Consider playing the *Happy Birthday* audio clip (allegro, 125 bpm); then replay the clip at one faster tempo marking and one slower tempo marking, so students can hear how the same song sounds at different tempos.

Have pairs complete the problems.

2 Monitor

Help students get started by asking them to, "Describe the differences between a grave (grah-vey) tempo and a prestissimo tempo? Can you think of a song you know that is slow and a song you know that is fast?"

Look for points of confusion:

- **Not knowing how to determine each song's tempo marking.** Ask, "Since beats per minute would be a ratio of beats to 1 minute, how could you determine that ratio?"
- **Struggling to set up a double number line.** Refer to the Lesson 12 Summary to review the setup. If more support is needed ask, "What number should be at the first tick mark of the 'minutes' number line? What would the corresponding number on the 'beats' number line represent?"

Look for productive strategies:

- Relating a given pair of beats and minutes to beats per minute as equivalent ratios.
- Correctly constructing double number lines to represent a tempo and song length.
- Choosing a song length that is not a whole number of minutes and determining the number of beats per second instead.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide two sample songs and play them for students as you complete Problem 1 as a whole class. Then model how to complete Problem 2a. Have students complete Problems 2b and 2c with their partners. Omit Problem 2d.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can "hear the math." They can listen to different musical tempos and experiment with adjusting equivalent ratios to alter the tempo.

Math Language Development

MLR1: Stronger and Clearer Each Time

Provide students with opportunities to clarify the use of *per* as "for every." Ask, "Why is it useful to know how many beats per minute? How did you use double number lines to solve this problem?" Give students time to revise their initial thinking.

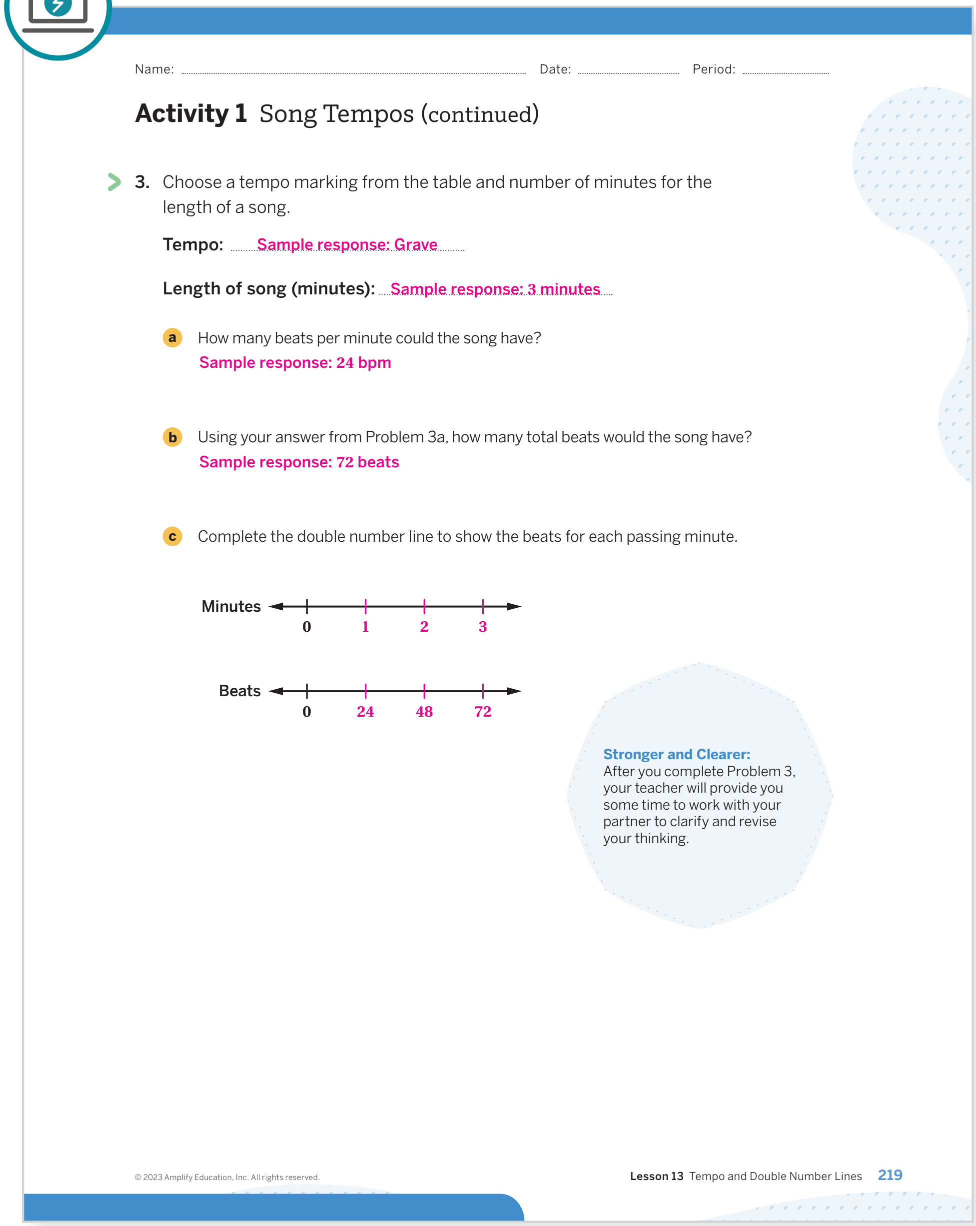
English Learners

Encourage students to refer to the class display as they work to clarify the use of the word *per*.

Pairs | 15 min

Activity 1 Song Tempos (continued)

Students use equivalent ratios to determine the beats per minute of different songs, and create a double number line to represent the beats per minute of a chosen tempo.



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Activity 1 Song Tempos (continued)

3. Choose a tempo marking from the table and number of minutes for the length of a song.

   Tempo: Sample response: Grave

   Length of song (minutes): Sample response: 3 minutes

   a. How many beats per minute could the song have?
   Sample response: 24 bpm

   b. Using your answer from Problem 3a, how many total beats would the song have?
   Sample response: 72 beats

   c. Complete the double number line to show the beats for each passing minute.

Stronger and Clearer: After you complete Problem 3, your teacher will provide you some time to work with your partner to clarify and revise your thinking.

3 Connect

Display the correct responses for Problem 2 and the blank number lines from the Activity 1 PDF.

Have pairs of students share how they determined the tempos of the songs in Problems 2a–2d. Then have them share their chosen tempos and song lengths for Problem 3, how they determined the total beats in the song, and the equivalent ratios they represented on their double number lines.

Highlight that for a context, such as "beats per minute," students can use a double number line to determine and represent equivalent ratios, and the representation is a good visual model of elapsing time and the corresponding amount of the second quantity, such as beats.

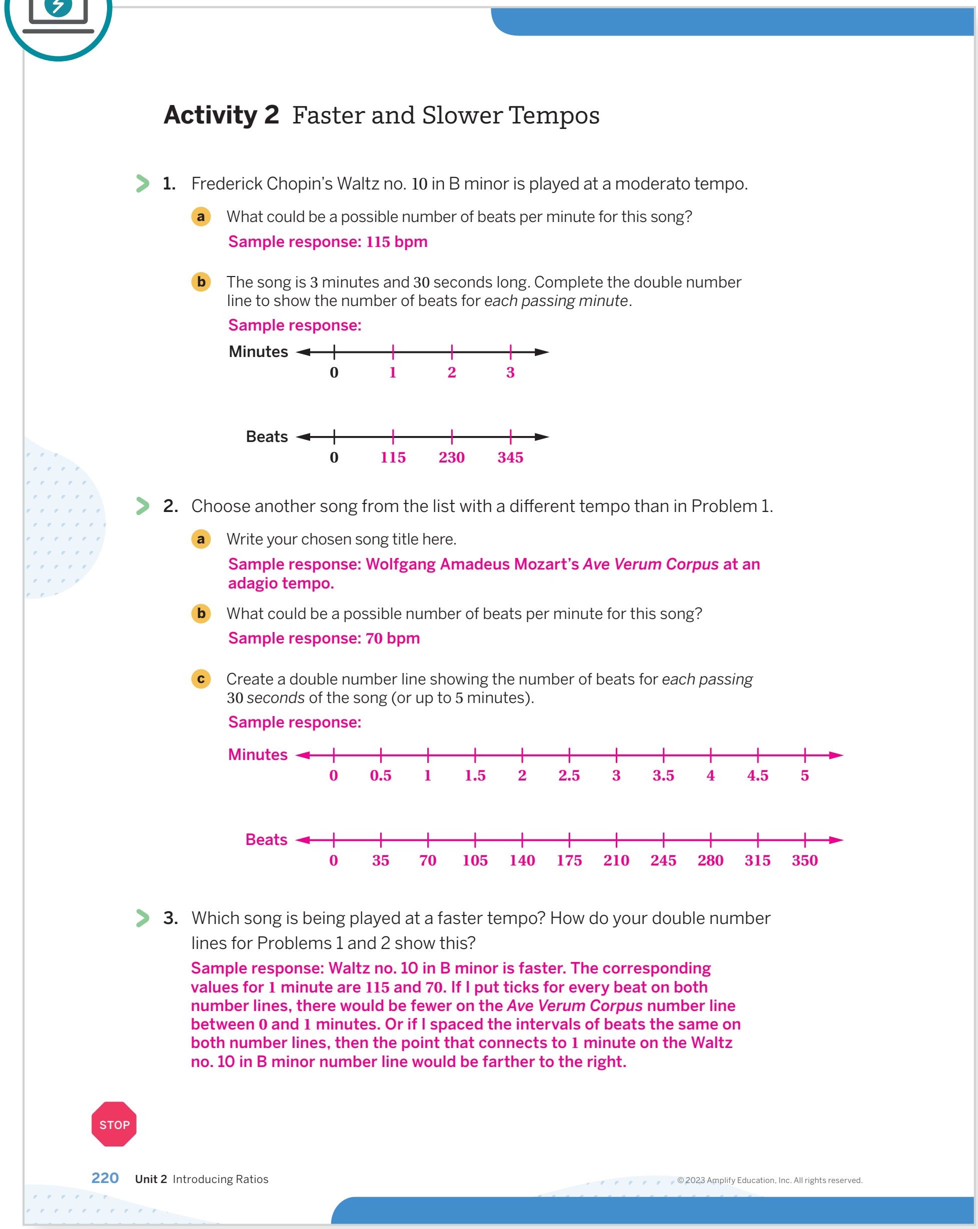
Ask,

- "Do you think a table would be more useful here than a double number line? Explain."
- (If time allows) "What would the other equivalent ratio containing a 1 represent in this context?" minutes per beat
  - » "How could you show that on the double number line?" You could divide to get $1 \div 24 = \frac{1}{24}$, so the ratio is $\frac{1}{24}$: 1 and if you partition each number line into 24 equal parts between 0 and the first tick, it would be shown by the new first ticks. (You could also think of 1 minute as 60 seconds and divide to get $60 \div 24 = 2.5$. That means each beat is 2.5 seconds apart and the ratio for seconds per beat is 1 : 2.5.)
  - » Display the Activity 1 PDF (answers) and demonstrate how to represent one or both of these solutions.

Independent | 15 min

Activity 2 Faster and Slower Tempos

Students practice creating more double number lines for beats per minute and tempo, and also begin to look at comparing two ratios by using these representations.



Activity 2 Faster and Slower Tempos

1. Frederick Chopin's Waltz no. 10 in B minor is played at a moderato tempo.

   a. What could be a possible number of beats per minute for this song?

   Sample response: 115 bpm

   b. The song is 3 minutes and 30 seconds long. Complete the double number line to show the number of beats for *each passing minute*.

   Sample response:

Minutes	0	1	2	3
Beats	0	115	230	345

2. Choose another song from the list with a different tempo than in Problem 1.

   a. Write your chosen song title here.

   Sample response: Wolfgang Amadeus Mozart's *Ave Verum Corpus* at an adagio tempo.

   b. What could be a possible number of beats per minute for this song?

   Sample response: 70 bpm

   c. Create a double number line showing the number of beats for *each passing 30 seconds* of the song (or up to 5 minutes).

   Sample response:

Minutes	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Beats	0	35	70	105	140	175	210	245	280	315	350

3. Which song is being played at a faster tempo? How do your double number lines for Problems 1 and 2 show this?

   Sample response: Waltz no. 10 in B minor is faster. The corresponding values for 1 minute are 115 and 70. If I put ticks for every beat on both number lines, there would be fewer on the *Ave Verum Corpus* number line between 0 and 1 minutes. Or if I spaced the intervals of beats the same on both number lines, then the point that connects to 1 minute on the Waltz no. 10 in B minor number line would be farther to the right.

STOP

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1 Launch

Display the Activity 2 PDF. Explain to students they should choose a song from the list for Problem 2. If time and resources allow, consider playing a clip of a few of the songs so students can hear the different tempos.

2 Monitor

Help students get started by having them refer to the chart in Activity 1 and locating the *moderato* row. Ask, "What is a bpm in the range?"

Look for points of confusion:

- **Having trouble determining the beats for 30 seconds in Problem 2.** Ask, "How many minutes is 30 seconds? How many beats correspond to 1 minute? So what is half of that?"

Look for productive strategies:

- Using a double number line to accurately determine and to represent equivalent ratios of beats to minutes and half-minutes.
- Explaining how there would be more ticks for beats (if all were shown) between each tick for whole minutes on the double number line representing the faster song.

3 Connect

Display select students' double number lines.

Have students share how they determined the values to include on their double number lines, and how they used the double number lines to explain which song is faster.

Highlight that, similarly to single number lines, fractions and decimals can also be represented on double number lines for equivalent ratios that include those types of numbers. Ratios containing a 1 can be used to compare rates and determine equivalent ratios.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

After students complete Problem 1, have them use their double number line to represent 0.5 minutes and the corresponding beats. Then have them add the other 30-second intervals, up to 3.5 minutes. Omit Problem 2.

Accessibility: *Guide Processing and Visualization*

Select the second song for students to use in Problem 2. Play both songs at the start of the activity so they can hear the differences in the tempos. Play the songs again at the end of the activity and reinforce the connection between the tempos they hear and the number lines created.

Math Language Development

MLR8: Discussion Supports

During the Launch, review the following terms from previous lessons: *double number line*, *parallel lines*, *tick mark*, *equal increments*, *equivalent ratios*, and "line up." Use visuals to support understanding of these terms in the context of this problem.

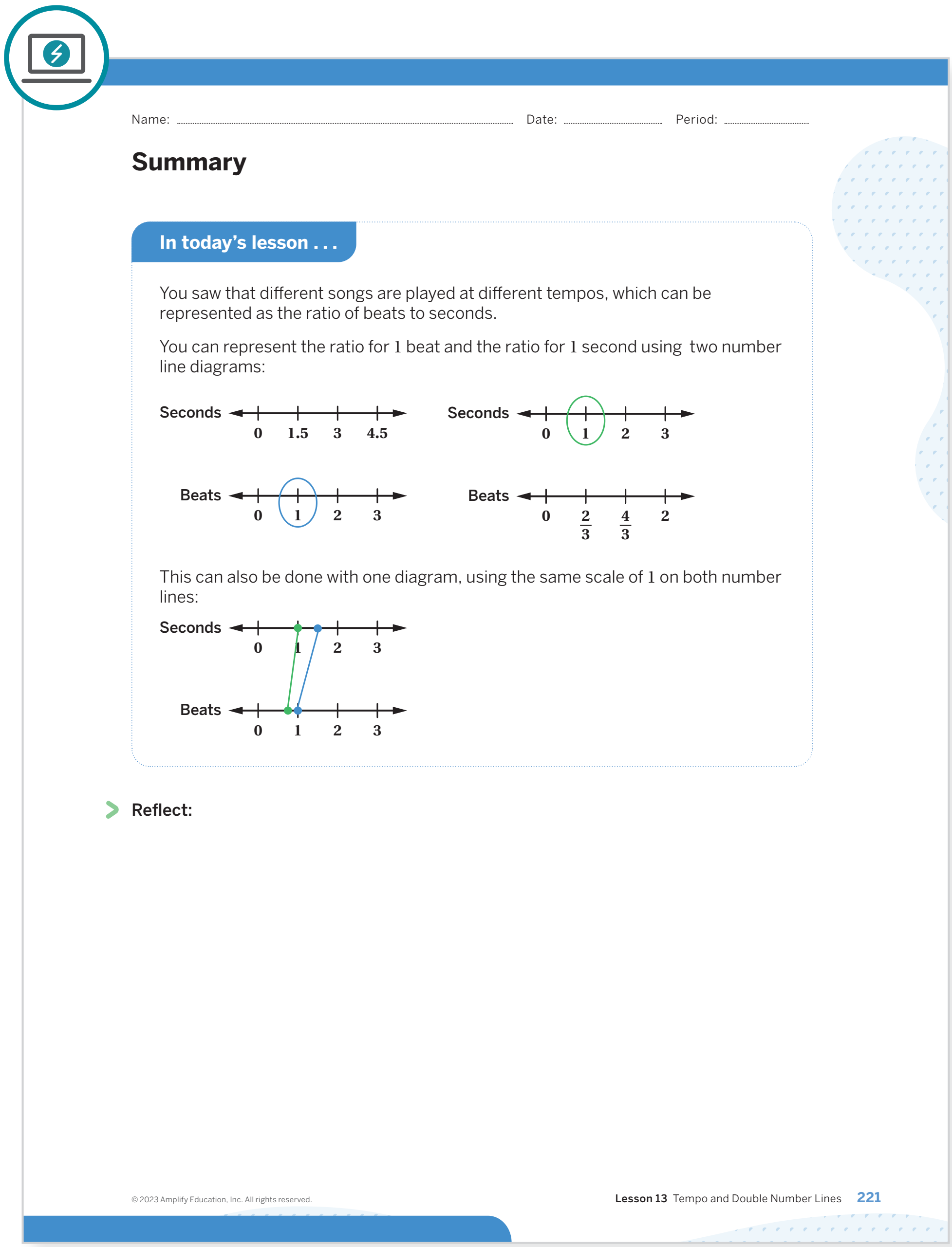
English Learners

Encourage students to refer to the class display as they review the terms from previous lessons.

Whole Class | 5 min

Summary

Review and synthesize all of the different ways double number lines can be created and used to represent contexts involving equivalent ratios.



Name: Date: Period:

Summary

In today's lesson . . .

You saw that different songs are played at different tempos, which can be represented as the ratio of beats to seconds.

You can represent the ratio for 1 beat and the ratio for 1 second using two number line diagrams:

This can also be done with one diagram, using the same scale of 1 on both number lines:

Reflect:

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Synthesize

Display the double number lines from the Summary.

Highlight that when creating double number lines:

- Each line should have evenly spaced tick marks and be labeled by the quantity it represents, including units of measure.
- With different scales, equivalent ratios should be aligned vertically, and can be represented by the tick marks themselves.
- With the same scales, equivalent ratios must be identified by corresponding points, and can be connected by segments or shown using color coding or different marks for the "points" (such as circles, triangles, squares, etc.).

Ask:

- "What are some important things to pay attention to when you create a double number line?" Making sure tick marks vertically align and are appropriately spaced, the math corresponds to data, and labels are correct.
- "What other scenarios could be represented using double number lines?" Walking speeds, recipes, or making gift bags.
- "What would an inaccurate interpretation/ representation on the double number line look like? Why would it be incorrect?" No vertical alignment, top line and bottom line do not correspond to each other, incorrect spacing, or incorrect units.

Have students share how they would interpret the information presented on the number line to help guide them in interpreting the number of seconds per beats in a given song.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

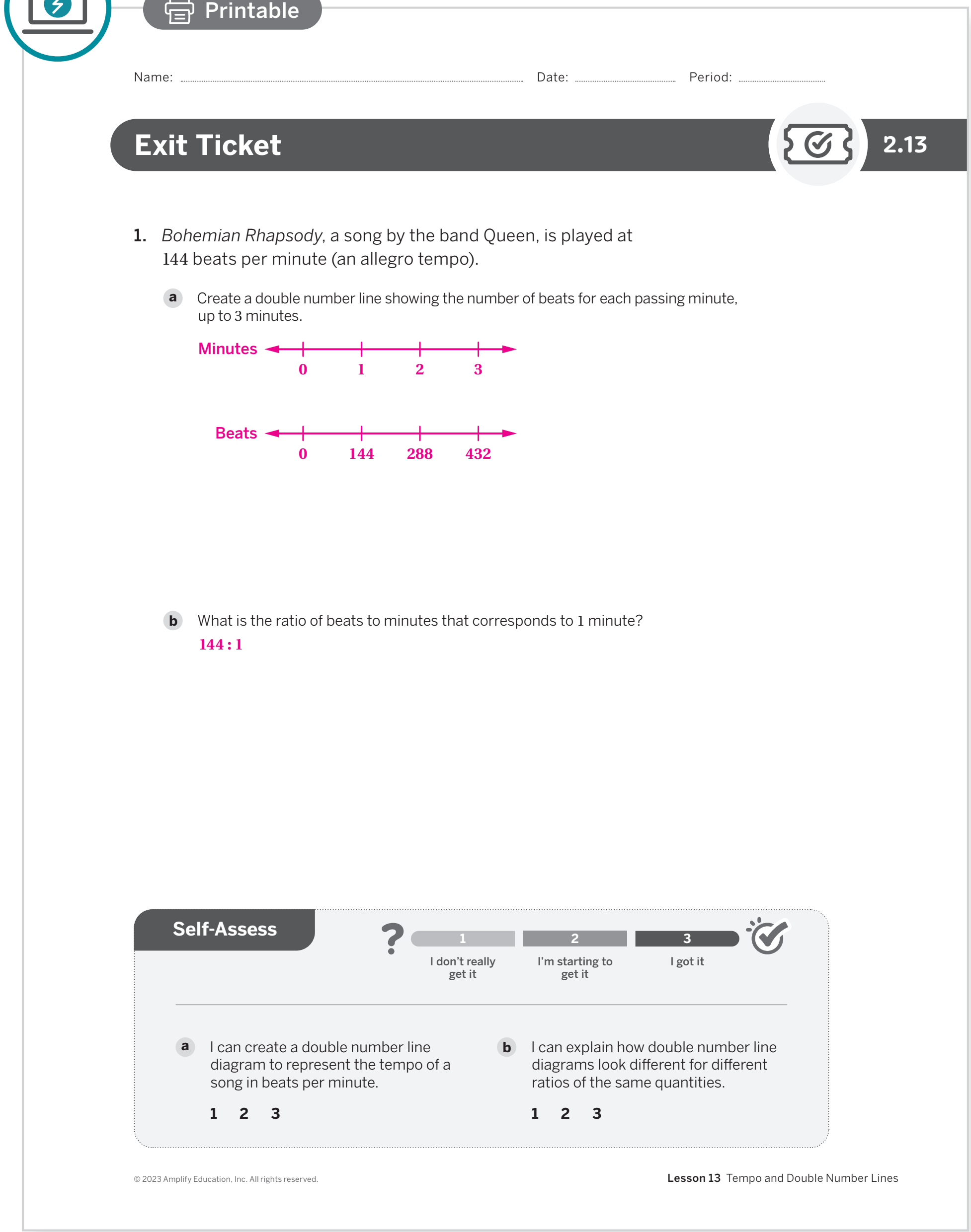
- "How might you think differently about beats in a song the next time you listen to music?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of determining the beats per minute of a song.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 2.13

1. *Bohemian Rhapsody*, a song by the band Queen, is played at 144 beats per minute (an allegro tempo).

   a Create a double number line showing the number of beats for each passing minute, up to 3 minutes.

   Minutes: 0, 1, 2, 3

   Beats: 0, 144, 288, 432

   b What is the ratio of beats to minutes that corresponds to 1 minute?

   144 : 1

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can create a double number line diagram to represent the tempo of a song in beats per minute.

1 2 3

b I can explain how double number line diagrams look different for different ratios of the same quantities.

1 2 3

Lesson 13 Tempo and Double Number Lines

Success looks like . . .

- **Goal:** Drawing and labeling a double number line diagram from scratch, with parallel lines and equally-spaced tick marks.
  - » Creating a double number line for the number of beats per minute in part a.
- **Goal:** Using double number line diagrams to find a wider range of equivalent ratios.
  - » Determining the number of beats for the first minute of a song in part b.

Suggested next steps

If students are unclear how to draw and label the double number line, consider:

- Asking, "How should you label each line? How should the tick marks be spaced and labeled?"
- Reviewing Activity 1, pointing to the labels and equally spaced tick marks.
- Assigning Practice Problem 1.

If students do not recall what ratio containing a 1 means or cannot determine it, consider:

- Referring back to Lesson 7.
- Ask students what other examples they could think of that would use one "unit" and how the word per relates to ratios containing a 1.

Professional Learning

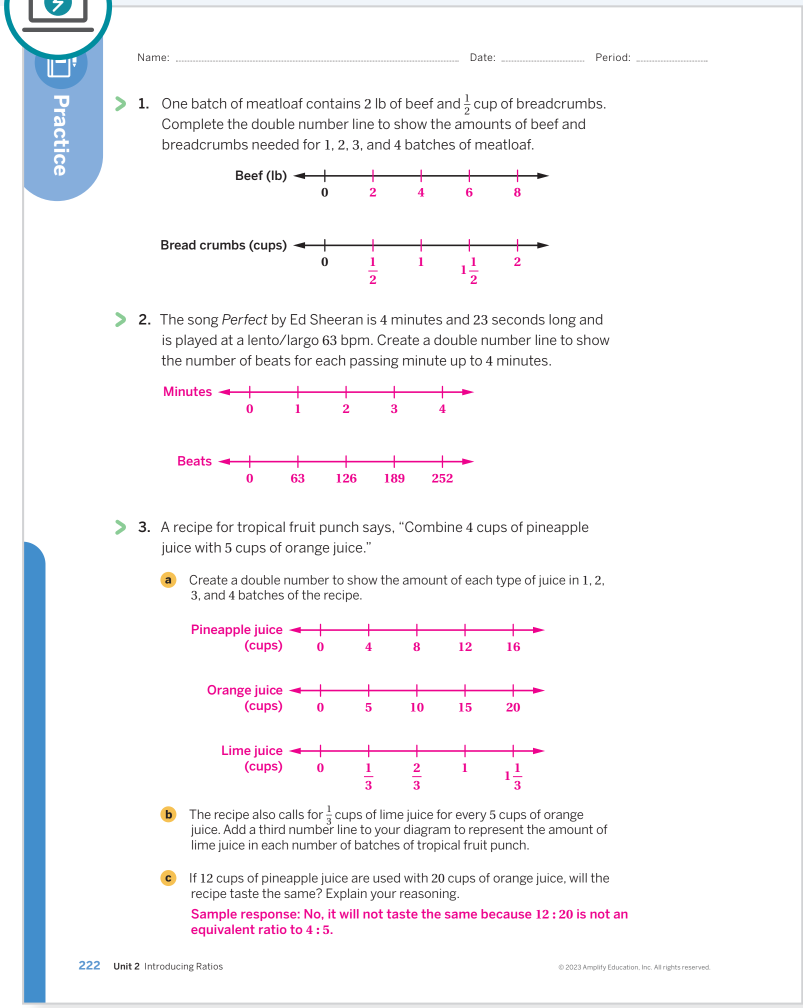
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What different ways did students approach incorporating double number lines to musical tempos? What does that tell you about similarities and differences among your students?
- The focus of this lesson was creating and interpreting different double number lines in a musical context. How did it go? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. One batch of meatloaf contains 2 lb of beef and $\frac{1}{2}$ cup of breadcrumbs. Complete the double number line to show the amounts of beef and breadcrumbs needed for 1, 2, 3, and 4 batches of meatloaf.

2. The song *Perfect* by Ed Sheeran is 4 minutes and 23 seconds long and is played at a lento/largo 63 bpm. Create a double number line to show the number of beats for each passing minute up to 4 minutes.

3. A recipe for tropical fruit punch says, "Combine 4 cups of pineapple juice with 5 cups of orange juice."

a. Create a double number to show the amount of each type of juice in 1, 2, 3, and 4 batches of the recipe.

b. The recipe also calls for $\frac{1}{3}$ cups of lime juice for every 5 cups of orange juice. Add a third number line to your diagram to represent the amount of lime juice in each number of batches of tropical fruit punch.

c. If 12 cups of pineapple juice are used with 20 cups of orange juice, will the recipe taste the same? Explain your reasoning.

Sample response: No, it will not taste the same because 12 : 20 is not an equivalent ratio to 4 : 5.

222 **Unit 2** Introducing Ratios

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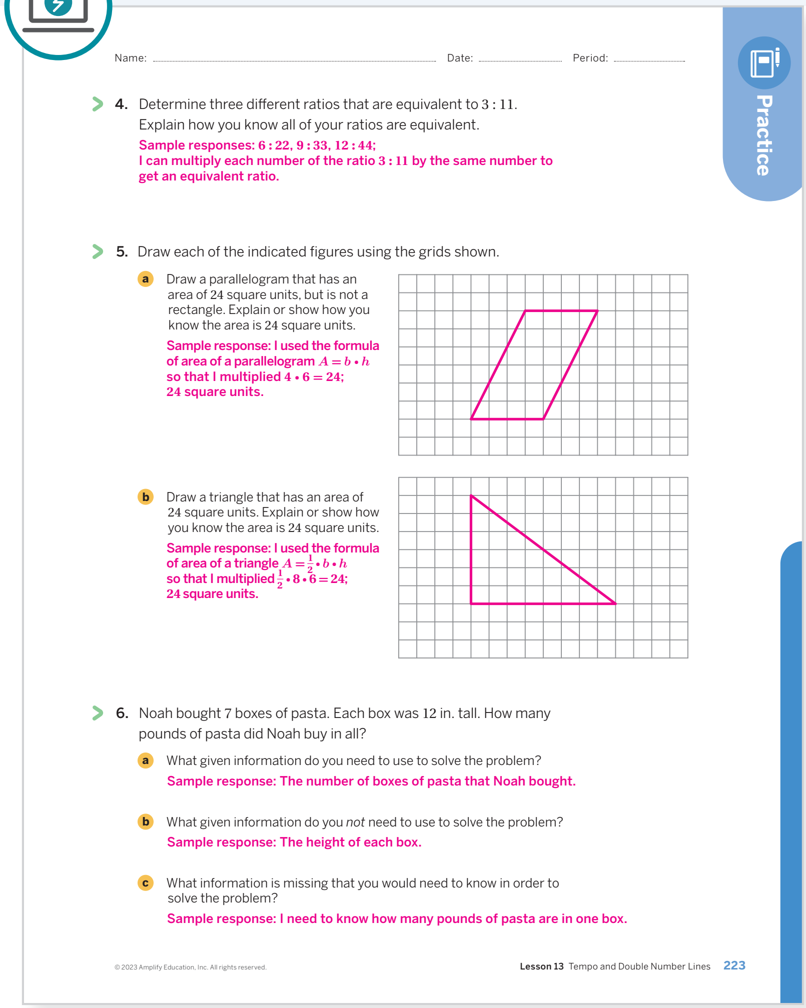

Name: Date: Period:

4. Determine three different ratios that are equivalent to 3 : 11. Explain how you know all of your ratios are equivalent.

Sample responses: 6 : 22, 9 : 33, 12 : 44;
I can multiply each number of the ratio 3 : 11 by the same number to get an equivalent ratio.

5. Draw each of the indicated figures using the grids shown.

a. Draw a parallelogram that has an area of 24 square units, but is not a rectangle. Explain or show how you know the area is 24 square units.

Sample response: I used the formula of area of a parallelogram $A = b \cdot h$ so that I multiplied $4 \cdot 6 = 24$; 24 square units.

b. Draw a triangle that has an area of 24 square units. Explain or show how you know the area is 24 square units.

Sample response: I used the formula of area of a triangle $A = \frac{1}{2} \cdot b \cdot h$ so that I multiplied $\frac{1}{2} \cdot 8 \cdot 6 = 24$; 24 square units.

6. Noah bought 7 boxes of pasta. Each box was 12 in. tall. How many pounds of pasta did Noah buy in all?

a. What given information do you need to use to solve the problem?

Sample response: The number of boxes of pasta that Noah bought.

b. What given information do you *not* need to use to solve the problem?

Sample response: The height of each box.

c. What information is missing that you would need to know in order to solve the problem?

Sample response: I need to know how many pounds of pasta are in one box.

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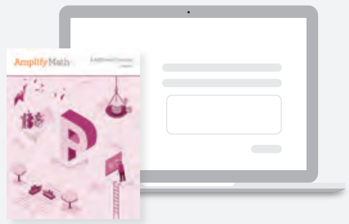
Lesson 13 Tempo and Double Number Lines 223

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activities 1 and 2	2
Spiral	4	Unit 2 Lesson 6	2
	5	Unit 1 Lesson 6	2
Formative	6	Unit 2 Lesson 14	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 3

Solving Ratio Problems

Whole Class

In this Sub-Unit, students use equivalent ratios to determine missing values and compare ratios in real-world problems, with an emphasis on practical applications to recipes.

SUB-UNIT

3 Solving Ratio Problems

Narrative Connections

Who brought Italy to India and back again?

In the 1980s, Italian cuisine was rare in Kolkata, India. And yet, for 10-year-old Ritu Dalmia, there was nothing better. She had gotten a taste for it after a school trip to Italy. For a month, she and her classmates sampled dishes like spaghetti pomodoro. For Dalmia, it was love-at-first-taste.

This love would start her on a journey many decades long, spanning multiple countries.

She opened MezzaLuna, one of Delhi's first Italian restaurants. Two years later, Dalmia headed to London to open Vama, a successful, high-end Indian restaurant. Five years after that, she returned to India to open another Italian restaurant — Diva. Diva was so successful that offshoots sprouted up, including Diva Cafe, DIVA Piccola, and Latitude 28. Not one to rest on her laurels, Dalmia returned to the source — Italy — to open Cittamani. This exciting new restaurant fused Indian cuisine with Italian ingredients.

Dalmia's passion has brought new tastes and flavors to those who might not otherwise have the opportunity to try them. Whether you're a home cook or a globe-hopping celebrity chef, the right ingredients in the right amounts are important to executing a meal. But to get the recipe exactly right, ratios are the key ingredient!

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore ratios within recipes and food in the following places:

- **Lesson 15, Activity 1:** All-Natural Food Coloring
- **Lesson 16, Activity 1:** Comparing Chili Peppers
- **Lesson 19, Activity 2:** Metric Recipes

UNIT 2 | **LESSON 14**

Solving Equivalent Ratio Problems

Let's practice identifying needed information to solve ratio problems.

Focus

Goals

1. **Language Goal:** Determine what information is needed to solve a problem involving equivalent ratios. Ask questions to elicit that information. **(Speaking and Listening, Writing)**
2. **Language Goal:** Choose and create representations to solve equivalent ratio problems involving a missing value. Explain the solution strategy by using the chosen representation. **(Speaking and Listening)**

Coherence

Today

Using the ***Info Gap*** routine, students determine what information is necessary to solve equivalent ratio problems involving a missing value, and they ask appropriate questions to elicit that information. This routine allows students to refine the language they use and to ask increasingly more precise questions until they get the information they need to make sense of the problem. Students solve for the missing value by using any method and representation they choose, and they explain both their representations and their thinking.

Previously

In Lessons 6–13, students have been working with equivalent ratios to solve real-world and mathematical problems. They have used multiple representations — diagrams, tables, and double number lines — to depict ratio relationships between quantities and to generate equivalent ratios.

Coming Soon

In Lesson 15, students will explore part-part-whole ratios by using the relationship between the quantities that are the parts and the total amount (as another quantity) to solve problems.

Rigor

- Students build **conceptual understanding** of equivalent ratios with missing values.
- Students develop **procedural fluency** with equivalent ratios.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	25 min	5 min	5 min
Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per pair

Math Language Development

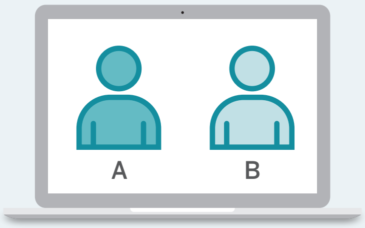
Review word

- *equivalent ratios*

Amps | Featured Activity

Activity 1
Interactive Info Gap

Students use an interactive version of the ***Info Gap*** routine to solve equivalent ratio problems.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated when their questions do not yield the information needed to solve the problem. Have them reflect on the information their question *did* yield before examining the information they still need. It may be helpful to use a sentence frame such as "I now know . . . , but I still need to know . . . , so I can ask . . ." Encourage students to brainstorm one or two more precise questions that will lead to the missing information.

Modifications to Pacing

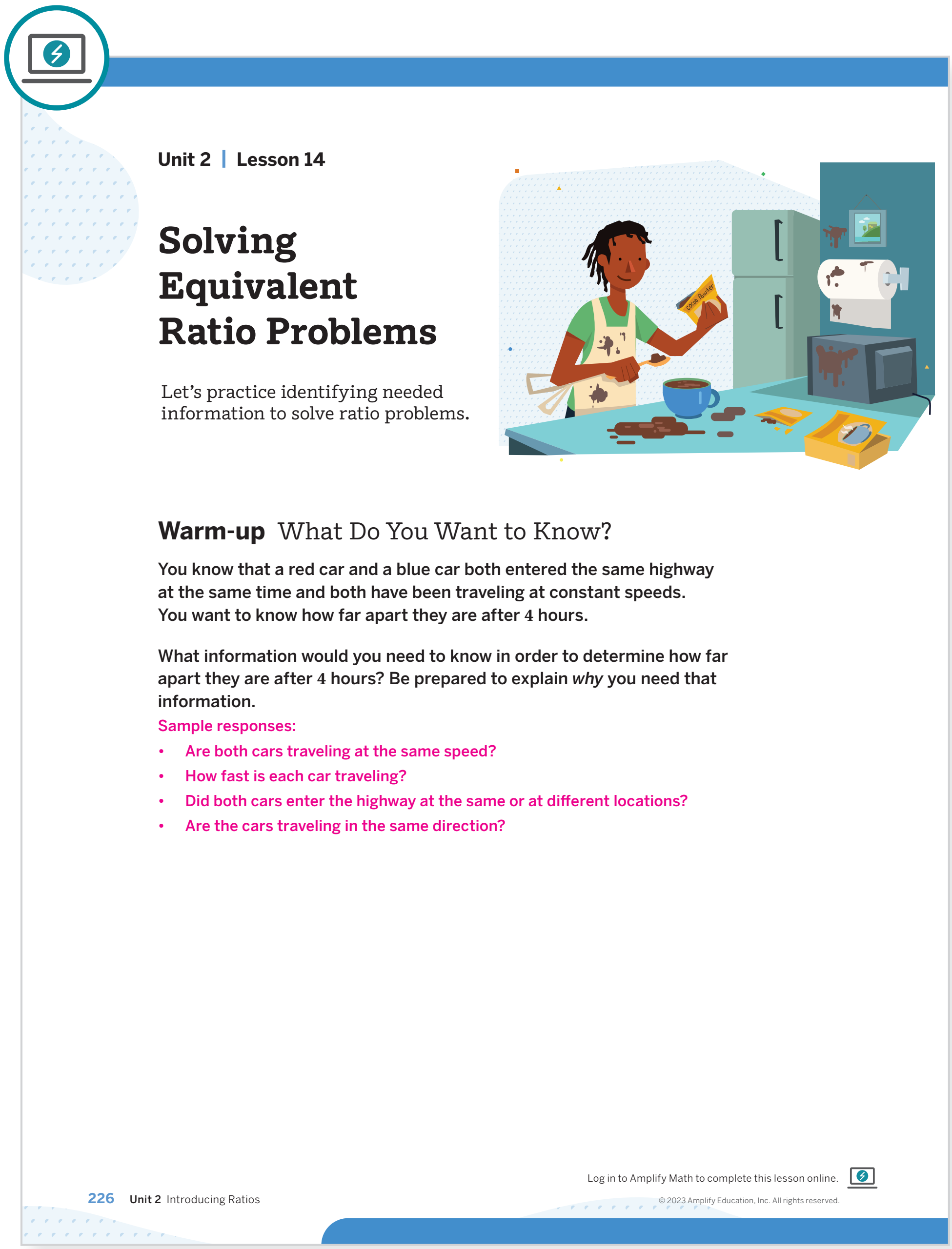
You may want to consider this additional modification if you are short on time.

- Omit the **Warm-up** and make **Activity 1** a whole-class activity. Conduct the ***Info Gap*** routine with you holding both the Problem and Data cards. Have students work in pairs to identify the information they need to know and solve the problems.

Independent | 10 min

Warm-up What Do You Want to Know?

Students begin to identify and ask for information they need to solve an equivalent ratio problem about the distance between two cars, preparing them for the ***Info Gap*** routine.



Unit 2 | Lesson 14

Solving Equivalent Ratio Problems

Let's practice identifying needed information to solve ratio problems.

Warm-up What Do You Want to Know?

You know that a red car and a blue car both entered the same highway at the same time and both have been traveling at constant speeds. You want to know how far apart they are after 4 hours.

What information would you need to know in order to determine how far apart they are after 4 hours? Be prepared to explain *why* you need that information.

Sample responses:

- Are both cars traveling at the same speed?
- How fast is each car traveling?
- Did both cars enter the highway at the same or at different locations?
- Are the cars traveling in the same direction?

226 Unit 2 Introducing Ratios

Log in to Amplify Math to complete this lesson online.

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1 Launch

Explain that *constant speed* means the cars neither speed up nor slow down at any time.

2 Monitor

Help students get started by having them identify the question asked and the information that is given. Ask, "Do you have enough information to know how far apart they are?"

Look for points of confusion:

- **Identifying irrelevant information.** Have students provide a sample answer to their own question, and ask, "Does that help you answer the question?"

Look for productive strategies:

- Identifying relevant but general information (e.g., car speed), or relevant and precise information (e.g., how fast each car is moving).

3 Connect

Have students share the information they need and why they need it. Record their questions for the class to see, and provide only the information explicitly requested.

- One car is traveling 5 mph faster than the other car.
- The red car is traveling faster than the blue car.
- The blue car is traveling at 60 mph.
- The red car is traveling at 65 mph.
- Both cars entered the highway at the same location.
- Both cars are traveling in the same direction.

Ask, after each question is answered, whether they have enough information to solve the problem. When they do, have them solve the problem and share their strategies.

Highlight that when identifying missing information, start with the question, and ask, "What else do I need to know to answer this question?"

Power-up

To power up students' ability to identify needed, extra, and missing information in a given problem, have students complete:

Priya purchased 6 bags of cherries. Each bag is 6 inches wide. How many pounds of cherries did she buy?

What additional information would be sufficient to determine the answer to the question? Select *all* that apply.

[✓ A.] The number of cherries in each bag and the weight of one cherry.

B. The weight of one cherry.

C. The cost of one pound of cherries.

[✓ D.] The weight of each bag of cherries.

[✓ E.] How much money she spent in total and the cost of one pound of cherries.

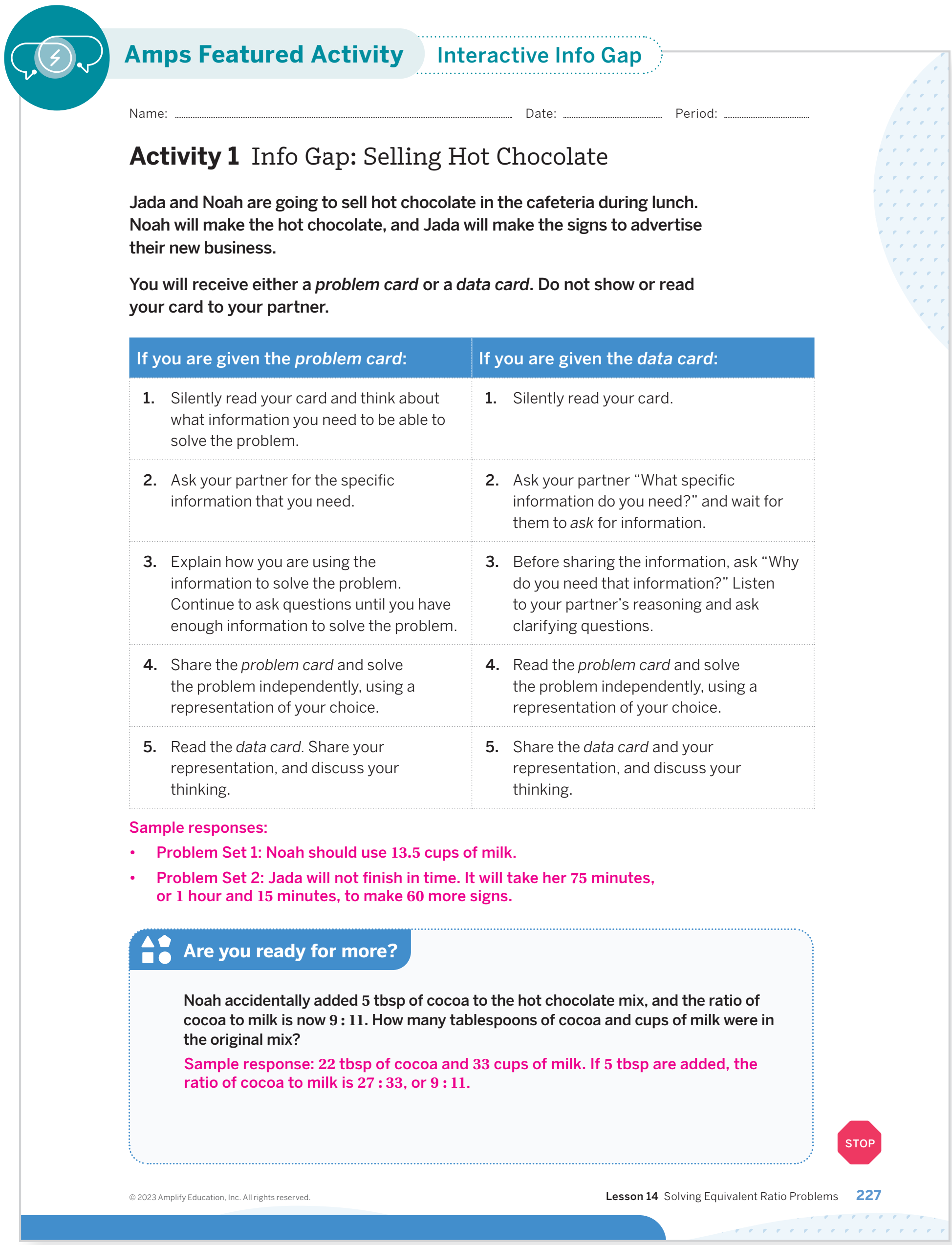
Use: Before the Warm-up

Informed by: Performance on Lesson 13, Practice Problem 6.

Pairs | 25 min

Activity 1 Info Gap: Selling Hot Chocolate

Students participate in the ***Info Gap*** routine to solve equivalent ratio problems about the ingredients for making hot chocolate and the time it takes to make marketing posters.



Amps Featured Activity Interactive Info Gap

Name: ______ Date: ______ Period: ______

Activity 1 Info Gap: Selling Hot Chocolate

Jada and Noah are going to sell hot chocolate in the cafeteria during lunch. Noah will make the hot chocolate, and Jada will make the signs to advertise their new business.

You will receive either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given the *problem card*:	If you are given the *data card*:
1. Silently read your card and think about what information you need to be able to solve the problem.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner "What specific information do you need?" and wait for them to *ask* for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask "Why do you need that information?" Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently, using a representation of your choice.	**4.** Read the *problem card* and solve the problem independently, using a representation of your choice.
5. Read the *data card*. Share your representation, and discuss your thinking.	**5.** Share the *data card* and your representation, and discuss your thinking.

Sample responses:

- **Problem Set 1: Noah should use 13.5 cups of milk.**
- **Problem Set 2: Jada will not finish in time. It will take her 75 minutes, or 1 hour and 15 minutes, to make 60 more signs.**

Are you ready for more?

Noah accidentally added 5 tbsp of cocoa to the hot chocolate mix, and the ratio of cocoa to milk is now 9 : 11. How many tablespoons of cocoa and cups of milk were in the original mix?

Sample response: 22 tbsp of cocoa and 33 cups of milk. If 5 tbsp are added, the ratio of cocoa to milk is 27 : 33, or 9 : 11.

STOP

Lesson 14 Solving Equivalent Ratio Problems 227

1 Launch

Review the ***Info Gap*** routine directions. Give each pair of students the Set 1 Problem and Data Cards from the Activity 1 PDF. Once pairs complete the routine with Set 1, give them Set 2, and have them switch roles.

2 Monitor

Help students get started by asking, "What information do you have? What information do you need?"

Look for points of confusion:

- **Asking irrelevant questions.** Ask, "What information do you need?"
- **Misinterpreting the meaning of numbers or associating quantities incorrectly.** Ask, "Do those values represent the same quantity? How can you revise your model?"

Look for productive strategies:

- Asking precise questions to get a ratio of milk to cocoa powder and the amount of cocoa powder Noah has for Set 1, and to get a ratio of signs made to minutes, the number of signs left to make, and the amount of time to finish for Set 2.
- Creating a table or diagram to represent the equivalent ratios with a missing value, and solving by using multiplication/division to scale up/down.

3 Connect

Have students share how they used their chosen representation to solve.

Highlight how using multiplication and division are similar or different in each scenario. Explain that "scaling up" should be used when the third given value is a multiple of the other corresponding given value (Set 2), and "scaling down" to a ratio containing a 1 can be used when the third given value is not a multiple of the other corresponding given value (Set 1).

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate how to use the ***Info Gap*** routine by modeling a think aloud using a sample situation (or using Problem Set 1). Display the questions you ask so students can reference them as they complete the activity. If you use Problem Set 1 for the think aloud, have students complete the activity for Problem Set 2.

Math Language Development

MLR4: Information Gap

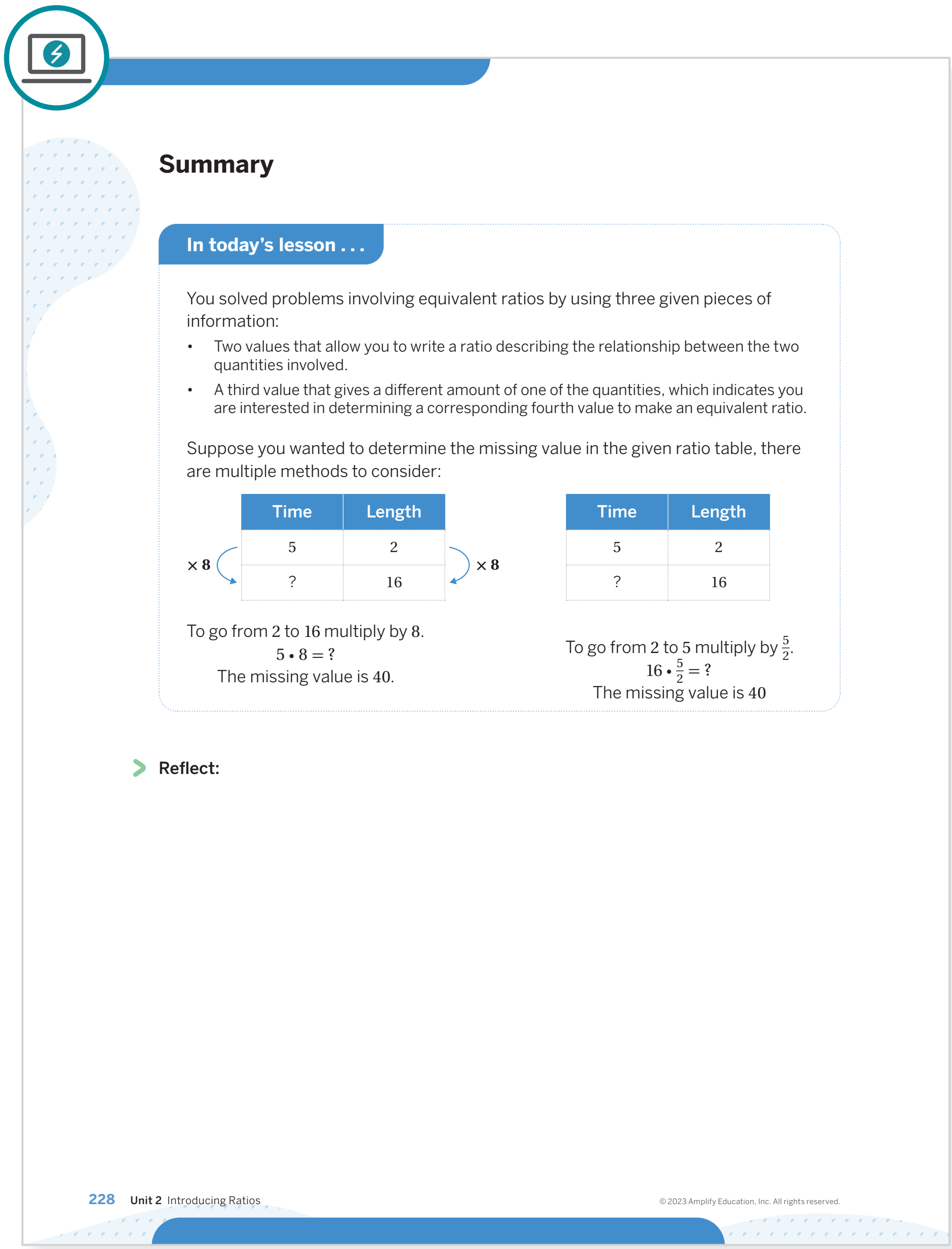
Display questions or question starters for students who need a starting point, such as:

- "Can you tell me . . . (specific piece of information)?"
- "Why do you need to know . . . (that piece of information)?"

Whole Class | 5 min

Summary

Review and synthesize the type of information required to solve equivalent ratio problems involving missing values.



Summary

In today's lesson . . .

You solved problems involving equivalent ratios by using three given pieces of information:

- Two values that allow you to write a ratio describing the relationship between the two quantities involved.
- A third value that gives a different amount of one of the quantities, which indicates you are interested in determining a corresponding fourth value to make an equivalent ratio.

Suppose you wanted to determine the missing value in the given ratio table, there are multiple methods to consider:

Time	Length
5	2
?	16

To go from 2 to 16 multiply by 8.

$5 \bullet 8 = ?$

The missing value is 40.

Time	Length
5	2
?	16

To go from 2 to 5 multiply by $\frac{5}{2}$.

$16 \bullet \frac{5}{2} = ?$

The missing value is 40

Reflect:

Synthesize

Display the table from the Summary for students to reference.

Ask, "How can you use the given information to determine the missing value in the table?" I use the given values for ingredients A and B to write a ratio of 5 to 2. Then, I can create an equivalent ratio with 16 as the new quantity for ingredient B. Since 16 is a multiple of 2, I can scale up by multiplying by 8. I need 40 cups of ingredient B because $5 \bullet 8 = 40$.

Highlight:

- When solving problems involving equivalent ratios, students are often given three values and need to determine a fourth value.
- Problems will describe the relationship between two quantities (e.g., cups of milk and tablespoons of cocoa). Students can write a ratio to represent this relationship by using two pieces of given information.
- Problems will also provide a third value which is a different amount of *one* of the quantities in the students' ratio (e.g., She needs to use 9 tbsp of cocoa powder).
- Determining how much of the fourth (missing) quantity is needed to maintain the same relationship as in the first ratio (e.g., How much milk does she need?) can be done by using equivalent ratios.
- This results in two equivalent ratios (e.g., 3 : 2 is equivalent to 13.5 : 9).

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

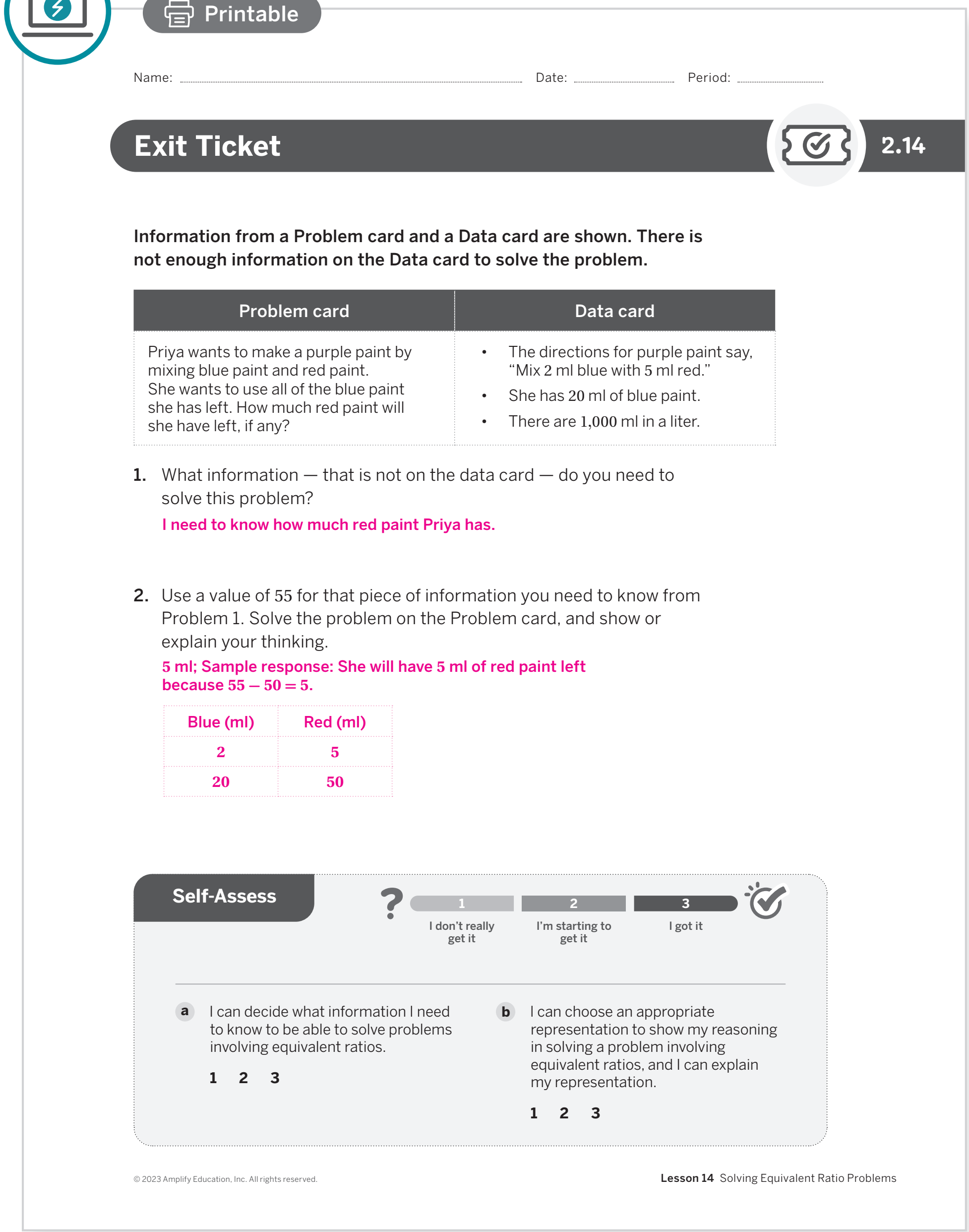
- "How did you know what information you needed so that you could answer the questions on the problem cards?"
- "What makes a question effective?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of solving equivalent ratio problems with missing values by identifying needed information and using given values to solve.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 2.14

Information from a Problem card and a Data card are shown. There is not enough information on the Data card to solve the problem.

Problem card	Data card
Priya wants to make a purple paint by mixing blue paint and red paint. She wants to use all of the blue paint she has left. How much red paint will she have left, if any?	• The directions for purple paint say, "Mix 2 ml blue with 5 ml red." • She has 20 ml of blue paint. • There are 1,000 ml in a liter.

1. What information — that is not on the data card — do you need to solve this problem?

   I need to know how much red paint Priya has.

2. Use a value of 55 for that piece of information you need to know from Problem 1. Solve the problem on the Problem card, and show or explain your thinking.

   5 ml; Sample response: She will have 5 ml of red paint left because 55 – 50 = 5.

Blue (ml)	Red (ml)
2	5
20	50

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a) I can decide what information I need to know to be able to solve problems involving equivalent ratios.

1 2 3

b) I can choose an appropriate representation to show my reasoning in solving a problem involving equivalent ratios, and I can explain my representation.

1 2 3

Success looks like . . .

- **Language Goal:** Determining what information is needed to solve a problem involving equivalent ratios. Asking questions to elicit that information. **(Speaking and Listening, Writing)**
  - » Determining the additional information needed to make purple paint in Problem 1.
- **Language Goal:** Choosing and creating representations to solve equivalent ratio problems involving a missing value. Explaining the solution strategy using the chosen representation. **(Speaking and Listening)**
  - » Determining the amount of paint left with a chosen representation, such as a table, in Problem 2.

Suggested next steps

If students are unable to identify the missing piece of information, consider:

- Reviewing the key points in the Summary, and asking:
  - » "What information can you use to create an equivalent ratio? What value is still missing?"
  - » "Will that missing value answer the question being asked? What else would you need to know? Do you have that information?"

If students say Priya needs 50 ml of red paint for Problem 2, consider:

- Asking, "What quantity does 20 ml of blue paint represent? So, then, what does 50 ml of red paint represent in the problem? Does that answer the question being asked on the problem card? What does the 55 represent in the problem?"

Professional Learning

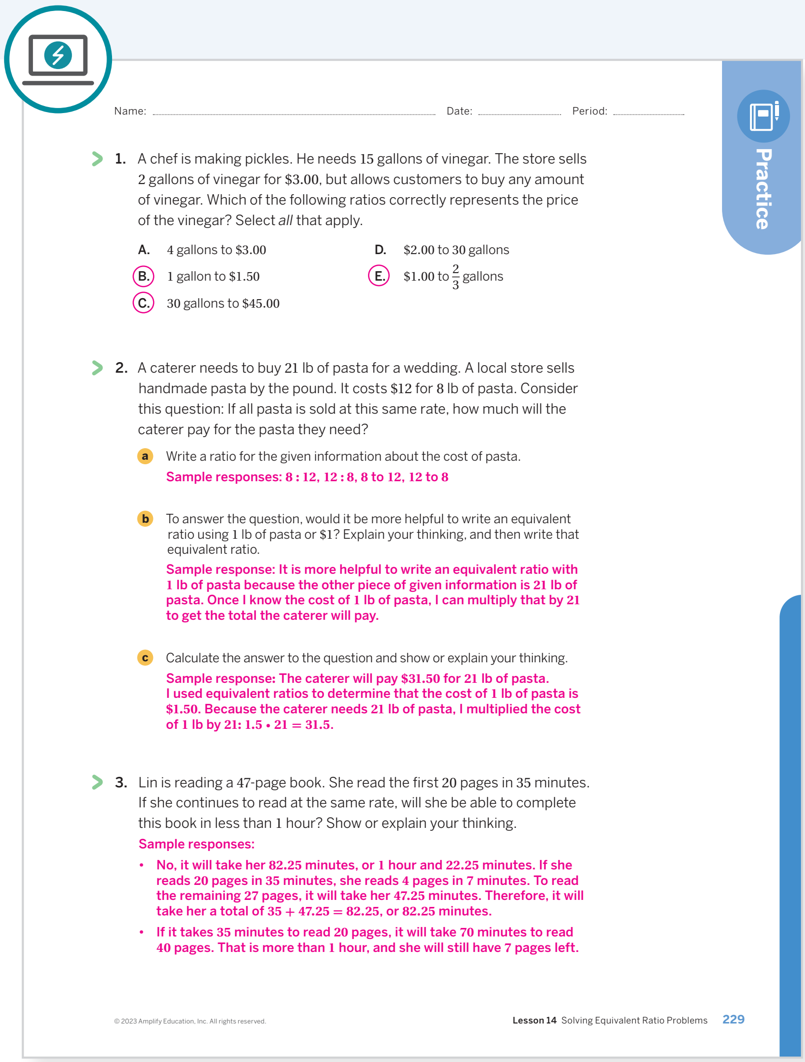
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In earlier lessons, students developed various strategies to represent and solve ratio problems. How did that support their work today?
- What different ways did students approach identifying and asking for missing information? What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. A chef is making pickles. He needs 15 gallons of vinegar. The store sells 2 gallons of vinegar for $3.00, but allows customers to buy any amount of vinegar. Which of the following ratios correctly represents the price of the vinegar? Select *all* that apply.

   A. 4 gallons to $3.00
   [✓ B.] 1 gallon to $1.50
   [✓ C.] 30 gallons to $45.00
   D. $2.00 to 30 gallons
   [✓ E.] $1.00 to $\frac{2}{3}$ gallons

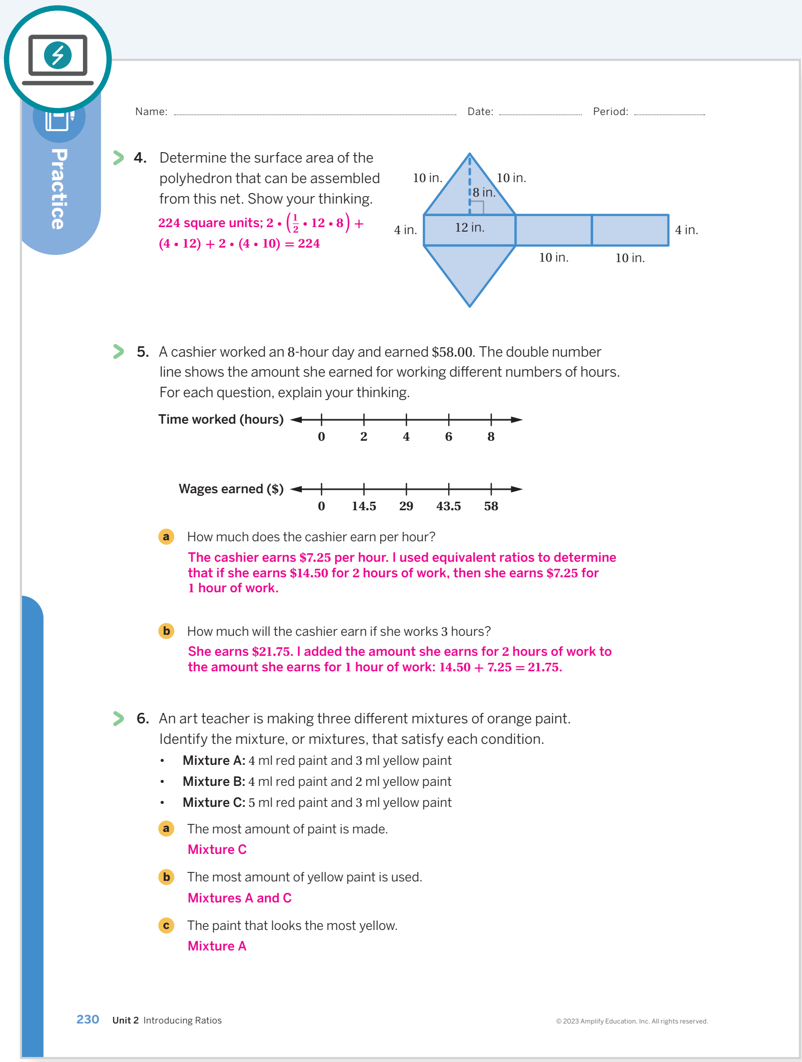
2. A caterer needs to buy 21 lb of pasta for a wedding. A local store sells handmade pasta by the pound. It costs $12 for 8 lb of pasta. Consider this question: If all pasta is sold at this same rate, how much will the caterer pay for the pasta they need?

   a. Write a ratio for the given information about the cost of pasta.
   Sample responses: 8 : 12, 12 : 8, 8 to 12, 12 to 8

   b. To answer the question, would it be more helpful to write an equivalent ratio using 1 lb of pasta or $1? Explain your thinking, and then write that equivalent ratio.
   Sample response: It is more helpful to write an equivalent ratio with 1 lb of pasta because the other piece of given information is 21 lb of pasta. Once I know the cost of 1 lb of pasta, I can multiply that by 21 to get the total the caterer will pay.

   c. Calculate the answer to the question and show or explain your thinking.
   Sample response: The caterer will pay $31.50 for 21 lb of pasta. I used equivalent ratios to determine that the cost of 1 lb of pasta is $1.50. Because the caterer needs 21 lb of pasta, I multiplied the cost of 1 lb by 21: $1.5 \cdot 21 = 31.5$.

3. Lin is reading a 47-page book. She read the first 20 pages in 35 minutes. If she continues to read at the same rate, will she be able to complete this book in less than 1 hour? Show or explain your thinking.
   Sample responses:
   - **No, it will take her 82.25 minutes, or 1 hour and 22.25 minutes. If she reads 20 pages in 35 minutes, she reads 4 pages in 7 minutes. To read the remaining 27 pages, it will take her 47.25 minutes. Therefore, it will take her a total of $35 + 47.25 = 82.25$, or 82.25 minutes.**
   - **If it takes 35 minutes to read 20 pages, it will take 70 minutes to read 40 pages. That is more than 1 hour, and she will still have 7 pages left.**



Practice

Name: Date: Period:

4. Determine the surface area of the polyhedron that can be assembled from this net. Show your thinking.
   224 square units; $2 \cdot \left(\frac{1}{2} \cdot 12 \cdot 8\right) + (4 \cdot 12) + 2 \cdot (4 \cdot 10) = 224$

5. A cashier worked an 8-hour day and earned $58.00. The double number line shows the amount she earned for working different numbers of hours. For each question, explain your thinking.

   a. How much does the cashier earn per hour?
   The cashier earns $7.25 per hour. I used equivalent ratios to determine that if she earns $14.50 for 2 hours of work, then she earns $7.25 for 1 hour of work.

   b. How much will the cashier earn if she works 3 hours?
   She earns $21.75. I added the amount she earns for 2 hours of work to the amount she earns for 1 hour of work: $14.50 + 7.25 = 21.75$.

6. An art teacher is making three different mixtures of orange paint. Identify the mixture, or mixtures, that satisfy each condition.
   - **Mixture A:** 4 ml red paint and 3 ml yellow paint
   - **Mixture B:** 4 ml red paint and 2 ml yellow paint
   - **Mixture C:** 5 ml red paint and 3 ml yellow paint

   a. The most amount of paint is made.
   Mixture C

   b. The most amount of yellow paint is used.
   Mixtures A and C

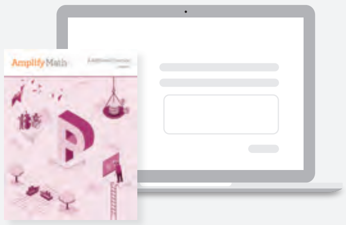
   c. The paint that looks the most yellow.
   Mixture A

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 16	2
	5	Unit 2 Lesson 12	2
Formative	6	Unit 2 Lesson 15	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 2 | **LESSON 15**

Part-Part-Whole Ratios

Let's look at situations where you can add the quantities in a ratio together.

Focus

Goals

1. **Language Goal:** Choose and create diagrams to help solve problems involving ratios with a total amount. Explain the solution method. **(Speaking and Listening)**
2. **Language Goal:** Choose to start with the parts or the total in solving ratio problems, and justify the choice. **(Speaking and Listening)**
3. **Language Goal:** Explain why the sum of the quantities makes sense in certain contexts and not in others. **(Speaking and Listening)**

Coherence

Today

Students consider situations in which the sum of the quantities makes sense in context. They write ratios describing the relationship between the individual parts (part-part), each part and the whole (part-whole), and all parts and the whole (part-part-whole). Applying previous work with equivalent ratios, students represent and solve problems involving the total amount as well as the component parts. They consider when it is best to start with the whole to determine the equivalent component parts, or when it is best to start with the parts to determine the equivalent whole.

Previously

In Lesson 14, students determined what information is needed to solve equivalent ratio problems. They solved for missing values, explaining both their representations and thinking for others to understand.

Coming Soon

In Lesson 16, students will determine specific equivalent ratios with shared values in order to compare ratios, allowing them to also determine whether two situations involve something happening at the same rate.

Rigor

- Students build **conceptual understanding** of part-part-whole ratios.
- Students continue to develop **procedural fluency** with equivalent ratios with missing values.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (optional, as needed)

Math Language Development

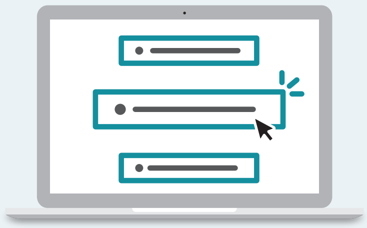
Review word

- *equivalent ratios*

Amps | Featured Activity

Activity 1
Mixing Liquids

Students simulate mixing water and food coloring. They can see the mixture they create in real time.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel like giving up because they feel like not enough information is given as they try to make sense of the relationships among the nickels, dimes, and quarters in Activity 2. Encourage students to list out the information that is given and write their own questions that they believe they could answer using that information. Then push them forward to persevere, as necessary, by asking, "How could you use those questions or answers to be able to fill in one complete row of the table (that does not have to include a total of 500)?"

Modifications to Pacing

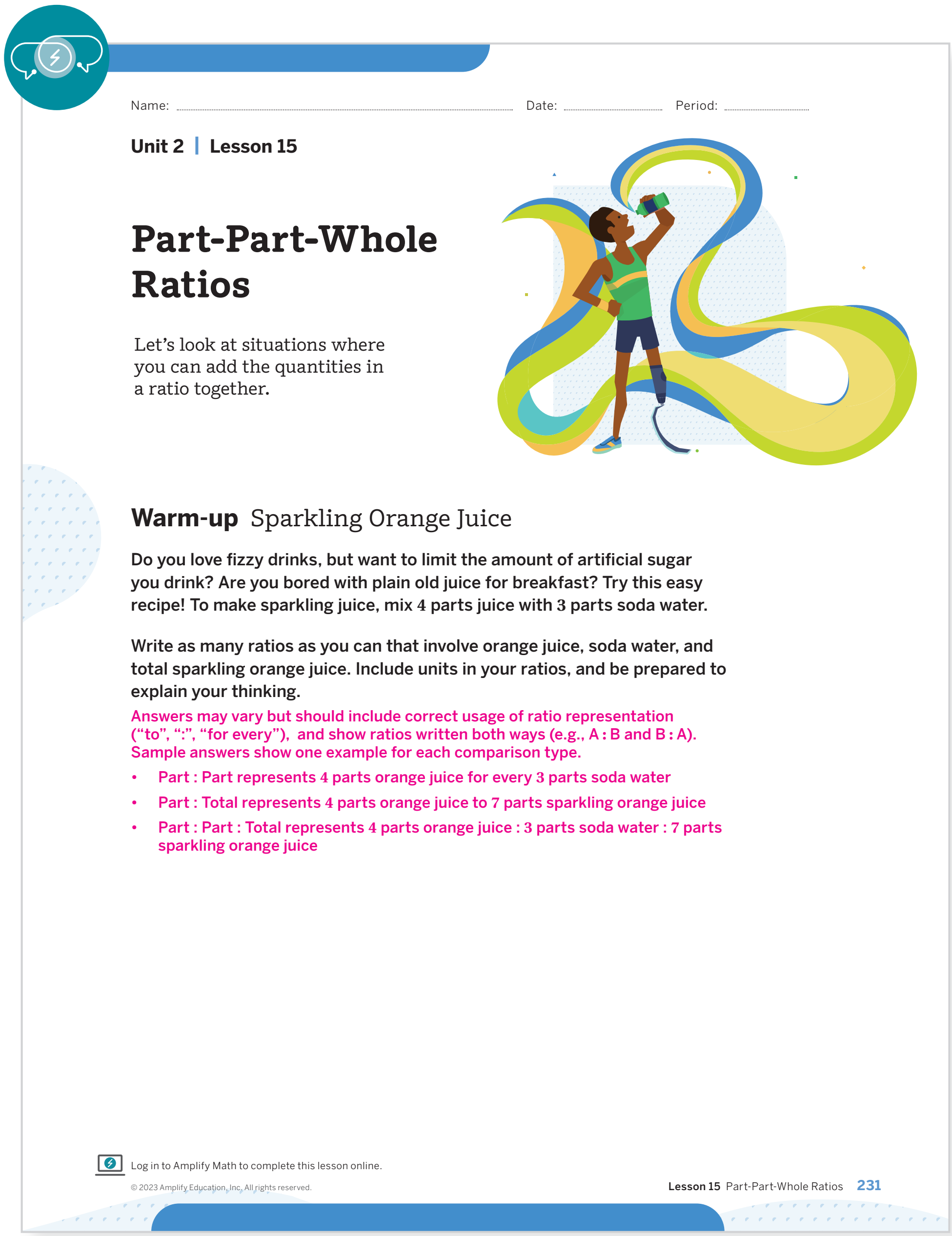
You may want to consider these additional modifications if you are short on time.

- Omit the **Warm-up**, but have students pause after completing Problem 1a in Activity 1 to highlight the additive relationship between the component parts and the total, showing them how to write this as a part : part : whole ratio.
- In **Activity 1**, omit Problem 2b.
- In **Activity 2**, have students ignore the 500 total coins at first and ask them to simply identify *any* possible total number of coins that Han could have. Then discuss the case of 500 total coins as a class.

Independent | 5 min

Warm-up Sparkling Orange Juice

Students are introduced to ratios involving parts and wholes by writing ratios to describe the relationships among the two ingredients (parts) in a combined sparkling juice (whole).



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Unit 2 | Lesson 15

Part-Part-Whole Ratios

Let's look at situations where you can add the quantities in a ratio together.

Warm-up Sparkling Orange Juice

Do you love fizzy drinks, but want to limit the amount of artificial sugar you drink? Are you bored with plain old juice for breakfast? Try this easy recipe! To make sparkling juice, mix 4 parts juice with 3 parts soda water.

Write as many ratios as you can that involve orange juice, soda water, and total sparkling orange juice. Include units in your ratios, and be prepared to explain your thinking.

Answers may vary but should include correct usage of ratio representation ("to", ":", "for every"), and show ratios written both ways (e.g., A : B and B : A). Sample answers show one example for each comparison type.

- **Part : Part represents 4 parts orange juice for every 3 parts soda water**
- **Part : Total represents 4 parts orange juice to 7 parts sparkling orange juice**
- **Part : Part : Total represents 4 parts orange juice : 3 parts soda water : 7 parts sparkling orange juice**

Log in to Amplify Math to complete this lesson online.

Lesson 15 Part-Part-Whole Ratios 231

1 Launch

Set an expectation for the amount of time that students have to work individually on the activity.

2 Monitor

Help students get started by having students draw a model of the recipe, and asking, "What relationships do you notice?"

Look for points of confusion:

- **Not considering the combined total.** Ask, "What is this recipe making with those two ingredients? How much sparkling orange juice will the recipe make?"

Look for productive strategies:

- Writing ratios that describe the relationships among the two ingredients (part to part), among each ingredient and the combined juice (part to whole), and among the parts and the whole.

3 Connect

Have students share their ratios. Display their responses in three *unlabeled* groups (part : part, part : whole, part : part : whole).

Ask, "What do the ratios in each group have in common?"

Highlight that when two or more quantities have the same units (e.g., cups to cups or parts to parts), ratios can describe how the quantities relate to one another *and*, when the context allows, how an individual quantity relates to the combined total (or whole), which is just another quantity. Note also that a ratio can associate quantities of the same "units" with the word *parts*, instead of a particular unit of measure such as cups or inches.

Math Language Development

MLR2: Collect and Display

Organize student responses on the class display/anchor chart into three groups: part to part, part to whole, and part to part to whole.

English Learners

Add diagrams to the display so that students can visualize the three different groups.

Power-up

To power up students' ability to identify parts and wholes and use them to solve problems, have students complete:

Three painters created their favorite shade of pink:

Painter A		Painter B		Painter C	
Red	**White**	**Red**	**White**	**Red**	**White**
3	4	2	3	6	8

1. Determine the ratio of red paint to total paint for each painter.

Painter A: 3:7 | Painter B: 2:5 | Painter A: 3:7

2. Which painter has the lightest shade of pink paint? Painter B
3. Which two painters created the same shade of pink paint? Painters A and C

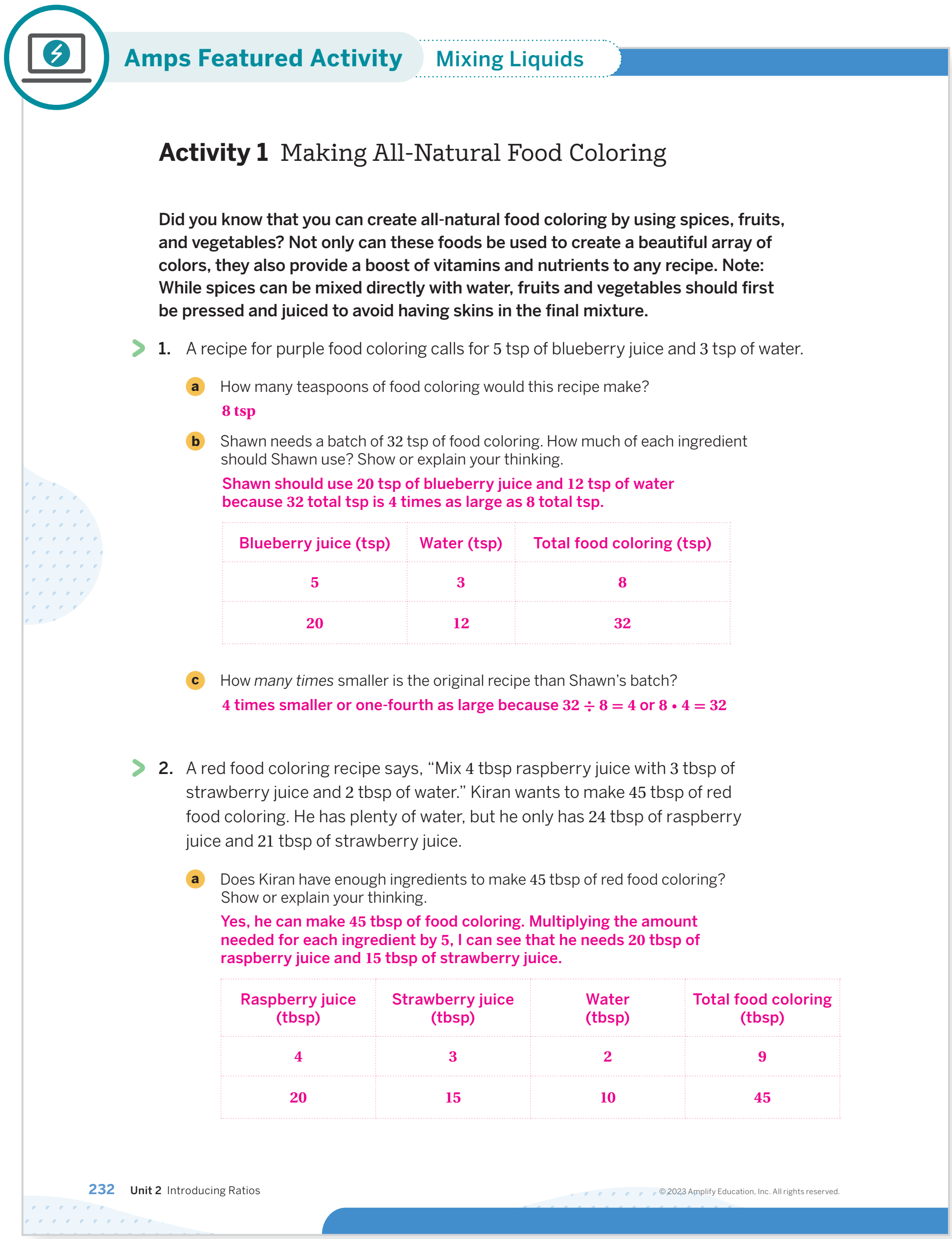
Use: Before the Warm-up
Informed by: Performance on Lesson 14, Practice Problem 6.

Pairs | 15 min

Activity 1 Making All-Natural Food Coloring

Students reason about recipes involving ratios of two or three ingredients and their sums to solve equivalent ratio problems involving part-part-whole ratios.

Amps Featured Activity Mixing Liquids



Activity 1 Making All-Natural Food Coloring

Did you know that you can create all-natural food coloring by using spices, fruits, and vegetables? Not only can these foods be used to create a beautiful array of colors, they also provide a boost of vitamins and nutrients to any recipe. Note: While spices can be mixed directly with water, fruits and vegetables should first be pressed and juiced to avoid having skins in the final mixture.

1. A recipe for purple food coloring calls for 5 tsp of blueberry juice and 3 tsp of water.

   a. How many teaspoons of food coloring would this recipe make?

   8 tsp

   b. Shawn needs a batch of 32 tsp of food coloring. How much of each ingredient should Shawn use? Show or explain your thinking.

   Shawn should use 20 tsp of blueberry juice and 12 tsp of water because 32 total tsp is 4 times as large as 8 total tsp.

Blueberry juice (tsp)	Water (tsp)	Total food coloring (tsp)
5	3	8
20	12	32

   c. How *many times* smaller is the original recipe than Shawn's batch?

   4 times smaller or one-fourth as large because $32 \div 8 = 4$ or $8 \cdot 4 = 32$

2. A red food coloring recipe says, "Mix 4 tbsp raspberry juice with 3 tbsp of strawberry juice and 2 tbsp of water." Kiran wants to make 45 tbsp of red food coloring. He has plenty of water, but he only has 24 tbsp of raspberry juice and 21 tbsp of strawberry juice.

   a. Does Kiran have enough ingredients to make 45 tbsp of red food coloring? Show or explain your thinking.

   Yes, he can make 45 tbsp of food coloring. Multiplying the amount needed for each ingredient by 5, I can see that he needs 20 tbsp of raspberry juice and 15 tbsp of strawberry juice.

Raspberry juice (tbsp)	Strawberry juice (tbsp)	Water (tbsp)	Total food coloring (tbsp)
4	3	2	9
20	15	10	45

232 Unit 2 Introducing Ratios

1 Launch

Explain that mixing solids and liquids does not always yield a units to units total mixture (e.g., 1 cup + 1 cup ≠ 2 cups). That is why these recipes are presented as mixing two liquids, fruit juice and water, instead of amounts of whole fruit and water.

2 Monitor

Help students get started by having them create a diagram or table with the given information, and asking, "What relationships does this show? Where does it show the total teaspoons of food coloring?"

Look for points of confusion:

- **Representing Problem 1b without the total.** Ask, "What does the 32 tsp represent? How can you show that in your representation?"
- **Adding the given parts rather than starting with the total in Problem 2a.** Ask, "Would the color be the same if you used all of the raspberries and strawberries? Why?"
- **Thinking they need to use up all of the ingredients in Problem 2b.** Ask, "How many times larger is this batch than the original recipe?" Remind them that the multiplier must be the same for every quantity.

Look for productive strategies:

- Creating a clear and labeled table or diagram, and using it to explain their thinking to their partner.
- Recognizing when to start with the whole to determine the parts (Problems 1b and 2a) and when to start with the parts to determine the whole (Problem 2b).

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problem 1, and, if time is available, have them work on Problem 2b.

Extension: *Math Enrichment*

Have students complete the following problem:
How much of this same red food coloring could Kiran make if he had 50 cups of raspberry juice, 40 cups of strawberry juice, and 30 cups of water? 112.5 cups; He does not have the quantities in the same ratio to make $50 + 40 + 30$, or 120 cups. Because $50 \div 4 = 12.5$, multiply 3 and 2 each by 12.5. The sum of these products is $50 + 37.5 + 25 = 112.5$.

Math Language Development

MLR7: Compare and Connect

Have students compare their solution strategies with 2–3 partners and encourage them to make connections between strategies based on whether they started with wholes or parts.

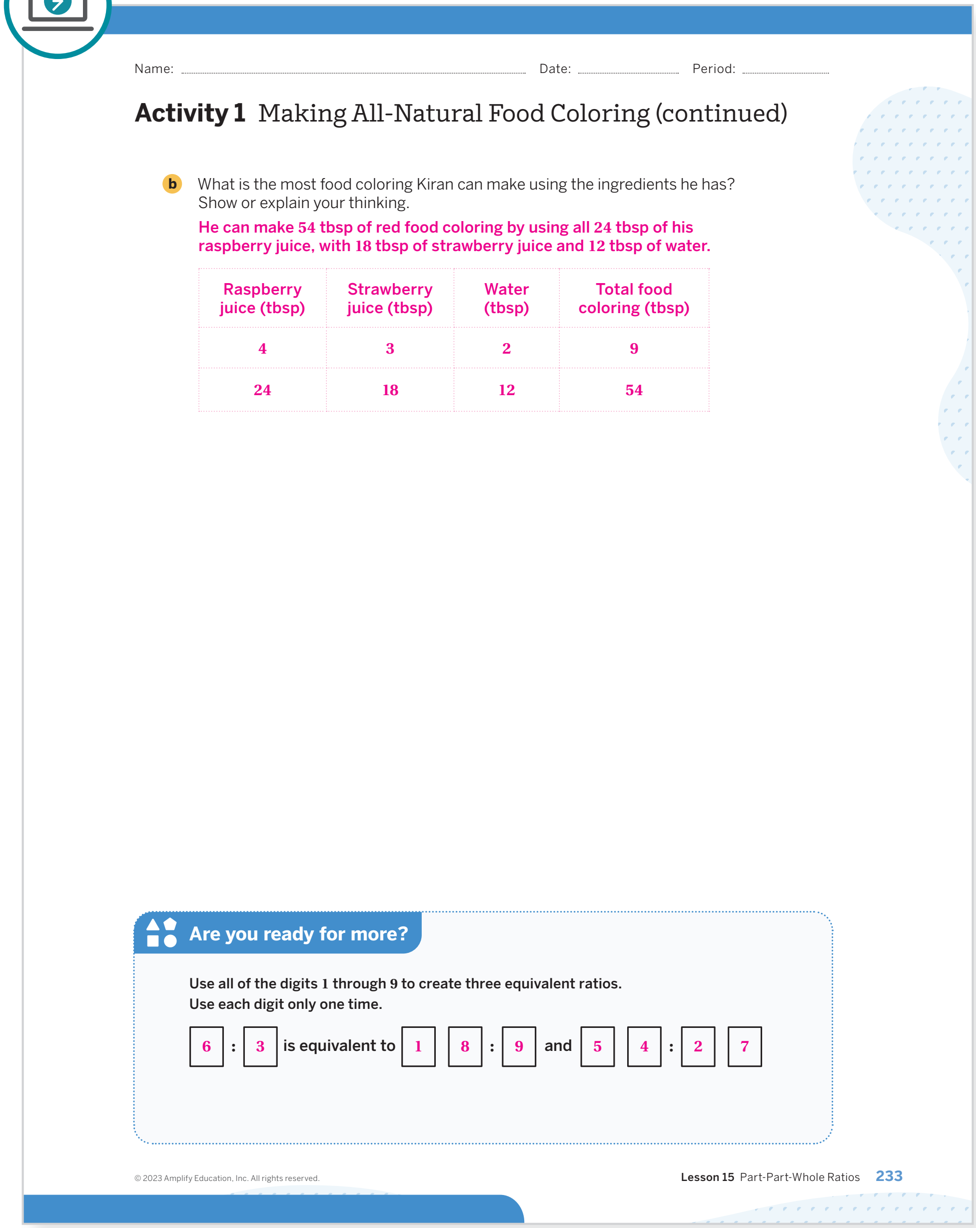
English Learners

Help students organize their findings by asking, "What would the headings of the table be?" Ask them to highlight the phrases in the problems that help them determine the headings.

Pairs | 15 min

Activity 1 Making All-Natural Food Coloring (continued)

Students reason about recipes involving ratios of two or three ingredients and their sums to solve equivalent ratio problems involving part-part-whole ratios.



Name: ______________ Date: ________ Period: ________

Activity 1 Making All-Natural Food Coloring (continued)

b What is the most food coloring Kiran can make using the ingredients he has? Show or explain your thinking.

He can make 54 tbsp of red food coloring by using all 24 tbsp of his raspberry juice, with 18 tbsp of strawberry juice and 12 tbsp of water.

Raspberry juice (tbsp)	Strawberry juice (tbsp)	Water (tbsp)	Total food coloring (tbsp)
4	3	2	9
24	18	12	54

Are you ready for more?

Use all of the digits 1 through 9 to create three equivalent ratios. Use each digit only one time.

6 : 3 is equivalent to 18 : 9 and 54 : 27

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3 Connect

Have students share how they solved each problem, focusing on when and why they started with the whole or with the parts.

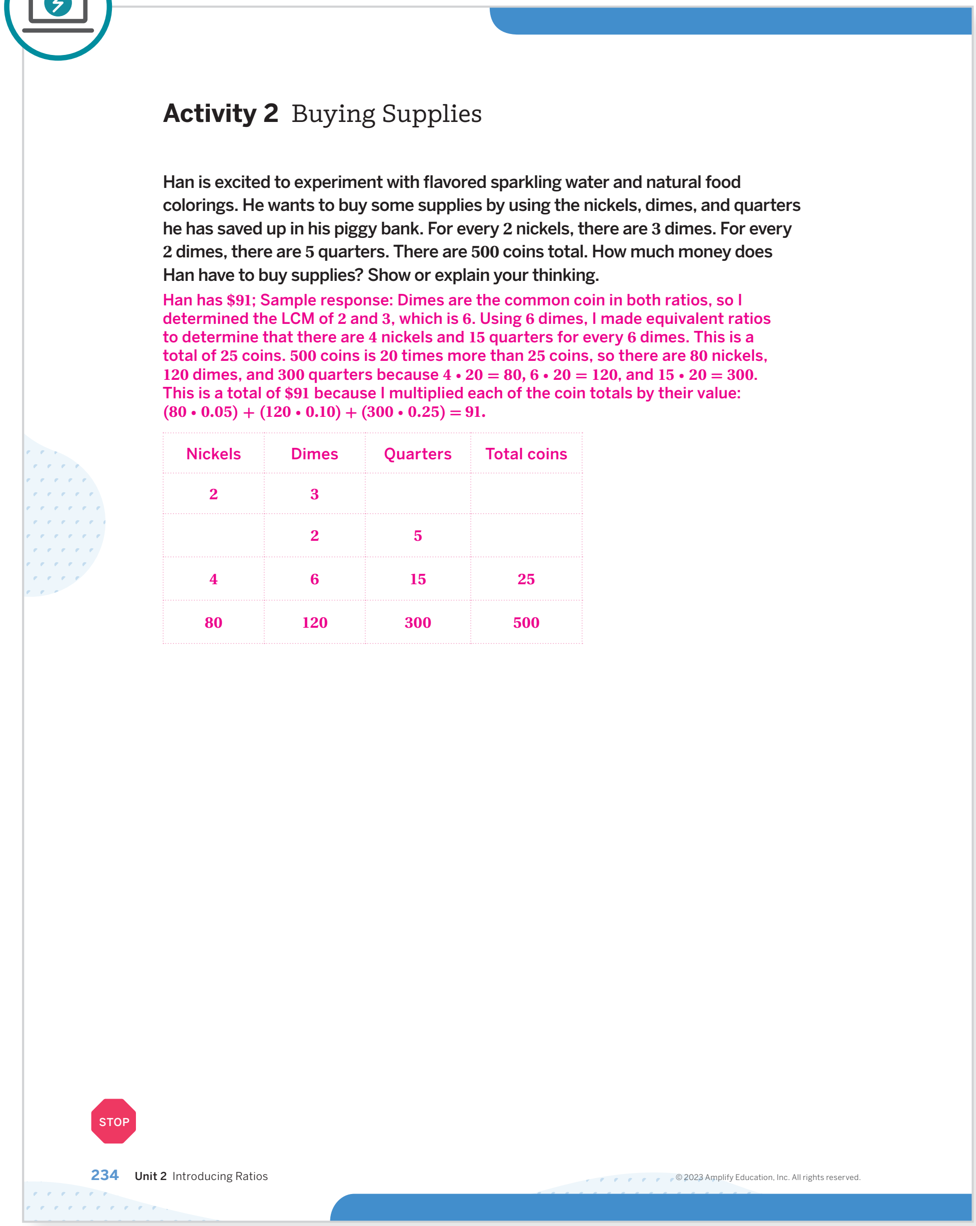
Ask, "Why did Kiran have strawberries left over in Problem 2b?" You need the same multiplier for each quantity in an equivalent ratio, so he only has enough raspberries to make the recipe 6 times larger.

Highlight that the given information helps students determine where to start — with the whole to determine missing parts, or with the parts to determine a missing whole. Using that information in either case, students can write an equivalent ratio to determine the missing value.

Pairs | 15 min

Activity 2 Buying Supplies

Students apply part-part-whole ratio reasoning to a scenario involving three types of coins and a total amount of money.



Activity 2 Buying Supplies

Han is excited to experiment with flavored sparkling water and natural food colorings. He wants to buy some supplies by using the nickels, dimes, and quarters he has saved up in his piggy bank. For every 2 nickels, there are 3 dimes. For every 2 dimes, there are 5 quarters. There are 500 coins total. How much money does Han have to buy supplies? Show or explain your thinking.

Han has \$91; Sample response: Dimes are the common coin in both ratios, so I determined the LCM of 2 and 3, which is 6. Using 6 dimes, I made equivalent ratios to determine that there are 4 nickels and 15 quarters for every 6 dimes. This is a total of 25 coins. 500 coins is 20 times more than 25 coins, so there are 80 nickels, 120 dimes, and 300 quarters because $4 \cdot 20 = 80$, $6 \cdot 20 = 120$, and $15 \cdot 20 = 300$. This is a total of \$91 because I multiplied each of the coin totals by their value: $(80 \cdot 0.05) + (120 \cdot 0.10) + (300 \cdot 0.25) = 91$.

Nickels	Dimes	Quarters	Total coins
2	3		
	2	5	
4	6	15	25
80	120	300	500

STOP

234 Unit 2 Introducing Ratios

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1 Launch

Read the problem as a class, and ask, "What is this problem asking you to determine?" The total value of the 500 coins. "What do you need to know to solve the problem?" How many nickels, dimes, and quarters he has.

2 Monitor

Help students get started by asking, "What ratios do you know? What ratios do you need to determine?"

Look for points of confusion:

- **Ineffectively using a pair of two-column tables.** Ask, "Is there a way to combine the given information into one table? How can you make one complete row?"
- **Not determining the common quantity of dimes to create one ratio of nickels to dimes to quarters.** Ask, "What quantity do the two ratios have in common? How can you make the ratios use the same number of dimes?"

Look for productive strategies:

- Using one representation (e.g., a four-column table) to show the relationship between each type of coin and the total number of coins.
- Using the LCM for dimes to establish a ratio of nickels to dimes to quarters to total coins.

3 Connect

Have students share how their representations helped them interpret and solve the problem.

Highlight that when two given ratios share a quantity, students can use the LCM to determine a common value for the shared quantity (e.g., 6 dimes). Then you can determine equivalent ratios involving parts or totals or both, to determine the necessary missing values.

Ask, "Instead of using the LCM, what would the GCF of the amount of dimes tell you?" The number of nickels and quarters for every 1 dime.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with copies of the Activity 2 PDF to help them organize their thinking and work in a single table. Consider asking students to first ignore the statement that there are 500 coins total, and to begin by listing possible combinations of coins that maintain the ratio relationships described. After they have listed 3 or 4 combinations, ask them to find the totals to see if any have a total of 500 coins. Then have them continue to find more combinations until they find the one with a total number of 500 coins.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that Han has coins in an assortment of nickels, dimes and quarters.
- **Read 2:** Students should highlight or underline the phrase 500 *coins total* and the given ratio relationships.
- **Read 3:** Ask students to create diagrams or tables to represent the relationships among the quantities. Then ask them to plan their solution strategy.

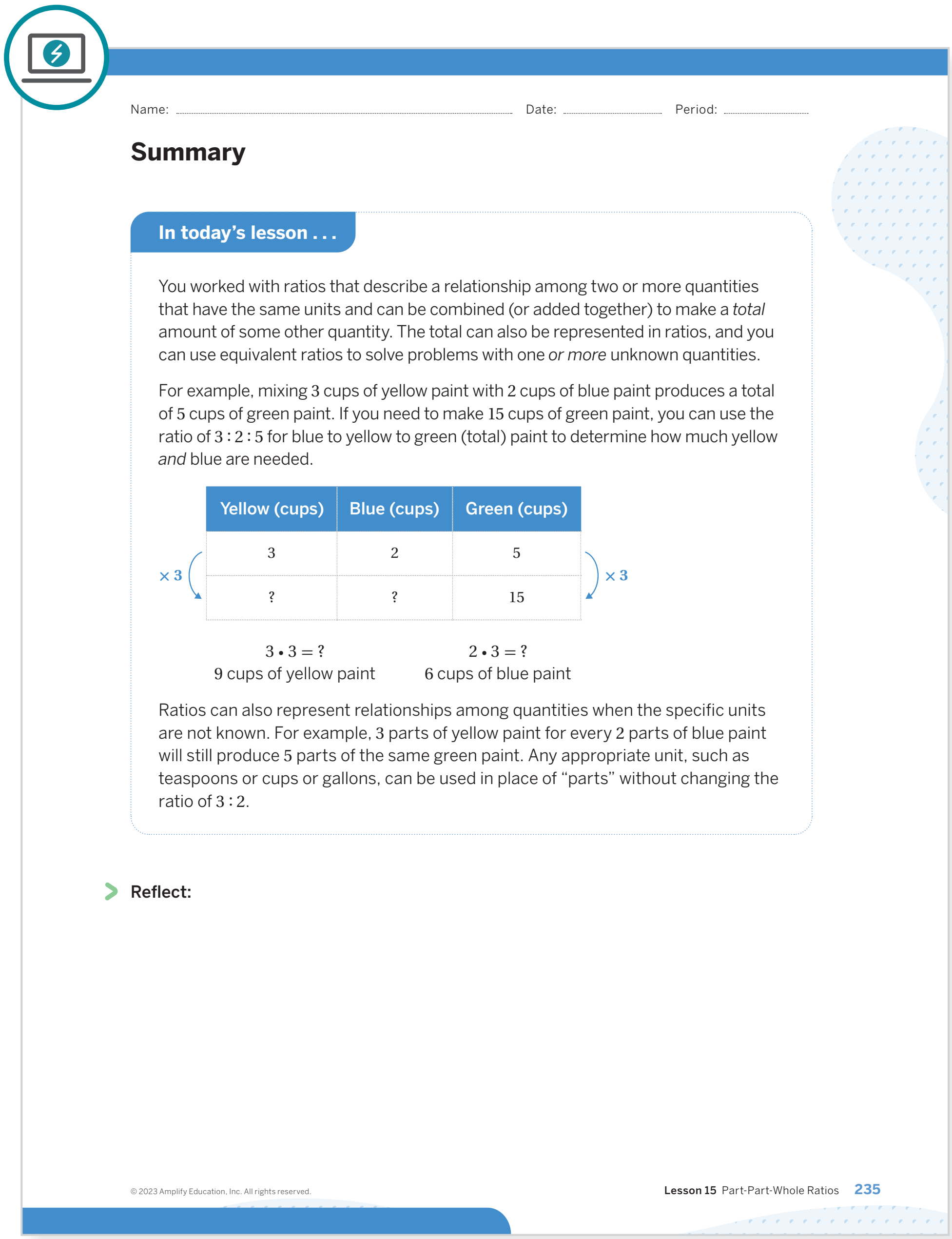
English Learners

Emphasize the difference between the total *number* of coins being 500 and the total dollar amount.

Whole Class | 5 min

Summary

Review and synthesize how to determine missing values in part-part-whole ratios.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You worked with ratios that describe a relationship among two or more quantities that have the same units and can be combined (or added together) to make a *total* amount of some other quantity. The total can also be represented in ratios, and you can use equivalent ratios to solve problems with one *or more* unknown quantities.

For example, mixing 3 cups of yellow paint with 2 cups of blue paint produces a total of 5 cups of green paint. If you need to make 15 cups of green paint, you can use the ratio of 3 : 2 : 5 for blue to yellow to green (total) paint to determine how much yellow *and* blue are needed.

Yellow (cups)	Blue (cups)	Green (cups)
3	2	5
?	?	15

× 3 × 3

$3 \cdot 3 = ?$
9 cups of yellow paint

$2 \cdot 3 = ?$
6 cups of blue paint

Ratios can also represent relationships among quantities when the specific units are not known. For example, 3 parts of yellow paint for every 2 parts of blue paint will still produce 5 parts of the same green paint. Any appropriate unit, such as teaspoons or cups or gallons, can be used in place of "parts" without changing the ratio of 3 : 2.

Reflect:

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Synthesize

Highlight that, in order for the problem to make sense to consider the whole as a relevant quantity itself, all of the other quantities (parts) must use the same units. For example, in Activity 1 Problem 2, all of the ingredients were measured in tablespoons, and the total food coloring created was simply equal to the result of adding those amounts together. When working with part-part or part-whole ratios in the same context, the information each ratio tells students is different. But, if students combine that information to form a part-part-whole ratio, then some of the same information is presented differently. This will be explored more in the next lessons.

Ask:

- "Could you consider the whole if you have cups of water and tablespoons of juice? Why or why not?" Yes, because both ingredients are liquid, and you would just need to first convert the tablespoons to cups, or cups to tablespoons, so the units are then the same.
- "Does it make sense to say if one car is driving 60 mph and another is driving 30 mph, then together they are driving 90 mph?" Sample responses: Yes, in total they would be traveling 90 miles in 1 hour. *or* No, it does not make sense because you also need to add the hours, so it would be 90 miles in 2 hours, which is equivalent to 45 miles in 1 hour — which is really the *average rate* of the two cars instead. **Note:** no response should be considered correct or incorrect at this point, but this question is really intended to elicit student thinking and have them explain and justify their reasoning.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

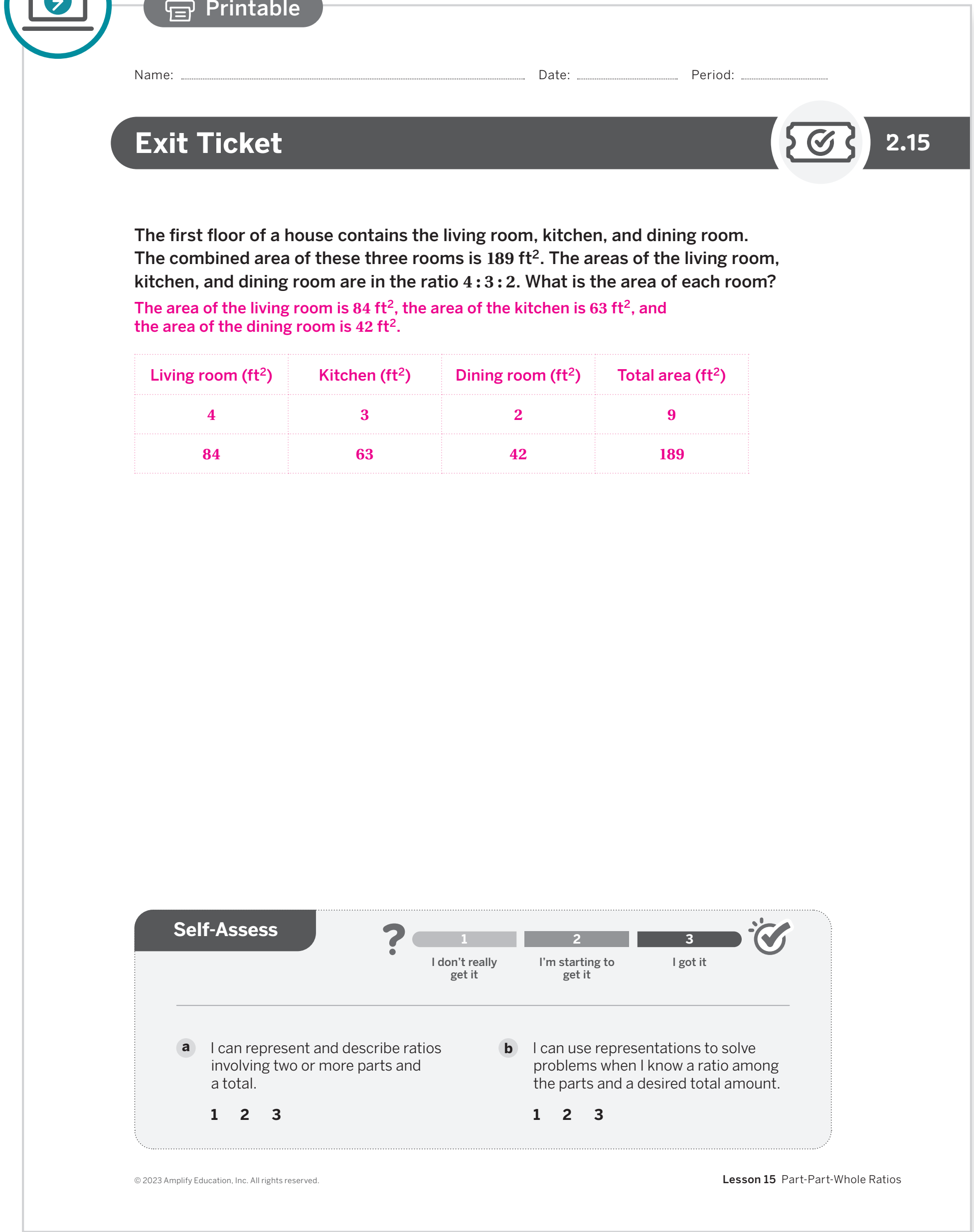
- "How were the ratios you worked with today similar to or different than the ratios you worked with in previous lessons?"
- "When does the total amount matter in a comparison?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the missing parts when given the total amount of an equivalent ratio.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket

2.15

The first floor of a house contains the living room, kitchen, and dining room. The combined area of these three rooms is 189 ft^2. The areas of the living room, kitchen, and dining room are in the ratio $4:3:2$. What is the area of each room?

The area of the living room is 84 ft^2, the area of the kitchen is 63 ft^2, and the area of the dining room is 42 ft^2.

Living room (ft^2)	Kitchen (ft^2)	Dining room (ft^2)	Total area (ft^2)
4	3	2	9
84	63	42	189

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can represent and describe ratios involving two or more parts and a total.

1 2 3

b I can use representations to solve problems when I know a ratio among the parts and a desired total amount.

1 2 3

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Lesson 15 Part-Part-Whole Ratios

Success looks like . . .

- **Language Goal:** Choosing and creating diagrams to help solve problems involving ratios and the total amount. Explaining the solution method. **(Speaking and Listening)**
    - » Using a representation, such as a four-column table, to show the relationship between the area of each room and the total area of the first floor of the house.
- **Language Goal:** Choosing to start with the parts or the total in solving ratio problems, and justifying the choice. **(Speaking and Listening)**
- **Language Goal:** Explaining why the sum of the quantities makes sense in certain contexts and not in others. **(Speaking and Listening)**

Suggested next steps

If students solve incorrectly, consider:

- Asking, "What information do you know? Does that represent a part or the whole? What do you need to know? How can you show all of this information in a table?"
- Reviewing Activity 1, Problem 1b and asking, "How did you solve this problem? How can you use that to help you solve the problem on the Exit Ticket?"

Professional Learning

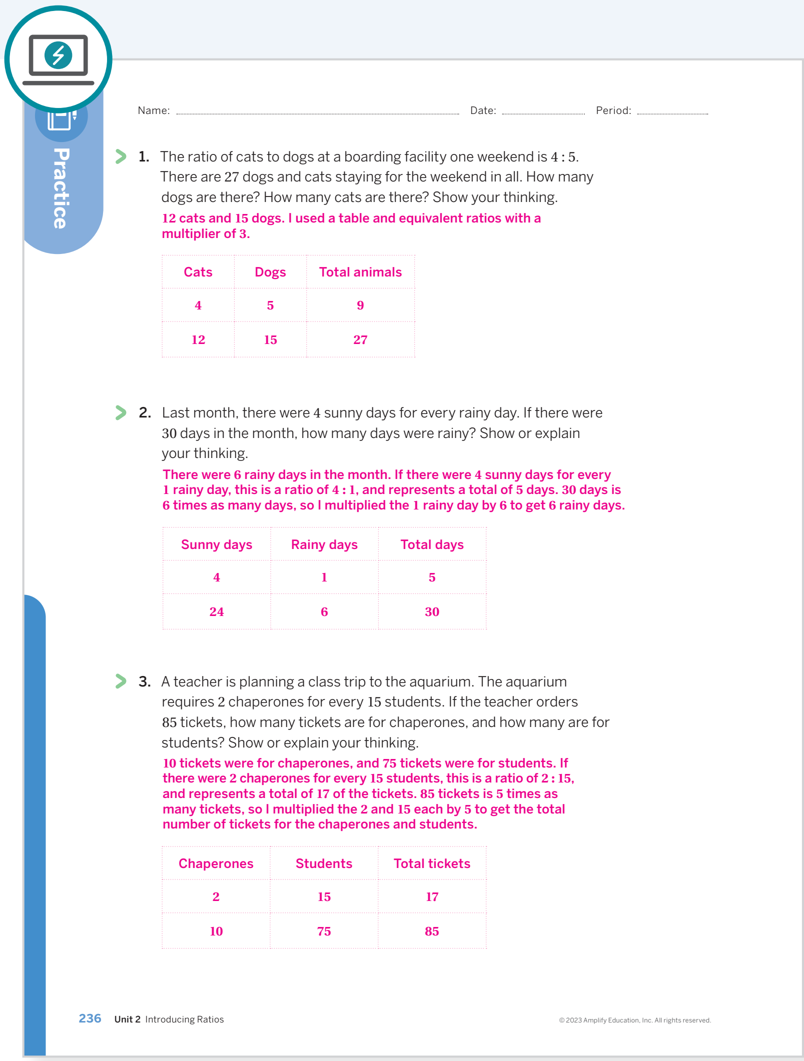
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The instructional goal of this lesson was to solve problems involving ratios with a total amount. How well did students accomplish this? What did you specifically do to help students accomplish it?
- During the discussion about Activity 2, how did you encourage each student to share their understandings? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. The ratio of cats to dogs at a boarding facility one weekend is 4 : 5. There are 27 dogs and cats staying for the weekend in all. How many dogs are there? How many cats are there? Show your thinking.

12 cats and 15 dogs. I used a table and equivalent ratios with a multiplier of 3.

Cats	Dogs	Total animals
4	5	9
12	15	27

2. Last month, there were 4 sunny days for every rainy day. If there were 30 days in the month, how many days were rainy? Show or explain your thinking.

There were 6 rainy days in the month. If there were 4 sunny days for every 1 rainy day, this is a ratio of 4 : 1, and represents a total of 5 days. 30 days is 6 times as many days, so I multiplied the 1 rainy day by 6 to get 6 rainy days.

Sunny days	Rainy days	Total days
4	1	5
24	6	30

3. A teacher is planning a class trip to the aquarium. The aquarium requires 2 chaperones for every 15 students. If the teacher orders 85 tickets, how many tickets are for chaperones, and how many are for students? Show or explain your thinking.

10 tickets were for chaperones, and 75 tickets were for students. If there were 2 chaperones for every 15 students, this is a ratio of 2 : 15, and represents a total of 17 of the tickets. 85 tickets is 5 times as many tickets, so I multiplied the 2 and 15 each by 5 to get the total number of tickets for the chaperones and students.

Chaperones	Students	Total tickets
2	15	17
10	75	85

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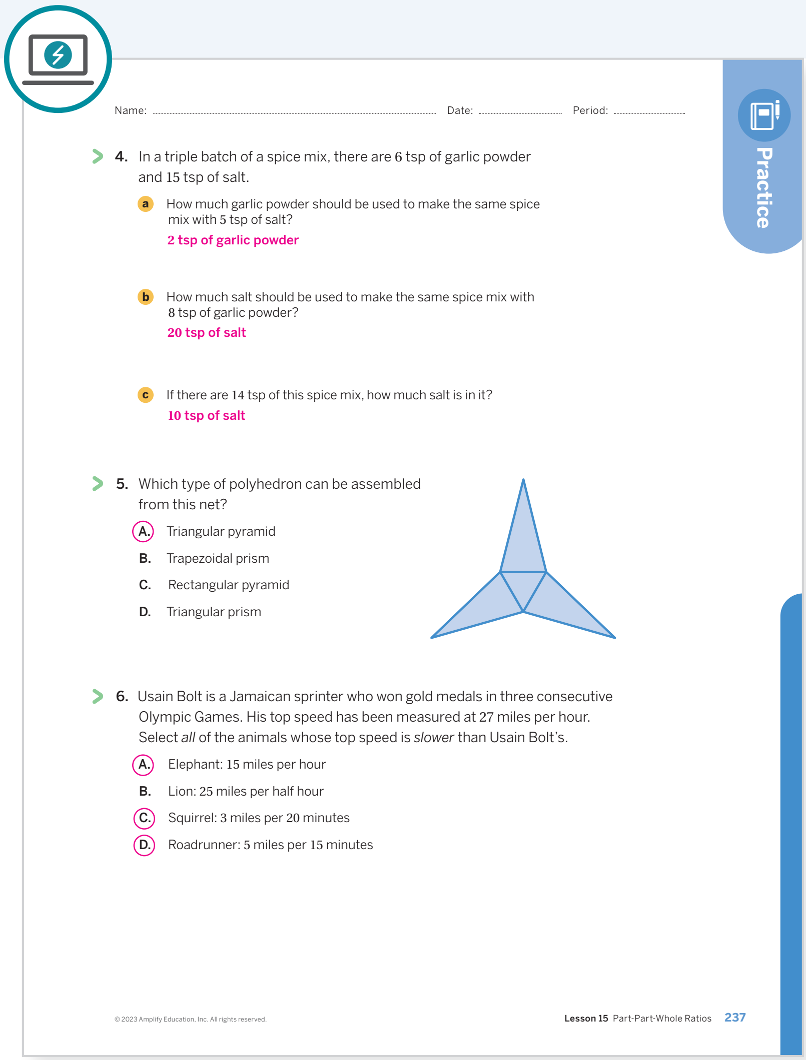

Name: Date: Period:

Practice

4. In a triple batch of a spice mix, there are 6 tsp of garlic powder and 15 tsp of salt.
   - a How much garlic powder should be used to make the same spice mix with 5 tsp of salt?
     2 tsp of garlic powder
   - b How much salt should be used to make the same spice mix with 8 tsp of garlic powder?
     20 tsp of salt
   - c If there are 14 tsp of this spice mix, how much salt is in it?
     10 tsp of salt

5. Which type of polyhedron can be assembled from this net?
   - [✓ A.] Triangular pyramid
   - B. Trapezoidal prism
   - C. Rectangular pyramid
   - D. Triangular prism

6. Usain Bolt is a Jamaican sprinter who won gold medals in three consecutive Olympic Games. His top speed has been measured at 27 miles per hour. Select *all* of the animals whose top speed is *slower* than Usain Bolt's.
   - [✓ A.] Elephant: 15 miles per hour
   - B. Lion: 25 miles per half hour
   - [✓ C.] Squirrel: 3 miles per 20 minutes
   - [✓ D.] Roadrunner: 5 miles per 15 minutes

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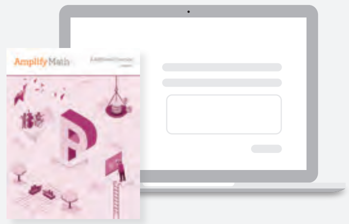
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	**1**	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	**4**	Unit 2 Lesson 13	2
	5	Unit 1 Lesson 16	1
Formative ⚡	**6**	Unit 2 Lesson 16	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 2 | **LESSON 16**

Comparing Ratios

Let's compare ratios.

Focus

Goals

1. **Language Goal:** Choose and create representations to help compare two ratios. **(Speaking and Listening)**
2. **Language Goal:** Justify that two situations do not happen at the same rate by determining a ratio to describe each situation where the two ratios share one value but not the other, i.e., $a : b$ and $a : c$, or $x : z$ and $y : z$. **(Speaking and Listening)**

Coherence

Today

Students investigate whether two ratios are equivalent, or if not, which one represents *more* or *less* of something, such as by comparing two scenarios and asking whether they are happening "at the same rate." Students create equivalent ratios in which one quantity has the same value to compare relative speed or taste — spiciness, sweetness, sourness. In each context, the values have been purposely chosen so that students can reason in at least two ways: using common multiples or using ratios containing a 1. They explain, justify, and compare their strategies, examining when one strategy may be more efficient than others.

Note that, in this lesson and Lesson 17, *same rate* means the ratios are equivalent (*rate* will be more thoroughly and formally explored in Unit 3).

Previously

In Lessons 6–13, students worked with equivalent ratios and used multiplicative reasoning, to eventually solve for missing values in Lessons 14–15. They also began to see the phrase *same rate* as synonymous with *equivalent*.

Coming Soon

In Lesson 17, students will extend their work with comparing ratios to include situations in which the total parts are not the same and a "whole" quantity must be considered.

Rigor

- Students build **conceptual understanding** of comparing ratios.
- Students continue to develop **procedural fluency** with equivalent ratios.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- Activity 2 PDF, pre-cut cards, one set per group

Math Language Development

Review word

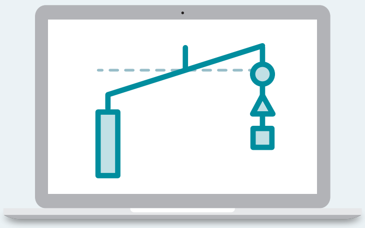
- *equivalent ratios*

Amps | Featured Activity

Activity 1

Comparing Chilis

Students can choose from a table, a double number line, or free sketch to order chili pepper powders from most to least spicy. You can see and compare their work within and across representation types.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may be able to explain their strategy clearly, but they may struggle to justify why they chose their strategy in the first place. Encourage students to listen to the justifications of students who used a different strategy, and ask, "Did you consider their strategy? If so, why did you choose your strategy instead? If not, hearing it now do you think you would have rather used their strategy, and why?"

Modifications to Pacing

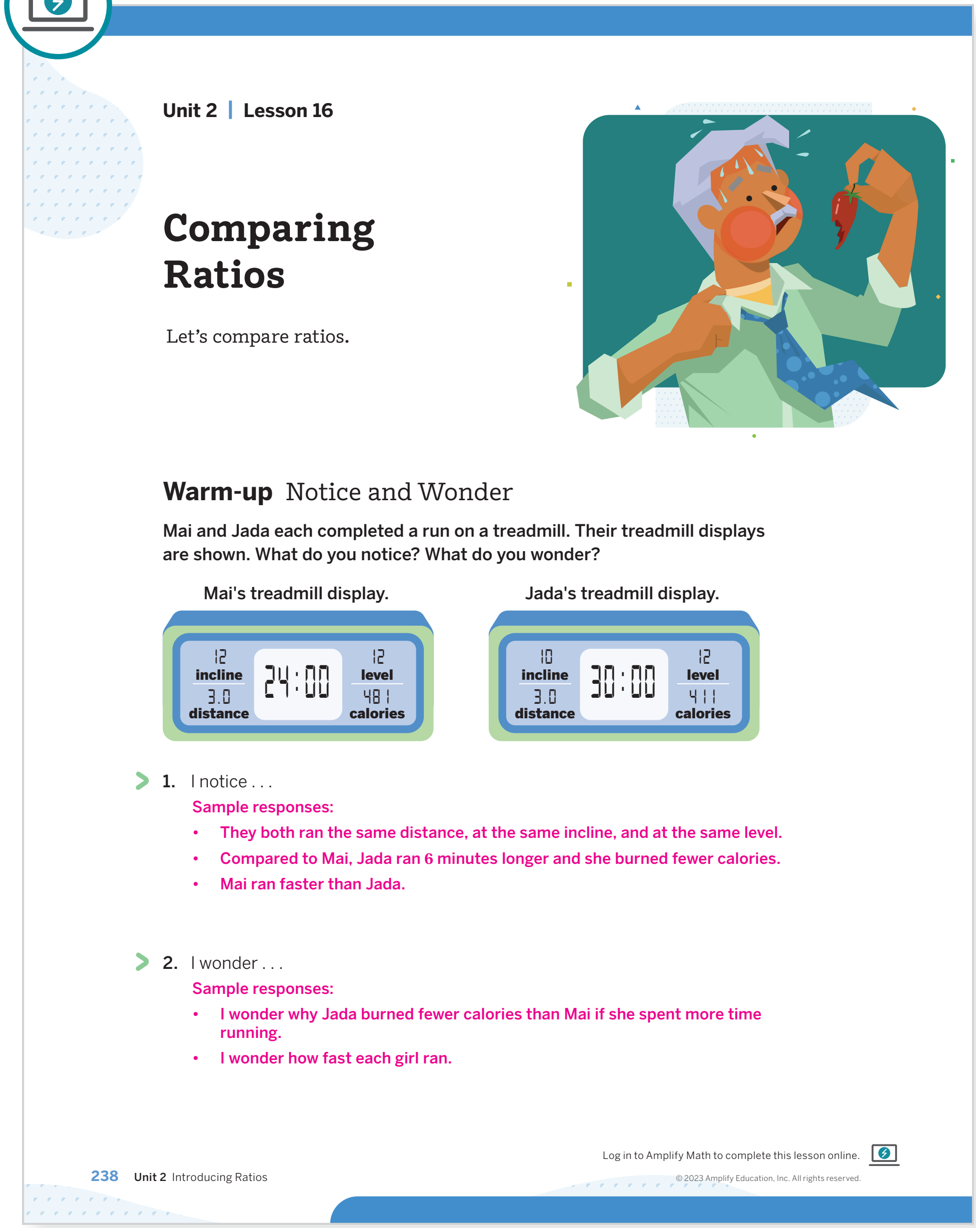
You may want to consider this additional modification if you are short on time.

- Change **Activity 2** into a partner activity. Partners create one recipe to compare against one you provide. Alternatively, **Activity 2** may be omitted entirely. If omitted, ask students to evaluate the efficiency of each strategy — common multiples or ratios containing a 1 — during the discussion at the conclusion of Activity 1.

Independent | 5 min

Warm-up Notice and Wonder

Students are introduced to comparing ratios by reasoning about the information shown on the treadmill displays for two runners who ran the same distance in different times.



Unit 2 | Lesson 16

Comparing Ratios

Let's compare ratios.

Warm-up Notice and Wonder

Mai and Jada each completed a run on a treadmill. Their treadmill displays are shown. What do you notice? What do you wonder?

Mai's treadmill display.

12 **incline** 3.0 **distance** 24:00 12 **level** 481 **calories**

Jada's treadmill display.

10 **incline** 3.0 **distance** 30:00 12 **level** 411 **calories**

1. I notice . . .

Sample responses:

- They both ran the same distance, at the same incline, and at the same level.
- Compared to Mai, Jada ran 6 minutes longer and she burned fewer calories.
- Mai ran faster than Jada.

2. I wonder . . .

Sample responses:

- I wonder why Jada burned fewer calories than Mai if she spent more time running.
- I wonder how fast each girl ran.

Log in to Amplify Math to complete this lesson online.

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1 Launch

Conduct the ***Notice and Wonder*** routine with the two treadmill displays.

2 Monitor

Help students get started by asking, "What is the same about their workouts? What is different?"

Look for points of confusion:

- **Focusing on the extraneous information (e.g., incline, level, or calories).** Ask, "What other values are on the treadmill display? Are any the same for each girl?"

Look for productive strategies:

- Recognizing that the distance, incline, and level are the same, but the time and calories are different.
- Understanding that the girls ran at different speeds because it took different amounts of time to run the same distance.

3 Connect

Have students share what they noticed and wondered, focusing on how the times are different but the distance is the same.

Ask, "If each person ran a constant speed, meaning neither girl sped up or slowed down the entire time, who ran faster? How do you know?" Mai ran faster because she ran the same distance as Jada but in less time.

Highlight that speed is a ratio of distance to time. When comparing ratios, one quantity must have the same value in each ratio. Sometimes one value is already the same, such as both girls are running 3 miles, and sometimes students need to use equivalent ratios to make one the same. Then students can compare the other quantity. For example, they can compare the time it took each girl to run 3 miles. They could also compare the distance each girl ran in the same amount of time, such as, how far Mai would run in 30 minutes.

Power-up

To power up students' ability to compare ratios (speeds) using different units, have students complete:

1. Each animal traveled a certain distance in a certain amount of time. For each animal, determine its speed in miles per hour.
   a. Cheetah: 40 miles in half an hour. 80 mph
   b. Peregrine falcon: 80 miles in one third hour. 240 mph
   c. Lion: 100 miles in two hours. 50 mph
2. Which animal is the fastest? Peregrine falcon

Use: Before the Warm-up.

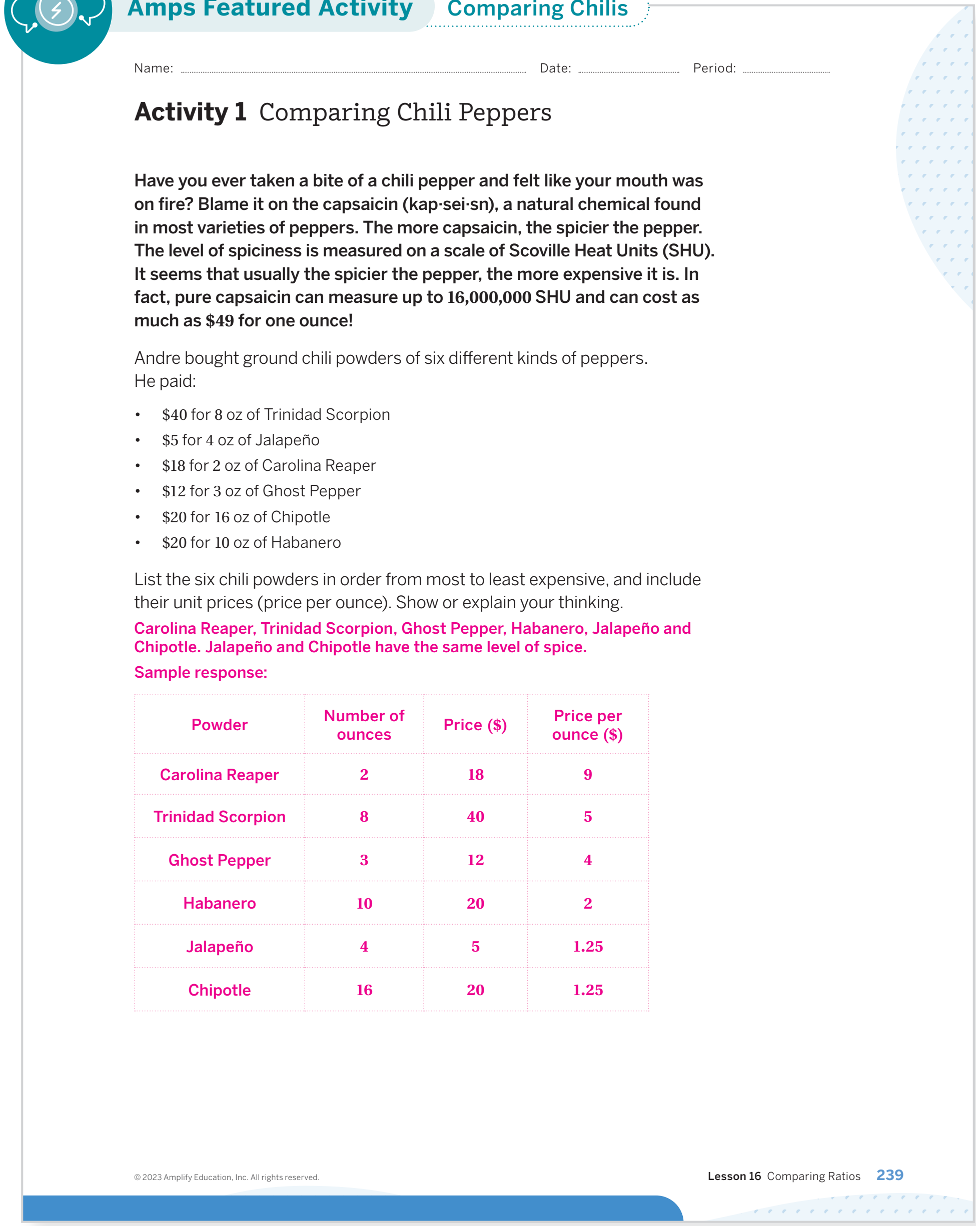
Informed by: Performance on Lesson 15, Practice Problem 6.

Pairs | 15 min

Activity 1 Comparing Chili Peppers

Students compare the spiciness of six ground chili powders (based also on their cost) by creating equivalent ratios in which one quantity has the same value.

Amps Featured Activity Comparing Chilis



Name: ______ Date: ______ Period: ______

Activity 1 Comparing Chili Peppers

Have you ever taken a bite of a chili pepper and felt like your mouth was on fire? Blame it on the capsaicin (kap·sei·sn), a natural chemical found in most varieties of peppers. The more capsaicin, the spicier the pepper. The level of spiciness is measured on a scale of Scoville Heat Units (SHU). It seems that usually the spicier the pepper, the more expensive it is. In fact, pure capsaicin can measure up to 16,000,000 SHU and can cost as much as $49 for one ounce!

Andre bought ground chili powders of six different kinds of peppers.
He paid:

- $40 for 8 oz of Trinidad Scorpion
- $5 for 4 oz of Jalapeño
- $18 for 2 oz of Carolina Reaper
- $12 for 3 oz of Ghost Pepper
- $20 for 16 oz of Chipotle
- $20 for 10 oz of Habanero

List the six chili powders in order from most to least expensive, and include their unit prices (price per ounce). Show or explain your thinking.

Carolina Reaper, Trinidad Scorpion, Ghost Pepper, Habanero, Jalapeño and Chipotle. Jalapeño and Chipotle have the same level of spice.

Sample response:

Powder	Number of ounces	Price ($)	Price per ounce ($)
Carolina Reaper	2	18	9
Trinidad Scorpion	8	40	5
Ghost Pepper	3	12	4
Habanero	10	20	2
Jalapeño	4	5	1.25
Chipotle	16	20	1.25

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What information do you know? What are you trying to determine? What do you need to know that will help you?"

Look for points of confusion:

- **Using only one quantity (prices or ounces) instead of ratios.** Refer to the Warm-up and ask, "How did you know Mai ran faster? Can one quantity be made the same here?"
- **Comparing only in pairs, or struggling to compare all peppers at once.** Ask, "How can you compare one pair of peppers? Can you use the same strategy to compare a third pepper? All peppers?"

Look for productive strategies:

- Using multiples to compare (e.g., Trinidad Scorpion, Jalapeno, Carolina Reaper, and Chipotle because 16 is a multiple of 2, 4, and 8).
- Comparing all at once by determining a ratio where the second value is 1 (unit price) or using LCM to make equivalent ratios.

3 Connect

Have students share their strategies, focusing on how they used multiples or ratios containing a 1 to compare.

Highlight that to compare, one value in the ratios needs to be the same. Students can create equivalent ratios like this using common multiples, common factors, or ratios containing a 1.

Display the Activity 1 PDF.

Ask, "Using the chart, would you say it is true that spicier chili powders cost more? Why or why not?" Sample responses: Overall, yes, but within each SHU range some peppers may be spicier than others. Prices may change.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide a blank table for students to use to help organize their thinking. Ask, "How can this table help you solve this problem?"

Extension: *Interdisciplinary Connections*

Mention that an American pharmacist, Wilbur Scoville, developed a way to test the spiciness of peppers in 1912. Capsaicin oil was extracted from a dried pepper and then sugar water was added until taste testers could no longer detect the heat. A Scoville Heat Unit of 5,000, for example, means that the extracted oil must be diluted with sugar water 5,000 times before the taste tester could not detect any heat! **(Science, History)**

Math Language Development

MLR6: Three Reads

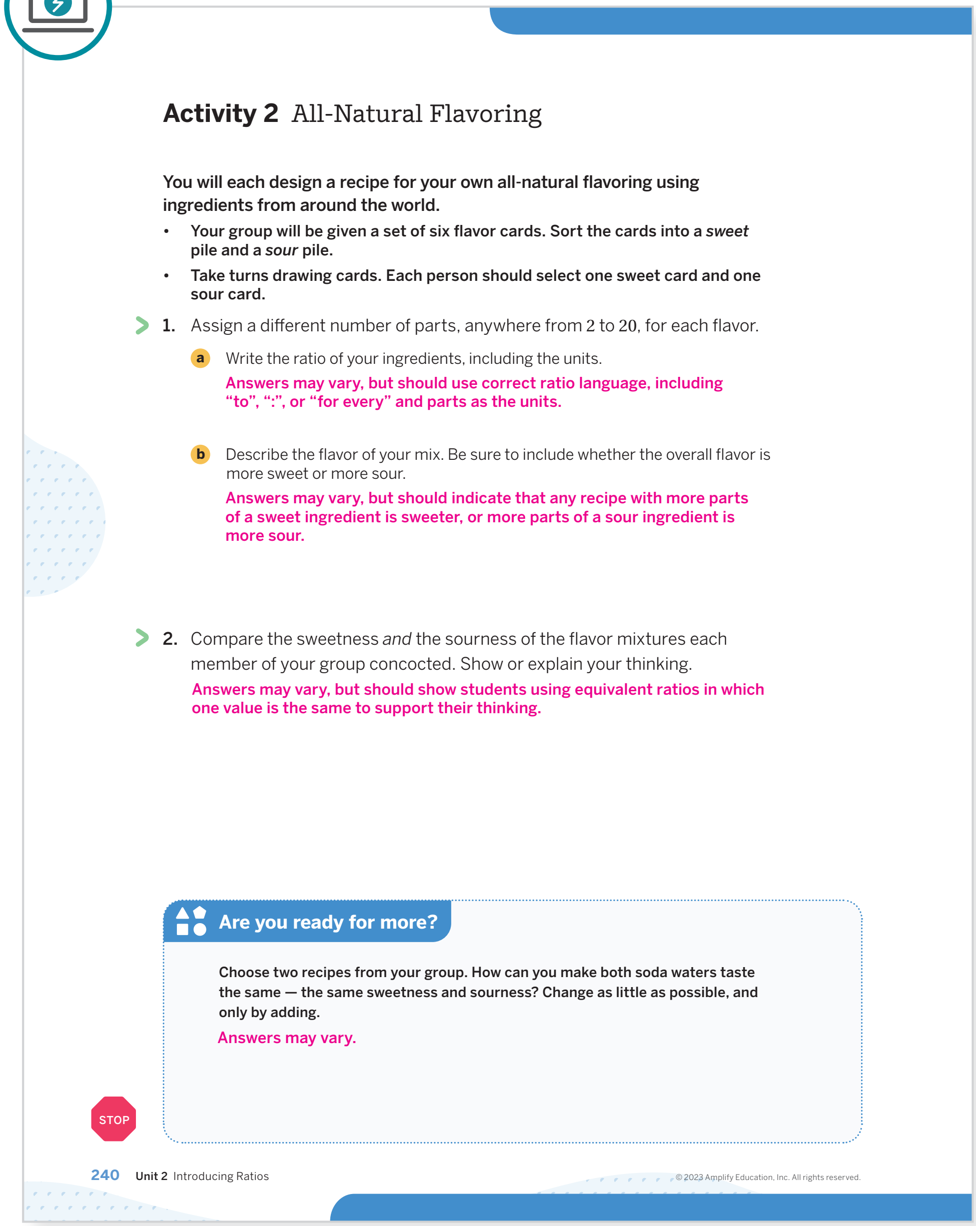
Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that Andre bought different amounts of six types of chili powders.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as $49 for one ounce.
- **Read 3:** Ask students how they can represent these relationships they found in Read 2. Then ask them to plan their solution strategy.

Small Groups | 15 min

Activity 2 All-Natural Flavoring

Students use sweet and sour flavor cards to concoct their own all-natural flavoring mixes, and use equivalent ratios to compare the flavors among their group.



Activity 2 All-Natural Flavoring

You will each design a recipe for your own all-natural flavoring using ingredients from around the world.

- **Your group will be given a set of six flavor cards. Sort the cards into a *sweet* pile and a *sour* pile.**
- **Take turns drawing cards. Each person should select one sweet card and one sour card.**

1. Assign a different number of parts, anywhere from 2 to 20, for each flavor.
   - a. Write the ratio of your ingredients, including the units.
     Answers may vary, but should use correct ratio language, including “to”, “:”, or “for every” and parts as the units.
   - b. Describe the flavor of your mix. Be sure to include whether the overall flavor is more sweet or more sour.
     Answers may vary, but should indicate that any recipe with more parts of a sweet ingredient is sweeter, or more parts of a sour ingredient is more sour.

2. Compare the sweetness *and* the sourness of the flavor mixtures each member of your group concocted. Show or explain your thinking.
   Answers may vary, but should show students using equivalent ratios in which one value is the same to support their thinking.

Are you ready for more?

Choose two recipes from your group. How can you make both soda waters taste the same — the same sweetness and sourness? Change as little as possible, and only by adding.

Answers may vary.

STOP

240 **Unit 2** Introducing Ratios

1 Launch

Give each group one set of cards from the Activity 2 PDF. Read the directions aloud, explaining that the flavorings are liquids that when added to soda water make flavored soda. Clarify that they will only compare the sweetness or sourness of the flavor mix.

2 Monitor

Help students get started by asking, “How many parts of each flavor do you want? What ratio represents your ingredients?”

Look for points of confusion:

- **Misinterpreting their ratios in Problem 1b.** Ask, “If a recipe is 2 parts sweet and 2 parts sour, how can you change it to be sweeter? More sour? Looking at your recipe, is it more sweet or sour? Why?”
- **Incorrectly comparing across recipes.** Refer to Activity 1, and ask, “How did you compare the chili peppers? How could that help you here?”

Look for productive strategies:

- Recognizing that between recipes sweet or sour parts need to be compared in ratios (or to the total).
- Using common multiples or ratios containing a 1 to create equivalent ratios in which one quantity is the same, and then comparing the other quantities.

3 Connect

Have groups of students share how they used common multiples or ratios containing a 1 to compare the flavors, and why they chose their strategies. Consider including using totals to compare (which is useful for Lesson 17).

Highlight that common multiples are helpful when comparing ratios, especially when corresponding values are multiples. Common multiples may not be efficient when students have to compare many ratios (Activity 1), because the LCM may be large. Ratios containing a 1 are helpful when comparing many values at once, but could result in fractions or decimals.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Provide students with a list of values to choose from for their ingredient parts. These values should lend themselves to using multiples, rather than ratios containing a 1, e.g., 2, 3, 4, 6, 8, 12. Consider demonstrating how to describe the flavor of a sample mix, such as *for every 2 parts monk fruit, there are 3 parts tamarind*. Ask students whether this flavor mix is more sweet or more sour, and how they know.

Math Language Development

MLR8: Discussion Supports

Provide sentence frames for students to use as they compare each natural flavoring. For example:

- “If ________, then ________ because . . .”
- “Both ________ and ________ are alike/different because . . .”
- “That could/could not be true because . . .”
- “This method is more/less efficient because . . .”

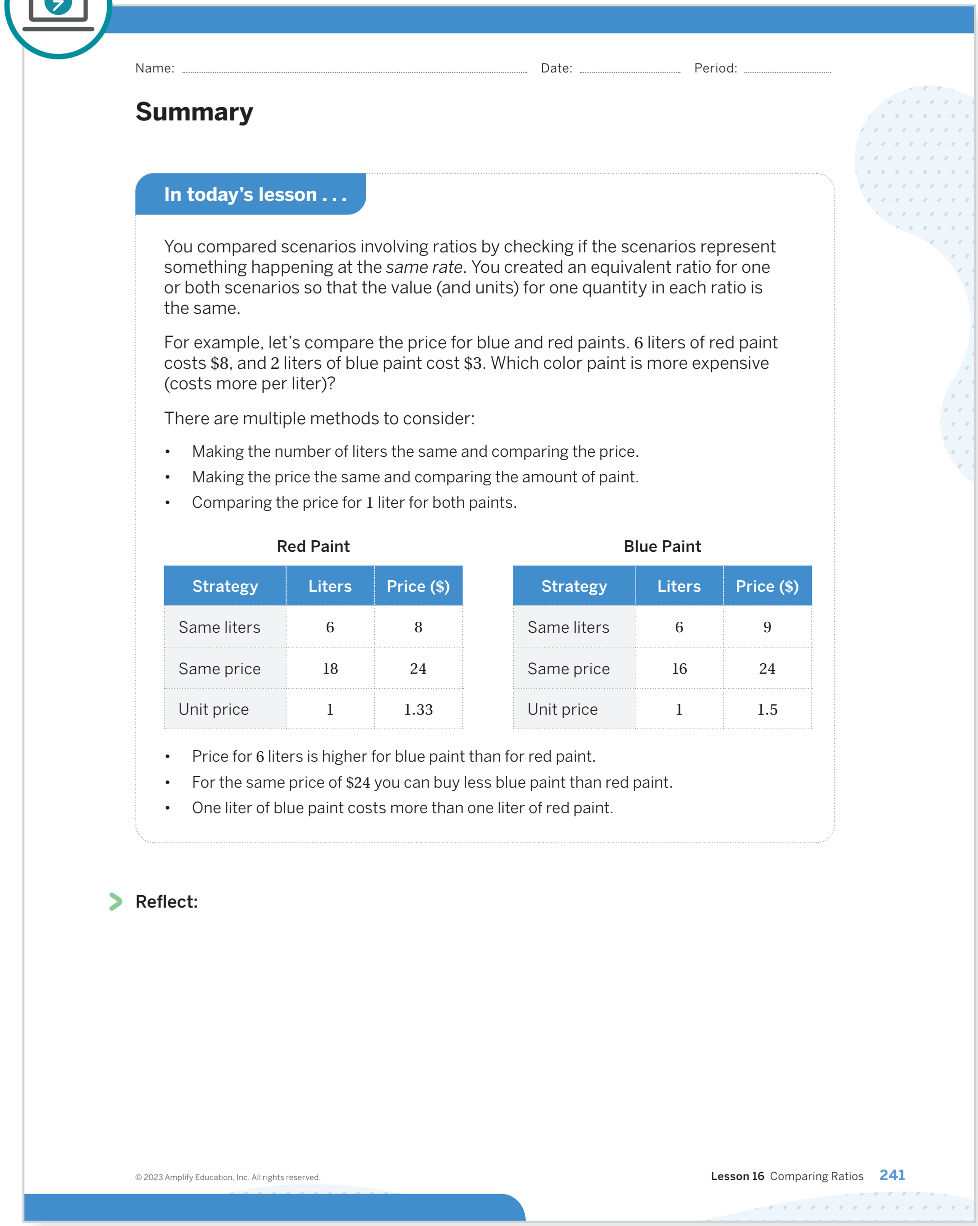
English Learners

Emphasize the difference between *sweet* and *more sweet*, or *sour* and *more sour.*

Whole Class | 5 min

Summary

Review and synthesize how to determine whether two ratios are equivalent.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You compared scenarios involving ratios by checking if the scenarios represent something happening at the *same rate*. You created an equivalent ratio for one or both scenarios so that the value (and units) for one quantity in each ratio is the same.

For example, let's compare the price for blue and red paints. 6 liters of red paint costs $8, and 2 liters of blue paint cost $3. Which color paint is more expensive (costs more per liter)?

There are multiple methods to consider:

- Making the number of liters the same and comparing the price.
- Making the price the same and comparing the amount of paint.
- Comparing the price for 1 liter for both paints.

Red Paint

Strategy	Liters	Price ($)
Same liters	6	8
Same price	18	24
Unit price	1	1.33

Blue Paint

Strategy	Liters	Price ($)
Same liters	6	9
Same price	16	24
Unit price	1	1.5

- Price for 6 liters is higher for blue paint than for red paint.
- For the same price of $24 you can buy less blue paint than red paint.
- One liter of blue paint costs more than one liter of red paint.

Reflect:

Synthesize

Ask:

- "What does it mean for ratios to have a common value?" There is the same amount of corresponding quantities (e.g., sweetness, sourness, price, ounces) in each ratio.
- "How can you make ratios with a common value?" I determine which quantity I want to make the same, and I can use common multiples or unit ratios to make equivalent ratios.
- "How does having a common value help you compare?" I can just compare the value of the other quantity.

Highlight that students can always use common multiples or ratios containing a 1 to create the equivalent ratios, but sometimes one strategy is more efficient than the other. If two ratios are not equivalent, students can use the same strategies to determine which ratio represents more or less of one of the quantities relative to the other, by simply comparing the values for the quantity that is not the same.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

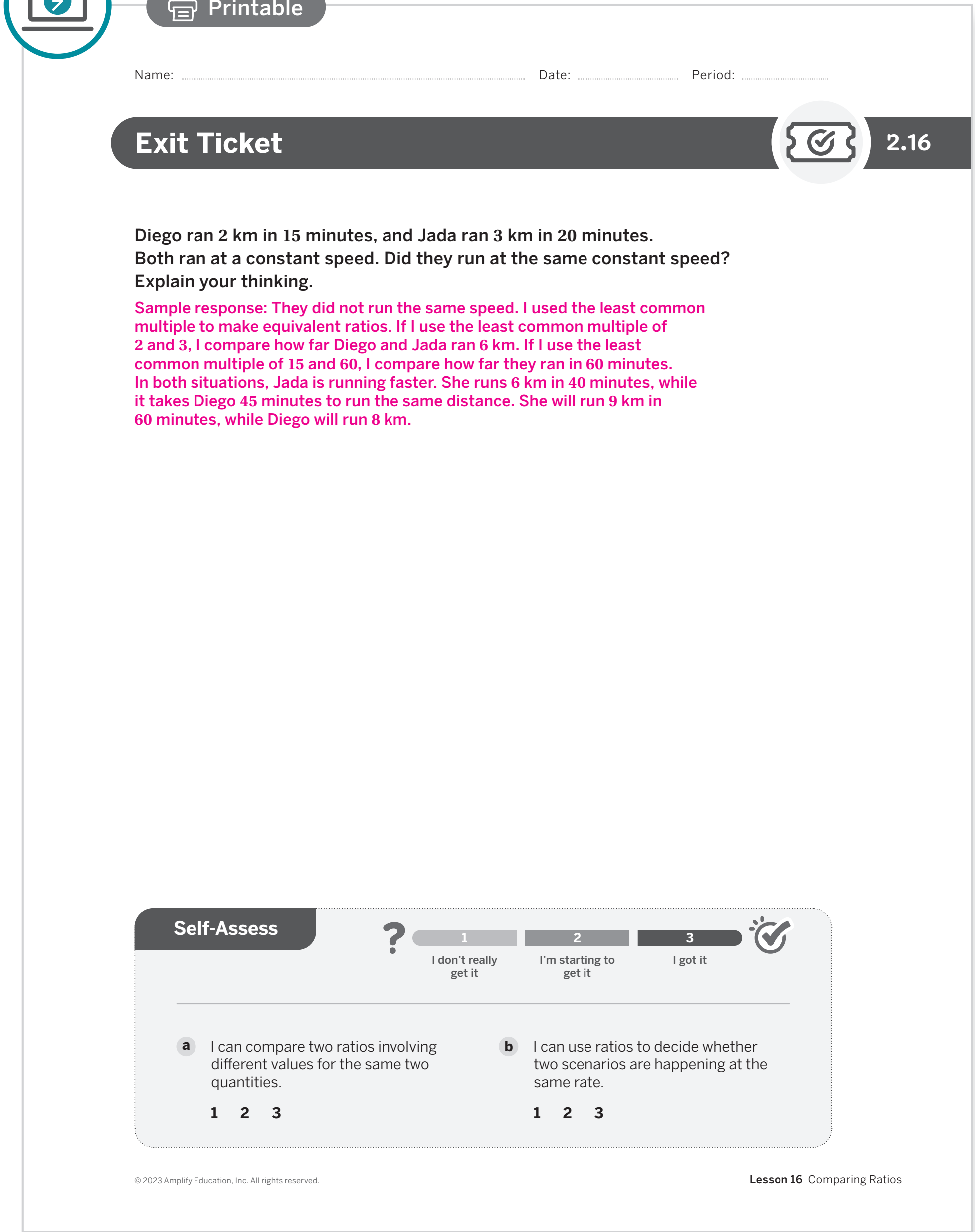
- "What did you find interesting from today's lesson?"
- "How did you use ratios to compare quantities today?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of comparing ratios by creating equivalent ratios to determine whether two people ran at the same speed.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 2.16

Diego ran 2 km in 15 minutes, and Jada ran 3 km in 20 minutes. Both ran at a constant speed. Did they run at the same constant speed? Explain your thinking.

Sample response: They did not run the same speed. I used the least common multiple to make equivalent ratios. If I use the least common multiple of 2 and 3, I compare how far Diego and Jada ran 6 km. If I use the least common multiple of 15 and 60, I compare how far they ran in 60 minutes. In both situations, Jada is running faster. She runs 6 km in 40 minutes, while it takes Diego 45 minutes to run the same distance. She will run 9 km in 60 minutes, while Diego will run 8 km.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a) I can compare two ratios involving different values for the same two quantities.

1 2 3

b) I can use ratios to decide whether two scenarios are happening at the same rate.

1 2 3

Lesson 16 Comparing Ratios

Success looks like . . .

- **Language Goal:** Choosing and creating representations to help compare two situations and explain whether they happen at the same rate. **(Speaking and Listening)**
- **Language Goal:** Justifying that two situations do not happen at the same rate by determining a ratio to describe each situation where the two ratios share one value but not the other, i.e., $a : b$ and $a : c$, or $x : z$ and $y : z$. **(Speaking and Listening)**
  - » Determining that Diego and Jada did not run at the same speed by using least common multiples.

Suggested next steps

If students say they ran the same speed, or they provide an incorrect or incomplete justification for why they did not run the same speed, consider:

- Reviewing the Warm-up and asking, "How did you know that Mai ran faster? Are any of the quantities in this example the same? How can you create equivalent ratios in which one quantity is the same?"

Professional Learning

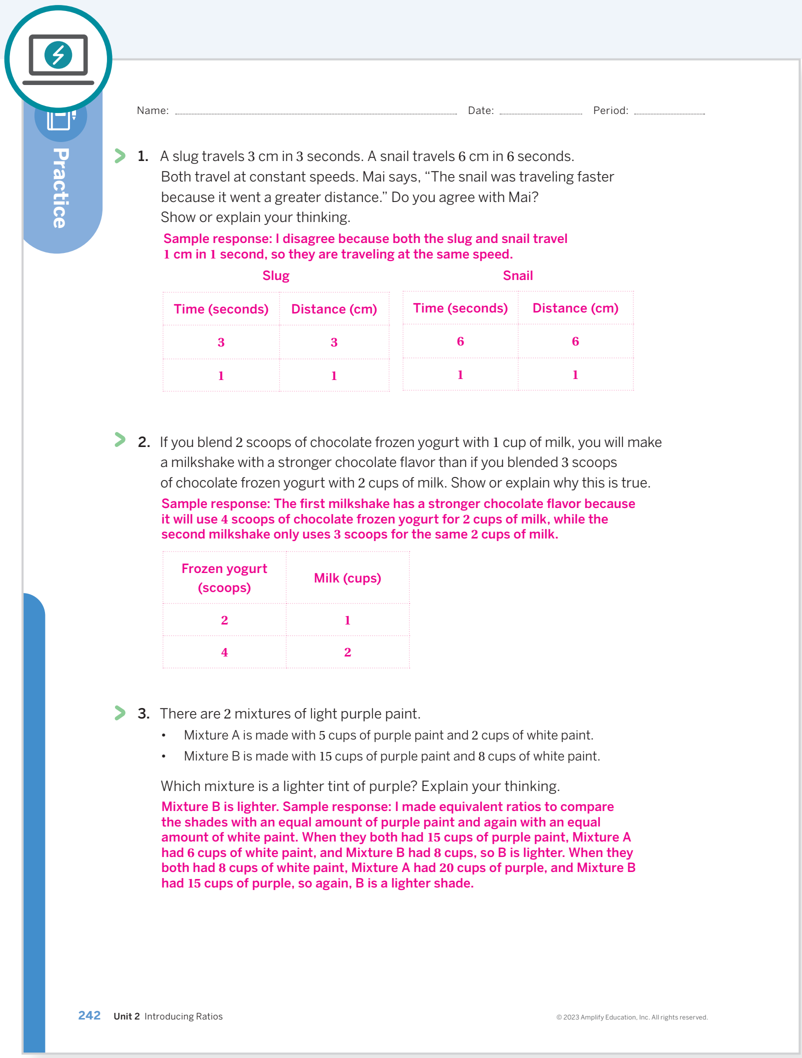
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How was the ratio comparison from today's lesson similar to or different from previous lessons in which students compared quantities?
- What did you see in the way some students approached Activity 1 that you would like other students to try? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. A slug travels 3 cm in 3 seconds. A snail travels 6 cm in 6 seconds. Both travel at constant speeds. Mai says, "The snail was traveling faster because it went a greater distance." Do you agree with Mai? Show or explain your thinking.

Sample response: I disagree because both the slug and snail travel 1 cm in 1 second, so they are traveling at the same speed.

Slug

Time (seconds)	Distance (cm)
3	3
1	1

Snail

Time (seconds)	Distance (cm)
6	6
1	1

2. If you blend 2 scoops of chocolate frozen yogurt with 1 cup of milk, you will make a milkshake with a stronger chocolate flavor than if you blended 3 scoops of chocolate frozen yogurt with 2 cups of milk. Show or explain why this is true.

Sample response: The first milkshake has a stronger chocolate flavor because it will use 4 scoops of chocolate frozen yogurt for 2 cups of milk, while the second milkshake only uses 3 scoops for the same 2 cups of milk.

Frozen yogurt (scoops)	Milk (cups)
2	1
4	2

3. There are 2 mixtures of light purple paint.
   - Mixture A is made with 5 cups of purple paint and 2 cups of white paint.
   - Mixture B is made with 15 cups of purple paint and 8 cups of white paint.

   Which mixture is a lighter tint of purple? Explain your thinking.

   Mixture B is lighter. Sample response: I made equivalent ratios to compare the shades with an equal amount of purple paint and again with an equal amount of white paint. When they both had 15 cups of purple paint, Mixture A had 6 cups of white paint, and Mixture B had 8 cups, so B is lighter. When they both had 8 cups of white paint, Mixture A had 20 cups of purple, and Mixture B had 15 cups of purple, so again, B is a lighter shade.

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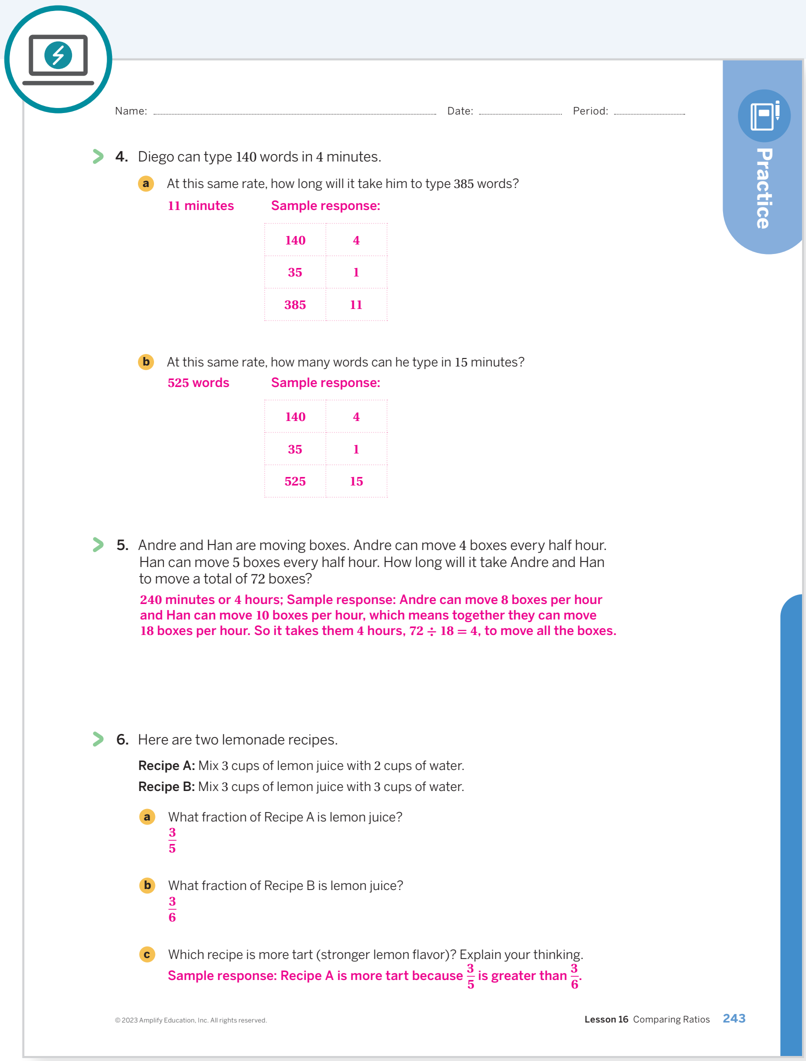

Name: Date: Period:

Practice

4. Diego can type 140 words in 4 minutes.

   a At this same rate, how long will it take him to type 385 words?

   11 minutes **Sample response:**

140	4
35	1
385	11

   b At this same rate, how many words can he type in 15 minutes?

   525 words **Sample response:**

140	4
35	1
525	15

5. Andre and Han are moving boxes. Andre can move 4 boxes every half hour. Han can move 5 boxes every half hour. How long will it take Andre and Han to move a total of 72 boxes?

   240 minutes or 4 hours; Sample response: Andre can move 8 boxes per hour and Han can move 10 boxes per hour, which means together they can move 18 boxes per hour. So it takes them 4 hours, $72 \div 18 = 4$, to move all the boxes.

6. Here are two lemonade recipes.

   Recipe A: Mix 3 cups of lemon juice with 2 cups of water.

   Recipe B: Mix 3 cups of lemon juice with 3 cups of water.

   a What fraction of Recipe A is lemon juice?

   $\frac{3}{5}$

   b What fraction of Recipe B is lemon juice?

   $\frac{3}{6}$

   c Which recipe is more tart (stronger lemon flavor)? Explain your thinking.

   Sample response: Recipe A is more tart because $\frac{3}{5}$ is greater than $\frac{3}{6}$.

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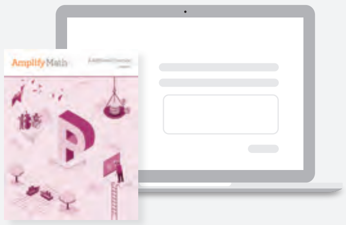
Lesson 16 Comparing Ratios 243

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-Up, Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 14	2
	5	Unit 2 Lesson 15	2
Formative ⚡	6	Unit 2 Lesson 17	2

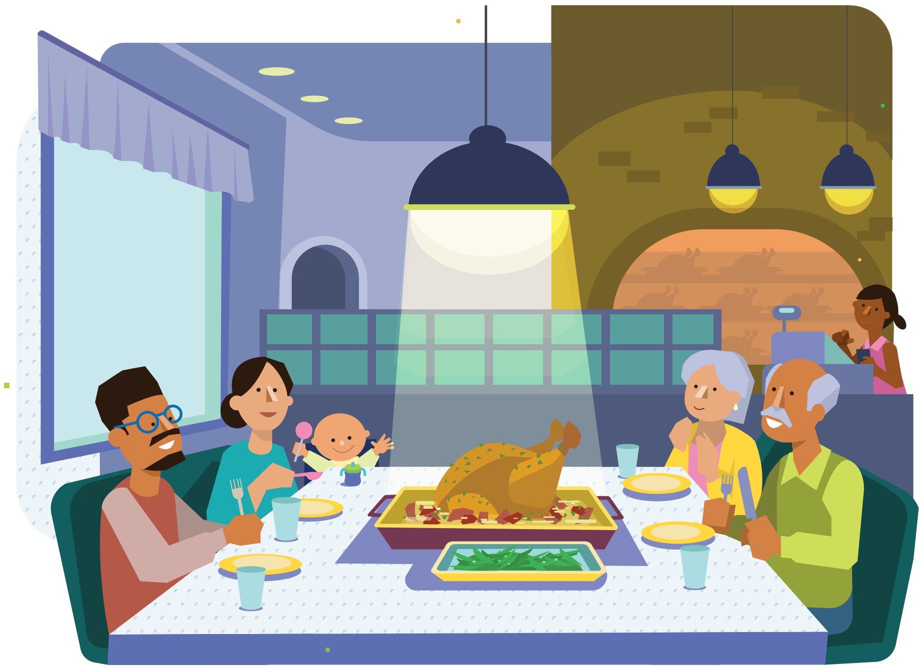
⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 2 | **LESSON 17**



More Comparing and Solving

Let's practice using ratios to solve more problems.

Focus

Goals

1. **Language Goal:** Understand when a question requires a direct comparison between quantities or a ratio comparison. **(Speaking and Listening)**
2. Choose and create representations to help solve comparison problems involving equivalent ratios and total amounts.

Coherence

Today

Students continue their work with ratio comparisons. First, they distinguish between direct comparison of quantities and ratio comparisons in the context of time and speed. They also consider when a ratio comparison requires equivalent ratios with common values versus when common values are not necessary. Students then investigate comparisons with part-part-whole ratios, recognizing that the common total, rather than common parts, can make some comparisons more efficient.

Previously

In Lessons 14–15, students solved equivalent ratio problems including those involving part-part-whole ratios. In Lesson 16, they began to solve ratio comparison problems.

Coming Soon

In Lessons 18–19, students will extend their work with equivalent ratios to convert units of measurement both within and between measurement systems.

Rigor

- Students build **conceptual understanding** of comparing ratios when the total parts are different.
- Students develop **procedural fluency** with comparing ratios.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice
Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

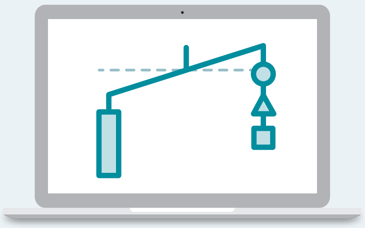
Review word

- *equivalent ratios*

Amps | **Featured Activity**

Activity 1
Real-Time Feedback

Students compare different chefs' progress according time and speed, and receive real-time feedback on their work.



Building Math Identity and Community
Connecting to Mathematical Practices

Students may become frustrated in Activity 2 when they try to compare recipe flavors by using the structure of a ratio and making two quantities the same in all three ratios. Encourage students to shift their perspective by asking, "Thinking about when you have worked with recipes in previous lessons, was there another quantity (perhaps not given to you directly) that you sometimes considered in making comparisons?"

Modifications to Pacing

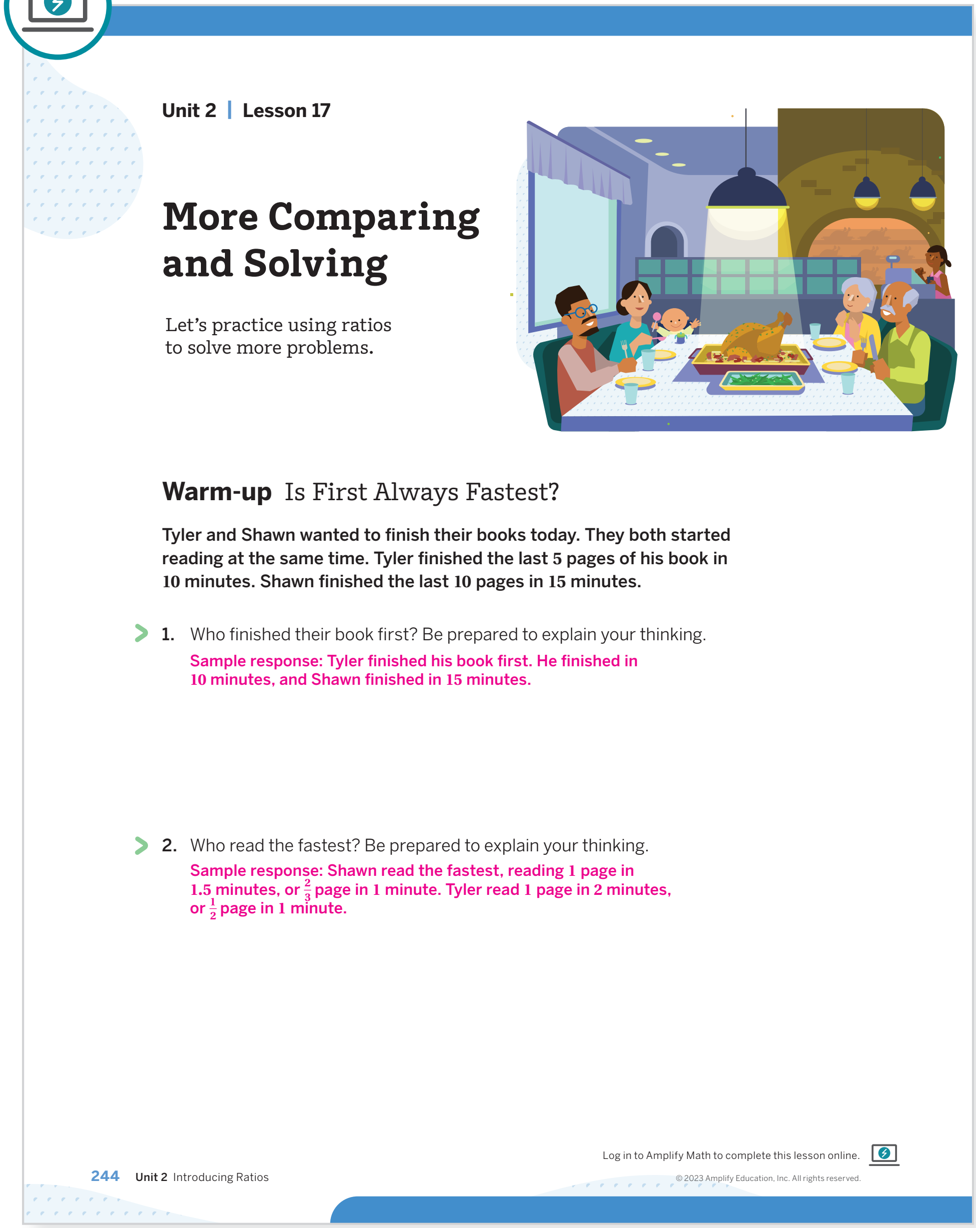
You may want to consider these additional modifications if you are short on time.

- Present the **Warm-up** to the whole class, and simply ask students to explain how their strategy to solve would differ for the question in Problems 1 and 2, and why.
- In **Activity 1**, Problem 1 may be omitted.
- In **Activity 2**, Problem 1 may be omitted.

Independent | 5 min

Warm-up Is First Always Fastest?

Students distinguish between direct comparison and ratio comparison to reason about who finishes their reading first as opposed to who reads the fastest.



Unit 2 | Lesson 17

More Comparing and Solving

Let's practice using ratios to solve more problems.

Warm-up Is First Always Fastest?

Tyler and Shawn wanted to finish their books today. They both started reading at the same time. Tyler finished the last 5 pages of his book in 10 minutes. Shawn finished the last 10 pages in 15 minutes.

1. Who finished their book first? Be prepared to explain your thinking.
   Sample response: Tyler finished his book first. He finished in 10 minutes, and Shawn finished in 15 minutes.

2. Who read the fastest? Be prepared to explain your thinking.
   Sample response: Shawn read the fastest, reading 1 page in 1.5 minutes, or $\frac{2}{3}$ page in 1 minute. Tyler read 1 page in 2 minutes, or $\frac{1}{2}$ page in 1 minute.

Log in to Amplify Math to complete this lesson online.

244 Unit 2 Introducing Ratios

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1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by asking, "What information do you know? What do you need to know?"

Look for points of confusion:

- **Not making a direct comparison of 10 to 15 for Problem 1.** Ask, "If they start at 12:00, what time does Tyler finish? What time does Shawn finish?"
- **Not using ratio comparison to determine who reads fastest.** Refer to the Warm-up from Lesson 16 and ask, "How did you know Mai ran the fastest? How can you use that same thinking here?"

Look for productive strategies:

- Recognizing that "finishing first" is a direct comparison of end times and does not require a ratio comparison.
- Recognizing that "fastest" is a ratio of pages read to time, and using equivalent ratios with a common value to compare.

3 Connect

Have students share their responses, focusing on how and why they used a direct comparison for Problem 1 and ratio comparison for Problem 2.

Highlight that determining "first" is a direct comparison of one quantity (time). Determining "fastest" is a comparison of speed, which is a ratio of pages read to time. Therefore, students are comparing ratios.

Ask, "What else could you compare directly in this scenario? What other comparisons might require ratios?"

Power-up

To power up students' ability to compare ratios to solve problems, have students complete:

Here are two mixtures for sky blue paint:
Mixture A: Mix 3 parts blue paint with 5 parts white paint.
Mixture B: Mix 3 parts blue paint with 3 parts white paint.

1. Write a ratio comparing blue paint to total paint in each mixture:
   - **a.** Mixture A: 3 : 8
   - **b.** Mixture B: 3 : 6
2. What fraction of each mixture is blue paint?
   - **a.** Mixture A: $\frac{3}{8}$
   - **b.** Mixture B: $\frac{3}{6}$

Use: Before the Warm-up.

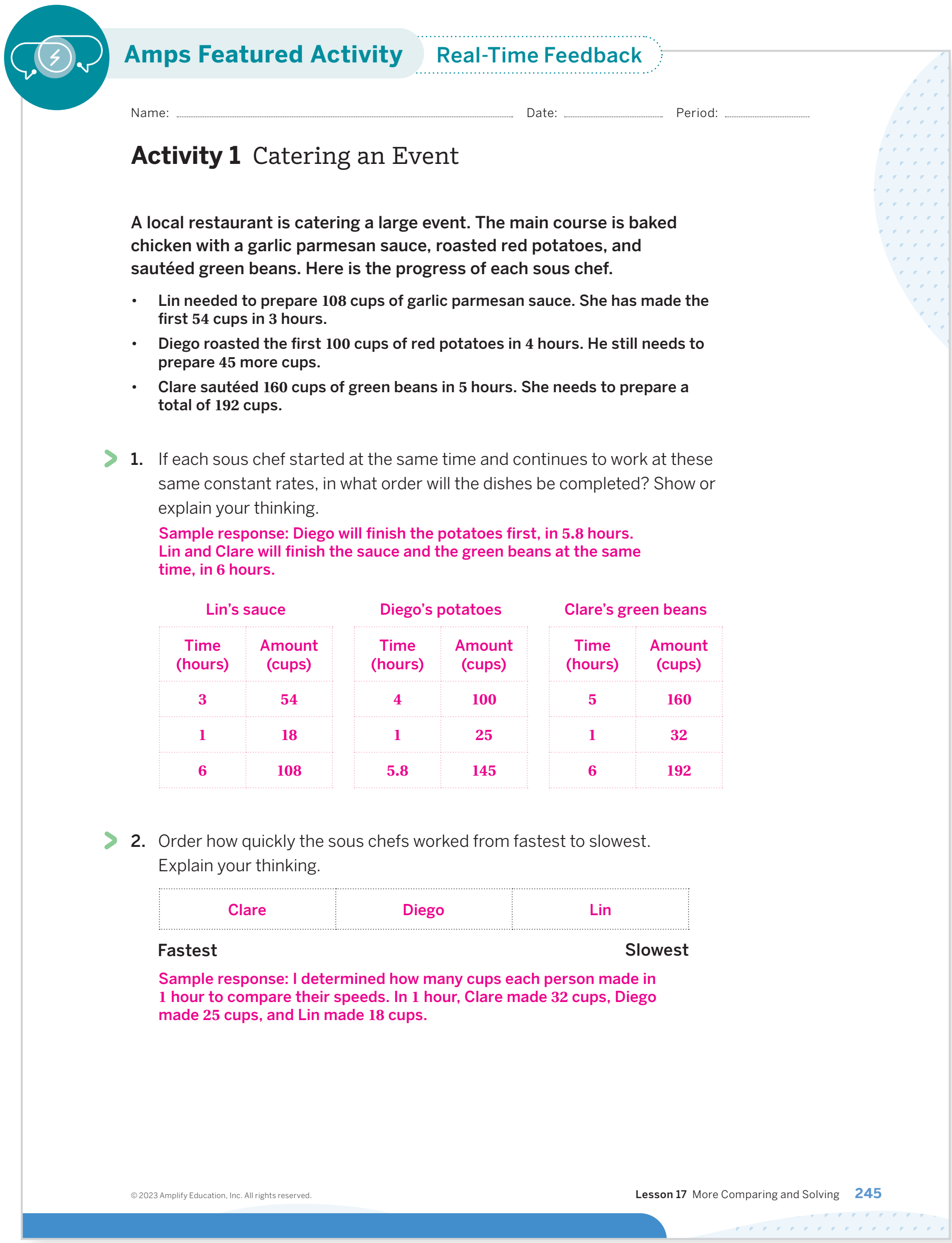
Informed by: Performance on Lesson 16, Practice Problem 6.

Pairs | 15 min

Activity 1 Catering an Event

Students determine when a common value is necessary for comparison as they determine who finished their meal prep first and who worked the fastest.

Amps Featured Activity Real-Time Feedback



Name: Date: Period:

Activity 1 Catering an Event

A local restaurant is catering a large event. The main course is baked chicken with a garlic parmesan sauce, roasted red potatoes, and sautéed green beans. Here is the progress of each sous chef.

- **Lin needed to prepare 108 cups of garlic parmesan sauce. She has made the first 54 cups in 3 hours.**
- **Diego roasted the first 100 cups of red potatoes in 4 hours. He still needs to prepare 45 more cups.**
- **Clare sautéed 160 cups of green beans in 5 hours. She needs to prepare a total of 192 cups.**

1. If each sous chef started at the same time and continues to work at these same constant rates, in what order will the dishes be completed? Show or explain your thinking.

Sample response: Diego will finish the potatoes first, in 5.8 hours. Lin and Clare will finish the sauce and the green beans at the same time, in 6 hours.

Lin's sauce

Time (hours)	Amount (cups)
3	54
1	18
6	108

Diego's potatoes

Time (hours)	Amount (cups)
4	100
1	25
5.8	145

Clare's green beans

Time (hours)	Amount (cups)
5	160
1	32
6	192

2. Order how quickly the sous chefs worked from fastest to slowest. Explain your thinking.

Clare	Diego	Lin

Fastest **Slowest**

Sample response: I determined how many cups each person made in 1 hour to compare their speeds. In 1 hour, Clare made 32 cups, Diego made 25 cups, and Lin made 18 cups.

1 Launch

Explain that each sous chef is responsible for only one component of the final dish.

2 Monitor

Help students get started by asking, "What information do you know? Could a table help you get started?"

Look for points of confusion:

- **Using common values to determine who finished first.** Ask, "Do all three have the same amount of work to complete?"
- **Assuming that finished first also means fastest.** Refer to the Warm-up and ask, "Why did you say Shawn read the fastest if Tyler finished first? How can you use that here?"

Look for productive strategies:

- Using equivalent ratios without common values to determine the elapsed time for each person (Problem 1), and with common values to determine who worked the fastest (Problem 2).
- Using LCM or ratios containing a 1 to create equivalent ratios with common values.

3 Connect

Have students share their responses and strategies, focusing on how they determined if and when they needed a common value.

Highlight that both problems required equivalent ratios to compare, but they were used differently. Problem 1 did not need a common value because it was a direct comparison of elapsed time while Problem 2 required a common value because it compared speed, which is a ratio.

Ask, "Which strategy was more efficient to compare speed — or a ratios containing a 1 or LCM? Why?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can choose from a table, a double number line, or a free sketch to make comparisons and help determine the order in which the dishes will be completed.

Extension: *Math Enrichment*

Have students imagine they are in charge of preparing the side salad. Have them decide on three values so that they finish after Diego but before Clare and Lin: **(1)** How many cups of salad they must prepare; **(2)** How many cups they have already prepared; **(3)** How long it took them to prepare those cups of salad. Ask, "Did Clare still work the fastest?"

Math Language Development

MLR6: Three Reads

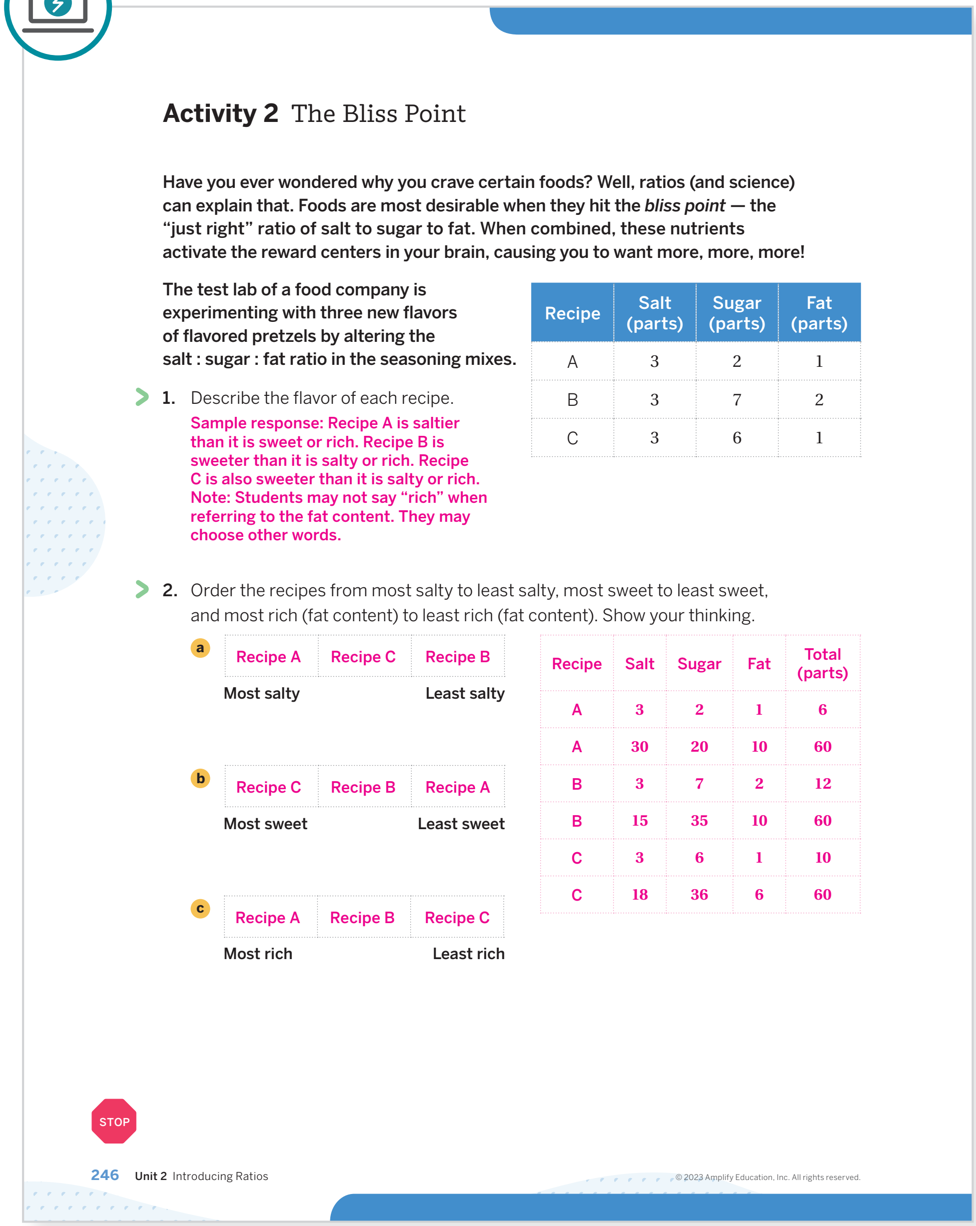
Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that three chefs are making different amounts of different dishes and that they have been working for different amounts of time. Explain that a *sous chef* is like an assistant chef to the head chef.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as Lin made 54 cups in 3 hours.
- **Read 3:** Ask students how they can represent these relationships they found in Read 2. Then ask them to plan their solution strategy.

Pairs | 15 min

Activity 2 The Bliss Point

Students create equivalent ratios and consider the quantity of total parts (the whole) in order to compare the flavors of three potato chip recipes.



Activity 2 The Bliss Point

Have you ever wondered why you crave certain foods? Well, ratios (and science) can explain that. Foods are most desirable when they hit the *bliss point* — the "just right" ratio of salt to sugar to fat. When combined, these nutrients activate the reward centers in your brain, causing you to want more, more, more!

The test lab of a food company is experimenting with three new flavors of flavored pretzels by altering the salt : sugar : fat ratio in the seasoning mixes.

Recipe	Salt (parts)	Sugar (parts)	Fat (parts)
A	3	2	1
B	3	7	2
C	3	6	1

1. Describe the flavor of each recipe.

   Sample response: Recipe A is saltier than it is sweet or rich. Recipe B is sweeter than it is salty or rich. Recipe C is also sweeter than it is salty or rich. Note: Students may not say "rich" when referring to the fat content. They may choose other words.

2. Order the recipes from most salty to least salty, most sweet to least sweet, and most rich (fat content) to least rich (fat content). Show your thinking.

a

Recipe A	Recipe C	Recipe B

Most salty Least salty

b

Recipe C	Recipe B	Recipe A

Most sweet Least sweet

c

Recipe A	Recipe B	Recipe C

Most rich Least rich

Recipe	Salt	Sugar	Fat	Total (parts)
A	3	2	1	6
A	30	20	10	60
B	3	7	2	12
B	15	35	10	60
C	3	6	1	10
C	18	36	6	60

STOP

1 Launch

Explain that there is no one *bliss point* for all foods and all people, so companies use taste tests to determine a *bliss range*. Also explain that fats, such as butter and oil add "richness" to a dish, so the term *rich* is used to describe the flavor.

2 Monitor

Help students get started by asking, "What information do you know? What do you need to know?"

Look for points of confusion:

- **Inaccurately describing the flavors.** Have students focus on one recipe at a time, and ask, "Which flavor would be the strongest in this recipe? Why?"
- **Comparing two ratios (e.g., salt to sugar) at a time.** Have students compare the sweetness of Recipes B and C by comparing sugar to salt, then by comparing sugar to fat. Ask, "Why were the results different? How can you use total parts to create equivalent ratios?"

Look for productive strategies:

- Understanding that they need to use a ratio, rather than direct comparison, because a recipe's flavor is determined by the mix of ingredients.
- Comparing flavors by making equivalent ratios in which the total parts are a shared value.

3 Connect

Have students share their responses, focusing on why they needed a common total in order to compare.

Highlight that totals often help when comparing ratios with two or more parts. Common totals were needed here because you cannot make two quantities the same in all three ratios.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Complete Problem 1 as a whole class discussion to ensure that students connect the terms *saltier, sweeter,* and *richer* to the amount of *salt, sugar,* and *fat,* respectively, in foods. Then have students complete Problem 2.

Extension: *Math Enrichment*

Have pairs of students write their own bliss point ratio for a flavor of potato chips and then compare the flavors with each other.

Math Language Development

MLR3: Critique, Correct, Clarify

Before students begin Problem 2, present a flawed response, such as, "All recipes use 3 parts salt, so they are equally salty." Have them critique the statement, work to correct it, and then clarify the reasoning used.

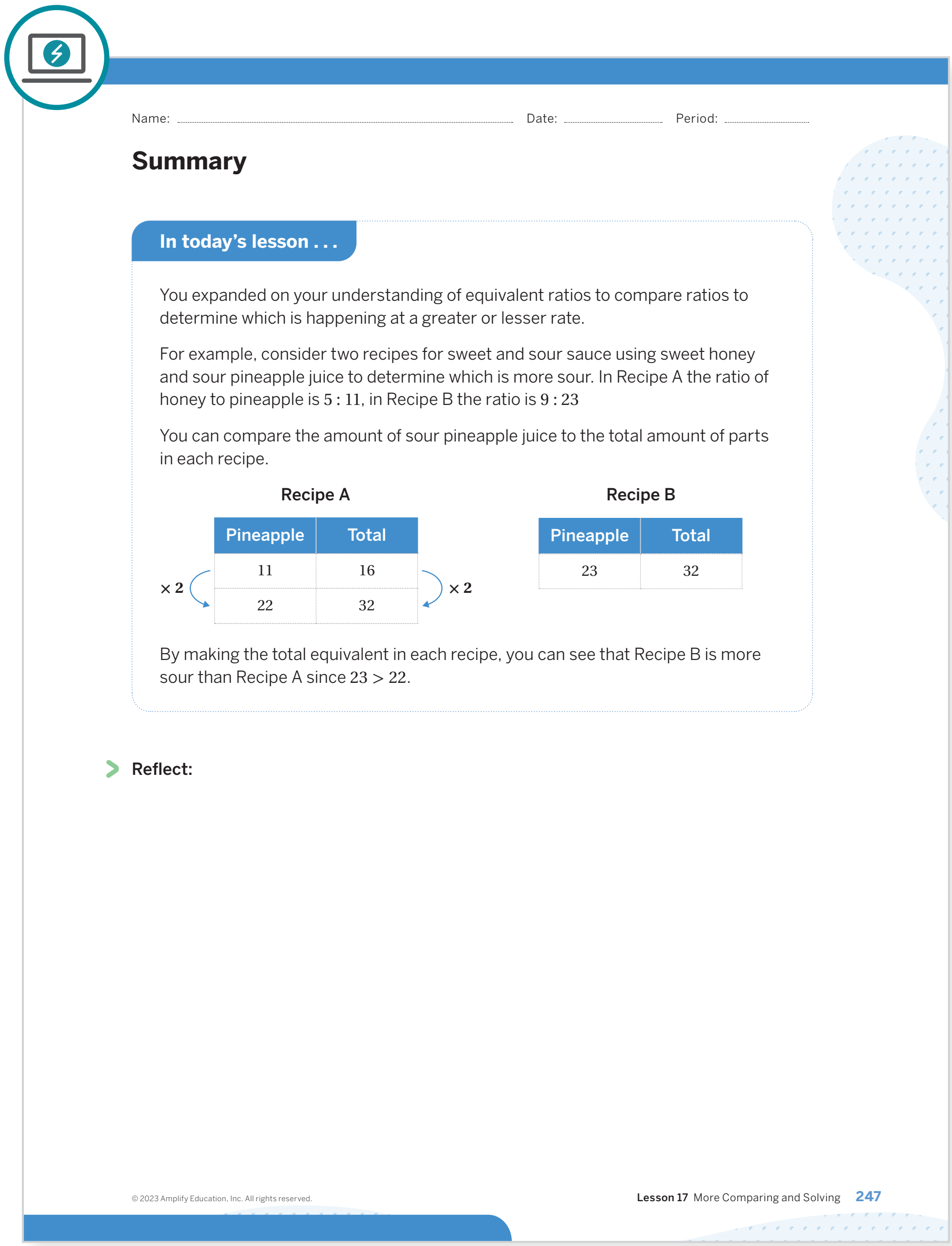
English Learners

Emphasize how the term *saltier* means the same as saying "more salty." Similarly, *richer* and *more rich* both express the same idea as it relates to the fat content in foods.

Whole Class | 5 min

Summary

Review and synthesize the types of comparison and ratio strategies that can be used to compare among two or more quantities and their totals.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You expanded on your understanding of equivalent ratios to compare ratios to determine which is happening at a greater or lesser rate.

For example, consider two recipes for sweet and sour sauce using sweet honey and sour pineapple juice to determine which is more sour. In Recipe A the ratio of honey to pineapple is 5 : 11, in Recipe B the ratio is 9 : 23

You can compare the amount of sour pineapple juice to the total amount of parts in each recipe.

Recipe A

Pineapple	Total
11	16
22	32

× 2 × 2

Recipe B

Pineapple	Total
23	32

By making the total equivalent in each recipe, you can see that Recipe B is more sour than Recipe A since 23 > 22.

Reflect:

Synthesize

Ask,

- "What does it mean for ratios to have a common value?" There is the same amount of corresponding quantities in each ratio.
- "How do you make a common value when you have three or more quantities?" I use common multiples or ratios containing a 1 to determine a common value for total parts.
- "When might it be useful or necessary to consider a total quantity? Explain your thinking." Sometimes it makes calculations and determining common values more efficient. Other times I may have more than two quantities and cannot determine multiple common values in order to just look at a direct comparison of one quantity.

Highlight that when comparing, sometimes students can compare two values directly, and sometimes they must use a ratio. When comparing ratios, sometimes students need a common value and sometimes they do not. Sometimes students can compare in pairs, and other times they need to consider the total.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

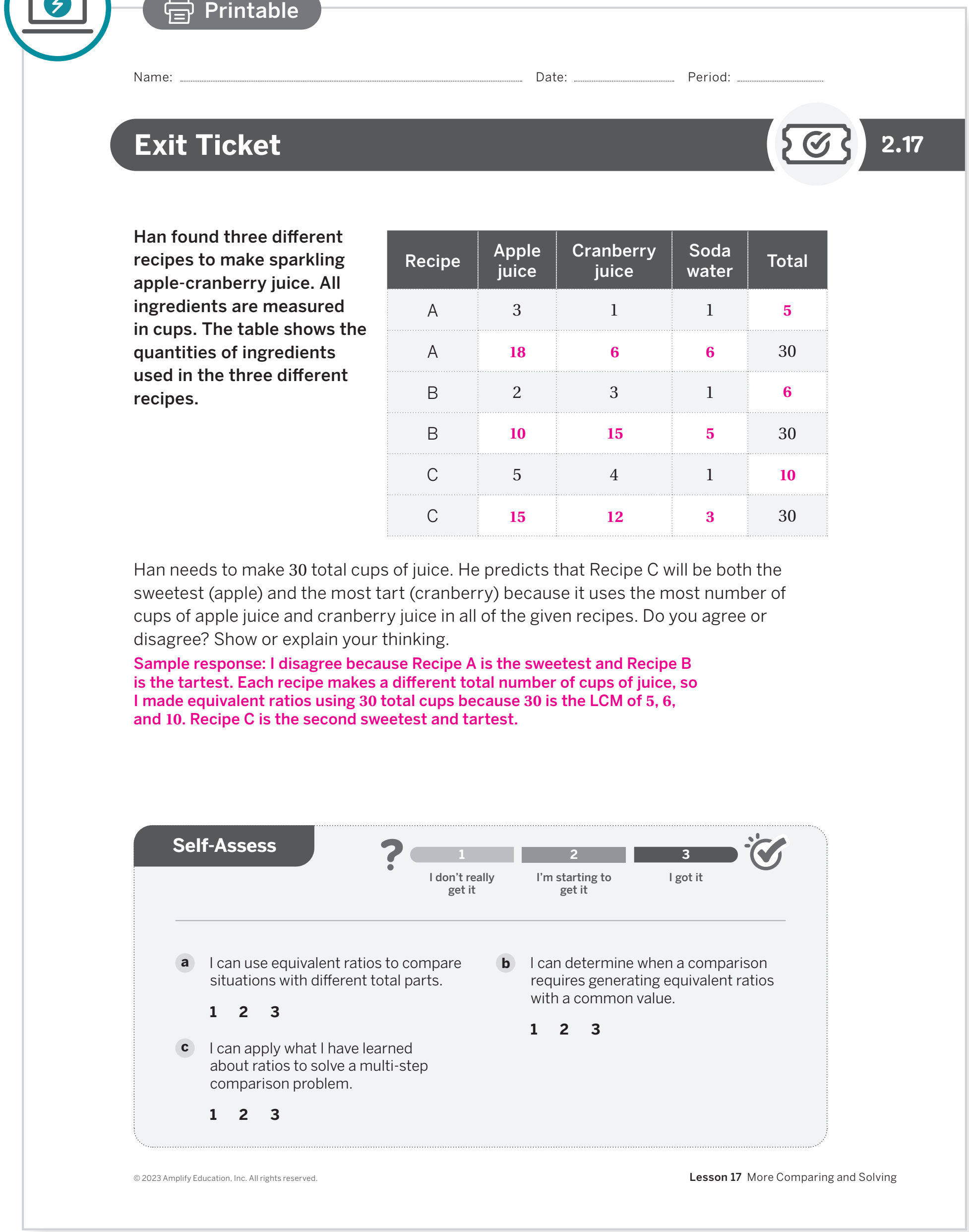
- "Were any strategies more useful than others when comparing ratios in these activities? Why?"
- "Is first always fastest? Why not?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by comparing the flavor of three juice recipes by using a common value for total parts.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket

2.17

Han found three different recipes to make sparkling apple-cranberry juice. All ingredients are measured in cups. The table shows the quantities of ingredients used in the three different recipes.

Recipe	Apple juice	Cranberry juice	Soda water	Total
A	3	1	1	**5**
A	**18**	**6**	**6**	30
B	2	3	1	**6**
B	**10**	**15**	**5**	30
C	5	4	1	**10**
C	**15**	**12**	**3**	30

Han needs to make 30 total cups of juice. He predicts that Recipe C will be both the sweetest (apple) and the most tart (cranberry) because it uses the most number of cups of apple juice and cranberry juice in all of the given recipes. Do you agree or disagree? Show or explain your thinking.

Sample response: I disagree because Recipe A is the sweetest and Recipe B is the tartest. Each recipe makes a different total number of cups of juice, so I made equivalent ratios using 30 total cups because 30 is the LCM of 5, 6, and 10. Recipe C is the second sweetest and tartest.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can use equivalent ratios to compare situations with different total parts.

1 2 3

b I can determine when a comparison requires generating equivalent ratios with a common value.

1 2 3

c I can apply what I have learned about ratios to solve a multi-step comparison problem.

1 2 3

Success looks like . . .

- **Language Goal:** Understanding when a question requires a direct comparison between quantities or a ratio comparison. **(Speaking and Listening)**
  - » Comparing amounts of juice in three different recipes by using equivalent ratios.
- **Goal:** Choosing and creating representations to help solve comparison problems involving equivalent ratios and total amounts.

Suggested next steps

If students agree with Han or attempt to solve by using multiple two-ratio comparisons, consider:

- Reviewing Activity 2, and asking:
  - » "Did the 7 parts sugar make Recipe B sweeter than Recipe C? Why not?" No, because when I made the equivalent ratios with a common value of 60 parts, Recipe C used 36 parts sweet and B used 35 parts.
  - » "How did you order the sweetness for all three recipes?" I made equivalent ratios in which the total parts were the same, and then compared the number of sweet parts. Remind students that comparing two ratios at a time can result in inconsistent conclusions.

Professional Learning

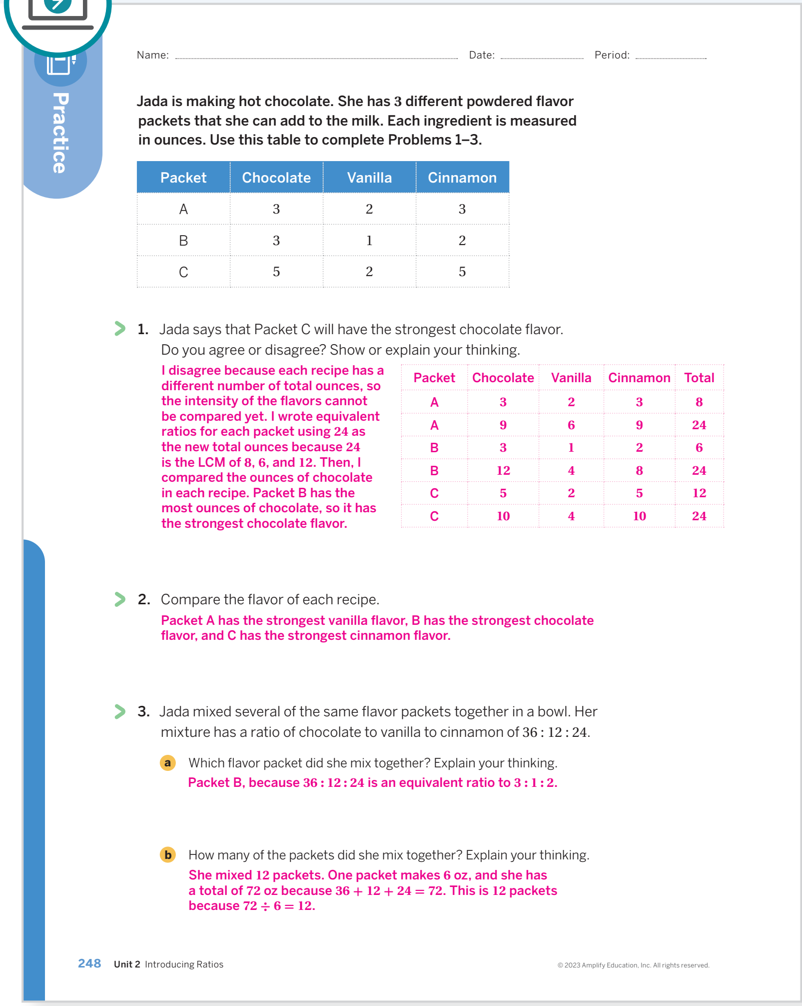
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In earlier lessons, students compared quantities using ratio comparison. How did that support them as they distinguished between when to use direct versus ratio comparison in Activity 1?
- What trends do you see in participation? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

Jada is making hot chocolate. She has 3 different powdered flavor packets that she can add to the milk. Each ingredient is measured in ounces. Use this table to complete Problems 1–3.

Packet	Chocolate	Vanilla	Cinnamon
A	3	2	3
B	3	1	2
C	5	2	5

1. Jada says that Packet C will have the strongest chocolate flavor. Do you agree or disagree? Show or explain your thinking.

   I disagree because each recipe has a different number of total ounces, so the intensity of the flavors cannot be compared yet. I wrote equivalent ratios for each packet using 24 as the new total ounces because 24 is the LCM of 8, 6, and 12. Then, I compared the ounces of chocolate in each recipe. Packet B has the most ounces of chocolate, so it has the strongest chocolate flavor.

Packet	Chocolate	Vanilla	Cinnamon	Total
A	3	2	3	8
A	9	6	9	24
B	3	1	2	6
B	12	4	8	24
C	5	2	5	12
C	10	4	10	24

2. Compare the flavor of each recipe.

   Packet A has the strongest vanilla flavor, B has the strongest chocolate flavor, and C has the strongest cinnamon flavor.

3. Jada mixed several of the same flavor packets together in a bowl. Her mixture has a ratio of chocolate to vanilla to cinnamon of $36 : 12 : 24$.

   a. Which flavor packet did she mix together? Explain your thinking.

   Packet B, because $36 : 12 : 24$ is an equivalent ratio to $3 : 1 : 2$.

   b. How many of the packets did she mix together? Explain your thinking.

   She mixed 12 packets. One packet makes 6 oz, and she has a total of 72 oz because $36 + 12 + 24 = 72$. This is 12 packets because $72 \div 6 = 12$.

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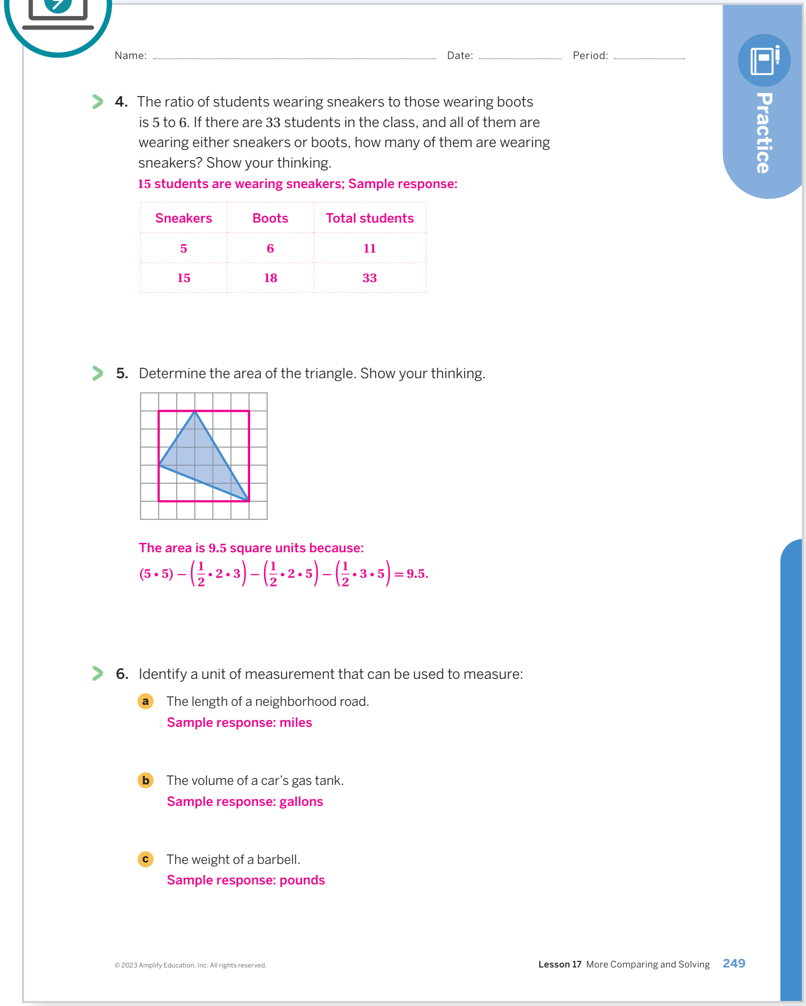

Practice

Name: Date: Period:

4. The ratio of students wearing sneakers to those wearing boots is 5 to 6. If there are 33 students in the class, and all of them are wearing either sneakers or boots, how many of them are wearing sneakers? Show your thinking.

   15 students are wearing sneakers; Sample response:

Sneakers	Boots	Total students
5	6	11
15	18	33

5. Determine the area of the triangle. Show your thinking.

   The area is 9.5 square units because:

   $$(5 \cdot 5) - \left(\frac{1}{2} \cdot 2 \cdot 3\right) - \left(\frac{1}{2} \cdot 2 \cdot 5\right) - \left(\frac{1}{2} \cdot 3 \cdot 5\right) = 9.5.$$

6. Identify a unit of measurement that can be used to measure:

   a. The length of a neighborhood road.

   Sample response: miles

   b. The volume of a car's gas tank.

   Sample response: gallons

   c. The weight of a barbell.

   Sample response: pounds

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Lesson 17 More Comparing and Solving 249

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 15	2
	5	Unit 1 Lesson 10	2
Formative ⚡	6	Unit 2 Lesson 18	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 2 | **LESSON 18**

Measuring With Different-Sized Units

Let's measure the length, volume, or weight of an object by using different units.

Focus

Goals

1. **Language Goal:** Generalize that it takes more of a smaller unit or fewer of a larger unit to measure the same quantity. **(Speaking and Listening, Writing)**
2. **Language Goal:** Given a measurement in one unit, estimate what would be the same amount expressed in a different unit, and explain the reasoning. **(Speaking and Listening)**

Coherence

Today

Students review standard units of length, volume and weight by measuring familiar objects and determining the best unit of measurement to use for each one. They explore different attributes and corresponding units of measurement for a given object, taking turns to analyze and critique the ideas and explanations of a peer. Students also interact with different types of measurement on an experiential level, coming to recognize that it takes more of a smaller unit and less of a larger unit to measure the same quantity. This idea is an important foundation for converting units of measurement by using ratio reasoning in Lesson 19.

Previously

In Lesson 17, students compared ratios in which the quantities had different total parts, requiring multiple steps and determining equivalent ratios with common values.

Coming Soon

In Lesson 19, students will connect the ideas about measurement from this lesson with ratio concepts in earlier lessons to converting units between measurement systems — metric and U.S. Customary.

Rigor

- Students measure objects in different units, building **conceptual understanding** that the smaller a measurement is, the greater the number will be as compared to a larger unit.
- Students **apply** their understanding of different units of measurement to decide how and when to use a given unit when measuring an object.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1 (optional)	Activity 2	Summary	Exit Ticket
5 min	10 min	30 min	5 min	5 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

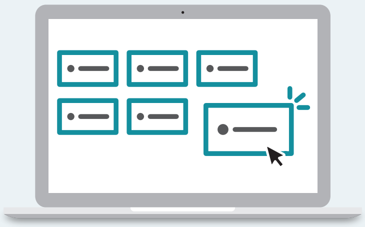
Materials

- Exit Ticket
- Additional Practice
- pre-cut slips of Warm-up measurement units (optional; not provided)
- ruler
- scale
- four 1-liter bottles
- four 1-quart bottles
- one 1-gallon jug
- select objects to be measured (textbook, stapler, other items of your choice)

Amps | **Featured Activity**

Warm-up
Digital Card Sort

Students match various units to their corresponding attributes by dragging and connecting them on screen.



Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

At first, students may feel upset if they are not finding any patterns or structure in Activity 2 – that is the smaller the unit, the greater amount of it it will take to measure a given quantity. Encourage them to persist as they look for structure and have confidence in their thinking, which could be correct but not supported by incorrect measurements. Suggest students measure again, or allow a different group member to measure to check their results.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

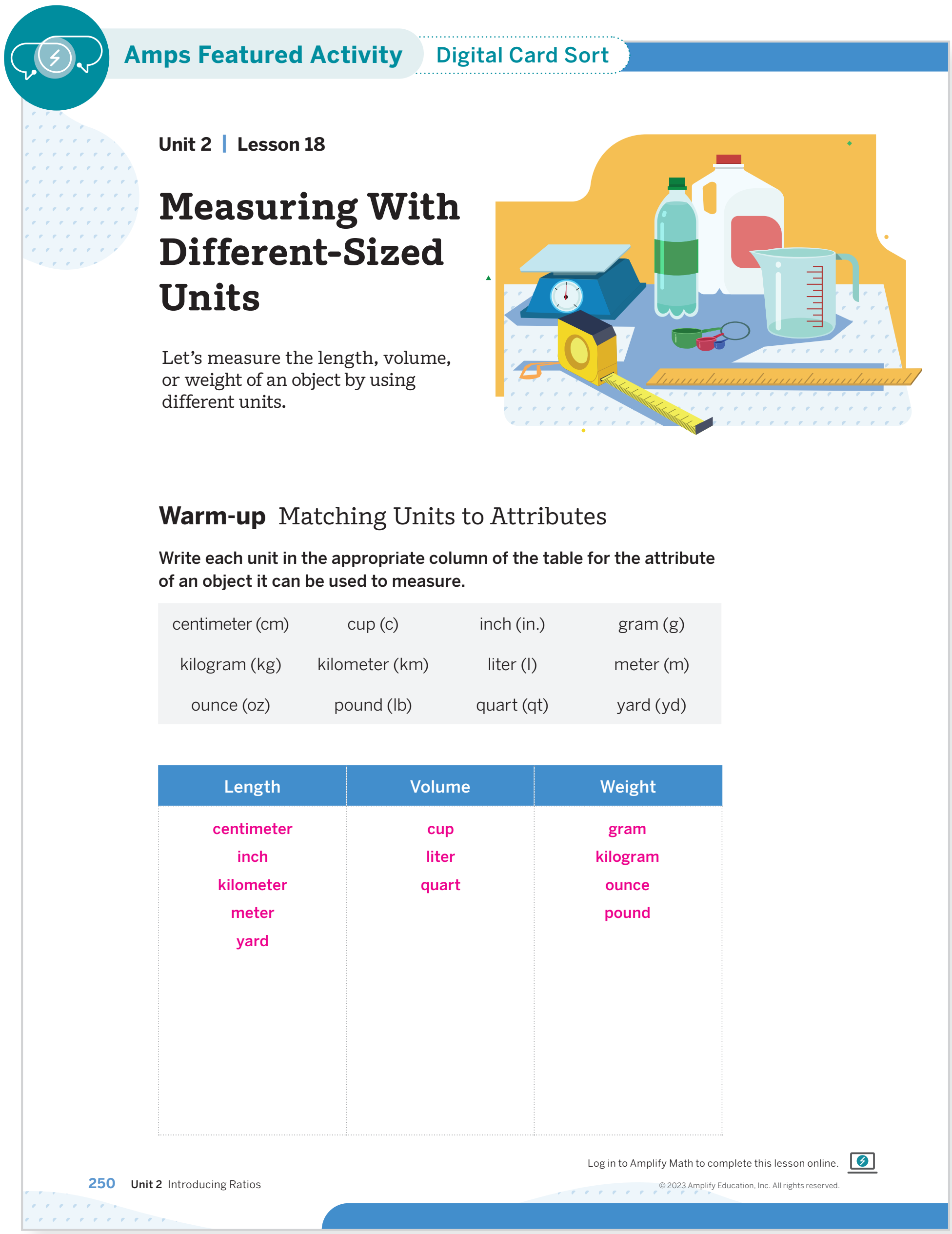
- Optional **Activity 1** may be omitted.
- In **Activity 2**, fewer objects may be weighed at Station 2, or you may choose to have groups interact with fewer physical stations.

Independent | 5 min

Warm-up Matching Units to Attributes

Students categorize units of measurement for length, volume, and weight to prepare for measurement conversion in subsequent activities.

Amps Featured Activity Digital Card Sort



Unit 2 | Lesson 18

Measuring With Different-Sized Units

Let's measure the length, volume, or weight of an object by using different units.

Warm-up Matching Units to Attributes

Write each unit in the appropriate column of the table for the attribute of an object it can be used to measure.

centimeter (cm)	cup (c)	inch (in.)	gram (g)
kilogram (kg)	kilometer (km)	liter (l)	meter (m)
ounce (oz)	pound (lb)	quart (qt)	yard (yd)

Length	Volume	Weight
centimeter inch kilometer meter yard	cup liter quart	gram kilogram ounce pound

Log in to Amplify Math to complete this lesson online.

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1 Launch

Consider creating pre-cut slips containing each unit for students to paste into the table.

2 Monitor

Help students get started by asking them to provide examples of different contexts where length, volume, and weight would be measured.

Look for points of confusion:

- **Writing units under the wrong category.** Give an example of an object that would be measured by using the given unit.
- **Writing ounces in the volume category.** Clarify that ounces here refers to weight, as opposed to *fluid ounces*, which refers to volume.

Look for productive strategies:

- Writing each unit only once in the correct column, and possibly being able to provide an example of how each unit would be used.

3 Connect

Have students share under which attribute category each unit was placed, one unit at a time.

Ask, "What are some other units of length, weight and volume that are not included in the list? In which categories would they be?" Sample responses: Pint (volume), tons (weight), millimeter (length)

Highlight that different units can be used to measure the same real-world object depending on whether students are measuring its length, weight, or volume. While each unit is used for a particular attribute, an object can be measured with different units within that category depending on which is needed or more appropriate (e.g., a car would be measured in tons, but a cracker would be measured in grams).

Power-up

To power up students' ability to categorize or identify appropriate units to measure length, weight, or volume, have students complete:

Match each unit of measure with the object being measured.

__a__ The volume of a container of milk.

__c__ The length of a fence.

__b__ The amount of baking soda in a recipe.

a. gallons

b. teaspoons

c. meters

Use: Before Activity 1.

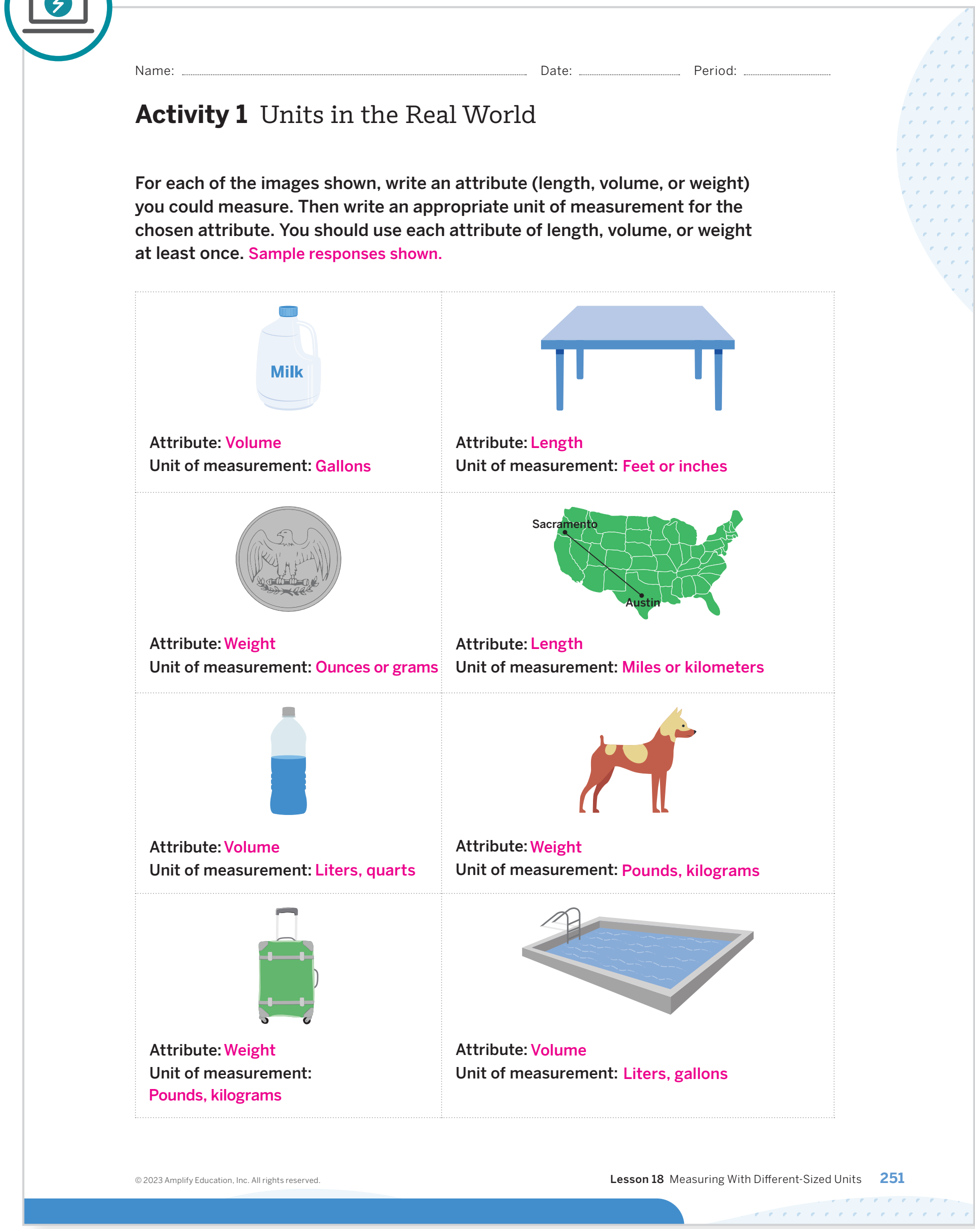
Informed by: Performance on Lesson 17, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 7 and 8.

Optional

Pairs | 10 min

Activity 1 Units in the Real World

Students assign one attribute (length, volume, or weight) and an appropriate unit of measure for each image, preparing them for the hands-on experiments in Activity 2.



Name: ______ Date: ______ Period: ______

Activity 1 Units in the Real World

For each of the images shown, write an attribute (length, volume, or weight) you could measure. Then write an appropriate unit of measurement for the chosen attribute. You should use each attribute of length, volume, or weight at least once. Sample responses shown.

Attribute: Volume Unit of measurement: Gallons	Attribute: Length Unit of measurement: Feet or inches
Attribute: Weight Unit of measurement: Ounces or grams	Attribute: Length Unit of measurement: Miles or kilometers
Attribute: Volume Unit of measurement: Liters, quarts	Attribute: Weight Unit of measurement: Pounds, kilograms
Attribute: Weight Unit of measurement: Pounds, kilograms	Attribute: Volume Unit of measurement: Liters, gallons

1 Launch

Ensure students understand the meaning of the term *attribute*. Note that there can be more than one correct answer for each object. Then have pairs use the ***Take Turns*** routine to complete the table. At the end of the activity, have each pair share with another pair.

2 Monitor

Help students get started by having them clarify what each image is or having them start with a familiar image.

Look for points of confusion:

- **Providing an incorrect unit for a chosen attribute.** Refer back to the Warm-up chart. If applicable, consider asking, "Which of your responses makes more sense to you, the attribute or the unit?"

Look for productive strategies:

- Providing an appropriate unit for each attribute (such as measuring the distance on the map in feet, as opposed to inches) and being able to justify why each specific unit was chosen.

3 Connect

Have partners share responses with other sets of partners.

Ask:

- "Why is it possible for the images to have more than one answer?" Different contexts require different information
- "What is another object that you could measure the [volume/weight/length] of, and what unit would be appropriate?"

Highlight that different units are used to measure different attributes of an object. While there are different units, some are more appropriate than others and the context should be taken into account to get the most accurate and helpful measurement.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students choose four of the images for which to write an attribute and appropriate unit of measurement. Allowing them to choose the images will help promote a greater sense of participation and ownership. Consider also chunking this task into smaller, more manageable parts by displaying one image at a time for students to complete.

Extension: *Math Enrichment*

Have students estimate a measurement for some or all of the images presented and include an explanation that supports their estimated measurement.

Math Language Development

MLR3: Critique, Correct, Clarify

Show a card with a conceptual error, such as attributing volume to measure the table. Ask students to identify and analyze the error, and write a justification of the revision. This will help students understand the differences between the attributes of length, volume, and weight.

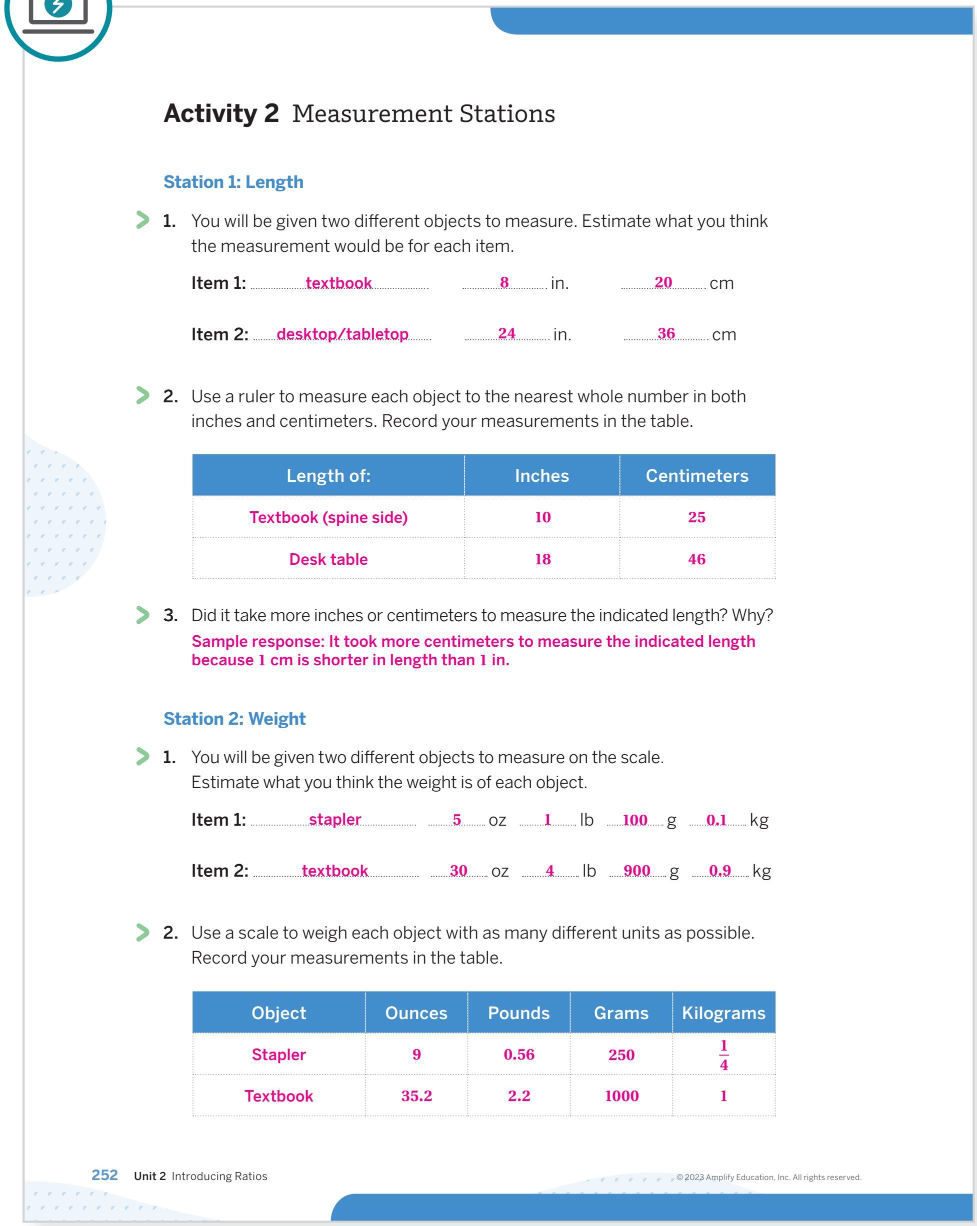
English Learners

Provide time for students to give and receive feedback in order to clarify their revised justification.

Small Groups | 30 min

Activity 2 Measurement Stations

Students experiment with measuring different attributes of various objects by using different units to relate the unit size to the measurement.



Activity 2 Measurement Stations

Station 1: Length

1. You will be given two different objects to measure. Estimate what you think the measurement would be for each item.

Item 1: textbook 8 in. 20 cm

Item 2: desktop/tabletop 24 in. 36 cm

2. Use a ruler to measure each object to the nearest whole number in both inches and centimeters. Record your measurements in the table.

Length of:	Inches	Centimeters
Textbook (spine side)	10	25
Desk table	18	46

3. Did it take more inches or centimeters to measure the indicated length? Why?
Sample response: It took more centimeters to measure the indicated length because 1 cm is shorter in length than 1 in.

Station 2: Weight

1. You will be given two different objects to measure on the scale. Estimate what you think the weight is of each object.

Item 1: stapler 5 oz 1 lb 100 g 0.1 kg

Item 2: textbook 30 oz 4 lb 900 g 0.9 kg

2. Use a scale to weigh each object with as many different units as possible. Record your measurements in the table.

Object	Ounces	Pounds	Grams	Kilograms
Stapler	9	0.56	250	$\frac{1}{4}$
Textbook	35.2	2.2	1000	1

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1 Launch

Arrange students in groups of 3–4 and explain each station, as well as the protocol for rotating.

- **Station 1 (length):** Provide access to two objects such as a textbook and a desk. Specify whether they can measure any length or should measure a certain length. Have students measure the same length in both centimeters and inches and record their findings.
- **Station 2 (weight):** Provide two objects that are noticeably different in weight, such as a stapler and a textbook, that students can weigh by using a scale. If the scale does not provide all four units listed in the Student Edition, have students weigh the objects in whatever units are available and modify the chart as needed.
- **Station 3 (volume):** Provide a gallon container filled with water, four empty clear (labeled) quart bottles, and four empty clear (labeled) liter bottles. Consider coloring the water with a few drops of blue food coloring. Have students determine the numbers of liters and quarts equivalent to a gallon by pouring the water from the larger container to the smaller ones. Alternatively, students can watch the *Pouring Water* video.

2 Monitor

Help students get started by asking, "What will you be measuring? How will you do it?"

Look for points of confusion:

- **Struggling to estimate measurements.** Ask, "What is something you know the [length, weight, volume] of? Do you think this measurement would be more or less?"
- **Having difficulty choosing or maintaining one level of precision.** Clarify that it is okay to round or estimate, but it should be done consistently.

Look for productive strategies:

- Noticing that if students measure the same quantity with different units, it will take more of the smaller unit and less of the larger unit to express the measurement.

Activity 2 continued ›

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

Instead of having students measure, provide sample measurements for several objects as well as displaying how long an inch is compared to a centimeter for Problems 1–3. The goal of these problems is for students to notice that 1 cm is shorter than 1 in., which is why it takes more centimeters to measure the length of an object than inches.

Math Language Development

MLR8: Discussion Supports — Press for Reasoning

Encourage students to solidify their understanding by asking peers to elaborate on their responses during the Connect. Provide sentence frames, such as:

- "I agree/disagree because . . ."
- "How do you know . . ."
- "Can you give an example?"

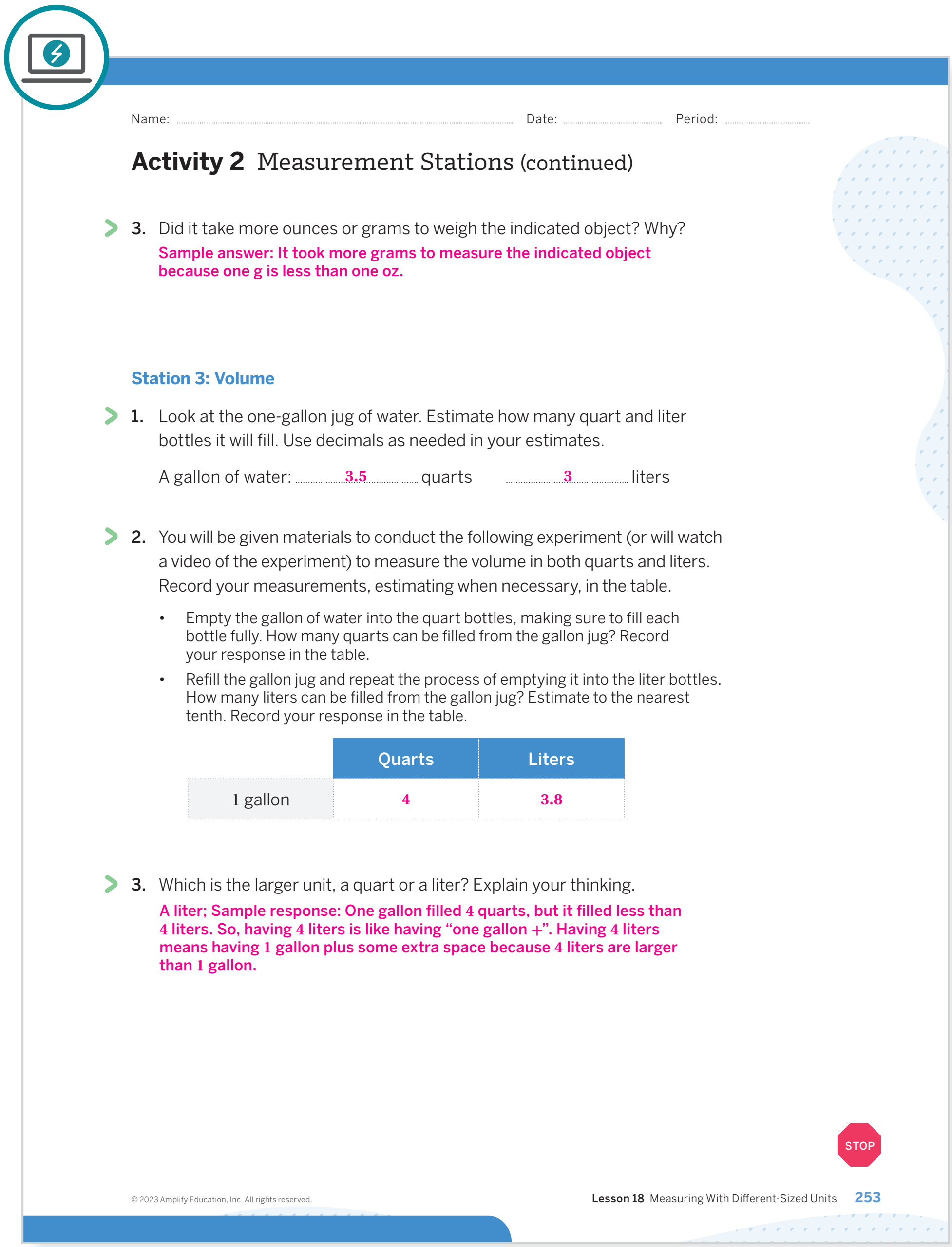
English Learners

Ask students who are more familiar with metric measures to share their experiences as they navigate daily life in the U.S., where Customary measures are more common.

Small Groups | 30 min

Activity 2 Measurement Stations (continued)

Students experiment with measuring different attributes of various objects by using different units to relate the unit size to the measurement.



Name: ______ Date: ______ Period: ______

Activity 2 Measurement Stations (continued)

3. Did it take more ounces or grams to weigh the indicated object? Why?
Sample answer: It took more grams to measure the indicated object because one g is less than one oz.

Station 3: Volume

1. Look at the one-gallon jug of water. Estimate how many quart and liter bottles it will fill. Use decimals as needed in your estimates.

A gallon of water: **3.5** quarts **3** liters

2. You will be given materials to conduct the following experiment (or will watch a video of the experiment) to measure the volume in both quarts and liters. Record your measurements, estimating when necessary, in the table.

- Empty the gallon of water into the quart bottles, making sure to fill each bottle fully. How many quarts can be filled from the gallon jug? Record your response in the table.
- Refill the gallon jug and repeat the process of emptying it into the liter bottles. How many liters can be filled from the gallon jug? Estimate to the nearest tenth. Record your response in the table.

	Quarts	Liters
1 gallon	**4**	**3.8**

3. Which is the larger unit, a quart or a liter? Explain your thinking.
A liter; Sample response: One gallon filled 4 quarts, but it filled less than 4 liters. So, having 4 liters is like having “one gallon +”. Having 4 liters means having 1 gallon plus some extra space because 4 liters are larger than 1 gallon.

STOP

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Lesson 18 Measuring With Different-Sized Units 253

3 Connect

Have students share their responses, noting whether any of their findings were surprising and why. Allow students to also ask their classmates questions about their processes and results.

Ask:

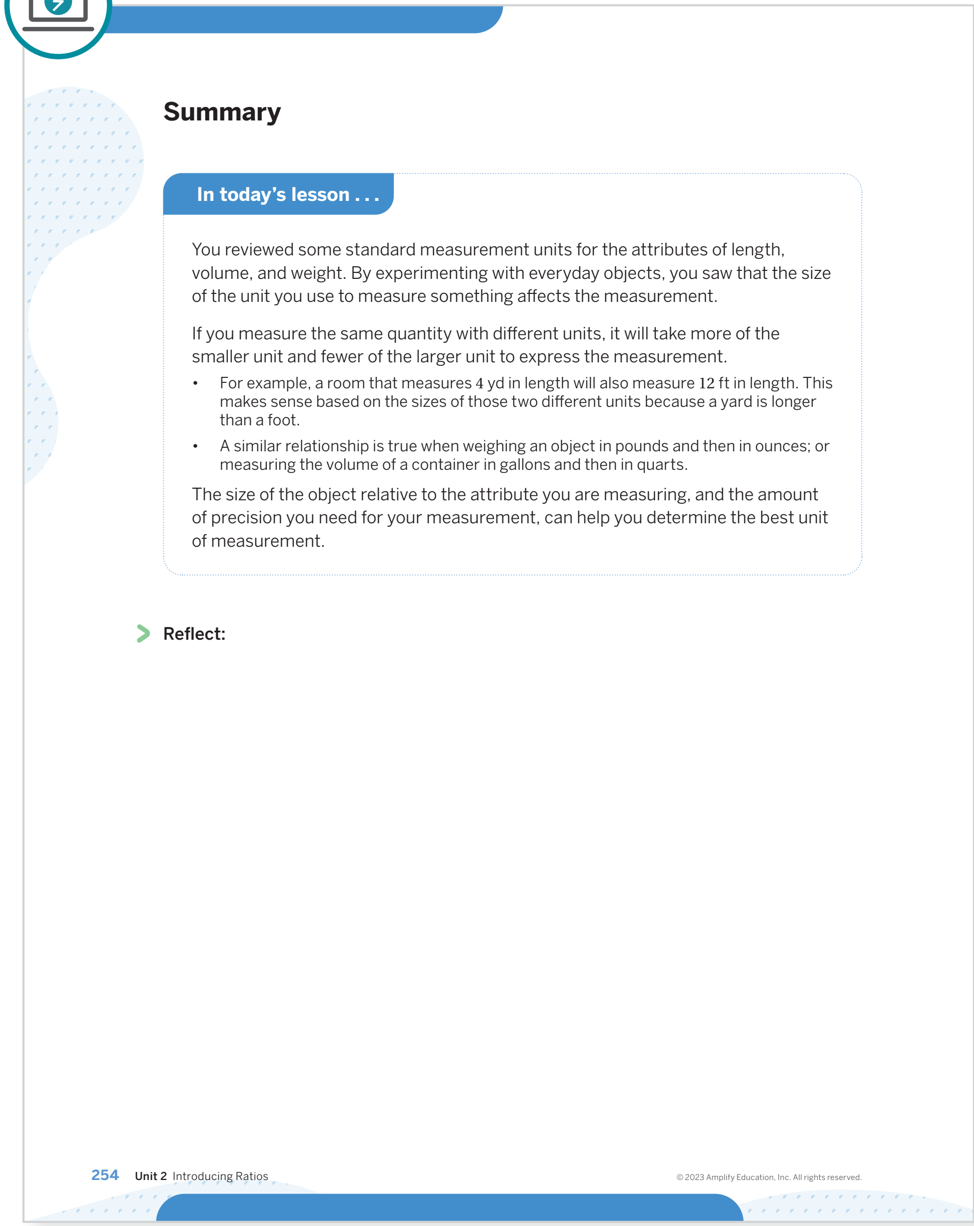
- “Why might choosing a larger or smaller unit be better for measuring something?” If the object’s measure is relatively large or small, then choosing a unit that is too small or too large would give really large or really small values, and those might not be very helpful for understanding or comparing with other measurements.
- (optional) “Did anyone notice any patterns among the values for the different units in any of your tables of results?” Sample response: The number of centimeters was about 2.5 times the number of inches for the measured lengths of both objects.

Highlight that different countries use different systems of measurement (e.g., metric and U.S. Customary). In the metric system, some units that are used are grams, meters, and liters. In the U.S. Customary System, some units that are used are ounces, inches, and gallons. Regardless of the measurement system or systems being used, it will always take more of the smaller unit than the larger unit to measure an attribute of a given object.

Whole Class | 5 min

Summary

Review and synthesize the idea that different units of measurement can be used for different objects and scenarios.



Summary

In today's lesson . . .

You reviewed some standard measurement units for the attributes of length, volume, and weight. By experimenting with everyday objects, you saw that the size of the unit you use to measure something affects the measurement.

If you measure the same quantity with different units, it will take more of the smaller unit and fewer of the larger unit to express the measurement.

- For example, a room that measures 4 yd in length will also measure 12 ft in length. This makes sense based on the sizes of those two different units because a yard is longer than a foot.
- A similar relationship is true when weighing an object in pounds and then in ounces; or measuring the volume of a container in gallons and then in quarts.

The size of the object relative to the attribute you are measuring, and the amount of precision you need for your measurement, can help you determine the best unit of measurement.

Reflect:

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Synthesize

Ask,

- "When might you want to know the length, weight or volume of an object?" See Activity 1 for examples.
- "How is knowing that different units give different measures helpful or important when measuring objects?" I might need to know a measurement in a specific unit; I might only have a measuring device that shows a particular unit; Different countries use different units.
- How do you think the experiments and results from today relate to ratios?" You can use the conversion ratio between units to determine larger/smaller amounts.

