# Amplify Math

# Grade 8

## Volume 1: Units 1–4

**Teacher Edition**

## About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.



Cover illustration by Caroline Hadilaksono.


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# Dear Teacher,

Thank you for choosing Amplify Math. Our team has worked hard to ensure you have strong, easy-to-follow instructional supports that get all students talking and thinking together about grade-level math.

You can learn more about the program design in the pages that follow, but we wanted to call out a few things that really drive our point home about the student experience:

### Make math social

The student experience is **social and collaborative**. Your students will have plenty of opportunities to talk through their reasoning and work with their peers to come to new understandings. Thanks to our **partnership with Desmos**, you can kick off these social math experiences both offline and while logged in.

### Power-ups

Your priority is teaching grade-level math to each and every student in your classroom, every day. To help with that, we've developed **Power-ups** to provide just-in-time support for your students.

### Narrative

We kick off each sub-unit with a short, **engaging narrative** about historical or current-day events or phenomena. That way, your students can see the relevance of math in their everyday lives.

### Featured Mathematicians

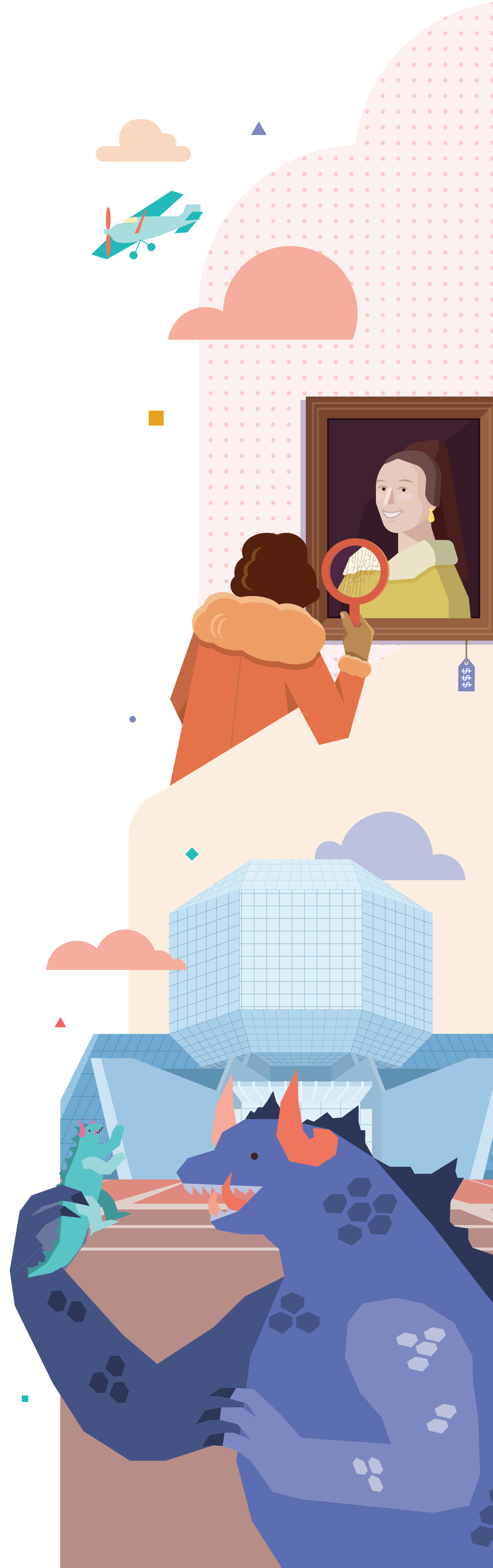
It's important to us that students see themselves in our materials. To that end, we've woven in **the work of innovative mathematical thinkers**. We've also included some of their personal stories, so that students can see themselves mirrored in the living history of mathematics.

### Data

We provide plenty of **data to help you drive your instruction** and talk about student performance with your colleagues, as well as with caregivers who may not have had the best experiences with math when they were in school.

We hope you see the quality work of our team of editors and advisors in the program.

Sincerely,
The Amplify Math Team



# Acknowledgments

## Program Advisors

Amplify gratefully acknowledges the outstanding contributions and work of distinguished program advisors who have been integral to the development of Amplify Math. This product is testimony to their expertise, understanding of student learning needs, and dedication to rigorous and equitable mathematics instruction.

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## Educator Advisory Board

Amplify gratefully acknowledges the outstanding contributions and work of esteemed members of our Educator Advisory Board. This product exhibits their respected observations and reflections.

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## Field Trials

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Leadership Learning Academy, Utah

Lusher Charter School, Louisiana

Memphis Grizzlies Preparatory Charter School, Tennessee

Saddleback Valley Unified School District, California

San Juan Unified School District, California

Santa Paula Unified School District, California

Silver Summit Academy, Utah

Streetsboro City Schools, Ohio

West Contra Costa Unified School District, California

Wyoming City Schools, Ohio

Young Women's Leadership School of Brooklyn, New York

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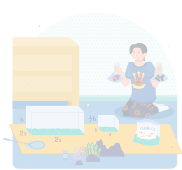
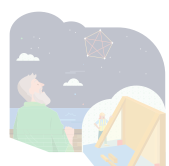
# Program Scope and Sequence

Volume 1

| | Unit 1 | Unit 2 | Unit 3 | Unit 4 |
|---|---|---|---|---|
| **Grade 6**<br>160 days total | Area and Surface Area<br>20 Instructional Days<br>3 Assessment Days<br>23 days total | Introducing Ratios<br>20 Instructional Days<br>2 Assessment Days<br>22 days total | Rates and Percentages<br>15 Instructional Days<br>2 Assessment Days<br>17 days total | Dividing Fractions<br>17 Instructional Days<br>3 Assessment Days<br>20 days total |
| **Grade 7**<br>153 days total | Scale Drawings<br>13 Instructional Days<br>2 Assessment Days<br>15 days total | Introducing Proportional Relationships<br>17 Instructional Days<br>2 Assessment Days<br>19 days total | Measuring Circles<br>12 Instructional Days<br>2 Assessment Days<br>14 days total | Percentages<br>13 Instructional Days<br>2 Assessment Days<br>15 days total |
| **Grade 8**<br>145 days total | <br>Rigid Transformation and Congruence<br>18 Instructional Days<br>3 Assessment Days<br>21 days total | <br>Dilations and Similarity<br>12 Instructional Days<br>2 Assessment Days<br>14 days total | <br>Linear Relationships<br>19 Instructional Days<br>2 Assessment Days<br>21 days total | <br>Linear Equations and Systems of Linear Equations<br>17 Instructional Days<br>2 Assessment Days<br>19 days total |
| **Algebra 1**<br>157 days total | Linear Equations, Inequalities, and Systems<br>26 Instructional Days<br>3 Assessment Days<br>29 days total | Data Analysis and Statistics<br>22 Instructional Days<br>3 Assessment Days<br>25 days total | Functions and Their Graphs<br>22 Instructional Days<br>3 Assessment Days<br>25 days total | Introducing Exponential Functions<br>22 Instructional Days<br>3 Assessment Days<br>25 days total |

| Unit 5 | Unit 6 | Unit 7 | Unit 8 |
| --- | --- | --- | --- |
| **Arithmetic in Base Ten**<br>14 Instructional Days<br>2 Assessment Days<br>16 days total | **Expressions and Equations**<br>19 Instructional Days<br>2 Assessment Days<br>21 days total | **Rational Numbers**<br>19 Instructional Days<br>2 Assessment Days<br>21 days total | **Data Sets and Distributions**<br>17 Instructional Days<br>3 Assessment Days<br>20 days total |
| **Rational Number Arithmetic**<br>20 Instructional Days<br>3 Assessment Days<br>23 days total | **Expressions, Equations, and Inequalities**<br>23 Instructional Days<br>3 Assessment Days<br>26 days total | **Angles, Triangles, and Prisms**<br>18 Instructional Days<br>3 Assessment Days<br>21 days total | **Probability and Sampling**<br>17 Instructional Days<br>3 Assessment Days<br>20 days total |
|  |  |  |  |
| **Functions and Volume**<br>21 Instructional Days<br>3 Assessment Days<br>24 days total | **Exponents and Scientific Notation**<br>15 Instructional Days<br>2 Assessment Days<br>17 days total | **Irrationals and the Pythagorean Theorem**<br>16 Instructional Days<br>2 Assessment Days<br>18 days total | **Associations in Data**<br>9 Instructional Days<br>2 Assessment Days<br>11 days total |
| **Introducing Quadratic Functions**<br>23 Instructional Days<br>3 Assessment Days<br>26 days total | **Quadratic Equations**<br>24 Instructional Days<br>3 Assessment Days<br>27 days total | | |

Volume 1

# Unit 1 Rigid Transformations and Congruence

Students begin Grade 8 by joining talented architects, artists, and mathematicians in the study of two-dimensional figures. Equipped with their geometry toolkits, students manipulate familiar figures with new methods, and make key discoveries along the way.

**Unit Narrative:**
The Art of Transformation



**Sub-Unit Narrative:**
How do you make a piece of cardboard come alive?
Pack your geometry toolkits for a transformational journey into the movement of figures.



**Sub-Unit Narrative:**
How can a crack make a piece of art priceless?
Something special happens when you perform rigid transformations on a figure.



**Sub-Unit Narrative:**
What's got 10 billion galaxies and goes great with maple syrup?
Construct a triangle from a straight angle and cut two parallel lines to see what angle relationships you notice.

# Unit 2 Dilations and Similarity

Students explore a new type of transformation, dilations, and practice using dilations to create and recognize similar figures. Students' understanding of the characteristics of these similar figures, of similar triangles specifically, will serve as the foundation for their study of the slope of a line.

**Unit Narrative:**
More Than Meets the Eye



**Sub-Unit Narrative:**
Would you put poison in your eye?
Shrink and stretch objects on and off the plane and study the characteristics of the figures you dilate.




**Sub-Unit Narrative:**
Do you really get what you pay for?
Learn how some companies use dilations to create similar, and slightly smaller, sized packaging, in a process called "shrinkflation."

# Unit 3 Linear Relationships

Students make connections between the rate of change, slope, and the constant of proportionality, drawing on previous knowledge to explore an exciting new relationship: the linear relationship.

**Unit Narrative:**
A Straight Change



**Sub-Unit Narrative:**
How fast is a geography teacher?
On your mark, get set, go! Use your understanding of slope to show how a geography teacher shocked the world with her record setting speed.



**Sub-Unit Narrative:**
How did a coal mine help build America's most famous amusement park?
Use linear relationships to collect as many coins as you can at Honest Carl's Funtime World amusement park.



**Sub-Unit Narrative:**
How did a 16-year-old take down a Chicago Bull?
Create equations from linear relationships and find how a 16-year-old was able to beat Michael Jordan in a game of basketball.

# Unit 4 Linear Equations and Systems of Linear Equations

Students begin this unit by developing algebraic methods for solving linear equations with variables on both sides of the equation. They then use these algebraic methods, along with graphs and tables, to solve systems of linear equations.

**Unit Narrative:**
The Path the Mind Takes




**Sub-Unit Narrative:**
Who was the Father of Algebra?
When traders in 9th century Baghdad needed a better system for solving problems, a mathematician developed a new method he called “al-jabr” or algebra.




**Sub-Unit Narrative:**
How is anesthesia like buying live lobsters?
Now that you have practiced solving equations, take a closer look at how you can use linear equations to solve everyday problems.

Volume 2

# Unit 5 Functions and Volume

By the end of this unit, students will have derived the formulas for the volumes of cylinders, cones, and spheres. But it all starts with a deep dive into the concept of what makes a relationship a function at the beginning of the unit.

Unit Narrative:
Pumping up the Volume on Functions

PRE-UNIT READINESS ASSESSMENT

LAUNCH



## Sub-Unit 1 Representing and Interpreting Functions



Sub-Unit Narrative:
Who has the better camera: you or your grandparents?
Learn how functions can help you tell stories.

MID-UNIT ASSESSMENT

## Sub-Unit 2 Cylinders, Cones, and Spheres



Sub-Unit Narrative:
Who invented the waffle cone?
Use your prior knowledge about finding the volume of rectangular prisms to derive formulas for finding the volumes of cylinders, cones, and spheres.

CAPSTONE



END-OF-UNIT ASSESSMENT

# Unit 6 Exponents and Scientific Notation

Unit Narrative:
From Teeny-Tiny to Downright Titanic

This unit is about the numbers so large and so small that students must develop new ways of working with them. Students deepen their knowledge of exponents before exploring how powers of 10 and scientific notation can be used to write and work with numbers as small as the mass of a bacterium or as large as the number of atoms in the Universe.

PRE-UNIT READINESS ASSESSMENT

LAUNCH



## Sub-Unit 1 Exponent Rules ..... 621



Sub-Unit Narrative:
How many carbs are in a game of chess?
You probably already know a thing or two about exponents, but what happens when you multiply or divide expressions with exponents?

## Sub-Unit 2 Scientific Notation ..... 669



Sub-Unit Narrative:
Who should we call when we run out of numbers?
You'll work with numbers that are super small and incredibly large. But you won't waste your time writing pesky zeros!

CAPSTONE



END-OF-UNIT ASSESSMENT

# Unit 7 Irrationals and the Pythagorean Theorem

Students study rational and irrational numbers using geometry and expressions before exploring a proof of the Pythagorean Theorem.

Unit Narrative:
The Mystery of the Pythagoreans



Sub-Unit Narrative:
How rational were the Pythagoreans?
Find out if every number can be represented by a fraction.




Sub-Unit Narrative:
What do the President of the United States and Albert Einstein have in common?
Uncover a special property of right triangles when you explore one of the nearly 500 proofs of the Pythagorean Theorem.

# Unit 8 Associations in Data

What makes a cat logo consumer friendly? Is there a relationship between eye distance and height for a species of krill? Are adults just as likely to ride a bike as kids? Did the hole in the ozone layer have an association with skin cancer rates in Australia? In this unit, students will grapple with these questions and more, as they discover new ways to represent associations in data and build their data literacy.

**Unit Narrative:**
Data and the Ozone Layer



**Sub-Unit Narrative:**
Who is the biggest mover and shaker in the Antarctic Ocean?
Explore the ozone hole using scatter plots, while learning about the different kinds of associations data can have.

# Get all students talking and thinking about grade-level math.

Amplify Math was designed around the idea that core math needs to serve 100% of students in accessing grade-level math every day. To that end, the program delivers:

## 1 Productive discourse made easier to facilitate and more accessible for students

### Clean and clear lesson design

The lessons all include straightforward "1, 2, 3 step" guidance for launching and facilitating discussions around the tasks. Thoughtful and specific differentiation supports are included for every activity. Every lesson ends with a summary and reflection moment, an Exit Ticket, and a practice problem set.

### Narrative and storytelling

All students ask "Why do I need to know this? When am I ever going to use this in the real world?" Amplify Math helps students make the connections with math and their everyday lives to help them see and appreciate the relevance of the math they're figuring out in class. Throughout the units, students will be introduced to historical and current narratives that show the many places mathematics inhabits in our world and how the work they do in class connects to our history and their own reality.

## 2 Flexible, social problem-solving experiences online

### Social learning experiences online

By partnering with Desmos, we've been able to deliver digital lessons, which we call **Amps**, that get students thinking, talking, revising, and celebrating their ideas. As students work in the interactive slides, new functionality may appear and they will often be asked to justify their actions and thinking. All of this is made visible to the teacher in real time.

### Automatically differentiated activities

Our **Power-ups** automatically provide differentiated activities to students who need pre-requisite support, based on performance on past problems and assessments. They're available in this Teacher Edition, too. Phil Daro partnered with us on this feature to ensure we were giving all students, even the ones who might be three years behind in math, but only 15 minutes behind the day's lesson, the chance to experience success in math.

## 3 Real-time insights, data, and reporting that inform instruction

### Teacher orchestration tools

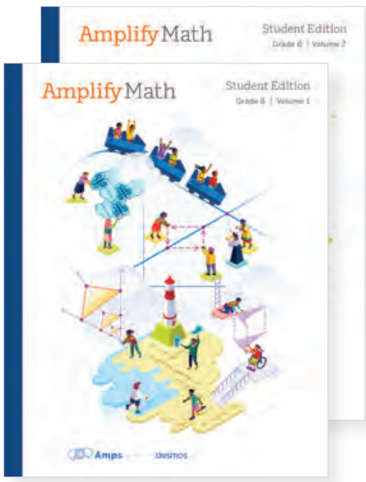
Once a teacher launches an Amp, students will be automatically moved to the lesson of the day and will see the interactive screens. Teachers will have the ability not only to pace the lesson the way they want to, but also to see student work in real time. The orchestration tools offer teachers ways to overlay student work to spot misconceptions and also the ability to spotlight student work anonymously to discuss with the class.

### Embedded and standalone assessments

Amplify Math includes both a suite of standalone assessments and embedded assessments that allow teachers and leaders insights into where students are and how they might best be supported. The full reporting suite covers student and class performance based on work done in lessons, Exit Tickets, and practice sets, performance by standards, and performance on Interim assessments.

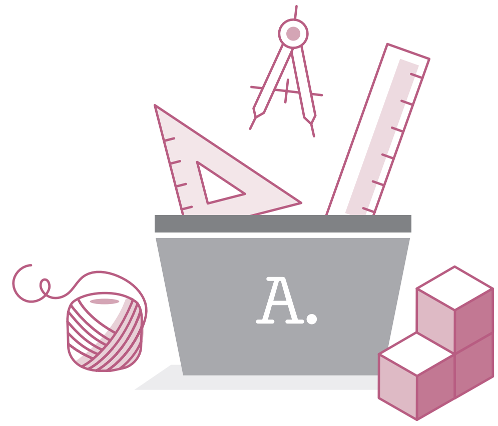
# Amplify Math resources

## Student Materials



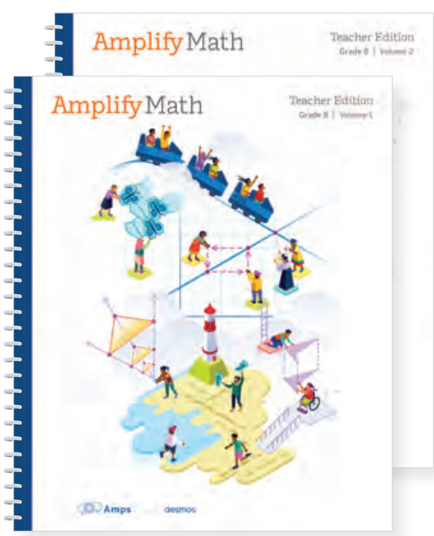

Student workbooks, 2 volumes

Amps, our exclusive collection of digital lessons powered by desmos

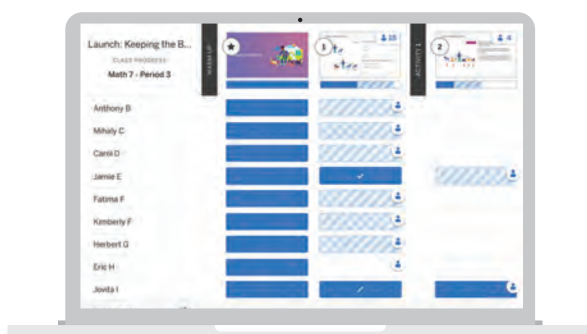



Hands-on manipulatives (optional)

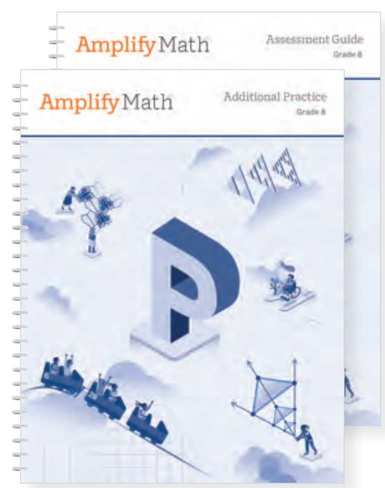
## Teacher Materials




Teacher Edition, 2 volumes



Digital Teacher Edition and classroom monitoring tools




Additional Practice and Assessment Guide blackline masters

# Program architecture

## Course

Interim Assesments

| UNIT 1 | UNIT 2 | UNIT 3 | UNIT 4 | UNIT 5 | UNIT 6 | UNIT 7 | UNIT 8 |
|---|---|---|---|---|---|---|---|
| 18 lessons | 12 lessons | 19 lessons | 17 lessons | 21 lessons | 15 lessons | 16 lessons | 9 lessons |

***Note:*** Interim assessments may be administered according to your district/school's timeline; this depiction is just one of many possible administrations.

## Unit

A Pre-Unit Readiness Assessment

A Mid-Unit Assessment

End-of-Unit Assessment A

LAUNCH

Sub-Unit 1

Sub-Unit 2

Sub-Unit 3

CAPSTONE

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

***Note:*** The number of sub-units and lessons vary from unit to unit; this depiction shows the general structure of a unit.

## Lesson

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket | Practice |
|---|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min | timing varies |

***Note:*** The number of activities vary from lesson to lesson; this depiction shows the general structure of a lesson.

**Key:**
Independent
Small Groups
Pairs
Whole Class

# Navigating This Program

## Lesson Brief

UNIT 1 | **LESSON 3**

# Symmetry and Reflection

Let's describe ways figures reflect on the plane.

### Focus

**Goals**

1. **Language Goal:** Describe the movement of figures informally and formally using the terms *reflection*, *line of reflection*, *image*, and *preimage*. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Identify the features that determine a reflection. **(Speaking and Listening, Reading and Writing)**

### Rigor

- Students build **conceptual understanding** of how figures can be flipped or reflected on a plane.
- Students build **fluency** in using precise mathematical vocabulary to describe reflections.

### Coherence

**Today**

Students begin by studying different figures to review lines of symmetry. They move into drawing and measuring reflected triangles, coming to understand that the line of reflection lies halfway between the two triangles and is perpendicular to the line segments that connect the corresponding vertices.

**Previously**

In Lesson 2, students described the features that identified translations and rotations.

**Coming Soon**

In Lesson 4, students will translate, reflect, and rotate figures on a grid.



**Lesson goals**, **coherence mapping**, and a breakdown for how **conceptual understanding**, **procedural fluency**, and **application** are addressed are included for each lesson.

LESSON BRIEF | WARM-UP | ACTIVITIES | SUMMARY | EXIT TICKET | PRACTICE

**Suggested timing** for the lesson and each activity is included for quick reference.

The benefits of teaching one or more of the activities **online** are outlined for each lesson.

Every lesson pacing guide includes **modification** suggestions.

**Building Math Identity and Community** supports for teachers are included in the Lesson Brief. Student supports appear online and in the printed Student Edition.

## Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Activity 3 | Summary | Exit Ticket |
|---|---|---|---|---|---|
| 5 min | 15 min | 8 min | 8 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Pairs | Whole Class | Independent |

**In this Warm-up, students build on their understanding of symmetry from Grade 4.*

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

### Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: rulers, tracing paper, protractors (optional)

### Math Language Development

**New words**

- *image*
- *line of reflection*
- *orientation*
- *preimage**
- *prime notation*
- *reflection*

**Review words**

- *corresponding points*
- *perpendicular*
- *symmetry*
- *vertex*

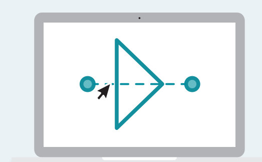
*Students may confuse *preimage* and *image* throughout the unit when discussing the original image and the transformed image. Highlight the prefix *pre* in *preimage* indicates the original image.

**Amps** | Featured Activity

**Activity 1**

**Real-Time Reflections**

When students adjust the line of reflection, an animation shows the reflected image, giving students an opportunity to revise their response, if needed.



Amps powered by desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students may not want to make the effort required to use precise units and measuring tools to measure the exact distance of corresponding points to the line of reflection. Ask them to identify what the stumbling block is. By identifying the cause of their negative emotions, students will be able to form a plan that will help them regulate their behavior in response. For example, they might just need a peer to remind them how to use and read measurements on a ruler.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, Problem choices D, E, and F may be omitted.
- **Activity 3**, Problem 1 may be omitted. In this activity, students practice drawing reflections. Students will have other opportunities to practice drawing reflections in the Practice.

19B Unit 1 Rigid Transformations and Congruence

# Navigating This Program

## Lesson

The **student-facing** content is presented to the left.

A short **description of the activity and its targeted goal** is outlined at the top.

**Easy 1-2-3 guidance** for teachers shortens the amount of time required to plan. The "look for" prompts are helpful to scan while teaching.

**Differentiation supports**, including our alternative warm-ups called Power-ups, provide practical guidance for scaffolding or extending the learning for all students. Differentiation supports, including our just-in-time supports called Power-ups, provide practical guidance for scaffolding or extending the learning for all students.

Pairs | 8 min

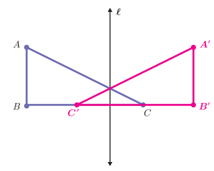
### Activity 3 Drawing Reflections

Students practice drawing reflections, strengthening their understanding of how the line of reflection relates to the corresponding points in the preimage and image.

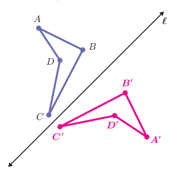
Name: ______ Date: ______ Period: ______

**Activity 3** Drawing Reflections

1. Reflect Triangle $ABC$ across line $\ell$. Use $A'$, $B'$, and $C'$ to indicate vertices in the image that correspond to the points $A$, $B$, and $C$ in the preimage.




2. Reflect Polygon $ABCD$ across line $\ell$. Use $A'$, $B'$, $C'$, and $D'$ to indicate vertices in the image that correspond to the points $A$, $B$, $C$, and $D$ in the preimage.




STOP



**Lesson 3** Symmetry and Reflection 23

**1 Launch**

Have students use a ruler to draw the reflection of each figure and only use tracing paper to check their work.

**2 Monitor**

**Help students get started** by having them draw a perpendicular line from point $A$ to the line $\ell$ in Problem 1, and then measure the distance from point $A$ to the line $\ell$.

**Look for points of confusion:**

- **Drawing a reflected point the same distance from the line as point $A$, but not perpendicular to line $\ell$ in Problem 2.** Use a protractor, or corner of an index card or paper, to help students create a right angle formed by line $\ell$ and point $A$.

**Look for productive strategies:**

- Using rulers to measure the distance from each point in the preimage to the line of reflection.
- Only using tracing paper to check their reflected image after it is drawn.

**3 Connect**

**Display** correct student drawings.

**Have students share** the strategies they used for drawing each image.

**Highlight** that an image is determined by the preimage and placement of the line of reflection. The line of reflection may not always be strictly vertical (as in Problem 1) or horizontal. The line of reflection may be slanted (as in Problem 2).

**Differentiated Support**

**Accessibility:** ***Vary Demands to Optimize Challenge***

If students need more processing time, have them focus on completing Problem 1, and only work on Problem 2 as time allows.

**Accessibility:** ***Optimize Access to Tools***

Provide access to tracing paper, should students wish to use it during the activity.

**Extension:** ***Math Enrichment***

Have students draw their own reflections and lines of reflections that satisfy the given criteria.

- Draw the reflection of a preimage in which the image overlaps the preimage.
- Draw the reflection of a preimage in which the image touches exactly one of the vertices of the preimage.
- Draw the reflection of a preimage in which the image touches exactly one of the sides of the preimage.

**Lesson 3** Symmetry and Reflection 23

LESSON BRIEF | WARM-UP | ACTIVITIES | SUMMARY | EXIT TICKET | PRACTICE

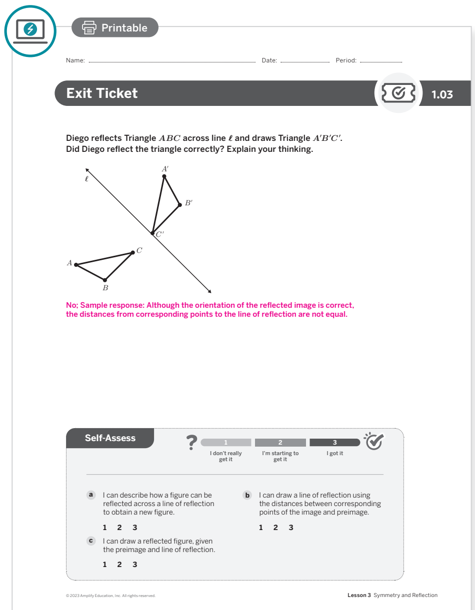
Each lesson ends with an **Exit Ticket** which includes a self assessment for students.

Independent | 5 min

## Exit Ticket

Students demonstrate their understanding of reflection by critiquing the work of another student and constructing a viable argument.

Printable




### Success looks like . . .

- **Language Goal:** Describing the movement of figures informally and formally using the terms *reflection*, *line of reflection*, *image*, and *preimage*. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Identifying the features that determine a reflection. **(Speaking and Listening, Reading and Writing)**
  - » Explaining why the Diego's reflection is incorrect.

### Suggested next steps

**If students think that Diego's reflection is correct, consider:**

- Rev...

### Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

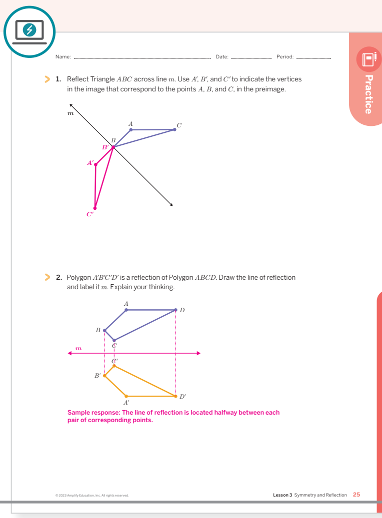
**Points to Ponder . . .**

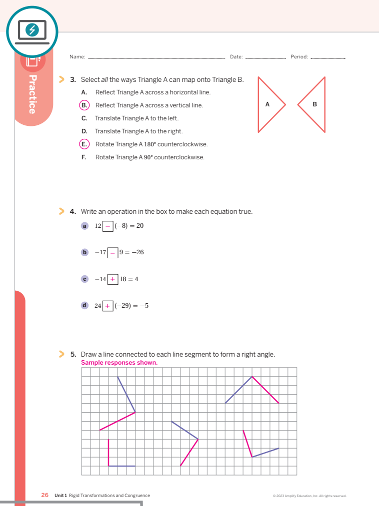
- How did students attend to precision when describing reflections? How are you helping students become self-aware of their progress and growth in this area?
- What different ways did students approach drawing reflections? What does that tell you about similarities and differences among your students?

A targeted set of 4-6 **practice problems** are included online and in the print Student Edition. Each set includes at least one spiral review problem and one formative problem as a prerequisite check for the next lesson.

## Practice

Independent





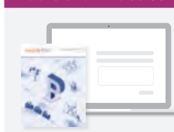
In the **Additional Practice book**, students will find a worked out example and four to eight practice problems per lesson.

### Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| | 1 | Activity 1 | 1 |
| On-lesson | 2 | Activity 2 | 1 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Grade 7 | 2 |
| Formative | 5 | Unit 1 Lesson 4 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

### Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

25–26 Unit 1 Rigid Transformations and Congruence

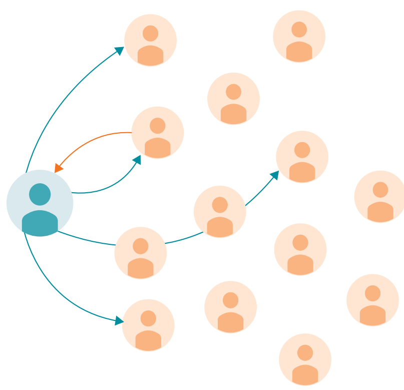
# Social, collaborative digital experiences

Digital lessons, when designed the right way, can be powerful in their ability to surface student thinking and spark interesting and productive discussions. To bring our vision of what digital lessons can and should be to life, we've partnered with Desmos to create our complete library of **Amps**—social, collaborative lessons powered by Desmos technology that recast technology from simply mirroring what can be done in a workbook to presenting captivating scenarios where students work together and interact with the mathematics in real time.

Amps
POWERED BY desmos

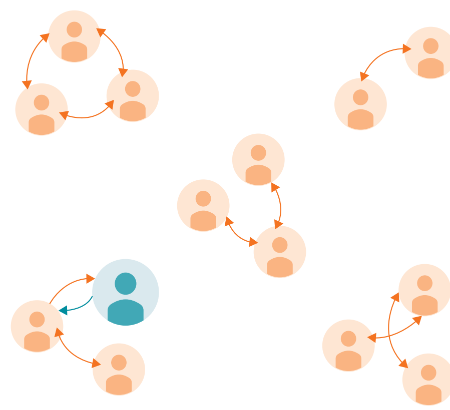
## 1 Launch

Teachers launch an activity and ensure students understand what's being asked.

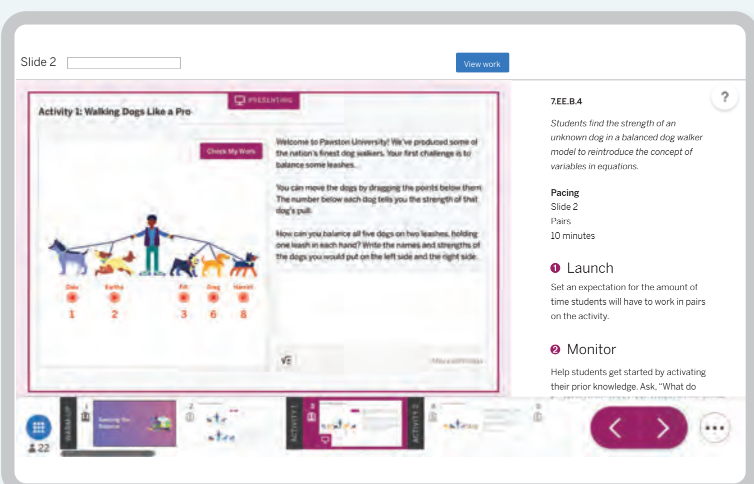


## 2 Monitor

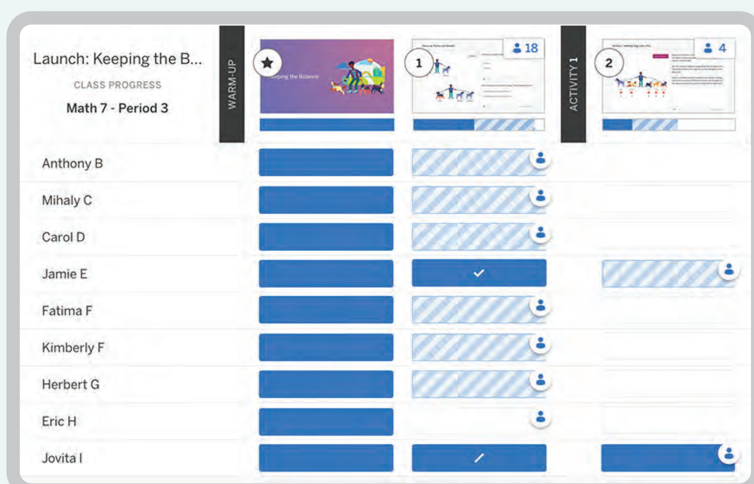
Students interact with each other to discuss and work out strategies for solving a problem.



### Teacher experience




When you launch a lesson, you'll have access to **easy-to-skim teacher notes and all of the controls necessary** to manage the lesson.

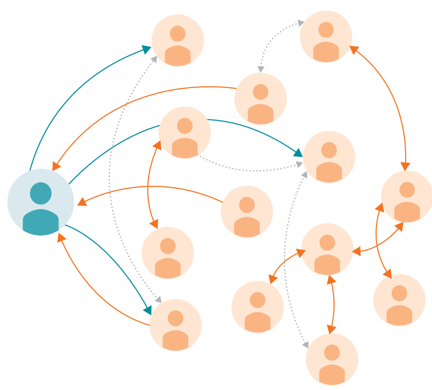



After students have started working you can access the Class Progress screen to **see where students are in the lesson and even control which problems they have access to.**

When you launch an **Amp**, you will be kickstarting small group and whole class discussions where students can see how their thinking can impact a situation and learn how their peers are justifying their actions and decisions.

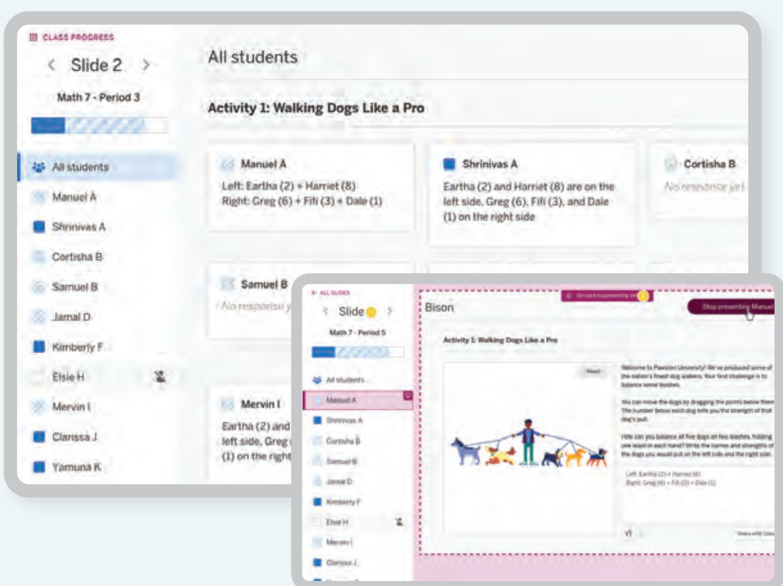
## 3 Connect

Students construct viable arguments and critique each other's reasoning, then synthesize with the teacher at the end.

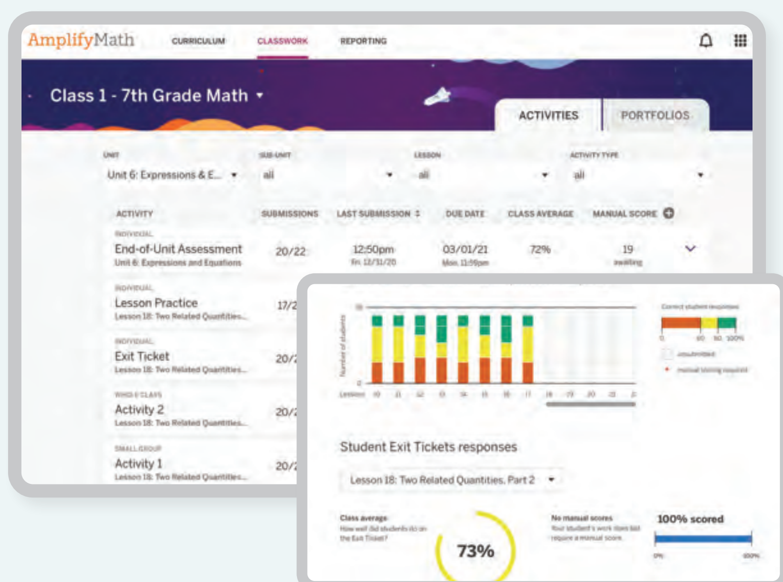


## 4 Review

After class, teachers can provide feedback on submitted student work and run reports.




**All student responses can be viewed easily** on the All Students screen. You can often view a composite view of responses and spotlight student work anonymously.




After students complete work that's ready for grading, you can head to Classwork to **quickly provide feedback.**

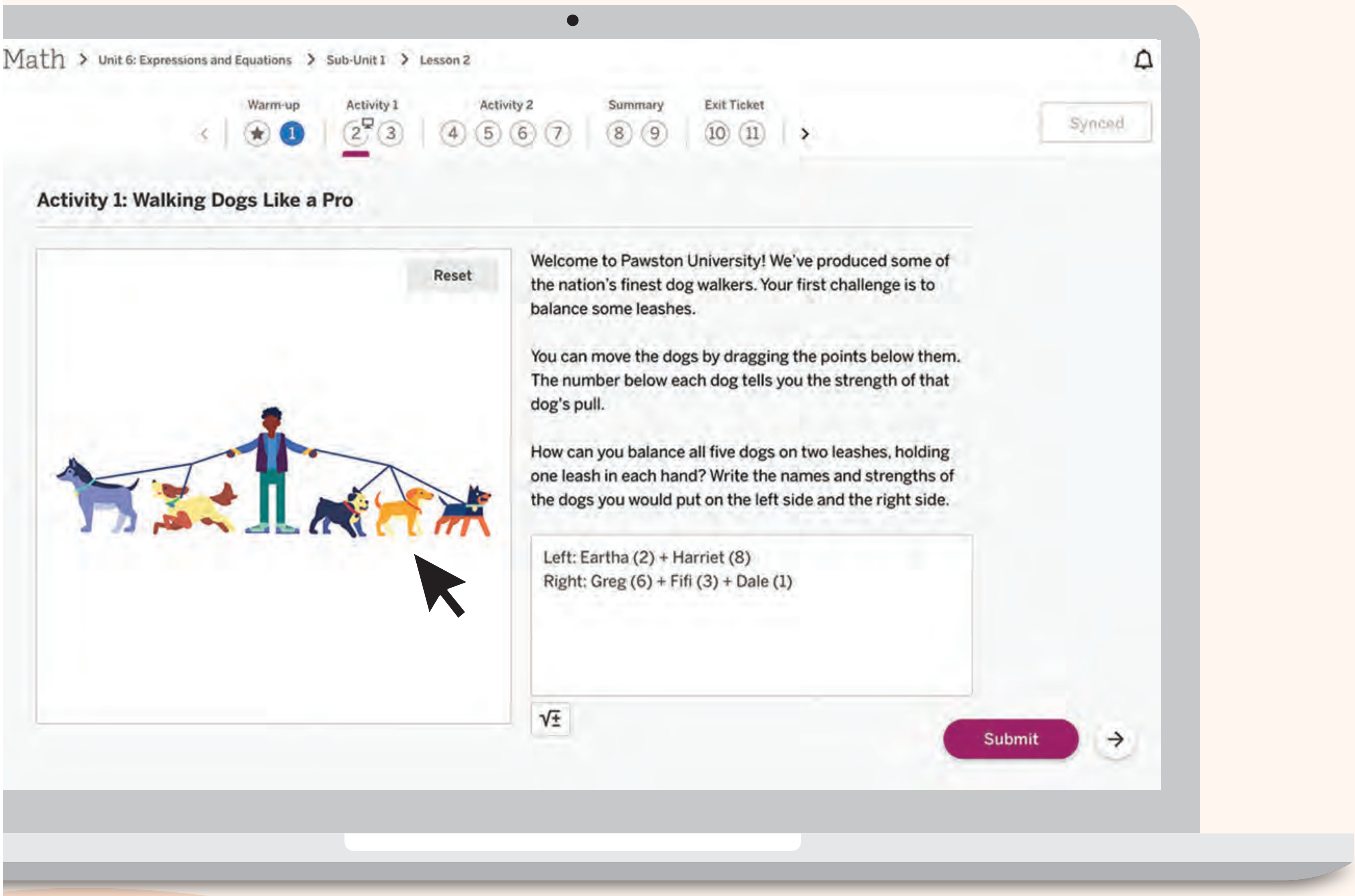
Once students have completed an Exit Ticket, a practice problem set, or an assessment, you can **run reports at the class, student, and standards levels to check in on student progress.**

# Connecting everyone in the classroom

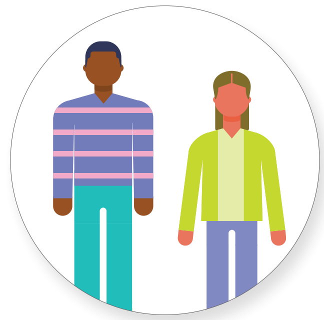
The student experience is interactive and responsive. As students manipulate the interactive elements of the Amps, they will be asked to justify their thinking, and often they will get to see how their peers are thinking, too. All along the way, teachers can monitor work in real time.

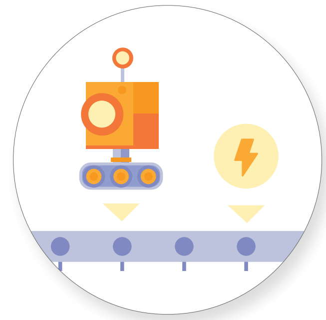
## Student experience

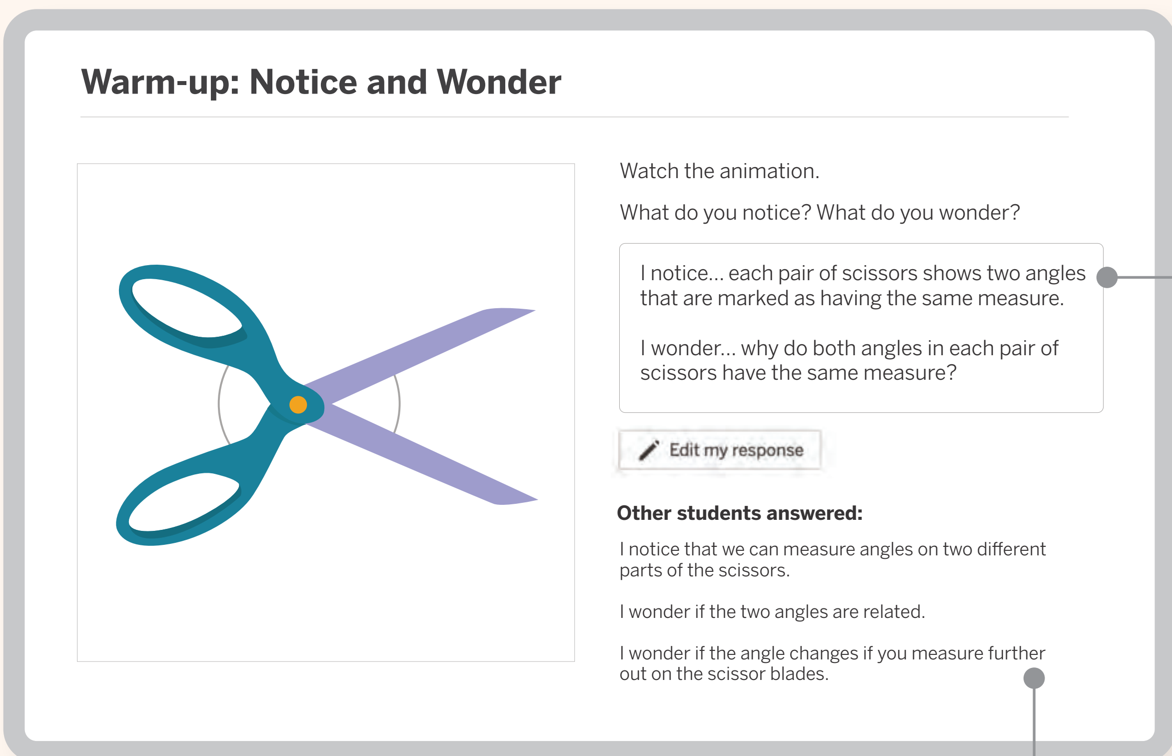
The student experience is intuitive and engaging, offering students **low floors and high ceilings** as they engage with the lesson content.



Many of the lessons feature rich, visual moments that students can interact with. Amps are meant to feel intuitive, immersive, and sometimes even playful.

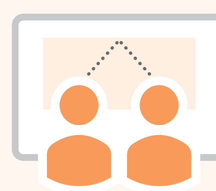








As students work, the slides change, prompting students to **describe their strategies**. Teachers can see student work in real time and spotlight responses anonymously to support in-class discussion.



When working online, students will sometimes **see their peers' thinking** on their own screens. These connection points are great for sparking discussions ahead of the synthesis moment in the lesson.

# Routines in Amplify Math

Routines help you and your students to maintain a sense of familiarity and structure throughout the school year. As the year progresses, routines free up time you would otherwise spend giving new directions for each activity.

| Routine | What is it? | Where is it? |
|---|---|---|
| *Turn and Talk* | Turn and Talk can be done anytime, needs little preparation, and only takes a minute or two. Students turn and talk to their partner about another student's thinking, eventually deciding whether they agree or not and why. The students then switch roles. Eventually you will call on a few students to say what they think, or report what their partner said. | Use anytime students are working |
| *Ask Three Before Me* | Do you find yourself responding to all of your students' questions? This routine saves you time and empowers your students' agency and voice, while allowing them to view each other as knowledgeable resources. The routine is simple: When a student has a question, let them know they should ask three other students before they ask you. | |
| *Go Find a Good Idea* | When students are stuck and productive struggle has stalled, direct them to get up and walk around the room to find a good idea in the written work of other students. Encourage them to ask questions of other students and to explain their work. Then, they should bring a good idea back and continue working with it, citing from whom they got it. | |
| *Notice and Wonder* | Students are shown some media or a mathematical representation. They are prompted with "What do you notice? What do you wonder?," and are given a few minutes to think and share with a partner. Then you then ask several students to share what they noticed and wondered, recording responses for all to see.<br>***Note:*** Notice and Wonder and I Notice/I Wonder are trademarks of NCTM and the Math Forum and used in these materials with permission. | Warm-ups, Activity launches |
| *Math Talks and Strings* | Typically, one problem is displayed at a time. Problems are purposefully chosen to elicit different approaches, often in a way that builds from one problem to the next. These problems could be numerical or algebraic in nature.<br>Give students a few moments to quietly think and give you a signal when they have an answer and a strategy, then select some to share different strategies for each problem. Ask questions like "Who thought about it a different way?" | Warm-ups |
| *Which One Doesn't Belong?* | Students are asked "Which one doesn't belong?" when presented with several figures, diagrams, graphs, or expressions. The best part is that every answer choice is correct! It's the similarities and differences among the options that are mathematically meaningful here. Prompt students to explain their rationale for deciding which option doesn't belong, and give them opportunities to make their rationale more precise. | Warm-ups |
| *Card Sort* | A card sort uses cards or slips of paper that can be manipulated and moved around. Individually, in pairs, or in small groups students put things into categories or groups based on shared characteristics or connections. | Activities |
| *Find and Fix* | Students are presented with the work of another hypothetical student whose work contains a flaw or error. They first identify any errors, then show or explain how to correct these errors, and explain why the person may have made the error. | Activities |
| *Group Presentations and Gallery Tours* | Instruct students—typically in groups—to create a visual display of their work, such as how they solved a problem with mathematical modeling, invented a new problem, designed a simulation or experiment, or organized and displayed data.<br>In the Gallery Tour version of this routine, student work is captured on a piece of paper, a poster, or on an assigned portion of the board. Students then move around the room to observe, record notes or questions on their own paper, or write on each other's work (posing clarifying questions, giving kudos, or identifying portions they may disagree with). You lead a discussion, allowing students to respond to questions or critiques of their work. | Activities |
| *Info Gap* | One partner receives a problem card with a math question that doesn't have enough given information, and the other partner receives a data card with information relevant to the problem card. Students ask each other questions, such as "What information do you need?" and "Why do you need this information?" and are expected to explain what they will do with the information. | Activities |

# Math language development

All students, including English Learners, benefit from math lessons that are designed with strong math language development supports. Working in close partnership with the **English Learners Success Forum** (ELSF), the Amplify curriculum writers ensured the math language development strategies and supports in the program were clear, useful, and appropriate for all students. This was achieved by infusing the instruction with research-based Math Language Routines (MLRs) and by providing sentence frames where appropriate, both in the teacher language provided for each task and in the differentiation supports section. ELSF has helped review all student-facing content to ensure it's developmentally appropriate and reflects the fact that math is a new language students are mastering.

ENGLISH LEARNERS SUCCESS FORUM

**The math language development story starts at the unit level where teachers will see new vocabulary and a correlation of MLRs to lessons listed in the unit planning materials. In the Unit Study materials, one of the MLRs is highlighted under the Focus on Differentiated support section.**

## Embedded language development support

- **Course level:** The course design centers the development of communication skills.
- **Unit level:** Teachers will understand how language development progresses throughout the unit.
- **Lesson level:** Each lesson includes definitions of new vocabulary and language goals.
- **Activities:** Math Language Routines support the development of new vocabulary and activities often include additional supports based on language demands.
- **Assessments:** Suggested next steps offer guidance for teachers to support students' mathematical language development, based on their performance on assessment items.

## Sentence frames

Sentence frames can give a student a sensible jumping off point for verbalizing or writing out an idea. Amplify Math leans on sentence frames that all serve one or more language functions, including describing a topic, explaining it, justifying thinking, generalizing, critiquing, representing, and interpreting ideas.

## Math Language Routines

The Math Language Routines deployed throughout the Teacher Edition:

**MLR1:** Stronger and Clearer Each Time

**MLR2:** Collect and Display

**MLR3:** Critique, Correct, Clarify

**MLR4:** Information Gap

**MLR5:** Co-craft Questions

**MLR6:** Three Reads

**MLR7:** Compare and Connect

**MLR8:** Discussion Supports

Some routines adapted from Zwiers, J. (2014). Building academic language: Meeting Common Core Standards across disciplines, grades 5–12 (2nd ed.). San Francisco, CA: Jossey-Bass.

UNIT 1

# Rigid Transformations and Congruence

Students begin Grade 8 by joining talented architects, artists, and mathematicians in the study of two-dimensional figures. Equipped with their geometry toolkits, students manipulate familiar figures with new methods, and make key discoveries along the way.

**Essential Questions**

- What happens to a figure as you move it around a two-dimensional plane?
- What does it mean for two figures to be "the same"?
- Do the measures of the interior angles of a triangle really add up to 180°?
- *(By the way, can you spot a fraudulent painting of the Mona Lisa?)*

# Key Shifts in Mathematics

## Focus

### In this unit . . .

Students explore the properties of rigid transformations — translations, rotations, and reflections — and use these properties to reason about plane figures. Students learn that angles and distances are preserved when rigid transformations are performed, and that two figures are congruent if they can be mapped onto one another using rigid transformations. With the understanding that lines can also be transformed, students reason that when two parallel lines are cut by a transversal, the alternate interior angles formed are congruent. By deconstructing a straight angle, students also discover that the sum of the angle measures in a triangle is $180°$.

## Coherence

### Previously . . .

Students began studying geometry in kindergarten and continued exploring shapes throughout elementary school. Fast forward to Grade 7, where students discovered that angle measures are preserved in scaled copies. They also saw that areas increase or decrease proportionally to the square of the scale factor. Their study of scaled copies was limited to pairs of figures with the same orientation.

### Coming soon . . .

In the next unit, students will study a new type of transformation: dilations. With an understanding of dilations and scale factor, students will develop informal arguments for proving similar triangles, arguments they will build on in later years in high school. The study of dilations and similarity provides background for understanding the slope of a line in the coordinate plane.

## Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

### Conceptual Understanding

Students experiment with rigid transformations to explore how and why side lengths and angle measures are preserved (Lesson 9). Students discover why a triangle must be composed of angle measures that add to $180°$ (Lesson 16).

### Procedural Fluency

Over the first part of the unit, through Practice and Additional Practice, students develop fluency as they perform rigid transformations with figures. They also gain valuable practice measuring with precision using tools from their geometry toolkits.

### Application

Students examine different patterns formed by tiling on an Omani mosque and create their own border patterns (Lesson 18).

# The Art of Transformation

SUB-UNIT

1

Lessons 2–8

## Rigid Transformations

Students first explore ***transformations*** on the plane, without the added structure of a grid or coordinate system. In later lessons, they use the precision of a grid and coordinates to further their understanding of ***translations***, ***rotations***, and ***reflections***.

**Narrative:** The world's first animated feature film was created using geometric transformations.

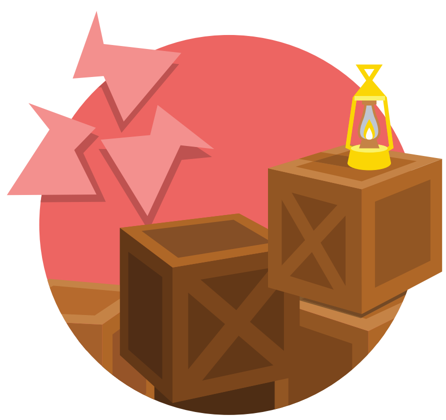
SUB-UNIT

2

Lessons 9–12

## Rigid Transformations and Congruence

Equipped with their geometry toolkits, students explore what it means for two objects or figures to be "the same" and develop the mathematical vocabulary—***congruence*** — to precisely describe when two figures are "the same."



**Narrative:** Spotting forgeries of artistic works involves an understanding of congruent polygons.

# Launch

Lesson 1

## Tessellations

Students create patterns with shapes, drawing inspiration from the tiles of an Omani palace, the artwork of M.C. Escher, and the pentagons of Marjorie Rice. You will want to display these ***tessellations*** for all to see.

SUB-UNIT

# 3

Lessons 13–17

## Angles in a Triangle

Turns out, lines and angles can also be transformed. Students encounter parallel lines and ***transversals***, exploring the measures of the ***alternate interior angles*** that are formed. They establish a framework that will help them understand dilations, similarity, and slope in upcoming units.

**Narrative:** Discover what the sum of the angles in a triangle tells us about our Universe.

# Capstone

Lesson 18

## Creating a Border Pattern Using Transformations

Students apply what they have learned about transformations to study and create border patterns.

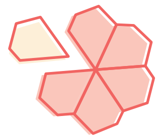
# Unit at a Glance

**Spoiler Alert:** Translations, rotations, and reflections are all examples of rigid transformations, meaning they preserve a figure's shape and size when the figure is transformed.

## Assessment

**A** **Pre-Unit Readiness Assessment**

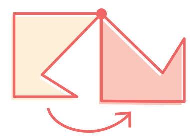
## Launch Lesson



**1** **Tessellations**

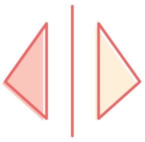
Create patterns using tessellations.

## Sub-Unit 1: Rigid Transformations



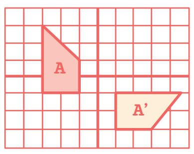
**2** **Moving on the Plane** •

Describe and identify translations and rotations.



**3** **Symmetry and Reflection** •

Describe and identify reflections.



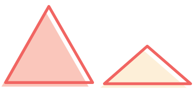

**8** **Describing Transformations**

Create a drawing on a coordinate plane of a transformed object using verbal descriptions.

## Assessment

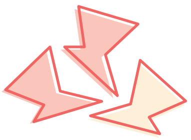
**A** **Mid-Unit Assessment**

## Sub-Unit 2: Rigid Transformations and Congruence



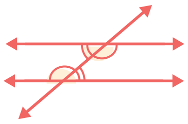
**9** **No Bending or Stretching**

Identify and draw sequences of rigid transformations.



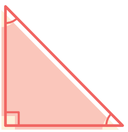
**10** **What Is the Same?**

Define the term *congruent* using rigid transformations, side lengths, and angle measures.



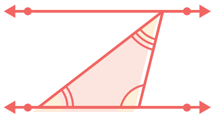
**15** **Alternate Interior Angles**

Calculate angle measures using alternate interior, adjacent, vertical, and supplementary angle relationships.



**16** **Adding the Angles in a Triangle**

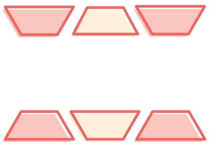
Deconstruct a straight angle and create a triangle to find the sum of the measures of its interior angles.



**17** **Parallel Lines and the Angles in a Triangle**

Argue why the sum of the interior angle measures of any triangle always adds to 180°.

## Capstone Lesson



**18** **Creating a Border Pattern Using Transformations**

Study a tile pattern that adorns a famous mosque. Then create one.

## Key Concepts

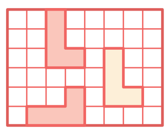
**Lesson 5:** Describe and perform a sequence of transformations.
**Lesson 10:** Define and determine congruence.
**Lesson 16:** Make a discovery about the interior angles of a triangle.

## Pacing

**18 Lessons:** 45 min each
**3 Assessments:** 45 min each

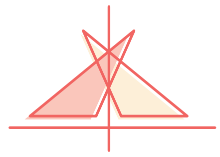
**Full Unit:** 21 days
● **Modified Unit:** 18 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



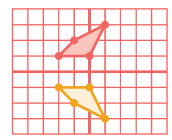
### 4 Grid Moves

Perform translations, rotations, and reflections on a grid by drawing and labeling the resulting image.



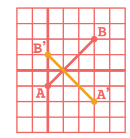
### 5 Making the Moves

Explain the sequence of transformations that maps one image onto another.



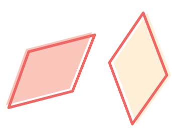
### 6 Coordinate Moves (Part 1) ●

Generalize the process for reflecting or translating any point on the coordinate plane.



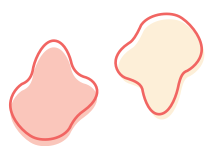

### 7 Coordinate Moves (Part 2) ●

Perform coordinate moves, now with rotations. Generalize the process for rotating any point on the coordinate plane.



### 11 Congruent Polygons

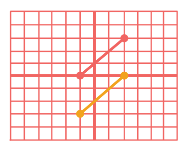
Determine whether two polygons are congruent.



### 12 Congruence (optional) ●

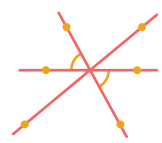
Explore ideas of congruence with other figures.

## Sub-Unit 3: Angles in a Triangle



### 13 Line Moves

Rotate a line segment $180^\circ$ using centers of the midpoint, a point on the segment, and a point not on the segment.



### 14 Rotation Patterns

Rotate angles to discover a new way to show that vertical angles have the same measure.

## Assessment

### A End-of-Unit Assessment

## ● Modifications to Pacing

**Lessons 2–3:** Early lessons on transformations help students see the movements without the restrictions of a grid. If pressed for time, you may choose to combine Lessons 2 and 3 and have students work with translations, rotations, and reflections in one lesson.

**Lessons 6–7:** Transformations with coordinates are intended to be taught over two lessons so that students have time to internalize these concepts, but if needed, these lessons can be combined into one.

**Lesson 12:** Lesson 12 may be omitted as no new standards are introduced. Consider adding non-polygons from Lesson 12 to Lesson 11.

# Unit Supports

## Math Language Development

| Lesson | New Vocabulary |
|---|---|
| 1 | tessellation |
| 2 | angle of rotation<br>center of rotation<br>rotation<br>translation |
| 3 | image<br>line of reflection<br>orientation<br>preimage<br>prime notation<br>reflection |
| 4 | transformation |
| 5 | sequence of transformations |
| 9 | rigid transformation |
| 10 | congruent |
| 15 | alternate interior angle<br>transversal |
| 17 | exterior angle<br>Triangle Sum Theorem |

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

| Lesson(s) | Mathematical Language Routines |
|---|---|
| 10 | **MLR1:** Stronger and Clearer Each Time |
| 1–3, 5, 7, 9, 10, 13, 15, 16 | **MLR2:** Collect and Display |
| 7 | **MLR3:** Critique, Correct, Clarify |
| 8 | **MLR4:** Information Gap |
| 12 | **MLR5:** Co-craft Questions |
| 1, 4–7, 12–14, 16 | **MLR7:** Compare and Connect |
| 3, 4, 6, 9, 11, 12–15, 17, 18 | **MLR8:** Discussion Supports |

## Materials

**Every lesson includes:**

Exit Ticket

Additional Practice

**Additional required materials include:**

| Lesson(s) | Materials |
|---|---|
| 1, 10, 17, 18 | colored pencils |
| 2–18 | geometry toolkits<br>• ruler<br>• protractor<br>• tracing paper<br>• index card |
| 8 | graph paper |
| 1 | pattern blocks |
| 1, 2, 4, 5, 7, 8, 16, 18 | PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs. |
| 1, 18 | plain sheets of paper |
| 1, 16 | scissors |

## Instructional Routines

Activities throughout this unit include the following instructional routines:

| Lesson(s) | Instructional Routines |
|---|---|
| 1, 2, 15, 18 | Notice and Wonder |
| 3 | Which One Doesn't Belong? |
| 4 | True or False |
| 6 | Partner Problems |
| 8 | Info Gap |
| 1, 16, 18 | Gallery Tour |
| 3, 10–12, 14, 16 | Poll the Class |
| 6, 7, 9, 12 | Think Pair Share |

# Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

| Assessments | When to Administer |
|---|---|
| **Pre-Unit Readiness Assessment**<br>This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit. | Prior to Lesson 1 |
| **Exit Tickets**<br>Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned. | End of each lesson |
| **Mid-Unit Assessment**<br>This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit. | After Lesson 8 |
| **End-of-Unit Assessment**<br>This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide. | After Lesson 18 |

**Amps**
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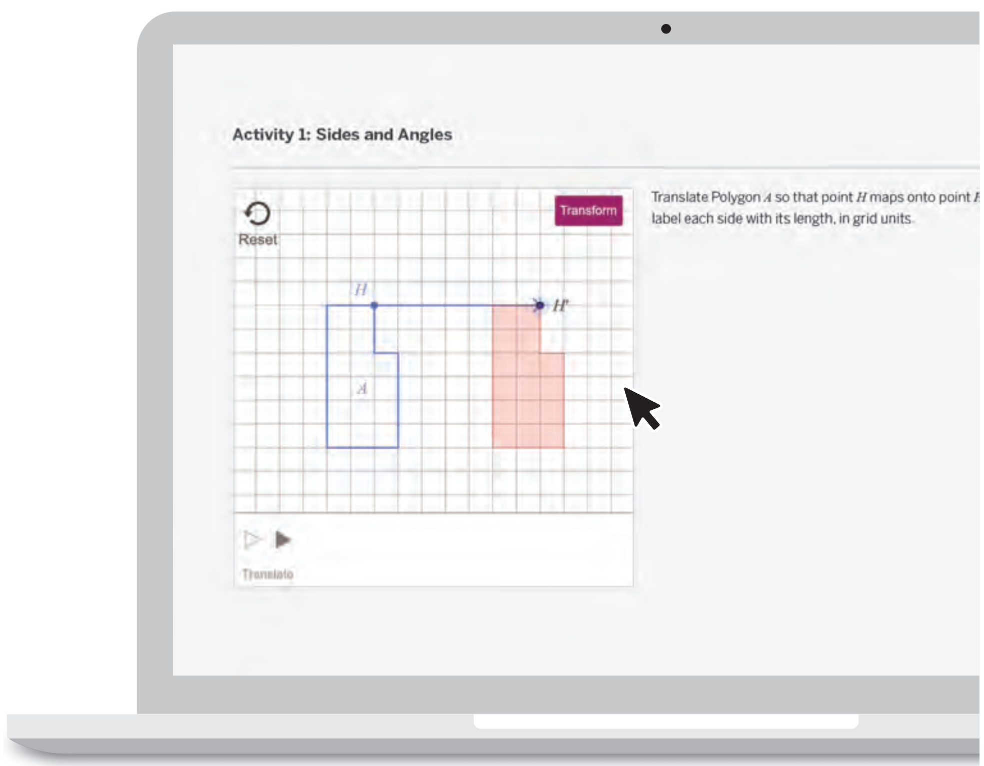
## Social & Collaborative Digital Moments

### Featured Activity

**Sides and Angles**

Put on your student hat and work through **Lesson 9, Activity 1:**




**Points to Ponder . . .**

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology into your classroom?

### Other Featured Activities:

- Frog Dance **(Lesson 2)**
- Rotations in Different Directions **(Lesson 7)**
- Digital Tessellations **(Lesson 1)**
- Transformation Golf **(Lessons 5–9)**

# Unit Study

## Professional Learning

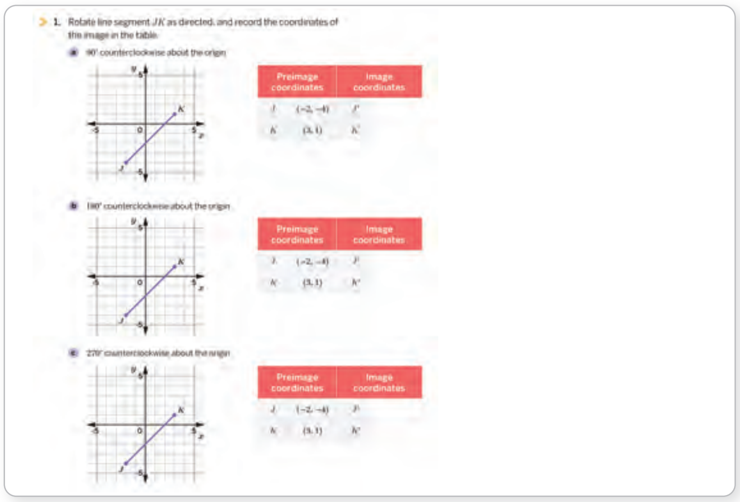
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

## Anticipating the Student Experience With Fawn Nguyen

**Sub-Unit 1** introduces three types of rigid transformations — translation, reflection, and rotation. Students learn to perform the different transformations on a preimage based on given instructions. In turn, they are asked to describe the transformation, or sequence of transformations, that would map a preimage to its image. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

### Do the Math

Put on your student hat and tackle these problems from **Lesson 7, Activity 2:**




Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

**Points to Ponder . . .**

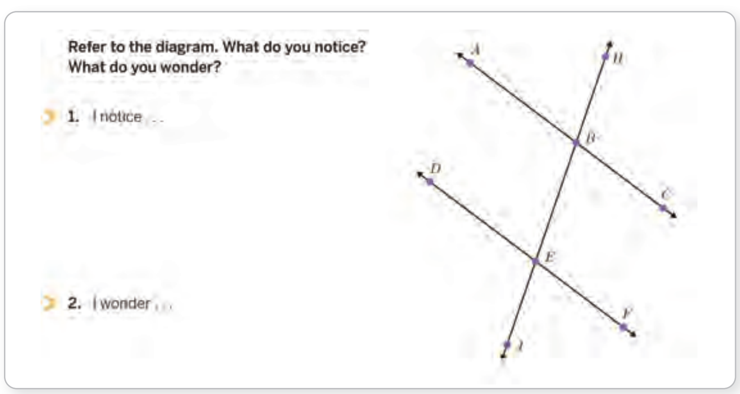
- What was it like to engage in this problem as a learner?
- Some students find rotations more challenging than other rigid transformations. What strategy did you use to complete this activity?
- What implications might this have for your teaching in this unit?

## Focus on Instructional Routines

### Notice and Wonder

**Rehearse . . .**

How you'll facilitate the ***Notice and Wonder*** instructional routine in **Lesson 15, Warm-up:**




**Points to Ponder . . .**

- What is the mathematical value of a good "I wonder . . ." statement? How can you encourage students to think deeply about these?

**This routine . . .**

- Makes a mathematical task accessible to all students with these two approachable questions.
- Provides students with an entry point into the mathematics and/or context of a problem.
- Piques students' curiosity about the mathematics and/or context of a problem.
- Helps students build their sense-making and observation skills.

**Anticipate . . .**

- What student statements will you be looking for as you monitor student progress during the Warm-up? How will you determine how to sequence those statements during the discussion?
- How can you help a student who does not know what to write for the "I notice . . ." or "I wonder . . ." prompts?
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

## Strengthening Your Effective Teaching Practices

**Establish mathematics goals to focus learning.**

### This effective teaching practice . . .

- Establishes clear goals for both you and your students so that you both know what the lesson is targeting. If you know the target, you know whether or not you hit it.
- Provides a benchmark, which will help you to make instructional decisions based on your students' performance.

### Points to Ponder . . .

- How will understanding the target goals for each lesson or activity help you when planning how to spend your instructional time?
- How can you use the lesson goals to know whether you need to redirect instruction or provide additional support?

## Math Language Development

### MLR7: Compare and Connect

MLR7 appears in Lessons 1, 4–7, 12–14, and 16.

- In Lesson 6, have students share strategies for finding the coordinates of images with the class, and then prompt students to reflect on the strategies of their peers.
- In Lesson 14, as students share what they noticed about the rotation of a line, ask them to consider what changes and what stays the same when $180^\circ$ rotations are applied to the figures. This will help them make deeper connections.
- **English Learners:** Use gestures to demonstrate what it looks like to slide, turn, or flip an object or figure.

### Point to Ponder . . .

- How can you help students make connections or comparisons to previous lessons or learnings that may be challenging for students to recall at first?

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Opportunities to vary the demands of a task or activity appear in Lessons 1–18.

- Chunking a complex task into smaller, more manageable parts allows students to focus on one part at a time.
- Providing pre-created copies of transformations instead of having students perform the physical transformations themselves, if the goal of the activity is to use the transformations to understand a connected mathematical concept.
- Some students may benefit from more processing time. When restricting the number of tasks or problems students need to complete, consider allowing them to choose which problem(s) to complete. Students are often more engaged when they have choice.

### Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to vary the demands of a particular task or activity? What clues will you gather from your students?

## Unit Assessments

Use the results of the Pre-Unit Readiness Assessment to understand your students' prior knowledge and determine their prerequisite skills.

### Look Ahead . . .

- Review and unpack the Mid- and End-of-Unit Assessments, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

### Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with performing and describing transformations in this unit? Do you think your students will generally:
  - have more difficulty with one of the transformations over the others?
  - struggle to be precise with their language? Or struggle to use their geometry toolkits effectively?
  - be unable to identify which transformations are part of a sequence of transformations?
  - find it more challenging to perform transformations or describe transformations?

## Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their social awareness and self-management.

### Points to Ponder . . .

- Are students able to set goals that help them show whether two figures are congruent? Can they stay focused on the task at hand, controlling their impulses in order to achieve that goal?
- Are students able to analyze each situation understanding the results of each choice of transformation? Can they evaluate their work to draw a conclusion about the congruence of figures?

# Tessellations

Let's discover patterns with shapes.

## Focus

### Goals

1. Create tessellations using pattern blocks or triangles.
2. **Language Goal:** Describe patterns in tessellations. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students look at historical examples to learn about tessellations. They apply what they discover about tessellations by using pattern blocks to make their own tessellations. Working with a partner, students then explore the relationship of triangles in tessellations as they consider that any type of triangle can be used to make a tessellation.

### Previously

Students began their study of shapes in kindergarten, learning about their names and attributes in later elementary grades. In Grade 5, students began classifying two-dimensional shapes based on their attributes. In Grade 6, students explored triangles in greater depth as they learned how to find the area and its relation to a rectangle.

### Coming Soon

Students will more formally describe and perform transformations of points, lines, and figures, discovering that some transformations create congruent figures. They will analyze and determine whether two figures are congruent by using rigid transformations or by measuring sides and angles. At the end of the unit, students will explore the relationship between intersecting lines and angles, and will consider the interior angles of a triangle in greater depth, as well as determine that any triangle can be used for a tessellation.

## Rigor

- Students experiment with slides, flips, and turns to build **conceptual understanding** of patterns among shapes.
- Students **apply** geometric patterns to artwork.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 3 min | 15 min | 15 min | 5 min | 8 min |
| Independent | Small Groups | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- plain sheets of paper
- colored pencils
- pattern blocks or *Pattern Blocks* PDF, pre-cut, one per group
- scissors

### Math Language Development

**New word**

- *tessellation*

**Review words**

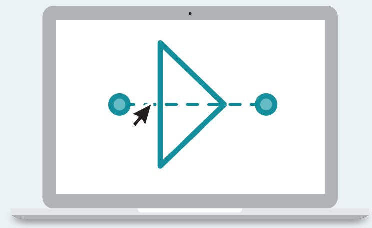
- *polygon*
- *quadrilateral*

**Amps** | Featured Activity

**Activity 1**

### Digital Tessellations

Students can create tessellations digitally using draggable pattern blocks on a virtual canvas. Consider printing them to post around the classroom.



Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Some students may compare themselves to their peers and think that their tessellations are not as artistic or neat as others in their small group. Announce beforehand that students will be entering these activities with a variety of artistic skills and interests. Point out that the goal of the lesson is for students to create their own unique tessellation, using the structure of the pattern blocks, that will not be judged on artistic ability, while still encouraging students to be as creative and neat as they can.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- **Activity 1** may be omitted.
- In **Activity 2**, have pre-cut triangles ready for students to use.

Independent | 3 min

## **Warm-up** Notice and Wonder

Students analyze a tessellation by M.C. Escher to build curiosity around the concept of tessellations and patterns of geometric shapes.

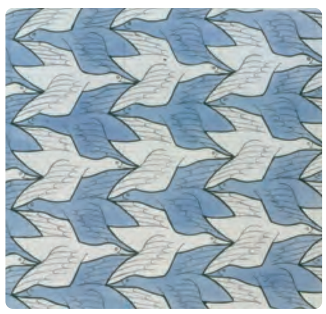
Unit 1 | Lesson 1 – Launch

### Tessellations

Let's discover patterns with shapes.

#### **Warm-up** Notice and Wonder

The artwork shown was created by the Dutch artist Maurits C. (M.C.) Escher (1898–1972). What do you notice? What do you wonder?



M.C. Escher's "Two Birds"© 2020 The M.C. Escher Company-The Netherlands. All rights reserved. www.mcescher.com

1. I notice . . .
   Sample responses:
   - There are repeated shapes or figures in the painting.
   - The shapes look like birds.
   - Some birds look like they are flying in one direction, and other birds are flying in the opposite direction.
   - The birds fit together perfectly. There are no gaps between birds and no overlap.
2. I wonder . . .
   Sample responses:
   - What inspired the artist to make this painting?
   - What other shapes or animals can fit a pattern like this?
   - Did the artist create any other drawings or paintings that are similar?

4 **Unit 1** Rigid Transformations and Congruence

Log in to Amplify Math to complete this lesson online.



### 1 Launch

Consider displaying other works by M.C. Escher to introduce the artist to students and pique curiosity. Conduct the ***Notice and Wonder*** routine using the artwork shown.

### 2 Monitor

**Help students get started** by asking them what repeating object or animal they see in the artwork.

**Look for points of confusion:**

- **Not realizing the birds flying in either direction are the exact same shape.** Have students trace a bird flying in one direction onto a sheet of paper and overlay it on top of a bird flying in the other direction.

**Look for productive strategies:**

- Noticing all of the birds are the exact same shape.
- Noticing that the birds fit together perfectly.
- Extending their thinking by asking themselves what other animals or objects could create similar patterns.

### 3 Connect

**Display** the image of M.C. Escher's artwork.

**Have students share** what they noticed and wondered about the artwork with a partner before sharing with the whole class.

**Highlight** student responses that connected M.C. Escher's work to geometric shapes or patterns. Then highlight student questions about the pattern and methods used to make the pattern.

**Define** a ***tessellation*** as any pattern made of repeating shapes that completely covers a plane, without any gaps or overlaps.

**Ask**, "How does M.C. Escher's artwork show a tessellation? How do you know that there are no gaps? No overlaps?"

## Differentiated Support

**Extension: *Interdisciplinary Connections***

Have students explore the M.C. Escher website created by the M.C. Escher Foundation and The M.C. Escher Company. Have them read M.C. Escher's biography and/or his route to fame. Alternatively, you may wish to read these sections with students or provide a summary. As time permits, allow them to explore the online gallery which contains selected works by M.C. Escher. Particular ones students may be interested in are the following categories: Most Popular, Mathematical, Impossible Constructions, and Transformation Prints. Consider having them choose one of his works and describe what they see, using their own words. **(Art)**

Small Groups | 15 min

# Activity 1 Tessellate

Students experiment with pattern blocks to create a tessellation to understand how a pattern of shapes can fill a plane without any gaps or overlaps.

**Amps Featured Activity** Digital Tessellations

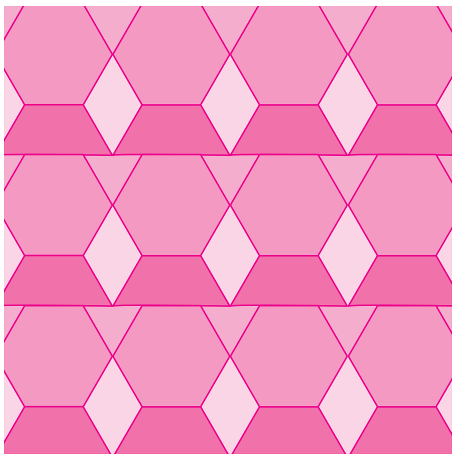
Name: ________________ Date: ________ Period: ________

**Activity 1** Tessellate

**You will be given a set of pattern blocks. Use them to create a tessellation of your own.**

Draw a sketch of your tessellation here.

Sample response shown.







## 1 Launch

Distribute pattern blocks, plain pieces of paper, and colored pencils. If pattern blocks are not available, distribute pre-cut shapes from the *Pattern Blocks* PDF. Show students an example of a completed tessellation using pattern blocks. Then show them a non-example, one with gaps or overlaps.

## 2 Monitor

**Help students get started** by having them show you how they can outline a block as their initial shape.

**Look for points of confusion:**

- **Thinking they must use only one type of a shape in their tessellations**. Tell students that they can use several different shapes and provide an example of tessellation that uses a triangle and a rhombus.
- **Not realizing that they can experiment with the position or orientation of the shapes.** Show an example of sliding a shape, flipping a shape, or rotating a shape to help students get started thinking about the different ways they can manipulate the shapes to create a tessellation.

## 3 Connect

**Display** several examples of student tessellations.

**Have students share** how they created their tessellations.

**Highlight** interesting strategies that were used or patterns that were created. Highlight ideas of shapes that have been "slid," "flipped," or "turned." Ask students to point out any examples of symmetry among the patterns.

**Ask**, "How can the pattern you made be used to fill the whole plane?"

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge, Optimize Access to Technology***

Consider demonstrating how to create a tessellation using the pattern blocks for students to reference. Alternatively, have students use the Amps slides for this activity, in which they can create tessellations digitally using draggable pattern blocks on a virtual canvas. Consider printing student work to display around the classroom.

**Extension: *Math Enrichment***

Have students create a second tessellation that highlights a different pattern among the pattern blocks.

## Math Language Development

***MLR7: Compare and Connect***

Display multiple examples of students' tessellations, and invite students to share what they notice. During the discussion, amplify language students use to communicate about geometric features of tessellations, e.g., no gaps or overlaps, or the pattern can be extended by sliding the shapes to the right or left.

**English Learners**

Use gestures to amplify language as students discuss geometric features and patterns.

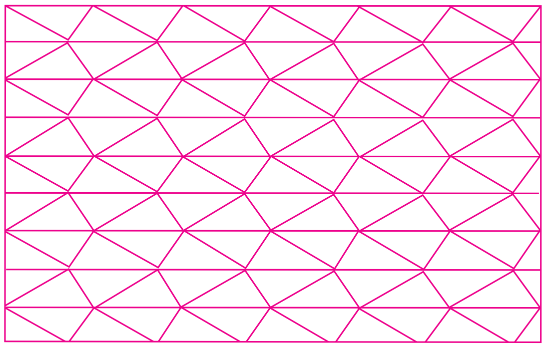
Pairs | 15 min

# Activity 2 Triangle Tessellations

Students create a tessellation using only triangles to see that any triangle can be used to create tessellations.

## Activity 2 Triangle Tessellations

1. You will be given a plain sheet of paper and scissors. Draw a triangle and cut it out. Exchange triangles with your partner, and create a tessellation using your partner's triangle. Draw a sketch of your tessellation here.
   **Sample response shown.**



2. Can you think of a triangle that does not work for making tessellations?
   **Sample response: No; regardless of what type of triangle I draw, I can always find a way to make a tessellation.**

**Collect and Display:** As you share your response, your teacher will add the math language you use to a class display. You will continue to add and refer to this display throughout the unit.

**Are you ready for more?**

1. Draw a quadrilateral and cut it out. Exchange quadrilaterals with your partner, and create a tessellation using your partner's quadrilateral.
2. Can you think of a quadrilateral that does not work for making tessellations?
   **Sample response: No; regardless of what type of quadrilateral I draw, I can always find a way to make a tessellation.**

STOP

6 **Unit 1** Rigid Transformations and Congruence



### 1 Launch

Distribute a sheet of plain paper to each student and a pair of scissors to each pair of students.

### 2 Monitor

**Help students get started** by having students outline their partner's triangle on their own paper.

**Look for points of confusion:**

- **Not understanding how to create a tessellation using their partner's triangle.** Have students slide, flip, or turn the triangle until they can create a shape made up of several triangles which then can be repeated.

**Look for productive strategies:**

- Attempting to create tessellations using different kinds of triangles, e.g., acute, obtuse, right, equilateral, isosceles, or scalene.
- Making a conjecture that any triangle can be used to create a tessellation.

### 3 Connect

**Display** students' tessellations around the room, and conduct the ***Gallery Tour*** routine so that students can view each other's tessellations.

**Have students share** the different strategies they used to create their tessellations. Select different students who used different strategies, such as sliding, flipping, or turning the triangle to create different patterns.

**Ask**, "Can you think of a triangle that cannot be used to create a tessellation?" If students claim there is a triangle that cannot be used to create a tessellation, have them draw it and ask the class to attempt to create a tessellation.

**Highlight** that at the end of the unit, students will have an opportunity to prove whether any triangle can be used to create a tessellation.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Provide pre-cut triangles for students to manipulate and arrange without having to draw their own triangles and cut them out.

### Extension: *Math Enrichment*

Let students know that a shape that can create a tessellation on its own, without gaps or overlaps, is a shape that can *tessellate the plane*. Have students explore other shapes to determine if they can tessellate the plane. Ask them to think of a shape that cannot tessellate the plane. Sample response: A circle cannot tessellate the plane because it is impossible to place circles next to each other without gaps.

## Math Language Development

### *MLR2: Collect and Display*

As students share strategies used to create their tessellations, create a class display to collect and display language used to describe *sliding*, *flipping*, and *turning* the tesselations. Encourage students to refer to this class display in future discussions about transformations in this unit.

### English Learners

Use gestures to emphasize what it looks like to slide, turn, and flip.

Whole Class | 5 min

# Summary The Art of Transformation

Review the curiosity and perseverance involved in creating tessellations, and pique student excitement for the upcoming unit.

Narrative Connections

Unit 1 Rigid Transformations and Congruence

## The Art of Transformation

Her name was Marjorie Rice, and she had a secret.

On the outside, she looked like any suburban housewife. She had a home in San Diego, a husband, and five children. But while her husband was at work, and the children were at school, Rice's other life emerged.

She'd already been caught several times, sketching at the kitchen counter. But what might have been dismissed as idle doodling was something more. Marjorie Rice was on the hunt for pentagons.

It began when she read an article in *Scientific American* on ***tessellations***. It had long been believed that every pentagon that could be tessellated had already been documented. But the article announced the discovery of new tessellating pentagons uncovered by a computer scientist living in California.

Inspired, and with barely any math education, Rice set off to work.

Over the 1975 holiday season, she worked tirelessly, at her kitchen table, hiding her work from family and friends. In February 1976, she wrote to the article's author, Martin Gardner, and showed him her discoveries. Gardner then forwarded her findings to Dr. Doris Schattschneider, a mathematician at Moravian College who confirmed her work:

Marjorie Rice had found a new tessellating pentagon.

By 1977, Rice would discover three more tessellating pentagons, and nearly 60 unique patterns for tiling them. Using her ingenuity and intuition on how to use transformations to duplicate shapes, Marjorie Rice shook the math world — reminding them there's always room for an amateur.

**Welcome to Unit 1.**





### Narrative Connections

Read the narrative aloud as a class or have students read it individually.

### Synthesize

**Display** the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

**Have students share** what they learned about tessellations and the geometric figures used to create tessellations. Then have them share what they hope to learn more about in this unit.

**Highlight** that students will continue to explore patterns with geometric shapes and figures in this unit. Mention that students can turn in their completed tessellations or continue working on them outside of class. Consider posting them around the room for the duration of this unit.

**Formalize vocabulary: *tessellation***

### Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. To help students engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful today when creating a tessellation?"
- "Were any strategies or tools not helpful? Why?"

## Math Language Development

### *MLR2: Collect and Display*

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this lesson. Ask them to review and reflect on any terms and phrases related to the term *tessellation* that were added to the display during the lesson.

# Exit Ticket

Students demonstrate their understanding of tessellations by recreating a butterfly pattern from one of Majorie Rice's tessellations.

Printable

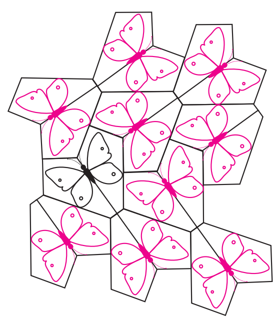
Name: ______________ Date: ________ Period: ________

## Exit Ticket 1.01

Here is an example of the pentagon tessellations Marjorie Rice discovered and how it can be used to make butterfly patterns.


Majorie Rice

Recreate Marjorie Rice's tessellation pattern using this butterfly pattern started for you.
See students' work.



**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can create my own tessellation using pattern blocks.
1 2 3

b. I can create a tessellation from any type of triangle.
1 2 3


Lesson 1 Tessellations

## Success looks like . . .

- **Goal:** Creating tessellations using pattern blocks or triangles.
  - » Recreating the tessellation pattern.
- **Language Goal:** Describing patterns in tessellations. **(Speaking and Listening, Writing)**

## Suggested next steps

**If students are unable to identify the pattern, consider:**

- Showing how the butterfly shape is composed of two tessellated pentagons.
- Reviewing Activity 2.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Point to Ponder . . .

- What different strategies did your students use in creating tessellations? Were some of your students more comfortable trying new strategies? How can you encourage all of your students to try new strategies and ideas?

# Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. Imagine a friend, family member, or future student who has not yet learned about tessellations. What would you do and say to teach them about tessellations?

   Sample response: I would have them use a pattern block and make repeated shapes that do not overlap and that touch on all sides. I would have them use many different colors. If multiple people were creating these tessellations, then I would have an art gallery tour so everyone could see each other's patterns.

2. Below is one example of the many complex tile patterns found in the famous 14th century Moorish palace of Alhambra, located in Spain. M.C. Escher visited the palace before making many of his drawings and paintings. Describe what you see.

Anneke Bart/SLU

   Sample response: I see some examples of repeated use of a polygon, similar to a tessellation.





Name: ______ Date: ______ Period: ______

Practice

3. Compare each of the following values using the symbols >, =, or <.

   a. $4$ > $-4$
   b. $11$ < $15$
   c. $-11$ > $-15$
   d. $-(-6)$ = $6$
   e. $8.01$ > $8$
   f. $-8.01$ < $-8$
   g. $-2.5$ = $-\frac{10}{4}$
   h. $-\frac{2}{3}$ > $-\frac{3}{2}$
   i. $-\frac{1}{5}$ = $-\frac{2}{10}$
   j. $-\frac{3}{4}$ < $\frac{3}{4}$

4. Rectangle $ABCD$ is drawn on a coordinate plane.

   a. If point $A$ is placed at $(2, 3)$, point $B$ at $(4, 3)$, and point $C$ at $(4, -3)$, what could be the location of point $D$?

      Point $D$ could be located at $(2, -3)$.

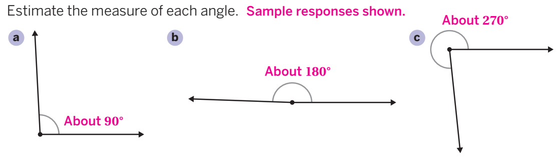
   b. Find the length of segment $CD$.

      2 units

   c. Find the area and perimeter of Rectangle $ABCD$.

      The area is 12 square units, and the perimeter is 16 units.

5. Estimate the measure of each angle. Sample responses shown.








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 1 | 2 |
| Spiral | 3 | Grade 6 | 1 |
| | 4 | Grade 6 | 2 |
| Formative | 5 | Unit 1 Lesson 2 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Sub-Unit 1
# Rigid Transformations

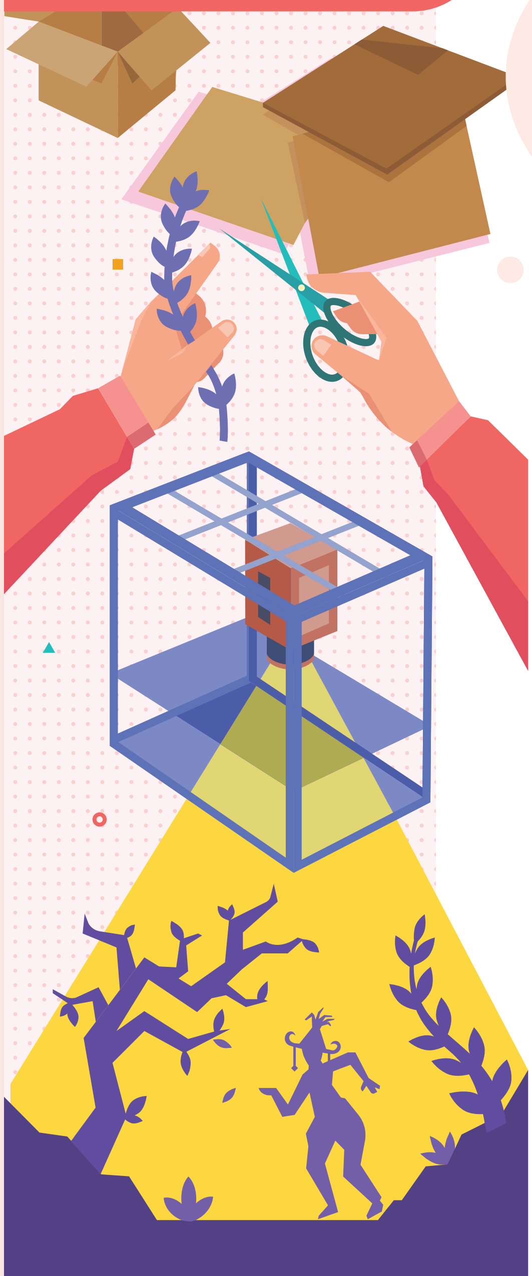
Whole Class

Students begin by studying examples of transformations in the plane. Then, students attend to precision with transformations using the structure of a grid and the coordinates of points.

SUB-UNIT

1 Rigid Transformations

Narrative Connections



## How do you make a piece of cardboard come alive?

### Before Walt Disney, there was Lotte Reiniger.

As a girl living in Berlin, Reiniger was clever with a pair of scissors. She cut intricate figures out of the cardboard from old soap boxes. For many kids, this was a way to pass the time. But for Reiniger, it was something more.

Her interest in puppets led her into the world of German art and cinema. By the time she was twenty, she started making her own films.

Her most famous achievement is *The Adventures of Prince Achmed*. It was the world's first animated full-length feature film — ten years before Disney's *Snow White.*

With a staff of just five people, Reiniger constructed elaborate paper puppets. Then, using a camera of her own invention, she would lay the puppets out and change their position frame-by-frame. It was a long and tedious process, but when you ran the images through a film projector, it came out as a single fluid movement.

By changing the position of solid figures, Reiniger turned a piece of cardboard into a flapping wing, a gesturing arm, or a sorcerer casting a spell. With only a pair of scissors, her imagination, and clever uses of geometric transformations, Reiniger changed the world of animation forever.





### Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students will continue to see the connections between animation and step-by-step, geometric transformations in the following places:

- **Lesson 2, Warm-up:** Notice and Wonder
- **Lesson 2, Activity 1:** Frog Dance
- **Lesson 5, Activity 1:** Make that Move

# Moving on the Plane

Let's describe ways figures can move on the plane.

## Focus

### Goals

1. **Language Goal:** Describe the movement of figures informally and formally using the terms *clockwise*, *counterclockwise*, *translation*, and *rotation.* **(Speaking and Listening, Writing)**

2. **Language Goal:** Identify the features that determine a translation or rotation. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students are introduced to movements of figures on a plane. They use informal language to describe the movements, and then are introduced to the formal mathematical language, *translation* and *rotation*. Students attend to precision when describing these movements of figures.

### Previously

In Lesson 1, students created tessellations using pattern blocks and triangles. They informally described the patterns found in their tessellations, and in tessellations from works of art and famous math historians.

### Coming Soon

In Lesson 3, students will learn the features that classify a reflection on a plane and use precise mathematical language to describe the reflection.

## Rigor

- Students build **conceptual understanding** of how figures can slide or turn on the plane.
- Students build **fluency** in using precise mathematical vocabulary to describe translations and rotations.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 20 min | 13 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per pair
- Activity 1 PDF, *Translations* and *Rotations*, for display
- geometry toolkits: rulers, tracing paper, protractors (optional)

### Math Language Development

**New words**

- ***angle of rotation***
- ***center of rotation***
- ***rotation***
- ***translation***

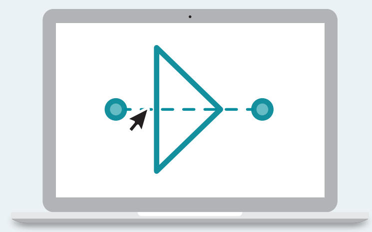
**Review words**

- *clockwise*
- *corresponding*
- *counterclockwise*
- *vertex*

**Amps** | **Featured Activity**

### Activity 2
### Interactive Geometry

Students view an animation of their predicted response (translation or rotation), giving them a chance to reflect and revise as needed.



**Amps** POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students may struggle to describe the precise moves of the Frog Dance using their developing math language. Have them ask clarifying questions, consider their partner's perspective, and be aware of their partner's thoughts and feelings in order to strengthen the effectiveness of communication.

### Modifications to Pacing

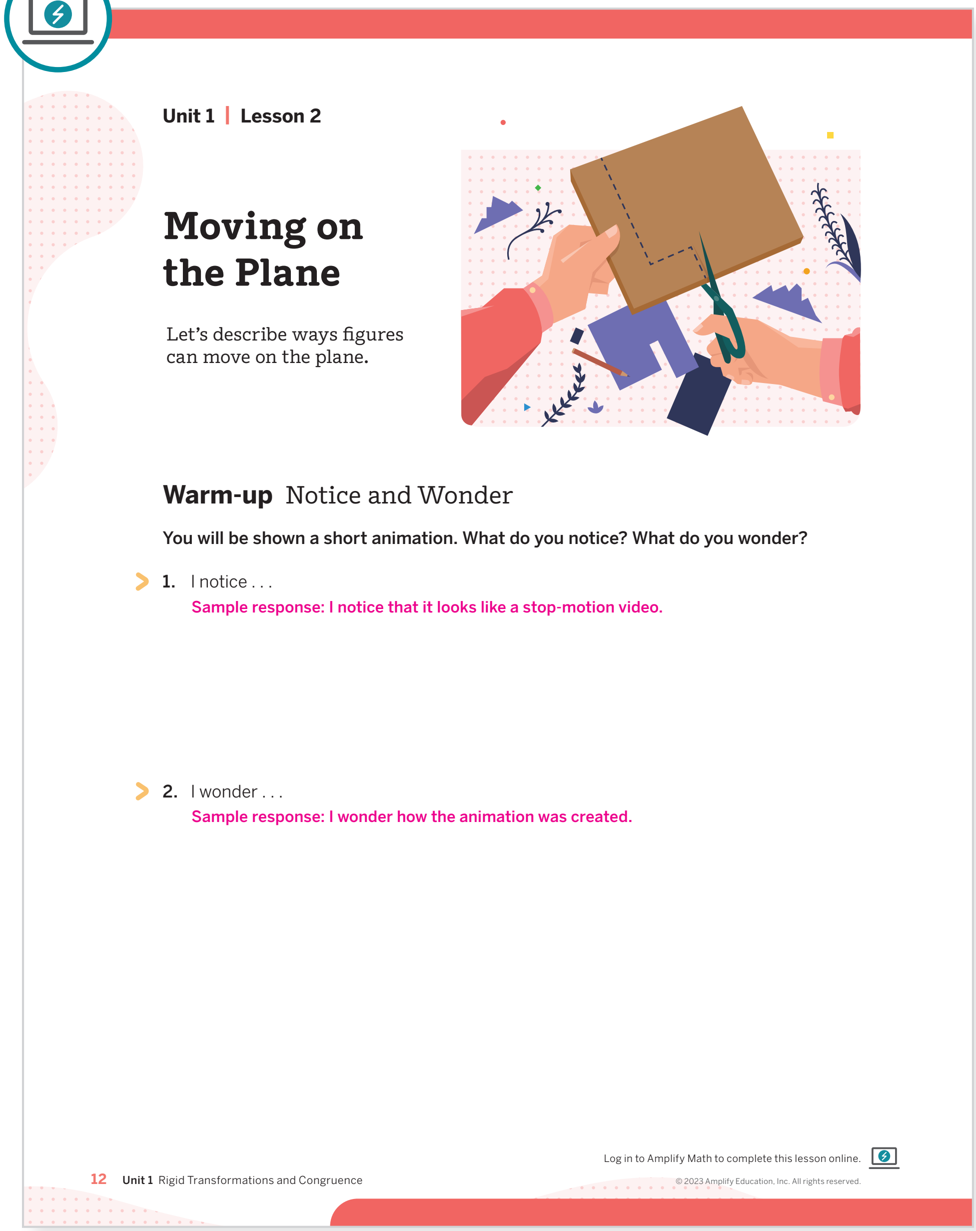
You may want to consider this modification if you are short on time.

- In **Activity 1**, have students work in pairs, and choose one dance move to describe.

Independent | 5 min

## **Warm-up** Notice and Wonder

Students watch an animation as an introduction to movement of figures on the plane.


Unit 1 | Lesson 2

# Moving on the Plane

Let's describe ways figures can move on the plane.

**Warm-up** Notice and Wonder

You will be shown a short animation. What do you notice? What do you wonder?

1. I notice . . .
   Sample response: I notice that it looks like a stop-motion video.

2. I wonder . . .
   Sample response: I wonder how the animation was created.

Log in to Amplify Math to complete this lesson online.

12 Unit 1 Rigid Transformations and Congruence



### 1 Launch

Have students watch Lotte Reiniger's Prince Frog 1961 video. Conduct the ***Notice and Wonder*** routine using the animation.

### 2 Monitor

**Help students get started** by asking them what part of the animation stands out to them.

**Look for productive strategies:**

- Noticing the rigid movements of the objects in the animation.
- Noticing the animation resembles a stop-motion or claymation video.

### 3 Connect

**Have students share** what they notice and wonder. Record responses for all to see.

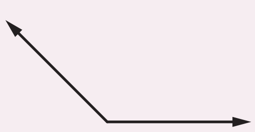
**Ask**, "What math do you see in the animation?" How do you think Lotte Reiniger created her animation?"

**Highlight** that Lotte Reiniger often used silhouette movements in her animations. She was able to use this technique because the parts of the characters stay the same, but the positions change. These types of moments are made using different types of symmetry.

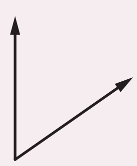
## Power-up

**To power up students' ability to estimating angles, have students complete:**

Recall that a circle measures 360°, a straight line measures 180°, and a right angle measures 90°. For each angle determine if it measures greater or less than 90°, then approximate its measure.

1. 

   a. Greater or less than 90°?
   Greater than 90°
   
   b. Approximate measure:
   Sample response: About 135°

2. 

   a. Greater or less than 90°?
   Less than 90°
   
   b. Approximate measure:
   Sample response: About 55°

**Use:** Before Activity 2
**Informed by:** Performance on Lesson 1, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 7

Pairs | 20 min

# Activity 1 Frog Dance

Students observe and describe movements of figures using informal language, and then are introduced to the precise mathematical language: *rotation* and *translation*.

Name: ________ Date: ________ Period: ________

## Activity 1 Frog Dance

Plan ahead: How will you use this opportunity to build a relationship with your partner?

**You will be given a sheet with three sets of dancing frog images.**

1. Arrange the sheet so that you and your partner can both see them right-side up. Choose one player to start the game.
   - The starting player mentally chooses Dance A, B, or C and describes the dance to the other player.
   - The other player identifies the dance as Dance A, B, or C, based on the starting player's description.
2. After one round, trade roles. When you have described all three dances, come to an agreement on the words or phrases you can use to describe the moves in each dance.
3. Complete the tables on this and the next page to write a final description of the moves for each dance.

Sample responses are shown.

Dance A:

| From . . . | To . . . | Description of moves |
|---|---|---|
| Frame 1 | Frame 2 | The frog moves (or slides) to the right. |
| Frame 2 | Frame 3 | The frog turns (or rotates) to the side, so the crown is facing the right. |
| Frame 3 | Frame 4 | The frog stays on its side, but moves or (slides) up. |
| Frame 4 | Frame 5 | The frog moves (slides) to the left. |
| Frame 5 | Frame 6 | The frog turns (or rotates) to the side, so the crown is facing up. |





## 1 Launch

Distribute the Activity 1 PDF to each pair of students.

## 2 Monitor

**Help students get started** by asking students what words they can use to describe the movement from Frame 1 to Frame 2.

**Look for points of confusion:**

- **Not understanding what words they can use to describe the movements.** Ask them if they are thinking from the perspective of an observer or from the perspective of the frog. Have them think of words from their everyday lives that describe movement, such as "move to the right," "turn," etc.
- **Not describing the movements with enough detail.** Ask them if the frog is facing the same direction each time and where in the square the frog is located.

**Look for productive strategies:**

- Using the words *slide* or *turn* to describe translations and rotations, respectively.
- Precisely describing how the frog is sliding or turning.

Activity 1 continued ›

## Differentiated Support

**Accessibility: *Guide Visualization and Processing, Optimize Access to Tools, Vary Demands to Optimize Challenge***

Complete Dance A together as a class and demonstrate — or ask a student volunteer to demonstrate — the frog's movements by using hand gestures or an inanimate object. Provide access to colored pencils or highlighters for students to mark the location of the crown, or other identifier, to assist them in tracking the frog's movements.

## Math Language Development

***MLR2: Collect and Display***

Collect and add to the class display the new vocabulary terms *translation* and *rotation*. Connect these to the previously collected terms *slide* and *turn*.

**English Learners**

When discussing the definition of a *rotation*, the term about is likely to be unfamiliar in this context to many students. Highlight that rotating something *about* a point means to rotate it *around* a point.

Pairs | 20 min

# Activity 1 Frog Dance (continued)

Students observe and describe movements of figures using informal language, and then are introduced to the precise mathematical language: *rotation* and *translation*.

**Activity 1** Frog Dance (continued)

Dance B:

| From . . . | To . . . | Description of moves |
|---|---|---|
| Frame 1 | Frame 2 | The frog moves (or slides) to the right. |
| Frame 2 | Frame 3 | The frog turns (or rotates) to the side, so the crown is facing the right. |
| Frame 3 | Frame 4 | The frog stays on its side, but moves (or slides) to the left. |
| Frame 4 | Frame 5 | The frog moves (or slides) up. |
| Frame 5 | Frame 6 | The frog turns (or rotates), so the crown is facing up. |

Dance C:

| From . . . | To . . . | Description of moves |
|---|---|---|
| Frame 1 | Frame 2 | The frog moves (or slides) to the right. |
| Frame 2 | Frame 3 | The frog turns (or rotates) to the side, so the crown is facing the left. |
| Frame 3 | Frame 4 | The frog moves (or slides) to the left. |
| Frame 4 | Frame 5 | The frog moves (or slides) up. |
| Frame 5 | Frame 6 | The frog turns (or rotates), so the crown is facing up. |

14 Unit 1 Rigid Transformations and Congruence



## 3 Connect

**Have pairs of students share** their final descriptions for each dance. Record phrases that students used in two categories, those that describe translations and those that describe rotations.

**Display** the Activity 1 PDF, *Translations and Rotations*.

**Define** a ***translation*** as a movement that slides a figure without turning it. Then define a ***rotation*** as a movement that turns a figure a certain angle (called the ***angle of rotation***) about a point (called the ***center of rotation***).

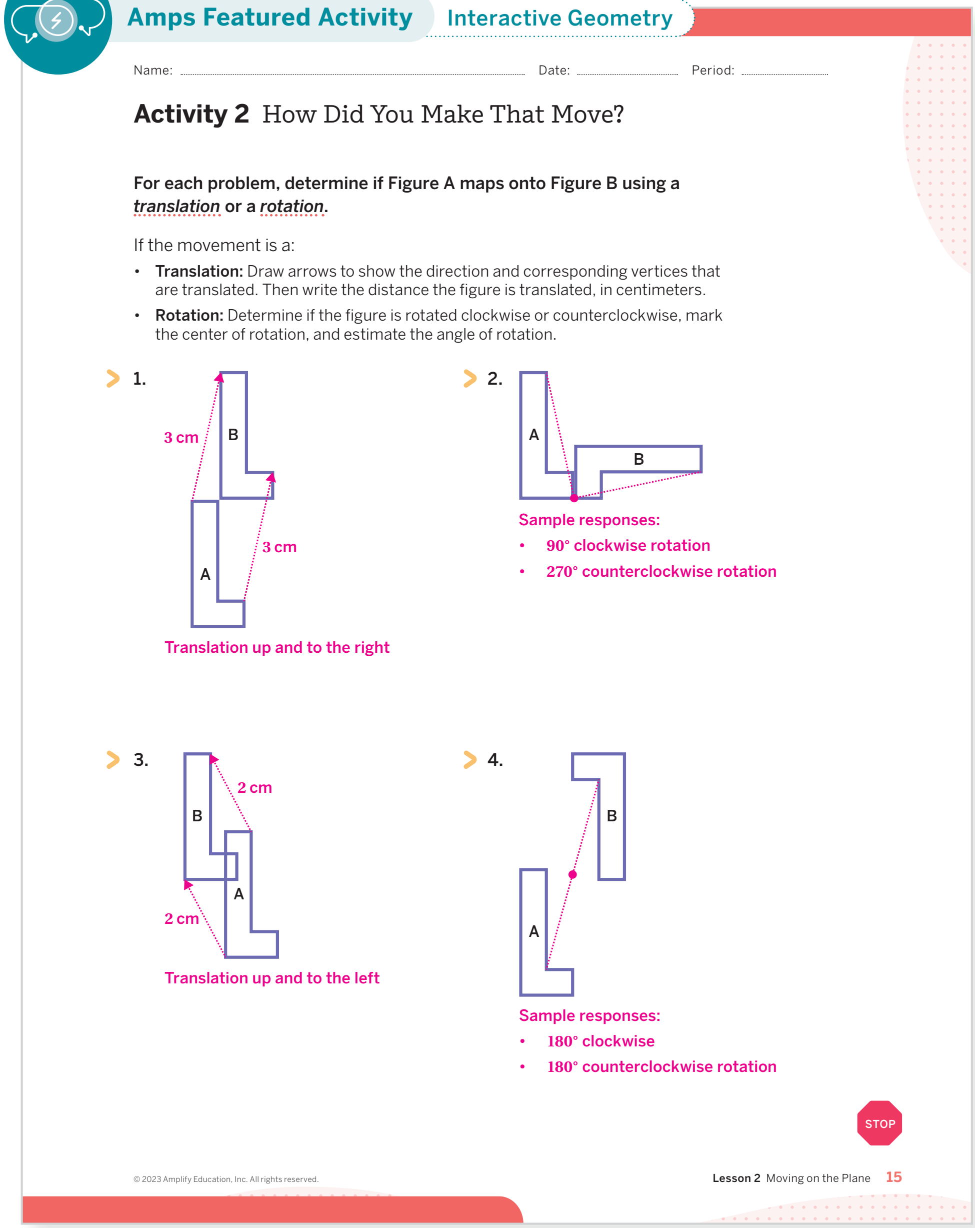
**Highlight** that in a translation, each point in the figure moves the same distance in the same direction. The matching point in the original figure and translated figure are called *corresponding points*. In a rotation, each point in the figure travels along a circle around the center, forming the same angle. To describe a rotation, students need to provide the direction, *clockwise* or *counterclockwise*, the center of rotation, and the angle of rotation, usually measured in degrees.

Pairs | 13 min

# Activity 2 How Did You Make That Move?

Students identify and describe a translation or rotation to practice using precise language when describing these moves.

Amps Featured Activity | Interactive Geometry



Name: Date: Period:

## Activity 2 How Did You Make That Move?

**For each problem, determine if Figure A maps onto Figure B using a *translation* or a *rotation*.**

If the movement is a:

- **Translation:** Draw arrows to show the direction and corresponding vertices that are translated. Then write the distance the figure is translated, in centimeters.
- **Rotation:** Determine if the figure is rotated clockwise or counterclockwise, mark the center of rotation, and estimate the angle of rotation.

1.



Translation up and to the right

2.



Sample responses:

- 90° clockwise rotation
- 270° counterclockwise rotation

3.



Translation up and to the left

4.



Sample responses:

- 180° clockwise
- 180° counterclockwise rotation

STOP





## 1 Launch

As students translate figures for the first time in Problems 1 and 3, allow them to describe the translations that include diagonal lines. In later lessons, students will describe translations using a combination of vertical and horizontal lines. Provide access to geometry toolkits.

## 2 Monitor

**Help students get started** by having them trace Figure A onto tracing paper. Then tell them to move the tracing paper so that Figure A maps onto, or matches, Figure B.

**Look for points of confusion:**

- **Struggling to identify the center of rotation.** Have students use their pencil to hold down the tracing paper and test different centers of rotation, while turning the tracing paper in a circular motion to map Figure A onto Figure B.
- **Not understanding how to identify the angle of rotation.** Have students draw the angle using corresponding points on Figure A and Figure B and the center of rotation as the vertex. Then have them estimate the angle or use their protractor to find the exact angle.

**Look for productive strategies:**

- Noticing Problem 2 can be rotated 90° clockwise or 270° counterclockwise.
- Noticing an 180° clockwise rotation has the same result as an 180° counterclockwise rotation.

## 3 Connect

**Ask**, "Can there be more than one response for Problem 2? Problem 4?"

**Highlight** that when describing a translation, both direction and distance need to be provided. When describing a rotation, the direction, the center of rotation, and the angle of rotation need to be provided.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1 and 2, and only work on Problems 3 and 4 as time allows. Additionally, to assist students with organizational skills, create a checklist with the features needed to describe rotations and translations. Have students refer to this checklist each time they need to describe these movements.

### Extension: *Math Enrichment*

Have students draw figures that show both a translation and a rotation. Have them trade papers with a partner and then record the precise language that describes each movement.

Whole Class | 5 min

# Summary

Review and synthesize the mathematical language used to describe how figures move on a plane (translations and rotations).

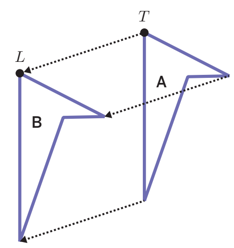
## Summary

**In today's lesson . . .**

You described how a figure moves in a plane.

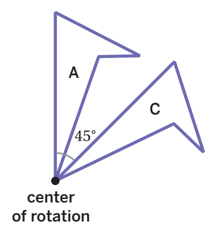
A ***translation*** slides a figure without turning it. Every point in the figure moves the same distance in the same direction. A translation can be described by two points.

- For example, if a translation maps point $T$ onto point $L$, it moves the entire figure the same distance and direction as the distance and direction from point $T$ to point $L$. The distance and direction of a translation can be shown by an arrow.
- Figure A was translated down and to the left, as shown by the arrows. Figure B is a translation of Figure A.




A ***rotation*** turns a figure about a point, called the ***center of rotation***. Every point on the figure travels along the path of a circle around the center of rotation to form the ***angle of rotation***. The rotation can be:

- **clockwise:** traveling in the same direction as the hands of a clock, or
- **counterclockwise:** traveling in the opposite direction as the hands on a clock.




A rotation can be described by an angle, a center, and the direction of the rotation.

- For example, Figure A was rotated 45° clockwise around the center of rotation shown. Figure C is a rotation of Figure A.

If one point on the original figure moves to another point on the new figure, they are *corresponding points*.

Reflect:




## Synthesize

**Have students share** how they describe translations and rotations in their own words. Some students may call translations “slides” and rotations “turns”.

**Ask:**

- “What do you need to include when describing a translation?” distance and direction; For example, translate a figure 3 units to the right.
- “What do you need to include when describing a rotation?” direction, center of rotation, and angle of rotation; For example, rotate a figure 90° counterclockwise about point $A$.

**Display** the Summary from the Student Edition.

**Formalize vocabulary:**

- *angle of rotation*
- *center of rotation*
- *rotation*
- *translation*

**Highlight** that a translation or rotation changes the position of a figure without changing its shape or size. Students can choose any pair of corresponding points to describe the movement.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- “What happens to a figure as you move it around a two-dimensional plane?”

## Math Language Development

### *MLR2: Collect and Display*

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *angle of rotation*, *center of rotation*, *rotation*, and *translation* that were added to the display during the lesson.

Independent | 5 min

## Exit Ticket

Students demonstrate their understanding of translations and rotations by precisely describing each movement of a figure.

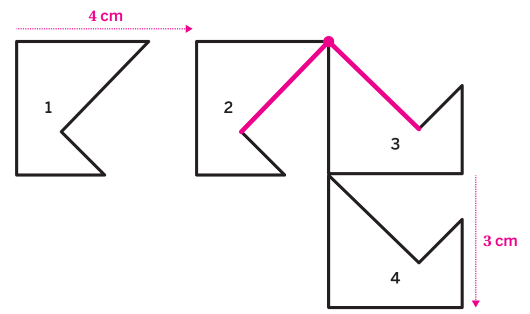
Printable

Name: ______________ Date: ________ Period: ________

### Exit Ticket 1.02

**Several movements of a figure are shown. For each movement:**

- **Decide whether it shows a translation or rotation.**
- **Show or precisely describe the movement. Include the distance, in centimeters, and angle of rotation, if applicable. You will need a centimeter ruler.**




1. Movement from Figure 1 to Figure 2:
   **Translation; 4 cm to the right**

2. Movement from Figure 2 to Figure 3:
   **Rotation; 90° counterclockwise or 270° clockwise about the marked point**

3. Movement from Figure 3 to Figure 4:
   **Translation; 3 cm down**

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can describe how a figure slides and turns to move from one position and/or location to another.
1 2 3

b. I know the difference between a translation and rotation.
1 2 3





### Success looks like . . .

- **Language Goal:** Describing the movement of figures informally and formally using the terms *clockwise*, *counterclockwise*, *translation*, and *rotation.* **(Speaking and Listening, Writing)**
  - » Describing the movement from Figure 2 to Figure 3 with appropriate vocabulary in Problem 2.
- **Language Goal:** Identifying the features that determine a translation or rotation. **(Speaking and Listening, Writing)**

### Suggested next steps

**If students do not describe all of the movements of the figure or do not include the distance or angle measure, consider:**

- Providing a checklist for students to use as a reminder of what it means to precisely describe a figure's movement.
- Assigning Practice Problem 3.
- Reassessing after Lesson 4.

### Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- What did students find frustrating about describing the movements in the Frog Dance? What helped them work through this frustration?
- How did you encourage each student to listen to one another's descriptions?

### Math Language Development

**Language Goal: Describing the movement of figures informally and formally using the terms *clockwise, counterclockwise, translation*, and *rotation*.**

Reflect on students' language development toward this goal.

- How did students begin to informally describe the movement of figures in this lesson? What language did they use?
- How has their use of language progressed after being introduced to the terms *clockwise*, *counterclockwise*, *translation*, and *rotation*? How can you support them in using their developing math language?

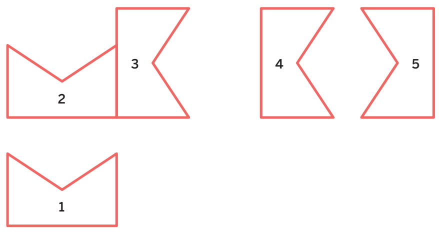
# Practice

Independent

Name: ______________________ Date: __________ Period: __________

Practice

1. You will need a centimeter ruler. The diagram shows several moves of a figure. Determine if each statement is *true* or *false*.




a. From Figure 1 to Figure 2, the figure is translated 3 cm up.
**True**

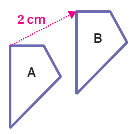
b. From Figure 2 to Figure 3, the figure is rotated $90°$ counterclockwise.
**False**

c. From Figure 3 to Figure 4, the figure is translated 4 cm to the right.
**True**

d. From Figure 4 to Figure 5, the figure is rotated $180°$ clockwise.
**True**

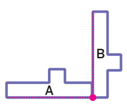
2. For each movement from Figure A to Figure B:
   - Decide whether it shows a translation or rotation.
   - Show or precisely describe the movement. Include the distance, in centimeters, and angle of rotation, if applicable.

a.



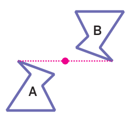

**Translation up and to the right**

b.




**$90°$ clockwise or $270°$ counterclockwise rotation about the marked point**

c.




**$180°$ rotation about the marked point**

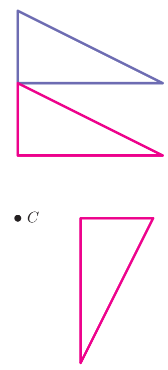




Name: ______________________ Date: __________ Period: __________

Practice

3. You will need a centimeter ruler. Translate the triangle shown 2 cm down, and then rotate the newly translated triangle $90°$ clockwise about point $C$.




4. Evaluate each expression.

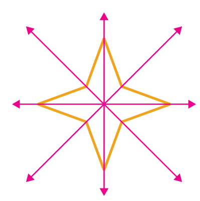
a. $-5 \bullet (-2.4)$ **$= 12$**

b. $-7.4 \div 10$ **$= -0.74$**

c. $-\frac{4}{7} \div (-2)$ **$= \frac{2}{7}$**

d. $4 \bullet \left(-\frac{3}{8}\right)$ **$= -\frac{3}{2}$**

5. Draw *all* the lines of symmetry for the figure shown. How many total lines of symmetry are there?
**4 lines**







## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Grade 7 | 1 |
| Formative | 5 | Unit 1 Lesson 3 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Symmetry and Reflection

Let's describe ways figures reflect on the plane.

## Focus

### Goals

1. **Language Goal:** Describe the movement of figures informally and formally using the terms *reflection*, *line of reflection*, *image*, and *preimage*. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Identify the features that determine a reflection. **(Speaking and Listening, Reading and Writing)**

## Coherence

### Today

Students begin by studying different figures to review lines of symmetry. They move into drawing and measuring reflected triangles, coming to understand that the line of reflection lies halfway between the two triangles and is perpendicular to the line segments that connect the corresponding vertices.

### Previously

In Lesson 2, students described the features that identified translations and rotations.

### Coming Soon

In Lesson 4, students will translate, reflect, and rotate figures on a grid.

## Rigor

- Students build **conceptual understanding** of how figures can be flipped or reflected on a plane.
- Students build **fluency** in using precise mathematical vocabulary to describe reflections.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Activity 3 | Summary | Exit Ticket |
|---|---|---|---|---|---|
| 5 min | 15 min | 8 min | 8 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice
Independent

### Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: rulers, tracing paper, protractors (optional)

### Math Language Development

**New words**

- *image*
- *line of reflection*
- *orientation*
- *preimage**
- *prime notation*
- *reflection*

**Review words**

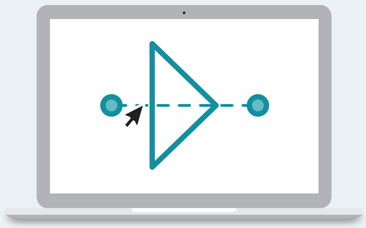
- *corresponding points*
- *perpendicular*
- *symmetry*
- *vertex*

*Students may confuse *preimage* and *image* throughout the unit when discussing the original image and the transformed image. Highlight the prefix *pre* in *preimage* indicates the original image.

**Amps** | **Featured Activity**

### Activity 1
### Real-Time Reflections

When students adjust the line of reflection, an animation shows the reflected image, giving students an opportunity to revise their response, if needed.



Amps POWERED BY desmos

### Building Math Identity and Community
Connecting to Mathematical Practices

Students may not want to make the effort required to use precise units and measuring tools to measure the exact distance of corresponding points to the line of reflection. Ask them to identify what the stumbling block is. By identifying the cause of their negative emotions, students will be able to form a plan that will help them regulate their behavior in response. For example, they might just need a peer to remind them how to use and read measurements on a ruler.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, Problem choices D, E, and F may be omitted.
- **Activity 3**, Problem 1 may be omitted. In this activity, students practice drawing reflections. Students will have other opportunities to practice drawing reflections in the Practice.

Pairs | 5 min

# **Warm-up** Which One Doesn't Belong?

Students compare four figures to review the characteristics and vocabulary that describe reflection symmetry.

Name: ____________ Date: ________ Period: ________

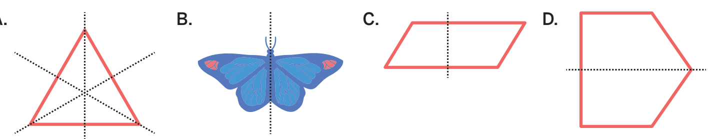
Unit 1 | Lesson 3

## Symmetry and Reflection

Let's describe ways figures reflect on the plane.

### **Warm-up** Which One Doesn't Belong?

Study the figures. Which figure does not belong with the others? Explain your thinking.




Sample responses:

- Choice A is the only figure with multiple lines of symmetry. It is also the only figure that appears to be an equilateral polygon.
- Choice B is the only figure that is not a polygon. It is also the only figure that is a photo of a real-world object.
- Choice C is the only figure where the marked line of symmetry is not correct.
- Choice D is the only figure that has a horizontal line of symmetry, as the figures are shown.

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Conduct the *Which One Doesn't Belong?* routine. Because there is no single correct response, attend to students' explanations, encourage use of math terminology, and ensure the reasons given are mathematically sound.

## 2 Monitor

**Help students get started** by asking them to choose any figure and identify what makes it different from the other figures.

**Look for points of confusion:**

- **Not realizing that the figure in choice C does not show the correct line of symmetry.** Ask students what would happen if they folded the figure along this line.

**Look for productive strategies:**

- Identifying any one figure that is different. Each figure has at least one characteristic that makes it different from the others.
- Noticing that the dotted line shows the line of symmetry in choices A, B, and D and that choice C does not show the correct line of symmetry.

## 3 Connect

**Have pairs of students share** their responses. Use the *Poll the Class* routine to see which figure students chose as not belonging. Begin from choices A, B, and D, and end with choice C. Have students share their explanations for why their chosen figure does not belong. If students do not notice the incorrect line of symmetry drawn in choice C, ask them if all the lines of symmetry are drawn correctly on all of the figures.

**Highlight** that choices A, B, and D have *reflection symmetry* because if students were to fold the figures along the lines of symmetry, each half of the figure looks exactly the same.

## Math Language Development

*MLR8: Discussion Supports*

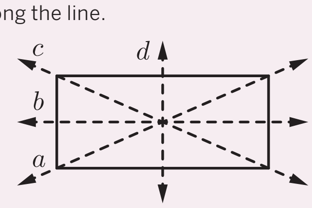
To support student understanding of lines of symmetry, have them demonstrate using folding gestures with their hands as they think about each figure.

## Power-up

**To power up students' ability to draw lines of symmetry, have students complete:**

Recall that a *line of symmetry* is a line that divides a figure into two halves that match up exactly when the figure is folded along the line.

Determine which lines are lines of symmetry in the given rectangle. Select *all* that apply.




A. Line $a$ C. Line $c$

B. Line $b$ D. Line $d$

**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 2, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 1

Pairs | 15 min

# Activity 1 Mirror Image

Students draw the reflection of a triangle to see how the line of reflection is related to the line segments between corresponding points.

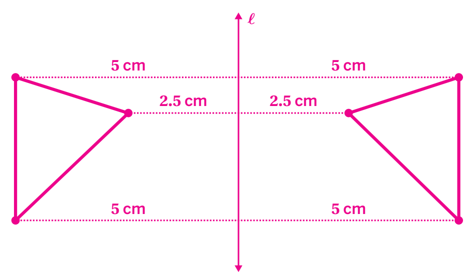
**Amps Featured Activity** Real-Time Reflections

## Activity 1 Mirror Image

**You will need tracing paper from your geometry toolkit.**

1. Use the tracing paper to complete the following steps.
   - Draw a vertical line in the middle of the paper, and label it $\ell$. On one side of the line, draw a triangle.
   - Fold the paper along the line. Retrace the triangle on the other side of the line.
   - Unfold the paper. You should now have two triangles that are mirror images of each other.
   - Draw a dotted line segment to connect each of the corresponding points of the two triangles.
   - Measure and label the distance from each point on the original triangle to its corresponding point on the new triangle.

   **Sample response shown.**




2. How is the line $\ell$ related to each dotted line segment you drew?

   **Sample responses:**
   - **Line $\ell$ is perpendicular to all of the dotted line segments, which represent the distances marked from each point to line $\ell$.**
   - **Line $\ell$ is located halfway between the corresponding points. In other words, the points on line $\ell$ are midpoints of the dotted line segments representing these distances.**

20 **Unit 1** Rigid Transformations and Congruence



## 1 Launch

Have students complete Problem 1 individually, share their drawing with a partner, and then complete Problem 2 with their partner. Provide access to geometry toolkits for the duration of the lesson.

## 2 Monitor

**Help students get started** by modeling how to draw the triangles and measure the distances between corresponding points.

**Look for points of confusion:**

- **Noticing the distances between the corresponding dotted line segments and the line of reflection are the same, but not noticing they are perpendicular.** Have students use a protractor to measure the angle formed by the line of reflection and each dotted line segment.

## 3 Connect

**Have students share** how the line of reflection is related to the dotted line segments they drew.

**Define** new vocabulary words. Say, "A ***reflection*** flips each point across a ***line of reflection*** to a point on the opposite side of the line. The term ***image*** describes the new figure created and the original figure is called the ***preimage***. To tell the figures apart, label the *corresponding vertices* of the image using a tick mark; this notation is called ***prime notation***." Have students label the *vertices* of the preimage as $A$, $B$, $C$ and the corresponding points in the image $A'$, $B'$, $C'$.

**Highlight** that the line of reflection lies halfway between the preimage and its image, and is perpendicular to the line segments connecting the corresponding points.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students perform the physical actions described in Problem 1, consider providing pre-created copies to pairs and have them either begin with measuring the distances or provide distances labeled and have them complete Problem 2. The goal of this activity is for students to notice the relationships between the line of reflection and the distances marked between corresponding points, not to physically perform the actions themselves.

## Math Language Development

### *MLR2: Collect and Display*

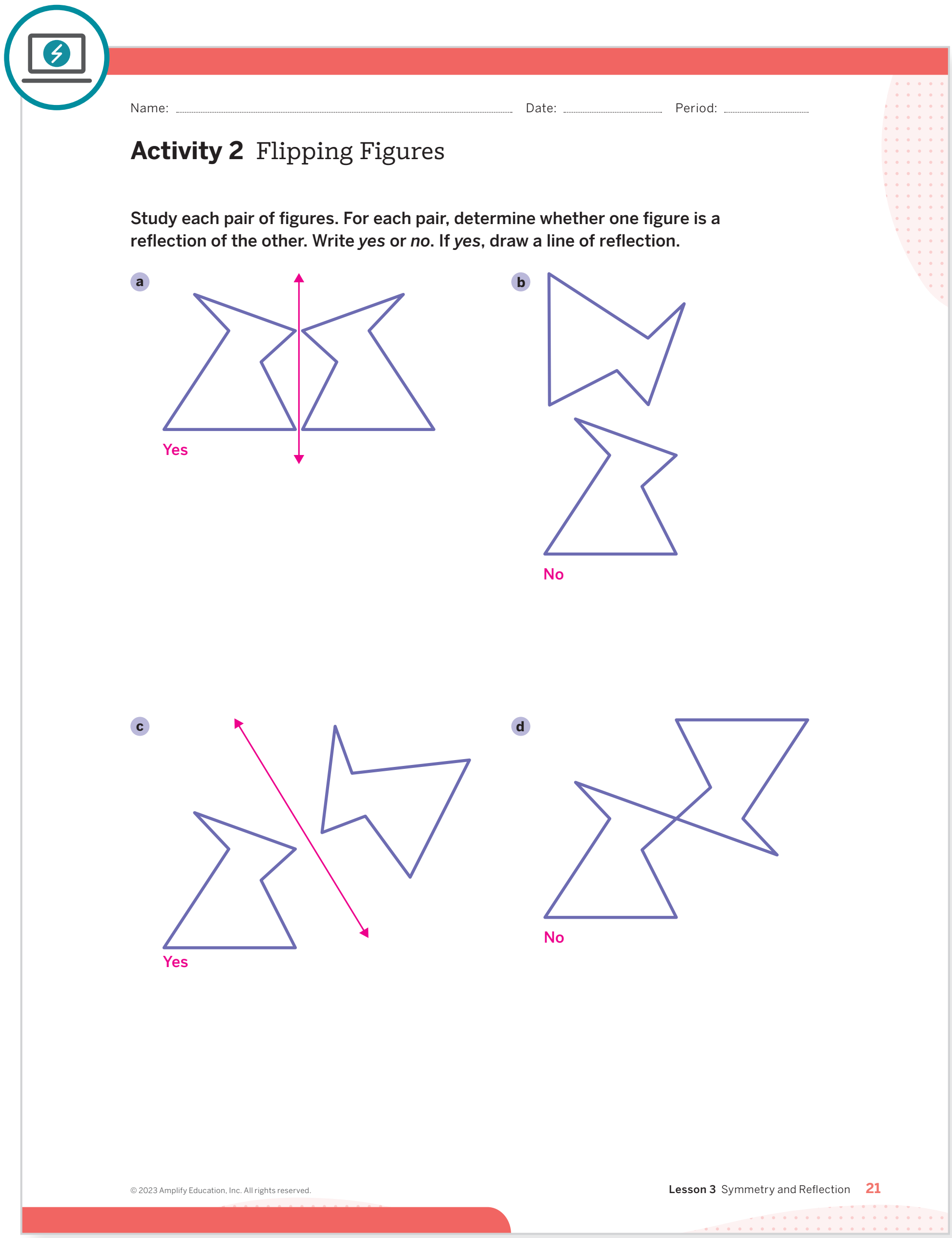
Collect and add to the class display the new vocabulary terms *reflection*, *line of reflection*, *image*, *preimage*, and *prime notation*. Connect reflection to the previously collected term *flip*.

### English Learners

Use physical manipulatives, such as a mirror, to demonstrate how the mirror acts as a *line of reflection*. Use the mirrors reflection to discuss the *preimage* and *image*.

# Activity 2 Flipping Figures

Students identify the characteristics that determine a reflection, building understanding that a reflection changes the orientation of a figure.



Name: ______ Date: ______ Period: ______

## Activity 2 Flipping Figures

Study each pair of figures. For each pair, determine whether one figure is a reflection of the other. Write *yes* or *no*. If *yes*, draw a line of reflection.

a Yes

b No

c Yes

d No





## 1 Launch

Activate students' background knowledge by asking them to describe what is the same and what is different when looking at a reflection of themselves in the mirror.

## 2 Monitor

**Help students get started** by reviewing the characteristics of a line of reflection.

**Look for points of confusion:**

- **Thinking that part d shows a line of reflection.** Have students use tracing paper and folding to check their thinking.
- **Thinking that part f shows rotation, not reflection.** Use tracing paper to show students that a rotation of the preimage would result in the image facing a different direction.

**Look for productive strategies:**

- Noticing translation or rotation in parts b, d, and e.
- Measuring the distances between corresponding points to draw the line of reflection.
- Noticing that reflected figures are mirror images of each other.

**Activity 2 continued** ›

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge***

If students need more processing time, have them focus on completing parts a–c, and only work on parts d–f as time allows.

**Extension: *Math Enrichment***

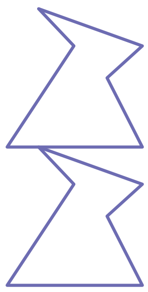
Have students identify translations or rotations for parts b, d, and e. Ask them to describe the movements using precise mathematical language.

# Activity 2 Flipping Figures (continued)

Students identify the characteristics that determine a reflection, building understanding that a reflection changes the orientation of a figure.

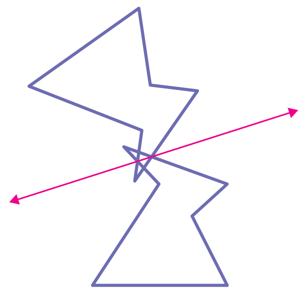
## Activity 2 Flipping Figures (continued)

e



No

f



Yes





## 3 Connect

**Have students share** their strategies for knowing which figures show a reflection.

**Ask**, "How can you differentiate a reflection from a translation or a rotation?" A reflection changes the direction of the figure, or the way it "faces."

**Define** the term ***orientation*** as to how the relative points on a figure are arranged. Have students label the vertices of the preimage in part a as $A$, $B$, $C$, and so on, going clockwise around the figure. Then have them label the image's corresponding vertices using prime notation, $A'$, $B'$, $C'$, and so on. Point out that the direction of the corresponding vertices are reversed in the image. This is an example of how the orientation of the figure has been reversed.

**Highlight** that a rotation and a translation preserve a figure's orientation, while a reflection does not.

Pairs | 8 min

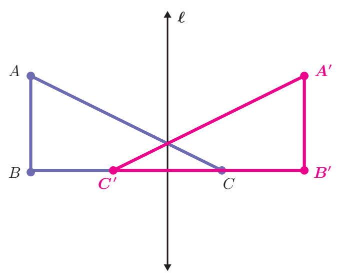
# Activity 3 Drawing Reflections

Students practice drawing reflections, strengthening their understanding of how the line of reflection relates to the corresponding points in the preimage and image.

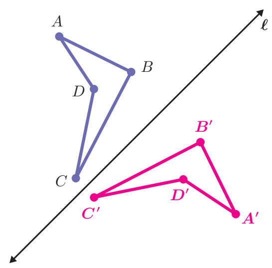
Name: ____________ Date: ________ Period: ________

## Activity 3 Drawing Reflections

1. Reflect Triangle $ABC$ across line $\ell$. Use $A'$, $B'$, and $C'$ to indicate vertices in the image that correspond to the points $A$, $B$, and $C$ in the preimage.




2. Reflect Polygon $ABCD$ across line $\ell$. Use $A'$, $B'$, $C'$, and $D'$ to indicate vertices in the image that correspond to the points $A$, $B$, $C$, and $D$ in the preimage.




STOP





### 1 Launch

Have students use a ruler to draw the reflection of each figure and only use tracing paper to check their work.

### 2 Monitor

**Help students get started** by having them draw a perpendicular line from point $A$ to the line $\ell$ in Problem 1, and then measure the distance from point $A$ to the line $\ell$.

**Look for points of confusion:**

- **Drawing a reflected point the same distance from the line as point $A$, but not perpendicular to line $\ell$ in Problem 2.** Use a protractor, or corner of an index card or paper, to help students create a right angle formed by line $\ell$ and point $A$.

**Look for productive strategies:**

- Using rulers to measure the distance from each point in the preimage to the line of reflection.
- Only using tracing paper to check their reflected image after it is drawn.

### 3 Connect

**Display** correct student drawings.

**Have students share** the strategies they used for drawing each image.

**Highlight** that an image is determined by the preimage and placement of the line of reflection. The line of reflection may not always be strictly vertical (as in Problem 1) or horizontal. The line of reflection may be slanted (as in Problem 2).

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problem 1, and only work on Problem 2 as time allows.

### Accessibility: *Optimize Access to Tools*

Provide access to tracing paper, should students wish to use it during the activity.

### Extension: *Math Enrichment*

Have students draw their own reflections and lines of reflections that satisfy the given criteria.

- Draw the reflection of a preimage in which the image overlaps the preimage.
- Draw the reflection of a preimage in which the image touches exactly one of the vertices of the preimage.
- Draw the reflection of a preimage in which the image touches exactly one of the sides of the preimage.

Whole Class | 5 min

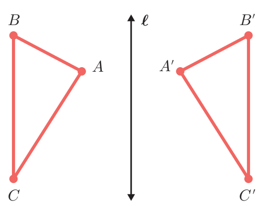
# Summary

Review and synthesize the features of reflection and the mathematical language used to describe how figures can be reflected in a plane.

## Summary

**In today's lesson . . .**

You explored how to precisely reflect a figure over a line. A ***reflection*** moves every point on a figure to a point directly on the opposite side of the ***line of reflection***. The new point is the same distance from the line as its corresponding point in the original figure. The ***orientation*** of the vertices is reversed.




The term ***image*** describes the new figure created by moving the original figure. The original figure is called the ***preimage***.

In the diagram, the vertices of the image are labeled using ***prime notation***, $A'$, $B'$, and $C'$. This notation is read "$A$ prime", "$B$ prime", and "$C$ prime". These represent the vertices in the image and correspond to the vertices $A$, $B$, and $C$ in the preimage.

Reflect:




## Synthesize

**Have students share** what reflection means, using their own words.

**Ask:**

- "What do you need to include when describing a reflection?" The line of reflection across which a figure is reflected.
- "How do the corresponding vertices of the preimage and image compare to the line of reflection?" They are located the same distance to the line of reflection.
- "Does a reflection change or preserve the orientation of the preimage?" A reflection changes the orientation of the preimage. The orientation of the image is reversed.

**Display** the Summary from the Student Edition.

**Formalize vocabulary:**

- *image*
- *line of reflection*
- *orientation*
- *preimage*
- *prime notation*
- *reflection*

**Highlight** that a reflection across a line moves each point to a point directly on the opposite side of the line of symmetry. The new point is the same distance from the line of symmetry as the original point of the figure. Prime notation can be used to label the image to clearly see which points in the image correspond to the points in the preimage.

## Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful today when identifying and drawing reflections? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

## Math Language Development

***MLR2: Collect and Display***

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *image*, *line of reflection*, *orientation*, *preimage*, *prime notation*, and *reflection* that were added to the display during the lesson.

Independent | 5 min

# Exit Ticket

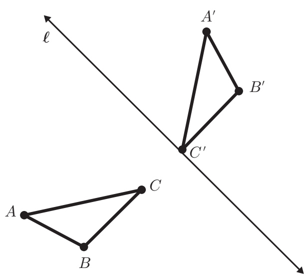
Students demonstrate their understanding of reflection by critiquing the work of another student and constructing a viable argument.

Printable

Name: ______________________ Date: ____________ Period: ____________

## Exit Ticket 1.03

Diego reflects Triangle $ABC$ across line $\ell$ and draws Triangle $A'B'C'$. Did Diego reflect the triangle correctly? Explain your thinking.




No; Sample response: Although the orientation of the reflected image is correct, the distances from corresponding points to the line of reflection are not equal.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can describe how a figure can be reflected across a line of reflection to obtain a new figure.
1 2 3

b. I can draw a line of reflection using the distances between corresponding points of the image and preimage.
1 2 3

c. I can draw a reflected figure, given the preimage and line of reflection.
1 2 3



**Lesson 3** Symmetry and Reflection

## Success looks like . . .

- **Language Goal:** Describing the movement of figures informally and formally using the terms *reflection*, *line of reflection*, *image*, and *preimage*. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Identifying the features that determine a reflection. **(Speaking and Listening, Reading and Writing)**
  - » Explaining why the Diego's reflection is incorrect.

## Suggested next steps

**If students think that Diego's reflection is correct, consider:**

- Reviewing Activity 3.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- How did students attend to precision when describing reflections? How are you helping students become self-aware of their progress and growth in this area?
- What different ways did students approach drawing reflections? What does that tell you about similarities and differences among your students?

## Math Language Development

**Language Goal: Describing the movement of figures informally and formally using the terms *reflection*, *line of reflection*, *image*, and *preimage*.**

Reflect on students' language development toward this goal.

- How have students progressed in their use of mathematical vocabulary to describe the movement of figures, particularly related to reflections?

**Sample descriptions for the Exit Ticket problem:**

| Emerging | Expanding |
| --- | --- |
| One triangle is farther away than the other. | The distances from corresponding points to the line of reflection are not equal. |

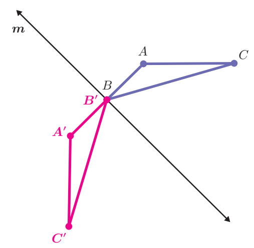
# Practice

Independent

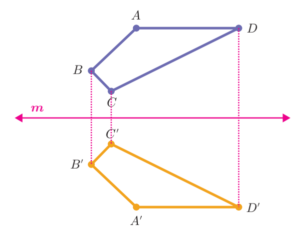
Name: ____________ Date: ________ Period: ________

Practice

1. Reflect Triangle $ABC$ across line $m$. Use $A'$, $B'$, and $C'$ to indicate the vertices in the image that correspond to the points $A$, $B$, and $C$, in the preimage.




2. Polygon $A'B'C'D'$ is a reflection of Polygon $ABCD$. Draw the line of reflection and label it $m$. Explain your thinking.




**Sample response: The line of reflection is located halfway between each pair of corresponding points.**

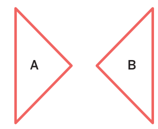




Name: ____________ Date: ________ Period: ________

Practice

3. Select *all* the ways Triangle A can map onto Triangle B.




   **A.** Reflect Triangle A across a horizontal line.
   **B.** Reflect Triangle A across a vertical line.
   **C.** Translate Triangle A to the left.
   **D.** Translate Triangle A to the right.
   **E.** Rotate Triangle A $180°$ counterclockwise.
   **F.** Rotate Triangle A $90°$ counterclockwise.

4. Write an operation in the box to make each equation true.

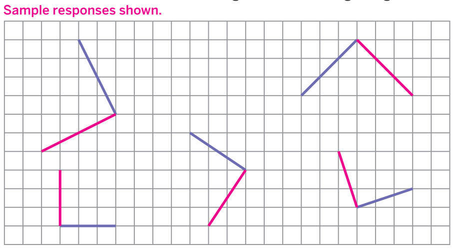
   a. $12 \boxed{-} (-8) = 20$

   b. $-17 \boxed{-} 9 = -26$

   c. $-14 \boxed{+} 18 = 4$

   d. $24 \boxed{+} (-29) = -5$

5. Draw a line connected to each line segment to form a right angle.
**Sample responses shown.**







## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 1 |
| | 2 | Activity 2 | 1 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Grade 7 | 2 |
| Formative | 5 | Unit 1 Lesson 4 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Grid Moves

Let's transform some figures on grids.

## Focus

### Goals

1. **Language Goal:** Describe the moves needed to perform a transformation. **(Speaking and Listening)**
2. Draw and label the image and corresponding points of figures that have been translated, reflected, or rotated.
3. **Language Goal:** Draw the image of a figure that results from translations, rotations, and reflections in square grids, and justify that the image is a transformation of the original figure. **(Speaking and Listening)**

## Coherence

### Today

Students perform translations, reflections, and rotations on a square grid. Students may begin to notice how the structure of the grid helps them draw images resulting from certain moves, but may choose to continue to use tracing paper to check their work. Students are introduced to a new term — *transformation* — to describe the different moves.

### Previously

In Lessons 2 and 3, students learned the names for the single moves of a figure — *translation*, *reflection*, and *rotation* — and learned how to identify and construct them. They also used prime notation to label images, such as labeling the image of a point $P$ as $P'$.

### Coming Soon

In Lesson 5, students will perform a sequence of transformations on a preimage to produce an image.

## Rigor

- Students build **conceptual understanding** of how the structure of a grid helps them perform and identify transformations.
- Students build **fluency** in using precise mathematical vocabulary to describe transformations.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Independent | Independent | Independent | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Translations, Rotations, and Reflections*
- geometry toolkits: rulers, protractors or index cards, tracing paper

### Math Language Development

**New word**

- ***transformation***

**Review words**

- *clockwise*
- *corresponding*
- *counterclockwise*
- *preimage*
- *image*
- *reflection*
- *rotation*
- *translation*

**Amps** | Featured Activity

**Activity 1**

### Formative Feedback for Students

Instead of just being told whether they are correct or incorrect, students *see* the consequences of their response, and can work out for themselves any errors that need corrected.

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### Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel lost as they transition to the coordinate plane, and forget the tools available to them in their geometry toolkits. Encourage them to use any of the available tools to help increase their mathematical understanding, such as tracing paper or rulers.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 1 may be omitted.
- Assign **Activity 2** to students in groups of three, having each student complete one transformation.

Independent | 5 min

## Warm-up True or False

Students examine whether three movements each show a reflection, to strengthen their understanding of the characteristics of reflections.

Name: ____________ Date: ________ Period: ________

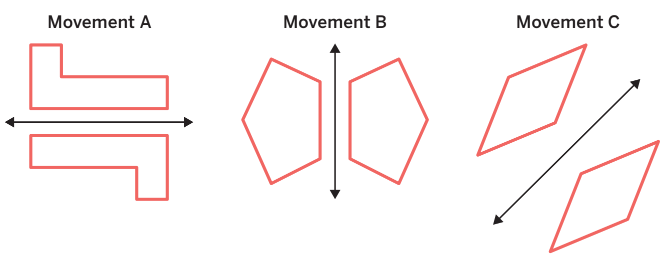
**Unit 1 | Lesson 4**

### Grid Moves

Let's transform some figures on grids.

**Warm-up** True or False

Shawn thinks the following movements are all examples of *reflections* across the line shown. Do you agree with Shawn? Be prepared to explain your thinking.




Sample response: I do not agree with Shawn. While Movement B shows a reflection, the other images cannot be made by only a reflection. Movement A is a rotation. Movement C is a translation, but the image can also be made by reflecting the preimage, and then translating it.

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Set an expectation for the amount of time students will have to work individually on the activity. Provide access to geometry toolkits for the duration of the lesson.

### 2 Monitor

**Help students get started** by asking which figures are mirror images of each other.

**Look for points of confusion:**

- **Thinking Movement C illustrates a reflection.** Have students use tracing paper or a ruler to determine that while the quadrilaterals are identical, lines drawn between corresponding points are *not* perpendicular to the line, indicating that a reflection has not taken place.

**Look for productive strategies:**

- Using tracing paper or a ruler to verify whether a reflection has taken place.

### 3 Connect

**Have students share** their strategies for determining which movement shows a reflection. Include incorrect strategies, and encourage student feedback on each other's strategies.

**Highlight** that reflections maintain the same distance between corresponding points and the line of reflection.

### Differentiated Support

**Accessibility: *Guide Visualization and Processing, Optimize Access to Tools***

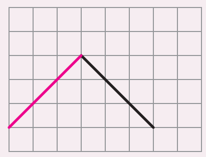
Consider providing copies of each movement in the Warm-up for students to physically manipulate. For example, they could fold each movement over the line of reflection to determine if the movement is a reflection.

### Power-up

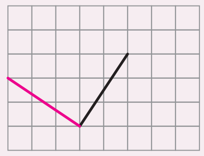
**To power up students' ability to construct right angles, have students complete:**

Recall that a *right angle* is an angle that measures 90° and that it is formed by two perpendicular lines or rays. Using a protractor or the corner of an index card, connect a line to each segment to form a right angle. Sample responses shown.

1.



2.



**Use:** Before Activity 1
**Informed by:** Performance on Lesson 3, Practice Problem 5

Independent | 15 min

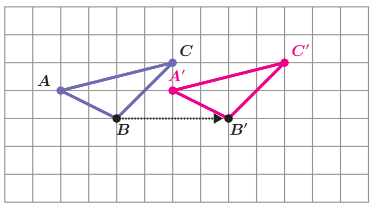
# Activity 1 Transformation Information

Students transform figures that are on a grid to see that the properties of a grid can help them draw the transformed images. They are introduced to a new term: *transformation*.