Highlight that the work students recorded today represents "experimental values" for measurements, and some human error could have been involved or at certain points they may have needed to estimate or round, even when using a measuring device. Since every standard unit of measure for any attribute represents a precise size, the relationship between the sizes of two units can be used to determine the exact measurement of an object in one unit if it is known in a different unit, which will be done by using ratios in the next lesson.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

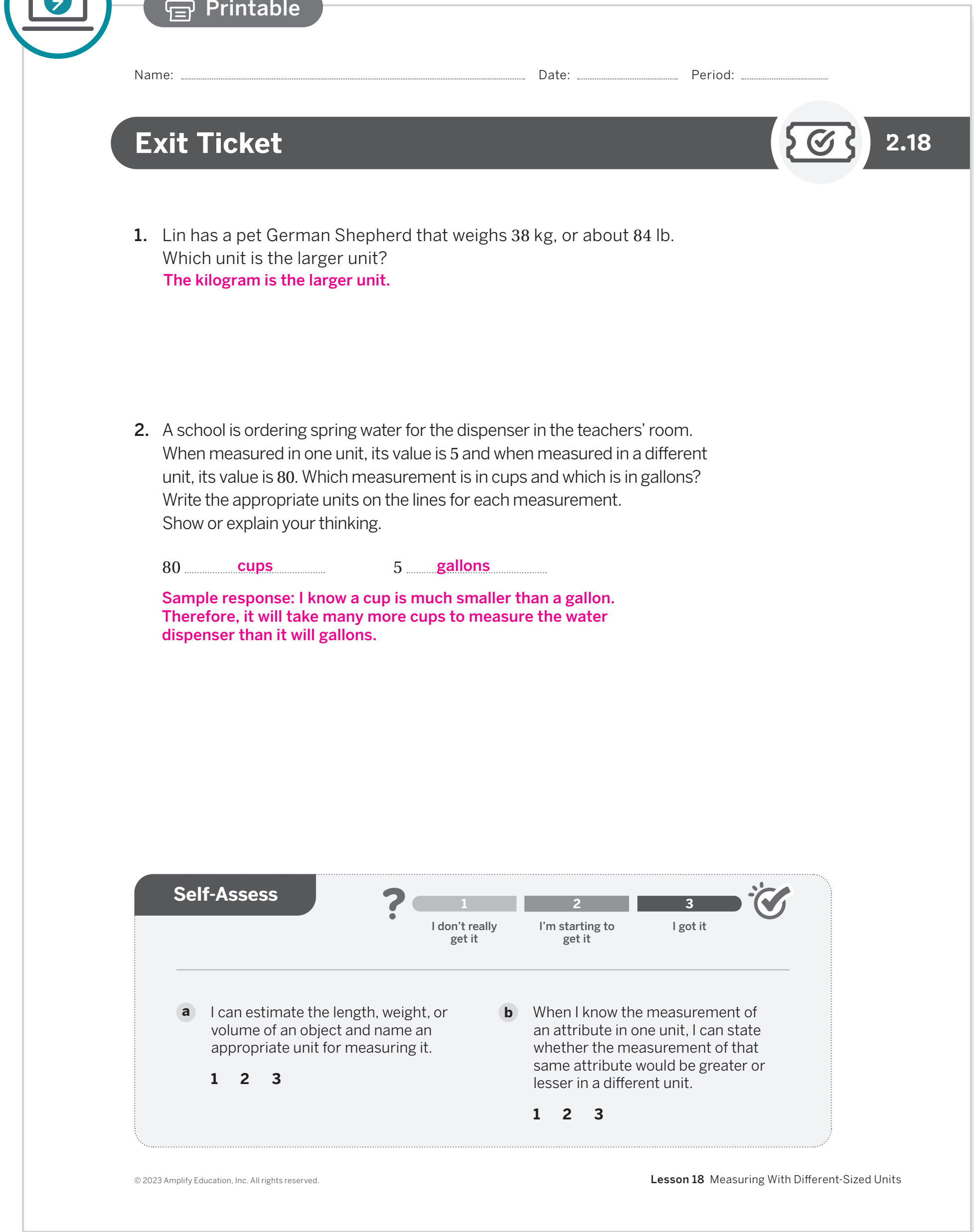
- "How does using different units affect the outcome of measuring the same object?"
- "What is important to consider when deciding the unit to measure an object?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by thinking through real-world examples and deciding the correct units of measurements.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 2.18

1. Lin has a pet German Shepherd that weighs 38 kg, or about 84 lb. Which unit is the larger unit?
   The kilogram is the larger unit.

2. A school is ordering spring water for the dispenser in the teachers' room. When measured in one unit, its value is 5 and when measured in a different unit, its value is 80. Which measurement is in cups and which is in gallons? Write the appropriate units on the lines for each measurement. Show or explain your thinking.

   80 **cups** 5 **gallons**

   Sample response: I know a cup is much smaller than a gallon. Therefore, it will take many more cups to measure the water dispenser than it will gallons.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can estimate the length, weight, or volume of an object and name an appropriate unit for measuring it.

1 2 3

b. When I know the measurement of an attribute in one unit, I can state whether the measurement of that same attribute would be greater or lesser in a different unit.

1 2 3

Lesson 18 Measuring With Different-Sized Units

Success looks like . . .

- **Language Goal:** Generalizing that it takes more of a smaller unit or fewer of a larger unit to measure the same quantity. **(Speaking and Listening, Writing)**
  - » Understanding that it takes less kilograms than pounds to represent the weight of German Shepherd in Problem 1.
- **Language Goal:** Given a measurement in one unit, estimating what would be the same amount expressed in a different unit, and explaining the reasoning. **(Speaking and Listening)**

Suggested next steps

If students confuse pounds and kilograms in Problem 1, consider:

- Asking, "Which weighs more, one kilogram or one pound?"
- Reviewing Activity 2, showing that it takes more of a smaller unit than a larger unit to weigh the same object. Then ask, "How could that help you here?"
- Assigning Practice Problem 3.

If students confuse gallons and cups in Problem 2, consider:

- Asking, "Which is the smaller unit? How does that help you determine the answer?"
- Showing the gallon jug from Activity 2 and asking, "What is something you know that is measured in cups?"

Professional Learning

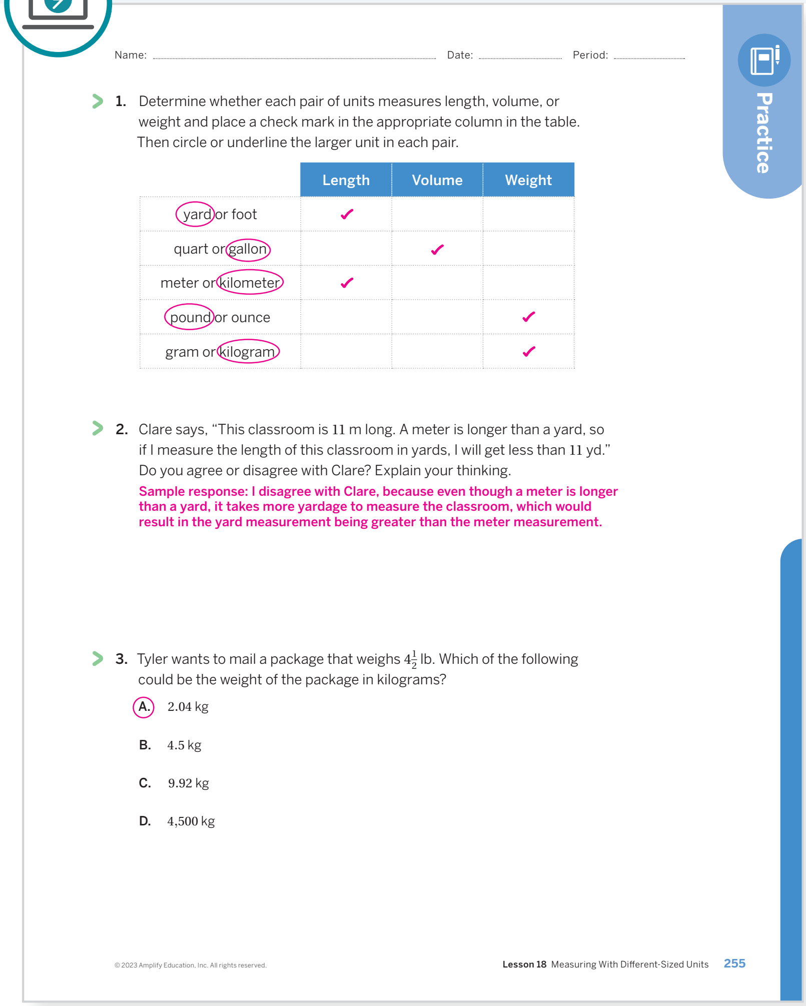
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did Activity 2 change or illustrate what you know about your students as learners?
- How do you think your students are progressing in their understanding of how the unit size and type affect the measurement? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Determine whether each pair of units measures length, volume, or weight and place a check mark in the appropriate column in the table. Then circle or underline the larger unit in each pair.

	Length	Volume	Weight
[✓ yard] or foot	✓		
quart or [✓ gallon]		✓	
meter or [✓ kilometer]	✓		
[✓ pound] or ounce			✓
gram or [✓ kilogram]			✓

2. Clare says, "This classroom is 11 m long. A meter is longer than a yard, so if I measure the length of this classroom in yards, I will get less than 11 yd." Do you agree or disagree with Clare? Explain your thinking.
Sample response: I disagree with Clare, because even though a meter is longer than a yard, it takes more yardage to measure the classroom, which would result in the yard measurement being greater than the meter measurement.

3. Tyler wants to mail a package that weighs $4\frac{1}{2}$ lb. Which of the following could be the weight of the package in kilograms?

[✓ A.] 2.04 kg

B. 4.5 kg

C. 9.92 kg

D. 4,500 kg

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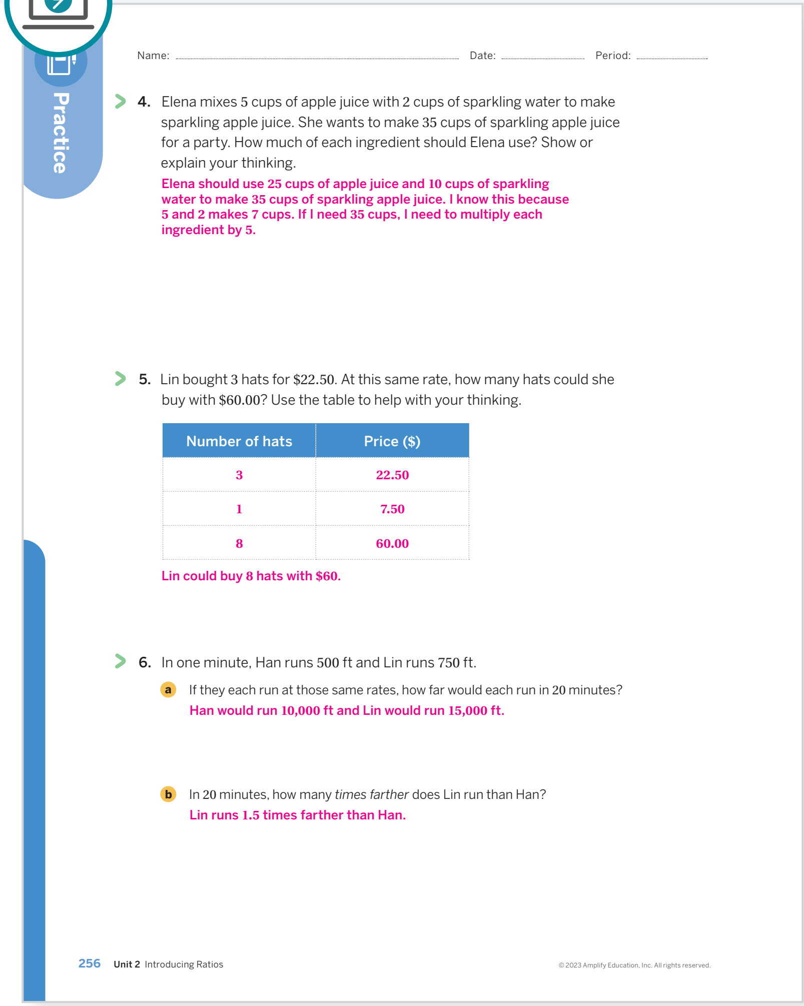
Lesson 18 Measuring With Different-Sized Units 255



Name: Date: Period:

Practice

4. Elena mixes 5 cups of apple juice with 2 cups of sparkling water to make sparkling apple juice. She wants to make 35 cups of sparkling apple juice for a party. How much of each ingredient should Elena use? Show or explain your thinking.
Elena should use 25 cups of apple juice and 10 cups of sparkling water to make 35 cups of sparkling apple juice. I know this because 5 and 2 makes 7 cups. If I need 35 cups, I need to multiply each ingredient by 5.

5. Lin bought 3 hats for $22.50. At this same rate, how many hats could she buy with $60.00? Use the table to help with your thinking.

Number of hats	Price ($)
3	22.50
1	7.50
8	60.00

Lin could buy 8 hats with $60.

6. In one minute, Han runs 500 ft and Lin runs 750 ft.

a. If they each run at those same rates, how far would each run in 20 minutes?
Han would run 10,000 ft and Lin would run 15,000 ft.

b. In 20 minutes, how many *times farther* does Lin run than Han?
Lin runs 1.5 times farther than Han.

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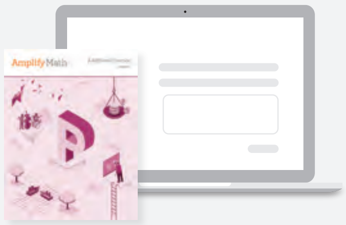
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up, Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 15	2
	5	Unit 2 Lesson 11	2
Formative ⚡	6	Unit 2 Lesson 19	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

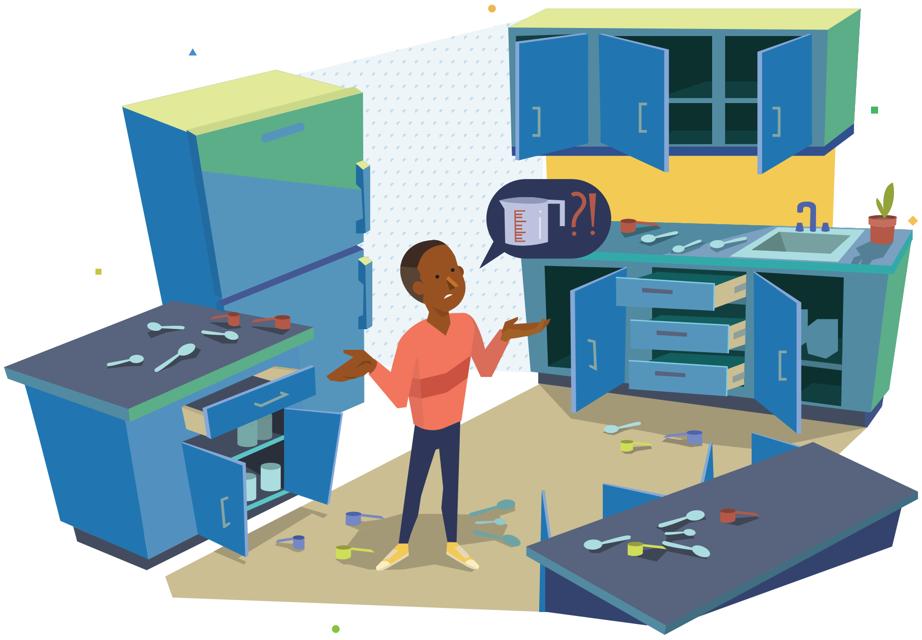


For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 2 | **LESSON 19**

Converting Units

Let's convert measurements to different units.



Focus

Goals

1. Choose and create a double number line diagram or a table to solve problems involving unit conversion.
2. **Language Goal:** Explain how to use a "rate per 1" to solve problems involving unit conversion. **(Speaking and Listening)**
3. Recognize that when two or more things are measured in the same two different units, the pairs of measurements are equivalent ratios.

Coherence

Today

Students progress to converting units that may be in different systems of measurement, using ratio reasoning and their choice of representations and strategies, such as double number lines, tables, or multiplication or division to determine equivalent ratios and missing values. They practice these skills, checking for accuracy, and think about how to use different tools in some real-world scenarios of measuring out recipe ingredients.

Previously

In Grades 4 and 5, students began converting units of measurements by multiplying and dividing, but only using units within the same measurement system. In Lesson 18 of this unit, students measured the same objects by using different units to see that the same measurement takes more of a smaller unit and less of a larger unit, preparing them to relate those differences as common factors in equivalent ratios.

Coming Soon

In Lesson 20, students will revisit Fermi problems from Lesson 1, now fully equipped to reason about them using ratio reasoning.

Rigor

- Students develop **procedural fluency** to convert between units.
- Students **apply** equivalent ratios to converting measurements.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

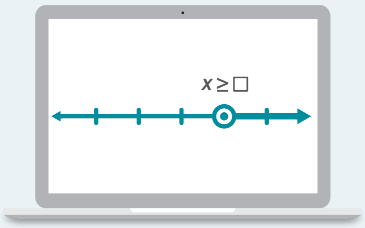
- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair
- calculators

Amps | Featured Activity

Activities 1

Digital Double Number Lines

Students can easily create and manipulate double number line diagrams to help guide their thinking.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel confused about the significance of knowing how to convert measurements between systems or within the same system as they struggle to reason through conversions. Have them engage in metacognitive functions by asking them to ask themselves, "Why are conversion strategies important? Why might one way not be able to be used all the time? How have I been able to overcome difficulties and mental blocks like this in the past?"

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have pairs divide the workload so each converts only half of the ingredients in Problem 1.
- In **Activity 2**, have pairs divide the workload so each converts only half of the ingredients, making sure however that both students do some volumes (milliliters) and some weights (grams).

Independent | 5 min

Warm-up Road Trip

Students analyze a speed limit sign and begin to think about why units are important and how the same quantity can take on different values in different units.

Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 19

Converting Units

Let's convert measurements to different units.

Warm-up Road Trip

Elena and her mom are visiting England from the United States. One day, Elena's mom was driving on the highway at a speed of 60 miles per hour when she got pulled over by the police for speeding. Outside the car, Elena noticed this road sign.

1. What do you think happened?
   Sample response: The speed limit sign is 80 km/h, not miles per hour. A kilometer is a shorter distance than a mile so Elena's mom was actually going above the speed limit.

2. Where else might Elena and her mom run into similar issues while they are exploring England?
   Sample response: They might go to the grocery store and think that fruits and vegetables are weighed by the pound, which are actually weighed in grams or kilograms.

Log in to Amplify Math to complete this lesson online.

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Lesson 19 Converting Units 257

1 Launch

Ensure students understand the term *maximum*. Set an expectation for the amount of time students will have to work independently on Problems 1 and 2.

2 Monitor

Help students get started by asking, "Have you ever seen a sign like this? What does the number 80 represent?"

Look for points of confusion:

- **Assuming measurement units are the same in all countries.** Highlight some of the different, related units of the metric and U.S. Customary systems and explain that most of the world uses the metric system.
- **Thinking only about speed for Problem 2, not knowing to switch attributes to weight, capacity, etc.** Ask, "What else might Elena and her mom do in England that might involve measurements?"

Look for productive strategies:

- Recognizing the importance of attributing units when representing information, and considering those when making claims or performing operations.

3 Connect

Have students share their responses to Problems 1 and 2.

Highlight that there are many different units that can be used to measure the same attribute of an object, and there are different systems of measurement used to do so. Students will see in the next activities how conversions using ratios can always be made to go between different units.

Ask, "Why else, or in what other context might it be important to pay attention to different units?"

Math Language Development

MLR7: Compare and Connect

Ask students to share their approaches to determine whether or not Elena's mom was speeding. This will help students reason about when it is important to convert one unit of measure to another.

English Learners

If you have not already done so in Lesson 18, Activity 2, ask students who are more familiar with metric measures to share their experiences as they navigate daily life in the U.S., where Customary measures are more common.

Power-up

To power up students' ability to coordinate units of time and distance to compare rates, have students complete:

In 10 minutes, Clare walked 12 blocks and Jada walked 10 blocks.

1. How many blocks did each of them walk in 1 min?
   Clare: 1.2 blocks
   Jada: 1 block
2. How many times farther did Clare walk than Jada?
   1.2 times farther

Use: Before Activity 1.

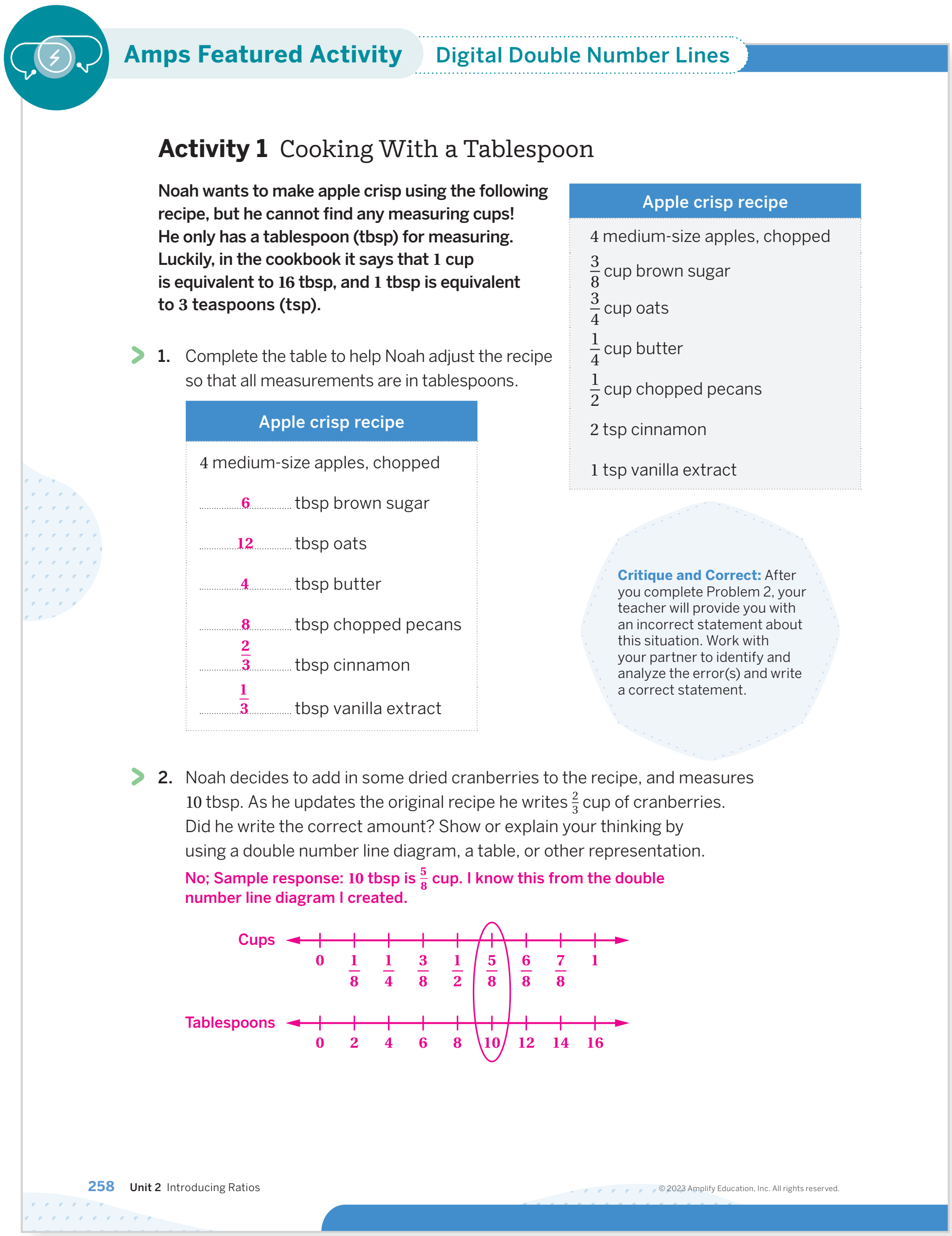
Informed by: Performance on Lesson 18, Practice Problem 6.

Pairs | 15 min

Activity 1 Cooking With a Tablespoon

Students relate measurement conversions within the same measurement system (cups and tablespoons) to equivalent ratios.

Amps Featured Activity Digital Double Number Lines



Activity 1 Cooking With a Tablespoon

Noah wants to make apple crisp using the following recipe, but he cannot find any measuring cups! He only has a tablespoon (tbsp) for measuring. Luckily, in the cookbook it says that 1 cup is equivalent to 16 tbsp, and 1 tbsp is equivalent to 3 teaspoons (tsp).

Apple crisp recipe

4 medium-size apples, chopped
$\frac{3}{8}$ cup brown sugar
$\frac{3}{4}$ cup oats
$\frac{1}{4}$ cup butter
$\frac{1}{2}$ cup chopped pecans
2 tsp cinnamon
1 tsp vanilla extract

1. Complete the table to help Noah adjust the recipe so that all measurements are in tablespoons.

Apple crisp recipe

4 medium-size apples, chopped
6 tbsp brown sugar
12 tbsp oats
4 tbsp butter
8 tbsp chopped pecans
$\frac{2}{3}$ tbsp cinnamon
$\frac{1}{3}$ tbsp vanilla extract

Critique and Correct: After you complete Problem 2, your teacher will provide you with an incorrect statement about this situation. Work with your partner to identify and analyze the error(s) and write a correct statement.

2. Noah decides to add in some dried cranberries to the recipe, and measures 10 tbsp. As he updates the original recipe he writes $\frac{2}{3}$ cup of cranberries. Did he write the correct amount? Show or explain your thinking by using a double number line diagram, a table, or other representation.

No; Sample response: 10 tbsp is $\frac{5}{8}$ cup. I know this from the double number line diagram I created.

258 Unit 2 Introducing Ratios

1 Launch

Use the *Think-Pair-Share* routine to have students work together on the problems.

2 Monitor

Help students get started by asking, "What information do you need to know? What information do you know? How can you use that?"

Look for points of confusion:

- **Saying 0 tbsp for cinnamon and vanilla extract because they are less than one tablespoon.** Ask, "Could a fraction of a tablespoon be used? How could you determine that fraction?"
- **Having trouble explaining the conversion method used for Problem 2.** Ask, "How might you use a double number line or table?"

Look for productive strategies:

- Recognizing when a measurement would be less than a tablespoon and when it would be more.
- Writing the given conversions as ratios containing a 1 and using those to determine necessary equivalent ratios.

3 Connect

Have students share their conversion strategies, focusing on when they converted smaller units to larger units and vice versa. Record examples as it is helpful.

Ask, "How did you use ratios specifically in your conversions?"

Highlight that within the same measurement system, it is generally true that each larger unit corresponds to a whole number of smaller units. If students can determine a ratio containing a 1, they can then set up equivalent ratios to determine larger or smaller amounts of a given quantity, even when those values themselves might be fractional or decimal amounts.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Consider bringing in a set of measuring cups that show how 1 cup, 1 tbsp, and 1 tsp compare in size to one another. Consider demonstrating, using water or another substance, how 1 tbsp is equivalent to 3 tsp, and how 1 cup is equivalent to 16 tbsp.

Extension: *Math Enrichment*

Have students complete the following problem:

How could you adjust the table you created in Problem 1 so that the measurements in tablespoons for every ingredient are whole numbers? Triple the recipe.

Math Language Development

MLR3: Critique, Correct, Clarify

Present an incorrect solution and explanation. For example, "Noah used zero cups of cinnamon because 2 tsp is less than 1 tbsp." Ask students to critique the solution and reasoning, propose a corrected solution, and clarify the reasoning they use.

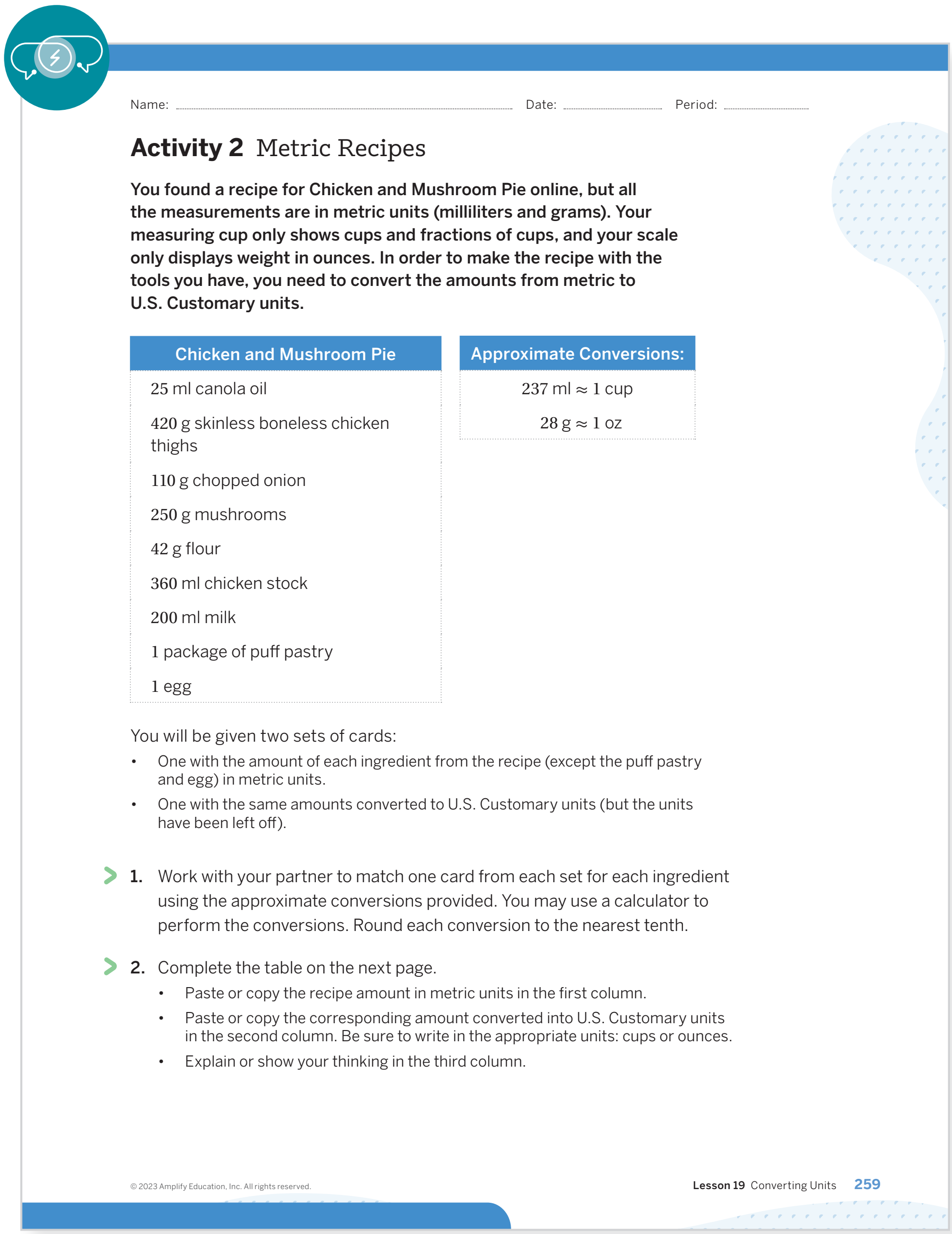
English Learners

Encourage students to refer to the class anchor chart to support their use of appropriate mathematical language in their improved response.

Pairs | 15 min

Activity 2 Metric Recipes

Students extend ratio reasoning to convert a recipe between measurement systems (metric and U.S. Customary), reinforcing their understanding of “how much per 1.”



Name: ____________ Date: ________ Period: ________

Activity 2 Metric Recipes

You found a recipe for Chicken and Mushroom Pie online, but all the measurements are in metric units (milliliters and grams). Your measuring cup only shows cups and fractions of cups, and your scale only displays weight in ounces. In order to make the recipe with the tools you have, you need to convert the amounts from metric to U.S. Customary units.

Chicken and Mushroom Pie
25 ml canola oil
420 g skinless boneless chicken thighs
110 g chopped onion
250 g mushrooms
42 g flour
360 ml chicken stock
200 ml milk
1 package of puff pastry
1 egg

Approximate Conversions:
237 ml ≈ 1 cup
28 g ≈ 1 oz

You will be given two sets of cards:

- One with the amount of each ingredient from the recipe (except the puff pastry and egg) in metric units.
- One with the same amounts converted to U.S. Customary units (but the units have been left off).

1. Work with your partner to match one card from each set for each ingredient using the approximate conversions provided. You may use a calculator to perform the conversions. Round each conversion to the nearest tenth.

2. Complete the table on the next page.
   - Paste or copy the recipe amount in metric units in the first column.
   - Paste or copy the corresponding amount converted into U.S. Customary units in the second column. Be sure to write in the appropriate units: cups or ounces.
   - Explain or show your thinking in the third column.

1 Launch

Keep students in the same pairs and distribute pre-cut cards from the Activity 2 PDF to every pair. Note that ounces is a unit of weight here, not volume (which would be in fluid ounces). Consider also discussing why some of the recipe measurements are in grams and some are in milliliters. Provide access to calculators as needed.

2 Monitor

Help students get started by having them choose an ingredient card and asking, “Which conversion values will you use? Can you set up a calculation?”

Look for points of confusion:

- **Focusing more on matching than converting.** Explain that using estimation can be helpful in some examples, and may narrow options, but calculations should be done to check or determine final actual matches.
- **Not knowing how to perform the conversions when values are not factors or multiples.** Ask, “Could you set up a ratio box for two equivalent ratios? What operation do you need to do?” Then remind them they can use a calculator.

Look for productive strategies:

- Using estimation strategies to eliminate unreasonably large or small amounts.
- Knowing which measurements to multiply and which to divide, and recognizing the same operation can be applied to every same type of conversion.
- Being able to convert amounts in multiple ways — using a calculator, setting up a double number line or table, or using mental math strategies (e.g., for 42 g of flour, I know that half of 28 is 14 and 28 + 14 = 42, so I think I would need 1.5 oz; I can use my calculator to check, 42 ÷ 28 = 1.5).

Activity 2 continued ›

Differentiated Support

Accessibility: *Activate Background Knowledge, Guide Processing and Visualization*

To illustrate the relationship between milliliters and cups, consider bringing in an empty liter bottle, a 1-cup measuring cup, and some water. Show how there are about $4\frac{1}{2}$ cups of water in 1 liter. Then display the following: 1 liter = 1,000 ml ≈ $4\frac{1}{4}$ cups. Because 1,000 ÷ 4 ≈ 235, and these numbers were rounded, it is reasonable that there are about 237 ml in 1 cup.

Accessibility: *Vary Demands to Optimize Challenge*

Assign each pair of students with the task of converting either the milliliters to cups, or the grams to ounces. Then have pairs of students, who each performed different conversions, share their conversions with each other.

Accessibility: *Optimize Access to Tools*

Consider providing copies of blank tables or blank double numbers lines to assist students as they perform the conversions.

Math Language Development

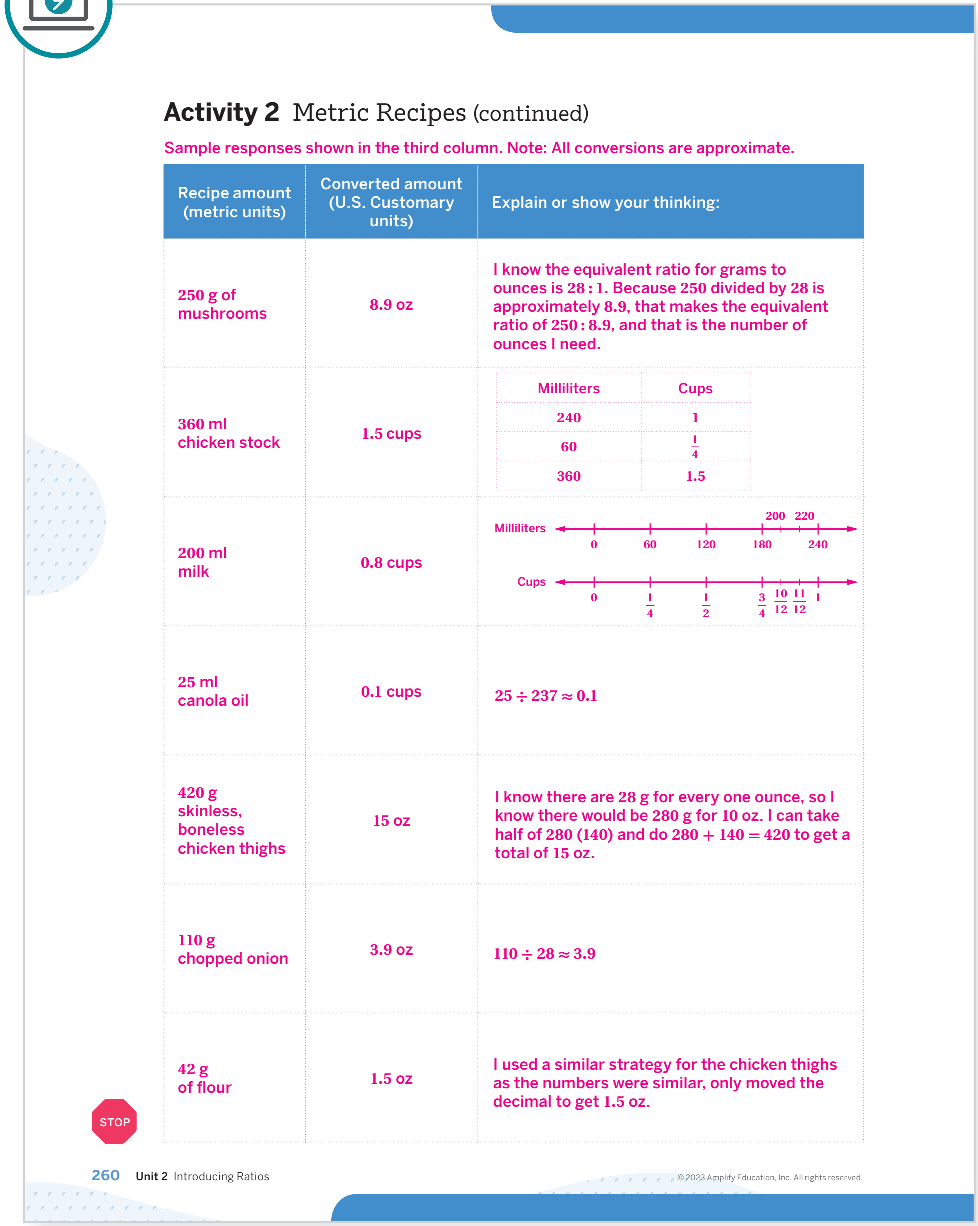
MLR2: Collect and Display

As students share the strategies they used to perform the conversions, listen for and scribe the words and phrases they use. Highlight key vocabulary that students use in the discussion, while continuing to refer to other representations to make sense of them as needed.

Pairs | 15 min

Activity 2 Metric Recipes (continued)

Students extend ratio reasoning to convert a recipe between measurement systems (metric and U.S. Customary), reinforcing their understanding of “how much per 1.”



Activity 2 Metric Recipes (continued)

Sample responses shown in the third column. Note: All conversions are approximate.

Recipe amount (metric units)	Converted amount (U.S. Customary units)	Explain or show your thinking:
250 g of mushrooms	8.9 oz	I know the equivalent ratio for grams to ounces is 28 : 1. Because 250 divided by 28 is approximately 8.9, that makes the equivalent ratio of 250 : 8.9, and that is the number of ounces I need.
360 ml chicken stock	1.5 cups	Milliliters | Cups 240 | 1 60 | $\frac{1}{4}$ 360 | 1.5
200 ml milk	0.8 cups	Milliliters: 0, 60, 120, 180, 200, 220, 240 Cups: 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{10}{12}$, $\frac{11}{12}$, 1
25 ml canola oil	0.1 cups	$25 \div 237 \approx 0.1$
420 g skinless, boneless chicken thighs	15 oz	I know there are 28 g for every one ounce, so I know there would be 280 g for 10 oz. I can take half of 280 (140) and do $280 + 140 = 420$ to get a total of 15 oz.
110 g chopped onion	3.9 oz	$110 \div 28 \approx 3.9$
42 g of flour	1.5 oz	I used a similar strategy for the chicken thighs as the numbers were similar, only moved the decimal to get 1.5 oz.

STOP

260 Unit 2 Introducing Ratios

3 Connect

Display a blank table for showing correct matches.

Have students share one match at a time and the strategies or representations they used to perform the conversions. If time allows, have others share different thinking or representations for the same result.

Ask:

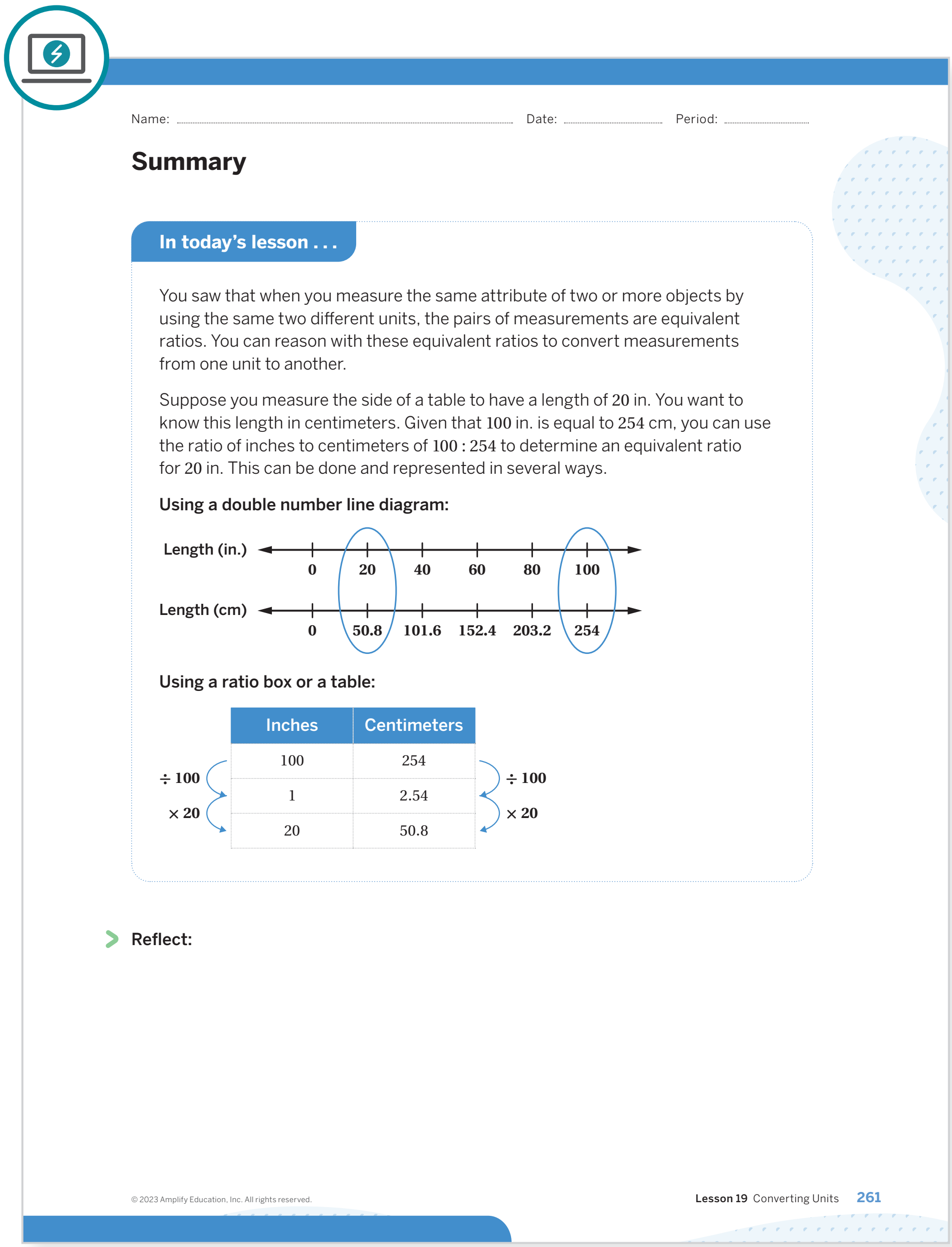
- “Did you use the same conversion strategy for each ingredient? Why or why not?”
- “How do you know whether to multiply or divide?” I used the ratios containing a 1 to see which quantity matched the 1 and compared that to what I was given and needed to know.

Highlight that the conversions given were ratios containing a 1 telling you “how much per 1,” which students have seen are a useful tool in determining any equivalent ratio. However in this case, the ratios did not have a 1 corresponding to the same (metric) units that were given in the recipe, so students could not just multiply to determine the equivalent conversions. All of the tools and strategies developed in this unit could be helpful in visualizing the relationships, and once students determined the calculation necessary (division, or multiplication by a unit fraction or decimal), then the same calculation could be used for every conversion between the same two units.

Whole Class | 5 min

Summary

Review and synthesize how using a given conversion for "how much per 1" to write equivalent ratios relates to converting between same or different system measurements.



Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that when you measure the same attribute of two or more objects by using the same two different units, the pairs of measurements are equivalent ratios. You can reason with these equivalent ratios to convert measurements from one unit to another.

Suppose you measure the side of a table to have a length of 20 in. You want to know this length in centimeters. Given that 100 in. is equal to 254 cm, you can use the ratio of inches to centimeters of 100 : 254 to determine an equivalent ratio for 20 in. This can be done and represented in several ways.

Using a double number line diagram:

Length (in.)	0	20	40	60	80	100
Length (cm)	0	50.8	101.6	152.4	203.2	254

Using a ratio box or a table:

Inches	Centimeters
100	254
1	2.54
20	50.8

÷ 100

× 20

÷ 100

× 20

Reflect:

Synthesize

Display the tables with inches and centimeters from the Summary.

Ask:

- "How does knowing 'how much per 1' help you convert between units of measurement?" I can divide or multiply 1 : 2.54 depending if I need a larger or smaller amount.
- "How do the pairs of numbers in the table represent equivalent ratios? How can you use equivalent ratios to convert between units of measurement?" Use the ratio of 1 : 2.54 to multiply by 100 to get 254 and by 20 to get 50.8
- "Are any of the conversion strategies you saw today more efficient? Less efficient? Explain."

Highlight that two measurements of the same object in different units form equivalent ratios, and students can use all of the familiar tools (tables, double number line diagrams) when thinking about converting units of measure. If students know a rate of "how much per 1" that relates the two units, they can use it to convert one measurement to the other by multiplication or division, regardless of the values or whether the units come from the same measurement system or different measurement systems.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

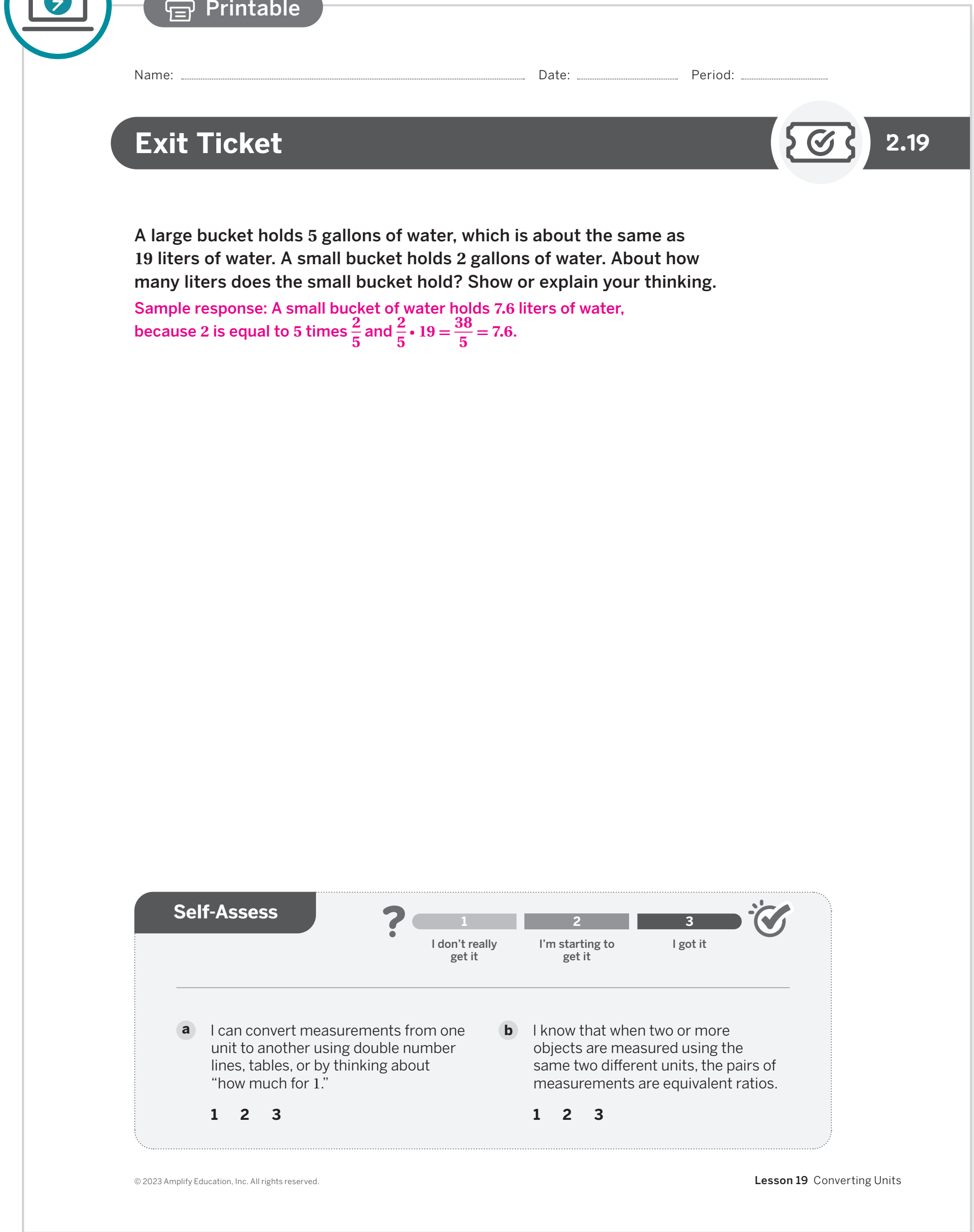
- "What is a strategy you can use to convert measurements from one unit to another?"
- "How do equivalent ratios help you when converting measurements?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of unit conversions by using equivalent ratios to convert gallons to liters.

Printable



Name: Date: Period:

Exit Ticket

2.19

A large bucket holds 5 gallons of water, which is about the same as 19 liters of water. A small bucket holds 2 gallons of water. About how many liters does the small bucket hold? Show or explain your thinking.

Sample response: A small bucket of water holds 7.6 liters of water, because 2 is equal to 5 times $\frac{2}{5}$ and $\frac{2}{5} \cdot 19 = \frac{38}{5} = 7.6$.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can convert measurements from one unit to another using double number lines, tables, or by thinking about "how much for 1."

1 2 3

b. I know that when two or more objects are measured using the same two different units, the pairs of measurements are equivalent ratios.

1 2 3

Lesson 19 Converting Units

Success looks like . . .

- **Goal:** Choosing and creating a double number line diagram or table to solve problems involving unit conversion.
- **Language Goal:** Explaining how to use a "rate per 1" to solve problems involving unit conversion. **(Speaking and Listening)**
- **Goal:** Recognizing that when two or more things are measured in the same two different units, the pairs of measurements are equivalent ratios.
  - » Using equivalent ratios and the ratio 5 gallons to 19 liters to determine the number of liters equivalent to 2 gallons.

Suggested next steps

If students are unsure how to make the conversion, consider:

- Having students create a double number line or a table, and asking, "How can you use multiplication or division to create an equivalent ratio?"
- Reviewing Problem 2 of Activity 1.
- Assigning Practice Problem 1.

If students get hung up thinking they need to know "how much per 1" for liters or gallons, consider:

- Asking, "Why do you think that? What information can you use from the problem to help you determine that equivalency?"
- Reviewing the Summary from Lesson 14 about the information necessary to determine a missing value, and how to use that to set up equivalent ratios.

Professional Learning

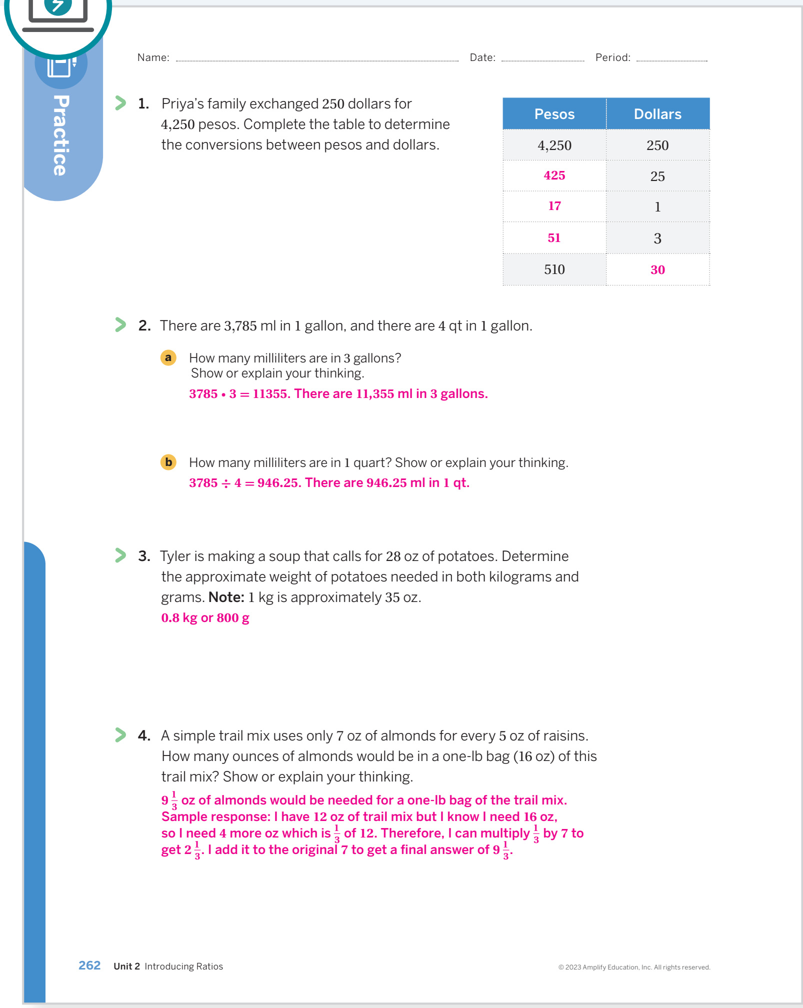
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students worked on the recipe conversions?
- During the discussions, how did you encourage each student to share their understanding? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Priya's family exchanged 250 dollars for 4,250 pesos. Complete the table to determine the conversions between pesos and dollars.

Pesos	Dollars
4,250	250
425	25
17	1
51	3
510	30

2. There are 3,785 ml in 1 gallon, and there are 4 qt in 1 gallon.

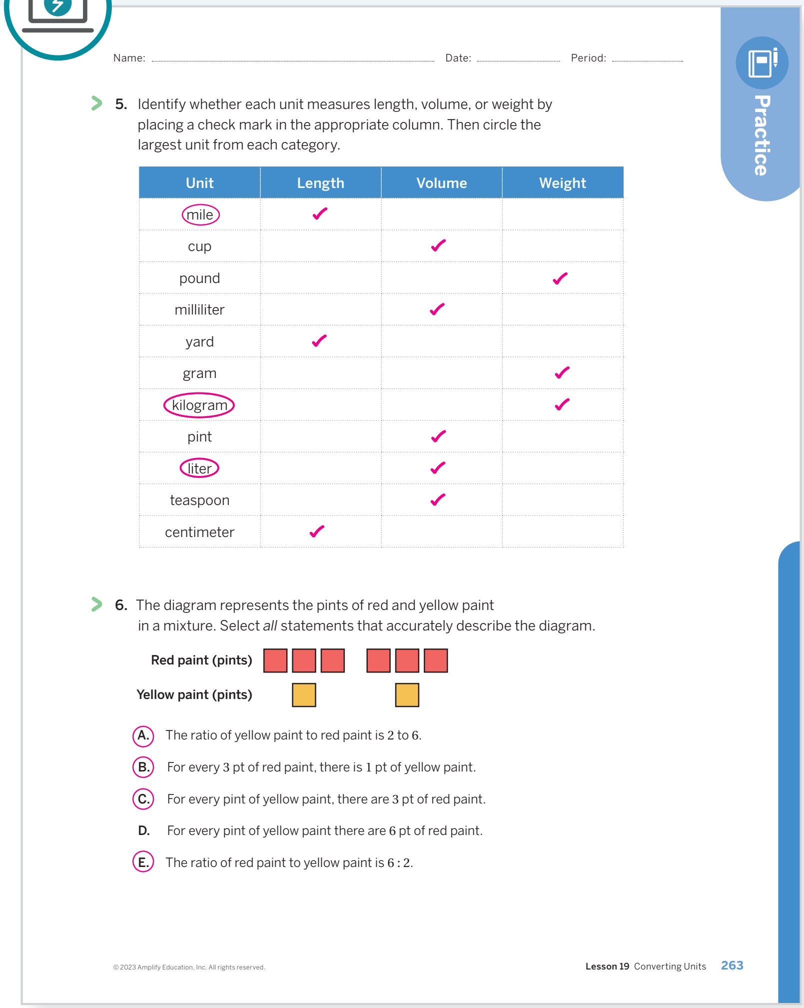
   a. How many milliliters are in 3 gallons? Show or explain your thinking.
   $3785 \cdot 3 = 11355$. There are 11,355 ml in 3 gallons.

   b. How many milliliters are in 1 quart? Show or explain your thinking.
   $3785 \div 4 = 946.25$. There are 946.25 ml in 1 qt.

3. Tyler is making a soup that calls for 28 oz of potatoes. Determine the approximate weight of potatoes needed in both kilograms and grams. **Note:** 1 kg is approximately 35 oz.
0.8 kg or 800 g

4. A simple trail mix uses only 7 oz of almonds for every 5 oz of raisins. How many ounces of almonds would be in a one-lb bag (16 oz) of this trail mix? Show or explain your thinking.
$9\frac{1}{3}$ oz of almonds would be needed for a one-lb bag of the trail mix. Sample response: I have 12 oz of trail mix but I know I need 16 oz, so I need 4 more oz which is $\frac{1}{3}$ of 12. Therefore, I can multiply $\frac{1}{3}$ by 7 to get $2\frac{1}{3}$. I add it to the original 7 to get a final answer of $9\frac{1}{3}$.

262 **Unit 2** Introducing Ratios



Practice

Name: Date: Period:

5. Identify whether each unit measures length, volume, or weight by placing a check mark in the appropriate column. Then circle the largest unit from each category.

Unit	Length	Volume	Weight
[✓ mile]	✓		
cup		✓	
pound			✓
milliliter		✓	
yard	✓		
gram			✓
[✓ kilogram]			✓
pint		✓	
[✓ liter]		✓	
teaspoon		✓	
centimeter	✓		

6. The diagram represents the pints of red and yellow paint in a mixture. Select *all* statements that accurately describe the diagram.

[✓ A.] The ratio of yellow paint to red paint is 2 to 6.
[✓ B.] For every 3 pt of red paint, there is 1 pt of yellow paint.
[✓ C.] For every pint of yellow paint, there are 3 pt of red paint.
D. For every pint of yellow paint there are 6 pt of red paint.
[✓ E.] The ratio of red paint to yellow paint is 6 : 2.

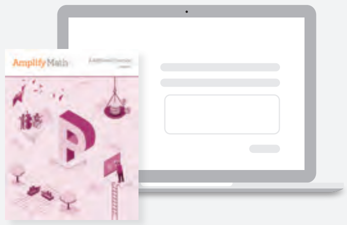
Lesson 19 Converting Units 263

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 7	2
	5	Unit 2 Lesson 18	2
Formative ⚡	6	Unit 2 Lesson 20	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

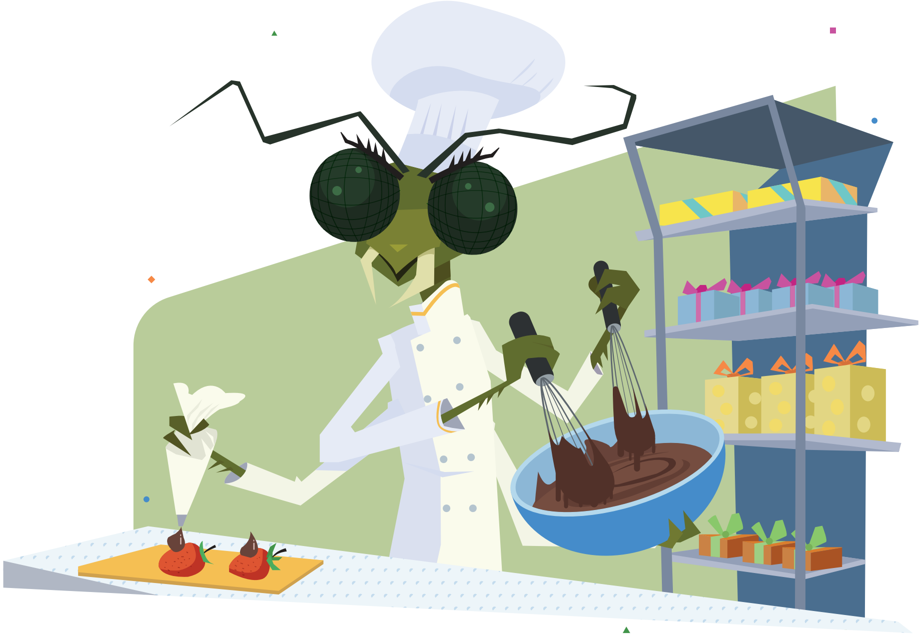
Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

More Fermi Problems

Let's solve a Fermi problem.



Focus

Goals

1. **Language Goal:** Apply ratio reasoning to an unfamiliar problem. **(Speaking and Listening, Writing)**
2. **Language Goal:** Decide what information is needed to solve real-world ratio problems. **(Speaking and Listening)**
3. **Language Goal:** Make simplifying assumptions about a real-world problem. **(Speaking and Listening, Writing)**

Coherence

Today

Students apply ratio reasoning to solve one of the following Fermi problems:

- How many sticky notes will it take to cover the Washington Monument?
- How many insect fragments are allowed to be in the world's largest chocolate bar?
- If a radio station played your favorite song non-stop for the rest of your life, how many times would you hear it?

Each problem requires students to break down larger questions into more manageable sub-questions to make sense of the problem. They also require making simplifying assumptions, estimates, and decisions about which quantities are important and what mathematics to use. Students should speak precisely as they report estimates and describe quantities and units.

Previously

Throughout this unit, students developed an understanding of *ratio* — the invariant multiplicative relationship between two quantities. They used several different representations to model ratio relationships and developed multiple strategies to solve equivalence and comparison problems.

Coming Soon

In Unit 3, students will extend their work with ratios to focus on the concept of rate, particularly unit rates (per 1) and percentages (per 100). They will also begin to graph ratio relationships.

Rigor

- Students build **fluency** solving equivalent ratios with missing values.
- Students **apply** ratio reasoning to solve Fermi problems.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2 (optional)	Summary	Exit Ticket
5 min	30 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

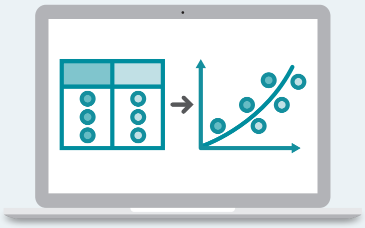
Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (instructions)
- Activity 1 – PDF, *Washington Monument*, one per group
- Activity 1 PDF (data cards), pre-cut cards, one set per group
- calculators
- computers (optional)
- materials for creating a visual display

Amps | **Featured Activity**

Activity 1
Using Work From Previous Slides

In later slides, students can build on their work from previous slides. It's their work, so they get to hold onto it!



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel uncomfortable making assumptions and rough estimations during Part 1 of Activity 1. Model how to simplify a problem from personal experience by walking through a thought process to arrive at estimates. For example, say, "If I want to know how many texts I send each year, I would start by considering the most and least texts I send in a day. Some days I send 50 texts, and other days I send none. So, an average of 25 seems like a reasonable number to use." Consider having students then participate with their own example, giving reasonable high, low, and final estimates.

Modifications to Pacing

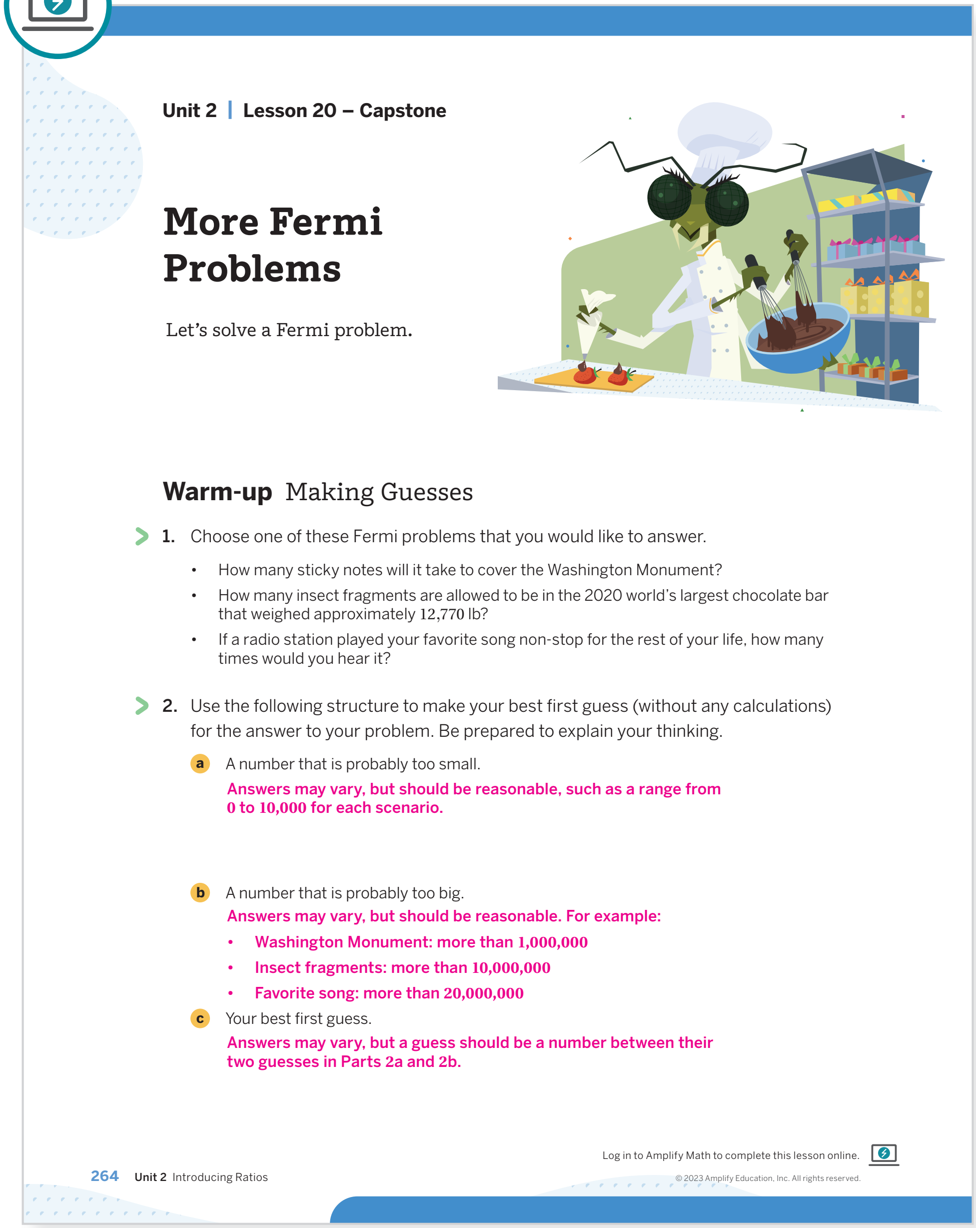
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** can be treated as part of the Launch for **Activity 1**. The discussion of estimates can be done as a whole class. You may also consider omitting the poster creation and the ***Gallery Tour*** routine.
- Optional **Activity 2** may be omitted.

Independent | 5 min

Warm-up Making Guesses

Students choose a Fermi problem to solve. They begin making sense of the problem by estimating values that are too high and too low before determining a best guess.



Unit 2 | **Lesson 20 – Capstone**

More Fermi Problems

Let's solve a Fermi problem.

Warm-up Making Guesses

1. Choose one of these Fermi problems that you would like to answer.
    - How many sticky notes will it take to cover the Washington Monument?
    - How many insect fragments are allowed to be in the 2020 world's largest chocolate bar that weighed approximately 12,770 lb?
    - If a radio station played your favorite song non-stop for the rest of your life, how many times would you hear it?
2. Use the following structure to make your best first guess (without any calculations) for the answer to your problem. Be prepared to explain your thinking.
    - a A number that is probably too small.
      Answers may vary, but should be reasonable, such as a range from 0 to 10,000 for each scenario.
    - b A number that is probably too big.
      Answers may vary, but should be reasonable. For example:
        - **Washington Monument: more than 1,000,000**
        - **Insect fragments: more than 10,000,000**
        - **Favorite song: more than 20,000,000**
    - c Your best first guess.
      Answers may vary, but a guess should be a number between their two guesses in Parts 2a and 2b.

Log in to Amplify Math to complete this lesson online.

264 **Unit 2** Introducing Ratios

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1 Launch

Explain that students will use their chosen problem for the rest of the lesson.

2 Monitor

Help students get started by asking, "Which question interests you the most? Why?"

Look for points of confusion:

- **Struggling to make a best guess.** Explain that best guesses are usually values between a guess that is too high and one that is too low. Ask, "What sounds more reasonable, a number closer to your too-low or too-high estimate, or a number right in the middle? Why?"

Look for productive strategies:

- Using simpler forms of the original problem to make reasonable estimations (e.g., number of sticky notes to cover a desk, number of insect parts allowed in 1 pound, how many times a song plays in an hour).
- Making a best guess that is between their too low and too high guesses.

3 Connect

Have students share and compare their guesses with other students who chose the same problem.

Ask, "How did you make your best guesses?"

Highlight that these questions are all Fermi problems, like the ones students saw in Lesson 1 of this unit. Explain that Fermi problems are often very difficult or impossible to measure directly, but relatively accurate answers can be estimated with simplified facts and calculations.

Differentiated Support

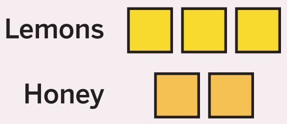
Accessibility: *Vary the Task Demands*

Write three values on the board: 100, 100,000, and 100,000,000. For each of the three problems, have students explain whether they think each value is too low, too high, or a reasonable estimate of the answer. Explain that when students make rough estimations, it can help to use a familiar context or a more manageable example. For instance, it would not make sense that 100 sticky notes would cover an entire monument because it would take more than 100 sticky notes to cover the board in the classroom. Have students complete Problem 2 before working on Activity 1.

Power-up

To power up students' ability to describe ratios from a visual model have students complete:

Determine if each statement is true or false based on the diagram.



a. The ratio of lemons to honey is equivalent to 6 : 4. True

b. For every 3 parts of honey there are 2 parts of lemon. False

c. The ratio of honey to lemons is 2 : 3. True

d. For every teaspoon of honey there are 1.5 teaspoons of lemon. True

Use: Before Activity 1.

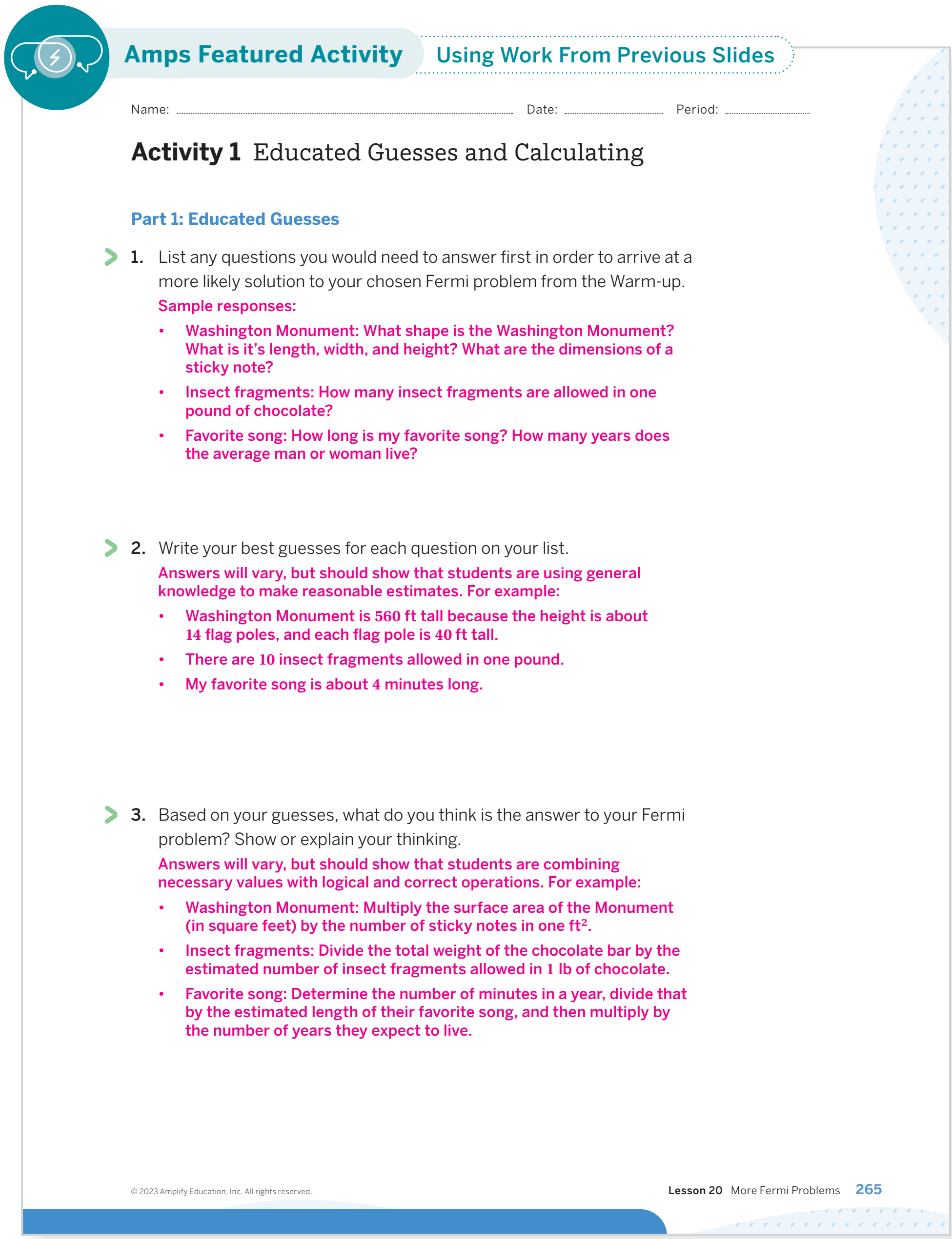
Informed by: Performance on Lesson 19, Practice Problem 6

Pairs | 30 min

Activity 1 Educated Guesses and Calculating

Students break down their Fermi problem into smaller questions that can be measured or estimated. They apply ratio reasoning to solve with estimated values and then with given values.

Amps Featured Activity Using Work From Previous Slides



Name: ________ Date: ________ Period: ________

Activity 1 Educated Guesses and Calculating

Part 1: Educated Guesses

1. List any questions you would need to answer first in order to arrive at a more likely solution to your chosen Fermi problem from the Warm-up.
   Sample responses:
   - **Washington Monument: What shape is the Washington Monument? What is it's length, width, and height? What are the dimensions of a sticky note?**
   - **Insect fragments: How many insect fragments are allowed in one pound of chocolate?**
   - **Favorite song: How long is my favorite song? How many years does the average man or woman live?**

2. Write your best guesses for each question on your list.
   Answers will vary, but should show that students are using general knowledge to make reasonable estimates. For example:
   - **Washington Monument is 560 ft tall because the height is about 14 flag poles, and each flag pole is 40 ft tall.**
   - **There are 10 insect fragments allowed in one pound.**
   - **My favorite song is about 4 minutes long.**

3. Based on your guesses, what do you think is the answer to your Fermi problem? Show or explain your thinking.
   Answers will vary, but should show that students are combining necessary values with logical and correct operations. For example:
   - **Washington Monument: Multiply the surface area of the Monument (in square feet) by the number of sticky notes in one ft^2.**
   - **Insect fragments: Divide the total weight of the chocolate bar by the estimated number of insect fragments allowed in 1 lb of chocolate.**
   - **Favorite song: Determine the number of minutes in a year, divide that by the estimated length of their favorite song, and then multiply by the number of years they expect to live.**

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Lesson 20 More Fermi Problems 265

1 Launch

Explain that students will prepare a visual representation of their work for the ***Gallery Tour*** routine at the end of the activity. Consider allowing the use of a calculator to focus work time on process and reasoning rather than on calculations. Review the Activity 1 PDF (instructions).

2 Monitor

Help students get started by asking, "What information do you know?"

Look for points of confusion:

- **Missing a critical piece of information (Part 1, Problem 1).** Ask, "What quantities do you need to know to be able to solve? Can you measure, calculate or assume something about it?"
- **Making an unreasonable estimation (Part 1, Problem 2).** Ask, "In the Warm-up, can you use high and low estimates to make a best guess?"
- **Misusing values or missing a step in the solution process (Part 1, Problem 3 or Part 2, Problem 4).** Draw attention to establishing ratios by asking, "If you wanted to know [e.g., how many square inches in a square foot], how could you do this?"
- **Calculating with incompatible units of measure (Part 1, Problem 3 or Part 2, Problem 4).** Ask, "How can you use ratios to convert all similar quantities to the same unit of measure?".

Look for productive strategies:

- Considering what information they know or can be measured, calculated, or assumed.
- Simplifying problems to make reasonable estimations (Part 1, Problem 2; see Sample responses).
- Using equations, tables, and diagrams to represent relationships between quantities and solve (see Sample responses).

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide physical objects to represent some of the abstract concepts, such as: sticky notes and a ruler for monument; a bar of chocolate for insects; or, a clock for the song.

Extension: *Math Enrichment*

Have students consider the largest and smallest possible answers for their problem. Ask, "What values on the data card are most likely estimates or averages — meaning they could change? What do you think is a minimum for those values? What about a maximum? How could any changes affect your final answer?"

Math Language Development

MLR1: Stronger and Clearer Each Time

Have pairs that are working on the same Fermi problem share their work for Part 1. Encourage listeners to provide feedback to help the speakers strengthen and clarify language. Consider rotating and repeating. Then have pairs refine and revise their original work.

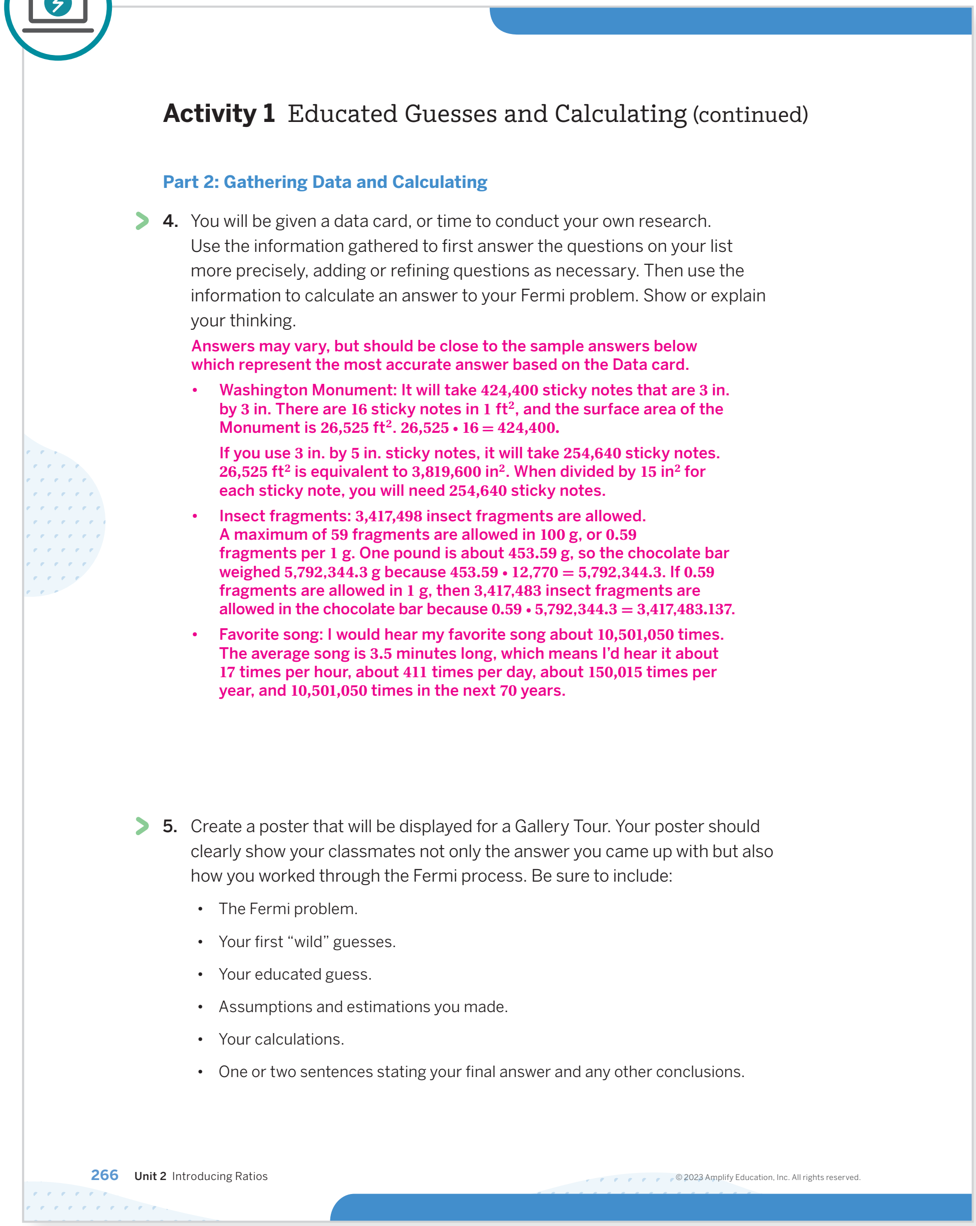
English Learners

Consider allowing students to respond to Problems 1–3 in their primary language, and then craft a final response in English.

Pairs | 30 min

Activity 1 Educated Guesses and Calculating (continued)

Students break down their Fermi problem into smaller questions that can be measured or estimated. They apply ratio reasoning to solve with estimated values and then with given values.



Activity 1 Educated Guesses and Calculating (continued)

Part 2: Gathering Data and Calculating

4. You will be given a data card, or time to conduct your own research. Use the information gathered to first answer the questions on your list more precisely, adding or refining questions as necessary. Then use the information to calculate an answer to your Fermi problem. Show or explain your thinking.

Answers may vary, but should be close to the sample answers below which represent the most accurate answer based on the Data card.

- **Washington Monument: It will take 424,400 sticky notes that are 3 in. by 3 in. There are 16 sticky notes in 1 ft^2, and the surface area of the Monument is 26,525 ft^2. $26,525 \cdot 16 = 424,400$.**

  If you use 3 in. by 5 in. sticky notes, it will take 254,640 sticky notes. 26,525 ft^2 is equivalent to 3,819,600 in^2. When divided by 15 in^2 for each sticky note, you will need 254,640 sticky notes.
- **Insect fragments: 3,417,498 insect fragments are allowed. A maximum of 59 fragments are allowed in 100 g, or 0.59 fragments per 1 g. One pound is about 453.59 g, so the chocolate bar weighed 5,792,344.3 g because $453.59 \cdot 12,770 = 5,792,344.3$. If 0.59 fragments are allowed in 1 g, then 3,417,483 insect fragments are allowed in the chocolate bar because $0.59 \cdot 5,792,344.3 = 3,417,483.137$.**
- **Favorite song: I would hear my favorite song about 10,501,050 times. The average song is 3.5 minutes long, which means I'd hear it about 17 times per hour, about 411 times per day, about 150,015 times per year, and 10,501,050 times in the next 70 years.**

5. Create a poster that will be displayed for a Gallery Tour. Your poster should clearly show your classmates not only the answer you came up with but also how you worked through the Fermi process. Be sure to include:

- The Fermi problem.
- Your first "wild" guesses.
- Your educated guess.
- Assumptions and estimations you made.
- Your calculations.
- One or two sentences stating your final answer and any other conclusions.

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3 Connect

Display the posters for the same question in groups around the room, and conduct the ***Gallery Tour*** routine.

Have students share how different groups solved the same problem in a similar or different way, focusing on why there were (most likely) a variety of responses for Problems 3 and 4.

Ask,

- "What are some examples of when you had to make the problem simpler in order to proceed? How did you make it simpler?" Sample responses: I thought about a familiar context that I could see or apply numbers to (e.g., the number of sticky notes to cover my desk). I wrote a ratio containing a 1 to make the problem smaller before scaling up (e.g., the number of sticky notes that cover one square foot, instead of the entire surface area of the monument). I assumed the sticky notes could be cut into pieces as needed, there was at least one bug part per pound of chocolate, and that I would live until I am 90 because I am really healthy.
- "How did you use ratio reasoning to solve your problem?" Sample responses: I determined the relationship between two quantities (e.g., square inches to square feet, sticky notes per square foot, seconds per year, the number of times the song would play per day, the number of grams per pound). Then, I determined an equivalent ratio in which the other number in that ratio is what I used to ___ (or was my answer to ___).

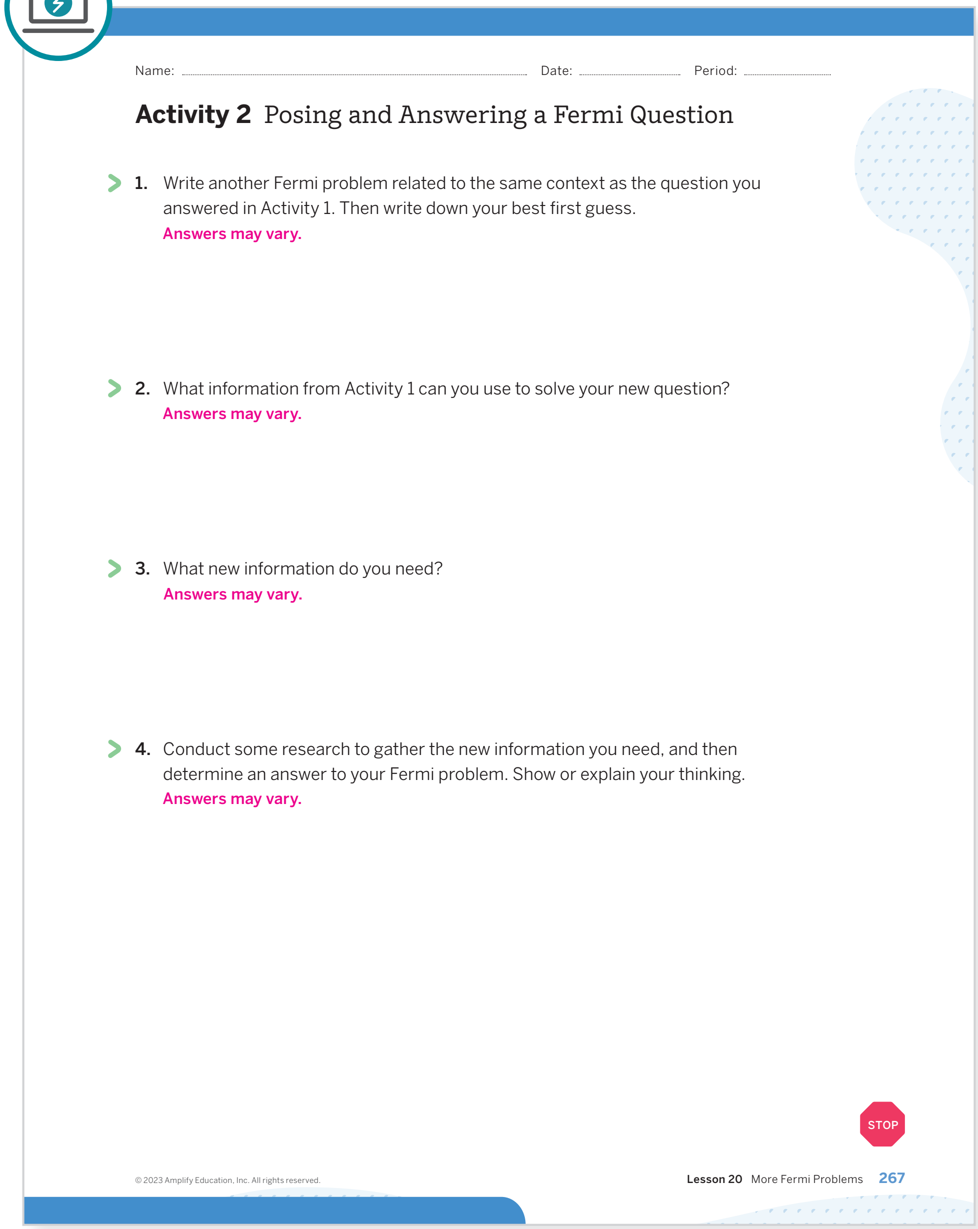
Highlight that Fermi problems, like most real-world problems, involve at least two quantities. When values cannot be measured directly, then students must estimate or calculate using other information they know. These problems also often require them to make the problem simpler in order to proceed — making simplifying assumptions, identifying additional information needed, and using mathematical modeling and estimations. As a result, there is not always one exact answer to a Fermi problem — just a "best answer," based on the values students determined or chose to use. Even when there is a most likely (or correct) answer, they can come very close with estimates — depending on the level of precision required or requested.

Optional

Pairs | 20 min

Activity 2 Posing and Answering a Fermi Problem

Students extend their work from Activity 1 to write their own Fermi problem that requires research, estimation, and ratio reasoning in order to determine an answer.



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Activity 2 Posing and Answering a Fermi Question

1. Write another Fermi problem related to the same context as the question you answered in Activity 1. Then write down your best first guess.
   Answers may vary.

2. What information from Activity 1 can you use to solve your new question?
   Answers may vary.

3. What new information do you need?
   Answers may vary.

4. Conduct some research to gather the new information you need, and then determine an answer to your Fermi problem. Show or explain your thinking.
   Answers may vary.

STOP

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Lesson 20 More Fermi Problems 267

1 Launch

Explain that students will brainstorm their own Fermi problem related to their context in Activity 1. Remind them that Fermi problems include at least two quantities to measure or calculate. Provide access to computers or other resources to conduct research for Problem 4.

2 Monitor

Help students get started by asking, "What else could you measure or calculate related to your topic?"

Look for points of confusion:

- **Writing a problem that does not use two quantities that are in a ratio relationship.** Ask, "How can you edit your new question to involve at least two quantities?"
- **Missing necessary information (Problems 2–3).** Have students review their data card from Activity 1, and ask, "What information will be useful to solve your new problem? What new information do you need to know?"

Look for productive strategies:

- Writing a question that requires ratio reasoning be applied between at least two quantities, and identifying any new information needed to determine an answer.
- Using and adjusting their ratios and representations from Activity 1 to help solve their new problem.

3 Connect

Have students share their questions and how they used ratio reasoning to determine an answer. Consider pairing groups who explored the same original question together.

Highlight that not all Fermi problems can be solved by using only ratios but nearly any seemingly impossible problem can be solved, or very closely estimated.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students choose from the following questions:

- How much would it cost to cover the Washington Monument with sticky notes?
- How many insect fragments have I eaten (in chocolate) this past year?
- How long would it take to play your song 1 billion times?

Extension: *Math Enrichment*

Instead of building on the same contexts in this lesson, have students research, write, and solve their own Fermi problem about any topic of their choice.

Math Language Development

MLR8: Discussion Supports — Restate It!

Present non-examples of Fermi problems, such as, "How many students are in our classroom right now?" Ask pairs of students to rewrite the problem as a Fermi problem and identify the changes needed.

English Learners

Illustrate how the non-examples are not Fermi problems by providing the numerical answer to each non-example during the discussion.

Whole Class | 5 min

Unit Summary

Review and synthesize how mathematical modeling, such as estimations, ratio reasoning and representations, can help solve Fermi problems.

Narrative Connections

Unit Summary

Ratios are everywhere. They are in the paint on the walls, the food on your plate, even in the rhythms of your music. They give things their consistency and balance. The wrong ratio of red to blue can mean the difference between fuchsia and mauve. A band that's off-rhythm can cause any dancer to stumble.

Much like you saw with Oobleck, the ratios in paint colors, song tempos, and recipes are a constant dance between quantities. In many ways, you've known these quantities all your life. You can taste when soup has too much salt. You can see when the shade is off when mixing colors.

Expressing these ratios mathematically lets you be more precise than with "feel" alone.

While the problems you explored throughout this unit have been light-hearted, your process mirrored that of mathematicians, scientists, business leaders, and policy experts who are working on some of the world's most complex problems. From managing the world's ecosystems and preparing disaster relief efforts to starting a new business and launching spaceships to Mars, ratios play a key role in the world (and Universe) around us.

See you in Unit 3.

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Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Highlight that the process used today can also be used to approach problems in the students' own lives. For example, "How long will it take me to afford a new gaming system if I work part-time in a restaurant?"

Ask:

- "What are some actions you can take to make this problem easier to solve?" Sample responses: I can assume that I am a waiter/waitress who can work 10 hours per week. I can estimate the average hourly wage, the number of tables I serve per hour, how much I can make in tips per table, and how much the typical gaming system costs. I use equivalent ratios and multiplication to determine how many hours I need to work to afford the gaming system. Then, I can determine how many weeks it will take to work that many hours.
- "How could you apply this same type of reasoning to your own life?" Answers may vary but should include two quantities that can be measured or calculated.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

- "How can ratios help you estimate solutions to seemingly impossible real-world problems?"

Differentiated Support

Extension: ***Math Enrichment***

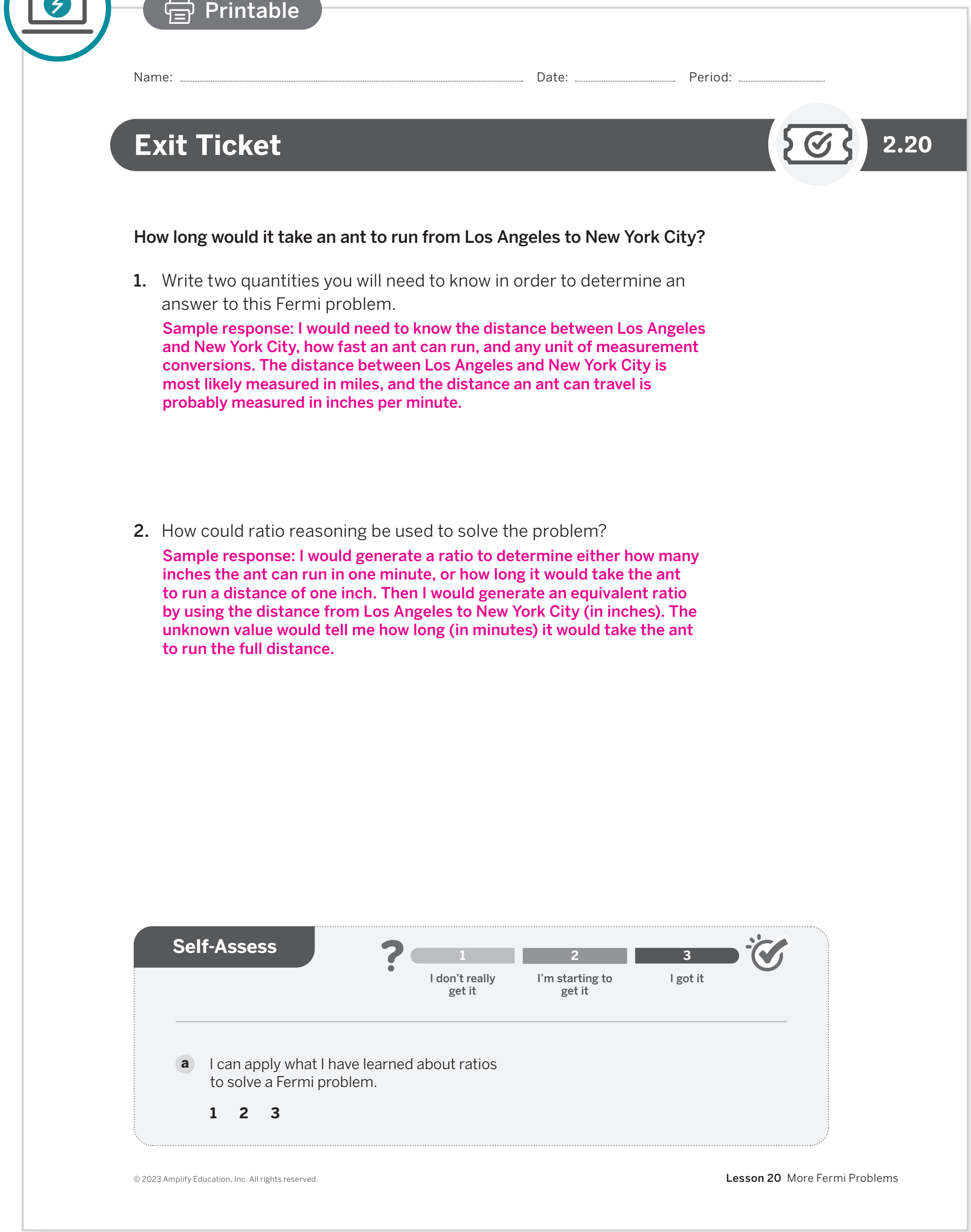
Bring the discussion back to Enrico Fermi and summarize his life and contributions that students learned about in this unit. One of Enrico Fermi's famous quotes is, "Before I came here, I was confused about this subject. Having listened to your lecture, I am still confused. But on a higher level." Let students know that it is common, and, in fact necessary, for mathematicians and scientists to continually ask questions. Ask students to write about a time in which they generated more questions as they further explored or learned about a certain mathematical idea.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying necessary quantities and explaining how to use ratios to solve a Fermi problem.

Printable



Name: ____________________ Date: __________ Period: __________

Exit Ticket 2.20

How long would it take an ant to run from Los Angeles to New York City?

1. Write two quantities you will need to know in order to determine an answer to this Fermi problem.
   Sample response: I would need to know the distance between Los Angeles and New York City, how fast an ant can run, and any unit of measurement conversions. The distance between Los Angeles and New York City is most likely measured in miles, and the distance an ant can travel is probably measured in inches per minute.

2. How could ratio reasoning be used to solve the problem?
   Sample response: I would generate a ratio to determine either how many inches the ant can run in one minute, or how long it would take the ant to run a distance of one inch. Then I would generate an equivalent ratio by using the distance from Los Angeles to New York City (in inches). The unknown value would tell me how long (in minutes) it would take the ant to run the full distance.

Self-Assess

?	1	2	3
	I don't really get it	I'm starting to get it	I got it

a I can apply what I have learned about ratios to solve a Fermi problem.
1 2 3

Success looks like . . .

- **Language Goal:** Applying reasoning developed throughout this unit to an unfamiliar problem. **(Speaking and Listening, Writing)**
  - » Applying ratio reasoning to an ant's travels in Problems and 2.
- **Language Goal:** Deciding what information is needed to solve a real-world problem. **(Speaking and Listening)**
  - » Determining the information needed to determine the time it takes an ant to run between two cities in Problem 1.
- **Language Goal:** Making simplifying assumptions about a real-world problem. **(Speaking and Listening, Writing)**
  - » Making assumptions that use equivalent ratios to determine the amount of time the ant would travel in Problems 1 and 2.

Suggested next steps

If students are unable to identify two necessary quantities for Problem 1, consider:

- Reviewing the types of questions used to address Activity 1, Problem 1, such as, "What information or quantities do I know? What can I measure? What can I calculate? What can I assume?"

If students are unable to identify at least one way they will use ratio reasoning to solve (Problem 2), consider:

- Asking, "What units would you expect the distance from LA and NYC to be reported in? Would you use the same units to report how fast an ant runs? Why not? How could you use ratio reasoning to help you think through this part of the problem?"

Professional Learning

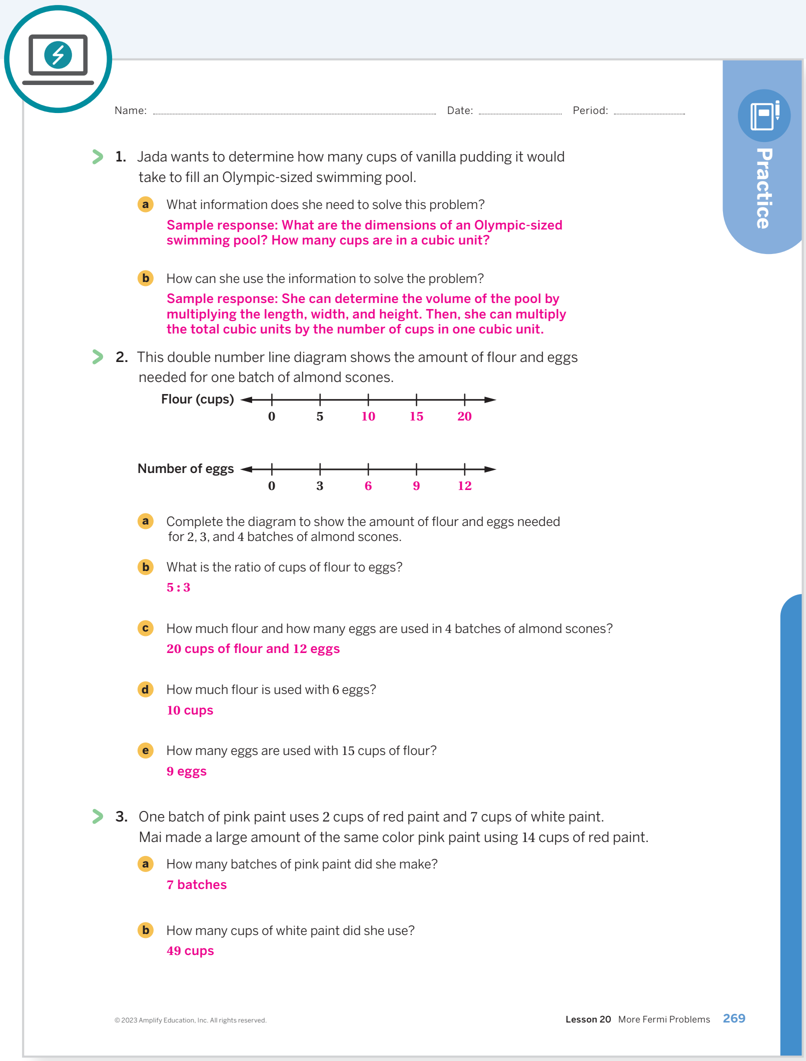
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? An instructional goal for this lesson was that students apply ratio reasoning to an unfamiliar problem. How well did students accomplish this? What did you specifically do to help students accomplish this?
- In what ways have your students gotten better at breaking larger questions into more manageable sub-questions and modeling with mathematics? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Jada wants to determine how many cups of vanilla pudding it would take to fill an Olympic-sized swimming pool.
   - a What information does she need to solve this problem?
     Sample response: What are the dimensions of an Olympic-sized swimming pool? How many cups are in a cubic unit?
   - b How can she use the information to solve the problem?
     Sample response: She can determine the volume of the pool by multiplying the length, width, and height. Then, she can multiply the total cubic units by the number of cups in one cubic unit.
2. This double number line diagram shows the amount of flour and eggs needed for one batch of almond scones.

   Flour (cups): 0, 5, 10, 15, 20

   Number of eggs: 0, 3, 6, 9, 12

   - a Complete the diagram to show the amount of flour and eggs needed for 2, 3, and 4 batches of almond scones.
   - b What is the ratio of cups of flour to eggs?
     5 : 3
   - c How much flour and how many eggs are used in 4 batches of almond scones?
     20 cups of flour and 12 eggs
   - d How much flour is used with 6 eggs?
     10 cups
   - e How many eggs are used with 15 cups of flour?
     9 eggs
3. One batch of pink paint uses 2 cups of red paint and 7 cups of white paint. Mai made a large amount of the same color pink paint using 14 cups of red paint.
   - a How many batches of pink paint did she make?
     7 batches
   - b How many cups of white paint did she use?
     49 cups

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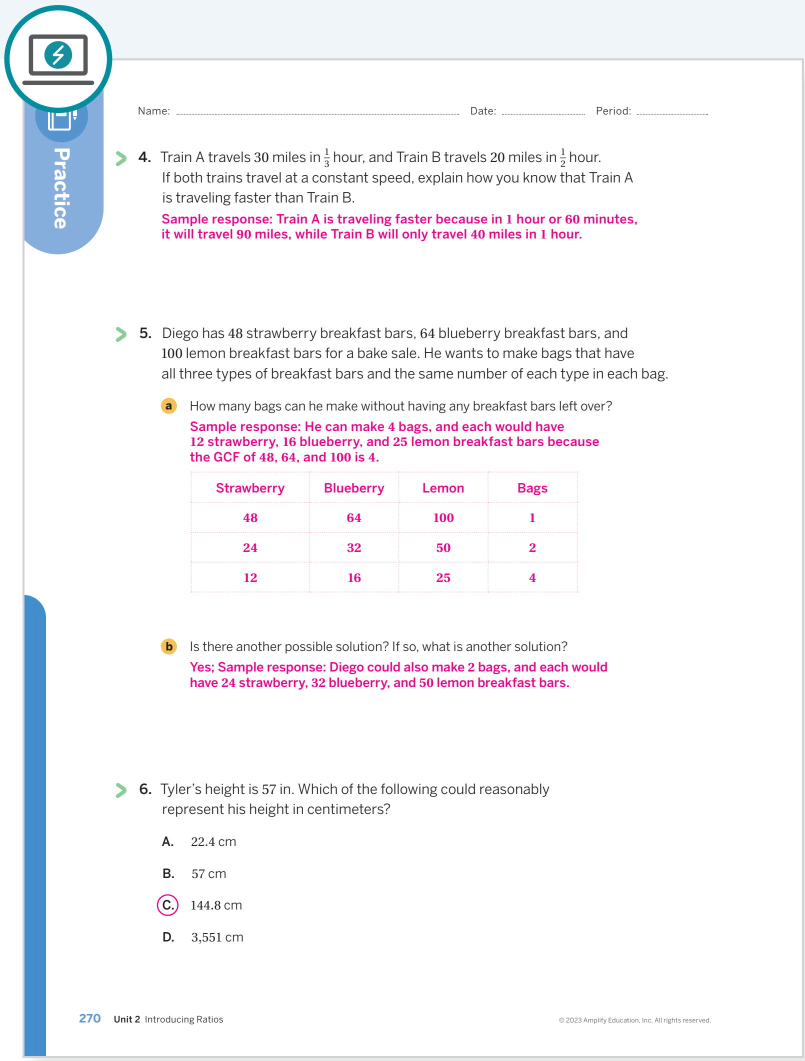
Lesson 20 More Fermi Problems 269



Name: Date: Period:

4. Train A travels 30 miles in $\frac{1}{3}$ hour, and Train B travels 20 miles in $\frac{1}{2}$ hour. If both trains travel at a constant speed, explain how you know that Train A is traveling faster than Train B.
   Sample response: Train A is traveling faster because in 1 hour or 60 minutes, it will travel 90 miles, while Train B will only travel 40 miles in 1 hour.
5. Diego has 48 strawberry breakfast bars, 64 blueberry breakfast bars, and 100 lemon breakfast bars for a bake sale. He wants to make bags that have all three types of breakfast bars and the same number of each type in each bag.
   - a How many bags can he make without having any breakfast bars left over?
     Sample response: He can make 4 bags, and each would have 12 strawberry, 16 blueberry, and 25 lemon breakfast bars because the GCF of 48, 64, and 100 is 4.

Strawberry	Blueberry	Lemon	Bags
48	64	100	1
24	32	50	2
12	16	25	4

   - b Is there another possible solution? If so, what is another solution?
     Yes; Sample response: Diego could also make 2 bags, and each would have 24 strawberry, 32 blueberry, and 50 lemon breakfast bars.
6. Tyler's height is 57 in. Which of the following could reasonably represent his height in centimeters?
   - A. 22.4 cm
   - B. 57 cm
   - [✓ C.] 144.8 cm
   - D. 3,551 cm

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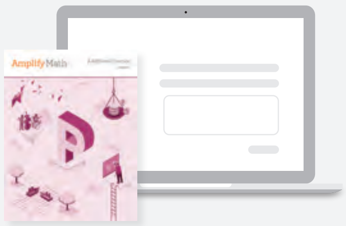
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
Spiral	2	Unit 2 Lesson 12	2
	3	Unit 2 Lesson 5	2
	4	Unit 2 Lesson 16	2
	5	Unit 2 Lesson 11	2
	6	Unit 2 Lesson 18	2

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 3

Rates and Percentages

Students understand the concept of unit rate in the contexts of constant price and speed, recognizing that equivalent ratios have the same unit rates. They use several visual and algebraic representations of percentages to determine missing percentages, parts, and wholes.

Essential Questions

- How are the terms *same rate*, *constant rate*, and *unit rate* similar and different?
- What is the relationship between unit rates and percentages?
- How are percentages used to estimate and compare quantities?
- *(By the way, if you're a day late and a dollar short, do you need more time or more money?)*

Key Shifts in Mathematics

Focus

In this unit . . .

Students continue to work with ratios and equivalent ratios, adding coordinate graphing to their list of available representations. They formalize their understanding of rate, largely emphasizing unit rates in the contexts of constant price or speed. Then students explore the concept of percentages as rates per 100, again leveraging equivalent ratios in order to develop algorithms for determining unknown percentages, parts, and wholes. They also compare quantities with different totals using percentages.

Coherence

Previously . . .

In Unit 2, students were introduced to the concept of ratios and equivalent ratios. They used ratio tables and double number lines to determine missing values and compare ratios. Students also began to think about constant rates or things happening at the same rate, using ratios to perform measurement conversions and calculate constant prices and speeds.

Coming soon . . .

In Unit 4, students will divide whole numbers and fractions by fractions, for which their understanding of equivalent ratios and unit rates can be leveraged as one possible way of thinking. In Unit 6, students will revisit graphing equivalent ratios, and also writing equations to represent all the points corresponding to a ratio relationship. In Grade 7, students will extend ratio reasoning to proportional thinking, determining the constant of proportionality for a proportional relationship; they will also solve multi-step percentage problems, such as percent increase or decrease, and work with interest.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Equivalent ratios have the same unit rates and can be graphed (Lessons 2–6). Percentages are rates per 100 (Lessons 8–9). Students recognize equivalent ratios can be used to determine any unknown in a percentage problem (Lessons 11–12).

Procedural Fluency

To determine a unit rate, the corresponding values for two quantities in a ratio relationship can be divided (Lessons 5–7). Benchmark percentages correspond to benchmark fractions (Lesson 10), and an algorithm can be derived to determine a missing percentage (Lesson 9), part (Lesson 11), or whole (Lesson 12) in a percentage problem.

Application

To compare two different constant rates, such as unit prices or speed, unit rates can be used (Lesson 7). Percentages can be used to determine discounts (Lesson 13), to compare ratios or subgroups of a population (Lesson 14), or to determine the outcome of an election (Lesson 15).

Stand and Be Counted

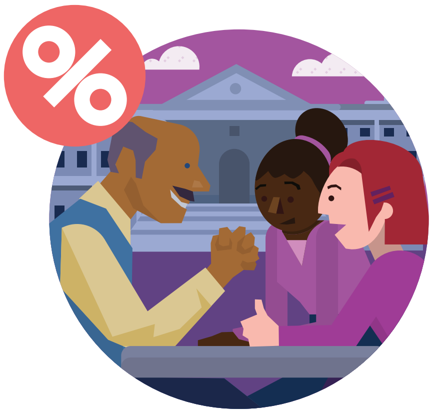
SUB-UNIT

1

Lessons 2–7

Rates

Students build on the notion of things happening “at the same rate” from the previous unit to investigate price and speed more deeply, and to consider the concept of rate more broadly. In particular, they see the utility of ***unit rates*** for comparison and determining missing values in ratio contexts.



Narrative: From planning a school event to running a race, rates are a great tool for measuring and comparing things.

SUB-UNIT

2

Lessons 8–14

Percentages

Students are introduced to the concept of ***percentages*** as rates per 100, recognizing that equivalent ratios can be leveraged to generalize algorithms for determining missing parts, wholes, and percentages. Similar to unit rates, they also use percentages to make comparisons, such as for subgroups of a population, and to think about fair representation.

Narrative: Percentage is a helpful way to compare values and understand populations, and even make decisions.

Launch

Lesson 1

Choosing Representation for Student Council

Students consider the meaning of “fair representation” in the context of student government at a middle school. They apply ratio concepts from the previous unit to compare votes and to distribute elected positions equitably.

Capstone

Lesson 15

Voting for a School Mascot

Students bring together all of the mathematical ideas of the unit and revisit fair representation in a voting context about selecting a school mascot.

Unit at a Glance

Spoiler Alert: To determine a given percent of a given number, you can move the decimal point two places to the left in the percent and multiply by the number. This works because of place value and the definition of a percent as a rate per 100.

Assessment

A **Pre-Unit Readiness Assessment**

Launch Lesson

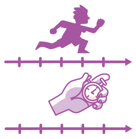
1 **Choosing Representation for Student Council**

Apply ratio reasoning to think about fair representation.

Sub-Unit 1: Rates

2 **How Much for One?** ●

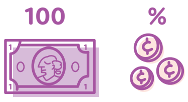
Unit price is the cost of one item.



3 **Constant Speed** ●

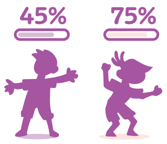
Speed can also be described in terms of unit rates of distance per time.

Sub-Unit 2: Percentages



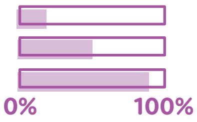
8 **What Are Percentages?**

Percentages are a rate per 100.



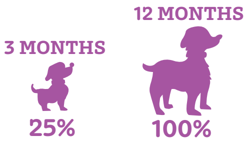
9 **Determining Percentages**

Equivalent ratios and corresponding representations can be used to determine the percent that one amount is relative to another.



10 **Benchmark Percentages** ●

Certain percentages correspond to common fractions.



11 **This Percent of That**

Any percent of a given whole can be determined by using an algorithm involving multiplication and division.

Assessment

A **End-of-Unit Assessment**

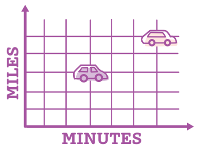
Key Concepts

Lesson 4: Two ratios are equivalent if they have the same rate per 1.
Lesson 9: To determine what percent one number is of another, divide the two numbers and multiply by 100.
Lesson 13: To determine any missing value in percentage problems, multiply and divide according to an algorithm.

Pacing

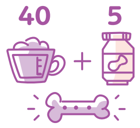
15 Lessons: 45 min each*
2 Assessments: 45 min each
Full Unit: 17–18 days
● **Modified Unit:** 11–14 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.
*The pacing of Lesson 5, in its entirety, is 75 minutes.



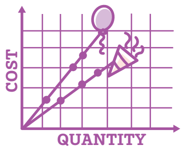
4 Comparing Speeds

Equivalent ratios and unit rates can be used to compare speeds, including by graphing them on the coordinate plane.



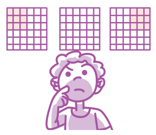
5 Interpreting Rates

Every ratio has two unit rates, but sometimes one is more useful.



6 Comparing Rates

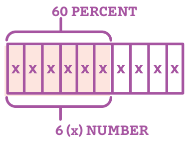
Unit rates and their graphs are efficient for comparing ratios and rates.



7 Solving Rate Problems ●

Practice and apply understanding of unit rate to solve problems involving constant rates and comparison.

Capstone Lesson

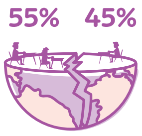


12 This Percent of What

When a percentage and the corresponding part are known, the whole can be determined by using an algorithm or a related tape diagram.

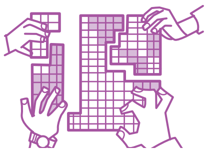
13 Solving Percentage Problems ●

Equivalent ratios, or a particular algorithm, can be used to solve percentage problems, depending on the given information.



14 If Our Class Were the World ●

Ratios and percentages are used to make sense of the world's population at the scale of a middle-school class.



15 Voting for a School Mascot ●

Unit rates and percentages can be used to determine and to interpret voting results.

● Modifications to Pacing

Lessons 2–3: These lessons can be omitted as they serve as a bridge between the content of Unit 2 and the truly new content of this unit. If students struggled with Unit 2, Lessons 16 and 17, it is highly recommended that these are not omitted.

Lesson 7: This lesson can be omitted because it is largely practice and application of the skills and concepts from previous lessons.

Lesson 10: It is not recommended that this entire lesson be omitted, although that is an option. Working with benchmark percentages and establishing a connection between common fractions and percentages will be beneficial in the lessons that follow, and would especially benefit struggling students. However, it could be sufficient to just have students complete Activity 1, and ideally address the Summary.

Lessons 14–15: Either of these two lessons that are different applications of percentages could be omitted, but it is not recommended that you omit both.

Unit Supports

Math Language Development

Lesson	New Vocabulary
2	rate
2	unit rate
8	percentage

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
1, 4, 6, 10–12, 15	**MLR1:** Stronger and Clearer Each Time
2, 4, 5, 8, 11	**MLR2:** Collect and Display
3, 8, 10, 12, 15	**MLR3:** Critique, Correct, Clarify
1, 2, 3	**MLR5:** Co-craft Questions
3, 4, 6–10, 13, 15	**MLR7:** Compare and Connect
1, 2, 5, 13, 14	**MLR8:** Discussion Supports

Materials

Every lesson includes:

- Exit Ticket
- Additional Practice

Additional required materials include:

Lesson(s)	Materials
7, 11, 14	calculators
3	masking tape
3	meter sticks
3, 6–10, 12–15	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
2, 15	rulers
3	stopwatches
3	string
14	tools for creating a visual display

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routines
14	Gallery Tour
7	Mix and Mingle
1, 4, 12	Notice and Wonder
3, 14	Number String
2, 13	Number Talk
7	Take Turns
4, 5, 6, 8, 10, 11, 12, 14, 15	Think-Pair-Share
4	Which One Doesn't Belong?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's ***Performance Task*** is available in the Assessment Guide.	After Lesson 14

Amps POWERED BY desmos

Social & Collaborative Digital Moments

Featured Activity

The Mascot Vote

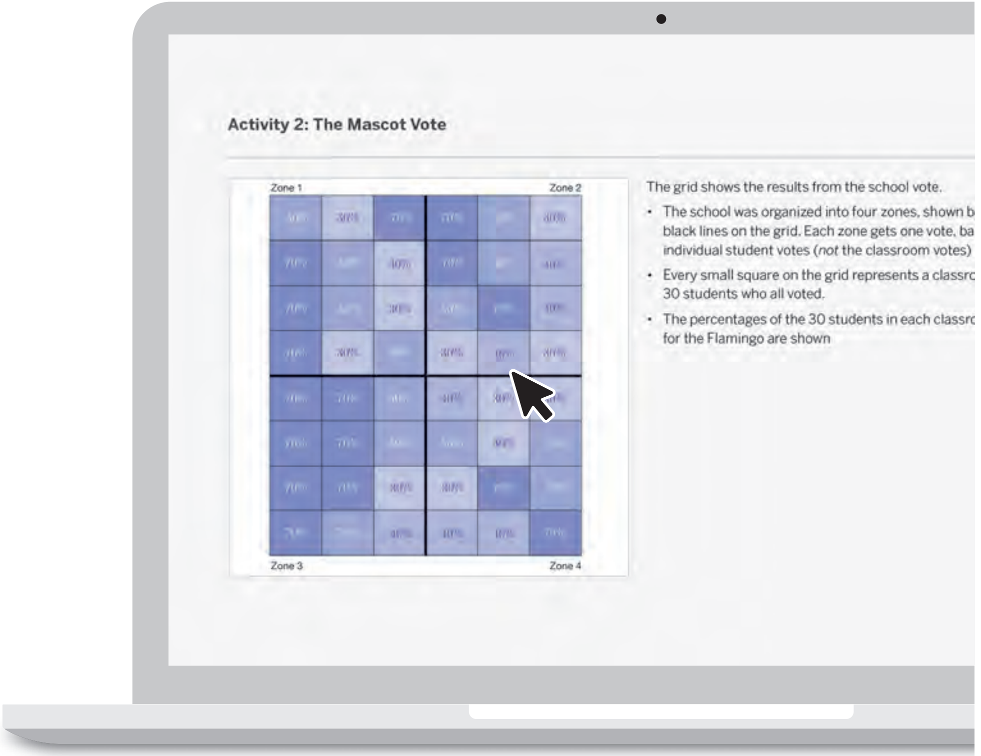
Put on your student hat and work through **Lesson 15, Activity 2:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Moving for 10 Seconds **(Lesson 3)**
- Puppies Grow Up **(Lesson 11)**
- What's the Better Deal? **(Lesson 13)**
- If Our Class Were the World **(Lesson 14)**



Activity 2: The Mascot Vote

The grid shows the results from the school vote.

- The school was organized into four zones, shown b black lines on the grid. Each zone gets one vote, ba individual student votes (*not* the classroom votes)
- Every small square on the grid represents a classro 30 students who all voted.
- The percentages of the 30 students in each classro for the Flamingo are shown

Unit Study
Professional Learning

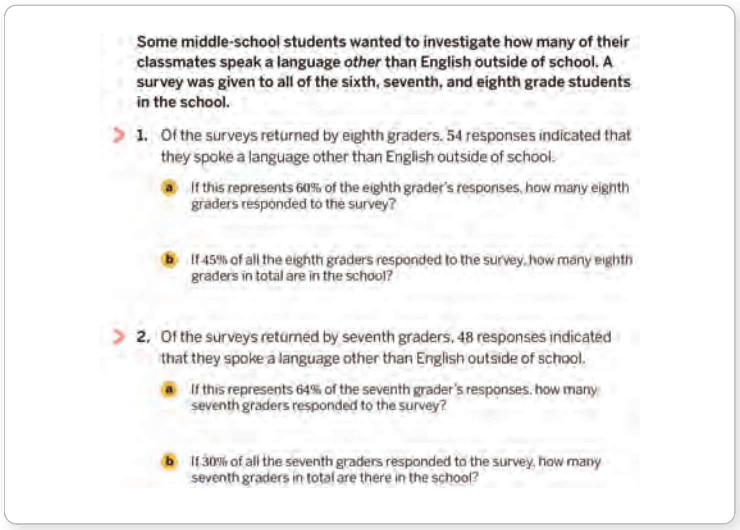
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 2 introduces students to percentages. The work prior to this was about rates and unit rates, including the use of ratio tables and graphs. Students understand that something other than rates is needed when looking at groups of a population. They learn about benchmark percentages, meaning how to find "this percent of that," and "this percent of what." Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from Lesson 12, Activity 2:



Some middle-school students wanted to investigate how many of their classmates speak a language *other* than English outside of school. A survey was given to all of the sixth, seventh, and eighth grade students in the school.

1. Of the surveys returned by eighth graders, 54 responses indicated that they spoke a language other than English outside of school.
   - a If this represents 60% of the eighth grader's responses, how many eighth graders responded to the survey?
   - b If 45% of all the eighth graders responded to the survey, how many eighth graders in total are in the school?
2. Of the surveys returned by seventh graders, 48 responses indicated that they spoke a language other than English outside of school.
   - a If this represents 64% of the seventh grader's responses, how many seventh graders responded to the survey?
   - b If 30% of all the seventh graders responded to the survey, how many seventh graders in total are there in the school?

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

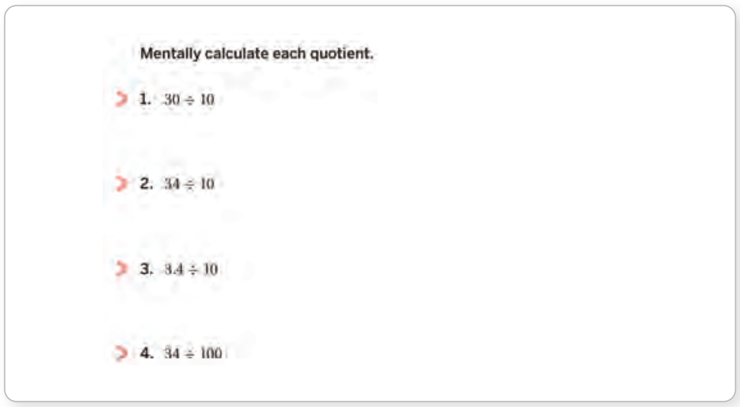
- What was it like to engage in this problem as a learner?
- What strategies (ratio tables, double number lines, tape diagrams, etc.) might your students use? Is there one that is more efficient than another?
- For each of the three problems, what two strategies might you suggest to your students if they struggled with where to begin?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Number String

Rehearse . . .

How you'll facilitate the *Number String* instructional routine in Lesson 3, Warm-up:



Mentally calculate each quotient.

1. $30 \div 10$
2. $34 \div 10$
3. $3.4 \div 10$
4. $34 \div 100$

Points to Ponder . . .

- How will you draw out the connections between these expressions, especially when students are not able to make them?

This routine . . .

- Presents a set of related problems, intentionally chosen to support students in building either or both conceptual understanding and fluency.
- Helps students recognize structure and relationships among numbers and operations.
- Supports students in using multiple strategies, and presents the opportunity for them to see and hear other possible strategies or ways of thinking and reasoning.

Anticipate . . .

- Students will attempt each problem in isolation, and may even be successful.
- Students may try to employ the same thinking or strategy across all problems, even if it is not applicable or more efficient methods exist.
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Facilitate meaningful mathematical discourse.

This effective teaching practice . . .

- Ensures that there is a shared understanding of the mathematical ideas students have explored and discovered during each lesson's activities.
- Allows students to listen to and critique the strategies and conclusions of others.

Points to Ponder . . .

- How can you establish a classroom environment in which diverse approaches to solving problems are cultivated?
- Some students may not not know how to dive deeper into discussions about mathematics. How can you model these discussions?

Math Language Development

MLR1: Stronger and Clearer Each Time

MLR1 appears in Lessons 1, 4, 6, 10–12, 15.

- In these lessons, opportunities are provided to have students first craft an initial draft of their response to a particular problem. Students then share their responses with 2–3 partners to receive feedback and then revise or refine their original response.
- Often, specific suggestions are provided to help reviewing partners look for clarity in the responses. For example:
  - » In Lesson 4, consider displaying the suggested questions so that reviewers look for how the responses indicate how the graph shows which group is working at a faster rate.
  - » In Lesson 10, reviewers are encouraged to ask how the response includes more information than just the percent symbol being removed.

Point to Ponder . . .

- How can you help your students grow in both giving and receiving feedback? How will you structure your classroom culture so that there is an expected norm in which your students feel supported, not criticized?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **End-of-Unit Assessment**, noting the concepts and skills assessed.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with determining and using unit rates and percentages throughout the unit? Do you think your students will generally:
  - » encounter difficulties in solving problems due to unfinished learning about ratios carrying over from the previous unit?
  - » rely too heavily on algorithms that lead to misapplications and misunderstandings?
  - » understand the concepts and ratio relationships, but struggle with full execution when dealing with non-whole number quantities?

Fostering Diverse Thinking

Use this opportunity for students to connect mathematics to the world around them:

- In Lesson 14, students consider how the projected world population in 2050 might have an effect on their results from the *If Our Class Were the World* activity. Based on a projected increasing urban population (70% by 2050) and an increasing population of older adults, they consider how our society might need to adapt to address these needs.

Point to Ponder . . .

- How can I help increase my students' awareness of the importance of social and environmental sustainability in a rapidly changing world?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their self-awareness and self-management skills.

Points to Ponder . . .

- Are students able to exercise patience and persist in order to understand the underlying conceptual structure of real-world and mathematical rate and percentage problems with different unknowns, rather than jumping straight to a familiar, but potentially incorrect, algorithm?
- Are students able to monitor their own work to step back and assess the reasonableness of solutions, considering that underlying rates and percentages provide sufficient information to estimate solutions?

Choosing Representation for Student Council

Let's think about representation and what is fair.

Focus

Goals

1. **Language Goal:** Apply reasoning about fractions and ratios to analyze voting situations involving two choices. **(Speaking and Listening, Writing)**
2. **Language Goal:** Apply reasoning about fractions and ratios to describe or critique representation of subgroups of a population. **(Speaking and Listening, Writing)**
3. **Language Goal:** Comprehend and explain the term *majority*. **(Speaking and Listening)**

Coherence

Today

Students think mathematically about two guiding ideas that set the stage for the unit: fairness and representation. They first consider the distribution of representation by grade among the leadership positions on the student council. Students then rank three classes relative to the agreement or disagreement based on votes for and against an issue. Students can draw some conclusions based on counts, but must use fractions or ratio reasoning to fully analyze the results. Students are then given the opportunity to present and justify any reorganization of the makeup of the student council they choose, using the enrollment of the school by grade. They can again make claims that are qualitative or based on simple counts, but should also leverage their work with ratios from Unit 2 to establish and describe fair representation.

Previously

In Unit 2, students developed a foundational understanding of ratios and equivalent ratios.

Coming Soon

In Lesson 2, students will begin to formalize the concept of *unit rate* in the likely familiar context of unit price.

Rigor

- Students **apply** their ratio reasoning from the previous unit to think about fair representation in elections.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

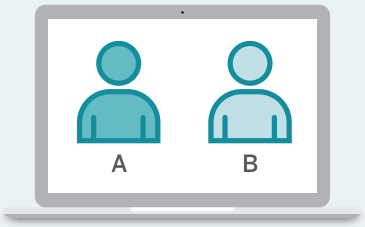
- *equivalent ratios*
- *ratio*

Amps | Featured Activity

Activity 2

Interactive Charts

Students can label an interactive chart to demonstrate fair representation, and can use it to share their thinking.



Building Math Identity and Community

Connecting to Mathematical Practices

As students begin to apply fractions and ratios in new situations, they may not be motivated to solve familiar problems in Activity 1. Have students spend some time setting goals for the new unit. Encourage them to use these goals to motivate themselves to focus and to seek new understanding.

Modifications to Pacing

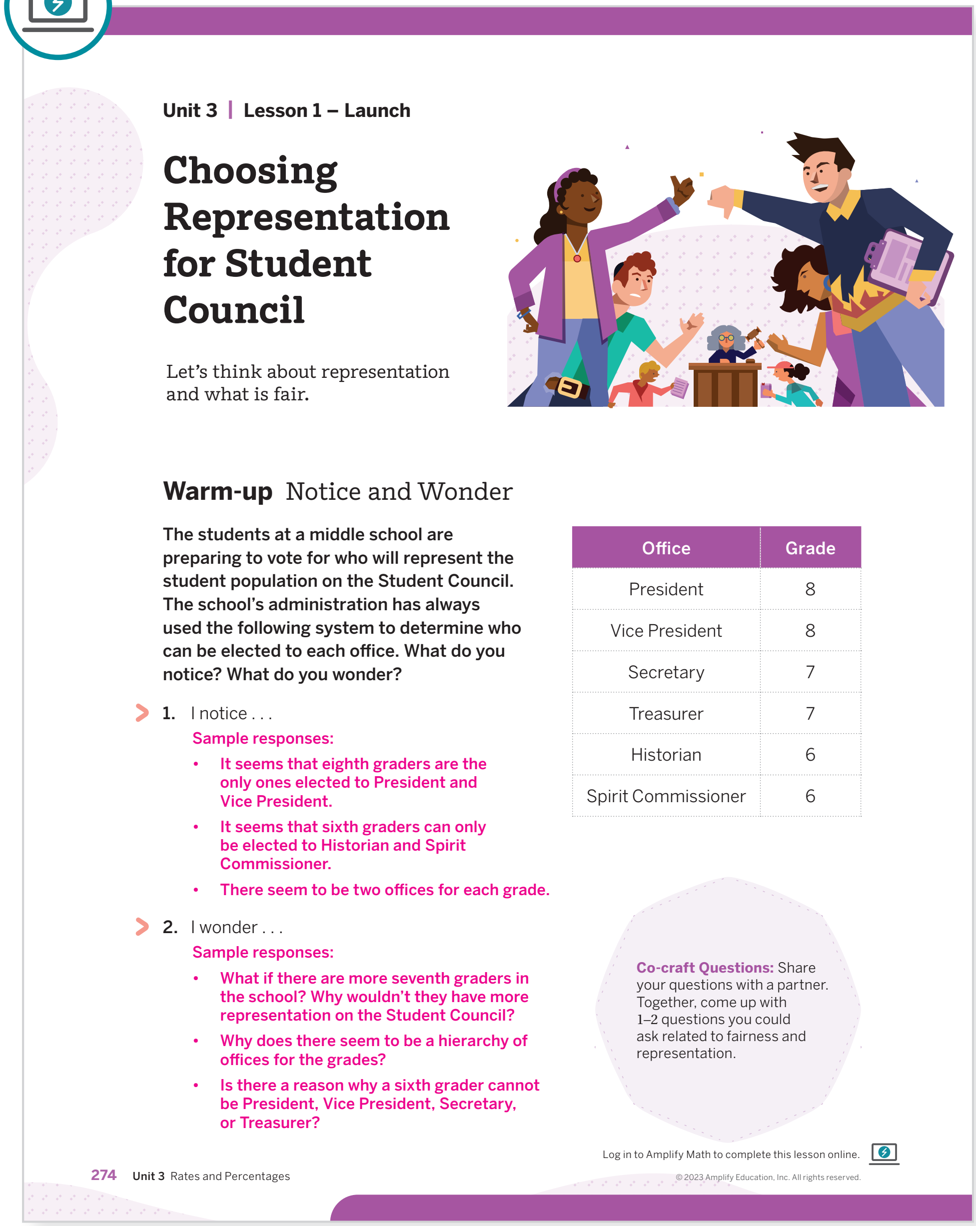
You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be omitted.

Independent | 5 min

Warm-up Notice and Wonder

Students begin to think about fair representation by using the context of elected positions on a middle school student council.



Unit 3 | Lesson 1 – Launch

Choosing Representation for Student Council

Let's think about representation and what is fair.

Warm-up Notice and Wonder

The students at a middle school are preparing to vote for who will represent the student population on the Student Council. The school's administration has always used the following system to determine who can be elected to each office. What do you notice? What do you wonder?

Office	Grade
President	8
Vice President	8
Secretary	7
Treasurer	7
Historian	6
Spirit Commissioner	6

1. I notice . . .

   Sample responses:
   - It seems that eighth graders are the only ones elected to President and Vice President.
   - It seems that sixth graders can only be elected to Historian and Spirit Commissioner.
   - There seem to be two offices for each grade.

2. I wonder . . .

   Sample responses:
   - What if there are more seventh graders in the school? Why wouldn't they have more representation on the Student Council?
   - Why does there seem to be a hierarchy of offices for the grades?
   - Is there a reason why a sixth grader cannot be President, Vice President, Secretary, or Treasurer?

Co-craft Questions: Share your questions with a partner. Together, come up with 1–2 questions you could ask related to fairness and representation.

Log in to Amplify Math to complete this lesson online.

274 Unit 3 Rates and Percentages

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1 Launch

Conduct the ***Notice and Wonder*** routine using the paragraph and the table.

2 Monitor

Help students get started by asking, "How many different positions are there? How many different grades are represented?"

Look for productive strategies:

- Noticing that each grade has two representatives and that there seems to be a hierarchy of positions.
- Wondering whether it is fair to have equal numbers of positions or who should have more representation.

3 Connect

Have individual students share what they noticed and wondered from the data in the table. Record class observations.

Ask, "Do you think it's fair? Why or why not?"

Highlight that there is more than one way to consider what is fair, who is represented, and how. These ideas, in this same context of student government, will be the focus of the other activities in this lesson.

Math Language Development

MLR5: Co-craft Questions

After students complete Problem 2, have them share their questions or what they wondered with a partner. Ask them to work together to generate 1–2 questions they could ask related to fairness and representation. Students will revisit this context and their questions during the upcoming activities in this lesson.

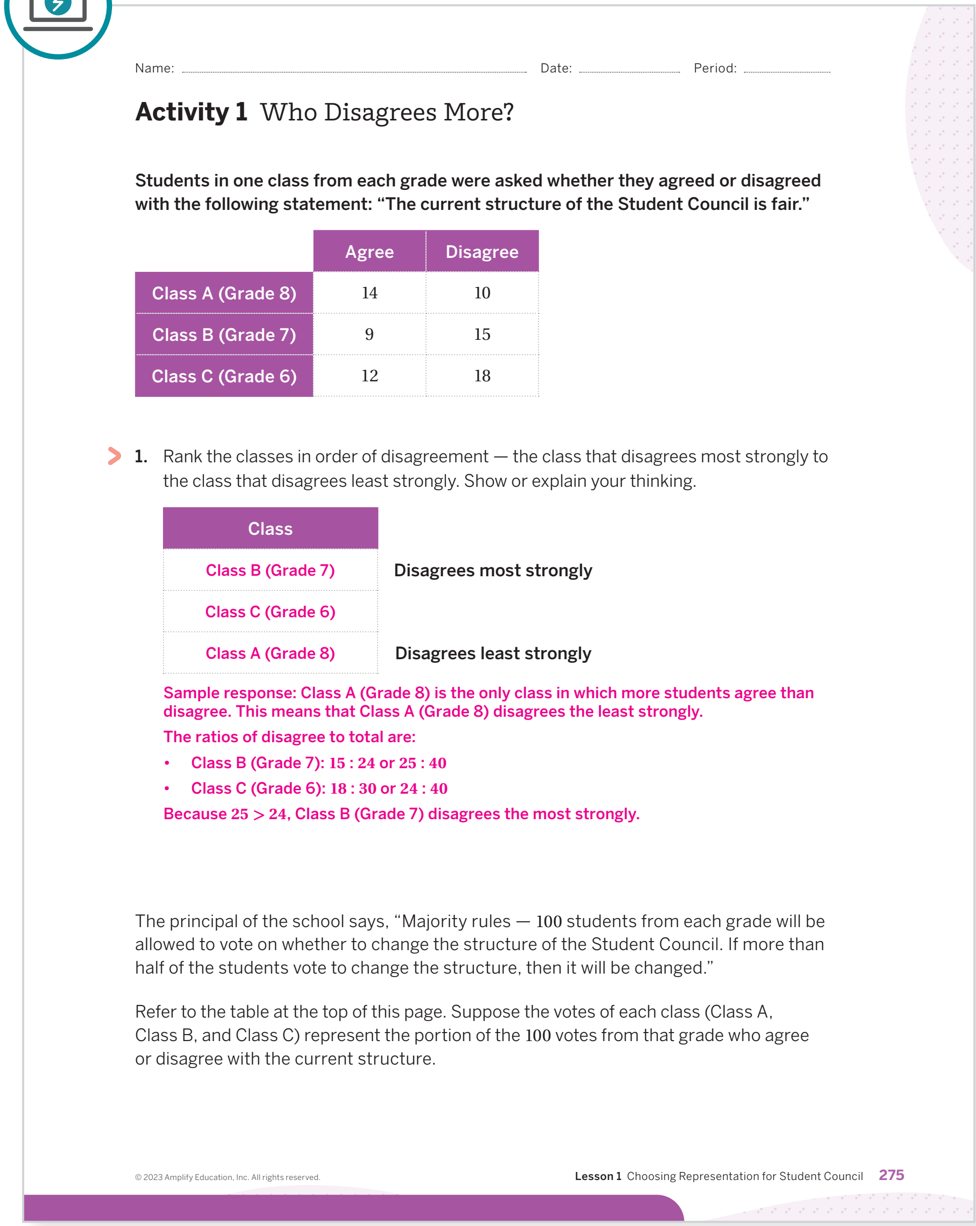
English Learners

Model for students an example of a question related to fairness based on the table. For example, "Should a student who has recently transferred to the school be allowed to run for office?"

Small Groups | 15 min

Activity 1 Who Disagrees More?

Students use their understanding of fractions and apply ratio reasoning from the previous unit to compare and rank how much three classes of students disagree with a statement.



Name: ______________________ Date: __________ Period: __________

Activity 1 Who Disagrees More?

Students in one class from each grade were asked whether they agreed or disagreed with the following statement: "The current structure of the Student Council is fair."

	Agree	Disagree
Class A (Grade 8)	14	10
Class B (Grade 7)	9	15
Class C (Grade 6)	12	18

1. Rank the classes in order of disagreement — the class that disagrees most strongly to the class that disagrees least strongly. Show or explain your thinking.

Class	
Class B (Grade 7)	**Disagrees most strongly**
Class C (Grade 6)	
Class A (Grade 8)	**Disagrees least strongly**

Sample response: Class A (Grade 8) is the only class in which more students agree than disagree. This means that Class A (Grade 8) disagrees the least strongly.

The ratios of disagree to total are:

- **Class B (Grade 7): 15 : 24 or 25 : 40**
- **Class C (Grade 6): 18 : 30 or 24 : 40**

Because 25 > 24, Class B (Grade 7) disagrees the most strongly.

The principal of the school says, "Majority rules — 100 students from each grade will be allowed to vote on whether to change the structure of the Student Council. If more than half of the students vote to change the structure, then it will be changed."

Refer to the table at the top of this page. Suppose the votes of each class (Class A, Class B, and Class C) represent the portion of the 100 votes from that grade who agree or disagree with the current structure.

1 Launch

Arrange students in groups of three. They will work with this same group to complete Activities 1 and 2.

2 Monitor

Help students get started by asking, "Does the class of eighth graders agree or disagree overall? What about the seventh graders? Sixth graders?"

Look for points of confusion:

- **Ranking the classes based solely on the number of students who disagreed (Problem 1)**. Ask, "Is the total number of students in each class the same? How could that impact how you interpret the results?"
- **Not knowing what to do with Class A because they agree overall (Problem 1)**. Ask the question a different way, "*How much* of the class disagrees?"
- **Thinking some votes in Problem 2 cannot be determined or including fraction or decimal votes.** Remind students that the values represent students and must be whole numbers. Ask, "How many votes need to be decided? How could the data from the corresponding class in Problem 1 help you?"

Look for points of confusion:

- Using equivalent fractions with a common total number of students to determine the order for ranking the classes in Problem 1.
- Using equivalent ratios with a common value to determine the order for ranking the classes in Problem 1.
- Using equivalent ratios to determine the values for the keep and change votes in Problem 2 with rounding decimals or applying repeated reasoning in addition to qualitative interpretations as necessary, such as for the seventh and eighth grades.
- Recognizing that 300 students get a vote, so a majority is more than 150 students.

Activity 1 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display the table from the Warm-up at the beginning of the activity. Display or read the statement aloud, "The current structure of the Student Council is fair." Before presenting the table that shows the number of students who agree or disagree with the statement, use the ***Poll the Class*** routine to ask students if they agree or disagree with the statement. This will allow students greater engagement in the activity. Suggest that students determine the total number of students in each class as they consider how they will approach Problem 1.

Math Language Development

MLR8: Discussion Supports

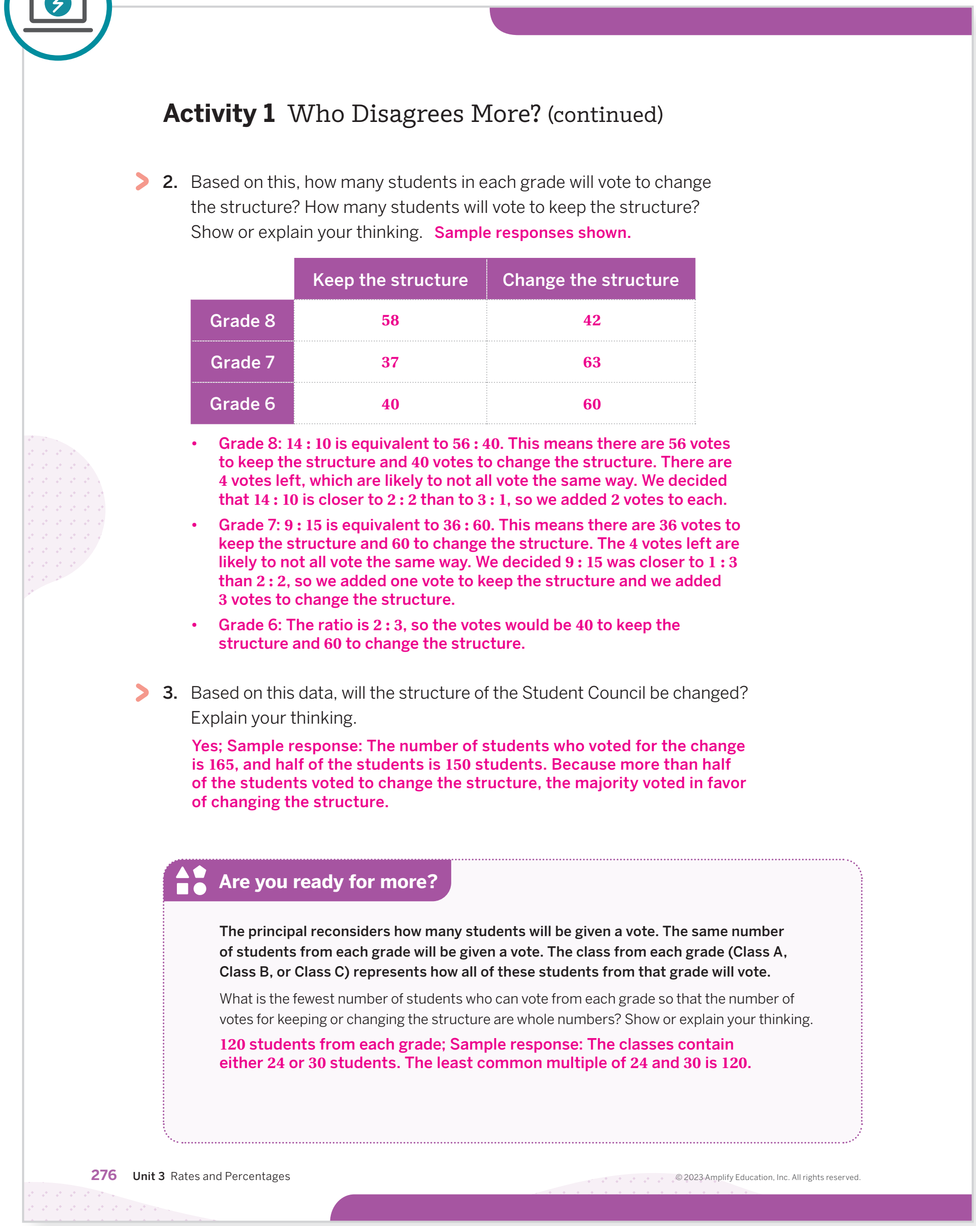
During the Connect, as students share how they determined the missing values in the table from Problem 2, emphasize student reasoning around how they used equivalent ratios. Ask, "How did using equivalent ratios help you determine how many students in each grade will vote to change the structure?" Sample response: I looked for equivalent ratios that had a sum that was as close to 100 as possible.

English Learners

Display the ratios 14 : 10 and 56 : 40 for Grade 6 and annotate them as *equivalent ratios*. Do the same for Grades 7 and 8.

Activity 1 Who Disagrees More? (continued)

Students use their understanding of fractions and apply ratio reasoning from the previous unit to compare and rank how much three classes of students disagree with a statement.



Activity 1 Who Disagrees More? (continued)

2. Based on this, how many students in each grade will vote to change the structure? How many students will vote to keep the structure? Show or explain your thinking. **Sample responses shown.**

	Keep the structure	Change the structure
Grade 8	58	42
Grade 7	37	63
Grade 6	40	60

- Grade 8: 14 : 10 is equivalent to 56 : 40. This means there are 56 votes to keep the structure and 40 votes to change the structure. There are 4 votes left, which are likely to not all vote the same way. We decided that 14 : 10 is closer to 2 : 2 than to 3 : 1, so we added 2 votes to each.
- Grade 7: 9 : 15 is equivalent to 36 : 60. This means there are 36 votes to keep the structure and 60 to change the structure. The 4 votes left are likely to not all vote the same way. We decided 9 : 15 was closer to 1 : 3 than 2 : 2, so we added one vote to keep the structure and we added 3 votes to change the structure.
- Grade 6: The ratio is 2 : 3, so the votes would be 40 to keep the structure and 60 to change the structure.

3. Based on this data, will the structure of the Student Council be changed? Explain your thinking.

Yes; Sample response: The number of students who voted for the change is 165, and half of the students is 150 students. Because more than half of the students voted to change the structure, the majority voted in favor of changing the structure.

Are you ready for more?

The principal reconsiders how many students will be given a vote. The same number of students from each grade will be given a vote. The class from each grade (Class A, Class B, or Class C) represents how all of these students from that grade will vote.

What is the fewest number of students who can vote from each grade so that the number of votes for keeping or changing the structure are whole numbers? Show or explain your thinking.

120 students from each grade; Sample response: The classes contain either 24 or 30 students. The least common multiple of 24 and 30 is 120.

276 **Unit 3** Rates and Percentages

3 Connect

Display the table from Problem 2.

Have groups of students share how they determined the missing values in the table, focusing on the seventh and eighth grade values, how they handled fraction or decimal values, and how they assigned remaining votes.

Ask:

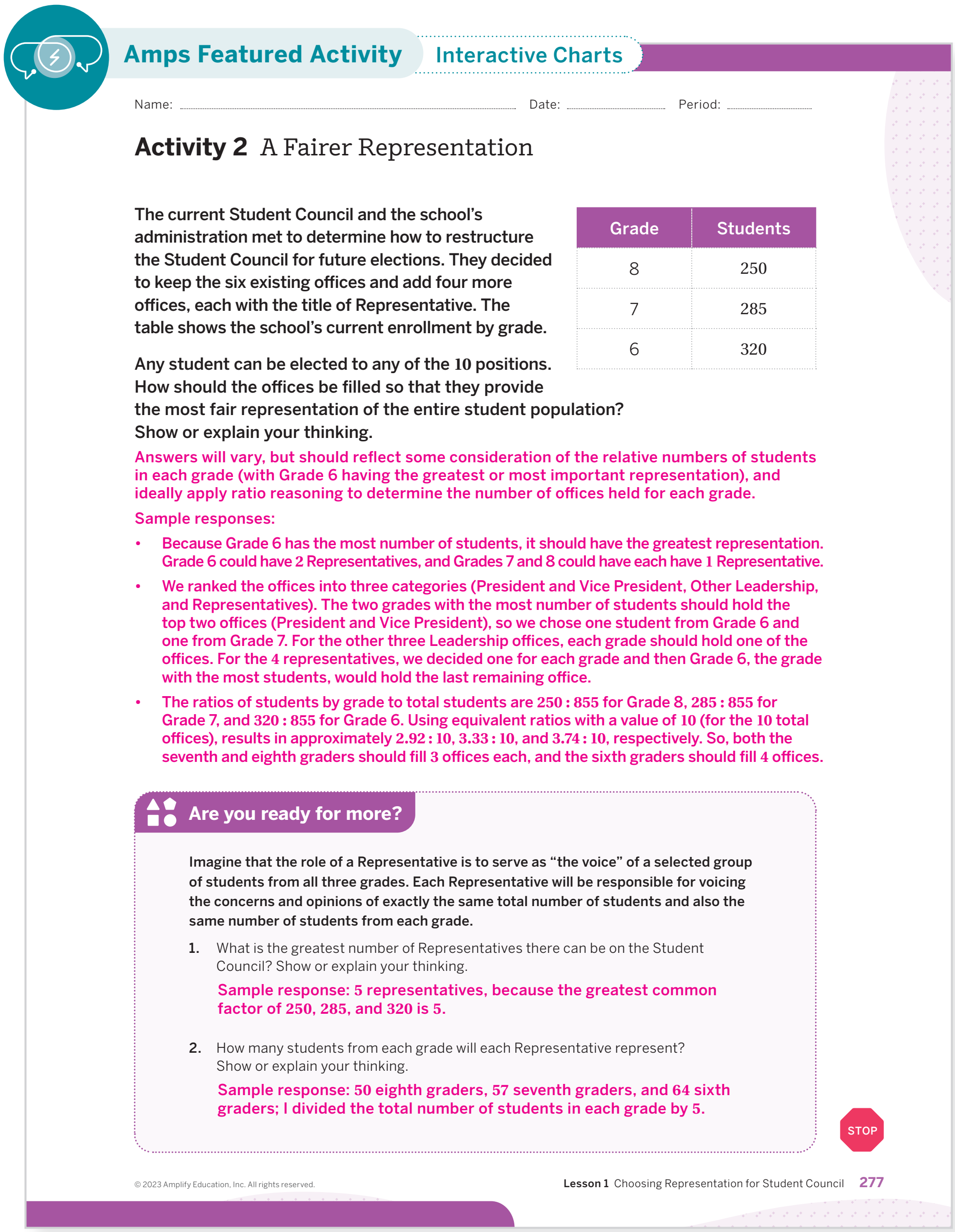
- "Will all of the students be happy with the results? Who might not be?"
- "If the principal said a 'supermajority' of two-thirds of the votes was needed, would the outcome be the same?"

Highlight that equivalent ratios could be used in two different ways: to compare ratios (Problem 1) and to determine missing values relative to a known total (Problem 2). Because equivalent ratios are related by common factors, they preserve the relationship between the votes. This means that if a class agreed, then a grade would agree, and therefore the class and the grade would both agree just as strongly. Many elections (or votes on issues) rely on a simple majority of more than half, but others require a supermajority of two-thirds or three-fourths.

Small Groups | 15 min

Activity 2 A Fairer Representation

Students revisit fair representation on the middle school Student Council by using current enrollment data to help determine the most fair and representative way to fill ten positions.



Amps Featured Activity Interactive Charts

Name: ______ Date: ______ Period: ______

Activity 2 A Fairer Representation

The current Student Council and the school's administration met to determine how to restructure the Student Council for future elections. They decided to keep the six existing offices and add four more offices, each with the title of Representative. The table shows the school's current enrollment by grade.

Grade	Students
8	250
7	285
6	320

Any student can be elected to any of the 10 positions. How should the offices be filled so that they provide the most fair representation of the entire student population? Show or explain your thinking.

Answers will vary, but should reflect some consideration of the relative numbers of students in each grade (with Grade 6 having the greatest or most important representation), and ideally apply ratio reasoning to determine the number of offices held for each grade.

Sample responses:

- **Because Grade 6 has the most number of students, it should have the greatest representation. Grade 6 could have 2 Representatives, and Grades 7 and 8 could have each have 1 Representative.**
- **We ranked the offices into three categories (President and Vice President, Other Leadership, and Representatives). The two grades with the most number of students should hold the top two offices (President and Vice President), so we chose one student from Grade 6 and one from Grade 7. For the other three Leadership offices, each grade should hold one of the offices. For the 4 representatives, we decided one for each grade and then Grade 6, the grade with the most students, would hold the last remaining office.**
- **The ratios of students by grade to total students are 250 : 855 for Grade 8, 285 : 855 for Grade 7, and 320 : 855 for Grade 6. Using equivalent ratios with a value of 10 (for the 10 total offices), results in approximately 2.92 : 10, 3.33 : 10, and 3.74 : 10, respectively. So, both the seventh and eighth graders should fill 3 offices each, and the sixth graders should fill 4 offices.**

Are you ready for more?

Imagine that the role of a Representative is to serve as "the voice" of a selected group of students from all three grades. Each Representative will be responsible for voicing the concerns and opinions of exactly the same total number of students and also the same number of students from each grade.

1. What is the greatest number of Representatives there can be on the Student Council? Show or explain your thinking.

   Sample response: 5 representatives, because the greatest common factor of 250, 285, and 320 is 5.

2. How many students from each grade will each Representative represent? Show or explain your thinking.

   Sample response: 50 eighth graders, 57 seventh graders, and 64 sixth graders; I divided the total number of students in each grade by 5.

STOP

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Lesson 1 Choosing Representation for Student Council 277

1 Launch

Keep students in the same groups. Read the problem and answer any questions, making it clear *who* is elected is not important, only what grade they are in.

2 Monitor

Help students get started by asking, "Can there be an equal number of positions from all three grades? How would you decide which grades should have more or less?"

Look for points of confusion:

- **Not considering or using the enrollment data.** Ask, "What if there were 50 eighth grade students and 400 students in both the sixth and seventh grades? Would that change your thinking?"

Look for productive strategies:

- Listing the number of positions for each grade level by simple qualitative analysis, such as sixth grade has more students, so it should get one more position.
- Listing the number of positions for each grade level by comparing ratios of the grade levels.
- Considering both the number of positions and relative roles of each, such as by using a weighted system.

3 Connect

Ask, or poll the groups:

- "Should one grade have more representation?"
- "Should certain positions only be for certain people, meaning by grade?"

Have groups of students share the distribution of representation by grade that they determined, and explain how they made decisions and determined their final numbers. Allow others to respond with any constructive criticism.

Highlight that ratios and equivalent ratios are one way to describe fair representation.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use interactive tables and sketches to help organize their thinking.

Accessibility: *Vary Demands to Optimize Challenge*

Chunk this task into smaller, more manageable parts by having students begin with *either* the original six positions or only the four new Representative positions. Have them use the numbers to make decisions and claims, and then repeat for the other set of positions.

Math Language Development

MLR1: Stronger and Clearer Each Time

Use this routine to support students in their written explanation for how the offices should be filled so that they provide a fair representation. Give students time to write a draft response before meeting with 2–3 partners to give and receive feedback. After receiving feedback, allow students time to write an improved response.

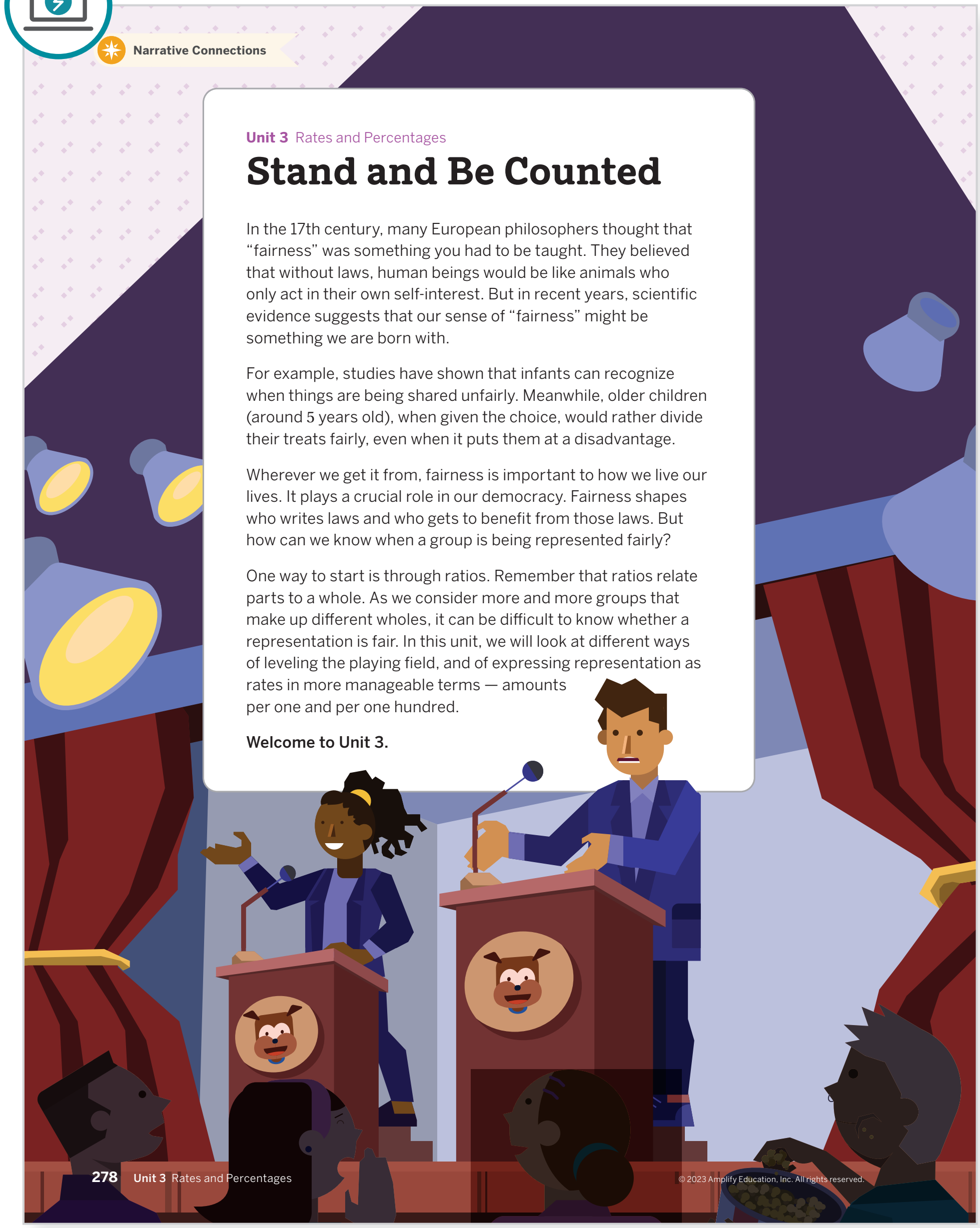
English Learners

Allow students to write their first draft in their primary language and use structured pairing with peers who speak the same primary language to give and receive feedback.

Whole Class | 5 min

Summary Stand and Be Counted

Review and synthesize what fair representation could mean and how ratio reasoning can be used to help make decisions about fairness and representation.



Narrative Connections

Unit 3 Rates and Percentages

Stand and Be Counted

In the 17th century, many European philosophers thought that "fairness" was something you had to be taught. They believed that without laws, human beings would be like animals who only act in their own self-interest. But in recent years, scientific evidence suggests that our sense of "fairness" might be something we are born with.

For example, studies have shown that infants can recognize when things are being shared unfairly. Meanwhile, older children (around 5 years old), when given the choice, would rather divide their treats fairly, even when it puts them at a disadvantage.

Wherever we get it from, fairness is important to how we live our lives. It plays a crucial role in our democracy. Fairness shapes who writes laws and who gets to benefit from those laws. But how can we know when a group is being represented fairly?

One way to start is through ratios. Remember that ratios relate parts to a whole. As we consider more and more groups that make up different wholes, it can be difficult to know whether a representation is fair. In this unit, we will look at different ways of leveling the playing field, and of expressing representation as rates in more manageable terms — amounts per one and per one hundred.

Welcome to Unit 3.

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Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Ask:

- "Do you think our Student Council is fair? If not, how could we make it more fair in our school?"
- "How do you think the ideas of fair representation on a Student Council relate to other elected positions or issues?"
- "Aside from grade levels, what are some other ways you could describe groups of students (or a general population) that would make sense to think about when considering fair representation?"

Highlight how there may not be one universally-agreed-upon way to determine fairness, but it's important to think about representation. Students can use population numbers and ratios to consider those things.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

- "How did you use the work on ratios from Unit 2 in this lesson?"
- "How do words, such as *fair* and *representation*, relate to numbers and math?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of fair representation by analyzing a table and summarizing the data.

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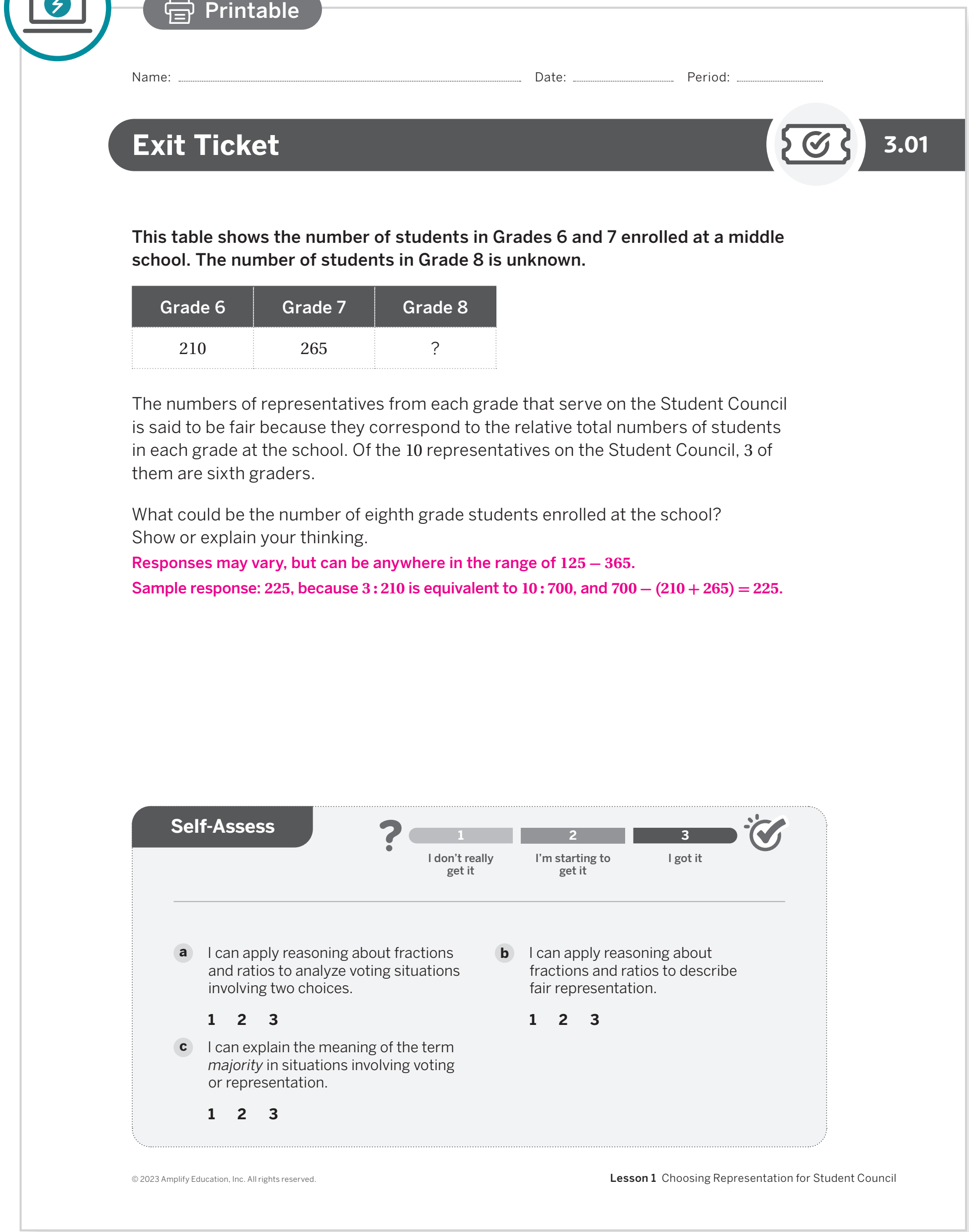


Name: ______ Date: ______ Period: ______

Exit Ticket 3.01

This table shows the number of students in Grades 6 and 7 enrolled at a middle school. The number of students in Grade 8 is unknown.

Grade 6	Grade 7	Grade 8
210	265	?

The numbers of representatives from each grade that serve on the Student Council is said to be fair because they correspond to the relative total numbers of students in each grade at the school. Of the 10 representatives on the Student Council, 3 of them are sixth graders.

What could be the number of eighth grade students enrolled at the school? Show or explain your thinking.

Responses may vary, but can be anywhere in the range of $125 - 365$.

Sample response: 225, because $3 : 210$ is equivalent to $10 : 700$, and $700 - (210 + 265) = 225$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can apply reasoning about fractions and ratios to analyze voting situations involving two choices.

1 2 3

b I can apply reasoning about fractions and ratios to describe fair representation.

1 2 3

c I can explain the meaning of the term *majority* in situations involving voting or representation.

1 2 3

Lesson 1 Choosing Representation for Student Council

Success looks like . . .

- **Language Goal:** Applying reasoning about fractions and ratios to analyze voting situations involving two choices. **(Speaking and Listening, Writing)**
- **Language Goal:** Applying reasoning about fractions and ratios to describe or critique representation of subgroups of a population. **(Speaking and Listening, Writing)**
  - » Calculating ratios to determine the number of eighth grade students at the school.
- **Language Goal:** Comprehending and explaining the term *majority*. **(Speaking and Listening)**

Suggested next steps

If students cannot determine an appropriate number of eighth grade students, consider:

- Asking, "Can you use the given information to determine how many total students might be in the school? How does that help you think about the number of eighth graders?"
- Reviewing the ratio strategies used in Activity 1 or Activity 2.
- Assigning Practice Problem 2.

Professional Learning

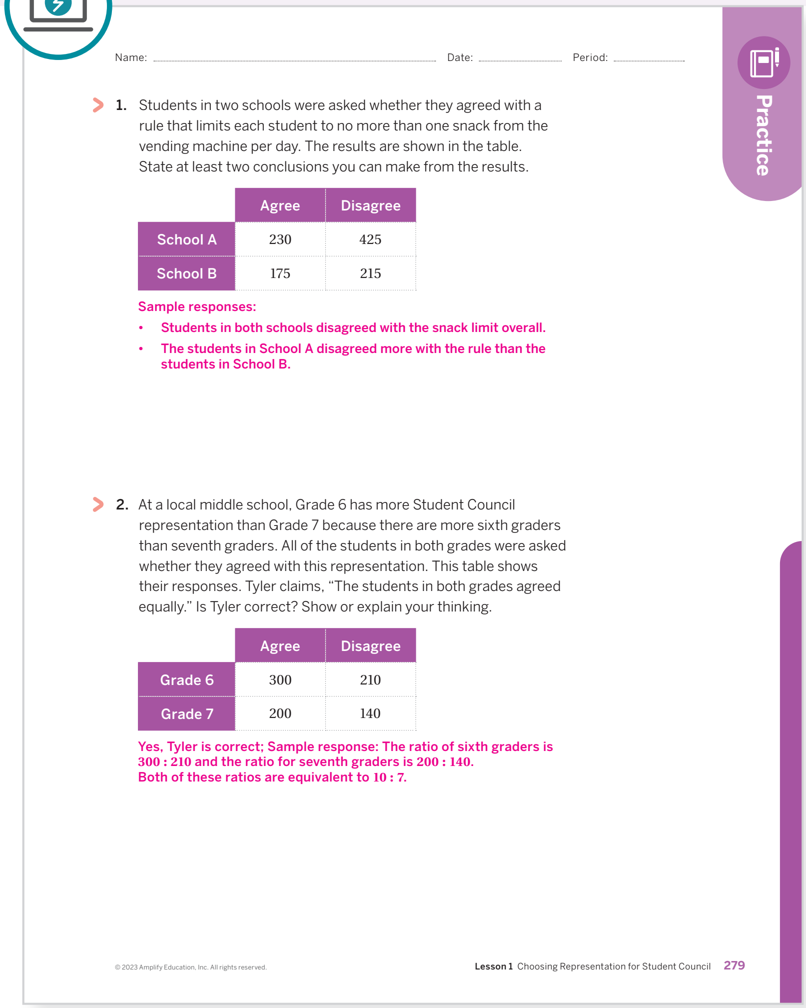
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The instructional goal for this lesson was to apply reasoning about ratios and percentages to analyze voting situations involving two choices. How well did students accomplish this? What did you specifically do to help students accomplish it?
- What trends do you see in participation?

Practice

Independent



Name: Date: Period:

Practice

1. Students in two schools were asked whether they agreed with a rule that limits each student to no more than one snack from the vending machine per day. The results are shown in the table. State at least two conclusions you can make from the results.

	Agree	Disagree
School A	230	425
School B	175	215

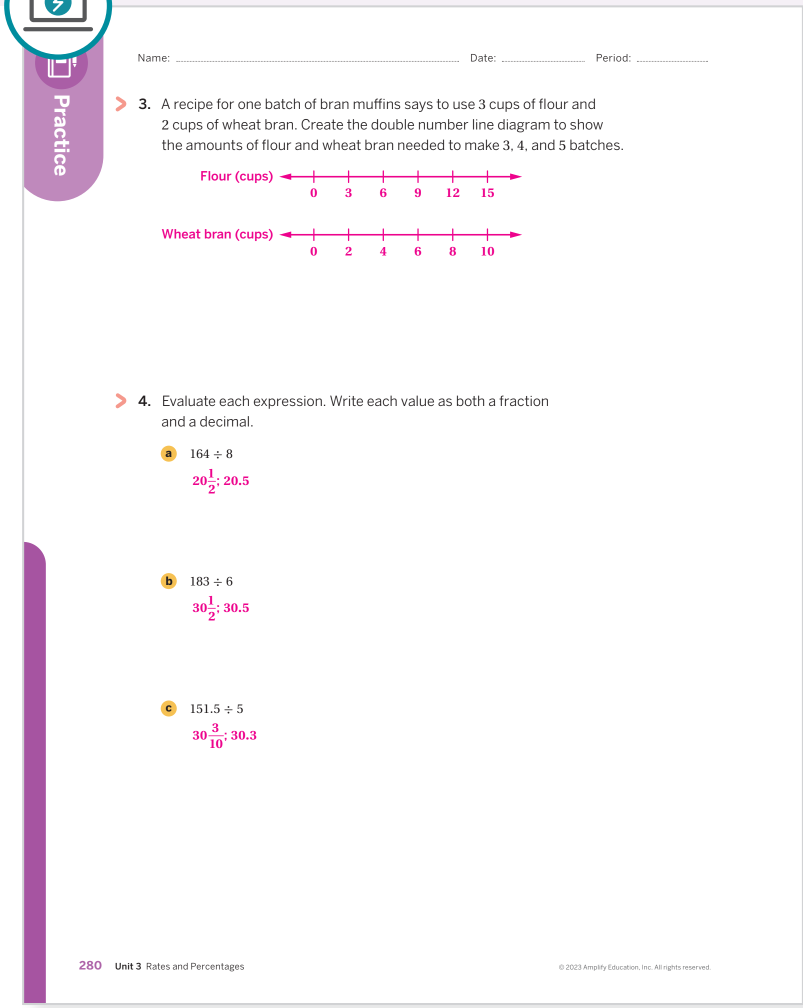
Sample responses:

- **Students in both schools disagreed with the snack limit overall.**
- **The students in School A disagreed more with the rule than the students in School B.**

2. At a local middle school, Grade 6 has more Student Council representation than Grade 7 because there are more sixth graders than seventh graders. All of the students in both grades were asked whether they agreed with this representation. This table shows their responses. Tyler claims, "The students in both grades agreed equally." Is Tyler correct? Show or explain your thinking.

	Agree	Disagree
Grade 6	300	210
Grade 7	200	140

Yes, Tyler is correct; Sample response: The ratio of sixth graders is 300 : 210 and the ratio for seventh graders is 200 : 140. Both of these ratios are equivalent to 10 : 7.



Name: Date: Period:

Practice

3. A recipe for one batch of bran muffins says to use 3 cups of flour and 2 cups of wheat bran. Create the double number line diagram to show the amounts of flour and wheat bran needed to make 3, 4, and 5 batches.

Flour (cups)	0	3	6	9	12	15
Wheat bran (cups)	0	2	4	6	8	10

4. Evaluate each expression. Write each value as both a fraction and a decimal.

a. $164 \div 8$

$20\frac{1}{2}$; 20.5

b. $183 \div 6$

$30\frac{1}{2}$; 30.5

c. $151.5 \div 5$

$30\frac{3}{10}$; 30.3

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
Spiral	3	Unit 2 Lesson 12	2
Formative	4	Unit 3 Lesson 2	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

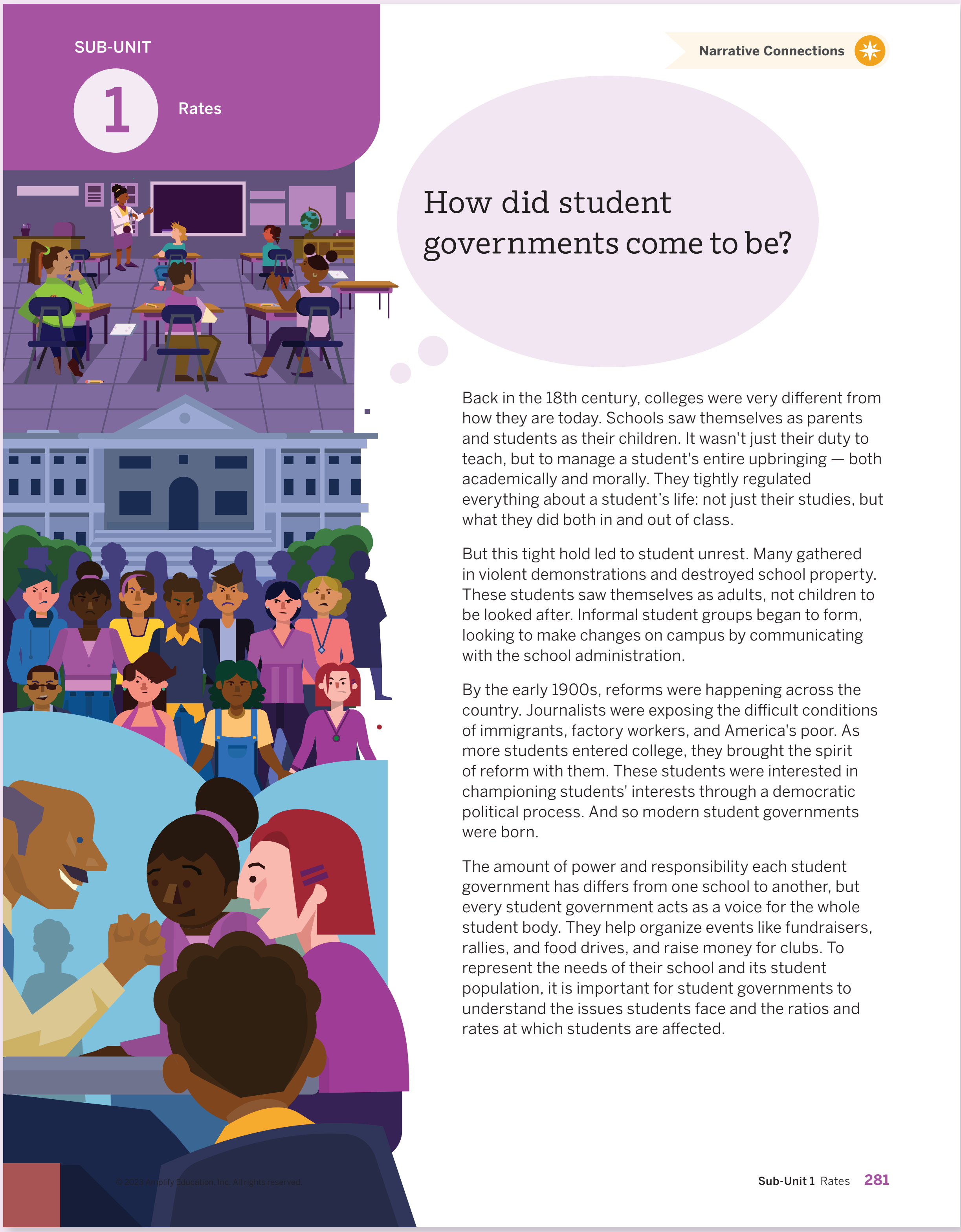
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 1
Rates

Whole Class

In this Sub-Unit, students build on prior work with price and speed to expand their understanding of rate, moving to missing value and comparison problems encountered by a student council.



SUB-UNIT

1 Rates

Narrative Connections

How did student governments come to be?

Back in the 18th century, colleges were very different from how they are today. Schools saw themselves as parents and students as their children. It wasn't just their duty to teach, but to manage a student's entire upbringing — both academically and morally. They tightly regulated everything about a student's life: not just their studies, but what they did both in and out of class.

But this tight hold led to student unrest. Many gathered in violent demonstrations and destroyed school property. These students saw themselves as adults, not children to be looked after. Informal student groups began to form, looking to make changes on campus by communicating with the school administration.

By the early 1900s, reforms were happening across the country. Journalists were exposing the difficult conditions of immigrants, factory workers, and America's poor. As more students entered college, they brought the spirit of reform with them. These students were interested in championing students' interests through a democratic political process. And so modern student governments were born.

The amount of power and responsibility each student government has differs from one school to another, but every student government acts as a voice for the whole student body. They help organize events like fundraisers, rallies, and food drives, and raise money for clubs. To represent the needs of their school and its student population, it is important for student governments to understand the issues students face and the ratios and rates at which students are affected.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how rates can guide a student government's decision-making in the following places:

- **Lesson 2, Activity 2:** Profits From School Spirit Sales
- **Lesson 6, Activity 1:** Planning a Celebration
- **Lesson 7, Activity 2:** Card Sort: Who Is Offering a Better Deal?

UNIT 3 | **LESSON 2**

How Much for One?

Let's use ratios to describe how much items cost.

Focus

Goals

1. **Language Goal:** Calculate equivalent ratios between prices and quantities and present the solution method using multiple representations. **(Speaking and Listening)**
2. **Language Goal:** Calculate unit price and express it using the word *per.* **(Speaking and Listening, Writing)**
3. Understand the phrase *at this rate* indicates that equivalent ratios are involved.

Coherence

Today

Students are introduced to the concept of unit rate in the likely familiar context of unit price, using the word *per* to refer to the cost of one item. The phrases *same rate* and *at this rate* are used to indicate that all ratios of price to quantity or amount in a scenario are equivalent. In determining unit prices in a variety of scenarios, they notice that unit prices are useful for computing prices of other amounts. Students choose from double number lines and other ratio representations to support their reasoning. They also continue to use precision in stating the units for the numbers in ratios.

Previously

In Unit 2, students used double number lines to determine equivalent ratios, and they were introduced to the term *rate* informally.

Coming Soon

In Lesson 3, students will continue to explore unit rates and how equivalent ratios and ratios in which one quantity is 1 can be used to determine unknown amounts within the context of constant speed.

Rigor

- Students build on their **conceptual understanding** of rates from Unit 2 where the term *same rate* referred to equivalent ratios and price.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- rulers

Math Language Development

New words

- *rate**
- *unit rate*

Review words

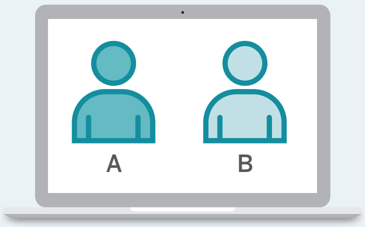
- *double number line*
- *per*

*Students may confuse the noun form of *rate* with the verb form *to rate*. Be ready to address the differences between them and let them know that this unit focuses on the mathematical definition.

Amps | Featured Activity

Activity 2
Using Work From Previous Slides

In the second activity, students use the rates they computed in the first activity. It's their work, so they get to hold onto it!



Building Math Identity and Community

Connecting to Mathematical Practices

With new vocabulary and concepts related to sales, students may be lacking in self-confidence as they approach Activity 2. Remind students that once they have determined how to find the total profit for one item, they can use a similar structure to determine the total profit for any item. Explain that the pattern they see is the same pattern that will be used for any profit problems in the future.

Modifications to Pacing

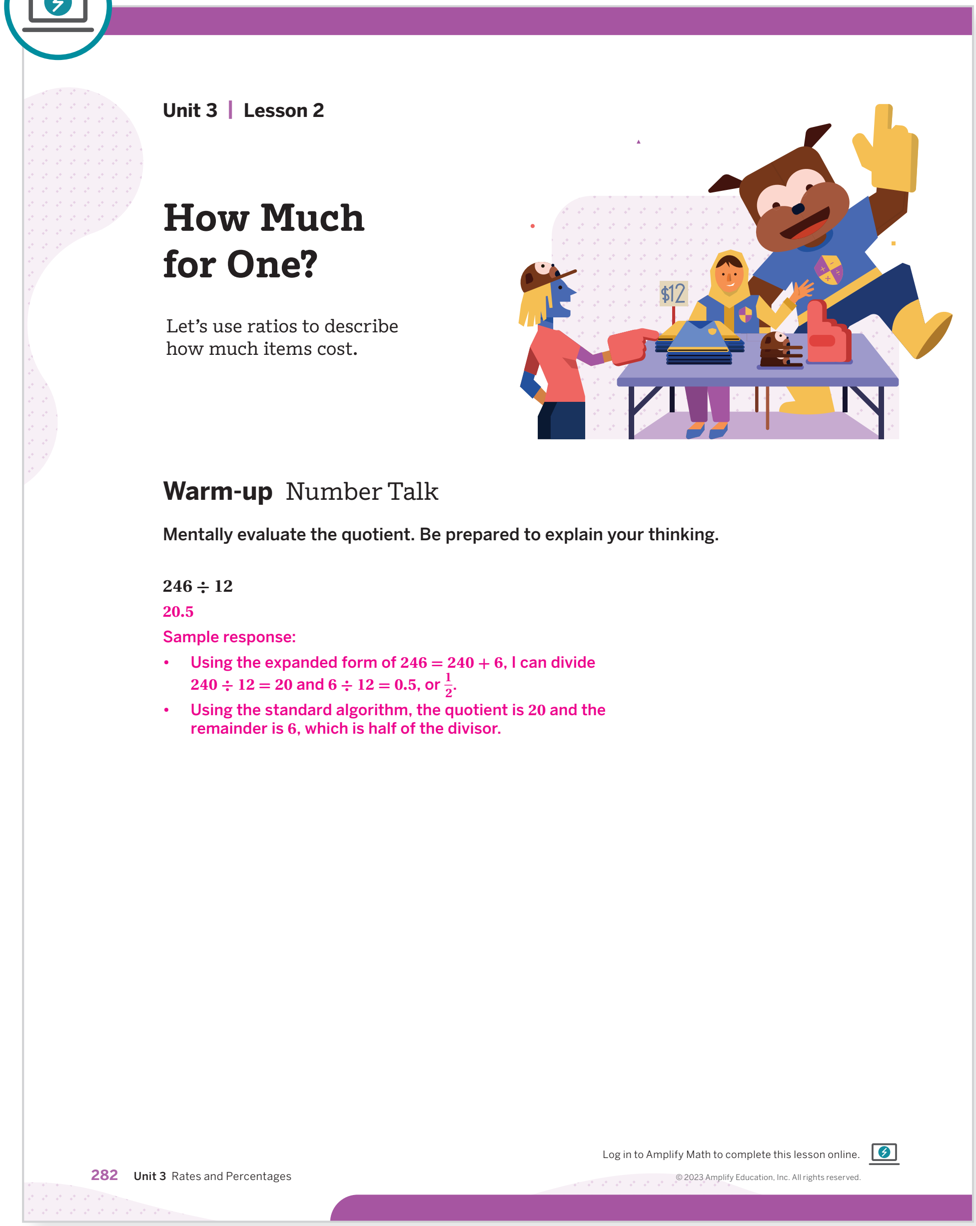
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 2 may be omitted. Omitting Problem 2 in this activity will also effectively omit Problem 2 in Activity 2.
- In **Activity 2**, Problem 2 may be omitted.

Independent | 10 min

Warm-up Number Talk

Students mentally compute a division problem and interpret the remainder as a fraction or decimal to find structure and pattern in division.



Unit 3 | Lesson 2

How Much for One?

Let's use ratios to describe how much items cost.

Warm-up Number Talk

Mentally evaluate the quotient. Be prepared to explain your thinking.

$246 \div 12$

20.5

Sample response:

- **Using the expanded form of $246 = 240 + 6$, I can divide $240 \div 12 = 20$ and $6 \div 12 = 0.5$, or $\frac{1}{2}$.**
- **Using the standard algorithm, the quotient is 20 and the remainder is 6, which is half of the divisor.**

Log in to Amplify Math to complete this lesson online.

282 Unit 3 Rates and Percentages

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1 Launch

Display the expression for the class.

2 Monitor

Help students get started by having them write 246 in expanded form.

Look for points of confusion:

- **Struggling to interpret the remainder as a decimal or fraction.** Ask, "What can you do with that remainder 6? How does it relate to the divisor? How can the remainder be shown as a fraction?"

Look for points of confusion:

- Visualizing the dividend as $240 + 6$ and evaluating $240 \div 12$ and then $6 \div 12$.
- Recognizing that dividing 6 by 12 gives the fraction $\frac{6}{12}$, which is equivalent to $\frac{1}{2}$ or 0.5.
- Mentally extending the dividend to be 246.0, and then dividing 24 by 12, and 6 (or 6.0) by 12.

3 Connect

Have individual students share strategies used to divide mentally, focusing on what they did with the remainder of 6. Record student strategies as they are shared.

Ask, "How are the different ways of representing the remainder related?" They all show one half in some way.

Highlight that the remainder can be represented as a fraction or a decimal. The fraction $\frac{1}{2}$ is the same as the decimal 0.5.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share strategies for how they divided mentally, ask, "What did you do with the remainder of 6?" Encourage students to add on to their classmates' responses and use the mathematical terms they have learned in prior grades related to division: *quotient*, *remainder*, *divisor*, *dividend*, etc.

English Learners

Provide time for students to formulate what they will say with a partner before they share with the whole class.

Power-up

To power up students' ability to determine the quotient of two whole numbers, have students complete:

Recall that a division expression can be rewritten as a fraction. For example $1 \div 2 = \frac{1}{2}$.

Rewrite each division problem as a fraction, then simplify each fraction using common factors. Finally, write your fraction as an equivalent decimal.

- $12 \div 8$ $\frac{12}{8}$; $\frac{3}{2}$; 1.5
- $20 \div 50$ $\frac{20}{50}$; $\frac{2}{5}$; 0.2

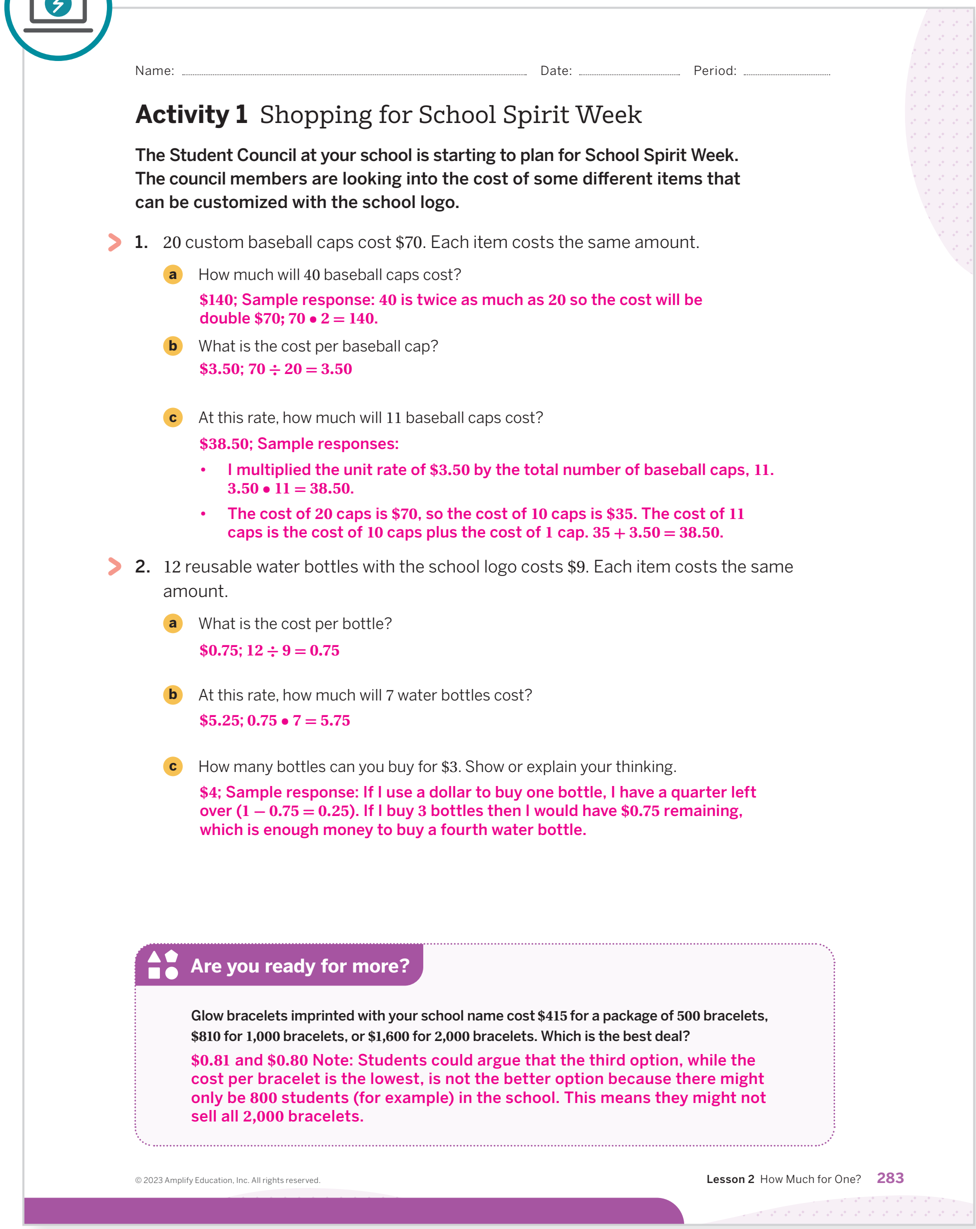
Use: Before the Warm-up.

Informed by: Performance on Lesson 2, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 2.

Pairs | 10 min

Activity 1 Shopping for School Spirit Week

Students determine the unit price of various items and use unit price as a constant rate to generate equivalent ratios.



Name: ____________ Date: ________ Period: ________

Activity 1 Shopping for School Spirit Week

The Student Council at your school is starting to plan for School Spirit Week. The council members are looking into the cost of some different items that can be customized with the school logo.

1. 20 custom baseball caps cost $70. Each item costs the same amount.

   a. How much will 40 baseball caps cost?

   $140; Sample response: 40 is twice as much as 20 so the cost will be double $70; 70 • 2 = 140.

   b. What is the cost per baseball cap?

   $3.50; 70 ÷ 20 = 3.50

   c. At this rate, how much will 11 baseball caps cost?

   $38.50; Sample responses:

   - **I multiplied the unit rate of $3.50 by the total number of baseball caps, 11. 3.50 • 11 = 38.50.**
   - **The cost of 20 caps is $70, so the cost of 10 caps is $35. The cost of 11 caps is the cost of 10 caps plus the cost of 1 cap. 35 + 3.50 = 38.50.**

2. 12 reusable water bottles with the school logo costs $9. Each item costs the same amount.

   a. What is the cost per bottle?

   $0.75; 12 ÷ 9 = 0.75

   b. At this rate, how much will 7 water bottles cost?

   $5.25; 0.75 • 7 = 5.75

   c. How many bottles can you buy for $3. Show or explain your thinking.

   $4; Sample response: If I use a dollar to buy one bottle, I have a quarter left over (1 − 0.75 = 0.25). If I buy 3 bottles then I would have $0.75 remaining, which is enough money to buy a fourth water bottle.

Are you ready for more?

Glow bracelets imprinted with your school name cost $415 for a package of 500 bracelets, $810 for 1,000 bracelets, or $1,600 for 2,000 bracelets. Which is the best deal?

$0.81 and $0.80 Note: Students could argue that the third option, while the cost per bracelet is the lowest, is not the better option because there might only be 800 students (for example) in the school. This means they might not sell all 2,000 bracelets.

Lesson 2 How Much for One? 283

1 Launch

Have students use the ***Think-Pair-Share*** routine. Give 1–2 minutes of individual think time before they share initial thoughts and then complete the problems with a partner.

2 Monitor

Help students get started by asking, "What is the ratio of ____ to dollars?"

Look for points of confusion:

- **Struggling to understand how the given numbers work together to represent the problem.** Encourage students to use a double number line to relate unit rates and equivalent ratios.
- **Not interpreting solutions that are not whole numbers.** Refer back to the Warm-up where a remainder was shown as a fraction or decimal.

Look for productive strategies:

- Representing the problems by using double number lines or tables.
- Using equivalent ratios to solve.
- Dividing the total cost by the number of items to determine the cost per one, and then multiplying to determine the cost of more than one item.
- Using ratio and rate language, such as *per, for each,* and *at that rate*.

3 Connect

Have students share their strategies and representations.

Highlight representations students used, such as double number lines. If none were used, choose one scenario to show how a double number line relates to unit price.

Define *unit rate* as how much one quantity changes when the other changes by 1. In the context of price, the cost of any number of items will be equivalent to the unit rate of price per item.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students whether they have ever had to determine the cost of one item when the cost of several items are known. Ask them what strategies they used to determine the unit cost and what assumption(s) they needed to make. Sample responses: Divide the total cost by the number of items. I need to assume the cost of each item is always the same.

Accessibility: *Vary Demands to Optimize Challenge*

Have students focus on Problems 1b, 2b, and 3b. Rephrase the questions to ask for the ratio, where the number of items is 1. Remind them that each ratio should be equivalent to the given ratio.

Math Language Development

MLR5: Co-craft Questions

During the Launch, have students read the introductory scenario for Problem 1. Ask, "What mathematical questions could you ask about this scenario?" Support students' use of conversation skills to generate and refine their questions collaboratively by seeking clarity and revoicing oral responses.

English Learners

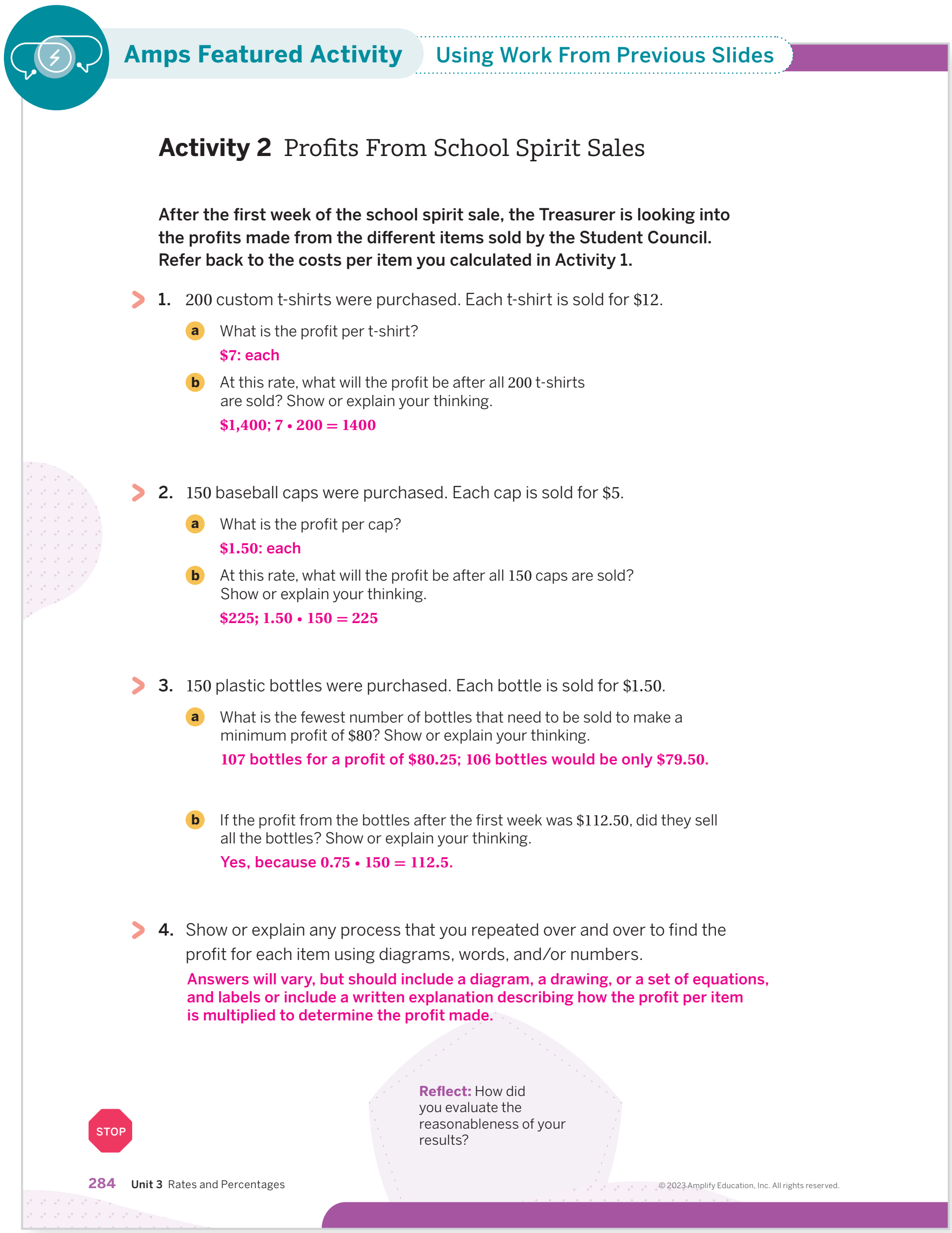
Before students begin to co-craft their own questions, display sample mathematical questions for Problem 1, such as, "If 20 t-shirts cost $100, how much will 40 t-shirts cost? 10 t-shirts?"

Pairs | 15 min

Activity 2 Profits from School Spirit Sales

Students further connect the ideas of cost and unit rate by looking at simple profits made from sales of the School Spirit Week items from Activity 1.

Amps Featured Activity Using Work From Previous Slides



Activity 2 Profits From School Spirit Sales

After the first week of the school spirit sale, the Treasurer is looking into the profits made from the different items sold by the Student Council. Refer back to the costs per item you calculated in Activity 1.

1. 200 custom t-shirts were purchased. Each t-shirt is sold for $12.

- **a** What is the profit per t-shirt?
  $7: each
- **b** At this rate, what will the profit be after all 200 t-shirts are sold? Show or explain your thinking.
  $1,400; 7 • 200 = 1400

2. 150 baseball caps were purchased. Each cap is sold for $5.

- **a** What is the profit per cap?
  $1.50: each
- **b** At this rate, what will the profit be after all 150 caps are sold? Show or explain your thinking.
  $225; 1.50 • 150 = 225

3. 150 plastic bottles were purchased. Each bottle is sold for $1.50.

- **a** What is the fewest number of bottles that need to be sold to make a minimum profit of $80? Show or explain your thinking.
  107 bottles for a profit of $80.25; 106 bottles would be only $79.50.
- **b** If the profit from the bottles after the first week was $112.50, did they sell all the bottles? Show or explain your thinking.
  Yes, because 0.75 • 150 = 112.5.

4. Show or explain any process that you repeated over and over to find the profit for each item using diagrams, words, and/or numbers.
Answers will vary, but should include a diagram, a drawing, or a set of equations, and labels or include a written explanation describing how the profit per item is multiplied to determine the profit made.

STOP

Reflect: How did you evaluate the reasonableness of your results?

284 **Unit 3** Rates and Percentages

1 Launch

Clarify that *profit* means "the difference between the cost to purchase an item and the amount for which the item is sold."

2 Monitor

Help students get started by asking, "What are the amounts you need to know to determine the profit made from one t-shirt?"

Look for points of confusion:

- **Using sale price and not profit.** Ask, "What are you trying to find here?" The total profit.
- **Not attending to the difference between knowing the amount sold versus the profit (Problem 3).** Ask, "Which do you know here, bottles or dollars? How would your calculation be different?"
- **Thinking how many bottles at the *price* of $1.50 they would need to sell for an $80 profit.** Ask the reminding question, "What does profit mean?"

Look for productive strategies:

- Multiplying profit per one item by number sold.
- Recognizing that profit per one and profit for more than one are equivalent ratios. If students correctly subtract cost from sales, ask them to connect their work to ratios and rates.

3 Connect

Display each problem, one at a time.

Have pairs of students share the strategies used for solving the problems, focusing on the use of equivalent ratios, double number lines, or tables. Note if a student uses the term *unit profit* and ask what this means.

Ask, "How is profit per item similar to unit price?"

Highlight the repeated structure of multiplying the profit per item by the number of items sold. The profit per item is the common factor to determine the total profit from each item.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

For Problem 1a, suggest that the t-shirts were sold for $6 instead, which means a profit of $1 per shirt. Have students explain how they would determine a response for Problem 1b in that case. Then have them attempt Problems 1–3 as written. Have students use the Amps slides for this activity, in which they can use interactive tables and sketches to help organize their thinking.

Extension: *Math Enrichment*

Have students come up with a fourth spirit item to sell, and have them state a reasonable cost for purchasing the item. Then ask them to determine the sale price if the goal is to make a profit of $400 from selling 125 items. Answers may vary.

Math Language Development

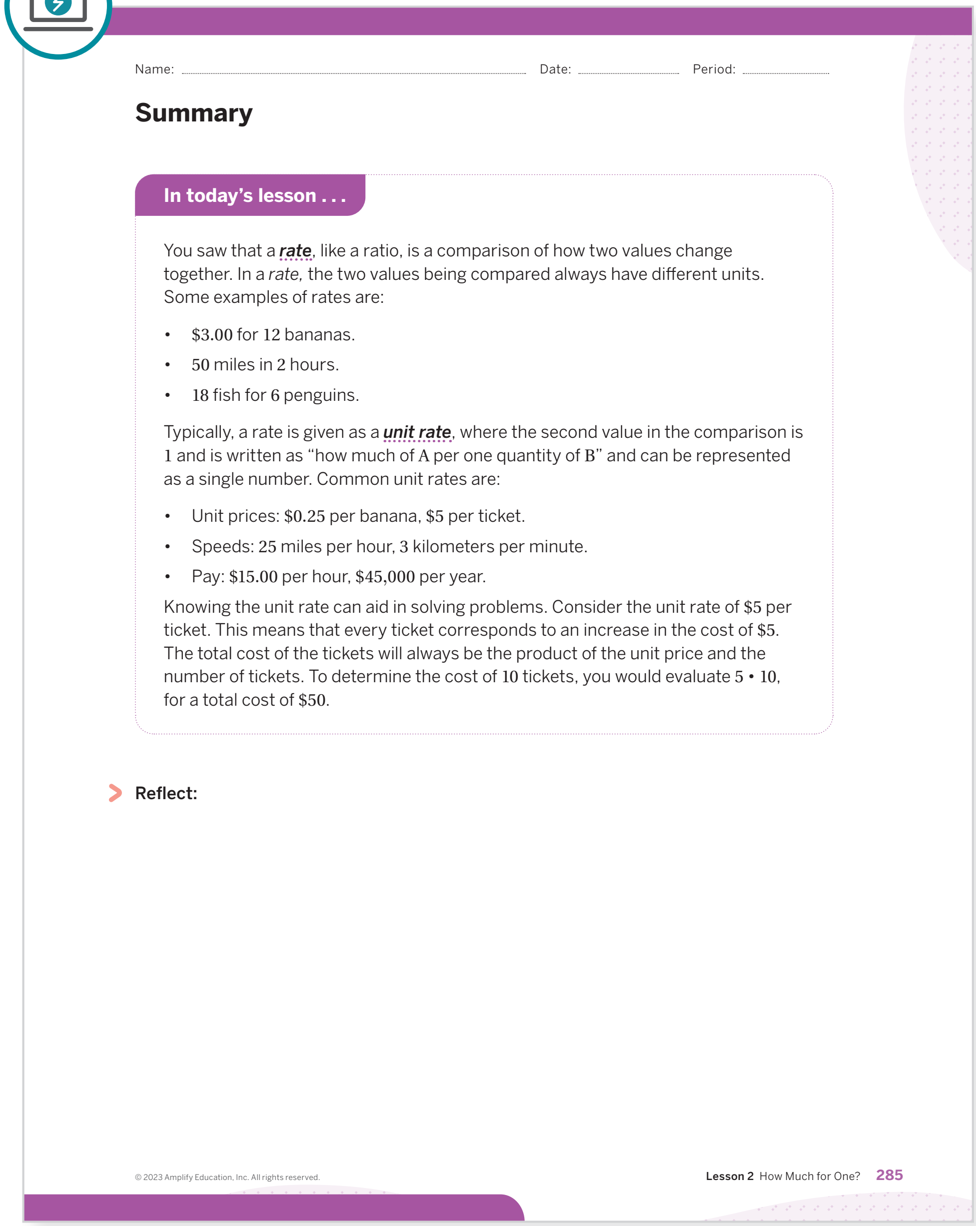
MLR2: Collect and Display

While students work, circulate to each pair and listen for students' use of *per* and *at this rate*, noting how the uses are different. *Per* refers to one item, and *at this rate* refers to multiples of an item and is the same as calculating equivalent ratios. Start a class display of mathematical words and phrases related to rates, such as *per* and *at this rate*. Encourage students to refer to the display during future discussions in this unit.

Whole Class | 5 min

Summary

Review and synthesize possible strategies for determining a unit price and how unit price can be used in scenarios involving buying multiple items.



Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that a ***rate***, like a ratio, is a comparison of how two values change together. In a *rate*, the two values being compared always have different units. Some examples of rates are:

- $3.00 for 12 bananas.
- 50 miles in 2 hours.
- 18 fish for 6 penguins.

Typically, a rate is given as a ***unit rate***, where the second value in the comparison is 1 and is written as "how much of A per one quantity of B" and can be represented as a single number. Common unit rates are:

- Unit prices: $0.25 per banana, $5 per ticket.
- Speeds: 25 miles per hour, 3 kilometers per minute.
- Pay: $15.00 per hour, $45,000 per year.

Knowing the unit rate can aid in solving problems. Consider the unit rate of $5 per ticket. This means that every ticket corresponds to an increase in the cost of $5. The total cost of the tickets will always be the product of the unit price and the number of tickets. To determine the cost of 10 tickets, you would evaluate 5 • 10, for a total cost of $50.

Reflect:

Synthesize

Display the Summary.

Ask:

- "What does unit price mean in this scenario?" The cost for one item.
- "Once you know the unit price, what else can you determine?" The cost of any number of items, or the number of items that can be purchased for any amount of dollars.

Have students share how they used the unit rate.

Highlight the strategies used: division, multiplication, and equivalent ratios. You may want to point out to students that, by multiplying, they are finding part of an equivalent ratio.

Define *rate* (in general) as a comparison of how two quantities change together.

Formalize vocabulary:

- *rate*
- *unit rate*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How were equivalent ratios helpful in this lesson?"

Math Language Development

MLR2: Collect and Display

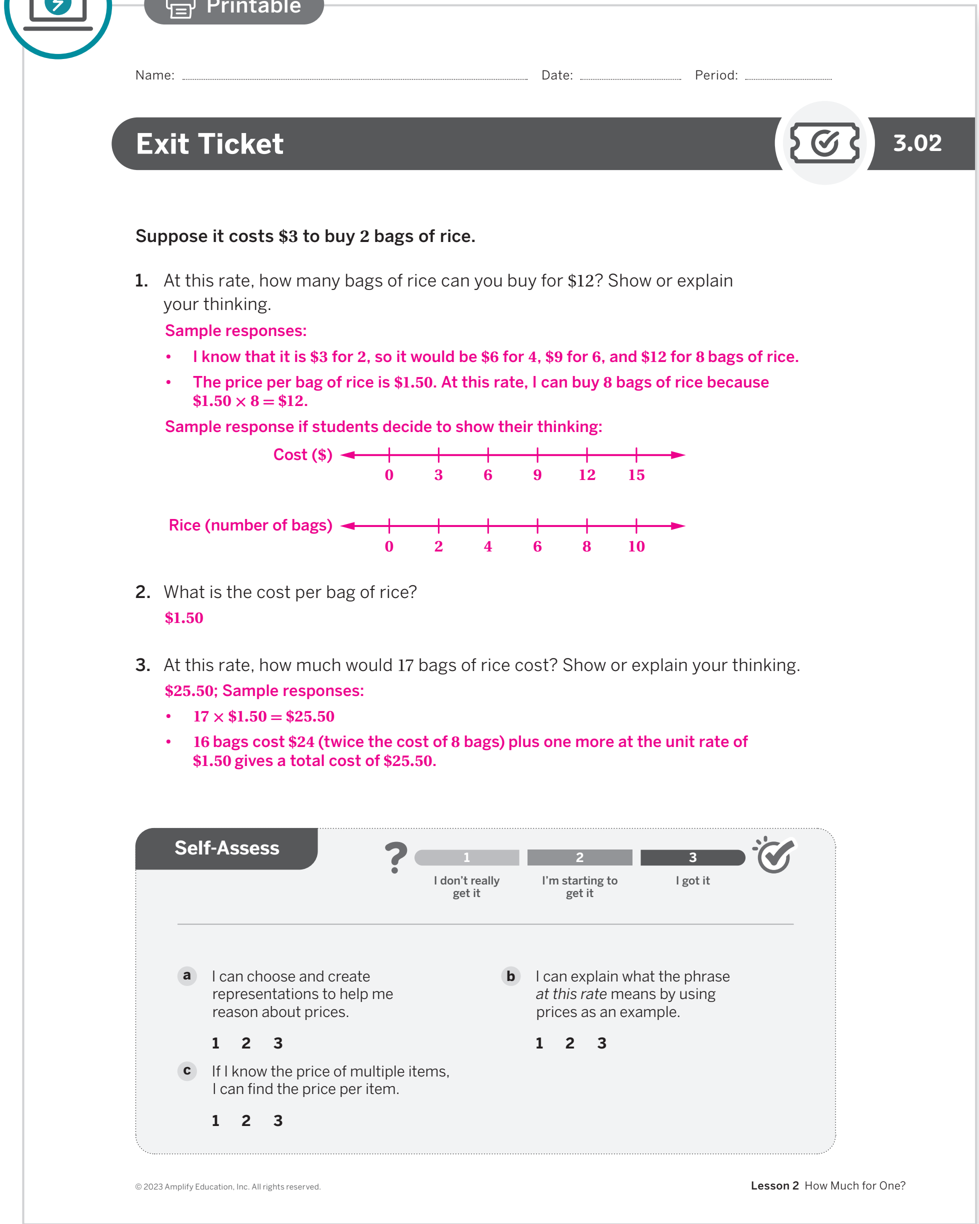
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this lesson. Ask them to review and reflect on any terms and phrases related to the terms *rate* and *unit rate* that were added to the display during the lesson. Add a double number line diagram to the class display, highlighting a rate and the unit rate.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of unit rate in the context of price by generating equivalent ratios.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.02

Suppose it costs $3 to buy 2 bags of rice.

1. At this rate, how many bags of rice can you buy for $12? Show or explain your thinking.

   Sample responses:
   - **I know that it is $3 for 2, so it would be $6 for 4, $9 for 6, and $12 for 8 bags of rice.**
   - **The price per bag of rice is $1.50. At this rate, I can buy 8 bags of rice because $1.50 × 8 = $12.**

   Sample response if students decide to show their thinking:

Cost ($)	0	3	6	9	12	15
Rice (number of bags)	0	2	4	6	8	10

2. What is the cost per bag of rice?

   $1.50

3. At this rate, how much would 17 bags of rice cost? Show or explain your thinking.

   $25.50; Sample responses:
   - **$17 \times \$1.50 = \$25.50$**
   - **16 bags cost $24 (twice the cost of 8 bags) plus one more at the unit rate of $1.50 gives a total cost of $25.50.**

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can choose and create representations to help me reason about prices.
1 2 3

b. I can explain what the phrase *at this rate* means by using prices as an example.
1 2 3

c. If I know the price of multiple items, I can find the price per item.
1 2 3

Lesson 2 How Much for One?

Success looks like . . .

- **Language Goal:** Calculating equivalent ratios between prices and quantities and presenting the solution method using multiple representations. **(Speaking and Listening)**
  - » Determining equivalent ratios between cost and number of bags of ice in Problem 1.
- **Language Goal:** Calculating unit price and expressing it using the word *per.* **(Speaking and Listening, Writing)**
- **Goal:** Understanding the phrase *at this rate* indicates that equivalent ratios are involved.

Suggested next steps

If students have difficulty with Problem 1, consider:

- Asking, "If you set up $3 for 2 bags as a ratio, how would that be written? How can that help you determine how many bags you can get for $12?"

If students have difficulty with Problem 2, consider:

- Referring back to Activity 1 and asking, "What was a strategy you used to find the cost per t-shirt?"

If students have difficulty in determining an equivalent ratio with an odd number of bags, consider:

- Briefly discussing the meaning of the word *per* and asking, "Now that you know the price per bag, how can you determine the cost for 17 bags?"
- Showing a double number line using a scale of 1 for bags of rice and having students help fill in values for cost.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students were asked to "solve unit rate problems including those involving unit pricing." Where in your students' work today did you see or hear evidence of them doing this?
- How do you think this will help them as they move toward solving problems involving constant speed? Is there language used in this lesson that you can use when teaching about unit rate and constant speed?

Math Language Development

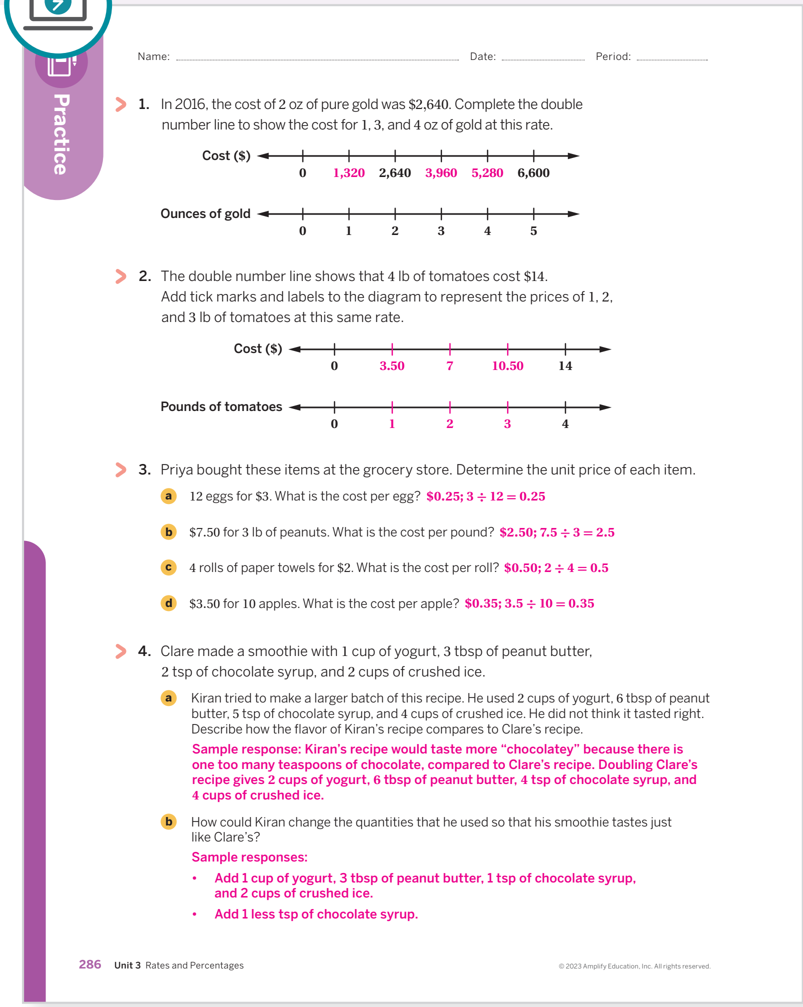
Language Goal: Calculating unit price and expressing it using the word *per.*

Reflect on students' language development toward this goal.

- Do students understand the meanings of the phrase per as they describe unit rates or unit prices? How did using the *Collect and Display* routine in Activity 2 help them?
- What other strategies can you use to help students understand and use this language?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. In 2016, the cost of 2 oz of pure gold was $2,640. Complete the double number line to show the cost for 1, 3, and 4 oz of gold at this rate.

Cost ($)	0	1,320	2,640	3,960	5,280	6,600
Ounces of gold	0	1	2	3	4	5

2. The double number line shows that 4 lb of tomatoes cost $14. Add tick marks and labels to the diagram to represent the prices of 1, 2, and 3 lb of tomatoes at this same rate.

Cost ($)	0	3.50	7	10.50	14
Pounds of tomatoes	0	1	2	3	4

3. Priya bought these items at the grocery store. Determine the unit price of each item.

   a. 12 eggs for \$3. What is the cost per egg? **\$0.25; $3 \div 12 = 0.25$**

   b. \$7.50 for 3 lb of peanuts. What is the cost per pound? **\$2.50; $7.5 \div 3 = 2.5$**

   c. 4 rolls of paper towels for \$2. What is the cost per roll? **\$0.50; $2 \div 4 = 0.5$**

   d. \$3.50 for 10 apples. What is the cost per apple? **\$0.35; $3.5 \div 10 = 0.35$**

4. Clare made a smoothie with 1 cup of yogurt, 3 tbsp of peanut butter, 2 tsp of chocolate syrup, and 2 cups of crushed ice.

   a. Kiran tried to make a larger batch of this recipe. He used 2 cups of yogurt, 6 tbsp of peanut butter, 5 tsp of chocolate syrup, and 4 cups of crushed ice. He did not think it tasted right. Describe how the flavor of Kiran's recipe compares to Clare's recipe.

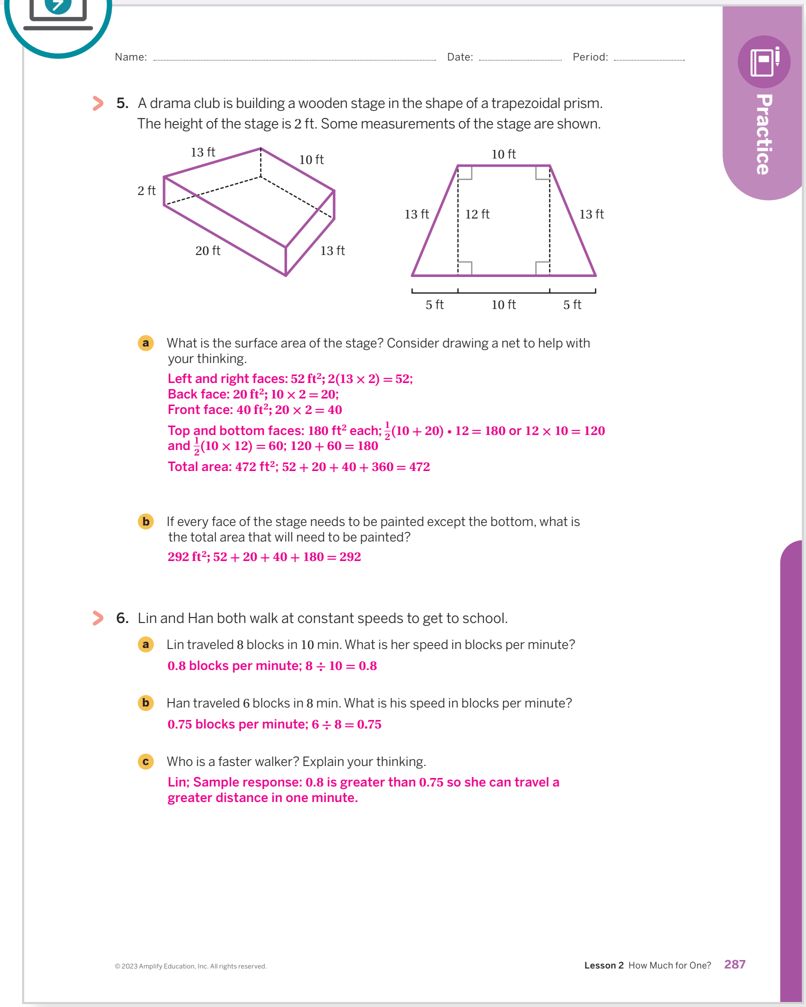
   Sample response: Kiran's recipe would taste more "chocolatey" because there is one too many teaspoons of chocolate, compared to Clare's recipe. Doubling Clare's recipe gives 2 cups of yogurt, 6 tbsp of peanut butter, 4 tsp of chocolate syrup, and 4 cups of crushed ice.

   b. How could Kiran change the quantities that he used so that his smoothie tastes just like Clare's?

   Sample responses:

   - **Add 1 cup of yogurt, 3 tbsp of peanut butter, 1 tsp of chocolate syrup, and 2 cups of crushed ice.**
   - **Add 1 less tsp of chocolate syrup.**

286 **Unit 3** Rates and Percentages



Practice

Name: ________ Date: ________ Period: ________

5. A drama club is building a wooden stage in the shape of a trapezoidal prism. The height of the stage is 2 ft. Some measurements of the stage are shown.

   a. What is the surface area of the stage? Consider drawing a net to help with your thinking.

   Left and right faces: 52 ft^2; $2(13 \times 2) = 52$;
   Back face: 20 ft^2; $10 \times 2 = 20$;
   Front face: 40 ft^2; $20 \times 2 = 40$

   Top and bottom faces: 180 ft^2 each; $\frac{1}{2}(10 + 20) \cdot 12 = 180$ or $12 \times 10 = 120$ and $\frac{1}{2}(10 \times 12) = 60$; $120 + 60 = 180$

   Total area: 472 ft^2; $52 + 20 + 40 + 360 = 472$

   b. If every face of the stage needs to be painted except the bottom, what is the total area that will need to be painted?

   292 ft^2; $52 + 20 + 40 + 180 = 292$

6. Lin and Han both walk at constant speeds to get to school.

   a. Lin traveled 8 blocks in 10 min. What is her speed in blocks per minute?

   0.8 blocks per minute; $8 \div 10 = 0.8$

   b. Han traveled 6 blocks in 8 min. What is his speed in blocks per minute?

   0.75 blocks per minute; $6 \div 8 = 0.75$

   c. Who is a faster walker? Explain your thinking.

   Lin; Sample response: 0.8 is greater than 0.75 so she can travel a greater distance in one minute.

Lesson 2 How Much for One? 287

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 4	2
	5	Unit 1 Lesson 16	2
Formative ⚡	6	Uint 3 Lesson 3	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 3 | **LESSON 3**

Constant Speed

Let's use ratios to determine how fast objects or people move.

Focus

Goals

1. **Language Goal:** Calculate the distance an object travels in 1 unit of time and express it by using a phrase, such as *meters per second*. **(Speaking and Listening, Writing)**
2. For an object moving at a constant speed, use a double number line diagram to represent equivalent ratios between the distance traveled and elapsed time.
3. **Language Goal:** Justify which of two objects is moving faster, by identifying that it travels more distance in the same amount of time or that it travels the same distance in less time. **(Speaking and Listening, Writing)**

Coherence

Today

Students continue to reason with unit rate, now in the context of constant speed. They measure the time it takes them to travel a predetermined distance — first moving slowly, and then moving quickly. Students then calculate and compare the speeds they traveled in meters per second. Double number lines are used to represent the association between distance and time, and to convey the idea of constant speed as a set of equivalent ratios. Students come to understand that, like price, speed can be described using the terms *per* and *at this rate*. The idea of a constant speed relating the quantities of distance and time is foundational to the general idea of constant rate, and is important in developing students' abilities to reason abstractly about quantities.

Previously

In Lesson 2, students worked with unit rates involving unit prices.

Coming Soon

In Lessons 4–7, students will continue to explore and use unit rates to interpret, compare, and determine missing values.

Rigor

- Students continue to build on their **conceptual understanding** of rate by connecting equivalent ratios to constant speed.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Independent	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- *Double Number Line* PDF (for display)
- meter sticks
- masking tape
- stopwatches
- string

Math Language Development

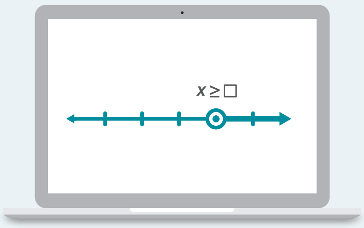
Review words

- *per*
- *rate*
- *unit rate*

Amps | **Featured Activity**

Activity 1
Interactive Double Number Lines

Students use digital timers and use interactive double number lines to organize their thinking.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may not be able to mathematically discern the difference between their tasks in Activity 1 and in Activity 2. Discuss with students how the activities are alike and how they are different to help them organize their thinking. Consider different visual representations that help students shift from the quantitative reasoning to the qualitative reasoning required for the interpretation of the data.

Modifications to Pacing

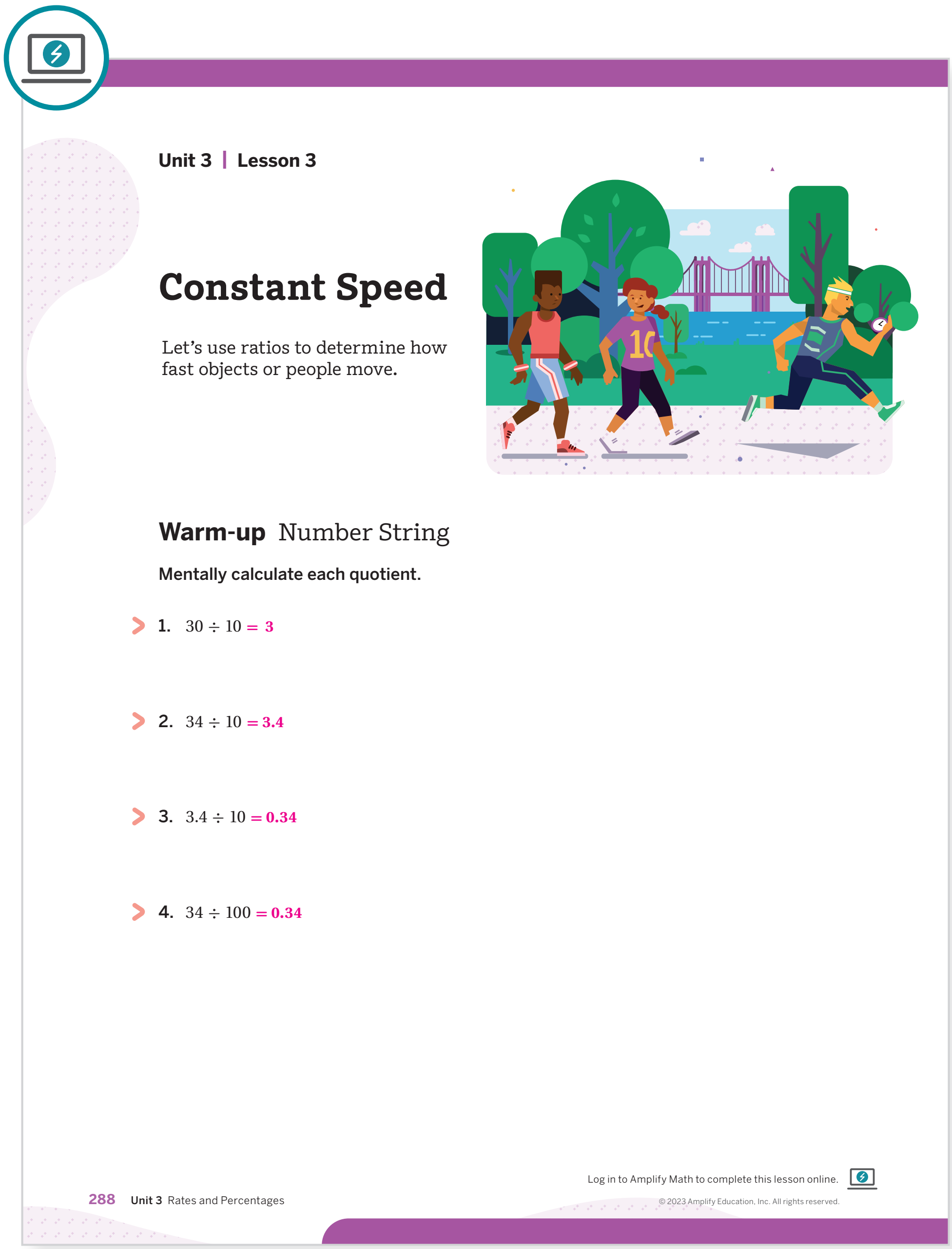
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, reduce the number of measurements by having students alternate walking and timing — only one will walk slowly and the other will walk quickly.
- In **Activity 2**, have students focus on Problems 1, 2, and 4a.

Independent | 5 min

Warm-up Number String

Students use place value and the structure of base ten numbers to mentally calculate related quotients.



Unit 3 | **Lesson 3**

Constant Speed

Let's use ratios to determine how fast objects or people move.

Warm-up Number String

Mentally calculate each quotient.

1. $30 \div 10 = 3$
2. $34 \div 10 = 3.4$
3. $3.4 \div 10 = 0.34$
4. $34 \div 100 = 0.34$

Log in to Amplify Math to complete this lesson online.

288 **Unit 3** Rates and Percentages

1 Launch

Display one problem at a time and give students 30 seconds of individual think time before conducting a 30-second discussion for each.

2 Monitor

Help students get started by asking, "How is the value of the 3 different in 34 and 3.4? What are the values of the digits in each number?"

Look for points of confusion:

- **Not connecting the base ten structure of dividing by 10.** "The quotient of 30 divided by 10 can be interpreted as 'from 30, you can make exactly 3 groups of 10, with 0 left over.' What would the result of dividing 34 by 10 tell you?"

Look for productive strategies:

- Referring to the previous problems to explain a solution.
- Understanding the connection between base ten place value structures and division by powers of 10.
- Seeing the problems as multiplying by $\frac{1}{10}$.
- Seeing the problems as a missing factor, e.g., $x \cdot 10 = 34$.

3 Connect

Have individual students share their strategies for each problem, focusing on how they used place value understanding and the previous problems to inform their solution. Record student strategies next to each problem.

Highlight strategies based on place value and use of precise language, such as, "dividing by 10 is the same as multiplying by one tenth," to explain the underlying concept of why it appears that "the decimal point moved" left or right.

Ask a student to explain how the problems and solutions are connected.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, after students share their strategies and reasonings for how the problems are related, display an incorrect statement, such as, "7.2 divided by 100 is 0.72 because the same digits 7 and 2 are in the quotient and 0.72 is less than 7.2." Ask:
Critique: "Why is this statement incorrect? What is incorrect about the reasoning used?"
Correct and Clarify: Have students write a corrected statement, including reasoning. Then have them explain how they know their statement is correct.

Power-up

To power up students' ability to compare to constant rates, have students complete:

Recall that calculating a unit rate can help you compare speeds.

1. Determine the speed each slug crawls in ft per minute.
   a. Slug A crawled 5 ft in 2 minutes. $5 \div 2 = 2.5$ 2.5 ft per minute.
   b. Slug B crawled 8 ft in 3 minutes. $8 \div 3 = 2\frac{2}{3}$ $2\frac{2}{3}$ ft per minute.
2. Which slug is traveling at a faster speed? Slug A.

Use: Before Activity 1.

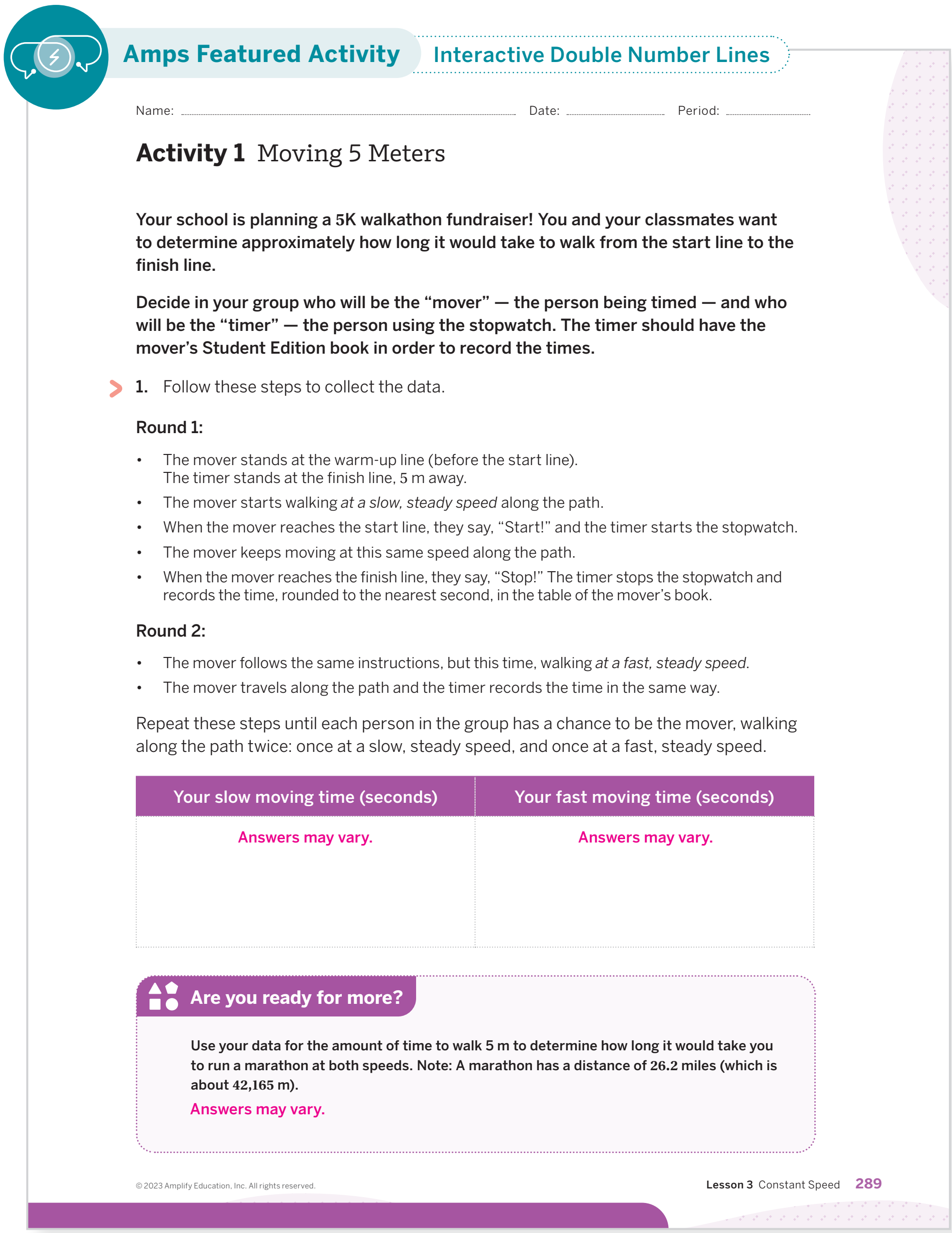
Informed by: Performance on Lesson 2, Practice Problem 6 and and Pre-Unit Readiness Assessment, Problem 5.

Small Groups | 20 min

Activity 1 Moving 5 Meters

Students think about rate in the context of constant speed by measuring the time it takes them to walk 5 meters and using equivalent ratios to estimate other times and distances.

Amps Featured Activity Interactive Double Number Lines



Name: ______ Date: ______ Period: ______

Activity 1 Moving 5 Meters

Your school is planning a 5K walkathon fundraiser! You and your classmates want to determine approximately how long it would take to walk from the start line to the finish line.

Decide in your group who will be the "mover" — the person being timed — and who will be the "timer" — the person using the stopwatch. The timer should have the mover's Student Edition book in order to record the times.

1. Follow these steps to collect the data.

Round 1:

- The mover stands at the warm-up line (before the start line).
  The timer stands at the finish line, 5 m away.
- The mover starts walking *at a slow, steady speed* along the path.
- When the mover reaches the start line, they say, "Start!" and the timer starts the stopwatch.
- The mover keeps moving at this same speed along the path.
- When the mover reaches the finish line, they say, "Stop!" The timer stops the stopwatch and records the time, rounded to the nearest second, in the table of the mover's book.

Round 2:

- The mover follows the same instructions, but this time, walking *at a fast, steady speed*.
- The mover travels along the path and the timer records the time in the same way.

Repeat these steps until each person in the group has a chance to be the mover, walking along the path twice: once at a slow, steady speed, and once at a fast, steady speed.

Your slow moving time (seconds)	Your fast moving time (seconds)
Answers may vary.	Answers may vary.

Are you ready for more?

Use your data for the amount of time to walk 5 m to determine how long it would take you to run a marathon at both speeds. Note: A marathon has a distance of 26.2 miles (which is about 42,165 m).

Answers may vary.

1 Launch

Prior to the lesson, use tape to prepare four 5-meter paths. Divide the class into four groups and assign each group to one of the four paths. If necessary, explain that a 5K is a 5-kilometer race.

2 Monitor

Help students get started by asking, "How did you convert meters to kilometers in the last unit?" Consider referring back to Unit 2, Lesson 19.

Look for points of confusion:

- **Having difficulty estimating the distance traveled in 1 second.** Encourage students to mark their double number line by using 1 second intervals, to help cue division.
- **Multiplying the time to walk 5 m by 5,000.** "What do you need to multiply the 5 by to get to 5 kilometers?"

Look for productive strategies:

- Using the language from Lesson 2 of *per* and *at this rate* when describing relationships on a double number line.
- Thinking of Problem 2 in steps by determining the time for 10 m and then 1,000 m to get to 5,000 m.
- Reasoning abstractly by representing speed with equivalent ratios.
- Writing and evaluating a correct expression.
  If students have difficulty rounding to tenths of a second, consider using the Warm-up to review rounding, or have them round to the nearest second instead.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Suggest students think of 5 m as 500 cm. This will allow them to work primarily with whole numbers, as decimals will be less likely. The activity goal is to understand constant speed as an example of a rate.

Accessibility: *Guide Processing and Visualization*

Begin with a physical demonstration of the activity to show one volunteer walking while another volunteer records their time using a stopwatch.

Accessibility: *Activate Prior Knowledge*

Remind students they have previously worked with double number lines, including those that related distance to time.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies, ask, "What is the same and what is different about these approaches?" Help students connect the strategies by asking, "Where do you see the measurement of speed, ______ meters per second, in each strategy?" This will help students connect the concept of rate to a visual representation of that rate.

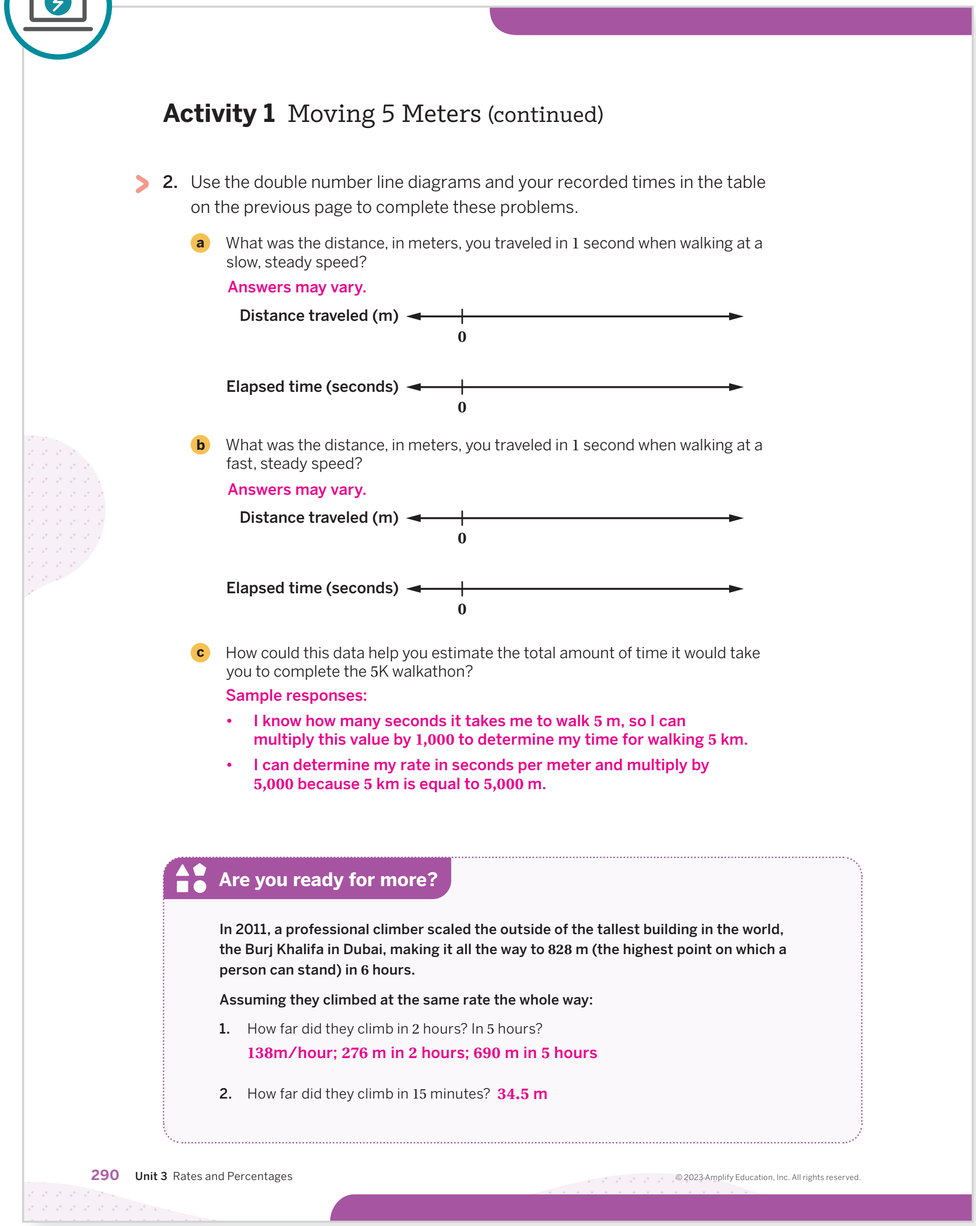
English Learners

Consider a physical demonstration of what a steady speed (constant speed) looks like versus a non-constant speed. Display the phrases *steady speed* and *constant speed* while a volunteer walks at a sample rate.

Small Groups | 20 min

Activity 1 Moving 5 Meters (continued)

Students think about rate in the context of constant speed by measuring the time it takes them to walk 5 meters and using equivalent ratios to estimate other times and distances.



Activity 1 Moving 5 Meters (continued)

2. Use the double number line diagrams and your recorded times in the table on the previous page to complete these problems.

   a. What was the distance, in meters, you traveled in 1 second when walking at a slow, steady speed?

      Answers may vary.

      Distance traveled (m) 0

      Elapsed time (seconds) 0

   b. What was the distance, in meters, you traveled in 1 second when walking at a fast, steady speed?

      Answers may vary.

      Distance traveled (m) 0

      Elapsed time (seconds) 0

   c. How could this data help you estimate the total amount of time it would take you to complete the 5K walkathon?

      Sample responses:

      - **I know how many seconds it takes me to walk 5 m, so I can multiply this value by 1,000 to determine my time for walking 5 km.**
      - **I can determine my rate in seconds per meter and multiply by 5,000 because 5 km is equal to 5,000 m.**

Are you ready for more?

In 2011, a professional climber scaled the outside of the tallest building in the world, the Burj Khalifa in Dubai, making it all the way to 828 m (the highest point on which a person can stand) in 6 hours.

Assuming they climbed at the same rate the whole way:

1. How far did they climb in 2 hours? In 5 hours?

   138m/hour; 276 m in 2 hours; 690 m in 5 hours

2. How far did they climb in 15 minutes? **34.5 m**

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3 Connect

Display the *Double Number Lines* PDF and label the number lines with distance traveled (meters) and elapsed time (seconds) for reference when discussing strategies.

Have groups of students share strategies used to calculate the distance traveled in 1 second, including what they did when a quotient had many decimal places.

Highlight that when time and distance are represented on a double number line, it is showing the object traveling at a constant speed or a constant rate. This means that all of the ratios of meters traveled to seconds elapsed (or miles traveled to hours elapsed) are equivalent. The object does not move faster or slower at any time. The intervals on the double number line show this steady rate.

Note: Consider revisiting the definition of ***unit rate*** as how much one quantity changes when the other changes by 1. In the context of speed, it is how much distance is covered (e.g., meters) per 1 unit of time (e.g., 1 second). As with constant price in Lesson 2, a unit rate for constant speed can be used to determine how far something travels in a given amount of time or how long it will take something to travel a given distance.

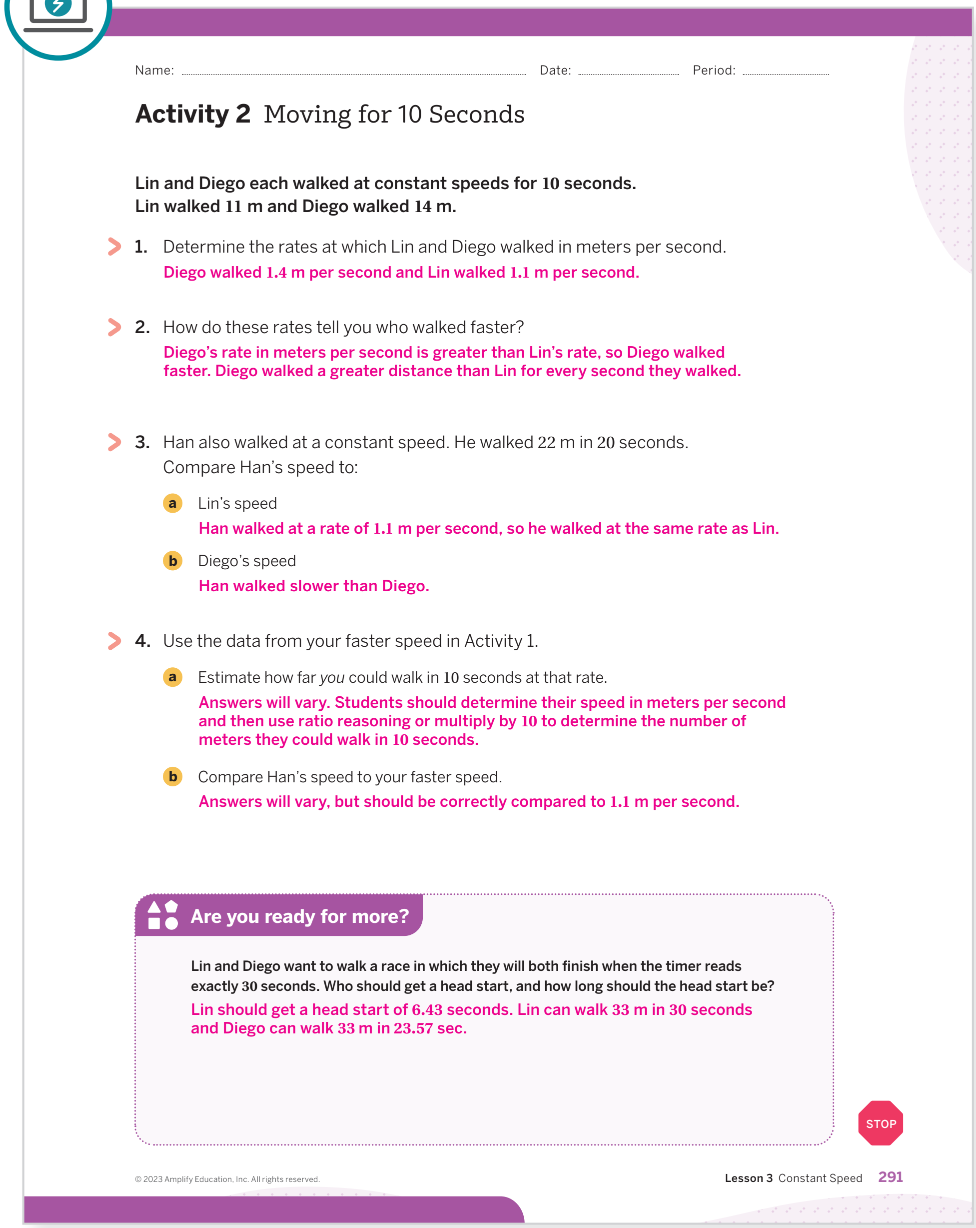
Ask:

- "How are the diagrams representing someone moving slowly and someone moving quickly different? How are they similar?"
- "How could this also show the distance traveled in one second?" (to transition to Activity 2)

Pairs | 10 min

Activity 2 Moving for 10 Seconds

Students continue to think about and to compare walking speeds as rates of meters per second, but in this activity, they are working with a fixed time, rather than a fixed distance.



Name: ______ Date: ______ Period: ______

Activity 2 Moving for 10 Seconds

Lin and Diego each walked at constant speeds for 10 seconds. Lin walked 11 m and Diego walked 14 m.

1. Determine the rates at which Lin and Diego walked in meters per second.
   Diego walked 1.4 m per second and Lin walked 1.1 m per second.

2. How do these rates tell you who walked faster?
   Diego's rate in meters per second is greater than Lin's rate, so Diego walked faster. Diego walked a greater distance than Lin for every second they walked.

3. Han also walked at a constant speed. He walked 22 m in 20 seconds. Compare Han's speed to:
   - a Lin's speed
     Han walked at a rate of 1.1 m per second, so he walked at the same rate as Lin.
   - b Diego's speed
     Han walked slower than Diego.

4. Use the data from your faster speed in Activity 1.
   - a Estimate how far *you* could walk in 10 seconds at that rate.
     Answers will vary. Students should determine their speed in meters per second and then use ratio reasoning or multiply by 10 to determine the number of meters they could walk in 10 seconds.
   - b Compare Han's speed to your faster speed.
     Answers will vary, but should be correctly compared to 1.1 m per second.

Are you ready for more?

Lin and Diego want to walk a race in which they will both finish when the timer reads exactly 30 seconds. Who should get a head start, and how long should the head start be?

Lin should get a head start of 6.43 seconds. Lin can walk 33 m in 30 seconds and Diego can walk 33 m in 23.57 sec.

STOP

Lesson 3 Constant Speed 291

1 Launch

Say, "In the last activity, you all traveled the same distance, but in different times. In this activity the amount of time will be the same, but the distances may be different." Have students work individually to complete the problem and then compare responses with a partner.

2 Monitor

Help students get started by asking, "What information is the same for Lin and Diego? What information is different? How can you show this?"

Look for points of confusion:

- **Struggling to organize the information given.** Ask, "What information do you know? What do you need to determine?"
- **Confusing distance and time, or thinking the farther distance means a slower speed.** Have students use diagrams to aid their thinking.

Look for productive strategies:

- Creating a visual representation that clearly represents their thinking to others, such as a ratio table, double number line, or other diagram.
- Using units of "meters per second" for unit rates.

3 Connect

Have individual students share different strategies used to solve, focusing on the use of unit rates to compare.

Ask:

- "What were you solving for to be able to compare the speeds?"
- "Is this a unit rate? How do you know?"

Highlight that *meters per second* is a unit for measuring speed. It tells how many meters an object goes in one second.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Problems 1, 2, and 4a.

Accessibility: *Guide Processing and Visualization*

Consider displaying the information using a diagram, such as drawing a triple number line diagram that shows the time, in seconds, on the bottom number line, Lin's distance on the middle number line, and Diego's distance on the top number line.

Math Language Development

MLR5: Co-craft Questions

During the Launch, read the scenario aloud and ask students to write 2–3 mathematical questions about the scenario. Invite students to share their questions with a partner before sharing with the whole class. Ask students to use the phrase *at a constant speed* in at least one of their questions so that they must reason about its mathematical meaning.

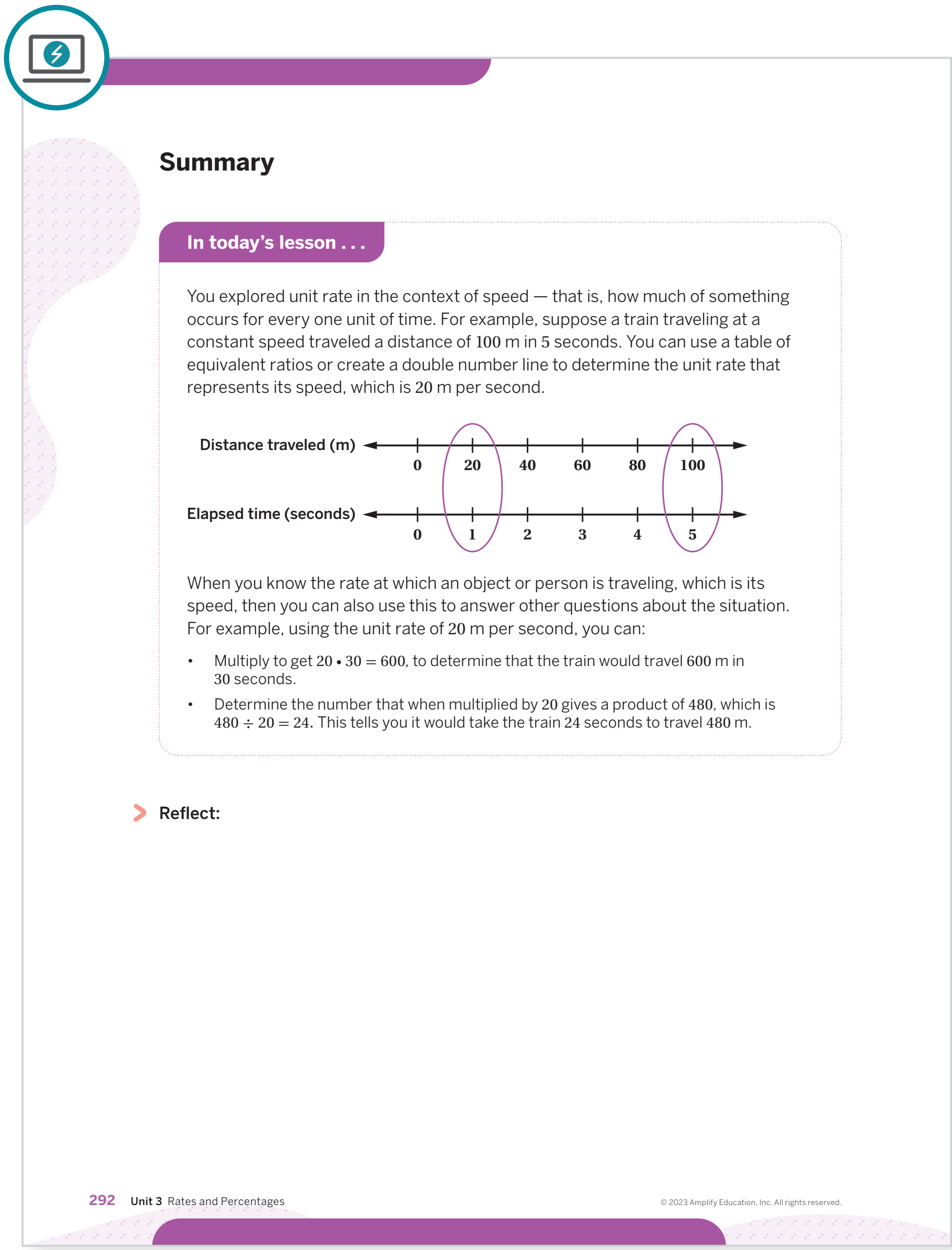
English Learners

Display a sample mathematical question, such as "Why did Diego walk a greater distance?" or "Who walked at a greater constant speed?"

Whole Class | 5 min

Summary

Review and synthesize the strategies for determining meters per second, connecting to unit price from the previous lesson.



Summary

In today's lesson . . .

You explored unit rate in the context of speed — that is, how much of something occurs for every one unit of time. For example, suppose a train traveling at a constant speed traveled a distance of 100 m in 5 seconds. You can use a table of equivalent ratios or create a double number line to determine the unit rate that represents its speed, which is 20 m per second.

When you know the rate at which an object or person is traveling, which is its speed, then you can also use this to answer other questions about the situation. For example, using the unit rate of 20 m per second, you can:

- Multiply to get $20 \cdot 30 = 600$, to determine that the train would travel 600 m in 30 seconds.
- Determine the number that when multiplied by 20 gives a product of 480, which is $480 \div 20 = 24$. This tells you it would take the train 24 seconds to travel 480 m.

Reflect:

292 Unit 3 Rates and Percentages

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Synthesize

Display or refer to the double number line showing distance traveled and elapsed time.

Formalize vocabulary: ***unit rate***

Note: This term was formalized in Lesson 2. With students only having worked with the context of constant price, revisiting it here with constant speed helps ensure generalization.

Highlight in both Activities 1 and 2, one goal was to determine the values corresponding to x meters in 1 second, which could then be used to determine other values of distance and time corresponding to those constant unit rates.

Ask, "How were your strategies today similar to what you did in Lesson 2 with unit price?"

Have students share how unit rates for constant speed and constant price are related. Focus on responses connected to strategies for determining unit rate and how knowing the unit rate (the "per one") makes it possible to determine any missing values using equivalent ratios.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

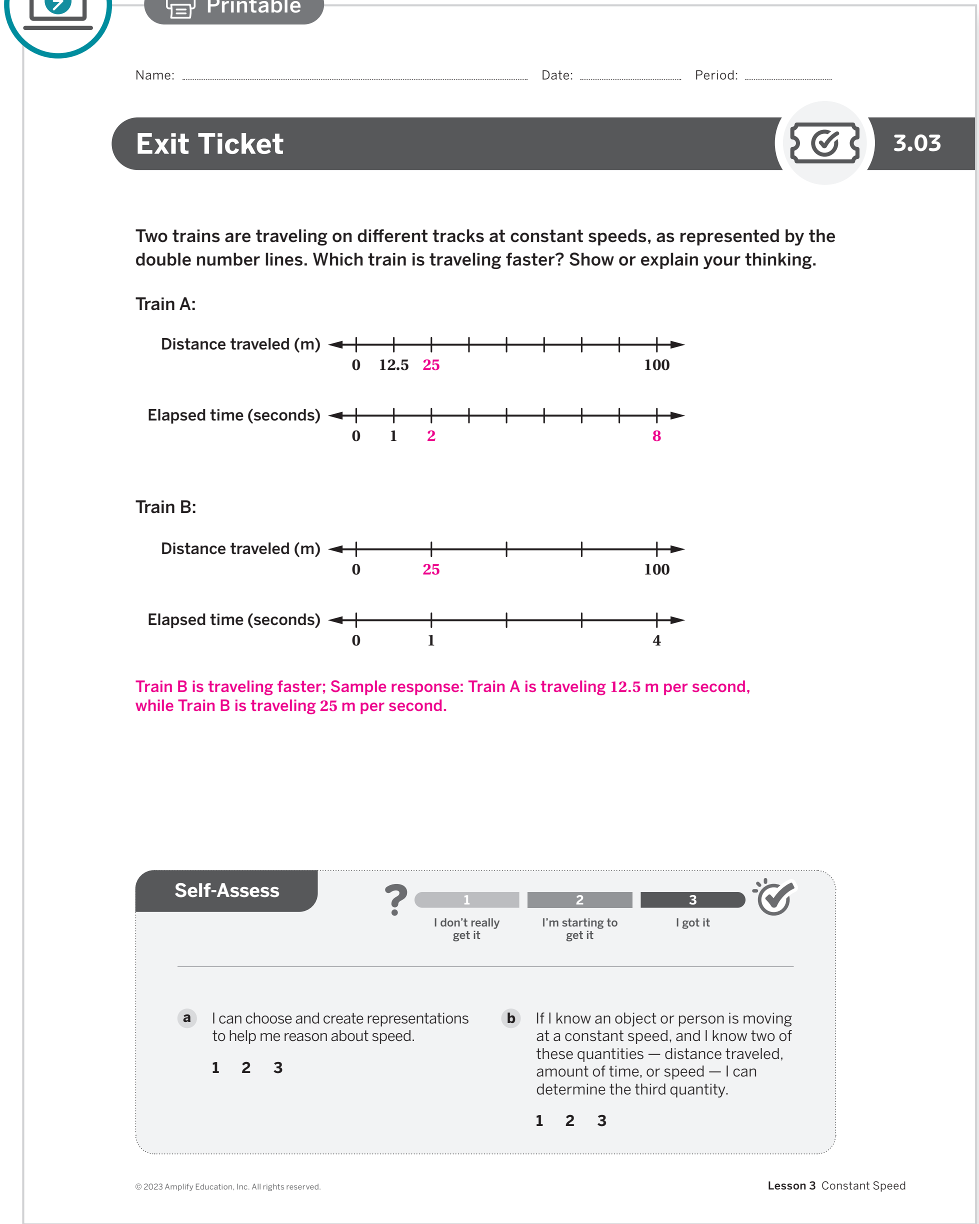
- "How does what you did today connect to what you did with unit price in Lesson 2?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by solving a problem with constant speed.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.03

Two trains are traveling on different tracks at constant speeds, as represented by the double number lines. Which train is traveling faster? Show or explain your thinking.

Train A:

Distance traveled (m): 0, 12.5, 25, 100

Elapsed time (seconds): 0, 1, 2, 8

Train B:

Distance traveled (m): 0, 25, 100

Elapsed time (seconds): 0, 1, 4

Train B is traveling faster; Sample response: Train A is traveling 12.5 m per second, while Train B is traveling 25 m per second.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can choose and create representations to help me reason about speed.

1 2 3

b If I know an object or person is moving at a constant speed, and I know two of these quantities — distance traveled, amount of time, or speed — I can determine the third quantity.

1 2 3

Lesson 3 Constant Speed

Success looks like . . .

- **Language Goal:** Calculating the distance an object travels in 1 unit of time and expressing it by using a phrase, such as *meters per second*. **(Speaking and Listening, Writing)**
  - » Determining the distance that each train travels in 1 second.
- **Goal:** For an object moving at a constant speed, using a double number line diagram to represent equivalent ratios between the distance traveled and elapsed time.
- **Language Goal:** Justifying which of two objects is moving faster, by identifying that it travels more distance in the same amount of time or that it travels the same distance in less time. **(Speaking and Listening, Writing)**

Suggested next steps

If students confuse what they are trying to solve for, consider:

- Referring back to Activity 2. Ask, "How did you know that Diego was walking faster?"
- Asking, "To compare the constant speeds of both trains, what information do you already know and what information do you need to know?" I know Train A's meters per second. I need to know Train B's meters per second.

If students are not sure what to divide for Train B, consider:

- Referring to the elapsed time (seconds) number line and asking:
  - » "How can you get from 4 to 1?" Divide 4 by 4 which equals 1.
  - » "How does this inform what you can do on the 'distance traveled (meters)' number line?"

Professional Learning

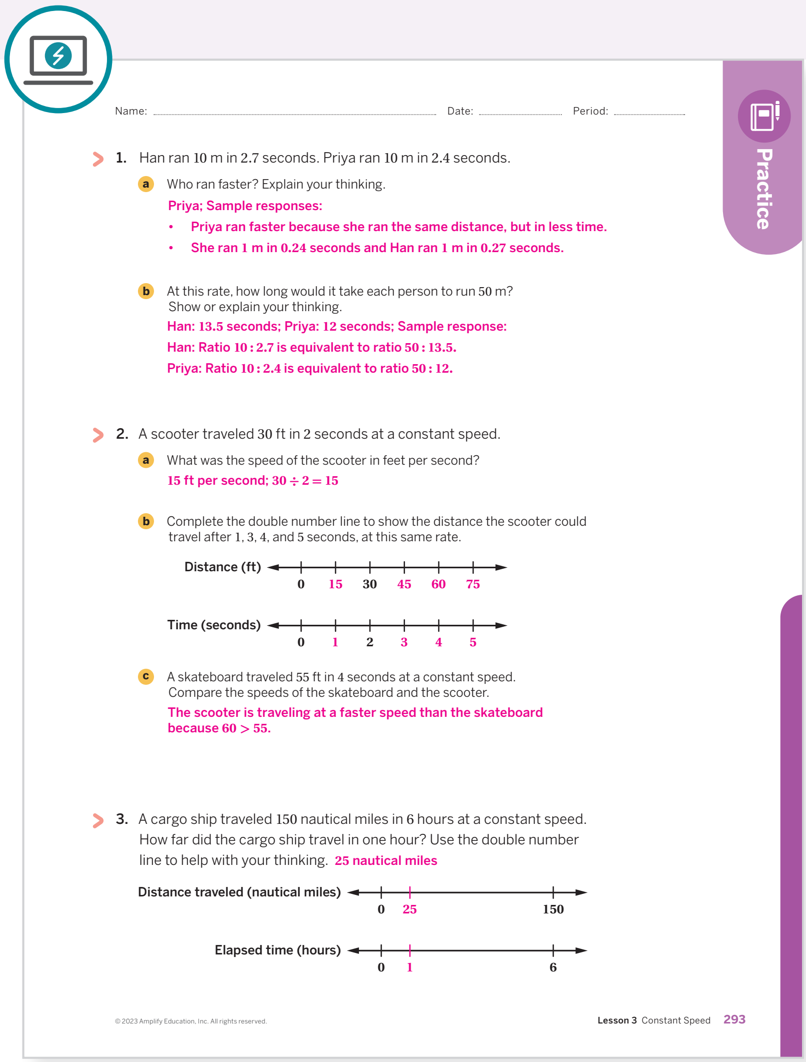
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students worked with meters per second. How did that build on the earlier work students did with unit price?
- What might you change the next time you teach this activity?

Practice

Independent



Name: Date: Period:

1. Han ran 10 m in 2.7 seconds. Priya ran 10 m in 2.4 seconds.
   - a. Who ran faster? Explain your thinking.
     Priya; Sample responses:
     - **Priya ran faster because she ran the same distance, but in less time.**
     - **She ran 1 m in 0.24 seconds and Han ran 1 m in 0.27 seconds.**
   - b. At this rate, how long would it take each person to run 50 m? Show or explain your thinking.
     Han: 13.5 seconds; Priya: 12 seconds; Sample response:
     Han: Ratio 10 : 2.7 is equivalent to ratio 50 : 13.5.
     Priya: Ratio 10 : 2.4 is equivalent to ratio 50 : 12.

2. A scooter traveled 30 ft in 2 seconds at a constant speed.
   - a. What was the speed of the scooter in feet per second?
     15 ft per second; $30 \div 2 = 15$
   - b. Complete the double number line to show the distance the scooter could travel after 1, 3, 4, and 5 seconds, at this same rate.

Distance (ft)	0	15	30	45	60	75
Time (seconds)	0	1	2	3	4	5

   - c. A skateboard traveled 55 ft in 4 seconds at a constant speed. Compare the speeds of the skateboard and the scooter.
     The scooter is traveling at a faster speed than the skateboard because $60 > 55$.

3. A cargo ship traveled 150 nautical miles in 6 hours at a constant speed. How far did the cargo ship travel in one hour? Use the double number line to help with your thinking. **25 nautical miles**

Distance traveled (nautical miles)	0	25	150
Elapsed time (hours)	0	1	6

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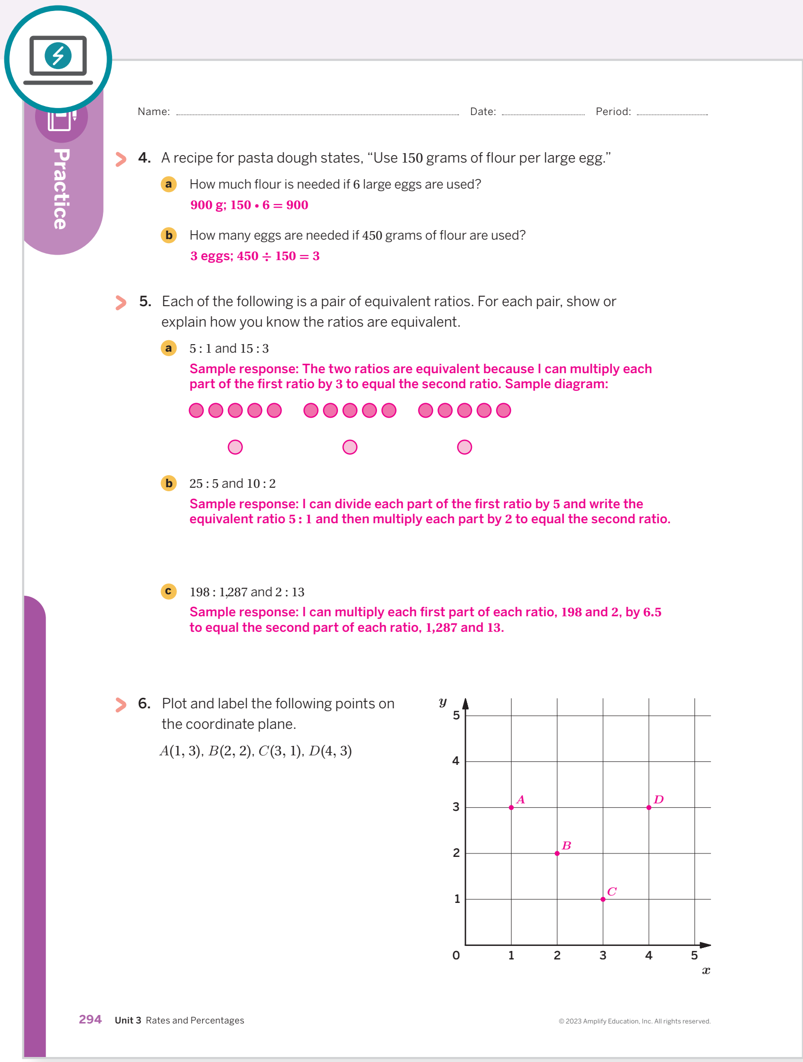
Lesson 3 Constant Speed 293



Name: Date: Period:

4. A recipe for pasta dough states, "Use 150 grams of flour per large egg."
   - a. How much flour is needed if 6 large eggs are used?
     900 g; $150 \cdot 6 = 900$
   - b. How many eggs are needed if 450 grams of flour are used?
     3 eggs; $450 \div 150 = 3$

5. Each of the following is a pair of equivalent ratios. For each pair, show or explain how you know the ratios are equivalent.
   - a. 5 : 1 and 15 : 3
     Sample response: The two ratios are equivalent because I can multiply each part of the first ratio by 3 to equal the second ratio. Sample diagram:
   - b. 25 : 5 and 10 : 2
     Sample response: I can divide each part of the first ratio by 5 and write the equivalent ratio 5 : 1 and then multiply each part by 2 to equal the second ratio.
   - c. 198 : 1,287 and 2 : 13
     Sample response: I can multiply each first part of each ratio, 198 and 2, by 6.5 to equal the second part of each ratio, 1,287 and 13.

6. Plot and label the following points on the coordinate plane.
   $A(1, 3)$, $B(2, 2)$, $C(3, 1)$, $D(4, 3)$

294 Unit 3 Rates and Percentages

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 4	2
	5	Unit 2 Lesson 6	2
Formative ⚡	6	Unit 3 Lesson 4	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

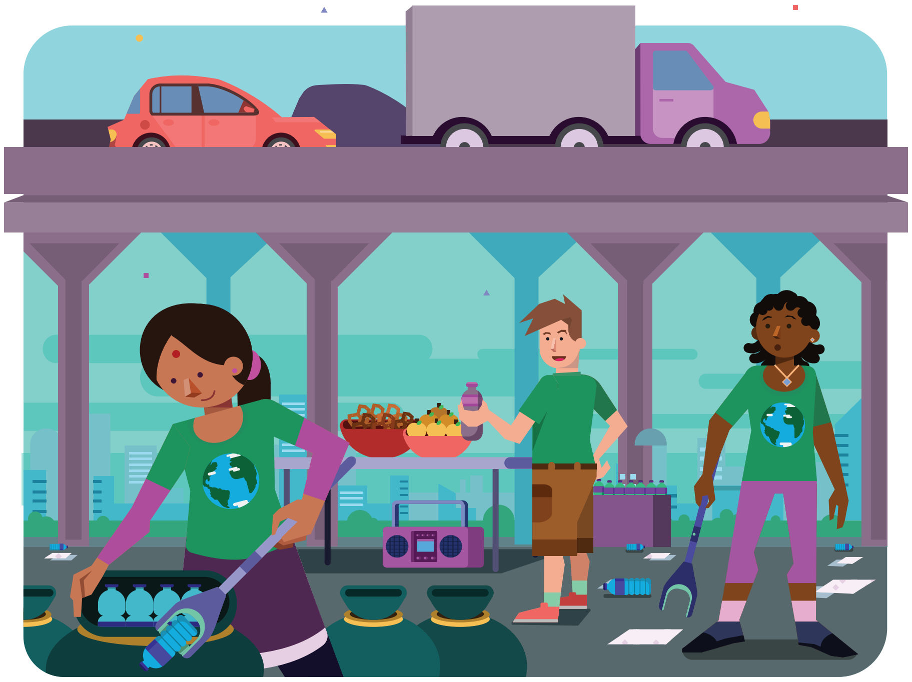
Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 3 | **LESSON 4**

Comparing Speeds

Let's use graphs to represent ratios and to compare speeds.



Focus

Goals

1. **Language Goal:** Explain that if two ratios have the same rate per 1, then they are equivalent ratios. **(Speaking and Listening, Writing)**
2. **Language Goal:** Recognize that calculating how much for 1 of the same unit is a useful strategy for comparing rates. Express these rates by using the word *per* and specifying the unit. **(Speaking and Listening, Writing)**
3. **Language Goal:** Plot pairs of values from tables that represent equivalent ratios and interpret points on a graph of equivalent ratios by using rate language. **(Speaking and Listening, Writing)**
4. **Language Goal:** Justify comparisons of speeds by using tables or graphs. **(Speaking and Listening, Writing)**

Coherence

Today

Students solidify their understanding that when two ratios are associated with the same unit rate, then they are equivalent ratios. They determine whether two ratios representing distance and time are equivalent, and use unit rates to compare speeds. Students also begin to plot points on a coordinate plane to represent ratios, recognizing that equivalent ratios can be connected by a straight line that goes through the origin. They use their graphs to interpret and justify comparisons of rate, specifically speed.

Previously

In Lessons 2 and 3, students used equivalent ratios to determine unit rates and solved constant price and speed problems.

Coming Soon

In Lesson 5, students will identify the two unit rates associated with any ratio as they continue to graph ratios and rates.

Rigor

- Students continue to build **conceptual understanding** of equivalent ratios and unit rates by plotting points to represent them graphically on the coordinate plane.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Individual	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- straightedges, one per student (optional)

Math Language Development

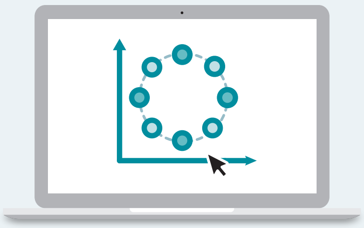
Review words

- *coordinate plane*
- *equivalent ratios*
- *origin*
- *unit rate*

Amps | Featured Activity

Activity 2
Interactive Graphs

Students use an interactive graph to work with unit rates.



Building Math Identity and Community

Connecting to Mathematical Practices

Knowing that they will get a chance to share their responses with a partner might indicate to students they do not need to actively make sense of the problems on their own. Prior to beginning Activity 1, lead students in a discussion about why giving 100% effort is necessary. Guide students to understand that others are depending on them, too, and that two people can learn from their individual efforts.

Modifications to Pacing

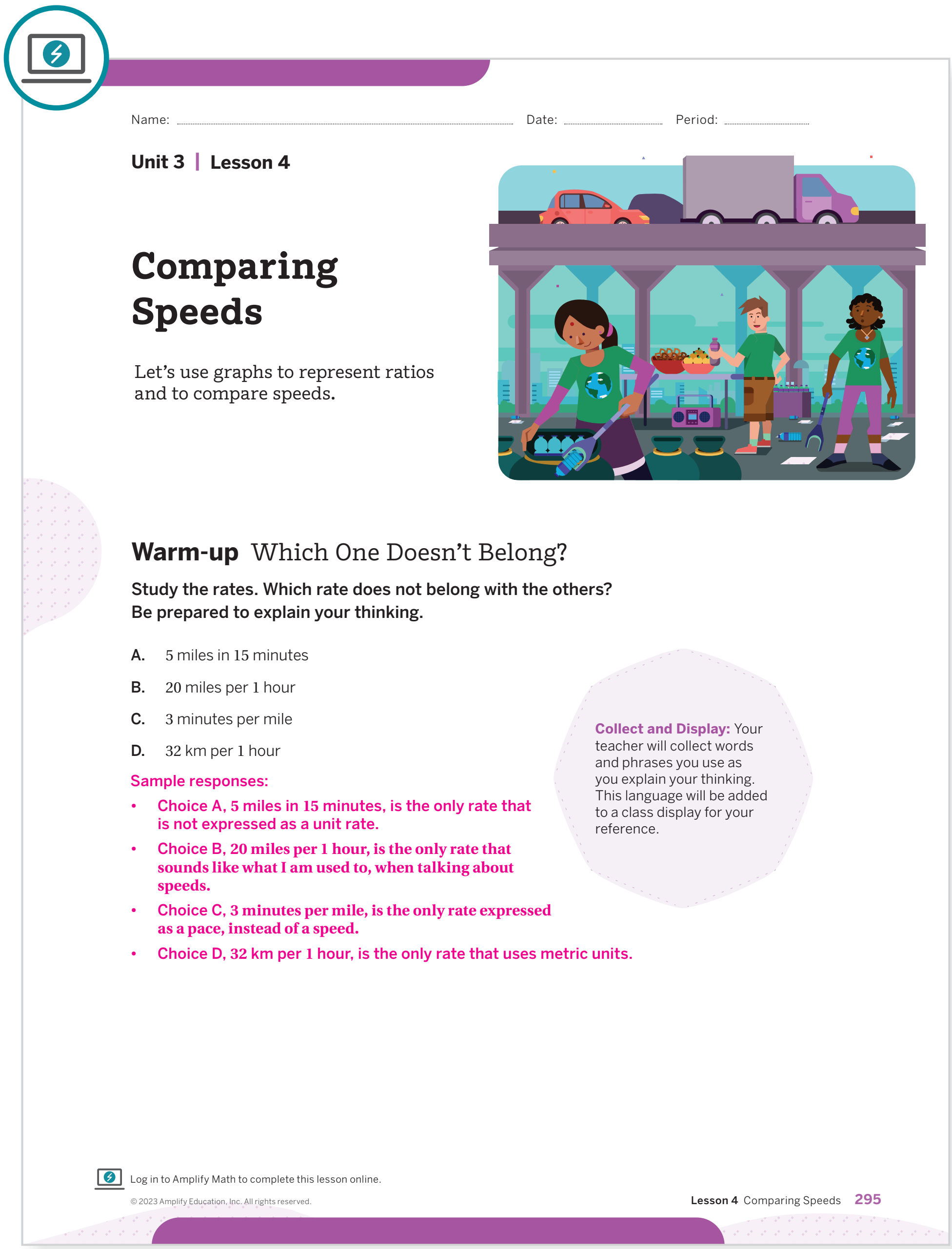
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Part 1 may be omitted, but students will still use the given table for Part 2.
- In **Activity 2**, Problem 4 can be discussed as a class. Consider asking students targeted questions, such as, “You said Group A was moving faster in Problem 1. How does the graph show that?”

Individual | 5 min

Warm-up Which One Doesn't Belong?

Students compare four rates to use reasoning and to hold mathematical conversations.



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Unit 3 | Lesson 4

Comparing Speeds

Let's use graphs to represent ratios and to compare speeds.

Warm-up Which One Doesn't Belong?

Study the rates. Which rate does not belong with the others? Be prepared to explain your thinking.

A. 5 miles in 15 minutes

B. 20 miles per 1 hour

C. 3 minutes per mile

D. 32 km per 1 hour

Sample responses:

- Choice A, 5 miles in 15 minutes, is the only rate that is not expressed as a unit rate.
- Choice B, 20 miles per 1 hour, is the only rate that sounds like what I am used to, when talking about speeds.
- Choice C, 3 minutes per mile, is the only rate expressed as a pace, instead of a speed.
- Choice D, 32 km per 1 hour, is the only rate that uses metric units.

Collect and Display: Your teacher will collect words and phrases you use as you explain your thinking. This language will be added to a class display for your reference.

Log in to Amplify Math to complete this lesson online.

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Lesson 4 Comparing Speeds 295

1 Launch

Conduct the ***Which One Doesn't Belong?*** routine.

2 Monitor

Help students get started by reminding them there is no one correct answer and there could be a reason every statement may not belong.

Look for productive strategies:

- Determining that the three rates involving miles are equivalent. **Note:** all four rates are equivalent, when considering rounding for 32 km.

3 Connect

Have individual students share which one they decided doesn't belong and why. Encourage the use of mathematical language, such as *per, unit rate, for each,* or *for every.* As each option is identified, ask whether anyone else chose the same option but for a different reason.

Highlight that there are many different ways to write and to describe the same rate by using different language, different units, and equivalent ratios.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their responses, listen for and amplify mathematical language, such as *per*, *unit rate*, *for each*, or *for every*. Add these phrases to the class display for students' reference during future class discussions.

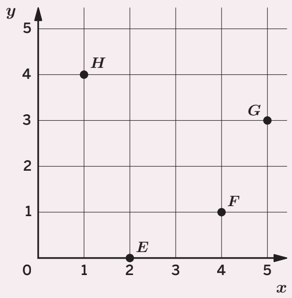
English Learners

Fluent English speakers may use the phrases *for every* and *for each* interchangeably. *For every* is usually reserved when there are more than two items. *For each* can be used regardless of the quantity of items. Allow students to use either phrase, however, you may want to consistently use *for each*, to avoid any confusion.

Power-up

To power up students' ability to plot points on a coordinate plane, have students complete:

Recall that the first number in a coordinate pair describes the horizontal distance from the origin, and the second number describes the vertical distance. Identify which point is located at each of the coordinates:



a. $(5, 3)$ G

b. $(4, 1)$ F

c. $(2, 0)$ E

d. $(1, 4)$ H

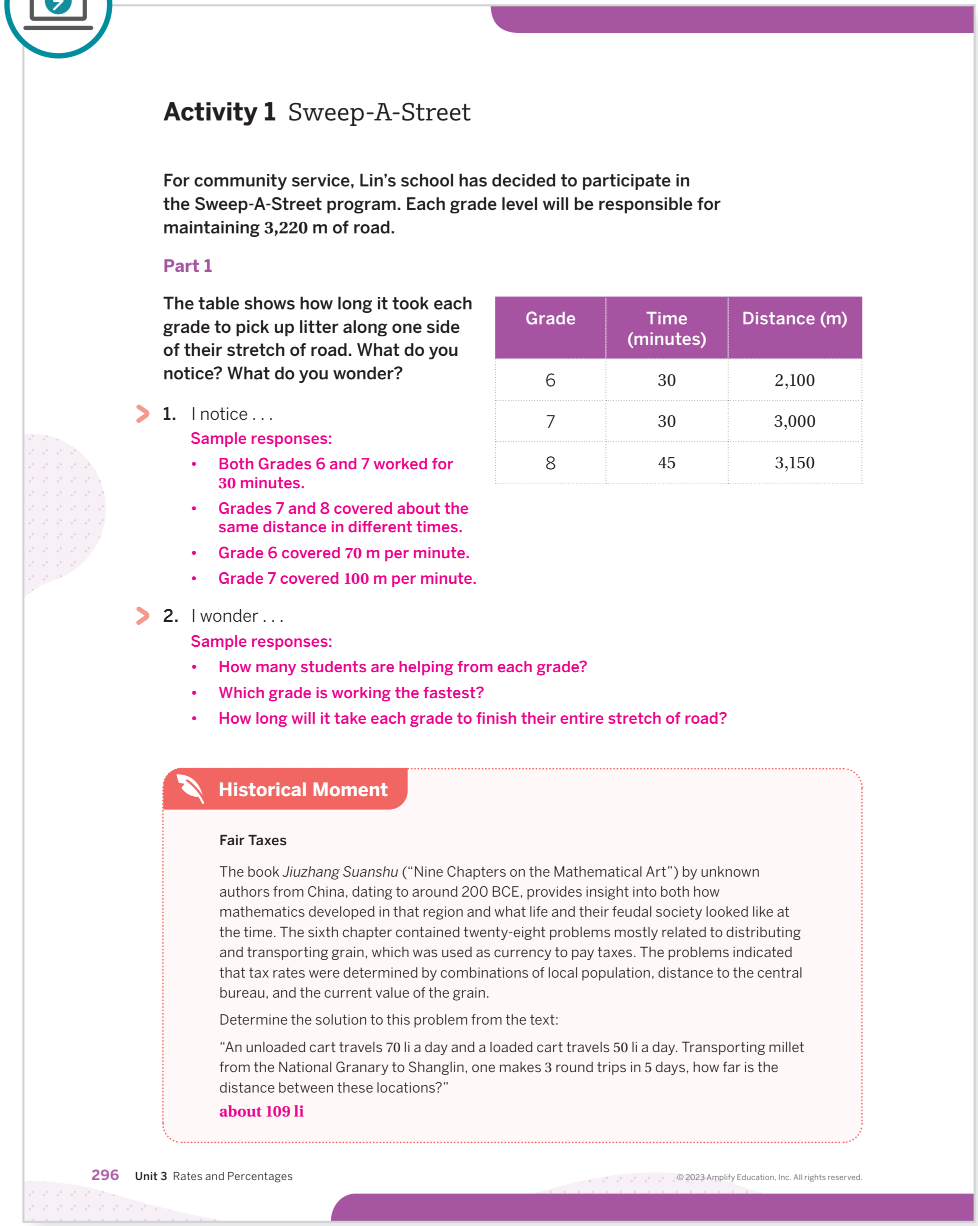
Use: Before Activity 1

Informed by: Performance on Lesson 3, Practice Problem 6

Pairs | 15 min

Activity 1 Sweep-A-Street

Students plot points on a coordinate plane to represent ratio relationships, relating them to equivalent ratios and constant rates.



Activity 1 Sweep-A-Street

For community service, Lin's school has decided to participate in the Sweep-A-Street program. Each grade level will be responsible for maintaining 3,220 m of road.

Part 1

The table shows how long it took each grade to pick up litter along one side of their stretch of road. What do you notice? What do you wonder?

Grade	Time (minutes)	Distance (m)
6	30	2,100
7	30	3,000
8	45	3,150

1. I notice . . .
   Sample responses:
   - Both Grades 6 and 7 worked for 30 minutes.
   - Grades 7 and 8 covered about the same distance in different times.
   - Grade 6 covered 70 m per minute.
   - Grade 7 covered 100 m per minute.
2. I wonder . . .
   Sample responses:
   - How many students are helping from each grade?
   - Which grade is working the fastest?
   - How long will it take each grade to finish their entire stretch of road?

Historical Moment

Fair Taxes

The book *Jiuzhang Suanshu* ("Nine Chapters on the Mathematical Art") by unknown authors from China, dating to around 200 BCE, provides insight into both how mathematics developed in that region and what life and their feudal society looked like at the time. The sixth chapter contained twenty-eight problems mostly related to distributing and transporting grain, which was used as currency to pay taxes. The problems indicated that tax rates were determined by combinations of local population, distance to the central bureau, and the current value of the grain.

Determine the solution to this problem from the text:

"An unloaded cart travels 70 li a day and a loaded cart travels 50 li a day. Transporting millet from the National Granary to Shanglin, one makes 3 round trips in 5 days, how far is the distance between these locations?"

about 109 li

296 Unit 3 Rates and Percentages

1 Launch

Conduct the ***Notice and Wonder*** routine for Part 1, and have students share responses with the class. Then have students conduct the ***Think-Pair-Share*** routine for Part 2. Allow 2–3 minutes for students to complete Problems 3–4 independently and then have them share their responses with a partner before completing Problems 5–6 together.

2 Monitor

Help students get started on Part 2 by asking, "What are the units for each axis? How would you interpret a point plotted at (10, 500)?"

Look for points of confusion:

- **Thinking that the eighth graders worked faster because they covered more distance in Part 1.** Ask, "Do all of the distances reflect the same amount of time?"
- **Thinking that the sixth and seventh graders have the same rate because they line up vertically on the graph.** Ask, "If two points line up vertically, what does that mean is the same?"

Look for productive strategies:

- Plotting points correctly on the graph.
- Using a straight line through the origin to connect equivalent ratios.
- Noticing that there is a pair of equivalent ratios in the graph in Part 2, Problem 5 by calculating unit rates.
- Multiplying to determine the total meters that the eighth graders cleaned up in Problem 6a.
- Explaining that the sixth and eighth graders have worked at the same rate and therefore have the same point on the coordinate plane in Problem 6b.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Tools*

Suggest students use a ruler to help line up points to see the equivalent ratios on the graph.

Extension: *Math Enrichment*

Have students complete the following problem: "At these same rates, how many meters could the sixth graders clean in 45 minutes? The seventh graders?" Sixth graders: 3,150 m; Seventh graders: 4,500 m

Math Language Development

MLR7: Compare and Connect

During the Connect, after you have discussed how the graph shows equivalent ratios, ask them where they might see the unit rate for each grade on the graph, and why the unit rates are the same for two of the grades.

English Learners

Pair English Learners with a native speaker. This will give them an opportunity to hear how a native speaker uses the language.

Historical Moment

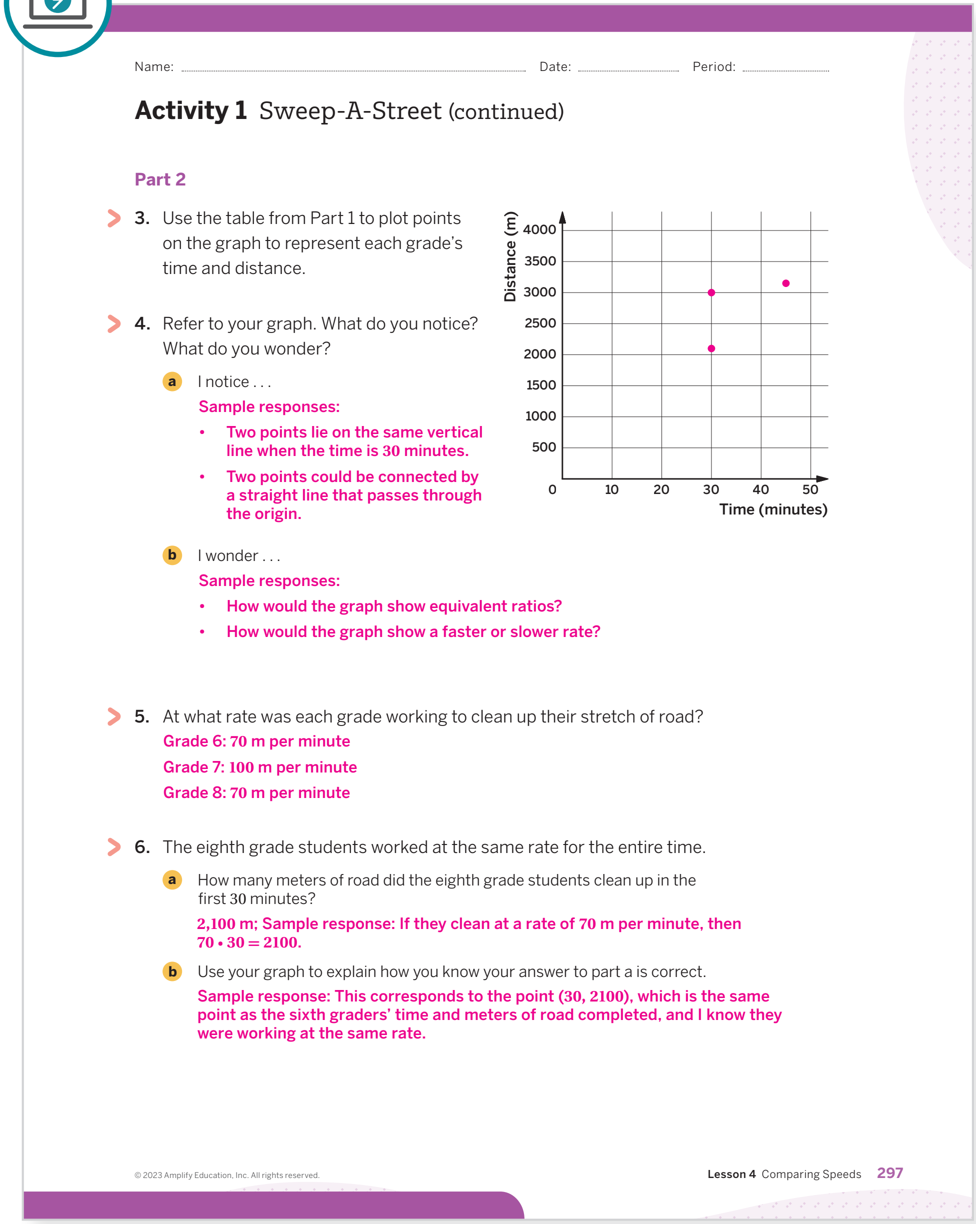
Fair Taxes

Have students complete the Historical Moment, in which they learn about how the ancient text, "Nine Chapters on the Mathematical Art," which provides insight as to what tax rates might have been like in ancient China, around 200 BCE. Tell students that the *li* is a traditional Chinese unit of distance. While its length has varied over time, its length is standardized today as equivalent to $\frac{1}{2}$ km, or 500 m.

Pairs | 15 min

Activity 1 Sweep-A-Street (continued)

Students plot points on a coordinate plane to represent ratio relationships, relating them to equivalent ratios and constant rates.



Name: ______________________ Date: __________ Period: __________

Activity 1 Sweep-A-Street (continued)

Part 2

3. Use the table from Part 1 to plot points on the graph to represent each grade's time and distance.

4. Refer to your graph. What do you notice? What do you wonder?

   a. I notice . . .

   Sample responses:
   - Two points lie on the same vertical line when the time is 30 minutes.
   - Two points could be connected by a straight line that passes through the origin.

   b. I wonder . . .

   Sample responses:
   - How would the graph show equivalent ratios?
   - How would the graph show a faster or slower rate?

5. At what rate was each grade working to clean up their stretch of road?

   Grade 6: 70 m per minute

   Grade 7: 100 m per minute

   Grade 8: 70 m per minute

6. The eighth grade students worked at the same rate for the entire time.

   a. How many meters of road did the eighth grade students clean up in the first 30 minutes?

   2,100 m; Sample response: If they clean at a rate of 70 m per minute, then $70 \cdot 30 = 2100$.

   b. Use your graph to explain how you know your answer to part a is correct.

   Sample response: This corresponds to the point (30, 2100), which is the same point as the sixth graders' time and meters of road completed, and I know they were working at the same rate.

Lesson 4 Comparing Speeds 297

3 Connect

Display a blank graph. Ask, "What do the x- and y-axes represent on this graph?" The x-axis is the number of minutes. The y-axis is the number of meters.

Have pairs of students share where they plotted the points on the graph and then share their answers to Problems 5 and 6 from Part 2. If no students mention it, draw a line to connect the equivalent ratios to support the discussion. The best time for this may be before or after asking the following questions.

Ask:

- "What do you think the graph would look like if you added more points to represent other times and distances for each grade?" They would all fit to the same straight lines, the same line for sixth and eighth grade, and then a different one for seventh grade.
- "How would the unit rates for each grade be represented on the graph?" The points would have an x-coordinate of 1 and it would be the same point for both the sixth and eighth graders.
- "Why do you think both lines connecting the equivalent ratios pass through the origin?" On double number lines, I always start both with a 0 and if I think about how I can subtract equivalent ratios, like $45 - 30$ and $3{,}150 - 2{,}100$ to get $15 : 1{,}050$. Then if I subtract again, $15 - 15$ and $1{,}050 - 1{,}050$ you get 0 s. Also, if I multiply both values in a ratio by 0 then I get 0 for both.

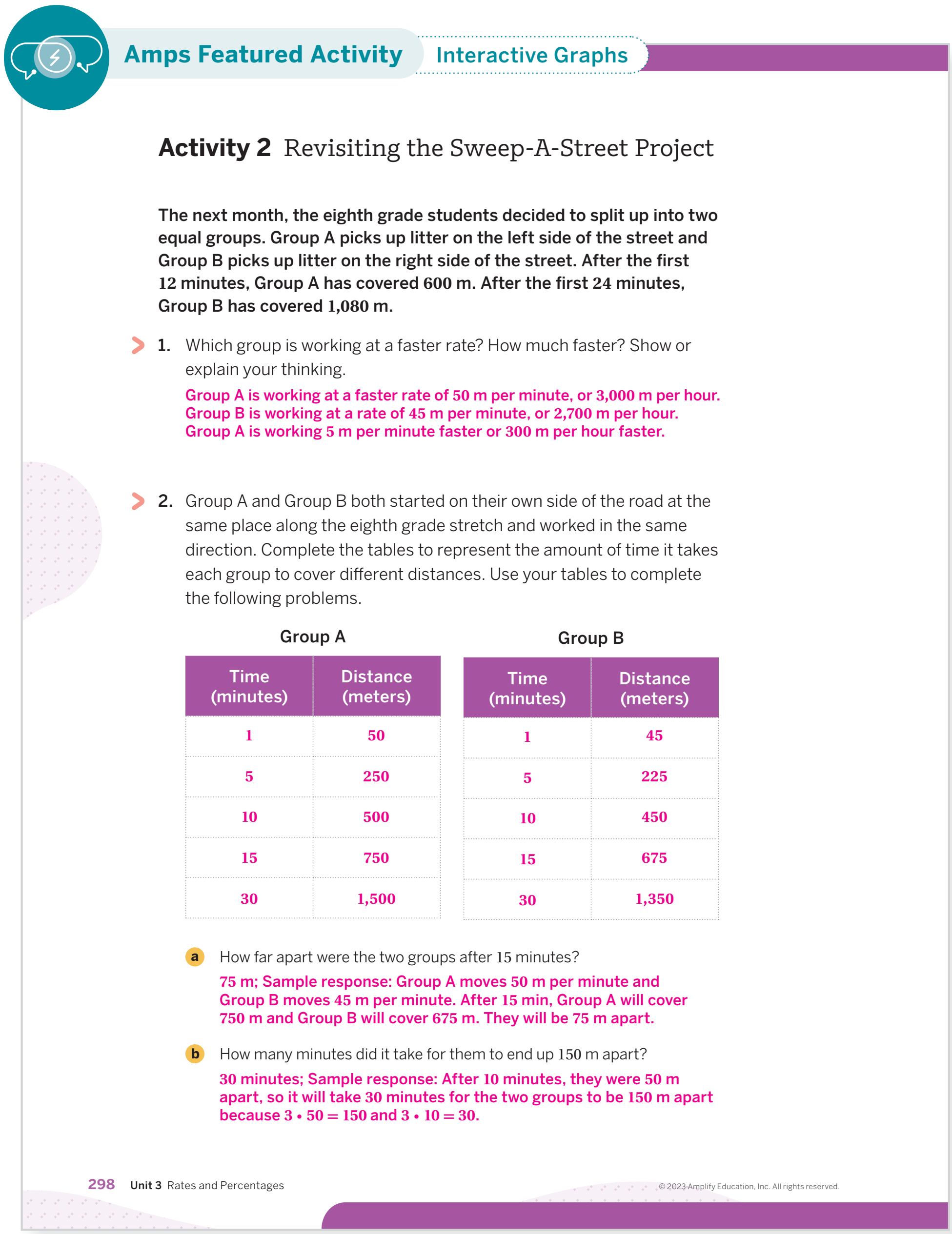
Highlight that speeds can be compared by determining the rate per 1, which can be done by dividing. In this example, the number of meters was divided by the number of minutes to determine the number of meters per minute. Equivalent ratios always have the same unit rate. Ratios can also be represented by plotting points on a graph. The points representing equivalent ratios will follow a pattern, which is a straight line that goes through the origin. Any point not on that line is not an equivalent ratio.

Pairs | 15 min

Activity 2 Revisiting the Sweep-A-Street Project

Students determine unit rates and use tables and graphs of equivalent ratios to compare unit rates and to interpret information about a scenario involving constant speeds.

Amps Featured Activity Interactive Graphs



Activity 2 Revisiting the Sweep-A-Street Project

The next month, the eighth grade students decided to split up into two equal groups. Group A picks up litter on the left side of the street and Group B picks up litter on the right side of the street. After the first 12 minutes, Group A has covered 600 m. After the first 24 minutes, Group B has covered 1,080 m.

1. Which group is working at a faster rate? How much faster? Show or explain your thinking.
   Group A is working at a faster rate of 50 m per minute, or 3,000 m per hour. Group B is working at a rate of 45 m per minute, or 2,700 m per hour. Group A is working 5 m per minute faster or 300 m per hour faster.

2. Group A and Group B both started on their own side of the road at the same place along the eighth grade stretch and worked in the same direction. Complete the tables to represent the amount of time it takes each group to cover different distances. Use your tables to complete the following problems.

Group A

Time (minutes)	Distance (meters)
1	50
5	250
10	500
15	750
30	1,500

Group B

Time (minutes)	Distance (meters)
1	45
5	225
10	450
15	675
30	1,350

a. How far apart were the two groups after 15 minutes?
75 m; Sample response: Group A moves 50 m per minute and Group B moves 45 m per minute. After 15 min, Group A will cover 750 m and Group B will cover 675 m. They will be 75 m apart.

b. How many minutes did it take for them to end up 150 m apart?
30 minutes; Sample response: After 10 minutes, they were 50 m apart, so it will take 30 minutes for the two groups to be 150 m apart because $3 \cdot 50 = 150$ and $3 \cdot 10 = 30$.

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1 Launch

Allow students 1 minute to think of a strategy for completing Problems 1 and 2 before sharing and working with a partner. Pause for a whole class discussion after Problem 2. Then have pairs continue to complete Problems 3–4.

2 Monitor

Help students get started by asking, "What do you need to know to be able to compare their rates?" The same corresponding unit rates (e.g., meters per minute) of each group.

Look for points of confusion:

- **Not plotting the points correctly on the coordinate plane.** Ask students how they would plot on the graph. They should move across the x-axis first and then up the y-axis. "Over and then up" to plot the points.
- **Struggling to determine the unit rate for each group.** Ask students how they can find the unit rate. By dividing the number of meters by the number of minutes.
- **Struggling to determine how far apart the two groups will be after 15 minutes.** Ask students how they can use the unit rate that they found in Problem 1 to determine how far apart they will be.
- **Incorrect values in Problem 2.** Students need to have correct values in order to support meaningful work in Problems 3 and 4.

Look for productive strategies:

- Determining equivalent ratios and using unit rates to fill in the tables with useful shared values for each group. If students are generating equivalent ratios but cannot answer the questions in Problem 2, ask, "What values would be helpful or needed there?"
- Plotting the points determined in their tables on the coordinate plane, attending to time and distance coordinates correctly.
- Explaining what the horizontal and vertical differences mean in context and how it relates to comparison.

Activity 2 continued >

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Problems 1, 2a, and 3. They can still participate in the class discussion for Problem 4.

Extension: *Math Enrichment*

Have students construct another graph of the rates of Groups A and B with the labels of the axes reversed. Ask them to explain how this still shows that the same group was moving faster.

Math Language Development

MLR1: Stronger and Clearer Each Time

Provide students time to individually develop a draft response for Problem 4, in which they refer directly to their graphs. Have them meet with 2–3 partners to give and receive feedback on their drafts. Encourage partners to use these questions as they provide feedback:

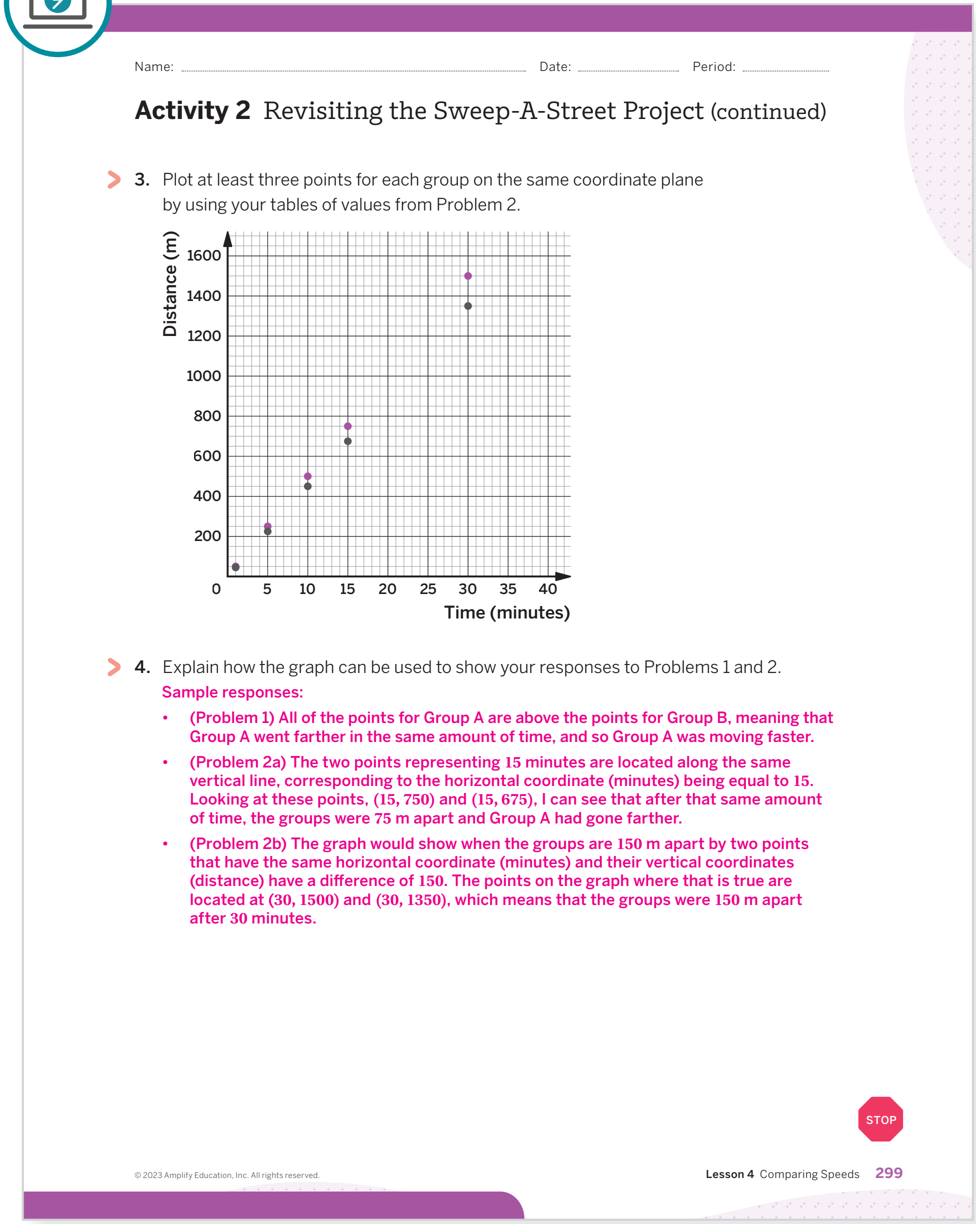
- "Does the response include how the graph shows which group is working at a faster rate?"
- "Does the response include how the graph shows how far apart the groups were after 15 minutes?"

Have students write an improved response, based on the feedback received.

Pairs | 15 min

Activity 2 Revisiting the Sweep-A-Street Project (continued)

Students determine unit rates and use tables and graphs of equivalent ratios to compare unit rates and to interpret information about a scenario involving constant speeds.



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Activity 2 Revisiting the Sweep-A-Street Project (continued)

3. Plot at least three points for each group on the same coordinate plane by using your tables of values from Problem 2.

4. Explain how the graph can be used to show your responses to Problems 1 and 2.

Sample responses:

- **(Problem 1) All of the points for Group A are above the points for Group B, meaning that Group A went farther in the same amount of time, and so Group A was moving faster.**
- **(Problem 2a) The two points representing 15 minutes are located along the same vertical line, corresponding to the horizontal coordinate (minutes) being equal to 15. Looking at these points, (15, 750) and (15, 675), I can see that after that same amount of time, the groups were 75 m apart and Group A had gone farther.**
- **(Problem 2b) The graph would show when the groups are 150 m apart by two points that have the same horizontal coordinate (minutes) and their vertical coordinates (distance) have a difference of 150. The points on the graph where that is true are located at (30, 1500) and (30, 1350), which means that the groups were 150 m apart after 30 minutes.**

STOP

3 Connect

Display the blank graph from Problem 3.

Have pairs of students share their responses to Problems 1–3 and plot their points on the graph for all to see. Then have students share their responses to Problem 4, going back through in the order of Problem 1, 2a, and 2b. If no students suggest it, consider drawing lines to highlight common x-values (minutes) or y-values (distance) or to connect equivalent ratios for each rate when it is appropriate for supporting discussion.

Note: It is not an expectation of this grade that students use the term *slope* to describe lines. Interpreting relative positions of points and the "steepness" of corresponding lines by using the language of ratios and rates is accessible.

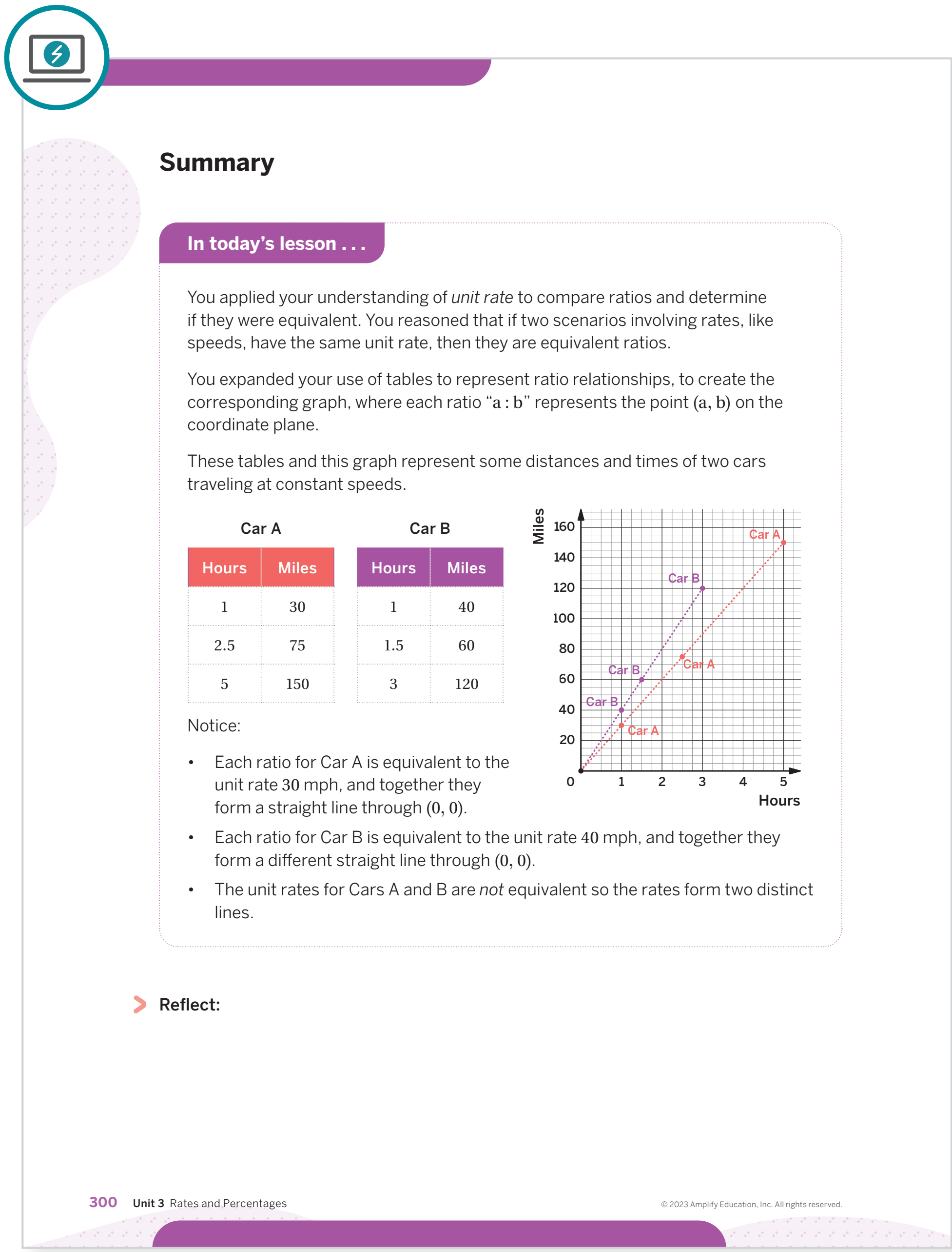
Ask, "If the points representing each rate are connected by a line, how could you use those lines to describe which group is moving faster?"

Highlight that students' work in Activity 1 showed that the graph of equivalent ratios follow a pattern of a straight line through the origin. Such a graph of points representing ratios can be used to determine whether any two ratios are equivalent. These types of graphs can also be used to identify a corresponding unit rate as a point that has a coordinate of 1 and would be on the same line through the origin.

Whole Class | 5 min

Summary

Review and synthesize how equivalent ratios that are plotted on a graph follow a pattern and can be connected by a straight line that goes through the origin.



Summary

In today's lesson . . .

You applied your understanding of *unit rate* to compare ratios and determine if they were equivalent. You reasoned that if two scenarios involving rates, like speeds, have the same unit rate, then they are equivalent ratios.

You expanded your use of tables to represent ratio relationships, to create the corresponding graph, where each ratio "$a : b$" represents the point (a, b) on the coordinate plane.

These tables and this graph represent some distances and times of two cars traveling at constant speeds.

Car A

Hours	Miles
1	30
2.5	75
5	150

Car B

Hours	Miles
1	40
1.5	60
3	120

Notice:

- Each ratio for Car A is equivalent to the unit rate 30 mph, and together they form a straight line through $(0, 0)$.
- Each ratio for Car B is equivalent to the unit rate 40 mph, and together they form a different straight line through $(0, 0)$.
- The unit rates for Cars A and B are *not* equivalent so the rates form two distinct lines.

Reflect:

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Synthesize

Display the tables and graph showing distances and times for Cars A and B.

Ask:

- "Looking at the graph, what would be another pair of values for hours and miles that represents the constant speed of Car A? Does that make sense according to the table?"
- "What about for Car B?"

Have students share responses to the questions one at a time, focusing on those who indicate that all points for each car should still be on the same line and that the corresponding values in the table would both be the same multiple of the values in the unit rate row.

Highlight that two equivalent ratios, such as those representing a constant speed, have the same unit rates. The patterns in values of equivalent ratios from tables, where each pair of values shares a common factor, can be seen on a graph in the pattern of a straight line through the origin, because each coordinate increases or decreases by the same factor. To compare two rates such as constant speeds, unit rates can be used because they share a common value of 1. A graph also shows this comparison by looking at the corresponding points on the lines for each speed where the value for one coordinate is 1.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

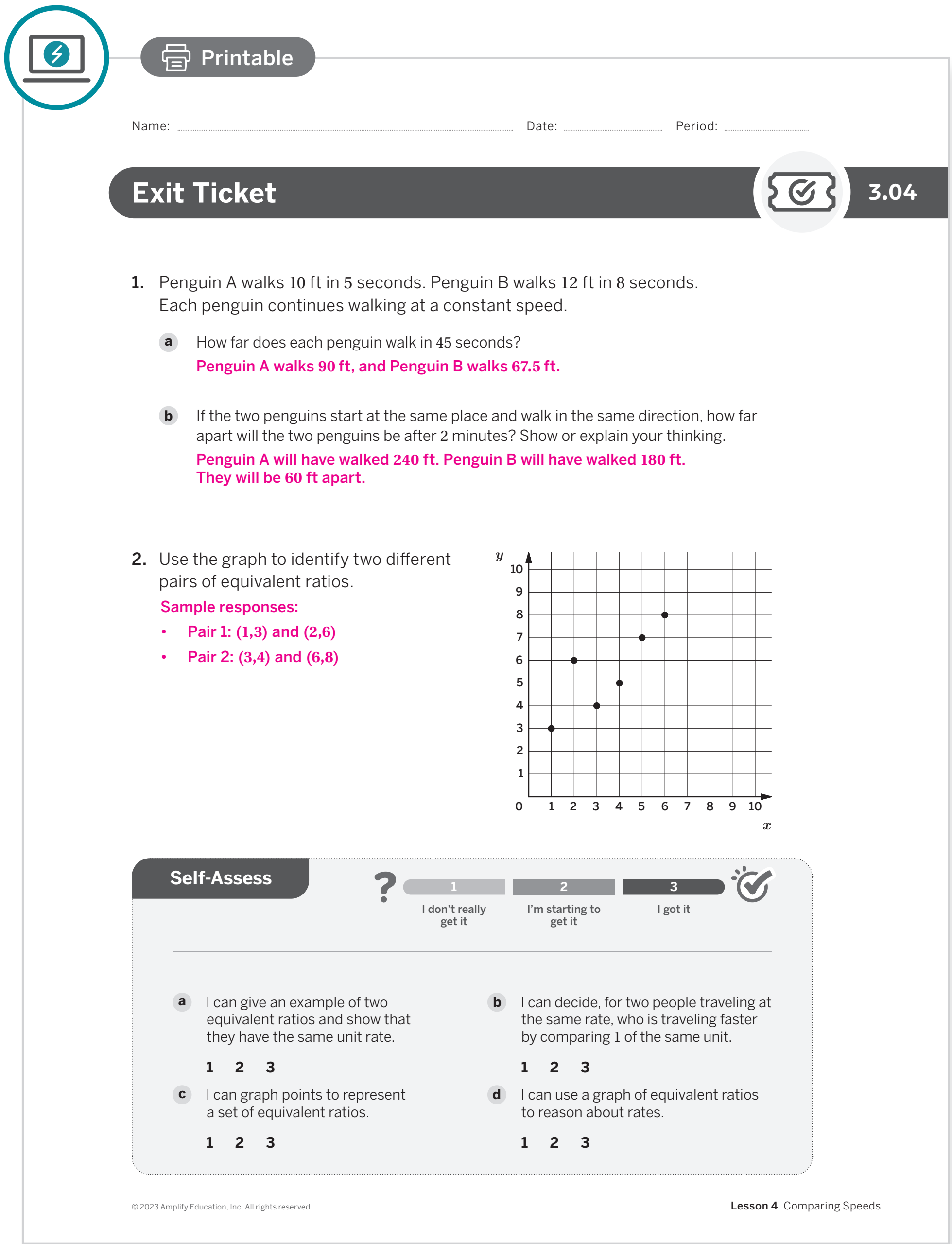
- "What strategies or tools did you find helpful today when graphing the values in a ratio table?"
- "Were any strategies or tools not helpful? Why?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how the relationships among equivalent ratios, corresponding unit rates, and graphs of those ratios can be used to compare rates.

Printable



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Exit Ticket 3.04

1. Penguin A walks 10 ft in 5 seconds. Penguin B walks 12 ft in 8 seconds. Each penguin continues walking at a constant speed.

   a How far does each penguin walk in 45 seconds?

   Penguin A walks 90 ft, and Penguin B walks 67.5 ft.

   b If the two penguins start at the same place and walk in the same direction, how far apart will the two penguins be after 2 minutes? Show or explain your thinking.

   Penguin A will have walked 240 ft. Penguin B will have walked 180 ft. They will be 60 ft apart.

2. Use the graph to identify two different pairs of equivalent ratios.

   Sample responses:
   - **Pair 1: (1,3) and (2,6)**
   - **Pair 2: (3,4) and (6,8)**

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can give an example of two equivalent ratios and show that they have the same unit rate.

1 2 3

b I can decide, for two people traveling at the same rate, who is traveling faster by comparing 1 of the same unit.

1 2 3

c I can graph points to represent a set of equivalent ratios.

1 2 3

d I can use a graph of equivalent ratios to reason about rates.

1 2 3

Lesson 4 Comparing Speeds

Success looks like . . .

- **Language Goal:** Explaining that if two ratios have the same rate per 1, then they are equivalent ratios. **(Speaking and Listening, Writing)**
- **Language Goal:** Recognizing that calculating how much for 1 of the same unit is a useful strategy for comparing rates. Expressing these rates by using the word *per* and specifying the unit. **(Speaking and Listening, Writing)**
- **Language Goal:** Plotting pairs of values from tables that represent equivalent ratios and interpreting points on a graph of equivalent ratios using rate language. **(Speaking and Listening, Writing)**
  - » Identifying two different pairs of equivalent rates on a graph in Problem 2.
- **Language Goal:** Justifying comparisons of speeds by using tables or graphs. **(Speaking and Listening, Writing)**

Suggested next steps

If students struggle to find the rate of the penguins for Problem 1a, consider:

- Reviewing how to determine and use unit rate in Activity 1.
- Assigning Practice Problem 1.

If students use addition to solve Problem 1b, consider:

- Reviewing Problem 2a in Activity 2.

If students struggle with identifying the points in the coordinate plane in Problem 2, consider:

- Reviewing identifying equivalent ratios and plotting the points in Activity 1, Part 2.

Professional Learning

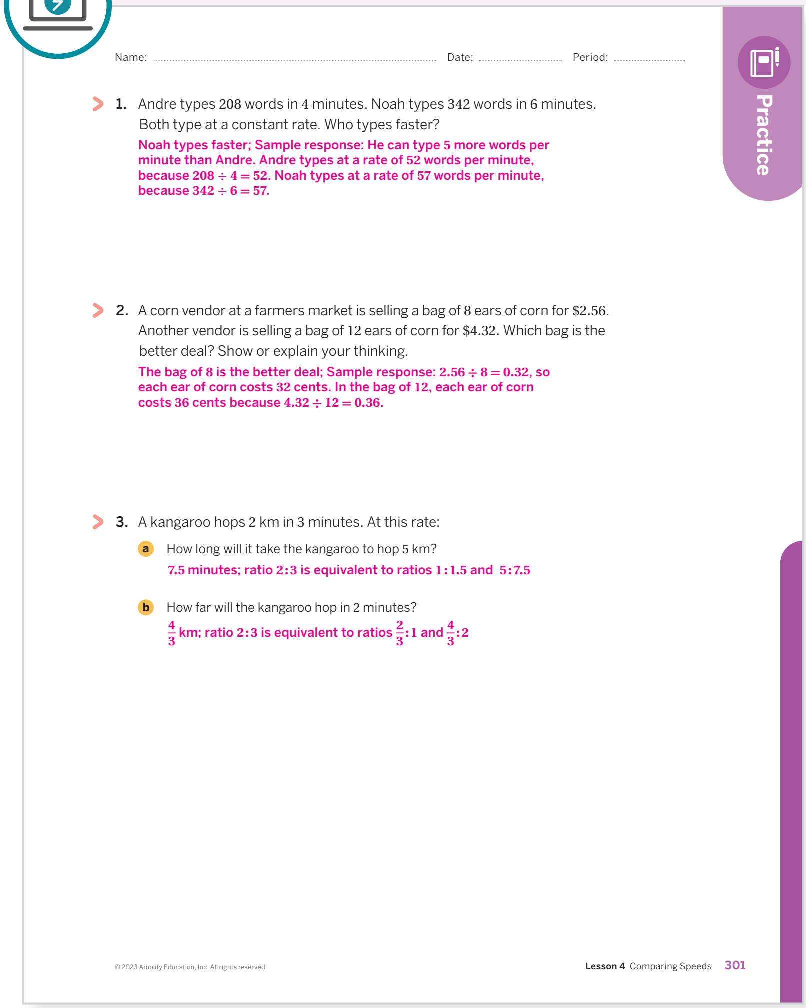
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What different ways did students approach graphing ratios? What does that tell you about similarities and differences among your students?
- What did interpreting graphs of ratios on the coordinate plane reveal about your students as learners?

Practice

Independent



Name: Date: Period:

1. Andre types 208 words in 4 minutes. Noah types 342 words in 6 minutes. Both type at a constant rate. Who types faster?

   Noah types faster; Sample response: He can type 5 more words per minute than Andre. Andre types at a rate of 52 words per minute, because $208 \div 4 = 52$. Noah types at a rate of 57 words per minute, because $342 \div 6 = 57$.

2. A corn vendor at a farmers market is selling a bag of 8 ears of corn for $2.56. Another vendor is selling a bag of 12 ears of corn for $4.32. Which bag is the better deal? Show or explain your thinking.

   The bag of 8 is the better deal; Sample response: $2.56 \div 8 = 0.32$, so each ear of corn costs 32 cents. In the bag of 12, each ear of corn costs 36 cents because $4.32 \div 12 = 0.36$.

3. A kangaroo hops 2 km in 3 minutes. At this rate:

   a. How long will it take the kangaroo to hop 5 km?

   7.5 minutes; ratio $2:3$ is equivalent to ratios $1:1.5$ and $5:7.5$

   b. How far will the kangaroo hop in 2 minutes?

   $\frac{4}{3}$ km; ratio $2:3$ is equivalent to ratios $\frac{2}{3}:1$ and $\frac{4}{3}:2$

Practice

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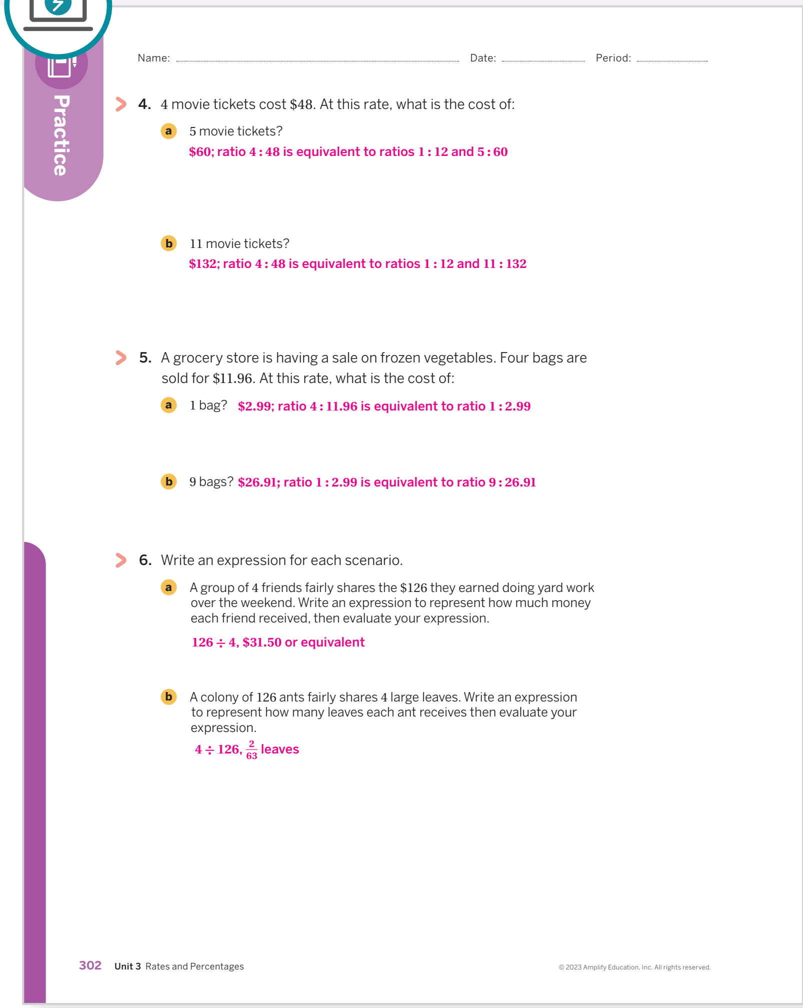
Lesson 4 Comparing Speeds 301



Name: Date: Period:

Practice

4. 4 movie tickets cost $48. At this rate, what is the cost of:

   a. 5 movie tickets?

   $60; ratio $4:48$ is equivalent to ratios $1:12$ and $5:60$

   b. 11 movie tickets?

   $132; ratio $4:48$ is equivalent to ratios $1:12$ and $11:132$

5. A grocery store is having a sale on frozen vegetables. Four bags are sold for $11.96. At this rate, what is the cost of:

   a. 1 bag? **$2.99; ratio $4:11.96$ is equivalent to ratio $1:2.99$**

   b. 9 bags? **$26.91; ratio $1:2.99$ is equivalent to ratio $9:26.91$**

6. Write an expression for each scenario.

   a. A group of 4 friends fairly shares the $126 they earned doing yard work over the weekend. Write an expression to represent how much money each friend received, then evaluate your expression.

   $126 \div 4$, $31.50 or equivalent

   b. A colony of 126 ants fairly shares 4 large leaves. Write an expression to represent how many leaves each ant receives then evaluate your expression.

   $4 \div 126$, $\frac{2}{63}$ leaves

302 Unit 3 Rates and Percentages

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 3	2
	5	Unit 3 Lesson 2	2
Formative	6	Unit 3 Lesson 5	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 3 | **LESSON 5**

Interpreting Rates

Let's explore unit rates and their graphs some more.

Focus

Goals

1. **Language Goal:** Calculate and interpret the two unit rates associated with a ratio, i.e., $\frac{a}{b}$ and $\frac{b}{a}$ for the ratio $a : b$. **(Speaking and Listening, Writing)**
2. **Language Goal:** Choose which unit rate to use to solve a given problem and explain the choice. **(Speaking and Listening, Writing)**
3. **Language Goal:** Relate the two unit rates for a given ratio relationship to a graph of corresponding points. **(Speaking and Listening, Writing)**

Coherence

Today

Students determine the two unit rates, $\frac{a}{b}$ and $\frac{b}{a}$, associated with a ratio $a : b$. They recognize that while both unit rates describe amounts of both quantities, each describes the amount of one quantity per 1 of a different second quantity. Students also explain which unit rate may be more useful or efficient for solving rate problems depending on the given information and the missing value they are trying to determine. They continue to graph points to represent equivalent ratios and are able to identify both unit rates on the graph.

Previously

In Lesson 4, students determined unit rates and saw that two equivalent ratios have the same unit rates. They compared unit rates and also graphed points representing equivalent and non-equivalent ratios.

Coming Soon

In Lesson 6, students will extend their work with comparing rates and corresponding graphs. They will generalize the use of unit rates and graphs for comparing rates, discovering that the steepness of the lines can be used to describe rates as being "more" or "less" of one quantity.

Rigor

- Students build **procedural skills** for determining unit rates using tables and graphs.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Small Groups	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

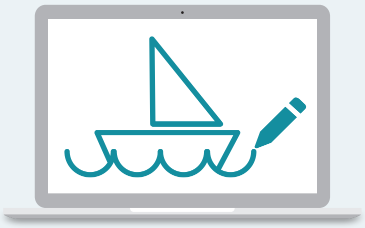
Review word

- *unit rate*

Amps | Featured Activity

Activity 2
Digital Sketches

Students sketch on top of a graph to show their thinking about the prices of raffle tickets.



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Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students may think that constructing their own argument is sufficient. Remind them that they can learn from each other by listening to other students' arguments. They also can help each other by correcting and critiquing arguments. This requires engagement by all students, and will benefit everyone. By considering and critiquing the reasoning of others, students will develop a deeper understanding of the mathematics.

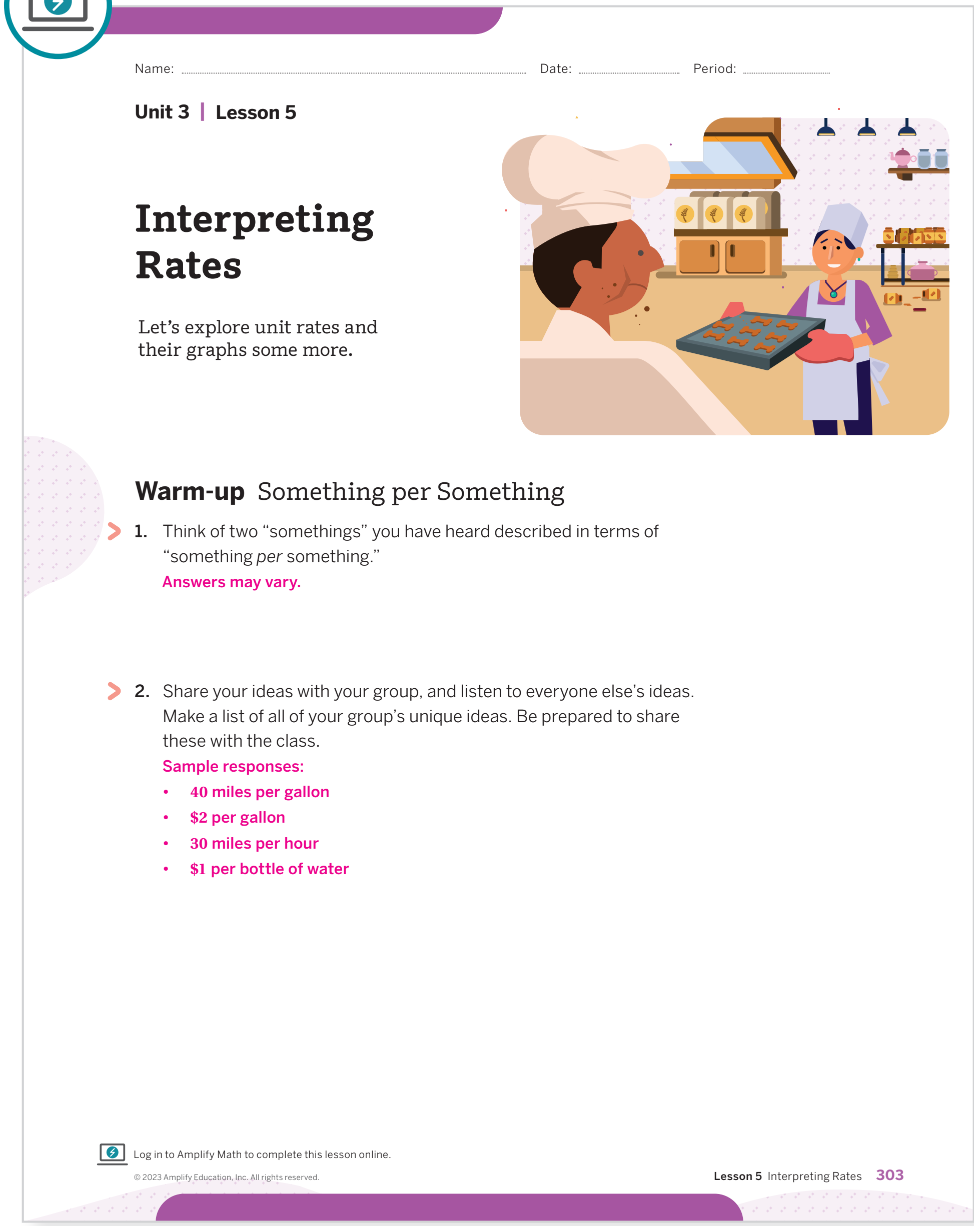
Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1,** omit Problem 4.
- In **Activity 2,** omit Problem 4.

Warm-up Something per Something

Students activate background knowledge of familiar examples of rates (“something per something”), preparing them to determine and interpret unit rates in this lesson.



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Unit 3 | Lesson 5

Interpreting Rates

Let’s explore unit rates and their graphs some more.

Warm-up Something per Something

1. Think of two “somethings” you have heard described in terms of “something *per* something.”
   Answers may vary.

2. Share your ideas with your group, and listen to everyone else’s ideas. Make a list of all of your group’s unique ideas. Be prepared to share these with the class.
   Sample responses:
   - **40 miles per gallon**
   - **$2 per gallon**
   - **30 miles per hour**
   - **$1 per bottle of water**

Log in to Amplify Math to complete this lesson online.

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Lesson 5 Interpreting Rates 303

1 Launch

Arrange students in small groups. Give students 1 minute of think time and then time to share with their group before discussing examples as a class.

2 Monitor

Help students get started by asking them to think outside of math class, “Where have you heard or used the word *per* before?”

Look for productive strategies:

- Listing a rate of something per more than one of something else.
- Listing an example, such as “40 miles per gallon” to mean a unit rate where the 1 is implied.

3 Connect

Have groups of students share their list of ideas with the class. Record the responses. If time permits, after all groups have an opportunity to share, consider asking students whether any new ideas have come to mind, or whether they can think of similar examples that might use different units.

Highlight that all of the examples are rates because they all tell how much of one thing *per* how much of another thing. Often the second quantity does not include a number, and the units are stated as singular (e.g., gallon, hour, etc.), and that means it is a unit rate of *per* 1. Statements such as these describing a rate, are also generally understood to describe a constant rate, even though they can be read as a specific ratio.

Power-up

To power up students’ ability to write expressions to represent scenarios involving rates in contextual scenarios, have students complete:

A bakery uses 120 eggs to make 15 identical cakes. Select the expression that represents how many eggs were in each cake.

A. $120 \cdot 15$

B. $120 - 15$

[✓ **C.**] $120 \div 15$

D. $15 \div 120$

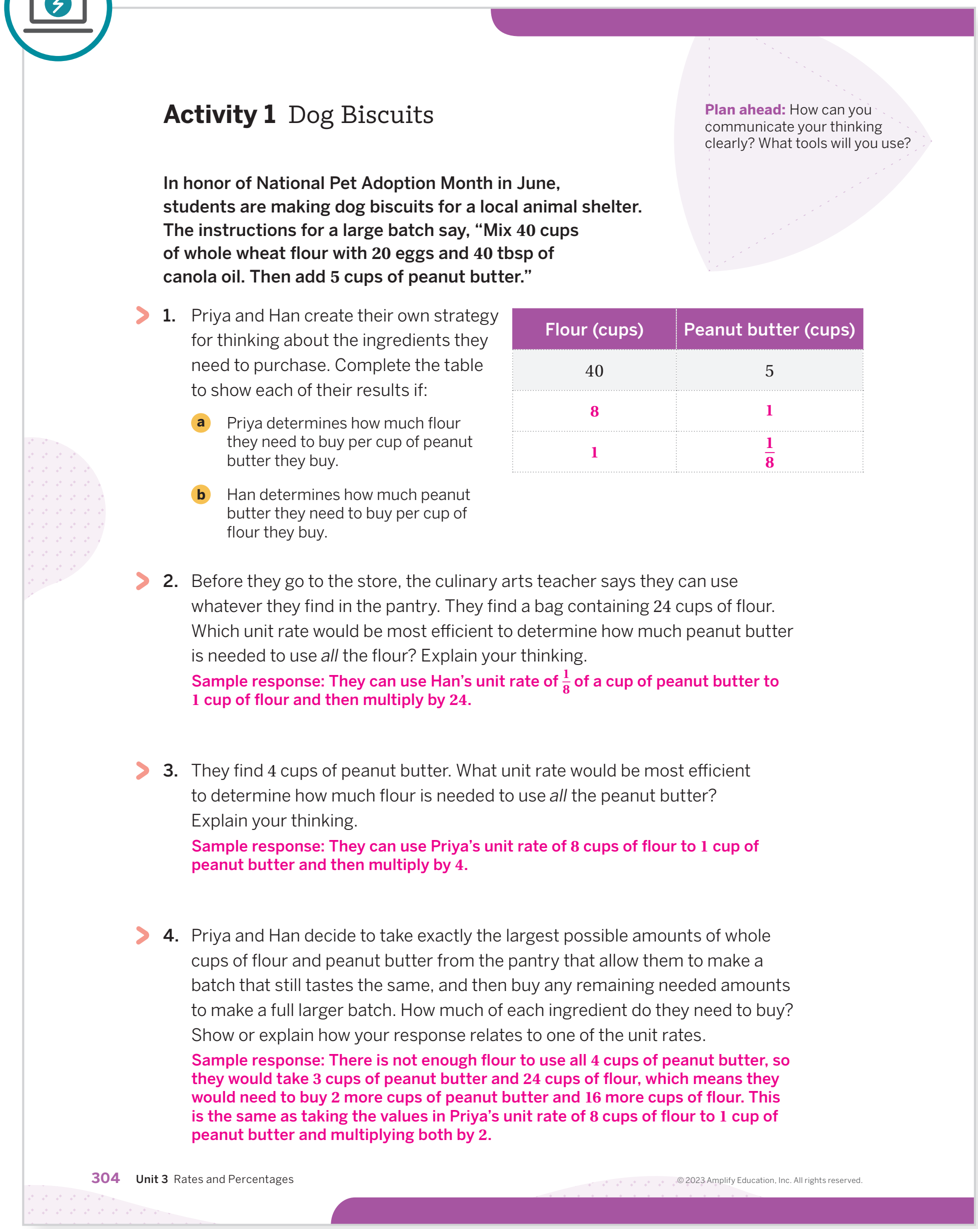
Use: Before Activity 1.

Informed by: Performance on Lesson 2, Practice Problems 6 and Pre-Unit Readiness Assessment, Problem 8.

Pairs | 15 min

Activity 1 Dog Biscuits

Students determine the two unit rates for a single context and choose which one to use to solve problems involving different unknown equivalent ratios.



Activity 1 Dog Biscuits

Plan ahead: How can you communicate your thinking clearly? What tools will you use?

In honor of National Pet Adoption Month in June, students are making dog biscuits for a local animal shelter. The instructions for a large batch say, "Mix 40 cups of whole wheat flour with 20 eggs and 40 tbsp of canola oil. Then add 5 cups of peanut butter."

1. Priya and Han create their own strategy for thinking about the ingredients they need to purchase. Complete the table to show each of their results if:
   - a. Priya determines how much flour they need to buy per cup of peanut butter they buy.
   - b. Han determines how much peanut butter they need to buy per cup of flour they buy.

Flour (cups)	Peanut butter (cups)
40	5
8	1
1	$\frac{1}{8}$

2. Before they go to the store, the culinary arts teacher says they can use whatever they find in the pantry. They find a bag containing 24 cups of flour. Which unit rate would be most efficient to determine how much peanut butter is needed to use *all* the flour? Explain your thinking.
   Sample response: They can use Han's unit rate of $\frac{1}{8}$ of a cup of peanut butter to 1 cup of flour and then multiply by 24.

3. They find 4 cups of peanut butter. What unit rate would be most efficient to determine how much flour is needed to use *all* the peanut butter? Explain your thinking.
   Sample response: They can use Priya's unit rate of 8 cups of flour to 1 cup of peanut butter and then multiply by 4.

4. Priya and Han decide to take exactly the largest possible amounts of whole cups of flour and peanut butter from the pantry that allow them to make a batch that still tastes the same, and then buy any remaining needed amounts to make a full larger batch. How much of each ingredient do they need to buy? Show or explain how your response relates to one of the unit rates.
   Sample response: There is not enough flour to use all 4 cups of peanut butter, so they would take 3 cups of peanut butter and 24 cups of flour, which means they would need to buy 2 more cups of peanut butter and 16 more cups of flour. This is the same as taking the values in Priya's unit rate of 8 cups of flour to 1 cup of peanut butter and multiplying both by 2.

304 **Unit 3** Rates and Percentages

1 Launch

Have students use the *Think-Pair-Share* routine. Provide them 1 minute of individual think time. Then have them complete the activity with a partner. Activate background knowledge by asking whether anyone knows what a culinary arts class is.

2 Monitor

Help students get started by asking, "How are Han and Priya thinking differently? How many cups of peanut butter is Priya thinking about? How many cups of flour is Han thinking about?"

Look for points of confusion:

- **Confusing the unit rates or the operations using them.** Suggest students add more rows to the table to represent other equivalent ratios they are trying to determine. Ask, "Would you be multiplying or dividing them? What by what?"

Look for productive strategies:

- Dividing to determine both unit rates (Problem 1).
- Explaining responses by referencing either unit rate and a corresponding factor or divisor.
- Recognizing the more efficient unit rate to use based on which quantity is known (e.g., knowing x cups of peanut butter, if the unit rate for 1 cup of peanut butter is used, and then multiplying both values by x).

3 Connect

Display the completed table for students to check.

Have pairs of students share how and why they chose and used unit rates to determine different amounts of flour and peanut butter that can be used in the recipe.

Highlight that there are always two unit rates that can be used to solve constant-rate problems. Sometimes one is more efficient, depending on the missing value being determined.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Problems 1–3. For Problems 2–3, encourage them to determine the corresponding amounts of flour any way they can first, such as by adding more rows of equivalent ratios to the table. If time permits, have them connect their responses back to one of the unit rates.

Extension: *Math Enrichment*

Have students determine six other unit rates or ratios in which eggs are one of the ingredients. Without units, the six ratios are: (a) flour : eggs, $2:1$, $1:\frac{1}{2}$; (b) eggs : oil, $1:2$, $\frac{1}{2}:1$; (c) eggs : peanut butter, $4:1$, $1:\frac{1}{4}$

Math Language Development

MLR8: Discussion Supports—Press for Details

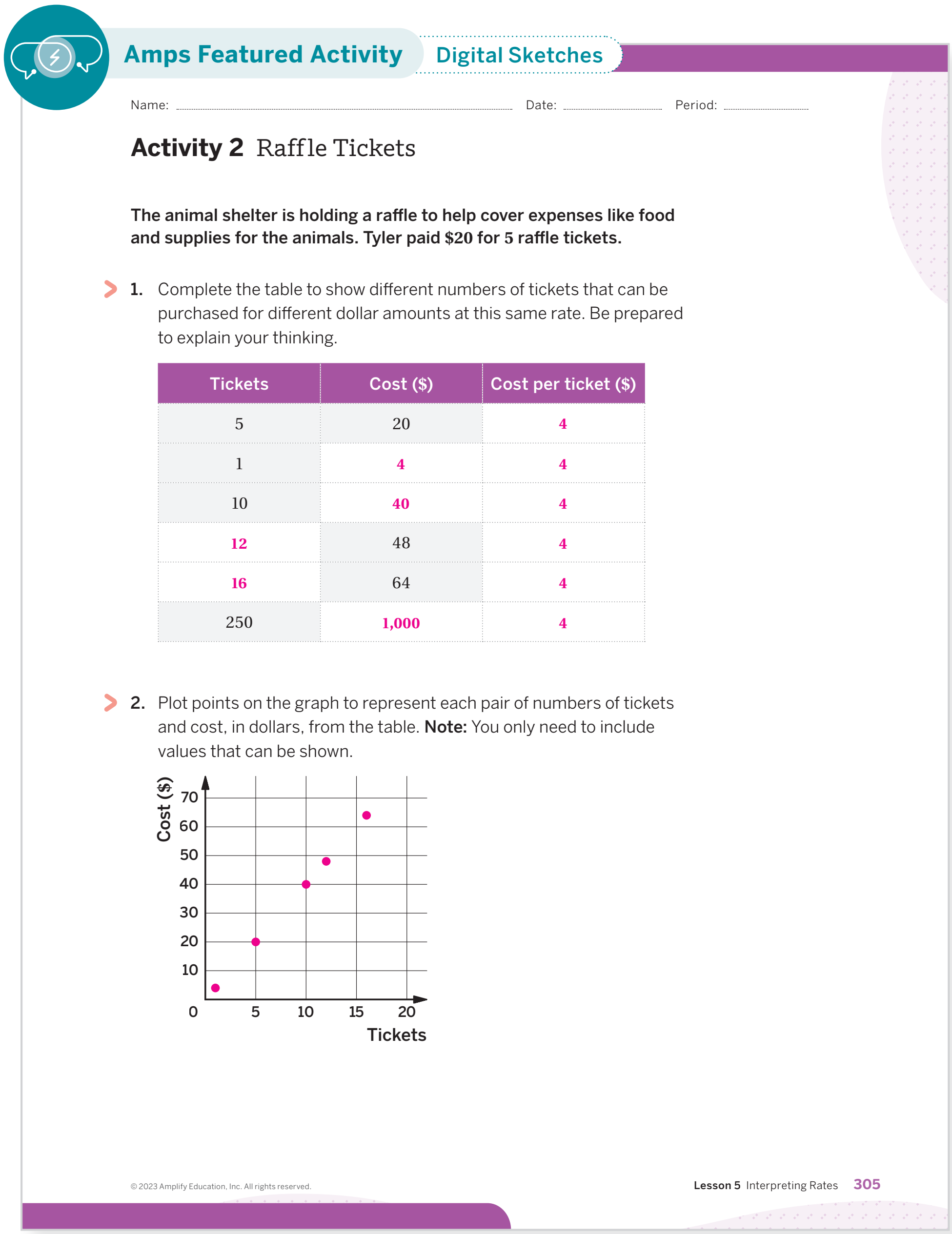
During the Connect, as students share how they used unit rates to solve Problems 2–4, press for details in their reasoning by asking the following questions:

- "Why did you use this unit rate and not the other unit rate?"
- "In Problem 2, if you used the unit rate of 8 cups of flour to 1 cup of peanut butter, what would you need to do to determine how much peanut butter to use?" Sample response: Find an equivalent ratio. Because $8 \cdot 3 = 24$, I can multiply 1 by 3.

Pairs | 15 min

Activity 2 Raffle Tickets

Students use a constant unit rate to determine missing values, representing the unit rate in its own column of a table, and then identify both unit rates on a graph of equivalent ratios.


Amps Featured Activity Digital Sketches

Name: ______ Date: ______ Period: ______

Activity 2 Raffle Tickets

The animal shelter is holding a raffle to help cover expenses like food and supplies for the animals. Tyler paid $20 for 5 raffle tickets.

1. Complete the table to show different numbers of tickets that can be purchased for different dollar amounts at this same rate. Be prepared to explain your thinking.

Tickets	Cost ($)	Cost per ticket ($)
5	20	4
1	4	4
10	40	4
12	48	4
16	64	4
250	1,000	4

2. Plot points on the graph to represent each pair of numbers of tickets and cost, in dollars, from the table. **Note:** You only need to include values that can be shown.

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Lesson 5 Interpreting Rates 305

1 Launch

Have students use the *Think-Pair-Share* routine. Provide them 1 minute of individual think time. Then have them complete the activity with a partner.

2 Monitor

Help students get started by asking, "What is different about this table? How will the "Cost per ticket" column be related to the other columns?"

Look for points of confusion:

- **Not recognizing the "Cost per ticket" column as the unit rate corresponding to the other values in a row.** Have students determine the other missing value using equivalent ratios first. Then have them determine "Cost per ticket" for the second row representing 1 ticket. Ask, "Should cost per ticket be the same or different for the other rows of values? How can you use those other values in each row to determine or to justify the value for "Cost per ticket?"
- **Using the "Cost per ticket" value as one of the coordinates when plotting points on the coordinate plane.** Ask, "What do the labels on the graph say the quantity for each axis is? Where are those values found in your table?"

Look for productive strategies:

- Recognizing that the "Cost per ticket" column is 4 in every row because that represents a constant unit rate, and using this to efficiently determine the other missing value in each row.
- Recognizing a unit rate can be seen on a graph as the two values in the coordinates of an individual point, where the value of one coordinate is 1.

Activity 2 continued >

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Problems 1 and 2. If time allows, have them add and complete another row in the table, where the value for "Dollars" is 1.

Extension: *Math Enrichment*

Ask students to use tables of graphs to find a counterexample that illustrates why this statement is not always true: "If the graph of a set of ordered pairs falls on a straight line, then the ordered pairs show equivalent ratios." False; The points $(0, 3)$, $(1, 4)$, $(2, 5)$, and $(3, 6)$ fall on a straight line, but they do not represent equivalent ratios.

Math Language Development

MLR2: Collect and Display

Collect and display student responses for Problems 1 and 2 and use these responses to highlight connections as they share how their graph represents both unit rates.

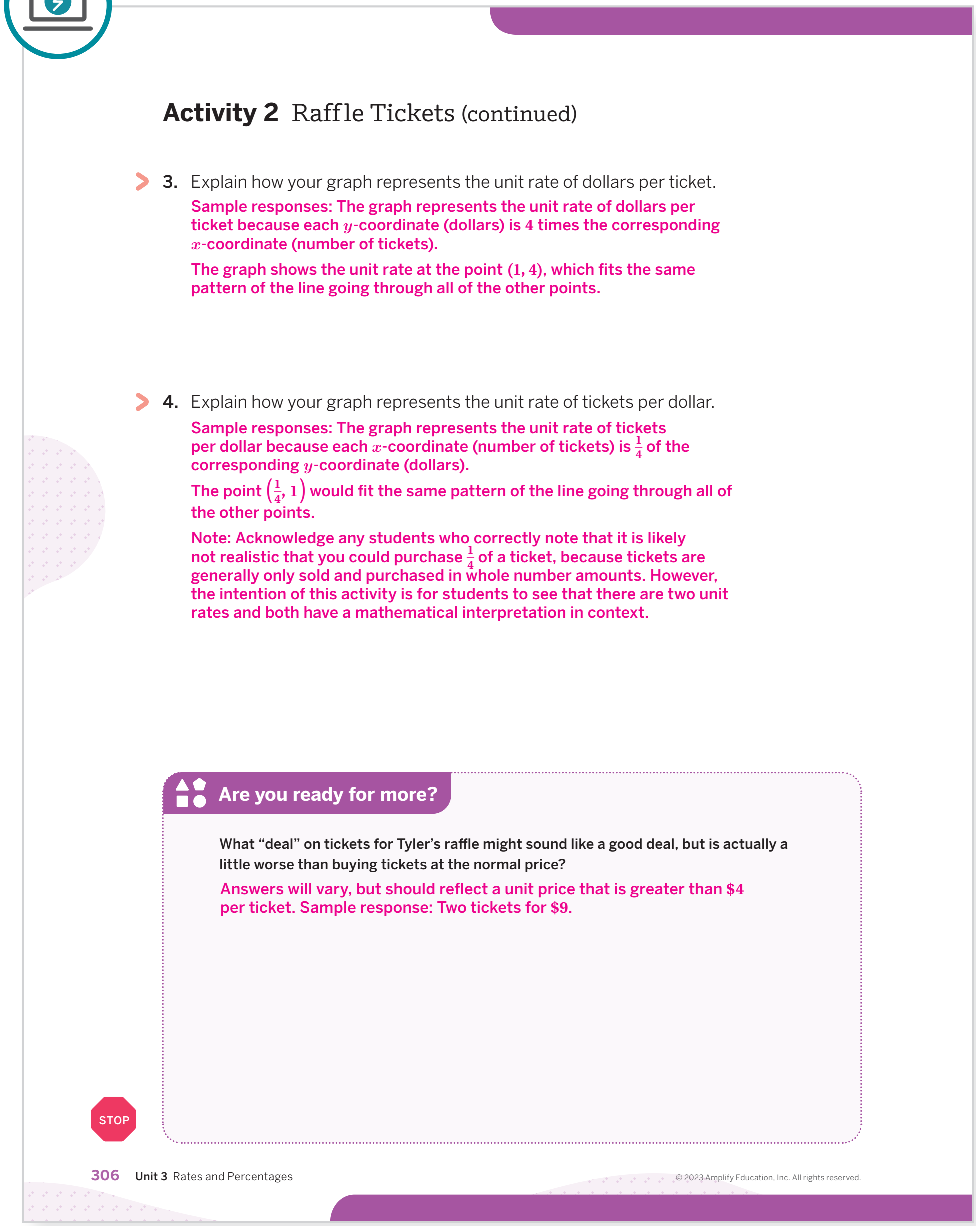
English Learners

Highlight the connections between the table and graph and add these representations to the class display with annotations emphasizing the connections. For example, highlight the ordered pair $(1, 4)$ and how it is represented in the table and the graph.

Pairs | 15 min

Activity 2 Raffle Tickets (continued)

Students use a constant unit rate to determine missing values, representing the unit rate in its own column of a table, and then identify both unit rates on a graph of equivalent ratios.



Activity 2 Raffle Tickets (continued)

3. Explain how your graph represents the unit rate of dollars per ticket.

   Sample responses: The graph represents the unit rate of dollars per ticket because each y-coordinate (dollars) is 4 times the corresponding x-coordinate (number of tickets).

   The graph shows the unit rate at the point $(1, 4)$, which fits the same pattern of the line going through all of the other points.

4. Explain how your graph represents the unit rate of tickets per dollar.

   Sample responses: The graph represents the unit rate of tickets per dollar because each x-coordinate (number of tickets) is $\frac{1}{4}$ of the corresponding y-coordinate (dollars).

   The point $\left(\frac{1}{4}, 1\right)$ would fit the same pattern of the line going through all of the other points.

   Note: Acknowledge any students who correctly note that it is likely not realistic that you could purchase $\frac{1}{4}$ of a ticket, because tickets are generally only sold and purchased in whole number amounts. However, the intention of this activity is for students to see that there are two unit rates and both have a mathematical interpretation in context.

Are you ready for more?

What "deal" on tickets for Tyler's raffle might sound like a good deal, but is actually a little worse than buying tickets at the normal price?

Answers will vary, but should reflect a unit price that is greater than $4 per ticket. Sample response: Two tickets for $9.

STOP

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- Recognizing a unit rate can be seen on a graph as one value (for example, seeing the unit rate for dollars per ticket simply as the value 4), representing the relationship between the two coordinates of every point, which can be described using multiplication.
- Recognizing a unit rate can be seen on a graph as describing how to generate more points or move from point to point when going from 1 to 2 to 3 and so on, along one axis. **Note:** This is not an expectation of this grade and can be acknowledged as correct but all students do not need to understand this yet.

3 Connect

Display the table and graph to capture student responses from Problems 1–2, and then to be referenced as students share their responses from Problems 3–4.

Have pairs of students share their responses and explanations to each problem, one at a time, focusing on the unit rate of dollars per ticket throughout.

Ask:

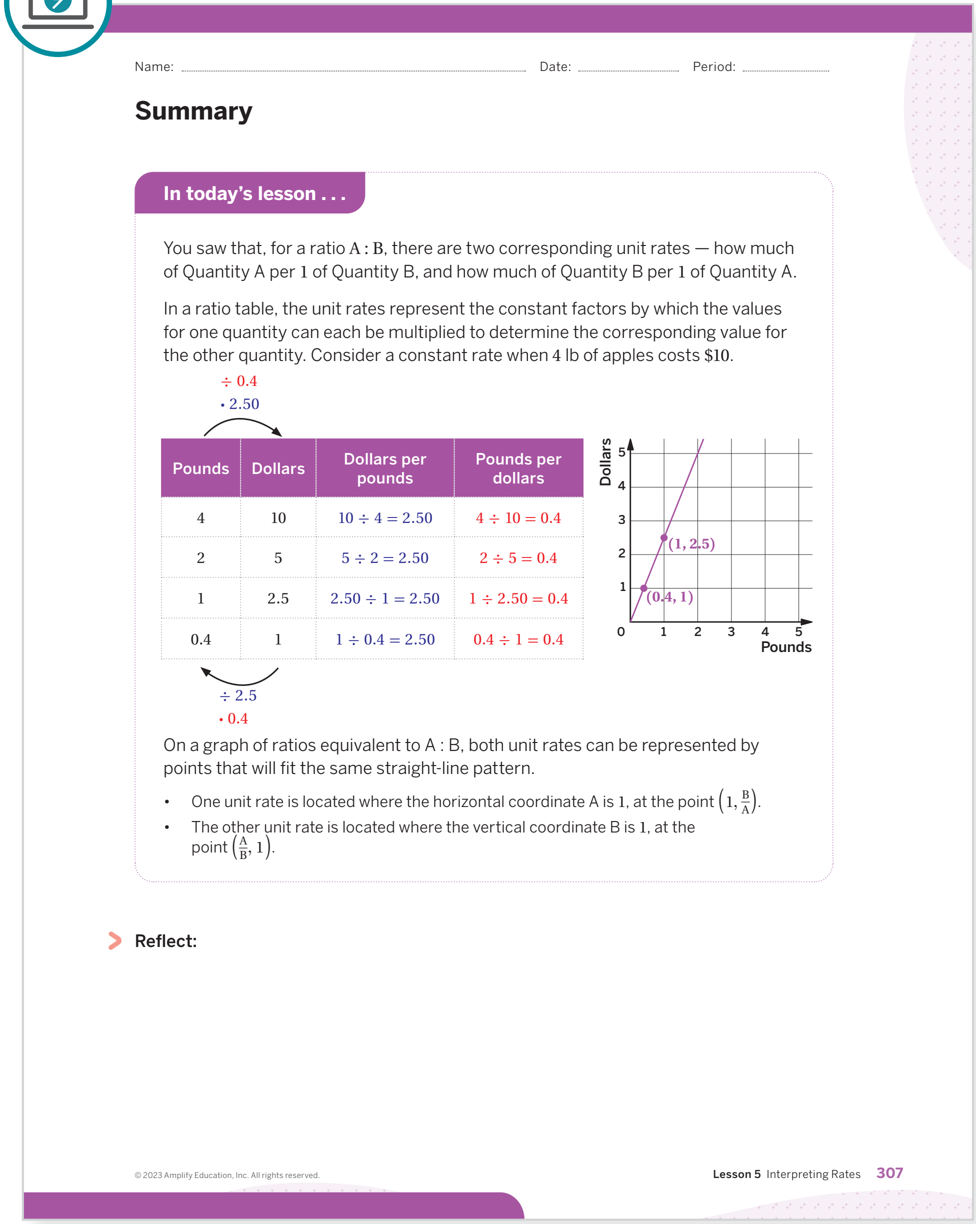
- "How would you know if another point added to the graph has the same unit rate of 4 dollars per ticket?" It would be an equivalent ratio, so it should line up in a straight line with the other points that are on the graph.
- "Would the point representing the unit rate of $\frac{1}{4}$ of a ticket per dollar fit the same pattern?" Yes, because it's the other unit rate for this scenario and an equivalent ratio.

Highlight that, because all of the values in the table represent equivalent ratios, they have the same unit rate, and the points fit the pattern of a straight line through the origin. Both unit rates can be represented as points on the graph, or as describing the relationship between the values of every point, depending on which way you are relating them (e.g., tickets to dollars, or dollars to tickets).

Whole Class | 5 min

Summary

Review and synthesize that every ratio and its set of equivalent ratios have two unit rates, which can be calculated, graphed, and used to determine missing values.



Name: __________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that, for a ratio $A : B$, there are two corresponding unit rates — how much of Quantity A per 1 of Quantity B, and how much of Quantity B per 1 of Quantity A.

In a ratio table, the unit rates represent the constant factors by which the values for one quantity can each be multiplied to determine the corresponding value for the other quantity. Consider a constant rate when 4 lb of apples costs $\$10$.

$\div 0.4$

$\cdot 2.50$

Pounds	Dollars	Dollars per pounds	Pounds per dollars
4	10	$10 \div 4 = 2.50$	$4 \div 10 = 0.4$
2	5	$5 \div 2 = 2.50$	$2 \div 5 = 0.4$
1	2.5	$2.50 \div 1 = 2.50$	$1 \div 2.50 = 0.4$
0.4	1	$1 \div 0.4 = 2.50$	$0.4 \div 1 = 0.4$

$\div 2.5$

$\cdot 0.4$

On a graph of ratios equivalent to A : B, both unit rates can be represented by points that will fit the same straight-line pattern.

- One unit rate is located where the horizontal coordinate A is 1, at the point $\left(1, \frac{B}{A}\right)$.
- The other unit rate is located where the vertical coordinate B is 1, at the point $\left(\frac{A}{B}, 1\right)$.

Reflect:

Synthesize

Display the tables and the graph.

Highlight how the tables show that for the same sets of equivalent ratios, each of the two associated unit rates can be seen as a factor that relates the values for both quantities in each row, by multiplying or dividing. Then reiterate that both unit rates can also be represented as points on the graph, and both will fit the same straight-line pattern as the rest of the equivalent ratios.

Ask, "Using the given relationships of 4 lb of apples costs $\$10$:"

- "How would you compute the number of pounds of apples for 1 dollar?" $4 \div 10$
- "How would you compute the dollars for 1 lb of apples?" $10 \div 4$
- "For what types of problems might multiplying by $\frac{10}{4}$ be more efficient?" Determining the number of dollars when I know the number of pounds of apples.
- "For what types of problems might multiplying by $\frac{4}{10}$ be more efficient?" Determining the number of pounds when I know the dollars.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

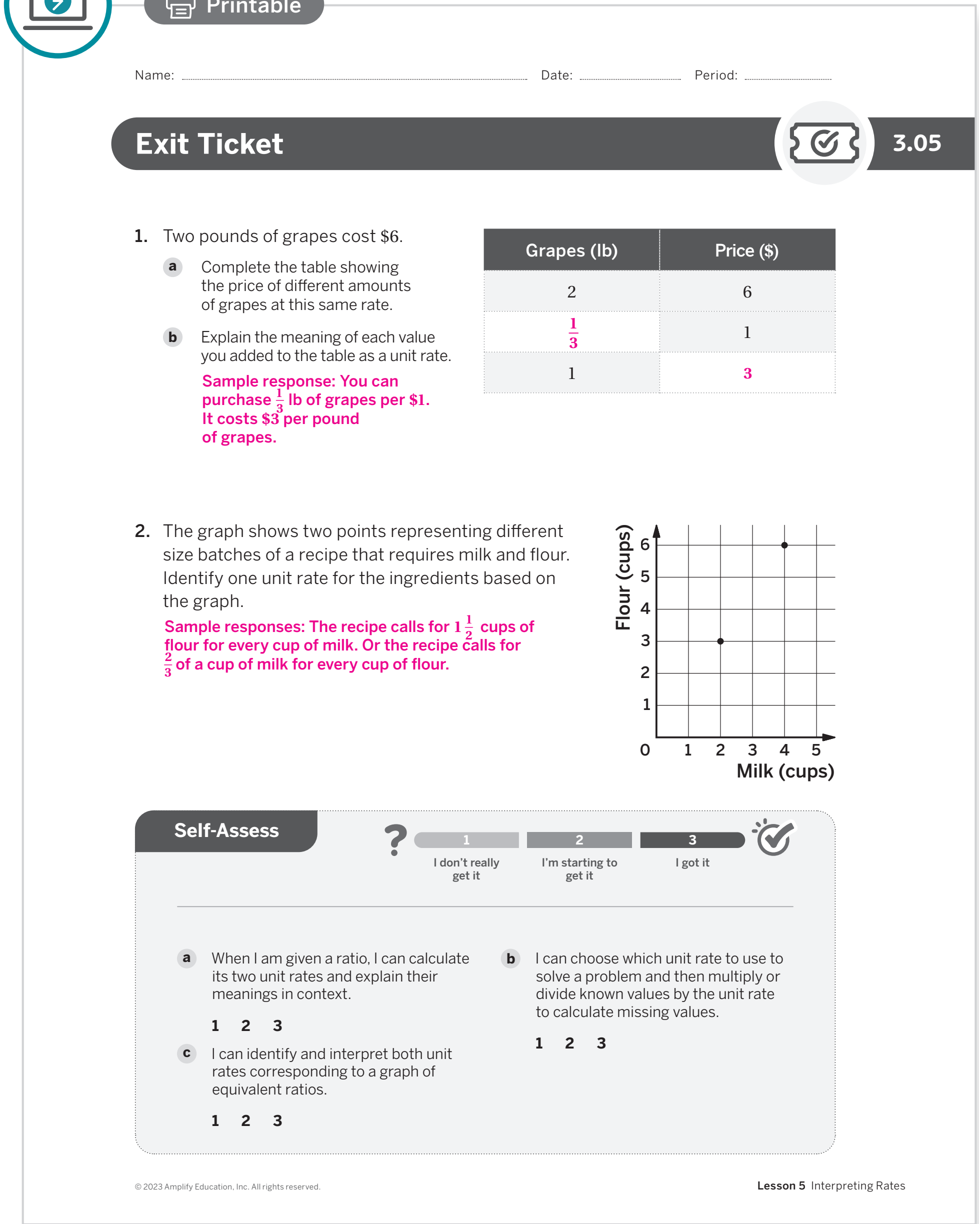
- "What strategies or tools did you find helpful today when working with unit rates and graphs? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of unit rates by determining the two unit rates associated with a ratio involving price and relating them to a graph.

Printable



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Exit Ticket 3.05

1. Two pounds of grapes cost $6.

a Complete the table showing the price of different amounts of grapes at this same rate.

b Explain the meaning of each value you added to the table as a unit rate.

Sample response: You can purchase $\frac{1}{3}$ lb of grapes per $1. It costs $3 per pound of grapes.

Grapes (lb)	Price ($)
2	6
$\frac{1}{3}$	1
1	3

2. The graph shows two points representing different size batches of a recipe that requires milk and flour. Identify one unit rate for the ingredients based on the graph.

Sample responses: The recipe calls for $1\frac{1}{2}$ cups of flour for every cup of milk. Or the recipe calls for $\frac{2}{3}$ of a cup of milk for every cup of flour.

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a When I am given a ratio, I can calculate its two unit rates and explain their meanings in context.

1 2 3

b I can choose which unit rate to use to solve a problem and then multiply or divide known values by the unit rate to calculate missing values.

1 2 3

c I can identify and interpret both unit rates corresponding to a graph of equivalent ratios.

1 2 3

Lesson 5 Interpreting Rates

Success looks like . . .

- **Language Goal:** Calculating and interpreting the two unit rates associated with a ratio, i.e., $\frac{a}{b}$ and $\frac{b}{a}$ for the ratio $a : b$. **(Speaking and Listening, Writing)**
- **Language Goal:** Choosing which unit rate to use to solve a given problem and explaining the choice. **(Speaking and Listening, Writing)**
- **Language Goal:** Relating the two unit rates for a given ratio relationship to a graph of corresponding points. **(Speaking and Listening, Writing)**
  - » Identifying one of the two unit rates from the graph of milk and flour in Problem 2.

Suggested next steps

If students struggle with completing the table in Problem 1a, consider:

- Reviewing Activity 1, Problem 1.
- Asking, "How can you multiply or divide to get from 6 to 1? from 2 to 1?"
- Assigning Practice Problem 1.

If students struggle with interpreting the unit rates in Problem 1b, consider asking:

- "What unit goes with each value that you determined? What is the corresponding value for the other quantity?"

If students struggle with interpreting a unit rate from the graph in Problem 2, consider asking:

- "What units go with each plotted point on the graph? How could you write those as a ratio?"
- "Could you create a table using the plotted points to help you determine a unit rate?"

Professional Learning

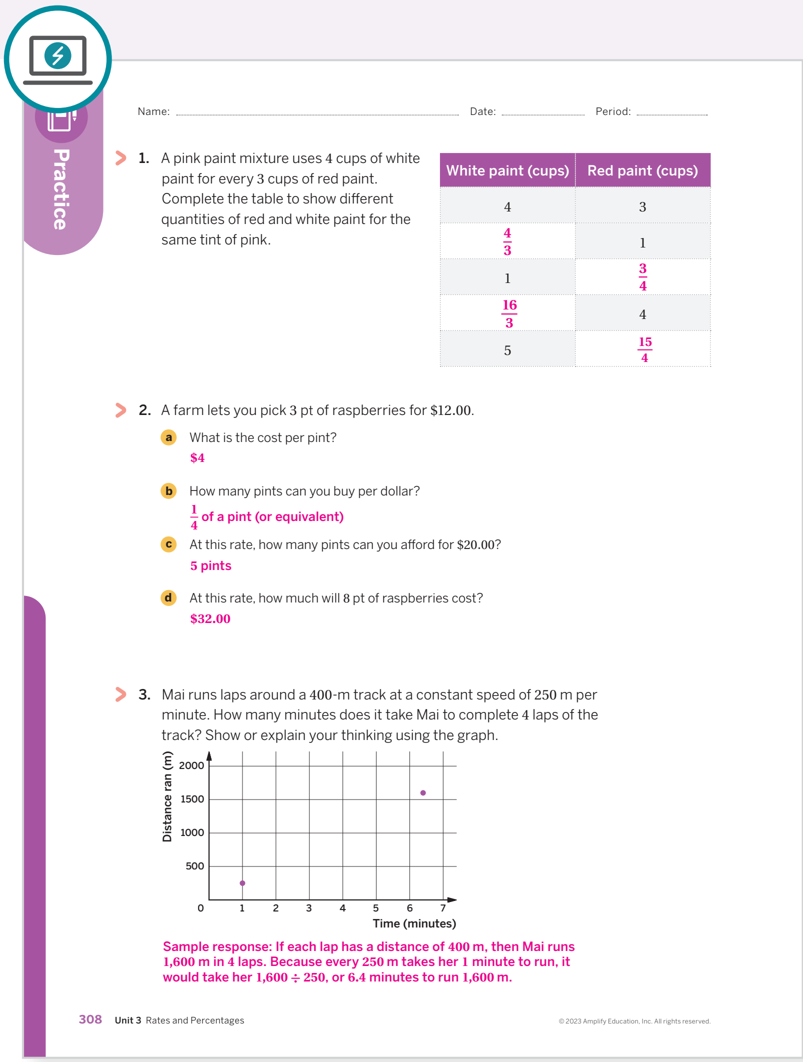
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How was graphing unit rates from today's lesson similar to or different from Lesson 4?
- What did students find frustrating about graphing unit rates? What helped them work through this frustration?

Practice

Independent



Practice

Name: Date: Period:

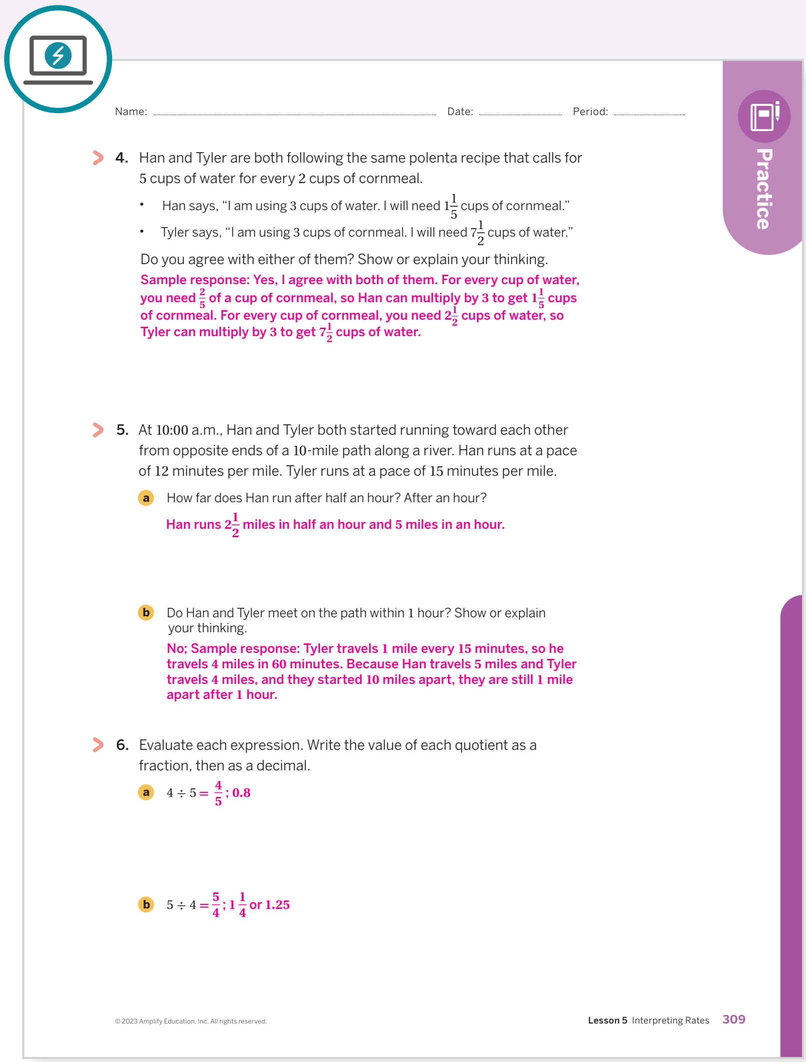
1. A pink paint mixture uses 4 cups of white paint for every 3 cups of red paint. Complete the table to show different quantities of red and white paint for the same tint of pink.

White paint (cups)	Red paint (cups)
4	3
$\frac{4}{3}$	1
1	$\frac{3}{4}$
$\frac{16}{3}$	4
5	$\frac{15}{4}$

2. A farm lets you pick 3 pt of raspberries for $12.00.
   - a What is the cost per pint?
     $4
   - b How many pints can you buy per dollar?
     $\frac{1}{4}$ of a pint (or equivalent)
   - c At this rate, how many pints can you afford for $20.00?
     5 pints
   - d At this rate, how much will 8 pt of raspberries cost?
     $32.00

3. Mai runs laps around a 400-m track at a constant speed of 250 m per minute. How many minutes does it take Mai to complete 4 laps of the track? Show or explain your thinking using the graph.

Sample response: If each lap has a distance of 400 m, then Mai runs 1,600 m in 4 laps. Because every 250 m takes her 1 minute to run, it would take her 1,600 ÷ 250, or 6.4 minutes to run 1,600 m.



Practice

Name: Date: Period:

4. Han and Tyler are both following the same polenta recipe that calls for 5 cups of water for every 2 cups of cornmeal.
   - Han says, "I am using 3 cups of water. I will need $1\frac{1}{5}$ cups of cornmeal."
   - Tyler says, "I am using 3 cups of cornmeal. I will need $7\frac{1}{2}$ cups of water."

   Do you agree with either of them? Show or explain your thinking.

   Sample response: Yes, I agree with both of them. For every cup of water, you need $\frac{2}{5}$ of a cup of cornmeal, so Han can multiply by 3 to get $1\frac{1}{5}$ cups of cornmeal. For every cup of cornmeal, you need $2\frac{1}{2}$ cups of water, so Tyler can multiply by 3 to get $7\frac{1}{2}$ cups of water.

5. At 10:00 a.m., Han and Tyler both started running toward each other from opposite ends of a 10-mile path along a river. Han runs at a pace of 12 minutes per mile. Tyler runs at a pace of 15 minutes per mile.
   - a How far does Han run after half an hour? After an hour?
     Han runs $2\frac{1}{2}$ miles in half an hour and 5 miles in an hour.
   - b Do Han and Tyler meet on the path within 1 hour? Show or explain your thinking.
     No; Sample response: Tyler travels 1 mile every 15 minutes, so he travels 4 miles in 60 minutes. Because Han travels 5 miles and Tyler travels 4 miles, and they started 10 miles apart, they are still 1 mile apart after 1 hour.

6. Evaluate each expression. Write the value of each quotient as a fraction, then as a decimal.
   - a $4 \div 5 = \frac{4}{5}$; 0.8
   - b $5 \div 4 = \frac{5}{4}; 1\frac{1}{4}$ or 1.25

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 13	2
	5	Unit 3 Lesson 4	2
Formative	6	Unit 3 Lesson 6	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 3 | **LESSON 6**

Comparing Rates

Let's use graphs to compare rates.

Focus

Goals

1. **Language Goal:** Explain that if two ratios are equivalent, they have the same rate per 1. **(Speaking and Listening)**
2. **Language Goal:** Use a table or a graph to compare the rates of two scenarios involving the same units. **(Speaking and Listening, Writing)**

Rigor

- Students use graphs to further their **conceptual understanding** of comparing rates.
- Students continue to build **procedural fluency** with determining unit rates.

Coherence

Today

Students extend their previous work and generalize rate comparisons using variables. In doing so, they solidify two key understandings that have been built up from the previous unit:

- When both quantities in a ratio are multiplied by the same factor, the result is an equivalent ratio;
- When two ratios have the same unit rates, they are equivalent ratios.

Students then discover that, in general, they can use patterns in the points of graphs of ratio relationships (i.e., the lines) to compare rates. They begin to recognize that the "steepness" of the lines that fit to each set of equivalent ratios show greater and lesser rates; the greater rate relative to the quantity on the horizontal axis has a steeper line, and the greater rate relative to the quantity on the vertical axis has a flatter line.

Previously

In Lessons 4–5, students determined both unit rates $\left(\frac{a}{b} \text{ or } \frac{b}{a}\right)$ for a ratio, and they saw that two ratios are equivalent if they have the same unit rate.

Coming Soon

In Lesson 7, students will solve problems in which they compare prices or speeds, applying what they know about unit rates.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (for display)

Math Language Development

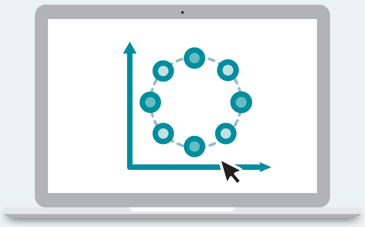
Review word

- *unit rate*

Amps | **Featured Activity**

Activity 2
Overlay Graphs

Each student can plot their points, showing the relationship between the number of items and price. When you overlay the results, your class will see that more expensive items have a steeper line.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may not realize that their work can be affected by their emotions. During Activity 2, they might find themselves feeling like they are at a roadblock trying to understand how to build the structures of the graphs. Their optimism can be improved by explaining to themselves the meaning of the problem and looking for anything that they understand and/or know how to do. Remind them to connect the table with the graphs as a starting point for building a structure.

● Modifications to Pacing

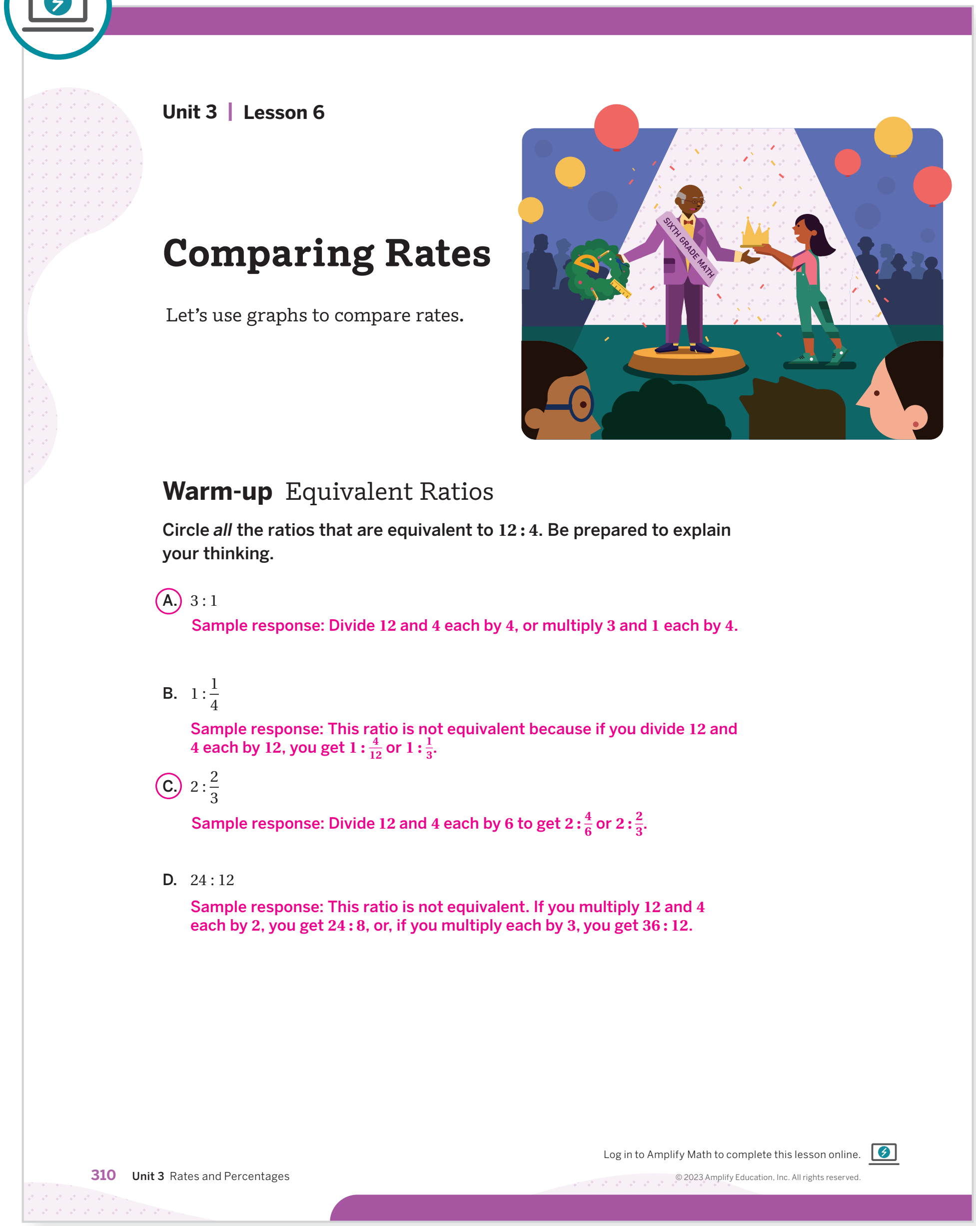
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- Present **Activity 2** to the whole class. Using $b = 2$ and $c = 4$, have students direct you in plotting five points for each item, one of which should be the unit price. Then have students use the *Think-Pair-Share* routine to complete Part 2, Problem 3.

Independent | 5 min

Warm-up Equivalent Ratios

Students determine whether given ratios are equivalent, activating their prior knowledge that ratios are equivalent when both quantities are multiplied or divided by the same factor.



Unit 3 | **Lesson 6**

Comparing Rates

Let's use graphs to compare rates.

Warm-up Equivalent Ratios

Circle *all* the ratios that are equivalent to $12 : 4$. Be prepared to explain your thinking.

[✓ A.] $3 : 1$

Sample response: Divide 12 and 4 each by 4, or multiply 3 and 1 each by 4.

B. $1 : \frac{1}{4}$

Sample response: This ratio is not equivalent because if you divide 12 and 4 each by 12, you get $1 : \frac{4}{12}$ or $1 : \frac{1}{3}$.

[✓ C.] $2 : \frac{2}{3}$

Sample response: Divide 12 and 4 each by 6 to get $2 : \frac{4}{6}$ or $2 : \frac{2}{3}$.

D. $24 : 12$

Sample response: This ratio is not equivalent. If you multiply 12 and 4 each by 2, you get $24 : 8$, or, if you multiply each by 3, you get $36 : 12$.

Log in to Amplify Math to complete this lesson online.

310 **Unit 3** Rates and Percentages

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by activating their prior knowledge. Ask, "How do you know whether two ratios are equivalent?"

Look for points of confusion:

- **Not multiplying or dividing by the same factor.** Have students create a table for the two ratios and label the multiplier for each column. Ask, "Is this the same multiplier? What does that tell you about equivalence?"
- **Not knowing how to work with ratios involving fractions.** Ask, "What would 12 be divided by to get this whole number? So what should you do with the 4?"

Look for productive strategies:

- Determining whether a ratio is equivalent by relating 12 to the first quantity in each ratio by using multiplication or division.
- Recognizing equivalent fractions (e.g., $\frac{2}{3} = \frac{4}{6}$ in Choice C). If students calculate correctly but do not recognize the equivalent fractions, ask, "Could you write your fraction another way?"

3 Connect

Have students share their responses and strategies, focusing on how they used a consistent factor to multiply or divide both quantities.

Highlight that two ratios are equivalent when both quantities are multiplied or divided by the same factor.

Math Language Development

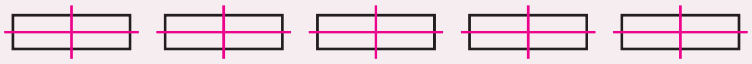
Accessibility: *Optimize Access to Tools*

Provide access to manipulatives students could use as they complete the Warm-up, such as counters or cubes. Because Choices B and C contain fractions, suggest that students create a table of values to help determine if these ratios are equivalent to $12 : 4$.

Power-up

To power up students' ability to represent the quotient of two whole number values as fractions and decimals, have students complete:

Recall that division expressions can be interpreted as sharing something in a fair way. $5 \div 4$ can be thought of as: How can 5 granola bars be shared fairly among 4 people? Determine how much of a granola bar each person will receive.



Sample response: Each person will receive $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$, or $\frac{5}{4}$ of a granola bar.

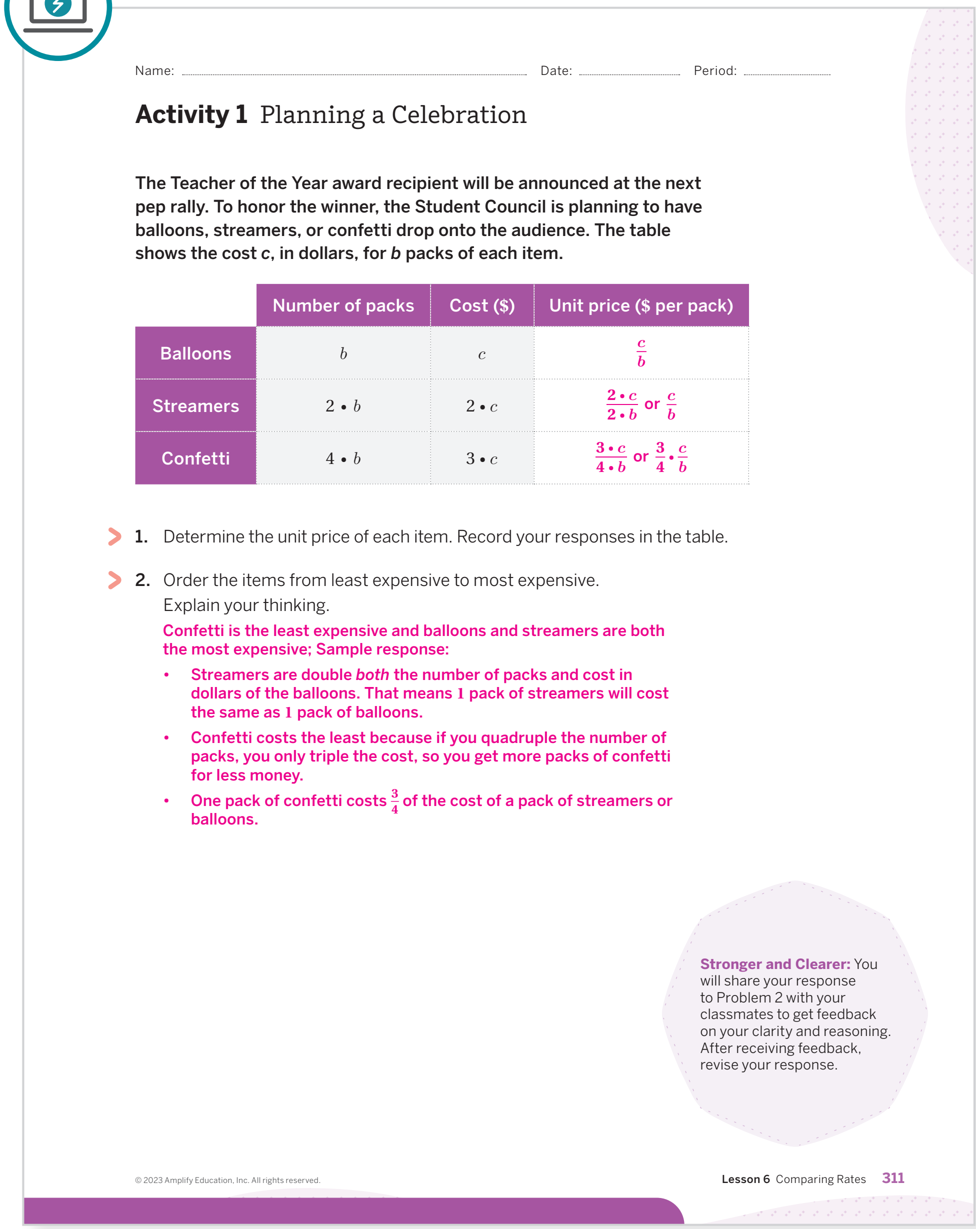
Use: Before Activity 1.

Informed by: Performance on Lesson 5, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4.

Pairs | 10 min

Activity 1 Planning a Celebration

Students generalize how to determine unit prices by using variables, and then use that to compare unit prices in the context of different packs of celebratory items.



Name: ____________ Date: ________ Period: ________

Activity 1 Planning a Celebration

The Teacher of the Year award recipient will be announced at the next pep rally. To honor the winner, the Student Council is planning to have balloons, streamers, or confetti drop onto the audience. The table shows the cost *c*, in dollars, for *b* packs of each item.

	Number of packs	Cost ($)	Unit price ($ per pack)
Balloons	b	c	$\frac{c}{b}$
Streamers	$2 \bullet b$	$2 \bullet c$	$\frac{2 \bullet c}{2 \bullet b}$ or $\frac{c}{b}$
Confetti	$4 \bullet b$	$3 \bullet c$	$\frac{3 \bullet c}{4 \bullet b}$ or $\frac{3}{4} \bullet \frac{c}{b}$

1. Determine the unit price of each item. Record your responses in the table.

2. Order the items from least expensive to most expensive. Explain your thinking.

   Confetti is the least expensive and balloons and streamers are both the most expensive; Sample response:

   - **Streamers are double *both* the number of packs and cost in dollars of the balloons. That means 1 pack of streamers will cost the same as 1 pack of balloons.**
   - **Confetti costs the least because if you quadruple the number of packs, you only triple the cost, so you get more packs of confetti for less money.**
   - **One pack of confetti costs $\frac{3}{4}$ of the cost of a pack of streamers or balloons.**

Stronger and Clearer: You will share your response to Problem 2 with your classmates to get feedback on your clarity and reasoning. After receiving feedback, revise your response.

1 Launch

Have students use the *Think-Pair-Share* routine. Give them 1–2 minutes to complete the first row of the table independently. Then have them compare their answers with a partner and complete the table and Problem 2.

2 Monitor

Help students get started by asking, "Using the information in the table, what operation would be used to determine the cost per pack?"

Look for points of confusion:

- **Writing the unit prices as $\frac{b}{c}$.** Remind students that while ratios have two unit rates, this context asks for cost per pack. Ask, "Which unit rate do you have represented?"
- **Struggling to use the packs : cost rates to order the items**. Ask, "How are the expressions for packs related? What about the expressions for cost? Which relationships are the same? Different?"

Look for productive strategies:

- Determining unit price by representing division as a fraction, such as $\frac{c}{b}$.
- Recognizing that multiplying both quantities by the same factor results in an equivalent ratio, but multiplying each by a different factor does not.
- Understanding that confetti will always cost the least because its unit price is equal to the others multiplied by a fraction less than 1 (in this case, $\frac{3}{4}$).

3 Connect

Have pairs of students share how they used division to determine the unit prices, and how they used the unit prices to order the items. Encourage students to use precise, contextual language.

Highlight that balloons and streamers cost the same because their unit prices can both be simplified to $\frac{c}{b}$. Confetti costs the least because the unit price of $\frac{c}{b}$ is being multiplied by a fraction less than 1 (in this case, $\frac{3}{4}$) .

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students use sample values for the variables, such as $b = 2$ and $c = 4$ to complete the table. Then ask these questions:

- "What do you notice about the unit prices for the three items?"
- "Would that be the same for any value chosen for b and c? Why or why not?"

Extension: *Math Enrichment*

Have students add a celebratory item to the table that would cost more than all three previously listed items. Have them use variables for packs, cost, and unit price, and ask them to explain how they know their item costs more. Answers should reflect less than 1 pack per dollar.

Math Language Development

MLR1: Stronger and Clearer Each Time

Have students write a draft response to Problem 2 and then share their responses with 2–3 partners. Have reviewing partners use these prompts to assist the authors in clarifying their draft responses.

- "How do you know _____ costs the least? The most?"
- "How could you use values to show your thinking?"

Have students use the feedback they received to improve their response.

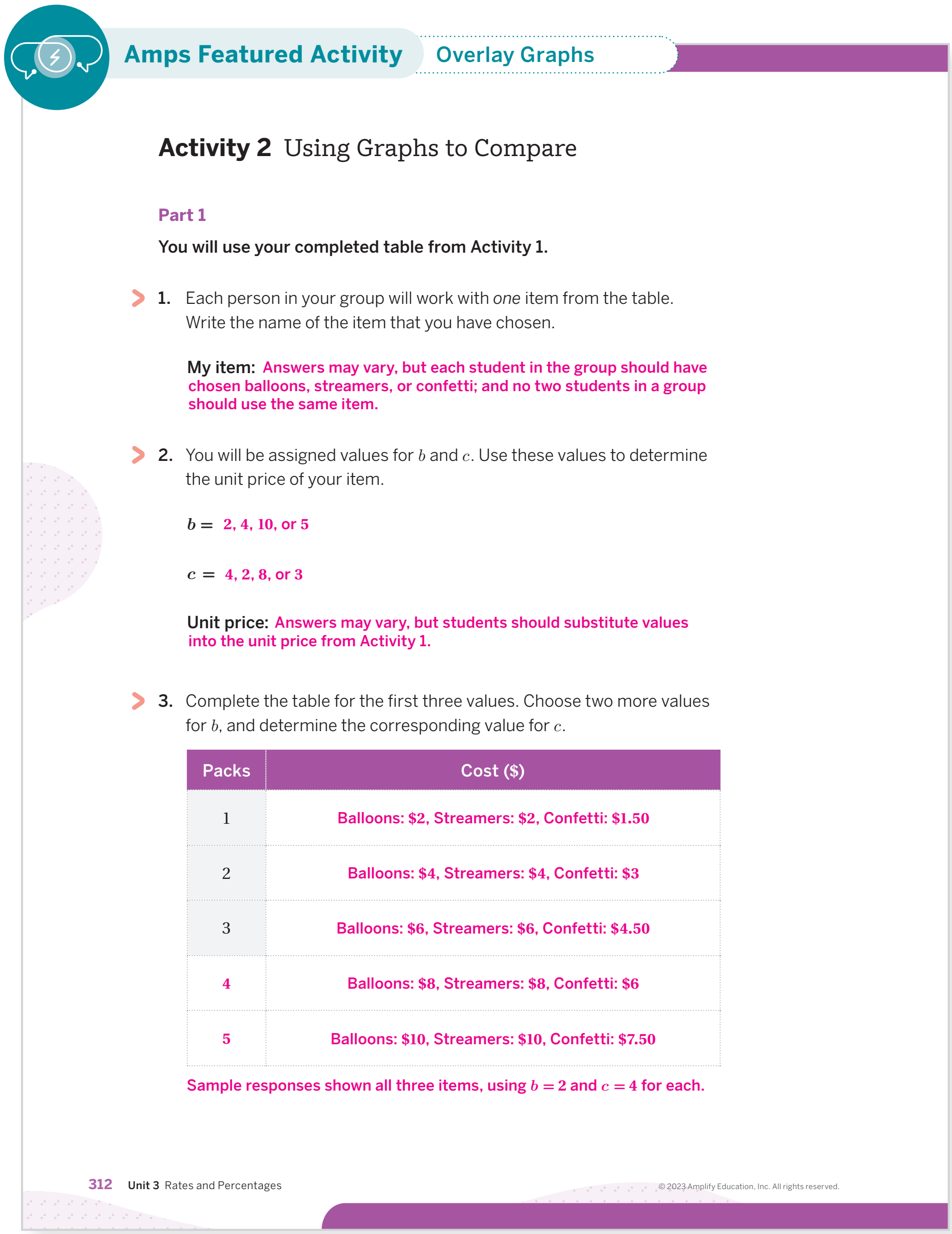
English Learners

Display the phrase "least expensive" and "least cost" so that students know these phrases express the same concept. Repeat for "most expensive" and "greatest cost."

Small Groups | 20 min

Activity 2 Using Graphs to Compare

Students extend their work from Activity 1 to graph the rate of each item, discovering that the steeper the line, the greater the unit rate.



Amps Featured Activity Overlay Graphs

Activity 2 Using Graphs to Compare

Part 1

You will use your completed table from Activity 1.

1. Each person in your group will work with *one* item from the table. Write the name of the item that you have chosen.

My item: **Answers may vary, but each student in the group should have chosen balloons, streamers, or confetti; and no two students in a group should use the same item.**

2. You will be assigned values for b and c. Use these values to determine the unit price of your item.

$b =$ **2, 4, 10, or 5**

$c =$ **4, 2, 8, or 3**

Unit price: **Answers may vary, but students should substitute values into the unit price from Activity 1.**

3. Complete the table for the first three values. Choose two more values for b, and determine the corresponding value for c.

Packs	Cost ($)
1	**Balloons: $2, Streamers: $2, Confetti: $1.50**
2	**Balloons: $4, Streamers: $4, Confetti: $3**
3	**Balloons: $6, Streamers: $6, Confetti: $4.50**
4	**Balloons: $8, Streamers: $8, Confetti: $6**
5	**Balloons: $10, Streamers: $10, Confetti: $7.50**

Sample responses shown all three items, using $b = 2$ and $c = 4$ for each.

312 **Unit 3** Rates and Percentages

1 Launch

Arrange students in groups of three. To avoid repeating decimals, assign groups one of the following sets of values for b and c (ordered from most accessible to most challenging): $b = 2$ and $c = 4$; $b = 4$ and $c = 2$; $b = 10$ and $c = 8$; $b = 5$ and $c = 3$. **Note**: In Part 2, students will use graphs to compare rates focusing on the lines that connect equivalent ratios. It is not an expectation of this grade that students understand this as the slope of the lines or as relative changes in vertical and horizontal position (e.g., rise and run), but interpreting the "steepness" of the lines with the language of ratios and rates is accessible.

2 Monitor

Help students get started by asking, **Part 1:** "What was the unit rate of your item in Activity 1?"; **Part 2:** "What is an appropriate scale for your graph?" Explain whether it is acceptable that only the first three points fit on their graph.

Look for points of confusion:

- **Incorrectly using the unit price from Activity 1 (Problems 2–3).** Have students substitute the values for b and c into the *unit* price expression. Ask, "How can you use that value?"
- **Not knowing how to interpret the lines for balloons and streamers being on top of each other.** Ask, "What do you know about the unit rates for balloons and streamers? Does the graph make sense?"
- **Misinterpreting the lines on the graph.** Ask, "In Activity 1, which items were the same price? Different prices? What do their lines on the graph show you?"

Look for productive strategies:

- Substituting b and c values into the unit price expression, and using division to determine the unit price.
- Relating same unit rate to same line.
- Recognizing that a line that is "above" or "steeper than" another means the item is more expensive (confetti).

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can plot their points to see the graphed results. You can overlay the class results so that students can see the more expensive items are represented by steeper lines.

Extension: *Math Enrichment*

Have students add two more celebratory items to their graphs — one that is more expensive and one that is less expensive — than the streamers, balloons, and confetti.

Math Language Development

MLR7: Compare and Connect

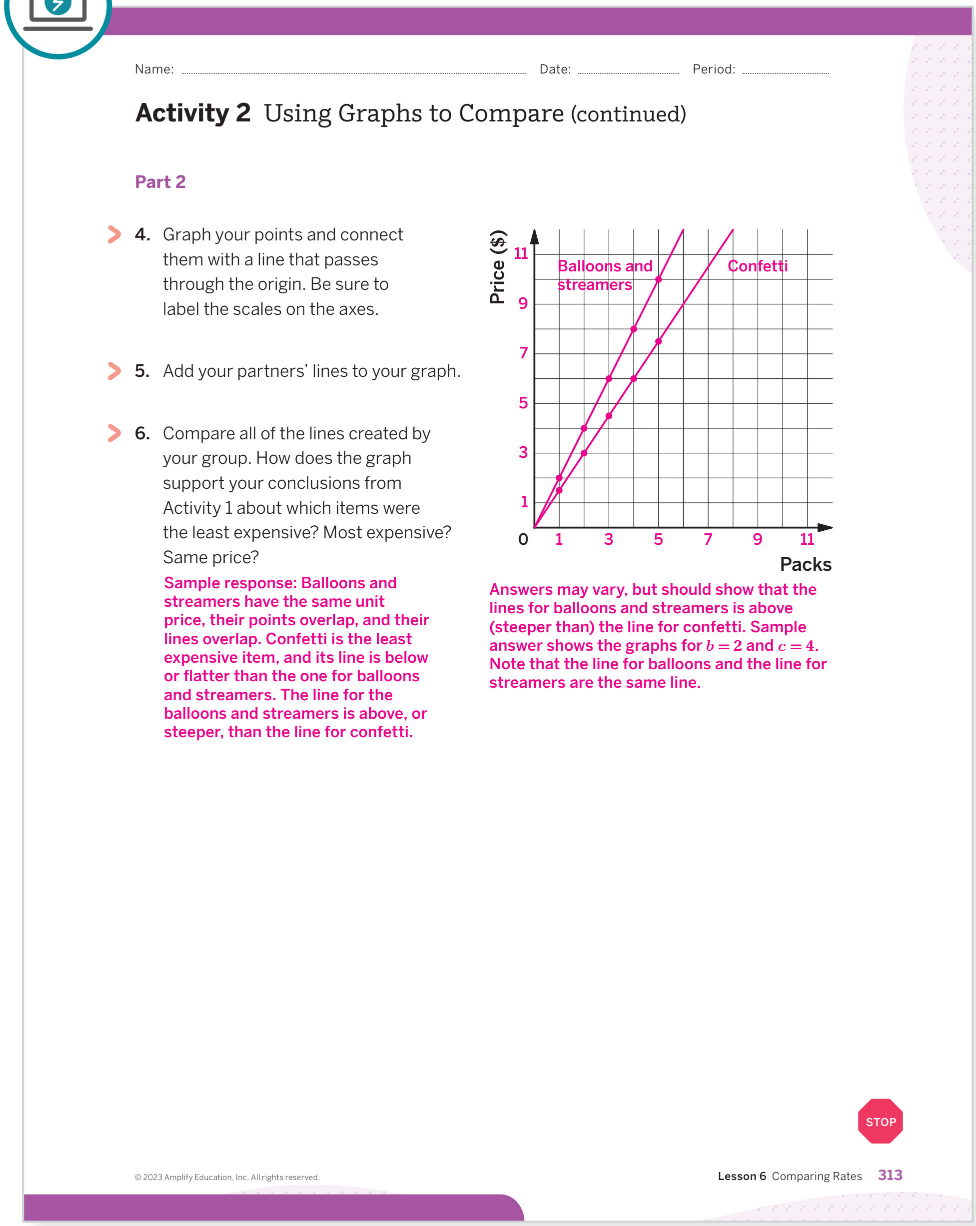
During the Connect, as you display the graphs from the Activity 2 PDF, look for students using mathematical terms as they compare the graphs. For example, "On the first graph, the graph for balloons and streamers is *steeper.* This means that the balloons and streamers had a *greater cost per pack* because each point on the steeper graph has a higher cost than its corresponding point on the graph for confetti."

English Learners

Annotate each graph with which relationship is steeper and has a greater unit rate. Circle or highlight how the axes are reversed in each graph.

Activity 2 Using Graphs to Compare (continued)

Students extend their work from Activity 1 to graph the rate of each item, discovering that the steeper the line, the greater the unit rate.



Name: Date: Period:

Activity 2 Using Graphs to Compare (continued)

Part 2

4. Graph your points and connect them with a line that passes through the origin. Be sure to label the scales on the axes.

5. Add your partners' lines to your graph.

6. Compare all of the lines created by your group. How does the graph support your conclusions from Activity 1 about which items were the least expensive? Most expensive? Same price?

Sample response: Balloons and streamers have the same unit price, their points overlap, and their lines overlap. Confetti is the least expensive item, and its line is below or flatter than the one for balloons and streamers. The line for the balloons and streamers is above, or steeper, than the line for confetti.

Answers may vary, but should show that the lines for balloons and streamers is above (steeper than) the line for confetti. Sample answer shows the graphs for $b = 2$ and $c = 4$. Note that the line for balloons and the line for streamers are the same line.

STOP

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Lesson 6 Comparing Rates 313

3 Connect

Have groups of students share and compare their graphs with at least one other group before sharing with the class and discussing what each graph had in common. Consider pairing groups that used different values of b and c.

Display the Activity 2 PDF. Show one graph at a time.

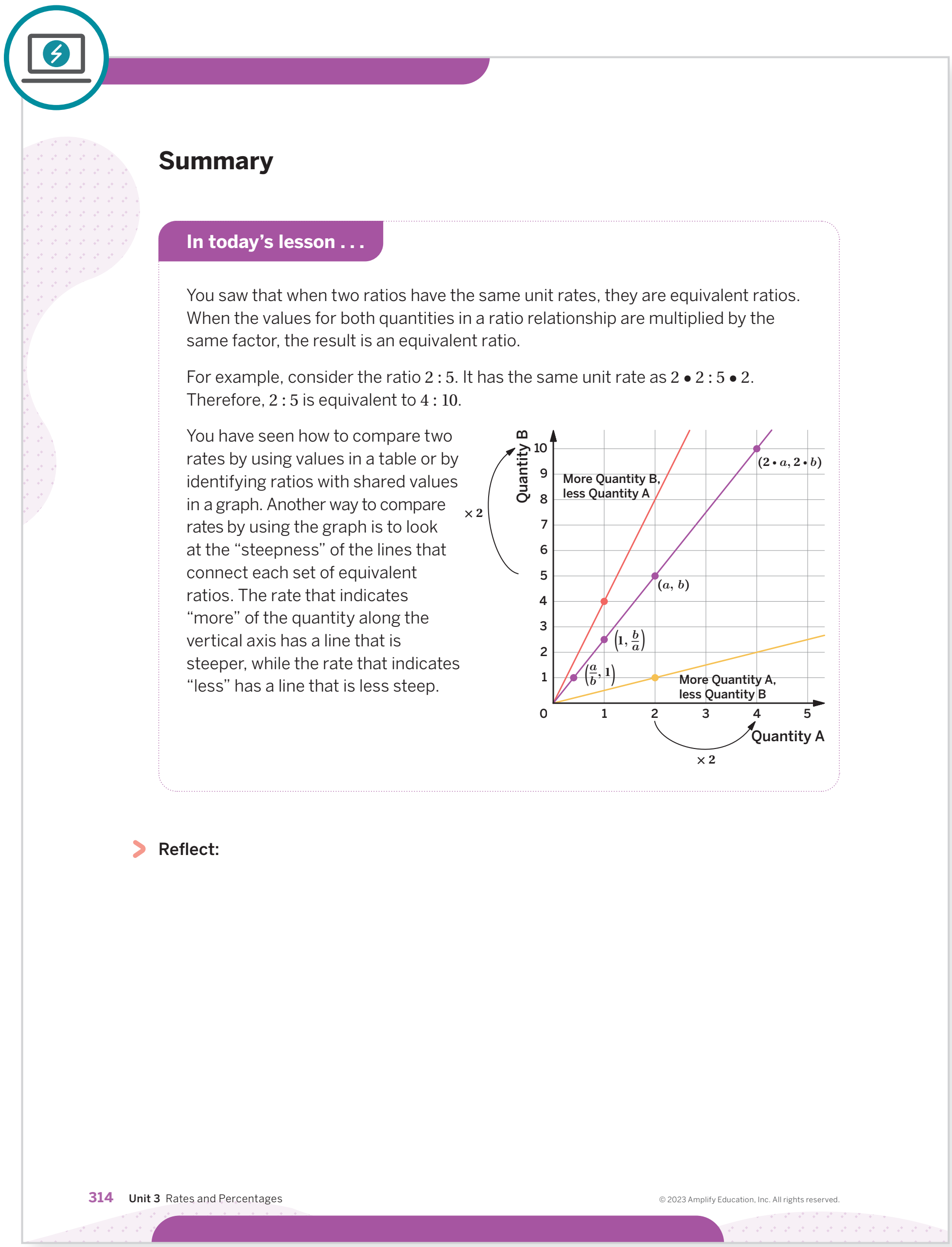
Ask, "In the second graph, does it still make sense that the steepest line is for the most expensive item? Why or why not?" No, the steepest line is now the least expensive item (confetti) because the values on the axes switched. When cost is on the horizontal axis, a higher cost is represented by a flatter line because you get fewer bags for more money.

Highlight that the relative placement of the lines representing two rates can be used to compare which rate is "more" of one quantity. The rate that is "more" of the quantity on the vertical axis has a line that is above the line for the other rate ("steeper"); and the rate that is "less" would be below ("flatter"). Similarly, the rate that is "more" of the horizontal quantity corresponds to the line that is "flatter" (below), and the rate that is "less" corresponds to the line that is "steeper" (above).

Whole Class | 5 min

Summary

Review and synthesize how to use graphs to represent and compare rates, making connections to equivalent ratios from the previous unit.



Summary

In today's lesson . . .

You saw that when two ratios have the same unit rates, they are equivalent ratios. When the values for both quantities in a ratio relationship are multiplied by the same factor, the result is an equivalent ratio.

For example, consider the ratio $2 : 5$. It has the same unit rate as $2 \bullet 2 : 5 \bullet 2$. Therefore, $2 : 5$ is equivalent to $4 : 10$.

You have seen how to compare two rates by using values in a table or by identifying ratios with shared values in a graph. Another way to compare rates by using the graph is to look at the "steepness" of the lines that connect each set of equivalent ratios. The rate that indicates "more" of the quantity along the vertical axis has a line that is steeper, while the rate that indicates "less" has a line that is less steep.

Reflect:

314 Unit 3 Rates and Percentages

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Synthesize

Display the graph from the Summary.

Ask:

- "What is the value of a? b? s?" $a = 2$, $b = 5$, $s = 2$
- "Think back to the packs and prices from Activity 2. Imagine Quantity A on this graph is packs and Quantity B is price. What would a graph for a more expensive item look like? a less expensive item?" Sample responses: More expensive items will have a line that is stepper than the others, and every point will be above the others. The line will be in the top left of the graph. If the item is less expensive, it is the least steep or flattest, and all of the points are below the others. The line is located in the bottom right of the graph.

Highlight that both tables and graphs are tools to compare rates. The information presented in a table can be used to make a graph; and likewise, the information from a graph can be used to make a table. Graphs are particularly useful because the lines produced provide a visual confirmation that a rate is greater than, less than, or equivalent to another rate.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

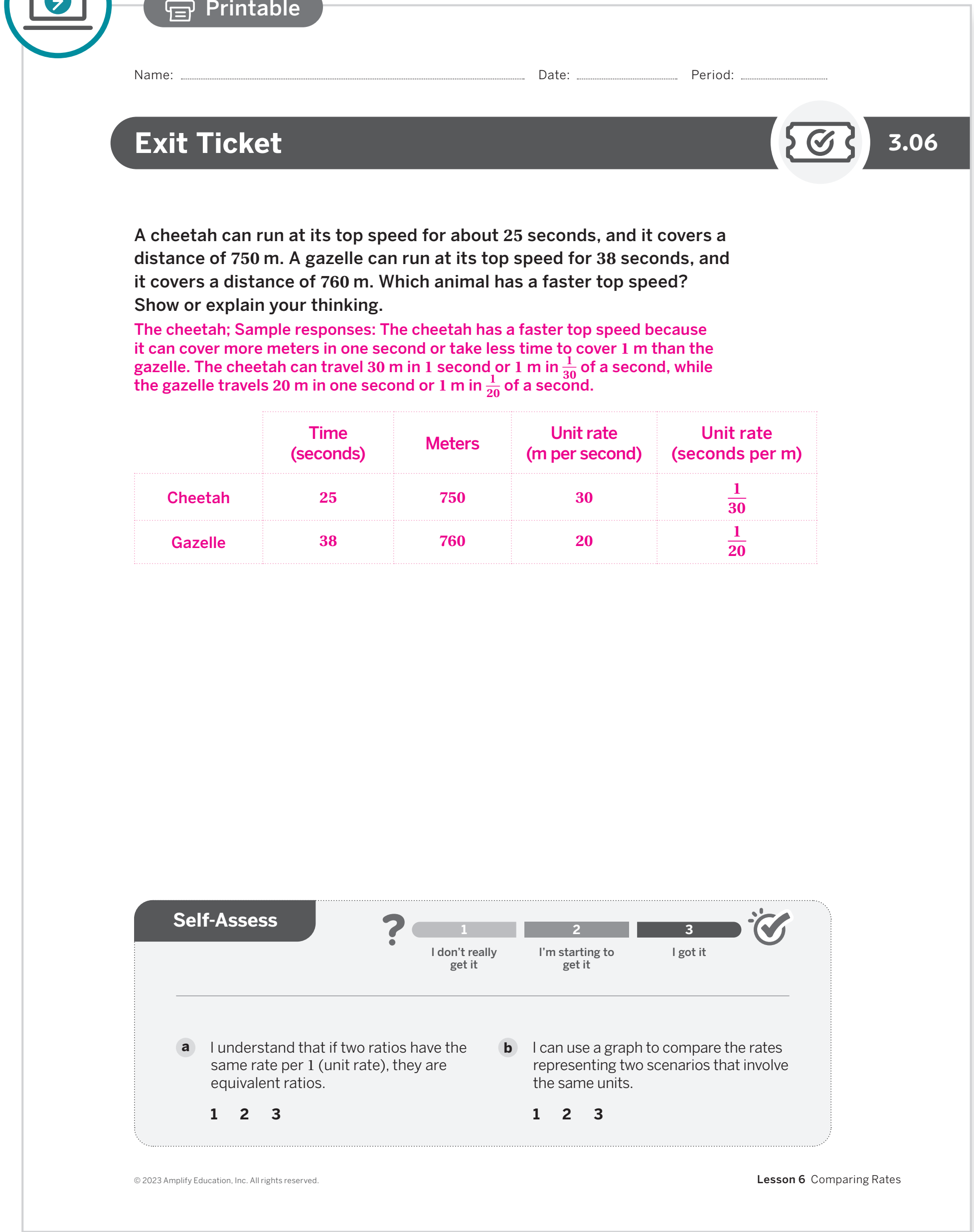
- "What are some strategies you can use to compare rates?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining and comparing unit rates.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 3.06

A cheetah can run at its top speed for about 25 seconds, and it covers a distance of 750 m. A gazelle can run at its top speed for 38 seconds, and it covers a distance of 760 m. Which animal has a faster top speed? Show or explain your thinking.

The cheetah; Sample responses: The cheetah has a faster top speed because it can cover more meters in one second or take less time to cover 1 m than the gazelle. The cheetah can travel 30 m in 1 second or 1 m in $\frac{1}{30}$ of a second, while the gazelle travels 20 m in one second or 1 m in $\frac{1}{20}$ of a second.

	Time (seconds)	Meters	Unit rate (m per second)	Unit rate (seconds per m)
Cheetah	25	750	30	$\frac{1}{30}$
Gazelle	38	760	20	$\frac{1}{20}$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I understand that if two ratios have the same rate per 1 (unit rate), they are equivalent ratios.

1 2 3

b I can use a graph to compare the rates representing two scenarios that involve the same units.

1 2 3

Lesson 6 Comparing Rates

Success looks like . . .

- **Language Goal:** Explaining that if two ratios are equivalent, they have the same rate per 1. **(Speaking and Listening)**
- **Language Goal:** Using a table or a graph graph to compare the rates of two scenarios involving the same units. **(Speaking and Listening, Writing)**
  - » Calculating the unit rate (speed) for each animal to determine which has a faster top speed.

Suggested next steps

If students incorrectly determine the unit rate of meters per second, consider:

- Referring to Activity 1 and asking, "How did you determine the unit price of the balloons? How can you use that here? What unit rate will that tell you — meters per second or seconds per meter? Why?"

If students incorrectly interpret the unit rate of seconds per meter, consider:

- Reminding them that a unit rate is not a fraction. Ask, "What does each quantity in the rate $\frac{1}{20}$ mean? What does the quantity in the rate $\frac{1}{30}$ mean? How do they help you compare?"

Professional Learning

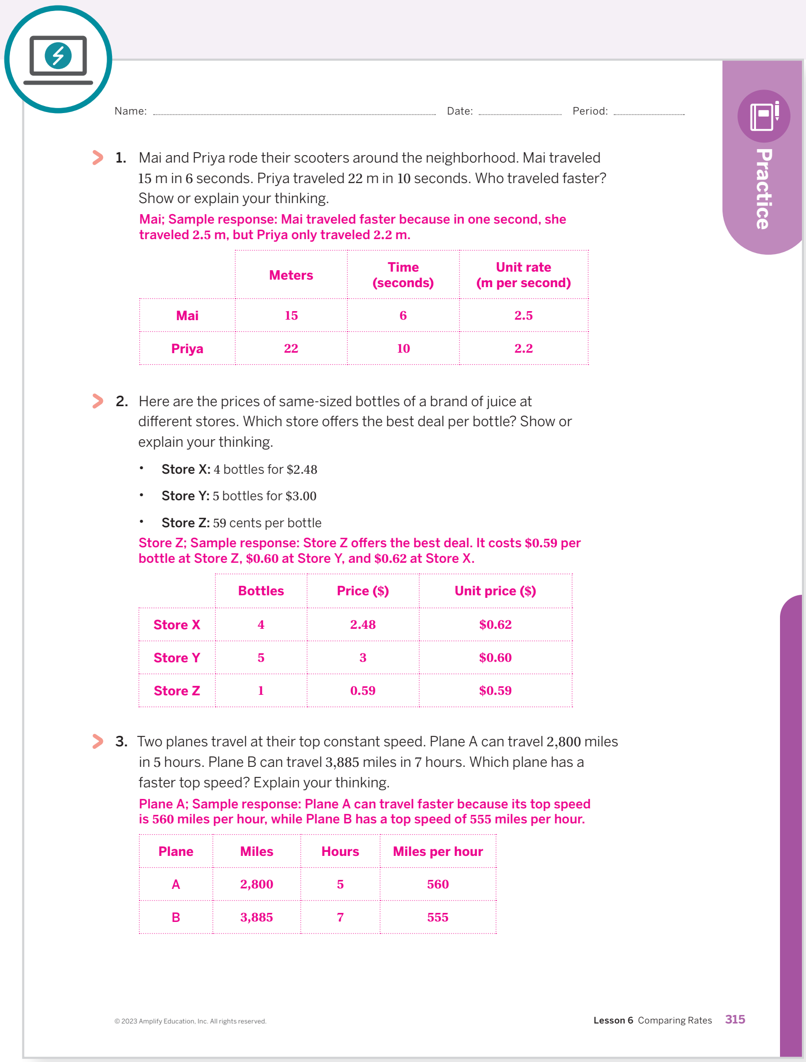
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students used graphs to compare rates. How did this build on the students' earlier work with writing, graphing, and comparing rates?
- What did students find frustrating about Activity 1? What helped them work through this frustration? What might you change for the next time you teach this lesson to minimize frustrations but not eliminate productive struggle?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Mai and Priya rode their scooters around the neighborhood. Mai traveled 15 m in 6 seconds. Priya traveled 22 m in 10 seconds. Who traveled faster? Show or explain your thinking.

Mai; Sample response: Mai traveled faster because in one second, she traveled 2.5 m, but Priya only traveled 2.2 m.

	Meters	Time (seconds)	Unit rate (m per second)
Mai	15	6	2.5
Priya	22	10	2.2

2. Here are the prices of same-sized bottles of a brand of juice at different stores. Which store offers the best deal per bottle? Show or explain your thinking.

- **Store X:** 4 bottles for $2.48
- **Store Y:** 5 bottles for $3.00
- **Store Z:** 59 cents per bottle

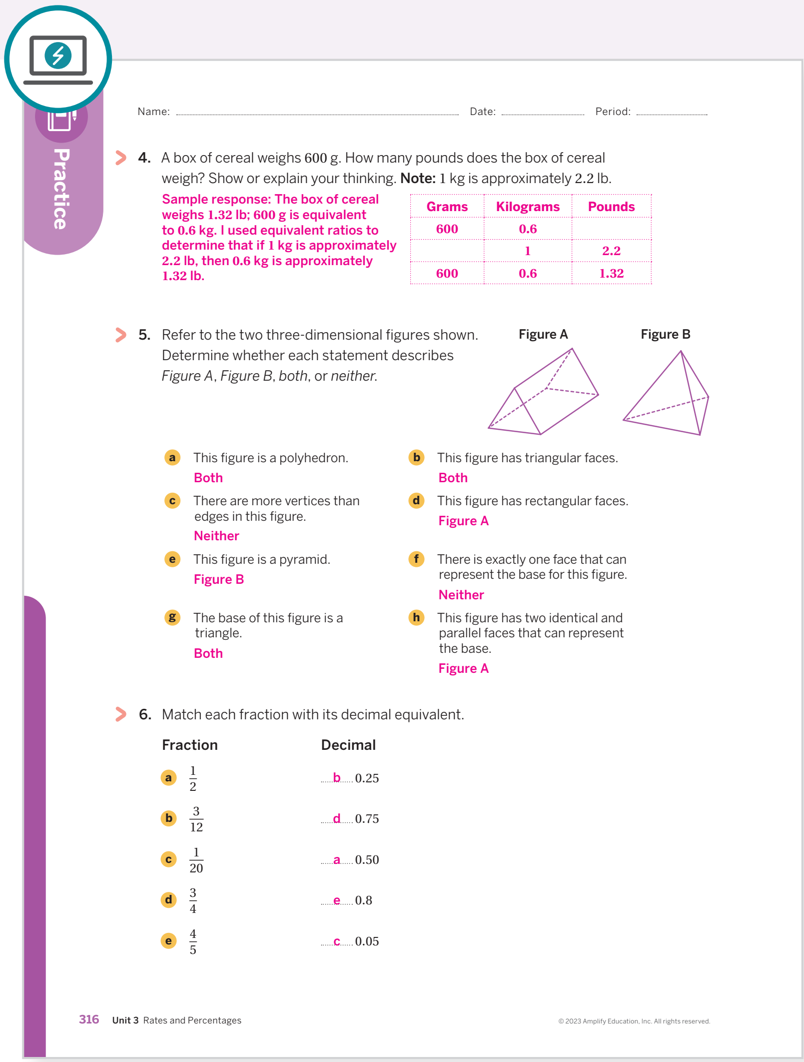
Store Z; Sample response: Store Z offers the best deal. It costs $0.59 per bottle at Store Z, $0.60 at Store Y, and $0.62 at Store X.

	Bottles	Price ($)	Unit price ($)
Store X	4	2.48	$0.62
Store Y	5	3	$0.60
Store Z	1	0.59	$0.59

3. Two planes travel at their top constant speed. Plane A can travel 2,800 miles in 5 hours. Plane B can travel 3,885 miles in 7 hours. Which plane has a faster top speed? Explain your thinking.

Plane A; Sample response: Plane A can travel faster because its top speed is 560 miles per hour, while Plane B has a top speed of 555 miles per hour.

Plane	Miles	Hours	Miles per hour
A	2,800	5	560
B	3,885	7	555



Practice

Name: ________ Date: ________ Period: ________

4. A box of cereal weighs 600 g. How many pounds does the box of cereal weigh? Show or explain your thinking. **Note:** 1 kg is approximately 2.2 lb.

Sample response: The box of cereal weighs 1.32 lb; 600 g is equivalent to 0.6 kg. I used equivalent ratios to determine that if 1 kg is approximately 2.2 lb, then 0.6 kg is approximately 1.32 lb.

Grams	Kilograms	Pounds
600	0.6	
	1	2.2
600	0.6	1.32

5. Refer to the two three-dimensional figures shown. Determine whether each statement describes *Figure A*, *Figure B*, *both*, or *neither*.

a. This figure is a polyhedron.
Both

b. This figure has triangular faces.
Both

c. There are more vertices than edges in this figure.
Neither

d. This figure has rectangular faces.
Figure A

e. This figure is a pyramid.
Figure B

f. There is exactly one face that can represent the base for this figure.
Neither

g. The base of this figure is a triangle.
Both

h. This figure has two identical and parallel faces that can represent the base.
Figure A

6. Match each fraction with its decimal equivalent.

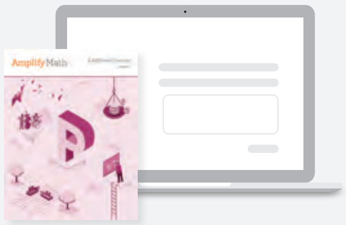
Fraction	Decimal
a $\frac{1}{2}$	**b** 0.25
b $\frac{3}{12}$	**d** 0.75
c $\frac{1}{20}$	**a** 0.50
d $\frac{3}{4}$	**e** 0.8
e $\frac{4}{5}$	**c** 0.05

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 2 Lesson 19	2
	5	Unit 1 Lesson 16	2
Formative ⚡	6	Unit 3 Lesson 7	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

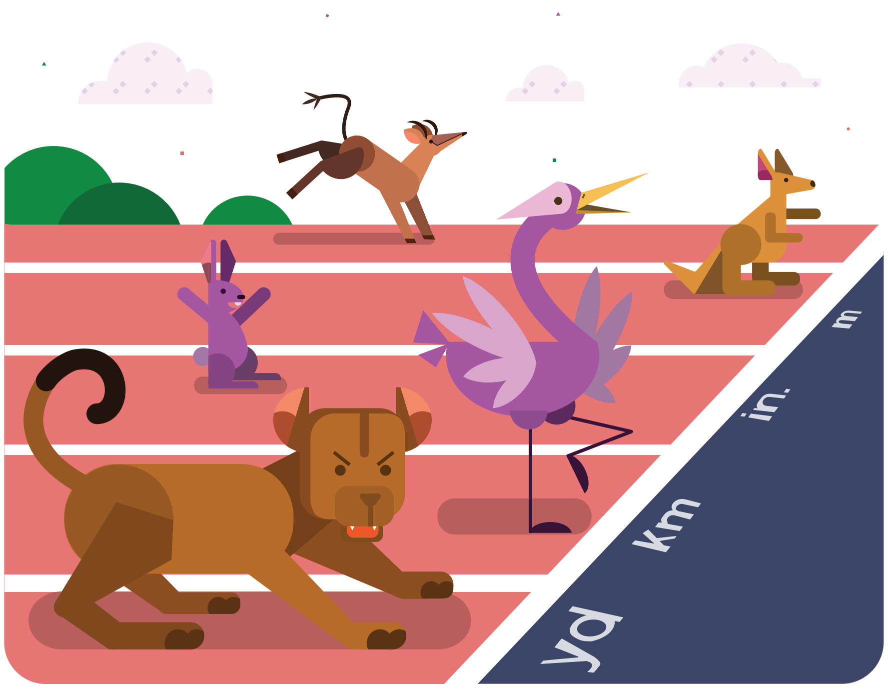


For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 3 | **LESSON 7**

Solving Rate Problems

Let's use unit rates to compare constant speeds and prices.



Focus

Goals

1. **Language Goal:** Apply reasoning about ratios and rates to convert and compare speeds expressed in different units. **(Writing)**
2. **Language Goal:** Apply reasoning about ratios and rates to justify which price is a better deal. **(Speaking and Listening)**

Coherence

Today

Students build fluency with unit rates in solving problems involving speed and price. They order the top speeds of several animals from fastest to slowest by using multiple unit rates — converting measurements to the same unit and determining speeds. Students also determine which store offers the best deals by comparing unit prices or using unit prices to generate equivalent ratios. In both scenarios, they have to first choose which unit rates to use, then divide to determine the desired unit rates, and lastly multiply or divide by those unit rates to determine missing values. Students also choose appropriate and helpful representations without being prompted by pulling from prior experiences with tables, diagrams, and graphs.

Previously

Over several previous lessons, students have discovered that in order to compare quantities in a ratio relationship, one quantity must be the same in both ratios.

Coming Soon

In Lesson 8, students will extend their work with the concept of rates to more broadly include percentages, as a rate per 100.

Rigor

- Students build **fluency** with dividing to determine unit rates.
- Students **apply** reasoning about ratios and rates to compare speeds and prices.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (for display)
- Activity 1 PDF (answers)
- Activity 2 PDF, pre-cut cards, one set per pair
- *Measurement Conversions* PDF (for display)
- calculators (optional)

Math Language Development

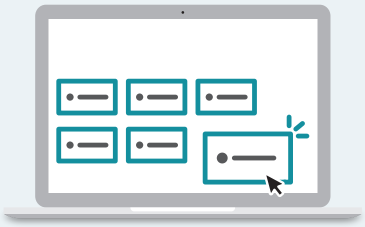
Review word

- *unit rate*

Amps | **Featured Activity**

Activity 2
Digital Partners

Students work in pairs to determine which store offers the best deal on each item included in community care packages.



Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students may rush to judgment about which is the better deal. Have students explain how they plan to control their impulses as they approach this activity without jumping to an unsupported conclusion. The sharing of information will help students make sense of the problems and encourage them to realize that they need unit rates to compare the deals properly.

Modifications to Pacing

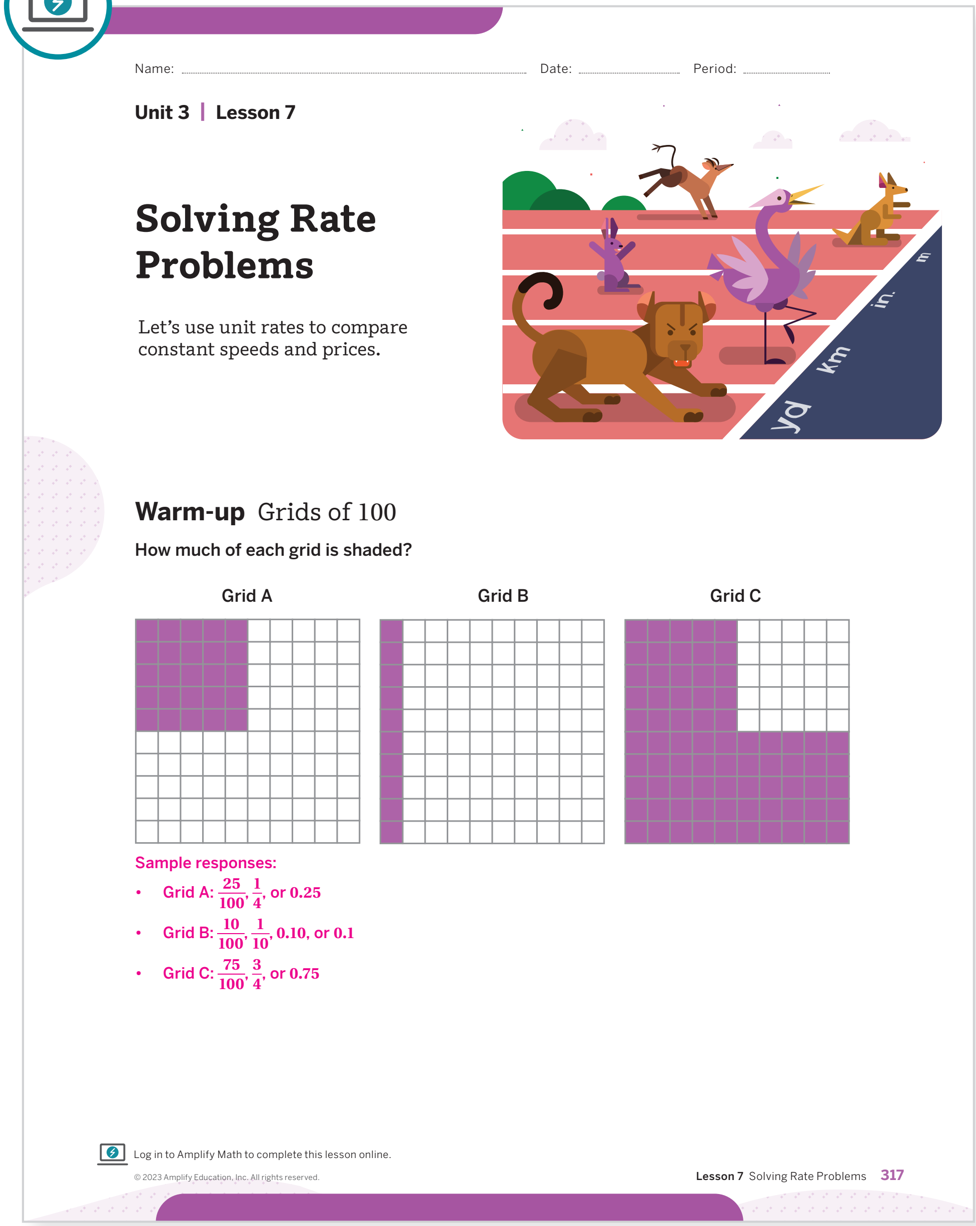
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, have students choose three of the six animals to compare, one of which should be the kangaroo or ostrich.
- In **Activity 2**, assign one card to each student. Have them determine and explain which is the better deal.

Independent | 5 min

Warm-up Grids of 100

Students activate their prior knowledge to name the shaded portion of a 100-grid, allowing them to connect fractions and decimals to rates per 1, and preparing for rates per 100.



Name: ____________ Date: ________ Period: ________

Unit 3 | **Lesson 7**

Solving Rate Problems

Let's use unit rates to compare constant speeds and prices.

Warm-up Grids of 100

How much of each grid is shaded?

Grid A Grid B Grid C

Sample responses:

- Grid A: $\frac{25}{100}$, $\frac{1}{4}$, or 0.25
- Grid B: $\frac{10}{100}$, $\frac{1}{10}$, 0.10, or 0.1
- Grid C: $\frac{75}{100}$, $\frac{3}{4}$, or 0.75

Log in to Amplify Math to complete this lesson online.

Lesson 7 Solving Rate Problems 317

1 Launch

Tell students you will show them three 10 × 10 grids which represent 1 whole. Explain that you will show them each grid for 3 seconds, hide it, and then show it again. Their job is to determine how much is shaded.

2 Monitor

Help students get started by asking, "What would the area be for one shaded row/column?"

Look for points of confusion:

- **Trying to count the shaded squares**. Ask, "Do you see groups of shaded squares? How does that help you determine the total shaded area?"
- **Not reporting their answers as fractions or decimals (e.g., "one column")**. Ask, "How many individual squares are shaded for what you said? How could you then describe the total number of squares if all 100 squares represent one whole?"

Look for productive strategies:

- Using the structure of the hundreds grid to reason (e.g., one row/column represents $\frac{10}{100}$, or the grid can be cut into fourths which represents $\frac{25}{100}$).
- Naming shaded parts as fractions and decimals in equivalent forms. Acknowledge if students use percentages, but do not push this form, as it will be the focus of the next several lessons.

3 Connect

Have students share the different ways they visualized and named the shaded portion of each image, focusing on both fractions and decimals used to name the shaded portion out of 100.

Ask, "How does your thinking here relate to rates?"

Highlight how each fraction or decimal named can be thought of as a rate per 100.

Power-up

To power up students' ability to convert between fractions and decimals, have students complete:

Recall that creating an equivalent fraction with a denominator of 10 or 100 can help when converting a fraction to a decimal. Determine an equivalent fraction with a denominator of 10 or 100 for each of the fractions.

1. $\frac{3}{5} = \frac{6}{10}$
2. $\frac{3}{20} = \frac{15}{100}$
3. $\frac{7}{25} = \frac{28}{100}$

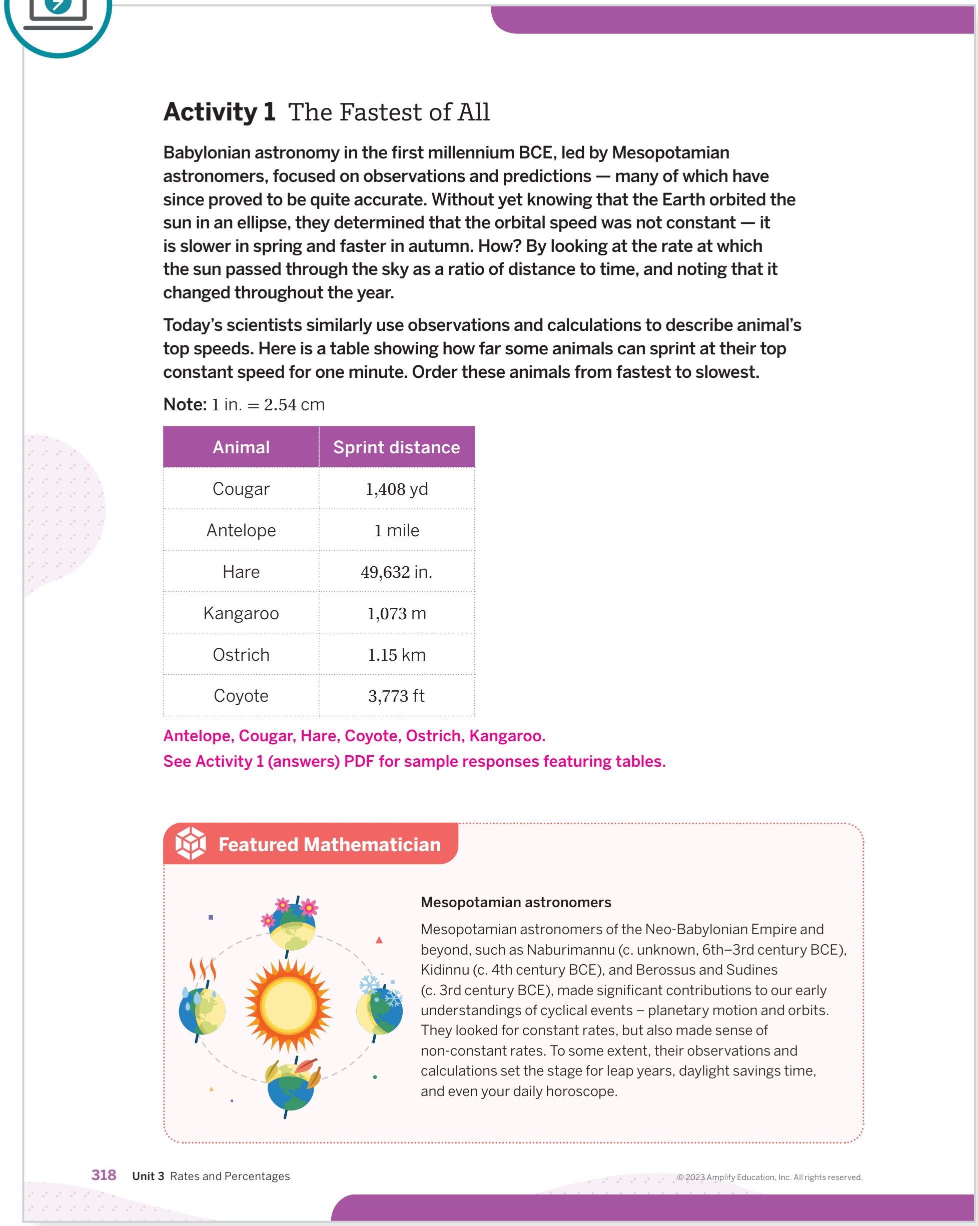
Use: Before the Warm-up

Informed by: Performance on Lesson 6, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4.

Pairs | 15 min

Activity 1 The Fastest of All

Students apply reasoning about ratios and rates to convert measurements and to compare speeds when distances are expressed in different units.



Activity 1 The Fastest of All

Babylonian astronomy in the first millennium BCE, led by Mesopotamian astronomers, focused on observations and predictions — many of which have since proved to be quite accurate. Without yet knowing that the Earth orbited the sun in an ellipse, they determined that the orbital speed was not constant — it is slower in spring and faster in autumn. How? By looking at the rate at which the sun passed through the sky as a ratio of distance to time, and noting that it changed throughout the year.

Today's scientists similarly use observations and calculations to describe animal's top speeds. Here is a table showing how far some animals can sprint at their top constant speed for one minute. Order these animals from fastest to slowest.

Note: 1 in. = 2.54 cm

Animal	Sprint distance
Cougar	1,408 yd
Antelope	1 mile
Hare	49,632 in.
Kangaroo	1,073 m
Ostrich	1.15 km
Coyote	3,773 ft

Antelope, Cougar, Hare, Coyote, Ostrich, Kangaroo.

See Activity 1 (answers) PDF for sample responses featuring tables.

Featured Mathematician

Mesopotamian astronomers

Mesopotamian astronomers of the Neo-Babylonian Empire and beyond, such as Naburimannu (c. unknown, 6th–3rd century BCE), Kidinnu (c. 4th century BCE), and Berossus and Sudines (c. 3rd century BCE), made significant contributions to our early understandings of cyclical events – planetary motion and orbits. They looked for constant rates, but also made sense of non-constant rates. To some extent, their observations and calculations set the stage for leap years, daylight savings time, and even your daily horoscope.

318 Unit 3 Rates and Percentages

1 Launch

Have students use the ***Mix and Mingle*** routine. Give them 1 minute to think about a plan for comparing the distances, and then have them share ideas with several other students. They will complete the activity with an assigned partner. Provide calculators as needed. If requested, share the *Measurement Conversions* PDF.

2 Monitor

Help students get started by asking, "What units are used? Can you compare these distances?"

Look for points of confusion:

- **Not converting all distances to the same unit**. Remind students that rate comparison requires the same units.
- **Not knowing when or how to perform multiple conversions**. Ask, "What information do you know? If you use that unit rate, your distance is now in what unit? Are you done?"

Look for productive strategies:

- Choosing a unit for distance that makes sense in context and requires fewer conversions (e.g., feet, yards, or meters). If students convert to inches, miles, or kilometers, consider redirecting them to more appropriate units by asking, "Is there a unit that would require fewer conversions? Why?"
- Recognizing when conversions between U.S. Customary and metric units require multiple conversions, and organizing their work to keep track of intermediate steps.

3 Connect

Have students share which units they converted to and why, followed by how they used unit rates to order the animals from fastest to slowest. If any students rounded values, have them share how it led to the ostrich and coyote running the same speed.

Highlight that the order remained the same, no matter which common unit was used. Use the Activity 2 PDF (answers) for display as needed.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students focus on the distances reported in U.S. Customary units: cougar, antelope, hare, and coyote. Display or provide the *Measurement Conversions* PDF.

Extension: *Math Enrichment*

Tell students that a jaguar is faster than a cougar, but slower than an antelope. Ask them to determine a reasonable number of kilometers a jaguar could run in 1 minute. Responses should be between 1.29 km and 1.6 km.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share, ask them to note any strategies that were similar or different. Some students may have chosen to convert all measurements to feet, while other students may have chosen to use inches or another measurement unit. Draw their attention to how unit rates were used, regardless of which measurement unit was preferred.

English Learners

Have students highlight the unit rate in each representation.

Featured Mathematician

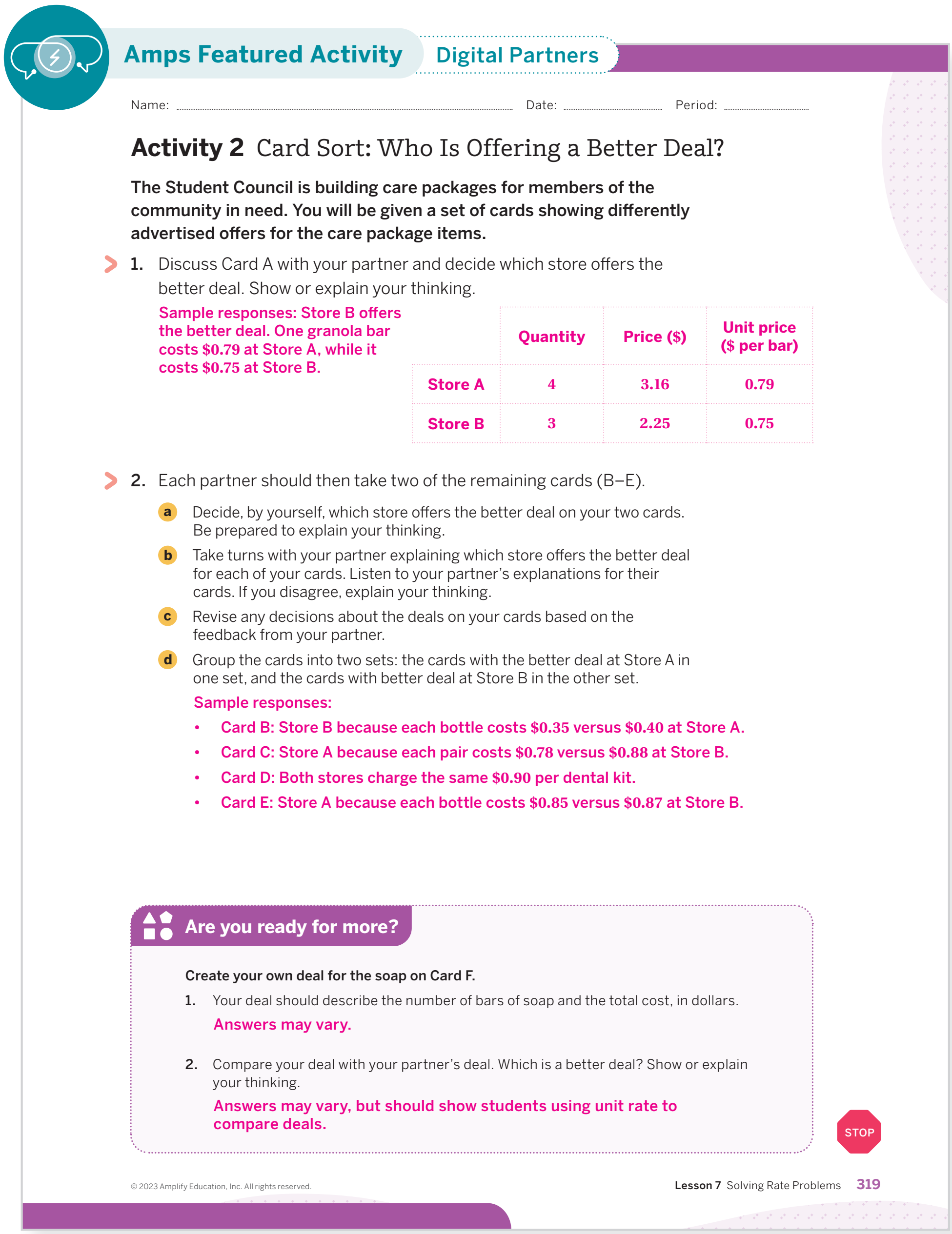
Mesopotamian astronomers

Have students read about the featured group of mathematicians, who were ancient Babylonian mathematicians and astronomers that studied the rate of the ecliptic, among many other things.

Pairs | 15 min

Activity 2 Card Sort: Who Is Offering a Better Deal?

Students apply reasoning about ratios and rates to to determine which store offers a better deal on items for community care packages.



Amps Featured Activity Digital Partners

Name: ________ Date: ________ Period: ________

Activity 2 Card Sort: Who Is Offering a Better Deal?

The Student Council is building care packages for members of the community in need. You will be given a set of cards showing differently advertised offers for the care package items.

1. Discuss Card A with your partner and decide which store offers the better deal. Show or explain your thinking.

   Sample responses: Store B offers the better deal. One granola bar costs $0.79 at Store A, while it costs $0.75 at Store B.

	Quantity	**Price ($)**	**Unit price ($ per bar)**
Store A	4	3.16	0.79
Store B	3	2.25	0.75

2. Each partner should then take two of the remaining cards (B–E).

   a. Decide, by yourself, which store offers the better deal on your two cards. Be prepared to explain your thinking.

   b. Take turns with your partner explaining which store offers the better deal for each of your cards. Listen to your partner's explanations for their cards. If you disagree, explain your thinking.

   c. Revise any decisions about the deals on your cards based on the feedback from your partner.

   d. Group the cards into two sets: the cards with the better deal at Store A in one set, and the cards with better deal at Store B in the other set.

   Sample responses:
   - **Card B: Store B because each bottle costs $0.35 versus $0.40 at Store A.**
   - **Card C: Store A because each pair costs $0.78 versus $0.88 at Store B.**
   - **Card D: Both stores charge the same $0.90 per dental kit.**
   - **Card E: Store A because each bottle costs $0.85 versus $0.87 at Store B.**

Are you ready for more?

Create your own deal for the soap on Card F.

1. Your deal should describe the number of bars of soap and the total cost, in dollars.

   Answers may vary.

2. Compare your deal with your partner's deal. Which is a better deal? Show or explain your thinking.

   Answers may vary, but should show students using unit rate to compare deals.

STOP

Lesson 7 Solving Rate Problems 319

1 Launch

Give each pair one set of pre-cut cards A–E. Reserve Card F for students ready for more. Have students use the ***Take Turns*** routine for Problem 2b.

2 Monitor

Help students get started by having them look at Card A and asking, "What needs to be true to make a rate comparison?"

Look for points of confusion:

- **Struggling to perform the decimal division or multiplication**. Consider reviewing these skills or providing a calculator as needed.

Look for productive strategies:

- Determining a common multiple for the number of items by using the multiplier to determine the price for the common number of items and by comparing the new prices.
- Determining the unit rate for one store, applying it to the number of items available from the other store, and comparing the costs for the same number of items.
- Comparing deals by determining the unit prices.
- Comparing deals by determining the unit rates per dollar.

3 Connect

Have students share which store offers the better deal for each item, focusing on how they used different representations and equivalent ratios or unit rates to compare. Consider showing examples of strategies not presented.

Highlight that when comparing rates, one quantity must be the same in both rates. Show how each strategy makes one quantity the same (1 for unit rates).

Ask, "Was the same strategy the most efficient in every case?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Have pairs of students work together on Cards A–E, rather than just on Card A. Consider providing them with a table, such as the following, that they can use to complete as they examine each card.

	Quantity	Price ($)	Unit price ($ per ____)
Store A			
Store B			

Math Language Development

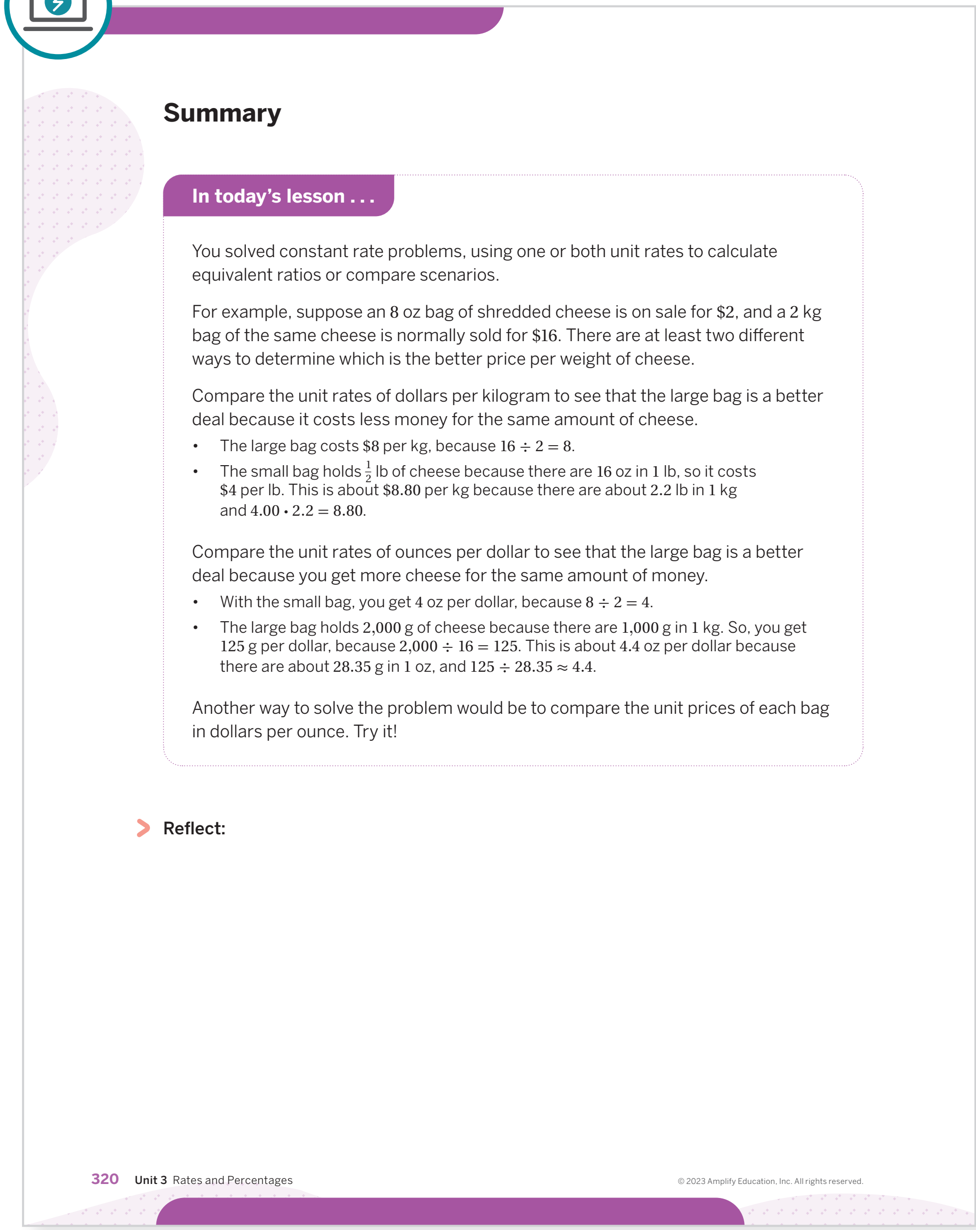
MLR7: Compare and Connect

During the Connect, ask students to compare how their different representations help them determine which stores offer the better deal. Encourage them to make connections between how they used equivalent ratios or unit rates. To help facilitate a realistic discussion about unit rates, ask, "Does it matter which store has the better deal when the differences are so small? If not, when would it matter?" Sample response: If I am purchasing only a few items, it may not matter as much. If I am purchasing a large quantity of items, such as hundreds of bottles of water for a community event, it will make a difference.

Whole Class | 5 min

Summary

Review and synthesize how to compare rates, connecting unit rates to work with equivalent ratios from the previous unit.



Summary

In today's lesson . . .

You solved constant rate problems, using one or both unit rates to calculate equivalent ratios or compare scenarios.

For example, suppose an 8 oz bag of shredded cheese is on sale for $2, and a 2 kg bag of the same cheese is normally sold for $16. There are at least two different ways to determine which is the better price per weight of cheese.

Compare the unit rates of dollars per kilogram to see that the large bag is a better deal because it costs less money for the same amount of cheese.

- The large bag costs $8 per kg, because $16 \div 2 = 8$.
- The small bag holds $\frac{1}{2}$ lb of cheese because there are 16 oz in 1 lb, so it costs $4 per lb. This is about $8.80 per kg because there are about 2.2 lb in 1 kg and $4.00 \cdot 2.2 = 8.80$.

Compare the unit rates of ounces per dollar to see that the large bag is a better deal because you get more cheese for the same amount of money.

- With the small bag, you get 4 oz per dollar, because $8 \div 2 = 4$.
- The large bag holds 2,000 g of cheese because there are 1,000 g in 1 kg. So, you get 125 g per dollar, because $2{,}000 \div 16 = 125$. This is about 4.4 oz per dollar because there are about 28.35 g in 1 oz, and $125 \div 28.35 \approx 4.4$.

Another way to solve the problem would be to compare the unit prices of each bag in dollars per ounce. Try it!

Reflect:

320 Unit 3 Rates and Percentages

Synthesize

Highlight that when comparing rates, one approach is to compare unit rates. Depending on the context, one unit rate (e.g., price per one item) may be preferable or more meaningful than the other (e.g., number of items per one dollar). Double number lines, ratio tables, and graphs can all be used to help determine a unit rate and solve related problems.

Ask:

- "Why are unit rates an efficient method to compare?" Since a unit rate makes one quantity have a value of 1, and 1 is a common factor for every pair of numbers, you can always determine the unit rate of any ratio.
- "When might you choose to not use unit rates?" When one quantity is a multiple or factor of the other or the division results in fractions and decimals that are difficult to compute or compare.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

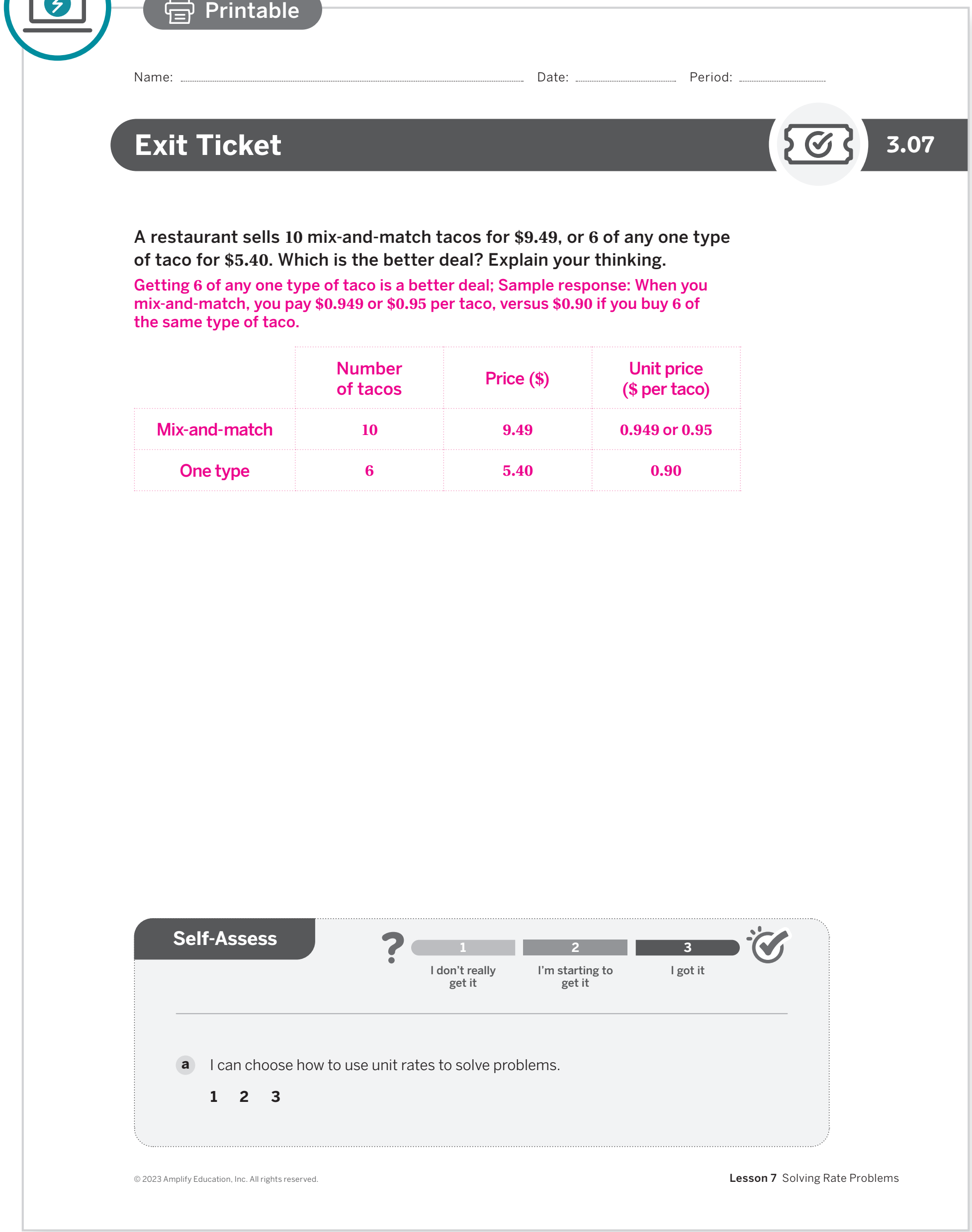
- "How are the terms *same rate*, *constant rate*, and *unit rate* similar? How are they different?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining which taco deal is better.

Printable



Name: __________ Date: __________ Period: __________

Exit Ticket 3.07

A restaurant sells 10 mix-and-match tacos for $9.49, or 6 of any one type of taco for $5.40. Which is the better deal? Explain your thinking.

Getting 6 of any one type of taco is a better deal; Sample response: When you mix-and-match, you pay $0.949 or $0.95 per taco, versus $0.90 if you buy 6 of the same type of taco.

	Number of tacos	Price ($)	Unit price ($ per taco)
Mix-and-match	10	9.49	0.949 or 0.95
One type	6	5.40	0.90

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can choose how to use unit rates to solve problems.

1 2 3

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Lesson 7 Solving Rate Problems

Success looks like . . .

- **Language Goal:** Applying reasoning about ratios and rates to convert and compare speed expressed in different units. **(Writing)**
- **Language Goal:** Applying reasoning about ratios and rates to justify which price is a better deal. **(Speaking and Listening)**
  - » Determining which set of tacos is a better deal using unit prices.

Suggested next steps

If students determine that the mix-and-match deal is better, consider:

- Reviewing Activity 2. Ask, "What were some of the ways you could compare the items for the care packages? Could you use one of those strategies here?"

Professional Learning

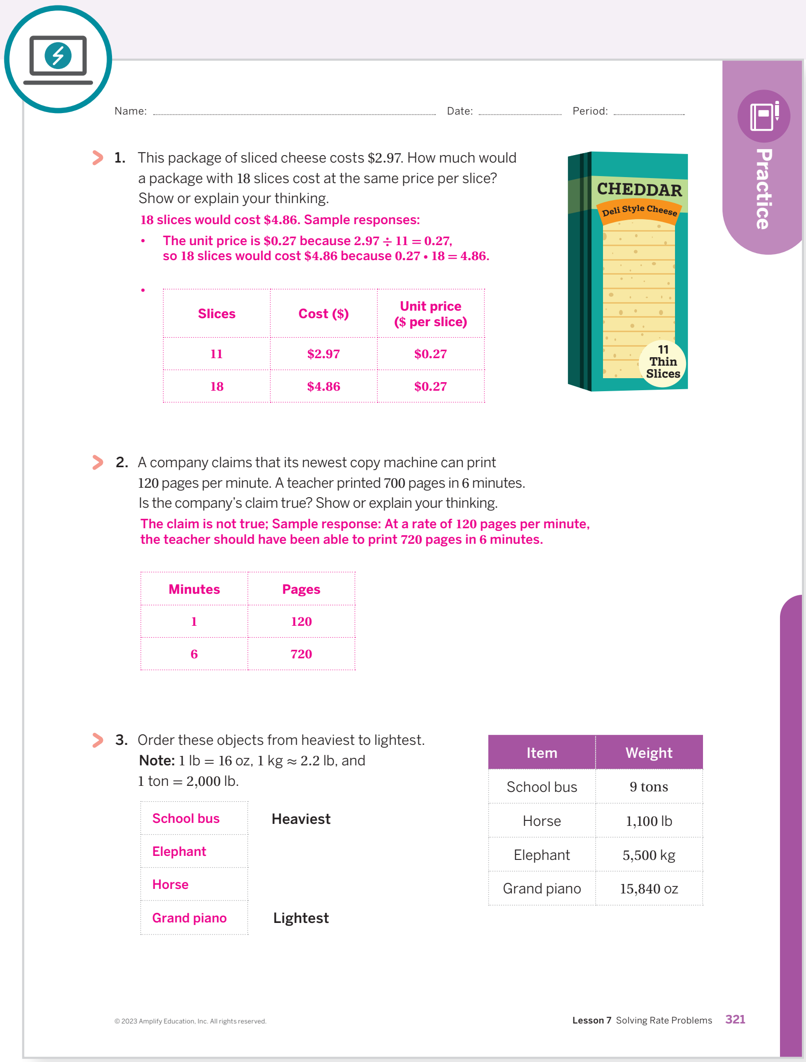
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students were asked to use ratio and rate reasoning to solve real-world problems. Where in your students' work today did you see or hear evidence of them doing this?
- How did students make sense of the problems when no scaffolding was given to them? How are you helping students become aware of how they are progressing in this area? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. This package of sliced cheese costs $2.97. How much would a package with 18 slices cost at the same price per slice? Show or explain your thinking.

18 slices would cost $4.86. Sample responses:

- **The unit price is $0.27 because $2.97 \div 11 = 0.27$, so 18 slices would cost $4.86 because $0.27 \cdot 18 = 4.86$.**
-

Slices	Cost ($)	Unit price ($ per slice)
11	$2.97	$0.27
18	$4.86	$0.27

2. A company claims that its newest copy machine can print 120 pages per minute. A teacher printed 700 pages in 6 minutes. Is the company's claim true? Show or explain your thinking.

The claim is not true; Sample response: At a rate of 120 pages per minute, the teacher should have been able to print 720 pages in 6 minutes.

Minutes	Pages
1	120
6	720

3. Order these objects from heaviest to lightest.
Note: 1 lb = 16 oz, 1 kg ≈ 2.2 lb, and 1 ton = 2,000 lb.

Item	Weight
School bus	9 tons
Horse	1,100 lb
Elephant	5,500 kg
Grand piano	15,840 oz

School bus	**Heaviest**
Elephant	
Horse	
Grand piano	**Lightest**

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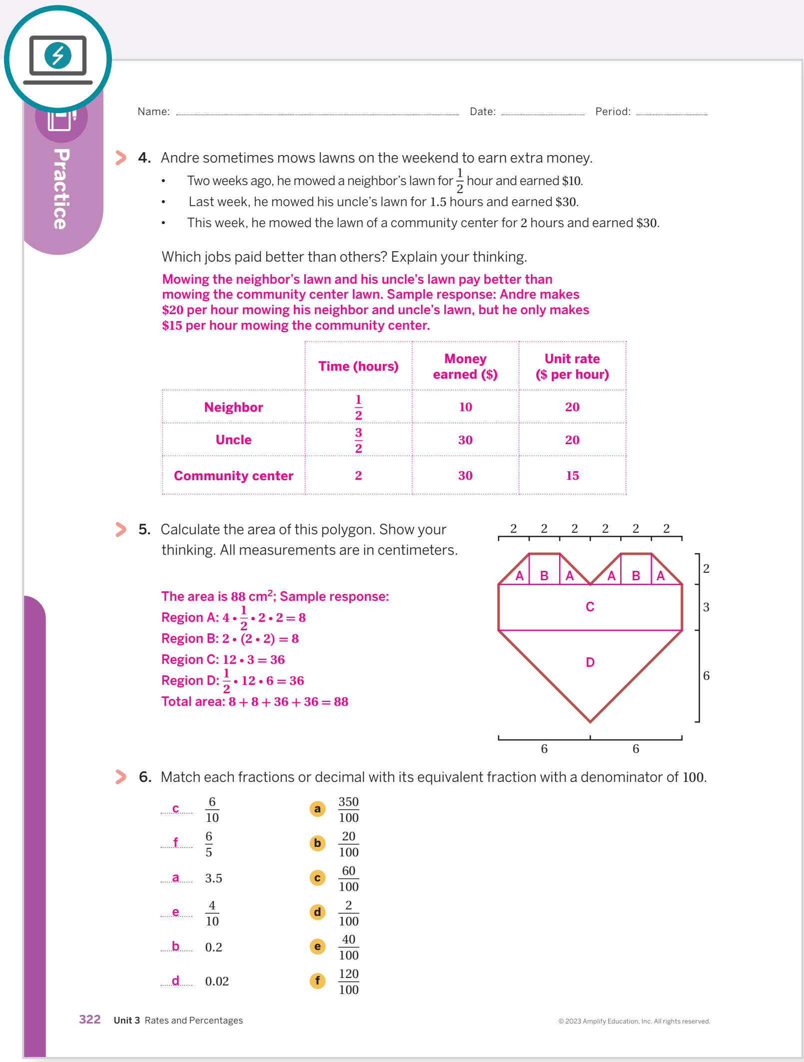
Lesson 7 Solving Rate Problems 321



Practice

Name: Date: Period:

4. Andre sometimes mows lawns on the weekend to earn extra money.

- Two weeks ago, he mowed a neighbor's lawn for $\frac{1}{2}$ hour and earned $10.
- Last week, he mowed his uncle's lawn for 1.5 hours and earned $30.
- This week, he mowed the lawn of a community center for 2 hours and earned $30.

Which jobs paid better than others? Explain your thinking.

Mowing the neighbor's lawn and his uncle's lawn pay better than mowing the community center lawn. Sample response: Andre makes $20 per hour mowing his neighbor and uncle's lawn, but he only makes $15 per hour mowing the community center.

	Time (hours)	Money earned ($)	Unit rate ($ per hour)
Neighbor	$\frac{1}{2}$	10	20
Uncle	$\frac{3}{2}$	30	20
Community center	2	30	15

5. Calculate the area of this polygon. Show your thinking. All measurements are in centimeters.

The area is 88 cm^2; Sample response:
Region A: $4 \cdot \frac{1}{2} \cdot 2 \cdot 2 = 8$
Region B: $2 \cdot (2 \cdot 2) = 8$
Region C: $12 \cdot 3 = 36$
Region D: $\frac{1}{2} \cdot 12 \cdot 6 = 36$
Total area: $8 + 8 + 36 + 36 = 88$

6. Match each fractions or decimal with its equivalent fraction with a denominator of 100.

c	$\frac{6}{10}$	a	$\frac{350}{100}$
f	$\frac{6}{5}$	b	$\frac{20}{100}$
a	3.5	c	$\frac{60}{100}$
e	$\frac{4}{10}$	d	$\frac{2}{100}$
b	0.2	e	$\frac{40}{100}$
d	0.02	f	$\frac{120}{100}$

322 **Unit 3** Rates and Percentages

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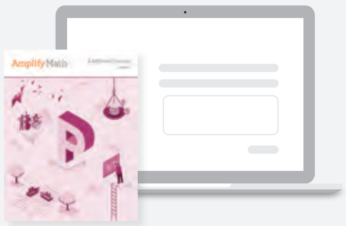
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 2 Lesson 4	2
	5	Unit 1 Lesson 13	2
Formative ⚡	6	Unit 3 Lesson 8	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 2

Percentages

Whole Class

In this Sub-Unit, students encounter percentages, as rates per 100, and use algorithms for determining missing values in percentage problems, such as thinking about fair representation and looking at subgroups of a population.

SUB-UNIT 2 Percentages

Narrative Connections

What can a corpse teach us about governing?

Wander the streets of Central London long enough and you will come across University College London. Within its walls, you'll find a curious sight at the Student Center: the preserved corpse of Jeremy Bentham!

Born in 1748, Bentham worked as an attorney and law critic. In many ways he was ahead of his time. He wrote extensively, pushing for prison reform, universal suffrage, and the decriminalization of homosexuality. But Bentham's greatest claim to fame is as the father of "utilitarianism."

Utilitarianism is a philosophy that argues that society should act to create the greatest amount of happiness for the greatest number of people. This was a startling revelation in 19th century Europe. European society was organized in a strict class structure. Bentham suggested that a pauper's happiness was worth the same as that of a noble or a prince.

This philosophy helped justify democracy as a form of government. It argued for governments to be responsible for the well-being of all its people.

While unit rates are useful for representing things that change, describing the change per one unit is less practical when looking at groups of a population. For example, to identify an issue's supporters and critics, there is another kind of ratio we can use—per one hundred. In the next lessons, you will see how this does a better job at representing the different individuals within society.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how percentages can help represent different populations around the school in the following places:

- **Lesson 12, Activity 2:** Multilingual Middle Schoolers
- **Lesson 13, Activity 1:** Reporting on Audience Size
- **Lesson 14, Activity 2:** If Our Class Were the World

UNIT 3 | **LESSON 8**

What Are Percentages?

Let’s learn about percentages.

Focus

Goals

1. **Language Goal:** Comprehend the term *percentage* and the symbol % to mean *a rate per 100*. **(Speaking and Listening, Writing)**
2. Draw and label a double number line diagram to represent percentages of a given whole and to determine corresponding amounts or percentages.

Coherence

Today

Students are introduced to percentages as a way to describe how much of a quantity compares to a given whole, corresponding to a rate per 100. They begin with the context of money, relating the values of different coins to the value of a dollar, where a dollar (or 100 cents) represents 100%, and so the number of cents corresponds to the percentage of a dollar. Students then use double number lines to determine percentages and parts of a given whole that is not 100. In doing so, they use the equivalent ratio reasoning they have developed to think about rates per 100. To strongly communicate and reiterate this relationship, double number lines are the primary representation for the first several lessons exploring percentages. However, if students prefer to reason by using tables, and eventually by multiplying or dividing by unit rates, they should not be discouraged from doing so.

Previously

In Lessons 1–7, students developed an understanding of rates as a unit-per-unit comparison of two quantities. They also saw that equivalent ratios represent the same rate.

Coming Soon

In Lesson 9, students will use double number lines to determine the percentage when given a part and the whole.

Rigor

- Students use money as the context to build **conceptual understanding** of percentages as rates per 100.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- *Hundred Squares* PDF
- *Double Number Lines*: *Percentage Problems* PDF

Math Language Development

New word

- *percentage (percent)**

Review words

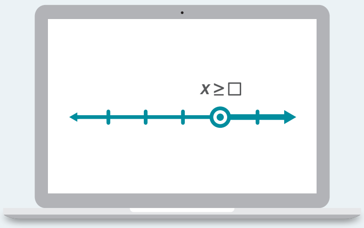
- *rate*
- *unit rate*

*Students may already be familiar with percentages and percents. While the term *percentage* is used to describe these rates in general terms and the term *percent* is used to describe a specific rate, allow students to use these terms interchangeably. They may also just use the term *percent*. The mathematical goal is to understand these as rates per 100.

Amps | **Featured Activity**

Activity 2
Interactive Double Number Lines

Students can manipulate double number lines to plot and reason about parts and percentages.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

While completing Activity 1, students must focus on consistent and appropriate use of new vocabulary, such as *percent*, and with visual displays, such as double number lines. Through self-regulation of their own thoughts and behaviors, students will be able to draw connections among different representations related to percents. Ask students to identify how they can help each other be both controlled and precise in their approaches to the activity.

Modifications to Pacing

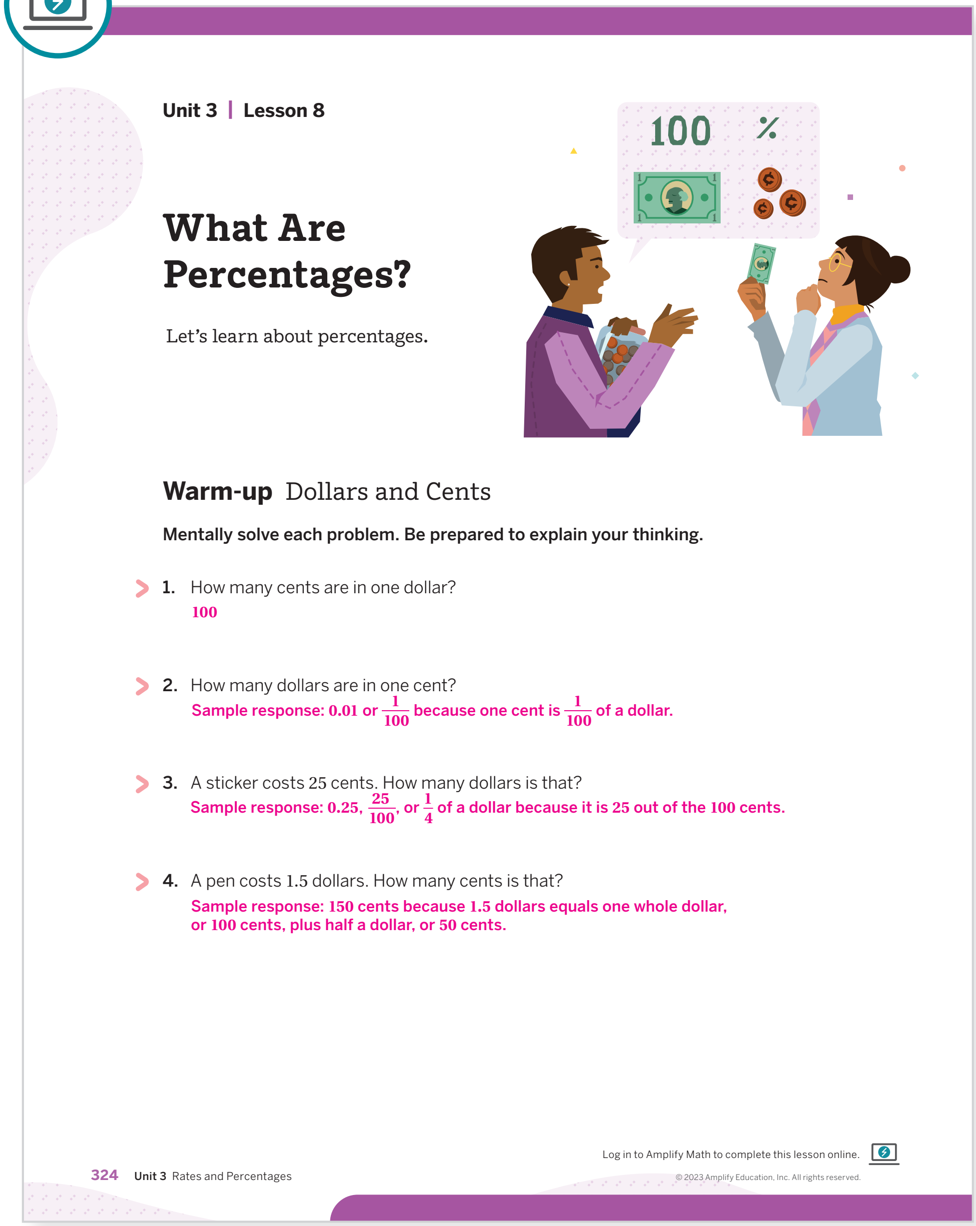
You may want to consider these additional modifications if you are short on time.