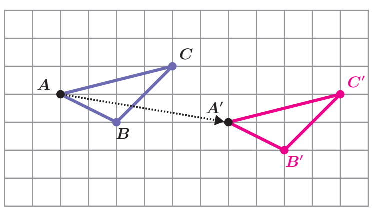
**Amps Featured Activity** Formative Feedback for Students

### Activity 1 Transformation Information

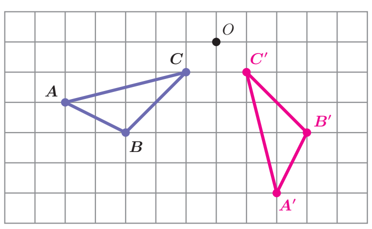
1. Translate Triangle $ABC$, so that point $B$ maps onto point $B'$. Label the corresponding points on the image with $A'$, $B'$, and $C'$.




2. Translate Triangle $ABC$, so that point $A$ maps onto point $A'$. Label the corresponding points on the image with $A'$, $B'$, and $C'$.




3. Rotate Triangle $ABC$ $90°$ *counterclockwise* about point $O$. Label the corresponding points on the image with $A'$, $B'$, and $C'$.







## 1 Launch

**Note:** The term *transformation* has not been introduced yet. Have students complete the activity individually, before sharing their responses with a partner.

## 2 Monitor

**Help students get started** by reviewing the terms and characteristics of translations, rotations, and reflections.

**Look for points of confusion:**

- **Not translating all the points the same distance in Problem 2.** Ask students to describe their reasoning for Problem 1, and how their answer would change if they were to translate down instead. Say, "In Problem 2, you need to move each point 1 unit down *and* 6 units to the left." Remind students that they can use tracing paper to check their work.
- **Struggling to rotate the triangle in Problem 3.** Have students estimate and draw a $90°$ angle from each vertex of the triangle, using point $O$ as the center of rotation.
- **Struggling to reflect the triangle in Problem 4.** Have students find the distance from each vertex to the line of symmetry and draw the reflected vertex using the same distance. Students may use a protractor or index card to construct lines perpendicular to the line of reflection.
- **Forgetting to label the image coordinates.** Remind students that each point in the image corresponds to a point in the preimage, and should be labeled using prime notation.

**Look for productive strategies:**

- Using the grid units to help decide where to draw the transformed figures.
- Using a ruler or counting units on the grid to measure distance between corresponding points for reflections.

Activity 1 continued >

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge, Activate Prior Knowledge***

If students need more processing time, have them focus on completing Problems 1, 3 and 4, and only work on Problem 2 as time allows. Consider also beginning with a physical demonstration using tracing paper to perform each type of transformation. This will support connections between what students learned in prior lessons and transformations on grids in this lesson.

## Math Language Development

***MLR8: Discussion Supports***

To support student understanding of lines of symmetry, have them demonstrate using folding gestures with their hands as they consider each figure.

**English Learners**

As you perform the think-aloud and model the mathematical language used, utilize gestures to connect the language to physical movements.

Independent | 15 min

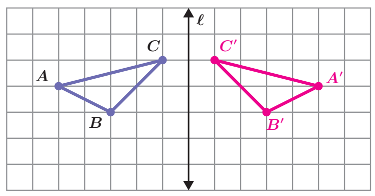
# Activity 1 Transformation Information (continued)

Students transform figures that are on a grid to see that the properties of a grid can help them draw the transformed images. They are introduced to a new term: *transformation*.

Name: ______ Date: ______ Period: ______

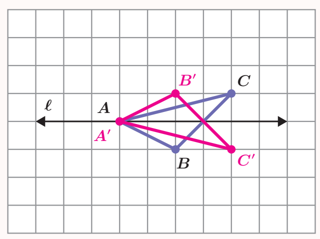
## Activity 1 Transformation Information (continued)

4. Reflect Triangle $ABC$ across line $\ell$. Label the corresponding points on the image with $A'$, $B'$, and $C'$.

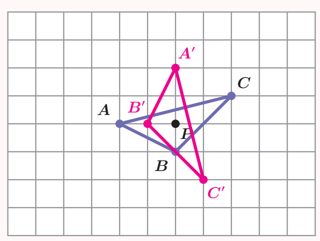



**Are you ready for more?**

1. Reflect Triangle $ABC$ across line $\ell$. Label the corresponding points on the image with $A'$, $B'$, and $C'$.




2. Rotate Triangle $ABC$ 90° *clockwise* about point $P$. Label the corresponding points on the image with $A'$, $B'$, and $C'$.






Lesson 4 Grid Moves 29

## 3 Connect

**Have students share** the strategies they used to transform the images. Focus on students who used tracing paper and students who used the grid units to draw the transformations.

**Ask:**

- "How do the translations in Problems 1 and 2 differ?" In Problem 1, the triangle is translated in one direction (to the right). In Problem 2, the triangle is translated in two directions (down and to the right).
- "When rotating a figure, how does the orientation of the image vertices compare to the orientation of the preimage vertices, relative to the center of rotation?" The orientation is reversed.
- "Can you think of one word that you can use to describe any of these types of movements?" Sample responses: move, change, transform

**Define** the term ***transformation*** as a rule for moving or changing figures on the plane. Translations, reflections, and rotations are all examples of transformations.

**Highlight** how the structure of the grid can help students perform each transformation.

Independent | 15 min

# Activity 2 Identifying Transformations

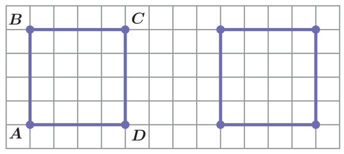
Students identify how various transformations for some figures can result in the same image.

## Activity 2 Identifying Transformations

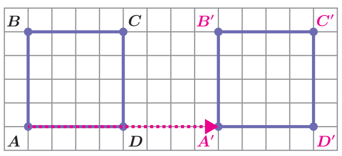
**Square $ABCD$ is drawn on a grid, and a transformation has been applied.**

Kiran, Clare, and Noah each see different transformations. For each student:

- Label the vertices with the correct letters to show why each response is correct.
- Describe how each transformation maps the original figure onto the new figure.
- Be sure to include a line of reflection and a center of rotation, when necessary.

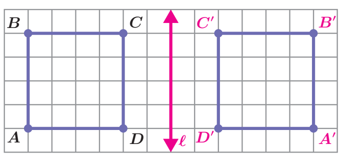



1. Kiran: "I see a translation."



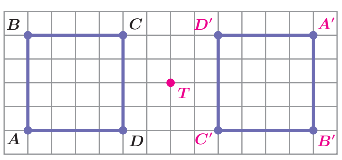

The square is translated 8 units to the right.

2. Clare: "I see a reflection."




The square is reflected across line $\ell$.

3. Noah: "I see a rotation."




Sample response: The square is rotated 180° about point $T$.

STOP





## 1 Launch

Give students 1 minute to discuss strategies with a partner before working individually on the activity.

## 2 Monitor

**Help students get started** by asking what kind of transformation they see in the image.

**Look for points of confusion:**

- **Thinking Kiran's translation moves four units to the right because point $A'$ is four units to the right of point $D$.** Ask students to identify the pairs of corresponding points, and then calculate the distance between each pair.
- **Struggling to visualize the center of rotation in Problem 3.** Model how to find the center of rotation by holding tracing paper down with the point of a pencil and spinning the tracing paper around that point.
- **Labeling corresponding points incorrectly in Problems 2 and 3.** Make sure students have labeled vertices on their tracing paper.

**Look for productive strategies:**

- Finding multiple ways to describe a rotation for Problem 3.
- Using tracing paper to help with labeling the image or understanding the transformation.

## 3 Connect

**Have individual students share** their strategies.

**Highlight** that different transformations can produce similar images. To understand the specific transformation described, it is helpful to label the coordinates of the image compared to the preimage, any line of reflection, or center of rotation.

**Ask**, "Are there other shapes besides a square for which different transformations can produce a similar image?" Sample responses: yes, a regular octagon, a circle

## Differentiated Support

### Accessibility: *Guide Visualization and Processing, Optimize Access to Tools*

Have students assign a different color to each of the vertices of the preimage and use colored pencils or highlighters to label the corresponding vertices in the image with the same colors. Provide access to tracing paper, should students wish to use it during the activity.

### Extension: *Math Enrichment*

Challenge students to generate their own examples of an image that could be created by performing more than one transformation of a preimage.

## Math Language Development

### *MLR7: Compare and Connect*

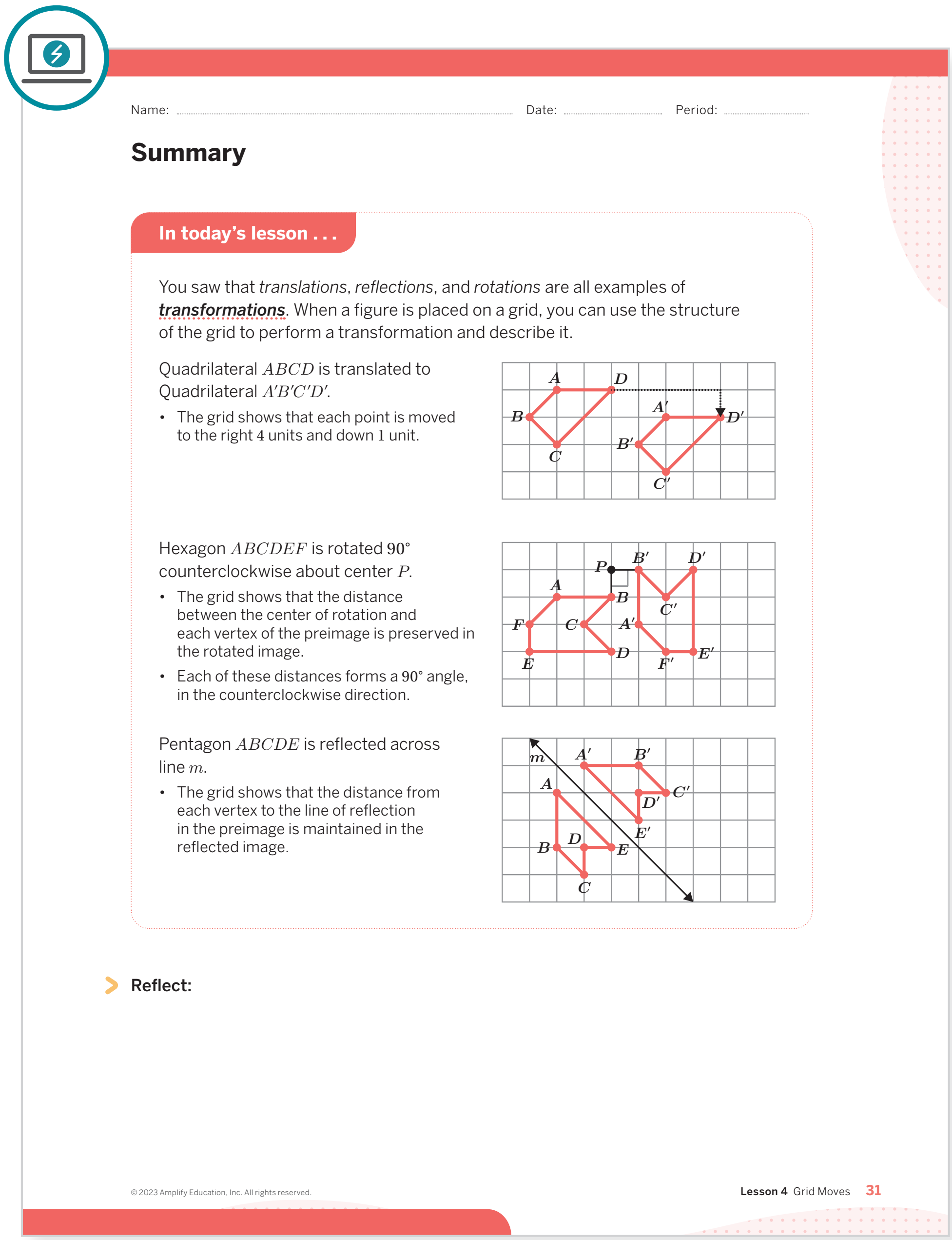
Have students share and compare their strategies for transforming the square and connect these strategies for transforming any regular polygon.

### English Learners

Encourage students to use tracing paper to assist them as they label each image.

# Summary

Review and synthesize how grids can assist with performing or identifying transformations.



Name: ______ Date: ______ Period: ______

**Summary**

**In today's lesson . . .**

You saw that *translations*, *reflections*, and *rotations* are all examples of ***transformations***. When a figure is placed on a grid, you can use the structure of the grid to perform a transformation and describe it.

Quadrilateral $ABCD$ is translated to Quadrilateral $A'B'C'D'$.

- The grid shows that each point is moved to the right 4 units and down 1 unit.

Hexagon $ABCDEF$ is rotated $90°$ counterclockwise about center $P$.

- The grid shows that the distance between the center of rotation and each vertex of the preimage is preserved in the rotated image.
- Each of these distances forms a $90°$ angle, in the counterclockwise direction.

Pentagon $ABCDE$ is reflected across line $m$.

- The grid shows that the distance from each vertex to the line of reflection in the preimage is maintained in the reflected image.

**Reflect:**

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**Lesson 4** Grid Moves 31

## Synthesize

**Display** the Anchor Chart PDF, *Translations, Rotations, and Reflections* for students to reference throughout this unit.

**Formalize vocabulary: *transformation***

**Ask:**

- "What are some important things to keep in mind when performing a translation, rotation, or reflection?" Sample responses: Translations can occur in more than one direction. Reflections reverse the orientation of a figure, and are always across a line of reflection. Rotations are always about a certain center of rotation.
- "What is something new that you learned today about translations, rotations, or reflections?" Sample response: I learned that these are all types of transformations.

**Highlight** how the structure of the grid can be used to perform and identify each type of transformation. Because the size of each grid square is the same, students can use the grid to count the number of units a figure is translated or the distance each corresponding point is to a line of reflection. Similarly, students can use the corners of grid squares to verify right angles. While not all distances or angles can be counted or measured using grid units, the grid is still a valuable tool for performing and identifying transformations.

## Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

- "How do transformations on a grid differ from transformations that are not on a grid? How are they similar?"

## Math Language Development

### *MLR2: Collect and Display*

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *transformation* that were added to the display during the lesson.

Independent | 5 min

# Exit Ticket

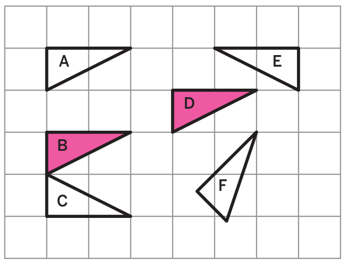
Students demonstrate their understanding by identifying which figures represent only a translation.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 1.04

Which of these triangles are translations of Triangle A, with no rotation or reflection applied? Shade *all* that apply, and explain your thinking.




Sample response: Triangles B and D are the only images that share the exact features of Triangle A, and have not been rotated or reflected.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can decide which type of transformation(s) will map one figure onto another.
1 2 3

b. I can draw and label the image and corresponding points of figures that have been translated, reflected, or rotated.
1 2 3

c. I can use grids to transform figures.
1 2 3





## Success looks like . . .

- **Language Goal:** Describing the moves needed to perform a transformation. **(Speaking and Listening)**
  - » Explaining which translations of Triangle A are not the result of a rotation or reflection.
- **Goal:** Drawing and labeling the image and corresponding points of figures that have been translated, reflected, or rotated.
- **Language Goal:** Drawing the image of a figure that results from translations, rotations, and reflections in square grids and justifying that the image is a transformation of the original figure. **(Speaking and Listening)**

## Suggested next steps

**If students have trouble identifying Triangles $B$ and $D$ as translations, consider:**

- Having them use tracing paper and point out that the paper does not have to be turned or flipped over to map Triangle $A$ onto Triangle $B$ or $D$.
- Reviewing Problems 1 and 2 from Activity 1.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- How did students transition from working with transformations without grids to working with grids in this lesson? Are your students comfortable with using grids? How might you alter your instruction if they are not comfortable?
- How are students progressing in their conceptual development of understanding how to describe and perform translations, reflections, and rotations? Is one or more of these more challenging to them? What strategies can you use to help them further develop their understanding?

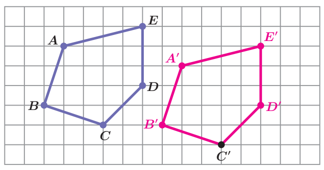
# Practice

Independent

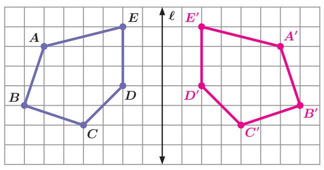
Name: ______ Date: ______ Period: ______

1. Complete each of the transformations described.

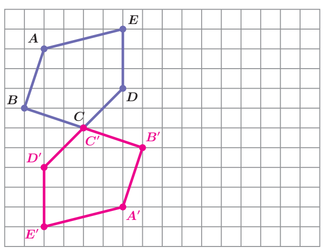
a. Draw the translated image of Pentagon $ABCDE$, so that point $C$ maps onto point $C'$. Label the corresponding points on the image with $A'$, $B'$, $C'$, $D'$, and $E'$.




b. Draw the reflection of Pentagon $ABCDE$ across line $\ell$. Label the corresponding points on the image with $A'$, $B'$, $C'$, $D'$, and $E'$.




c. Draw the rotation of Pentagon $ABCDE$ clockwise 180° about point $C$. Label the corresponding points on the image with $A'$, $B'$, $C'$, $D'$, and $E'$. Tracing paper and a ruler may be useful here.



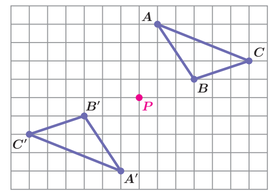




Name: ______ Date: ______ Period: ______

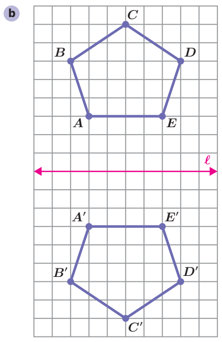
2. Describe each transformation that has occurred. Draw any points or lines that are used in each transformation.

a.



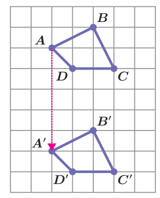

180° rotation about point $P$

b.




reflection across line $\ell$

c.



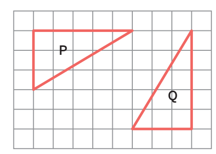

translation 5 units down

3. Match each expression with an equivalent expression.

| Expressions | Equivalent expressions |
|---|---|
| a. $-3x - 7$ | b $-0.9 + 5.7x$ |
| b. $-3.4 + 5.7x + 2.5$ | d $-\frac{1}{2}(6x - 14)$ |
| c. $1.8x - 5.9 + 3.9x$ | c $5.7x - 5.9$ |
| d. $-3x + 7$ | a $\left(-\frac{7}{2} - \frac{3}{2}x\right) \cdot 2$ |

4. Identify the single transformation that will map Triangle P onto Triangle Q.

A. Translation
B. Reflection
C. Rotation
D. None of the above







## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 2 |
| Spiral | 3 | Grade 7 | 1 |
| Formative | 4 | Unit 1 Lesson 5 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Making the Moves

Let's draw and describe translations, reflections, and rotations.

## Focus

### Goals

1. **Language Goal:** Draw a transformation of a figure using information given orally. **(Speaking and Listening)**
2. **Language Goal:** Explain the sequence of transformations that maps one figure onto another. **(Speaking and Listening)**

## Coherence

### Today

Students take turns providing verbal descriptions of transformations that have occurred, and drawing images based on these verbal descriptions. They come to understand that sometimes there is no single translation, rotation, or reflection that will map one figure to another, resulting in the need for a sequence of transformations.

### Previously

In Lesson 4, students were introduced to the term *transformation* and began to explore transformations on a square grid.

### Coming Soon

In Lessons 6–7, students will perform transformations on the coordinate plane, noticing what happens to the coordinates of transformed points. They will be able to describe the effect of transformations on the coordinates of the transformed points.

## Rigor

- Students build **conceptual understanding** that sometimes, a sequence of transformations is necessary to map one figure onto another.
- Students build **fluency** in using precise mathematical vocabulary to describe a sequence of transformations.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 12 min | 5 min | 7 min |
| Independent | Pairs | Independent | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

## Materials

- Exit Ticket
- Additional Practice
- Power-up PDF, as needed
- Power-up PDF (answers), as needed
- Activity 1 PDF (teacher directions and demo card)
- Activity 1 PDF (cards), one set of cards per pair
- geometry toolkits: rulers, protractors or index cards, tracing paper

## Math Language Development

**New word**

- ***sequence of transformations***

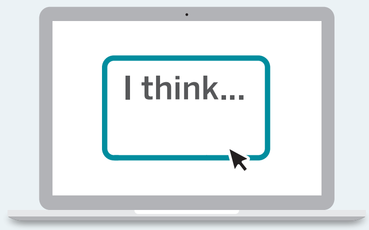
**Review words**

- *clockwise*
- *counterclockwise*
- *image*
- *preimage*
- *rotation*
- *reflection*
- *transformation*
- *translation*

**Amps** | Featured Activity

### Activity 2
### See Student Thinking

Students are asked to explain their thinking behind mapping a preimage onto an image, and these explanations are available to you digitally, in real time.

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### Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel overly challenged as they describe the transformation on their card in Activity 1; they may give up or feel stuck as they use their developing mathematical language to provide precise verbal descriptions. Help them practice taking control of their own impulses by asking them to identify what they know to be true about the figures, and use mathematical language to express their thoughts.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted as students have had prior opportunities to practice drawing the line of reflection.
- **Activity 1** may be omitted. In this activity, students come to understand how the coordinate plane can provide more specific information about transformations than a square grid.

Independent | 5 min

# **Warm-up** Finding the Line of Reflection

Students analyze a preimage and image to determine the placement for a line of reflection.

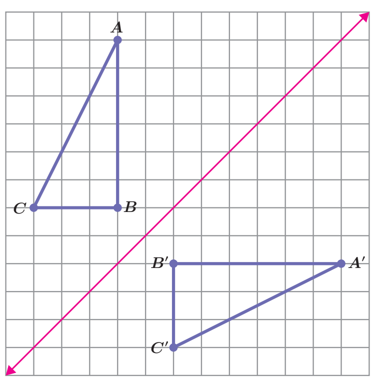
Unit 1 | Lesson 5

## Making the Moves

Let's draw and describe translations, reflections, and rotations.

### **Warm-up** Finding the Line of Reflection

**Triangle $ABC$ has been reflected to create Triangle $A'B'C'$. Draw the line of reflection.**




Log in to Amplify Math to complete this lesson online.



34 **Unit 1** Rigid Transformations and Congruence

## 1 Launch

Set an expectation for the amount of time students will have to work individually on the activity. Provide access to geometry toolkits for the duration of the lesson.

## 2 Monitor

**Help students get started** by asking them, "Imagine folding a paper so that Triangle $ABC$ maps onto Triangle $A'B'C'$. Where would that fold line be?"

**Look for points of confusion:**

- **Thinking that lines of reflection must be horizontal or vertical.** Have students use tracing paper to trace the image and preimage and then fold the paper so the triangles are mapped onto one another. By unfolding the paper, they should see the line of reflection is slanted.

**Look for productive strategies:**

- Finding the distance between corresponding points and finding the midpoints of these distances to draw the line of reflection. Once two midpoints are found, students can draw the line of reflection connecting them.

## 3 Connect

**Have individual students share** their strategies for finding the line of reflection.

**Highlight** how the grid allows for distances to be observed without the need for a ruler or other measurement tool.

**Ask**, "How many points do you need to construct to find the line of reflection?

## Differentiated Support

**Accessibility: *Guide Visualization and Processing, Optimize Access to Tools***

Consider providing copies of Triangle $ABC$ from the Warm-up for students to physically manipulate. For example, they could experiment folding the grid to determine where the line of reflection might be.

**Extension: *Math Enrichment***

Ask students how they know the line of reflection does not intersect any of the vertices or side lengths of Triangle $ABC$. If the line of reflection intersected a vertex (or side) of Triangle $ABC$, then the image and preimage of that vertex (or side) would be in the same location.

## Power-up

**To power up students' ability to describe transformations:**

Provide students with a copy of the Power-up PDF.

**Use:** Before Activity 1

**Informed by:** Performance on Lesson 4, Practice Problem 4

Pairs | 15 min

# **Activity 1** Make That Move

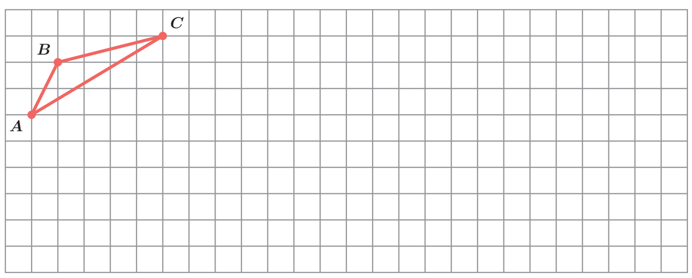
Students describe and draw transformations to understand limitations of a square grid and to develop a need for the coordinate plane.

Name: ________ Date: ________ Period: ________

**Activity 1** Make That Move

**You will be given a set of cards. You and your partner will take turns describing and drawing transformations. Decide which student will draw first. The other student will describe the transformation on their card.**

| When describing . . . | When drawing . . . |
|---|---|
| • Your goal is to describe your transformation as accurately as possible using *only* your words. • You may offer verbal feedback to your partner once they have completed their initial sketch. | • Your goal is to listen carefully to your partner, and recreate the transformation they describe. • Draw the transformation your partner describes on the grid shown. |




Answers may vary, based on the card chosen from the Activity 1 PDF.





## 1 Launch

Distribute the cards from the Activity 1 PDF (cards). Specific directions and a demonstration card can be found on the Activity 1 PDF (teacher directions and demo card).
**Note:** The Activity 1 PDF needs to be distributed for both print and digital lessons.

## 2 Monitor

**Help students get started** by modeling the routine for this activity using the demo card provided.

**Look for points of confusion:**

- **Struggling to use only words as they describe the transformations.** Encourage students to visualize each movement and then choose the words that can describe the movement. It may be helpful to display words students use to describe the movements.

**Look for productive strategies:**

- Using precise descriptions, such as specific points, lines, or angles.
- Responding to constructive feedback to revise sketches.

## 3 Connect

**Ask:**

- "What elements of your partner's description were helpful as you were sketching?"
- "What are some details or ideas that could have helped you describe your transformation?" Listen for student ideas that suggest the need for a coordinate plane, such as specific locations of points.

**Highlight** how using precise mathematical language can assist when performing certain geometric actions, such as transformations. Say, "In future lessons, you will learn how to make your descriptions more clear."

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, assign them Card Set 1A and 1B.

### Accessibility: *Clarify Language and Symbols*

Remind students that they can refer to the class display or the Anchor Chart PDF, *Translations, Rotations, and Reflections,* when providing directions to their partner.

### Extension: *Math Enrichment*

Have students describe more than one method for achieving the transformation on their card.

## Math Language Development

### *MLR2: Collect and Display*

As students describe the transformation of Triangle $ABC$, listen for and collect vocabulary and phrases they use to describe reflections, rotations, and translations.

### English Learners

As you add to the class display, use gestures to highlight the differences between each transformation.

Independent | 12 min

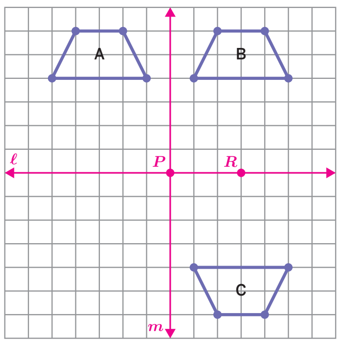
# Activity 2 A to B to C

Students practice describing transformations, leading them to see that in some cases, multiple transformations are necessary to map one figure onto another.

Amps Featured Activity | See Student Thinking

### Activity 2 A to B to C

**Refer to Figures A, B, and C. For each problem, describe a transformation that could map each figure onto the other. Draw any points or lines that are used in your transformation.**




1. A transformation that maps Figure A onto Figure B.
   Sample responses:
   - Translate 6 units to the right, or
   - Reflect Figure A across line $m$.

2. A transformation that maps Figure B onto Figure C.
   Sample responses:
   - Reflect Figure B across line $\ell$, or
   - Rotate Figure B 180° clockwise or counterclockwise around point $R$.

3. A transformation that maps Figure A onto Figure C.
   Sample responses:
   - Translate 6 units to the right, and then reflect across line $\ell$, or
   - Reflect Figure A across line $m$, and then rotate around point $R$.
   - Rotate Figure A 180° clockwise or counterclockwise around point $P$.
   - Reflect Figure A across line $m$, and then reflect across line $\ell$.

**Compare and Connect:** After completing Problems 1–3, create a visual display of your chosen strategy or strategies. Then compare your strategy with a partner.

**Are you ready for more?**

Describe a *single* transformation that maps Figure A directly onto Figure C.

Figure A can be rotated 180° clockwise or counterclockwise about point $P$ to map onto Figure C.

STOP

36 Unit 1 Rigid Transformations and Congruence



## 1 Launch

Set an expectation for the amount of time students will have to work individually on the activity before sharing their responses with a partner.

## 2 Monitor

**Help students get started** by reminding them to describe their transformations precisely.

**Look for points of confusion:**

- **Thinking there is no transformation that will map Figure A onto Figure C.** Allow students to think this way at this point, because the concept of multiple transformations has not been discussed yet. Make a note of this thinking, and encourage them to share during the Connect.

**Look for productive strategies:**

- Thinking strategically about the properties of the shape that indicate which transformation has taken place, such as identifying corresponding segments or angles.
- Identifying and drawing lines of reflection and centers of rotation on the grid.
- Finding multiple ways to map Figure A onto Figure B, and Figure B onto Figure C.

## 3 Connect

**Have students share** different strategies for mapping Figure A onto Figure B, Figure B onto Figure C, and Figure A onto Figure C.

**Highlight** that there are many ways to map Figure A onto Figure C, including a single *transformation* or several *transformations*.

**Note:** The term ***sequence of transformations*** will be formally defined in the Summary.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Provide students with specific guidance for Problems 1 and 2. Ask them to describe a translation that can map Figure A onto Figure B in Problem 1. Then ask them to describe a reflection that can map Figure B onto Figure C in Problem 2.

### Extension: *Math Enrichment*

Encourage students to find multiple sequences of transformations for Problem 3.

## Math Language Development

### *MLR7: Compare and Connect*

Ask students to prepare a visual display of their chosen strategy or strategies. Then have students compare their strategy with a partner.

### English Learners

Provide students time to formulate a response before sharing their strategy with their partner. Display sentence frames to support conversation, such as:

- "To map Figure A onto Figure B, I ______ because . . ."
- "I noticed ______, so I . . ."
- I agree/disagree because . . ."

Whole Class | 5 min

## Summary

Review and synthesize how sometimes more than one transformation is needed to map one figure onto another.

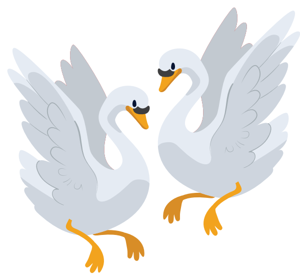
Name: ______ Date: ______ Period: ______

### Summary

**In today's lesson . . .**

You described and performed transformations that map one figure onto another.

In some cases, mapping one image onto another requires more than one transformation. To map one bird onto the other bird in the following image, a reflection and a translation are needed.



When more than one transformation is applied to a preimage, that series of moves is called a ***sequence of transformations***. There can be more than one sequence of transformations that maps a preimage to an image.

**Reflect:**





### Synthesize

**Display** the Summary from the Student Edition.

**Ask:**

- "Can you imagine a single translation, rotation, or reflection that would map one bird onto the other?" No, a single transformation will not map one bird onto the other. In this case, there needs to be a translation and a reflection.
- "How does this compare with the image from Activity 2?" Each mapping in Activity 2 can be performed with a single transformation, which is not possible with these birds.

**Highlight** that to map one bird onto the other more than one transformation is needed. Define the term ***sequence of transformations*** as two or more transformations performed in a particular order.

**Formalize vocabulary:** ***sequence of transformations***

### Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

- "Can every preimage be mapped onto an image using a single transformation?"

## Math Language Development

### *MLR2: Collect and Display*

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *sequence of transformations* that were added to the display during the lesson.

Independent | 7 min

# Exit Ticket

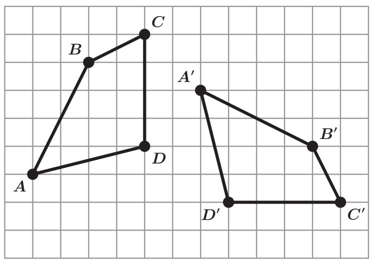
Students demonstrate their understanding by describing a sequence of transformations.

Printable

Name: ______________ Date: ________ Period: ________

## Exit Ticket 1.05

Describe a sequence of transformations that maps Quadrilateral $ABCD$ onto Quadrilateral $A'B'C'D'$.




Sample responses:

- Rotate Quadrilateral $ABCD$ 90° clockwise about point $D$, and then translate the resulting image 2 units down and 3 units right.
- Translate Quadrilateral $ABCD$ 6 units right and 3 units up, and then rotate the resulting image 90° clockwise about point $A$.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can use the terms *translation*, *reflection*, and *rotation* to precisely describe transformations.
1 2 3

b I can explain the sequence of transformations that takes a preimage to its image.
1 2 3





## Success looks like . . .

- **Language Goal:** Drawing a transformation of a figure using information given orally. **(Speaking and Listening)**
- **Language Goal:** Explaining the sequence of transformations that maps one figure onto another. **(Speaking and Listening)**
  » Explaining the sequence that maps Quadrilateral $ABCD$ onto Quadrilateral $A'B'C'D'$.

## Suggested next steps

**If students struggle to identify or describe a sequence of transformations, consider:**

- Asking, "Do you think a translation may have occurred? A reflection? A rotation? Why or why not?"
- Reviewing Activity 2.
- Assigning Practice Problems 1 and 2.

**If students do not identify a center of rotation, consider:**

- Demonstrating the different images after a $90°$ rotation using different centers of rotation.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- How did students grapple with mapping Figure A onto Figure C in Activity 2? Did they see a need for multiple transformations?
- How are students progressing in their ability to precisely describe transformations? Are they leaving out important details, such as the line of reflection or angle of rotation? How can you help them see the need for precise and detailed descriptions?

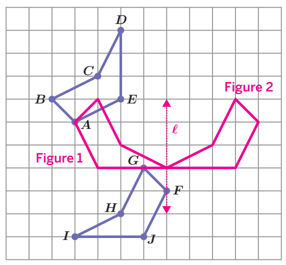
# Practice

Independent

Practice

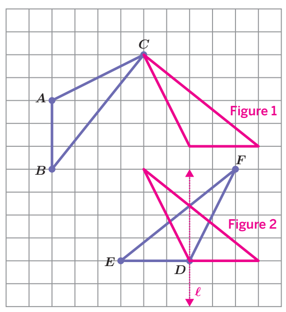
Name: ______ Date: ______ Period: ______

1. Describe a sequence of transformations that maps Polygon $ABCDE$ onto Polygon $FGHIJ$.




Sample response: Rotate $ABCDE$ 90° clockwise about point $A$ to create Figure 1. Reflect Figure 1 across line $\ell$ to create Figure 2. Translate Figure 2 left 4 units and down 3 units to create $FGHIJ$.

2. Describe a sequence of transformations that maps Triangle $ABC$ onto Triangle $DEF$.




Sample response: Rotate Triangle $ABC$ 90° counterclockwise about point $C$ to create Figure 1. Translate Figure 1 down 5 units to create Figure 2. Reflect Figure 2 across line $\ell$ to create Triangle $DEF$.





Name: ______ Date: ______ Period: ______

Practice

3. Select *all* the sequences of transformations that would return a triangle to its original position.

A. Reflect a triangle across line $m$, and then reflect the image across line $m$ again.

B. Translate a triangle 1 unit to the right, then 4 units to the left, and then 3 units to the right.

C. Reflect a triangle across line $\ell$, and then reflect the image across a different line.

D. Rotate a triangle 90° counterclockwise around point $C$, and then rotate the image 270° counterclockwise around the same point.

4. Solve each equation. Show your thinking.

a. $12 + 0.5x = 21.5$

$12 + 0.5x - 12 = 21.5 - 12$

$0.5x = 9.5$

$0.5x \div 0.5 = 9.5 \div 0.5$

$x = 19$

b. $-5(x - 2) = 30$

$-5(x - 2) \div (-5) = 30 \div (-5)$

$x - 2 = -6$

$x - 2 + 2 = -6 + 2$

$x = -4$

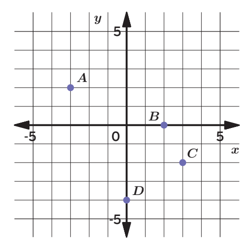
5. Identify the coordinates of the points graphed on the coordinate plane.

a. Point $A$ ( −3 , 2 )

b. Point $B$ ( 2 , 0 )

c. Point $C$ ( 3 , −2 )

d. Point $D$ ( 0 , −4 )








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| | 2 | Activity 2 | 2 |
| Spiral | 3 | Unit 1 Lesson 4 | 2 |
| | 4 | Grade 7 | 1 |
| Formative | 5 | Unit 1 Lesson 6 | 1 |

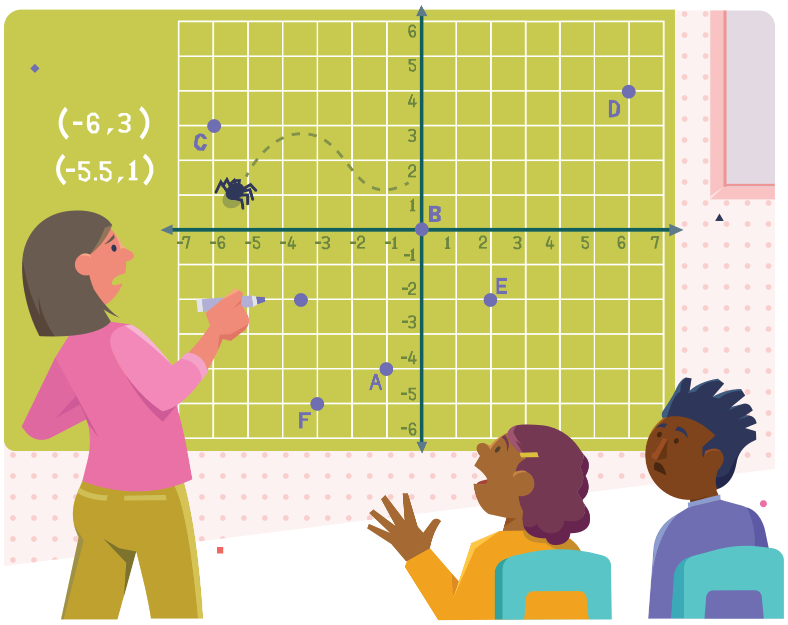
**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 1 | **LESSON 6**

# Coordinate Moves (Part 1)




Let's transform some figures and see what happens to the coordinates of the points.

## Focus

### Goals

1. **Language Goal:** Generalize the process to translate any point on the coordinate plane. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Generalize the process to reflect any point on the coordinate plane. **(Speaking and Listening, Reading and Writing)**

## Coherence

### Today

Students continue to investigate the effects of transformations. They use coordinates to describe preimages and their images under translations and reflections on the coordinate plane. Students describe the effect of translations and reflections using coordinates.

### Previously

Students used the structure of a grid to describe transformations and sequences of transformations.

### Coming Soon

In Lesson 7, students will use coordinates to describe preimages and their images under rotations on the coordinate plane. They will describe the effect of rotations using coordinates. In Lesson 8, they will develop coordinate notations to describe the effects of these transformations.

## Rigor

- Students build **conceptual understanding** by investigating the patterns among coordinates for translations and reflections.
- Students build **fluency** in describing the effect of translations and reflections using coordinates.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Activity 3 (optional) | Summary | Exit Ticket |
|---|---|---|---|---|---|
| 5 min | 10 min | 15 min | 15 min | 5 min | 5 min |
| Independent | Independent | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: rulers, tracing paper

### Math Language Development

**Review words**

- *corresponding*
- *coordinate plane*
- *image*
- *preimage*
- *reflection*
- *transformation*
- *translation*

**Amps** | **Featured Activity**

**Activities 1 and 2**

### Interactive Graphs

Students can drag points to create transformations of a preimage. You can overlay student answers to provide immediate feedback.

### Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel disengaged when asked to make predictions *before* performing the transformations in Activities 1 and 2. Encourage them to persist as they look for structure. For example, ask them to examine the coordinates of the preimages and their corresponding images, and look for any patterns that emerge.

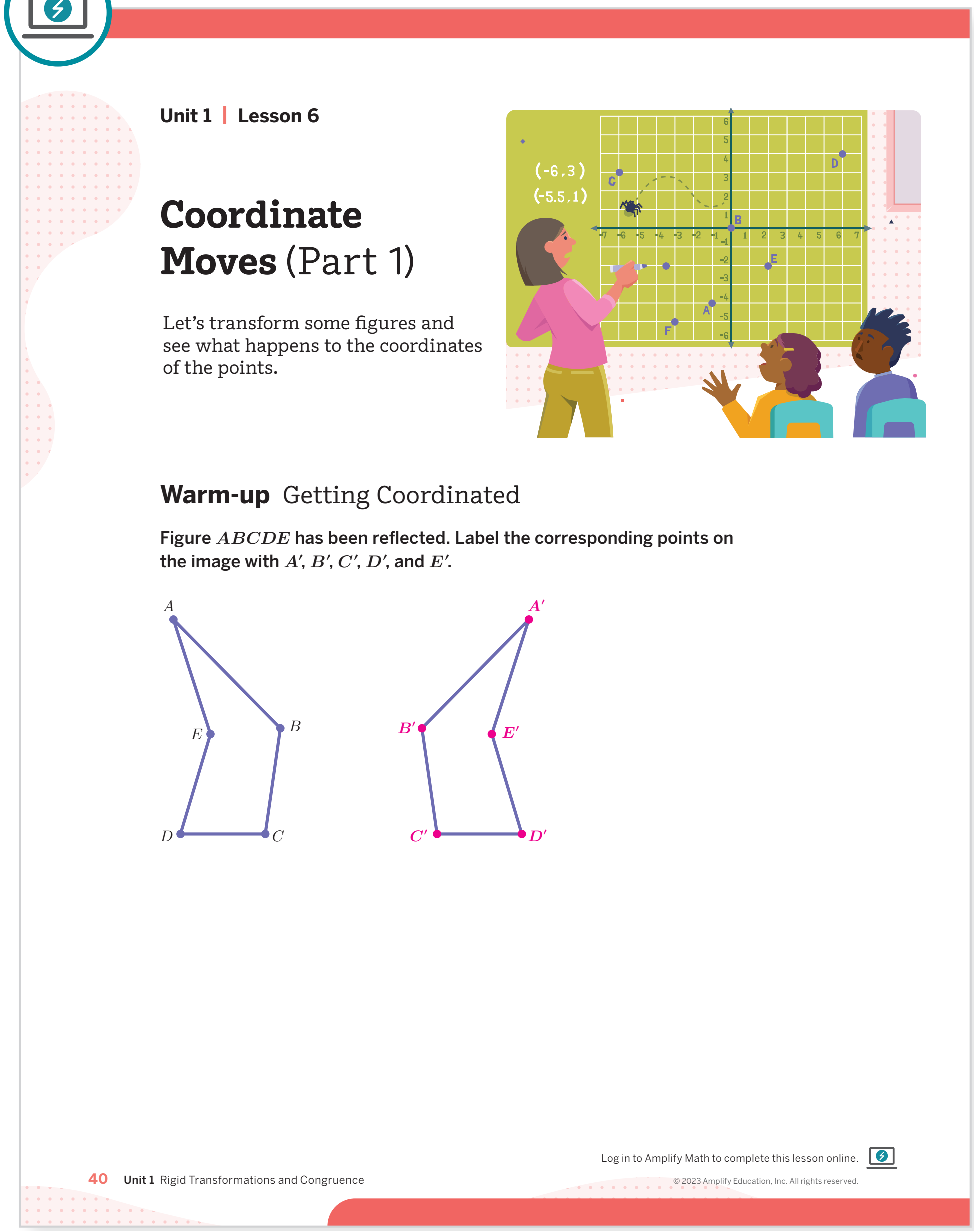
### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted as it provides additional practice for identifying corresponding points after a reflection.
- In **Activity 2**, have students complete the task only for two points.
- Optional **Activity 3** may be omitted.

# Warm-up Getting Coordinated

Students practice identifying corresponding points between a preimage and an image to further see the need for a coordinate plane to help identify specific coordinates.


Unit 1 | Lesson 6

## Coordinate Moves (Part 1)

Let's transform some figures and see what happens to the coordinates of the points.

### Warm-up Getting Coordinated

Figure $ABCDE$ has been reflected. Label the corresponding points on the image with $A'$, $B'$, $C'$, $D'$, and $E'$.



Log in to Amplify Math to complete this lesson online.

40 Unit 1 Rigid Transformations and Congruence

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## 1 Launch

Set an expectation for the amount of time students will have to work individually on the activity. Provide access to geometry toolkits for the duration of the lesson.

## 2 Monitor

**Help students get started** by asking them to name a pair of corresponding points.

**Look for points of confusion:**

- **Labeling the image of point $E$ as $B'$ by thinking the transformation is a translation.** Ask them if they can map the preimage onto the image by just moving it to the right.

**Look for productive strategies:**

- Identifying corresponding points by drawing a line of reflection, using tracing paper, or folding the page.

## 3 Connect

**Have students share** their strategies for labeling the coordinates of the image.

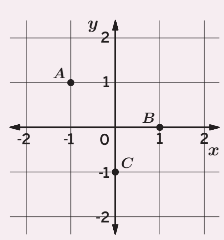
**Ask:**

- "How can you tell this is a reflection and not another type of transformation?" The orientation of the corresponding points is reversed.
- "Where would the line of reflection be?" Halfway between each pair of corresponding points, perpendicular to the distances between them.
- "What might help you describe the position of the line?" Sample response: If I knew the exact coordinates of the vertices of the preimage and image, I could find coordinates that the line of reflection would pass through.

## Power-up

**To power up students' ability to identify coordinates, have students complete:**

Recall that in a coordinate pair $(x, y)$, the $x$-value indicates the horizontal (left/right) direction from the origin while the $y$-value indicates the vertical (up/down) direction from the origin. Determine which point matches each coordinate pair.




<u>B</u> $(1, 0)$

<u>C</u> $(0, -1)$

<u>A</u> $(-1, 1)$

**Use:** Before Activity 1

**Informed by:** Performance on Lesson 5, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 3

Independent | 10 min

# Activity 1 Translating Points on the Coordinate Plane

Students translate points on the coordinate plane, and look for patterns in how the coordinates of the point change.

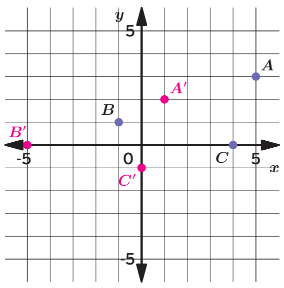
**Amps Featured Activity** Interactive Graphs

Name: ____________ Date: ________ Period: ________

### Activity 1 Translating Points on the Coordinate Plane

**Refer to the graph showing points $A$, $B$, and $C$.**

1. Translate points $A$, $B$, and $C$ to the left 4 units and down 1 unit. Draw the image of these points in the graph. Label the points in the image as $A'$, $B'$, and $C'$, respectively.




2. Write the coordinates of each point in the table.

| Preimage coordinates | | Image coordinates | |
|---|---|---|---|
| $A$ | (5, 3) | $A'$ | (1, 2) |
| $B$ | (−1, 1) | $B'$ | (−5, 0) |
| $C$ | (4, 0) | $C'$ | (0, −1) |

3. Compare the coordinates of the original points with the coordinates of their images. What do you notice?
Translating left 4 units resulted in subtracting 4 from each original $x$-coordinate. Translating down 1 unit resulted in subtracting 1 from each original $y$-coordinate.

**Are you ready for more?**

How would the coordinates change if you translated the points 1 unit up and 4 units to the right instead?
The values of $x$ and $y$ would increase, instead of decrease. I would need to add 1 to each $y$-coordinate and 4 to each $x$-coordinate, instead of subtract.





## 1 Launch

Activate students' prior knowledge by asking how a coordinate of a point is written. Have students conduct the *Think-Pair-Share* routine for Problems 1–2, and then discuss Problem 3 as a whole class.

## 2 Monitor

**Help students get started** by asking, "How can you use the grid to translate point $A$ 4 units to the left?"

**Look for points of confusion:**

- **Confusing the order of the coordinates.** Remind students that coordinate pairs are written in the form $(x, y)$, and they can remember this by remembering that $x$ comes before $y$, as in the alphabet.

**Look for productive strategies:**

- Identifying a pattern and using it to predict the coordinates of the image *before* graphing.

## 3 Connect

**Display** correct student work.

Highlight how moving left changes the $x$-coordinate, and moving down changes the $y$-coordinate.

**Ask:**

- "Where would the image of a point be if you translate it 3 units up and 4 units down?" The image would be located one unit down from the preimage.
- "Why do you subtract when moving left or down?" Left is the negative direction along the $x$-axis, and down is the negative direction along the $y$-axis.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on translating points $A$ and $C$. As time permits, and after they have successfully completed Problems 1 and 2 for these two points, have them complete the activity with the remaining point $B$.

## Math Language Development

### *MLR8: Discussion Supports*

Support students in the whole class discussion for Problem 3. Provide time for them to rehearse their responses before sharing. Ask students to annotate their table in Problem 2 with directional phrases that indicate their movement and how the $x$-coordinates changed or the $y$-coordinates changed. For example, for point $A$, have students write *move left; $x$ changes* and *move down; $y$ changes*.

### English Learners

Highlight, through the use of gestures, what movement along the axes looks like and emphasize how the values change by subtracting when moving left and down.

Pairs | 15 min

# Activity 2 Reflecting Points on the Coordinate Plane

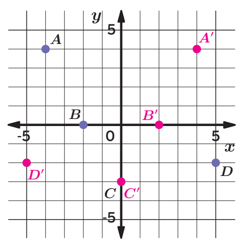
Students reflect points on the coordinate plane, and look for patterns in how the coordinates of the point change.

**Amps Featured Activity** Interactive Graphs

## Activity 2 Reflecting Points on the Coordinate Plane

**Transforming points and figures using coordinates allows you to be very precise. When they studied which shapes of billiard tables resulted in special bouncing patterns for billiard balls, mathematicians Alex Wright and Maryam Mirzakhani first transformed the tables using multiple reflections.**

1. Refer to the graph showing Points $A$, $B$, $C$, and $D$.




a. Reflect points $A$, $B$, $C$, and $D$ across the $y$-axis. Plot and label the resulting points $A'$, $B'$, $C'$, and $D'$, respectively.

b. Write the coordinates of each point in the table.

c. Compare the coordinates of the preimage points with the coordinates of their images. What do you notice?

**Reflecting a point across the $y$-axis changes the sign of the $x$-coordinate. The $x$-coordinate of the image has the opposite sign as the $x$-coordinate of the preimage. The $y$-coordinate stays the same.**

| Preimage coordinates | | Image coordinates | |
|---|---|---|---|
| $A$ | (−4, 4) | $A'$ | (4, 4) |
| $B$ | (−2, 0) | $B'$ | (2, 0) |
| $C$ | (0, −3) | $C'$ | (0, −3) |
| $D$ | (5, −2) | $D'$ | (−5, −2) |

**Featured Mathematician**


**Maryam Mirzakhani**

Maryam Mirzakhani grew up in Iran, where she was the first female student to earn a gold medal at the International Math Olympiad. She moved to the U.S. to complete her graduate work, becoming a professor at Princeton University and later Stanford University. She was awarded the Fields Medal in 2014, one of the highest honors in mathematics, for her study of moduli spaces. She and her colleagues used this work – along with geometric transformations and precise coordinates – to prove that certain quadrilateral shapes, such as billiard tables, have special "orbit closures." Mirzakhani died from breast cancer at the age of 40. Today, numerous schools, prizes, and other establishments bear her name.

Jan Vondrák





## 1 Launch

Ask students to decide who will complete Problem 1 and who will complete Problem 2. Set an expectation for the amount of time students will have to work individually on their problem, and then have them compare their responses with their partner.

## 2 Monitor

**Help students get started** by having them measure the distance from each point to the $y$-axis by counting the number of grid squares, and then counting the same amount on the other side of the $y$-axis to find the reflected point.

**Look for points of confusion:**

- **Not knowing how to reflect point $C$ in Problem 1 or point $B$ in Problem 2.** Ask students, "How far from the $y$-axis is point $C$ in Problem 1? How far away from the $x$-axis is point $B$ in Problem 2?"

**Look for productive strategies:**

- Noticing the pattern of changing the sign of the $x$-coordinate in Problem 1 and the $y$-coordinate in Problem 2.

Activity 2 continued ›

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge***

If students need more processing time, have them focus on reflecting points $A$, $C$, and $D$ in Problems 1 and 2.

## Math Language Development

***MLR7: Compare and Connect***

After students present their strategies for reflecting points, ask them to consider what is the same and what is different about the strategies used. Draw their attention to the different ways students reasoned to find the reflected coordinates.

**English Learners**

Encourage students to use gestures when reasoning about the reflected points.

## Featured Mathematician

**Maryam Mirzakhani**

Have students read about featured mathematician Maryam Mirzakhani, who earned one of the highest honors in mathematics for her study of moduli spaces.

Pairs | 15 min

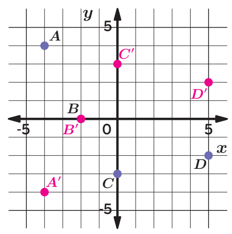
# Activity 2 Reflecting Points on the Coordinate Plane (continued)

Students reflect points on the coordinate plane, and look for patterns in how the coordinates of the point change.

Name: ______ Date: ______ Period: ______

## Activity 2 Reflecting Points on the Coordinate Plane (continued)

2. Refer to the graph showing points $A$, $B$, $C$, and $D$.




a. Reflect points $A$, $B$, $C$, and $D$ across the $x$-axis. Draw the image of these points in the graph. Label the points in the image as $A'$, $B'$, $C'$, and $D'$, respectively.

b. Write the coordinates of each point in the table.

| Preimage coordinates | | Image coordinates | |
|---|---|---|---|
| $A$ | (−4, 4) | $A'$ | (−4, −4) |
| $B$ | (−2, 0) | $B'$ | (−2, 0) |
| $C$ | (0, −3) | $C'$ | (0, 3) |
| $D$ | (5, −2) | $D'$ | (5, 2) |

c. Compare the coordinates of the preimage points with the coordinates of their images. What do you notice?
**Reflecting a point across the $x$-axis changes the sign of the $y$-coordinate. The $y$-coordinate of the image has the opposite sign as the $y$-coordinate of the preimage. The $x$-coordinate stays the same.**





## 3 Connect

**Have pairs of students share** different strategies for finding the coordinates of the image. Begin with students who used the structure of the grid without using the axes of the coordinate plane, followed by students who noticed the pattern of changing the sign of a coordinate.

**Highlight** the similarities between the coordinates of the preimage and the image, and how the line of reflection affects which coordinates are changed, and which coordinates remain the same.

**Ask:**

- "How is reflecting on the coordinate plane similar to reflecting on a grid? How is it different?"
- "Are some points more challenging to reflect than others? Why or why not?"
- "How did changing the line of reflection affect the coordinates of the image?"
- "Why does reflecting across the $y$-axis change the sign of the $x$-coordinate?" Reflecting across the $y$-axis means the point is now on the other side of the vertical $y$-axis. The $x$-coordinates on either side of the vertical $y$-axis have opposite signs.

Optional

Pairs | 15 min

# Activity 3 Partner Problems: Predicting Placement

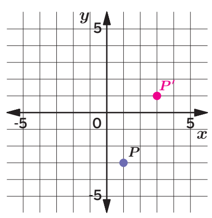
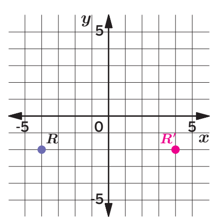
Students predict the coordinates of points that are translated or reflected to strengthen understanding of the patterns they discovered in earlier activities of this lesson.

## Activity 3 Partner Problems: Predicting Placement

**With your partner, decide who will complete Column A and who will complete Column B. You each will work on the problems in your column; however, you should have the same responses as your partner. If you do not have the same responses, rework your partner's problem and discuss any errors.**

Plan ahead: In what ways will you take on the perspective of your partner during this activity?

**For each column, perform the transformation as described. Then write the coordinates of the image.**

| Column A | Column B |
|---|---|
| 1. Translate point $P$ to the right 2 units and then up 4 units, and label the resulting image as $P'$. What are the coordinates of point $P'$?  $P'($ 3 , 1 $)$ | Point $P'$ is the result of translating point $P(1, -3)$ 2 units right and 4 units up. What are the coordinates of point $P'$? $P'($ 3 , 1 $)$ |
| 2. Point $R'$ is the result of reflecting the point $R(-4, -2)$ across the $y$-axis. What are the coordinates of point $R'$? $R'($ 4 , −2 $)$ | Reflect point $R$ across the $y$-axis and label the resulting image as point $R'$. What are the coordinates of point $R'$?  $R'($ 4 , −2 $)$ |

STOP

44 Unit 1 Rigid Transformations and Congruence



## 1 Launch

Conduct the ***Partner Problems*** routine.

## 2 Monitor

**Help students get started** by asking, "What patterns did you notice during Activities 1 and 2?"

**Look for points of confusion:**

- **Thinking they have to complete both problems from each column.** Explain that if a student and their partner arrive at the same response for their respective problems, they can move to the next problem. They only have to work on their partner's problem if they have different responses.

**Look for productive strategies:**

- Referencing patterns from previous activities to make predictions about the coordinates after the translations and reflections.

## 3 Connect

**Have students share** any problems in which they did not have the same response as their partner, and how they came to an agreement of their final response.

**Ask:**

- "Did anyone learn a new strategy from their partner?"
- "If you know the coordinates of a point and the transformation that occurs, do you need to refer to a graph to know the coordinates of the image?"

**Highlight** strategies students used to find the coordinates without graphing.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, suggest that they complete Column A if they are more confident with reflections and Column B if they are more comfortable with translations. Consider using intentional grouping to pair students so that one student feels confident about reflections and can help assist their partner.

Whole Class | 5 min

## Summary

Review and synthesize how the coordinates of a point changes after a translation or reflection on the coordinate plane.

Name: ______ Date: ______ Period: ______

### Summary

**In today's lesson . . .**

You performed translations and reflections on the coordinate plane, and observed how the coordinates of the transformed points changed under each of these types of transformations.

You can use coordinates to describe the position of points and find patterns in the coordinates of transformed points.

You can describe a translation by expressing it as a sequence of horizontal and vertical translations.

| Translating a point to the left or right . . . | Translating a point up or down . . . |
|---|---|
| changes the value of the $x$-coordinate.<br>**Example:** Preimage: $(3, -5)$<br>• If the point is translated to the left 2 units, the image is $(1, -5)$.<br>• If the point is translated to the right 2 units, the image is $(5, -5)$. | changes the value of the $y$-coordinate.<br>**Example:** Preimage: $(3, -5)$<br>• If the point is translated up 2 units, the image is $(3, -3)$.<br>• If the point is translated down 2 units, the image is $(3, -7)$. |

Reflecting a point across an axis changes the sign of one coordinate.

| Reflecting a point across the $x$-axis . . . | Reflecting a point across the $y$-axis . . . |
|---|---|
| changes the sign of the $y$-coordinate. The $x$-coordinate remains the same.<br>**Example:** Preimage: $(3, -5)$<br>Image: $(3, 5)$ | changes the sign of the $x$-coordinate. The $y$-coordinate remains the same.<br>**Example:** Preimage: $(3, -5)$<br>Image: $(-3, -5)$ |

**Reflect:**





### Synthesize

**Highlight** the patterns that students generated during the course of the lesson.

**Ask:**

- "What are some advantages to knowing the coordinates of points when you are performing transformations?"
- "How does translating a point to the left or right affect the coordinates of the point? Up or down?"
- "What changes did you see when reflecting points across the $x$-axis? The $y$-axis?"
- "If the point $(-8, -5)$ undergoes the following transformations, what would be the coordinates of the image?"
  - » Translation left a units and down b units (-8 - a, -5 - b)
  - » Translation right a units and up b units (-8 + a, -5 + b)
  - » Reflection across the x-axis (-8, 5)
  - » Reflection across the y-axis (8, -5)

### Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

- "How can knowing the coordinate of a point help you translate or reflect it?"

Independent | 5 min

# Exit Ticket

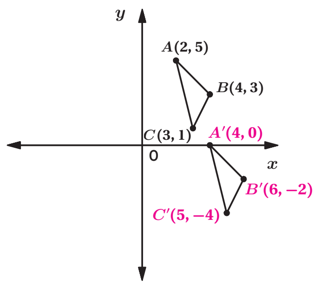
Students demonstrate their understanding of translations and reflections on the coordinate plane by finding the missing coordinates of an image.

Printable

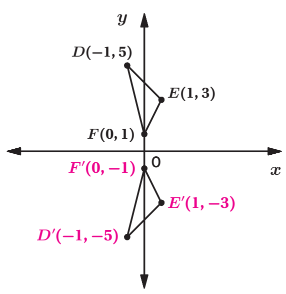
Name: ______ Date: ______ Period: ______

## Exit Ticket 1.06

1. Triangle $ABC$ is translated 5 units down and 2 units to the right to create Triangle $A'B'C'$. Label the coordinates of the image.




2. Triangle $DEF$ is reflected across the $x$-axis to create Triangle $D'E'F'$. Label the coordinates of the image.




**Self-Assess**

1 I don't really get it — 2 I'm starting to get it — 3 I got it

a. I can apply translations to points on a grid if I know their coordinates. 1 2 3

b. I can apply reflections to points on a grid if I know their coordinates. 1 2 3



**Lesson 6** Coordinate Moves (Part 1)

## Success looks like . . .

- **Language Goal:** Generalizing the process to translate any point on the coordinate plane. **(Speaking and Listening, Reading and Writing)**
  - » Determining the coordinates of a translated image from the coordinates of the original image.
- **Language Goal:** Generalizing the process to reflect any point on the coordinate plane. **(Speaking and Listening, Reading and Writing)**
  - » Determining the coordinates of a reflected image from the coordinates of the original image.

## Suggested next steps

**If students struggle with finding the coordinates of the image in Problem 1, consider:**

- Reviewing integer operations.

**If students struggle with finding the coordinates in Problem 2, consider:**

- Reviewing Problem 2 from Activity 2.
- Assigning Practice Problem 3.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- How well do your students understand the patterns of the coordinates for translations and reflections? Do you need to review integer operations with them to better understand the patterns for translations?
- Are they able to explain why the $x$-coordinates change signs when reflecting across the $y$-axis, and why the $y$-coordinates change signs when reflecting across the $x$-axis? How can you help them see that this makes sense?

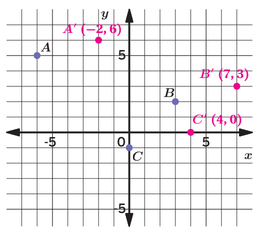
# Practice

Independent

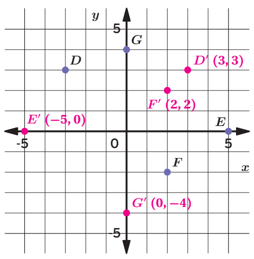
Practice

Name: Date: Period:

1. Points $A(-6, 5)$, $B(3, 2)$, and $C(0, -1)$ are plotted on the coordinate plane. What are the coordinates of $A$, $B$, and $C$ after a translation 4 units to the right and 1 unit up? Plot these points on the grid, and label them $A'$, $B'$, and $C'$. Include the coordinates of the images in your labels.




2. Points $D(-3, 3)$, $E(5, 0)$, $F(2, -2)$, and $G(0, 4)$ are plotted on the coordinate plane.




a. What are the coordinates of points $D$ and $E$ after a reflection across the $y$-axis? Plot these points on the grid, and label them $D'$ and $E'$. Include the coordinates of the images in your labels.

$D'(3, 3)$; $E'(-5, 0)$

b. What are the coordinates of points $F$ and $G$ after a reflection across the $y$-axis? Plot these points on the grid, and label them $F'$ and $G'$. Include the coordinates of the images in your labels.

$F'(2, 2)$; $G'(0, -4)$

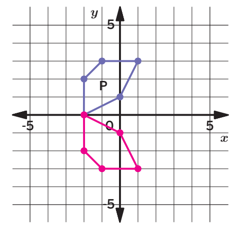




Name: Date: Period:

3. Pentagon P is reflected across the $x$-axis. Predict the coordinates of the image by completing the table. Check your predictions by graphing the image.

| Preimage coordinates | Image coordinates |
|---|---|
| $(-2, 0)$ | $(-2, 0)$ |
| $(-2, 2)$ | $(-2, -2)$ |
| $(-1, 3)$ | $(-1, -3)$ |
| $(1, 3)$ | $(1, -3)$ |
| $(0, 1)$ | $(0, -1)$ |




4. For each statement, explain how you would find the measure of the missing angle.

a. Two angles are complementary, and you are given the measure of one of these angles.

**Subtracting the known angle from 90° would give the unknown complementary angle.**

b. Two angles are supplementary, and you are given the measure of one of these angles.

**Subtracting the known angle from 180° would give the unknown supplementary angle.**

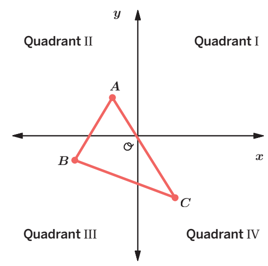
5. Triangle $ABC$ is rotated 90° clockwise about the origin to create Triangle $A'B'C'$. In which quadrant would point $C'$ be located?

A. Quadrant I

B. Quadrant II

C. Quadrant III

D. Quadrant IV








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 3 | 2 |
| Spiral | 4 | Grade 7 | 1 |
| Formative | 5 | Unit 1 Lesson 7 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 1 | **LESSON 7**

# Coordinate Moves (Part 2)

Let's transform some more figures and see what happens to the coordinates of the points.

## Focus

### Goal

1. **Language Goal:** Generalize the process to rotate any point on the coordinate plane. **(Speaking and Listening, Reading and Writing)**

## Coherence

### Today

Students continue to investigate the effects of transformations. They use coordinates to describe preimages and their images under rotations on the coordinate plane. Students describe the effect of rotations using coordinates.

### Previously

In Lesson 6, students learned how to translate and reflect figures or points on the coordinate plane. In Lesson 4, students learned how to rotate figures or points on a grid.

### Coming Soon

In Lesson 8, students will explore sequences of transformations on the coordinate plane, and develop coordinate notations to describe the effects of these transformations.

## Rigor

- Students build **conceptual understanding** by investigating the patterns among coordinates for rotations.
- Students build **fluency** in describing the effect of rotations using coordinates.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 12 min | 20 min | 5 min | 7 min |
| Pairs | Independent | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

## Materials

- Exit Ticket
- Additional Practice
- Power-up PDF, as needed
- Power-up PDF (answers), as needed
- Activity 2 PDF (for display)
- geometry toolkits: rulers, tracing paper

## Math Language Development

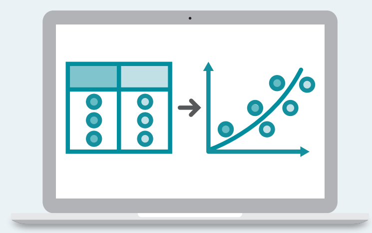
**Review words**

- *clockwise*
- *coordinate plane*
- *corresponding*
- *counterclockwise*
- *image*
- *preimage*
- *rotation*
- *origin*
- *transformation*

**Amps** | **Featured Activity**

### Activity 1
### Using Work From Previous Slides

Students make observations about a rotation. In the next slide, students use their observations to construct an image and check their understanding. It's their work, so they get to hold onto it!



Amps POWERED BY desmos

## Building Math Identity and Community

Connecting to Mathematical Practices

At first, students may feel lost if they do not make the connection between the direction of the rotation and the effect on the coordinates in Activity 2. Encourage them to persist as they look for structure. For example, ask them to examine the coordinates of the preimages and their corresponding images, and look for any patterns that emerge.

## Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted as it provides additional practice identifying corresponding points after a rotation.
- In **Activity 2**, students may work in groups of three and complete one part each from Problem 1.
- In the **Exit Ticket**, omit Problem 1.

Pairs | 5 min

# **Warm-up** Rotating Coordinates

Students will make observations about a square that has been rotated to strengthen their conceptual understanding of rotation.

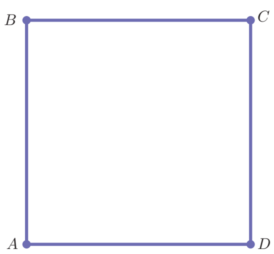
Unit 1 | Lesson 7

## **Coordinate Moves** (Part 2)

Let's transform some more figures and see what happens to the coordinates of the points.

### **Warm-up** Rotating Coordinates

Square $ABCD$ has been rotated in such a way that the image coincided with the preimage.




What could the angle of rotation be? Find as many possible answers as you can.

Sample responses: 90°, 180°, 270°, 360°



Log in to Amplify Math to complete this lesson online.



## 1 Launch

Conduct the ***Think-Pair-Share*** routine as students work. Provide access to geometry toolkits for the duration of the lesson.

## 2 Monitor

**Help students get started** by asking them to identify a possible center of rotation.

**Look for points of confusion:**

- **Struggling to visualize a possible rotation.** Encourage students to use tracing paper to create an image.
- **Suggesting only one correct answer.** Challenge students to look for multiple angles of rotation.

**Look for productive strategies:**

- Finding multiple possible angles of rotation.
- Describing measurements of possible angles using the formula $90k$.

## 3 Connect

**Have students share** their strategies for finding the center of rotation and possible angle of rotation.

**Highlight** that when determining an angle of rotation, many correct responses are possible, but all of them are multiples of $90^{\circ}$.

## Differentiated Support

**Accessibility: *Guide Processing and Visualization***

Have students rotate their Student Edition until the preimage is in the same orientation as the image, keeping track of the movements of the vertices of the preimage. Consider demonstrating for one angle of rotation, such as $90^{\circ}$, to help students visualize the rotation. Ask them if there are any other rotations. **Note:** At this point, students are not expected to know the angles of rotations.

## Power-up

**To power up students' ability to make sense of rotations:**

Provide students with a copy of the Power-up PDF.

**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 6, Practice Problem 5

Independent | 12 min

# Activity 1 Rotations of a Point

Students will make observations about points that have been rotated, and use their observations to predict a pattern in how the coordinates of the points are changed.

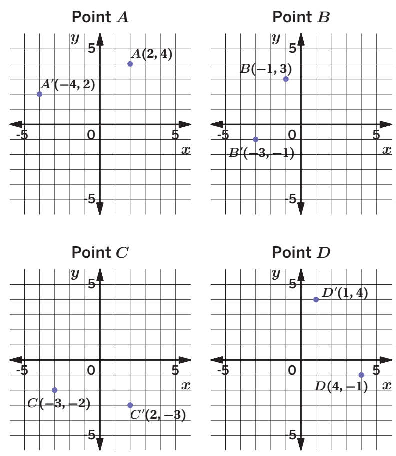
**Amps Featured Activity** Using Work From Previous Slides

Name: ______________ Date: ________ Period: ________

## Activity 1 Rotations of a Point

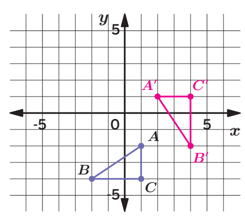
1. Each of these points has been rotated 90° counterclockwise about the origin. Compare the coordinates of the original points with the coordinates of their images. What do you notice?

   Sample responses:
   - The $x$- and $y$-coordinates are switched, and the $x$-coordinate of the image has the opposite sign of the $y$-coordinate of the preimage.
   - The $x$-coordinate of the image is the opposite of the $y$-coordinate of the preimage. The $y$-coordinate of the image is the $x$-coordinate of the preimage.




2. Use the pattern you noticed to predict the coordinates of Triangle $A'B'C'$ after rotating Triangle $ABC$ 90° counterclockwise about the origin. Record your predictions in the table. Then check your predictions by plotting your points on the coordinate plane.

| Preimage coordinates | | Image coordinates | |
|---|---|---|---|
| $A$ | $(1, -2)$ | $A'$ | $(2, 1)$ |
| $B$ | $(-2, -4)$ | $B'$ | $(4, -2)$ |
| $C$ | $(1, -4)$ | $C'$ | $(4, 1)$ |




**Critique and Correct:** Your teacher will display an incorrect rotation. With a partner, determine why it is incorrect and then correct it.



Lesson 7 Coordinate Moves (Part 2) 49

### 1 Launch

Complete Problem 1 as a whole class, and then have students complete Problem 2 independently.

### 2 Monitor

**Help students get started** by asking them to examine the coordinates of each pair of points given at the start of the activity and look for similarities.

**Look for points of confusion:**

- **Rotating first, and then recording the coordinates of points in Problem 2.** This problem is designed for students to test whether their observations in Problem 1 hold true for polygons. Have them make their predictions first.
- **Graphing the image incorrectly in Problem 2.** Have students use their geometry toolkits to check their work. Encourage students to mark the origin to ensure correct alignment when rotating the tracing paper.

**Look for productive strategies:**

- Checking their work using tracing paper or an index card.
- Noticing that 90° rotations about the origin move a point to a neighboring quadrant.

### 3 Connect

**Highlight** the pattern among the coordinates when rotating a point 90° counterclockwise about the origin. The $x$-coordinate of the image is the opposite of the $y$-coordinate of the preimage. The $y$-coordinate of the image is the $x$-coordinate of the preimage.

**Ask** students if they think patterns exist for other degrees and directions of rotations about the origin.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on rotating points $B$ and $C$ in Problem 2. As time permits, and after they have successfully completed the rotation for each, have them rotate the remaining point $A$.

## Math Language Development

### *MLR3: Critique, Correct, Clarify*

During the discussion of Problem 2, display an incorrectly performed rotation. For example, provide the image coordinates for point $C$ as $(4, -1)$. Ask these questions:

- ***Critique:*** "How do you know that these could not be the coordinates of the reflection of point $C$? Is the $x$-coordinate correct? Is the $y$-coordinate correct?"
- ***Correct:*** "What should be the correct coordinates of the image?"
- ***Clarify:*** Describe in your own words how a 90° counterclockwise rotation about the origin affects the coordinates of a point.

Pairs | 20 min

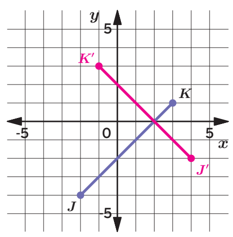
# Activity 2 Rotations in Different Directions

Students rotate a line segment using different angles of rotation and in different directions to look for patterns and draw conclusions about the coordinates of the images.

### Activity 2 Rotations in Different Directions

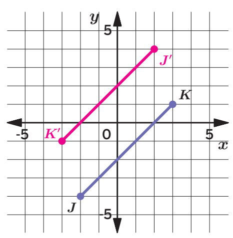
1. Rotate line segment $JK$ as directed, and record the coordinates of the image in the table.

a. 90° counterclockwise about the origin



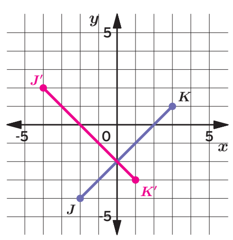

| Preimage coordinates | | Image coordinates | |
|---|---|---|---|
| $J$ | $(-2, -4)$ | $J'$ | **(4, −2)** |
| $K$ | $(3, 1)$ | $K'$ | **(−1, 3)** |

b. 180° counterclockwise about the origin




| Preimage coordinates | | Image coordinates | |
|---|---|---|---|
| $J$ | $(-2, -4)$ | $J'$ | **(2, 4)** |
| $K$ | $(3, 1)$ | $K'$ | **(−3, −1)** |

c. 270° counterclockwise about the origin




| Preimage coordinates | | Image coordinates | |
|---|---|---|---|
| $J$ | $(-2, -4)$ | $J'$ | **(−4, 2)** |
| $K$ | $(3, 1)$ | $K'$ | **(1, −3)** |




## 1 Launch

Have each partner choose three rotations each to complete from Problem 1. Then have them share their responses with each other before completing Problem 2 together.

## 2 Monitor

**Help students get started** by referring them back to Activity 1 and reminding them of their observations about how the coordinates changed when rotating a point 90° counterclockwise about the origin.

**Look for points of confusion:**

- **Struggling with rotating about the origin.** Remind students that they may use their geometry toolkits as needed. They may find tracing paper to be particularly helpful.
- **Thinking that the same pattern among the coordinates applies even when the angle of rotation or direction changes.** Have them test their predictions by actually rotating each endpoint using tracing paper to see that the direction and angle of rotation affects the resulting coordinates of the image.
- **Not noticing when rotated points are in the wrong quadrant.** Students may find it helpful to imagine the entire plane rotating to become new quadrants.

**Look for productive strategies:**

- Predicting the quadrant placement of points before plotting the rotated images.
- Seeing the connection between a 90° rotation in one direction and a 270° rotation in the opposite direction.
- Noticing that rotating a line segment 90° or 270° produces perpendicular line segments, while rotating 180° produces parallel line segments. Highlight this concept in the Connect section.

Activity 2 continued >

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them complete Problems 1b and 1e and only work on the others as time allows.

### Extension: *Math Enrichment*

Ask students to explain *why* rotating a point 180° either clockwise or counterclockwise results in the same image. Sample response: A full circle measures 360° and regardless in which direction a point is rotated, rotating it 180° will result in the image rotating halfway around a full circle.

## Math Language Development

### *MLR7: Compare and Connect*

Have students compare different types of rotations. For example, how does rotating 90° clockwise compare to rotating 90° counterclockwise?

Pairs | 20 min

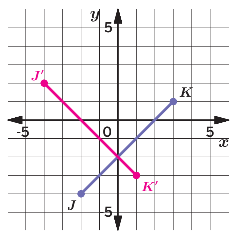
# Activity 2 Rotations in Different Directions (continued)

Students rotate a line segment using different angles of rotation and in different directions to look for patterns and draw conclusions about the coordinates of the images.

Name: ______ Date: ______ Period: ______

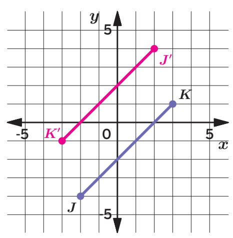
## Activity 2 Rotations in Different Directions (continued)

d. 90° clockwise about the origin



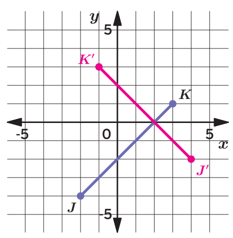

| Preimage coordinates | | Image coordinates | |
|---|---|---|---|
| $J$ | $(-2, -4)$ | $J'$ | $(-4, 2)$ |
| $K$ | $(3, 1)$ | $K'$ | $(1, -3)$ |

e. 180° clockwise about the origin




| Preimage coordinates | | Image coordinates | |
|---|---|---|---|
| $J$ | $(-2, -4)$ | $J'$ | $(2, 4)$ |
| $K$ | $(3, 1)$ | $K'$ | $(-3, -1)$ |

f. 270° clockwise about the origin




| Preimage coordinates | | Image coordinates | |
|---|---|---|---|
| $J$ | $(-2, -4)$ | $J'$ | $(4, -2)$ |
| $K$ | $(3, 1)$ | $K'$ | $(-1, 3)$ |

2. What observations can you make about the images and their coordinates?

Sample responses:

- The 180° rotations have the same image coordinates, regardless of whether the direction is clockwise or counterclockwise.
- The 90° clockwise rotation has the same image as the 270° counterclockwise rotation.
- The 90° counterclockwise rotation has the same image as the 270° clockwise rotation.

STOP





## 3 Connect

**Have groups of students share** similarities or patterns they discovered among the coordinates of the images for Problem 1.

Sample responses:

- I noticed that in Problems b and e, the image line segments were both parallel to the preimage line segments. The angle of rotation was the same, 180°, even though the directions were different.
- I noticed that each of the following problems produced the same image:

  Problems a and f

  Problems b and e

  Problems c and d

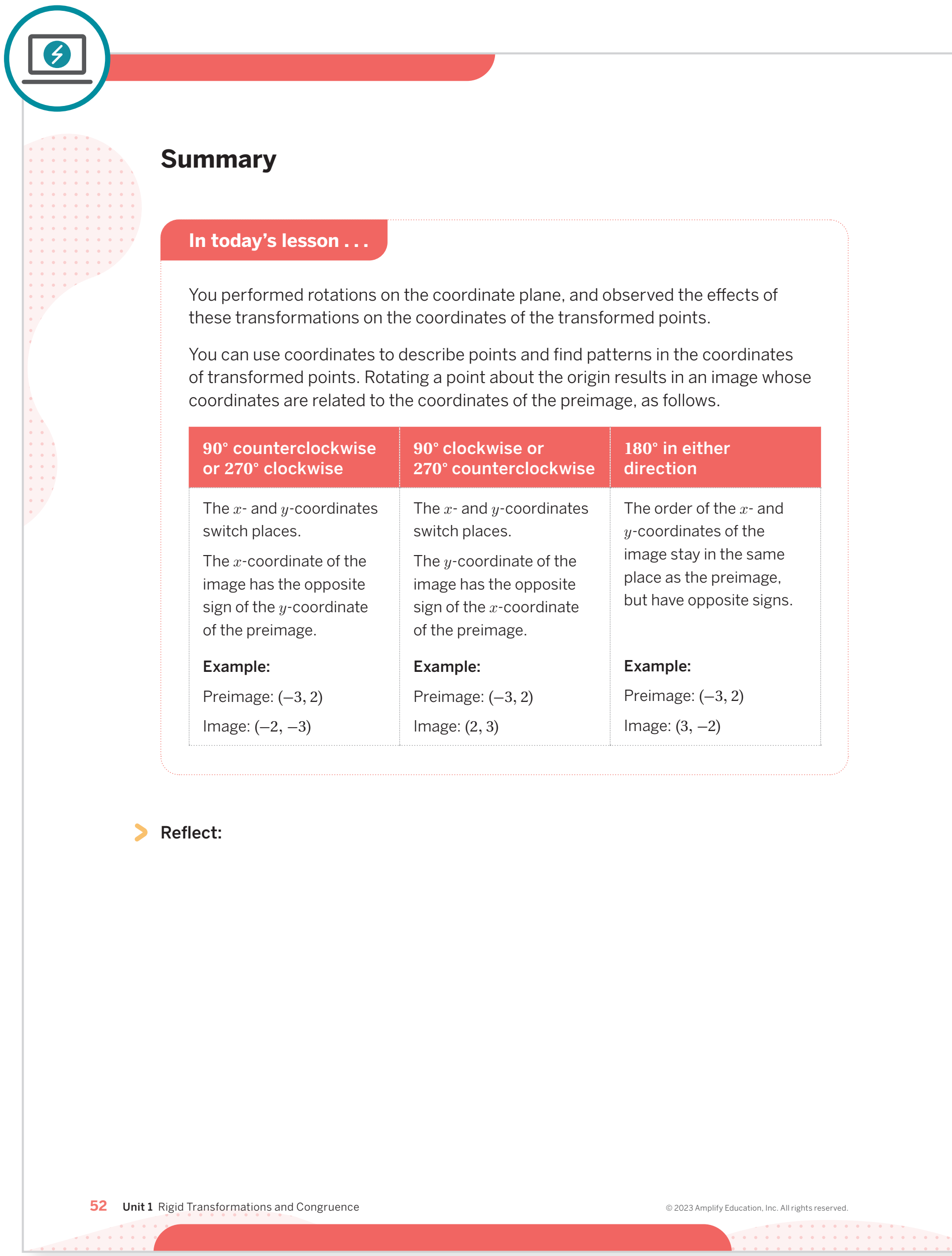
**Display** the Activity 2 PDF.

**Highlight** that rotating 180° in either direction produces parallel line segments. This will be helpful to students as they study parallel lines in future lessons. Rotating 90° or 270° in either direction produces perpendicular line segments. Students may notice that rotating 90° in one direction results in the same image as rotating 270° in the opposite direction. This will be discussed further during the Summary.

Whole Class | 5 min

# Summary

Review and synthesize how the coordinates of points change after a rotation on the coordinate plane.



## Summary

**In today's lesson . . .**

You performed rotations on the coordinate plane, and observed the effects of these transformations on the coordinates of the transformed points.

You can use coordinates to describe points and find patterns in the coordinates of transformed points. Rotating a point about the origin results in an image whose coordinates are related to the coordinates of the preimage, as follows.

| 90° counterclockwise or 270° clockwise | 90° clockwise or 270° counterclockwise | 180° in either direction |
|---|---|---|
| The $x$- and $y$-coordinates switch places.<br>The $x$-coordinate of the image has the opposite sign of the $y$-coordinate of the preimage. | The $x$- and $y$-coordinates switch places.<br>The $y$-coordinate of the image has the opposite sign of the $x$-coordinate of the preimage. | The order of the $x$- and $y$-coordinates of the image stay in the same place as the preimage, but have opposite signs. |
| **Example:**<br>Preimage: $(-3, 2)$<br>Image: $(-2, -3)$ | **Example:**<br>Preimage: $(-3, 2)$<br>Image: $(2, 3)$ | **Example:**<br>Preimage: $(-3, 2)$<br>Image: $(3, -2)$ |

Reflect:

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## Synthesize

**Highlight** the patterns that students generated during the course of the lesson.

**Ask:**

- "What are some advantages to knowing the coordinates of points when performing transformations?"
- "When rotating $180°$, does it matter whether the rotation is clockwise or counterclockwise?" No, the resulting image is the same no matter in which direction the rotation occurred.
- "What similarities did you see when rotating $90°$ in one direction versus rotating $270°$ in the opposite direction?" They have the same effect on the preimage and produce the same image.
- "If the point $(-8, -5)$ undergoes the following transformations, what would be the coordinates of the image?" **Note:** You may find it helpful to display a coordinate plane with the point $(-8, -5)$ plotted for students to reference.
  - » Rotation $90°$ clockwise $(-5, 8)$
  - » Rotation $270°$ counterclockwise $(-5, 8)$
  - » Rotation $90°$ counterclockwise $(5, -8)$
  - » Rotation $270°$ clockwise $(5, -8)$
  - » Rotation $180°$ clockwise $(8, 5)$
  - » Rotation $180°$ counterclockwise $(8, 5)$

## Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

- "How can knowing the coordinates of points help you rotate a point or figure?"

Independent | 7 min

# Exit Ticket

Students demonstrate their understanding of rotations on the coordinate plane by describing the degree and direction of rotations that have occurred.

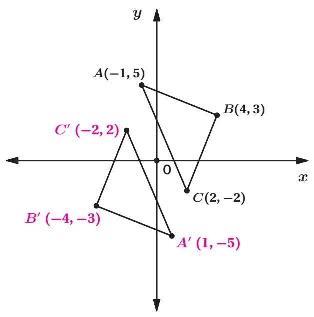
Printable

Name: ______________ Date: __________ Period: __________

## Exit Ticket 1.07

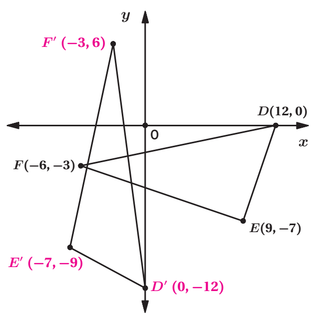
**The following triangles have been rotated about the origin. Indicate the degree and direction of each rotation, and label the coordinates of the image.**

1.




Sample responses:
- 180° clockwise
- 180° counterclockwise

2.




Sample responses:
- 90° clockwise
- 270° counterclockwise

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can apply rotations to points on a grid if I know their coordinates.

1 2 3



**Lesson 7** Coordinate Moves (Part 2)

## Success looks like . . .

- **Language Goal:** Generalizing the process to rotate any point on the coordinate plane. **(Speaking and Listening, Reading and Writing)**
  - » Giving the degree, direction, and coordinates of a rotated image in Problems 1 and 2.

## Suggested next steps

**If students have trouble identifying the rotation in Problem 1, consider:**

- Reviewing Activity 2, Problems 1b and 1e.

**If students have trouble identifying the rotation in Problem 2, consider:**

- Reviewing Activity 2, Problems 1c and 1d.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- How well do your students understand the patterns of the coordinates for rotations? Are they able to explain why a 90° rotation in one direction results in the same image as a 270° rotation in the opposite direction? How can you help reinforce this understanding?
- Are students able to explain why a 180° rotation results in the same image, regardless of the direction of the rotation?

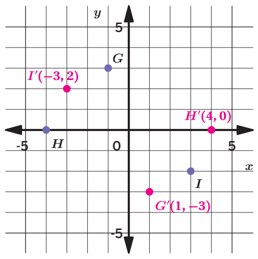
# Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. Points $G(-1, 3)$, $H(-4, 0)$, and $I(3, -2)$ are plotted on the coordinate plane. What are the coordinates of $G$, $H$, and $I$ after a rotation $180°$ about the origin? Plot these points on the grid, and label them $G'$, $H'$, and $I'$. Include the coordinates of the images in your labels.




2. Point $P(5, 3)$ is rotated $270°$ clockwise about the origin, and the image is labeled $P'$. Which of the following are the coordinates of point $P'$?

A. $(-5, -3)$ B. $(-3, -5)$ C. $(3, -5)$ D. $(-3, 5)$

3. Triangle $XYZ$ has been rotated about the origin to create Triangle $X'Y'Z'$. The following table shows the coordinates of the vertices. Indicate the degree and direction of the rotation that maps Triangle $XYZ$ onto Triangle $X'Y'Z'$.

| Preimage coordinates | | Image coordinates | |
|---|---|---|---|
| $X$ | $(0, 5)$ | $X'$ | $(5, 0)$ |
| $Y$ | $(-2, 1)$ | $Y'$ | $(1, 2)$ |
| $Z$ | $(4, 3)$ | $Z'$ | $(3, -4)$ |

90° clockwise rotation about the origin, or 270° counterclockwise rotation about the origin

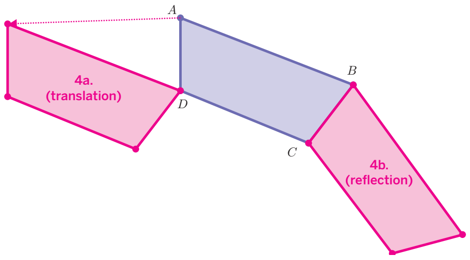




Name: ______ Date: ______ Period: ______

Practice

4. Draw the image of Quadrilateral $ABCD$ after each transformation indicated.




a. A translation that maps point $B$ onto point $D$.

b. A reflection across line segment $BC$.

5. Write five expressions that have a value of $\frac{3}{5}$, according to the following criteria.
Sample responses shown.

a. One expression must be a sum. $\frac{1}{5}+\frac{2}{5}$

b. One expression must be a difference. $1\frac{2}{5}-\frac{4}{5}$

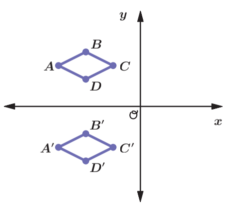
c. One expression must be a product. $3 \cdot \frac{1}{5}$

d. One expression must be a quotient. $6 \div 10$

e. One expression must involve at least two operations. $2\left(\frac{1}{10}+\frac{2}{10}\right)$

6. Mai says that Quadrilateral $A'B'C'D'$ is the image of Quadrilateral $ABCD$ after a reflection across the $x$-axis. Do you agree with Mai? Explain your thinking.

Sample response: No. The corresponding points do not fit a reflection across the $x$-axis. The image is a translation.








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 1 Lesson 2 | 2 |
| | 5 | Grade 7 | 2 |
| Formative | 6 | Unit 1 Lesson 8 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Describing Transformations

Let's transform polygons on the coordinate plane.


## Focus

### Goals

1. **Language Goal:** Create a drawing on a coordinate plane of a transformed object using verbal descriptions. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Identify what information is needed to transform a polygon and ask questions to elicit that information. **(Speaking and Listening)**

## Coherence

### Today

Students apply a sequence of transformations to a polygon on the coordinate plane. They use the ***Info Gap*** routine to request information from their partner, and explain *why* they need each piece of information. Students also explore patterns and discover rules for these transformations.

### Previously

In Lessons 6 and 7, students practiced applying individual transformations and sequences of transformations to figures on the coordinate plane.

### Coming Soon

In Lesson 9, students will begin to see that translations, rotations, and reflections preserve lengths and angle measures, and lay the groundwork for identifying congruent figures.

## Rigor

- Students build **fluency** in using precise mathematical vocabulary to describe a sequence of transformations.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 10 min | 7 min | 5 min |
| Pairs | Pairs | Independent | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Power-up PDF, as needed
- Power-up PDF (answers), as needed
- Activity 1 PDF, pre-cut cards
- Activity 1 PDF (answers)
- *Info Gap Routine* PDF (for display)
- geometry toolkits: rulers, tracing paper
- graph paper (optional)

### Math Language Development

**Review words**

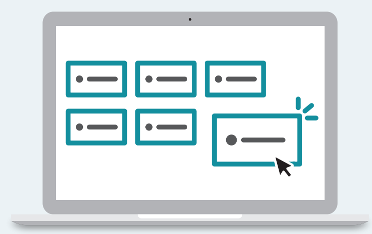
- *translation*
- *rotation*
- *reflection*
- *transformation*
- *sequence of transformations*
- *corresponding points*

**Amps** | Featured Activity

**Activity 1**

**Interactive Graphs**

Students can drag points to create transformations of a preimage. You can overlay student answers to provide immediate feedback.



### Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated as they ask questions and receive limited information in response; they may be unsure of how to plan a solution pathway that helps them phrase their questions to receive the information they need. Encourage them to reflect on what information would be necessary to perform the entire sequence of transformations, and organize their thinking by recording their questions and the answers they receive.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted, as it provides additional practice understanding rotations and the center of rotation.
- In **Activity 1**, have students complete only the first Problem and Data card.

Pairs | 5 min

# **Warm-up** Center of Rotation

Students examine a rotation on the coordinate plane, identifying the center of rotation and understanding its importance in describing a rotation.

Name: ______________ Date: ________ Period: ________

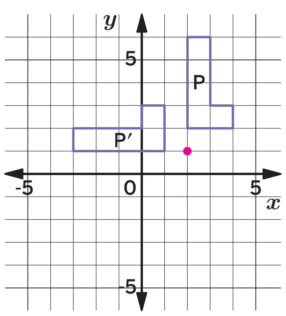
**Unit 1 | Lesson 8**

## Describing Transformations

Let's transform polygons on the coordinate plane.


### **Warm-up** Center of Rotation

**Andre performs a 90° counterclockwise rotation of Polygon P and creates the image Polygon P′, but he does not indicate the center of the rotation. What is the center of rotation?**




The center of rotation is (2, 1).

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Have students work individually to complete the Warm-up, and then have them share responses with a partner. Provide access to geometry toolkits for the duration of the lesson.

## 2 Monitor

**Help students get started** by demonstrating a rotation using a center of rotation that is *not* the origin.

**Look for points of confusion:**

- **Thinking that the center of rotation can only be at the origin.** Have students rotate the polygon in the Warm-up using the origin as the center to realize that the image is not the same as the image shown.

**Look for productive strategies:**

- Using tracing paper to perform the rotation and test possible locations where the center of rotation would need to be for the preimage to be mapped onto the image.
- Drawing a line segment that connects two corresponding points, and constructing a perpendicular line through its center to locate the center of rotation.

## 3 Connect

**Have students share** their strategies for locating the center of rotation.

**Ask,** "Why is it important that you know the precise center of rotation?"

**Highlight** how moving the center of rotation shifts the location of the image.

## Power-up

**To power up students' ability to identifying corresponding vertices:**

Provide students with a copy of the Power-up PDF.

**Use:** Before Activity 1

**Informed by:** Performance on Lesson 7, Practice Problem 6

Pairs | 15 min

# **Activity 1** Info Gap: Transformation Information

Students use the *Info Gap* routine to apply a sequence of transformations to a polygon.

**Amps Featured Activity** Interactive Graphs

**Activity 1** Info Gap: Transformation Information

**You will be given either a *problem card* or a *data card*.**
**Do not show or read your card to your partner.**

| If you are given the *problem card*: | If you are given the *data card*: |
|---|---|
| **1.** Silently read your card, and think about what information you need to be able to solve the problem. | **1.** Silently read your card. |
| **2.** Ask your partner for the specific information that you need. | **2.** Ask your partner, "What specific information do you need?" and wait for them to ask for information. |
| **3.** Explain how you will use the information to solve the problem.<br>Continue to ask questions until you have enough information to solve the problem. | **3.** Before sharing the information, ask, "Why do you need that information?"<br>Listen to your partner's reasoning, and ask clarifying questions. |
| **4.** Share the problem card with your partner, and solve the problem independently. | **4.** Read the problem card your partner shares with you, and solve the problem independently. |
| **5.** Read the data card your partner shares with you. Discuss the reasoning each of you used to solve the problem. | **5.** Share the data card with your partner. Discuss the reasoning each of you used to solve the problem. |

**Pause here so your teacher can review your work. You will be given a new set of cards to repeat the activity, this time trading roles with your partner.**

**Are you ready for more?**

**Sometimes two transformations, one performed after the other, can be described as a single transformation. For example:**

- Translating a point 2 units up, followed by translating the image 3 units up can be described as translating the original point 5 units up.
- Rotating a point $20°$ counterclockwise about the origin, followed by rotating the image $80°$ clockwise about the origin can be described as rotating the original point $60°$ clockwise around the origin.

Find a single transformation that gives the same result as reflecting a point across the $x$-axis, followed by reflecting the image across the $y$-axis.

**Rotating the original point $180°$ about the origin either clockwise or counterclockwise.**

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## 1 Launch

Model the ***Info Gap*** routine and display the *Info Gap Routine* PDF. Distribute pre-cut cards from the Activity 1 PDF to each pair of students. Start by distributing the first set and distribute the second set after you have checked student work.

## 2 Monitor

**Help students get started** by explaining that they may need several rounds of discussion to determine the information they need.

**Look for points of confusion:**

- **Asking questions that are not sufficiently precise.** Encourage them to find out *which* transformations they need to perform, and then find out the information they need for each transformation.

**Look for productive strategies:**

- Successfully determining or remembering to ask which transformations were applied, the order in which the transformations were applied, and what information is needed to describe a translation, rotation, or reflection.

## 3 Connect

**Have pairs of students share** the images they produced for each problem card.

**Ask:**

- "Was the order in which the transformations were applied important? Why?"
- "If this same problem was placed on a grid without coordinates, how would you talk about the points?"
- "How did using coordinates help in talking about the problem?"

**Highlight** that one advantage of the coordinate plane is that it allows students to precisely communicate information about transformations.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. Consider using the following questions during the think-aloud.

- "I wonder what the transformation(s) were. I think I should ask if there was a specific kind of transformation. I will ask if there was a translation. If there was, then I will ask if the figure was moved up, down, left, or right, and how many units in these directions."
- "I wonder if more than one transformation was performed. I think I should ask if there were two transformations, and in what order they were performed."

## Math Language Development

### *MLR4: Information Gap*

Display prompts for students who benefit from a starting point, such as:

- "Can you tell me . . . (specific piece of information)?"
- "Why do you need to know . . . (that piece of information)?"

### English Learners

Consider providing sample questions students could ask, such as:

- Was there a translation? How many units and in what direction?
- Was there a rotation? What was the angle of rotation? What was the direction of rotation? What was the center of rotation?
- Was there a reflection? Across which axis?
- In what order were the transformations performed?

Independent | 10 min

# Activity 2 Transformation Rules

Students explore patterns between transformations and the effects on coordinates, and find rules for transformations on the coordinate plane.

Name: ______________ Date: ________ Period: ________

## Activity 2 Transformation Rules

Complete the table with a partner. The first row has been completed for you.

| Written description | Transformation rule | Example |
|---|---|---|
| Translation $a$ units right and $b$ units up | $(x, y) \rightarrow (x + a, y + b)$ | $(-3, 2) \rightarrow (2, 9)$ Translate 5 units right and 7 units up |
| Translation $a$ units left and $b$ units down | $(x, y) \rightarrow (x - a, y - b)$ | $(-3, 2) \rightarrow (-8, -5)$ Translate 5 units left and 7 units down |
| Rotation 90° counterclockwise about the origin | $(x, y) \rightarrow (-y, x)$ | Sample response: $(4, -3) \rightarrow (3, 4)$ |
| Rotation 90° clockwise about the origin | $(x, y) \rightarrow (y, -x)$ | Sample response: $(-11, -6) \rightarrow (-6, 11)$ |
| Rotation 180° about the origin | $(x, y) \rightarrow (-x, -y)$ | Sample response: $(-9, -7) \rightarrow (9, 7)$ |
| Reflection across the $x$-axis | $(x, y) \rightarrow (x, -y)$ | Sample response: $(-12, -3) \rightarrow (-12, 3)$ |
| Reflection across the $y$-axis | $(x, y) \rightarrow (-x, y)$ | $(7, -1) \rightarrow (-7, -1)$ |

STOP





### 1 Launch

Encourage students to look back at their notes and strategies from the previous two lessons, as they will now collect all their findings in one useful table.

**Note:** In the digital version of this lesson, the table is replaced by a card sort.

### 2 Monitor

**Help students get started** by reviewing the first row of the table together.

**Look for points of confusion:**

- **Writing descriptions that are sequences of transformations.** While these descriptions may be technically correct, they are not the most efficient. During the Connect, compare these descriptions with single transformations, and ask students to make comparisons.

**Look for productive strategies:**

- Using notes and strategies from previous lessons as a reference when completing the table. Highlight these notes and strategies during the Connect.
- Using grid paper to verify whether the rule or the example they have written accurately depicts the transformation.

### 3 Connect

**Have students share** their table entries.

**Highlight** that each rule can describe a single transformation that maps the preimage onto the image, and that these rules can be used to find the coordinates of an image *without* needing to perform the transformation.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Provide the written descriptions and have students focus on providing the remaining rules and examples. Consider having students first only focus on the translation rows. After ensuring they understand, have them complete the reflection rows next and the rotation rows last.

### Accessibility: *Guide Processing and Visualization*

Some students may find it challenging to remember the transformation rules. Remind them that they can always determine or confirm the rule by graphing a point, applying the transformation, and verifying how the coordinates of the image compare to the preimage.

## Math Language Development

### *MLR7: Compare and Connect*

To address the point of confusion some students may have in writing out descriptions that are sequences of transformations, use this routine to have students compare their descriptions to the single transformations. Highlight how the descriptions connect to the notation for the transformation rules.

### English Learners

Model the transformations being described with visual examples.

Whole Class | 7 min

# Summary

Review and synthesize how coordinate notation can be used to describe the effect of transformations on the coordinate plane.

### Summary

**In today's lesson . . .**

You described transformations using coordinates. You also discovered rules to describe the changes to coordinates created by these transformations.

| Transformation | Rule |
|---|---|
| Translation $a$ units right and $b$ units up | $(x, y) \to (x + a, y + b)$ |
| Translation $a$ units left and $b$ units down | $(x, y) \to (x - a, y - b)$ |
| Reflection across the $x$-axis | $(x, y) \to (x, -y)$ |
| Reflection across the $y$-axis | $(x, y) \to (-x, y)$ |
| Rotation $90°$ clockwise about the origin | $(x, y) \to (y, -x)$ |
| Rotation $90°$ counterclockwise about the origin | $(x, y) \to (-y, x)$ |
| Rotation $180°$ about the origin | $(x, y) \to (-x, -y)$ |

When you perform a sequence of transformations, the order of the transformations can be important. Two translations may be performed in any order, and the image is the same. However, when performing a translation and a reflection, changing the order of the transformations will change the location of the image on the coordinate plane.

Reflect:

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## Synthesize

**Display** the table from the Summary of the Student Edition.

**Ask:**

- "Which of the transformation rules listed in the table do you find most challenging to understand? What can you do to help your understanding?"
- "Think of a moment in today's lesson in which your partner used precise language — what did they say? How did it help you?"
- "Why do you think it might be helpful to use coordinates to describe transformations?"
- "Why is it important to be precise when communicating about transformations?"

**Highlight** that precise verbal and written descriptions ensure that we are accurately and effectively describing transformations. To describe a transformation, the following information is needed.

- Translation: the direction of the translation and how many units to move in each direction
- Rotation: the center of rotation, the angle of rotation, and the direction of the rotation
- Reflection: the line of reflection

## Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful when applying and describing transformations of a figure?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by providing the information necessary to perform a sequence of transformations.

Printable

Name: ______________ Date: ________ Period: ________

## Exit Ticket 1.08

Jada applies two transformations to a polygon on the coordinate plane. One of the transformations is a translation, and the other is a reflection.

What information does Jada need to provide to communicate the transformations she has used?

Jada needs to provide the horizontal and vertical distances for the translation, the line of reflection, and the order in which the transformations should be performed.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can apply transformations to a polygon on a grid if I know the coordinates of its vertices. 1 2 3

b. I can describe transformations on the coordinate plane by using a rule that changes the coordinates. 1 2 3





## Success looks like . . .

- **Language Goal:** Creating a drawing on a coordinate plane of a transformed object using verbal descriptions. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Identifying what information is needed to transform a polygon. Asking questions to elicit that information. **(Speaking and Listening)**
  - » Providing the information needed by Jada to explain her transformations.

## Suggested next steps

**If students do not identify all the necessary information to communicate the transformations used, consider:**

- Reviewing Lesson 5.
- Assigning Practice Problem 3.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- What worked and didn't work today? Where in your students' notes and discussions did you observe evidence of them making sense of problems or persevering in solving problems?
- What did students find challenging about Activity 1? What helped them work through the challenge? What might you change the next time you teach this lesson?

## Math Language Development

**Language Goal: Identifying what information is needed to transform a polygon. Asking questions to elicit that information.**

Reflect on students' language development toward this goal.

- What are some examples of developing questions and how can you help students be more precise in the questions they ask?

**Sample questions for the Exit Ticket problem:**

| Emerging | Expanding |
|---|---|
| How far did the polygon move? | What are the horizontal and vertical distances for the translation? |
| How was it reflected? | What is the line of reflection? |

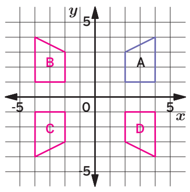
# Practice

Independent

Name: ______________________ Date: __________ Period: __________

Practice

> **1.** Consider Trapezoid A.




ⓐ Draw Trapezoid B, the reflection of Trapezoid A, using the $y$-axis as the line of reflection.

ⓑ Draw Trapezoid C, the reflection of Trapezoid B, using the $x$-axis as the line of reflection.

ⓒ Draw Trapezoid D, the reflection of Trapezoid C, using the $y$-axis as the line of reflection.

> **2.** The point $(-4, 1)$ is transformed using the following rules. Write the coordinates of each image, and describe the transformation that has occurred.

ⓐ $(x, y) \rightarrow (-y, x)$
$(-1, -4)$; rotation 90° counterclockwise about the origin

ⓑ $(x, y) \rightarrow (-x, y)$
$(4, 1)$; reflection across the $y$-axis

ⓒ $(x, y) \rightarrow (x - 5, y + 7)$
$(-9, 8)$; translation 5 units left and 7 units up

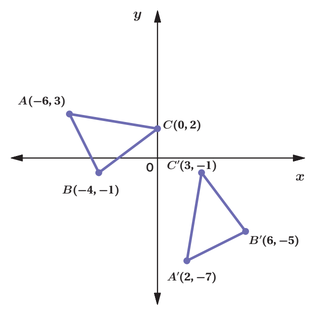




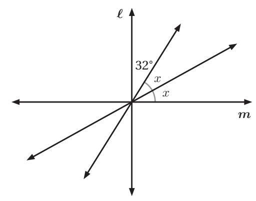
Name: ______________________ Date: __________ Period: __________

Practice

> **3.** Describe a sequence of transformations that maps Triangle $ABC$ onto Triangle $A'B'C'$.
> Sample response: Rotate ABC 90° counterclockwise about the origin, then translate the image to the right 5 units and down 1 unit.



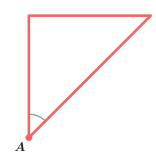

> **4.** Line $\ell$ is perpendicular to line $m$. Find the value of $x$.
> $90 - 32 = 58$, $58 \div 2 = 29$, so $x = 29°$




> **5.** Use your ruler and protractor to make some measurements for the given triangle.

ⓐ What is the measure of angle $A$ to the nearest degree?
45°

ⓑ What is the perimeter of the triangle, to the nearest centimeter?
12 cm








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 1 | 2 |
| Spiral | 4 | Grade 7 | 2 |
| Formative ⚡ | 5 | Unit 1 Lesson 9 | 1 |

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Sub-Unit 2

# Rigid Transformations and Congruence

Whole Class

Equipped with their geometry toolkits, students explore what it means for two figures to be the same and are formally introduced to the meaning of the term congruence.

SUB-UNIT

2 Rigid Transformations and Congruence

Narrative Connections

## How can a crack make a piece of art priceless?

As World War II came to a close, American troops stormed a Nazi-held salt mine. The mine was being used to store the stolen art collection of the Nazi marshal, Hermann Goering. Among Goering's most prized possessions was a painting from the 17th century Dutch master Johannes Vermeer.

The Allies traced the painting's sale to a Dutch art dealer, Han van Meegeren. Selling cultural treasures to the Nazis was punishable by death. But after his arrest, van Meegeren made an astonishing claim:

"It's not a *Vermeer*," he said. "I painted it myself!"

He claimed he was a master forger. For years he'd been passing off his own work as that of other artists, swindling Goering to the tune of $7 million.

To test van Meegeren's claim, a commission of experts examined the suspected forgery, studying the cracks on the painting's surface.

Over time, all paintings develop something called *craquelure*. It is a network of cracks that form on the paint as it dries. The chemicals in the paint, where the painting was made, even the material of the canvas all affect how the craquelure appears. This gives each painting its own sort of fingerprint.

By examining the craquelure and comparing it to the craquelure of authentic paintings from Vermeer's period, the commission was able to confirm the *Vermeer* was indeed a forgery. The courts dismissed van Meegeren's treason charge. Instead he was sentenced to a year in prison for committing forgery.





### Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students will continue to explore whether two different pieces of art or graphical designs are "the same" in the following places:

- **Lesson 9, Warm-up:** Can You Spot the Fake?
- **Lesson 10, Activity 1:** Are They the Same?
- **Lesson 12, Activity 3:** Astonished Faces

# No Bending or Stretching

Let's compare measurements before and after translations, rotations, and reflections.

## Focus

### Goals

1. **Language Goal:** Comprehend that the term *rigid transformation* refers to a transformation in which all pairs of corresponding distances and angle measures in the preimage and the image are the same. **(Speaking and Listening)**
2. Draw and label a diagram of the image of a polygon under a rigid transformation, including calculating the side lengths and angle measures.
3. **Language Goal:** Identify a sequence of rigid transformations given a preimage and its image. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students begin to see that translations, rotations, and reflections preserve lengths and angle measures, and for the first time, they call them *rigid transformations*. As students experiment with measuring corresponding sides and angles in a polygon and its image, they use the structure of the grid as well as appropriate geometric tools, including protractors, rulers, and tracing paper.

### Previously

In earlier lessons, students talked about corresponding points of a preimage and its image after a transformation.

### Coming Soon

In Lesson 10, students will understand that they can call two figures *congruent* if the figures can be obtained by a sequence of rigid transformations.

## Rigor

- Students build **conceptual understanding** of rigid transformations and their effects on side lengths and angle measures.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 20 min | 10 min | 5 min | 5 min |
| Independent | Pairs | Independent | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice
Independent

### Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: rulers, protractors, tracing paper, index cards

### Math Language Development

**New word**

- ***rigid transformation***

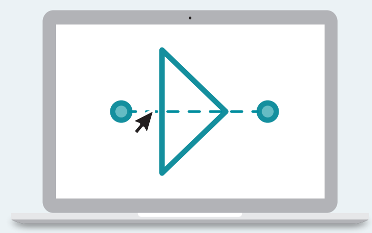
**Review words**

- *corresponding*
- *reflection*
- *rotation*
- *translation*

**Amps** | **Featured Activity**

**Activity 1**
**Interactive Transformations**

Students manipulate polygons and measure angles with interactive tools.



### Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students may feel a range of confidence levels using the grid and selecting mathematical tools. Ask students to seek out other students who are more comfortable working with these tools and who can help them gain more confidence.

### Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- **Activity 2** may be omitted and used as practice at a later time.

Independent | 5 min

# **Warm-up** Can You Spot the Fake?

Students analyze the cracking patterns in the paint of two images to explain why one image is a fake. This prepares them for thinking about how they can verify whether two figures are the same.



## No Bending or Stretching

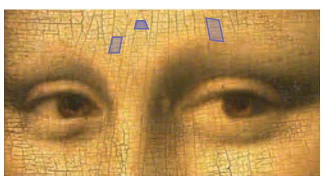
Let's compare measurements before and after translations, rotations, and reflections.

### **Warm-up** Can You Spot the Fake?

**Art experts and historians are always on the hunt for counterfeit art. One technique used to detect counterfeits is to study the *craquelure*, the natural patterns created by paint cracking over time. It is very difficult to fake craquelure!**

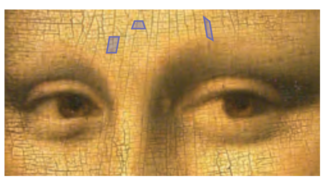
Consider the two images of the famous Mona Lisa painting. The first image is real, the second, a counterfeit. Focus your attention on the highlighted polygons formed by the cracking of the paint. How can you tell the second image is a counterfeit? Use any appropriate tool to support your claim.

Real



Public Domain

Counterfeit



Public Domain

Sample response: I can see that one of the polygons in the second image has different side lengths and different angle measures, which means it must not be the same.

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Activate students' background knowledge by asking, "What can be faked? In art, it is particularly difficult to spot a fake. Why might someone want to determine whether artwork is a fake? Today, you will learn about one technique used to identify fake art." Provide access to geometry toolkits for the duration of the lesson.

## 2 Monitor

**Help students get started** by pointing out a pair of congruent polygons in the artwork and asking, "How can you check to see if these are the same?"

**Look for points of confusion:**

- **Writing a justification that is not sufficient.** Challenge students to use their geometry tools, such as a ruler or protractor, to be more precise in their explanations.

**Look for productive strategies:**

- Students making use of rulers and protractors to support their claims.

## 3 Connect

**Display** student work making use of strategically-selected mathematical tools to support their claim.

**Have students share** how they were able to confirm that the image is a fake using their geometry tools.

**Highlight** that precise measurements can help confirm whether two shapes are the same or different.

## Math Language Development

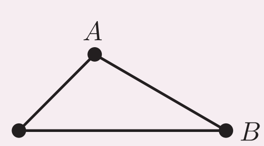
### Extension: *Interdisciplinary Connections*

Preview the online resource "Math Professor Helps Uncover Art Fakes" from NPR Morning Edition that highlights how a college math professor has used mathematics and computer programming to help determine art forgeries.The computer program analyzes pen strokes and compares them to known pen strokes of the Flemish artist Pieter Bruegel. Decide if you would like to read the article together with your students or provide a summary. Facilitate a class discussion on how Daniel Rockmore's personal interest in art merged with his mathematical interests. Ask students if they think that math can be related to any of their interests, such as sports, music, nature, etc. **(Art)**

## Power-up

**To power up students' ability to measure with a protractor and a ruler, have students complete:**

Recall that acute angles measure less than $90^\circ$ while obtuse angles measure greater than $90^\circ$. For each angle, determine whether it is *acute* or *obtuse*, then use a protractor to determine its angle measure.




1. Angle $A$
   a. Acute or obtuse? Obtuse  b. Angle measure? $105^\circ$
2. Angle $B$
   a. Acute or obtuse? Acute  b. Angle measure? $30^\circ$

**Use:** Before Activity 1
**Informed by:** Performance on Lesson 8, Practice Problem 5, and Pre-Unit Readiness Assessment, Problem 8

Pairs | 20 min

# Activity 1 Sides and Angles

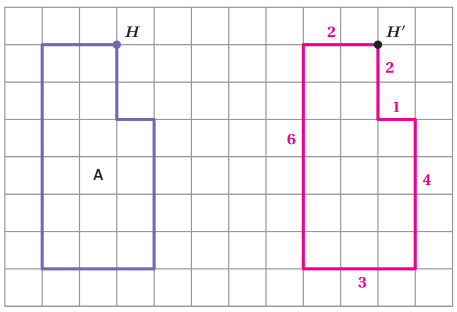
Students perform a translation, a rotation, and a reflection to discover how each transformation affects the side lengths and angle measures of the transformed image.

**Amps Featured Activity** Interactive Transformations

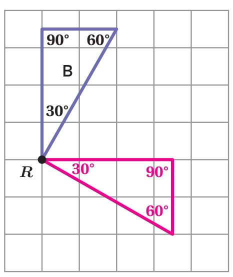
Name: ______ Date: ______ Period: ______

**Activity 1** Sides and Angles

1. Translate Polygon A so that point $H$ maps onto point $H'$. In the image, label each side with its length, in grid units.




2. Rotate Triangle B 90° clockwise using point $R$ as the center of rotation. In the image, label each angle with its measure, in degrees. Verify the angle measures using your protractor.




**Reflect:** How did using tools from your geometry toolkit help deepen your understanding of transformations?





## 1 Launch

Say, "You will investigate how each transformation — translation, rotation, or reflection — affects the side lengths and angle measures of a figure. You will select your own tools to use from your geometry toolkits."

## 2 Monitor

**Help students get started** by demonstrating how to use a ruler to begin to translate Polygon $A$ to Polygon $A'$ in Problem 1. Show students how to use tracing paper to rotate the figure in Problem 2.

**Look for points of confusion:**

- **Counting grid squares to find diagonal lengths in Problem 3.** Ask them if they think the length of one diagonal of one grid square is the same length as one side of the grid square. Demonstrate for students how to use their ruler to find accurate diagonal lengths.
- **Thinking the polygons in Problem 3 do not have the same side lengths or angle measures due to rounding errors or measuring inaccuracies.** Have students use tracing paper to trace one polygon and map it onto the other to verify the side lengths and angle measures are the same.

**Look for productive strategies:**

- Selecting appropriate tools from their geometry toolkits strategically.
- Mentioning corresponding side lengths and angle measures in their explanations for Problem 4.

**Activity 1 continued**

## Differentiated Support

### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can manipulate the polygons and measure angles using interactive tools. This will alleviate any risks of measurement errors and allow students to notice the side lengths and angle measures of the transformed images, without having to physically measure them using rulers or protractors.

### Extension: *Math Enrichment, Interdisciplinary Connections*

Mention that rigid transformations are also called *isometries*. An *isometry* is a transformation that preserves distance. The prefix *iso-* means *same* and *metry* means *distance*. Ask students how they could use the meaning of this prefix *iso-* to help remember what the properties of an isosceles triangle. An isosceles triangle has two sides that are the *same* length. **(Language Arts)**

## Math Language Development

### *MLR8: Discussion Supports*

As students describe their approaches, connect the terms *corresponding sides* and *corresponding angles* to students' explanations by using different types of sensory inputs, such as demonstrating the transformation or inviting students to do so, using the images and using hand gestures.

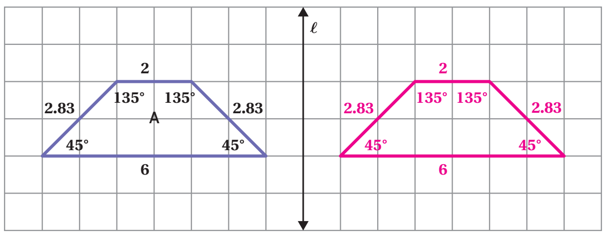
Pairs | 20 min

# Activity 1 Sides and Angles (continued)

Students perform a translation, a rotation, and a reflection to discover how each transformation affects the side lengths and angle measures of the transformed image.

**Activity 1** Sides and Angles (continued)

**3.** Reflect Polygon A across line $\ell$. In the image, label each side length, in grid units. Then label each angle measure, in degrees.




**4.** What did you notice about the side lengths and angle measures of each transformed polygon in Problems 1, 2, and 3? What conclusions can you make about the three types of transformations?

Sample response: The corresponding side lengths and corresponding angle measures were the same after each translation, rotation, and reflection. These three types of transformations keep corresponding side lengths and corresponding angle measures the same.





## 3 Connect

**Display** correct student work for Problems 1, 2, and 3.

**Have pairs of students share** how they performed the given transformations for each problem and what they found for their side lengths and angle measurements. Have the class share whether they agree after each explanation, before discussing what conclusions can be made about the transformations.

**Ask,** "Based on the measurements you found for the corresponding sides and corresponding angles, what conclusions can you make about these three transformations?"

**Highlight** that the corresponding side lengths and corresponding angle measures are preserved (kept the same) in each of the three transformations.

**Define** that a ***rigid transformation*** is a move that does not change any of the measurements of a figure. Translations, rotations, and reflections are all examples of rigid transformations, as is any sequence of these.

Independent | 10 min

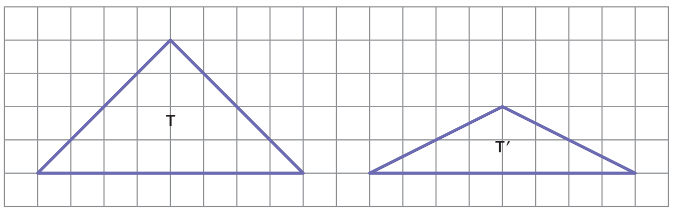
# Activity 2 Rigid Transformations

Students determine whether a sequence of rigid transformations maps one figure onto another to further their understanding about how rigid transformations preserve side lengths and angle measures.

Name: ______ Date: ______ Period: ______

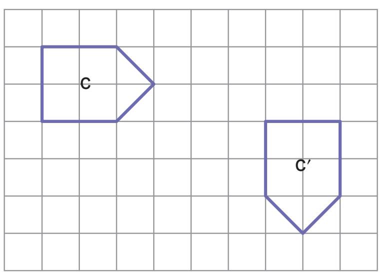
## Activity 2 Rigid Transformations

1. Is there a sequence of ***rigid transformations*** that maps Triangle T onto Triangle T′? Explain your thinking.




Sample response: No, there is no sequence of rigid transformations that maps Triangle T onto Triangle T′. Rigid transformations result in images in which corresponding angles have the same measure, but the corresponding angle measures on these two triangles are not the same.

2. Is there a sequence of rigid transformations that maps Pentagon C onto Pentagon C′? Explain your thinking.




Sample response: Yes, this is an example of a sequence of rigid transformations. I can see that Polygon C has been rotated and translated to map onto Polygon C′, and these transformations are examples of rigid transformations.

STOP





## 1 Launch

Have students conduct the *Think-Pair-Share* routine by giving 4 minutes of quiet work time and 2 minutes to discuss their responses with a partner.

## 2 Monitor

**Help students get started** by asking what they notice about the angle measurements in Triangle T compared to Triangle T′.

**Look for points of confusion:**

- **Using insufficient justification in their explanations.** Ask students to be specific in describing either the corresponding sides or corresponding angles to support their thinking.

**Look for productive strategies:**

- Describing the sequence of transformations that takes Pentagon C to Pentagon C′ in Problem 2.
- Using measures of corresponding sides and corresponding angles as evidence. **Note:** This method will be further developed in Lesson 10.

## 3 Connect

**Display** examples of student work.

**Have students share** their thinking.

**Ask**, "You know that rigid transformations preserve side lengths and angle measures. Can you also say that if two figures have the same side lengths and angle measures, there *must* be a rigid transformation that maps one figure onto the other?"

**Highlight** that for Problem 1, it is enough to say that if the shapes are not the same size, there is no rigid transformation that maps one figure onto the other. For Problem 2, because students can recreate the rigid transformations, they can say the figures are the same.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problem 1, and only work on Problem 2 as they have time available. Consider providing the side and angle measurements for each pair of figures so that students can focus on analyzing the measurements, as opposed to doing the measuring themselves.

## Math Language Development

### *MLR2: Collect and Display*

Have students share their work with a partner. As they discuss with a partner, listen for and collect the language they use to describe each transformation. Record students' words on a visual display and update it throughout the remainder of the lesson.

### English Learners

Include annotated drawings of the transformations on the class display so that students can connect the descriptions, words, and phrases to visual depictions of transformations.

Whole Class | 5 min

# Summary

Review and synthesize how a rigid transformation preserves side lengths and angle measures of an image.

### Summary

**In today's lesson . . .**

You discovered that the translations, rotations, reflections, and sequences of these motions you have learned about so far are all examples of rigid transformations. A ***rigid transformation*** is a move that does not change measurements — side lengths or angle measures — from the preimage to the image.

Earlier, you learned that a preimage and its image have corresponding points. A preimage and its image also have corresponding sides and corresponding angles. When a preimage is transformed using a rigid transformation, corresponding sides have the same lengths and corresponding angles have the same measures.

**Reflect:**





## Synthesize

**Ask:**

- "By studying two figures, how could you tell that one is not the image of the other under a rigid transformation?" Sample response: If the corresponding side lengths are not the same, or if the corresponding angle measures are not the same, then a rigid transformation has not occurred.
- "What are the three types of rigid transformations?" translation, reflection, rotation
- "If a figure has undergone a sequence of rigid transformations to map onto another figure, what can you say about the two figures?" Sample response: The two figures have the same side lengths and the same angle measures.

**Formalize vocabulary:** ***rigid transformation***

**Highlight** when a preimage is transformed using a rigid transformation, corresponding sides have the same lengths and corresponding angles have the same measure. Translations, rotations, and reflections are all examples of rigid transformations. Sequences of these transformations are also rigid transformations.

## Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful today when identifying a rigid transformation?"

## Math Language Development

***MLR2: Collect and Display***

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *rigid transformation* that were added to the display during the lesson.

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by determining the side lengths and angle measures of a polygon after a rigid transformation has been performed.

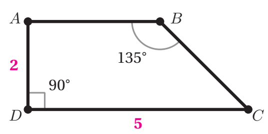
Printable

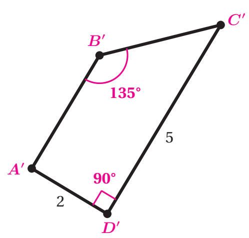
Name: ______________ Date: ________ Period: ________

## Exit Ticket 1.09

**Trapezoid $A'B'C'D'$ is the image of Trapezoid $ABCD$ after a rigid transformation has been performed.**

1. Label all the vertices on Trapezoid $A'B'C'D'$.
2. On both figures, label all known side lengths and angle measures.




**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can describe the effects of a rigid transformation on the side lengths and angle measures of a polygon.

1 2 3





## Success looks like . . .

- **Language Goal:** Comprehending that the term *rigid transformation* refers to a transformation in which all pairs of corresponding distances and angle measures in the preimage and the image are the same. **(Speaking and Listening)**
- **Goal:** Drawing and labeling a diagram of the image of a polygon under a rigid transformation, including calculating the side lengths and angle measures.
  - » Labeling the side lengths and angle measures of two trapezoids related by a rigid transformation.
- **Language Goal:** Identifying a sequence of rigid transformations given a preimage and its image. **(Speaking and Listening, Writing)**

## Suggested next steps

**If students do not label the corresponding vertices correctly for Problem 1, consider:**

- Reviewing strategies for rotating an object from Activity 1, Problem 2.

**If students mislabel angle or side measures in Problem 2, consider:**

- Reviewing Activity 2, Problem 2.
- Reviewing the definition of a rigid transformation.

## Professional Learning

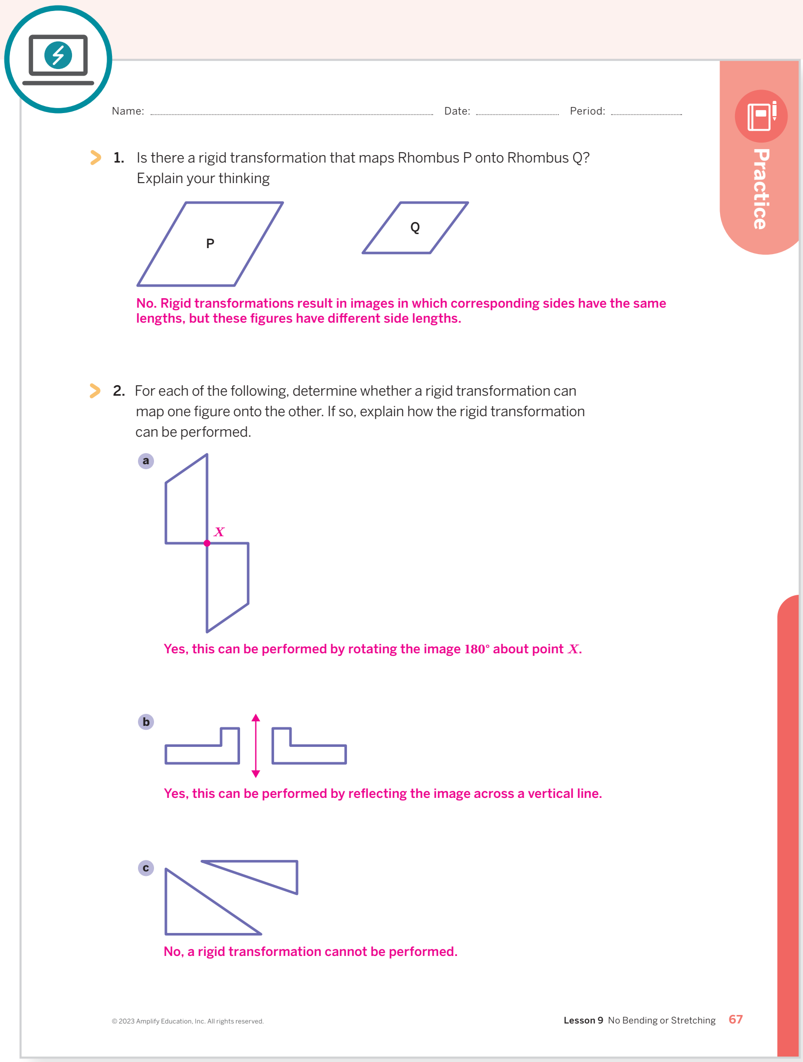
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Point to Ponder . . .

- How well do your students understand rigid transformations? How well can they describe the effects of rigid transformations on the side lengths and angle measures of transformed figures?
- What might you change for the next time you teach this lesson?

# Practice

Independent


Name: Date: Period:

Practice

1. Is there a rigid transformation that maps Rhombus P onto Rhombus Q? Explain your thinking.



No. Rigid transformations result in images in which corresponding sides have the same lengths, but these figures have different side lengths.

2. For each of the following, determine whether a rigid transformation can map one figure onto the other. If so, explain how the rigid transformation can be performed.

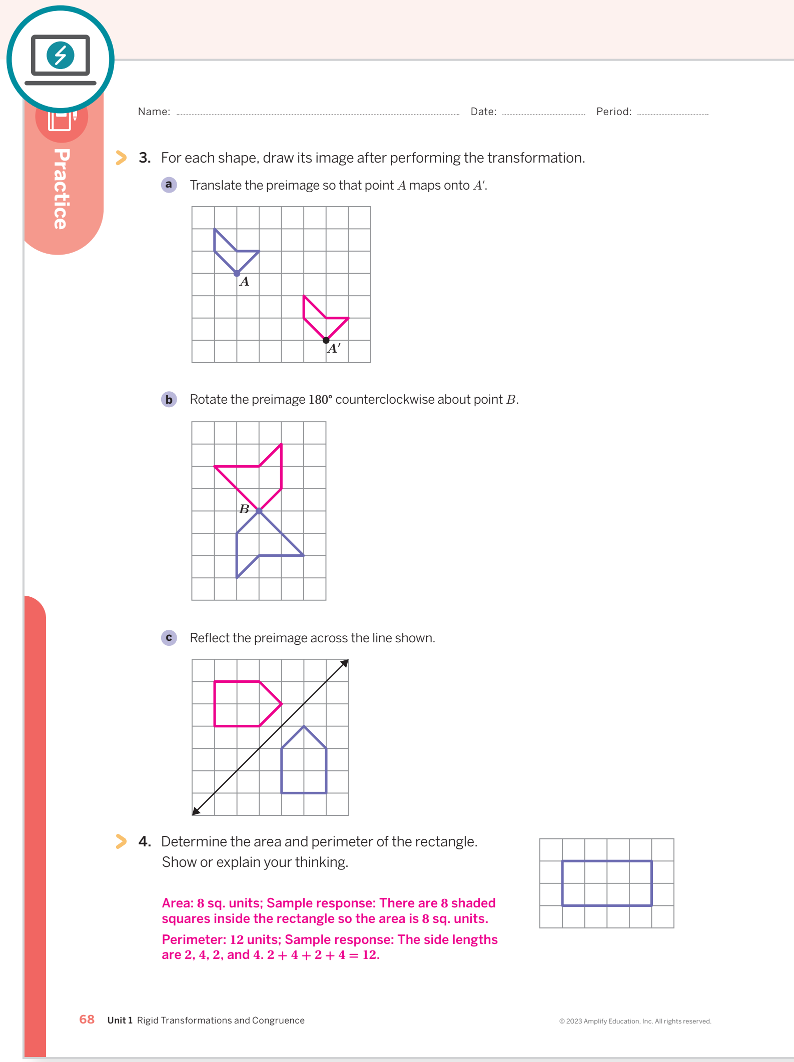
   a. 

   Yes, this can be performed by rotating the image 180° about point $X$.

   b. Yes, this can be performed by reflecting the image across a vertical line.

   c. No, a rigid transformation cannot be performed.





Name: Date: Period:

Practice

3. For each shape, draw its image after performing the transformation.

   a. Translate the preimage so that point $A$ maps onto $A'$.

   b. Rotate the preimage 180° counterclockwise about point $B$.

   c. Reflect the preimage across the line shown.

4. Determine the area and perimeter of the rectangle. Show or explain your thinking.

Area: 8 sq. units; Sample response: There are 8 shaded squares inside the rectangle so the area is 8 sq. units.
Perimeter: 12 units; Sample response: The side lengths are 2, 4, 2, and 4. $2 + 4 + 2 + 4 = 12$.




## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| | 2 | Activity 1 | 2 |
| Spiral | 3 | Unit 1 Lesson 4 | 1 |
| Formative | 4 | Unit 1 Lesson 10 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 1 | LESSON 10

# What Is the Same?

Let's decide whether shapes are the same.


## Focus

### Goals

1. **Language Goal:** Use the term *congruent* to describe two figures that can be mapped onto each other by using a sequence of rigid transformations. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Comprehend that congruent figures have corresponding side lengths, corresponding angle measures, and areas that are equal. **(Speaking and Listening, Reading and Writing)**
3. **Language Goal:** Comprehend that figures with the same area and perimeter may or may not be congruent. **(Speaking and Listening, Reading and Writing)**

## Coherence

### Today

Students explore what it means for shapes to be the "same" and learn that the term *congruent* is a mathematical way to talk about figures being the "same." They understand that two figures are congruent if there is a sequence of rigid transformations that maps one onto the other. They realize that figures that are congruent can have different orientations, but corresponding side lengths and corresponding angle measures are equal.

### Previously

In Lesson 9, students learned that translations, rotations, and reflections are examples of rigid transformations. They saw that rigid transformations preserve side lengths and angle measures. In elementary grades, deciding whether two shapes are the "same" usually involves making sure that they are the same general shape and same size. As shapes become more complex and students use new ways to measure their attributes, such as side lengths and angle measures, this surfaces the need for a more precise way to talk about shapes being the "same."

### Coming Soon

In Lesson 11, students will build on their understanding of congruent figures by testing whether two figures are congruent.

## Rigor

- Students build **conceptual understanding** of what it means for two figures to be *congruent*.

## Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Pairs | Independent | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

### Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- colored pencils
- geometry toolkits: rulers, protractors, tracing paper

### Math Language Development

**New word**

- *congruent*

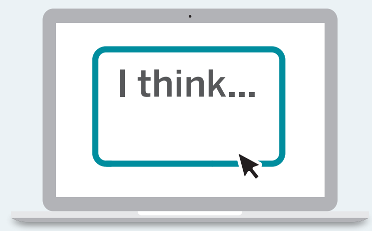
**Review words**

- *corresponding*
- *orientation*
- *reflection*
- *rigid transformation*
- *rotation*
- *translation*

**Amps** | Featured Activity

### Activity 1
### See Student Thinking

Students manipulate and compare figures to determine if they are congruent and explain their thinking. These explanations are available to you digitally, in real time.




Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students may act as though their explanation is the only correct explanation and may not listen as actively to their peers' arguments. Provide students a thinking question before they share, such as, "As your classmates share, consider what your argument has in common and listen for arguments that reach the same conclusion from a different perspective."

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted. It serves to get students thinking about the orientation of figures.
- **Activity 1** may be shortened to have students analyze parts a and b.
- In **Activity 2**, Problem 5 may be omitted and addressed later in Lesson 11.

Pairs | 5 min

## **Warm-up** Find the Right Hands

Students find all the matching right hands to reinforce the concept of orientation and mirror images.

Name: ______ Date: ______ Period: ______

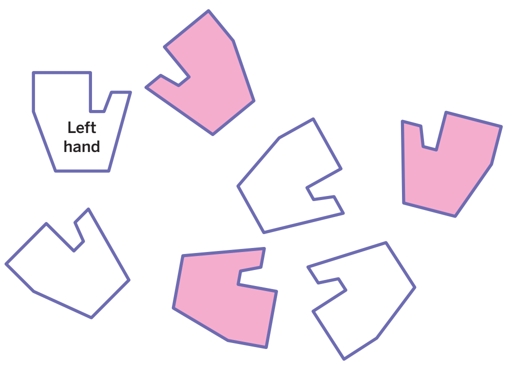
**Unit 1 | Lesson 10**

### What Is the Same?

Let's decide whether shapes are the same.

**Warm-up** Find the Right Hands

**A person's hands are mirror images of each other. In the diagram, a left hand is labeled, where the palm of the hand is facing down. Shade all of the right hands where the palm is facing down.**




Log in to Amplify Math to complete this lesson online.





### 1 Launch

Activate background knowledge by asking students what they notice about their left and right hands.

### 2 Monitor

**Help students get started** by holding up both of your hands and pointing out that a person's hands are mirror images of each other.

**Look for points of confusion:**

- **Thinking they should only see reflections.** Point out that the hands have also been rotated.
- **Shading some of the left hands because they think the palms can face up or down.** Ask students if the palms of the hands are facing up or down by having them reread the directions.

### 3 Connect

**Display** student work.

**Have students share** the ways in which the left and right hands are the same, and the ways in which they are different.

**Ask**, "Is the image of a left hand the same as the image of a right hand? Are figures that are mirror images of each other the same or different?"

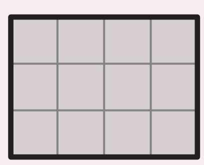
**Highlight** that the side lengths and angles for the left and right hands match up, but that a left hand will only perfectly match a right hand if it is flipped, or reflected. Connect this to what students learned in Lesson 9 — that rigid transformations, such as reflections, preserve side lengths and angle measures, even if the orientation is reversed. Announce that in today's lesson, students will build on this idea of what makes two figures "the same" and give mathematical meaning to the word "same."

## Power-up

**To power up students' ability to determine the perimeter and the area of a rectangle, have students complete:**

Recall that the *perimeter* of a rectangle is the total length of the edges, while its area is the number of square units that cover it.

Determine the perimeter and the area of the rectangle.



Perimeter: 14 units

Area: 12 square units

**Use:** Before Activity 2
**Informed by:** Performance on Lesson 9, Practice Problem 4, and Pre-Unit Readiness Assessment, Problem 6

Independent | 15 min

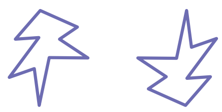
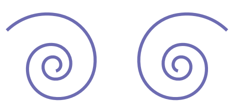
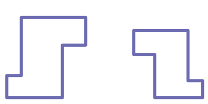
# Activity 1 Are They the Same?

Students decide whether pairs of figures are "the same," leading them to see the need for a precise meaning of what makes two figures "the same." The term *congruent* is introduced.

**Amps Featured Activity** See Student Thinking

**Activity 1** Are They the Same?

For each pair of figures, decide whether they are the same. Explain your thinking.

| Pair | Are they the same? | Explain your thinking. |
|---|---|---|
| a  | Yes | The figures are the same; the figure on the right is the image of the left figure after a reflection and translation. |
| b  | No | The figures are not the same; they have congruent sides, but the angle measures do not match. |
| c  | Yes | The figures are the same; the figure on the right is the image of the figure on the left after a reflection. |
| d  | No | The figures are not the same; the figure on the left is larger than the figure on the right. |

70 **Unit 1** Rigid Transformations and Congruence



## 1 Launch

Provide access to geometry toolkits for the duration of the lesson.

## 2 Monitor

**Help students get started** by showing how to use tracing paper to determine whether the pair of figures in part a are the same.

**Look for productive strategies:**

- Performing rigid transformations on the figures in parts a and c to determine they are the same.
- Noting the figures in part d are different sizes and, therefore, not "the same."

## 3 Connect

**Display** a student's table showing the correct responses for each pair of figures.

**Have students share** how they can explain why the figures in parts a and c are the same and why the figures in parts b and d are not the same.

**Ask**, "What do you mean by 'the same'? Are there any pairs of figures for which you found it challenging to determine if they are 'the same'?" Answers may vary as students may not always agree on what makes two figures "the same."

**Define** that two figures are ***congruent*** if one figure can be mapped onto the other by a sequence of rigid transformations. Let students know that instead of using the phrase "the same," they will use the term "congruent" moving forward. Introduce the congruent symbol (≅) along with an example of how it is used.

**Highlight** that one way to prove that two figures are congruent is to describe the sequence of rigid transformations that maps one figure onto the other. Have students determine the rigid transformations that produce the congruent figures in parts a and c.

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge***

If students need more processing time, have them focus on parts a and b and only work on parts c and d as time allows.

**Extension: *Math Enrichment***

Explain to students that co- and con- in the term *congruent* is a Latin root which means *together*. Have students come up with other words that have this same root. The words do not have to be mathematical words. Sample responses: *converse, coordinate, corresponding, costar, coworker, connect*

Pairs | 15 min

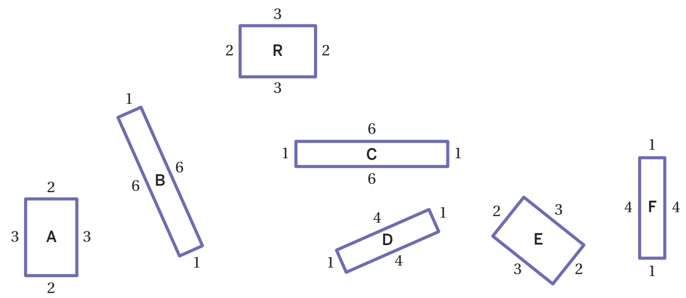
## Activity 2 Area, Perimeter, and Congruence

Students investigate the areas and perimeters of a group of rectangles to discover that figures of the same overall shape (e.g., rectangles) are not necessarily congruent.

Name: ______________ Date: ________ Period: ________

### Activity 2 Area, Perimeter, and Congruence

Study the rectangles shown. You will need access to your geometry toolkit.




1. Which of these rectangles have the same area as Rectangle R, but different perimeters? Explain your thinking.
   Rectangle R has an area of 6 square units and a perimeter of 10 units. Rectangles B and C have the same area, 6 square units, but different perimeters. Rectangles B and C both have perimeters of 14 units.

2. Which rectangles have the same perimeter as Rectangle R, but different areas? Explain your thinking.
   Rectangle R has an area of 6 square units and a perimeter of 10 units. Rectangles D and F have the same perimeter, 10 units, but different areas. Rectangles D and F both have areas of 4 square units.

3. Which have the same area and the same perimeter as Rectangle R? Explain your thinking.
   Rectangle R has an area of 6 square units and a perimeter of 10 units. Rectangles A and E have the same perimeter and area.

4. Using your geometry tools, decide which rectangles are ***congruent***. Shade congruent rectangles with the same color.
   Students should shade Rectangles A, R, and E the same color. They should shade Rectangles B and C the same color. They should shade Rectangles D and F the same color.




### 1 Launch

Let students know that they will investigate the perimeters and areas of a group of rectangles in which there may or may not be congruent figures. Distribute colored pencils.

### 2 Monitor

**Help students get started** by asking them to explain how to find the perimeter and area of Rectangle R.

**Look for points of confusion:**

- **In Problem 4, thinking that because all rectangles have the same overall shape, they are all congruent.** Remind students of the definition of congruent from Activity 1.
- **In Problem 5, thinking that if rectangles with the same area and same perimeter are congruent, then *any* two polygons with the same area and same perimeter are congruent.** Ask students to see if they can produce a sequence of rigid transformations that maps one of the figures in Problem 5 onto the other.

**Look for productive strategies:**

- Organizing their work by finding and recording the area and perimeter for each rectangle in Problems 1–3.

Activity 2 continued >

### Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge***

Provide the areas and perimeters already calculated so that students can focus on comparing the rectangles. Consider also chunking this task into smaller, more manageable parts. For example, provide students with a subset of the rectangles with which to begin and introduce the remaining rectangles once they have completed their initial set.

### Math Language Development

***MLR1: Stronger and Clearer Each Time***

Provide students time to meet with 2–3 students to share and receive feedback on their responses. Display prompts for feedback that will help them strengthen their ideas and clarify their language. For example:

- "How was a sequence of transformations used to . . .?"
- "What properties do the shapes share?"
- "What was different and what was the same about each pair?"

**English Learners**

Strategically pair students with partners who speak the same primary language. Allow students to share and receive feedback in their primary language.

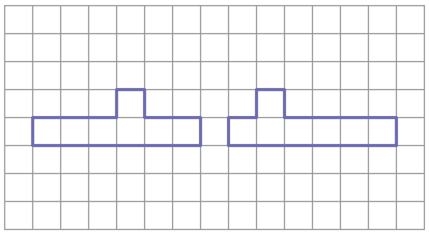
Pairs | 15 min

# Activity 2 Area, Perimeter, and Congruence (continued)

Students investigate the areas and perimeters of a group of rectangles to discover that figures of the same overall shape (e.g., rectangles) are not necessarily congruent.

## Activity 2 Area, Perimeter, and Congruence (continued)

5. These polygons have the same perimeter and the same area. Are they congruent? Explain your thinking.

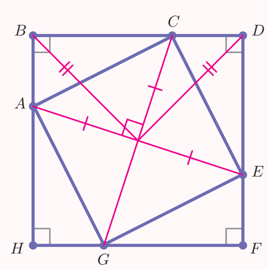


No, they are not congruent. One polygon cannot be mapped onto the other polygon using rigid transformations, which means they are not congruent.

**Are you ready for more?**

Figure $BDFH$ is a square. Points $A$, $C$, $E$, and $G$ are selected and marked so that the lengths of the bold line segments are the same. Is Figure $ACEG$ also a square? Explain your thinking.

Sample response: Yes. Rotate Triangle $ABC$ 90° clockwise using the center of Square $BDFH$ as the center of rotation. Segment $AC$ will map onto segment $CE$. As the rotation continues about the center of the square, segment $AC$ will map onto segment $EG$ and then onto segment $GA$, proving that all four segments have the same length and all four angles are 90°.




STOP





## 3 Connect

**Ask:**

- "Do congruent rectangles have the same perimeter? Explain your thinking." Sample response: Yes, congruent rectangles will have the same perimeter because their side lengths will be the same.
- "Do congruent rectangles have the same area? Explain your thinking." Sample response: Yes, congruent rectangles will have the same area because their side lengths will be the same.
- "Are rectangles with the same perimeter always congruent? Why or why not?" Sample response: No, Rectangles R, D, and F have the same perimeter, yet Rectangle R is not congruent to either Rectangle D or Rectangle F.
- "Are rectangles with the same area always congruent? Why or why not?" Sample response: No, Rectangles R, B, and C have the same area, yet Rectangle R is not congruent to either Rectangle B or Rectangle C.
- "If two figures have the same perimeter and same area, are they congruent?" Sample response: Not necessarily. The figures in Problem 5 have the same perimeter and area, but one cannot be mapped onto the other by using rigid transformations.

**Display** the figures in Problem 5.

**Have students share** whether they think the figures are congruent by using the ***Poll the Class*** routine. Have students share their thinking with a partner before sharing with the class.

**Highlight** that measuring perimeter and area is a strategy that can be used to show that two figures are *not* congruent if these measures differ. If these measures are the same, more work is needed to decide whether the two figures are congruent. Point out that *polygons* with the same area and same perimeter are not necessarily congruent, as shown in Problem 5. However, *rectangles* with the same area and same perimeter will always be congruent.

Whole Class | 5 min

# Summary

Review and synthesize what it means for two figures to be congruent and how congruence is related to rigid transformations.

Name: ______________ Date: __________ Period: __________

## Summary

**In today's lesson . . .**

You explored what it means for two figures to be congruent. This is a new term for an idea you already know about and have been using. Two figures are ***congruent*** if one figure maps onto the other figure exactly by using a sequence of rigid transformations. The congruence symbol $\cong$ can be used to show two figures are congruent. For example, $\triangle ABC \cong \triangle DEF$ means that the two triangles are congruent. The statement is read "Triangle $ABC$ is congruent to Triangle $DEF$".

Here are some other facts about congruent figures:

- You do not need to check *all* the measurements to prove two figures are congruent. Instead, you can find a sequence of rigid transformations that maps one figure onto the other. If you can find such a sequence, then the figures are congruent.
- Two figures that are exact mirror images of each other are congruent. This means there must be a *reflection* in the sequence of transformations that maps one figure onto the other.
- Because two congruent polygons have the same area and the same perimeter, one way to show that two polygons are *not* congruent is to show that they have different perimeters or different areas.

**Reflect:**





## Math Language Development

### *MLR2: Collect and Display*

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *congruent* that were added to the display during the lesson.

## Synthesize

Have students share their best definition of the term congruent.