- Present a completed table in **Activity 1** to the whole class. Have students use the ***Think-Pair-Share*** routine to complete Problems 2–4.
- In **Activity 2**, Problem 3 may be omitted.

Independent | 5 min

Warm-up Dollars and Cents

Students activate their prior knowledge by reasoning about monetary amounts as decimals and fractions. This prepares them to consider rates per 100 in the next activity.



Unit 3 | Lesson 8

What Are Percentages?

Let's learn about percentages.

Warm-up Dollars and Cents

Mentally solve each problem. Be prepared to explain your thinking.

1. How many cents are in one dollar?
   100
2. How many dollars are in one cent?
   Sample response: 0.01 or $\frac{1}{100}$ because one cent is $\frac{1}{100}$ of a dollar.
3. A sticker costs 25 cents. How many dollars is that?
   Sample response: 0.25, $\frac{25}{100}$, or $\frac{1}{4}$ of a dollar because it is 25 out of the 100 cents.
4. A pen costs 1.5 dollars. How many cents is that?
   Sample response: 150 cents because 1.5 dollars equals one whole dollar, or 100 cents, plus half a dollar, or 50 cents.

Log in to Amplify Math to complete this lesson online.

324 Unit 3 Rates and Percentages

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1 Launch

Display the problems one at a time for all to see, and have students mentally solve each.

2 Monitor

Help students get started by asking, "Which coin is worth one cent? How many pennies does it take to equal a dollar?"

Look for points of confusion:

- **Thinking there are no dollars in one cent (Problem 2)**. Ask, "What fraction of a dollar does one cent represent?"

Look for productive strategies:

- Recognizing that one dollar is equivalent to 100 cents, and therefore any number of cents less than 100 represents a fraction of a dollar, and any number of dollars greater than 1 represents more than 100 cents.

3 Connect

Have students share their responses and strategies for each problem, ensuring both fraction and decimal amounts are shown.

Display the *Hundred Squares* PDF.

Ask:

- "What does each grid represent? Each square?" Each grid is 1 whole, \$1 or 100 cents. Each square is 1 cent or $\frac{1}{100}$.
- "How can you represent each answer from Problems 1–4 on the hundreds grid?" Answers may vary, but there should be totals of 100, 1, 25, and 150 squares shaded.

Highlight that ratio language can be used to describe the relationship between cents and dollars. For example, in Problem 3, there are 25 cents for every 100 cents (which is equal to 1 dollar).

Differentiated Support

Accessibility: *Guide Processing and Visualization, Activate Background Knowledge*

Consider bringing in 1 U.S. dollar and 100 pennies (in a plastic bag), or showing a visual that illustrates the number of pennies that are in 1 U.S. dollar. For students who may not be as familiar with U.S. currency, display the relationship that there are 100 pennies in 1 U.S. dollar.

Power-up

To power up students' ability to write fractions and decimal values as fractions with a denominator of 100, have students complete:

Recall that to create an equivalent fraction you can multiply the denominator and numerator by the same number. Determine which number to multiply by for each of the following sets of equivalent fractions.

1. $\frac{8 \cdot \boxed{2}}{50 \cdot \boxed{2}} = \frac{\boxed{16}}{100}$
2. $\frac{7 \cdot \boxed{5}}{20 \cdot \boxed{5}} = \frac{\boxed{35}}{100}$

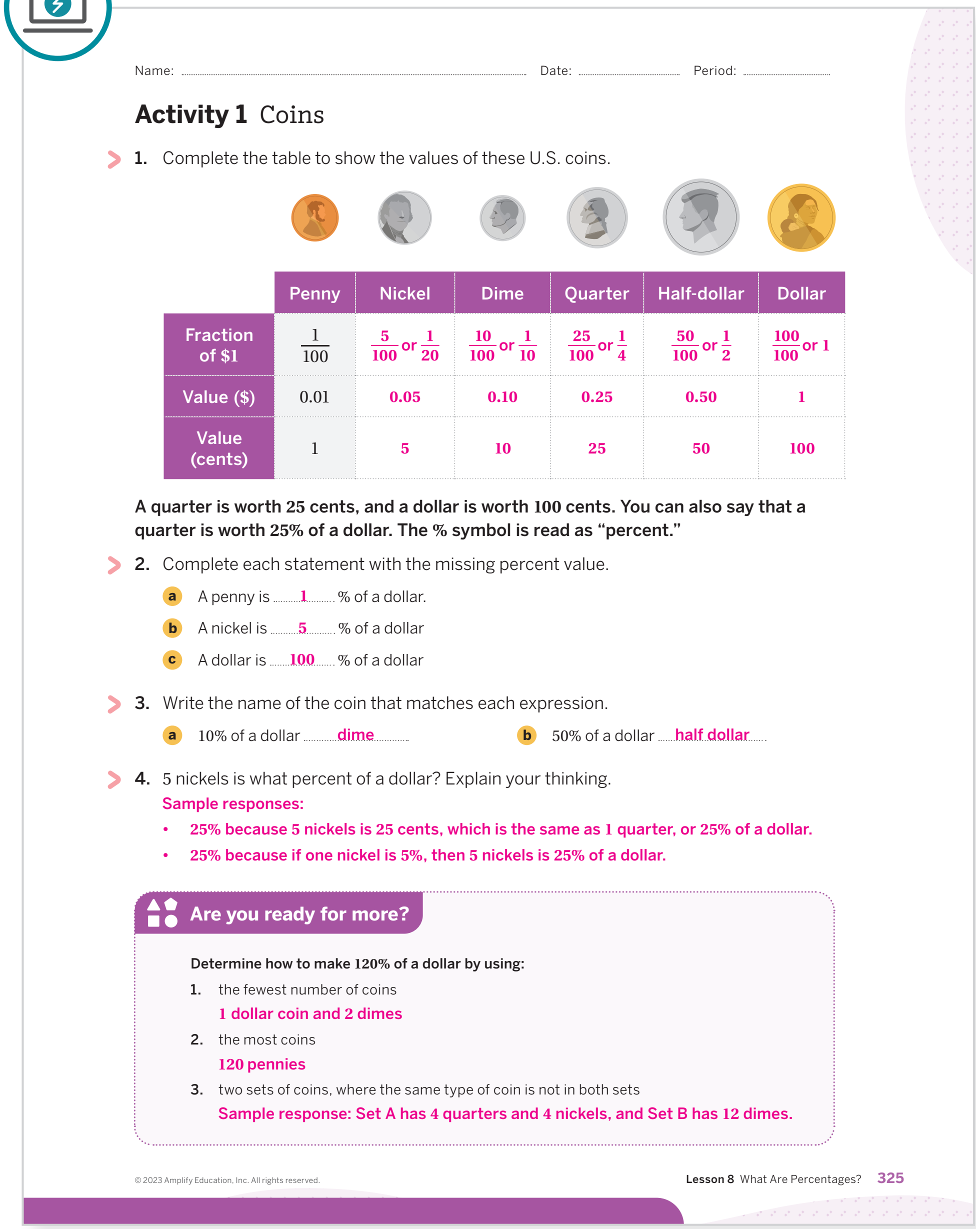
Use: Before the Warm-up

Informed by: Performance on Lesson 7, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 7.

Pairs | 15 min

Activity 1 Coins

Students relate coin values to fractions and percents of 1 dollar, which will be connected to the definition of a percentage as a rate per 100.



Name: .... Date: .... Period: ....

Activity 1 Coins

1. Complete the table to show the values of these U.S. coins.

	Penny	Nickel	Dime	Quarter	Half-dollar	Dollar
Fraction of $1	$\frac{1}{100}$	$\frac{5}{100}$ or $\frac{1}{20}$	$\frac{10}{100}$ or $\frac{1}{10}$	$\frac{25}{100}$ or $\frac{1}{4}$	$\frac{50}{100}$ or $\frac{1}{2}$	$\frac{100}{100}$ or 1
Value ($)	0.01	0.05	0.10	0.25	0.50	1
Value (cents)	1	5	10	25	50	100

A quarter is worth 25 cents, and a dollar is worth 100 cents. You can also say that a quarter is worth 25% of a dollar. The % symbol is read as "percent."

2. Complete each statement with the missing percent value.
   - a A penny is 1 % of a dollar.
   - b A nickel is 5 % of a dollar
   - c A dollar is 100 % of a dollar

3. Write the name of the coin that matches each expression.
   - a 10% of a dollar dime
   - b 50% of a dollar half dollar

4. 5 nickels is what percent of a dollar? Explain your thinking.
   Sample responses:
   - **25% because 5 nickels is 25 cents, which is the same as 1 quarter, or 25% of a dollar.**
   - **25% because if one nickel is 5%, then 5 nickels is 25% of a dollar.**

Are you ready for more?

Determine how to make 120% of a dollar by using:

1. the fewest number of coins
   1 dollar coin and 2 dimes
2. the most coins
   120 pennies
3. two sets of coins, where the same type of coin is not in both sets
   Sample response: Set A has 4 quarters and 4 nickels, and Set B has 12 dimes.

1 Launch

Point out the half-dollar and dollar coins. Have students use the *Think-Pair-Share* routine, giving 3 minutes to work independently on Problems 1–2 before comparing and completing the remaining problems with a partner.

2 Monitor

Help students get started by giving them a copy of the *Hundred Squares* PDF and asking, "How would you shade the grid to represent the value of one penny? How does this relate to the values given in each row of the table?"

Look for points of confusion:

- **Using the number of coins as the percent**. Ask, "Is *one* quarter *one* percent of a dollar? Why not?" Reiterate that the percentage is the value.

Look for productive strategies:

- Writing the value of each coin as a fraction and as a corresponding decimal value in dollars. If students simplify fractions and cannot represent them as decimals, ask, "What equivalent fraction could help?"
- Relating percentages of a dollar to coin values.
- Using ratio reasoning for multiple coins (Problem 4).

3 Connect

Have students share their responses and strategies, focusing on precise uses of language or notation**.**

Define a ***percentage*** as a rate per 100. Note that when the percentage is known, such as 75%, it is read as "seventy-five percent," *not* "seventy-five percentage."

Display the *Double Number Lines: Percentage Problems* PDF. Mark 0, 50, and 100 on both number lines.

Highlight that students may be familiar with seeing and using the word *percent*, and while there are nuances between *percentage* and *percent*, students may use these terms interchangeably.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Activate Background Knowledge*

Consider bringing in several types of U.S. coins: pennies, nickels, dimes, quarters, half-dollars, and dollars. For students who may not be as familiar with U.S. currency, display these relationships, whether numerically or with visual images.

- There are 100 pennies in 1 U.S. dollar.
- There are 20 nickels in 1 U.S. dollar.
- There are 10 dimes in 1 U.S. dollar.
- There are 4 quarters in 1 U.S. dollar.
- There are 2 half-dollars in 1 U.S. dollar.
- There is 1 dollar coin that is equivalent to 1 U.S. dollar bill.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect statement, such as "5 nickels is 5% of a dollar because there are 5 coins." Ask:

Critique: "Why is this statement incorrect?" Listen for distinctions between value of coins and number of coins.
Correct and Clarify: Have students write a corrected statement. Then have them explain how they know their statement is correct.

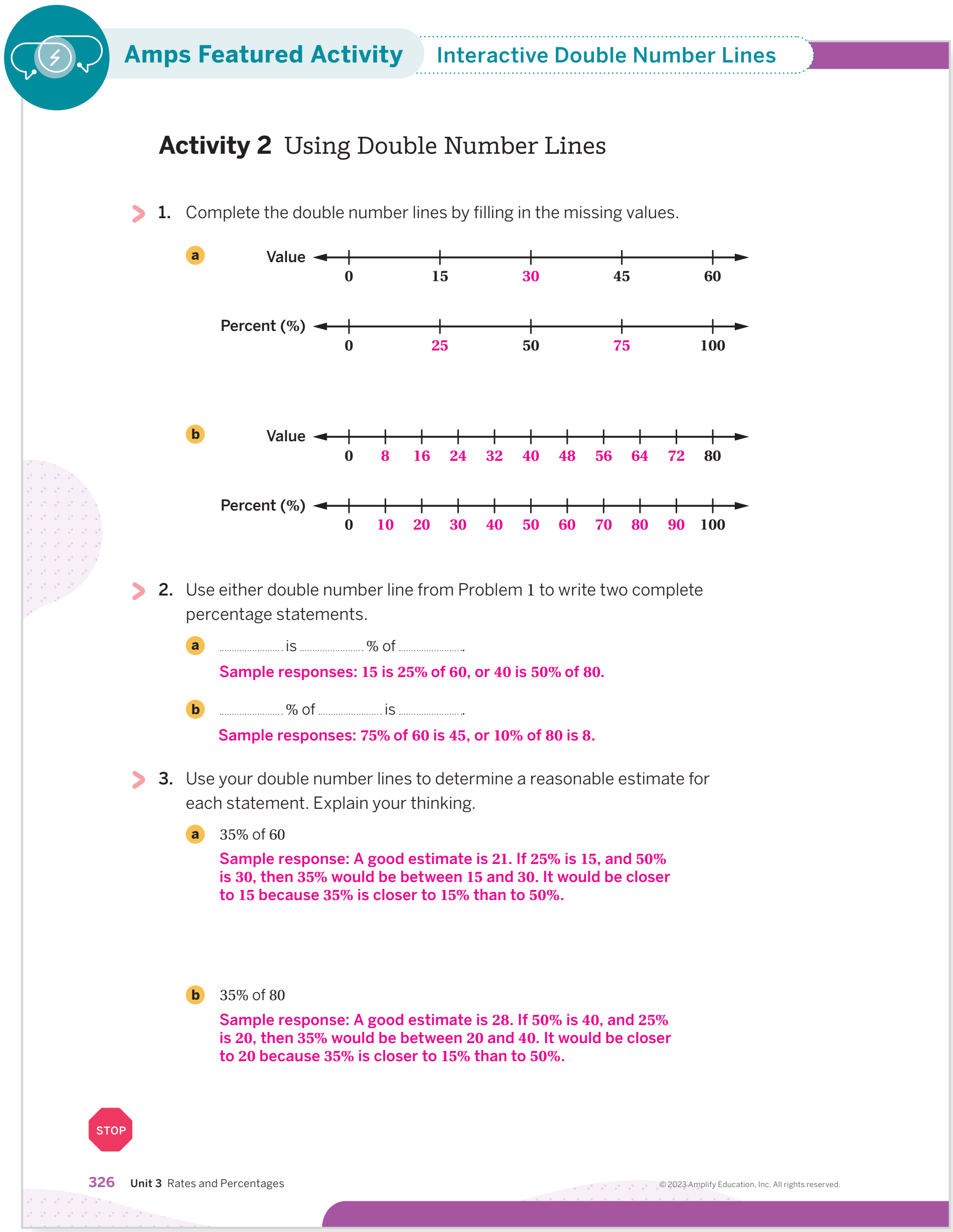
English Learners

Physically display 5 nickels or images of 5 nickels. Write *nickel* next to each one and display the completed table from Problem 1.

Activity 2 Using Double Number Lines

Students apply previous work with equivalent ratios and double number lines to reason about percentages of wholes other than 100.

Amps Featured Activity Interactive Double Number Lines



Activity 2 Using Double Number Lines

1. Complete the double number lines by filling in the missing values.

a

Value	0	15	30	45	60
Percent (%)	0	25	50	75	100

b

Value	0	8	16	24	32	40	48	56	64	72	80
Percent (%)	0	10	20	30	40	50	60	70	80	90	100

2. Use either double number line from Problem 1 to write two complete percentage statements.

a ______ is ______ % of ______.

Sample responses: 15 is 25% of 60, or 40 is 50% of 80.

b ______ % of ______ is ______.

Sample responses: 75% of 60 is 45, or 10% of 80 is 8.

3. Use your double number lines to determine a reasonable estimate for each statement. Explain your thinking.

a 35% of 60

Sample response: A good estimate is 21. If 25% is 15, and 50% is 30, then 35% would be between 15 and 30. It would be closer to 15 because 35% is closer to 15% than to 50%.

b 35% of 80

Sample response: A good estimate is 28. If 50% is 40, and 25% is 20, then 35% would be between 20 and 40. It would be closer to 20 because 35% is closer to 15% than to 50%.

STOP

326 Unit 3 Rates and Percentages

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "How many equal intervals are between 0 and 100? What can you divide by to determine the first unknown percent?"

Look for points of confusion:

- **Thinking that the value matches the percentage.** Ask, "Does it make sense in Problem 1a that 45 on the top would align with 45 on the bottom?"
- **Struggling to place values appropriately in Problem 2.** Ask, "What quantity should get a % symbol? What value is the whole (the one something is a *percent of*)?"
- **Not knowing how to estimate for Problem 3.** Ask, "Where would 35% be on each bottom number line? What are the nearby values on each top number line?"

Look for productive strategies:

- Using multiplication and division to reason about the number lines independently.
- Using equivalent ratios to reason about the number lines together.

3 Connect

Have students share their responses and strategies for Problems 1–3, one at a time, focusing on how they used division or multiplication and equivalent ratios.

Ask, "Why would 35% of 60 not be the same as 35% of 80?"

Highlight that percentages can be expressed as rate per 100. Double number lines represent equivalent ratios where the whole corresponds to 100%. Percentages can also be multiplied or divided as equivalent ratios to determine unknown values. In Problem 1a, 15 : 25 is equivalent to 60 : 100, which means "15 is 25% of 60."

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Problems 1 and 2. In Problem 2, consider providing them with a value they can use for the whole, while they use the double number line to determine a related part and percent.

Alternatively, provide students with completed double number lines for Problem 1 and have them focus on Problems 2 and 3.

Extension: *Math Enrichment*

Have students create a double number line diagram to determine 70% of 30 using as few tick marks as possible.

Math Language Development

MLR7: Compare and Connect

As students complete Problem 1, pair students who used division with students who used ratio reasoning. Have them review each other's double number lines and ask them to discuss and compare the strategies they used. Ask them to discuss, "What are the advantages of each strategy?"

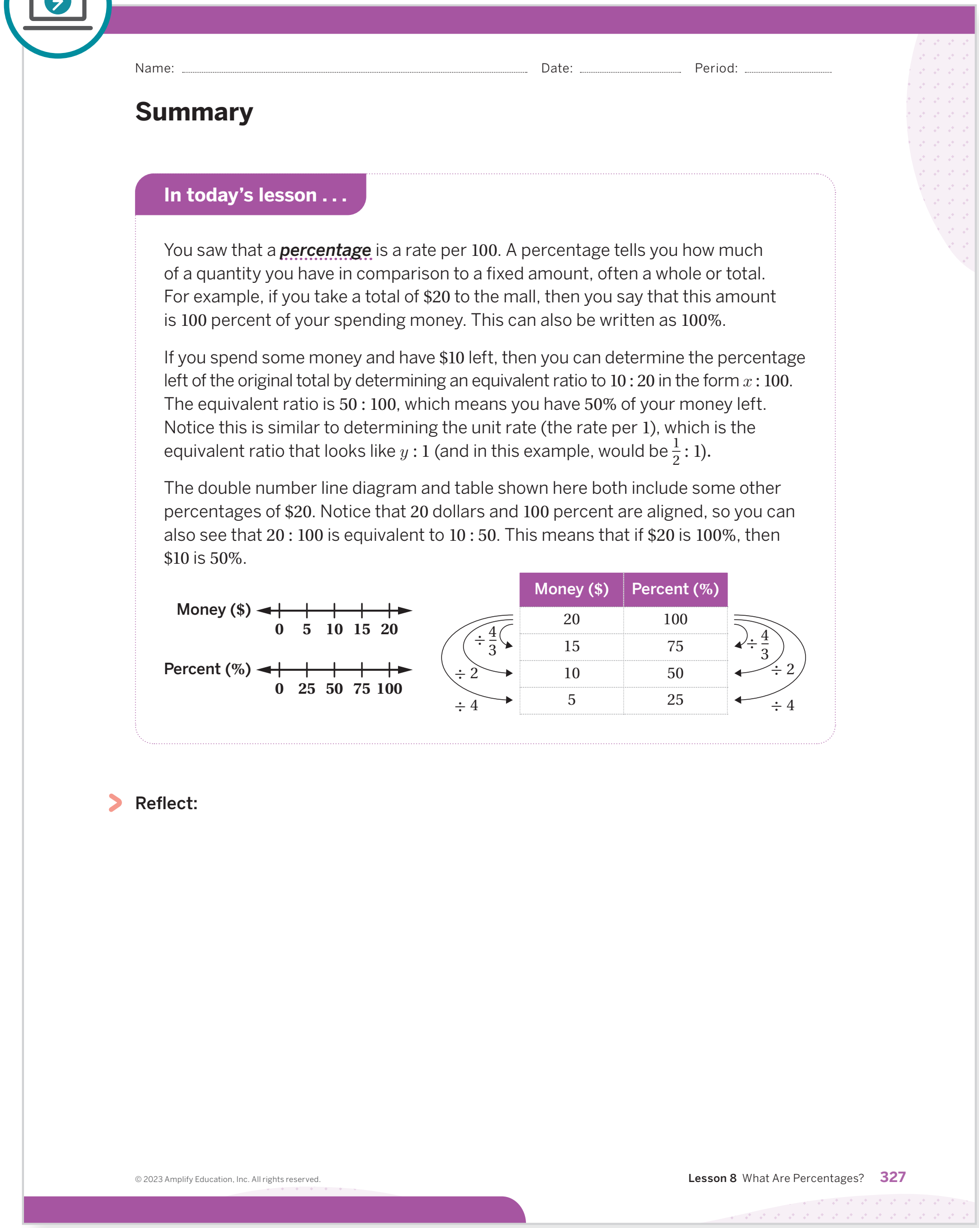
English Learners

Encourage students to refer to the class display and model for students how to connect ratio reasoning to the double number line. For example, illustrate how the number line shows 25% of 60 is 15.

Whole Class | 5 min

Summary

Review and synthesize the connection between percentages and ratios, as well as equivalent ratios and determining unknown values or percentages.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that a ***percentage*** is a rate per 100. A percentage tells you how much of a quantity you have in comparison to a fixed amount, often a whole or total. For example, if you take a total of \$20 to the mall, then you say that this amount is 100 percent of your spending money. This can also be written as 100%.

If you spend some money and have \$10 left, then you can determine the percentage left of the original total by determining an equivalent ratio to 10 : 20 in the form $x : 100$. The equivalent ratio is 50 : 100, which means you have 50% of your money left. Notice this is similar to determining the unit rate (the rate per 1), which is the equivalent ratio that looks like $y : 1$ (and in this example, would be $\frac{1}{2} : 1$).

The double number line diagram and table shown here both include some other percentages of \$20. Notice that 20 dollars and 100 percent are aligned, so you can also see that 20 : 100 is equivalent to 10 : 50. This means that if \$20 is 100%, then \$10 is 50%.

Money (\$)	Percent (%)
20	100
15	75
10	50
5	25

Reflect:

Synthesize

Ask, "How is a percentage a ratio?" Sample response: A percentage is an equivalent ratio involving a value of 100 that corresponds to the whole or total. For example, a quarter is worth 25% of a dollar because you have 25 cents for every 100 cents.

Formalize vocabulary: ***percentage***

Highlight that, because a percentage represents a rate, equivalent ratios can be used to solve percentage problems. Emphasize that ratio tables and double number lines are still useful representations for determining those equivalent ratios.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did coins help you think and talk about percentages?"

Math Language Development

MLR2: Collect and Display

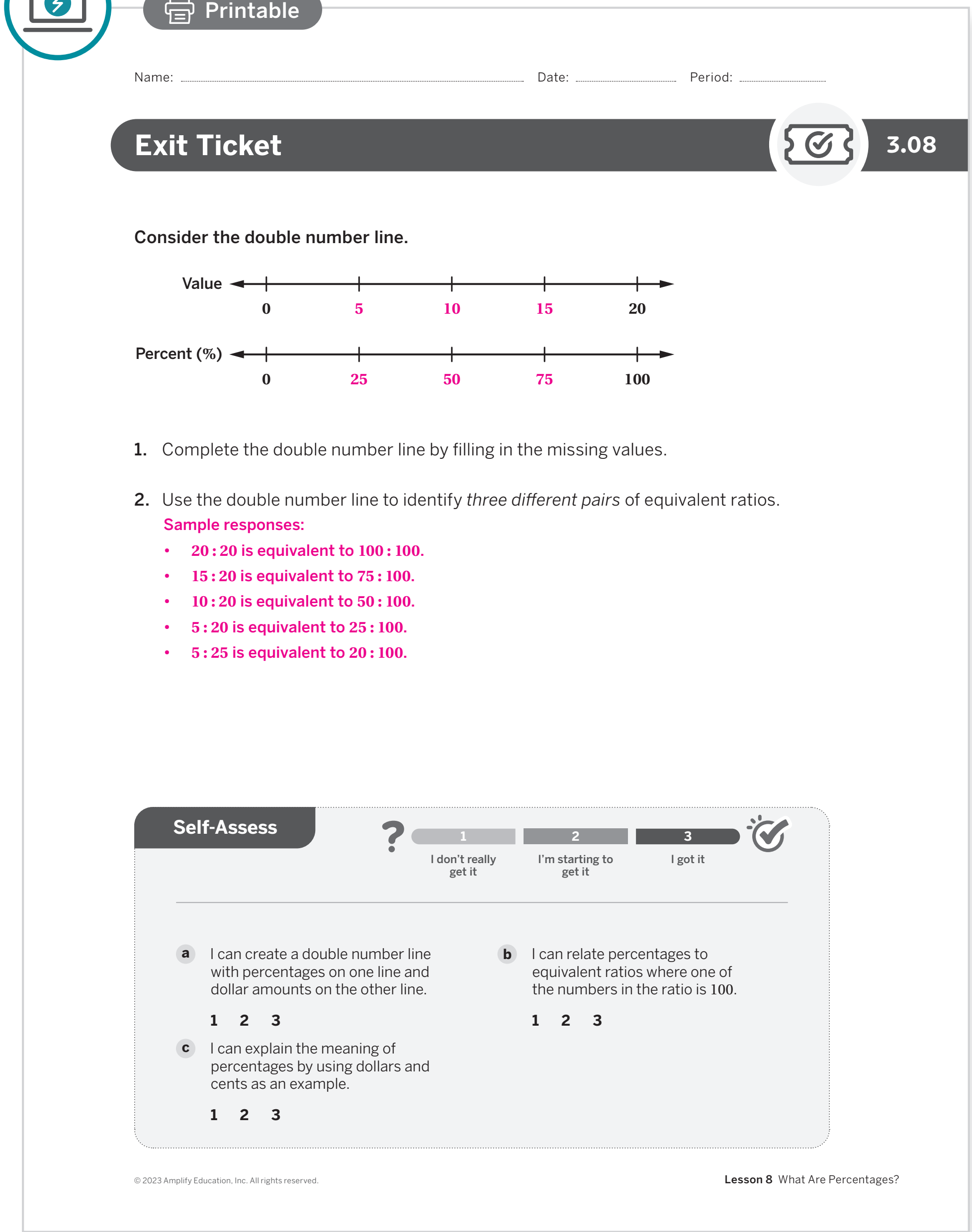
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the term *percentage* (or *percent*) that were added to the display during the lesson. Add visual diagrams, such as ratio tables and double number lines that illustrate percentages to the class display.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using a double number line to determine unknown amounts and percentages, and to identify three pairs of equivalent ratios.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.08

Consider the double number line.

Value: 0, 5, 10, 15, 20

Percent (%): 0, 25, 50, 75, 100

1. Complete the double number line by filling in the missing values.
2. Use the double number line to identify *three different pairs* of equivalent ratios.

Sample responses:

- $20 : 20$ is equivalent to $100 : 100$.
- $15 : 20$ is equivalent to $75 : 100$.
- $10 : 20$ is equivalent to $50 : 100$.
- $5 : 20$ is equivalent to $25 : 100$.
- $5 : 25$ is equivalent to $20 : 100$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can create a double number line with percentages on one line and dollar amounts on the other line. 1 2 3

b. I can relate percentages to equivalent ratios where one of the numbers in the ratio is 100. 1 2 3

c. I can explain the meaning of percentages by using dollars and cents as an example. 1 2 3

Lesson 8 What Are Percentages?

Success looks like . . .

- **Language Goal:** Comprehending the term *percentage* and the symbol % to mean *a rate per 100*. **(Speaking and Listening, Writing)**
- **Goal:** Drawing and labeling a double number line diagram to represent percentages of a given whole and to determine corresponding amounts or percentages.
  - » Creating a double number line to determine pairs of equivalent ratios in Problem 2.

Suggested next steps

If students write the incorrect values on the number line, consider:

- Reviewing Activity 2, Problem 1 and asking, "What strategies did you use to complete the double number lines? How did you use division or equivalent ratios to fill in missing values here?"

If students use the double number lines to say that $5 : 25$ is equivalent to $10 : 50$ (or other similar correct statements of equivalence), consider asking:

- "Is there another set of equivalent ratios that compare the values on the top number line to those on the bottom?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and what didn't work today? In Unit 2, students used double number lines to represent and explore ratio relationships between two quantities. How did that support their work with percentages today?
- Which groups of students did and did not have their ideas seen and heard today? What might you change for the next time you teach this lesson to ensure more voices are heard?

Math Language Development

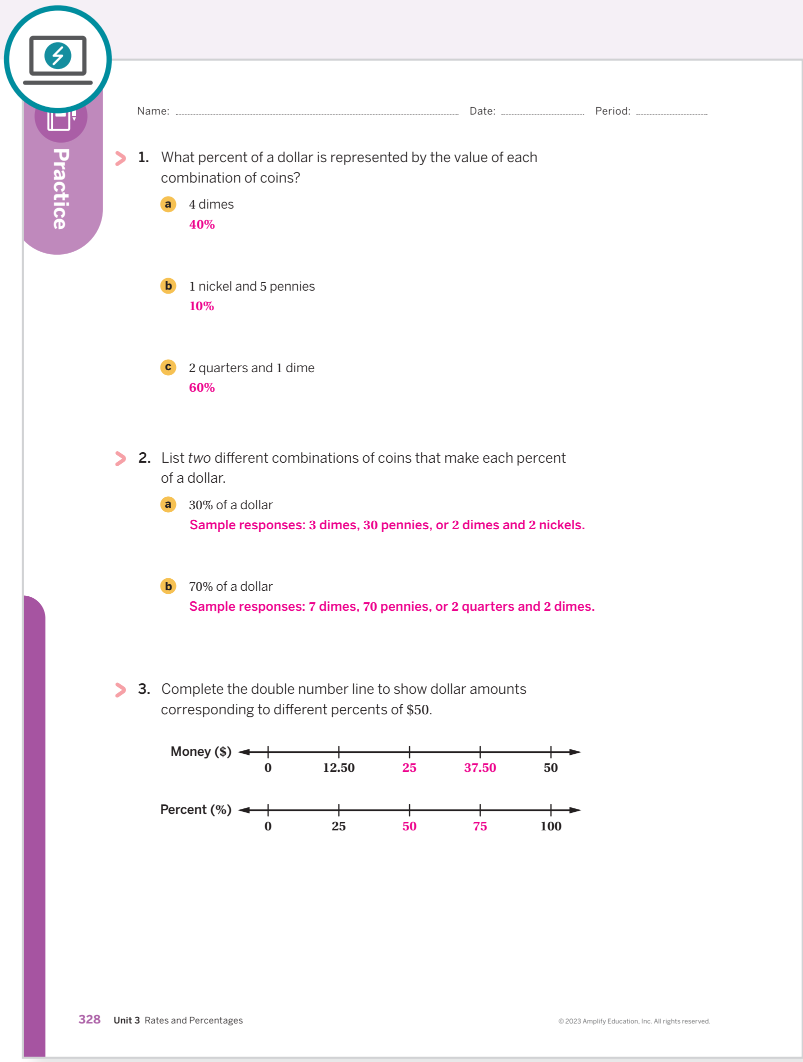
Language Goal: Comprehending the term *percentage* and the symbol % to mean *a rate per 100*.

Reflect on students' language development toward this goal.

- Students may have been familiar with percentages prior to this unit. How did they begin to describe percentages? How have they progressed in understanding and describing a percentage as a *rate*?
- How can you help them be more precise in their descriptions?

Practice

Independent



Practice

Name: .... Date: .... Period: ....

1. What percent of a dollar is represented by the value of each combination of coins?

   a. 4 dimes
   40%

   b. 1 nickel and 5 pennies
   10%

   c. 2 quarters and 1 dime
   60%

2. List *two* different combinations of coins that make each percent of a dollar.

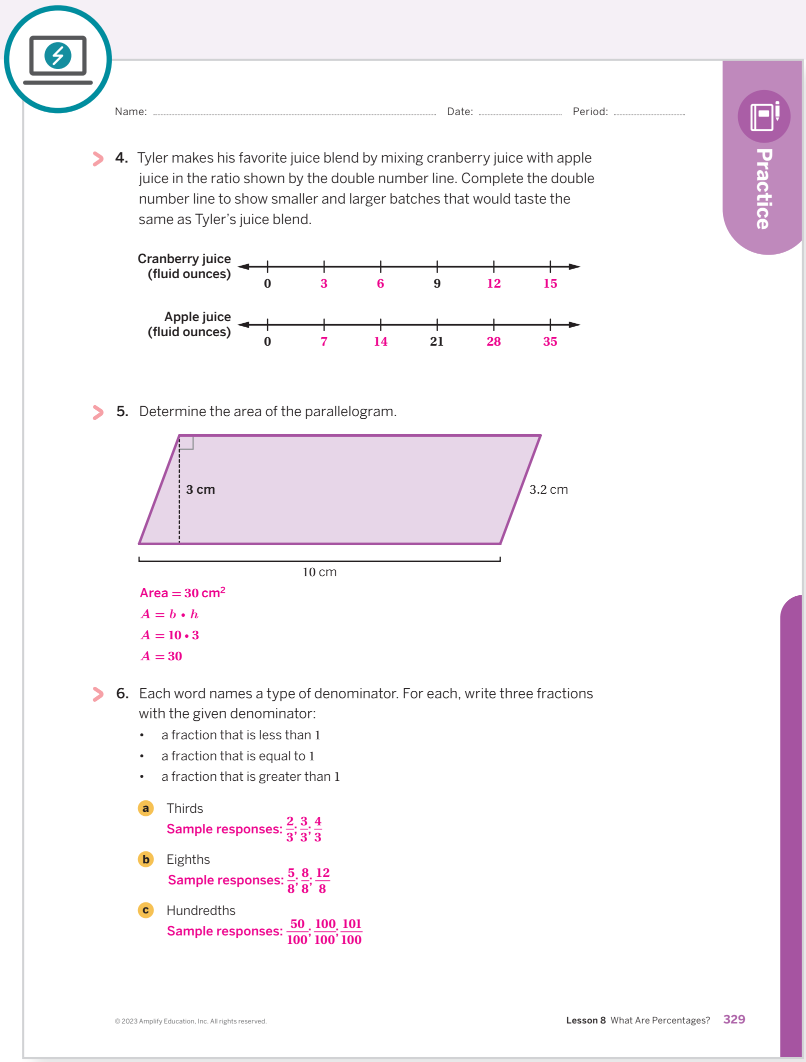
   a. 30% of a dollar
   Sample responses: 3 dimes, 30 pennies, or 2 dimes and 2 nickels.

   b. 70% of a dollar
   Sample responses: 7 dimes, 70 pennies, or 2 quarters and 2 dimes.

3. Complete the double number line to show dollar amounts corresponding to different percents of $50.

Money ($)	0	12.50	25	37.50	50
Percent (%)	0	25	50	75	100

328 Unit 3 Rates and Percentages



Practice

Name: .... Date: .... Period: ....

4. Tyler makes his favorite juice blend by mixing cranberry juice with apple juice in the ratio shown by the double number line. Complete the double number line to show smaller and larger batches that would taste the same as Tyler's juice blend.

Cranberry juice (fluid ounces)	0	3	6	9	12	15
Apple juice (fluid ounces)	0	7	14	21	28	35

5. Determine the area of the parallelogram.

Area = 30 cm²

$A = b \cdot h$

$A = 10 \cdot 3$

$A = 30$

6. Each word names a type of denominator. For each, write three fractions with the given denominator:
   - a fraction that is less than 1
   - a fraction that is equal to 1
   - a fraction that is greater than 1

   a. Thirds
   Sample responses: $\frac{2}{3}, \frac{3}{3}, \frac{4}{3}$

   b. Eighths
   Sample responses: $\frac{5}{8}, \frac{8}{8}, \frac{12}{8}$

   c. Hundredths
   Sample responses: $\frac{50}{100}, \frac{100}{100}, \frac{101}{100}$

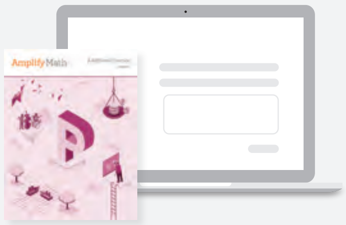
Lesson 8 What Are Percentages? 329

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 12	2
	5	Unit 1 Lesson 6	2
Formative ⚡	6	Unit 3 Lesson 9	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 3 | **LESSON 9**

Determining Percentages

Let's determine percentages in general.

Focus

Goals

1. **Language Goal:** Critique or justify statements about percentages and equivalent numerical expressions. **(Speaking and Listening)**
2. **Language Goal:** Generalize a process for determining the percentage that p is of w and justify why this can be abstracted as $\frac{p}{w} \cdot 100$. **(Speaking and Listening)**

Coherence

Today

Students determine what percentage one amount is relative to another, including percentages greater than 100%. They first rely on double number lines and ratio reasoning to determine what percentage one value is of another. They also connect fractions and percentages in equivalent ratios, recognizing that both representations describe the relationship between a part and the whole. This connection means when the part is greater than, less than, or equal to the whole, the corresponding percentage is greater than, less than, or equal to 100%. In the context of durations of time, students recognize a repeated process of division (part divided by whole) and multiplication (quotient times 100) results in determining a percentage given the part and the whole. This leads them to develop a generalized expression for determining a percentage, $\frac{p}{w} \cdot 100$.

Previously

In Lesson 8, students were introduced to percentages as a rate per 100, and they used double number lines to determine unknown amounts and percentages of different totals.

Coming Soon

In Lesson 10, students will build on previous fraction work to further explore the connection between benchmark percentages and common fractions, for example, "$\frac{3}{4}$ of" or "75% of" a number.

Rigor

- Students use double number lines to build their **conceptual understanding** of the meaning of percentages greater than 100.
- Students leverage their understanding of percentages as ratios to build **procedural skills** for determining a percentage by dividing.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- *Double Number Lines: Percentage Problems* PDF

Math Language Development

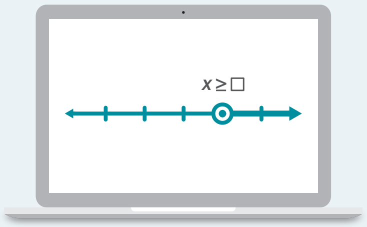
Review words

- *percent*
- *percentage*

Amps | Featured Activity

Activity 1
Interactive Double Number Lines

Students can use double number lines to plot and reason about percentages greater than and less than 100%.



Building Math Identity and Community

Connecting to Mathematical Practices

Feeling overwhelmed, students might forget to approach Activity 2 with a growth mindset. While their understanding of percentages might be a bit shaky, students should express what they need in order to solidify their understanding. Emphasize that they should ask questions that help them better understand how to be successful next time. Encourage them to look for the regular and repeated reasoning for calculating percents that will reinforce their future success.

Modifications to Pacing

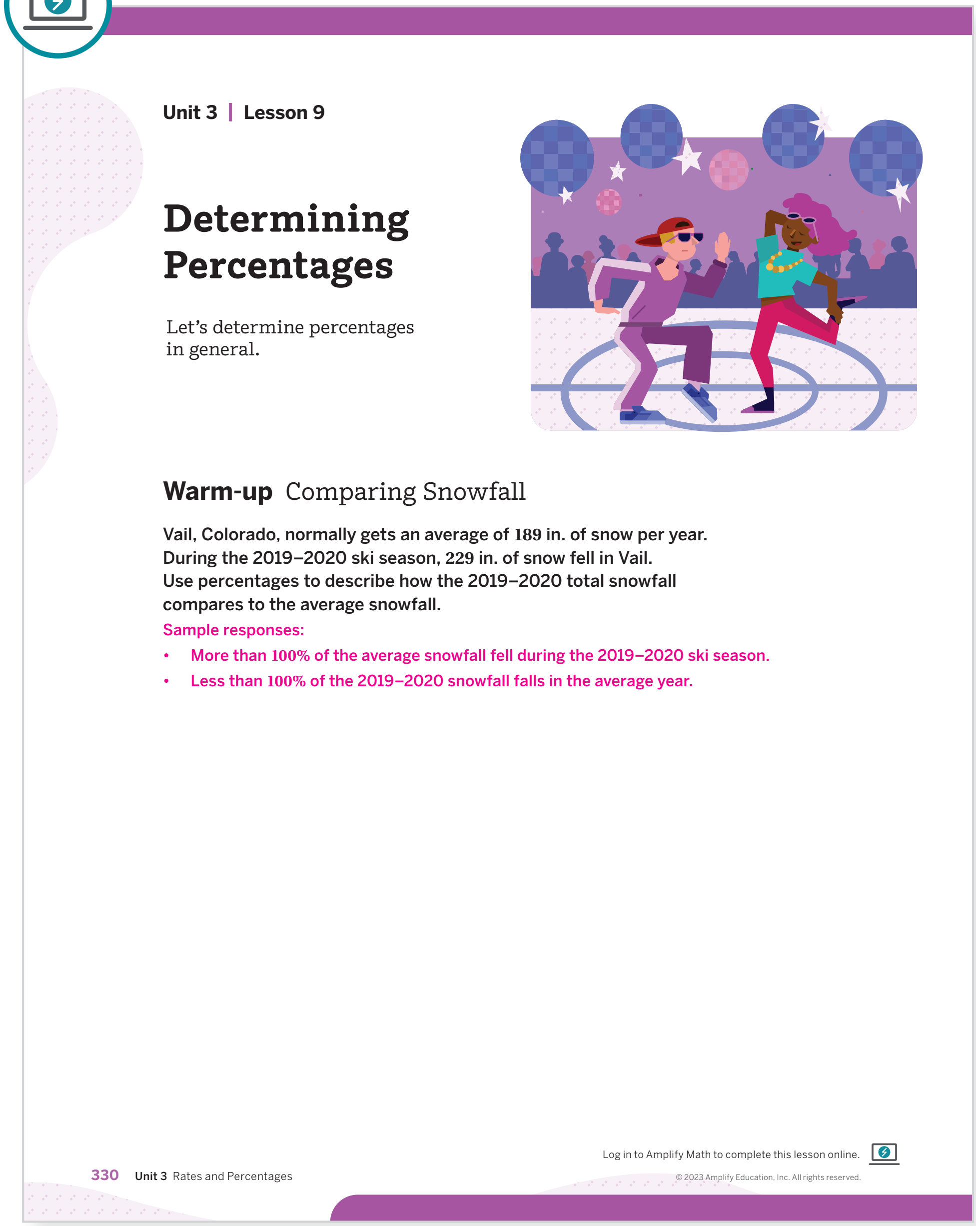
You may want to consider these additional modifications if you are short on time:

- The **Warm-up** may be omitted.
- In **Activity 1**, Problem 2 may be omitted.
- In **Activity 2**, have students complete Problem 1 with a partner. Briefly review the answers as a class. Then present Problem 2 to the class and crowdsource answers. Show Problem 3 and its answers, and ask students to connect the expressions to their work in Problem 1. Problem 4 may be omitted.

Independent | 5 min

Warm-up Comparing Snowfall

Students are introduced to percentages greater than 100% as they compare a single value to an average in the context of snowfall totals.



Unit 3 | Lesson 9

Determining Percentages

Let's determine percentages in general.

Warm-up Comparing Snowfall

Vail, Colorado, normally gets an average of 189 in. of snow per year. During the 2019–2020 ski season, 229 in. of snow fell in Vail. Use percentages to describe how the 2019–2020 total snowfall compares to the average snowfall.

Sample responses:

- More than 100% of the average snowfall fell during the 2019–2020 ski season.
- Less than 100% of the 2019–2020 snowfall falls in the average year.

Log in to Amplify Math to complete this lesson online.

330 Unit 3 Rates and Percentages

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1 Launch

Tell students that exact percentages are not needed.

2 Monitor

Help students get started by having them verbally compare the snowfall totals. Ask, "What is 100% in the problem? What would be an amount of snowfall that is greater than 100%? Less than 100%?"

Look for points of confusion:

- **Thinking the 2019–20 snowfall is less than or equal to 100% of the average snowfall.** Ask, "What does 100% of something mean? How much would it have snowed in 2019–20 if the total were 100% of the average snowfall? Less than 100%?"

Look for productive strategies:

- Recognizing the average snowfall of 189 in. is the value associated with 100% in this scenario.
- Reasoning about the comparison as either: the 2019–20 snowfall is more than 100% of the average snowfall or the average snowfall is less than 100% of the 2019–20 snowfall.

3 Connect

Have students share their responses, being sure to include those who compared using percentages greater than 100%, and those who compared using percentages less than 100%.

Ask, "If the 2018–19 snowfall total was 100% of the 2019–20 total, how much snow fell in 2018–19? What if the 2018–19 snowfall was 100% of the average snowfall total?" 229 in.; 189 in.

Highlight that comparing the two snowfall amounts using percentages is still really the same as comparing the two quantities. If Quantities A and B are equal, then A is 100% of B, and B is 100% of A. If A is greater than B, then A is greater than 100% of B, and B is less than 100% of A.

Differentiated Support

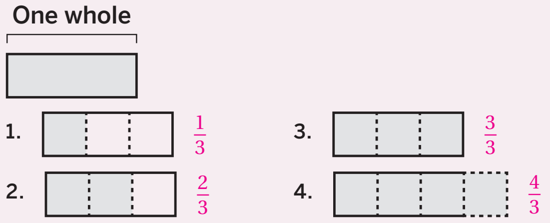
Accessibility: *Guide Processing and Visualization*

Provide the information in the introductory text visually, such as in a bar graph, table, or tape diagram where the whole represents 189 in. of snow.

Power-up

To power up students' ability to write fractions that are greater than, equal to, or less than one whole, have students complete:

Identify the fraction of the bar that is shaded.



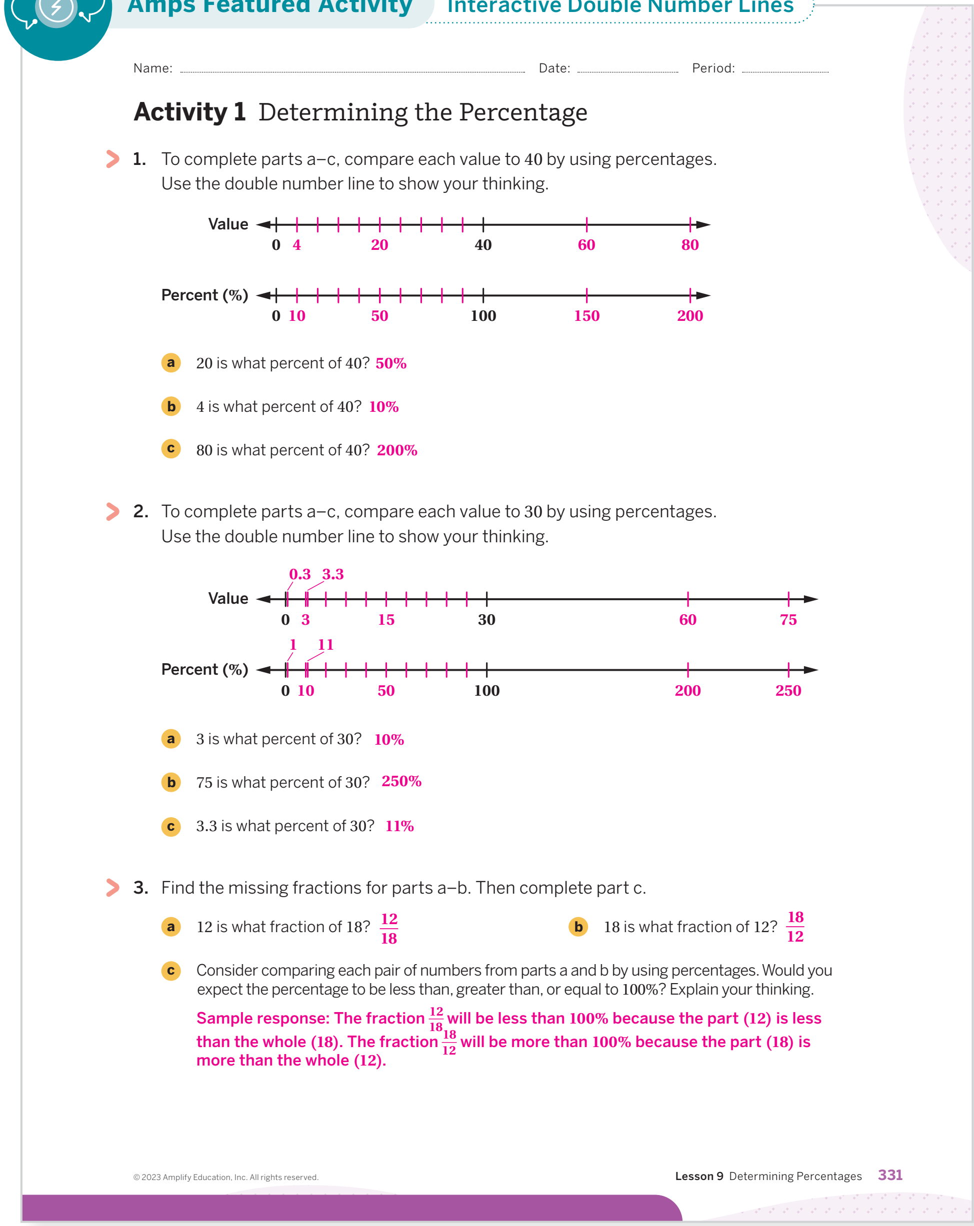
Use: Before Activity 1.
Informed by: Performance on Lesson 8, Practice Problem 6.

Pairs | 15 min

Activity 1 Determining the Percentage

Students use double number lines to determine what percent one amount is relative to another, connecting percentages to previous work with equivalent ratios and fractions.

Amps Featured Activity Interactive Double Number Lines



Name: ________ Date: ________ Period: ________

Activity 1 Determining the Percentage

1. To complete parts a–c, compare each value to 40 by using percentages. Use the double number line to show your thinking.

- a 20 is what percent of 40? 50%
- b 4 is what percent of 40? 10%
- c 80 is what percent of 40? 200%

2. To complete parts a–c, compare each value to 30 by using percentages. Use the double number line to show your thinking.

- a 3 is what percent of 30? 10%
- b 75 is what percent of 30? 250%
- c 3.3 is what percent of 30? 11%

3. Find the missing fractions for parts a–b. Then complete part c.

- a 12 is what fraction of 18? $\frac{12}{18}$
- b 18 is what fraction of 12? $\frac{18}{12}$
- c Consider comparing each pair of numbers from parts a and b by using percentages. Would you expect the percentage to be less than, greater than, or equal to 100%? Explain your thinking.

  Sample response: The fraction $\frac{12}{18}$ will be less than 100% because the part (12) is less than the whole (18). The fraction $\frac{18}{12}$ will be more than 100% because the part (18) is more than the whole (12).

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity. Have copies of the *Double Number Lines: Percentage Problems* PDF available for those who struggle to coordinate multiple responses on one diagram.

2 Monitor

Help students get started by asking, "What values do you know? Not know?"

Look for points of confusion:

- **Struggling to determine or plot values greater than 100% (e.g., Problem 1c).** Ask, "What is the whole? How do you go from 40 to 80? How does that help you locate 80 and determine its corresponding percentage on the double number line?"
- **Struggling to determine the percentage for 3.3 (Problem 2c).** Ask, "Knowing that 10% of 30 is 3, what is 5%? 1%? How can this help you?"

Look for productive strategies:

- Using division, multiplication, and ratio reasoning to plot values on the double number lines.
- Using additive reasoning with percentages for 3.3, e.g., 10% is 3, so 1% is 0.3, and because $3 + 0.3 = 3.3$, then 3.3 represents $10\% + 1\% = 11\%$.
- Recognizing that "part greater than whole" means greater than 100%, and "part less than whole" means less than 100%.

3 Connect

Have students share their responses and strategies, focusing on how they used both ratio (or multiplicative) reasoning as well as additive reasoning for Problems 1–2, and the relationship among parts, wholes, and percentages to reason about Problem 3.

Highlight that fractions also represent division — part divided by the whole. With fractions, the whole is 1, and with percentages, the whole is 100%. The fraction $\frac{12}{18}$ is less than 1, so 12 is less than 100% of 18; similarly, the fraction $\frac{18}{12}$ is greater than 1, so 18 is greater than 100% of 12.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Consider altering the values in Problem 1 so that the total is 200 and the parts are 100, 20, and 40. Provide students with a copy of the *Double Number Lines: Percentage Problems* PDF to help them organize and show their thinking.

Extension: *Math Enrichment*

Have students determine each of the following.

1. 104 is what percentage of 40? 260%
2. 81.3 is what percentage of 30? 271%

Math Language Development

MLR7: Compare and Connect

During the Connect as students share their responses and strategies, focus their attention on Problems 1c and 2b where the percents are greater than 100. Ask pairs of students to compare how they approached these problems and to make connections between how they used ratio reasoning to solve.

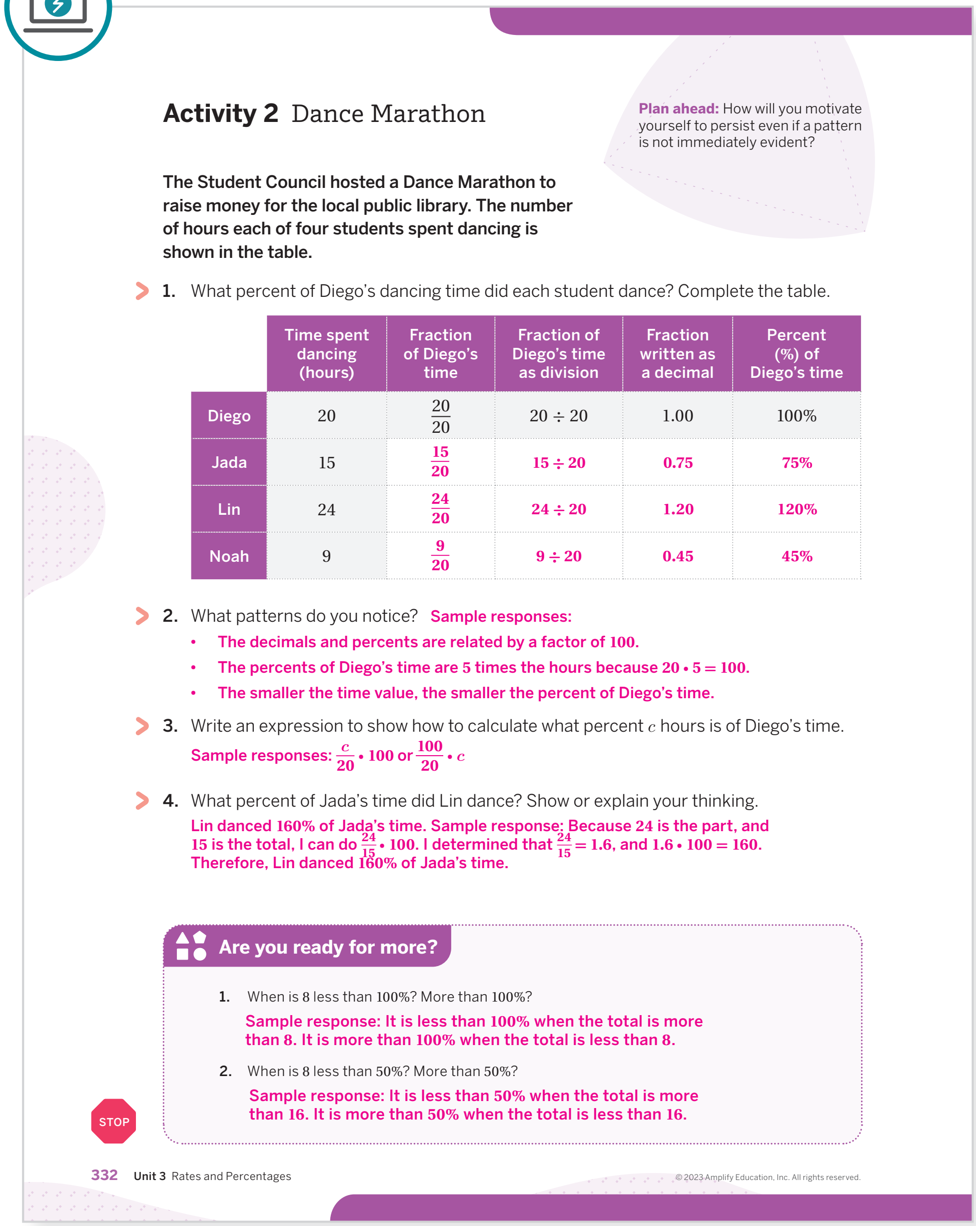
English Learners

Consider annotating the double number lines by drawing a circle around 40 and 100 in Problem 1 and 30 and 100 in Problem 2 and writing the term "whole" to help illustrate why the percents in Problems 1c and 2b are greater than one whole.

Pairs | 15 min

Activity 2 Dance Marathon

Students use repeated reasoning to develop a general expression for determining the percentage one amount is relative to another, in the context of time for a Dance Marathon.



Activity 2 Dance Marathon

Plan ahead: How will you motivate yourself to persist even if a pattern is not immediately evident?

The Student Council hosted a Dance Marathon to raise money for the local public library. The number of hours each of four students spent dancing is shown in the table.

1. What percent of Diego's dancing time did each student dance? Complete the table.

	Time spent dancing (hours)	Fraction of Diego's time	Fraction of Diego's time as division	Fraction written as a decimal	Percent (%) of Diego's time
Diego	20	$\frac{20}{20}$	20 ÷ 20	1.00	100%
Jada	15	$\frac{15}{20}$	15 ÷ 20	0.75	75%
Lin	24	$\frac{24}{20}$	24 ÷ 20	1.20	120%
Noah	9	$\frac{9}{20}$	9 ÷ 20	0.45	45%

2. What patterns do you notice? **Sample responses:**
   - **The decimals and percents are related by a factor of 100.**
   - **The percents of Diego's time are 5 times the hours because $20 \cdot 5 = 100$.**
   - **The smaller the time value, the smaller the percent of Diego's time.**

3. Write an expression to show how to calculate what percent c hours is of Diego's time.
   Sample responses: $\frac{c}{20} \cdot 100$ or $\frac{100}{20} \cdot c$

4. What percent of Jada's time did Lin dance? Show or explain your thinking.
   Lin danced 160% of Jada's time. Sample response: Because 24 is the part, and 15 is the total, I can do $\frac{24}{15} \cdot 100$. I determined that $\frac{24}{15} = 1.6$, and $1.6 \cdot 100 = 160$. Therefore, Lin danced 160% of Jada's time.

Are you ready for more?

1. When is 8 less than 100%? More than 100%?
   Sample response: It is less than 100% when the total is more than 8. It is more than 100% when the total is less than 8.
2. When is 8 less than 50%? More than 50%?
   Sample response: It is less than 50% when the total is more than 16. It is more than 50% when the total is less than 16.

STOP

332 **Unit 3** Rates and Percentages

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What information do you know? Not know?"

Look for points of confusion:

- **Incorrectly representing fractions with division or as decimals.** Ask, "Which value is being divided by the other? What denominators are related to decimals? How can you use equivalent fractions to get one of those denominators?"
- **Struggling to determine the percentages.** Point to the fractions and ask, "How can you use ratios to determine the percentages?"
- **Struggling to write an expression (Problem 3).** Have students add a row to the table for c hours as the time. Then have them complete the row.

Look for productive strategies:

- Using prior knowledge of fractions as division, equivalent fractions, and decimals.
- Using double number lines, ratio reasoning, or unit rates to determine the unknown percentages.
- Recognizing that they repeatedly divided the part by the whole (20) and multiplied by 100, and generalizing this as $\frac{p}{20} \cdot 100$.
- Recognizing that decimal values and the corresponding percentages look very similar, but the position of the decimal point is different.

3 Connect

Have students share their responses to Problem 1, focusing on how they used ratio reasoning to complete the final column of the table. Then have students share their responses for Problems 2–4.

Highlight that the expression $\frac{p}{w} \cdot 100$ can be used to determine what percent one amount (p, the part) is of another (w, the whole).

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

In Problem 1, allow students to choose one row (Jada, Lin, or Noah) to complete. Before moving on to Problem 2, display the table and ask students who completed different rows to share their responses. Then have students complete Problems 2–4.

Math Language Development

MLR7: Compare and Connect

During the Connect, highlight the connections between the fraction-decimal equivalents shown in the table and how the decimal values compare to the percent of Diego's time. Consider asking:

- "Which value is considered the whole? Why?"
- "Can you write an equivalent fraction for each student with a denominator of 100? How do the numerators compare to the percent of Diego's time?"
- "Can you draw a double number line to show how each fraction of Diego's time is related to its corresponding percent of Diego's time?"

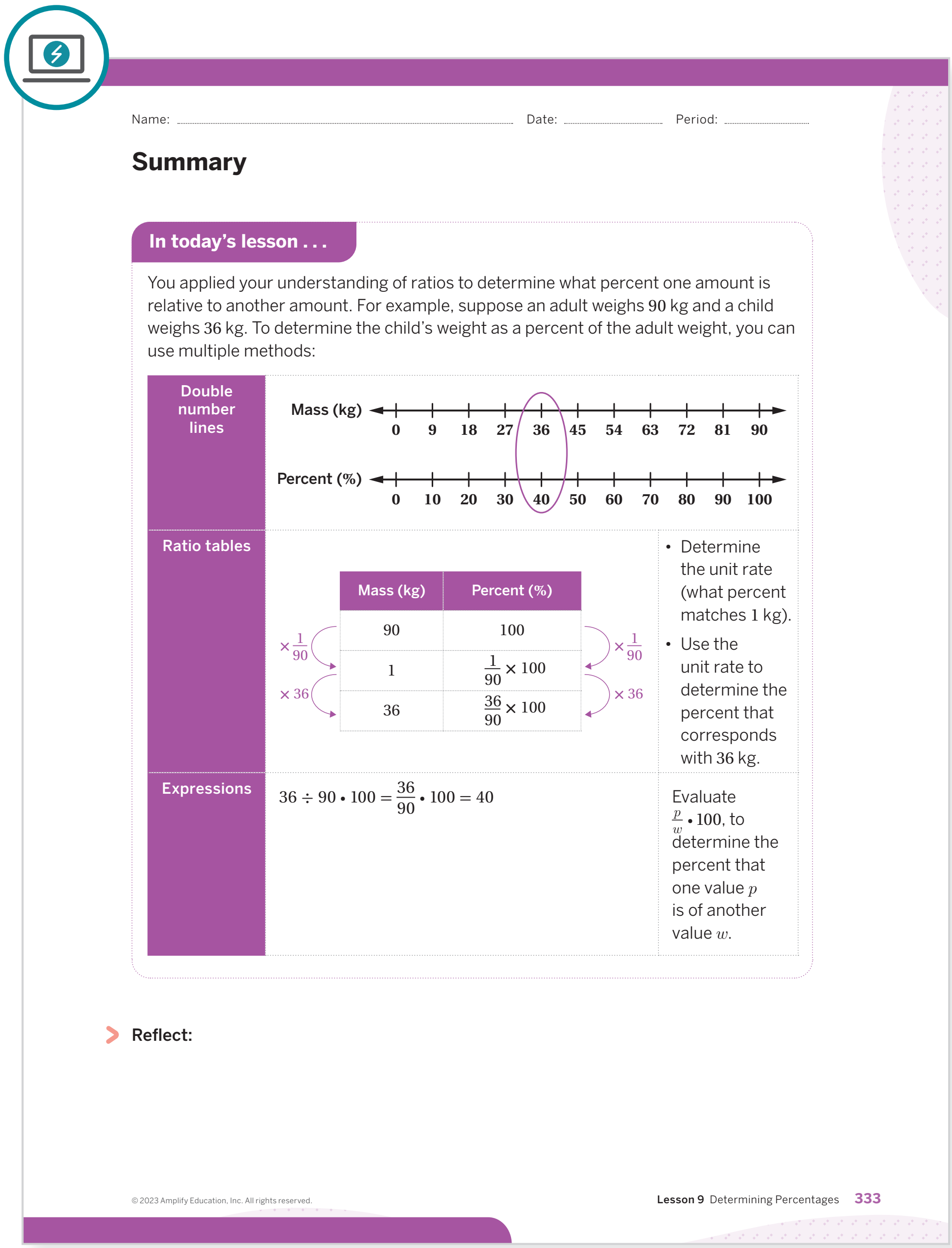
English Learners

Annotate a double number line that illustrates, for example, how $\frac{9}{20}$ corresponds with 45% of Diego's time.

Whole Class | 5 min

Summary

Review and synthesize how to use the relationship between part, whole, and percent to determine the percent one amount is in relation to another.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You applied your understanding of ratios to determine what percent one amount is relative to another amount. For example, suppose an adult weighs 90 kg and a child weighs 36 kg. To determine the child's weight as a percent of the adult weight, you can use multiple methods:

<table>
<tr><td>Double number lines</td><td colspan="2">

Mass (kg)	0	9	18	27	36	45	54	63	72	81	90
Percent (%)	0	10	20	30	40	50	60	70	80	90	100

</td></tr>
<tr><td>Ratio tables</td><td>

	Mass (kg)	Percent (%)	
$\times \frac{1}{90}$	90	100	$\times \frac{1}{90}$
$\times 36$	1	$\frac{1}{90} \times 100$	$\times 36$
	36	$\frac{36}{90} \times 100$	

</td><td>

- Determine the unit rate (what percent matches 1 kg).
- Use the unit rate to determine the percent that corresponds with 36 kg.

</td></tr>
<tr><td>Expressions</td><td>$36 \div 90 \cdot 100 = \frac{36}{90} \cdot 100 = 40$</td><td>Evaluate $\frac{p}{w} \cdot 100$, to determine the percent that one value p is of another value w.</td></tr>
</table>

Reflect:

Synthesize

Ask, "How would your process be similar and different when determining what percent 80 is of 56 versus determining what percent 56 is of 80?" In both examples, I would divide the part by the whole and multiply by 100. In the first example, the part is 80 and the whole was 56, so the percentage will be greater than 100%. These values were reversed in the second example, so the percentage will be less than 100%.

Highlight that different parts correspond to different percentages of the same whole, and the same percentage of different wholes does not correspond to the same part. For example, in Activity 1, it was shown that 10% of 30 is 3, but 10% of 40 is 4. When the part and the whole are known, the percentage of the whole that the part represents (or corresponds to) can be determined by dividing the two values and then multiplying by 100. This is true and always works because a percentage is a rate per 100.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

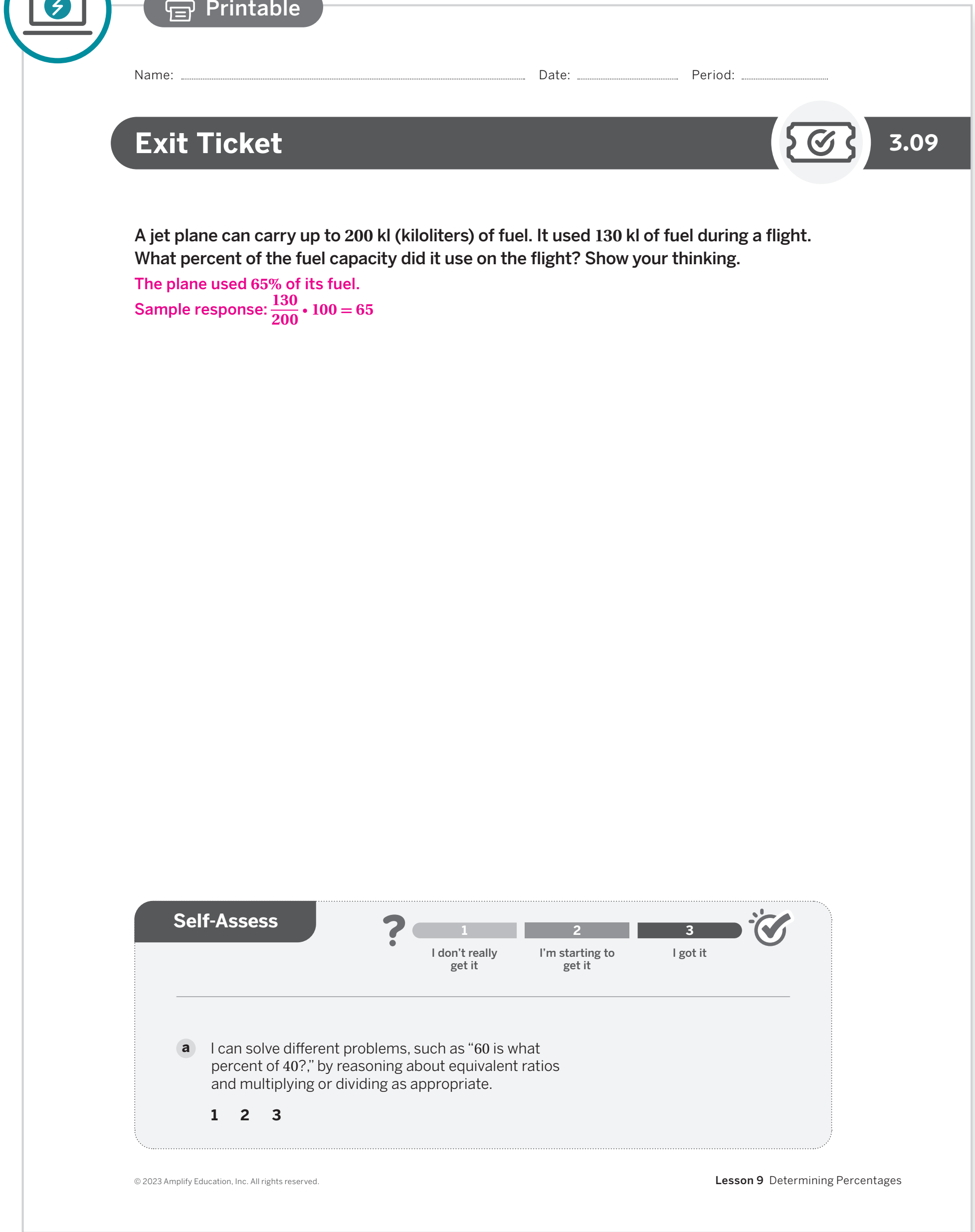
- "What does it mean for a percentage to be more than 100%? Less than 100%?"
- "How does this understanding relate to your prior work with fractions?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the percentage of the fuel capacity a jet plane used.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 3.09

A jet plane can carry up to 200 kl (kiloliters) of fuel. It used 130 kl of fuel during a flight. What percent of the fuel capacity did it use on the flight? Show your thinking.

The plane used 65% of its fuel.

Sample response: $\frac{130}{200} \cdot 100 = 65$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can solve different problems, such as "60 is what percent of 40?," by reasoning about equivalent ratios and multiplying or dividing as appropriate.

1 2 3

Lesson 9 Determining Percentages

Success looks like . . .

- **Language Goal:** Critiquing or justifying statements about percentages and equivalent numerical expressions. **(Speaking and Listening)**
- **Language Goal:** Generalizing a process for determining the percentage that p is of w and justifying why this can be abstracted as $\frac{p}{w} \cdot 100$. **(Speaking and Listening)**
  - » Solving for the percent of fuel capacity used by a plane by determining $\frac{p}{w} \cdot 100$.

Suggested next steps

If students are unable to determine the percentage, or they incorrectly use the expression $\frac{p}{w} \cdot 100$, consider:

- Reviewing Activity 1, Problem 1, and asking, "How can you use a double number line to help you think about the problem? What value is the whole? The part?"
- Reviewing Activity 2, Problems 3 and 4, and asking, "Where in the expression is the part? Where is the whole? How can that help you here?"

Professional Learning

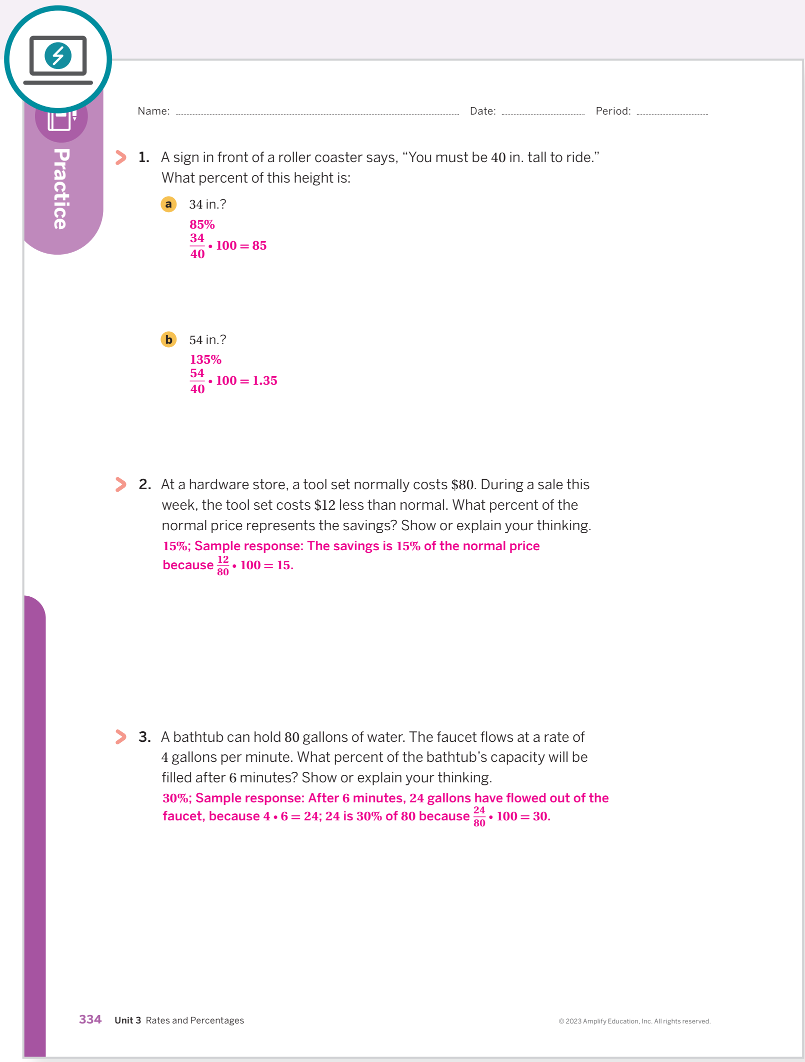
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How was the use of double number lines in Activity 1 similar to or different from their use in Lesson 8, Activity 2? How did students use the similarities to work efficiently?
- Think about the questions you asked students today and what the students said or did as a result of the questions. Which question was the most effective? What made it so effective? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. A sign in front of a roller coaster says, “You must be 40 in. tall to ride.” What percent of this height is:

   a 34 in.?

   85%

   $\frac{34}{40} \cdot 100 = 85$

   b 54 in.?

   135%

   $\frac{54}{40} \cdot 100 = 1.35$

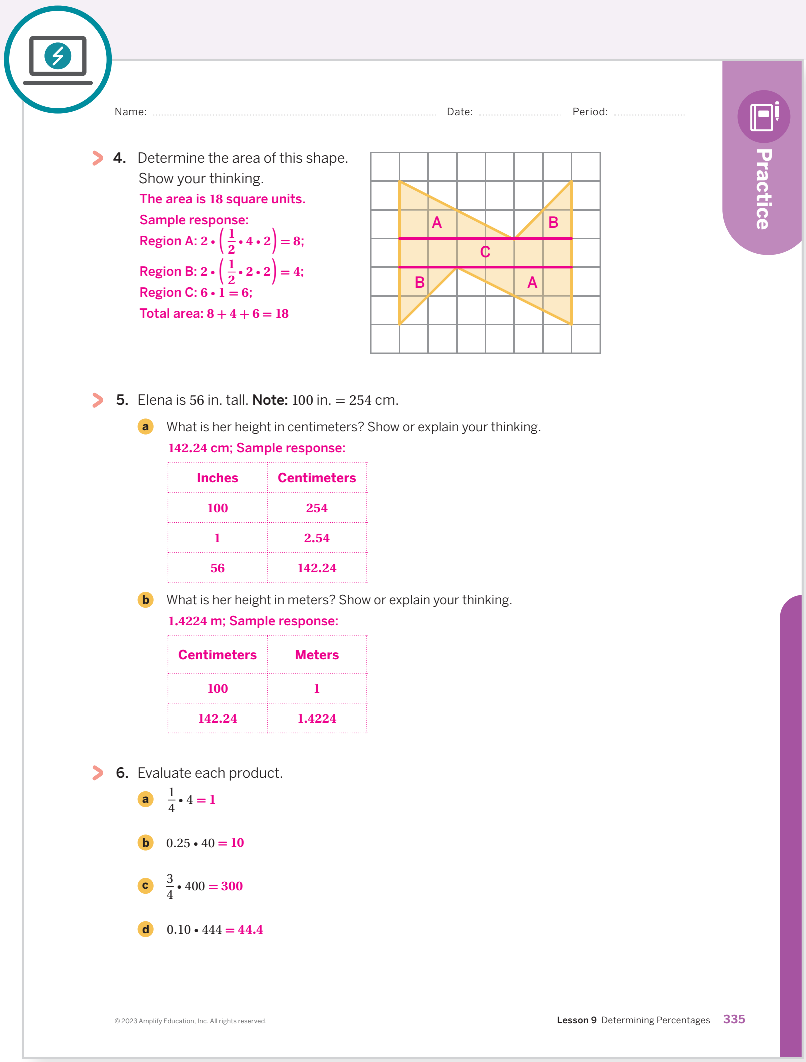
2. At a hardware store, a tool set normally costs $80. During a sale this week, the tool set costs $12 less than normal. What percent of the normal price represents the savings? Show or explain your thinking.

   15%; Sample response: The savings is 15% of the normal price because $\frac{12}{80} \cdot 100 = 15$.

3. A bathtub can hold 80 gallons of water. The faucet flows at a rate of 4 gallons per minute. What percent of the bathtub’s capacity will be filled after 6 minutes? Show or explain your thinking.

   30%; Sample response: After 6 minutes, 24 gallons have flowed out of the faucet, because $4 \cdot 6 = 24$; 24 is 30% of 80 because $\frac{24}{80} \cdot 100 = 30$.

334 **Unit 3** Rates and Percentages



Name: ______ Date: ______ Period: ______

4. Determine the area of this shape. Show your thinking.

   The area is 18 square units.

   Sample response:

   Region A: $2 \cdot \left(\frac{1}{2} \cdot 4 \cdot 2\right) = 8$;

   Region B: $2 \cdot \left(\frac{1}{2} \cdot 2 \cdot 2\right) = 4$;

   Region C: $6 \cdot 1 = 6$;

   Total area: $8 + 4 + 6 = 18$

5. Elena is 56 in. tall. **Note:** 100 in. = 254 cm.

   a What is her height in centimeters? Show or explain your thinking.

   142.24 cm; Sample response:

Inches	Centimeters
100	254
1	2.54
56	142.24

   b What is her height in meters? Show or explain your thinking.

   1.4224 m; Sample response:

Centimeters	Meters
100	1
142.24	1.4224

6. Evaluate each product.

   a $\frac{1}{4} \cdot 4 = 1$

   b $0.25 \cdot 40 = 10$

   c $\frac{3}{4} \cdot 400 = 300$

   d $0.10 \cdot 444 = 44.4$

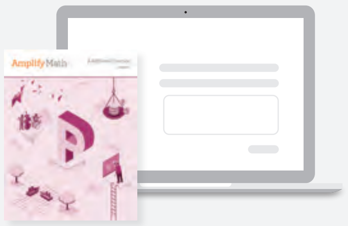
Lesson 9 Determining Percentages 335

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1–2	2
	2	Activities 1–2	2
	3	Activities 1–2	2
Spiral	4	Unit 1 Lesson 5	2
	5	Unit 2 Lesson 19	2
Formative ⚡	6	Unit 3 Lesson 10	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 3 | **LESSON 10**

Benchmark Percentages

Let's use fractions to make sense of some common percentages.

Focus

Goals

1. **Language Goal:** Explain how to solve problems involving the percentages 10%, 25%, 50%, or 75% by reasoning about the fractions $\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$. **(Speaking and Listening, Writing)**
2. **Language Goal:** Generalize processes for calculating 10%, 25%, 50%, and 75% of a quantity. **(Speaking and Listening)**

Coherence

Today

Students connect benchmark percentages (multiples of 1% and 5%) to common fractions. They begin by using ratio and multiplicative reasoning to evaluate benchmark percentages that correspond to unit fractions. This leads to generalizations, such as 50% of a quantity being equivalent to $\frac{1}{2}$ of that quantity. The use of the word *of* explicitly reinforces that percentages are ratios (rates per 100) and not numbers, but that corresponding fractions and percentages are equivalent operators (or factors). Students also work with multiples of benchmark percentages, including some percentages greater than 100% and less than 1%. This gives students further connections to multiplication, division, equivalent ratios, and also addition of fractions — recognizing percentages of the same whole are also additive. By the end of this lesson, students should be able to identify and to use several fraction operators for determining benchmark percentages of a number.

Previously

In Lessons 8–9, students developed an understanding of percentages as rates per 100 and used double number lines to represent percentages.

Coming Soon

In Lessons 11–12, students will generalize processes for determining an unknown part or an unknown whole in a percentage problem.

Rigor

- Students leverage their understanding of fraction multiplication to build **procedural skills** for determining the benchmark percentage of a number.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (answers)
- *Double Number Lines: Percentage Problems* PDF (as needed)

Math Language Development

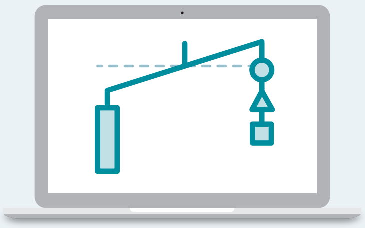
Review words

- *percent*
- *percentage*

Amps | Featured Activity

Activity 2
Real-Time Feedback

Students can check their percentage calculations as they work their way though problems.



Building Math Identity and Community

Connecting to Mathematical Practices

As students share their responses with a partner, they may forget to actively listen, and thus might not be able to precisely communicate during discussions. Remind students that by listening well, they can help improve their own understanding and their own level of precision, as they communicate their thoughts. Review what it means to actively listen and encourage students to practice active listening habits.

Modifications to Pacing

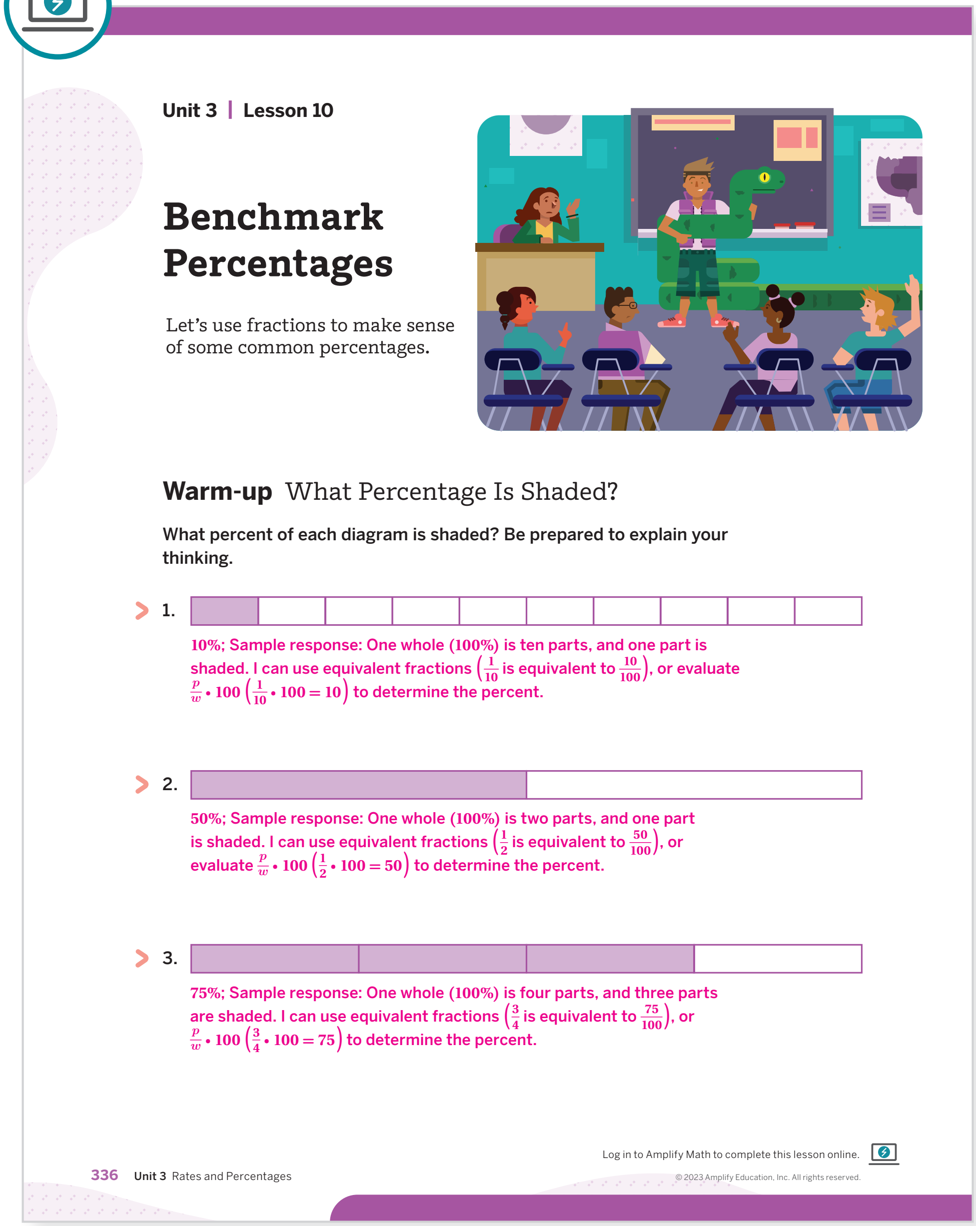
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 4 may be omitted.
- In **Activity 2**, assign pairs of students 1–3 animals to focus on for Problems 1 and 2. Have pairs of students share their responses and strategies with the class.

Independent | 5 min

Warm-up What Percent Is Shaded?

Students identify percentages represented by shaded tape diagrams, introducing them to the relationship between benchmark percentages and common fractions.



Unit 3 | Lesson 10

Benchmark Percentages

Let's use fractions to make sense of some common percentages.

Warm-up What Percentage Is Shaded?

What percent of each diagram is shaded? Be prepared to explain your thinking.

1. 10%; Sample response: One whole (100%) is ten parts, and one part is shaded. I can use equivalent fractions $\left(\frac{1}{10}\right.$ is equivalent to $\left.\frac{10}{100}\right)$, or evaluate $\frac{p}{w} \cdot 100$ $\left(\frac{1}{10} \cdot 100 = 10\right)$ to determine the percent.

2. 50%; Sample response: One whole (100%) is two parts, and one part is shaded. I can use equivalent fractions $\left(\frac{1}{2}\right.$ is equivalent to $\left.\frac{50}{100}\right)$, or evaluate $\frac{p}{w} \cdot 100$ $\left(\frac{1}{2} \cdot 100 = 50\right)$ to determine the percent.

3. 75%; Sample response: One whole (100%) is four parts, and three parts are shaded. I can use equivalent fractions $\left(\frac{3}{4}\right.$ is equivalent to $\left.\frac{75}{100}\right)$, or $\frac{p}{w} \cdot 100$ $\left(\frac{3}{4} \cdot 100 = 75\right)$ to determine the percent.

Log in to Amplify Math to complete this lesson online.

336 **Unit 3** Rates and Percentages

1 Launch

Read the directions to the class, emphasizing that their responses should be *percents* for the shaded parts, where the entire "bar" is the whole.

2 Monitor

Help students get started by activating prior knowledge. Ask, "What fraction of the whole is shaded? How else can you express that amount?"

Look for points of confusion:

- **Not seeing the total number of parts as the whole that corresponds to 100%.** Ask, "If I shaded the whole, or 100% of the bar, how many parts would I shade? What does that tell you about x parts and 100%?"

Look for productive strategies:

- Recognizing that while the three wholes are the same size and each represents 100%, they are divided into different numbers of parts, so the wholes that correspond to 100% could be thought of as 10, 2, or 4.
- Determining the percent by using equivalent fractions, the expression $\frac{p}{w} \cdot 100$, or equivalent ratios (such as on double number lines).

3 Connect

Have students share their responses and strategies, one diagram at a time, focusing on how they used ratio reasoning or $\frac{p}{w} \cdot 100$ to determine the percent.

Ask, "How can you use fractions to name the size of the shaded portions in each diagram?" $\frac{1}{10}, \frac{1}{2}, \frac{3}{4}$

Highlight that each shaded portion can be named as a percent of the whole or as a fraction of the whole. For example, Problem 3 represents $\frac{3}{4}$ of the whole or 75% of the whole.
Note: Students will extend these connections to expressions and calculations in Activity 1.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies, draw connections between ratio reasoning and the expression $\frac{p}{w} \cdot 100$, where p represents the part and w represents the whole. Consider labeling the top increments of each diagram with the parts of the whole, as fractions. Then label the bottom increments of each diagram with the corresponding percentages of each whole.

English Learners

Annotate the diagrams to show how the *whole*, *part*, and *percent* are represented.

Power-up

To power up students' ability to multiply whole numbers by benchmark fractions and decimals, have students complete:

Recall that when multiplying fractions the numerators are multiplied and the denominators are multiplied. Calculate the products:

1. $\frac{25}{100} \cdot \frac{4}{1} = \frac{100}{100}$ or 1
2. $\frac{2}{10} \cdot \frac{3}{1} = \frac{6}{10}$ or $\frac{3}{5}$
3. $\frac{8}{10} \cdot \frac{5}{1} = \frac{40}{10}$ or 4

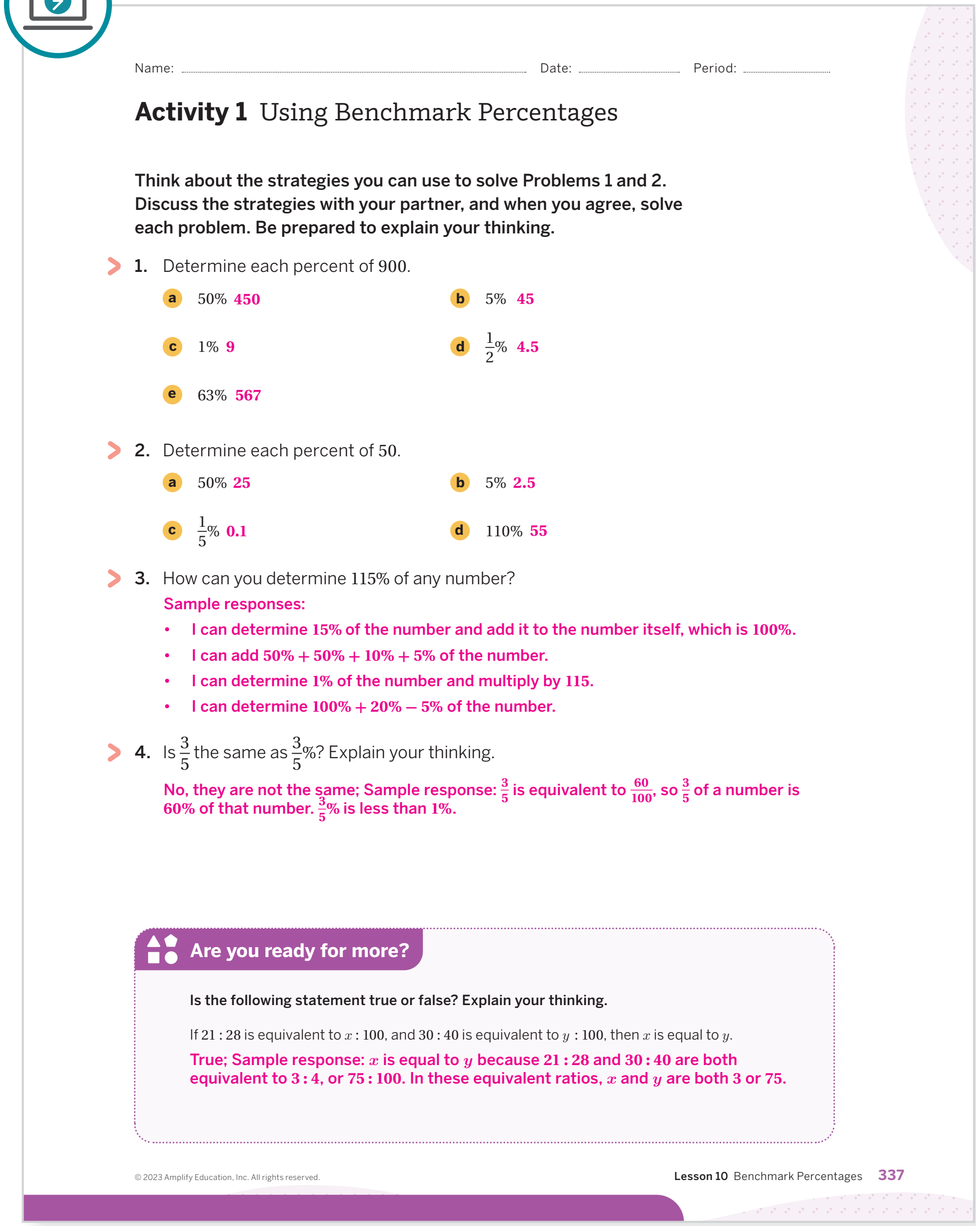
Use: Before Activity 1.

Informed by: Performance on Lesson 9, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3.

Pairs | 15 min

Activity 1 Using Benchmark Percentages

Students determine various benchmark percents of 900 and 50 by relating percentages to equivalent ratios, coming to recognize that percentages are additive.



Name: Date: Period:

Activity 1 Using Benchmark Percentages

Think about the strategies you can use to solve Problems 1 and 2. Discuss the strategies with your partner, and when you agree, solve each problem. Be prepared to explain your thinking.

1. Determine each percent of 900.
   - a 50% **450**
   - b 5% **45**
   - c 1% **9**
   - d $\frac{1}{2}$% **4.5**
   - e 63% **567**
2. Determine each percent of 50.
   - a 50% **25**
   - b 5% **2.5**
   - c $\frac{1}{5}$% **0.1**
   - d 110% **55**
3. How can you determine 115% of any number?

   Sample responses:
   - **I can determine 15% of the number and add it to the number itself, which is 100%.**
   - **I can add 50% + 50% + 10% + 5% of the number.**
   - **I can determine 1% of the number and multiply by 115.**
   - **I can determine 100% + 20% – 5% of the number.**
4. Is $\frac{3}{5}$ the same as $\frac{3}{5}$%? Explain your thinking.

   No, they are not the same; Sample response: $\frac{3}{5}$ is equivalent to $\frac{60}{100}$, so $\frac{3}{5}$ of a number is 60% of that number. $\frac{3}{5}$% is less than 1%.

Are you ready for more?

Is the following statement true or false? Explain your thinking.

If 21 : 28 is equivalent to x : 100, and 30 : 40 is equivalent to y : 100, then x is equal to y.

True; Sample response: x is equal to y because 21 : 28 and 30 : 40 are both equivalent to 3 : 4, or 75 : 100. In these equivalent ratios, x and y are both 3 or 75.

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Lesson 10 Benchmark Percentages 337

1 Launch

Have students use the ***Think-Pair-Share*** routine. Give them 1 minute to think about their strategies. Then have them share with a partner and complete the activity.

2 Monitor

Help students get started by having them show 50% of 900 on a double number line.

Look for points of confusion:

- **Struggling to reason about fractional percentages.** Ask, "How can you use 1% of the number to help you?"

Look for productive strategies:

- Using equivalent ratios or double number lines to evaluate each problem independently of the others. Consider asking, "Can you mentally compute 50% of a number? What about 5%?"
- Using previous problems to solve new ones by applying ratio, additive, or multiplicative reasoning (e.g., if 50% of 900 is 450, then because 50 ÷ 10 = 5, 5% of 900 is equal to 450 ÷ 10).

3 Connect

Have students share how they used ratio, multiplicative, or additive reasoning to complete Problems 1–2. Then have them share their responses to Problems 3–4.

Ask, "Why are 50% of 900 and 50% of 50 not equal to the same value?"

Highlight that, because percentages are equivalent ratios, a part and a whole can be related to a corresponding percentage by multiplication and division. This also means that different parts of the same whole are related in the same way as the corresponding percentages. For example, since 50 is 5 times 10, the part corresponding to 50% will be 5 times the part corresponding to 10%.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

Provide access to copies of the *Double Number Lines: Percentage Problems* PDF for students to use for Problems 1 and 2.

Accessibility: *Vary Demands to Optimize Challenge*

Consider changing the whole in Problem 1 to 100 and have students respond to parts a–e using this new whole. Before beginning Problem 2, ask students to predict how their responses to Problems 1a–b will compare to their responses for Problems 2a–b.

Math Language Development

MLR1: Stronger and Clearer Each Time

Before the Connect, as time allows, have students share their responses to Problems 3 and 4 with another pair of students. Ask partners to review responses and provide feedback that can be used to improve the clarity of the responses. Consider displaying these sample questions:

- "How do you know your method in Problem 3 will work for any number?"
- "Does the response to Problem 4 include more explanation than just the percent symbol being removed?"

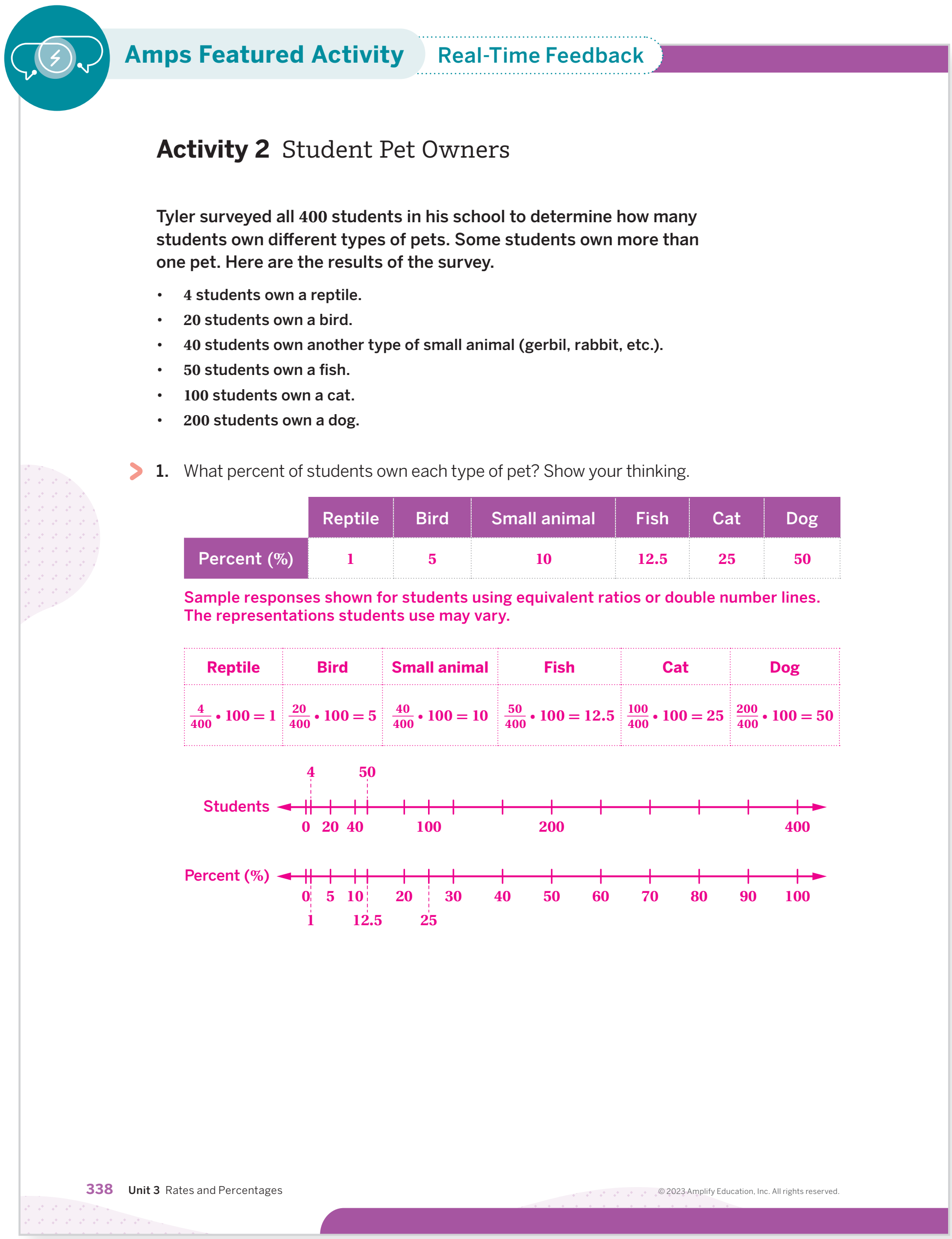
Have students revise their responses to Problems 3 and 4 after receiving feedback.

Pairs | 15 min

Activity 2 Student Pet Owners

Students solve percentage problems in the context of student pet ownership, and through repeated reasoning they connect benchmark percentages to common fractions.

Amps Featured Activity Real-Time Feedback



Activity 2 Student Pet Owners

Tyler surveyed all 400 students in his school to determine how many students own different types of pets. Some students own more than one pet. Here are the results of the survey.

- **4 students own a reptile.**
- **20 students own a bird.**
- **40 students own another type of small animal (gerbil, rabbit, etc.).**
- **50 students own a fish.**
- **100 students own a cat.**
- **200 students own a dog.**

1. What percent of students own each type of pet? Show your thinking.

	Reptile	Bird	Small animal	Fish	Cat	Dog
Percent (%)	1	5	10	12.5	25	50

Sample responses shown for students using equivalent ratios or double number lines. The representations students use may vary.

Reptile	Bird	Small animal	Fish	Cat	Dog
$\frac{4}{400} \cdot 100 = 1$	$\frac{20}{400} \cdot 100 = 5$	$\frac{40}{400} \cdot 100 = 10$	$\frac{50}{400} \cdot 100 = 12.5$	$\frac{100}{400} \cdot 100 = 25$	$\frac{200}{400} \cdot 100 = 50$

338 Unit 3 Rates and Percentages

1 Launch

Note the term *representative* in Problem 2 and ensure students have an understanding of its meaning here.

2 Monitor

Help students get started by asking, "What information do you know? What information do you need?"

Look for points of confusion:

- **Not recognizing that the whole is 400.** Ask, "How many students were polled?"
- **Not understanding a decimal percentage (fish).** Ask, "If 4 represents 1%, what would 2 represent?"
- **Using $\frac{p}{w} \cdot 100$ to determine the number of students who own each type of pet (Problem 2).** Ask, "What does $\frac{p}{w} \cdot 100$ help you determine? Is that the information you need in this problem?"

Look for productive strategies:

- Using double number lines, equivalent ratios, or $\frac{p}{w} \cdot 100$ to determine percentages.
- Using ratio, additive, and multiplicative reasoning to determine how many students own each animal.
- Rounding 137.5 fish owners to 137 or 138 because 137.5 makes mathematical, but not contextual, sense.

Activity 2 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students create a table, such as the following, to help them determine the percent of students who own each type of pet in Problem 1. Consider providing a blank pre-created table instead of having students create their own.

Pet	Number of students	Fraction of the total number of students	Decimal equivalent	Percent of the total number of students

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect statement for Problem 1, such as "200% of students own a dog because $\frac{200}{100} \cdot 100 = 200$." Ask:

Critique: "Why is this statement incorrect?" Listen for those who recognize the whole is 400 and for those who reason that 200% does not make sense in this context.

Correct and Clarify: Have students write a corrected statement. Then have them explain how they know their statement is correct.

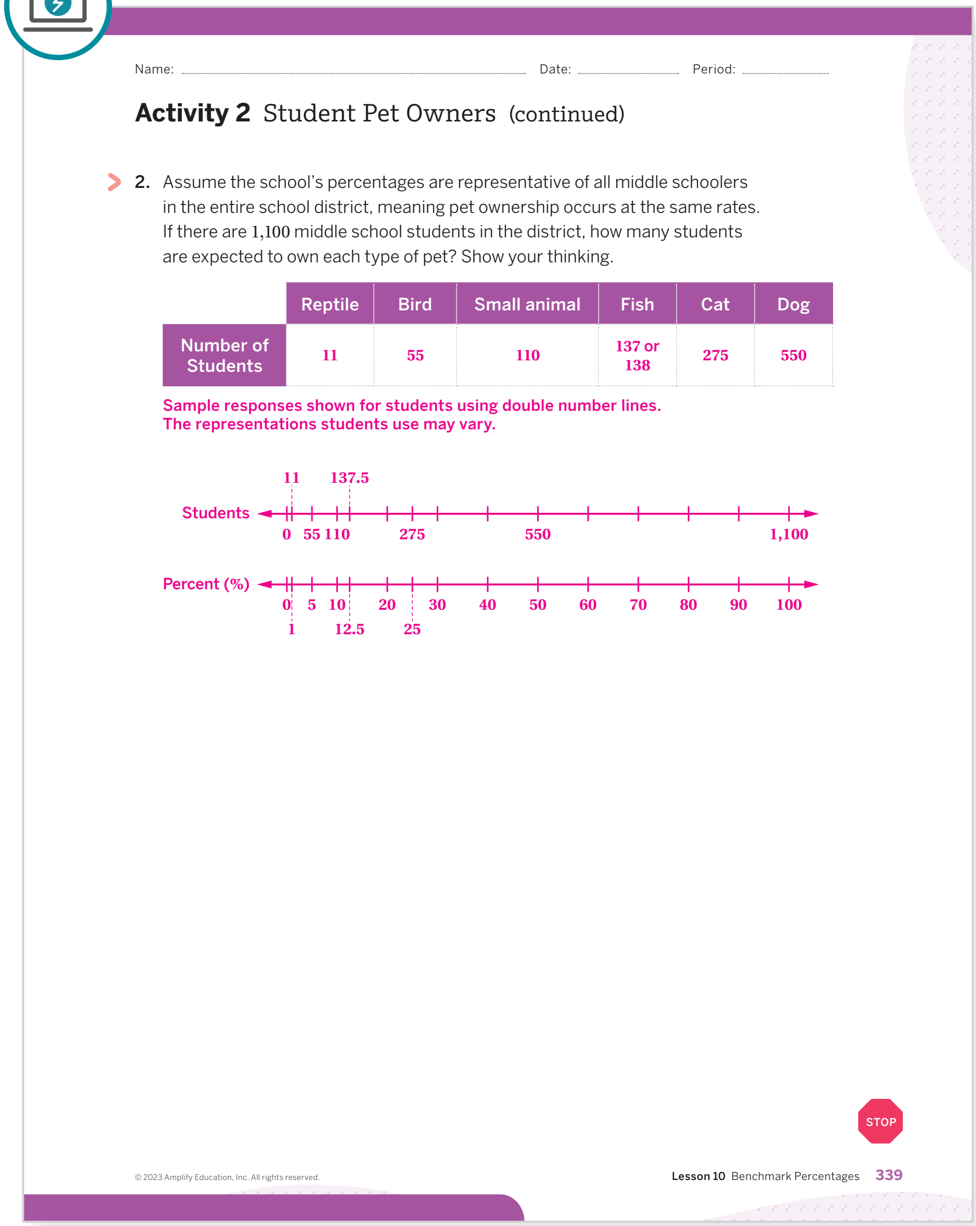
English Learners

Annotate the given value of 400 in the problem as the *whole*.

Pairs | 15 min

Activity 2 Student Pet Owners (continued)

Students solve percentage problems in the context of student pet ownership, and through repeated reasoning they connect benchmark percentages to common fractions.



Name: ______ Date: ______ Period: ______

Activity 2 Student Pet Owners (continued)

2. Assume the school's percentages are representative of all middle schoolers in the entire school district, meaning pet ownership occurs at the same rates. If there are 1,100 middle school students in the district, how many students are expected to own each type of pet? Show your thinking.

	Reptile	Bird	Small animal	Fish	Cat	Dog
Number of Students	11	55	110	137 or 138	275	550

Sample responses shown for students using double number lines. The representations students use may vary.

STOP

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Lesson 10 Benchmark Percentages 339

3 Connect

Have students share their responses and strategies, focusing on how their strategies differed between Problems 1 and 2.

Display the Activity 2 PDF (answers).

Ask:

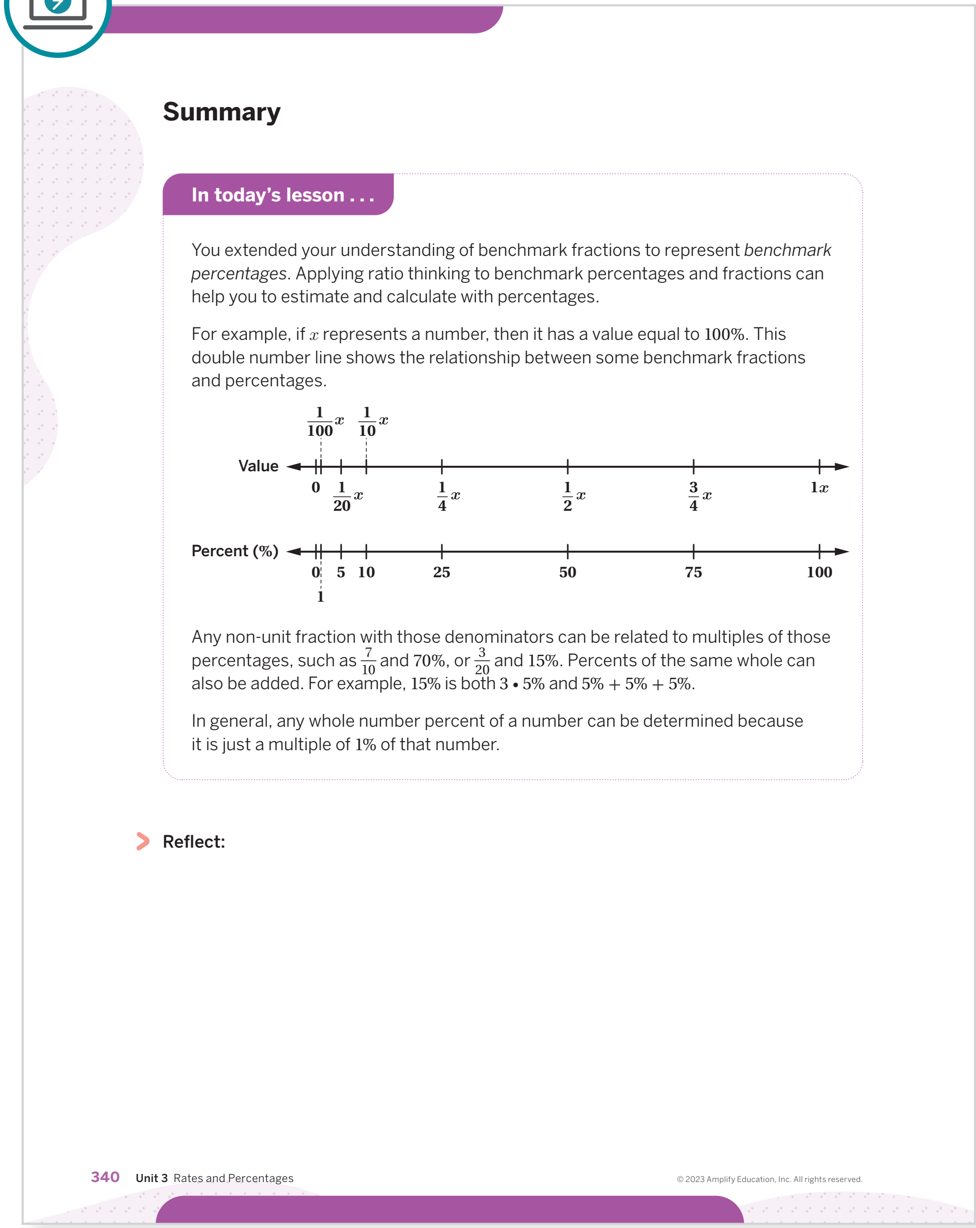
- "How is your thinking represented in this table?" Sample response: Instead of using $\frac{p}{w} \cdot 100$, the table converts the $\frac{p}{w}$ to an equivalent fraction with a denominator of 100. The percentage is the numerator of the equivalent fraction, or the numerator times 100, as in the algorithm.
- "Which fractions in the second row can be simplified further?" $\frac{5}{100}=\frac{1}{20}$, $\frac{10}{100}=\frac{1}{10}$, $\frac{25}{100}=\frac{1}{4}$, and $\frac{50}{100}=\frac{1}{2}$

Highlight that students can use the relationship between benchmark fractions and their corresponding benchmark percentages to efficiently determine what percent represents a given part out of a whole. Specifically, if they recognize that the part divided by the whole is $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{10}$ they know the corresponding percentage is 50%, 25%, or 10% respectively.

Whole Class | 5 min

Summary

Review and synthesize the relationship between benchmark percentages and common fractions, and how that relationship can be used to solve percentage problems.



Summary

In today's lesson . . .

You extended your understanding of benchmark fractions to represent *benchmark percentages*. Applying ratio thinking to benchmark percentages and fractions can help you to estimate and calculate with percentages.

For example, if x represents a number, then it has a value equal to 100%. This double number line shows the relationship between some benchmark fractions and percentages.

Any non-unit fraction with those denominators can be related to multiples of those percentages, such as $\frac{7}{10}$ and 70%, or $\frac{3}{20}$ and 15%. Percents of the same whole can also be added. For example, 15% is both $3 \cdot 5\%$ and $5\% + 5\% + 5\%$.

In general, any whole number percent of a number can be determined because it is just a multiple of 1% of that number.

Reflect:

Synthesize

Display the table in the Student Summary.

Highlight that:

- 10% of a number will always be equal to $\frac{1}{10}$ of that number.
- 25% of a number will always be equal to $\frac{1}{4}$ of that number.
- 50% of a number will always be equal to $\frac{1}{2}$ of that number.

Benchmark percentages and their corresponding values (the parts they represent relative to a given whole) can be added, subtracted, multiplied, and divided to determine other percentages of the same whole.

Ask, "How can you use the fact that 25% of a number is equivalent to $\frac{1}{4}$ of that number to determine 75% of a number?" 75% of a number is equivalent to $\frac{3}{4}$ of that number because $75 = 25 \cdot 3$, and $\frac{1}{4} \cdot 3 = \frac{3}{4}$.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

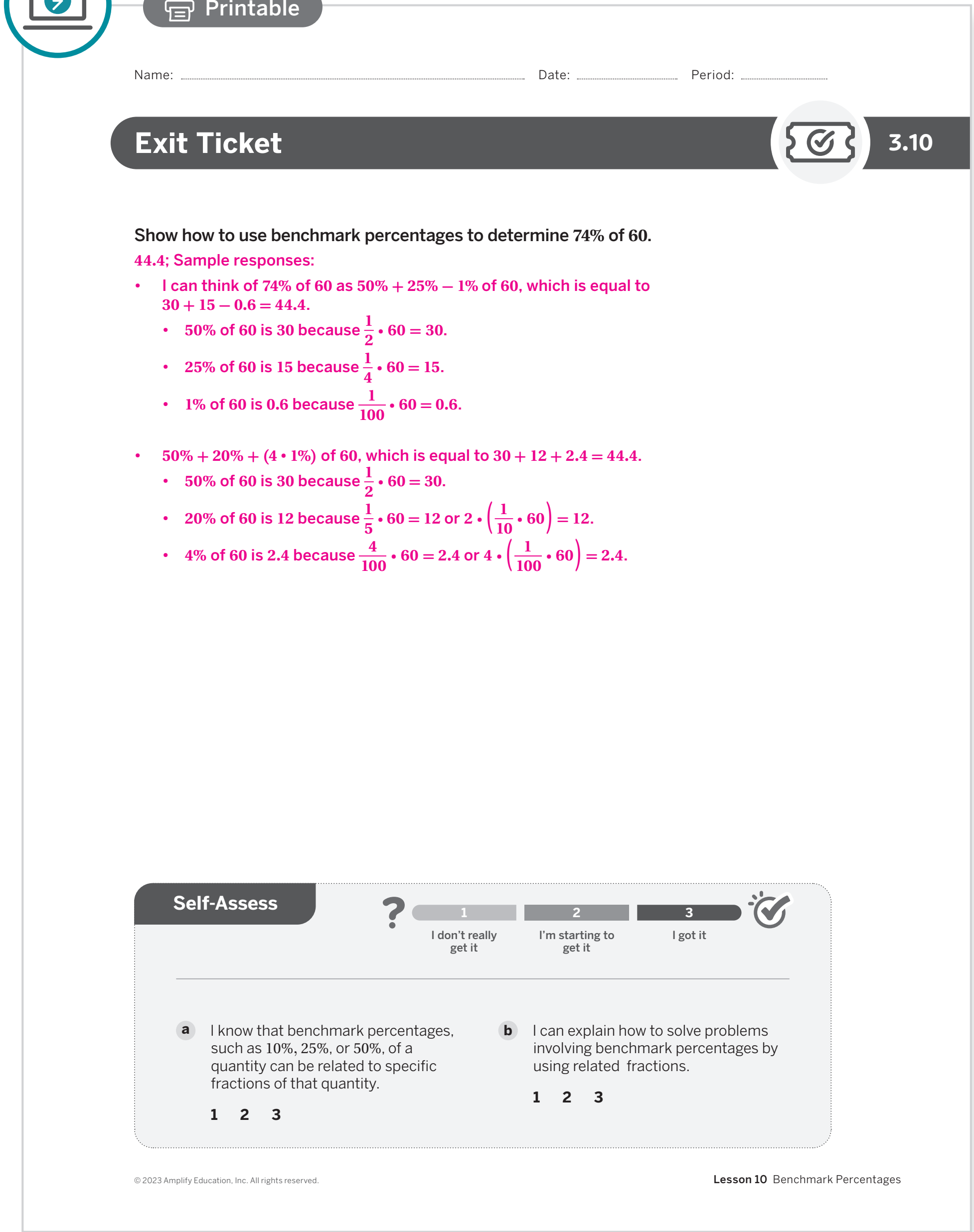
- "How are fractions and percentages similar? How are they different?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using benchmark fractions to determine 74% of 60.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket

3.10

Show how to use benchmark percentages to determine 74% of 60.

44.4; Sample responses:

- I can think of 74% of 60 as $50\% + 25\% - 1\%$ of 60, which is equal to $30 + 15 - 0.6 = 44.4$.
  - 50% of 60 is 30 because $\frac{1}{2} \cdot 60 = 30$.
  - 25% of 60 is 15 because $\frac{1}{4} \cdot 60 = 15$.
  - 1% of 60 is 0.6 because $\frac{1}{100} \cdot 60 = 0.6$.
- $50\% + 20\% + (4 \cdot 1\%)$ of 60, which is equal to $30 + 12 + 2.4 = 44.4$.
  - 50% of 60 is 30 because $\frac{1}{2} \cdot 60 = 30$.
  - 20% of 60 is 12 because $\frac{1}{5} \cdot 60 = 12$ or $2 \cdot \left(\frac{1}{10} \cdot 60\right) = 12$.
  - 4% of 60 is 2.4 because $\frac{4}{100} \cdot 60 = 2.4$ or $4 \cdot \left(\frac{1}{100} \cdot 60\right) = 2.4$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I know that benchmark percentages, such as 10%, 25%, or 50%, of a quantity can be related to specific fractions of that quantity.

1 2 3

b I can explain how to solve problems involving benchmark percentages by using related fractions.

1 2 3

Lesson 10 Benchmark Percentages

Success looks like . . .

- **Language Goal:** Explaining how to solve problems involving the percentages 10%, 25%, 50%, and 75% by reasoning about the fractions $\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$. **(Speaking and Listening, Writing)**
  - » Explaining how to determine 74% of 60 using benchmark percentages.
- **Language Goal:** Generalizing processes for calculating 10%, 25%, 50%, and 75% of a quantity. **(Speaking and Listening)**

Suggested next steps

If students do not know which benchmark percentages to use, or how to use fractions related to benchmark percentages, consider:

- Reviewing Activity 1, Problems 1e, 2d, and 3. Ask, "What benchmark percentages used in Activity 1 could you use here? How can you think about those benchmark percentages as fractions?"

If students attempt to solve by multiplying $\frac{74}{100} \cdot 60$, consider:

- Asking, "Can you achieve the same result using benchmark fractions?"
- Reviewing Activity 1, Problem 1.

Professional Learning

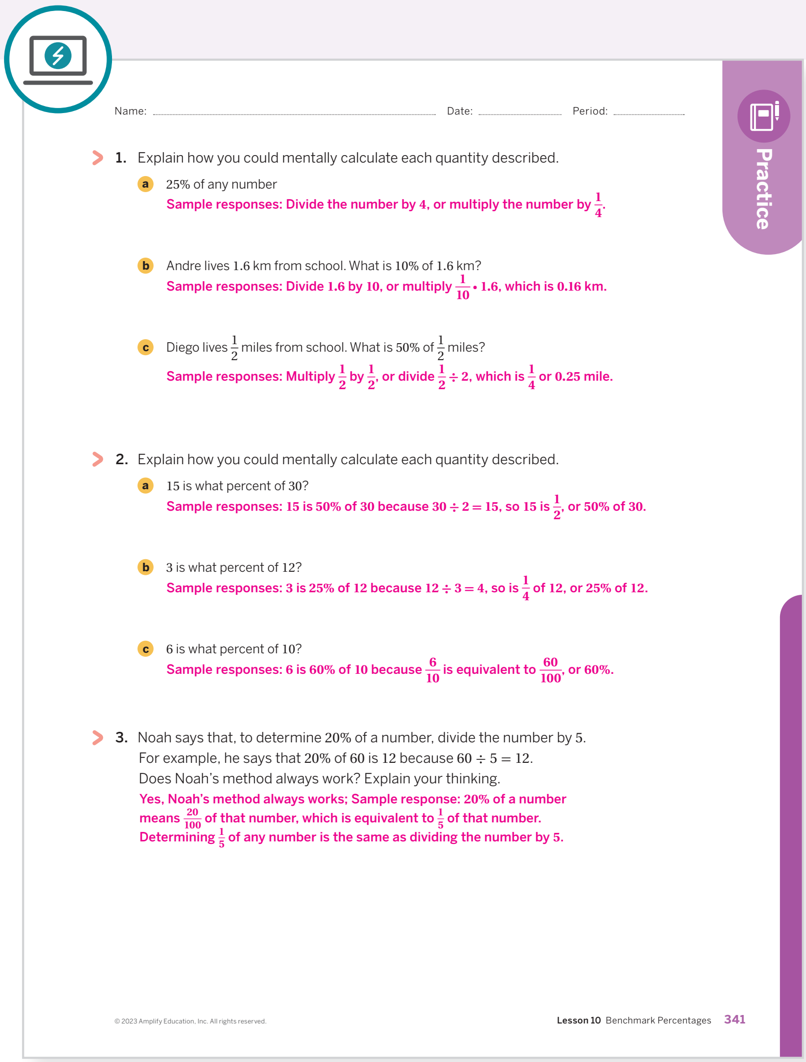
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? An instructional goal for this lesson was for students to solve problems involving benchmark percentages by reasoning about benchmark fractions. How well did students accomplish this? What did you specifically do to help students accomplish it?
- What did you see in the way some students approached Activity 1 that you would like other students to try? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Explain how you could mentally calculate each quantity described.
   - a 25% of any number
     Sample responses: Divide the number by 4, or multiply the number by $\frac{1}{4}$.
   - b Andre lives 1.6 km from school. What is 10% of 1.6 km?
     Sample responses: Divide 1.6 by 10, or multiply $\frac{1}{10} \cdot 1.6$, which is 0.16 km.
   - c Diego lives $\frac{1}{2}$ miles from school. What is 50% of $\frac{1}{2}$ miles?
     Sample responses: Multiply $\frac{1}{2}$ by $\frac{1}{2}$, or divide $\frac{1}{2} \div 2$, which is $\frac{1}{4}$ or 0.25 mile.

2. Explain how you could mentally calculate each quantity described.
   - a 15 is what percent of 30?
     Sample responses: 15 is 50% of 30 because $30 \div 2 = 15$, so 15 is $\frac{1}{2}$, or 50% of 30.
   - b 3 is what percent of 12?
     Sample responses: 3 is 25% of 12 because $12 \div 3 = 4$, so is $\frac{1}{4}$ of 12, or 25% of 12.
   - c 6 is what percent of 10?
     Sample responses: 6 is 60% of 10 because $\frac{6}{10}$ is equivalent to $\frac{60}{100}$, or 60%.

3. Noah says that, to determine 20% of a number, divide the number by 5.
   For example, he says that 20% of 60 is 12 because $60 \div 5 = 12$.
   Does Noah's method always work? Explain your thinking.
   Yes, Noah's method always works; Sample response: 20% of a number means $\frac{20}{100}$ of that number, which is equivalent to $\frac{1}{5}$ of that number. Determining $\frac{1}{5}$ of any number is the same as dividing the number by 5.

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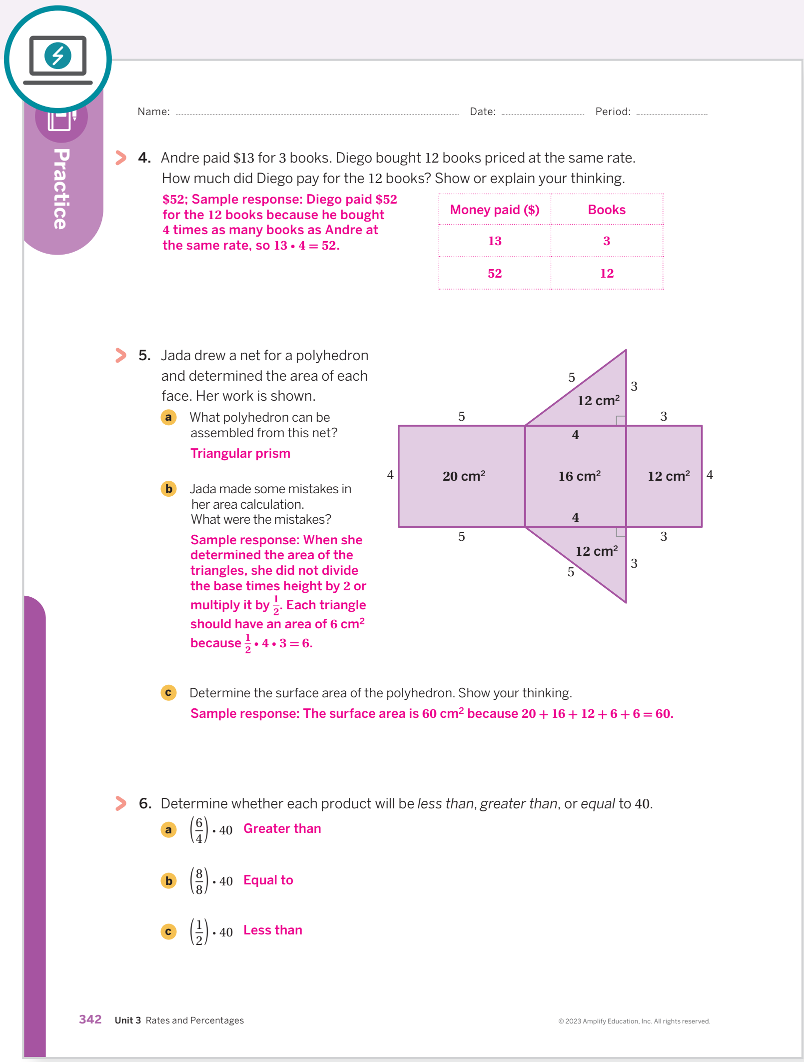
Lesson 10 Benchmark Percentages 341



Practice

Name: Date: Period:

4. Andre paid \$13 for 3 books. Diego bought 12 books priced at the same rate.
   How much did Diego pay for the 12 books? Show or explain your thinking.
   \$52; Sample response: Diego paid \$52 for the 12 books because he bought 4 times as many books as Andre at the same rate, so $13 \cdot 4 = 52$.

Money paid ($)	Books
13	3
52	12

5. Jada drew a net for a polyhedron and determined the area of each face. Her work is shown.
   - a What polyhedron can be assembled from this net?
     Triangular prism
   - b Jada made some mistakes in her area calculation.
     What were the mistakes?
     Sample response: When she determined the area of the triangles, she did not divide the base times height by 2 or multiply it by $\frac{1}{2}$. Each triangle should have an area of 6 cm^2 because $\frac{1}{2} \cdot 4 \cdot 3 = 6$.
   - c Determine the surface area of the polyhedron. Show your thinking.
     Sample response: The surface area is 60 cm^2 because $20 + 16 + 12 + 6 + 6 = 60$.

6. Determine whether each product will be *less than*, *greater than*, or *equal* to 40.
   - a $\left(\frac{6}{4}\right) \cdot 40$ Greater than
   - b $\left(\frac{8}{8}\right) \cdot 40$ Equal to
   - c $\left(\frac{1}{2}\right) \cdot 40$ Less than

342 **Unit 3** Rates and Percentages

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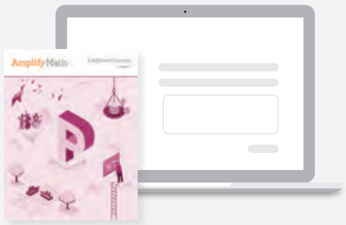
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 16	2
	5	Unit 1 Lesson 16	2
Formative	6	Unit 3 Lesson 11	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 3 | **LESSON 11**

This Percent of That

Let's determine any percent of any number.

Focus

Goals

1. **Language Goal:** Comprehend a phrase, such as "$n\%$ of w," to refer to the value that makes a ratio with w that is equivalent to $n : 100$. **(Speaking and Listening, Writing)**
2. **Language Goal:** Explain how to solve problems such as "$n\%$ of w is ?" and "$n\%$ of ? is p." **(Speaking and Listening, Writing)**
3. **Language Goal:** State explicitly what one is finding the percentage of. **(Speaking and Listening, Writing)**

Coherence

Today

Students generalize a process for determining any percentage of any quantity (the whole or total). They may choose to continue to think of percentages as rates and work with ratio representations, such as double number lines or tables, as they progress toward developing an algorithm. Students associate all percentages, not just benchmarks, with fractions that have denominators of 100, which can be used as operators to multiply the whole by the percentage to determine the corresponding part. They also see how percentages can be represented and be visualized by using tape diagrams. This will prepare them to work with tape diagrams in the next lesson.

Previously

In Lesson 10, students built upon their understanding of fractions from Grades 4 and 5 to relate benchmark percentages (10%, 25%, 50%, and 75%) to common fractions.

Coming Soon

In Lesson 12, students will represent percentages by using tape diagrams, and in particular, they will focus on the third and final type of percentage problem — determining the whole, given the percentage and the part.

Rigor

- Students develop **procedural skills** for determining any percentage of a given whole.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos ⋮ **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- calculators
- *Double Number Lines: Percentage Problems* PDF (as needed)
- *Tape Diagrams* PDF (as needed)

Math Language Development

Review word

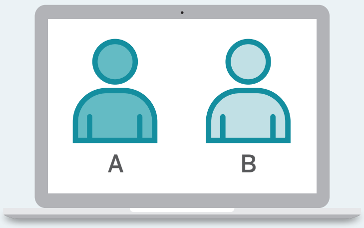
- *percentage (percent)*

Amps ⋮ Featured Activity

Activity 1

Puppies *Really* Grow Up

Students receive real-time validation and feedback on calculation and response inputs by watching an animated puppy grow accordingly.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students complete Activity 2, they may think they need to draw tape diagrams to determine the responses for each row of the table and thus, feel overwhelmed. Suggest they look for repeated reasoning to help them come up with an algorithm that will be more efficient than drawing tape diagrams.

Modifications to Pacing

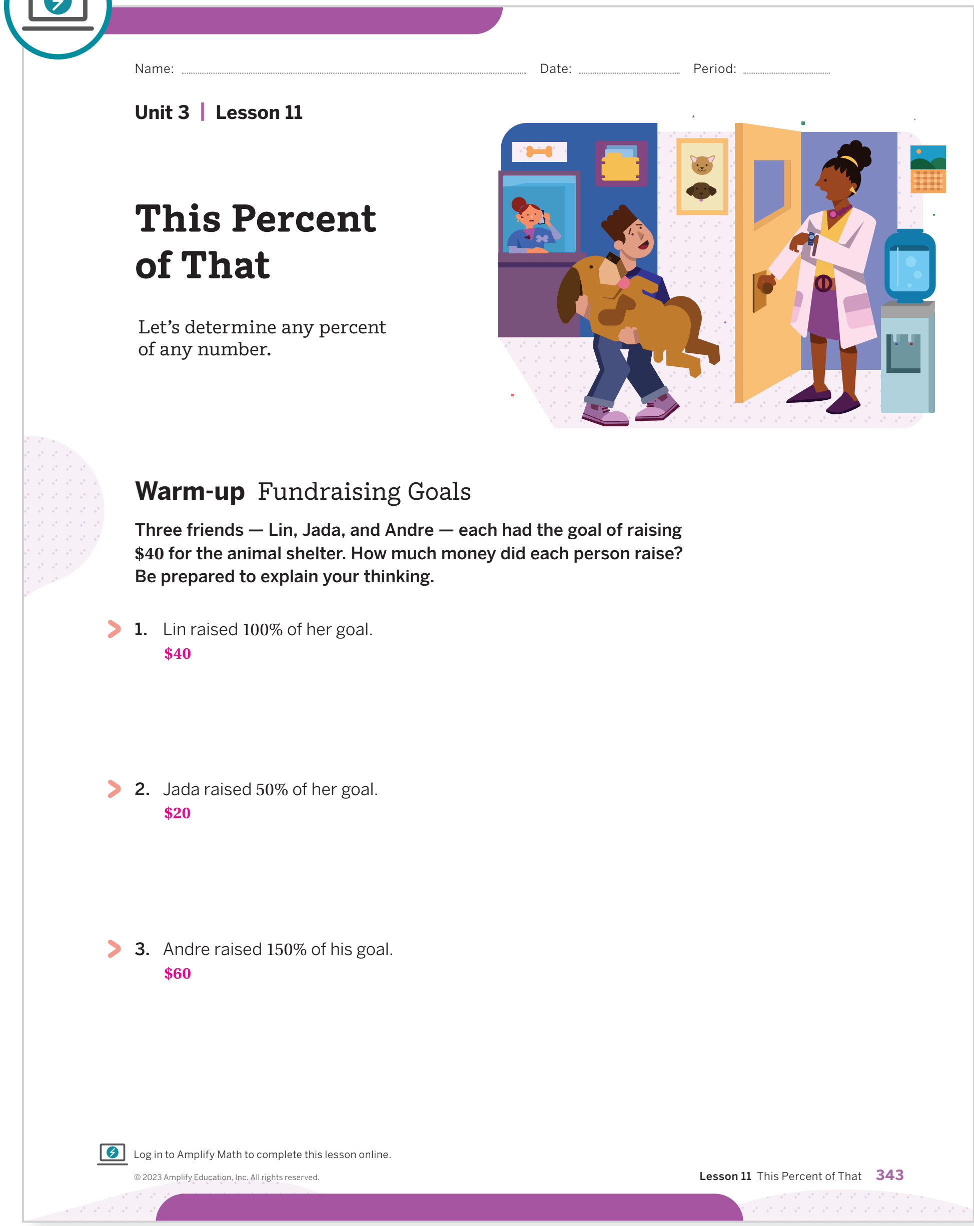
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, you may choose to give fixed percentages for Problem 1, including some *but not all* benchmarks and multiples. Problem 3 may also be omitted.

Independent | 5 min

Warm-up Fundraising Goals

Students determine three related benchmark percentages of a fundraising goal to remind them of connections between fractions, addition and multiplication, and percentages.



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Unit 3 | Lesson 11

This Percent of That

Let's determine any percent of any number.

Warm-up Fundraising Goals

Three friends — Lin, Jada, and Andre — each had the goal of raising $40 for the animal shelter. How much money did each person raise? Be prepared to explain your thinking.

1. Lin raised 100% of her goal.
   $40
2. Jada raised 50% of her goal.
   $20
3. Andre raised 150% of his goal.
   $60

Log in to Amplify Math to complete this lesson online.

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Lesson 11 This Percent of That 343

① Launch

Activate students' background knowledge by asking, "Have you ever seen something that looks like a thermometer or a bar tracking progress toward a goal, such as for a fundraiser?"

② Monitor

Help students get started by asking, "What does 100% of a goal mean? What has to be true about percentages less than 100%? Greater than 100%?"

Look for points of confusion:

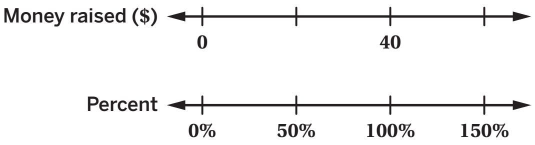
- **Thinking all three students had a goal of $40 total.** Reiterate that *each* student had a goal of $40.

Look for productive strategies:

- Creating a double number line or a ratio table to determine how much money each student raised.
- Using benchmark fractions, percentages, and their multiples to determine the amount of money raised, and checking the reasonableness of their answers as being less than, greater than, or equal to $40.

③ Connect

Display a double number line, such as this:



Have individual students share how they determined the missing parts for each corresponding percentage, focusing on those who used benchmark percentages, fractions, and multiplication.

Ask, "What would 200% of the goal be?"

Highlight that students now know several ways to represent and to calculate benchmark percentages of a number, which means determining the missing part when the whole and percentage are known.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

Provide access to copies of the *Double Number Lines: Percentage Problems* PDF for students to use during the Warm-up. Ask them what value represents the whole.

Power-up

To power up students' ability to relate the product to the size of the fraction factor relative to 1, have students complete:

a. $\frac{3}{1} \cdot \frac{2}{3} = 2$

b. $\frac{3}{1} \cdot \frac{3}{3} = 3$

c. $\frac{3}{1} \cdot \frac{4}{3} = 4$

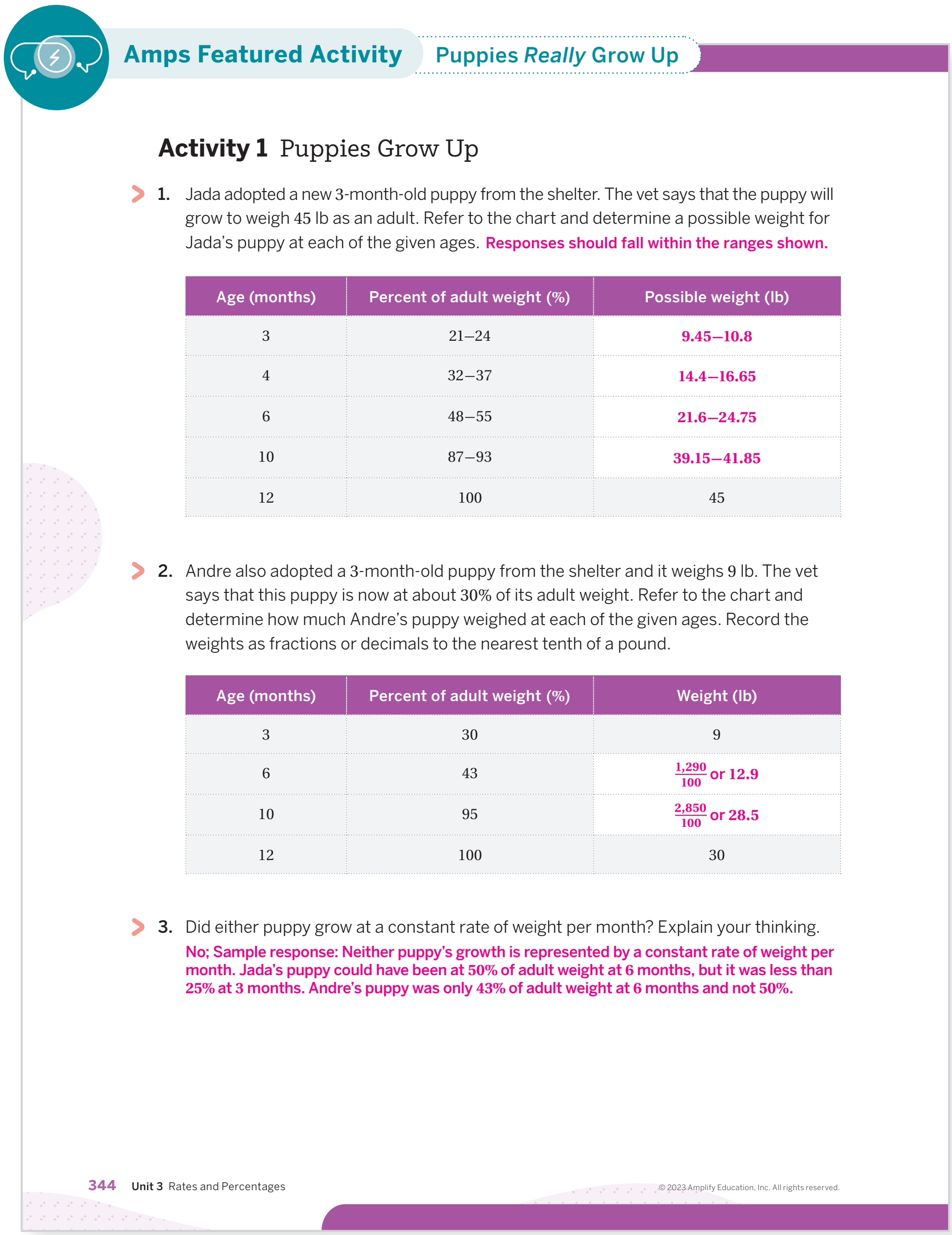
d. Will $3 \cdot \frac{3}{4}$ be greater than, equal to, or less than 3? Less than 3.

Use: Before the Warm-up.
Informed by: Performance on Lesson 10, Practice Problem 6.

Pairs | 20 min

Activity 1 Puppies Grow Up

Students identify and determine percentages of a whole that are not all benchmarks or multiples, leading to a generalized process for determining a missing part.



Amps Featured Activity Puppies *Really* Grow Up

Activity 1 Puppies Grow Up

1. Jada adopted a new 3-month-old puppy from the shelter. The vet says that the puppy will grow to weigh 45 lb as an adult. Refer to the chart and determine a possible weight for Jada's puppy at each of the given ages. **Responses should fall within the ranges shown.**

Age (months)	Percent of adult weight (%)	Possible weight (lb)
3	21–24	**9.45–10.8**
4	32–37	**14.4–16.65**
6	48–55	**21.6–24.75**
10	87–93	**39.15–41.85**
12	100	45

2. Andre also adopted a 3-month-old puppy from the shelter and it weighs 9 lb. The vet says that this puppy is now at about 30% of its adult weight. Refer to the chart and determine how much Andre's puppy weighed at each of the given ages. Record the weights as fractions or decimals to the nearest tenth of a pound.

Age (months)	Percent of adult weight (%)	Weight (lb)
3	30	9
6	43	$\frac{1,290}{100}$ **or 12.9**
10	95	$\frac{2,850}{100}$ **or 28.5**
12	100	30

3. Did either puppy grow at a constant rate of weight per month? Explain your thinking.
No; Sample response: Neither puppy's growth is represented by a constant rate of weight per month. Jada's puppy could have been at 50% of adult weight at 6 months, but it was less than 25% at 3 months. Andre's puppy was only 43% of adult weight at 6 months and not 50%.

344 Unit 3 Rates and Percentages

1 Launch

Have students use the ***Think-Pair-Share*** routine. Give them 2 minutes to read Problem 1 and think about a strategy for completing the table. Then have them complete Problems 1–3 with a partner. Provide access to calculators.

2 Monitor

Help students get started by asking, "What benchmark percentages might help you here?"

Look for points of confusion:

- **Thinking they need to determine more than one weight for each age in Problem 1.** Clarify only one weight is needed and they can choose any percentage in the given range.

Look for productive strategies:

- Using equivalent ratios or benchmark percentages, such as 10%, and multiples.
- Multiplying the whole by a fraction out of 100 to determine the part.
- Reasoning with percentages to show that neither puppy grew at a constant rate.

3 Connect

Display the tables from Problems 1 and 2.

Have pairs of students share their chosen percentages for Problem 1 and how they determined the corresponding weights for Problems 1 and 2. Then have pairs share their explanations for Problem 3.

Highlight that any percentage of a number (the whole) can be determined by relating the percentage to a fraction with a denominator of 100 and then multiplying by the whole, to determine the missing part.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can receive real-time validation and feedback on their calculations and responses by watching an animated puppy grow accordingly.

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

Provide access to copies of the *Double Number Lines: Percentage Problems* PDF for students to use. Ask them what value represents the whole in this context. Remind them of benchmark percentages, corresponding fractions, and how to add percentages.

Math Language Development

MLR1: Stronger and Clearer Each Time

Before the Connect, as time allows, have students share their responses to Problem 3 with another pair of students. Ask partners to review responses and provide feedback that can be used to improve the clarity of the responses. Consider displaying these sample questions:

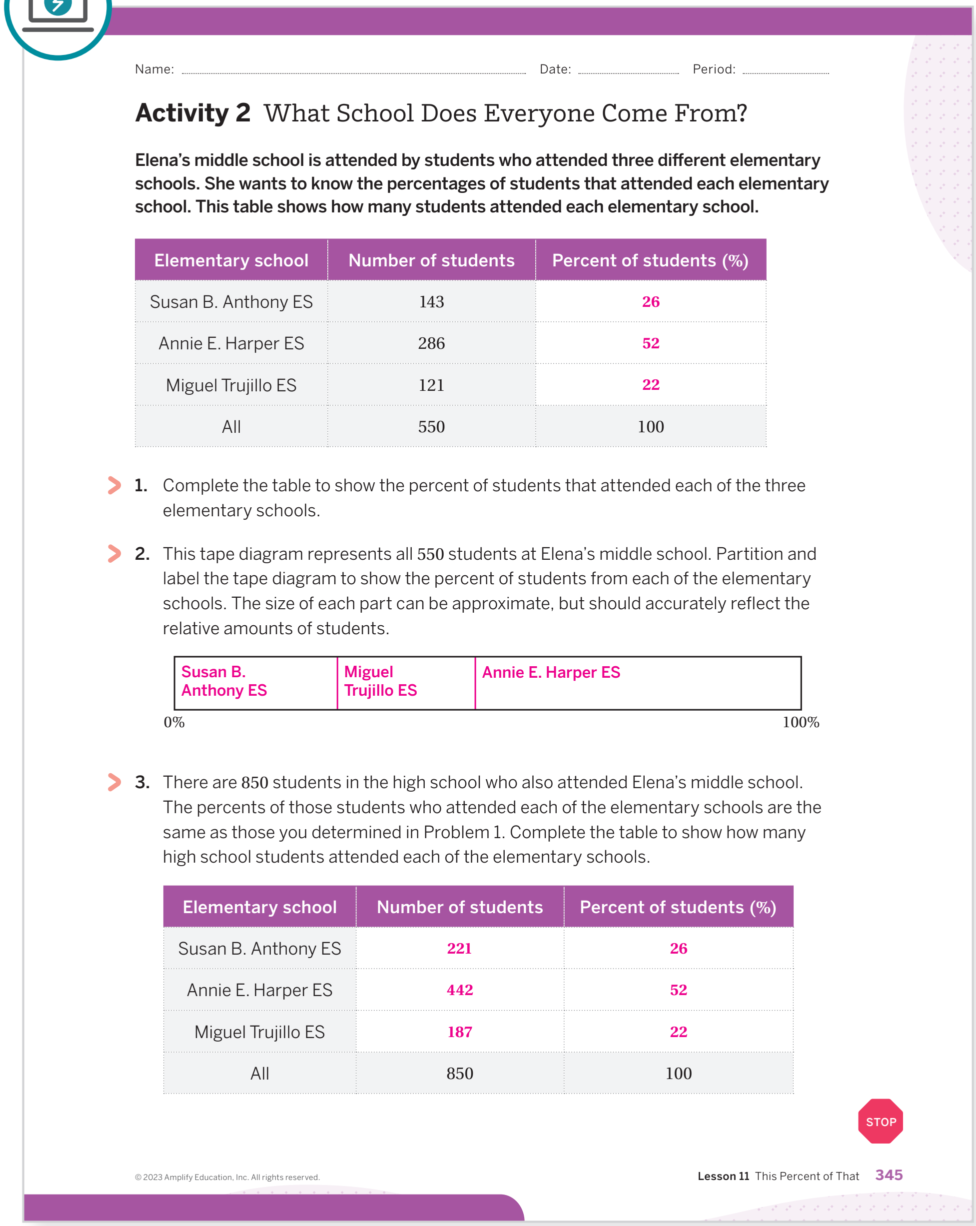
- "Does the explanation include mathematical examples of why the puppy did or did not grow at a constant rate?"
- "Does the response include information about both puppies, and not just one?"

Have students revise their responses after receiving feedback.

Pairs | 10 min

Activity 2 What School Does Everyone Come From?

Students represent percentages using a tape diagram, and practice determining missing percentages and missing parts in the context of feeder elementary schools.



Name: ________ Date: ________ Period: ________

Activity 2 What School Does Everyone Come From?

Elena's middle school is attended by students who attended three different elementary schools. She wants to know the percentages of students that attended each elementary school. This table shows how many students attended each elementary school.

Elementary school	Number of students	Percent of students (%)
Susan B. Anthony ES	143	26
Annie E. Harper ES	286	52
Miguel Trujillo ES	121	22
All	550	100

1. Complete the table to show the percent of students that attended each of the three elementary schools.

2. This tape diagram represents all 550 students at Elena's middle school. Partition and label the tape diagram to show the percent of students from each of the elementary schools. The size of each part can be approximate, but should accurately reflect the relative amounts of students.

Susan B. Anthony ES | Miguel Trujillo ES | Annie E. Harper ES

0% 100%

3. There are 850 students in the high school who also attended Elena's middle school. The percents of those students who attended each of the elementary schools are the same as those you determined in Problem 1. Complete the table to show how many high school students attended each of the elementary schools.

Elementary school	Number of students	Percent of students (%)
Susan B. Anthony ES	221	26
Annie E. Harper ES	442	52
Miguel Trujillo ES	187	22
All	850	100

STOP

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Lesson 11 This Percent of That 345

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What do you know? What do you need to know?"

Look for points of confusion:

- **Misapplying an algorithm to determine percentages in Problem 1.** Have students check the reasonableness of their answers, such as by using the percent for a number like 275.
- **Not knowing how to partition the tape diagram in Problem 2.** Ask, "What fraction of the whole is the same as 50%? How would you show that?"
- **Struggling to get started with the empty table in Problem 3.** Ask, "How can you use the percentages from Problem 1 to help you?"

Look for productive strategies:

- Applying the algorithm $\frac{p}{w} \cdot 100$ to determine the percentages in Problem 1.
- Relating benchmark percentages and corresponding fractions, such as 50% and $\frac{1}{2}$, or 25% and $\frac{1}{4}$.
- Developing and applying an algorithm such as $\frac{p}{100} \cdot w$ to determine the number of students.

3 Connect

Display the tables for Problems 1 and 3 and consider drawing a blank tape diagram.

Have pairs of students share first how they determined the missing percentages in Problem 1, focusing on algorithms. Have students guide you in constructing a corresponding tape diagram. Then have more pairs share how they determined the missing parts for Problem 3.

Highlight that, to determine a missing part given any whole w and any percentage p, multiply $\frac{p}{100} \cdot w$.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools, Vary Demands to Optimize Challenge*

Provide copies of the *Double Number Lines: Percentage Problems* PDF and the *Tape Diagrams* PDF. If students need more processing time, have them focus on Problem 3, giving them the percentages from Problem 1.

Extension: *Math Enrichment*

Tell students that there are two middle schools that feed into the high school and there are 1,615 students in the high school. Ask them to determine the approximate percent of the total high school students that come from Susan. B. Anthony ES. About 13.7%

Math Language Development

MLR2: Collect and Display

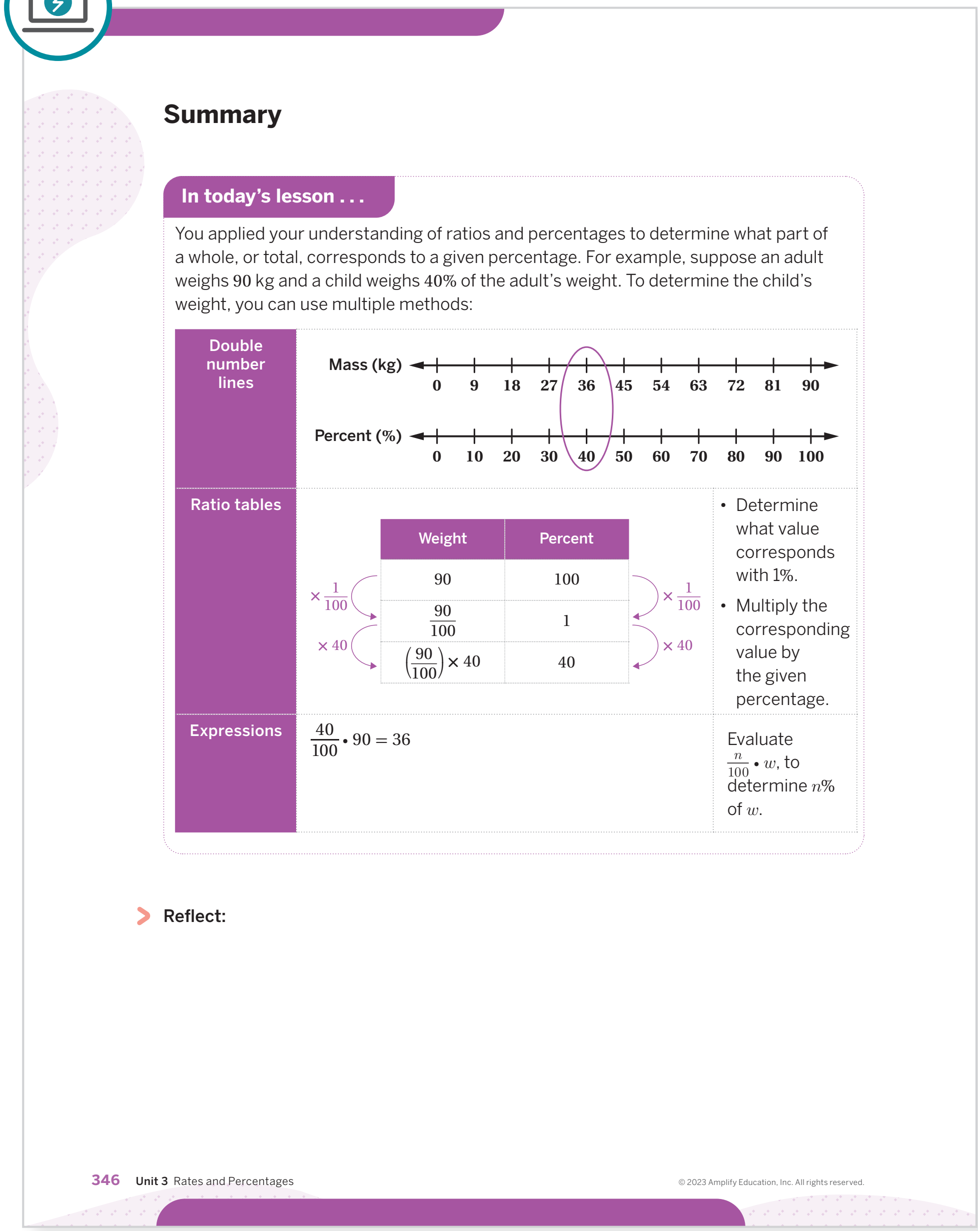
During the Connect, listen for and collect language students use to generalize the process of determining missing parts in percentage problems. Record these on a visual display. Examples could include: "identify the whole," "determine the fraction of the whole," "determine the percent by dividing the numerator of the fraction by the denominator and multiplying by 100," "draw a double number line," "use the formula $\frac{p}{w} \cdot 100$," etc.

English Learners

Include tape diagrams and/or double number lines on the visual display to support students' sense-making.

Summary

Review and synthesize how to determine a missing part in a percentage problem, given the percentage and the whole.



Summary

In today's lesson . . .

You applied your understanding of ratios and percentages to determine what part of a whole, or total, corresponds to a given percentage. For example, suppose an adult weighs 90 kg and a child weighs 40% of the adult's weight. To determine the child's weight, you can use multiple methods:

Double number lines	Mass (kg) 0, 9, 18, 27, 36, 45, 54, 63, 72, 81, 90 Percent (%) 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100	
Ratio tables	$\times \frac{1}{100}$, $\times 40$	- Determine what value corresponds with 1%. - Multiply the corresponding value by the given percentage.
Expressions	$\frac{40}{100} \cdot 90 = 36$	Evaluate $\frac{n}{100} \cdot w$, to determine n% of w.

Weight	Percent
90	100
$\frac{90}{100}$	1
$\left(\frac{90}{100}\right) \times 40$	40

Reflect:

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Synthesize

Highlight that students have now seen how to use the ratio relationships in percentage problems to determine either a missing percent or a missing part. In general, to determine a missing part, the percent is represented by a factor that is a fraction with a denominator of 100 and that is multiplied by the whole to determine the corresponding part. **Note:** The final case, determining a missing whole, will be addressed in the next lesson.

- "How does the fraction $\frac{n}{100}$ relate to a decimal?"
- "What if you are trying to determine 12.5% of a number – how could you write that factor as a decimal? How could you write that factor as a fraction that does not include a decimal in the numerator?"

Have individual students share responses to the questions, focusing on those who make connections to place value, the location of the decimal point, and corresponding denominators.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is determining a missing part different from determining a missing percentage? Are there any ways they are the same?"
- "How are benchmark percentages and fractions useful for determining non-benchmark values? Are there any benchmark percentages you found yourself using or thinking about more than others?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of determining missing parts in percentage problems when given the whole and a non-benchmark percent.

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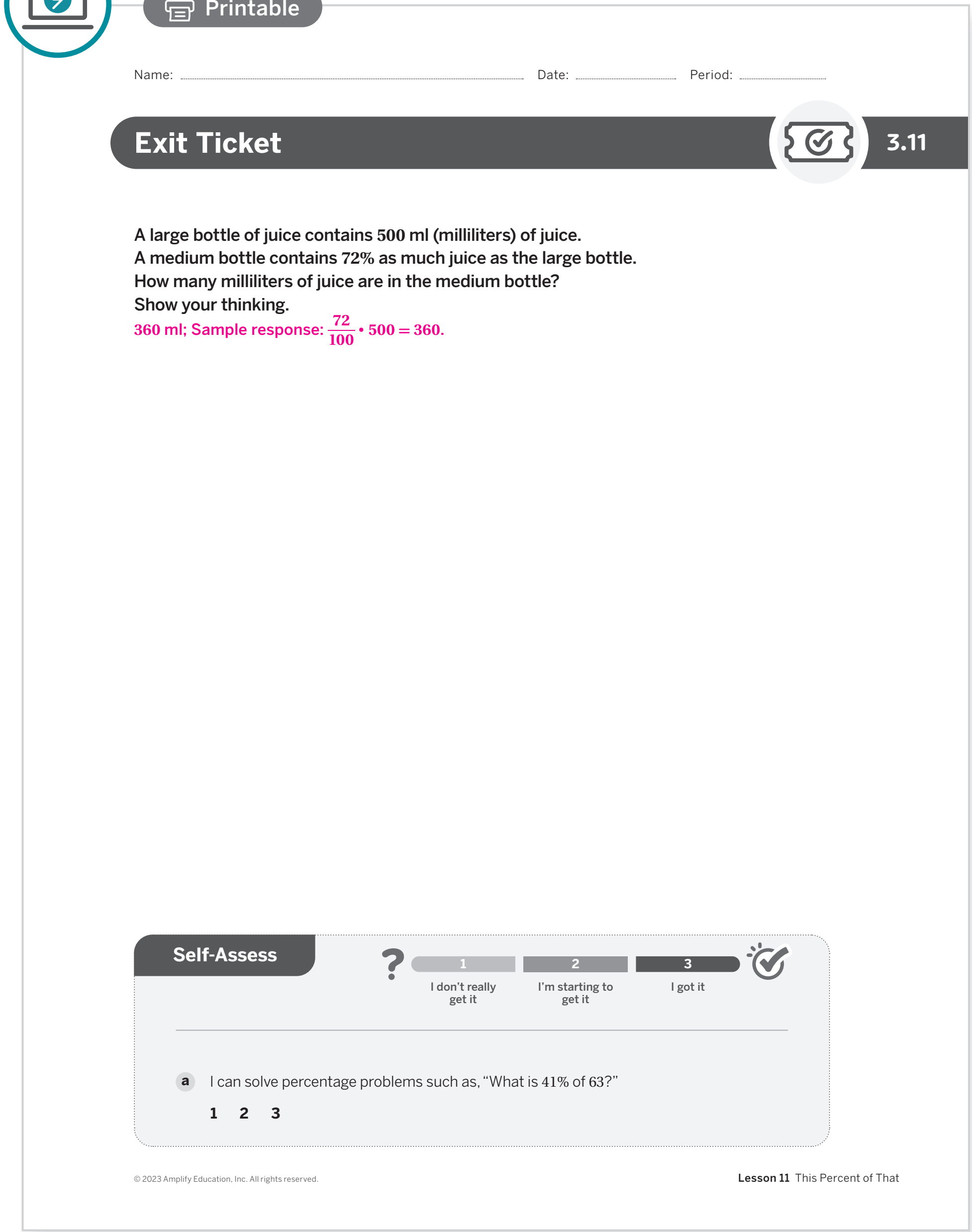

Name: ______________ Date: ________ Period: ________

Exit Ticket 3.11

A large bottle of juice contains 500 ml (milliliters) of juice.
A medium bottle contains 72% as much juice as the large bottle.
How many milliliters of juice are in the medium bottle?
Show your thinking.

360 ml; Sample response: $\frac{72}{100} \cdot 500 = 360$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can solve percentage problems such as, "What is 41% of 63?"

1 2 3

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Lesson 11 This Percent of That

Success looks like . . .

- **Language Goal:** Comprehending a phrase, such as "n% of w," to refer to the value that makes a ratio with w that is equivalent to $n : 100$. **(Speaking and Listening, Writing)**
  - » Evaluating 72% of 500.
- **Language Goal:** Explaining how to use a table to solve problems such as "n% of w is ?" and "n% of ? is p." **(Speaking and Listening, Writing)**
- **Language Goal:** Stating explicitly what one is finding the percentage of. **(Speaking and Listening, Writing)**

Suggested next steps

If students incorrectly determine the missing part, consider:

- Asking, "Could you solve this using a ratio table or a double number line?"
- Reviewing the strategies from Activities 1 and 2, and showing the general algorithm.
- Assigning Practice Problem 1.

Professional Learning

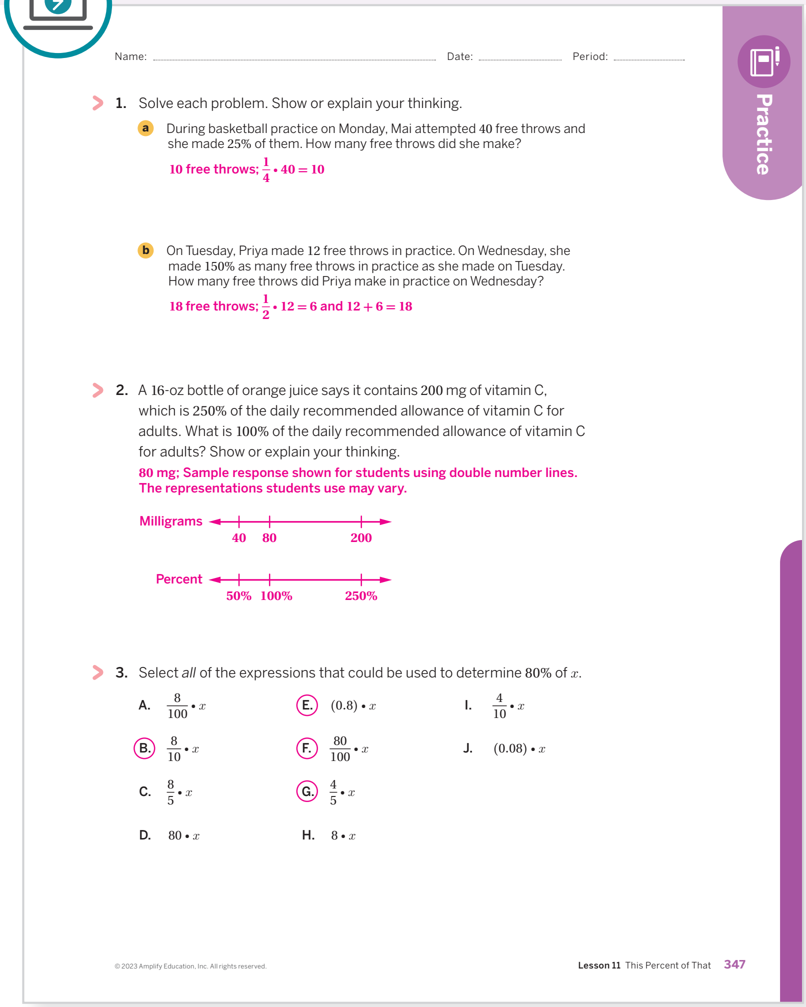
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In earlier lessons, students worked with benchmark percentages. How did that support determining missing parts by using other percentages today?
- In what ways have your students developed efficiencies for working with percentages and solving related problems?

Practice

Independent



Practice

Name: .... Date: .... Period: ....

1. Solve each problem. Show or explain your thinking.
   - **a** During basketball practice on Monday, Mai attempted 40 free throws and she made 25% of them. How many free throws did she make?

     10 free throws; $\frac{1}{4} \cdot 40 = 10$
   - **b** On Tuesday, Priya made 12 free throws in practice. On Wednesday, she made 150% as many free throws in practice as she made on Tuesday. How many free throws did Priya make in practice on Wednesday?

     18 free throws; $\frac{1}{2} \cdot 12 = 6$ and $12 + 6 = 18$

2. A 16-oz bottle of orange juice says it contains 200 mg of vitamin C, which is 250% of the daily recommended allowance of vitamin C for adults. What is 100% of the daily recommended allowance of vitamin C for adults? Show or explain your thinking.

   80 mg; Sample response shown for students using double number lines. The representations students use may vary.

   Milligrams: 40, 80, 200

   Percent: 50%, 100%, 250%

3. Select *all* of the expressions that could be used to determine 80% of x.

A. $\frac{8}{100} \cdot x$

[✓ B.] $\frac{8}{10} \cdot x$

C. $\frac{8}{5} \cdot x$

D. $80 \cdot x$

[✓ E.] $(0.8) \cdot x$

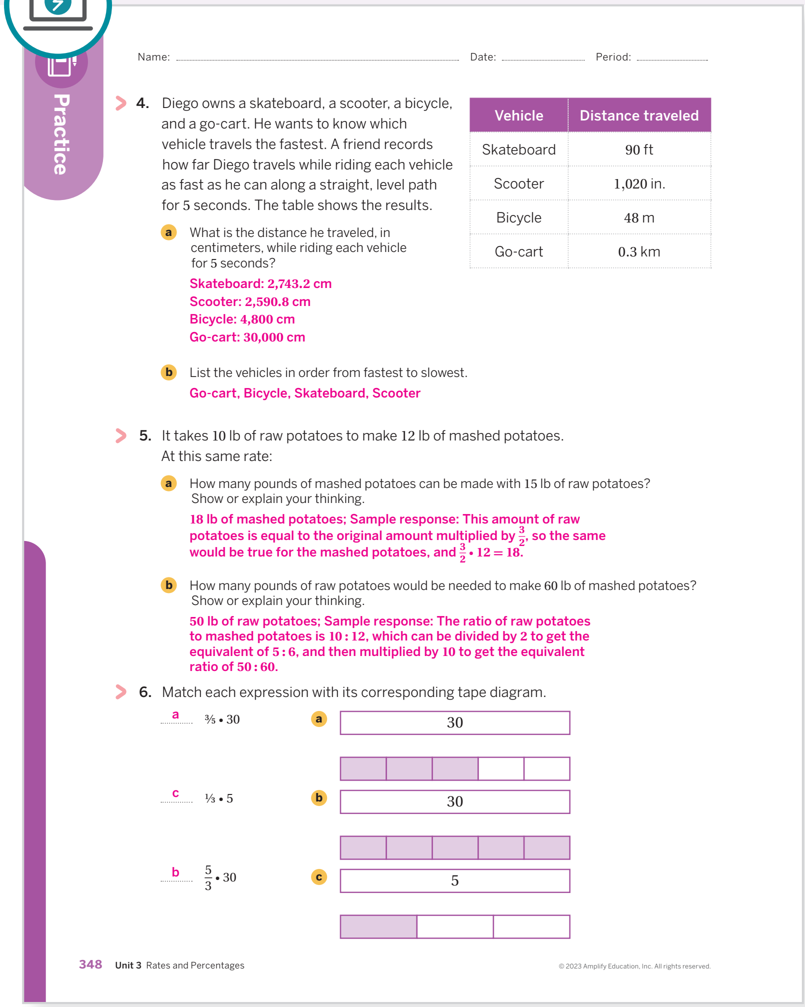
[✓ F.] $\frac{80}{100} \cdot x$

[✓ G.] $\frac{4}{5} \cdot x$

H. $8 \cdot x$

I. $\frac{4}{10} \cdot x$

J. $(0.08) \cdot x$



Practice

Name: .... Date: .... Period: ....

4. Diego owns a skateboard, a scooter, a bicycle, and a go-cart. He wants to know which vehicle travels the fastest. A friend records how far Diego travels while riding each vehicle as fast as he can along a straight, level path for 5 seconds. The table shows the results.

Vehicle	Distance traveled
Skateboard	90 ft
Scooter	1,020 in.
Bicycle	48 m
Go-cart	0.3 km

   - **a** What is the distance he traveled, in centimeters, while riding each vehicle for 5 seconds?

     Skateboard: 2,743.2 cm
     Scooter: 2,590.8 cm
     Bicycle: 4,800 cm
     Go-cart: 30,000 cm
   - **b** List the vehicles in order from fastest to slowest.

     Go-cart, Bicycle, Skateboard, Scooter

5. It takes 10 lb of raw potatoes to make 12 lb of mashed potatoes. At this same rate:
   - **a** How many pounds of mashed potatoes can be made with 15 lb of raw potatoes? Show or explain your thinking.

     18 lb of mashed potatoes; Sample response: This amount of raw potatoes is equal to the original amount multiplied by $\frac{3}{2}$, so the same would be true for the mashed potatoes, and $\frac{3}{2} \cdot 12 = 18$.
   - **b** How many pounds of raw potatoes would be needed to make 60 lb of mashed potatoes? Show or explain your thinking.

     50 lb of raw potatoes; Sample response: The ratio of raw potatoes to mashed potatoes is $10:12$, which can be divided by 2 to get the equivalent of $5:6$, and then multiplied by 10 to get the equivalent ratio of $50:60$.

6. Match each expression with its corresponding tape diagram.

a ...... $\frac{3}{5} \cdot 30$

c ...... $\frac{1}{3} \cdot 5$

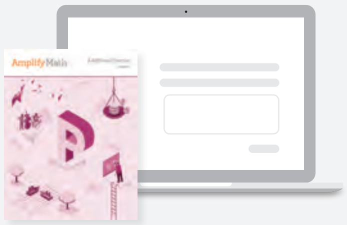
b ...... $\frac{5}{3} \cdot 30$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 7	2
	5	Unit 3 Lesson 6	2
Formative ⚡	6	Unit 3 Lesson 12	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, see the **Grade 6 Additional Practice**.

UNIT 3 | **LESSON 12**

This Percent of What

Let's use tape diagrams to represent percentages and to determine an unknown whole.

Focus

Goals

1. **Language Goal:** Draw and label a tape diagram to represent a situation involving percentages. **(Writing)**
2. **Language Goal:** Comprehend a sentence, such as "n% of ? is p" to refer to the value that makes a ratio with p that is equivalent to $n : 100$. **(Speaking and Listening, Writing)**
3. **Language Goal:** Explain how to solve problems, such as "n% of ? is p." **(Speaking and Listening, Writing)**

Coherence

Today

Students use tape diagrams to represent percentage problems, recognizing connections between percentages and fractions. They continue to see that, when reasoning about percentages, it is important to indicate the whole as 100%, just as it is important to indicate the whole when working with fractions. Students see that tape diagrams are particularly useful in solving problems of the form "n% of ? is p," meaning they know the part and the percentage and the missing value they are determining is the whole. Through repeated reasoning with these types of percentage scenarios, students develop another general algorithm for calculating the whole, $\frac{p}{n} \cdot 100$.

Previously

Students used double number lines and ratio reasoning to develop algorithms for working with percentage problems involving a missing percentage in Lessons 8–9, and a missing part in Lessons 10–11.

Coming Soon

In Lesson 13, students will continue to work with percentage problems by finding any of the three possible missing values and choosing appropriate representations and strategies. Later in Grade 7, students will solve multi-step percentage problems.

Rigor

- Students build **conceptual understanding** of percentages by using tape diagrams.
- Students develop **procedural skills** for determining a missing whole in percentage problems.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- *Percentage Algorithms* PDF
- *Tape Diagrams* PDF

Math Language Development

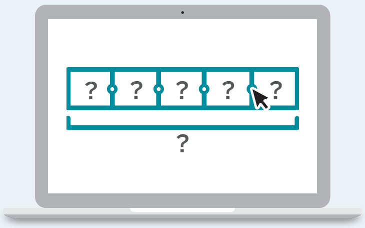
Review words

- *percent*
- *percentage*
- *tape diagram*

Amps | **Featured Activity**

Activity 1
Digital Tape Diagrams

Students can sketch on tape diagrams to help them determine missing values.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

At first, students may not immediately be able to identify an expression to model the visual pattern and might want to quit before really getting started. Encourage students to set a goal of identifying what they do know about the pattern and build on that goal by using what they know about the structure of expressions to determine the correct expression. Students can repeat until they have solved the problem. By looking only one step ahead, a task can seem much more manageable.

Modifications to Pacing

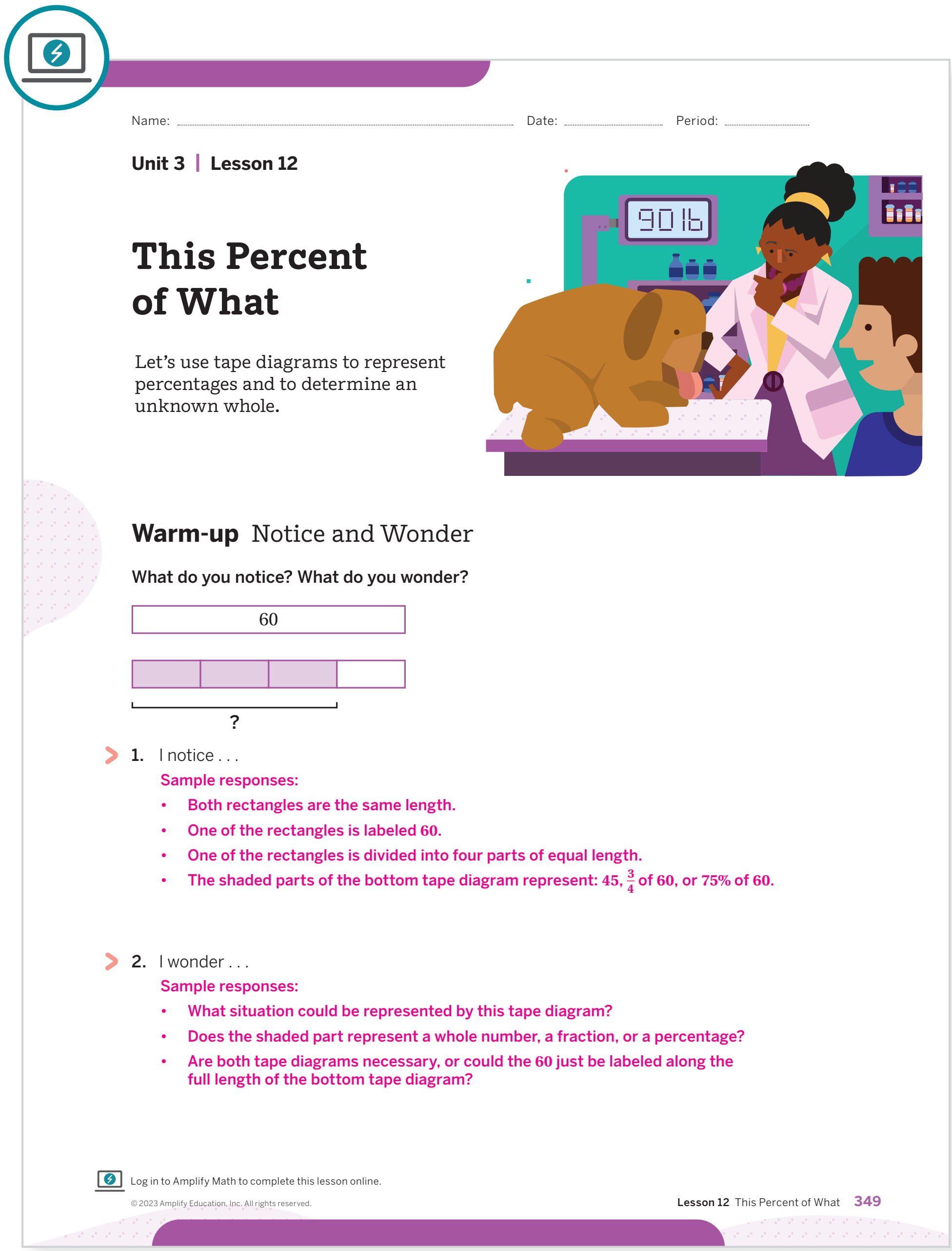
You may want to consider these additional modifications if you are short on time:

- The **Warm-up** may be omitted.
- In **Activity 1**, Problem 2 may be omitted.
- In **Activity 2**, Problems 1b, 2b, and 3b may be omitted.

Independent | 5 min

Warm-up Notice and Wonder

Students analyze a tape diagram to foster flexible thinking about the connections between parts, wholes, fractions, and percentages.



Name: ____________ Date: ________ Period: ________

Unit 3 | Lesson 12

This Percent of What

Let's use tape diagrams to represent percentages and to determine an unknown whole.

Warm-up Notice and Wonder

What do you notice? What do you wonder?

1. I notice . . .

   Sample responses:
   - Both rectangles are the same length.
   - One of the rectangles is labeled 60.
   - One of the rectangles is divided into four parts of equal length.
   - The shaded parts of the bottom tape diagram represent: 45, $\frac{3}{4}$ of 60, or 75% of 60.

2. I wonder . . .

   Sample responses:
   - What situation could be represented by this tape diagram?
   - Does the shaded part represent a whole number, a fraction, or a percentage?
   - Are both tape diagrams necessary, or could the 60 just be labeled along the full length of the bottom tape diagram?

Log in to Amplify Math to complete this lesson online.

Lesson 12 This Percent of What 349

1 Launch

Conduct the *Notice and Wonder* routine with the tape diagram. Record class observations.

2 Monitor

Help students get started by suggesting they try to think about the diagram in different ways: whole numbers, fractions, or percentages.

Look for productive strategies:

- Noticing both rectangles are the same length, but one has been divided into four equal parts with three parts shaded.
- Noticing that the shaded area represents $\frac{3}{4}$ of 60, or 45; or determining each shaded part represents 15.
- Interpreting the shaded area as representing 75% of 60.
- Making up a scenario or question represented by the diagram, such as, "There are 60 students in the concert band, and I want to know if $\frac{3}{4}$ of them play brass or woodwind instruments. How many is that?"

3 Connect

Display the diagram for students to reference as they share what they noticed and wondered.

Have individual students share one thing they noticed or wondered at a time, bringing as many students and interpretations into the conversation as possible. Allow students to share new ideas inspired by others' responses as well.

Highlight that the "?" could represent a fraction or a percentage of the whole, which is also just a number (in this case, the whole number 45).

Ask, if it has not yet been presented, "What if the entire pink shaded area in the bottom rectangle represents the whole, or 100%? How does that change your thinking and interpretation of the diagram?"

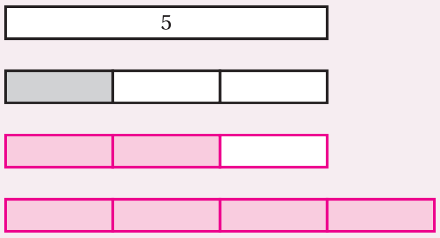
Power-up

To power up student's ability to represent expressions with tape diagrams, have students complete:

The tape diagram shown represents $\frac{1}{3} \cdot 5$. Sketch two more tape diagrams to represent

a. $\frac{2}{3} \cdot 5$

b. $\frac{4}{3} \cdot 5$



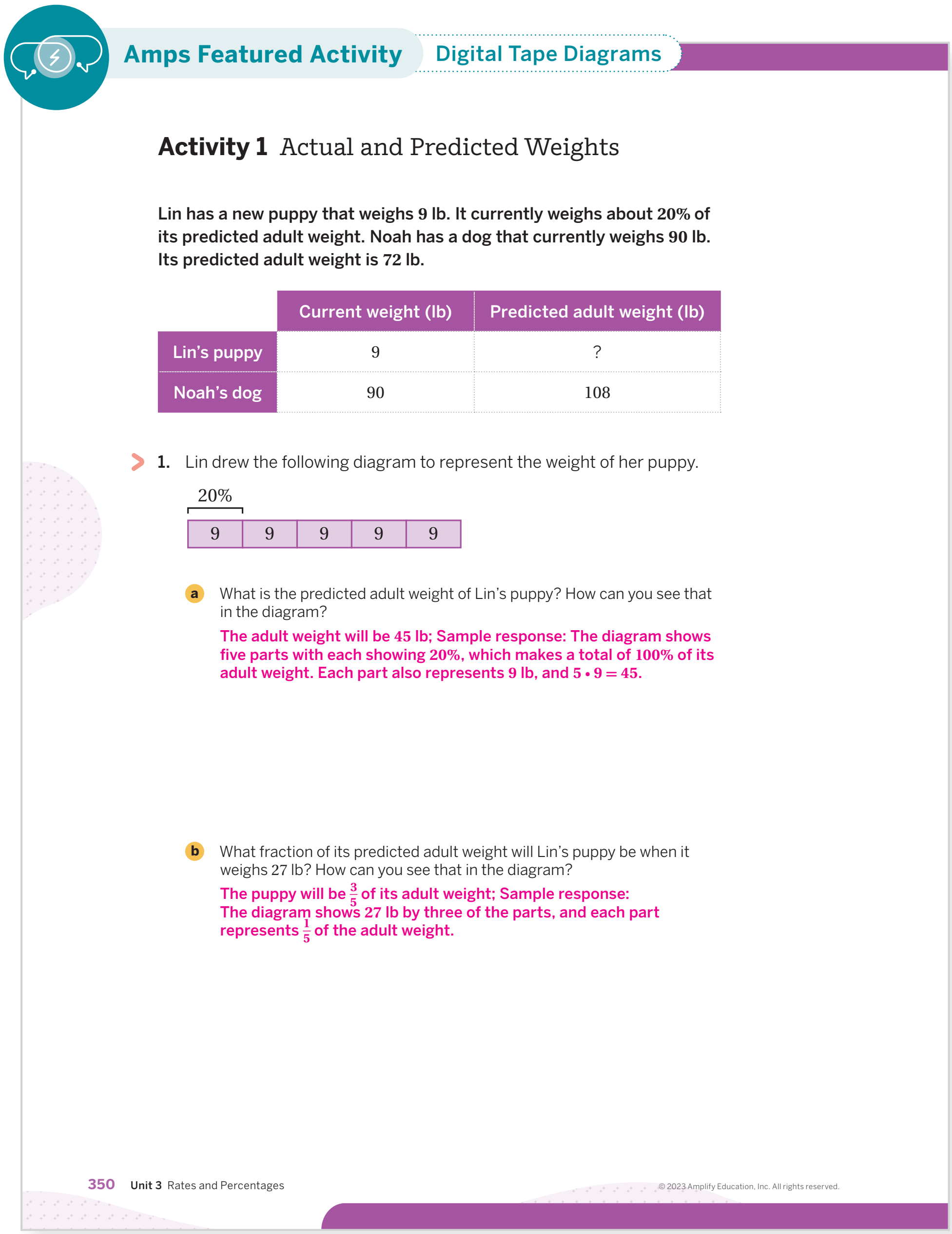
Use: Before Activity 1.

Informed by: Performance on Lesson 11, Practice Problem 6, and Pre-Unit Readiness Assessment, Problem 6.

Pairs | 15 min

Activity 1 Actual and Predicted Weights

Students use tape diagrams to relate and to interpret parts, wholes, fractions, and percentages.



Amps Featured Activity Digital Tape Diagrams

Activity 1 Actual and Predicted Weights

Lin has a new puppy that weighs 9 lb. It currently weighs about 20% of its predicted adult weight. Noah has a dog that currently weighs 90 lb. Its predicted adult weight is 72 lb.

	Current weight (lb)	Predicted adult weight (lb)
Lin's puppy	9	?
Noah's dog	90	108

1. Lin drew the following diagram to represent the weight of her puppy.

20%				
9	9	9	9	9

a. What is the predicted adult weight of Lin's puppy? How can you see that in the diagram?

The adult weight will be 45 lb; Sample response: The diagram shows five parts with each showing 20%, which makes a total of 100% of its adult weight. Each part also represents 9 lb, and $5 \cdot 9 = 45$.

b. What fraction of its predicted adult weight will Lin's puppy be when it weighs 27 lb? How can you see that in the diagram?

The puppy will be $\frac{3}{5}$ of its adult weight; Sample response: The diagram shows 27 lb by three of the parts, and each part represents $\frac{1}{5}$ of the adult weight.

350 **Unit 3** Rates and Percentages

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1 Launch

Have students use the *Think-Pair-Share* routine. Give them 1 minute to think about Problem 1 before working with a partner to share and complete Problems 1a and 1b. Pause for a whole class discussion before students move on to Problem 2. Then have students repeat the *Think-Pair-Share* routine similarly for Problem 2. Provide or make available copies of the Activity 1 PDF.

2 Monitor

Help students get started by asking, "How is the given information about Lin's puppy from the table represented in the diagram? What else does the diagram show?"

Look for points of confusion:

- **Using incorrect parts and wholes.** Have students clearly label 100% on all of their diagrams and consider also having them write the words *part* and *whole* beside corresponding given and missing values.
- **Losing track of the relevant information while using one tape diagram to solve multiple problems.** Have students use the *Tape Diagrams* PDF to draw a new tape diagram to represent each problem, partitioning and labeling all values appropriately.

Look for productive strategies:

- Recognizing that the part and percentage are known in Problem 1a, so the missing value for predicted adult weight represents the whole.
- Relating benchmark fractions and benchmark percentages, such as $\frac{1}{5}$ and 20%, and using those and their multiples to determine other missing values.
- Using percentages to compare two weights by identifying a part and a whole.
- Flexibly shifting thinking about what the part and whole are, and relating them to tape diagrams by using fractions or percentages.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide copies of the *Tape Diagrams* PDF for students to use if they choose during the activity.

Accessibility: *Guide Processing and Visualization*

Prior to students beginning Problem 1, consider only providing information about Lin's puppy and only introduce information about Noah's dog before students begin Problem 2.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students respond to Problem 1, have them meet with another pair of students for feedback. Consider displaying these questions: "Does the explanation include where the predicted adult weight is shown on the diagram? Where is the fraction of this weight shown on the diagram?"
Have students revise their responses after receiving feedback.

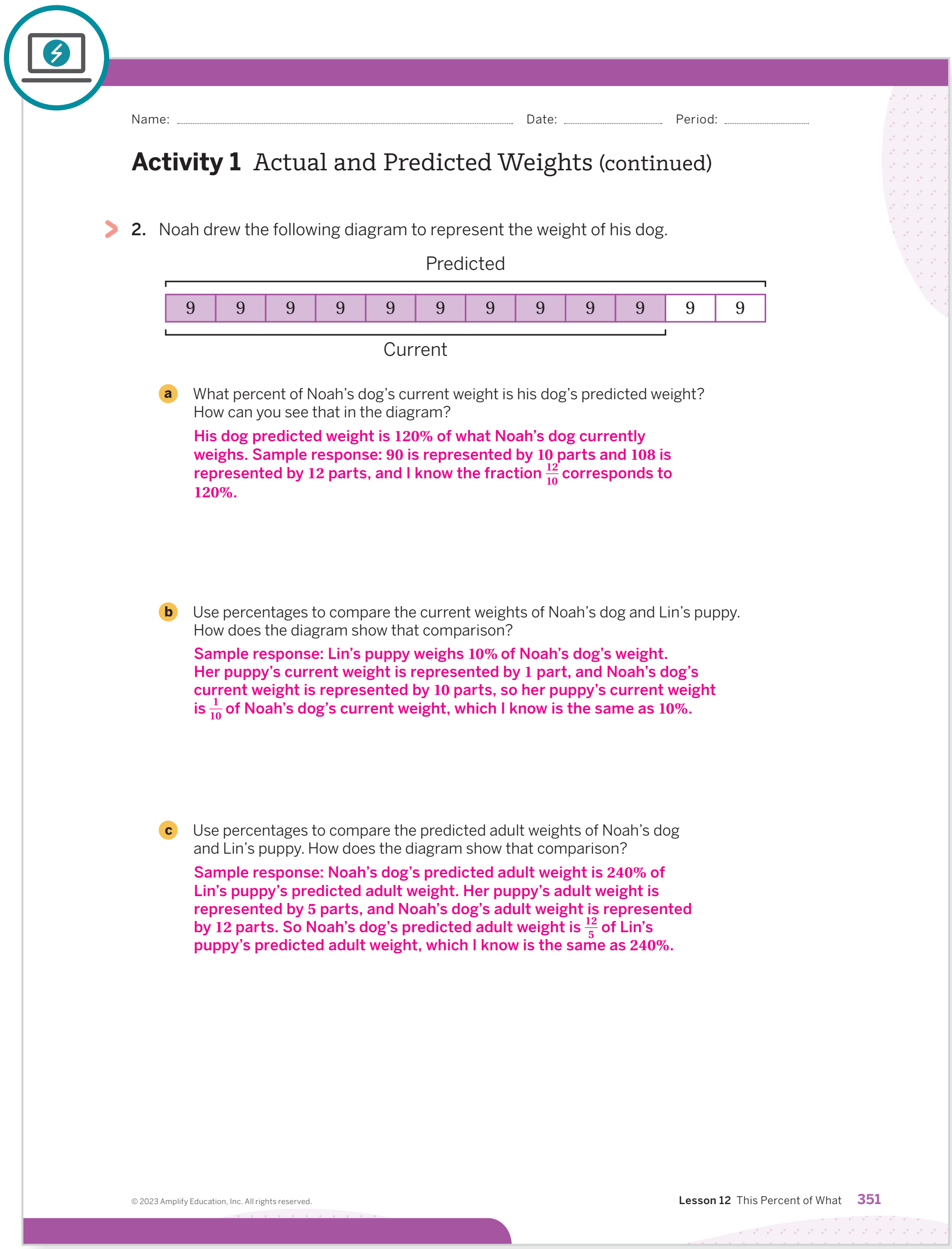
English Learners

Have students annotate the number line with "adult weight" and draw a circle around three sections of 9 to indicate 27 lb and $\frac{3}{5}$ of the adult weight.

Pairs | 15 min

Activity 1 Actual and Predicted Weights (continued)

Students use tape diagrams to relate and to interpret parts, wholes, fractions, and percentages.



Name: Date: Period:

Activity 1 Actual and Predicted Weights (continued)

2. Noah drew the following diagram to represent the weight of his dog.

a. What percent of Noah's dog's current weight is his dog's predicted weight? How can you see that in the diagram?

His dog predicted weight is 120% of what Noah's dog currently weighs. Sample response: 90 is represented by 10 parts and 108 is represented by 12 parts, and I know the fraction $\frac{12}{10}$ corresponds to 120%.

b. Use percentages to compare the current weights of Noah's dog and Lin's puppy. How does the diagram show that comparison?

Sample response: Lin's puppy weighs 10% of Noah's dog's weight. Her puppy's current weight is represented by 1 part, and Noah's dog's current weight is represented by 10 parts, so her puppy's current weight is $\frac{1}{10}$ of Noah's dog's current weight, which I know is the same as 10%.

c. Use percentages to compare the predicted adult weights of Noah's dog and Lin's puppy. How does the diagram show that comparison?

Sample response: Noah's dog's predicted adult weight is 240% of Lin's puppy's predicted adult weight. Her puppy's adult weight is represented by 5 parts, and Noah's dog's adult weight is represented by 12 parts. So Noah's dog's predicted adult weight is $\frac{12}{5}$ of Lin's puppy's predicted adult weight, which I know is the same as 240%.

Lesson 12 This Percent of What 351

3 Connect

Display the Activity 1 PDF.

Have pairs of students share how they determined each missing value and how the corresponding diagram shows their result and interpretation, focusing on identifying the part and whole that correspond to a percentage.

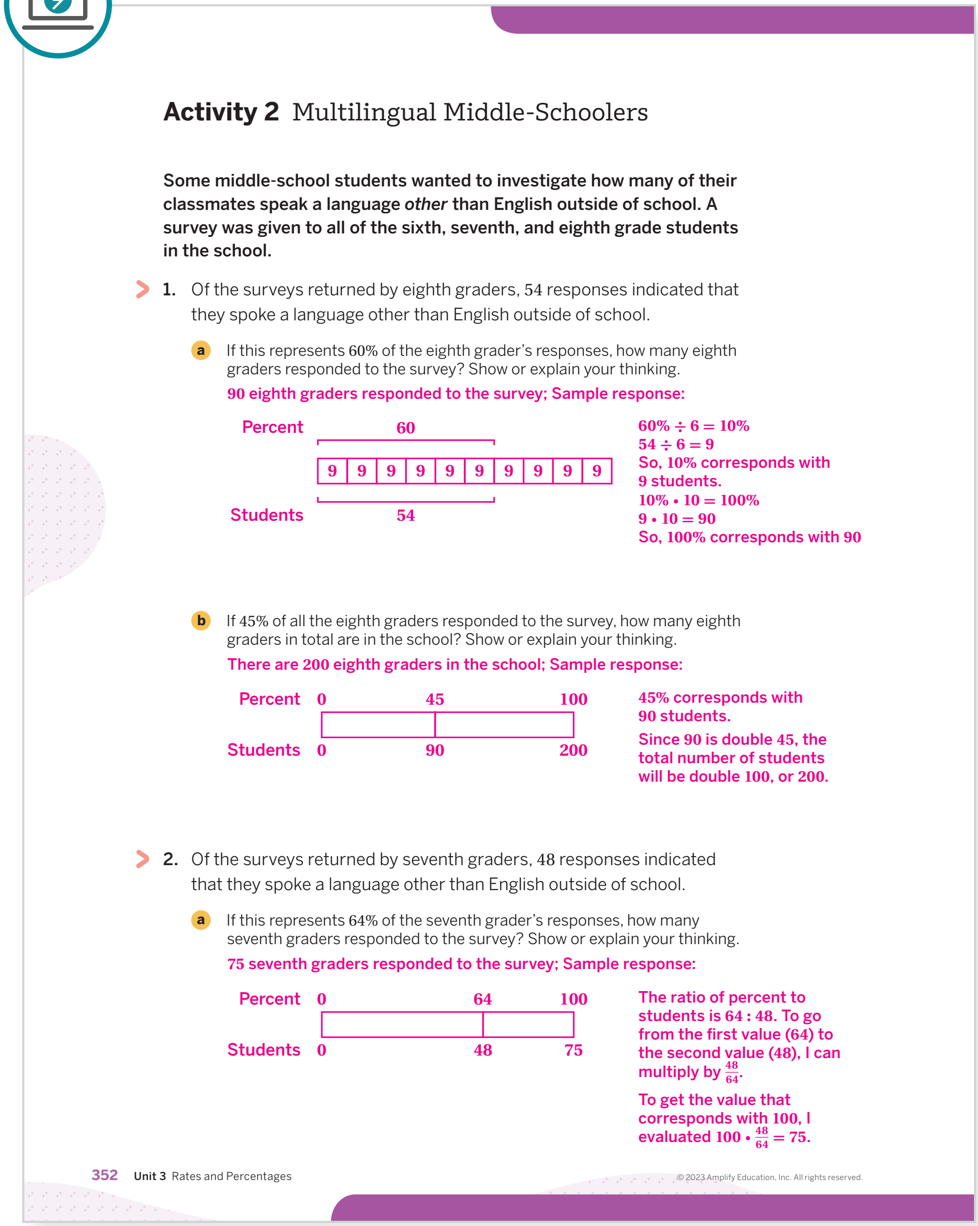
Ask, "How are the two diagrams similar? How are they different?"

Highlight how tape diagrams, which are familiar for representing fractions, make sense as representations for percentage problems because they show parts and wholes. Each partitioned area along a tape diagram represents both a percentage and, when known, a value for a quantity. It is important to label the whole as 100% clearly, especially when working with percentages greater than 100%. Clarify that in some cases it useful to use benchmark percentages to partition their diagrams to make sense of the problem, while in other cases, they may need to use the second model since benchmark percentages aren't given.

Pairs | 15 min

Activity 2 Multilingual Middle-Schoolers

Students interpret responses to a survey of language usage, leading them to develop a process for determining a missing whole when given a part and percentage.



Activity 2 Multilingual Middle-Schoolers

Some middle-school students wanted to investigate how many of their classmates speak a language *other* than English outside of school. A survey was given to all of the sixth, seventh, and eighth grade students in the school.

1. Of the surveys returned by eighth graders, 54 responses indicated that they spoke a language other than English outside of school.

   a. If this represents 60% of the eighth grader's responses, how many eighth graders responded to the survey? Show or explain your thinking.

   90 eighth graders responded to the survey; Sample response:

$60\% \div 6 = 10\%$
$54 \div 6 = 9$
So, 10% corresponds with 9 students.
$10\% \cdot 10 = 100\%$
$9 \cdot 10 = 90$
So, 100% corresponds with 90

   b. If 45% of all the eighth graders responded to the survey, how many eighth graders in total are in the school? Show or explain your thinking.

   There are 200 eighth graders in the school; Sample response:

45% corresponds with 90 students.

Since 90 is double 45, the total number of students will be double 100, or 200.

2. Of the surveys returned by seventh graders, 48 responses indicated that they spoke a language other than English outside of school.

   a. If this represents 64% of the seventh grader's responses, how many seventh graders responded to the survey? Show or explain your thinking.

   75 seventh graders responded to the survey; Sample response:

The ratio of percent to students is 64 : 48. To go from the first value (64) to the second value (48), I can multiply by $\frac{48}{64}$.

To get the value that corresponds with 100, I evaluated $100 \cdot \frac{48}{64} = 75$.

1 Launch

Explain that *multilingual* means "using more than one language." Consider activating background knowledge by polling your class about their use of other languages. Share that in 2017, 21.8% of U.S. residents reported using a language other than English at home. Make copies of the *Tape Diagrams* PDF available.

2 Monitor

Help students get started by asking students to identify what is known and what is unknown. Have them write percentage statements of the form "n% of ? is p."

Look for points of confusion:

- **Thinking the given number of responses is the whole.** Encourage students to use a tape diagram or other representation to help visualize and organize their thinking.
- **Using the given number of responses as the part for both questions in each scenario.** Have students use a tape diagram or other representation to see that "positive" responses < all responses < all students, so the number of responses is the part when all students form the whole.

Look for productive strategies:

- Recognizing that in all problems, the part and percentage are known, and the missing value is the whole.
- Using tape diagrams or other representations to organize their thinking and determine the missing values.
- Developing a process or algorithm to determine a missing whole, given a part and corresponding percentage.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Tools, Guide Processing and Visualization*

Provide copies of the *Tape Diagrams* PDF for students to use if they choose during the activity. Consider demonstrating how to create a tape diagram to represent Problem 1a. Provide colored pencils and ask students to color code where 60% corresponds to 54 students.

Math Language Development

MLR3: Critique, Correct, Clarify

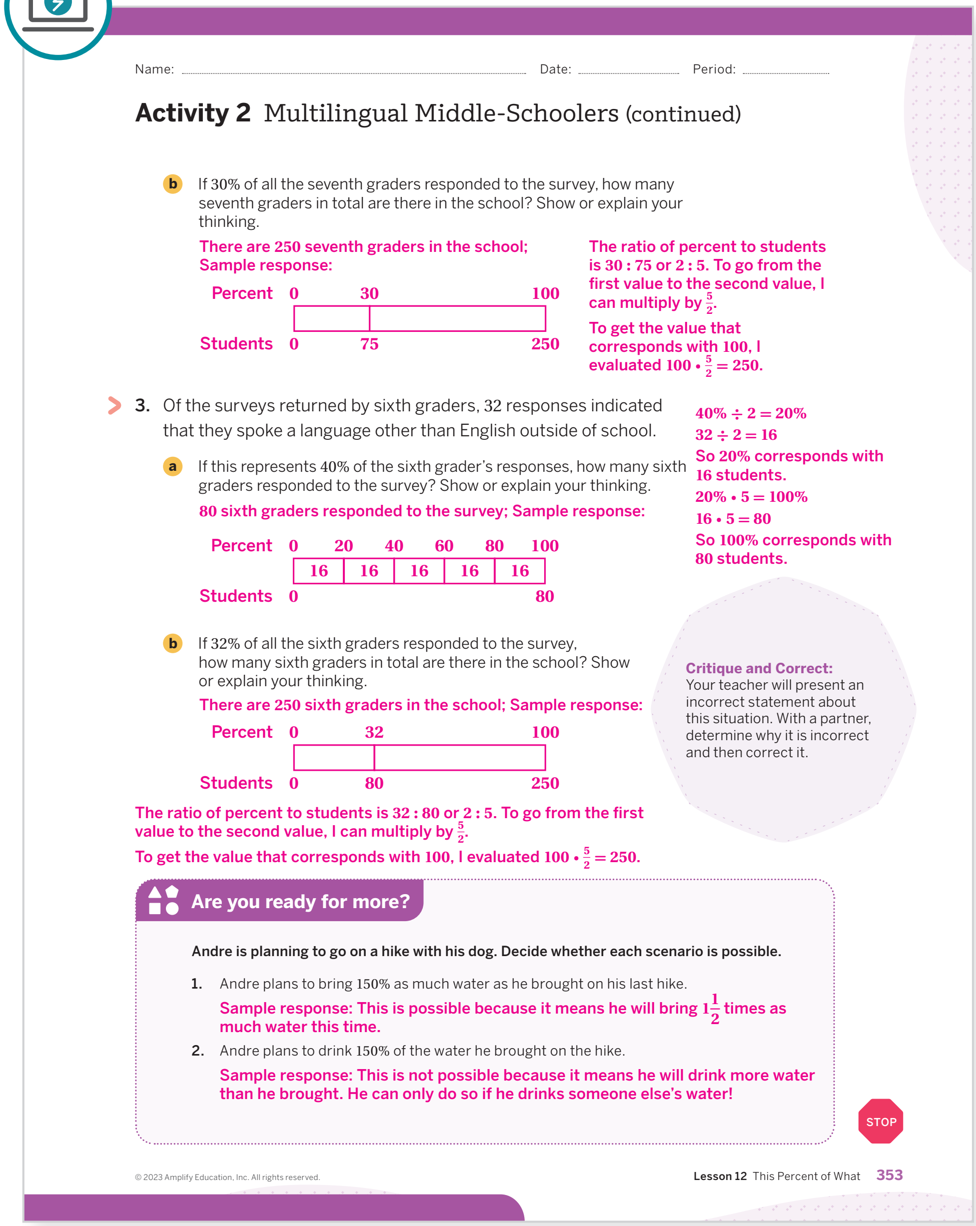
During the Connect, display an incorrect statement for Problem 3, such as, "If 32% of the sixth graders responded to the survey, then this means that there are about 25 students in the school, because 32% of 80 is about 25." Ask:

Critique: "Why is this statement incorrect?" Listen for those who recognize that it doesn't make sense to only have 25 students in the school, if 80 students responded to the survey.
Correct and Clarify: Have students write a corrected statement. Then have them explain how they know their statement is correct.

Pairs | 15 min

Activity 2 Multilingual Middle-Schoolers (continued)

Students interpret responses to a survey of language usage, leading them to develop a process for determining a missing whole when given a part and percentage.



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Activity 2 Multilingual Middle-Schoolers (continued)

b If 30% of all the seventh graders responded to the survey, how many seventh graders in total are there in the school? Show or explain your thinking.

There are 250 seventh graders in the school; Sample response:

Percent	0	30	100
Students	0	75	250

The ratio of percent to students is 30 : 75 or 2 : 5. To go from the first value to the second value, I can multiply by $\frac{5}{2}$.

To get the value that corresponds with 100, I evaluated $100 \cdot \frac{5}{2} = 250$.

3. Of the surveys returned by sixth graders, 32 responses indicated that they spoke a language other than English outside of school.

a If this represents 40% of the sixth grader's responses, how many sixth graders responded to the survey? Show or explain your thinking.

80 sixth graders responded to the survey; Sample response:

Percent	0	20	40	60	80	100
		16	16	16	16	16
Students	0					80

$40\% \div 2 = 20\%$
$32 \div 2 = 16$
So 20% corresponds with 16 students.
$20\% \cdot 5 = 100\%$
$16 \cdot 5 = 80$
So 100% corresponds with 80 students.

b If 32% of all the sixth graders responded to the survey, how many sixth graders in total are there in the school? Show or explain your thinking.

There are 250 sixth graders in the school; Sample response:

Percent	0	32	100
Students	0	80	250

The ratio of percent to students is 32 : 80 or 2 : 5. To go from the first value to the second value, I can multiply by $\frac{5}{2}$.

To get the value that corresponds with 100, I evaluated $100 \cdot \frac{5}{2} = 250$.

Critique and Correct:
Your teacher will present an incorrect statement about this situation. With a partner, determine why it is incorrect and then correct it.

Are you ready for more?

Andre is planning to go on a hike with his dog. Decide whether each scenario is possible.

1. Andre plans to bring 150% as much water as he brought on his last hike.
   Sample response: This is possible because it means he will bring $1\frac{1}{2}$ times as much water this time.
2. Andre plans to drink 150% of the water he brought on the hike.
   Sample response: This is not possible because it means he will drink more water than he brought. He can only do so if he drinks someone else's water!

STOP

Lesson 12 This Percent of What 353

3 Connect

Display student representations or blank tape diagrams from the *Tape Diagrams* PDF as needed.

Have individual students share how they determined each missing value, focusing on repeated reasoning across the problems for each of the three grades.

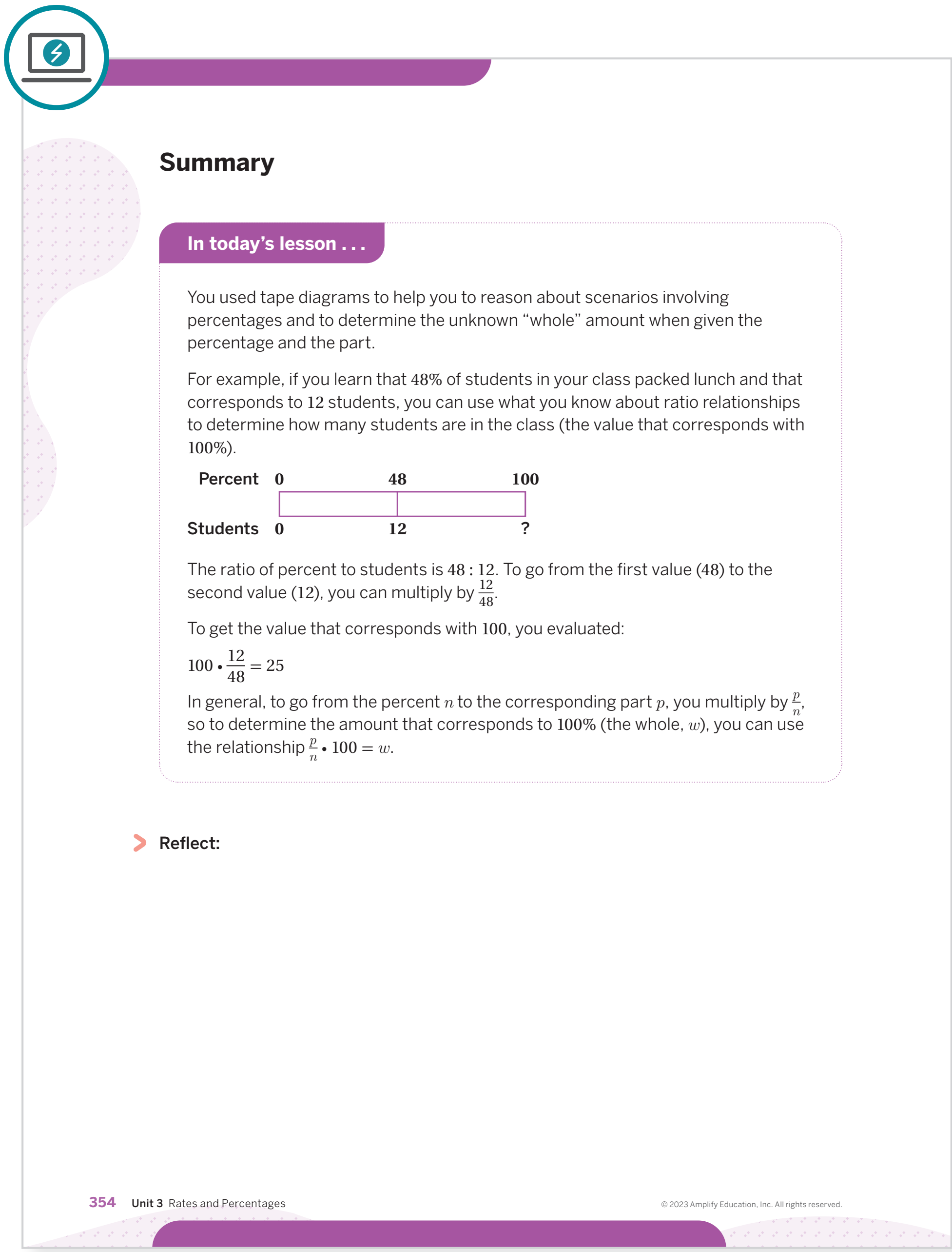
Highlight that when the part and percentage are known, a missing whole can be determined by multiplying the part by 100 and then dividing by the value corresponding to the percentage (per 100). This algorithm can be written as $p \cdot \frac{100}{n}$ or $\frac{p}{n} \cdot 100$.

Ask, "Why is it true that, for each grade, there were greater percentages of "positive" responses (part a's), but the total numbers of students were greater (part b's)?"

Whole Class | 5 min

Summary

Review and synthesize how tape diagrams can be used to represent and to make sense of percentage problems, and also the process for determining a missing whole.



Summary

In today's lesson . . .

You used tape diagrams to help you to reason about scenarios involving percentages and to determine the unknown "whole" amount when given the percentage and the part.

For example, if you learn that 48% of students in your class packed lunch and that corresponds to 12 students, you can use what you know about ratio relationships to determine how many students are in the class (the value that corresponds with 100%).

The ratio of percent to students is $48 : 12$. To go from the first value (48) to the second value (12), you can multiply by $\frac{12}{48}$.

To get the value that corresponds with 100, you evaluated:

$$100 \bullet \frac{12}{48} = 25$$

In general, to go from the percent n to the corresponding part p, you multiply by $\frac{p}{n}$, so to determine the amount that corresponds to 100% (the whole, w), you can use the relationship $\frac{p}{n} \bullet 100 = w$.

Reflect:

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Synthesize

Display the *Percentage Algorithms* PDF and tape diagram from the Summary.

Highlight that students have now seen percentage problems involving missing percentages, missing parts, and missing wholes. Tables, double number lines, and tape diagrams can be used to represent all three scenarios, and there is a process that can be followed to calculate each type of missing value. Sometimes, thinking about benchmark percentages and doing mental calculations may still be more efficient.

Ask:

- "How is determining the "whole" similar to or different from determining the percentage or the part?"
- "How would you use the algorithm to determine what number 28 is 140% of?"

Have students share their responses, noting how the three algorithms are related. Consider also having students share how to construct a corresponding tape diagram for each from a blank tape diagram.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

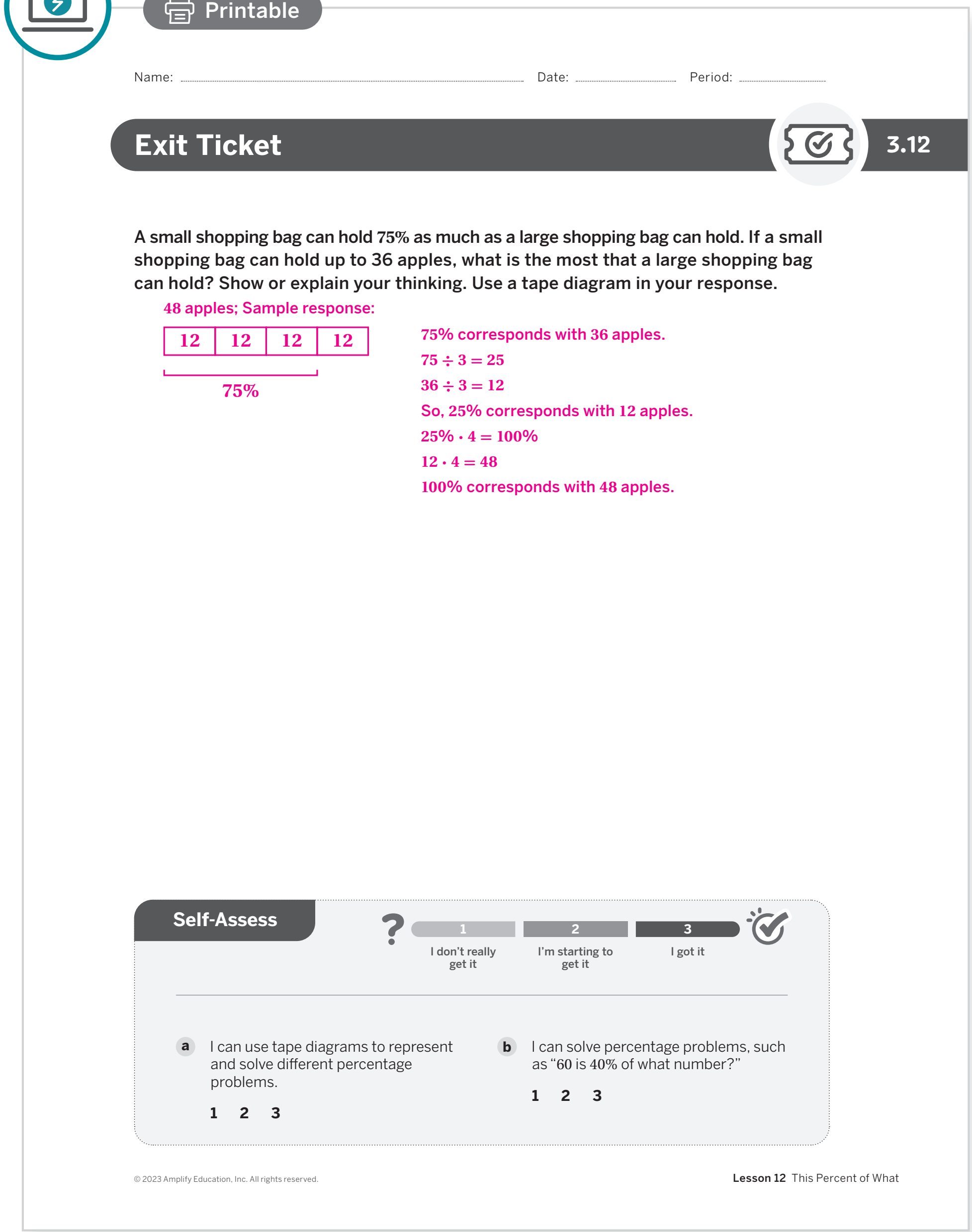
- "How is determining a missing whole similar to determining a missing part or percentage? How is it different?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of using tape diagrams to represent percentages and determining a missing whole.

Printable



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Exit Ticket 3.12

A small shopping bag can hold 75% as much as a large shopping bag can hold. If a small shopping bag can hold up to 36 apples, what is the most that a large shopping bag can hold? Show or explain your thinking. Use a tape diagram in your response.

48 apples; Sample response:

12	12	12	12

75%

75% corresponds with 36 apples.

$75 \div 3 = 25$

$36 \div 3 = 12$

So, 25% corresponds with 12 apples.

$25\% \cdot 4 = 100\%$

$12 \cdot 4 = 48$

100% corresponds with 48 apples.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can use tape diagrams to represent and solve different percentage problems.

1 2 3

b I can solve percentage problems, such as "60 is 40% of what number?"

1 2 3

Lesson 12 This Percent of What

Success looks like . . .

- **Language Goal:** Drawing and labeling a tape diagram to represent a situation involving percentages. **(Writing)**
  - » Creating a tape diagram to show reasoning involving percentages in Problem 2.
- **Language Goal:** Comprehending a sentence, such as "n% of ? is p." to refer to the value that makes a ratio with p that is equivalent to $n : 100$. **(Speaking and Listening, Writing)**
  - » Determining "n% of ? is p." in Problem 1.
- **Language Goal:** Explaining how to solve problems, such as "n% of ? is p." **(Speaking and Listening, Writing)**

Suggested next steps

If students cannot determine how much a large bag holds in Problem 1, consider:

- Reviewing the strategies from Activity 1.
- Assigning Practice Problem 3.
- Asking, "How could you set up a number sentence to show what you're solving for?"

If students struggle to create a diagram to illustrate the answer in Problem 2, consider:

- Walking through the steps of creating the diagram. Ask,
- "How many partitions would you need?"
- "How would you show 75% on the diagram?"
- Assigning Practice Problem 1.

Professional Learning

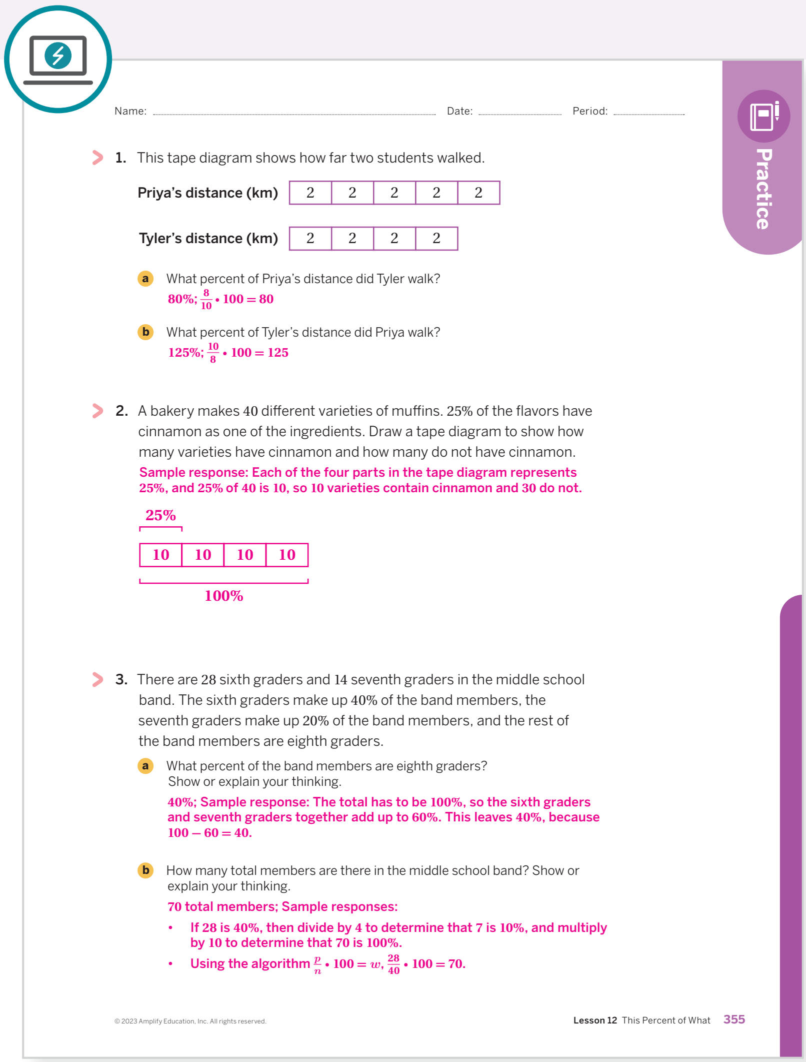
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- One focus of this lesson was using tape diagrams to understand percentages. How did that go?
- Thinking about the questions you asked students today and what students said or did as a result of the questions, which questions were the most effective?

Practice

Independent



Name: Date: Period:

Practice

1. This tape diagram shows how far two students walked.

Priya's distance (km)	2	2	2	2	2
Tyler's distance (km)	2	2	2	2	

a. What percent of Priya's distance did Tyler walk?
80%; $\frac{8}{10} \cdot 100 = 80$

b. What percent of Tyler's distance did Priya walk?
125%; $\frac{10}{8} \cdot 100 = 125$

2. A bakery makes 40 different varieties of muffins. 25% of the flavors have cinnamon as one of the ingredients. Draw a tape diagram to show how many varieties have cinnamon and how many do not have cinnamon.
Sample response: Each of the four parts in the tape diagram represents 25%, and 25% of 40 is 10, so 10 varieties contain cinnamon and 30 do not.

25%

10	10	10	10

100%

3. There are 28 sixth graders and 14 seventh graders in the middle school band. The sixth graders make up 40% of the band members, the seventh graders make up 20% of the band members, and the rest of the band members are eighth graders.

a. What percent of the band members are eighth graders? Show or explain your thinking.
40%; Sample response: The total has to be 100%, so the sixth graders and seventh graders together add up to 60%. This leaves 40%, because $100 - 60 = 40$.

b. How many total members are there in the middle school band? Show or explain your thinking.
70 total members; Sample responses:
- **If 28 is 40%, then divide by 4 to determine that 7 is 10%, and multiply by 10 to determine that 70 is 100%.**
- **Using the algorithm $\frac{p}{n} \cdot 100 = w$, $\frac{28}{40} \cdot 100 = 70$.**

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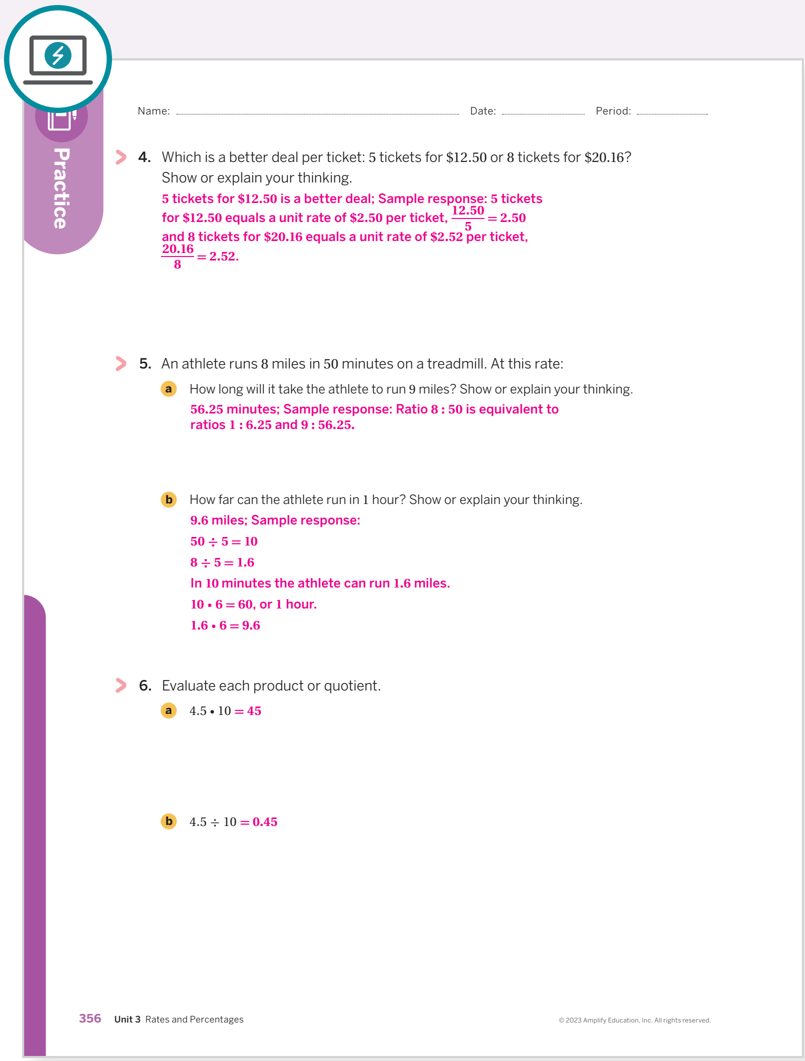
Lesson 12 This Percent of What 355



Name: Date: Period:

Practice

4. Which is a better deal per ticket: 5 tickets for \$12.50 or 8 tickets for \$20.16? Show or explain your thinking.
5 tickets for \$12.50 is a better deal; Sample response: 5 tickets for \$12.50 equals a unit rate of \$2.50 per ticket, $\frac{12.50}{5} = 2.50$ and 8 tickets for \$20.16 equals a unit rate of \$2.52 per ticket, $\frac{20.16}{8} = 2.52$.

5. An athlete runs 8 miles in 50 minutes on a treadmill. At this rate:

a. How long will it take the athlete to run 9 miles? Show or explain your thinking.
56.25 minutes; Sample response: Ratio 8 : 50 is equivalent to ratios 1 : 6.25 and 9 : 56.25.

b. How far can the athlete run in 1 hour? Show or explain your thinking.
9.6 miles; Sample response:
$50 \div 5 = 10$
$8 \div 5 = 1.6$
In 10 minutes the athlete can run 1.6 miles.
$10 \cdot 6 = 60$, or 1 hour.
$1.6 \cdot 6 = 9.6$

6. Evaluate each product or quotient.

a. $4.5 \cdot 10 = 45$

b. $4.5 \div 10 = 0.45$

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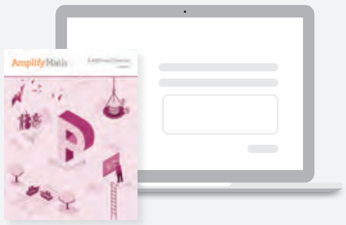
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 7	2
	5	Unit 3 Lesson 4	2
Formative	6	Unit 3 Lesson 13	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 3 | **LESSON 13**

Solving Percentage Problems

Let's solve more percentage problems.

Focus

Goals

1. Apply reasoning about percentages to solve more complex real-world problems involving percentages.
2. **Language Goal:** Explain the solution methods by using multiple representations to solve problems involving percentages. **(Speaking and Listening, Writing)**
3. Determine what information is needed to solve a problem involving percentages.

Coherence

Today

Students practice solving more percentage problems, but with less support. They have opportunities to choose appropriate abstract or quantitative representations and strategies seen throughout the unit. Drawing a double number line is still a good strategy, but students may opt for tables or even more abbreviated reasoning methods, such as using algorithms to write and to evaluate expressions or equations. The students work to solve applications of percentages in real-world scenarios, such as reporting data in the media and determining the best deal when presented with a variety of discounting methods.

Previously

In Lessons 8–12, students saw that a percentage is a rate per 100. They used double number line diagrams to develop generalized processes for solving problems involving percentages and three different potential missing values: the percent, the part, and the whole.

Coming Soon

In Lesson 14, students will use ratios and percentages of the world's population across a variety of demographics to determine how the class would look when the percentages of the students' demographic categories match that of the entire world's.

Rigor

- Students work with all three types of percentage problems to solidify **procedural skills** for determining missing values.
- Students **apply** their understanding of percentages to different real-world scenarios, such as discounted items.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (instructions, for display)
- *Prize Cards* PDF, pre-cut cards
- *Double Number Lines: Percentage Problems* PDF (as needed)
- *Percentage Algorithms* PDF
- *Tape Diagrams* PDF (as needed)

Math Language Development

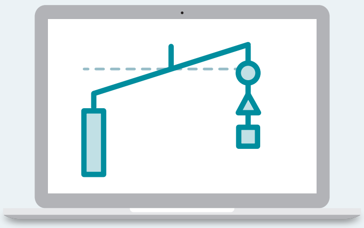
Review words

- *percent*
- *percentage*

Amps | **Featured Activity**

Activity 2
Virtual Game Show

Students calculate prices using percentages, and then make decisions on a virtual game show based on their calculations.



Amps
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Building Math Identity and Community

Connecting to Mathematical Practices

Throughout these activities, students might be overwhelmed by the process of determining the part, whole, and percent in each problem, as each quantity may vary in how it is expressed in a verbal description. As students reason about the quantities, have them take a step back and consider how to motivate themselves to persist. They should think about ways to search for and identify patterns even when they are not obvious.

Modifications to Pacing

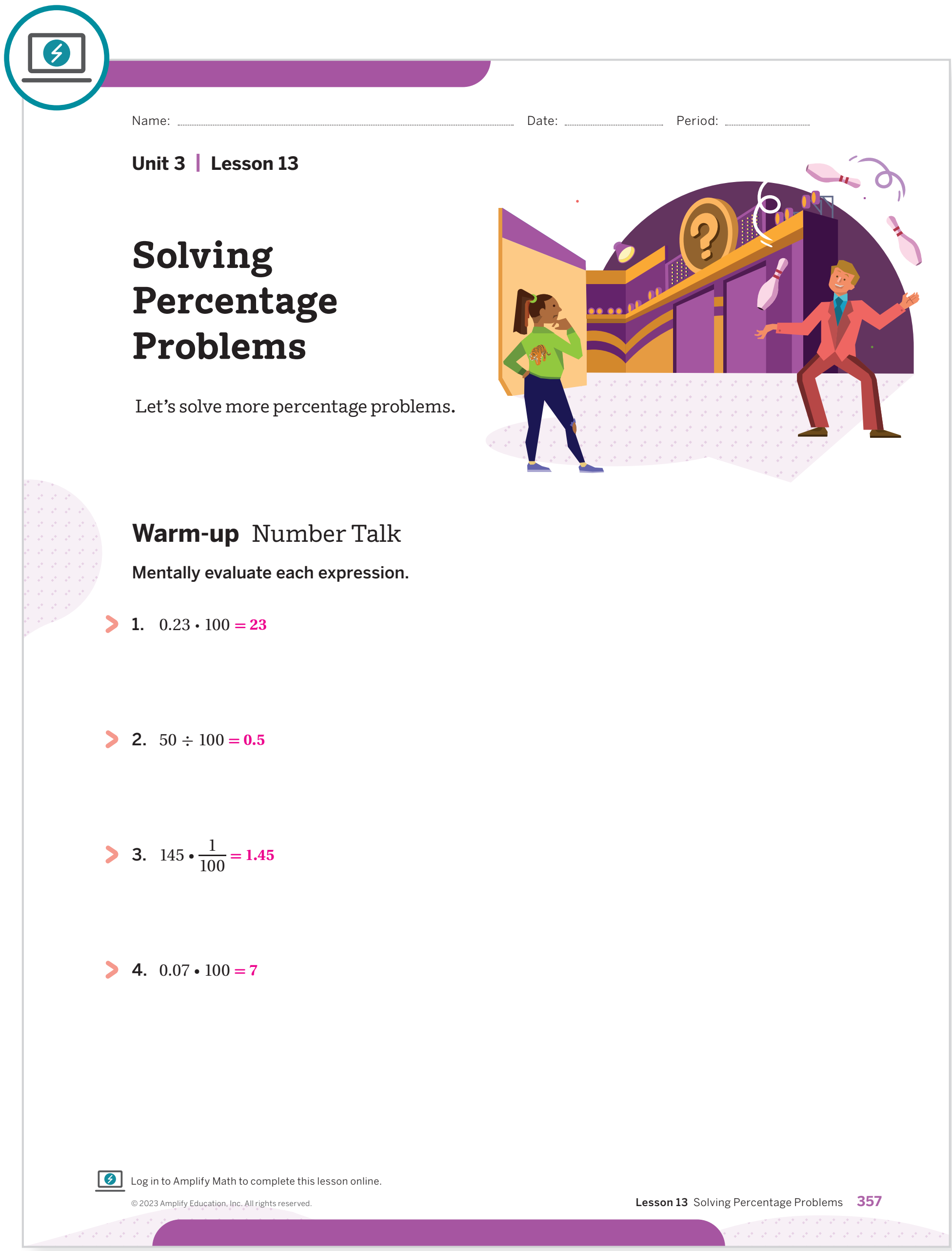
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, do not have students write the headlines. This activity may be done as a whole class, as well.

Independent | 5 min

Warm-up Number Talk

Students review the connections between place value and multiplication and division by 100 to help with their calculations involving percentages in the rest of the lesson.



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Unit 3 | Lesson 13

Solving Percentage Problems

Let's solve more percentage problems.

Warm-up Number Talk

Mentally evaluate each expression.

1. $0.23 \cdot 100 = 23$

2. $50 \div 100 = 0.5$

3. $145 \cdot \frac{1}{100} = 1.45$

4. $0.07 \cdot 100 = 7$

Log in to Amplify Math to complete this lesson online.

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Lesson 13 Solving Percentage Problems 357

1 Launch

Conduct the ***Number Talk*** routine. Display one problem at a time, keeping previous problems displayed. Give students 30 seconds of think time for each problem.

2 Monitor

Help students get started by suggesting they check the reasonableness of their answers by asking themselves questions like, "Should the result be greater than or less than: the first number in the expression? 100? 1?"

Look for points of confusion:

- **"Moving" the decimal point in the wrong direction or the wrong number of places.** Ask, "How do place values change when you multiply/divide by 10?"
- **Using 100 (as in Problem 1) instead of $\frac{1}{100}$ in Problem 3.** Ask, "How is dividing by one hundredth different from dividing by 100?"

Look for productive strategies:

- Recognizing and using the relationship between multiplication and division.
- Understanding how place value affects the location of the decimal when multiplying or dividing by 100.

3 Connect

Have individual students share their responses, collecting a variety of strategies for each problem.

Ask:

- "How is multiplying by $\frac{1}{100}$ related to division?"
- "What is important to remember when dividing a one digit number by 100?"

Highlight that, depending on the given information, multiplying and dividing by 100 is often necessary when working with percentages because percentages are a rate per 100.

Math Language Development

MLR8: Discussion Supports

During the Connect, consider asking these additional probing questions:

- "For which expressions is the result 100 times greater than the first factor or the dividend?" The expressions in Problems 1 and 4.
- "For which expressions is the result 100 times less than the first factor or the dividend?" The expressions in Problems 2 and 3.
- "Will multiplying a value by 100 or by $\frac{1}{100}$ produce a product that is 100 times greater than the value?" Multiplying by 100. "100 times less?" Multiplying by $\frac{1}{100}$.
- "Will dividing a value by 100 produce a quotient that is 100 times greater or 100 times less than the original value?" 100 times less

Power-up

To power up students' ability to connect place value to multiplication and division of decimals values by 10, have students complete:

Recall that multiplication and division are related operations, and knowing the solution to one problem can help you solve a related problem.

a. $3.2 \cdot 10 = 32$

b. $3.2 \cdot 1 = 3.2$

c. $3.2 \cdot \frac{1}{10} = 0.32$

d. $3.2 \div 10 = 0.32$

e. $3.2 \div 1 = 3.2$

f. $3.2 \div \frac{1}{10} = 32$

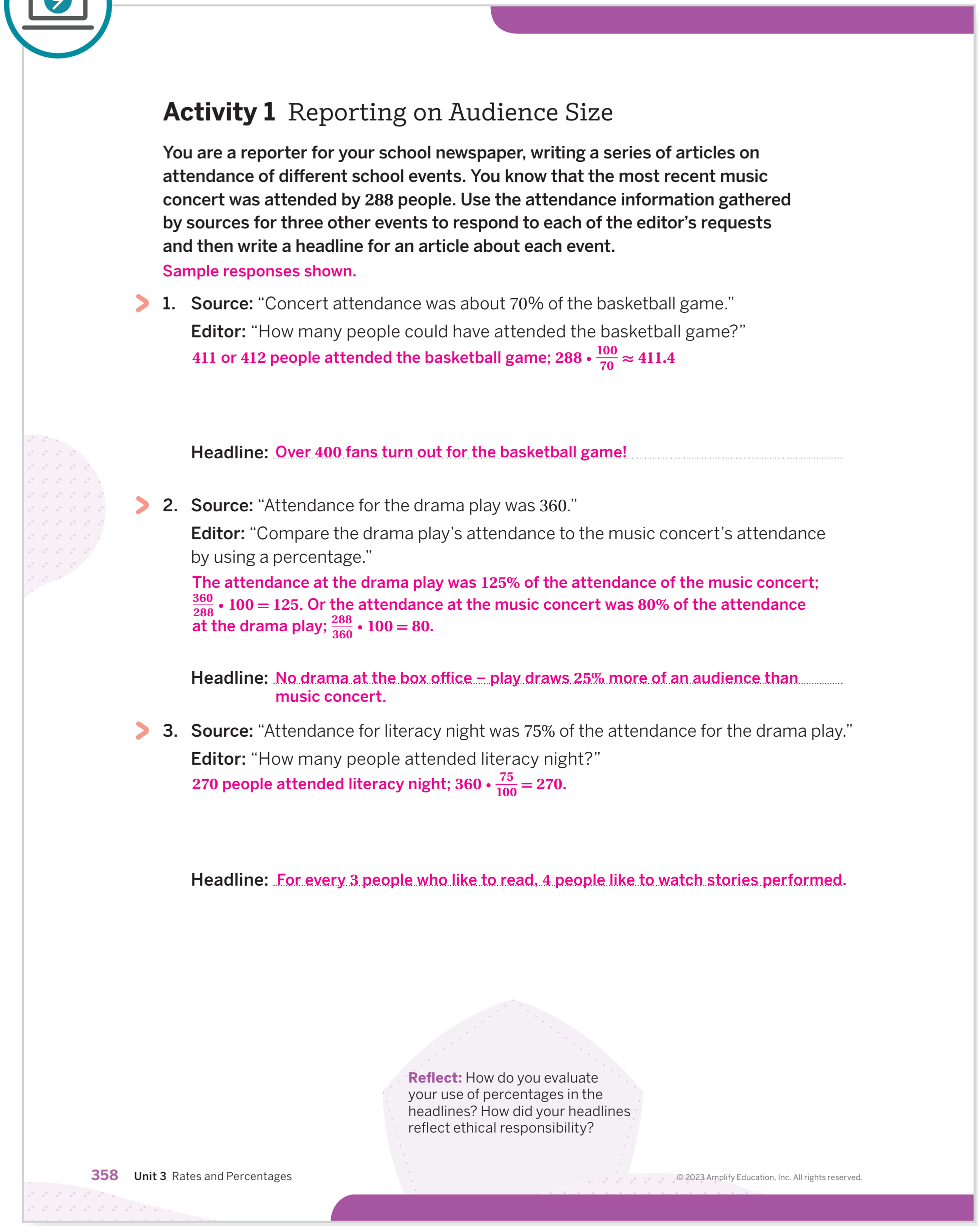
Use: Before the Warm-up.