**Ask:**

- "Are a figure and its mirror image congruent? Why or why not?" Yes, the mirror image is a reflection (rigid transformation) of the figure.
- "How can you determine whether two figures are congruent?" Recreate a sequence of rigid transformations, measure corresponding side lengths, measure corresponding angles
- "What are some ways to know that two figures are not congruent?" If a sequence of rigid transformations cannot map one figure onto the other, if corresponding side lengths are not the same, if corresponding angle measures are not the same, if the figures have different areas or perimeters
- "What are some characteristics that are shared by congruent figures?" Corresponding side lengths are the same and corresponding angle measures are the same.

**Highlight** that the term *congruent* does not precisely mean "same shape, same size," but that figures are congruent when there is a sequence of translations, rotations, and reflections (rigid transformations) that map one figure onto the other. Discuss the symbols used to represent triangle and congruence.

**Formalize vocabulary: *congruent***

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does it mean for two figures to be 'the same'?"

Independent | 5 min

# Exit Ticket

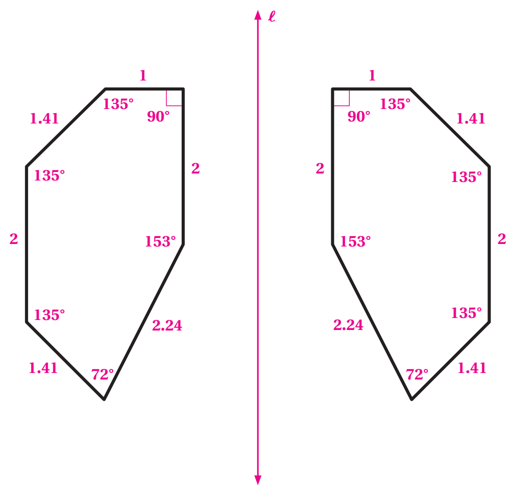
Students demonstrate their understanding by determining whether two figures are congruent.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 1.10

Are these figures congruent? Explain your thinking.
You may want to use your geometry toolkit.




Yes, these figures are congruent.

Sample response:

- A reflection across line ℓ maps one figure onto the other, so the figures are congruent.
- The corresponding side lengths and corresponding angle measures are the same, which means the figures are congruent.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can use rigid transformations or angle measurements and side lengths to decide whether two figures are congruent.
1 2 3





## Success looks like . . .

- **Language Goal:** Using the term *congruent* to describe two figures that can be mapped onto each other by using a sequence of rigid transformations. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Comprehending that congruent figures have corresponding side lengths, corresponding angle measures, and areas that are equal. **(Speaking and Listening, Reading and Writing)**
  - » Verifying whether the two figures are congruent.
- **Language Goal:** Comprehending that figures with the same area and perimeter may or may not be congruent. **(Speaking and Listening, Reading and Writing)**

## Suggested next steps

**If students do not describe rigid transformations, side lengths, or angle measures to determine congruence, consider:**

- Reviewing strategies to determine congruence from Activity 1.
- Assigning Practice Problem 3.
- Asking, "How can you determine whether two figures are congruent?"

**If students think the figures are not congruent because the orientation is reversed, consider:**

- Reviewing the Warm-up and reminding students that a reflection is a type of rigid transformation.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- How did your students transition from thinking about what it means for two figures to be the "same" and congruent figures? Are they comfortable in using the term *congruent* moving forward?
- What are the go-to strategies your students are using to determine whether two figures are congruent? Are they thinking about rigid transformations?

## MLR Math Language Development

**Language Goal: Comprehending that congruent figures have corresponding side lengths, corresponding angle measures, and areas that are equal.**

Reflect on students' language development toward this goal.

- Do students' responses to the Exit Ticket problem include mathematical language, such as:
  - » Identifying a reflection which is a rigid transformation?
  - » Indicating that corresponding side lengths and angle measures are equal?
- How can you help students be more precise in their justifications as to whether two given figures are congruent?

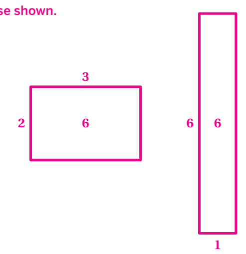
# Practice

Independent

Name: Date: Period:

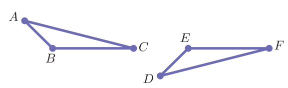
1. If two rectangles have the same perimeter, do they have to be congruent? Explain your thinking.
**Sample response: No. A rectangle with a length of 3 cm and width of 2 cm will have the same perimeter as a rectangle with a length of 4 cm and a width of 1 cm. However, their corresponding sides do not have the same length, so the rectangles will not be congruent.**

2. Draw two rectangles that have the same area, but are *not* congruent.
**Sample response shown.**



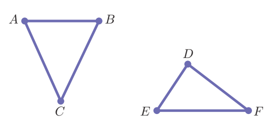

3. For each pair of triangles, decide whether the statement about congruence is *true* or *false*. Explain your thinking.

a. $\triangle ABC \cong \triangle DEF$




**True; Sample response: Triangle $ABC$ can be reflected across segment $BC$ and translated to the right to map onto Triangle $DEF$.**

b. $\triangle ABC \cong \triangle DEF$




**False; Sample response: Triangle $ABC$ and Triangle $DEF$ have different angle measures, so they cannot be congruent.**



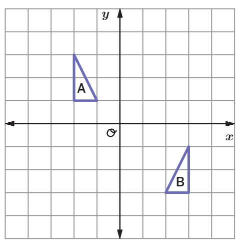

Name: Date: Period:

4. Find the coordinates of the image of point $A(2, -5)$ after each transformation.

a. Point $A(2, -5)$ is reflected across the $x$-axis. What are the coordinates of the image?
**$(2, 5)$**

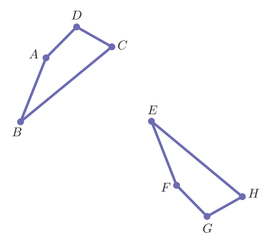
b. Point $A(2, -5)$ is reflected across the $y$-axis. What are the coordinates of the image?
**$(-2, -5)$**

5. Prove that Triangle A and Triangle B are congruent by describing a sequence of rigid transformations that they could have used to map Triangle A onto Triangle B. Explain your thinking.




**Sample response: Reflect Triangle A across the $y$-axis. Then translate the figure down 4 units and to the right 1 unit.**

6. A series of rigid transformations was used to map one of these polygons onto the other. Which of the following statement(s) are true? Select *all* that apply.




A. $\angle A \cong \angle G$
B. $\angle A \cong \angle F$
C. Segment $AD$ is congruent to segment $FG$.
D. Segment $AD$ is congruent to segment $GH$.
E. Segment $BD$ is congruent to segment $EG$.




## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 1 | 2 |
| Spiral | 4 | Unit 1 Lesson 3 | 1 |
| | 5 | Unit 1 Lesson 6 | 2 |
| Formative | 6 | Unit 1 Lesson 11 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Congruent Polygons

Let's decide whether two figures are congruent.

## Focus

### Goals

1. **Language Goal:** Compare and contrast side lengths, angle measures, and areas using rigid transformations to explain why two figures are, or are not, congruent. **(Speaking and Listening, Reading and Writing)**

2. **Language Goal:** Critique arguments about whether two figures with the same corresponding sides lengths may be non-congruent figures. **(Speaking and Listening)**

3. **Language Goal:** Justify that two polygons on a grid are congruent by describing a sequence of rigid transformations that maps one polygon onto the other. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students use rigid transformations that show two figures are congruent and construct arguments for why two figures are not congruent. They come to understand that, for many shapes, simply having corresponding side lengths that are equal will not guarantee the figures are congruent.

### Previously

In Lesson 10, students defined what it means for two figures to be congruent and began to apply this meaning to determine if two figures are congruent.

### Coming Soon

In Lesson 12, students will apply their understanding of congruence to different types of figures, such as ovals.

## Rigor

- Students continue to build **conceptual understanding** of what it means for two polygons to be congruent.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Pairs | Independent | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: rulers, protractors, tracing paper

### Math Language Development

**Review words**

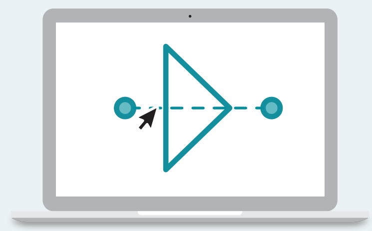
- *congruent*
- *corresponding*
- *orientation*
- *reflection*
- *rigid transformation*
- *rotation*
- *translation*

**Amps** | **Featured Activity**

**Activity 1**

### Digital Geometry Tools

Students use digital geometry tools to determine whether two polygons are congruent.



Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students may feel defeated if they struggle to precisely describe their thinking. Have them use their geometry tools and consider assigning strategic partners so that students feel more supported in accurately describing the rigid movements of congruent figures.

### Modifications to Pacing

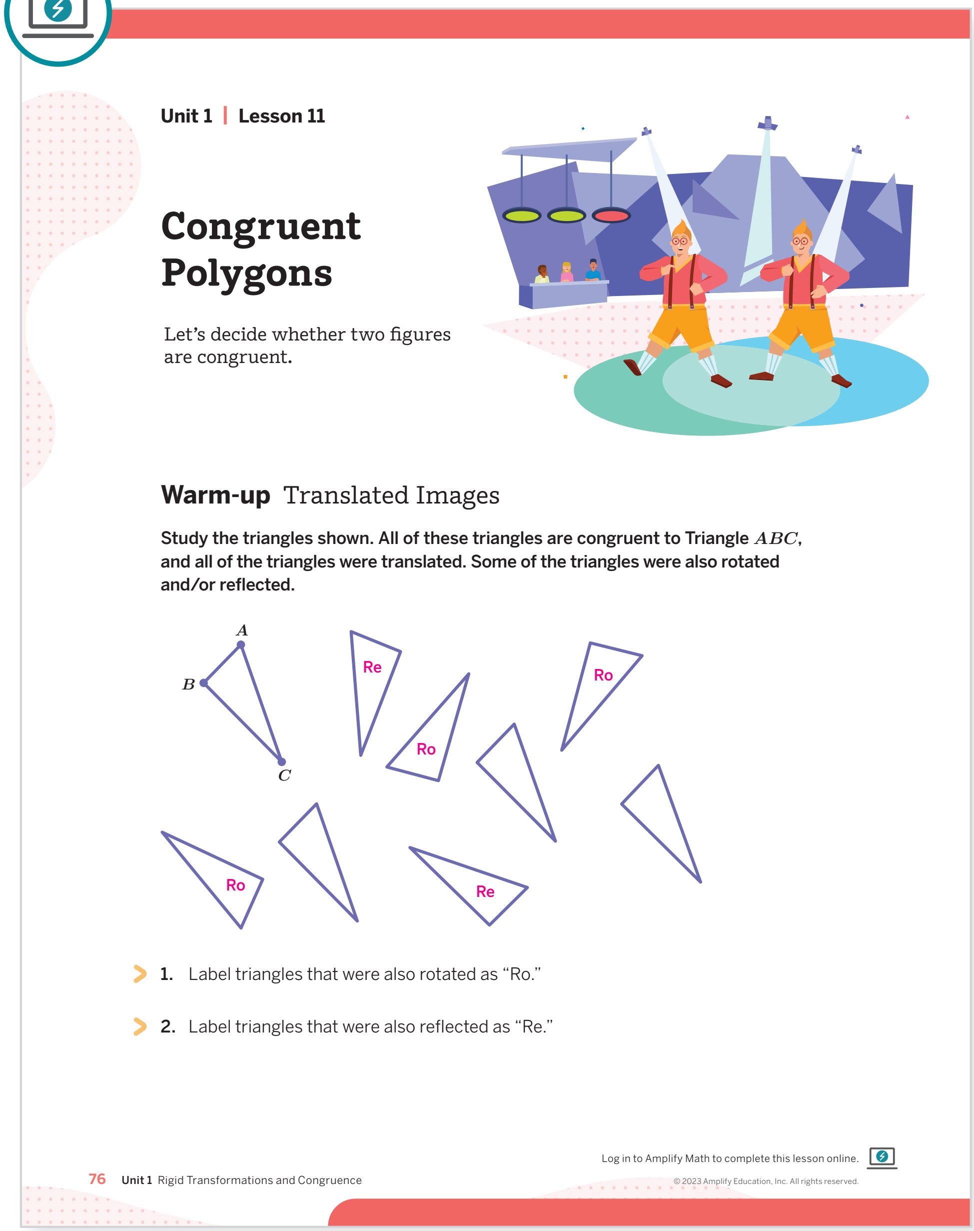
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted. It provides an opportunity for students to think about figures that have undergone more than one rigid transformation.
- In **Activity 1**, you may omit parts c and d as they are additional examples of figures that may or may not be congruent.
- In **Activity 2**, you may omit Problem 2.

Pairs | 5 min

# Warm-up Translated Images

Students examine a set of congruent triangles to determine the type of transformation performed for each triangle.



Unit 1 | Lesson 11

## Congruent Polygons

Let's decide whether two figures are congruent.

### Warm-up Translated Images

**Study the triangles shown. All of these triangles are congruent to Triangle $ABC$, and all of the triangles were translated. Some of the triangles were also rotated and/or reflected.**



1. Label triangles that were also rotated as "Ro."
2. Label triangles that were also reflected as "Re."

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Provide access to geometry toolkits for the duration of the lesson.

## 2 Monitor

**Help students get started** by selecting a triangle and demonstrating how it was transformed using tracing paper.

**Look for points of confusion:**

- **Thinking the triangles that were reflected were actually rotated.** Ask students if they can demonstrate the reflection using tracing paper, and, when they are not able to, ask what transformation is needed to achieve the resulting triangle.

**Look for productive strategies:**

- Using tracing paper to note each transformation.
- Labeling corresponding vertices to determine whether a reflection has occurred that reverses the orientation of the triangle.

## 3 Connect

**Display** student work showing correct responses.

**Have students share** their strategies for determining the type(s) of transformations performed for each triangle.

**Ask**, "Which transformation did you recognize first? Which was the most challenging? Why?"

**Highlight** that if an image is translated, it will have the same direction. Rotations usually change the direction of an image and reflections usually change the orientation of the image. Being able to quickly recognize these three types of transformations will be useful when planning out a sequence of transformations to prove congruence.

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge, Optimize Access to Tools***

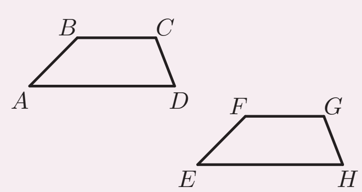
Use a think-aloud to model and demonstrate to students how you would determine which triangles were only translations of Triangle $ABC$ first. Have students cross those triangles out so that they can focus on the remaining ones. Then model how you would use tracing paper to determine one triangle that is an example of a rotation and one that is an example of a reflection. Have students determine the remaining ones.

## Power-up

**To power up students' ability to name corresponding part of congruent polygons, have students complete:**

Trapezoid $ABCD$ is translated to be Trapezoid $EFGH$. Fill in each blank with the congruent, corresponding side or angle.

1. $\angle A \cong$ $\angle E$
2. segment $BC$ $\cong$ segment $FG$
3. $\angle C$ $\cong \angle G$




**Use:** Before Activity 1
**Informed by:** Performance on Lesson 10, Practice Problem 6

# Activity 1 Congruent Pairs

Students determine whether pairs of polygons on a coordinate plane are congruent to understand that both side lengths and angle measures must be preserved for figures to be congruent.

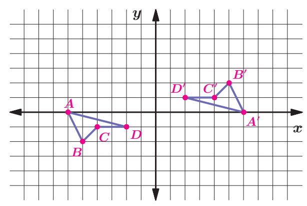
**Amps Featured Activity** Digital Geometry Tools

Name: ______ Date: ______ Period: ______

**Activity 1** Congruent Pairs

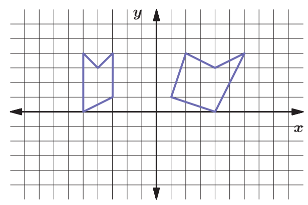
For each pair of figures, decide whether they are congruent. Explain your thinking. If they are congruent, label the corresponding vertices.

a



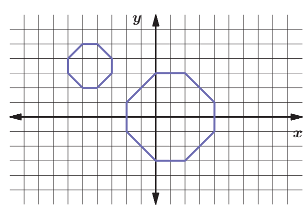

Yes, these figures are congruent because the corresponding side lengths and angle measures are equal.

b



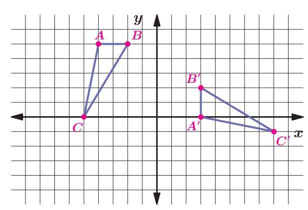

No, these figures are not congruent. The figure on the right has two right angles, while the figure on the left does not have any right angles.

c




No, these figures are not congruent because their sides do not have the same lengths.

d




Yes, these figures are congruent because the corresponding side lengths and angle measures are equivalent.

**Discussion Support:** Pay attention to the strategies you used and be ready to share them. As your classmates share, be ready to restate their ideas in your own words.





## 1 Launch

Ask, "What are the ways you can determine whether two figures are congruent?"

## 2 Monitor

**Help students get started** by asking if they can perform a transformation to map one figure onto the other in part a.

**Look for points of confusion:**

- **Visually determining congruence or using tracing paper and saying informally "they look the same."** Have students explain congruence in terms of rigid transformations. Alternatively, have students measure side lengths and angles to check congruence.

**Look for productive strategies:**

- Using both ways of checking congruence: rigid transformations and measuring side lengths and angle measures.

## 3 Connect

**Display** all four pairs of figures and use the *Poll the Class* routine to see which students thought which pairs of figures were congruent.

**Have students share** how they can check whether each pair of figures is congruent by using rigid transformations. Start by having students who measured the side lengths and angle measures share their thinking. Then call on students who used transformations; sequence the transformation strategies by those who used the greatest number of transformations to those who used the least number.

**Ask**, "What happens if you try to use rigid transformations to map one figure onto the other in part b?"

**Highlight** that when two figures are congruent, there is a rigid transformation that matches one shape up perfectly with the other.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, allow them to focus on completing two of the four problems in this activity. Consider allowing them to choose which problems they would like to complete. Once students have successfully completed the problems, invite them to share their responses with a partner prior to a whole class discussion.

### Extension: *Math Enrichment*

Have students find a second way to prove the figures are congruent, either by describing transformations or by determining the measures of side lengths and angles.

## Math Language Development

### *MLR8: Discussion Supports—Revoicing*

As students present their strategies during the Connect discussion, encourage them to restate and revoice their peers' ideas. Consider having each student describe the previously shared strategy in their own words, before sharing their own strategy.

### English Learners

Use hand gestures to illustrate the rigid transformations. Connect the terms used by displaying a visual similar to the following: *translation, rotation, reflection = rigid transformations → congruent*.

Pairs | 15 min

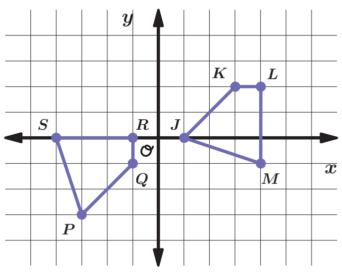
# Activity 2 Are You Sure They Are Congruent?

Students critique arguments to determine the best reasoning for deciding whether two polygons are congruent.

### Activity 2 Are You Sure They Are Congruent?

**Students in a different class are asked to determine if two polygons are congruent.**

1. Priya is trying to persuade her classmates that the polygons shown are congruent. Which argument is most convincing?




A. Both figures have 4 sides and an area of 5.5 square units.

B. I can map one figure onto the other by translating Polygon $JKLM$ down 3 units and to the left 4 units. Then I can reflect the image across segment $QP$.

C. When I measure the side lengths of each polygon, I get the same measurements.

D. When I measure the angles of each polygon, the angle measurements show $m\angle S = m\angle J$, $m\angle R = m\angle K$, $m\angle Q = m\angle L$, and $m\angle P = m\angle M$.

Explain your thinking.

Argument B is the most convincing. Two polygons are congruent if one can be mapped onto the other using a sequence of rigid transformations, such as translation and reflection.





## 1 Launch

Set an expectation for the amount of time students will have to work in pairs.

## 2 Monitor

**Help students get started** by asking them whether they can determine if the two polygons in Problem 1 are congruent by using rigid transformations.

**Look for points of confusion:**

- **Thinking that if both figures have the same area (Problem 1), then they are congruent.** Show students an example of two polygons with the same area and same side lengths, and ask whether they are congruent. Display Problem 5 from Lesson 10, Activity 2, if needed.
- **Thinking that if both figures have the same side length measures (Problem 2), then they are congruent.** Have students try to perform a sequence of rigid transformations to map one figure onto the other.

**Look for productive strategies:**

- Using both rigid transformations and features of the figures, for example, angle and side length measures, to determine whether the figures are congruent.

Activity 2 continued >

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge***

If students need more processing time, have them focus on completing Problem 1, and only work on Problem 2 as time allows.

## Math Language Development

***MLR8: Discussion Supports—Restate It!***

During the Connect discussion, revoice student ideas to demonstrate mathematical language used by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.

**English Learners**

Highlight complex phrases, such as "if two figures have different corresponding side lengths, then they are *not* congruent. However, the *converse* is not true; just because two figures have the same side lengths, it does not necessarily mean they are congruent." If time allows, address converse statements about congruent angles.

Pairs | 15 min

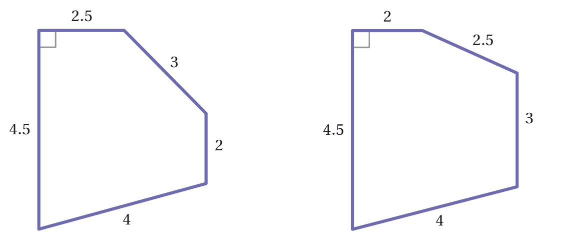
# Activity 2 Are You Sure They Are Congruent? (continued)

Students critique arguments to determine the best reasoning for deciding whether two polygons are congruent.

Name: ______________________ Date: __________ Period: __________

## Activity 2 Are You Sure They Are Congruent? (continued)

2. Andre studied the two figures shown and noticed that the side lengths of each figure are equivalent. Is this enough to claim the figures are congruent? Explain your thinking.



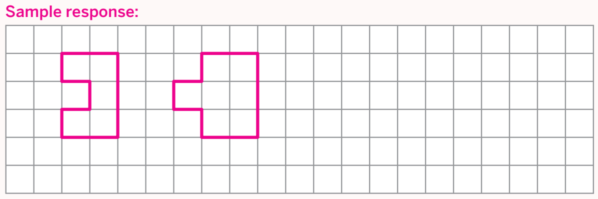

**No. These figures are not congruent. One figure cannot be mapped onto the other using a sequence of rigid transformations because the order of the sides is different. The side with length 3 is between side lengths 2 and 2.5 in the figure on the left, but that same side is between side lengths 2.5 and 4 in the figure on the right.**

**Are you ready for more?**

A polygon has 8 sides: five sides that each have a length of 1 unit, two sides that each have a length of 2 units, and one side that has a length of 3 units. All sides lie on grid lines. Draw a polygon with these side lengths on the grid shown here.

Is there a second polygon, not congruent to your first, that also has these side lengths? If so, draw this polygon on the grid shown here.

Sample response:



STOP





## 3 Connect

**Display** each problem, discussing each one before moving on to the next problem.

**Have pairs of students share** which argument they thought was most convincing for Problem 1 by using the ***Poll the Class*** routine.

**Ask:**

- "Why was the argument in Problem 1, choice B the most convincing argument?" Rigid transformations preserve side lengths and angle measures.
- "Did you use any measurements (length, area, angle measures) to help decide whether the polygons are congruent?" Answers may vary.
- "Why was the argument in Problem 1, choice A not a convincing argument?" A lot of different figures can have 4 sides and an area of 5.5 square units and not be congruent.
- "Would arguments C and D, if used together, in Problem 1 be a convincing way to prove congruence?" Yes, because if the side lengths and angle measures are the same, then I know that the figures are congruent.
- "In general, when proving congruence, what types of arguments are most convincing?" Arguments that demonstrate the specific rigid transformations or arguments that describe both the side lengths and angle measures being equal.
- "In Problem 2, why is it not enough for Andre to claim that the figures are congruent if their side lengths are the same?" Two figures can have the same side lengths without being congruent, as demonstrated by the figures in Problem 2.

**Highlight** that, as in the previous activity, if two figures have different side lengths, then they are not congruent. However, the converse is not true — just because two figures have the same side lengths, it does not necessarily mean they are congruent. The same is true for angles — congruent angle measures alone are not enough to prove congruence.

Whole Class | 5 min

# Summary

Review and synthesize how to determine whether two polygons are congruent.

## Summary

**In today's lesson . . .**

You applied the definition of congruence to polygons. You learned that:

- Two polygons are *congruent* when there is a sequence of translations, rotations, and reflections that map one polygon onto the other.
- Two polygons are *not congruent* if they have different side lengths, different angle measures, or different areas.

Even if two polygons have the same side lengths, they might not be congruent. With four sides of the same length, for example, you can create many different rhombuses that are not congruent to one another because the angles may be different.

**Reflect:**

80 **Unit 1** Rigid Transformations and Congruence



## Synthesize

**Ask:**

- "How do you know whether two polygons are congruent?"
- "How do you know whether two polygons are not congruent?"
- "If you know two polygons have different side lengths, is that enough to determine that the polygons are *not* congruent?" Yes
- "If you know two polygons have the same side lengths, is that enough to determine that the polygons are congruent?" No
- "If you know two polygons have the same angle measures, is that enough to prove congruence?" It is not enough.

**Have students share** an example of polygons that have the same side lengths, but are not congruent. Then have students share an example of polygons that have the same angle measures, but are not congruent.

**Highlight** that even if two figures have the same side lengths, they may not be congruent. With four sides of the same length, for example, students can construct many different rhombuses that are not congruent to one another, because the angle measures are different.

## Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful today when identifying congruent polygons?"

Independent | 5 min

# Exit Ticket

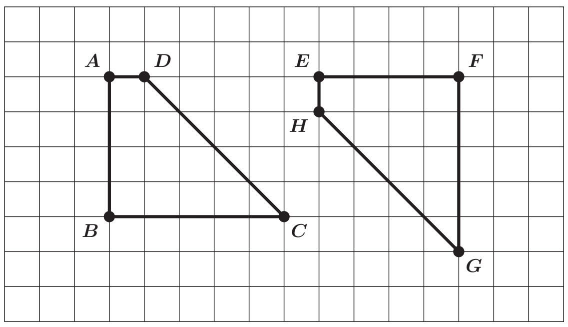
Students demonstrate their understanding of congruent polygons by describing a sequence of transformations that proves two polygons are congruent.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 1.11

Describe a sequence of transformations that shows that Quadrilateral $ABCD$ is congruent to Quadrilateral $EFGH$.




Sample response: Rotate Quadrilateral $ABCD$ 90° clockwise about Point $A$. Reflect the image across segment $AD$. Translate the image $ABCD$ 6 units to the right.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can decide whether two figures are congruent by using rigid transformations.

1 2 3



**Lesson 11** Congruent Polygons

## Success looks like . . .

- **Language Goal:** Comparing and contrasting side lengths, angle measures, and areas using rigid transformations to explain why two figures are, or are not, congruent. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Critiquing arguments about whether two figures with corresponding sides lengths the same may be non-congruent figures. **(Speaking and Listening)**
- **Language Goal:** Justifying that two polygons on a grid are congruent by describing a sequence of rigid transformations that maps one polygon onto the other. **(Speaking and Listening, Writing)**
  - » Describing a sequence of transformation from Quadrilateral $ABCD$ to Quadrilateral $EFGH$.

## Suggested next steps

**If students use informal language to explain that both polygons are congruent, consider:**

- Reviewing Activity 1.
- Assigning Practice Problem 2.
- Asking, "How can you use the grid and your knowledge of transformations to more specifically describe why the polygons are congruent?"

**If students are incorrectly describing the sequence of rigid transformations, consider:**

- Providing tracing paper or other tools from the geometry toolkits.
- Reviewing how to perform and describe each type of rigid transformation.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Point to Ponder . . .

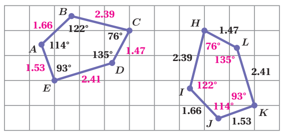
- How did your students approach Activity 2? Are they progressing in their understanding of what it means for two figures or polygons to be congruent, beyond informal observations that figures "look like the same size and shape"?
- What might you change for the next time you teach this lesson?

# Practice

Independent

Name: ______ Date: ______ Period: ______

1. Refer to Pentagons $ABCDE$ and $JIHLK$.



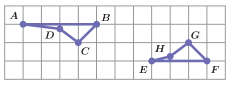

a. Show that the two pentagons are congruent by describing a sequence of rigid transformations that can map one figure onto the other.

**Rotate Pentagon $ABCDE$ 90° counterclockwise about point C. Then translate the image 3 units to the right.**

b. Label the side lengths of Pentagon $ABCDE$ and the angle measures of Pentagon $JIHLK$.

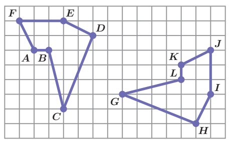
2. For each pair of figures, decide whether they are congruent. Explain your thinking.

a.



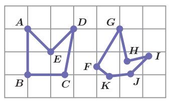

**No; Polygon $ABCD$ is not congruent to Polygon $EFGH$ because $AB$ has a length of 4 units and $EF$ has a length of 3 units. $AB$ and $EF$ are the longest sides of each polygon and need to be the same length for the polygons to be congruent.**

b.




**Yes; When Figure $ABCDEF$ is rotated 90° clockwise about Point $C$ and translated 1 unit up and 4 units to the right, it will map onto Figure $KLGHIJ$.**

c.




**No; The figures cannot be mapped onto each figure by using sequence of rigid transformations because Polygon $ABCDE$ has 5 sides and Polygon $FGHIJK$ has 6 sides.**

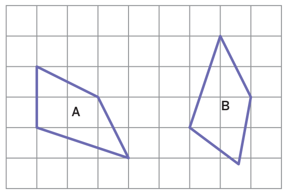




Practice

Name: ______ Date: ______ Period: ______

3. Lin says that she can map Polygon A onto Polygon B using *only* rotations and translations. Do you agree with Lin? Explain your thinking.




**No, a reflection must have been performed because the orientation of the two figures is different.**

4. Point $(3, -5)$ was transformed using different transformations. Match the transformations described with the coordinates of the images.

| Transformation | Image coordinates |
|---|---|
| a. Translated 2 units up and 4 units to the left | **e** $(-3, 5)$ |
| b. Reflected across the $x$-axis | **f** $(5, -9)$ |
| c. Rotated 90° counterclockwise about the origin | **b** $(3, 5)$ |
| d. Reflected across the $y$-axis | **d** $(-3, -5)$ |
| e. Rotated 180° about the origin | **c** $(5, 3)$ |
| f. Translated 4 units down and 2 units to the right | **a** $(-1, -3)$ |

5. Kiran says it is more challenging to determine if two ovals are congruent than if two triangles are congruent. Do you agree with this statement? Explain your thinking.

**Sample response: Yes, because ovals do not have clear sides or angles to measure like a polygon, I agree that it will be more difficult to determine if two ovals are congruent.**





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 1 | 2 |
| Spiral | 3 | Unit 1 Lesson 2 | 2 |
| | 4 | Unit 1 Lesson 7 | 1 |
| Formative | 5 | Unit 1 Lesson 12 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students that need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Optional

UNIT 1 | LESSON 12

# Congruence

Let's find ways to test congruence of polygons and other interesting figures.

## Focus

### Goals

1. Determine whether figures are congruent by measuring the distances between corresponding points.
2. Draw and label corresponding points on congruent figures.
3. **Language Goal:** Justify that in congruent figures, the corresponding distances between pairs of points are equal. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students explore the idea that the distance between any pair of corresponding points of congruent figures must be the same. Because there are too many pairs of points to consider, this is mainly a criterion for showing that two figures are not congruent; that is, if there is a pair of points on one figure where the points are a different distance apart than the corresponding points on another figure, then those figures are not congruent. For congruent figures built out of several different parts (for example, a collection of circles) the distances between all pairs of points must be the same.

### Previously

So far, students have mainly looked at congruence for polygons. The line segments in polygons provide easily-defined distances and angles to measure and compare.

### Coming Soon

In high school, students will build on what they know about determining congruence of polygons and other figures, such as ovals, and focus more specifically on finding ways of determining congruence of triangles.

## Rigor

- Students develop **conceptual understanding** about the distances between corresponding points of congruent figures.
- Students apply their understanding of congruence to determine whether two faces are congruent.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Activity 3 | Summary | Exit Ticket |
|---|---|---|---|---|---|
| 5 min | 12 min | 10 min | 8 min | 5 min | 5 min |
| Independent | Independent | Independent | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: rulers, tracing paper, protractor

### Math Language Development

**Review words**

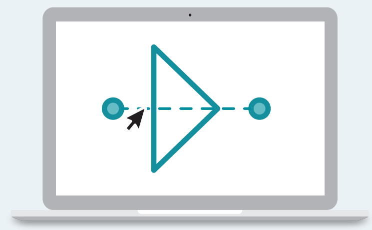
- *congruent*
- *rigid transformation*
- *translation*
- *rotation*
- *reflection*
- *corresponding*
- *orientation*

**Amps** | **Featured Activity**

**Activity 2**

**Interactive Geometry**

Students use digital geometry tools to explore congruence with non-polygons.



Amps powered by desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

At first, students may feel lost trying to make conjectures or justify their reasoning about congruence with non-polygons. Ask students to consider what is different about the figures they are studying today and encourage them to explain their thinking by first talking about what they find challenging about determining a justification for congruence. That level of metacognition will help students identify a different approach to the activity.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted and used as a formative practice problem in Lesson 11.
- **Activity 3** may be omitted as it reinforces the concepts learned in Activity 2.

Independent | 5 min

## Warm-up Not Just the Vertices

Students locate corresponding points (non-vertices) to better understand a figure's structure, preparing them for testing congruence among curved figures in the upcoming activities.

Name: ______ Date: ______ Period: ______

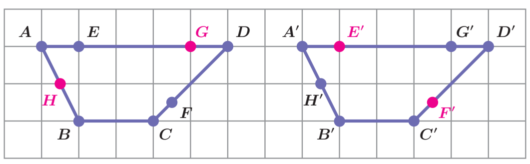
Unit 1 | Lesson 12

### Congruence

Let's find ways to test congruence of polygons and other interesting figures.

#### Warm-up Not Just the Vertices

Trapezoid $ABCD$ is congruent to Trapezoid $A'B'C'D'$.




1. Draw and label the points on Trapezoid $A'B'C'D'$ that correspond to points $F$ and $E$.

2. Draw and label the points on Trapezoid $ABCD$ that correspond to points $G'$ and $H'$.

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

### 2 Monitor

**Help students get started** by asking, "Which point corresponds to point $A$?"

**Look for points of confusion:**

- **Not realizing that corresponding points don't have to be vertices.** Have students describe the transformation that maps point $E$ onto point $E'$, for example, and ask them how that transformation compares to the one that maps point $A$ onto point $A'$.
- **Mislabeling the points.** Remind students that the order of points matters. Show how point $A$ corresponds to point $A'$ and have students relabel any points they have mislabeled.

### 3 Connect

**Display** student work showing correct responses.

**Have students share** how they identified the corresponding points.

**Highlight** that when two figures are congruent, *every* point on one figure has a corresponding point on the other figure.

**Ask**, "How can you use points on the two polygons to determine whether the polygons are congruent?" Sample response: I can find the lengths of line segments between two points and see that all corresponding lengths are the same.

## Differentiated Support

**Accessibility: *Optimize Access to Tools***

Provide access to tracing paper to assist students in identifying corresponding points.

## Power-up

**To power up students' ability to determine congruence, have students complete:**

Select *all* of the true statements about two congruent polygons.

A. Their areas are the same.

B. Their angles are the same measure.

C. Their perimeters are the same.

D. One polygon can be mapped onto the other by a series or rotations, reflections, and translations.

**Use:** Before the Warm-up
**Informed by:** Performance on Lesson 11, Practice Problem 5 and Exit Ticket

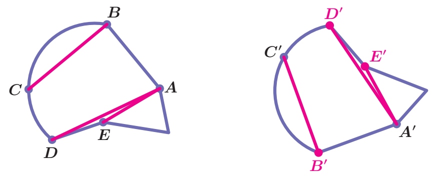
Independent | 12 min

# **Activity 1** Corresponding Points in Congruent Figures

Students locate corresponding points on figures and connect and measure line segments to deepen their understanding of congruence as they apply the concept to curved shapes.

**Activity 1** Corresponding Points in Congruent Figures

**Here are two congruent shapes with some corresponding points labeled.**




1. Label the points corresponding to $B$, $D$, and $E$ with $B'$, $D'$, and $E'$.

2. Draw line segments $AD$ and $A'D'$ and measure them. Repeat for segments $AE$ and $A'E'$ and for segments $BC$ and $B'C'$. What do you notice?
   **Each pair of line segments has the same length.**

3. Do you think there could be a pair of corresponding segments with different lengths? Explain your thinking.
   **Sample response: No. Any pair of corresponding segments will necessarily have the same lengths because if the lengths were different the figures would not be the same.**




## 1 Launch

Have students conduct *Think-Pair-Share* routine.

## 2 Monitor

**Help students get started** by demonstrating the transformations that map one figure onto the other.

**Look for points of confusion:**

- **Thinking there could be corresponding side lengths of different lengths in Problem 3.** Have students draw an example of what they are thinking. Then have them perform rigid transformations to see if the figure they drew is congruent to the preimage in the activity.

## 3 Connect

**Display** correct student work.

**Have students share** how they located the corresponding points, starting with students who identified corresponding parts of each figure to help label points, followed by students who performed rigid transformations. Have students share their thinking behind their responses to Problems 2 and 3.

**Ask**, "Which strategy would have worked best to locate point $C'$ had it not been marked?"

**Highlight** that performing rigid transformations matches the shapes up perfectly. This method allows students to locate the corresponding point on the image for any point on the preimage. Identifying key features only works for points such as $A$, $B$, $D$, and $E$, which are vertices and can be identified by the parts of the figures that are "joined" at these points.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge, Guide Visualization and Processing*

If students need more processing time, have them focus on completing Problems 1 and 2, and only work on Problem 3 as time allows. Provide students with the images on grid paper to assist in labeling corresponding parts and measuring line segments.

### Extension: *Math Enrichment*

Have students use tracing paper to create a new figure that is either congruent to the shape in the activity, or slightly different.

## Math Language Development

### *MLR7: Compare and Connect*

Call students' attention to the different strategies used to match figures to identify corresponding points. As students respond to the Ask question from the Connect, consider asking these follow-up questions to help them clarify their thinking.

- "If you identify corresponding points, how would you locate point $C'$?"
- "If you use transformations, how would you locate point $C'$?"

### English Learners

Use hand gestures to illustrate how rigid transformations could be used to locate point $C'$.

Independent | 10 min

# Activity 2 Congruent Ovals

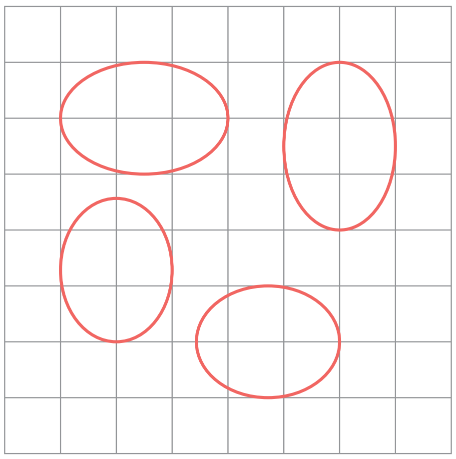
Students begin to explore the subtleties of congruence for curved shapes.

**Amps Featured Activity** Interactive Geometry

Name: ______ Date: ______ Period: ______

## Activity 2 Congruent Ovals

Four ovals are shown. Are any of the ovals shown congruent to one another? Explain your thinking.



Sample response: The top two ovals are congruent to each other and the bottom two ovals are congruent to each other. To best determine if the ovals in each pair are congruent, I can trace one of the top pair of ovals on tracing paper and check that it can be mapped onto the other oval. I can repeat this for the bottom pair of ovals.



Lesson 12 Congruence 85

## 1 Launch

Give students three minutes of quiet work time, and then invite them to share their reasoning with a partner, followed by a whole-class discussion. Provide access to geometry toolkits for the duration of the lesson.

## 2 Monitor

**Help students get started** by asking what they notice that is different about these figures than ones they have previously studied.

**Look for points of confusion:**

- **Not knowing how to precisely determine congruence for curved shapes**. Let students know that ovals or curved shapes can be more challenging than polygons, yet their geometry tools can help them. Have students use tracing paper to find congruent ovals.

**Look for productive strategies:**

- Using precise language of transformations as students attempt to move one traced oval to match up perfectly with another.

## 3 Connect

**Display** the ovals.

**Have students share** which ovals are congruent and how they know they are congruent, starting with students who used measurements for length and width, followed by students who described a sequence of rigid transformations.

**Ask**, “What is different about determining congruence with ovals than with polygons?”

**Highlight** that using transformations is essential when showing that two of the ovals match up because, unlike polygons, these shapes are not determined by a finite list of vertices and side lengths.

## Differentiated Support

**Accessibility: *Activate Prior Knowledge***

Connect this new concept to one with which students have experienced prior success. For example, review the criteria used to determine congruence for polygons so that students can transfer these strategies in determining congruence for curved shapes.

## Math Language Development

***MLR5: Co-craft Questions***

Ask, “What mathematical questions could you ask about this situation?” The purpose of this routine is to allow students to make sense of a context before feeling pressure to produce responses, and to develop students’ awareness of the language used in mathematics.

**English Learners**

Consider using a think-aloud strategy to model how to craft a mathematical question about the situation before having students craft their own.

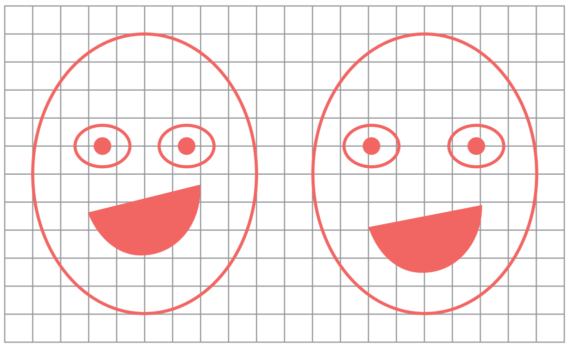
Pairs | 8 min

# Activity 3 Astonished Faces

Students determine whether two faces are congruent to understand that while individual parts of two figures may be congruent, the entire figures may not be congruent.

**Activity 3** Astonished Faces

Are these faces congruent? Explain your thinking.



Sample response: No. While the individual components of the faces are congruent, for example the eyes and mouth, the faces as a whole are not congruent. For example, the mouths can be mapped to each other using a translation, as can the eyes. But the distance between the eyes and the distance between the mouth and the eyes are not the same for each face. For these two figures to be congruent, all corresponding points, and the distance between those points, must be the same.

STOP

86 **Unit 1** Rigid Transformations and Congruence



## 1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

## 2 Monitor

**Help students get started** by having them find a pair of points on each figure that will help test congruence.

**Look for points of confusion:**

- **Thinking the two faces are congruent if all the individual parts of the face are congruent.** Have students draw a segment between a pair of corresponding points on the mouth and eyes of each figure, measure the segments, and ask what they notice.

**Look for productive strategies:**

- Selecting corresponding points between the figures, noting that different translations are used for each, and using that information to show the faces are not congruent.

## 3 Connect

**Display** the two faces and use the ***Poll the Class*** routine to see which students think the faces are congruent and which students think the faces are not congruent.

**Have pairs of students share** what strategies they used to determine whether the faces were congruent.

**Ask**, "The size and shape of the mouths and eyes are the same, so why are these two figures *not* congruent?"

**Highlight** that even though the individual parts of the two faces are congruent, the two faces as a whole are not congruent. For the two figures to be congruent, the same transformation has to apply to all parts of the figure.

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge, Optimize Access to Tools***

Provide access to tracing paper for students to use during the activity. Consider chunking this task into smaller, more manageable parts. For example, present one section of the face at a time and monitor students to ensure they are making progress throughout the activity.

## Math Language Development

***MLR8: Discussion Supports—Revoicing***

As pairs share their results and reasoning, revoice their ideas using terms such as *congruent figures*. Invite students to use the terms when describing their results and sharing their strategies.

**English Learners**

Encourage students to refer to the class display of key terms and phrases to assist them in the discussion.

Whole Class | 5 min

## Summary

Review and synthesize how to check whether two non-polygonal figures are congruent.

Name: ______ Date: ______ Period: ______

### Summary

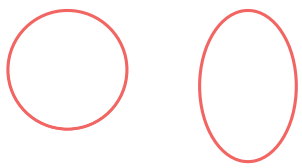
**In today's lesson . . .**

You explored different ways to show congruence between sets of polygons and other interesting figures.

To show that two figures are congruent, you can map one figure onto the other by a sequence of rigid transformations. This is true even for figures with curved sides. Distances between corresponding points on congruent figures are always equivalent, even for curved shapes.

To show two figures are *not* congruent, you can find parts of the figures that would correspond if the figures were congruent, but in reality have different measurements.

Here is an example of two figures that are *not* congruent.



**Reflect:**





### Synthesize

**Display** the Summary from the Student Edition.

**Ask**, "How can you best explain why these two figures are not congruent?"

Sample responses:

- The distance from the top to the bottom in one figure is different from the distance from the top to the bottom in the other figure.
- By performing rigid transformations, I am not able to map one figure onto the other.

**Have students share** responses to this question with their partners before sharing with the whole group. Start by calling on students who can explain how to use distances on the figures to determine they are not congruent. Then have students share how rigid transformations would prove these figures are not congruent.

**Highlight** that for two figures to be congruent, the distance between pairs of corresponding points must be the same.

### Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

- "How could you use the distance within a figure to help determine whether it is congruent to another figure?"

Independent | 5 min

# Exit Ticket

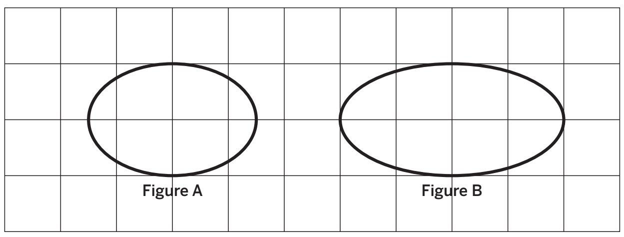
Students demonstrate their understanding by determining whether two figures are congruent.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 1.12

Are Figures A and B congruent? Explain your thinking.




No; Sample response: If you draw a line segment to represent the width of Figure A and Figure B, Figure A would have a width of approximately 3 units and Figure B would have a width of 4 units. This means these two figures cannot be congruent.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can use distances between points to decide whether two figures are congruent.

1 2 3





## Success looks like . . .

- **Goal:** Determining whether figures are congruent by measuring the distances between corresponding points.
  - » Measuring the widths of the two figures to determine whether they are congruent.
- **Goal:** Drawing and labeling corresponding points on congruent figures.
- **Language Goal:** Justifying that, in congruent figures, the corresponding distances between pairs of points are equal. **(Speaking and Listening, Writing)**

## Suggested next steps

**If students use informal language to state that the two figures "look different and are not the same," consider:**

- Asking, "What strategies can you use to check whether these two figures are congruent?"
- Reviewing Activity 1.
- Assigning Practice Problem 2.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Point to Ponder . . .**

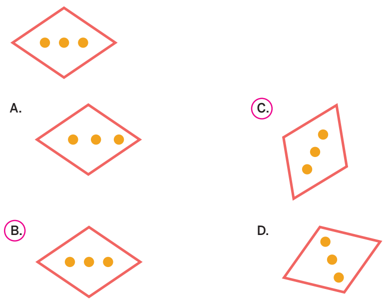
- How did students approach Activity 1? Did any of your students experience frustration when trying to determine whether any of the ovals were congruent? If so, what helped them work through their frustration?
- What might you change for the next time you teach this lesson?

# Practice

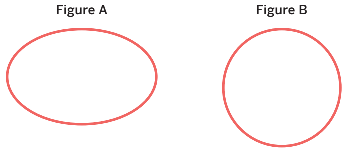
Independent

Name: ______ Date: ______ Period: ______

1. Which of these figures are congruent to the figure shown? Select *all* that apply.




2. Consider Figures A and B. Show, using measurements, that these two figures are not congruent.

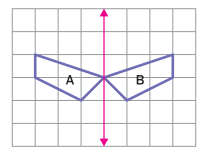



Sample response: Measurements for Figure A show a slightly longer width and slightly shorter height. Figure B is a circle with a height and width (diameter) that are the same. Because the heights and widths across the figures are not the same, the figures are not congruent.

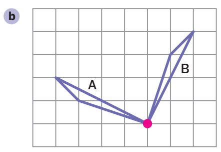



Name: ______ Date: ______ Period: ______

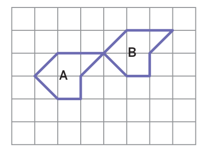
3. For each pair of polygons, describe the transformation that maps Polygon A onto Polygon B.

a


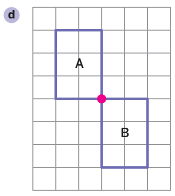

Reflect Polygon A across the vertical line that passes through the vertex shared by both polygons.

b



Rotate Polygon A 90° clockwise about the vertex shared by both polygons.

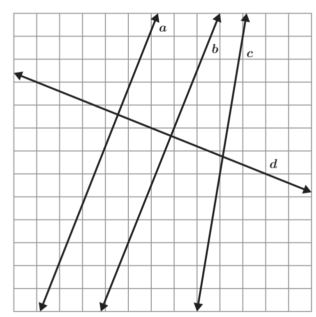
c



Translate Polygon A up 1 unit and to the right 3 units.

d



Sample response: Rotate Polygon A 180° about the vertex shared by both polygons.

4. Refer to the four lines shown.




a. Name a pair of lines that appear to be parallel.

Lines $a$ and $b$

b. Name a pair of lines that appear to be perpendicular.

- Lines $a$ and $d$
- Lines $b$ and $d$




## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activities 2 and 3 | 1 |
| | 2 | Activities 2 and 3 | 2 |
| Spiral | 3 | Unit 1 Lesson 4 | 1 |
| Formative | 4 | Unit 1 Lesson 13 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

**Sub-Unit 3**

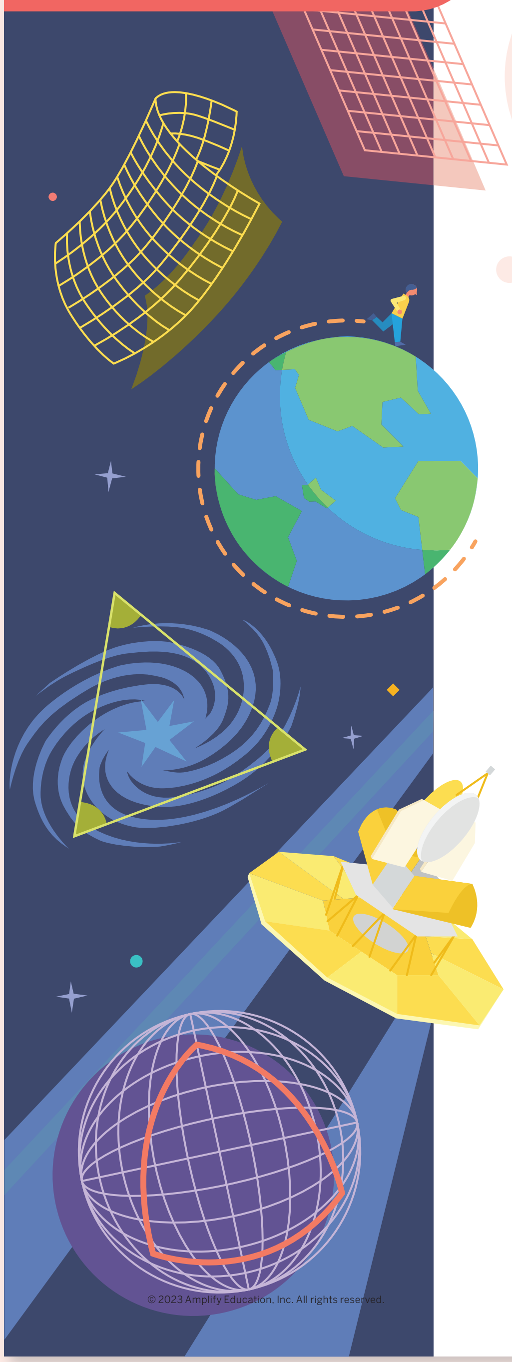
# Angles in a Triangle

Whole Class

Students consider parallel lines and transversals and study the measures of the alternate interior angles that are formed. These concepts help students build a framework for understanding dilations, similarity, and slope in later units.

SUB-UNIT

3 Angles in a Triangle





Narrative Connections

## What has 10 billion galaxies and goes great with maple syrup?

There have been a few theories about how our Universe is shaped. Some thought it was open like a saddle. Others thought it was round like a football. Some have even suggested that the Universe is shaped like a doughnut!

But, how do you actually find out?

Before we answer that, let's start with something a little bit smaller — Earth! Standing on Earth's surface, the world certainly *appears* flat. But there are ways to prove it isn't. For example, you could start walking (or swimming) in any direction, and eventually you'd wind up where you started.

Another way is to pick three points, thousands of miles apart, on Earth's surface. On a flat surface, the interior angles of the triangle formed by those points will always add up to — *spoiler alert!* — 180 degrees. But on a curved surface, like Earth's, the angles end up being something greater. The bigger the triangle on Earth, the greater the sum of its three angles.

Physicists have done the same thing with the Universe. With an assist from a specially designed spacecraft called the WMAP, NASA scientists effectively plotted a gigantic triangle across the Universe, then measured its angles. And what did they find?

It turned out the sum of the angles were very close to 180 degrees. So while the idea of a doughnut Universe sounds scrumptious, we'll have to settle for a Universe that's flat as a pancake (give or take a little curvature).

For now, let's see what else we can discover about the angles, lines, and triangles.

Sub-Unit 3 Angles in a Triangle 91

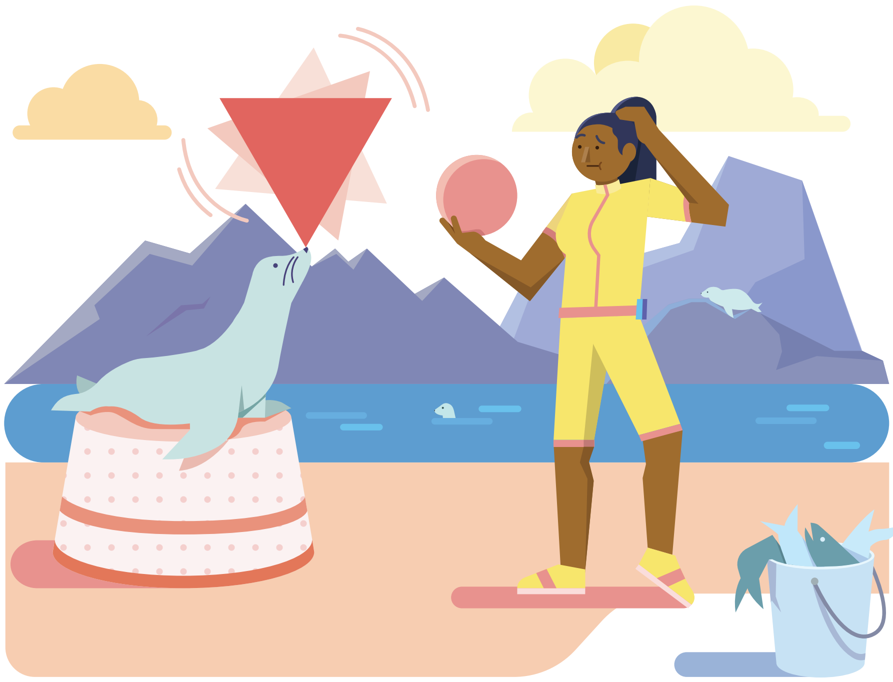
### Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students will closely inspect lines and angles — just as physicists did to determine the the shape of the Universe — in the following places:

- **Lesson 14, Activity 2:** Solving for Unknown Angles
- **Lesson 16, Activity 2:** Tear It Up
- **Lesson 18, Activity 1:** How Is It Made?

# Line Moves

Let's transform some lines.



## Focus

### Goals

1. Draw and label rotations of $180°$ of a line segment about the midpoint, a point on the segment, and a point not on the segment.
2. **Language Goal:** Generalize the outcome when rotating a line segment $180°$. **(Speaking and Listening, Writing)**
3. **Language Goal:** Describe observations of lines and parallel lines under rigid transformations, including lines that are taken to lines and parallel lines that are taken to parallel lines. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students rotate line segments $180°$ and apply rigid transformations on parallel lines. When students compare their application of a rigid transformation with their peers, they begin to see that lines are taken to lines and parallel lines are taken to parallel lines.

### Previously

In Lesson 12, students explored the idea that the distance between any pair of corresponding points of congruent figures must be the same.

### Coming Soon

In Lesson 14, students will investigate how a $180°$ rotation about a point of two intersecting lines rotates each angle to an angle that is vertical to its preimage.

## Rigor

- Students build **conceptual understanding** about how rigid transformations affect lines, line segments, and parallel lines.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 10 min | 15 min | 12 min | 5 min | 6 min |
| Pairs | Small Groups | Small Groups | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice
Independent

### Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: rulers, tracing paper

### Math Language Development

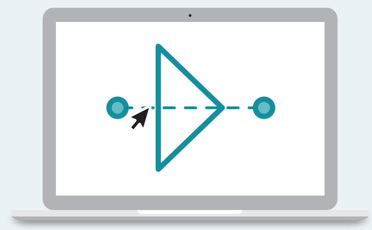
**Review words**

- *angle of rotation*
- *center of rotation*
- *rigid transformation*
- *rotation*

**Amps** | **Featured Activity**

**Activity 1**
### Interactive Geometry

Students can drag points to create transformations of a preimage. You can overlay student answers to provide immediate feedback.



Amps powered by desmos

### Building Math Identity and Community
Connecting to Mathematical Practices

Students may start to lose focus as they look for structure when rotating line segments. Encourage them to persist as they look for patterns. For example, have them pause and focus on one step at a time. Have them use resources, such as tracing paper, to regain motivation.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, provide the final image and focus on Problem 4.
- In **Activity 1**, you may arrange students in groups of three and assign students Problem 1a, 1b, or 1c.

Pairs | 10 min

## Warm-up Rotating a Triangle

Students examine several rotations of an isosceles triangle to reinforce the idea that applying a rigid transformation on a figure preserves its side length and angle.

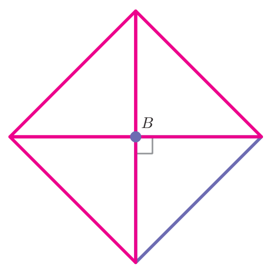
Unit 1 | Lesson 13

### Line Moves

Let's transform some lines.

**Warm-up** Rotating a Triangle

**Refer to the isosceles right triangle shown.**




1. Rotate the isosceles right triangle $90°$ clockwise about point $B$. Draw the image.
2. Rotate the original isosceles right triangle $180°$ about point $B$. Draw the image.
3. Rotate the original isosceles right triangle $270°$ clockwise about point $B$. Draw the image.
4. What do you notice?
   **Sample response: The resulting figure that includes the original triangle and its three images is a square.**

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Activate students' prior knowledge by asking them about the features of an isosceles right triangle. Provide access to geometry toolkits for the duration of the lesson.

### 2 Monitor

**Help students get started** by reminding them that an isosceles right triangle has a right angle and two sides of equal lengths.

**Look for productive strategies:**

- Using tracing paper to help them rotate the triangle.
- Using the right angle in the triangle to help with each rotation.
- Noticing the resulting image is a square.

### 3 Connect

**Have pairs of students share** their conclusions about rotating an isosceles right triangle.

**Highlight** that rotating the isosceles right triangle $90°$ interchanges the four copies of the triangle. The lengths and angle measures of the figure are preserved under the rotation.

**Ask:**

- "What do you notice about the figure?" Sample response: It is a square. I know this because the isosceles right triangle maps onto itself, so all of the sides of the figure are the same. Because one angle of the triangle measures $90°$, I know the sum of the other two angles in each triangle measures $90°$ and therefore each angle in the figure also measures $90°$.
- "What do you know about the two opposite sides?" The opposite side lengths are equal in length and parallel.

### Differentiated Support

**Accessibility: *Optimize Access to Tools***

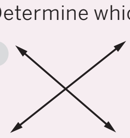
Have students use tracing paper to rotate the figure. Consider demonstrating how to use the tracing paper to rotate the figure in Problem 1. Then have students complete Problems 2–4 with their partner.

### Power-up

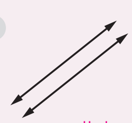
**To power up students' ability to identify parallel and perpendicular lines, have students complete:**

Recall that *parallel lines* are a pair of lines that never intersect. *Perpendicular lines* are a pair of lines that create a right angle when they intersect.

Determine which pair of lines are parallel and which pair of lines are perpendicular.

a 

perpendicular

b 

parallel

**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 12, Practice Problem 4 and Pre-Unit Readiness Assessment, Problem 2

Small Groups | 15 min

# Activity 1 Rotating a Segment

Students explore special cases of rotating a line segment 180°, seeing that this rotation produces a parallel segment the same length as the original.

**Amps Featured Activity** Interactive Geometry

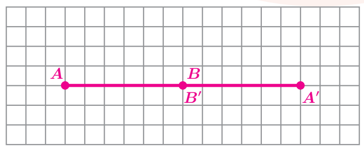
Name: ________ Date: ________ Period: ________

## Activity 1 Rotating a Segment

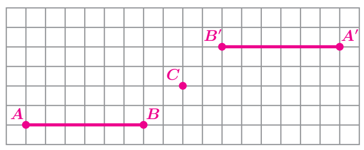
**Plan ahead:** What can you do to make sure you have an optimistic attitude before beginning this activity?

1. For each grid, draw and label a line segment $AB$. Then perform the indicated transformation.

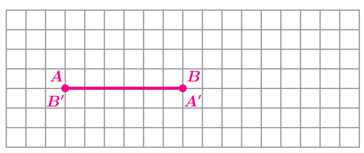
   a. Rotate segment $AB$ 180° about point $B$ and label the resulting image $A'B'$.
   **Sample response shown.**




   b. Draw and label a point $C$ that is not on the line segment $AB$. Rotate segment $AB$ 180° about point $C$ and label the resulting image $A'B'$.
   **Sample response shown.**




   c. Rotate segment $AB$ 180° about its midpoint and label the resulting image $A'B'$.
   **Sample response shown.**




2. What do you notice when you rotate a line segment 180° about a point?
   **Sample response: The image is the same length as the preimage. The image is parallel to the preimage.**





## 1 Launch

Set an expectation for the amount of time students will have to work on the activity.

## 2 Monitor

**Help students get started** by telling them to draw a vertical, horizontal, or diagonal line segment with points $A$, $B$, and $C$ on the cross section of the grid. Suggest that students draw the line and point toward the center of the grid to ensure the image after the rotation can be drawn on the grid.

**Look for points of confusion:**

- **Not being sure of the midpoint.** Remind students that this point is halfway between points $A$ and $B$. Encourage students to measure the line segment or use the grid to help them locate the midpoint.
- **Not being sure of the patterns.** Have students compare their line segments with each member in their group to look for and make use of structure.

**Look for productive strategies:**

- Drawing diagonal segments.
- Noticing the line segment remains the same length.
- Noticing they are performing a rigid transformation.

## 3 Connect

**Display** the different line segments created and the image under each rotation in Problem 1.

**Ask**, "What is the same about your line segment and image and the line segment and image for each person in your group?"

**Have groups of students share** what they noticed when rotating a line segment 180°.

**Highlight** that a 180° rotation produces an image that is the same length and is parallel to or on the same line as the preimage.

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge, Optimize Access to Technology***

Have students draw a horizontal or vertical line segment (not diagonal) to use during the activity. Alternatively, have students use the Amps slides for this activity, in which they see an animated movement of line segments when they are rotated 180° about different centers of rotation.

**Extension: *Math Enrichment***

Have students explore different locations for point $C$ in Problem 1b and describe what they notice.

## Math Language Development

***MLR8: Discussion Supports***

Use this routine to support whole class discussion as students discuss whether it is necessary to specify the direction of a 180° rotation. After each student shares, call on other students to restate what was shared using developing mathematical language, e.g., *rotation, line segment, midpoint*, etc.

**English Learners**

Use a *Think-Pair-Share* strategy to allow students to rehearse with a peer before sharing out with the whole class.

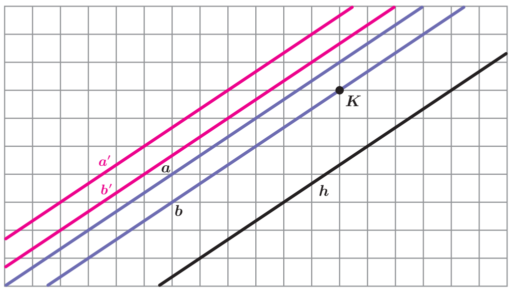
Small Groups | 12 min

# Activity 2 Parallel Lines

Students perform three different transformations on a set of parallel lines to see that parallel lines are taken to parallel lines under a rigid transformation.

### Activity 2 Parallel Lines

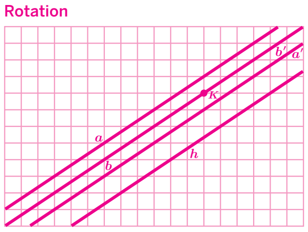
The diagram shows parallel lines $a$ and $b$, point $K$, and line $h$.



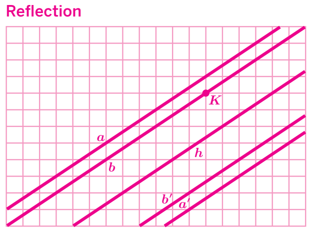

1. Have each member in your group choose one of the following transformations to perform. Perform the transformation on the grid provided here.
   - Translate lines $a$ and $b$ 3 units up and 2 units to the right.
     **Response shown on graph above.**
   - Rotate lines $a$ and $b$ 180° about point $K$.
     **Response shown on graph below.**
   - Reflect lines $a$ and $b$ across line $h$.
     **Response shown on graph below.**

2. Discuss each transformation with your group. What do you notice about the image of two parallel lines under a rigid transformation?
   **The lines remain parallel and the distance between them is the same.**

Rotation




Reflection




STOP




## 1 Launch

Activate students' prior knowledge by asking them what they know about parallel lines.

## 2 Monitor

**Help students get started** by having each student choose a different transformation to perform.

**Look for points of confusion:**

- **Having trouble performing their transformation.** Have students trace line $a$ and line $b$ on tracing paper.
- **Having trouble determining the distance between the lines.** Have students determine the shortest distance by drawing and measuring a perpendicular segment from a point on line $a$ to a point on line $b$.

**Look for productive strategies:**

- Noticing the lines remain parallel and the distance between them remains the same. Select these students to share during the Connect.

## 3 Connect

**Display** student work showing one example of each transformation.

**Have groups of students share** what they noticed about parallel lines under a rigid transformation.

**Highlight** that when a rigid transformation is performed on parallel lines, the lines remain parallel and the distance between the lines stays the same. Illustrate this idea using hand gestures.

## Differentiated Support

**Extension:** ***Math Enrichment, Interdisciplinary Connections***

Tell students that the geometry they are learning is called Euclidean geometry. Another type of geometry, spherical geometry, is the geometry of the two-dimensional surface of a sphere. Point out that spherical geometry best describes the geometry of Earth. Consider bringing in an inflatable plastic sphere, or a balloon, and illustrate these principles of spherical geometry. (Science)

- **Straight lines are actually great circles** that go around the entire sphere. Draw sample lines on the sphere to illustrate this concept.
- **There are no parallel lines.** Draw sample lines on the sphere to illustrate why the longitude lines of Earth are not actually parallel.

## Math Language Development

***MLR7: Compare and Connect***

Have students compare with their groups what they noticed about parallel lines under rigid transformations. Encourage them to make connections between each others' observations. As students share, emphasize that the lines remain parallel and the distance between the lines remains the same.

**English Learners**

Use hand gestures to illustrate that the lines remain parallel and the distances remain the same.

Whole Class | 5 min

## Summary

Review and synthesize the outcome of rotating a line segment 180° and performing rigid transformations on two parallel lines.

Name: ______ Date: ______ Period: ______

### Summary

**In today's lesson . . .**

You applied a $180°$ rotation to a line segment and discovered the following:

When the center of rotation is . . .

- the midpoint of the line segment, the segment maps onto itself, except the endpoints are switched.
- an endpoint of the line segment, the segment together with its image form a segment twice as long as the original.
- not a point on the line segment, the image is parallel to the original segment.

You also applied different rigid transformations to parallel lines. A rigid transformation of two parallel lines results in two parallel lines that are the same distance apart as the original two lines.

**Reflect:**





### Synthesize

**Have students share** what they noticed when they rotated a line segment $180°$ and applied a rigid transformation on two parallel lines.

**Highlight:**

- Highlight that a $180°$ rotation produces an image that is the same length and is parallel to or on the same line as the preimage.
- A rigid transformation of two parallel lines results in two parallel lines that are the same distance apart as the original two parallel lines.

### Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful today when rotating lines? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

### Math Language Development

***MLR2: Collect and Display***

Capture the language discussed during the Synthesize section using the class display. For example, "A $180°$ rotation produces an image that is the same length and is parallel to or on the same line as the preimage" should be added to the display and students should be encouraged to refer to this during future discussions.

Independent | 6 min

# Exit Ticket

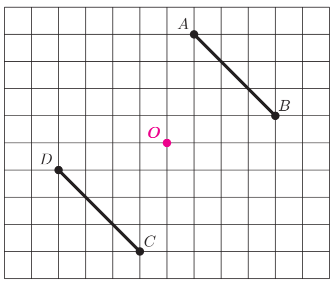
Students demonstrate their understanding by locating the center of rotation of a line segment that is rotated 180° and identifying the outcome of the rotation.

Printable

Name: ______________ Date: __________ Period: __________

## Exit Ticket 1.13

Line segment $AB$ is rotated 180° about point $O$ — not shown — to create line segment $CD$.




1. Draw the center of rotation and label it as point $O$. Show or explain your thinking.
   Point $O$ is halfway between the corresponding points of the line segment, at a 180° angle.

2. Which statements are true about line segment $AB$ and its image? Select *all* that apply.
   - A. Line segment $AB$ is parallel to line segment $CD$.
   - B. Line segment $AB$ has the same length as line segment $CD$.
   - C. Line segment $AB$ is longer than line segment $CD$.
   - D. Point $A$ corresponds to point $D$.
   - E. Point $A$ corresponds to point $C$.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can describe the images after rotating a line segment 180° using different centers of rotation.
1 2 3

b I can describe the effects of a rigid transformation on a pair of parallel lines.
1 2 3



**Lesson 13** Line Moves

## Success looks like . . .

- **Goal:** Drawing and labeling 180° rotations of a line segment about the midpoint, a point on the segment, and a point not on the segment.
- **Language Goal:** Generalizing the outcome when rotating a line segment 180°. **(Speaking and Listening, Writing)**
- **Language Goal:** Describing observations of lines and parallel lines under rigid transformations, including lines that are taken to lines and parallel lines that are taken to parallel lines. **(Speaking and Listening, Writing)**
  - » Selecting the statements that are true about line segment $AB$ and its image in Problem 2.

## Suggested next steps

**If students do not correctly draw the center of rotation, consider:**

- Having them draw lines to connect corresponding points.
- Having them use tracing paper to trace line segment $AB$ and rotating it using different centers of rotation
- Reviewing Activity 1.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- What resources did students use as they worked on rotating line and line segments 180°? Which resources were especially helpful?
- In earlier lessons, students rotated figures on a grid. How did that support how students rotated lines and line segments 180°?

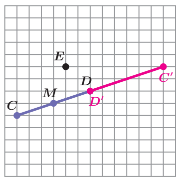
# Practice

Independent

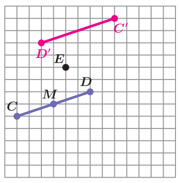
Name: Date: Period:

1. The diagram in each of parts a, b, and c shows line segment $CD$ and point $E$ that is *not* on $CD$.

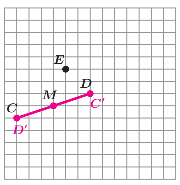
   a. Rotate segment $CD$ $180°$ about point $D$ and label the resulting image $C'D'$.




   b. Rotate segment $CD$ $180°$ about point $E$ and label the resulting image $C'D'$.




   c. Rotate segment $CD$ $180°$ about point $M$ and label the resulting image $C'D'$.

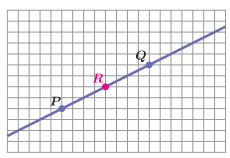



2. Points $P$ and $Q$ are plotted on the line shown.

   a. Label point $R$ so that a $180°$ rotation with center $R$ maps point $P$ onto point $Q$ and maps point $Q$ onto point $P$.

   b. Is there more than one point $R$ that satisfies the conditions in part a?

   No








Practice

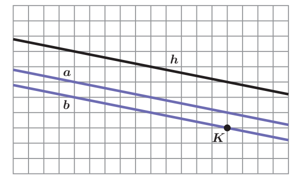
Name: Date: Period:

3. The diagram shows parallel lines $a$ and $b$, point $K$, and line $h$.

   - Elena rotates line $a$ about point $K$, and then reflects the image across line $h$. She labels the final image $a'$.
   - Jada rotates line $b$ about point $K$, and then reflects the image across line $h$. She labels the final image $b'$.

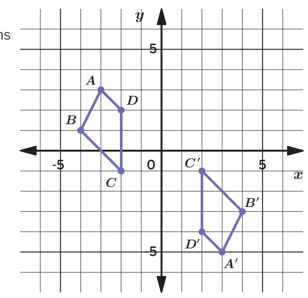
   What is true about lines $a'$ and $b'$?

   **Sample response: The distance between $a'$ and $b'$ is the same as the distance between $a$ and $b$. Lines $a'$ and $b'$ are parallel.**




4. The graph shows two quadrilaterals. Describe a sequence of transformations that maps Quadrilateral $ABCD$ onto Quadrilateral $A'B'C'D'$.

   **Sample response: Rotate $180°$ about point $C$, and then translate the image four units to the right.**




5. The diagram shows intersecting lines.

   a. List *all* the pairs of vertical angles.

   **Pair 1: $\angle ABE$ and $\angle CBD$**

   **Pair 2: $\angle ABC$ and $\angle EBD$**

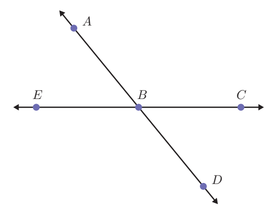
   b. List *all* the pairs of supplementary angles.

   **Pair 1: $\angle ABE$ and $\angle ABC$**

   **Pair 2: $\angle ABC$ and $\angle CBD$**

   **Pair 3: $\angle CBD$ and $\angle EBD$**

   **Pair 4: $\angle EBD$ and $\angle ABE$**








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 1 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 1 Lesson 8 | 3 |
| Formative | 5 | Unit 1 Lesson 14 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 1 | **LESSON 14**

# Rotation Patterns

Let's rotate some angles.

## Focus

### Goals

1. Comprehend that congruent, vertical angles are formed when an angle is rotated $180°$ about the intersection point of two intersecting lines.
2. **Language Goal:** Generalize that vertical angles are congruent using informal arguments about $180°$ rotations of lines, line segments, or angles. **(Speaking and Listening)**

## Coherence

### Today

Students apply a $180°$ rotation of two intersecting lines to justify that vertical angles are congruent. Students look for and make use of structure when they are presented with intersecting lines and are asked to determine unknown angle measures.

### Previously

In Lesson 13, students rotated line segments and applied rigid transformations to parallel lines.

### Coming Soon

In Lesson 15, students will investigate parallel lines intersected by a transversal and will justify that alternate interior angles are congruent.

## Rigor

- Students build **conceptual understanding** that vertical angles are congruent by using rigid transformations.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 20 min | 12 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: protractors, rulers, tracing paper

### Math Language Development

**Review words**

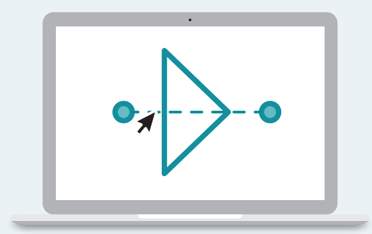
- *angle of rotation*
- *center of rotation*
- *congruent*
- *rotation*
- *rigid transformation*
- *straight angle*
- *vertical angles*

**Amps** | Featured Activity

**Activity 1**

### Interactive Geometry

Students can drag points to create transformations of a preimage. You can overlay student answers to provide immediate feedback.



Amps powered by desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel lost as they try to look for and make use of structure when determining missing angle measures. Have them engage in metacognitive functions by asking them to think about their own thinking process. For example, have them look at the diagram and list everything they notice before they determine any missing angle measures.

### Modifications to Pacing

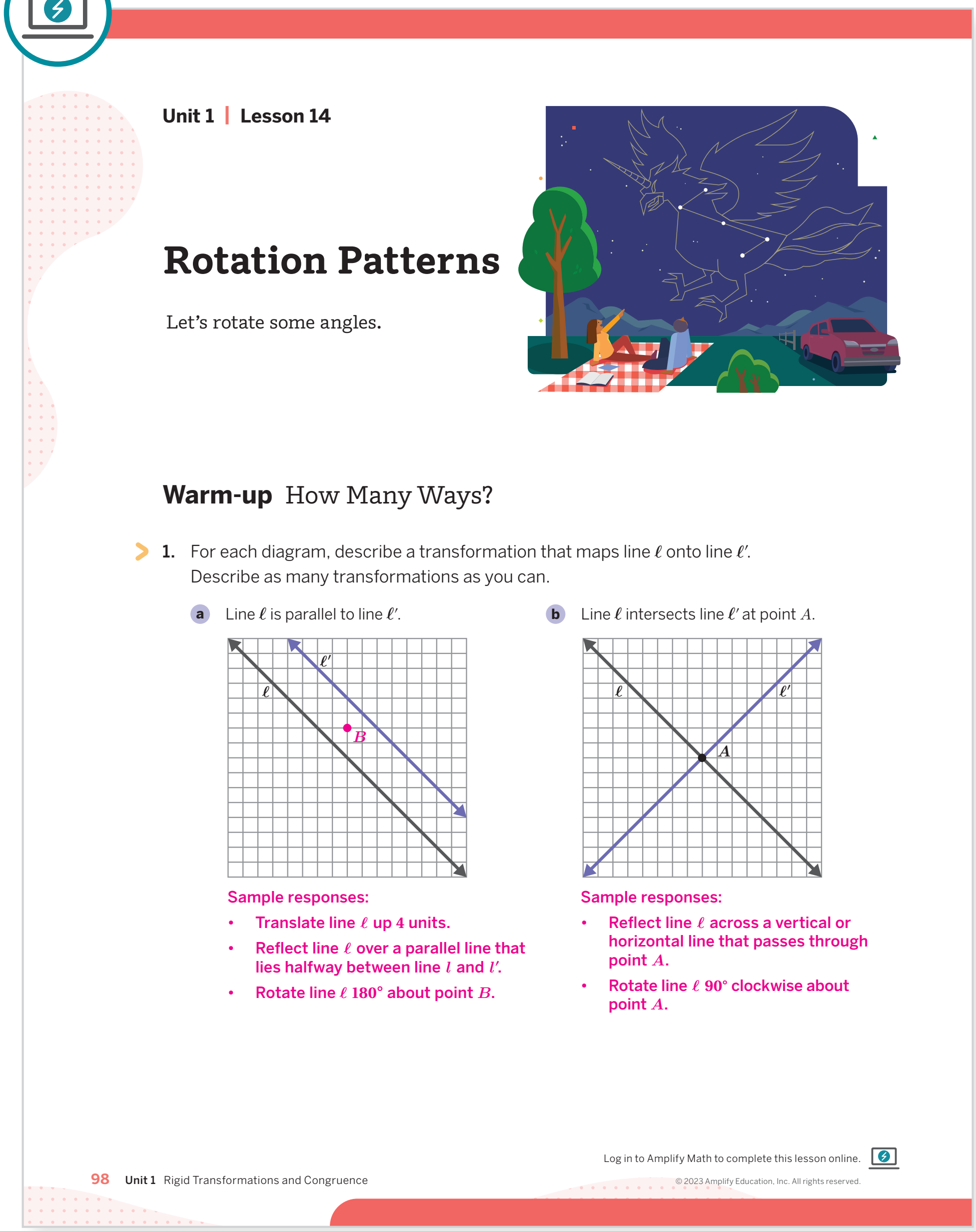
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted. It reinforces the idea that multiple transformations can result in the same image.
- In **Activity 2**, Problem 2 may be omitted.

Pairs | 5 min

## Warm-up How Many Ways?

Students describe transformations to understand that different transformations can result in the same image.


Unit 1 | Lesson 14

**Rotation Patterns**

Let's rotate some angles.

**Warm-up** How Many Ways?

1. For each diagram, describe a transformation that maps line $\ell$ onto line $\ell'$. Describe as many transformations as you can.

a Line $\ell$ is parallel to line $\ell'$.

Sample responses:
- Translate line $\ell$ up 4 units.
- Reflect line $\ell$ over a parallel line that lies halfway between line $\ell$ and $\ell'$.
- Rotate line $\ell$ 180° about point $B$.

b Line $\ell$ intersects line $\ell'$ at point $A$.

Sample responses:
- Reflect line $\ell$ across a vertical or horizontal line that passes through point $A$.
- Rotate line $\ell$ 90° clockwise about point $A$.

Log in to Amplify Math to complete this lesson online.

98 Unit 1 Rigid Transformations and Congruence

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### 1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity. Provide access to geometry toolkits for the Warm-up and Activity 1.

### 2 Monitor

**Help students get started** by reminding them to describe the specific transformation including details about the line of reflection, center of rotation, or direction of translation.

**Look for points of confusion:**

- **Having trouble describing the line of reflection.** Have students draw and label the line of reflection before describing it.
- **Questioning the angle of rotation for part b.** Allow access to a protractor or have students estimate the angle.

**Look for productive strategies:**

- Noticing that line $\ell'$ is 4 units above line $\ell$ in part a and describing this as a translation.
- Noticing that line $\ell$ can be rotated to map onto line $\ell'$ in part b.
- Seeing different types of transformations that result in the same image for parts a and b.

### 3 Connect

**Have pairs of students share** the transformations they wrote for each image. Use the *Poll the Class* routine to see which students thought of the same transformations.

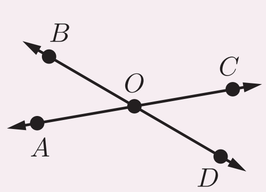
**Ask**, "Will a single translation work for the image in part b? Explain your thinking." No, a translation of the line will be parallel to the original line or the same line.

**Highlight** that sometimes different transformations of a preimage can result in the same image.

## Power-up

**To power up students' ability to recognize vertical and supplementary angles, have students complete:**

1. Identify whether each pair of angles are *vertical* or *supplementary*.
   a $\angle AOB$ and $\angle BOC$ supplementary
   b $\angle AOB$ and $\angle DOC$ vertical
   c $\angle AOD$ and $\angle BOC$ vertical
   d $\angle DOC$ and $\angle BOC$ supplementary




2. If $m\angle AOB = 40°$, determine each angle measure.
   a $m\angle BOC =$ 140°
   b $m\angle COD =$ 40°
   c $m\angle DOA =$ 140°

**Use:** Before Activity 1
**Informed by:** Performance on Lesson 13, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 4

Pairs | 20 min

# Activity 1 Let's Do Some 180s

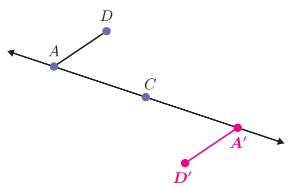
Students apply their understanding of 180° rotations to informally demonstrate the alternate interior angle theorem and vertical angle theorem.

Amps Featured Activity | Interactive Geometry

Name: ______ Date: ______ Period: ______

## Activity 1 Let's Do Some 180s

1. The figure shows a line with points $A$ and $C$ on the line and a segment $AD$ where point $D$ is not on the line.




a. Rotate the figure $180°$ about point $C$. Label the points corresponding to $A$ and $D$ with $A'$ and $D'$.

b. What do you know about the relationship between $\angle CAD$ and $\angle CA'D'$? Show or explain your thinking.

**Sample responses:**

- **The angle measures are the same because a rigid transformation does not change angle measures.**
- **I used a protractor to measure the angles and found that both angle measures are 55°.**





## 1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

## 2 Monitor

**Help** students get started by reminding them that when rotating a line segment $180°$ about a point on the segment, the image of the segment will be on the same line.

**Look for points of confusion:**

- **Not understanding that rotating the figure includes both segments $CA$ and $AD$ in Problem 1.** Because students have been rotating one segment at a time up to this point, explain that the entire figure includes both segments. Encourage them to use tracing paper to help visualize the rotation.

**Look for productive strategies:**

- For Problem 1, noticing that the measures of $\angle CAD$ and $\angle CA'D'$ are the same because they applied a rigid transformation.
- For Problem 1, noticing that segment line $AD$ is parallel to segment line $A'D'$ because they rotated the figure $180°$.
- For Problem 2, noticing line $AA'$ is a straight line and line $DD'$ is a straight line, so $\angle AOD$ and $\angle A'OD'$ are vertical angles.

Activity 1 continued >

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students perform the rotations, provide the rotated figures for Problems 1a and 2a and have students focus on completing Problems 1b and 2b.

### Extension: *Math Enrichment*

While this activity focused on performing rotations of $180°$ to prepare students for understanding vertical angles, ask students to rotate the figure in Problem 1 $90°$ clockwise about point $C$. Ask, "Do the angle measures still stay the same? Why or why not?" Yes, it does not matter what the angle of rotation is. Any rotation is a rigid transformation.

## Math Language Development

### *MLR7: Compare and Connect*

Use this routine as students share what they noticed during the Connect discussion. Ask them to consider what changes and what stays the same when $180°$ rotations are applied to the figures. Consider asking, "Why did the angle measures stay the same?" A rotation is a rigid transformation, which does not change angle measures (or distances).

### English Learners

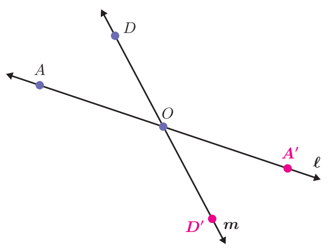
Consider displaying a visual similar to the following: *translation, rotation, reflection = rigid transformations → angle measures and distances stay the same.*

# Activity 1 Let's Do Some 180s (continued)

Students apply their understanding of 180° rotations to informally demonstrate the alternate interior angle theorem and vertical angle theorem.

**Activity 1** Let's Do Some 180s (continued)

2. The figure shows two lines $\ell$ and $m$ that intersect at a point $O$. Point $A$ is on the line $\ell$ and point $D$ is on the line $m$.




a. Rotate the figure $180°$ about point $O$. Label the points corresponding to $A$ and $D$ with $A'$ and $D'$.

b. What do you know about the relationship between the angles in the figure? Explain or show your thinking.

**Sample responses:**

- **The angle measures are the same because a rigid transformation does not change the angle measures.**
- **I used a protractor to measure the angles and found $m\angle AOD$ and $m\angle A'OD'$ both have a measure of $45°$ and $m\angle DOA'$ and $m\angle D'OA$ both have a measure of $135°$.**





## 3 Connect

**Display** correct student work for Problems 1a and 2a.

**Have pairs of students share** the relationships they found between the angle measures of the preimage and image and their reasoning about these relationships. Start by having students share who measured the angles, and then have students share who used rigid transformations.

**Ask:**

- "How do you know segment line $AD$ is parallel to segment line $A'D'$ in Problem 1?" A rigid transformation was performed, so the angle measures are preserved. This means that $\angle DAC$ and $\angle D'A'C$ have equal measures, which means the lines are parallel.
- "How do you know that line $AA'$ and line $DD'$ are straight lines in Problem 2?" A rigid transformation was performed, so angle measures are preserved. Because there are two pairs of vertical angles and the full circle measures $360°$, I know these are straight lines, each measuring $180°$.
- "How do you classify angles like $\angle AOD$ and $\angle A'OD'$ in Problem 2?" Vertical angles
- "How many pairs of vertical angles do you see in the figure for Problem 2?" Two pairs of vertical angles

**Highlight** that two pairs of congruent angles, called *vertical angles*, are formed when two lines intersect. Vertical angles have the same measure.

Pairs | 12 min

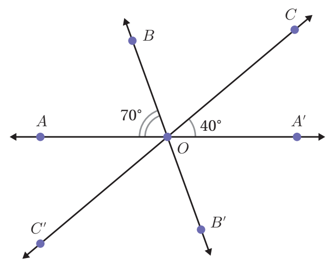
# Activity 2 Solving for Unknown Angles

Students examine three intersecting lines that pass through the same intersection point to discover that each pair of vertical angles have the same measure.

Name: ______ Date: ______ Period: ______

## Activity 2 Solving for Unknown Angles

**Points $A$, $B$, and $C$ are located at different distances from point $O$. The points $A$, $B$, and $C$ are each rotated 180° about point $O$ creating the images of points $A'$, $B'$, and $C'$.**




*The figure may not be drawn to scale.*

1. Name a segment that has the same length as segment $AO$. Explain your thinking.
   **Sample response: Segment $A'O$ has the same length as segment $AO$ because a rotation is a rigid transformation.**

2. List all the angles that have a measure of 40°. Explain your thinking.
   **Sample response: $\angle AOC'$ and $\angle A'OC$; Because $\angle A'OC$ has a given measure of 40°, I know $\angle AOC'$ has the same measure because they are vertical angles.**

3. List all the angles with a measure of 70°. Explain your thinking.
   $\angle AOB, \angle A'OB', \angle BOC, \angle B'OC'$
   - $\angle AOB$ and $\angle A'OB'$ are vertical angles.
   - The measure of $\angle BOC$ is 70° because $180 - 70 - 40 = 70$. $\angle B'OC'$ and $\angle BOC$ are vertical angles.

STOP





## 1 Launch

Students should not have access to geometry toolkits, but instead should apply what they know about vertical angles, straight angles, and rigid transformations to complete the activity.

## 2 Monitor

**Help students get started** by asking them to identify a pair of vertical angles in the diagram.

**Look for points of confusion:**

- **Only listing $\angle AOB$ and $\angle A'OB'$ for Problem 3.** Ask students to determine the remaining angles in the diagram, and then revisit this problem.
- **Not being sure how to determine the measure of $\angle BOC$.** Ask students to look for angles that form a straight line. Remind them that a straight angle measures 180°.

**Look for productive strategies:**

- Noticing that $m\angle AOA'$ is 180° and using this to determine $m\angle BOC$.
- Noticing that the sum of all the angle measures is 360° and connecting this to the measure of a full circle.

## 3 Connect

**Have** pairs of students share their strategies for determining side lengths and angle measures that are the same.

**Ask**, "What strategies did you use to determine $m\angle BOC$?" Sample response: I noticed that $\angle AOA'$ is a straight angle so its measure is 180°. I subtracted 70° and 40° from 180°, which means $m\angle BOC$ is 70°.

**Highlight** that when you know there are intersecting lines and you know at least one angle measure, you can determine the measures of other angles by looking for vertical angles and straight angles.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge, Guide Visualization and Processing, Optimize Access to Tools*

If students need more processing time, have them focus on completing Problems 1 and 2 and only work on Problem 3 as time allows. Consider providing colored pencils for students to use to color code the vertical angle pairs.

### Extension: *Math Enrichment*

Provide a similar diagram, but change the given angle measures to 65° for $\angle AOB$ and 35° for $\angle AOC$. Have students use what they know about the measures of straight angles and vertical angles to determine all of the angle measures in the diagram.

## Math Language Development

### *MLR8: Discussion Supports*

As pairs of students share their strategies for determining side lengths and angle measures, highlight the chain of reasoning involved in determining the missing measures. For example, ask "Building on what you know about intersecting lines and straight angles, how can you determine the missing measures?"

### English Learners

Provide sentence frames for students to use to explain how they determined the angle measures, such as:

- I know that ___ and ___ have the same measure because they are vertical angles.

Whole Class | 5 min

# Summary

Review and synthesize how vertical angles can be proven congruent by reasoning about rigid transformations.

### Summary

**In today's lesson . . .**

You rotated intersecting lines $180^\circ$ about their point of intersection. Because a rotation is a rigid transformation that preserves angle measures, the vertical angles are congruent.

Reflect:





## Synthesize

**Ask**, "How does a $180^\circ$ rotation affect the angle measures for a pair of intersecting lines?"

**Have students share** what they noticed when intersecting lines are rotated $180^\circ$.

**Highlight** that a rotation of two intersecting lines about the point of intersection rotates each angle to an angle that is vertical to its preimage. Since rotation is a rigid transformation that preserves angle measures, the vertical angles must have the same measure.

## Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

- "How did you apply your understanding of rotations to help you determine an angle measure?"

Independent | 5 min

# Exit Ticket

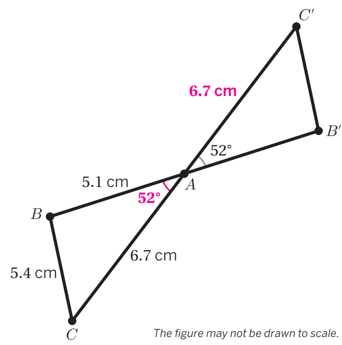
Students demonstrate their understanding by identifying congruent sides and angles when a figure is rotated about 180° about a point.

Printable

Name: ______________________ Date: __________ Period: __________

## Exit Ticket 1.14

**Triangle $ABC$ is rotated 180° about point $A$ to create Triangle $AB'C'$.**




*The figure may not be drawn to scale.*

1. What is the length of segment $AC'$? Explain your thinking.
   **6.7 cm; Because a rotation is a rigid transformation, I know the side lengths and angle measures of the preimage and image are congruent.**

2. Name a pair of vertical angles. What are their angle measures?
   **$\angle BAC$ and $\angle B'AC'$; They both measure 52°.**

3. Name two different angles that are also congruent.
   **Sample responses: $\angle ABC$ and $\angle AB'C'$; $\angle ACB$ and $\angle AC'B'$**

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. If I am given a pair of vertical angles and know the angle measure of one of them, I can calculate the angle measure of the other.
1 2 3

 **Lesson 14** Rotation Patterns

## Success looks like . . .

- **Goal:** Comprehending that congruent, vertical angles are formed when an angle is rotated 180° about the intersection point of two intersecting lines.
  » Explaining that side lengths and angle measures are preserved under a rotation in Problems 1, 2, and 3.
- **Language Goal:** Generalizing that vertical angles are congruent using informal arguments about 180° rotations of lines, line segments, or angles. **(Speaking and Listening)**
  » Explaining why the vertical angles are congruent in Problem 2.

## Suggested next steps

**If students do not know that the length of segment $AC'$ is the same as the length of segment $AC$ in Problem 1, consider:**

- Providing tracing paper to show the 180° rotation.
- Asking them to identify the corresponding line segments in the image.
- Reviewing the fact that rigid transformations preserve side lengths.

**If students do not identify a pair of vertical angles and their measures in Problem 2, consider:**

- Drawing different diagrams of intersecting lines and having students identify the vertical angles.
- Reviewing the fact that rigid transformations preserve angle measures.
- Reviewing Activity 1.

**If students do not identify congruent angles in Problem 3, consider:**

- Reviewing the fact that rigid transformations preserve angle measures.
- Reassessing after Lesson 15.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- The instructional goal for this lesson is to comprehend that congruent, vertical angles are formed when an angle is rotated 180° about the intersection point of two intersecting lines. How well did your students comprehend this concept? What did you specifically do to help students comprehend it?
- Thinking about the questions you asked students today and what they said or did as a result of your questions, which question was the most effective? The least effective? How might you alter your questioning techniques moving forward?

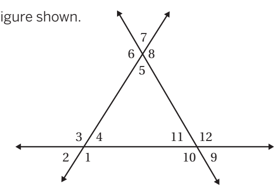
# Practice

Independent

Name: ______________ Date: ________ Period: ________

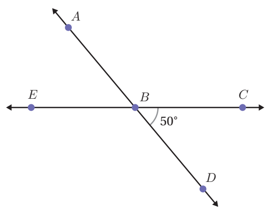
Practice

1. List *all* the pairs of congruent angles in the figure shown.




$\angle 1 \cong \angle 3$
$\angle 2 \cong \angle 4$
$\angle 5 \cong \angle 7$
$\angle 6 \cong \angle 8$
$\angle 9 \cong \angle 11$
$\angle 10 \cong \angle 12$

2. Use the figure to calculate the measure of each angle. Explain your thinking.




*The figure may not be drawn to scale.*

a. $m\angle ABC = 130°$
$\angle ABC$ and $\angle CBD$ are supplementary angles.

b. $m\angle EBD = 130°$
$\angle EBD$ and $\angle CBD$ are supplementary angles.

c. $m\angle ABE = 50°$
$\angle ABE$ and $\angle CBD$ are vertical angles.

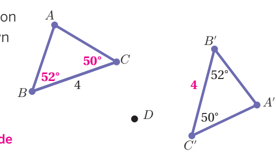




Name: ______________ Date: ________ Period: ________

Practice

3. $\triangle A'B'C'$ is an image of $\triangle ABC$ after a rotation about point $D$. The figures may not be drawn to scale.




a. What is the length of side $B'C'$? Explain your thinking.
Sample response: A rotation is a rigid transformation, so the corresponding side lengths are equivalent. The length of side $B'C'$ is 4 units because it is congruent to side $BC$.

b. What is $m\angle ABC$? Explain your thinking.
Sample response: A rotation is a rigid transformation, so the corresponding angles have equal measures. $m\angle ABC = 52°$ because $\angle ABC$ is congruent to $\angle A'B'C'$.

c. What is $m\angle ACB$? Explain your thinking.
Sample response: A rotation is a rigid transformation, so the corresponding angles have equal measures. $m\angle ACB = 50°$ because $\angle ACB$ is congruent to $\angle A'C'B'$.

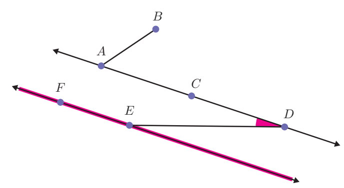
4. The point $(-4, 1)$ is rotated 180° counterclockwise about the origin. What are the coordinates of the image?

A. $(-1, -4)$ B. $(-1, 4)$ C. $(4, 1)$ **D.** $(4, -1)$

5. Refer to the figure shown.

a. Highlight or shade line $FE$.

b. Highlight or shade $\angle CDE$.








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 1 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 1 | 2 |
| Spiral | 4 | Unit 1 Lesson 7 | 2 |
| Formative | 5 | Unit 1 Lesson 15 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 1 | **LESSON 15**

# Alternate Interior Angles

Let's explore why some angles are always congruent.


## Focus

### Goals

1. Determine angle measures using alternate interior, adjacent, vertical, and supplementary angle relationships to solve problems.
2. **Language Goal:** Justify that alternate interior angles formed by a transversal connecting two parallel lines are congruent using properties of rigid motions. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students look for and make use of structure by exploring the relationship between angles formed when two parallel lines are intersected by a transversal. Students discover that alternate interior angles are congruent using rigid transformations and angle relationships.

### Previously

In Lesson 14, students applied their understanding of rigid transformations when they rotated intersecting lines $180°$ in order to establish the fact that vertical angles are congruent.

### Coming Soon

In Lesson 16, students will justify that the sum of the interior angle measures of a triangle is $180°$ using rigid transformations.

## Rigor

- Students build **conceptual understanding** about angle relationships formed when parallel lines are intersected by a transversal.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Activity 3 | Summary | Exit Ticket |
|---|---|---|---|---|---|
| 5 min | 8 min | 12 min | 8 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice
Independent

### Materials

- Exit Ticket
- Additional Practice
- protractors

### Math Language Development

**New words**

- *alternate interior angle*
- *transversal*

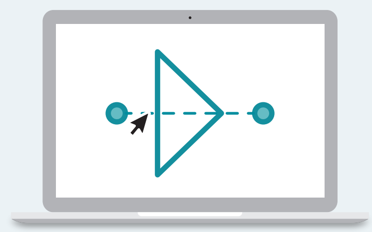
**Review words**

- *rotation*
- *supplementary angles*
- *translation*
- *vertical angles*

**Amps** | Featured Activity

**Activity 2**
**Angle Countdown**

In real time, students are informed how many more angles they should measure.



Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

At first, students may not see a clear path to finding the requested angle measurements and might want to quit before really getting started. Encourage students to set a goal of initially analyzing the structure of each figure to mark what they do know about the angle relationships or measures given. Students can repeat until they have solved the problem. By looking only one step ahead, a task can seem much more manageable.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** can be omitted.
- In **Activity 2**, have students complete Problem 3.
- **Activity 3** may be omitted as students extend their understanding to study a diagram in which there are two transversals intersecting a pair of parallel lines.

Independent | 5 min

## **Warm-up** Notice and Wonder

Students study intersecting lines to prepare them for learning about angle relationships that are formed when parallel lines are intersected by a transversal in upcoming activities.

Name: ______________________ Date: __________ Period: __________

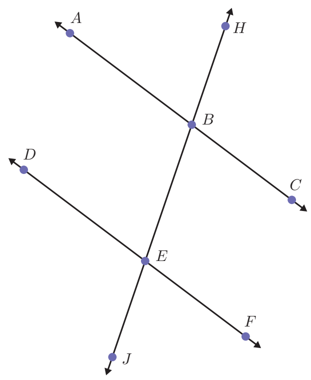
**Unit 1 | Lesson 15**

# Alternate Interior Angles

Let's explore why some angles are always congruent.

### **Warm-up** Notice and Wonder

**Refer to the diagram. What do you notice? What do you wonder?**




1. I notice . . .

   **Sample responses:**
   - **I notice that there is a pair of parallel lines.**
   - **The parallel lines are intersected by a third line.**
   - **Eight angles are formed by the intersection of the three lines.**
   - **Some of the angle measures appear to be the same. Others are supplementary.**

2. I wonder . . .

   **Sample responses:**
   - **If the third line was rotated, but still intersected the two parallel lines, would eight angles still be formed? Would some of the angle measures still be congruent or supplementary?**
   - **Does this relationship always happen when two parallel lines are intersected by a third line?**
   - **Would these relationships change if the two lines were not parallel?**

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Conduct the ***Notice and Wonder*** routine using the figure.

### 2 Monitor

**Help students get started** by asking them what makes this figure similar to and different from the other figures they have seen in this unit.

**Look for productive strategies:**

- Noticing a line that intersects lines $AC$ and $DF$.
- Noticing that eight angles are formed and that some of those angles appear to be the same size.
- Wondering if lines $AC$ and $DF$ are parallel.

### 3 Connect

**Have students share** what they noticed and wondered. Record and display their responses for all to see.

**Define** a ***transversal*** (or transversal line) is a line that intersects two or more lines.

**Highlight** that a transversal can pass through two or more lines that may or may not be parallel.

**Ask**, "Does anyone notice anything interesting about the angles in the figure?" **Note:** Students will investigate these angles in the next activity.

## MLR Math Language Development

***MLR8: Discussion Supports***

As you define the term *transversal* during the Connect discussion, emphasize that a transversal is a line that intersects two or more lines. The two lines do *not* have to be parallel. Consider drawing and labeling some examples, such as the following:

- 2 parallel lines intersected by 1 transversal.
- 2 parallel lines intersected by 2 different transversals.
- 2 intersecting lines (but not parallel) intersected by 1 transversal.
- 2 intersecting lines (but not parallel) intersected by 2 different transversals.

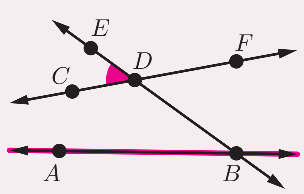
## Power-up

**To power up students' ability to identifying lines and angles in diagrams, have students complete:**

Recall that one way that lines can be named is by identifying two points on the line, and angles can be named using three, where the vertex is the middle point.

**1.** Highlight or shade the line $AB$.

**2.** Highlight or shade $\angle CDE$.




**Use:** Before Activity 1

**Informed by:** Performance on Lesson 14, Practice Problem 5

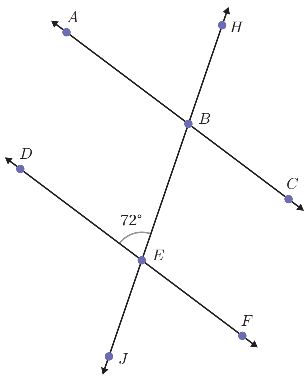
Pairs | 8 min

# Activity 1 Alternate Interior Angles

Students explore the relationships between angles formed when two parallel lines are intersected by a transversal line to learn that alternate interior angles are congruent.

**Activity 1** Alternate Interior Angles

**You will be given a protractor. Refer to the diagram. Lines $AC$ and $DF$ are parallel. They are intersected by transversal $JH$.**




1. Use your protractor to measure the seven missing angle measures.
   **$m\angle JEF$, $m\angle ABH$, and $m\angle EBC$ all measure 72°.**
   **$m\angle DEJ$, $m\angle BEF$, $m\angle ABE$, and $m\angle HBC$ all measure 108°.**

2. What do you notice when a transversal intersects a pair of parallel lines?
   **Sample responses:**
   - **Eight angles are formed.**
   - **Two sets of congruent angles are formed with four congruent angles in each set.**
   - **The congruent angles seem to be in the same position along the parallel lines related to the transversal.**
   - **Some of the angle pairs are supplementary.**




## 1 Launch

Have students complete Problem 1 individually. Then have them share their responses with a partner before completing Problem 2. Provide access to geometry toolkits for this activity only.

## 2 Monitor

**Help students get started** by having them label the measure of $\angle FEJ$.

**Look for points of confusion:**

- **Not knowing how to determine missing angle measures in the figure.** Encourage students to look for supplementary angles. Allow students to check their work using a protractor.

**Look for productive strategies:**

- Noticing that there are only two different angle measurements.
- Remembering that a 180° rotation about the midpoint of segment $EB$ produces congruent angles.

## 3 Connect

**Have pairs of students share** what they noticed about the angle measures. Begin with students who used a protractor to measure the angles, and then have students share who used angle relationships and transformations.

**Define *alternate interior angles*.** Say, "Alternate interior angles are created when a pair of parallel lines are intersected by a transversal. These angles lie inside the parallel lines and on *opposite (alternate)* sides of the transversal."

**Highlight** that alternate interior angles are congruent.

**Ask**, "How can you show that alternate interior angles are congruent using rigid transformations?" A 180° rotation about the midpoint of segment $EB$ produces congruent angles.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Provide the angle measure for $\angle CBE$ as 108° to assist students as they begin the activity. Provide colored pencils or highlighters for students to use to highlight the angles $\angle DEB$, $\angle JEF$, $\angle ABH$, and $\angle EBC$ to emphasize the congruent measures.

### Extension: *Math Enrichment*

Have students draw a pair of nonparallel lines intersected by a transversal and use a protractor to measure the angles to see if the same type of angle relationships are formed.

## Math Language Development

### *MLR8: Discussion Supports*

Use this routine to amplify students' mathematical uses of language when describing and demonstrating transformations used for showing that alternate interior angles are congruent.

### English Learners

Allow pairs of students to rehearse together before sharing with the whole class.

Pairs | 12 min

# Activity 2 Three, Five, Seven

Students determine angle measures using angle relationships to understand how angles are related when two parallel lines are intersected by a transversal.

**Amps Featured Activity** Angle Countdown

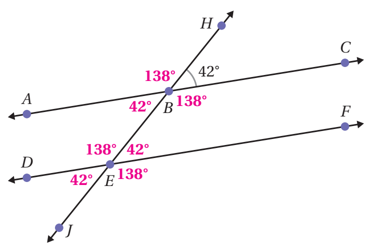
Name: ____________ Date: ________ Period: ________

## Activity 2 Three, Five, Seven

**In each diagram, line $AC$ is parallel to line $DF$. The lines are intersected by transversal $HJ$. The figures may not be drawn to scale.**

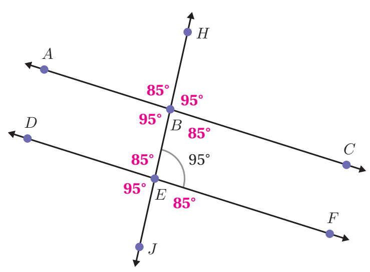
1. Determine *any three* angle measures that are not currently labeled.

   The diagram shows all seven missing angle measures. Student responses should indicate three of these angle measures.

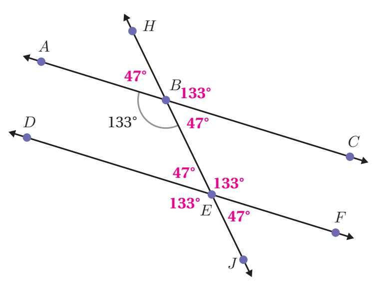



2. Determine *any five* angle measures that are not currently labeled.

   The diagram shows all seven missing angle measures. Student responses should indicate five of these angle measures.




3. Determine *all* seven angle measures that are not currently labeled.








## 1 Launch

Collect geometry toolkits. Set an expectation for the amount of time students will have to work in pairs on the activity.

## 2 Monitor

**Help students get started** by asking them to identify any pair of congruent angles.

**Look for points of confusion:**

- **Not knowing how to determine a missing angle measure.** Remind students that alternate interior angles are congruent and supplementary angles have a sum of $180°$. Ask them what they can do with this information.
- **Labeling congruent angle measurements because the angles "look like the same size".** Ask students how they can be certain that the angles do not differ in measure by one degree. Encourage them to use rigid transformations and angle relationships to determine the angle measures.

**Look for productive strategies:**

- Using alternate interior, vertical, and supplementary angles to determine the measure of missing angles.

## 3 Connect

**Display** student work showing the angle measures they determined.

**Have students share** their strategies for determining the missing angle measures.

**Ask**, "What were some angle relationships you used to find missing measures?" vertical angles, supplementary angles, alternate interior angles

**Highlight** that although each figure has seven angles, there are only two different angle measures. This happens when a transversal intersects a pair of parallel lines.

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization***

If students need more processing time, have them focus on Problem 3, and only work on Problems 1 and 2 as time allows. Consider highlighting the three angles congruent to the given angle to assist students in processing the visual information.

**Extension: *Math Enrichment***

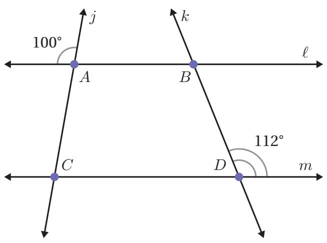
Provide a diagram similar to the one in Problem 3, but label the given angle measure $x$. Have students write expressions that represent the angle measures of the remaining seven angles. Students should write the expressions $x$ and $180 - x$ in the correct locations.

# Activity 3 Double Transversals

Students apply what they know about angle relationships and extend it to analyze a diagram in which two transversals intersect a pair of parallel lines.

## Activity 3 Double Transversals

**Refer to the diagram. Lines $\ell$ and $m$ are parallel. The lines are intersected by two transversals, $j$ and $k$.**



*The figure may not be drawn to scale.*

What is the sum of $m\angle CAB$, $m\angle ABD$, $m\angle BDC$, and $m\angle DCA$? Show or explain your thinking.

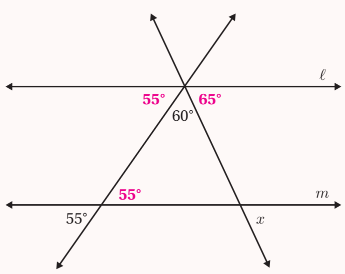
$m\angle CAB + m\angle ABD + m\angle BDC + m\angle DCA = 360°$.

Sample response:

- I know $m\angle CAB = 100°$ because it forms a vertical angle pair with the angle labeled $100°$.
- I know $m\angle ABD = 112°$ because $\angle ABD$ and the angle labeled $112°$ are alternate interior angles.
- I know $m\angle BDC = 68°$ because it is supplementary to the angle labeled $112°$.
- I know $m\angle DCA = 80°$ because $\angle DCA$ is congruent to its alternate interior angle, which is supplementary to the angle labeled $100°$.

**Are you ready for more?**

Refer to the diagram. Parallel lines $\ell$ and $m$ are intersected by two transversals that cross line $l$ at the same point. Determine the measure of angle $x$.



*The figure may not be drawn to scale.*

$x = 65°$

STOP

108 **Unit 1** Rigid Transformations and Congruence



## 1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

## 2 Monitor

Help students get started by asking them which angles have a measure of $100°$.

**Look for points of confusion:**

- **Thinking that $m\angle DCA = 112°$.** Remind students that only lines $\ell$ and $m$ are parallel.
- **Struggling to understand the more complex diagram.** Show one parallel line and transversal at a time. Use a sheet of paper to cover line $j$. Have students record the angle measures, and then repeat the process with line $k$.

**Look for productive strategies:**

- Using strategies they have learned, such as supplementary angle pairs and alternate interior angles to determine the 14 missing angle measures.

## 3 Connect

**Display** student work showing the angle measures they determined.

**Have students share** their strategies for how they determined the angle measures.

**Ask**, "Without measuring, why is $m\angle DCA$ not equal to $112°$?" If $m\angle DCA = 112°$, then lines $j$ and $k$ must be parallel.

**Highlight** that if a pair of parallel lines is intersected by two transversals, students can calculate the missing angles by studying one transversal at a time.

## Differentiated Support

**Accessibility: *Guide Processing and Visualization***

Provide the angle measure for $\angle DCA$ as $80°$ to assist students as they begin the activity.

**Extension: *Math Enrichment***

Have students make a conjecture about the sum of the angle measures in any trapezoid.

## Math Language Development

***MLR2: Collect and Display***

As students discuss with a partner, listen for and collect vocabulary, phrases, and gestures they use to describe the diagrams. Record these onto the class visual display and update it throughout the lesson. Remind students to borrow language from the display as needed in future discussions.

Whole Class | 5 min

# Summary

Review and synthesize the relationship between angles formed when two parallel lines are intersected by a transversal.

Name: ______ Date: ______ Period: ______

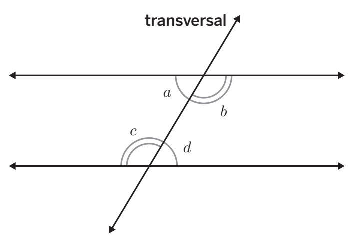
## Summary

**In today's lesson . . .**

You explored the relationship between angles formed when two parallel lines are intersected by a transversal.

A ***transversal*** is a line that intersects two or more lines. ***Alternate interior angles*** are formed when a pair of parallel lines are intersected by a transversal. Interior angles are located inside the parallel lines and alternate angles are located on opposite sides of the transversal. So, alternate interior angles are located both inside the parallel lines and on opposite sides of the transversal. Alternate interior angles are congruent.

The diagram shows two pairs of alternate interior angles. Angles $a$ and $d$ are one pair of alternate interior angles, and are therefore congruent. Angles $b$ and $c$ are another pair of alternate interior angles, and are therefore congruent.




Reflect:





## Synthesize

**Display** the Summary from the Student Edition.

**Have students share** the strategies they used in this lesson to determine angle measures when two parallel lines are intersected by a transversal.

**Highlight** that vertical angles are always congruent and when parallel lines are intersected by a transversal, alternate interior angles are congruent.

**Formalize vocabulary:**

- *alternate interior angle*
- *transversal*

**Ask**, "How many angle measures can you determine if you are given a pair of parallel lines, a transversal intersecting those lines, and one angle measure?" All the angle measures can be found using angle relationships.

**Note:** You may choose to introduce the term *corresponding angles*. Use the figure from the Warm-up to highlight corresponding angles, such as $\angle HBC$ and $\angle BEF$.

## Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful today when determining an angle measure? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

## Math Language Development

### *MLR2: Collect and Display*

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *congruent* that were added to the display during the lesson.

Independent | 5 min

# Exit Ticket

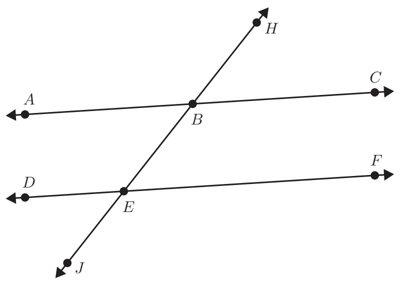
Students demonstrate their understanding by identifying congruent angles formed when two parallel lines are intersected by a transversal.

Printable

Name: ______________ Date: ________ Period: ________

## Exit Ticket 1.15

Lines $AC$ and $DF$ are parallel and are intersected by transversal $HJ$.




1. List *all* the angles that are congruent to $\angle ABH$.
   $\angle EBC, \angle DEB, \angle JEF$

2. List *all* the angles that are congruent to $\angle ABE$.
   $\angle HBC, \angle BEF, \angle DEJ$

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a) If I am given two parallel lines intersected by a transversal, I can identify alternate interior angles and use that information to calculate missing angle measurements.

1 2 3





## Success looks like . . .

- **Goal:** Determining angle measures using alternate interior, adjacent, vertical, and supplementary angle relationships to solve problems.
  - » Determining angles congruent to a given angle using these relationships in Problems 1 and 2.
- **Language Goal:** Justifying that alternate interior angles formed by a transversal connecting two parallel lines are congruent using properties of rigid motions. **(Speaking and Listening, Writing)**

## Suggested next steps

**If students do not list all the congruent angles, or list incorrect angles, for Problems 1 and 2, consider:**

- Providing one angle measure to students and then reassessing by having them determine the remaining angle measures.
- Reviewing Activity 1.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- During the discussion for Activity 3, how did you encourage your students to listen to one another's strategies?
- How did students self-manage today? How are you helping them become aware of their progress with the mathematical concepts in this unit?

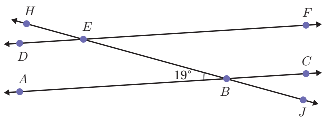
# Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. Line $AC$ is parallel to line $DF$. The lines are intersected by transversal $HJ$.




*The figure may not be drawn to scale.*

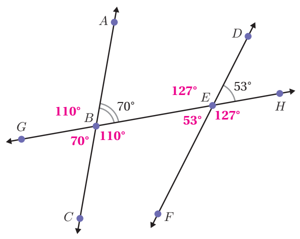
a. What is $m\angle FEB$? Explain your thinking.

**Sample response: $m\angle FEB = 19°$ because $\angle ABE$ and $\angle FEB$ are alternate interior angles.**

b. Explain why $m\angle DEH$ has a measure of $19°$.

**Sample response: $\angle DEH$ and $\angle FEB$ are vertical angles, and $\angle FEB$ and $\angle EBA$ are alternate interior angles. This means that they are all congruent angles and each angle has a measure of $19°$.**

2. The diagram shows three lines with two given angle measures. Determine the measure of the six missing angles. Note that lines $AC$ and $DF$ are not parallel.



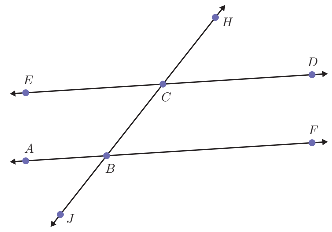

*The figure may not be drawn to scale.*




Name: ______ Date: ______ Period: ______

Practice

3. Line $ED$ is parallel to $AF$. Lin claims that if she knows the measure of one angle, she is able to determine the measure of the remaining seven angles. Do you agree or disagree with Lin? Explain your thinking.

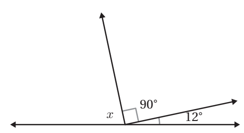



**Sample response: I agree with Lin because the measure of the known angle and its vertical angle are the same. The known angle and its adjacent angle are supplementary. Because the lines are parallel, the corresponding angles on each line are congruent, which can be proved by using rigid transformations.**

4. The point $(1, 3)$ is reflected across the $x$-axis, and then again across the $y$-axis. What are the coordinates of points of the final image?

**$(-1, -3)$**

5. Determine the measure of the missing angle.




*The figure may not be drawn to scale.*

**$180 - 90 - 12 = 78$**
**$x = 78°$**




## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 1 Lesson 6 | 1 |
| Formative | 5 | Unit 1 Lesson 16 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Adding the Angles in a Triangle

Let's explore the interior angles of triangles.

## Focus

### Goals

1. **Language Goal:** Comprehend that a straight angle can be decomposed into three angles to construct a triangle. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Justify that the sum of the interior angle measures of a triangle is $180°$. **(Speaking and Listening, Writing)**

## Rigor

- Students begin to build **conceptual understanding** that the sum of the angle measures in any triangle is $180°$.

## Coherence

### Today

Students examine the relationships among the interior angles of a triangle. They explore different triangle angle sums and observe that if a straight angle is decomposed into three angles, it appears that the three angles can be used to create a triangle.

### Previously

In Lesson 15, students explored the relationship angles formed when two parallel lines are cut by a transversal and found that alternate interior angles are congruent.

### Coming Soon

In Lesson 17, students will continue to explore the interior angles of a triangle and conclude that *any* triangle has an interior angle sum of $180°$.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 10 min | 20 min | 5 min | 5 min |
| Independent | Pairs | Small Groups | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per group
- Activity 2 PDF, *Making a Triangle* (for display)
- geometry toolkits: protractors, rulers
- scissors

### Math Language Development

**Review words**

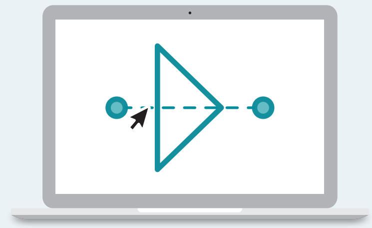
- *interior angle*
- *transversal*

**Amps** | Featured Activity

**Activity 1**

### Interactive Geometry

Students drag points to create different triangles, seeing how the angles and their measures change, but how the sum of the angle measures remains $180°$.



Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students who are more confident with this mathematical concept may be able to lead discussions within their groups. Have these students help their peers in using the structure of a straight angle to help discover the sum of the measures of the interior angles in a triangle. Remind them to add to the conversation in ways that are helpful, but to also "step back" to give other voices a chance to share.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- **Activity 1** may be shortened by having students complete Problems 1 and 2.
- **Activity 2** may be shortened by having students complete Problems 1 and 3.

# Warm-up Three Triangles

Students visually inspect three triangles to predict the triangle with the greatest interior angle sum and test their predictions in the next activity.

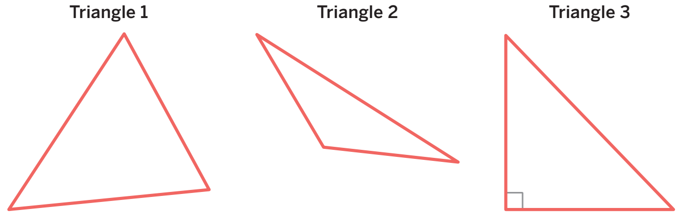
Unit 1 | Lesson 16

## Adding the Angles in a Triangle

Let's explore the interior angles of triangles.

### Warm-up Three Triangles

Refer to the three triangles shown.




Which triangle do you think has the greatest sum of the three interior angle measures? Explain your thinking.

Sample response: I think Triangle 1 has the greatest interior angle sum because it looks like the largest triangle.
While the mathematically correct answer is that every triangle has the same interior angle sum of 180°, accept all student responses at this point, provided they justify their thinking.
The interior angle sum will be discovered in this lesson.

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Set an expectation for the amount of time students will have to work independently on the activity.

## 2 Monitor

**Help students get started** by asking them to estimate the angle measures for each triangle and reminding them that *interior angles* are the angles located *inside* the triangle.

**Look for points of confusion:**

- **Thinking they have to measure the angles.** Do not provide access to protractors, as students will be given the angle measures in Activity 1. Ask students to do a visual inspection and make a prediction.

**Look for productive strategies:**

- Noticing the right angle symbol represents an angle measure of 90°.
- Knowing that the sum of the interior angle measures for any triangle is 180°. Ask students to pause on sharing this until it is revealed in Activity 1, as other students are still exploring.

## 3 Connect

**Have students share** which triangle they thought had the greatest interior angle sum by using the *Poll the Class* routine. Record the number of students who chose each triangle.

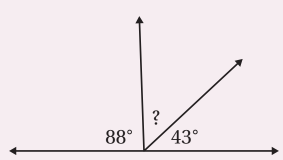
**Ask**, "Do you think the side lengths of the triangle affect the sum of its interior angles?"

## Power-up

**To power up students' ability to determine the measure of an unknown angle in a straight-angle diagram, have students complete:**

Recall that when two or more angles form a straight line, their sum is 180°.

1. Which expressions can be used to determine the measure of the unknown angle? Select *all* that apply.
   A. $180 - (88 + 43)$
   B. $88 + 43$
   C. $90 - 43$
   D. $180 - 88 - 43$




2. What is the measure of the unknown angle? 49°

**Use:** Before Activity 2
**Informed by:** Performance on Lesson 15, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 5

Pairs | 10 min

# Activity 1 Find All Three

Students determine the sum of the interior angles of different triangles to generalize that the sum of the interior angles in any triangle is 180°.

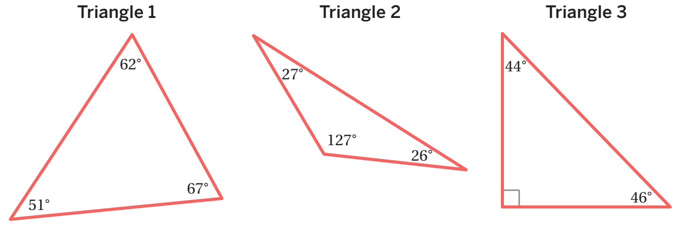
**Amps Featured Activity** Interactive Geometry

Name: ______ Date: ______ Period: ______

## Activity 1 Find All Three

**Refer to the same triangles from the Warm-up, now with their interior angle measures labeled. The figures may not be drawn to scale.**

Triangle 1 Triangle 2 Triangle 3




1. Determine the sum of the interior angle measures for each triangle.
   a. Triangle 1: 180°
   b. Triangle 2: 180°
   c. Triangle 3: 180°

2. What do you notice about the sum of the interior angle measures?
   The three triangles all have an interior angle sum of 180°.

3. Draw a different triangle. Make a prediction for the sum of the interior angle measures.
   Triangles may vary, but students should begin to discover and predict the sum of the interior angles of any triangle will be 180°.

4. Measure the interior angles. Was your prediction correct?
   Sample response: Yes, my prediction was correct.



Lesson 16 Adding the Angles in a Triangle 113

## 1 Launch

Activate students' prior knowledge by asking them how they can determine the greatest interior sum of a triangle. Have students work in pairs to complete Problems 1 and 2. If time allows, have students complete Problems 3 and 4 individually. Then have them compare their work with a partner. Provide access to geometry toolkits for Problems 3 and 4.

## 2 Monitor

**Help students get started** by asking them how they can find the sum of the interior angles of the triangles given.

**Look for points of confusion:**

- **Thinking they do not have enough information to determine the sum of the angle measures for Triangle 3.** Ask students what the square in the lower left corner represents. a 90° angle

**Look for productive strategies:**

- Noticing the sum of the interior angle measures in each of the three triangles is 180°.

## 3 Connect

**Have pairs of students share** what they noticed about the sum of the interior angle measures for each of the three triangles in Problem 1. Have students share how the triangle they drew in Problem 1 compares to the triangles from Problem 1.

**Ask** students if the side lengths affect the sum of the interior angles. If they think that a triangle with longer sides will have an interior angle sum greater than 180°, ask them to draw a triangle with longer sides and have them use a protractor to measure the angles to check their thinking.

**Highlight** that the sum of the interior angles of each of these triangles, including the one they drew, is 180°.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Triangles 1 and 2, and only complete the activity for Triangle 3 as time allows. Alternatively, consider providing a different triangle already drawn and labeled for students to use in Problems 3 and 4 instead of having them draw the triangles and measure the angles.

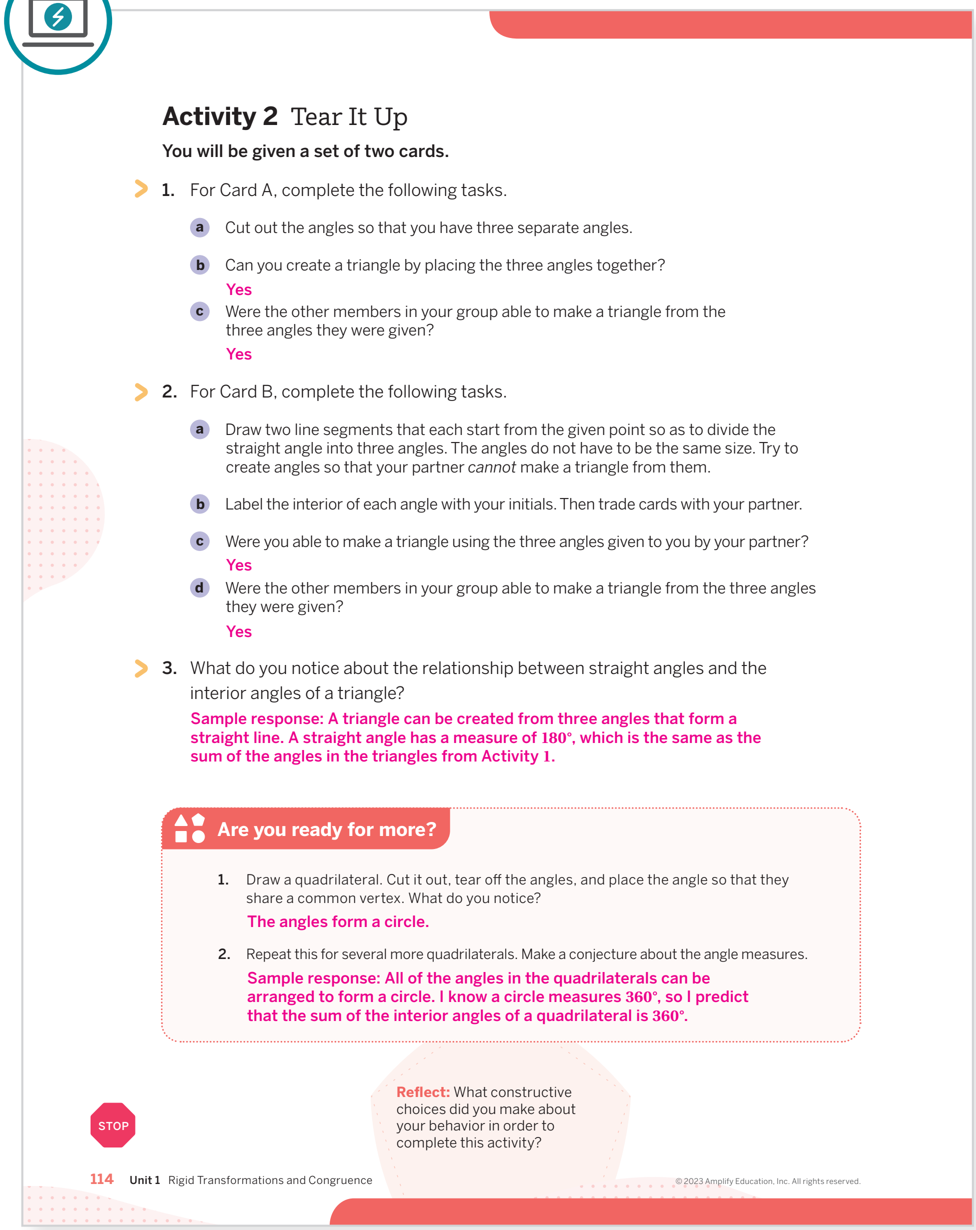
### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can drag points to create different triangles and see how the angle measures change, yet the sum of the angle measures remains the same.

Small Groups | 20 min

# Activity 2 Tear It Up

Students experiment with angles to discover that three angles with measures that have a sum of $180°$ can be used to form a triangle.



## Activity 2 Tear It Up

**You will be given a set of two cards.**

1. For Card A, complete the following tasks.
   a. Cut out the angles so that you have three separate angles.
   b. Can you create a triangle by placing the three angles together?
   **Yes**
   c. Were the other members in your group able to make a triangle from the three angles they were given?
   **Yes**
2. For Card B, complete the following tasks.
   a. Draw two line segments that each start from the given point so as to divide the straight angle into three angles. The angles do not have to be the same size. Try to create angles so that your partner *cannot* make a triangle from them.
   b. Label the interior of each angle with your initials. Then trade cards with your partner.
   c. Were you able to make a triangle using the three angles given to you by your partner?
   **Yes**
   d. Were the other members in your group able to make a triangle from the three angles they were given?
   **Yes**
3. What do you notice about the relationship between straight angles and the interior angles of a triangle?
   **Sample response: A triangle can be created from three angles that form a straight line. A straight angle has a measure of $180°$, which is the same as the sum of the angles in the triangles from Activity 1.**

**Are you ready for more?**

1. Draw a quadrilateral. Cut it out, tear off the angles, and place the angle so that they share a common vertex. What do you notice?
   **The angles form a circle.**
2. Repeat this for several more quadrilaterals. Make a conjecture about the angle measures.
   **Sample response: All of the angles in the quadrilaterals can be arranged to form a circle. I know a circle measures $360°$, so I predict that the sum of the interior angles of a quadrilateral is $360°$.**

STOP

**Reflect:** What constructive choices did you make about your behavior in order to complete this activity?

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## 1 Launch

Provide each group with a set of cards from the Activity 2 PDF. Tell students they will start with Card A. Provide access to rulers and scissors.

## 2 Monitor

**Help students get started** by displaying the Activity 2 PDF, *Making a Triangle*. For Problem 1, show students how a triangle is formed using the three angles. Tell students that they may draw extra lines to join the angles and form a triangle. This is indicated by the dotted line. For Problem 2, show students how to create three angles by drawing two line segments from the given point.

**Look for points of confusion:**

- **Thinking that the three angles cannot be rearranged to form a triangle.** Encourage students to rotate the angles. If students still do not find a way to form a triangle, have them work with a peer helper.

**Look for productive strategies:**

- Noticing each set of three angles can be rearranged to form a triangle.
- Noticing that a straight angle has a measure of $180°$ and connecting this to what they discovered about the interior angles of a triangle from Activity 1.

## 3 Connect

**Display** the different triangles created from the sets of angles. If time allows, conduct the ***Gallery Tour*** routine so students can compare the different angles and triangles they formed from Card B.

**Have groups of students share** what they notice about straight angles and the interior angles of a triangle.

**Highlight** that if a straight angle is decomposed into three angles, it appears that the three angles can be used to form a triangle.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Cut out the angles for Card A for students or provide possible divisions of the straight angle for Card B, so that they can focus on creating triangles.

### Extension: *Math Enrichment*

Building on the Extension from Lesson 13, ask students if the sum of the angles of any triangle in spherical geometry is also $180°$. Using the sphere you used earlier, draw lines to form a triangle in which each angle measures $90°$. Ask students what they notice. Draw other triangles to illustrate the following principle of spherical geometry: The sum of the angle measures of a triangle is always greater than $180°$.

## Math Language Development

### *MLR7: Compare and Connect*

As students prepare their work for discussion, look for those who successfully construct and create triangles. Encourage students to explain how they arranged the angles. Emphasize the language used to make sense of each set of angles and the language used to determine that the sum of the interior angle measures of a triangle is $180°$.

### English Learners

Provide sentence frames to support student conversation, such as:

- "To arrange the triangles, first I _____ because . . ."
- "I noticed that _____, so I . . ."

Whole Class | 5 min

# Summary

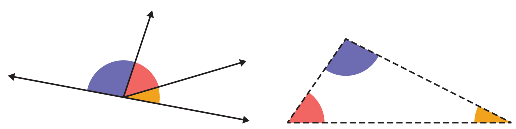
Review and synthesize the connection between the measure of a straight angle and the sum of the interior angle measures of a triangle.

Name: ______ Date: ______ Period: ______

## Summary

**In today's lesson . . .**

You investigated the interior angles of a triangle. You found that the sum of the angles inside the triangles you investigated in this lesson is $180°$. You may wonder if this relationship is true for *all* triangles and so, you will continue to explore this in the next lesson. You also found that any three angles that have a sum of $180°$ can be used to form a triangle.



**Reflect:**





## Synthesize

**Have students share** what they observed about the sum of the interior angle measures of a triangle and the measure of a straight line.

**Display** the Summary from the Student Edition.

**Highlight** that the sum of the interior angle measures in any triangle is $180°$. Students will prove why this is true in Lesson 17.

**Ask:**

- "When you know the measures of two angles inside a triangle, how can you find the measure of the third angle?" Subtract the sum of the two known angle measures from $180°$.
- "Are there three angle measures that *cannot* be used to form a triangle?" Yes, if the sum of the three angle measures is less than or greater than $180°$, a triangle cannot be formed.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Do the measures of the interior angles of a triangle really add up to $180°$?"

## Math Language Development

### *MLR2: Collect and Display*

Capture the language discussed during the Synthesize section using the class display. For example, "The sum of the interior angle measures in any triangle is 180°" should be added to the display and students should be encouraged to refer to this during future discussions.

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by selecting three angle measures that could be the interior angle measures of a triangle.

Printable

Name: Date: Period:

## Exit Ticket 1.16

Select *three* of the following measures that could be angles in the *same* triangle.

A. 40°
B. 180°
C. 35°
D. 90°
E. 20°
F. 120°

Explain your thinking.

Sample response: I know these could be the angles in a triangle because 40°, 20°, and 120°, add up to 180°.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I know that any three angles that have a sum of 180° can be used to form a triangle.

1 2 3





## Success looks like . . .

- **Language Goal:** Comprehending that a straight angle can be decomposed into three angles to construct a triangle. **(Speaking and Listening, Reading and Writing)**
  - » Selecting three angle measures that make up a straight angle.
- **Language Goal:** Justifying that the sum of the interior angle measures of a triangle is 180°. **(Speaking and Listening, Writing)**

## Suggested next steps

**If students do not select three measures that have a sum of 180°, consider:**

- Providing a calculator.
- Reviewing Activity 1.
- Reassessing after Lesson 17.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- How did students approach Activity 2? Did any of your students experience frustration when trying to form triangles from the sets of three angles? If so, what helped them work through their frustration?
- In this lesson, students found the sum of the three interior angle measures of triangles. How will this understanding support their learning of alternate interior angles in future lessons?

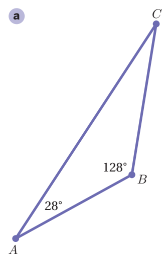
# Practice

Independent

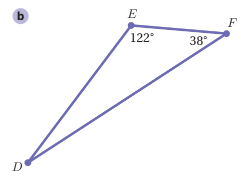
Name: ______ Date: ______ Period: ______

Practice

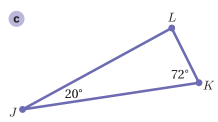
1. For each triangle, write a possible measure for the third angle.
The figures may not be drawn to scale.

a


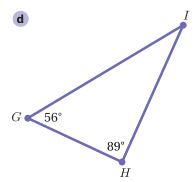
$180 - 28 - 128 = 24$,
$m\angle C = 24°$

b


$180 - 122 - 38 = 20$,
$m\angle D = 20°$

c


$180 - 20 - 72 = 88$,
$m\angle L = 88°$

d


$180 - 56 - 89 = 35$,
$m\angle I = 35°$

2. Which of the following sets of angles are possible for a triangle? Select *all* that apply.

A. 60°, 60°, 60°
B. 90°, 90°, 45°
C. 30°, 40°, 50°
D. 90°, 45°, 45°
E. 120°, 30°, 30°

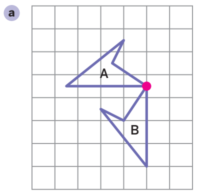
3. Can there be a triangle with two right angles? Explain your thinking.
No; Sample response: The sum of the three interior angles should be 180°. Because two right angles already have a sum of 180°, the measure of the third angle must be 0°, which is not possible for a triangle.



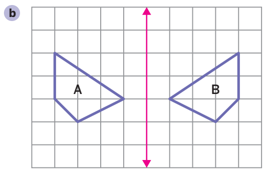

Name: ______ Date: ______ Period: ______

Practice

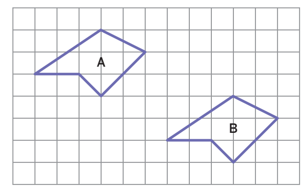
4. For each pair of polygons, describe the transformation that maps Polygon A onto Polygon B.

a


Sample response: Rotate Polygon A 90° counterclockwise about the bottom right vertex of Polygon A.

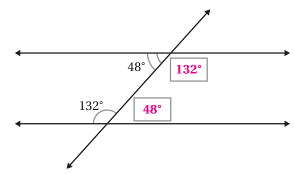
b


Sample response: Reflect Polygon A across a vertical line that lies halfway between the two polygons.

c


Sample response: Translate Polygon A 6 units right and 3 units down.

5. Refer to the figure, which shows two parallel lines intersected by a transversal. Determine the two missing angle measures indicated.



*The figure may not be drawn to scale.*




## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 1 |
| | 2 | Activity 1 | 1 |
| | 3 | Activity 1 | 2 |
| Spiral | 4 | Unit 1 Lesson 5 | 2 |
| Formative | 5 | Unit 1 Lesson 17 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Parallel Lines and the Angles in a Triangle

Let's investigate why the angles in a triangle add up to 180°.

## Focus

### Goal

1. **Language Goal:** Generalize the Triangle Sum Theorem using the congruence of alternate interior angles when parallel lines are cut by a transversal. **(Speaking and Listening, Reading and Writing)**

## Rigor

- Students develop **conceptual understanding** that the sum of the angle measures in any triangle is 180°.
- Students **apply** their understanding to solve angle puzzles.

## Coherence

### Today

Students continue exploring the interior angles of a triangle. Using their knowledge of angle relationships, students construct an argument for *why* the sum of the angle measures in *any* triangle is 180°, and then apply their understanding to solve challenging angle puzzles.

### Previously

In Lesson 16, students found that a straight angle can be decomposed into three angles to form a triangle, reasoning about the sum of the measures of the three angles.

### Coming Soon

In Lesson 18, students will apply what they have learned about rigid transformations to create unique border patterns.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice
Independent

### Materials

- Exit Ticket
- Additional Practice
- colored pencils
- geometry toolkits: protractors, rulers

### Math Language Development

**New words**

- ***exterior angle***
- ***Triangle Sum Theorem***

**Review words**

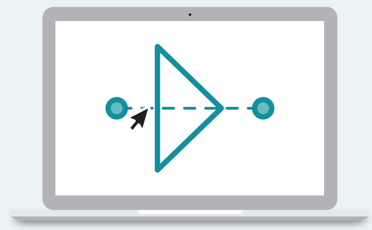
- *alternate interior angles*
- *straight angle*
- *tessellation*
- *transversal*

**Amps** | **Featured Activity**

**Activity 1**

### Interactive Geometry

Students drag points on a pair of parallel lines to create different triangles. Given the seven angle measures, students can look for patterns using different triangles.



Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students may resist thinking deeply about *why* the angles of any triangle has a sum of $180^\circ$. Ask them to resist accepting this rule without persevering in their attempts to make sense of why it is true. Ask them to engage in metacognitive functions, i.e., thinking about their own thinking process. For example, have them conduct their own ***Notice and Wonder*** routine for Activity 1, which will help them record their thought processes.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted. It serves to reinforce student understanding of congruent angles.
- **Activity 2** may be shortened by having students complete Puzzles 1 and 2.

Pairs | 5 min

## Warm-up Matching Angles

Students apply rigid transformations to identify congruent angles, preparing them for reasoning about angles formed when parallel lines are cut by a transversal.

Unit 1 | Lesson 17

### Parallel Lines and the Angles in a Triangle

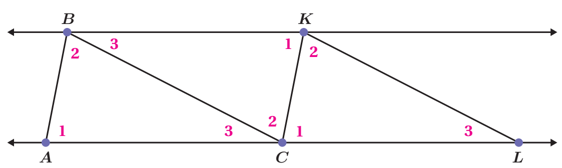
Let's investigate why the angles in a triangle add up to 180°.

**Warm-up** Matching Angles

**You will need colored pencils for this Warm-up.**

The figure shows two congruent triangles $ABC$ and $CKL$ placed between two parallel lines.

Shade *all* the congruent angles using different colors for each pair of congruent angles.




Angles labeled with the number 1 are congruent.
Angles labeled with the number 2 are congruent.
Angles labeled with the number 3 are congruent.

Log in to Amplify Math to complete this lesson online.



118 Unit 1 Rigid Transformations and Congruence

### 1 Launch

Provide students with colored pencils.

### 2 Monitor

**Help students get started** by asking them how they can apply their knowledge of rigid transformations and angle relationships to identify congruent angles in the figure.

**Look for points of confusion:**

- **Only coloring the angles inside the shaded triangles.** Ask students to look for angles *outside* of the shaded triangles.
- **Not recognizing the sides of the triangle as transversals.** Have students extend the sides of the triangles to make this connection.

**Look for productive strategies:**

- Using their knowledge of rigid transformations to identify congruent angles.
- Identifying alternate interior angles.
- Noticing the triangle in the center is congruent to the two shaded triangles.

### 3 Connect

**Have pairs of students share** which angles they identified as congruent and why they thought they were congruent.

**Highlight** that the figure shows two parallel lines and that the triangles each have two sides that can be thought of as transversals.

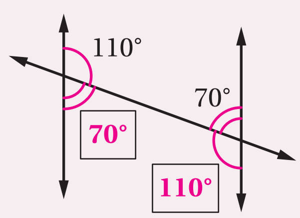
**Ask** students what they notice about the triangle in the center. Remind students that in Lesson 1, they created a tessellation using a triangle. Then ask students why any triangle can be used for a tessellation. Students will test their theories in the next activity.

### Power-up

**To power up students' ability to determine unknown angles using relationships among alternate interior angles, have students complete:**

Recall that when two parallel lines are intersect by a transversal, alternate interior angles are congruent.

1. In two different colors, mark the two pairs of alternate interior angles in the diagram.
2. Determine the two missing angle measurements indicated.




**Use:** Before Activity 1
**Informed by:** Performance on Lesson 16, Practice Problem 5

Pairs | 15 min

# Activity 1 Triangles and Parallel Lines

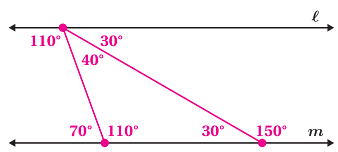
Students create their own triangle using points from two parallel lines to generalize the Triangle Sum Theorem.

Amps Featured Activity | Interactive Geometry

Name: ______ Date: ______ Period: ______

## Activity 1 Triangles and Parallel Lines

**You will need a protractor. The figure shows two parallel lines, $\ell$ and $m$.**




Sample response shown.

1. Draw two points on line $m$ and one point on line $\ell$. Connect the points to create a triangle. Measure and label the seven angles that are formed.

2. Compare your drawing with a partner. What patterns do you notice? List as many patterns as you can.

   Sample responses:
   - There are three straight angles formed that each have a sum of their measures of 180°. (The angle formed by the angles 110°, 40°, and 30°; the angle formed by the angles 110° and 70°; the angle formed by the angles 30° and 150°).
   - The interior angle measures in the triangle have a sum of 180°.
   - All of the alternate interior angles have the same measure (the two angles labeled 110° and the two angles labeled 30°)

3. Explain how the figure demonstrates why the sum of the angle measures in *any* triangle is 180°.

   Sample response: No matter how the angles are drawn to create different triangles, alternate interior angles are always congruent and straight angles always have a measure of 180°. The three angles that form the straight angle at the top have the same measure as the interior angles of any triangle that is drawn.

**Discussion Support:** As your classmates share their observations, refer to the class display. Restate your classmates' ideas using the math language you are learning.

**Are you ready for more?**

**Using a ruler, create at least three different quadrilaterals. Use a protractor to measure the four interior angles of each quadrilateral.**

1. What is the sum of these four angle measures?

   360°

2. How can you use your knowledge about triangles to verify the sum of the angles in any quadrilateral?

   Sample response: Because a quadrilateral can be partitioned into two triangles, the sum of the interior angle measures of the quadrilateral is the same as the sum of the angle measures in the two triangles, which is $180° + 180° = 360°$.





### 1 Launch

Provide access to geometry toolkits.

### 2 Monitor

**Help students get started** by modeling how to form a triangle by plotting three points on the parallel lines.

**Look for points of confusion:**

- **Not understanding why the triangle has a sum of 180°**. Write a list of vocabulary words, such as *rotation*, *transversal*, and *alternate interior angles*, to help their thinking. Ask them to record angle measurements.

**Look for productive strategies:**

- Noticing alternate interior angles are congruent.
- Noticing the interior angles in the triangle have a sum of 180°.
- Noticing the three angles formed on line $\ell$ have a sum of 180°.

### 3 Connect

**Display** different triangles students created with their labeled angle measures.

**Have pairs of students share** the patterns they noticed in their figures.

**Ask**, "How does your figure show *why* the sum of the angles in *any* triangle is 180°?" The three angles formed on the top line, which is a straight angle, have the same measure as the three angles of the triangle.

**Highlight** that line $\ell$ is parallel to the side of the triangle on line $m$ even if the vertices of the triangle shift. Using knowledge of alternate interior angles, the three angles on line $\ell$ are congruent to the angles in the triangle. This demonstrates that the sum of the angles in *any* triangle is 180°.

**Say** that a theorem is a mathematical statement that has been shown to be true. Then define the ***Triangle Sum Theorem*** as a theorem that states the sum of of the interior angles of any triangle is 180°.

## Differentiated Support

### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can drag points along the parallel lines to create different triangles to note patterns among the angle measurements.

### Accessibility: *Vary Demands to Optimize Challenge*

Provide 2–3 triangles already drawn for students to use, including labeling the angle measurements. Have students use the triangles to complete Problems 2 and 3. This will allow students to focus on the goals of the activity without having to do the drawing and measuring themselves.

## Math Language Development

### *MLR8: Discussion Supports—Revoicing*

Use this routine to support whole class discussion. For each observation that is shared, ask students to restate and/or revoice what they heard using mathematical language.

### English Learners

Provide sentence frames to support student conversation, such as:

- "Using angle relationships, I know that ___, so that must mean ___."

# Activity 2 Angle Puzzles

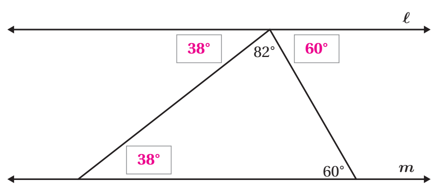
Students apply their understanding of the Triangle Sum Theorem and angle relationships to solve challenging angle puzzles.

**Activity 2** Angle Puzzles

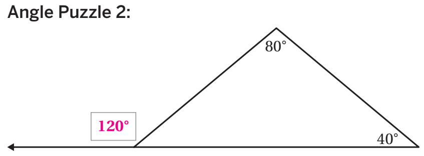
Determine the missing angle measures for each angle puzzle. The figures may not be drawn to scale.

Angle Puzzle 1:

Line $\ell$ is parallel to line $m$.




Angle Puzzle 2:







## 1 Launch

Collect geometry toolkits. Allow students to complete each angle puzzle at their own pace.

## 2 Monitor

**Help students get started** by telling them to visually inspect Puzzle 1, looking for angle relationships before they calculate any missing angles.

**Look for points of confusion:**

- **Having trouble completing a puzzle.** Tell students that they may need to determine one angle measure to help them determine another angle measure. Using Angle Puzzle 1, model how to determine the missing angle measures. Voice your thought process aloud so that students can model this process of thinking as they complete each puzzle.
- **Thinking they do not have enough information to find the missing angle measure that lies outside the triangle in Puzzle 2.** Ask them how they can find the unknown angle measure inside the triangle and whether that will help them find the missing angle measure outside the triangle.

**Look for productive strategies:**

- Looking for structure among the angle relationships in each angle puzzle.
- Using alternate interior angles, straight angles, and Triangle Sum Theorem to help them determine missing angle measures.
- Rechecking their calculations by adding the angle measures in a triangle, or by adding the angle measures that form a straight angle.

Activity 2 continued ›

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge, Guide Visualization and Processing***

If students need more processing time, have them focus on Puzzles 1 and 2, and only complete Puzzles 3 and 4 as time allows.

**Accessibility: *Clarify Vocabulary and Symbols***

Maintain a display of important terms and information, including diagrams, such as:

- A straight angle measures $180°$.
- Alternate interior angles are congruent.
- The sum of the angle measures in a triangle is $180°$.

**Extension: *Math Enrichment***

Have students create their own angle puzzle, and then trade puzzles with a partner. Each student should complete their partner's puzzle.

Pairs | 15 min

# Activity 2 Angle Puzzles (continued)

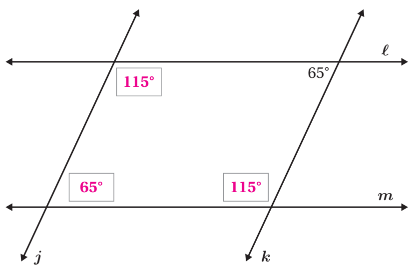
Students apply their understanding of the Triangle Sum Theorem and angle relationships to solve challenging angle puzzles.

Name: ______________________ Date: __________ Period: __________

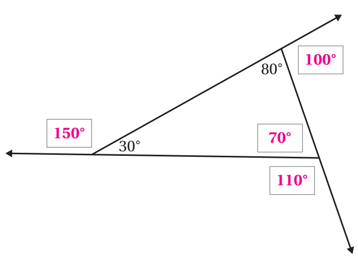
## Activity 2 Angle Puzzles (continued)

**Angle Puzzle 3:**

Line $\ell$ is parallel to line $m$ and line $j$ is parallel to line $k$.




**Angle Puzzle 4:**




STOP



**Lesson 17** Parallel Lines and the Angles in a Triangle 121

## 3 Connect

**Display** correct student work for each puzzle.

**Have students share** their strategies for determining the missing angle measures.

**Ask:**

- "Is there only one way to solve each puzzle? Explain your thinking." No; answers may vary.
- "What strategies did you use to determine the missing angle measures in Puzzle 3?" Answers may vary.
- "What do you notice about the figure in Puzzle 4?" Sample response: The measure of each exterior angle equals the sum of the measures of the remote interior angles.

**Define** an ***exterior angle*** as an angle between a side of a polygon and an extended adjacent side. Say that the missing angle in Puzzle 2 is an exterior angle to the triangle.

**Highlight** that when parallel lines are cut by transversals, angle relationships and the Triangle Sum Theorem can help students determine missing angle measures.

Whole Class | 5 min

# Summary

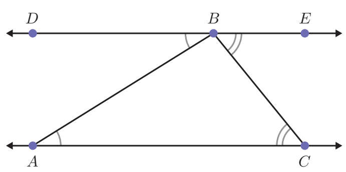
Review and synthesize how the Triangle Sum Theorem can be demonstrated using parallel lines and transversals.

### Summary

**In today's lesson . . .**

You applied what you learned about angle relationships, rigid transformations, and parallel lines to informally establish the ***Triangle Sum Theorem***. This theorem tells you that the sum of the three interior angles in *any* triangle is always $180°$.

Refer to parallel lines $DE$ and $AC$. You know that $m\angle ABD = m\angle BAC$ and $m\angle ACB = m\angle CBE$ because the angles in each angle pair are alternate interior angles. You also know that angles $\angle ABD$, $\angle ABC$, and $\angle CBE$ form a straight angle, so their measures add up to $180°$. Therefore, the sum of the interior angles of any triangle is $180°$.




Reflect:




## Synthesize

**Ask**, "In your own words and using the triangle shown in the Summary, explain how you know that the sum of the angles in *any* triangle is $180°$."
See students' responses. Look for evidence of correct reasoning, understanding of angle relationships, and mathematical terminology, such as *parallel lines*, *straight angle*, *alternate interior angles*, *congruent*, etc.

**Display** the Summary from the Student Edition.

**Highlight** that by applying understanding of alternate interior angles and straight angles, students are able to generalize that any triangle has an interior angle sum of $180°$.

**Formalize vocabulary:**

- *exterior angle*
- *Triangle Sum Theorem*

## Reflect

After synthesizing the concepts of the lesson, allow a few moments for student reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help students engage in meaningful reflection, consider asking:

- "How can the relationship between the interior angles of a triangle and a straight angle help you to determine an unknown angle measure?"

## Math Language Development

### *MLR2: Collect and Display*

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *Triangle Sum Theorem* that were added to the display during the lesson.

Independent | 5 min

# Exit Ticket

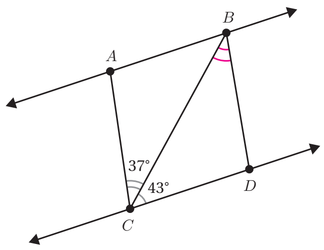
Students demonstrate their understanding by determining a missing angle measure.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 1.17

Line $AB$ is parallel to line $CD$. What is the measure of $\angle CAB$? Show or explain your thinking.




*The figure may not be drawn to scale.*

Sample response: $m\angle CAB = 100°$; I know that $\angle BCD$ and $\angle ABC$ are alternate interior angles, so they have the same measure. I know $m\angle ACB + m\angle CBA + m\angle BAC = 180°$, because $\angle ACB$, $\angle CBA$, and $\angle CAB$ are interior angles of a triangle. Because $37 + 43 + m\angle CAB = 180$, I know that $m\angle CAB = 100°$.

**Self-Assess**

? 1 I don't really get it — 2 I'm starting to get it — 3 I got it

a. I can explain, using diagrams, why the sum of the angles in any triangle is 180°.
1 2 3

b. If I know two of the angle measures in a triangle, I can determine the third angle measure.
1 2 3





## Success looks like . . .

- **Language Goal:** Generalizing the Triangle Sum Theorem using the congruence of alternate interior angles when parallel lines are cut by a transversal. **(Speaking and Listening, Reading and Writing)**
  - » Applying the Triangle Sum Theorem to determine the measure of $\angle CAB$.

## Suggested next steps

**If students do not calculate the angle correctly, consider:**

- Highlighting $\angle DCB$ and $\angle CBA$ to emphasize alternate interior angles.
- Reviewing the terms *straight angles*, *alternate interior angles*, and the *Triangle Sum Theorem*.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- During the discussion in Activity 1, how did you encourage each student to share their understandings?
- What challenges did students encounter as they worked on the angle puzzles? How did they work through them?

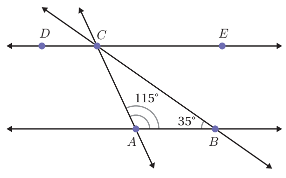
# Practice

Independent

Name: ______________________ Date: __________ Period: __________

Practice

1. The diagram shows parallel lines $AB$ and $DE$.



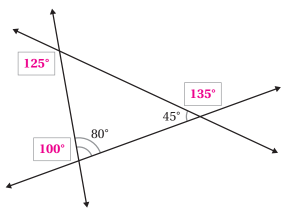

*The figure may not be drawn to scale.*

a. What is $m\angle ACD$?
115°

b. What is $m\angle ECB$?
35°

c. What is $m\angle ACB$?
30°

2. Three intersecting lines are shown.




*The figure may not be drawn to scale.*

a. Determine the three missing angle measures.
Correct responses shown on diagram.

b. What is the sum of these three angle measures?
360°

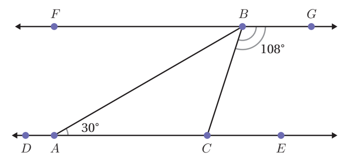




Name: ______________________ Date: __________ Period: __________

Practice

3. Line $DE$ is parallel to line $FG$. Is it possible to determine all five angle measures with the given information? Show or explain your thinking.

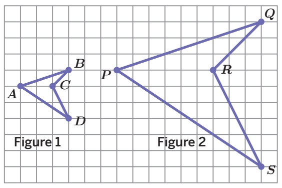



*The figure may not be drawn to scale.*

Yes; Sample response:

- $\angle DAB$ and $\angle BAC$ are supplementary angles, so $m\angle DAB = 150°$.
- $\angle BAC$ and $\angle FBA$ are alternate interior angles, so $m\angle FBA = 30°$.
- $\angle FBA$, $\angle ABC$, and $\angle CBG$ form a straight line measuring 180°, so $m\angle ABC = 42°$.
- $\angle BAC$, $\angle ABC$, and $\angle BCA$ are interior angles in a triangle, so $m\angle BCA = 108°$.
- $\angle BCA$ and $\angle BCE$ are supplementary angles, so $m\angle BCE = 72°$.

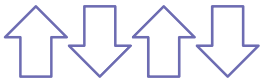
4. The two figures shown are scaled copies of each other.




a. What scale factor is used to take Figure 1 to Figure 2?
3

b. What scale factor is used to take Figure 2 to Figure 1?
$\frac{1}{3}$

5. Describe a transformation that maps the pattern shown onto itself. A protractor and a ruler may be useful here.



A 180° rotation about a point located in the center of the pattern.





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 1 | 2 |
| Spiral | 4 | Grade 7 | 2 |
| Formative | 5 | Unit 1 Lesson 18 | 2 |

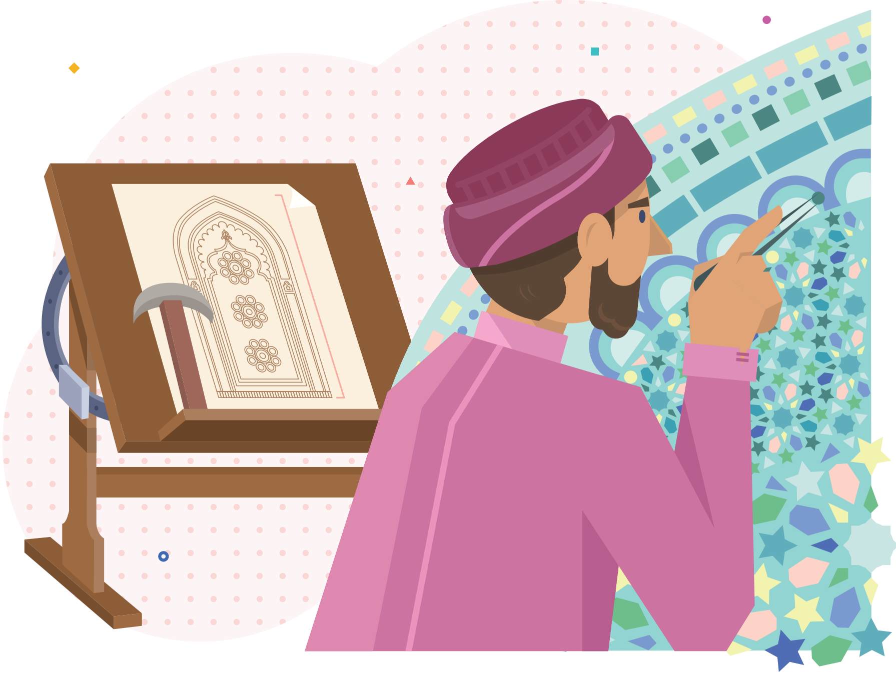
**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice.**

# Creating a Border Pattern Using Transformations

Let's create borders using transformations.



## Focus

### Goals

1. Create a border pattern using rigid transformations.
2. **Language Goal:** Explain the rigid transformations needed to map a design onto itself. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students use the language of transformations to create, describe, and investigate patterns on a plane by creating their own border pattern. Students model with mathematics as they apply transformations to design their own border patterns.

### Previously

Throughout this unit, students applied reflections, rotations, and translations of figures on a plane, square grid, and coordinate plane.

### Coming Soon

In Unit 2, students will investigate dilations and understand the similarity of figures in terms of rigid transformations and dilations.

## Rigor

- Students **apply** their understanding of rigid transformations to study Islamic art and create their own border pattern.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 5 min | 25 min | 5 min | 5 min |
| Independent | Independent | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice
Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (for display)
- colored pencils
- geometry toolkits: protractors, rulers, tracing paper
- plain sheets of paper

### Math Language Development

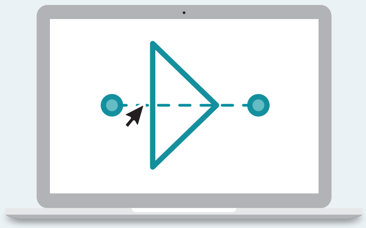
**Review words**

- *reflection*
- *rotation*
- *translation*
- *transformation*

**Amps** | Featured Activity

**Activity 2**
**Interactive Geometry**

Students experiment with creating border patterns by sketching a preimage and selecting different buttons to apply rigid transformations.



Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel uncomfortable with their artistic ability as they draw their preimage in Activity 2. Help them build their confidence by having them include their own personal interests in their design. As students create their border patterns, find positive examples to encourage all students. Students may be more comfortable describing the transformations that model pre-created border patterns. Consider having them research border patterns in art or architecture and describe the mathematics that model them.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted as students will analyze the same image in Activity 1.
- In **Activity 1**, have students look for examples of one type of transformation, instead of all three.

Independent | 5 min

## Warm-up Notice and Wonder

Students study an image with a complex pattern to see how transformations are applied in Islamic art.

Name: ______________________ Date: __________ Period: __________

Unit 1 | Lesson 18 – Capstone

### Creating a Border Pattern Using Transformations

Let's create borders using transformations.

**Warm-up** Notice and Wonder

**Consider the image shown. What do you notice? What do you wonder?**

1. I notice . . .
   Sample responses:
   - The pattern shows translations, reflections, and rotations of figures.
   - The pattern shows symmetry.

2. I wonder . . .
   Sample responses:
   - Can other geometric patterns or art be created from transformations?
   - Where is this pattern located?


Hussain Warraich/Shutterstock.com

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Conduct the *Notice and Wonder* routine using the image.

### 2 Monitor

**Help students get started** by asking how this image relates to the math they have been studying in this unit.

**Look for productive strategies:**

- Noticing reflection, rotation, and translation symmetry.

### 3 Connect

**Have students share** what they noticed and wondered about the image. Record student observations.

**Ask** students how transformations of figures can be seen in this image, if students have not already mentioned this idea.

**Highlight** that the patterns in the image are created using transformations. This pattern is on a wall in the Sultan Qaboos Grand Mosque in Muscat, Oman. These patterns are frequently seen in Islamic art where geometric designs are used to create complex patterns.

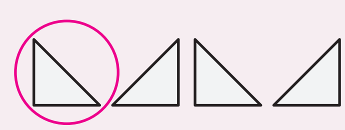
## Differentiated Support

**Extension: *Interdisciplinary Connections***

Have students explore the beautiful interior and exterior of the Sultan Qaboos Grand Mosque in Muscat, Oman by exploring the official website of the mosque. Students can virtually move from room to room, zoom in and out, and rotate to see a full $360°$ view of each room. As they explore, ask them to point out the rigid transformations they see. **(Art, Architecture)**

## Power-up

**To power up students' ability to describe a transformation of multiple figures, have students complete:**



1. Identify a repeating shape in the pattern by circling it.
2. Which of the following describes how you can map the pattern onto itself.
   **A.** $180°$ rotation about a point located in the center.
   **B.** Reflections about a horizontal line through the center of all four figures.
   **C.** Reflection about a vertical line between the second and third triangles.

**Use:** Before Activity 1
**Informed by:** Performance on Lesson 16, Practice Problem 5

Independent | 5 min

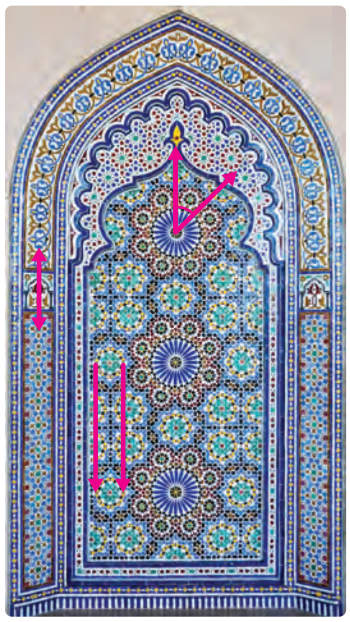
# Activity 1 How Is It Made?

Students analyze the image from the Warm-up to look for examples of transformations.

**Activity 1** How Is It Made?

**The pattern you saw in the Warm-up is from a wall in the Sultan Qaboos Grand Mosque in Muscat, Oman.**

Find a single pattern or multiple patterns within the image that have rotation, translation, or reflection symmetry. Show or describe how each transformation is produced.



Hussain Warraich/Shutterstock.com

a. rotation

**Sample response: 45° rotation about its center**

b. translation

**Sample response: The top circular image is translated down.**

c. reflection

**Sample response: reflection across a vertical line**





## 1 Launch

Activate students' background knowledge by asking them where they might see patterns and designs that apply transformations.
**Note:** Provide access to geometry toolkits for the duration of the lesson.

## 2 Monitor

**Help students get started** by having them choose one specific pattern within the image and describing the type of transformation seen in the pattern.

**Look for productive strategies:**

- Determining all three transformations in one pattern.
- Drawing multiple lines of symmetry for a pattern.
- Noticing the border pattern created by transformations.

## 3 Connect

**Have students share** which patterns they chose and the transformations that were applied in their chosen pattern.

**Highlight** that the image is created from different patterns that apply rigid transformations.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, allow them to focus on completing two of the three transformations in this activity. Consider allowing them to choose which problems they would like to complete. Alternatively, preselect the patterns that demonstrate each type of transformation and display them. Have students show or describe how each transformation is applied.

## Math Language Development

### *MLR8: Discussion Supports—Restate It!*

Use this routine to support whole class discussion. For each pattern and transformation that is shared, ask students to restate what they heard using developing mathematical language. Call their attention to any words or phrases that helped to clarify the original statement. This provides more students with an opportunity to produce language as they interpret the reasoning of others.

Pairs | 25 min

# Activity 2 Designing a Border Pattern

Students design a border pattern to apply their knowledge of transformations.

Amps Featured Activity | Interactive Geometry

Name: ______ Date: ______ Period: ______

## Activity 2 Designing a Border Pattern

**In this activity, you will design your own border pattern. Border patterns have been studied by mathematicians, such as John Conway. You will be given a plain sheet of paper to use, starting with Problem 2.**

1. Design a preimage for your border in this box.
   Sample response shown.

2. Trace your preimage on the sheet of paper.
   a. Apply a series of rigid transformations to create your border pattern. Draw a sketch of your border pattern here.
   Sample response shown.

   b. Trade borders with a partner. Describe a transformation of your partner's border pattern that maps the pattern onto itself. Write as many specific transformations as you can.
   Sample responses:
   - The pattern can be mapped onto itself by a translation to the right.
   - The pattern can be mapped onto itself by a reflection across a horizontal line.

   c. If there is time remaining in the activity, color your border pattern.

**Featured Mathematician**

**John Horton Conway**

John Horton Conway was a British mathematician known for his playful attitude toward mathematics. Among his many contributions, he provided names for the seven groups of symmetric, infinitely long border (or "frieze") patterns: hop, step, sidle, spinning hop, spinning sidle, jump, and spinning jump. In 2020, Conway passed away due to complications from COVID-19.

"John Horton Conway" by Thane Plambeck, courtesy of Flickr, (https://www.flickr.com/photos/thane/20366806/) is licensed under the Creative Commons Attribution 2.0 Generic license, https://creativecommons.org/licenses/by/2.0/

STOP



**Lesson 18** Creating a Border Pattern Using Transformations 127

## 1 Launch

Display the Activity 2 PDF. For each border, point out how each pattern is made using transformations. Keep this PDF displayed as a reference for students while they design their border patterns. Provide access to colored pencils and blank sheets of paper.

## 2 Monitor

**Help students get started** by challenging them to design a preimage that has geometric patterns or has meaningful images.

**Look for points of confusion:**

- **Having trouble designing a preimage.** Provide specific directions. For example say, "First, in the middle of the square, draw an icon or image that represents you. This may be your favorite hobby, food, sport, or animal. Next, in each of the four corners, write your first initial facing in any direction. Last, add a design in the remaining space."
- **Having trouble drawing their border.** Students may find it helpful to retrace their preimage on tracing paper and use it to draw their border.

**Look for productive strategies:**

- Turning the paper to produce rotations, flipping the paper to produce reflections, and sliding the paper to produce translations.
- Creating border patterns that intersect each other.
- Creating a circular border.

## 3 Connect

**Display** the different border patterns created by students. If time allows, use the ***Gallery Tour*** routine by setting up areas around the classroom to display student work and have students look for transformations in each border.

**Have pairs of students share** the types of transformations that are applied in the border patterns of other students.

**Highlight** that transformations can be applied to create complex patterns in art or architecture.

## Differentiated Support

### Extension: *Math Around the World*

Have students explore the online site "19th Century Navajo Weaving at ASM" from the Arizona State Museum. They should examine the transformations of different Navajo woven blankets found on this site, such as Chief's-style Blankets, Sarapes, Transitional Period blankets, Moqui Stripe Patterns, and Eye Dazzlers. Have students choose a woven blanket and describe the transformations used. Consider printing copies of these blankets they can use to annotate the transformations as they describe them.

## Featured Mathematician

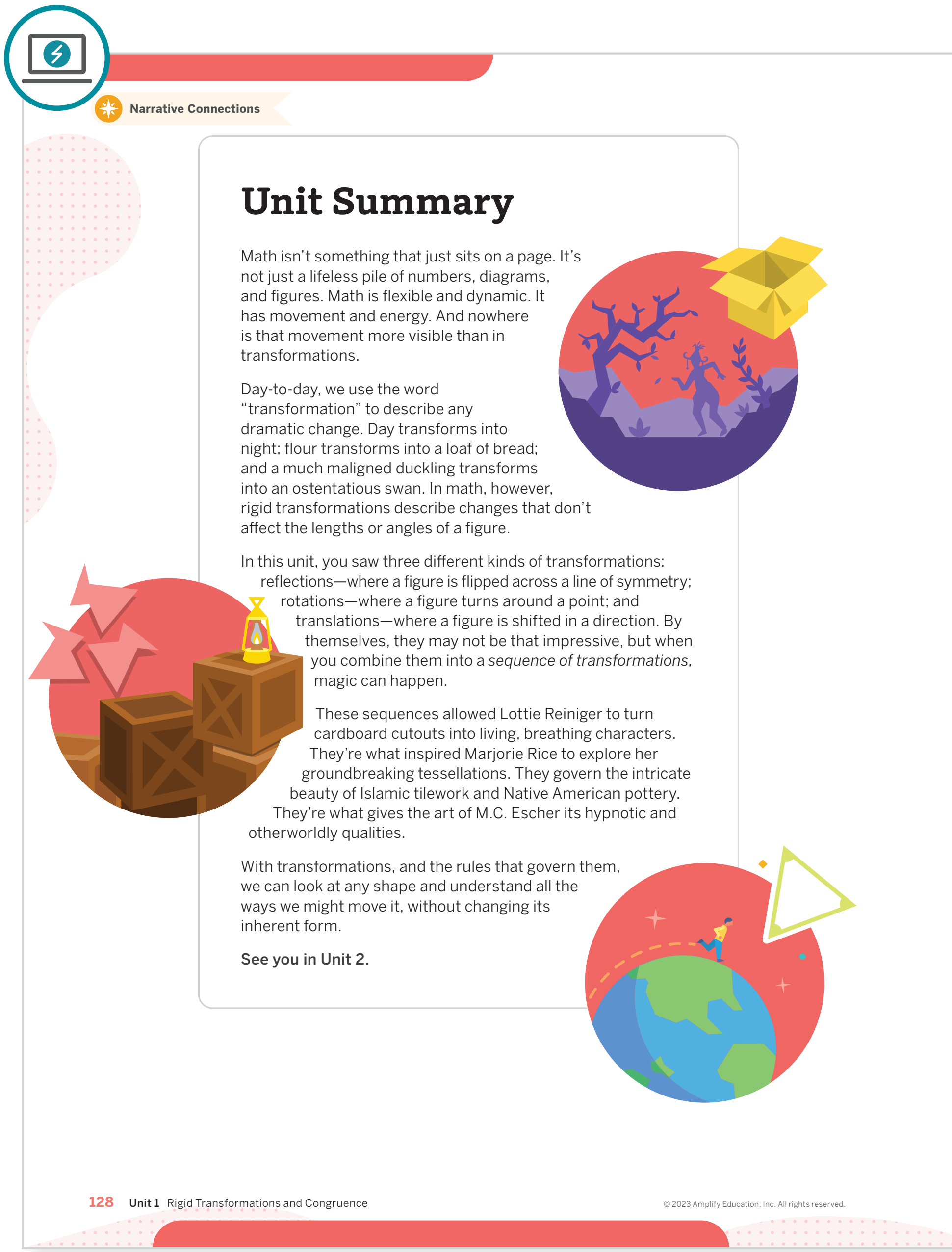
### John Horton Conway

Have students read about mathematician John H. Conway, who used footprint patterns to describe all two-dimensional designs that are repetitive in one direction.

Whole Class | 5 min

# Unit Summary

Review and synthesize how rigid transformations can be applied to create designs.



Narrative Connections

## Unit Summary

Math isn't something that just sits on a page. It's not just a lifeless pile of numbers, diagrams, and figures. Math is flexible and dynamic. It has movement and energy. And nowhere is that movement more visible than in transformations.

Day-to-day, we use the word "transformation" to describe any dramatic change. Day transforms into night; flour transforms into a loaf of bread; and a much maligned duckling transforms into an ostentatious swan. In math, however, rigid transformations describe changes that don't affect the lengths or angles of a figure.

In this unit, you saw three different kinds of transformations: reflections—where a figure is flipped across a line of symmetry; rotations—where a figure turns around a point; and translations—where a figure is shifted in a direction. By themselves, they may not be that impressive, but when you combine them into a *sequence of transformations*, magic can happen.

These sequences allowed Lottie Reiniger to turn cardboard cutouts into living, breathing characters. They're what inspired Marjorie Rice to explore her groundbreaking tessellations. They govern the intricate beauty of Islamic tilework and Native American pottery. They're what gives the art of M.C. Escher its hypnotic and otherworldly qualities.

With transformations, and the rules that govern them, we can look at any shape and understand all the ways we might move it, without changing its inherent form.

**See you in Unit 2.**

128 Unit 1 Rigid Transformations and Congruence

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### Narrative Connections

Read the narrative aloud as a class or have students read it individually.

### Synthesize

**Display** the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

**Have students share** how border patterns are produced using rigid transformations.

**Ask**, "Where have you seen border patterns?"

**Highlight** that rigid transformations have been used over thousands of years to create complex patterns. These patterns can be seen in architecture, embroidery, pottery, and in many other places.

### Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

- "Did anything surprise you while reading the narratives of this unit?"
- "Is there anything you would like to learn more about these topics? What are some steps you can take to learn more?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by connecting applications of rigid transformations to their everyday lives.

Printable

Name: ______________ Date: ________ Period: ________

## Exit Ticket 1.18

**Aside from border patterns, where have you seen an application of transformations?**

Sample responses: paintings, architecture, wallpaper, rugs, company logos, borders around photo frames

**Self-Assess**

| 1 | 2 | 3 |
|---|---|---|
| I don't really get it | I'm starting to get it | I got it |

a I can repeatedly use rigid transformations to make interesting patterns of figures.

1 2 3





## Success looks like . . .

- **Goal:** Creating a border pattern using rigid transformations.
- **Language Goal:** Explaining the rigid transformations needed to map a design onto itself. **(Speaking and Listening, Writing)**
  - » Describing transformations they have seen in their everyday lives.

## Suggested next steps

**If students are not sure where they have seen an application of transformations, consider:**

- Having them look around the classroom to see if they recognize any transformations. Encourage them to look at floor or ceiling tiles, patterns on clothing, or artwork.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- In this lesson, students designed border patterns. How did that build on the earlier work students did with drawing transformations of a figure?
- What surprised you as your students worked on their border patterns?

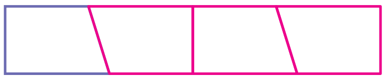
# Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. Refer to the trapezoid shown.



   a. Use rigid transformations on the trapezoid to design a border pattern.

   b. Describe the rigid transformations you used.
   Sample response: I used 180° rotations and translations of the preimage.

2. Refer to the national flag of Trinidad and Tobago. Describe a transformation that maps the triangle in the lower left corner onto the triangle in the upper right corner.

Public Domain

Sample response: 180° rotation about a point in the center of the flag.

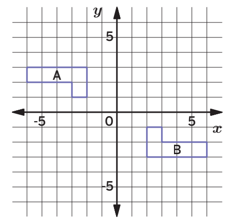




Name: ______ Date: ______ Period: ______

Practice

3. Polygon A is congruent to Polygon B. Describe a transformation or sequence of transformations that maps Polygon A onto Polygon B.
   Sample response: Reflect Polygon A across the $x$-axis, and then reflect its image across the $y$-axis.

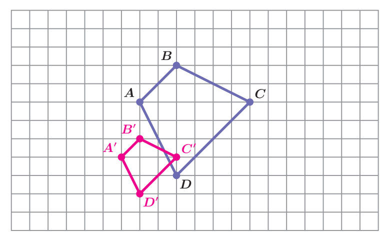



4. Write *all* of the possible combinations of three angle measures, from the following list, that can be the interior angle measures of a triangle.

| | | | | | | |
|---|---|---|---|---|---|---|
| 60° | 20° | 100° | 40° | 110° | 50° | 30° |

20°, 60°, 100°
20°, 50°, 110°
30°, 50°, 100°
30°, 40°, 110°

5. On the grid, draw a scaled copy of Quadrilateral $ABCD$, using a scale factor of $\frac{1}{2}$.








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| Spiral | 2 | Unit 1<br>Lesson 5 | 2 |
| | 3 | Unit 1<br>Lesson 2 | 2 |
| | 4 | Unit 1<br>Lesson 16 | 2 |
| | 5 | Grade 7 | 2 |

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 2

# Dilations and Similarity

Students explore a new type of transformation — dilations — and practice using dilations to create and recognize similar figures. Students' understanding of the characteristics of these similar figures, of similar triangles specifically, will serve as the foundation for their study of the slope of a line.

**Essential Questions**

- What does it mean to dilate a figure?
- How can you identify whether two figures are similar?
- How can similar triangles be used to find the slope of a line?
- *(By the way, can you create an optical illusion that will trick your teacher's eyes?)*

# Key Shifts in Mathematics

## Focus

### In this unit . . .

Students move beyond rigid transformations to discover the properties of dilations in the first sub-unit. Students practice identifying dilations using rulers or grids to precisely identify the scale factor that takes one image to the other. Students explore the characteristics of figures after they have been dilated by looking at the angle and side measures. This leads students to a formal definition of the word *similar* in Lesson 6, kicking off the second sub-unit. In this sub-unit, students apply concepts of proportional reasoning to similar figures — in particular similar triangles. Their work with similar triangles will lead them to their first introduction to slope in Lesson 11.

## Coherence

### Previously . . .

In Unit 1, students studied rigid transformations. Students gained experience identifying and creating a sequence of rigid transformations using mathematical tools and the structure of a grid. Students deconstructed a straight angle to create a triangle, confirming that the interior angles of a triangle measure to a sum of $180^\circ$.

### Coming soon . . .

In this unit, students meet slope. In Unit 3, students really get to know slope. Using what they have learned about proportional relationships in Grade 7, students will learn that the constant of proportionality is the same as the rate of change or the slope of a line. Before long, they will see that not all lines represent proportional relationships. They will study these nonproportional linear relationships for the rest of the unit, exploring representations of linear relationships and using equations to describe lines and real-world context. This prepares students to examine systems of linear relationships in Unit 4.

## Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

### Conceptual Understanding

Students review the concept of scaled copies (Lesson 1), before being formally introduced to dilations (Lesson 2). With an understanding of dilations, students later examine the relationship between scaled copies and similar figures (Lesson 7).

### Procedural Fluency

Students build key skills dilating polygons on a grid (Lesson 5). Students use proportional side lengths of similar triangles to find unknown side lengths (Lesson 10).

### Application

Students apply their knowledge of dilations to find missing information, such as the center of dilation, scale factor, or images of dilation (Lesson 3).

# More Than Meets the Eye

SUB-UNIT

1

Lessons 2–5

## Dilations

Students explore another type of transformation — ***dilations*** — and connect dilations to the rigid transformations they previously studied. They discover how artists use dilations to create perspective drawings and the illusion of 3D imagery.

**Narrative:** The pupils of your eyes dilate in response to light. But there is more to dilation than meets the eye.

SUB-UNIT

2

Lessons 6–11

## Similarity

By investigating the properties of dilated figures, students discover that dilated figures are ***similar***. They formalize the special properties of dilated figures and learn that similar right triangles can be used to find ***slope***, which will be of further importance in upcoming units.


**Narrative:** Understanding similarity and proportional reasoning can help you combat *shrinkflation*.

## Launch

Lesson 1

### Projecting and Scaling

Students look at standard paper sizes as scaled copies of each other to discover that the relationship of each paper size can be represented by a proportional pattern.

## Capstone

Lesson 12

### Optical Illusions

Students apply concepts they learned about transformations and dilations to create optical illusions on a grid.

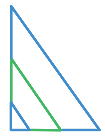
# Unit at a Glance

**Spoiler Alert:** Pairs of triangles with at least two congruent angle measures must be similar to each other.

## Assessment

A **Pre-Unit Readiness Assessment**

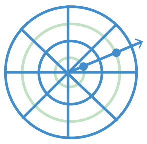
## Launch Lesson



1 **Projecting and Scaling**

Uncover a pattern in the relationship between standard paper sizes.

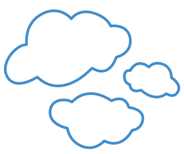
## Sub-Unit 1: Dilations



2 **Circular Grids**

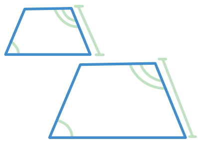
Grids do not have to be square to be useful. First learn to dilate a figure by using a circular grid.

## Sub-Unit 2: Similarity



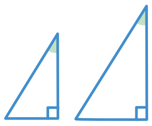
6 **Similarity**

Find similar figures by creating a sequence of transformations with dilations.



7 **Similar Polygons**

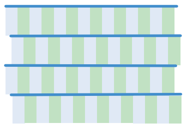
Explore what it means for two polygons to be similar by looking at their side length and angle measures.



8 **Similar Triangles**

Uncover special properties of similar triangles.

## Capstone Lesson



12 **Optical Illusions**

Identify and create patterns with optical illusions.

## Assessment

A **End-of-Unit Assessment**

## Key Concepts

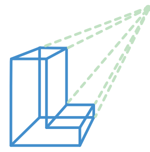
**Lesson 4:** The scale factor between an image and its preimage depends on the center of dilation.
**Lesson 6:** A dilation with a scale factor greater than or less than 1 creates a scaled copy that is similar to its preimage.
**Lesson 9:** Proportional reasoning can be used to determine missing side lengths of similar triangles.

## Pacing

**12 Lessons:** 45 min each
**2 Assessments:** 45 min each

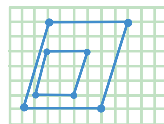
**Full Unit:** 14 days
● **Modified Unit:** 12 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



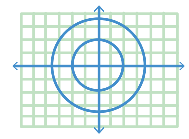
### 3 Dilations on a Plane ●

Apply dilations to points on a plane without the structure of a grid.



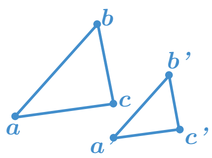
### 4 Dilations on a Square Grid ●

Apply dilations to polygons on a grid without coordinates.



### 5 Dilations With Coordinates

Apply dilations to polygons, this time on a coordinate plane.

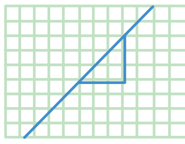



### 9 Ratios of Side Lengths in Similar Triangles ●

Develop strategies for determining missing lengths of similar triangles.

### 10 The Shadow Knows ●

Don't have a tall enough ruler? Use the shadow of a lamppost to measure its height.



### 11 Meet Slope

Use similar right triangles to find the slope of a line.

## ● Modifications to Pacing

**Lessons 3–4:** Lessons 3 and 4 both work with dilations on a square grid. They can be combined if necessary.

**Lessons 9–10:** Students can learn to use known side lengths of similar triangles to determine missing side lengths by engaging in a combination of activities from Lessons 9 and 10.

# Unit Supports

## Math Language Development

| Lesson | New Vocabulary |
|---|---|
| 2 | center of dilation<br>dilation |
| 6 | similar |
| 11 | slope<br>slope triangles |

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

| Lesson(s) | Mathematical Language Routines |
|---|---|
| 6 | **MLR1:** Stronger and Clearer Each Time |
| 1, 2, 5–9. 11, 12 | **MLR2:** Collect and Display |
| 3, 7, 8 | **MLR3:** Critique, Correct, Clarify |
| 5 | **MLR4:** Information Gap |
| 3, 5, 6, 9, 10, 12 | **MLR7:** Compare and Connect |
| 2, 4, 7–11 | **MLR8:** Discussion Supports |

## Materials

Every lesson includes:

Exit Ticket   Additional Practice

Additional required materials include:

| Lesson(s) | Materials |
|---|---|
| 1 | A4 Paper and US Letter Paper |
| 12 | black markers |
| 12 | black pens |
| 1, 9, 10 | calculators |
| 4 | colored pencils |
| 2–8, 12 | geometry toolkits |
| 12 | graph paper |
| 7 | glue |
| 3, 5–8, 10–12 | PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs. |
| 7 | plain sheets of paper |
| 1, 11 | rulers |
| 1 | scissors |

## Instructional Routines

Activities throughout this unit include the following instructional routines:

| Lesson(s) | Instructional Routines |
|---|---|
| 1, 2, 3, 10, 11 | Notice and Wonder |
| 2, 6, 7, 9, 10, 12 | Think-Pair-Share |
| 2 | Partner Problems |
| 5 | Info Gap |
| 6, 7, 9, 12 | Poll the Class |
| 8 | Card Sort |
| 6, 9 | Which One Doesn't Belong? |
| 12 | Gallery Tour |

# Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

| Assessments | When to Administer |
|---|---|
| **Pre-Unit Readiness Assessment**<br>This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit. | Prior to Lesson 1 |
| **Exit Tickets**<br>Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned. | End of each lesson |
| **End-of-Unit Assessment**<br>This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide. | After Lesson 12 |

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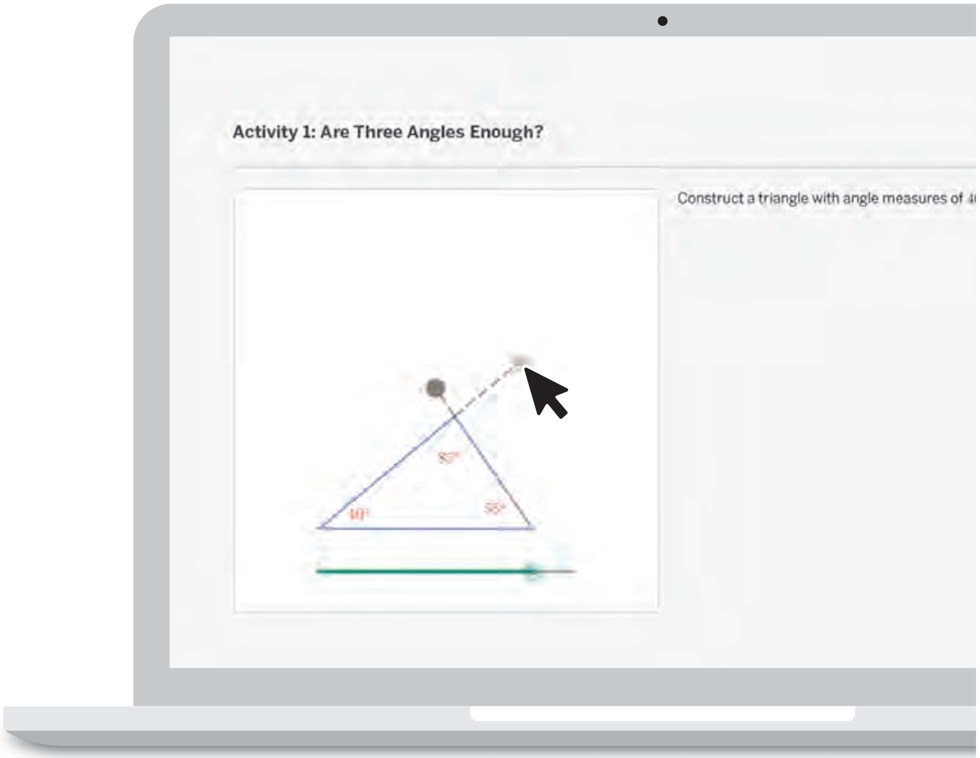
## Social & Collaborative Digital Moments

### Featured Activity

**Are Three Angles Enough?**

Put on your student hat and work through **Lesson 8, Activity 1:**




Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

**Other Featured Activities:**

- Dilations on a Grid **(Lesson 4)**
- Info Gap: Make My Dilation **(Lesson 5)**
- Are They Similar? **(Lesson 6)**
- Four Challenges **(Lesson 10)**

# Unit Study
## Professional Learning

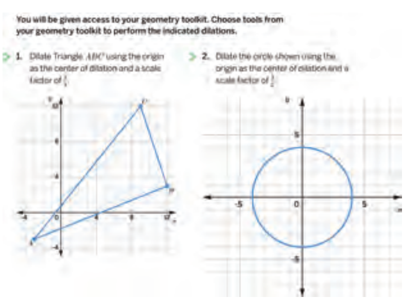
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

## Anticipating the Student Experience With Fawn Nguyen

**Unit 2** introduces dilations with coordinates. This work begins with the idea of scaling and ratios. Students work on dilations of polygons on circular grids, square grids, and coordinate planes. Eventually this understanding equips them for work with similar figures and slope. Prepare yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

### Do the Math

Put on your student hat and tackle these problems from **Lesson 5, Activity 2:**


You will be given access to your geometry toolkit. Choose tools from your geometry toolkit to perform the indicated dilations.

1. Dilate Triangle $ABC$ using the origin as the center of dilation and a scale factor of
2. Dilate the circle shown using the origin as the center of dilation and a scale factor of

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

**Points to Ponder . . .**

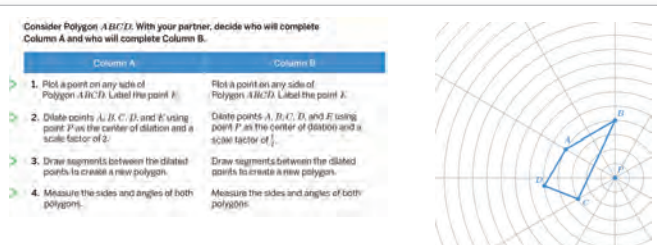
- What was it like to engage in this problem as a learner?
- A key understanding with dilation is knowing the relationship between the center of dilation and the scale factor. What strategies might help students grasp these concepts?
- In Problem 1, the coordinates of the image can be found by multiplying the coordinates of the preimage by the scale factor. Why does this strategy *not* work in Problem 2?
- What implications might this have for your teaching in this unit?

## Focus on Instructional Routines

### Think-Pair-Share

**Rehearse . . .**

How you'll facilitate the ***Think-Pair-Share*** instructional routine in **Lesson 2, Activity 2:**


Consider Polygon $ABCD$. With your partner, decide who will complete Column A and who will complete Column B.

| Column A | Column B |
|---|---|
| 1. Plot a point on any side of Polygon $ABCD$. Label the point $E$. | Plot a point on any side of Polygon $ABCD$. Label the point $E$. |
| 2. Dilate points $A$, $B$, $C$, $D$, and $E$ using point $P$ as the center of dilation and a scale factor of 2. | Dilate points $A$, $B$, $C$, $D$, and $E$ using point $P$ as the center of dilation and a scale factor of $\frac{1}{2}$. |
| 3. Draw segments between the dilated points to create a new polygon. | Draw segments between the dilated points to create a new polygon. |
| 4. Measure the sides and angles of both polygons. | Measure the sides and angles of both polygons. |



**Points to Ponder . . .**

- How can you use the partner sharing portion of this routine to help you facilitate full-class discussion?
- What mathematical thinking can you be listening for when students speak with their partners?

**This routine . . .**

- Provides students independent time to think about the task and prepare a plan before sharing with a partner.
- Gives students a low-stakes opportunity to share their ideas with a partner before sharing with the whole class.
- Allows teachers to eavesdrop on student thinking as they share with their partners, enabling teachers to pre-select students to share during whole-class discussion.
- Creates ample opportunity for collaboration.

**Anticipate . . .**

- How can you ensure the "think" time and the "pair" time are each used effectively?
- How will you encourage and support students who either do not want to work independently or do not want to work with a partner?
- If you *have not* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

## Strengthening Your Effective Teaching Practices

**Implement tasks that promote reasoning and problem solving.**

### This effective teaching practice . . .

- Provides opportunities for students to engage in low floor, high ceiling tasks that allow for multiple entry points and a variety of solution strategies.
- Requires the use of reasoning and problem solving strategies as opposed to merely requiring the use of established procedures or skills.

### Points to Ponder . . .

- Where do you see opportunities to highlight different strategies that students used to approach the tasks in this unit?
- How can you encourage students to consider the alternative approaches that other students may have used?

## Math Language Development

### MLR2: Collect and Display

MLR2 appears in Lessons 1, 2, 5–9, 11, and 12.

- In Lesson 2, as students share their responses, you can highlight and collect terms and phrases they use to describe dilations, such as *scale factor, center of dilation*, and *scaled copy*.
- Throughout the unit, as students formalize the new vocabulary they are learning, have them refer to the class display to continually review and reflect on new terms that are added.
- **English Learners:** Add diagrams or illustrations to the class display so that students can visualize the terms or phrases. Consider also using hand gestures to illustrate some terms, such as *vertical, horizontal, parallel*, or *slope*.

### Point to Ponder . . .

- How will you encourage or guide students toward using their developing math language to describe dilations and similar figures?

## Differentiated Support

### Accessibility: *Guide Processing and Visualization, Optimize Access to Tools, Optimize Access to Technology*

Opportunities to provide visual support, physical manipulatives, or the use of technology appear in Lessons 2–5, 8–11.

- Throughout the unit, students will select tools from their geometry toolkits to perform dilations or draw and measure figures. Options are provided to assist students in selecting certain tools.
- In Lesson 9, students can use the Amps slides for Activity 1, in which they can modify the side lengths of a triangle using different scale factors. Animations appear to help them visualize the effects.
- Use color coding and annotation to illustrate student thinking, such as color coding corresponding side lengths or angles of similar figures.

### Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to use technology or when to suggest students use color coding or certain tools to help them make sense of dilations and similar figures?

## Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

### Look Ahead . . .

- Review and unpack the **End-of-Unit Assessment**, noting the concepts and skills assessed in it.
- With your student hat on, complete each problem.

### Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with solving one-step equations with a variable throughout the unit? Do you think your students will generally:
  - struggle to find the scale factor or center of dilation?
  - have difficulty using grids and mathematical tools precisely?
  - be unsure about how to identify if two figures are similar?

## Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their self-awareness and self-management skills.

### Points to Ponder . . .

- What are their strengths and what do they know about transforming figures that they can build upon and use to begin reasoning algebraically?
- Are students able to exercise patience and persist in order to recreate a sequence of transformations that confirms two figures are similar?

# Projecting and Scaling

Let's explore scaling.

## Focus

### Goals

1. **Language Goal:** Describe the features of scaled copies of a rectangle. **(Speaking and Listening)**
2. Identify rectangles that are scaled copies of one another.

## Coherence

### Today

Students cut and arrange rectangles from two different paper sizes — US Letter and A4 — to model the properties of scaled copies. Students reason abstractly as they rearrange the two sets of rectangles so that each set shares an angle, observing that when the rectangles are scaled copies of one another, the opposite vertices all lie on the same line. They connect the meaning of the aligned vertices when they calculate the ratio of the side lengths for all the rectangles, seeing that the rectangles created from the A4 paper produce scaled copies.

### Previously

In Grade 7, students examined scaled copies. For polygons, they identified that the side lengths of scaled copies are proportional, and the constant of proportionality relating the original lengths to the corresponding lengths in the scaled copy is the scale factor.

### Coming Soon

In Lesson 2, students will come to understand and use the term *dilation*. They will recognize that a dilation is determined by a point called the *center of dilation* and a number called the *scale factor*. In Lesson 9, students will revisit ratios and scale factors as they study the side lengths of similar triangles.

## Rigor

- Students build **conceptual understanding** of scaled copies.

# Pacing Guide

Suggested Total Lesson Time ~ **45 min**

| Warm-up | Activity 1 | Summary | Exit Ticket |
| --- | --- | --- | --- |
| 5 min | 30 min | 5 min | 5 min |
| Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

## Materials

- Exit Ticket
- Additional Practice
- calculators
- rulers marked with millimeters
- scissors
- US Letter paper, one sheet per pair
- A4 paper, one sheet per pair

**Note:** As an alternative option to A4 paper, cut a sheet with the same dimensions as A5 (148 mm by 210 mm). If this option is chosen, provide half of the US Letter instead of the full sheet.

## Math Language Development

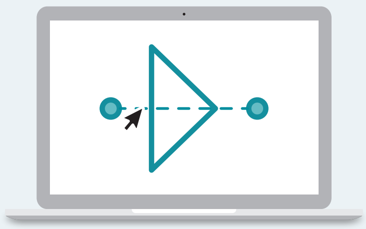
**Review words**

- *proportional relationship*
- *ratio*
- *scaled copy*
- *scale factor*

**Amps** | **Featured Activity**

### Activity 1
### Using Work From Previous Slides

Ratios students enter are shown to them on a later slides to assist with comparisons.



Amps POWERED BY desmos

## Building Math Identity and Community

Connecting to Mathematical Practices

Students may resist thinking quantitatively or abstractly when they relate the set of rectangles to the ratio of the side lengths in Activity 1. Have them engage in metacognitive functions by asking them to think about their own thinking process. For example, have them conduct their own ***Notice and Wonder*** routine, which will help them record their thought processes.

## Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted. Its purpose is to get students thinking about the different sizes of paper that are commonly used.
- In **Activity 1**, pre-cut the rectangles and offer the calculations for the side lengths of all the rectangles. This will allow students to focus on the relationships observed.

Pairs | 5 min

# **Warm-up** Notice and Wonder

Students compare two sheets of standard paper to notice their difference in size and to prepare them for analyzing their dimensions more closely in Activity 1.



## Projecting and Scaling

Let's explore scaling.

### **Warm-up** Notice and Wonder

**You will be given two sheets of paper. What do you notice? What do you wonder?**

1. I notice . . .
   Sample responses:
   - The papers are different sizes.
   - One paper is both wider and longer than the other paper.
   - One paper looks like the same size of paper that I am used to using.

2. I wonder . . .
   Sample responses:
   - Why standard-sized papers can be different sizes.
   - When would you use one size paper than the other?
   - Are these papers scaled copies of each other?

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Distribute one sheet of US Letter paper and one sheet of A4 paper to each pair of students. Tell them that both papers are a standard size, but for different countries. The US Letter is commonly used in North and South America, while A4 paper is commonly used in Europe and Asia. Then conduct the ***Notice and Wonder*** routine.

## 2 Monitor

**Help students get started** by asking them to visually compare the size of each paper.

**Look for points of confusion:**
- **Trying to measure the paper.** Have students visually compare the papers by placing one on top of the other.

**Look for productive strategies:**
- Noticing that the A4 paper is taller.
- Noticing that the A4 paper is narrower.
- Wondering why the papers are different sizes.

## 3 Connect

**Have students share** what they noticed and wondered. Record and display their responses for all to see.

**Display** the measurements of each paper. US Letter paper measures 216 mm by 280 mm and A4 paper measures 210 mm by 297 mm.

**Ask**, "What do you think the 4 in A4 represents? When someone decides on the dimensions of standard items, such as paper, what do they need to consider?"

**Note:** The A4 paper is part of the A series paper sizes, where each paper is half the size of the previous. For example, the A4 paper is half the size of the A3 paper, the A5 paper is half the size of the A4 paper, and so on. The A4 paper represents 4 half cuts of the A0 paper. The American National Standards Institute and International Organization for Standardization are two organizations that develop different standards.

Pairs | 30 min

# Activity 1 Sorting Rectangles

Students create different sets of rectangles to explore the properties of scaled copies.

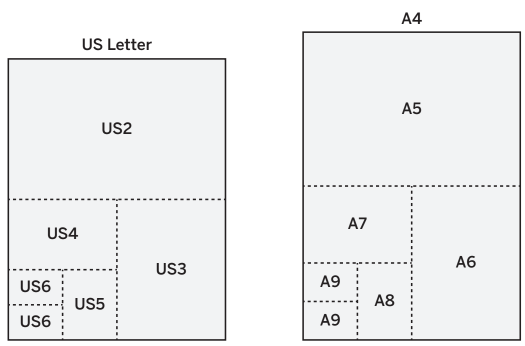
**Amps Featured Activity** Using Work From Previous Slides

Name: ______ Date: ______ Period: ______

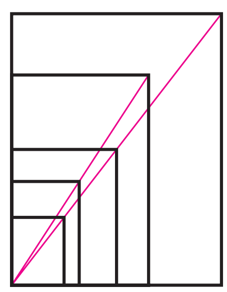
**Activity 1** Sorting Rectangles

**You will be given the materials for this activity.**

1. Cut each sheet of paper in half. Then cut each sheet in half again. Continue cutting each sheet in half, as illustrated in the diagram. Label each rectangle as shown.




2. For each sheet of paper, stack the rectangles so that they all line up at a corner, as shown in the diagram. You will have two stacks of rectangles. What do you notice?
**Sample responses: The rectangles created from the A4 paper look like scaled copies of each other, but the rectangles created from the US Letter paper do not look like scaled copies of each other. The bottom left corner and all of the top right corners of the A4 rectangles form a straight line, but the rectangles cut from the US Letter paper do not.**







## 1 Launch

Provide access to rulers, scissors, and calculators.

## 2 Monitor

**Help students get started** by modeling how to cut the paper. Hold the paper in a portrait orientation and cut it horizontally in half. Folding and creasing the paper may help provide a straight line for cutting. Remind students to label each rectangle immediately after cutting.

**Look for points of confusion:**

- **Not noticing any patterns for Problem 2.** Have students draw a line from the bottom left corner to the top right corner of the largest rectangle to help them see which corners align.
- **Writing different measurements and calculations than their partner or writing the ratios in a reverse order.** Have students compare their measurements and calculations with their peers and check for correctness after students complete Problem 5 to help them complete Problem 6.
- **Thinking that the ratio of the sides is the same as the scale factor.** Revisit this during the Connect. Additionally, this concept will be further explored throughout the unit.
- **Thinking that the scale factor of the successive rectangle is 2.** Because the paper is rotated each time, the scale factor of the rectangles compared in portrait orientation is not 2.

**Look for productive strategies:**

- Noticing the same ratios for the rectangles where the top right corners create a straight line.
- Noticing "one type of rectangle" from the A4 paper, and "two types of rectangles" from the US Letter paper.
- Noticing that the rectangles created from the A4 paper are scaled copies of each other.

**Activity 1 continued >**

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Have students complete the activity first using only the A4 paper and omit Problem 3. It is more important for them to notice that the rectangles created from the A4 paper are scaled copies.

### Extension: *Math Enrichment*

Show students other sizes of paper, such as Legal or Tabloid. Have them experiment to see whether rectangles created from these sizes of paper are also scaled copies.

| Size | Dimensions (mm) | Dimensions (in.) |
|---|---|---|
| Legal | $216 \times 356$ | $8.5 \times 14$ |
| Tabloid | $432 \times 279$ | $11 \times 17$ |

## Math Language Development

### *MLR2: Collect and Display*

During the Connect, as students share, collect the language they use. Ask them if there are more mathematically precise ways to say the same idea. For example, "the rectangles are the same, but smaller" can be restated as "the ratios of the corresponding side lengths are equivalent." Add mathematical words and phrases to a class display and encourage students to refer to the display during future discussions in this unit.

### English Learners

Have students perform the visual test described in the Connect section to make sense of the rectangles as scaled copies.

Pairs | 30 min

# Activity 1 Sorting Rectangles (continued)

Students create different sets of rectangles to explore the properties of scaled copies.

## Activity 1 Sorting Rectangles (continued)

3. Record the dimensions, to the nearest millimeter, for each rectangle created from US Letter paper.

| Rectangle | Length of short side (mm) | Length of long side (mm) | Ratio of long side to short side |
|---|---|---|---|
| Full sheet | 216 | 280 | 1.3 |
| US2 | 140 | 216 | 1.5 |
| US3 | 108 | 140 | 1.3 |
| US4 | 70 | 108 | 1.5 |
| US5 | 54 | 70 | 1.3 |
| US6 | 35 | 54 | 1.5 |

4. Record the dimensions, in millimeters, for each rectangle created from A4 paper.

| Rectangle | Length of short side (mm) | Length of long side (mm) | Ratio of long side to short side |
|---|---|---|---|
| Full sheet | 210 | 297 | 1.4 |
| A5 | 149 | 210 | 1.4 |
| A6 | 105 | 149 | 1.4 |
| A7 | 74 | 105 | 1.4 |
| A8 | 53 | 74 | 1.4 |
| A9 | 37 | 53 | 1.4 |

5. Calculate each ratio of the long side to the short side. Write the ratios in the table, rounding to the nearest tenth.

6. What do you notice about the ratios for the rectangles?
**Sample response: The side-length ratios are all the same for the rectangles made from the A4 paper, but the ratios are not all the same for the side lengths of the rectangles cut from the US Letter paper.**

STOP

**Collect and Display:** Your teacher will collect words and phrases you use. This language will be added to a class display for your reference.

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## 3 Connect

**Have students share** what they noticed about the ratios and rectangles.

### Highlight

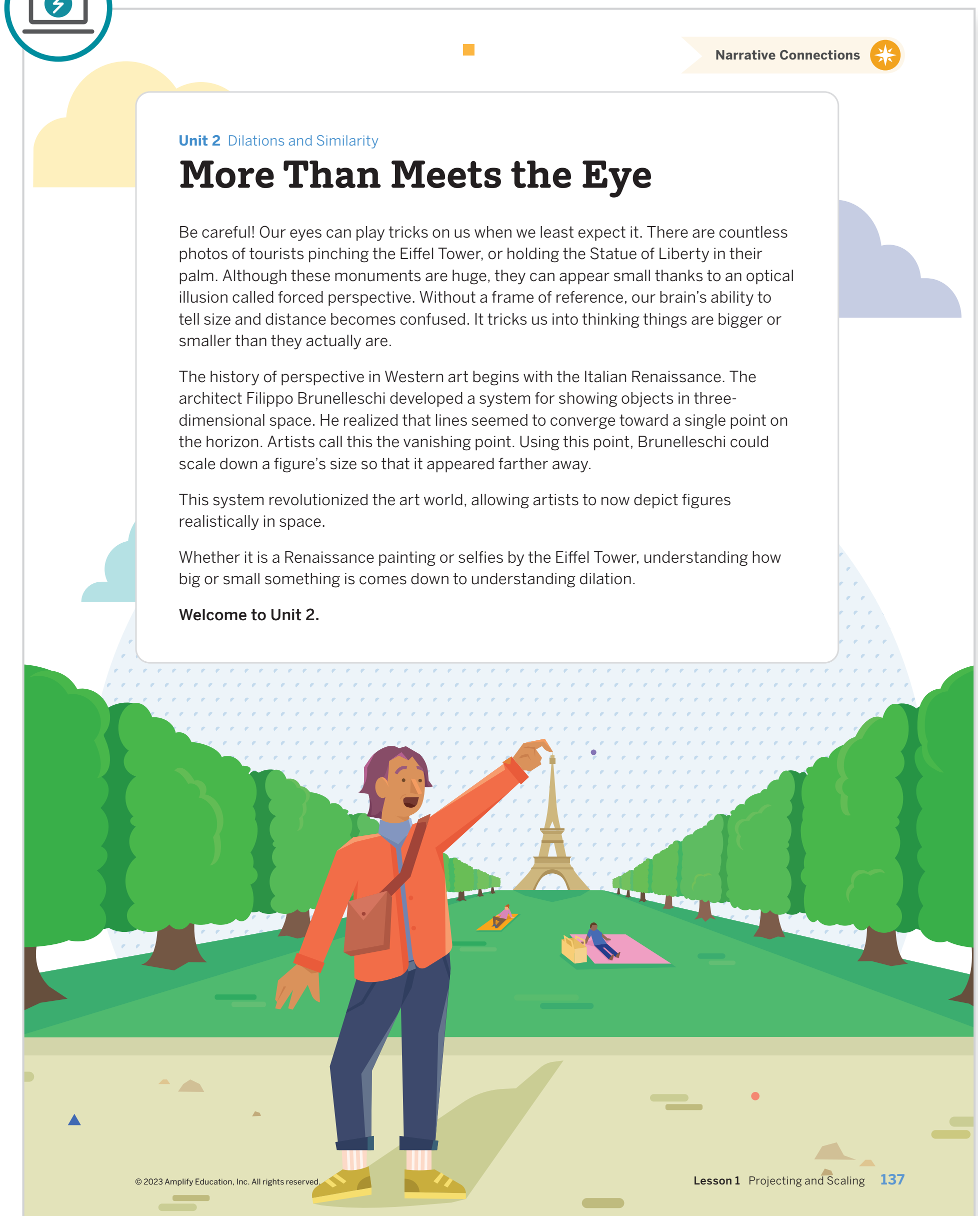
- For the US Letter paper, there are two groups of rectangles: one where the sides have a ratio of 1.5 and the other where the sides have a ratio of 1.3. When aligned at one corner, the rectangles with a ratio of 1.5 have opposite vertices that lie on the same line, and the rectangles with a ratio of 1.3 have opposite vertices that lie on the same line.
- When the A4 rectangles are aligned at one corner, the opposite vertices of *all* the rectangles lie on the same line, and *all* the ratios between the sides are equivalent.
- The types of rectangles created from the A4 paper are called *scaled copies*.
- A visual test may help students decide whether or not two cut-out figures are scaled copies of one another. The visual test involves holding each figure at a different distance from the eye and checking if it is possible to make the two figures match up exactly.

**Ask:**

- "How can you show that any two rectangles from the A4 sheet are scaled copies?" Sample response: The side lengths of one rectangle can be multiplied by the same number, called the *scale factor*, to get the corresponding sides of the second rectangle.
- "What scale factor takes A8 to the full A4 sheet?" 4 Emphasize that the scale factor is different than the ratio of the dimensions within the same rectangle.

# **Summary** More Than Meets the Eye

Review and synthesize the properties of a scaled copy.




## Narrative Connections

Read the narrative aloud as a class or have students read it individually.

## Synthesize

**Display** the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

**Have students share** how they can identify a scaled copy in their own words.

**Highlight** that a scaled copy is a copy of a figure where every length in the original figure is multiplied by the same number. This number is known as the *scale factor.*

**Ask**, "How can you use the scale factor to draw a scaled copy of a rectangle?" Sample response: Multiply the side lengths of the original rectangle by the scale factor to draw the new rectangle.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

- "What strategies or tools can help you identify scaled copies?"

Independent | 5 min

# Exit Ticket

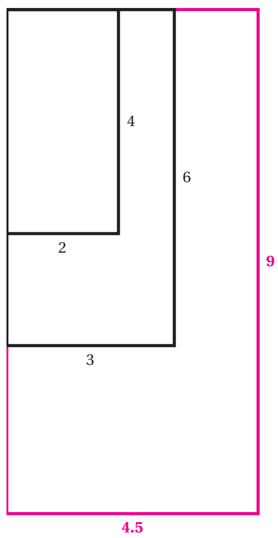
Students demonstrate their understanding of scaled copies by making observations and using those observations to draw a scaled copy of a rectangle.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 2.01

Refer to the rectangles shown.




1. Describe how you know that the rectangles are scaled copies of one another.
   Sample responses:
   - The ratio of the width to length in each rectangle is equivalent.
   - The longer side of the larger rectangle is $\frac{3}{2}$ of the longer side of the smaller rectangle. The shorter side of the larger rectangle is also $\frac{3}{2}$ of the shorter side of the smaller rectangle.
2. What is the scale factor that maps the smaller rectangle onto the larger rectangle?
   $\frac{3}{2}$
3. Use the same scale factor to draw a scaled copy of the larger rectangle, and then label the side lengths.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can decide whether one rectangle is a scaled copy of another rectangle.
1 2 3

b. I can describe features of a scaled copy of a rectangle.
1 2 3


**Lesson 1** Projecting and Scaling

## Success looks like . . .

- **Language Goal:** Describing features of scaled copies of a rectangle. **(Speaking and Listening)**
  » Explaining why the rectangles are scaled copies of each other in Problem 1.
- **Goal:** Identifying rectangles that are scaled copies of one another.

## Suggested next steps

**If students do not know why the rectangles are scaled copies of one another or cannot draw a rectangle that is a scaled copy, consider:**

- Reviewing scaled copies and scale factor.
- Having them find the ratio of the dimensions.

**If students think the scale factor is 2, consider:**

- Reassessing after Lesson 9, where this topic will be further explored.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

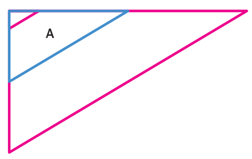
- What was especially satisfying about using rectangles from paper to look for scaled copies?
- Which groups of students did and did not have their ideas seen and heard today?

# Practice

Independent

Name: ______ Date: ______ Period: ______

**1.** Refer to Polygon A.




a. Draw a scaled copy of Polygon A using a scale factor of $\frac{1}{4}$.

b. Draw a scaled copy of Polygon A using a scale factor of 2.

**2.** Triangle $ABC$ is a scaled copy of Triangle $DEF$.
Side $AB$ measures 12 cm and is the longest side of Triangle $ABC$.
Side $DE$ measures 8 cm and is the longest side of Triangle $DEF$.

a. Triangle $ABC$ is a scaled copy of Triangle $DEF$ with what scale factor?
$\frac{12}{8} = \frac{3}{2}$ **(or equivalent)**

b. Triangle $DEF$ is a scaled copy of Triangle $ABC$ with what scale factor?
$\frac{8}{12} = \frac{2}{3}$ **(or equivalent)**

**3.** Rectangle A measures 12 cm by 3 cm. Rectangle B is a scaled copy of Rectangle A. Select *all* the measurement pairs that could be the dimensions of Rectangle B.

A. 6 cm by 1.5 cm

B. 10 cm by 1 cm

C. 18 cm by 4.5 cm

D. 6 cm by 1 cm

E. 80 cm by 20 cm




Name: ______ Date: ______ Period: ______

**4.** Which of these sets of angle measures could be the three interior angle measures of a triangle?

A. 40°, 50°, 60°

B. 50°, 60°, 70°

C. 60°, 70°, 80°

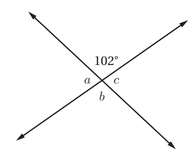
D. 70°, 80°, 90°

**5.** The diagram shows two intersecting lines. Determine the missing angle measures.

$a = 78°$

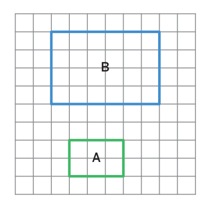
$b = 102°$

$c = 78°$




*The figure may not be drawn to scale.*

**6.** Refer to Rectangles A and B.




a. What scale factor maps Rectangle A onto Rectangle B?
**2**

b. What scale factor maps Rectangle B onto Rectangle A?
$\frac{1}{2}$




## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 1 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 1 | 2 |
| Spiral | 4 | Unit 1 Lesson 16 | 2 |
| | 5 | Unit 1 Lesson 14 | 2 |
| Formative | 6 | Unit 2 Lesson 2 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

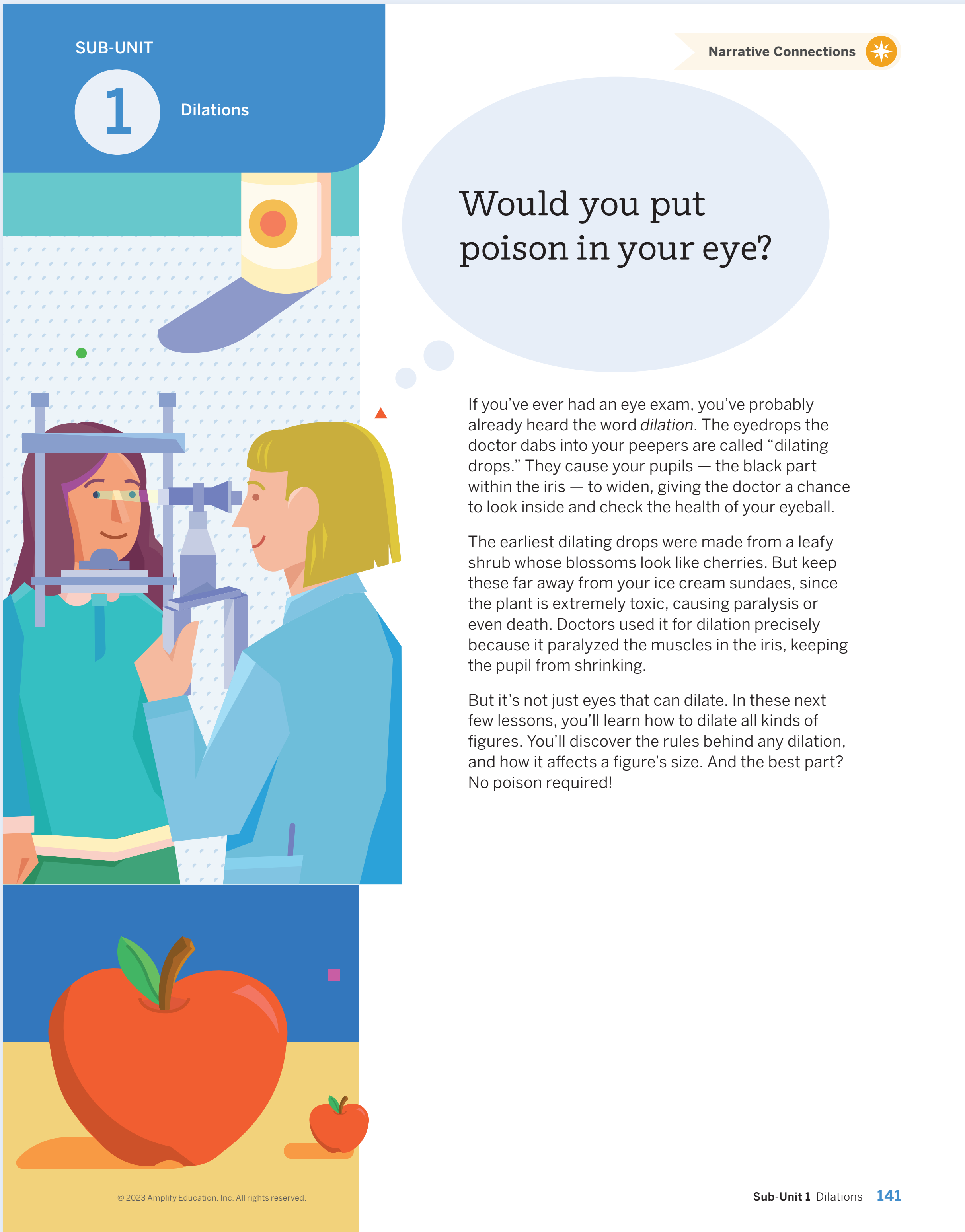
## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Sub-Unit 1
# Dilations

Whole Class

In this Sub-Unit, students study dilations and make connections to the rigid transformations they studied in Unit 1, before uncovering how artists used dilations to create perspective drawing and the illusion of 3D imagery.


SUB-UNIT

1 Dilations

Narrative Connections

## Would you put poison in your eye?

If you've ever had an eye exam, you've probably already heard the word *dilation*. The eyedrops the doctor dabs into your peepers are called "dilating drops." They cause your pupils — the black part within the iris — to widen, giving the doctor a chance to look inside and check the health of your eyeball.

The earliest dilating drops were made from a leafy shrub whose blossoms look like cherries. But keep these far away from your ice cream sundaes, since the plant is extremely toxic, causing paralysis or even death. Doctors used it for dilation precisely because it paralyzed the muscles in the iris, keeping the pupil from shrinking.

But it's not just eyes that can dilate. In these next few lessons, you'll learn how to dilate all kinds of figures. You'll discover the rules behind any dilation, and how it affects a figure's size. And the best part? No poison required!



Sub-Unit 1 Dilations 141

### Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students will explore the connections between dilations, the human eye, and what we can see in the following places:

- **Lesson 2, Activity 1:** A Droplet on the Surface
- **Lesson 3, Activity 3:** Perspective Drawing
- **Lesson 4, Warm-up:** Estimating a Scale Factor

# Circular Grids

Let's dilate figures on circular grids.

## Focus

### Goals

1. **Language Goal:** Comprehend the term *dilation* as a transformation of a figure that produces scaled copies of that figure. **(Speaking and Listening)**
2. Create dilations of polygons using a circular grid, given a scale factor and the center of dilation.
3. **Language Goal:** Explain how a dilation affects the size, side lengths, and angles of polygons. **(Speaking and Listening)**
4. **Language Goal:** Explain the effect of the scale factor and its distance from the center of dilation. **(Speaking and Listening)**

## Coherence

### Today

Students are formally introduced to dilations and a method for producing dilations using a circular grid. Students notice that a dilation produces a scaled copy and describes how a scale factor affects the size, side lengths, and angles of a polygon.

### Previously

In Lesson 1, students were introduced to the general idea of a dilation as a method for producing scaled copies of geometric figures.

### Coming Soon

In this Sub-unit, students will apply dilations to points without a grid, and then move to applications on a square grid, solidifying their understanding of the relationship between a polygon and its dilated image.

## Rigor

- Students build **conceptual understanding** of dilations.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice | Independent

### Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: rulers, protractors

### Math Language Development

**New words**

- *center of dilation*
- *dilation*

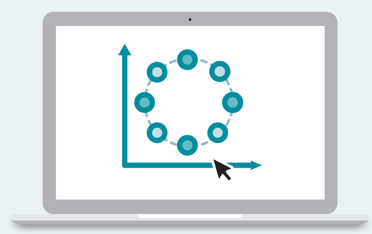
**Review words**

- *scaled copy*
- *scale factor*
- *optical illusion*

**Amps** | Featured Activity

### Activity 2
Overlay Graphs

You can overlay all student-created dilations and provide immediate feedback.



Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students who are more confident with the work in Activity 2 may be able to lead discussions within their groups about the structure of scaled copies after a dilation of the vertices of the original polygon. Remind students to "step up" if they have something to add to the conversation, but also to "step back" to give other voices a chance to share.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, have students only complete the dilations using two points.

Independent | 5 min

# Warm-up Notice and Wonder

Students analyze an optical illusion as an introduction to circular grids.

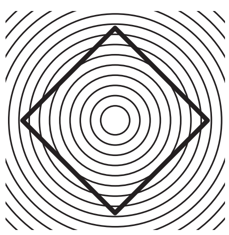
Unit 2 | Lesson 2

## Circular Grids

Let's dilate figures on circular grids.

### Warm-up Notice and Wonder

**Consider the following optical illusion. What do you notice? What do you wonder?**



1. I notice . . .
   Sample responses:
   - There are multiple circles with the same center.
   - The circles seem to be equally spaced apart.
   - There is a darker shape in the middle that looks like it might be a polygon (square).
2. I wonder . . .
   Sample responses:
   - Is the shape in the center really a polygon?
   - Is the shape in the center really a square?
   - Are the "sides" of the shape in the center straight line segments?

Log in to Amplify Math to complete this lesson online.

142 Unit 2 Dilations and Similarity



## 1 Launch

Conduct the ***Notice and Wonder*** routine.

## 2 Monitor

**Help students get started** by asking what shapes they notice in the image.

**Look for productive strategies:**
- Noticing that all the circles have the same center.
- Noticing that the distances between the circles are the same.
- Noticing that the radii of the circles are increasing.

## 3 Connect

**Have students share** what they noticed and wondered. Record and display their responses for all to see.

**Ask**, "How can you check whether the figure is a square?" Sample response: Use a ruler and protractor to check for straight, equal sides and right angles.

**Highlight** that *optical illusions* trick people's brains into thinking that the distance and size of the objects are different from what they actually are. Tell students that at the end of the unit they will create their own optical illusions, but in order to do so, they will need to learn how to make sense of the "tricks" used to create them.

## Power-up

**To power up students' ability to identify a scale factor, have students complete:**

Recall that the *scale factor* between two figures is the value that the original figure's side lengths are multiplied by to produce the scaled copy.

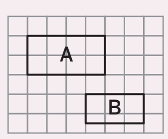



Figure B is a scaled copy of Figure A.

1. What is the length across the top of Figure A. 4 units
2. What is the length across the top of Figure B? 3 units
3. Is the *scale factor* used to map Figure A onto Figure B greater than or less than 1? Less than 1
4. What is the *scale factor* used to map Figure A onto Figure B? $\frac{3}{4}$

**Use:** Before Activity 2
**Informed by:** Performance on Lesson 1, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 1

Pairs | 15 min

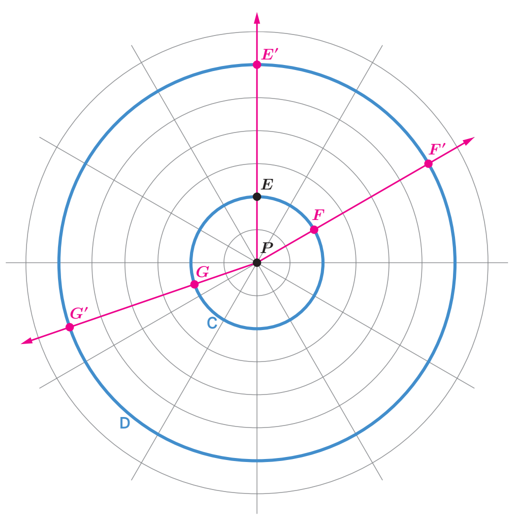
# Activity 1 A Droplet on the Surface

Students examine the ratios of the distance of points on a circle that maps to another circle to gain an understanding of the terms *dilation* and *center of dilation*.

Name: ______ Date: ______ Period: ______

## Activity 1 A Droplet on the Surface

**In the circular grid, the distance from one circle to the next is the same. The radius of the innermost circle is 1 unit. The radius of each successive circle is 1 unit more than the radius of the previous circle. All the circles share the same center, point $P$. Circle C, Circle D, and point $E$ are marked on the grid.**




1. Plot two more points on Circle C. Label them $F$ and $G$.

**1–3. Sample responses shown on grid.**

2. Draw rays from point $P$ through the three points on Circle C. Extend the rays past Circle D.
3. Plot points where the ray intersects Circle D. Label the corresponding points $E'$, $F'$, and $G'$.
4. In the table, write the distance, in units, from point $P$ to each point you drew.
5. Find the ratio of the distances from the image of each point to the preimage point. What do you notice?
**The ratios are all equivalent to 3. Sample responses shown in table.**

| Distance from point $P$ | | | |
|---|---|---|---|
| Point $E$ | 2 | Point $E'$ | 6 |
| Point $F$ | 2 | Point $F'$ | 6 |
| Point $G$ | 2 | Point $G'$ | 6 |





### 1 Launch

Ask students to imagine the circles that are formed when a pebble is dropped in a still pond. Provide access to rulers.

### 2 Monitor

**Help students get started** by asking, "How can you use the grid to determine the distance from point $P$ to $E$?" Count the number of circles starting from $P$.

**Look for points of confusion:**

- **Not remembering how to draw a ray in Problem 2.** Remind students that a ray begins at a point and continues infinitely in one direction.
- **Questioning how to find the ratios in Problem 5.** Ask, "How do the distances of the points on Circle D compare to the distances of the points on Circle C?" Emphasize the order when writing ratios.

### 3 Connect

**Have students share** their observations.

**Highlight** that because the ratios of the distances from the points on Circle D to Circle C are all equivalent to 3, Circle D is a scaled copy of Circle C using the scale factor of 3.

**Define** the terms ***dilation*** and ***center of dilation***. Activate background knowledge that students may have heard the word *dilation* in the eye doctor's office. A *dilation* is a transformation in which each point on a figure moves along a ray and changes its distance from a fixed point. This point is called the *center of dilation*. All the original distances are multiplied by the same scale factor. Because a dilation is a transformation, the corresponding points are labeled using prime notation.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Provide copies of a pre-plotted, pre-labeled circular grid for Problems 1–3. Then have students complete Problems 4 and 5. This will allow them to grasp the concepts of this activity without actually doing the plotting and labeling themselves.

### Extension: *Math Enrichment*

Have students determine the circumference and area of Circles C and D and describe what they notice. Circle C's circumference: about 12.56; Circle C's area: about 12.56. Circle D's circumference: about 37.68 (about 3 times that of Circle C); Circle D's area: about 113.04 (about 9 times that of Circle C).

## Math Language Development

### *MLR8: Discussion Supports—Press for Details*

During the Connect, as students share their observations, press for details in their reasoning. If they say, for example, "The distances are all either 2 or 6," ask them to clarify what this means for the two circles. Consider revoicing to highlight developing language and amplify precise language, such as, "The ratio of the distances from the points on Circle D to Circle C is 3."

### English Learners

Use gestures, such as pointing to each circle or each distance, as you highlight mathematical language.

Pairs | 15 min

# Activity 2 Partner Problems: A Quadrilateral on a Circular Grid

Students dilate points on a polygon to see how the scale factor affects the image, coming to an understanding that dilating *only* the vertices produces the same image.

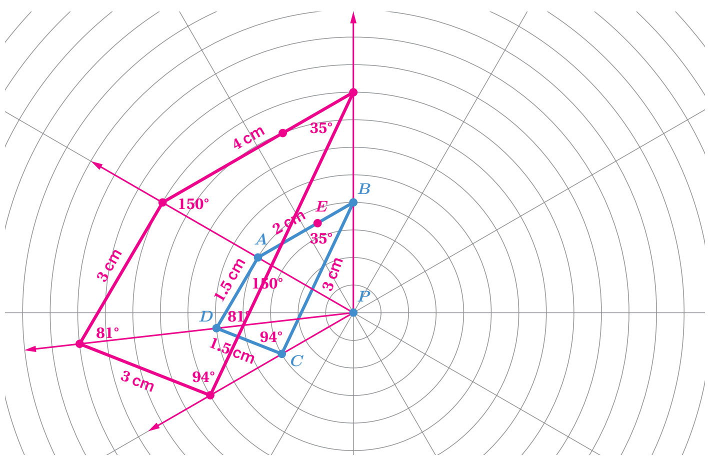
Amps Featured Activity | Overlay Graphs

### Activity 2 Partner Problems: A Quadrilateral on a Circular Grid

**Consider Polygon $ABCD$. With your partner, decide who will complete Column A and who will complete Column B.**

| Column A | Column B |
|---|---|
| **1.** Plot a point on any side of Polygon $ABCD$. Label the point $E$. | Plot a point on any side of Polygon $ABCD$. Label the point $E$. |
| **2.** Dilate points $A$, $B$, $C$, $D$, and $E$ using point $P$ as the center of dilation and a scale factor of 2. | Dilate points $A$, $B$, $C$, $D$, and $E$ using point $P$ as the center of dilation and a scale factor of $\frac{1}{2}$. |
| **3.** Draw segments between the dilated points to create a new polygon. | Draw segments between the dilated points to create a new polygon. |
| **4.** Measure the sides and angles of both polygons. | Measure the sides and angles of both polygons. |

Column A response:







## 1 Launch

Have students explain *dilation* and *center of dilation* in their own words to strengthen their understanding. Conduct the ***Think-Pair-Share*** routine. With a partner, have students choose either Column A *or* Column B to complete individually before sharing their responses with their partner. Provide access to geometry toolkits.

## 2 Monitor

**Help students get started** by telling them that the scale factor will help them determine the distance of each corresponding point from the center of dilation.

**Look for points of confusion:**

- **Having trouble dilating any points.** Have students draw rays from point $P$ through each point and count the distance.
- **Not knowing how to dilate using the scale factor of $\frac{1}{2}$.** Have students determine the distance from point $P$ to each point and then calculate half of that distance to plot the dilated points.

**Look for productive strategies:**

- Noticing that dilating the vertices of the original polygon produces a scaled copy of the figure.

Activity 2 continued >

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Consider one of these alternative approaches to this activity.

- Have pairs complete one column and then meet with another pair who completed the other column to respond to Problem 5.
- Provide copies of pre-dilated polygons and have students respond to Problems 5 and 6.
- Have students use the Amps slides for this activity, in which they can digitally dilate the polygons.

### Extension: *Math Enrichment*

Have students determine the scale factor that takes the polygon in Column A to the polygon in Column B. $\frac{1}{4}$

## Math Language Development

### *MLR2: Collect and Display*

During the Connect, as students share their methods and observations, collect and display phrases and images that they use to describe dilations of polygons. For example, *scale factor, distance, center of dilation, scaled copy, vertices*, etc.

### English Learners

To support student understanding, use gestures to show how a dilation changes a figure's size. Be sure to use gestures that illustrate how an image is enlarged and also how an image is reduced.

# Activity 2 Partner Problems: A Quadrilateral on a Circular Grid (continued)

Pairs | 15 min

Students dilate points on a polygon to see how the scale factor affects the image, coming to an understanding that dilating *only* the vertices produces the same image.

Name: ______ Date: ______ Period: ______

### Activity 2 Partner Problems: A Quadrilateral on a Circular Grid (continued)

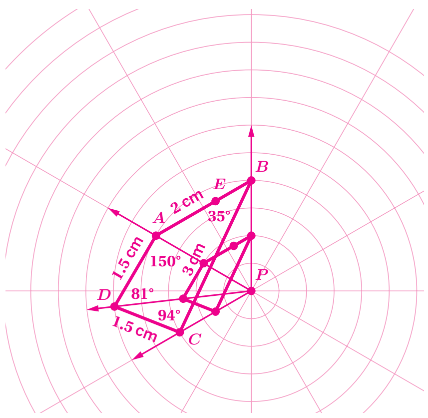
5. Compare your work with your partner. How does the dilation affect the polygon's size, side lengths, and angles? List as many observations as you can.

   Sample responses:
   - The image of the polygon is smaller if it is dilated by a scale factor less than 1 and is larger if the scale factor is greater than 1.
   - The corresponding sides are twice or half the length of the original polygon, depending on the scale factor.
   - The corresponding sides are parallel and the corresponding angles are congruent.

6. Andre says that to dilate Polygon $ABCD$, he can just dilate the vertices and connect them. Do you agree? Why or why not?

   Sample response: I agree. I noticed that when point $E$ was dilated, the image was plotted on the corresponding side of the dilated polygon at double (or half) the distance as the original point.

Column B response:




STOP





## 3 Connect

**Display** correct student work.

**Have students share** their methods of dilations and observations of the dilated figures.

**Ask**, "How does the scale factor affect a point's distance from the center of dilation? Do the resulting polygons produce scaled copies?"

**Highlight** that a scaled copy is produced during a dilation. To dilate a polygon, there is no need to dilate points that are not vertices that lie on each side of the polygon. Instead, students can dilate just the vertices and then connect them. The size of the image depends on the size of the scale factor.

Whole Class | 5 min

# Summary

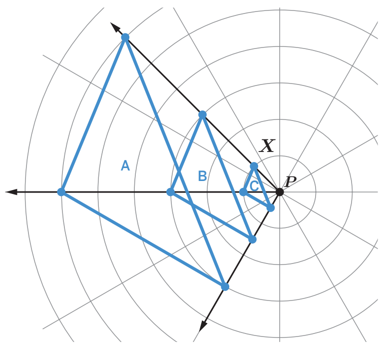
Review and synthesize how dilations are transformations that produce scaled copies and how dilations can be performed on a circular grid.

## Summary

**In today's lesson . . .**

You performed dilations of a figure. A ***dilation*** is a transformation which is defined by a fixed point $P$, called the ***center of dilation***, and a scale factor $k$. In the figure shown, the dilation moves each point $X$ into a point $X'$ along ray $PX$ such that its distance from a fixed point changes by the scale factor. A scale factor greater than $1$ produces a larger scaled copy, and a scale factor less than $1$ produces a smaller scaled copy.

- Triangle B is dilated using point $P$ as the center of dilation and a scale factor of 2 to produce Triangle A.
- Triangle B is dilated using point $P$ as the center of dilation and a scale factor of $\frac{1}{3}$ to produce Triangle C.




**Reflect:**




## Synthesize

**Have students share** what a dilation is in their own words.

**Ask:**

- "What happens to a figure if it is dilated by a scale factor that is greater than 1? Less than 1?"
- "How do the size, side lengths, and angles of the original figure compare with the dilated figure?"

**Highlight:**

- Polygons can be dilated by dilating the vertices and drawing segments between them. This produces a scaled copy, which means that the corresponding side lengths are proportional and corresponding angles are congruent.
- A scale factor greater than 1 will enlarge the side lengths at the same scale, where the corresponding sides are parallel on the circular grid.
- A scale factor less than 1 will reduce the side lengths at the same scale, where the corresponding sides are parallel on the circular grid.

**Formalize vocabulary:**

- *center of dilation*
- *dilation*

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does it mean to dilate a figure?"

## Math Language Development

### *MLR2: Collect and Display*

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *center of dilation* and *dilation* that were added to the display during the lesson.

Independent | 5 min

# Exit Ticket

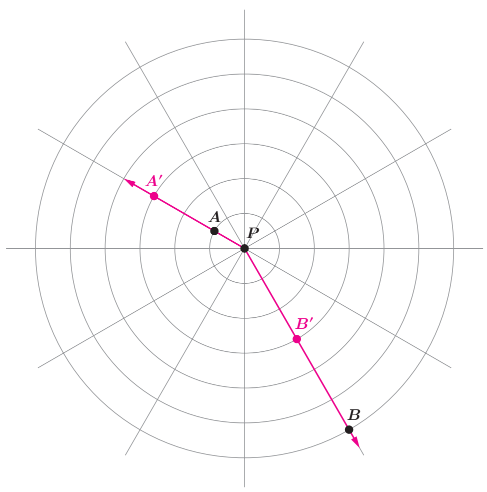
Students demonstrate their understanding by dilating points with a scale factor greater than 1 and less than 1.

Printable

Name: ______________ Date: ________ Period: ________

## Exit Ticket 2.02

**Refer to the circular grid with plotted points $A$, $P$, and $B$.**




1. Dilate point $A$ using point $P$ as the center of dilation and a scale factor of 3. Label the new point $A'$.

2. Dilate point $B$ using point $P$ as the center of dilation and a scale factor of $\frac{1}{2}$. Label the new point $B'$.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can apply dilations to figures on a circular grid when the center of dilation is the center of the grid.

1 2 3



**Lesson 2** Circular Grids

## Success looks like . . .

- **Language Goal:** Comprehending the term *dilation* as a transformation of a figure that produces scaled copies of that figure. **(Speaking and Listening)**
  » Dilating points $A$ and $B$ with different scale factors.
- **Goal:** Creating dilations of polygons using a circular grid, given a scale factor and the center of dilation.
- **Language Goal:** Explaining how a dilation affects the size, side lengths, and angles of polygons. **(Speaking and Listening)**
- **Language Goal:** Explaining the effect of the scale factor and its distance from the center of dilation. **(Speaking and Listening)**

## Suggested next steps

**If students do not dilate the points correctly, consider:**

- Reviewing scale factors using Activity 2.
- Asking students to draw rays, and then asking them to dilate the points.
- Reassessing after Lesson 3.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- How did the circular grid set students up to develop their understanding of dilations?
- During Partner Problems in Activity 2, how did you encourage each student to listen to one another's strategies?

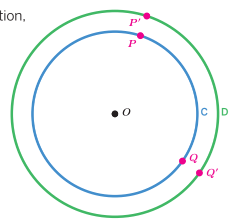
# Practice

Independent

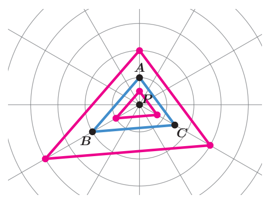
Name: ______________ Date: ________ Period: ________

**Practice**

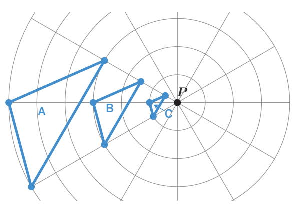
1. Refer to Circles C and D. Point $O$ is the center of dilation, and the dilation maps Circle C onto Circle D.
   - a. Plot a point on Circle C and label it $P$. Plot point $P'$, the image of $P$, after the dilation.
     Sample responses shown.
   - b. Plot a point on Circle D and label it $Q'$. Plot point $Q$ so it will map onto point $Q'$ after the dilation.
     Sample responses shown.




2. Refer to Triangle $ABC$.
   - a. Dilate Triangle $ABC$ using point $P$ as the center of dilation and a scale factor of 2.
   - b. Dilate Triangle $ABC$ using point $P$ as the center of dilation and a scale factor of $\frac{1}{2}$.
   - c. How do the sides and angles of the two dilated triangles compare to each other? List as many observations as you can.
     Sample responses:
     - The side lengths of the largest triangle are 4 times the side lengths of the smallest triangle.
     - The corresponding angle measures are the same for all of the triangles.




3. Triangles A, B, and C are scaled copies of each other. One of the triangles was dilated using point $P$ as the center of dilation and a scale factor of 2. The same triangle was dilated using point $P$ as the center of dilation and a scale factor of $\frac{1}{3}$. Which is the original triangle? Explain your thinking.
   Triangle B is the original triangle. A scale factor greater than 1 will produce a larger triangle, and a scale factor less than 1 will produce a smaller triangle.



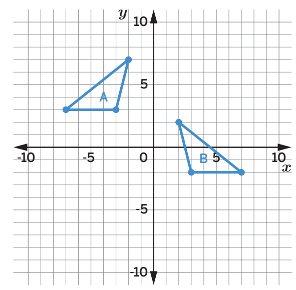





Name: ______________ Date: ________ Period: ________

**Practice**

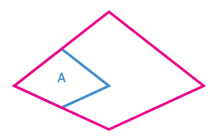
4. Describe a sequence of transformations that you could use to show that Triangles A and B are congruent.
   Sample response: Reflect Triangle A across the $y$-axis, and then translate it down 5 units.

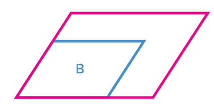



5. Mai makes trail mix by combining 3 cups of raisins with 7 cups of oats.
   - a. How many cups of raisins should be added to 1 cup of oats?
     $\frac{3}{7}$ cups
   - b. What is the constant of proportionality?
     $\frac{3}{7}$

| Oats (cups) | Raisins (cups) |
|---|---|
| 1 | ? |
| 7 | 3 |

6. Consider Polygons A and B.
   - a. Draw a scaled copy of Polygon A using a scale factor of 2.
   - b. Draw a scaled copy of Polygon B using a scale factor of $\frac{3}{2}$.








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 1 Lesson 11 | 2 |
| | 5 | Grade 7 | 2 |
| Formative | 6 | Unit 2 Lesson 3 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Dilations on a Plane

Let's dilate figures without a grid.

## Focus

### Goals

1. Create a dilation of a figure, given a scale factor and the center of dilation.
2. Identify the center, scale factor, and image of a dilation without a circular grid.

## Coherence

### Today

Students apply dilations to points on a plane without the structure of a grid. They practice identifying centers of dilation, scale factors, and images of dilation. Students must think about the dilations in terms of the given information and make decisions about which measurement tools will help them accomplish their goals.

### Previously

In Lesson 2, students were formally introduced to dilations and they explored how to perform a dilation on a circular grid.

### Coming Soon

In Lesson 4, students will begin to dilate figures using the structure of a square grid.

## Rigor

- Students build **conceptual understanding** of dilations on a plane.
- Students **apply** their knowledge of dilations to find missing information, such as the center of dilation, scale factor, or images of dilation.

## Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Activity 3 (optional) | Summary | Exit Ticket |
|---|---|---|---|---|---|
| 5 min | 15 min | 15 min | 15 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Independent | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF (for display)
- Anchor Chart PDF, *Dilations*
- geometry toolkits: rulers or index cards

### Math Language Development

**Review words**

- *dilation*
- *center of dilation*
- *scale factor*
- *scaled copy*

**Amps** | Featured Activity

**Exit Ticket**

**Real-Time Exit Ticket**

Check in real time if your students can identify the center of dilation, preimage, and image using a digital Exit Ticket.

Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel lost as they attempt dilations without the structure of a grid. Encourage students to look back at their work from Unit 1, and consider the tools they have available (rulers, etc.) to assist in understanding.

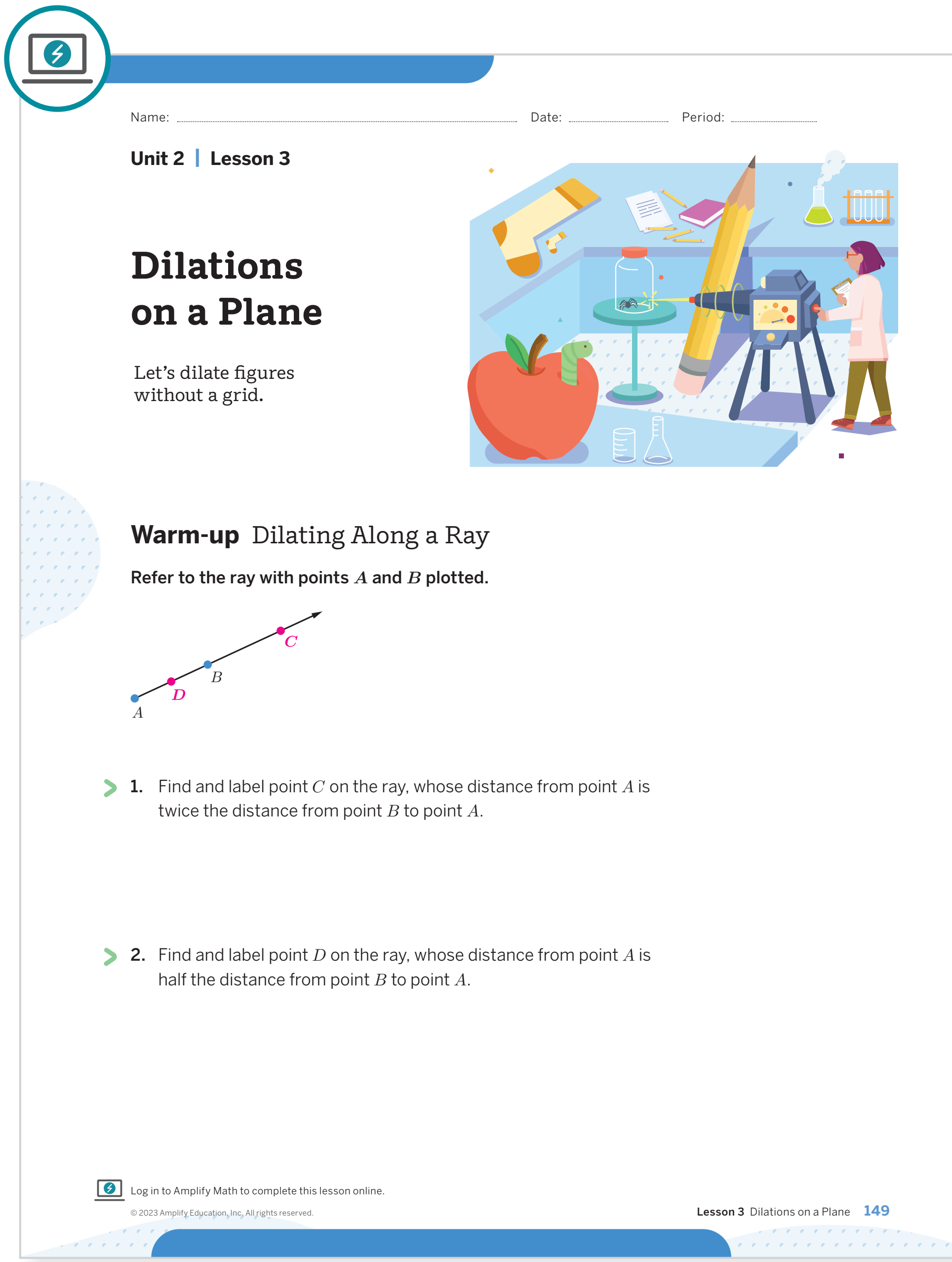
### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- Omit Problems 2 and 3 from **Activity 1**.
- Optional **Activity 3** may be omitted.

# **Warm-up** Dilating Along a Ray

Students find the image of a point along a ray to understand how to perform dilations using only measurement tools, without the structure of a grid.


Name: Date: Period:

Unit 2 | Lesson 3

## Dilations on a Plane

Let's dilate figures without a grid.

**Warm-up** Dilating Along a Ray

**Refer to the ray with points $A$ and $B$ plotted.**



1. Find and label point $C$ on the ray, whose distance from point $A$ is twice the distance from point $B$ to point $A$.

2. Find and label point $D$ on the ray, whose distance from point $A$ is half the distance from point $B$ to point $A$.

Log in to Amplify Math to complete this lesson online.

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Lesson 3 Dilations on a Plane 149

## 1 Launch

Review the terms *center of dilation* and *scale factor*. Provide access to geometry toolkits for the duration of this lesson.

## 2 Monitor

**Help students get started** by asking them to explain what it means to "dilate a point."

**Look for points of confusion:**

- **Estimating the placement of points $C$ and $D$ without the use of a measurement tool.** This is acceptable at this point in the lesson. Ask these students to share first during the Connect, and then ask if there is a more precise method to guarantee the exact location of the images.
- **Switching the placement of points $C$ and $D$.** Ask, "What does it mean for a point to be twice the distance away? Is it closer or farther away?"

**Look for productive strategies:**

- Using a ruler to measure distances.

## 3 Connect

**Have students share** their methods for plotting points $C$ and $D$. Begin with students who approximated the locations without any measurement tool, followed by students who used a ruler. Finally, ask (or show) how students can find point $D$ by marking the length of the ray on an index card or slip of paper and then folding it in half.

**Highlight** the similarities and differences of performing dilations on a circular grid.

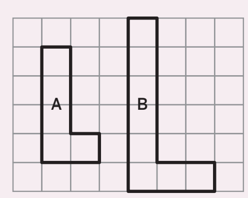
**Ask**, "How do rays help you dilate points on a plane?"

## Power-up

**To power up students' ability to identify scaled figures, have students complete:**

Recall that a *scaled copy* is a copy of a figure where every length in the original figure is multiplied by the same value to determine the corresponding lengths in the copy.

Compare the lengths in Figure A with the lengths in Figure B to determine if they are scaled copies. Be prepared to explain your thinking.




No, they are not scaled copies; Sample response: To go from the left side of Figure A to Figure B you multiply by 1.5 but to go from the top of Figure A to FIgure B you multiply by 1.

**Use:** Before Activity 2
**Informed by:** Performance on Lesson 3, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 7

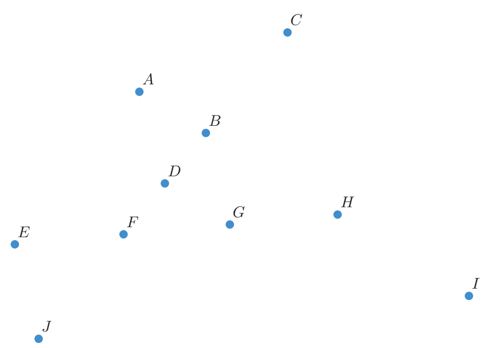
Pairs | 15 min

# Activity 1 Dilation Obstacle Course

Students identify missing information to reinforce the idea that the preimage, image, and center of dilation of a point are along the same line.

### Activity 1 Dilation Obstacle Course

Refer to the points $A$ through $J$ shown.




1. Dilate point $B$ using point $A$ as the center of dilation and a scale factor of 5. Which point is its image?
   **The image is point $I$.**
2. Dilate point $G$ using point $H$ as the center of dilation, so that its image is point $E$. What scale factor did you use?
   **I used a scale factor of 3.**
3. Dilate point $E$ using point $H$ as the center of dilation, so that its image is point $G$. What scale factor did you use?
   **I used a scale factor of $\frac{1}{3}$.**
4. Using point $B$ as the center of dilation, dilate point $H$ so that its image is itself. What scale factor did you use?
   **The scale factor must equal 1 for a preimage to map onto itself.**

**Are you ready for more?**

Tyler and Diego want to find a center of dilation in order to dilate point $F$ so that its image is point $B$. Tyler thinks the center is point $J$, while Diego thinks it is point $C$. Who is correct? Explain your thinking.

**They are both correct; Sample response: The center of dilation can be point $J$ if the scale factor is greater than 1, or point $C$ if the scale factor is less than 1.**




## 1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

## 2 Monitor

**Help students get started** by revisiting the Warm-up, and ask students to describe the process for dilating a point. Help students make the connection that drawing rays would be beneficial in this activity.

**Look for points of confusion:**

- **Switching the scale factors in Problems 2 and 3.** Ask students to locate the center of dilation and the preimage, place a finger on the preimage, and then move it to the image. Ask, "Did your finger get closer to the center of dilation, or farther away? What does that imply about the scale factor?"

**Look for productive strategies:**

- Identifying the center of dilation first and using it to draw a ray through the preimage.
- Using an index card to mark units of distance.

## 3 Connect

**Ask:**

- "What do you notice about your responses for Problems 2 and 3?" It is acceptable at this point if students do not recognize the reciprocals as this idea will be revisited in Activity 2.
- "Why must the scale factor be 1 in Problem 4?" Any number times 1 is itself.

**Highlight** that the preimage, image, and center of dilation always fall on a ray, with the center of dilation as the endpoint.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Some students may be distracted by all of the points. The goal of this activity is for students to realize that the preimage, image, and center of dilation of a point all lie on the same line. Display or provide copies of the image with only the relevant points for each problem.

### Accessibility: *Optimize Access to Tools*

Suggest that students use a ruler or index card to help measure distances.

### Extension: *Math Enrichment*

Have students draw and label a point $P$ on a separate sheet of paper. Ask them to draw and label a point $Q$ so that $Q$ is 1 in. away from point $P$. Have them perform the following dilations.

- Dilate point $Q$ using point $P$ as the center of dilation and a scale factor of 3. Label the image point $Y$. How far is point $Y$ from point $P$? 3 in. From point $Q$? 2 in.
- Dilate point $P$ using point $Q$ as the center of dilation and a scale factor of 3. Label the image point $Z$. How far is point $Z$ from point $P$? 2 in. From point $Q$? 3 in. From point $Y$? 5 in.

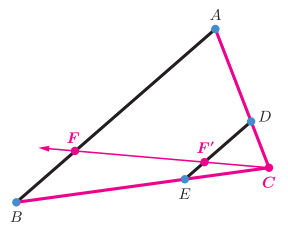
# Activity 2 Dilating a Line Segment

Students examine the dilation of a line segment to find the center of dilation and explore the effects of the size of the scale factor on a figure.

Name: ______________ Date: ________ Period: ________

### Activity 2 Dilating a Line Segment

**Mai dilated line segment $AB$ to create the image, segment $DE$, but erased her center of dilation.**




1. Use a ruler to find and draw Mai's center of dilation. Label it point $C$.

2. What is the scale factor of the dilation? Explain or show your thinking.
   The scale factor is $\frac{1}{3}$; Sample response: To find the scale factor, I divided the length of segment $CD$ by the length of segment $CA$.

3. Choose a point on segment $AB$ and label it point $F$. Find the precise location of point $F'$, the image of the dilation of point $F$. Explain or show your thinking.
   Locations for point $F$ may vary. Sample response: I drew a ray beginning at point $C$ that passes through point $F$ and intersects segment $DE$. The point where the ray and segment $DE$ intersect is point $F'$.

4. How would the scale factor change if segment $DE$ is the preimage and segment $AB$ is the result of the dilation?
   The scale factor would be 3; Sample response: Because segment $AB$ is longer than segment $DE$, the scale factor must be greater than 1. To find the scale factor, I divided the length of segment $CA$ by the length of segment $CD$.




## 1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

## 2 Monitor

**Help students get started** by asking them to discuss which point is the image of point $A$ and which is the image of point $B$ with their partner.

**Look for points of confusion:**

- **Not knowing how to find the center of dilation.** Have students draw a ray that starts from point $B$ and passes through point $E$.
- **Thinking the scale factor is 3 in Problem 2.** Ask whether the preimage is larger or smaller than the image, and what effect they expect this to have on the scale factor.

**Look for productive strategies:**

- Finding the scale factor by dividing the length of line segment $DE$ by the length of line segment $AB$, or by dividing the length of line segment $CD$ (or line segment $CE$) by the length of line segment $CA$ (or line segment $CB$).
- Measuring the distance from point $F$ to an endpoint, and multiplying this distance by the scale factor to find the location of point $F'$.
- Recognizing that the scale factor in Problem 4 is a reciprocal of the scale factor in Problem 2.

## 3 Connect

**Have pairs of students share** their strategies for finding the center of dilation and the scale factor. Select students who measured the lengths of the line segments and students who measured the distances from the center of dilation.

**Ask**, "How did you decide where to place point $F'$ on line segment $DE$?" Mention that finding a point of intersection on a ray and using a scale factor are both valid methods.

**Highlight** that the scale factor is the ratio of the image to the preimage. If the preimage and image are switched, the new scale factor is the reciprocal of the original scale factor.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Demonstrate how to draw rays to find the center of dilation in Problem 1. Show how the rays intersect in only one point and that this point is the center of dilation.

### Extension: *Math Enrichment*

Ask students to dilate line segment $AB$ using the same scale factor and a different center of dilation, and compare their resulting image with line segment $DE$. Student responses may vary.

## Math Language Development

### *MLR7: Compare and Connect*

During the Connect, as students share their strategies, have them compare the different solution pathways for finding the scale factor. Connect these strategies to the ratio of the image to the preimage, and what the reciprocal of that ratio means. Amplify the language students use to describe how the scale factor affects the size of the image and its distance from the center of dilation.

### English Learners

Encourage students to borrow from the class display as they use their developing mathematical language.

Independent | 15 min

# Activity 3 Perspective Drawing

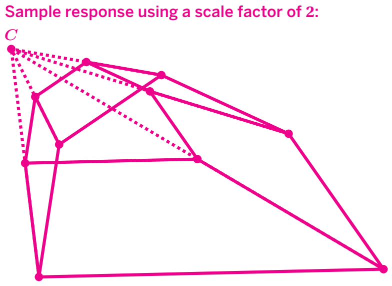
Students use dilations to create the illusion of three dimensions on a two-dimensional plane.

### Activity 3 Perspective Drawing

**A perspective drawing is an optical illusion that allows for an image printed or drawn on two-dimensional paper to have a three-dimensional look. You will use dilations to create a perspective drawing.**

1. In the space provided at the bottom of this page, draw a polygon.
2. Choose a point outside the polygon to use as the center of dilation. Label it point $C$.
3. Using your center as point $C$ and a scale factor of your choosing, dilate the polygon. Record the scale factor you use.
4. Draw a segment that connects each of the original vertices with its image. This will allow your diagram to look like a three-dimensional drawing! If time allows, you can shade the sides to make it look more realistic.
5. Compare your drawing with the drawings of other students. Talk about these questions.
   - What is the same, and what is different?
   - How do the choices you made affect the final drawing?
   - Was your dilated polygon closer to point $C$ than to the original polygon, or farther away? How do you decide this?

**Answers may vary.**
**Sample response using a scale factor of 2:**




**Reflect:** What good choices did you make about your personal behavior today?

STOP





## 1 Launch

**Ask**, "How could you use dilations to create artwork?" Display the Activity 3 PDF featuring examples of artwork with and without perspective. Ask students to compare the two examples and conduct the ***Notice and Wonder*** routine.

## 2 Monitor

**Help students get started** by demonstrating how to create a perspective drawing.

**Look for points of confusion:**
- **Thinking their center of dilation must be in a specific location.** Any point is fine as a center of dilation, but the effect on what the image looks like may vary. Encourage students to try different locations for the center of dilation and observe the effect on their drawing.

**Look for productive strategies:**
- Creating rays through the center of dilation and each vertex of their polygon.
- Multiplying the distance from the center to each vertex by the scale factor.

## 3 Connect

**Display** several student drawings with the same scale factor, but a different location for the center of dilation to demonstrate that the point of view or perspective on them is different.

**Ask:**
- "What is the effect on the image when the scale factor is greater than 1?" The image is larger than the preimage and farther away from the center of dilation.
- "What is the effect on the image when the scale factor is less than 1?" The image is smaller than the preimage and closer to the center of dilation.
- "What is the effect on the image when the scale factor is equal to 1?" The image maps onto itself.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Provide a polygon for students to use, such as a quadrilateral or pentagon.

### Extension: *Interdisciplinary Connections*

Consider showing students examples of 1-point perspective drawing and illustrating where the vanishing point is located. Tell them the vanishing point is where the parallel lines seem to converge (meet at a point). Ask them to describe how vanishing points relate to dilations. **(Art)** Sample response: The vanishing point is the center of dilation. Rays can extend from the vanishing point to create images of dilations of objects, where the preimage is closer to the vanishing point.

## Math Language Development

### *MLR3: Critique, Correct, Clarify*

During the Connect, display the following three incorrect statements:

- When the scale factor is greater than 1, the image maps onto itself.
- When the scale factor equals 1, the image is larger than the preimage and farther away from the center of dilation.
- When the scale factor is less than 1, the image is smaller than the preimage and farther away from the center of dilation.

Have pairs of students critique these statements, write corrected statements, and clarify their reasoning as to how they corrected them.

Whole Class | 5 min

# Summary

Review and synthesize dilations on a plane without a grid.

Name: ______ Date: ______ Period: ______

## Summary

**In today's lesson . . .**

You performed dilations on a plane using a center of dilation and a scale factor. You explored how to find the center and scale factor given a preimage and its dilated image.

You can determine the scale by finding the ratio of the distance between the image and the center of dilation, and the distance between the preimage and the center. Scale factors that are greater than 1 create images that are farther away from the center of dilation, while scale factors less than 1 create images that are closer to the center of dilation.

Reflect:





## Synthesize

**Display** Part 1 of the Anchor Chart, *Dilations*. Tell students that over the course of this unit, they will return to this anchor chart to update it with their new understandings.

**Ask:**

- "How would you explain the steps for dilating a point on a plane without the structure of a grid?"
- "What must be true about the preimage, center of dilation, and image?" They must be along the same line, and the center of dilation cannot be between the preimage and the image.
- "What is the relationship between the scale factor that maps the preimage onto the image, and the scale factor that maps the image onto the preimage? The scale factors are reciprocals.

**Highlight:**

- The scale factor is the ratio that determines the size of an image, including whether the original figure is enlarged or reduced.
- The placement of the center of dilation affects the placement of the image.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful today when dilating a point? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding of dilations on a plane by identifying the center of dilation, preimage, and image.

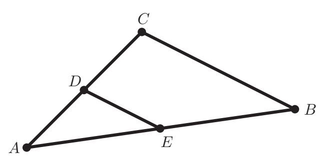
**Amps Featured Activity** Real-Time Exit Ticket

Printable

Name: ______________________ Date: __________ Period: __________

## Exit Ticket 2.03

Lin drew a triangle and a dilation of the triangle with a scale factor of $\frac{1}{2}$.




1. Which point is the center of dilation? Explain your thinking.
   Point $A$ is the center of dilation; Sample response: There are two triangles: Triangle $AED$ and Triangle $ABC$. Point $D$ lies on line segment $AC$ and point $E$ lies on line segment $AB$. Point $A$ is the point of intersection of line segment $AC$ and line segment $AB$.

2. Which triangle is the preimage, and which triangle is the dilated image? Explain your thinking.
   Triangle $ABC$ is the preimage, and Triangle $AED$ is the dilation; Sample response: I know the image must be smaller than the preimage because the scale factor is less than 1.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can apply a dilation to a figure using measurement tools. 1 2 3

b I can identify the center, scale factor, and image of a dilation without a grid. 1 2 3



Lesson 3 Dilations on a Plane

### Success looks like . . .

- **Goal:** Creating a dilation of a figure, given a scale factor and the center of dilation.
- **Goal:** Identifying the center, scale factor, and image of a dilation without a circular grid.
  - » Determine the center of dilation and image of the dilation in Problems 1 and 2.

### Suggested next steps

**If students do not identify point $A$ as the center of dilation, consider:**

- Reviewing Problem 3 from Activity 1.
- Reviewing Problem 1 from Activity 2.
- Assigning Practice Problem 2.

**If students identify the preimage and image incorrectly in Problem 2, consider:**

- Reviewing Problem 4 from Activity 2.

It is okay if students do not name the image so that the letters correspond to the preimage. This idea will be revisited in more depth in future coursework.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- What worked and didn't work today? In what ways did Activity 2 go as planned?
- During the discussion about Activity 1, how did you encourage each student to share their understandings? What might you change for the next time you teach this lesson?

# Practice

Independent

Name: ________ Date: ________ Period: ________

**1.** Segment $AB$ measures 3 cm. Point $O$ is the center of dilation. How long is the image of $AB$ after a dilation with:

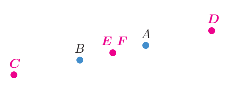
a. A scale factor of 5? **$3 \cdot 5 = 15$; 15 cm**

b. A scale factor of 3.7? **$3 \cdot 3.7 = 11.1$; 11.1 cm**

c. A scale factor of $\frac{1}{5}$? **$3 \cdot \frac{1}{5} = \frac{3}{5}$; $\frac{3}{5}$ cm**

d. A scale factor of $s$? **$3 \cdot s = 3s$; $3s$ cm**

**2.** Refer to points $A$ and $B$. Plot the points for each dilation described.




a. Point $C$ is the image of point $B$ using point $A$ as the center and a scale factor of 2.

b. Point $D$ is the image of point $A$ using point $B$ as the center and a scale factor of 2.

c. Point $E$ is the image of point $B$ using point $A$ as the center and a scale factor of $\frac{1}{2}$.

d. Point $F$ is the image of point $A$ using point $B$ as the center and a scale factor of $\frac{1}{2}$.

**3.** Rectangle A has a length of 12 in. and a width of 8 in. Rectangle B has a length of 15 in. and a width of 10 in. Rectangle C has a length of 30 in. and a width of 15 in.

a. Is Rectangle A a scaled copy of Rectangle B? If so, what is the scale factor? If not, why not?

**Yes, because $\frac{12}{15} = \frac{8}{10} = \frac{4}{5}$. The scale factor is $\frac{4}{5}$.**

b. Is Rectangle B a scaled copy of Rectangle A? If so, what is the scale factor? If not, why not?

**Yes, because $\frac{15}{12} = \frac{10}{8} = \frac{5}{4}$. The scale factor is $\frac{5}{4}$.**

c. Explain how you know that Rectangle C is *not* a scaled copy of Rectangle B.

**Sample responses:**

- **Rectangle C's length is twice as great as Rectangle B's length, but the width was not doubled.**
- **$\frac{15}{10}$ is not equivalent to $\frac{30}{15}$.**

d. Is Rectangle A a scaled copy of Rectangle C? If so, what is the scale factor? If not, why not?

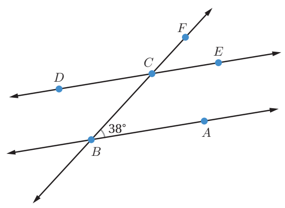
**No; Sample response: Rectangle A is a scaled copy of Rectangle B, but Rectangle B is not a scaled copy of Rectangle C, so Rectangle A is also not a scaled copy of Rectangle C.**




Practice

Name: ________ Date: ________ Period: ________

**4.** Consider parallel lines $AB$ and $DE$ and transversal $BC$. Calculate the measure of each of the indicated angles. Explain your thinking.




*The figure may not be drawn to scale.*

a. m$\angle BCD$ = **38°**

**Sample response: $\angle BCD$ and $\angle ABC$ are alternate interior angles.**

b. m$\angle ECF$ = **38°**

**Sample response: $\angle BCD$ and $\angle ECF$ are vertical angles.**

c. m$\angle DCF$ = **142°**

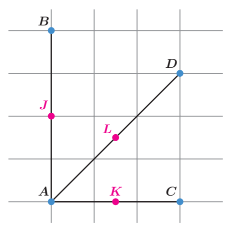
**Sample response: $\angle DCF$ and $\angle ECF$ are supplementary angles.**

**5.** Consider segments $AB$, $AD$, and $AC$.

a. Plot the point in the middle of segment $AB$ and label it point $J$.

b. Plot the point in the middle of segment $AC$ and label it point $K$.

c. Plot the point in the middle of segment $AD$ and label it point $L$.







## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 1 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 1 Lesson 15 | 2 |
| Formative | 5 | Unit 2 Lesson 4 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Dilations on a Square Grid

Let's dilate figures on a square grid.


## Focus

### Goals

1. Create a dilation of a polygon on a square grid, given a scale factor and center of dilation.
2. **Language Goal:** Identify the image of a figure on a square grid, given a scale factor and the center of dilation. **(Speaking and Listening)**

## Coherence

### Today

Students apply dilations to polygons on a grid without coordinates. The grid offers a way of measuring distances between points, especially points that lie at the intersection of grid lines.

### Previously

In Lesson 3, students studied dilations on a plane without the structure of a grid.

### Coming Soon

In Lesson 5, students use coordinates to more precisely describe dilations on a grid.

## Rigor

- Students develop **conceptual understanding** for how the structure of a grid can be used to make dilations.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- colored pencils
- geometry toolkits: rulers, index cards

### Math Language Development

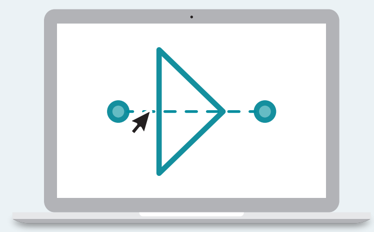
**Review words**

- *dilation*
- *center of dilation*
- *scale factor*
- *scaled copy*

**Amps** | **Featured Activity**

### Activity 1
### Interactive Geometry

Students drag vertices to represent dilations of polygons on a grid.



Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students may want to stick with the methods they learned in Lesson 3 rather than investing in the new tools presented in this one. Validate students' feelings and methods, and encourage them to practice using the structure of the square grid today so that they can have more strategies from which to choose for future problems. Pair these students with partners who can support a better understanding of how to use the structure of the square grid.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, Problem 2 may be omitted.
- In **Activity 2**, Problems 3 and 4 may be omitted.

Independent | 5 min

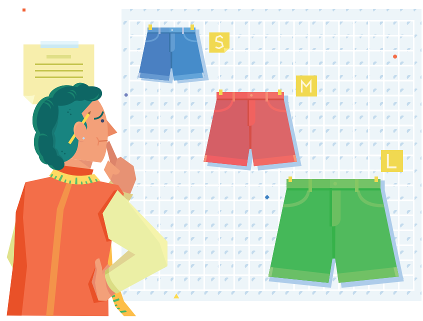
# **Warm-up** Estimating a Scale Factor

Students estimate a scale factor without a grid to better see the usefulness of a grid in the upcoming activities.

Unit 2 | Lesson 4

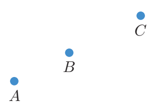
## Dilations on a Square Grid

Let's dilate figures on a square grid.




### **Warm-up** Estimating a Scale Factor

**Point $C$ is the dilation of point $B$, with point $A$ as the center of dilation. Estimate the scale factor. Explain your thinking.**




**Sample response: The scale factor should be little more than 2 and a little less than 2.5, so I'm estimating about 2.3.**

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Tell students they will estimate the scale factor for a dilation without the use of a ruler.

## 2 Monitor

**Help students get started** by reminding them of the definition of scale factor and having them consider the distance from point $A$ to point $B$.

**Look for points of confusion:**

- **Guessing a scale factor.** Clarify that estimating is not the same as guessing and ask, "Between which two numbers do you think the scale factor is? Can you be more precise?".
- **Struggling** to find an estimate without a grid. Ask students how they can estimate a scale when there is not one provided, and encourage them to develop a strategy for estimation.

**Look for productive strategies:**

- Using marks or informal grid lines to make an estimate.

## 3 Connect

**Display** the Warm-up from the Student Edition.

**Ask:**

- "Is the scale factor greater than 1? 2? 3? Explain your thinking."
- "What made this Warm-up challenging? Without the use of a ruler, what might be helpful in finding the scale factor going forward?"

**Highlight** that students will continue to explore dilations and identify scale factors, but this time using the structure of a square grid.

## Differentiated Support

**Accessibility: *Optimize Access to Tools***

Provide access to students' geometry toolkits and suggest that students use rulers or index cards to informally estimate the scale factor.

**Extension: *Math Enrichment***

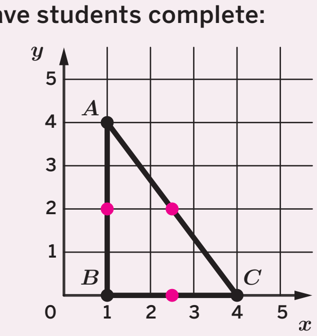
Tell students that point $D$ is the dilation of point $B$ with point $A$ as the center of dilation and a scale factor of $\frac{1}{2}$. Point $E$ is the dilation of point $D$ with point $A$ as the center of dilation and a scale factor of $\frac{1}{2}$. Ask students to describe the location of points $D$ and $E$, without plotting the dilations. Point $D$ is halfway between points $A$ and $B$. Point $E$ is $\frac{1}{4}$ the distance between points $A$ and $B$.

## Power-up

**To power up students' ability to determine the point in the middle of a line segment on a grid, have students complete:**

Recall that in order to determine the point in the middle of a line segment on a grid, you can use tools such as counting boxes on the grid, a ruler, or an index card.

1. Plot point $Q$ in the middle of segment $AB$.
2. Plot point $R$ in the middle of segment $BC$.
3. Plot point $S$ in the middle of segment $AC$.




**Use:** Before Activity 1
**Informed by:** Performance on Lesson 3, Practice Problem 5

Pairs | 15 min

# Activity 1 Dilations on a Grid

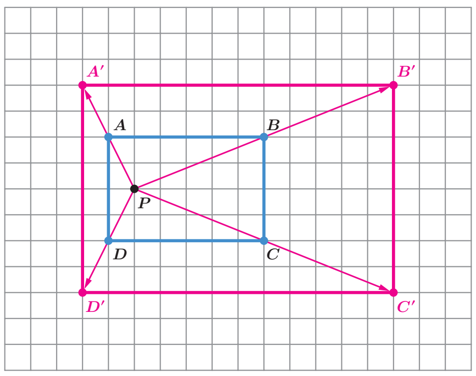
Students perform dilations on a square grid to see how the structure of the grid can be particularly helpful.

Amps Featured Activity | Interactive Geometry

Name: ______ Date: ______ Period: ______

**Activity 1** Dilations on a Grid

1. Dilate Rectangle $ABCD$ by a scale factor of 2 with point $P$ as the center of dilation. Label the corresponding vertices in the image as $A'$, $B'$, $C'$, and $D'$.








## 1 Launch

Say, "You should use any tool you find useful, but I challenge you to see if you can be precise *without* the use of a ruler." Provide access to geometry toolkits for the duration of the lesson.

## 2 Monitor

**Help students get started** by saying, "Use your ruler to find a point that is twice as far from point $P$ as point $A$ is from point $P$."

**Look for points of confusion:**

- **Struggling to make precise measurements.** Ask, "Do you need to know the precise measurements to draw a segment that is twice as long?" Encourage students to use an index card.
- **Dilating a point in only a vertical or horizontal direction.** Ask, "How far is this point from the center? Make sure you dilate in the vertical and horizontal direction."
- **Questioning where to place a point that does not lie on the corners of the gridlines in Problem 2.** Ask, "How could you use your ruler to find a distance that is $\frac{1}{2}$ of the distance between the center and the point $Q$? How could you use the grid to confirm this location?"

**Look for productive strategies:**

- Using the grid or a ruler to create dilations.
- Using the grid to create dilations.

Activity 1 continued >

## Differentiated Support

### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use a digital geometry toolkit to perform dilations on a grid.

### Extension: *Math Enrichment*

Ask students how the image of Rectangle $ABCD$ would be similar to and different from the image in Problem 1 if the center of dilation was point $D$ instead of point $P$. Have them experiment with different centers of dilation, providing them with graph paper as needed. The locations of the images vary, but the sizes of the images are the same.

## Math Language Development

### *MLR8: Discussion Supports*

During the Connect, as students respond to the first Ask question, highlight the language they use, such as "draw a ray from the center of dilation through each preimage point" or "use an index card to measure the distance from the center of dilation through each preimage point." Encourage the use of developing math language, such as *preimage*, *image*, *distance*, and *center of dilation*.

### English Learners

During the discussion, point to or annotate terms as you say them: *image*, *preimage*, *distance*, *center of dilation*.

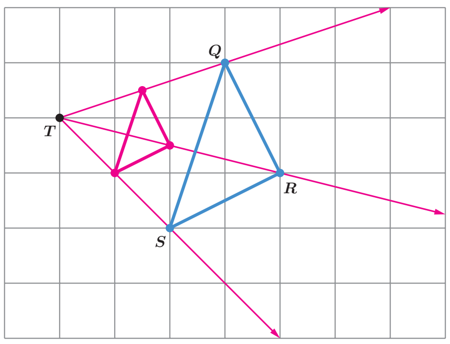
Pairs | 15 min

# Activity 1 Dilations on a Grid (continued)

Students perform dilations on a square grid to see how the structure of the grid can be particularly helpful.

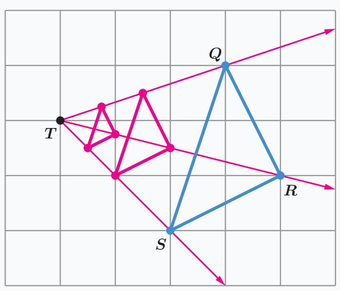
**Activity 1** Dilations on a Grid (continued)

2. Dilate Triangle $QRS$ by a scale factor of $\frac{1}{2}$ with point $T$ as the center of dilation.




**Are you ready for more?**

Dilate Triangle $QRS$ by a scale factor of $\frac{1}{4}$ with point $T$ as the center of dilation.







## 3 Connect

**Display** student work showing correct responses to Problems 1 and 2.

**Have students share** the methods they used to create the dilations in Problem 1 and Problem 2. Sequence responses by first asking students who used a ruler to share, and then asking students who used the grid to share.

**Ask:**

- "How can you use the structure of the grid to create a dilation?"
- "In Problem 1, Is Rectangle $A'B'C'D'$ a scaled copy of Rectangle $ABCD$? What do you know about the sides and angles of the image and preimage?"

**Highlight** that descriptive measurements such as "two up and one over" can be multiplied by the scale factor to create a dilation.

Pairs | 15 min

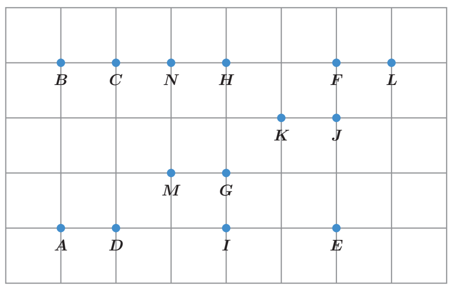
# **Activity 2** Dilation Obstacle Course . . . on a Grid!

Students apply their new understanding of how to use a grid to create and identify dilations.

Name: ____________ Date: ________ Period: ________

**Activity 2** Dilation Obstacle Course . . . on a Grid!

Refer to points $A$ through $L$ shown.




1. Using point $I$ as the center of dilation, dilate point $G$ so that its image is point $H$. What scale factor did you use?
   3

2. Suppose point $F$ is an image of point $N$ after a dilation. Compare the scale factors with point $B$ and point $C$ as the centers of dilation.
   **Sample response: For the position of the given image, the closer the dilated point is to the center of dilation, the greater the scale factor will be. For example, the scale factor with point $C$ as the center is 4. The scale factor with point $B$ as the center is 2.5.**

3. To dilate point $K$ so that its image is point $A$, what point could be the center of dilation, and what would be the scale factor?
   **Point $L$ could be the center, with a scale factor of 3.**

4. Points $D$, $E$, and $J$ can be used to form Triangle $DEJ$. Points $D$, $I$, and $G$ can be used to form Triangle $DIG$. Identify the center of dilation and the scale factor that map Triangle $DEJ$ onto Triangle $DIG$.
   **Point $D$ is the center of dilation, with a scale factor of $\frac{1}{2}$.**

STOP





## 1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity. Distribute colored pencils.

## 2 Monitor

**Help students get started** by asking, "How can you describe the distance of point $G$ to point $I$, the center?"

**Look for points of confusion:**

- **Finding a scale factor of 2 for Problem 1.** Have students find the distance from point $G$ to point $I$, and ask them to find how many times that length would need to scale to reach point $H$.
- **Struggling to find a scale factor that is not a whole number in Problem 2.** Make sure students use the correct distances from the center for each point, and ask students how they can use division to find the scale factor between distances.

## 3 Connect

**Display** the points on the grid.

**Have students share** how they found their solutions to Problems 1–4.

**Highlight** how the scale factor between two fixed points changes based on the point used as the center of dilation in Problem 2.

**Ask:**

- "In Problem 2, what happens if you move point $C$ closer to points $N$ and $F$ in your image? Will the scale factor be less than or greater than 4?" The scale factor will be greater. **Note:** If the point and its image are fixed, then the distance between the center and the point and the scale factor are inversely proportional.
- "Could point $M$ be the center of dilation in Problem 3? Why or why not?" No; Sample response: The point and its image should be on the same side of the center.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Have students use colored pencils to color code the points that are relevant to each problem, to help reduce distractor points. Alternatively, display or provide copies of points that are only relevant to each problem at a time.

### Extension: *Math Enrichment*

As a follow-up to Problem 4, ask students to identify any other pairs of polygons that can be formed by the points on the grid such that one polygon is a dilation of the other. Have them justify their thinking.

## Math Language Development

### *MLR8: Discussion Supports—Revoicing*

During the Connect, as students share their solutions to Problem 4, revoice their ideas by restating a statement as a question. This will help clarify, demonstrate mathematical language, and involve more students. For example, if a student says "The two triangles share point $D$ and the smaller one is half the larger one," consider asking:

- "What do you mean by 'half'? Are you comparing the areas of the triangles or the side lengths?"
- "In terms of dilations, what do you mean by 'the two triangles share point $D$'?"

Whole Class | 5 min

# Summary

Review and synthesize how the structure of the grid can be helpful in creating and identifying dilations.

## Summary

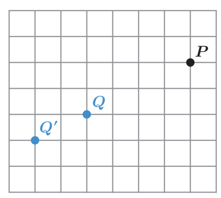
**In today's lesson . . .**

You explored how square grids can be useful for showing dilations. The grid is helpful, especially when the center of dilation and the point(s) being dilated are marked at the intersections of grid squares. Rather than using a ruler to measure the distance between the points, you can count grid squares.

For example, suppose you want to dilate point $Q$ with a scale factor of $\frac{3}{2}$ and point $P$ as the center of dilation.

Because point $Q$ is 4 grid squares to the left and 2 grid squares down from point $P$, the dilated image will be 6 grid squares to the left and 3 grid squares down from point $P$. Can you see why?

The dilated image is marked as point $Q'$ in the grid shown.




> **Reflect:**




## Synthesize

**Have students share** what was different about their methods for working with dilations today than the previous lesson, without the structure of a grid.

**Ask:**

- "How can you perform dilations on a square grid?"
- "What else might help you be more precise when working with dilations on a grid?"
  Sample responses: Using coordinates or using a ruler or index card to verify measurements.

**Highlight** that in the next lesson, students will learn to be even more precise and descriptive as they work with dilations on the coordinate plane.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "In what ways was the grid helpful in making dilations? "

Independent | 5 min

# Exit Ticket

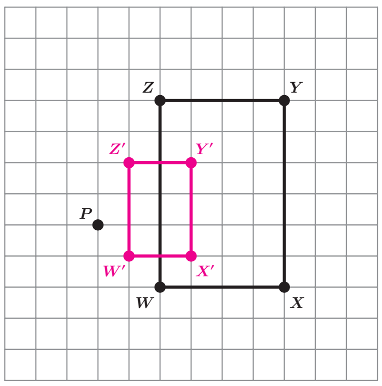
Students demonstrate their understanding by dilating a rectangle on a square grid given a point of center and a scale factor.

Printable

Name: ______________ Date: ________ Period: ________

## Exit Ticket 2.04

Dilate Rectangle $WXYZ$ with a scale factor of $\frac{1}{2}$ and point $P$ as the center of dilation. Label the vertices of the image $W'X'Y'Z'$.




**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can apply dilations to figures on a square grid.

1 2 3



**Lesson 4** Dilations on a Square Grid

## Success looks like . . .

- **Goal:** Creating a dilation of a polygon on a square grid, given a scale factor and center of dilation.
- **Language Goal:** Identifying the image of a figure on a square grid, given a scale factor and center of dilation. **(Speaking and Listening)**
  - » Drawing the image of Rectangle $WXYZ$ on a square grid.

## Suggested next steps

**If students cannot create an accurate dilation, consider:**

- Reviewing strategies from Activity 1, Problem 1.
- Assigning Practice Problem 1.

**If students appear to not be comfortable using the structure of the grid to find dilations, consider:**

- Letting them know that multiple methods are valid and encouraging them to try using the structure of the grid so they are comfortable with this approach.
- Assigning Practice Problem 1.

**If students have the correct image, but the name is incorrect, consider:**

- Asking students to be precise, and explain why it is important to name images correctly.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?
- The instructional goal for this lesson was to create dilations and identify scaled images on a grid without the use of coordinates.
  How well did students accomplish this? What did you specifically do to help students accomplish it?

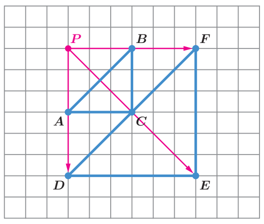
# Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

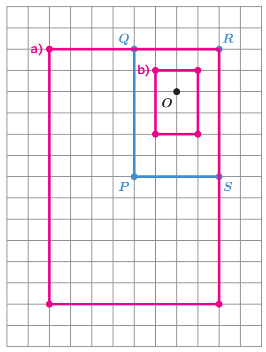
1. Triangle $ABC$ can be mapped to Triangle $DFE$ using a dilation. What are the center and the scale factor of the dilation? Label the center as point $P$. **The scale factor is 2.**




2. Consider Rectangle $PQRS$. Sketch the image of Quadrilateral $PQRS$ under the following dilations:

   a. The dilation centered at point $R$ with a scale factor of 2.

   b. The dilation centered at point $O$ with a scale factor of $\frac{1}{2}$.



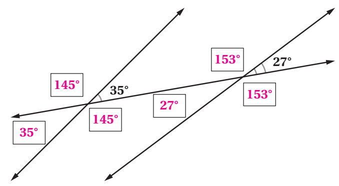





Name: ______ Date: ______ Period: ______

Practice

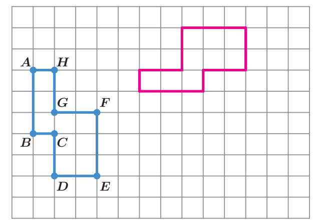
3. The diagram shows three lines, along with some angle measures. Determine the missing angle measures.




*The figure may not be drawn to scale.*

4. Construct an image of the polygon after performing the following sequence of transformations:

   - Translate the polygon 5 units to the right and 1 unit down.
   - Rotate the result $90^\circ$ counterclockwise about point $A'$.




5. Point $B$ has coordinates $(-2, -5)$. After a translation 4 units down, a reflection across the $y$-axis, and a translation 6 units up, what are the coordinates of the image?
**(2, −3)**





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 1 |
| Spiral | 3 | Unit 1 Lesson 15 | 1 |
| | 4 | Unit 1 Lesson 5 | 1 |
| Formative | 5 | Unit 2 Lesson 5 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available


For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Dilations With Coordinates

Let's look at dilations on the coordinate plane.

## Focus

### Goal

1. **Language Goal:** Describe and apply dilations to polygons on the coordinate plane, given the coordinates of the vertices and the center of dilation. **(Speaking and Listening, Reading and Writing)**

## Rigor

- Students build **conceptual understanding** of dilations on the coordinate plane.
- Students create dilated images of polygons to build **procedural fluency**.

## Coherence

### Today

Students apply dilations to polygons on a coordinate plane. The coordinates allow for more precise descriptions of dilations.

### Previously

Students performed dilations on polygons using a plane in Lesson 3, and using a grid without coordinates in Lesson 4.

### Coming Soon

In Lesson 6, students will define similarity using sequences of transformations.

## Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 20 min | 10 min | 7 min | 5 min |
| Independent | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

### Materials

- Exit Ticket
- Additional Practice
- *Info Gap Routine* PDF
- Power-up PDF, as needed
- Power-up PDF (answers), as needed
- Activity 1 PDF, one per pair
- Activity 1 PDF, (answers)
- Anchor Chart PDF, *Translations, Rotations, and Reflections* (optional, from Unit 1)
- Anchor Chart PDF, *Dilations* (optional)
- geometry toolkit: rulers, protractors

### Math Language Development

**Review words**

- *dilation*
- *center of dilation*
- *scaled copy*
- *scale factor*

**Amps** | Featured Activity

**Exit Ticket**

**Real-time Exit Ticket**

Check in real time if your students can describe dilated figures on the coordinate plane using a digital Exit Ticket.

### Building Math Identity and Community

Connecting to Mathematical Practices

Students may get stuck thinking they need to use the precise terms for the dilations in their descriptions during Activity 2. Encourage these students to describe their dilations in a way that makes sense to them and to look for things they know about the specific points, lines, or angles on their card to help them.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, Problems 2 and 4 may be omitted.

Independent | 5 min

# Warm-up Different Dilations

Students will compare the same dilation on a square grid and on a coordinate plane to determine which structure allows for more precise descriptions.

Name: ______________________ Date: __________ Period: __________

**Unit 2 | Lesson 5**

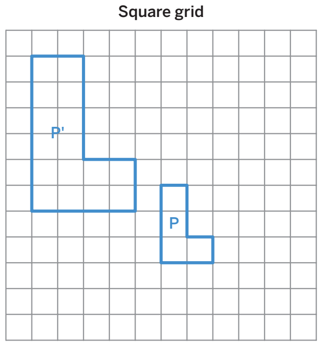
## Dilations With Coordinates

Let's look at dilations on the coordinate plane.

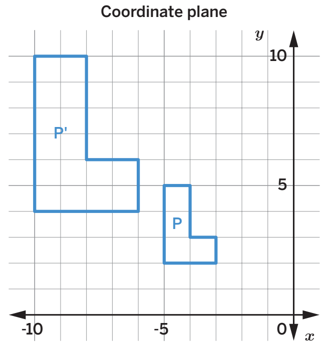
### Warm-up Describing Dilation

Consider the dilation of Polygon P on the square grid shown, and on the coordinate plane shown. Which one would you choose if you were asked to describe the dilation? Explain your thinking.

Square grid




Coordinate plane




Answers may vary. Sample responses:

- The square grid allows me to describe how the polygon is dilated, without worrying about its placement on the plane.
- With the coordinate plane, I can use coordinates to describe the vertices of the polygon, its image, and the exact location of the center of dilation.

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Set an expectation for the amount of time students will have to work individually on the activity.

## 2 Monitor

**Help students get started** by asking, "What is different about the two dilations?"

**Look for points of confusion:**

- **Thinking there is only one correct response.** Students can select either dilation, as long as they provide reasons to justify their selection.
- **Thinking the square grid and the coordinate plane show the same exact information.** Ask students how they can identify or describe the location of one of the vertices of Polygon P using the square grid.

**Look for productive strategies:**

- Describing the features of either the square grid or the coordinate plane that assist with performing or describing dilations.
- Coming up with reasons to justify their selection.

## 3 Connect

**Display** the two dilations side-by-side.

**Have students share** their reasons for choosing the dilation on either the square grid or the coordinate plane.

**Highlight** how the use of coordinates can assist with performing and describing dilations with more precision.

## Math Language Development

*MLR7: Compare and Connect*

During the Connect discussion, have students compare the square grid to the coordinate plane. Ask, "What is the same about the two grids? What is different?" Look for and amplify language students use, such as *coordinates*, *ordered pairs*, *axes labels/scales*, etc. Encourage students to justify their preference for which grid they would use.

### English Learners

Annotate the coordinate plane with the mathematical language students use to describe how it is different from the square grid.

## Power-up

**To power up students' ability to determine the coordinates of an image after a series of transformation:**

Provide students with a copy of the Power-up PDF.

**Use:** Before the Warm-up
**Informed by:** Performance on Lesson 4, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 2

Pairs | 20 min

# **Activity 1** Info Gap: Make My Dilation

Students take turns describing and drawing dilations in order to practice precise communication.

**Activity 1** Info Gap: Make My Dilation

**Plan ahead:** How will you show that you are listening to your partner? When might you need to use clarifying questions?

**You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.**

| If you are given a *problem card*: | If you are given a *data card*: |
|---|---|
| **1.** Silently read your card, and think about what information you need to be able to solve the problem. | **1.** Silently read your card. |
| **2.** Ask your partner for the specific information that you need. | **2.** Ask your partner, "*What specific information do you need?*" and wait for them to ask for information. |
| **3.** Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem. | **3.** Before sharing the information, ask, "*Why do you need that information?*" Listen to your partner's reasoning, and ask clarifying questions. |
| **4.** Share the problem card and solve the problem independently. | **4.** Read the problem card, and solve the problem independently. |
| **5.** Read the data card and discuss your thinking. | **5.** Share the data card and discuss your thinking. |

**Pause here so your teacher can review your work. You will be given a new set of cards. Repeat the activity, trading roles with your partner.**




## 1 Launch

Model the ***Info Gap*** routine and display the *Info Gap Routine* PDF. Distribute pre-cut cards from the Activity 1 PDF to each pair of students. Start by distributing the first set and distribute the second set after you have checked student work. Provide access to geometry toolkits for the duration of the lesson.

## 2 Monitor

**Help students get started** by explaining that they may need several rounds of discussion to determine the information they need.

**Look for points of confusion:**

- **Not knowing how to find information about dilations.** Say, "Define a dilation in your own words. What information do you have? What information do you need?"
- **Forgetting to ask for the preimage coordinates from the data card.** Remind students that a dilation is performed on a preimage to create an image, and that they will need the vertices of the preimage or image from their partner.

**Look for productive strategies:**

- Using precise descriptions in terms of specific points.
- Responding to constructive feedback to revise sketches.

## 3 Connect

**Ask:**

- "Which elements of your partner's descriptions were helpful when you were sketching?"
- "If there had been no coordinate grid at all, would you still have been able to request or provide the needed information to perform the transformation?"

**Highlight** how using precise mathematical language allows for greater accuracy and clarity when performing certain geometric actions, such as dilations.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. Consider using the following questions during the think-aloud.

- "Polygon $ABCD$ is the preimage, but I don't know where the polygon is located on the coordinate plane. I think I should ask for the coordinates of the vertices of the preimage."
- "I wonder which polygon is larger, or whether they are the same size. I think I should ask for the scale factor."
- "I don't know where to draw the rays that show the dilation. I think I should ask for the center of dilation."

## Math Language Development

### *MLR4: Information Gap*

Display prompts for students who benefit from a starting point, such as:

- "Can you tell me . . . (specific piece of information)?"
- "Why do you need to know . . . (that piece of information)?"

### English Learners

Consider providing sample questions students could ask, such as the following:

- What is the center of dilation?
- What is the scale factor?
- What are the coordinates of point $A$? Point $B$?, etc.

Pairs | 10 min

# Activity 2 Dilate It!

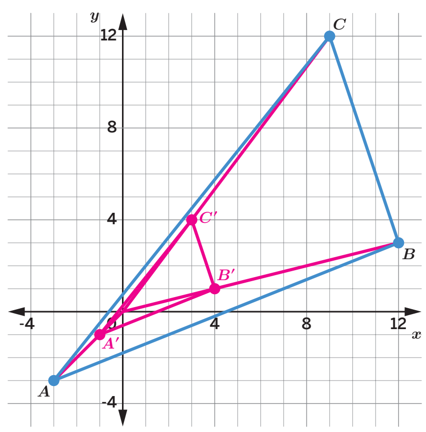
Students draw dilations in the coordinate plane, including those not centered at the origin or that involve preimages that are not polygons, to compare the strategies used.

Name: ______ Date: ______ Period: ______

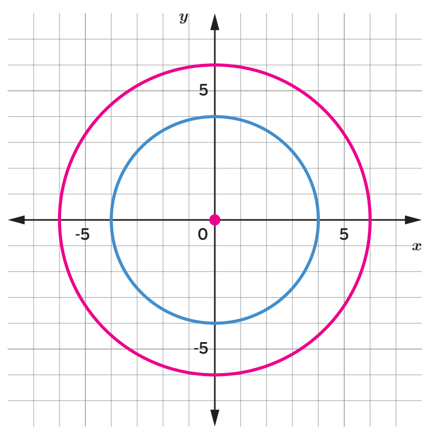
**Activity 2** Dilate It!