Informed by: Performance on Lesson 12, Practice Problem 6.

Pairs | 10 min

Activity 1 Reporting on Audience Size

Students interpret three scenarios to determine different missing values in percentage problems — part, whole, and percent.



Activity 1 Reporting on Audience Size

You are a reporter for your school newspaper, writing a series of articles on attendance of different school events. You know that the most recent music concert was attended by 288 people. Use the attendance information gathered by sources for three other events to respond to each of the editor's requests and then write a headline for an article about each event.

Sample responses shown.

1. **Source:** "Concert attendance was about 70% of the basketball game."

   Editor: "How many people could have attended the basketball game?"

   411 or 412 people attended the basketball game; $288 \cdot \frac{100}{70} \approx 411.4$

   Headline: Over 400 fans turn out for the basketball game!

2. **Source:** "Attendance for the drama play was 360."

   Editor: "Compare the drama play's attendance to the music concert's attendance by using a percentage."

   The attendance at the drama play was 125% of the attendance of the music concert; $\frac{360}{288} \cdot 100 = 125$. Or the attendance at the music concert was 80% of the attendance at the drama play; $\frac{288}{360} \cdot 100 = 80$.

   Headline: No drama at the box office – play draws 25% more of an audience than music concert.

3. **Source:** "Attendance for literacy night was 75% of the attendance for the drama play."

   Editor: "How many people attended literacy night?"

   270 people attended literacy night; $360 \cdot \frac{75}{100} = 270$.

   Headline: For every 3 people who like to read, 4 people like to watch stories performed.

Reflect: How do you evaluate your use of percentages in the headlines? How did your headlines reflect ethical responsibility?

358 **Unit 3** Rates and Percentages

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1 Launch

Consider displaying, distributing, or having copies available of the *Percentage Algorithms* PDF.

2 Monitor

Help students get started by asking, "What information do you have? What do you need to determine?"

Look for points of confusion:

- **Incorrectly setting up an algorithm.** Have them refer to the *Percentage Algorithms* PDF.

Look for productive strategies:

- Using equivalent ratios or other rate reasoning.
- Identifying the missing value as a part, whole, or percentage, and applying the proper known representation, expression or algorithm correctly.
- Recognizing and using benchmark percentages.
- Understanding that a fraction of a person could be interpreted by rounding up.

3 Connect

Have pairs of students share their responses and different strategies for each problem, focusing on ratio reasoning, and then allow several pairs to share their headlines, one problem at a time.

Ask, "How can it be that attendance at the music concert was 70% and attendance at literacy night was 75%, but more people attended the music concert?"

Highlight that each of the three possible missing values to be determined in percentage problems were presented here: Problem 1 – whole, Problem 2 – percentage, and Problem 3 – part. In each case equivalent ratios could be used, but there are also efficient strategies (or algorithms) to determine each quantity more directly.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Optimize Access to Tools*

The audience size of the music concert could be changed from 288 to 300, which simplifies the dependent calculations from problem to problem, thus making a double number line more accessible. If you choose to alter this value, provide copies of the *Double Number Lines: Percentage Problems* PDF for students to use during the activity.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies, make sure you hear from students with different strategies for each problem. Encourage students to make comparisons and connections between when they are able to use familiar percentages and when they cannot.

English Learners

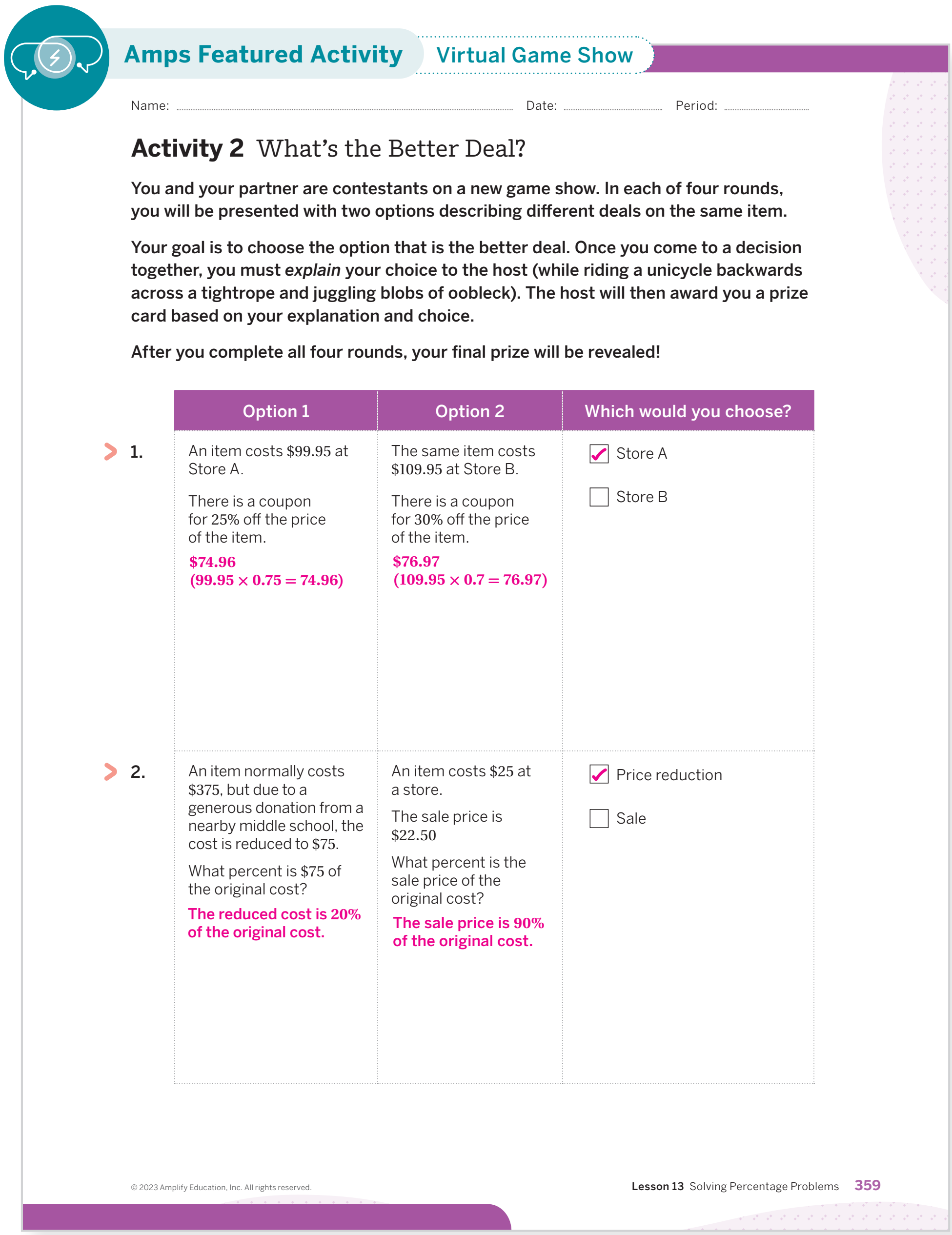
Have students refer to the class display to support their use of mathematical language.

Pairs | 20 min

Activity 2 What's the Better Deal?

Students continue to practice determining missing parts, wholes, and percentages in a game show setting involving different types of retail discounts.

Amps Featured Activity Virtual Game Show



Name: ______ Date: ______ Period: ______

Activity 2 What's the Better Deal?

You and your partner are contestants on a new game show. In each of four rounds, you will be presented with two options describing different deals on the same item.

Your goal is to choose the option that is the better deal. Once you come to a decision together, you must *explain* your choice to the host (while riding a unicycle backwards across a tightrope and juggling blobs of oobleck). The host will then award you a prize card based on your explanation and choice.

After you complete all four rounds, your final prize will be revealed!

	Option 1	Option 2	Which would you choose?
1.	An item costs $99.95 at Store A. There is a coupon for 25% off the price of the item. **$74.96** **(99.95 × 0.75 = 74.96)**	The same item costs $109.95 at Store B. There is a coupon for 30% off the price of the item. **$76.97** **(109.95 × 0.7 = 76.97)**	☑ Store A ☐ Store B
2.	An item normally costs $375, but due to a generous donation from a nearby middle school, the cost is reduced to $75. What percent is $75 of the original cost? **The reduced cost is 20% of the original cost.**	An item costs $25 at a store. The sale price is $22.50 What percent is the sale price of the original cost? **The sale price is 90% of the original cost.**	☑ Price reduction ☐ Sale

1 Launch

Reference the first page of the Activity 2 PDF (instructions) to explain how each pair will participate in the game show. Have copies of the *Prize Cards* PDF available to distribute. Calculators may be made available. Before starting the activity consider asking, "If an item on sale is 30% off, what percentage of the normal price would you pay?" 70%

2 Monitor

Help students get started by asking, "What do you know and what do you need to determine?" Consider also suggesting pairs work with "friendlier" values first to determine a process before using the given values.

Look for points of confusion:

- **Not identifying what is being solved for.** Ask, "What do you need to determine in this problem: the part, whole, or percent?"
- **Getting stuck trying to use a double number line.** Refer to the *Percentage Algorithms* PDF.
- **Having trouble understanding and organizing the information given in each problem.** Have students reread the problem and help them organize the information given.
  - » Problem 1: Assuming that the greater percentage off must result in the better deal.
  - » Problem 2: Thinking that the price was reduced *by* $75 instead of being reduced *to* $75.
  - » Problem 3: Forgetting to account for the full price of the item in Problem 3, Store C, or thinking 33% off at Store D is for only the second pair.
  - » Problem 4: Comparing just the original prices for a one month supply.

Look for productive strategies:

- Identifying the unknown and choosing an appropriate algorithm, representation, or strategy.
- Distinguishing percent off from percent paid.

Activity 2 continued >

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider rounding dollar amounts, such as $100 in Problem 1 and $110 in Problem 2. This will still allow students to participate in the mathematical goal of the activity, but will simplify calculations.

Accessibility: *Optimize Access to Technology, Optimize Access to Tools*

Have students use the Amps slides for this activity, in which they can select from a menu of digital tools to show their thinking, such as double number lines, tables, tape diagrams, or free-form sketches.
If you choose to use the print version for this activity, provide copies of the *Double Number lines: Percentage Problems* and *Tape Diagrams* PDFs.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share how they determined the better deal, ask them to make connections between the various representations.
Ask:

- "Can you think of another way or another representation you can use to verify your response?"
- "Which representation do you think is the most efficient? Why?"
- "Which representation(s) help you visualize the relationships?"

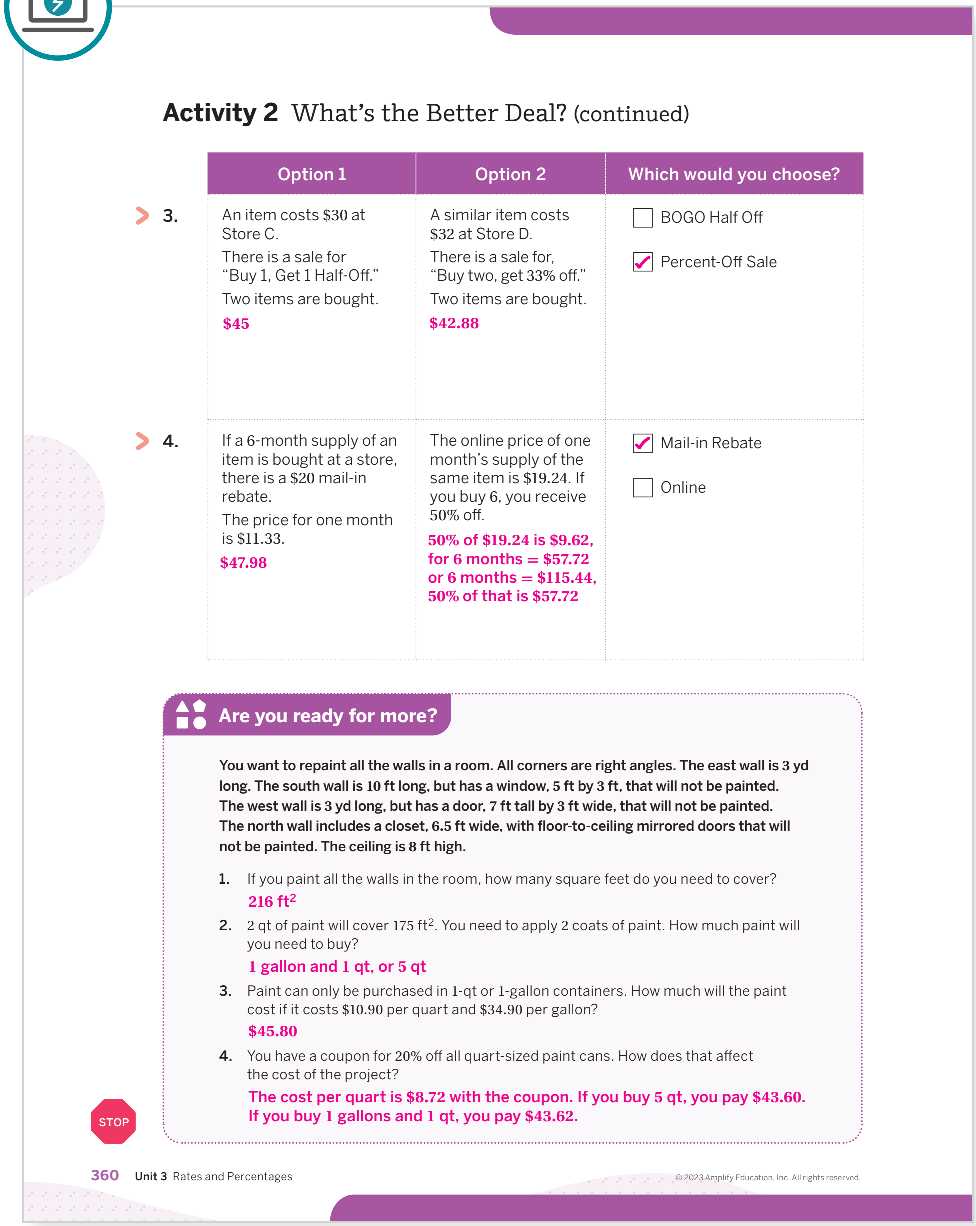
English Learners

Display and annotate various representations that can be used to determine the better deal for one of the problems.

Pairs | 20 min

Activity 2 What's the Better Deal? (continued)

Students continue to practice determining missing parts, wholes, and percentages in a game show setting involving different types of retail discounts.



Activity 2 What's the Better Deal? (continued)

	Option 1	Option 2	Which would you choose?
3.	An item costs \$30 at Store C. There is a sale for "Buy 1, Get 1 Half-Off." Two items are bought. \$45	A similar item costs \$32 at Store D. There is a sale for, "Buy two, get 33% off." Two items are bought. \$42.88	☐ BOGO Half Off ☑ Percent-Off Sale
4.	If a 6-month supply of an item is bought at a store, there is a \$20 mail-in rebate. The price for one month is \$11.33. \$47.98	The online price of one month's supply of the same item is \$19.24. If you buy 6, you receive 50% off. 50% of \$19.24 is \$9.62, for 6 months = \$57.72 or 6 months = \$115.44, 50% of that is \$57.72	☑ Mail-in Rebate ☐ Online

Are you ready for more?

You want to repaint all the walls in a room. All corners are right angles. The east wall is 3 yd long. The south wall is 10 ft long, but has a window, 5 ft by 3 ft, that will not be painted. The west wall is 3 yd long, but has a door, 7 ft tall by 3 ft wide, that will not be painted. The north wall includes a closet, 6.5 ft wide, with floor-to-ceiling mirrored doors that will not be painted. The ceiling is 8 ft high.

1. If you paint all the walls in the room, how many square feet do you need to cover?
   216 ft^2
2. 2 qt of paint will cover 175 ft^2. You need to apply 2 coats of paint. How much paint will you need to buy?
   1 gallon and 1 qt, or 5 qt
3. Paint can only be purchased in 1-qt or 1-gallon containers. How much will the paint cost if it costs \$10.90 per quart and \$34.90 per gallon?
   \$45.80
4. You have a coupon for 20% off all quart-sized paint cans. How does that affect the cost of the project?
   The cost per quart is \$8.72 with the coupon. If you buy 5 qt, you pay \$43.60. If you buy 1 gallons and 1 qt, you pay \$43.62.

STOP

360 Unit 3 Rates and Percentages

3 Connect

Display the final prize images from the second page of the Activity 2 PDF (instructions).

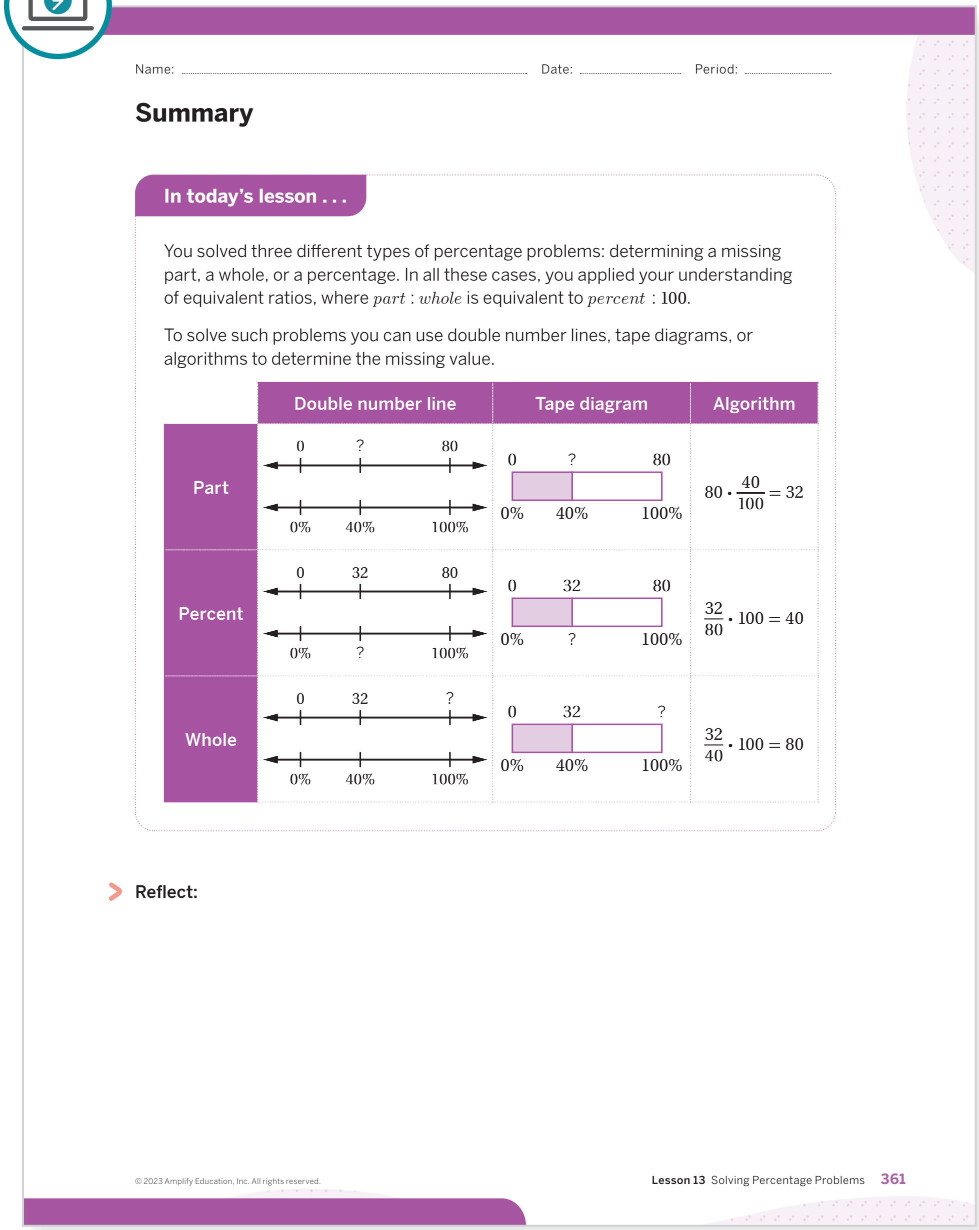
Have pairs of students share first, for the options in each round, "How were you thinking about the meaning of 'a better deal'?" Then have pairs share explanations of their choices, including those that chose the more obvious/correct option and those that made convincing arguments for the other option. Emphasize how students determined their strategy or the algorithm to use based on the information given and the information they were trying to determine. Connect any representations some groups used (such as double number lines or tables) to the expressions, algorithms, or calculations of other groups.

Highlight that the representations used up until this point are helpful to visualize the math, but sometimes it is more efficient to use an algorithm.

Ask, "How was this activity the same or different from Activity 2 in Lesson 7, which was also about deals and prices, but before we discussed percentages?" Both were about finding the better deal, but, in Lesson 7, I used unit price and today I used percentages.

Summary

Review and synthesize strategies used for finding part, whole, and percent in percentage problems.



Name: ____________ Date: ________ Period: ________

Summary

In today's lesson . . .

You solved three different types of percentage problems: determining a missing part, a whole, or a percentage. In all these cases, you applied your understanding of equivalent ratios, where $part : whole$ is equivalent to $percent : 100$.

To solve such problems you can use double number lines, tape diagrams, or algorithms to determine the missing value.

	Double number line	Tape diagram	Algorithm
Part	0, ?, 80 0%, 40%, 100%	0, ?, 80 0%, 40%, 100%	$80 \cdot \frac{40}{100} = 32$
Percent	0, 32, 80 0%, ?, 100%	0, 32, 80 0%, ?, 100%	$\frac{32}{80} \cdot 100 = 40$
Whole	0, 32, ? 0%, 40%, 100%	0, 32, ? 0%, 40%, 100%	$\frac{32}{40} \cdot 100 = 80$

Reflect:

Synthesize

Display the three representations, and also display or reference the *Percentage Algorithms* PDF as needed.

Ask:

- "What were some ways you found helpful for identifying missing values in percentage problems?"
- "What are some benefits or using each representation to solve problems involving percentages 0?"

Have individual students share responses to the questions, referencing the diagrams in the Summary and the *Percentage Algorithms* PDF as necessary.

Highlight how the mathematics developed over the course of this unit so far, from rates and unit rates per 1 to percentages per 100, are all related by the larger concept of ratios (and specifically equivalent ratios), which was seen in real-world applications in this lesson. These problems have many possible representations, which are all connected to algorithms in some way; and those representations make determining and communicating answers more accessible and clear.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

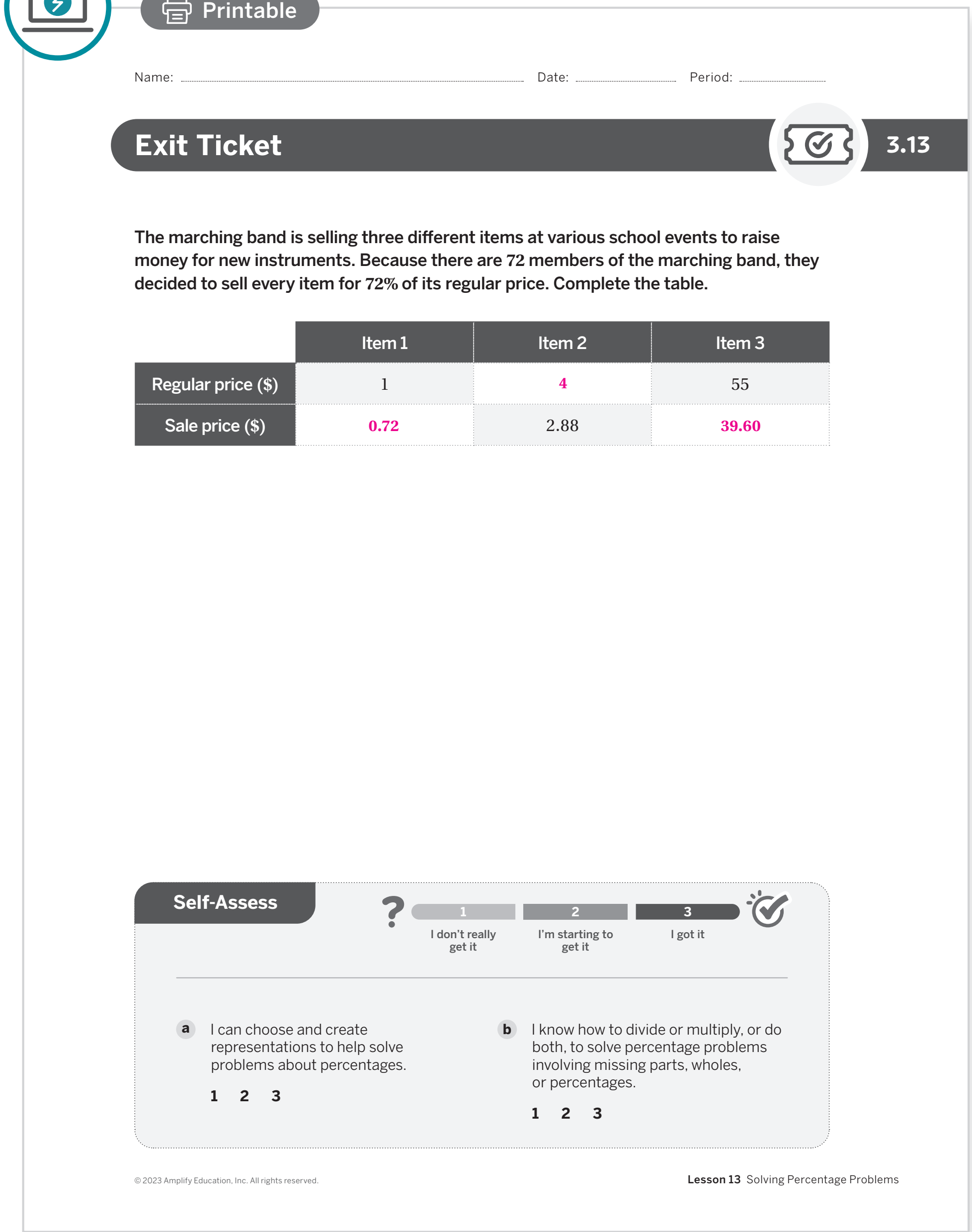
- "Up until this lesson, we worked with each algorithm separately. How did you reason with the problems to determine which algorithm or strategy to use?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of solving percentage problems for missing values by determining sale prices of three items.

Printable



Name: Date: Period:

Exit Ticket 3.13

The marching band is selling three different items at various school events to raise money for new instruments. Because there are 72 members of the marching band, they decided to sell every item for 72% of its regular price. Complete the table.

	Item 1	Item 2	Item 3
Regular price ($)	1	4	55
Sale price ($)	0.72	2.88	39.60

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can choose and create representations to help solve problems about percentages.

1 2 3

b I know how to divide or multiply, or do both, to solve percentage problems involving missing parts, wholes, or percentages.

1 2 3

Lesson 13 Solving Percentage Problems

Success looks like . . .

- **Goal:** Applying reasoning about percentages to solve more complex real-world problems involving percentages.
  - » Completing the table by computing the unknown values sale prices.
- **Language Goal:** Explaining the solution methods by using multiple representations to solve problems involving percentages. **(Speaking and Listening, Writing)**
- **Goal:** Determining what information is needed to solve a problem involving percentages.

Suggested next steps

If students use double number lines to solve a problem, consider referring back to Lesson 11 and Lesson 12 and the algorithms generalized as a result of those lessons, consider:

- Having students recalculate the problems using those algorithms. As they do, students can describe and connect the steps that are performed to what is happening in the double number lines.

If students have difficulty identifying the missing value, consider:

- Referring back to Lessons 11 and 12 and reviewing what the part is and what the whole is.

Professional Learning

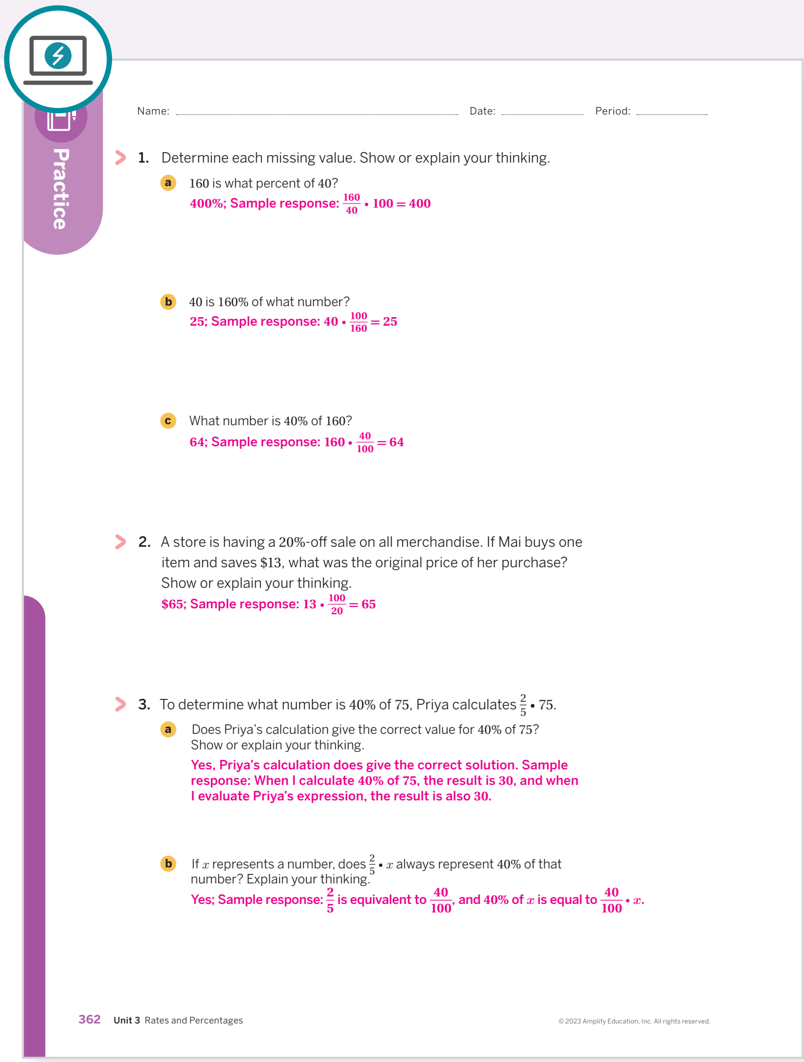
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In earlier lessons, students relied heavily on visual representations to solve percentage problems. How did that support using algorithms in this lesson? Were students able to connect the visual representations to the algorithms?
- What did the interactions during Activity 2 reveal about your students as cooperative learners? How will you use this information to guide future cooperative activities?

Practice

Independent



Name: Date: Period:

1. Determine each missing value. Show or explain your thinking.
   - a) 160 is what percent of 40?
     400%; Sample response: $\frac{160}{40} \cdot 100 = 400$
   - b) 40 is 160% of what number?
     25; Sample response: $40 \cdot \frac{100}{160} = 25$
   - c) What number is 40% of 160?
     64; Sample response: $160 \cdot \frac{40}{100} = 64$
2. A store is having a 20%-off sale on all merchandise. If Mai buys one item and saves $13, what was the original price of her purchase? Show or explain your thinking.
   $65; Sample response: $13 \cdot \frac{100}{20} = 65$
3. To determine what number is 40% of 75, Priya calculates $\frac{2}{5} \cdot 75$.
   - a) Does Priya's calculation give the correct value for 40% of 75? Show or explain your thinking.
     Yes, Priya's calculation does give the correct solution. Sample response: When I calculate 40% of 75, the result is 30, and when I evaluate Priya's expression, the result is also 30.
   - b) If x represents a number, does $\frac{2}{5} \cdot x$ always represent 40% of that number? Explain your thinking.
     Yes; Sample response: $\frac{2}{5}$ is equivalent to $\frac{40}{100}$, and 40% of x is equal to $\frac{40}{100} \cdot x$.

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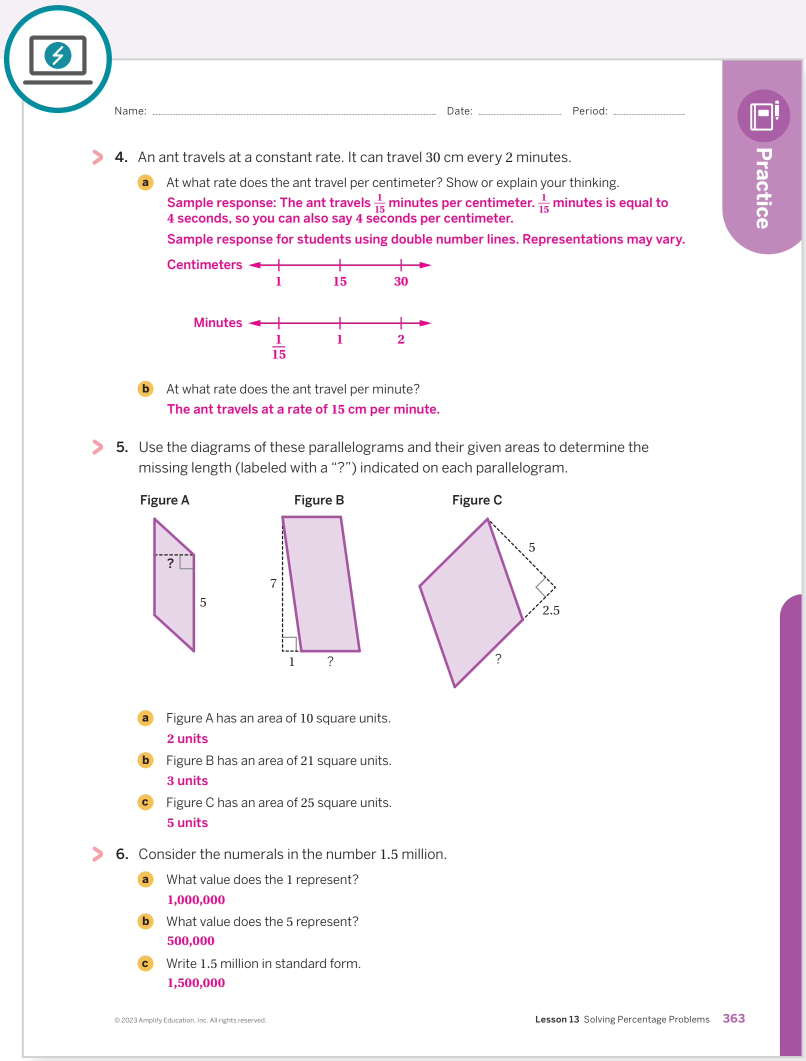

Name: Date: Period:

4. An ant travels at a constant rate. It can travel 30 cm every 2 minutes.
   - a) At what rate does the ant travel per centimeter? Show or explain your thinking.
     Sample response: The ant travels $\frac{1}{15}$ minutes per centimeter. $\frac{1}{15}$ minutes is equal to 4 seconds, so you can also say 4 seconds per centimeter.
     Sample response for students using double number lines. Representations may vary.
   - b) At what rate does the ant travel per minute?
     The ant travels at a rate of 15 cm per minute.
5. Use the diagrams of these parallelograms and their given areas to determine the missing length (labeled with a "?") indicated on each parallelogram.
   - a) Figure A has an area of 10 square units.
     2 units
   - b) Figure B has an area of 21 square units.
     3 units
   - c) Figure C has an area of 25 square units.
     5 units
6. Consider the numerals in the number 1.5 million.
   - a) What value does the 1 represent?
     1,000,000
   - b) What value does the 5 represent?
     500,000
   - c) Write 1.5 million in standard form.
     1,500,000

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Lesson 13 Solving Percentage Problems 363

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 4	2
	5	Unit 1 Lesson 8	1
Formative	6	Unit 3 Lesson 14	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 3 | **LESSON 14**

If Our Class Were the World

Let's use percentages to better understand our world.

Focus

Goals

1. Apply reasoning about percentages and equivalent ratios to analyze and to approximate characteristics of the world's population.
2. **Language Goal:** Present a comparison that uses the number of students in the class to represent the proportion of the world's population with a particular characteristic. **(Speaking and Listening, Writing)**

Coherence

Today

Students observe ratios and percentages of different populations in the world and then use those to determine what their class would be like if the ratios were equivalent and the percentages were the same. They work with many percentages that are not whole numbers, using knowledge gained in earlier lessons of the unit. While working with these extremely large numbers for the world population and the relatively small scale of whole numbers of students in their class, they make choices about rounding and the significance of different place values. This sometimes results in corresponding percentage values in context being close to, but not exactly, equivalent ratios. Students communicate their work and results clearly by creating graphical displays.

Previously

In Lessons 8–12, students explored percentages and the relationships among percentages, parts, and wholes, ultimately developing algorithms for determining any missing piece of information. In Lesson 13, students solved problems involving each of three possible missing values.

Coming Soon

In Lesson 15, students will apply rates and percentages in a capstone scenario involving vote-counting methods.

Rigor

- Students **apply** the concepts of equivalent ratios, rate, and percentages in relating demographics of the world's population to the scale of their class.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Pairs	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- *Percentage Algorithms* PDF
- *Place Value Chart* PDF
- calculators
- tools for creating a visual display

Math Language Development

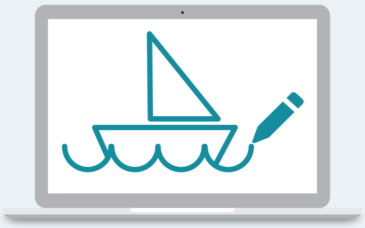
Review words

- *percent*
- *percentage*

Amps | **Featured Activity**

Activity 2
Create a Visual Display

Students will be able to digitally create a display to explain their thinking.



Building Math Identity and Community

Connecting to Mathematical Practices

As students engage in Activities 1 and 2, they may not fully understand the advantages and disadvantages of using a relatively small class size to represent large world populations. They may not see the point of these activities and feel less engaged. As students reason about the quantities and the percentages they use to represent the relationships, model for them how to pause and think about how to interpret these quantitative relationships in context.

Modifications to Pacing

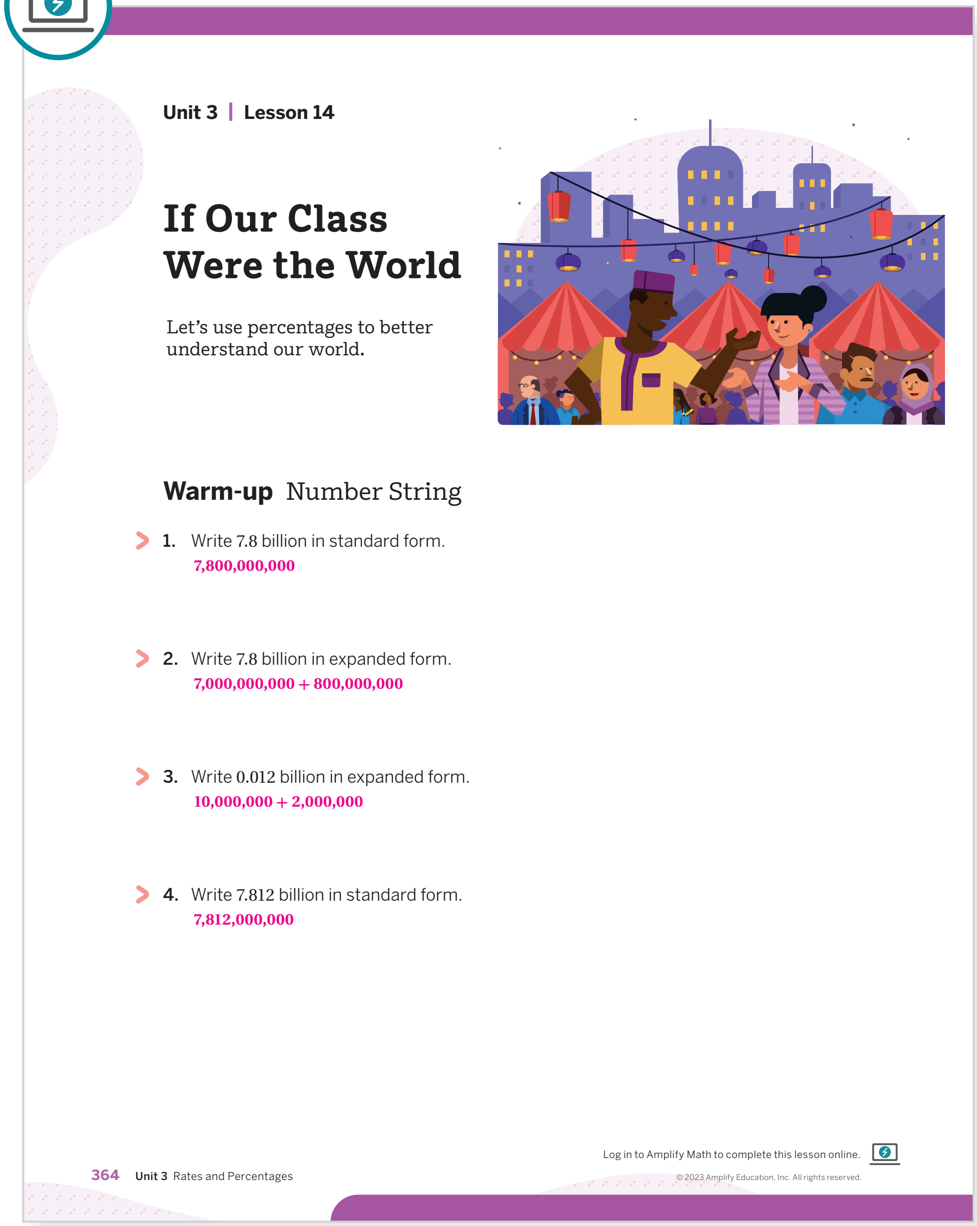
You may want to consider this additional modification if you are short on time.

- **Activity 1** may be completed as a whole class or omitted.

Pairs | 5 min

Warm-up Number String

Students review place value words and standard and expanded forms of base ten numbers to help prepare them for working with and interpreting large numbers.



Unit 3 | Lesson 14

If Our Class Were the World

Let's use percentages to better understand our world.

Warm-up Number String

1. Write 7.8 billion in standard form.
   7,800,000,000
2. Write 7.8 billion in expanded form.
   7,000,000,000 + 800,000,000
3. Write 0.012 billion in expanded form.
   10,000,000 + 2,000,000
4. Write 7.812 billion in standard form.
   7,812,000,000

Log in to Amplify Math to complete this lesson online.

364 Unit 3 Rates and Percentages

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1 Launch

Have students use the *Think-Pair-Share* routine. Give them 2 minutes of individual work time, followed by 1 minute to compare and discuss.

2 Monitor

Help students get started by activating prior knowledge. Ask, "How would 7.8 billion be represented by using a place value chart?"

Look for points of confusion:

- **Confusing the terms *standard form* and *expanded form*.** Review the difference between the two forms.
- **Writing incorrect numbers, based on the number of digits.** Refer to a place value chart and ask, "How many digits are there in a billion?"

Look for productive strategies:

- Connecting the decimal number in billions to standard and expanded forms by using place value understanding.
- Understanding that 0.012 billion is one-hundredth and two-thousandths of a billion $\left(\frac{1}{100}+\frac{2}{1000}\right)$.

3 Connect

Have pairs of students share their responses and how they determined them, noting the differences between standard and expanded forms, and connecting place value to the location of decimal points, numbers of zeros, and fractions involving powers of 10.

Ask:

- "What connections can you make between the corresponding decimal numbers of billions and the standard forms of those values?"
- "Why do you think decimals were used?"

Highlight that the number 7.8 billion represents the population of the world, which will be used as the basis for the next two activities.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Display or provide copies of the *Place Value Chart* PDF for students to use throughout this lesson. Activate prior knowledge from elementary grades about place value, standard form, and expanded form. Consider displaying a sample number, such as 3.42 million, written in both standard and expanded form:

Standard form: 3,420,000
Expanded form: 3,000,000 + 400,000 + 20,000

Power-up

To power up students' ability to understand place value in multi-digit numbers written as decimals of a larger place value unit, have students complete:

Recall that the number 500,000 can be written in other forms, including 500 thousand, or 0.5 million.

1. In the number 1.3 million, what value does the 3 represent? 300,000
2. In the number 25.7 million, what value does the 5 represent? 5,000,000

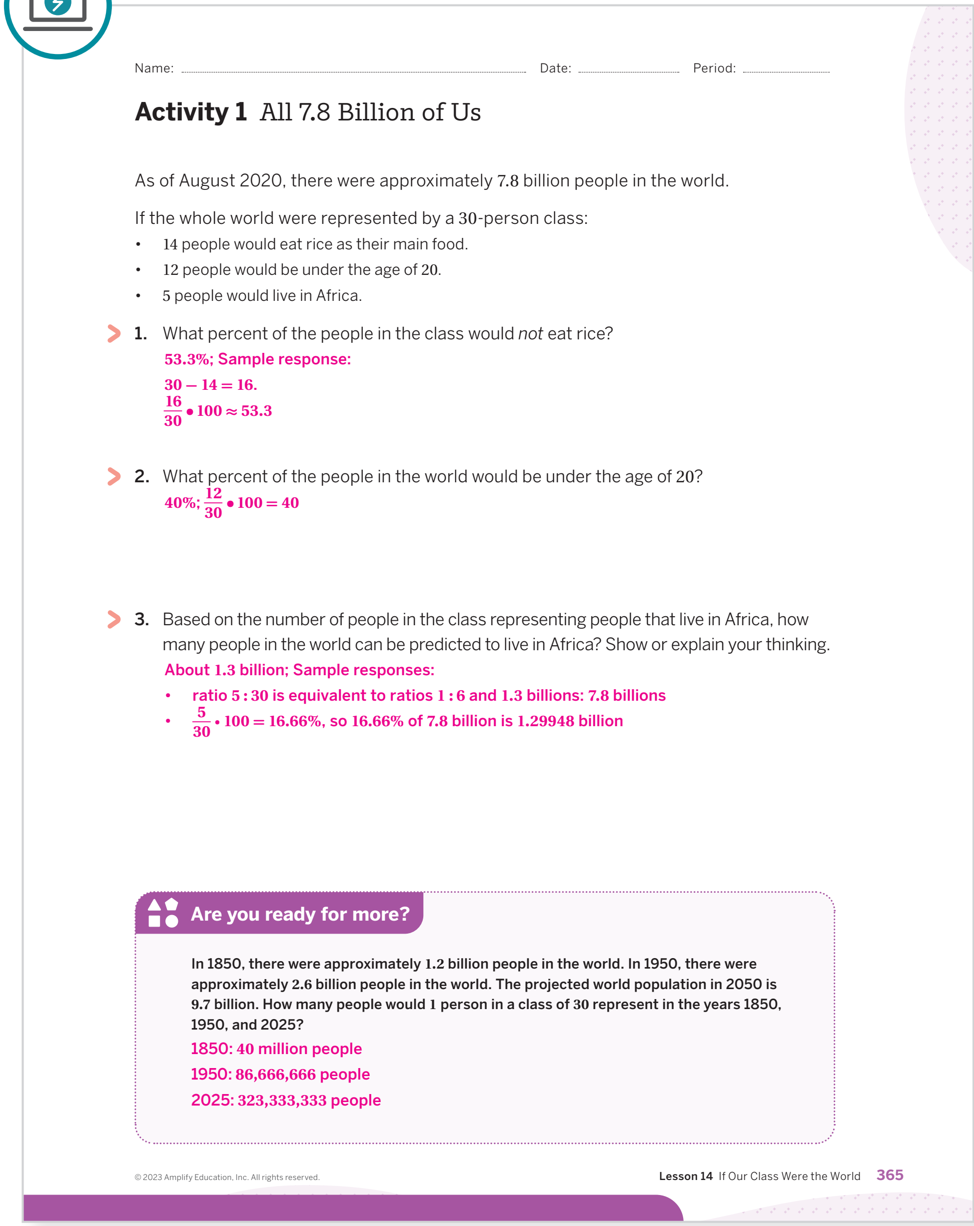
Use: Before the Warm-up.

Informed by: Performance on Lesson 13, Practice Problem 6.

Small Groups | 10 min

Activity 1 All 7.8 Billion of Us

Students observe three descriptions of the world's population represented by ratios, requiring them to consider approximately equivalent ratios.



Name: __________ Date: __________ Period: __________

Activity 1 All 7.8 Billion of Us

As of August 2020, there were approximately 7.8 billion people in the world.

If the whole world were represented by a 30-person class:

- 14 people would eat rice as their main food.
- 12 people would be under the age of 20.
- 5 people would live in Africa.

1. What percent of the people in the class would *not* eat rice?

   53.3%; Sample response:

   $30 - 14 = 16.$

   $\frac{16}{30} \bullet 100 \approx 53.3$

2. What percent of the people in the world would be under the age of 20?

   $40\%; \frac{12}{30} \bullet 100 = 40$

3. Based on the number of people in the class representing people that live in Africa, how many people in the world can be predicted to live in Africa? Show or explain your thinking.

   About 1.3 billion; Sample responses:

   - **ratio 5 : 30 is equivalent to ratios 1 : 6 and 1.3 billions: 7.8 billions**
   - $\frac{5}{30} \bullet 100 = 16.66\%$**, so 16.66% of 7.8 billion is 1.29948 billion**

Are you ready for more?

In 1850, there were approximately 1.2 billion people in the world. In 1950, there were approximately 2.6 billion people in the world. The projected world population in 2050 is 9.7 billion. How many people would 1 person in a class of 30 represent in the years 1850, 1950, and 2025?

1850: 40 million people

1950: 86,666,666 people

2025: 323,333,333 people

Lesson 14 If Our Class Were the World 365

1 Launch

Ask, "What would be a reasonable approximate percentage of people who eat rice as their main food?" A reasonable percentage would be just under 50% because 15 out of 30 is exactly 50%. Provide access to calculators.

2 Monitor

Help students get started by asking students to explain what is the part and what is the whole.

Look for points of confusion:

- **Setting up expressions incorrectly.** Refer to the *Percentage Algorithms* PDF, or to corresponding lessons: missing percentage (Lesson 9), missing part (Lesson 11), or missing whole (Lesson 12).
- **Interpreting the decimal forms incorrectly in Problem 3.** Have students write the standard form of the decimal numbers or use a place value chart to help interpret them.

Look for productive strategies:

- Connecting ratios of the population to the percentages of population to solve for missing values.
- Writing and evaluating correct expressions. If students encounter numbers with lots of places to the right of the decimal, consider having them round to the nearest tenth.
- Recognizing equivalent ratios (12 : 30 and 2 : 5) and the corresponding percentage (40%).

3 Connect

Have groups of students share their responses and strategies for solving the problems.

Ask, "Is the ratio for the world population exactly *equivalent* to the population for people in the class?" No, because you have to round in many cases.

Highlight that the same percentages can represent both small values and large values, such as small class sizes and entire populations of people in the world.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If working with the decimal notations is too challenging, consider having students work with smaller whole numbers that are representative of the decimal notations, such as 7,800 million, instead of 7.8 billion.

Math Language Development

MLR8: Discussion Supports—Revoicing

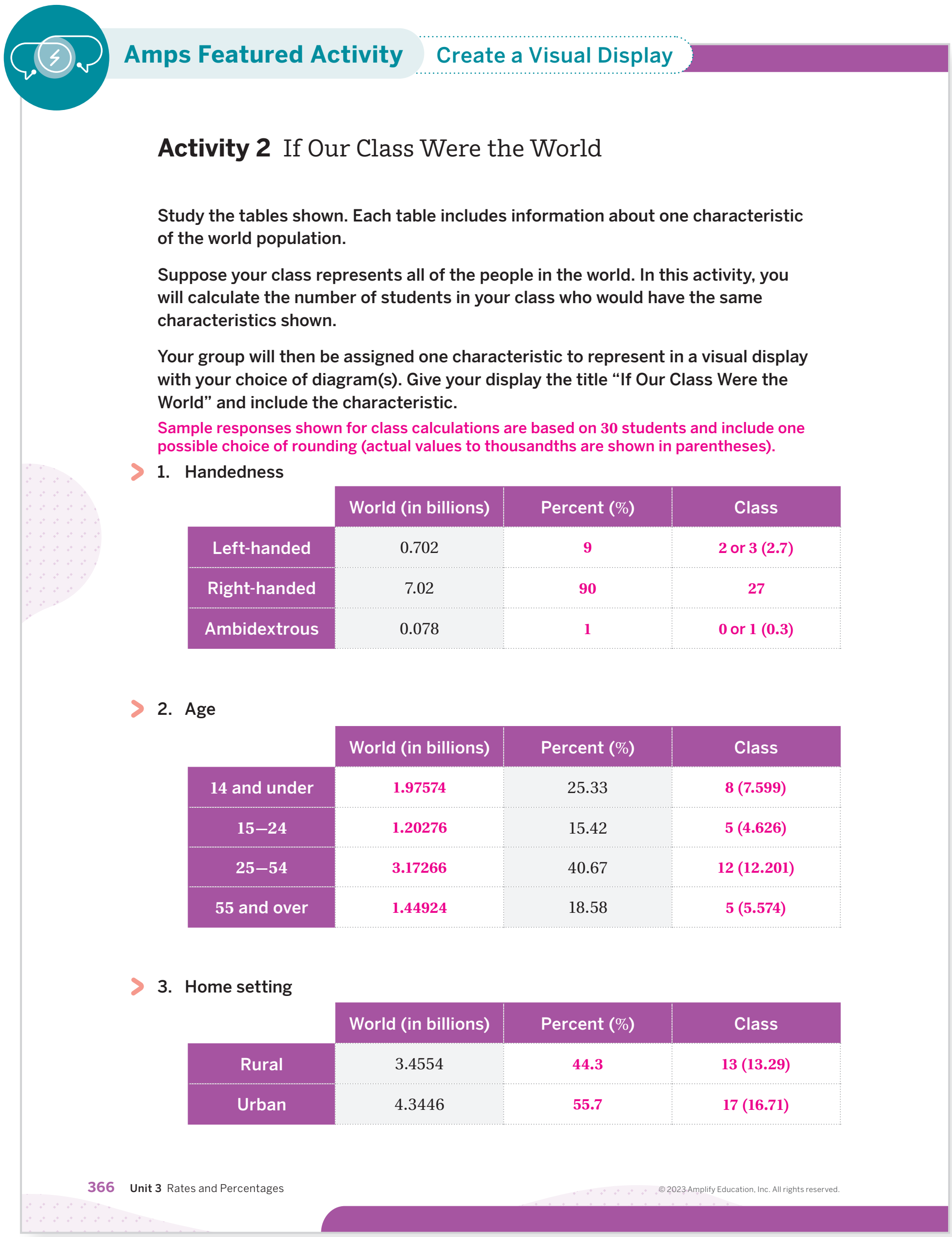
During the Connect, as students describe their strategies for predicting the number of people that live in Africa, revoice student ideas to demonstrate the use of mathematical language. Request details in students' explanations by challenging ideas, asking them to elaborate on ideas, or asking them to provide examples.

Small Groups | 20 min

Activity 2 If Our Class Were the World

Students apply what they have learned about percentages and equivalent ratios to analyze and to interpret five characteristics of the world's population at the scale of their class.

Amps Featured Activity Create a Visual Display



Activity 2 If Our Class Were the World

Study the tables shown. Each table includes information about one characteristic of the world population.

Suppose your class represents all of the people in the world. In this activity, you will calculate the number of students in your class who would have the same characteristics shown.

Your group will then be assigned one characteristic to represent in a visual display with your choice of diagram(s). Give your display the title "If Our Class Were the World" and include the characteristic.

Sample responses shown for class calculations are based on 30 students and include one possible choice of rounding (actual values to thousandths are shown in parentheses).

1. Handedness

	World (in billions)	Percent (%)	Class
Left-handed	0.702	9	2 or 3 (2.7)
Right-handed	7.02	90	27
Ambidextrous	0.078	1	0 or 1 (0.3)

2. Age

	World (in billions)	Percent (%)	Class
14 and under	1.97574	25.33	8 (7.599)
15–24	1.20276	15.42	5 (4.626)
25–54	3.17266	40.67	12 (12.201)
55 and over	1.44924	18.58	5 (5.574)

3. Home setting

	World (in billions)	Percent (%)	Class
Rural	3.4554	44.3	13 (13.29)
Urban	4.3446	55.7	17 (16.71)

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1 Launch

Provide access to calculators. Explain that each group will be assigned one characteristic to present to the class at the end. Each presentation should illustrate their group's solution and explain their process for determining the missing values. **Note:** Sample responses are based on a 30-student class. Your actual class size could be used instead, but you should work these out prior to the activity and consider the values that will arise.

2 Monitor

Help students get started by asking for Problem 1, "How could you determine the percentage when the total population and a part of the population is given?"

Look for points of confusion:

- **Still struggling to interpret decimal numbers of billions.** Refer back to the Warm-up or Activity 1, Problem 3 and ask, "How did you determine the values represented by these decimal numbers?"
- **Having difficulty calculating with decimals.** Have students write out the expression and then use a calculator to evaluate.

Look for productive strategies:

- Determining correct non-whole numbers of students in the class. Encourage students to consider their value in context by asking, "Can there be a fraction of a student?"
- Recognizing that balancing out the number of students from the class with the percentages sometimes requires rounding numbers of students up or down, and sometimes it is worth considering a "floor" or "ceiling" type of rounding. The class numbers should always total the number of students (or 30, if using the sample class).
- Using algorithms or ratio relationships to accurately solve for the missing values.
- Creating a visual display that accurately presents their thinking, calculations, and interpretation of solutions.

Activity 2 continued ›

Fostering Diverse Thinking

What Will the World Population Look Like in 2050?

Preview "Shifting Demographics" data from the United Nations, which may be found online. Decide if you would like your students to explore the site or if you would like to provide a summary. Highlight the following information with:

- The world population is predicted to be 9.7 billion by 2050. Half of the growth is expected to come from just 9 countries. The population of sub-Saharan Africa is projected to double, while Europe's population is likely to decrease.
- By 2050, adults 65 and older are projected to outnumber teens and young adults, ages 15 to 24.
- By 2050, almost 70% of the world's population is expected to live in urban centers.

Even though percent information is not provided for age or continent, consider having students make educated guesses as to how the tables might change in Problems 2–4 to represent the world population in 2050.

Facilitate a class discussion by asking, "How do you think countries should prepare for this population growth?"

Small Groups | 20 min

Activity 2 If Our Class Were the World (continued)

Students apply what they have learned about percentages and equivalent ratios to analyze and to interpret five characteristics of the world's population at the scale of their class.

Name: ______ Date: ______ Period: ______

Activity 2 If Our Class Were the World (continued)

Sample responses shown for class calculations are based on 30 students and include one possible choice of rounding (actual values to thousandths are shown in parentheses).

4. Continent

	World (in billions)	Percent (%)	Class
Europe	0.74802	9.59	3 (2.877)
Asia	4.64412	59.54	18 (17.862)
Africa	1.3416	17.20	5 (5.16)
North America	0.5928	7.60	2 (2.28)
South America	0.43134	5.53	1 (1.659)
Oceania	0.0429	0.55	1 (0.165)
Antarctica	0	0	0

5. Most spoken language

	World (in billions)	Percent (%)	Class
English	1.287	16.5	5 (4.95)
Hindi	0.6474	8.3	3 (2.49)
Mandarin Chinese	1.1388	14.6	4 (4.38)
Spanish	0.546	7	2 (2.1)
French	0.2808	3.6	1 (1.08)
Arabic	0.2808	3.6	1 (1.08)
Other	3.6192	46.4	14 (13.92)

STOP

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3 Connect

Display all groups' visual displays of "If Our Class Were the World" using the ***Gallery Tour*** routine.

Ask:

- "How did you handle interpreting fractions of people into the answers?" I had to understand when to round up or down based on the other numbers.
- "Were you surprised by any of your results?" Numbers that represented some populations, such as 0.3 for ambidextrous, are small, but when thought of in billions, they are actually very large. Accounting for a whole number of people can be challenging when balancing out the distribution of characteristics. If something is not 0, but should not round to 1, then you have to decide if it's fair to say 0 and to make it look like they don't exist at all.
- "In what ways do you think our class is actually representative of the world?" From the characteristics of the activity, possibly home setting.
- "In what ways is it not?" Responses may vary. Sample response: Age, place of residence, access to clean water, etc.

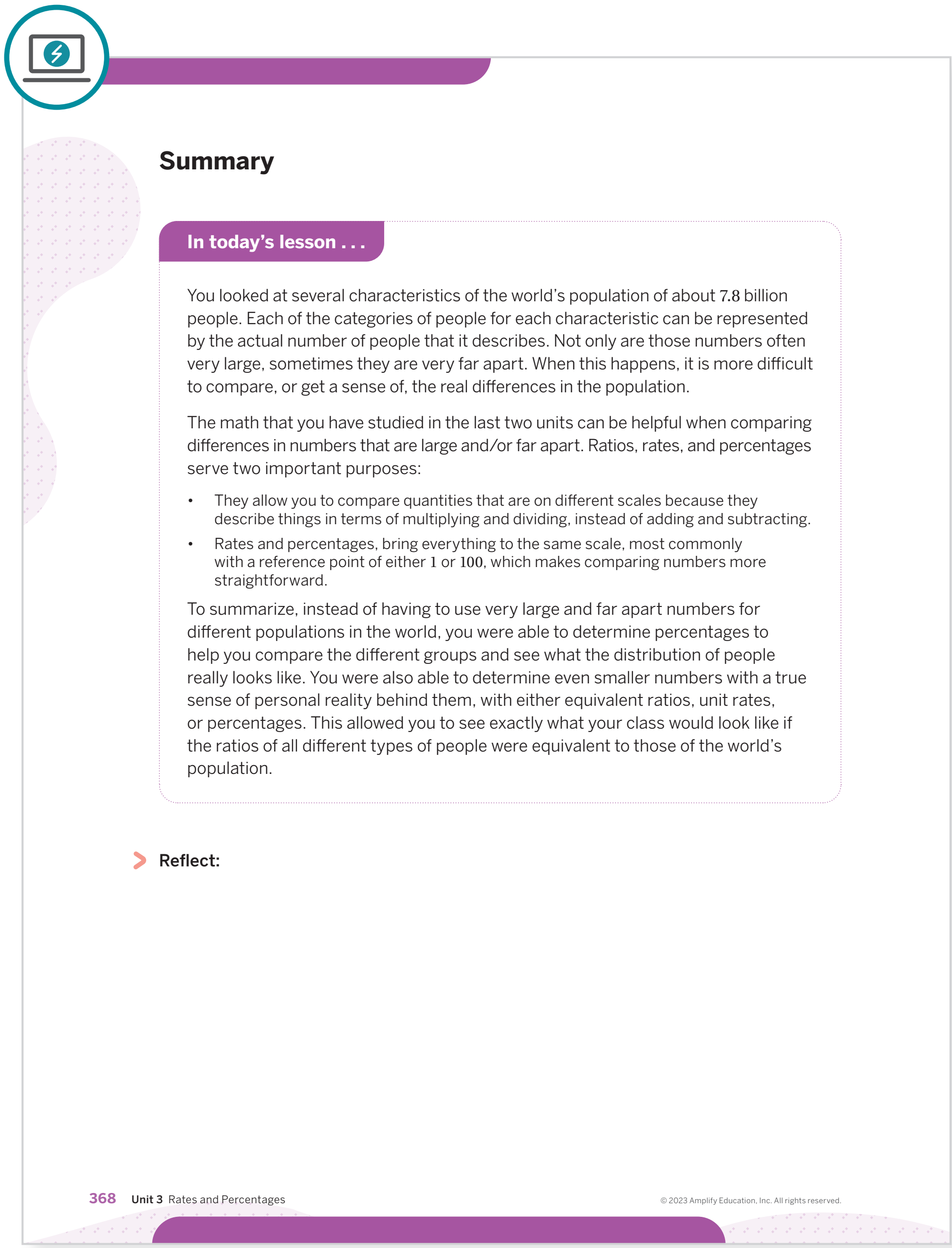
Note: Consider encouraging students to think about and discuss other possible demographic information not included in the activity for which their class may or may not be representative of the world's population.

Highlight the fact that having widely varying percentages and then considering those on a scale as small as a class limits the possible whole-number values. This can make accounting for, and representing, each characteristic difficult.

Whole Class | 5 min

Summary

Review and synthesize that using ratios and percentages can be used to connect us to the world.



Summary

In today's lesson . . .

You looked at several characteristics of the world's population of about 7.8 billion people. Each of the categories of people for each characteristic can be represented by the actual number of people that it describes. Not only are those numbers often very large, sometimes they are very far apart. When this happens, it is more difficult to compare, or get a sense of, the real differences in the population.

The math that you have studied in the last two units can be helpful when comparing differences in numbers that are large and/or far apart. Ratios, rates, and percentages serve two important purposes:

- They allow you to compare quantities that are on different scales because they describe things in terms of multiplying and dividing, instead of adding and subtracting.
- Rates and percentages, bring everything to the same scale, most commonly with a reference point of either 1 or 100, which makes comparing numbers more straightforward.

To summarize, instead of having to use very large and far apart numbers for different populations in the world, you were able to determine percentages to help you compare the different groups and see what the distribution of people really looks like. You were also able to determine even smaller numbers with a true sense of personal reality behind them, with either equivalent ratios, unit rates, or percentages. This allowed you to see exactly what your class would look like if the ratios of all different types of people were equivalent to those of the world's population.

Reflect:

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Synthesize

Ask:

- "What was the same and what was different about using the percentage algorithms when working with large numbers, such as the world population?"
- "How could you use rate language or unit rates to communicate your same calculations and findings?"

Have students share how they used algorithms related to rates, ratios, and percentages while calculating with large numbers, such as the world population.

Highlight that communicating and interpreting the results can be interesting and challenging when dealing with populations because there cannot realistically be a fraction of a person.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

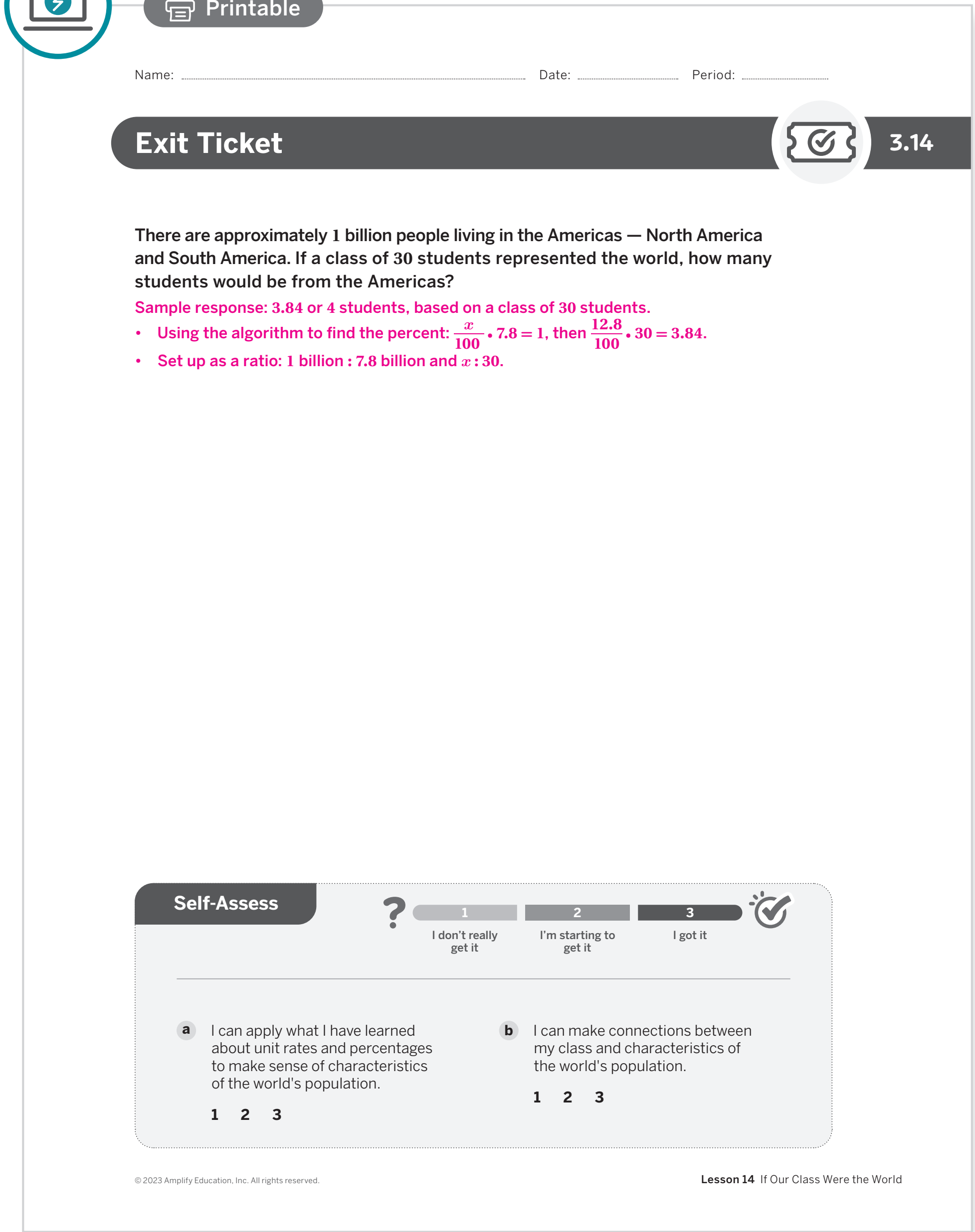
- "How are percentages used to estimate and to compare quantities?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the equivalent percentage and number of students to the population of the Americas.

Printable



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Exit Ticket 3.14

There are approximately 1 billion people living in the Americas — North America and South America. If a class of 30 students represented the world, how many students would be from the Americas?

Sample response: 3.84 or 4 students, based on a class of 30 students.

- Using the algorithm to find the percent: $\frac{x}{100} \cdot 7.8 = 1$, then $\frac{12.8}{100} \cdot 30 = 3.84$.
- Set up as a ratio: 1 billion : 7.8 billion and $x : 30$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can apply what I have learned about unit rates and percentages to make sense of characteristics of the world's population.

1 2 3

b. I can make connections between my class and characteristics of the world's population.

1 2 3

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Lesson 14 If Our Class Were the World

Success looks like . . .

- **Goal:** Applying reasoning about percentages and equivalent ratios to analyze and approximate characteristics of the world's population.
- **Language Goal:** Presenting a comparison that uses the number of students in the class to represent the proportion of the world's population with a particular characteristic. **(Speaking and Listening, Writing)**
  - » Calculating the number of students who would be from the Americas.

Suggested next steps

If students fail to recognize that there is a first step to determine the percentage represented by 1 billion people in the world, consider:

- Asking students to identify the three parts of the equation: the part (1 billion), the whole (7.8 billion), and the percentage (unknown). Then ask students, "What do you first need to determine?"

If students incorrectly set up the expression to determine the percentage, consider:

- Referring back to Activity 2, Problem 1 and ask, "How did you set up the expression to determine the percentage of left-handed people?"

If students incorrectly complete the equivalent ratio as 7.8 : 30, consider:

- Focusing on the reasonableness of the number 7.8 in this case. If the ratio of the world population is stated as 1 : 7.8, then determining equivalent ratios could give 2 : 15.6, 3 : 23.4, and 4 : 31.2, and so could 7.8 : 30 be correct? To simplify the doubling, tripling, etc., for students, you may also consider rounding 7.8 to 8.

Professional Learning

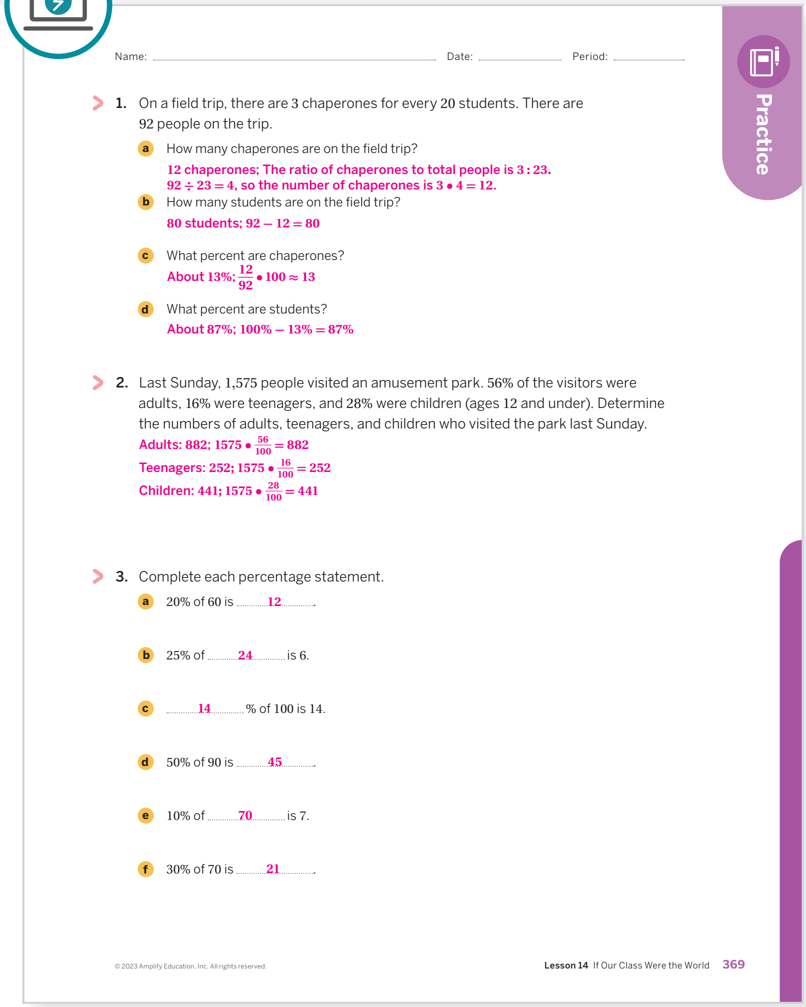
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- A goal for this lesson was to have students connect their class to the global population. How well did students accomplish this? What did you specifically do to help students accomplish it?
- Knowing where students need to be by the end of this unit, how did the visual displays from Activity 2 influence that future goal?

Practice

Independent



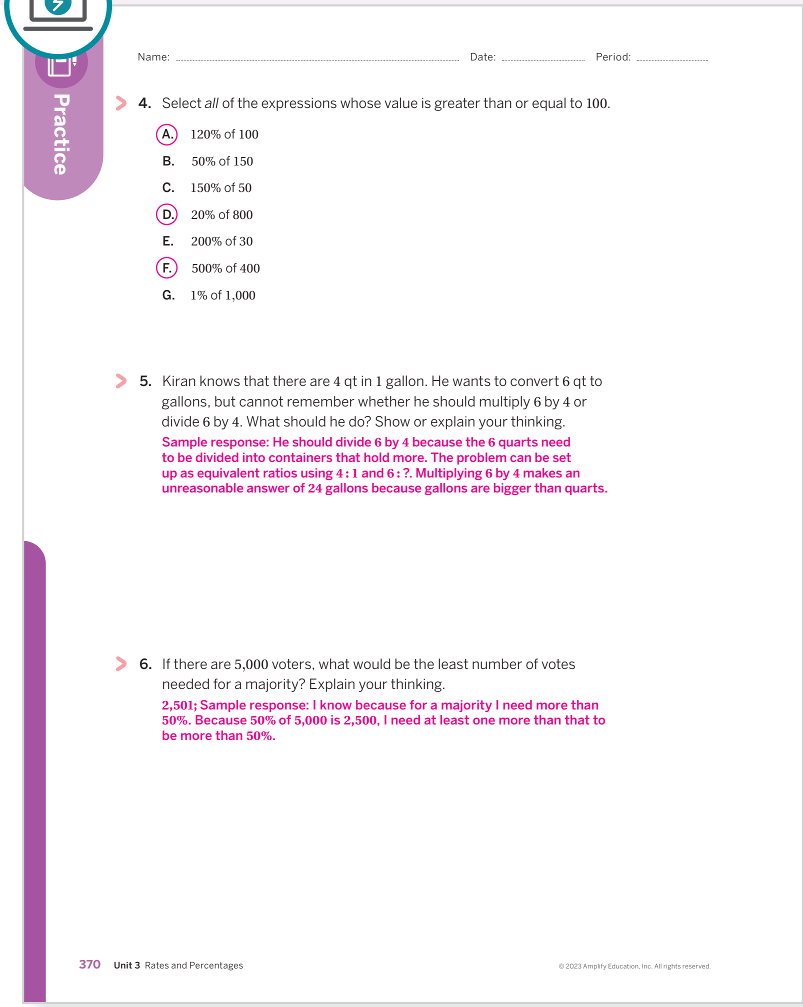
Name: Date: Period:

Practice

1. On a field trip, there are 3 chaperones for every 20 students. There are 92 people on the trip.
   - a How many chaperones are on the field trip?
     12 chaperones; The ratio of chaperones to total people is 3 : 23. $92 \div 23 = 4$, so the number of chaperones is $3 \bullet 4 = 12$.
   - b How many students are on the field trip?
     80 students; $92 - 12 = 80$
   - c What percent are chaperones?
     About 13%; $\frac{12}{92} \bullet 100 \approx 13$
   - d What percent are students?
     About 87%; $100\% - 13\% = 87\%$

2. Last Sunday, 1,575 people visited an amusement park. 56% of the visitors were adults, 16% were teenagers, and 28% were children (ages 12 and under). Determine the numbers of adults, teenagers, and children who visited the park last Sunday.
   Adults: 882; $1575 \bullet \frac{56}{100} = 882$
   Teenagers: 252; $1575 \bullet \frac{16}{100} = 252$
   Children: 441; $1575 \bullet \frac{28}{100} = 441$

3. Complete each percentage statement.
   - a 20% of 60 is **12**.
   - b 25% of **24** is 6.
   - c **14** % of 100 is 14.
   - d 50% of 90 is **45**.
   - e 10% of **70** is 7.
   - f 30% of 70 is **21**.



Name: Date: Period:

Practice

4. Select *all* of the expressions whose value is greater than or equal to 100.
   - [✓ A.] 120% of 100
   - B. 50% of 150
   - C. 150% of 50
   - [✓ D.] 20% of 800
   - E. 200% of 30
   - [✓ F.] 500% of 400
   - G. 1% of 1,000

5. Kiran knows that there are 4 qt in 1 gallon. He wants to convert 6 qt to gallons, but cannot remember whether he should multiply 6 by 4 or divide 6 by 4. What should he do? Show or explain your thinking.
   Sample response: He should divide 6 by 4 because the 6 quarts need to be divided into containers that hold more. The problem can be set up as equivalent ratios using 4 : 1 and 6 : ?. Multiplying 6 by 4 makes an unreasonable answer of 24 gallons because gallons are bigger than quarts.

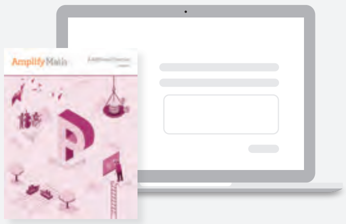
6. If there are 5,000 voters, what would be the least number of votes needed for a majority? Explain your thinking.
   2,501; Sample response: I know because for a majority I need more than 50%. Because 50% of 5,000 is 2,500, I need at least one more than that to be more than 50%.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	1
Spiral	4	Unit 3 Lesson 13	1
	5	Unit 2 Lesson 19	2
Formative ⚡	6	Unit 3 Lesson 15	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

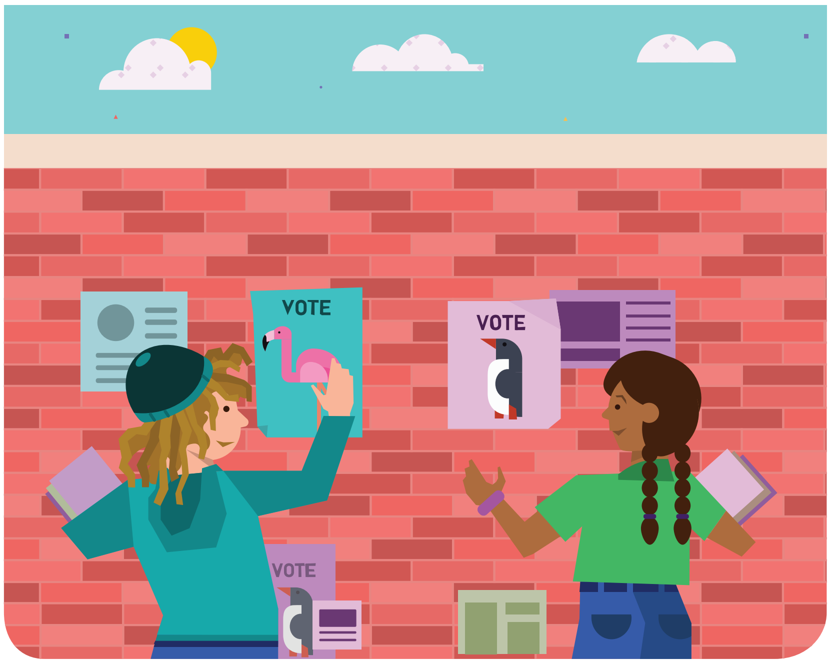
Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Voting for a School Mascot

Let's think about different ways of voting.



Focus

Goals

1. **Language Goal:** Apply reasoning about ratios and percentages to analyze voting situations. **(Speaking and Listening, Writing)**
2. **Language Goal:** Compare and contrast different voting systems. **(Speaking and Listening, Writing)**
3. **Language Goal:** Suggest and critique whether or not a method for distributing votes is fair. **(Speaking and Listening, Writing)**

Coherence

Today

In this capstone lesson, students use unit rates and percentages to determine the results of a vote for a new school mascot. Using two different vote-counting methods, and several different ways in which rules and boundaries can be defined, they recognize how the same votes can be interpreted to yield different results. In both methods, determining results and establishing how to define boundaries, students use the precise language and the mathematics developed throughout the unit.

Previously

Students further developed their understanding of ratios, exploring rates and unit rates in Lessons 2–7, and percentages in Lessons 8–14.

Coming Soon

In Grade 7, students will continue to build toward the concepts of slope and linear functions as they extend their understanding of ratios and rates to proportional relationships. They will solve multi-step ratio, rate, and percentage problems.

Rigor

- Students are given opportunities to represent and solve multi-step problems using percentages to solidify their **procedural skills**.
- Students **apply** reasoning with percentages and ratios in a real-world voting context.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, one per pair, plus extra copies
- rulers

Math Language Development

Review word

- *percentage*

Amps | Featured Activity

Activity 2
Interactive Voting Map

Students can sketch boundaries to form new zones on a voting map.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might become overwhelmed by all of the interrelated moving parts as they try to construct boundaries on the grid. Encourage students to manage their time and energy by focusing on smaller, short-term goals first, stopping to check in on overall progress after each step and before proceeding to the next.

Modifications to Pacing

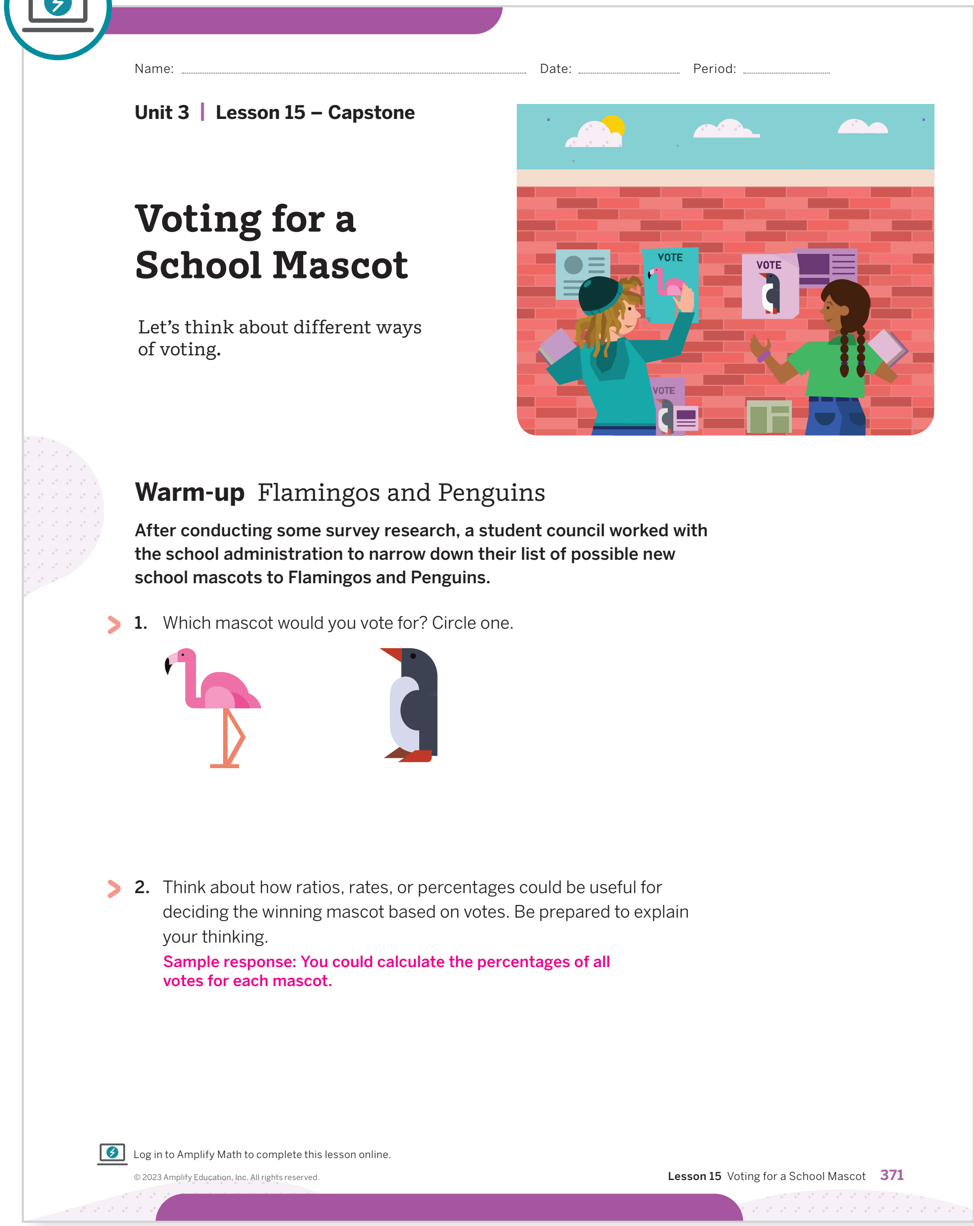
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted, but the context of voting for either the Flamingo or the Penguin for the lesson will need to be established in Activity 1.
- In **Activity 2**, pairs could focus on only completing Problems 1 and 2. Problem 3 could be omitted, or you could consider working out a plan, and even executing it, as a whole class.

Pairs | 5 min

Warm-up Flamingos and Penguins

Students consider how two voting methods involve ratios, rates, and percentages. The scenario presented also sets up the context for the entire lesson.



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Unit 3 | Lesson 15 – Capstone

Voting for a School Mascot

Let's think about different ways of voting.

Warm-up Flamingos and Penguins

After conducting some survey research, a student council worked with the school administration to narrow down their list of possible new school mascots to Flamingos and Penguins.

1. Which mascot would you vote for? Circle one.

2. Think about how ratios, rates, or percentages could be useful for deciding the winning mascot based on votes. Be prepared to explain your thinking.
   Sample response: You could calculate the percentages of all votes for each mascot.

Log in to Amplify Math to complete this lesson online.

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1 Launch

Activate students' background knowledge by asking what they know about mascots, what they represent and how they are used, or how and why they may be chosen. Have students make a choice in Problem 1 and then use the *Think-Pair-Share* routine for Problem 2.

2 Monitor

Help students get started by asking, "What types of numbers are associated with voting?"

Look for productive strategies:

- Recognizing that percentages can be used to describe groups of a population (e.g., $x\%$ for the flamingo).
- Understanding that anything greater than 50% is the (simple) majority of voters.
- Explaining that ratios can also be used to represent the results (e.g., $2:3$ in favor of the penguin).
- Thinking that results could be made easier to count by determining the number of votes for each mascot per 100 voters.
- Using the language of the unit, such as *per*, *unit rate*, or *percentage*.

3 Connect

Have pairs of students share how they see ratios, rates, or percentages as being useful in the scenario, focusing on examples they use to explain their thinking.

Highlight that this scenario is the context students will be using for the entire lesson.

Power-up

To power up students' ability to connect percentages or rates to voting, have students complete:

Recall that, in voting, the term "majority" can mean "the greater number of votes." In an election that involved 200 voters and two choices, which amounts of votes below would be considered a majority? Select *all* that apply.

A. 99 votes

[✓ B.] 51% of votes

[✓ C.] 101 votes

D. $\frac{2}{5}$ of votes

[✓ E.] $\frac{4}{5}$ of votes

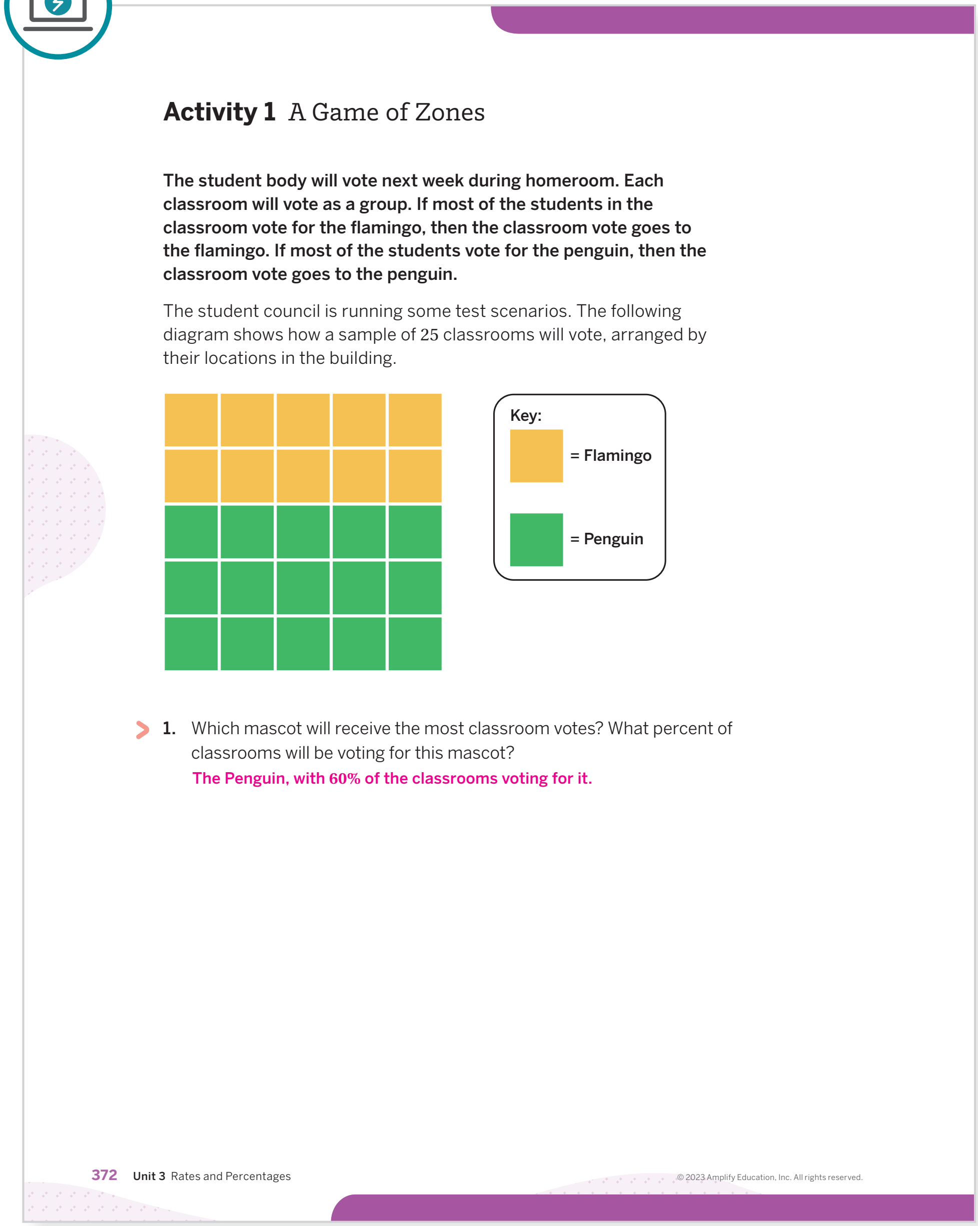
Use: Before the Warm-up.

Informed by: Performance on Lesson 14, Practice Problem 6.

Pairs | 15 min

Activity 1 A Game of Zones

Students use ratios and percentages to determine different outcomes based on the same votes but using two different vote counting methods involving grouping votes.



Activity 1 A Game of Zones

The student body will vote next week during homeroom. Each classroom will vote as a group. If most of the students in the classroom vote for the flamingo, then the classroom vote goes to the flamingo. If most of the students vote for the penguin, then the classroom vote goes to the penguin.

The student council is running some test scenarios. The following diagram shows how a sample of 25 classrooms will vote, arranged by their locations in the building.

Key:
🟨 = Flamingo
🟩 = Penguin

1. Which mascot will receive the most classroom votes? What percent of classrooms will be voting for this mascot?
   The Penguin, with 60% of the classrooms voting for it.

1 Launch

Ensure students understand that each square represents the votes of all students in the classroom but only counts as one vote for tallying, and the colors correspond to the mascot receiving the most votes in the classroom. Then set an expectation for the amount of time pairs will have to complete the activity.

2 Monitor

Help students get started by asking, "How many classrooms voted for each mascot?"

Look for points of confusion:

- **Thinking in the zone voting method each classroom's vote still counts the same (Problems 2 and 3).** Clarify that, similar to how all students in a classroom vote but each classroom only gets one vote, all classrooms in the zone do vote, but the zone only gets one vote. Consider suggesting that students could label the zones and organize how the votes from each were cast in a table.
- **Thinking each zone must include at least one classroom that voted for each mascot (Problem 3).** Have students review the zones from Problem 2a.

Look for productive strategies:

- Determining the winning percent for the penguin in Problem 1 using a standard algorithm, such as dividing $15 \div 25$ and multiplying by 100.
- Recognizing that the flamingo can win 3 zones but no more, and using that to first build three zones with 3 or more yellow classrooms, leaving the remaining green classrooms to populate the other two zones entirely.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider illustrating how the Zone Vote counting method would work for Problem 2a. Label the zones from top to bottom as Zones 1–5 and ask these questions:

- "In Zone 1, how many classrooms are in the zone?" 5 classrooms
- "How many of the Zone 1 classrooms voted for the Flamingo? the Penguin?" 5 and 0
- "Which mascot does Zone 1 vote for?" Flamingo

Extension: *Math Enrichment*

Have students consider a grid of 100 classrooms following the same layout and voting pattern but in a 10×10 grid (with each column containing 4 yellow squares at the top). Ask, "What is the most number of five-classroom zones the Penguin could win? Ten-classroom zones?"

Math Language Development

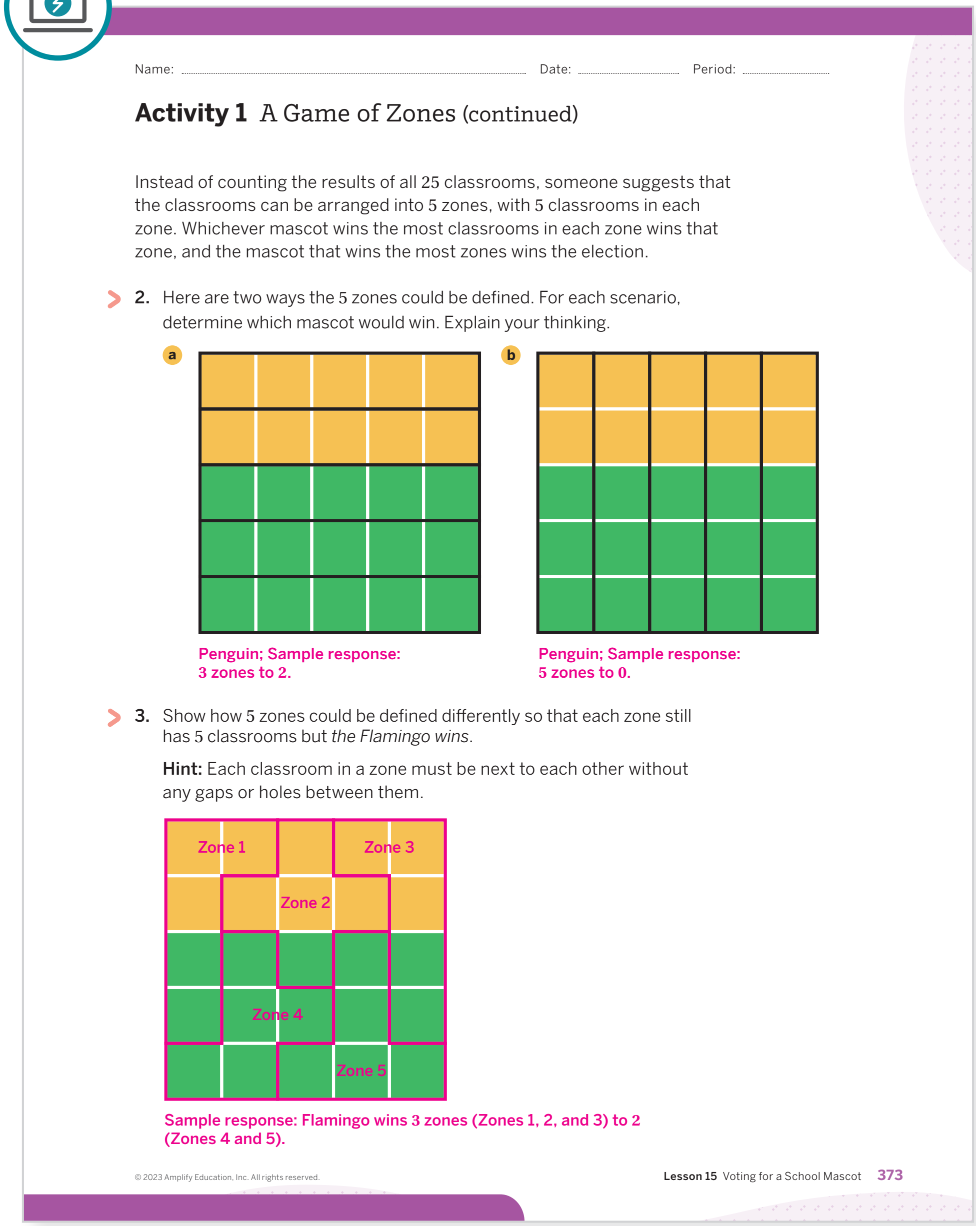
MLR3: Critique, Correct, Clarify

Use this routine to help students identify an error in a comparison statement. Display the following statement before discussing the final problem: "If two zones contain the same number of classrooms, they must result in the same outcome." Have students discuss with a partner why this statement is incorrect. Listen for students to notice that the statement does not take into account the number of votes "for" each mascot. Therefore, the information given is incomplete.

Pairs | 15 min

Activity 1 A Game of Zones (continued)

Students use ratios and percentages to determine different outcomes based on the same votes but using two different vote counting methods involving grouping votes.



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Activity 1 A Game of Zones (continued)

Instead of counting the results of all 25 classrooms, someone suggests that the classrooms can be arranged into 5 zones, with 5 classrooms in each zone. Whichever mascot wins the most classrooms in each zone wins that zone, and the mascot that wins the most zones wins the election.

2. Here are two ways the 5 zones could be defined. For each scenario, determine which mascot would win. Explain your thinking.

   a. Penguin; Sample response: 3 zones to 2.

   b. Penguin; Sample response: 5 zones to 0.

3. Show how 5 zones could be defined differently so that each zone still has 5 classrooms but *the Flamingo wins*.

   Hint: Each classroom in a zone must be next to each other without any gaps or holes between them.

Sample response: Flamingo wins 3 zones (Zones 1, 2, and 3) to 2 (Zones 4 and 5).

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3 Connect

Display an unmarked version of the classroom grid.

Have pairs of students share responses for each problem, in order, and be sure to allow multiple groups to share their zones for Problem 3, having each explain their strategy.

Highlight that the two methods used the same numbers, but had different results. Also, point out that, even in the zone method with the same rules, the way in which the zones were constructed could also lead to different results.

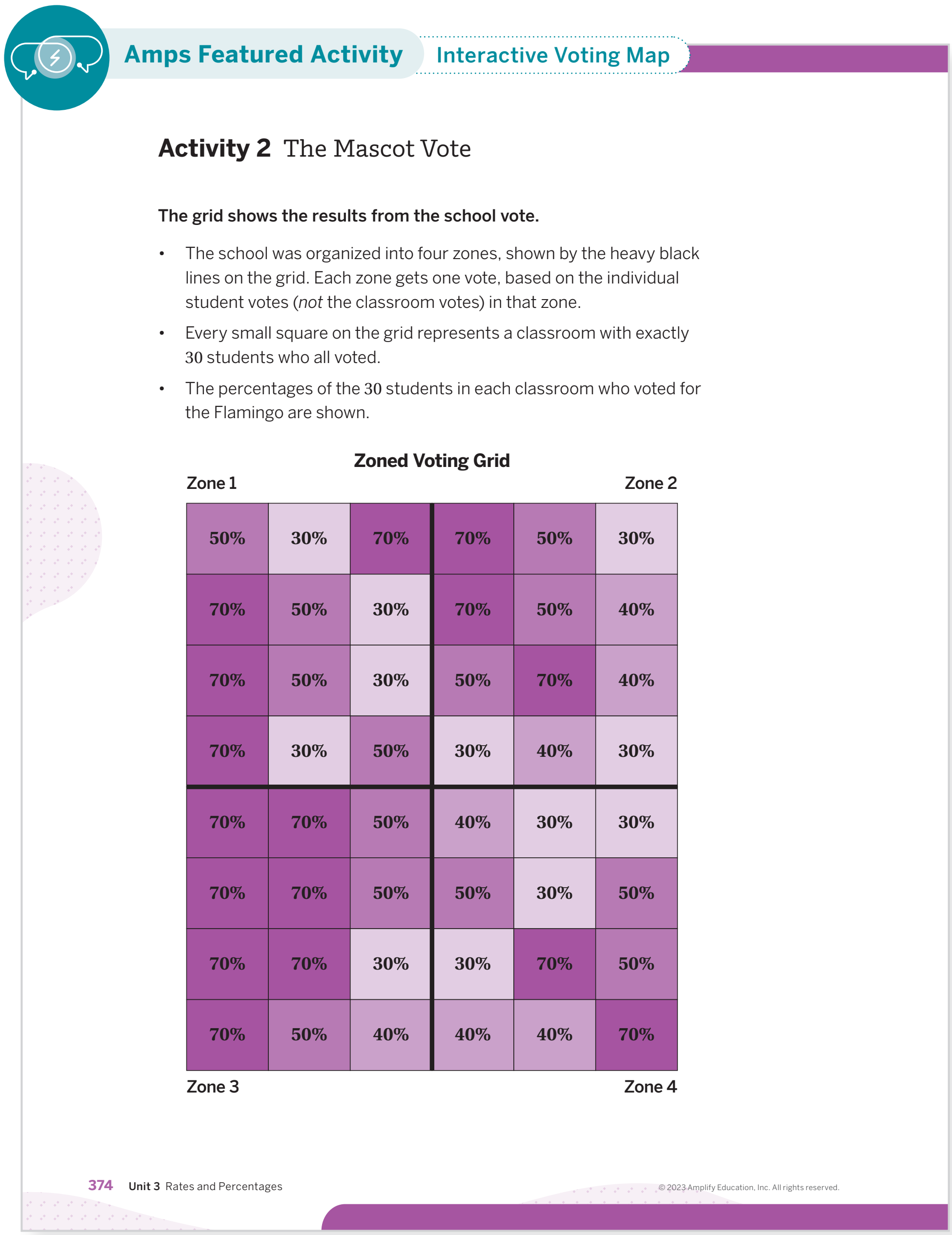
Ask, "What might be some advantages and disadvantages of voting using zones?" One advantage might be that there are fewer votes to tally in the end, so it could be more efficient. One disadvantage might be that the minority could win the overall vote, and, likewise, that means the majority would lose.

Pairs | 15 min

Activity 2 The Mascot Vote

Students use ratios and percentages to calculate the results of the vote, and then they redesign the zones of the classroom grid to change the results.

Amps Featured Activity Interactive Voting Map



Activity 2 The Mascot Vote

The grid shows the results from the school vote.

- The school was organized into four zones, shown by the heavy black lines on the grid. Each zone gets one vote, based on the individual student votes (*not* the classroom votes) in that zone.
- Every small square on the grid represents a classroom with exactly 30 students who all voted.
- The percentages of the 30 students in each classroom who voted for the Flamingo are shown.

Zoned Voting Grid

Zone 1 — Zone 2 (top); Zone 3 — Zone 4 (bottom)

50%	30%	70%	70%	50%	30%
70%	50%	30%	70%	50%	40%
70%	50%	30%	50%	70%	40%
70%	30%	50%	30%	40%	30%
70%	70%	50%	40%	30%	30%
70%	70%	50%	50%	30%	50%
70%	70%	30%	30%	70%	50%
70%	50%	40%	40%	40%	70%

374 **Unit 3** Rates and Percentages

1 Launch

Read the rules for the Zoned Voting Grid aloud and offer any necessary clarifications. Distribute one copy of the Activity 2 PDF to each pair for drawing their own boundaries in Problem 3. Then have pairs complete the activity together.

2 Monitor

Help students get started by asking, "How many students are in each class? So how many students voted for the Flamingo in the upper left classroom, where it received 70% of the votes?"

Look for points of confusion:

- **Thinking the exact percentages do not matter for the zone vote (Problem 2).** Remind students that in this method of the zone vote, all individual votes in the zone count. Ask, "How many individual student votes would be needed to win a zone?"
- **Forgetting to balance out any changes in votes that moved zones (Problem 3).** For example, if a square is shifted to another zone, its votes must be subtracted from its original zone. This must also be reflected in the total possible votes and what is needed to determine the outcome of the zone vote.

Look for productive strategies:

- Calculating the number of votes in each square and then adding to determine the totals, perhaps recognizing that the calculations of the four percents — 70%, 50%, 40%, and 30% — only need to be done one time.
- Totaling the number of voting squares and multiplying by 30 to determine the total number of voters per zone.
- Recognizing that, for example, "70% of 30 + 70% of 30 + 70% of 30" is equivalent to "70% of 30 + 30 + 30" or "70% of 30 • 3."
- Modifying the original zone boundaries to meet the criteria (e.g., recognizing that Zone 3 can remain a Flamingo vote despite losing many votes).
- Creating completely new zones one at a time and checking to make sure they meet the criteria.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can manipulate the boundaries to form new zones on a voting map.

Accessibility: *Vary Demands to Optimize Challenge*

Have students focus on manipulating the boundaries between Zones 1 and 3 to "flip" Zone 1 to be a vote for the Flamingo.

Extension: *Math Enrichment*

Distribute additional copies of the Activity 2 PDF and present students with one of the following challenges:

- Show two ways that six zones with at least 175 students in each could result in a flamingo win, and then a penguin win.
- Construct their own zoning rules so the Flamingo always wins no matter how the boundaries are drawn.

Math Language Development

MLR7: Compare and Connect

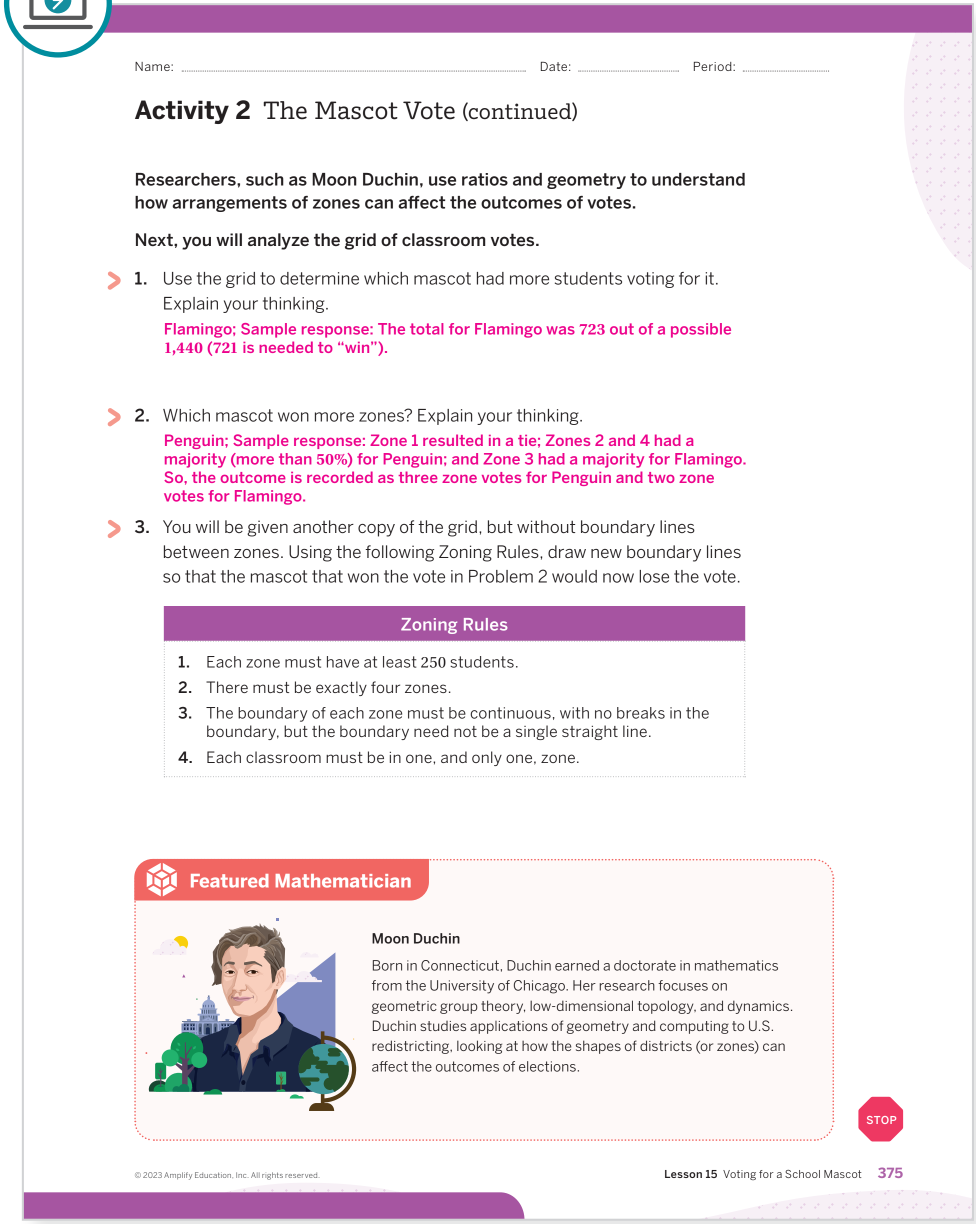
During the Connect, prompt students to verbally explain any connections they see between the voting grid from Activity 1 and the voting grid they created in this activity. Ask, "What is the same about the grids? What is different?"

English Learners

As students share connections between the grids, point to and highlight areas on the grid that are referenced during the discussion.

Activity 2 The Mascot Vote (continued)

Students use ratios and percentages to calculate the results of the vote, and then they redesign the zones of the classroom grid to change the results.



Name: Date: Period:

Activity 2 The Mascot Vote (continued)

Researchers, such as Moon Duchin, use ratios and geometry to understand how arrangements of zones can affect the outcomes of votes.

Next, you will analyze the grid of classroom votes.

1. Use the grid to determine which mascot had more students voting for it. Explain your thinking.
   Flamingo; Sample response: The total for Flamingo was 723 out of a possible 1,440 (721 is needed to "win").
2. Which mascot won more zones? Explain your thinking.
   Penguin; Sample response: Zone 1 resulted in a tie; Zones 2 and 4 had a majority (more than 50%) for Penguin; and Zone 3 had a majority for Flamingo. So, the outcome is recorded as three zone votes for Penguin and two zone votes for Flamingo.
3. You will be given another copy of the grid, but without boundary lines between zones. Using the following Zoning Rules, draw new boundary lines so that the mascot that won the vote in Problem 2 would now lose the vote.

Zoning Rules

1. Each zone must have at least 250 students.
2. There must be exactly four zones.
3. The boundary of each zone must be continuous, with no breaks in the boundary, but the boundary need not be a single straight line.
4. Each classroom must be in one, and only one, zone.

Featured Mathematician

Moon Duchin

Born in Connecticut, Duchin earned a doctorate in mathematics from the University of Chicago. Her research focuses on geometric group theory, low-dimensional topology, and dynamics. Duchin studies applications of geometry and computing to U.S. redistricting, looking at how the shapes of districts (or zones) can affect the outcomes of elections.

Lesson 15 Voting for a School Mascot 375

3 Connect

Have pairs of students share their responses and thinking for Problems 1 and 2, and then have several pairs share how they drew the boundary lines, focusing on strategies they used to make sure the rules were being followed and the desired outcome would be achieved.

Highlight that there were many possible ways to draw the zones to ensure the Flamingo won. Likewise, there are many possible ways the same could be done to ensure the Penguin would win.

Ask:

- "Do you think it is possible to know how students or classrooms and zones will vote ahead of time? How?" Yes; by polling students.
- "Do you think there is a way of establishing how the zones can be defined so that one mascot would win regardless of how the boundaries are drawn?" Yes, if the zones were much less equal in size, or, if most of the votes for the intended loser were placed in just a few zones.

Featured Mathematician

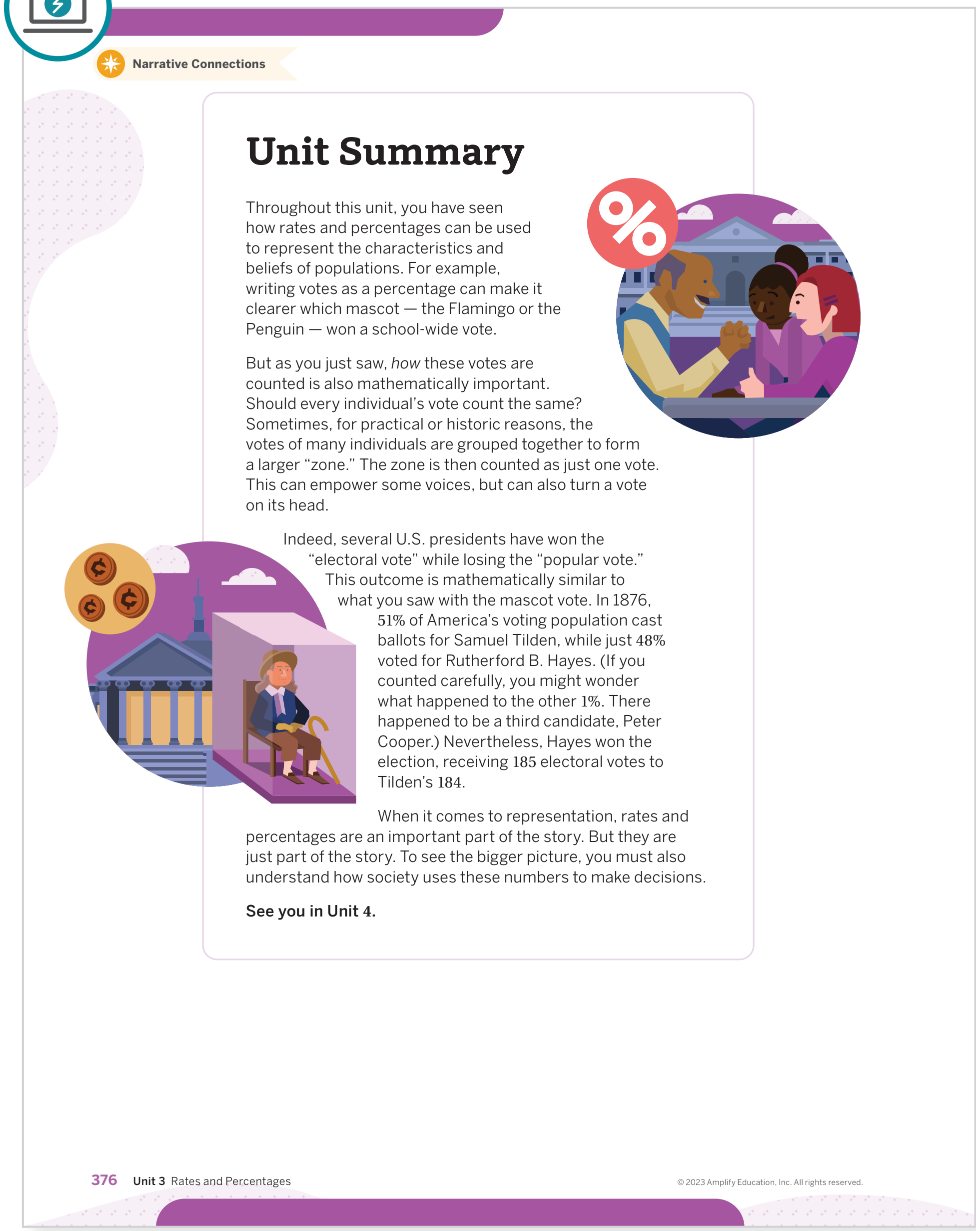
Moon Duchin

Have students read about Moon Duchin, who uses her background in geometric group theory and topology to study relationships between redistricting and election outcomes.

Whole Class | 5 min

Unit Summary

Review and synthesize how rates and percentages are used in voting scenarios.



Narrative Connections

Unit Summary

Throughout this unit, you have seen how rates and percentages can be used to represent the characteristics and beliefs of populations. For example, writing votes as a percentage can make it clearer which mascot — the Flamingo or the Penguin — won a school-wide vote.

But as you just saw, *how* these votes are counted is also mathematically important. Should every individual's vote count the same? Sometimes, for practical or historic reasons, the votes of many individuals are grouped together to form a larger "zone." The zone is then counted as just one vote. This can empower some voices, but can also turn a vote on its head.

Indeed, several U.S. presidents have won the "electoral vote" while losing the "popular vote." This outcome is mathematically similar to what you saw with the mascot vote. In 1876, 51% of America's voting population cast ballots for Samuel Tilden, while just 48% voted for Rutherford B. Hayes. (If you counted carefully, you might wonder what happened to the other 1%. There happened to be a third candidate, Peter Cooper.) Nevertheless, Hayes won the election, receiving 185 electoral votes to Tilden's 184.

When it comes to representation, rates and percentages are an important part of the story. But they are just part of the story. To see the bigger picture, you must also understand how society uses these numbers to make decisions.

See you in Unit 4.

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Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Ask:

- "How can ratios, rates, or percentages make determining the winner of an election more efficient?" If you know how many votes are possible then, while all votes should be counted, once one side reaches a majority, the winner is known. You can also track trends and the changing ratios of tallied votes to determine what rate of votes for one side or the other is needed to win from that point on.
- "Which vote counting method do you think is the most fair? Why?"

Highlight the connections between the mathematics — rates and percentages can represent constant values that remain the same for populations large and small — in the context of voting for a mascot in this lesson, and those of:

- Other voting scenarios within schools and for a student council (Lesson 1 in particular, but also Lessons 2–7).
- Shared characteristics or preferences of subgroups within a population (Lessons 8–13).
- The demographics of the world population (Lesson 14).

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. To help them engage in meaningful reflection, consider asking,

- "What is the relationship between unit rates and percentages?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of ways to use ratios, rates, and percentages by reflecting on how they were used in this lesson.

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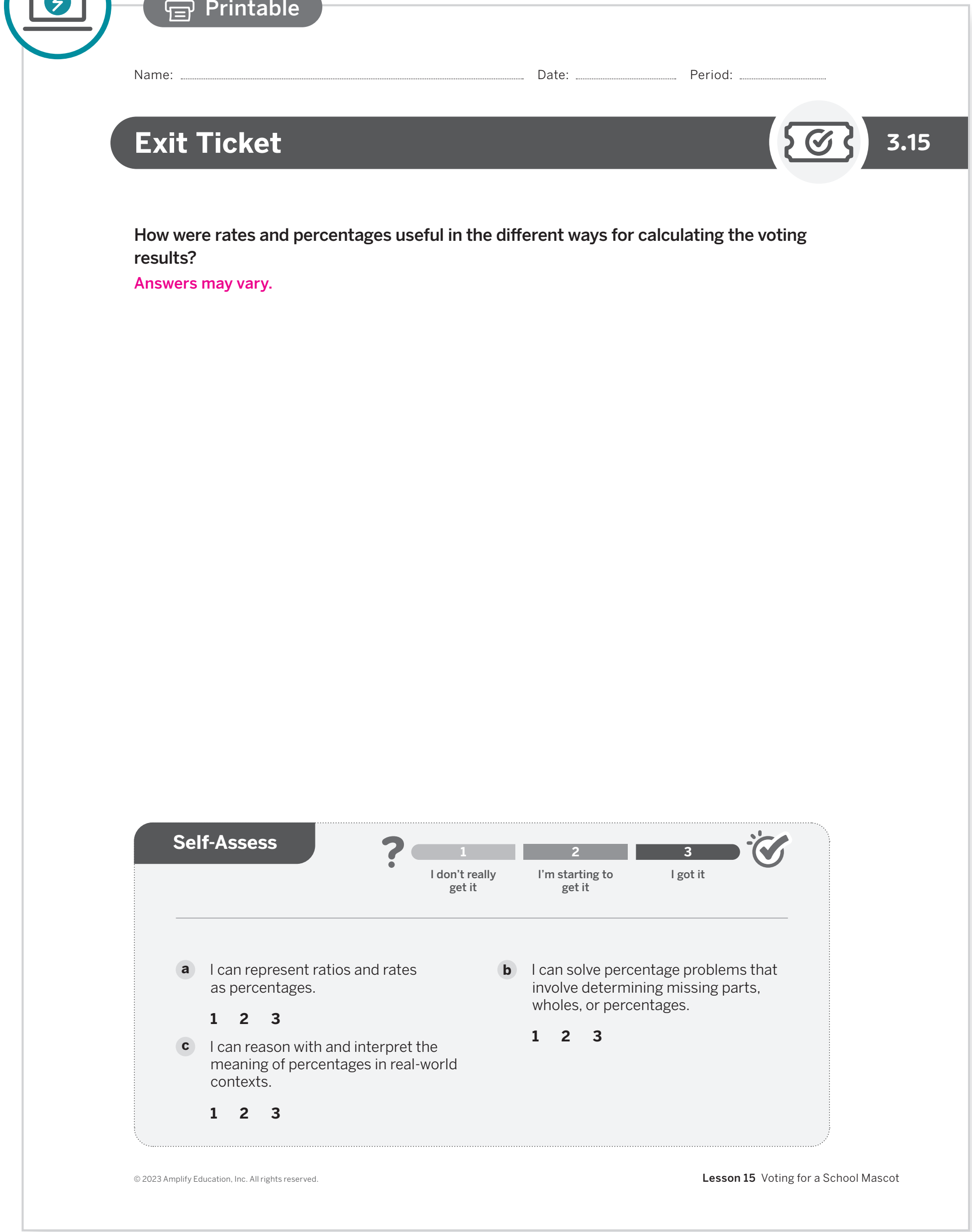


Name: ______ Date: ______ Period: ______

Exit Ticket 3.15

How were rates and percentages useful in the different ways for calculating the voting results?

Answers may vary.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a. I can represent ratios and rates as percentages.

1 2 3

b. I can solve percentage problems that involve determining missing parts, wholes, or percentages.

1 2 3

c. I can reason with and interpret the meaning of percentages in real-world contexts.

1 2 3

Success looks like . . .

- **Language Goal:** Applying reasoning about ratios and percentages to analyze voting situations. **(Speaking and Listening, Writing)**
  - » Explaining how they determined voting results using rates and percentages.
- **Language Goal:** Comparing and contrasting different voting systems. **(Speaking and Listening, Writing)**
- **Language Goal:** Suggesting and critiquing whether or not a method for distributing votes is fair. **(Speaking and Listening, Writing)**

Suggested next steps

If students write about less mathematically focused ideas, such as "they helped me solve the problems," consider:

- Referring them to the anchor charts from the unit and remind them how those mathematical concepts were used in the lesson.

Professional Learning

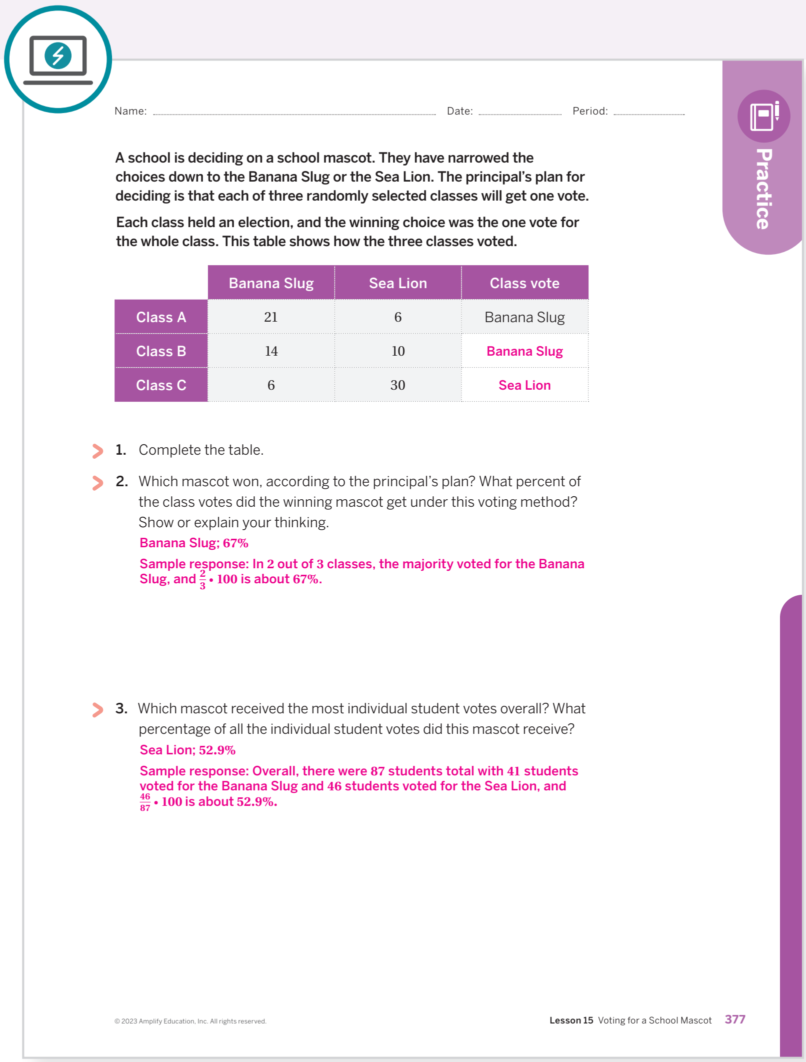
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- Reflect upon the unit as a whole. How did it build on and connect to the previous two units? Think about the mathematics and your students as learners.
- Have you changed any ideas you used to have about ratios, rates, and percentages?

Practice

Independent



Practice

Name: Date: Period:

A school is deciding on a school mascot. They have narrowed the choices down to the Banana Slug or the Sea Lion. The principal's plan for deciding is that each of three randomly selected classes will get one vote.

Each class held an election, and the winning choice was the one vote for the whole class. This table shows how the three classes voted.

	Banana Slug	Sea Lion	Class vote
Class A	21	6	Banana Slug
Class B	14	10	**Banana Slug**
Class C	6	30	**Sea Lion**

1. Complete the table.
2. Which mascot won, according to the principal's plan? What percent of the class votes did the winning mascot get under this voting method? Show or explain your thinking.

   Banana Slug; 67%

   Sample response: In 2 out of 3 classes, the majority voted for the Banana Slug, and $\frac{2}{3} \cdot 100$ is about 67%.

3. Which mascot received the most individual student votes overall? What percentage of all the individual student votes did this mascot receive?

   Sea Lion; 52.9%

   Sample response: Overall, there were 87 students total with 41 students voted for the Banana Slug and 46 students voted for the Sea Lion, and $\frac{46}{87} \cdot 100$ is about 52.9%.

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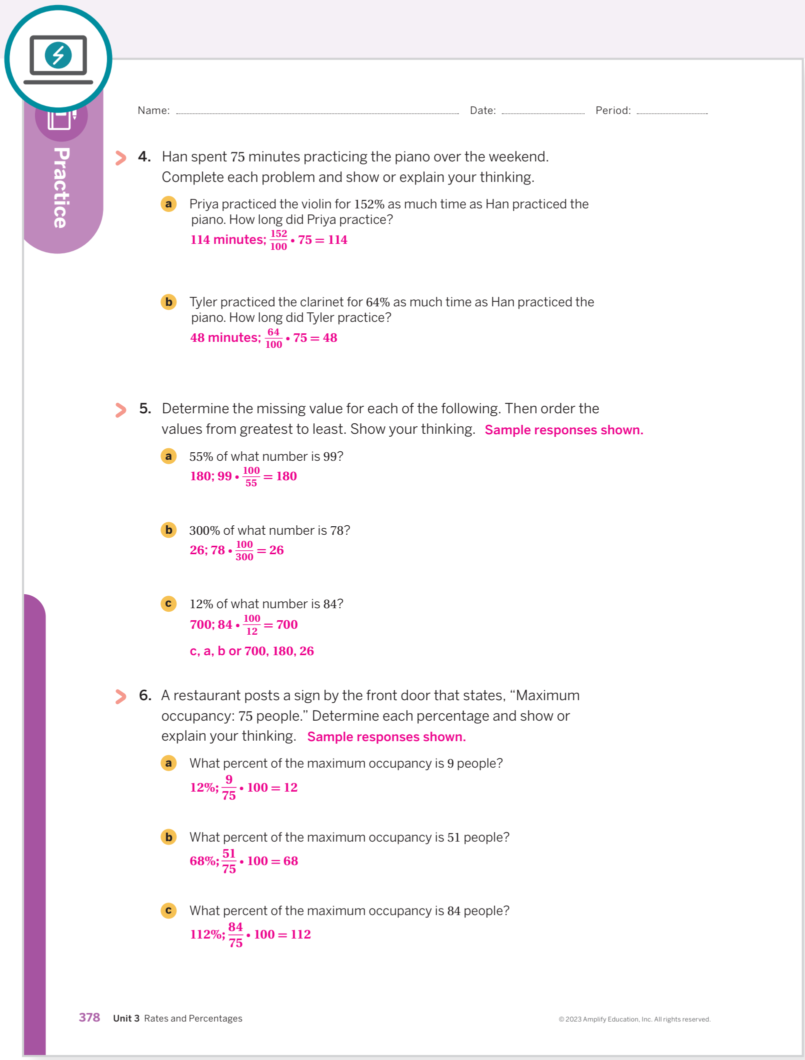
Lesson 15 Voting for a School Mascot 377



Practice

Name: Date: Period:

4. Han spent 75 minutes practicing the piano over the weekend. Complete each problem and show or explain your thinking.
   - a Priya practiced the violin for 152% as much time as Han practiced the piano. How long did Priya practice?

     114 minutes; $\frac{152}{100} \cdot 75 = 114$
   - b Tyler practiced the clarinet for 64% as much time as Han practiced the piano. How long did Tyler practice?

     48 minutes; $\frac{64}{100} \cdot 75 = 48$
5. Determine the missing value for each of the following. Then order the values from greatest to least. Show your thinking. **Sample responses shown.**
   - a 55% of what number is 99?

     180; $99 \cdot \frac{100}{55} = 180$
   - b 300% of what number is 78?

     26; $78 \cdot \frac{100}{300} = 26$
   - c 12% of what number is 84?

     700; $84 \cdot \frac{100}{12} = 700$

     c, a, b or 700, 180, 26
6. A restaurant posts a sign by the front door that states, "Maximum occupancy: 75 people." Determine each percentage and show or explain your thinking. **Sample responses shown.**
   - a What percent of the maximum occupancy is 9 people?

     12%; $\frac{9}{75} \cdot 100 = 12$
   - b What percent of the maximum occupancy is 51 people?

     68%; $\frac{51}{75} \cdot 100 = 68$
   - c What percent of the maximum occupancy is 84 people?

     112%; $\frac{84}{75} \cdot 100 = 112$

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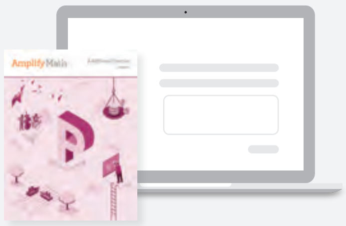
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 13	2
	5	Unit 3 Lesson 13	1
	6	Unit 3 Lesson 9	2

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 4

Dividing Fractions

Students extend their understanding of partitive and quotitive division from whole numbers to fractions. They use this along with the relationship between multiplication and division to construct models and develop an algorithm for dividing fractions, and they apply it to problems involving lengths, areas, and volumes.

Essential Questions

- How can dividing by the same fraction be interpreted in two different ways?
- How is dividing by a fraction related to multiplying fractions?
- What does it mean when a quantity represents a fractional number of equal-sized groups?
- *(By the way, how many tanks could a fish tank fill if a fish tank could fill tanks?)*

Key Shifts in Mathematics

Focus

In this unit . . .

Students use the two interpretations of division – quotitive (how many groups) and partitive (how much in a group) – to solve problems where fractions represent the total, the amount in a group, or the number of groups. They use tape diagrams and other models along with the relationship between multiplication and division to solve for unknowns in division problems, including measurement problems involving fractional lengths. Students also develop generalizable strategies for working with fractions, such as identifying common denominators and using the standard algorithm.

Coherence

Previously . . .

In Grades 3–5, students worked with multiplication and division of whole numbers. In Grade 3, they identified the two interpretations of division and established the relationship between multiplication and division. Additionally in Grade 3, students related the concept of area to multiplication, specifically in determining a formula for the area of a rectangle. In Grade 5, students divided whole numbers and unit fractions and discovered that fractions can be interpreted as division. Also in Grade 5, students determined the volume of a rectangular prism with whole-number dimensions. This was revisited in Unit 1 of this grade, along with the discovery of a formula for the area of a triangle.

Coming soon . . .

In Unit 5, students will develop a standard algorithm for dividing decimals and whole numbers. In Grade 7, students will compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Division can be used to answer "How many groups?" and "How many in each group?" (Lessons 2, 5, 8–9) A division situation can be represented by both multiplication and division equations (Lessons 3, 5–6, 8–9) and models, such as tape diagrams (Lessons 2–10). Common denominators, whole number division and multiplication, and related quotients or ratios can all be used to divide fractions (Lessons 7, 10, 12).

Procedural Fluency

The standard algorithm for dividing fractions of multiplying by the reciprocal of the divisor is an efficient and generalizable way to determine any quotient involving one or more fractions (Lesson 11).

Application

Strategies for dividing fractions, such as the standard algorithm, can be used in any context in which there are equal-sized groups and either the number of groups or the size of each group is unknown. Examples include determining unit rates and geometric measurement problems (Lessons 12–17).

Crossing the Fractional Divide

SUB-UNIT

1

Lessons 2–4

Rates

Students enter the mysterious Spöklik Furniture store, looking for clues to find their lost friend Maya. To exit the maze-like Housewares department, students must solve a problem by estimating and comparing several quotients, reminding them of the relationship between fractions and division. Along the way, students revisit other division concepts using whole numbers and unit fractions, while also encountering some non-unit fractions. They distinguish two interpretations of division — partitive and quotitive — and represent both types using multiplication and division equations.

Narrative: Find a missing friend at Spöklik Furniture, where not everything is what it seems.

SUB-UNIT

2

Lessons 5–12

Percentages

Students search for more clues of their missing friend in the Spöklik Showroom. On their journey, they encounter a Spöklik employee trying to determine the length of a bolt presented as a continued fraction. These lessons set them up for being able to extend their thinking to that level. By first exploring both interpretations of division with any fractions in any position, and through models such as tape diagrams, students develop general procedures for dividing fractions, including identifying common denominators and using the standard algorithm of multiplying by the ***reciprocal***.

Narrative: Use fractions to build furniture in the Spöklik Showroom with some ghostly companions.

Launch

Lesson 1

Seeing Fractions

Students look at different images of rectangular areas partitioned into several other smaller rectangles of different sizes and orientations. They activate their prior knowledge of fractions representing relationships between parts to the whole to identify as many different fractions as possible. This exercise draws students' attention to units and highlights what represents a whole, which sets them up to think more about fractions and division in this unit. Their friend Maya is certainly hoping they know what to do.

SUB-UNIT

3

Lessons 8–16

Percentages

Students are relieved to have found their friend Maya, but the adventure is far from over. The next step is sneaking past the guards at the checkout area with Maya's dog, Penny, hidden in a box that contained a pyramid-shaped statue. Students must figure out how much packing material to remove from the box so that it can close with Maya's dog safely hidden inside it. This puzzle represents a similar application of fraction division as in the lessons, with the students dividing fractional lengths and determining unknown fractional lengths in rectangular areas and volumes.

Narrative: Make your way out of Spöklik Furniture, and measure with fractions as you go.

Capstone

Lesson 17

Now, Where Was That Bus?

Just when students begin to think their Spöklik nightmare is over, they realize that Maya has no idea where to catch the bus out of the spooky store. Both the ticket she's holding and the parking garage signage are cryptic, to say the least. Students use clues embedded throughout the unit in their Student Edition, and also apply everything they have learned about dividing fractions to help decipher these final clues and help Maya get to the bus stop, and finally, to home.

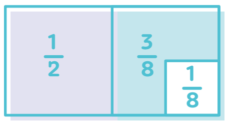
Unit at a Glance

Spoiler Alert: Just as with other fractional operations, dividing fractions can be performed by using an algorithm — multiplying the dividend by the reciprocal of the divisor.

Assessment

A **Pre-Unit Readiness Assessment**

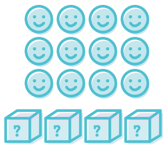
Launch Lesson



1 **Seeing Fractions**

Identify fractional parts and their referent whole in pattern images.

Sub-Unit 1: Interpreting Division Scenarios

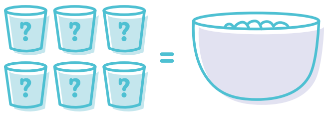


2 **Meanings of Division** ●

Use models to review and to build upon previous work with partitive and quotitive division.

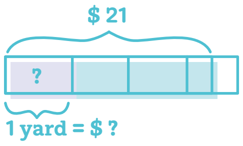
3 **Relating Multiplication and Division** ●

Use the relationship between multiplication and division to evaluate contexts involving division with fractions.



8 **How Much in Each Group?** (Part 1)

Model and determine "How much in each group?" when a fraction of a group is known.

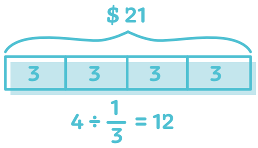


9 **How Much in Each Group?** (Part 2) ●

Distinguish between different types of equal-sized group problems by writing and modeling them.

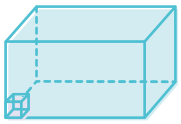
Assessment

A **Mid-Unit Assessment**



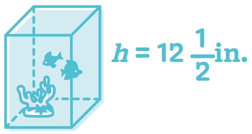
10 **Dividing by Unit and Non-Unit Fractions**

Develop an algorithm for fraction division in two steps.



15 **Volume of Prisms**

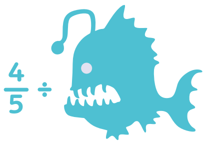
Divide fractions to determine missing dimensions when given the volume of a cube or a rectangular prism.



16 **Fish Tanks Inside of Fish Tanks**

Divide fractions to solve more complex volume problems.

Capstone Lesson



17 **Now, Where Was That Bus?** ●

Apply understanding of dividing fractions to decipher a code in the unit's context of the Spöklik Furniture store.

Assessment

A **End-of-Unit Assessment**

Key Concepts

Lesson 6: Division with fractions answers “how many groups?”
Lesson 8: Division with fractions answers “how much in one group?”
Lesson 11: To divide by a fraction, you can multiply the dividend by the reciprocal of the divisor.

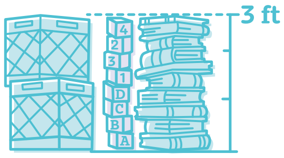
Pacing

17 Lessons: 45 min each
3 Assessments: 45 min each

Full Unit: 20 days
● **Modified Unit:** 14–15 days

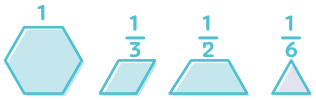
Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

Sub-Unit 2: Division with Fractions



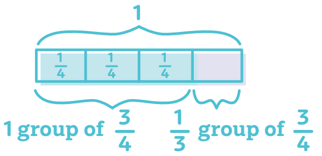
4 Size of Divisor and Size of Quotient ●

Understand how the size of a divisor for any given dividend affects the size of the quotient.



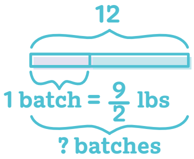
5 How Many Groups?

Use pattern blocks to model multiplication and division with fractions.



6 Using Diagrams to Determine the Number of Groups

Draw and use tape diagrams to build conceptual understanding of dividing by a non-unit fraction.



7 Dividing With Common Denominators

Use common denominators to divide any fractions.

Sub-Unit 3: Fractions in Lengths, Areas, and Volumes

$$\frac{5}{2} \div \frac{3}{4} = \frac{5}{2} \cdot \frac{4}{3}$$

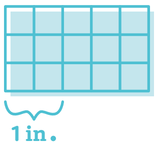
11 Using an Algorithm to Divide Fractions

Formalize a one-step algorithm for fraction division.

$$\frac{2}{3} \div \frac{4}{5} = 10 \div 12$$

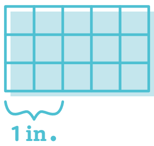
12 Related Quotients ●

Divide any fractions by writing and evaluating related quotients. Optional work includes relating fraction division to ratios and unit rates.



13 Fractional Lengths

Divide fractions to solve problems involving fractional lengths, including multiplicative comparisons.



14 Area With Fractional Side Lengths

Divide fractions to determine missing dimensions when given the area of a rectangle or a triangle.

● Modifications to Pacing

Lessons 2–3: These lessons are largely a review of the two interpretations of division (Lesson 2) and the relationship between multiplication and division (Lesson 3) from earlier grades. You may consider merging them into one lesson by simply having students complete the Lesson 2 Warm-up, and Activity 1 of Lesson 3. They may also be omitted entirely, but students would benefit from the key concepts of each being addressed somehow prior to Lesson 5, and that lesson’s Warm-up may require extra time.

Lesson 4: The focus of this lesson is determining how the size of the divisor affects the size of the quotient, which is not an explicit expectation for the grade, and so, it really serves to develop a deeper understanding of division. You may consider omitting this lesson entirely, or solely focusing on Activity 2.

Lesson 9: This lesson may be omitted. It serves mostly as practice distinguishing between the two interpretations of division — having students model and solve problems, as well as writing their own.

Lesson 12: The focus of this lesson is dividing fractions by writing and evaluating related division expressions with the same quotient, which is not an explicit expectation for the grade. This lesson may be omitted, understanding that the opportunity for students to connect to their previous work with ratios and unit rates may be lost.

Lesson 17: This capstone lesson may be omitted, but, in addition to offering a fun and challenging application of all of the work of the unit, this particular lesson also offers closure to the narrative story embedded throughout this unit.

Unit Supports

Math Language Development

Lesson	New Vocabulary
10	reciprocal

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
6, 11, 15	**MLR1:** Stronger and Clearer Each Time
1, 4, 10–12	**MLR2:** Collect and Display
1, 5, 12	**MLR3:** Critique, Correct, Clarify
13	**MLR4:** Information Gap
13	**MLR5:** Co-craft Questions
7, 14, 16, 17	**MLR6:** Three Reads
2, 5–8, 10, 14, 15, 17	**MLR7:** Compare and Connect
2–4, 7–9, 11, 12, 14, 16	**MLR8:** Discussion Supports

Materials

Every lesson includes:

- Exit Ticket
- Additional Practice

Additional required materials include:

Lesson(s)	Materials
15	$\frac{1}{2}$-in. cubes
16	calculator
7, 9–11	colored pencils
7, 15	geometry toolkits
14	graph paper
2	index cards
5–6	pattern blocks
1, 4–6, 12–14, 17	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
16	ruler
14	straightedge

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routines
2, 9	Gallery Tour
13	Info Gap
4, 12	Number Talk
10	Take Turns
1, 4–12, 15	Think-Pair-Share
1	Turn and Talk

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 9
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 17

Amps

POWERED BY desmos

Social & Collaborative Digital Moments

Featured Activity

Dividing Fractions by Fractions

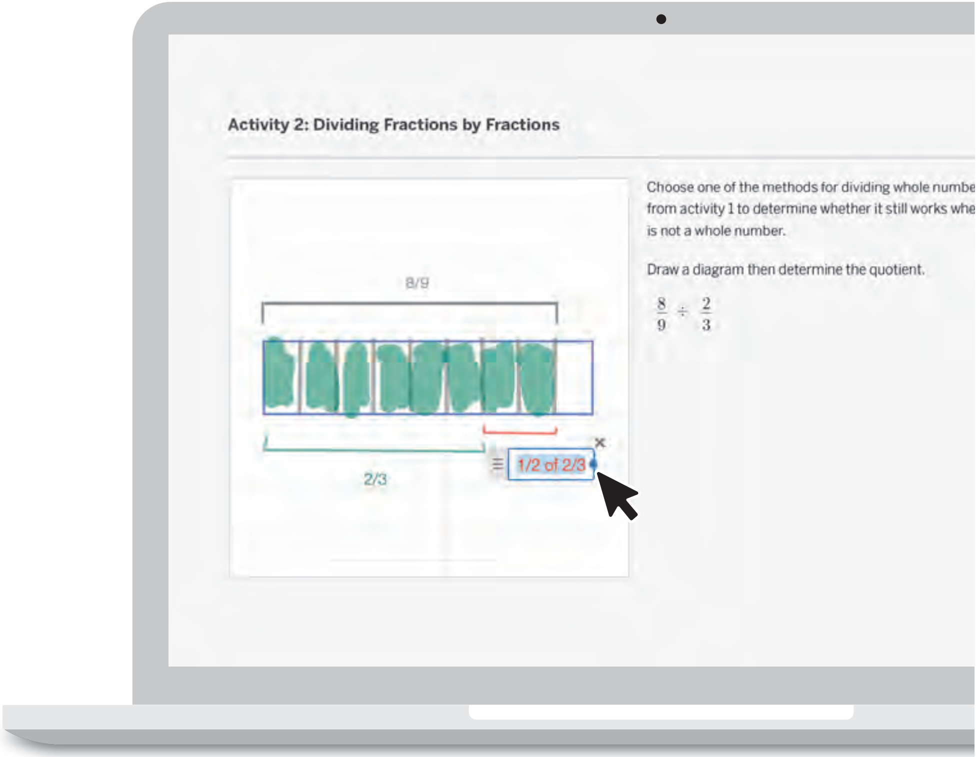
Put on your student hat and work through **Lesson 10, Activity 2:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Representing Fractional-Sized Groups **(Lesson 6)**
- Fractions of Ropes **(Lesson 7)**
- A Scenario of Your Own **(Lesson 9)**
- Exploring the Fraction Division Algorithm **(Lesson 11)**
- Volume of Cubes and Prisms **(Lesson 15)**



Activity 2: Dividing Fractions by Fractions

Choose one of the methods for dividing whole numbe
from activity 1 to determine whether it still works whe
is not a whole number.

Draw a diagram then determine the quotient.

$\frac{8}{9} \div \frac{2}{3}$

Unit Study

Professional Learning

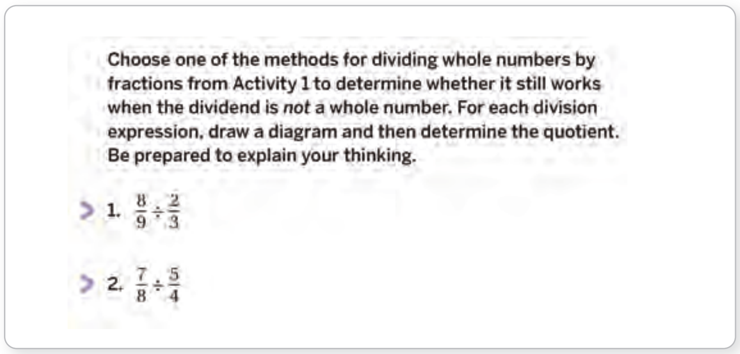
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 2 introduces students to division with fractions. Students previously worked with division scenarios and interpreted each with a multiplication equation and a division equation. The consistent application of writing a story problem, sketching a diagram, and writing equations for each division problem allows students to practice the skills while deepening their conceptual understanding. Students experience a progression of division problems, from dividing by unit and non-unit fractions to exploring the division algorithm, and then connecting to ratio tables. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from Lesson 10, Activity 2:



Choose one of the methods for dividing whole numbers by fractions from Activity 1 to determine whether it still works when the dividend is *not* a whole number. For each division expression, draw a diagram and then determine the quotient. Be prepared to explain your thinking.

1. $\frac{8}{9} \div \frac{2}{3}$

2. $\frac{7}{8} \div \frac{5}{4}$

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

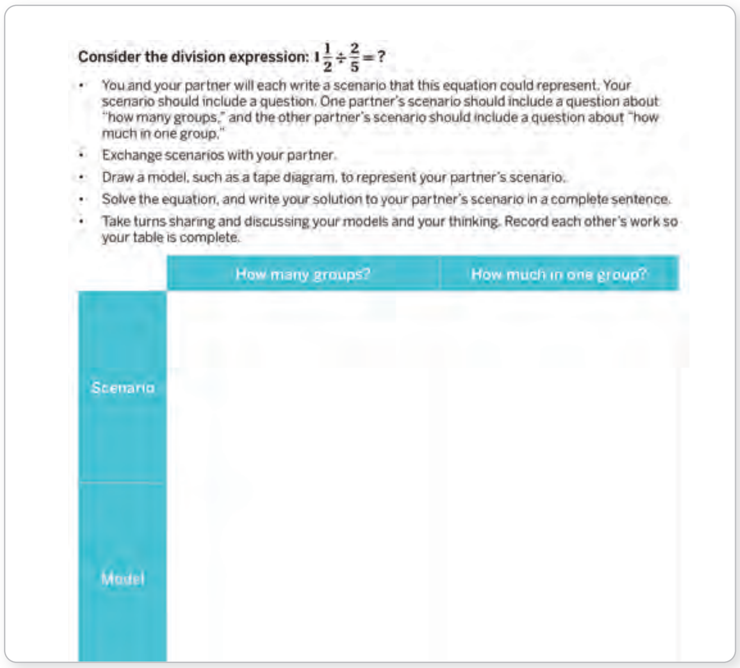
- What was it like to engage in this problem as a learner?
- Activity 2 asks students to choose one of the methods from Activity 1 to determine if it still works for dividing by fractions. Other than drawing tape diagrams, students may also use a ratio table, double number lines, common denominator, etc. What is your go-to strategy and when did you first learn to use it?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Gallery Tour

Rehearse . . .

How you'll facilitate the *Gallery Tour* instructional routine in Lesson 9, Activity 2:



Consider the division expression: $1\frac{1}{2} \div \frac{2}{5} = ?$

- You and your partner will each write a scenario that this equation could represent. Your scenario should include a question. One partner's scenario should include a question about "how many groups," and the other partner's scenario should include a question about "how much in one group."
- Exchange scenarios with your partner.
- Draw a model, such as a tape diagram, to represent your partner's scenario.
- Solve the equation, and write your solution to your partner's scenario in a complete sentence.
- Take turns sharing and discussing your models and your thinking. Record each other's work so your table is complete.

	How many groups?	How much in one group?
Scenario		
Model		

Points to Ponder . . .

- How will you organize the display of artifacts — by pairs or by types of division? What will you prompt students to look for and provide feedback on in the work of their peers?

This routine . . .

- Allows students to see and discuss multiple strategies, representations, and solution paths.
- Provides a low-stakes environment for students to give and receive feedback on peer work.
- Gives students ownership of checking their understanding.

Anticipate . . .

- Students may be hesitant to offer critical feedback.
- Students may have difficulty making sense of work different from their own.
- If you have not used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you have used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Elicit and Use Evidence of Student Thinking.

This effective teaching practice . . .

- Helps you assess student progress toward the mathematical goals and objectives of the lessons and units. By knowing your students' current levels, you can help them get to where they need to be!
- Allows you to adjust your instruction, based upon student responses, so that you can support your students and extend their learning.

Points to Ponder . . .

- How and when will I use student responses from this unit's assessments to help adjust my instruction?
- Where are there moments during the lessons' activities in which I can informally elicit evidence of my students' thinking? How will I approach these moments?

Math Language Development

MLR8: Discussion Supports

MLR8 appears in Lessons 2–4, 7–9, 11, 12, 14, and 16.

- Throughout the unit, sentence frames are provided for you to display to your students. Students can use these prompts to help frame their responses and add structure and organization to their thinking.
- In Lesson 4, further probing questions are provided so that you can press for details in student reasoning as students compare how the quotient of a division expression changes as the divisor increases.
- **English Learners:** Provide wait time to allow students to formulate a response before sharing with others.

Point to Ponder . . .

- During class discussions in this unit, how will you know when to probe further to assess student understanding and encourage your students to use their developing mathematical vocabulary?

Differentiated Support

Accessibility: Optimize Access to Tools, Optimize Access to Technology

Opportunities to provide visual support, physical manipulatives, or the use of technology appear in Lessons 1–17.

- In Lessons 5 and 6, provide access to pattern blocks or copies of pattern blocks to help guide student thinking. In Lesson 6, distribute the *Fraction Strips* PDF during the Warm-up to help them visualize fractional parts of 1 whole.
- Multiple opportunities are provided for students to use the Amps slides for activities in which they can create and interact with digital tape diagrams. Alternatively, provide blank tape diagrams for students to use partition and label from the *Tape Diagrams* PDF.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide whether to use concrete physical manipulatives — such as pattern blocks and fraction strips — or technology (through the Amps slides for each activity) to support students' understanding of division with fractions?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **Mid-** and **End-of-Unit Assessments**, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with solving one-step equations with a variable throughout the unit? Do you think your students will generally:
    - miss the underlying concept of balance and mathematical equality?
    - simply struggle with the concept of variables and unknowns?
    - be fully ready to solve procedurally and efficiently, but misapply the properties of equality?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their self-management and responsible decision making skills.

Points to Ponder . . .

- Are students able to regulate their emotions and control their impulses so that they can stay focused on the task at hand? Do they set goals and motivate themselves to achieve the goals? Do students apply organizational skills to help them manage stress?
- Are students able to communicate effectively? When they speak, do they use precise language? Do they listen at least as much as they speak? Is their listening active with the purpose of either receiving help or understanding well enough to provide help?

Seeing Fractions

Let's look for fractions in different patterns.

Focus

Goals

1. Use spatial reasoning to identify fractional parts of a given whole, or to identify the whole, given a fractional part.
2. **Language Goal:** Use mathematical language precisely and flexibly to describe parts, wholes, and fractions. **(Speaking and Listening)**

Rigor

- Students **apply** prior knowledge of fractions to identify fractional parts and their whole in pattern images.

Coherence

Today

Students apply their prior knowledge of fractions to identify fractional parts and their referent whole in pattern images. They recognize that, depending on the whole identified, the same part may represent multiple different fractions. As they engage in a friendly competition, students must justify their thinking and attend to precision as they name both the fractional part and the whole.

Previously

In Grades 3–5, students developed several understandings of fractions, including a whole partitioned into equal parts. They related fractions to division. Students also performed addition, subtraction, and multiplication with fractions, mixed numbers, and whole numbers. Division of fractions in Grade 5 was limited to whole numbers and unit fractions.

Coming Soon

In Lessons 2–4, students review the connections between multiplication and division, and between fractions and division, preparing them to divide fractions throughout this unit.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	25 min	5 min	5 min
Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (optional)
- Activity 1 PDF, one per group
- Activity 1 PDF (answers)
- *Hidden Word* PDF (for display)
- *Hidden Word* PDF (answer, for display)

Amps | **Featured Activity**

Activity 1
Spirit of Competition

Small groups of students face off in a friendly competition among classmates to see who can identify the most unique fractions represented in a puzzle.

Building Math Identity and Community

Connecting to Mathematical Practices

Students may resort to speaking disrespectfully to each other as they try to persuade the other team to give them the bonus point in Activity 1. Explain that there are standards for behavior and expectations for the way they speak to each other. Remind students that mathematically proficient students use precise language and reasoning to convince others. Ask students to identify ways they can respectfully communicate their ideas.

Modifications to Pacing

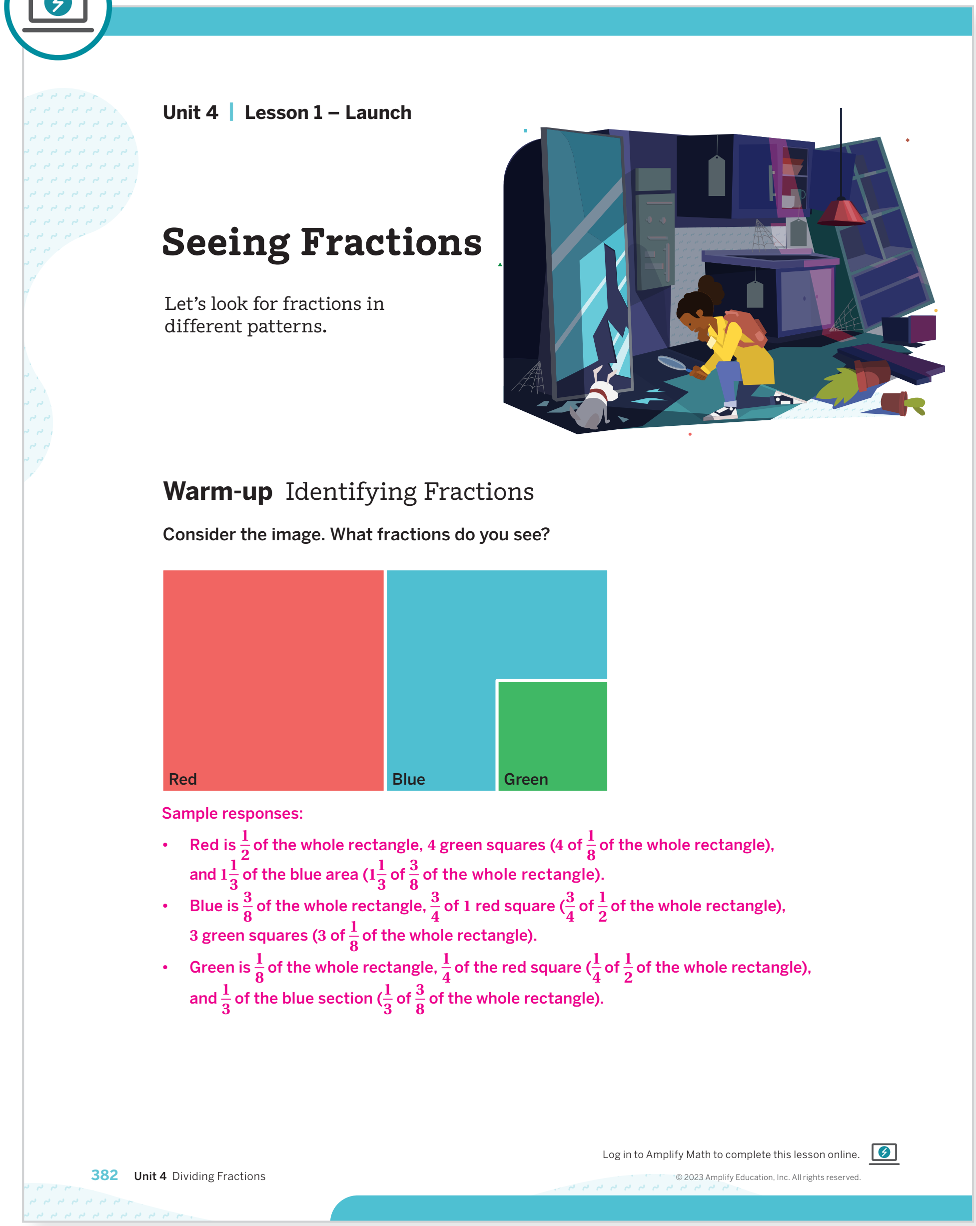
You may want to consider these additional modifications if you are short on time.

- Make the **Warm-up** a whole-class activity. Have students use the *Turn and Talk* routine before sharing with the whole class.
- In **Activity 1**, consider eliminating Part 1, Problem 2. Have small groups of students generate the list together before competing against another group. If time is very short, you may consider eliminating the competition entirely, although this will definitely limit the fun!

Pairs | 10 min

Warm-up Identifying Fractions

Students identify fractional parts in an image, seeing that the fractions used to describe any given part can differ depending on the corresponding whole.



Unit 4 | Lesson 1 – Launch

Seeing Fractions

Let's look for fractions in different patterns.

Warm-up Identifying Fractions

Consider the image. What fractions do you see?

Sample responses:

- Red is $\frac{1}{2}$ of the whole rectangle, 4 green squares (4 of $\frac{1}{8}$ of the whole rectangle), and $1\frac{1}{3}$ of the blue area ($1\frac{1}{3}$ of $\frac{3}{8}$ of the whole rectangle).
- Blue is $\frac{3}{8}$ of the whole rectangle, $\frac{3}{4}$ of 1 red square ($\frac{3}{4}$ of $\frac{1}{2}$ of the whole rectangle), 3 green squares (3 of $\frac{1}{8}$ of the whole rectangle).
- Green is $\frac{1}{8}$ of the whole rectangle, $\frac{1}{4}$ of the red square ($\frac{1}{4}$ of $\frac{1}{2}$ of the whole rectangle), and $\frac{1}{3}$ of the blue section ($\frac{1}{3}$ of $\frac{3}{8}$ of the whole rectangle).

Log in to Amplify Math to complete this lesson online.

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1 Launch

Use the *Think-Pair-Share* routine. Give students 2 minutes to work independently before sharing and comparing with a partner.

2 Monitor

Help students get started by activating prior knowledge. Ask, "What is a fraction? What might a fraction represent in this image?"

Look for points of confusion:

- **Incorrectly naming fractions.** Ask, "What is the part? What is the whole? How many of those parts compose one whole?" If needed, ask, "Could you divide up the whole further to help with, or to check, your thinking?"

Look for productive strategies:

- Considering different wholes (e.g., the large rectangle, each part, or a combination of parts).
- Using precise language to identify both the fractional part and its related whole (e.g., one-fourth of the red square or one-fourth of one-half of the whole rectangle).

3 Connect

Display the figure, and record student responses for all to see as they share.

Have students share their responses and reasoning, ensuring they see multiple fractions for each part. Encourage them to use precise language to name both the fractional part and the corresponding whole.

Ask, "Why can the same part be represented by more than one fraction?"

Highlight two ways to name the part and whole (e.g., the green square is one-fourth of the red square; the green square is one-fourth of one-half of the whole rectangle), and how switching the part and whole leads to different results (e.g., the red square is 4, or $\frac{4}{1}$, of the green square).

Math Language Development

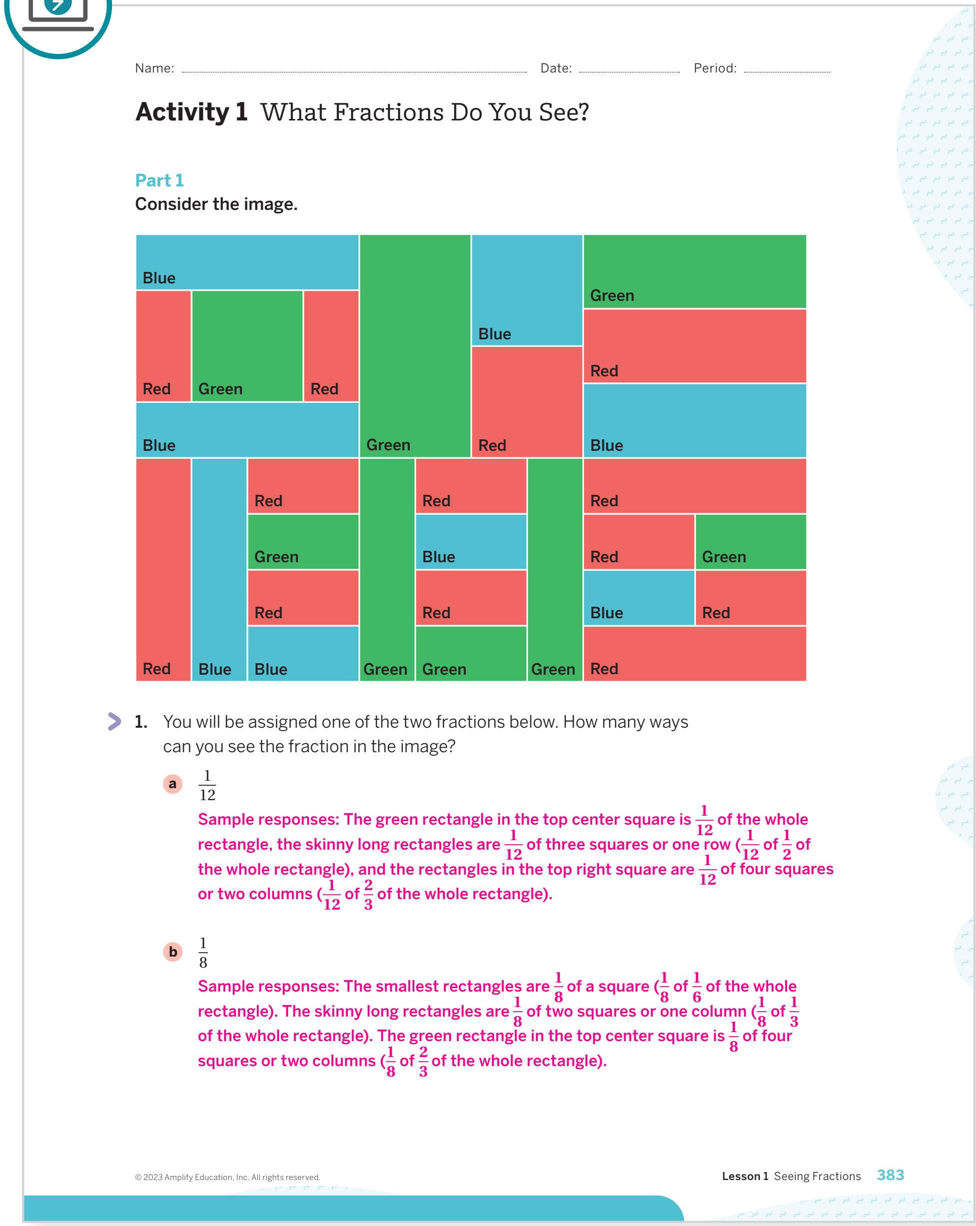
MLR2: Collect and Display

While students discuss their responses with a partner, circulate and listen for the language they use to describe the fractions they see in the image. Collect this language and add it to a class display that students can refer to throughout the unit. Examples of terms and phrases students might say are: *part, whole, whole rectangle, largest rectangle, fractional parts,* etc.

Small groups | 25 min

Activity 1 What Fractions Do You See?

Students examine a more complex image, further exploring the relationship among the part, the whole, and the fraction.



Name: ____________ Date: ________ Period: ________

Activity 1 What Fractions Do You See?

Part 1

Consider the image.

1. You will be assigned one of the two fractions below. How many ways can you see the fraction in the image?

   a $\frac{1}{12}$

   Sample responses: The green rectangle in the top center square is $\frac{1}{12}$ of the whole rectangle, the skinny long rectangles are $\frac{1}{12}$ of three squares or one row ($\frac{1}{12}$ of $\frac{1}{2}$ of the whole rectangle), and the rectangles in the top right square are $\frac{1}{12}$ of four squares or two columns ($\frac{1}{12}$ of $\frac{2}{3}$ of the whole rectangle).

   b $\frac{1}{8}$

   Sample responses: The smallest rectangles are $\frac{1}{8}$ of a square ($\frac{1}{8}$ of $\frac{1}{6}$ of the whole rectangle). The skinny long rectangles are $\frac{1}{8}$ of two squares or one column ($\frac{1}{8}$ of $\frac{1}{3}$ of the whole rectangle). The green rectangle in the top center square is $\frac{1}{8}$ of four squares or two columns ($\frac{1}{8}$ of $\frac{2}{3}$ of the whole rectangle).

Lesson 1 Seeing Fractions 383

1 Launch

- **Part 1:** Assign Problem 1a to one-half of the class and 1b to the other half. Note that Problem 1a may be more accessible. Give students 3 minutes to work independently before sharing with the class.
- **Part 2:** Arrange students in groups of 3. Provide each group with one copy of the Activity 1 PDF. Consider making extra copies for groups to use as needed. Give groups 8 minutes to work before tallying and sharing their scores.
- **Part 3:** Pair two groups together, and give them 5–7 minutes to compare lists and to tally their new scores.

2 Monitor

Help students get started by asking, "What do you notice about this image?"

Look for points of confusion:

- **Incorrectly naming fractions.** Ask, "What is the part? The whole? Do you know how many of that part would fit in one whole? How could you determine that so that you can write a corresponding fraction?"

Look for productive strategies:

- Recognizing that one or more shapes or colors can represent a part or the whole.
- Choosing a part and determining all permutations for different wholes, or choosing a whole and considering each part in relation to that whole.
- Using equipartitioning, multiplicative reasoning, or operations with fractions.
- Using precise language to identify both the fractional part and the related whole to convince the opposing team why they deserve a bonus point.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

Before having students begin Part 1, ask them where they see each of the following fractions in the image: Sample responses are provided.

$\frac{1}{2}$: In the top middle square, the green rectangle is $\frac{1}{2}$ of that square.

$\frac{1}{3}$: Each rectangle in the top right square is $\frac{1}{3}$ of that square.

$\frac{1}{6}$: There are 6 squares that make up the image, so each square is $\frac{1}{6}$ of the whole.

Provide access to extra copies of the image, markers, and pairs of scissors should students choose to use them to help make sense of the fractional parts of the image.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display the following incomplete response for Problem 1. Then ask the following questions.

"The smallest rectangles are $\frac{1}{8}$."

Critique: "What is incorrect or incomplete about this statement?"

Correct and Clarify: "How would you correct or further clarify this statement?"

Listen for students who correctly identify the whole (e.g., $\frac{1}{8}$ *of* $\frac{1}{6}$ *of the whole rectangle* or $\frac{1}{8}$ *of a square*).

English Learners

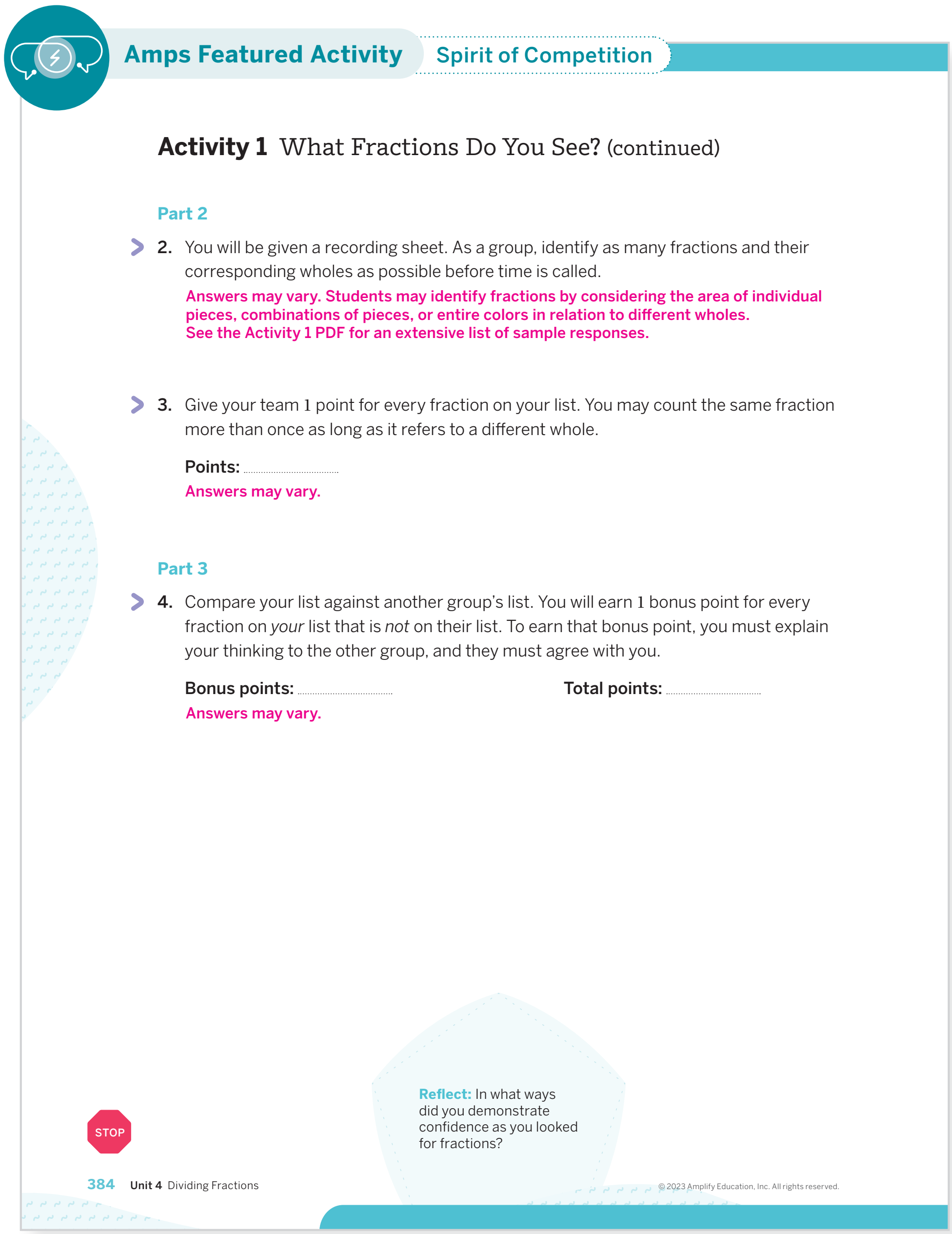
Annotate the squares with the fractional amounts as students correct the statement.

Small groups | 25 min

Activity 1 What Fractions Do You See? (continued)

Students examine a more complex image, further exploring the relationship among the part, the whole, and the fraction.

Amps Featured Activity Spirit of Competition



Activity 1 What Fractions Do You See? (continued)

Part 2

2. You will be given a recording sheet. As a group, identify as many fractions and their corresponding wholes as possible before time is called.
   Answers may vary. Students may identify fractions by considering the area of individual pieces, combinations of pieces, or entire colors in relation to different wholes. See the Activity 1 PDF for an extensive list of sample responses.

3. Give your team 1 point for every fraction on your list. You may count the same fraction more than once as long as it refers to a different whole.

   Points: ....
   Answers may vary.

Part 3

4. Compare your list against another group's list. You will earn 1 bonus point for every fraction on *your* list that is *not* on their list. To earn that bonus point, you must explain your thinking to the other group, and they must agree with you.

   Bonus points: .... **Total points:** ....
   Answers may vary.

STOP

Reflect: In what ways did you demonstrate confidence as you looked for fractions?

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3 Connect

Have each group of students share one fraction that earned them a bonus point in Part 3, focusing on identifying both the fraction and the related whole. Record their responses on the board, and continue eliciting responses from each group, as time allows, or until there are no more unique fractions to share.

Display the Activity 1 PDF (answers) to the class, one page at a time.

Ask:

- For the first page, "How do the fractions relate to your previous work with multiples?" Sample response: When I look by row, the denominators are multiples of the denominator in the first column. Unlike in the previous work where subsequent multiples became larger, I see that the multiples represent smaller pieces here.
- For the second page, "This image shows that relative to the entire rectangle, green is $\frac{47}{144}$, blue is $\frac{41}{144}$, and red is $\frac{56}{144}$. How could you use these fractions to determine what fraction each color is of a row? A column? 1 square?" Sample response: I first think about how the new whole (row, column, or square) is related to the whole rectangle. Then I can use ratio thinking or division to determine the new denominator. For example, if there are 144 total parts in one large rectangle, then there are 72 parts in one row because there are 2 rows and $144 \div 2 = 72$. Likewise, there are 48 parts in 1 column because $144 \div 3 = 48$, and 24 parts in 1 square because $144 \div 6 = 24$.

Highlight the importance of the whole when identifying fractions. For example, depending on the whole, the same part can represent many different fractions, or the same fraction can also represent multiple different-sized parts.

Differentiated Support

Extension: ***Math Around the World, Interdisciplinary Connections***

Let students know that many cultures around the world developed concepts of fractions independently from one another. For example, as early as 1800 BC, Egyptian mathematicians used unit fraction format to write all of their fractions. For example, they would write the fraction $\frac{3}{4}$ as the sum of the unit fractions $\frac{1}{2}$ and $\frac{1}{4}$. This practice of writing fractions did not allow for repeating a particular unit fraction as they were writing a sum. **(History)**

Remind students they learned about unit fractions in elementary grades. Ask students to use the same method the Egyptian mathematicians used for writing fractions using unit fractions, for the following fractions:

$\frac{5}{8} = \frac{1}{2} + \frac{1}{8}$

$\frac{6}{10} = \frac{1}{2} + \frac{1}{10}$

$\frac{7}{12} = \frac{1}{3} + \frac{1}{4}$

Let students know there will be other extension opportunities to explore how other cultures used fractions later in this unit.

Whole Class | 5 min

Summary Crossing the Fractional Divide

Review and synthesize how fractions represent part of a whole, connecting students' work to division and to the larger work of the unit.

Unit 4 Dividing Fractions

Crossing the Fractional Divide

In most of the units in this course, we talk about the different ways math touches almost every part of our lives — art, history, technology, and current events.

In *this* unit, we'll be talking about fractions — specifically, dividing fractions. You may remember that you use fractions to represent quantities that are not whole numbers, but are instead located *between* whole numbers. But while the idea of fractions might seem intuitive to you, operating with them — adding, subtracting, multiplying, and, yes, dividing them — requires more calculation than whole numbers did. And *understanding* what you are doing at each step often requires careful thought.

But don't worry! With practice and patience, you will get a feel for how to operate with fractions, expanding on the kinds of numbers at your disposal.

With all this mind, let's try something a little different in this unit . . .

Over the next few lessons, you will be reading a story. And, in this story, you will encounter different problems. Some of them will be quite tough, taking your understanding of working with fractions to new depths.

Just remember to relax and breathe. You might want to team up with a friend, or come back to these problems with fresh eyes. With patience and perseverance, you will come out on the other side stronger and more comfortable with dividing fractions.

So, when you're ready, turn the page.

Welcome to Unit 4.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display and read the Summary. Then, display the *Hidden Word* PDF. After students share their guesses with the class, display the *Hidden Word* (answer) PDF.

Ask students to identify the hidden word in the image. Consider having them use the ***Turn and Talk*** routine before sharing with the class. *Obelus* (spelled with one letter in each of the six squares, starting in the top left square and moving to the right, row by row).

Highlight that an obelus is the division symbol they have used for many years (÷). Explain that the symbol was first used in 1659 and is predominantly used in English-speaking countries. Most other countries use some form of the fraction bar for division, such as the bar in $\frac{1}{2}$. This bar is called a *vinculum*.

Have students share how they used division in their work today. Sample responses: Fractions are a way to represent division. Saying that one square is one-sixth of the large rectangle is the same as saying one square takes up the same area as one rectangle divided into six equal parts.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

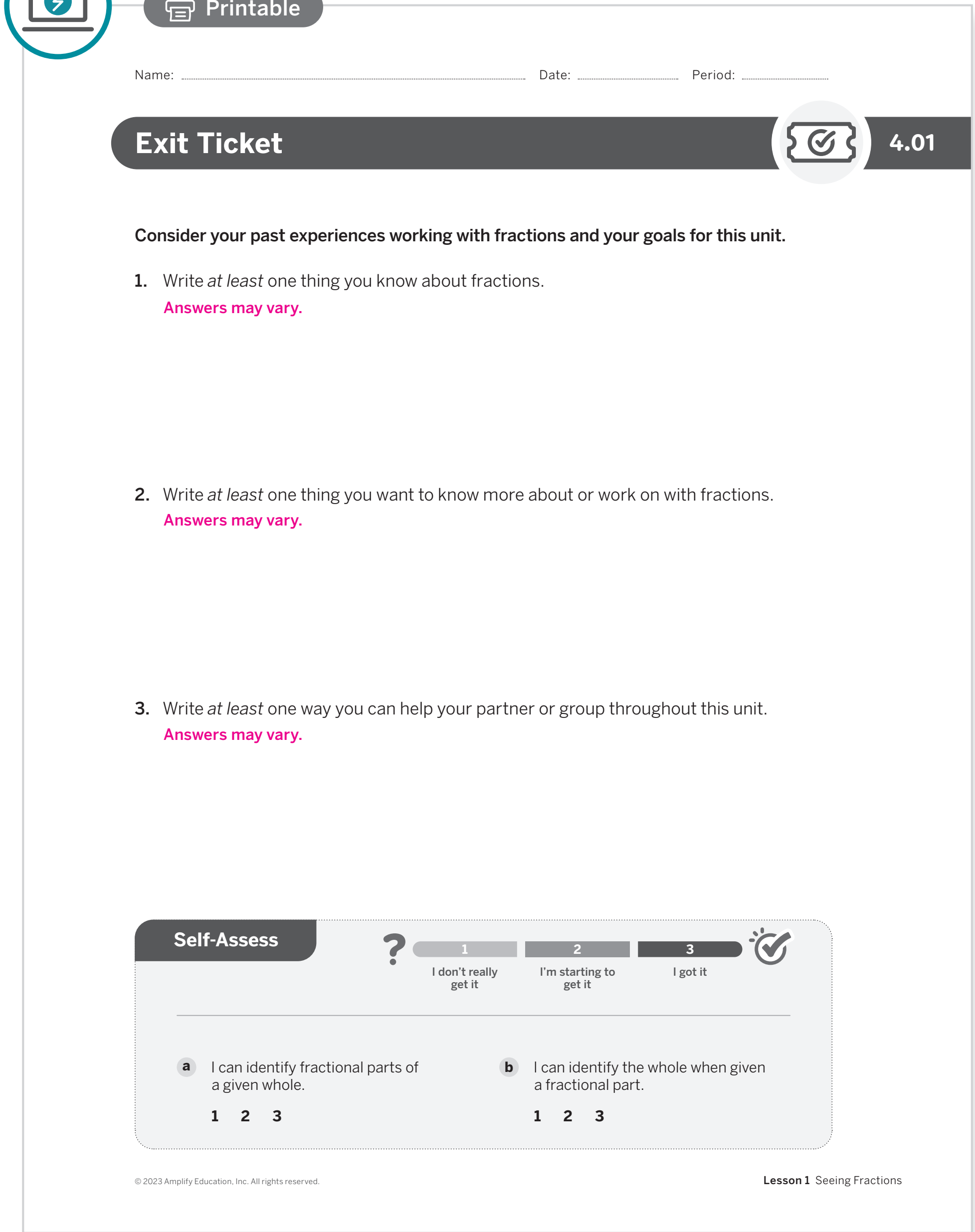
- "How were you able to use your prior knowledge about fractions today?"
- "What is something new you learned about fractions today?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using $\frac{1}{2}$ to describe the total area for three different colors in an image.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 4.01

Consider your past experiences working with fractions and your goals for this unit.

1. Write *at least* one thing you know about fractions.
   Answers may vary.

2. Write *at least* one thing you want to know more about or work on with fractions.
   Answers may vary.

3. Write *at least* one way you can help your partner or group throughout this unit.
   Answers may vary.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can identify fractional parts of a given whole.
1 2 3

b I can identify the whole when given a fractional part.
1 2 3

Lesson 1 Seeing Fractions

Success looks like . . .

- **Goal:** Using spatial reasoning to identify fractional parts of a given whole, or to identify the whole, given a fractional part.
- **Language Goal:** Using mathematical language precisely and flexibly to describe parts, wholes, and fractions. **(Speaking and Listening)**
  - » Writing about what they know or want to know about fractions in Problems 1 and 2.

Suggested next steps

If students are unable to write one thing they know about fractions, consider:

- Asking one or all of the following questions: "What is a fraction? Where do you see or use fractions in the real world? What have you learned to do with fractions in previous grades?"

If students are unable to write one thing they want to know more about or to work on with fractions, consider:

- Asking, "Do you know everything there is to know about fractions? Do you know all the ways fractions are used in math and the real world?"

If students are unable to write one way to help their partner or group, consider:

- Asking, "What made you a successful group member in today's lesson? How was a classmate helpful to you today?"

Professional Learning

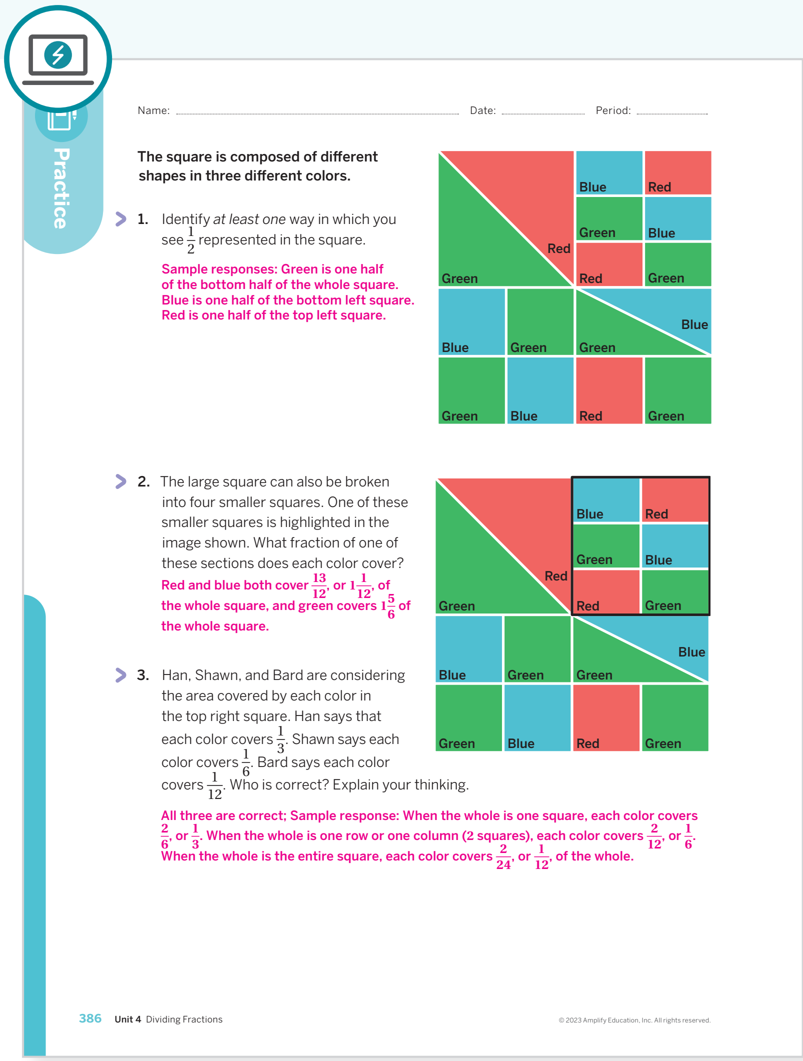
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students identified fractional parts and their corresponding wholes. How will that support their upcoming work with fraction division?
- What did you see in the way some students approached Activity 1 that you would like other students to try? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

The square is composed of different shapes in three different colors.

1. Identify *at least one* way in which you see $\frac{1}{2}$ represented in the square.

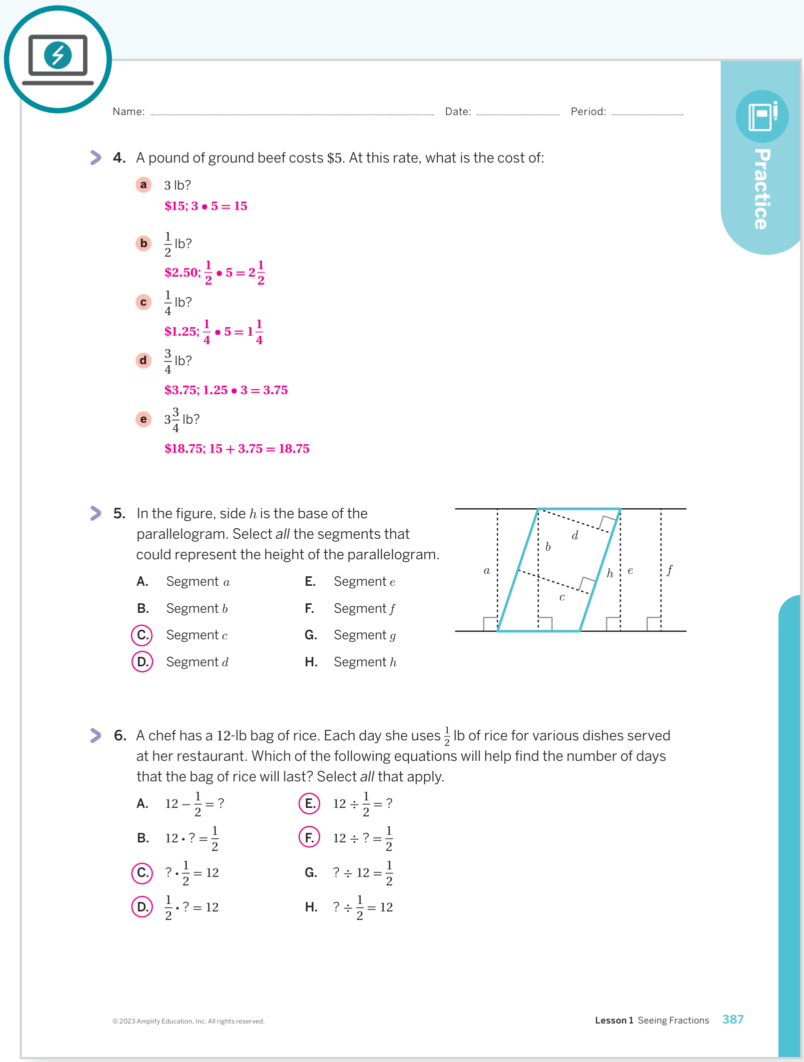
   Sample responses: Green is one half of the bottom half of the whole square. Blue is one half of the bottom left square. Red is one half of the top left square.

2. The large square can also be broken into four smaller squares. One of these smaller squares is highlighted in the image shown. What fraction of one of these sections does each color cover?

   Red and blue both cover $\frac{13}{12}$, or $1\frac{1}{12}$, of the whole square, and green covers $1\frac{5}{6}$ of the whole square.

3. Han, Shawn, and Bard are considering the area covered by each color in the top right square. Han says that each color covers $\frac{1}{3}$. Shawn says each color covers $\frac{1}{6}$. Bard says each color covers $\frac{1}{12}$. Who is correct? Explain your thinking.

   All three are correct; Sample response: When the whole is one square, each color covers $\frac{2}{6}$, or $\frac{1}{3}$. When the whole is one row or one column (2 squares), each color covers $\frac{2}{12}$, or $\frac{1}{6}$. When the whole is the entire square, each color covers $\frac{2}{24}$, or $\frac{1}{12}$, of the whole.



Practice

Name: Date: Period:

4. A pound of ground beef costs $5. At this rate, what is the cost of:
   - a 3 lb?

     $\$15; 3 \bullet 5 = 15$
   - b $\frac{1}{2}$ lb?

     $\$2.50; \frac{1}{2} \bullet 5 = 2\frac{1}{2}$
   - c $\frac{1}{4}$ lb?

     $\$1.25; \frac{1}{4} \bullet 5 = 1\frac{1}{4}$
   - d $\frac{3}{4}$ lb?

     $\$3.75; 1.25 \bullet 3 = 3.75$
   - e $3\frac{3}{4}$ lb?

     $\$18.75; 15 + 3.75 = 18.75$

5. In the figure, side h is the base of the parallelogram. Select *all* the segments that could represent the height of the parallelogram.

   A. Segment a
   B. Segment b
   [✓ C.] Segment c
   [✓ D.] Segment d
   E. Segment e
   F. Segment f
   G. Segment g
   H. Segment h

6. A chef has a 12-lb bag of rice. Each day she uses $\frac{1}{2}$ lb of rice for various dishes served at her restaurant. Which of the following equations will help find the number of days that the bag of rice will last? Select *all* that apply.

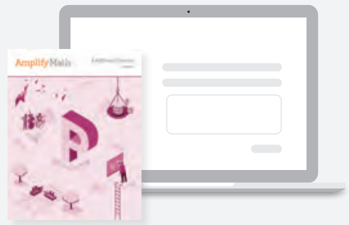
   A. $12 - \frac{1}{2} = ?$
   B. $12 \cdot ? = \frac{1}{2}$
   [✓ C.] $? \cdot \frac{1}{2} = 12$
   [✓ D.] $\frac{1}{2} \cdot ? = 12$
   [✓ E.] $12 \div \frac{1}{2} = ?$
   [✓ F.] $12 \div ? = \frac{1}{2}$
   G. $? \div 12 = \frac{1}{2}$
   H. $? \div \frac{1}{2} = 12$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 4	2
	5	Unit 1 Lesson 7	2
Formative ⚡	6	Unit 4 Lesson 2	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 1
Interpreting Division Scenarios

Whole Class

In this Sub-Unit, students revisit other division concepts using whole numbers and unit fractions, while also encountering some non-unit fractions. They distinguish two interpretations of division – partitive and quotitive, and represent both types using multiplication and division equations.

SUB-UNIT

1 Interpreting Division Scenarios

Narrative Connections

Welcome to Spöklik Furniture

An eeriness settles over Spöklik Furniture. Once upon a time, people came here for new couches, fine china, and silver serving spoons. Now, it's nothing but an abandoned old warehouse — full of cobwebs and unsold furniture . . . So, what was your friend Maya doing out here? She texted, asking to meet here. Now, you find her phone lying by the entrance, battery dead. You walk past the shopping carts to the store's first section: Housewares.

Clicking on your flashlight, you start your search. After a few minutes of wandering, the place starts to feel like a maze! You try to trace your steps back to the entrance, but end up going in circles. Suddenly, you hear a pair of voices:

"Martha! We can't afford *that*!"

"Oh, live a little, George! You don't want the Albees calling us cheap, do you?"

Two figures appear, pushing a shopping cart. Martha cradles a crystal salad bowl in her arms. "You there!" she coos, gesturing toward you. "Be a darling and lend us a hand."

"What's the problem?" you ask.

"We're buying a house warming gift for our friends, the Albees. The trouble is, we can't decide what to get. If the gift is too cheap, it'll be a scandal. But if it's too expensive, George will have a fit. **We need an item that costs between 100 and 1,000 spök-bucks**, but these prices are *so* confusing . . ."

Looking at their cart, you see that, instead of showing the price, each tag is printed with a strange division problem. George and Martha look at you with imploring eyes. Maybe if you help them, they can help you get out of this place . . .

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to encounter stories involving division at Spöklik Furniture in the following places:

- **Lesson 3, Activity 1:** Multiplication or Division?
- **Lesson 4, Activity 1:** How Many Does It Take?

Highlight the question posed in the narrative and consider having a brief discussion to ensure all students understand the question. While some students may be able to determine an answer now, they should all be equipped to answer it by the end of the Sub-Unit. Consider establishing a process for students to submit responses privately at any point during the next few class sessions, and then hold a discussion after Lesson 4.

UNIT 4 | **LESSON 2**

Meanings of Division

Let's explore ways to think about division.

Focus

Goals

1. Interpret and create tape diagrams that represent situations involving equal-sized groups.
2. **Language Goal:** Recognize there are two different ways to interpret a division expression, i.e., asking "how many groups?" or "how many in each group?" **(Speaking and Listening, Writing)**

Coherence

Today

Students revisit the two different meanings of division by writing and solving word problems. They come to differentiate the two types of division: "how many groups" (quotitive) and "how many in each group" (partitive). Students review the importance of having equal-sized groups in real-world scenarios and by using and drawing diagrams to support their reasoning when interpreting division equations and evaluating quotients.

Previously

In earlier grades, students wrote and solved division word problems involving a given number of groups or a given size of each group.

Coming Soon

In Lesson 3, students will explore the relationship between multiplication and division, especially as it relates to fractions.

Rigor

- Students use visual models to build upon their **conceptual understanding** of partitive and quotitive division.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	25 min	5 min	5 min
Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

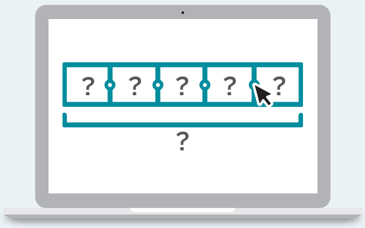
- Exit Ticket
- Additional Practice
- *Tape Diagrams* PDF (as needed)
- index cards

Amps | Featured Activity

Activity 1

Dynamic Tape Diagrams

Students can create digital tape diagrams. You can overlay all of the diagrams to see similarities and differences at a glance.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students complete the ***Gallery Tour*** routine in Activity 1, they might become distracted and unfocused. Encourage students to write the purpose of the activity on a card and carry it with them so that they can remind themselves of the purpose of the walk. Students should use this to keep themselves on track to achieve their personal and academic goals.

Modifications to Pacing

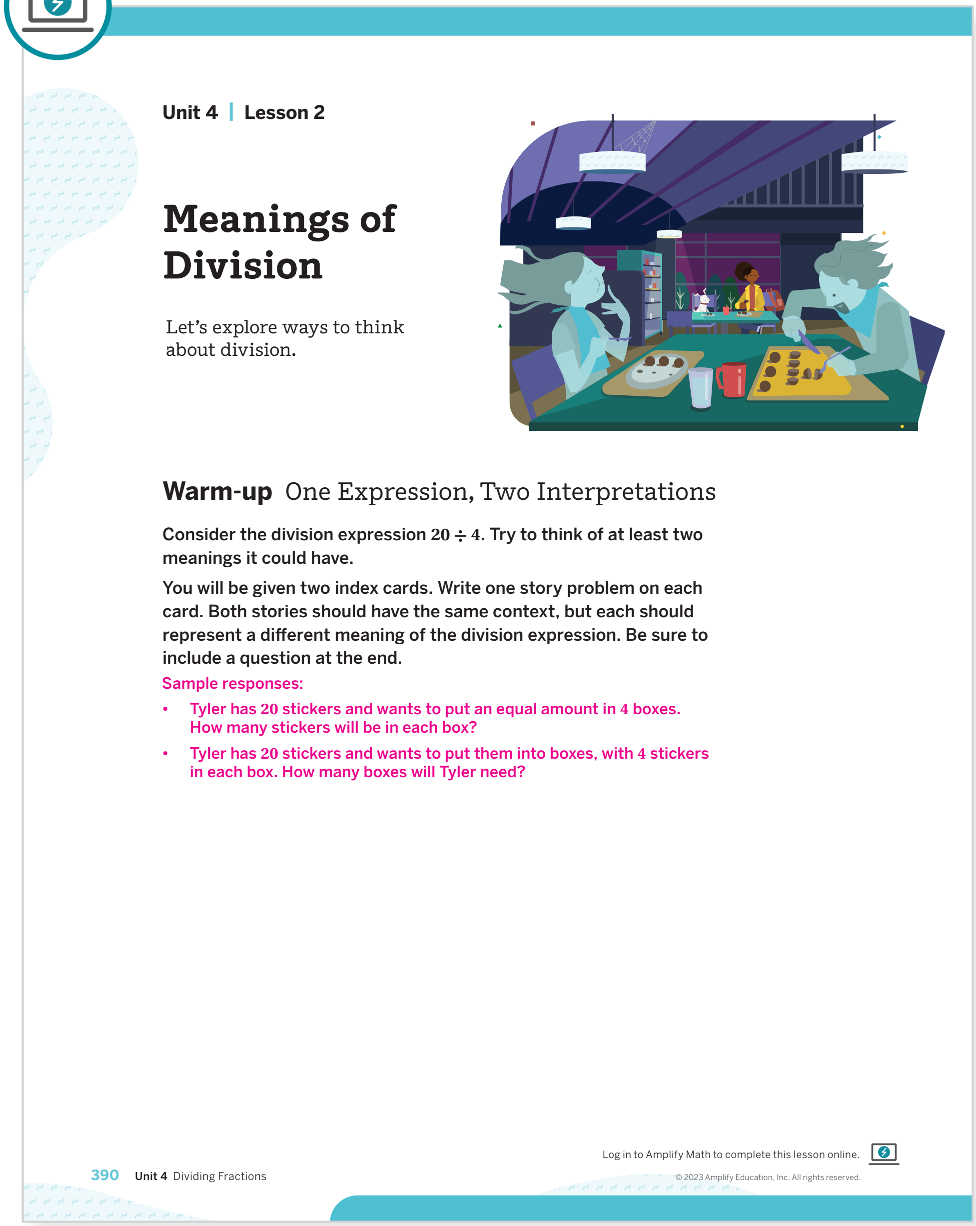
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, have partners write only one story together.
- In **Activity 1**, have students only create one diagram for each problem in Part 1, and have students only create a diagram for Problem 5 in Part 2.

Pairs | 10 min

Warm-up One Expression, Two Interpretations

Students write two scenarios to represent a division expression, preparing them to explore the two interpretations of division.



Unit 4 | Lesson 2

Meanings of Division

Let's explore ways to think about division.

Warm-up One Expression, Two Interpretations

Consider the division expression $20 \div 4$. Try to think of at least two meanings it could have.

You will be given two index cards. Write one story problem on each card. Both stories should have the same context, but each should represent a different meaning of the division expression. Be sure to include a question at the end.

Sample responses:

- Tyler has 20 stickers and wants to put an equal amount in 4 boxes. How many stickers will be in each box?
- Tyler has 20 stickers and wants to put them into boxes, with 4 stickers in each box. How many boxes will Tyler need?

390 Unit 4 Dividing Fractions

Log in to Amplify Math to complete this lesson online.

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1 Launch

Distribute two index cards to each pair.

2 Monitor

Help students get started by activating students' prior knowledge and asking, "What does $20 \div 4$ mean? How could you use familiar objects from around school or home to represent that expression?"

Look for points of confusion:

- **Writing scenarios that reflect** only one meaning of division. Ask, "Can you draw two diagrams that represent $20 \div 4$ in two different ways?"

Look for productive strategies:

- Recognizing that division has two meanings either by writing one scenario showing "how many groups" (quotitive) and the other showing "how many in each group" (partitive), or drawing two different models.

3 Connect

Have students share their scenarios.

Display and organize the cards into two categories (but do not say yet what the categories represent): "how many groups" and "how many in each group."

Ask, "What is the same about all of the scenarios in this category of cards? How might you title this category? What about the other category – what is the same and how might you title it?"

Highlight that division represents creating equal-sized groups. Use the categories of cards displayed to identify the interpretations of *how many groups* of a certain size can be made (quotitive) and *how much is in each group* of a certain number of groups (partitive). Then add to the display by demonstrating how to represent each interpretation using a tape diagram.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you display and organize the scenarios into the two categories described, draw attention to the language used in the scenarios that indicate to which category it might belong. For the sample response scenarios provided, highlight the language *how many* ___ *in each* ___ as indicating one category and *how many* ___ as indicating the other category.

English Learners

Annotate, circle, or otherwise highlight these key terms and phrases in the scenarios students wrote.

Power-up

To power up students' ability to differentiate between situations involving division and multiplication with fractions and whole numbers, have students complete:

Match each statement with the expression that represents it.

__b__ The number of $\frac{1}{2}$ cup servings in 6 cups of rice.	a. $6 \cdot \frac{1}{2}$
__a__ The total amount of water is 6 bottles with $\frac{1}{2}$ liters in each.	b. $6 \div \frac{1}{2}$

__b__ A 6 m rope is cut into $\frac{1}{2}$ m lengths.

__a__ Jad ran half the distance of Han, who ran a total of 6 km.

Use: Before the Warm-up.

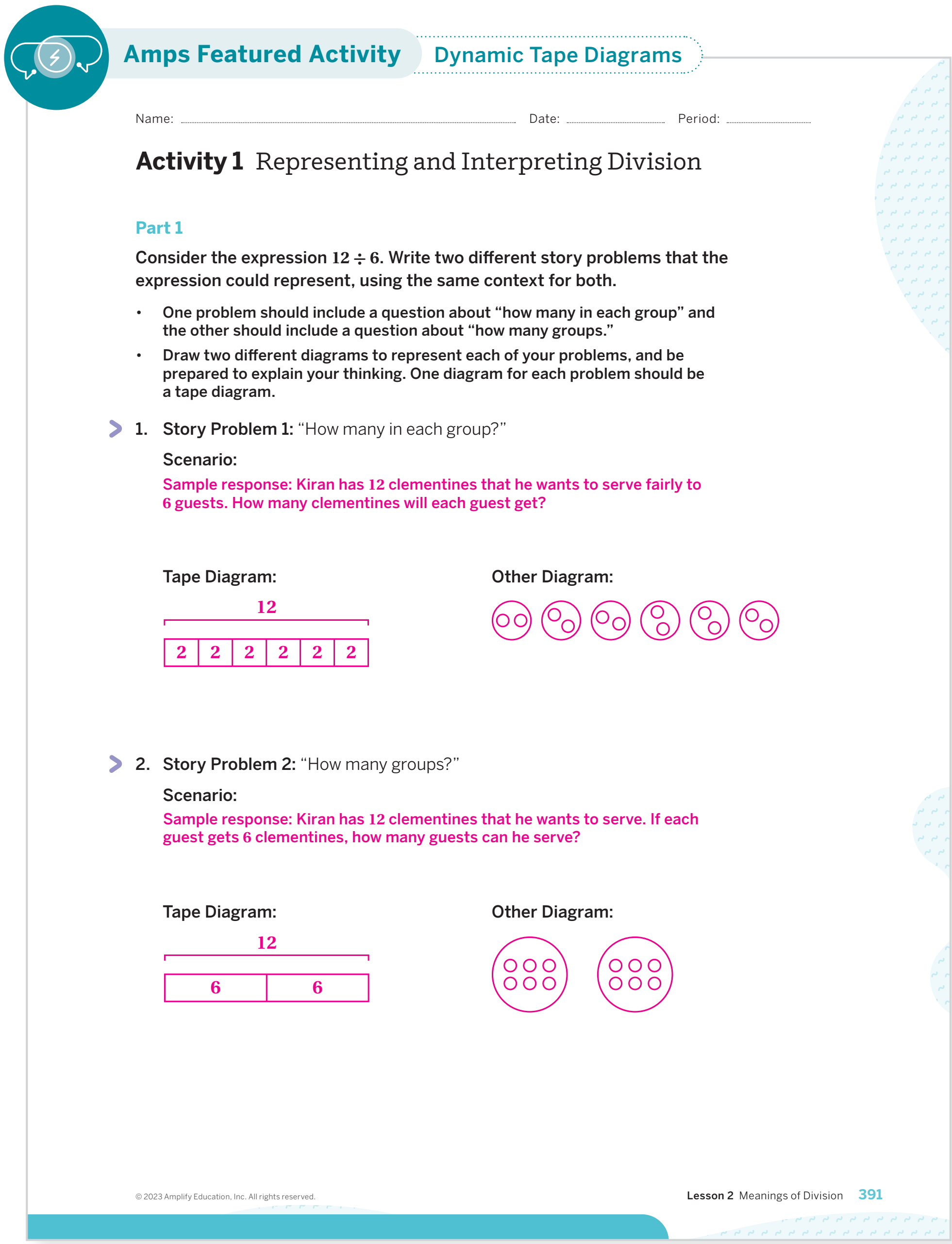
Informed by: Performance on Lesson 1, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 2.

Pairs | 25 min

Activity 1 Representing and Interpreting Division

Students further develop their understanding of partitive and quotitive division by creating their own scenarios and diagrams to represent division expressions.

Amps Featured Activity Dynamic Tape Diagrams



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Activity 1 Representing and Interpreting Division

Part 1

Consider the expression $12 \div 6$. Write two different story problems that the expression could represent, using the same context for both.

- One problem should include a question about "how many in each group" and the other should include a question about "how many groups."
- Draw two different diagrams to represent each of your problems, and be prepared to explain your thinking. One diagram for each problem should be a tape diagram.

1. **Story Problem 1:** "How many in each group?"

   Scenario:

   Sample response: Kiran has 12 clementines that he wants to serve fairly to 6 guests. How many clementines will each guest get?

   Tape Diagram:

   12

   2 | 2 | 2 | 2 | 2 | 2

   Other Diagram:

2. **Story Problem 2:** "How many groups?"

   Scenario:

   Sample response: Kiran has 12 clementines that he wants to serve. If each guest gets 6 clementines, how many guests can he serve?

   Tape Diagram:

   12

   6 | 6

   Other Diagram:

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Lesson 2 Meanings of Division 391

1 Launch

Have students complete Part 1 in pairs. If needed, clarify that the "same context" means using the same units (the nouns or objects in the story problems). Have students pause after completing Part 1 and use the ***Gallery Tour*** routine to share student work. Then have pairs complete Part 2.

2 Monitor

Help students get started by asking, "What will your story be about? What will 12 represent? What will 6 represent?"

Look for points of confusion:

- **Writing two scenarios that both represent the same interpretation of division.** Reference the display from the Warm-up and ask, "Which category would each of your scenarios belong to?"
- **Struggling to represent division expressions using more than one type of diagram (Part 1).** Ask, "Can you think of a model you have used in the past that can show the same information differently?"
- **Mismatching the diagrams and scenarios.** Ask, "What is the difference between 'how many groups are there' and 'how many there are in each group'? How would their representations look different?"
- **Writing 6 as a solution or not knowing how to divide by $\frac{1}{2}$ (Part 2, Problem 5).** Explain it in a context that might be familiar to students, such as eating half sandwiches.

Look for productive strategies:

- Representing both interpretations of a division expression with diagrams and scenarios, and associating the proper units with the dividend, divisor, and quotient in each.
- Using diagrams and numerical reasoning to determine the quotients and explain them in context.
- Drawing an accurate model to show how to divide by $\frac{1}{2}$ in Part 2, Problem 5.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide students with copies of blank tape diagrams, such as from the *Tape Diagrams* PDF. Direct students to only work with the tape diagrams that do not include percentages.

Accessibility: *Vary Demands to Optimize Challenge, Optimize Access to Technology*

Instead of having students draw their own diagrams or write their own stories in Part 1, provide the sample responses on separate slips of paper and have students sort them into each category. Alternatively, have students use the Amps slides for this activity, in which they can create digital tape diagrams.

Math Language Development

MLR8: Discussion Supports—Revoicing

During the Connect, as students share what they noticed during the ***Gallery Tour***, ask them to restate what they heard for each diagram and scenario using their own understanding. Then ask the original speaker whether their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement, such as the *size of each group* or *how many groups*.

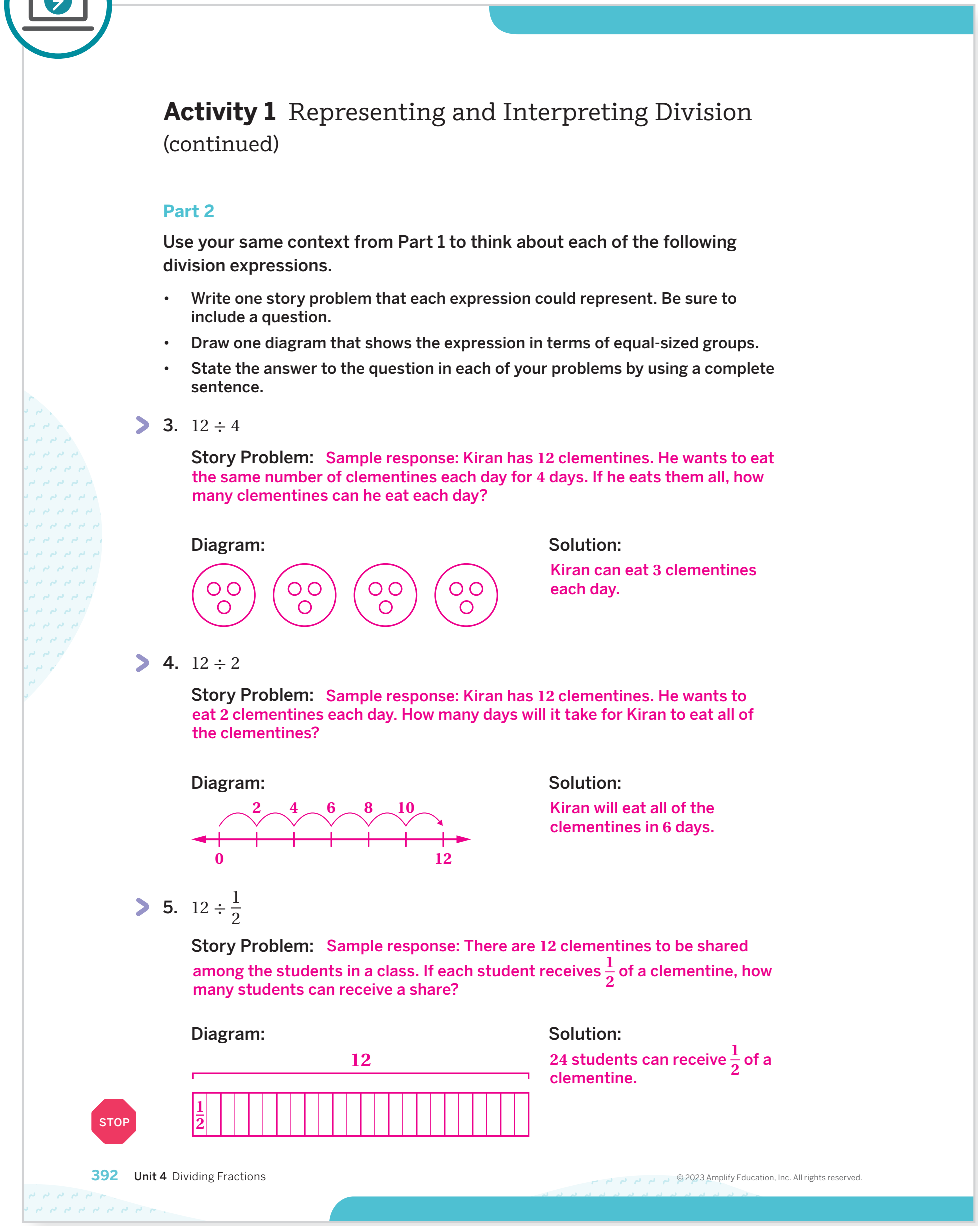
English Learners

Provide sufficient wait time for students so that they have time to formulate what they will say before sharing with the whole group.

Pairs | 25 min

Activity 1 Representing and Interpreting Division (continued)

Students further develop their understanding of partitive and quotitive division by creating their own scenarios and diagrams to represent division expressions.



Activity 1 Representing and Interpreting Division (continued)

Part 2

Use your same context from Part 1 to think about each of the following division expressions.

- Write one story problem that each expression could represent. Be sure to include a question.
- Draw one diagram that shows the expression in terms of equal-sized groups.
- State the answer to the question in each of your problems by using a complete sentence.

3. $12 \div 4$

Story Problem: Sample response: Kiran has 12 clementines. He wants to eat the same number of clementines each day for 4 days. If he eats them all, how many clementines can he eat each day?

Diagram:

Solution: Kiran can eat 3 clementines each day.

4. $12 \div 2$

Story Problem: Sample response: Kiran has 12 clementines. He wants to eat 2 clementines each day. How many days will it take for Kiran to eat all of the clementines?

Diagram:

Solution: Kiran will eat all of the clementines in 6 days.

5. $12 \div \frac{1}{2}$

Story Problem: Sample response: There are 12 clementines to be shared among the students in a class. If each student receives $\frac{1}{2}$ of a clementine, how many students can receive a share?

Diagram:

Solution: 24 students can receive $\frac{1}{2}$ of a clementine.

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3 Connect

Have students share what they noticed that was similar and what was different about the scenarios or diagrams during the ***Gallery Tour*** (conducted earlier). Then have students share their scenarios and diagrams for each expression from Part 2, adding to the display to continue to distinguish between partitive and quotitive scenarios.

Ask:

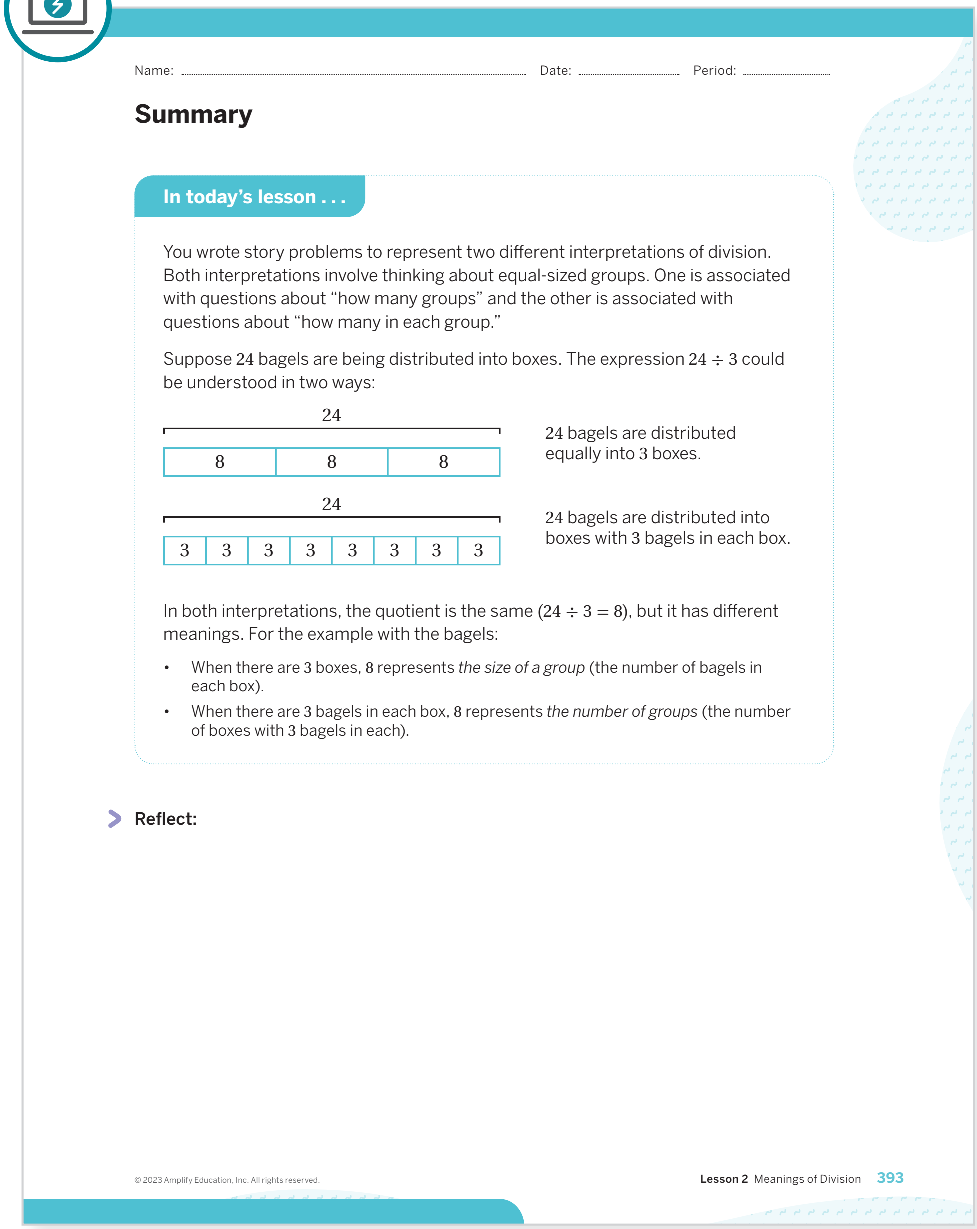
- "How can the same division expression be interpreted in two different ways?" "How many groups?" or "How much in each group?"
- "How do you decide what is known and what is unknown? How does that inform how a diagram might look different?" The total is what is being divided, and most of the time represents the full length of the tape diagram. If I know the number of groups, then I can partition the tape diagram into that many parts, or, if I know the amount in one group, I can start building the tape diagram from copies of that amount.

Highlight that division can be interpreted as a way to determine two different values — the size of each group (when the number of groups and a total amount are known), or how many groups (when the total amount and the size of each group is known).

Whole Class | 5 min

Summary

Review and synthesize the two interpretations of division.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You wrote story problems to represent two different interpretations of division. Both interpretations involve thinking about equal-sized groups. One is associated with questions about "how many groups" and the other is associated with questions about "how many in each group."

Suppose 24 bagels are being distributed into boxes. The expression 24 ÷ 3 could be understood in two ways:

24

8	8	8

24 bagels are distributed equally into 3 boxes.

24

3	3	3	3	3	3	3	3

24 bagels are distributed into boxes with 3 bagels in each box.

In both interpretations, the quotient is the same ($24 \div 3 = 8$), but it has different meanings. For the example with the bagels:

- When there are 3 boxes, 8 represents *the size of a group* (the number of bagels in each box).
- When there are 3 bagels in each box, 8 represents *the number of groups* (the number of boxes with 3 bagels in each).

Reflect:

Synthesize

Display the diagrams from the Student Edition.

Ask:

- "How are the diagrams similar? How are they different?"
- "What questions could the first diagram help you answer? What about the second diagram?"

Highlight that division involves creating equal-sized groups. Typically, either the number of groups or the size of the groups is known, in addition to the total amount being divided up (the dividend). The interpretations and models do not impact the resulting quotient or missing value, but they can be useful tools for understanding a scenario and arriving at a solution as the types of numbers being divided change to fractions.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

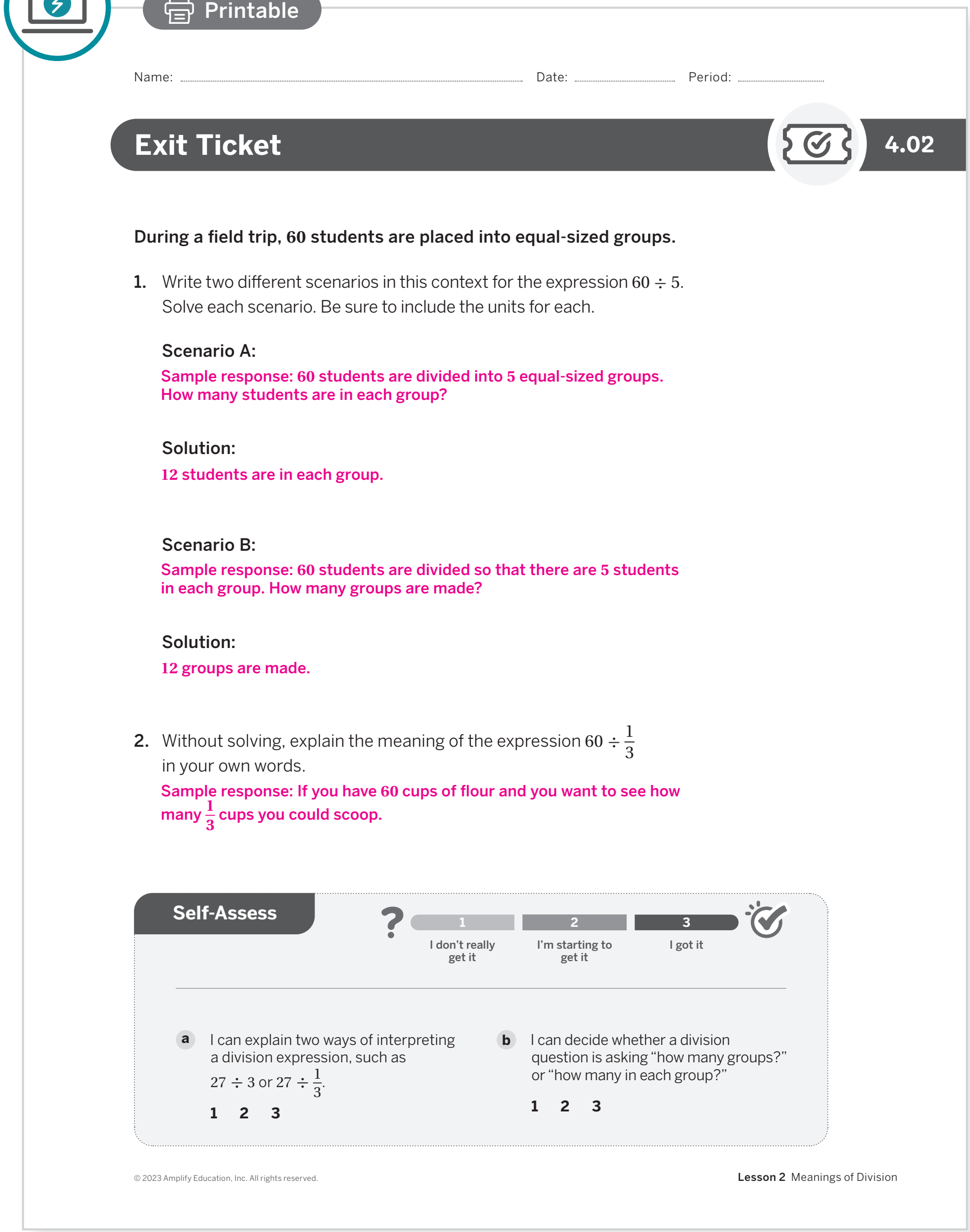
- "How can dividing by the same fraction be interpreted two different ways?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the different meanings of division by describing an expression in two ways.

Printable



Name: Date: Period:

Exit Ticket 4.02

During a field trip, 60 students are placed into equal-sized groups.

1. Write two different scenarios in this context for the expression 60 ÷ 5. Solve each scenario. Be sure to include the units for each.

   Scenario A:

   Sample response: 60 students are divided into 5 equal-sized groups. How many students are in each group?

   Solution:

   12 students are in each group.

   Scenario B:

   Sample response: 60 students are divided so that there are 5 students in each group. How many groups are made?

   Solution:

   12 groups are made.

2. Without solving, explain the meaning of the expression $60 \div \frac{1}{3}$ in your own words.

   Sample response: If you have 60 cups of flour and you want to see how many $\frac{1}{3}$ cups you could scoop.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can explain two ways of interpreting a division expression, such as $27 \div 3$ or $27 \div \frac{1}{3}$.

1 2 3

b I can decide whether a division question is asking "how many groups?" or "how many in each group?"

1 2 3

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Lesson 2 Meanings of Division

Success looks like . . .

- **Goal:** Interpreting and creating tape diagrams that represent situations involving equal-sized groups.
- **Language Goal:** Recognizing there are two different ways to interpret a division expression, i.e., asking "how many groups?" or "how many in each group?" **(Speaking and Listening, Writing)**
  - » Writing questions for each interpretation of division in Problem 1.

Suggested next steps

If students have trouble thinking about the expression in more than one way, consider:

- Referencing the display from the Warm-up and asking, "How can you use the examples from the two categories to help you?"
- Reviewing Activity 1, Part 1 and ask, "How do those diagrams represent the given expression differently?"

If students have trouble explaining the expression in Problem 2, consider:

- Asking students, "What is an example of something that could have a value of a $\frac{1}{3}$? When might you have 60 of something and you would want to know how many of those $\frac{1}{3}$ parts could fit inside?"
- Reviewing Activity 1, Part 2, Problem 3.
- Drawing a model.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students explore the two meanings of division. How will that support future work of dividing fractions?
- In what ways did Activity 1 go as planned? What might you change for the next time you teach this lesson?

Math Language Development

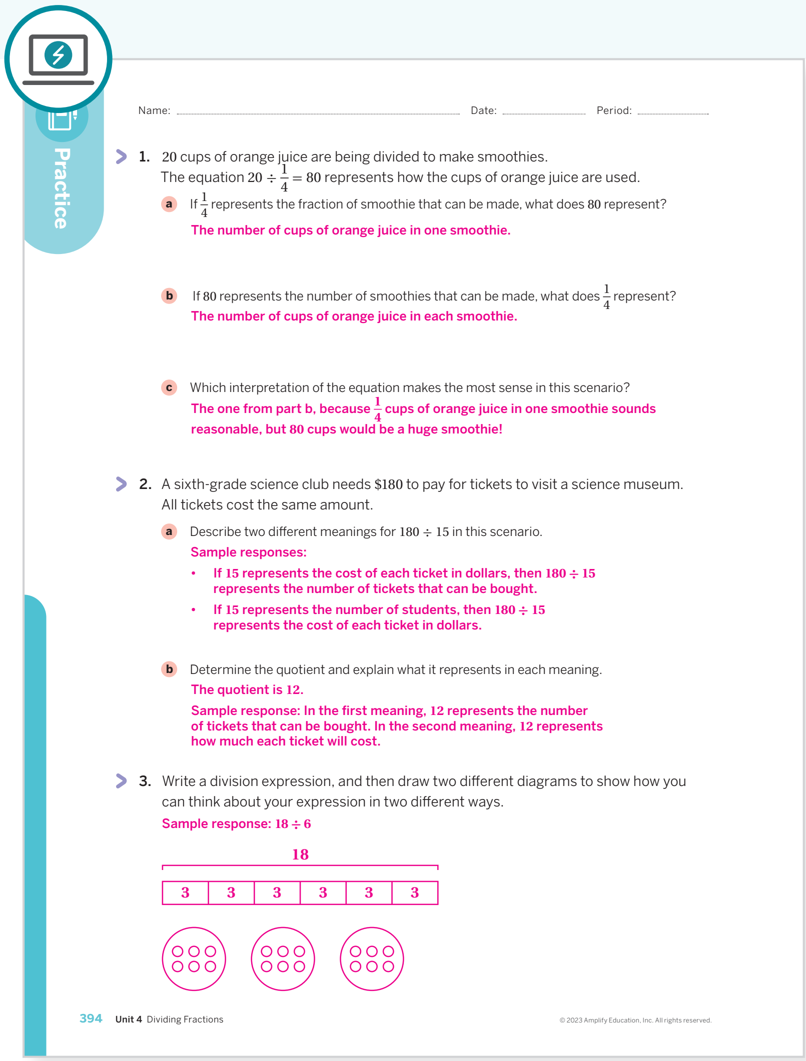
Language Goal: Recognizing there are two different ways to interpret a division expression, i.e., asking "how many groups?" or "how many in each group?"

Reflect on students' language development toward this goal.

- Do students' responses to Problem 1 of the Exit Ticket demonstrate they understand the two different meanings of division?
- What support can you provide to help them to be more precise in their written scenarios?

Practice

Independent



Practice

Name: Date: Period:

1. 20 cups of orange juice are being divided to make smoothies. The equation $20 \div \frac{1}{4} = 80$ represents how the cups of orange juice are used.
   - **a** If $\frac{1}{4}$ represents the fraction of smoothie that can be made, what does 80 represent?
     The number of cups of orange juice in one smoothie.
   - **b** If 80 represents the number of smoothies that can be made, what does $\frac{1}{4}$ represent?
     The number of cups of orange juice in each smoothie.
   - **c** Which interpretation of the equation makes the most sense in this scenario?
     The one from part b, because $\frac{1}{4}$ cups of orange juice in one smoothie sounds reasonable, but 80 cups would be a huge smoothie!

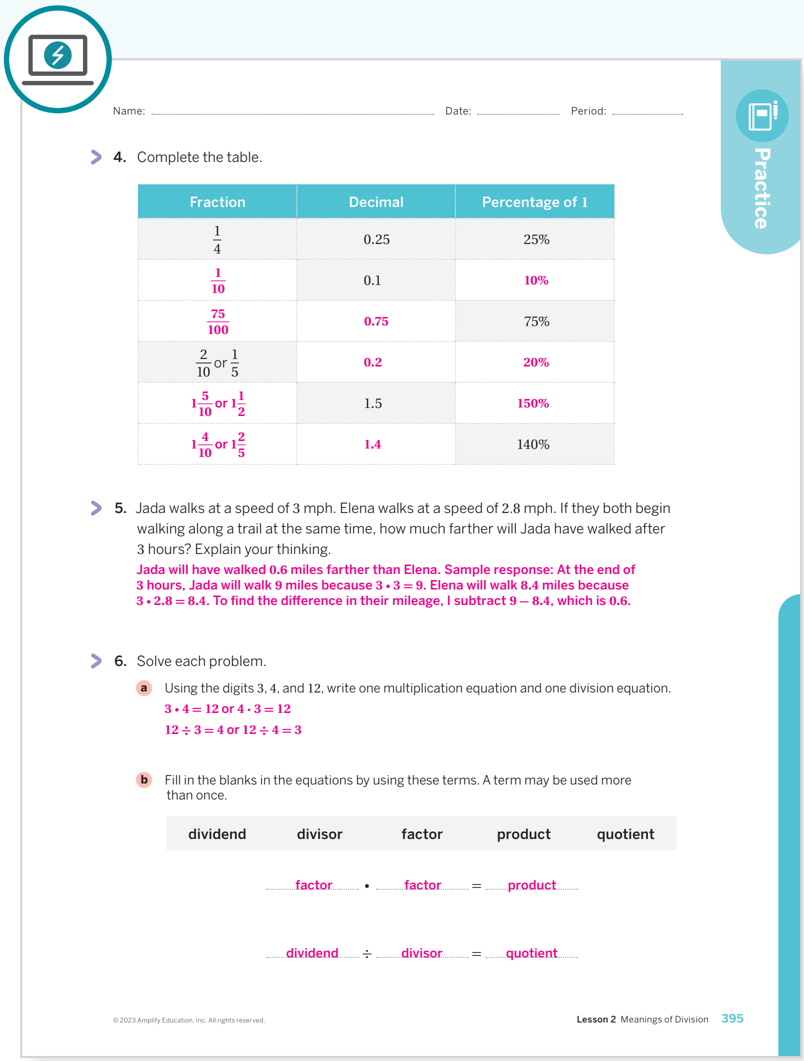
2. A sixth-grade science club needs $180 to pay for tickets to visit a science museum. All tickets cost the same amount.
   - **a** Describe two different meanings for $180 \div 15$ in this scenario.
     Sample responses:
     - **If 15 represents the cost of each ticket in dollars, then $180 \div 15$ represents the number of tickets that can be bought.**
     - **If 15 represents the number of students, then $180 \div 15$ represents the cost of each ticket in dollars.**
   - **b** Determine the quotient and explain what it represents in each meaning.
     The quotient is 12.
     Sample response: In the first meaning, 12 represents the number of tickets that can be bought. In the second meaning, 12 represents how much each ticket will cost.

3. Write a division expression, and then draw two different diagrams to show how you can think about your expression in two different ways.
   Sample response: $18 \div 6$

18

3 3 3 3 3 3

394 Unit 4 Dividing Fractions



Practice

Name: Date: Period:

4. Complete the table.

Fraction	Decimal	Percentage of 1
$\frac{1}{4}$	0.25	25%
$\frac{1}{10}$	0.1	10%
$\frac{75}{100}$	0.75	75%
$\frac{2}{10}$ or $\frac{1}{5}$	0.2	20%
$1\frac{5}{10}$ or $1\frac{1}{2}$	1.5	150%
$1\frac{4}{10}$ or $1\frac{2}{5}$	1.4	140%

5. Jada walks at a speed of 3 mph. Elena walks at a speed of 2.8 mph. If they both begin walking along a trail at the same time, how much farther will Jada have walked after 3 hours? Explain your thinking.
   Jada will have walked 0.6 miles farther than Elena. Sample response: At the end of 3 hours, Jada will walk 9 miles because $3 \cdot 3 = 9$. Elena will walk 8.4 miles because $3 \cdot 2.8 = 8.4$. To find the difference in their mileage, I subtract $9 - 8.4$, which is 0.6.

6. Solve each problem.
   - **a** Using the digits 3, 4, and 12, write one multiplication equation and one division equation.
     $3 \cdot 4 = 12$ or $4 \cdot 3 = 12$
     $12 \div 3 = 4$ or $12 \div 4 = 3$
   - **b** Fill in the blanks in the equations by using these terms. A term may be used more than once.

     dividend **divisor** **factor** **product** **quotient**

     factor • factor = product

     dividend ÷ divisor = quotient

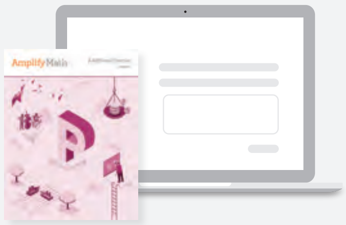
Lesson 2 Meanings of Division 395

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Warm-up	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 13	2
	5	Unit 3 Lesson 4	2
Formative ⚡	6	Unit 4 Lesson 3	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

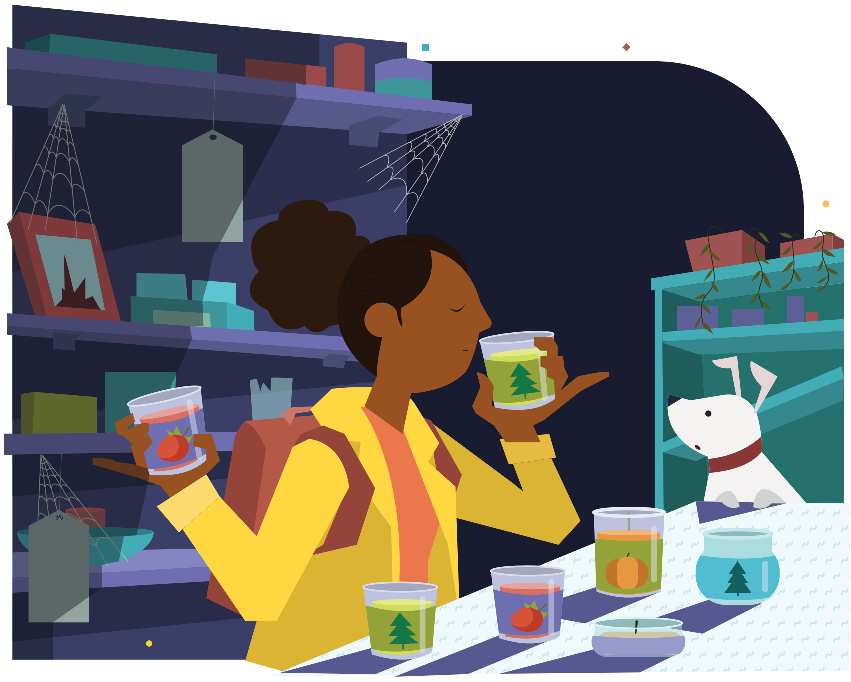


For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 4 | **LESSON 3**

Relating Division and Multiplication

Let's review how division and multiplication are related.



Focus

Goals

1. **Language Goal:** Generate multiplication and division equations to represent a situation involving fractions, and relate the equations to a diagram. **(Speaking and Listening)**
2. **Language Goal:** Explain how to determine the unknown quantity in a multiplication or division situation involving fractions. **(Speaking and Listening, Writing)**
3. **Language Goal:** Identify the unknown quantity in a situation (i.e., the number of groups, the amount in one group, or the total amount) and generate corresponding equations. **(Speaking and Listening, Writing)**

Coherence

Today

Students interpret division situations in story problems that involve equal-sized groups. They draw diagrams and write both division equations and multiplication equations to make sense of the relationship between known and unknown quantities, including fractions. Students then estimate and solve the story problems by using their diagrams and equations.

Previously

In Grades 3–5, students explored the relationship between multiplication and division. In Lesson 2, students explored the two different meanings of division — "how many groups?" and "how much in one group?" — relating division expressions and their interpretations to real-world scenarios.

Coming Soon

Students will continue to practice their estimation skills and will revisit the connection between division and fractions by determining whether a quotient is less than 1, close to 1, or much greater than 1.