**You will be given access to your geometry toolkit. Choose tools from your geometry toolkit to perform the indicated dilations.**

1. Dilate Triangle $ABC$ using the origin as the center of dilation and a scale factor of $\frac{1}{3}$.




2. Dilate the circle shown using the origin as the center of dilation and a scale factor of $\frac{3}{2}$.








## 1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

## 2 Monitor

**Help students get started** by dilating vertex $A$ from Problem 1 together.

**Look for points of confusion:**

- **Multiplying the coordinates by the scale factor.** This strategy works for Problems 1–3, so direct students to look closely at Problem 4. Ask students to locate the center identified in the problem, and compare the distances from the center to a vertex on the preimage, and then from the center to the corresponding vertex on the image.
- **Dividing the distance by the scale factor in Problem 3.** Ask students if the image should be larger or smaller than the preimage, based on the given scale factor.

**Look for productive strategies:**

- Using the scale factor to determine the length of the sides for the image.
- Dilating only one point and using it to complete the image in Problem 2.

**Activity 2 continued >**

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

If students need more processing time, consider having them focus on Problems 1 and 3. Have students use colored pencils or highlighters to color code the information in the text of each problem that indicates the center of dilation in one color and the scale factor in another color.

### Extension: *Math Enrichment*

Ask students to explain why multiplying the coordinates of the preimage by the scale factor *only* works when the center of dilation is the origin. When the center of dilation is the origin, the rays that connect the preimage points to the image create a proportional relationship.

## Math Language Development

### *MLR2: Collect and Display*

During the Connect, display Part 1 of the Unit 2 Anchor Chart, *Dilations*. Draw students' attention to the Anchor Chart and to the class display you started in Lesson 1. As students discuss their strategies for Problem 2, collect the language they use to describe how they dilated the circle — such as radius or diameter — and add this to the class display.

### English Learners

As you add terms, such as a *radius* and *diameter* to the class display, draw visuals to help distinguish the terms.

Pairs | 15 min

# Activity 2 Dilate It! (continued)

Students draw dilations in the coordinate plane, including those not centered at the origin or that involve preimages that are not polygons, to compare the strategies used.

## Activity 2 Dilate It! (continued)

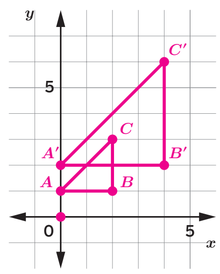
3. Triangle $ABC$ has vertices located at $A(0, 1)$, $B(2, 1)$, and $C(2, 3)$. Triangle $A'B'C'$ is the result of dilating Triangle $ABC$ using the origin as the center of dilation and a scale factor of 2.

| Preimage | | Image | |
|---|---|---|---|
| Point $A$ | (0, 1) | Point $A'$ | **(0, 2)** |
| Point $B$ | (2, 1) | Point $B'$ | **(4, 2)** |
| Point $C$ | (2, 3) | Point $C'$ | **(4, 6)** |

a. Predict the coordinates of Triangle $A'B'C'$ and record them in the table.

b. Draw Triangle $ABC$ and its image on the coordinate plane. Use the images to verify whether your predictions were correct. Explain your thinking.

**Sample response: My predictions were correct. I multiplied the coordinates of the preimage by the scale factor, and the resulting coordinates are the vertices of Triangle $A'B'C'$.**




4. Parallelogram $ABCD$ has vertices $A(-5, 5)$, $B(0, 5)$, $C(3, 3)$, and $D(-2, 3)$. Parallelogram $A'B'C'D'$ is the result of dilating Parallelogram $ABCD$ using point $A$ as the center of dilation and a scale factor of $\frac{1}{2}$.

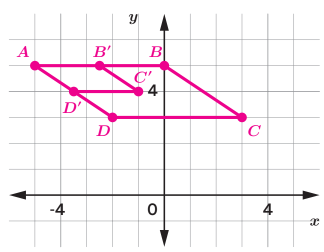
| Preimage | | Image | |
|---|---|---|---|
| Point $A$ | (−5, 5) | Point $A'$ | **(−2.5, 2.5)** |
| Point $B$ | (0, 5) | Point $B'$ | **(0, 2.5)** |
| Point $C$ | (3, 3) | Point $C'$ | **(1.5, 1.5)** |
| Point $D$ | (−2, 3) | Point $D'$ | **(−1, 1.5)** |

a. Predict the coordinates of Parallelogram $A'B'C'D'$ and record them in the table.

**An incorrect sample response is shown.**

b. Draw Parallelogram $ABCD$ and its image on the coordinate plane. Use the images to verify whether your predictions were correct. Explain your thinking.

**Sample response: My predictions were not correct. Multiplying the preimage coordinates by the scale factor did not result in the coordinates that make Parallelogram $A'B'C'D'$.**




STOP

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## 3 Connect

**Have pairs of students share** their strategies for creating the dilated images.

**Ask**, "Did you need or use a different strategy when attempting Problem 2?"

**Note:** Problem 2 is important to highlight because students are asked to dilate a circle, and circles do not contain straight lines. This makes it challenging to verify that the image is the correct size. Students may use the radius or diameter — instead of side lengths — to check their work.

**Highlight:**

- When dilating a point using the origin as the center of dilation, multiply the coordinates by the scale factor.
- When dilating a point using a center of dilation that is *not* the origin, the structure of the coordinate plane helps to find the distance from the center of dilation to a point on the preimage and a corresponding point on the image.
- To draw the image, dilate each vertex and connect the points, or dilate one point and use the scale factor to determine the side lengths and the placement of the other vertices.

Whole Class | 7 min

## Summary

Review and synthesize how dilations can be performed on a coordinate plane and the essential information needed to describe a dilation: coordinates, center, and scale factor.

Name: ______ Date: ______ Period: ______

### Summary

**In today's lesson . . .**

You dilated polygons on a coordinate plane.

Performing a dilation of a polygon requires three essential pieces of information:

1. The coordinates of the vertices
2. The coordinates of the center of dilation
3. The scale factor of the dilation

With this information, you can precisely describe any dilation of a figure.

**Reflect:**



**Lesson 5** Dilations With Coordinates 167

### Synthesize

**Ask:**

- "How are coordinates useful when describing and drawing dilations?" Sample response: The use of coordinates precisely communicates information about the location of the center, preimage, and image.
- "How do dilations compare to the transformations you saw in Unit 1?" Sample response: Translations, rotations, and reflections create images that are congruent to the preimage. Images of dilations may be larger or smaller than the preimage.

**Highlight** that the coordinate plane allows students to communicate geometric information precisely, pointing out the following:

- Students can specify the exact location of the preimage and the center of dilation using coordinates.
- Students can also use the grid to locate the corresponding points on the image.

### Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How does your strategy for dilating on a coordinate plane differ from dilating on a square grid?"

Independent | 5 min

# Exit Ticket

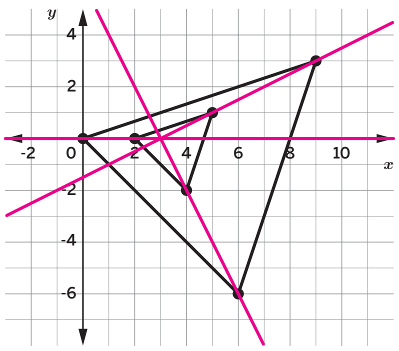
Students demonstrate their understanding by describing a dilation that has taken place on a coordinate plane.

Amps Featured Activity | Real-time Exit Ticket | Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 2.05

The smaller triangle is dilated to create the larger triangle. Describe this dilation. Be sure to include all the information another person would need to perform the dilation.




Sample response: The preimage has vertices located at the points (2, 0), (5, 1), and (4, −2). The triangle was dilated using the point (3, 0) as the center of dilation and a scale factor of 3, which results in the larger triangle with vertices located at points (0, 0), (9, 3), and (6, −6), respectively.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can perform dilations of a polygon on a coordinate plane if I know the coordinates of the vertices, the center of dilation, and the scale factor.
1 2 3

b. I can find the center of dilation and the scale factor of a dilation on a coordinate plane.
1 2 3





## Success looks like . . .

- **Language Goal:** Describing and applying dilations to polygons on the coordinate plane, given the coordinates of the vertices and the center of dilation. **(Speaking and Listening, Reading and Writing)**
  - » Describing the dilation of the smaller triangle to the larger triangle.

## Suggested next steps

**If students struggle to describe the dilation, consider:**

- Reviewing Activity 2.
- Assigning Practice Problem 2.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- What worked and didn't work today? What was especially satisfying about the *Info Gap* routine from Activity 1?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

# Practice

Independent

Practice

Name: Date: Period:

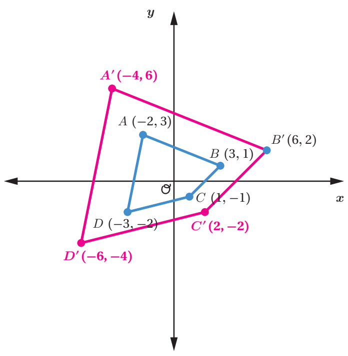
1. Quadrilateral $ABCD$ is dilated with the origin as the center of dilation, taking point $B$ to point $B'$.

   a. What is the scale factor of the dilation?

   The scale factor is 2.

   b. Draw Quadrilateral $A'B'C'D'$.

   c. Label the coordinate of points for $A'$, $C'$, and $D'$.



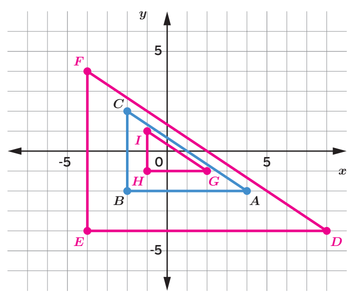

2. Consider Triangle $ABC$ on the coordinate plane.

   a. Using the origin as the center and a scale factor of 2, draw the dilation of Triangle $ABC$. Label the image Triangle $DEF$.

   b. Using the origin as the center and a scale factor of $\frac{1}{2}$, draw the dilation of Triangle $ABC$. Label the image Triangle $GHI$.

   c. Is Triangle $GHI$ a dilation of Triangle $DEF$? If yes, identify the center of dilation and scale factor.

   Yes, Triangle $GHI$ is a dilation of Triangle $DEF$. The center of the dilation is the origin and the scale factor is $\frac{1}{4}$.








Name: Date: Period:

Practice

3. Use what you know about the interior angle measures of triangles to complete these problems.

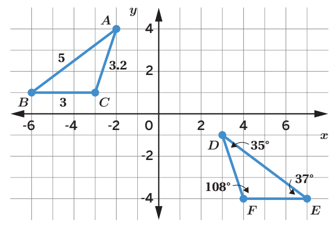
   a. Triangle $JKL$ is a right triangle, and the measure of angle $J$ is $28°$. What are the measures of the other two angles?

   The measures of the other two angles are 90° and 62°.

   b. Triangle $PQR$ is an obtuse triangle, and the measure of angle $Q$ is $72°$. What are the measures of the other two angles?

   Answers may vary. The sum of the other two angle measures should equal 108°, and one angle measure should be greater than 90°.

4. Triangles $ABC$ and $DEF$ are shown on the coordinate plane.




   a. Show that $\Delta ABC \cong \Delta DEF$.

   Sample response: Reflect Triangle $ABC$ across the $y$-axis, and translate the image 1 unit to the right and 5 units down.

   b. Find the side lengths of Triangle $DEF$.

   $DE = 5$, $EF = 3$, $FD = 3.2$

   c. Find the angle measures of Triangle $ABC$.

   $m\angle A = 35°$, $m\angle B = 37°$, $m\angle C = 108°$





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 2 |
| Spiral | 3 | Unit 1 Lesson 16 | 2 |
| Formative | 4 | Unit 2 Lesson 6 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Sub-Unit 2
# Similarity

Whole Class

In this Sub-Unit, students discover that dilated figures are similar to each other and that these similar figures have special, sometimes even eye-popping, characteristics.

SUB-UNIT

2 Similarity


Narrative Connections

## Do you really get what you pay for?

Roll through the aisles of your supermarket and you might think everything looks perfectly normal. But take a closer look and you might notice something a bit off about your favorite products.

For years, many manufacturers have been slowly reducing the size of their products. It's nothing dramatic. Maybe 10 percent here or there, shrinking the size of a box of cereal, or a carton of juice. But while the amount you get is shrinking, the price you pay actually remains the same.

This is called *shrinkflation*. Through subtle changes in packaging, manufacturers can get away with selling less of their product for the same cost. Most of the time, you'll find this in junk food and beverages, but it also happens with other household goods, like toilet paper.

A standard sheet of toilet paper used to be 4.5 by 4.5 inches. Now, many brands have shrunk that down to 4 by 4 inches. Some even make toilet paper that's 3.9 by 4 inches. This might not seem like a huge difference, but these shrinkages can really add up.

So how do we protect ourselves from overpriced, postage-sized TP? One strategy is to always check the unit price of a good. That way you know how much you're paying per ounce.

When we're dealing with tiny dilations in packaging, it's hard to know exactly when you're being duped. That's why it's important to have an accurate way of measuring, and knowing when things are exactly the same or when they are merely similar . . .





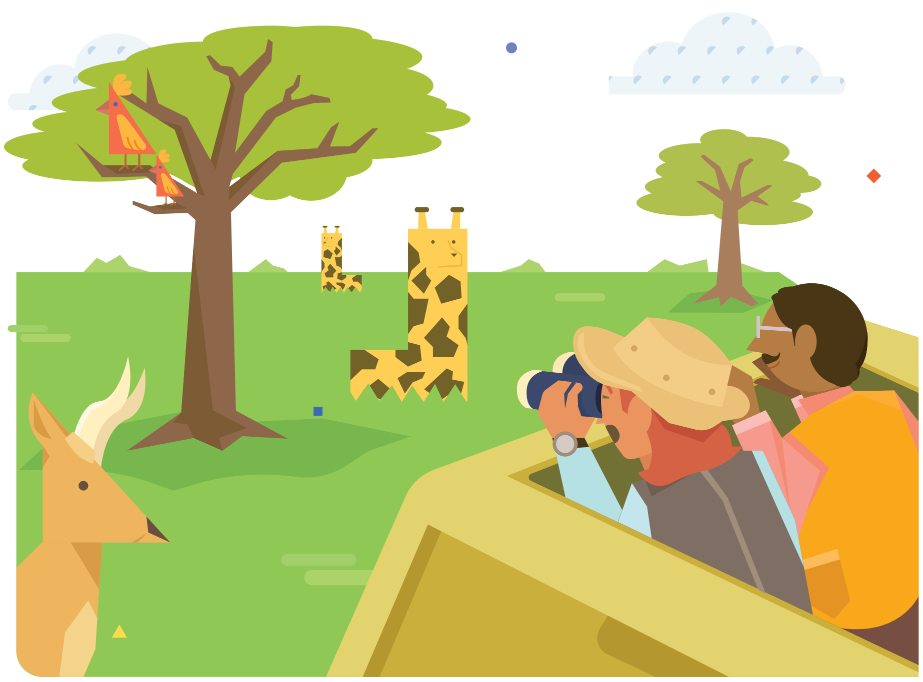
### Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students will closely inspect whether two or more figures are similar in the following places:

- **Lesson 6, Activity 2:** Are They Similar?
- **Lesson 7, Activity 1:** Different Dilations
- **Lesson 8, Activity 3:** Card Sort: Similar or Not?

# Similarity

Let's explore similar figures.



## Focus

### Goals

1. **Language Goal:** Comprehend that the phrase *similar figures* means there is a sequence of translations, rotations, reflections, or dilations that maps one figure onto the other. **(Speaking and Listening, Writing)**

2. **Language Goal:** Justify the similarity of two figures using a sequence of transformations that maps one figure onto the other. **(Speaking and Listening)**

## Rigor

- Students build **conceptual understanding** of the relationship between similar figures and the sequence of translations, rotations, reflections, or dilations that map one figure onto another.

## Coherence

### Today

Students learn that two figures are *similar* if there is a sequence of translations, rotations, reflections, or dilations that maps one figure onto another. They draw sketches of similar figures under different transformations and come to understand that there can be multiple sequences of transformations that demonstrate the similarity of figures.

### Previously

In Unit 1, students learned that two figures are congruent when there is a sequence of rigid transformations that maps one figure onto another. So far in this unit, students have explored the term *dilation* as a transformation in which each point on a figure moves along a line and changes its distance from a fixed point.

### Coming Soon

In Lesson 7, students will identify similar figures as scaled copies and investigate the properties of similar figures.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 7 min | 15 min | 15 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Power-up PDF, as needed
- Power-up PDF (answers), as needed
- Anchor Chart PDF, *Dilations*
- geometry toolkits: rulers, protractor, tracing paper

### Math Language Development

**New words**

- *similar**

**Review words**

- *center of dilation*
- *dilation*
- *reflection*
- *rotation*
- *translation*
- *sequence of transformations*

*Students may be familiar with the everyday use of the term *similar*, resembling without being identical. Let them know that this everyday use will be a good starting point as they explore mathematical similarity.

**Amps** | **Featured Activity**

### Activity 2
### See Student Thinking

After students read their peer responses, give them a chance to reflect and notice that more than one correct sequence of transformations can be applied to an image to show that two figures are similar.


Amps
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### Building Math Identity and Community

Connecting to Mathematical Practices

Students may recognize that different sequences of transformations may be applied in Activity 2. As students share their work with partners, highlight positive examples of student discussions where they use mathematically precise language to communicate their different sequences of transformations. Emphasize the importance of listening to others' perspectives, especially when the sequences were different, but the end results were the same.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- **Activity 1** may be omitted. It serves as an opportunity for students to sketch similar figures using scale factors greater than 1 or less than 1.
- In **Activity 2**, Problems 5 and 6 may be omitted.

Independent | 7 min

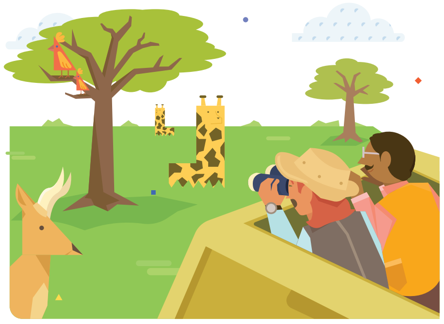
# Warm-up Which One Doesn't Belong?

Students compare four images to reason that two figures are *similar* when there is a sequence of translations, reflections, rotations, or dilations that maps one figure onto the other.

Unit 2 | Lesson 6

## Similarity

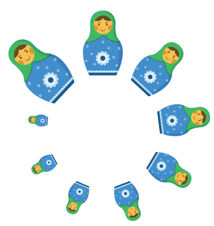
Let's explore similar figures.



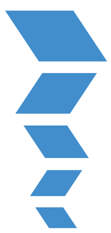
### Warm-up Which One Doesn't Belong?

**Study the images. Which image does not belong with the others? Explain your thinking.**

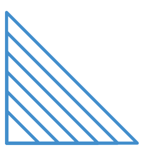
A.



C.



B.



D.

Sample responses:
- Choice A is the only image that shows a rotation.
- Choice C is the only image that shows a reflection.
- Choice B is the only image that does not show a reflection, rotation, or translation. It appears that Choice B might only show dilations.
- Choice D is the only image that shows a translation.

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Conduct the ***Which One Doesn't Belong?*** routine. Encourage students to look for at least one reason why each image might not belong with the others.

### 2 Monitor

**Help students get started** by asking them to choose one image and identify what makes it different from the others.

**Look for productive strategies:**
- Noticing that all of the choices apply a dilation.
- Thinking of transformations as they generate reasons why each image might not belong with the others.

### 3 Connect

**Have students share** what all the choices have in common. They all apply a dilation.

**Define** the term ***similar***. Say, "Two figures are *similar* if one figure can be mapped onto the other by a sequence of translations, rotations, reflections, or dilations. One aspect that all of the choices have in common is that they all show sets of similar figures." Revisit Choices A, B, and C, and ask students to identify the transformations that were applied to show similarity.

**Highlight** Choice D. Tell students that each successive image is slightly reduced in size. Some companies use this method, known as *shrinkflation*, to shrink the size of a product, but keep the price the same.

**Ask**, "How are similarity and transformations related to shrinkflation?"

## Differentiated Support

**Extension: *Math Around the World, Interdisciplinary Connections***

Mention that the images in Choice A are Russian nesting dolls, called *matryoshka*. The first matryoshka was created in 1892, however nesting dolls actually originated in China. Chinese artisans created nesting boxes between 794 and 1185 CE, which were used for storage and decoration. Nesting dolls next made their way to Japan in the form of *Shichifukujin* dolls and other wooden products around the 15th century. The Russian matryoshka dolls were inspired by the Shichifukujin dolls. Consider showing images of Chinese nesting boxes, Japanese nesting dolls, and Russian nesting dolls and ask students to describe the mathematics they see. **(History)**

## Power-up

**To power up students' ability to describe the rigid transformations that result in two congruent figures:**

Provide students with a copy of the Power-up PDF.

**Use:** Before Activity 1

**Informed by:** Performance on Lesson 5, Practice Problem 4 and Pre-Unit Readiness Assessment, Problem 3

Pairs | 15 min

# Activity 1 Creating Similar Figures

Students create rough sketches of similar figures to help them understand how different transformations affect a figure's image and learn that any two congruent figures are similar.

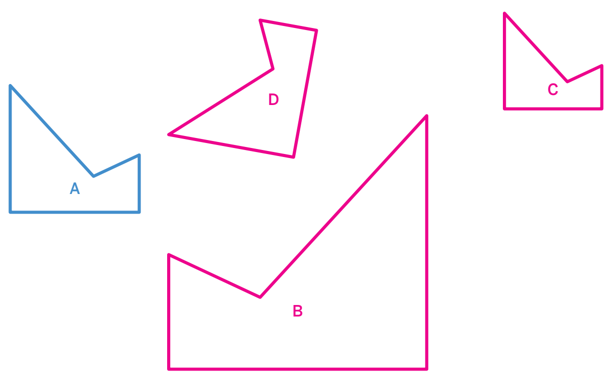
Name: ______________ Date: ________ Period: ________

**Activity 1** Creating Similar Figures

**Complete the following problems using Figure A and the space provided on this page.**

1. Sketch the image of Figure A using a reflection and a dilation with a scale factor greater than 1. Label your sketch Figure B.
2. Sketch the image of Figure A using a dilation with a scale factor less than 1. Label your sketch Figure C.
3. Sketch the image of Figure A using a translation and rotation, but no dilation. Label your sketch Figure D.

**Sample response shown.**




**Compare and Connect:** Your teacher will display some of your classmates' images. What similarities do you see in the images sketched and the transformations that were used?





## 1 Launch

Tell students they do not need to draw precise measurements. However, they need to indicate clearly whether the image is larger or smaller than the original figure. Provide access to geometry toolkits for the duration of this lesson.

## 2 Monitor

**Help students get started** by reminding them how to sketch a reflection, rotation, and translation.

**Look for points of confusion:**

- **Thinking that they have to choose an exact scale factor or measure exact angles.** Explain that precise measurements are not needed, only rough sketches of the images.

**Look for productive strategies:**

- Using hand gestures to help students visualize and draw each transformation.
- Selecting and using tools, including using their fingers or other objects, to measure side lengths and distances.

## 3 Connect

**Display** some student sketches that are not exact, but capture the main features of the figure. Ask other students to identify the transformations used.

**Ask:**

- "Are all the figures similar?"
- "Does a dilation need to occur for a pair of figures to be similar?"

**Highlight** that the images of all the problems are similar to Figure A. A single transformation or multiple transformations can be applied to produce similar images, including ones that do not apply a dilation. Emphasize that Figures A and D are congruent and they are also similar. The scale factor that maps Figure A onto Figure D is 1.

## Differentiated Support

### Accessibility: *Activate Prior Knowledge*

Review rigid transformations. Demonstrate how to sketch an example of each type of rigid transformation of Figure A and leave them displayed for students to reference during the activity. Label your examples with the type of transformation: translation, rotation, or reflection.

### Accessibility: *Vary Demands to Optimize Challenge*

Chunk this task into smaller, more manageable parts by asking students to first reflect Figure A in Problem 1. Then ask them to dilate the result using a scale factor greater than 1. In Problem 3, first ask students to translate the figure. Then ask them to rotate the result.

## Math Language Development

### *MLR7: Compare and Connect*

During the Connect, as students respond to the Ask questions, draw connections between the terms *translation, rotation, reflection,* and *dilation* and whether these transformations result in *congruent* and/or *similar* figures.

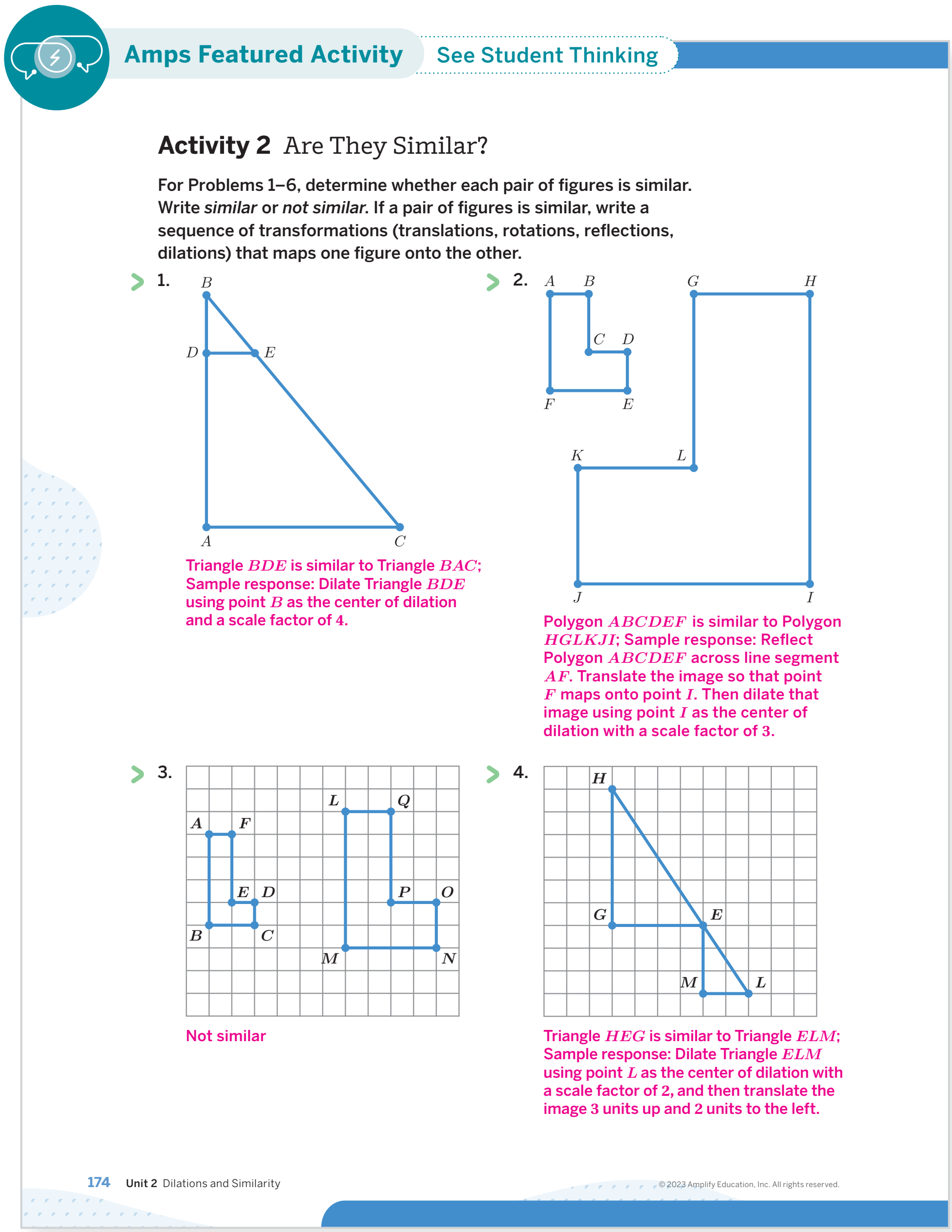
### English Learners

Annotate Figures B, C, and D as either congruent *and* similar, or *only* similar. Use hand gestures to illustrate that Figures B and C are only similar (not congruent).

Pairs | 15 min

# Activity 2 Are They Similar?

Students determine whether a sequence of transformations maps one figure onto the other to discover similar figures.


Amps Featured Activity | See Student Thinking

**Activity 2** Are They Similar?

For Problems 1–6, determine whether each pair of figures is similar. Write *similar* or *not similar*. If a pair of figures is similar, write a sequence of transformations (translations, rotations, reflections, dilations) that maps one figure onto the other.

1. Triangle $BDE$ is similar to Triangle $BAC$; Sample response: Dilate Triangle $BDE$ using point $B$ as the center of dilation and a scale factor of 4.

2. Polygon $ABCDEF$ is similar to Polygon $HGLKJI$; Sample response: Reflect Polygon $ABCDEF$ across line segment $AF$. Translate the image so that point $F$ maps onto point $I$. Then dilate that image using point $I$ as the center of dilation with a scale factor of 3.

3. Not similar

4. Triangle $HEG$ is similar to Triangle $ELM$; Sample response: Dilate Triangle $ELM$ using point $L$ as the center of dilation with a scale factor of 2, and then translate the image 3 units up and 2 units to the left.

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## 1 Launch

Have students use the ***Think-Pair-Share*** routine as they complete each problem. Students should use their geometry toolkits or a grid to verify the sequence of transformations using precise measurements. Remind students to use a ruler when they need to find the scale factor of figures not on a grid.

## 2 Monitor

**Help students get started** by having them match corresponding sides of the triangles, measure them, and find the ratio to determine the scale factor for Problem 1.

**Look for points of confusion:**

- **Having trouble determining the sequence of transformations.** Have students refer to the Unit 1 and Unit 2 anchor charts showing the characteristics of each transformation. Allow access to tracing paper for struggling students.
- **Thinking that Problem 1 is not similar because only a dilation is applied.** Tell students if *any* transformation is applied, the figures are similar.
- **Thinking that the figures in Problems 3 and 5 are similar.** Have students calculate the side lengths of the figures, and then compare the ratios of the corresponding sides.
- **Thinking Problem 6 is not similar because the figures are congruent.** Remind students that an enlargement or reduction is not necessary in order for the figures to be similar. If *any* transformation is applied, the figures are similar.

**Look for productive strategies:**

- Noticing that there are multiple sequences of transformations that can map one figure onto another.

Activity 2 continued ›

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge, Activate Prior Knowledge*

For students who need more processing time, have them focus on Problems 1, 3, and 4. Display the following for students to use as a guide:

- **Congruent and similar:** Translations, Rotations, Reflections
- **Similar:** Dilations

### Accessibility: *Optimize Access to Tools*

Provide copies of the figures so that students can cut them out and manipulate them to help them visualize the transformations. Suggest that students use a ruler to measure distances when not on a grid.

## Math Language Development

### *MLR1: Stronger and Clearer*

During the Connect, as students share their responses for Problem 2, have them individually write an initial draft of their sequence of transformations. Have them share their responses with 2–3 partners. Partners should provide feedback by asking clarifying questions, such as, "How did you know to reflect the figure?", "What is the line of reflection?", and "How did you know the scale factor is 3?" After receiving feedback, provide students time to write improved responses.

### English Learners

Allow pairs of students who speak the same primary language to provide feedback to each other.

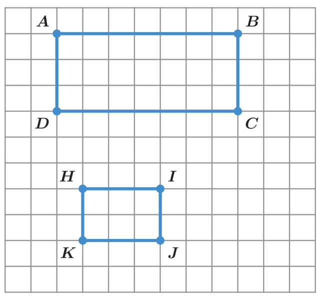
# Activity 2 Are They Similar? (continued)

Students determine whether a sequence of transformations maps one figure onto the other to discover similar figures.

Name: ______ Date: ______ Period: ______

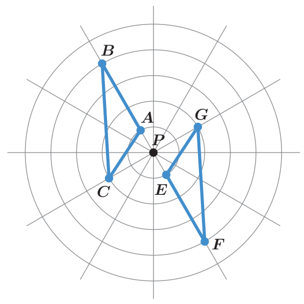
## Activity 2 Are They Similar? (continued)

5.




Not similar

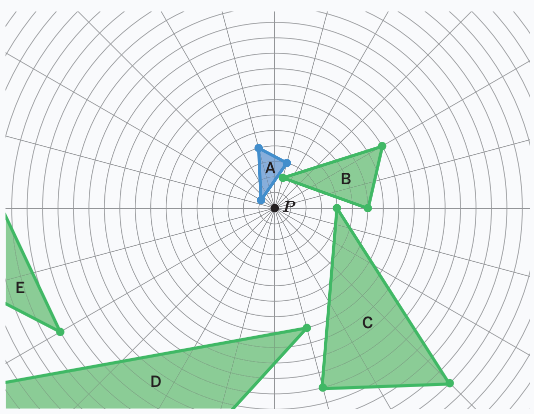
6.




Triangle $ABC$ is similar to Triangle $EFG$; Sample response: Rotate Triangle $ABC$ $180^\circ$ about point $P$.

**Are you ready for more?**

The same sequence of transformations that maps Triangle A onto Triangle B is used to map Triangle B onto Triangle C, and so on. Describe a sequence of transformations that could be this sequence.




Dilate each triangle using point $P$ as the center of dilation and a scale factor of 2, and then rotate the result clockwise $75^\circ$ about point $P$.

STOP





## 3 Connect

**Have pairs of students share** their responses. Conduct the ***Poll the Class*** routine to see which pairs they identified as similar figures. Focusing on Problem 2, select students who wrote different transformations to share their sequence of transformations.

**Ask:**

- "After hearing your classmates' sequences of transformations for Problem 2, what conclusions can you make about proving two figures are similar?" There can be different sequences of transformations to show that two figures are similar.
- "How do you know that the polygons in Problem 3 are *not* similar?" **Note:** Consider demonstrating a possible sequence of transformations, such as translating Polygon $ABCDEF$ so that point $B$ maps onto point $M$, and then dilating by the scale factor 2 using point M as the center of dilation. All of the points, except $A$ and $F$, map onto Polygon $LMNOPQ$, so the figures are not similar.

**Highlight** that there can be many correct sequences of transformations that show that two figures are similar. In order to show that two figures are similar, it is enough to show a sequence of transformations that maps one figure onto the other.

Whole Class | 5 min

# Summary

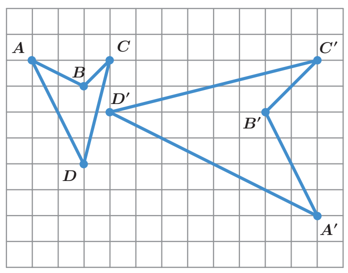
Review and synthesize how two figures are similar if a sequence of translations, rotations, reflections, or dilations can be applied to map one figure onto another.

### Summary

**In today's lesson . . .**

You saw that two figures are ***similar*** if one figure can be mapped onto the other by a sequence of transformations. There may be many correct sequences of transformations, but you only need to describe one to show that two figures are similar.

The symbol ~ indicates that two figures are similar. In the diagram, Polygon $ABCD$ ~ Polygon $A'B'C'D'$. Here is one sequence of transformations that maps Polygon $ABCD$ onto Polygon $A'B'C'D'$.




**Step 1** Dilate Polygon $ABCD$ using point $D$ as the center of dilation and a scale factor of 2.

**Step 2** Translate the image so that point $D$ maps onto point $D'$.

**Step 3** Rotate the new image 90° clockwise about point $D'$.

**Step 4** Reflect the new image across a horizontal line that contains points $D'$ and $B'$.

**Reflect:**




## Synthesize

**Have students share** how they can identify whether two figures are similar, using their own words.

**Highlight** that two figures are *similar* if there is a sequence of translations, rotations, reflections, or dilations that maps one figure onto the other. Introduce the similarity symbol (~) and display the congruent symbol (≅) for comparison. Highlight that corresponding points in the image are labeled using prime notation. For example, point $A'$ is the image of point $A$. Display and reference Part 2 of the Anchor Chart PDF, *Dilations*.

**Formalize vocabulary: *similar***

**Display** Part 2 of the Anchor Chart PDF, *Dilations*. Introduce the similarity symbol and show students how to use the symbol.

**Ask**, "What is the same and different about two figures that are similar versus two figures that are congruent?" Sample response: Two figures that are congruent are also similar, by using a scale factor of 1. Figures that are congruent use a sequence of rigid transformations (translations, rotations, or reflections) to map one figure onto the other. Figures that are similar — but not congruent — include dilations where the scale factor is not equal to 1.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does it mean to say two figures are similar?"

## Math Language Development

### *MLR2: Collect and Display*

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *similar* that were added to the display during the lesson.

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by analyzing a student's incorrect description of a sequence of transformations and explaining how to correct it.

Printable

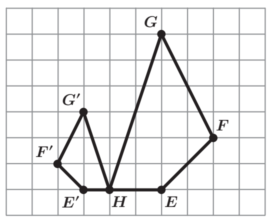
Name: ______ Date: ______ Period: ______

## Exit Ticket 2.06

Noah provides the following sequence of transformations to show that Polygon $EFGH \sim$ Polygon $E'F'G'H$.
Polygon $EFGH$ *is similar to* Polygon $E'F'G'H$.

**Step 1:** Dilate Polygon $EFGH$ using point $H$ as the center of dilation and a scale factor of $\frac{1}{2}$.

**Step 2:** Reflect the result across segment $GH$.




Is Noah's method correct? If not, explain how you could fix it.

Noah's method is not correct; Sample response: Instead of reflecting across segment $GH$, he should reflect the figure across a vertical line that passes through point $H$.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can apply a sequence of transformations to one figure to obtain a similar figure. 1 2 3

b. I can use a sequence of transformations to show that two figures are similar. 1 2 3


**Lesson 6** Similarity

## Success looks like . . .

- **Language Goal:** Comprehending that the phrase *similar figures* means there is a sequence of translations, rotations, reflections, or dilations that maps one figure onto the other. **(Speaking and Listening, Writing)**
- **Language Goal:** Justifying the similarity of two figures using a sequence of transformations that maps one figure onto the other. **(Speaking and Listening)**
  - » Explaining whether Noah's sequence of transformations produces a similar image.

## Suggested next steps

**If students think Noah's method is correct, consider:**

- Highlighting line segment $GH$.
- Asking students to write the sequence of transformations that maps one figure onto the other.
- Reviewing Activity 2.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- What resources did students use as they worked on selecting pairs of similar figures in Activity 2? Which resources were especially helpful?
- The instructional goal for this lesson was to comprehend that the phrase similar figures means there is a sequence of translations, rotations, reflections, and dilations that maps one figure onto the other. How well did students accomplish this? What did you specifically do to help students accomplish it?

## Math Language Development

**Language Goal: Comprehending that the phrase *similar figures* means there is a sequence of translations, rotations, reflections, or dilations that maps one figure onto the other.**

Reflect on students' language development toward this goal.

- How have students initially described the similarity of figures? Have they progressed in their descriptions of similar figures in this lesson to begin to describe them as they relate to transformations?
- How have the language routines used in this lesson helped students develop their mathematical language related to similar figures?

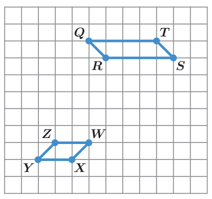
# Practice

Independent

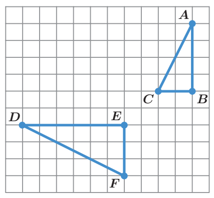
Name: ______ Date: ______ Period: ______

1. Which of the following shows a pair of similar figures?

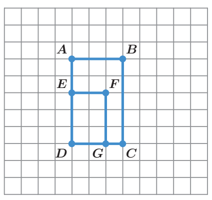
A.




C.

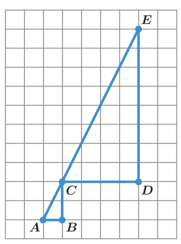



B.



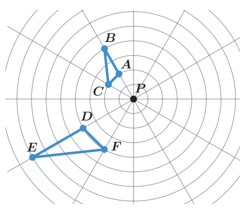

2. Show that each pair of figures is similar by identifying a sequence of transformations that maps the smaller figure onto the larger one.

a




Sample response: Translate Triangle $ABC$ so that point $A$ maps onto point $C$. Dilate the image using point $C$ as the center of dilation and a scale factor of 4.

b




Sample response: Rotate Triangle $ABC$ 90° counterclockwise about point $P$. Dilate the image using point $P$ as the center of dilation and a scale factor of 2.

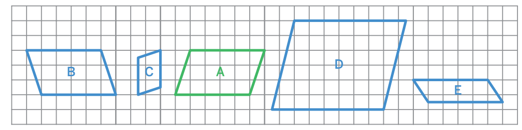




Practice

Name: ______ Date: ______ Period: ______

3. The Rhomboid Snack Group produces granola bars. They want to start producing granola bars in different sizes, but can only do so if they are *similar* to the original bar. Figure A shows the size of the original granola bar. Which granola bar design should they choose? List *all* that apply. Explain your thinking.




**Figures B and C; Sample response: They are the only figures that are similar to Figure A. Figure B can be produced using a reflection. Figure C can be produced using a dilation and rotation.**

4. Each table represents a proportional relationship. Complete each table to show the missing values.

a

| $a$ | $b$ |
|---|---|
| 3.5 | 7 |
| $\frac{1}{2}$ | **1** |
| **4.2** | 8.4 |

b

| $x$ | $y$ |
|---|---|
| 2 | 0.31 |
| 6 | **0.93** |
| 18 | **2.79** |

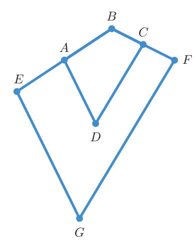
5. Polygon $ABCD$ is a scaled copy of Polygon $EBFG$ with a scale factor of $\frac{1}{2}$. Which of the following is *not* true?

A. Angle $BCD$ is congruent to angle $BFG$.

B. Segment $AD$ is half as long as segment $EG$.

C. The measure of angle $EGF$ is twice as great as the measure of angle $ADC$.

D. Segment $EG$ is twice as long as segment $AD$.








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 1 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 1 | 2 |
| Spiral | 4 | Grade 7 | 1 |
| Formative | 5 | Unit 2 Lesson 7 | 2 |

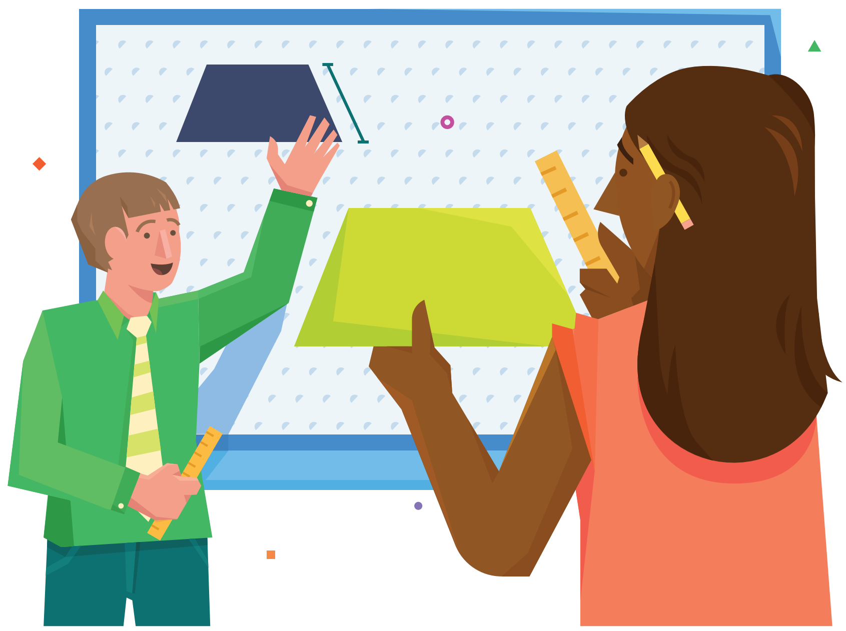
**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Similar Polygons

Let's study the sides and angles of similar polygons.



## Focus

### Goals

1. **Language Goal:** Comprehend the phrase *similar polygons* to mean polygons that have corresponding proportional side lengths and corresponding congruent angles. **(Speaking and Listening, Writing)**
2. **Language Goal:** Critique arguments that claim two polygons are similar. **(Speaking and Listening)**
3. **Language Goal:** Justify the similarity of two polygons given their angle measures and side lengths. **(Speaking and Listening)**

## Rigor

- Students build **conceptual understanding** of the relationship between scaled copies and similar figures.

## Coherence

### Today

Today students make the explicit connection that scaled copies can be obtained by a sequence of transformations and are therefore similar figures. Students understand that in order to determine whether two figures are similar, they can check whether they are scaled copies. They also critique the reasoning of others to determine which properties are necessary to determine similarity.

### Previously

In Lesson 6, students defined similar figures as those that can be achieved by a sequence of transformations that may include dilation.

### Coming Soon

In Lesson 8, students will come to understand that two triangles are similar if they have two congruent corresponding angles.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Power-up PDF, as needed
- Power-up PDF (answers), as needed
- Activity 1 PDF, pre-cut cards, two per student
- plain sheets of paper, one per student
- glue
- geometry toolkits: tracing paper, rulers, protractors

### Math Language Development

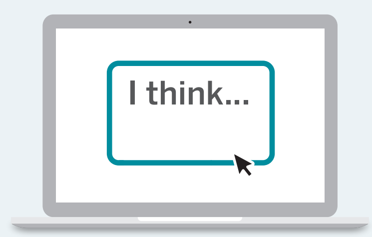
**Review words**

- *dilation*
- *proportional*
- *scaled copies*
- *sequence of transformations*
- *similar*
- *congruent*

**Amps** | Featured Activity

### Activity 2
### See Student Thinking

Students are asked to explain their thinking when they determine whether two figures are similar, and these explanations are available to you digitally, in real time.




Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

As students share their responses with a partner, they may forget to actively listen, and thus might not be able to critique the reasoning used. Remind students that by listening well, they can help improve their own understanding of another person's reasoning, which will help them think more deeply about the mathematics. Review what it means to actively listen and encourage students to practice active listening habits.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted and addressed at a later time.
- In **Activity 1**, have students work together to find a sequence of transformations for only one partner's pair of figures.

Pairs | 5 min

## **Warm-up** Sometimes, Always, Never

Students compare congruence and similarity to determine that two congruent figures are always similar, but two similar figures are not necessarily congruent.

Name: ______________ Date: ________ Period: ________

Unit 2 | Lesson 7

### Similar Polygons

Let's study the sides and angles of similar polygons.

**Warm-up** Sometimes, Always, Never

Determine whether each statement is *always*, *sometimes*, or *never* true.

1. If two figures are congruent, then they are similar.
   This is always true.
2. If two figures are similar, then they are congruent.
   This is sometimes true.
3. If two figures are not congruent, then they are not similar.
   This is sometimes true.
4. If two figures are not similar, then they are congruent.
   This is never true.

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Have students use the ***Think-Pair-Share*** routine as they complete the Warm-up. Pause for discussion after each problem.

### 2 Monitor

**Help students get started** by asking them to explain the process for creating congruent and similar figures.

**Look for points of confusion:**

- **Thinking Statements 1 and 2 are never true.** Display two congruent figures. Ask, "Is there a scale factor you can use that would produce a figure that does not change in size?"
- **Thinking Statement 4 is sometimes true.** Ask students to draw a pair of non-similar figures that are congruent.

**Look for productive strategies:**

- Drawing examples to support their reasoning.

### 3 Connect

**Display** a nested Venn diagram with a section labeled *congruent* inside of the section labeled *similar*. Throughout the lesson, have students suggest properties that are true for both congruence and similarity, true for only one, or true for neither.

**Have students share** their responses by using the ***Poll the Class*** routine. If there is disagreement on a response, ask students with opposing answers to explain their thinking to come to an agreement on a response.

**Highlight** that all congruent figures are similar, but not all similar figures are congruent. If there appears to be some confusion about this, encourage students to think about the scale factor that would be applied to one figure to map to a congruent figure.

## MLR Math Language Development

*MLR2: Collect and Display*

Consider displaying a nested Venn diagram, labeled *Congruent* inside of the section labeled *Similar* throughout this lesson, or add it to the class display. This will serve as a visual reminder that all congruent figures are also similar. Throughout this lesson, consider asking students to suggest properties that could be added to the nested Venn diagram.

**English Learners**

Include a visual of two congruent figures and two similar (but not congruent) figures on the nested Venn diagram.

## Power-up

**To power up students' ability to identify properties of scaled copies:**

Provide students with a copy of the Power-up PDF.

**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 6, Practice Problem 5

Pairs | 15 min

# Activity 1 Different Dilations

Students apply a sequence of transformations to scaled copies of a polygon to discover that any two scaled copies are similar.

## Activity 1 Different Dilations

**Although it doesn't always seem that way, humans often behave in predictable ways. Dr. Hannah Fry, a research mathematician, has been fascinated with the idea of predicting human behavior such as how humans perceive similar but slightly different objects.**

**You and your partner will be given a set of cards with scaled copies of parallelograms and a plain sheet of paper.**

1. Choose two of the cards and verify that they are scaled copies of each other. Explain your thinking here.
   **Answers may vary. Students should make measurements using rulers and protractors, and conclude that the two figures have equal corresponding angle measures and that corresponding side lengths are scaled by the same scale factor (proportional).**
2. Glue your cards anywhere on the separate sheet of paper.
3. Switch papers with your partner, and ask them to show that your two parallelograms are similar.
   **Answers may vary, but should mention a sequence of transformations that include a dilation using the scale factor chosen to create the scaled copy.**

**Featured Mathematician**

**Hannah Fry**

Have you ever wondered how streaming services know what shows to recommend? Many aspects of our lives are now influenced by algorithms designed to interpret and predict human behavior. Like so many of us, English mathematician Hannah Fry wants to understand why people do the things they do. She has worked with physicists, mathematicians, computer scientists, architects, and geographers to understand human behavior through pattern recognition. Her work studying the patterns of human behavior through mathematics has touched on many aspects of society, from shopping and dating to crime and terrorism.

"Hannah Fry" by Sebastiaan ter Burg, courtesy of Flickr, licensed under CC BY 2.0.(https://www.flickr.com/photos/31013861@N00/36638999274)




## 1 Launch

Distribute one plain sheet of paper and two cards from the Activity 1 PDF to each student. Provide access to geometry toolkits and glue.

## 2 Monitor

**Help students get started** by asking, "What are the properties of scaled copies?" The corresponding side lengths are proportional and the corresponding angles are congruent.

**Look for points of confusion:**

- **Not knowing how to show that the figures are similar.** Remind students that they can use a sequence of transformations.
- **Not knowing how to describe the sequence of transformations.** Encourage students to use the tracing paper from their geometry toolkits to match a pair of corresponding vertices, and describe how one point would map onto the other.

**Look for productive strategies:**

- Selecting and using tracing paper to map the sequence of transformations.

## 3 Connect

**Display** selected student work with labeled congruent angles, proportional side lengths, and a written sequence of transformations.

**Ask:**

- "Is there a way to place the cards on the paper so that there is no sequence of transformations for the scaled figures?" No. **Note:** If a student says yes, ask them to demonstrate, and invite the class to try and find a sequence.
- "What does this imply about scaled copies and similarity?" Two figures are similar if they are scaled copies of each other.

**Highlight** that students do not need to perform a sequence of transformations to prove that two figures are similar if they can prove that they are scaled copies.

## Math Language Development

### *MLR8: Discussion Supports—Press for Details*

During the Connect, as students respond to the second Ask question, press for details to solidify the connection between scaled copies and similar figures. For example, if a student says, "Scaled copies are similar," ask these follow-up questions to drive home the point:

- "What do you know about the corresponding angle measures of scaled copies? What does this tell you about the corresponding angle measures of similar figures?" The corresponding angle measures of scaled copies, and thus similar figures, are congruent.
- "What do you know about the corresponding side lengths of scaled copies? What does this tell you about the corresponding side lengths of similar figures?" The corresponding side lengths of scaled copies, and thus similar figures, are proportional.

## Featured Mathematician

### Hannah Fry

Have students read about featured mathematician Hannah Fry, a researcher who uses mathematics to analyze and predict human behavior. Her research draws from and influences many different fields, including computer science and geography.

Pairs | 15 min

# Activity 2 Are You Sure They Are Similar?

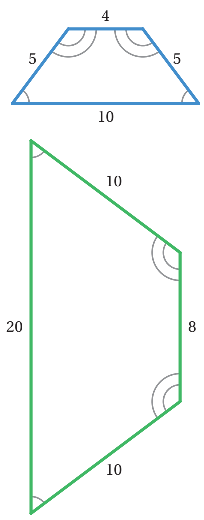
Students critique the reasoning of others to test which properties must be present to determine similarity.

**Amps Featured Activity** See Student Thinking

Name: ______________ Date: ________ Period: ________

## Activity 2 Are You Sure They Are Similar?

1. Clare is trying to persuade her classmates that the polygons shown are similar. Which argument is the most convincing? Explain your thinking.




   **A.** The corresponding side lengths are proportional.

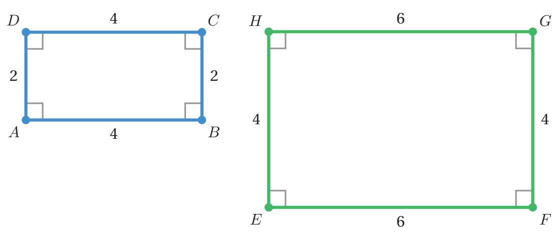
   **B.** The corresponding angles are congruent.

   **C.** The figures are scaled copies.

   **D.** The smaller figure has been rotated.

   Sample response: Argument C is the most convincing. Similar figures have proportional side lengths and congruent angles. Any two scaled copies can be mapped onto each other by a sequence of transformations, so scaled copies will always be similar. We cannot say two figures are similar just because the side lengths are proportional, or just because angles are congruent.

2. Jada studied the two figures shown and noticed that the angles are all congruent and the side lengths of each figure differ by 2 units. Is this enough to claim the figures are similar?




   No, it is not enough; Sample response: I need to multiply all the side lengths of one polygon by the same scale factor to get the side lengths of the other polygon. Because there is no scale factor that dilates Rectangle $ABCD$ onto Rectangle $EFGH$, the rectangles are not similar.

STOP





## 1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

## 2 Monitor

**Help students get started** by asking, "Using the results from the previous activity, how can you determine whether two figures are similar?"

**Look for points of confusion:**

- **Choosing argument D in Problem 1.** Ask students if it is sufficient to describe the mapping as a rotation.
- **Thinking that Jada is correct in Problem 2.** Have students think about the relationship of the corresponding side lengths in similar figures, and ask if there is a scale factor that would map Rectangle $ABCD$ onto Rectangle $EFGH$.

## 3 Connect

**Display** both pairs of figures.

**Have pairs of students share** their thinking. Select students who disagreed with each statement to convince the students who agreed using mathematical reasoning.

**Ask:**

- "If you know two polygons have corresponding congruent angles, can you say the two polygons are similar?"
- "If you know two polygons have corresponding proportional side lengths, can you say the two polygons are similar?"

**Highlight** that similar polygons have corresponding congruent angles *and* corresponding proportional side lengths.

## Differentiated Support

### Accessibility: *Clarify Vocabulary and Symbols*

Before students complete Problem 1, display and discuss important vocabulary that they will need to access in the problem, such as *corresponding, proportional, scaled copies,* and *rotated*.

### Extension: *Math Enrichment*

Have students show and describe two different ways they could alter the figures in Problem 2 so that they would be similar. Sample responses:

- Alter the side lengths of Rectangle $EFGH$ so that the longer sides each measure 8 units.
- Alter the side lengths of Rectangle $EFGH$ so that the shorter sides each measure 3 units.

## Math Language Development

### *MLR3: Critique, Correct, Clarify*

Consider introducing Problem 2 using this routine and tell students that Jada claims this information is enough to show the figures are similar. Let them know her statement is incorrect. Ask these questions:

- ***Critique:*** "How do you know that this information is not enough to claim the figures are similar?"
- ***Correct:*** "How would you correct Jada's claim? Are the figures similar?"
- ***Clarify:*** "Write a corrected claim that Jada could use to determine whether *any two figures* are similar. How do you know your claim is correct?"

Whole Class | 5 min

# Summary

Review and synthesize the properties of similar figures and how to show whether two figures are similar.

### Summary

**In today's lesson . . .**

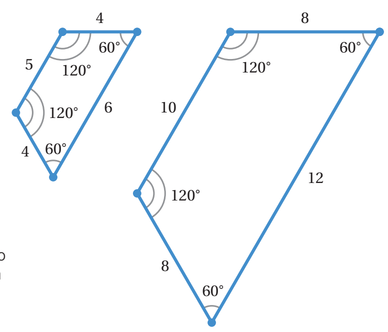
You explored the properties of figures to determine similarity.

When two polygons are similar:

- Every angle and side in one polygon has a corresponding part in the other polygon.
- All pairs of corresponding angles have the same measure.
- Corresponding sides are related by a single scale factor. Each side length in one polygon is multiplied by the scale factor to get the corresponding side length in the other polygon.
- A sequence of transformations can be applied to one polygon to map onto another polygon.

To show two polygons are similar, you can show they are scaled copies of each other.

- For example, you can examine the angle measures of these trapezoids and conclude that corresponding angles are congruent.
- Then you can determine that side lengths are proportional because each side length of the smaller trapezoid can be multiplied by 2 to get the corresponding side length of the larger trapezoid.
- Because these trapezoids meet the criteria for being scaled copies, they must be similar.




**Reflect:**




## Synthesize

**Display** the cards from Activity 1.

**Ask:**

- "How can you determine whether two figures are similar? What arguments will be the most convincing?" By describing a sequence of transformations that maps one figure to the other, or by checking whether corresponding side lengths are proportional and corresponding angles are congruent.
- "How can you determine whether two figures are *not* similar?"
- "For which figures is it enough to know that the lengths of corresponding sides are proportional to show that the figures are similar?" Rectangles
- "For which figures is it enough to know that the corresponding angles are congruent to show that the figures are similar?" Rhombuses

  **Note:** Mention to students that they will explore this idea further in the next lesson.

**Highlight** that students can use these strategies to verify whether two figures are similar:

- Describing a sequence of transformations that maps one figure onto the other
- Determining whether the figures are scaled copies of one another by checking whether side lengths are proportional and corresponding angles are congruent.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you identify whether two figures are similar?"

## Math Language Development

### *MLR2: Collect and Display*

If you chose to display a nested Venn diagram as suggested in the Warm-up, provide students time to refer back to this diagram during the Summary. Encourage students to add to the display any words, phrases, and images about congruence and similarity that have not yet been added.

Independent | 5 min

# Exit Ticket

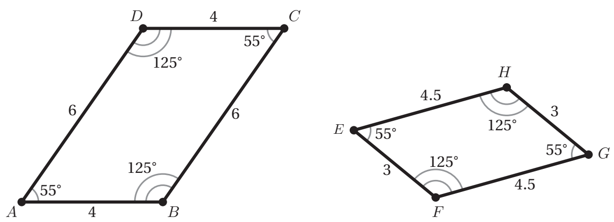
Students demonstrate their understanding by explaining how they know two figures are similar.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 2.07

**Is Quadrilateral $ABCD \sim$ Quadrilateral $EFGH$? Explain your thinking.**
**Note: The figures may not be drawn to scale.**




Sample response: I know these figures are similar because all corresponding angles are equal, and the side lengths of Quadrilateral $ABCD$ can be multiplied by a scale factor of $\frac{3}{4}$ to equal the side lengths of Quadrilateral $EFGH$.

**Self-Assess**

1 I don't really get it — 2 I'm starting to get it — 3 I got it

a. I can use angle measures and side lengths to conclude that two polygons are not similar.
1 2 3

b. I know the relationship between angle measures and side lengths in similar polygons
1 2 3



**Lesson 7** Similar Polygons

## Success looks like . . .

- **Language Goal:** Comprehending the phrase *similar polygons* to mean polygons that have corresponding proportional side lengths and corresponding congruent angles. **(Speaking and Listening, Writing)**
- **Language Goal:** Critiquing arguments that claim two polygons are similar. **(Speaking and Listening)**
- **Language Goal:** Justifying the similarity of two polygons given their angle measures and side lengths. **(Speaking and Listening)**
  - » Explaining whether the two quadrilaterals are similar.

## Suggested next steps

**If students are not able to show the similarity between the figures, consider:**

- Reviewing Activity 2 for arguments that do not verify similarity of figures.
- Assigning Practice Problem 3.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- Have you changed any ideas you used to have about similarity as a result of today's lesson?
- What did students find frustrating about creating a sequence of transformations in Activity 1? What helped them work through this frustration? What might you change for the next time you teach this lesson?

## Math Language Development

**Language Goal: Comprehending the phrase *similar polygons* to mean polygons that have corresponding proportional side lengths and corresponding congruent angles.**

Reflect on students' language development toward this goal.

- Have students progressed in their descriptions of similar figures and justifications of whether two polygons are similar? Are they using mathematical language such as:
  - » Corresponding side lengths are proportional?
  - » The ratios of corresponding side lengths are equal?
  - » Corresponding angles are congruent?

**Use:** Before the Warm-up
**Informed by:** Performance on Lesson 6, Practice Problem 5.

# Practice

Independent

Name: ______________ Date: ________ Period: ________

Practice

1. Triangle $DEF$ is a dilation of Triangle $ABC$ with scale factor of 2. In Triangle $ABC$, the greatest angle measures 82°. What is the greatest angle measure in Triangle $DEF$?

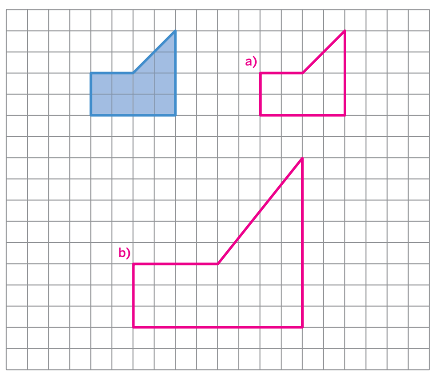
   A. 41°

   B. 82°

   C. 84°

   D. 164°

2. Consider the following polygon. **Sample response shown.**




   a. Draw a polygon that is similar, but could be mistaken for being *not* similar.

   b. Draw a polygon that is *not* similar, but could be mistaken for a similar polygon.



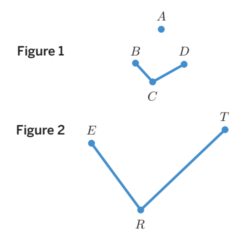


Name: ______________ Date: ________ Period: ________

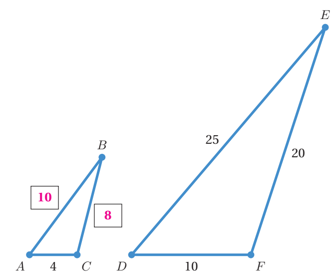
Practice

3. Lin claims that Figure 2 is a dilation of Figure 1 using point $A$ as the center of dilation. What are some ways you can convince Lin that her claim is *not* true?

   **Sample response: The measures of $\angle C$ and $\angle R$ are not equal, so Figure 2 cannot be a dilation of Figure 1.**

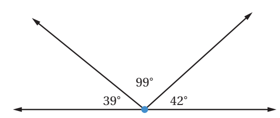



4. Triangle $ABC$ is a scaled copy of Triangle $DEF$ with a scale factor of $\frac{2}{5}$. Find the missing lengths of Triangle $ABC$.




5. The line shown has been partitioned into three angles. Is there a triangle with these three angle measures? Explain your thinking.

   **Sample response: Yes, the sum of the three angle measures in a straight line is 180°, which is the same as the sum of the interior angles in a triangle.**








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Grade 7 | 1 |
| Formative | 5 | Unit 2 Lesson 8 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Similar Triangles

Let's explore similar triangles.

## Focus

### Goals

1. **Language Goal:** Generalize a process for identifying similar triangles and justify that finding two pairs of congruent corresponding angles is sufficient to show similarity. **(Speaking and Listening)**

2. **Language Goal:** Justify that two triangles are similar by verifying that two pairs of corresponding angles are congruent. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students focus their study on triangles and determine whether or not they are similar by looking only at the corresponding angle measures. They understand that if two triangles share three corresponding angle measures, then they are similar, reasoning that because the sum of the angle measures in a triangle is $180°$, knowing two angle measures determines the third angle measurement. Students conclude that for triangles, all that is needed to deduce similarity is having two congruent corresponding angle pairs.

### Previously

In Lesson 7, students found that, in order to check whether two polygons are similar, it is important, in general, to check that corresponding angle measures are congruent and that corresponding side lengths are proportional.

### Coming Soon

In Lesson 9, students will find missing side lengths of similar triangles. In Unit 3, they use the similarity criterion to understand the concept of the slope of a line. Later on in high school, they will learn that three proportional side lengths (but not two) is also enough to deduce that two triangles are similar.

## Rigor

- Students build **conceptual understanding** by discovering how many corresponding congruent angle pairs are needed to definitively say that two triangles are congruent.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Activity 3 | Summary | Exit Ticket |
|---|---|---|---|---|---|
| 5 min | 10 min | 8 min | 12 min | 5 min | 5 min |
| Independent | Pairs | Independent | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, pre-cut cards, one set per pair
- geometry toolkits: tracing paper, protractors, rulers

### Math Language Development

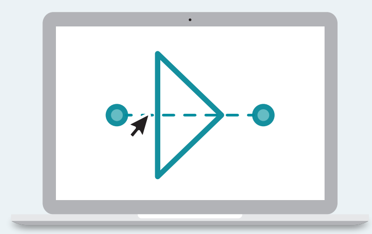
**Review words**

- *scale factor*
- *congruent*
- *corresponding*
- *similar*
- *dilation*
- *sequence of transformations*

**Amps** | **Featured Activity**

### Activity 1
## Digital Triangles

Students compare triangles they draw with those drawn by their peers to see. They will see how having three corresponding angle pairs that are congruent is sufficient evidence to prove two triangles similar.



Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students may be quick to accept informal or incomplete arguments about similar triangles in Activity 2. Encourage students to spend more time with the problem, draw a visual model, and discuss possible exceptions or counterarguments with a partner before coming to a conclusion.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted
- **Activity 2** may be omitted.

Independent | 5 min

## Warm-up Imagine a Triangle . . .

Students activate prior knowledge about the angles of a triangle to prepare for the reasoning needed about angle measures of triangles later in the lesson.

Name: ________ Date: ________ Period: ________

Unit 2 | Lesson 8

### Similar Triangles

Let's explore similar triangles.

**Warm-up** Imagine a Triangle . . .

**A triangle has an angle measure of 100°. What else must be true about the triangle? Select *all* the true statements.**

A. The triangle is isosceles.

B. The remaining two angle measures add to 80°.

C. One angle has a measure of 20° and the other has a measure of 60°.

D. The missing angles must each be acute.

E. It is an obtuse triangle.

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Activate students' prior knowledge by asking them to describe the features of acute, isosceles, and obtuse triangles.

### 2 Monitor

**Help students get started** by having them sketch a triangle and asking, "What is the sum of the interior angle measures of a triangle?"

**Look for points of confusion:**

- **Thinking the angle measures must be 20° and 60°.** Tell students those *could* be the angle measures, and ask, "Can you think of another pair of angle measures that would also work for this triangle?"
- **Not recognizing that both missing angles must be acute.** Remind them of the definition of *acute* angle and ask, "What is the sum of the missing angle measures? What is the maximum measure for one of those angles?"

**Look for productive strategies:**

- Drawing a visual model to test examples and counterexamples

### 3 Connect

**Have students share** with a partner their examples and counterexamples before sharing with the class.

**Highlight** student strategies that used visual models and detailed counterexamples to prove the false statements incorrect.

**Ask**, "Could this triangle have a right angle? Why or why not?" No; Sample response: The remaining angle measures must have a sum equal to 80°, so none of the angles in this triangle can measure 90°.

## Math Language Development

### *MLR3: Critique, Correct, Clarify*

During the Connect, display the incorrect statements, A and C. Use the following routine.

- ***Critique:*** Have students critique these statements as to why they are incorrect. Encourage the use of visual examples or counterexamples.
- ***Correct and Clarify:*** Have students write corrected statements. Ask them to clarify how they know their revised statement is correct.

### English Learners

The idea of a *counterexample* might be unfamiliar. Draw a triangle with angle measures of 100°, 30°, and 50°. Write the term *counterexample* next to this triangle to illustrate how this shows Choice C is not a true statement, because not all triangles with one angle measure of 100° has to have the other two angles measure 60° and 80°.

## Power-up

**To power up students' ability to determine whether three angles can form a triangle, have students complete:**

Recall that the sum of the measures of the angles in any triangle is 180°. Complete the table so that each set of three angles can form a triangle.

| | $m\angle A$ | $m\angle B$ | $m\angle C$ |
|---|---|---|---|
| Triangle E | 100° | 70° | 10° |
| Triangle F | 83° | 25° | 72° |

**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 7, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 5

Pairs | 10 min

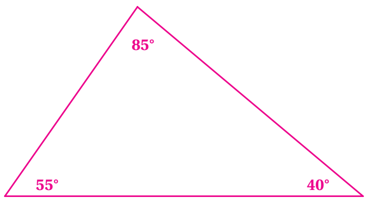
# Activity 1 Are Three Angles Enough?

Students create and compare triangles with congruent angles to see that the triangles are not necessarily congruent, but similar.

**Amps Featured Activity** Digital Triangles

### Activity 1 Are Three Angles Enough?

1. Construct a triangle with angle measures of 40°, 55°, and 85°.
   Sample responses shown.




2. Compare your triangle to a partner's triangle.
   a. Are your triangles congruent?
      Sample responses:
      - No. While the angle measures are the same, the corresponding side lengths are not.
      - (Unlikely) Yes. All angle measures and side lengths are the same.
   b. Are your triangles similar? Explain your thinking.
      Sample responses:
      - Yes, because they have three congruent angles and all the corresponding sides are proportional.
      - Yes, because I can map one onto the other by applying a sequence of rigid transformations and a dilation.

186 Unit 2 Dilations and Similarity


## 1 Launch

Provide access to geometry toolkits for Activities 1, 2, and 3. Collect the toolkits prior to the Exit Ticket.

## 2 Monitor

**Help students get started** by having them draw a 40° angle and helping them draw a side length and second angle.

**Look for points of confusion:**

- **Thinking that their triangle is the "same" as their partner's because the angle measures are the same.** Have students measure the side lengths and compare them.
- **Informally describing that their triangles are similar with a dilation.** Have them use tracing paper to map one angle to the other and then ask students to use their ruler to find side lengths and a scale factor that proves the dilation works.

## 3 Connect

**Display** the animation from the Activity 1 Amps slides showing triangles mapped onto each other.

**Have students share** what they noticed about the relationships between their triangle and their partner's triangle.

**Highlight** that if triangles share three corresponding congruent angles, then they are similar.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Provide students with two pre-cut triangles that have the three congruent angle measures and different side lengths labeled. This will allow them to access the mathematical goal of the activity, without having to actually construct the triangles themselves.

### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can interact with a digital animation that illustrates the mathematical goal of this activity.

## Math Language Development

### *MLR2: Collect and Display*

While students work, circulate and listen to the language they use to describe whether their triangles are congruent or similar, such as *corresponding angle measures, congruent, corresponding side lengths, proportional, rigid transformations, dilation,* etc. Record these words and phrases and add them to the class display. Encourage students to use these words and phrases during the Connect discussion.

### English Learners

Add visual examples of these words and phrases to the class display.

Independent | 8 min

# Activity 2 Is One Angle Enough?

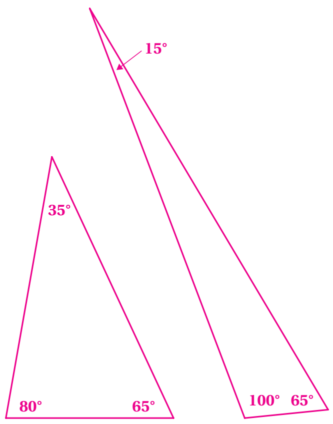
Students consider two triangles with one corresponding congruent angle to determine if knowing this is enough to determine whether the triangles are similar.

Name: ______________ Date: ______ Period: ______

## Activity 2 Is One Angle Enough?

**Andre drew a triangle with one angle that measured 65°. Bard drew a triangle with one angle that measured 65°. Can Andre and Bard guarantee they drew similar triangles? If yes, explain why. If not, show an example.**

**Sample response: No, there could be two triangles that each have an angle measuring 65° but one could have remaining angle measures of 80° and 35° and the other could have remaining angle measures of 100° and 15°.**








### 1 Launch

Ask, "How many corresponding congruent angle pairs, would you guess, are needed, at minimum, to prove similarity: 1, 2, or 3?"

### 2 Monitor

**Help students get started** by asking, "What must be true about the other two angles of the triangle?"

**Look for points of confusion:**

- **Thinking the two triangles will be similar.** Have students draw two 65° angles. Ask, "Can you come up with two different combinations for the remaining two angles?"

**Look for productive strategies:**

- Drawing an accurate example of two triangles — with one corresponding congruent angle — that are not similar.

### 3 Connect

**Display** the animation from the Activity 2 Amps slides that shows the two triangles mapped onto each other.

**Ask**, "Is knowing that two triangles share one congruent corresponding angle enough to determine they must be similar?"

**Have pairs of students share** the different ways they can show the triangles are not similar, either by describing dilations and scale factor or by finding side lengths and looking for proportional relationships.

**Highlight** that knowing one congruent corresponding angle is not enough to determine whether two triangles are similar, because knowing only one angle means the other two angle measures may not be congruent. When two triangles have different angle measures, they are not similar.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Alter the activity by displaying different triangles that each have one angle measure of 65°. Include some triangles that are similar, and others that are not. Ask students to determine whether the triangles are similar.

### Extension: *Math Enrichment*

Have students complete the following problem:
Andre and Bard each drew a *right triangle* with one angle measuring 65°. Is this enough information to guarantee similar triangles? Explain your thinking. Yes, because they are right triangles, I can determine that all three corresponding angle pairs are congruent.

## Math Language Development

### *MLR8: Discussion Supports*

During the Connect, as students respond to the Ask question, display the triangles from the sample response. Alternatively, display two triangles that fit the given criteria, but are not similar. Ask students to explain what information would need to change in order for the triangles to be similar. Emphasize that the sample response is considered a *counterexample* that shows why one congruent angle is not sufficient information to determine whether two triangles are similar.

### English Learners

Annotate the two triangles by indicating the one pair of congruent angles, yet the triangles are *not* similar.

Pairs | 12 min

# Activity 3 Card Sort: Similar or Not?

Students sort pairs of triangles into categories to realize that knowing two pairs of corresponding angles are congruent is sufficient to determine whether the triangles are similar.

## Activity 3 Card Sort: Similar or Not?

**You will be provided with a set of cards. Each card contains two triangles.**

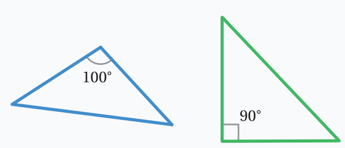
1. Sort the cards into three groups:
   - Triangles that are *similar.*
   - Triangles that are *not similar.*
   - Triangles for which you *do not have enough information* to determine whether they are similar.

| Similar | Not similar | Not enough information |
|---|---|---|
| Card 2, Card 3, Card 5 | Card 6 | Card 1, Card 4 |

2. Select a card for which you decided did not have enough information to determine whether the triangles are similar. Explain what other information would be needed.
   Sample response: Cards 1 and 4 show only one angle measure in each triangle. I would need to know if there are at least two pairs of angles that are congruent for me to determine whether the triangles are similar.

**Are you ready for more?**

**Tyler and Elena wanted to determine in which category to place the following pair of triangles. Tyler said there was not enough information to determine whether they are similar. Elena says there is enough information and she knows the triangles are *not* similar. Do you agree with Tyler or Elena? Explain your thinking.**




Sample response: Elena is correct. Because one triangle has an angle measuring 100°, the other two angle measures must each be less than 80°, which means it cannot also have a 90° angle. The two triangles can only have at most one angle that has the same measure, and, therefore, are not similar.

STOP

188 Unit 2 Dilations and Similarity


## 1 Launch

Say, "You already know that if you have one pair of corresponding angles that are congruent between two triangles, it is not sufficient to say the triangles are similar. If you know that all three corresponding angle pairs are congruent, then it is sufficient to say the triangles are similar. What about knowing two corresponding angle pairs? Let's find out." Distribute the cards from the Activity 3 PDF to each pair of students.

## 2 Monitor

**Help students get started** by having them begin with Card 2 and determining the third angle measure.

**Look for points of confusion:**

- **Thinking the triangles on Card 1 are similar.** Say, "Think back to Activity 2. Why is knowing one pair of angles not enough to determine they are similar?"
- **Thinking the triangles on Card 5 are not similar.** Ask, "Have you confirmed what the missing angle measure is? What does that measure tell you?"

## 3 Connect

**Ask**, "How many pairs of corresponding congruent angles are sufficient to determine two triangles are similar? Why?"

**Highlight** that two pairs of corresponding congruent angles are sufficient to determine that two triangles are similar. When triangles share two pairs of corresponding congruent angles, they actually share three pairs of corresponding congruent angles because the measure of the unknown third angle must be the same value for both triangles.

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge***

Consider one of these alternative approaches to this activity.

- If students need more processing time, have them focus on only sorting Cards 1, 2, 4, and 5. Then omit Problem 2.
- Have students work with all of the cards, but first ask them to sort the cards by the number of corresponding angle pairs they know are congruent.

Whole Class | 5 min

# Summary

Review and synthesize how determining two congruent corresponding angle pairs is sufficient evidence for showing that two triangles are similar.

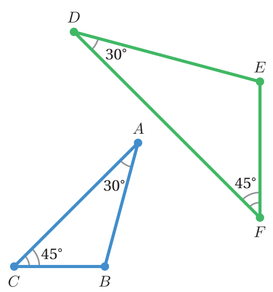
Name: ______ Date: ______ Period: ______

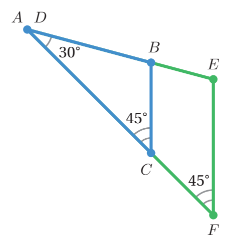
## Summary

**In today's lesson . . .**

You further developed your understanding that two polygons are similar when there is a sequence of translations, rotations, reflections, and dilations taking one polygon to the other. You saw that when the polygons are triangles, you only need to check that both triangles have two congruent angles to show they are similar.

For example, Triangle $ABC$ and Triangle $DEF$ each have a 30° angle and a 45° angle. You can translate Triangle $ABC$ so that point $A$ maps onto point $D$, and then rotate the resulting triangle so that the two 30° angles are aligned.




Now a dilation with center $A$ and appropriate scale factor will map point $C$ onto point $F$. This dilation also maps point $B$ onto point $E$, showing that $\Delta ABC \sim \Delta DEF$.

**Reflect:**





### Synthesize

**Display** the Summary from the Student Edition.

**Ask:**

- "How can you show two triangles are similar using transformations?"
- "How can you show two triangles are similar using side lengths and angles?"
- "How can you show two triangles are similar using only angles?"
- "Does what you learned today apply to other types of polygons? Are two corresponding congruent angle pairs sufficient to determine similarity with a quadrilateral? Why or why not?" Two angles are not sufficient to determine similarity with quadrilaterals. The two angle criteria is specific to triangles because if you know that two corresponding angle pairs are congruent, then the third angle pair of the triangles will also have the same measure. Quadrilaterals have four angles, so knowing only two pairs is not sufficient.

**Have students share** the ways students can determine whether two triangles are similar.

**Highlight** that today students learned a special feature specific to triangles — that knowing two corresponding angle pairs are congruent is sufficient information to know the two triangles are similar.

### Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Which ways of determining whether two triangles are similar are you most comfortable with? Which ones, if any, are you least comfortable with?"

Independent | 5 min

# Exit Ticket

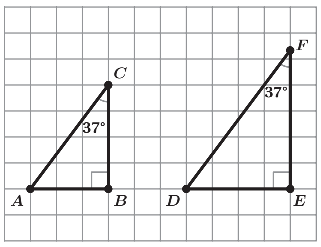
Students demonstrate their understanding by determining whether two triangles are similar.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 2.08

Is $\triangle ABC \sim \triangle DEF$? Explain your thinking.




Yes, they are similar; Sample response:

- They have two angles with the same measure.
- One triangle can be translated so that its right angle is mapped onto the right angle of the other triangle. The triangle can then be dilated to map onto the other triangle, which proves that they are similar.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I know how to decide whether two triangles are similar by studying their angle measures.

1 2 3



**Lesson 8** Similar Triangles

## Success looks like . . .

- **Language Goal:** Generalizing a process for identifying similar triangles and justifying that finding two pairs of congruent corresponding angles is sufficient to show similarity. **(Speaking and Listening)**
- **Language Goal:** Justifying that two triangles are similar by checking that two pairs of corresponding angles are congruent. **(Speaking and Listening, Writing)**
  - » Explaining whether the two triangles are similar.

## Suggested next steps

**If students use other methods besides analyzing the congruent corresponding angles to prove similarity, consider:**

- Assigning Practice Problem 1.
- Monitoring these students first and asking, "How can you use just the angle measures to determine whether two triangles are similar?"

**If students are using informal or insufficient rationale for proving similarity, consider:**

- Reviewing Activity 1.
- Having students review the Summary in their Student Editions.
- Assigning Practice Problem 1.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- Which groups of students did and didn't have their ideas seen and heard today?
- What different ways did students approach justifying if two triangles were similar? What does that tell you about similarities and differences among your students?

# Practice

Independent

Name: Date: Period:

Practice

**1.** In each pair of triangles, some of the angle measures are given. Determine whether the triangles are similar, not similar, or if there is not enough information to determine whether they are similar. Place a check mark in the appropriate column.

| Triangle pairs | Similar | Not Similar | Not enough information |
|---|---|---|---|
| Triangle A: 53°, 71°<br>Triangle B: 53°, 71° | ✓ | | |
| Triangle C: 90°, 33°<br>Triangle D: 90°, 57° | ✓ | | |
| Triangle E: 63°, 45°<br>Triangle F: 14°, 71° | | ✓ | |
| Triangle G: 100°<br>Triangle H: 70° | | | ✓ |

**2.** In the space provided, draw two equilateral triangles that are *not* congruent.

**Sample response:**

**a** Measure the side lengths and angles of your triangles. Are the two triangles similar? Why or why not?

**Sample response: Yes, all angles are congruent, which means the triangles are similar.**

**b** Do you think two equilateral triangles will *always*, *sometimes*, or *never* be similar? Explain your thinking.

**Always; Sample response: Because an equilateral triangle will always have three angles that measure 60°, two equilateral triangles will always have at least two angles that have the same measurements (in fact, they have three), and so, they will always be similar.**

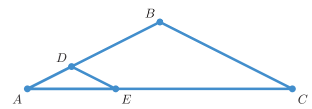




Name: Date: Period:

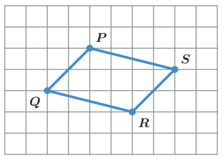
Practice

**3.** In the figure, segment $BC$ is parallel to segment $DE$. Explain why $\Delta ABC \sim \Delta ADE$.




**Sample response: Because segment $BC$ and segment $DE$ are parallel and segment $AB$ is a transversal, $\angle ABC$ is congruent to $\angle ADE$. $\angle DAE$ is congruent to $\angle BAC$. This means both triangles have two congruent angles and therefore are similar.**

**4.** Quadrilateral $PQRS$ is a parallelogram. Let Quadrilateral $P'Q'R'S'$ be the image of Quadrilateral $PQRS$ after performing a dilation centered at a point $O$ (not shown) with a scale factor of 3. Which of the following is true?




A. $P'Q' = PQ$

B. $P'Q' = 3PQ$

C. $3P'Q' = PQ$

D. The relationship of segment $PQ$ to segment $P'Q'$ cannot be determined from the information given.

**5.** Simplify each fraction.

**a** $\frac{8}{12} = \frac{2}{3}$

**b** $\frac{25}{10} = \frac{5}{2}$

**c** $\frac{3}{3} = 1$

**d** $-\frac{9}{24} = -\frac{3}{8}$





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 2<br>Lesson 3 | 1 |
| Formative | 5 | Unit 2<br>Lesson 9 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available


For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Ratios of Side Lengths in Similar Triangles

Let's use similarity to determine side lengths in similar triangles.


## Focus

### Goals

1. Calculate unknown side lengths in similar triangles using two methods: using the ratios of side lengths *within* the triangles and using the scale factor between *similar* triangles.
2. **Language Goal:** Generalize that the ratios of corresponding side lengths in similar triangles are equal. **(Speaking and Listening)**

## Coherence

### Today

Students will discover that the ratio of a pair of side lengths in one triangle will equal the ratio of the corresponding side lengths in a similar triangle. While this fact is not limited to triangles, this lesson focuses on the particular case of triangles before students formally learn about the slope of a line in Lesson 11. Students then apply their understanding by constructing viable solutions using two different methods when solving for an unknown side length given similar triangles.

### Previously

In Lesson 1, students calculated the ratio of lengths of different rectangles to discover properties of scaled copies. In Lesson 7, students learned that similar figures are scaled copies, and that as a result, there is a scale factor that they can use to multiply all of the side lengths in one polygon to find the corresponding side lengths in a similar polygon.

### Coming Soon

In Lesson 10, students will apply their understanding of similar triangles by predicting the height of a tall object given the heights and shadows of proportional figures.

## Rigor

- Students build **conceptual understanding** by comparing the ratios of corresponding side lengths of similar triangles.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 20 min | 10 min | 5 min | 5 min |
| Independent | Small Groups | Small Groups | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice
Independent

### Materials
- Exit Ticket
- Additional Practice
- calculators

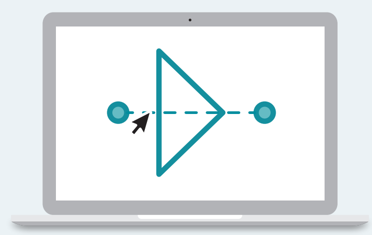
### Math Language Development

**Review words**
- *corresponding*
- *dilation*
- *ratio*
- *scale factor*
- *similar*

**Amps** | **Featured Activity**

**Activity 1**
### See Student Thinking

Students explain what they notice about ratios of corresponding sides within similar figures.



### Building Math Identity and Community
Connecting to Mathematical Practices

In Activity 1, students might feel a sense of frustration if they do not immediately see the structure of the ratios in the table for Problem 2, because they wrote the ratios as decimal numbers. Help them see that by writing numbers in different forms, they can look for the structure to notice mathematical relationships.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, have students choose one row to complete in Problem 1.
- In **Activity 2**, Problem 2 may be omitted.

Independent | 5 min

## Warm-up Which One Doesn't Belong?

Students review different ways ratios are written and simplified, in preparation for the upcoming activities in which they will use ratios to study similar figures.

Unit 2 | Lesson 9

# Ratios of Side Lengths in Similar Triangles

Let's use similarity to determine side lengths in similar triangles.


### Warm-up Which One Doesn't Belong?

Study these ratios. Which ratio does not belong with the others? Explain your thinking.

A. $2:10$

C. $\frac{2}{5}$

B. $-10$ to $-25$

D. $\frac{4}{10}$ to $\frac{10}{10}$

Sample responses:

- Choice A is the only ratio that is not equivalent to $2:5$.
- Choice B is the only ratio using negative numbers.
- Choice C is the only ratio that is written as one number, a fraction.
- Choice D is the only ratio that compares two fractions.

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Conduct the ***Which One Doesn't Belong?*** routine. Encourage students to find at least one reason for why each ratio doesn't belong with the others.

### 2 Monitor

**Help students get started** by asking them to choose any one ratio and identify what makes it different from the other ratios.

**Look for points of confusion:**

- **Not knowing how to write a ratio as a fraction.** Demonstrate by writing the number left of the colon as the numerator and the number to the right of the colon as the denominator.
- **Not realizing the ratio in choice B is equivalent to $2:5$.** Tell students that another way to find the ratio is to calculate the quotient. Remind them of the rules of signed numbers.
- **Not knowing how to simplify the ratio in choice D.** Have students simplify $\frac{4}{10}$, and then $\frac{10}{10}$ before comparing the ratio.

**Look for productive strategies:**

- Simplifying or dividing the ratios to compare them.
- Noticing that all choices, except A, are equivalent to the ratio 2 to 5.

### 3 Connect

**Have students share** their responses. Use the ***Poll the Class*** routine to see which ratio they chose. Select students to explain their thinking.

**Ask** students to simplify each ratio and share their strategies.

**Highlight** that one way to compare ratios is to simplify them. By simplifying the ratios, or finding the quotients, it can be more straightforward to see that choice A is the only ratio that is not equivalent to $2:5$ or $0.4$.

### Math Language Development

***MLR2: Collect and Display***

During the Connect, listen for words and phrases that students use to share their reasoning for why certain ratios do not belong with the others. Display these words and phrases, such as *equivalent, not equivalent, negative, fraction*, etc.

**English Learners**

If students are not familiar with the term *simplify*, illustrate what it means to simplify a ratio by providing examples.

### Power-up

**To power up students' ability to simplify fractions, have students complete:**

Recall that in order to completely simplify fractions, you divide the numerator and denominator by their greatest common factor (GCF). For each fraction, first determine the GCF between the numerator and denominator, then rewrite each fraction in simplest form.

| Fraction | GCF | Simplest form |
|---|---|---|
| $\frac{18}{27}$ | 9 | $\frac{2}{3}$ |
| $-\frac{16}{28}$ | 4 | $-\frac{4}{7}$ |

**Use:** Before Activity 1
**Informed by:** Performance on Lesson 8, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 8

Small Groups | 20 min

# **Activity 1** Ratios of Side Lengths Within Similar Triangles

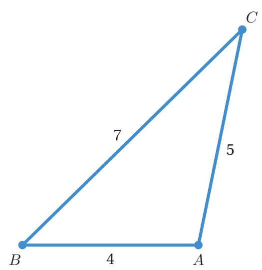
Students explore the ratios of side lengths within the same triangle to see that the same ratio is preserved in similar triangles.

**Amps Featured Activity** See Student Thinking

Name: ____________ Date: ________ Period: ________

### **Activity 1** Ratios of Side Lengths Within Similar Triangles

**Triangle $ABC$ is similar to each of Triangles $DEF$, $GHI$, and $JKL$.**
**Note that Triangles $DEF$, $GHI$, and $JKL$ are not shown.**




**1.** The scale factor for the dilation that maps Triangle $ABC$ onto each triangle is shown in the table. Determine the side lengths of Triangles $DEF$, $GHI$, and $JKL$. Record them in the table.

| Triangle | Scale factor | Length of short side | Length of medium side | Length of long side |
|---|---|---|---|---|
| $ABC$ | 1 | 4 | 5 | 7 |
| $DEF$ | 2 | **8** | **10** | **14** |
| $GHI$ | 3 | **12** | **15** | **21** |
| $JKL$ | $\frac{1}{2}$ | **2** | **$\frac{5}{2}$ or 2.5** | **$\frac{7}{2}$ or 3.5** |

**Pause here so your teacher can review your work.**




## 1 Launch

Have students work individually to complete Problem 1. Check their work before having them complete Problems 2–4 in groups of three. Provide access to calculators for the duration of the lesson.

## 2 Monitor

**Help students get started** by having them identify the short, medium, and long side of Triangle $ABC$. Students may find it helpful to label the sides using the letters "S," "M," and "L."

**Look for points of confusion:**

- **Not noticing the equivalent ratios for Problem 3.** Encourage students to simplify the ratios so that they can better see their equivalence.
- **Not writing the same ratios for each column.** Have students check each other's work in their small groups.

**Look for productive strategies:**

- Noticing the same ratios for each column.

**Activity 1 continued ›**

## Differentiated Support

### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can enter the side lengths of a triangle using different scale factors. When they do so, an animation appears, allowing them to visualize a triangle's size, based on the scale factor.

## Math Language Development

### *MLR8: Discussion Supports—Press for Details*

During the Connect, press for details in students' reasoning as to why the ratio of the medium side to the long side for any triangle similar to $\Delta ABC$ will always be $\frac{5}{7}$. Display the table from Problem 1, and ask these follow-up questions to help solidify this concept:

- "What multiplication expressions can you write to represent the lengths of the medium and long sides of $\Delta DEF$? Of $\Delta GHI$?" **$\Delta DEF$: $5 \cdot 2$; $7 \cdot 2$. $\Delta GHI$: $5 \cdot 3$; $7 \cdot 3$**
- "What do you notice? Use a math term from this unit in your response." **The second factor of each expression is the scale factor.**
- "What expressions would you write to represent the length of the medium side and the length of the long side for any triangle similar to $\Delta ABC$, with a scale factor of $s$?" **$5s$; $7s$**

Small Groups | 20 min

# Activity 1 Ratios of Side Lengths Within Similar Triangles (continued)

Students explore the ratios of side lengths within the same triangle to see that the same ratio is preserved in similar triangles.

## Activity 1 Ratios of Side Lengths Within Similar Triangles (continued)

2. With your group members, decide who will complete Column A, Column B, and Column C. For all four triangles, find and record the ratio of the indicated side lengths given for each column.

| Triangle | Column A<br>Ratio of long side to short side | Column B<br>Ratio of long side to medium side | Column C<br>Ratio of medium side to short side |
|---|---|---|---|
| $ABC$ | $\frac{7}{4}$ or 1.75 | $\frac{7}{5}$ or 1.4 | $\frac{5}{4}$ or 1.25 |
| $DEF$ | $\frac{14}{8}$ or 1.75 | $\frac{14}{10}$ or 1.4 | $\frac{10}{8}$ or 1.25 |
| $GHI$ | $\frac{21}{12}$ or 1.75 | $\frac{21}{15}$ or 1.4 | $\frac{15}{12}$ or 1.25 |
| $JKL$ | $\frac{7}{4}$ or 1.75 | $\frac{14}{10}$ or 1.4 | $\frac{5}{4}$ or 1.25 |

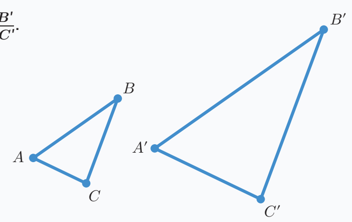
3. What do you notice about the ratios?
Sample response: The ratios within each column are the same.

4. Compare your results with your group members and then complete your table with your group's completed ratios.

**Are you ready for more?**

$\triangle ABC \sim \triangle A'B'C'$. Explain why $\frac{AB}{BC} = \frac{A'B'}{B'C'}$.

There is a scale factor, $s$, that is multiplied by the side lengths of Triangle $ABC$ to get the side lengths of Triangle $A'B'C'$. Therefore, $A'B' = AB \cdot s$ and $B'C' = BC \cdot s$.
So, $\frac{A'B'}{B'C'} = \frac{AB \cdot s}{BC \cdot s} = \frac{AB}{BC}$.




194 Unit 2 Dilations and Similarity



## 3 Connect

**Display** correct student work for Problem 2.

**Have groups of students share** what they noticed about the structure of the ratios.

**Ask:**

- "Are all of the short sides corresponding in all of the triangles? Medium? Long sides?" Yes
- "What is the ratio of the medium side to the long side in Triangle $ABC$?" $\frac{5}{7}$
- "Will the ratio of the medium side to the long side be $\frac{5}{7}$ for any triangle similar to Triangle $ABC$? Explain your thinking." Yes. A triangle similar to Triangle $ABC$ will have side lengths $4s$, $5s$, and $7s$ for some (positive) scale factor $s$. The ratio of the medium side to the long side will always be $\frac{5s}{7s} = \frac{5}{7}$.

**Highlight** that the ratios of corresponding pairs of side lengths in any set of similar triangles are equal.

Small Groups | 10 min

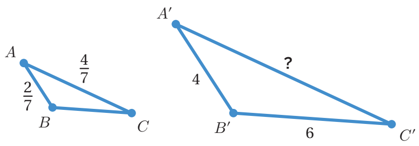
# Activity 2 Completing the Missing Steps

Students calculate an unknown side length to see that they can use the scale factor or internal ratios to solve for a missing length given a pair of similar triangles.

Name: ________ Date: ________ Period: ________

## Activity 2 Completing the Missing Steps

In the diagram, $\triangle ABC \sim \triangle A'B'C'$. Bard and Elena both started to calculate the unknown side length $A'C'$. The first two steps for each student's method is shown.




*The figures may not be drawn to scale.*

**1.** Complete the missing Step 3 for each student.

| Bard | Elena |
|---|---|
| **Step 1:** Create a ratio table. | **Step 1:** Create a ratio table. |
| Long side $(AC)$: $\frac{4}{7}$ \| ? <br> Short side $(AB)$: $\frac{2}{7}$ \| 4 <br> $\times 14$ | Long side $(AC)$: $\frac{4}{7}$ \| ? <br> Short side $(AB)$: $\frac{2}{7}$ \| 4 <br> $\times 2$ |
| **Step 2:** Calculate the scale factor: $4 \div \frac{2}{7} = 14$ | **Step 2:** Determine the ratio of the long side to the short side in Triangle $ABC$: $\frac{4}{7} \div \frac{2}{7} = 2$ |
| **Step 3:** Multiply $AC$ by the scale factor, 14, to determine $A'C'$, $\frac{4}{7} \cdot 14 = 8$. | **Step 3:** Because the ratio of $AC$ to $AB$ is 2, the ratio of the corresponding sides $A'C'$ to $A'B'$ is also 2, $4 \cdot 2 = 8$. |
| **Length of** $A'C'$: $A'C' = 8$ | **Length of** $A'C'$: $A'C' = 8$ |

**2.** Use either Bard's or Elena's method to determine $BC$.

| | | |
|---|---|---|
| Medium side $(BC)$ | ? | 6 |
| Short side $(AB)$ | $\frac{2}{7}$ | 4 |

Bard's method: Divide $B'C'$ by the scale factor 14, $\frac{6}{14} = \frac{3}{7}$. So, $BC = \frac{3}{7}$.

Elena's method: $B'C' \div A'B' = \frac{3}{2}$, so $BC$ is $\frac{3}{2}$ times the length of $AB$, $\frac{2}{7} \cdot \frac{3}{2} = \frac{3}{7}$.

STOP



Lesson 9 Ratios of Side Lengths in Similar Triangles 195

## 1 Launch

Have students use the ***Think-Pair-Share*** routine. Give them 3 minutes of individual think time, and then complete Bard's steps with their partners. Repeat the routine for Elena's steps.

## 2 Monitor

**Help students get started** by reminding them that prime notation can help them identify corresponding sides. Then have them identify the short, medium, and long side for each triangle.

**Look for points of confusion:**

- **Questioning how Bard arrived at the scale factor.** Have students study the second column in the table or compare the short sides in each triangle.
- **Questioning Elena's ratio.** Have students study the second row in the table or compare the long and short sides in Triangle $ABC$.

**Look for productive strategies:**

- Noticing that using either method results in the same length of side $A'C'$.

## 3 Connect

**Have groups of students share** how they completed the steps for Bard and Elena.

**Highlight** that there are different methods to calculate an unknown side length of similar triangles. Students can use the scale factor *between* the triangles or use the ratio of corresponding side lengths *within* the triangles.

**Ask** students to explain their methods for Problem 2 and why they selected a certain method.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Bard's steps in Problem 1 and use Bard's method to determine the length of side $BC$ in Problem 2. Alternatively, consider altering the side lengths in $\triangle ABC$ so that they are whole numbers or decimals, such as 1.5 and 3.

### Extension: *Math Enrichment*

Ask students to critique Han's method.

**Han:** I compared the two short sides ($AB$ and $A'B'$). Because $4 \div \frac{2}{7} = 14$, I multiplied the longer side $AC$ by 14 to obtain $A'C'$, which is $\frac{4}{7} \times 14 = 8$.

**Han's method is correct, as it compares side lengths between triangles.**

## Math Language Development

### *MLR7: Compare and Connect*

During the Connect, compare and contrast the different methods Bard and Elena used for calculating an unknown side length of similar triangles. Draw connections to how the scale factor and ratio of long to short side is shown in each ratio table. Emphasize language such as "the scale factor *between* the triangles" or "the ratio of corresponding side lengths *within* the triangles."

### English Learners

Use hand gestures to distinguish the phrases "*between* the triangles" and "*within* the triangles."

Whole Class | 5 min

# Summary

Review and synthesize how to use the ratio of corresponding side lengths of a triangle to determine unknown side lengths in similar triangles.

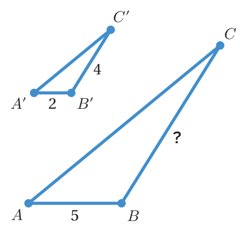
## Summary

**In today's lesson . . .**

You discovered that the ratio of a pair of side lengths in one triangle is equal to the ratio of the corresponding side lengths in a similar triangle.

For a pair of similar triangles, you can calculate the missing side length by using the ratios of side lengths *within* a triangle or by using the scale factor *between* the triangles.

Suppose you know $\triangle ABC \sim \triangle A'B'C'$. Here are two methods you can use to determine side $BC$.



*The figures may not be drawn to scale.*

**Method 1: Using the scale factor *between* the triangles**

Because you need to determine the length of side $BC$, find the ratio of the lengths of the corresponding sides $AB$ to $A'B'$ to determine the scale factor. The ratio is 5 : 2, so the scale factor is 2.5. Multiply the length of side $B'C'$ by the scale factor to determine the length of side $BC$, $4 \cdot 2.5 = 10$.

**Method 2: Using ratio of sides *within* one triangle**

In Triangle $A'B'C'$, the ratio of the medium side to the short side is 4 : 2, or 2. This means that the medium side is twice the length of the short side in both triangles. Therefore, the length of side $BC$ is twice the length of side $AB$, $5 \cdot 2 = 10$.

Reflect:




## Synthesize

**Have students share** the advantages or disadvantages for each method. Ask them whether they have a preferred method, and if so, why.

**Highlight:**

- The ratios of pairs of corresponding side lengths in similar triangles are equal.
- To determine an unknown side of similar triangles, students can either use the ratios of side lengths *within* the triangles or the scale factor *between* the similar triangles.

**Display** the two triangles from the Summary in the Student Edition.

**Ask:**

- "How can you find the scale factor using sides $AB$ and $A'B'$? How can you determine the length of side $BC$ using the scale factor?"
- "How many times longer is side $B'C'$ than side $A'B'$? How can you use this to determine the length of side $BC$?"

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies did you find helpful today when calculating a side length given similar triangles?"

Independent | 5 min

# Exit Ticket

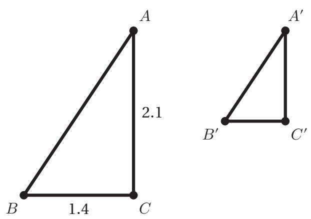
Students demonstrate their understanding by calculating the ratio of side lengths of similar triangles.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 2.09

$\triangle ABC \sim \triangle A'B'C'$. Determine the ratio of $A'C'$ to $B'C'$.
Show or explain your thinking.




*The figures may not be drawn to scale.*

Sample response: Because the triangles are similar, I know that $\frac{AC}{BC} = \frac{A'C'}{B'C'}$. $\frac{AC}{BC} = \frac{2.1}{1.4}$, so $\frac{A'C'}{B'C'}$ is 1.5.

**Self-Assess**

? 1 I don't really get it — 2 I'm starting to get it — 3 I got it

a. I can determine whether two triangles are similar by looking at the ratios of the lengths of corresponding sides.
1 2 3

b. I can determine the missing side length in a pair of similar triangles by using the scale factor between triangles or by using the ratios of side lengths within one triangle.
1 2 3





## Success looks like . . .

- **Goal:** Calculating unknown side lengths in similar triangles using the ratios of side lengths within the triangles and the scale factor between similar triangles.
- **Language Goal:** Generalizing that the ratios of corresponding side lengths in similar triangles are equal. **(Speaking and Listening)**
  - » Determining the ratio of sides for two similar triangles.

## Suggested next steps

**If students cannot determine the ratio of $A'C'$ to $B'C'$, consider:**

- Reviewing Activity 1.
- Having them use a scale factor to determine side lengths, and then compare the ratios.
- Reassessing after Lesson 10.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- During the discussion about completing Bard and Elena's method in Activity 2, how did you encourage each student to share their understandings?
- In this lesson, students explored the ratios of side lengths within the same triangle to see that the same ratio is preserved in similar triangles. How will that support their understanding of slope in Lesson 11?

## Math Language Development

**Language Goal: Generalizing that the ratios of corresponding side lengths in similar triangles are equal.**

Reflect on students' language development toward this goal.

- In what ways did students use their developing math language to justify their response to the Exit Ticket problem?
- What support do they still need in order to be more precise in their justifications?

**Sample justifications:**

| Emerging | Expanding |
|---|---|
| The ratios are the same. | The ratios of corresponding side lengths in similar triangles are equal. |

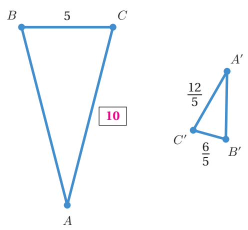
# Practice

Independent

Name: Date: Period:

Practice

1. In the diagram, $\Delta ABC \sim \Delta A'B'C'$. Determine the length of side $AC$. Explain your thinking.




*The figures may not be drawn to scale.*

Sample response:

| | | |
|---|---|---|
| $BC$ | 5 | $\frac{6}{5}$ |
| $AC$ | ? | $\frac{12}{5}$ |

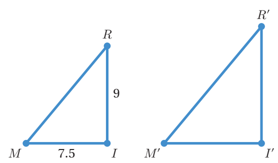
- I used the corresponding sides to find the scale factor: $5 \div \frac{6}{5} = \frac{25}{6}$. Then I used the scale factor to find the missing side $\frac{12}{5} \cdot \frac{25}{6} = 10$.
- I used the ratio of $A'C'$ and $B'C'$: $\frac{12}{5} \div \frac{6}{5} = 2$. Then I used it to find the missing side $5 \cdot 2 = 10$.

2. In the diagram, $\Delta MIR \sim \Delta M'I'R'$. Determine the ratio of $I'R'$ and $M'I'$. Explain your thinking.

Sample response: Because the triangles are similar, I know that $\frac{I'R'}{M'I'} = \frac{IR}{MI}$.

$\frac{IR}{MI} = \frac{9}{7.5} = 1.2$,

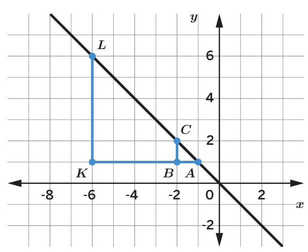
so $\frac{I'R'}{M'I'} = 1.2$.




3. Determine a center and a scale factor for a dilation that would map Triangle $ABC$ onto Triangle $AKL$.

**Center:** $(-1, 1)$

**Scale factor:** 5



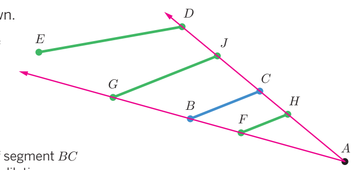





Name: Date: Period:

Practice

4. Refer to the line segments shown.




a. Which segment is a dilation of segment $BC$ using point $A$ as the center of dilation and a scale factor of $\frac{2}{3}$?

$FH$

b. Which segment is a dilation of segment $BC$ using point $A$ as the center of dilation and a scale factor of $\frac{3}{2}$?

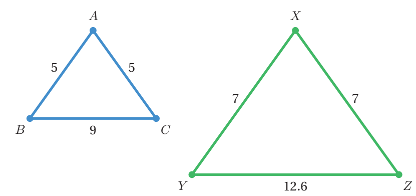
$GJ$

c. Which segment is not a dilation of segment $BC$? Explain your thinking.

Segment $DE$ is not a dilation of segment $BC$. Sample response:

- Point $E$ is not on the same ray as points $A$ and $B$.
- Segment $DE$ is not parallel to segment $BC$.

5. Triangle $ABC$ is similar to Triangle $XYZ$. What is the scale factor that maps Triangle $ABC$ onto Triangle $XYZ$? Explain your thinking.




*The figures may not be drawn to scale.*

Sample response: The scale factor is 1.4. I found the ratio of two corresponding sides, $\frac{12.6}{9} = 1.4$.





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| | 2 | Activity 1 | 2 |
| Spiral | 3 | Unit 2 Lesson 5 | 1 |
| | 4 | Unit 2 Lesson 3 | 2 |
| Formative | 5 | Unit 2 Lesson 10 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available


For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 2 | **LESSON 10**


# The Shadow Knows

Let's use shadows to determine the height of a figure.

## Focus

### Goals

1. **Language Goal:** Calculate the unknown heights of figures by using proportional reasoning and explain the solution method. **(Speaking and Listening)**
2. **Language Goal:** Justify why the relationship between the height of figures and the length of their shadows cast by the Sun is approximately proportional. **(Speaking and Listening)**
3. Calculate the unknown side lengths of similar triangles using proportional reasoning.

## Rigor

- Students strengthen their **fluency** in calculating unknown side lengths using proportional reasoning.
- Students **apply** their understanding of similar triangles and proportional relationships.

## Coherence

### Today

Students examine the length of shadows of different figures. They apply their understanding of similar triangles and proportional relationships to estimate the height of a tall figure.

### Previously

In Lesson 9, students used the ratios of side lengths in similar triangles to find missing side lengths.

### Coming Soon

In Lesson 11, students will learn how similar triangles can be used to determine the slope of a line.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Dilations*
- calculators

### Math Language Development

**Review words**

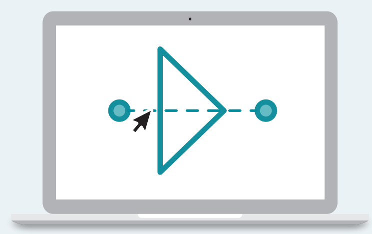
- *similar*

**Amps** | Featured Activity

**Exit Ticket**

**Real-Time Exit Ticket**

Check in real-time whether your students can calculate side lengths of similar figures.



Amps

POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

At first, students may feel lost if they do not notice any relationships between the figure's height and the length of its shadow as they think about how to use mathematics to model the problem in Activity 1. Help them practice taking control of their own learning by suggesting they seek out support from 2–3 sources as a general guideline when they feel lost.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, have students only complete the first three challenges.

Independent | 5 min

## **Warm-up** Notice and Wonder

Students analyze images to see how a figure's shadow changes when the Sun's rays strike at different angles.


Name: Date: Period:

Unit 2 | Lesson 10

# The Shadow Knows

Let's use shadows to determine the height of a figure.

### **Warm-up** Notice and Wonder

**Consider the following images. What do you notice? What do you wonder?**

1. I notice . . .
   Sample responses:
   - Three of the images appear to be at different times during the day, and one image is at night.
   - As the day continues, each shadow gets longer.
2. I wonder . . .
   Sample responses:
   - Why are the shadows different lengths?
   - Why is there no shadow in the first image?

Log in to Amplify Math to complete this lesson online.





### 1 Launch

To pique student interest, start by telling students the following riddle: "Everyone has it, but no one can lose it. What is it?" Your shadow. Then conduct the ***Notice and Wonder*** routine using the images from the Warm-up.

### 2 Monitor

**Help students get started** by observing how the shadows change in each image.

**Look for productive strategies:**
- Noticing the different shadow lengths depending on the figure's height.
- Noticing the shadows appear to become longer proportionally in the second and third image.
- Wondering about the relationship between the shadow length and the Sun.

### 3 Connect

**Display** the images from the Warm-up and ask students to share what they noticed and wonder.

**Highlight** that the shadow length depends on the figure's height and time of day.

**Ask:**
- "What causes shadows?" The Sun's rays move in a straight line and when an object or person blocks the path of light, a shadow appears. Using the third image, draw three straight lines so that it touches the top of each figure and the ground. Tell students that because the Sun is very far away, the rays that reach the Earth are approximately parallel. Keep this image for display for students to reference as they complete Activity 1.

## Math Language Development

*MLR8: Discussion Supports — Annotate It!*

During the Connect discussion, as students respond to the Ask question, illustrate how the Sun's rays move in a straight line and how shadows are formed by drawing or annotating on the displays in the Warm-up.

**English Learners**

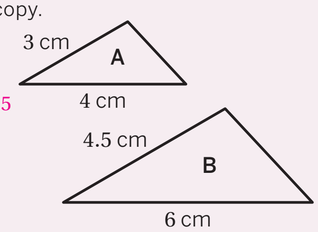
Allow students to record what they noticed and wondered in their primary language, before participating in the class discussion. To support student understanding, invite them to use gestures when describing what they noticed and wondered.

## Power-up

**To power up students' ability to determine the scale factor given two similar triangles, have students complete:**

Recall that a *scaled copy* is a copy of a figure where every length in the original figure is multiplied by the same value to determine the corresponding lengths in the copy.

Triangle A is similar to Triangle B.




**1.** What is the scale factor that maps Triangle A onto Triangle B? 1.5

**2.** What is the scale factor that maps Triangle B onto Triangle A? $\frac{2}{3}$

**Use:** Before Activity 1
**Informed by:** Performance on Lesson 9, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 6

Pairs | 15 min

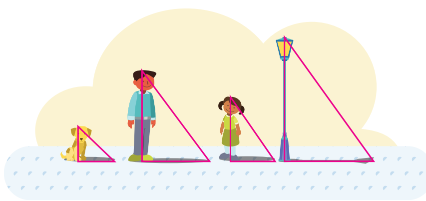
# Activity 1 Figures and Shadows

Students apply what they know about proportional relationships and the length of a shadow to find the height of a figure that is difficult to measure directly.

### Activity 1 Figures and Shadows

**Study the image. The table lists the height of each person, dog, and lamppost, and their shadow.**

| | Height (in.) | Shadow length (in.) |
|---|---|---|
| Mocha | 43 | 29 |
| Mr. Mendez | 72 | 48 |
| Mai | 51 | 34 |
| Lamppost | ? | 114 |



*The figures may not be drawn to scale.*

1. What relationships do you notice between each person's or object's height and the length of its shadow?
   **Each person's or object's height is approximately 1.5 times longer than their shadow. The shadow lengths are approximately two-thirds the height of each person or object.**

2. Explain *why* the ratios of the height of each person or object to the length of their shadow are approximately the same.
   **The triangles formed by the shadows, heights, and the sun's rays are similar.**

3. Determine the height of the lamppost. Explain your thinking.
   **Because $114 \cdot 1.5 = 171$, the height of the lamppost is about 171 in. (or 14 ft 3 in.).**

**Historical Moment**

Over 2,000 years ago, the ancient Greek mathematician Eratosthenes also studied shadows closely (in a slightly different way). He used his study of shadows to estimate the circumference of Earth with an error of less than 2%!

200 Unit 2 Dilations and Similarity


## 1 Launch

Tell students that when an object is too tall to measure directly, they can determine its height by using the length of its shadow. Provide access to calculators.

## 2 Monitor

**Help students get started** by having them find the ratio of each figure's height to its shadow length.

**Look for points of confusion:**

- **Not knowing how to find the height of the lamppost.** Tell students to use the ratio of the height and shadow for the three given figures to calculate the height of the lamppost.
- **Not knowing why the relationship between the height of each figure and length of their shadows is approximately proportional.** Have students draw right triangles using the figure, the ground, and the Sun's rays and label any given measurements to help emphasize the connection of similar triangles.

## 3 Connect

**Display** an image where students drew a right triangle using a person or the lamppost, the ground, and the Sun's rays. If no student drew a triangle, draw the triangle to emphasize similarity.

**Have pairs of students share** what they noticed about the relationship between each figure's height and length of its shadow. Draw an arrow between the columns in the table to show this relationship. Then have them share their strategies for finding the height and why it works.

**Highlight** that because the four figures and their shadows create similar triangles, students can use proportional reasoning to calculate the height of the lamppost.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Consider demonstrating how to annotate or label Mocha's height and shadow length to provide a visual reference before students begin the activity. Suggest that students add another column to their table that shows the ratio of each height to shadow length to assist them with Problem 2.

## Math Language Development

### *MLR7: Compare and Connect*

During the Connect, as students share what they noticed, connect their strategies for determining the height of the lamppost to the idea of proportional relationships. Illustrate why the triangles formed by the height of each person or object and their shadow length are similar triangles.

## Historical Moment

### Studying Shadows

Have students read about how Eratosthenes used shadows to estimate the circumference of Earth with incredible accuracy.

Pairs | 15 min

# Activity 2 Four Challenges

Students use proportional reasoning to calculate unknown side lengths to develop procedural fluency.

Name: ______ Date: ______ Period: ______

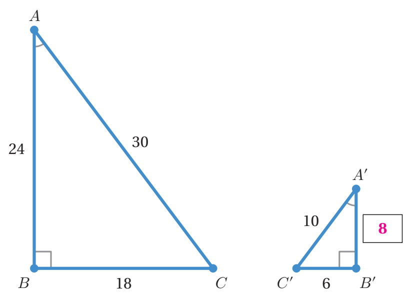
## Activity 2 Four Challenges

**Plan ahead:** What will you do to stay focused on the challenges? How will you control your impulses?

**For each challenge, determine the missing side length and explain your thinking. The figures may not be drawn to scale.**

**Challenge 1:**

$\Delta ABC \sim \Delta A'B'C'$

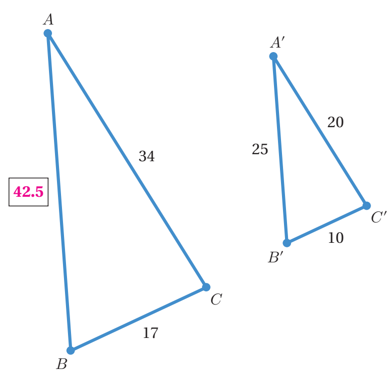



Sample response:

- I used the corresponding sides to find the scale factor that maps $\Delta ABC$ on to $\Delta A'B'C'$, $\frac{10}{30} = \frac{1}{3}$. Then I used the scale factor to determine the missing side length, $24 \cdot \frac{1}{3} = 8$.
- I used the ratio of $AB$ and $AC$: $\frac{24}{30} = \frac{4}{5}$. Then I used the ratio to find the missing side length, $10 \cdot \frac{4}{5} = 8$.

**Challenge 2:**

$\Delta ABC \sim \Delta A'B'C'$




Sample response:

- I used the corresponding sides to find the scale factor that maps $\Delta A'B'C'$ onto $\Delta ABC$: $\frac{34}{20} = 1.7$. Then I used the scale factor to find the missing side length, $25 \cdot 1.7 = 42.5$.
- I used the ratio of $A'B'$ and $A'C'$: $\frac{25}{20} = \frac{5}{4}$. Then I used the ratio to find the missing side length, $34 \cdot \frac{5}{4} = 42.5$.





## 1 Launch

Have students use the *Think-Pair-Share* routine as they complete each challenge. Have students discuss and resolve any discrepancies or disagreements. Provide access to calculators.

## 2 Monitor

**Help students get started** by having them identify a pair of corresponding sides in Challenge 1.

**Look for points of confusion:**

- **Not knowing how to calculate the missing side length for Challenges 1 and 2.** Use Challenge 1 to demonstrate to students two different strategies: creating a ratio table and using the scale factor. Then have students choose one of these strategies to solve Challenge 2.
- **Thinking the length of segment *YC* is 24 or the length of line segment *XB* is 19.2 in Challenge 4.** Have students redraw the figures as two separate triangles to help them understand that 24 is the length of line segment $AC$ and 19.2 is the length of line segment $AB$.
- **Not knowing why the triangles are similar in Challenge 4.** Remind students about the relationship between angles formed when parallel lines are intersected by a transversal from Unit 1.

**Look for productive strategies:**

- Using different methods to solve each challenge.
- Using a different method than their partner to solve a problem, but arriving at the same solution.

Activity 2 continued ›

## Differentiated Support

### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity. After they submit their response digitally, an animated illustration appears, allowing them to see their numbers "come to life."

### Accessibility: *Vary Demands to Optimize Challenge, Guide Visualization and Processing*

If students need more processing time, have them focus on Challenges 1 and 2. Provide colored pencils or highlighters and suggest students color code corresponding sides or angles the same color.

## Math Language Development

### *MLR7: Compare and Connect*

During the Connect, as students share the strategies they used, annotate or display these two strategies:

- Use ratios *within* triangles.
- Use ratios *between* triangles.

Ask students to determine which strategy they used by using the language "ratios *within* triangles" or "ratios *between* triangles." Highlight that using ratios between triangles utilizes the scale factor.

Pairs | 15 min

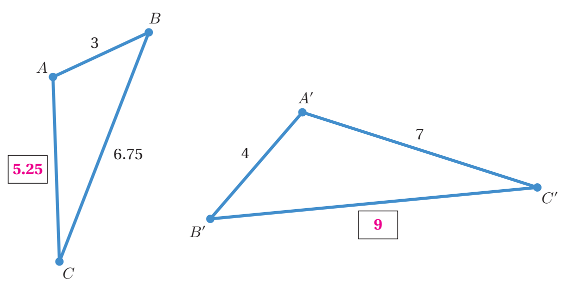
# Activity 2 Four Challenges (continued)

Students use proportional reasoning to calculate unknown side lengths to develop procedural fluency.

### Activity 2 Four Challenges (continued)

**Challenge 3:**

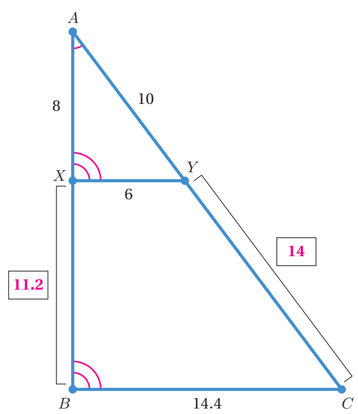
$\triangle ABC \sim \triangle A'B'C'$




Sample response: Corresponding side lengths of similar triangles are proportional. I used the corresponding sides to find the scale factor that maps $\triangle ABC$ onto $\triangle A'B'C'$: $\frac{4}{3}$. Then I used the scale factor to find the missing sides: $6.75 \cdot \frac{4}{3} = 9$ and $7 \div \frac{4}{3} = 5.25$.

**Challenge 4:**

Segment $XY$ is parallel to segment $BC$.




Sample response: $\triangle ABC \sim \triangle AXY$ because two corresponding angles are congruent. I used the corresponding sides to find the scale factor that maps $\triangle AXY$ onto $\triangle ABC$: $\frac{14.4}{6} = 2.4$.

To find segment $XB$, I multiplied segment $AX$ by the scale factor and calculated the length of side $AB$: $8 \cdot 2.4 = 19.2$. Then I subtracted segment $AX$ from side $AB$ to determine segment $XB$: $19.2 - 8 = 11.2$.

To find segment $YC$, I multiplied segment $AY$ by the scale factor and calculated the length of side $AC$: $10 \cdot 2.4 = 24$. Then I subtracted segment $AY$ from side $AC$ to determine segment $YC$: $24 - 10 = 14$.

STOP





## 3 Connect

**Have pairs of students share** the strategies they used to solve Challenges 1 and 2. Select students who solved for the missing side lengths in different ways.

**Highlight** that students can use the ratios within the triangle or corresponding side lengths between the similar triangles to determine the unknown side length.

**Ask**, "After you solve each problem, how can you verify that your solution is reasonable?" Sample response: I can compare corresponding side lengths. As an example, use Challenge 2 to show that if students wrote 21 as the length of side $AB$, they can compare the sides to see that the side lengths in Triangle $ABC$ are greater than the corresponding side lengths in Triangle $A'B'C'$. Therefore the length of side $AB$ should be greater than the length of its corresponding side $A'B'$, which is 25.

# Summary

Review and synthesize how proportional relationships can be used to find the height of a figure that is difficult to measure directly.

Name: ______ Date: ______ Period: ______

## Summary

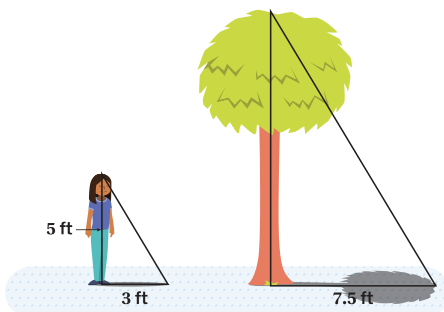
**In today's lesson . . .**

You examined the lengths of shadows of different figures. Because all of the figures were perpendicular to the ground and the sun's rays were cast on each figure at the same angle, you found that the right triangles that resulted from the figures and their shadows formed similar triangles.

The height of the tree shown can be determined by applying knowledge of similar triangles.

The ratio of the person's height to their shadow is $\frac{5}{3}$. This means the ratio of the tree's height to its shadow is also $\frac{5}{3}$.

So, you know the height of the tree is $\frac{5}{3} \cdot 7.5$, or 12.5 ft.




*The figures may not be drawn to scale.*

**Reflect:**



Lesson 10 The Shadow Knows 203

## Synthesize

**Display** the Summary from the Student Edition.

**Ask:**

- "How can you use your height and shadow length to find the height of a tall tree?" Sample response: I can compare the ratio of the height and shadow of each object and then use proportional reasoning. Share with students that in the 6th century BC, Thales of Miletus measured the height of the great pyramid at Giza by comparing its shadows!
- "If the position of the Sun changed, would you still be able to use shadows to find the height of the lamppost? Explain your thinking." Yes; Sample response: The triangles formed by the height of the object, ground, and the Sun's rays would still be similar. I can use ratios to calculate the height of the lamppost.

**Have students share** their strategies for finding an unknown side length when they are given two similar triangles.

**Highlight** that students can use proportional reasoning to make predictions about quantities that are difficult or impossible to measure directly. Use Part 3 of the Anchor Chart PDF, *Dilations* to review how to calculate unknown side lengths in similar triangles using ratios.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you use similar triangles to determine the height of a tall object?"

Independent | 5 min

# Exit Ticket

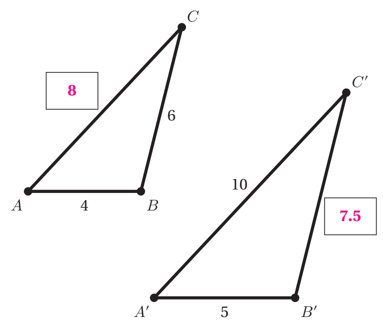
Students demonstrate their understanding by using proportional reasoning to determine missing side lengths.

**Amps Featured Activity** Real-Time Exit Ticket | Printable

Name: ____________ Date: ________ Period: ________

## Exit Ticket 2.10

$\triangle ABC \sim \triangle A'B'C'$. Determine the missing side lengths. Explain your thinking.




*The figures may not be drawn to scale.*

Sample response: I know that the corresponding side lengths of similar triangles are proportional. I used the corresponding sides to find the scale factor that maps $\triangle ABC$ onto $\triangle A'B'C'$, which is $\frac{5}{4} = 1.25$. Then I used the scale factor to find the missing sides, $6 \cdot 1.25 = 7.5$ and $10 \div 1.25 = 8$.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can model a real-world context with similar triangles to find the height of an unknown figure.

1 2 3



Lesson 10 The Shadow Knows

### Success looks like . . .

- **Language Goal:** Calculating the unknown heights of figures by using proportional reasoning and explaining the solution method. **(Speaking and Listening)**
- **Language Goal:** Justifying why the relationship between the height of a figure and the length of its shadows cast by the Sun is approximately proportional. **(Speaking and Listening)**
- **Goal:** Calculating the unknown side lengths of similar triangles using proportional reasoning.
  - » Using proportional reasoning to determine the length of segment $AC$ and segment $B'C'$.

### Suggested next steps

**If students calculate both side lengths incorrectly, consider:**

- Changing the scale factor to a whole number and reassessing.
- Providing the scale factor to students and reassessing.
- Reviewing the ratios of side lengths in similar triangles.

**If students calculated only one side length correctly, consider:**

- Asking students if they would multiply or divide by the scale factor to determine the second side length.
- Having them check their solutions informally by comparing the corresponding side lengths.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is?
- What challenges did students encounter as they worked on Activity 2? How did they work through them?

# Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

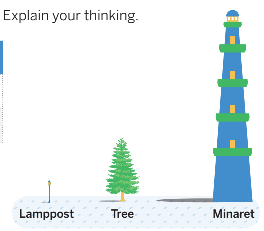
1. Determine the height of the tree and minaret. Explain your thinking.

| | Lamppost | Tree | Minaret |
|---|---|---|---|
| Height (ft) | 15 | 50 | 140 |
| Shadow (ft) | 6 | 20 | 56 |

Sample response: The ratio of the lamppost's height to its shadow is $\frac{15}{6} = \frac{5}{2}$.

Tree: $20 \cdot \frac{5}{2} = 50$ ft

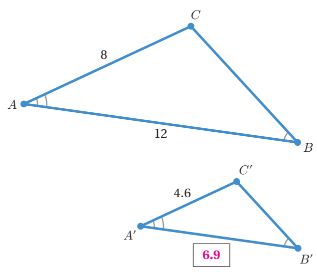
Minaret: $56 \cdot \frac{5}{2} = 140$ ft




*The figures may not be drawn to scale.*

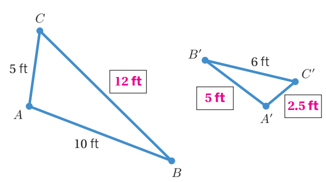
2. In the diagram, $\Delta ABC \sim \Delta A'B'C'$. Determine the missing side length. Explain your thinking.

Sample response: I found the ratio of side $AB$ to side $AC$, $\frac{12}{8} = \frac{3}{2}$. Then I used the ratio to determine side $A'B'$, $4.6 \cdot \frac{3}{2} = 6.9$.




*The figures may not be drawn to scale.*

3. $\Delta ABC \sim \Delta A'B'C'$. The scale factor that maps Triangle $ABC$ onto Triangle $A'B'C'$ is $\frac{1}{2}$. Determine the missing side lengths.




$5 \cdot \frac{1}{2} = 2.5$

$10 \cdot \frac{1}{2} = 5$

$6 \div \frac{1}{2} = 12$

*The figures may not be drawn to scale.*



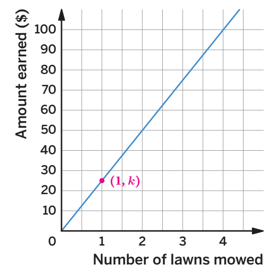


Practice

Name: ______ Date: ______ Period: ______

4. The graph shows the amount Tyler earns based on the number of lawns he mows. Label the point $(1, k)$ on the graph, find the value of $k$, and explain its meaning.

$k = 25$; This means that Tyler will earn $25 when he mows one lawn.

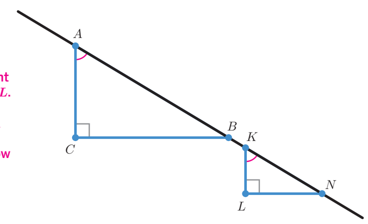



5. A rectangle has a length of 6 units and a width of 4 units. Which of the following statements tells you that Quadrilateral $ABCD$ is *not* similar to this rectangle? Select *all* that apply.

A. $AB = BC$
B. $m\angle ABC = 105°$
C. $AB = 8$
D. $BC = 8$
E. $BC = 2 \cdot AB$
F. $2 \cdot AB = 3 \cdot BC$

6. Segments $AC$ and $KL$ are parallel. Segments $CB$ and $LN$ are parallel. Show that $\Delta ABC \sim \Delta KNL$.

Sample response: Both triangles are right triangles, angle $C$ is congruent to angle $L$. Line segment $AN$ is a transversal for segments $AC$ and $KL$, which makes $\angle A$ congruent to $\angle K$. If I know at least two pairs of angles are congruent, then I know that both triangles are similar.








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Grade 7 | 2 |
| | 5 | Unit 2 Lesson 7 | 3 |
| Formative | 6 | Unit 2 Lesson 11 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Meet Slope

Let's explore the slope of a line.

## Focus

### Goals

1. **Language Goal:** Comprehend the term *slope* to mean the numerical value that represents the ratio of the vertical distance and the horizontal distance between any two points on a line. **(Speaking and Listening)**
2. **Language Goal:** Draw a line on a coordinate plane given its slope and describe observations about lines with the same slope. **(Speaking and Listening, Writing)**
3. **Language Goal:** Justify that all "slope triangles" that lie on one line are similar by using transformations or by using the idea that if two pairs of corresponding angles are congruent, then the triangles are similar. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students learn about the slope of the line and how it is connected to what they have learned about similar triangles.

### Previously

In Lesson 10, students used proportional relationships between similar triangles to find missing side lengths.

### Coming Soon

In Lesson 12, students will use concepts from Units 1 and 2 to identify and create patterns with optical illusions. In Unit 3, students will use slope to write equations for lines.

## Rigor

- Students build **conceptual understanding** of the slope of a line.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 10 min | 15 min | 10 min | 5 min | 5 min |
| Independent | Pairs | Independent | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Power-up PDF, as needed
- Power-up PDF (answers), as needed
- Anchor Chart PDF, *Slope*
- rulers or index cards

### Math Language Development

**New words**

- ***slope***
- ***slope triangles***

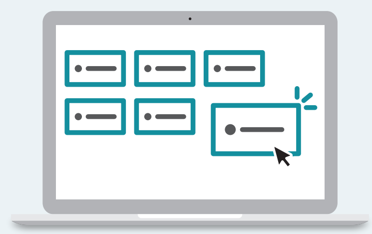
**Review words**

- *similar*

**Amps** | **Featured Activity**

### Activity 1
## Digital Card Sort

Students match slopes to their lines and receive real-time feedback as they work.



Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students may struggle in actively listening to their classmates' strategies and arguments for describing similar triangles and how they relate to the definition of slope. Have students be explicit about responding to and building off their classmates' responses, and highlight students who are revoicing or incorporating others' opinions in their own responses. Emphasize the need to think critically before incorporating another person's idea. Students should critique the reasoning used before determining whether they agree with it.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have students complete 4 of the 6 graphs.
- In **Activity 2**, Problem 2 may be omitted.

Independent | 10 min

# Warm-up Notice and Wonder

Students examine a set of similar triangles where one side of each triangle lies on the same line, to build understanding about the slope of a line.

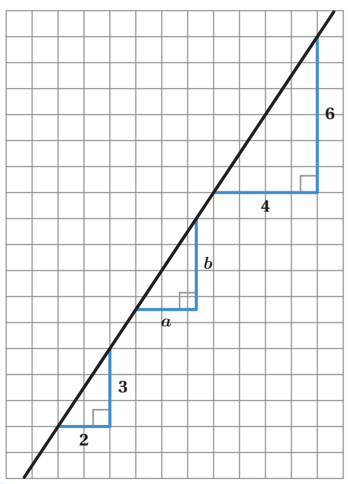
Unit 2 | Lesson 11

## Meet Slope

Let's explore the slope of a line.

### Warm-up Notice and Wonder

Study the image. What do you notice? What do you wonder?




1. I notice . . .
   Sample responses:
   - There are 3 triangles.
   - The largest triangle has side lengths that are twice as long as the side lengths of the smallest triangle.
   - All of the triangles appear to be scaled copies of each other.
   - The longest sides of each triangle lie on the same line.
2. I wonder . . .
   Sample responses:
   - What are the values of $a$ and $b$?
   - Can I draw other triangles where the longest side lies on this line and have those triangles also be scaled copies of these triangles?

**Co-craft Questions:** Share your questions with a partner. Together, come up with 1–2 questions that you think you might be able to answer.

Log in to Amplify Math to complete this lesson online.

206 Unit 2 Dilations and Similarity



## 1 Launch

Conduct the ***Notice and Wonder*** routine.

## 2 Monitor

**Help students get started** by asking them what they notice about corresponding angles.

**Look for productive strategies:**

- Noticing the triangles are all similar by describing a sequence of transformations or by using what they know about angles formed when parallel lines are cut by a transversal.

## 3 Connect

**Display** the triangles.

**Have students share** their responses.

**Ask:**

- "How do you know that all three triangles are similar?" Sample response: All three triangles can be verified they are similar by analyzing the congruent angles formed when two parallel lines (line segments) are cut by the transversal.
- "What is the exact value of $\frac{b}{a}$? How do you know this?" $\frac{b}{a}$ is $\frac{3}{2}$ because all three triangles are similar.

**Define** the term ***slope***. Explain that right triangles, like the ones shown, can be constructed when there is a slanted line to serve as the side opposite the right angle. These triangles are called ***slope triangles***, where one side length is horizontal and one side length is vertical, and the quotient of the length of the vertical side and the horizontal side for the triangles are always the same. This numerical value that represents the ratio is called the ***slope*** of the line. **Note:** In later units, students will come to understand how slope can be defined as the quotient of the vertical distance divided by the horizontal distance as one moves along the line from left to right and therefore could be negative.

## Math Language Development

### *MLR5: Co-craft Questions*

After students individually record what they noticed and wondered, ask them to share their responses with a partner. Ask them to work together to co-craft 1–2 mathematical questions that they think they might be able to answer, or that they would like to answer, by the end of this lesson.

### English Learners

Model crafting 1–2 mathematical questions that could be asked about the triangles before having pairs of students co-craft their own questions.

## Power-up

**To power up students' ability to determine that two triangles are scaled copies using angle relationships related to parallel lines and a transversal:**

Provide students with a copy of the Power-up PDF.

**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 10, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4

Pairs | 15 min

# Activity 1 Different Slopes, Different Lines

Students match various lines with different slopes to strengthen their understanding of the slope of a line.

Amps Featured Activity | Digital Card Sort


Name: Date: Period:

**Activity 1** Different Slopes, Different Lines

Study the images shown on each of these graphs. Match each line with its corresponding slope. Draw a line in the empty grid for Graph 6 that has a slope of $\frac{1}{5}$.

| Slope | Graph |
| --- | --- |
| $\frac{1}{3}$ | 2 |
| 2 | 4 |
| 1 | 3 |
| 0.25 | 5 |
| $\frac{3}{2}$ | 1 |
| $\frac{1}{5}$ | 6 |







## 1 Launch

Display the Anchor Chart PDF, *Slope*.

## 2 Monitor

**Help students get started** by asking, "What does the slope of a line mean? How can you find it?"

**Look for points of confusion:**

- **Thinking there is not a matching slope for Graphs 2 or 3 because the ratio is not yet simplified.** Ask students to look for equivalent ratios.
- **Reversing the ratio, for example thinking the slope of the line in Graph 1 is $\frac{2}{3}$.** Remind students that slope describes the steepness of the line. Ask, "Do you think that $\frac{2}{3}$ or $\frac{3}{2}$ would describe a steeper slope? Why do you think so?" Highlight that the slope ratio is vertical side length to horizontal side length.
- **Struggling to draw a triangle for Graphs 4 or 5.** Have students examine two places on the line where the line crosses an intersection of grid lines.
- **Struggling to draw a line for Graph 6.** Have students draw a triangle with sides that have a ratio of $\frac{1}{5}$ and ask them about the slope.

## 3 Connect

**Display** student work showing correct matches.

**Have students share** how they can use slope triangles to find the slopes of lines.

**Ask**, "Why is the slope of the line on Graph 4 $\frac{1}{2}$ and not 2?"

**Highlight** that given a slope, students can draw a right triangle using vertical and horizontal lengths that correspond to the slope ratio, and then extend the longest side of the right triangle to create a line with that slope. Show that for Graph 6, two different scaled triangles will result in the same slope, because their sides have the same ratio.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Graphs 1, 2, and 3. Tell them to match these graphs with the slopes $\frac{1}{3}$, 1, and $\frac{3}{2}$ from the table.

### Extension: *Math Enrichment*

Challenge students to draw a line with a slope of 1.25 for Graph 6. Students should draw a line with a slope of $\frac{5}{4}$, which is equivalent to 1.25.

## Math Language Development

### *MLR2: Collect and Display*

During the Connect, as students share how they found the slopes of the lines, listen for the mathematical language they use, such as *ratio*, *vertical*, *horizontal*, *distance*, *steeper*, *less steep*, or *side length*. Display the language they use or add it to the class display. Include visuals, such as comparing a steeper line to a less steep line.

### English Learners

As students use these terms, use hand gestures to illustrate several of them, such as *vertical*, *horizontal*, *steeper*, or *less steep*.

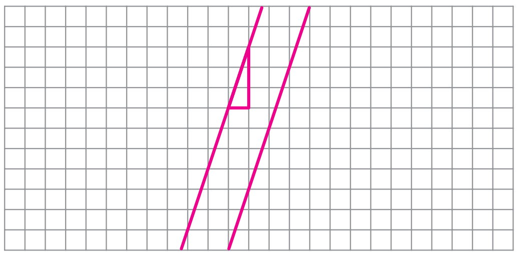
Independent | 10 min

# Activity 2 Multiple Lines With the Same Slope

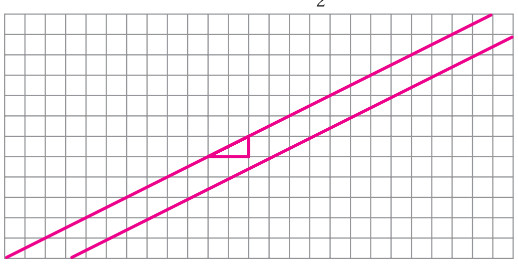
Students draw lines with given slopes to come to understand that lines with the same slope are parallel.

### Activity 2 Multiple Lines with the Same Slope

1. Draw two lines that each have a slope of 3. **Sample response shown.**



2. Draw two lines that each have a slope of $\frac{1}{2}$. **Sample response shown.**



3. What do you notice about the lines you drew in Problems 1 and 2?
**Sample response: I notice that when two lines have the same slope, they are parallel.**

STOP





## 1 Launch

Provide access to rulers or index cards.

## 2 Monitor

**Help students get started** by having them restate the definition of slope in their own words.

**Look for points of confusion:**

- **Not knowing how to draw a line with the slope of 3.** Have students draw a triangle with sides that have a vertical to horizontal length ratio of 3.
- **Drawing a negative slope.** Display these alongside positive slopes and ask students to compare and contrast. Activate background knowledge by using terms such as "uphill" and "downhill." **Note:** Positive and negative slopes will be further discussed in Unit 3.

**Look for productive strategies:**

- Using slope triangles to verify the lines drawn have the correct slope.
- Counting horizontal and vertical grid unit distances to find slope.
- Drawing lines that are parallel and making the connection to the fact that the lines have the same slope.

## 3 Connect

**Display** student work for Problems 1 and 2.

**Have students share** how they can use slope triangles or horizontal and vertical distances to draw the slope of a line. Then ask them to share their responses to Problem 3.

**Highlight** that lines with the same slope are parallel.

**Ask**, "How does the slope of the line relate to its steepness?"

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Provide pre-drawn lines for students to use for Problems 1 and 2, and have them focus on analyzing them to respond to Problem 3.

### Extension: *Math Enrichment*

Ask students these questions and have them explain their thinking.

- "What is the slope of a horizontal line?" 0; If I draw a triangle between two points on the line, the vertical distance is 0.
- "What is the slope of a vertical line?" It doesn't exist; If I draw a triangle between two points on the line, the vertical distance always varies, but the horizontal distance is 0.

## Math Language Development

### *MLR8: Discussion Supports*

During the Connect, as students share their responses to Problem 3, provide sentence frames to help them formulate their thoughts, such as:

- "Two lines with the same slope are ____ because . . ."
- "The lines with slopes of 3 are ____ than the lines with slopes of $\frac{1}{2}$, because . . ."

Listen for and amplify language used to complete these sentence frames, such as "the ratios of the slope triangles are equivalent, so the lines are always the same steepness" for the first sentence frame. Connect "equivalent ratios of slope triangles" with "same steepness."

# Summary

Review and synthesize what the slope of a line means, and how slope triangles can be used to find the slope of a line.

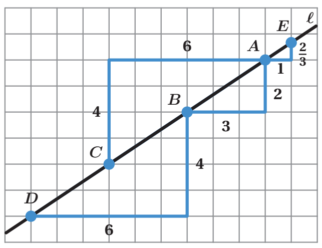
Name: ______ Date: ______ Period: ______

## Summary

**In today's lesson . . .**

You used similar triangles to discover the slope of a line.

The four triangles shown are all examples of ***slope triangles***. One side of a slope triangle lies on the line $\ell$, one side is a vertical line segment, and one side is a horizontal line segment.




The ***slope*** of the line $\ell$ is the numerical value that represents the ratio of the length of the vertical side and the length of the horizontal side of any of these slope triangles. The slope of the line $\ell$ can be written as $\frac{4}{6}$, $\frac{2}{3}$, or any equivalent value.

**Reflect:**





## Synthesize

**Display** the Summary from the Student Edition.

**Ask:**

- "How can you use a slope triangle to find the slope of a line?" Find the ratio of the length of the vertical side to the length of the horizontal side.
- "Does it matter which two points you use to create a slope triangle? Why or why not?" No, it does not matter; Sample response: Any two slope triangles are similar. So, the ratios of the two corresponding sides — vertical to horizontal — will always be equivalent.
- "Why are any two slope triangles that lie on the same line similar?" Sample response: Slope triangles are right triangles. The remaining angles form congruent corresponding angles. So, all of the corresponding angle measures of two slope triangles are the same.

**Formalize vocabulary:**

- *slope*
- *slope triangles*

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can similar triangles be used to find the slope of a line?"

## Math Language Development

### *MLR2: Collect and Display*

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *slope* and *slope triangles* that were added to the display during the lesson.

Independent | 5 min

# Exit Ticket

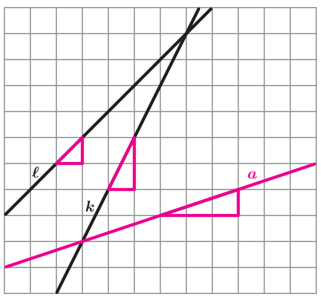
Students demonstrate their understanding of slope and slope triangles by finding the slope of a line.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 2.11

Refer to lines $\ell$ and $k$ on the grid shown.




1. Which line has a slope of 1?
   Line $\ell$ has a slope of 1.

2. Which line has a slope of 2?
   Line $k$ has a slope of 2.

3. Graph a line whose slope is $\frac{1}{3}$. Label this line $a$.
   Sample response shown.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can draw a line on a grid with a given slope. 1 2 3

b I can find the slope of a line on a grid. 1 2 3



**Lesson 11** Meet Slope

## Success looks like . . .

- **Language Goal:** Comprehending the term *slope* to mean the ratio of the vertical distance and the horizontal distance between any two points on a line. **(Speaking and Listening)**
  - » Determining the slope of a given line by using the ratio in Problems 1 and 2.
- **Language Goal:** Drawing a line on a coordinate plane given its slope and describing observations about lines with the same slope. **(Speaking and Listening, Writing)**
  - » Graphing a line with slope of $\frac{1}{3}$ in Problem 3.
- **Language Goal:** Justifying that all "slope triangles" that lie on one line are similar by using transformations or by using the idea that if two pairs of corresponding angles are congruent then the triangles are similar. **(Speaking and Listening, Writing)**

## Suggested next steps

**If students are having difficulty drawing slope triangles to find the slopes of the lines, consider:**

- Reviewing Activity 1.
- Assigning Practice Problem 1.

**If students are having difficulty drawing lines with a slope of $\frac{1}{3}$, consider:**

- Asking students what they know about slope and how it relates to vertical and horizontal distance.
- Reviewing the images of slope triangles from the Summary in the Student Edition.
- Assigning Practice Problem 2.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

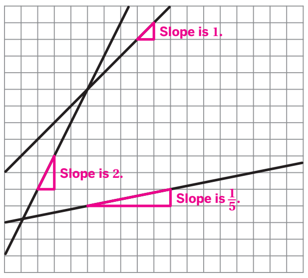
- What challenges did students encounter as they worked on finding the slope of a line? How did they work through them?
- In this lesson, students were introduced to the slope of a line. Thinking about where students need to be by the end of Unit 3, how did this introduction support their learning?

# Practice

Independent

Name: Date: Period:

**1.** Of the three lines shown, one has a slope of $1$, one has a slope of $2$, and one has a slope of $\frac{1}{5}$. Label each line with its slope.

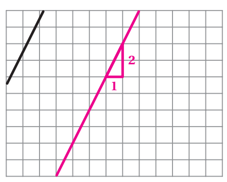



**2.** Lin drew a line with a slope of $\frac{1}{3}$. Shawn drew a line with a slope of $\frac{1}{2}$. Who drew a steeper slope? Explain your thinking.

**Sample response: Shawn drew a steeper slope. Both lines have a slope with the same vertical distance, but Lin's slope has a greater horizontal distance, which means it is less steep than Shawn's line.**

**3.** Refer to the line shown. Draw a second line with the same slope. What is the slope of each line? Explain your thinking.

**Sample response: Lines that have the same slope are parallel. The slope of each line is 2.**








Name: Date: Period:

**4.** Triangle A has side lengths $3$, $4$, and $5$. Triangle B has side lengths $6$, $7$, and $8$.

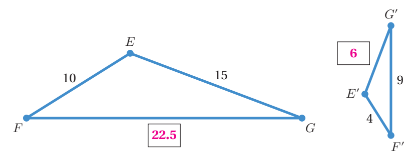
a. Explain how you know that Triangle B is *not* similar to Triangle A.

**Sample response: The shortest side in Triangle B is twice as long as the shortest side in Triangle A, but the longest side is only 1.6 times as long. These different ratios mean the triangles cannot be similar**

b. Give possible sides lengths of a triangle that would be similar to Triangle A.

**Sample response: 6, 8, and 10**

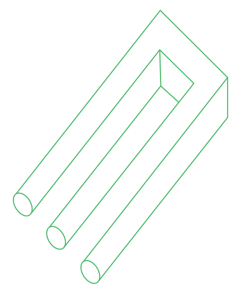
**5.** In the diagram, $\Delta EFG \sim \Delta E'F'G'$. Determine the missing values. Explain your thinking.




*The figures may not be drawn to scale*

**Sample response: I know that the corresponding side lengths of similar triangles are multiplied by the same scale factor. I used the corresponding sides to find the scale factor that takes $\Delta E'F'G'$ to $\Delta EFG$, which is $\frac{10}{4} = 2.5$. Then I used the scale factor to find the missing side lengths, $9 \cdot 2.5 = 22.5$ and $15 \div 2.5 = 6$.**

**6.** The illustration shown is often referred to as an "impossible trident." What do you notice? What do you wonder?



a. I notice . . .

**Sample response: The trident appears to have three tines at one end. At the other end, there appears to be only two tines connected at the base.**

b. I wonder . . .

**Sample response: How is this illusion created?**





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 1 |
| | 2 | Activity 1 | 1 |
| | 3 | Activity 1 | 2 |
| Spiral | 4 | Unit 2 Lesson 6 | 2 |
| | 5 | Unit 2 Lesson 9 | 2 |
| Formative | 6 | Unit 2 Lesson 12 | 1 |

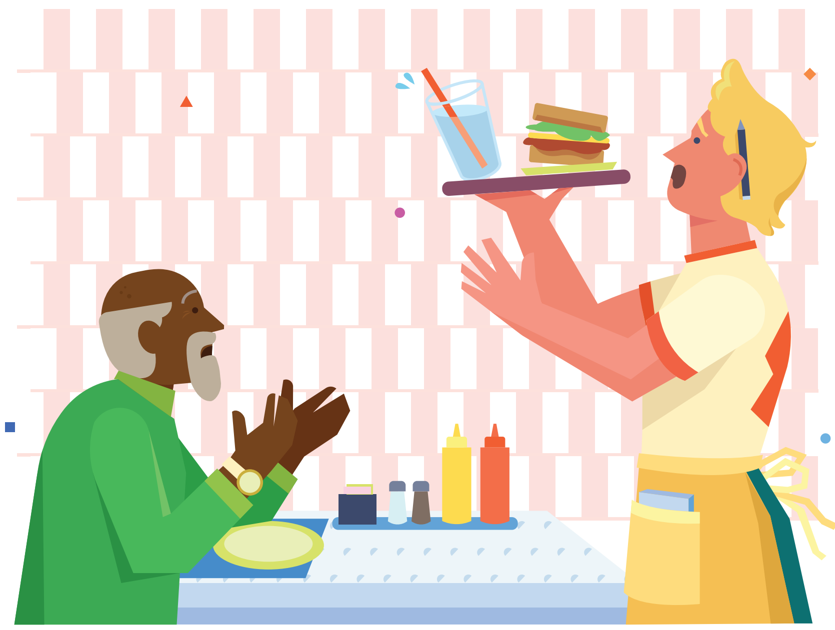
**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available


For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Optical Illusions

Let's create drawings that trick the eye.



## Focus

### Goals

1. **Language Goal:** Identify patterns in optical illusions. **(Speaking and Listening, Writing)**
2. Create optical illusions using the structure of a grid.

## Coherence

### Today

Students identify patterns in optical illusions and draw connections to concepts studied in Units 1 and 2. Students will create their own optical illusions as they generalize informal ideas about what makes an illusion effective.

### Previously

Throughout Unit 2, students explored ideas about similar triangles and dilations. In Unit 1, students studied rigid motions and worked with patterns using tessellations.

### Coming Soon

In Unit 3, students will connect what they have learned about similar triangles to develop an understanding of slope.

## Rigor

- Students strengthen their **conceptual understanding** of transformations as they explore the patterns shown in optical illusions.
- Students **apply** concepts learned from Units 1 and 2 to create optical illusions.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 10 min | 20 min | 5 min | 5 min |
| Independent | Pairs | Independent | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Power-up PDF, as needed
- Power-up PDF (answers), as needed
- Activity 2 PDF, one page per student (as needed)
- graph paper
- geometry toolkits: rulers, protractors, index cards
- black markers
- black pens

### Math Language Development

**Review words**

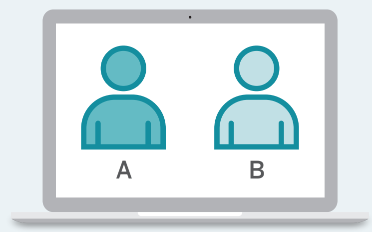
- *dilation*
- *transversal*
- *optical illusion*

**Amps** | Featured Activity

**Activity 2**

**Digital Collaboration**

Students create optical illusions and digitally share and collaborate with a peer.




Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

**Self-awareness:** Students may feel discouraged or inadequate if they are unable to see or create an optical illusion. Remind students that there is no one correct way to view an artwork or an illusion and validate all observations and interpretations. Encourage students to study the images looking for mathematical structure, even if they are unable to see the actual illusion. Provide more structure for students who would benefit from assistance in creating their artwork.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, the Gallery Tour can be omitted. Instead, display examples of student work and facilitate a whole class discussion.

Independent | 5 min

# Warm-up The Cafe Wall Illusion

Students study a geometric pattern to find an optical illusion and draw connections to math studied in Units 1 and 2.

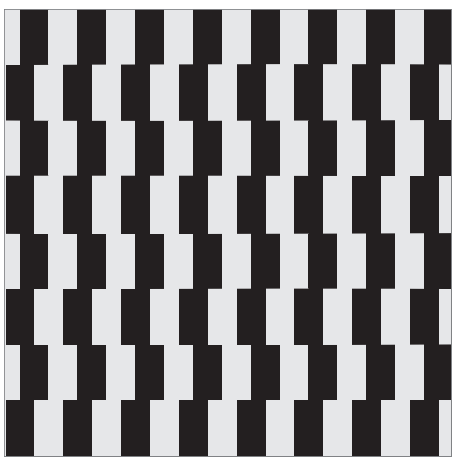


## Optical Illusions

Let's create drawings that trick the eye.

### Warm-up The Cafe Wall Illusion

Are the horizontal lines parallel or sloped? Explain your thinking and construct an argument to prove your response.



Sample response: The lines appear sloped but when I used my ruler, I saw that the lines were straight. If I draw a vertical line through the parallel lines, I can see it forms right angles at each point of intersection, meaning it must be a perpendicular transversal intersecting parallel lines.

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Provide access to geometry toolkits for the duration of the lesson.

## 2 Monitor

**Help students get started** by asking, "What tools could you use to verify whether the lines are parallel?"

**Look for points of confusion:**

- **Insufficiently addressing both what they notice and what they wonder.** Encourage students to think more deeply about connections or questions and provide examples of what they notice.
- **Quickly selecting a response (parallel or sloped) without providing justification.** Ask students to defend their response. Ask, "Why do you think so?" or "Tell me more about what you see."

**Look for productive strategies:**

- Using rulers or straight edges to verify the lines are parallel.
- Drawing a transversal and measuring the angles formed to verify the lines are parallel.

## 3 Connect

**Display** the illustration.

**Have students share** whether they think the lines are parallel using the ***Poll the Class*** routine. Have students explain their thinking and sequence their responses, ending with students who used their geometry toolkits to verify the lines are parallel.

**Highlight** that if students draw a transversal that is perpendicular to the lines, each intersecting angle will be a right angle, meaning all of the lines are parallel to each other.

**Ask**, "Why do you think it looks like the lines are *not* parallel?"

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge***

It is not essential that students see the optical illusions in this lesson. Encourage them to have fun and explore what they do see, or to listen to how others perceive an illusion. Validate all perspectives of the artwork and illusions presented in this lesson.

## Power-up

**To power up students' ability to identify an optical illusion:**

Provide students with a copy of the Power-up PDF.

**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 11, Practice Problem 6

Pairs | 10 min

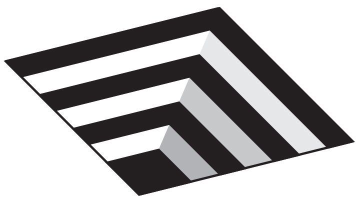
# Activity 1 Is That a Hole in the Paper?

Students study an optical illusion to gain insight into how illusions work and how they could be created.

Name: ______ Date: ______ Period: ______

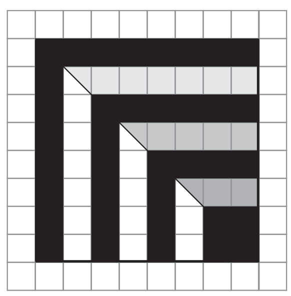
**Activity 1** Is That a Hole in the Paper?

**Consider this illustration.**



1. Do you see an illusion? If so, describe the illusion and why you think it happens. If not, describe what you see.
   **Sample response: Yes, I see an illusion that makes it look as though there is a hole in the paper. This is an example of a perspective drawing, creating an optical illusion that makes an image printed on paper have a three-dimensional look.**

2. A grid was used to create the illusion. Study the grid. What math do you see?
   **Sample response: I see squares of different scale factors leading to the smallest square in the bottom corner.**



3. How could a grid be useful in designing a pattern such as this?
   **Sample response: It appears as if every other grid line was shaded black. The dimensions of each square increase by one each time as the length and width each increase by one.**



Lesson 12 Optical Illusions 213

## 1 Launch

Conduct the *Think-Pair-Share* routine.

## 2 Monitor

**Help students get started** by activating their prior knowledge. Ask them to describe what they see using mathematical vocabulary they have learned in this unit, or in prior units or grades.

**Look for points of confusion:**

- **Not recognizing the illusion.** Have students view the image from a different angle. Point out that it is okay and normal to not see an illusion, even if others do see it. Have them describe what they do see.

**Look for productive strategies:**

- Making connections to dilations, scale factors, or other topics and concepts in geometry.

## 3 Connect

**Display** the illustration of the optical illusion from the Student Edition.

**Have pairs of students share** what they notice and what connections they can make to the mathematical concepts they studied in Units 1 and 2.

**Ask**, "If you were to ask someone to recreate this illusion, what might the directions sound like?"

**Highlight** that the black and white pattern and the gray shading work together to help create the illusion of a three-dimensional perspective. Identify how a grid could be used to make this illusion.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students do not see the illusion, reinforce that this is okay and normal. Not everyone sees an illusion. Students can still describe the math that they see in the image, even if they do not see the illusion itself.

## Math Language Development

### *MLR7: Compare and Connect*

During the Connect, as students share the connections they notice between the illusion and the mathematical concepts they have learned in Units 1 and 2, press for details in their reasoning. For example, if a student says, "I see three rays or lines that meet at a point," ask them what math term they have learned in this unit that describes that point. Center of dilation

### English Learners

Annotate the illusion with the math terms students use, such as *center of dilation*.

Independent | 20 min

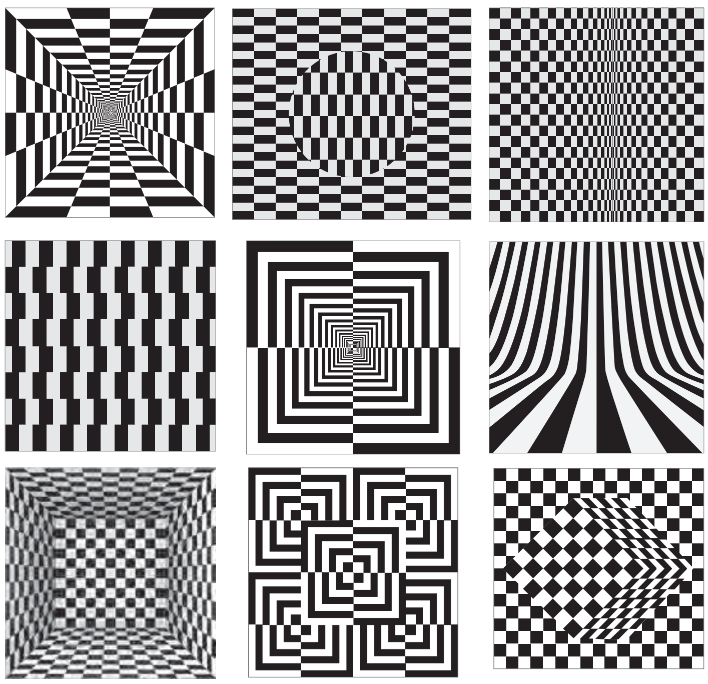
# Activity 2 Optical Illusions

Students use the structure of a grid to design their own optical illusions.

Amps Featured Activity | Digital Collaboration

## Activity 2 Optical Illusions

**Prominent mathematician and physicist Roger Penrose, along with many others in the fields of math, science, and art, have long tried to create optical illusions. And now you get to join them! Here are some examples of optical illusions to consider.**



GreenBelka/Shutterstock.com, art_of_sun/Shutterstock.com, ScottMurph/Shutterstock.com, shooarts/Shutterstock.com, Master3D/Shutterstock.com

1. You will be given materials. Create your own optical illusion to see if you can trick your classmates' eyes!
   Answers may vary.





## 1 Launch

Display the illusions from the Student Edition. Ask, "What makes these illusions effective? What math do you see?" Highlight how students can use lines and alternating black-and-white patterns to create illusions. Distribute black markers, pens, and graph paper. Give students 10 minutes to create their illusions and then conduct the ***Gallery Tour*** routine.

## 2 Monitor

**Help students get started** by suggesting they recreate one of the examples if they do not have an idea of their own they want to create. You may wish to provide other examples of optical illusions.

**Look for points of confusion:**

- **Not being able to create or recreate an illusion.** Have students select one illusion from the Activity 2 PDF and follow the instructions.
- **Working without precision or neatness.** Ask, "What do you notice about lines and precision in the illusions you have seen so far?" Encourage students to use a straightedge and the structure of the grid to help them create neat lines and patterns.

**Look for productive strategies:**

- Recreating an illusion from an illustration given, using the structure of the grid and other strategies, with precision.
- Creating their own illusion using strategies or concepts from Units 1 and 2.

**Activity 2 continued ›**

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Consider allowing students to choose one of these options.

- Allow students to select an illusion from the Activity 2 PDF. Encourage students to follow the steps provided as a guide to creating their own illusion.
- Provide a partially-completed illusion to students and have them complete it.
- Students can recreate one of the illusions provided in the activity.
- It is not essential that students draw an illusion during this activity. Allow them to choose to analyze one of the given illusions and record the mathematics they see in the image.

## Math Language Development

### *MLR2: Collect and Display*

As students engage in the ***Gallery Tour***, record the mathematical language they use to describe the illusions. Consider annotating the illusions with these math terms and phrases, such as *parallel*, *similar*, *scale factor*, *dilation*, and *center of dilation*.

Independent | 20 min

# Activity 2 Optical Illusions (continued)

Students use the structure of a grid to design their own optical illusions.

Name: ______ Date: ______ Period: ______

## Activity 2 Optical Illusions (continued)

2. You will now take part in a Gallery Tour of your peers' work. Record any notes in the table.

| What patterns can you see in the artwork? | What makes optical illusions work best? |
|---|---|
| Sample response: I can see alternating black and white squares. | Sample response: Clear, neat lines and patterns can help make the images come to life. Shading helps create the effect of a three-dimensional image. |

3. What connections do you see to topics you have learned about in Units 1 and 2?
Optical illusions, such as tessellations, use repeated shapes and transformations of these shapes to create an artistic effect. Unlike tessellations, optical illusions typically contain at least one pattern that serves to trick the eye into seeing something different.

**Featured Mathematician**

**Sir Roger Penrose**

English mathematician and physicist Sir Roger Penrose turned just 27 in 1958, the year he and his father published a paper on an "impossible triangle," first devised by Swedish mathematician Oscar Reutersvärd in 1934. Now commonly known as the Penrose Triangle, the triangle appears to contain a combination of properties that cannot be realized by any three-dimensional object in space. Roger Penrose has since won many awards for his contributions on topics such as black holes or the relationship between consciousness and physics.



**Reflect:** How did you take the perspective of others as you created your optical illusion?

STOP





## 3 Connect

**Have pairs of students share** their observations from the ***Gallery Tour***.

**Display** examples of the illusions students created.

**Ask:**

- "As you looked at the illusions that others created, did you get any new ideas about how to make an optical illusion?"
- "What are some mathematical questions that others could ask about your artwork?"
- "If someone else wanted to create an illusion, what advice would you have for them?"

**Highlight** examples of illusions that were created by students who attended to precision by accurately recreating an illusion from an example. Then highlight illusions that were created by students who used mathematical thinking to create their own illusions.

## Differentiated Support

### Extension: *Math Enrichment, Interdisciplinary Connections*

Have students use the internet, or another source, to view images of several of the optical illusions listed here. Alternatively, show these images to students. Let students know many artists, including M.C. Escher, have used similar optical illusions or impossible figures in their artwork. **(Art)**

- Müller-Lyer
- Penrose Stairs
- Impossible Trident
- Impossible triangle
- Four-sided impossible figure

## Featured Mathematician

### Sir Roger Penrose

Have students read about featured mathematician Sir Roger Penrose, who co-published with his father a paper about the "impossible triangle," just one of his many contributions to the field of math. He is also well known for his work on physics and consciousness.

Whole Class | 5 min

# Unit Summary

Review and synthesize the patterns and mathematics that are found in optical illusions.


Narrative Connections

## Unit Summary

Things are not always what they seem.

At the start of this unit, you met Renaissance architect Filippo Brunelleschi. He introduced a linear perspective into the world of art. With a system of grid lines that came together at a "vanishing point," he could craft a sense of space and depth within a two dimensional canvas.

Using precise dilations, Brunelleschi blurred the line between what was real and what was a painting. Since Brunelleschi's time, dilation has enriched our culture. It has contributed to art, architecture, and design. But not all changes have been for the better.

Dilations can also play tricks on us. Optical illusions can trick the brain into thinking things are farther or closer than they actually are. Additionally, consumer goods have been subject to "shrinkflation." By using congruent angles and proportional corresponding sides, companies release packaging that is mathematically *similar* to the original, but is in fact imperceptibly smaller.

By understanding the rules for dilation, you can see through these tricks. While there is nothing wrong with using your eyes and trusting your intuition, math gives you a way to be precise in your measurements and to know exactly how and when the wool is being pulled over your eyes.

See you in Unit 3.

216 Unit 2 Dilations and Similarity

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## Narrative Connections

Read the narrative aloud as a class or have students read it individually.

## Synthesize

**Display** the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

**Ask**, "How can optical illusions be used in real-life?" Sample responses: the fashion industry, architecture, advertising, company logos, artistic designs, and artwork.

**Have students share** what they found most interesting in this lesson and how it connects to the first two geometry units.

**Highlight** that this is the end of Unit 2. Students will next see how geometry can be used to better help them make key connections in Unit 3, their first unit in Grade 8 on algebraic concepts.

## Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

- "Can you think of any other examples of optical illusions, either in art or in the real world?"
- "How are dilations used in perspective drawings to create the illusion of three-dimensional space?"

Independent | 5 min

# Exit Ticket

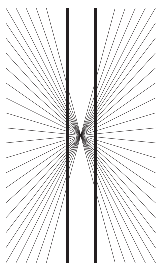
Students demonstrate their understanding of optical illusions by describing the math they see in a famous illustration.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 2.12

**Consider the illustration shown here, known as the Hering Illusion.**



Peter Hermes Furian/ Shutterstock.com

1. Are the vertical lines parallel or bending? Construct an argument to justify your response.
   **Sample response: The lines appear to be bending, but when I use a straight edge, I can see they are each straight and parallel to each other.**

2. What math do you see?
   **Sample response: I see lines that appear to all intersect at one point in the center. The illustration looks like a perspective drawing with the center point farther away than the edges.**
   **The two vertical lines appear to bow out at the center and are intersected by all of the lines that connect to the center.**

**Self-Assess**

| 1 | 2 | 3 |
|---|---|---|
| I don't really get it | I'm starting to get it | I got it |

a. I can identify, describe, and draw optical illusions.
1 2 3





## Success looks like . . .

- **Language Goal:** Identifying patterns in optical illusions. **(Speaking and Listening, Writing)**
  » Explaining the structure of the lines in the Hering Illusion.
- **Goal:** Creating optical illusions using the structure of a grid.

## Suggested next steps

**If students are unable to see any mathematics in the illustration, consider:**

- Reviewing examples of mathematics that were discussed in Activities 1 and 2.
- Asking, "What do you see?" instead of "What math do you see?" and helping students describe their thinking by providing them with math vocabulary that can be used to describe what they see.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

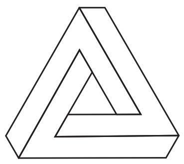
- What was especially satisfying about seeing students work with optical illusions?
- Which groups of students did and did not have their ideas seen and heard today?

# Practice

Independent

Name: ________ Date: ________ Period: ________

1. The triangle shown is a Penrose Triangle. What do you notice? What do you wonder?



smx12/Shutterstock.com

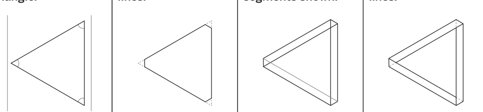
a. I notice . . .
Sample response: The triangle looks three-dimensional in two different ways.

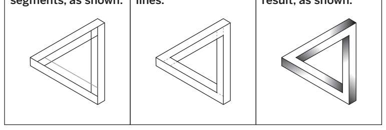
b. I wonder . . .
Sample response: How is a Penrose Triangle created?

c. Create your own Penrose Triangle by following these steps:

**Steps**

| | | | |
|---|---|---|---|
| **1** Draw an equilateral triangle. | **2** Erase on the dotted lines. | **3** Draw the three line segments shown. | **4** Erase on the dotted lines. |
| **5** Draw three more line segments, as shown. | **6** Erase on the dotted lines. | **7** Shade the final result, as shown. | |





**Draw your triangle here**

Answers may vary.

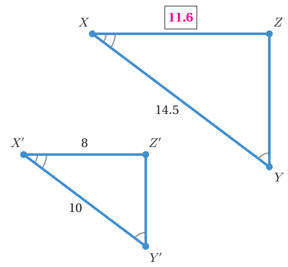




Practice

Name: ________ Date: ________ Period: ________

2. In the diagram, $\Delta XYZ \sim \Delta X'Y'Z'$. Find the missing length of side $XZ$. Explain your thinking.

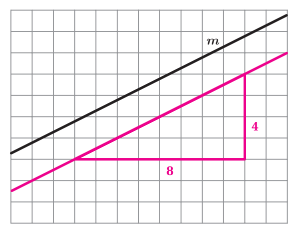



*The figures may not be drawn to scale.*

11.6; Sample response: Because the two triangles are similar, their side lengths must be proportional. I used the corresponding sides to find the scale factor $\frac{14.5}{10} = 1.45$. Sides $XZ$ and $X'Z'$ are corresponding sides, which means $8 \cdot 1.45$ represents the length of side $XZ$; side $XZ = 11.6$.

3. What strategy can you use to determine the slope of line $m$ shown? Explain your thinking.

Sample response: I drew a line that goes through the nodes of the grid and parallel to the line $m$. I drew a slope triangle for the line and found the slope, $\frac{4}{8} = \frac{1}{2}$. I know that parallel lines have the same slope, so the line $m$ also has the slope $\frac{1}{2}$.








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| Spiral | 2 | Unit 2 Lesson 9 | 1 |
| | 3 | Unit 2 Lesson 11 | 3 |

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

## UNIT 3

# Linear Relationships

Students make connections between the rate of change, slope, and the constant of proportionality, drawing on previous knowledge to explore an exciting new relationship: the linear relationship.

**Essential Questions**

- What does the slope of a line tell you about the line?
- What can proportional relationships teach you about linear relationships?
- What does it mean for an ordered pair to be a solution to a linear equation?
- *(By the way, did a 16-year-old really beat Michael Jordan in a game of one-on-one basketball?)*

# Key Shifts in Mathematics

## Focus

### In this unit . . .

Students begin by revisiting different representations of proportional relationships. Students make connections between the slope and the constant of proportionality, drawing on previous knowledge to explore a new type of relationship: the linear relationship. They discover some lines are not proportional, but linear, and spend time studying the features of linear relationships. The unit concludes with two lessons that involve graphing equations in two unknowns, and then finding and interpreting their solutions.

## Coherence

### Previously . . .

At the end of the previous unit on dilations, students learned the terms *slope* and *slope triangle*, used the similarity of slope triangles on the same line to understand that any two distinct points on a line determine the same slope. Students learned about proportional relationships in Grades 6 and 7. In Grade 7, students were formally introduced to the equation $y = kx$ and developed strategies for identifying and creating representations of proportional relationships in graphs, tables, and equations.

### Coming soon . . .

In Unit 4, students will continue their study of linear relationships. To start, students will solve linear equations in one variable, building key procedural fluency they will apply to later lessons in the unit. After developing an understanding for solving linear equations in one variable, students will explore systems of linear equations.

## Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

### Conceptual Understanding

Students build a conceptual understanding of the slope (Lesson 7) and the $y$-intercept (Lesson 9) of a linear relationship in context. Students develop an understanding for what a solution means by considering solutions to equations and lines while weighing appropriate restrictions for a given context (Lesson 16).

### Procedural Fluency

Students practice identifying the unit rate for proportional relationships (Lesson 4). After learning about linear relationships, students practice determining whether a relationship is linear or nonlinear (Lesson 8). Finally, students practice finding the slope of line given two points (Lesson 14).

### Application

Students use their knowledge of proportional relationships to create a display for different models of electronic racing toys using different representations (Lesson 6). Later, students use their algebraic understanding to write the equation of a line using two points (Lesson 14).

# A Straight Change

SUB-UNIT

1

Lessons 2–6

### Proportional Relationships

Students activate their prior understanding of ratios and proportional relationships to make connections between a proportional relationship and its unit rate. They discover that the slope of the line representing a proportional relationship has the same value as its unit rate and use similar triangles to determine the slope.

**Narrative:** Running at a constant rate results in a special kind of relationship between distance and time.

SUB-UNIT

2

Lessons 7–15

### Linear Relationships

Students determine the height of their teacher — you — as measured in cups. This begins their exploration of nonproportional ***linear relationships*** and how they can be represented in graphs, tables, equations, and verbal descriptions.

**Narrative:** The thrill of a roller coaster ride is all about the slope between two points.

# Launch Lesson

Lesson 1

## Visual Patterns

Students explore patterns with shapes and numbers to bridge the geometric thinking they used in Units 1 and 2 with the algebraic thinking they will use in Unit 3 and beyond.

SUB-UNIT

# 3

Lessons 16–18

## Linear Equations

Students explore what it means for an ordered pair to be a solution to a problem involving a linear relationship. They use graphs, tables, and equations to justify their thinking.

**Narrative:** Linear equations can help you sink the winning basket.

# Capstone

Lesson 19

## Rogue Planes

Students discover that the coordinate plane has gone rogue as they match equations with lines on rotated planes.

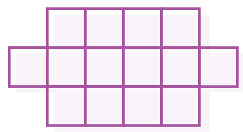
# Unit at a Glance

**Spoiler Alert:** A solution to a linear relationship is an ordered pair, $(x, y)$, that makes the equation true and whose point can also be found on the line of the equation, with coordinates $(x, y)$.

## Assessment

A **Pre-Unit Readiness Assessment**

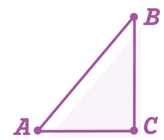
## Launch Lesson



1 **Visual Patterns**

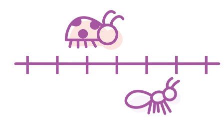
Examine visual patterns, and draw conclusions about how the patterns grow.

## Sub-Unit 1: Proportional Relationships




2 **Proportional Relationships**

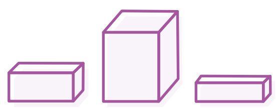
Make connections about the slope of a line and the constant of proportionality by measuring and representing heart rate data.



3 **Understanding Proportional Relationships**

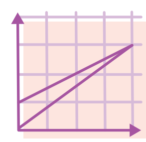
Build fluency skills by graphing proportional relationships from animations and verbal descriptions.

## Linear Relationships



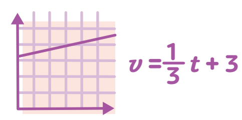
8 **Comparing Relationships**

Justify whether the values in a given table could or could not represent a linear relationship.



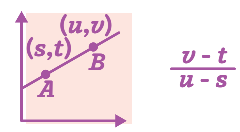
9 **More Linear Relationships**

Identify and interpret the positive vertical intercept and slope of the graph of a linear relationship




10 **Representations of Linear Relationships**

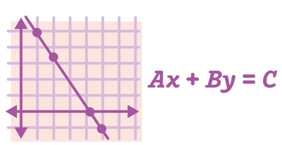
Create an equation that represents a linear relationship in context.




11 **Writing Equations for Lines Using Two Points**

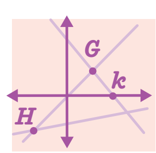
Use two points to find the slope and write the equation of a line.

## Linear Equations




16 **Solutions to Linear Equations**

Find solutions to equations that are of the form $Ax + By = C$.




17 **More Solutions to Linear Equations**

Study of the relationship between a linear equation in two variables, its solution set, and its graph.

$y = Ax + B$

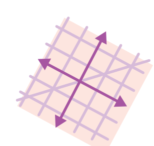
or

$Cx + Dy = E$

18 **Coordinating Linear Relationships**

Discover that the equations $Ax + By = C$ and $y = mx + b$ can both be used to represent the same situation.

## Capstone Lesson



19 **Rogue Planes**

Something weird is happening with the coordinate planes in this lesson. Match lines to equations using these rogue planes.

## Key Concepts

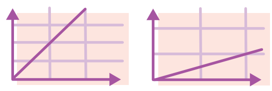
**Lesson 2:** The slope of the line of a proportional relationship is the constant of proportionality.
**Lesson 7:** Linear relationships have a constant rate of change and an initial value.
**Lesson 16:** A solution to a linear equation is any ordered pair that makes the equation true and whose point lies on the line of the graphed equation.

## Pacing

**19 Lessons:** 45 min each
**2 Assessments:** 45 min each

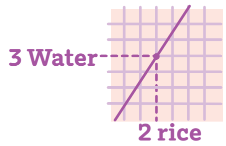
**Full Unit:** 21 days
● **Modified Unit:** 18 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



### 4 Graphs of Proportional Relationships

Examine and compare proportional relationships with and without scaled axes.




### 5 Representing Proportional Relationships

Create an equation and a graph to represent proportional relationships, including an appropriate scale and axes.

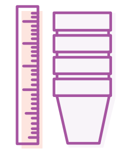
$15(2) = 30$

$15(0.2) = 3$

### 6 Comparing Proportional Relationships ●

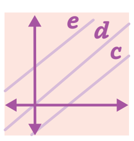
Present a comparison of two proportional relationships using multiple other representations.

## Sub-Unit 2:



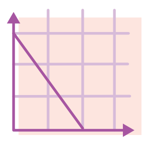
### 7 Introducing Linear Relationships

Learn about linear relationships with stacked cups.



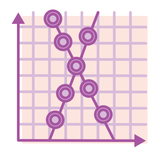

### 12 Translating to $y = mx + b$

Translate lines and see what happens to the equations of the lines.



### 13 Slopes Don't Have to Be Positive

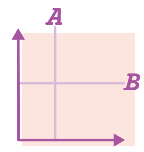
Explore lines with a negative slope.



### 14 Writing Equations for Lines Using Any Two Points, Revisited

Write equations, but this time with negative slopes.

## Sub-Unit 3:




### 15 Equations for All Kinds of Lines

Write equations for vertical and horizontal lines.

## Assessment

### A End-of-Unit Assessment

## ● Modifications to Pacing

**Lessons 2–3:** These lessons address the same standard and can be combined if students have a strong understanding of proportional relationships from Grade 7.

**Lesson 6:** The final lesson in the sub-unit, Lesson 6 can be omitted if students have demonstrated sufficient mastery of proportional relationships.

**Lesson 19:** This lesson presents an engaging way for students to practice coordinating lines and equations, but can be omitted, if needed as its mathematical content is not required for students to learn.

# Unit Supports

## Math Language Development

| Lesson | New vocabulary |
|---|---|
| 7 | initial value<br>linear relationship<br>rate of change |
| 9 | vertical intercept<br>$y$-intercept |
| 13 | horizontal intercept<br>$x$-intercept |

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

| Lesson(s) | Mathematical Language Routines |
|---|---|
| 2, 5, 13, 18 | **MLR1:** Stronger and Clearer Each Time |
| 1, 2, 7, 9, 10, 13, 15 | **MLR2:** Collect and Display |
| 14, 17 | **MLR3:** Critique, Correct, Clarify |
| 5, 13, 18 | **MLR4:** Information Gap |
| 4, 11, 13, 15 | **MLR5:** Co-craft Questions |
| 16 | **MLR6:** Three Reads |
| 1, 5, 6, 8, 9, 12, 14–16 | **MLR7:** Compare and Connect |
| 4, 6–9, 11, 12, 17 | **MLR8:** Discussion Supports |

## Materials

Every lesson includes:

Exit Ticket    Additional Practice

| Lesson(s) | Additional required materials | |
|---|---|---|
| 5, 8, 11, 18 | calculators | |
| 1, 15 | colored pencils | |
| 19 | geometry toolkits | |
| 5 | graph paper | |
| 6 | graph paper<br>poster paper | markers<br>sticky notes |
| 1, 2, 4–10, 12–15, 17, 18 | PDFs are required for these lessons. Refer to each lesson to see which activities require PDFs. | |
| 2–8, 10–16, 18 | rulers | |
| 17 | plain sheets of paper | |
| 7 | stackable cups | |
| 10 | marbles | 100 ml graduated cylinders |

## Instructional Routines

Activities throughout this unit include these instructional routines:

| Lesson(s) | Instructional Routines |
|---|---|
| 4, 8, 9, 12 | Card Sort |
| 6 | Gallery Tour |
| 5, 13, 18 | Info Gap |
| 6 | Number Talk |
| 10 | Partner Problems |
| 8, 14, 15 | Poll the Class |
| 2, 3, 5, 13, 15, 16, 19 | Think-Pair-Share |
| 14 | Two Truths and a Lie |
| 1, 15 | Which One Doesn't Belong |
| 4, 9 | Would You Rather? |

# Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

| Assessments | When to Administer |
|---|---|
| **Pre-Unit Readiness Assessment**<br>This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit. | Prior to Lesson 1 |
| **Exit Tickets**<br>Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned. | End of each lesson |
| **End-of-Unit Assessment**<br>This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide. | After Lesson 19 |

Amps
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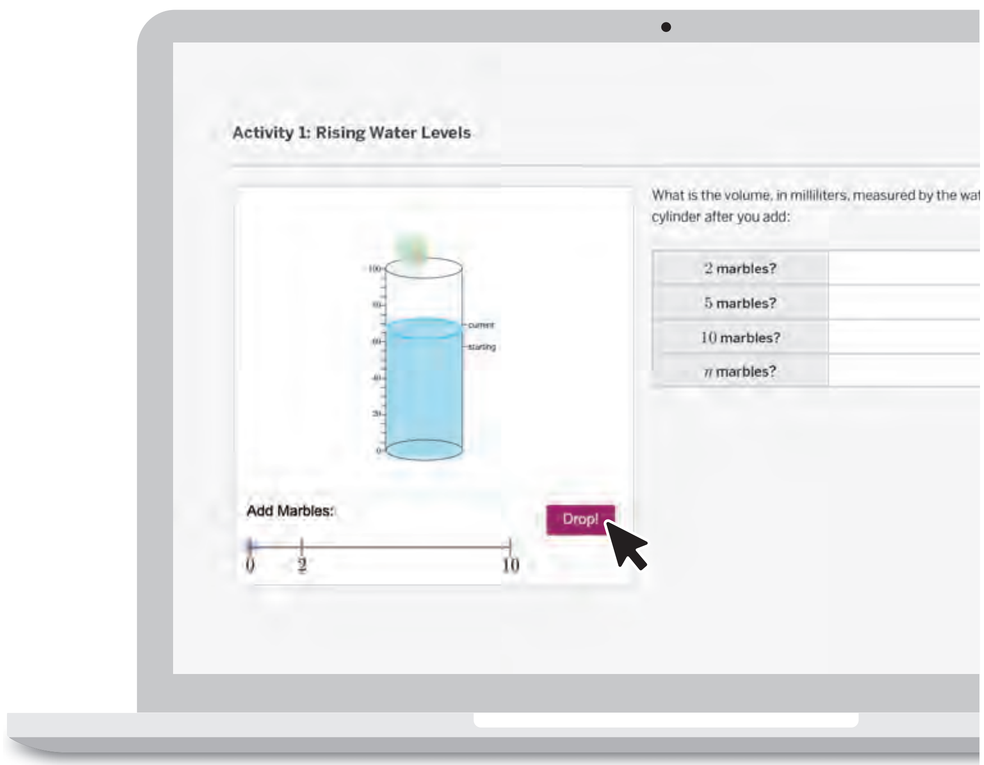
## Social & Collaborative Digital Moments

### Featured Activity

**Rising Water Levels**

Put on your student hat and work through **Lesson 10, Activity 1:**




Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

### Other Featured Activities:

- Rogue Planes **(Lesson 19)**
- Traveling Bugs **(Lesson 3)**
- Coin Collector **(Lesson 14)**
- Card Sort: Slopes, Vertical Intercepts, and Graphs **(Lesson 9)**

# Unit Study
## Professional Learning

This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

## Anticipating the Student Experience With Fawn Nguyen

**Sub-Unit 2**, introduces students to representing linear relationships. They begin to explore nonproportional linear relationships where the graphs do not pass through the origin. Students interpret the meaning of the $y$-intercept and the slope of the line in various real-world contexts. They learn to graph and write equations in slope intercept form, $y = mx + b$, from two given points. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

### Do the Math

Put on your student hat and tackle these problems from **Lesson 14, Activity 2:**

**The Coin Collector arcade game at Honest Carl's Funtime World requires a player to control a character that moves along a straight line to collect coins. The fewer lines a player uses, the more points they earn.**

For each graph shown, draw lines to collect coins. Label each line with a number (1, 2, 3, etc.), and then write the equation for each line.

**Note:** You may not need to use all of the space provided for the equations. Additionally, you may add more equations, as needed.

**Round 1:**

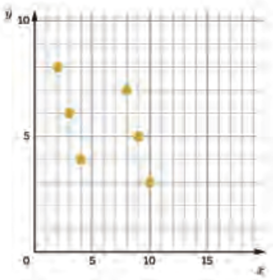
**Equations:**

Line 1:

Line 2:

Line 3:

Line 4:




Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

#### Points to Ponder . . .

- What was it like to engage in this problem as a learner?
- In this task, students are most likely writing the equations in slope-intercept form, $y = mx + b$. They will learn the standard form of $Ax + By = C$ later in the unit. How do you support students' learning of the different forms?
- What implications might this have for your teaching in this unit?

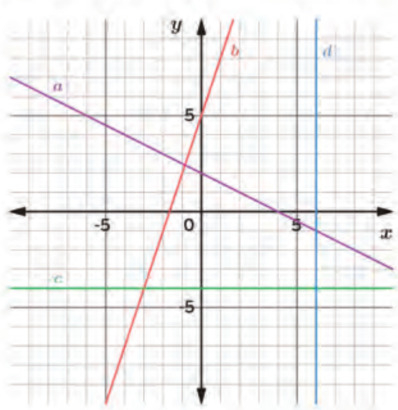
## Focus on Instructional Routines

### Which One Doesn't Belong?

#### Rehearse . . .

How you'll facilitate the ***Which One Doesn't Belong?*** instructional routine in **Lesson 15, Warm-up:**

**Study the four lines shown. Which line does not belong? Explain your thinking.**




#### Points to Ponder . . .

- The discussion works best when students select a variety of answer choices. Which answer choice do you think students will most likely choose? How can you encourage a range of answer choices while you monitor?

#### This routine . . .

- Fosters a need to define terms carefully and use words precisely.
- Highlights similarities and differences in mathematical concepts.
- Can be done individually or collaboratively.
- Provides a low-floor entry point where all possible answer choices can be validated.

#### Anticipate . . .

- How will you sequence student responses in your class discussion?
- How can you frame the routine so that students know this is different from a multiple choice problem with one "correct" answer?
- If you haven't used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you have used this routine before, what went well? What would you want to refine?

## Strengthening Your Effective Teaching Practices

**Support productive struggle in learning mathematics.**

### This effective teaching practice . . .

- Provides students with the opportunity to wrestle with mathematical concepts and relationships before you intervene, which builds student confidence and perseverance.
- Allows you greater opportunities to monitor student progress, probe for student understanding, and offer differentiation support.

### Points to Ponder . . .

- How comfortable are you with allowing students the time to wrestle with mathematical ideas, before you intervene?
- When is the right moment to intervene? What can you look for as you monitor student work and student conversations to know when they are engaging in productive struggle or unproductive struggle?

## Math Language Development

### MLR8: Discussion Supports

MLR8 appears in Lessons 4, 6–9, 11, 12, 17.

- Throughout the unit, sentence frames are provided for you to display to your students. Students can use these prompts to help frame their responses and add structure and organization to their thinking.
- In Lesson 11, further probing questions are provided so that you can ask your students for further clarification or to press for details in their reasoning.
- **English Learners:** Provide students the opportunity to rehearse what they will say with a partner before they share with the whole class.

### Point to Ponder . . .

- During class discussions in this unit, how will you know when to press for details or probe further to assess for understanding? What clues will you look for from your students' responses?
- How will you decide when to display or provide sentence frames to help students by providing a structure for their responses?

## Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

### Look Ahead . . .

- Review and unpack the **End-of-Unit Assessment**, noting the concepts and skills assessed.
- With your student hat on, complete each problem.

### Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with solving one-step equations with a variable throughout the unit? Do you think your students will generally:
  - » Have difficulty calculating slope from a line? From two points?
  - » Struggle to write an equation from a context?
  - » Find one representation of linear relationships more challenging than the other?
  - » Have trouble with graphing, labeling axes, and using appropriate scales?

## Differentiated Support

### Accessibility: Guide Processing and Visualization, Optimize Access to Technology

Opportunities to provide visual support, guide student processing, or provide the use of technology appear in Lessons 1–19.

- Throughout the unit, consider providing pre-completed graphs so that students can focus on analyzing the relationships, without having to construct the graphs themselves.
- Display or provide copies of the Anchor Chart PDFs, Representations of Linear Relationships and Slope (from Unit 2) for students to reference throughout the unit.
- Use color coding and annotations to highlight how the slope and vertical intercept appear in a verbal description, graph, table of values, and equation.
- In Lesson 10, use the Amps slides for Activity 1, in which students can see the rising water levels as marbles are added to a virtual cylinder.

### Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to provide a pre-completed graph or suggest students create a table to help illustrate the relationship between quantities?

## Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their self-awareness and self-management skills.

### Points to Ponder . . .

- What are their strengths and what do they know about numerical reasoning that they can build upon and use to begin reasoning algebraically?
- Are students able to exercise patience and persist in order to understand the underlying conceptual structure of one-variable equations, rather than asking for or jumping to a procedural shortcut?


# Visual Patterns

Let's explore patterns in shapes and numbers.

## Focus

### Goal

1. **Language Goal:** Write an algebraic expression to describe a visual pattern. **(Writing)**

## Rigor

- Students examine visual patterns to build **conceptual understanding** of how to write expressions to describe change.

## Coherence

### Today

Students examine visual patterns, and draw conclusions about how the patterns grow. Students write expressions that describe patterns, and make conclusions about what each term of an expression represents.

### Previously

In Grade 7, students examined equivalent expressions, proportional and nonproportional linear relationships, and pattern growth.

### Coming Soon

Students begin this unit by deepening their knowledge of proportional relationships. In Lesson 2, they will make connections about the constant of proportionality and the slope of a line. In Lesson 7, students will discover a relationship that is not proportional, but linear. In Lessons 8–19, students will continue studying linear relationships, proportional or nonproportional, and explore what it means to be a solution to an equation. By the end of the unit, students will be able to represent real-world linear relationships using equations, tables, and graphs.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 12 min | 15 min | 5 min | 7 min |
| Independent | Independent | Small Groups | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- Activity 2 PDF, one set per group
- colored pencils (optional)

### Math Language Development

**Review words**

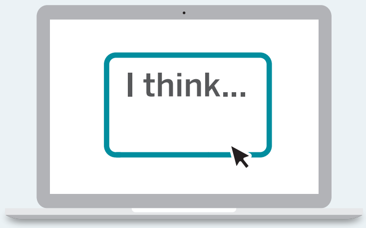
- *nonproportional relationship*
- *proportional relationship*

**Amps** | Featured Activity

**Activity 1**

### See Student Thinking

Students are asked to explain their thinking as they describe and extend visual patterns, and these explanations are available to you digitally, in real time.




Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

At first, students may not immediately be able to identify an expression to model the visual pattern and might want to quit before really getting started in either Activity. Encourage students to set a goal of identifying what they do know about the pattern and build on that goal by using what they know to determine the expression. Students can repeat this process until they have solved the problem. By looking only one step ahead, a task can seem much more manageable.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, have students work in pairs on their patterns.

Independent | 5 min

# **Warm-up** Which One Doesn't Belong?

Students analyze patterns to prepare for examining pattern growth in the upcoming activities.

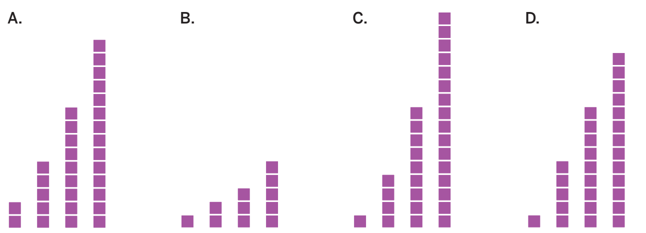
Unit 3 | Lesson 1 – Launch

## Visual Patterns

Let's explore patterns in shapes and numbers.


### **Warm-up** Which One Doesn't Belong?

Examine the following patterns. Which pattern does not belong? Explain your thinking.




A. 2, 5, 9, 14, . . .
B. 1, 2, 3, 5, . . .
C. 1, 4, 9, 16, . . .
D. 1, 5, 9, 13, . . .

Sample responses:

- Pattern A is the only one that does not start with 1.
- Pattern B is the only one that does not have 9 as the third step.
- Pattern C is the only one that represents squared numbers (or does not have 5 somewhere in the pattern).
- Pattern D is the only one that increases by the same amount for each step.

**Collect and Display:** As you share your thinking, your teacher will collect the math language you use to add to a class display. You will continue to add and refer to this display throughout the unit.

Log in to Amplify Math to complete this lesson online.

222 Unit 3 Linear Relationships



## 1 Launch

Conduct the ***Which One Doesn't Belong?*** routine.

## 2 Monitor

**Help students get started** by asking, "What feature do three of the patterns share, but one does not?"

**Look for points of confusion:**

- **Thinking that there can only be one explanation for why a pattern does not belong.** During the Connect, have these students record different explanations from classmates to support their choice.

**Look for productive strategies:**

- Finding multiple pieces of evidence to support their choice.
- Finding evidence for more than one choice.

## 3 Connect

**Have individual students share** their evidence for which pattern does not belong. Select students who can provide a different explanation for the same answer choice. Consider finishing the discussion on pattern D, and ask whether students have seen a pattern like this before.

**Highlight** that each pattern has multiple steps, with the number of blocks increasing in every step. In particular, highlight that pattern D is the only pattern showing the same increase for each step.

**Ask**, "How can you track the changes from one step to the next in a pattern?"

## Math Language Development

### *MLR2: Collect and Display*

During the Connect, collect informal language students to describe which pattern doesn't belong and add this language to a class display. Highlight words, such as *change*, *growth*, and *rate*. Continue adding to the display during Activity 1.

### English Learners

Emphasize that patterns can be seen *within* each representation and they can be seen across representations. For example, students might describe Pattern A's growth from one step to the next, but they can also describe how Patterns B, C, D all begin with 1 square.

Independent | 12 min

# Activity 1 What Comes Next?

Students draw subsequent figures in a visual pattern to see how it changes, and then write an equation to describe the $n$th figure of the pattern.

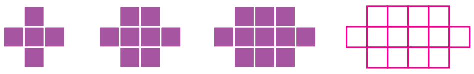
**Amps Featured Activity** See Student Thinking

Name: ______________________ Date: ____________ Period: ____________

## Activity 1 What Comes Next?

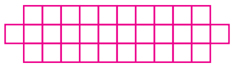
Consider the following pattern.

Figure 1 Figure 2 Figure 3 Figure 4



1. Study the pattern and draw a sketch of Figure 4. How many squares are in Figure 4?
   **14 squares**

2. What does Figure 10 look like? Draw or describe Figure 10 here. How many squares are in Figure 10?



**Sample response: Figure 10 consists of a rectangle with a height of 3 squares and a width of 10 squares, with one square in the middle on the left and one on the right. There are 32 squares in Figure 10.**

Pause here while your class shares sketches.





## 1 Launch

Display the pattern from the Student Edition. Have students complete Problems 1 and 2 individually. Then have them share their responses with a partner before completing Problems 3–5.

## 2 Monitor

**Help students get started** by asking them to locate the shape of Figure 1 in Figure 2, and the shape of Figure 2 in Figure 3.

**Look for points of confusion:**

- **Thinking that they must draw every square for Figure 10.** Let students know that a "sketch" is a rough drawing and does not need to be accurate.
- **Interpreting the three additional squares in every figure as addition in their expression for Problem 5.** Ask, "Which operation describes repeated addition? How can you represent that in the expression?"
- **Struggling to write an expression for Problem 5.** Remove 2 squares from each figure of the pattern, and ask students to write the expression for this updated pattern. Then ask them to modify their expression to match the original pattern.

**Look for productive strategies:**

- Completing the table by adding 3 to the number of squares for each successive figure, instead of counting every square in the figure.

Activity 1 continued ›

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Change Figure 10 to Figure 6 in Problem 2 and omit Figure 26 from the table in Problem 4.

### Accessibility: *Guide Processing and Visualization*

Suggest students analyze each column in each figure and describe how it changes or grows. Consider demonstrating how to highlight or circle each column to help students visualize each column.

### Extension: *Math Enrichment*

Ask students to explain how each term of the expression $3n + 2$ relates to the pattern of figures. Ask:

- "Where do you see 2 in the pattern? $3n$?" There are 2 squares on the left and right sides of the columns. Figure $n$ has $n$ columns with 3 squares in each column.
- "If the expression $4n + 4$ represents a similar pattern of figures, what might each figure look like? How might the pattern grow?" Sample response: Figure $n$ would consist of $n$ columns with 4 squares in each column and with 2 squares on either side of the columns for a total of 4 squares on the sides.

Independent | 12 min

# Activity 1 What Comes Next? (continued)

Students draw subsequent figures in a visual pattern to see how it changes, and then write an equation to describe the $n$th figure of the pattern.

### Activity 1 What Comes Next? (continued)

3. Describe how the pattern is growing.
Sample response: For each next figure, a column of 3 squares is added. The number of squares in each next figure increases by 3.

4. Complete the table to show the number of squares for different figure numbers.

| Figure | 1 | 2 | 3 | 4 | 10 | 26 |
|---|---|---|---|---|---|---|
| Number of squares | 5 | 8 | 11 | 14 | 32 | 80 |

5. Write an expression that represents the number of squares in Figure $n$.
$3n + 2$ (or equivalent)

**Are you ready for more?**

What does Figure 0 look like? Draw or describe Figure 0 here.

Sample response: □□; Figure 0 would not have any columns of 3 squares in the middle.





## 3 Connect

**Have individual students share** their sketches for Figures 4 and 10, and their descriptions of how the pattern grows.

**Ask**, "What is changing as the pattern grows from one figure to the next?" The total number of squares is increasing by 3 every time. Then display the Activity 1 PDF.

**Highlight** that their expressions from Problem 5 should correctly predict the number of squares in a given figure. Ask students to use their expression to verify the last column in the table.

Small Groups | 15 min

# Activity 2 Sketchy Patterns

Students explore new patterns and make comparisons to identify proportional and nonproportional relationships.

Name: ______ Date: ______ Period: ______

**Activity 2** Sketchy Patterns

**You will be given a sheet with a new pattern on it and some problems about the pattern. Use the pattern to respond to the problems. Record your responses on the sheet.**

After you have completed the problems on your sheet, compare your pattern with your group members. What is different? What is the same?

**Sample responses:**

- **Each of our patterns grows by adding the same number each time to result in the next figure. However, the same number that is added each time is not the same number for each pattern.**

  **In Pattern A, the number added each time is 2.**

  **In Pattern B, the number added each time is 4.**

  **In Pattern C, the number added each time is 3.**

  **In Pattern D, the number added each time is 4.**

- **All of the expressions we wrote include multiplication, but only Pattern B can be simplified to an expression that does not also use addition.**

STOP





## 1 Launch

Group students by four, and distribute one pattern from the Activity 2 PDF to each member.

## 2 Monitor

**Help students get started** by having them look back at Activity 1. Ask, "Are there any parts of your pattern that stay the same? Can you describe what changes from one figure to the next?"

**Look for points of confusion:**

- **Thinking that there is only one expression to describe the way the pattern is growing.** Encourage students to color-code their expressions and patterns.

**Look for productive strategies:**

- Recognizing whether a pattern is proportional or nonproportional.
- Rewriting expressions in equivalent forms, and identifying how each term can be seen in the pattern.

## 3 Connect

**Display** each pattern and ask students to suggest expressions to represent the pattern, emphasizing equivalent expressions as they are shared. In each case, finish the discussion with the expression that resembles $mx + b$.

**Have groups of students share** their observations from comparing each pattern.

**Ask,** "What is the same for these patterns? What is different?" In every pattern, each figure adds the same number, but they have different "starting points." **Note:** The term *initial value* will be defined later in this unit.

**Highlight** that only Pattern B can be written with one term that is the product of a coefficient and a variable. Remind students that this is a *proportional relationship*.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Instead of giving one pattern from the Activity 2 PDF to each group member, assign each group one pattern. After group members have analyzed their pattern and before the Connect, have them share their patterns with another group and explain how they determined the pattern and wrote the expression.

### Extension: *Math Enrichment*

Have students write an equation in two variables that represents each pattern. Students should define their variables and graph each equation, including labeling and scaling their axes. An example equation for Pattern A is $s = 2n + 6$, where $s$ represents the total number of squares and $n$ represents the figure number.

## Math Language Development

### *MLR7: Compare and Connect*

During the Connect, as students respond to the Ask question, highlight the language students use to describe the similarities and differences among the four patterns. For example, they may use the term "starting point" or "constant" to describe the initial value in each pattern, which is also the constant in the expression. Draw connections between the starting point and the constant term in the expression as well as between the value that is added each time and the coefficient of $n$ in the expression.

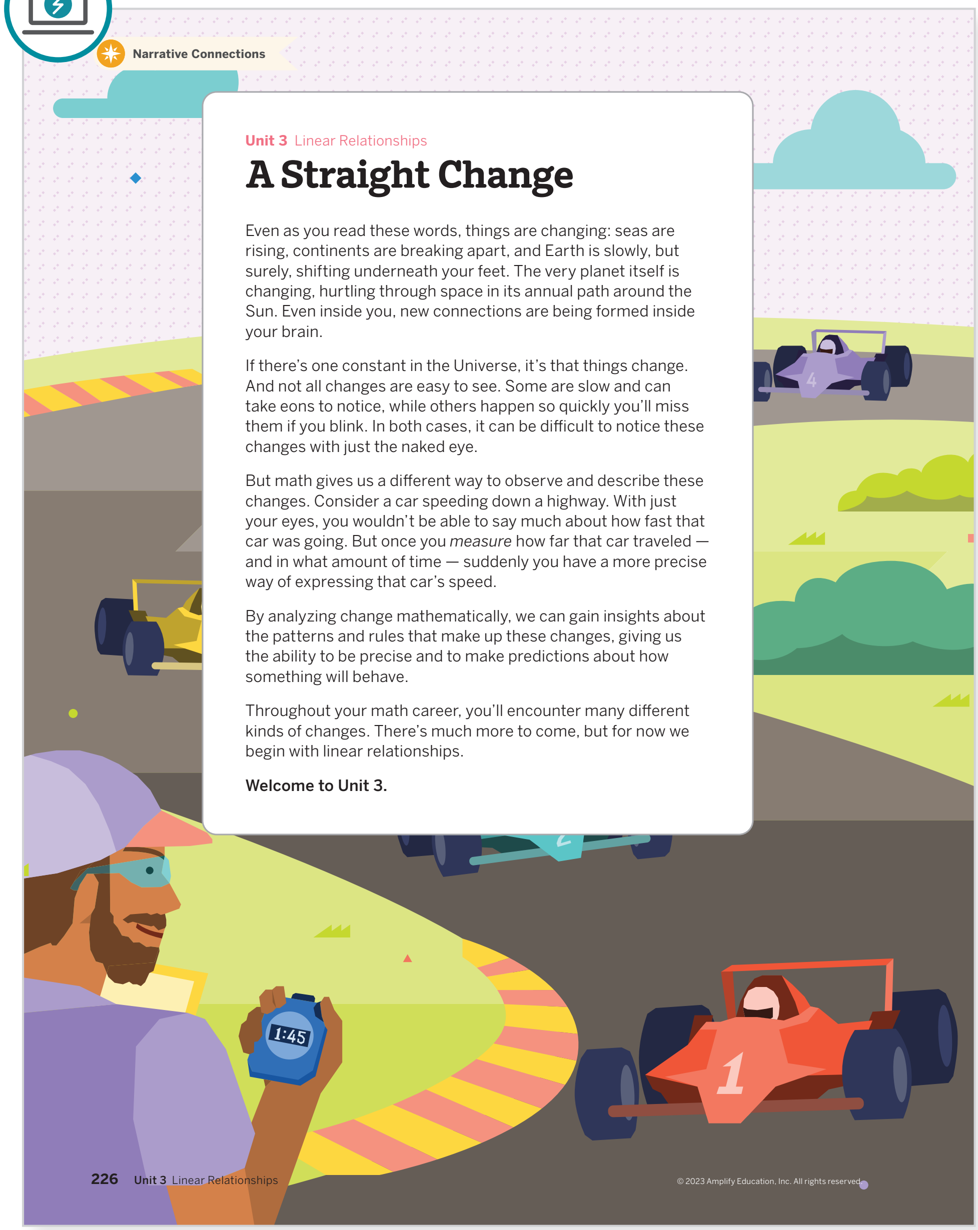
### English Learners

Annotate the table and the expression with the starting point and the value that is added each time.

Whole Class | 5 min

# Summary A Straight Change

Review and synthesize how visual patterns grow and change.


Narrative Connections

Unit 3 Linear Relationships

## A Straight Change

Even as you read these words, things are changing: seas are rising, continents are breaking apart, and Earth is slowly, but surely, shifting underneath your feet. The very planet itself is changing, hurtling through space in its annual path around the Sun. Even inside you, new connections are being formed inside your brain.

If there's one constant in the Universe, it's that things change. And not all changes are easy to see. Some are slow and can take eons to notice, while others happen so quickly you'll miss them if you blink. In both cases, it can be difficult to notice these changes with just the naked eye.

But math gives us a different way to observe and describe these changes. Consider a car speeding down a highway. With just your eyes, you wouldn't be able to say much about how fast that car was going. But once you *measure* how far that car traveled — and in what amount of time — suddenly you have a more precise way of expressing that car's speed.

By analyzing change mathematically, we can gain insights about the patterns and rules that make up these changes, giving us the ability to be precise and to make predictions about how something will behave.

Throughout your math career, you'll encounter many different kinds of changes. There's much more to come, but for now we begin with linear relationships.

**Welcome to Unit 3.**



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## Narrative Connections

Read the narrative aloud as a class or have students read it individually.

## Synthesize

**Display** the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

**Ask**, "How did you describe patterns in today's lesson?" Sample response: We made tables and wrote expressions.

**Highlight** that students will continue to examine relationships and explore how patterns change in this unit.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful today when writing expressions to describe how patterns change? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

Independent | 7 min

# Exit Ticket

Students demonstrate their understanding by writing an expression from a pattern.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 3.01

Consider the following pattern.

Figure 1 Figure 2 Figure 3

1. Describe how the pattern is growing.
   As the pattern grows, the heights of the left and right columns of each succeeding figure increase by 1 square.

2. Write an expression that represents the number of squares in Figure $n$.
   Sample response: $2n + 2$

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can analyze a pattern of figures and predict what a future figure will look like.
1 2 3

b I can write an expression that represents the number of objects in the $n$th figure of a pattern.
1 2 3



**Lesson 1** Visual Patterns

## Success looks like . . .

- **Language Goal:** Writing an algebraic expression to describe a visual pattern. **(Writing)**
  » Writing an algebraic expression for the number of squares in Figure $n$.

## Suggested next steps

**If students struggle to come up with an expression, consider:**

- Asking, "What would Figure 4 of this pattern look like? What would Figure $n$ of this pattern look like?"
- Assigning Practice Problem 1.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- What worked and did not work today? Which students' ideas were you able to highlight during Activity 2?
- The instructional goal for this lesson was for students to write expressions to describe visual patterns. How well did students accomplish this? What did you specifically do to help students accomplish it? What might you change for the next time you teach this lesson?

# Practice

Independent

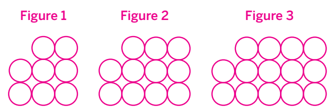
Name: Date: Period:

Practice

**1.** Kiran is describing a pattern. He says, "In Figure 1, there are 8 circles. In Figure 2, there are 11 circles. In Figure 3, there are 14 circles."

**a** Draw what Figures 1, 2, and 3 could look like.

**Sample responses shown.**

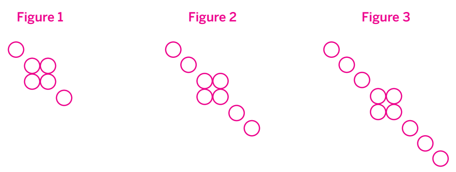



**b** Write an expression that represents the number of circles in Figure $n$.

**Sample response: $3n + 5$**

**2.** Design your own pattern of objects in which the number of objects in Figure $n$ can be represented by the expression $2n + 4$. Draw Figures 1–3.

**Sample response:**




**3.** For each row, decide whether the expression in Column A is equivalent to the expression in Column B. If they are not equivalent, change the expression in Column B so that it is equivalent to the expression in Column A.

| Column A | Column B | Equivalent? | If not equivalent, change Column B to . . . |
|---|---|---|---|
| $3x - 2x + 0.5$ | $1.5x$ | No | Sample response: $x + 0.5$ |
| $6(x + 4) - 2(x + 5)$ | $2(2x + 7)$ | Yes | |
| $3(x + 4) - 2(x - 4)$ | $x + 4$ | No | Sample response: $x + 20$ |
| $20\left(\frac{2}{5}x + \frac{3}{4}y - \frac{1}{2}\right)$ | $\frac{1}{2}(16x + 30y - 20)$ | Yes | |

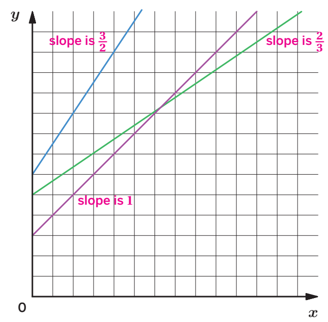




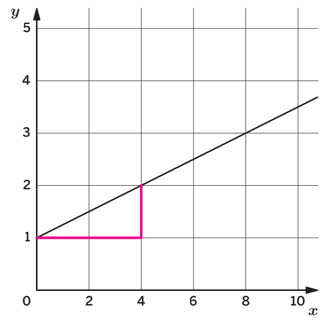
Name: Date: Period:

Practice

**4.** Of the three lines shown, one has a slope of 1, one has a slope of $\frac{2}{3}$, and one has a slope of $\frac{3}{2}$. Label each line with its correct slope.




**5.** Determine the slope of the line shown. Show or explain your thinking.




$\frac{1}{4}$; Sample response: I drew a slope triangle on the line. The vertical side length is 1 unit and the horizontal side length is 4 units so the slope is $\frac{1}{4}$.





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| | 2 | Activity 2 | 2 |
| Spiral | 3 | Grade 7 | 2 |
| | 4 | Unit 2 Lesson 11 | 2 |
| Formative | 5 | Unit 3 Lesson 2 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

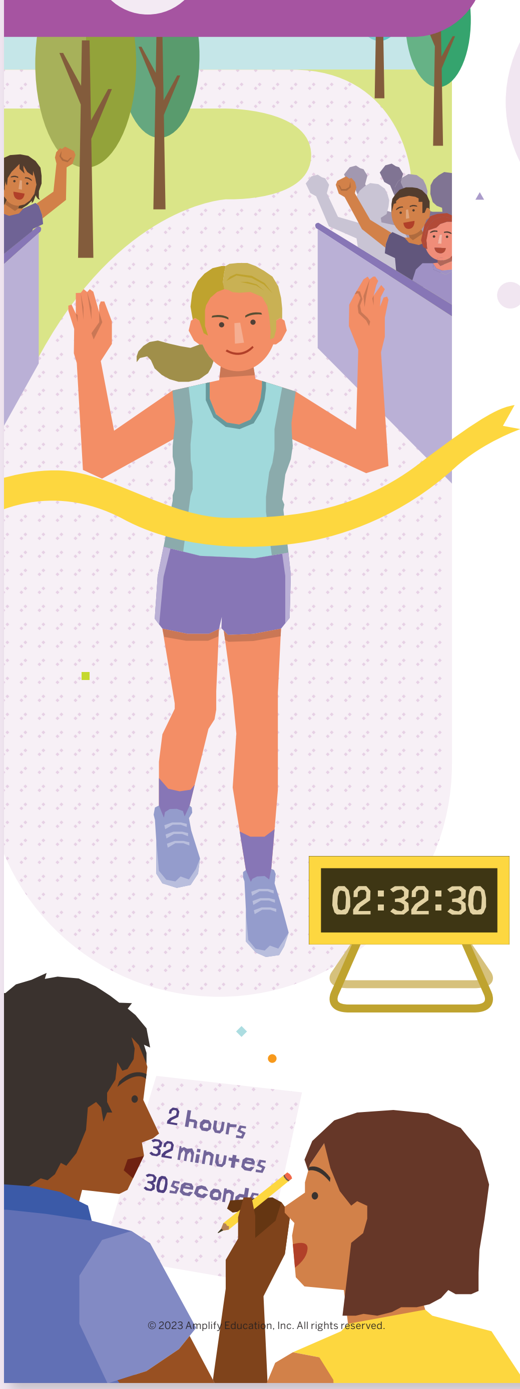
## Sub-Unit 1
# Proportional Relationships

Whole Class

In this Sub-Unit, students make connections between a proportional relationship and the slope of its line, its unit rate, and similar triangles that can be used to determine the slope.

SUB-UNIT

1 Proportional Relationships




Narrative Connections

## How fast is a geography teacher?

On October 22, 1978, Grete Waitz ran her first marathon in New York City. A school geography teacher from Norway, Waitz had never run more than 13 miles at a time. But after some encouragement from her husband, she decided to tackle the New York City Marathon's 26.2 miles.

Few people had heard of Waitz since she hadn't yet raced in America. And when this newcomer appeared in the lead, spectators were shocked. She not only won the race by 10 minutes, she set a world record of 2 hours 32 minutes 30 seconds in the process.

Still, the last ten miles had tested Waitz's endurance. At the finish line, she took off her sneakers, chucked them at her husband, and vowed to never run a marathon again. But the very next year she broke that vow. She returned to New York to beat her previous world record, in a time of 2:27:33. She had become the first woman to run a marathon in under two-and-a-half hours.

Grete Waitz's record-setting victories not only changed attitudes about women's racing —they changed the face of the New York City Marathon itself, and helped popularize it around the world. Her 10-minute margin of victory in 1978 transformed her from being a pacesetter to being a champion.

Whether you're calculating speed using a graph or an equation, figuring out the relationship between distance and time is essential for both the racer and the spectator. When the race begins, which one would you like to be?





### Narrative Connections

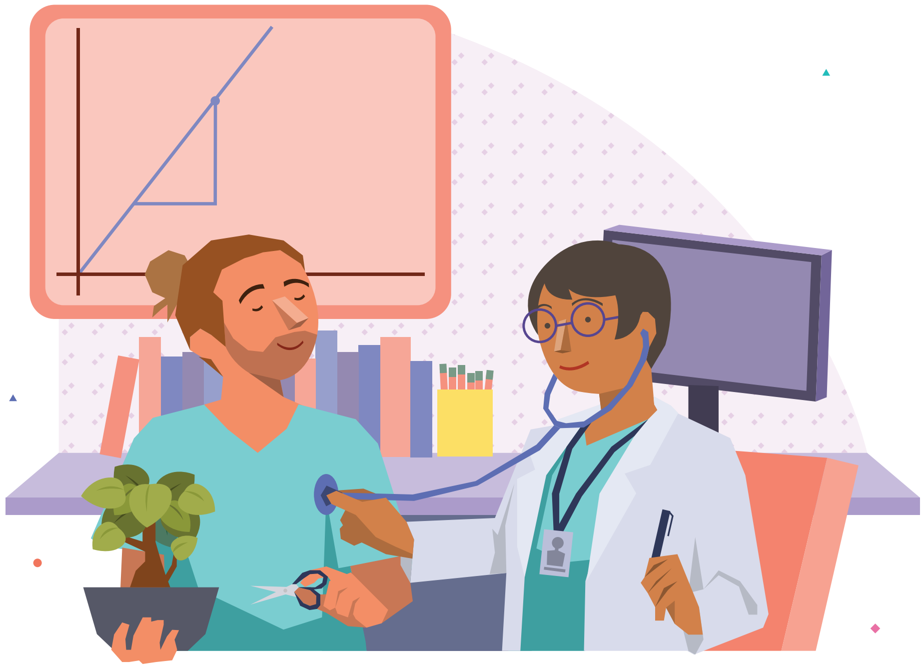
Read the narrative aloud as a class or have students read it individually. Students continue to explore real-world applications of proportional relationships in the following places:

- **Lesson 2, Activities 1–3:** Graphing Heart Rates, The Equation of a Line, Scale Factor
- **Lesson 3, Activities 1–2:** Moving Through Representations; Twice as Fast, Twice as Slow
- **Lesson 4, Activity 1:** Calculating the Rate
- **Lesson 5, Activities 1–2:** Representations of Proportional Relationships, Info Gap: Proportional Relationship
- **Lesson 6, Activity 1:** Gallery Tour

UNIT 3 | **LESSON 2**

# Proportional Relationships

Let's explore the connection between points that lie on the line of a proportional relationship and the slope of the line.



## Focus

### Goals

1. Create an equation relating the quotient of the vertical and horizontal side lengths of a slope triangle to the slope of a line.
2. Comprehend that for the equation of a proportional relationship given by $y = kx$, $k$ represents the unit rate.
3. **Language Goal:** Justify whether a point is on the line of a proportional relationship by determining whether the ratio of the vertical distance to the horizontal distance (from the origin to the point) equals the slope of the line. **(Speaking and Listening)**

## Coherence

### Today

Students examine how their heart rate can be represented as a proportional relationship in a table and on a graph. When graphed, students make connections about the slope of the line and the constant of proportionality. They justify whether a point is on the line and what it means in context.

### Previously

In Unit 2, students were introduced to the slope of a line. In Grade 7, students learned about proportional relationships and the constant of proportionality.

### Coming Soon

Students will continue studying proportional relationships through the first part of this unit, before learning about linear relationships in Lesson 7.

## Rigor

- Students build **conceptual understanding** of proportional relationships by exploring how their heart rates can be represented in a table and on a graph.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Activity 3 | Summary | Exit Ticket |
|---|---|---|---|---|---|
| 5 min | 10 min | 10 min | 10 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Independent | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Anchor Chart, *Slope* (from Unit 2)
- rulers

### Math Language Development

**Review words**

- *constant of proportionality*
- *proportional relationship*
- *slope*
- *unit rate*

**Amps** | **Featured Activity**

**Activity 1**

**Interactive Graph**

Students plot points and connect them with a line on an interactive graph.

Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

When graphing their heart rates in Activity 1, students might find themselves at a roadblock, where the data does not fit the graph, and start to doubt themselves. Ask students how they can change the precision of the graph to accommodate all of the data, still representing it accurately. By solving the problem themselves with skills or knowledge that they already have, students will gain self-confidence.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 2 may be omitted.
- In **Activity 2**, Problem 1 may be discussed briefly in the launch before focusing on Problems 2–4.
- In **Activity 3**, consider doing only one problem.

Independent | 5 min

# Warm-up Heart Rate

Students find their pulse to explore the relationship between time and heart rate.

Unit 3 | Lesson 2

## Proportional Relationships

Let's explore the connection between points that lie on the line of a proportional relationship and the slope of the line.

### Warm-up Heart Rate

1. Find your pulse. Count the number of heartbeats in 20 seconds and complete the first row in the table.
Sample response shown.

| Time | Number of heartbeats |
|---|---|
| 20 seconds | 30 |
| 1 minute | 90 |

2. Assume the number of heartbeats per second remains constant. Based on your response to Problem 1, predict the number of heartbeats you will have in 1 minute.
Sample response: The constant of proportionality is $\frac{3}{2}$ because $20 \times \frac{3}{2} = 30$. This means that I can multiply 60 by $\frac{3}{2}$ to obtain 90. So, I predict I will have 90 heartbeats in 1 minute.

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Activate prior knowledge by asking students what they know about heart rates and if they know how to locate their pulse. Have students share how to find their pulse, assist where needed, and make sure everyone is ready before starting the timer. Ask students how they think their heart rate might change after running a race. Then display a timer for 20 seconds to begin the activity. **Note:** Provide access to rulers throughout the duration of this lesson.

## 2 Monitor

**Help students get started** by showing them multiple ways of finding their pulse.

**Look for points of confusion:**

- **Not being able to find their heart rate in beats per minute.** Ask how many seconds are in 1 minute, and prompt students to think about how they can use ratios to find the number of heartbeats.
- **Incorrectly counting the number of heartbeats in 20 seconds.** Ask students to count aloud for you or a partner, and consider modeling how to count heartbeats. Provide a range for expected heartbeats, anywhere from 10 to 40. Then run the timer a second time.

## 3 Connect

**Ask**, "How did you find your heart rate in beats per minute, as it is typically measured? How could you find your heart rate in beats per second?"

**Highlight** strategies using ratios or extending the table to find a heart rate out of 60 seconds.

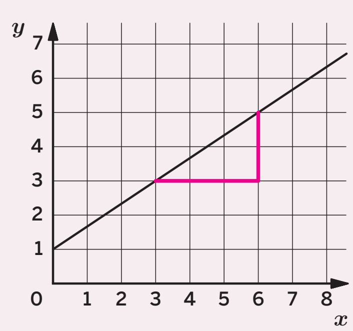
**Have students share** if they think the heart rate represents a proportional relationship without revealing the answer. Use student answers discussing graphs to transition to Activity 1.

## Power-up

**To power up students' ability to determine the slope of a line, have students complete:**

Recall that in order to determine the *slope* of a line you can draw a *slope triangle* then calculate the ratio of its vertical side length to its horizontal side length.

1. Draw a slope triangle for the line shown.
Sample response shown.
2. Use your slope triangle to determine the slope of the line. $\frac{2}{3}$ or equivalent




**Use:** Before Activity 1

**Informed by:** Performance on Lesson 1, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 4

# Activity 1 Graphing Heart Rates

Students graph their heart rate data to see that the slope of the line is the same as the constant of proportionality and that the slope can be used to describe their heart rate.

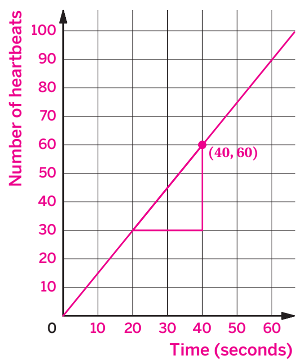
**Amps Featured Activity** Interactive Graph

Name: ______ Date: ______ Period: ______

## Activity 1 Graphing Heart Rates

**When you are at rest, your heart pumps the least amount of blood you need because you are not moving or exercising. Your resting heart rate is normally between 60 (beats per minute) and 100 (beats per minute). Doctors care about heart rates because they can be indicators of good health.**

1. Use your results from the Warm-up to graph the number of heartbeats you counted in 20 seconds and your prediction for the number of heartbeats in 60 seconds. Be sure to create a scale for your graph. Sample response shown.




2. Draw a line connecting your two points. What is the slope of this line? What does it represent within the context of the scenario?
   Sample response: I can draw a slope triangle with a vertical distance of 30 and a horizontal distance of 20. The slope of the line is $\frac{30}{20} = \frac{3}{2}$, or 1.5. It means my heart beats 1.5 times per second.

3. Plot an additional point on the line and label the coordinates of the point on the graph. What does this point represent within the context of the scenario?
   Sample response: I plotted the point (40, 60). The point (40, 60) means that at 40 seconds, I would count 60 heartbeats.



Lesson 2 Proportional Relationships 231

## 1 Launch

Preview Problem 2, and ask students to recall the definition of *slope* and strategies for finding the slope of a line.

## 2 Monitor

**Help students get started** by helping them create a scale.

**Look for points of confusion:**

- **Creating an inaccurate scale.** Explain why the scale does not work and help students find an appropriate scale.
- **Not being able to find the slope.** Show students how to draw slope triangles, and ask them to restate the definition of *slope*. Point students to the Unit 2 Anchor Chart PDF, *Slope*.
- **Confusing $x$ and $y$ in Problem 3.** Ask students to match the variables with the axes labels. Remind students that coordinates of points are written as $(x, y)$.

**Look for productive strategies:**

- Drawing slope triangles.

## 3 Connect

**Display** student graphs with varying slopes.

**Have students share** how they found the slope and their responses for Problem 2.

**Ask:**

- "How is the slope of the line related to the constant of proportionality or unit rate?
- "Looking at the graph, how can you tell this relationship is proportional?"
- "Can you use time in minutes as the label on the $x$-axis? Would that change the slope?" Yes, the slope would show beats per minute and would have a different value.

**Highlight** that the slope of the line represents the number of beats per second, which is the same as the constant of proportionality (or unit rate) for proportional relationships.

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization***

Provide a graph with the scale and axes already labeled. Consider previewing Problem 2 with students to review the meaning of the slope of a line and slope triangles. Display the slope formula.

## Math Language Development

***MLR2: Collect and Display***

During the Connect, as students respond to the Ask questions, highlight the mathematical language students use, such as *proportional, constant of proportionality*, and *slope*. Add these words and phrases to a class display and encourage students to refer to the display during future discussions in this unit.

**English Learners**

Add visual examples of each word to the class display.

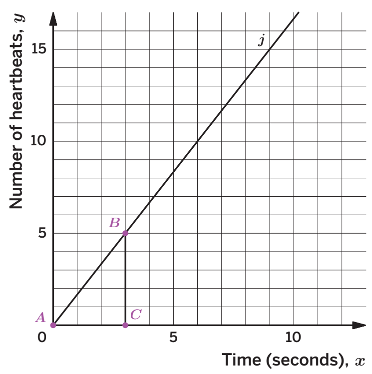
Pairs | 10 min

# Activity 2 The Equation of the Line

Students examine a graph showing heartbeats per second to come up with a rule for finding other points on the line.

### Activity 2 The Equation of a Line

**Kiran is training to swim in the 100-meter freestyle race. He measures his heart rate after he swims one length of the pool. He plots the point $B$ as shown on the graph and draws line $j$ to connect points $B$ and the origin.**




1. What are the coordinates of point $B$? What do they represent in context?
   $B(3, 5)$; At 3 seconds, Kiran's heart had beat 5 times.

2. Is the point $(9, 16)$ on the line? Explain your thinking.
   No; Sample response: The ratio of the vertical distance to the horizontal distance is not the same as the slope of the line, $\frac{16}{9}$ does not equal $\frac{5}{3}$.

3. Is the point $(15, 25)$ on the line? Explain your thinking.
   Yes; Sample response: The ratio of the vertical distance to the horizontal distance is the same as the slope of the line; $\frac{25}{15} = \frac{5}{3}$.

4. Suppose you know the $x$- and $y$-coordinates of a point. Write an equation that could be used to confirm the point is on line $j$.
   Sample response: $\frac{y}{x} = \frac{5}{3}$




## 1 Launch

Conduct the ***Think-Pair-Share*** routine.

## 2 Monitor

**Help students get started** by asking them what they notice about the labels of the axes so they can describe the point in context.

**Look for points of confusion:**

- **Questioning whether a point is on the line in Problems 2 and 3.** Place a point on the line, draw a slope triangle, and ask students if they see a connection between the side lengths and the coordinates of the point.
- **Not being able to write an equation in Problem 4.** Ask students how they found the slope, and then ask, "What must be true of any two points on this line when dividing the values of $y$ and $x$?"

## 3 Connect

**Display** the Activity 2 graph.

**Have students share** their responses to Problems 1–3. Then discuss Problem 4 gradually moving to more abstract thinking:

- Explaining division of the value of $y$ by the value of $x$.
- Describing a ratio of $y : x$ being equal to $5 : 3$ because they represent corresponding side lengths of similar triangles.
- The equation $\frac{y}{x} = \frac{5}{3}$ or $y = \frac{5}{3}x$.

**Ask:**

- "How does dividing $\frac{y}{x}$ determine the slope?"
- "Why will any point on the line with coordinates $(x, y)$ satisfy the equation $\frac{y}{x} = \frac{5}{3}$?"
- "How can you use the equation to find the number of Kiran's heartbeats in 23 seconds?"

**Highlight** that the slope is the same as the constant of proportionality (or unit rate) for proportional relationships.

## Differentiated Support

### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can build upon their work from Activity 1 by using the heart rate from Activity 1 in this activity.

## Math Language Development

### *MLR1: Stronger and Clearer Each Time*

Use this routine to support students in their written explanations for Problems 2 and 3. Provide students time to decide whether each of the two points lie on the line. Have them write 1–2 sentences explaining their thinking. Have students share their explanations with 2–3 partners to receive feedback. After receiving feedback, give students time to improve their response.

### English Learners

Highlight 1–2 ideas that would make a good explanation or justification. This will help support students in building metalinguistic awareness as they make sense of their written work.

Independent | 10 min

# Activity 3 Scale Factor

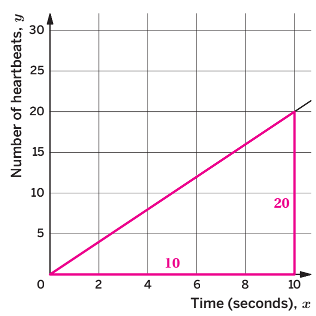
Students study two graphs with different scales to better understand the definition of slope.

Name: ______ Date: ______ Period: ______

**Activity 3** Scale Factor

**Kiran studied some data from similar swimmers with whom he might compete. Help him determine the slope and equation of each line.**

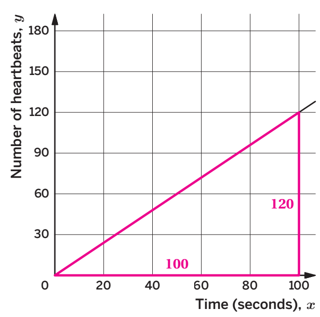
**1.** Jada's heart rate data




a. Slope: $\frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{20}{10} = 2$

b. Equation: $y = 2x$

**2.** Tyler's heart rate data




a. Slope: $\frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{120}{100} = \frac{6}{5}$

b. Equation: $y = \frac{6}{5}x$

STOP





## 1 Launch

Set a time expectation for students to work independently on the activity.

## 2 Monitor

**Help students get started** by asking what they notice about the scales on each axis.

**Look for points of confusion:**

- **Counting vertical and horizontal grid units without referencing the scales.** Prompt students to identify the horizontal distance and vertical distance of their triangles by looking at the scale.
- **Thinking the slopes are the same because the lines look the same.** Help ensure students can find the numeric value of each slope, and ask them why the slopes appear the same, even though they can see their values are different.

**Look for productive strategies:**

- Labeling slope triangles with horizontal and vertical distances, based on the scale of the axes.

## 3 Connect

**Display** student work showing slope triangles correctly labeled with horizontal and vertical distances.

**Have students share** how they found the slope and equation of each line.

**Ask:**

- "These lines appear to have the same steepness, but do they have the same slope?"
- "Which swimmer has a faster heart rate? Explain your thinking."
- "Why might you have different scales on each axis?"

**Highlight** that students must keep the scale on each axis in mind when finding the slope. The slope is the vertical change divided by the horizontal change, using the distances given by the scales on each axis.

## Differentiated Support

**Accessibility: *Guide Processing and Visualization***

Have students annotate the horizontal and vertical axis in each graph to help them pay attention to how the scales on the axes vary. Display the slope formula for students to reference in this activity and the general form of the equation for the line of a proportional relationship, $y = kx$, where $k$ represents the constant of proportionality. Consider displaying the following statement: For a proportional relationship, the slope has the same value as the constant of proportionality.

**Extension: *Math Enrichment***

Ask students to sketch the graph of Jada's heart rate data from Problem 1 on the same coordinate plane as Tyler's heart rate data in Problem 2. Have them explain how they determined how to graph the line representing Jada's heart rate. Then ask them to compare the two graphs. Sample response: I plotted the point $(10, 20)$ representing Jada's heart rate data on the graph in Problem 2. Jada's line lies slightly above Tyler's line, which means she had a greater heart rate.

Whole Class | 5 min

# Summary

Review and synthesize the relationship between points that lie on the line of a proportional relationship and the slope of the line.

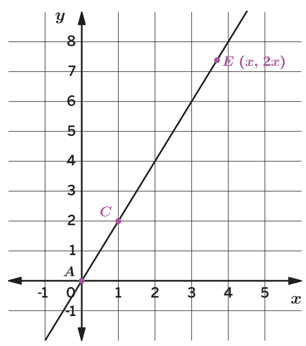
## Summary

**In today's lesson . . .**

You found your resting heart rate. You collected your heart rate data in a table, and then represented it on the coordinate plane. The relationship between time and heartbeats was proportional and could be represented by a graph with the equation $y = kx$ where $k$ is the constant of proportionality. For proportional relationships, the slope of the line that represents the relationship has the same value as the constant of proportionality. This value is also the unit rate.

Consider the line shown.

- The slope of the line shown is 2. For point $C$, the ratio of the vertical distance, 2, to the horizontal distance, 1, is equal to 2 : 1, or 2.
- The constant of proportionality is 2 and is represented in the equation $y = 2x$. The equation tells you the $y$-values are always twice the $x$-values.
- The unit rate is 2 because the point $(1, 2)$ lies on the graph of the line.




**Reflect:**





## Synthesize

**Display** the Summary from the Student Edition.

**Have students share** one point that is on the line and one point that is not on the line and ask how they know whether each point is on the line.

**Highlight** that students can use the graph or the equation to determine that a point is on the line.

**Ask:**

- "How do you know that the equation $y = 2x$ represents a proportional relationship?" Sample response: The equation is in the form $y = kx$, where $k$ is the constant of proportionality. In this case, $k = 2$.
- "What does the value 2 represent in the equation?" Sample response: $k$ is the constant of proportionality

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does the slope of a line tell you about the line?".

Independent | 5 min

# Exit Ticket

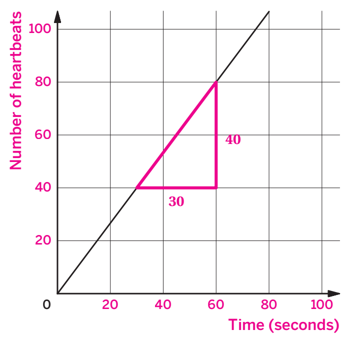
Students demonstrate their understanding by finding the slope of a line that represents a proportional relationship and writing an equation to represent the line.

Printable

Name: ____________ Date: ________ Period: ________

## Exit Ticket 3.02

Elena drew the following line to represent her heart rate of 80 beats per minute.




1. Create a scale that could be used to represent this scenario. Label each axis.
   **Sample scale and axes labels shown.**

2. What is the slope of Elena's line? Explain your thinking.
   **$\frac{4}{3}$; Sample response: When I draw a slope triangle, I can find the ratio of the vertical distance to the horizontal distance, which is $\frac{40}{30}$, or $\frac{4}{3}$.**

3. Write an equation to represent Elena's line. Show or explain your thinking.
   **Sample response: $\frac{y}{x} = \frac{4}{3}$ or $y = \left(\frac{4}{3}\right)x$, when $y$ represents the number of heartbeats and $x$ represents the time in seconds.**

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can decide whether a point lies on the line of a proportional relationship by finding the ratio of the vertical distance to the horizontal distance and comparing it to the slope of the line.
1 2 3

b. I can write the equation of a line that represents a proportional relationship by understanding that, for proportional relationships, the slope is the same as the constant of proportionality.
1 2 3



Lesson 2 Proportional Relationships

## Success looks like . . .

- **Goal:** Creating an equation relating the quotient of the vertical and horizontal side lengths of a slope triangle to the slope of a line.
- **Goal:** Comprehending that for the equation of a proportional relationship given by $y = kx$, $k$ represents the unit rate.
- **Language Goal:** Justifying whether a point is on the line of a proportional relationship by determining whether the ratio of the vertical distance to the horizontal distance (from the origin to the point) equals the slope of the line. **(Speaking and Listening)**

## Suggested next steps

**If students are unable to correctly scale or label their graph, consider:**

- Reviewing the graph from Activity 2.

**If students are unable to find the slope of the line, consider:**

- Reviewing how to use slope triangles to find the slope from Activity 1.
- Assigning Practice Problem 1.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- Knowing where students need to be by the end of this unit, how did relating slope to the constant of proportionality (or unit rate) influence that future goal?
- Which groups of students did and did not have their ideas seen and heard today?

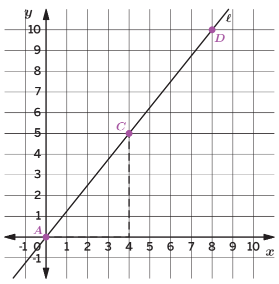
# Practice

Independent

Name: Date: Period:

Practice

1. Line $\ell$ is shown in the coordinate plane.



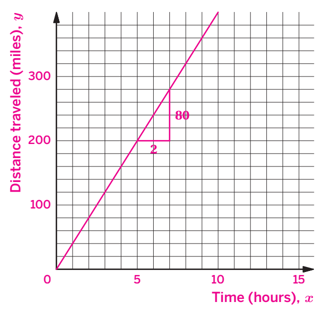
a. What are the coordinates of points $C$ and $D$?
Point $C(4, 5)$
Point $D(8, 10)$

b. Is the point $(20, 24)$ on line $\ell$? Explain your thinking.
No, $\frac{24}{20}$ does not equal $\frac{5}{4}$.

c. Is the point $(16, 20)$ on line $\ell$? Explain your thinking.
Yes, $\frac{20}{16} = \frac{5}{4}$.

d. Write an equation that would allow you to test whether any point $(x, y)$ is on line $\ell$.
Sample response: $\frac{y}{x} = \frac{5}{4}$

2. From rest, a car travels at a constant rate. After 2 hours, the car traveled 80 miles.




a. Graph the line showing the relationship between the car's distance and time. Be sure to create a scale and label the axes.

b. What is the slope of the line, and what does it represent in context?
The slope is $\frac{80}{2} = 40$, which means the car travels 40 miles for every 1 hour (or 40 mph).

c. What is the equation of the line?
$y = 40x$





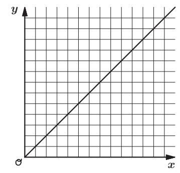
Name: Date: Period:

Practice

3. Which of the following *does not* represent a proportional relationship?

A. From rest, Clare walks at a constant speed of 3 mph.

B.



C.

| $x$ | $y$ |
|---|---|
| 2 | 4 |
| 5 | 10 |
| 6 | 12 |

D.

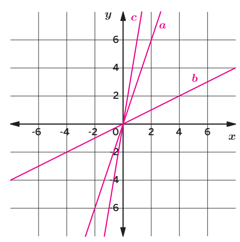
| $x$ | $y$ |
|---|---|
| 3 | 5 |
| 4 | 6 |
| 5 | 7 |

4. Draw and label a line that has each indicated slope.

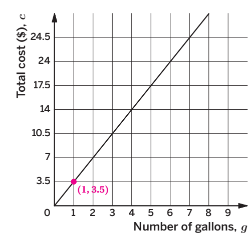
a. Line $a$ has a slope of 3.

b. Line $b$ has a slope of $\frac{1}{2}$.

c. Line $c$ has a slope of 5.




5. The graph shows the cost of gas per gallon at a local gas station. Write an equation that relates the total cost $c$ to the number of gallons of gas $g$ purchased.
$c = 3.5g$








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| | 2 | Activity 2 | 2 |
| Spiral | 3 | Grade 7 | 1 |
| | 4 | Unit 2 Lesson 11 | 1 |
| Formative | 5 | Unit 3 Lesson 3 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Understanding Proportional Relationships

Let's study some graphs of proportional relationships.

## Focus

### Goals

1. **Language Goal:** Create graphs and equations of proportional relationships in context, using an appropriate scale. **(Reading and Writing)**
2. **Language Goal:** Interpret diagrams or graphs of proportional relationships in context. **(Reading and Writing)**

## Coherence

### Today

Students match graphs to animations of movement. Then they analyze the constant of proportionality and the slope of a graph in context. Attending to precision in labeling axes, choosing an appropriate scale, and drawing lines are skills exercised in this lesson.

### Previously

In Lesson 2, students reviewed proportional relationships represented in a table and on a graph, and made connections between the slope of the line and the constant of proportionality (or unit rate) for proportional relationships.

### Coming Soon

In Lesson 4, students will see the importance of labeling the scale in determining the information that can be interpreted from a graph.

## Rigor

- Students build on their **conceptual understanding** of interpreting proportional relationships.

## Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 10 min | 10 min | 15 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

### Materials

- Exit Ticket
- Additional Practice
- rulers

### Math Language Development

**Review words**

- *constant of proportionality*
- *proportional relationship*
- *slope*
- *unit rate*

**Amps** | **Featured Activity**

**Warm-up**

**Animated Traveling Bugs**

Students view an animation of traveling insects to prepare for comparing features of proportional relationships.

Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

As students share their responses with a partner, they may forget to actively listen, and thus might not be able to precisely communicate during discussions. Remind students that by listening well, they can help improve their own understanding and their own level of precision as they communicate their thoughts. Review what it means to actively listen and encourage students to practice active listening habits.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted, but students will need to know which line represents the ant and which line represents the ladybug before proceeding to Activity 1.
- In **Activity 1**, Problem 2 may be omitted.

Independent | 10 min

# Warm-up Traveling Bugs

Students view an animation of two traveling insects, and use the information to label corresponding lines on a coordinate plane.

**Amps Featured Activity** Animated Traveling Bugs

Name: ____________ Date: ________ Period: ________

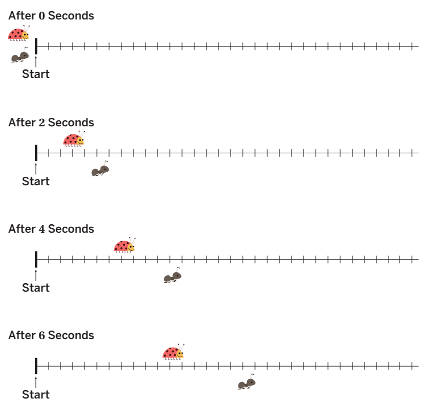
Unit 3 | Lesson 3

## Understanding Proportional Relationships

Let's study some graphs of proportional relationships.

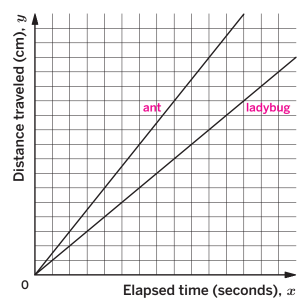
### Warm-up Traveling Bugs

**Iowa State University's Department of Entomology hosts a traveling Insect Olympics with events that include Roach Races, Cockroach Pulls, and the Jumping Stick Jump. Lin decided to create her own race to see which insect travels faster, a ladybug or an ant. The diagrams with tick marks show the positions of the ladybug and the ant at different times. Each tick mark represents 1 cm.**




You will watch an animation that illustrates the relationship between distance and time for both insects. This relationship is represented by the following graph. Which line represents which insect? Label each line. Explain your thinking.

**Sample response: The ant travels faster than the ladybug, so the top line must represent the ant and the bottom line must represent the ladybug.**




Log in to Amplify Math to complete this lesson online.





### 1 Launch

Display the online animation *Traveling Bugs*. Provide students with rulers for the duration of the lesson.

### 2 Monitor

**Help students get started** by asking, "How can a coordinate plane help you track how distance changes over time?"

**Look for points of confusion:**

- **Misidentifying which line represents which insect.** Ask, "How long does it take the ladybug to travel 12 cm? The ant?" Ladybug: 6 seconds; ant: 4 seconds. Ask students what effect this would have if they were to place these points on the graph. The ladybug's point would be farther to the right, because time is on the $x$-axis.

**Look for productive strategies:**

- Counting along the minor grid lines to mark the position of each insect. Students will be provided with a scale for the axes in Activity 1.

### 3 Connect

**Have students share** their strategies for identifying which line represents each insect.

**Highlight** that even without knowing the scale, they can recognize which line represents a faster speed because a greater distance is traveled in the same amount of time.

**Ask**, "Can you use this graph to find specific information about how far each insect travels in a certain amount of time? Why or why not?" The $x$- and $y$-axes can be scaled so that I know the value of specific points on the line.

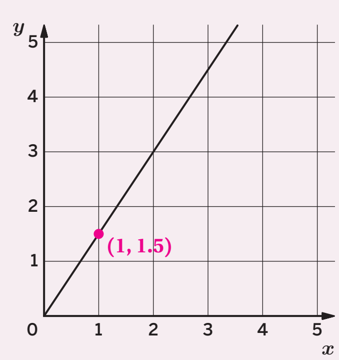
## Power-up

**To power up students' ability to write an equation to represent a proportional relationship from a graph, have students complete:**

Recall that a proportional relationship can be modeled by the equation $y = kx$ where $k$ is the constant of proportionality.

1. Identify the point $(1, k)$ on the graph, where $k$ represents the constant of proportionality.
   $(1, 1.5)$

2. Write the equation of the line in the form $y = kx$.
   $y = 1.5x$



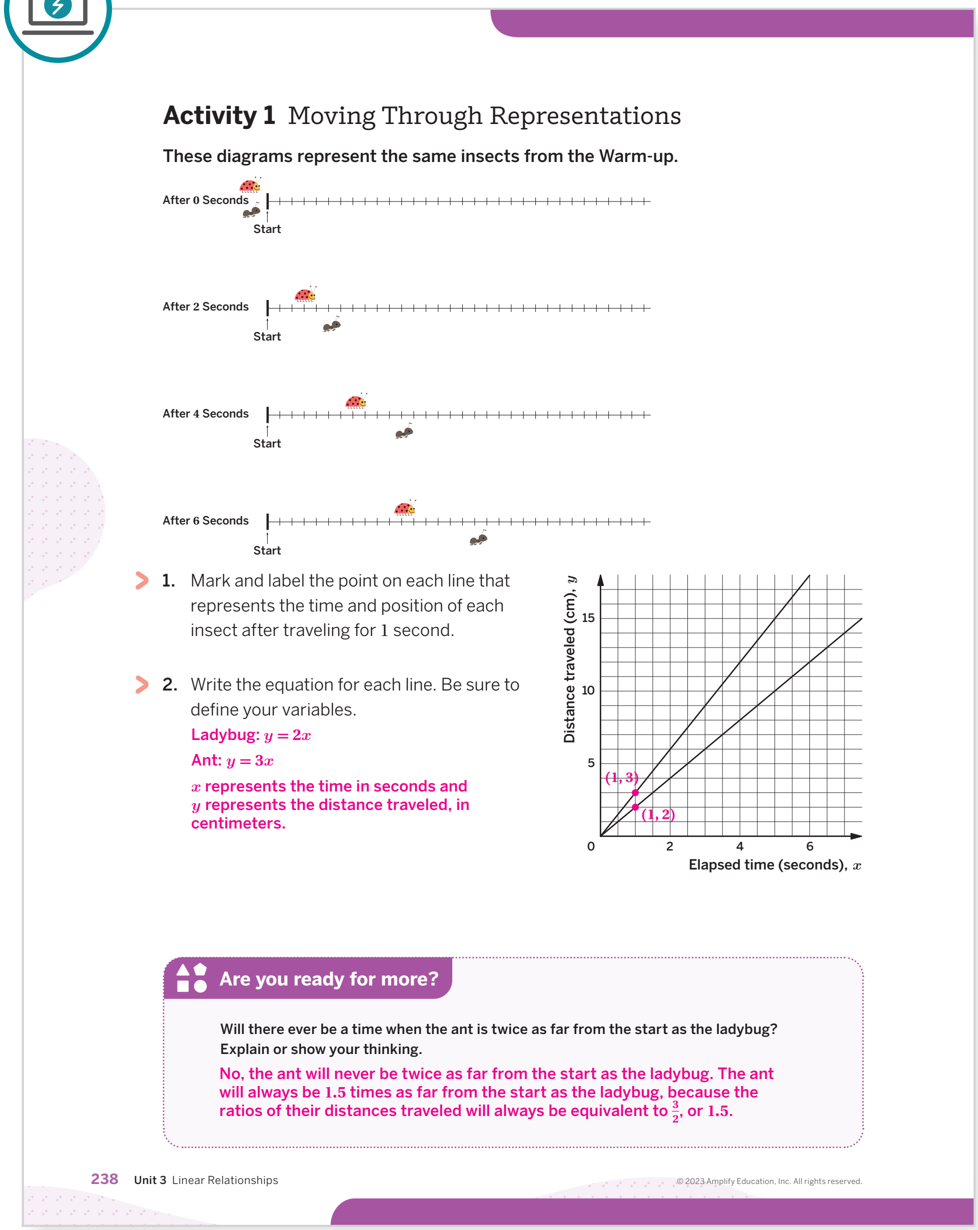

**Use:** Before Activity 1

**Informed by:** Performance on Lesson 2, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 2

Pairs | 10 min

# Activity 1 Moving Through Representations

Students interpret a graph representing the traveling speed of two insects to create equations that represent each insect's movement.



## Activity 1 Moving Through Representations

These diagrams represent the same insects from the Warm-up.



1. Mark and label the point on each line that represents the time and position of each insect after traveling for 1 second.

2. Write the equation for each line. Be sure to define your variables.
   Ladybug: $y = 2x$
   Ant: $y = 3x$
   $x$ represents the time in seconds and $y$ represents the distance traveled, in centimeters.



**Are you ready for more?**

Will there ever be a time when the ant is twice as far from the start as the ladybug? Explain or show your thinking.

No, the ant will never be twice as far from the start as the ladybug. The ant will always be 1.5 times as far from the start as the ladybug, because the ratios of their distances traveled will always be equivalent to $\frac{3}{2}$, or 1.5.

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## 1 Launch

Ask students to label the lines based on the discussion from the Warm-up. Have students work individually to complete the problems, and then have them share their responses with a partner.

## 2 Monitor

**Help students get started** by asking, "What do the points on the line represent in this context?"

**Look for points of confusion:**

- **Plotting the points incorrectly in Problem 1.** Ask students to look carefully at the scale of the axes when plotting points.
- **Defining variables incorrectly in Problem 2.** Ask students to identify the $x$- and $y$-axes on the graph, and read the corresponding labels. Have students use the points from Problem 1 to verify their variables.

**Look for productive strategies:**

- Marking points on the line that correspond to information from the diagram in order to find the scale.

## 3 Connect

**Have pairs of students share** their scales and equations.

**Ask:**

- "What features of the tick-mark diagrams, lines, and equations can you identify that would allow someone to figure out which insect is moving faster?"
- "What do the values 2 and 3 in your equations represent on the graph?"

**Highlight** that one way to find the slope of a line representing a proportional relationship is to find $y$ for which the point $(1, y)$ is on the line.

## Differentiated Support

**Accessibility:** ***Vary Demands to Optimize Challenge, Guide Processing and Visualization***

Provide the graph pre-labeled with the points described in Problem 1. Have students complete Problem 2. Display the slope formula for students to reference and the general form of the equation for the line of a proportional relationship, $y = kx$, where $k$ represents the constant of proportionality.

**Extension:** ***Math Enrichment***

As a follow-up to the *Are you ready for more?* problem, have students complete the following problem:

At 1 second, the ant is 1 cm away from the ladybug. When will the ant be twice as far from the ladybug? Three times as far? The ant will be twice as far from the ladybug, 2 cm, at 2 seconds. The ant will be three times as far from the ladybug, 3 cm, at 3 seconds.

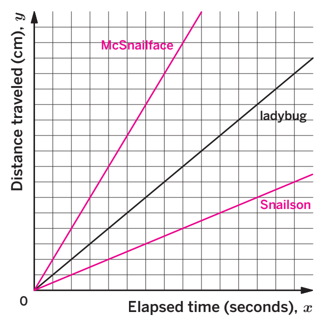
# Activity 2 Twice as Fast, Twice as Slow

Students consider additional animals in the context of traveling speed, and make connections between different representations of proportional relationships.

Name: ______ Date: ______ Period: ______

## Activity 2 Twice as Fast, Twice as Slow

**Tyler provides two snails to race against the ladybug in Lin's race. Refer to the graph of the ladybug.**




1. The first snail, Snaily McSnailface, travels twice as fast as the ladybug. Plot the snail's positions on the coordinate plane, label the line, and write an equation for the line.
   $y = 4x$

2. The second snail, Sally Snailson, travels twice as slow as the ladybug. Plot the snail's positions on the coordinate plane, label the line, and write an equation for the line.
   $y = x$

3. Order the equations from least constant of proportionality to greatest constants of proportionality.

| | | |
|---|---|---|
| $y = x$ | $y = 2x$ | $y = 4x$ |
| Least | | Greatest |

4. Compare the steepness of the lines and the constants of proportionality of their corresponding equations. What do you notice?
   Sample response: As the constants of proportionality (slope) increases, the line becomes steeper.

**Reflect:** How did you motivate yourself to stay focused throughout the activity?

STOP





## 1 Launch

Have students use the ***Think-Pair-Share*** routine for this activity.

## 2 Monitor

**Help students get started** by asking, "What does it mean to move twice as fast? To move twice as slow?"

**Look for points of confusion:**

- **Graphing lines that do not pass through the origin.** Ask, "How far has each snail traveled in one second? In five seconds? In zero seconds?"
- **Struggling to compare $y = x$ with the other equations in Problem 3.** Remind students that $y = x$ can be rewritten as $y = 1x$.

**Look for productive strategies:**

- Noticing that both new graphs represent proportional relationships.
- Noticing that the equation $y = x$ looks different than expected because the scales of the axes are not the same.

## 3 Connect

**Have students share** their strategies for graphing the lines and writing equations for Problems 1 and 2. Lead with students who use scales and exact values, and follow with those who use proportional reasoning for distances.

**Ask:**

- "If you add another snail that is faster than Snaily McSnailface, where should its line be? What can you tell about the constant of proportionality in its equation?"
- "What is the meaning of the point $(0, 0)$ in this context?"

**Highlight** that as the constant of proportionality increases, the line becomes steeper because there is a greater vertical change for an equal amount of horizontal change.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Provide a pre-completed and pre-labeled graph showing the lines representing Snaily McSnailface and Sally Snailson. Ask students to determine how each snail's rate compares to the ladybug. Then have them write the equations in Problems 1 and 2, before proceeding with the rest of the activity.

### Extension: *Math Enrichment*

Have students complete the following problem:

How many times faster is Snaily McSnailface than Sally Snailson? Explain your thinking. Snaily McSnailface is traveling 4 times faster than Sally Snailson because the constant of proportionality is 4 times greater for Snaily McSnailface.

Whole Class | 5 min

# Summary

Review and synthesize how graphs of proportional relationships can be interpreted within context, and how the scale of a graph can affect the interpretation.

## Summary

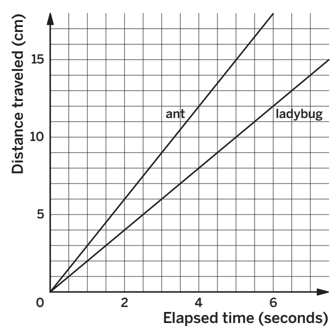
**In today's lesson . . .**

You made sense of the proportional relationship between distance and time using graphs.

When creating graphs to represent proportional relationships in context, it is important to label the axes and the scale. Without these, it is difficult to interpret the graphs in a meaningful way.

Consider the graph shown.

- Even without the scale, you can determine that the top line has a greater slope because it is steeper.
- With the scale, you can determine precisely how much faster one insect travels compared to the other insect. Then you can use these values to answer questions about the distance and length of time that each insect traveled.




**Reflect:**

240 Unit 3 Linear Relationships


## Synthesize

**Display** the Summary from the Student Edition.

**Ask:**

- "What would the graph of an insect traveling 3 times faster than the ant look like?" It would be a straight line passing through the points $(0, 0)$, $(1, 9)$, and $(2, 18)$.
- "What equation would represent the traveling speed of this new insect? What is the slope and what does it represent in this context?" $y = 9x$; the slope, $9$, means that the new insect travels $9$ cm per second.

**Highlight** that when two proportional relationships are represented on a graph, students can draw conclusions about the constant of proportionalities (unit rates) by examining the steepness of the lines.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies did you find helpful when determining the equations for the lines?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by graphing a proportional relationship, writing an equation to represent the relationship, and interpreting the unit rate.

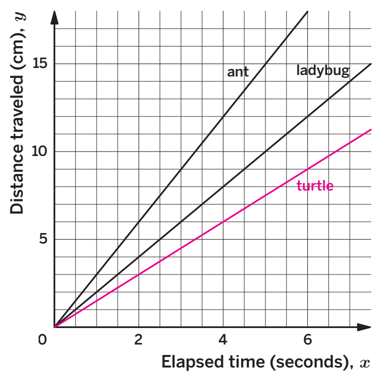
Printable

Name: ______________ Date: ________ Period: ________

## Exit Ticket 3.03

The graph represents the positions of two insects in a race.

1. On the same coordinate plane, draw a line that represents a turtle who travels half as fast as the ant. Label the line, and write an equation of the line.
   **Sample response: $y = \frac{3}{2}x$**
   **$x$ represents the time in seconds and $y$ represents the distance traveled, in centimeters.**




2. Explain how your line illustrates that the turtle travels half as fast as the ant.
   **Sample response: The turtle's unit rate is half the ant's unit rate, which means it takes the turtle twice as long to travel the same distance as the ant. For example, the ant travels 6 cm in 2 seconds, while the turtle travels 6 cm in 4 seconds.**

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can graph a proportional relationship to represent a real-world scenario.
1 2 3

b. I can write the equation of a proportional relationship by finding the unit rate.
1 2 3

c. I can use the slope to compare the rates of travel for different animals.
1 2 3



**Lesson 3** Understanding Proportional Relationships

## Success looks like . . .

- **Language Goal:** Creating graphs and equations of proportional relationships in context, using an appropriate scale. **(Reading and Writing)**
- **Language Goal:** Interpreting diagrams or graphs of proportional relationships in context. **(Reading and Writing)**

## Suggested next steps

**If students do not correctly draw a line or identify the equation in Problem 1, consider:**

- Reviewing strategies from Activity 2.
- Assigning Practice Problem 1.

**If students have trouble with their explanation in Problem 2, consider:**

- Asking them to demonstrate what it means to "move half as fast."

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- Knowing where students need to be by the end of this unit, how did Problems 1 and 2 from Activity 2 influence that future goal?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

# Practice

Independent

Name: ________ Date: ________ Period: ________

Practice

1. A mobile gaming service offers 6 games for $14.40.

   a. If the total cost is proportional to the number of games purchased, write an equation that represents the relationship between total cost $y$ and the number of games $x$. Show or explain your thinking.

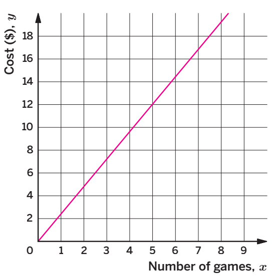
   **$y = 2.4x$; Sample response: The unit rate is $\frac{14.4}{6} = 2.4$. This is the constant of proportionality because the relationship is proportional.**

   b. Graph this relationship on the coordinate plane.

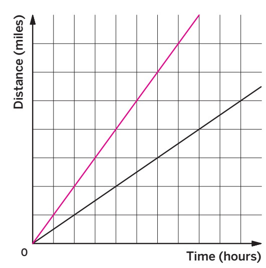
   c. How many games can be purchased for $42? Show or explain your thinking.

   **17 games; Sample response: Substitute 42 for $y$ in the equation $y = 2.4x$. Then solve for $x$.**

   $$42 = 2.4x$$
   $$42 \div 2.4 = 2.4x \div 2.4$$
   $$17.5 = x$$




2. Priya jogs at a constant speed. The relationship between her distance and time is shown on the graph. Diego rides his bicycle at a constant speed that is twice as fast as Priya. Graph the relationship between Diego's distance and time on the same coordinate plane.





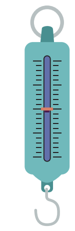



Name: ________ Date: ________ Period: ________

Practice

3. The table shows a proportional relationship between the weight on a spring scale and the distance the spring has stretched. Some of the values are missing.

| Distance (cm) | Weight (newtons) |
|---|---|
| 20 | 28 |
| 55 | **77** |
| **100** | 140 |
| 1 | **$\frac{7}{5}$** |



   a. Complete the table.

   b. Describe the scales you could use on the $x$- and $y$-axes of a coordinate plane that would show all the distances and weights in the table.

   **Sample response: From 0 to 100 on the horizontal axis (distance) and from 0 to 140 on the vertical axis (weight).**

4. Solve each equation. Show your thinking.

   a. $2x + 60 = 100$

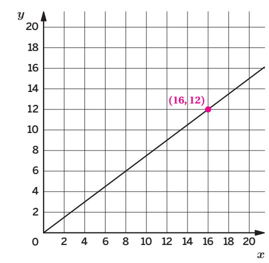
   $$2x + 60 - 60 = 100 - 60$$
   $$2x = 40$$
   $$2x \div 2 = 40 \div 2$$
   $$x = 20$$

   b. $45 = 34 + \frac{1}{2}x$

   $$45 - 34 = 34 + \tfrac{1}{2}x - 34$$
   $$11 = \tfrac{1}{2}x$$
   $$11 \div \tfrac{1}{2} = x$$
   $$22 = x$$

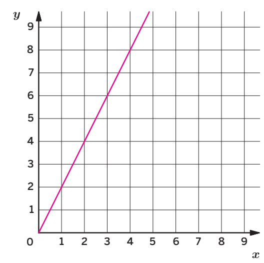
5. Think about what you have learned about graphs and equations of proportional relationships.

   a. Write an equation that represents the following graph.




   Equation: $y = \frac{3}{4}x$

   b. Graph the relationship that is represented by the equation $y = 2x$.








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 2 |
| Spiral | 3 | Grade 7 | 2 |
| | 4 | Grade 7 | 1 |
| Formative | 5 | Unit 3 Lesson 4 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 3 | **LESSON 4**

# Graphs of Proportional Relationships

Let's think about scale.

## Focus

### Goals

1. Compare graphs that represent the same proportional relationship using differently-scaled axes.
2. **Language Goal:** Create graphs representing the same proportional relationship using differently-scaled axes. **(Reading and Writing)**

## Coherence

### Today

Students see that there are many successful ways to set up and scale axes in order to graph a proportional relationship. Students examine and compare proportional relationships with and without scaled axes, and sort graphs based on the proportional relationship they represent.

### Previously

In Lesson 3, students represented a real-world context in a graph, and wrote equations from graphs. They examined how the constant of proportionality affects the steepness of a graphed line.

### Coming Soon

In Lessons 5 and 6, students will compare different representations of proportional relationships.

## Rigor

- Students strengthen their **fluency** in identifying the constant of proportionality and writing equations for proportional relationships.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Independent | Independent | Small Groups | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

## Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 2 PDF, pre-cut cards
- rulers

## Math Language Development

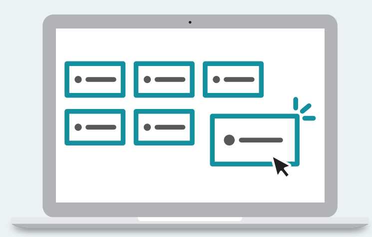
**Review words**

- *constant of proportionality*
- *proportional relationship*
- *unit rate*

**Amps** | Featured Activity

### Activity 2
### Digital Card Sort

Students match equivalent proportional relationships graphed on differently-scaled axes by dragging and connecting them on screen.



Amps POWERED BY desmos

## Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might become anxious about sharing how their thinking changed because it might be different than someone else's response. Encourage students to celebrate differences. They should all consider the growth others show during the activity and express appreciation for their efforts.

## Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, you may reduce the number of cards distributed to each group.

Independent | 5 min

## **Warm-up** Would You Rather?

Students examine two graphs with limited information and try to predict who will win a race.

Name: ______ Date: ______ Period: ______

**Unit 3 | Lesson 4**

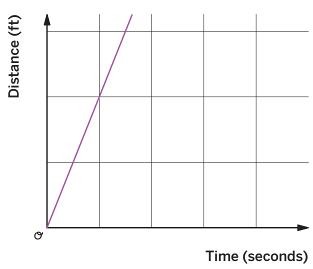
# Graphs of Proportional Relationships

Let's think about scale.

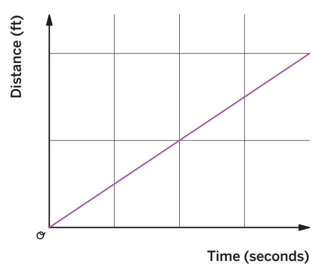
### **Warm-up** Would You Rather?

**These graphs show the relationship between distance and time for two competitors running a race. Each runner maintains a constant speed.**

**Diego**




**Priya**




If you had to predict a winner based on these graphs, would you choose Diego or Priya? Explain your thinking.

**Answers may vary. Ideally, students will say there is not enough information because a scale is not provided for either graph. It is acceptable at this point if students choose Diego or Priya as long as they can justify their choice.**

Log in to Amplify Math to complete this lesson online.



**Lesson 4** Graphs of Proportional Relationships 243

### 1 Launch

Conduct the ***Would You Rather?*** routine as students make their predictions. Provide students with rulers for the duration of the lesson.

### 2 Monitor

**Help students get started** by asking what evidence they could use to support their claim.

**Look for productive strategies:**

- Recognizing that an accurate prediction cannot be made without knowing the scale of the axes.

### 3 Connect

**Have individual students share** their predictions with the class. Ask for evidence from students who think Diego will win, students who think Priya will win, and students who are undecided.

**Ask**, "What information would help you make your prediction?" Sample responses:

- It would help to know how fast Diego and Priya are running.
- It would help to know the scales for each graph.

**Highlight** that without knowing the scale of the axes, it is not possible to determine the speed of either runner.

## Math Language Development

*MLR5: Co-craft Questions*

Before revealing the question in the Warm-up, display the introductory text and the two graphs. Ask students to work with a partner to write 1-2 mathematical questions they have about the graph and context. Ask pairs of students to share their questions with the class.

### English Learners

Display a sample question, such as "Is Diego really running at a faster rate?" or "What are the scales on the axes?"

## Power-up

**To power up students' ability to graph a proportional relationship from an equation:**

Provide students with a copy of the Power-up PDF.

**Use:** Before Activity 1

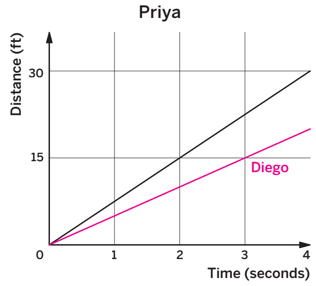
**Informed by:** Performance on Lesson 3, Practice Problem 5

# Activity 1 Calculating the Rate

Students re-examine the graphs from the Warm-up, now with additional information , to determine whether their prediction still holds true.

### Activity 1 Calculating the Rate

**The graphs from the Warm-up are shown, now with additional information.**


1. Calculate Diego's speed in feet per second, and use it to write an equation for the number of feet $y$ traveled in $x$ seconds. Show or explain your thinking.

   **5 ft per second; $y = 5x$; Sample response: Using the point $(2, 10)$, I can get a speed of 5 ft per second. This is the constant of proportionality (slope) for the proportional relationship.**

2. Calculate Priya's speed in feet per second, and use it to write an equation for the number of feet $y$ traveled in $x$ seconds. Show or explain your thinking.

   **7.5 ft per second; $y = 7.5x$; Sample response: Using the point $(2, 15)$, I can get a speed of 7.5 ft per second. This is the constant of proportionality (slope) for the proportional relationship.**

3. Does this new information change your thinking about who will win the race?

   **Sample response: Yes, now I know that Diego is not as fast as Priya, so if their speeds remain constant, Priya will win the race.**

**Pause here while your class shares responses.**

4. Choose either Diego or Priya. Graph one runner's line on the other runner's graph, and compare the steepness of the lines. What do you notice?

   **Sample response: When the lines for both runners are on the same graph, Priya's line is steeper than Diego's line.**

**Are you ready for more?**

Han and Clare start out $1,000$ ft apart and travel toward each other. Han is traveling at $20$ ft per second, and Clare is traveling at $10$ ft per second. How long will it take them to meet?

**It will take them just over 33 seconds to meet.**

244 Unit 3 Linear Relationships



## 1 Launch

Activate prior knowledge by asking, "How can you determine the unit rate from the graph of a proportional relationship?"

## 2 Monitor

**Help students get started** by asking, "If the race distance is $60$ m, how long would it take each runner to travel this distance?"

**Look for points of confusion:**

- **Not knowing how to calculate speed in Problems 1 and 2.** Remind students that they can choose any point on the line and divide the $y$-coordinate by the $x$-coordinate to find the unit rate.
- **Relying on the appearance of the lines instead of the information about the scale of the axes in Problem 3.** Select a point that each line passes through (e.g., $(2, 10)$ for Diego and $(2, 15)$ for Priya), and ask students what each point means in context.

**Look for productive strategies:**

- Recognizing that the unit rate corresponds to the value of $y$ of the point $(1, y)$ on each graph.

## 3 Connect

**Have individual students share** their equations from Problems 1 and 2, and how their thinking changed from the Warm-up.

**Ask:**

- "Does the winner of the race depend on the length of the race?"
- "What do you notice about the scales of the axes?"

**Display** the animation of Diego and Priya running the race. Ask, "If Priya is running faster than Diego, why does the slope of the line representing her speed appear less steep?"

**Highlight** that the steepness of the line is determined by the scale of the axes. Have students complete Problem 4 so they can make a direct comparison of Diego's and Priya's speed.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Consider demonstrating how to calculate Diego's rate and how to use the rate to write an equation for the line. Use a think-aloud approach. Then have students calculate Priya's rate and write an equation for the line for Problem 2. Display the general form of the equation for the line of a proportional relationship, $y = kx$, where $k$ represents the constant of proportionality.

## Math Language Development

### *MLR8: Discussion Supports*

During the Connect, focus students' attention on the difference between *steepness* and *rate of change*, given the different scales on the axes. Ask, "What do you notice about the steepness of the graph and the scale of the graph?"

### English Learners

When referencing the steepness of the graph use gestures to model what is meant by the term *steep*. When discussing the *scales* of the graph, annotate the graph to highlight where the scales are found and what they represent.

Small Groups | 15 min

# **Activity 2** Card Sort: Proportional Relationships

Students sort cards based on the proportional relationship they represent, and then write an equation representing each relationship to build fluency.

**Amps Featured Activity** Digital Card Sort

Name: ______ Date: ______ Period: ______

## **Activity 2** Card Sort: Proportional Relationships

**You will be given 12 cards. Each card represents one of five possible proportional relationships.**

1. Sort the cards into groups based on what proportional relationship they represent. Record your groupings in the table.
   **Sample response shown in table. Students may determine different group numbers for each set of cards.**
2. Write an equation for each group that can represent each card in the group. Record the equation in the table.
   **Sample responses shown.**

| Group | Card(s) in this group . . . | Equation that can represent each card in the group . . . |
|---|---|---|
| 1 | C, D, G, K | $y = \frac{7}{2}x$ |
| 2 | B, E, H | $y = 3x$ |
| 3 | F, J | $y = \frac{5}{2}x$ |
| 4 | I, L | $y = \frac{4}{3}x$ |
| 5 | A | $y = \frac{1}{4}x$ |

STOP





## 1 Launch

Arrange students in groups of four and distribute one set of cards per group from the Activity 2 PDF.

## 2 Monitor

**Help students get started** by asking, "How can you determine whether two different graphs represent the same proportional relationship?" By finding the constant of proportionality (unit rate).

**Look for points of confusion:**

- **Grouping cards by the steepness of the graphs.** Ask students whether a point from the line of one card would lie on the line of another card.
- **Writing the reciprocal of the unit rate.** Remind students that one way to think about the unit rate is "the change in $y$ for every unit change in $x$."

**Look for productive strategies:**

- Identifying a point and verifying that the line from each card in a group passes through that point.

## 3 Connect

**Have groups of students share** their strategies for grouping the cards and the matching equations.

**Highlight** that the scale of the axes a graph is drawn on can be misleading about the actual relationship between the two variables if you just look at the steepness of the line, without paying attention to the numbers on the axes.

**Ask**, "Does the graph from Card A look like what you would expect for the equation $y = \frac{1}{4}x$?" Have students identify another card where the equation does not match their initial expectation from examining the graph, and select a few students to share the graph they chose and explain their thinking.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Provide students with a subset of the cards to sort, such as Cards A, B, E, F, H, and J.

### Extension: *Math Enrichment*

Have students create additional graphs for the following equations:

$y = \frac{1}{4}x$ $\quad y = \frac{5}{2}x$ $\quad y = \frac{4}{3}x$

## Math Language Development

### *MLR8: Discussion Supports*

During the Connect, to support students in producing statements about proportional relationships, provide sentence frames for them to use when they describe the reasoning for their matches. For example:

- "Card _____ and Card _____ match/don't match because _____."
- "Card _____ matches with Card _____ because they have the same slope."

Encourage the use of relevant vocabulary, such as *slope* and *unit rate*.

### English Learners

As each match is shared, annotate the graphs with their slope and/or corresponding equation.

Whole Class | 5 min

# Summary

Review and synthesize how the scale of the axes can influence the appearance of a graph.

## Summary

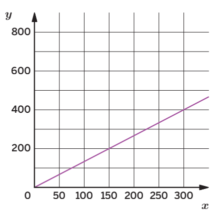
**In today's lesson . . .**

You explored how the scale of the axes can influence the appearance of a graph.

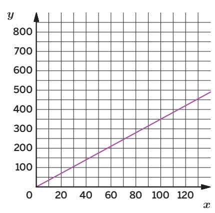
If you want to compare two proportional relationships, using graphs with different scales can be misleading about which proportional relationship has a greater constant of proportionality (or slope).

For example, consider these three graphs.

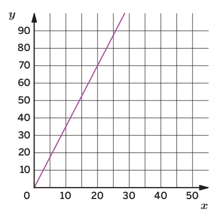
**Graph A**




**Graph B**




**Graph C**




Without looking carefully, you might conclude that the slopes of the lines of Graphs A and B are much closer to one another than the slope of the line of Graph C. However, by finding the unit rate using a point from each line, you can determine that the slope of the lines of Graphs B and C are actually equivalent, and less than the slope of the line of Graph A.

Reflect:




## Synthesize

**Have students share** something that surprised them from today's lesson.

**Ask**, "When do you think it might be reasonable or important to use different scales?" Sample response: Sometimes, we choose specific ranges for the axes in order to see specific information, and those choices can have an impact on how information appears in a graph.

**Highlight** that the $x$- and $y$-axes do not need to share the same scale, and can change depending on what information students want to highlight in the graph.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did manipulating the scale of the axes affect the graphs of proportional relationships?"

Independent | 5 min

# Exit Ticket

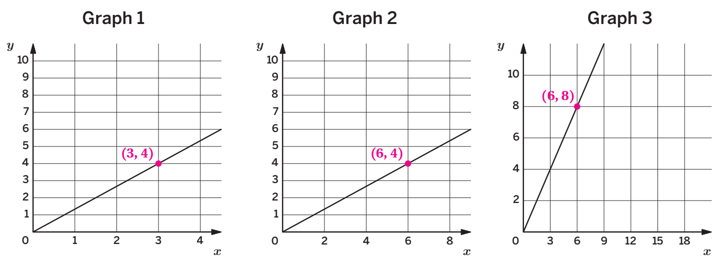
Students demonstrate their understanding by determining whether three lines graphed using differently-scaled axes represent the same proportional relationship.

Printable

Name: ______________________ Date: __________ Period: __________

## Exit Ticket 3.04

Consider the following three graphs.




1. Does Graph 2 represent the same relationship as Graph 1? Explain your thinking.
   No; Sample response: The constant of proportionality (slope) for Graph 1 is $\frac{4}{3}$, and the constant of proportionality (slope) for Graph 2 is $\frac{2}{3}$.

2. Does Graph 3 represent the same relationship as Graph 1? Explain your thinking.
   Yes; Sample response: The constant of proportionality (slope) for Graphs 1 and 3 are both equal to $\frac{4}{3}$.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can write the equation representing a proportional relationship from a graph.
1 2 3

b. I can tell when two graphs represent the same proportional relationship, even if the scales are different.
1 2 3





## Success looks like . . .

- **Goal:** Comparing graphs that represent the same proportional relationship using differently-scaled axes.
- **Language Goal:** Creating graphs representing the same proportional relationship using differently-scaled axes. **(Reading and Writing)**

## Suggested next steps

**If students think Graph 1 and Graph 2 represent the same proportional relationship, consider:**

- Asking, "What coordinate points can you confidently identify from the graphs? How does that help you find the constant of proportionality?"
- Reviewing strategies for finding the constant of proportionality for a line from Activity 2.

**If students think Graph 1 and Graph 3 represent different proportional relationships, consider:**

- Assigning Practice Problem 2.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- What worked and did not work today? Which routines enabled all students to think mathematically in today's lesson?
- During the discussion about the card sort from Activity 2, how did you encourage each student to share their understandings? What might you change the next time you teach this lesson?

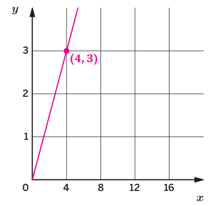
# Practice

Independent

Name: Date: Period:

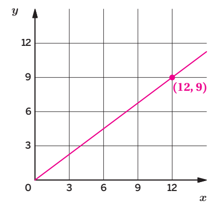
Practice

1. Two coordinate planes with different scales are shown. Graph the equation $y = 0.75x$ on each coordinate plane. Explain your thinking.

a 


**Explanation:**

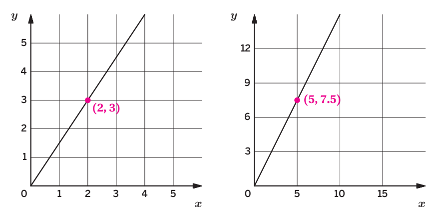
Sample response: Because $0.75 = \frac{3}{4}$, the line should pass through the point $(4, 3)$. The line also passes through the origin. Draw a line connecting $(0, 0)$ and $(4, 3)$.

b 


**Explanation:**

Sample response: Because $0.75 = \frac{3}{4} = \frac{9}{12}$, the line should pass through the point $(12, 9)$. The line also passes through the origin. Draw a line connecting $(0, 0)$ and $(12, 9)$.

2. A water tank is filled at a constant rate. The two graphs shown represent the same proportional relationship between the volume of water $y$ and the amount of time $x$ that has passed, in minutes.




a Write an equation that represents the relationship between volume $y$ and time $x$.

$y = \frac{3}{2}x$

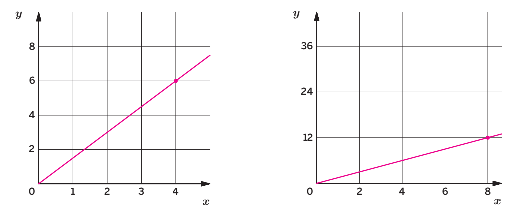




Name: Date: Period:

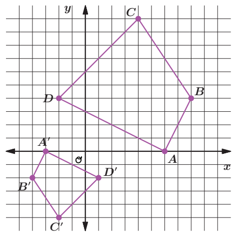
Practice

b Draw the missing lines on the following graphs to show the same proportional relationship as part a.



3. Describe a sequence of rotations, reflections, translations, and/or dilations that show that Quadrilateral $ABCD$ is similar to Quadrilateral $A'B'C'D'$. Be specific, stating the amount and direction of a translation, a line of reflection, the center and angle of a rotation, and the center and scale factor of a dilation.

Sample response: Dilate Quadrilateral $ABCD$ using the origin as the center of dilation with a scale factor of $\frac{1}{2}$. Then rotate the image 180° using the origin as the center of rotation.




4. Given the graph of a line, describe how you can tell whether the line's slope is greater than 1, equal to 1, or less than 1.

Sample response:
- When a line's slope is equal to 1, the $y$-values increase by 1 every time the $x$-values increase by 1.
- When a line's slope is greater than 1, the $y$-values increase by more than 1 every time the $x$-values increase by 1.
- When a line's slope is less than 1, the $y$-values increase by less than 1 every time the $x$-values increase by 1.

5. Write an equation that represents a proportional relationship and whose line passes through the point $(25, 15)$. Show or explain your thinking.

The constant of proportionality is $\frac{15}{25} = \frac{3}{5}$. So, the equation is $y = \frac{3}{5}x$.





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 2 |
| Spiral | 3 | Unit 2 Lesson 5 | 2 |
| | 4 | Unit 3 Lesson 2 | 2 |
| Formative | 5 | Unit 3 Lesson 5 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

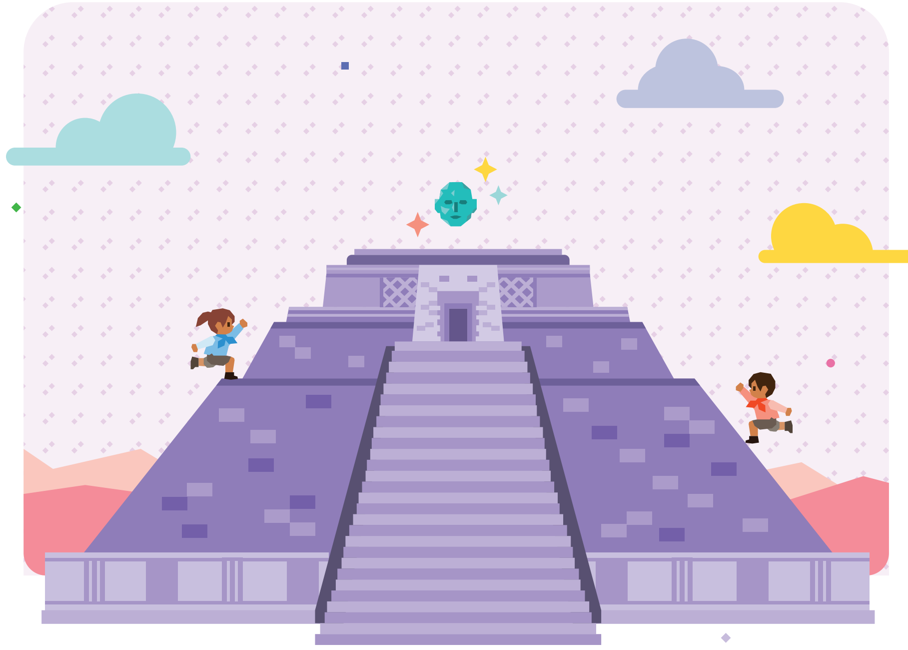
## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 3 | **LESSON 5**

# Representing Proportional Relationships

Let's look at representations of proportional relationships.



## Focus

### Goals

1. Create an equation and a graph with appropriate scale and axes labels to represent proportional relationships.
2. **Language Goal:** Determine what information is needed to create graphs that represent proportional relationships. Ask questions to elicit that information. **(Speaking and Listening)**

## Coherence

### Today

Students create multiple representations of proportional relationships. For each representation, they identify key features, such as the constant of proportionality and relate how they know that each representation describes the same situation.

### Previously

In Lessons 3 and 4, students labeled the scale of each axis, and examined the effect of the scale on the appearance of the graphed line.

### Coming Soon

In Lesson 6, students will create visual displays for different representations of pairs of proportional relationships.

## Rigor

- Students create and interpret multiple representations of proportional relationships to build **procedural skills**.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 10 min | 12 min | 15 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

## Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards
- Activity 2 PDF (answers)
- *Info Gap Routine* PDF (for display)
- Anchor Chart PDF, *Representations of Proportional Relationships*
- calculators
- graph paper
- rulers

## Math Language Development

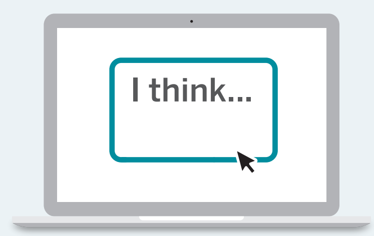
**Review words**

- *constant of proportionality*
- *proportional*
- *unit rate*

**Amps** | Featured Activity

### Activity 1
### See Student Thinking

Students are asked to create multiple representations for a proportional relationship, and these representations are available to you digitally, in real time.




Amps POWERED BY desmos

## Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might think that one good argument is enough. Remind students that they can learn from each other. They should listen to others' arguments, as they build their sets of evidence. They also can help each other by looking for errors in the thinking so that a correction can be made. This requires engagement by all students.

## Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problems 1 and 2 may be omitted.
- In **Activity 2**, have students only complete the first set of cards.

Pairs | 10 min

## Warm-up Defining Variables

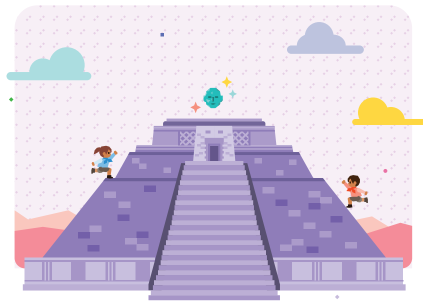
Students examine three situations, define variables, and calculate unit rates to prepare for representing proportional relationships in Activity 1.

Name: ______ Date: ______ Period: ______

**Unit 3 | Lesson 5**

# Representing Proportional Relationships

Let's look at representations of proportional relationships.



### Warm-up Defining Variables

**Consider each of the following situations. For each situation, define variables to represent the quantities needed to calculate the indicated unit rate.**

1. A rice pilaf recipe calls for 3 cups of water for every 2 cups of rice. Calculate the unit rate to describe how the amount of ingredients should change in order to prepare any amount of pilaf. **Sample responses:**
   - **If $x$ represents the number of cups of rice and $y$ represents the number of cups of water, then the unit rate is $3 : 2 = 1.5$, or 1.5 cups of water per cup of rice.**
   - **If $x$ represents the number of cups of water and $y$ represents the number of cups of rice, then the unit rate is $2 : 3 = \frac{2}{3}$, or $\frac{2}{3}$ cups of rice per cup of water.**
2. A tank is filled with water at a constant rate. After 20 minutes, there are 35 liters of water in the tank. Calculate the unit rate to describe how the amount of water should change after any amount of time. **Sample responses:**
   - **If $x$ represents the time in minutes and $y$ represents the number of liters of water; Then the unit rate is $35 : 20 = 1.75$, or 1.75 liters of water per minute.**
   - **If $x$ represents the number of liters of water and $y$ represents the time in minutes, then the unit rate is $20 : 35 = \frac{7}{4}$, or $\frac{7}{4}$ minutes per liter of water. Note: While this representation may not be a typical response, it is plausible.**
3. While walking a dog, Shawn and the dog are both walking at a constant rate. When Shawn has walked 120 steps, the dog has walked 480 steps. Calculate the unit rate to describe how the amount of steps should change. **Sample responses:**
   - **If $x$ represents Shawn's steps and $y$ represents the dog's steps, then the unit rate is $480 : 120 = 4 : 1 = 4$, or 4 of the dog's steps for every one of Shawn's steps.**
   - **If $x$ represents the dog's steps and $y$ represents Shawn's steps, then the unit rate is $120 : 480 = 1 : 4 = \frac{1}{4}$, or $\frac{1}{4}$ of Shawn's steps per every one of the dog's steps.**

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Conduct the ***Think-Pair-Share*** routine.

### 2 Monitor

**Help students get started** by modeling how to complete Problem 1.

**Look for points of confusion:**

- **Thinking there are no variables in a given situation.** Have students identify known values in the situation, and whether these can change in relation to each other.
- **Interpreting the unit rate as distance over time in Problem 3.** Ask, "Where do you see these units in the situation?"

**Look for productive strategies:**

- Circling or underlining the variables in each situation.
- Finding two possible unit rates for Problems 1 and 3.

### 3 Connect

**Have pairs of students share** their variables and unit rates for each situation. In particular, look for students who have determined different unit rates for the same problem.

**Highlight** that the variables should be defined by the two quantities in the problem that change in relation to each other, and that the unit rate shows specifically how much one quantity changes when the other increases by one unit. Show that in Problem 1, 1.5 represents the cups of water needed for 1 cup of rice, and $\frac{2}{3}$ represents the cups of rice needed for 1 cup of water.

**Ask**, "Which situation posed the biggest challenge for defining the variables?"

## Math Language Development

*MLR7: Compare and Connect*

During the Connect, identify pairs of students who determined different ways of writing the unit rates for the same problem. In Problem 1, some students might write the unit rate as "per cup of rice" while others might write the unit rate as "per cup of water." Ask students to compare how these ways of writing the unit rate are different and encourage them to share their strategies for how they defined the variables and used those to determine the unit rate.

### English Learners

Use color coding to annotate how the variables are defined and the language used, such as "per cup of rice."

## Power-up

**To power up students' ability to writing equations for a proportional relationship from a point, have students complete:**

Recall that the constant of proportionality $k$ can be calculated using the relationship $k = \frac{y}{x}$.

For each point, determine the constant of proportionality then write the equation of the proportional relationship that passes through it.

1. $(8, 12)$
   $k = \frac{3}{2}$;
   $y = \frac{3}{2}x$
2. $(9, 3)$
   $k = \frac{1}{3}$;
   $y = \frac{1}{3}x$

**Use:** Before Activity 1

**Informed by:** Performance on Lesson 4, Practice Problem 5

Pairs | 12 min

# Activity 1 Representations of Proportional Relationships

Students create multiple representations for a proportional relationship to see how the constant of proportionality can be identified in each representation.

Amps Featured Activity | See Student Thinking

### Activity 1 Representations of Proportional Relationships

**Jada and Noah are practicing for the 100-meter dash. While each runs at a constant rate, they noticed they each take a different number of steps to travel the same distance. When Noah takes 10 steps, Jada takes 8 steps. When Noah takes 15 steps, Jada takes 12 steps. Solve these problems to describe the relationship between the number of steps Jada takes and the number of steps Noah takes.**

1. Define your variables. Sample response shown.

   Let $x$ represent the number of steps Jada takes.

   Let $y$ represent the number of steps Noah takes.

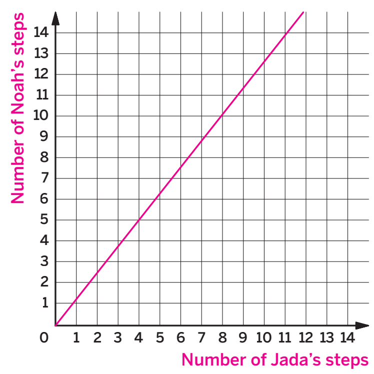
2. Create a table, a graph, and an equation to represent this situation.

   Table:

| Jada | Noah |
|---|---|
| 4 | 5 |
| 8 | 10 |
| 12 | 15 |
| 20 | 25 |

   Equation: $y = \frac{5}{4}x$

   Graph:




3. Find the constant of proportionality in each representation. Explain your thinking.

   Table: The constant of proportionality, $\frac{5}{4}$, is the number of Noah's steps divided by the corresponding number of Jada's steps.

   Graph: The constant of proportionality, $\frac{5}{4}$, is the value of $y$ when $x = 1$ on the graph.

   Equation: The constant of proportionality, $\frac{5}{4}$, is the coefficient of $x$ in the equation.

4. What does the constant of proportionality mean in this context?

   Sample response: The constant of proportionality in this context represents the number of steps Noah takes each time Jada takes one step.

250 Unit 3 Linear Relationships



## 1 Launch

Discuss Problem 1 as a class, and then arrange students in pairs to complete Problems 2–4. **Note:** Provide students with rulers for the duration of the lesson.

## 2 Monitor

**Help students get started** by asking, "Can you predict how many steps Noah will take if you know how many Jada takes?"

**Look for points of confusion:**

- **Reversing the constant of proportionality when creating the table or graph.** Ask students whether their table or graph is supported by the variables they defined in Problem 1.

## 3 Connect

**Have pairs of students share** their tables, graphs, and equations. Compare students who chose $x$ to represent Jada's steps, to students who chose $x$ to represent Noah's steps, and show how their choice affects the constant of proportionality.

**Highlight** how the constant of proportionality can be found in each representation, and that this has the same value as the unit rate, if the relationship is proportional.

**Ask:**

- "Which representation was more challenging to identify or calculate the constant of proportionality? Why?"
- "Is the point $(0, 0)$ on the line for this relationship? What does the point $(0, 0)$ represent in this context?"
- "How can you tell that the equation, description, graph, and table all represent the same situation?" Sample response: In each representation, we can always find how many steps Noah took, if we know how many steps Jada took.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Instruct different pairs of students to define the variables differently. For example, tell one pair of students to let $x$ represent the number of steps Jada takes. Tell a different pair of students to let $x$ represent the number of steps Noah takes. After completing the activity, have these pairs of students compare their tables, graphs, and equations.

## Math Language Development

### *MLR1: Stronger and Clearer Each Time*

After students complete Problem 3, provide them time to meet with 2–3 partners to share their responses and give and receive feedback. Encourage reviewers to ask clarifying questions such as:

- "How did you identify the constant of proportionality?"
- "Did the scales for the axes cause any confusion?"

Have students write a final draft response, based on the feedback they received.

### English Learners

Allow pairs of students who speak the same primary language to provide feedback to each other.

Pairs | 15 min

# Activity 2 Info Gap: Proportional Relationships

Students complete the ***Info Gap*** routine to identify the information that is necessary to create graphs of a proportional relationship.

Name: ______ Date: ______ Period: ______

## Activity 2 Info Gap: Proportional Relationships

**Competitions such as the Olympics or the New York City Marathon attract some of the best athletes in the world. You are about to learn more about two famous achievements of world-class athletes Jesse Owens and Grete Waitz.**


lazyllama/Shutterstock.com

**You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.**

| If you are given a *problem card*: | If you are given a *data card*: |
|---|---|
| **1.** Silently read your card, and think about what information you need to be able to solve the problem. | **1.** Silently read your card. |
| **2.** Ask your partner for the specific information that you need. | **2.** Ask your partner, *"What specific information do you need?"* and wait for them to ask for information. |
| **3.** Explain how you are using the information to solve the problem.<br>Continue to ask questions until you have enough information to solve the problem. | **3.** Before sharing the information, ask, *"Why do you need that information?"*<br>Listen to your partner's reasoning, and ask clarifying questions. |
| **4.** Share the problem card, and solve the problem independently in the space provided on this page. | **4.** Read the problem card, and solve the problem independently in the space provided on this page. |
| **5.** Read the data card and discuss your thinking. | **5.** Share the data card and discuss your thinking. |

**Pause here so your teacher can review your work. You will be given a new set of cards. Repeat the activity, trading roles with your partner.**

STOP





### 1 Launch

Distribute calculators, graph paper, and the cards from the Activity 2 PDF. Display the *Info Gap Routine* PDF and model the ***Info Gap*** routine with students. Have students read the narratives from the Problem Cards together before working on the problems.

### 2 Monitor

**Help students get started** by explaining that they may need several rounds of discussion to determine the information they need.

**Look for points of confusion:**

- **Not knowing what information to ask for to create the graph.** Ask, "What information is necessary to graph a proportional relationship? What information do you have? What information do you need?"
- **Not knowing how to scale the axes before answering the question from the Problem Card.** Encourage students to answer the question on their card first, and then think about how to scale their graph.

**Look for productive strategies:**

- Asking increasingly more precise questions until they get the information they need.

### 3 Connect

**Have pairs of students share** their graphs and responses to the Problem Cards.

**Ask:**

- "Other than the answer, what information would have been nice to have?"
- "How did you decide to label the two axes?"
- "How did you decide to scale the axes?"
- "Where can you see the constant of proportionality on the graphs you created?"

**Highlight** that the slope of the line will change, depending on which value you place on which axis.

## Differentiated Support

**Accessibility: *Guide Processing and Visualization***

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. Consider using the following during the think-aloud.

- "I need to sketch a graph that shows the distance and time that Jesse Owens ran when he set the world record. I will ask for the distance he ran first. Then I will ask for the time it took him to run this distance."

## Math Language Development

***MLR4: Information Gap***

Display prompts for students who benefit from a starting point, such as:

- "Can you tell me . . . (specific piece of information)?"
- "Why do you need to know . . . (that piece of information)?"

**English Learners**

Consider providing sample questions students could ask for Problem Card 1, such as the following:

- What distance did Jesse Ownes run when set the world record?
- How long did it take Jesse Owens to run this distance?

Whole Class | 5 min

# Summary

Review and synthesize how proportional relationships can be represented in multiple ways, and how the constant of proportionality can be determined from each representation.

## Summary

**In today's lesson . . .**

You explored how proportional relationships can be represented in multiple ways.

Proportional relationships can be represented with written descriptions, equations, graphs, and tables. Which representation you choose depends on the purpose. The constant of proportionality can be determined in each representation. Remember the constant of proportionality has the same value as the slope of the line and the relationship's unit rate.

**Written Description**

A bakery recipe calls for 27 g of honey for every 6 g of flour.

**Equation**

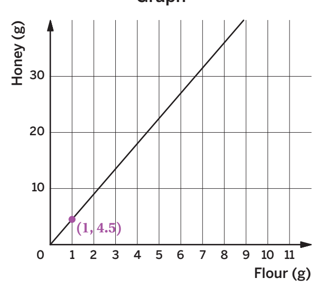
$y = 4.5x$, where $y$ represents the number of grams of honey and $x$ represents the number of grams of flour.

**Constant of Proportionality 4.5**

**Table**

| Flour (g), $x$ | Honey (g), $y$ |
|---|---|
| 1 | 4.5 |
| 6 | 27 |
| 10 | 45 |

**Graph**




> Reflect:




## Synthesize

**Display** the Summary from the Student Edition.

**Highlight** that each representation of proportional relationships calls attention to different features of the proportional relationship.

**Have students share** how they would find the constant of proportionality in each representation, and then display the Anchor Chart PDF, *Representations of Proportional Relationships*.

**Ask:**

- "The proportional relationship $y = 5.5x$ includes the point $(18, 99)$ on its graph. How could you choose a scale for a pair of axes with a $10$ by $10$ grid to show this point?" Have each grid line represent $10$ or $20$ units.
- "What are some things you learned about graphing today that will help you in the future?" Answers may vary.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Which representation of proportional relationships do you find the most challenging to create or interpret?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by writing an equation and sketching a graph of a proportional relationship, and then using either representation to solve a problem.