Rigor

- Students **apply** their understanding of the relationship between the operations of multiplication and division of whole numbers from earlier grades to scenarios involving division with fractions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

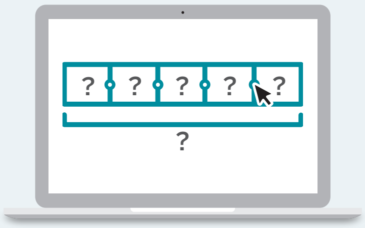
- *dividend*
- *divisor*
- *factor*
- *product*
- *quotient*

Amps | Featured Activity

Activities 1 and 2

Dynamic Tape Diagrams

Students can create digital tape diagrams. You can overlay them all to see similarities and differences at a glance.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may not see that there are multiple ways to represent the problem in Activity 1. After identifying the problem, students should use the diagram and equation to help them analyze the situation and ultimately solve the problem. After completing the activity, have them reflect on how effective their solution strategies were.

Modifications to Pacing

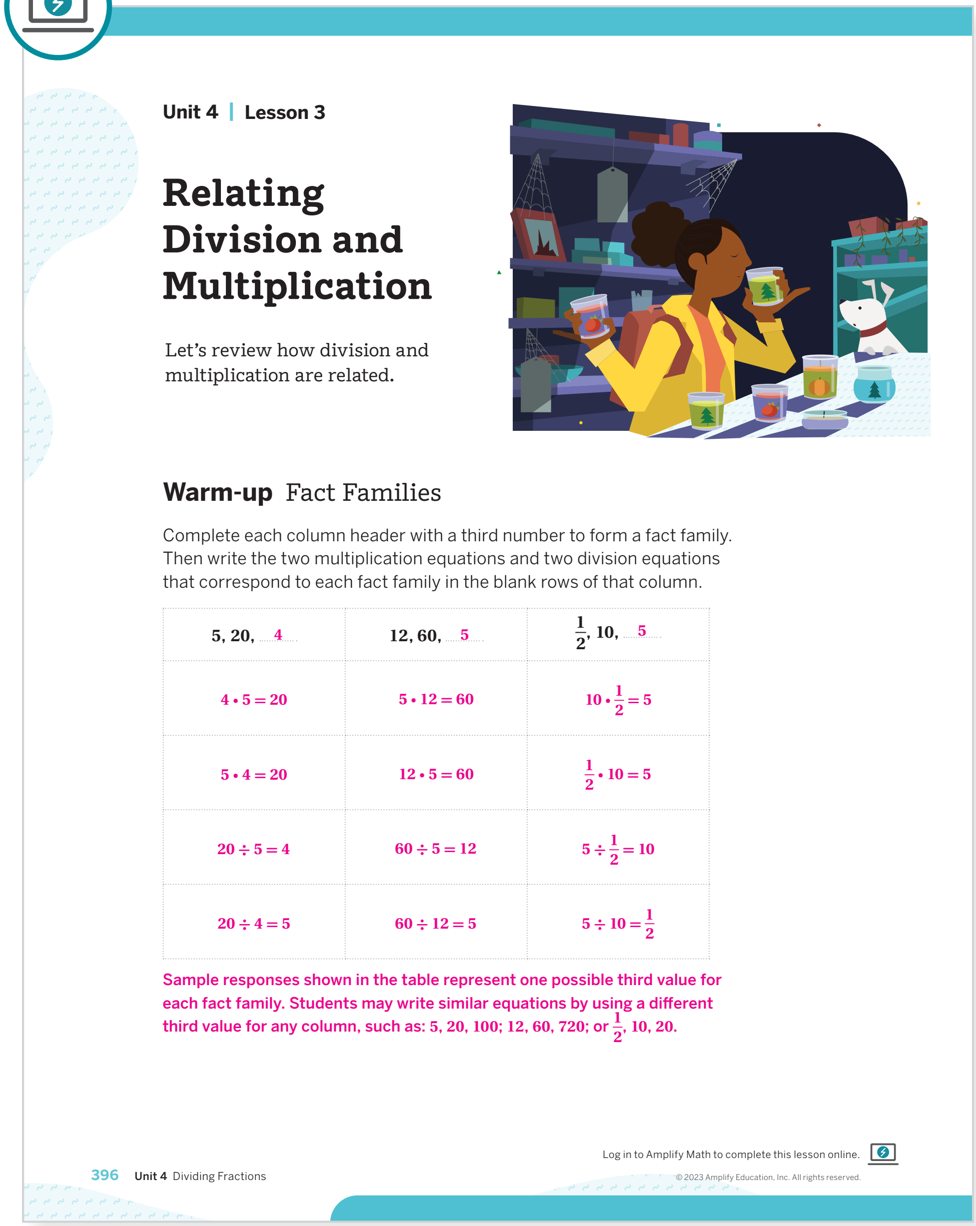
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, choose Problem 1 or 2 and skip the estimation.

Independent | 5 min

Warm-up Fact Families

Students activate prior knowledge of multiplication and division fact families, preparing them to divide with fractions.



Unit 4 | Lesson 3

Relating Division and Multiplication

Let's review how division and multiplication are related.

Warm-up Fact Families

Complete each column header with a third number to form a fact family. Then write the two multiplication equations and two division equations that correspond to each fact family in the blank rows of that column.

5, 20, 4	12, 60, 5	$\frac{1}{2}$, 10, 5
$4 \cdot 5 = 20$	$5 \cdot 12 = 60$	$10 \cdot \frac{1}{2} = 5$
$5 \cdot 4 = 20$	$12 \cdot 5 = 60$	$\frac{1}{2} \cdot 10 = 5$
$20 \div 5 = 4$	$60 \div 5 = 12$	$5 \div \frac{1}{2} = 10$
$20 \div 4 = 5$	$60 \div 12 = 5$	$5 \div 10 = \frac{1}{2}$

Sample responses shown in the table represent one possible third value for each fact family. Students may write similar equations by using a different third value for any column, such as: 5, 20, 100; 12, 60, 720; or $\frac{1}{2}$, 10, 20.

Log in to Amplify Math to complete this lesson online.

396 Unit 4 Dividing Fractions

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by asking, "What is a fact family? What should go in the blanks?

Look for points of confusion:

- **Struggling to determine the third value to complete the fact families.** Ask, "How are the two values related? How would you set up a division or multiplication equation with the given numbers?"

Look for productive strategies:

- Relating products to dividends, or factors to quotients or divisors, using the relationship between multiplication and division.
- Explaining how they determined the third value in each fact family by using multiplication or division.
- Recognizing dividing by a unit fraction as the same as multiplying by its whole-number denominator.

3 Connect

Have students share the third value they determined to complete each fact family, and then all of the related equations, being sure to allow for all possible answers.

Ask, "Does the relationship between multiplication and division change for different types of numbers, such as fractions or decimals? Why or why not?"

Highlight that every fact family includes two multiplication equations and two related division equations, regardless of the types of numbers involved, because the factors (or divisor and quotient) always correspond to the number of equal-sized groups and the size of those groups.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they have worked with fact families in prior grades. A fact family consists of three numbers that are used together to create a set of math facts. Those math facts can be addition, subtraction, multiplication, or division. Consider displaying the following as an example of a fact family.

$3 \cdot 4 = 12$
$4 \cdot 3 = 12$
$12 \div 4 = 3$
$12 \div 3 = 4$

Power-up

To power up students' ability to reason about the relationship between factors and multiples, have students complete:

Recall that a *factor* is a number that divides evenly into a given whole number. A *multiple* is a number that is the product of a given number and a whole number.

In each statement complete the missing word with *factor*, *multiple*, or *factor and multiple*.

1. 4 is a multiple of 2.
2. 12 is a factor and multiple of 12.
3. 4 is a factor of 8.
4. 18 is a multiple of 6.

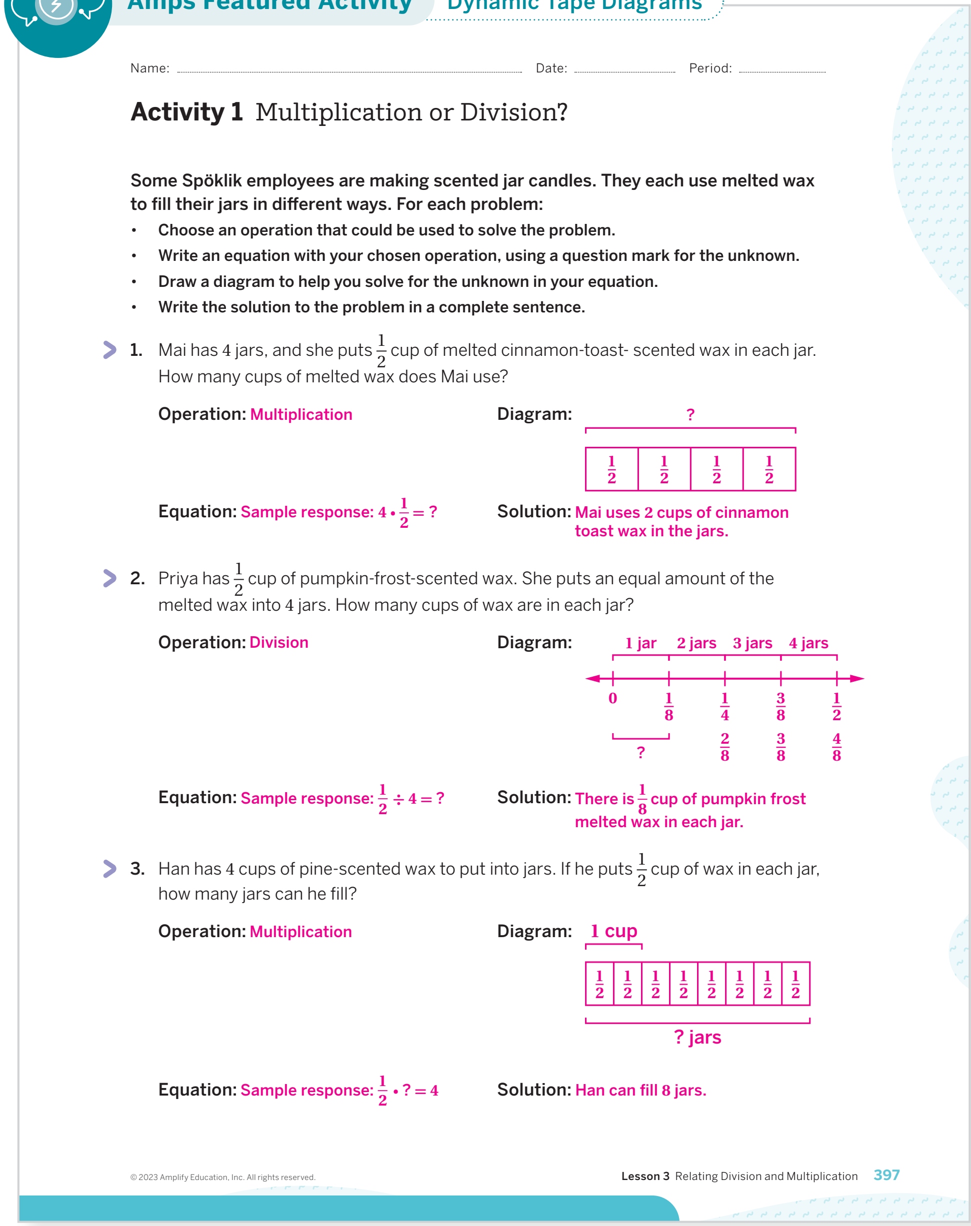
Use: Before the Warm-up.
Informed by: Performance on Lesson 2, Practice Problem 6.

Pairs | 15 min

Activity 1 Multiplication or Division?

Students determine whether scenarios are best represented by multiplication or division and then write equations and create diagrams to help answer related questions.

Amps Featured Activity Dynamic Tape Diagrams



Name: ______ Date: ______ Period: ______

Activity 1 Multiplication or Division?

Some Spöklik employees are making scented jar candles. They each use melted wax to fill their jars in different ways. For each problem:

- **Choose an operation that could be used to solve the problem.**
- **Write an equation with your chosen operation, using a question mark for the unknown.**
- **Draw a diagram to help you solve for the unknown in your equation.**
- **Write the solution to the problem in a complete sentence.**

1. Mai has 4 jars, and she puts $\frac{1}{2}$ cup of melted cinnamon-toast- scented wax in each jar. How many cups of melted wax does Mai use?

Operation: Multiplication

Diagram:

Equation: Sample response: $4 \cdot \frac{1}{2} = ?$

Solution: Mai uses 2 cups of cinnamon toast wax in the jars.

2. Priya has $\frac{1}{2}$ cup of pumpkin-frost-scented wax. She puts an equal amount of the melted wax into 4 jars. How many cups of wax are in each jar?

Operation: Division

Diagram:

Equation: Sample response: $\frac{1}{2} \div 4 = ?$

Solution: There is $\frac{1}{8}$ cup of pumpkin frost melted wax in each jar.

3. Han has 4 cups of pine-scented wax to put into jars. If he puts $\frac{1}{2}$ cup of wax in each jar, how many jars can he fill?

Operation: Multiplication

Diagram:

Equation: Sample response: $\frac{1}{2} \cdot ? = 4$

Solution: Han can fill 8 jars.

Lesson 3 Relating Division and Multiplication 397

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What information do you know, and what do you want to know? Which operation could help?"

Look for points of confusion:

- **Writing the same equations or drawing the same diagrams for more than one scenario.** Have students identify the total, and then ask, "What does this other given value represent?"
- **Struggling to divide $\frac{1}{2}$ into four equal parts (Problem 2).** Have students fold a piece of paper in half and label each side as $\frac{1}{2}$. Then fold those halves into halves and ask, "What fraction is each part now?" Repeat to make $\frac{1}{8}$.

Look for productive strategies:

- Identifying the given information as number of groups or size of each group in each scenario, and then creating diagrams and writing equations.

3 Connect

Have students share their responses and strategies, focusing on students who used different operations and how all of their equations and diagrams are related. Display the different models used, including tape diagrams.

Ask:

- "Did you determine your equation first or draw your diagram first? How did that help you determine the other representation and then solve the problem?"
- For the tape diagrams, "What does the number in each 'box' represent?"

Highlight that when given two pieces of information in an equal-sized groups scenario, the relationship can be represented using either multiplication or division.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Replace the fraction $\frac{1}{2}$ with 2 and have students complete the problems using this new value. This will allow them to still access the activity goal of determining whether multiplication or division best represents the scenarios.

Extension: *Math Enrichment*

Have students complete the following problem:
Write three scenarios, each using the values 6 and $2\frac{2}{3}$. One scenario should be multiplication and two should use division. Draw a diagram to solve the problem in each scenario. Then represent each scenario with an equation. Answers will vary.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their responses and strategies, draw their attention to the connections between the representations (words, diagrams, and equations) for each problem. Highlight how the values and relationships are shown in each representation. For example, show how the 4 jars are represented in Problem 1 in the verbal description, diagram, and equation. Point out that there are 4 equal groups of $\frac{1}{2}$ shown in the equation and diagram.

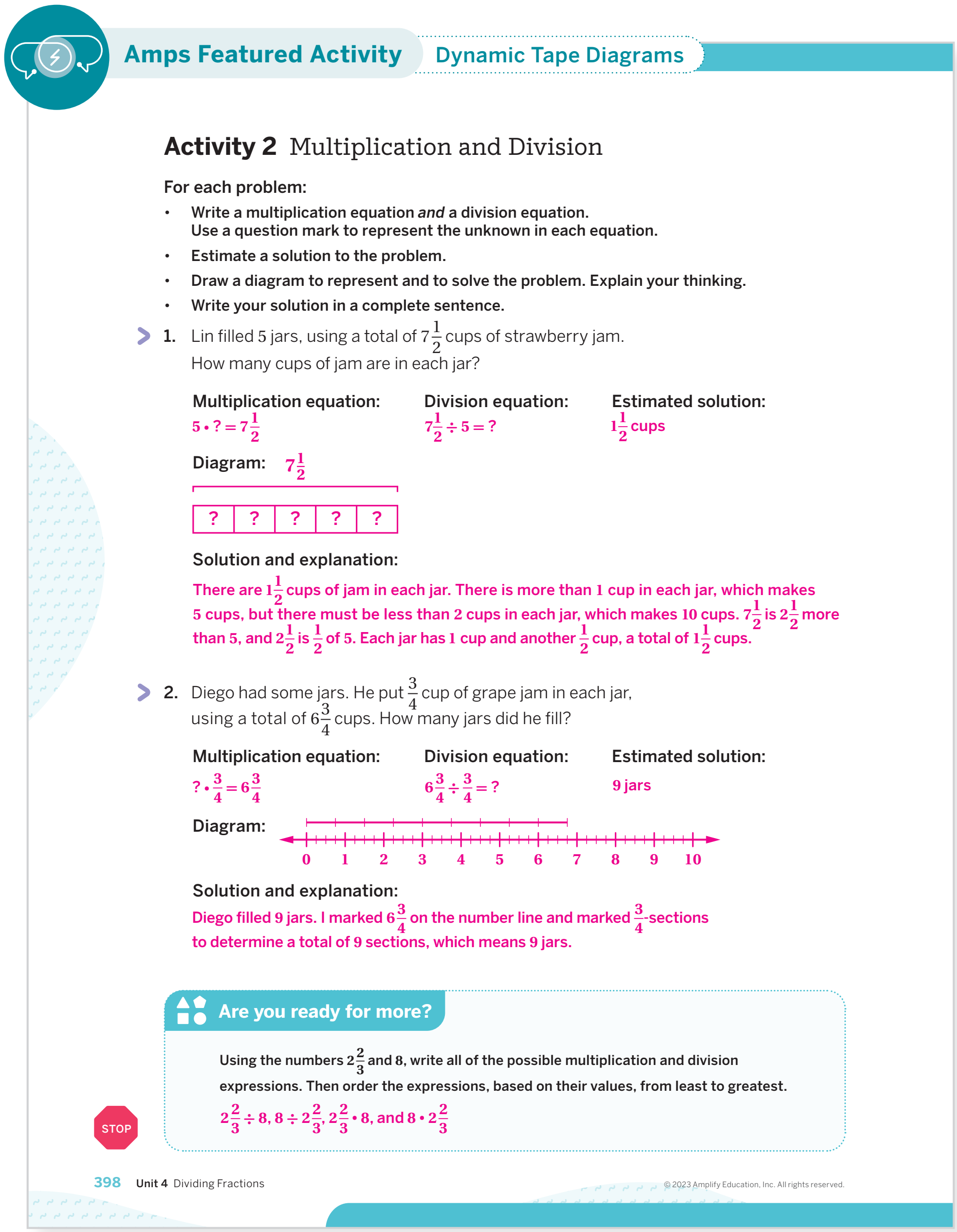
English Learners

Use hand gestures and/or annotations as you illustrate how each value is shown in each representation.

Pairs | 15 min

Activity 2 Multiplication and Division

Students now write both a division equation and a multiplication equation to represent a scenario, and they use either or both to create a diagram and to solve the problem.



Amps Featured Activity Dynamic Tape Diagrams

Activity 2 Multiplication and Division

For each problem:

- **Write a multiplication equation *and* a division equation. Use a question mark to represent the unknown in each equation.**
- **Estimate a solution to the problem.**
- **Draw a diagram to represent and to solve the problem. Explain your thinking.**
- **Write your solution in a complete sentence.**

1. Lin filled 5 jars, using a total of $7\frac{1}{2}$ cups of strawberry jam. How many cups of jam are in each jar?

Multiplication equation: $5 \cdot ? = 7\frac{1}{2}$

Division equation: $7\frac{1}{2} \div 5 = ?$

Estimated solution: $1\frac{1}{2}$ cups

Diagram:

Solution and explanation:

There are $1\frac{1}{2}$ cups of jam in each jar. There is more than 1 cup in each jar, which makes 5 cups, but there must be less than 2 cups in each jar, which makes 10 cups. $7\frac{1}{2}$ is $2\frac{1}{2}$ more than 5, and $2\frac{1}{2}$ is $\frac{1}{2}$ of 5. Each jar has 1 cup and another $\frac{1}{2}$ cup, a total of $1\frac{1}{2}$ cups.

2. Diego had some jars. He put $\frac{3}{4}$ cup of grape jam in each jar, using a total of $6\frac{3}{4}$ cups. How many jars did he fill?

Multiplication equation: $? \cdot \frac{3}{4} = 6\frac{3}{4}$

Division equation: $6\frac{3}{4} \div \frac{3}{4} = ?$

Estimated solution: 9 jars

Diagram:

Solution and explanation:

Diego filled 9 jars. I marked $6\frac{3}{4}$ on the number line and marked $\frac{3}{4}$-sections to determine a total of 9 sections, which means 9 jars.

Are you ready for more?

Using the numbers $2\frac{2}{3}$ and 8, write all of the possible multiplication and division expressions. Then order the expressions, based on their values, from least to greatest.

$2\frac{2}{3} \div 8$, $8 \div 2\frac{2}{3}$, $2\frac{2}{3} \cdot 8$, and $8 \cdot 2\frac{2}{3}$

STOP

398 Unit 4 Dividing Fractions

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by having them draw and use a diagram to help determine the operation.

Look for points of confusion:

- **Having trouble estimating an answer.** Ask students to try to come up with a range of solutions by giving estimates that would be too "large" and too "small."
- **Solving using an incorrect operation.** Ensure students have correct diagrams, and then ask, "Does your solution make sense?"

Look for productive strategies:

- Relating values in multiplication and division equations to create diagrams and solve for unknowns: dividend relates to the product, and divisor and quotient relate to the factors.
- Relating each part of a diagram to an equation *and* the context, and coordinating those representations to determine the unknown.

3 Connect

Have students share how their diagrams and equations are related and represent the problems, focusing on strategies for representing fractional values.

Ask:

- "When can you use a different operation than what is in your equation to solve?"
- "How does the position of the unknown in your equations affect your thinking about diagrams?"

Highlight that the relationship between multiplication and division can be used to help determine the unknown in a multiplication equation or corresponding division equation. To solve a multiplication equation, use division. To solve a division equation, use multiplication.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide the following checklist to help students make sense of each problem before they write their equations.

Sample response provided for Problem 1.

☐ What are the three quantities? number of jars, number of cups of jam total, number of cups of jam in each jar

☐ Using words, write a sentence that describes how these quantities are related using multiplication. number of cups of jam in each jar × the number of jars = number of cups total

☐ Using words, write a sentence that describes how these quantities are related using division. number of cups total ÷ number of jars = number of cups of jam in each jar

Math Language Development

MLR8: Discussion Supports—Annotate It!

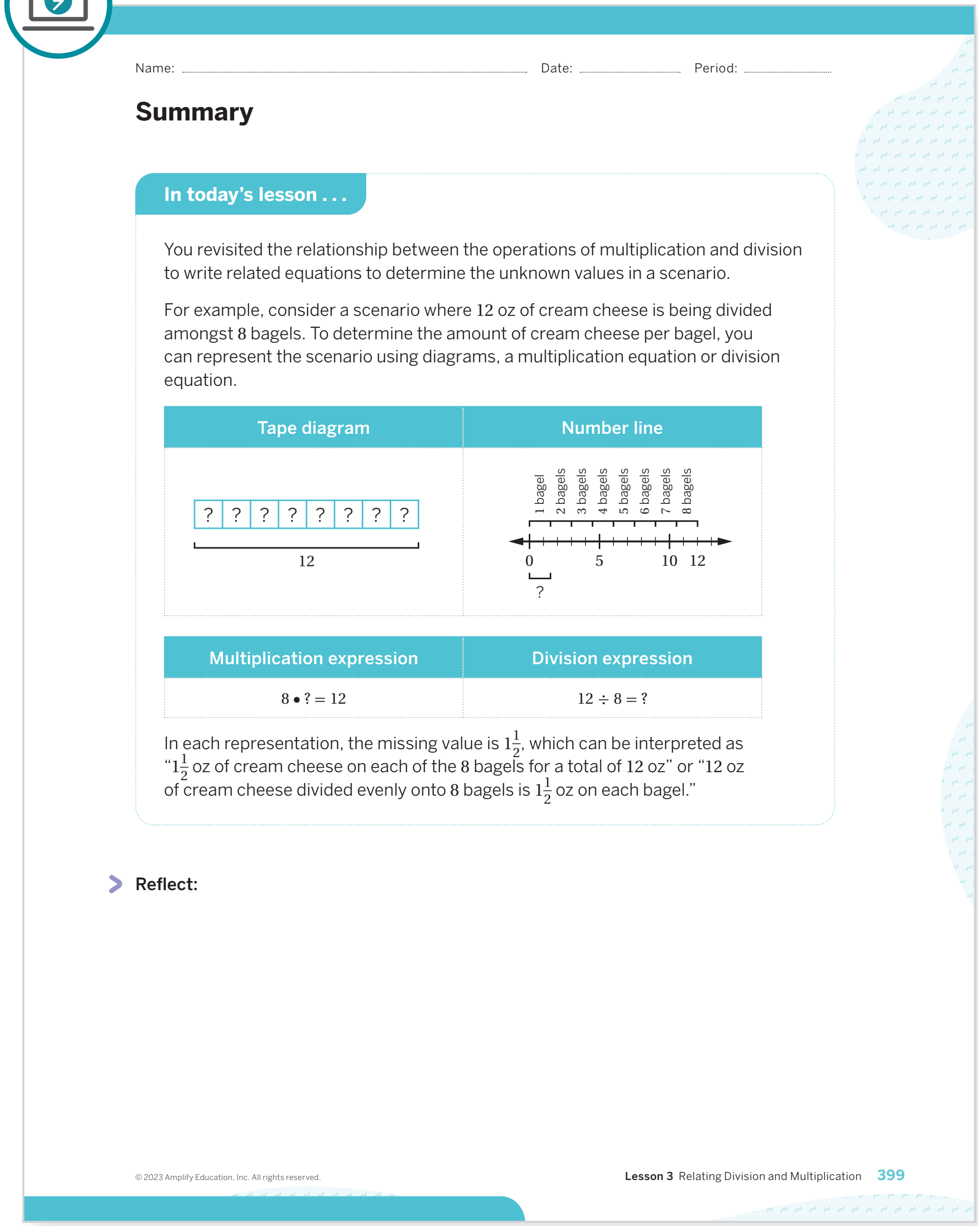
During the Connect, as students share how their diagrams and equations are related, draw their attention to where they see the unknown in each representation. Then ask students how the unknown in the multiplication equation corresponds with the unknown in the division equation.

Guide them to use mathematical vocabulary they have previously learned, such as *factor* and *quotient*. Add a multiplication and corresponding division equation to the class display and annotate the terms *factor*, *product*, *dividend*, *divisor*, and *quotient*.

Whole Class | 5 min

Summary

Review and synthesize how multiplication and division are related, and that the types of numbers involved do not affect that relationship or the interpretation of a problem.



Name: ______________ Date: ________ Period: ________

Summary

In today's lesson . . .

You revisited the relationship between the operations of multiplication and division to write related equations to determine the unknown values in a scenario.

For example, consider a scenario where 12 oz of cream cheese is being divided amongst 8 bagels. To determine the amount of cream cheese per bagel, you can represent the scenario using diagrams, a multiplication equation or division equation.

Tape diagram	Number line
? ? ? ? ? ? ? ? 12	1 bagel, 2 bagels, 3 bagels, 4 bagels, 5 bagels, 6 bagels, 7 bagels, 8 bagels 0 5 10 12 ?
Multiplication expression	**Division expression**
8 • ? = 12	12 ÷ 8 = ?

In each representation, the missing value is $1\frac{1}{2}$, which can be interpreted as "$1\frac{1}{2}$ oz of cream cheese on each of the 8 bagels for a total of 12 oz" or "12 oz of cream cheese divided evenly onto 8 bagels is $1\frac{1}{2}$ oz on each bagel."

Reflect:

Synthesize

Ask:

- "What is similar about multiplication and division scenarios? What is different?"
- "Are all division scenarios the same? Explain your thinking." No. The unknown could be number of groups or amount in each group
- "Describe how the terms factor, dividend, divisor, product and quotient are related." Sample response: A dividend is also a product; a divisor and a quotient are also factors.

Highlight that the same diagram can be used to represent either a division scenario or a multiplication scenario, because the corresponding equations are related. To determine an unknown value in such scenarios, sometimes one type of diagram or one type of equation is more efficient, but either can be used to help with thinking through solving equal-sized groups problems, and no matter what the types of numbers involved are: whole numbers, fractions, or decimals.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

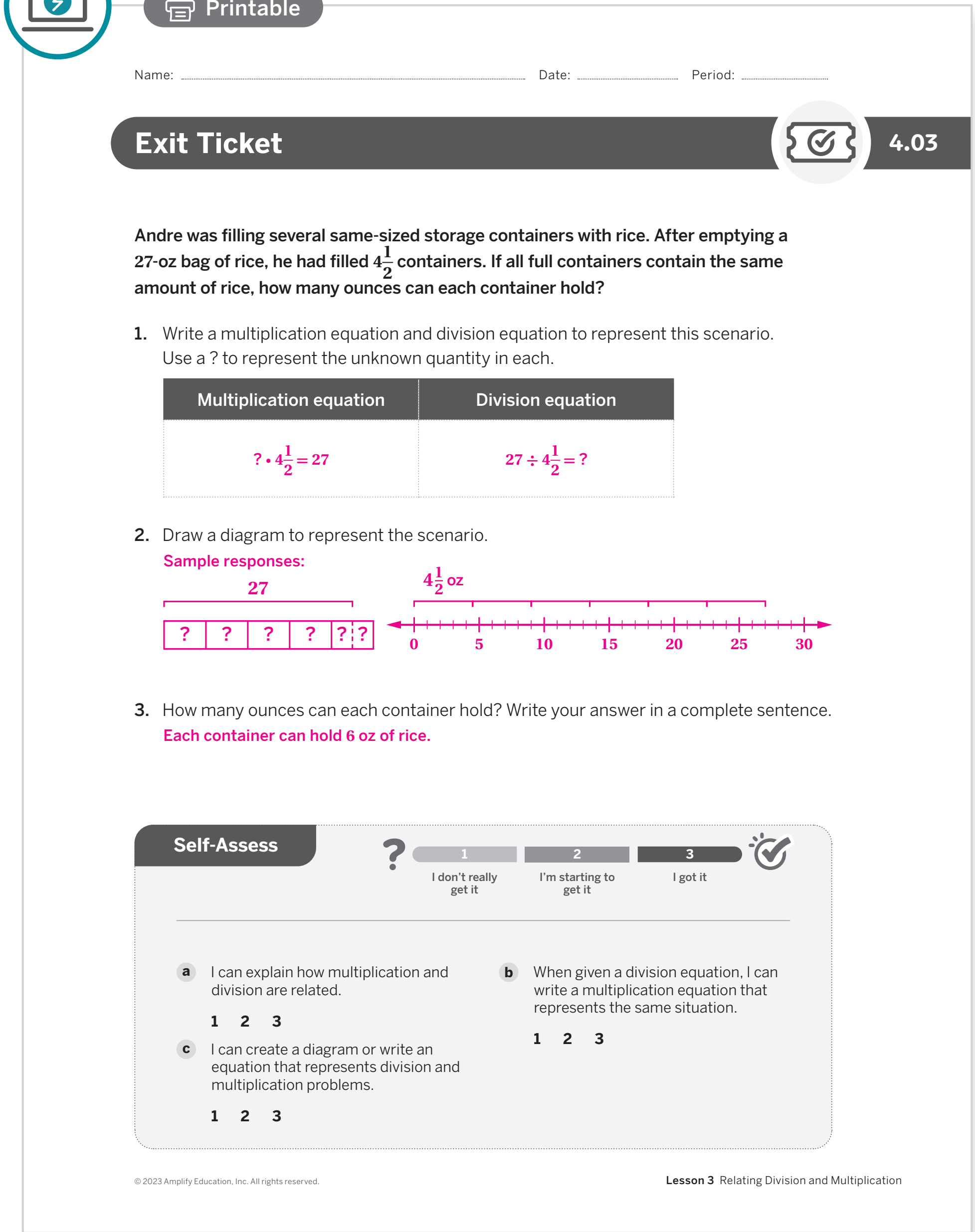
- "How can you use two related equations to determine an unknown?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of representing and solving a division scenario by writing a division and a multiplication equation and by drawing a diagram.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 4.03

Andre was filling several same-sized storage containers with rice. After emptying a 27-oz bag of rice, he had filled $4\frac{1}{2}$ containers. If all full containers contain the same amount of rice, how many ounces can each container hold?

1. Write a multiplication equation and division equation to represent this scenario. Use a ? to represent the unknown quantity in each.

Multiplication equation	Division equation
$? \cdot 4\frac{1}{2} = 27$	$27 \div 4\frac{1}{2} = ?$

2. Draw a diagram to represent the scenario.

Sample responses:

3. How many ounces can each container hold? Write your answer in a complete sentence.

Each container can hold 6 oz of rice.

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can explain how multiplication and division are related.
1 2 3

b When given a division equation, I can write a multiplication equation that represents the same situation.
1 2 3

c I can create a diagram or write an equation that represents division and multiplication problems.
1 2 3

Lesson 3 Relating Division and Multiplication

Success looks like . . .

- **Language Goal:** Generating multiplication and division equations to represent a situation involving fractions, and relating the equations to a diagram. **(Speaking and Listening)**
  - » Writing a multiplication and division equation to determine the number of ounces in each container of rice and drawing a diagram in Problems 1 and 2.
- **Language Goal:** Explaining how to determine the unknown quantity in a multiplication or division situation involving fractions. **(Speaking and Listening, Writing)**
- **Language Goal:** Identifying the unknown quantity in a situation (i.e., the number of groups, the amount in one group, or the total amount) and generating corresponding equations. **(Speaking and Listening, Writing)**
  - » Identifying the unknown quantity as the number of ounces of rice in each container and writing a multiplication and division equation in Problems 1 and 2.

Suggested next steps

If students can only write one type of equation correctly for Problem 1, consider:

- Reviewing the problems and equations from Activity 2.
- Asking, "How can you use a [multiplication / division] equation to create a corresponding [division / multiplication] equation?"
  If $3 \cdot 4 = 12$, then I can write $12 \div 4 = 3$.

If students solve by multiplying $4\frac{1}{2}$ by 27, consider:

- Having students reread the problem and asking, "Does this solution make sense?"
- Asking, "Can you describe the scenario in your own words? What are the knowns and what are the unknowns?"

Professional Learning

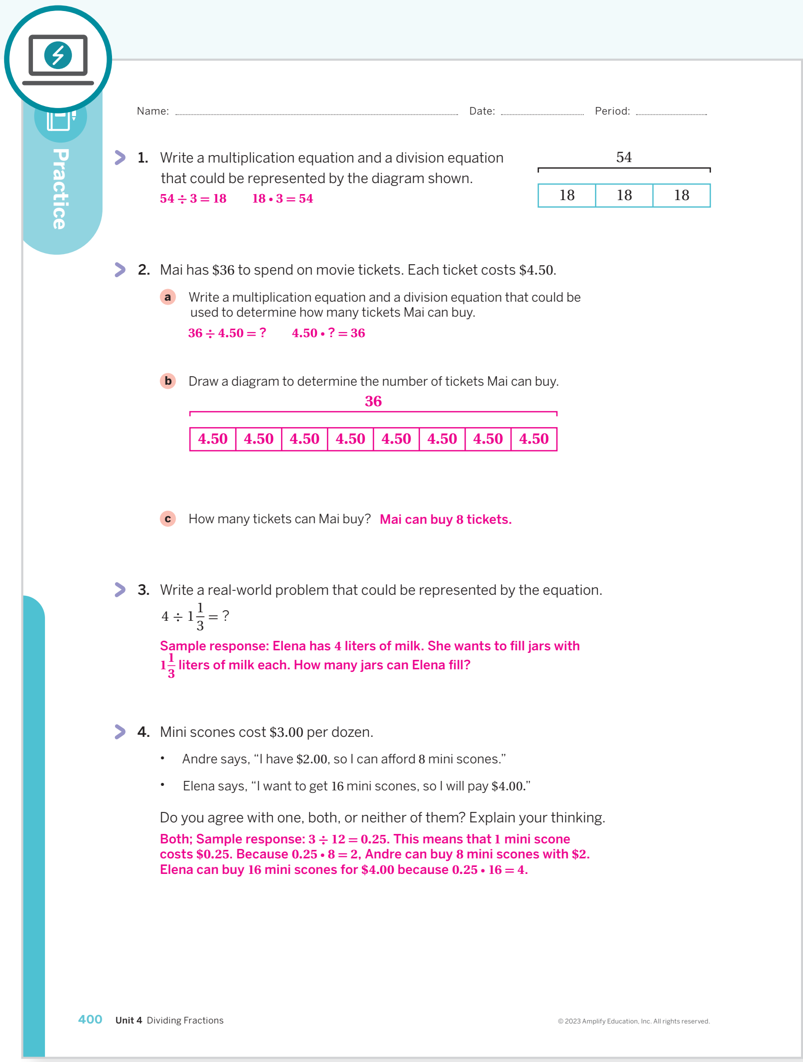
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students used multiplication and division to divide fractions. How did that build on the earlier work students did with determining "how many groups?" or "how many in each group?"
- Did students find anything frustrating in Activity 2? What helped them work through this frustration and how might you do anything different the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

1. Write a multiplication equation and a division equation that could be represented by the diagram shown.

   54

18	18	18

   $54 \div 3 = 18$ $18 \cdot 3 = 54$

2. Mai has $36 to spend on movie tickets. Each ticket costs $4.50.

   a. Write a multiplication equation and a division equation that could be used to determine how many tickets Mai can buy.

   $36 \div 4.50 = ?$ $4.50 \cdot ? = 36$

   b. Draw a diagram to determine the number of tickets Mai can buy.

   36

4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50

   c. How many tickets can Mai buy? **Mai can buy 8 tickets.**

3. Write a real-world problem that could be represented by the equation.

   $4 \div 1\frac{1}{3} = ?$

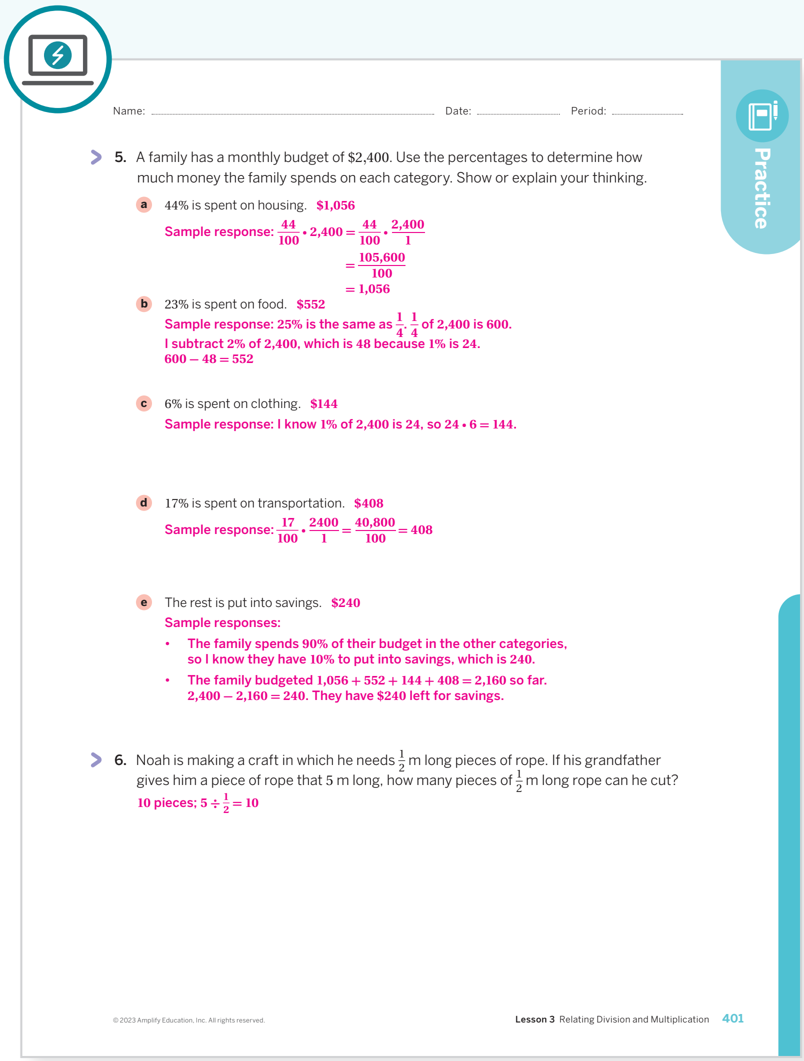
   Sample response: Elena has 4 liters of milk. She wants to fill jars with $1\frac{1}{3}$ liters of milk each. How many jars can Elena fill?

4. Mini scones cost $3.00 per dozen.

   - Andre says, "I have $2.00, so I can afford 8 mini scones."
   - Elena says, "I want to get 16 mini scones, so I will pay $4.00."

   Do you agree with one, both, or neither of them? Explain your thinking.

   Both; Sample response: $3 \div 12 = 0.25$. This means that 1 mini scone costs $0.25. Because $0.25 \cdot 8 = 2$, Andre can buy 8 mini scones with $2. Elena can buy 16 mini scones for $4.00 because $0.25 \cdot 16 = 4$.



Name: ________ Date: ________ Period: ________

5. A family has a monthly budget of $2,400. Use the percentages to determine how much money the family spends on each category. Show or explain your thinking.

   a. 44% is spent on housing. **$1,056**

   Sample response: $\frac{44}{100} \cdot 2{,}400 = \frac{44}{100} \cdot \frac{2{,}400}{1}$
   $= \frac{105{,}600}{100}$
   $= 1{,}056$

   b. 23% is spent on food. **$552**

   Sample response: 25% is the same as $\frac{1}{4}$. $\frac{1}{4}$ of 2,400 is 600. I subtract 2% of 2,400, which is 48 because 1% is 24. $600 - 48 = 552$

   c. 6% is spent on clothing. **$144**

   Sample response: I know 1% of 2,400 is 24, so $24 \cdot 6 = 144$.

   d. 17% is spent on transportation. **$408**

   Sample response: $\frac{17}{100} \cdot \frac{2400}{1} = \frac{40{,}800}{100} = 408$

   e. The rest is put into savings. **$240**

   Sample responses:

   - **The family spends 90% of their budget in the other categories, so I know they have 10% to put into savings, which is 240.**
   - **The family budgeted $1{,}056 + 552 + 144 + 408 = 2{,}160$ so far. $2{,}400 - 2{,}160 = 240$. They have $240 left for savings.**

6. Noah is making a craft in which he needs $\frac{1}{2}$ m long pieces of rope. If his grandfather gives him a piece of rope that 5 m long, how many pieces of $\frac{1}{2}$ m long rope can he cut?

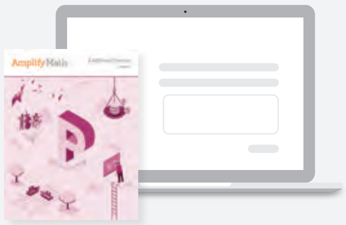
   10 pieces; $5 \div \frac{1}{2} = 10$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
	3	Activity 1, 2	2
Spiral	4	Unit 3 Lesson 6	2
	5	Unit 3 Lesson 13	2
Formative ⚡	6	Unit 4 Lesson 4	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Size of Divisor and Size of Quotient

Let's explore quotients of different sizes.

Focus

Goals

1. **Language Goal:** Explain how the sizes of dividends and divisors affect quotients with common values. **(Speaking and Listening, Writing)**
2. Estimate quotients and generalize how the dividend and divisor affect the size of a quotient relative to 1 (i.e., a number greater than 1, a fraction less than 1, or a value that is close to 1.

Coherence

Today

Students explore the relationships between the numbers in a division equation. They see that they can estimate the size of the quotient by reasoning about the relative sizes of the divisor and the dividend. Students first relate lesser divisors to greater quotients, and greater divisors to lesser quotients. Students then recognize that when the divisor is less than the dividend, the quotient is greater than 1; and when the divisor is greater than the dividend, the quotient is less than 1. They also determine that greater differences result in quotients much greater or much less than 1. Regardless of which is greater, when the dividend and divisor are approximately equal, the quotient is close to 1. Note: This lesson has the first "clue" for the Capstone activity.

Previously

Students explored the two meanings of division: "how many groups?" and "how many in each group?" in Lesson 2 and then the relationships between multiplication and division in Lesson 3.

Coming Soon

In Lesson 5, students will use pattern blocks as a basis for exploring division problems with non-unit fraction divisors. The focus of Lessons 5–7 will be on quotitive division and on determining "how many groups."

Rigor

- Students build **conceptual understanding** of how the size of a divisor, for any given dividend, affects the size of the quotient.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

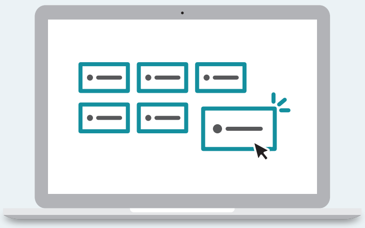
Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, one set per pair

Amps | **Featured Activity**

Activity 2
Digital Card Sort

Students estimate quotients and order the expressions by dragging them on screen.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might be frustrated that there is not enough time to complete each of the division problems. Assure them that the task is possible without actually doing the division. Ask them to use the structure of the division problems themselves to reason about the size of the quotients and then order them. Walk through an example with students so that they can regulate their emotions before beginning the timed task.

Modifications to Pacing

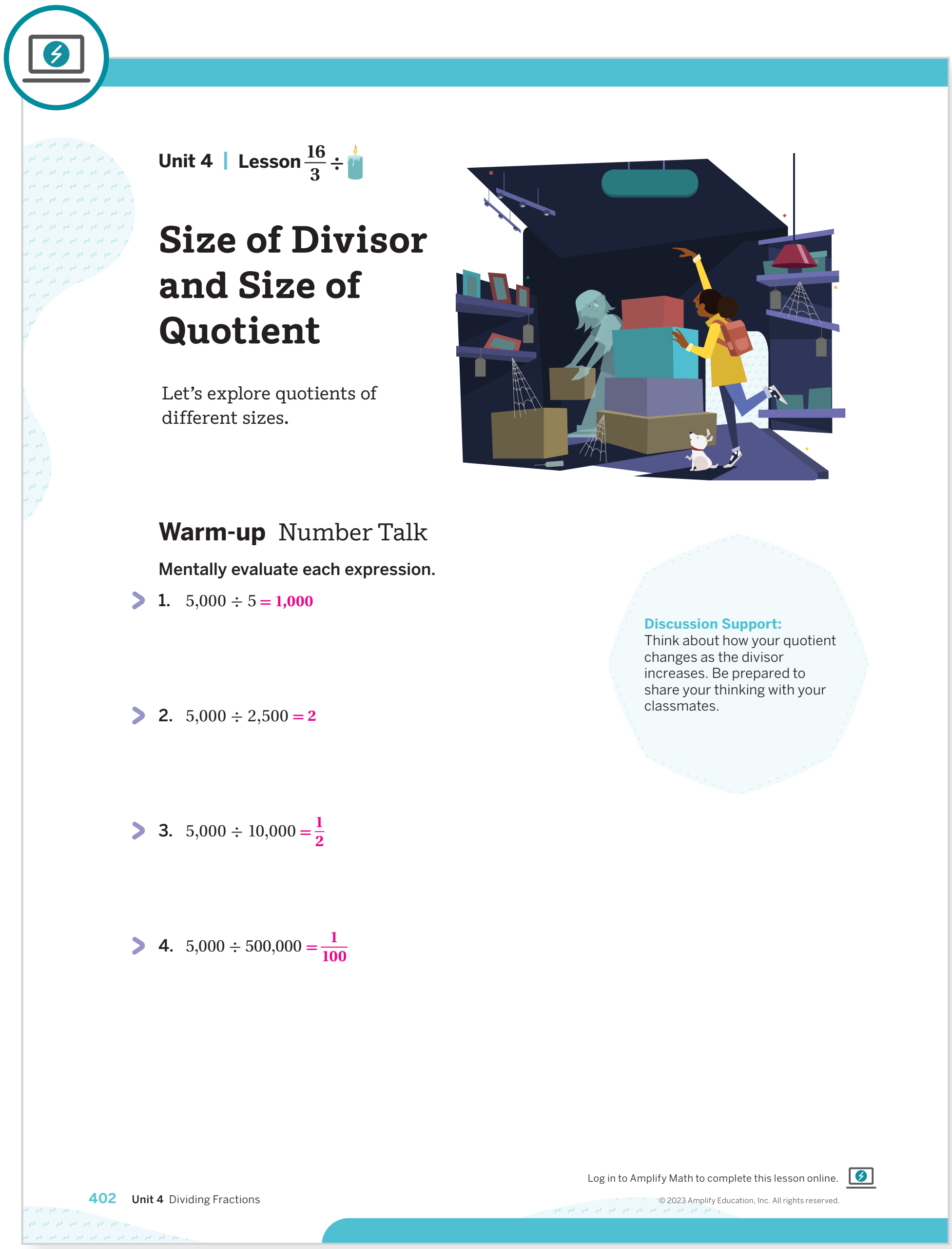
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have students only complete the first problem.
- In **Activity 2**, distribute only the first column of cards in each set (students should still do both sets).

Independent | 5 min

Warm-up Number Talk

Students mentally evaluate four expressions with the same dividend, recognizing that the greater the divisor, the smaller the quotient, and vice versa.



Unit 4 | Lesson $\frac{16}{3} \div$

Size of Divisor and Size of Quotient

Let's explore quotients of different sizes.

Warm-up Number Talk

Mentally evaluate each expression.

1. $5{,}000 \div 5 = 1{,}000$
2. $5{,}000 \div 2{,}500 = 2$
3. $5{,}000 \div 10{,}000 = \frac{1}{2}$
4. $5{,}000 \div 500{,}000 = \frac{1}{100}$

Discussion Support: Think about how your quotient changes as the divisor increases. Be prepared to share your thinking with your classmates.

402 Unit 4 Dividing Fractions

Log in to Amplify Math to complete this lesson online.

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1 Launch

Use the *Number Talk* routine, revealing each expression one at a time.

2 Monitor

Help students get started by asking, "What is $5 \div 5$? 1 $50 \div 5$? 10 How can those quotients help you think about the first expression?"

Look for points of confusion:

- **Thinking quotients cannot be determined when the divisor is greater.** Ask, "Could you find the value if the order were reversed? How might that help with the given order?"

Look for productive strategies:

- Recognizing the relationships between one expression and the next, and using those to help evaluate subsequent expressions.
- Using place-value understanding to attend to and account for zeros while operating with basic facts involving the non-zero numbers.

3 Connect

Display the four expressions.

Have students share their strategies for determining the quotients and display their explanations.

Ask:

- "Which values are the same in all of the expressions? Which values are different?"
- "How does the way the dividend changed relate to the differences in the quotients?"

Highlight that dividing can be interpreted as equipartitioning a quantity (represented by the dividend) into a number of equal-sized groups (the divisor), and when there are more groups, the "share" for each group becomes less, and by the same factor (e.g., 2 times as many groups results in 2 times less in a share). If there are fewer objects than groups, then the share is a fraction less than 1.

Math Language Development

MLR8: Discussion Supports

Let students know they should think about how their quotient changes as the divisor increases. During the Connect, as they share their strategies, ask them to explain their thinking. Amplify language students use that relates to equipartitioning a quantity, such as *equal-sized groups*. Ask:

- "If the divisor increases and the dividend stays the same, will the number of equal-sized groups be greater, less, or the same?"
- "Without calculating, which of these expressions has a greater quotient, $5{,}000 \div 25$ or $5{,}000 \div 125$?"

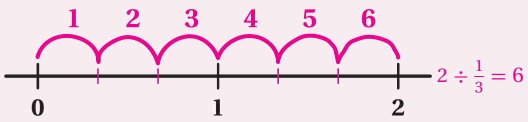
English Learners

Provide students the opportunity to rehearse and formulate what they will say with a partner before sharing with the class.

Power-up

To power up students' ability to make sense of fractional parts, have students complete:

Use the number line to simplify the expression $2 \div \frac{1}{3}$.



Use: Before Activity 1

Informed by: Performance on Lesson 3, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4

Pairs | 15 min

Activity 1 How Many Does It Take?

Students use estimates for the heights of several familiar objects to divide and to estimate how many of each are needed to make a tower of a given height.

Name: ______ Date: ______ Period: ______

Activity 1 How Many Does It Take?

Two clerks are working on a display in the Spöklik kitchen and dining sections. To help show the differences in the heights of different tables, drawers, and shelves, they plan to position towers of cube-shaped objects in, on, or under the showroom pieces.

1. Estimate how many of each object it would take to build a tower from the floor to the bottom of a breakfast table that is 3 ft tall. Be prepared to explain your thinking.
   - a Milk crates
     Sample response: 3
     Kari Marttila/Shutterstock.com
   - b ABC blocks
     Sample response: 30
     MidoSemsem/Shutterstock.com
   - c Dice
     Sample response: 60
     xpixel/Shutterstock.com
   - d Puzzle cubes
     Sample response: 12
     gd_project/Shutterstock.com

2. You want to stack cookbooks vertically to fill a shelf with a height of 72 cm.
   - a If the spine of each cookbook is 2 cm thick, write and evaluate an expression to determine how many cookbooks you need to fill the shelf.
     $72 \div 2$; I need to stack 36 cookbooks.
   - b If the spine of each cookbook is $1\frac{1}{2}$ cm thick, write and evaluate an expression to determine how many cookbooks you need to fill the shelf.
     $72 \div 1\frac{1}{2}$; I need to stack 48 cookbooks.

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Lesson 4 Size of Divisor and Size of Quotient 403

1 Launch

Display the phrase, "dividend ÷ divisor = quotient" and keep it visible for the remainder of the lesson. Have students use the *Think-Pair-Share* routine to complete Problem 1, and then complete Problems 2–3 together.

2 Monitor

Help students get started by asking, "What do you need to know about each object?"

Look for points of confusion:

- **Writing multiplication expressions or equations.** While multiplication could be used, these should be written as division. Have students draw a diagram.
- **Not being able to evaluate $72 \div 1\frac{1}{2}$ in Problem 2b.** Suggest students convert to the decimal 1.5 or first try to evaluate $72 \div \frac{1}{2}$.

Look for productive strategies:

- Drawing diagrams showing each object's estimated or exact measurement to determine how many fit.
- Using the estimated relative sizes of two objects to determine how many more or less are needed.

3 Connect

Display the images, or the objects and a yardstick (if available).

Have students share how they estimated in Problem 1, and their expressions and calculations for Problems 2–3.

Ask, "What do you think is the relationship between the height of the object and the number of those objects in a stack of a certain height?"

Highlight that the height of each object is the *divisor* and the number of objects is the *quotient*. For the same total height (*dividend*), the greater the divisor and the lesser the quotient; and vice versa.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider one of the alternative approaches to this activity.

- Provide students with estimates of the height of each object in Problem 1. This will allow them to access the activity goal, without the added task of estimation.
- Bring in actual objects for Problem 1 and display them. Allow students to handle them. Decide if you would like them to estimate their heights or have them measure.

Extension: *Math Enrichment*

Have students reconsider Problem 2, this time with a shelf that has a height of $\frac{72}{3}$ cm.

Math Language Development

MLR8: Discussion Supports

Encourage students to refer to the class display, highlighting the use of the terms *divisor, dividend*, and *quotient* in a division equation.

English Learners

Ask the entire class to chorally repeat the phrases that include these terms in context, such as "The height of the table is the dividend. The height of [each object] is the divisor. The number of [each object] is the quotient." This will provide English Learners the opportunity to listen to and speak the words in context.

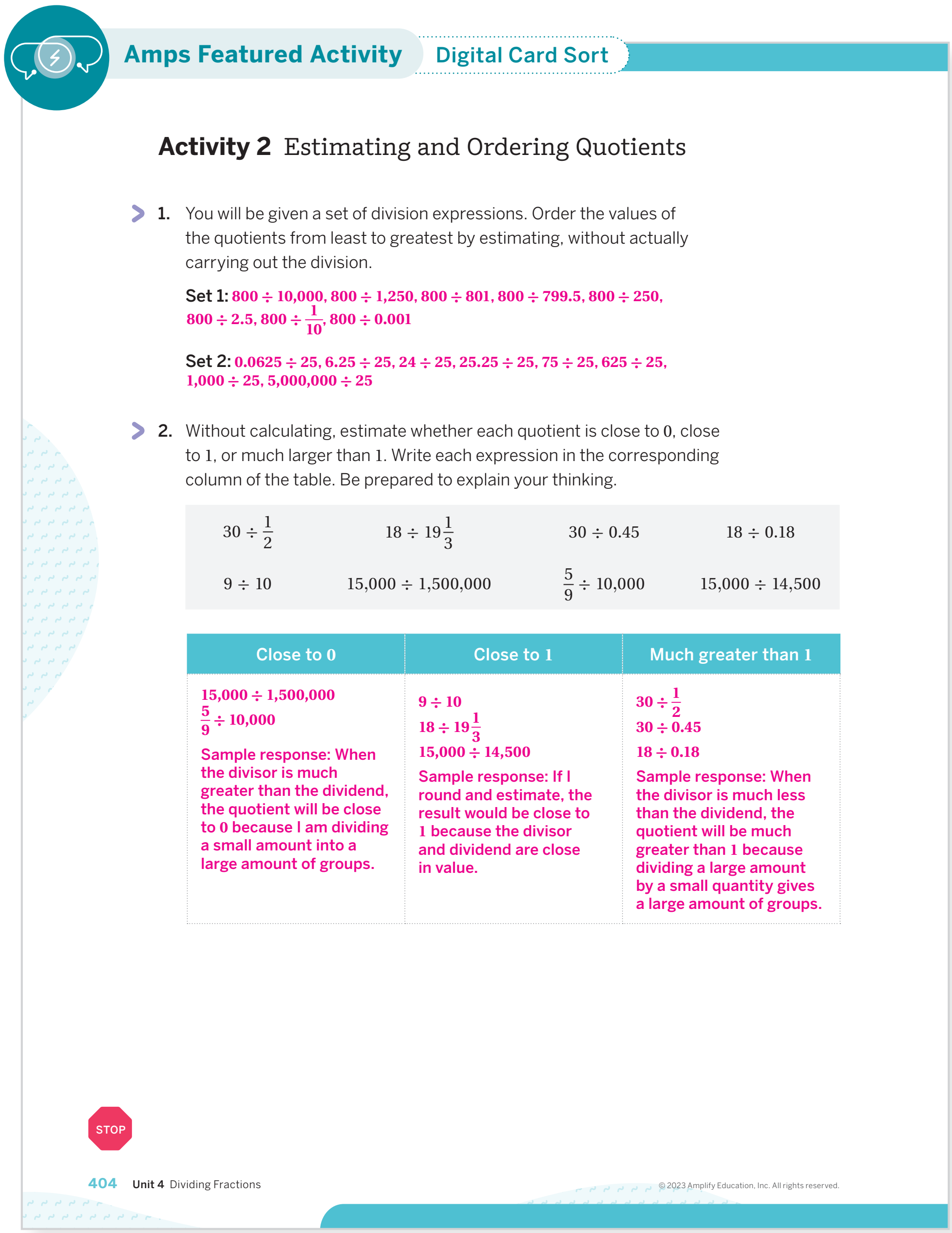
Note: In Problem 2, point to the spine of a Student Edition to illustrate what the term *spine* means, as it relates to a book.

Pairs | 15 min

Activity 2 Estimating and Ordering Quotients

Students apply the relationship between dividends, divisors and quotients to order expressions.

Amps Featured Activity Digital Card Sort



Activity 2 Estimating and Ordering Quotients

1. You will be given a set of division expressions. Order the values of the quotients from least to greatest by estimating, without actually carrying out the division.

   Set 1: $800 \div 10{,}000$, $800 \div 1{,}250$, $800 \div 801$, $800 \div 799.5$, $800 \div 250$, $800 \div 2.5$, $800 \div \frac{1}{10}$, $800 \div 0.001$

   Set 2: $0.0625 \div 25$, $6.25 \div 25$, $24 \div 25$, $25.25 \div 25$, $75 \div 25$, $625 \div 25$, $1{,}000 \div 25$, $5{,}000{,}000 \div 25$

2. Without calculating, estimate whether each quotient is close to 0, close to 1, or much larger than 1. Write each expression in the corresponding column of the table. Be prepared to explain your thinking.

$30 \div \frac{1}{2}$	$18 \div 19\frac{1}{3}$	$30 \div 0.45$	$18 \div 0.18$
$9 \div 10$	$15{,}000 \div 1{,}500{,}000$	$\frac{5}{9} \div 10{,}000$	$15{,}000 \div 14{,}500$

Close to 0	Close to 1	Much greater than 1
$15{,}000 \div 1{,}500{,}000$ $\frac{5}{9} \div 10{,}000$ Sample response: When the divisor is much greater than the dividend, the quotient will be close to 0 because I am dividing a small amount into a large amount of groups.	$9 \div 10$ $18 \div 19\frac{1}{3}$ $15{,}000 \div 14{,}500$ Sample response: If I round and estimate, the result would be close to 1 because the divisor and dividend are close in value.	$30 \div \frac{1}{2}$ $30 \div 0.45$ $18 \div 0.18$ Sample response: When the divisor is much less than the dividend, the quotient will be much greater than 1 because dividing a large amount by a small quantity gives a large amount of groups.

STOP

404 Unit 4 Dividing Fractions

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1 Launch

Keep students in pairs and distribute one set of cards from the Activity 2 PDF to each partner. Allow 3 minutes of independent work time to order their sets, without calculating, and then 2 minutes to share their thinking with a partner. Pause for a class discussion comparing this problem to the Warm-up, noting how the types of numbers in the expressions do not impact the relationships. Then have pairs complete Problem 2 together.

2 Monitor

Help students get started by asking, "What do you notice about all the dividends (Set 1) or divisors (Set 2)? How is this like Activity 1?"

Look for points of confusion:

- **Struggling to estimate quotients involving decimals or fractions.** Ask, "Which number is greater? What does that tell you about the quotient?"

Look for productive strategies:

- Using the relative sizes of the dividend and divisor, and place value, to order and categorize the quotients.

3 Connect

Have pairs share their categorizations of the expressions from Problem 2, one at a time, and explain their thinking and how it evolved as they worked with more and more expressions.

Highlight that when the dividend and divisor are equal, the quotient is equal to 1, and so, comparing the relative sizes of the dividend and divisor is the same as comparing the quotient to 1. If divisor < dividend, then quotient > 1, and if divisor > dividend, then quotient < 1. Or if they are "close," then the quotient is "close to 1."

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider these alternative approaches to this activity.

- In Problem 1, consider providing students with a smaller subset of cards in which to order. Consider introducing the remaining cards after they have ordered the initial set.
- In Problem 2, have students categorize the following expressions from the Set 1 cards in Problem 1 first.

$800 \div 10{,}000 \qquad 800 \div 801 \qquad 800 \div \frac{1}{10}$

Then have them use these expressions to help them make sense of the relationship between the dividends and divisors for each category.

Math Language Development

MLR2: Collect and Display

As students work, circulate and listen to the language they use to order and sort the expressions. Add terms and phrases they use to the class display, such as "the divisor is close to the dividend," "the divisor is greater than the dividend," and "the divisor is less than the dividend." During the Connect, draw connections between these relationships and the relative size of the quotient.

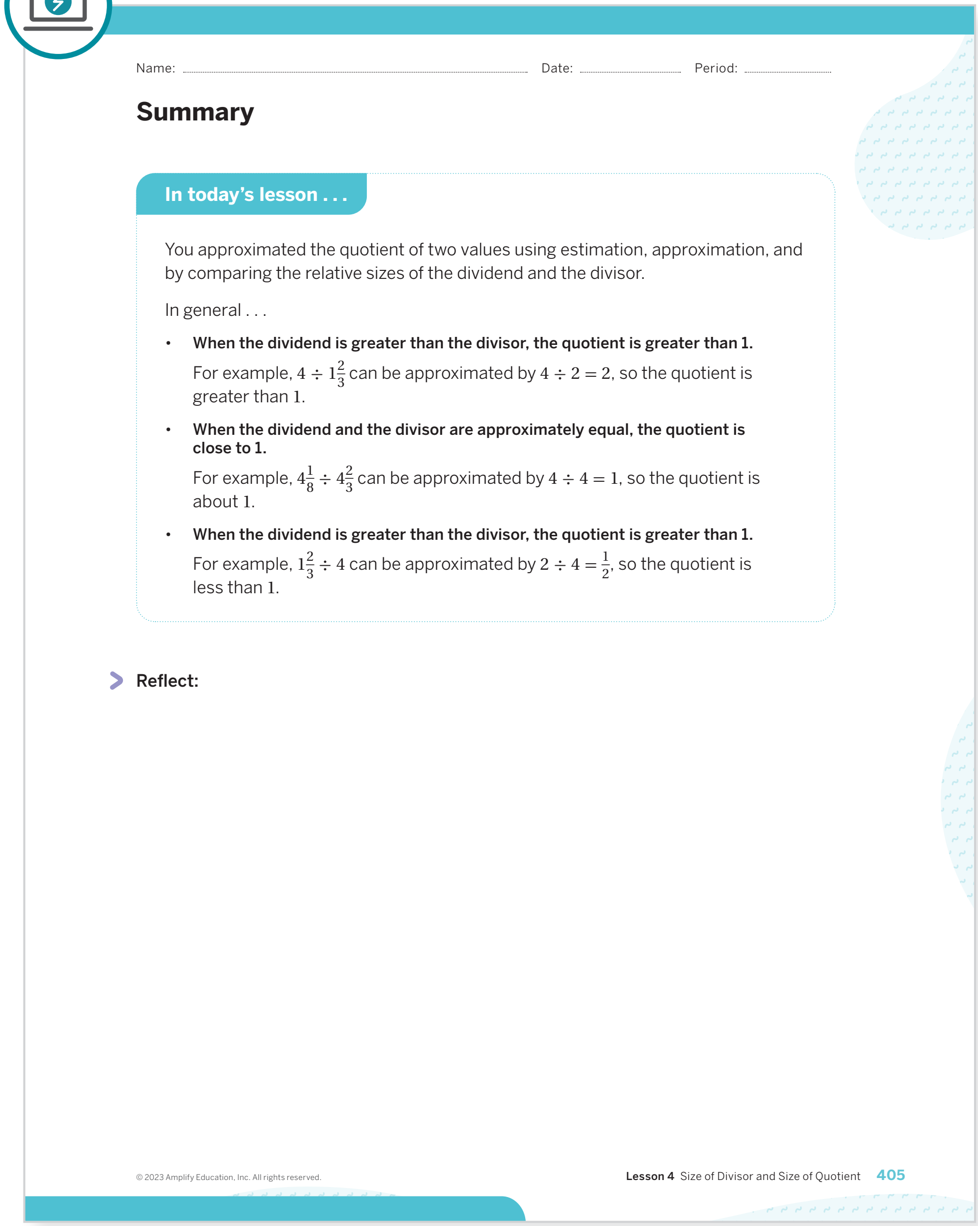
English Learners

Annotate the table with these terms and phrases. For example, annotate the *Close to* 1 column with the phrase "the divisor is close to the dividend."

Whole Class | 5 min

Summary

Review and synthesize the ways in which relative values of divisors and dividends affect the size of the related quotients.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You approximated the quotient of two values using estimation, approximation, and by comparing the relative sizes of the dividend and the divisor.

In general . . .

- **When the dividend is greater than the divisor, the quotient is greater than 1.**

  For example, $4 \div 1\frac{2}{3}$ can be approximated by $4 \div 2 = 2$, so the quotient is greater than 1.
- **When the dividend and the divisor are approximately equal, the quotient is close to 1.**

  For example, $4\frac{1}{8} \div 4\frac{2}{3}$ can be approximated by $4 \div 4 = 1$, so the quotient is about 1.
- **When the dividend is greater than the divisor, the quotient is greater than 1.**

  For example, $1\frac{2}{3} \div 4$ can be approximated by $2 \div 4 = \frac{1}{2}$, so the quotient is less than 1.

> **Reflect:**

Synthesize

Highlight the ways in which quotients can be estimated by considering the relative sizes of the *dividend* and the *divisor* and also by comparing values to those in related quotients.

Ask:

- "What happens to the quotient if you divide the same number by other numbers that are less and less?" The quotient will be greater and greater.
- "What makes a quotient close to 0? Can it ever equal 0?" When the dividend is very small compared to the divisor. Yes, but only if the dividend is exactly zero, will the quotient be zero.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

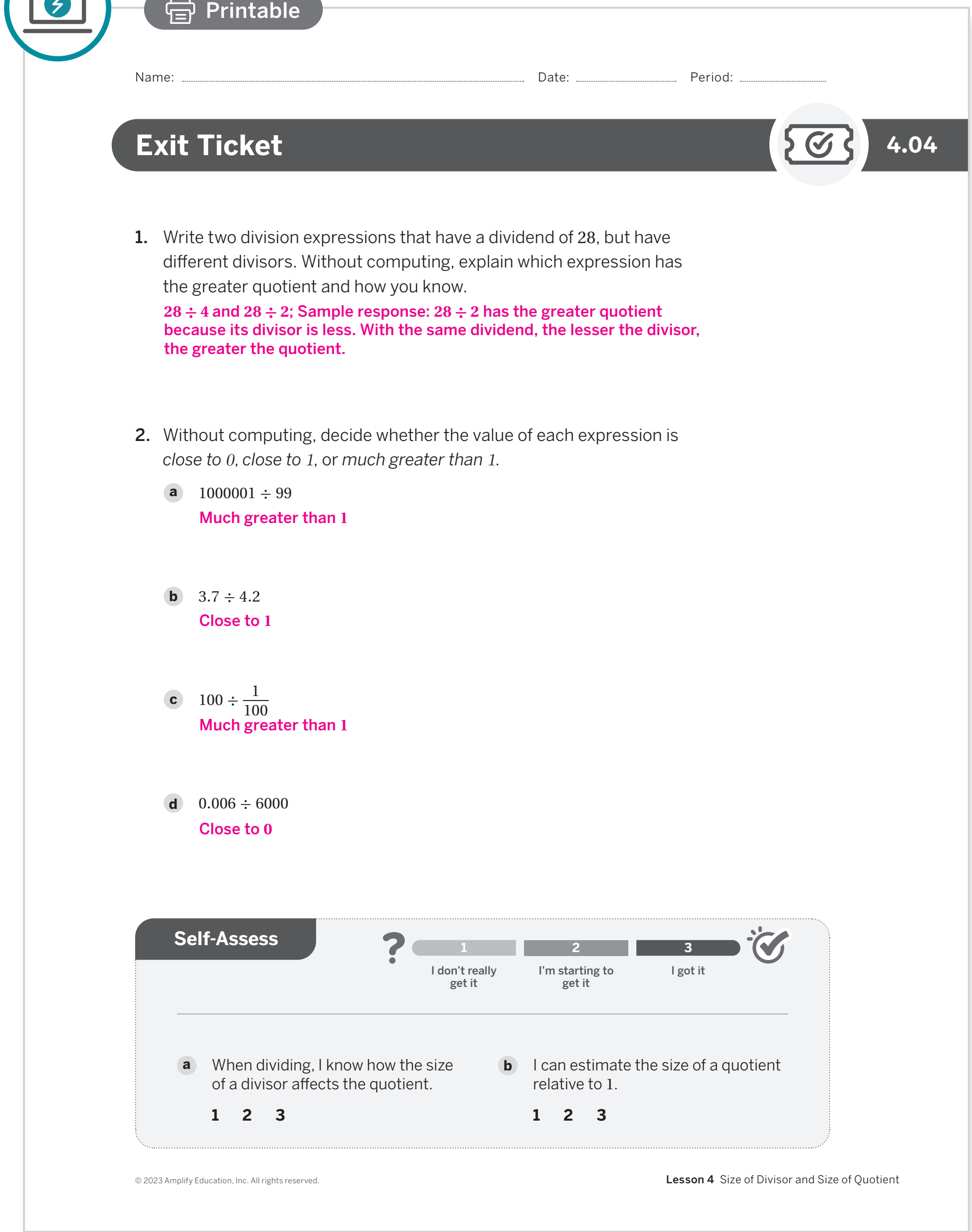
- "When estimating a quotient in an expression, what values and relationships are important to consider? Why?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of estimating quotients by evaluating different expressions.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket

4.04

1. Write two division expressions that have a dividend of 28, but have different divisors. Without computing, explain which expression has the greater quotient and how you know.

   28 ÷ 4 and 28 ÷ 2; Sample response: 28 ÷ 2 has the greater quotient because its divisor is less. With the same dividend, the lesser the divisor, the greater the quotient.

2. Without computing, decide whether the value of each expression is *close to 0*, *close to 1*, or *much greater than 1*.

   a. $1000001 \div 99$

   Much greater than 1

   b. $3.7 \div 4.2$

   Close to 1

   c. $100 \div \frac{1}{100}$

   Much greater than 1

   d. $0.006 \div 6000$

   Close to 0

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a. When dividing, I know how the size of a divisor affects the quotient.

1 2 3

b. I can estimate the size of a quotient relative to 1.

1 2 3

Lesson 4 Size of Divisor and Size of Quotient

Success looks like . . .

- **Language Goal:** Explaining how the sizes of dividends and divisors affect quotients with common values. **(Speaking and Listening, Writing)**
  - » Writing two different division expressions with a dividend of 28 and then comparing their quotients in Problem 1.
- **Goal:** Estimating quotients and generalizing how the dividend and divisor affect the size of a quotient relative to 1 (i.e., a number greater than 1, a fraction less than 1, or a value that is close to 1.
  - » Determining the value of a division expression in relation to 1 in Problem 2.

Suggested next steps

If students write expressions with divisors of 28 or cannot compare the relative size of the two quotients in Problem 1, consider:

- Reviewing the terms *dividend and divisor.*
- Having students list the factor pairs for 28 and choose one pair to use to write two division expressions. Then ask, "Which quotient is greater? What is true about its divisor compared to the other quotient?"
- Assigning Practice Problem 3.

If students cannot categorize the expressions in Problem 2 without computing, consider:

- Having them round the values and estimate the quotients mentally. Review strategies from Activity 2, Problem 2. Ask, "What do you know about the size of the quotient as it relates to the size of the dividend or the divisor?"
- Assigning Practice Problem 2.

Professional Learning

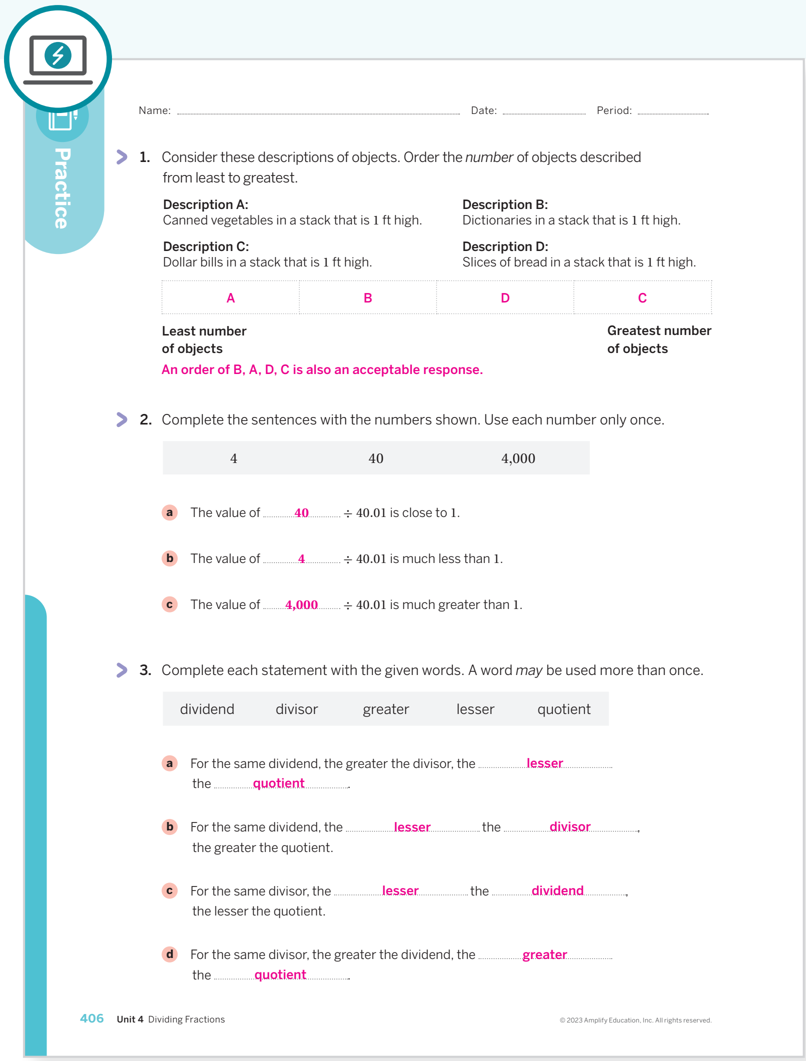
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The focus of this lesson was thinking about strategies to estimate quotients. How did it go?
- What challenges did students encounter as they did their estimations in Activity 2? How did they work through them and what might you keep and what might you change the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Consider these descriptions of objects. Order the *number* of objects described from least to greatest.

Description A:
Canned vegetables in a stack that is 1 ft high.

Description B:
Dictionaries in a stack that is 1 ft high.

Description C:
Dollar bills in a stack that is 1 ft high.

Description D:
Slices of bread in a stack that is 1 ft high.

A	B	D	C

Least number of objects **Greatest number of objects**

An order of B, A, D, C is also an acceptable response.

2. Complete the sentences with the numbers shown. Use each number only once.

4	40	4,000

a. The value of 40 ÷ 40.01 is close to 1.

b. The value of 4 ÷ 40.01 is much less than 1.

c. The value of 4,000 ÷ 40.01 is much greater than 1.

3. Complete each statement with the given words. A word *may* be used more than once.

dividend	divisor	greater	lesser	quotient

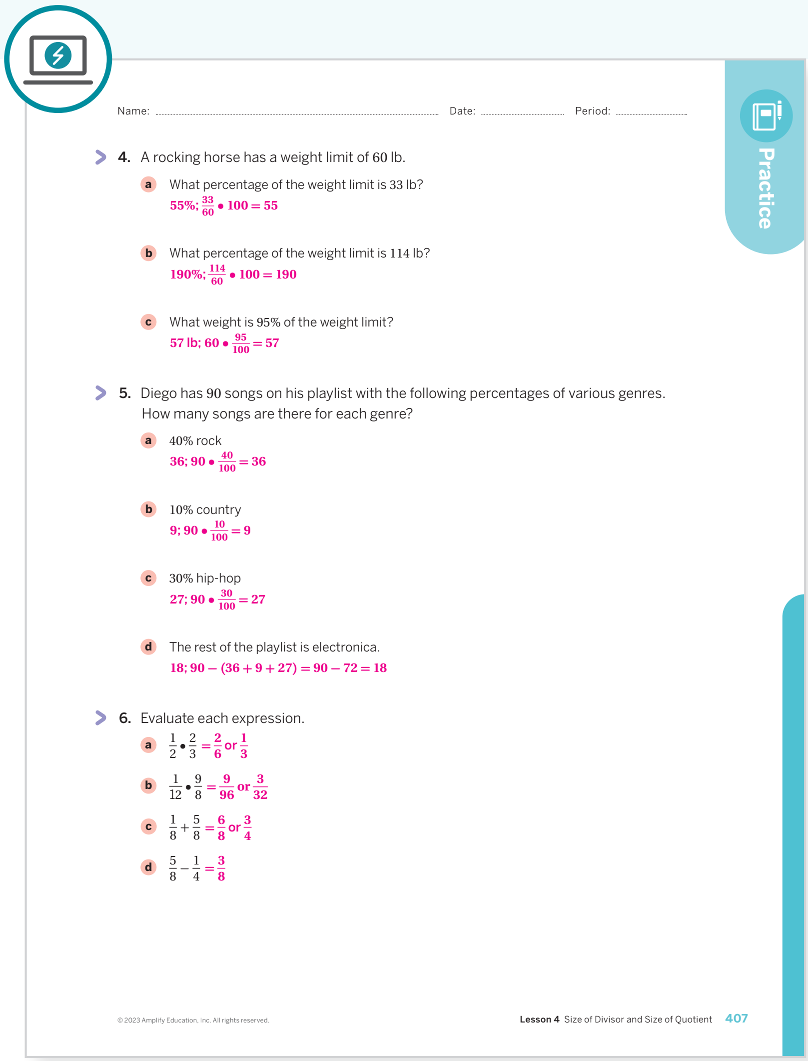
a. For the same dividend, the greater the divisor, the lesser the quotient.

b. For the same dividend, the lesser the divisor, the greater the quotient.

c. For the same divisor, the lesser the dividend, the lesser the quotient.

d. For the same divisor, the greater the dividend, the greater the quotient.

406 **Unit 4** Dividing Fractions



Practice

Name: Date: Period:

4. A rocking horse has a weight limit of 60 lb.

a. What percentage of the weight limit is 33 lb?
$55\%; \frac{33}{60} \bullet 100 = 55$

b. What percentage of the weight limit is 114 lb?
$190\%; \frac{114}{60} \bullet 100 = 190$

c. What weight is 95% of the weight limit?
$57 \text{ lb}; 60 \bullet \frac{95}{100} = 57$

5. Diego has 90 songs on his playlist with the following percentages of various genres. How many songs are there for each genre?

a. 40% rock
$36; 90 \bullet \frac{40}{100} = 36$

b. 10% country
$9; 90 \bullet \frac{10}{100} = 9$

c. 30% hip-hop
$27; 90 \bullet \frac{30}{100} = 27$

d. The rest of the playlist is electronica.
$18; 90 - (36 + 9 + 27) = 90 - 72 = 18$

6. Evaluate each expression.

a. $\frac{1}{2} \bullet \frac{2}{3} = \frac{2}{6} \text{ or } \frac{1}{3}$

b. $\frac{1}{12} \bullet \frac{9}{8} = \frac{9}{96} \text{ or } \frac{3}{32}$

c. $\frac{1}{8} + \frac{5}{8} = \frac{6}{8} \text{ or } \frac{3}{4}$

d. $\frac{5}{8} - \frac{1}{4} = \frac{3}{8}$

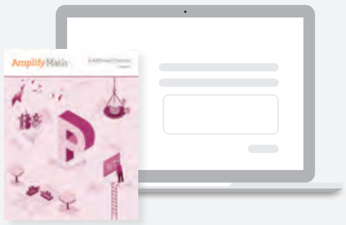
Lesson 4 Size of Divisor and Size of Quotient 407

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 9	2
	5	Unit 3 Lesson 13	2
Formative	6	Unit 4 Lesson 5	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 2

Division With Fractions

Whole Class

In this Sub-Unit, students explore both interpretations of division with any fractions in any position, and through models like tape diagrams, they develop general procedures for dividing fractions, including common denominators and the standard algorithm of multiplying by the reciprocal.

SUB-UNIT 2 Division With Fractions

Narrative Connections

Spöklik Furniture: The Showroom

"Excellent!," Martha says. "The Albees will *adore* this. You've been so wonderful — much more helpful than that girl with the ghastly yellow jacket!" *Yellow jacket . . .? Maya's jacket is yellow*!

"We passed her in the showroom," George says. "Just through there." He points to a set of doors. *Odd. That wasn't there before . . .* You race for the door.

Spöklik's showroom is as grand as it is confusing: a massive warehouse with model living rooms, bedrooms, kitchens, and bathrooms. Once, shoppers wandered through this maze of displays, looking for furniture or decoration ideas. Now, the place has a strange, lonely quality. Lamplight from the model rooms spills out into the wide aisles as you step quietly, searching for Maya.

Suddenly, there is a crash! Turning the corner, you find a ghostly woman in coveralls sitting in the aisle, surrounded by tools and furniture parts. Her name tag reads: SAMIRA, SPÖKLIK TEAM MEMBER.

'PICKLES!', she swears, scattering a pile of dowels with a kick. She looks up, noticing you. "Sorry. Didn't mean to scare you. It's just that I've been building this thing for a *lifetime* now . . ." She gestures to a loose pile of boards and you realize you have no idea what it's supposed to be — a bookshelf? A dresser? A bed? "Maybe if we put our heads together, I can finally get this thing built. All I need is a Number 42 serrated flange bolt with a struntprat stem."

Samira points to the picture on her instructions. "See? The stem needs to be *this* long. The problem is I can't make heads or tails of this number!"

How long is the bolt Samira needs?

Sub-Unit 2 Division With Fractions 409

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to divide fractions in problems involving the Spöklik Showroom in the following places:

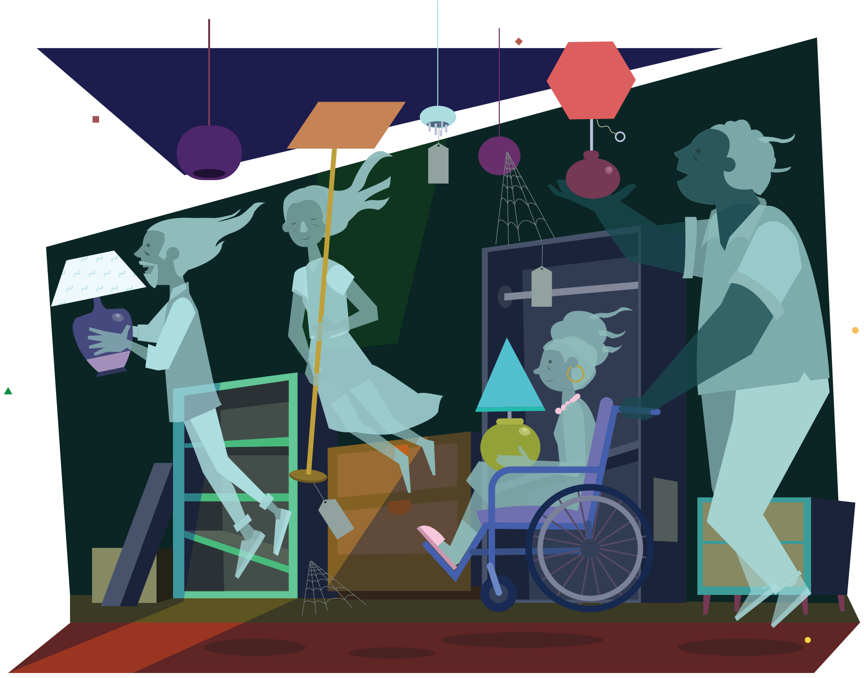
- **Lesson 9, Activity 1:** Reupholstering a Chair
- **Lesson 12, Activity 2:** Using Related Quotients

Highlight the question posed in the narrative and consider having a brief discussion to ensure all students understand the question. While some students may be able to determine an answer now, they should all be equipped to answer it by the end of the sub-unit. Allow students to submit responses privately at any point during the next several class sessions, and then hold a discussion after Lesson 12.

UNIT 4 | **LESSON 5**

How Many Groups?

Let's use blocks and diagrams to think about division with fractions.



Focus

Goals

1. **Language Goal:** Use pattern blocks or diagrams to represent and to solve multiplication equations in which the size of a group is not a whole number. **(Writing)**
2. **Language Goal:** Create a diagram to represent and solve a problem asking, "how many groups?" in which the divisor is a unit fraction, and explain the solution method. **(Speaking and Listening, Writing)**

Coherence

Today

Students begin a series of three lessons that focus on the "how many groups?" (quotitive) interpretation of division from the earlier foundational lessons, but now involving non-unit fractions. In this lesson, the shift from whole numbers and unit fractions begins when the "size of the group" (the divisor) takes on non-unit fractional values. Students can reorient to fractions and relationships using pattern blocks in an optional activity first, or they can go straight to applying that reasoning with pattern blocks to represent multiplication and division equations and to answer questions of the form, "How many of this fraction are there in this other number?"

Previously

In Lessons 2–4, students reviewed the two interpretations of division and several relationships among the values in division and multiplication equations, mostly with whole numbers and unit fractions.

Coming Soon

In Lesson 6, students will continue to work with quotitive division situations, and will leverage tape diagrams and other models to explore similar "how many groups?" questions, but with non-unit fraction dividends and quotients as well.

Rigor

- Students use pattern blocks to build **conceptual understanding** of multiplication and division with fractions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1 (optional)	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one page per pair
- Activity 1 PDF (answers)
- *Pattern Blocks* PDF (optional)
- *pattern blocks*

Amps | Featured Activity

Activity 1
Interactive Pattern Blocks

Students use interactive pattern blocks to compare fractional parts of shapes.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might impulsively assign values to each pattern block in Activity 2. Remind students that each shape represents an equivalent fraction and have them look closely at the pattern to discern the value that each block represents. Students may want to think aloud as they complete Problem 2 so they can hear how values are repeated and develop a strategy for using pattern blocks to visualize the problem.

Modifications to Pacing

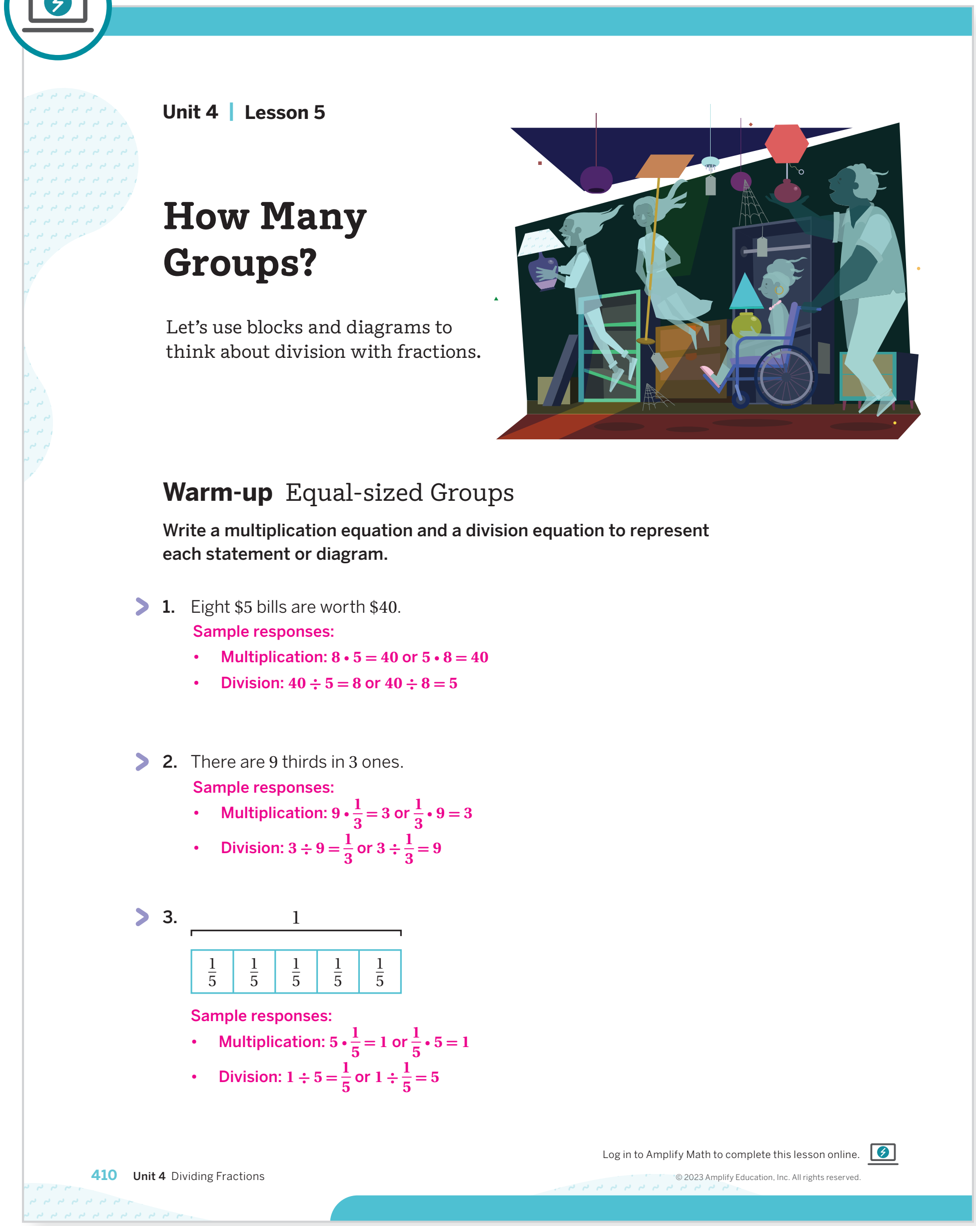
You may want to consider these additional modifications if you are short on time.

- Optional **Activity 1** may be skipped.
- In **Activity 2**, Problems 1a and 2a may be omitted.

Independent | 5 min

Warm-up Equal-sized Groups

Students activate prior knowledge by writing multiplication and division equations to represent statements and diagrams, which include some fractional values.



Unit 4 | Lesson 5

How Many Groups?

Let's use blocks and diagrams to think about division with fractions.

Warm-up Equal-sized Groups

Write a multiplication equation and a division equation to represent each statement or diagram.

1. Eight \$5 bills are worth \$40.
   Sample responses:
   - **Multiplication: $8 \cdot 5 = 40$ or $5 \cdot 8 = 40$**
   - **Division: $40 \div 5 = 8$ or $40 \div 8 = 5$**

2. There are 9 thirds in 3 ones.
   Sample responses:
   - **Multiplication: $9 \cdot \frac{1}{3} = 3$ or $\frac{1}{3} \cdot 9 = 3$**
   - **Division: $3 \div 9 = \frac{1}{3}$ or $3 \div \frac{1}{3} = 9$**

3.

   Sample responses:
   - **Multiplication: $5 \cdot \frac{1}{5} = 1$ or $\frac{1}{5} \cdot 5 = 1$**
   - **Division: $1 \div 5 = \frac{1}{5}$ or $1 \div \frac{1}{5} = 5$**

Log in to Amplify Math to complete this lesson online.

1 Launch

Set an expectation for the amount of time students will have to work independently on the activity.

2 Monitor

Help students get started by asking, "What three numbers are represented? Can you relate those by using multiplication? By using division?"

Look for points of confusion:

- **Struggling to frame repeated addition as multiplication.** Refer to one of their equivalent addition statements and ask, "How many same-sized groups are being added?"
- **Unable to identify the 5 in Problem 3.** Ask students to write any equation they can for the diagram.

Look for productive strategies:

- Writing addition equations first, and then translating those to multiplication equations.
- Using the relationship between multiplication and division to write a division equation.

3 Connect

Have individual students share their multiplication and division equations. Show at least one multiplication and one division equation for every problem.

Ask, "Why are there 4, and only 4, total equations using the two operations possible for every problem?"

Highlight that multiplication and division can both be used to represent equal-sized groups scenarios, and the relationships among the values and the equations are still the same, even when some of the values are fractions.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students first create an equation to represent each statement or diagram, and then think about how they can use their addition equations to write multiplication equations, and lastly, division equations.

Extension: *Math Enrichment*

Challenge students to draw their own tape diagrams (that illustrate repeated addition, multiplication, or division) and trade them with a partner. Each partner should write a multiplication equation and division equation to represent the diagram.

Power-up

To power up students' ability to evaluate expressions with fractions, have students complete:

Recall that when adding and subtracting fractions, the fractions need to have the same denominators. When multiplying fractions they do not.

Simplify each expression.

1. $\frac{2}{3} + \frac{1}{6} = \frac{5}{6}$
2. $\frac{2}{3} - \frac{1}{6} = \frac{1}{2}$ or equivalent
3. $\frac{2}{3} \cdot \frac{1}{6} = \frac{1}{9}$ or equivalent

Use: Before Activity 2.

Informed by: Performance on Lesson 4, Practice Problems 6 and Pre-Unit Readiness Assessment, Problem 6.

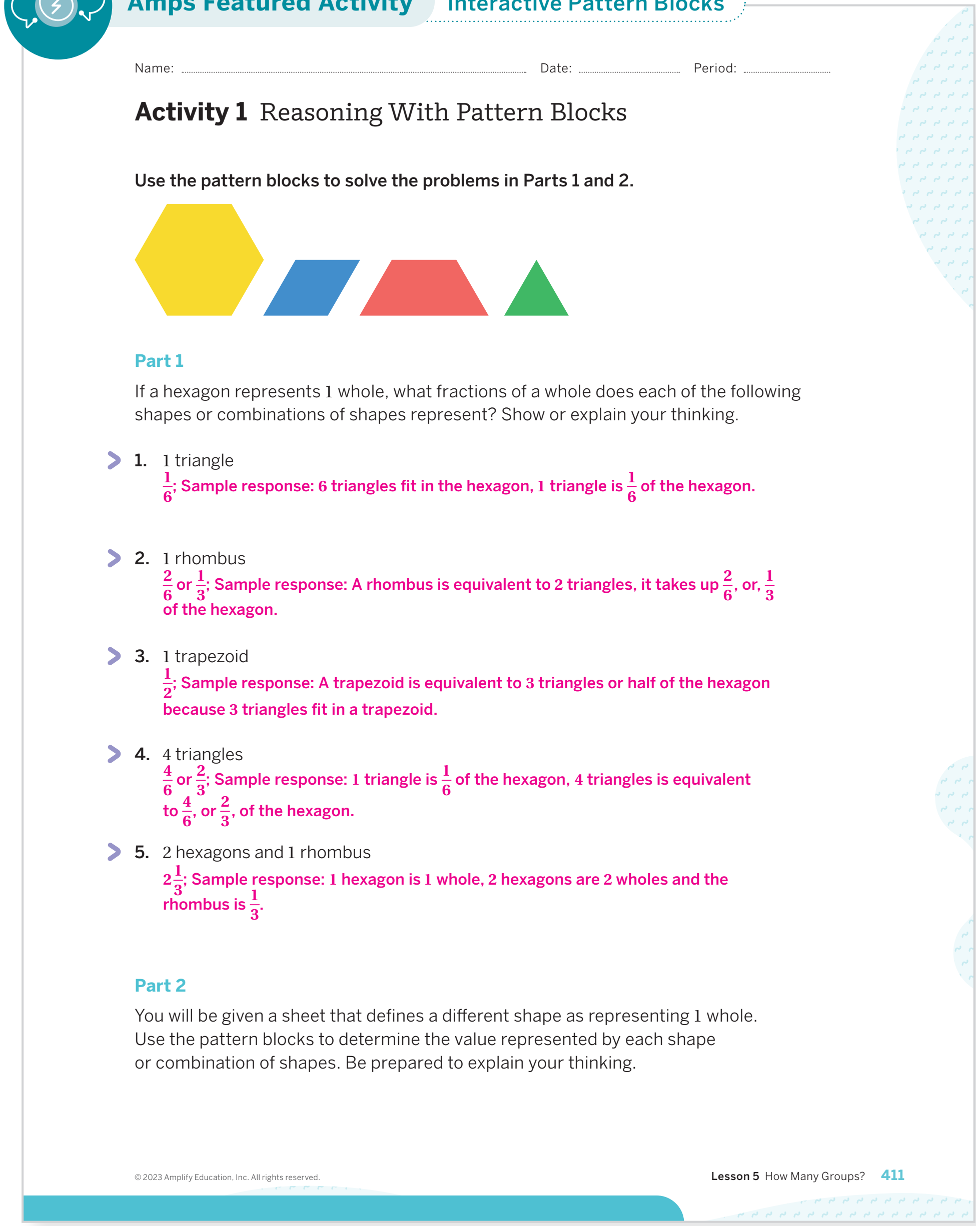
Optional

Pairs | 15 min

Activity 1 Reasoning With Pattern Blocks

Students use the areas of geometric shapes to begin to review equipartitioning and fractions, establishing a concrete basis for "how many groups" division problems.

Amps Featured Activity Interactive Pattern Blocks



Name: ______ Date: ______ Period: ______

Activity 1 Reasoning With Pattern Blocks

Use the pattern blocks to solve the problems in Parts 1 and 2.

Part 1

If a hexagon represents 1 whole, what fractions of a whole does each of the following shapes or combinations of shapes represent? Show or explain your thinking.

1. 1 triangle
   $\frac{1}{6}$; Sample response: 6 triangles fit in the hexagon, 1 triangle is $\frac{1}{6}$ of the hexagon.
2. 1 rhombus
   $\frac{2}{6}$ or $\frac{1}{3}$; Sample response: A rhombus is equivalent to 2 triangles, it takes up $\frac{2}{6}$, or, $\frac{1}{3}$ of the hexagon.
3. 1 trapezoid
   $\frac{1}{2}$; Sample response: A trapezoid is equivalent to 3 triangles or half of the hexagon because 3 triangles fit in a trapezoid.
4. 4 triangles
   $\frac{4}{6}$ or $\frac{2}{3}$; Sample response: 1 triangle is $\frac{1}{6}$ of the hexagon, 4 triangles is equivalent to $\frac{4}{6}$, or $\frac{2}{3}$, of the hexagon.
5. 2 hexagons and 1 rhombus
   $2\frac{1}{3}$; Sample response: 1 hexagon is 1 whole, 2 hexagons are 2 wholes and the rhombus is $\frac{1}{3}$.

Part 2

You will be given a sheet that defines a different shape as representing 1 whole. Use the pattern blocks to determine the value represented by each shape or combination of shapes. Be prepared to explain your thinking.

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Lesson 5 How Many Groups? 411

1 Launch

Have students work in pairs on Part 1 before sharing their strategies and answers with the class.Then give each pair one page from the Activity 1 PDF. Give pairs a few minutes to work on Part 2, and then have students use the *Gallery Tour* routine to view other groups' work.

2 Monitor

Help students get started by asking, "How many triangles can fit on top of the hexagon? How can you represent this as a fraction?"

Look for points of confusion:

- **Struggling to identify the fraction when the reference shape in Part 2 cannot tile another given shape.** Ask, "Is there a different shape you could use to cover both of these? How would that help you identify the fraction?"

Look for productive strategies:

- Laying the smaller shapes on top of the larger shapes to determine the fraction as parts relative to whole.
- Using other relationships among the shapes, such as always referencing triangles, to determine new fractions.

3 Connect

Display students' work from the *Gallery Tour* for them to reference.

Have partners share strategies they leveraged from Part 1 to work with their shape in Part 2, as well as any different strategies they needed.

Highlight that, in each case, the value associated with a shape or combination of shapes, which is the part, is always relative to the size of the shape defined as 1 whole. The shape representing the part is also the size of a group, and determining its value is the same as determining "how many groups" are needed to create an area of 1.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Assign students the *Trapezoid* page of the Activity 1 PDF.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use interactive pattern blocks to compare fractional parts of shapes.

Extension: *Math Enrichment*

Have students describe how they could use pattern blocks to represent the fraction $\frac{4}{6}$. Sample responses:

- 2 rhombuses if the hexagon represents 1 whole
- 4 triangles if the trapezoid represents 1 whole

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display the following incorrect statement that reflects a possible common misunderstanding. Then ask the following questions.

"The fraction for the rhombus is 3 because 3 rhombuses fit inside the hexagon."

Critique: "Do you agree with this statement? Why or why not?"

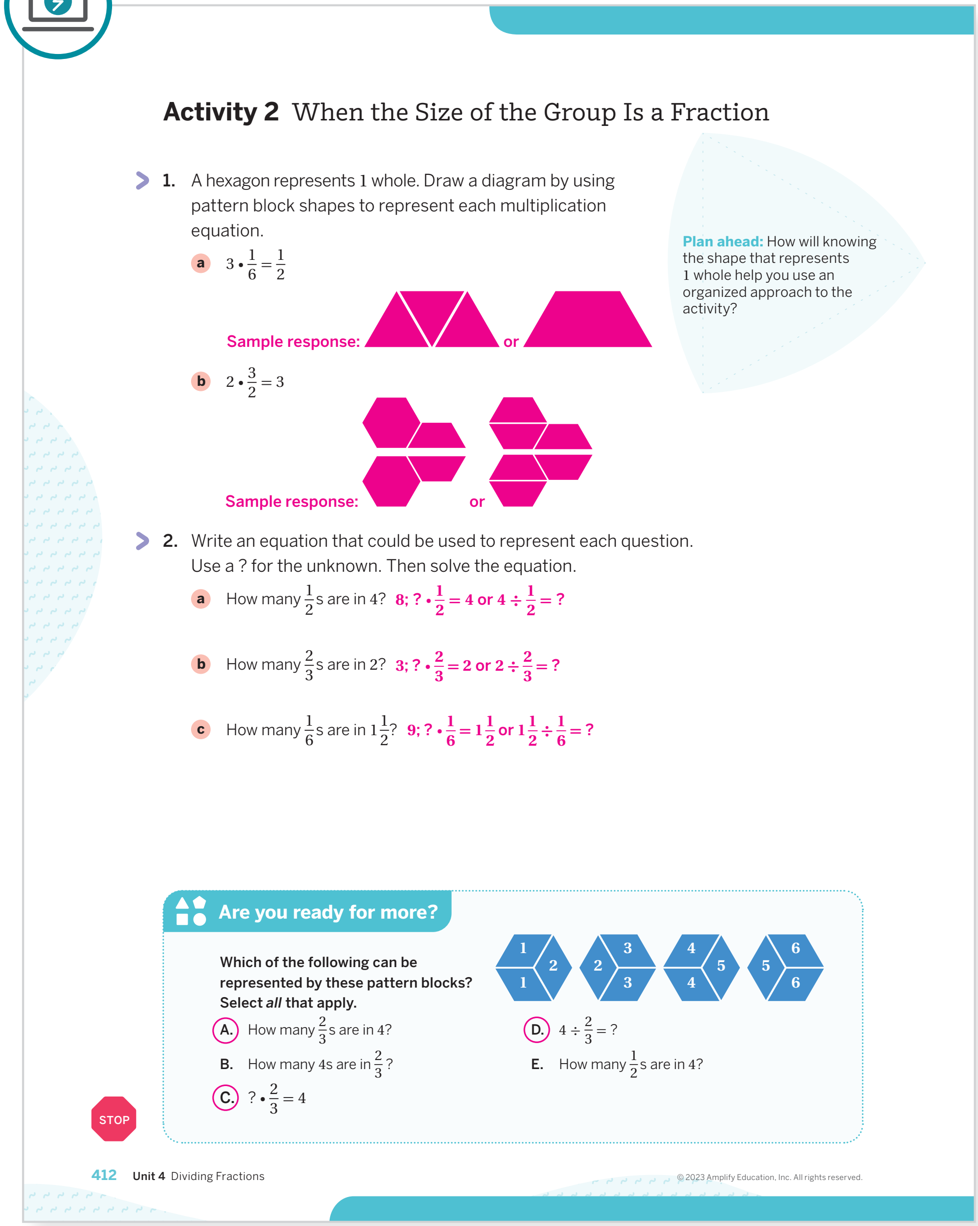
Correct and Clarify: "How would you correct or further clarify this statement?"

Listen for students who recognize that the hexagon represents 1 whole, and the rhombus is smaller than the hexagon. This means the fraction representing the rhombus is less than 1.

Pairs | 15 min

Activity 2 When the Size of the Group Is a Fraction

Students use pattern blocks to represent multiplication and division equations related to a number of equal-sized groups that have fractional values.



Activity 2 When the Size of the Group Is a Fraction

1. A hexagon represents 1 whole. Draw a diagram by using pattern block shapes to represent each multiplication equation.

   a $3 \cdot \frac{1}{6} = \frac{1}{2}$

   Sample response: or

   b $2 \cdot \frac{3}{2} = 3$

   Sample response: or

Plan ahead: How will knowing the shape that represents 1 whole help you use an organized approach to the activity?

2. Write an equation that could be used to represent each question. Use a ? for the unknown. Then solve the equation.

   a How many $\frac{1}{2}$s are in 4? **8; $? \cdot \frac{1}{2} = 4$ or $4 \div \frac{1}{2} = ?$**

   b How many $\frac{2}{3}$s are in 2? **3; $? \cdot \frac{2}{3} = 2$ or $2 \div \frac{2}{3} = ?$**

   c How many $\frac{1}{6}$s are in $1\frac{1}{2}$? **9; $? \cdot \frac{1}{6} = 1\frac{1}{2}$ or $1\frac{1}{2} \div \frac{1}{6} = ?$**

Are you ready for more?

Which of the following can be represented by these pattern blocks? Select *all* that apply.

[✓ A.] How many $\frac{2}{3}$s are in 4?

B. How many 4s are in $\frac{2}{3}$?

[✓ C.] $? \cdot \frac{2}{3} = 4$

[✓ D.] $4 \div \frac{2}{3} = ?$

E. How many $\frac{1}{2}$s are in 4?

STOP

412 Unit 4 Dividing Fractions

1 Launch

Have students use the *Think-Pair-Share* routine. Provide them 3 minutes of individual work time for Problem 1. Then have partners compare diagrams and complete Problem 2 together.

2 Monitor

Help students get started by asking, "Can you read the equations in Problems 1a and 1b by using words, such as *equal groups*?"

Look for points of confusion:

- **Struggling to represent $\frac{3}{2}$ in Problem 1b.** Ask, "What represents $\frac{1}{2}$ if the hexagon is 1 whole? So what would $\frac{3}{2}$ look like?"

Look for productive strategies:

- Identifying and using shapes that represent equivalent fractions (e.g., 1 trapezoid and 3 triangles).
- Making copies of the fraction factor or divisor as the size of a group, as in repeated addition, until reaching the target value for either number of groups or total.
- Recognizing that questions like those in Problem 2 can be represented as division or missing factor multiplication, and can be solved using pattern blocks or fraction arithmetic.

3 Connect

Display pattern blocks for reference and recreate student representations for discussion.

Have pairs of students share their diagrams for Problem 1, drawing attention to any equivalent fractions. Then have pairs share their equations and strategies for solving Problem 2, including using pattern blocks.

Highlight that multiplication can be interpreted as a number of groups of a certain size, which also corresponds to dividing a total into a number of groups of that size. Pattern blocks are helpful in visualizing groups that include some fractions of a whole.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Allow students to use pattern blocks or copies of pattern blocks to help them make sense of each question in Problem 2.

Accessibility: *Guide Processing and Visualization*

Demonstrate how the equation in Problem 1 can be written as a question, "How many equal groups of $\frac{1}{6}$ are in $\frac{1}{2}$?" Show how the answer to that question is the factor 3. Display this question, the equation, and a corresponding diagram for students to reference as they complete activity.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their diagrams for Problem 1, draw their attention to connections between the equations and the pattern block diagrams. Ask them how they could write the equations in words, such as, "How many equal groups of _____ are in _____?" Then as they share their responses to Problem 2, ask them how the structure of the questions helped them write the equations.

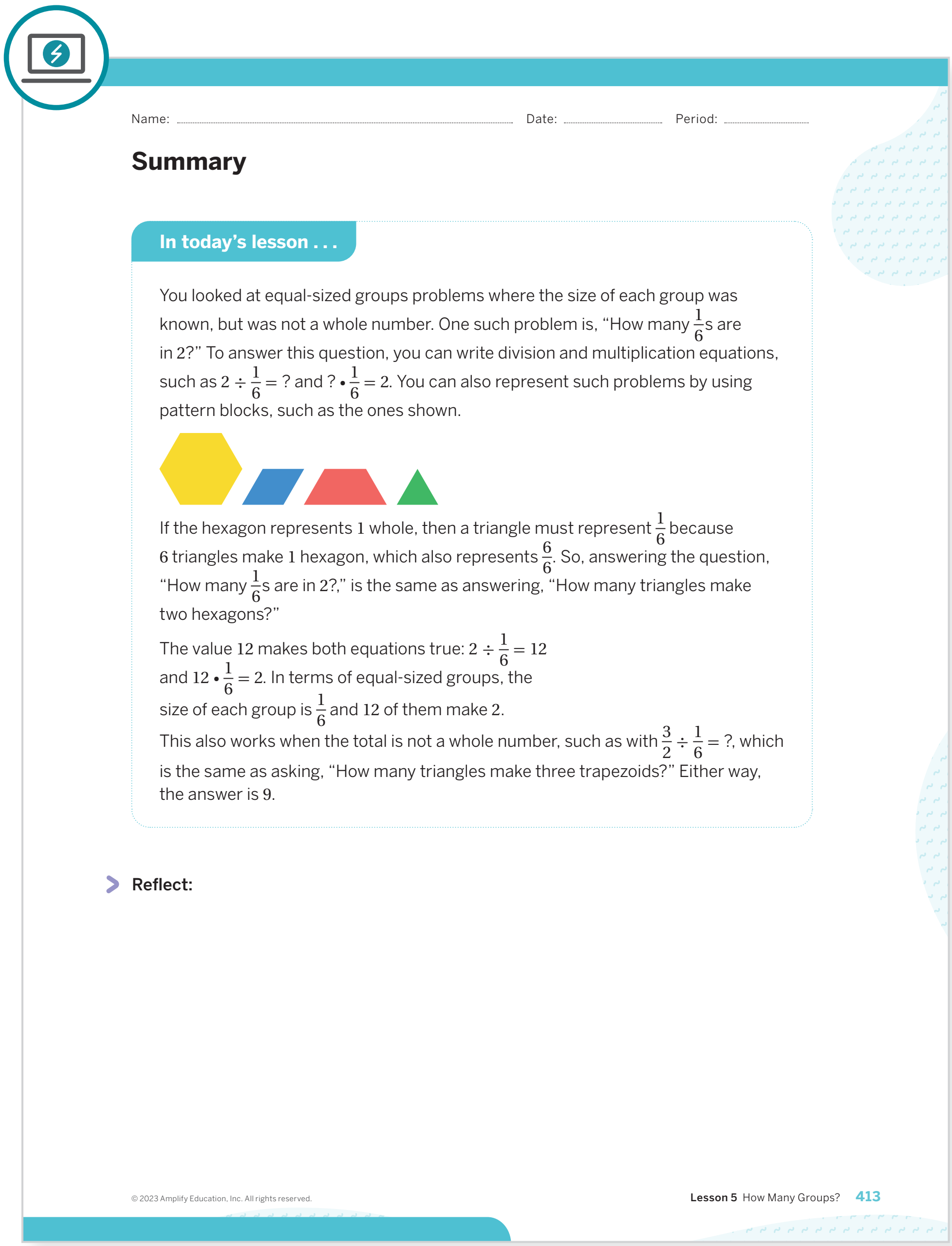
English Learners

Consider annotating the question "How many equal groups of _____ are in _____?" with the terms *factor, product, dividend, divisor,* and *quotient*.

Whole Class | 5 min

Summary

Review and synthesize how the relationship between multiplication and division can be used to solve equal-sized groups problems with groups of fractions.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You looked at equal-sized groups problems where the size of each group was known, but was not a whole number. One such problem is, "How many $\frac{1}{6}$s are in 2?" To answer this question, you can write division and multiplication equations, such as $2 \div \frac{1}{6} = ?$ and $? \cdot \frac{1}{6} = 2$. You can also represent such problems by using pattern blocks, such as the ones shown.

If the hexagon represents 1 whole, then a triangle must represent $\frac{1}{6}$ because 6 triangles make 1 hexagon, which also represents $\frac{6}{6}$. So, answering the question, "How many $\frac{1}{6}$s are in 2?," is the same as answering, "How many triangles make two hexagons?"

The value 12 makes both equations true: $2 \div \frac{1}{6} = 12$ and $12 \cdot \frac{1}{6} = 2$. In terms of equal-sized groups, the size of each group is $\frac{1}{6}$ and 12 of them make 2.

This also works when the total is not a whole number, such as with $\frac{3}{2} \div \frac{1}{6} = ?$, which is the same as asking, "How many triangles make three trapezoids?" Either way, the answer is 9.

Reflect:

Synthesize

Ask:

- "What fractions can be represented with a standard set of pattern blocks?"
- "What is a fraction that cannot be represented with a standard set of pattern blocks? What would be a good model for representing an equal-sized groups problem where that fraction is the size of a group?"

Have students share responses and draw diagrams to the questions. If time allows, write a problem by using a suggested fraction as a class and represent its solution using the suggested model.

Highlight that how multiplication and division are used to model and solve equal sized group problems does not change even when the group size or the whole are fractions. The relationship between multiplication and division also remains the same when working with fractions.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

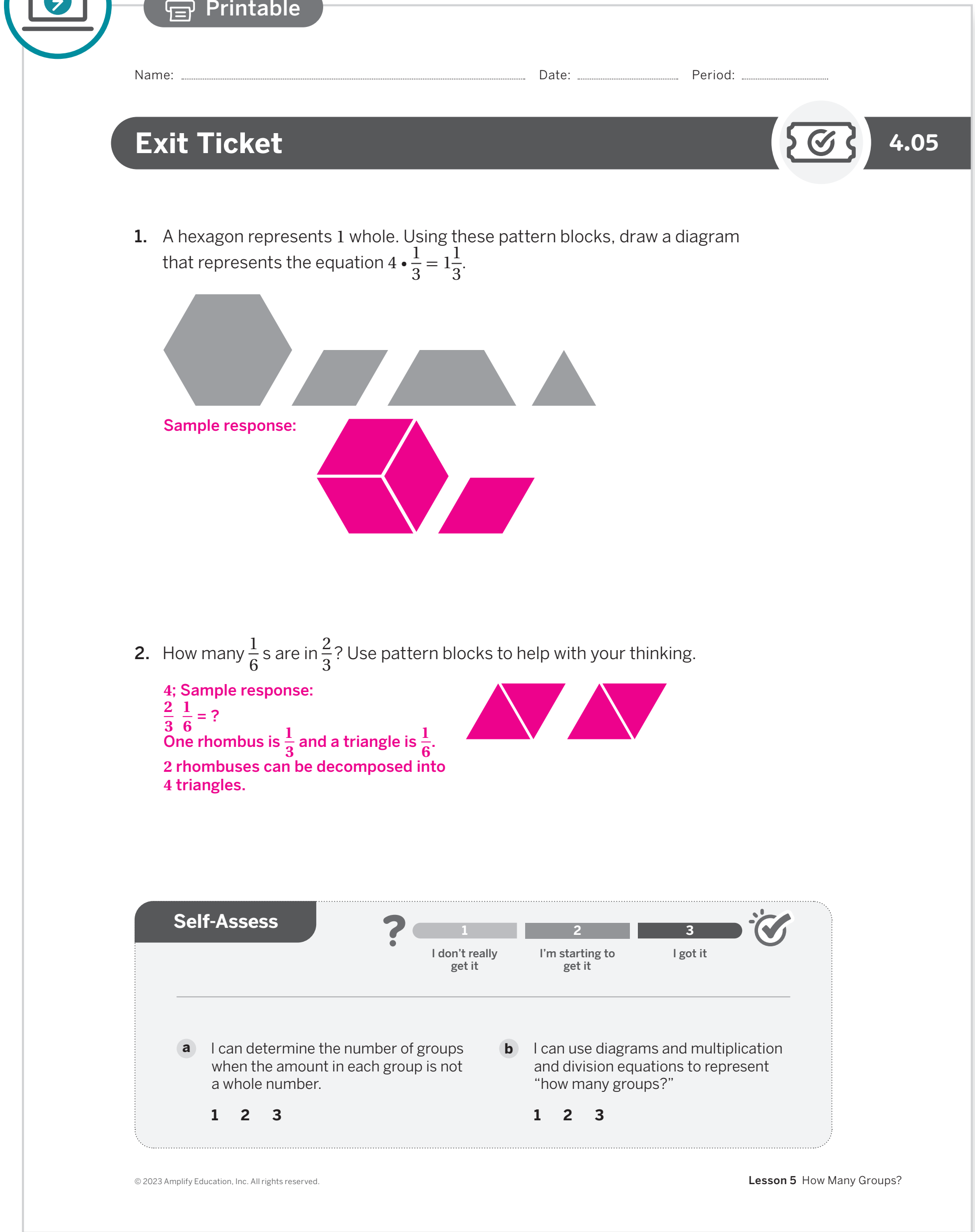
- "How did you use pattern blocks to think about fractions? About multiplication? About division?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of multiplication and division with fractions using pattern blocks to help guide their thinking.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 4.05

1. A hexagon represents 1 whole. Using these pattern blocks, draw a diagram that represents the equation $4 \cdot \frac{1}{3} = 1\frac{1}{3}$.

Sample response:

2. How many $\frac{1}{6}$s are in $\frac{2}{3}$? Use pattern blocks to help with your thinking.

4; Sample response:
$\frac{2}{3}\ \frac{1}{6} = ?$
One rhombus is $\frac{1}{3}$ and a triangle is $\frac{1}{6}$.
2 rhombuses can be decomposed into 4 triangles.

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can determine the number of groups when the amount in each group is not a whole number.
1 2 3

b I can use diagrams and multiplication and division equations to represent "how many groups?"
1 2 3

Success looks like . . .

- **Language Goal:** Using pattern blocks or diagrams to represent and solve multiplication equations in which the size of a group is not a whole number. **(Writing)**
  - » Drawing a diagram of pattern blocks to represent a multiplication equation in Problem 1.
- **Language Goal:** Creating a diagram to represent and solve a problem asking, "how many groups?" in which the divisor is a unit fraction, and explaining the solution method. **(Speaking and Listening, Writing)**
  - » Using pattern blocks to determine how many groups of $\frac{1}{6}$s are in $\frac{2}{3}$ in Problem 2.

Suggested next steps

If students do not draw a diagram that shows 4 groups of $\frac{1}{3}$ in Problem 1, consider:

- Reviewing Problem 1 from Activity 2.
- Asking them to read the equation by using words, such as *equal groups*, and to identify the shape that corresponds to the size of a group.

If students cannot determine the solution for Problem 2, consider:

- Reviewing the Problem 2 from Activity 2.
- Asking, "How many $\frac{1}{6}$s are in $\frac{1}{3}$? How can that help your thinking? Can you draw a diagram using pattern blocks?"
- Assigning Practice Problem 1.

Professional Learning

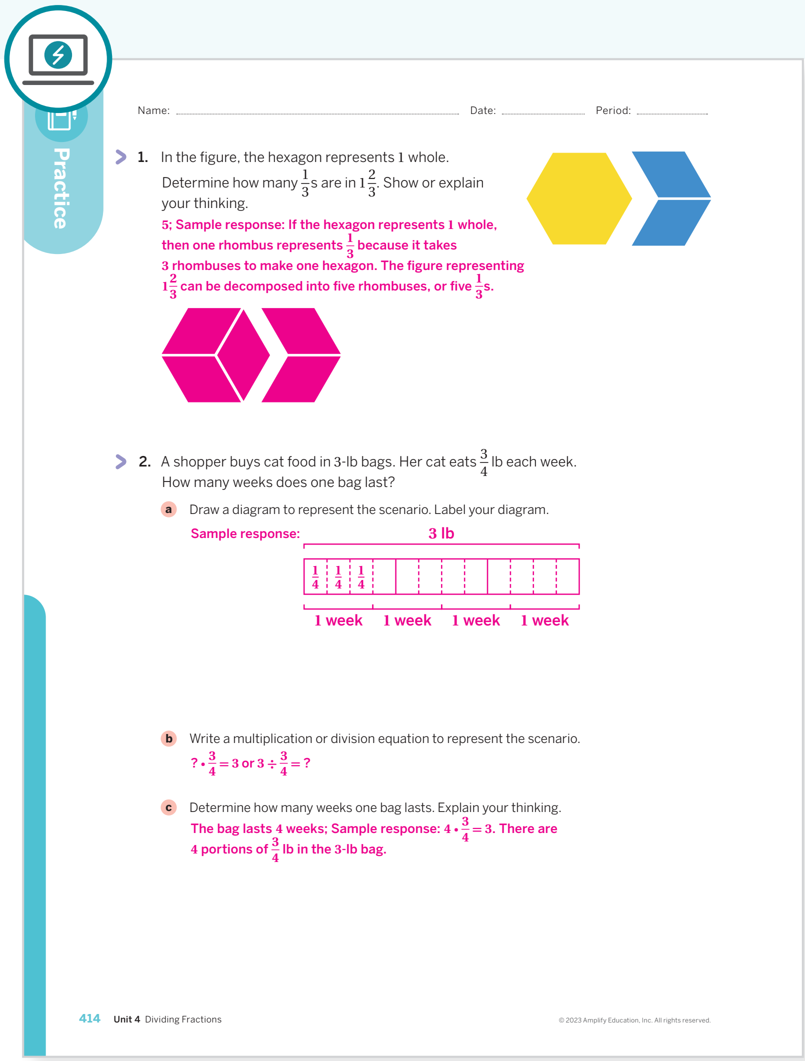
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did reasoning about division with fractions and pattern blocks reveal about your students as learners?
- What trends do you see in participation? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. In the figure, the hexagon represents 1 whole.

   Determine how many $\frac{1}{3}$s are in $1\frac{2}{3}$. Show or explain your thinking.

   5; Sample response: If the hexagon represents 1 whole, then one rhombus represents $\frac{1}{3}$ because it takes 3 rhombuses to make one hexagon. The figure representing $1\frac{2}{3}$ can be decomposed into five rhombuses, or five $\frac{1}{3}$s.

2. A shopper buys cat food in 3-lb bags. Her cat eats $\frac{3}{4}$ lb each week. How many weeks does one bag last?

   a. Draw a diagram to represent the scenario. Label your diagram.

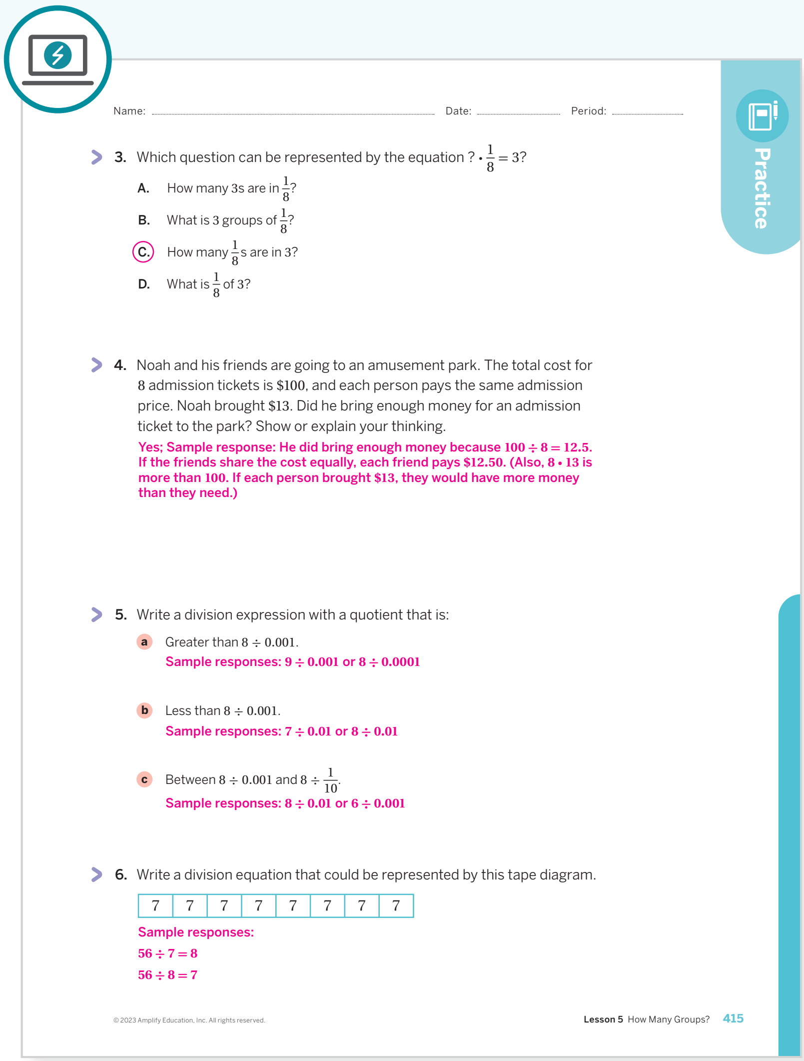
   Sample response:

   b. Write a multiplication or division equation to represent the scenario.

   $? \cdot \frac{3}{4} = 3$ or $3 \div \frac{3}{4} = ?$

   c. Determine how many weeks one bag lasts. Explain your thinking.

   The bag lasts 4 weeks; Sample response: $4 \cdot \frac{3}{4} = 3$. There are 4 portions of $\frac{3}{4}$ lb in the 3-lb bag.



Name: Date: Period:

3. Which question can be represented by the equation $? \cdot \frac{1}{8} = 3$?

   A. How many 3s are in $\frac{1}{8}$?

   B. What is 3 groups of $\frac{1}{8}$?

   [✓ C.] How many $\frac{1}{8}$s are in 3?

   D. What is $\frac{1}{8}$ of 3?

4. Noah and his friends are going to an amusement park. The total cost for 8 admission tickets is \$100, and each person pays the same admission price. Noah brought \$13. Did he bring enough money for an admission ticket to the park? Show or explain your thinking.

   Yes; Sample response: He did bring enough money because $100 \div 8 = 12.5$. If the friends share the cost equally, each friend pays \$12.50. (Also, $8 \cdot 13$ is more than 100. If each person brought \$13, they would have more money than they need.)

5. Write a division expression with a quotient that is:

   a. Greater than $8 \div 0.001$.

   Sample responses: $9 \div 0.001$ or $8 \div 0.0001$

   b. Less than $8 \div 0.001$.

   Sample responses: $7 \div 0.01$ or $8 \div 0.01$

   c. Between $8 \div 0.001$ and $8 \div \frac{1}{10}$.

   Sample responses: $8 \div 0.01$ or $6 \div 0.001$

6. Write a division equation that could be represented by this tape diagram.

7	7	7	7	7	7	7	7

   Sample responses:

   $56 \div 7 = 8$

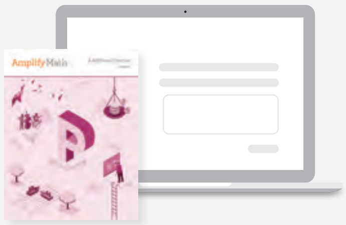
   $56 \div 8 = 7$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 2	2
	5	Unit 4 Lesson 4	2
Formative ⚡	6	Unit 4 Lesson 6	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 4 | **LESSON 6**

Using Diagrams to Determine the Number of Groups

Let’s use blocks and diagrams to understand more about division with fractions.

Focus

Goals

1. **Language Goal:** Coordinate multiplication and division equations with pattern block diagrams in which a different shape represents one whole. **(Writing)**
2. **Language Goal:** Create a diagram to represent and solve a problem asking, “how many groups?” in which the divisor is a non-unit fraction, and explain the solution method. **(Speaking and Listening, Writing)**
3. **Language Goal:** Identify or generate a multiplication or division equation that represents a given situation involving a fractional divisor. **(Writing)**

Coherence

Today

Students continue to work with quotitive division situations that ask “how many groups?” but are presented as “how many of this in that?” Unlike the previous lesson, sometimes the quotient is no longer a whole number. Students use pattern blocks again to help them first identify the size of a group (the divisor), recognizing also the importance of identifying what represents a whole (or a value of 1). Next, they coordinate those with a target value (the dividend) to write division expressions that represent the related quotient. Students then use tape diagrams and other models to help with their thinking about these types of questions and determining quotients and “naming” the result as a fraction relative to the size of a group.

Previously

In Lesson 5, students represented “how many groups?” (quotitive) division problems involving fractions by using pattern blocks and equations.

Coming Soon

In Lesson 7, students will use common denominators to generalize a division strategy that compares the sizes of the dividend and divisor relative to the same unit fraction.

Rigor

- Students draw and use tape diagrams to build **conceptual understanding** of division with fractions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

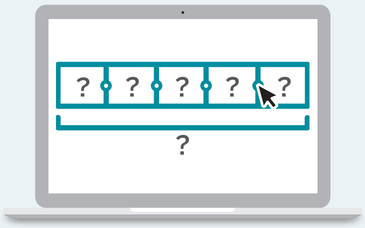
Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, *Fraction Strips*
- *Pattern Blocks* PDF (optional)
- *Tape Diagrams* PDF (as needed)
- pattern blocks

Amps | **Featured Activity**

Activity 2
Digital Diagrams

Students can sketch on digital tape diagrams to solve problems. You can overlay them all to see similarities and differences at a glance.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might not want to take the time to draw a precise diagram in Activity 2. Explain that the diagrams can help them make sense of the problems and check to see if their answer is reasonable. Precision helps their work be more consistent, allowing their results to be more successful.

Modifications to Pacing

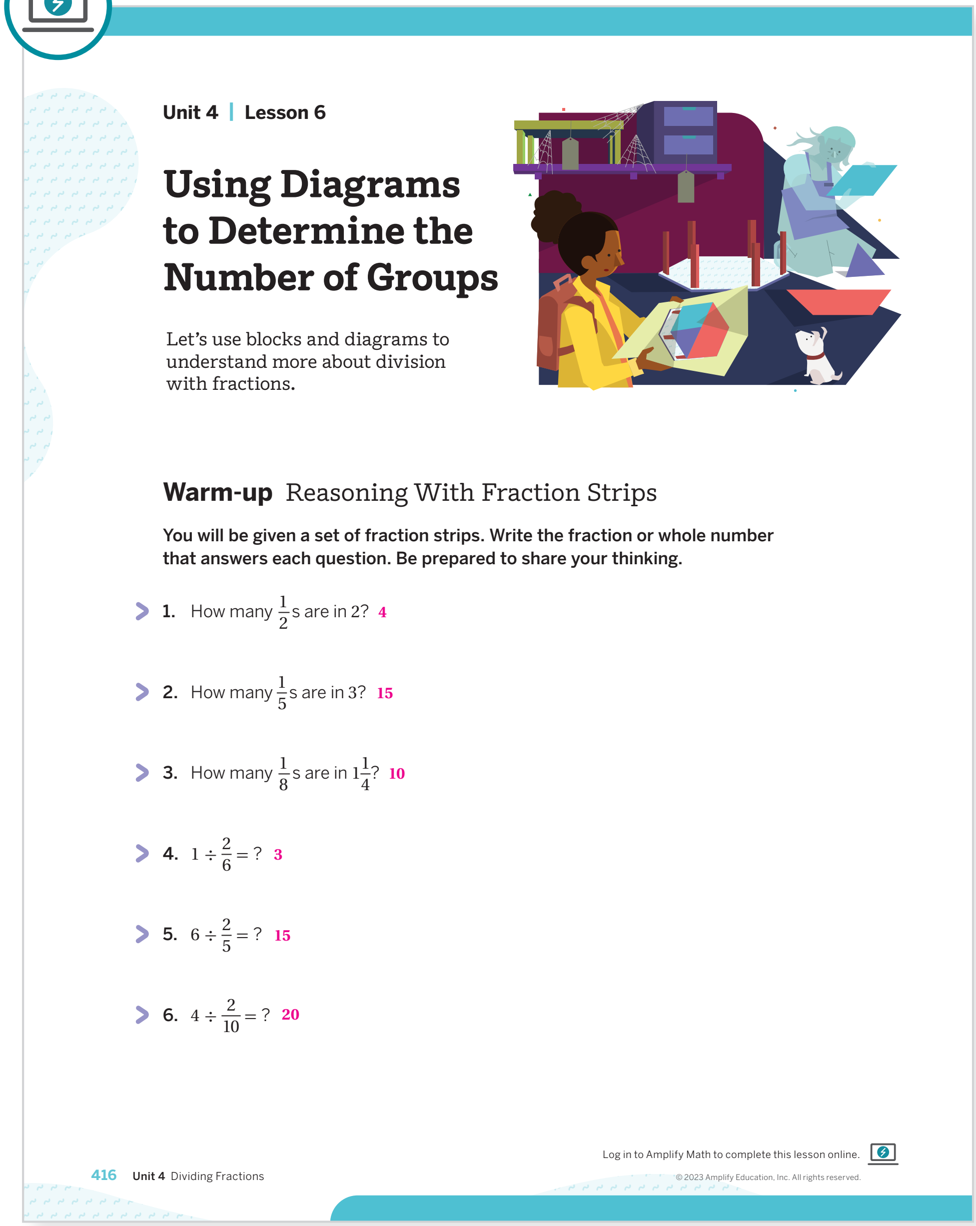
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problems 2, 4, and 6 may be omitted.
- **Activity 1** may be done as a whole class.
- In **Activity 2**, assign only one of Problems 2a–c to each pair (**Note:** these are ordered from most to least accessible).

Pairs | 10 min

Warm-up Reasoning With Fraction Strips

Students use fraction strips to reason about division in terms of equal-sized groups, and as being related to questions about "how many of this in that?"



Unit 4 | Lesson 6

Using Diagrams to Determine the Number of Groups

Let's use blocks and diagrams to understand more about division with fractions.

Warm-up Reasoning With Fraction Strips

You will be given a set of fraction strips. Write the fraction or whole number that answers each question. Be prepared to share your thinking.

1. How many $\frac{1}{2}$s are in 2? 4
2. How many $\frac{1}{5}$s are in 3? 15
3. How many $\frac{1}{8}$s are in $1\frac{1}{4}$? 10
4. $1 \div \frac{2}{6} = ?$ 3
5. $6 \div \frac{2}{5} = ?$ 15
6. $4 \div \frac{2}{10} = ?$ 20

Log in to Amplify Math to complete this lesson online.

416 Unit 4 Dividing Fractions

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1 Launch

Distribute fraction strips from the Warm-up 1 PDF, *Fraction Strips* to each pair. Set an expectation for the amount of time pairs will have to work on the activity.

2 Monitor

Help students get started by pointing to the $\frac{1}{2}$ strip and asking, "How can you describe the size of one of these compared to the size of the 1 strip?" Repeat for other fractions as needed.

Look for points of confusion:

- **Not being able to extrapolate for a number of wholes that is greater than the number of physical models that they are using (Problems 5–6).** Ask, "How many more fraction strips would you need to make another whole?"
- **Not being able to relate the division expressions in Problems 4–6 to similar questions in Problems 1–3.** Have students write a division expression for one of the first problems, and then ask, "How could you do the opposite and write a similar question for this division expression?"

Look for productive strategies:

- Recognizing that for each unit fraction, the number of strips equivalent to 1 whole is the same as the denominator.
- Combining or drawing additional fraction strips in order to represent more wholes.
- Identifying and using equivalent fractions.

3 Connect

Have pairs of students share how they interpreted Problems 4–6, and how they reasoned about dividing by non-unit fractions.

Ask, if equivalent fraction strategies have not been shared, "For Problem 6, how else could you think about $\frac{2}{10}$ if you did not have the tenths strips?

Highlight that "how many of this in that" questions also represent the "how many groups" type of division.

Differentiated Support

Accessibility: *Optimize Access to Tools, Guide Processing and Visualization, Activate Prior Knowledge*

Students previously worked with fraction strips in elementary grades. As you distribute the Warm-up PDF, *Fractions Strips*, remind them that each row of strips represents 1 whole. Provide access to scissors to allow students to cut the strips so that they can physically manipulate them. Consider demonstrating how to use fraction strips to model the question in Problem 1. Ask:

- "How many $\frac{1}{2}$s are in 1 whole?" 2
- "How many $\frac{1}{2}$ would be in 2 wholes?" Twice this amount, 4

Power-up

To power up students' ability to write a division sentence to represent a diagram, have students complete:

Determine which equations are modeled by the tape diagram. Select *all* that apply.

4	4	4	4	4	4

[✓ **A.**] $4 \cdot 6 = 24$

B. $4 + 6 = 10$

[✓ **C.**] $4 + 4 + 4 + 4 + 4 + 4 = 24$

[✓ **D.**] $24 \div 6 = 4$

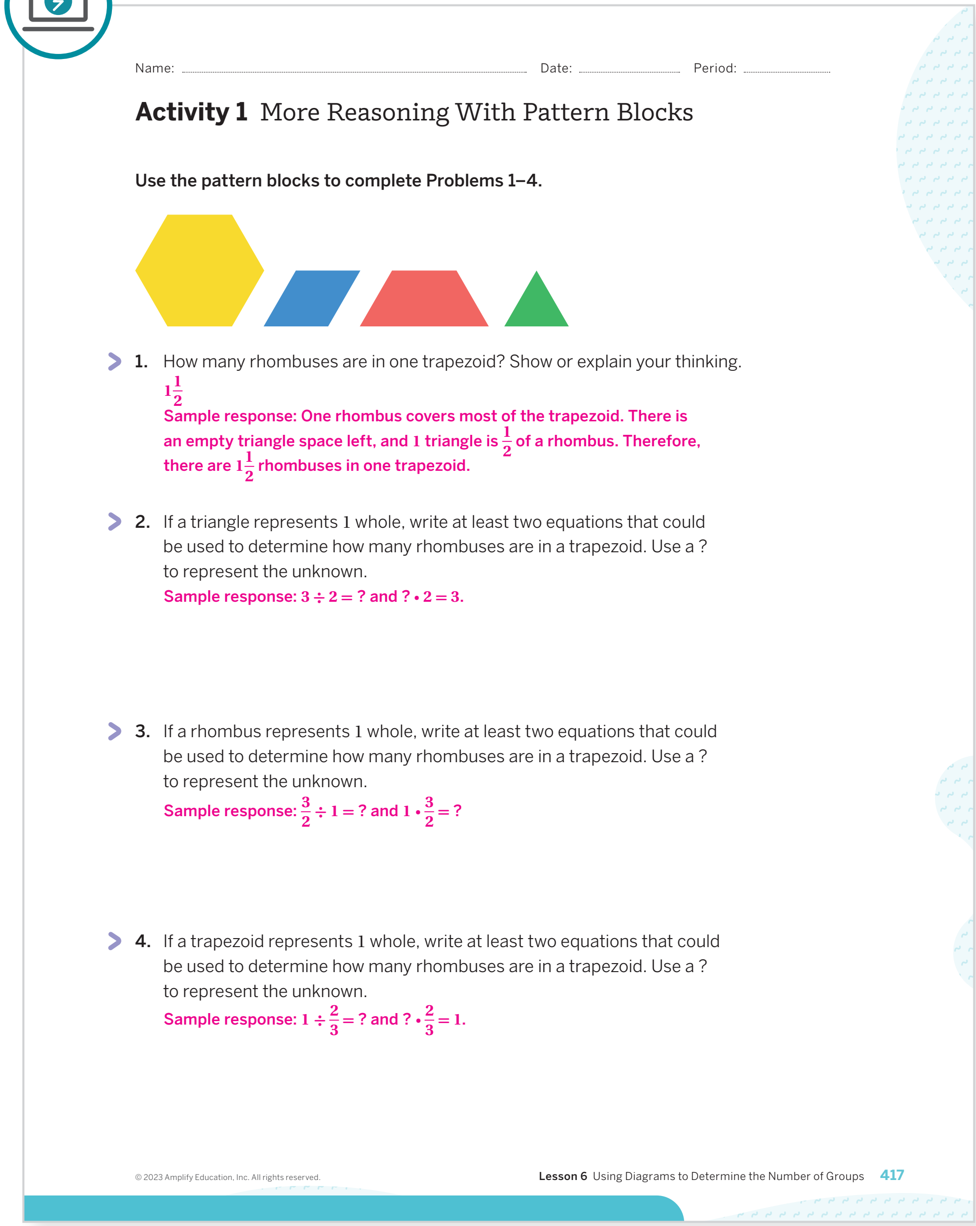
Use: Before Activity 1.

Informed by: Performance on Lesson 5, Practice Problem 6.

Pairs | 10 min

Activity 1 More Reasoning With Pattern Blocks

Students use pattern blocks to help them describe a quotient that is not a whole number, and write several equations to relate the same quotient to different wholes.



Name: Date: Period:

Activity 1 More Reasoning With Pattern Blocks

Use the pattern blocks to complete Problems 1–4.

1. How many rhombuses are in one trapezoid? Show or explain your thinking.
   $1\frac{1}{2}$
   Sample response: One rhombus covers most of the trapezoid. There is an empty triangle space left, and 1 triangle is $\frac{1}{2}$ of a rhombus. Therefore, there are $1\frac{1}{2}$ rhombuses in one trapezoid.

2. If a triangle represents 1 whole, write at least two equations that could be used to determine how many rhombuses are in a trapezoid. Use a ? to represent the unknown.
   Sample response: $3 \div 2 = ?$ and $? \cdot 2 = 3$.

3. If a rhombus represents 1 whole, write at least two equations that could be used to determine how many rhombuses are in a trapezoid. Use a ? to represent the unknown.
   Sample response: $\frac{3}{2} \div 1 = ?$ and $1 \cdot \frac{3}{2} = ?$

4. If a trapezoid represents 1 whole, write at least two equations that could be used to determine how many rhombuses are in a trapezoid. Use a ? to represent the unknown.
   Sample response: $1 \div \frac{2}{3} = ?$ and $? \cdot \frac{2}{3} = 1$.

Lesson 6 Using Diagrams to Determine the Number of Groups 417

1 Launch

Distribute pattern blocks. Have students use the *Think-Pair-Share* routine. Give them 1 minute to respond to Problem 1 before sharing with a partner, and then completing Problems 2–4 together.

2 Monitor

Help students get started by asking, “Is the solution going to be a whole number? What other shape might help you?”

Look for points of confusion:

- **Struggling to name the “remainder” as one-half.** Ask, “How many triangles are in a trapezoid?”
- **Having difficulty writing equations because they cannot name the value of each shape.** Ask, “Does this shape represent a value less than or greater than 1 whole? How could you determine its value?” Then have them review the language and equations from the Warm-up.

Look for productive strategies:

- Determining that $1\frac{1}{2}$ rhombuses fit in a trapezoid by referencing the triangle.
- Recognizing the rhombus is the dividend.
- Using the relationship between multiplication and division to determine a second equation from a first.
- Using previous equations and the relationships between the sizes of individual shapes to write or check new equations.

3 Connect

Have pairs of students share how they determined a solution to Problem 1, and then their equations for Problems 2–4, focusing on how their thinking and the results changed based on the different given wholes.

Highlight that regardless of what defines a whole, the quotient of two values relative to any whole will always be the same.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide access to physical pattern blocks should students choose to use them during this activity.

Accessibility: *Guide Processing and Visualization*

Tell students that, in Problems 2, 3, and 4, the “whole” is redefined. Consider providing copies of pattern blocks that students can annotate each new shape as the “whole” or “1.”

Extension: *Math Enrichment*

Have students revisit Problems 1–4, but this time, have them consider the question, “How many hexagons are in one triangle?” There is $\frac{1}{6}$ of a hexagon in one triangle.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 1, have them share their responses with their partner. Ask partners to review responses and provide feedback that can be used to improve the clarity of the responses. Consider displaying these sample questions:

- “Does the response clearly show why the solution is greater than 1?”
- “Does the response reference the triangle?”

Have students revise their responses after receiving feedback.

English Learners

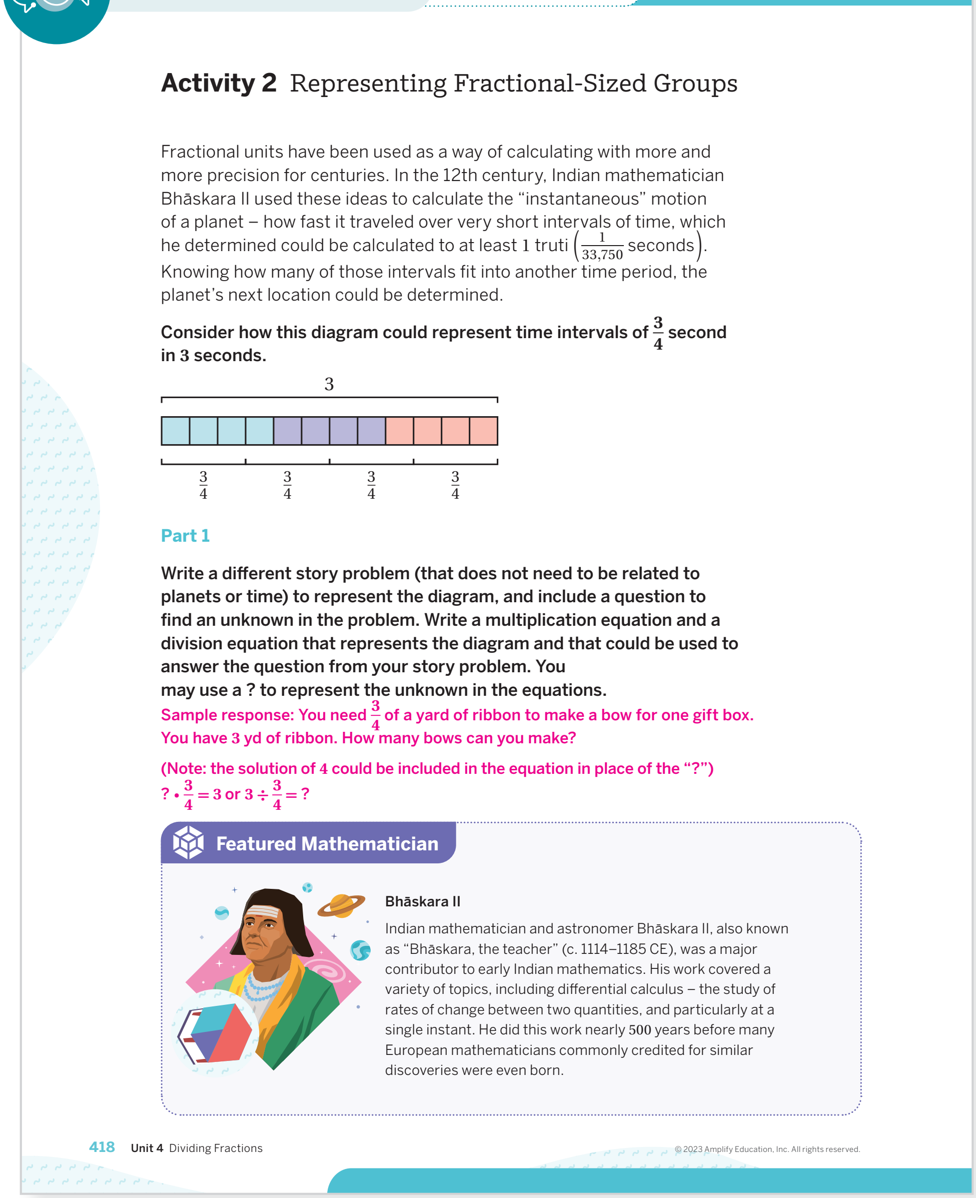
Pair students who speak the same primary language together to provide written and oral feedback to each other.

Pairs | 15 min

Activity 2 Representing Fractional-Sized Groups

Students create tape diagrams to represent equal-sized groups of fractions and determine how many, including fractions of a group, fit into another given value.

Amps Featured Activity Digital Diagrams



Activity 2 Representing Fractional-Sized Groups

Fractional units have been used as a way of calculating with more and more precision for centuries. In the 12th century, Indian mathematician Bhāskara II used these ideas to calculate the "instantaneous" motion of a planet – how fast it traveled over very short intervals of time, which he determined could be calculated to at least 1 truti $\left(\frac{1}{33{,}750}\text{ seconds}\right)$. Knowing how many of those intervals fit into another time period, the planet's next location could be determined.

Consider how this diagram could represent time intervals of $\frac{3}{4}$ second in 3 seconds.

Part 1

Write a different story problem (that does not need to be related to planets or time) to represent the diagram, and include a question to find an unknown in the problem. Write a multiplication equation and a division equation that represents the diagram and that could be used to answer the question from your story problem. You may use a ? to represent the unknown in the equations.

Sample response: You need $\frac{3}{4}$ of a yard of ribbon to make a bow for one gift box. You have 3 yd of ribbon. How many bows can you make?

(Note: the solution of 4 could be included in the equation in place of the "?")
$? \cdot \frac{3}{4} = 3$ or $3 \div \frac{3}{4} = ?$

Featured Mathematician

Bhāskara II

Indian mathematician and astronomer Bhāskara II, also known as "Bhāskara, the teacher" (c. 1114–1185 CE), was a major contributor to early Indian mathematics. His work covered a variety of topics, including differential calculus – the study of rates of change between two quantities, and particularly at a single instant. He did this work nearly 500 years before many European mathematicians commonly credited for similar discoveries were even born.

418 Unit 4 Dividing Fractions

1 Launch

Give pairs a few minutes to work on Part 1 and then pause for a whole class discussion before having pairs continue to work together on Part 2.

2 Monitor

Help students get started by asking, "What values do you see represented in the diagram? How are they related?"

Look for points of confusion:

- **Writing a story problem that does not match the diagram or does not represent multiplication or division (Part 1).** Have students first interpret the diagram relative to a "how many of this in that" statement.
- **Creating tape diagrams that do not preserve the same scale of 1 relative to both other values being represented (Part 2).** Have students start their diagram by only representing whole numbers, with each part of the tape marking representing 1s as they need. Then ask:
  - » "What values do you need to represent?" $\frac{3}{4}$ and 1
  - » "Which represents the amount in one group?" $\frac{3}{4}$
  - » "How can you equipartition the parts of the tape to be able to represent both of those values?" Partition the tape into fourths.

Look for productive strategies:

- Decomposing whole numbers and fractions to unit fractions to build their tape diagrams.
- Creating an appropriate diagram but not knowing how to name fractional parts of the quotient. Nudge these students to compare the "remainder" to the size of one group.
- Using either the multiplication equation or the division equation to write the other corresponding equation.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use interactive tape diagrams to represent a given statement.

Accessibility: *Optimize Access to Tools*

For Part 2, provide blank tape diagrams for students to use to partition and label. Consider making copies of the *Tape Diagrams* PDF, being sure to direct students to only using the blank tape diagrams that are not labeled with percentages.

Math Language Development

MLR7: Compare and Connect

During the Connect, display the multiplication and division equations, tape diagrams, and corresponding questions from Part 2. Draw students' attention to how the value after the phrase "are in ____" represents the total of the tape diagram, the *product* in the multiplication equation, and the *dividend* in the division equation. Annotate these terms on the displays. Use a similar process to illustrate how the value after the phrase "how many ____" is represented.

Featured Mathematician

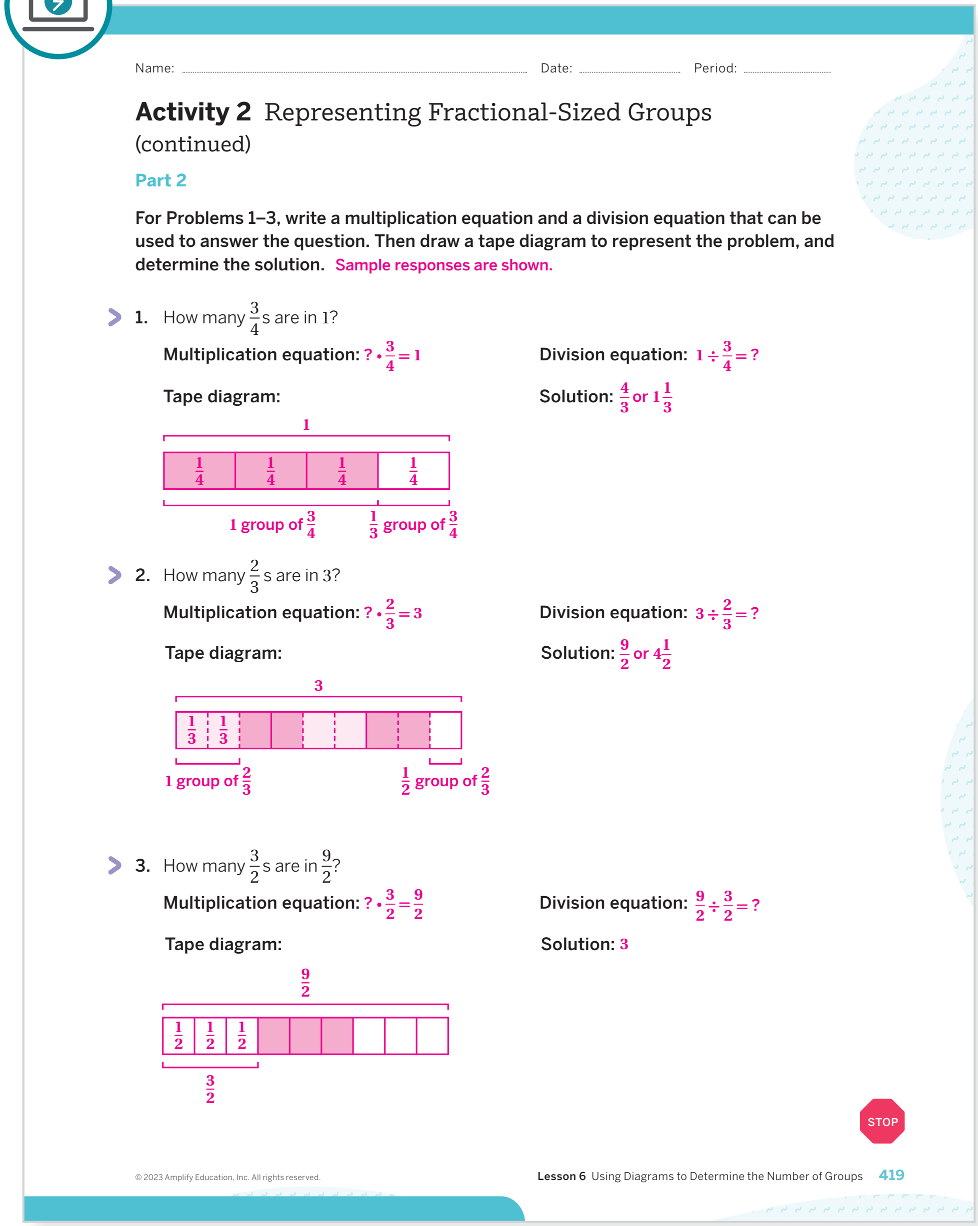
Bhāskara II

Have students read about featured mathematician Bhāskara II, a 12th century Indian mathematician and astronomer who used fractions and early calculus concepts to describe planetary motion.

Pairs | 15 min

Activity 2 Representing Fractional-Sized Groups (continued)

Students create tape diagrams to represent equal-sized groups of fractions and determine how many, including fractions of a group, fit into another given value.



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Activity 2 Representing Fractional-Sized Groups (continued)

Part 2

For Problems 1–3, write a multiplication equation and a division equation that can be used to answer the question. Then draw a tape diagram to represent the problem, and determine the solution. Sample responses are shown.

1. How many $\frac{3}{4}$s are in 1?

   Multiplication equation: $? \cdot \frac{3}{4} = 1$

   Division equation: $1 \div \frac{3}{4} = ?$

   Tape diagram:

   Solution: $\frac{4}{3}$ or $1\frac{1}{3}$

2. How many $\frac{2}{3}$s are in 3?

   Multiplication equation: $? \cdot \frac{2}{3} = 3$

   Division equation: $3 \div \frac{2}{3} = ?$

   Tape diagram:

   Solution: $\frac{9}{2}$ or $4\frac{1}{2}$

3. How many $\frac{3}{2}$s are in $\frac{9}{2}$?

   Multiplication equation: $? \cdot \frac{3}{2} = \frac{9}{2}$

   Division equation: $\frac{9}{2} \div \frac{3}{2} = ?$

   Tape diagram:

   Solution: 3

3 Connect

Display blank or completed tape diagrams for Part 2. Keep the diagram for Problem 1 available for further discussion later.

Have students share how they constructed their diagrams, noting any differences in process or results, and focusing on how they identified which value represents the size of a group. Then have them share their equations and solutions.

Ask, "If you substitute the solution for Problem 1 into the corresponding division equation, you get $1 \div \frac{3}{4} = \frac{4}{3}$, which is equivalent to $1\frac{1}{3}$. Where do you see each of these values in this diagram: $1, \frac{3}{4}, \frac{4}{3}, 1\frac{1}{3}$?"

Highlight that while Problem 3 from Part 2 may have appeared the most complex, the diagram was relatively straightforward to make because the dividend and divisor had common denominators. In all of the diagrams, the partitioning was essentially creating common denominators in order to show how many times the divisor goes into the dividend. (Consider sharing that the next lesson will really focus on this strategy, without having to use diagrams.)

Differentiated Support

Extension: ***Math Around the World, Interdisciplinary Connections***

One of the oldest written records of the use of fractions and decimals were from Babylonian mathematicians, around 2000 BCE. They used a base 60 place value system, whereas we use a base 10 place value system today. Display the following table which compares the base 10 place value system we use today with the base 60 place value system used by Babylonian mathematicians.

Base 10 system		
Tens	Ones	Tenths

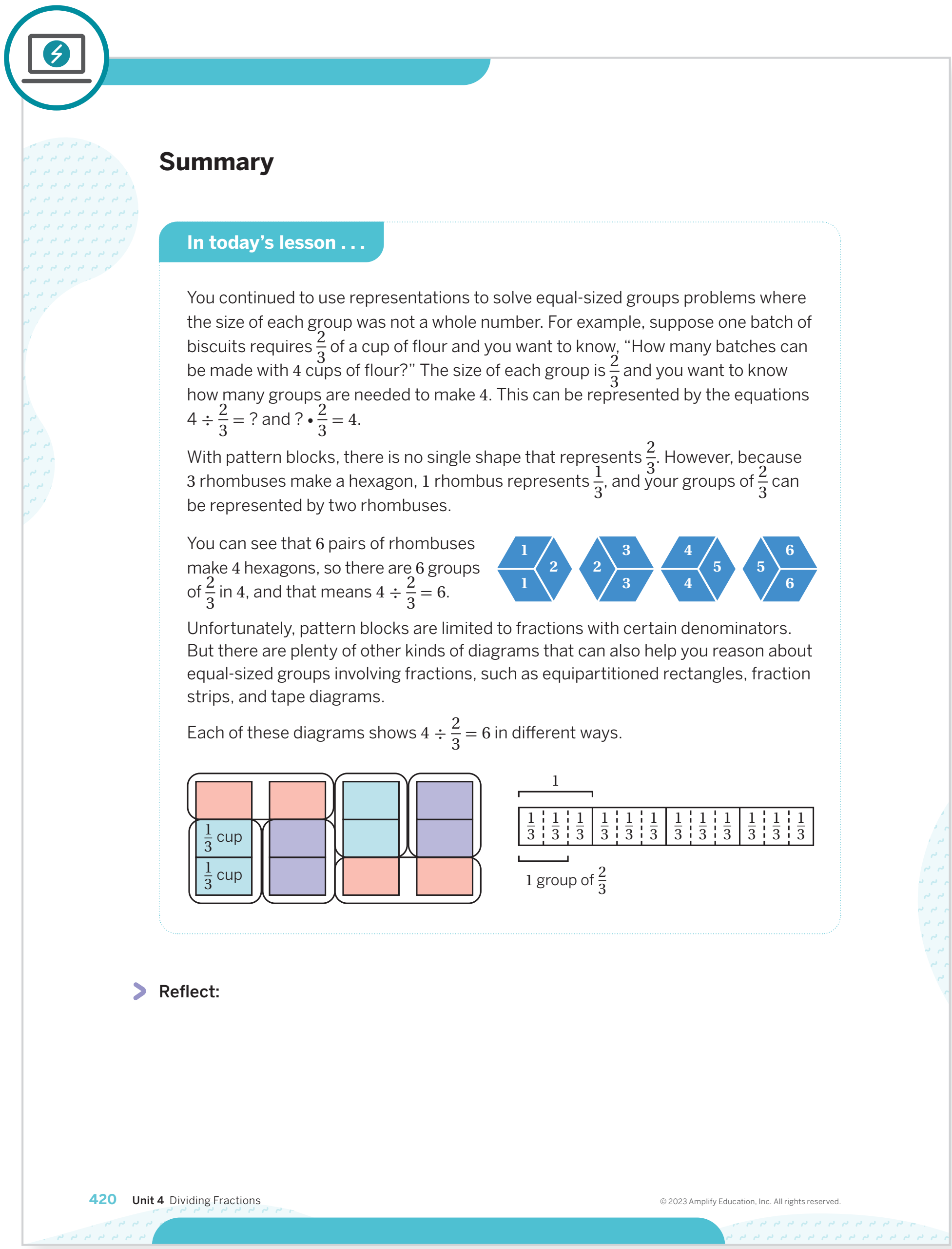
Base 60 system		
Sixties	Ones	Sixtieth

Tell students that the fraction $\frac{1}{2}$ in base 10 is written as $\frac{5}{10}$ or 0.5. However, in base 60, the fraction $\frac{1}{2}$ is written as $\frac{30}{60}$, or 30 in the sixtieth place. Babylonian mathematicians also did not have a symbol for the decimal point, so distinguishing between whole numbers and fractions or decimals was sometimes challenging. Their symbol for 30 could mean several different quantities, for example, 30 sixties, 30 ones, or 30 sixtieths. **(History)**

Whole Class | 5 min

Summary

Review and synthesize the big ideas about quotitive division with fractions across the past two lessons, focusing on the language, equations, and diagrams of equal-sized groups.



Summary

In today's lesson . . .

You continued to use representations to solve equal-sized groups problems where the size of each group was not a whole number. For example, suppose one batch of biscuits requires $\frac{2}{3}$ of a cup of flour and you want to know, "How many batches can be made with 4 cups of flour?" The size of each group is $\frac{2}{3}$ and you want to know how many groups are needed to make 4. This can be represented by the equations $4 \div \frac{2}{3} = ?$ and $? \cdot \frac{2}{3} = 4$.

With pattern blocks, there is no single shape that represents $\frac{2}{3}$. However, because 3 rhombuses make a hexagon, 1 rhombus represents $\frac{1}{3}$, and your groups of $\frac{2}{3}$ can be represented by two rhombuses.

You can see that 6 pairs of rhombuses make 4 hexagons, so there are 6 groups of $\frac{2}{3}$ in 4, and that means $4 \div \frac{2}{3} = 6$.

Unfortunately, pattern blocks are limited to fractions with certain denominators. But there are plenty of other kinds of diagrams that can also help you reason about equal-sized groups involving fractions, such as equipartitioned rectangles, fraction strips, and tape diagrams.

Each of these diagrams shows $4 \div \frac{2}{3} = 6$ in different ways.

Reflect:

420 Unit 4 Dividing Fractions

Synthesize

Display the equation $4 \div \frac{2}{3} = ?$

Ask:

- "Would you expect the quotient here to be less than 1 or greater than 1? Why?" Greater than 1 because $4 > \frac{2}{3} = ?$.
- "Would a tape diagram showing sixths be helpful in determining this quotient? How would that look different from the tape diagram showing thirds?" Yes, but it's not necessary, because it would just show twice as many parts but would represent the same relative number of groups

Highlight that, so far, students have been investigating fraction division problems for one of the two types of division: "how many groups?" They have also seen this asked as "how many of this in that?" and have been able to determine those quotients using related multiplication and division equations and diagrams. During the past two lessons, they have also seen not only whole numbers and unit fractions, but also non-unit fractions. Students have also worked with different examples of those types of values as the dividend, the divisor, and the quotient.

Reflect

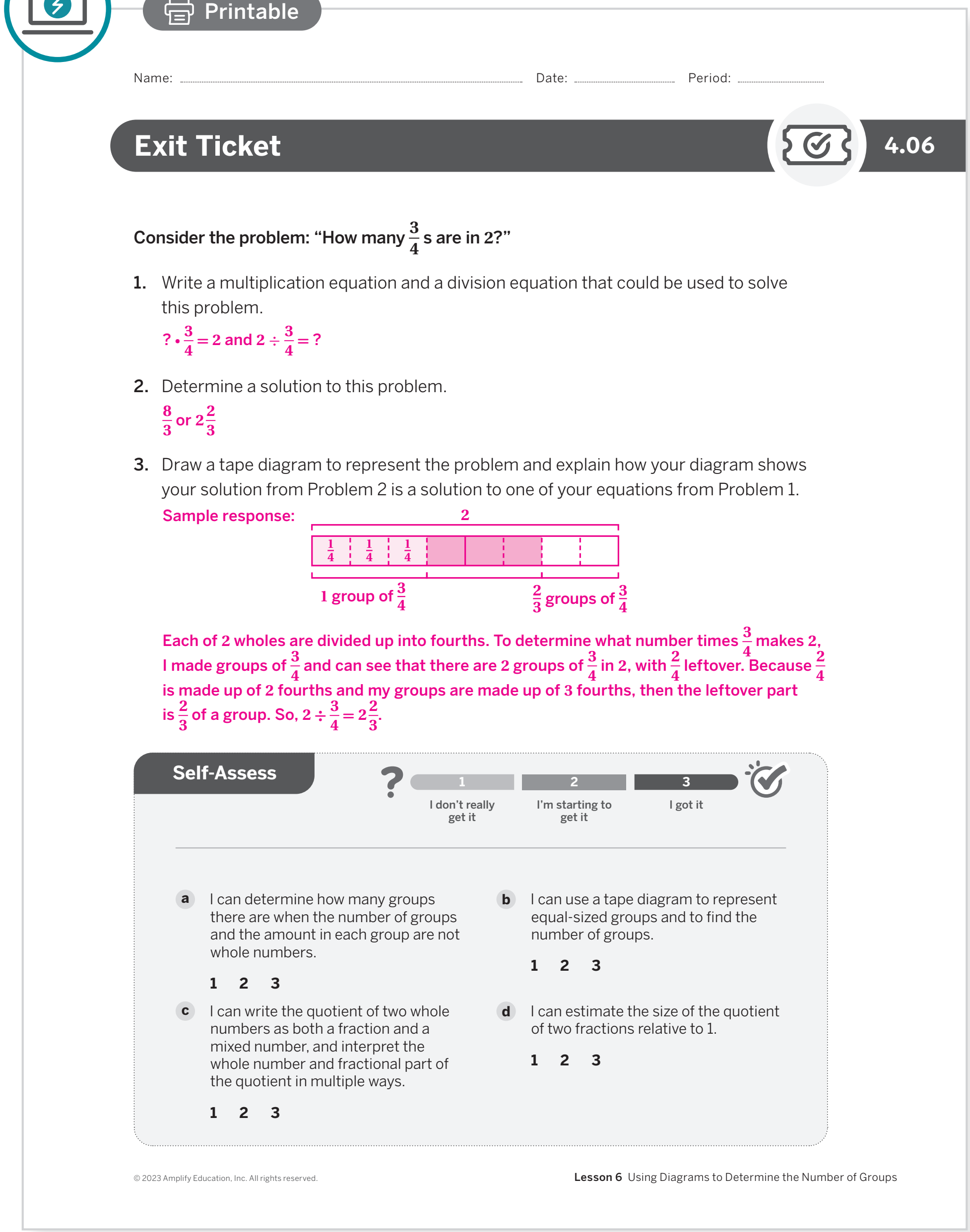
After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is dividing by a fraction related to multiplying fractions?"

Exit Ticket

Students demonstrate their understanding of how to represent and to determine an unknown number of groups when the amount in each group is not a whole number.

Printable



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Exit Ticket 4.06

Consider the problem: "How many $\frac{3}{4}$s are in 2?"

1. Write a multiplication equation and a division equation that could be used to solve this problem.
   $? \cdot \frac{3}{4} = 2$ and $2 \div \frac{3}{4} = ?$
2. Determine a solution to this problem.
   $\frac{8}{3}$ or $2\frac{2}{3}$
3. Draw a tape diagram to represent the problem and explain how your diagram shows your solution from Problem 2 is a solution to one of your equations from Problem 1.
   Sample response:

Each of 2 wholes are divided up into fourths. To determine what number times $\frac{3}{4}$ makes 2, I made groups of $\frac{3}{4}$ and can see that there are 2 groups of $\frac{3}{4}$ in 2, with $\frac{2}{4}$ leftover. Because $\frac{2}{4}$ is made up of 2 fourths and my groups are made up of 3 fourths, then the leftover part is $\frac{2}{3}$ of a group. So, $2 \div \frac{3}{4} = 2\frac{2}{3}$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can determine how many groups there are when the number of groups and the amount in each group are not whole numbers.
   1 2 3

b. I can use a tape diagram to represent equal-sized groups and to find the number of groups.
   1 2 3

c. I can write the quotient of two whole numbers as both a fraction and a mixed number, and interpret the whole number and fractional part of the quotient in multiple ways.
   1 2 3

d. I can estimate the size of the quotient of two fractions relative to 1.
   1 2 3

Lesson 6 Using Diagrams to Determine the Number of Groups

Success looks like . . .

- **Language Goal:** Coordinating multiplication and division equations with pattern block diagrams in which a different shape represents one whole. **(Writing)**
- **Language Goal:** Creating a diagram to represent and solve a problem asking, "how many groups?" in which the divisor is a non-unit fraction, and explaining the solution method. **(Speaking and Listening, Writing)**
  - » Determining how many $\frac{3}{4}$s are in 2 by drawing a tape diagram and then explaining how the diagram shows the solution in Problem 3.
- **Language Goal:** Identifying or generating a multiplication or division equation that represents a given situation involving a fractional divisor. **(Writing)**

Suggested next steps

If students cannot write appropriate equations for Problem 1, consider:

- Reviewing the equations in Activity 2.

If students cannot determine the quotient in Problem 2, consider:

- Ensuring they can draw a proper diagram for Problem 3, and have them use that to help them.

If students cannot draw an appropriate diagram, or cannot name the fractional part of the quotient, in Problem 3, consider:

- Reviewing the diagrams in Activity 2.
- Asking, "How many fourths are in $\frac{2}{4}$? $\frac{3}{4}$?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students determined how many groups there are when the number of groups and the amount in each group are not whole numbers. How did that build on the earlier work students did with dividing with whole numbers?
- Which groups of students did and didn't have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Math Language Development

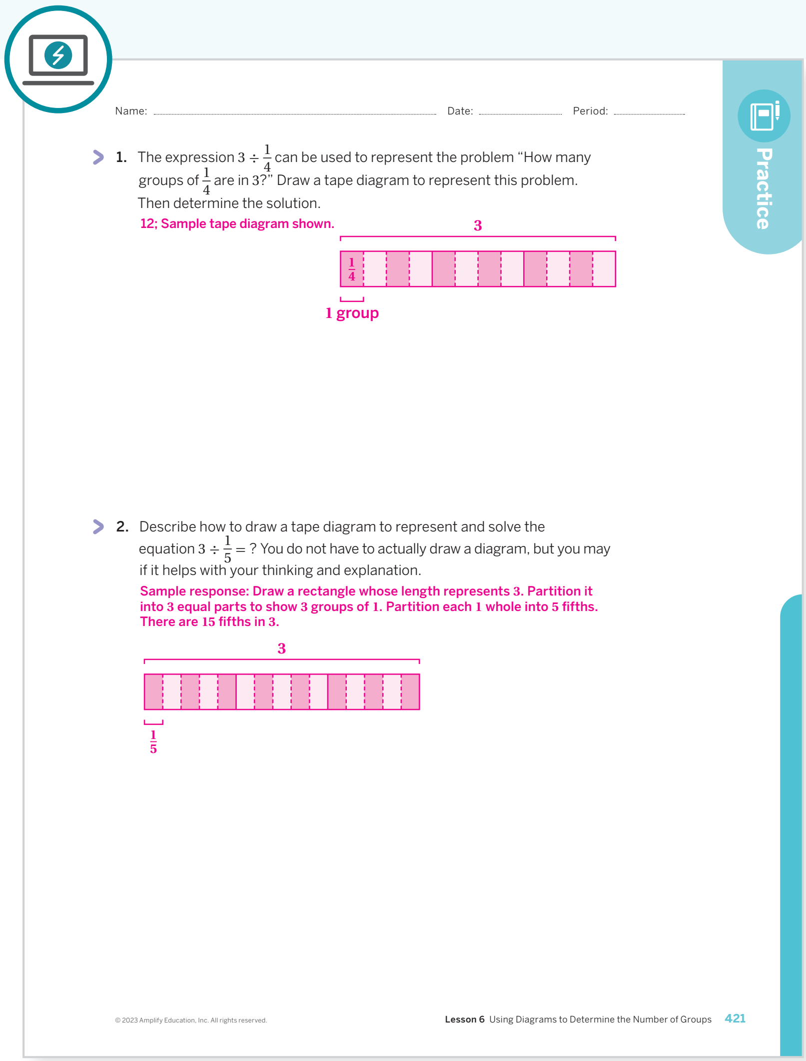
Language Goal: Creating a diagram to represent and solve a problem asking, "how many groups?" in which the divisor is a non-unit fraction, and explaining the solution method.

Reflect on students' language development toward this goal.

- How have students progressed in interpreting and describing division problems involving "How many groups?"
- What strategies can you use to help students understand and use this language?

Practice

Independent



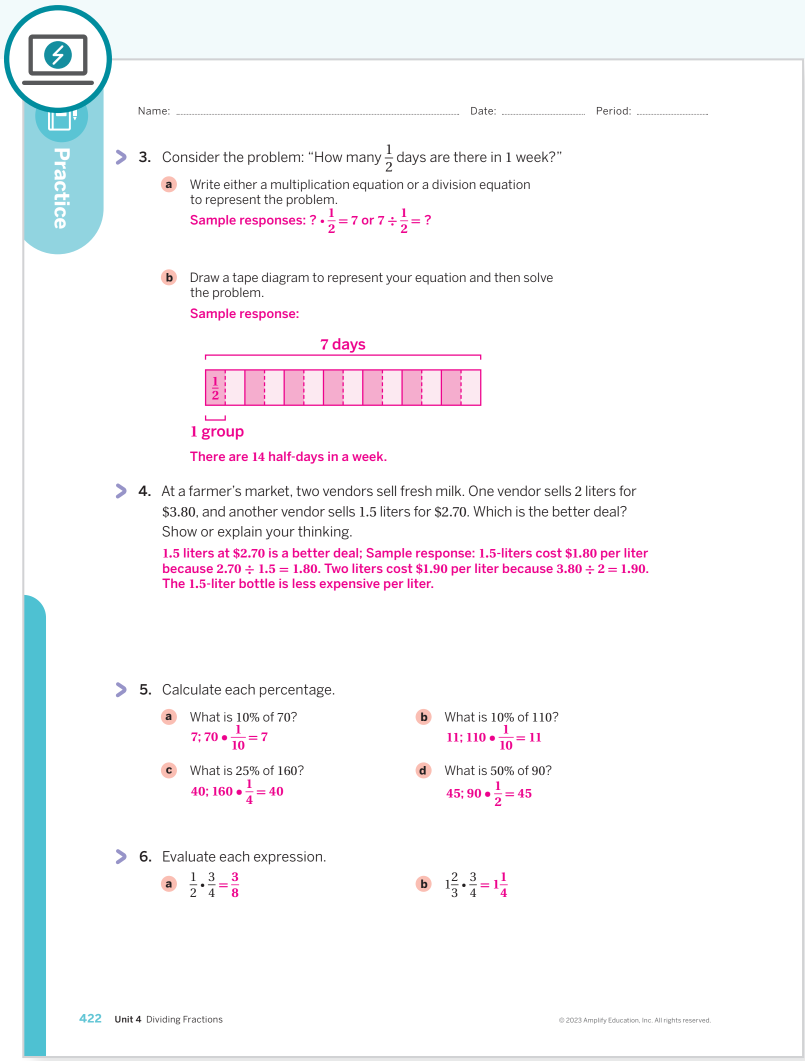
Name: Date: Period:

1. The expression $3 \div \frac{1}{4}$ can be used to represent the problem "How many groups of $\frac{1}{4}$ are in 3?" Draw a tape diagram to represent this problem. Then determine the solution.

12; Sample tape diagram shown.

2. Describe how to draw a tape diagram to represent and solve the equation $3 \div \frac{1}{5} = ?$ You do not have to actually draw a diagram, but you may if it helps with your thinking and explanation.

Sample response: Draw a rectangle whose length represents 3. Partition it into 3 equal parts to show 3 groups of 1. Partition each 1 whole into 5 fifths. There are 15 fifths in 3.



Name: Date: Period:

3. Consider the problem: "How many $\frac{1}{2}$ days are there in 1 week?"
   - a Write either a multiplication equation or a division equation to represent the problem.
     Sample responses: $? \cdot \frac{1}{2} = 7$ or $7 \div \frac{1}{2} = ?$
   - b Draw a tape diagram to represent your equation and then solve the problem.
     Sample response:

There are 14 half-days in a week.

4. At a farmer's market, two vendors sell fresh milk. One vendor sells 2 liters for \$3.80, and another vendor sells 1.5 liters for \$2.70. Which is the better deal? Show or explain your thinking.

1.5 liters at \$2.70 is a better deal; Sample response: 1.5-liters cost \$1.80 per liter because $2.70 \div 1.5 = 1.80$. Two liters cost \$1.90 per liter because $3.80 \div 2 = 1.90$. The 1.5-liter bottle is less expensive per liter.

5. Calculate each percentage.
   - a What is 10% of 70?
     7; $70 \bullet \frac{1}{10} = 7$
   - b What is 10% of 110?
     11; $110 \bullet \frac{1}{10} = 11$
   - c What is 25% of 160?
     40; $160 \bullet \frac{1}{4} = 40$
   - d What is 50% of 90?
     45; $90 \bullet \frac{1}{2} = 45$

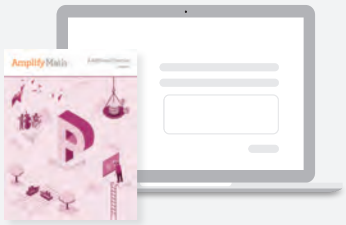
6. Evaluate each expression.
   - a $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$
   - b $1\frac{2}{3} \cdot \frac{3}{4} = 1\frac{1}{4}$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 4	2
	5	Unit 3 Lesson 13	2
Formative ⚡	6	Unit 4 Lesson 7	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 4 | **LESSON 7**

Dividing With Common Denominators

Let's think about dividing things into groups using common denominators.

Focus

Goals

1. Rewrite and evaluate a division expression by using fractions with a common denominator.
2. **Language Goal:** Explain why the quotient of two fractions with common denominators is the same as the quotient of the numerators. **(Speaking and Listening, Writing)**
3. Divide a unit fraction by a unit fraction.

Coherence

Today

In this lesson, students extend the previous work to include cases where the number of groups is a fraction less than 1. In these situations, the question becomes "what fraction of a group?" as they work through disagreements. Students notice that they can use the same reasoning strategies as with situations with a whole number of groups, because the structure (number of groups) • (size of a group) = (total amount) is the same as before. They write multiplication equations of this form and the corresponding division equations while using common denominators.

Previously

In Lesson 6, students worked with division situations involving questions, such as "how many groups?" or "how many of this in that?"

Coming Soon

In Lesson 8, students interpret division expressions as a way to answer "how much in a group?"

Rigor

- Students write multiplication and division equations to build **conceptual understanding** of dividing with common denominators in this lesson.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

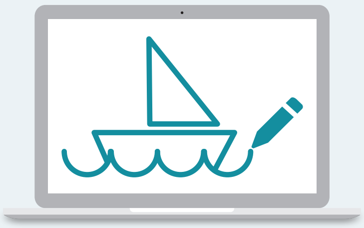
Materials

- Exit Ticket
- Additional Practice
- *Tape Diagrams* PDF (as needed)
- colored pencils

Amps | **Featured Activity**

Activity 1
Dynamic Ropes

Students can use interactive ropes to compare and contrast length by using multiplication and division.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

When working with a partner in Activity 1, students might be overly-confident of their own solutions and disregard the thoughts of others. Remind students that communicating their own reasoning, as well as considering the reasoning of others will allow them to compare each argument and develop a logical solution. Listening to the reasoning of others helps students clarify and improve their own arguments.

Modifications to Pacing

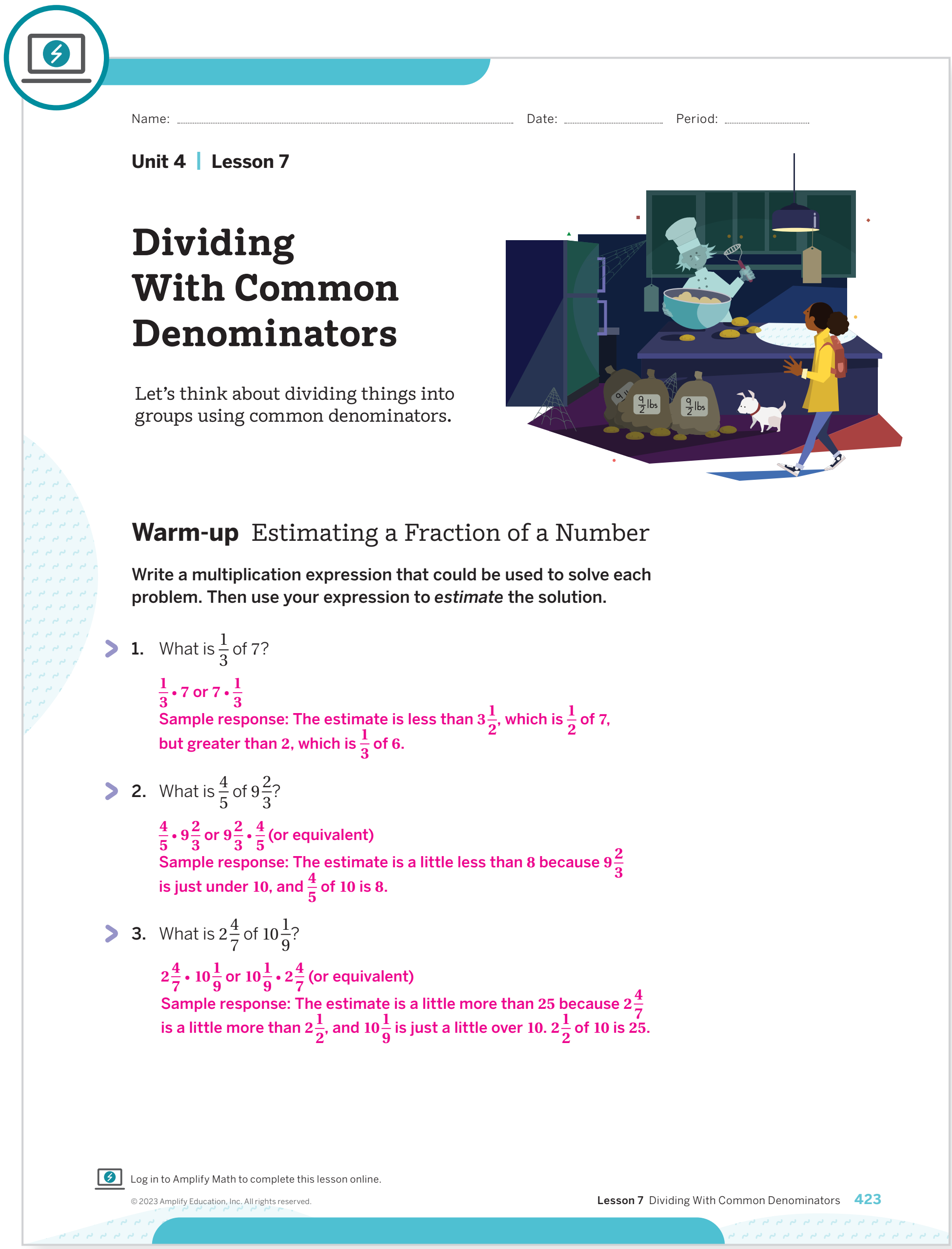
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 3 may be omitted.
- In **Activity 2**, Problems 1a and 2a may be omitted.

Independent | 5 min

Warm-up Estimating a Fraction of a Number

Students estimate the value of a fraction of a number by using what they know about the size of the given fraction to explore division problems in which the quotient is less than 1 whole.



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Unit 4 | Lesson 7

Dividing With Common Denominators

Let's think about dividing things into groups using common denominators.

Warm-up Estimating a Fraction of a Number

Write a multiplication expression that could be used to solve each problem. Then use your expression to *estimate* the solution.

1. What is $\frac{1}{3}$ of 7?

   $\frac{1}{3} \cdot 7$ or $7 \cdot \frac{1}{3}$
   Sample response: The estimate is less than $3\frac{1}{2}$, which is $\frac{1}{2}$ of 7, but greater than 2, which is $\frac{1}{3}$ of 6.

2. What is $\frac{4}{5}$ of $9\frac{2}{3}$?

   $\frac{4}{5} \cdot 9\frac{2}{3}$ or $9\frac{2}{3} \cdot \frac{4}{5}$ (or equivalent)
   Sample response: The estimate is a little less than 8 because $9\frac{2}{3}$ is just under 10, and $\frac{4}{5}$ of 10 is 8.

3. What is $2\frac{4}{7}$ of $10\frac{1}{9}$?

   $2\frac{4}{7} \cdot 10\frac{1}{9}$ or $10\frac{1}{9} \cdot 2\frac{4}{7}$ (or equivalent)
   Sample response: The estimate is a little more than 25 because $2\frac{4}{7}$ is a little more than $2\frac{1}{2}$, and $10\frac{1}{9}$ is just a little over 10. $2\frac{1}{2}$ of 10 is 25.

Log in to Amplify Math to complete this lesson online.

Lesson 7 Dividing With Common Denominators 423

① Launch

Show students one problem at a time and allow them to work for 1 minute. Discuss strategies for solving before moving on to the next problem.

② Monitor

Help students get started by asking, "What does the word *of* mean in math?"

Look for points of confusion:

- **Not estimating the fractions to more friendly numbers.** Have students use friendly numbers, such as $\frac{1}{2}$ of 7 in Problem 1 or $\frac{4}{5}$ of 10 in Problem 2.

Look for productive strategies:

- Using benchmark fractions to help estimate.
- Deciding if the quotient will be less or more than what was estimated.
- Rounding to an easy whole number to help estimate.

③ Connect

Have individual students share their strategies and solutions for the estimated quotients.

Ask:

- "Did anyone solve the problem the same way, but can explain it differently?"
- "Did anyone solve the problem in a different way?"

Highlight that students can determine the exact value of a fraction of a number by multiplying the fraction and the number. It does not matter whether the number is a whole number, mixed number, or another fraction.

Math Language Development

MLR8: Discussion Supports

After students complete Problem 1 and during the discussion of the strategies they used, listen for students who connected the idea of "What is ______ of ______?" to multiplication. Consider showing questions involving simpler fractions or whole numbers, such as:

- "What is $\frac{1}{2}$ of 12?"
- "What is $\frac{1}{3}$ of 12?"

Ask students how they could write a multiplication expression for these simpler questions. Then have them proceed with the rest of the Warm-up.

Power-up

To power up students' ability to multiply fractions, have students complete:

Recall that when multiplying fractions, it is helpful to rewrite mixed numbers as improper fractions. For example, you may want to rewrite $1\,\frac{1}{2} \cdot \frac{1}{3}$ as $\frac{3}{2} \cdot \frac{1}{3}$ in order to determine the product.

Determine each product. Show your thinking.

a. $1\,\frac{1}{2} \cdot \frac{1}{3} = \frac{3}{2} \cdot \frac{1}{3}$
$= \frac{3 \cdot 1}{2 \cdot 3}$
$= \frac{3}{6}$ or $\frac{1}{2}$

b. $\frac{3}{5} \cdot 2\,\frac{1}{7} = \frac{3}{5} \cdot \frac{15}{7}$
$= \frac{3 \cdot 15}{5 \cdot 7}$
$= \frac{45}{35} \quad \frac{\div 5}{\div 5}$
$= \frac{9}{7}$ or $1\,\frac{2}{7}$

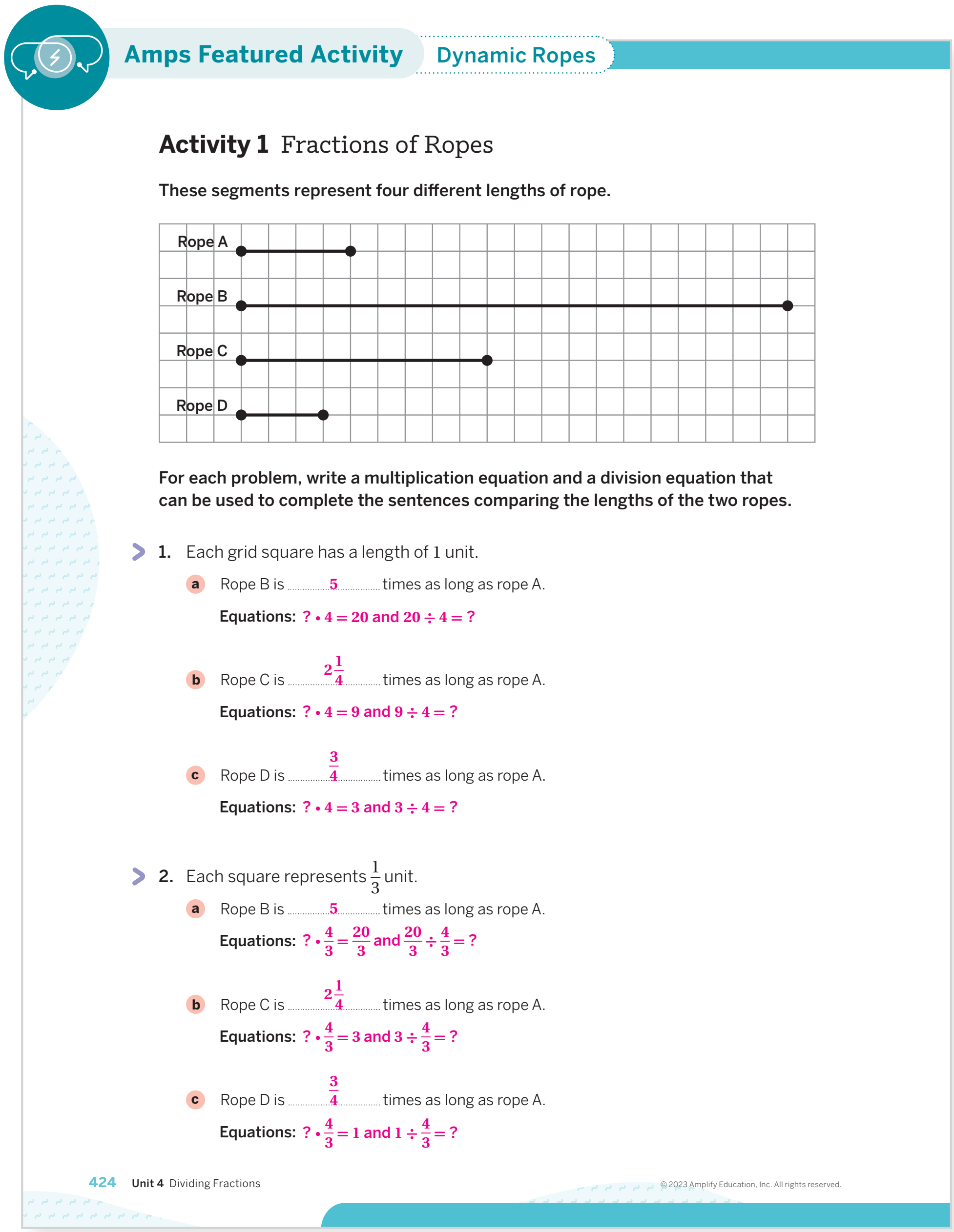
Use: Before the Warm-up
Informed by: Performance on Lesson 6, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4.

Pairs | 15 min

Activity 1 Fractions of Ropes

Students use fractions of ropes to transition their thinking from “how many groups?” to “what fraction of a group?”

Amps Featured Activity Dynamic Ropes



Activity 1 Fractions of Ropes

These segments represent four different lengths of rope.

For each problem, write a multiplication equation and a division equation that can be used to complete the sentences comparing the lengths of the two ropes.

1. Each grid square has a length of 1 unit.
   - a Rope B is 5 times as long as rope A.
     Equations: $? \cdot 4 = 20$ and $20 \div 4 = ?$
   - b Rope C is $2\frac{1}{4}$ times as long as rope A.
     Equations: $? \cdot 4 = 9$ and $9 \div 4 = ?$
   - c Rope D is $\frac{3}{4}$ times as long as rope A.
     Equations: $? \cdot 4 = 3$ and $3 \div 4 = ?$
2. Each square represents $\frac{1}{3}$ unit.
   - a Rope B is 5 times as long as rope A.
     Equations: $? \cdot \frac{4}{3} = \frac{20}{3}$ and $\frac{20}{3} \div \frac{4}{3} = ?$
   - b Rope C is $2\frac{1}{4}$ times as long as rope A.
     Equations: $? \cdot \frac{4}{3} = 3$ and $3 \div \frac{4}{3} = ?$
   - c Rope D is $\frac{3}{4}$ times as long as rope A.
     Equations: $? \cdot \frac{4}{3} = 1$ and $1 \div \frac{4}{3} = ?$

424 Unit 4 Dividing Fractions

1 Launch

Give students 2 minutes of think time and then a couple minutes to compare their responses with a partner and discuss any disagreements about Rope A. Give a few more minutes to complete the rest of the activity as a pair.

2 Monitor

Help students get started by asking them to determine the lengths in units of Rope A and Rope B. Ask, “How can that help you compare the lengths?” I can compare the number of units.

Look for points of confusion:

- **Struggling to create a multiplication and division equation.** Have students label the lengths of the ropes on the diagram. Then use those numbers to create the equations.
- **Struggling to use the fractions correctly in Problem 2.** Ask students how Problems 1 and 2 are different and how can they use Problem 1 to help write Problem 2.

Look for productive strategies:

- Labelling the lengths of each line on the diagram.
- Creating the division equation from the multiplication equation.

3 Connect

Display the equation: $? \cdot \frac{4}{3} = \frac{20}{3}$.

Have individual students share how they created a corresponding division equation.

Highlight that the ropes are fractional parts and that the pair of equations represent a situation with a fractional group. Problem 2 begins to show the use of the common denominators.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use interactive ropes to compare and contrast length by using multiplication and division.

Accessibility: *Guide Processing and Visualization*

Suggest students determine the length of each rope in Problems 1 and 2 before they begin writing their equations.

Extension: *Math Enrichment*

Have students revisit the same three questions, this time with each square representing $\frac{1}{4}$ unit.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share how they created a corresponding division equation for the multiplication equation $? \cdot \frac{4}{3} = \frac{20}{3}$, highlight the connections between each term in the equation.

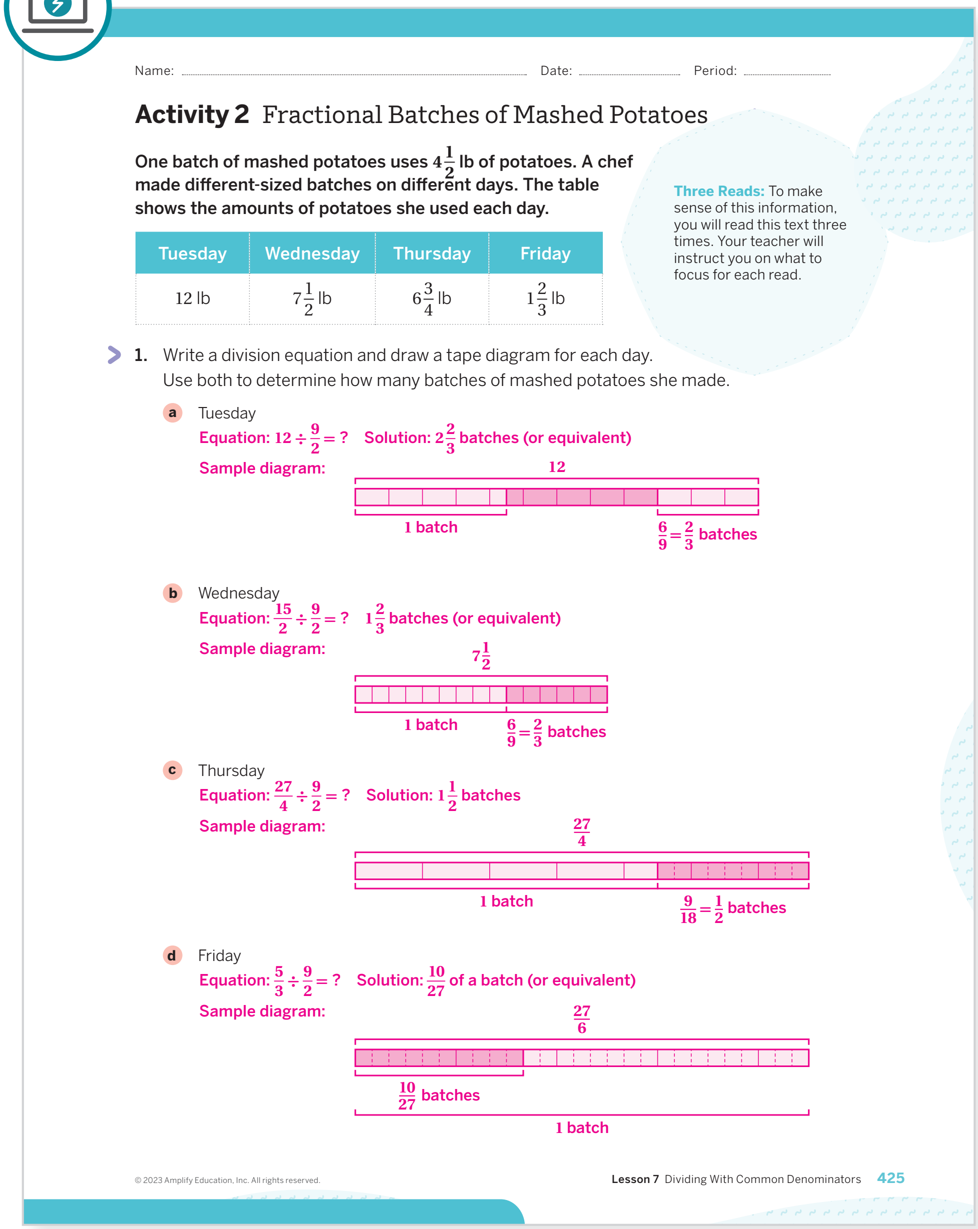
- *Product → Dividend*
- *One Factor → Divisor*
- *Other Factor → Quotient*

Multiplication	Division
$? \cdot \frac{4}{3} = \frac{20}{3}$	$\frac{20}{3} \div \frac{4}{3} = ?$

Pairs | 15 min

Activity 2 Fractional Batches of Mashed Potatoes

Students make sense of quotients that are less than 1 and greater than 1 in the same context and generalize their reasoning to solve division problems without contexts.



Name: ________ Date: ________ Period: ________

Activity 2 Fractional Batches of Mashed Potatoes

One batch of mashed potatoes uses $4\frac{1}{2}$ lb of potatoes. A chef made different-sized batches on different days. The table shows the amounts of potatoes she used each day.

Tuesday	Wednesday	Thursday	Friday
12 lb	$7\frac{1}{2}$ lb	$6\frac{3}{4}$ lb	$1\frac{2}{3}$ lb

Three Reads: To make sense of this information, you will read this text three times. Your teacher will instruct you on what to focus for each read.

1. Write a division equation and draw a tape diagram for each day. Use both to determine how many batches of mashed potatoes she made.

   a. Tuesday

   Equation: $12 \div \frac{9}{2} = ?$ Solution: $2\frac{2}{3}$ batches (or equivalent)

   Sample diagram:

   12

   1 batch

   $\frac{6}{9} = \frac{2}{3}$ batches

   b. Wednesday

   Equation: $\frac{15}{2} \div \frac{9}{2} = ?$ $1\frac{2}{3}$ batches (or equivalent)

   Sample diagram:

   $7\frac{1}{2}$

   1 batch

   $\frac{6}{9} = \frac{2}{3}$ batches

   c. Thursday

   Equation: $\frac{27}{4} \div \frac{9}{2} = ?$ Solution: $1\frac{1}{2}$ batches

   Sample diagram:

   $\frac{27}{4}$

   1 batch

   $\frac{9}{18} = \frac{1}{2}$ batches

   d. Friday

   Equation: $\frac{5}{3} \div \frac{9}{2} = ?$ Solution: $\frac{10}{27}$ of a batch (or equivalent)

   Sample diagram:

   $\frac{27}{6}$

   $\frac{10}{27}$ batches

   1 batch

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Lesson 7 Dividing With Common Denominators 425

1 Launch

Have students use the *Think-Pair-Share* routine. Provide them 1 minute of individual think time to think about how they will set up an equation for Problem 1a. Then have them use colored pencils to draw diagrams and complete the activity with a partner.

2 Monitor

Help students get started by asking them what parts of the equation they are given and what they need to still determine.

Look for points of confusion:

- **Struggling to represent a situation with a tape diagram.** Have students represent only one quantity or number at a time, nudging them to start with the dividend.
- **Not sure how to set up the equation for each scenario.** Ask, Which value should be the dividend? Which should be the divisor?

Look for productive strategies:

- Writing equations and using those to set up their tape diagrams, and then solving.
- Building tape diagrams using successive partitionings, and applying it across the whole tape.
- Recognizing the efficiency and utility of identifying a common denominator first.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1b and 2b. This will provide them with opportunities to work with different types of fractions.

Accessibility: *Optimize Access to Tools, Guide Processing and Visualization*

Provide students with access to blank tape diagrams they can use to partition and label for Problems 1 and 2. Suggest they use colored pencils to annotate the different parts of the diagram that correspond to the equations to help with their thinking. Consider making copies of the *Tape Diagrams* PDF, being sure to direct students to only use the blank tape diagrams that are not labeled with percentages.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that a chef made different sized-batches of mashed potatoes, using different amounts of potatoes each day.
- **Read 2:** Ask students to name the given quantities and relationships, such as *one batch uses $4\frac{1}{2}$ lb of potatoes.*
- **Read 3:** Ask students to preview Problem 1 and brainstorm strategies for how to write a division equation or draw a tape diagram.

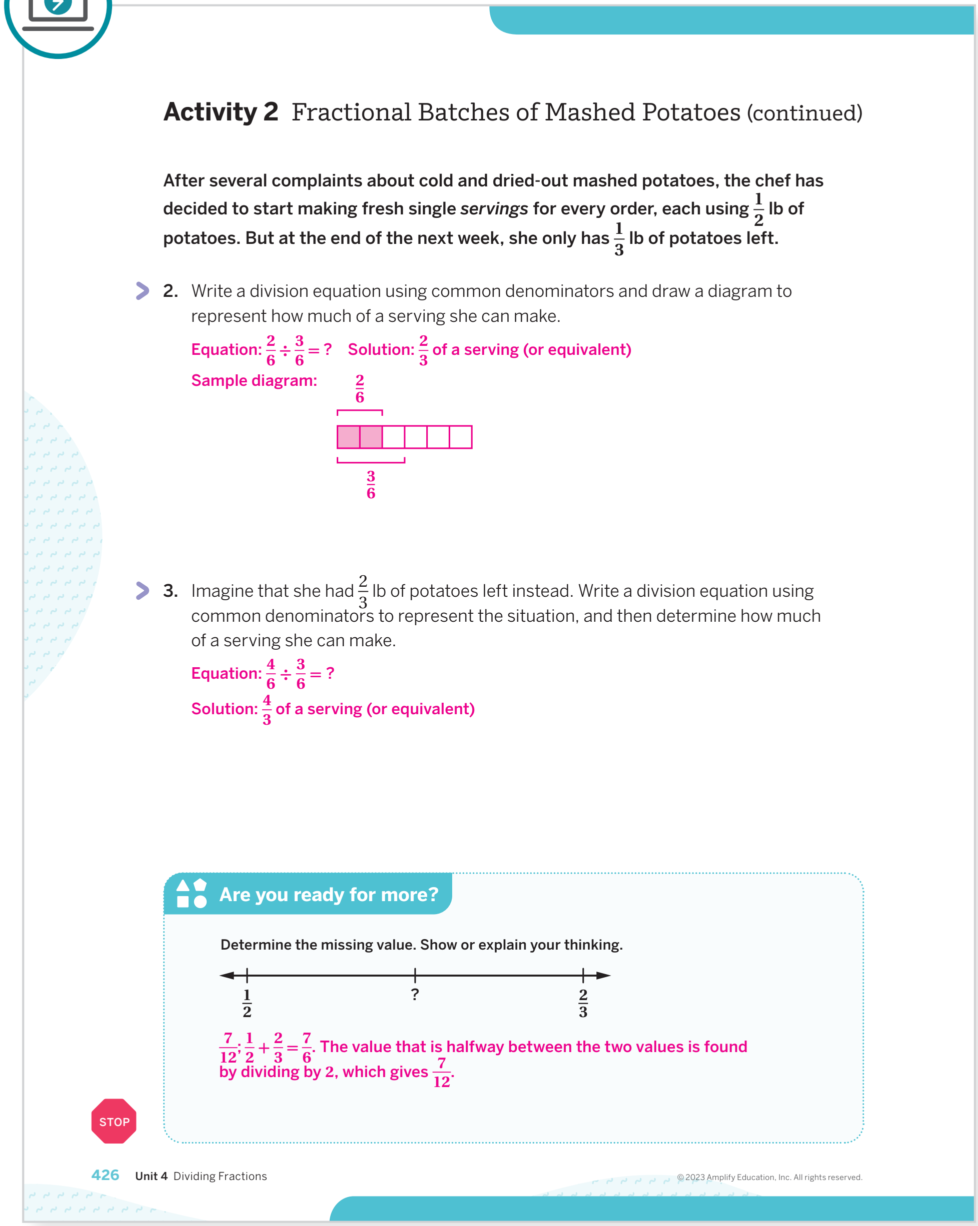
English Learners

Draw a picture or use images from the internet to illustrate what a batch of mashed potatoes might look like.

Pairs | 15 min

Activity 2 Fractional Batches of Mashed Potatoes (continued)

Students make sense of quotients that are less than 1 and greater than 1 in the same context and generalize their reasoning to solve division problems without contexts.



Activity 2 Fractional Batches of Mashed Potatoes (continued)

After several complaints about cold and dried-out mashed potatoes, the chef has decided to start making fresh single *servings* for every order, each using $\frac{1}{2}$ lb of potatoes. But at the end of the next week, she only has $\frac{1}{3}$ lb of potatoes left.

2. Write a division equation using common denominators and draw a diagram to represent how much of a serving she can make.

   Equation: $\frac{2}{6} \div \frac{3}{6} = ?$ Solution: $\frac{2}{3}$ of a serving (or equivalent)

   Sample diagram:

3. Imagine that she had $\frac{2}{3}$ lb of potatoes left instead. Write a division equation using common denominators to represent the situation, and then determine how much of a serving she can make.

   Equation: $\frac{4}{6} \div \frac{3}{6} = ?$

   Solution: $\frac{4}{3}$ of a serving (or equivalent)

Are you ready for more?

Determine the missing value. Show or explain your thinking.

$\frac{7}{12}$; $\frac{1}{2} + \frac{2}{3} = \frac{7}{6}$. The value that is halfway between the two values is found by dividing by 2, which gives $\frac{7}{12}$.

STOP

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3 Connect

Display either completed diagrams for all problems or blank diagrams to populate with student thinking.

Have individual students share their solutions and strategies for each of the problems, focusing on those who used, or unknowingly ended up with, common denominators. Record an equivalent expression using common denominators for each problem.

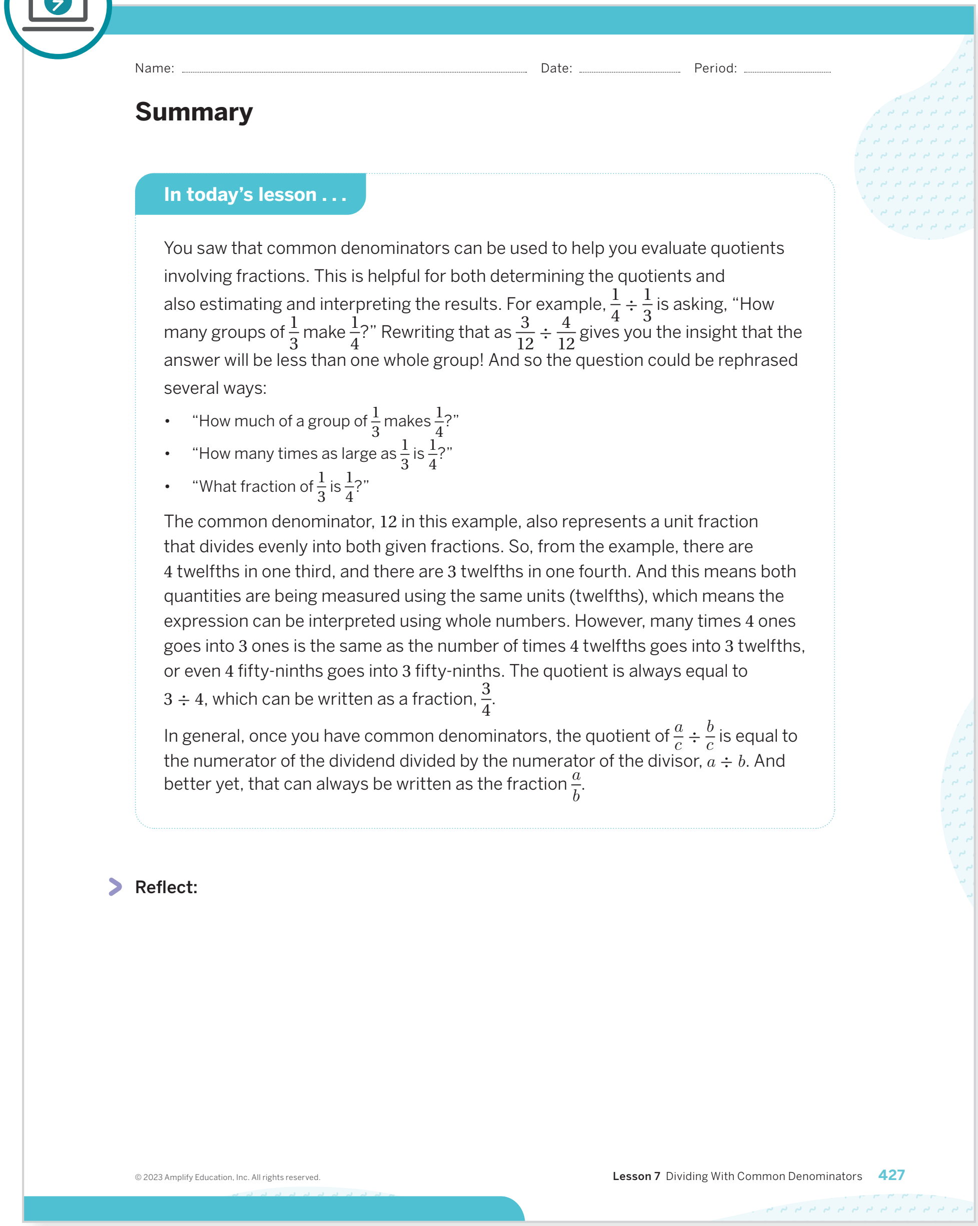
Ask, "What patterns or relationships do you see in the common denominator expression and their quotients?" Dividends with lesser numerators result in quotients less than 1. The quotients are equal to the fraction containing both numerators.

Highlight that students generalized their reasoning to solve division problems where the quotient is less than 1 without the contexts.

Whole Class | 5 min

Summary

Review and synthesize how a division problem can represent the idea of equal-sized groups, but may represent a total amount that is less than the size of one full group.



Name: Date: Period:

Summary

In today's lesson . . .

You saw that common denominators can be used to help you evaluate quotients involving fractions. This is helpful for both determining the quotients and also estimating and interpreting the results. For example, $\frac{1}{4} \div \frac{1}{3}$ is asking, "How many groups of $\frac{1}{3}$ make $\frac{1}{4}$?" Rewriting that as $\frac{3}{12} \div \frac{4}{12}$ gives you the insight that the answer will be less than one whole group! And so the question could be rephrased several ways:

- "How much of a group of $\frac{1}{3}$ makes $\frac{1}{4}$?"
- "How many times as large as $\frac{1}{3}$ is $\frac{1}{4}$?"
- "What fraction of $\frac{1}{3}$ is $\frac{1}{4}$?"

The common denominator, 12 in this example, also represents a unit fraction that divides evenly into both given fractions. So, from the example, there are 4 twelfths in one third, and there are 3 twelfths in one fourth. And this means both quantities are being measured using the same units (twelfths), which means the expression can be interpreted using whole numbers. However, many times 4 ones goes into 3 ones is the same as the number of times 4 twelfths goes into 3 twelfths, or even 4 fifty-ninths goes into 3 fifty-ninths. The quotient is always equal to $3 \div 4$, which can be written as a fraction, $\frac{3}{4}$.

In general, once you have common denominators, the quotient of $\frac{a}{c} \div \frac{b}{c}$ is equal to the numerator of the dividend divided by the numerator of the divisor, $a \div b$. And better yet, that can always be written as the fraction $\frac{a}{b}$.

Reflect:

Synthesize

Highlight the key points from the Student Edition Summary, and emphasize that using common denominators is a strategy that will work for any division expression.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

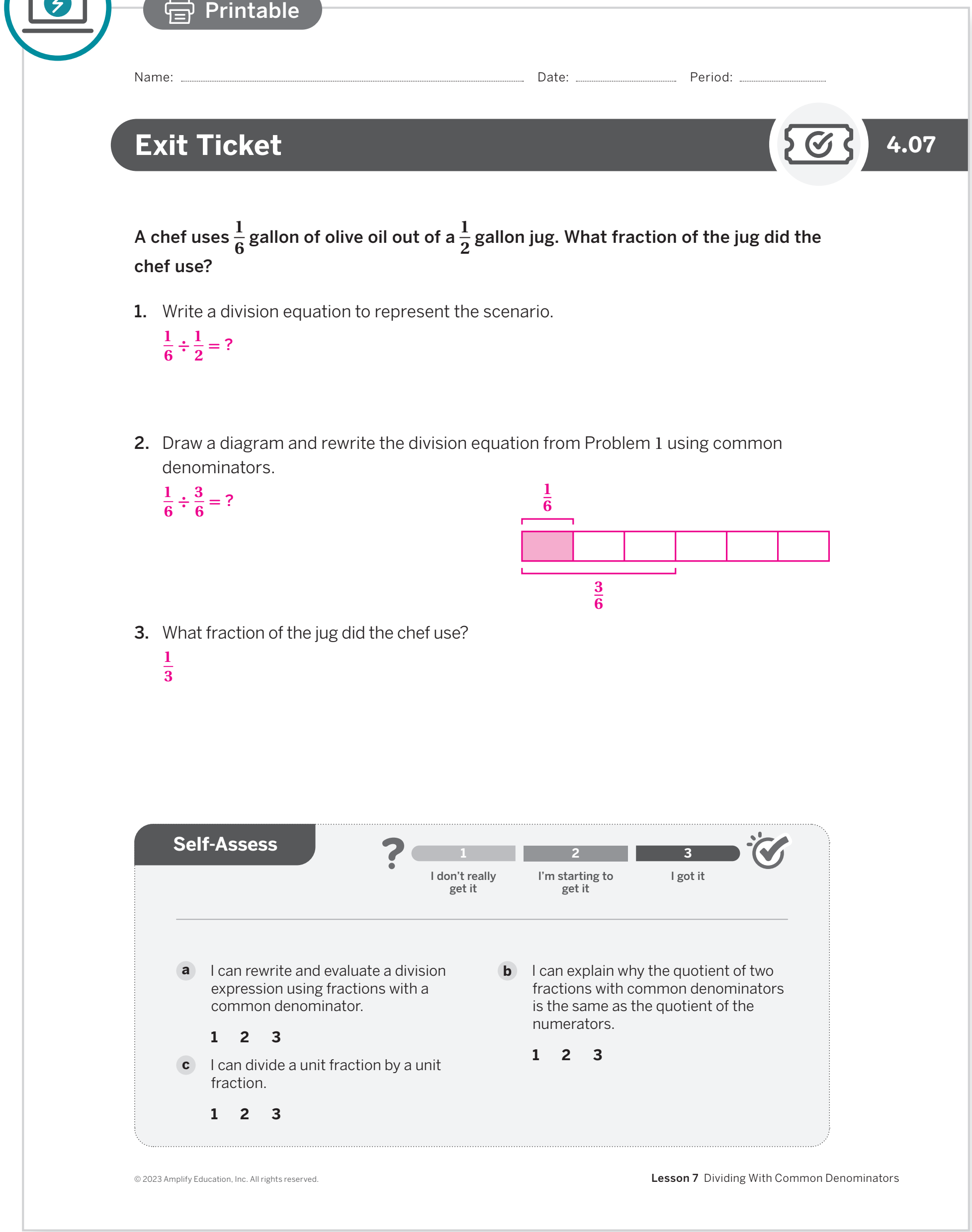
- "How can you tell whether a division situation involves less than one whole group?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by representing the equation on a tape diagram.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 4.07

A chef uses $\frac{1}{6}$ gallon of olive oil out of a $\frac{1}{2}$ gallon jug. What fraction of the jug did the chef use?

1. Write a division equation to represent the scenario.

   $\frac{1}{6} \div \frac{1}{2} = ?$

2. Draw a diagram and rewrite the division equation from Problem 1 using common denominators.

   $\frac{1}{6} \div \frac{3}{6} = ?$

   $\frac{1}{6}$

   $\frac{3}{6}$

3. What fraction of the jug did the chef use?

   $\frac{1}{3}$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can rewrite and evaluate a division expression using fractions with a common denominator.
1 2 3

b. I can explain why the quotient of two fractions with common denominators is the same as the quotient of the numerators.
1 2 3

c. I can divide a unit fraction by a unit fraction.
1 2 3

Lesson 7 Dividing With Common Denominators

Success looks like . . .

- **Goal:** Rewriting and evaluating a division expression using fractions with a common denominator.
  - » Rewriting the expression $\frac{1}{6} \div \frac{1}{2}$ using a common denominator of 6.
- **Language Goal:** Explaining why the quotient of two fractions with common denominators is the same as the quotient of the numerators. **(Speaking and Listening, Writing)**
- **Goal:** Dividing a unit fraction by a unit fraction.
  - » Determining how many $\frac{1}{6}$ gallon are within $\frac{1}{2}$ gallon.

Suggested next steps

If students write the multiplication or division equation incorrectly in Problem 1, consider:

- Reviewing writing equations from Activity 2.
- Assigning Practice Problem 2.
- Asking, "What are the two values you are using for this question? How could you write the multiplication equation using those two values?"

If students struggle to draw the tape diagram in Problem 2, consider:

- Asking for the total and how it could be drawn on the tape diagram.

Professional Learning

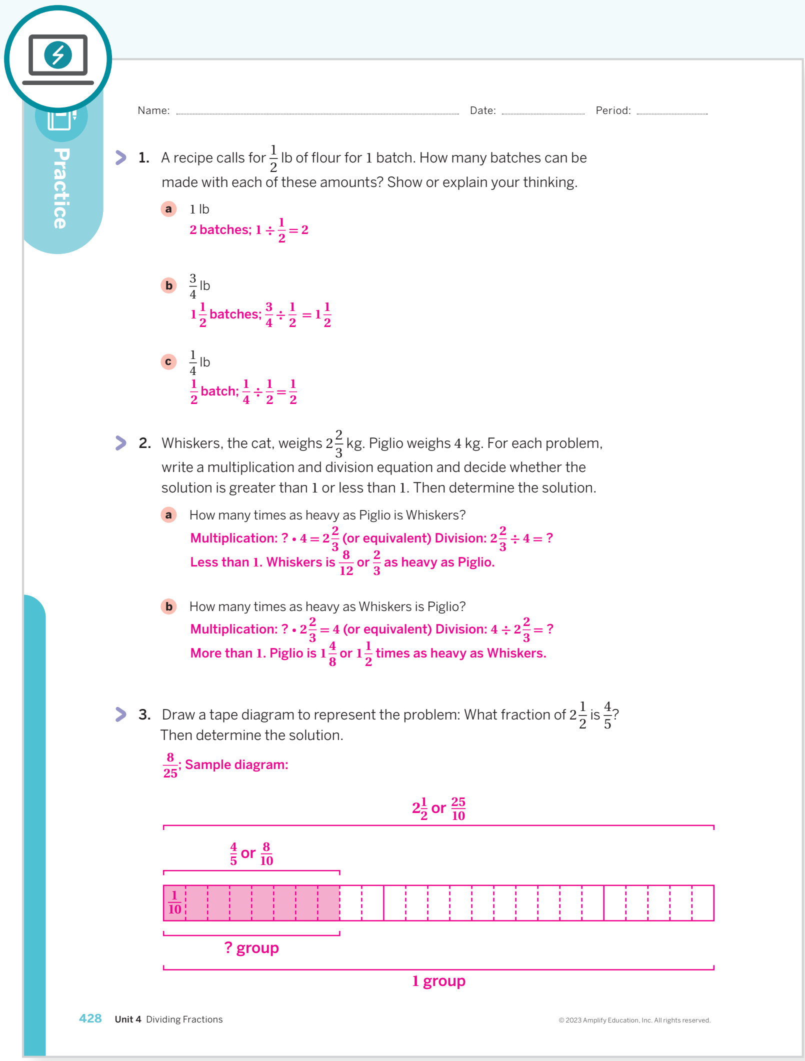
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students used multiplication with fractions and drew diagrams to represent the equations. How will that support division with fractions?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

Independent



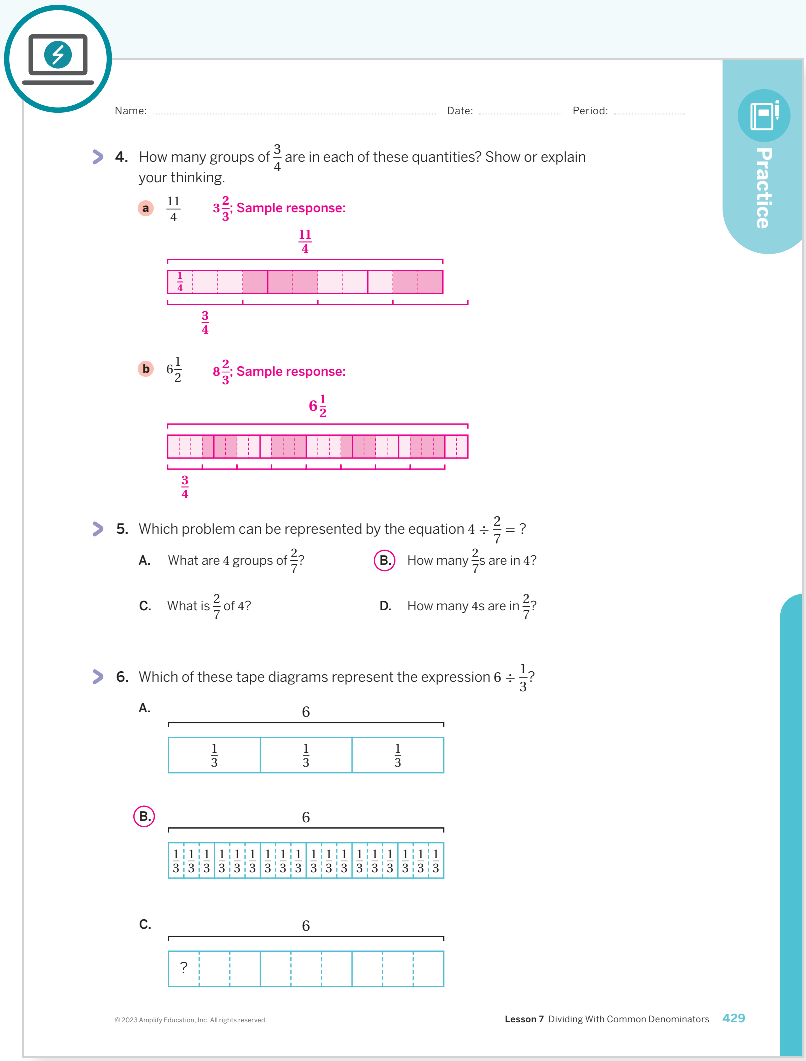
Practice

Name: Date: Period:

1. A recipe calls for $\frac{1}{2}$ lb of flour for 1 batch. How many batches can be made with each of these amounts? Show or explain your thinking.
   - a. 1 lb
     2 batches; $1 \div \frac{1}{2} = 2$
   - b. $\frac{3}{4}$ lb
     $1\frac{1}{2}$ batches; $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$
   - c. $\frac{1}{4}$ lb
     $\frac{1}{2}$ batch; $\frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$

2. Whiskers, the cat, weighs $2\frac{2}{3}$ kg. Piglio weighs 4 kg. For each problem, write a multiplication and division equation and decide whether the solution is greater than 1 or less than 1. Then determine the solution.
   - a. How many times as heavy as Piglio is Whiskers?
     Multiplication: $? \bullet 4 = 2\frac{2}{3}$ (or equivalent) Division: $2\frac{2}{3} \div 4 = ?$
     Less than 1. Whiskers is $\frac{8}{12}$ or $\frac{2}{3}$ as heavy as Piglio.
   - b. How many times as heavy as Whiskers is Piglio?
     Multiplication: $? \bullet 2\frac{2}{3} = 4$ (or equivalent) Division: $4 \div 2\frac{2}{3} = ?$
     More than 1. Piglio is $1\frac{4}{8}$ or $1\frac{1}{2}$ times as heavy as Whiskers.

3. Draw a tape diagram to represent the problem: What fraction of $2\frac{1}{2}$ is $\frac{4}{5}$? Then determine the solution.
   $\frac{8}{25}$; Sample diagram:



Practice

Name: Date: Period:

4. How many groups of $\frac{3}{4}$ are in each of these quantities? Show or explain your thinking.
   - a. $\frac{11}{4}$ **$3\frac{2}{3}$; Sample response:**
   - b. $6\frac{1}{2}$ **$8\frac{2}{3}$; Sample response:**

5. Which problem can be represented by the equation $4 \div \frac{2}{7} = ?$
   - A. What are 4 groups of $\frac{2}{7}$?
   - [✓ B.] How many $\frac{2}{7}$s are in 4?
   - C. What is $\frac{2}{7}$ of 4?
   - D. How many 4s are in $\frac{2}{7}$?

6. Which of these tape diagrams represent the expression $6 \div \frac{1}{3}$?
   - A.
   - [✓ B.]
   - C.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 6	2
	5	Unit 4 Lesson 5	2
Formative	6	Unit 4 Lesson 8	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 4 | **LESSON 8**

How Much in Each Group? (Part 1)

Let's look at division problems that help to determine the size of one group.

Focus

Goals

1. **Language Goal:** Create a tape diagram to represent and solve a problem asking, "How much in 1 group?" where the dividend, divisor, and quotient may be fractions, and explain the solution method. **(Speaking and Listening, Writing)**
2. Write multiplication and division equations to represent a problem asking, "How much in 1 group?"

Coherence

Today

Students will work with scenarios where the number of groups is known, but the size of each group is unknown. In some cases, the number of groups may be a fraction of one group. They write and interpret division expressions as a way to answer "how much in one group?" (partitive division) questions. Students recognize that the same tools — multiplication and division equations, and tape diagrams — can be used to solve these problems as for "how much in one group?" questions, because the structure of equal-sized groups scenarios still applies. They apply this reasoning to determine both whole-number and fractional quotients.

Previously

In Lessons 5-6, students explored division situations in which the number of groups was unknown. Students wrote equations and drew diagrams to detemine the number of groups.

Coming Soon

In Lesson 9, students will write and solve partitive division problems to determine "how much in 1 group". Students will identify when a given scenario represents either a partitive or quotitive division.

Rigor

- Students use tape diagrams, number lines and other models to develop **conceptual understanding** of "how much in a group?"

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

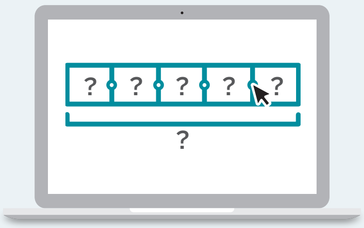
Materials

- Exit Ticket
- Additional Practice

Amps | **Featured Activity**

Activity 2
Digital Diagrams

Students can sketch on digital tape diagrams to solve problems. You can overlay them all to see similarities and differences at a glance.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might not believe that they need tape diagrams to solve these problems. After the activity, spend some time reflecting on how the tape diagrams made the structure of the problems more obvious. Ask students to reflect on their self-efficacy and how the diagrams were more useful than they had first believed.

Modifications to Pacing

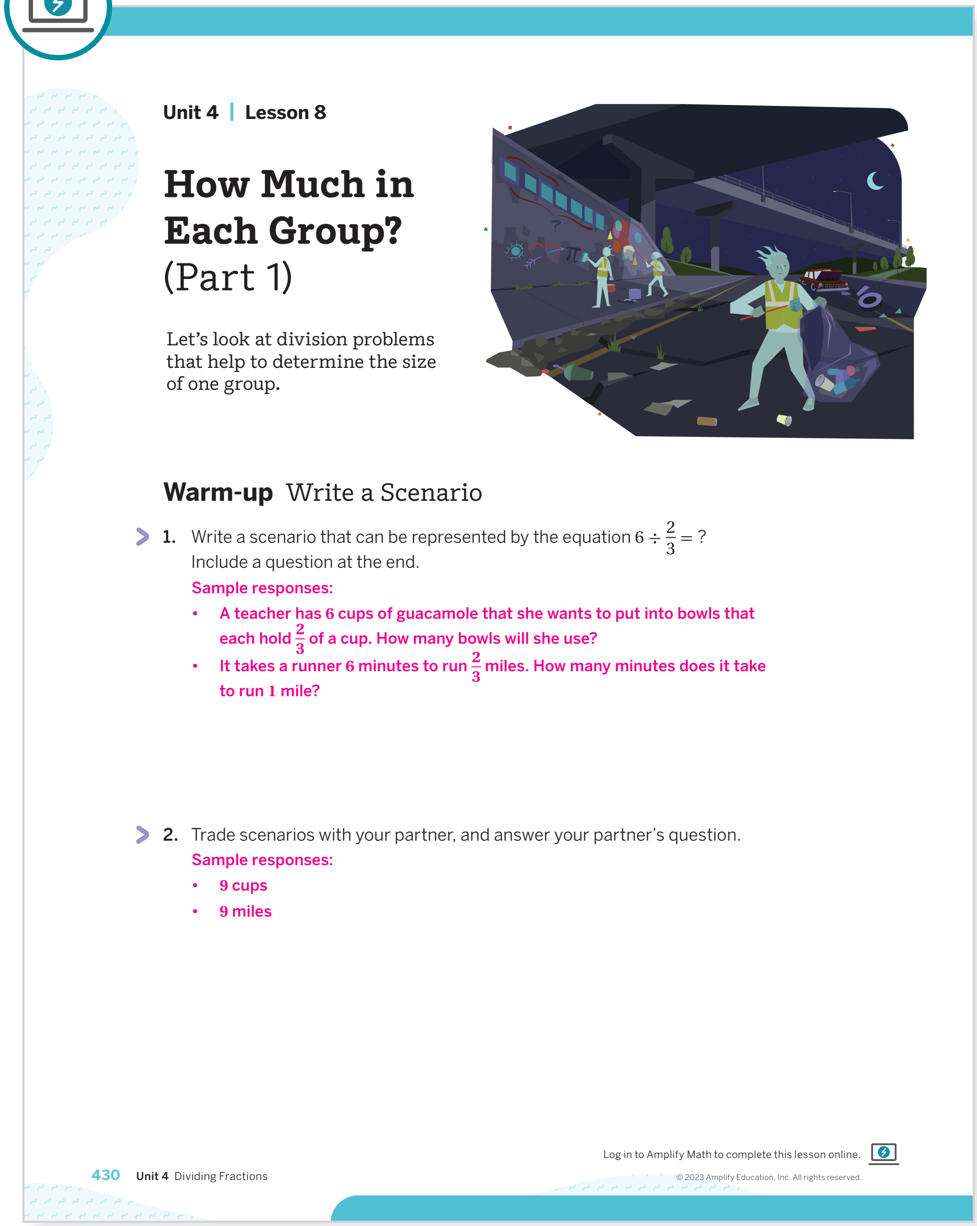
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Part 2 may be omitted, but you may want to consider extending the discussion of Part 1 to include related questions and their answers, and/or corresponding equations, for each diagram.
- In **Activity 2**, Problem 1 may be omitted.

Pairs | 5 min

Warm-up Write a Scenario

Students interpret a division expression by writing a corresponding scenario as a prelude to partitive division.



Unit 4 | Lesson 8

How Much in Each Group? (Part 1)

Let's look at division problems that help to determine the size of one group.

Warm-up Write a Scenario

1. Write a scenario that can be represented by the equation $6 \div \frac{2}{3} = ?$ Include a question at the end.

   Sample responses:
   - A teacher has 6 cups of guacamole that she wants to put into bowls that each hold $\frac{2}{3}$ of a cup. How many bowls will she use?
   - It takes a runner 6 minutes to run $\frac{2}{3}$ miles. How many minutes does it take to run 1 mile?

2. Trade scenarios with your partner, and answer your partner's question.

   Sample responses:
   - 9 cups
   - 9 miles

Log in to Amplify Math to complete this lesson online.

430 Unit 4 Dividing Fractions

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1 Launch

Set an expectation for the amount of time that pairs of students will have to work on the activity.

2 Monitor

Help students get started by asking, "What model can you draw to help think about this problem?"

Look for points of confusion:

- **Struggling to think of a context.** Ask, "What does $6 \div \frac{2}{3}$ mean? Is there a related multiplication equation that could help you determine a context?"
- **Not knowing how to divide to solve the equation.** Ask, "What is a related multiplication expression that could help you?"

Look for productive strategies:

- Using $\frac{2}{3}$ to write a partitive (not quotitive, as they did in earlier lessons) scenario.
- Drawing a diagram or using a related multiplication equation to determine the quotient.

3 Connect

Display the equation.

Have pairs of students share their scenarios and how it corresponds to the division equation. Then have students share the quotient and relate it to one or more of the scenarios.

Ask:

- "How did you think of a context?"
- "Did anyone use a model to determine the solution? If so, which model did you use and how did it help?"

Highlight that, regardless of the context of each scenario, the solution is the same because they are all based on the same division equation. Some scenarios represented "how many groups?" while others represented "how much in each group?," which will be the focus of the other activities in this lesson.

Math Language Development

MLR7: Compare and Connect

During the Connect, as pairs of students share their scenarios and relate them to the division equation, draw their attention to the types of questions that are asked in the scenarios that they and their classmates wrote. Consider asking these follow-up questions:

- "What value represents the dividend in your scenario? How did you know this should be the dividend?"
- "What corresponding multiplication equation can you write? Did anyone use this as a strategy before thinking of a scenario?"

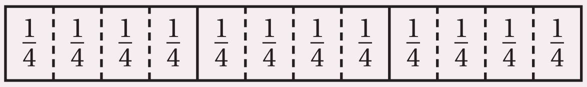
English Learners

Annotate the *dividend, divisor,* and *quotient* in a few of the scenarios.

Power-up

To power up students' ability to identify how many fractional pieces are in a whole, have students complete:

Which two equations represent the tape diagram?



[✓ **A.**] $3 \div \frac{1}{4} = 12$ **C.** $3 \div 4 = \frac{1}{4}$

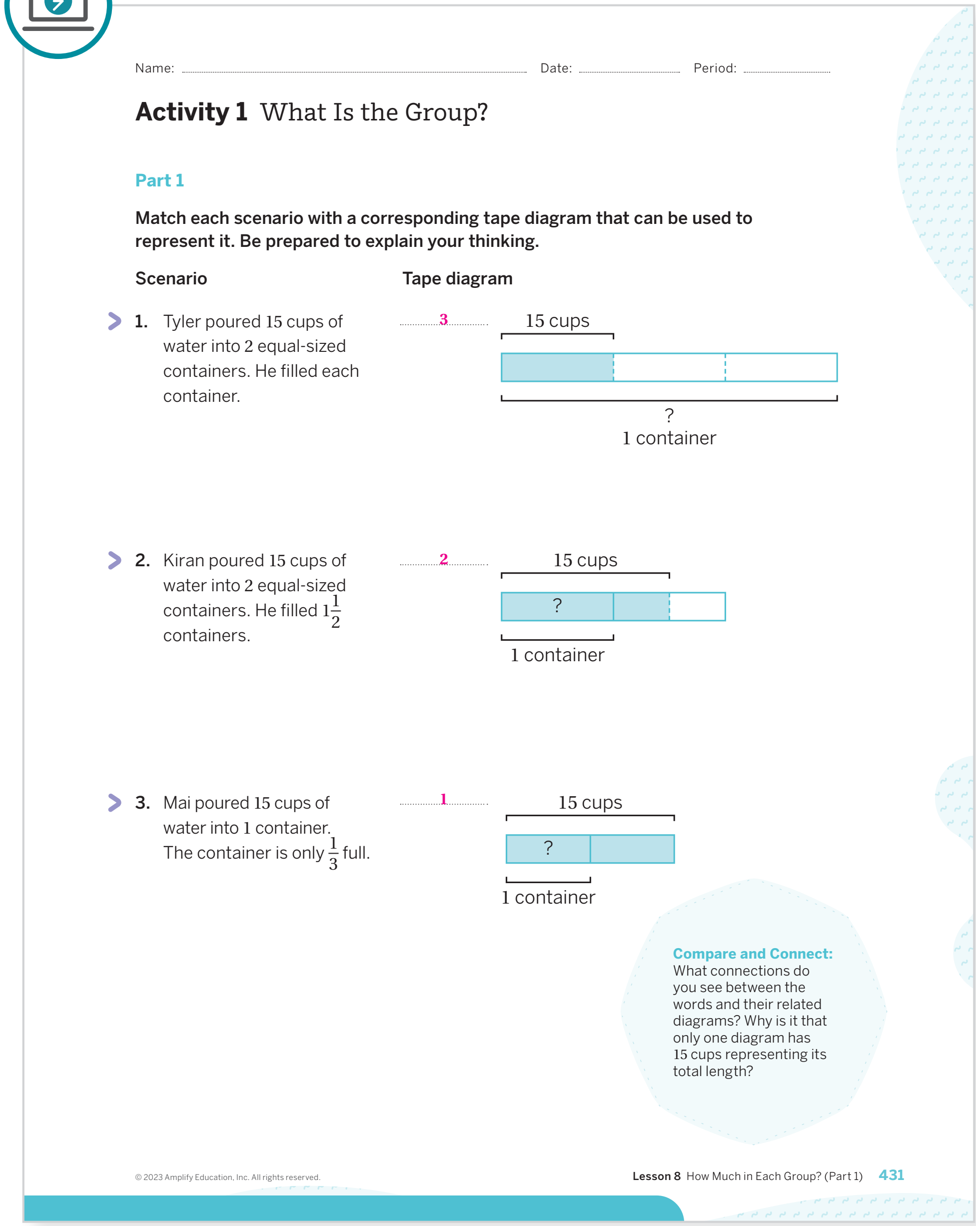
B. $\frac{1}{4} \div 3 = 12$ [✓ **D.**] $3 \div 12 = \frac{1}{4}$

Use: Before Activity 1.
Informed by: Performance on Lesson 7, Practice Problem 6.

Pairs | 15 min

Activity 1 What Is the Group?

Students coordinate among scenarios, diagrams, and equations to determine the amount in one group, including when a given value corresponds to a fraction of a group.



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Activity 1 What Is the Group?

Part 1

Match each scenario with a corresponding tape diagram that can be used to represent it. Be prepared to explain your thinking.

Scenario		Tape diagram
1. Tyler poured 15 cups of water into 2 equal-sized containers. He filled each container.	3	15 cups ? 1 container
2. Kiran poured 15 cups of water into 2 equal-sized containers. He filled $1\frac{1}{2}$ containers.	2	15 cups ? 1 container
3. Mai poured 15 cups of water into 1 container. The container is only $\frac{1}{3}$ full.	1	15 cups ? 1 container

Compare and Connect: What connections do you see between the words and their related diagrams? Why is it that only one diagram has 15 cups representing its total length?

Lesson 8 How Much in Each Group? (Part 1) 431

1 Launch

Have students conduct the *Think-Pair-Share* routine, giving them 2–3 minutes to complete Problem 1, before comparing and discussing the answers with a partner and then completing Part 2 together.

2 Monitor

Help students get started by asking, "How is the size of one group represented in each tape diagram? How can you relate that to a given piece of information in one of the scenarios?"

Look for points of confusion:

- **Interpreting the scenarios by using "how many groups" thinking.** Have students quickly review a scenario from Lesson 5 or 6. Ask, "How are these different?"
- **Confusing units of cups and containers.** Have students add units to the values in their equations and read them aloud.
- **Struggling to solve $15 \div 1\frac{1}{2} = ?$ (Scenario 2).** Ask, "How many half-containers are in 1 whole container? How could that help you?"
- **Thinking the size of one group must always be less than the given total of 15 cups (Scenario 3).** Ask, "What does it say 15 cups represents?" Then point to the first tape diagram and ask, "Does it look like it makes sense for there to be more than 15 cups in 1 container?"

Look for productive strategies:

- Writing 2 questions to reflect one full container as the unknown "size of a group" for each scenario.
- Using the tape diagrams to help determine the unknown number of cups in one container.
- Coordinating the relationships between scenarios and diagrams with equations, noticing patterns as values are placed.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest students annotate each scenario with how many containers, or how much of one container, was filled in each problem. Then ask them if this helps them determine the matching tape diagram.

Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students write the questions, equations, and answers in Part 2, provide these on different slips of paper. Have students sort the slips as to which scenario they belong. Alternatively, just provide the questions on slips of paper. Then ask students to write the equations and answers to the question.

Math Language Development

MLR7: Compare and Connect

While students work, point out the Compare and Connect question in their Student Edition. Ask them to think about this question as they progress through the activity. During the Connect, revisit this question and ask students to share their thinking. Emphasize how both containers were filled in Scenario 1, which illustrates why its corresponding tape diagram represented the total length.

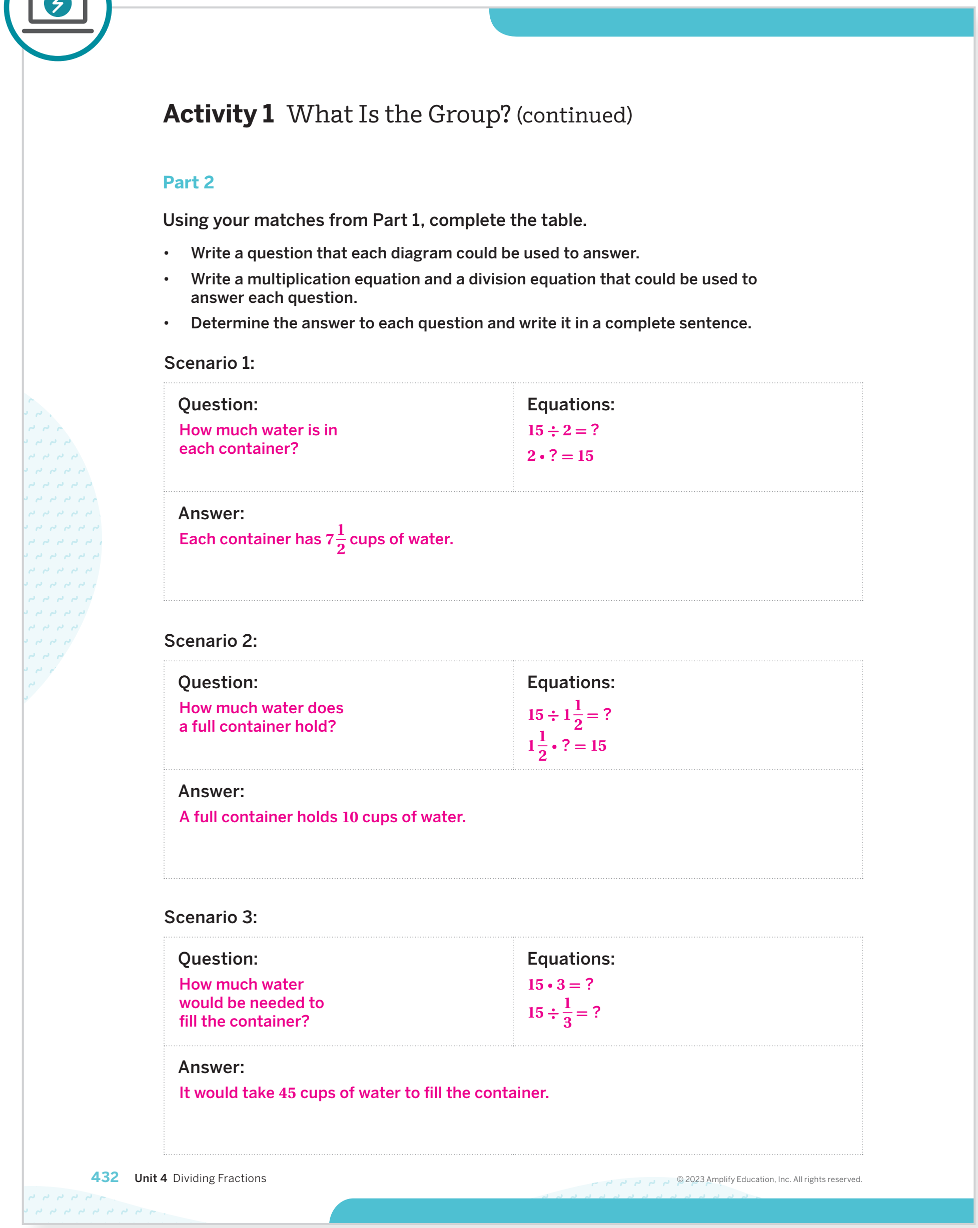
English Learners

Use color coding to highlight how 15 cups in each scenario relates to the size of 1 container, both in words and in the tape diagram.

Pairs | 15 min

Activity 1 What Is the Group? (continued)

Students coordinate among scenarios, diagrams, and equations to determine the amount in one group, including when a given value corresponds to a fraction of a group.



Activity 1 What Is the Group? (continued)

Part 2

Using your matches from Part 1, complete the table.

- Write a question that each diagram could be used to answer.
- Write a multiplication equation and a division equation that could be used to answer each question.
- Determine the answer to each question and write it in a complete sentence.

Scenario 1:

Question:	Equations:
How much water is in each container?	$15 \div 2 = ?$ $2 \cdot ? = 15$
Answer: Each container has $7\frac{1}{2}$ cups of water.	

Scenario 2:

Question:	Equations:
How much water does a full container hold?	$15 \div 1\frac{1}{2} = ?$ $1\frac{1}{2} \cdot ? = 15$
Answer: A full container holds 10 cups of water.	

Scenario 3:

Question:	Equations:
How much water would be needed to fill the container?	$15 \cdot 3 = ?$ $15 \div \frac{1}{3} = ?$
Answer: It would take 45 cups of water to fill the container.	

3 Connect

Display the tape diagrams.

Have students share their questions for each scenario and explain their thinking. Then have them share their process for how they determined their answers.

Ask:

- "What information was missing from each of the tape diagrams? And what units correspond to that?" Size of a group; cups
- "How do the diagrams represent both a division equation and a multiplication equation? Explain your thinking." The total of 15 cups can always be interpreted as either the dividend or the product.

Highlight that for both "how many groups?" (in previous lessons) and "how many in one group?" (in this lesson), the quotient is the same for any given expression. Only the known and unknown differ, and they may have different units. Tape diagrams and equations are both useful tools for answering either of these questions.

Differentiated Support

Extension: *Math Around the World, Interdisciplinary Connections*

Earlier Extensions in this unit explored how ancient Egyptian and Babylonian mathematicians represented and worked with fractions. Tell students that around 30 BCE, historical records showed that the Chinese mathematicians worked with fractions, including addition, subtraction, and multiplication of fractions. They used a base 10 system and placed the numerator above the denominator, both of which we use today. They did not separate the numerator and denominator with a horizontal line as we do today.

Similarly, around 500 CE, there is historical evidence that the Hindu mathematicians used fractions in a similar way, placing the numerator above the denominator with no horizontal line to separate them.

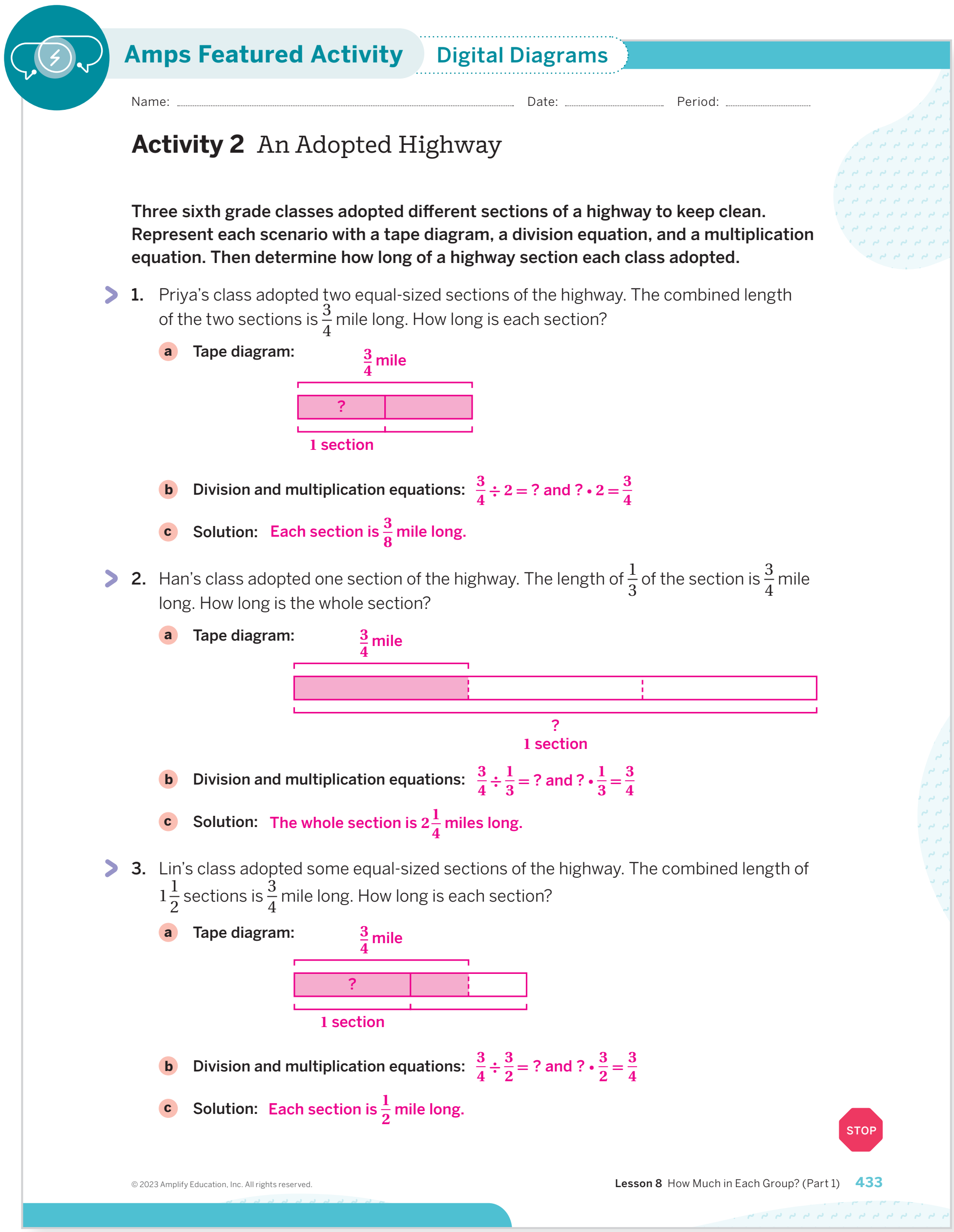
Around 1200 CE, Arab mathematicians introduced the horizontal line that separates the numerator from the denominator. Many sources attribute this notation to the Arab mathematician al-Hassar. This same notation later appeared in Fibonacci's writings in the 13th century. **(History)**

Pairs | 15 min

Activity 2 An Adopted Highway

Students interpret scenarios involving cleaning sections of a highway to draw tape diagrams and to write equations to answer questions about “how much in one section?”

Amps Featured Activity Digital Diagrams



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Activity 2 An Adopted Highway

Three sixth grade classes adopted different sections of a highway to keep clean. Represent each scenario with a tape diagram, a division equation, and a multiplication equation. Then determine how long of a highway section each class adopted.

1. Priya's class adopted two equal-sized sections of the highway. The combined length of the two sections is $\frac{3}{4}$ mile long. How long is each section?
   - **a** **Tape diagram:** $\frac{3}{4}$ mile ? 1 section
   - **b** **Division and multiplication equations:** $\frac{3}{4} \div 2 = ?$ and $? \cdot 2 = \frac{3}{4}$
   - **c** **Solution:** Each section is $\frac{3}{8}$ mile long.
2. Han's class adopted one section of the highway. The length of $\frac{1}{3}$ of the section is $\frac{3}{4}$ mile long. How long is the whole section?
   - **a** **Tape diagram:** $\frac{3}{4}$ mile ? 1 section
   - **b** **Division and multiplication equations:** $\frac{3}{4} \div \frac{1}{3} = ?$ and $? \cdot \frac{1}{3} = \frac{3}{4}$
   - **c** **Solution:** The whole section is $2\frac{1}{4}$ miles long.
3. Lin's class adopted some equal-sized sections of the highway. The combined length of $1\frac{1}{2}$ sections is $\frac{3}{4}$ mile long. How long is each section?
   - **a** **Tape diagram:** $\frac{3}{4}$ mile ? 1 section
   - **b** **Division and multiplication equations:** $\frac{3}{4} \div \frac{3}{2} = ?$ and $? \cdot \frac{3}{2} = \frac{3}{4}$
   - **c** **Solution:** Each section is $\frac{1}{2}$ mile long.

STOP

Lesson 8 How Much in Each Group? (Part 1) 433

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the three problems.

2 Monitor

Help students get started by asking, “What information is known? What is unknown? How can you use that to help you draw a diagram?”

Look for points of confusion:

- **Struggling to draw a correct diagram with fractions of a group.** Ask, “To which scenario from Activity 1 is this similar? Can you use its corresponding diagram to help you?”
- **Writing incorrect equations, or miscalculating unknowns.** Ensure students have a correct diagram, and then ask: “How can you describe what this diagram shows in words? What are you trying to determine, and what operation would help you do that? Does your solution look like it makes sense?”

Look for productive strategies:

- Recognizing that every diagram includes $\frac{3}{4}$ mile, but, in each problem, it is associated with a different number of sections or groups.
- Using diagrams and equations to represent and solve the problem in context.

3 Connect

Have students share how they modeled each problem using both diagrams and equations, and how they determined their solutions.

Ask, “What is similar and what is different in each of your tape diagrams?”

Highlight how these three types of diagrams can be used to solve any “how much in one group” problem involving fractions, when the number of groups is a whole number, or a fraction less than or greater than 1.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital tape diagrams. After they create their digital tape diagrams, you can use the digital technology to overlay them and see their similarities and differences.

Extension: *Math Enrichment*

Provide the following as Scenario 4 and have students model it with a tape diagram and equations. Then ask them to solve the problem.

Scenario 4: Another class cleaned $1\frac{1}{2}$ miles of highway, which is $\frac{3}{4}$ of their adopted section. How long is their adopted section?” 2 miles

Math Language Development

MLR8: Discussion Supports

While students work, display the following sentence frames to help them coordinate the quantities and relationships in each scenario.

Scenarios 1 and 3: “There are _____ sections. Each section is _____ miles long. The total length is _____ miles long.”

Scenario 2: “There is one section. _____ of this section is _____ long. The total length of this section is _____ miles.”

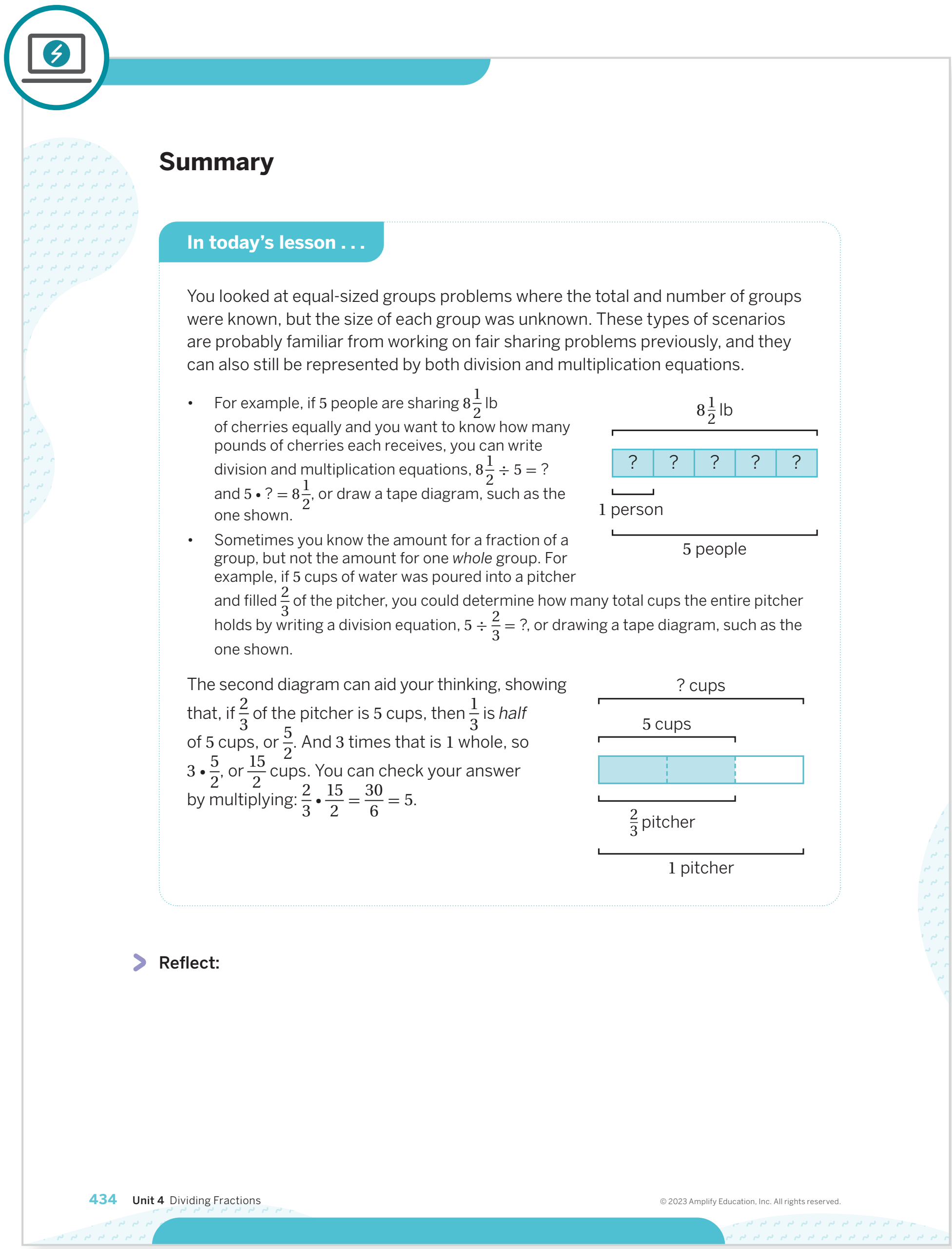
English Learners

Annotate key words and phrases, such as *equal-sized sections, combined length, how long is each section,* etc.

Whole Class | 5 min

Summary

Review and synthesize how to determine "how much in one group?" by dividing fractions in different types of problems.



Summary

In today's lesson . . .

You looked at equal-sized groups problems where the total and number of groups were known, but the size of each group was unknown. These types of scenarios are probably familiar from working on fair sharing problems previously, and they can also still be represented by both division and multiplication equations.

- For example, if 5 people are sharing $8\frac{1}{2}$ lb of cherries equally and you want to know how many pounds of cherries each receives, you can write division and multiplication equations, $8\frac{1}{2} \div 5 = ?$ and $5 \bullet ? = 8\frac{1}{2}$, or draw a tape diagram, such as the one shown.
- Sometimes you know the amount for a fraction of a group, but not the amount for one *whole* group. For example, if 5 cups of water was poured into a pitcher and filled $\frac{2}{3}$ of the pitcher, you could determine how many total cups the entire pitcher holds by writing a division equation, $5 \div \frac{2}{3} = ?$, or drawing a tape diagram, such as the one shown.

The second diagram can aid your thinking, showing that, if $\frac{2}{3}$ of the pitcher is 5 cups, then $\frac{1}{3}$ is *half* of 5 cups, or $\frac{5}{2}$. And 3 times that is 1 whole, so $3 \bullet \frac{5}{2}$, or $\frac{15}{2}$ cups. You can check your answer by multiplying: $\frac{2}{3} \bullet \frac{15}{2} = \frac{30}{6} = 5$.

Reflect:

434 Unit 4 Dividing Fractions

Synthesize

Display the tape diagrams from the Student Edition Summary.

Ask:

- "What is similar and what is different about these two diagrams?"
- "How can you use tape diagrams like these to help you determine an unknown amount in one group?"

Highlight that sometimes the amount of multiple groups or a fraction of a group is known, but not the amount in *one* group. In either case, there is always a corresponding division equation whose unknown quotient represents the amount in one group.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

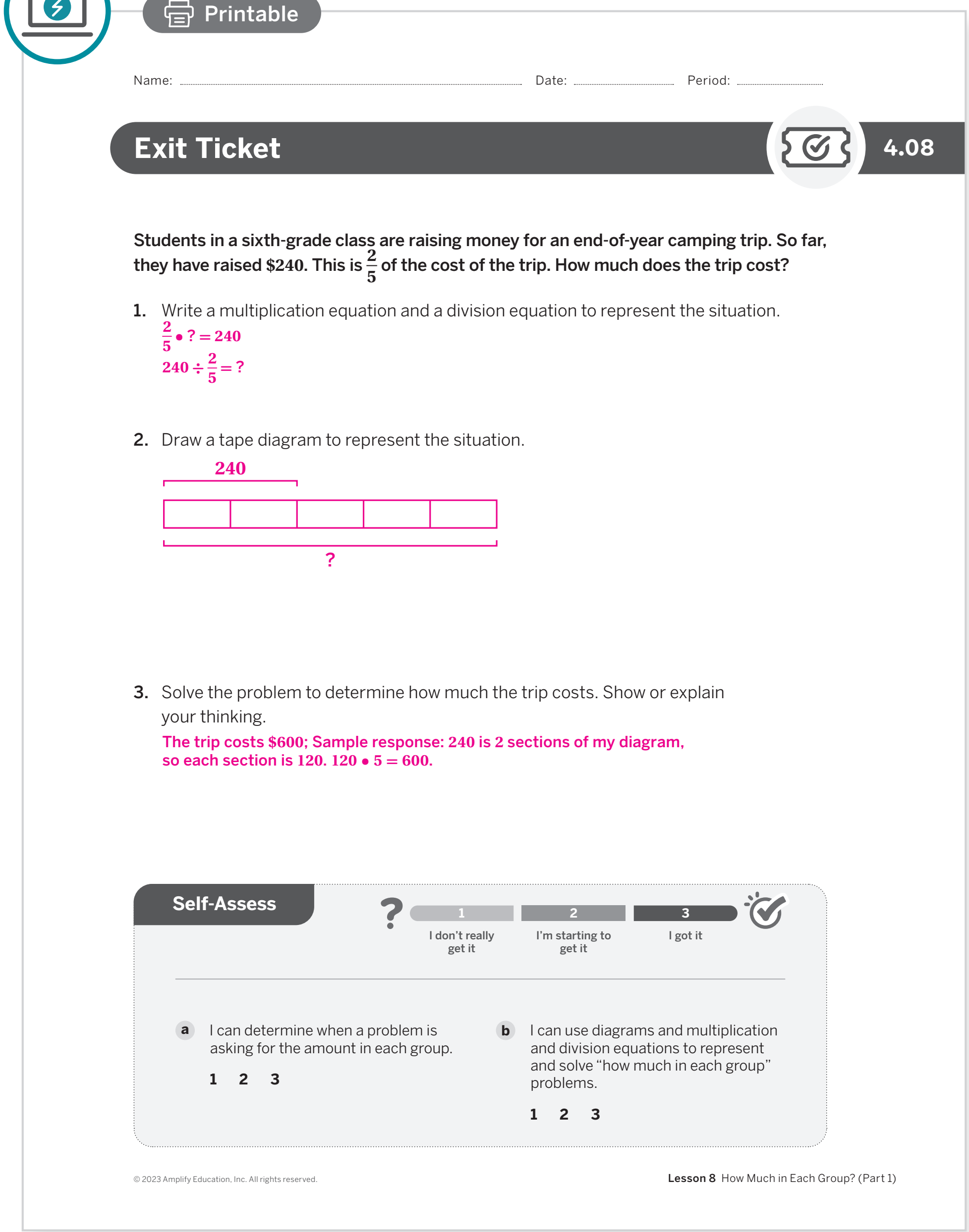
- "How can dividing by the same fraction be interpreted in two different ways?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of how to determine the size of one group by writing equations, drawing tape diagrams, and then solving for an unknown.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 4.08

Students in a sixth-grade class are raising money for an end-of-year camping trip. So far, they have raised $240. This is $\frac{2}{5}$ of the cost of the trip. How much does the trip cost?

1. Write a multiplication equation and a division equation to represent the situation.
   $\frac{2}{5} \bullet ? = 240$
   $240 \div \frac{2}{5} = ?$

2. Draw a tape diagram to represent the situation.

3. Solve the problem to determine how much the trip costs. Show or explain your thinking.
   The trip costs $600; Sample response: 240 is 2 sections of my diagram, so each section is 120. $120 \bullet 5 = 600$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can determine when a problem is asking for the amount in each group.
1 2 3

b I can use diagrams and multiplication and division equations to represent and solve "how much in each group" problems.
1 2 3

Lesson 8 How Much in Each Group? (Part 1)

Success looks like . . .

- **Language Goal:** Creating a tape diagram to represent and solve a problem asking, "How much in 1 group?" where the dividend, divisor, and quotient may be fractions, and explain the solution method. **(Speaking and Listening, Writing)**
  - » Drawing a tape diagram to determine the cost of 1 trip.
- **Goal:** Writing multiplication and division equations to represent a problem asking, "How much in 1 group?"
  - » Writing an equation to represent the cost on 1 trip.

Suggested next steps

If students have difficulty writing one or both equations for Problem 1, consider:

- Asking, "What is known? What is unknown? How can you use that information to write an equation and draw a diagram?"
- Reviewing Activity 2 to show how the diagrams and scenarios corresponded to one another.

If students have trouble drawing the tape diagram, consider:

- Asking, "What is known? What is unknown? How can you represent that information in a tape diagram?"
- Referencing the second tape diagram from the Student Edition Summary to use as a model.

Professional Learning

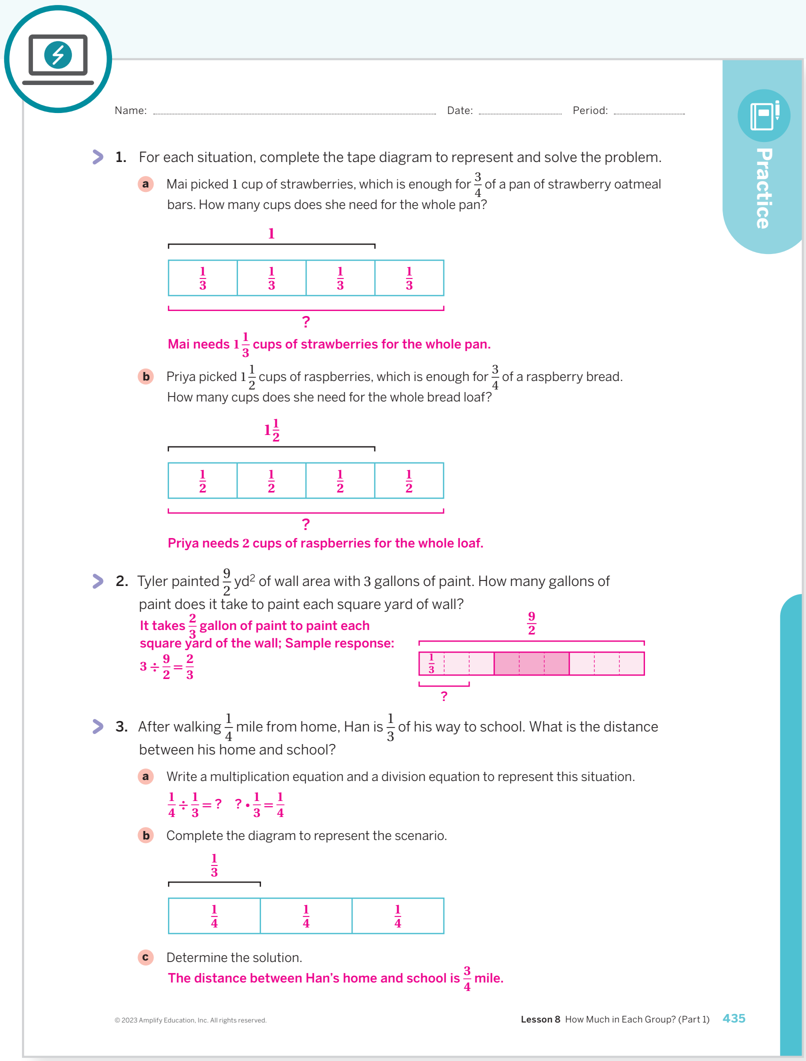
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- 6.NS.A.1 asks students to determine quotients of fractions using models and equations. Where in your students' work did you see evidence of them doing this today? What might you repeat or do differently the next time you teach this lesson?
- What worked and didn't work today? During Activity 2, how did you encourage each student to listen to one another's strategies?

Practice

Independent



Name: Date: Period:

Practice

1. For each situation, complete the tape diagram to represent and solve the problem.
   - a Mai picked 1 cup of strawberries, which is enough for $\frac{3}{4}$ of a pan of strawberry oatmeal bars. How many cups does she need for the whole pan?

Mai needs $1\frac{1}{3}$ cups of strawberries for the whole pan.

   - b Priya picked $1\frac{1}{2}$ cups of raspberries, which is enough for $\frac{3}{4}$ of a raspberry bread. How many cups does she need for the whole bread loaf?

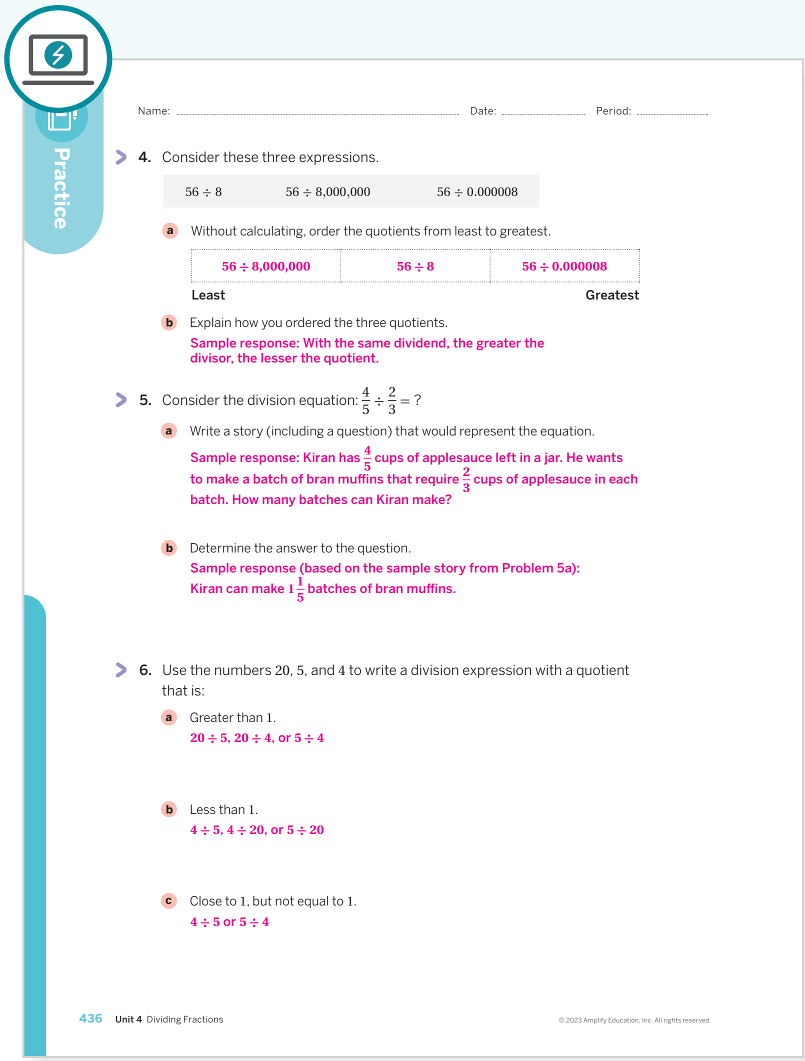
Priya needs 2 cups of raspberries for the whole loaf.

2. Tyler painted $\frac{9}{2}$ yd^2 of wall area with 3 gallons of paint. How many gallons of paint does it take to paint each square yard of wall?

It takes $\frac{2}{3}$ gallon of paint to paint each square yard of the wall; Sample response:
$3 \div \frac{9}{2} = \frac{2}{3}$

3. After walking $\frac{1}{4}$ mile from home, Han is $\frac{1}{3}$ of his way to school. What is the distance between his home and school?
   - a Write a multiplication equation and a division equation to represent this situation.
     $\frac{1}{4} \div \frac{1}{3} = ?$ $? \cdot \frac{1}{3} = \frac{1}{4}$
   - b Complete the diagram to represent the scenario.

   - c Determine the solution.
     The distance between Han's home and school is $\frac{3}{4}$ mile.



Name: Date: Period:

Practice

4. Consider these three expressions.

$56 \div 8$ $56 \div 8{,}000{,}000$ $56 \div 0.000008$

   - a Without calculating, order the quotients from least to greatest.

$56 \div 8{,}000{,}000$	**$56 \div 8$**	**$56 \div 0.000008$**

Least **Greatest**

   - b Explain how you ordered the three quotients.
     Sample response: With the same dividend, the greater the divisor, the lesser the quotient.

5. Consider the division equation: $\frac{4}{5} \div \frac{2}{3} = ?$
   - a Write a story (including a question) that would represent the equation.
     Sample response: Kiran has $\frac{4}{5}$ cups of applesauce left in a jar. He wants to make a batch of bran muffins that require $\frac{2}{3}$ cups of applesauce in each batch. How many batches can Kiran make?
   - b Determine the answer to the question.
     Sample response (based on the sample story from Problem 5a): Kiran can make $1\frac{1}{5}$ batches of bran muffins.

6. Use the numbers 20, 5, and 4 to write a division expression with a quotient that is:
   - a Greater than 1.
     $20 \div 5$, $20 \div 4$, or $5 \div 4$
   - b Less than 1.
     $4 \div 5$, $4 \div 20$, or $5 \div 20$
   - c Close to 1, but not equal to 1.
     $4 \div 5$ or $5 \div 4$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activities 1, 2	2
	2	Activities 1, 2	2
	3	Activities 1, 2	2
Spiral	4	Unit 4 Lesson 4	2
	5	Unit 4 Lesson 7	2
Formative ⚡	6	Unit 4 Lesson 9	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 4 | **LESSON 9**

How Much in Each Group?
(Part 2)

Let's practice dividing fractions in real-world scenarios.

Focus

Goals

1. **Language Goal:** Interpret a situation involving equal-sized groups, and generate mathematical questions that could be asked about it. **(Reading and Writing, Speaking and Listening)**
2. **Language Goal:** Solve a problem involving division of fractions, and present the solution method. **(Reading and Writing, Speaking and Listening)**
3. **Language Goal:** Compare and contrast strategies for solving problems about "how many groups?" and "how much in one group?" **(Speaking and Listening)**

Coherence

Today

Students practice determining the amount in one group by interpreting, representing, and solving real-world division problems. They write their own division story problems representing both number of groups and amount in each group scenarios. Students make sense of these problems by creating models to help solve them. As students move back and forth between the contexts, the abstract equations, and the diagrams that represent the problems, they reason abstractly and quantitatively.

Previously

In Lesson 8, students were introduced to "how much in one group?" They represented story problems using models and equations and then used this work to solve division problems.

Coming Soon

In Lessons 10 and 11, students will develop a general algorithm for dividing fractions.

Rigor

- Students **apply** their understanding of "how much in one group?" versus "how many groups?" by writing their own scenarios, and using diagrams and equations to solve problems.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	5 min	5 min
Pairs	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

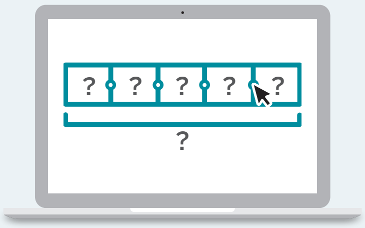
Materials

- Exit Ticket
- Additional Practice

Amps | **Featured Activity**

Activity 1
Digital Diagrams

Students can sketch on digital tape diagrams to solve problems. You can overlay them all to see similarities and differences at a glance.



Amps
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Building Math Identity and Community
Connecting to Mathematical Practices

Students might struggle to understand the problem in Activity 1. Encourage students to use organizational tools and representations to help them make sense of the problem. Remind them to use their classmates as resources, too. Students can encourage each other as they persevere. Such encouragement will motivate students to do their best and finish the task.

Modifications to Pacing

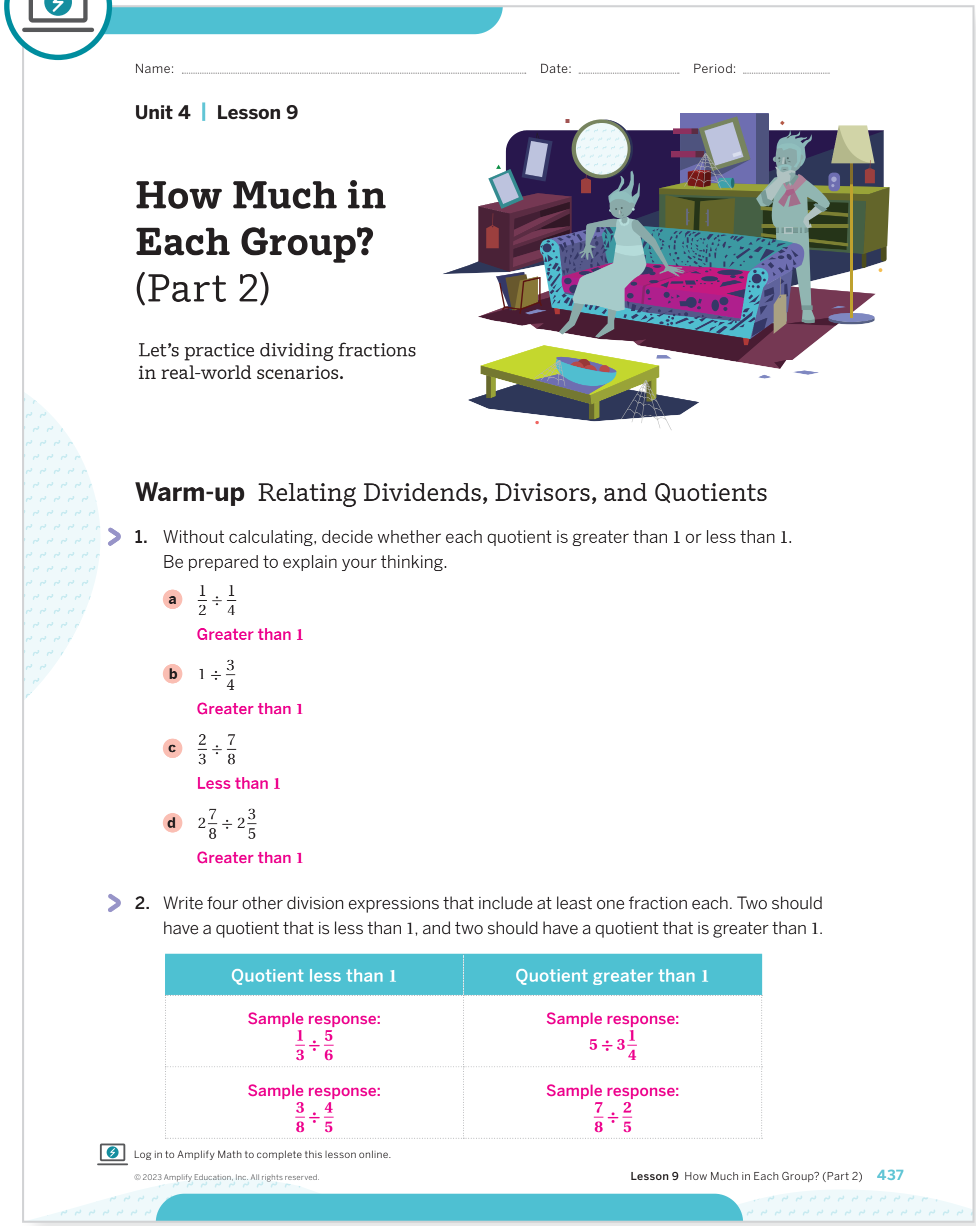
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, have different pairs of students work with each of the first three scenarios (quotitive), but all pairs should also work with the fourth scenario (partitive).
- In **Activity 2**, have different pairs work with one type of division, rather than having each pair work with both types.

Pairs | 10 min

Warm-up Relating Dividends, Divisors, and Quotients

Students use the interpretation of fractions as division to determine whether a quotient is less than or greater than 1.



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Unit 4 | **Lesson 9**

How Much in Each Group?
(Part 2)

Let's practice dividing fractions in real-world scenarios.

Warm-up Relating Dividends, Divisors, and Quotients

1. Without calculating, decide whether each quotient is greater than 1 or less than 1. Be prepared to explain your thinking.
   - a $\frac{1}{2} \div \frac{1}{4}$
     Greater than 1
   - b $1 \div \frac{3}{4}$
     Greater than 1
   - c $\frac{2}{3} \div \frac{7}{8}$
     Less than 1
   - d $2\frac{7}{8} \div 2\frac{3}{5}$
     Greater than 1

2. Write four other division expressions that include at least one fraction each. Two should have a quotient that is less than 1, and two should have a quotient that is greater than 1.

Quotient less than 1	Quotient greater than 1
Sample response: $\frac{1}{3} \div \frac{5}{6}$	Sample response: $5 \div 3\frac{1}{4}$
Sample response: $\frac{3}{8} \div \frac{4}{5}$	Sample response: $\frac{7}{8} \div \frac{2}{5}$

Log in to Amplify Math to complete this lesson online.

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Lesson 9 How Much in Each Group? (Part 2) 437

1 Launch

Activate prior knowledge by drawing a number line and asking students how to plot halves, fourths, and eighths. Have students conduct the *Think-Pair-Share* routine, giving them 2 minutes to complete Problem 1, before sharing answers with a partner and completing Problem 2 together.

2 Monitor

Help students get started by asking, "Can you use whole numbers to write a division expression whose quotient is less than 1? How might your thinking in determining that expression help you here?"

Look for points of confusion

- **Thinking they need to actually divide (and perhaps cannot).** Ask, "Which is greater: the dividend or the divisor? What does that tell you about the quotient?"
- **Struggling to come up with division expressions for Problem 2 that include fractions.** Ask, "Can you use whole numbers first, and then replace either the dividend or the divisor with a fraction that does not affect the relative size of the quotient?"

Look for productive strategies:

- Recognizing that regardless of the values or types of numbers, when a dividend is greater than a divisor, the quotient is greater than 1; and vice versa.

3 Connect

Display a blank table like the one from Problem 2.

Have students share their answers and thinking for Problem 1, followed by placing each expression in the table. Then collect expressions for Problem 2 in the appropriate columns as well, drawing attention to different examples where the fractional values are dividends and/or divisors.

Highlight that regardless of the values or types of numbers, comparing the divisor to the dividend can help make estimations and determine whether quotients are reasonable.

Differentiated Support

Accessibility: ***Activate Prior Knowledge, Guide Processing and Visualization***

Display two division expressions using whole numbers, such as 4 ÷ 12 and 15 ÷ 3. Ask them how they know, without dividing, whether the quotient will be greater than 1 or less than 1. Then ask them if they can use similar reasoning to analyze the problems in the Warm-up.

Power-up

To power up students' ability to determine whether the quotient of two values is less than, approximately equal to, or greater than 1, ask, have students complete:

Order the expressions from least quotient to greatest quotient.

$8 \div 4$	$8 \div 8$	$8 \div \frac{1}{2}$	$8 \div 15$	$8 \div 7$
Least				**Greatest**
$8 \div 15$	$8 \div 8$	$8 \div 7$	$8 \div 4$	$8 \div \frac{1}{2}$

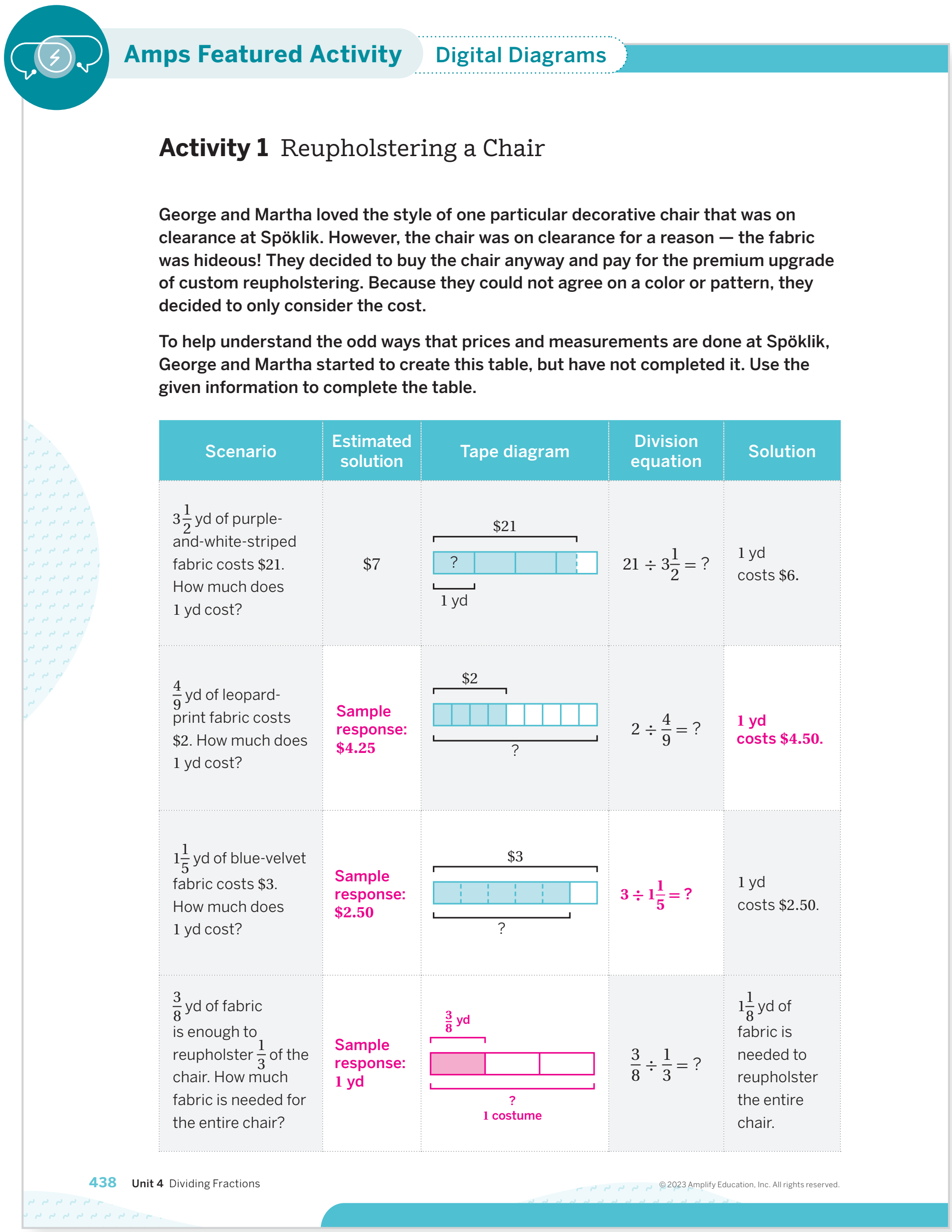
Use: Before the Warm-up.
Informed by: Performance on Lesson 8, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 1.

Independent | 15 min

Activity 1 Reupholstering a Chair

Students model and solve several division problems about "how much in one group" across a variety of cases involving fractional amounts.

Amps Featured Activity Digital Diagrams



Activity 1 Reupholstering a Chair

George and Martha loved the style of one particular decorative chair that was on clearance at Spöklik. However, the chair was on clearance for a reason — the fabric was hideous! They decided to buy the chair anyway and pay for the premium upgrade of custom reupholstering. Because they could not agree on a color or pattern, they decided to only consider the cost.

To help understand the odd ways that prices and measurements are done at Spöklik, George and Martha started to create this table, but have not completed it. Use the given information to complete the table.

Scenario	Estimated solution	Tape diagram	Division equation	Solution
$3\frac{1}{2}$ yd of purple-and-white-striped fabric costs \$21. How much does 1 yd cost?	\$7	\$21 ? 1 yd	$21 \div 3\frac{1}{2} = ?$	1 yd costs \$6.
$\frac{4}{9}$ yd of leopard-print fabric costs \$2. How much does 1 yd cost?	**Sample response: \$4.25**	\$2 ?	$2 \div \frac{4}{9} = ?$	**1 yd costs \$4.50.**
$1\frac{1}{5}$ yd of blue-velvet fabric costs \$3. How much does 1 yd cost?	**Sample response: \$2.50**	\$3 ?	$3 \div 1\frac{1}{5} = ?$	1 yd costs \$2.50.
$\frac{3}{8}$ yd of fabric is enough to reupholster $\frac{1}{3}$ of the chair. How much fabric is needed for the entire chair?	**Sample response: 1 yd**	$\frac{3}{8}$ yd ? 1 costume	$\frac{3}{8} \div \frac{1}{3} = ?$	$1\frac{1}{8}$ yd of fabric is needed to reupholster the entire chair.

438 Unit 4 Dividing Fractions

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1 Launch

Set an expectation for the amount of time students will have to work independently on the activity. Explain what they will share at the end, and how.

2 Monitor

Help students get started by referencing the first example row. Ask, "What is each part of the tape diagram representing?"

Look for points of confusion:

- **Confusing the given number of groups (e.g., $\frac{4}{9}$ yards) as the size of a group.** Have students name all of the values in their tape diagram with units to help them see the mismatch.
- **Switching the dividend and divisor in their equations.** Ask, "What is the total amount?"
- **Still struggling to execute a strategy for determining the solutions.** Have students use a correctly drawn tape diagram to identify how the unknown value for one group is represented, and then ask, "If each part of the diagram represents 1, does that look correct? If not, what should they be?"

Look for productive strategies:

- Using the example to help them draw diagrams, write equations, and solve the other problems.
- Recognizing the last problem is different, because the groups are chairs rather than yards/fabric.
- Interpreting similarities and differences in all scenarios to estimate, draw diagrams, and write and solve equations.

3 Connect

Have individual students share some or all of their work and thinking with at least two other classmates, taking turns for each row.

Display some of the solutions, depending on students' needs.

Highlight that in these scenarios the number of groups was a fraction, which often require additional partitions in the diagrams.

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Ask students if they are familiar with the term *reupholstering* or what it means to reupholster furniture. Consider displaying images of furniture before and after they were reupholstered to help students visualize this context.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital tape diagrams. After they create their digital tape diagrams, you can use the digital technology to overlay them and see their similarities and differences.

Accessibility: *Vary Demands to Optimize Challenge*

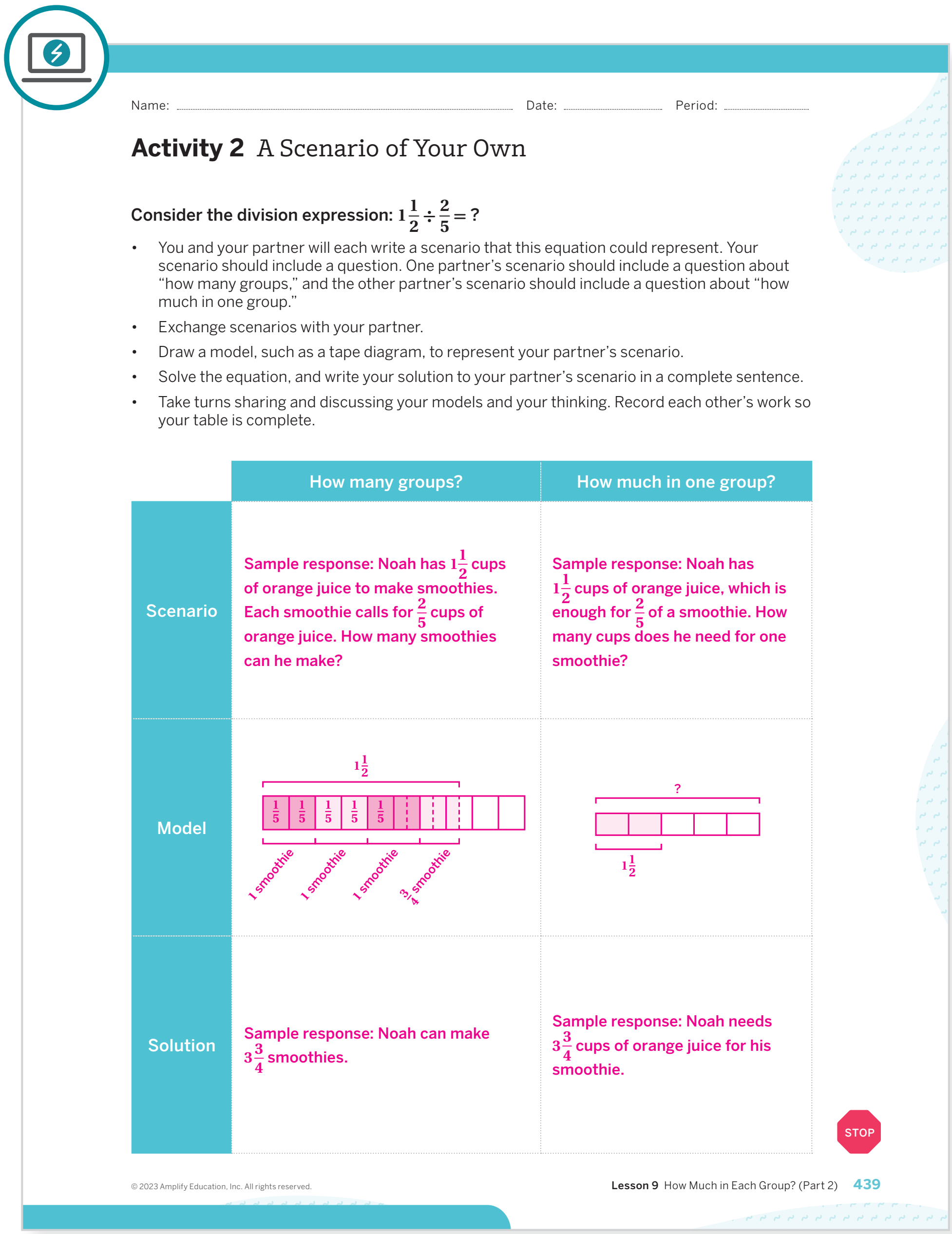
Consider one of these alternative approaches to this activity:

- Have students focus on completing entire rows, even if they are only able to complete one or two rows.
- Have students focus on completing entire columns, such as the tape diagram column and division equation column. This will help you determine where any misconceptions may occur.
- Have students complete the second and fourth rows first, which do not require mixed numbers.

Pairs | 10 min

Activity 2 A Scenario of Your Own

Students distinguish between the two interpretations of division by creating and solving their own real-world story problems, and by drawing appropriate diagrams.



Name: ________ Date: ________ Period: ________

Activity 2 A Scenario of Your Own

Consider the division expression: $1\frac{1}{2} \div \frac{2}{5} = ?$

- You and your partner will each write a scenario that this equation could represent. Your scenario should include a question. One partner's scenario should include a question about "how many groups," and the other partner's scenario should include a question about "how much in one group."
- Exchange scenarios with your partner.
- Draw a model, such as a tape diagram, to represent your partner's scenario.
- Solve the equation, and write your solution to your partner's scenario in a complete sentence.
- Take turns sharing and discussing your models and your thinking. Record each other's work so your table is complete.

	How many groups?	How much in one group?
Scenario	Sample response: Noah has $1\frac{1}{2}$ cups of orange juice to make smoothies. Each smoothie calls for $\frac{2}{5}$ cups of orange juice. How many smoothies can he make?	Sample response: Noah has $1\frac{1}{2}$ cups of orange juice, which is enough for $\frac{2}{5}$ of a smoothie. How many cups does he need for one smoothie?
Model		
Solution	Sample response: Noah can make $3\frac{3}{4}$ smoothies.	Sample response: Noah needs $3\frac{3}{4}$ cups of orange juice for his smoothie.

1 Launch

Help pairs determine who will write the scenario for each column. Consider using the *Gallery Tour* routine at the end so students can see a variety of scenarios.

2 Monitor

Help students get started by asking, "What should the values in the equation represent — total, size of one group, or number of groups — for your type of scenario?"

Look for points of confusion:

- **Writing a scenario that does not match the assigned type of division.** Have students review Activity 1 (how much in one group) or Lessons 5–6 (how many groups).

Look for productive strategies:

- Drawing a model that accurately represents the correct type of scenario, but struggling to name the fractional part of the quotient. Ask, "How could more partitions help?"
- Determining the correct quotient for their partner's scenario using their model, and writing their solution with the appropriate units in context.

3 Connect

Display students' work around the room for the *Gallery Tour*, if using that routine. Otherwise, consider displaying two blank tape diagrams to model each type of scenario.

Have students share their thinking for writing each type of problem.

Highlight that students have been developing strategies for representing and solving these two different types of division problems, and in the next lessons, they will generalize the mathematics.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital tape diagrams. After they create their digital tape diagrams, you can use the digital technology to overlay them and see their similarities and differences.

Extension: *Math Enrichment*

Ask students to use the quotient they determined for $1\frac{1}{2} \div \frac{2}{5} = ?$ and the relationship between multiplication and division to solve the equation $\frac{2}{5} \div 1\frac{1}{2} = ?$ $\frac{4}{15}$

Math Language Development

MLR8: Discussion Supports

During the *Gallery Tour*, have students study the different scenarios written to look for similarities in the language used. Display these questions as students study the different scenarios:

- "What kinds of words or phrases do you see in the "how many groups?" scenarios?"
- "What kinds of words or phrases do you see in the "how much in one group?" scenarios?"

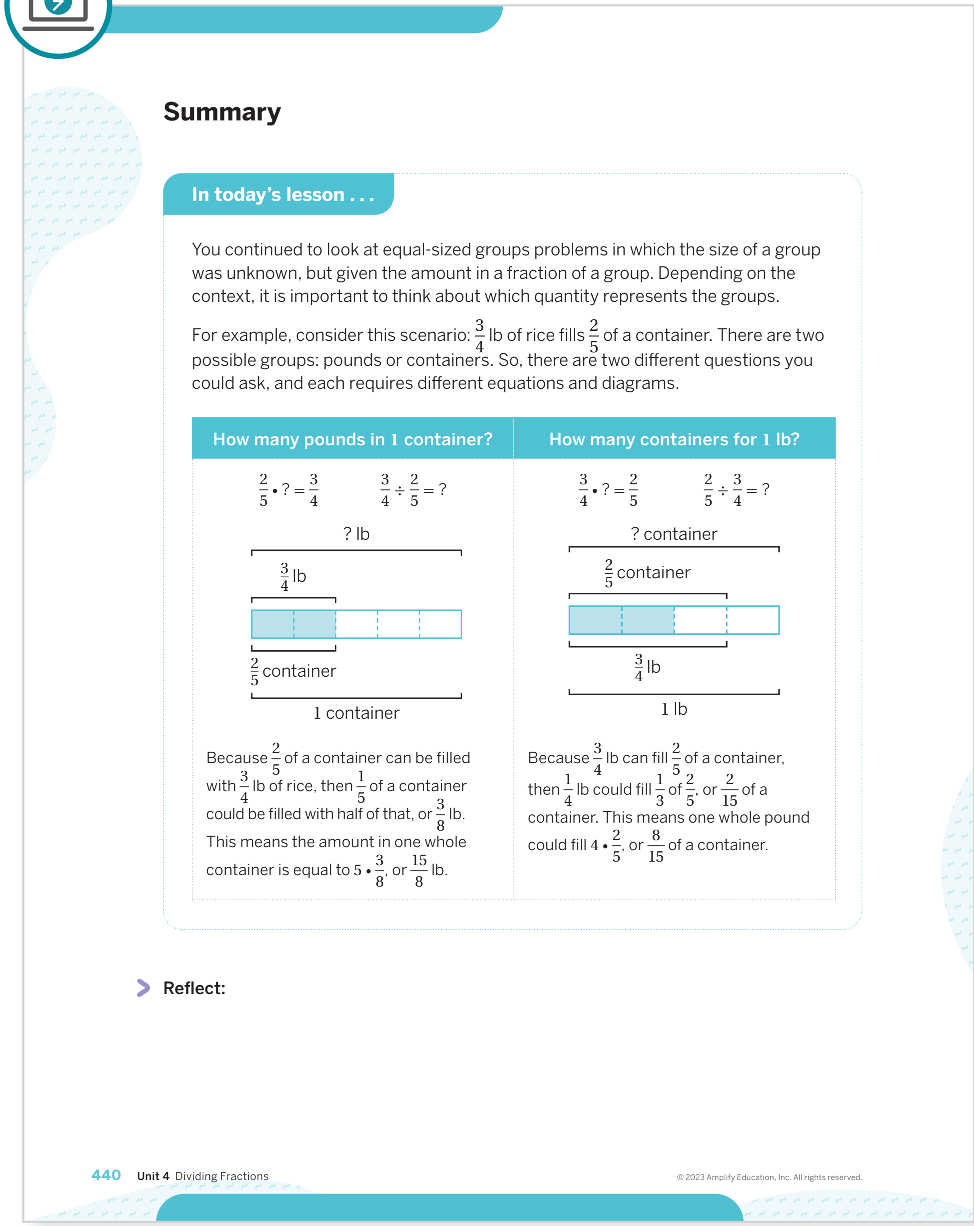
English Learners

Annotate any common words and phrases, such as *each, how many, how many/much for one,* etc.

Whole Class | 5 min

Summary

Review and synthesize how to analyze a division problem and draw a model that helps determine the size of one group.



Summary

In today's lesson . . .

You continued to look at equal-sized groups problems in which the size of a group was unknown, but given the amount in a fraction of a group. Depending on the context, it is important to think about which quantity represents the groups.

For example, consider this scenario: $\frac{3}{4}$ lb of rice fills $\frac{2}{5}$ of a container. There are two possible groups: pounds or containers. So, there are two different questions you could ask, and each requires different equations and diagrams.

How many pounds in 1 container?	How many containers for 1 lb?
$\frac{2}{5} \cdot ? = \frac{3}{4}$ $\quad$ $\frac{3}{4} \div \frac{2}{5} = ?$	$\frac{3}{4} \cdot ? = \frac{2}{5}$ $\quad$ $\frac{2}{5} \div \frac{3}{4} = ?$
? lb $\frac{3}{4}$ lb $\frac{2}{5}$ container 1 container	? container $\frac{2}{5}$ container $\frac{3}{4}$ lb 1 lb
Because $\frac{2}{5}$ of a container can be filled with $\frac{3}{4}$ lb of rice, then $\frac{1}{5}$ of a container could be filled with half of that, or $\frac{3}{8}$ lb. This means the amount in one whole container is equal to $5 \cdot \frac{3}{8}$, or $\frac{15}{8}$ lb.	Because $\frac{3}{4}$ lb can fill $\frac{2}{5}$ of a container, then $\frac{1}{4}$ lb could fill $\frac{1}{3}$ of $\frac{2}{5}$, or $\frac{2}{15}$ of a container. This means one whole pound could fill $4 \cdot \frac{2}{5}$, or $\frac{8}{15}$ of a container.

Reflect:

Synthesize

Display the two tape diagram images from the Student Edition Summary.

Ask:

- "How does each diagram represent the dividend and the divisor?"
- "How can you determine when the unknown in a story problem is referring to 'size of one group' and when the unknown is referring to 'the number of groups'?"

Highlight that sometimes it is not always obvious whether a division problem involves determining the number of groups or the size of one group. There may be two wholes to keep track of and two possible questions that could be asked. The problem needs to be carefully analyzed in order to determine the unknown.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

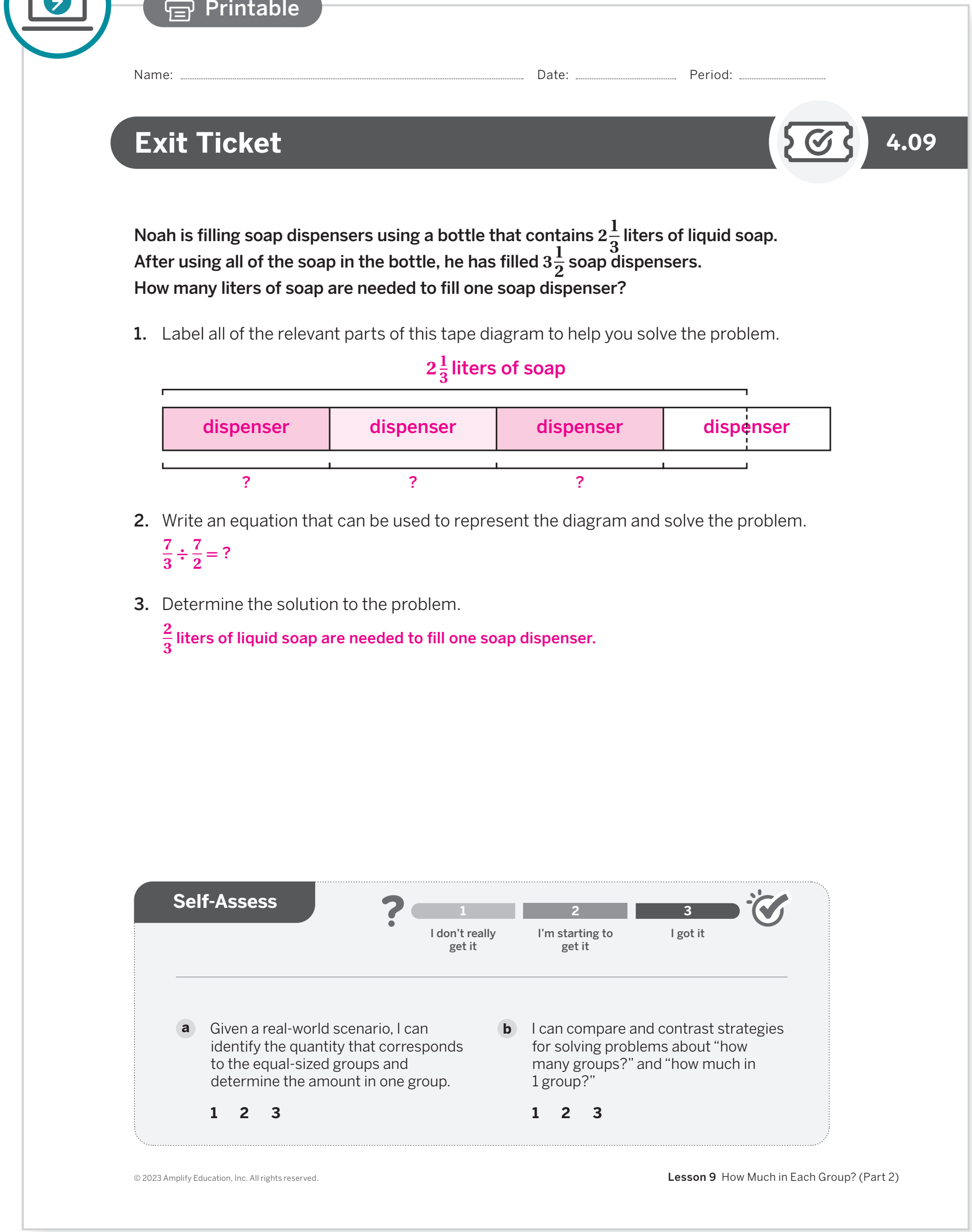
- "What does it mean when a quantity represents a fractional number of equal-sized groups?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by creating a model and writing an equation to determine the size of one group in a division problem.

Printable



Name: Date: Period:

Exit Ticket 4.09

Noah is filling soap dispensers using a bottle that contains $2\frac{1}{3}$ liters of liquid soap. After using all of the soap in the bottle, he has filled $3\frac{1}{2}$ soap dispensers. How many liters of soap are needed to fill one soap dispenser?

1. Label all of the relevant parts of this tape diagram to help you solve the problem.

$2\frac{1}{3}$ liters of soap

dispenser	dispenser	dispenser	dispenser
?	?	?	

2. Write an equation that can be used to represent the diagram and solve the problem.

$\frac{7}{3} \div \frac{7}{2} = ?$

3. Determine the solution to the problem.

$\frac{2}{3}$ liters of liquid soap are needed to fill one soap dispenser.

Self-Assess

? 1 I don't really get it 2 I'm starting to get it 3 I got it

a Given a real-world scenario, I can identify the quantity that corresponds to the equal-sized groups and determine the amount in one group.

1 2 3

b I can compare and contrast strategies for solving problems about "how many groups?" and "how much in 1 group?"

1 2 3

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Lesson 9 How Much in Each Group? (Part 2)

Success looks like . . .

- **Language Goal:** Interpreting a situation involving equal-sized groups, and generating mathematical questions that could be asked about it. **(Reading and Writing, Speaking and Listening)**
- **Language Goal:** Solving a problem involving division of fractions, and presenting the solution method. **(Reading and Writing, Speaking and Listening)**
  - » Using a tape diagram to determine the number of liters of soap needed for one dispenser in Problems 1, 2, and 3.
- **Language Goal:** Comparing and contrasting strategies for solving problems about "how many groups?" and "how much in one group?" **(Speaking and Listening)**

Suggested next steps

If students have trouble labeling the tape diagram in Problem 1, consider:

- Asking, "What is being divided in this scenario? Where do you see that in the tape diagram? What does each part of the diagram represent?"
- Reviewing Activity 1.

If students reverse the dividend and divisor for Problem 2, consider:

- Asking, "What is the total amount that is being divided?"
- Assigning Practice Problem 3.

If students are unsure how to determine the solution in Problem 3, consider:

- Reviewing Activity 2.
- Suggesting they try the strategy of converting to common denominators.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In having the students write story problems and create corresponding tape diagrams, what did the work in Activity 2 teach you about your students as learners?
- In what ways did Activity 1 go as planned? What did not go as planned and what might you change for the next time you teach this lesson?

Math Language Development

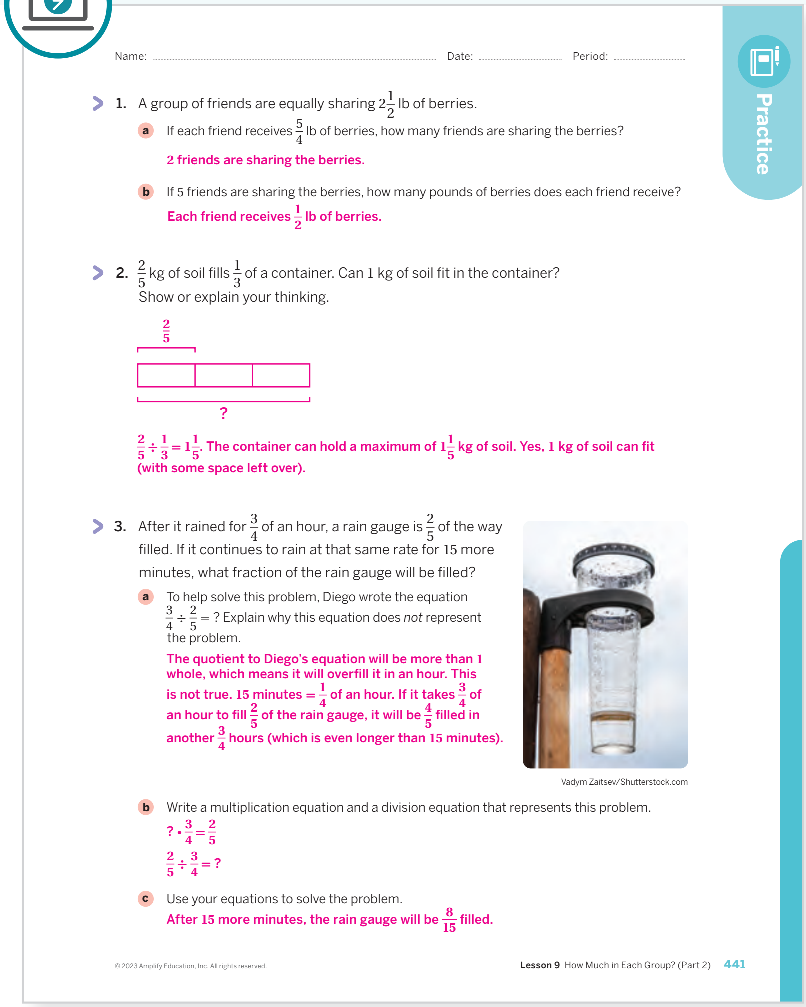
Language Goal: Comparing and contrasting strategies for solving problems about "how many groups?" and "how much in one group?"

Reflect on students' language development toward this goal.

- How have students progressed in using the language of "how many groups?" and "how much in one group?" throughout this unit so far?
- How did using the *Gallery Tour* routine in Activity 2 help them compare and contrast the language used for each type of division scenario?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. A group of friends are equally sharing $2\frac{1}{2}$ lb of berries.
   - **a** If each friend receives $\frac{5}{4}$ lb of berries, how many friends are sharing the berries?

     2 friends are sharing the berries.
   - **b** If 5 friends are sharing the berries, how many pounds of berries does each friend receive?

     Each friend receives $\frac{1}{2}$ lb of berries.

2. $\frac{2}{5}$ kg of soil fills $\frac{1}{3}$ of a container. Can 1 kg of soil fit in the container? Show or explain your thinking.

   $\frac{2}{5}$

   ?

   $\frac{2}{5} \div \frac{1}{3} = 1\frac{1}{5}$. The container can hold a maximum of $1\frac{1}{5}$ kg of soil. Yes, 1 kg of soil can fit (with some space left over).

3. After it rained for $\frac{3}{4}$ of an hour, a rain gauge is $\frac{2}{5}$ of the way filled. If it continues to rain at that same rate for 15 more minutes, what fraction of the rain gauge will be filled?

   Vadym Zaitsev/Shutterstock.com
   - **a** To help solve this problem, Diego wrote the equation $\frac{3}{4} \div \frac{2}{5} = ?$ Explain why this equation does *not* represent the problem.

     The quotient to Diego's equation will be more than 1 whole, which means it will overfill it in an hour. This is not true. 15 minutes $= \frac{1}{4}$ of an hour. If it takes $\frac{3}{4}$ of an hour to fill $\frac{2}{5}$ of the rain gauge, it will be $\frac{4}{5}$ filled in another $\frac{3}{4}$ hours (which is even longer than 15 minutes).
   - **b** Write a multiplication equation and a division equation that represents this problem.

     $? \cdot \frac{3}{4} = \frac{2}{5}$

     $\frac{2}{5} \div \frac{3}{4} = ?$
   - **c** Use your equations to solve the problem.

     After 15 more minutes, the rain gauge will be $\frac{8}{15}$ filled.

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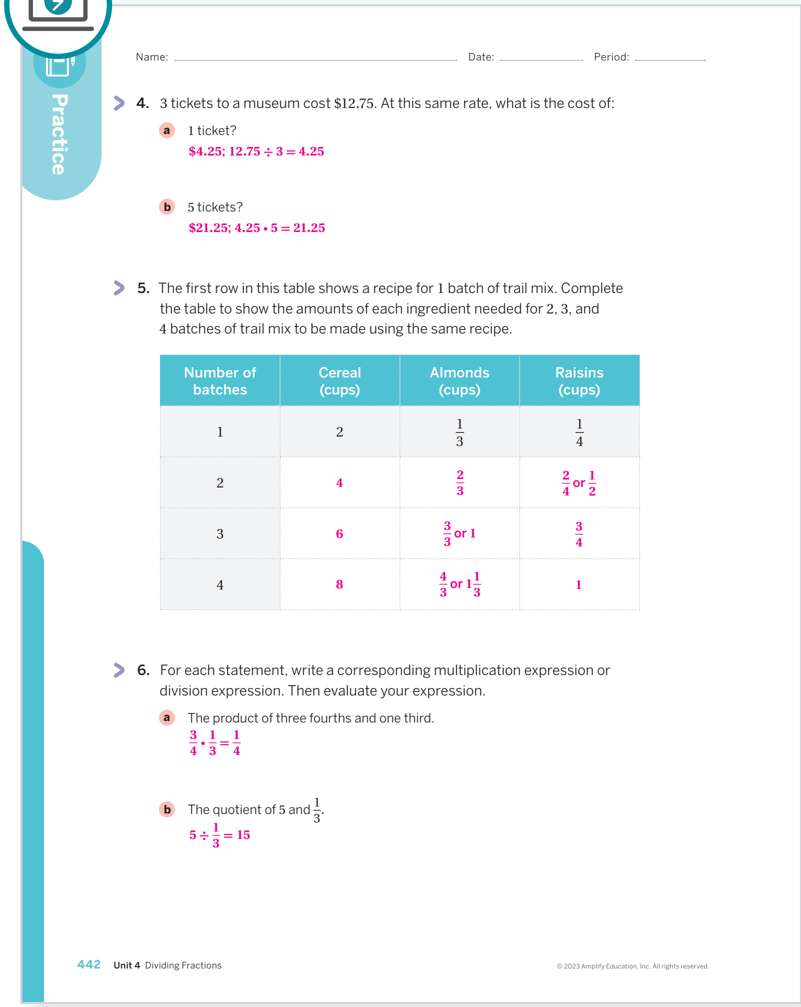
Lesson 9 How Much in Each Group? (Part 2) 441



Practice

Name: ______ Date: ______ Period: ______

4. 3 tickets to a museum cost $12.75. At this same rate, what is the cost of:
   - **a** 1 ticket?

     $4.25; 12.75 ÷ 3 = 4.25
   - **b** 5 tickets?

     $21.25; 4.25 • 5 = 21.25

5. The first row in this table shows a recipe for 1 batch of trail mix. Complete the table to show the amounts of each ingredient needed for 2, 3, and 4 batches of trail mix to be made using the same recipe.

Number of batches	Cereal (cups)	Almonds (cups)	Raisins (cups)
1	2	$\frac{1}{3}$	$\frac{1}{4}$
2	**4**	**$\frac{2}{3}$**	**$\frac{2}{4}$ or $\frac{1}{2}$**
3	**6**	**$\frac{3}{3}$ or 1**	**$\frac{3}{4}$**
4	**8**	**$\frac{4}{3}$ or $1\frac{1}{3}$**	**1**

6. For each statement, write a corresponding multiplication expression or division expression. Then evaluate your expression.
   - **a** The product of three fourths and one third.

     $\frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4}$
   - **b** The quotient of 5 and $\frac{1}{3}$.

     $5 \div \frac{1}{3} = 15$

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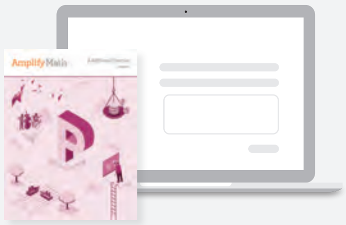
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 2	2
	5	Unit 2 Lesson 7	2
Formative ⚡	6	Unit 4 Lesson 10	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

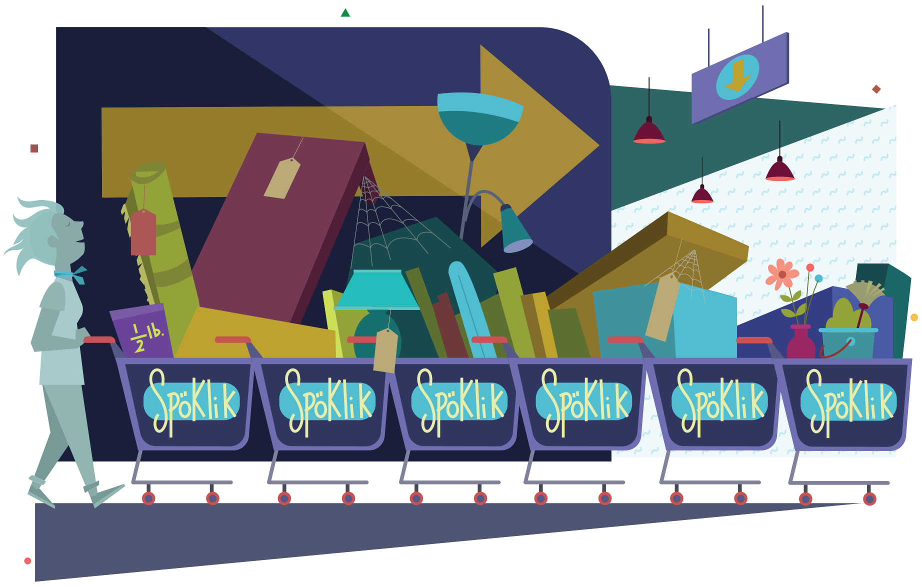
Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Dividing by Unit and Non-Unit Fractions

Let's look for patterns when we divide by fractions.



Focus

Goals

1. **Language Goal:** Interpret and critique explanations of how to divide by a fraction. **(Speaking and Listening, Writing)**
2. **Language Goal:** Use a tape diagram to represent dividing by a unit fraction $\frac{1}{b}$ and explain why this is the same as multiplying by b. **(Speaking and Listening, Writing)**
3. **Language Goal:** Use a tape diagram to represent dividing by a non-unit fraction $\frac{a}{b}$ and explain why this produces the same result as multiplying the number by b and dividing by a. **(Speaking and Listening, Writing)**

Coherence

Today

Students develop a general rule for dividing by fractions. They use tape diagrams to first represent and evaluate sequenced quotients of whole numbers divided by unit fraction and non-unit fraction divisors with the same denominator. Students should notice patterns in their processes of interpreting quotients and creating diagrams, and then recognize how these patterns are also related to the values in the corresponding division expressions. They generalize these same relationships to quotients with fraction dividends as well.

Previously

In Lesson 9, students distinguished between the two types of division problems and practiced interpreting, representing, and solving both kinds of division problems.

Coming Soon

In Lesson 11, students will establish and apply a general algorithm for dividing fractions by fractions.

Rigor

- Students create tape diagrams to build **conceptual understanding** of general rules that can be applied when dividing by fractions, preparing them to determine the standard algorithm of multiplying the dividend by the reciprocal of the divisor.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- *Tape Diagrams* PDF (as needed)
- colored pencils

Math Language Development

New word

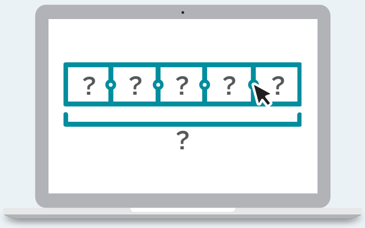
- ***reciprocal (of a whole number)****

*The term *reciprocal* will be defined further for the general case in Lesson 11.

Amps | Featured Activity

Activity 2
Digital Diagrams

Students can sketch on digital tape diagrams to solve problems. You can overlay them all to see similarities and differences at a glance.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might not understand the division of whole numbers by fractions conceptually in Activity 1. Their foundational understanding of division might be challenged by a quotient that is greater than the dividend. Encourage students to have a growth attitude, thinking or saying that it does not make sense to them yet. Then ask them to search for similarities in the process of dividing by a unit fraction and a non-unit fraction, so that they can recognize the repeated reasoning being used.

Modifications to Pacing

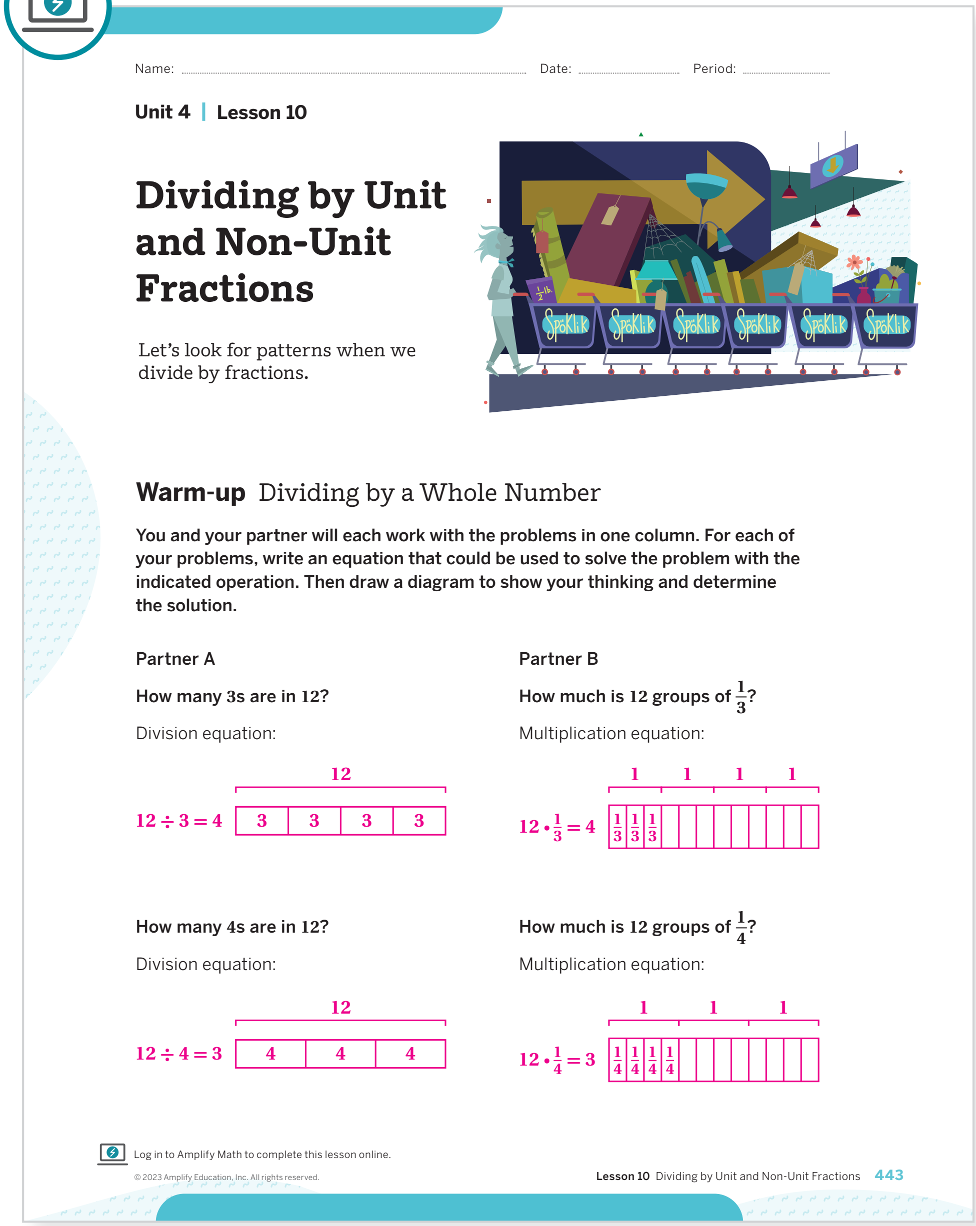
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, assign one set to pairs to complete together.
- In **Activity 1**, assign Problem 1 to half of the pairs, and Problem 2 to the other half. **Note:** Problems 3–4 will then need to be done with the whole class, after sharing and discussing diagrams and answers from both Problems 1 and 2.

Pairs | 10 min

Warm-up Dividing by a Whole Number

Students use tape diagrams to activate prior knowledge and revisit the idea that dividing by a whole number is equivalent to multiplying by a unit fraction.



Name: ______________ Date: ________ Period: ________

Unit 4 | Lesson 10

Dividing by Unit and Non-Unit Fractions

Let's look for patterns when we divide by fractions.

Warm-up Dividing by a Whole Number

You and your partner will each work with the problems in one column. For each of your problems, write an equation that could be used to solve the problem with the indicated operation. Then draw a diagram to show your thinking and determine the solution.

Partner A	Partner B
How many 3s are in 12?	How much is 12 groups of $\frac{1}{3}$?
Division equation:	Multiplication equation:
$12 \div 3 = 4$	$12 \cdot \frac{1}{3} = 4$
How many 4s are in 12?	How much is 12 groups of $\frac{1}{4}$?
Division equation:	Multiplication equation:
$12 \div 4 = 3$	$12 \cdot \frac{1}{4} = 3$

Log in to Amplify Math to complete this lesson online.

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Lesson 10 Dividing by Unit and Non-Unit Fractions 443

1 Launch

Have each pair of students identify who will be Partner A and who will be Partner B. Give students 4 minutes to complete their two problems, and then have pairs use the *Take Turns* routine to share and to discuss one problem at a time.

2 Monitor

Help students get started by having them write equations like ____ ÷ ____ = ____ or ____ • ____ = ____.

Look for points of confusion:

- **Partner A not recognizing 12 as the whole.** Have them draw their tape diagrams with 12 as the whole.
- **Partner B not recognizing they need 12 parts.** Remind them that 12 groups means 12 parts.

Look for productive strategies:

- Recognizing that 12 is the whole and needs to be divided into 3 and 4 parts (Partner A).
- Recognizing that 12 is the number of parts (Partner B).

3 Connect

Display students' tape diagrams and solutions.

Have individual students share how they approached drawing diagrams and solving each problem. Then have pairs share what they noticed about the related division and multiplication problems.

Highlight that dividing by a whole number is the same as multiplying by the corresponding unit fraction with that whole number as the denominator.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share what they noticed, draw their attention to how the divisor in the division equation is related to the second factor in the multiplication equation. Later, in this lesson, students will learn this term as the *reciprocal*. For now, have them share what they notice and describe what they notice using their own words. For example, they may say:

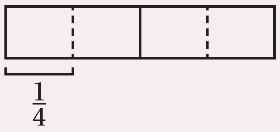
- "The numbers are the same, but in the multiplication equation, it's a fraction with that number in the denominator and 1 in the numerator."
- "The numbers are flipped." Press students for what they mean by the term "flipped."

Power-up

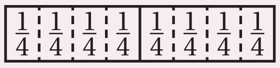
To power up students' ability to multiply and divide by a unit fraction, have students complete:

Determine which tape diagram models the expression $2 \div \frac{1}{4}$, then determine the quotient.

A.



[✓ C.]



B.

$2 \div \frac{1}{4} = 8.$

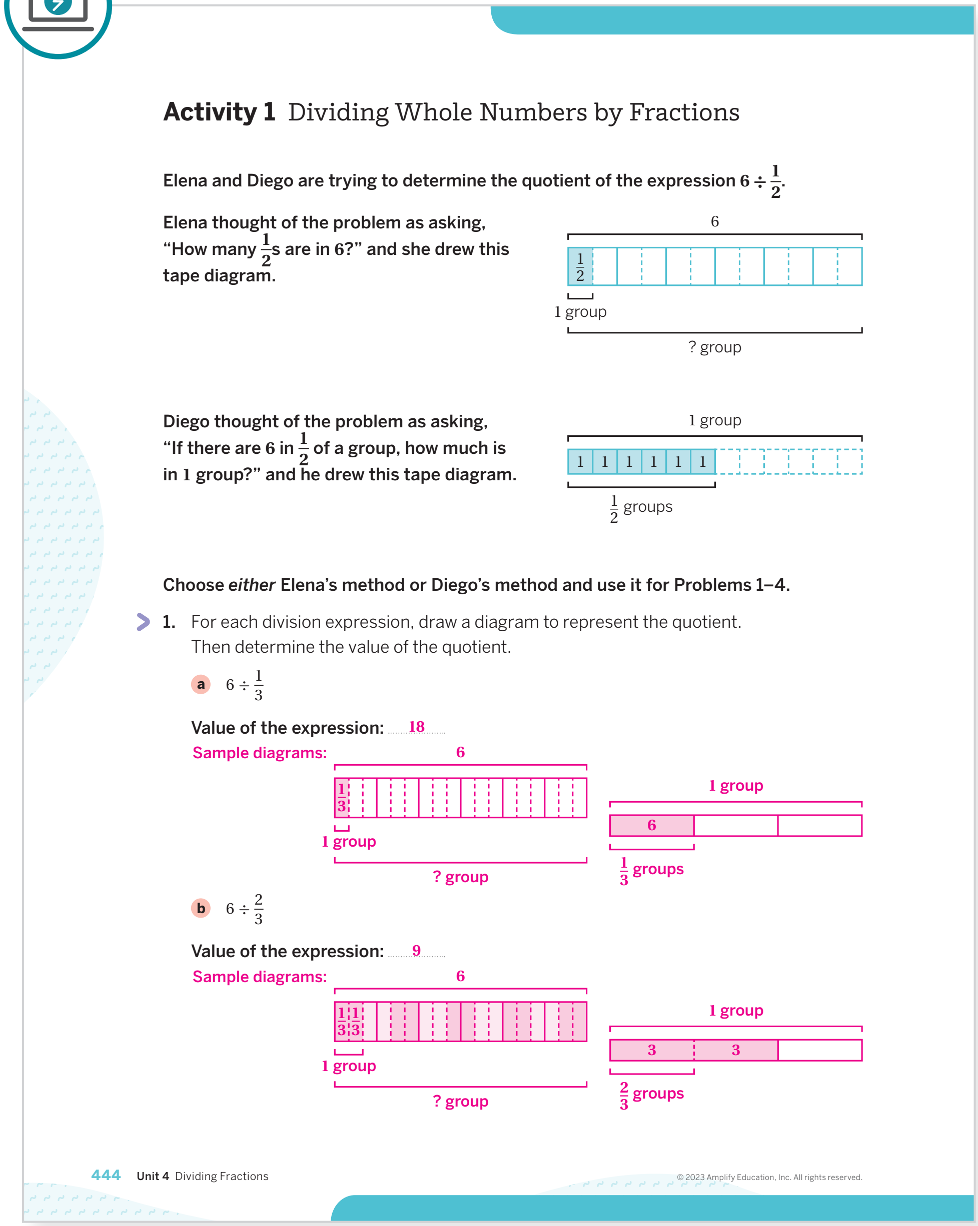
Use: Before the Warm-up

Informed by: Performance on Lesson 9, Practice Problem 6.

Pairs | 15 min

Activity 1 Dividing Whole Numbers by Fractions

Students use tape diagrams and the meanings of division to divide whole numbers by unit and non-unit fractions with the same denominator, which helps to see patterns.



Activity 1 Dividing Whole Numbers by Fractions

Elena and Diego are trying to determine the quotient of the expression $6 \div \frac{1}{2}$.

Elena thought of the problem as asking, "How many $\frac{1}{2}$s are in 6?" and she drew this tape diagram.

Diego thought of the problem as asking, "If there are 6 in $\frac{1}{2}$ of a group, how much is in 1 group?" and he drew this tape diagram.

Choose *either* Elena's method or Diego's method and use it for Problems 1–4.

1. For each division expression, draw a diagram to represent the quotient. Then determine the value of the quotient.

a $6 \div \frac{1}{3}$

Value of the expression: 18

Sample diagrams:

b $6 \div \frac{2}{3}$

Value of the expression: 9

Sample diagrams:

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1 Launch

Make sure pairs understand that they should choose one method to use for all problems, and work together by using that same method. Consider providing colored pencils to help students organize and show their thinking in their tape diagrams.

2 Monitor

Help students get started by asking, "How did [Elena/Diego] represent $\frac{1}{2}$? How can you do something similar for $\frac{1}{3}$?"

Look for points of confusion:

- **Not knowing how to apply their chosen method to non-unit fractions.** Have students look at their completed diagrams for $\frac{1}{3}$ and ask, "Do you see where $\frac{2}{3}$ is represented in the diagram?"
- **Thinking a divisor of $\frac{a}{b}$ results in a greater quotient than a divisor of $\frac{1}{b}$.** For example, ask, "Is $\frac{2}{3}$ less than or greater than $\frac{1}{3}$? Would dividing by 2 result in a quotient that is less than or greater than the quotient from dividing by 1?"

Look for productive strategies:

- Thinking appropriately about partitive or quotitive division to create diagrams and to determine the quotients.
- Recognizing that they can use their unit-fraction diagrams to help them create their non-unit fraction diagrams, and likewise, to determine the quotients.
- Noticing that the process for determining a quotient with a unit fraction divisor is always the same.
- Noticing that the process for determining a quotient with a non-unit fraction divisor is always the same, and is also always related to the quotient involving the unit fraction divisor with the same denominator.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, suggest they use Elena's method and focus on Problems 1 and 4.

Extension: *Math Enrichment*

Have students choose a method to evaluate the expression $1 \div \frac{6}{5}$ and explain their thinking. $\frac{5}{6}$; Sample response: Using Diego's method, I can determine how much is in 1 group if there is 1 in $\frac{6}{5}$ of a group.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies, highlight strategies in which students used their tape diagrams representing division by a unit fraction to help them make sense of division by a non-unit fraction. Remind students that they worked with unit fractions in prior grades. Ask them to state in their own words the difference between a unit fraction and a non-unit fraction.

English Learners

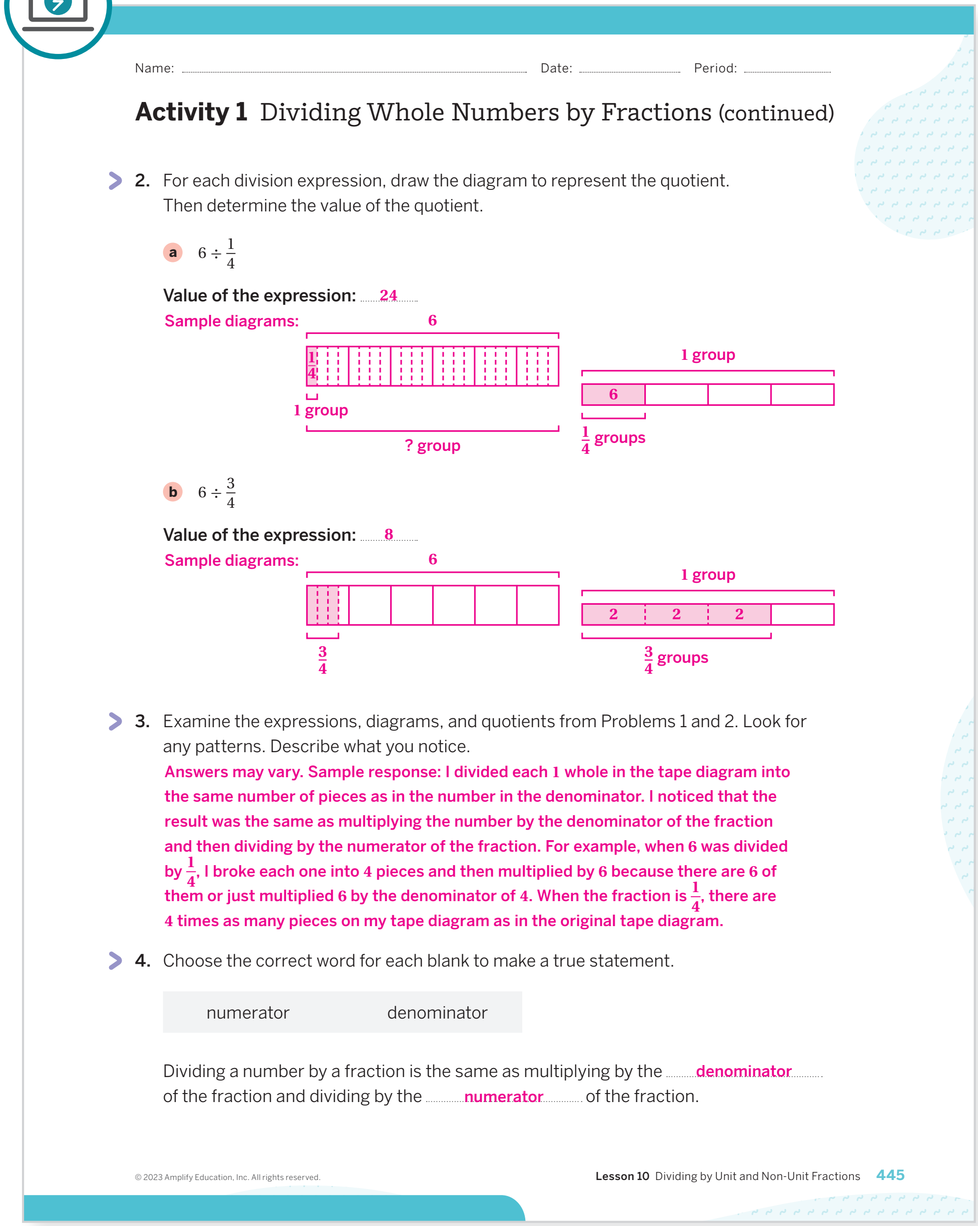
As you discuss Problem 4, include visual examples of multiplication and division equations, such as:

$6 \div \frac{2}{3} = 9$ $\quad$ $(6 \bullet 3) \div 2 = 9$ $\quad$ $6 \bullet \frac{3}{2} = 9$

Pairs | 15 min

Activity 1 Dividing Whole Numbers by Fractions (continued)

Students use tape diagrams and the meanings of division to divide whole numbers by unit and non-unit fractions with the same denominator, which helps to see patterns.



Name: Date: Period:

Activity 1 Dividing Whole Numbers by Fractions (continued)

2. For each division expression, draw the diagram to represent the quotient. Then determine the value of the quotient.

a $6 \div \frac{1}{4}$

Value of the expression: 24

Sample diagrams:

b $6 \div \frac{3}{4}$

Value of the expression: 8

Sample diagrams:

3. Examine the expressions, diagrams, and quotients from Problems 1 and 2. Look for any patterns. Describe what you notice.

Answers may vary. Sample response: I divided each 1 whole in the tape diagram into the same number of pieces as in the number in the denominator. I noticed that the result was the same as multiplying the number by the denominator of the fraction and then dividing by the numerator of the fraction. For example, when 6 was divided by $\frac{1}{4}$, I broke each one into 4 pieces and then multiplied by 6 because there are 6 of them or just multiplied 6 by the denominator of 4. When the fraction is $\frac{1}{4}$, there are 4 times as many pieces on my tape diagram as in the original tape diagram.

4. Choose the correct word for each blank to make a true statement.

numerator	denominator

Dividing a number by a fraction is the same as multiplying by the **denominator** of the fraction and dividing by the **numerator** of the fraction.

3 Connect

Display blank tape diagrams with a length of 6, which are to be completed for each problem.

Have pairs share their strategies and solutions to the problems, followed by the patterns they noticed and then the completed true statement from Problem 4.

Ask as many of the following questions as time permits:

- "Which interpretation of division did Elena/Diego use?"
- "What are the similarities and differences between dividing by a unit fraction and a non-unit fraction?"
- "How do your tape diagrams show that dividing by a fraction is the two-step process of multiplying by the denominator and dividing by the numerator?"
- "Did you divide by the numerator when the divisor was a unit fraction? How is this represented?"
- "Do you always have to multiply by the denominator first?"

Define: The ***reciprocal of a whole number*** is the unit fraction whose denominator is the whole number. For example, $\frac{1}{b}$ and $\frac{b}{1}$, or b, are reciprocals.

Note: The product of a number and its reciprocal is 1.

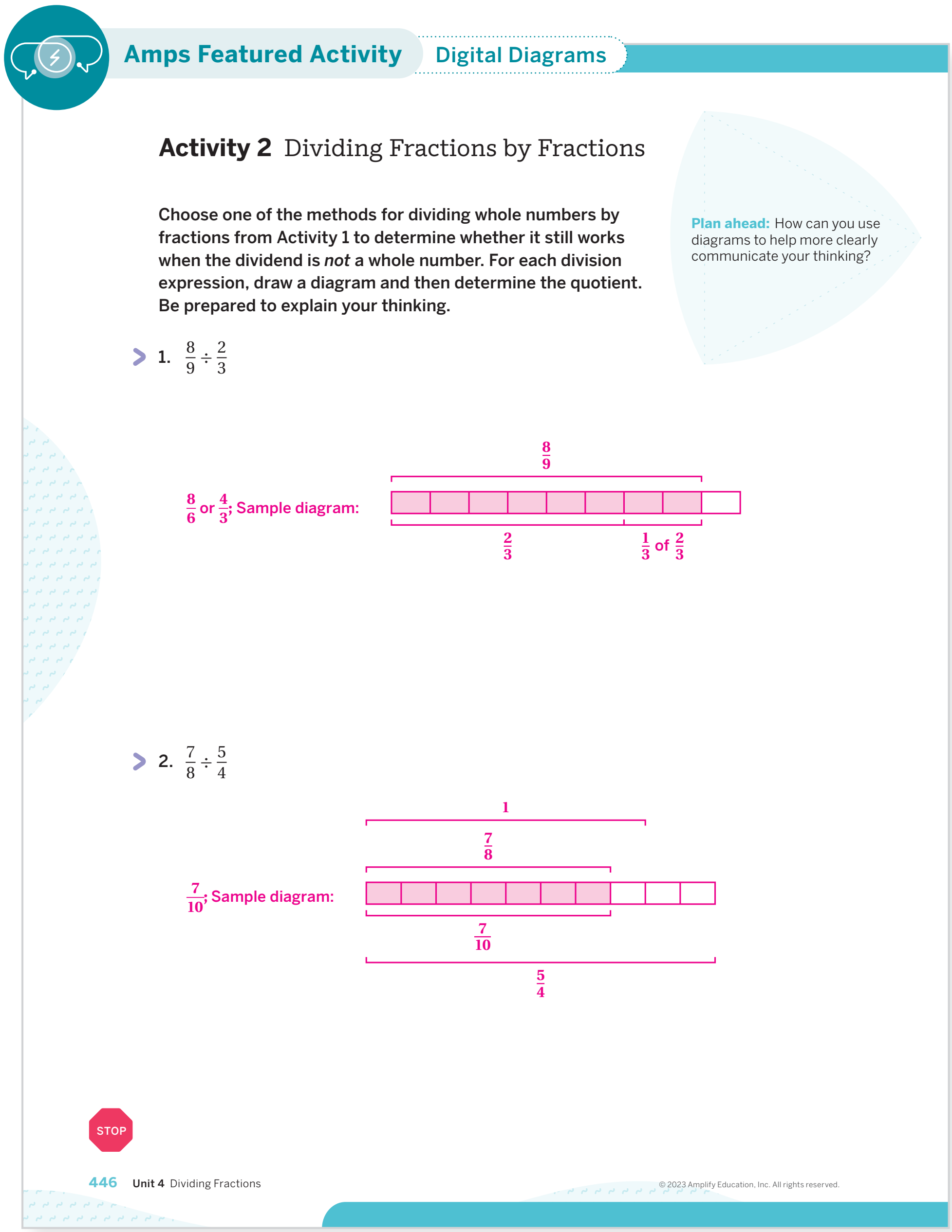
Highlight that dividing a whole number by a unit fraction is the same as multiplying the whole number by the denominator, and that dividing by a non-unit fraction can start the same, but the result must be then divided by the numerator.

Pairs | 10 min

Activity 2 Dividing Fractions by Fractions

Students apply the patterns and the rule they determined for dividing whole numbers by fractions in Activity 1 to determine that those also generalize to any quotients.

Amps Featured Activity Digital Diagrams



Activity 2 Dividing Fractions by Fractions

Choose one of the methods for dividing whole numbers by fractions from Activity 1 to determine whether it still works when the dividend is *not* a whole number. For each division expression, draw a diagram and then determine the quotient. Be prepared to explain your thinking.

Plan ahead: How can you use diagrams to help more clearly communicate your thinking?

1. $\frac{8}{9} \div \frac{2}{3}$

$\frac{8}{6}$ or $\frac{4}{3}$; Sample diagram:

2. $\frac{7}{8} \div \frac{5}{4}$

$\frac{7}{10}$; Sample diagram:

STOP

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1 Launch

Have students use the ***Think-Pair-Share*** routine. Provide 2 minutes of individual work time to construct a diagram for Problem 1. Then have students share and complete the activity with a partner.

2 Monitor

Help students get started by asking, "How can you represent $\frac{8}{9}$ by using a tape diagram? Can you determine an equivalent fraction to $\frac{2}{3}$ that might be helpful?"

Look for points of confusion:

- **Not knowing how to coordinate the different denominators.** Ask, "Can you determine equivalent fractions with the same denominators?"

Look for productive strategies:

- Using common denominators to help construct diagrams.
- Applying the same steps and strategy from Activity 1, thinking about dividing by the related unit fraction first, and creating diagrams in two steps.
- Thinking of the divisor as the size of one group in naming fractional parts of groups in the quotients.

3 Connect

Display two blank tape diagrams.

Have students share how they constructed their diagrams and determined the quotients.

Ask:

- "Do both methods from Activity 1 still apply?"
- "Does the general statement or rule from Activity 1 still apply?"

Highlight that the rule for dividing by a fraction, which is the same as multiplying by the denominator and dividing by the numerator, still applies when the dividend is a fraction.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create and interact with digital tape diagrams to model the division of fractions.

Accessibility: *Optimize Access to Tools*

Provide blank tape diagrams for students to use to partition and label. Consider making copies of the *Tape Diagrams* PDF, being sure to direct students to only using the blank tape diagrams that are not labeled with percentages.

Extension: *Math Enrichment*

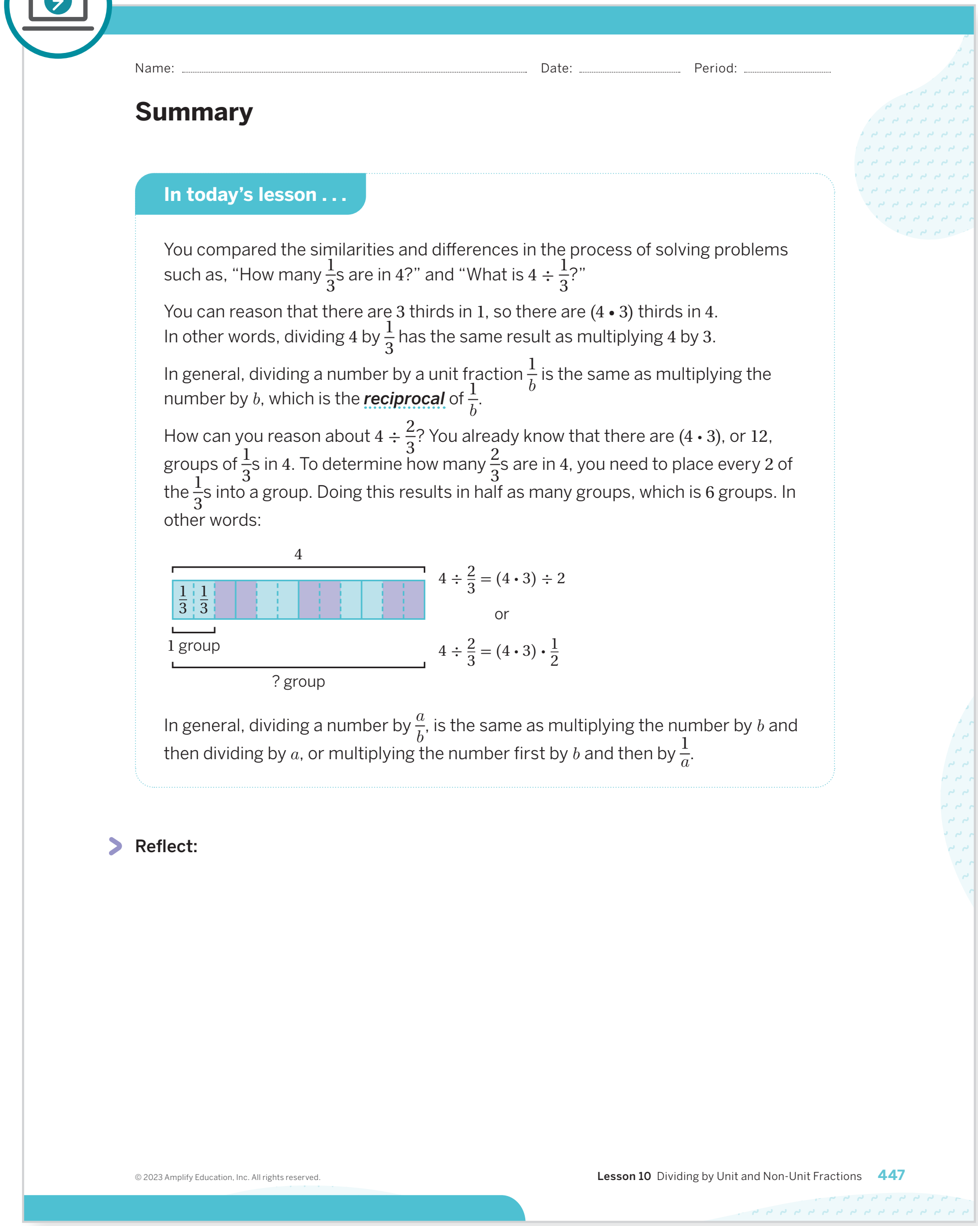
Have students explain the similarities and differences between each of the following:

- Dividing a number by $\frac{5}{4}$.
- Dividing the same number by $\frac{4}{5}$.
- Multiplying the same number $\frac{4}{5}$.
- Multiplying the same number by $\frac{5}{4}$.

Whole Class | 5 min

Summary

Review and synthesize the general rule that dividing by a fraction is the same as multiplying by the denominator and dividing by the numerator.



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Summary

In today's lesson . . .

You compared the similarities and differences in the process of solving problems such as, "How many $\frac{1}{3}$s are in 4?" and "What is $4 \div \frac{1}{3}$?"

You can reason that there are 3 thirds in 1, so there are $(4 \cdot 3)$ thirds in 4. In other words, dividing 4 by $\frac{1}{3}$ has the same result as multiplying 4 by 3.

In general, dividing a number by a unit fraction $\frac{1}{b}$ is the same as multiplying the number by b, which is the ***reciprocal*** of $\frac{1}{b}$.

How can you reason about $4 \div \frac{2}{3}$? You already know that there are $(4 \cdot 3)$, or 12, groups of $\frac{1}{3}$s in 4. To determine how many $\frac{2}{3}$s are in 4, you need to place every 2 of the $\frac{1}{3}$s into a group. Doing this results in half as many groups, which is 6 groups. In other words:

$$4 \div \frac{2}{3} = (4 \cdot 3) \div 2$$

or

$$4 \div \frac{2}{3} = (4 \cdot 3) \cdot \frac{1}{2}$$

In general, dividing a number by $\frac{a}{b}$, is the same as multiplying the number by b and then dividing by a, or multiplying the number first by b and then by $\frac{1}{a}$.

Reflect:

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Lesson 10 Dividing by Unit and Non-Unit Fractions 447

Synthesize

Ask:

- "How are the quotients of $4 \div \frac{1}{3}$ and $4 \div \frac{2}{3}$ related?"
  $4 \div \frac{1}{3}$ is 2 times as large as $4 \div \frac{2}{3}$.
- "Without actually dividing, could you now also determine the quotient of $4 \div \frac{4}{3}$? What would it be?"
  Yes, it would be half the quotient of $4 \div \frac{2}{3}$, so it would be 3.

Formalize vocabulary: ***reciprocal (of a whole number)***

Highlight that all of the learning and experiences of students in the first several lessons of this unit (and from earlier grades) — relationships between multiplication and division, and also unit and non-unit fractions, as well as the interpretations of division — all contributed to their ability to recognize the general rules seen in this lesson for determining any quotients with fraction divisors of any kind.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What did you notice about the result of dividing a number by a non-unit fraction?"

Math Language Development

MLR2: Collect and Display

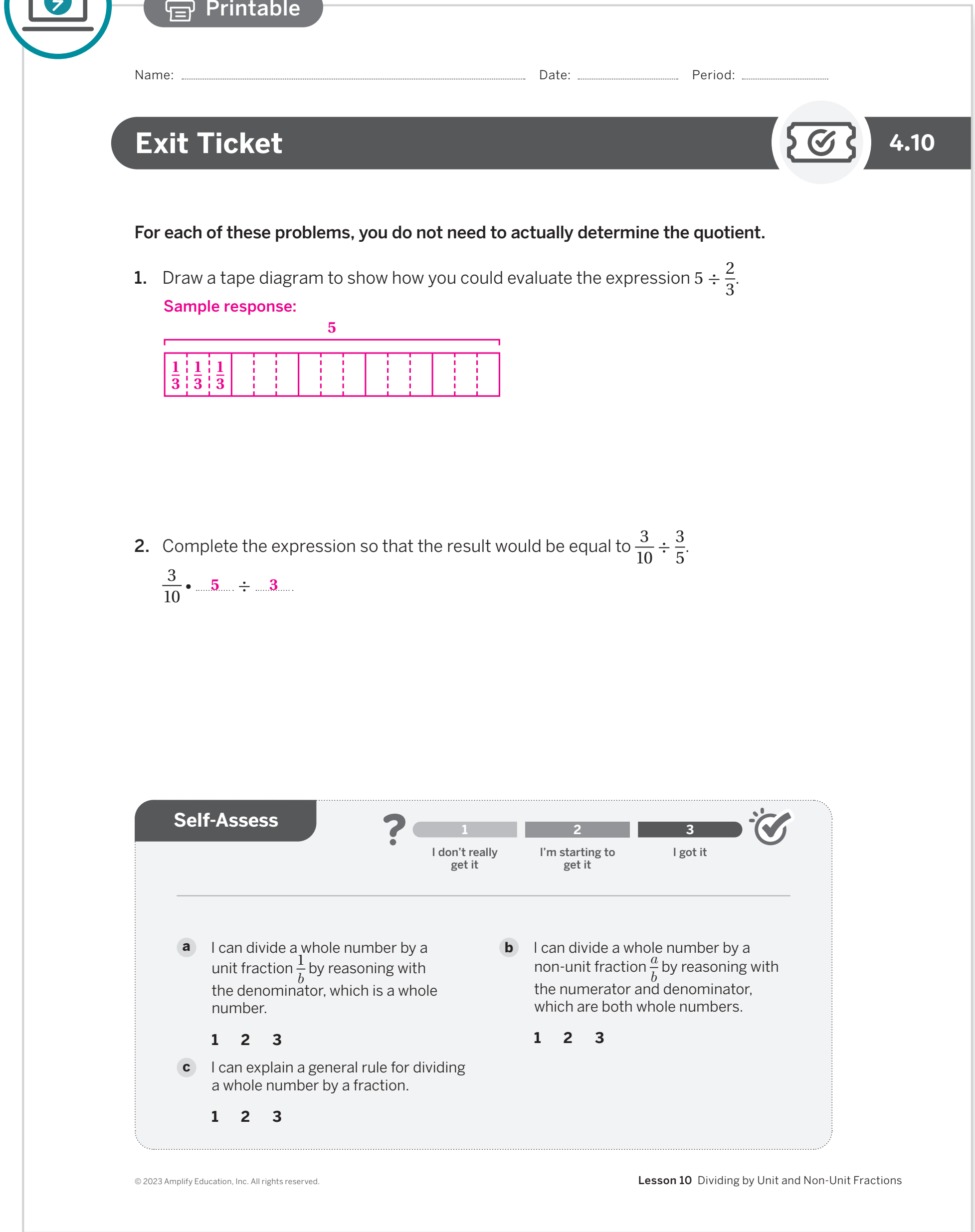
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term reciprocal (*of a whole number*) that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of dividing with fractions by using a diagram and by using multiplication.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 4.10

For each of these problems, you do not need to actually determine the quotient.

1. Draw a tape diagram to show how you could evaluate the expression $5 \div \frac{2}{3}$.

Sample response:

2. Complete the expression so that the result would be equal to $\frac{3}{10} \div \frac{3}{5}$.

$\frac{3}{10} \bullet$ **5** $\div$ **3**

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can divide a whole number by a unit fraction $\frac{1}{b}$ by reasoning with the denominator, which is a whole number.

1 2 3

b. I can divide a whole number by a non-unit fraction $\frac{a}{b}$ by reasoning with the numerator and denominator, which are both whole numbers.

1 2 3

c. I can explain a general rule for dividing a whole number by a fraction.

1 2 3

Lesson 10 Dividing by Unit and Non-Unit Fractions

Success looks like . . .

- Language **Goal:** Interpreting and critiquing explanations of how to divide by a fraction. **(Speaking and Listening, Writing)**
- Language **Goal:** Using a tape diagram to represent dividing by a unit fraction $\frac{1}{b}$ and explaining why this is the same as multiplying by b. **(Speaking and Listening, Writing)**
- Language **Goal:** Using a tape diagram to represent dividing by a non-unit fraction $\frac{a}{b}$ and explaining why this produces the same result as multiplying the number by b and dividing by a. **(Speaking and Listening, Writing)**
  - » Drawing a tape diagram to represent $5 \div \frac{2}{3}$ in Problem 1.

Suggested next steps

If students cannot create an appropriate diagram for Problem 1, consider:

- Asking, "How could you create a diagram for dividing $5 \div \frac{1}{3}$ first and then modify that to represent this expression?"
- Reviewing Activity 1.
- Assigning Practice Problem 1.

If students put the numerator and denominator in the wrong parts of the fraction in Problem 2, consider:

- Reviewing the general rule from Activity 1.

Professional Learning

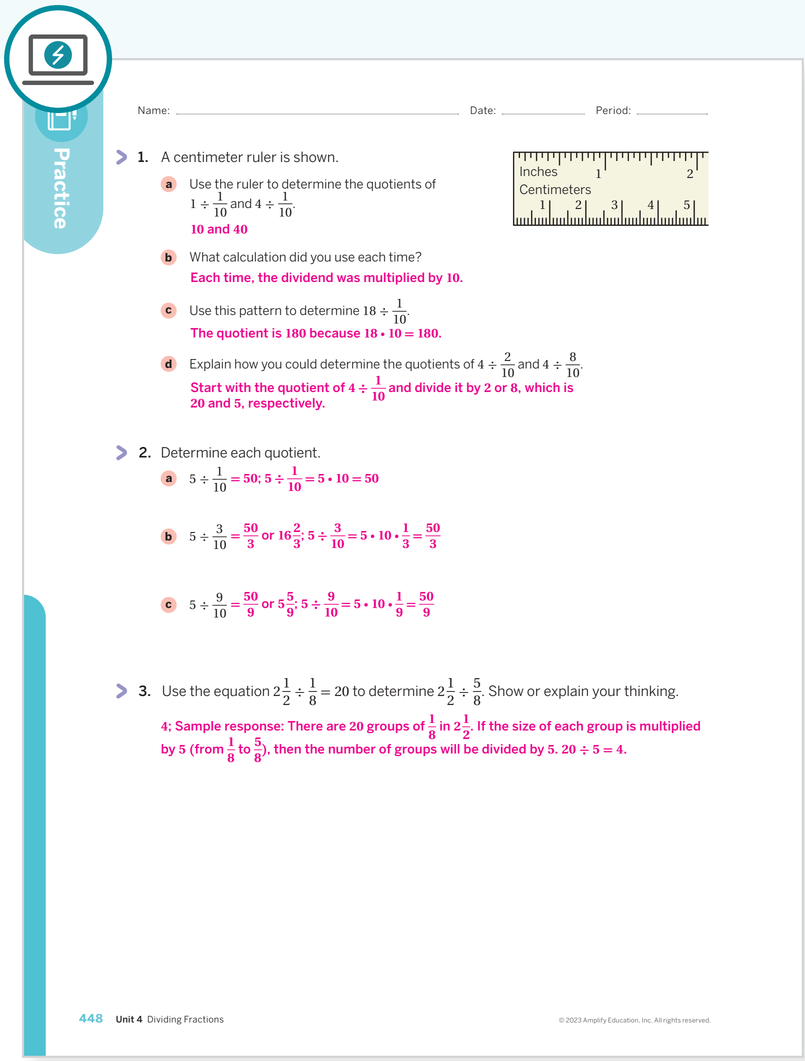
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did the work of pattern recognition and making generalizations reveal about your students as learners?
- What challenges did students encounter as they worked with tape diagrams to divide by fractions? How did they work through them? What might you change for the next time you teach this lesson?

Practice

Independent



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1. A centimeter ruler is shown.
   - **a** Use the ruler to determine the quotients of $1 \div \frac{1}{10}$ and $4 \div \frac{1}{10}$.
     10 and 40
   - **b** What calculation did you use each time?
     Each time, the dividend was multiplied by 10.
   - **c** Use this pattern to determine $18 \div \frac{1}{10}$.
     The quotient is 180 because $18 \cdot 10 = 180$.
   - **d** Explain how you could determine the quotients of $4 \div \frac{2}{10}$ and $4 \div \frac{8}{10}$.
     Start with the quotient of $4 \div \frac{1}{10}$ and divide it by 2 or 8, which is 20 and 5, respectively.

2. Determine each quotient.
   - **a** $5 \div \frac{1}{10} = 50; 5 \div \frac{1}{10} = 5 \cdot 10 = 50$
   - **b** $5 \div \frac{3}{10} = \frac{50}{3}$ or $16\frac{2}{3}; 5 \div \frac{3}{10} = 5 \cdot 10 \cdot \frac{1}{3} = \frac{50}{3}$
   - **c** $5 \div \frac{9}{10} = \frac{50}{9}$ or $5\frac{5}{9}; 5 \div \frac{9}{10} = 5 \cdot 10 \cdot \frac{1}{9} = \frac{50}{9}$

3. Use the equation $2\frac{1}{2} \div \frac{1}{8} = 20$ to determine $2\frac{1}{2} \div \frac{5}{8}$. Show or explain your thinking.
   4; Sample response: There are 20 groups of $\frac{1}{8}$ in $2\frac{1}{2}$. If the size of each group is multiplied by 5 (from $\frac{1}{8}$ to $\frac{5}{8}$), then the number of groups will be divided by 5. $20 \div 5 = 4$.

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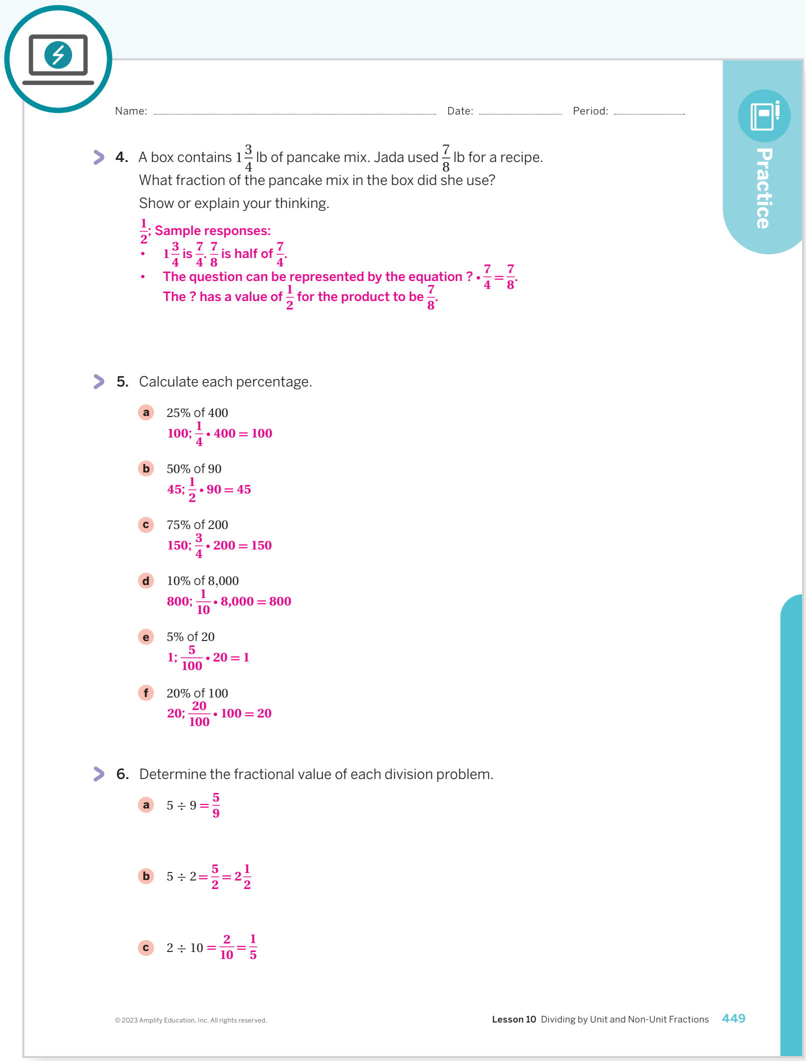

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4. A box contains $1\frac{3}{4}$ lb of pancake mix. Jada used $\frac{7}{8}$ lb for a recipe. What fraction of the pancake mix in the box did she use?
   Show or explain your thinking.
   $\frac{1}{2}$; Sample responses:
   - **$1\frac{3}{4}$ is $\frac{7}{4}$. $\frac{7}{8}$ is half of $\frac{7}{4}$.**
   - **The question can be represented by the equation $? \cdot \frac{7}{4} = \frac{7}{8}$. The ? has a value of $\frac{1}{2}$ for the product to be $\frac{7}{8}$.**

5. Calculate each percentage.
   - **a** 25% of 400
     $100; \frac{1}{4} \cdot 400 = 100$
   - **b** 50% of 90
     $45; \frac{1}{2} \cdot 90 = 45$
   - **c** 75% of 200
     $150; \frac{3}{4} \cdot 200 = 150$
   - **d** 10% of 8,000
     $800; \frac{1}{10} \cdot 8{,}000 = 800$
   - **e** 5% of 20
     $1; \frac{5}{100} \cdot 20 = 1$
   - **f** 20% of 100
     $20; \frac{20}{100} \cdot 100 = 20$

6. Determine the fractional value of each division problem.
   - **a** $5 \div 9 = \frac{5}{9}$
   - **b** $5 \div 2 = \frac{5}{2} = 2\frac{1}{2}$
   - **c** $2 \div 10 = \frac{2}{10} = \frac{1}{5}$

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Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 7	2
	5	Unit 3 Lesson 13	2
Formative ⚡	6	Unit 4 Lesson 11	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 4 | **LESSON 11**

Using an Algorithm to Divide Fractions

Let's divide fractions using the rule we learned.

Focus

Goals

1. Coordinate different strategies for dividing by a fraction.
2. **Language Goal:** Determine the quotient of two fractions, and explain the solution method. **(Speaking, Writing)**
3. **Language Goal:** Generalize a process for dividing a number n by a fraction $\frac{a}{b}$, and justify why this can be abstracted as $n \cdot \frac{b}{a}$. **(Speaking, Writing)**

Coherence

Today

Students complete the process of determining an algorithm for dividing any number by a fraction. They calculate quotients by using the steps they observed previously and compare them to quotients found by reasoning with a tape diagram while observing the structure. Through multiple examples, they connect the relationships among multiplication and division, fractions as division, and interpretations and representations of division to develop the algorithm: to divide by $\frac{a}{b}$, multiply by $\frac{b}{a}$.

Previously

In Lesson 10, students began developing a general algorithm for dividing fractions.

Coming Soon

In Lesson 12, students will write and solve related expressions to explore other generalizable methods for evaluating quotients involving fraction divisors.

Rigor

- Students divide with fractions to develop **procedural skills** of the algorithm of division with fractions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- colored pencils

Math Language Development

New word

- *reciprocal**

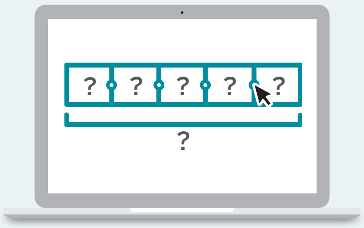
*The term *reciprocal* was defined for whole numbers in Lesson 10.

Amps | **Featured Activity**

Activity 1

Digital Diagrams

Students can sketch on digital tape diagrams to solve problems. You can overlay them all to see similarities and differences at a glance.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might be intimidated by the algorithm for dividing fractions. Ask students how they will regulate their emotions so that they can work towards their goal. Remind them that organization can calm them as it provides a structure within to work rather than chaos. Ask how they will organize their work to stay focused, and then help them to rely on their previous strategies to simplify the steps in the problem.

Modifications to Pacing

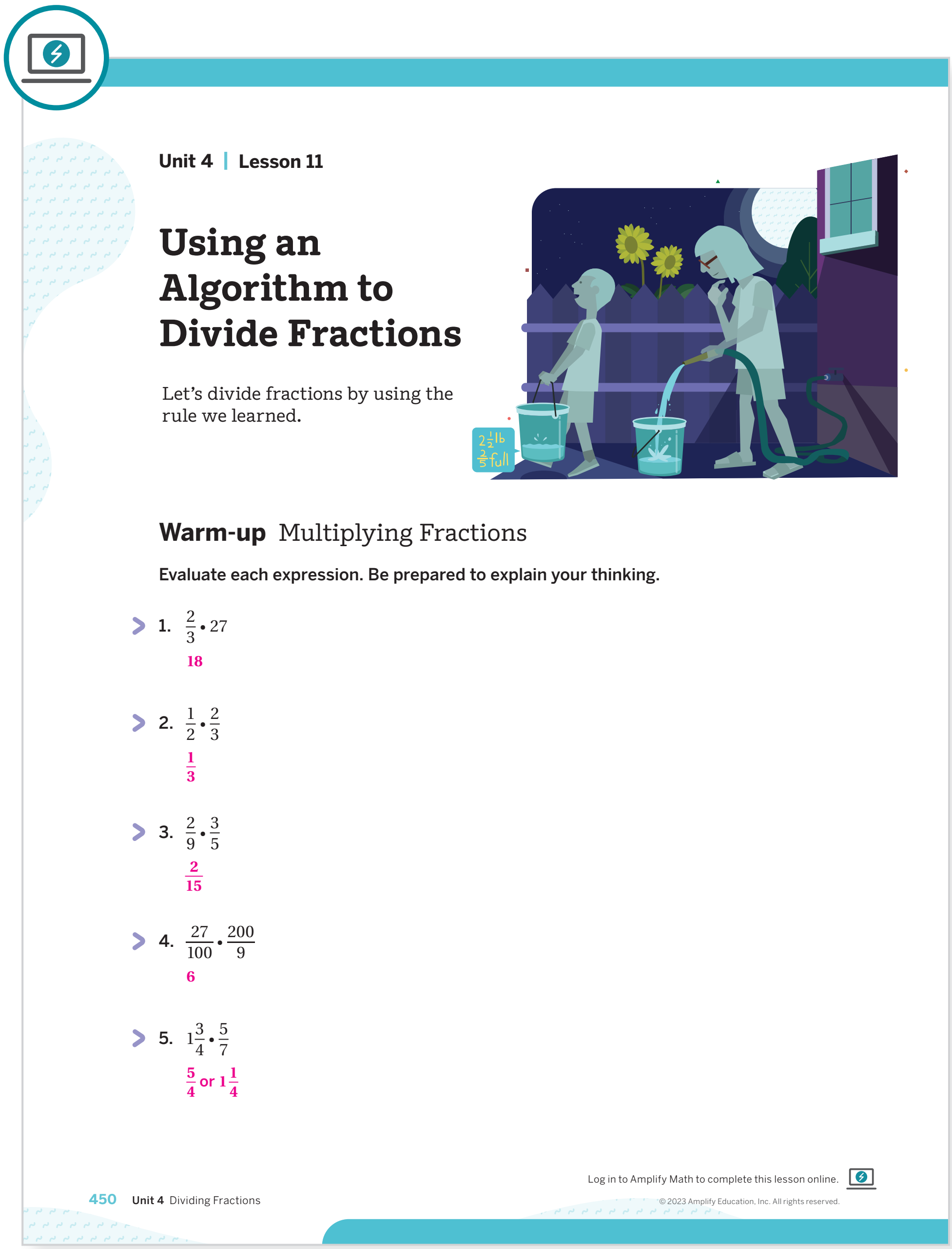
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problems 4–5 may be omitted.
- In **Activity 1**, Problem 3 may be discussed as a whole class.
- In **Activity 2**, have students choose 2 of the 4 problems.

Independent | 5 min

Warm-up Multiplying Fractions

Students revisit multiplication of fractions in preparation for dividing with fractions.



Unit 4 | **Lesson 11**

Using an Algorithm to Divide Fractions

Let's divide fractions by using the rule we learned.

Warm-up Multiplying Fractions

Evaluate each expression. Be prepared to explain your thinking.

1. $\frac{2}{3} \cdot 27$

   $\mathbf{18}$

2. $\frac{1}{2} \cdot \frac{2}{3}$

   $\frac{\mathbf{1}}{\mathbf{3}}$

3. $\frac{2}{9} \cdot \frac{3}{5}$

   $\frac{\mathbf{2}}{\mathbf{15}}$

4. $\frac{27}{100} \cdot \frac{200}{9}$

   $\mathbf{6}$

5. $1\frac{3}{4} \cdot \frac{5}{7}$

   $\frac{\mathbf{5}}{\mathbf{4}}$ **or** $\mathbf{1\frac{1}{4}}$

Log in to Amplify Math to complete this lesson online.

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1 Launch

Set an expectation for the amount of time for students to work on completing the Warm-up.

2 Monitor

Help students get started by asking, "How do you multiply fractions?"

Look for points of confusion:

- **Misapplying an algorithm for dividing.** Acknowledge this as correct for that operation, but point out that this is multiplication and ask, "Do you know a similar rule for multiplying instead?"

Look for productive strategies:

- Multiplying numerators and denominators.
- Converting whole numbers or mixed numbers to improper fractions.
- Simplifying fractions, including reducing values between factors in the product using common factors.

3 Connect

Display each expression.

Have individual students share their solutions and strategies.

Highlight that to multiply by a fraction, it is helpful for both factors to be written as fractions, and then the numerators can be multiplied and the denominators can be multiplied. While some fractions, or products can be simplified to equivalent fractions, that is not always necessary and does not change the value of the product.

Differentiated Support

Accessibility: ***Activate Prior Knowledge***

Remind students they previously multiplied fractions in elementary grades. Prior to beginning the Warm-up, have them list as many strategies as they can for multiplying two fractions, such as $\frac{1}{2} \cdot \frac{3}{4}$. Then ask them to preview the Warm-up problems to determine which strategies they think they will use.

Power-up

To power up students' ability to represent fractions as an expression with division, ask, have students complete:

Recall that the fraction $\frac{a}{b}$ can be rewritten as $a \div b$. Match each fractions with its equivalent division expression.

b $\frac{3}{2}$	**a.** $3 \div 8$
e $\frac{2}{3}$	**b.** $3 \div 2$
a $\frac{3}{8}$	**c.** $1 \div 2$
c $\frac{1}{2}$	**d.** $4 \div 2$
d $\frac{4}{2}$	**e.** $2 \div 3$

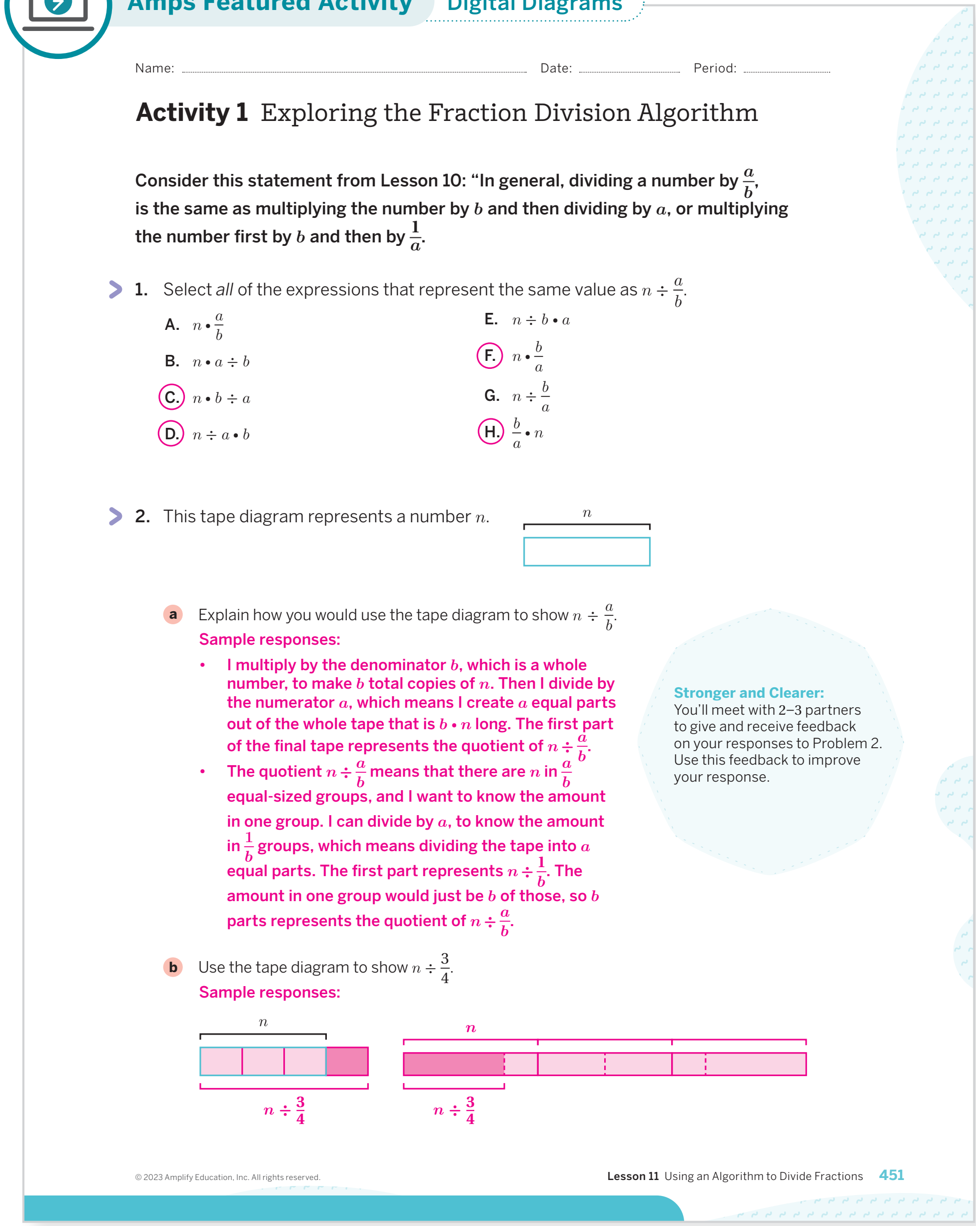
Use: Before Activity 1.
Informed by: Performance on Lesson 10, Practice Problem 6

Pairs | 20 min

Activity 1 Exploring the Fraction Division Algorithm

Students apply the patterns and general rule from the previous lesson to expressions involving letters, which lead them to the standard algorithm for dividing fractions.

Amps Featured Activity Digital Diagrams



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Activity 1 Exploring the Fraction Division Algorithm

Consider this statement from Lesson 10: "In general, dividing a number by $\frac{a}{b}$, is the same as multiplying the number by b and then dividing by a, or multiplying the number first by b and then by $\frac{1}{a}$.

1. Select *all* of the expressions that represent the same value as $n \div \frac{a}{b}$.

A. $n \cdot \frac{a}{b}$
B. $n \cdot a \div b$
[✓ C.] $n \cdot b \div a$
[✓ D.] $n \div a \cdot b$
E. $n \div b \cdot a$
[✓ F.] $n \cdot \frac{b}{a}$
G. $n \div \frac{b}{a}$
[✓ H.] $\frac{b}{a} \cdot n$

2. This tape diagram represents a number n.

a. Explain how you would use the tape diagram to show $n \div \frac{a}{b}$.

Sample responses:

- **I multiply by the denominator b, which is a whole number, to make b total copies of n. Then I divide by the numerator a, which means I create a equal parts out of the whole tape that is $b \cdot n$ long. The first part of the final tape represents the quotient of $n \div \frac{a}{b}$.**
- **The quotient $n \div \frac{a}{b}$ means that there are n in $\frac{a}{b}$ equal-sized groups, and I want to know the amount in one group. I can divide by a, to know the amount in $\frac{1}{b}$ groups, which means dividing the tape into a equal parts. The first part represents $n \div \frac{1}{b}$. The amount in one group would just be b of those, so b parts represents the quotient of $n \div \frac{a}{b}$.**

Stronger and Clearer: You'll meet with 2–3 partners to give and receive feedback on your responses to Problem 2. Use this feedback to improve your response.

b. Use the tape diagram to show $n \div \frac{3}{4}$.

Sample responses:

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Lesson 11 Using an Algorithm to Divide Fractions 451

① Launch

Consider having students pause for discussion after completing Problem 1. Tell them that for all of these problems, the letters can represent any whole numbers, but, if it is helpful, students may choose to work with actual numbers in place of the letters first.

Note: It is expected that many students will not have time or will not be able to determine all of the correct responses for Problem 3, and that is okay.

② Monitor

Help students get started by asking, "Do you remember how a fraction can be written as division? What do you remember about the general rules you determined in the previous lesson?"

Look for points of confusion:

- **Not knowing how to describe making a diagram when there are letters instead of numbers (Problem 2a).** Have students look ahead to Problem 2a and use $\frac{3}{4}$ to explain the specific case first.
- **Thinking division is commutative (Problems 1 and 3).** Ask, "Is 4 ÷ 2 the same as 2 ÷ 4?"

Look for productive strategies:

- Determining equivalent expressions by using the relationship between multiplication and division, the division interpretation of fractions, and the properties of operations.
- Applying the general strategies and interpretations of division from the previous lesson to represent the steps of multiplying by the denominator and dividing by the numerator in either order, along with using a tape diagram.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students choose numerical values to represent the numbers a, b, $\frac{a}{b}$, and $\frac{1}{a}$ in the statement at the top of the activity page. Then have them substitute those values into the problems in this activity to help them make sense of the relationships, using concrete values.

Extension: *Math Enrichment*

As a follow-up to Problem 3, have students draw diagrams or write an explanation that demonstrates why $\frac{c}{d} \div \frac{a}{b} = \frac{b}{a} \div \frac{d}{c}$.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 2, have them share their responses with their partner. Ask partners to review responses and provide feedback that can be used to improve the clarity of the responses. Consider displaying these sample questions:

- "Does the response make sense to you?"
- "What suggestions do you have for improvement?"

Have students revise their responses after receiving feedback.

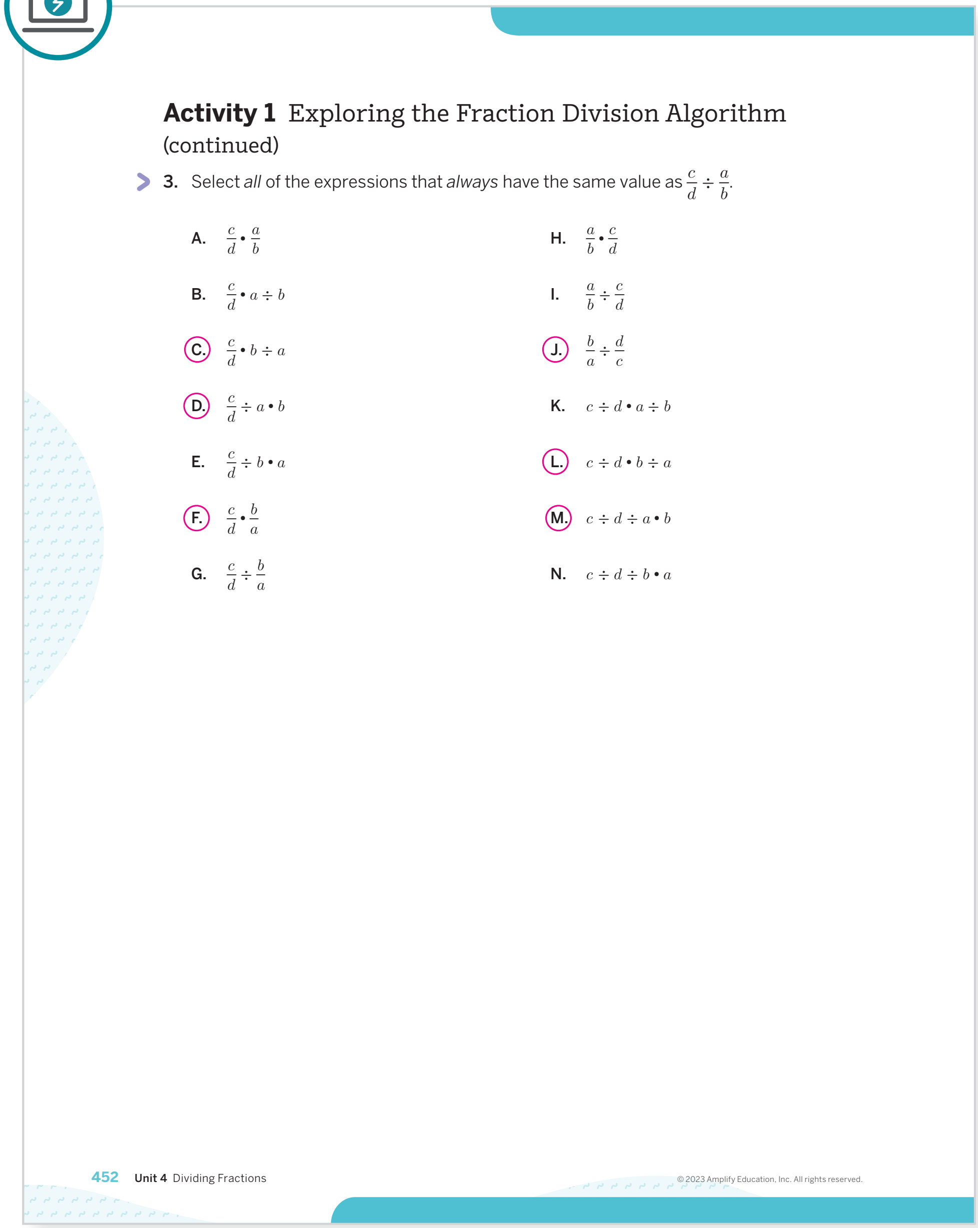
English Learners

Allow English Learners, and all students, to include visual drawings in their explanations which can help illustrate this concept for both the writer and the reader.

Pairs | 20 min

Activity 1 Exploring the Fraction Division Algorithm (continued)

Students apply the patterns and general rule from the previous lesson to expressions involving letters, which lead them to the standard algorithm for dividing fractions.



Activity 1 Exploring the Fraction Division Algorithm (continued)

3. Select *all* of the expressions that *always* have the same value as $\frac{c}{d} \div \frac{a}{b}$.

A. $\frac{c}{d} \cdot \frac{a}{b}$

B. $\frac{c}{d} \cdot a \div b$

[✓ C.] $\frac{c}{d} \cdot b \div a$

[✓ D.] $\frac{c}{d} \div a \cdot b$

E. $\frac{c}{d} \div b \cdot a$

[✓ F.] $\frac{c}{d} \cdot \frac{b}{a}$

G. $\frac{c}{d} \div \frac{b}{a}$

H. $\frac{a}{b} \cdot \frac{c}{d}$

I. $\frac{a}{b} \div \frac{c}{d}$

[✓ J.] $\frac{b}{a} \div \frac{d}{c}$

K. $c \div d \cdot a \div b$

[✓ L.] $c \div d \cdot b \div a$

[✓ M.] $c \div d \div a \cdot b$

N. $c \div d \div b \cdot a$

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3 Connect

Display all of the expressions for Problem 1, then a tape diagram for Problem 2, and finally all of the expressions for Problem 3.

Have students share how they determined which expressions were equivalent, focusing on F (in both Problems 1 and 3). Have students also share their thinking for Problem 2a and how they then applied it to Problem 2b.

Highlight that dividing any number by a fraction $\frac{a}{b}$ is not only the same as multiplying by b and dividing by a (or multiplying by $\frac{1}{a}$), but it is also equivalent to multiplying by the fraction $\frac{b}{a}$, which is called the *reciprocal*.

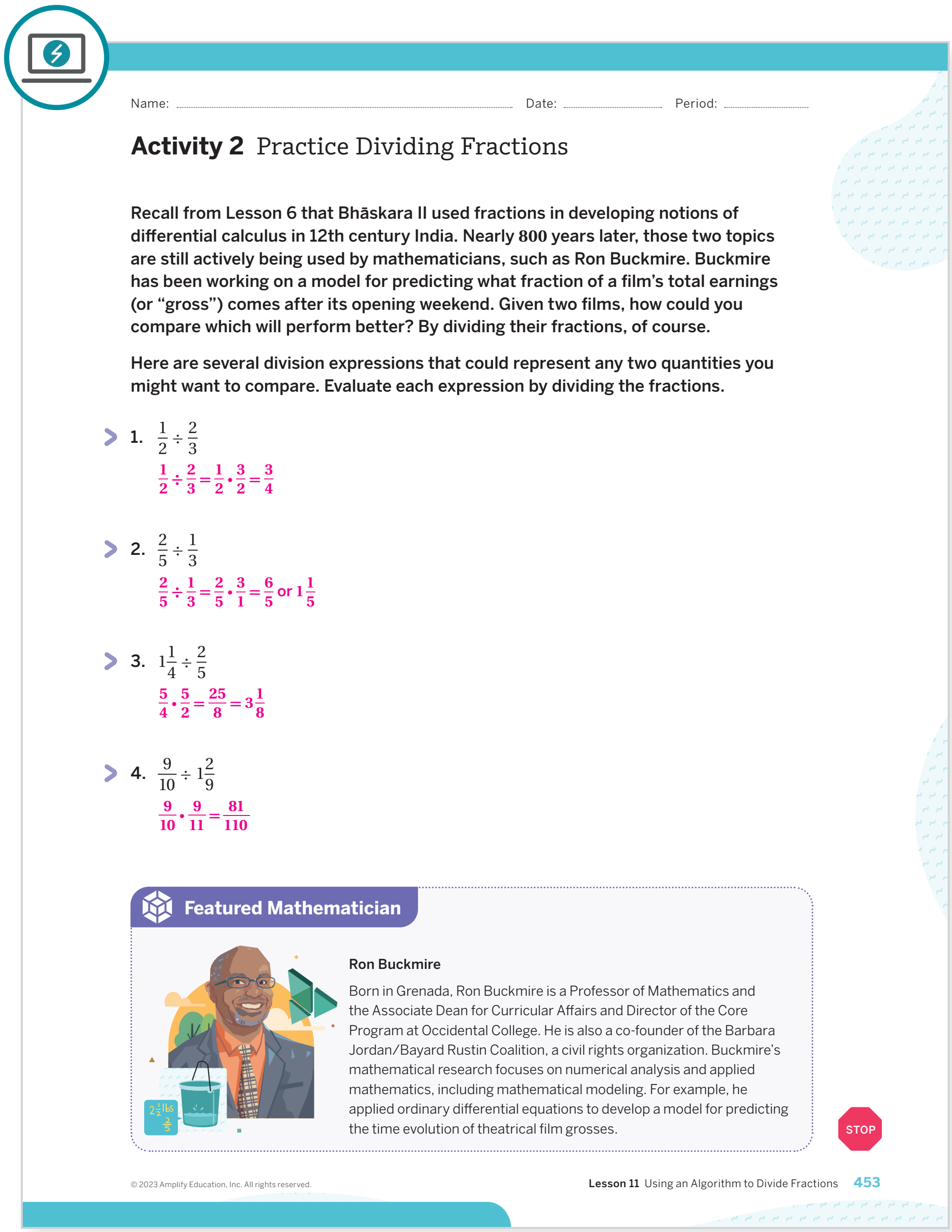
Define vocabulary: The ***reciprocal*** of a number is the fraction whose numerator is the denominator of the number and whose denominator is the numerator of the number. For example, $\frac{a}{b}$ and $\frac{b}{a}$ are reciprocals. Also, $\frac{1}{b}$ and $\frac{b}{1}$, or b, are reciprocals.

Note: The product of a number and its reciprocal is 1.

Pairs | 10 min

Activity 2 Practice Dividing Fractions

Students use the algorithm to divide fractions by fractions.



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Activity 2 Practice Dividing Fractions

Recall from Lesson 6 that Bhāskara II used fractions in developing notions of differential calculus in 12th century India. Nearly 800 years later, those two topics are still actively being used by mathematicians, such as Ron Buckmire. Buckmire has been working on a model for predicting what fraction of a film's total earnings (or "gross") comes after its opening weekend. Given two films, how could you compare which will perform better? By dividing their fractions, of course.

Here are several division expressions that could represent any two quantities you might want to compare. Evaluate each expression by dividing the fractions.

1. $\frac{1}{2} \div \frac{2}{3}$

   $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$

2. $\frac{2}{5} \div \frac{1}{3}$

   $\frac{2}{5} \div \frac{1}{3} = \frac{2}{5} \cdot \frac{3}{1} = \frac{6}{5}$ or $1\frac{1}{5}$

3. $1\frac{1}{4} \div \frac{2}{5}$

   $\frac{5}{4} \cdot \frac{5}{2} = \frac{25}{8} = 3\frac{1}{8}$

4. $\frac{9}{10} \div 1\frac{2}{9}$

   $\frac{9}{10} \cdot \frac{9}{11} = \frac{81}{110}$

Featured Mathematician

Ron Buckmire

Born in Grenada, Ron Buckmire is a Professor of Mathematics and the Associate Dean for Curricular Affairs and Director of the Core Program at Occidental College. He is also a co-founder of the Barbara Jordan/Bayard Rustin Coalition, a civil rights organization. Buckmire's mathematical research focuses on numerical analysis and applied mathematics, including mathematical modeling. For example, he applied ordinary differential equations to develop a model for predicting the time evolution of theatrical film grosses.

STOP

1 Launch

Have students use the *Think-Pair-Share* routine. Provide 5 minutes of individual work time and then 1–2 minutes to compare solutions and strategies.

2 Monitor

Help students get started by asking, "What is the first step of the algorithm for dividing by a fraction?"

Look for points of confusion:

- **Multiplying without using the reciprocal.** Have students use the two-step process from Lesson 10 first instead. Then ask, "How can you relate division to a fraction?"

Look for productive strategies:

- Rewriting mixed numbers as improper fractions.
- Writing the equivalent expression of multiplying by the reciprocal of the divisor, and then "multiplying across."

3 Connect

Display all of the problems.

Have students share their strategies and solutions for the problems, emphasizing the repeated reasoning.

Highlight that the algorithm is often more efficient than drawing diagrams or using other general procedures, especially when the values of numerators and denominators are large or share no common factors.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide blank tape diagrams for students to use to partition and label. Consider making copies of the *Tape Diagrams* PDF, being sure to direct students to only use the blank tape diagrams that are not labeled with percentages.

Math Language Development

MLR8: Discussion Supports

During the Connect, display the following sentence frames for students to use when they share their strategies and solutions:

- "I know there are _____ $\frac{2}{3}$s in $\frac{1}{2}$ because . . . "
- "I drew a diagram like this because . . ."
- "First, I ___ because . . ."
- "I rewrote the division as multiplication like this, ____, because . . ."

This will help students produce statements that describe how to divide a number by any fraction.

Featured Mathematician

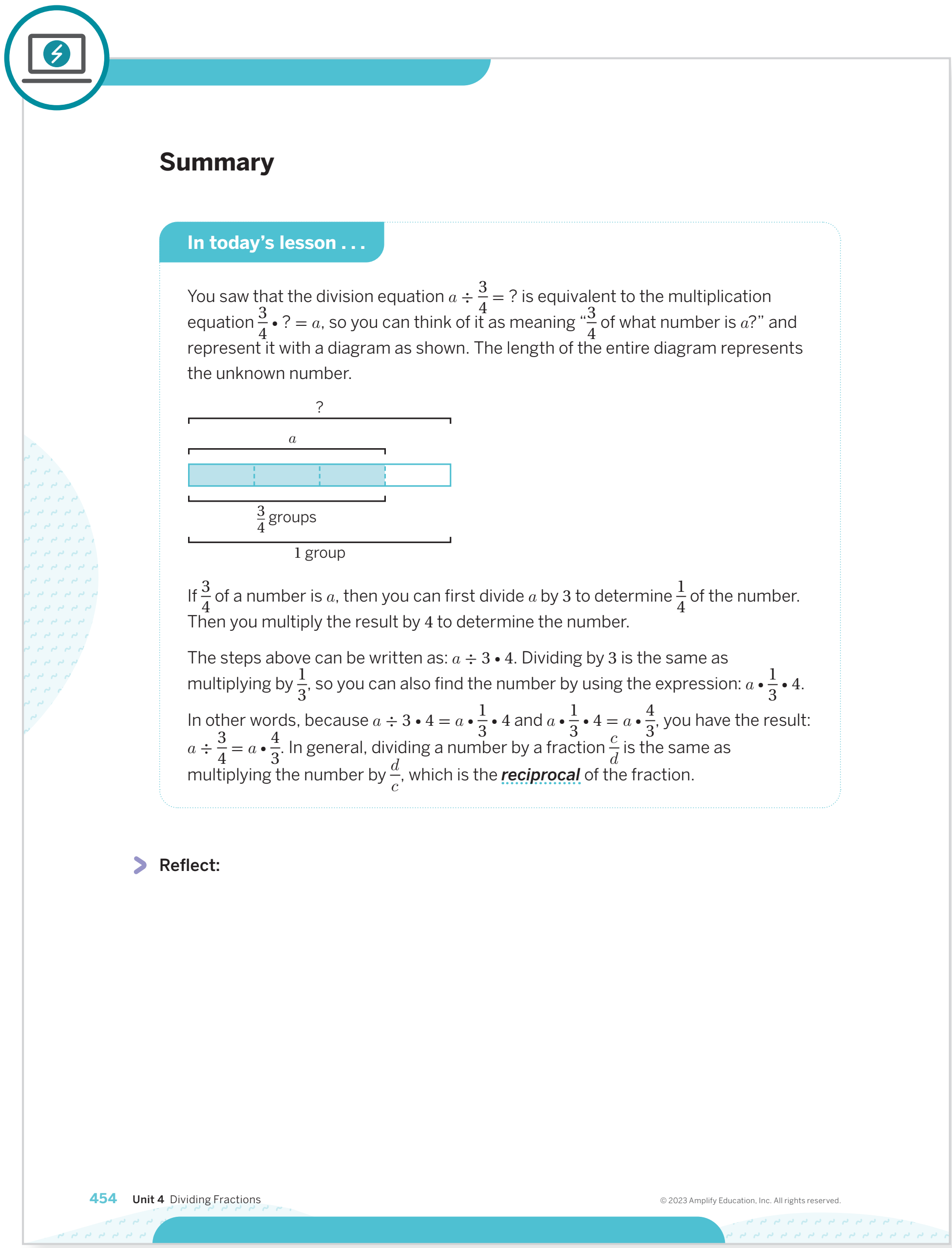
Ron Buckmire

Have students read about featured mathematician Ron Buckmire, a Professor of Mathematics at Occidental College, who studies and applies differential equations to model both mathematical and real-world problems, such as predicting the box office performance of motion pictures.

Whole Class | 5 min

Summary

Review and synthesize by discussing the process of using the algorithm to divide fractions.



Summary

In today's lesson . . .

You saw that the division equation $a \div \frac{3}{4} = ?$ is equivalent to the multiplication equation $\frac{3}{4} \bullet ? = a$, so you can think of it as meaning "$\frac{3}{4}$ of what number is a?" and represent it with a diagram as shown. The length of the entire diagram represents the unknown number.

If $\frac{3}{4}$ of a number is a, then you can first divide a by 3 to determine $\frac{1}{4}$ of the number. Then you multiply the result by 4 to determine the number.

The steps above can be written as: $a \div 3 \bullet 4$. Dividing by 3 is the same as multiplying by $\frac{1}{3}$, so you can also find the number by using the expression: $a \bullet \frac{1}{3} \bullet 4$.

In other words, because $a \div 3 \bullet 4 = a \bullet \frac{1}{3} \bullet 4$ and $a \bullet \frac{1}{3} \bullet 4 = a \bullet \frac{4}{3}$, you have the result: $a \div \frac{3}{4} = a \bullet \frac{4}{3}$. In general, dividing a number by a fraction $\frac{c}{d}$ is the same as multiplying the number by $\frac{d}{c}$, which is the ***reciprocal*** of the fraction.

Reflect:

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Synthesize

Formalize vocabulary: ***reciprocal***

Highlight that dividing by $\frac{a}{b}$ is equivalent to multiplying by b and then by $\frac{1}{a}$, or simply multiplying by $\frac{b}{a}$ (the *reciprocal* of $\frac{a}{b}$).

Ask, "Why do you think tape diagrams are useful for representing equations involving division of fractions?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is dividing by a fraction related to multiplying fractions?"

Math Language Development

MLR2: Collect and Display

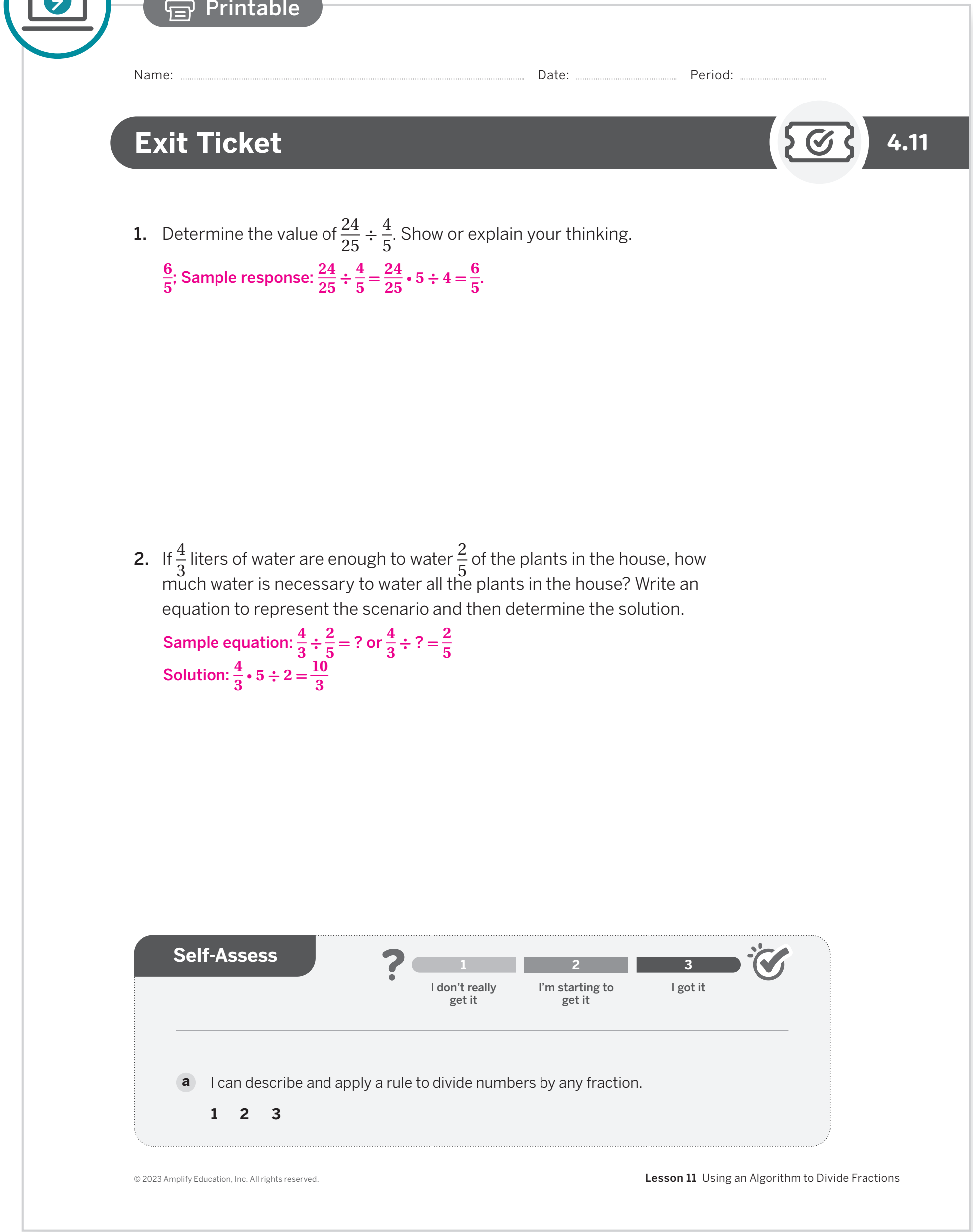
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *reciprocal* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of dividing with fractions by using the algorithm.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 4.11

1. Determine the value of $\frac{24}{25} \div \frac{4}{5}$. Show or explain your thinking.

   $\frac{6}{5}$; Sample response: $\frac{24}{25} \div \frac{4}{5} = \frac{24}{25} \cdot 5 \div 4 = \frac{6}{5}$.

2. If $\frac{4}{3}$ liters of water are enough to water $\frac{2}{5}$ of the plants in the house, how much water is necessary to water all the plants in the house? Write an equation to represent the scenario and then determine the solution.

   Sample equation: $\frac{4}{3} \div \frac{2}{5} = ?$ or $\frac{4}{3} \div ? = \frac{2}{5}$

   Solution: $\frac{4}{3} \cdot 5 \div 2 = \frac{10}{3}$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can describe and apply a rule to divide numbers by any fraction.

1 2 3

Lesson 11 Using an Algorithm to Divide Fractions

Success looks like . . .

- **Goal:** Coordinating different strategies for dividing by a fraction.
- **Language Goal:** Determining the quotient of two fractions, and explaining the solution method. **(Speaking and Listening, Writing)**
  - » Determining the value of $\frac{24}{25} \div \frac{4}{5}$ in Problem 1.
- **Language Goal:** Generalizing a process for dividing a number n by a fraction $\frac{a}{b}$, and justifying why this can be abstracted as $n \cdot \frac{b}{a}$. **(Speaking and Listening, Writing)**

Suggested next steps

If students cannot determine the quotient in Problem 1, consider:

- Reviewing the algorithm from Activity 1.
- Asking, "Is there a strategy other than the algorithm you could use instead?"
- Assigning Practice Problem 1.

If students struggle to write the equation in Problem 2, consider:

- Asking, "Which quantity would make the most sense to represent equal-sized groups?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did working with expressions involving letters reveal about your students as learners?
- What different ways did students approach dividing by fractions? What does that tell you about similarities and differences among your students?

Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. Select *all* the statements that provide the correct steps for evaluating the expression $\frac{14}{15} \div \frac{7}{5}$.

   [✓ A.] Multiply $\frac{14}{15}$ by 5, and then multiply by $\frac{1}{7}$.

   B. Divide $\frac{14}{15}$ by 5, and then multiply by $\frac{1}{7}$.

   C. Multiply $\frac{14}{15}$ by 7, and then multiply by $\frac{1}{5}$.

   [✓ D.] Multiply $\frac{14}{15}$ by 5, and then divide by 7.

2. Claire said that $\frac{4}{3} \div \frac{5}{2}$ is $\frac{10}{3}$ because $\frac{4}{3} \cdot 5 = \frac{20}{3}$ and $\frac{20}{3} \div 2 = \frac{10}{3}$. Explain why Clare's quotient and reasoning are incorrect. Determine the correct quotient.

   The correct quotient is $\frac{8}{15}$; Sample responses:

   - **Clare should have multiplied $\frac{4}{3}$ by 2 to determine how many groups of $\frac{1}{2}$ are in $\frac{4}{3}$ and then divide the result by 5.**
   - **Clare divided the fraction $\frac{4}{3}$ by the fraction $\frac{2}{5}$, instead of $\frac{5}{2}$.**

3. Determine the value of each of the following.

   a $\frac{8}{9} \div 4$

   $\frac{8}{9} \cdot \frac{1}{4} = \frac{2}{9}$

   b $\frac{3}{4} \div \frac{1}{2}$

   $\frac{3}{4} \cdot 2 = \frac{3}{2}$ or $1\frac{1}{2}$

   c $\frac{9}{2} \div \frac{3}{8}$

   $\frac{9}{2} \cdot \frac{8}{3} = 12$

   d $3\frac{1}{3} \div \frac{2}{9}$

   $\frac{10}{3} \cdot 9 \div 2 = 15$

Practice

Name: ______ Date: ______ Period: ______

4. Consider the problem: After charging for $\frac{1}{3}$ of an hour, a phone is at $\frac{2}{5}$ of its full power. How long will it take the phone to charge completely? Decide whether each equation can represent the situation. Write *yes* or *no*.

   a $\frac{1}{3} \cdot ? = \frac{2}{5}$

   No

   b $\frac{1}{3} \div \frac{2}{5} = ?$

   Yes

   c $\frac{2}{5} \div \frac{1}{3} = ?$

   No

   d $\frac{2}{5} \cdot ? = \frac{1}{3}$

   Yes

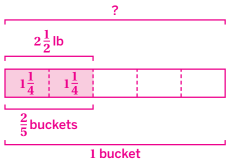
5. Elena and Noah are each filling a bucket with water. Noah's bucket is $\frac{2}{5}$ full and the water weighs $2\frac{1}{2}$ lb. How much does Elena's water weigh if her bucket is full and her bucket is identical to Noah's?

   a Write a multiplication and a division equation to represent the scenario.

   $\frac{2}{5} \cdot ? = 2\frac{1}{2}$ **(or equivalent);** $2\frac{1}{2} \div \frac{2}{5} = ?$

   b Draw a diagram to show the relationship between the quantities and determine the answer.

   Sample diagram:



6. Without calculating, determine how the expressions $98 \cdot 25$ and $(100 \cdot 25) - (2 \cdot 25)$ are related. Explain your thinking.

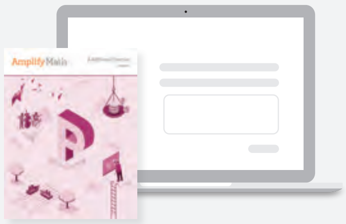
   Sample response: They are related expressions that give the same product. $98 \cdot 25$ means 98 groups of 25. $(100 \cdot 25) - (2 \cdot 25)$ means 100 groups of 25 minus 2 groups of 25. This is a total of 98 groups of 25 because $100 - 2 = 98$, which is the same as the first expression.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 9	2
	5	Unit 4 Lesson 8	2
Formative	6	Unit 4 Lesson 12	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

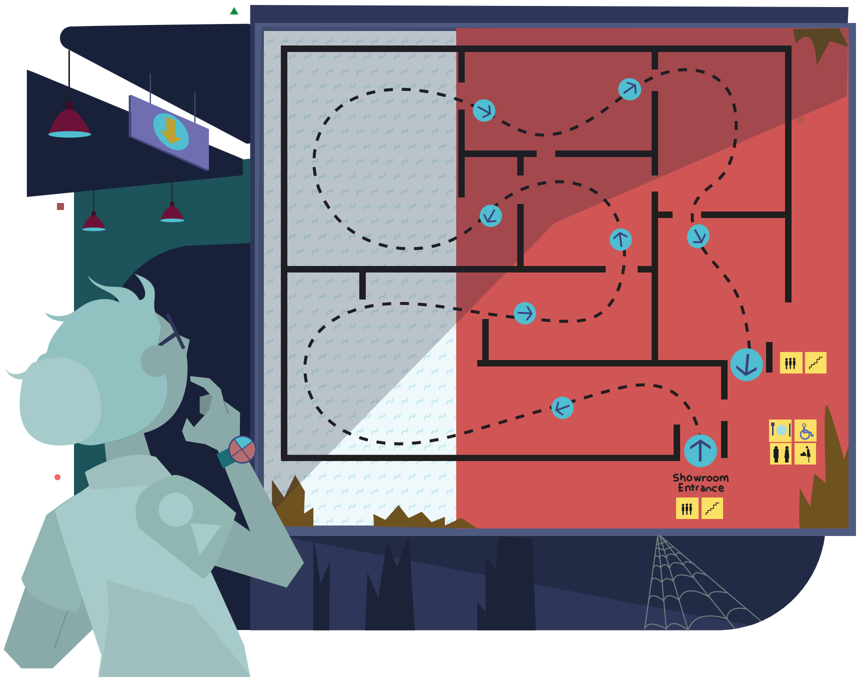


For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 4 | **LESSON 12**

Related Quotients

Let's solve division problems by using related quotients.



Focus

Goals

1. Given a division expression, write another related division expression with the same quotient.
2. **Language Goal:** Explain how two expressions are related and why this results in the same quotient. **(Speaking and Listening, Writing)**

Coherence

Today

Students write and evaluate related division expressions. They begin by mentally evaluating a multiplication expression, recognizing that related and original expressions result in the same answer. Students then evaluate a string of division expressions to recognize that any two division expressions will have the same quotient if the dividend and divisor are multiplied by the same factor. Students then apply this thinking in the context of a security guard investigating an area of a store, seeing that related expressions can often help them more efficiently attend to units and interpret quotients in context than using the algorithm. While it is optional, Activity 3 provides students the opportunity to make connections between their work with ratios and fraction division.
Note: This lesson has the second "clue" for the Capstone activity.

Previously

In Lessons 10–11, students generalized an algorithm for dividing fractions and multiplied by the reciprocal to divide a number by a fraction.

Coming Soon

In Lesson 13, students will use multiplication and division of fractions to solve problems involving fractional lengths and multiplicative comparison.

Rigor

- Students further their **conceptual understanding** of division by writing and evaluating related quotients.
- Students build **fluency** dividing whole numbers and fractions by fractions.
- Students **apply** their understanding of ratios to relate quotients of fractions as representing "how much in one group?" to corresponding unit ratios (optional Activity 3).

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	15 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF (optional)
- Activity 3 PDF (answers)

Math Language Development

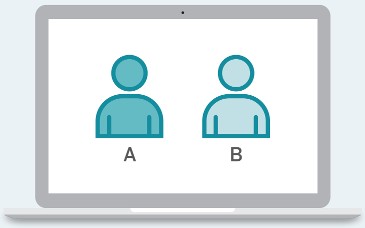
Review word

- *reciprocal*

Amps | Featured Activity

Activity 1
Using Work From Previous Slides

Students see the expressions and quotients they calculate in a table to explore how they are related.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

When working in pairs, students might misinterpret the quotient in the context of Activity 2. Highlight the importance of attending to the unit of each value, and set expectations for how partners can help one another determine precise information needed. Prior to the activity, work with students to set expectations for their behavior when working with a partner. Decide on a code word that classmates can use when they need to gently remind someone to adjust their behavior to show respect.

Modifications to Pacing

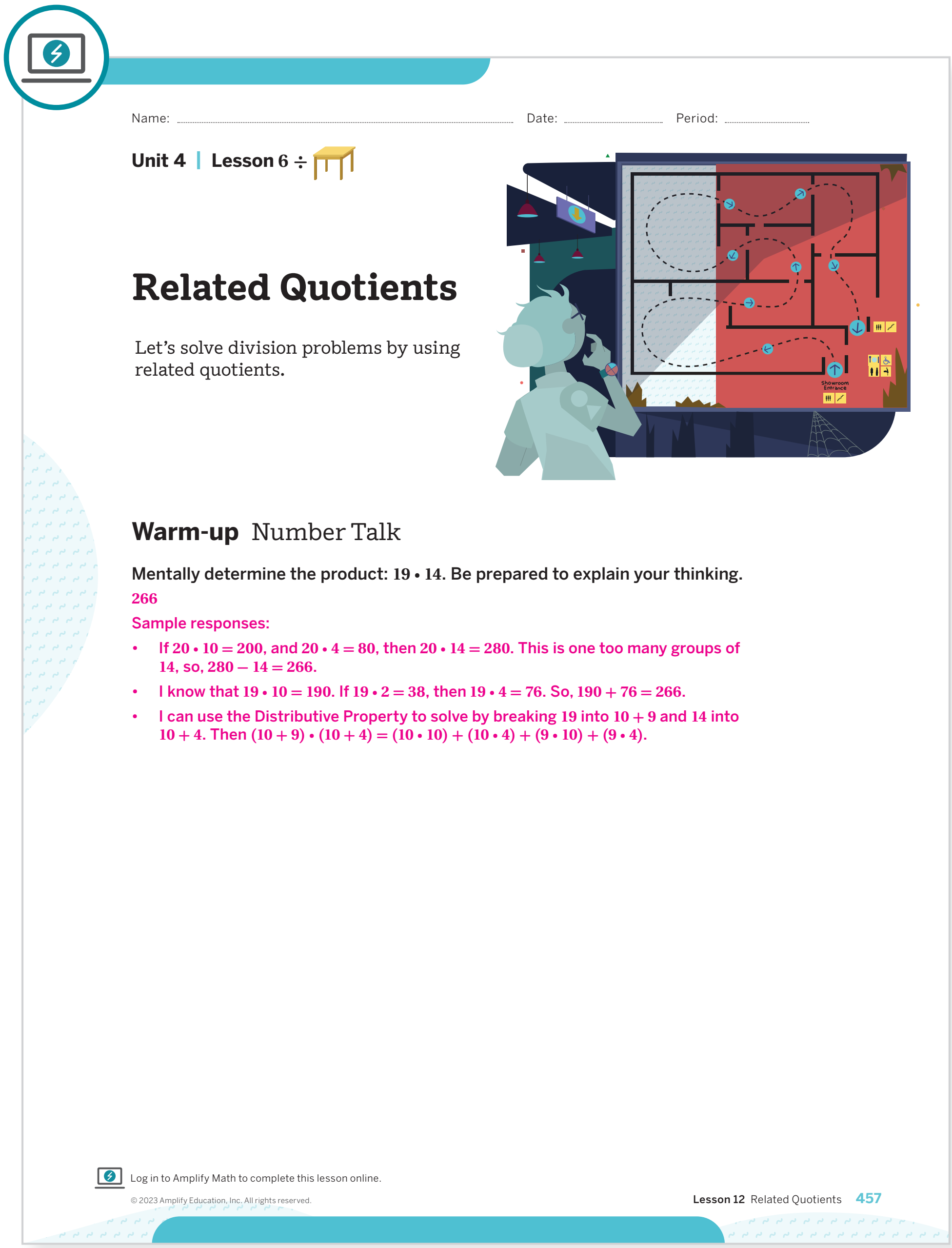
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have students complete Problems 1a–e with a partner. Problems 2 and 3 can also be done as a whole class.
- In **Activity 2**, have students work on each problem with their partner. Problems 3b–c may also be omitted.
- Optional **Activity 3** may be omitted.

Independent | 5 min

Warm-up Number Talk

Students use the properties of operations to mentally solve a multiplication problem, seeing the efficiency of using related expressions.



Name: ______ Date: ______ Period: ______

Unit 4 | **Lesson 6** ÷

Related Quotients

Let's solve division problems by using related quotients.

Warm-up Number Talk

Mentally determine the product: $19 \cdot 14$. Be prepared to explain your thinking.

266

Sample responses:

- If $20 \cdot 10 = 200$, and $20 \cdot 4 = 80$, then $20 \cdot 14 = 280$. This is one too many groups of 14, so, $280 - 14 = 266$.
- I know that $19 \cdot 10 = 190$. If $19 \cdot 2 = 38$, then $19 \cdot 4 = 76$. So, $190 + 76 = 266$.
- I can use the Distributive Property to solve by breaking 19 into $10 + 9$ and 14 into $10 + 4$. Then $(10 + 9) \cdot (10 + 4) = (10 \cdot 10) + (10 \cdot 4) + (9 \cdot 10) + (9 \cdot 4)$.

Log in to Amplify Math to complete this lesson online.

Lesson 12 Related Quotients 457

1 Launch

Conduct the *Number Talk* routine using the expression.

2 Monitor

Help students get started by activating prior knowledge. Ask, "How could you estimate the product?"

Look for points of confusion:

- **Adding only the products of the tens digits and ones digits.** Have students estimate the product, and ask, "Does your product make sense?"

Look for productive strategies:

- Mentally applying the algorithm. Encourage students to use an additional strategy to evaluate.
- Evaluating a simpler form of the problem by:
  - » rounding one or both factors before multiplying, then subtracting 14 and/or 20.
  - » decomposing one factor into its place value parts (e.g., $19 \cdot 14 = (19 \cdot 10) + (19 \cdot 4)$), or using the Distributive Property to decompose both factors into place value parts and adding the products.
  - » using the associative property to simplify $19 \cdot 4 = 19 \cdot 2 \cdot 2$ or $19 \cdot 4 = (19 \cdot 2) + (19 \cdot 2)$.

3 Connect

Have students share their products and how they made the expression "friendlier" to evaluate mentally.

Display student strategies as they share.

Ask, "Did the factors in the problem help you decide which strategy to use? How?"

Highlight how the "friendlier" expressions students used to solve are related to the original expression, and how they can be more efficient to solve. Note that related expressions result in the same answer.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their thinking for how they mentally determined the product, display the following sentence frames for them to use to help structure their thoughts.

- "First, I _____ because . . ."
- "I noticed _____, so I . . ."

As students share their strategies, emphasize the mathematical reasoning they used to break down the problem into "friendlier" parts.

English Learners

Provide students the opportunity to rehearse and formulate what they will say with a partner before they share with the whole class.

Power-up

To power up students' ability to determine how two expressions are related, ask, have students complete:

Match equivalent expressions.

_c_ $9 \cdot 8$	**a.** $(10 \cdot 8) + (2 \cdot 8)$
_d_ $8 \cdot 8$	**b.** $(10 \cdot 8) + (1 \cdot 8)$
_b_ $11 \cdot 8$	**c.** $(10 \cdot 8) - (1 \cdot 8)$
_a_ $12 \cdot 8$	**d.** $(10 \cdot 8) - (2 \cdot 8)$

Use: Before the Warm-up.

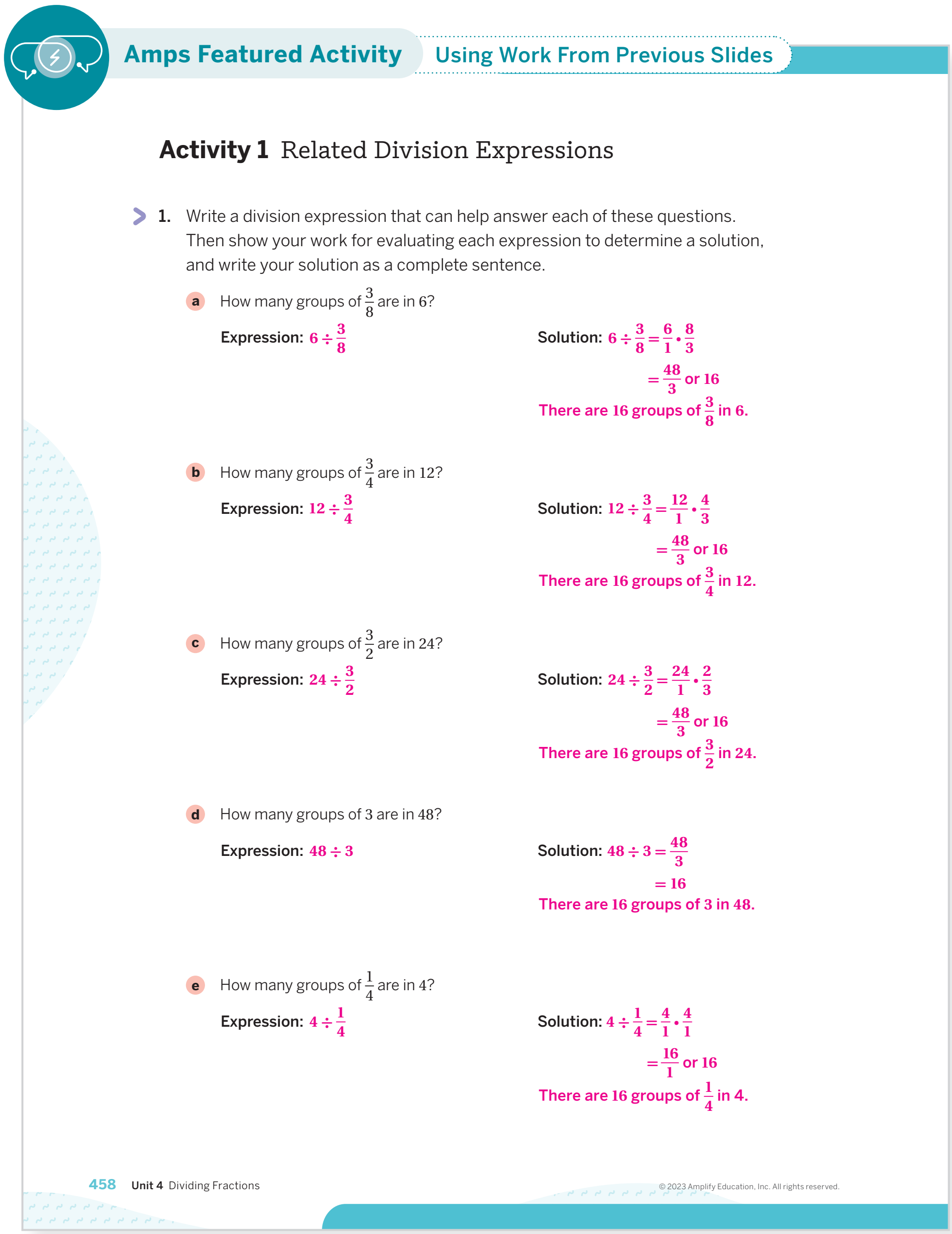
Informed by: Performance on Lesson 11, Practice Problem 6

Pairs | 15 min

Activity 1 Related Division Expressions

Students divide fractions using related expressions resulting from multiplying or dividing the dividend and divisor by the same factor, and recognize the quotient is the same.

Amps Featured Activity Using Work From Previous Slides



Activity 1 Related Division Expressions

1. Write a division expression that can help answer each of these questions. Then show your work for evaluating each expression to determine a solution, and write your solution as a complete sentence.

a How many groups of $\frac{3}{8}$ are in 6?

Expression: $6 \div \frac{3}{8}$

Solution: $6 \div \frac{3}{8} = \frac{6}{1} \cdot \frac{8}{3}$

$= \frac{48}{3}$ or 16

There are 16 groups of $\frac{3}{8}$ in 6.

b How many groups of $\frac{3}{4}$ are in 12?

Expression: $12 \div \frac{3}{4}$

Solution: $12 \div \frac{3}{4} = \frac{12}{1} \cdot \frac{4}{3}$

$= \frac{48}{3}$ or 16

There are 16 groups of $\frac{3}{4}$ in 12.

c How many groups of $\frac{3}{2}$ are in 24?

Expression: $24 \div \frac{3}{2}$

Solution: $24 \div \frac{3}{2} = \frac{24}{1} \cdot \frac{2}{3}$

$= \frac{48}{3}$ or 16

There are 16 groups of $\frac{3}{2}$ in 24.

d How many groups of 3 are in 48?

Expression: $48 \div 3$

Solution: $48 \div 3 = \frac{48}{3}$

$= 16$

There are 16 groups of 3 in 48.

e How many groups of $\frac{1}{4}$ are in 4?

Expression: $4 \div \frac{1}{4}$

Solution: $4 \div \frac{1}{4} = \frac{4}{1} \cdot \frac{4}{1}$

$= \frac{16}{1}$ or 16

There are 16 groups of $\frac{1}{4}$ in 4.

458 Unit 4 Dividing Fractions

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1 Launch

Have students use the *Think-Pair-Share* routine for Problem 1. Have one partner independently complete Problem 1a, b, and e, and the other partner independently complete Problem 1c, d, and e. Then have them share solutions and use both partners' work to complete Problems 2–3.

2 Monitor

Help students get started by having them draw a tape diagram to represent Problem 1a. Ask, "What expression does your model represent?"

Look for points of confusion:

- **Reversing the dividend and divisor in their expressions.** Ask, "In your expression, which value is the whole? The size of the group? Does that match the original question?"
- **Not recognizing the relationship(s) between the expressions.** Ask, "How did the dividend and divisor change from Problem 1a to 1b? Does the same change happen from each expression to the next?"
- **Struggling to write a related expression.** Ask, "How is the expression in Problem 1d related to Problem 3?"

Look for productive strategies:

- Using the algorithm or common denominators to evaluate Problems 1a–e.
- Recognizing that all of the given expressions in Problem 1 are related to each other because the dividends and divisors are multiplied or divided by the same value; therefore, the quotients are all the same.
- Recognizing that three-sixteenths is the same as $3 \div 16$ (in Problem 3), and using the relationship between multiplication and division to generate a related expression with a whole-number divisor and a whole-number dividend.

Activity 1 continued >

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Before students begin Problem 1a, display a question using whole numbers, such as "How many groups of 2 are in 6?" Ask students to write the corresponding expression and solution. Consider replacing the fractions in Problems 1a–1e with the unit fractions that have the same denominator. For example, replace $\frac{3}{8}$ with $\frac{1}{8}$ in Problem 1a.

Extension: *Math Enrichment*

Have students complete the following problem:
For Problems 1a–1e, how would the dividend and divisor change if you wanted to double the quotient each time? Halve the quotient?

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect statement for Problem 2, such as "Going from Problem 1a to 1b, the divisor is halved because 4 is half of 8." Ask:
Critique: "Why is this statement incorrect? Look for students who indicated that the divisor is doubled because fourths are twice (double) the size of eighths.

Correct and Clarify: Have students write a corrected statement. Then have them explain how they know their statement is correct.

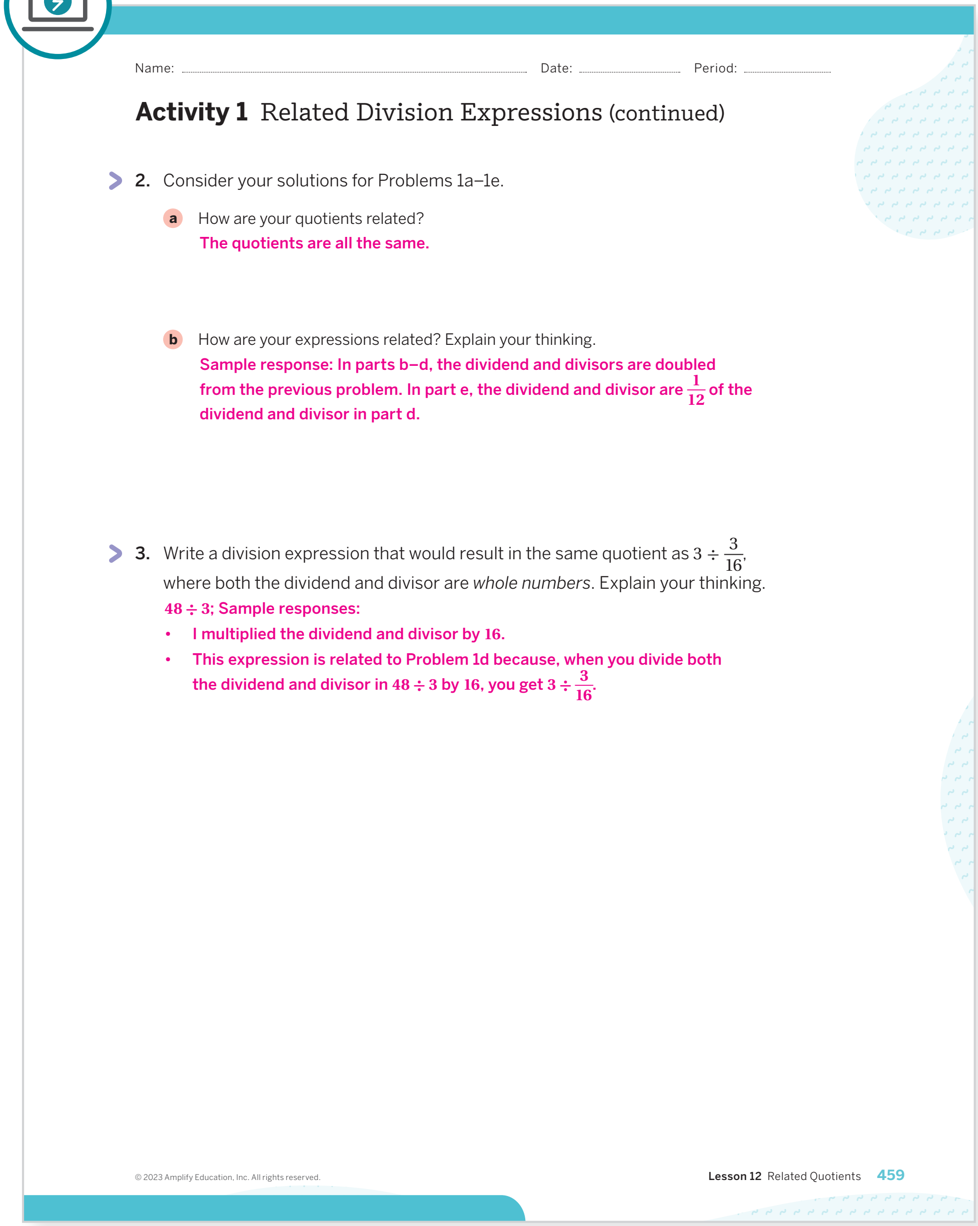
English Learners

In each statement, annotate the number that comes after the phrase "are in" as the *dividend* and the number that comes after the phrase "how many groups of" as the *divisor*.

Pairs | 15 min

Activity 1 Related Division Expressions (continued)

Students divide fractions using related expressions resulting from multiplying or dividing the dividend and divisor by the same factor, and recognize the quotient is the same.



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Activity 1 Related Division Expressions (continued)

2. Consider your solutions for Problems 1a–1e.

a How are your quotients related?

The quotients are all the same.

b How are your expressions related? Explain your thinking.

Sample response: In parts b–d, the dividend and divisors are doubled from the previous problem. In part e, the dividend and divisor are $\frac{1}{12}$ of the dividend and divisor in part d.

3. Write a division expression that would result in the same quotient as $3 \div \frac{3}{16}$, where both the dividend and divisor are *whole numbers*. Explain your thinking.

$48 \div 3$; Sample responses:

- **I multiplied the dividend and divisor by 16.**
- **This expression is related to Problem 1d because, when you divide both the dividend and divisor in $48 \div 3$ by 16, you get $3 \div \frac{3}{16}$.**

3 Connect

Display the solutions for Problem 1.

Have students share their solutions and strategies for Problems 2–3.

Ask:

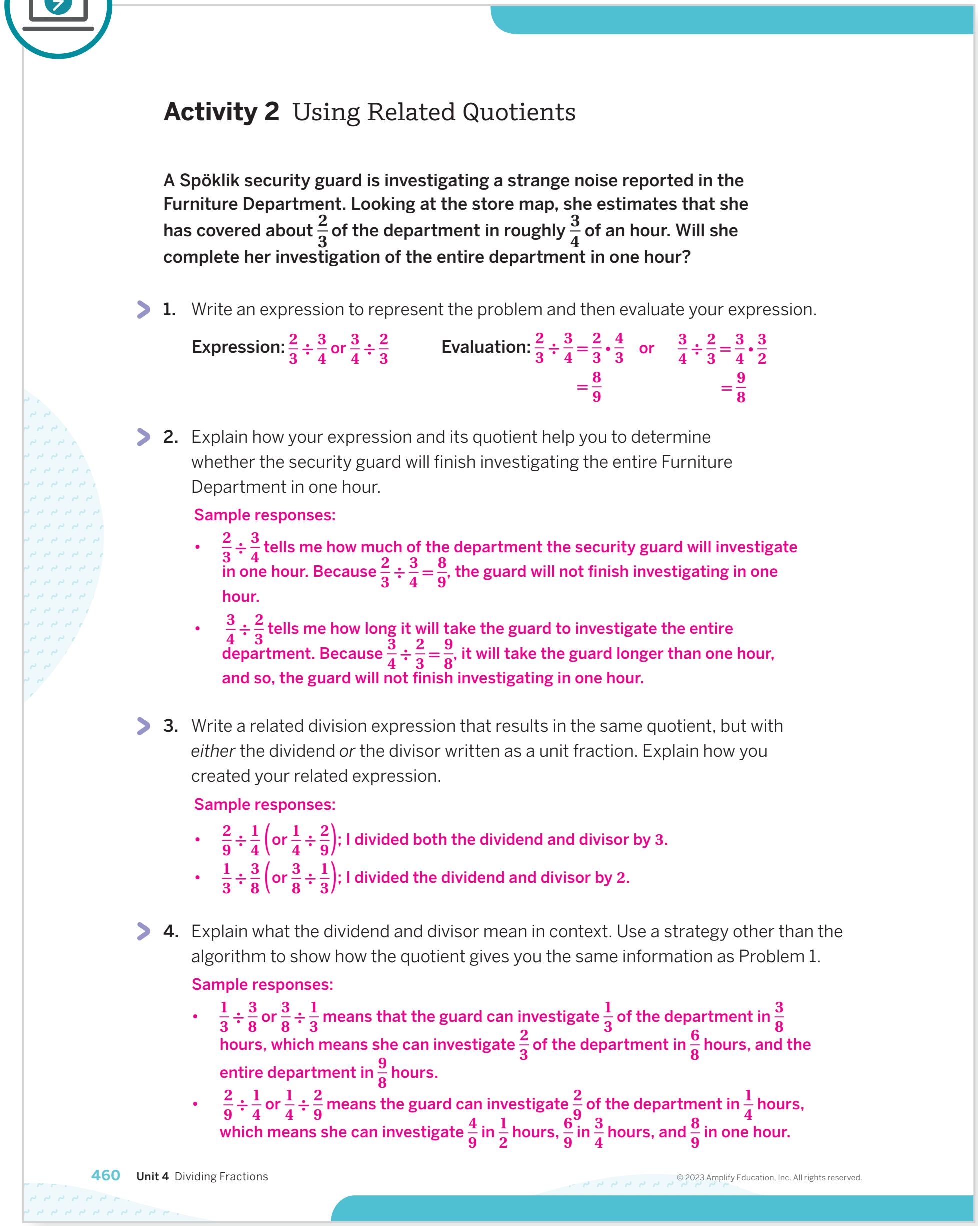
- "In Problem 3, which expression — the original one or the related one you created — can be solved in fewer steps?" The related expression can be solved in fewer steps.
- "How are related expressions similar to working with common denominators?" Sample response: You multiply both the dividend and divisor by the same value.

Highlight that any two division expressions that can be related by multiplying or dividing the dividend and divisor by the same value will result in the same quotient, including those involving fractional values.

Pairs | 15 min

Activity 2 Using Related Quotients

Students solve a real-world problem by writing a related division expression with a unit fraction, and recognize this as another universal strategy different from the algorithm.



Activity 2 Using Related Quotients

A Spöklik security guard is investigating a strange noise reported in the Furniture Department. Looking at the store map, she estimates that she has covered about $\frac{2}{3}$ of the department in roughly $\frac{3}{4}$ of an hour. Will she complete her investigation of the entire department in one hour?

1. Write an expression to represent the problem and then evaluate your expression.

   Expression: $\frac{2}{3} \div \frac{3}{4}$ or $\frac{3}{4} \div \frac{2}{3}$ **Evaluation:** $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9}$ or $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \cdot \frac{3}{2} = \frac{9}{8}$

2. Explain how your expression and its quotient help you to determine whether the security guard will finish investigating the entire Furniture Department in one hour.

   Sample responses:
   - $\frac{2}{3} \div \frac{3}{4}$ tells me how much of the department the security guard will investigate in one hour. Because $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$, the guard will not finish investigating in one hour.
   - $\frac{3}{4} \div \frac{2}{3}$ tells me how long it will take the guard to investigate the entire department. Because $\frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$, it will take the guard longer than one hour, and so, the guard will not finish investigating in one hour.

3. Write a related division expression that results in the same quotient, but with *either* the dividend *or* the divisor written as a unit fraction. Explain how you created your related expression.

   Sample responses:
   - $\frac{2}{9} \div \frac{1}{4} \left(\text{or } \frac{1}{4} \div \frac{2}{9}\right)$; I divided both the dividend and divisor by 3.
   - $\frac{1}{3} \div \frac{3}{8} \left(\text{or } \frac{3}{8} \div \frac{1}{3}\right)$; I divided the dividend and divisor by 2.

4. Explain what the dividend and divisor mean in context. Use a strategy other than the algorithm to show how the quotient gives you the same information as Problem 1.

   Sample responses:
   - $\frac{1}{3} \div \frac{3}{8}$ or $\frac{3}{8} \div \frac{1}{3}$ means that the guard can investigate $\frac{1}{3}$ of the department in $\frac{3}{8}$ hours, which means she can investigate $\frac{2}{3}$ of the department in $\frac{6}{8}$ hours, and the entire department in $\frac{9}{8}$ hours.
   - $\frac{2}{9} \div \frac{1}{4}$ or $\frac{1}{4} \div \frac{2}{9}$ means the guard can investigate $\frac{2}{9}$ of the department in $\frac{1}{4}$ hours, which means she can investigate $\frac{4}{9}$ in $\frac{1}{2}$ hours, $\frac{6}{9}$ in $\frac{3}{4}$ hours, and $\frac{8}{9}$ in one hour.

460 **Unit 4** Dividing Fractions

1 Launch

Set an expectation for the amount of time that pairs will work on the activity.

2 Monitor

Help students get started by asking, "What information do you know? What information do you need?"

Look for points of confusion:

- **Misinterpreting the quotient.** Ask, "What are the units for your divisor? Dividend? Quotient?"
- **Struggling to write a related expression with a unit fraction.** Ask, "How can you make the dividend have a numerator of 1? What do you need to do to the divisor?"

Look for productive strategies:

- Attending to the units of each value to interpret the expression and quotient in context.
- Making a related expression by dividing both the dividend and the divisor by the same numerator.
- Using multiplicative or ratio reasoning to evaluate the related expression.

3 Connect

Display the expressions $\frac{2}{3} \div \frac{3}{4}$ and $\frac{3}{4} \div \frac{2}{3}$.

Have students share their solutions for each problem, focusing on how both expressions represent the problem and lead to similar conclusions, but different quotients.

Highlight that neither two-thirds nor three-fourths divides evenly into 1, but every unit fraction does divide evenly into 1. Therefore, related expressions with unit fractions can be used for these problems.

Ask, "How does your work in this activity relate to unit rates?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest students draw a diagram to help organize their thinking about what it means to cover $\frac{2}{3}$ of the department in $\frac{3}{4}$ of an hour.

Extension: *Math Enrichment*

Ask students to solve this problem using ratios and explain their thinking. Sample response: For every $\frac{3}{4}$ of an hour, she covers $\frac{2}{3}$ of the department. This means she covers $\frac{1}{3}$ of the department in $\frac{1.5}{4}$ of an hour (halving each amount). Multiply by 3 to determine that she covers the entire department in $\frac{4.5}{4}$ hours, which is equal to $\frac{9}{8}$ hours.

Math Language Development

MLR8: Discussion Supports

As students share their solutions for Problem 2, ask, "How do you interpret $\frac{2}{3}$ divided by $\frac{3}{4}$ versus $\frac{3}{4}$ divided by $\frac{2}{3}$? How does your interpretation of which expression to use affect the quotient and the interpretation of the quotient?"

English Learners

Provide the following sentence frames for students to use when they explain their thinking.

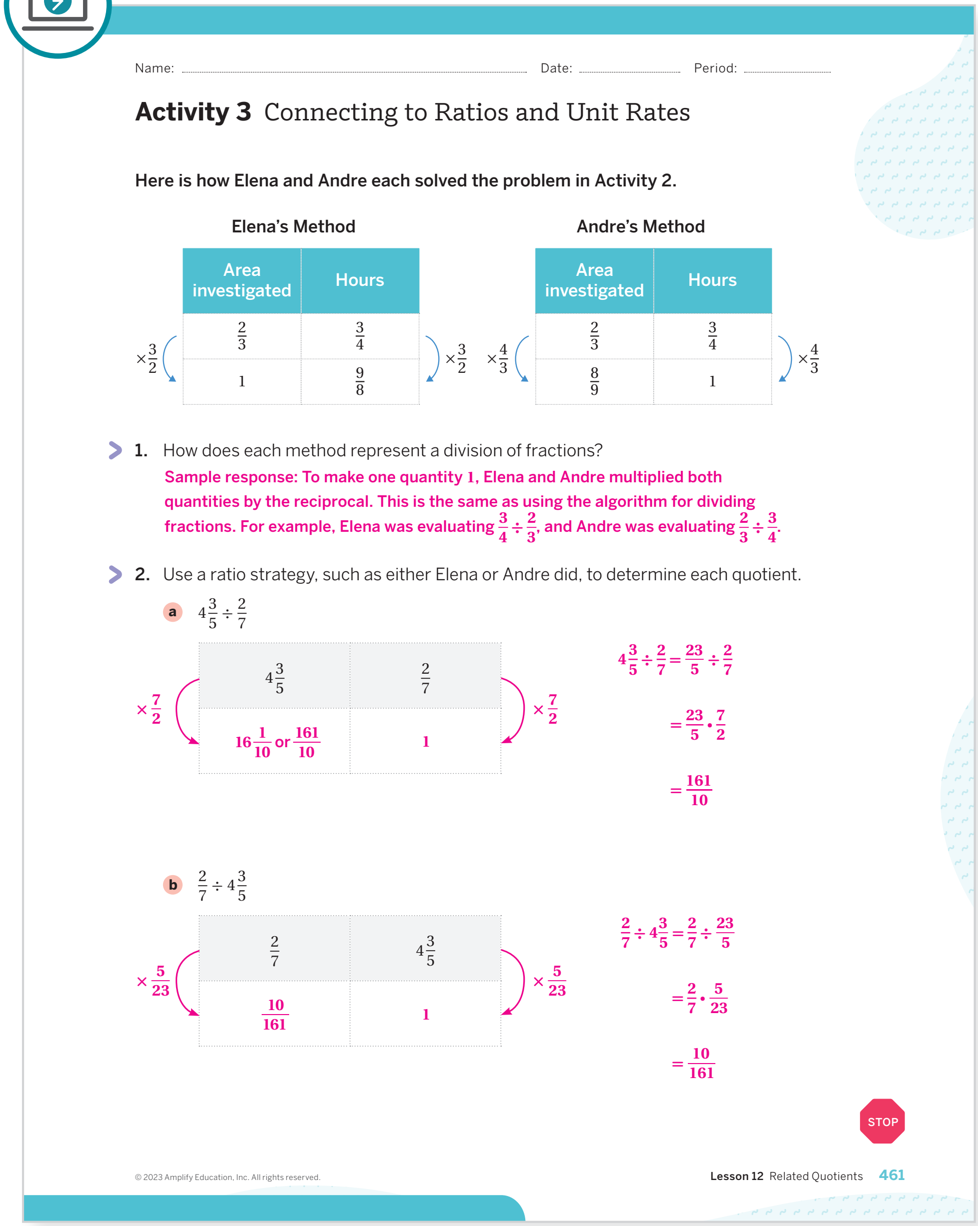
- "First, I _____ because . . ."
- "I noticed _____, so I . . ."
- "_____ corresponds to _____, so I . . ."

Optional

Pairs | 20 min

Activity 3 Connecting to Ratios and Unit Rates

Students connect their work in Activity 2 to their previous work with ratios and unit rates.



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Activity 3 Connecting to Ratios and Unit Rates

Here is how Elena and Andre each solved the problem in Activity 2.

Elena's Method

×$\frac{3}{2}$ | ×$\frac{3}{2}$

Area investigated	Hours
$\frac{2}{3}$	$\frac{3}{4}$
1	$\frac{9}{8}$

Andre's Method

×$\frac{4}{3}$ | ×$\frac{4}{3}$

Area investigated	Hours
$\frac{2}{3}$	$\frac{3}{4}$
$\frac{8}{9}$	1

1. How does each method represent a division of fractions?

   Sample response: To make one quantity 1, Elena and Andre multiplied both quantities by the reciprocal. This is the same as using the algorithm for dividing fractions. For example, Elena was evaluating $\frac{3}{4} \div \frac{2}{3}$, and Andre was evaluating $\frac{2}{3} \div \frac{3}{4}$.

2. Use a ratio strategy, such as either Elena or Andre did, to determine each quotient.

   a $4\frac{3}{5} \div \frac{2}{7}$

   ×$\frac{7}{2}$ | ×$\frac{7}{2}$

$4\frac{3}{5}$	$\frac{2}{7}$
$16\frac{1}{10}$ or $\frac{161}{10}$	1

$$4\frac{3}{5} \div \frac{2}{7} = \frac{23}{5} \div \frac{2}{7}$$
$$= \frac{23}{5} \cdot \frac{7}{2}$$
$$= \frac{161}{10}$$

   b $\frac{2}{7} \div 4\frac{3}{5}$

   ×$\frac{5}{23}$ | ×$\frac{5}{23}$

$\frac{2}{7}$	$4\frac{3}{5}$
$\frac{10}{161}$	1

$$\frac{2}{7} \div 4\frac{3}{5} = \frac{2}{7} \div \frac{23}{5}$$
$$= \frac{2}{7} \cdot \frac{5}{23}$$
$$= \frac{10}{161}$$

STOP

1 Launch

Have students use the *Think-Pair-Share* routine. Give 1 minute to consider the tables before sharing with their partner and completing Problems 1–4. Consider extending this activity by having students complete the additional problems on the Activity 3 PDF.

2 Monitor

Help students get started by asking, "How does the information in the tables relate to Activity 2?"

Look for points of confusion:

- **Misinterpreting the unit rate.** Have students read the values in the table using the rate language "per one."
- **Not relating multiplying by the reciprocal to the division algorithm.** Ask, "How can you use multiplication and division to describe breaking something in half? How does that help you here?"

Look for productive strategies:

- Connecting the values to their work in Activity 2, and recognizing that the two unit rates represent "how much of the department per hour?" and "how many hours per department?"
- Recognizing that multiplying by the reciprocal to generate an equivalent ratio with a 1 is the same as using the division algorithm.

3 Connect

Have students share their solutions and reasoning for each problem.

Highlight the connection between unit rate and the division algorithm (Problems 2–3). Consider using the ratio boxes to reinforce how multiplying by the reciprocal is the same as dividing by the original fraction.

Ask, "Will Andre and Elena's methods always work? Why or why not?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Use one or more of the following to help students make sense of this task:

- Provide blank tables for students to use in Problem 2 that have the column headers pre-labeled.
- Suggest students use colored pencils and annotate how Elena's method created the quantity 1 for the area investigated and Andre's method created the quantity 1 for the number of hours.
- Annotate Elena's method and Andre's method with their respective division expressions and highlight how the dividends and divisors relate to the quantities in each table.

Math Language Development

MLR2: Collect and Display

As students progress through the activity, note any mathematical vocabulary they use as they discuss with their partners, for example, *reciprocal*, *unit rate*, *per hour*, *per lawn*, etc. Add these terms to the class display and encourage students to refer to the display during future class discussions.

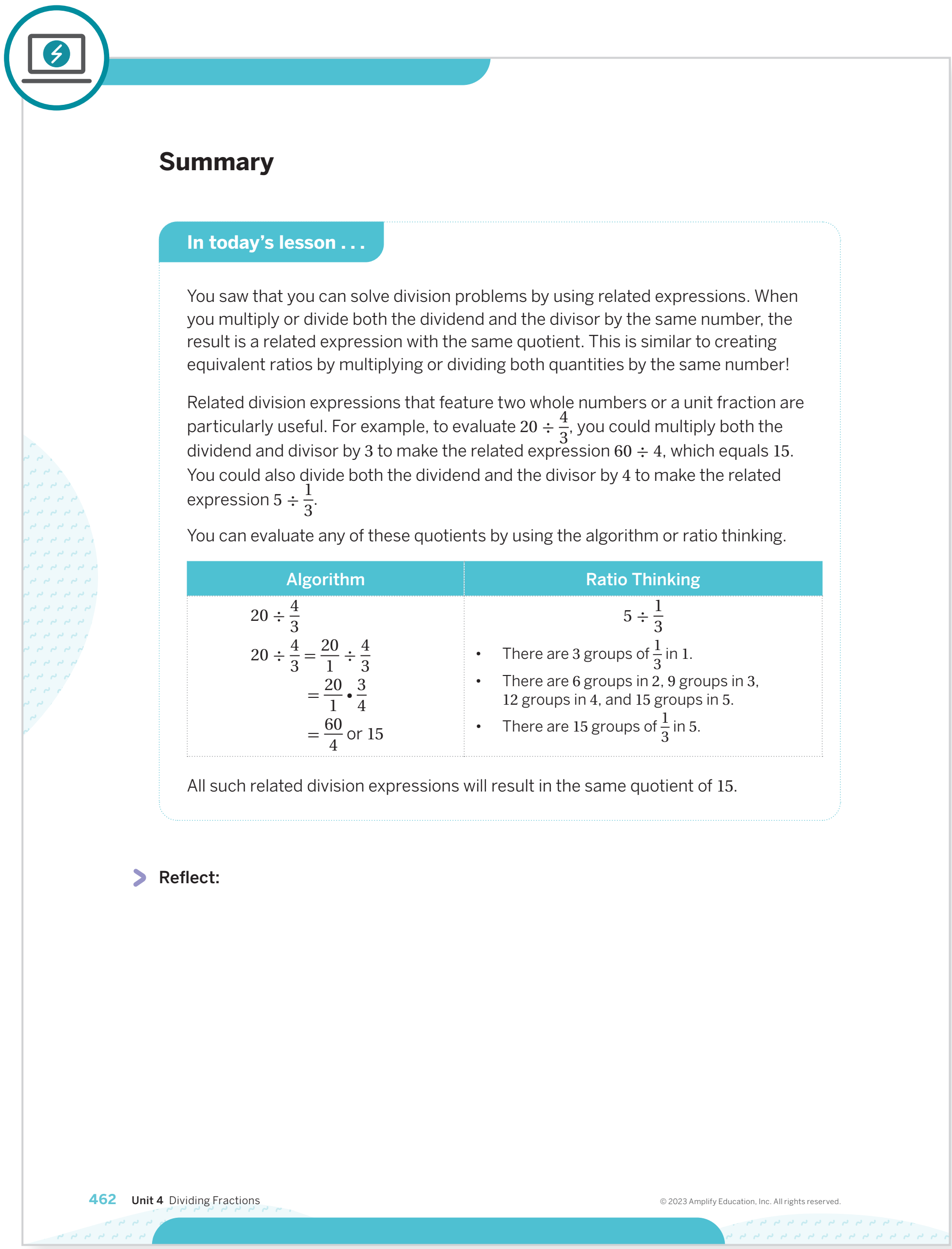
English Learners

During the Connect, as students share, annotate or label Elena's and Andre's ratio tables with the terms you added to the class display.

Whole Class | 5 min

Summary

Review and synthesize how to create and to use related expressions to solve fraction division problems.



Summary

In today's lesson . . .

You saw that you can solve division problems by using related expressions. When you multiply or divide both the dividend and the divisor by the same number, the result is a related expression with the same quotient. This is similar to creating equivalent ratios by multiplying or dividing both quantities by the same number!

Related division expressions that feature two whole numbers or a unit fraction are particularly useful. For example, to evaluate $20 \div \frac{4}{3}$, you could multiply both the dividend and divisor by 3 to make the related expression $60 \div 4$, which equals 15. You could also divide both the dividend and the divisor by 4 to make the related expression $5 \div \frac{1}{3}$.

You can evaluate any of these quotients by using the algorithm or ratio thinking.

Algorithm	Ratio Thinking
$20 \div \frac{4}{3}$ $20 \div \frac{4}{3} = \frac{20}{1} \div \frac{4}{3}$ $= \frac{20}{1} \cdot \frac{3}{4}$ $= \frac{60}{4}$ or 15	$5 \div \frac{1}{3}$ • There are 3 groups of $\frac{1}{3}$ in 1. • There are 6 groups in 2, 9 groups in 3, 12 groups in 4, and 15 groups in 5. • There are 15 groups of $\frac{1}{3}$ in 5.

All such related division expressions will result in the same quotient of 15.

Reflect:

Synthesize

Highlight that related expressions result in the same value as the original expression. In all the cases in this lesson, the qoutient was always the same.

Ask:

- "When are related expressions an efficient strategy?" Sample response: They are efficient when you make both the dividend and divisor a whole number, or when you make one of them a unit fraction.
- "What are some ways to create a related expression?" If you have a whole number divided by a fraction, multiply the dividend and divisor by the denominator, to get two whole numbers. If you have two fractions, divide both the dividend and divisor by the numerator of the divisor in order to get a unit fraction divisor.
- "How are related expressions and equivalent ratios similar?" Sample response: Both require you to multiply or divide two values or quantities by the same number.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

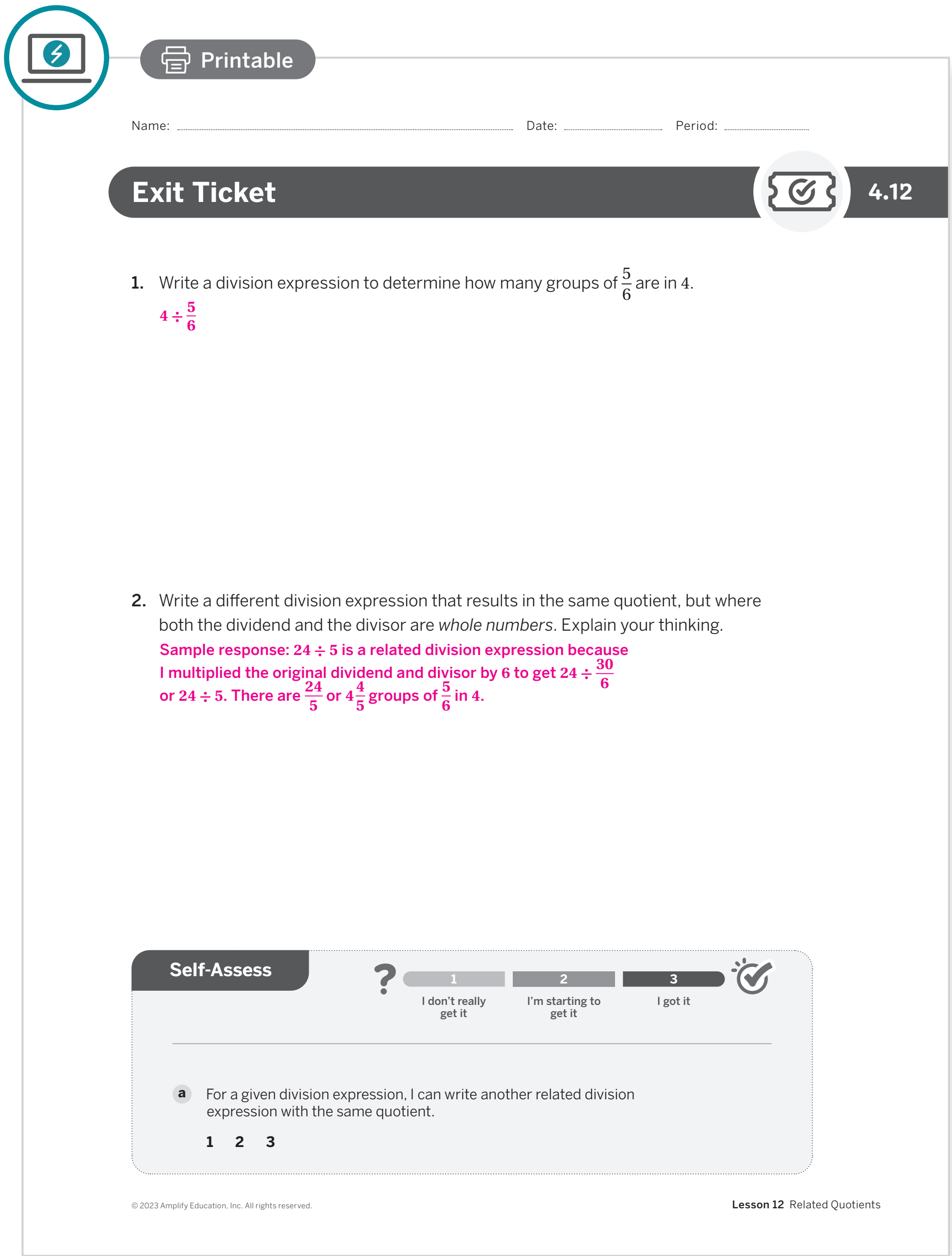
- "What surprised you about your work today?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing and evaluating related division expressions.

Printable



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Exit Ticket 4.12

1. Write a division expression to determine how many groups of $\frac{5}{6}$ are in 4.
   $4 \div \frac{5}{6}$

2. Write a different division expression that results in the same quotient, but where both the dividend and the divisor are *whole numbers*. Explain your thinking.
   Sample response: $24 \div 5$ is a related division expression because I multiplied the original dividend and divisor by 6 to get $24 \div \frac{30}{6}$ or $24 \div 5$. There are $\frac{24}{5}$ or $4\frac{4}{5}$ groups of $\frac{5}{6}$ in 4.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a For a given division expression, I can write another related division expression with the same quotient.

1 2 3

Lesson 12 Related Quotients

Success looks like . . .

- **Goal:** Given a division expression, writing another related division expression with the same quotient.
- **Language Goal:** Explaining how two expressions are related and why this results in the same quotient. **(Speaking and Listening, Writing)**
  - » Writing two different, but related, division expressions that determine the number of groups of $\frac{5}{6}$ in 4 in Problems 1 and 2.

Suggested next steps

If students $\frac{5}{6}$ write $\frac{5}{6} \div 4$ for Problem 1, consider:

- Asking, "What do $\frac{5}{6}$ and 4 mean in the original problem? In your expression? How can you edit your expression?"
- Having students model their expression and asking, "Does your model match the original problem?"

If students are unable to write an expression where the dividend and divisor are whole numbers, consider:

- Reviewing Activity 1, Problem 3. Ask, "How did you write an expression with a whole number dividend and divisor? How can you use that thinking here?"

Professional Learning

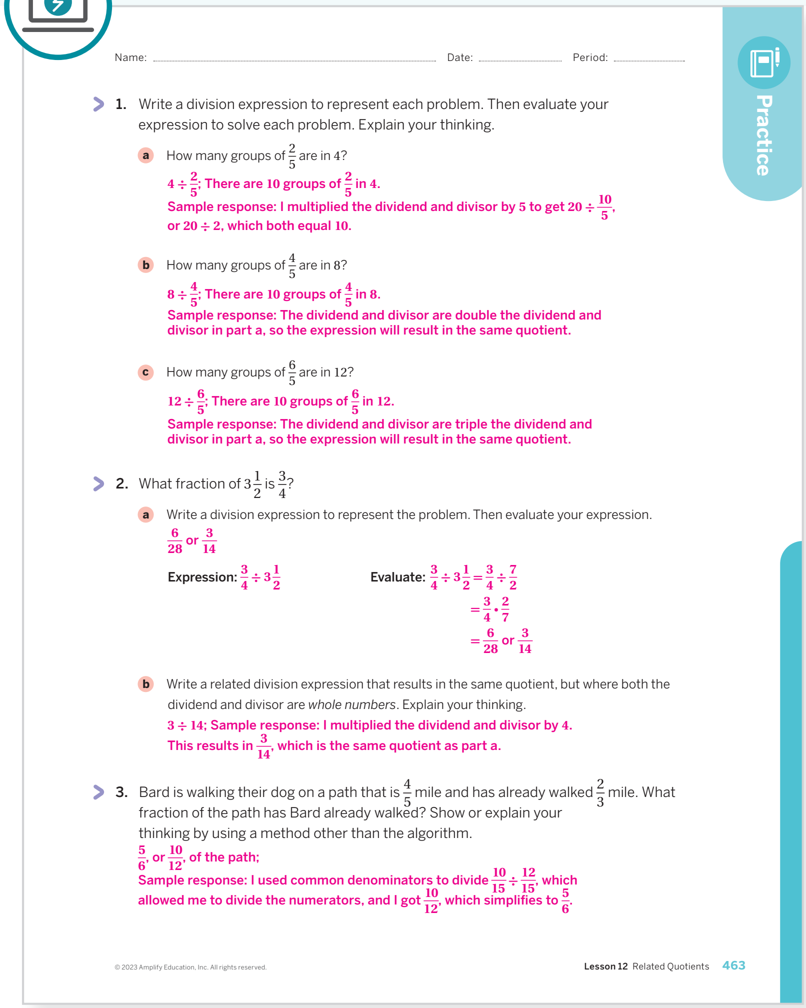
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students saw how related division expressions result in the same quotient. How will that support them as they identify and generate equivalent expressions with variables in Unit 6?
- What did you see in the way some students approached Activity 1 that you would like other students to try? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Write a division expression to represent each problem. Then evaluate your expression to solve each problem. Explain your thinking.
   - a How many groups of $\frac{2}{5}$ are in 4?

     $4 \div \frac{2}{5}$; There are 10 groups of $\frac{2}{5}$ in 4.
     Sample response: I multiplied the dividend and divisor by 5 to get $20 \div \frac{10}{5}$, or $20 \div 2$, which both equal 10.
   - b How many groups of $\frac{4}{5}$ are in 8?

     $8 \div \frac{4}{5}$; There are 10 groups of $\frac{4}{5}$ in 8.
     Sample response: The dividend and divisor are double the dividend and divisor in part a, so the expression will result in the same quotient.
   - c How many groups of $\frac{6}{5}$ are in 12?

     $12 \div \frac{6}{5}$; There are 10 groups of $\frac{6}{5}$ in 12.
     Sample response: The dividend and divisor are triple the dividend and divisor in part a, so the expression will result in the same quotient.

2. What fraction of $3\frac{1}{2}$ is $\frac{3}{4}$?
   - a Write a division expression to represent the problem. Then evaluate your expression.

     $\frac{6}{28}$ or $\frac{3}{14}$

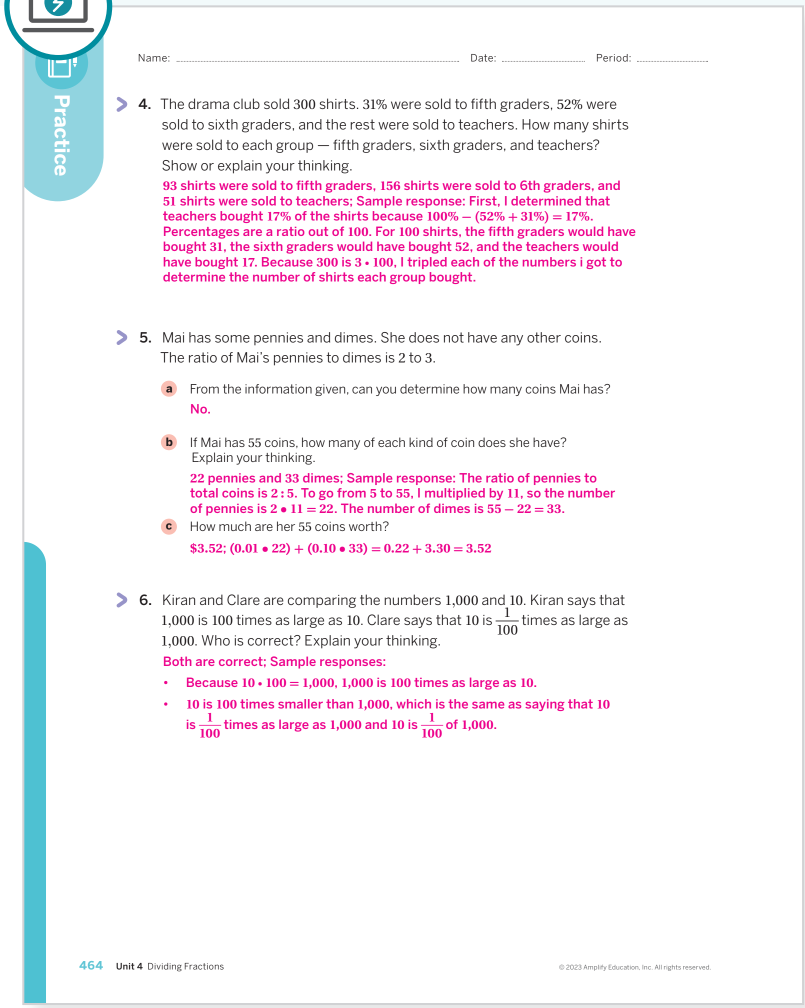
     Expression: $\frac{3}{4} \div 3\frac{1}{2}$

     Evaluate: $$\begin{aligned}\frac{3}{4} \div 3\frac{1}{2} &= \frac{3}{4} \div \frac{7}{2}\\ &= \frac{3}{4} \bullet \frac{2}{7}\\ &= \frac{6}{28} \text{ or } \frac{3}{14}\end{aligned}$$
   - b Write a related division expression that results in the same quotient, but where both the dividend and divisor are *whole numbers*. Explain your thinking.

     $3 \div 14$; Sample response: I multiplied the dividend and divisor by 4. This results in $\frac{3}{14}$, which is the same quotient as part a.

3. Bard is walking their dog on a path that is $\frac{4}{5}$ mile and has already walked $\frac{2}{3}$ mile. What fraction of the path has Bard already walked? Show or explain your thinking by using a method other than the algorithm.

   $\frac{5}{6}$, or $\frac{10}{12}$, of the path;
   Sample response: I used common denominators to divide $\frac{10}{15} \div \frac{12}{15}$, which allowed me to divide the numerators, and I got $\frac{10}{12}$, which simplifies to $\frac{5}{6}$.



Practice

Name: ______ Date: ______ Period: ______

4. The drama club sold 300 shirts. 31% were sold to fifth graders, 52% were sold to sixth graders, and the rest were sold to teachers. How many shirts were sold to each group — fifth graders, sixth graders, and teachers? Show or explain your thinking.

   93 shirts were sold to fifth graders, 156 shirts were sold to 6th graders, and 51 shirts were sold to teachers; Sample response: First, I determined that teachers bought 17% of the shirts because $100\% - (52\% + 31\%) = 17\%$. Percentages are a ratio out of 100. For 100 shirts, the fifth graders would have bought 31, the sixth graders would have bought 52, and the teachers would have bought 17. Because 300 is $3 \bullet 100$, I tripled each of the numbers i got to determine the number of shirts each group bought.

5. Mai has some pennies and dimes. She does not have any other coins. The ratio of Mai's pennies to dimes is 2 to 3.
   - a From the information given, can you determine how many coins Mai has?

     No.
   - b If Mai has 55 coins, how many of each kind of coin does she have? Explain your thinking.

     22 pennies and 33 dimes; Sample response: The ratio of pennies to total coins is $2:5$. To go from 5 to 55, I multiplied by 11, so the number of pennies is $2 \bullet 11 = 22$. The number of dimes is $55 - 22 = 33$.
   - c How much are her 55 coins worth?

     $\$3.52; (0.01 \bullet 22) + (0.10 \bullet 33) = 0.22 + 3.30 = 3.52$

6. Kiran and Clare are comparing the numbers 1,000 and 10. Kiran says that 1,000 is 100 times as large as 10. Clare says that 10 is $\frac{1}{100}$ times as large as 1,000. Who is correct? Explain your thinking.

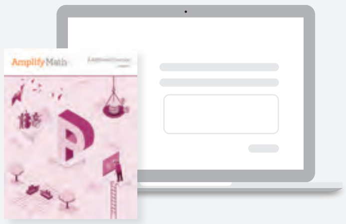
   Both are correct; Sample responses:
   - Because $10 \bullet 100 = 1{,}000$, 1,000 is 100 times as large as 10.
   - 10 is 100 times smaller than 1,000, which is the same as saying that 10 is $\frac{1}{100}$ times as large as 1,000 and 10 is $\frac{1}{100}$ of 1,000.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 13	2
	5	Unit 2 Lesson 17	2
Formative	6	Unit 4 Lesson 13	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Sub-Unit 3

Fractions in Lengths, Areas, and Volumes

Whole Class

In this Sub-Unit, students explore applications of fraction division in measurement contexts by dividing fractional lengths, and determining unknown fractional lengths in rectangular areas and volumes.

SUB-UNIT

3 Fractions in Lengths, Areas, and Volumes

Narrative Connections

Spöklik Furniture: Checking Out

"Is this what you were looking for?" a voice asks. You look up and your heart almost leaps out of your chest. You see a girl in a yellow jacket: Maya!

"Ah, perfect!" Samira says. Samira takes the bolt from Maya's outstretched hand. As Samira continues working, Maya throws her arms around you. Her dog, Penny, barks happily behind her.

"You two have been super helpful," Samira says. "Thanks for everything!"

"No problem," Maya says, "We'd better be going now!" Maya takes you by your wrist and, together, you make your way out through a stairwell.

"I'm so glad I found you. There's an exit through the checkout section, but the guards won't let me through with Penny. They're convinced she belongs in the store!"

At the bottom of the stairs, you step out into a massive room. Spectral shoppers hover in line, waiting to check out. Beyond the row of registers are the exit doors, watched over by the security guards Maya warned you about.

Suddenly, you spot a shopper toward the back of one of the lines. This shopper seems distracted by a set of tea towels. In their cart, you see a large cardboard box. Suddenly, an idea occurs to you: You can sneak Penny through by hiding her in the box!

You, Maya, and Penny quietly creep behind the cart. Opening the box, you find a strange sight: a jade statue shaped like a pyramid, surrounded by 3 squishy foam packets. **"We'll have to get rid of some of these packets for Penny to fit," Maya whispers. "But how many?"**

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to use fractions and division as they make their way out of Spöklik Furniture in the following places:

- **Lesson 13, Activity 1:** How Many Times as Tall or as Long?
- **Lesson 16, Activity 1:** Spöklik's Fish Tank
- **Lesson 17, Warm-up:** Hunting for Clues

Highlight the question posed in the narrative and consider having a brief discussion to ensure all students understand the question. While some students may be able to determine an answer now, they should all be equipped to answer it by the end of the Sub-Unit. Allow students to submit responses privately at any point during the next several class sessions, and then hold a discussion after Lesson 16.