Printable

Name: ______________ Date: ________ Period: ________

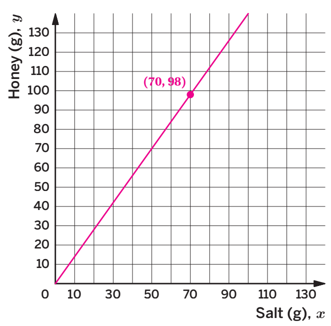
## Exit Ticket 3.05

The table shows the ratios of salt, honey, and flour in a baking recipe.

| Salt (g) | Honey (g) | Flour (g) |
|---|---|---|
| 10 | 14 | 4 |
| 25 | 35 | 10 |

1. Write an equation that represents the relationship between $x$ grams of salt and $y$ grams of honey. Show or explain your thinking.
   $y = \frac{7}{5}x$; The constant of proportionality is $\frac{14}{10} = \frac{7}{5}$.

2. Graph the relationship on the coordinate plane.
   Sample response shown.




3. How much honey is needed for 70 g of salt? Explain or show your thinking.
   98 g of honey are needed for 70 g of salt; Sample response: Substitute 70 for $x$ in the equation $y = \frac{7}{5}x$; $\frac{7}{5} \cdot 70 = 98$

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can create and interpret multiple representations of proportional relationships. 1 2 3

b. I can choose appropriate scales and labels for a coordinate plane in order to graph a proportional relationship. 1 2 3



Lesson 5 Representing Proportional Relationships

### Success looks like . . .

- **Goal:** Creating an equation and a graph with appropriate scale and axes labels to represent proportional relationships.
- **Language Goal:** Determining what information is needed to create graphs that represent proportional relationships. Asking questions to elicit that information. **(Speaking and Listening)**

### Suggested next steps

**If students use the values for flour to write their equation, consider:**

- Reviewing Activity 2.

**If students have trouble creating an appropriate scale for their grid, consider:**

- Reviewing strategies for determining the scale from Activities 1 and 2.
- Assigning Practice Problem 2.
- Asking, "Which ordered pairs should you expect to see on the graph?"

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- "What worked and did not work today? Which students' ideas were you able to highlight during Activity 2?"
- "What did students find frustrating about the *Info Gap* routine? What helped them work through this frustration? What might you change the next time you teach this lesson?"

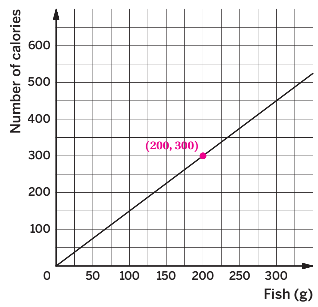
# Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. The graph shown represents the proportional relationship between grams of fish and number of calories.




a. Write an equation that represents this relationship. Let $x$ represent the number of grams of fish, and $y$ represent the number of calories. Show or explain your thinking.

$y = \frac{3}{2}x$ (or equivalent);

Sample response: The constant of proportionality is $\frac{300}{200} = \frac{3}{2}$.

b. Use your equation to complete the table.

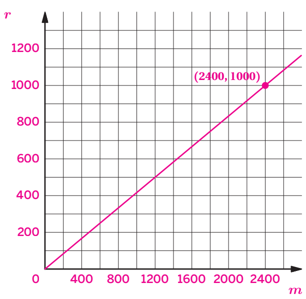
| Fish (g) | Number of calories |
|---|---|
| 1,000 | 1,500 |
| 1,334 | 2,001 |
| 1 | $\frac{3}{2}$ (or equivalent) |

2. Students at a middle school are selling raffle tickets for a school fundraiser. They collect \$24 for every 10 raffle tickets they sell.

a. Let $m$ represent the amount of money the students collect for selling $r$ raffle tickets. Write an equation that represents the relationship between $m$ and $r$.

$m = \frac{12}{5}r$ or $r = \frac{5}{12}m$

b. Graph this relationship on the coordinate plane. Place $r$ on the vertical axis and $m$ on the horizontal axis. Label the axes and provide an appropriate scale. Make sure the scale is large enough to see how much money the students would raise if they sell 1,000 tickets.








Name: ______ Date: ______ Period: ______

Practice

3. Solve each equation. Show your thinking.

a. $\frac{x+5}{7} = \frac{3}{4}$

$\frac{x+5}{7} \cdot 7 = \frac{3}{4} \cdot 7$

$x + 5 = \frac{21}{4}$

$x + 5 - 5 = \frac{21}{4} - 5$

$x = \frac{21}{4} - \frac{20}{4}$

$x = \frac{1}{4}$

b. $\frac{5}{2} = \frac{24-a}{8}$

$\frac{5}{2} \cdot 8 = \frac{24-a}{8} \cdot 8$

$20 = 24 - a$

$20 - 24 = 24 - a - 24$

$-4 = -a$

$4 = a$

4. Write an equation for each of the following proportional relationships. Be sure to define your variables. Show or explain your thinking.

a. Diego records his number of steps in the following table.

$y = 80x$, where $x$ represents the number of minutes and $y$ represents the total number of steps taken. The constant of proportionality is $\frac{2000}{25} = 80$.

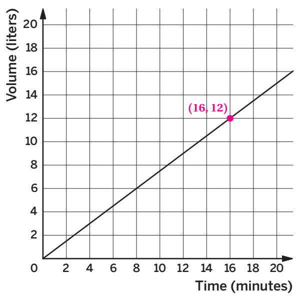
| Number of steps | Time (minutes) |
|---|---|
| 2,000 | 25 |
| 3,200 | 40 |

b. A recipe calls for 3 cups of bulgur for every 2 cups of water.

$y = 1.5x$, where $x$ represents the cups of bulgur and $y$ represents the cups of water. The constant of proportionality is $\frac{3}{2} = 1.5$.

c. Water is filling a container at a constant rate, as shown in the graph.

$y = \frac{3}{4}x$, where $x$ is the number of minutes and $y$ is the volume. The constant of proportionality is $\frac{12}{16} = \frac{3}{4}$.








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 2 |
| Spiral | 3 | Grade 7 | 2 |
| Formative | 4 | Unit 3 Lesson 6 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Comparing Proportional Relationships

Let's compare proportional relationships.


## Focus

### Goals

1. Compare the constant of proportionality for two proportional relationships, given multiple representations.
2. **Language Goal:** Interpret multiple representations of a proportional relationship in order to solve problems, and explain the solution method. **(Writing, Speaking and Listening)**
3. **Language Goal:** Compare two different proportional relationships using words and other representations. **(Speaking and Listening)**

## Coherence

### Today

Students expand on the work of the previous lesson by comparing two situations that are represented in different ways: a written description, a table of values, a graph, or an equation. Students move flexibly between representations and consider how to find the information they need from each type. They respond to context-related questions that compare the two situations and solve problems with the information they have extracted from each representation. Then they organize this information on a visual display and complete a ***Gallery Tour*** to view their classmates' work.

### Previously

In Lesson 5, students learned how to represent a proportional relationship in different ways, and how to find the constant of proportionality in each representation.

### Coming Soon

Starting in Lesson 7, students will begin to apply their knowledge of proportional relationships to explore linear relationships.

## Rigor

- Students **apply** their knowledge of proportional relationships to compare different models of electronic racing toys.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Summary | Exit Ticket |
|---|---|---|---|
| 10 min | 25 min | 5 min | 5 min |
| Independent | Small Groups | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

## Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 1 PDF, one per group
- Activity 1 PDF (answers)
- graph paper
- poster paper
- markers
- sticky notes
- rulers

## Math Language Development

**Review words**

- *constant of proportionality*
- *proportional relationship*
- *slope*
- *unit rate*

**Amps** | Featured Activity

**Exit Ticket**

**Real-Time Exit Ticket**

Check in real time if your students can compare proportional relationships using a digital Exit Ticket.

Amps POWERED BY desmos

## Building Math Identity and Community

Connecting to Mathematical Practices

Students may not know where to begin Activity 1 and may consequently feel disengaged. Encourage them to begin by thinking about how they can use mathematics to model which prototype is the fastest. Ask them to make a list of the types of representations they have used and decide how they will define the variables. Tell them that starting a list of items to consider is one way to help alleviate feeling overwhelmed.

## Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, the ***Gallery Tour*** may be replaced by a representative from each group presenting their poster.

Independent | 10 min

# **Warm-up** Number Talk

Students use mental math to find the values of several multiplication expressions in order to strengthen their number sense and fluency.

Name: ______ Date: ______ Period: ______

**Unit 3 | Lesson 6**

## Comparing Proportional Relationships

Let's compare proportional relationships.


### **Warm-up** Number Talk

**Mentally find the value of each product.**

1. $15 \cdot 2$ $= 30$
2. $15 \cdot 0.2$ $= 3$
3. $15 \cdot 0.5$ $= 7.5$
4. $15 \cdot 0.25$ $= 3.75$
5. $15 \cdot 2.25$ $= 33.75$

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Conduct the ***Number Talk*** routine.

## 2 Monitor

**Help students get started** by asking, "How can you use the value for the first product to find the value of the second product?"

**Look for points of confusion:**

- **Relying on traditional algorithms, even when attempting to solve problems mentally.** Ask students to describe a picture that could represent each product.
- **Not recognizing how knowing the values of $15 \cdot 0.2$ and $15 \cdot 0.05$ can be used to find $15 \cdot 0.25$.** Remind students that $0.25$ can be rewritten as $0.2 + 0.05$.

**Look for productive strategies:**

- Using fraction equivalencies when multiplying decimals.
- Using results from previous computations when evaluating later expressions.

## 3 Connect

**Have individual students share** their strategies for finding each product.

**Ask:**

- "Who can restate _____'s reasoning in a different way?"
- "Did anyone solve the problem the same way but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to _____'s strategy?"
- "Do you agree or disagree? Why?"

## Math Language Development

***MLR8: Discussion Supports***

During the Connect, as students share their strategies for mentally finding each product, display these sentence frames to support them.

- "First, I _____ because . . ."
- "I noticed _____, so I . . ."
- "Because I knew that _____, I was able to . . ."

Consider providing an example, such as "Because I knew that $15 \cdot 2 = 30$, I was able to determine the product of $15 \cdot 0.2$ by moving the decimal point in $30$ one place to the left, which is 3."

**English Learners**

Provide students the opportunity to rehearse what they will say with a partner before they share with the whole class.

## Power-up

**To power up students' ability to write equations of proportional relationships in different contexts:**

Provide students with a copy of the Power-up PDF.

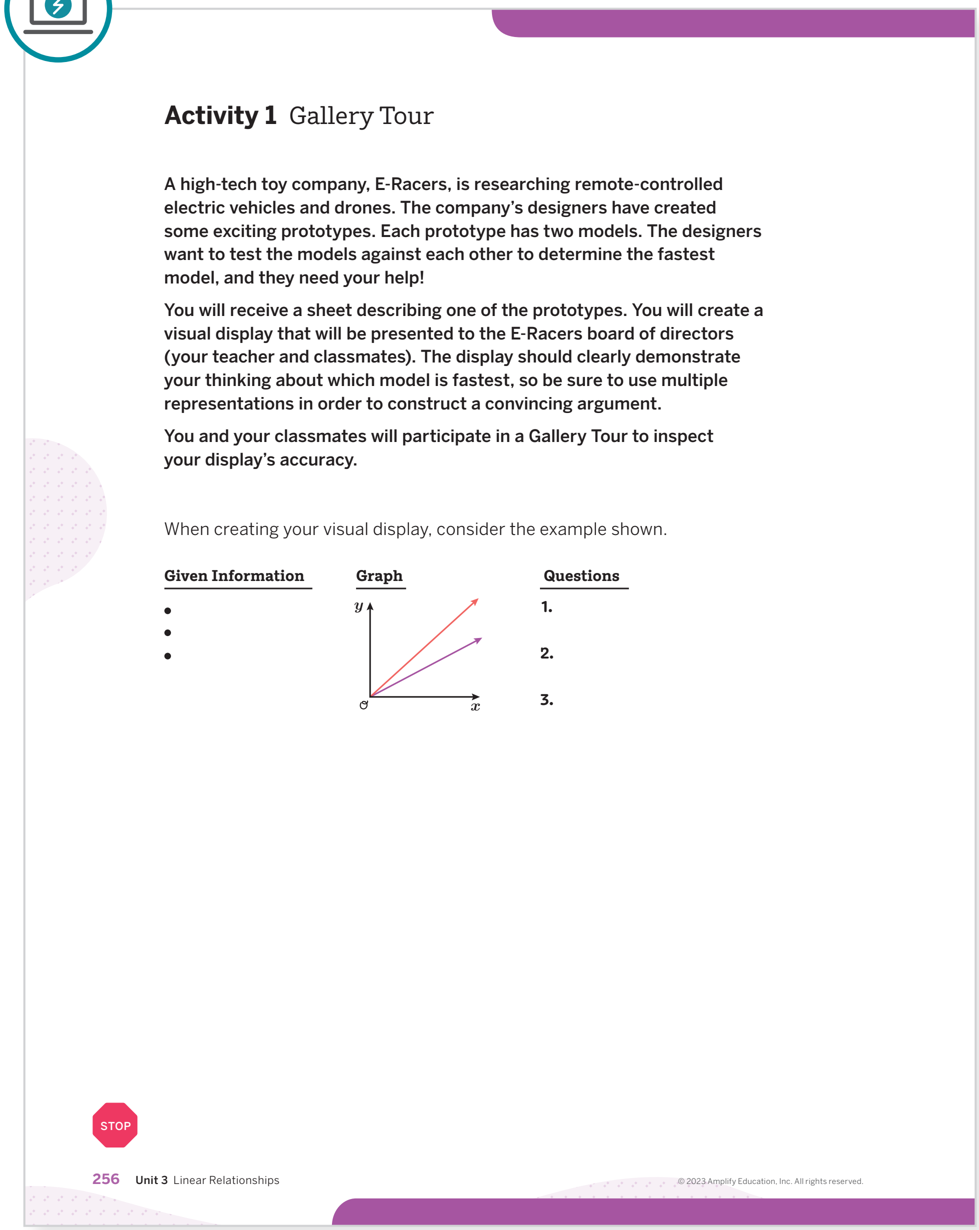
**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 5, Practice Problem 4 and Pre-Unit Readiness Assessment, Problems 2 and 3

Small Groups | 25 min

# Activity 1 Gallery Tour

Students create a visual display to demonstrate their ability to compare two proportional relationships that are represented in different ways.



**Activity 1** Gallery Tour

A high-tech toy company, E-Racers, is researching remote-controlled electric vehicles and drones. The company's designers have created some exciting prototypes. Each prototype has two models. The designers want to test the models against each other to determine the fastest model, and they need your help!

You will receive a sheet describing one of the prototypes. You will create a visual display that will be presented to the E-Racers board of directors (your teacher and classmates). The display should clearly demonstrate your thinking about which model is fastest, so be sure to use multiple representations in order to construct a convincing argument.

You and your classmates will participate in a Gallery Tour to inspect your display's accuracy.

When creating your visual display, consider the example shown.







## 1 Launch

Arrange students in groups of four and distribute the Activity 1 PDF so that each group receives one handout. After students have completed the problems, distribute materials to create their posters.

## 2 Monitor

**Help students get started** by asking, "What information do you need in order to create a visual representation?"

**Look for points of confusion:**

- **Creating visual displays using two different coordinate planes.** Remind students that in Lesson 4, they saw how graphing relationships on two different coordinate planes can lead to misinterpretations when comparing those relationships.
- **Switching the values for $x$ and $y$ when an equation is given in Prototypes 1 and 3.** Ask students to read the scenario again to determine which variable represents each quantity.

**Look for productive strategies:**

- Using a scale that highlights important information from both relationships.

## 3 Connect

Use the ***Gallery Tour*** routine to display student work.

**Have groups of students share** their feedback for the visual displays by using sticky notes.

**Ask**, "How did you decide what scale to use when you made your graph?"

**Highlight** that in order to compare the prototypes, the variables that are defined must be defined the same way for each prototype. For example, if $x$ represents the number of seconds for the Alpha model, $x$ must represent the number of seconds for the Beta model.

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization***

If students need more processing time, have them work only with Prototype 1. Consider providing them with a partially-completed table and graph with the table headers pre-labeled and the axes pre-labeled on the graph. Display the general form of the equation for the line of a proportional relationship, $y = kx$, where $k$ represents the constant of proportionality.

## Math Language Development

***MLR7: Compare and Connect***

During the ***Gallery Tour***, invite students to discuss the question, "What is the same and what is different?" about the representations on the posters. Look for opportunities to highlight representations that helped students complete the problems and decide which scales to use for the graph.

**English Learners**

Consider leaving the visual displays from the ***Gallery Tour*** displayed so that students can refer to them in future discussions.

Whole Class | 5 min

# Summary

Review and synthesize how to compare proportional relationships represented in different ways.

Name: ______ Date: ______ Period: ______

## Summary

**In today's lesson . . .**

You compared proportional relationships using different representations.

When you are given more than one proportional relationship — even if they are represented differently — you can find the constant of proportionality (or unit rate) from each representation and use it to compare the relationships.

For example, let's compare Clare's earnings to Jada's earnings. Clare's earnings are represented by an equation and information about Jada's earnings are shown in the table.

**Clare's earnings**

$y = 14.5x$, where $y$ represents the amount of money she earned for working $x$ hours

**Jada's earnings**

| Time worked (hours) | Earnings ($) |
|---|---|
| 7 | 92.75 |
| 37 | 490.25 |

**Constant of proportionality:** $14.5$ (the coefficient of $x$)

**Constant of proportionality:** $13.25$ (the ratio of earnings to corresponding time worked, $92.75 : 7$, or $13.25$)

Because $14.5 > 13.25$, Clare's earnings per hour are greater than Jada's earnings per hour.

**Reflect:**





## Synthesize

**Have students share** something they liked or want to know more about from another group's visual display from Activity 1.

**Highlight** how different proportional relationships can be compared, even when represented in different ways, by identifying the constant of proportionality for each relationship.

**Ask:**

- "How do visual displays help organize information?" Answers may vary.
- "When comparing proportional relationships, why is it important that the constant of proportionality represent the same relationship between the variables?" Sample response: There are two constants of proportionality for every proportional relationship. In order to compare two different proportional relationships using each constant of proportionality, it must represent the same relationship between the two quantities.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful today when comparing proportional relationships represented in different ways? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by drawing conclusions about two different salt water mixtures, both proportional relationships, yet represented in different ways.

**Amps Featured Activity** Real-Time Exit Ticket

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 3.06

**Some runners may drink a salt water mixture after races to replace their electrolytes. Two proportional recipes for mixtures of salt and water that each taste different are provided here.**

**Mixture A:** Information about Mixture A is shown in the table.

**Mixture B:** Mixture B is represented by the equation $y = 2.5x$, where $x$ represents the number of teaspoons of salt and $y$ represents the number of cups of water.

| Salt (tsp) | Water (cups) |
|---|---|
| 4 | 5 |
| 7 | $8\frac{3}{4}$ |
| 9 | $11\frac{1}{4}$ |

1. If you use 10 cups of water in your recipe, for which mixture would you use more salt? How much more? Show or explain your thinking.

   **Mixture A; 4 more teaspoons; Sample responses:**
   - **Mixture A would use 8 tsp of salt because 5 cups of water would need 4 tsp of salt, so 10 cups of water would need 8 tsp of salt.**
   - **Mixture B would use 4 tsp of salt because I can write the equation $10 = 2.5x$ and solve for $x$.**
   - **Mixture A would use 4 more teaspoons of salt than Mixture B for 10 cups of water.**

2. Which mixture tastes "saltier"? Explain your thinking.

   **Mixture A; Sample response: The constant of proportionality for Mixture A is $\frac{5}{4}$, or 1.25 cups of water per teaspoon of salt, and the constant of proportionality for Mixture B is 2.5 cups of water per teaspoon of salt. Mixture A uses less water per teaspoon, so the salt is more concentrated.**

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can compare proportional relationships represented in different ways: tables, graphs, and equations.

1 2 3





## Success looks like . . .

- **Goal:** Comparing the constant of proportionality for two proportional relationships, given multiple representations.
- **Language Goal:** Interpreting multiple representations of a proportional relationship in order to solve problems, and explaining the solution method. **(Writing, Speaking and Listening)**
- **Language Goal:** Comparing two different proportional relationships using words and other representations. **(Speaking and Listening)**
  - » Comparing Mixtures A and B in Problem 1 to determine which mixture uses more salt.

## Suggested next steps

**If students think Mixture B would use more salt in Problem 1, consider:**

- Asking, "If you made a table for Mixture B using the same quantities of salt from the table for Mixture A, what would that look like?"
- Reviewing strategies for comparing constants of proportionality from Activity 1.
- Assigning Practice Problem 1.

**If students think Mixture A would taste saltier in Problem 2, consider:**

- Asking, "Does something taste saltier when you use more salt, or less salt?"

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- What worked and did not work today? In this lesson, students compared proportional relationships in different representations. How will that support comparing functions?
- What did students find frustrating about creating their visual displays? What helped them work through this frustration? What might you change the next time you teach this lesson?.

## Math Language Development

**Language Goal: Comparing two different proportional relationships using words and other representations.**

Reflect on students' language development toward this goal.

- How did using the *Gallery Tour* routine in Activity 1 help students compare different proportional relationships? Would you change anything the next time you use this routine?
- How have students progressed in using the term *constant of proportionality* to describe proportional relationships that are expressed in different representations?

# Practice

Independent

Practice

Name: Date: Period:

1. A teacher wants to order soil for a school community garden. She collected information from two hauling companies.

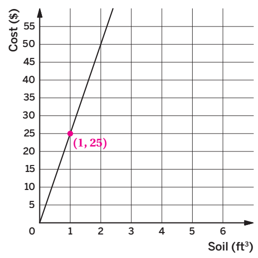
**Green Garden Supplies**

Green Garden Supplies provides their prices in the table shown.

| Soil ($ft^3$) | Cost (\$) |
|---|---|
| 8 | 196 |
| 20 | 490 |
| 26 | 637 |

**Happy Hauling Service**

Happy Hauling Service provides their prices in the graph shown.




a. Calculate the constant of proportionality for each relationship. What do they mean for each company?

**Green Garden Supplies: $\frac{196}{8} = 24.5$, so the constant of proportionality is 24.5.**

**Happy Hauling Service: The point $(1, 25)$ is on the line, so the constant of proportionality is 25.**

**Each constant of proportionality represents the cost in dollars per cubic foot.**

b. The teacher needs $40$ $ft^3$ of dirt delivered and she has a budget of \$1,000. Which company should she hire? Show or explain your thinking.

**Green Garden Supplies: $24.5 \cdot 40 = 980$**

**Happy Hauling Service: $25 \cdot 40 = 1000$**

**She should hire Green Garden Supplies because the cost is less expensive for the same volume of dirt to be delivered.**

2. Andre and Priya track the number of steps they walk, when they walk at a constant speed. Andre walks 6,000 steps in 50 minutes. Priya writes the equation $y = 118x$, where $y$ represents the number of steps and $x$ represents the number of minutes she walks, to describe her rate. For one week, they each walk at their same constant speeds for a total of 5 hours. Who walks more steps? How many more steps? Show or explain your thinking.

**Andre; 600 more steps; Sample response: Andre takes $\frac{6000}{50} = 120$ steps per minute, and Priya takes 118 steps per minute. Five hours is the same as 300 minutes, so Andre would take $120 \cdot 300 = 36{,}000$ steps, and Priya would take $118 \cdot 300 = 35{,}400$ steps. Andre walks 600 more steps than Priya.**

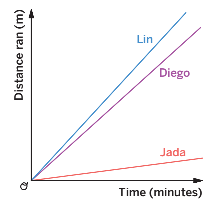



Name: Date: Period:

Practice

3. Lin runs twice as fast as Diego. Diego runs twice as fast as Jada. Could the following graph represent the speeds of Jada, Diego, and Lin? Explain your thinking.

**No; Sample response: In the graph, Lin appears to be running only slightly faster than Diego (not twice as fast), while Diego appears to be running much faster than Jada (much faster than twice as fast).**




4. The formula for converting temperature in degrees Celsius to degrees Fahrenheit is $C = \frac{5}{9}(F - 32)$. Use this formula to complete the table.

| Temperature (°F) | Temperature (°C) |
|---|---|
| 77 | 25 |
| 32 | **0** |
| **−0.4** | −18 |
| −40 | **−40** |

5. Shawn deposits some money into a bank account every week. After one week, the total account balance is \$11. After two weeks, the total account balance is \$21. After three weeks, the total account balance is \$31.

a. How much money does Shawn deposit each week?

**\$10**

b. How much money did Shawn have in the account initially, before the first week that money was deposited?

**\$1**




## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 1 | 2 |
| Spiral | 3 | Unit 3 Lesson 2 | 3 |
| | 4 | Grade 7 | 1 |
| Formative | 5 | Unit 3 Lesson 7 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Sub-Unit 2
# Linear Relationships

Whole Class

Students determine how many cups tall you are as they begin their exploration of nonproportional linear relationships represented as graphs, tables, equations, and written contexts.

SUB-UNIT

2 Linear Relationships

Narrative Connections

## How did a coal mine help build America's most famous amusement park?

Through the mountains of Carbon County, Pennsylvania there once ran a 9-mile stretch of railroad. This stretch was called the Mauch Chunk Switchback Railway. Here, cars filled with coal would roll down from the summit. Following them would be cars of mules. Their job was to drag the empty cars back to the top. Then someone at the coal company had an idea. What if instead of carrying coal, they carried passengers? So while the Mauch Chunk Switchback remained an operating coal rail during the day, in the evening it gave "pleasure rides" to visiting tourists.

Inventor LaMarcus Adna Thompson saw this and became fascinated. He applied the same design to something that required no mules and provided big thrills: a roller coaster!

Named the Switchback Railway, Thompson's invention opened to the public at Coney Island on June 16th, 1884. The novel experience drew huge crowds. Visitors came by the hundreds to climb a 45 ft tower, board a car, and roll along one track to another tower.

Modern roller coasters might be a far cry from the original model, with their death-defying heights and speeds exceeding 100 mph. But despite all the bells and whistles, the basic principles are the same. The speed of a car and the distance it travels depend on the steepness of the roller coaster's slopes. Learning to calculate the slope between two points can make the difference between a sleepy cruise and the ride of a lifetime.





### Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore the rate of change (slope) of nonproportional linear relationships in the following places:

- **Lesson 9, Activity 3:** Matching Equations
- **Lesson 10, Activities 1–2:** Rising Water Levels, Partner Problems
- **Lesson 12, Activity 1:** How Much More?
- **Lesson 13, Activities 1–3:** Noah's Game Card, Payback Plan, Info Gap: Making Designs
- **Lesson 15, Activity 2:** Han's Game Card

# Introducing Linear Relationships

Let's explore some relationships between two variables.


## Focus

### Goals

1. **Language Goal:** Compare and contrast proportional and nonproportional linear relationships. **(Speaking and Listening, Writing)**
2. **Language Goal:** Interpret the slope of the graph of a nonproportional linear relationship. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students use rate of change to explore linear relationships. They determine whether linear relationships are proportional. The meaning of the vertical intercept of the graph comes up briefly, but will be revisited more fully in Lesson 9.

### Previously

Students used tables, graphs, and equations to describe proportional relationships.

### Coming Soon

In Lesson 8, students will continue to learn about linear relationships. They will study what makes linear relationships special and will practice identifying linear and nonlinear relationships presented in tables and in context.

## Rigor

- Students build **conceptual understanding** of linear relationships by studying an example of a nonproportional relationship that has a constant rate of change.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 20 min | 15 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

## Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Slope* (from Unit 2)
- stackable cups (optional)
- rulers

## Math Language Development

**New words**

- ***initial value***
- ***linear relationship***
- ***rate of change***

**Review words**

- *proportional*

**Amps** | **Featured Activity**

### Activity 2
### Using Work From Previous Slides

Students use data from Activity 1 to create a graph that represents a linear relationship.

Amps
POWERED BY desmos

## Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students may not see the pattern as they analyze the table showing the height of the stack of cups and may feel overwhelmed. Consider asking them to write the height of the stack as an expression, such as $9.4 + 1.2$ for the second row, to help them see the repeated reasoning.

## Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 3 may be omitted.
- In **Activity 2**, Problems 2 and 3 may be omitted.

Pairs | 5 min

# Warm-up How Many Cups Tall Is Your Teacher?

Students visualize stacked cups to estimate height, predicting a pattern as the stack grows.

Unit 3 | Lesson 7

## Introducing Linear Relationships

Let's explore some relationships between two variables.


### Warm-up How Many Cups Tall Is Your Teacher?

**This biodegradable foam cup is shown in actual size. About how many more biodegradable foam cups would you have to stack, starting from the ground, to reach the top of your math teacher's head?**

Answers may vary.

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Activate students' background knowledge by asking for examples of when they needed to measure something using unexpected units. Display the image of the cup from the Warm-up and distribute rulers. Provide students with your height, in centimeters, rounded to the nearest ten.

## 2 Monitor

**Help students get started** by asking, "About how tall is the cup?"

**Look for points of confusion:**

- **Dividing your height by 10 to determine the number of cups that represent your height.** Ask them what would happen if you stacked one cup inside of the other, and determine whether the height of the stack would equal two entire cups. Demonstrate if you have cups available.

**Look for productive strategies:**

- Estimating the height of the cup to be 10 cm.
- Recognizing that the stack grows by the height of the cup's lip, not the height of the entire cup.

## 3 Connect

**Display** the image of the cup from the Warm-up.

**Highlight** a response that divides your height by the estimated height of a cup, 10 cm.

**Have students share** why this response underestimates the actual number of cups needed and what problem this response solves instead. This response shows cups being stacked end-over-end, which is not the same as cups being stacked by nesting inside one another.

**Ask** students what structures or strategies they might want to use to find the number of cups. Look for students who suggest using a table or graph as a way to transition to Activity 1.

## Power-up

**To power up students' ability to determine the initial value when given a constant rate of change, have students complete:**

The table shows Mai's bank account after a few weeks of working at her uncle's bakery. She earned the same amount of money each week, and did not deposit or withdraw any additional money from her account.

| Weeks | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Money | 100 | 150 | 200 | 250 |

1. How much money did she earn each week? $50
2. How much money did she have in her account before she began working at the bakery? $100

**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 6, Practice Problem 5

Pairs | 20 min

# Activity 1 Stacking Cups

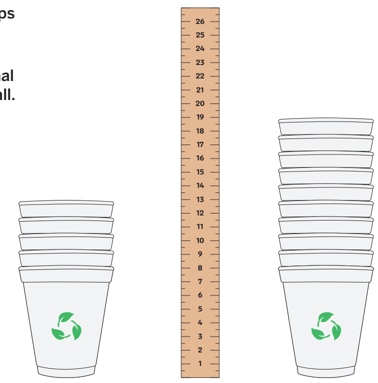
Students measure stacked cups and record their findings in a table, to explore a relationship that is linear, but nonproportional.

Name: ________ Date: ________ Period: ________

**Activity 1** Stacking Cups

**Let's compare two stacks of cups.**

- **One stack has one cup with 4 additional cups nested inside one another and is measured to be 13 cm tall.**
- **The other stack has one cup with 9 additional cups nested and is measured to be 19 cm tall.**



1. Complete the table.

| Number of additional cups | Height of stack (cm) |
|---|---|
| 1 | **9.4** |
| 2 | **10.6** |
| 4 | 13 |
| 9 | 19 |
| 14 | **25** |
| 19 | **31** |
| 29 | **43** |



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## 1 Launch

Tell students that they should consider the height of one whole cup as the starting point and record the height of the stack for each additional cup in the table.

## 2 Monitor

**Help students get started** by asking, "How many additional cups are in the taller stack than the shorter stack? How much height do those 6 cups add?"

**Look for points of confusion:**

- **Not knowing how to find other values in the table.** Ask students what is happening each time 6 cups are added. Then have them find out what happens if 3 cups are added, and ask if they can work backward from 5 cups to complete the table.
- **Thinking 2 cups must be twice as tall as 1 cup.** Ask students what is added each time a cup is added. Have students reflect on whether half the height of 2 additional cups makes sense, given what they know about the height of 1 cup from the Warm-up.
- **Dividing one variable by the other as if they are proportional.** Have students see that doing this does not produce a constant of proportionality between the two variables. Allow them to recognize that this must not be a proportional relationship and must be approached with a different strategy.
- **Not knowing how to determine the number of cups that represent your height.** Ask, "How would you find the height of 50 cups using the base and lip heights? How could you work backward from the total height to determine the number of cups needed?"

**Look for productive strategies:**

- Identifying a pattern that represents the height added to each subsequent term.
- Recognizing that the table does not represent a proportional relationship.

Activity 1 continued >

## Differentiated Support

**Accessibility:** ***Guide Visualization and Processing***

Have students annotate the image to show that the stack on the left has "one cup with 4 additional cups" and the stack on the right has "one cup with 9 additional cups."

**Accessibility:** ***Vary Demands to Optimize Challenge***

Consider providing a pre-completed table to students. Have them annotate the table to show how 1.2 cm are added to the height of the stack with each additional cup. Then have students complete Problems 2-4.

## Math Language Development

***MLR2: Collect and Display***

During the Connect, listen for the language students use to describe whether they think the relationship is proportional. Write these words and phrases on a visual display and update it throughout the remainder of the lesson. Encourage students to refer to this display and borrow from it as they use mathematical language during discussions.

**English Learners**

The phrase *additional cup* is key in this activity for students to understand. Highlight and define this phrase at the beginning of the activity. Emphasize that each stack starts with a base, or initial, cup and then *adds additional cups*.

Pairs | 20 min

# Activity 1 Stacking Cups (continued)

Students measure stacked cups and record their findings in a table, to explore a relationship that is linear, but nonproportional.

**Activity 1** Stacking Cups (continued)

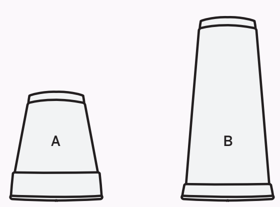
2. How much does each additional cup add to the height of the stack?
   Stacking 5 more cups adds 6 cm to the overall height, so adding one cup adds 1.2 cm to the overall height of the stack.

3. How many additional cups need to be stacked onto the original cup to reach a height of 1 m?
   1 m = 100 cm and $\frac{100-8.2}{1.2}=76.5$, which means 77 additional cups will be needed to reach 1 m.

4. How many biodegradable foam cups would you have to stack, starting from the ground, to reach the top of your math teacher's head?
   Sample response: If my teacher is 170 cm tall, then $170-8.2=161.8$ gives me the height in centimeters of additional cups I need to stack on top of the first cup. $\frac{161.8}{1.2}$ is a little less than 134, which means I will need 134 cups, in addition to the first cup. Some students may say they need 135 total cups.

**Are you ready for more?**

Consider these two different cups. Imagine more cups, like each one, are stacked on top of these. Which stack will be taller after 3 additional cups are stacked onto the first cup? After 100 additional cups? Explain your thinking.




The stack for Cup B will be taller after 3 additional cups, and the stack for Cup A will be taller after 100 additional cups. This is because Cup B is initially taller than Cup A, but Cup A increases by a greater value each time. After more and more cups are stacked, the stack for Cup A will be taller.

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## 3 Connect

**Display** student work showing the correct table.

**Have students share** what strategies worked and did not work in completing the table before discussing the number of cups students think are needed to represent your height.

**Ask:**

- "What patterns did you see in the data?"
- "How many parts of the cup are there? Which part has a greater impact on the height of the stack as more and more cups are added?"
- "Based on the values in the table, do you think this relationship is proportional? Why or why not?"
- "How did you determine the number of cups needed to reach my height, without actually stacking the cups?"

**Highlight** that this relationship is nonproportional because doubling the number of additional cups from 1 to 2 does not double the height of the stack. Demonstrate that while it is not possible to divide one variable by the other and get a constant of proportionality, there is a constant rate of change, the height of the cup's lip, being added each time.

**Define** the term ***rate of change*** as the amount that $y$ changes when $x$ increases by 1.

## Differentiated Support

**Extension:** ***Math Around the World***

In the Warm-up and Activity 1, students explored how many cups would be needed in a stack of cups to measure the height of you, their teacher. Tell them that different cultures around the world have used different units of measurement to measure distances. For example, the Inuit measured distance between locations in terms of the number of "sleeps" required to travel from one distance to another. They would measure and communicate about the distance between locations in terms of the number of stops to rest that were necessary. It was understood that this measure of the number of "sleeps" could be affected by weather, terrain, or the age, health, and experience of the travelers.

Facilitate a class discussion by asking these questions:

- "What do you see might be some advantages to measuring distance as the number of "sleeps" or number of stops of rest?" Sample response: It gives everyone an idea of how long the trip will take, more so than just measuring the distance in miles or kilometers.
- "Can you think of other ways to measure distance, other than U.S. customary or metric units of length?" Sample responses: The number of gallons of gas that would be used (if driving), the time it takes to walk/bicycle/drive given an average speed (e.g., a 15-minute walk, a 2-hour drive, etc).

Pairs | 15 min

# Activity 2 Graph It

Students graph their data to better understand the difference between linear and proportional relationships.

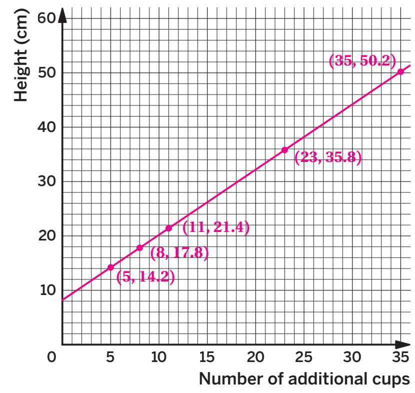
**Amps Featured Activity** Using Work From Previous Slides

Name: ______ Date: ______ Period: ______

**Activity 2** Graph It

**Plan ahead:** How can you clearly communicate the proof that your pattern works?

1. Refer to your table from Activity 1. Graph four points from your table on the coordinate plane shown.




2. What patterns do you notice?
   Sample responses:
   - I notice the points lie on a straight line with a positive slope, increasing by 6 cm for every 5 cups added.
   - I notice that each cup $x$ adds 1.2 cm to the overall height.
   - I notice the line does not pass through the origin, so the relationship is not proportional.

3. Prove your pattern works by plotting a new point. Show how the point's coordinates can be found using the pattern you described in Problem 2.
   Sample response: The point (24, 37) follows the pattern because adding 5 cups to 19 cups and 6 cm to 31 cm justifies a point located at (24, 37).





## 1 Launch

Remind students that the number of cups and the height of the stack is not a proportional relationship. This means it cannot be represented by the equation $y = kx$. Tell them that in this task, they will examine this relationship more carefully using a graph.

## 2 Monitor

**Help students get started** by showing them how to plot the first point on the graph, having them draw slope triangles between two points, and asking them how they can find the slope.

**Look for points of confusion:**

- **Not being able to see a pattern or only saying that the points fall on a line in Problem 2.** Ask students to find how their graph changes as the number of additional cups increases.
- **Not being able to describe the slope in context in Problem 4.** Point students to the pattern they found in Problem 2, and see if they can make a connection to the slope in context. Ensure students are referencing both variables, on the $x$- and $y$-axes, to describe the slope as a rate.
- **Not taking into account that the vertical intercept is not the origin.** Ask students what they notice about where the line crosses the $y$-axis and what they think this means. **Note:** Students will learn the definition of *vertical intercept* in Lesson 8. In this Activity, they will begin to explore its meaning in context.
- **Counting the number of grid squares to find the slope instead of using the scales on the axis.** Ask students to read the axes labels, and help them understand that the vertical or horizontal distance of their slope triangle is not the same as the number of grid squares.

**Look for productive strategies:**

- Being able to describe the pattern using an expression or an equation.
- Drawing slope triangles to find the slope.

Activity 2 continued ›

## Differentiated Support

**Accessibility:** ***Vary Demands to Optimize Challenge, Guide Processing and Visualization***

Consider providing a pre-completed graph, with points labeled, for students to analyze. Have them begin the activity with Problem 2. Display the Anchor Chart PDF, Slope, from Unit 2 for students to reference and suggest they determine the slope of the line graphed.

**Extension:** ***Math Enrichment***

Have students determine their own heights as measured by the number of stacked cups. Ask them to use the graph to estimate the number of stacked cups that represents their height.

## Math Language Development

***MLR8: Discussion Supports—Press for Details***

As students share their responses to Problem 6, invite them to use the following sentence frame.

- "The height of the stack is/is not proportional to the number of cups because . . ."

As partners share their thinking, press for details and the use of mathematical language by asking questions such as:

- "How does the graph support your thinking?"
- "Where on the graph do you see a constant rate of change?"
- This will help students use the graph to make sense of the data and interpret the slope in context.

Pairs | 15 min

# Activity 2 Graph It (continued)

Students graph their data to better understand the difference between linear and proportional relationships.

## Activity 2 Graph It (continued)

4. Connect the points on your graph to form a line. What is the slope of the line? What does the slope mean in this situation?
   I can draw a slope triangle with a vertical distance of 6 and a horizontal distance of 5, which means the slope of the line is $\frac{6}{5}$. Every additional cup adds $\frac{6}{5}$ cm to the overall height of the stack.

5. At what point does your line intersect the vertical axis? What do the coordinates of this point tell you about the stack of cups?
   The line intersects the vertical axis at the point $(0, 8.2)$. If there are 0 additional cups, the height of the initial cup is 8.2 cm.

6. What are some ways that you can tell that the number of cups is *not* proportional to the height of the stack?
   Sample responses:
   - The line does not pass through the origin.
   - The ratios of the vertical distance to the horizontal distance for the points that lie on the line are not equivalent. For example, $13 : 4$ is not equivalent to $19 : 9$.
   - There is an initial value, 8.2 cm, that is not equal to 0.

**Discussion Support:** How does the graph of the relationship support your response to Problem 6? How do the ordered pairs support your response?

STOP





## 3 Connect

**Display** student work showing a correct graph.

**Have students share** what patterns they noticed and how they know the line is nonproportional.

**Define** the term ***linear relationship*** as a relationship between two quantities in which there is a constant rate of change. This means that when one quantity increases by a certain amount, the other quantity increases or decreases by a proportional amount. Ask students why the stack of cups represents a linear relationship.Then ask students, "What is the height of 0 additional cups, and why is it not 0?"

**Define** the term ***initial value*** as the starting amount found in the context.

**Ask:**

- "What does the slope of the line represent in context?" The slope represents the change in the height of the stack of cups for each additional cup added.
- "How does the graph show that this relationship is linear, but nonproportional?" The graph is a straight line, but it does not pass through the origin.

**Highlight** that the slope of the line in a linear relationship, such as in a proportional relationship, is represented by the rate of change of the relationship.

Discuss that unlike proportional relationships, the graphs of linear relationships do not necessarily pass through the origin. Even if it is more accurate to represent a linear relationship with discrete points, students can connect these points and represent the relationship with a line. State that while not every point on the line makes sense in context, when drawn, it can help to see a pattern.

Whole Class | 5 min

# Summary

Review and synthesize how proportional relationships are always linear, but not all linear relationships are proportional.

Name: ______ Date: ______ Period: ______

## Summary

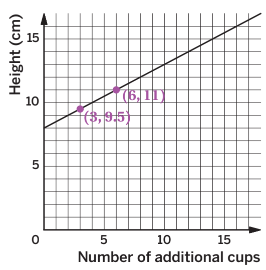
**In today's lesson . . .**

You encountered a linear relationship that was not proportional. A ***linear relationship*** is any relationship between two quantities in which there is a constant ***rate of change***. This means that when one quantity increases by a certain amount, the other quantity changes by a proportional amount.

A *proportional relationship* is a special type of linear relationship, but not all linear relationships are proportional.

For example, the graph displays the height, in centimeters, of the stacks for different additional numbers of cups.

- As the number of cups increases by $1$, the height of the stack increases by $0.5$ cm, which means the rate of change is $0.5$ cm per additional cup.
- You can see the line intersects the vertical axis at the point $(0, 8)$. This means if $0$ additional cups are added, the ***initial value*** of the cup has a height of $8$ cm.




The relationship shown is linear, but it is not proportional because the line does not pass through the origin. You can also see that the ratios of the vertical distance to the horizontal distance of the points are not equivalent. $9.5 : 3$ is not equivalent to $11 : 6$.

**Reflect:**



**Lesson 7** Introducing Linear Relationships 267

## Math Language Development

### *MLR2: Collect and Display*

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the terms *linear relationship, initial value,* and *rate of change* that were added to the display during the lesson.

## Synthesize

**Ask:**

- "What does the rate of change of a linear relationship tell you?"
- "How can you tell whether a linear relationship is proportional? From a graph? From a table? From a context?" From a graph, a linear relationship is proportional if it passes through the origin. From a table, a linear relationship is proportional if there is a constant ratio between the pairs of values. From a context, a linear relationship is proportional if the initial value is zero.
- "You saw today a linear relationship that was nonproportional. Can a proportional relationship also be linear?" All proportional relationships are linear because the graph of a proportional relationship is a straight line (that passes through the origin).

**Highlight** that there are linear relationships that are nonproportional and that all linear relationships, including those that are proportional, have a constant rate of change. Note that the rate of change of the linear relationship is the same value as the slope of a line representing the relationship. Stress that while some linear relationships are nonproportional, all proportional relationships are linear because all proportional relationships have a constant rate of change. In a proportional relationship, that constant rate of change is the constant of proportionality (or unit rate).

**Formalize vocabulary:**

- *linear relationship*
- *initial value*
- *rate of change*

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful today when finding the height of a stack of cups? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

Independent | 5 min

# Exit Ticket

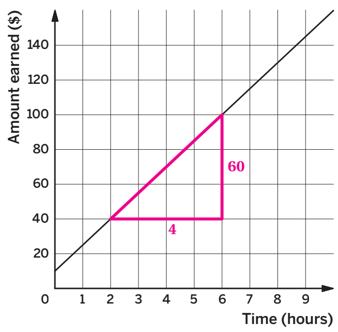
Students demonstrate their understanding by analyzing the graph of a nonproportional linear relationship and finding the constant rate of change.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 3.07

Andre charges a one-time fee for traveling to and from his babysitting jobs, plus an hourly rate. The graph shows the money he earned, based on the number of hours worked, for a recent all-day babysitting job.




1. Is the relationship a linear relationship? Is the relationship proportional? Explain your thinking.
   The relationship is linear, but not proportional, because while it has a constant rate of change, the line does not pass through the origin.

2. What hourly rate does Andre charge? Explain your thinking.
   Andre charges \$15 per hour; Sample response: By studying the graph, I can draw a slope triangle with a vertical distance of 60 and a horizontal distance of 4; $\frac{60}{4} = 15$.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can find the rate of change of a linear relationship by determining the slope of the line that represents the relationship.
1 2 3

b. I can determine if a graph represents a linear or nonlinear relationship.
1 2 3


**Lesson 7** Introducing Linear Relationships

## Success looks like . . .

- **Language Goal:** Comparing and contrasting proportional and nonproportional linear relationships. **(Speaking and Listening, Writing)**
- **Language Goal:** Interpreting the slope of a graph of a nonproportional linear relationship. **(Speaking and Listening, Writing)**

## Suggested next steps

**If students are confused about the relationship being linear and nonproportional, consider:**

- Reviewing Activity 3 for an example of a relationship that is linear, but nonproportional.
- Asking, "What do you notice about where the graph crosses the $y$-axis?"

**If students are unable to determine the rate of change, consider:**

- Asking, "Where can you see Andre's hourly rate represented on the graph?"
- Reviewing strategies for calculating the slope in Activity 2.
- Have students revisit this problem after Lesson 12.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- The focus of this lesson was the transition from proportional relationships to linear relationships that are nonproportional. How did this transition go?
- Which groups of students did and did not have their ideas seen and heard today?

# Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

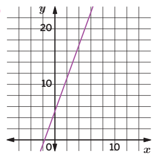
1. Choose *all* the relationships that are linear, but not proportional. Explain your thinking.

A. From rest, Diego walks at a constant speed of 5 km per hour.

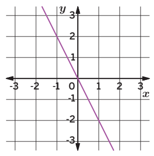
C. $y = 2x$

E. A giraffe is initially 3 ft tall and grows 6 in. every month for a year.

B.




D.




F.

| $x$ | $y$ |
|---|---|
| 2 | 4 |
| 3 | 5 |
| 5 | 7 |

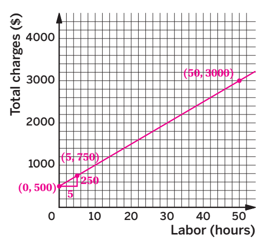
Choices B, E, and F are linear, but not proportional because they all show a constant rate of change and have an initial value that is not represented by (0, 0). Choices A, C, and D are all proportional because they have a constant of proportionality.

2. To paint a house, a painting company charges a flat fee of \$500 for supplies, plus \$50 for each hour of labor.

a. How much would the painting company charge to paint a house that requires 5 hours of labor? A house that requires 50 hours?

For 5 hours: \$750; For 50 hours: \$3,000

b. Draw a line representing the relationship between $x$, the number of hours it takes the painting company to paint the house, and $y$, the total cost of painting the house. Label the two points from part a on the graph.




c. Find the slope of the line you graphed. What does the slope mean in this context?

The slope of the line is 50, which is the same as the cost per hour, in dollars, that the painting company charges for labor.



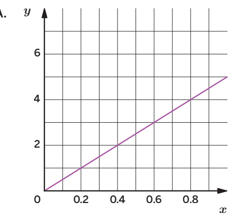


Name: ______ Date: ______ Period: ______

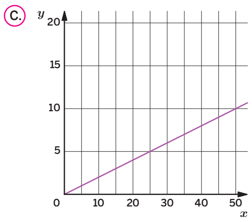
Practice

3. Which of these relationships has a slope that is *different* from the other three relationships? Explain your thinking.

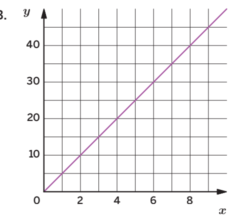
A.



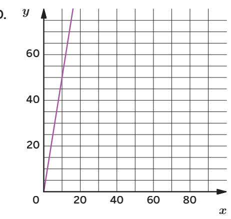

C.




B.




D.




Graph C is the only relationship that does not have a slope of 5. Graph C has a slope of $\frac{1}{5}$.

4. Which of the following tables does *not* represent a proportional relationship? Explain your thinking.

A.

| $x$ | $y$ |
|---|---|
| $-2$ | $-8$ |
| $\frac{1}{2}$ | 2 |
| 1 | 4 |

B.

| $x$ | $y$ |
|---|---|
| $-1$ | $-1$ |
| 1 | 1 |
| 2 | 2 |

C.

| $x$ | $y$ |
|---|---|
| 1 | 3 |
| 3 | 5 |
| 5 | 7 |

Table C does not represent a proportional relationship because there is no constant of proportionality such that $1k = 3$ and $3k = 5$.





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| | 2 | Activity 2 | 2 |
| Spiral | 3 | Unit 2 Lesson 11 | 2 |
| Formative | 4 | Unit 3 Lesson 8 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Comparing Relationships

Let's explore how linear relationships are different from other relationships.


## Focus

### Goals

1. **Language Goal:** Justify whether the values in a given table represent a linear relationship. **(Speaking and Listening)**

2. Compare rates of change for values in a table, and determine which table represents a linear relationship.

## Rigor

- Students develop **fluency** in determining whether a relationship is linear or nonlinear.

## Coherence

### Today

Students continue their work with linear relationships and determining the rate of change. They build on their understanding of proportional relationships to determine whether a table of values models a linear or nonlinear relationship.

### Previously

In Lesson 7, students discovered relationships that have a constant rate of change and are nonproportional.

### Coming Soon

In Lesson 9, students will explore how linear relationships are similar to and different from proportional relationships.

## Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 (optional) | Summary | Exit Ticket |
|---|---|---|---|---|
| 10 min | 25 min | 15 min | 5 min | 5 min |
| Pairs | Small Groups | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

### Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, *Analyzing Two Tables* (for display)
- Activity 2 PDF, pre-cut cards, one per pair
- calculators (optional)
- rulers

### Math Language Development

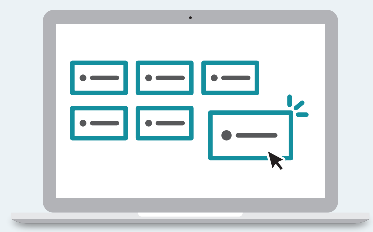
**Review words**

- *initial value*
- *linear relationship*
- *proportional relationship*
- *rate of change*

**Amps** | Featured Activity

**Activity 2**
### Digital Card Sort

Students match situations with graphs by dragging and connecting them on screen.



Amps powered by desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students may feel overwhelmed as they see the cards for the first time. In order to reduce their stress level, have students identify how all of the cards are alike. With this initial mini-task, students will feel like they have their feet under them. Then they can look at one card at a time with precision to distinguish its unique features and determine what type of relationship is modeled.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, provide the number of diagonals and have students complete Problem 2.
- Consider assigning **Activity 2** as additional practice.

Pairs | 10 min

# Warm-up Diagonals

Students complete a table of values as an introduction to nonlinear relationships.

Unit 3 | Lesson 8

## Comparing Relationships

Let's explore how linear relationships are different from other relationships.


### Warm-up Diagonals

Consider the following regular polygons.

Triangle Quadrilateral Pentagon Hexagon Heptagon

1. How many diagonals are present in each polygon? Record your responses in the table.

2. Could the relationship between the number of sides of a polygon and the number of diagonals be linear? Explain your thinking.
**No, there is not a linear relationship. As the number of sides increases by one, the number of diagonals does *not* increase by the same value each time. So, there is not a constant rate of change.**

| Polygon | Number of sides | Number of diagonals |
|---|---|---|
| Triangle | 3 | 0 |
| Quadrilateral | 4 | 2 |
| Pentagon | 5 | 5 |
| Hexagon | 6 | 9 |
| Heptagon | 7 | 14 |

Log in to Amplify Math to complete this lesson online.




270 Unit 3 Linear Relationships


## 1 Launch

Activate students' prior knowledge by asking them what they know about regular polygons and diagonals. Provide access to rulers.

## 2 Monitor

**Help students get started** by modeling how to draw a diagonal using the quadrilateral.

**Look for points of confusion:**

- **Not being sure of the number of diagonals in a triangle.** Have students complete the triangle last. Ask them to examine the number of diagonals in the other polygons and make use of their structure, and remind them that it is possible that there are no diagonals.
- **Thinking the pattern represents a linear relationship.** Remind students of the definition of a linear relationship. Then ask how the number of diagonals changes as the number of sides increases by 1.

## 3 Connect

**Have students share** their response for Problem 2.

**Highlight** that for a linear relationship, there must be a constant rate of change, which means that the change in one quantity divided by the change in the other quantity is constant. Stress that although the table shows a pattern, the rate of change is not constant, and therefore the relationship between the number of sides and the number of diagonals is not linear.

**Display** the Warm-up PDF, *Analyzing Two Tables*.

**Ask** students for values of $y$, one at a time, that make the tables represent a linear and nonlinear relationship. After each value, ask students if they need to write another value of $y$ to show whether the relationship is linear or nonlinear.

## Power-up

**To power up students' ability to recognize proportional and nonproportional relationships in tables, have students complete:**

Recall that if a table is modeling a proportional relationship, each ratio $y : x$ from the table is the same value, called the constant of proportionality.

1. What is the constant of proportionality from the first two columns of the table? 5
2. If the table is a proportional relationship, what is the value of the missing cell?
3. What is a possible value of the missing cell if the relationship is nonproportional? Answers may vary but may not be 25.

| $x$ | 1 | 3 | 5 |
|---|---|---|---|
| $y$ | 5 | 15 | 25 |

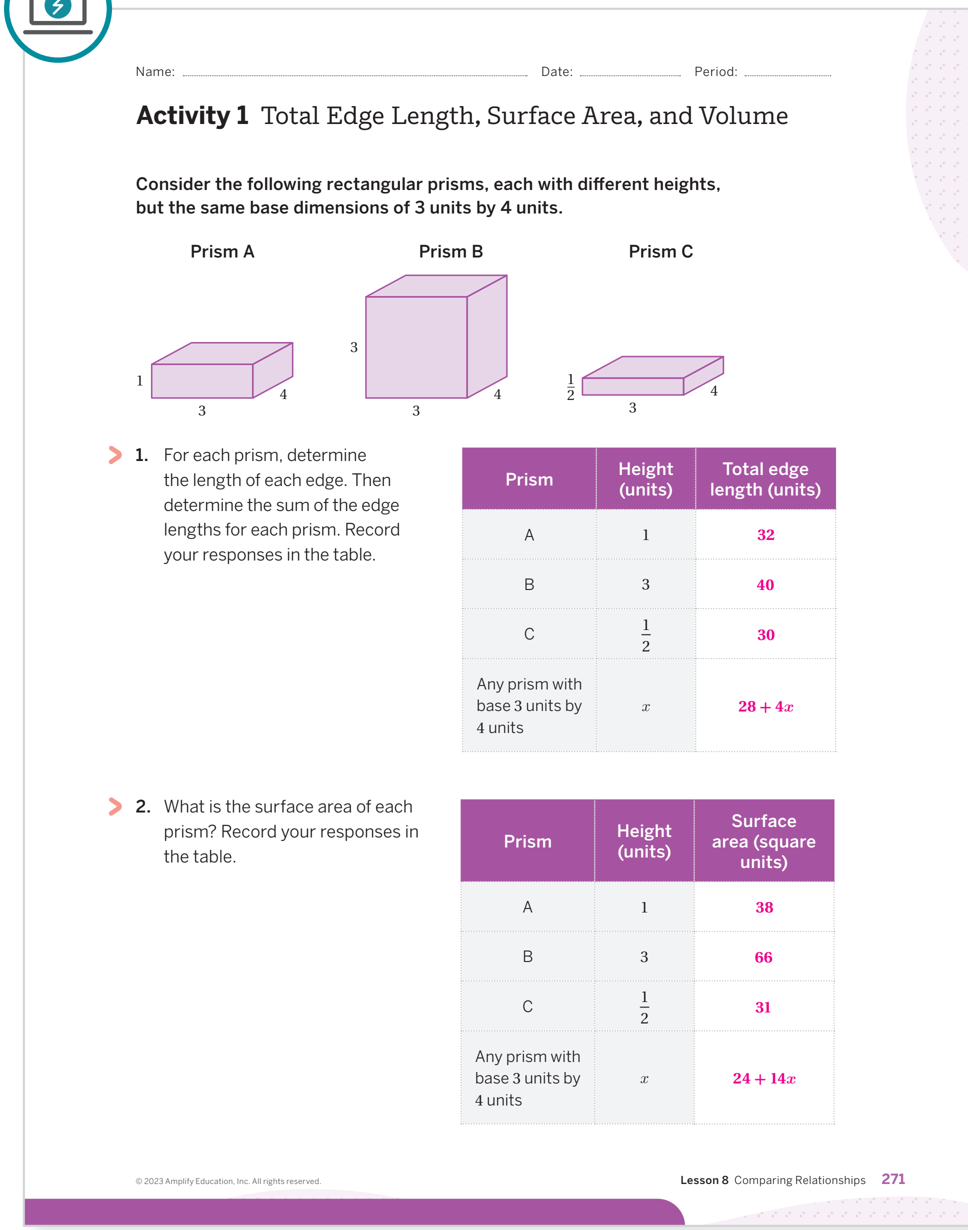
**Use:** Before Activity 2

**Informed by:** Performance on Lesson 7, Practice Problem 4 and Pre-Unit Readiness Assessment, Problem 1

Small Groups | 25 min

# Activity 1 Total Edge Length, Surface Area, and Volume

Students analyze data to see how the rate of change helps them identify a linear relationship.



Name: ______ Date: ______ Period: ______

**Activity 1** Total Edge Length, Surface Area, and Volume

**Consider the following rectangular prisms, each with different heights, but the same base dimensions of 3 units by 4 units.**



1. For each prism, determine the length of each edge. Then determine the sum of the edge lengths for each prism. Record your responses in the table.

| Prism | Height (units) | Total edge length (units) |
|---|---|---|
| A | 1 | **32** |
| B | 3 | **40** |
| C | $\frac{1}{2}$ | **30** |
| Any prism with base 3 units by 4 units | $x$ | **$28 + 4x$** |

2. What is the surface area of each prism? Record your responses in the table.

| Prism | Height (units) | Surface area (square units) |
|---|---|---|
| A | 1 | **38** |
| B | 3 | **66** |
| C | $\frac{1}{2}$ | **31** |
| Any prism with base 3 units by 4 units | $x$ | **$24 + 14x$** |





## 1 Launch

Activate students' prior knowledge by asking them how to determine the surface area and volume of a rectangular prism.

## 2 Monitor

**Help students get started** by providing a method or formula to determine the total edge length, surface area, and volume of a rectangular prism.

**Look for points of confusion:**

- **Having trouble completing the tables.** Provide students with a rectangular object, such as a tissue box, and small sticky notes. Have them label the edges of the object with the measurements provided in the problem to help with their thinking.
- **Thinking that the tables in Problems 2 and 3 represent a nonlinear relationship, as they talk about area and volume.** Remind students that, in a linear relationship, as one quantity increases by a set amount, the other quantity increases or decreases by a constant amount. For each table, have students find the difference between the values and compare the ratios to see that they are equivalent.
- **Thinking that the table in Problem 3 is not linear because it is proportional.** Remind students that a proportional relationship is also linear, but that not all linear relationships are proportional. Revisit with these students to check for understanding after Lesson 9.

**Look for productive strategies:**

- Noticing a constant value, in the tables for Problems 1 and 2, that represents the top and bottom faces of the rectangular prism.
- Calculating an additional height, for example a height of 2, to show that the increase is the same for each next value.

Activity 1 continued ›

## Differentiated Support

### Accessibility: *Activate Prior Knowledge*

Ask students what they know about the surface area and volume of rectangular prisms and what an *edge* of a prism means. Display the surface area and volume formulas. Consider demonstrating how to determine the sum of the edge lengths for Prism A.

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing the tables for Prisms A and B and then for any prism with base 3 units by 4 units.

## Math Language Development

### *MLR8: Discussion Supports*

During the Connect, as students share their explanations for Problem 4, display these sentence frames to help support them as they explain which relationship(s) are linear.

- "The relationship between ___ and ___ is linear because . . ."
- "The relationship between ___ and ___ is not linear because . . ."

### English Learners

Annotate the expressions $28 + 4x$, $24 + 14x$, and $12x$ in each table as *linear* and highlight the constant rate of change.

Small Groups | 25 min

# Activity 1 Total Edge Length, Surface Area, and Volume (continued)

Students analyze data to see how the rate of change helps them identify a linear relationship.

**Activity 1** Total Edge Length, Surface Area, and Volume (continued)

3. What is the volume of each prism? Record your responses in the table.

| Prism | Height (units) | Volume (cubic units) |
|---|---|---|
| A | 1 | 12 |
| B | 3 | 36 |
| C | $\frac{1}{2}$ | 6 |
| Any prism with base 3 units by 4 units | $x$ | $12x$ |

4. Consider these relationships for a rectangular prism with base dimensions 3 units by 4 units.
   - The relationship between height and total edge length.
   - The relationship between height and surface area.
   - The relationship between height and volume.

   Which of the relationships are linear? Explain your thinking.

   **All of the relationships are linear; Sample response: Each relationship is changing at a constant rate.**




## 3 Connect

**Have pairs of students share** their explanations for Problem 4. Use the ***Poll the Class*** routine to determine which students identify each table as linear.

**Ask**, "What is the rate of change in each relationship?" 4, 14, 12

**Highlight:**

- The rate of change is constant in each of the tables. This means that each table represents a linear relationship.
- Even if the values in the table are not consecutive, students can still determine whether the rate of change is constant by dividing the change in one quantity by the change in the other quantity. Point out how this is different when analyzing a table that represents a proportional relationship, where the rate of change is determined by $\frac{y}{x}$.
- All proportional relationships are linear, but not all linear relationships are proportional.

Optional

Pairs | 15 min

# Activity 2 Card Sort: Tables of Linear Relationships

Students sort cards to attend to precision and strengthen their fluency in identifying linear and nonlinear relationships from tables of values.

Amps Featured Activity | Digital Card Sort

Name: ____________ Date: ________ Period: ________

## Activity 2 Card Sort: Tables of Linear Relationships

**You will be given a set of cards. Each card contains a table with information about a relationship.**

1. Based on the information in each table, sort the cards by whether they represent *possible linear relationships* or *nonlinear relationships*. Record your card sort in the table.

| Possible linear relationships | Nonlinear relationships |
|---|---|
| Cards 1, 3, 4, 6 | Cards 2 and 5 |

2. For each card that represents a possible linear relationship, determine the rate of change. Explain the meaning of the rate of change in context. You may not need all of the rows in the table.

| Card | Rate of change | Explanation |
|---|---|---|
| Card 1 | $\frac{180-90}{4-2}=\frac{90-45}{2-1}=45$ | 45 miles per hour |
| Card 3 | $\frac{11.5-8}{3-2}=\frac{8-4.5}{2-1}=3.5$ | 3.5 additional inches per week |
| Card 4 | $\frac{6.5-4.5}{3-2}=\frac{3.5-2.5}{1.5-1}=2$ | 2 additional episodes per hour |
| Card 6 | $\frac{16.8-11.25}{4-2.5}=\frac{11.25-5.7}{2.5-1}=3.7$ | \$3.70 per additional pound |
| | | |
| | | |

STOP





## 1 Launch

Distribute one set of cards from the Activity 2 PDF to each pair of students. Then conduct the ***Card Sort*** routine.

## 2 Monitor

**Help students get started** by asking them how to determine a linear relationship from a table. Have students divide the change in one quantity by the change in the other quantity to help them sort the cards.

**Look for points of confusion:**

- **Thinking Card 2 is linear.** Remind students to check *every* row to see if there is a constant rate of change.
- **Thinking Cards 1 and 4 are not linear because the difference between the values in the right column are not the same.** Remind students that if the values in the table are not consecutive, they can compare the difference in the right column and the difference in the left column to look for an equivalent change.

**Look for productive strategies:**

- Simplifying the ratios that represent the change in one quantity to the change in the other quantity to look for a constant rate of change.

## 3 Connect

**Have pairs of students share** their strategies for determining the rate of change.

**Ask**, "What can you look for to determine whether a table of values could represent a linear relationship?"

**Highlight** that from a table, students can calculate the rate of change to identify whether the table represents a possible linear or nonlinear relationship.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Omit Cards 5 and 6 from the set. This will still allow students to access the mathematical goal of the activity, which is to strengthen their fluency in identifying linear and nonlinear relationships from tables of values.

### Extension: *Math Enrichment*

Provide students with two blank cards and have them label them Card 7 and Card 8. Ask them to create one table of values on each card, one that represents a linear relationship and one that does not. Have them trade their cards with another student and determine which table represents a linear relationship and why.

## Math Language Development

### *MLR7: Compare and Connect*

To begin the Connect, have students compare their strategies for determining whether a relationship is linear or nonlinear. Listen for words and phrases that indicate students were looking for a constant rate of change, such as "the same value was added each time," 'I checked every row,"and "I checked both columns, not just the right column."

### English Learners

Display these sentence frames to support students when they explain which relationships are linear or nonlinear.

- "_____ is linear because . . ."
- "_____ is nonlinear because . . ."

Whole Class | 5 min

# Summary

Review and synthesize how to identify linear and nonlinear relationships based on a table of values.

## Summary

**In today's lesson . . .**

You saw examples of linear and nonlinear relationships.

- *Linear relationships* have a constant rate of change.
- *Nonlinear relationships* do not have a constant rate of change.

Consider these tables which show the cost for bike rentals at two different companies. Bikes-R-Us charges a one-time rental fee *and* an hourly fee. Meadowland Bicycles posts their fees based on the number of hours.

**Bikes-R-Us**

| Time (hours) | Cost ($) |
|---|---|
| 1 | 14 |
| 3 | 26 |
| 5 | 38 |

Time: + 2, + 2; Cost: + 12, + 12

At Bikes-R-Us, the rate of change is $6 per hour.

Based on the information in the table, the relationship between the total cost and time is *linear*.

**Meadowland Bicycles**

| Time (hours) | Cost ($) |
|---|---|
| 1 | 10 |
| 3 | 26 |
| 5 | 38 |

Time: + 2, + 2; Cost: + 16, + 12

For Meadowland Bicycles, the cost for each additional hour varies.

There is no constant rate of change, so this is a *nonlinear relationship*.

Reflect:





## Synthesize

**Display** the Summary from the Student Edition.

**Have students share** their strategies for determining whether a relationship is linear from a table of values.

**Ask** students to create two tables, one representing a linear relationship and the other representing a nonlinear relationship. Ask them to explain how they created their tables, and how they know which one is linear and which one is nonlinear.

**Highlight** that students can identify whether a relationship expressed in a table is linear by looking for a constant rate of change.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies did you find helpful today when identifying a possible linear relationship from a table of values?"

Independent | 5 min

## Exit Ticket

Students demonstrate their understanding of linear and nonlinear relationships by determining whether relationships expressed as tables of values are linear or nonlinear.

Printable

Name: ______ Date: ______ Period: ______

### Exit Ticket 3.08

**A pizza parlor offers different varieties of pizza. Use the tables to complete the following problems.**

1. Does the relationship between the number of toppings and cost of the pizza represent a linear relationship? Explain your thinking.

   Yes, this relationship appears to be linear. As the number of toppings increases by 1, the cost of the pizza increases by \$1.50. The rate of change is constant.

| Number of toppings | Cost of pizza (\$) |
|---|---|
| 2 | 16.00 |
| 3 | 17.50 |
| 4 | 19.00 |
| 5 | 20.50 |

2. Does the diameter and cost of the pizza represent a linear relationship? Explain your thinking. If the relationship is linear, write an equation that represents the relationship.

   This relationship is nonlinear. As the diameter increases by 2 in., the cost of the pizza increases by \$3.50, then \$2.50, then \$2.25. The rate of change is not constant.

| Diameter (in.) | Cost of pizza (\$) |
|---|---|
| 8 | 11.00 |
| 10 | 14.50 |
| 12 | 16.00 |
| 14 | 18.25 |

**Self-Assess**

? 1 I don't really get it — 2 I'm starting to get it — 3 I got it

a. I can determine if a table represents a *linear* or *nonlinear* relationship.
1 2 3

b. I can determine the constant rate of change for a linear relationship from a table.
1 2 3





### Success looks like . . .

- **Language Goal:** Justifying whether the values in a given table represent a linear relationship. **(Speaking and Listening)**
  - » Writing a clear explanation for why the table in Problem 1 represents a linear relationship and the table in Problem 2 represents a nonlinear relationship.
- **Goal:** Comparing rates of change for values in a table and determining which table represents a linear relationship.

### Suggested next steps

**If students identify Problem 1 as nonlinear or Problem 2 as linear, consider:**

- Showing the rates of change for each table and asking them to compare them.
- Having them graph the values and asking them to identify whether the relationship is linear or nonlinear.
- Having students create their own linear and nonlinear tables.
- Reassessing after Lesson 9.

### Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- Which groups of students did or did not have their ideas seen and heard today?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?

### Math Language Development

**Language Goal: Justifying whether the values in a given table represent a linear relationship.**

Reflect on students' language development toward this goal.

- Do students' responses to the Exit Ticket Problems 1 and 2 provide a clear and accurate justification for why each relationship is linear or nonlinear? What mathematical vocabulary are they using?
- How can you help them be more precise in their justifications?

# Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. Based on the information in each table, decide whether it could represent a linear relationship. Show or explain your thinking.

a. Money in a savings account

| Time (months) | Money ($) |
|---|---|
| 3 | 60 |
| 5 | 80 |
| 8 | 110 |

Could be linear; Sample response: The rate of change is constant for the values in the table.
$\frac{80-60}{5-3}=\frac{110-60}{8-3}=10$

b. Cost of water usage

| Volume of water (gallons) | Cost ($) |
|---|---|
| 500 | 7.5 |
| 2,000 | 40 |
| 5,000 | 110 |

Nonlinear; Sample response: The rate of change is not constant for the values in the table.
$\frac{40-7.5}{2000-500}\neq\frac{110-4}{5000-2000}$

2. A taxi service charges $1.50 for the first $\frac{1}{10}$ mile, and then $0.15 for each additional $\frac{1}{10}$ mile.

a. Based on the description, do you think the relationship will be linear? Why or why not?

Sample response: I think it will be linear because the cost is increasing by $0.15 for every $\frac{1}{10}$ mile.

b. Complete the table with the missing information.

c. Determine if the relationship between distance traveled and total cost of the trip is a linear or nonlinear relationship. Show or explain your thinking.

Linear; Sample response: There is a constant rate of change.

$\frac{1.65-1.5}{\frac{2}{10}-\frac{1}{10}}=\frac{2.85-1.65}{1-\frac{2}{10}}=\frac{6-2.85}{3\frac{1}{10}-1}$

| Distance traveled (miles) | Total cost ($) |
|---|---|
| $\frac{1}{10}$ | 1.50 |
| $\frac{2}{10}$ | 1.65 |
| 1 | 2.85 |
| $3\frac{1}{10}$ | 6.00 |





Name: ______ Date: ______ Period: ______

Practice

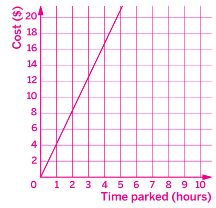
3. The equation $y = 4.2x$ could represent a variety of different real-world situations.

a. Write a description of a real-world situation that could be represented by this equation. Decide what quantities $x$ and $y$ represent in your situation.

Sample response: A parking garage charges $21 to park for 5 hours. $x$ represents the number of hours parked, and $y$ represents the total cost.

b. Create a table and a graph that represent the situation.

| Time parked (hours) | Cost ($) |
|---|---|
| 1 | 4.20 |
| 3 | 12.60 |
| 5 | 21 |



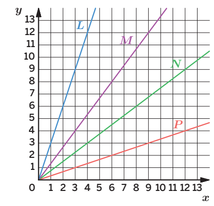

4. Match each equation with the graph of its line.

a. $y=\frac{1}{3}x$ P

b. $y=\frac{4}{3}x$ M

c. $y=3x$ L

d. $y=\frac{3}{4}x$ N



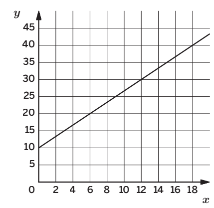

5. Refer to the line graphed on the coordinate plane. What is the slope of this line?

A. 10

B. $\frac{1}{3}$

C. $\frac{3}{5}$

D. $\frac{5}{3}$








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 1 |
| | 2 | Activity 1 | 2 |
| Spiral | 3 | Unit 3 Lesson 5 | 2 |
| | 4 | Unit 3 Lesson 3 | 1 |
| Formative | 5 | Unit 3 Lesson 9 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# More Linear Relationships

Let's explore some more linear relationships and their equations.


## Focus

### Goals

1. **Language Goal:** Describe how the slope and vertical intercept influence the graph of a line. **(Speaking and Listening, Writing)**
2. Identify and interpret the positive vertical intercept and slope of the graph of a linear relationship.
3. Identify and interpret the positive vertical intercept and slope of the equation of a linear relationship of the form $y = mx + b$.

## Coherence

### Today

Students continue to explore how linear relationships are similar to and different from proportional relationships. They learn about the term *vertical intercept*, match different real-world situations to their corresponding graphs, and then interpret the slope and vertical intercept in the situation being modeled. Students learn that the equation $y = mx + b$ can represent a linear relationship, and they make sense of problems by analyzing graphs and equations.

### Previously

In Sub-Unit 1, students explored proportional relationships. In Lesson 8, students identified linear relationships by calculating the rate of change given a table of values.

### Coming Soon

In Lesson 10, students will write an equation to represent a linear relationship given in context, and then interpret the slope and $y$-intercept.

## Rigor

- Students further their **conceptual understanding** of linear relationships by comparing different representations and interpreting the slope and $y$-intercept of a graph.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Activity 3 (optional) | Summary | Exit Ticket |
|---|---|---|---|---|---|
| 3 min | 15 min | 20 min | 10 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

## Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair
- Activity 2 PDF (answers)
- Anchor Chart PDF, *Representations of Linear Relationships*

## Math Language Development

**New words**

- ***vertical intercept***
- ***y-intercept***

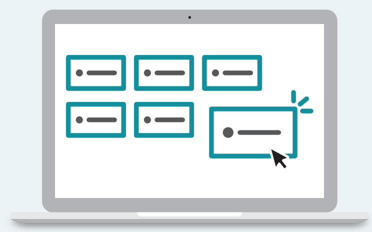
**Review words**

- *initial value*
- *linear relationship*
- *proportional relationship*
- *rate of change*
- *slope*

**Amps** | Featured Activity

### Activity 2
### Digital Card Sort

Students match real-world situations with graphs by dragging and connecting them on screen.



Amps POWERED BY desmos

## Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel lost if they do not know how to interpret the slope and vertical intercept. Encourage them to take control of their learning by suggesting they seek out support from 2–3 other sources as a general guideline when they feel frustrated.

## Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, provide Situation Cards A, B, C and Graph Cards 1, 2, and 6.
- Optional **Activity 3** may be omitted.

Independent | 3 min

# **Warm-up** Would You Rather?

Students compare two lines to draw attention to the slope and vertical intercept of each line.

Name: ______________ Date: ________ Period: ________

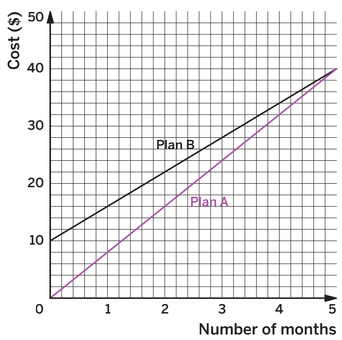
**Unit 3 | Lesson 9**

## More Linear Relationships

Let's explore some more linear relationships and their equations.


### **Warm-up** Would You Rather?

**The lines on the graph show the cost of two different subscription plans from Audio Line, a music streaming service.**




Which plan would you rather choose? Explain your thinking.

**Sample response: I would rather choose Plan A because it is less expensive than Plan B (at least up to 5 months).**

Log in to Amplify Math to complete this lesson online.



Lesson 9 More Linear Relationships 277

### 1 Launch

Activate students' background knowledge by asking whether anyone subscribes to a music streaming service. Provide information about how the pricing may work for a subscription plan. For example, some plans charge a monthly fee versus an annual fee. Then conduct the ***Would You Rather?*** routine.

### 2 Monitor

**Help students get started** by asking them the cost of each plan after 1 month and 3 months.

**Look for productive strategies:**

- Noticing that Plan B is more expensive up until 5 months.
- Noticing that Plan A is proportional and Plan B is nonproportional.

### 3 Connect

**Have students share** their responses.

**Highlight** that students can compare the cost of each plan based on the placement of each line.

**Ask:**

- "Which plan is proportional? Which plan is linear?" Both plans are linear, but Plan A is proportional and Plan B is nonproportional.
- "Do you think Plan A will always be cheaper? Why or why not?" Sample response: It appears that Plan A and Plan B cost the same amount for 5 months. Before 5 months, Plan A is less expensive, but is increasing at a faster rate than Plan B. I think that Plan A will be more expensive than Plan B after 5 months.

**Note:** In Activity 1, students will explore this scenario further.

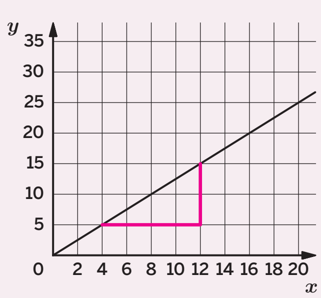
## Power-up

**To power up students' ability to calculate slope from a graph whose axes have a scale that is not equal to 1, have students complete:**

Recall that to determine the slope of a line, you can draw a slope triangle then calculate the ratio of its vertical side length to its horizontal side length. You must take the scale of each axis into consideration when determining each length.

1. Draw a slope triangle for the line shown. Sample response shown.

2. Determine the slope of the line. $\frac{5}{4}$ or equivalent




**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 8, Practice Problem 5

Pairs | 15 min

# Activity 1 Let's Compare

Students analyze two plans to determine the vertical intercept, and compare proportional and nonproportional relationships.

## Activity 1 Let's Compare

**Let's compare the two music subscription plans from the Warm-up.**

1. Complete the table showing the total cost for the first five months of each plan from Audio Line.

**Plan A:** Pay \$8 every month.

| Number of months, $x$ | Cost (\$), $y$ |
|---|---|
| 1 | 8 |
| 2 | 16 |
| 3 | 24 |
| 4 | 32 |
| 5 | 40 |

**Plan B:** Pay a one-time fee of \$10 to sign up, and then pay \$6 every month.

| Number of months, $x$ | Cost (\$), $y$ |
|---|---|
| 1 | 16 |
| 2 | 22 |
| 3 | 28 |
| 4 | 34 |
| 5 | 40 |

2. Write an equation that represents the cost $y$, in dollars, after $x$ months of service.

Plan A: $y = 8x$

Plan B: $y = 10 + 6x$

3. Diego wants to subscribe to one of these plans for 1 year. Which plan should he choose? Explain your thinking.
Sample response: Diego should choose Plan B. I substituted 12 for $x$ in each equation and found that Plan B will be less expensive after 1 year (12 months).
Plan A: $8(12) = 96$, \$96
Plan B: $10 + 6(12) = 82$, \$82




## 1 Launch

Tell students they will investigate the two plans from the Warm-up further using tables and equations.

## 2 Monitor

**Help students get started** by completing the cost of 1 month in each table together as a class.

**Look for points of confusion:**

- **Using reasoning to complete the table or equation for Plan B.** Remind students that because there is an initial sign-up fee of \$10, they should include this additional cost. In the equation, this will be represented by $+10$.
- **Being unsure of how to solve Problem 3.** Suggest that students continue the table or substitute 12 for $x$ in each equation.

## 3 Connect

**Display** the graph from the Warm-up.

**Have pairs of students share** how the initial sign-up fee and monthly cost appear in the table, graph, and equation.

**Define** the term ***vertical intercept*** as the point where the graph of a line intersects the vertical axis. Also known as the ***y-intercept***, it is the value of $y$ when the corresponding value of $x$ is 0.

**Ask:**

- "What is the vertical intercept for each plan? What does it represent?"
- "When do the plans cost the same?"
- "Is Plan A always cheaper?"

**Highlight** that the vertical intercept is located on the $y$-axis on the graph and is the constant in the equation. For a proportional relationship, the vertical intercept is 0 because the graph intersects the $y$-axis at the origin.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Demonstrate and encourage students to use color coding and annotations to highlight how the slope and vertical intercept appear in a verbal description, graph, table of values, and equation. Have them highlight the slope in each representation using one color and the vertical intercept in another color.

### Extension: *Math Enrichment*

Ask students which plan Diego should choose if he only wants to subscribe to one of these plans for 3 months, 5 months, or 8 months.
3 months: Plan A. They cost the same at 5 months. 8 months: Plan B.

## Math Language Development

### *MLR8: Discussion Supports*

During the Connect, support students in producing statements about the meaning of a vertical intercept by displaying sentence frames for them to use when they describe the reasoning for their matches. For example:

- "The vertical intercept ______ represents . . ."
- "The vertical intercept for Plan ______ is ______, so this tells me . . ."

### English Learners

Annotate the vertical intercept on the graph, verbal description, table of values, and equation. Use hand gestures to illustrate the meaning of "vertical."

Pairs | 20 min

# Activity 2 Card Sort: Slopes, Vertical Intercepts, and Graphs

Students sort cards to compare nonproportional linear and proportional relationships and to understand how the slope and vertical intercept appear in each.

**Amps Featured Activity** Digital Card Sort

Name: ______ Date: ______ Period: ______

**Activity 2** Card Sort: Slopes, Vertical Intercepts, and Graphs

**You will be given six cards describing different real-world situations and six cards containing graphs.**

1. Match each situation with its corresponding graph.

Situation A: Graph 2 Situation B: Graph 6 Situation C: Graph 1

Situation D: Graph 3 Situation E: Graph 5 Situation F: Graph 4

2. Select one proportional relationship and one nonproportional relationship. For each relationship you select, complete the following problems. Sample response shown.

a. How can you determine the slope from the graph? Show or explain your thinking.

| Proportional: Situation F | Nonproportional: Situation A |
|---|---|
| Because the $y$-values increase by 40 when the $x$-values increase by 1, the slope is 40. | Because the $y$-values increase by 10 when the $x$-values increase by 1, the slope is 10. |

b. Explain what the slope represents in the situation.

| Proportional: Situation F | Nonproportional: Situation A |
|---|---|
| The slope represents the amount Lin's mom pays each month. | The slope represents the cost per month. |

c. What is the vertical intercept? What does it tell you about the situation?

| Proportional: Situation F | Nonproportional: Situation A |
|---|---|
| (0, 0); This tells me that Lin's mom did not pay any money when the contract first started. | (0, 40); This tells me that the tablet costs $40. |

d. Write an equation that represents the situation.

| Proportional: Situation F | Nonproportional: Situation A |
|---|---|
| $y = 40x$ | $y = 40 + 10x$ |

**Compare and Connect:** After sharing your matches, examine Graphs 2 and 3. What does the value 40 represent in each situation? Discuss with your partner.



Lesson 9 More Linear Relationships 279

## 1 Launch

Distribute one set of cards from the Activity 2 PDF to each pair of students. Then conduct the *Card Sort* routine.

## 2 Monitor

**Help students get started** by drawing slope triangles to determine the slope on each graph card.

**Look for points of confusion:**

- **Not being able to match a graph to a situation.** Remind students to look at the scale on both axes when calculating the slope, and then simplify the slope.
- **Having trouble interpreting the slope and vertical intercept for Problems 2b and 2c.** Have students label the axes with each variable.

**Look for productive strategies:**

- Looking at the $y$-axis to identify whether a relationship is proportional or nonproportional.

## 3 Connect

**Display** student work showing the correct matches.

**Have students share** how they matched each situation to its graph.

**Define** the equation of a linear relationship as $y = mx + b$ where $m$ is the slope and $b$ is the vertical intercept.

**Ask**, "What equation represents a linear relationship with a vertical intercept of 0?" Sample responses: $y = mx$, $y = kx$, $y = mx + 0$.

**Highlight** that for a proportional relationship, the rate of change can be determined by the ratio of $y$ to $x$, but for a linear relationship, the slope is determined by the ratio of the vertical change to the horizontal change.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, limit the number of cards students need to sort by providing them with Situation Cards A, B, and C, and Graph Cards 1, 2, and 6.

### Extension: *Math Enrichment*

Ask students to select one proportional relationship and one nonproportional relationship. Have them explain how they could alter the situation so that the proportional relationship becomes nonproportional, and vice versa. Then have them explain how their corresponding graphs would change.

## Math Language Development

### *MLR7: Compare and Connect*

During the Connect, as students share their matches, call attention to the different ways the vertical intercept is represented graphically and within the context of each situation. Ask students to closely examine Graphs 2 and 3 and explain what the value 40 represents in each corresponding situation. Then ask them why the graphs look different. Sample response: The value 40 represents the cost of the tablet in both situations. The graphs look different because the slopes are different.

### English Learners

Annotate the vertical intercept on each graph with the phrases *initial value* and vertical *intercept*.

Optional

Pairs | 10 min

# Activity 3 Matching Equations

Students match an equation with its graph to see how the changing the coefficient of $x$ in the equation affects its line.

## Activity 3 Matching Equations

**The manager at Honest Carl's Funtime World compares two different roller coasters. The table gives the roller coasters' heights and speeds for the hill before the first drop.**

| Jack Rabbit | Thunderbolt |
|---|---|
| Starts from a platform with a height of 14 ft, and then climbs 4 ft per second. | Starts from a platform with a height of 4 ft, and then climbs 14 ft per second. |

1. Match each roller coaster with its equation, where $x$ represents the time in seconds and $y$ represents the height, in feet, of the roller coaster above the ground.

**Equation**

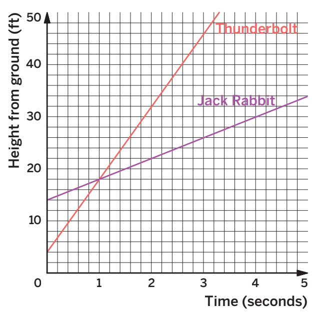
(a) Jack Rabbit — b $y = 4 + 14x$

(b) Thunderbolt — a $y = 4x + 14$

2. The manager of Honest Carl's Funtime World wants to know the height of each roller coaster after 3 seconds. How can she use the equation or graph to determine this?

**Sample response:**

- **For the equation, substitute 3 for $x$ into each equation.**
  **Jack Rabbit $14 + 4(3) = 26$; 26 ft**
  **Thunderbolt $4 + 14(3) = 46$; 46 ft**
- **For the graph, determine the $y$-value when the $x$-value is 3 for each line.**
  **Jack Rabbit (3, 26); 26 ft**
  **Thunderbolt (3, 46); 46 ft**




**Historical Moment**

***m* is for . . . slope?**

Why do we use the letter "*m*" to represent slope? Some speculate that it comes from the French word *monter*, which means to climb, while others speculate that it comes from the Latin word *montagne*, for mountain. Other countries even use different letters to represent the slope of a line! For example, Sweden uses the letter "*k*" and Uruguay uses the letter "*a*." Although the meaning of slope is the same across countries, the origin of why the letter "*m*" is used is still uncertain.

STOP

280 **Unit 3** Linear Relationships



## 1 Launch

Activate students' background knowledge by asking if they have ever been on a roller coaster. Ask students how the rollercoaster's height from the ground changes as the roller coaster climbs the first hill.

## 2 Monitor

**Help students get started** by labeling the vertical intercept of each line on the graph with their ordered pairs, $(0, 4)$ and $(0, 14)$.

**Look for points of confusion:**

- **Matching the wrong equations and lines.** Remind students that in the equations $y = mx + b$ and $y = b + mx$, $b$ represents the vertical intercept and $m$ represents the slope. Additionally, students may find it helpful to substitute different values for $x$ and match the corresponding values of $y$ on the graph.

**Look for productive strategies:**

- Using the context or graph to determine the matching equation.

## 3 Connect

**Ask:**

- "How do you know each relationship can be represented with a linear equation?"
- "How is the height of the platform represented in the equation and on the graph? What about the speed of the roller coaster?"
- "Which roller coaster travels at a faster rate? How can you tell, based on the graph and equation?"

**Highlight** that although the order of the slope and $y$-intercept in the equation do not affect the line, the value of the coefficient of $x$ *does* affect the slope of the line. For positive slopes, the greater the value of the coefficient, the steeper the line.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Suggest students use color coding and annotations to highlight the slope and vertical intercept of each equation and how they are represented in the corresponding graph and verbal description.

### Extension: *Math Enrichment*

Have students complete the following problem:
Suppose a roller coaster starts from a platform of 10 ft and then climbs an additional 312 ft in 60 seconds, at a constant speed. What equation represents the height $y$, in feet, of the roller coaster above the ground given the number of seconds $x$? **$y = 10 + 5.2x$ (or equivalent)**

## Historical Moment

### *m* is for . . . slope?

Have students read the *Historical Moment* to learn about some theories of why the letter *m* is commonly used to represent the slope. Be sure students also understand that the variable $m$ can also be used to label lines.

# Summary

Review and synthesize how the vertical intercept and slope of a linear relationship appear on a graph and in an equation.

Name: ______ Date: ______ Period: ______

## Summary

**In today's lesson . . .**

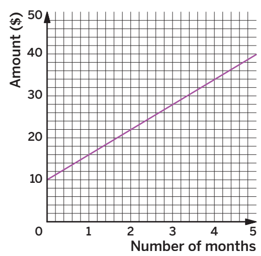
You used the slope and vertical intercept to interpret graphs of different real-world situations that represent linear relationships. The ***vertical intercept***, also called the ***$y$-intercept***, indicates where the line intersects the $y$-axis.

A linear equation can be represented using the form $y = mx + b$, where $m$ represents the slope and $b$ represents the $y$-intercept.

- For proportional linear relationships, the slope has the same value as the constant of proportionality (or unit rate).
- For nonproportional linear relationships, there is no constant of proportionality. The slope represents the constant rate of change.

Consider this graph of a line showing the amount of money paid for a music streaming service.

- The vertical intercept is $(0, 10)$. This means there was an initial cost of $10 for the service.
- The slope, 6, represents the cost of the plan per month. The equation $y = 10 + 6x$ represents the cost $y$ after $x$ months.




Reflect:





## Synthesize

**Display** the summary from the Student Edition and the Anchor Chart PDF, *Representations of Linear Relationships*.

**Have students share** how they can identify the vertical intercept and slope of a linear relationship from a graph and equation.

**Formalize vocabulary:**

- *vertical intercept*
- *$y$-intercept*

**Highlight** that the *vertical intercept* is the $b$-value in the equation $y = mx + b$ and the point $(0, b)$ on the graph where the line intersects the $y$-axis.

**Ask**, "How does a vertical intercept appear in a nonproportional linear relationship? How does it appear in a proportional relationship?" The vertical intercept is located at $(0, 0)$ for a proportional relationship. For a nonproportional relationship, the vertical intercept is located at $(0, b)$ for any value of $b$ that is not zero.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies did you find helpful today when identifying the vertical intercept and slope of a linear relationship?"

## Math Language Development

### *MLR2: Collect and Display*

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the terms *vertical intercept* and *$y$-intercept* that were added to the display during the lesson.

Independent | 5 min

# Exit Ticket

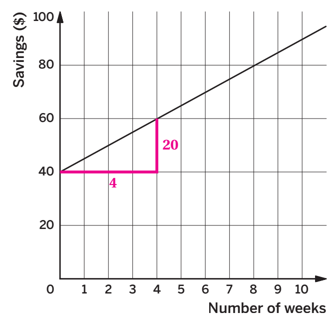
Students demonstrate their understanding by interpreting the slope and vertical intercept from the graph of a linear relationship.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 3.09

The graph shows the amount, in dollars, Andre saves in his piggy bank over time.




1. Determine the slope. Explain what it represents in this situation.
   The slope is 5. This means that Andre saves \$5 each week; Sample response: I can draw a slope triangle with a vertical distance of 20 and a horizontal distance of 4; $\frac{20}{4} = 5$.

2. Determine the vertical intercept. Explain what it represents in this situation.
   The vertical intercept is 40. This means that Andre started with \$40 in his piggy bank.

3. Write an equation that represents Andre's savings $y$ after $x$ weeks.
   $y = 5x + 40$ (or equivalent)

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can match graphs with the real-world situations they represent by identifying the slope and the vertical intercept.
1 2 3

b. I can interpret the slope and vertical intercept of a graph that represents a real-world situation.
1 2 3

c. I can use the general form of a linear equation, $y = mx + b$, to write an equation that represents a linear relationship.
1 2 3



**Lesson 9** More Linear Relationships

## Success looks like . . .

- **Language Goal:** Describing how the slope and vertical intercept influence the graph of a line. **(Speaking and Listening, Writing)**
- **Goal:** Identifying and interpreting the positive vertical intercept and slope of the graph of a linear relationship.
- **Goal:** Identifying and interpreting the positive vertical intercept and slope of the equation of a linear relationship in the form $y = mx + b$.

## Suggested next steps

**If students do not identify or interpret the slope correctly, consider:**

- Reviewing Activity 2, parts a and b.
- Drawing slope triangles.

**If students do not identify or interpret the vertical intercept correctly, consider:**

- Reviewing Activity 2, part c.
- Highlighting the $y$-axis.

**If students do not write the correct equation, consider:**

- Writing $y = \square x + \square$ and having them fill in the empty boxes with the slope and $y$-intercept.
- Reassessing after Lesson 10.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- In earlier lessons, students learned about proportional relationships. How did that support their understanding of linear relationships?
- What challenges did students encounter as they worked on Activity 1? How did they work through them?

# Practice

Independent

Practice

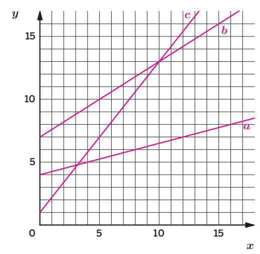
Name: ______ Date: ______ Period: ______

1. Create a graph that shows a linear relationship with each indicated slope. Then write an equation that represents the graph.

   a. Slope: $\frac{1}{4}$ $y=\frac{1}{4}x+4$ **Sample responses shown.**

   b. Slope: $\frac{3}{5}$ $y=\frac{3}{5}x+7$

   c. Slope: $\frac{6}{5}$ $y=\frac{6}{5}x+1$




2. Clare has a summer reading assignment. After reading the first 40 pages of a book, she plans to read 20 pages each day until she finishes the book. She creates the table and graph shown to track how many total pages she will read over the next few days. Clare claims after 7 days, she will have read 200 pages. Do you agree? Explain your thinking, based on the table and graph.

| Number of days, $x$ | Number of pages read, $y$ |
|---|---|
| 1 | 60 |
| 2 | 80 |
| 3 | 100 |
| 4 | 120 |
| 5 | 140 |

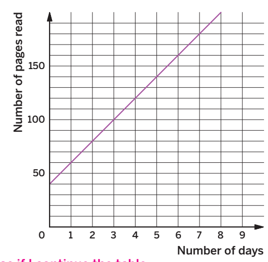



Table: **Sample response: I do not agree because if I continue the table, the pages read for 7 days will be 180.**

Graph: **Sample response: I do not agree because the point (7, 200) is not on the line.**




Name: ______ Date: ______ Period: ______

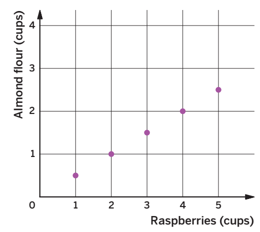
3. Explain what the slope and $y$-intercept represent in each real-world situation.

   a. The amount of money $y$ in a cash box after $x$ tickets are purchased for carnival games. The slope of the line is $\frac{1}{4}$ and the $y$-intercept is 8.
   **Sample response: The slope means that each ticket costs \$0.25. The $y$-intercept represents the amount, \$8, already in the cash box.**

   b. Han is graphing the relationship between the cost $y$ in dollars of a flower delivery and the number of flowers ordered, $x$. The slope of the line is 2, and the $y$-intercept is 3.
   **Sample response: The slope means that each flower costs 2. The $y$-intercept represents a flat delivery fee, tip, or other one-time fee.**

4. The table and graph show the amount of almond flour and fresh raspberries that are needed for each of Lin's and Noah's favorite raspberry lemon scone recipes.

**Lin's scone recipe**




**Noah's scone recipe**

| Raspberries (cups) | Almond flour (cups) |
|---|---|
| $\frac{3}{2}$ | 1 |
| 3 | 2 |
| $4\frac{1}{2}$ | 3 |

   a. If you have 6 cups of almond flour for each recipe, how many cups of raspberries would you need to make each recipe?
   Lin's recipe: **12 cups** Noah's recipe: **9 cups**

   b. If you have 5 cups of raspberries for each recipe, how many cups of almond flour do you need to make each recipe?
   Lin's recipe: **2.5 cups** Noah's recipe: **$3\frac{1}{3}$ cups**

5. Write the equation of a line that has a slope of 2 and a vertical intercept of 8.
   **$y=2x+8$**




## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 1 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 3 Lesson 6 | 2 |
| Formative | 5 | Unit 3 Lesson 10 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Representations of Linear Relationships

Let's write linear equations from context.


## Focus

### Goals

1. **Language Goal:** Create an equation that represents a linear relationship in context. **(Reading and Writing)**

2. **Language Goal:** Interpret the slope and vertical intercept of the graph of a line in context. **(Speaking and Listening)**

## Coherence

### Today

Students investigate the relationship between the total volume in a cylinder and the number of marbles added to the cylinder. They interpret the initial water volume as the vertical intercept and the slope as the rate of change, which is the amount by which the volume increases when one object is added. Students apply their understanding of linear relationships in context by writing and comparing equations with a partner.

### Previously

In Lesson 9, students explored the differences in a proportional and nonproportional relationship. They analyzed the vertical intercept and learned that the equation $y = mx + b$ represents a linear relationship.

### Coming Soon

In Lessons 11 and 14, students will develop a geometric and an algebraic method for writing the equation of a line given two points on the line.

## Rigor

- Students further their **conceptual understanding** of linear relationships by interpreting the slope and vertical intercept in a context.
- Students write equations given a description or a graph to develop **procedural fluency**.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 25 or 40 min* | 10 min | 5 min | 5 min |
| Pairs | Small Groups | Pairs | Whole Class | Independent |

*If using the digital version of Activity 1, the suggested pacing is 25 minutes. If using the print version, the suggested pacing is ~45 minutes.*

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Representations of Linear Relationships*
- rulers
- one 100 ml graduated cylinder filled with 60 ml of water per group
- 10–20 marbles (or identical objects that fit into the cylinder and do not float such as cubes, dice, etc.) per group

### Math Language Development

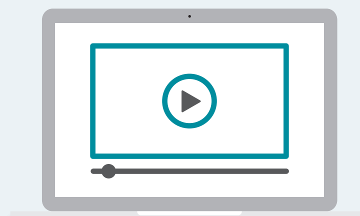
**Review words**

- linear relationship
- *slope*
- *vertical intercept*
- *$y$-intercept*

**Amps** | **Featured Activity**

### Activity 1
### Watch the Water Rise

Use this digital version of the activity to see rising water levels as marbles are added to a cylinder.



Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students who are more confident with this work may be able to lead discussions with their partner. Remind students to 'step up' if they have something to add to the conversation, but also to 'step back' to give other voices a chance to share.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- For **Activity 1**, begin with a demonstration using the digital version of the task. Record the measurements for all to see. After students have the information for Problem 1, have them complete the remaining problems in small groups.
- Omit **Activity 2**.

Pairs | 5 min

# **Warm-up** Can You Guess the Game?

Students watch an animation to pique their interest in a real-world linear relationship.

Unit 3 | Lesson 10

## Representations of Linear Relationships

Let's write linear equations from context.


**Warm-up** Can You Guess the Game?

You will be shown an animation of a game. How do you think the game is played?

Sample response: Each person adds a marble to a cylinder that contains some water. The person whose water reaches the top of their cylinder first is the winner.



Log in to Amplify Math to complete this lesson online.



### 1 Launch

Activate students' background knowledge by asking them if they have played a game at a carnival and how the games were played. Then display the animation, *Guess the Game*, from the Warm-up Amps slides.

### 2 Monitor

**Help students get started** by asking them to observe the number of marbles and water level.

**Look for productive strategies:**

- Noticing that as the number of marbles increases, the water level increases.
- Noticing that the goal of the game is for the water level to reach the top of the cylinder.

### 3 Connect

**Have students share** how they think the game is played.

**Ask**, "How can you describe the relationship between the water level and the number of marbles?" Answers may vary. Guide students towards realizing that the relationship is linear.

**Highlight** that each marble added increases the water level in the cylinder by the same amount, so the relationship is linear.

## Power-up

**To power up students' ability to write linear equations of the form $y = mx + b$, have students complete:**

Recall that linear equations can be written of the form $y = mx + b$ where $m$ represents the slope and $b$ represents the vertical intercept.

Write the equation of the line with a slope of $\frac{2}{3}$ and vertical intercept of 4.

$y = \frac{2}{3}x + 4$

**Use:** Before Activity 1

**Informed by:** Performance on Lesson 9, Practice Problem 5

Small Groups | 25 or 40 min*

# Activity 1 Rising Water Levels

Students analyze a linear relationship for data gathered in context and interpret the slope and vertical intercept of the equation that represents the relationship.

**Amps Featured Activity** Watch the Water Rise

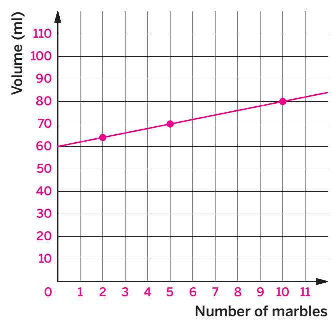
Name: ________ Date: ________ Period: ________

## Activity 1 Rising Water Levels

**You will be given the materials for this activity.**

1. What is the volume, in milliliters, measured by the water level in the cylinder after you add: **Sample response based on the digital version of this activity.**
   a. 2 marbles? **64 ml**
   b. 5 marbles? **70 ml**
   c. 10 marbles? **80 ml**
   d. $n$ marbles? **$60 + 2n$**

2. Use your responses from Problem 1 to plot three points on the graph. Then draw a line through the points. What patterns do you notice?
   **Sample responses:**
   - **As the number of marbles increases, the volume increases at a constant rate.**
   - **The relationship appears to be linear.**
   - **The line appears to intersect the $y$-axis at the point (0, 60).**




3. Write an equation for the volume $v$, after $n$ marbles are added to the cylinder. Explain what each number in your equation represents in this situation.
   **Sample response: $v = 2n + 60$; The value 2 represents the amount the volume increased in milliliters for each marble added to the cylinder. The value 60 represents the amount of water, in milliliters, before any marbles were added.**

4. If you wanted the water to reach the highest mark on the cylinder, how many marbles would you need to add? Explain your thinking.
   **Sample response: The highest mark on the cylinder represents 100 ml. I substituted 100 for $v$ in my equation and solved for $n$.**
   $v = 100$
   $100 = 2n + 60$
   $40 = 2n$
   $n = 20$
   **20 marbles**





### 1 Launch

Distribute one graduated cylinder filled with 60 ml of water along with 10–20 marbles and rulers to each group. For a shorter 25 minute activity, use the Activity 2 Amps slides. Have students label the $x$-axis using the variable $n$ to represent the number of marbles and the $y$-axis using the variable $v$ to represent the volume.

### 2 Monitor

**Help students get started** by modeling how to add marbles and read the water level on the cylinder.

**Look for points of confusion:**

- **Not knowing how to write the volume for $n$ marbles or how to write the equation for Problem 3.** Ask, "What was the initial volume? How much does one marble increase the volume?"
- **Not understanding why a line is drawn.** Tell students that although the intermediate points do not make sense for the context, it helps to better explore and understand the situation.
- **Not knowing how to solve Problem 4.** Have students use their equation and substitute the highest mark on the cylinder for $v$. Then have them solve the equation.

### 3 Connect

**Have groups of students share** their methods for writing the equation.

**Ask**, "How can you determine the slope and vertical intercept from the equation and graph? What do they represent in this context?"

**Highlight** that in this situation, the slope represents the volume increase in milliliters for each marble added to the cylinder, and the vertical intercept represents the initial amount of water in the cylinder, in milliliters.

## Differentiated Support

### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see the rising water levels as marbles are added to a virtual cylinder. This will allow them to access the mathematical goal of this activity, without having to use the actual physical objects and water.

### Extension: *Interdisciplinary Connections*

Ask students what happens when they add ice to a full glass of water. This is *water displacement*, which happens when an object is submerged in water, or another liquid. The object takes the place where the water used to be and the water level rises. The volume of the displaced water is equal to the volume of the submerged object. **(Science)**

## Math Language Development

### *MLR2: Collect and Display*

As students collect their data, listen for and record the vocabulary they use to describe what happens to the water level as they add additional marbles. Amplify phrases that relate to *volume, rate, slope,* and *vertical intercept*. Continue adding to this display in Activity 2.

### English Learners

Use gestures and pointing to connect mathematical terminology to the graph. For example, point to the vertical intercept and say, "The vertical intercept is 60." Annotate the graph by labeling the vertical intercept.

Pairs | 10 min

# Activity 2 Partner Problems

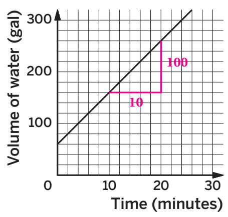
Students write a linear equation that represents a linear relationship in context to develop procedural fluency.

### Activity 2 Partner Problems

**With your partner, decide who will complete Column A and who will complete Column B. After each row, share your response with your partner. Although the problems in each row are different, your responses should be the same. If they are not the same, work together to correct any errors or resolve any disagreements.**

<table>
<tr><th>Column A</th><th>Column B</th></tr>
<tr><td>

1. The manager of a carnival fills a dunk tank with water. The graph shows the volume of water in the tank as it is filled.




Write an equation that gives the volume of water $w$ in the dunk tank after $t$ minutes.

Sample response: $w = 10t + 60$

</td><td>

A dunk tank at a carnival has 60 gallons of water in it when the manager begins to fill the tank with a hose. Water fills the tank at a rate of 10 gallons per minute.

Write an equation that gives the volume of water $w$ in the dunk tank after $t$ minutes.

Sample response: $w = 60 + 10t$

</td></tr>
<tr><td>

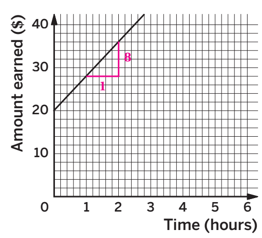
2. Han operates the bumper cars at a carnival. On Independence Day, he earned \$8 per hour, plus a \$20 bonus for working on a holiday.

Write an equation for the amount $a$ Han earned for working $h$ hours on Independence Day.

Sample response: $a = 8h + 20$

</td><td>

Han operates the bumper cars at a carnival. The graph shows the amount he earned working on Independence Day.




Write an equation for the amount $a$ Han earned for working $h$ hours on Independence Day.

Sample response: $a = 8h + 20$

</td></tr>
</table>

STOP

286 Unit 3 Linear Relationships



## 1 Launch

Activate students' background knowledge by asking them about activities at a carnival. Provide information about dunk tanks and bumper cars if needed. Then conduct the ***Partner Problems*** routine.

## 2 Monitor

**Help students get started** by reminding them to use the Anchor Chart PDF, *Representations of Linear Equations*, if they need help.

**Look for points of confusion:**

- **Writing the incorrect equation from a graph.** Have students draw a slope triangle and circle the vertical intercept. Then provide the equation $w = \square t + \square$ or $a = \square h + \square$ and have them complete the equation using the slope and vertical intercept.
- **Writing the incorrect equation from the description.** Consider providing the equation $y =$ rate of change $\cdot\ x\ +$ initial value.

**Look for productive strategies:**

- Simplifying slopes to check whether their equation is equivalent to their partner's equation.

## 3 Connect

**Ask,** "How does the slope and vertical intercept appear in a graph? How does the slope and vertical intercept appear in the equation?"

**Highlight:**

- The values $\frac{20}{10}$ and 2 both correctly define the slope, but the slope is often simplified in the equation.
- Linear equations are typically written in the form $y = mx + b$, but equations written in the form $y = b + mx$ are equivalent.

## Differentiated Support

**Accessibility: *Guide Processing and Visualization***

Display or provide copies of the Anchor Chart PDF, *Representations of Linear Relationships* for students to refer to as they complete this activity. Provide access to colored pencils and suggest students use color coding and annotations to color code the slope in one color and the vertical intercept in another color, across the various representations.

**Extension: *Math Enrichment***

Have students write a story context that can be represented by the equation $y = 30x + 100$.

Sample response: Priya opens a savings account and deposits \$100. Each month, she deposits \$30.

# Summary

Review and synthesize how to write an equation that represents a linear relationship in context.

Name: ______________________ Date: __________ Period: __________

## Summary

**In today's lesson . . .**

You wrote linear equations from different representations: verbal descriptions of real-world situations and graphs.

For example, in the marble activity, you graphed the relationship between the number of marbles and the volume of water in a cylinder. You interpreted the initial water volume as the $y$-intercept and the slope as the rate of change, or the amount the volume increased when one marble was added. Writing an equation helped you determine how many marbles are needed for the water to reach the top level of the cylinder.

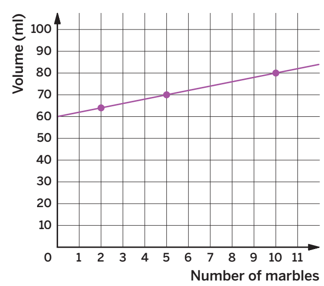
**Scenario:**

A cylinder contains 60 ml of water. Every marble that is added increases the volume of the water by 2 ml.

**Equation:**

$v = 2n + 60$, where $n$ represents the number of marbles and $v$ represents the volume of water, in milliliters.

**Graph:**




**Reflect:**





## Synthesize

**Display** the Summary from the Student Edition.

**Have students share** how to write an equation from a context in their own words.

**Highlight** that the slope represents the rate of change and the vertical intercept represents the initial amount, the value of $y$, when $x = 0$.

**Ask:**

- "Why can the line given in the Summary be written using a linear equation?" The graph is a straight line, so the relationship is linear.
- "How can writing a linear equation help you to solve a problem?" Sample response: Once I know the equation, I can substitute a known value for one quantity and solve the equation to find the corresponding unknown value for the other quantity.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What helped you write equations from a context?"

Independent | 5 min

## Exit Ticket

Students demonstrate their understanding by writing an equation that represents a linear relationship in context.

Printable

Name: ______ Date: ______ Period: ______

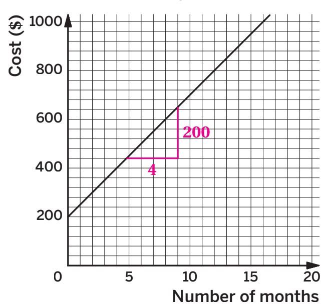
### Exit Ticket 3.10

**Elena pays $200 for a cell phone and $50 each month for service.**

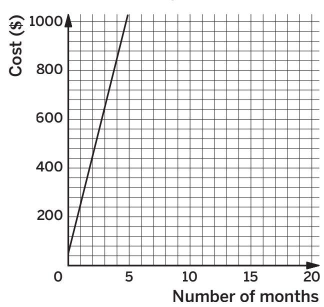
1. Write an equation for the cost $c$ after $n$ months of service for his phone.
   $c = 50n + 200$

2. Which graph shows the total cost of the phone after $n$ months of service? Explain your thinking.

Graph A




Graph B




Graph A; Sample response: The value $200 represents the initial cost, which is represented by the vertical intercept, the point located at (0, 200). The line has slope of $\frac{200}{4}$, which represents the rate Elena pays, $50 per month.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can represent a real-world situation using a linear equation or graph.
1 2 3

b. I understand the connections between a verbal description, graph, and equation of a linear relationship.
1 2 3



**Lesson 10** Representations of Linear Relationships

### Success looks like . . .

- **Language Goal:** Creating an equation that represents a linear relationship in context. **(Reading and Writing)**
- **Language Goal:** Interpreting the slope and vertical intercept of the graph of a line in context. **(Speaking and Listening)**

### Suggested next steps

**If students do not write the correct equation for Problem 1, consider:**

- Having them create a table showing the cost for the first 5 months, and then looking for a pattern.
- Reassessing after Lesson 12.

**If students do not choose the correct graph for Problem 2, consider:**

- Labeling the cell phone cost as the "initial cost" and highlighting the phrase "$50 each month."

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- In what ways did Activity 1 go as planned?
- In what ways in Activity 1 did unexpected things happen?

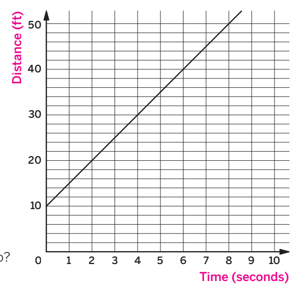
# Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. Consider the graph of the line shown.




a. Write a real-world situation that could be represented by the line. Label the axes on the graph.

**Sample response: Mai races her friend and gets a 10-ft head start. She runs at a speed of 5 ft per second.**

b. What equation describes the relationship between the two variables in your scenario?

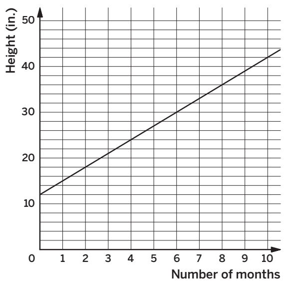
**Sample response: $d = 5t + 10$**

c. Explain what each number and variable in your equation represents in this situation.

**Sample response:**

- **The variable $d$ represents the distance.**
- **The number 10 represents the 10-ft head start.**
- **The number 5 represents the running rate in feet per second**
- **The variable $t$ represents the time in seconds.**

2. The graph shows the height $h$ in inches of a bamboo plant $n$ months after it has been planted.




a. Write an equation that gives the bamboo's height $h$ after $n$ months.

$h = 3n + 12$

b. After how many months will the bamboo plant be 66 in. tall? Show or explain your thinking.

**18 months; I substituted 66 for $h$ in my equation and solved for $n$.**

$66 = 3n + 12$

$54 = 3n$

$n = 18$





Name: ______ Date: ______ Period: ______

Practice

3. A taxi company charges an initial fee of \$2.50, and then \$2 per mile. Which equation represents the total cost $c$, after $x$ miles? Select *all* that apply.

A. $c = 2 + 2.50x$

B. $c = 2x + 2.50$

C. $x = 2 + 2.50c$

D. $x = 2c + 2.5$

E. $c = 2.5 + 2x$

4. Tyler and Jada each have a favorite banana bread recipe using slightly different amounts of mashed bananas and honey. The number of cups of mashed bananas is proportional to the number of cups of honey.

**Tyler's recipe:**

| Honey (cups) | Bananas (cups) |
|---|---|
| $\frac{1}{2}$ | $\frac{3}{4}$ |
| $2\frac{1}{2}$ | $3\frac{3}{4}$ |
| 3 | $4\frac{1}{2}$ |

**Jada's recipe:**

The relationship between the number of cups of mashed bananas $y$ and the number of cups of honey $x$ is represented by the equation $y = \frac{7}{4}x$.

a. If you have 4 cups of honey, how many cups of mashed bananas would you need to make each recipe?

**Tyler's recipe:** **6 cups**

**Jada's recipe:** **7 cups**

b. What is the rate of change for each recipe, and what does it mean within this context?

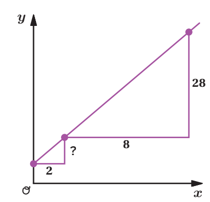
**Tyler's recipe:**

**The rate of change is $\frac{3}{2}$. This means there are $\frac{3}{2}$ cups of mashed bananas per 1 cup of honey.**

**Jada's recipe:**

**The rate of change is $\frac{7}{4}$. This means there are $\frac{7}{4}$ cups of mashed bananas per 1 cup of honey.**

5. Refer to the slope triangles shown. What is the unknown vertical side length? Explain your thinking.




**7**

**Sample response: The two triangles are similar. I determined the scale factor, 4, by comparing the ratio of corresponding side lengths. Then I used the scale factor to determine the unknown side length: $28 \div 4 = 7$.**





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 2 | 1 |
| Spiral | 4 | Unit 3 Lesson 6 | 1 |
| Formative | 5 | Unit 3 Lesson 11 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 3 | **LESSON 11**

# Writing Equations for Lines Using Two Points


Let's write an equation for a line that passes through two points.

## Focus

### Goals

1. Create an equation of a line with a positive slope on a coordinate plane using knowledge of similar triangles.
2. **Language Goal:** Justify that a point $(x, y)$ is on a line by verifying that the values of $x$ and $y$ satisfy the equation of the line. **(Speaking and Listening)**

## Coherence

### Today

Students extend their work with slope triangles to develop a method for calculating the slope using any two points on a line. They use a geometric method to write an equation of a line given two points on the line. Students then use their equations to justify whether a point is on the line.

### Previously

In Lesson 10, students created an equation that represented a linear relationship in context, and then interpreted the slope and $y$-intercept.

### Coming Soon

In Lesson 14, students will generate an algebraic method to determine the equation of line given two points on the line.

## Rigor

- Students write the equation of a line using two points and similar triangles to strengthen their **fluency** in writing linear equations.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Activity 3 (optional) | Summary | Exit Ticket |
|---|---|---|---|---|---|
| 10 min | 10 min | 15 min | 12 min | 5 min | 8 min |
| Pairs | Small Groups | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

**Amps** | Featured Activity

**Activity 3**
**Through the Tunnel**

Students enter an equation that will calculate the roller coaster's path, and revise their response as needed.

### Materials

- Exit Ticket
- Additional Practice
- calculators
- rulers

### Math Language Development

**Review words**

- *linear relationship*
- *slope*
- *$y$-intercept*
- *similar triangles*
- *vertical intercept*

### Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel lost if they do not make the connection between slope, similar triangles, and writing the equation of a line. Ask them to engage in metacognitive functions, i.e., thinking about their own thinking process. For example, have them conduct their own ***Notice and Wonder*** routine, which will help them record their thought processes.

### Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- Optional **Activity 3** may be assigned as additional practice.

Pairs | 10 min

# Warm-up Coordinates and Lengths in the Coordinate Plane

Students determine unknown coordinates of points using slope triangles to further explore the relationship between vertical and horizontal side lengths of slope triangles.

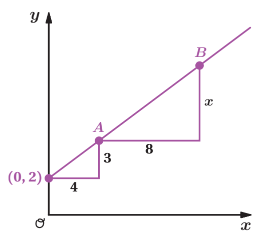
Unit 3 | Lesson 11

## Writing Equations for Lines Using Two Points

Let's write an equation for a line that passes through two points.


### Warm-up Coordinates and Lengths in the Coordinate Plane

Consider line $AB$ and the two slope triangles shown.




1. Determine the vertical side length $x$ of the larger triangle. Explain your thinking.

   $x = 6$; Sample response: The two triangles are similar, so the ratio of the vertical to horizontal side lengths of the triangles are equivalent: $\frac{3}{4} = \frac{x}{8}$, $x = 6$.

2. What are the coordinates of points $A$ and $B$? Explain your thinking.

   $A(4, 5)$; Sample response: The smaller triangle has a horizontal side length of 4 units. The $y$-coordinate of point $A$ is 5 because the vertical side length of the triangle is 3 units, and adding the 2 units that represent the distance from the $x$-axis yields a total distance from the $x$-axis of 5 units.

   $B(12, 11)$; Sample response: The $x$-value represents the sum of the horizontal side lengths of both triangles, $8 + 4 = 12$. The $y$-value represents the sum of the vertical side lengths of both triangles, plus the additional 2 units: $6 + 3 + 2 = 11$.

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Activate students' background knowledge by asking them to identify how slope triangles are used to analyze linear equations on a graph.

## 2 Monitor

**Help students get started** by asking them how to calculate an unknown side length given two similar triangles.

**Look for points of confusion:**

- **Not knowing how to determine the coordinates of points $A$ or $B$.** Ask students to determine the vertical distance between point $A$ and the $x$-axis.
- **Thinking that the coordinates of point $A$ are $(4, 3)$.** Tell students that the line does not represent a proportional relationship. Then highlight the vertical distance below the smaller triangle to emphasize the additional vertical distance of 2 units to the $x$-axis.

**Look for productive strategies:**

- Examining the horizontal lengths of the triangles to determine the $x$-coordinate.
- Examining the vertical lengths of the triangles to determine the $y$-coordinate.

## 3 Connect

**Have pairs of students share** their strategies for determining the coordinates of points $A$ and $B$.

**Highlight** that even if there are no gridlines on a graph, students can use the coordinates of points or lengths of similar triangles to determine unknown values.

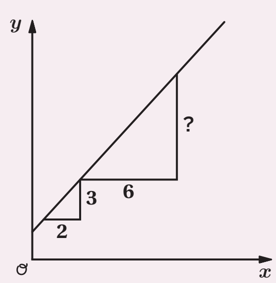
**Ask:**

- "Which coordinate value is affected by a horizontal change?" $x$ "Which coordinate value is affected by a vertical change?" $y$
- "How do you think you can write the equation of a line that is on a coordinate plane with no grid lines?"

## Power-up

**To power up students' ability to determine the unknown length in a slope triangle using reasoning about similar triangles, have students complete:**

Recall that the ratio of the vertical length to the horizontal length is constant for any slope triangle on a given line.




1. What is the ratio of the vertical length to the horizontal length in the smaller slope triangle? $\frac{3}{2}$
2. What is the unknown vertical length in the larger slope triangle? Be prepared to explain your thinking. 9; Sample response: The horizontal side of 2 is tripled to make 6 so 3 must be tripled to make a length of 9.

**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 10, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 7

Small Groups | 10 min

# Activity 1 Calculate the Slope

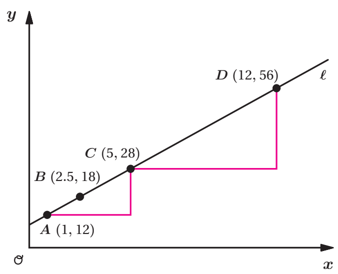
Students develop a method to calculate slope without a grid, seeing that they can use coordinates of *any* two points to calculate the slope of the line connecting these points.

Name: ______ Date: ______ Period: ______

**Activity 1** Calculate the Slope

**Plan ahead:** How will you encourage your partner to do their best work?

**Line $\ell$ is shown on the coordinate plane. Several points are marked on the line.**

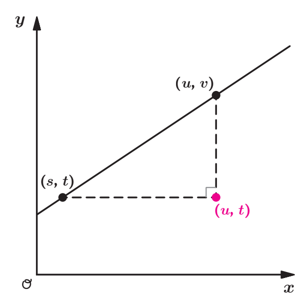



1. Choose two points on the line that are different from your partner. Using these two points, draw a slope triangle. Then determine the slope of the line $\ell$.

| | | | | | | |
|---|---|---|---|---|---|---|
| **Points:** | $A$ and $B$ | $A$ and $C$ | $A$ and $D$ | $B$ and $C$ | $B$ and $D$ | $C$ and $D$ |
| **Slope:** | $\frac{6}{1.5} = 4$ | $\frac{16}{4} = 4$ | $\frac{44}{11} = 4$ | $\frac{10}{2.5} = 4$ | $\frac{38}{9.5} = 4$ | $\frac{28}{7} = 4$ |

2. Compare your work with your partner. What do you notice?
   **Sample response: The slopes are equivalent.**

3. Describe a method for calculating the slope between *any* two points on a line. Use the diagram if it helps your thinking.
   **Sample response:**
   slope $= \frac{\text{vertical change}}{\text{horizontal change}} = \frac{v-t}{u-s}$








## 1 Launch

In groups of three, have each student choose a different pair of coordinates to complete the problems. Provide access to rulers for the duration of the lesson.

## 2 Monitor

**Help students get started** by asking them to calculate the difference in the values of $x$ to determine the horizontal length and the difference in the values of $y$ to determine the vertical length for each of their slope triangles.

**Look for points of confusion:**

- **Not observing patterns for Problem 2.** Have students simplify and compare any fractions.

**Look for productive strategies:**

- Noticing that they can use any two points on a line to calculate the slope of a line.
- Writing the slope as $\frac{v-t}{u-s}$ or $\frac{t-v}{s-u}$.

## 3 Connect

**Have groups of students share** what they noticed and their methods for calculating the slope between any two points.

**Ask**, "Does order matter when you subtract the values of $x$ and $y$?" No, as long as you follow the same order for subtracting coordinates.

**Highlight** that students can divide the difference in the values of $y$ by the difference in the values of $x$ using coordinates of *any* two points to calculate the slope. However, note that it is important to subtract the $x$-coordinates for the two points in the same order as the $y$-coordinates.

**Note:** If you would like to formally introduce the traditional formula invoking subscripts you may do so here, but students are encouraged to use a method that works for them.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Have students use points $A$ and $C$ for Problem 1. Then pause to facilitate a class discussion with Problem 2, using class responses to Problem 1.

### Extension: *Math Enrichment*

Have students use the diagram in Problem 3 to write a different, yet equivalent expression that represents the slope of the given line.
Sample response: $\frac{t-v}{s-u}$

## Math Language Development

### *MLR8: Discussion Supports—Press for Details*

During the Connect, as students respond to the Ask question, press for details in their reasoning. For example, if they say "The order does not matter," ask these additional questions, using points $A$ and $C$:

- "Subtract the $y$-values in any order. What are the possible differences?"
  $28 - 12 = 16$ and $12 - 28 = -16$
- "Subtract the $x$-values in any order. What are the possible differences?
  $5 - 1 = 4$ and $1 - 5 = -4$
- "What are the only two ways to determine the ratio of these differences so that the slope is 4?" Either $\frac{16}{4}$ or $\frac{-16}{-4}$.

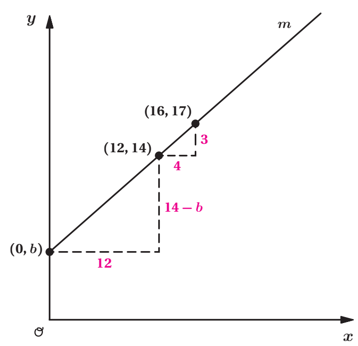
Pairs | 15 min

# Activity 2 Writing an Equation From Two Points

Students apply their knowledge of similar triangles to write an equation of a line, and then use the equation to check whether a point is on the line.

### Activity 2 Writing an Equation From Two Points

**Line $m$ is shown on the coordinate plane. Several points are marked on the line.**




1. Label the horizontal and vertical side lengths of each slope triangle so that they have a number or expression representing their lengths.

2. Use what you know about similar triangles to calculate the value of $b$. Show or explain your thinking.
   $b = 5$; Sample response: I used the ratio of side lengths of the smaller triangle, $\frac{3}{4}$, to find the vertical side in the larger triangle: $\frac{14-b}{12} = \frac{3}{4}$, so, $14 - b = 9$, $b = 5$.

3. Identify the slope and $y$-intercept. Then write an equation for the line.
   The slope is $\frac{3}{4}$, the $y$-intercept is 5. The equation for the line is $y = \frac{3}{4}x + 5$.

4. Are the following points on the line? Explain your thinking.
   a. $(24, 23)$
   Yes; Sample response: The equation $23 = 5 + \frac{3}{4}(24)$ is true.
   b. $(100, 80)$
   Yes; Sample response: The equation $80 = 5 + \frac{3}{4}(100)$ is true.
   c. $(60, 45)$
   No; Sample response: The equation $5 + \frac{3}{4}(60) = 45$ is not true.




## 1 Launch

Ask students what they need to know to write the equation of a line in the form $y = mx + b$. The slope and $y$-intercept. Have students complete Problems 1–3 in pairs, discuss the equation as a whole class, and then complete Problem 4 independently. Provide access to calculators for the duration of the lesson.

## 2 Monitor

**Help students get started** by having them calculate the vertical side length of the smaller slope triangle.

**Look for points of confusion:**

- **Not knowing how to write an expression for an unknown length in Problem 1.** Next to each calculated length, ask students how they determined the length. Use the expressions $17 - 4$, $16 - 12$, and $12 - 0$ to have students look for and make use of structure in order to write $14 - b$ for the unknown vertical length.
- **Having trouble determining the value of $b$.** Have students write an equation using the ratios of the horizontal and vertical side lengths of the similar slope triangles, then solve the equation for $b$.
- **Not being sure how to determine if a point is on a line.** Have students use the equation and substitute the value of $x$, and then compare the value of $y$ with their answer to check for equality.

## 3 Connect

**Have students share** their strategies for writing the equation of the line using similar triangles and coordinates of points.

**Highlight** that students can use two points and similar triangles to determine the slope and $y$-intercept of a line and write its equation. They can verify if a point is on the line by substituting the values of $x$ and $y$ in the equation to see if the equation holds true.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Demonstrate how to label the vertical and horizontal side lengths of the slope triangles. Consider writing each side length as a subtraction expression before simplifying it, so that students can visualize how to write the expression $14 - b$.

## Math Language Development

### *MLR5: Co-craft Questions*

Before revealing the problems in this activity, display the introductory text and the graph. Ask students to work with a partner to write 1–2 mathematical questions they have about the graph. Ask pairs of students to share their questions with the class.

### English Learners

Display a sample question, such as "What is the value of $b$?" or "What is the slope of the line?"

**Optional**

Pairs | 12 min

# Activity 3 Through the Tunnel

Students write the equation of a line using two points to develop procedural fluency.

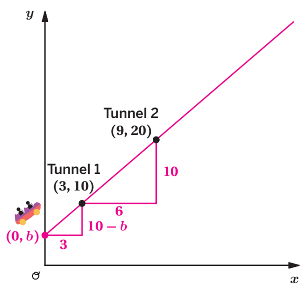
**Amps Featured Activity** Through the Tunnel

Name: ______ Date: ______ Period: ______

## Activity 3 Through the Tunnel

For each graph, write an equation that represents the line so that the roller coaster passes through the tunnels marked by the coordinates of points. Show or explain your thinking.

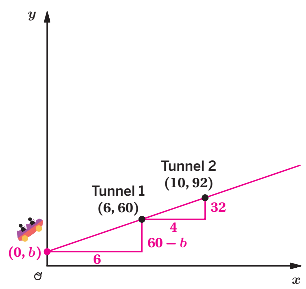
1.




$y = \frac{5}{3}x + 5$

Sample response: I drew slope triangles and used the ratio of side lengths of the larger triangle, $\frac{10}{6} = \frac{5}{3}$, to find the vertical side length in the smaller triangle: $\frac{10-b}{3} = \frac{5}{3}$, so $10 - b = 5$, $b = 5$.

2.




$y = 8x + 12$

Sample response: I drew slope triangles and used the ratio of side lengths of the smaller triangle, $\frac{32}{4} = 8$, to find the vertical side length in the larger triangle: $\frac{60-b}{6} = 8$, so $60 - b = 48$, $b = 12$.

**Are you ready for more?**

A line passes through the point $(2, 3)$ and has a slope of $\frac{1}{4}$. Write the coordinates for another point that lies on the same line. Then write an equation for the line.

Sample response: $(6, 4)$, $y = \frac{1}{4}x + 2.5$

STOP





## 1 Launch

Tell students their goal is to write an equation of a line so that the roller coaster passes through the tunnel.

## 2 Monitor

**Help students get started** by having them draw a line that passes through the points and labeling the $y$-intercept as $(0, b)$.

**Look for points of confusion:**

- **Having trouble calculating the $y$-intercept.** Have students draw two slope triangles using the $y$-intercept and two points, then label the vertical and horizontal lengths with a value or expression. Remind them to use their knowledge of similar triangles to determine an unknown value.

## 3 Connect

**Display** student work showing their responses. Discuss any discrepancies in student work and possible reasons for the discrepancies.

**Have students share** their strategies for writing the equations.

**Ask** students how they can calculate the slope of the line and use it to determine the $y$-intercept.

**Note:** Students will investigate another method to write the equation of a line using two points in Lesson 14.

**Highlight** that students can write an equation of a line if they know the coordinates of two points on the line.

## Differentiated Support

### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can enter an equation that will calculate the roller coaster's path and revise their equation as needed.

### Accessibility: *Guide Processing and Visualization*

Provide a checklist of steps students can use for this activity. For example:

**Step 1:** Draw a line through the points.

**Step 2:** Label the $y$-intercept $(0, b)$.

**Step 3:** Draw slope triangles and label the vertical and horizontal side lengths with a value or expression.

**Step 4:** Use your knowledge of similar triangles to calculate $b$.

**Step 5:** Write your equation using the slope and $y$-intercept.

Whole Class | 5 min

# Summary

Review and synthesize how to write an equation of a line using two points and similar triangles.

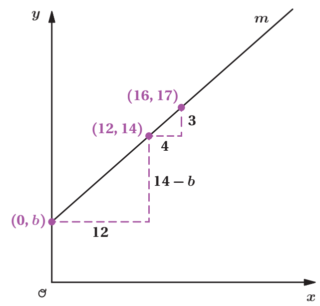
## Summary

**In today's lesson . . .**

You discovered a method for calculating the slope between any two points. You also applied your understanding of similar triangles to write the equation of a line passing through two given points.

For example, because the two triangles shown are similar, the ratios of corresponding side lengths are equivalent, $\frac{3}{4} = \frac{14-b}{12}$. Because $\frac{3}{4} = \frac{9}{12}$, this means that $14 - b = 9$ and $b = 5$. The $y$-intercept is 5.

Now you can use the slope, $\frac{3}{4}$, and $y$-intercept, 5, to write an equation for the line: $y = \frac{3}{4}x + 5$.




Reflect:




## Synthesize

**Have students share** how they can write an equation of a line using two points in their own words.

**Highlight** that students can draw a line and similar triangles to determine the slope and $y$-intercept of a line.

**Display** the Summary from the Student Edition.

**Ask**, "How do you know if a point is on the line?" Sample response: I can substitute the $x$ and $y$ values in the equation.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful today when writing an equation of a line?"

Independent | 8 min

# Exit Ticket

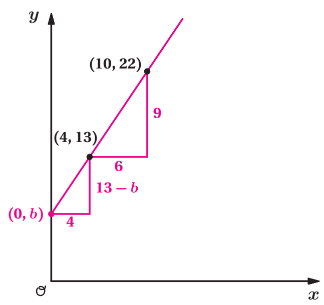
Students demonstrate their understanding of slope by writing an equation of a line given two points and determining whether an additional point is on that line.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 3.11

**Refer to the graph of the line shown.**




1. Write the equation for the line that passes through the points $(4, 13)$ and $(10, 22)$. Show or explain your thinking.
   **The equation for the line is $y = \frac{3}{2}x + 7$; Sample response: The slope is $\frac{9}{6} = \frac{3}{2}$. I used the ratio of side lengths in the larger triangle, $\frac{3}{2}$, to find the vertical side length in the smaller triangle: $\frac{13-b}{4} = \frac{3}{2}$, so $13 - b = 6$, which means $b = 7$. The $y$–intercept is 7.**

2. Is the point $(50, 82)$ on the line? Show or explain your thinking.
   **Yes; Sample response: The equation $82 = \frac{3}{2}(50) + 7$ is true.**

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can determine the slope of a line given two points that lie on the line.
1 2 3

b. I can write an equation for a line using slope triangles and use the equation to determine whether a point is on the line.
1 2 3





## Success looks like . . .

- **Goal:** Creating an equation of a line with positive slope on a coordinate plane using knowledge of similar triangles.
- **Language Goal:** Justifying that a point $(x, y)$ is on a line by verifying that the values of $x$ and $y$ satisfy the equation of the line. **(Speaking and Listening)**

## Suggested next steps

**If students calculate the slope incorrectly, consider:**

- Reviewing Activity 1.
- Having students draw slope triangles.

**If students do not write the correct $y$-intercept, consider:**

- Having students compare the similar triangles with labeled lengths to determine a scale factor.
- Reassessing after Lesson 14.

**If students do not know how to determine whether a point is on a line, consider:**

- Reviewing how to substitute values into an equation and solve the equation.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- During the discussion about calculating slope between two points, how did you encourage each student to share their understanding?
- What did students find frustrating when writing the equation of a line? What helped them work through this frustration?

# Practice

Independent

Name: Date: Period:

Practice

1. For each pair of points, calculate the slope of the line that passes through both points.

   a. (1, 1) and (7, 5)
   $\frac{5-1}{7-1}=\frac{4}{6}$ or $\frac{2}{3}$

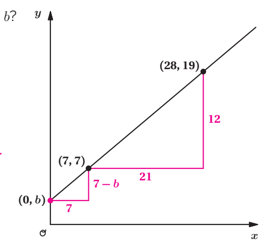
   b. (1, 1) and (5, 7)
   $\frac{7-1}{5-1}=\frac{6}{4}$ or $\frac{3}{2}$

   c. (2, 3) and (5, 7)
   $\frac{7-3}{5-2}=\frac{4}{3}$

2. Refer to the graph shown. What is the value of $b$? Show or explain your thinking.

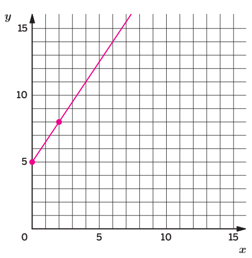
   **$b=3$; Sample response: First, I drew similar slope triangles and calculated the slope of each.**
   - **Slope of the larger triangle: $\frac{19-7}{28-7}=\frac{12}{21}=\frac{4}{7}$**
   - **Slope of the smaller triangle: $\frac{7-b}{7}$**

   **Then I used the ratio of the side lengths to determine the vertical side length in the smaller triangle: $\frac{4}{7}=\frac{7-b}{7}$, so $7-b=4$, $b=3$.**




3. Select *all* the points that are on the line that passes through the points (0, 5) and (2, 8). Use the graph to help with your thinking.

   A. (4, 11)
   B. (5, 10)
   C. (6, 14)
   D. (30, 50)
   E. (40, 60)








Name: Date: Period:

Practice

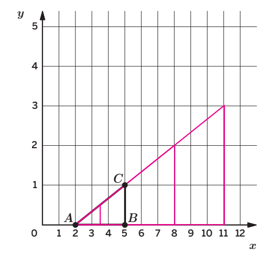
4. Consider Triangle $ABC$.

   a. Dilate Triangle $ABC$ using $A$ as the center of dilation and a scale factor of 2.

   b. Dilate Triangle $ABC$ using $A$ as the center of dilation and a scale factor of 3.

   c. Dilate Triangle $ABC$ using $A$ as the center of dilation and a scale factor of $\frac{1}{2}$.

   d. What are the coordinates of the image of point $C$ when Triangle $ABC$ is dilated using $A$ as the center of dilation and a scale factor $s$?
   $(2+3s, s)$




5. Andre says, "I found two figures that are congruent, so they cannot be similar." Diego says, "No, they are similar! The scale factor is 1." Which friend is correct? Use the definition of similarity to explain your thinking.

   **Diego is correct; Sample response: Congruent figures are also similar. Similar figures have corresponding side lengths that are proportional by a common scale factor. If they are congruent, then the side lengths have a scale factor of 1.**

6. Line segment $AB$ has a length of 4 units. It is translated up 5 units. Which of the statements is true about the image, $A'B'$, after the translation is applied? Select *all* that apply.

   A. Line segment $A'B'$ has a length of 5 units.
   B. Line segment $A'B'$ has a length of 4 units.
   C. Line segment $AB$ is congruent to line segment $A'B'$.
   D. Line segment $AB$ is longer than line segment $A'B'$.
   E. Line segment $A'B'$ is longer than line segment $AB$.





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 1 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 2 Lesson 5 | 3 |
| | 5 | Unit 2 Lesson 7 | 2 |
| Formative | 6 | Unit 3 Lesson 12 | 1 |

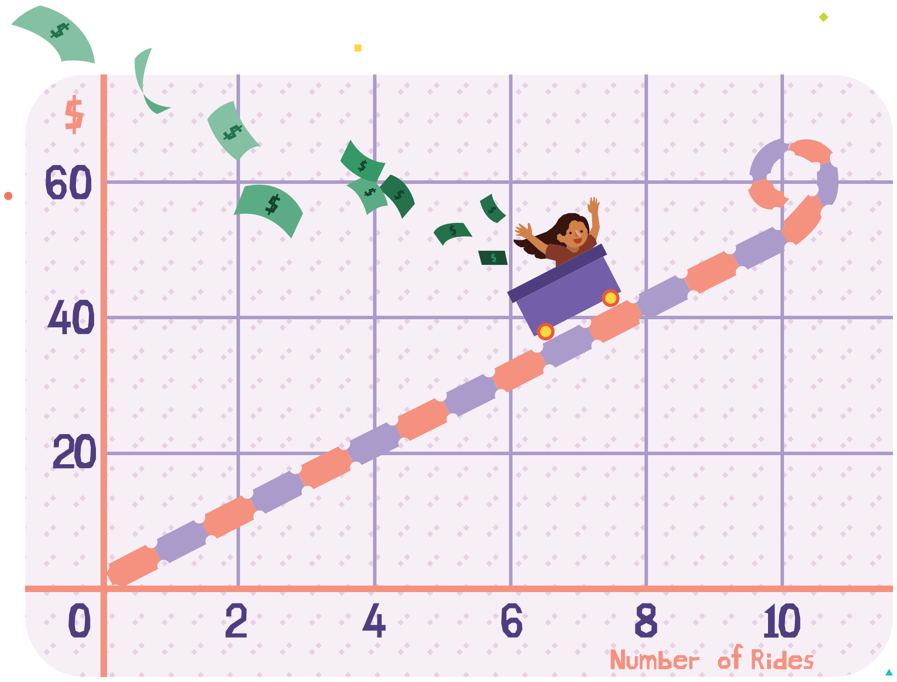
**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Translating to $y = mx + b$

Let's see what happens to the equations of translated lines.




## Focus

### Goals

1. Generalize that parallel lines have the same slope.
2. **Language Goal:** Connect features of the equation $y = b + mx$ to the graph, including lines with a negative $y$-intercept. **(Speaking and Listening)**
3. **Language Goal:** Create and compare graphs that represent linear relationships with the same rate of change, but different initial values. **(Speaking and Listening, Writing)**.

## Coherence

### Today

Students make sense of and apply the translations of lines in a new context. They see that any line in the coordinate plane can be considered a vertical translation of a proportional line, and they match lines presented in the form of an equation, graph, description, and table.

### Previously

In Lesson 7, students investigated the similarities and differences between linear and proportional relationships. In Lesson 10–11, students wrote an equation that represented a linear relationship with a positive slope.

### Coming Soon

In Lesson 13, students will be introduced to a negative slope, and they will interpret the slope in a context.

## Rigor

- Students **apply** their understanding of linear equations and graphs given a context.
- Students further their **conceptual understanding** of slope and $y$-intercept by analyzing different representations of the same linear relationship.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 17 min | 17 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** | Independent

## Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards
- Anchor Chart PDF, *Representations of Linear Relationships*
- rulers

## Math Language Development

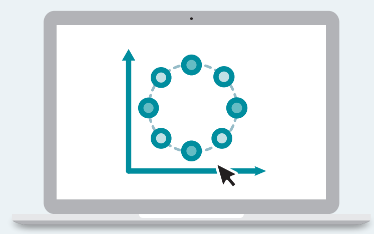
**Review words**

- *translation*
- *proportional relationship*
- *slope*
- *vertical intercept*
- *$y$-intercept*

**Amps** | **Featured Activity**

### Activity 1
### Overlay Graphs

Each student graphs a line based on a context. You can overlay student work to provide immediate feedback.



Amps POWERED BY desmos

## Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated as they try to match graphs, equations, tables, and descriptions. Encourage them to persist as they look for structure. For example, have them start by identifying the slope, and then the $y$-intercept of each line before moving on to tables and descriptions.

## Modifications to Pacing

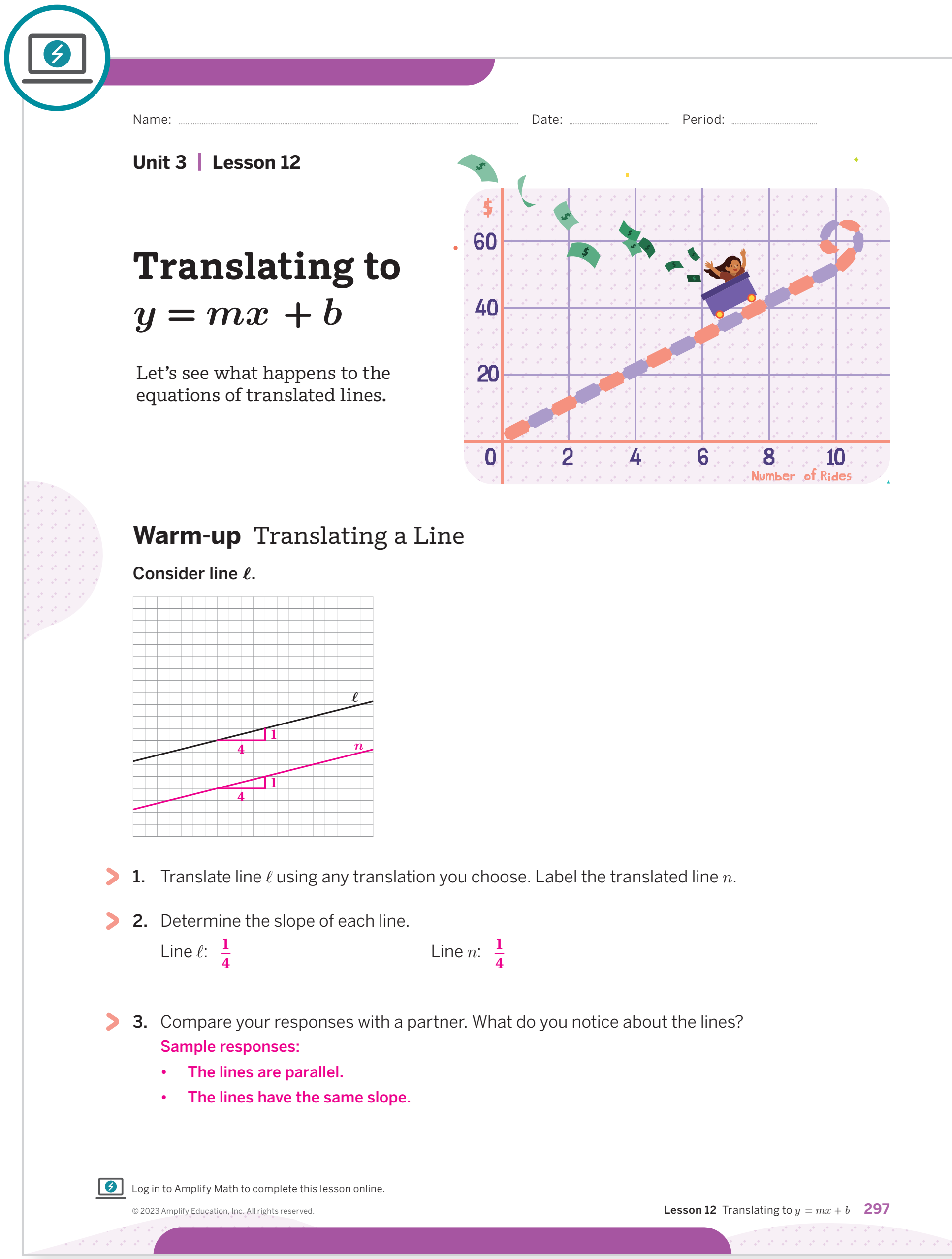
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, have students only complete the first two rows using Cards 1–6.

Pairs | 5 min

# Warm-up Translating a Line

Students translate a line, seeing that parallel lines have the same slope.



Name: ________ Date: ________ Period: ________

Unit 3 | Lesson 12

## Translating to $y = mx + b$

Let's see what happens to the equations of translated lines.



### Warm-up Translating a Line

Consider line $\ell$.



1. Translate line $\ell$ using any translation you choose. Label the translated line $n$.
2. Determine the slope of each line.
   Line $\ell$: $\frac{1}{4}$ Line $n$: $\frac{1}{4}$
3. Compare your responses with a partner. What do you notice about the lines?
   Sample responses:
   - The lines are parallel.
   - The lines have the same slope.

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Have students complete Problems 1 and 2 individually. Then have them share responses with a partner before completing Problem 3. Provide access to rulers for the duration of the lesson.

## 2 Monitor

**Help students get started** by asking them to translate the line in any distance and direction.

**Look for points of confusion:**

- **Not knowing how to translate a line.** Have students choose two points on line $\ell$ and provide a specific translation to apply. For example, have them translate the line 5 units up.