UNIT 4 | **LESSON 13**

Fractional Lengths

Let's solve problems about fractional lengths.

Focus

Goals

1. **Language Goal:** Apply division of fractions to solve problems involving fractional lengths, and explain the solution method. **(Speaking and Listening, Writing)**
2. **Language Goal:** Interpret a written question about multiplicative comparison, e.g., "how many times as long?", and write a division equation to represent it. **(Speaking and Listening, Writing)**

Coherence

Today

Students use division to solve problems involving fractional lengths. Building on their work from Lesson 7, they apply their understanding of the two interpretations of division — "how many groups?" and "how much in each group?" — to solve problems that involve a multiplicative comparison of distances or heights. Students then refine their understanding of mathematical language as they make sense of and solve a problem involving length and perimeter using the ***Info Gap*** routine. There is minimal scaffolding, which allows students to choose their strategies and representations for problem solving.

Previously

In Lesson 7, students saw multiplicative comparisons as asking "how many times as long?" and "what fraction of a group?" They linked these interpretations to both multiplication and division equations. In Lessons 10–11, they generalized an algorithm for dividing with fractions.

Coming Soon

In Lesson 14, students will solve problems involving the relationship between area and the side lengths of rectangles and triangles, in which these measurements are fractions.

Rigor

- Students build their **procedural fluency** of evaluating fraction division expressions.
- Students **apply** division with fractions to solve problems involving fractional lengths, including multiplicative comparisons.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per every three pairs
- *Tape Diagrams* PDF (as needed)
- graph paper (optional)

Amps | Featured Activity

Activity 1

Digital Diagrams

Students can sketch on digital tape diagrams to solve problems. You can overlay them all to see similarities and differences at a glance.

Building Math Identity and Community

Connecting to Mathematical Practices

As students begin to work on Activity 2, they might become overwhelmed and unable to see a starting place. Ask students to review how to model the situations in each set, considering they represent different ways of looking at division. Then have students create a diagram that would help them analyze those relationships. Focus on how modeling mathematics can help students to make sense of problems and feel more confident in solving them.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, Problem 2 may be omitted.
- Make **Activity 2** into a whole-class activity. Conduct the *Info Gap* routine with you holding both the problem and data cards for Set 2. Have students work in pairs to identify the information they need to know and solve the problems.

Independent | 5 min

Warm-up Comparing Paper Rolls

Students activate prior knowledge of multiplicative comparison as they write division and multiplication equations to compare the length of paper rolls.

Unit 4 | **Lesson 13**

Fractional Lengths

Let's solve problems about fractional lengths.

Warm-up Comparing Paper Rolls

The image shows bath tissue rolls and a paper towel roll. Let b represent the length of a bath tissue roll and let p represent the length of a paper towel roll. Write a multiplication equation and a division equation that represents the length of one roll in terms of the other.

$p = b \cdot 2\frac{1}{2}$ or $b = p \div 2\frac{1}{2}$

Log in to Amplify Math to complete this lesson online.

466 **Unit 4** Dividing Fractions

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1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by asking, "What do you notice about the rolls? How can you quantify that?"

Look for points of confusion:

- **Struggling to estimate the length of one paper towel roll.** Ask, "How can you divide the long roll to help you estimate its length?"
- **Struggling to write a division equation.** Ask, "How can you use your multiplication equation to write a division equation?"

Look for productive strategies:

- Estimating that one paper towel roll is $2\frac{1}{2}$ times as long as one bath tissue roll, and representing this relationship with the equation $p = b \cdot 2\frac{1}{2}$.
- Using the relationship between multiplication and division to represent the length of one bath tissue roll as $b = p \div 2\frac{1}{2}$.

3 Connect

Have students share their equations.

Display the equations $b = p \div 2\frac{1}{2}$ and $b = \frac{2}{5} \cdot p$.

Ask, "How are these equations similar? In the second equation, where did $\frac{2}{5}$ come from?"

Highlight how the equations use the relationship between multiplication and division. Because one paper towel roll is $2\frac{1}{2}$ times as long as one toilet paper roll, then one toilet paper roll is $\frac{2}{5}$ times as long as one paper towel roll.

Differentiated Support

Accessibility: *Activate Background Knowledge, Guide Processing and Visualization, Optimize Access to Tools*

Bring in bath tissue rolls and paper towel rolls and let students physically handle them to help them visualize their lengths. While students do not need to actually measure the lengths of these rolls in this Warm-up, provide access to rulers or other measuring tools for them to use if they choose to do so.

Power-up

To power up students' ability to compare the size of values using the concept of a is __ times the size of b", have students complete:

Use the place value chart to answer each question.

1,	0	0	0
Thousands	Hundreds	Tens	Ones

a. How many times larger is a digit one place value to the left in the chart? 10 times larger

b. How many times smaller is a digit one place value to the right in the chart? 10 times smaller

Use: Before the Warm-up

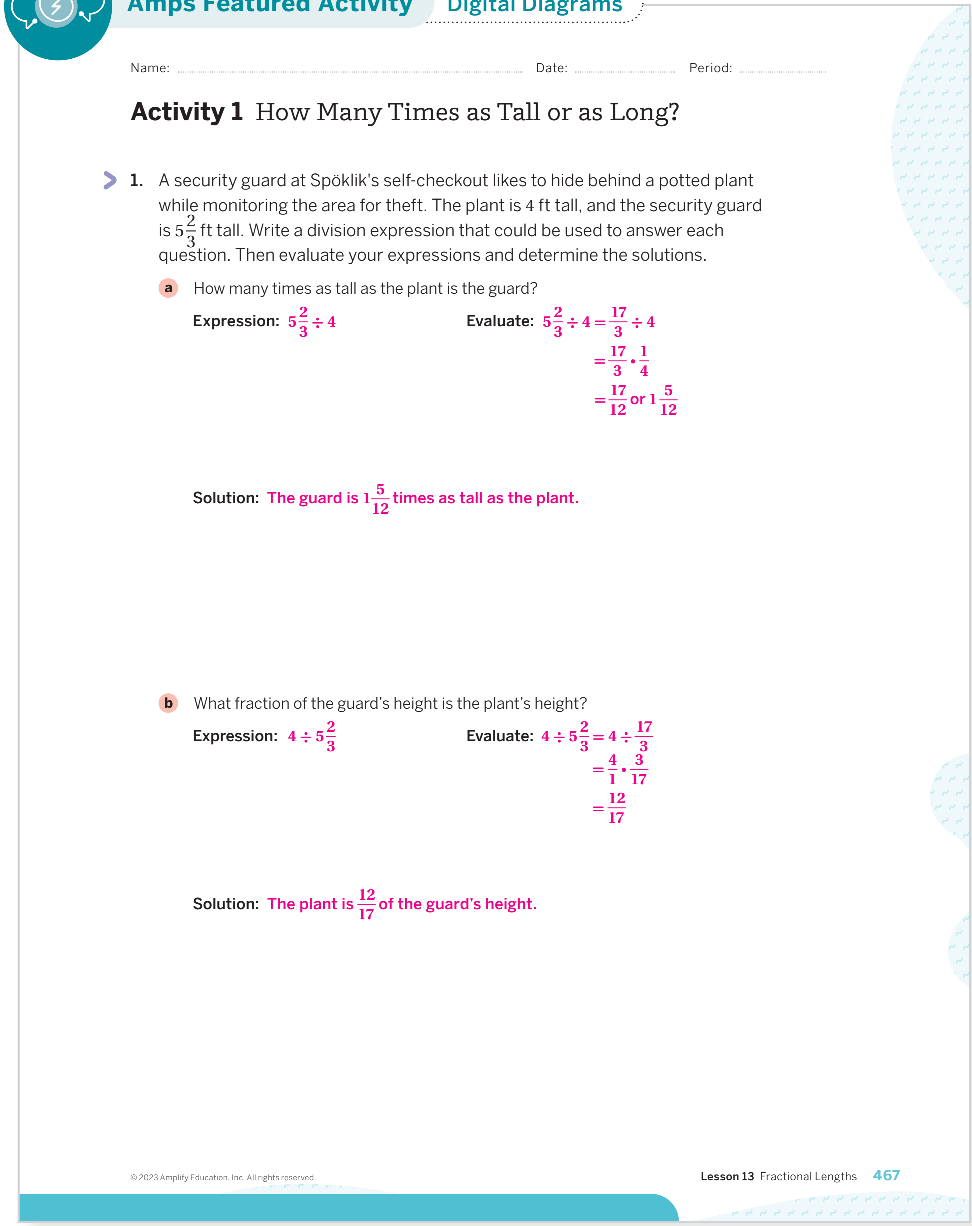
Informed by: Performance on Lesson 12, Practice Problem 6.

Pairs | 15 min

Activity 1 How Many Times as Tall or as Long?

Students extend their work with multiplicative comparison from Lesson 7, writing and evaluating division expressions to determine "how many times as tall or long?"

Amps Featured Activity Digital Diagrams



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Activity 1 How Many Times as Tall or as Long?

1. A security guard at Spöklik's self-checkout likes to hide behind a potted plant while monitoring the area for theft. The plant is 4 ft tall, and the security guard is $5\frac{2}{3}$ ft tall. Write a division expression that could be used to answer each question. Then evaluate your expressions and determine the solutions.

   a. How many times as tall as the plant is the guard?

   Expression: $5\frac{2}{3} \div 4$

   Evaluate: $5\frac{2}{3} \div 4 = \frac{17}{3} \div 4$
   $= \frac{17}{3} \cdot \frac{1}{4}$
   $= \frac{17}{12}$ or $1\frac{5}{12}$

   Solution: The guard is $1\frac{5}{12}$ times as tall as the plant.

   b. What fraction of the guard's height is the plant's height?

   Expression: $4 \div 5\frac{2}{3}$

   Evaluate: $4 \div 5\frac{2}{3} = 4 \div \frac{17}{3}$
   $= \frac{4}{1} \cdot \frac{3}{17}$
   $= \frac{12}{17}$

   Solution: The plant is $\frac{12}{17}$ of the guard's height.

1 Launch

Set an expectation for the amount of time that pairs will work on the activity.

2 Monitor

Help students get started by activating prior knowledge. Have them draw a diagram to represent Problem 1. Ask, "What multiplication equation represents your diagram? How can you rewrite that as a division equation?"

Look for points of confusion:

- **Struggling to write a division expression.** Ask, "What multiplication equation represents the problem? How can you use that to write a division expression?"
- **Reversing the dividend and divisor.** Ask, "Does it make sense that the guard is less than 1 times the height of the plant? Or that the plant is more than 1 times the height of the guard?"
- **Struggling to estimate in Problem 2.** Ask, "About how long is the guard's shift? What is half of that? Would half of 9.5 be more or less?"

Look for productive strategies:

- Using prior knowledge of the two interpretations of division and the relationship between multiplication and division to write and evaluate division expressions.
- Checking the reasonableness of their quotients by considering whether an answer greater than or less than 1 makes sense for a given comparison.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Tools*

Suggest students draw a diagram to help them make sense of each problem. Provide access to graph paper or copies of blank tape diagrams, such as from the *Tape Diagrams* PDF. Direct students to only work with the tape diagrams that do not include percentages.

Accessibility: *Guide Processing and Visualization*

Suggest that students complete Problem 1 using common denominators. Ask, "Think of a fraction that divides $5\frac{2}{3}$. Does that fraction divide 4? Does this help you complete Problem 1a? What about Problem 1b?"

Math Language Development

MLR5: Co-craft Questions

Display the first two sentences of Problem 1. Have students work with their partner to write 2–3 mathematical questions they have about this situation. Ask volunteers to share their questions with the class. Reveal the questions in parts a and b, and have students compare these questions with how they phrased their questions, if the questions are similar. For example, they may have written "Who is taller?" compared to "How many times as tall?"

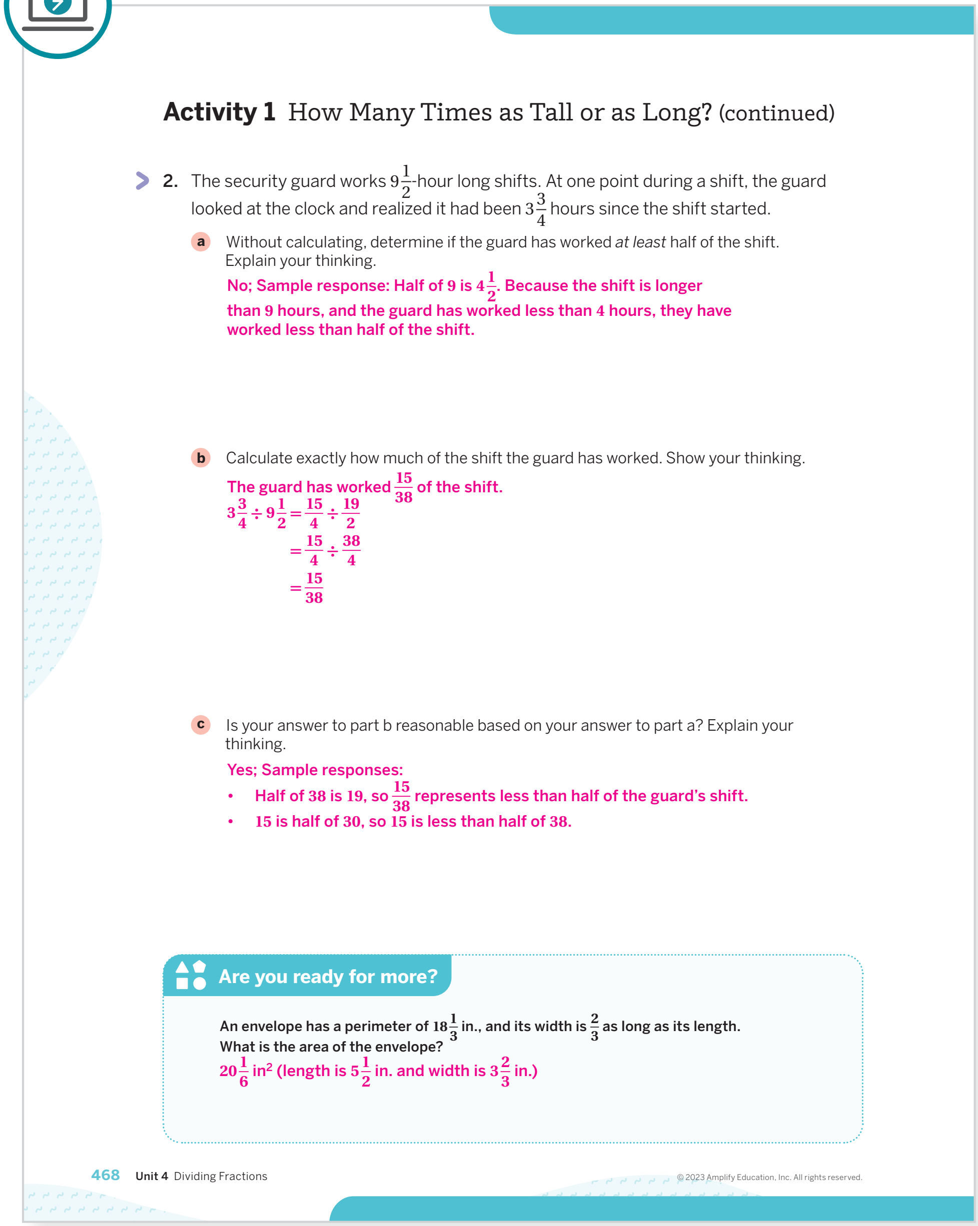
English Learners

Model for students an example of a mathematical question that they could ask about the situation. This will help support them in building metalinguistic awareness.

Pairs | 15 min

Activity 1 How Many Times as Tall or as Long? (continued)

Students extend their work with multiplicative comparison from Lesson 7, writing and evaluating division expressions to determine "how many times as tall or long?"



Activity 1 How Many Times as Tall or as Long? (continued)

2. The security guard works $9\frac{1}{2}$-hour long shifts. At one point during a shift, the guard looked at the clock and realized it had been $3\frac{3}{4}$ hours since the shift started.

   a. Without calculating, determine if the guard has worked *at least* half of the shift. Explain your thinking.

   No; Sample response: Half of 9 is $4\frac{1}{2}$. Because the shift is longer than 9 hours, and the guard has worked less than 4 hours, they have worked less than half of the shift.

   b. Calculate exactly how much of the shift the guard has worked. Show your thinking.

   The guard has worked $\frac{15}{38}$ of the shift.

   $$3\frac{3}{4} \div 9\frac{1}{2} = \frac{15}{4} \div \frac{19}{2}$$
   $$= \frac{15}{4} \div \frac{38}{4}$$
   $$= \frac{15}{38}$$

   c. Is your answer to part b reasonable based on your answer to part a? Explain your thinking.

   Yes; Sample responses:
   - **Half of 38 is 19, so $\frac{15}{38}$ represents less than half of the guard's shift.**
   - **15 is half of 30, so 15 is less than half of 38.**

Are you ready for more?

An envelope has a perimeter of $18\frac{1}{3}$ in., and its width is $\frac{2}{3}$ as long as its length. What is the area of the envelope?

$20\frac{1}{6}$ in^2 (length is $5\frac{1}{2}$ in. and width is $3\frac{2}{3}$ in.)

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3 Connect

Have students share their solutions and strategies, focusing on how they determined whether their solutions were reasonable.

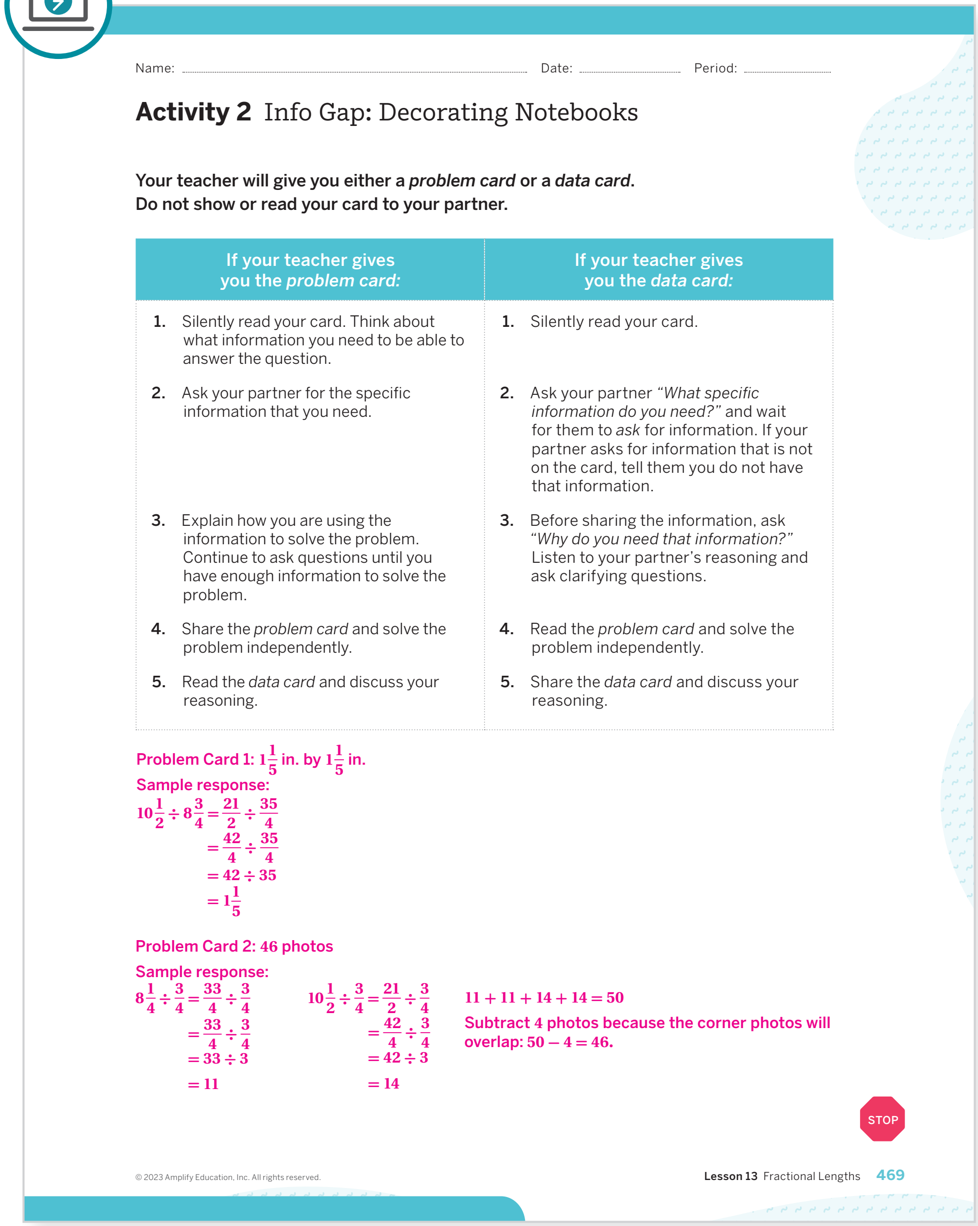
Ask, "Why were these problems solved using division? How does this relate to your work with the two interpretations of division?"

Highlight that a multiplicative comparison can be solved using division when the "times as many" or "fraction of" is unknown.

Pairs | 15 min

Activity 2 Info Gap: Decorating Notebooks

Students participate in the *Info Gap* routine by using division to solve problems about tiling photos with fractional side lengths.



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Activity 2 Info Gap: Decorating Notebooks

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card:*	If your teacher gives you the *data card:*
1. Silently read your card. Think about what information you need to be able to answer the question.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information. If your partner asks for information that is not on the card, tell them you do not have that information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask *"Why do you need that information?"* Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently.	**4.** Read the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.	**5.** Share the *data card* and discuss your reasoning.

Problem Card 1: $1\frac{1}{5}$ in. by $1\frac{1}{5}$ in.

Sample response:

$$\begin{aligned}10\frac{1}{2} \div 8\frac{3}{4} &= \frac{21}{2} \div \frac{35}{4}\\ &= \frac{42}{4} \div \frac{35}{4}\\ &= 42 \div 35\\ &= 1\frac{1}{5}\end{aligned}$$

Problem Card 2: 46 photos

Sample response:

$$\begin{aligned}8\frac{1}{4} \div \frac{3}{4} &= \frac{33}{4} \div \frac{3}{4}\\ &= \frac{33}{4} \div \frac{3}{4}\\ &= 33 \div 3\\ &= 11\end{aligned}$$

$$\begin{aligned}10\frac{1}{2} \div \frac{3}{4} &= \frac{21}{2} \div \frac{3}{4}\\ &= \frac{42}{4} \div \frac{3}{4}\\ &= 42 \div 3\\ &= 14\end{aligned}$$

$11 + 11 + 14 + 14 = 50$

Subtract 4 photos because the corner photos will overlap: $50 - 4 = 46$.

STOP

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① Launch

Review the *Info Gap* instructional routine. Give each pair of students the Set 1 Problem and Data cards from the Activity 2 PDF. Once the pairs complete the routine with Set 1, give them Set 2 and have partners switch roles.

② Monitor

Help students get started by asking, "What information do you have? What information do you need?"

Look for points of confusion:

- **Dividing the width of Tyler's notebook by $7\frac{3}{4}$ (Set 1).** Ask, "Where is Tyler placing the photos? Is that the length or the width?"
- **Dividing without knowing how Jada will arrange the photos (Set 2).** Ask, "Is Jada arranging her photos the same way Tyler did? Why is that important to know?"

Look for productive strategies:

- Asking precise questions to get the dimensions of the notebooks and photo arrangements for both card sets, the number of photos for Set 1, and the side length of the photos for Set 2.
- Recognizing that Set 1 asks "how large is each group?" and Set 2 asks "how many groups?" and representing the scenario with a division expression or diagram.
- Using the algorithm, common denominators, or related expressions to evaluate, and subtracting 4 photos in Set 2 to account for the overlapping photos at each corner.

③ Connect

Have students share their solutions and strategies.

Highlight how the use of division is similar and different for each problem card.

Ask, "How do your expressions and diagrams reflect these similarities and differences?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. Consider using the following during the think-aloud.

- "I need to determine the dimensions of each photo. I wonder if the photos are rectangles or squares."
- "I wonder how big Tyler's math notebook is. I will ask for the dimensions of the notebook."
- "I wonder how many photos Tyler used to decorate his notebook. I will ask for this number."

Math Language Development

MLR4: Information Gap

Display prompts for students who benefit from a starting point, such as:

- "Can you tell me . . . (specific piece of information)?"
- "Why do you need to know . . . (that piece of information)?"

English Learners

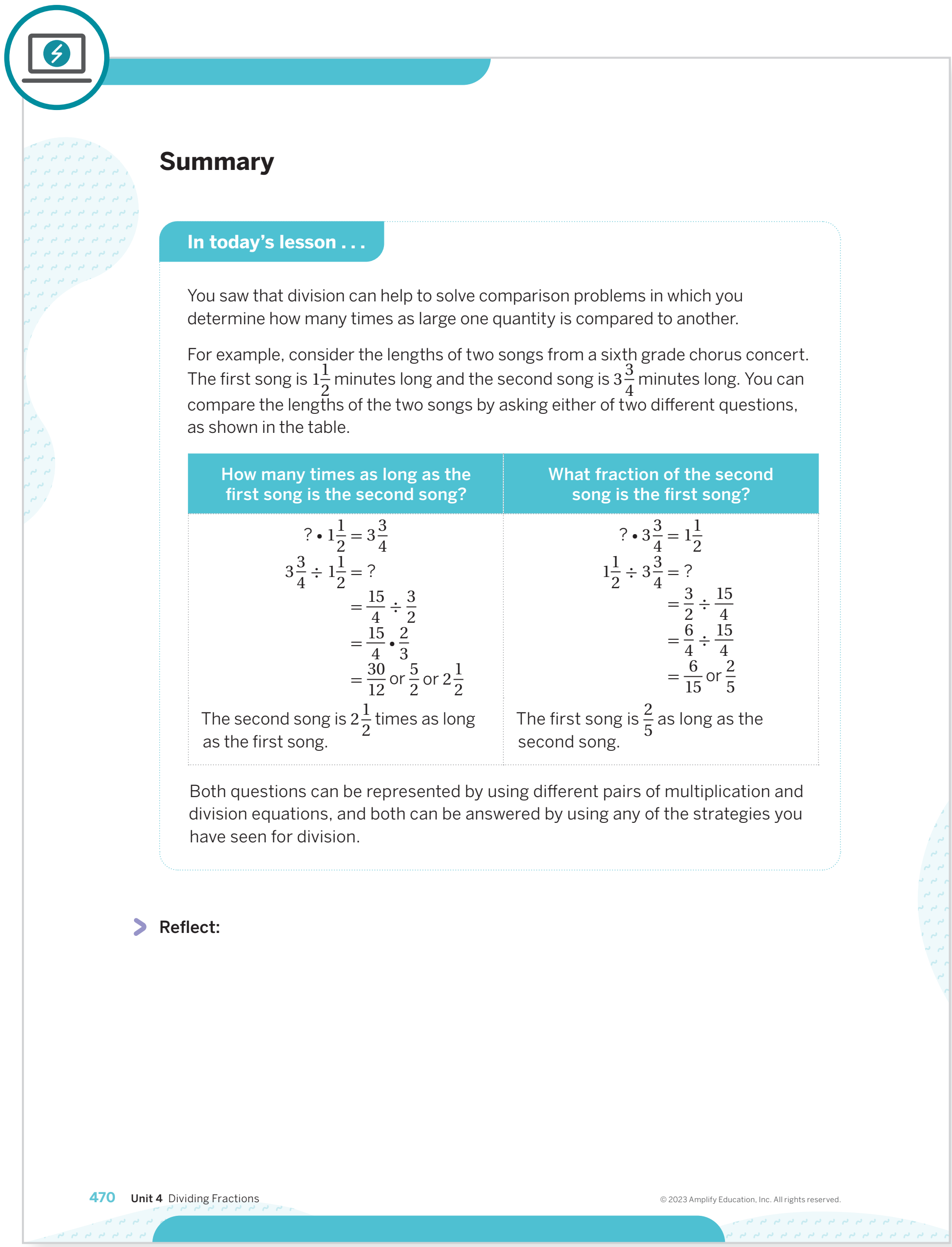
Consider providing sample questions students could ask for Problem Card 1, such as the following:

- What are the dimensions of Tyler's math notebook?
- How many photos did Tyler use to decorate his math notebook?

Whole Class | 5 min

Summary

Review and synthesize how to use division to solve problems involving fractional lengths, including comparison problems.



Summary

In today's lesson . . .

You saw that division can help to solve comparison problems in which you determine how many times as large one quantity is compared to another.

For example, consider the lengths of two songs from a sixth grade chorus concert. The first song is $1\frac{1}{2}$ minutes long and the second song is $3\frac{3}{4}$ minutes long. You can compare the lengths of the two songs by asking either of two different questions, as shown in the table.

How many times as long as the first song is the second song?	What fraction of the second song is the first song?
$? \bullet 1\frac{1}{2} = 3\frac{3}{4}$ $3\frac{3}{4} \div 1\frac{1}{2} = ?$ $= \frac{15}{4} \div \frac{3}{2}$ $= \frac{15}{4} \bullet \frac{2}{3}$ $= \frac{30}{12}$ or $\frac{5}{2}$ or $2\frac{1}{2}$	$? \bullet 3\frac{3}{4} = 1\frac{1}{2}$ $1\frac{1}{2} \div 3\frac{3}{4} = ?$ $= \frac{3}{2} \div \frac{15}{4}$ $= \frac{6}{4} \div \frac{15}{4}$ $= \frac{6}{15}$ or $\frac{2}{5}$
The second song is $2\frac{1}{2}$ times as long as the first song.	The first song is $\frac{2}{5}$ as long as the second song.

Both questions can be represented by using different pairs of multiplication and division equations, and both can be answered by using any of the strategies you have seen for division.

> Reflect:

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Synthesize

Highlight that any equal groups or multiplicative comparison problem can be written using both multiplication and division equations. However, the operation used to solve any given problem depends on what information is unknown. Division is used when the number or size of the groups (equal groups) or the "times as many" (comparison) is unknown.

Display the Summary from the Student Edition.

Ask, "How do the equations in the table represent two different equal-sized groups problems?" Sample response: Both equations represent a "how many groups" problem, where the known information is the size of one group and the total. I am solving the equations to determine the number of groups.

Reflect

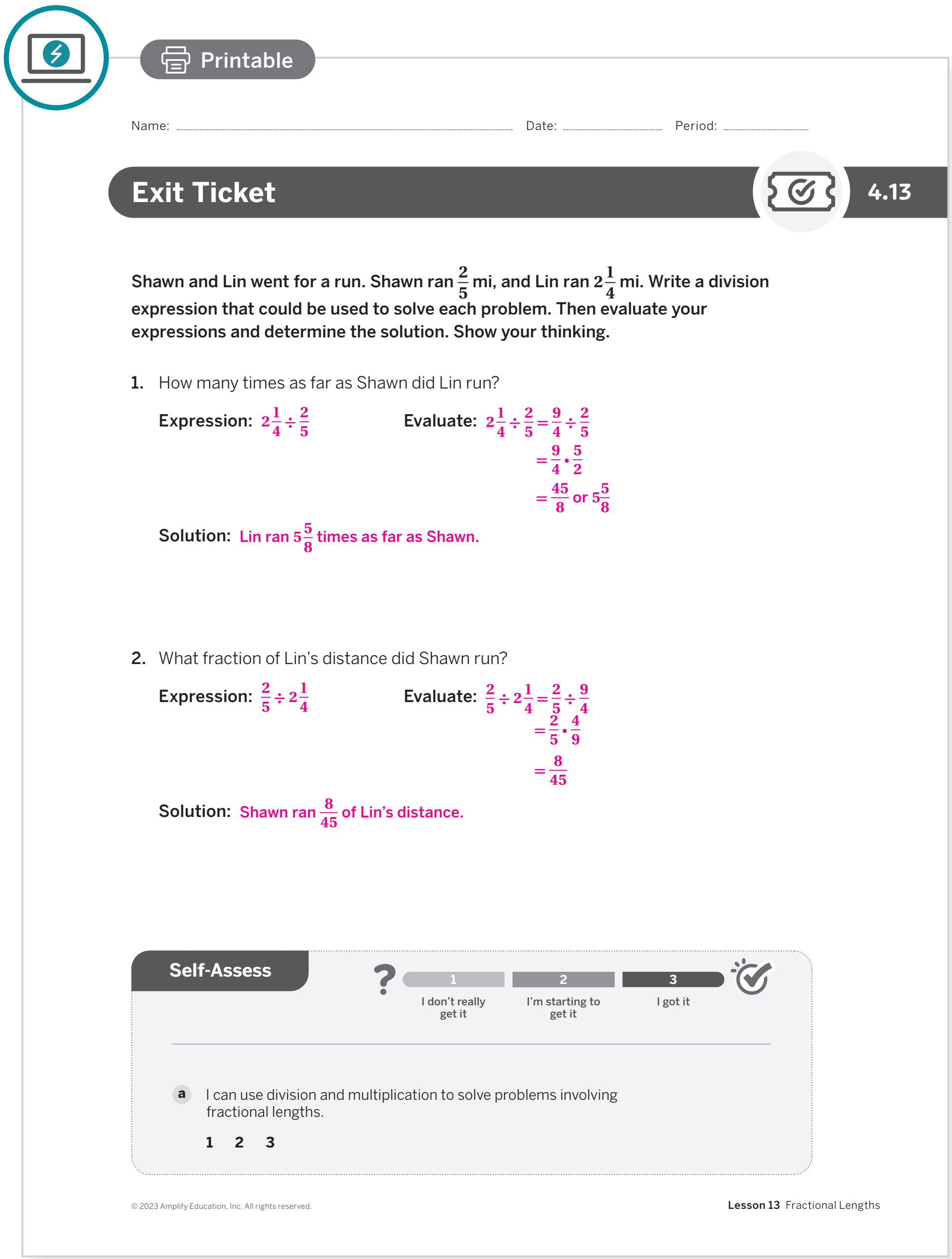
After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Which representations and strategies were the most helpful as you solved the problems today? What made them helpful?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing and evaluating expressions for a multiplicative comparison scenario involving fractional distances.



Printable

Name: ____________ Date: ________ Period: ________

Exit Ticket 4.13

Shawn and Lin went for a run. Shawn ran $\frac{2}{5}$ mi, and Lin ran $2\frac{1}{4}$ mi. Write a division expression that could be used to solve each problem. Then evaluate your expressions and determine the solution. Show your thinking.

1. How many times as far as Shawn did Lin run?

Expression: $2\frac{1}{4} \div \frac{2}{5}$

Evaluate: $2\frac{1}{4} \div \frac{2}{5} = \frac{9}{4} \div \frac{2}{5}$

$= \frac{9}{4} \cdot \frac{5}{2}$

$= \frac{45}{8}$ or $5\frac{5}{8}$

Solution: Lin ran $5\frac{5}{8}$ times as far as Shawn.

2. What fraction of Lin's distance did Shawn run?

Expression: $\frac{2}{5} \div 2\frac{1}{4}$

Evaluate: $\frac{2}{5} \div 2\frac{1}{4} = \frac{2}{5} \div \frac{9}{4}$

$= \frac{2}{5} \cdot \frac{4}{9}$

$= \frac{8}{45}$

Solution: Shawn ran $\frac{8}{45}$ of Lin's distance.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can use division and multiplication to solve problems involving fractional lengths.

1 2 3

Lesson 13 Fractional Lengths

Success looks like . . .

- **Language Goal:** Applying division of fractions to solve problems involving fractional lengths, and explaining the solution method. **(Speaking and Listening, Writing)**
  - » Solving for the fraction of Lin's distance run by Shawn in Problem 2.
- **Language Goal:** Interpreting a written question about multiplicative comparison, e.g., "how many times as long?", and writing a division equation to represent it. **(Speaking and Listening, Writing)**
  - » Solving how many times as far as Shawn did Lin run in Problem 1.

Suggested next steps

If students reverse the dividends and divisors in Problem 1, Problem 2, or both, consider:

- Having students write a multiplication equation first, then writing the related division equation.
- Reviewing how the expressions were written in Activity 1, Problems 1a–b.
- Asking, "Who ran a longer distance? Does it make sense that Lin would run $\frac{8}{45}$ of Shawn's distance? Why not? How can you edit your expressions?"

Professional Learning

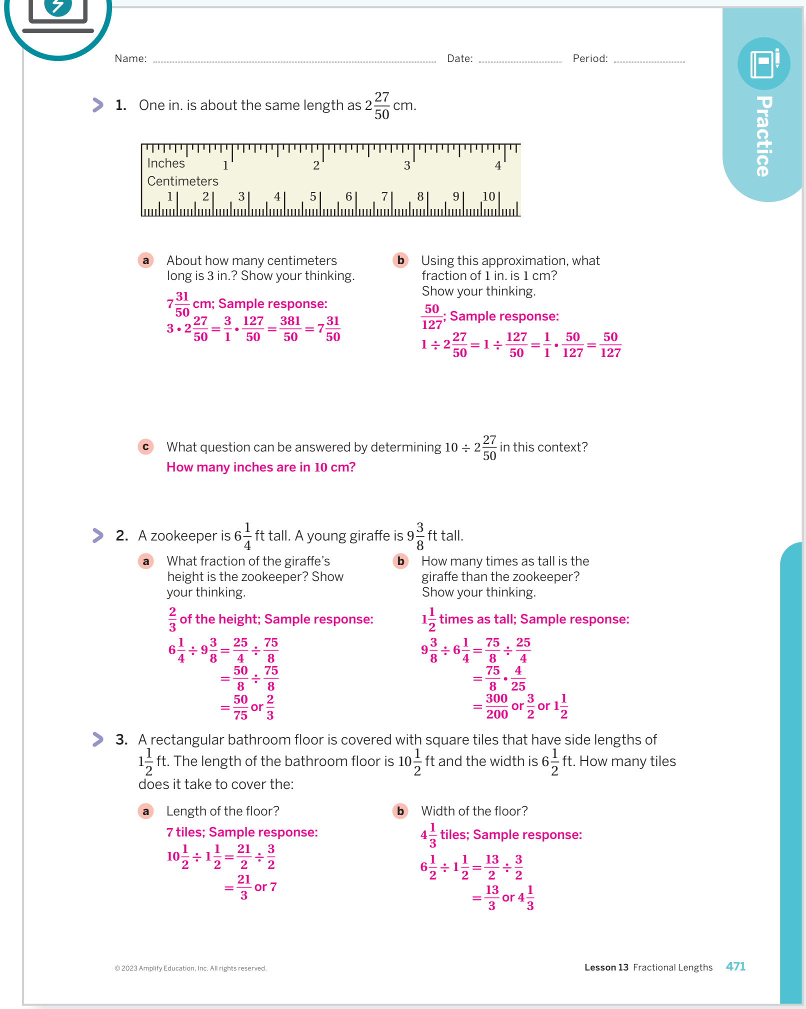
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- How was Activity 1 similar to or different from the ropes activity in Lesson 7? How did today's activity build upon this prior work?
- How did the *Info Gap* routine support students in using fraction division to solve fractional length problems? What might you change for the next time you teach this lesson?

Practice

Independent



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1. One in. is about the same length as $2\frac{27}{50}$ cm.

a. About how many centimeters long is 3 in.? Show your thinking.

$7\frac{31}{50}$ cm; Sample response:

$3 \cdot 2\frac{27}{50} = \frac{3}{1} \cdot \frac{127}{50} = \frac{381}{50} = 7\frac{31}{50}$

b. Using this approximation, what fraction of 1 in. is 1 cm? Show your thinking.

$\frac{50}{127}$; Sample response:

$1 \div 2\frac{27}{50} = 1 \div \frac{127}{50} = \frac{1}{1} \cdot \frac{50}{127} = \frac{50}{127}$

c. What question can be answered by determining $10 \div 2\frac{27}{50}$ in this context?

How many inches are in 10 cm?

2. A zookeeper is $6\frac{1}{4}$ ft tall. A young giraffe is $9\frac{3}{8}$ ft tall.

a. What fraction of the giraffe's height is the zookeeper? Show your thinking.

$\frac{2}{3}$ of the height; Sample response:

$$\begin{aligned} 6\frac{1}{4} \div 9\frac{3}{8} &= \frac{25}{4} \div \frac{75}{8} \\ &= \frac{50}{8} \div \frac{75}{8} \\ &= \frac{50}{75} \text{ or } \frac{2}{3} \end{aligned}$$

b. How many times as tall is the giraffe than the zookeeper? Show your thinking.

$1\frac{1}{2}$ times as tall; Sample response:

$$\begin{aligned} 9\frac{3}{8} \div 6\frac{1}{4} &= \frac{75}{8} \div \frac{25}{4} \\ &= \frac{75}{8} \cdot \frac{4}{25} \\ &= \frac{300}{200} \text{ or } \frac{3}{2} \text{ or } 1\frac{1}{2} \end{aligned}$$

3. A rectangular bathroom floor is covered with square tiles that have side lengths of $1\frac{1}{2}$ ft. The length of the bathroom floor is $10\frac{1}{2}$ ft and the width is $6\frac{1}{2}$ ft. How many tiles does it take to cover the:

a. Length of the floor?

7 tiles; Sample response:

$$\begin{aligned} 10\frac{1}{2} \div 1\frac{1}{2} &= \frac{21}{2} \div \frac{3}{2} \\ &= \frac{21}{3} \text{ or } 7 \end{aligned}$$

b. Width of the floor?

$4\frac{1}{3}$ tiles; Sample response:

$$\begin{aligned} 6\frac{1}{2} \div 1\frac{1}{2} &= \frac{13}{2} \div \frac{3}{2} \\ &= \frac{13}{3} \text{ or } 4\frac{1}{3} \end{aligned}$$

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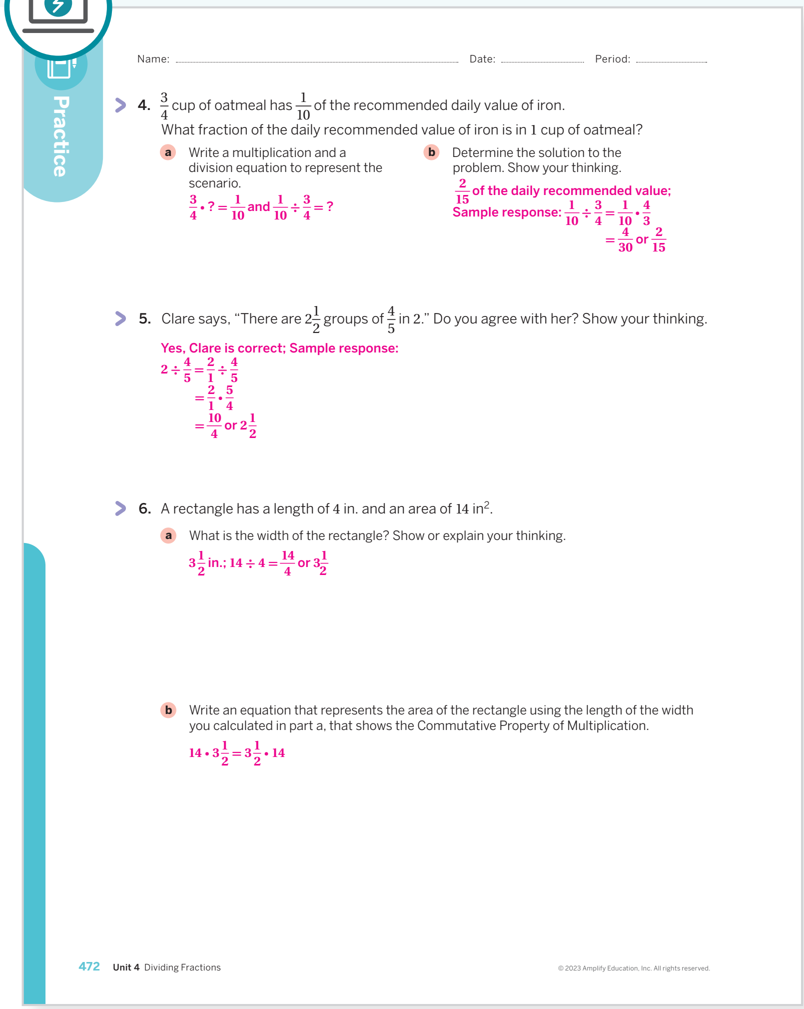

Name: ________ Date: ________ Period: ________

4. $\frac{3}{4}$ cup of oatmeal has $\frac{1}{10}$ of the recommended daily value of iron. What fraction of the daily recommended value of iron is in 1 cup of oatmeal?

a. Write a multiplication and a division equation to represent the scenario.

$\frac{3}{4} \cdot ? = \frac{1}{10}$ **and** $\frac{1}{10} \div \frac{3}{4} = ?$

b. Determine the solution to the problem. Show your thinking.

$\frac{2}{15}$ of the daily recommended value;

Sample response: $\frac{1}{10} \div \frac{3}{4} = \frac{1}{10} \cdot \frac{4}{3}$

$= \frac{4}{30}$ **or** $\frac{2}{15}$

5. Clare says, "There are $2\frac{1}{2}$ groups of $\frac{4}{5}$ in 2." Do you agree with her? Show your thinking.

Yes, Clare is correct; Sample response:

$$\begin{aligned} 2 \div \frac{4}{5} &= \frac{2}{1} \div \frac{4}{5} \\ &= \frac{2}{1} \cdot \frac{5}{4} \\ &= \frac{10}{4} \text{ or } 2\frac{1}{2} \end{aligned}$$

6. A rectangle has a length of 4 in. and an area of 14 in^2.

a. What is the width of the rectangle? Show or explain your thinking.

$3\frac{1}{2}$ **in.;** $14 \div 4 = \frac{14}{4}$ **or** $3\frac{1}{2}$

b. Write an equation that represents the area of the rectangle using the length of the width you calculated in part a, that shows the Commutative Property of Multiplication.

$14 \cdot 3\frac{1}{2} = 3\frac{1}{2} \cdot 14$

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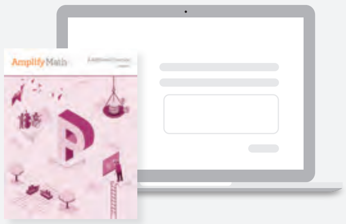
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 11	2
	5	Unit 4 Lesson 6	2
Formative	6	Unit 4 Lesson 14	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 4 | **LESSON 14**

Area With Fractional Side Lengths

Let’s explore the areas of rectangles and triangles with fractional side lengths.

Focus

Goals

1. Apply dividing by fractions to calculate the side length of a rectangle, given its area and the other side length.
2. **Language Goal:** Draw and label diagrams, and write related equations, to represent the area of a rectangle with fractional side lengths. **(Speaking and Listening)**
3. Apply dividing by fractions to calculate the base or height of a triangle, given its area and the other measurement.

Coherence

Today

Students solve problems involving areas of rectangles and triangles, but in cases where some lengths are fractions. They first get reoriented to rectangular area models by considering the diagrams of several related products involving mixed numbers, which can result in distributive thinking. Students then use the formulas for the areas of a rectangle and of a triangle, as well as the relationship between multiplication and division, to determine unknown lengths when another known length and the area of the shape are known. They also analyze and critique the work of another student on a similar task, and after determining what was incorrect, they correct the work by writing and solving a new equation.

Previously

In Lesson 13, students worked with fractional lengths, including perimeter. In Lessons 5–12, students developed several strategies for dividing with fractions. And in Unit 1, students determined the formulas for the area of a parallelogram and a triangle.

Coming Soon

In Lessons 14–15, students will continue to use division of fractions with geometric measurement, extending to finding the volume of rectangular prisms with fractional side lengths.

Rigor

- Students **apply** their understanding of dividing fractions to known geometric formulas, specifically areas of rectangles and triangles.
- Students strengthen their **fluency** in dividing fractions

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (optional)
- Activity 1 PDF (answers)
- Activity 1 PDF, *Are you ready for more?*
- Activity 1 PDF, *Are you ready for more?* (answer)
- straightedge
- graph paper

Math Language Development

Review words

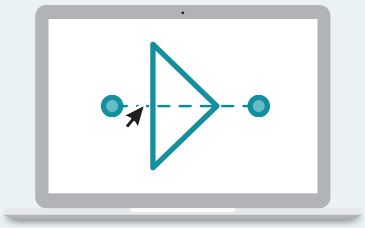
- *area*
- *squared*

Amps | Featured Activity

Activity 1

Tiling a Rectangle

Students can explore two different orientations of the tiles for a more visual experience.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might be uncomfortable sharing their responses in Activity 2. Have students identify ways that they can encourage each other. By focusing on the two-way interactions that will happen during these times (seek help and offer help, listen and speak) students can see that they both have something to give and gain as they build relationships with their peers.

Modifications to Pacing

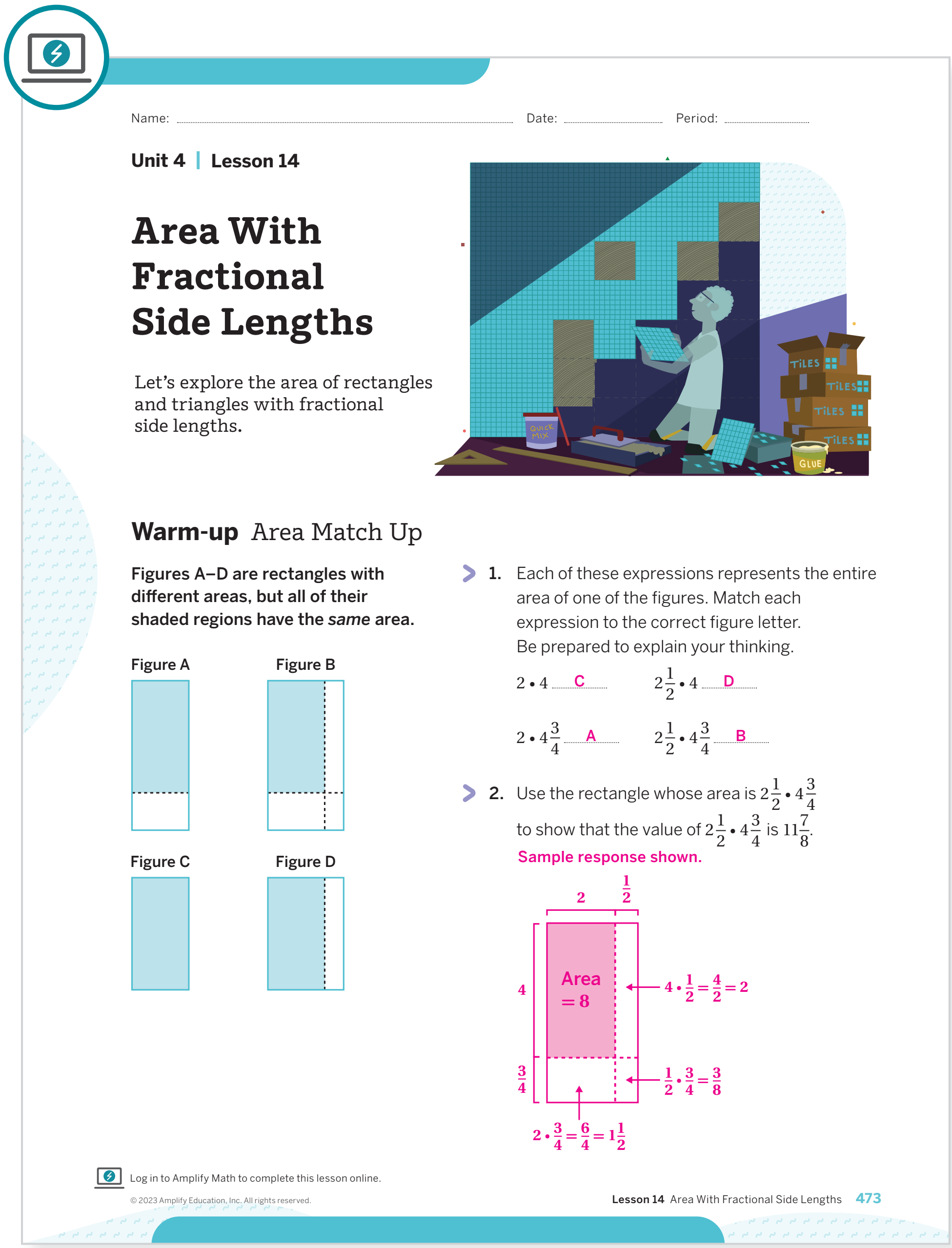
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 2 may be omitted.
- In **Activity 1**, Problem 1 can be done as a whole class and drawing a diagram for Problem 2 can be made optional.
- In **Activity 2**, Problem 1 may be omitted.

Independent | 10 min

Warm-up Area Match Up

Students interpret and match numerical expressions and diagrams to reinforce their understanding of area and of the relationship between multiplication and division.



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Unit 4 | **Lesson 14**

Area With Fractional Side Lengths

Let's explore the area of rectangles and triangles with fractional side lengths.

Warm-up Area Match Up

Figures A–D are rectangles with different areas, but all of their shaded regions have the *same* area.

Figure A

Figure B

Figure C

Figure D

1. Each of these expressions represents the entire area of one of the figures. Match each expression to the correct figure letter. Be prepared to explain your thinking.

$2 \cdot 4$ C

$2\frac{1}{2} \cdot 4$ D

$2 \cdot 4\frac{3}{4}$ A

$2\frac{1}{2} \cdot 4\frac{3}{4}$ B

2. Use the rectangle whose area is $2\frac{1}{2} \cdot 4\frac{3}{4}$ to show that the value of $2\frac{1}{2} \cdot 4\frac{3}{4}$ is $11\frac{7}{8}$.

Sample response shown.

Log in to Amplify Math to complete this lesson online.

1 Launch

Activate prior knowledge by asking, "What is the formula for the area of a rectangle?" $A = \ell \cdot w$

2 Monitor

Help students get started by prompting, "What do you know about Figure C?"

Look for points of confusion:

- **Not understanding how to match the white areas with the expression.** Have students label each blue rectangle with its dimensions and ask "Which is getting an additional piece in Figure _____? The length or the width?"
- **Mislabeling the diagrams.** Have students draw each piece separately and label the length and width. This is particularly important for Problem 2.

Look for productive strategies:

- Using the $\ell \cdot w$ structure in the expression and in the diagrams to make a match.
- Multiplying to get $2\frac{1}{2} \cdot 4\frac{3}{4} = 11\frac{7}{8}$ (Problem 2), but not showing how each of the 4 "parts" of Figure B map to that. Encourage students to relate their calculation to the diagram.

3 Connect

Display Figures A–D.

Ask, "Did you match the figures to the expressions or the expressions to the figures?"

Have individual students share their thinking and matches for Problem 1, and then how they determined the area of each section of Figure B for Problem 2.

Highlight that combining all of the partial products or areas of Figure B gives a sum of $11\frac{7}{8}$, which is the area of the entire rectangle.

MLR Math Language Development

MLR7: Compare and Connect

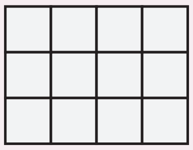
During the Connect, as students share their matches, ask, "What is the same or different about Figure B, compared to the other three figures? How do you see whole numbers or fractions greater than 1 represented in each area model?" This will help strengthen students' use of mathematical language and reasoning about area and multiplication of fractions.

English Learners

Annotate Figures A–D with their corresponding expressions from Problem 1. This will support students in making sense of the area models and the language used to describe the multiplication of fractions.

Power-up

To power up students' ability to write an equation that represents the Commutative Property of Multiplication, have students complete:



Recall that the Commutative Property of Multiplication says that the order in which you multiply two values does not change the product. For example, $6 \cdot 4 = 4 \cdot 6$.

Write an equation that represents the area model that models the Commutative Property of Multiplication.

$\underline{\ 4\ } \cdot \underline{\ 3\ } = \underline{\ 3\ } \cdot \underline{\ 4\ }$

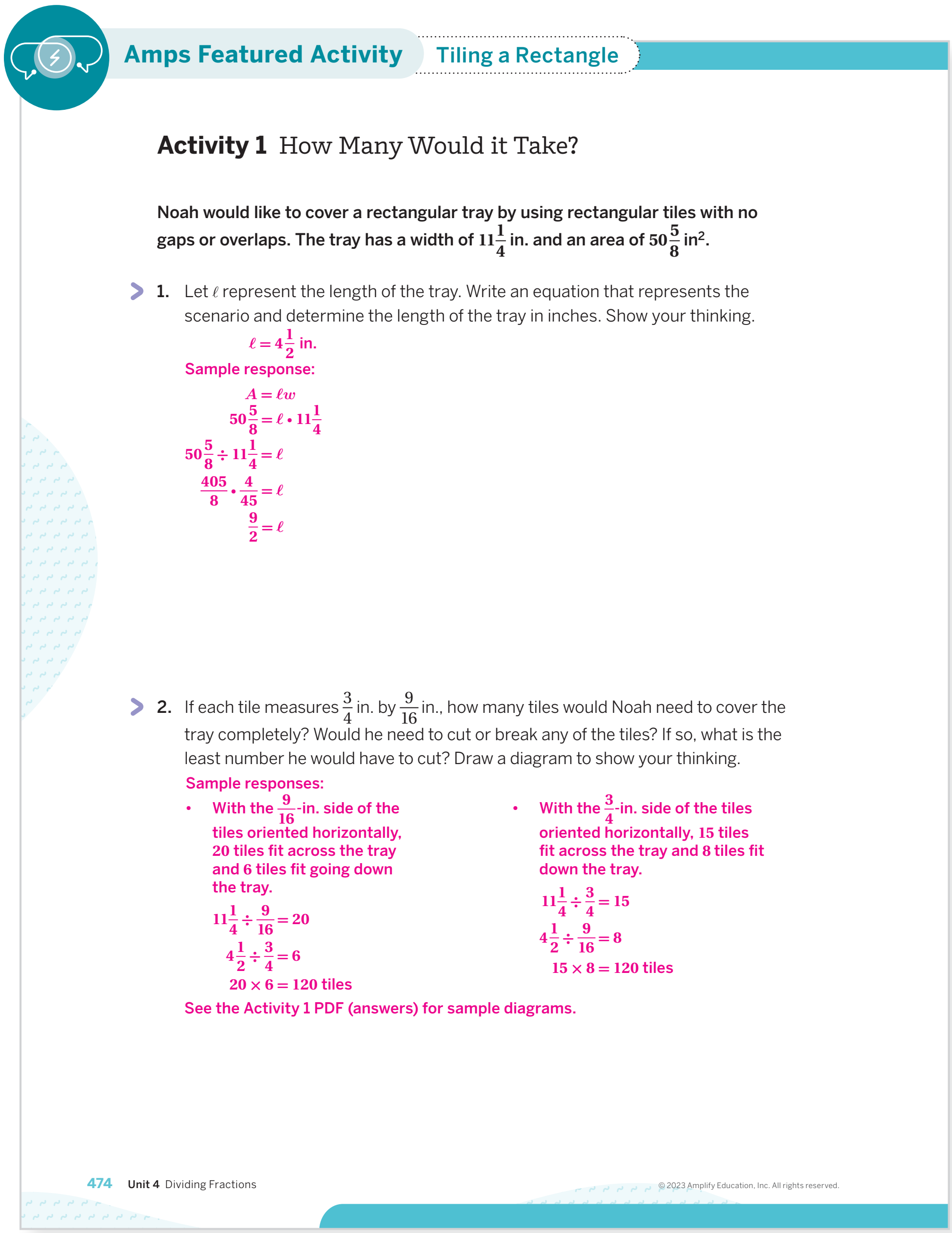
Use: Before the Warm-up.
Informed by: Performance on Lesson 13, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8.

Pairs | 15 min

Activity 1 How Many Would it Take?

Students divide fractional lengths representing the dimensions of a rectangular area and tiles needed to cover it, coordinating measurements in one and two dimensions.

Amps Featured Activity Tiling a Rectangle



Activity 1 How Many Would it Take?

Noah would like to cover a rectangular tray by using rectangular tiles with no gaps or overlaps. The tray has a width of $11\frac{1}{4}$ in. and an area of $50\frac{5}{8}$ in².

1. Let ℓ represent the length of the tray. Write an equation that represents the scenario and determine the length of the tray in inches. Show your thinking.

   $\ell = 4\frac{1}{2}$ in.

   Sample response:

   $$A = \ell w$$
   $$50\frac{5}{8} = \ell \cdot 11\frac{1}{4}$$
   $$50\frac{5}{8} \div 11\frac{1}{4} = \ell$$
   $$\frac{405}{8} \cdot \frac{4}{45} = \ell$$
   $$\frac{9}{2} = \ell$$

2. If each tile measures $\frac{3}{4}$ in. by $\frac{9}{16}$ in., how many tiles would Noah need to cover the tray completely? Would he need to cut or break any of the tiles? If so, what is the least number he would have to cut? Draw a diagram to show your thinking.

   Sample responses:

   - **With the $\frac{9}{16}$-in. side of the tiles oriented horizontally, 20 tiles fit across the tray and 6 tiles fit going down the tray.**

     $$11\frac{1}{4} \div \frac{9}{16} = 20$$
     $$4\frac{1}{2} \div \frac{3}{4} = 6$$
     $$20 \times 6 = 120 \text{ tiles}$$

   - **With the $\frac{3}{4}$-in. side of the tiles oriented horizontally, 15 tiles fit across the tray and 8 tiles fit down the tray.**

     $$11\frac{1}{4} \div \frac{3}{4} = 15$$
     $$4\frac{1}{2} \div \frac{9}{16} = 8$$
     $$15 \times 8 = 120 \text{ tiles}$$

   See the Activity 1 PDF (answers) for sample diagrams.

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1 Launch

Set an expectation for the amount of time that students will have to work individually on the activity. Tell students that they do not need to draw every tile for Problem 2.

2 Monitor

Help students get started by asking, "What information is Problem 1 asking you for? What information do you have? What do you need to know in order to solve the problem?"

Look for points of confusion:

- **Writing an equation representing a division of the wrong measurements.** Ask, "What part of this tray are you dividing?" Also consider having students explain their thinking by using a diagram.
- **Not knowing how to orientate or configure the tiles.** Have the student identify which is the $\frac{3}{4}$ side and the $\frac{9}{16}$ side of the tile. Ask "Which measurement do you want to go with the length of the tray? The width?"

Look for productive strategies:

- Noticing that the tiles divide evenly into the length and width of the tray, and that none would have to be cut to fit.
- Writing a division equation to accurately represent each aspect of the scenario.

3 Connect

Have pairs of students share their diagrams and explain how they approached the problem, including how the orientation of the tiles affected their calculations and solutions.

Display the Activity 1 PDF (answers).

Highlight that the answer to Problem 3 was the same, regardless of how the tiles were oriented, because multiplication is commutative.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can explore two different orientations of the tiles, allowing for a more visual experience.

Accessibility: *Guide Processing and Visualization*

Provide copies of the Activity 1 PDF, which includes a diagram of the tray and rectangular tile to support student thinking. Display the area formula for a rectangle, $A = \ell \cdot w$.

Extension: *Math Enrichment*

Have students complete the *Are you ready for more?* PDF, in which they will use ratio reasoning to analyze a pattern. This pattern is related to the Fibonacci sequence, but does not introduce this name to students.

Math Language Development

MLR6: Three Reads

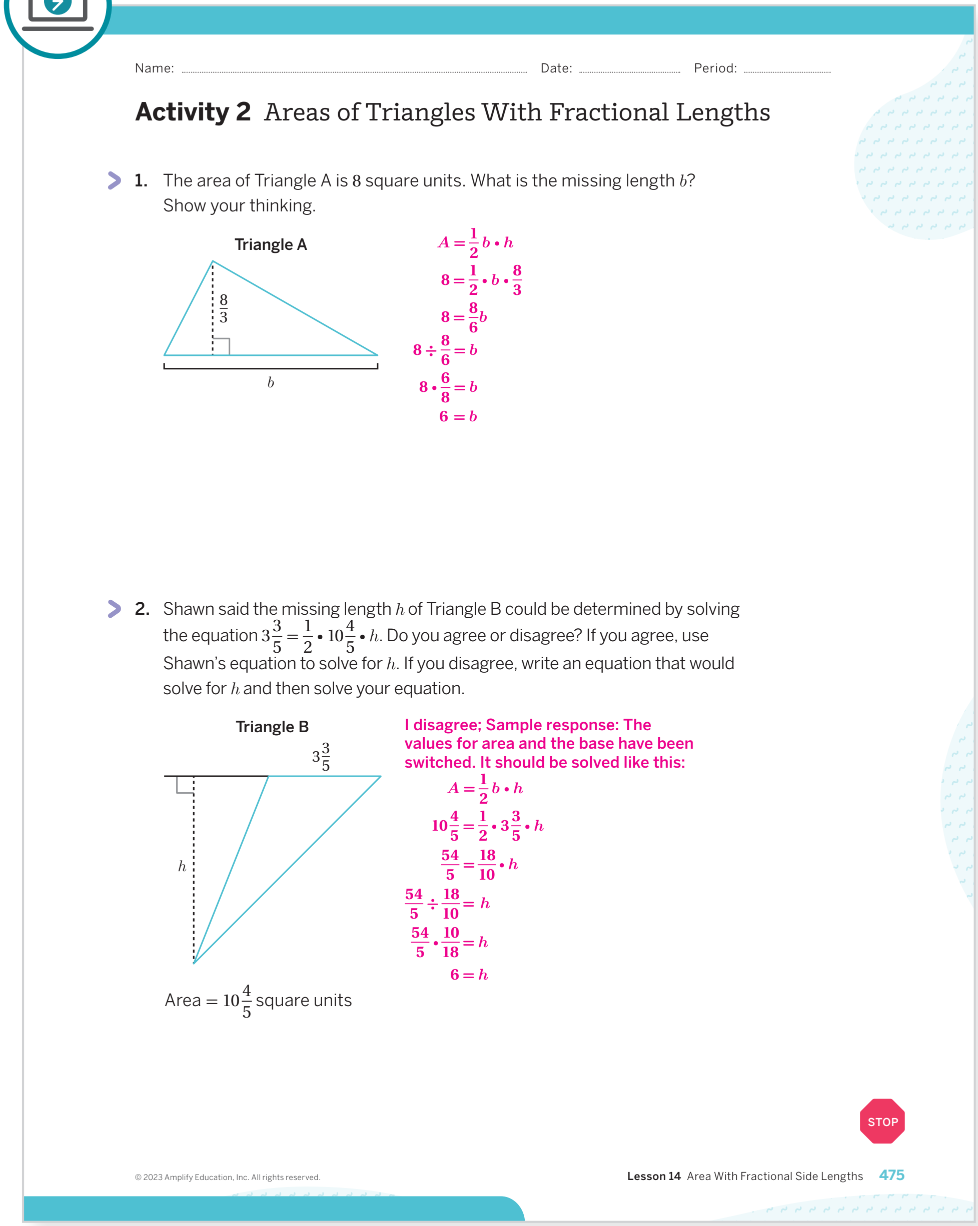
Use this routine to help students make sense of the introductory text and the text in Problems 1 and 2.

- **Read 1:** Students should understand that Noah wants to cover a rectangular tray with no gaps or overlaps and that this represents the area of the tray.
- **Read 2:** Ask students to name given quantities and relationships, such as the dimensions of the tray, or the dimensions of each tile.
- **Read 3:** Ask students to create diagrams or tables to represent the relationships among the quantities. Then ask them to plan their solution strategy to each of Problems 1 and 2.

Pairs | 10 min

Activity 2 Areas of Triangles With Fractional Lengths

Students apply their knowledge of division of fractions and the area formula to a triangle to calculate the missing lengths in triangles with fractional measurements.



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Activity 2 Areas of Triangles With Fractional Lengths

1. The area of Triangle A is 8 square units. What is the missing length b? Show your thinking.

Triangle A

$\frac{8}{3}$

b

$$A = \frac{1}{2} b \bullet h$$
$$8 = \frac{1}{2} \bullet b \bullet \frac{8}{3}$$
$$8 = \frac{8}{6} b$$
$$8 \div \frac{8}{6} = b$$
$$8 \bullet \frac{6}{8} = b$$
$$6 = b$$

2. Shawn said the missing length h of Triangle B could be determined by solving the equation $3\frac{3}{5} = \frac{1}{2} \bullet 10\frac{4}{5} \bullet h$. Do you agree or disagree? If you agree, use Shawn's equation to solve for h. If you disagree, write an equation that would solve for h and then solve your equation.

Triangle B

$3\frac{3}{5}$

h

Area $= 10\frac{4}{5}$ square units

I disagree; Sample response: The values for area and the base have been switched. It should be solved like this:

$$A = \frac{1}{2} b \bullet h$$
$$10\frac{4}{5} = \frac{1}{2} \bullet 3\frac{3}{5} \bullet h$$
$$\frac{54}{5} = \frac{18}{10} \bullet h$$
$$\frac{54}{5} \div \frac{18}{10} = h$$
$$\frac{54}{5} \bullet \frac{10}{18} = h$$
$$6 = h$$

STOP

Lesson 14 Area With Fractional Side Lengths 475

1 Launch

Activate prior knowledge by asking, "What is the formula for the area of a triangle?" $A = \frac{1}{2} \bullet b \bullet h$

2 Monitor

Help students get started by asking, "What information do you know that you can substitute into the formula?"

Look for points of confusion:

- **Not knowing how to determine a missing value of b or h in A = $\frac{1}{2} \bullet b \bullet h$.** Ask, "Thinking about the properties of multiplication, could you combine two factors (meaning $\frac{1}{2}$ and the given length)?"
- **Agreeing with Shawn.** Have students explain how they used Shawn's equation to solve for h.

Look for productive strategies:

- Using a copy of the triangle to compose a parallelogram.
- Recognizing what information is given to them to set up the equation and then solving the equation by using the algorithm for division.

3 Connect

Display the two problems, and solutions, one at a time.

Have pairs of students share their responses to Problem 2. Begin with students who reasoned concretely (e.g., composing a parallelogram), followed by those who reasoned symbolically (e.g., manipulating equations).

Ask, "What is similar and different about the two problems? Were you able to solve them the same way?" Yes, even though I was solving for a different length, b or h , they were solved the same way.

Highlight that what students learned about fractions and operations in this unit can help them reason more effectively about problems in other areas of mathematics, such as geometry.

Differentiated Support

Accessibility: *Activate Prior Knowledge, Clarify Vocabulary and Symbols*

Remind students that they previously determined the area of triangles using an area formula, but also composing two triangles to form a parallelogram. Ask students how they could compose a parallelogram in Problem 1 and then use the area formula for a parallelogram, $A = b \bullet h$, to determine the area formula for the triangle, $A = \frac{1}{2} \bullet b \bullet h$. Review with students what the terms *base* and *height* mean and how they are represented in the area formulas.

Math Language Development

MLR8: Discussion Supports

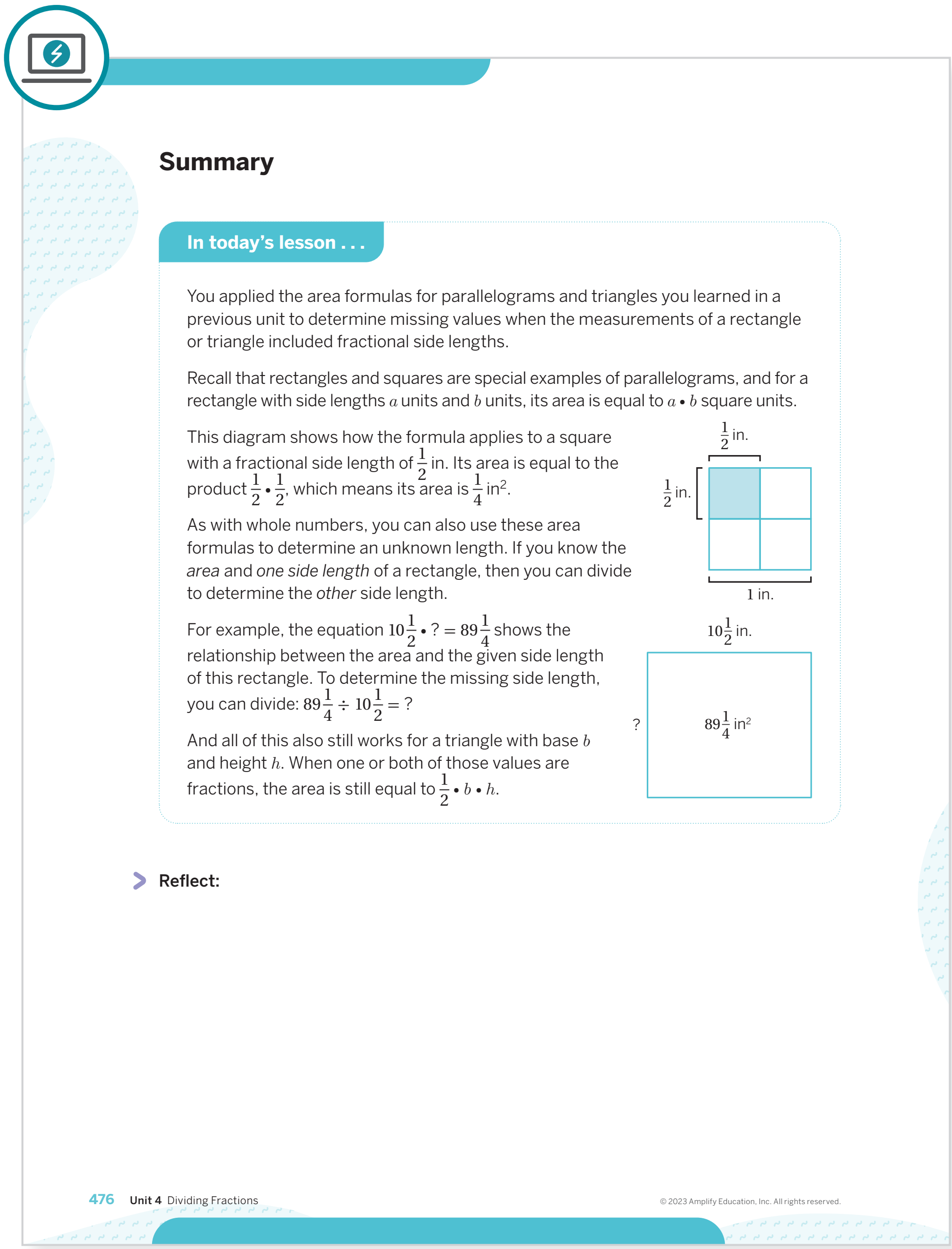
During the Connect, as students share their solutions and reasoning, encourage them to use mathematical language, such as *compose*, *rearrange*, *parallelogram*, *base*, *height*, and *area*. Invite students to chorally repeat the phrases that include these words in context. This will help students use more precise mathematical language that they previously learned when working with area.

English Learners

Annotate the triangles in Problems 1 and 2 with their respective bases and heights.

Summary

Review and synthesize how to determine fractional side lengths by using formulas when only the area and one length are known for rectangles and triangles.



Summary

In today's lesson . . .

You applied the area formulas for parallelograms and triangles you learned in a previous unit to determine missing values when the measurements of a rectangle or triangle included fractional side lengths.

Recall that rectangles and squares are special examples of parallelograms, and for a rectangle with side lengths a units and b units, its area is equal to $a \cdot b$ square units.

This diagram shows how the formula applies to a square with a fractional side length of $\frac{1}{2}$ in. Its area is equal to the product $\frac{1}{2} \cdot \frac{1}{2}$, which means its area is $\frac{1}{4}$ in^2.

As with whole numbers, you can also use these area formulas to determine an unknown length. If you know the *area* and *one side length* of a rectangle, then you can divide to determine the *other* side length.

For example, the equation $10\frac{1}{2} \cdot ? = 89\frac{1}{4}$ shows the relationship between the area and the given side length of this rectangle. To determine the missing side length, you can divide: $89\frac{1}{4} \div 10\frac{1}{2} = ?$

And all of this also still works for a triangle with base b and height h. When one or both of those values are fractions, the area is still equal to $\frac{1}{2} \cdot b \cdot h$.

Reflect:

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Synthesize

Display the two figures in the Summary.

Highlight that students used division to solve for missing length measurements in rectangles and triangles when the area and other necessary lengths were known, and included fractional values, which required dividing with fractions.

Ask:

- "How is determining an unknown base or height in a triangle different than determining an unknown side length in a rectangle?" In a triangle, there are three factors in the formula, because of the $\frac{1}{2}$.
- "What multiplication equation can you write to help you determine the height of a triangle that has a base of $\frac{5}{4}$ cm and an area of 10 cm^2? And then how would you determine the height?" $\frac{1}{2} \cdot \frac{5}{4} \cdot h = 10$; Sample responses:
  - I would multiply $\frac{1}{2} \cdot \frac{5}{4}$ to get $\frac{5}{8}$, and then divide 10 by $\frac{5}{8}$.
  - I can divide 10 by either $\frac{1}{2}$ or $\frac{5}{4}$ first, and then divide that result by the other factor.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

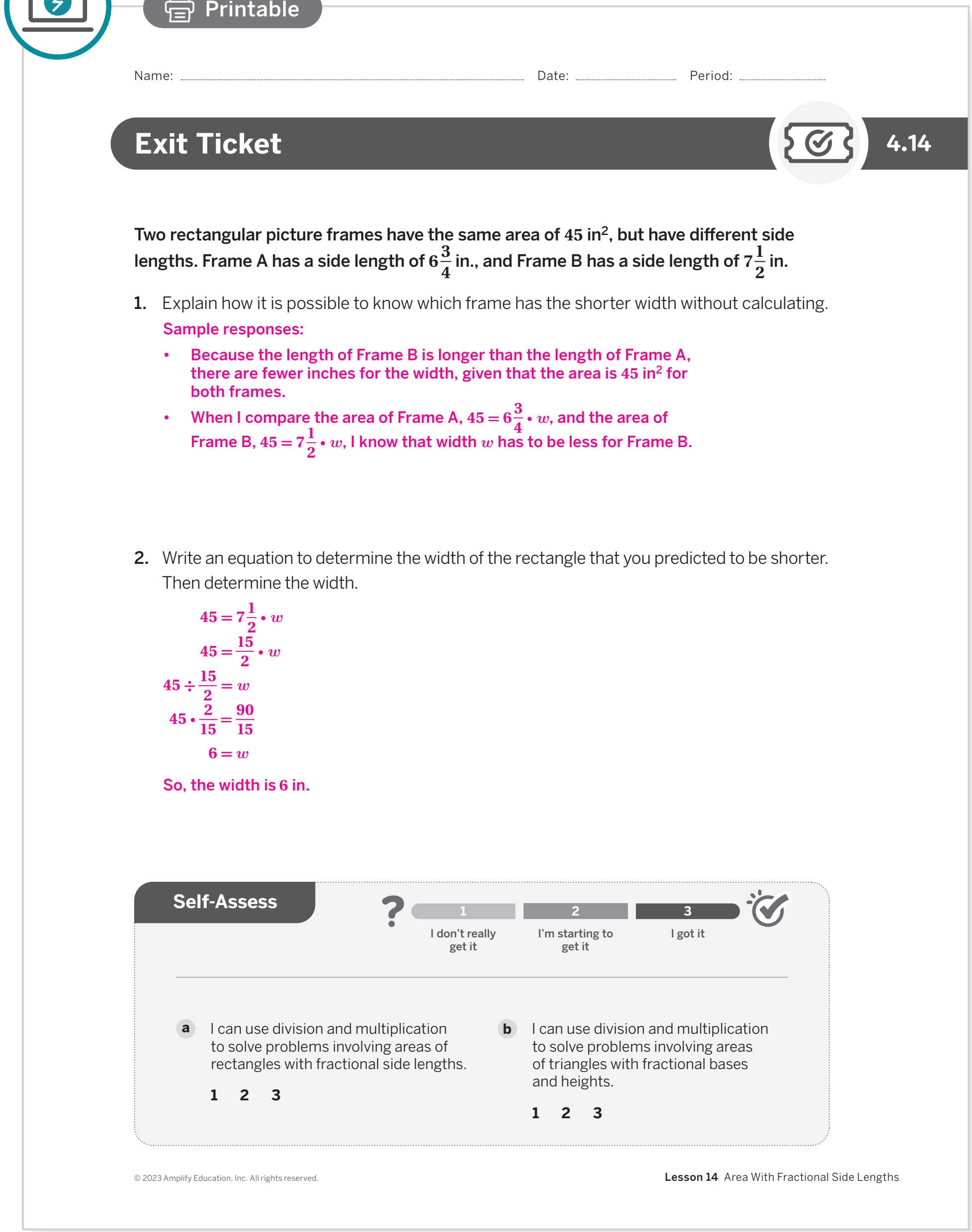
- "What connections can you make to Unit 1?"
- "How is the math the same? How is it different?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by comparing the dimensions of two frames with the same area.

Printable



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Exit Ticket 4.14

Two rectangular picture frames have the same area of 45 in^2, but have different side lengths. Frame A has a side length of $6\frac{3}{4}$ in., and Frame B has a side length of $7\frac{1}{2}$ in.

1. Explain how it is possible to know which frame has the shorter width without calculating.

   Sample responses:

   - **Because the length of Frame B is longer than the length of Frame A, there are fewer inches for the width, given that the area is 45 in^2 for both frames.**
   - **When I compare the area of Frame A, $45 = 6\frac{3}{4} \cdot w$, and the area of Frame B, $45 = 7\frac{1}{2} \cdot w$, I know that width w has to be less for Frame B.**

2. Write an equation to determine the width of the rectangle that you predicted to be shorter. Then determine the width.

$$45 = 7\frac{1}{2} \cdot w$$
$$45 = \frac{15}{2} \cdot w$$
$$45 \div \frac{15}{2} = w$$
$$45 \cdot \frac{2}{15} = \frac{90}{15}$$
$$6 = w$$

   So, the width is 6 in.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a. I can use division and multiplication to solve problems involving areas of rectangles with fractional side lengths.
1 2 3

b. I can use division and multiplication to solve problems involving areas of triangles with fractional bases and heights.
1 2 3

Lesson 14 Area With Fractional Side Lengths

Success looks like . . .

- **Goal:** Applying dividing by fractions to calculate the side length of a rectangle, given its area and the other side length.
  - » Solving for the unknown width of a rectangular frame using division of fractions in Problem 2.
- **Language Goal:** Drawing and labeling diagrams, and writing related equations, to represent the area of a rectangle with fractional side lengths. **(Speaking and Listening)**
  - » Writing an equation for the area of a rectangular frame in Problem 2.
- **Goal:** Applying dividing by fractions to calculate the base or height of a triangle, given its area and the other measurement.

Suggested next steps

If students think Frame A has the shorter width, consider:

- Having students write equations representing the two frames and asking, "What else must be true if the area is the same but one length is longer?"
- Having students sketch the two frames proportionally to scale, using the known measurements, and asking, "What do you notice about the relative lengths of the missing widths?"

If students have trouble determining the width, consider:

- Referring to Activity 1, Problem 1 and asking, "How did you determine the missing length of the tray?"

Professional Learning

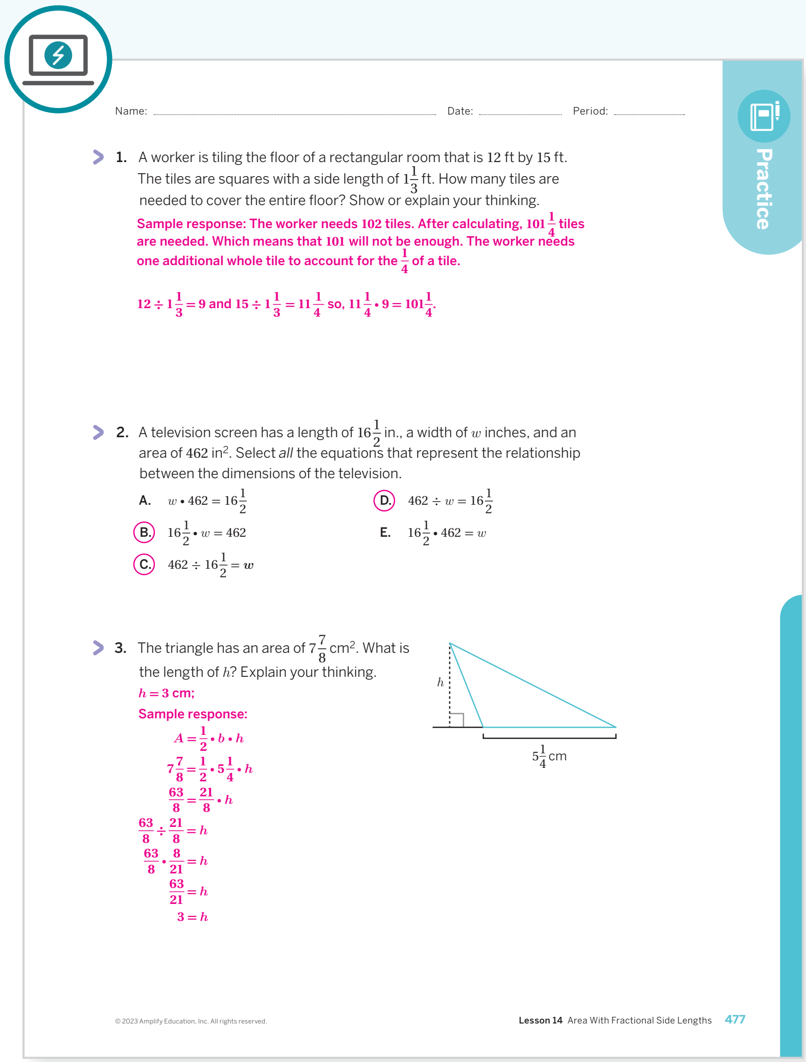
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? This lesson asked students to apply their understanding of dividing fractions to geometry from Unit 1. Where in your students' work today did you see or hear evidence of them reflecting back to that learning?
- Which students' ideas were you able to highlight during Activity 1? What might you change for the next time you teach this lesson to ensure that all strategies are represented?

Practice

Independent



Practice

Name: Date: Period:

1. A worker is tiling the floor of a rectangular room that is 12 ft by 15 ft. The tiles are squares with a side length of $1\frac{1}{3}$ ft. How many tiles are needed to cover the entire floor? Show or explain your thinking.

   Sample response: The worker needs 102 tiles. After calculating, $101\frac{1}{4}$ tiles are needed. Which means that 101 will not be enough. The worker needs one additional whole tile to account for the $\frac{1}{4}$ of a tile.

   $12 \div 1\frac{1}{3} = 9$ and $15 \div 1\frac{1}{3} = 11\frac{1}{4}$ so, $11\frac{1}{4} \cdot 9 = 101\frac{1}{4}$.

2. A television screen has a length of $16\frac{1}{2}$ in., a width of w inches, and an area of 462 in^2. Select *all* the equations that represent the relationship between the dimensions of the television.

   A. $w \cdot 462 = 16\frac{1}{2}$
   [✓ B.] $16\frac{1}{2} \cdot w = 462$
   [✓ C.] $462 \div 16\frac{1}{2} = w$
   [✓ D.] $462 \div w = 16\frac{1}{2}$
   E. $16\frac{1}{2} \cdot 462 = w$

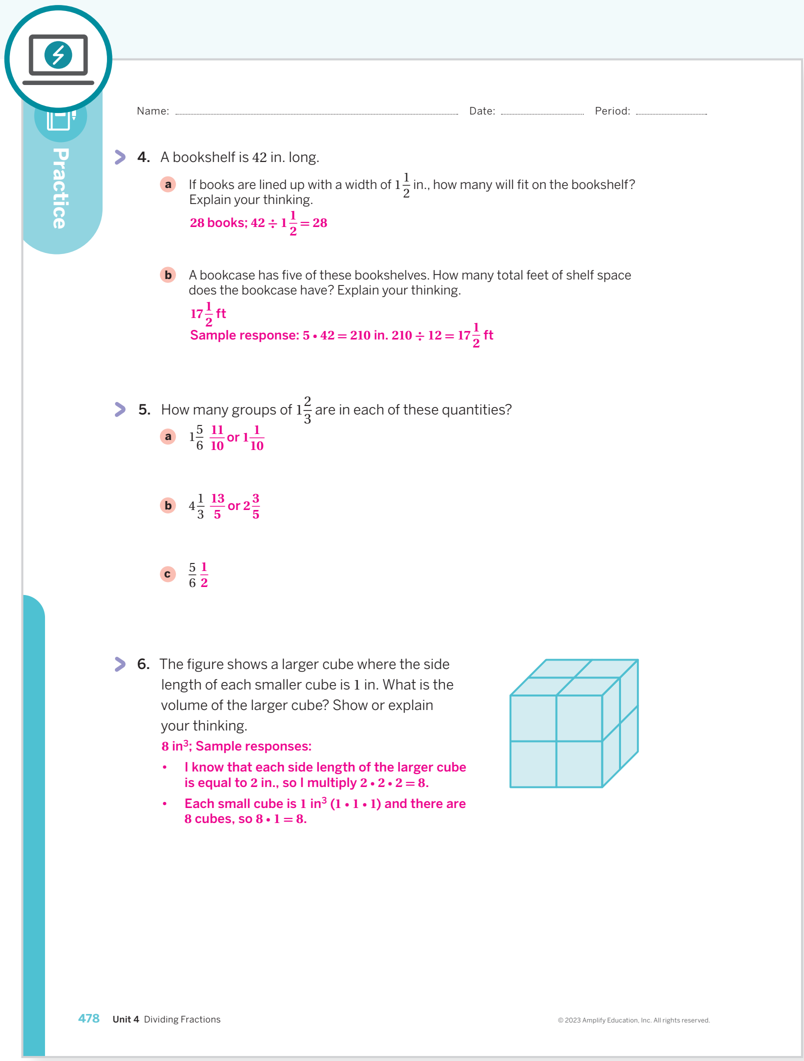
3. The triangle has an area of $7\frac{7}{8}$ cm^2. What is the length of h? Explain your thinking.

   $h = 3$ cm;

   Sample response:

$$A = \frac{1}{2} \cdot b \cdot h$$
$$7\frac{7}{8} = \frac{1}{2} \cdot 5\frac{1}{4} \cdot h$$
$$\frac{63}{8} = \frac{21}{8} \cdot h$$
$$\frac{63}{8} \div \frac{21}{8} = h$$
$$\frac{63}{8} \cdot \frac{8}{21} = h$$
$$\frac{63}{21} = h$$
$$3 = h$$

Lesson 14 Area With Fractional Side Lengths 477



Practice

Name: Date: Period:

4. A bookshelf is 42 in. long.

   a. If books are lined up with a width of $1\frac{1}{2}$ in., how many will fit on the bookshelf? Explain your thinking.

   28 books; $42 \div 1\frac{1}{2} = 28$

   b. A bookcase has five of these bookshelves. How many total feet of shelf space does the bookcase have? Explain your thinking.

   $17\frac{1}{2}$ ft

   Sample response: $5 \cdot 42 = 210$ in. $210 \div 12 = 17\frac{1}{2}$ ft

5. How many groups of $1\frac{2}{3}$ are in each of these quantities?

   a. $1\frac{5}{6}$ **$\frac{11}{10}$ or $1\frac{1}{10}$**

   b. $4\frac{1}{3}$ **$\frac{13}{5}$ or $2\frac{3}{5}$**

   c. $\frac{5}{6}$ **$\frac{1}{2}$**

6. The figure shows a larger cube where the side length of each smaller cube is 1 in. What is the volume of the larger cube? Show or explain your thinking.

   8 in^3; Sample responses:

   - **I know that each side length of the larger cube is equal to 2 in., so I multiply $2 \cdot 2 \cdot 2 = 8$.**
   - **Each small cube is 1 in^3 $(1 \cdot 1 \cdot 1)$ and there are 8 cubes, so $8 \cdot 1 = 8$.**

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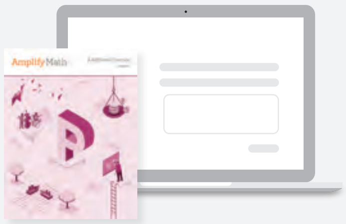
Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 13	2
	5	Unit 4 Lesson 13	1
Formative	6	Unit 4 Lesson 15	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

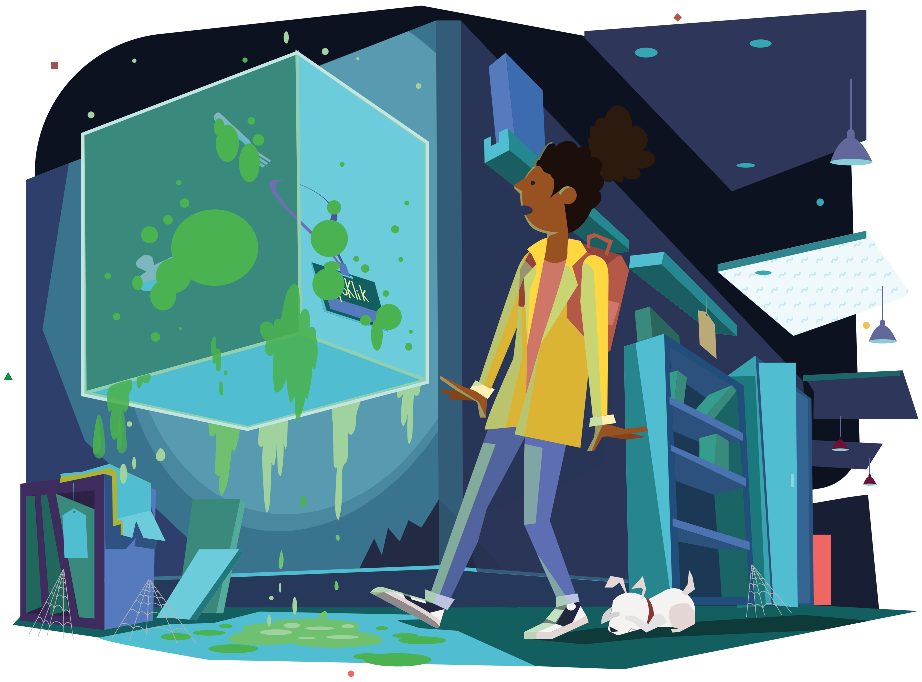


For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 4 | **LESSON 15**

Volume of Prisms

Let's explore the volume of prisms with fractional lengths.



Focus

Goals

1. **Language Goal:** Determine how to solve a problem involving the volume of a rectangular prism and fractional edge lengths. **(Speaking and Listening, Writing)**
2. Generalize that the volume of a rectangular prism with fractional edge lengths can be found by multiplying the edge lengths.
3. Generalize that it takes more smaller cubes or fewer larger cubes to fill the same volume.

Coherence

Today

Students extend their understanding of volume by considering a rectangular prism being packed with cubes with a unit fraction edge length. This reinforces the relationship between fractions and division, while preparing students to solve problems involving volumes of prisms with fractional edge lengths. Ultimately, students generalize that the volume of a rectangular prism with fractional edge lengths can still be determined by multiplying its edge lengths directly. Students analyze the work of others and draw their own conclusions about the relationship between the size of unit cube used and volume.

Previously

In Lesson 14, students focused on area with fractional measurements. In Unit 1, students generalized the formula for the volume of prisms.

Coming Soon

In Lesson 16, students will apply their understanding of volume for rectangular prisms with fractional edge lengths.

Rigor

- Students **apply** their understanding of dividing fractions to geometry, specifically to volume of cubes and rectangular prisms.
- Students strengthen their **fluency** with dividing fractions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- $\frac{1}{2}$-in. cubes

Math Language Development

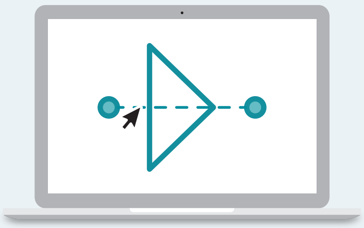
Review words

- *volume*
- *cubed*

Amps | **Featured Activity**

Activity 1
Real-Time Feedback

Students can check in real time whether their fractional volume calculations are correct.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not provide clear explanations in Activity 2. Have students reflect on their own responses by asking themselves whether or not it makes sense, how they can improve the explanation, and whether they can apply ideas of others.

Modifications to Pacing

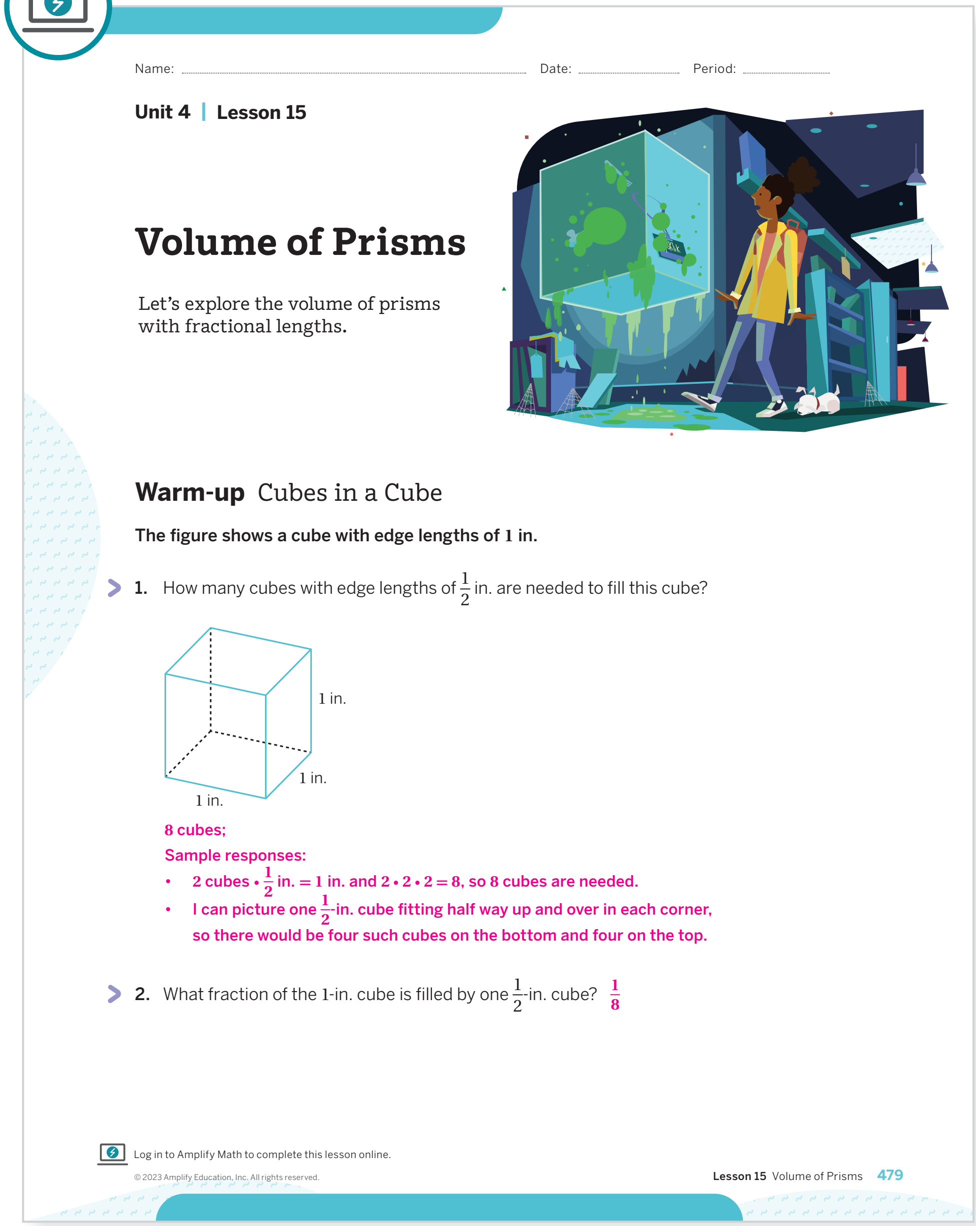
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have students only complete the first five rows of the table for Problem 1a, in addition to Problem 1b.
- **Activity 2** can be conducted as a whole class activity.

Independent | 5 min

Warm-up Cubes in a Cube

Students determine the number of $\frac{1}{2}$-in. cubes needed to fill a cube, which supports their thinking in determining how fractional edge lengths are related to volume in Activity 1.



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Unit 4 | **Lesson 15**

Volume of Prisms

Let's explore the volume of prisms with fractional lengths.

Warm-up Cubes in a Cube

The figure shows a cube with edge lengths of 1 in.

1. How many cubes with edge lengths of $\frac{1}{2}$ in. are needed to fill this cube?

8 cubes;
Sample responses:
- 2 cubes • $\frac{1}{2}$ in. = 1 in. and $2 \cdot 2 \cdot 2 = 8$, so 8 cubes are needed.
- I can picture one $\frac{1}{2}$-in. cube fitting half way up and over in each corner, so there would be four such cubes on the bottom and four on the top.

2. What fraction of the 1-in. cube is filled by one $\frac{1}{2}$-in. cube? $\frac{1}{8}$

Log in to Amplify Math to complete this lesson online.

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Lesson 15 Volume of Prisms 479

1 Launch

Have students refer to the image of the cube. Activate prior knowledge by asking, "If this cube has an edge length of 1 in., what is its volume, in cubic inches?" 1 in^3. "How do you know?"

2 Monitor

Help students get started by asking, "How many half inches are in 1 in.?"

Look for points of confusion:

- **Thinking the answer is 2 or 6 because two $\frac{1}{2}$ in. cubes equals one in.** Extend students' thinking to see this as being true for just one dimension and to apply it to all three dimensions.

Look for productive strategies:

- Recognizing that it takes two $\frac{1}{2}$-in. cubes to cover the length, width, and height of the cube, in a 3D array.
- Applying one of the formulas for calculating volume, $V = \ell \cdot w \cdot h$ or s^3.
- Visualizing the prism with two layers of four $\frac{1}{2}$-in. cubes, or as four towers of two $\frac{1}{2}$-in. cubes.

3 Connect

Have individual students share their responses and strategies, making sure that both the mathematical and visual strategies are shared.

Ask, "How did you determine the relationship between the edge lengths of the larger and smaller cubes?"

Highlight, that eight cubes are needed to fill the larger cube, so one $\frac{1}{2}$-in. cube is one out of the eight cubes, or $\frac{1}{8}$ of the large cube, whose volume is 1 in^3.

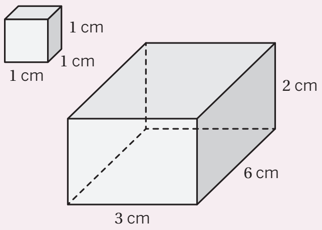
Differentiated Support

Accessibility: *Activate Prior Knowledge, Clarify Vocabulary and Symbols*

Remind students that they previously determined the volume of a cube by packing it with unit cubes. Display the volume formulas for a cube $V = \ell \cdot w \cdot h$ and $V = s^3$. Be sure students understand the meaning of the variables in each formula, and review with them what the terms *length, width, height*, and *side length* mean and how they are represented in the volume formulas. Remind students that either formula can be used when the figure is a cube, but only the first formula, $V = \ell \cdot w \cdot h$, can be used when the figure is a rectangular prism that is not a cube.

Power-up

To power up students' ability to connect the number of cubes in a solid to its volume, have students complete:



1. How many 1-cm cubes could be placed into the prism? 36 cubes
2. What does the value from Problem 1 represent?
   A. The prism's area
   B. The prism's surface area
   [✓ C.] The prism's volume
   D. The prism's dimensions

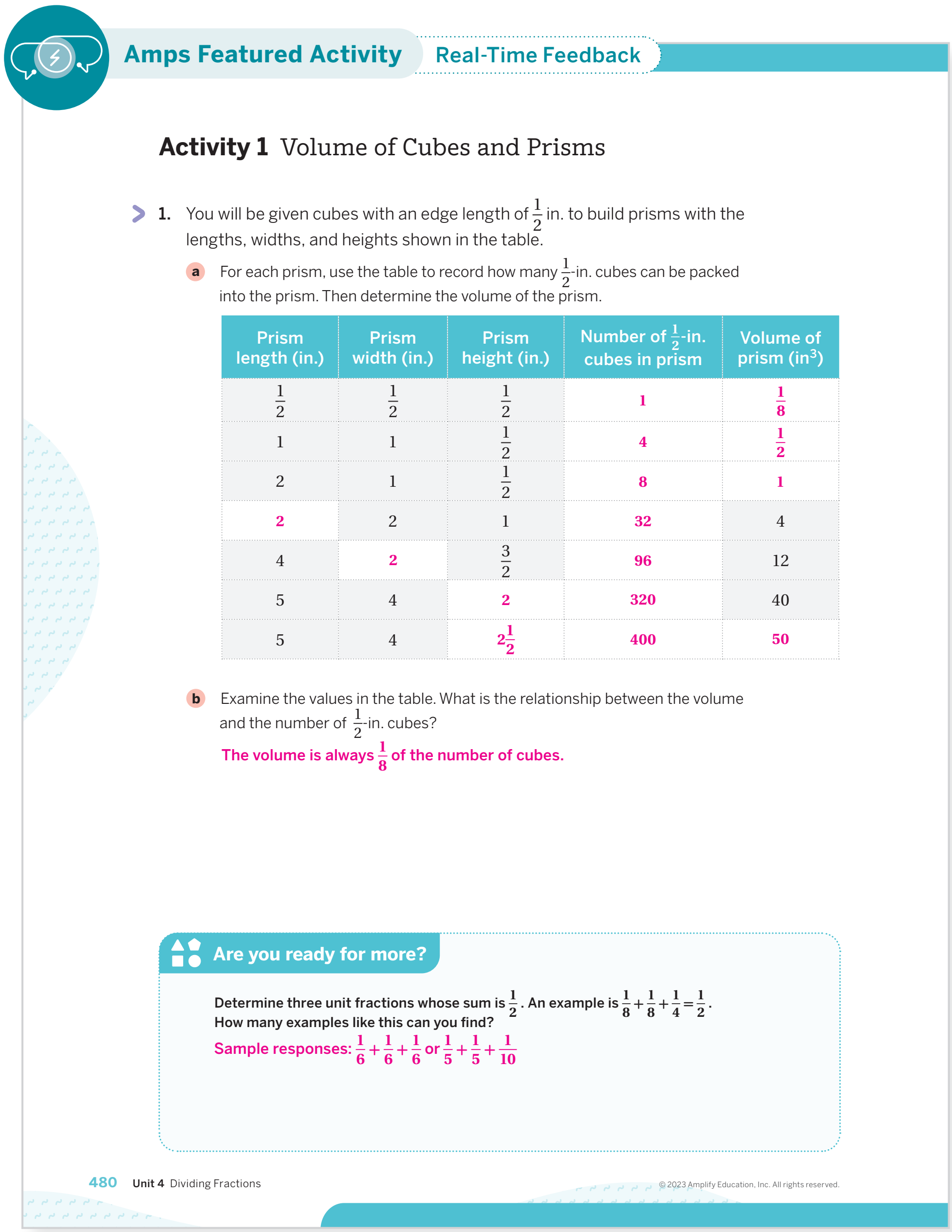
Use: Before the Warm-up.
Informed by: Performance on Lesson 14, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 7.

Pairs | 15 min

Activity 1 Volumes of Cubes and Prisms

Students generalize that the volume of a rectangular prism with fractional edge lengths is the product of the edge lengths, in inches.

Amps Featured Activity Real-Time Feedback



Activity 1 Volume of Cubes and Prisms

1. You will be given cubes with an edge length of $\frac{1}{2}$ in. to build prisms with the lengths, widths, and heights shown in the table.

   a For each prism, use the table to record how many $\frac{1}{2}$-in. cubes can be packed into the prism. Then determine the volume of the prism.

Prism length (in.)	Prism width (in.)	Prism height (in.)	Number of $\frac{1}{2}$-in. cubes in prism	Volume of prism (in^3)
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{8}$
1	1	$\frac{1}{2}$	4	$\frac{1}{2}$
2	1	$\frac{1}{2}$	8	1
2	2	1	32	4
4	2	$\frac{3}{2}$	96	12
5	4	2	320	40
5	4	$2\frac{1}{2}$	400	50

   b Examine the values in the table. What is the relationship between the volume and the number of $\frac{1}{2}$-in. cubes?

   The volume is always $\frac{1}{8}$ of the number of cubes.

Are you ready for more?

Determine three unit fractions whose sum is $\frac{1}{2}$. An example is $\frac{1}{8}+\frac{1}{8}+\frac{1}{4}=\frac{1}{2}$. How many examples like this can you find?

Sample responses: $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$ or $\frac{1}{5}+\frac{1}{5}+\frac{1}{10}$

480 Unit 4 Dividing Fractions

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1 Launch

Distribute at least eight $\frac{1}{2}$-in. cubes to each pair.

2 Monitor

Help students get started by activating prior knowledge by asking, "How do you calculate the volume of a cube?"

Look for points of confusion:

- **Confusing how to calculate the number of cubes versus the volume.** Have students make notations on the page using the first row and remind them how those values were calculated.
- **Not knowing how to determine missing lengths.** Ask, "What operation do you use to determine the volume? What is the opposite of that?"

Look for productive strategies:

- Generalizing that the volume of a rectangular prism with fractional edge lengths is the product of its edge lengths.
- Dividing the length, width, and height each by $\frac{1}{2}$ to determine the number of cubes in each prism.
- Noticing the relationship between the number of cubes and the volume across every prism is always a ratio of $1 : \frac{1}{8}$.

3 Connect

Display a completed table for students to check their responses.

Have pairs of students share the strategies used, focusing on how their strategies differed from the first three to the last four prisms.

Highlight that the relationship between the number of cubes and the volume is always $\frac{1}{8}$.

Ask, "In previous units and grades, you learned that the volume is the same as the number of unit cubes that can be packed into a rectangular prism. Does that same logic hold true here? Why or why not?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can construct prisms using cubes with a side length of $\frac{1}{2}$-in.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them only complete the first four rows of the table.

Accessibility: *Math Enrichment*

Ask students to describe the relationship between the volume of a cube and the number of $\frac{1}{4}$-in. cubes that would be needed to fill it. The volume is $\frac{1}{64}$ the number of cubes.

Math Language Development

MLR7: Compare and Connect

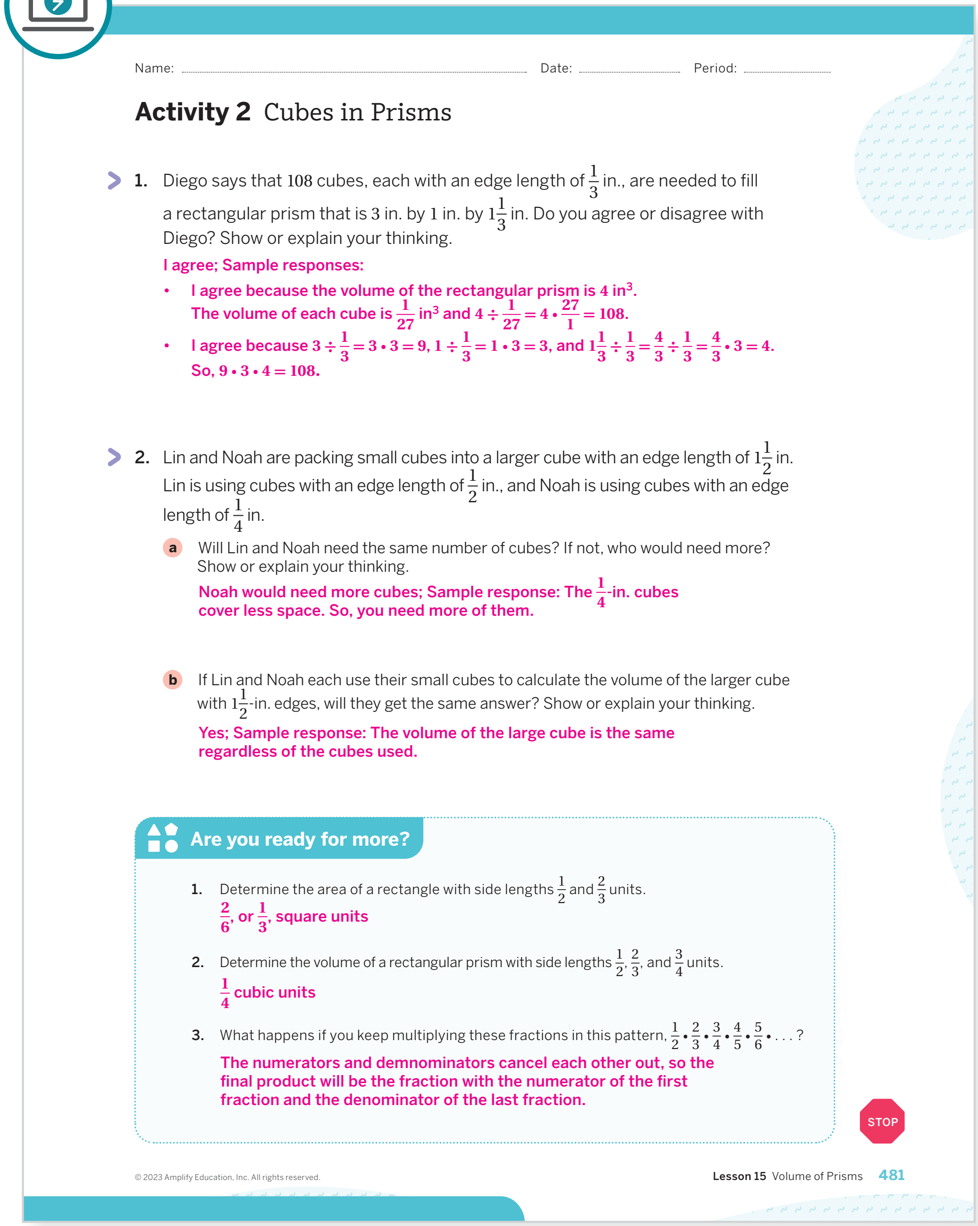
During the Connect, as students share the strategies they used and their responses to Problem 1b, draw connections between the number of $\frac{1}{2}$-in. cubes needed to build the prism and the prism's total volume. Ask:

- "If you used 1-in. cubes, what would be the relationship between the volume and the number of cubes needed?" The volume would be the same as the number of cubes.
- "Now, using $\frac{1}{2}$-in. cubes, is the volume greater or less than the number of cubes? Why do you think this is the case?" The volume is less than the number of cubes, because the volume of each cube is less than 1.

Small Groups | 15 min

Activity 2 Cubes in Prisms

Students explain why it takes more or less cubes of different fractional edge lengths to fill the same volume, and why the volume remains the same.



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Activity 2 Cubes in Prisms

1. Diego says that 108 cubes, each with an edge length of $\frac{1}{3}$ in., are needed to fill a rectangular prism that is 3 in. by 1 in. by $1\frac{1}{3}$ in. Do you agree or disagree with Diego? Show or explain your thinking.

   I agree; Sample responses:

   - **I agree because the volume of the rectangular prism is 4 in^3. The volume of each cube is $\frac{1}{27}$ in^3 and $4 \div \frac{1}{27} = 4 \cdot \frac{27}{1} = 108$.**
   - **I agree because $3 \div \frac{1}{3} = 3 \cdot 3 = 9$, $1 \div \frac{1}{3} = 1 \cdot 3 = 3$, and $1\frac{1}{3} \div \frac{1}{3} = \frac{4}{3} \div \frac{1}{3} = \frac{4}{3} \cdot 3 = 4$. So, $9 \cdot 3 \cdot 4 = 108$.**

2. Lin and Noah are packing small cubes into a larger cube with an edge length of $1\frac{1}{2}$ in. Lin is using cubes with an edge length of $\frac{1}{2}$ in., and Noah is using cubes with an edge length of $\frac{1}{4}$ in.

   a. Will Lin and Noah need the same number of cubes? If not, who would need more? Show or explain your thinking.

   Noah would need more cubes; Sample response: The $\frac{1}{4}$-in. cubes cover less space. So, you need more of them.

   b. If Lin and Noah each use their small cubes to calculate the volume of the larger cube with $1\frac{1}{2}$-in. edges, will they get the same answer? Show or explain your thinking.

   Yes; Sample response: The volume of the large cube is the same regardless of the cubes used.

Are you ready for more?

1. Determine the area of a rectangle with side lengths $\frac{1}{2}$ and $\frac{2}{3}$ units.

   $\frac{2}{6}$, or $\frac{1}{3}$, square units

2. Determine the volume of a rectangular prism with side lengths $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ units.

   $\frac{1}{4}$ cubic units

3. What happens if you keep multiplying these fractions in this pattern, $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdot \frac{5}{6} \cdot \ldots$?

   The numerators and demnominators cancel each other out, so the final product will be the fraction with the numerator of the first fraction and the denominator of the last fraction.

STOP

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1 Launch

Have students use the *Think-Pair-Share* routine, allowing 7–8 minutes of independent work time before sharing strategies and responses with a partner. Consider demonstrating how to quickly sketch a cube.

2 Monitor

Help students get started by saying, "Choose a dimension and ask yourself. 'How many thirds can fit into this length?'"

Look for points of confusion:

- **Struggling to visualize and keep track of the measurements of the prisms.** Encourage students to draw and label the measurements of the boxes described.

Look for productive strategies:

- Recognizing that they need to determine the volume of the rectangular prism in Problem 1.
- Identifying why cubes with a particular fractional edge length require more/fewer cubes, and why the volume, in cubic inches, is the same regardless of the size or number of cubes used.
- Recognizing that multiplying the edge lengths of the prism is an efficient way to determine its volume.

3 Connect

Have groups of students share their different strategies for determining the volume of the prism in Problem 1, and their reasoning for Problems 2a and 2b.

Ask:

- "Could the volume of the prism in Problem 1 be measured using 1 inch-cubes? Why or why not?"
- "Can unit cubes with any unit fractional edge length be used to determine volume? Why or why not?"

Highlight that any unit fraction that shares a common denominator with the measurement of each dimension would be helpful to use in determining volume.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

For Problem 1, suggest students draw a cube with measurement labels of 3 in., 1 in., and $1\frac{1}{3}$ in. Have them then label each dimension with the number of cubes of side length $\frac{1}{3}$ in. that are needed to complete that dimension length. For example, for the side length of 1 in., three cubes of side length $\frac{1}{3}$ in.are needed to complete that dimension length.

Math Language Development

MLR1: Stronger and Clearer Each Time

Before the Connect, as time allows, have groups share their responses to Problem 1 with another group. Ask groups to review responses and provide feedback that can be used to improve the clarity of the responses. Consider displaying these sample questions:

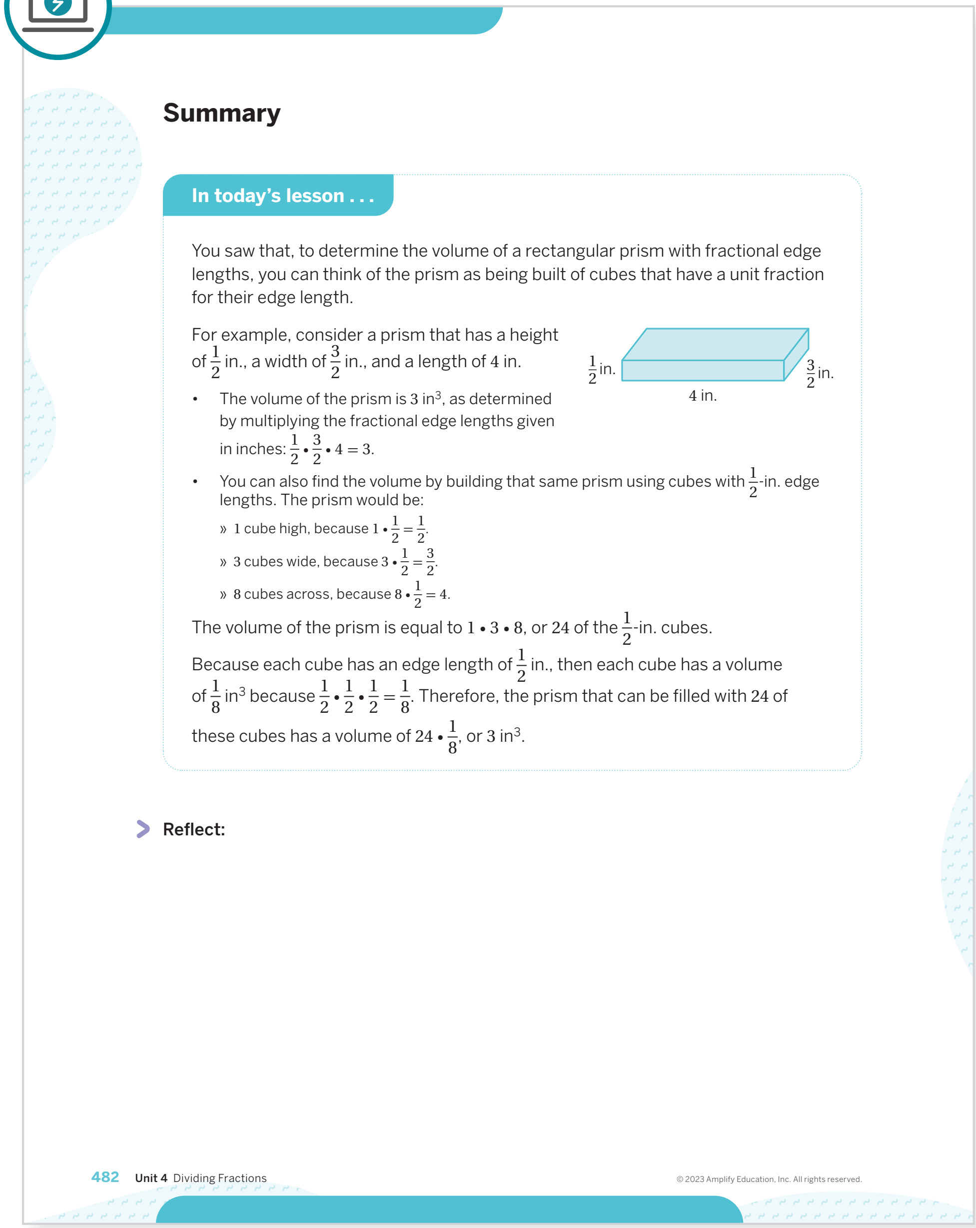
- "Does the response include an explanation, other than just agreeing or disagreeing?"
- "Does the explanation include calculations that illustrate whether 108 cubes is the correct number of cubes?"

Have students revise their responses after receiving feedback.

Whole Class | 5 min

Summary

Review and synthesize the relationship between volumes and measurements of unit cubes and prisms with fractional edge lengths, and how division of fractions can be used.



Summary

In today's lesson . . .

You saw that, to determine the volume of a rectangular prism with fractional edge lengths, you can think of the prism as being built of cubes that have a unit fraction for their edge length.

For example, consider a prism that has a height of $\frac{1}{2}$ in., a width of $\frac{3}{2}$ in., and a length of 4 in.

- The volume of the prism is 3 in^3, as determined by multiplying the fractional edge lengths given in inches: $\frac{1}{2} \cdot \frac{3}{2} \cdot 4 = 3$.
- You can also find the volume by building that same prism using cubes with $\frac{1}{2}$-in. edge lengths. The prism would be:
  - » 1 cube high, because $1 \cdot \frac{1}{2} = \frac{1}{2}$.
  - » 3 cubes wide, because $3 \cdot \frac{1}{2} = \frac{3}{2}$.
  - » 8 cubes across, because $8 \cdot \frac{1}{2} = 4$.

The volume of the prism is equal to $1 \cdot 3 \cdot 8$, or 24 of the $\frac{1}{2}$-in. cubes.

Because each cube has an edge length of $\frac{1}{2}$ in., then each cube has a volume of $\frac{1}{8}$ in^3 because $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$. Therefore, the prism that can be filled with 24 of these cubes has a volume of $24 \cdot \frac{1}{8}$, or 3 in^3.

Reflect:

Synthesize

Ask:

- "How was the process for working with fractional lengths and area in Lesson 14 similar to and different from working with fractional edge lengths and volume in this lesson?" In both cases, if the missing value is a length, you have to divide. When working with area, there are three quantities, and when working with volume, there are four quantities.
- "How was determining the volume or a missing edge length similar and different for prisms with fractional edge lengths versus whole-number edge lengths?" The formula is the same, and simply the types of numbers are different, which means the algorithms for calculations may be slightly different.

Highlight that what students know about fractions and operations can help determine the volume of rectangular prisms when the edge lengths are not whole numbers. Similar to area formulas, the same volume formulas can be used for two types of problems: determining volume given edge lengths (or height and the area of the base), or determining a missing edge length given volume and the other edge lengths (or the area of the base). Thinking about filling a volume by using unit fraction cubes rather than 1-unit cubes is just changing the "unit," similar to measuring a length in different units, such as inches or centimeters.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

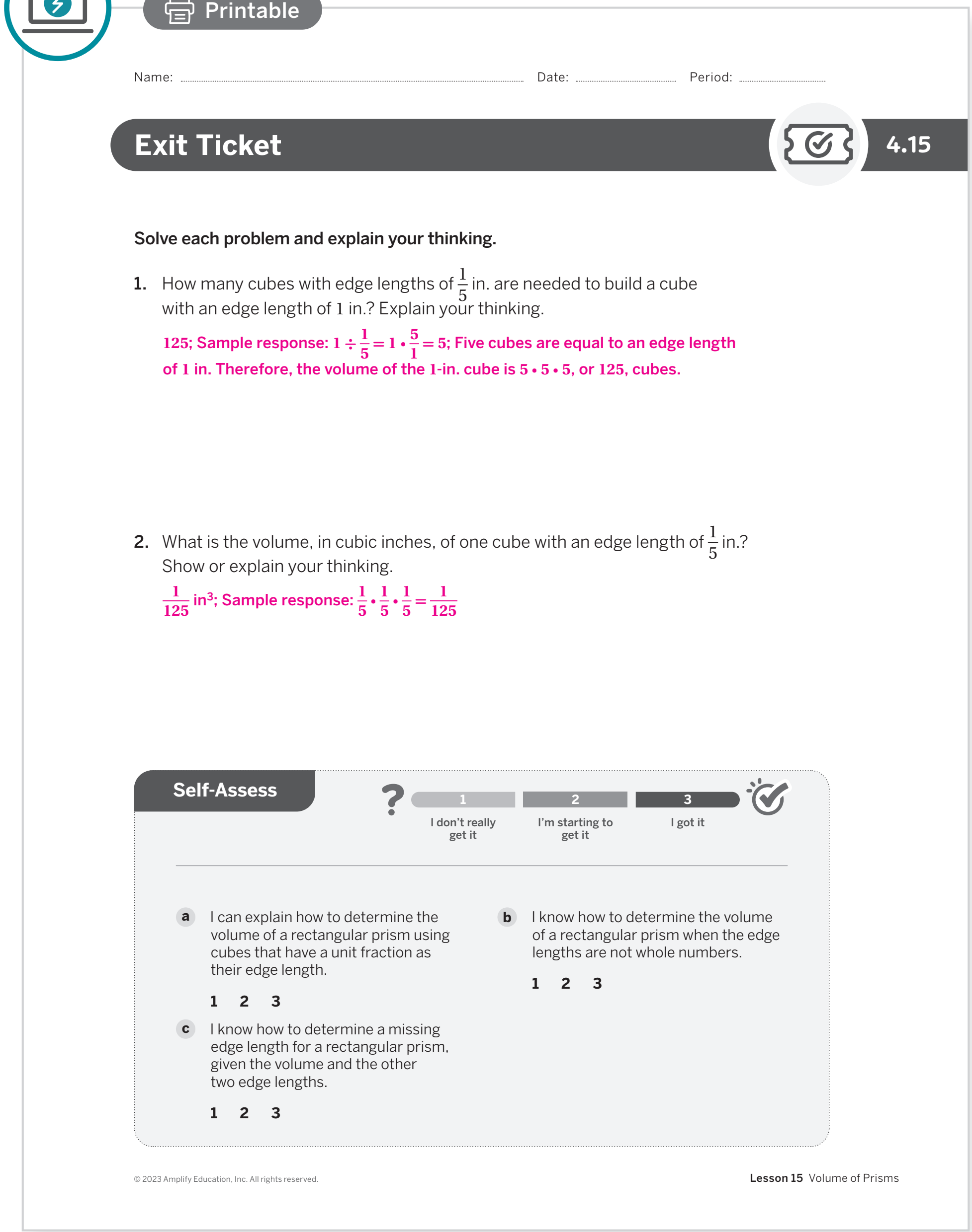
- "Were there certain parts of calculating a volume or an unknown length that you found challenging or were prone to making mistakes? If so, which parts?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of volume for rectangular prisms and unit cubes with fractional edge lengths.

Printable



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Exit Ticket 4.15

Solve each problem and explain your thinking.

1. How many cubes with edge lengths of $\frac{1}{5}$ in. are needed to build a cube with an edge length of 1 in.? Explain your thinking.

125; Sample response: $1 \div \frac{1}{5} = 1 \cdot \frac{5}{1} = 5$; Five cubes are equal to an edge length of 1 in. Therefore, the volume of the 1-in. cube is $5 \cdot 5 \cdot 5$, or 125, cubes.

2. What is the volume, in cubic inches, of one cube with an edge length of $\frac{1}{5}$ in.? Show or explain your thinking.

$\frac{1}{125}$ in^3; Sample response: $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{125}$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain how to determine the volume of a rectangular prism using cubes that have a unit fraction as their edge length.

1 2 3

b I know how to determine the volume of a rectangular prism when the edge lengths are not whole numbers.

1 2 3

c I know how to determine a missing edge length for a rectangular prism, given the volume and the other two edge lengths.

1 2 3

Success looks like . . .

- **Language Goal:** Determining how to solve a problem involving the volume of a rectangular prism and fractional edge lengths. **(Speaking and Listening, Writing)**
  - » Explaining how many cubes of $\frac{1}{5}$ in. are need to build a cube of 1 in. in Problem 1.
- **Goal:** Generalizing that the volume of a rectangular prism with fractional edge lengths can be found by multiplying the edge lengths.
  - » Determining the volume of a cube with edge length of $\frac{1}{5}$ in. by multiplying $\frac{1}{5}$ three times in Problem 2.
- **Goal:** Generalizing that it takes more smaller cubes or fewer larger cubes to fill the same volume.

Suggested next steps

If students have difficulty determining how many $\frac{1}{5}$-in. cubes fill the 1-inch prism, consider:

- Having them review the Warm-up.

If students have difficulty calculating the volume of the $\frac{1}{5}$-in. cube, consider:

- Referring them to the Summary example of the $\frac{1}{2}$-in. cubes and remind them of the formulas $V = \ell \cdot w \cdot h$ and $V = B \cdot h$.

Professional Learning

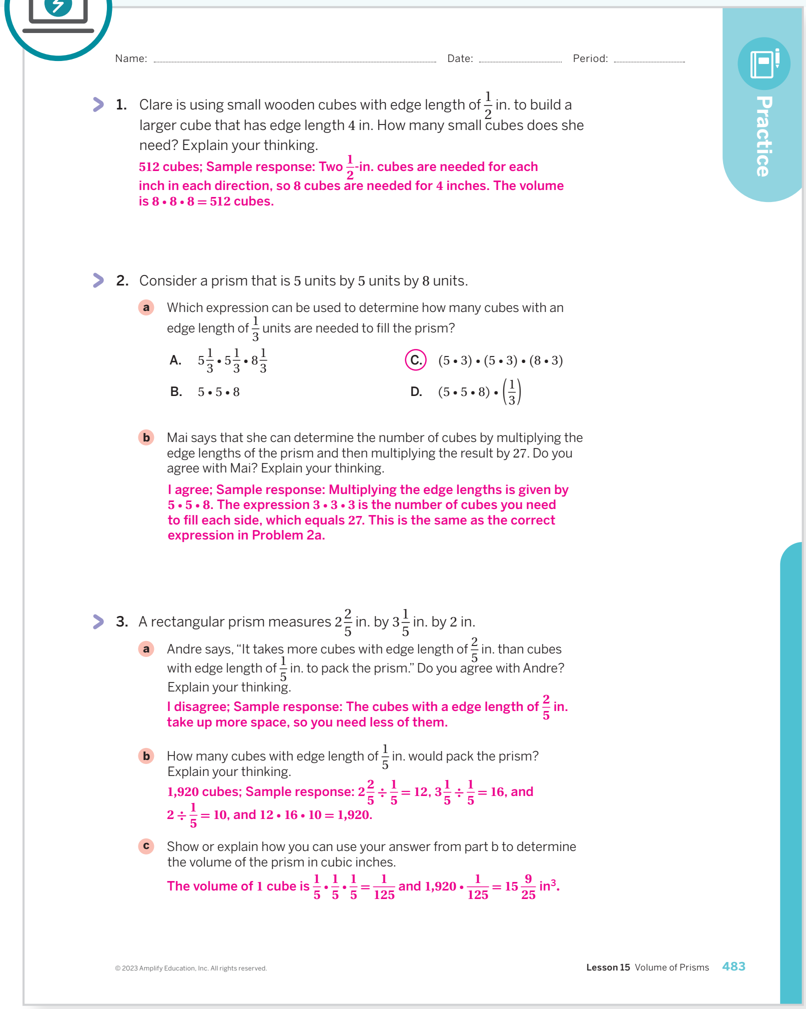
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?
- During the Activity 1 discussion, how did you encourage each student to share their understandings? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Clare is using small wooden cubes with edge length of $\frac{1}{2}$ in. to build a larger cube that has edge length 4 in. How many small cubes does she need? Explain your thinking.

512 cubes; Sample response: Two $\frac{1}{2}$-in. cubes are needed for each inch in each direction, so 8 cubes are needed for 4 inches. The volume is $8 \cdot 8 \cdot 8 = 512$ cubes.

2. Consider a prism that is 5 units by 5 units by 8 units.

   a. Which expression can be used to determine how many cubes with an edge length of $\frac{1}{3}$ units are needed to fill the prism?

   A. $5\frac{1}{3} \cdot 5\frac{1}{3} \cdot 8\frac{1}{3}$

   [✓ C.] $(5 \cdot 3) \cdot (5 \cdot 3) \cdot (8 \cdot 3)$

   B. $5 \cdot 5 \cdot 8$

   D. $(5 \cdot 5 \cdot 8) \cdot \left(\frac{1}{3}\right)$

   b. Mai says that she can determine the number of cubes by multiplying the edge lengths of the prism and then multiplying the result by 27. Do you agree with Mai? Explain your thinking.

   I agree; Sample response: Multiplying the edge lengths is given by $5 \cdot 5 \cdot 8$. The expression $3 \cdot 3 \cdot 3$ is the number of cubes you need to fill each side, which equals 27. This is the same as the correct expression in Problem 2a.

3. A rectangular prism measures $2\frac{2}{5}$ in. by $3\frac{1}{5}$ in. by 2 in.

   a. Andre says, "It takes more cubes with edge length of $\frac{2}{5}$ in. than cubes with edge length of $\frac{1}{5}$ in. to pack the prism." Do you agree with Andre? Explain your thinking.

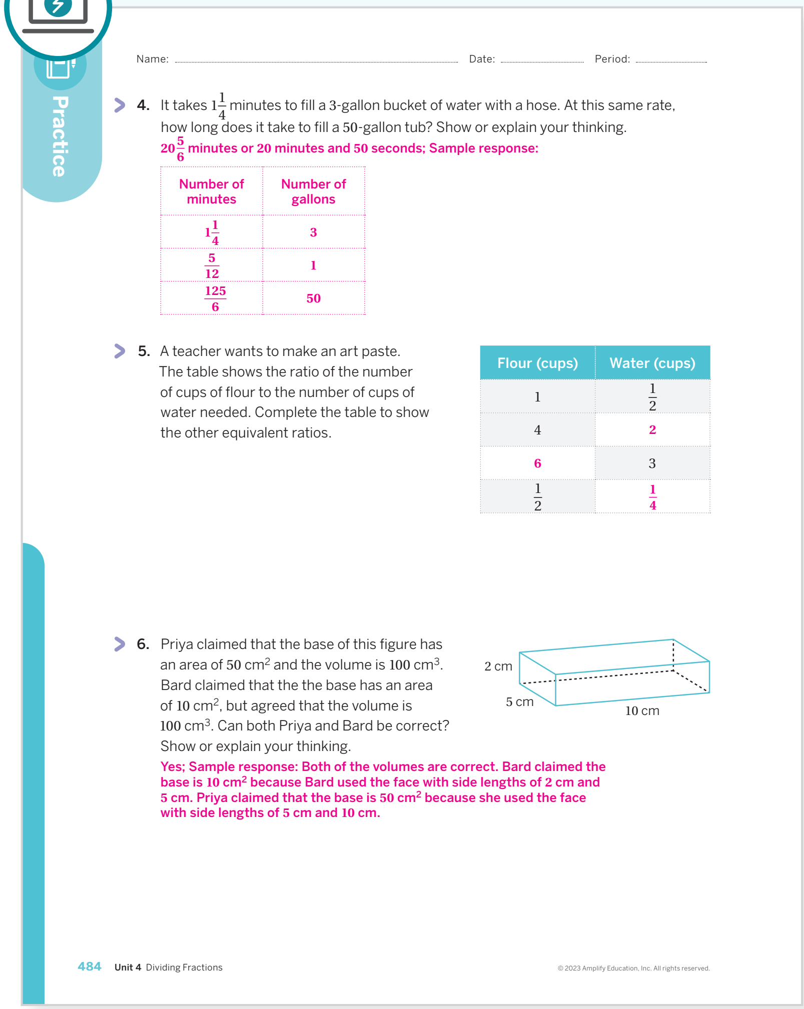
   I disagree; Sample response: The cubes with a edge length of $\frac{2}{5}$ in. take up more space, so you need less of them.

   b. How many cubes with edge length of $\frac{1}{5}$ in. would pack the prism? Explain your thinking.

   1,920 cubes; Sample response: $2\frac{2}{5} \div \frac{1}{5} = 12$, $3\frac{1}{5} \div \frac{1}{5} = 16$, and $2 \div \frac{1}{5} = 10$, and $12 \cdot 16 \cdot 10 = 1{,}920$.

   c. Show or explain how you can use your answer from part b to determine the volume of the prism in cubic inches.

   The volume of 1 cube is $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{125}$ and $1{,}920 \cdot \frac{1}{125} = 15\frac{9}{25}$ in^3.



Name: Date: Period:

Practice

4. It takes $1\frac{1}{4}$ minutes to fill a 3-gallon bucket of water with a hose. At this same rate, how long does it take to fill a 50-gallon tub? Show or explain your thinking.

$20\frac{5}{6}$ minutes or 20 minutes and 50 seconds; Sample response:

Number of minutes	Number of gallons
$1\frac{1}{4}$	3
$\frac{5}{12}$	1
$\frac{125}{6}$	50

5. A teacher wants to make an art paste. The table shows the ratio of the number of cups of flour to the number of cups of water needed. Complete the table to show the other equivalent ratios.

Flour (cups)	Water (cups)
1	$\frac{1}{2}$
4	**2**
6	3
$\frac{1}{2}$	**$\frac{1}{4}$**

6. Priya claimed that the base of this figure has an area of 50 cm^2 and the volume is 100 cm^3. Bard claimed that the the base has an area of 10 cm^2, but agreed that the volume is 100 cm^3. Can both Priya and Bard be correct? Show or explain your thinking.

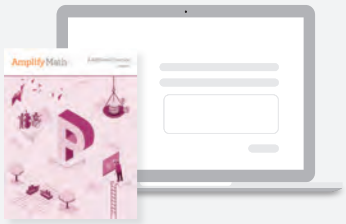
Yes; Sample response: Both of the volumes are correct. Bard claimed the base is 10 cm^2 because Bard used the face with side lengths of 2 cm and 5 cm. Priya claimed that the base is 50 cm^2 because she used the face with side lengths of 5 cm and 10 cm.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 14	2
	5	Unit 4 Lesson 13	2
Formative ⚡	6	Unit 4 Lesson 16	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

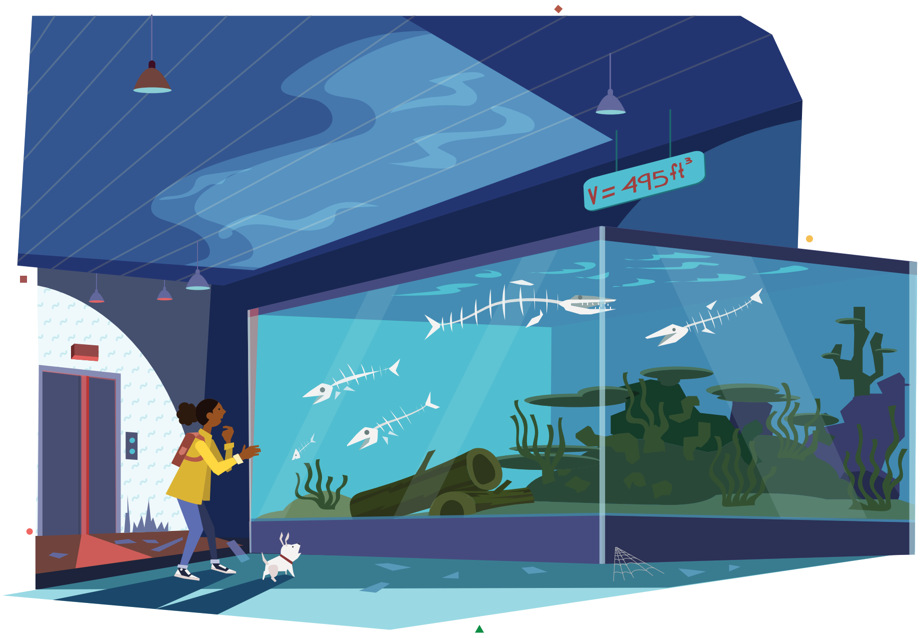


For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

UNIT 4 | **LESSON 16**

Fish Tanks Inside of Fish Tanks

Let’s look at the volume of some more prisms with fractional measurements.



Focus

Goals

1. Apply dividing by fractions to calculate one edge length of a rectangular prism, given its volume and the other two edge lengths.
2. **Language Goal:** Explain, using multiple representations, how to solve a problem involving the volume of a rectangular prism with fractional edge lengths. **(Speaking and Listening, Writing)**

Coherence

Today

Students apply what they have learned throughout the unit, specifically their understanding about volume of rectangular prisms with fractional edge lengths, in a hypothetical, but real-world context. They consider filling fish tanks with water from other fish tanks *and* fitting actual fish tanks inside of another fish tank. Students construct viable arguments about how problems should be solved, distinguishing from the context about whether they need to compare volumes or dimensions.
Note: This lesson has the last “clue” for the Capstone activity.

Previously

In Lessons 13–15, students used division with fractions for length, area, and volume problems. In Lesson 15, they specifically focused on the idea of filling prisms with cubes.

Coming Soon

In Lesson 17, the capstone of the unit, students will divide fractions to bring a conclusion to the fictional narrative that has surrounded this unit, helping Maya make her way to the bus that will take her home from Spöklik.

Rigor

- Students **apply** their understanding of dividing fractions to problems with volume.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

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For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- calculators

Math Language Development

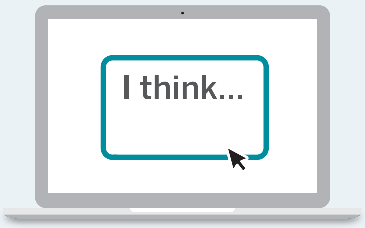
Review words

- *volume*
- *cubed*

Amps | Featured Activity

Activity 1
See Student Thinking

Students show their thinking, and you can see their step-by-step thinking to identify where misunderstandings might occur.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students might allow their negative self-talk to discourage them before they begin Activity 2. Ask guiding questions that will help students start building their personal explanations of the problem and looking for entry points to its solution. Help students recognize that they can find ways to use their strengths to start tackling a problem, even when, at first, it might seem impossible.

Modifications to Pacing

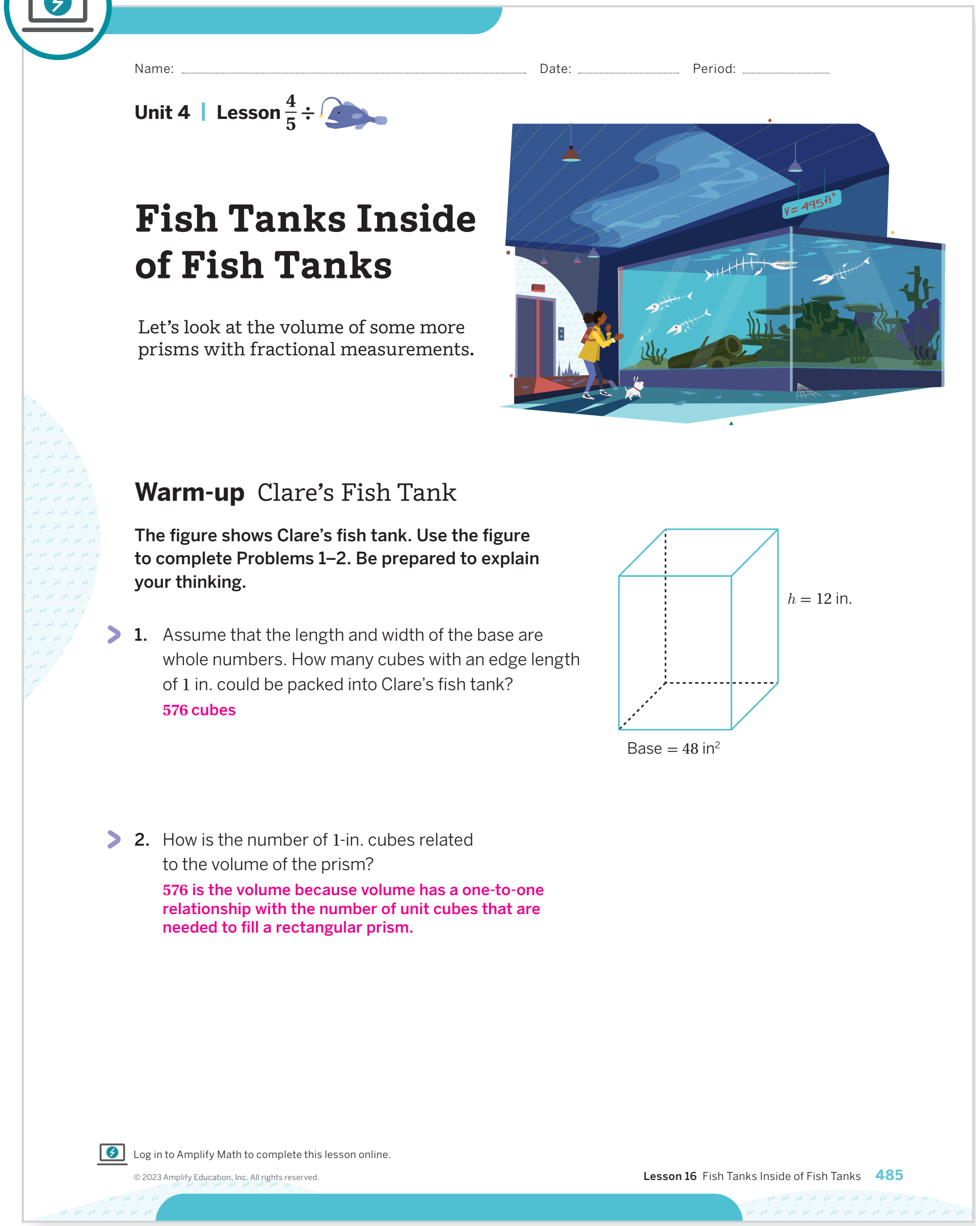
You may want to consider this additional modifications if you are short on time.

- **Activity 2** may be omitted.

Independent | 5 min

Warm-up Clare's Fish Tank

Students determine the number of cubes needed to fill a fish tank to reinforce the idea of using unit cubes and fractional-unit cubes to measure the volume of a rectangular prism.



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Unit 4 | Lesson $\frac{4}{5} \div$

Fish Tanks Inside of Fish Tanks

Let's look at the volume of some more prisms with fractional measurements.

Warm-up Clare's Fish Tank

The figure shows Clare's fish tank. Use the figure to complete Problems 1–2. Be prepared to explain your thinking.

1. Assume that the length and width of the base are whole numbers. How many cubes with an edge length of 1 in. could be packed into Clare's fish tank?
   576 cubes

2. How is the number of 1-in. cubes related to the volume of the prism?
   576 is the volume because volume has a one-to-one relationship with the number of unit cubes that are needed to fill a rectangular prism.

Log in to Amplify Math to complete this lesson online.

1 Launch

Activate prior knowledge by saying, "In the previous lesson, you saw how cubes could fit into a prism. You will be doing something similar in this lesson, but on a larger scale."

2 Monitor

Help students get started by asking, "Problem 1 is asking about how many unit cubes you can put *inside* the tank. What is it asking mathematically?" the volume of the tank

Look for points of confusion:

- **Thinking a measurement is missing (i.e., not understanding what the base measurement means).** Ask, "What does the base measurement tell you? What could the length and the width be?"

Look for productive strategies:

- Connecting the 1-in. cube dimensions to being the same as the volume.
- Recognizing that the base is made up of the length and the width. In this example, volume is represented by the equation $V = B \cdot h$, where b is the area of the base.

3 Connect

Have individual students share their responses, focusing on the connection of 1-in. cubes to volume.

Ask:

- "What could be some possible measurements in inches of the length and width?" 8 and 6, 2 and 24, 4 and 12
- "Which of these looks like it would make the most sense, based on the figure?" 8 and 6
- "If the cubes had an edge length of 2 in., would you need more or fewer cubes to fill the tank? What if they had an edge length of $\frac{1}{2}$ in.?"

Highlight that the number of 1-in. cubes is the same as the volume, which is written as 576 in^3.

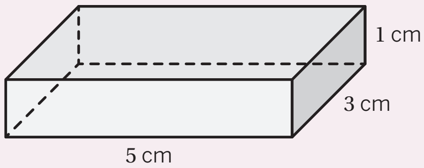
Power-up

To power up students' ability to identify the base as part of the volume calculations using the formula $V = B \cdot h$, have students complete:

Recall that the base of a prism is either of the two identical, parallel faces of a prism that are connected by a set of rectangular faces.

A rectangular prism is unique, in that it is a prism that has three pairs of bases. Identify the dimensions of each pair of bases of the given prism.

5 cm by 3 cm
5 cm by 1 cm
1 cm by 3 cm



Use: Before the Warm-up.

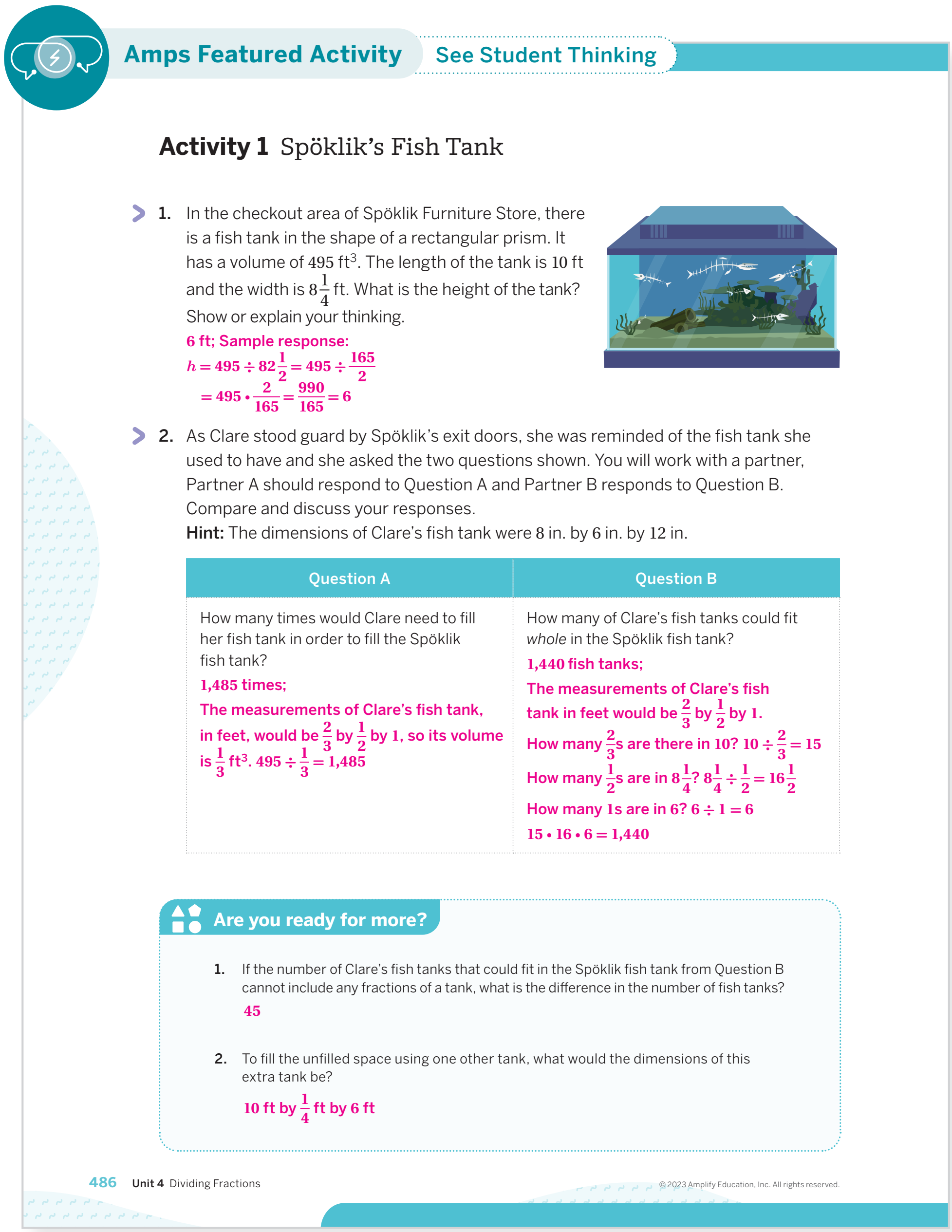
Informed by: Performance on Lesson 15, Practice Problem 6.

Pairs | 15 min

Activity 1 Spöklik's Fish Tank

Students solve problems involving the volume of two prisms, which serves as a bridge from the Warm-up to Activity 2.

Amps Featured Activity See Student Thinking



Activity 1 Spöklik's Fish Tank

1. In the checkout area of Spöklik Furniture Store, there is a fish tank in the shape of a rectangular prism. It has a volume of 495 ft^3. The length of the tank is 10 ft and the width is $8\frac{1}{4}$ ft. What is the height of the tank? Show or explain your thinking.

6 ft; Sample response:

$$h = 495 \div 82\frac{1}{2} = 495 \div \frac{165}{2}$$
$$= 495 \cdot \frac{2}{165} = \frac{990}{165} = 6$$

2. As Clare stood guard by Spöklik's exit doors, she was reminded of the fish tank she used to have and she asked the two questions shown. You will work with a partner, Partner A should respond to Question A and Partner B responds to Question B. Compare and discuss your responses.
Hint: The dimensions of Clare's fish tank were 8 in. by 6 in. by 12 in.

Question A	Question B
How many times would Clare need to fill her fish tank in order to fill the Spöklik fish tank? **1,485 times;** **The measurements of Clare's fish tank, in feet, would be $\frac{2}{3}$ by $\frac{1}{2}$ by 1, so its volume is $\frac{1}{3}$ ft^3. $495 \div \frac{1}{3} = 1{,}485$**	How many of Clare's fish tanks could fit *whole* in the Spöklik fish tank? **1,440 fish tanks;** **The measurements of Clare's fish tank in feet would be $\frac{2}{3}$ by $\frac{1}{2}$ by 1.** **How many $\frac{2}{3}$s are there in 10? $10 \div \frac{2}{3} = 15$** **How many $\frac{1}{2}$s are in $8\frac{1}{4}$? $8\frac{1}{4} \div \frac{1}{2} = 16\frac{1}{2}$** **How many 1s are in 6? $6 \div 1 = 6$** **$15 \cdot 16 \cdot 6 = 1{,}440$**

Are you ready for more?

1. If the number of Clare's fish tanks that could fit in the Spöklik fish tank from Question B cannot include any fractions of a tank, what is the difference in the number of fish tanks?

45

2. To fill the unfilled space using one other tank, what would the dimensions of this extra tank be?

10 ft by $\frac{1}{4}$ ft by 6 ft

486 **Unit 4** Dividing Fractions

1 Launch

Activate background knowledge by asking, "Have you ever seen a fish tank that is larger than what you would expect to have in a home? Maybe at a hotel, a mall, or a zoo?"

2 Monitor

Help students get started by having students draw a model of the Spöklik fish tank and ask, "How many of Clare's fish tanks could you fit in the width of the Spöklik fish tank?" 6

Look for points of confusion:

- **Not converting the units for Problem 2.** Ask, "Can you compare the measurements directly if one is in inches and one is in feet?"

Look for productive strategies:

- Distinguishing between *filling* the prism and *fitting* other prisms/cubes inside, and describing the effect that has on both the strategies used and the solutions.
- Connecting edge lengths in Question B in terms of "how many of the smaller tank's [widths] can you fit in the larger tank's [width]?"

3 Connect

Have groups of students share different representations used to solve the problems.

Ask, "What is the difference between *filling* a prism and *fitting* prisms inside a larger prism?"

Highlight that the two questions in Problem 2 are really asking for two different ways of seeing the volume of prisms. One is asking to connect the fluid volume of the water inside the tanks, whereas the other is connecting the solid dimensions of the prisms. These should always result in the same total measurement of volume, unless the context dictates otherwise (i.e., the prisms being fit inside do not go evenly along one or more dimensions).

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display two fish tank diagrams, either with the given dimensions pre-labeled, or ask students to label the given dimensions. Consider drawing the diagrams to scale as much as possible, to help students visualize that Clare's fish tank is much smaller than the fish tank at the store. Point out how the measurements for Clare's fish tank are given in inches.

Math Language Development

MLR6: Three Reads — Revoicing

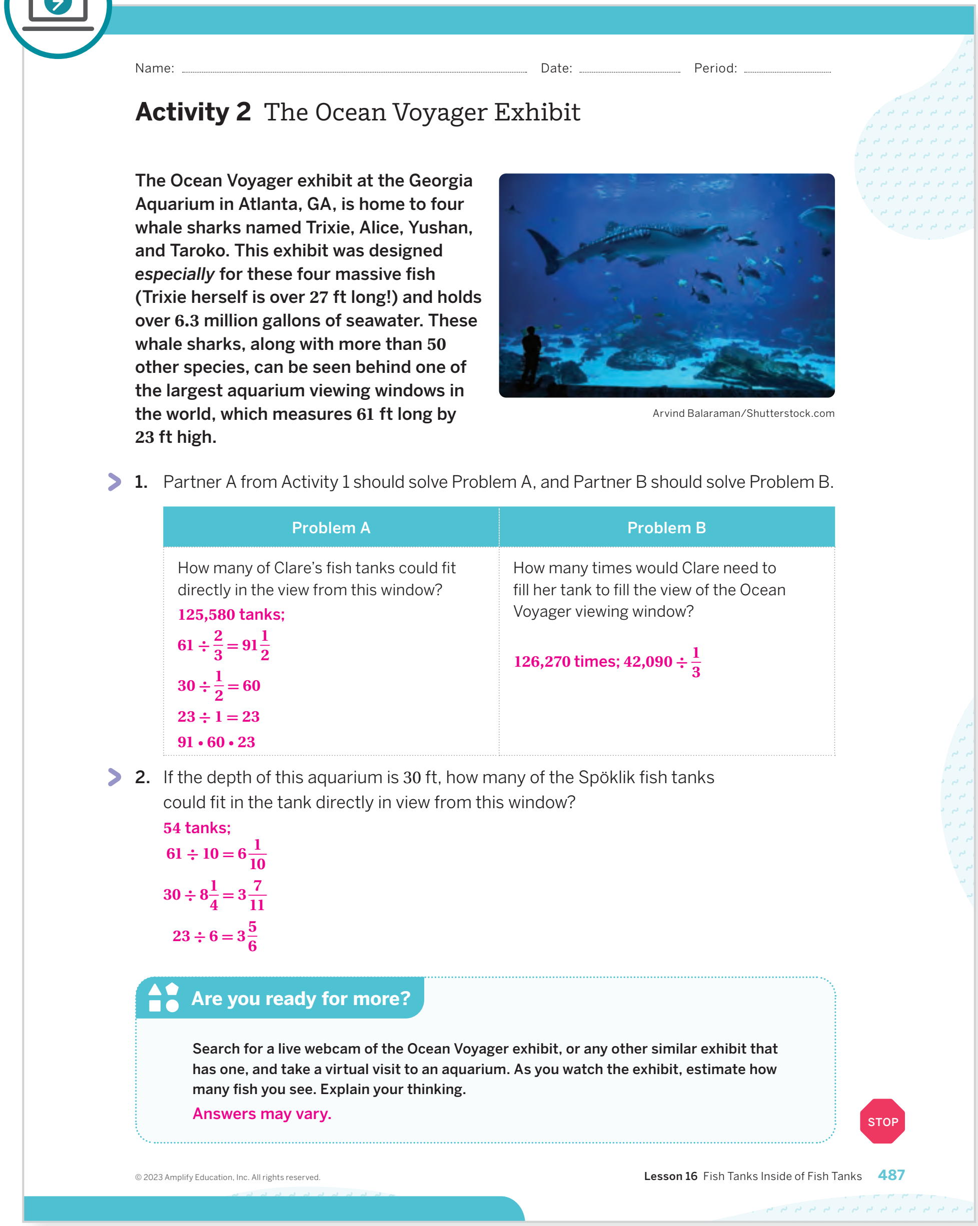
Use this routine to help students make sense of the text in Problems 1 and 2. Have students pre-read Problems 1 and 2 before completing each problem.

- **Read 1:** Students should understand that there are two fish tanks, the one in the checkout area of the store and Clare's fish tank that she used to have. The fish tanks have different dimensions.
- **Read 2:** Ask students to name the given quantities and relationships, such as the given dimensions or volume of the two fish tanks.
- **Read 3:** Ask students to brainstorm what is meant by the two different words used in Questions A and B: *fill* vs. *fit* and how these words might indicate what strategies they could use.

Pairs | 15 min

Activity 2 The Ocean Voyager Exhibit

Students solve more complex problems relating to fish tanks that are prisms of different sizes, given some information about their volumes and dimensions.



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Activity 2 The Ocean Voyager Exhibit

The Ocean Voyager exhibit at the Georgia Aquarium in Atlanta, GA, is home to four whale sharks named Trixie, Alice, Yushan, and Taroko. This exhibit was designed *especially* for these four massive fish (Trixie herself is over 27 ft long!) and holds over 6.3 million gallons of seawater. These whale sharks, along with more than 50 other species, can be seen behind one of the largest aquarium viewing windows in the world, which measures 61 ft long by 23 ft high.

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1. Partner A from Activity 1 should solve Problem A, and Partner B should solve Problem B.

Problem A	Problem B
How many of Clare's fish tanks could fit directly in the view from this window? 125,580 tanks; $61 \div \frac{2}{3} = 91\frac{1}{2}$ $30 \div \frac{1}{2} = 60$ $23 \div 1 = 23$ $91 \cdot 60 \cdot 23$	How many times would Clare need to fill her tank to fill the view of the Ocean Voyager viewing window? 126,270 times; $42{,}090 \div \frac{1}{3}$

2. If the depth of this aquarium is 30 ft, how many of the Spöklik fish tanks could fit in the tank directly in view from this window?

54 tanks;

$61 \div 10 = 6\frac{1}{10}$

$30 \div 8\frac{1}{4} = 3\frac{7}{11}$

$23 \div 6 = 3\frac{5}{6}$

Are you ready for more?

Search for a live webcam of the Ocean Voyager exhibit, or any other similar exhibit that has one, and take a virtual visit to an aquarium. As you watch the exhibit, estimate how many fish you see. Explain your thinking.

Answers may vary.

STOP

1 Launch

Read the introduction about The Ocean Voyager. Ensure students understand that Partner A and Partner B should not switch roles. Consider providing calculators.

2 Monitor

Help students get started by helping students visualize the size of the viewing window. Consider using a classroom wall to make a comparison.

Look for points of confusion:

- **Having trouble visualizing the sizes of the fish tanks.** Have students draw models of all three, with labels for all of the dimensions.
- **Thinking Problem 2 can also be solved using the formula $V = \ell \cdot w \cdot h$.** Remind students there is a difference between filling and fitting (from Activity 1), and ask, "Which applies here?"

Look for productive strategies:

- Using previous work from the Warm-up and Activity 1 (i.e., the volumes of the first two fish tanks).
- Using ratios to relate the corresponding dimensions. While this is not expected, it would show that students have a solid grasp of the multiplicative nature of ratios, fractions, and volume alike.

3 Connect

Have groups of students share their responses, focusing on how they determined which strategy to use to solve Problem 2 (relating volume or dimensions; *fill* vs. *fit*), and also explicitly stating when division was used.

Highlight how visualizing the filling and fitting can help determine the most appropriate strategy for solving these types of volume-related problems.

Differentiated Support

Accessibility: *Activate Background Knowledge, Guide Processing and Visualization*

Consider showing students images of the Georgia Aquarium to help them gain perspective on its size. If you drew (or had students draw) diagrams of Clare's fish tank from Activity 1, keep this diagram displayed throughout this activity for students to reference.

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing students with friendlier values for the viewing window of the aquarium, such as 60 ft long by 33 ft wide (depth) by 24 ft high.

Math Language Development

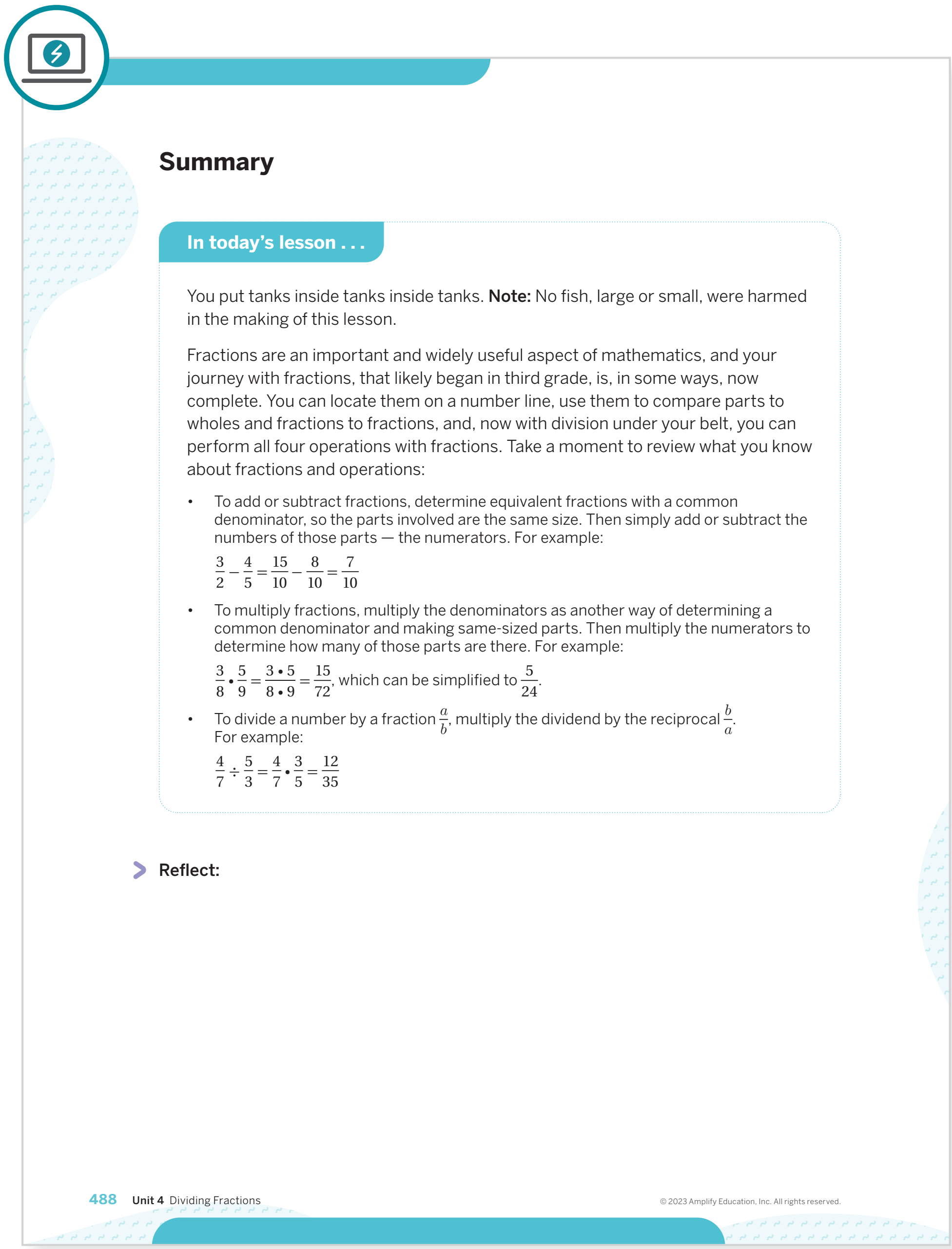
MLR8: Discussion Supports — Revoicing

During the Connect, as students share their responses and which strategies they used, ask them to restate and/or revoice a classmate's strategy in their own words. Consider first providing students time to restate what they hear with a partner, before sharing with the class. This will provide additional opportunities for all students to listen to and produce language describing strategies for determining the volume of rectangular prisms.

Whole Class | 5 min

Summary

Review and synthesize how much students have learned about fractions over many years, and how they have already seen where fractions may be useful in ratios and geometry.



Summary

In today's lesson . . .

You put tanks inside tanks inside tanks. **Note:** No fish, large or small, were harmed in the making of this lesson.

Fractions are an important and widely useful aspect of mathematics, and your journey with fractions, that likely began in third grade, is, in some ways, now complete. You can locate them on a number line, use them to compare parts to wholes and fractions to fractions, and, now with division under your belt, you can perform all four operations with fractions. Take a moment to review what you know about fractions and operations:

- To add or subtract fractions, determine equivalent fractions with a common denominator, so the parts involved are the same size. Then simply add or subtract the numbers of those parts — the numerators. For example:
  $\frac{3}{2} - \frac{4}{5} = \frac{15}{10} - \frac{8}{10} = \frac{7}{10}$
- To multiply fractions, multiply the denominators as another way of determining a common denominator and making same-sized parts. Then multiply the numerators to determine how many of those parts are there. For example:
  $\frac{3}{8} \cdot \frac{5}{9} = \frac{3 \cdot 5}{8 \cdot 9} = \frac{15}{72}$, which can be simplified to $\frac{5}{24}$.
- To divide a number by a fraction $\frac{a}{b}$, multiply the dividend by the reciprocal $\frac{b}{a}$. For example:
  $\frac{4}{7} \div \frac{5}{3} = \frac{4}{7} \cdot \frac{3}{5} = \frac{12}{35}$

Reflect:

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Synthesize

Have students share a memory they have of learning about fractions, from any grade. Focus on the positive memories of individual students and how they contributed to the learning of this unit. For those who share negative memories, encourage them to also consider how much they have learned and grown since.

Highlight that students are now equipped to perform all four operations with fractions, in both mathematical and real-world problems, which is timely — for Maya's sake — as one last challenge remains to help her find the bus home from Spöklik in the next and final lesson of the unit.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

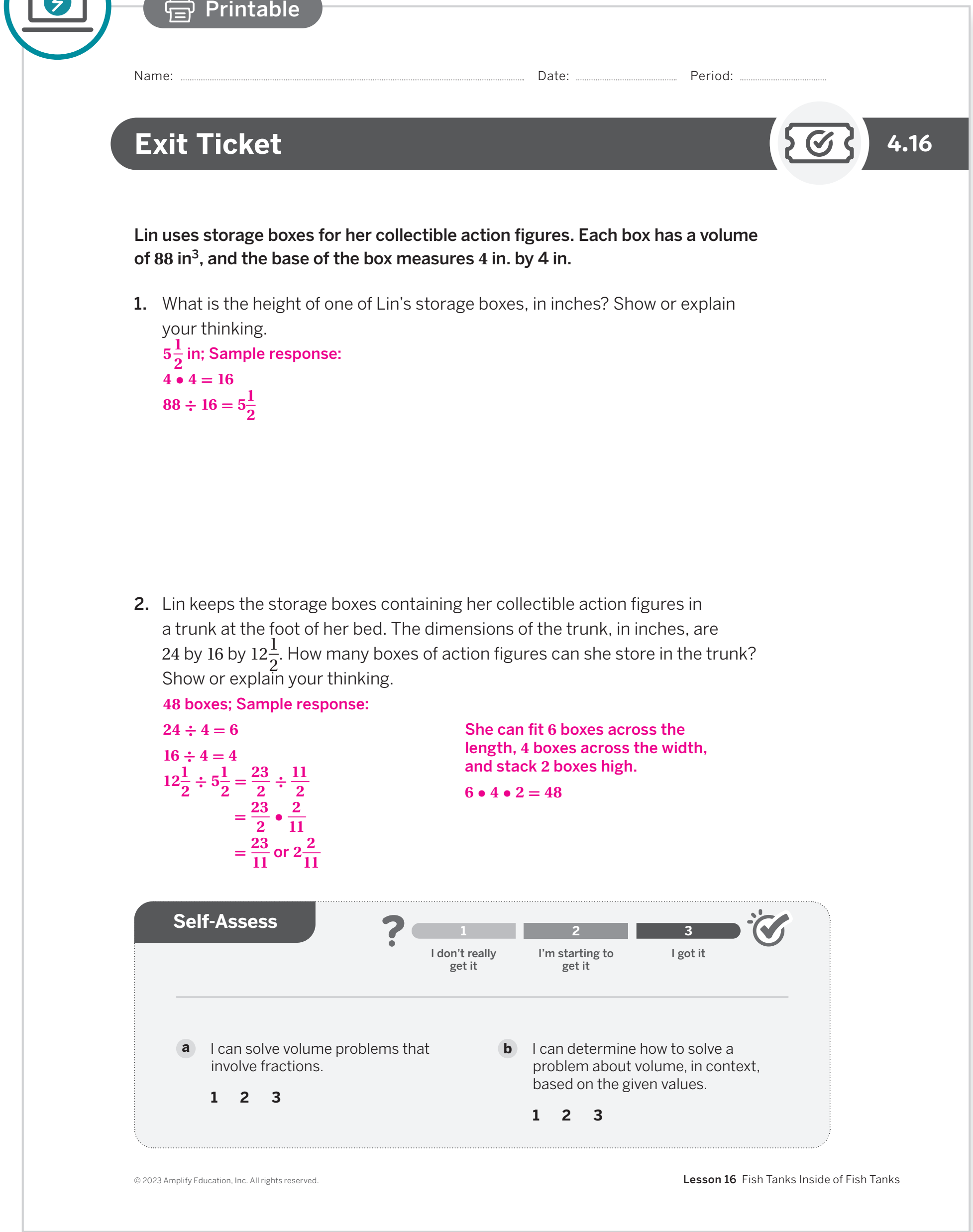
- "How many tanks could a fish tank fill if a fish tank could fit tanks?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using division to calculate a missing dimension and to determine the number of rectangular prisms that fit inside a larger rectangular prism.

Printable



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Exit Ticket

4.16

Lin uses storage boxes for her collectible action figures. Each box has a volume of 88 in^3, and the base of the box measures 4 in. by 4 in.

1. What is the height of one of Lin's storage boxes, in inches? Show or explain your thinking.
   $5\frac{1}{2}$ in; Sample response:
   $4 \bullet 4 = 16$
   $88 \div 16 = 5\frac{1}{2}$

2. Lin keeps the storage boxes containing her collectible action figures in a trunk at the foot of her bed. The dimensions of the trunk, in inches, are 24 by 16 by $12\frac{1}{2}$. How many boxes of action figures can she store in the trunk? Show or explain your thinking.
   48 boxes; Sample response:
   $24 \div 4 = 6$
   $16 \div 4 = 4$
   $$12\frac{1}{2} \div 5\frac{1}{2} = \frac{23}{2} \div \frac{11}{2} = \frac{23}{2} \bullet \frac{2}{11} = \frac{23}{11} \text{ or } 2\frac{2}{11}$$
   She can fit 6 boxes across the length, 4 boxes across the width, and stack 2 boxes high.
   $6 \bullet 4 \bullet 2 = 48$

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a I can solve volume problems that involve fractions.
1 2 3

b I can determine how to solve a problem about volume, in context, based on the given values.
1 2 3

Success looks like . . .

- **Goal:** Applying dividing by fractions to calculate one edge length of a rectangular prism, given its volume and the other two edge lengths.
  - » Determining the height of one storage box in Problem 1.
- **Language Goal:** Explaining, using multiple representations, how to solve a problem involving the volume of a rectangular prism with fractional edge lengths. **(Speaking and Listening, Writing)**

Suggested next steps

If students cannot accurately determine the missing height, consider:

- Having students write the equation to determine volume and then write the equation substituting the variables with the known information.
- Asking, "What did you do to determine that missing value in Activity 1, Problem 1?"

If students struggle to identify the strategy, consider:

- Asking, "Is this like a filling problem or a fitting problem?"
- Referring back to Activity 2, Problem 2 and asking how they knew what strategy to use.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? This lesson asked students to apply their understanding of fractions and operations. Where in your students' work today did you see or hear evidence of them doing this?
- What surprised you as your students worked on Activity 2? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______ Date: ______ Period: ______

1. A pool in the shape of a rectangular prism is being filled with water. The length of the pool is 24 ft and its width is 15 ft. When the height of the water in the pool is $1\frac{1}{3}$ ft, what is the volume of the amount of water in the pool?
   480 ft^3

2. To give their animals essential minerals and nutrients, farmers and ranchers often have a block of salt available for their animals.


AJSTUDIO PHOTOGRAPHY/
Shutterstock.com

   a. A rancher is ordering cube-shaped blocks of salt. The edge lengths of each block are $\frac{5}{12}$ ft. Is the volume of one block greater than or less than 1 ft^3? Explain your thinking.
   It is less than 1 ft^3 ; Sample response: The volume is $\frac{125}{1728}$ ft^3, which is less than 1 ft^3. or The edge length of a cube with a volume of 1 ft^3 would be 1 ft and $\frac{5}{12} < 1$.

   b. The box that contains the blocks of salt ordered measures $1\frac{1}{4}$ ft by $1\frac{2}{3}$ ft by $\frac{5}{6}$ ft. How many cubes-shaped blocks of salt can fit in the box? Show or explain your thinking.
   24 cubes; Sample response: The volume of the box is $\frac{125}{72}$ ft^3, and $\frac{125}{72} \div \frac{125}{1,728} = 24$.

3. Before refrigerators existed, some people had blocks of ice delivered to their homes. The delivery wagon contained a storage box that was a rectangular prism. Consider such a storage box that measures $7\frac{1}{2}$ ft by 6 ft by 6 ft. If the blocks of ice stored in the box were cubes with an edge length of $1\frac{1}{2}$ ft, how many blocks of ice could fit in the storage box?
   A. 270
   B. $3\frac{3}{4}$
   [✓ C.] 80
   D. 180

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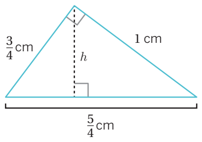
4. Consider the right triangle.



   a. What is the area of the triangle? Show your thinking.
   $A = \frac{3}{8}$ cm^2;
   Sample response: $A = \frac{1}{2} \cdot \frac{3}{4} \cdot 1$

   b. What is the height h, in centimeters, for the base that is $\frac{5}{4}$ cm long? Show your thinking.
   $h = \frac{3}{5}$ cm;
   Sample response: $\frac{3}{8} = \frac{1}{2} \cdot \frac{5}{4} \cdot h$

5. Consider a bucket that contains $11\frac{2}{3}$ gallons of water and is $\frac{5}{6}$ full.

   a. Write a multiplication equation *and* a division equation that could be used to determine how many gallons of water the bucket can hold when it is full.
   $\frac{5}{6} \cdot x = 11\frac{2}{3}$ $\qquad$ $11\frac{2}{3} \div \frac{5}{6} = x$

   b. How many gallons of water would be in the bucket when it is full?
   14 gallons

6. Priya's cat weighs $5\frac{1}{2}$ lb and her dog weighs $8\frac{1}{4}$ lb. For each part, estimate a number that would complete each comparison statement. Then find an exact solution. If any of your estimates were not close to the solution, explain why that may have happened.

   a. The cat is ____ as heavy as the dog.
   Estimate: **About $\frac{6}{8}$ or $\frac{3}{4}$**
   Explain: **Sample response: I rounded $5\frac{1}{2}$ up by $\frac{1}{2}$ and $8\frac{1}{4}$ down by $\frac{1}{4}$ so that explains the difference, but $\frac{3}{4}$ and $\frac{2}{3}$ are still pretty close.**
   Calculate: $5\frac{1}{2} \div 8\frac{1}{4}$
   $\frac{11}{2} \div \frac{33}{4}$
   $\frac{11}{2} \cdot \frac{4}{33} = \frac{2}{3}$

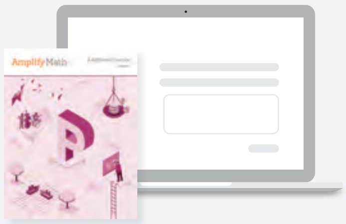
   b. The dog is ____ lb heavier than the cat.
   Estimate: **About 3 lb**
   Explain: **Sample response: The results are close.**
   Calculate: $8\frac{1}{4} - 5\frac{1}{2}$
   $7\frac{5}{4} - 5\frac{2}{4} = 2\frac{3}{4}$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 4 Lesson 14	2
	5	Unit 4 Lesson 11	2
Formative ⚡	6	Unit 4 Lesson 17	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

Now, Where Was That Bus?

Let's solve a location mystery by using fractions.

Focus

Goals

1. **Language Goal:** Apply operations with fractions to solve problems in a variety of situations, and explain the reasoning. **(Speaking and Listening, Writing)**
2. **Language Goal:** Generate an equation to represent a situation involving fractions, and justify the operation chosen. **(Speaking and Listening)**
3. **Language Goal:** Reflect upon the learning thus far and reevaluate areas of strength and areas for growth. **(Writing)**

Coherence

Today

In this Capstone lesson, students are presented with a scenario that requires them to multiply and to divide fractions with symbols inside of equations. Next, they apply their understanding of dividing fractions to decipher a numeric-based code to help Maya escape Spöklik's parking garage.

Previously

Students developed a conceptual understanding of fraction division in Lessons 2–11, and then applied this understanding to area and volume in Lessons 13–16.

Coming Soon

In Grade 7, students extend their understanding of positive fractions to rational numbers and the properties of operations.

Rigor

- Students **apply** multiplication and division of fractions to a contextual problem.
- Students efficiently, accurately, and flexibly demonstrate **fluency** with multiplying and dividing fractions.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Summary	Exit Ticket
10 min	25 min	5 min	5 min
Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

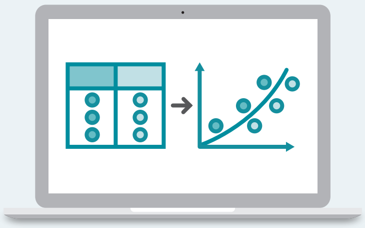
Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (optional)
- Activity 1 PDF (answers)

Amps | Featured Activity

Activity 1
Using Work From Previous Slides

Activity 1 builds from the Warm-up, and all information will carry over for ease of reference.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may not be motivated to complete Activity 1. Ask them to think about their thinking. By thinking metacognitively, they can consider what it is that is preventing them from achieving their goals. Then have them approach the problem from the opposite point of view, a more positive one. Encourage them to use the information that they have been given in this unit — values, symbols, operations — to help them get started. Remind them it is okay to change direction if something they try does not work.

Modifications to Pacing

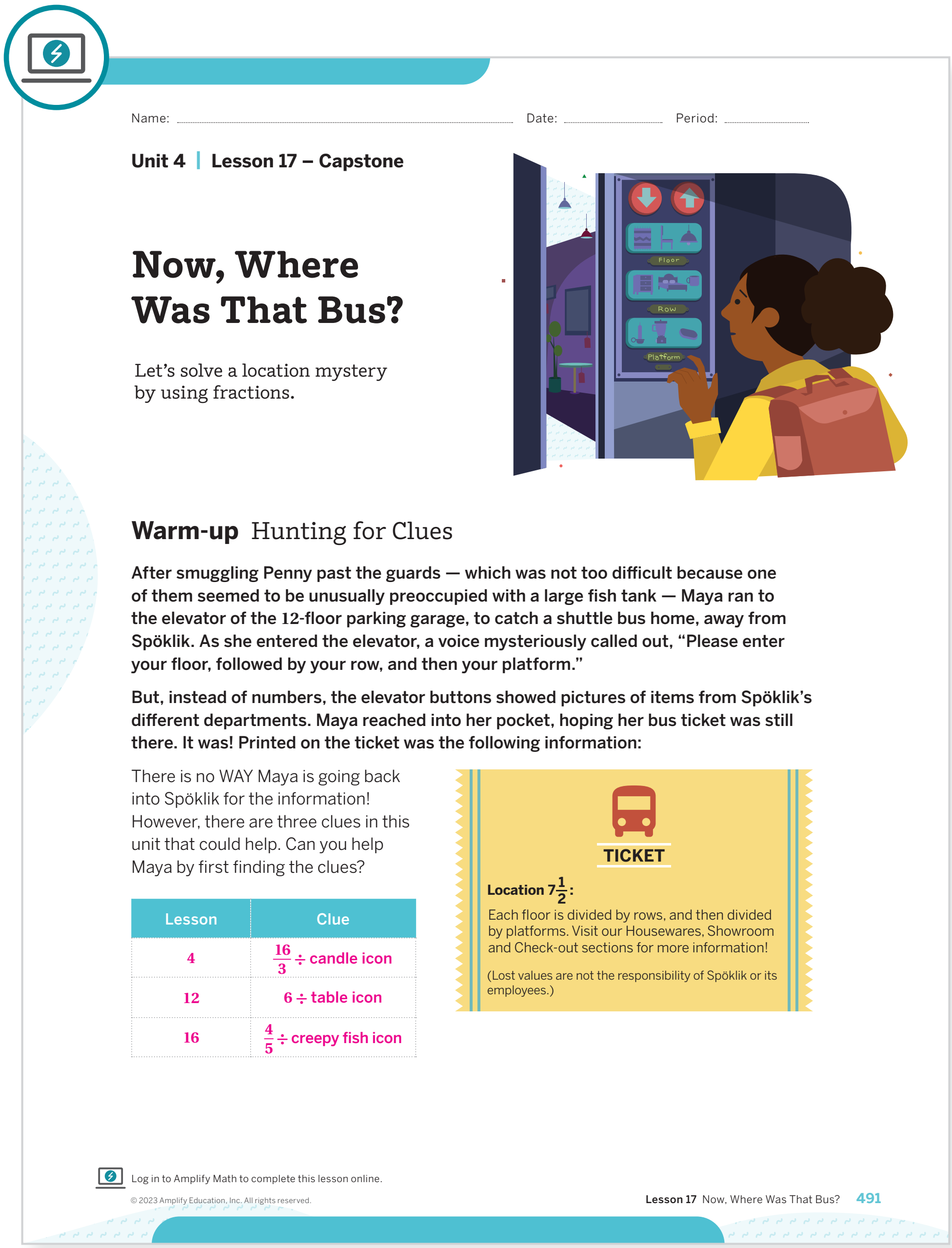
You may want to consider this additional modification if you are short on time.

- The **Warm-up** and **Activity 1** can be combined and done as a whole class. Read the Warm-up to the class, but have students record the clues in Activity 1 or in the Activity 1 PDF.

Pairs | 10 min

Warm-up Hunting for Clues

Students search for clues to be able to solve the mystery in Activity 1.



Name: ______ Date: ______ Period: ______

Unit 4 | Lesson 17 – Capstone

Now, Where Was That Bus?

Let's solve a location mystery by using fractions.

Warm-up Hunting for Clues

After smuggling Penny past the guards — which was not too difficult because one of them seemed to be unusually preoccupied with a large fish tank — Maya ran to the elevator of the 12-floor parking garage, to catch a shuttle bus home, away from Spöklik. As she entered the elevator, a voice mysteriously called out, "Please enter your floor, followed by your row, and then your platform."

But, instead of numbers, the elevator buttons showed pictures of items from Spöklik's different departments. Maya reached into her pocket, hoping her bus ticket was still there. It was! Printed on the ticket was the following information:

There is no WAY Maya is going back into Spöklik for the information! However, there are three clues in this unit that could help. Can you help Maya by first finding the clues?

TICKET

Location $7\frac{1}{2}$:

Each floor is divided by rows, and then divided by platforms. Visit our Housewares, Showroom and Check-out sections for more information!

(Lost values are not the responsibility of Spöklik or its employees.)

Lesson	Clue
4	$\frac{16}{3} \div$ candle icon
12	$6 \div$ table icon
16	$\frac{4}{5} \div$ creepy fish icon

Log in to Amplify Math to complete this lesson online.

Lesson 17 Now, Where Was That Bus? 491

1 Launch

Read the introduction aloud. Students may have noticed during earlier lessons that on the first page of three lessons, the lesson numbers were replaced with "unusual" representations: Lessons 4, 12, and 16. Encourage students to go back through their Student Edition to find the clues.

2 Monitor

Help students get started by asking "Was there anything you noticed that was a little bit different with any of the lesson numbers in this unit?"

Look for points of confusion:

- **Not writing the expressions (i.e., thinking only the symbols are the clues).** Ask, "Is the symbol the only important part of the clue?

Look for productive strategies:

- Recalling, or identifying, where the lesson numbers were written as an expression that included a symbol instead of as a number. Also possibly recognizing that the symbols represent items that could be found in the section of the store described in the corresponding Sub-Unit narrative.

3 Connect

Have individual students share the clues and where they found them.

Ask:

- "What do you need to do to turn these expressions into equations?" Set the expressions equal to a value.
- "What do you think these expressions are equal to?" The lesson numbers

Highlight that the clues will be used throughout the lesson in order to crack a code to help Maya.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text. Consider reading this text aloud to students the first time.

- **Read 1:** Students should understand there are clues hidden in this unit that will help them crack the code.
- **Read 2:** Ask students what information the bus ticket provides and what information is needed to enter in the elevator buttons.
- **Read 3:** Ask students to read the last paragraph and think about where they may have seen clues in this unit.

Power-up

To power-up students' ability to determine the operation needed for a comparison statement, have students complete:

Match each statement with the appropriate operation for comparing the two values.

_b_ A documentary on orangutans lasts 30 min and a podcast on chimpanzees lasts 42 min. How much longer is the podcast than the documentary?

_a_ A documentary on orangutans lasts 30 min and a podcast on chimpanzees lasts 42 min. How many times longer is the podcast than the documentary?

a. Division

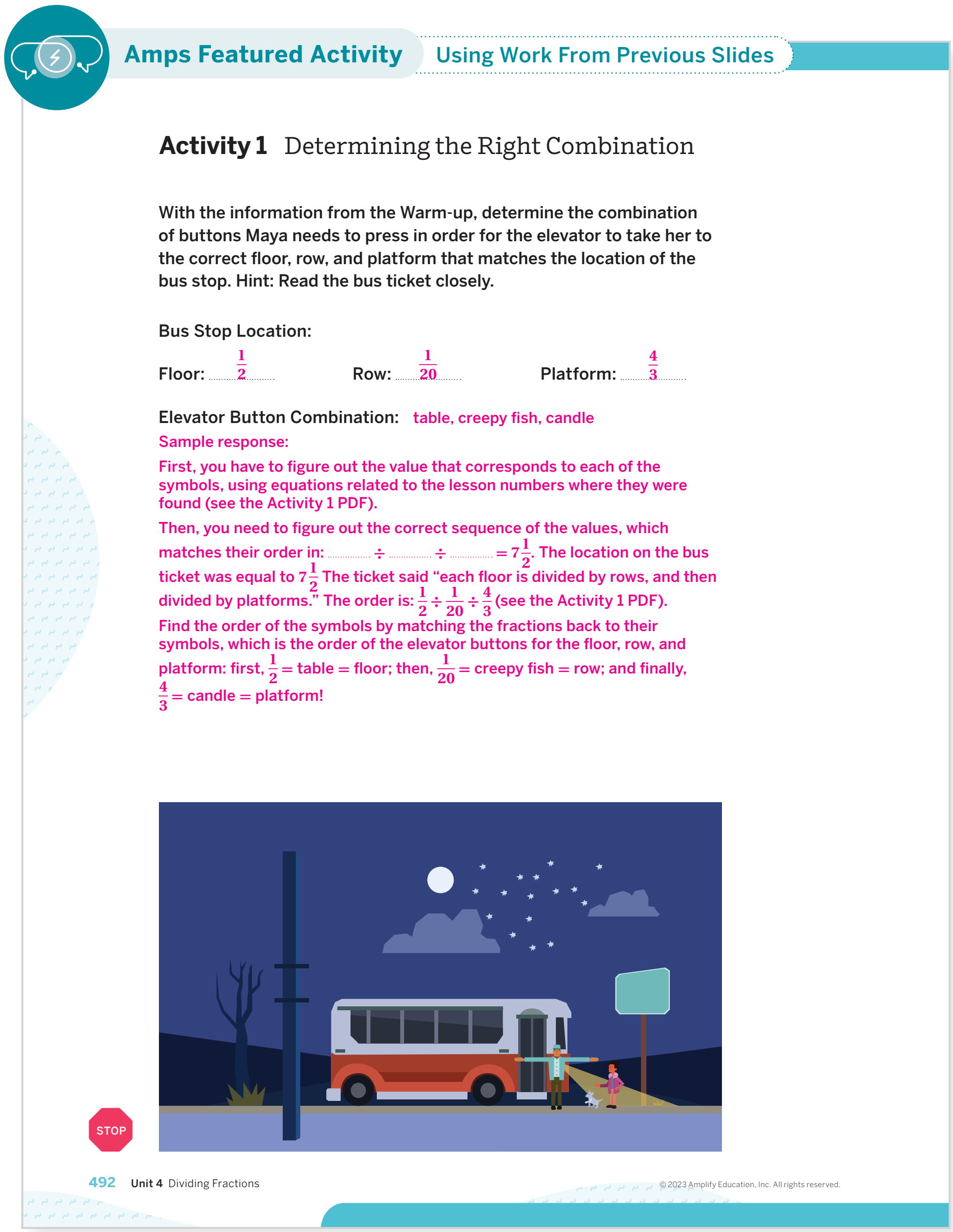
b. Subtraction

Use: Before the Warm-up.
Informed by: Performance on Lesson 16, Practice Problem 6.

Pairs | 25 min

Activity 1 Determining the Right Combination

Students divide fractions to solve equations for missing values that will help solve the mystery and determine the location of the bus stop.



Amps Featured Activity Using Work From Previous Slides

Activity 1 Determining the Right Combination

With the information from the Warm-up, determine the combination of buttons Maya needs to press in order for the elevator to take her to the correct floor, row, and platform that matches the location of the bus stop. Hint: Read the bus ticket closely.

Bus Stop Location:

Floor: $\frac{1}{2}$ Row: $\frac{1}{20}$ Platform: $\frac{4}{3}$

Elevator Button Combination: table, creepy fish, candle

Sample response:

First, you have to figure out the value that corresponds to each of the symbols, using equations related to the lesson numbers where they were found (see the Activity 1 PDF).

Then, you need to figure out the correct sequence of the values, which matches their order in: ...... $\div$ ...... $\div$ ...... $= 7\frac{1}{2}$. The location on the bus ticket was equal to $7\frac{1}{2}$ The ticket said "each floor is divided by rows, and then divided by platforms." The order is: $\frac{1}{2} \div \frac{1}{20} \div \frac{4}{3}$ (see the Activity 1 PDF).

Find the order of the symbols by matching the fractions back to their symbols, which is the order of the elevator buttons for the floor, row, and platform: first, $\frac{1}{2}$ = table = floor; then, $\frac{1}{20}$ = creepy fish = row; and finally, $\frac{4}{3}$ = candle = platform!

492 Unit 4 Dividing Fractions

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1 Launch

Say, "Now that you have this information, the location of the bus stop is going to involve the three symbols. You need to use all of the information given to you to figure out the combination, or sequence, of the elevator buttons that have to be pushed to get to the bus."

Note: If you sense that students are stuck, consider providing the Activity 1 PDF, as well as additional clues or information.

2 Monitor

Help students get started by asking, "Do you know the value of the symbols? How could you determine those?"

Look for points of confusion:

- **Calculating incorrectly for the symbol.** Have students check their solutions by substituting them back into the original equations.
- **Not knowing how to solve for the symbol.** Have students start with the Lesson 12 equation, which should be familiar. Ask, "6 divided by what number makes 12?" $\frac{1}{2}$

Look for productive strategies:

- Using the relationship between multiplication and division to solve for the values of the symbols.
- Applying general strategies or the algorithm to divide fraction in order to determine a final quotient of $7\frac{1}{2}$.
- Relating the order of the fractions in the division equation to their corresponding symbols, and determining that is the order for pressing those buttons.

3 Connect

Have pairs of students share their steps and strategies for determining their solutions.

Highlight that students used the information, including fractional values, symbols, and operations, to build their understanding and determine the solution.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with copies of the Activity 1 PDF to help them make sense of the task and organize their thinking. Mention that the clue in the third column is the corresponding equation that includes the symbol, yet this time using a ? to represent the value of the unknown symbol.

Consider demonstrating one incorrect combination of symbols to illustrate to students the goal of this task.

Math Language Development

MLR7: Compare and Connect

During the Launch, display one of the equations. Label the entire equation as "clue" and then label the candle, table, or creepy fish as "symbol" to help students organize the language being used in this task. During the Connect, ask students how they used the symbols to connect to the value of the unknown in each equation.

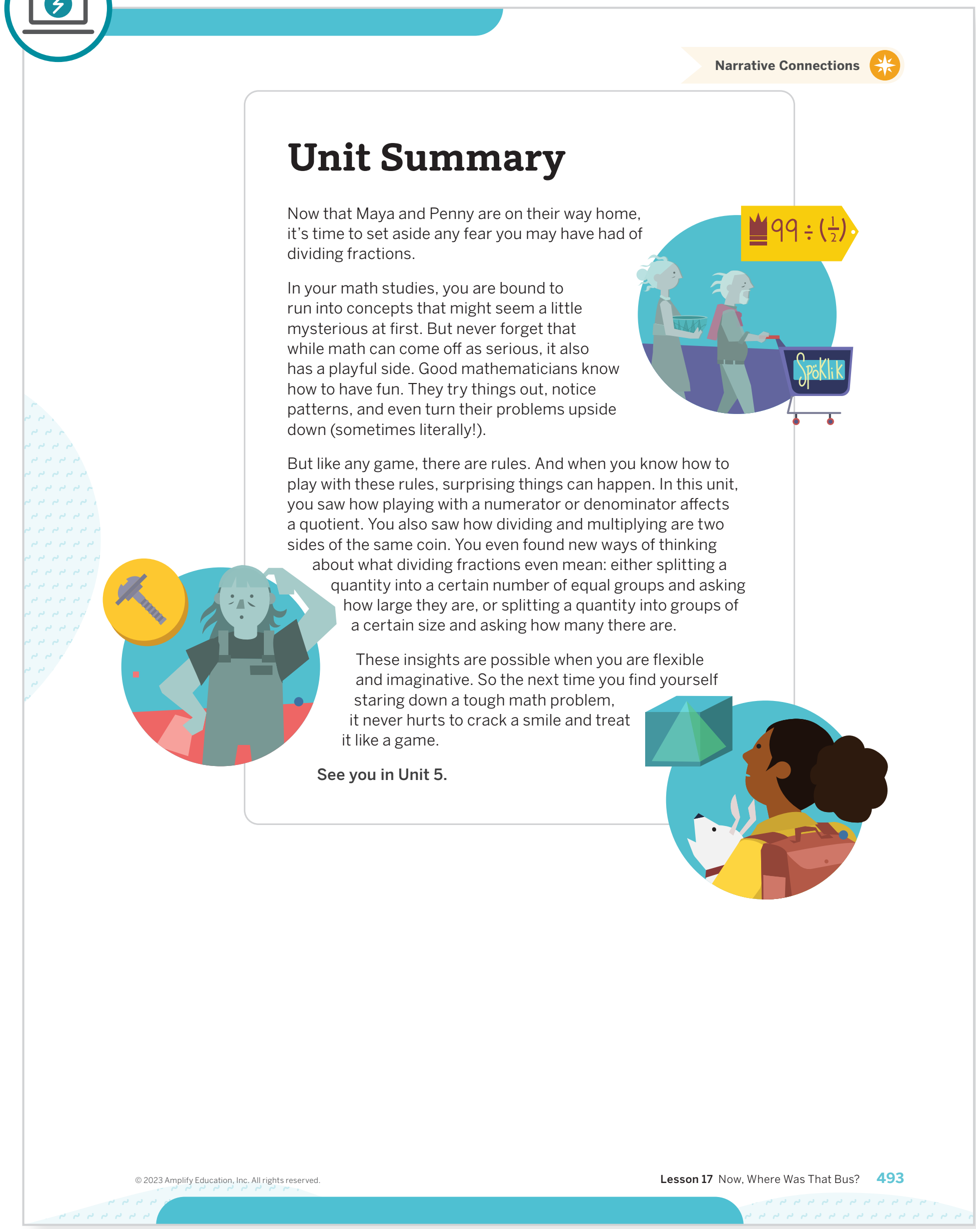
English Learners

Consider providing copies of the actual symbols of the candle, table, and creepy fish for students to use instead of using these words. Or suggest students use the letters C, T, and F.

Whole Class | 5 min

Unit Summary

Review and synthesize the way information, fractions, and calculations were used to solve the mystery.



Narrative Connections

Unit Summary

Now that Maya and Penny are on their way home, it's time to set aside any fear you may have had of dividing fractions.

In your math studies, you are bound to run into concepts that might seem a little mysterious at first. But never forget that while math can come off as serious, it also has a playful side. Good mathematicians know how to have fun. They try things out, notice patterns, and even turn their problems upside down (sometimes literally!).

But like any game, there are rules. And when you know how to play with these rules, surprising things can happen. In this unit, you saw how playing with a numerator or denominator affects a quotient. You also saw how dividing and multiplying are two sides of the same coin. You even found new ways of thinking about what dividing fractions even mean: either splitting a quantity into a certain number of equal groups and asking how large they are, or splitting a quantity into groups of a certain size and asking how many there are.

These insights are possible when you are flexible and imaginative. So the next time you find yourself staring down a tough math problem, it never hurts to crack a smile and treat it like a game.

See you in Unit 5.

Narrative Connections

Read the narrative aloud as a class or have students read it individually

Synthesize

Have students share something they enjoyed about the process of solving the mystery.

Highlight that students worked with a partner to find a solution. Collaboration can be a way to help others!

Ask, "How has your idea of working in a partnership developed or changed over the first four units?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

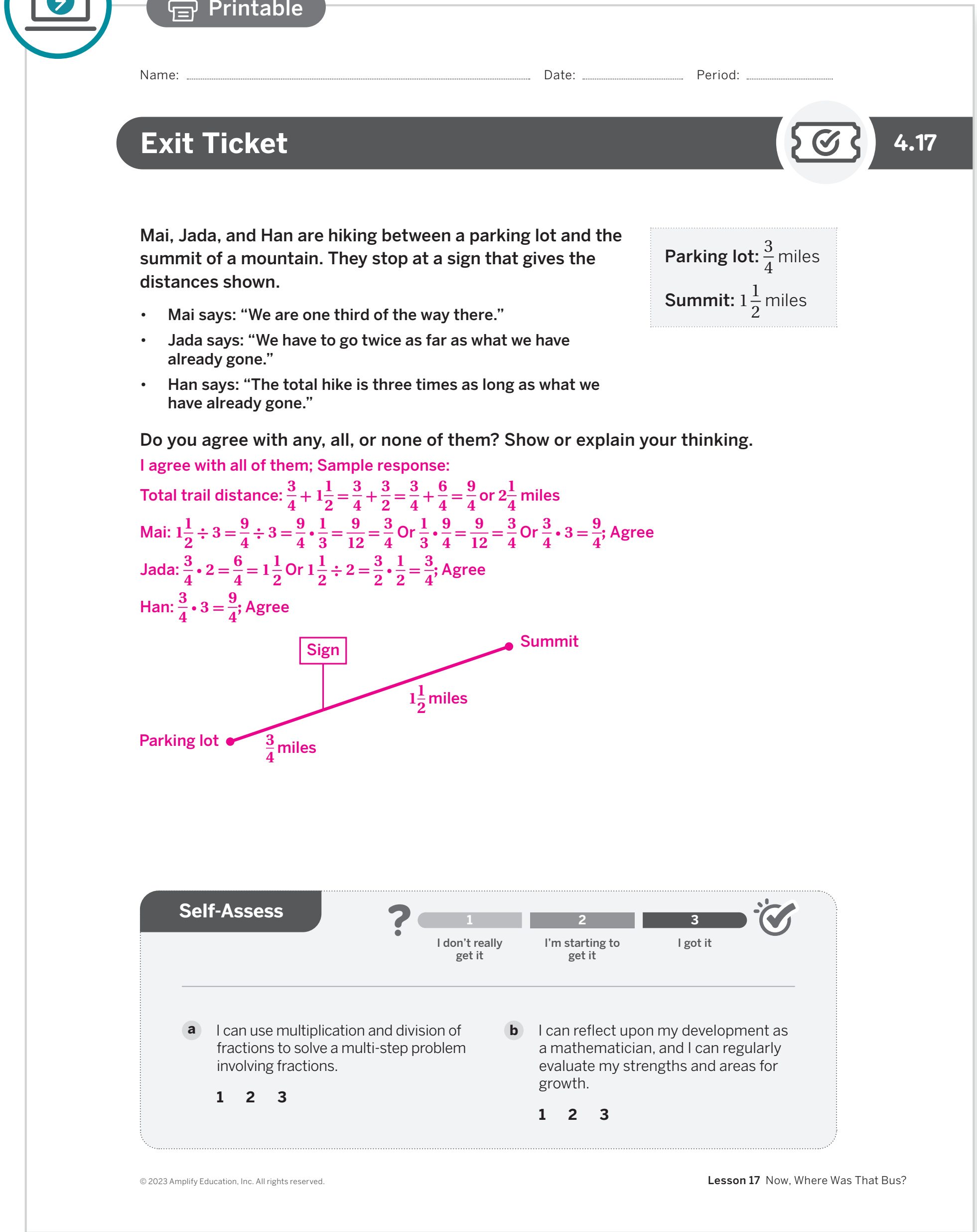
- "How did this one lesson represent all of your learning over the course of this unit?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by solving a multi-step, real-world problem involving multiplication and division of fractions.

Printable



Name: ________________ Date: ________ Period: ________

Exit Ticket 4.17

Mai, Jada, and Han are hiking between a parking lot and the summit of a mountain. They stop at a sign that gives the distances shown.

Parking lot: $\frac{3}{4}$ miles

Summit: $1\frac{1}{2}$ miles

- Mai says: "We are one third of the way there."
- Jada says: "We have to go twice as far as what we have already gone."
- Han says: "The total hike is three times as long as what we have already gone."

Do you agree with any, all, or none of them? Show or explain your thinking.

I agree with all of them; Sample response:

Total trail distance: $\frac{3}{4}+1\frac{1}{2}=\frac{3}{4}+\frac{3}{2}=\frac{3}{4}+\frac{6}{4}=\frac{9}{4}$ or $2\frac{1}{4}$ miles

Mai: $1\frac{1}{2}\div 3=\frac{9}{4}\div 3=\frac{9}{4}\cdot\frac{1}{3}=\frac{9}{12}=\frac{3}{4}$ Or $\frac{1}{3}\cdot\frac{9}{4}=\frac{9}{12}=\frac{3}{4}$ Or $\frac{3}{4}\cdot 3=\frac{9}{4}$; Agree

Jada: $\frac{3}{4}\cdot 2=\frac{6}{4}=1\frac{1}{2}$ Or $1\frac{1}{2}\div 2=\frac{3}{2}\cdot\frac{1}{2}=\frac{3}{4}$; Agree

Han: $\frac{3}{4}\cdot 3=\frac{9}{4}$; Agree

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can use multiplication and division of fractions to solve a multi-step problem involving fractions.

1 2 3

b I can reflect upon my development as a mathematician, and I can regularly evaluate my strengths and areas for growth.

1 2 3

Lesson 17 Now, Where Was That Bus?

Success looks like . . .

- **Language Goal:** Applying operations with fractions to solve problems in a variety of situations, and explain the reasoning. **(Speaking and Listening, Writing)**
    - » Solving problems involving distances on a mountain hike using fractions.
- **Language Goal:** Generating an equation to represent a situation involving fractions, and justifying the operation chosen. **(Speaking and Listening)**
- **Language Goal:** Reflecting upon the learning thus far and reevaluating areas of strength and areas for growth. **(Writing)**

Suggested next steps

If students do not know to first determine the distance from the parking lot to the summit (the total distance), consider:

- Having students draw and label a diagram representing the profile of the mountain to see that they need to add the $\frac{3}{4}$ and $1\frac{1}{2}$ first.
- The diagram should include where the sign is (or where the three hikers are seeing the sign).
- Asking, "What information do you need to know in order to solve this?"

If students choose an incorrect operation, consider:

- Having students make a tape diagram to represent the scenario and asking guiding questions such as:
    - » "Show me what Mai means in her statement. How would you represent that?"
    - » "How does that translate to an equation?" Make sure students say/write the values in the correct order, such as the total divided into thirds in Mai's example.
    - » "What operation does it sound like should be used?"

Professional Learning

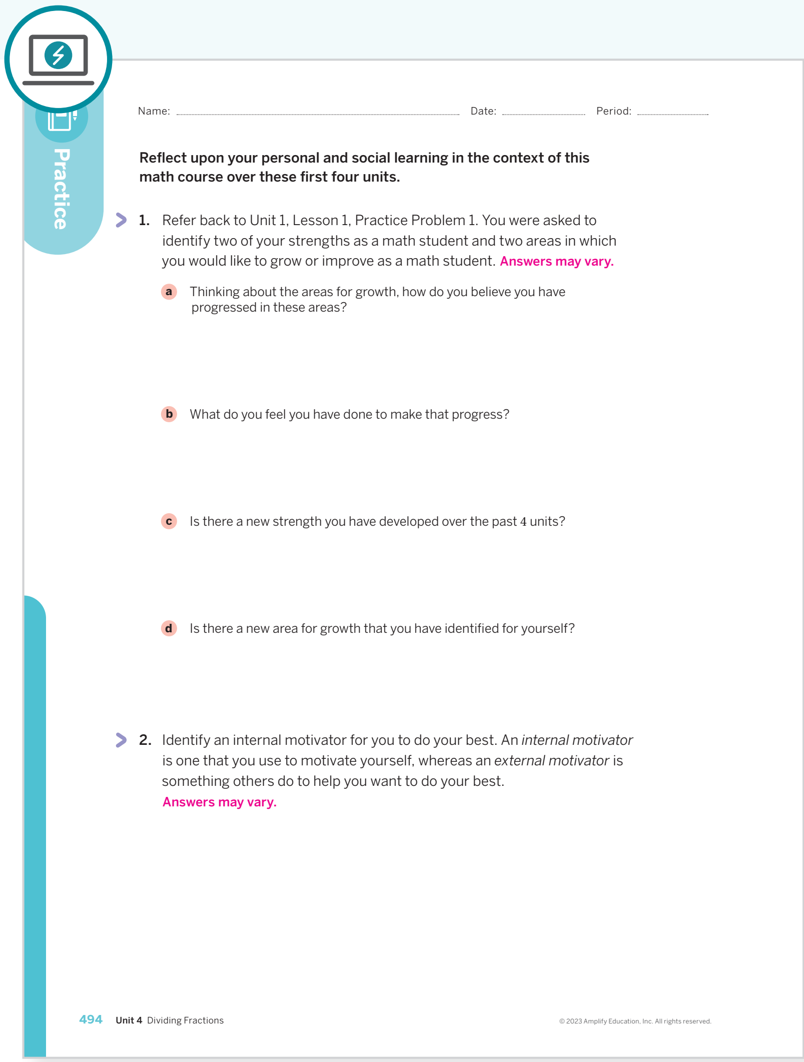
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What was especially satisfying about watching your students wrestle with the mystery today?
- What enabled all students to do math in today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ____________________ Date: ________ Period: ________

Reflect upon your personal and social learning in the context of this math course over these first four units.

1. Refer back to Unit 1, Lesson 1, Practice Problem 1. You were asked to identify two of your strengths as a math student and two areas in which you would like to grow or improve as a math student. **Answers may vary.**
   - a Thinking about the areas for growth, how do you believe you have progressed in these areas?
   - b What do you feel you have done to make that progress?
   - c Is there a new strength you have developed over the past 4 units?
   - d Is there a new area for growth that you have identified for yourself?
2. Identify an internal motivator for you to do your best. An *internal motivator* is one that you use to motivate yourself, whereas an *external motivator* is something others do to help you want to do your best.
   Answers may vary.

494 **Unit 4** Dividing Fractions

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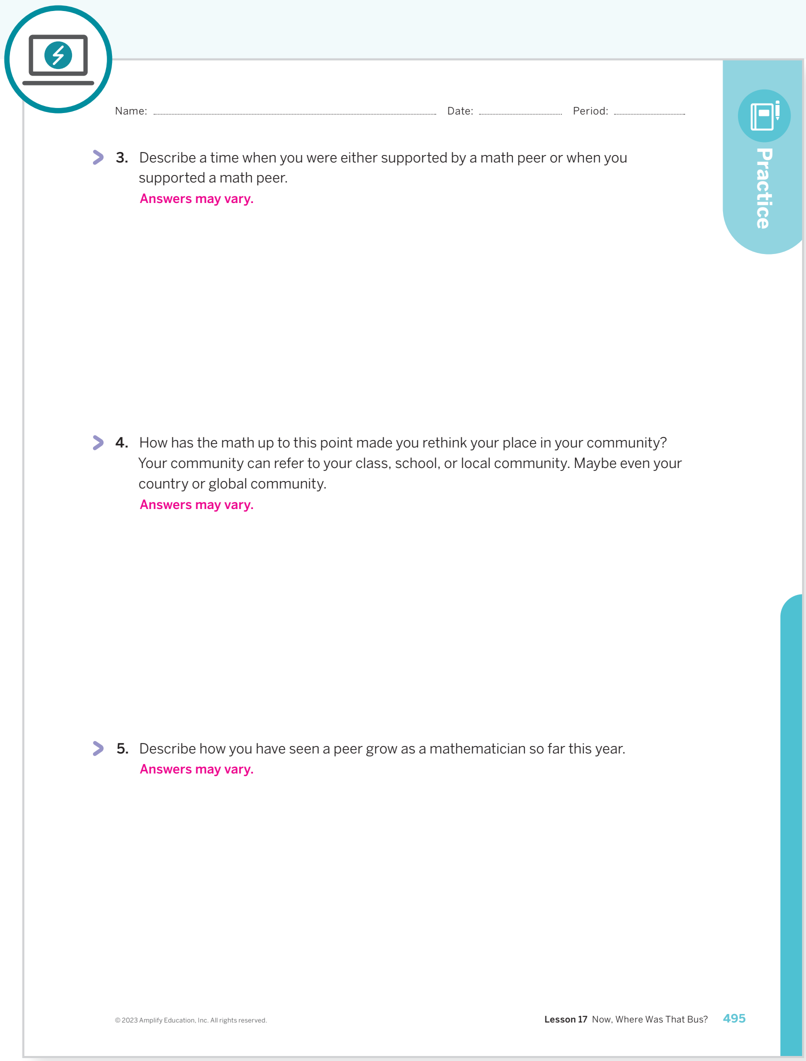

Practice

Name: ____________________ Date: ________ Period: ________

3. Describe a time when you were either supported by a math peer or when you supported a math peer.
   Answers may vary.
4. How has the math up to this point made you rethink your place in your community? Your community can refer to your class, school, or local community. Maybe even your country or global community.
   Answers may vary.
5. Describe how you have seen a peer grow as a mathematician so far this year.
   Answers may vary.

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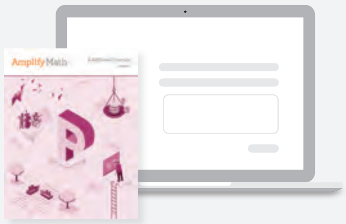
Lesson 17 Now, Where Was That Bus? 495

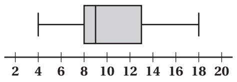
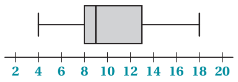
Because this is a mid-year check-in, no specific mathematical content standards are addressed in this lesson. Practice Problems 1–5 ask students to reflect upon their growth as a mathematician and collaborator, and consider what to focus on throughout the remainder of their year.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 6 Additional Practice**.

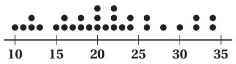
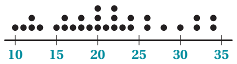
Glossary/Glosario

English	Español
A	**A**
absolute value The value that represents the distance between a number and zero. For example, because the distance between -3 and 0 is 3, the absolute value of -3 is 3, or $\|-3\| = 3$.	**valor absoluto** Valor que representa la distancia entre un número y cero. Por ejemplo, dado que la distancia entre -3 y 0 es 3, el valor absoluto de -3 es 3, o $\|-3\| = 3$.
Addition Property of Equality A property stating that if $a = b$, then $a + c = b + c$.	**Propiedad de igualdad en la suma** Propiedad que establece que si $a = b$, entonces $a + c = b + c$.
area The number of unit squares needed to fill a two-dimensional shape without gaps or overlaps.	**área** Número de unidades cuadradadas necesario para llenar una forma bidimensional sin dejar espacios vacíos ni superposiciones.
average The average of a set of values is their sum divided by the number of values in the set. The average represents a fair share, or a leveling out of the distribution, so that each value in the set has the same frequency.	**promedio** El promedio de una serie de valores es su suma dividida por la cantidad de valores en el conjunto. El promedio representa una repartición justa, o igualada, de la distribución, de manera que cada valor del conjunto tenga la misma frecuencia.
B	**B**
base (of an exponential expression) The number that is raised to a power. The power indicates the number of times the base is multiplied by itself.	**base (de una expresión exponencial)** Número que se eleva a una potencia. La potencia indica el número de veces que la base debe multiplicarse por sí mismo.
base (of a parallelogram) Any chosen side of the parallelogram.	**base (de un paralelogramo)** Cualquier lado escogido del paralelogramo.
base (of a prism) Either of the two identical, parallel faces of a prism that are connected by a set of rectangular faces.	**base (de un prisma)** Cualquiera de las dos caras idénticas y paralelas de un prisma que están conectadas por un conjunto de caras rectangulares.
base (of a pyramid) The face of a pyramid that is opposite from the vertex, where all the triangular faces connect.	**base (de una pirámide)** La cara de una pirámide que se opone al vértice, donde todas las caras triangulares se conectan.
base (of a triangle) Any chosen side of the triangle.	**base (de un triángulo)** Cualquier lado escogido del triángulo.
box plot A visual representation of the five-number summary for a numerical data set. 	**diagrama de cajas** Representación visual del resumen de cinco números de un conjunto de datos numéricos. 
C	**C**
categorical data Data that can be sorted into categories rather than counted, such as the different types of food bison eat or the colors of the rainbow.	**datos categóricos** Datos que pueden ser clasificados en categorías en vez de ser contados, como por ejemplo los diferentes tipos de comida que come un bisonte o los colores del arcoíris.
center A value that represents the typical value of a data set.	**centro** Valor que representa el valor típico de un conjunto de datos.

Glossary/Glosario

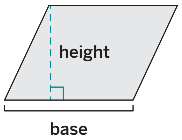
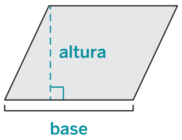
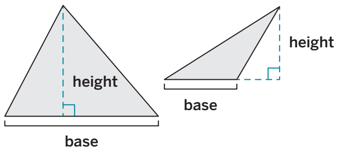
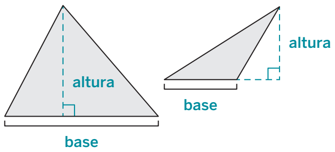
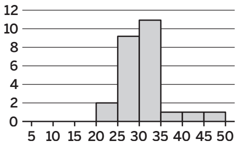
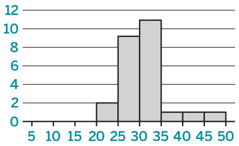
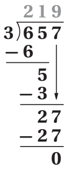
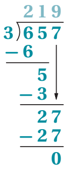
English	Español
coefficient A number that is multiplied by a variable, typically written in front of or "next to" the variable, often without a multiplication symbol.	**coeficiente** Número por el cual una variable es multiplicada, escrito comúnmente frente o junto a la variable sin un símbolo de multiplicación.
common factor A number that divides evenly into each of two or more given numbers.	**factor común** Número que divide en partes iguales cada número de entre dos o más números dados.
common multiple A number that is a multiple of two or more given numbers.	**múltiplo común** Número que es múltiplo de dos o más números dados.
compose To place together shapes or numbers, or to combine them.	**componer** Unir formas o números, o combinarlos.
coordinate plane A two-dimensional plane that represents all the ordered pairs (x, y), where x and y can both take on values that are positive, negative, or zero.	**plano de coordenadas** Plano bidimensional que representa todos los pares ordenados (x, y), donde tanto x como y pueden representar valores positivos, negativos o cero.
cubed The raising of a number to the third power (with an exponent of 3). This is read as that number, "cubed."	**al cubo** Un número elevado a la tercera potencia (con un exponente de 3) se lee como ese número "al cubo".

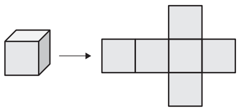
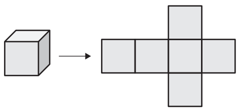
D

English	Español
decompose To take apart a shape or number.	**descomponer** Desmontar una forma o un número.
dependent variable In a relationship between two variables, the dependent variable represents the output values. The output values are unknown until the indicated calculations are performed on the independent variable.	**variable dependiente** En una relación entre dos variables, la variable dependiente representa los valores de salida. Los valores de salida son desconocidos hasta que se realizan los cálculos indicados sobre la variable independiente.
distribution A collection of all of the data values and their frequencies. A distribution can be described by its features when represented visually, such as in a dot plot.	**distribución** Una colección de todos los valores de datos y sus frecuencias. Una distribución puede ser descrita según sus características cuando es representada en forma visual, como por ejemplo en un diagrama de puntos.
Division Property of Equality A property stating that if $a = b$ and c does not equal 0, then $a \div c = b \div c$	**Propiedad de igualdad en la división** Propiedad que establece que si $a = b$ y c no equivale a 0, entonces $a \div c = b \div c$.
dot plot A representation of data in which the frequency of each value is shown by the number of dots drawn above that value on a horizontal number line. A dot plot can only be used to represent numerical data. 	**diagrama de puntos** Representación de datos en la cual la frecuencia de cada valor es equivalente al número de puntos que aparecen sobre dicho valor en una línea numérica horizontal. Un diagrama de puntos solo se puede usar para representar datos numéricos. 

English	Español
E	**E**
edge A line segment where two faces of a three-dimensional figure meet. The term *edge* can also refer to the side of a two-dimensional shape.	**arista** Segmento de una línea donde se encuentran dos caras de una figura tridimensional. *Arista* puede también referirse al lado de una forma bidimensional.
equation Two expressions with an equal sign between them. When the two expressions are equal, the equation is true. An equation can also be false when the values of the two expressions are not equal.	**ecuación** Dos expresiones con un signo de igual entre ellas. Cuando las dos expresiones son iguales, la ecuación es verdadera. Una ecuación también puede ser falsa, cuando los valores de las dos expresiones no son iguales.
equivalent If two mathematical quantities (especially fractions, ratios, or expressions) are equal in any form, then they are *equivalent*.	**equivalente** Si dos objetos matemáticos (especialmente fracciones, razones o expresiones) son iguales de cualquier manera, entonces son *equivalentes*.
equivalent expressions Two expressions whose values are equal when the same value is substituted into the variable for each expression.	**expresiones equivalentes** Dos expresiones cuyos valores son iguales cuando se sustituye el mismo valor en la variable de cada expresión.
equivalent fractions Two fractions that represent the same value or location on the number line.	**fracciones equivalentes** Dos fracciones que representan el mismo valor o la misma ubicación en la línea numérica.
equivalent ratios Any two ratios where the values for one quantity in each ratio can be multiplied or divided by the same number to get the values for the other quantity in each ratio.	**razones equivalentes** Dos razones para las cuales los valores de una cantidad en cada razón pueden ser multiplicados o divididos por el mismo número para obtener así los valores de la otra cantidad en cada razón.
exponent The number of times a factor is multiplied by itself.	**exponente** Número de veces que un factor es multiplicado por sí mismo.
expression A set of numbers, letters, operations, and grouping symbols that represent a quantity that can be calculated.	**expresión** Conjunto de números, letras, operaciones y símbolos de agrupamiento que representa una cantidad que puede ser calculada.
F	**F**
face One of many two-dimensional shapes that form the outer surface of a three-dimensional figure.	**cara** Una de muchas formas bidimensionales que forman la superficie externa de una figura tridimensional.
factor A number that divides evenly into a given whole number. For example, the factors of 15 are 1, 3, 5, and 15.	**factor** Número que divide de manera exacta a otro número dado. Por ejemplo, los factores de 15 son 1, 3, 5 y 15.
five-number summary The minimum, first quartile, median, third quartile, and maximum values of a data distribution.	**resumen de cinco números** El mínimo, el primer cuartil, la mediana, el tercer cuartil y los valores máximos de una distribución de datos.
frequency The number of times a value occurs in a data set.	**frecuencia** Número de veces que un valor está presente en un conjunto de datos.

Glossary/Glosario

English	Español
G	
greatest common factor The common factor of two or more given whole numbers whose value is the greatest (often abbreviated as "GCF").	**máximo factor común** Factor común de dos o más números enteros dados, cuyo valor es el mayor (comúnmente abreviado como "MFC").
H	
height (of a parallelogram) A segment measuring the shortest distance from the chosen base to the opposite side. 	**altura (de un paralelogramo)** Segmento que mide la distancia más corta desde la base escogida hasta el lado opuesto. 
height (of a triangle) A segment representing the distance between the base and the opposite vertex. 	**altura (de un triángulo)** Segmento que representa la distancia entre la base y el vértice opuesto. 
histogram A visual way to represent frequencies of numerical data values that have been grouped into intervals, called bins, along a number line. Bars are drawn above the bins where data exists, and the height of each bar reflects the frequency of the data values in that interval. 	**histograma** Forma visual de representar frecuencias de valores de datos que han sido agrupados en intervalos, llamados contenedores, a lo largo de una línea numérica. Se dibujan barras sobre los contenedores donde existen los datos, y la altura de cada barra refleja la frecuencia de los valores de datos en ese intervalo. 
I	
independent variable In a relationship between two variables, the independent variable represents the input values. Calculations are performed on the input values to determine the values of the dependent variable.	**variable independiente** En una relación entre dos variables, la variable independiente representa los valores de entrada. Se realizan cálculos con los valores de entrada para determinar los valores de la variable dependiente.
integers Whole numbers and their opposites.	**enteros** Números completos y sus opuestos.
interquartile range (IQR) A measure of spread (or variability) that is calculated as the difference between the third quartile (Q3) and the first quartile (Q1).	**rango intercuartil (RIC)** Medida de dispersión (es decir, de variabilidad) que es calculada mediante la diferencia entre el tercer cuartil (C3) y el primer cuartil (C1).
L	
least common multiple The common multiple of two or more given whole numbers whose value is the least (often abbreviated as "LCM").	**mínimo común múltiplo** Múltiplo común de dos o más números enteros dados, cuyo valor es el menor (comúnmente abreviado como "MCM").
long division A method that shows the steps for dividing base ten whole numbers and decimals, dividing one digit at a time, from left to right. 	**división larga** Método que muestra los pasos necesarios para dividir números enteros en base diez y decimales, por medio de la división de un dígito a la vez, de izquierda a derecha. 

English	Español
M	**M**
magnitude (of a number) The absolute value of a number, or the distance of a number from 0 on the number line.	**magnitud (de un número)** Valor absoluto de un número, o la distancia de un número con respecto al 0 en la línea numérica.
maximum The value in a data set that is the greatest.	**máximo** El valor más grande en un conjunto de datos.
mean A measure of center that represents the average of all values in a data set. The mean represents a fair share distribution or a balancing point of all of the values in the data set.	**media** Medida del centro que representa el promedio de todos los valores de un conjunto de datos. La media representa una distribución equitativa o un punto de equilibrio entre todos los puntos del conjunto de datos.
mean absolute deviation (MAD) A measure of spread (or variability) calculated by determining the average of the distances between each data value and the mean.	**desviación absoluta media (DAM)** Medida de dispersión (o variabilidad) que se calcula mediante la obtención del promedio de la distancia entre cada valor de datos y la media.
measure of center A single number used to summarize the typical value of a data set.	**medida de centro** Número individual que se utiliza para resumir el valor típico en un conjunto de datos.
measure of variability A single number used to summarize how the values in a data set vary.	**medida de variabilidad** Número individual que se utiliza para resumir cómo varían los valores en un conjunto de datos.
median The middle value in the data set when the values are listed in order from least to greatest. When there is an even number of data points, the median is the average of the two middle values.	**mediana** Valor medio de un conjunto de datos cuando sus valores están ordenados de menor a mayor. Cuando la cantidad de puntos de datos es par la mediana es el promedio de los dos valores medios.
minimum The value in a data set that is the least.	**mínimo** Valor que es el menor de un conjunto de datos.
mode The most frequently occurring value in a data set. A data set may have no mode, one mode, or more than one mode.	**modo** Valor que aparece con mayor frecuencia en un conjunto de datos. Un conjunto de datos puede tener un modo, más de un modo o ningún modo.
multiple A number that is the product of a given number and a whole number. For example, multiples of 7 include 7, 14, and 21.	**múltiplo** Número que es el producto de un número dado y un número entero. Por ejemplo, entre los múltiplos de 7 se incluyen 7, 14 y 21.
Multiplication Property of Equality A property stating that, if $a = b$, then $a \bullet c = b \bullet c$.	**Propiedad de igualdad en la multiplicación** Propiedad que establece que si $a = b$, entonces $a \bullet c = b \bullet c$.
N	**N**
negative number A number whose value is less than zero.	**número negativo** Número cuyo valor es menor que cero.
net A two-dimensional representation, or "flattening," of a three-dimensional solid's surface that shows all of its faces. 	**red** Representación bidimensional, o "aplanamiento", de la superficie de un sólido tridimensional, para mostrar todas sus caras. 
numerical data Numbers, quantities, or measurements that can be meaningfully compared.	**datos numéricos** Números, cantidades o medidas que pueden ser comparadas de manera significativa.

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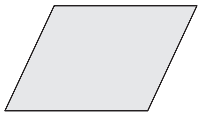
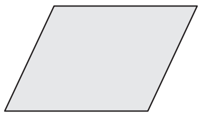
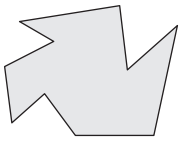
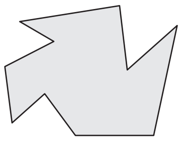
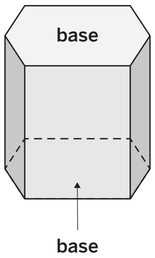
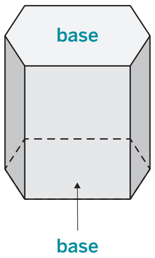
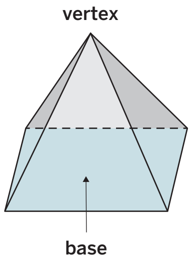
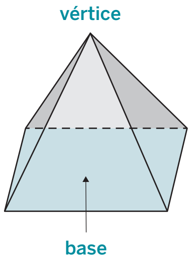
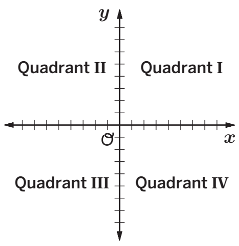
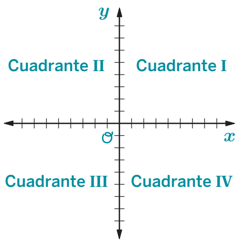
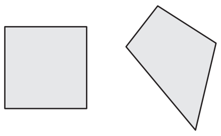
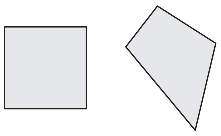
English	Español
O	**O**
opposite numbers Two numbers that are the same distance from 0, but are on different sides of the number line.	**números opuestos** Dos números que están a la misma distancia de 0, pero que están en lados diferentes de la línea numérica.
P	**P**
parallelogram A type of quadrilateral with two pairs of parallel sides. 	**paralelogramo** Tipo de cuadrilátero con dos pares de lados paralelos. 
per For each.	**por** Por cada uno de los elementos.
percentage A rate per 100. (A specific *percentage* is also called a *percent*, such as "70 percent.")	**porcentaje** Tasa por cada 100. (Un *porcentaje* específico también es llamado *por ciento*, como por ejemplo "70 por ciento".)
polygon A closed, two-dimensional shape with straight sides that do not cross each other. 	**polígono** Forma cerrada y bidimensional de lados rectos que no se entrecruzan. 
polyhedron A closed, three-dimensional shape with flat sides. (The plural of *polyhedron* is *polyhedra*.)	**poliedro** Forma cerrada y tridimensional de lados planos.
positive number A number whose value is greater than zero.	**número positivo** Número cuyo valor es mayor que cero.
prism A three-dimensional figure with two parallel, identical faces (called *bases*) that are connected by a set of rectangular faces. 	**prisma** Figura tridimensional con dos caras iguales y paralelas (llamadas *bases*) que se conectan entre sí a través de un conjunto de caras rectangulares. 
properties of equality Rules that apply to all equations. These include properties of addition, subtraction, multiplication, and division, which state that if an equation is true, then performing the same operation to both sides will result in an equivalent equation.	**propiedades de igualdad** Reglas que son aplicables a todas las ecuaciones. Incluyen las propiedades de suma, resta, multiplicación y división, las cuales señalan que si una ecuación es verdadera, al realizar la misma operación en ambos lados se obtendrá una ecuación equivalente.
pyramid A three-dimensional figure with one base and a set of triangular faces that meet at a singular vertex. 	**pirámide** Figura tridimensional con una base y un conjunto de caras triangulares que se conectan en un solo vértice. 

English	Español
Q	**Q**
quadrant Each of the four regions of the coordinate plane formed by the vertical and horizontal axes. The quadrants are labeled counterclockwise from top right to bottom right as I, II, III, IV. 	**cuadrante** Cada una de las cuatro regiones del plano de coordenadas formado por los ejes vertical y horizontal. Los cuadrantes se identifican en sentido contrario a las agujas del reloj, desde la parte superior derecha a la parte inferior derecha, como I, II, III y IV. 
quadrilateral A polygon with exactly four sides. 	**cuadrilátero** Polígono de exactamente cuatro lados. 
quartile One of three numbers (Q1, Q2, Q3) that divide an ordered data set into four sections so that each section contains 25% of data points.	**cuartil** Uno de los tres números (C1, C2, C3) que dividen un conjunto ordenado de datos en cuatro secciones, de manera que cada sección contenga el 25% de los puntos de datos.
R	**R**
range A measure of spread (or variability) that is calculated as the difference between the maximum and minimum values in the data set.	**rango** Medida de dispersión (o variabilidad) que es calculada mediante la diferencia entre los valores máximos y mínimos de un conjunto de datos.
rate A comparison of how two quantities change together.	**tasa** Comparación de cuánto cambian dos cantidades en conjunto.
ratio A comparison of two quantities, such that for every a units of one quantity, there are b units of another quantity.	**razón** Una comparación entre dos cantidades, de modo tal que por cada a unidades de una cantidad, hay b unidades de la otra cantidad
rational numbers The set of all the numbers that can be written as positive or negative fractions.	**números racionales** Conjunto que consta de todos los números que pueden ser escritos como fracciones positivas o negativas.
ratio relationship A relationship between quantities that establishes that the values for each quantity will always change together in the same way.	**relación de razón** Relación entre cantidades que establece que los valores para cada cantidad siempre cambiarán en conjunto de la misma manera.
ratio table A table of values organized in columns and rows that contains equivalent ratios.	**tabla de razones** Tabla de valores organizada en columnas y filas que contiene razones equivalentes.
reciprocal Two numbers whose product is 1 are *reciprocals* of each other. (When written in simplest fraction form, the numerator of each number corresponds to the denominator of the other number. For example, $\frac{3}{5}$ and $\frac{5}{3}$ are reciprocals.)	**recíproco/a** Dos números cuyo producto es 1 son *recíprocos* entre sí. (Al escribirlo en la forma de fracción más simple, el numerador de cada número corresponde al denominador del otro número. Por ejemplo, $\frac{3}{5}$ y $\frac{5}{3}$ son recíprocos.)
region The space inside a shape or figure.	**región** Espacio al interior de una forma o figura.

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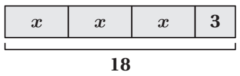
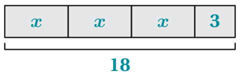
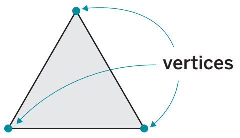
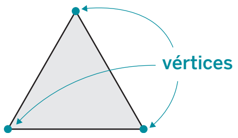
English	Español
S	**S**
sign (of a number) Indication of whether a number is positive or negative.	**signo (de un número)** Indicación de si un número es positivo o negativo.
solution to an equation A number that can be substituted in place of a variable to make an equation true.	**solución a una ecuación** Número que puede sustituir una variable para volver verdadera una ecuación.
solution to an inequality Any number that can be substituted in place of a variable to make an inequality true.	**solución a una desigualdad** Cualquier número que puede sustituir una variable para volver verdadera una desigualdad.
spread The variability of a distribution. A description of how the data values in the distribution vary from the center of the distribution.	**dispersión** Variabilidad de una distribución. Descripción de cuánto varían los valores en comparación con el centro de una distribución de datos.
squared The raising of a number to the second power (with an exponent of 2). This is read as that number, "squared."	**al cuadrado** Un número elevado a la segunda potencia (con un exponente de 2) se lee como ese número "al cuadrado".
statistical question A question that anticipates variability and can be answered by collecting data.	**pregunta estadística** Pregunta que anticipa variabilidad y que se puede responder mediante la recolección de datos.
Subtraction Property of Equality For rational numbers a, b, and c, if $a = b$, then $a - c = b - c$.	**Propiedad de igualdad en la resta** Para los números racionales a, b y c, si $a = b$, entonces $a - c = b - c$.
surface area The number of unit squares needed to cover all of the faces of a three-dimensional figure without gaps or overlaps.	**área de superficie** Número de unidades cuadradas necesarias para cubrir todas las caras de una figura tridimensional sin dejar espacios vacíos ni superposiciones.
T	**T**
tape diagram A model in which quantities are represented as lengths (of tape) placed end-to-end, and which can be used to show addition, subtraction, multiplication, or division. 	**diagrama de cinta** Modelo en el cual las cantidades están representadas como longitudes (de una cinta) colocadas de forma continua, y que pueden ser usadas para mostrar suma, resta, multiplicación y división. 
U	**U**
unit rate How much one quantity changes when the other changes by 1.	**tasa unitaria** Cuánto cambia cierta cantidad cuando la otra cambia por un factor de 1.
V	**V**
variability The spread of a distribution. A description of how the data values in the distribution vary from the center of the distribution.	**variabilidad** La dispersión de una distribución. Descripción de cuánto varían los valores en comparación con el centro de una distribución de datos.
variable A letter that represents an unknown number in an expression or equation.	**variable** Letra que representa un número desconocido en una expresión o ecuación.
vertex A point where two sides of a two-dimensional shape or two or more edges of a three-dimensional figure intersect. (The plural of *vertex* is *vertices*.) 	**vértice** Punto donde se intersecan dos lados de una forma bidimensional, o dos o más aristas de una figura tridimensional. 
volume The number of unit cubes needed to fill a three-dimensional figure without gaps or overlaps.	**volumen** Número de unidades cúbicas necesario para llenar una figura tridimensional sin dejar espacios vacíos ni superposiciones.

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