**Look for productive strategies:**

- Remembering that a translation of a line will produce a parallel line.
- Noticing that parallel lines have the same slope.

## 3 Connect

**Display** student work showing their completed graphs.

**Have pairs of students share** what they noticed about the two lines.

**Highlight** that when a line is translated on the coordinate plane, it produces a parallel line that has the same slope of the preimage.

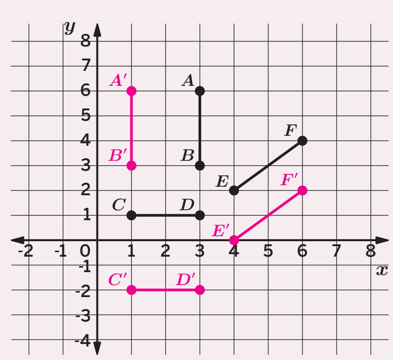
**Ask**, "Will a translated line always have the same slope as the preimage? Why or why not?" Yes, a translated line produces a line that is parallel, and parallel lines have the same slope.

## Power-up

**To power up students' ability to translate a line segment, have students complete:**

Recall that a *translation* slides a figure without changing its size or orientation.

1. Translate segment $AB$ 2 units to the left and label the new segment $A'B'$.
2. Translate the segment $CD$ 3 units down and label the new segment $C'D'$.
3. Translate the segment $EF$ 2 units down and label the new segment $E'F'$.





**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 11, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 5

Pairs | 17 min

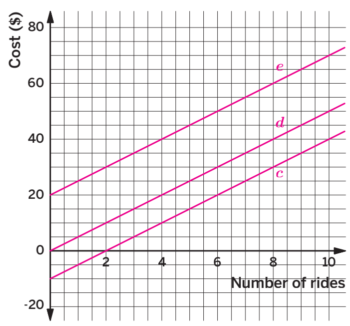
# Activity 1 How Much More?

Students make sense of and apply the translation of lines to three scenarios of a real-world context to see how the equation changes, based on the scenario.

**Amps Featured Activity** Overlay Graphs

### Activity 1 How Much More?

1. Noah wants to go to Honest Carl's Funtime World amusement park on Saturday. On weekends, the amusement park charges an admission fee of $20 per person and then $5 for each ride. Graph the relationship that represents the amount of money $y$ Noah would spend after $x$ rides. Label the line $e$.




2. On weekdays, Honest Carl's Funtime World offers a special deal where they do not charge an admission fee. On the same coordinate plane, graph the relationship that represents the amount of money $y$ Noah would spend after $x$ rides, if he goes there on a Wednesday. Label the line $d$.

3. Compare the two lines. How much more money does Noah spend after 2 rides if he goes to Honest Carl's Funtime World on a Saturday instead of a Wednesday? 4 rides? 8 rides? $x$ rides?
   **$20 more**

4. Write an equation for each line.
   **Weekend equation: $y = 5x + 20$**
   **Weekday equation: $y = 5x$**

5. Noah goes to Honest Carl's Funtime World on Wednesday and he has a coupon that can be used for 2 free rides. On the same coordinate plane, graph the relationship that represent the amount of money $y$ Noah would spend after $x$ rides, if he uses the coupon. Label the line $c$. Then write an equation for the line.
   $y = 5x - 10$

298 Unit 3 Linear Relationships



## 1 Launch

Activate students' background knowledge by asking them if they have ever paid admission to an amusement park or purchased ride or attraction tickets at a fair.

## 2 Monitor

**Help students get started** by asking them how much Noah would pay after 2 rides, 4 rides, and 6 rides.

**Look for points of confusion:**

- **Having trouble graphing the lines in Problems 1 and 2.** Have students create a table for the amount Noah pays for different numbers of rides, and then plot the points on the coordinate plane.
- **Having trouble writing the equations in Problem 4.** Have students draw a slope triangle and circle the vertical intercept. Then provide the equation $y = \square x + \square$ and have them complete the equation using the slope and vertical intercept.

## 3 Connect

**Have pairs of students share** their methods for graphing each scenario. Start with students who made a table and graphed, then students who plotted points directly, and lastly, students who wrote an equation before graphing.

**Ask:**

- "How does the price per ride affect the slope for each line?"
- "How does the fact that there is no weekday admission fee affect the line?"
- "How does the coupon for 2 free rides affect the equation?"

**Highlight** that the vertical intercept $-10$ represents Noah's coupon for 2 free rides. In the equation, the $b$-value is negative. On the graph, the vertical intercept is $-10$.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Consider one of these alternative approaches to this activity:

- Have students focus on completing Problems 1, 2, and 4.
- Provide a pre-completed graph for Problems 1 and 2 and launch the activity with a description of what these two lines mean within context. Have students begin the activity with Problem 3.

### Extension: *Math Enrichment*

Have students write an equation for a line that is parallel to $y = 2x$ and passes through any vertical intercept, $b$.
$y = 2x + b$

## Math Language Development

### *MLR7: Compare and Connect*

During the Connect, emphasize the connections between the equations and graphs. Ask:

- "Where does the term $5x$ in each equation come from?"
- "How is it represented on the graph?"

Problem 5 provides an opportunity to discuss the limitation of mathematical models. Highlight that the equation for Problem 5 does not work if Noah only goes on one ride as that would mean he gets paid 5, which is not realistic.

### English Learners

Use annotations to show how $5x$ is the same in each equation and how the slopes of the lines are the same.

Pairs | 17 min

# Activity 2 Card Sort: Translating a Line

Students sort cards to examine different representations of translated lines and to make connections to how the $y$-intercept appears in each representation.

Name: ______________________ Date: ____________ Period: ____________

**Activity 2** Card Sort: Translating a Line

**You will be given a set of cards. For each problem, determine the matching graph, equation, and table or description. Record the matching card numbers in the table.**

| | Graph | Equation | Table or description |
|---|---|---|---|
| **1.** The line $y = \frac{1}{2}x$ is translated up 1 unit. | Card 4 | Card 1 | Card 5 |
| **2.** The line $y = \frac{1}{2}x$ is translated down 1 unit. | Card 6 | Card 3 | Card 2 |
| **3.** The line $y = 2x$ is translated up 1 unit. | Card 7 | Card 12 | Card 9 |
| **4.** The line $y = 2x$ is translated down 1 unit. | Card 11 | Card 8 | Card 10 |

**Reflect:** How did you make constructive decisions in order to complete the activity successfully?

STOP





## 1 Launch

Display the Activity 2 Amps slides. Manipulate the point in two different places above the $x$-axis and two different places below the $x$-axis. Ask, "What changes and what stays the same?"

Distribute the cards from the Activity 2 PDF to each pair of students. Then conduct the *Card Sort* routine.

## 2 Monitor

**Help students get started** by graphing the line $y = \frac{1}{2}x$ for all to see, translating it up 1 unit, and asking students how the image of the line is similar to or different from the preimage.

**Look for points of confusion:**

- **Confusing the slope and $y$-intercept in the equation.** Have students refer to the Anchor Chart PDF, *Representations of Linear Relationships*.
- **Having trouble matching the verbal descriptions with the graph.** Have students match the equations and graphs first, and then use the equations to help them match the descriptions.

**Look for productive strategies:**

- Drawing the proportional line on the graph to help identify translations of a line.
- Noticing that the equations $y = mx + b$, $y = b + mx$, and $mx + b = y$ produce the same line.

## 3 Connect

**Ask**, "What clues did you look for to identify matching cards?"

**Have pairs of students share** their responses. Ensure that students understand that the equations $y = -1 + 2x$ and $y = 2x - 1$ are equivalent.

**Highlight** that translated lines will have the same slope, but different $y$-intercepts. Use Problem 2 to point out how the $y$-intercept appears on the graph, equation, and table.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Have students use color coding and/or annotations to highlight the slope and $y$-intercept in each matching representation.

### Extension: *Math Enrichment*

Without graphing, have students make a conjecture as to how the graphs of the equations $y = 3x + 8$ and $y = 3(x + 8)$ compare to the graph of the equation $y = 3x$. The graph of $y = 3x + 8$ is translated up 8 units, compared to the graph of $y = 3x$. The graph of $y = 3(x + 8)$ is translated up 24 units, compared to the graph of $y = 3x$ (due to the Distributive Property).

## Math Language Development

### *MLR8: Discussion Supports*

During the Connect, as students share their response to the Ask question, display sentence frames to support their reasoning, such as:

- "The equation on Card ____ matches the graph on Card ____ because . . ."
- "This description on Card ____ matches the graph on Card ____ because . . ."

### English Learners

Provide time for students to formulate their responses using the sentence frames before sharing with others.

Whole Class | 5 min

# Summary

Review and synthesize how the translation of a proportional line representing the equation $y = mx$ produces a parallel line represented by the equation $y = mx + b$.

### Summary

**In today's lesson . . .**

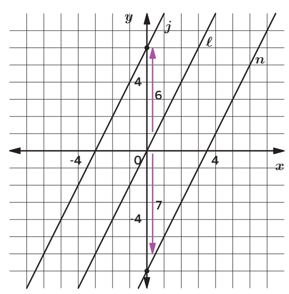
You investigated what happens to a line that represents a proportional relationship after a translation. A translation of a line that represents a proportional relationship creates a line that is parallel to the preimage, but changes the location of the vertical intercept.

The equation $y = mx$ represents a line that passes through the origin. The equation $y = mx + b$ represents a vertical translation of line $y = mx$ by $b$ units.

- If $b > 0$, the line is translated up.
- If $b < 0$, the line is translated down.

For example, the equation of line $\ell$ is $y = 2x$.

- Line $\ell$ is translated 6 units up to produce line $j$. So, the equation of line $j$ is $y = 2x + 6$.
- Line $\ell$ is translated 7 units down to produce line $n$. So, the equation of line $n$ is $y = 2x - 7$.




**Reflect:**




## Synthesize

**Display** the Summary from the Student Edition.

**Have students share** how they can determine whether the vertical intercept of a line is positive or negative from a graph and equation.

**Ask**, "How can you apply your knowledge of translations to draw a line?" Draw a proportional line with the same slope, and then translate it up the same number of units that are represented by the $b$-value in the equation $y = mx + b$.

**Note:** Students may plot the translated $y$-intercept first, and then use the slope to graph additional points.

**Highlight** that a proportional line that is translated up will have a positive vertical intercept, while a proportional line that is translated down will have a negative vertical intercept.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What can proportional relationships teach us about linear relationships?"

# Exit Ticket

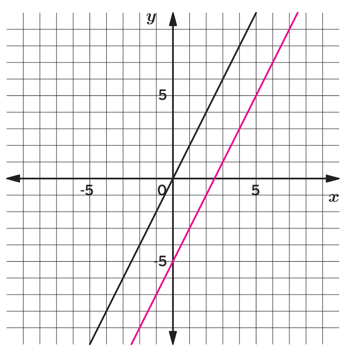
Students demonstrate their understanding of translated lines by comparing the graphs of two linear equations.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 3.12

**Consider the graph of $y = 2x$ shown on the coordinate plane.**




1. Draw the graph of $y = 2x - 5$ on the same coordinate plane.
2. What are the similarities and differences among the two graphs?
   **Sample response: Both graphs have a slope of 2. The graphs are different because the graph of $y = 2x - 5$ is translated down 5 units from the graph of $y = 2x$, so the $y$-intercepts are not the same.**

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I understand how the translation of a line representing a proportional relationship changes its equation. 1 2 3

b. I know that parallel lines have the same slope. 1 2 3


Lesson 12 Translating to $y = mx + b$

## Success looks like . . .

- **Goal:** Generalizing that parallel lines have the same slope.
- **Language Goal:** Connecting features of the equation $y = b + mx$ to the graph, including lines with a negative $y$-intercept. **(Speaking and Listening)**
- **Language Goal:** Creating and comparing graphs that represent linear relationships with the same rate of change, but different initial values. **(Speaking and Listening, Writing)**

## Suggested next steps

**If students do not know how the graph of $y = 2x$ compares to the graph of $y = 2x - 5$, consider:**

- Changing the $y$-intercept to positive 5.
- Reviewing Activity 2.
- Reassessing after Lesson 14.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- In what ways have your students become better at writing equations of a line?
- How did the *Card Sort* routine support students in comparing features of a linear relationship?

# Practice

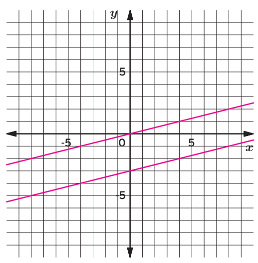
Independent

Name: ______ Date: ______ Period: ______

1. Select *all* the equations whose graphs have the same $y$-intercept.

A. $y = 3x - 8$
B. $y = -8x + 3$
C. $y = 3x + 8$
D. $y = 5x - 8$
E. $y = 2x - 8$
F. $y = 13x - 8$

2. Refer to the coordinate plane.




a. Graph the equations $y = \frac{1}{4}x$ and $y = \frac{1}{4}x - 3$.

b. How are the graphs the same? How are the graphs different?

**Sample response: Both graphs have a slope of $\frac{1}{4}$. The graph of $y = \frac{1}{4}x - 3$ is translated down 3 units from the graph of $y = \frac{1}{4}x$, so the $y$-intercepts are different.**

3. A cable company charges $70 per month for cable service to existing customers. For new customers, there is an additional one-time service fee of $100.

a. Write a linear equation representing the relationship between $x$, the number of months of service, and $y$, the total amount paid in dollars by a customer.

**Existing customer:** $y = 70x$

**New customer:** $y = 70x + 100$

b. When the two equations are graphed on the coordinate plane, how are the graphs similar?

**They are parallel lines. The graph of $y = 70x + 100$ is translated 100 units up from the graph of $y = 70x$.**





Practice

Name: ______ Date: ______ Period: ______

4. A mountain road is 5 miles long and gains elevation at a constant rate. After 2 miles, the elevation is 5,500 ft above sea level. After 4 miles, the elevation is 6,200 ft above sea level.

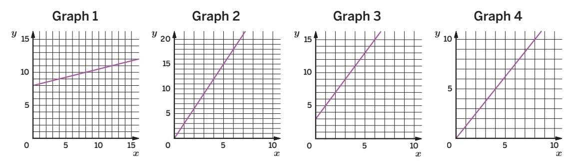
a. Determine the elevation of the road at the point where the road begins.

**4,800 ft above sea level**

b. What are the coordinates of the point on a graph that represent your response in part a? Let $y$ represent the elevation in feet and $x$ represent the distance along the road in miles.

**The point would be (0, 4800), located on the vertical axis.**

5. Consider Graphs 1–4 shown. For each real-world situation described, choose the graph that best represents it.




a. The perimeter $y$, in units, for an equilateral triangle with a side length of $x$ units. The slope of the line is 3.

**Graph 2**

b. The amount of money $y$ after $x$ tickets are purchased. The slope of the line is $\frac{1}{4}$.

**Graph 1**

c. The number of chapters $y$ read after $x$ days. The slope of the line is $\frac{5}{4}$.

**Graph 4**

d. The cost $y$, in dollars, of $x$ blueberry muffins ordered. The slope of the line is 2.

**Graph 3**

6. Calculate the slope of the line that passes through the points (4, 5) and (7, 6).

$\frac{6-5}{7-4} = \frac{1}{3}$





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 1 |
| | 2 | Activity 1 | 1 |
| | 3 | Activity 1 | 2 |
| Spiral | 4 | Unit 3 Lesson 10 | 2 |
| | 5 | Unit 3 Lesson 9 | 2 |
| Formative | 6 | Unit 3 Lesson 13 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

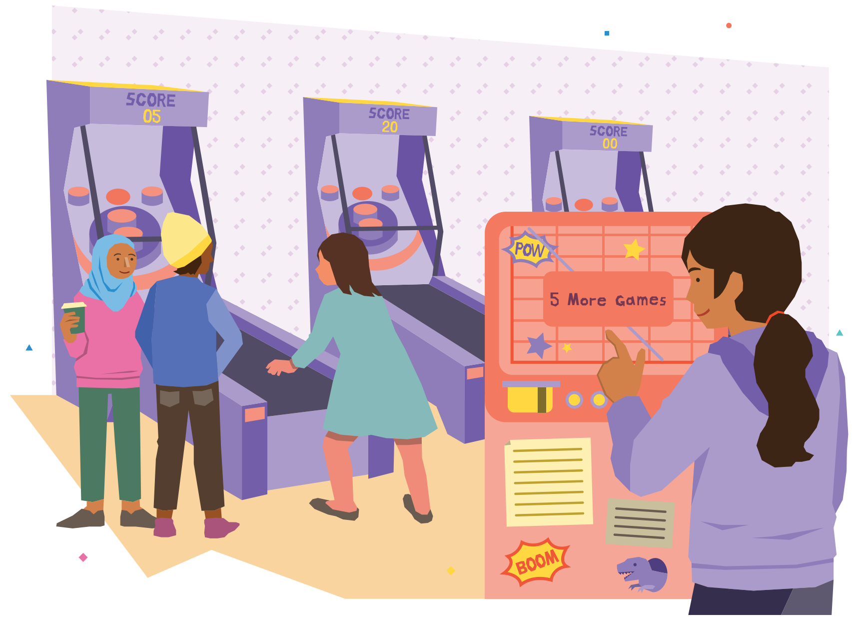
## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 3 | **LESSON 13**

# Slopes Don't Have to Be Positive

Let's find out what a negative slope means.




## Focus

### Goals

1. Create a graph of a line representing a linear relationship with a negative rate of change.
2. Interpret the slope of a decreasing line in context.
3. **Language Goal:** Identify and interpret the horizontal intercept of a graph of a linear relationship. **(Reading and Writing)**

## Rigor

- Students build their **conceptual understanding** of a negative slope.
- Students **apply** their understanding of slope to describe lines.

## Coherence

### Today

Students get their first glimpse of lines with a negative slope. They interpret a graph and reason that it makes sense for the slope to be negative in terms of the context. During their partner activity, students consider what information is sufficient to define and accurately communicate the position of a line on the coordinate plane.

### Previously

In Lesson 11, students applied their knowledge of similar triangles to write the equation of a line with a positive slope using two coordinates of points.

### Coming Soon

In Lesson 14, students will use an algebraic method to write the equation of a line using two points. In Lesson 15, students will extend their previous work to include equations for horizontal and vertical lines.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Activity 3 | Summary | Exit Ticket |
|---|---|---|---|---|---|
| 5 min | 10 min | 10 min | 15 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, pre-cut cards
- Anchor Chart PDF, *Representations of Linear Relationships* (as needed)
- Activity 3 PDF (answers)
- *Info Gap Routine* PDF (for display)
- rulers

### Math Language Development

**New words**

- ***horizontal intercept***
- ***$x$-intercept***

**Review words**

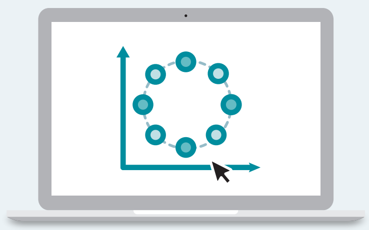
- *linear relationship*
- *slope*
- *vertical intercept*
- *$y$-intercept*

**Amps** | **Featured Activity**

**Activity 1**

**Overlay Graphs**

Each student graphs a line based on a context. You can overlay student work to provide immediate feedback.



Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 3, students might forget to pay attention to the details of the graph as they describe it and may feel deflated if they consistently miss the mark in their descriptions. Promote a healthy growth mindset by having students evaluate what did go right each time. Then ask them to work with their partners to determine how they could improve next time.

### Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- **Activity 3** may be omitted.

# Warm-up Same and Different

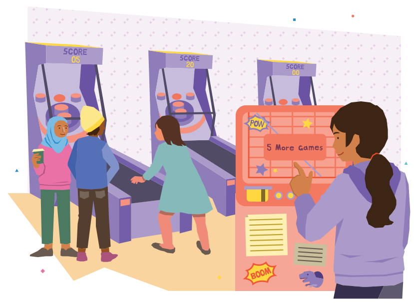
Students analyze two lines with opposite slopes as an introduction to lines with a negative slope.

Name: ______________ Date: ________ Period: ________

**Unit 3 | Lesson 13**

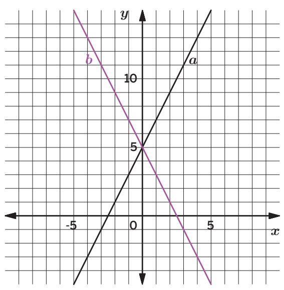
## Slopes Don't Have to Be Positive

Let's find out what a negative slope means.



### Warm-up Same and Different

Consider lines $a$ and $b$.




1. What is the same about the two lines?
   Sample responses:
   - Both lines have a $y$-intercept of 5.
   - Both lines have a slope triangle with a vertical side length of 2 units and a horizontal side length of 1 unit.
2. What is different about the two lines?
   Sample response: Moving from left to right on the coordinate plane, one line increases and the other line decreases.

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Conduct the ***Think-Pair-Share*** routine.

## 2 Monitor

**Help students get started** by asking them to visually inspect the lines, and then having them draw slope triangles.

**Look for points of confusion:**

- **Thinking the slope for line $b$ is 2.** Revisit with these students after Activity 3.

**Look for productive strategies:**

- Noticing the lines "lean" in different directions.
- Drawing slope triangles to determine the slope of each line.

## 3 Connect

**Have students share** what they noticed about the two lines.

**Highlight** that the slope triangles are the same for each line. If students did not draw slope triangles on the graphs, have them do so now or demonstrate for them. Emphasize that the slope triangles for each line have the same ratio of the vertical side length to the horizontal side length. Also highlight that when reading the graph from left to right, line $a$ goes up, while line $b$ goes down.

**Ask**, "What happens to the values of $y$ for each line as the values of $x$ increase?" Sample response: For line a, the $y$-values increase as the $x$-values increase. For line b, the $y$-values decrease as the $x$-values increase.

## Math Language Development

### *MLR2: Collect and Display*

During the Connect, as students share what is the same and what is different about the two lines, collect and display language students use referring to positive/negative slope and $x$- and $y$-intercepts. Leave the display up during the lesson and continue adding terms, phrases, and diagrams to support students' sense making about lines with negative slope.

### English Learners

Use hand gestures to illustrate how line $a$ increases from left to right and line $b$ decreases from left to right.

## Power-up

**To power up students' ability to calculate the slope from two points, have students complete:**

Recall that for any pair of points $(s, t)$ and $(u, v)$ the slope can be calculated using the relationship $slope = \frac{vertical\ change}{horizontal\ change} = \frac{v-t}{u-s}$.

Determine the slope between the points $(3, 5)$ and $(6, 11)$. Show your thinking. $2; \frac{11-5}{6-3} = \frac{6}{3}$

**Use:** Before Activity 2

**Informed by:** Performance on Lesson 12, Practice Problem 6

Pairs | 10 min

# Activity 1 Noah's Game Card

Students investigate changes on a game card to make sense of a negative slope and learn about the $x$-intercept of a line.

**Amps Featured Activity** Overlay Graphs

### Activity 1 Noah's Game Card

Noah loads a game card with $40 for the arcade at Honest Carl's Funtime World. Every time he plays a game, $2.50 is subtracted from the amount available on his card.

**1.** How much money, in dollars, is available on his card after Noah plays:

a. 1 game?
$37.50

b. 2 games?
$35

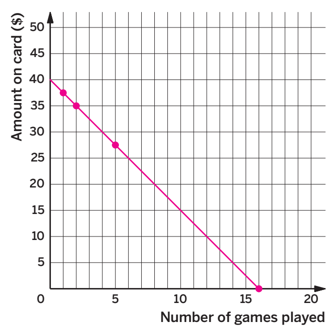
c. 5 games?
$27.50

d. $x$ games?
$(40 − 2.5$x$) or equivalent

**2.** Use your responses from Problem 1 to plot three points on the graph. Then draw a line through the points. What patterns do you notice?

Sample responses:

- The points are on the same line.
- As the number of games played increases, the value on the game card decreases at a constant rate.




**3.** How many games can Noah play before the game card runs out of money? Where do you see this number of games on your graph?
16 games; The number of games is the $x$-coordinate of the point $(16, 0)$.

304 Unit 3 Linear Relationships



## 1 Launch

Activate students' background knowledge by asking if they have ever used a game card at an arcade. If students are unfamiliar, provide some quick information about how a game card works. Provide access to rulers.

## 2 Monitor

**Help students get started** by asking them how they can determine the cost for the first 5 games.

**Look for points of confusion:**

- **Struggling to determine how much money is available after $x$ games.** Have students create a table using the values in Problems 1a–c. In the introduction to the activity, label $40 as "starting value" and underline "subtracted."
- **Having trouble understanding Problem 4.** Ask students to state the remaining value on the card when Noah runs out of money and ask them to relate this value to the graph.

## 3 Connect

**Display** student work showing the completed graph.

**Have students share** how the graph is similar to and different from the graphs they have seen so far in this unit.

**Highlight** the negative coefficient of $x$ and the decreasing line. **Note:** Students will further explore negative slope in Activity 2.

**Ask**, "What does the point on the vertical axis represent? What does the point on the horizontal axis represent?"

**Define** the term ***horizontal intercept*** as the point where the graph intersects the horizontal axis. Also known as the ***$x$-intercept***, it is the value of $x$ when the value of $y$ is $0$. The horizontal intercept in this problem is $(16, 0)$.

## Differentiated Support

### Accessibility: *Activate Background Knowledge*

Demonstrate how a game card works at an arcade by showing how $2.50 is subtracted from $40 each time a game is played. Consider illustrating this in a table.

### Extension: *Math Enrichment*

Have students solve the equation $40 - 2.5x = 0$ and ask them to explain how the solution relates to the context of the activity and the question in Problem 3. $x = 16$; The solution to the equation is the $x$-coordinate of the horizontal intercept of the graph (when $y = 0$).

## Math Language Development

### *MLR8: Discussion Supports*

During the Connect, as students share how the graph is similar to and different from the graphs they have seen so far in this unit, display sentence frames to support their thinking. For example:

- "The graphs in this unit have ___, while this graph has ___."
- "This graph is different because . . . "

Amplify language that describes the graph decreasing from left to right. Connect this to the negative coefficient in the expression in Problem 1d.

### English Learners

Annotate the negative coefficient of $x$ in the expression in Problem 1d and write *negative coefficient*.

Pairs | 10 min

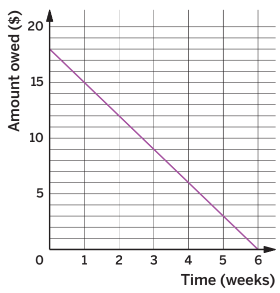
# Activity 2 Payback Plan

Students write the equation of a line to interpret a negative slope and $y$-intercept in context.

Name: ______ Date: ______ Period: ______

## Activity 2 Payback Plan

**Elena borrowed some money from her brother. She pays him back by giving him the same amount every week. The graph shows how much she owes him after each week.**




**Co-craft Questions:** Before completing this activity, preview Problem 3. What are some other questions you could ask about this graph? With your partner, think of 1–2 other questions you have about this situation.

1. Choose two points on the line and label the coordinates. Then calculate the slope of the line.
   **Sample response: Using the points (1, 15) and (4, 6) the slope is $\frac{6-15}{4-1} = -\frac{9}{3}$ or $-3$.**

2. Write an equation for the dollar amount $y$ owed after $x$ weeks. Then explain what each number in your equation represents in this situation.
   $y = -3x + 18$
   **The number $-3$ represents the amount Elena pays her brother each week.**
   **The number 18 represents the initial amount of money Elena borrowed from her brother.**

3. How much time will it take for Elena to pay back all the money she borrowed? Explain your thinking.
   **6 weeks; Sample response:**
   - **I used the graph and found the value of $x$ when $y$ (the amount owed) was 0.**
   - **I used the equation to substitute 0 for $y$ (the amount owed), and then solved the equation for $x$.**

   $0 = -3x + 18$
   $-18 = -3x$
   $x = 6$



Lesson 13 Slopes Don't Have to Be Positive 305

## 1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

## 2 Monitor

**Help students get started** by reminding them how to calculate the slope using the coordinates of two points.

**Look for points of confusion:**

- **Getting a positive value for the slope.** Have students identify if the numerator or denominator is positive or negative. Then ask them for the sign of a quotient when dividing numbers with opposite signs.
- **Not knowing how to write an equation.** Have students refer to the Anchor Chart, *Representations of Linear Relationships*.

## 3 Connect

**Ask:**

- "How do you know the sign of the slope based on the context?" Answers may vary.
- "How do you know the sign of the slope just by looking at the line?" If the line increases from left to right, the slope is positive. If the line decreases from left to right, the slope is negative.

**Have pairs of students share** why the slopes are equivalent even if they choose different points to calculate the slope. Then have students share how Problem 3 can be solved using the equation and graph.

**Highlight**

- If a line has a positive/negative slope, it will increase/decrease (from left to right) on a graph. The equation of a line with a positive/negative slope will have a positive/negative coefficient of $x$.
- If a line has a positive/negative vertical intercept, this point will be above/below the $x$-axis.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Provide students with two points to use in Problem 1, such as (4, 6) and (5, 3). Display the Anchor Chart PDF, *Representations of Linear Relationships* for students to reference throughout the activity.

### Extension: *Math Enrichment*

Have students use the equation from Problem 2 to find the value of $y$ when $x = 7$. Ask them to explain what this means in the context of the problem. $y = -3$; Sample response: At 6 weeks, Elena has paid back all she owed; the equation is not necessarily meaningful past 6 weeks. Or her brother will now owe her 3 at 7 weeks.

## Math Language Development

### *MLR5: Co-craft Questions*

Before revealing the problems in this activity, display the introductory text and the graph. Preview Problem 3 with students and ask them to work with a partner to write 1–2 additional questions they have about the graph or situation. Ask pairs of students to share their questions with the class.

### English Learners

Display a sample question, such as "How much money does Elena pay back her brother every week?"

Pairs | 15 min

# Activity 3 Info Gap: Making Designs

Students describe features of a line to practice recognizing the location of a line in a coordinate plane, and to distinguish between positive and negative slopes.

**Activity 3** Info Gap: Making Designs

**You will be given either a *design card* or a *blank graph card*. Do not show your card to your partner.**

| If you are given a *design card*: | If you are given a *blank graph card*: |
|---|---|
| **1.** Silently study the design and think about how you could communicate what your partner should draw.<br>Think about ways that you can describe what a line looks like, such as its slope or the points that it passes through. | **1.** Listen carefully as your partner describes each line, and draw each line based on their description. |
| **2.** Describe each line, one at a time, and give your partner time to draw each one. | **2.** You are not allowed to ask for more information about a line other than what your partner tells you. |
| **3.** Do not show your design card to your partner until they have finished drawing all the lines. | **3.** Do not show your drawing to your partner until you have finished drawing all the lines. |

When you and your partner are finished, place the drawing next to the card with the design, so that you and your partner can both see them. How is the drawing the same as the design? How is it different? Discuss any miscommunication that might have caused the drawing to look different from the design.

**Pause here so your teacher can review your work. When you are given a new set of cards, trade roles with your partner and repeat the activity.**

STOP





## 1 Launch

Explain to students they will describe some lines to a partner to try and get them to recreate a design. Give one partner Design Card 1 and the other partner a Blank Graph Card 1 from the Activity 1 PDF. Display the *Info Gap Routine* PDF and model the ***Info Gap*** routine. Arrange the room to ensure that the partner drawing the design cannot peek at the design from anywhere in the room. Once the first design has been successfully created, provide the second design and a blank graph to the other student in each partnership. Provide access to rulers.

## 2 Monitor

**Help students get started** by having them choose one line and describe the slope and $y$-intercept.

**Look for points of confusion:**

- **Forgetting to describe the slope as positive or negative.** Have students with the design card label each line with a plus or minus sign.

**Look for productive strategies:**

- Noticing parallel lines and using translations during their descriptions.
- Using coordinates, equations, or vertical and horizontal intercepts to describe the line.
- Visually inspecting the line to determine a positive or negative slope.

## 3 Connect

**Have pairs of students share** their methods for describing the lines. Start with students who used coordinates to describe the line placements, then students who used translations, then students who used equations or intercepts.

**Ask**, "What details were important to pay attention to?" Answers may vary. Some students may pay attention to coordinates or intercepts, while others look for parallel lines and translations.

**Highlight** that there are different methods to describe a line.

## Differentiated Support

**Accessibility: *Guide Processing and Visualization***

Display Design Card 1. Use a think-aloud to model Steps 1 and 2 for how to describe the location of line $a$, as if you were the recipient of that card. Consider using the following during the think-aloud.

- "In order to best describe the location of line $a$, I think I should provide the slope of the line and the $y$-intercept."
- "I could also provide the coordinates of two points that line $a$ passes through."

## Math Language Development

***MLR4: Information Gap***

Display these sentence frames for students who would benefit from a starting point, such as:

- "The slope of line ___ is ___."
- "The $y$-intercept of line ___ is ___."
- "Line ___ passes through points ___ and ___."

Whole Class | 5 min

# Summary

Review and synthesize how the sign of the slope affects the location of a line on a coordinate plane.

Name: ______________________ Date: __________ Period: __________

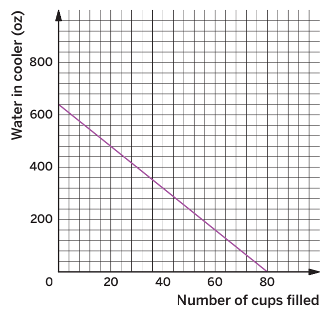
## Summary

**In today's lesson . . .**

You saw that the slope of a line can have a negative value. When a linear relationship has a negative slope, this means that as the $x$-values increase, the $y$-values decrease at a constant rate.

For example, the equation $a = -8n + 640$ represents the amount $a$ of water in a water cooler after $n$ cups are filled with water.

- The vertical intercept, $640$, represents the initial amount of water in the cooler.
- The slope, $-8$, tells you the rate of change in the amount of water each time a cup is filled. Because the slope is negative, the amount of water decreases.
- The ***horizontal intercept***, $80$, tells you that it takes $80$ cups of water to empty the water cooler.




Reflect:





## Synthesize

**Display** Design Card 1 from the Activity 3 PDF.

**Have students share** how they can identify if the slope of a line is positive or negative.

**Ask:**

- "How do you know if the slope of a line is positive or negative using coordinates of two points?" Divide the difference in the values of $y$ by the values of $x$ and then look at the sign.
- "How do you know if the slope of a line graphed on the coordinate plane is positive or negative by visual inspection?" If the line increases (from left to right) the slope is positive. If the line decreases (from left to right) the slope is negative.
- "How do you know if the slope of a line is positive or negative from a description?" For a positive slope, look for keywords such as increasing or adding values. For a negative slope, look for keywords such as decreasing or subtracting values.

**Highlight** that when a linear relationship has a negative slope, as the values of $x$ increase, the values of $y$ decrease at a constant rate.

**Formalize vocabulary:**

- *horizontal intercept*
- *$x$-intercept*

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does it mean for a slope to be negative?"

## Math Language Development

### *MLR2: Collect and Display*

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the terms *horizontal intercept* and *$x$-intercept* that were added to the display during the lesson.

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by graphing a line with a negative slope from a description and writing its equation.

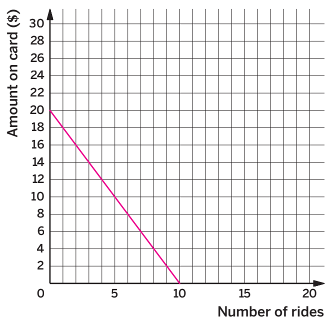
Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 3.13

**Kiran adds $20 to his public transportation fare card. Every time he rides public transportation, $2 is subtracted from the amount available on his card.**

1. Graph the relationship between the dollar amount $y$, available on the card after $x$ rides.




2. Write an equation that gives the dollar amount $y$ available on the card after $x$ rides.

$y = -2x + 20$

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can analyze the graph of a line and determine whether the slope is positive or negative.
1 2 3

b. I can write an equation to represent a real-world situation where the $y$-values decrease at a constant rate.
1 2 3



Lesson 13 Slopes Don't Have to Be Positive

## Success looks like . . .

- Creating a graph of a line representing a linear relationship with a negative rate of change.
- Interpreting the slope of a decreasing line in context.
- **Language Goal:** Identifying and interpreting the horizontal intercept of a graph of a linear relationship. **(Reading and Writing)**

## Suggested next steps

**If students do not draw the line correctly, consider:**

- Having them create a table for the first five rides and amount on the card, and then use the data to draw the line.
- Reviewing Activity 1.

**If students write the incorrect slope, consider:**

- Having them choose and label two points on the line to calculate the slope. Or have students visually inspect the slope of the line to determine if it is positive or negative.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- In earlier lessons, students calculated the slope using two points. How did that support their work calculating a negative slope?
- What challenges did students encounter as they worked on Activity 3, *Info Gap*? How did they work through the challenges?

# Practice

Independent

Practice

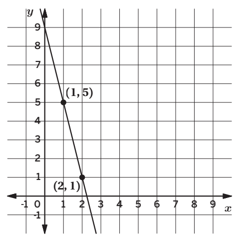
Name: ______________ Date: ________ Period: ________

1. During its flight, the elevation $e$, in feet, of an airplane and its time since takeoff are related by a linear equation. Consider the graph of such an equation, with time in minutes represented on the horizontal axis and elevation in feet on the vertical axis. For each situation, determine whether the slope is positive or negative.

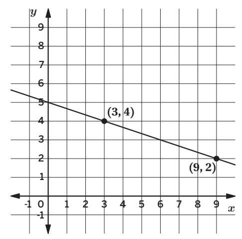
   a. The plane descends at a rate of $1{,}000$ ft per minute.
   **negative**

   b. The plane ascends at a rate of $2{,}000$ ft per minute.
   **positive**

2. Determine the slope of each line. Show your thinking.

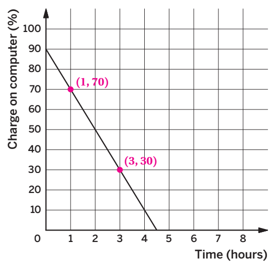
   a.



   $\frac{1-5}{2-1} = -\frac{4}{1}$ or $-4$

   b.



   $\frac{2-4}{9-3} = -\frac{2}{6}$ or $-\frac{1}{3}$

3. The graph shows the amount of charge $c$ left on Lin's computer after $h$ hours. Write an equation that describes the relationship between $c$ and $h$. Show or explain your thinking.




   **$c = 90 - 20h$; Sample response: I chose two points and calculated the slope: $\frac{70-30}{1-3} = -\frac{40}{2} = -20$. I saw the vertical intercept was $(0, 90)$. The equation for this line is $c = 90 - 20h$.**




Practice

Name: ______________ Date: ________ Period: ________

4. Elena and Diego both have part-time jobs. Elena's aunt pays her $1 for each call she makes to let people know about her aunt's new business. Diego washes his neighbor's windows and earns the same amount per window, as shown in the table. Select *all* the statements about their part-time jobs that are true.

| Number of windows | Amount earned ($) |
|---|---|
| 27 | 29.70 |
| 45 | 49.50 |
| 81 | 89.10 |

   A. Elena makes more money for making 10 calls than Diego makes for washing 10 windows.
   B. Diego makes more money for washing each window than Elena makes for making each call.
   C. Elena makes the same amount of money for 20 calls as Diego makes for 18 windows.
   D. Diego needs to wash 35 windows to make as much money as Elena makes for 40 calls.
   E. The equation $y = 1.10x$, where $y$ represents the number of dollars and $x$ represents the number of windows, represents Diego's situation.
   F. The equation $y = x$ represents Elena's situation, where $y$ represents the number of dollars and $x$ represents the number of calls.

5. A line passes through the points $(1, 1.5)$ and $(4, 6)$. Determine whether each point is also on the line. Place a check mark in the appropriate column.

| | On the line | Not on the line |
|---|---|---|
| $(5, 7.5)$ | ✓ | |
| $(80, 50)$ | | ✓ |
| $(100, 150)$ | ✓ | |

6. Which expression has a value of $-25$ when $a = -2$?

   A. $-10a + 5$
   B. $-10a - 5$
   C. $10a + 5$
   D. $10a - 5$




## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 1 |
| | 3 | Activity 2 | 1 |
| Spiral | 4 | Unit 3 Lesson 6 | 2 |
| | 5 | Unit 3 Lesson 2 | 2 |
| Formative | 6 | Unit 3 Lesson 14 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Writing Equations for Lines Using Any Two Points, Revisited

Let's write equations for lines.


## Focus

### Goals

1. Create an equation of a line with a positive or negative slope on the coordinate plane.

## Coherence

### Today

Students revisit how to write an equation of a line given two coordinates of points and develop an algebraic method to write the equation. They attend to precision as they apply their understanding in writing equations of lines with a positive or negative slope.

### Previously

In Lesson 11, students applied their knowledge of similar triangles to write the equation of a line with a positive slope using two coordinates of points.

### Coming Soon

In Lesson 15, students will extend their previous work to include equations for horizontal and vertical lines.

## Rigor

- Students **apply** their algebraic understanding to write the equation of a line using two points.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

## Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (for display)
- Anchor Chart PDF, *Representations of Linear Relationships* (as needed)
- Anchor Chart PDF, *Writing Linear Equations in $y = mx + b$ Form*
- rulers

## Math Language Development

**Review words**

- *slope*
- *$y$-intercept*

**Amps** | **Featured Activity**

### Activity 2
### Coin Game

Students enter an equation that will animate a line to collect coins.

## Building Math Identity and Community

Connecting to Mathematical Practices

Students might choose to draw the lines, but skip the step of writing the equation for the line in Activity 2. Have students consider constructive decisions they can make before starting the activity. Reflect on why skipping steps is not helpful to themselves. Challenge them to analyze the situation well so that they can minimize the work needed to achieve the goal.

## Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, round 3 may be omitted.

Pairs | 5 min

# Warm-up Two Truths and a Lie

Students analyze three statements to review methods that do or do not work for calculating the slope of a line.

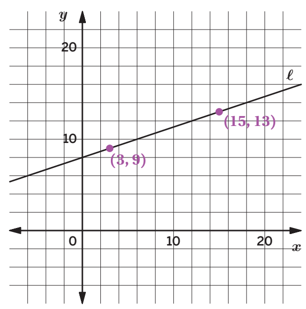
Unit 3 | Lesson 14

## Writing Equations for Lines Using Two Points, Revisited

Let's write equations for lines.


### Warm-up Two Truths and a Lie

**Line $\ell$ is shown on the coordinate plane. Two points are labeled on the line. Which of these three statements is a lie?**




A. The slope of the line can be calculated by evaluating the expression $\frac{13-9}{15-3}$.

B. The slope of the line can be calculated by evaluating the expression $\frac{9-13}{3-15}$.

C. The slope of the line can be calculated by evaluating the expression $\frac{13-9}{3-15}$.

**Explain your thinking.**

**Sample response:**
**The expressions in Choices A and B have values equivalent to $\frac{1}{3}$, while the expression in Choice C has a value that is equivalent to $-\frac{1}{3}$. The $x$-coordinates are not subtracted in the same order as the $y$-coordinates.**

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Conduct the *Two Truths and a Lie* routine.

## 2 Monitor

**Help students get started** by having them simplify each expression.

**Look for points of confusion:**

- **Thinking that Choice B is the false statement or thinking that Choice C is true.** Ask students to simplify the expression first and then look for the answer choice that is not equivalent to the others.

## 3 Connect

**Have students share** their responses. Use the *Poll the Class* routine to determine which choice each student selected.

**Ask**, "How do you know that Choices A and B are equivalent?" The quotient of two negative values is positive.

**Highlight** that to calculate the slope, it is important to subtract the $x$-coordinates of the two points in the same order as the $y$-coordinates are subtracted.

## Math Language Development

### *MLR3: Critique, Correct, Clarify*

During the Connect, after determining that Choice C is the lie, ask pairs to critique the statement in Choice C and identify the error. Have them write a corrected statement and explain how they know their statement is true. Ask them to share their corrected statements with the class. Highlight sense-making around the fact that the expression in Choice C yields a negative slope, but the line is increasing from left to right.

### English Learners

Students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

## Power-up

**To power up students' ability to substitute values into expressions and evaluating, have students complete:**

Recall that when a number is 'attached' to a variable in an expression, it represents the product of the number and the variable. For example, $3b$ represents $3 \cdot b$.

Evaluate the expression $6z + 8$ for $b = -5$. Show your thinking.
$-22$; $6(-5) + 8 = -30 + 8 = -22$

**Use:** Before Activity 2

**Informed by:** Performance on Lesson 13, Practice Problem 6, and Pre-Unit Readiness Assessment, Problem 8

Pairs | 15 min

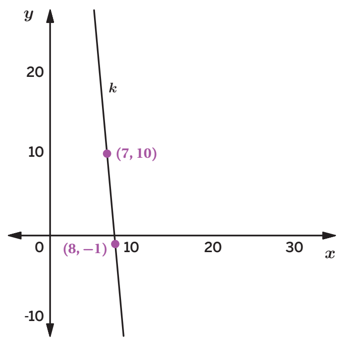
# Activity 1 Writing an Equation from Two Points, Revisited

Students develop an algebraic method to determine the equation of a line given two points.

Name: ______ Date: ______ Period: ______

## Activity 1 Writing an Equation from Two Points, Revisited

**Consider line $k$ with the two labeled points as shown.**




1. Calculate the slope of the line. Show your thinking.
   $\frac{10-(-1)}{7-8} = -\frac{11}{1} = -11$

2. Andre wants to write an equation for this line in the form $y = mx + b$, but he cannot see the $y$-intercept on the graph. He claims that because he knows the slope, he can calculate the value of $b$ by substituting the coordinates of one of the points, $(7, 10)$, for $x$ and $y$ in the equation $y = -11x + b$. His unfinished work is shown.

   **Andre's work:**
   For $m = -11$ and point $(7, 10)$:
   $y = mx + b$
   $10 = -11(7) + b$

   Finish Andre's work by solving the equation for $b$. Show your thinking.
   $10 = -77 + b$
   $b = 87$

3. Use the slope and $y$-intercept to write the equation for the line in the form $y = mx + b$.
   $y = -11x + 87$





## 1 Launch

Tell students that today they will investigate another method for writing the equation of a line. Have students complete Problem 1 individually. Then have them share their responses with a partner before completing Problems 2 and 3.

## 2 Monitor

**Help students get started** by reminding them to plot coordinates as $(x, y)$.

**Look for points of confusion:**

- **Not understanding Andre's work.** Highlight $x$ and 7 in one color and $y$ and 10 in another color to make the connection between the variables and values.
- **Not understanding why the final equation does not include 7 and 10 as $x$ and $y$.** Tell students that $x$ and $y$ represent *any* point on the line. Because the values can vary, they appear as variables. The slope and $y$-intercept of this line does not vary, so these values will appear as $-11$ and 87.

## 3 Connect

**Ask**, "If you substituted the point $(8, -1)$, would you arrive at the same equation?" As a class, substitute 8 and $-1$ into the equation to show that either point can be used to determine the $y$-intercept.

**Display** the Anchor Chart PDF, *Writing Linear Equations in $y = mx + b$ Form*.

**Highlight** that to write an equation for a line using two coordinates of points, first calculate the slope. Then substitute the coordinates of either point into the equation to solve for the $y$-intercept. Lastly, write the equation in $y = mx + b$ form using the slope and $y$-intercept.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Display the Anchor Chart PDF, *Representations of Linear Relationships* for students to reference throughout the activity. Consider providing students with the slope of the line in Problem 1 and have them begin the activity with Problem 2.

### Extension: *Math Enrichment*

Ask students how they know the $y$-intercept they found in Problem 2 is reasonable, given the graph shown at the beginning of the activity. Sample response: The vertical axis ends a little past 20 but the line does not intersect the vertical axis. If I were to extend the line and use the same scale, the line should intersect the vertical axis a little less than halfway between 80 and 100.

# Activity 2 Coin Collector

Students attend to precision and strengthen their fluency in writing equations of lines using two coordinates of points.

**Amps Featured Activity** Coin Game

### Activity 2 Coin Collector

**The Coin Collector arcade game at Honest Carl's Funtime World requires a player to control a character that moves along a straight line to collect coins. The fewer lines a player uses, the more points they earn.**

For each graph shown, draw lines to collect coins. Label each line with a number (1, 2, 3, etc.), and then write the equation for each line.

**Note:** You may not need to use all of the space provided for the equations. Additionally, you may add more equations, as needed. Sample responses shown.

**Round 1:**

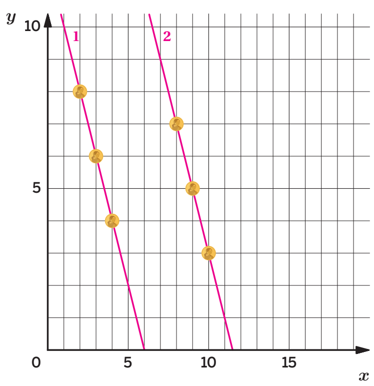
**Equations:**

Line 1: $y = -2x + 12$

Line 2: $y = -2x + 23$

Line 3:

Line 4:








## 1 Launch

Tell students that they are going to mimic playing the arcade game described in the prompt. The goal is to collect the coins using as few equations as possible. Use the Activity 2 PDF, to model different lines students can draw to collect coins. Provide access to rulers.

## 2 Monitor

**Help students get started** by instructing them to draw a line to collect the coins, and then label two points on that line. Next, have them use the method learned in Activity 1 to write the equation of the line.

**Look for points of confusion:**

- **Having trouble writing the equation.** Provide a list of steps. For example: First calculate the slope. Then choose any point and substitute its coordinates into the equation $y = mx + b$ to determine the $y$-intercept. Lastly, write the equation using the slope and $y$-intercept.

**Look for productive strategies:**

- Writing an equation with a positive or negative slope.
- Using any two points to write an equation for the line.

Activity 2 continued ›

## Differentiated Support

**Accessibility: *Optimize Access to Technology***

Have students use the Amps slides for this activity, in which they can enter an equation and view an animation of the line collecting the coins.

**Accessibility: *Vary Demands to Optimize Challenge***

If students need more processing time, have them focus on completing Rounds 1 and 2.

**Extension: *Math Enrichment***

Challenge students to collect all of the coins using only two equations for each round.

## Math Language Development

***MLR7: Compare and Connect***

During the Connect, ask students how the strategies used by their classmates are the same and how they are different. Consider asking:

- "How did you know if the coefficient of $x$ should be positive? Negative?"
- "How did you decide on a $y$-intercept?"

Have them discuss with their partner first and then ask pairs of students to share with the whole class.

**English Learners:**

Students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Pairs | 15 min

## **Activity 2** Coin Collector (continued)

Students attend to precision and strengthen their fluency in writing equations of lines using two coordinates of points.

Name: ______________________ Date: ____________ Period: ____________

### **Activity 2** Coin Collector (continued)

**Round 2:**

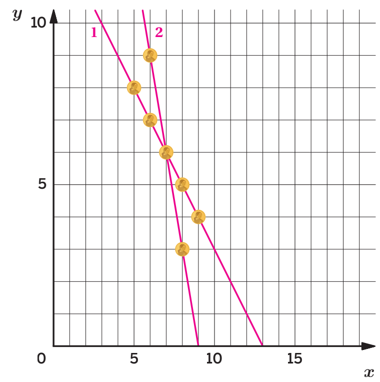
**Equations:**

Line 1: $y = -x + 13$

Line 2: $y = -3x + 27$

Line 3:

Line 4:




**Round 3:**

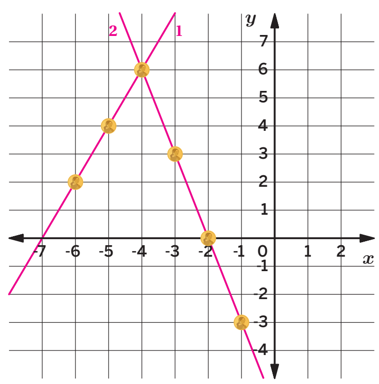
**Equations:**

Line 1: $y = 2x + 14$

Line 2: $y = -3x - 6$

Line 3:

Line 4:




STOP





### 3 **Connect**

**Display** the lines students drew for each round. Start with students who drew the most number of lines and progress to the students who drew the least number of lines.

**Have students share** their methods for writing equations for lines using two points.

**Highlight** that students do not need to see the vertical intercept to determine $b$. Only two points through which the line passes, even without a graph, are needed to write the equation of a line.

**Ask**, "How do you tell from a linear equation if its graph will have a positive or negative slope?"

Whole Class | 5 min

# Summary

Review and synthesize an algebraic method that can be used to write the equation of any line using two points through which the line passes.

## Summary

**In today's lesson . . .**

You wrote the equation of a line that passes through two points, including lines with a negative slope.

For example, to write an equation of a line that passes through the points $(1, 7)$ and $(2, 4)$, you can follow these steps.

1. Calculate the slope by finding the ratio of the difference in the $y$-coordinates to the difference in the $x$-coordinates: $\frac{7-4}{1-2} = -\frac{3}{1} = -3$. The slope is $-3$.
2. Substitute the slope and the coordinates of one of the points, for example $(1, 7)$, into the equation $y = mx + b$. Then solve for $b$.

| | |
|---|---|
| $7 = -3(1) + b$ | The slope is $-3$. The point is $(1, 7)$. |
| $7 = -3 + b$ | Multiply. |
| $10 = b$ | Add 3 to both sides. |

3. Write the equation in the form $y = mx + b$ using the slope, $-3$, and the $y$-intercept, $10$.

   The equation is $y = -3x + 10$.

Even if you used the other point $(2, 4)$, you would arrive at the same equation. Try it!

Reflect:




## Synthesize

**Have students share** how they can write the equation of any line using two points through which the lines passes in their own words.

**Highlight** that to write an equation using two points, first calculate the slope, and then substitute the coordinates of one of the points into the equation $y = mx + b$ to determine the $y$-intercept. Lastly, write the equation in the form $y = mx + b$.

**Ask:**

- "Will the slope of a line that passes through $(4, 10)$ and $(1, 8)$ be positive or negative?" Positive
- "What do you think a line will look like if the numerator in the slope is 0? What about if the denominator is 0?" **Note:** Students will explore these situations further in Lesson 15. Sample response: If the numerator is zero, then I think the line will be a flat horizontal line because there is no change. If the denominator is zero, then maybe the line is vertical because the $x$-values of a vertical line do not change.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies did you find helpful today when writing the equation of a line?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by writing the equation of a line given two points through which the line passes.

Printable

Name: ________ Date: ________ Period: ________

## Exit Ticket 3.14

**Without graphing, write an equation for the line that passes through the points $(2, 22)$ and $(5, 7)$. Show or explain your thinking.**

$y = -5x + 32$; Sample response:

Calculate the slope: $\frac{7-22}{5-2} = -\frac{15}{3} = -5$. The slope is $-5$.

Substitute the slope and the coordinates of one of the points into the equation $y = mx + b$.

Solve the equation for $b$.

$$7 = -5(5) + b$$
$$7 = -25 + b$$
$$32 = b$$

Write the equation in the form $y = mx + b$ using the slope, $-5$, and the $y$-intercept, $32$: $y = -5x + 32$.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can write an equation of a line with a positive or negative slope that passes through two points.

**1 2 3**



**Lesson 14** Writing Equations for Lines Using Two Points, Revisited

### Success looks like . . .

- **Goal:** Creating an equation of a line with a positive or negative slope on the coordinate plane.
  - » Writing an equation of a line that passes through given points.

### Suggested next steps

**If students write the incorrect slope, consider:**

- Reviewing Lesson 11, Activity 1.

**If students write the wrong $y$-intercept, consider:**

- Reviewing Activity 1.
- Reviewing how to substitute values in an equation.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- How was writing the equation of a line using an algebraic method similar to or different from writing the equation of a line using similar triangles from Lesson 11?
- What surprised you as your students worked on Activity 2?

# Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. Bard and Mai each write an equation of the line that passes through the points $(2, 9)$ and $(12, 14)$. They both calculate the slope as $\frac{1}{2}$.
   - Bard substitutes the point $(2, 9)$ to determine the $y$-intercept.
   - Mai substitutes the point $(12, 14)$ to determine the $y$-intercept.

   Each student's work is shown. Review their work and solutions. Find and fix any errors in each person's work.

   | Bard's work: | Mai's work: |
   | --- | --- |
   | $y = \frac{1}{2}x + b$ | $y = \frac{1}{2}x + b$ |
   | $9 = \frac{1}{2}(2) + b$ | $12 = \frac{1}{2}(14) + b$ |
   | $9 = 1 + b$ | $12 = 7 + b$ |
   | $b = 8$ | $b = 5$ |

   **Mai substituted the $x$- and $y$-values from the point $(12, 14)$ incorrectly. She reversed the $x$- and $y$-coordinates.**

   **Correct solution:**

   $y = \frac{1}{2}x + b$

   $14 = \frac{1}{2}(12) + b$

   $14 = 6 + b$

   $8 = b$

2. Write the equation of the line that passes through each pair of points. Show or explain your thinking.

   a. $(2, 14)$ and $(6, 26)$

   **Sample response:**

   **Slope:** $\frac{26 - 14}{6 - 2} = \frac{12}{4}$ **or 3**

   $14 = 3(2) + b$

   $14 = 6 + b$

   $b = 8$

   **Equation:** $y = 3x + 8$

   b. $(-5, 7)$ and $(1, 1)$

   **Sample response:**

   **Slope:** $\frac{1 - 7}{1 - (-5)} = -\frac{6}{6}$ **or** $-1$

   $1 = -1(1) + b$

   $1 = -1 + b$

   $b = 2$

   **Equation:** $y = -x + 2$





Name: ______ Date: ______ Period: ______

Practice

3. Clare added marbles to a container of water. When she added 5 marbles, the water level was 40 ml. When she added 7 marbles, the water level was 50 ml. Write an equation for the water level $y$ after $x$ marbles are added.

   **Sample response:**

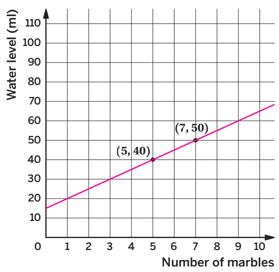
   **Slope: 5**

   $40 = 5(5) + b$

   $40 = 25 + b$

   $b = 15$

   $y = 5x + 15$

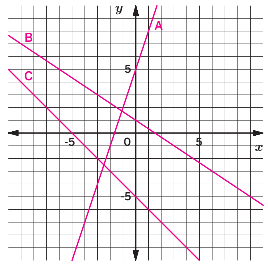



4. Graph each equation. Then label each line with its equation.

   a. **Equation A:** $y = 3x + 5$

   b. **Equation B:** $y = -\frac{2}{3}x + 1$

   c. **Equation C:** $y = -x - 5$



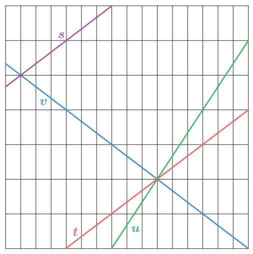

5. For each line, determine if the slope is positive or negative.

   a. line $s$ **positive**

   b. line $t$ **positive**

   c. line $u$ **positive**

   d. line $v$ **negative**








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
| --- | --- | --- | --- |
| On-lesson | 1 | Activity 1 | 1 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 3<br>Lessons 12–13 | 1 |
| Formative | 5 | Unit 3<br>Lesson 15 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

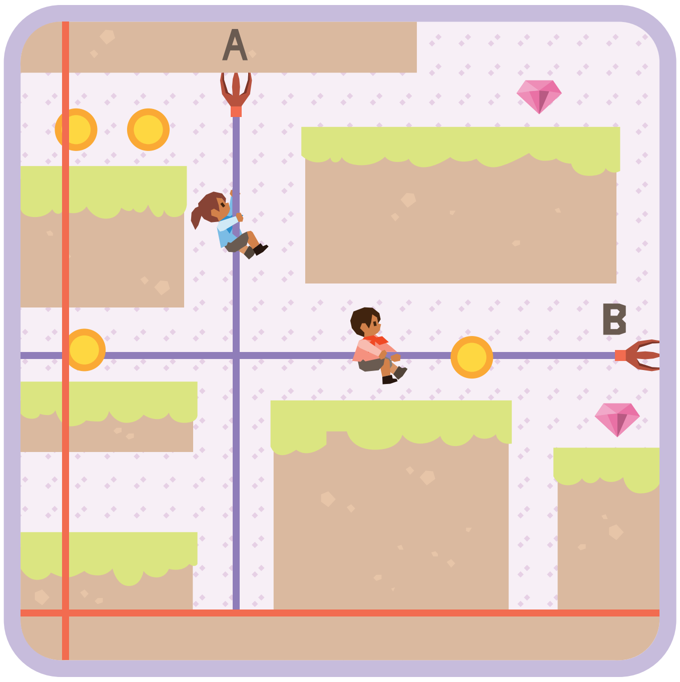
## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 3 | **LESSON 15**

# Equations for All Kinds of Lines

Let's write equations for vertical and horizontal lines.




## Focus

### Goals

1. Comprehend that for the graph of a vertical or horizontal line, one variable does not vary, while the other can have any value.
2. **Language Goal:** Generalize that a set of points of the form $(x, b)$ satisfies the equation $y = b$ and that a set of points of the form $(a, y)$ satisfies the equation $x = a$. **(Writing)**

## Coherence

### Today

Students extend their previous work to include equations for horizontal and vertical lines. They interpret the graph of a horizontal line and reason why the slope of zero makes sense in terms of the context. Students connect the equations of horizontal and vertical lines to their graphs, reasoning about why it makes sense that one variable remains constant, while the other variable can have any value. Students attend to precision as they apply their understanding in writing equations of lines with different slopes during a friendly competition.

### Previously

In Lesson 13, students wrote the equation of a line with a negative slope and interpreted the slope in context. In Lesson 14, students wrote equations of lines with a positive and negative slope using coordinates of points.

### Coming Soon

In Lesson 16, students start exploring linear equations that are not written in $y = mx + b$ form.

## Rigor

- Students build **conceptual understanding** of lines with a slope of zero as they interpret the graph of a horizontal line in context.
- Students build **conceptual understanding** of vertical and horizontal lines as they connect their equations to their graphs.
- Students write equations with different slopes to strengthen their **fluency** writing linear equations.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Activity 3 (optional) | Summary | Exit Ticket |
|---|---|---|---|---|---|
| 5 min | 15 min | 10 min | 15 min | 5 min | 5 min |
| Individual | Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

## Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 3 PDF (for display)
- colored pencils
- rulers

## Math Language Development

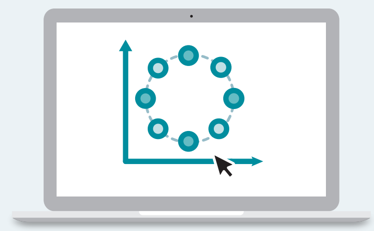
**Review words**

- *linear relationship*
- *horizontal*
- *vertical*
- *slope*

**Amps** | Featured Activity

### Activity 3
### Coin Game

Students enter an equation that will animate its corresponding line to help collect coins.



Amps POWERED BY desmos

## Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might feel uncomfortable with the new equations of lines with just one variable. Encourage students to step back and shift their perspective as they work through this activity. Students need to take control of their thoughts and focus them on determining why the equations are linear but have only one variable. With direct focus on this concept, students will be more likely to achieve success.

## Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- Optional **Activity 3** may be omitted.

Individual | 5 min

# **Warm-up** Which One Doesn't Belong?

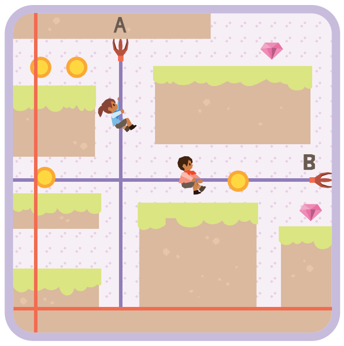
Students analyze four lines as an introduction to lines with a slope of zero.

Name: ______ Date: ______ Period: ______

**Unit 3 | Lesson 15**

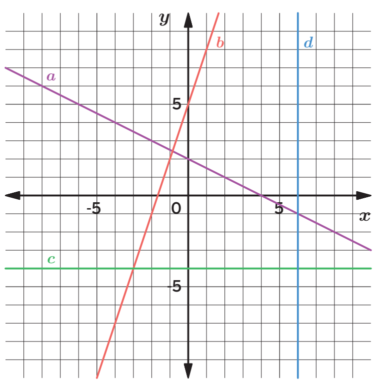
## Equations for All Kinds of Lines

Let's write equations for vertical and horizontal lines.




**Warm-up** Which One Doesn't Belong?

**Study the four lines shown. Which line does not belong? Explain your thinking.**




Sample response:

- Line $a$: This is the only line with a negative slope.
- Line $b$: This is the only line with a positive slope.
- Line $c$: This is the only line that is horizontal.
- Line $d$: This is the only line that is vertical.

Log in to Amplify Math to complete this lesson online.



Lesson 15 Equations for All Kinds of Lines 317

### 1 Launch

Conduct the ***Which One Doesn't Belong?*** routine.

### 2 Monitor

**Help students get started** by having them choose any one line and asking them what makes the line different from the others.

**Look for points of confusion:**

- **Thinking that line $d$ has a slope of zero.** Revisit slope with these students during the Connect.

**Look for productive strategies:**

- Visually inspecting the slope of each line.
- Noticing all of the lines have a different slope.

### 3 Connect

**Display** the graph from the Warm-up.

**Have students share** which line they chose. Use the ***Poll the Class*** routine to see which students selected each line.

**Ask** students if lines $a$ and $b$ have a positive or a negative slope. Then ask them about the slope of lines $c$ and $d$. Have students choose two points on the line to help them determine their response.

**Highlight** that lines $c$ and $d$ have a slope that is neither positive nor negative. Tell students that line $c$, a horizontal line, has a slope where the numerator is $0$ and the denominator is a non-zero number. Line $d$, a vertical line, has a slope where the numerator is a non-zero number and the denominator is $0$. Students will explore the equations for horizontal and vertical lines in the upcoming activities.

## Math Language Development

*MLR2: Collect and Display*

During the Connect, as students describe which line does not belong, collect and display the language they use that describes the slope, such as *negative*, *positive*, *horizontal*, and *vertical*. Add visual examples of each type of slope to the display. Keep the display up for the duration of this lesson.

**English Learners**

Use gestures to amplify the different types of slopes students describe.

## Power-up

**To power up students' ability to visually assess if the slope of line is positive or negative:**

Provide students with a copy of the Power-up PDF.

**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 14, Practice Problem 5

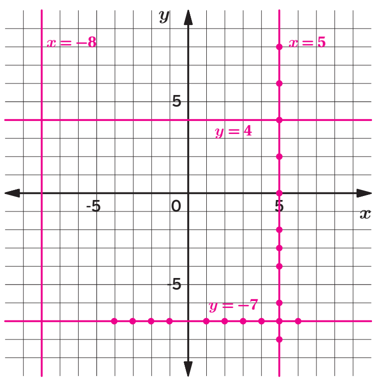
Pairs | 15 min

# Activity 1 All the Same

Students graph different points to write equations for horizontal and vertical lines and see that the variables for these lines are not dependent on one another.

### Activity 1 All the Same

Complete the following problems using the coordinate plane shown.




1. Plot at least 10 points whose $y$-coordinate is –7. What do you notice?
   Sample response: The points all lie on a horizontal line that intersects the $y$-axis at –7.

2. Study these equations. Which equation do you think represents *all* the points with a $y$-coordinate of –7?
   A. $x = -7$  B. $y = -7x$  C. $y = -7$  D. $x + y = -7$

3. Plot at least 10 points whose $x$-coordinate is 5. What do you notice?
   Sample response: The points all line on a vertical line that does not intersect the $y$-axis.

4. Study these equations. Which equation do you think represents *all* the points with a $x$-coordinate of 5?
   A. $x = 5$  B. $y = 5x$  C. $y = 5$  D. $x + y = 5$

5. Graph and label the equation $y = 4$ on the coordinate plane.

6. Graph and label the equation $x = -8$ on the coordinate plane.

318 Unit 3 Linear Relationships



## 1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity. Provide access to rulers for the duration of the lesson.

## 2 Monitor

**Help students get started** by plotting the point (5, –7) and labeling it for students to use as an example.

**Look for points of confusion:**

- **Plotting the points incorrectly.** Have students create a table labeled $x$ and $y$ and write their values in the table before plotting the points.
- **Thinking horizontal lines are written as $x = b$ and vertical lines are written as $y = a$.** Have students look at the coordinates of each point to see that vertical lines have the same values of $x$ and horizontal lines have the same values of $y$.

**Look for productive strategies:**

- Trying to use the slope and $y$-intercept to choose an equation.

## 3 Connect

**Display** student work showing the completed graph.

**Have students share** what they notice about the points, lines, and equations. Have them label each line with its equation.

**Ask**, "Why does the equation of a horizontal line not contain the variable $x$? Why does the equation of a vertical line not contain the variable $y$?" One of the two variables does not vary while the other can take any value.

**Highlight** that for a horizontal line, the value of $y$ is the same regardless of its value of $x$, and for a vertical line the value of $x$ is the same regardless of its value of $y$.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1-4. Alternatively, consider providing them with pre-completed graphs of the four equations. Have them determine 3–4 points that fall on each line and write the ordered pairs next the graph. Then have them complete Problems 2 and 4.

### Extension: *Math Enrichment*

Ask students to write the equations representing the two horizontal lines in slope-intercept form, $y = mx + b$. Then have them determine the value of the slope. $y = 0x + 4$ and $y = 0x + (-7)$; The slope of each line is 0.

Pairs | 10 min

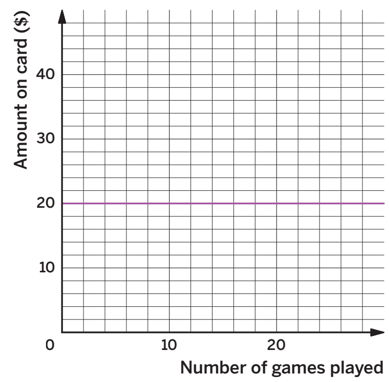
# Activity 2 Han's Game Card

Students interpret a graph of the situation and reason that it makes sense for the slope to be zero in terms of the context.

Name: ______ Date: ______ Period: ______

## Activity 2 Han's Game Card

The graph shows the available amount, in dollars, on Han's arcade game card at Honest Carl's Funtime World for one day.




1. Describe what happened to the available amount on Han's game card as the number of games played increased.
   Sample response: Han still had \$20 on the card after every game he played. The available amount on the card did not change.

2. What value makes sense for the slope of the line that represents the available amount on Han's game card? What does the slope represent in this situation?
   Sample response: The slope of the line is 0. This represents the amount he paid, which would be \$0 per game.

3. Write an equation that represents the available amount $y$ on the card after playing $x$ games.
   $y = 20$





### 1 Launch

Have students use the ***Think-Pair-Share*** routine.

### 2 Monitor

**Help students get started** by asking them what the points $(0, 20)$ and $(10, 20)$ represent in the context of the problem.

**Look for points of confusion:**

- **Writing the equation $x = 20$ in Problem 3.** Have students look back at the equations of horizontal lines in Activity 1. Ask them what the variable $x$ represents in this situation. If $x$ represents the number of games played, why would $x$ always equal 20?

**Look for productive strategies:**

- Noticing the available amount on the card remains the same.

### 3 Connect

**Have students share** what they noticed about the amount on Han's game card and what the slope represents in this situation.

**Highlight** that the slope of 0 means that no money is added or subtracted for each game he plays. The available amount Han has on his card will be the same regardless of the number of games he plays.

**Ask**, "How do you think it's possible for Han to have \$20 available on his card regardless of how many games he plays?" Sample responses: It is possible that the arcade offered free games for a period of time or that someone paid for his games that day.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Some students may think that because the amount on the card is not decreasing, that this means Han has not played any games. Show how the graph represents the number of games played has increased, and yet the amount on the card has not changed.

### Extension: *Math Enrichment*

Have students generate some other real-world examples of situations that could be represented by a horizontal line. Sample response: The dollar amount in a checking account as no money is deposited or withdrawn over several weeks.

## Math Language Development

### *MLR5: Co-craft Questions*

Before revealing the problems in this activity, display the introductory text and the graph. Ask them to work with their partner to write 1–2 questions they have about the graph or situation. Ask pairs of students to share their questions with the class.

### English Learners

Display a sample question, such as "Why is the amount on the card not decreasing?"

Optional

Pairs | 15 min

# Activity 3 Coin Collector, Revisited

Students attend to precision and strengthen their fluency in writing equations of lines, including lines that have a zero slope.

Amps Featured Activity | Coin Game

**Activity 3** Coin Collector, Revisited

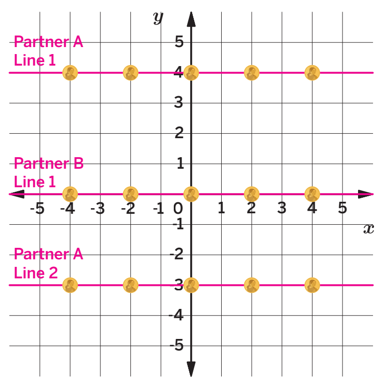
**During a two-player game of Coin Collector, players take turns moving a character along a straight line to collect coins. With your partner, determine who will be Partner A and who will be Partner B.**

For each round, take turns drawing lines and writing equations to collect the most coins. Then determine the total number of coins collected by each person.

**Round 1** Sample responses shown.

| Partner A's equations | Number of coins |
|---|---|
| Line 1: $y = 4$ | 5 |
| Line 2: $y = -3$ | 5 |
| Line 3: | |

| Partner B's equations | Number of coins |
|---|---|
| Line 1: $y = 0$ | 5 |
| Line 2: | |
| Line 3: | |




Partner A total: 10 coins    Partner B total: 5 coins

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## 1 Launch

Tell students they will revisit a version of the coin collector game in Lesson 14. Use the Activity 3 PDF to model how students will play the game in pairs. Tell students that once a coin is collected, it cannot be recollected to earn a point. Provide each partner with a different colored pencil to help them differentiate their lines.

## 2 Monitor

**Help students get started** by asking them to look for coins that are on the same vertical or horizontal line.

**Look for points of confusion:**

- **Writing the wrong equation.** Have students check each other's equations and lines.
- **Not realizing they can now write the equations of vertical or horizontal lines to collect coins.** Remind students that they now have more tools in their toolbox for writing equations to represent lines.

Activity 3 continued >

## Differentiated Support

### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can enter an equation and view an animation of the line collecting the coins.

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Rounds 1 and 2.

## Math Language Development

### *MLR7: Compare and Connect*

During the Connect, ask the following questions to help support students make connections between algebraic and graphical representations of horizontal and vertical lines.

- "What is the same and what is different about the equations of horizontal and vertical lines?"
- "How do equations of slanted lines compare to equations of horizontal or vertical lines?"

Consider displaying a graphic organizer, such as the following:

| Horizontal Lines | Vertical Lines | Slanted Lines |
|---|---|---|
| $y = \square$ | $x = \square$ | $y = \square x + \square$ |

Optional

Pairs | 15 min

# Activity 3 Coin Collector, Revisited (continued)

Students attend to precision and strengthen their fluency in writing equations of lines, including lines that have a zero slope.

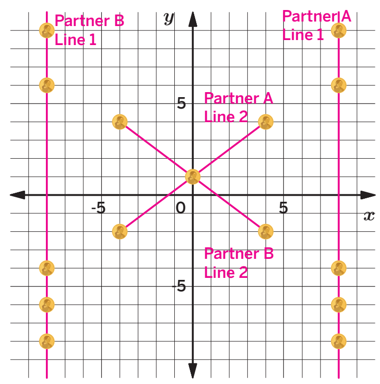
Name: ______ Date: ______ Period: ______

## Activity 3 Coin Collector, Revisited (continued)

**Round 2** Sample responses shown.

| Partner A's equations | Number of coins |
|---|---|
| Line 1: $x = 8$ | 5 |
| Line 2: $y = \frac{3}{4}x + 1$ | 3 |
| Line 3: | |

| Partner B's equations | Number of coins |
|---|---|
| Line 1: $x = -8$ | 5 |
| Line 2: $y = -\frac{3}{4}x + 1$ | 2 |
| Line 3: | |

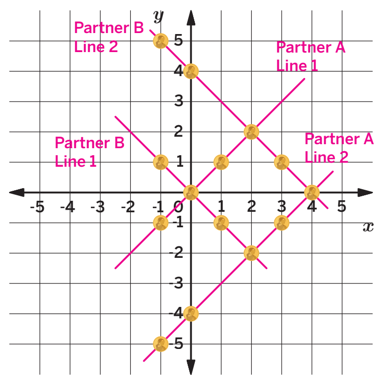



**Partner A total:** 8 coins **Partner B total:** 7 coins

**Round 3** Sample responses shown.

| Partner A's equations | Number of coins |
|---|---|
| Line 1: $y = x$ | 4 |
| Line 2: $y = x - 4$ | 4 |
| Line 3: | |

| Partner B's equations | Number of coins |
|---|---|
| Line 1: $y = -x$ | 3 |
| Line 2: $y = -x + 4$ | 3 |
| Line 3: | |




**Partner A total:** 8 coins **Partner B total:** 6 coins

STOP





## 3 Connect

**Display** the lines students drew for each round.

**Have pairs of students share** their strategies for collecting the most coins.

**Ask**, "What are some ways you can check whether your partner wrote the correct equation for a line they drew?" When $\square = a$ number, equations in the form $y = \square$ should be horizontal, equations in the form $x = \square$ should be vertical, and equations in the form $y = \square x + \square$ should be slanted. I can substitute the coordinates of a point into each equation to see if the equation is true.

**Highlight** that the equation of a horizontal or vertical line will only have one variable. Ask students to explain why this is true. Students should realize that for horizontal or vertical lines, one variable remains constant, while the other variable can have any value.

Whole Class | 5 min

# Summary

Review and synthesize how to write equations of horizontal and vertical lines.

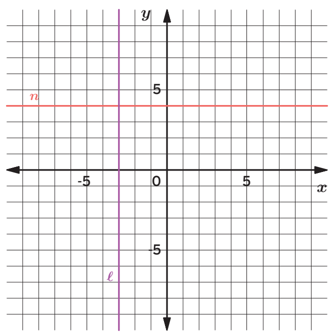
## Summary

**In today's lesson . . .**

You wrote equations for horizontal and vertical lines. In the coordinate plane . . .

- Horizontal lines represent situations where the $y$-values do not change when the $x$-values change. Horizontal lines have a slope of $0$.
- Vertical lines represent situations where the $x$-values do not change when the $y$-values change. Vertical lines have an undefined slope.

For example, the horizontal line $n$ shown is represented by the equation $y = 4$. The vertical line $\ell$ shown is represented by the equation $x = -3$.




**Reflect:**




## Synthesize

**Have students share** how they can tell a line will be horizontal, vertical, or neither from its equation.

**Highlight** that a set of points in the form $(x, b)$ satisfies the equation $y = b$ and that a set of points in the form $(a, y)$ satisfies the equation $x = a$.

**Ask**, "What do the lines with the equations $y = 3$, $x = 3$, and $y = 3x$ look like?" Sample response: $y = 3$ is a horizontal line that passes through every point that has a $y$-coordinate of $3$. $x = 3$ is a vertical line that passes through every point that has an $x$-coordinate of $3$. $y = 3x$ is a proportional line that passes through the points $(0, 0)$ and $(1, 3)$.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do the equations of horizontal and vertical lines compare to the equations of lines that are not horizontal or vertical?"

Independent | 5 min

# Exit Ticket

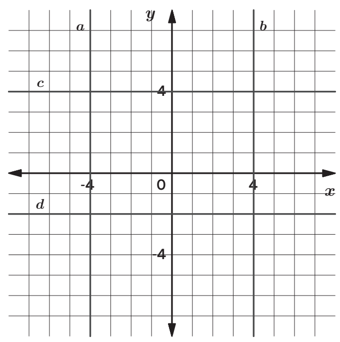
Students demonstrate their understanding by writing equations of vertical and horizontal lines.

Printable

Name: ______________________ Date: __________ Period: __________

## Exit Ticket 3.15

Consider the four lines shown on the coordinate plane. Write an equation for each line.




1. Line $a$: $x = -4$
2. Line $b$: $x = 4$
3. Line $c$: $y = 4$
4. Line $d$: $y = -2$

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can write equations to represent vertical and horizontal lines.
1 2 3





## Success looks like . . .

- **Goal:** Comprehending that for the graph of a vertical or horizontal line, one variable does not vary, while the other can take any value.
- **Language Goal:** Generalizing that a set of points of the form $(x, b)$ satisfies the equation $y = b$ and that a set of points of the form $(a, y)$ satisfies the equation $x = a$. **(Writing)**
  - » Writing the equation of the horizontal or vertical line by determining the form of the points on each line.

## Suggested next steps

**If students write the incorrect variable in the equation, consider:**

- Choosing and labeling several points on the line and asking them to look at the values of each variable. Ask them which variable remains the same, and how that should be expressed in the equation.
- Reviewing Activity 1.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- Which students' ideas were you able to highlight during Activity 2?
- In what ways have your students improved at interpreting the slope of a linear equation in context?

# Practice

Independent

Practice

Name: ________ Date: ________ Period: ________

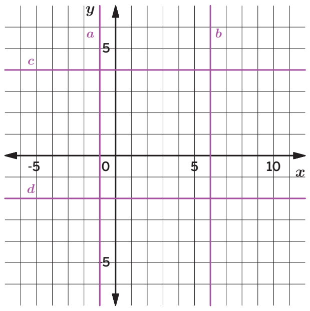
1. Consider the four lines shown on the coordinate plane. Write an equation for each line.

   a. Line $a$: $x = -1$

   b. Line $b$: $x = 6$

   c. Line $c$: $y = 4$

   d. Line $d$: $y = -2$



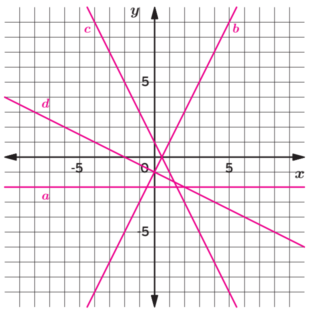

2. Use the coordinate plane to draw lines that meet the given criteria. Then write an equation for each line.

   a. Slope: 0
   $y$-intercept: $-2$
   $y = -2$

   b. Slope: 2
   $y$-intercept: $-1$
   $y = 2x - 1$

   c. Slope: $-2$
   $y$-intercept: 1
   $y = -2x + 1$

   d. Slope: $-\frac{1}{2}$
   $y$-intercept: $-1$
   $y = -\frac{1}{2}x - 1$








Practice

Name: ________ Date: ________ Period: ________

3. Lin's mom pays her smartphone bill each month. The table shows the amount of data used, in gigabytes (GB), and the total cost for several months. What equation relates the monthly cost $y$ to the amount of data $x$ Lin used?

| | January | February | March | April | May |
|---|---|---|---|---|---|
| Amount of data used (GB), $x$ | 1.2 | 2 | 3.2 | 1.5 | 4.2 |
| Cost ($), $y$ | 50 | 50 | 50 | 50 | 50 |

A. $y = 50$ B. $x = 50$ C. $y = 50x$ D. $x + y = 50$

4. A publisher wants to determine the thickness of a book they will print soon. The book has a front cover and a back cover, each of which have a thickness of $\frac{1}{4}$ in. The publisher can choose on what type of paper to print the book.

   a. Bond paper has a thickness of $\frac{1}{4}$ in. per 100 pages. Write an equation that gives the width $y$ of the book, if it has $x$-hundred pages printed on bond paper.
   $y = \frac{1}{4}x + \frac{1}{2}$

   b. Ledger paper has a thickness of $\frac{2}{5}$ in. per 100 pages. Write an equation that gives the width $y$ of the book, if it has $x$-hundred pages printed on ledger paper.
   $y = \frac{2}{5}x + \frac{1}{2}$

   c. If the publisher selects front and back covers with a thickness of $\frac{1}{3}$ in. each, how would this change the equations you wrote in parts a and b?
   For part a: $y = \frac{1}{4}x + \frac{2}{3}$
   For part b: $y = \frac{2}{5}x + \frac{2}{3}$

5. Consider the equation $\frac{1}{2} = \frac{x}{y}$. What is a possible solution to the equation for $x$ and $y$? Explain your thinking.
   Sample response: $x = 2$ and $y = 4$, because $\frac{1}{2} = \frac{2}{4}$ is a true statement.





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 1 |
| | 2 | Activity 3 | 1 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 3 Lesson 10 | 2 |
| Formative | 5 | Unit 3 Lesson 16 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Sub-Unit 3
# Linear Equations

Whole Class

Students explore what it means for an ordered pair to be a solution to a linear relationship, using a graph, table, or the equation to justify their thinking.

SUB-UNIT

3 Linear Equations

Narrative Connections


## How did a 16-year-old take down a Chicago Bull?

In 1987, 16-year-old Eric Barber had written to the TV show “SportsWorld,” regarding a featured segment called “Sports Fantasy,” which gave viewers the chance to act out their greatest sports dreams. For Barber, it was a face-off against one of the greatest basketball players of all time, Michael Jordan.

The twist? They would play in wheelchairs.

Born with scoliosis, Barber lost the use of his legs at the age of three, but that didn’t stop his love for the game. He learned to dribble and shoot on Chicago’s playgrounds, and at the age of 13 he was introduced to wheelchair basketball.

Jordan accepted the challenge. The day of the match, Barber was confident his experience would shut Jordan out. And at the start, it looked like Barber was right. He was soon leading 16–4. But Jordan quickly caught up, closing the gap to 18–14.

Finally, Barber sank the winning basket, thereby besting the legend with a final score of 20–14. Barber would go on to become a two-time bronze medal winning member of the U.S. Paralympic wheelchair basketball team.

As in the NBA, points in wheelchair basketball are scored depending on where a shot is taken from. Shots from half-court and beyond are worth three points, while shots within half-court are two points. In Barber and Jordan’s face-off, both competitors had several ways to score a winning basket. And linear equations can help model Barber’s path to victory.





## Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore real-world applications of linear equations in the following places:

- **Lesson 16, Activity 1:** Barber vs. Jordan
- **Lesson 18, Activity 1:** Representations of Linear Relationships
- **Lesson 18, Activity 2:** Info Gap: Linear Relationships

# Solutions to Linear Equations

Let's think about what the solution to a linear equation with two variables means.


## Focus

### Goals

1. Comprehend that the points that lie on the graph of an equation represent exactly the solution set of the equation of the line (i.e., that every point on the line is a solution, and any point *not* on the line is *not* a solution).
2. Create a graph and an equation in the form $Ax + By = C$ that represent a linear relationship.
3. Determine pairs of values that satisfy or do not satisfy a linear relationship using an equation or graph.

## Coherence

### Today

Students explore linear relationships as an equation with two variables and graph an equation in the form of $Ax + By = C$. They determine whether points on the graph of the equation represent solutions to the equation.

### Previously

Students have previously explored linear relationships in contexts where one variable depends on another, for example, distance depending on time.

### Coming Soon

In Lesson 17, students continue to work with linear equations in two variables by considering ordered pairs as solutions on a graph and by solving equations.

## Rigor

- Students build **conceptual understanding** of solutions that represent real-world scenarios using linear equations of the form of $Ax + By = C$.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Activity 3 (optional) | Summary | Exit Ticket |
|---|---|---|---|---|---|
| 5 min | 15 min | 15 min | 10 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Independent | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

## Materials

- Exit Ticket
- Additional Practice
- rulers

## Math Language Development

**Review words**

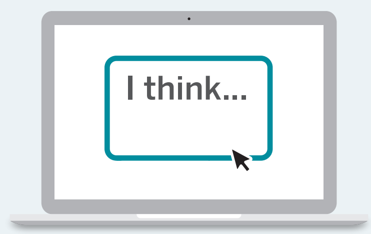
- *integer*
- *linear relationship*
- *ordered pair*

**Amps** | Featured Activity

**Activity 2**

### See Student Thinking

Students come up with length and width measures for rectangles by completing a table and updating a graph as you monitor their data in real time.




## Building Math Identity and Community

Connecting to Mathematical Practices

Some students may not be familiar with the kinds of baskets in basketball in Activity 1. Before the activity, encourage students to take on the perspective of someone who has never heard of basketball and have them explain what is needed to know for this exercise. By taking this approach, students all fully-understand the background in order to justify their models and make sense of the interpretations of them.

## Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 1 may be omitted.
- In **Activity 2**, have students only complete the first three columns of the table.

Independent | 5 min

# Warm-up Ordered Pairs

Students find a solution to an equation with two variables to see that they can solve for one variable only when another variable is fixed.

Unit 3 | Lesson 16

## Solutions to Linear Equations

Let's think about what the solution to a linear equation with two variables means.


### Warm-up Ordered Pairs

Choose a value for $x$ and a corresponding value for $y$ that makes the following equation true.

$3x + 2y = 24$

Sample response: $x = 4$, $y = 6$

Log in to Amplify Math to complete this lesson online.

326 Unit 3 Linear Relationships



## 1 Launch

Activate prior knowledge by asking, "How can you show that a pair of numbers is a solution to the equation?" Provide access to rulers for the duration of this lesson.

## 2 Monitor

**Help students get started** by asking what value they chose for $x$ and how they then solved the equation for $y$.

**Look for points of confusion:**

- **Thinking inverse operations alone can find the values of two different unknowns.** Remind students that this equation has two unknowns and have them find a value for $x$ before solving for $y$.

**Look for productive strategies:**

- Finding multiple solutions for $x$ and $y$.
- Finding non-integer solutions.

## 3 Connect

**Display** the equation from the Warm-up.

**Have students share** how they found their solutions by taking several different correct responses.

**Highlight** that the solutions to the equation with two variables are the values of $x$ and $y$ that make the equation true.

**Ask:**

- "Is it possible to find a solution without knowing either value of $x$ and $y$? Why or why not?" Yes; Sample response: I can use the guess-and-check method to try different values of $x$ and $y$ to make the equation true.
- "If you chose a new value for $x$, can you always then find a value for $y$?" Yes, by substituting the $x$-value and solving the equation for $y$.

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge***

Consider demonstrating how to determine the value of $y$ for a given value of $x$, such as $x = 0$. Then provide students with a sample $x$-value that they can use to get started, such as $x = 2$.

## Power-up

**To power up students' ability to determine a pair of values that would make an equation true, have students complete:**

Consider the equation $\frac{1}{3} = \frac{a}{b}$. Answers may vary.

1. What is a fraction that is equivalent to $\frac{1}{3}$?
2. Compare your fraction in Problem 1 to $\frac{a}{b}$. Which value in your fraction is equal to $a$?
3. Compare your fraction in Problem 1 to $\frac{a}{b}$. Which value in your fraction is equal to $b$?

**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 15, Practice Problem 5

Pairs | 15 min

# Activity 1 Barber vs. Jordan

Students write an equation representing a relationship between two quantities, and use the equation to find pairs of numbers that make it true.

Name: ____________ Date: ________ Period: ________

## Activity 1 Barber vs. Jordan

**Eric Barber competed in four Paralympic Games in Wheelchair Basketball, and won two bronze medals during his career. But he might be most known for a game of one-on-one he played against NBA basketball legend, Michael Jordan. For the match, it was decided the first player to score 20 points would win. The players could score baskets worth 2 points, two-pointers or 3 points, three-pointers. Both players would play while in a wheelchair.**

1. Determine the number of points Eric Barber scored if he made:
   a. 5 two-pointers and 2 three-pointers.
      **16 points**
   b. 4 two-pointers and 4 three-pointers.
      **20 points**
   c. 3 two-pointers and 1 three-pointer.
      **9 points**

2. Barber had an early lead and was winning 16 – 4, before Jordan began to catch up. What combinations of baskets could Barber have made to score his 16 points? Explain your thinking.
   **Sample response: He could have made 4 three-pointers and 2 two-pointers.**

3. Eric Barber eventually won the game 20 – 14. Use two variables to write an equation that represents possible combinations of two-pointers and three-pointers that would equal a total score of 20 points. Be sure to define your variables.
   **Sample response: Let $x$ represent the number of two-pointers made and let $y$ represent the number of three-pointers made. The equation is $2x + 3y = 20$.**





## 1 Launch

Conduct the ***Think-Pair-Share*** routine.

## 2 Monitor

**Help students get started** by asking, "How many points is 5 two-pointers worth?"

**Look for points of confusion:**

- **Not knowing how to start Problem 2.** Have students try to determine the greatest number of three-pointers Barber could have made.
- **Writing $x + y = 14$ for the equation where $x$ and $y$ represent the points from two- and three-pointers.** Ask students what unknowns they are trying to find and have them define their variables for those unknowns. Ask students to consider what steps they took in Problem 1 to find the total points.

**Look for productive strategies:**

- Precisely defining variables as the number of two- and three-point baskets made.

## 3 Connect

**Ask:**

- "How many combinations did you find for Problem 2? How did you know you found all of them?" Three combinations, only whole number values make sense in context.
- "Is the equation you wrote for Problem 3 a linear equation? Why or why not?"
- "How did you decide to define your variables?"
- "If $x$ represents the number of two-point baskets made, is it realistic for $x = 2.5$?"

**Highlight** how the scenario can be represented by a linear equation with two variables. Show different letters being used for variables to highlight that it does not matter which letters are used as long as they are defined precisely. Ask why defining $x$ as "baskets" is insufficient.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Consider providing a table, or suggest students create one, that they can use to organize the number of two-pointers and three-pointers that result in various scores for Problems 1 and 2. This will help them visualize the relationships to write the equation in Problem 3. For example:

| Number of points for Two-pointers | Number of points for Three-pointer | Total number of points |
|---|---|---|
| 2(____) | 3(____) | 2(____) + 3(____) |

## Math Language Development

### *MLR6: Three Reads*

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that Eric Barber and Michael Jordan played a game of wheelchair basketball.
- **Read 2:** Students should annotate the given quantities, such as the number of points scored for two-pointers and three-pointers.
- **Read 3:** Ask students to preview Problems 1-2 to brainstorm strategies to determine combinations of baskets made for given total scores.

### English Learners

Emphasize that a "two-pointer" is the actual basket that is made and 2 points is the score given.

Pairs | 15 min

# Activity 2 Rectangles

Students write an equation in the form of $Ax + By = C$ and graph the solutions to see that the points create a line.

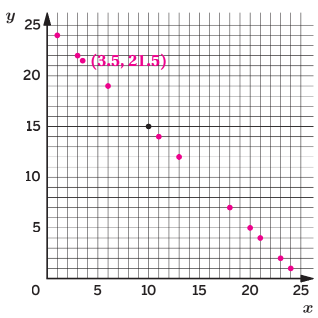
Amps Featured Activity   See Student Thinking

### Activity 2 Rectangles

1. There are many possible rectangles whose perimeter is 50 units. Complete the table with lengths $x$ and widths $y$ of at least five rectangles whose perimeter is 50 units. Sample response shown.

| Length, $x$ | 10 | 12 | 5 | 20 | 8 |
|---|---|---|---|---|---|
| Width, $y$ | 15 | 13 | 20 | 5 | 17 |

2. A rectangle with a length of 10 and a width of 15 is represented by the point $(10, 15)$. Plot the lengths $x$ and widths $y$ of the other rectangles whose perimeter is 50 units. What do you notice?
I noticed the points appear to form a pattern that can be represented by a line.




3. Let $x$ represent the length and let $y$ represent the width of a rectangle whose perimeter is 50 units. Write an equation that represents the relationship between $x$, $y$, and 50.
Sample response: $2x + 2y = 50$

4. Could one of these rectangles have a width of 3.5 units? Explain your thinking using the graph and the equation.
Yes; Sample response: The width could be 3.5 units if the length is 21.5 units; $2(21.5) + 2(3.5) = 50$. The graph would show a point located at $(21.5, 3.5)$.

328 Unit 3 Linear Relationships



## 1 Launch

Ask students to sketch a rectangle whose perimeter is 50 units and label the lengths of its sides. After giving them a minute to come up with their rectangle, ask them to share some of the lengths and widths they found.

## 2 Monitor

**Help students get started** by having them pick a length, and then sketch a rectangle to find the width.

**Look for points of confusion:**

- **Thinking the length and width will add to 50.** Remind students they have to consider all four sides when finding the perimeter.

**Look for productive strategies:**

- Writing the equations $2x + 2y = 50$ or $y = 25 - x$.

## 3 Connect

**Display** student work showing various points plotted in Problem 2.

**Have students share** what equations can be used to represent the context. Discuss how the equations $y = 25 - x$ or $2x + 2y = 50$ (or $x + y = 25$) represent the scenario.

**Ask:**

- "If you know an ordered pair is a solution to the equation, what does that look like on the graph?"
- "Is the ordered pair $(10, 10)$ a solution?"
- "Imagine a line connecting the points. Is the slope positive or negative? What does a negative slope mean in this context?"
- "What are the vertical and horizontal intercepts? What do they represent in context?"

**Highlight** that the slope is negative, which means that as the width increases, the length decreases. Show that any point on the line segment has a perimeter of 50 with side lengths equal to the $x$- and $y$-coordinates of the point.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Provide students with a partially completed table for Problem 1 with the lengths given. Have them determine the corresponding widths. Consider providing them with a pre-completed graph for Problem 2 and have them record what they notice. This will still allow them to access the goal of the activity without having to create the graph themselves.

### Extension: *Math Enrichment*

Ask students if the points $(0, 25)$ and $(25, 0)$ are solutions to the equation and make sense within the problem. They are solutions to the equation, but they do not make sense within the problem because a rectangle cannot have a length or width of 0 units.

## Math Language Development

### *MLR7: Compare and Connect*

During the Connect, as students share the equations they wrote, display the different equations that can accurately represent this situation and ask students how they compare with one another and how they relate to the graph. Consider asking these questions:

- Where do you see the $y$-intercept in the equation $y = 25 - x$? In the equation $x + y = 25$? In the equation $2x + 2y = 50$?
- If the perimeter is 50, why is the $y$-intercept 25?

### English Learners

Use color coding to annotate the equations and graphs with how they demonstrate the intercepts and slope, or how these values can be determined from the different equations.

Optional

Independent | 10 min

# Activity 3 Diophantine Equation

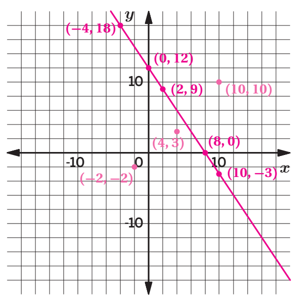
Students explore the properties of a Diophantine Equation, seeing a historical example that deepens their understanding of the solutions to a linear equation.

Name: ______________________ Date: __________ Period: __________

## Activity 3 Diophantine Equation

**A Diophantine equation is an equation involving two or more variables, such as $Ax + By = C$, where the solutions of interest are pairs of integers. For example, the equation $3x + 2y = 24$ would be a Diophantine equation if the only solutions that made sense were integer solutions.**

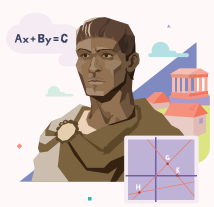
1. Plot at least five points that make the equation $3x + 2y = 24$ true. What do you notice about the points you have plotted?
   **Sample answer: I notice the points seem to form a straight line that has a negative slope.**




2. List three points that do not make the statement true. Using a different color, plot and label each point on the same coordinate plane. What do you notice about these points compared to your first set of points?
   **Sample response: These points do not lie on the line formed by the set of points that make the equation true.**

3. Is the ordered pair $(-2, 15)$ a solution to the Diophantine equation? Explain your thinking.
   **Yes; Sample responses:**
   - **$3(-2) + 2(15) = 24$ is a true statement.**
   - **The point $(-2, 15)$ is on the line.**

**Featured Mathematician**




**Diophantus**

The term *Diophantine* comes from the 3rd century mathematician Diophantus of Alexandria. Diophantus was known for his published work, *Arithmetica*, in which he detailed the solutions to different types of algebraic problems. At the time, algebraic symbols and variables had not yet been invented, so Diophantus had to use words to describe the "first unknown" or "second unknown."

STOP



**Lesson 16** Solutions to Linear Equations 329

## 1 Launch

Have students restate in their own words what a Diophantine Equation is after reading the introductory text. Ask students whether Activity 1 or 2 had a Diophantine Equation. Activity 1; There could only be positive integer solutions for the number of baskets.

## 2 Monitor

**Help** students get started by asking them to find a value of $x$ and a value of $y$ that makes an ordered pair a solution to the equation.

**Look for points of confusion:**

- Thinking they can only use positive values. Point out the word *integer* and ask students what they think it means. If needed, clarify that *integer* means positive or negative whole numbers.

**Look for productive strategies:**

- Noticing the pattern that shows adding 2 to the values of $x$ and subtracting 3 from the values of $y$.

## 3 Connect

**Display** student work showing points on the graph.

**Have students share** how they determined their ordered pairs.

**Ask:**

- "If you connect the points, will it form a line? Why or why not?"
- "Will the pattern extend into other quadrants besides quadrant I? How do you know?"

**Highlight** that any solution to the equation can be found on the line. Any point *not* on the line is *not* a solution.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Display the general form of a Diophantine equation and the given equation $3x + 2y = 24$ vertically aligned so that students can see the value of $A$ is 3, the value of $B$ is 2, and the value of $C$ is 24. Ask them to underline or highlight the phrase "whose solutions include pairs of integers" and explain that if the only solutions of interest (given by the context) are integer solutions, the equation is Diophantine. Otherwise, the equation is not Diophantine.

## Featured Mathematician

### Diophantus

Have students read about Diophantus, who detailed the solutions to different types of algebraic problems.

Whole Class | 5 min

# Summary

Review and synthesize what a solution to a linear equation in two variables represents and how solutions can be found.

## Summary

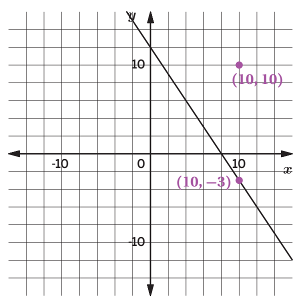
**In today's lesson . . .**

You saw that a solution to an equation with two variables is any ordered pair $(x, y)$ that makes the equation true.

You can think of pairs of numbers that are solutions to a linear equation as ordered pairs $(x, y)$ that represent points on the coordinate plane. These points form a line that represents all of the solutions to the equation. Only points that fall on the line are solutions to the equation. Points that do not fall on the line are *not* solutions to the equation.

For example, consider the linear equation $3x + 2y = 24$.

- The point $(10, -3)$ is on the line $3x + 2y = 24$. The ordered pair $(10, -3)$ is a solution to the equation $3x + 2y = 24$ because it makes the equation true; $3(10) + 2(-3) = 24$.
- The point $(10, 10)$ is not on the line. The ordered pair $(10, 10)$ is not a solution because $3(10) + 2(10) = 50$, not $24$.




Reflect:





## Synthesize

**Display** the Summary page from the Student Edition.

**Have students share** how they found solutions to equations and graphs in today's lesson.

**Ask:**

- "How are solutions to an equation represented on a graph?" Solutions to an equation are represented by points on the graph of the equation. For example, the point $(10, -3)$ is a solution to the equation $3x + 2y = 24$ because that point lies on the graph of the equation.
- "Is the ordered pair $(1.5, 2.5)$ a solution to the equation $3x + 2y = 24$? How can you be certain?" No; Sample response: The point $(1.5, 2.5)$ does not lie on the graph of the equation. I can verify this by substituting $1.5$ for $x$ and $2.5$ for y in the equation $3x + 2y = 24$. Because $3(1.5) + 2(2.5)$ does not equal 24, the ordered pair is not a solution to the equation.

**Highlight** that for the ordered pair to be a solution to an equation, the point must lie on the graph of the equation. When it is difficult to see on the graph, the coordinates of point $(x, y)$ can be substituted into the equation for values of $x$ and $y$ to see whether they make the equation true.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition.

To help them engage in meaningful reflection, consider asking:

- "What does it mean for an ordered pair to be a solution to a linear equation?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by identifying ordered pairs that make the equation true.

Printable

Name: Date: Period:

## Exit Ticket 3.16

Select *all* of the following ordered pairs that make the equation $x - 9y = 12$ true.

A. $(12, 0)$

B. $(0, 12)$

C. $(3, -1)$

D. $(0, -\frac{4}{3})$

E. $(-9, 12)$

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I know that the graph of an equation is a visual representation of all the solutions to the equation.
1 2 3

b. I understand that the solutions to an equation in two variables are the values of the variables that make the equation true.
1 2 3



**Lesson 16** Solutions to Linear Equations

## Success looks like . . .

- **Goal:** Comprehending that the points that lie on the graph of an equation represent exactly the solution set of the equation of the line (i.e., that every point on the line is a solution, and any point *not* on the line is *not* a solution).
- **Goal:** Creating a graph and an equation in the form $Ax + By = C$ that represent a linear relationship.
- **Goal:** Determining pairs of values that satisfy or do not satisfy a linear relationship using an equation or graph.
  - » Determining whether an ordered pair makes the equation true.

## Suggested next steps

**If students are unsure how to check if the ordered pairs are a solution, consider:**

- Reviewing Activity 2.
- Providing a graph of the line as a visual support for students.
- Assigning Practice Problem 1.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- During the discussion about Activity 2, how did you encourage each student to listen to one another's strategies?
- How did Activity 1 set students up to develop an understanding of the solutions to a linear equation?

# Practice

Independent

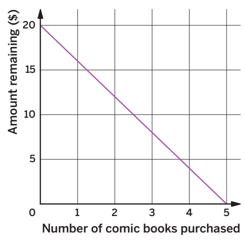
Name: ______ Date: ______ Period: ______

Practice

1. Select *all* of the ordered pairs $(x, y)$ that are solutions to the linear equation $2x + 3y = 6$.

   A. $(0, 2)$
   B. $(0, 6)$
   C. $(2, 3)$
   D. $(3, -2)$
   E. $(3, 0)$
   F. $(6, -2)$

2. The graph shows a linear relationship between $x$ and $y$. Suppose $x$ represents the number of comic books Priya buys at the store, all at the same price, and $y$ represents the amount of money, in dollars, she has after buying comic books.




   a. Find and interpret the $x$- and $y$-intercepts of this line
   
   $y$-intercept: Priya has \$20 before buying any comic books.
   
   $x$-intercept: If she buys 5 comic books, Priya will be out of money.

   b. Find and interpret the slope of this line.
   
   The slope of the line is $-4$. Using the points $(0, 20)$ and $(5, 0)$, I see that $\frac{20-0}{0-5} = \frac{20}{-5} = -4$. This means, for every comic Priya buys, she has 4 fewer dollars.

   c. Write an equation of this line.
   
   $y = -4x + 20$ or $y + 4x = 20$

   d. If Priya buys 3 comic books, how much money will she have left? Explain your thinking.
   
   \$8; Sample response: I substituted 3 for $x$ in the equation $y = -4x + 20$ and evaluated the expression to solve for $y$; $-4(3) + 20 = 8$

3. A container of fuel dispenses fuel at a rate of 5 gallons per minute. Let $y$ represent the amount of fuel remaining in the container, and $x$ represent the number of minutes that have passed since the fuel started dispensing. On the coordinate plane, will the slope of the line representing this relationship have a positive, negative, or zero slope? Explain your thinking.

   Negative; Sample response: The amount of fuel in the tank is decreasing.





Name: ______ Date: ______ Period: ______

Practice

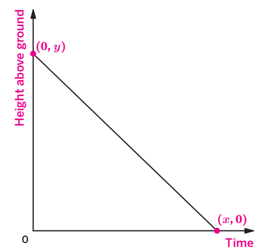
4. A sandwich store charges a delivery fee to bring lunch to an office building. One office pays \$33 for 4 turkey sandwiches. Another office pays \$61 for 8 turkey sandwiches. How much does each turkey sandwich add to the cost of the delivery? Explain your thinking.

   \$7; Sample response: The second office pays $\$61 - \$33 = \$28$ more for an additional $8 - 4 = 4$ sandwiches. This means the 4 extra sandwiches cost \$28, which means one sandwich costs $\$\frac{28}{4} = \$7$.

5. Match each pair of points with the slope of the line that passes through them.

| Pair of points | | Slope |
|---|---|---|
| a. $(9, 10)$ and $(7, 2)$ | a, d | $4$ |
| b. $(-8, -11)$ and $(-1, -5)$ | c | $-3$ |
| c. $(5, -6)$ and $(2, 3)$ | e | $-\frac{5}{2}$ |
| d. $(6, 3)$ and $(5, -1)$ | b | $\frac{6}{7}$ |
| e. $(4, 7)$ and $(6, 2)$ | | |

6. Refer to the graph shown.




   a. Write a story that matches the graph and label the axes
   
   Sample response: A plane descends from the sky at a constant speed until it lands.

   b. Label two points on the line, one where $x = 0$ and one where $y = 0$. Then explain what each point means in the context of the story.
   
   Sample response: The point where $x = 0$ represents the initial height of the plane before it begins to descend. The point where $y = 0$ represents the time it takes for the plane to land on the ground.





## Practice Problem Analysis

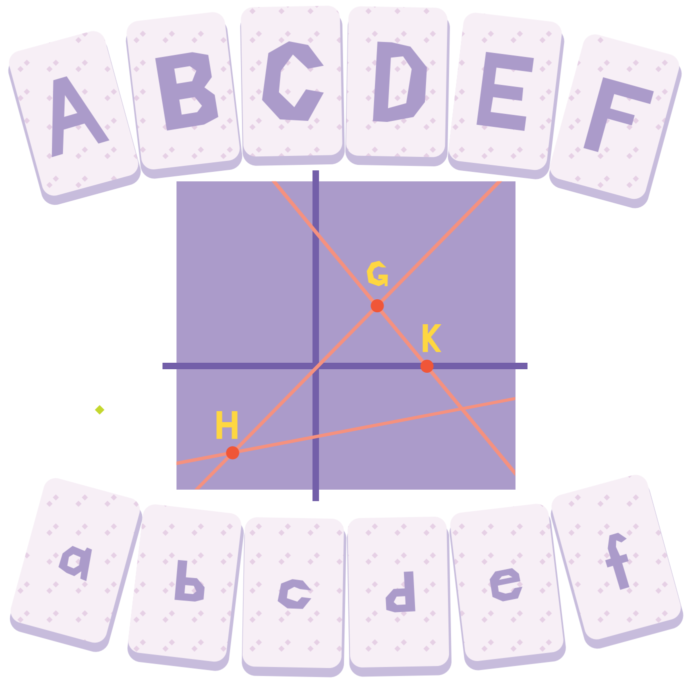
| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 1 |
| | 2 | Activity 2 | 2 |
| Spiral | 3 | Unit 3 Lesson 10 | 1 |
| | 4 | Unit 3 Lesson 7 | 1 |
| | 5 | Unit 3 Lesson 14 | 1 |
| Formative | 6 | Unit 3 Lesson 17 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# More Solutions to Linear Equations




Let's find solutions to more linear equations.

## Focus

### Goals

1. **Language Goal:** Calculate the solution to a linear equation given one variable, and explain the solution method. **(Listening and Speaking)**
2. Determine whether an ordered pair is a solution to an equation of a line using a graph of the line.

## Coherence

### Today

Students continue their study of the relationship between a linear equation in two variables, its solution set, and its graph. By considering equations where students can solve for either the value of $x$ or the value of $y$, they prepare for finding solutions to systems of equations, leading them to look at the structure of an equation and decide whether it may be more efficient to solve for one variable than another.

### Previously

In Lesson 16, students studied the set of solutions to a linear equation, the set of all values of $x$ and $y$ that make the linear equation true.

### Coming Soon

In Lesson 18, students continue their study of linear equations by looking at how linear equations can model scenarios in the real world.

## Rigor

- Students **apply** their understanding of linear relationships in graphs, equations, and tables to different contexts.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Independent | Small Groups | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, one set per pair
- plain sheets of paper

### Math Language Development

**Review words**

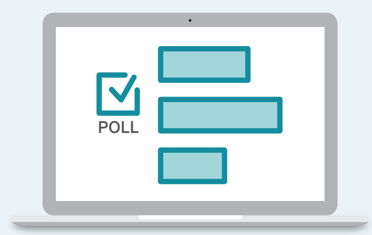
- *linear relationship*
- *ordered pair*
- *proportional relationship*
- *horizontal intercept*
- *vertical intercept*
- $y$*-intercept*
- $x$*-intercept*

**Amps** | Featured Activity

**Activity 1**

### Take a Digital Poll

Use real-time data to find out if your students think the statements in Activity 1 are true or false.




Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might feel their stress levels rise as they try to make use of the structure of the equation. Ask students to describe ways that they can control their stress and encourage students to participate in one of these exercises prior to starting the activity. Then have them set an academic goal in order to focus their energy in a productive way.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, have students only use Card pairs A–C.

Independent | 5 min

# Warm-up Intercepts

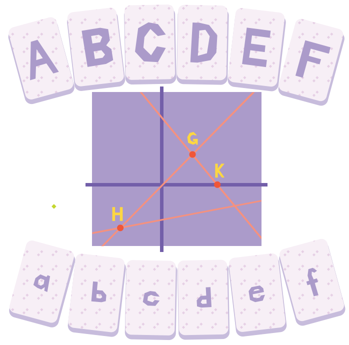
Students activate their prior knowledge about horizontal and vertical intercepts to prepare for identifying solutions on the graphs of linear equations.

Name: ______ Date: ______ Period: ______

Unit 3 | Lesson 17

## More Solutions to Linear Equations

Let's find solutions to more linear equations.




### Warm-up Intercepts

Find the horizontal and vertical intercepts for the line represented by each equation.

1. $y = -3x + 3$
   The horizontal intercept is $(1, 0)$.
   The vertical intercept is $(0, 3)$.

2. $2x + 5y = -10$
   The horizontal intercept is $(-5, 0)$.
   The vertical intercept is $(0, -2)$.

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Set an expectation for time to work independently on the activity.

## 2 Monitor

**Help students get started** by having them sketch the intercepts for each equation on a graph and consider what values must be zero for each.

**Look for points of confusion:**

- **Not knowing how to find the coordinates of the vertical and horizontal intercepts.** Ask students which value must be zero for one of the intercepts and have them substitute zero into the equation. Ask them how they can find the other variable's value.

**Look for productive strategies:**

- Drawing a sketch of the line to visualize their response.

## 3 Connect

**Display** correct student work for Problems 1 and 2.

**Have students share** how they found the vertical and horizontal intercepts.

**Ask:**

- "How can you know you have found the vertical and horizontal intercepts without graphing the line?"
- "How could you use the vertical and horizontal intercept to graph the line of the equation?"

**Highlight** that, no matter what form the equation is in, students can substitute $x = 0$ or $y = 0$ and solve for the other missing variable to find an intercept. Because they know an ordered pair that is a solution to the equation is also a point on the line, they can identify the horizontal and vertical intercepts without graphing the line.

## Differentiated Support

**Accessibility: *Optimize Access to Tools, Clarify Vocabulary and Symbols***

Provide access to graph paper, rulers, or graphing technology for students to use if they choose. Display the general form of a linear equation in slope-intercept form with the $y$-intercept and slope annotated for students to refer to as they work on Problem 1.

**Extension: *Math Enrichment***

Ask students to write an expression that gives the slope, $x$-intercept, and $y$-intercept of a line when the equation is written in the form $Ax + By = C$.
Slope: $-\frac{A}{B}$; $x$-intercept: $\frac{C}{A}$; $y$-intercept: $\frac{C}{B}$.

## Power-up

**To power up students' ability to identify horizontal intercepts, have students complete:**

Recall that when a point is located on the $x$-axis it will be of the form $(x, 0)$ and if it is on the $y$-axis it will be of the form $(0, y)$.

Determine whether each point will be located on the $x$-axis, the $y$-axis, or neither.

**a.** $(3, 0)$ $x$-axis
**b.** $(0, 3)$ $y$-axis
**c.** $(3, 3)$ neither
**d.** $(0, -3)$ $y$-axis

**Use:** Before the Warm-up
**Informed by:** Performance on Lesson 16, Practice Problem 6b and Pre-Unit Readiness Assessment, Problem 6

Small Groups | 15 min

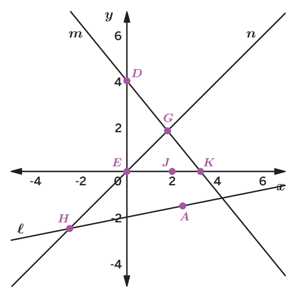
# Activity 1 True or False

Students determine whether different ordered pairs are solutions to increase their understanding of the relationship between a linear equation and its graph on the coordinate plane.

**Amps Featured Activity** Take a Digital Poll

## Activity 1 True or False?

Refer to the diagram shown for this activity. For each statement, decide if it is *true* or *false*. Explain your thinking.




| | True or False? | Explain your thinking. |
|---|---|---|
| 1. The ordered pair $(4, 0)$ is a solution to the equation that represents line $m$. | False | The point $(4, 0)$ does not lie on line $m$. |
| 2. The coordinates of point $G$ make both of the equations for line $m$ and line $n$ true. | True | Point $G$ lies at the intersection of lines $m$ and $n$, which means it lies on both lines and its coordinates are solutions to both equations. |
| 3. The ordered pair $(2, 0)$ makes both of the equations for line $m$ and line $n$ true. | False | The point $(2, 0)$ does not lie on either line and is, therefore, not a solution to either equation. |
| 4. There is no solution to the equation represented by line $\ell$ that has a $y$-value of 0. | False | While the graph doesn't show this value, I know that line $\ell$ will extend and intersect with the $x$-axis where $y = 0$, meaning $y = 0$ will be a solution. |

334 Unit 3 Linear Relationships



## 1 Launch

Set an expectation for the amount of time students have to work, in pairs, on the activity.

## 2 Monitor

**Help students get started** by asking what it means for an ordered pair to be a solution.

**Look for points of confusion:**

- **Thinking they can find the equations of the lines.** Have students identify what information they need to do this, and, if needed, remind them they cannot and do not need to find the equation with the information provided in the activity.

**Look for productive strategies:**

- Using the graph to identify a point is on the line and therefore a solution.

## 3 Connect

**Display** the correct response to each statement, and give students a few minutes to discuss any discrepancies with their partner.

**Have students share** how they evaluated each statement without using an equation.

**Ask:**

- "For Problem 1, if you had an equation of the line, how could you use it to confirm $(4, 0)$ is a solution?"
- "What is significant about Point $H$?"
- "Can you say that $x = 0$ is a solution to the equation for line $n$?"

**Highlight** that a solution to an equation in two variables is an ordered pair of numbers. Solutions to an equation lie on the graph of the equation. Discuss that the solution must represent both values of $x$ and $y$ to represent the coordinates of the point on the line.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students mark the points $(4, 0)$, $G$, and $(2, 0)$ to help them respond to Problems 1–3. Consider omitting Problem 4.

### Extension: *Math Enrichment*

Have students determine if the following statements are *true* or *false*.

- There is no ordered pair that is a solution to all three equations represented by lines $\ell$, $m$, and $n$. True.
- The intersection points $G$, $H$, and $K$ form a triangle whose side lengths lie on lines $\ell$, $m$, and $n$. False

## Math Language Development

### *MLR3: Critique, Correct, Clarify*

During the Connect, ask students to choose one of the statements they identified as false so that they become true statements. Ask these questions:

- ***Critique:*** "Which of the false statements will you choose to correct? Why did you identify this as a false statement?"
- ***Correct:*** "Write a corrected statement that is now true."
- ***Clarify:*** "How did you correct the statement? How do you know that the statement is now true?"

### English Learners

Have students cross out the part of the statement they are correcting and write the correction near it.

Pairs | 15 min

# **Activity 2** I'll Take an X, Please

Students are given equation cards and must ask for information about the value of $x$ or $y$ from a matching card to develop strategies for how to solve for one variable, given the other.

Name: ______ Date: ______ Period: ______

## **Activity 2** I'll Take an X, Please

**Plan ahead:** Make a plan for how you will listen to your partner without interrupting or only being focused on sharing your thoughts.

**You and your partner will be given six cards labeled A through F and six cards labeled a through f. In each pair of cards (for example, Cards A and a), there is an equation in one card and an ordered pair, $(x, y)$, that makes the equation true on the other card.**

| If you are given an *equation* card: | If you are given an *ordered pair* card: |
|---|---|
| **1.** Ask your partner for either the $x$-value or the $y$-value. Explain why you want this particular value. | **1.** Provide the value your partner requests. |
| **2.** Use the value your partner provides to find the value of the remaining unknown variable. Explain each step as you go. Show your calculations on a separate sheet of paper. | **2.** After your partner finds the remaining unknown variable, tell them if they are correct or incorrect. |
| **3.** If your value is correct, move onto the next set of cards. If your value is incorrect, look through your steps to find and correct any errors. | **3.** If your partner's value is correct, move onto the next set of cards. If your partner's value is incorrect, look through their steps to find and correct any errors. |

**Keep playing until you have completed Cards A through F.**

STOP



**Lesson 17** More Solutions to Linear Equations 335

## 1 Launch

Review the directions from the Activity 2 PDF and distribute the pre-cut cards. Activate students' prior knowledge about how they can solve for one variable ($x$ or $y$) if they are given an assigned value for the other. Consider demonstrating for the class using the equation $y = 5x - 11$ and $(1, -6)$ and a student volunteer.

## 2 Monitor

**Help students get started** by asking them to consider which variable's value will help them solve for the other variable more efficiently.

**Look for points of confusion:**

- **Not being strategic about which variable to request.** Remind students they can ask for the value of $x$ or $y$. Ask, "In the case of this equation, which variable would you rather know? Why?"

**Look for productive strategies:**

- Being strategic about which value to request so that solving for the other variable is as efficient as possible.

## 3 Connect

**Ask:**

- "How did you decide whether you requested the value of $x$ or the value of $y$?"
- "Which equations represent proportional relationships? How do you know? Which do not?" Cards C and F are proportional because they can be written as $y = mx$.
- "Once you have identified one solution to your equation, what are some ways you could find others?"

**Highlight** that all of the equations in this activity are linear, even if they are written in different forms. When an equation is already solved for one variable, e.g. $y = mx + b$, it requires less steps to solve for $y$ if given $x$.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Cards A–D.

### Accessibility: *Guide Processing and Visualization*

Display the equation Card a. Use a think-aloud to model Steps 1 and 2. Consider using the following during the think-aloud.

- "Could you give me the $x$-value? I would like to substitute the $x$-value into the equation so that I can solve the equation for $y$."
- "Now that I know the $x$-value is $10.5$, I can substitute that value into the equation $2(10.5) - y = 14$ and solve the equation for $y$. My result is that $y = 7$. Is that value correct?"

## Math Language Development

### *MLR8: Discussion Supports*

During the Connect, as students respond to the Ask questions, display the following sentence frames to help students frame their responses.

- "I decided to ask for the value of ___ because . . ."
- "The steps I took to determine the other value were . . ."

Encourage the listener to ask clarifying questions such as:

- "What would you do if you had chosen the other variable?"
- "Did you strategically choose to request one variable rather than the other, based on the structure of the equation?"
- "For which equation(s) did it require more work to solve for the other variable? Why?"

Whole Class | 5 min

# Summary

Review and synthesize how to find solutions to linear equations in two variables.

## Summary

**In today's lesson . . .**

You saw that no matter the form a linear equation is given, you can always determine solutions to the equation by starting with one value, and then solving for the other value.

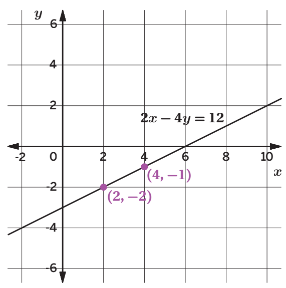
For example, consider the linear equation $2x - 4y = 12$.

- To determine a solution that has $x = 2$, you can substitute $x = 2$ into the equation and solve for $y$.

$$2(2) - 4y = 12$$
$$4 - 4y = 12$$
$$-4y = 8$$
$$y = -2$$

- To determine a solution that has $y = -1$, you can substitute $y = -1$ into the equation and solve for $x$.

$$2x - 4(-1) = 12$$
$$2x + 4 = 12$$
$$2x = 8$$
$$x = 4$$




> **Reflect:**




## Synthesize

**Display** the equation $3y + x = 12$ after students have read the Summary.

**Ask:**

- "What are some different strategies you can use to find a solution to the linear equation $3y + x = 12$?"

  Sample responses:

  - Graph the equation and find a point that lies on the graph. Verify the point by substituting its ordered pairs into the equation to make the equation true.
  - Substitute any value of one variable into the equation and solve the equation for the other variable.
  - Use the guess-and-check method to substitute values into the equation to make it true.
- "How do you know when you have found a solution to the equation $3y + x = 12$?" If the ordered pair $(x, y)$ makes the equation true, then it is a solution to the equation.
- "How can you find the horizontal and vertical intercepts using the equation?" To find the horizontal intercept, substitute zero for $y$ in the equation and solve for $x$. To find the vertical intercept, substitute zero for $x$ in the equation and solve for $y$.
- "How can you find the slope of the line?" Sample responses:
  - Find two ordered pairs that are solutions to the equation and then find the slope between those two points.
  - Graph the equation and find the slope from two points on the graph.
  - Rewrite the equation in the form $y = mx + b$.

**Highlight** different strategies for finding a solution to a linear equation in two variables.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "You have worked with linear relationships represented in written form, tables, equations, and graphs. Which are you most comfortable using to find solutions? Which are you least comfortable using?"

Independent | 5 min

# Exit Ticket

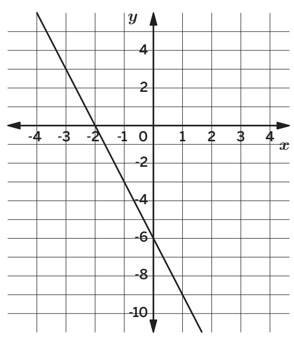
Students demonstrate their understanding of solutions to linear equations in two variables by using both the graph and equation to determine whether given ordered pairs are solutions.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 3.17

**The graph of the linear equation $3x + y = -6$ is shown.**




1. Is the ordered pair $(-2, 0)$ a solution to the equation $3x + y = -6$? Explain your thinking using both the graph and the equation.

   Yes; Sample response:
   - Graph: The point $(-2, 0)$ is on the line.
   - Equation: If I substitute $(-2, 0)$ into the equation: $3(-2) - (0) = -6$, I see that the ordered pair $(-2, 0)$ makes the equation true.

2. Is the ordered pair $(1, -8)$ a solution to the equation $3x + y = -6$? Explain your thinking using both the graph and the equation.

   No; Sample response:
   - Graph: The point $(1, -8)$ is not on the line.
   - Equation: If I substitute $(1, -8)$ into the equation: $3(1) - (-8) \neq 11$, I see that the ordered pair $(1, -8)$ does not make the equation true.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can find solutions to linear equations, in the form of $(x, y)$, when given either the $x$-value or the $y$-value.

1 2 3



**Lesson 17** More Solutions to Linear Equations

## Success looks like . . .

- **Language Goal:** Calculating the solution to a linear equation given one variable, and explaining the solution method. **(Listening and Speaking)**
- **Goal:** Determining whether a point is a solution to an equation of a line using a graph of the line.
  - » Determining whether a given ordered pair is a solution to an equation by using the graph.

## Suggested next steps

**If students are unable to determine whether an ordered pair is a solution using the graph, consider:**

- Reviewing Activity 1.
- Asking, "How do you know a point is the solution to a line?"

**If students are unable to determine whether an ordered pair is a solution using the equation, consider:**

- Asking, "What does it mean for an ordered pair to be a solution for the equation? How can you use substitution to determine whether an ordered pair is a solution?"
- Reviewing Activity 2.
- Assigning Practice Problem 2.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?
- Which groups of students did and didn't have their ideas seen and heard today?

# Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. For each equation, determine the value of $y$ when $x = -3$. Then determine the value of $x$ when $y = 2$.

a) $y = 6x + 8$

| $x$ | $y$ |
|---|---|
| $-3$ | **$-10$** |
| **$-1$** | $2$ |

b) $y = \frac{2}{3}x$

| $x$ | $y$ |
|---|---|
| $-3$ | **$-2$** |
| **$3$** | $2$ |

c) $y + x = 5$

| $x$ | $y$ |
|---|---|
| $-3$ | **$8$** |
| **$3$** | $2$ |

d) $y = \frac{3}{4}x - 2\frac{1}{2}$

| $x$ | $y$ |
|---|---|
| $-3$ | **$-\frac{19}{4}$** |
| **$6$** | $2$ |

2. State whether the following is *true* or *false*. Show or explain your thinking.

*The ordered pairs* $(6, 13)$, $(21, 33)$, *and* $(99, 137)$ *all lie on the same line. The equation of the line is* $y = \frac{4}{3}x + 5$.

**True; Sample response: All three ordered pairs make the equation true.**

$13 = \frac{4}{3}(6) + 5$ $\quad 33 = \frac{4}{3}(21) + 5$ $\quad 137 = \frac{4}{3}(99) + 5$

$13 = 13$ $\quad 33 = 33$ $\quad 137 = 137$

3. Consider the linear equation $y = \frac{1}{4}x + \frac{5}{4}$.

a) Are the ordered pairs $(1, 1.5)$ and $(12, 4)$ solutions to the equation? Show or explain your thinking.

**$(1, 1.5)$ is a solution because it makes the equation true. $(12, 4)$ is not a solution because $\left(\frac{1}{4}\right)(12) + \frac{5}{4} = 4.25$ which means $y$ would be $4.25$ and not $4$.**

b) Find the $x$-intercept of the graph of the equation. Show or explain your thinking.

**$(-5, 0)$; Sample response: The $x$-intercept is the value of $x$ when $y = 0$. When $y = 0$, the equation $0 = \frac{1}{4}x + \frac{5}{4}$ can be solved for $x$, where $x = -5$.**

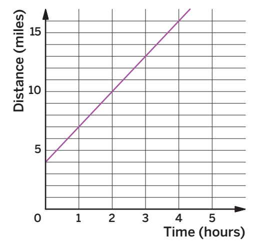




Name: ______ Date: ______ Period: ______

Practice

4. A group of hikers park their car at a trailhead and walk into the forest to a campsite. The next morning, they head out on a hike from their campsite, walking at a constant rate. The graph shows their distance $d$ in miles from their car after $h$ hours of hiking.




a) How far is the campsite from their car? Explain your thinking.

**4 miles; Sample response: This is the distance at $h = 0$ hours, meaning that is the location of their campsite.**

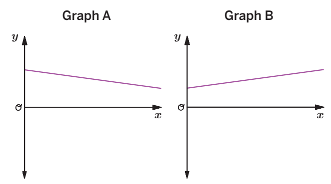
b) Write an equation that gives the distance $d$ from their car for any number of hours hiked $h$.

**$d = 3h + 4$**

c) After how many hours of hiking will they be $16$ miles from their car? Explain your thinking.

**4 hours; Sample response: Looking at the graph, I can see that the point on the line $(4, 16)$ represents the distance $16$ miles after $4$ hours hiking.**

5. Decide which graph best represents each of the following situations.




a) $y$ represents the weight of a kitten $x$ days after birth.

**Graph B**

b) $y$ represents the distance remaining in a car ride after $x$ hours of driving at a constant rate toward its destination.

**Graph A**

6. Write an equation to represent each relationship described.

a) Grapes cost $\$2.39$ per pound. Bananas cost $\$0.59$ per pound. You have $\$15$ to spend on $g$ pounds of grapes and $b$ pounds of bananas.

**$2.39g + 0.59b = 15$**

b) A savings account has $\$50$ in it at the start of the year and $\$20$ is deposited each week. After $x$ weeks, there are $y$ dollars in the account.

**$y = 20x + 50$**





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 1 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 3 Lesson 15 | 2 |
| | 5 | Unit 3 Lesson 12 | 1 |
| Formative | 6 | Unit 3 Lesson 18 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 3 | **LESSON 18**

# Coordinating Linear Relationships

Let's coordinate representations of linear relationships.

## Focus

### Goals

1. Coordinate between multiple representations of real-world linear relationships, including equations, graphs, verbal descriptions, and tables.

## Rigor

- Students **apply** their understanding of the multiple representations of linear relationships to a real-world problem.

## Coherence

### Today

Students apply what they have learned to solve real-world problems using the different representations of linear equations they have studied. Students see that both the equations $Ax + By = C$ and $y = mx + b$ can represent the same real-world situation.

### Previously

Students learned to represent linear relationships using equations of the form $Ax + By = C$ and $y = mx + b$. Students have also learned to create a graph of a linear relationship, and to coordinate the graph with the solutions for an equation.

### Coming Soon

In the culminating lesson of Unit 3, students will apply their understanding of linear relationships by orienting coordinate planes to lines in unusual ways. In Unit 4, students discover strategies for solving linear equations and will explore concepts related to systems of linear equations.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF
- Activity 2 PDF (answers)
- *Info Gap Routine* PDF (for display)
- calculators (optional)
- rulers

### Math Language Development

**Review words**

- *linear relationship*
- *ordered pairs*
- *proportional relationship*
- *horizontal intercept*
- *vertical intercept*
- $y$-*intercept*
- $x$-*intercept*

**Amps** | Featured Activity

### Exit Ticket
**Real-Time Exit Ticket**

Check in real time if your students can write and graph a linear equation to represent a real-world scenario, using a digital Exit Ticket that is automatically scored.

Amps POWERED BY desmos

### Building Math Identity and Community
Connecting to Mathematical Practices

Students might have a negative attitude about having to represent the linear relationship in several ways. Ask students to identify other times that the same information might be presented in different ways. Encourage them to “flip their thoughts” and look for the possible benefits of different kinds of models for the same information. This more optimistic viewpoint can help reduce students’ resistance to active participation.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, provide students with a table of values they can use to write their equations and make their graphs.
- In **Activity 2**, have students complete only the first set of cards.

Independent | 5 min

# **Warm-up** Hunting for Ordered Pairs

Students activate prior knowledge about solving linear equations to develop strategies for substituting values for one variable to find the other unknown variable.

Name: ______ Date: ______ Period: ______

**Unit 3 | Lesson 18**

## Coordinating Linear Relationships

Let's coordinate representations of linear relationships.

### **Warm-up** Hunting for Ordered Pairs

**Find four ordered pairs that are solutions to the equation, $\frac{1}{3}x + \frac{1}{2}y = 10$. Record the values in the table.** Sample response shown.

| $x$ | $y$ |
|---|---|
| 0 | 20 |
| 3 | 18 |
| 6 | 16 |
| 9 | 14 |

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Activate students' prior knowledge about how to find ordered pair solutions to a linear equation in two variables.

## 2 Monitor

**Help students get started** by having them identify a value for $x$ they can substitute into the equation.

**Look for points of confusion:**

- **Having difficulty solving for the other variable, based on the value they decided to substitute.** Ask students what they notice about the coefficients, and then ask them to identify what is challenging. Suggest they substitute 5 and 6 for $x$, and then ask which value would help ease calculations, and why.
- **Not being strategic about which values to substitute.** Validate correct solutions and pull students into discussion about more efficient methods of substitution.
- **Turning coefficient fractions to decimals.** Point out that $\frac{1}{3}$ is non-terminating and will need to be rounded, meaning the answer students get will not be precise. Encourage them to use this answer as an estimate and rework the problem using fractions.

**Look for productive strategies:**

- Substituting multiples of 3 for $x$ or multiples of 2 for $y$.

## 3 Connect

**Have students share** why they chose the values of $x$ and $y$ they did. Sequence responses by first choosing students who correctly worked with fractions and mixed numbers, followed by students who chose strategic multiples.

**Ask**, "Why is it preferable to have whole-number ordered pairs when working with a linear relationship?"

**Highlight** that when finding solutions to an equation in two variables, it can be helpful to be strategic about what values to substitute by looking at the coefficients.

## Power-up

**To power up students' ability to write linear equations to represent scenarios, have students complete:**

Match each context to the appropriate form of equation. After matching the equation, substitute the appropriate values to represent each scenario.

**a.** Tyler spent \$4 per pound on some strawberries and \$3 per pound on some grapes. He spent a total of \$12 on $x$ pounds of strawberries and $y$ pounds of grapes. $4x + 3y = 12$ ... a ... $Ax + By = C$

**b.** Noah read 3 books during the first week of summer. He made a goal of reading 4 books each month. How many books $y$ will he read after $x$ months? $y = 4x + 3$ ... b ... $y = mx + b$

**Use:** Before Activity 1

**Informed by:** Performance on Lesson 17, Practice Problem 6

Pairs | 15 min

# Activity 1 Representations of Linear Relationships

Students create multiple representations for a linear relationship, seeing how each representation can show the slope and initial value.

### Activity 1 Representations of Linear Relationships

Lin and Mai went on a canoeing trip. From their starting point, they will paddle and *portage* — hike while carrying their canoe — a total of 20 miles to arrive at their campsite before dark. They can paddle at a speed of 1.5 miles per hour and portage at a speed of 2 miles per hour.

Meg Wallace Photography/Shutterstock.com

Complete the following problems to describe the relationship between the number of hours they can paddle and the number of hours they can portage.

Sample responses shown.

1. Define your variables.

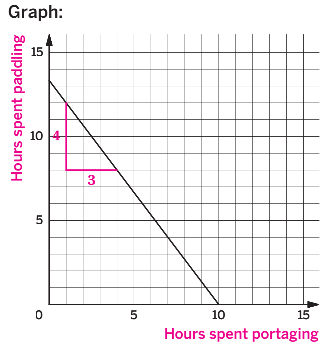
   Let $x$ represent the number of hours portaging, and let $y$ represent the number of hours paddling.

2. Create a table, graph a line, and write an equation to represent the situation.

Table:

| Hours spent portaging | Hours spent paddling |
|---|---|
| 1 | 12 |
| 4 | 8 |
| 7 | 4 |
| 10 | 0 |

Graph:




Equation:

$2x + 1.5y = 20$

or

$y = -\frac{4}{3}x + \frac{40}{3}$





## 1 Launch

Arrange students in groups of two. Ask for ideas for how to define the variables and complete Problem 1 with the class. Provide access to rulers.

## 2 Monitor

**Help students get started** by asking whether they would prefer to start from the table, graph, or equation. Suggest students write an equation first if they are unsure how to begin.

**Look for points of confusion:**

- **Not able to generate values in the table.** Have students begin by writing the equation. Then have students substitute values for $x$ into the equation to solve for $y$.
- **Not able to create a graph from an equation.** Have students first complete the table. Then ask students which values they can substitute for $x$ or $y$ to find coordinates for two points on the graph.

**Look for productive strategies:**

- Using an equation to first generate points on the graph, and then in the table.
- Using guess and check to create a table and then create the graph.
- Writing an equation in $y = mx + b$ form, based on the graph.
- Representing an equation in two forms, $y = mx + b$ and $Ax + By = C$.

**Activity 1 continued ›**

## Differentiated Support

### Accessibility: *Guide Visualization and Processing*

Provide a partially-completed table that shows 1 hour portaging corresponding with 12 hours paddling. Consider providing sample values for the number of hours spent portaging and ask them to determine the number of hours spent paddling. Display the general forms of linear equations: $y = mx + b$ and $Ax + By = C$.

### Extension: *Math Enrichment*

Ask students if they think the $x$- and $y$-intercepts make sense within the context of this problem. Sample response: No, I don't think they make sense because it would mean that they either spend 0 hours portaging or 0 hours paddling.

## Math Language Development

### *MLR1: Stronger and Clearer Each Time*

Give students time to write a draft explanation for Problem 4. Have them meet with 2–3 partners to share their responses and give and receive feedback. Reviewing partners should ask clarifying questions to make sense of their partners' drafts and offer suggestions for improvement. Consider providing sample questions, such as:

- "Does the response provide information about all four representations: equation, description, graph, and table?"
- "Does the response include any mathematical inaccuracies?"
- "Does the response make sense to you?"

Then have students write an improved response based on the feedback they received.

Pairs | 15 min

# **Activity 1** Representations of Linear Relationships (continued)

Students create multiple representations for a linear relationship, seeing how each representation can show the slope and initial value.

Name: ______ Date: ______ Period: ______

**Activity 1** Representations of Linear Relationships (continued)

3. How can you find the rate of change using the table and the graph? What does the rate of change mean within the context of this problem?

   Sample response:
   - Table: Find the change in hours spent paddling and divide that value by the change in hours spent portaging: $\frac{12-8}{1-4} = -\frac{4}{3}$.
   - Graph: The points $(10, 0)$ and $(7, 4)$ are on the line, which means the slope of the line is $\frac{4-0}{7-10} = -\frac{4}{3}$.

   This rate of change means that for every hour spent portaging, $\frac{4}{3}$ fewer hours were spent paddling.

4. Explain how you can tell that the equation, description, graph, and table all represent the same relationship.

   Sample response: The data in the table represent the coordinates of points found on the line in the coordinate plane. Because the points on the line are solutions to the equation for the line, when substituting coordinates $(4, 8)$ into the equation $2x + 1.5y = 20$, I see the points make the equation true $2(4) + 1.5(8) = 20$.

**Stronger and Clearer:** Share your response to Problem 4 with 2–3 partners. Ask each other clarifying questions and offer suggestions for improvement. Then revise your original response based on their feedback.





## 3 Connect

**Display** student work showing multiple correct representations.

**Have students share** what they chose to create first: the equation, the table, or the graph. Sequence responses starting with students who completed the table first and ending with students who wrote an equation first. If no students wrote an equation as $y = mx + b$, explore how to write this equation from the graph.

**Ask:**

- "How can you use the equation $2x + 1.5y = 20$ to create a graph and table?"
- "How can you test to see whether both equations are equivalent?" By substituting ordered pairs.
  **Note:** Students will learn how to solve linear equations in one variable algebraically in Unit 4.
- "How can you see the slope and initial value in each representation?"
- "Which representation do you think would be most helpful for Lin and Kiran?"

**Highlight** that an equation, a table, and a graph can all be used to represent the same linear relationship. Given one representation, the others can be created.

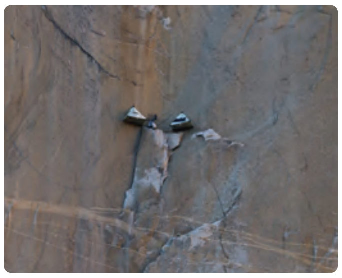
Pairs | 15 min

# Activity 2 Info Gap: Linear Relationships

Students complete the *Info Gap* routine to identify the information necessary to create graphs and equations of linear relationships.

## Activity 2 Info Gap: Linear Relationships

**Pictured here, rock climbers have set up their tents, suspended in air with ropes, as they rest and prepare to climb to the summit. You will be given either a problem card or data card describing a scenario related to rock climbing. Do not show or read your card to your partner.**



Wollertz/Shutterstock.com

| If you are given the *problem card:* | If you are given the *data card:* |
|---|---|
| 1. Silently read your card, and think about what information you need to be able to solve the problem. | 1. Silently read your card. |
| 2. Ask your partner for the specific information that you need. | 2. Ask your partner, "What specific information do you need?" and wait for them to ask for information. |
| 3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem. | 3. Before sharing the information, ask, "Why do you need that information?" Listen to your partner's reasoning, and ask clarifying questions. |
| 4. Share the problem card, and solve the problem independently in the space provided on this page. | 4. Read the problem card, and solve the problem independently in the space provided on this page. |
| 5. Read the data card, and discuss your reasoning. | 5. Share the data card, and discuss your reasoning. |

**Pause here so your teacher can review your work. You will be given a new set of cards. Repeat the activity, trading roles with your partner.**

STOP

342 Unit 3 Linear Relationships



### 1 Launch

Distribute the cards from the Activity 2 PDF. Display the *Info Gap Routine* PDF and model the *Info Gap* routine with students. Explain that they may need several rounds of discussion to determine the information they need.

### 2 Monitor

**Help students get started** by helping them label their axes and determine a scale.

**Look for points of confusion:**

- **Not knowing what information is required to create the graph.** Ask, "What information is necessary to graph a linear relationship? What information do you have? What information do you need?"

**Look for productive strategies:**

- Asking questions with more precision until students receive the information they need.

### 3 Connect

**Have pairs of students share** their graphs and responses to the problem cards.

**Ask:**

- "Other than the answer, what information would have been nice to have?"
- "How did you decide what to label the two axes?"
- "How did you decide to scale the axes?"
- "What ways can you tell that the slope for Problem Card 2 is negative?"
- "What is the equation of the line for each card? Take a few moments to find the equations with your partner."
- "Why did you decide to write the equation in the form you did?"

**Highlight** that when writing an equation from a graph, it may often be more efficient to write the equation in $y = mx + b$ form because the slope and the $y$-intercept can often be readily identified from the graph.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. Consider using the following questions during the think-aloud.

- "I know that the graph needs to show Jada's height compared to time. I think I should ask for the units. How is time measured? What are the units for the height?"
- "In order to graph the relationship, I need to know at least two points. I think I should ask for Jada's height after a certain number of minutes."

## Math Language Development

### *MLR4: Information Gap*

Display prompts for students who benefit from a starting point, such as:

- "Can you tell me . . . (specific piece of information)?"
- "Why do you need to know . . . (that piece of information)?".

### English Learners

Consider providing sample questions students could ask, such as the following:

- "What are the units that represent Jada's height above ground?"
- "What is Jada's height above ground after ___ minutes?"

Whole Class | 5 min

# Summary

Review and synthesize the multiple representations of linear relationships and how they can each be used to provide information about a context.

Name: ______ Date: ______ Period: ______

## Summary

**In today's lesson . . .**

You explored how linear relationships can be represented in multiple ways. Linear relationships can be represented with written descriptions, equations, graphs, and tables. Which representation you choose depends on the purpose. When creating representations, you can choose helpful values by paying attention to the context.

**Written description:**

An athlete wants to buy snack bars that cost \$2 each and hydration drinks that cost \$3 each. They have \$24 to spend.

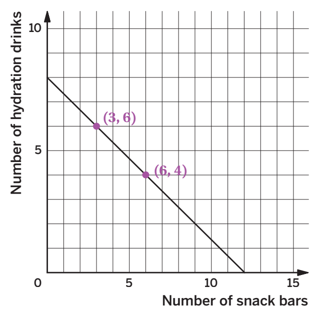
**Equation:**

$y = -\frac{2}{3}x + 8$ or $2x + 3y = 24$

**Table:**

| Number of snack bars, $x$ | Number of hydration drinks, $y$ |
|---|---|
| 6 | 4 |
| 3 | 6 |

**Graph:**




**Reflect:**





## Synthesize

**Display** the Summary from the Student Edition.

**Highlight** that each representation of linear relationships calls attention to different features of the linear relationship.

**Ask:**

- "How can you tell whether data in a table represents a linear relationship?" If there is a constant rate of change for all values, the data represent a linear relationship.
- "How can you tell whether a graph represents a linear relationship?" If the graph of the relationship is a straight line, then the relationship is linear.
- "How can you tell whether an equation represents a linear relationship?" If the equation can be written in the form $y = mx + b$ or $Ax + By = C$, then it is a linear relationship.
- "What are the similarities and differences you see in the two different equations representing the linear relationship shown in the Summary?" Sample responses:
  - Both equations use the variables $x$ and $y$.
  - There are different coefficients on the variables in the two equations.
  - There are different constants in the two equations.
  - One equation is written in the form $y = mx + b$. The other equation is written in the form $Ax + By = C$.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Which representation of linear relationships do you find the most challenging to create or interpret?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding of representations of linear relationships by writing an equation and creating a graph in context.

**Amps Featured Activity** Real-Time Exit Ticket

Printable

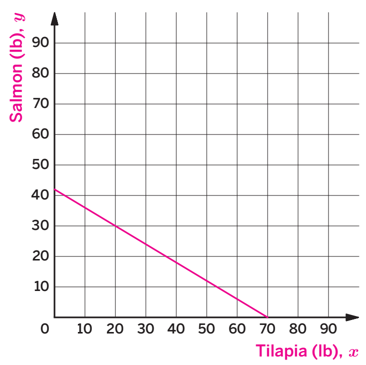
Name: ______ Date: ______ Period: ______

## Exit Ticket 3.18

**Each day, the Fabulous Fish Market orders tilapia, which costs $3 per pound, and salmon, which costs $5 per pound. The manager of the market budgets $210 daily to spend on this order.**

1. Define variables and write an equation representing the relationship between the number of pounds of each type of fish bought and how much the market spends.
   Sample response: Let $x$ be the number of pounds of tilapia, and let $y$ be the number of pounds of salmon; $3x + 5y = 210$.

2. Graph this relationship on the coordinate plane. Label the axes.




**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can write linear equations to reason about real-world situations.
1 2 3

b. I can coordinate between different representations of linear relationships in graphs, tables, and equations.
1 2 3



**Lesson 18** Coordinating Linear Relationships

## Success looks like . . .

- **Goal:** Coordinating between multiple representations of real-world linear relationships, including equations, graphs, verbal descriptions, and tables.
  - » Writing an equation for a verbal description and then graphing the relationship between the weight and cost in Problems 2 and 3.

## Suggested next steps

**If students have difficulty writing an equation from context, consider:**

- Reviewing Activity 1.
- Asking students to identify their total budget and determine what must add up to the total in context.
- Assigning Practice Problem 2.

**If students have difficulty creating a graph from the context and the equation, consider:**

- Reviewing Activity 1.
- Having students create a table of values and asking, "What value for $x$ could you substitute so that you can then solve for $y$?"
- Assigning Practice Problem 1.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- What challenges did students encounter as they worked on Activity 1? How did they work through them?
- Which teacher actions did you implement that made the *Info Gap* routine strong?

# Practice

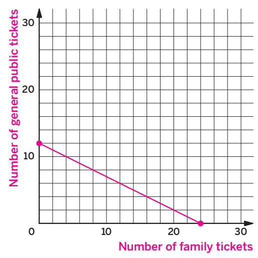
Independent

Name: Date: Period:

Practice

1. A high school theater is selling tickets to the school's spring musical. For family members of the cast, tickets cost \$5 each. For the general public, tickets cost \$10 each. The school earns \$120 from their ticket sales on opening night.

   a. Give two possible combinations of the number of family and general public tickets sold that earn a total of \$120 in sales.
   **20 family tickets and 2 general public tickets;**
   **4 family tickets and 10 general public tickets.**




   b. Write an equation to represent the relationship between the number of family tickets $x$ sold and the number of general public tickets $y$ sold.
   $5x + 10y = 120$

   c. Graph this relationship on the coordinate plane. Label the axes.

2. Jada is planning a cookout for her family. She wants to buy veggie burgers and determines 20 lb will be enough for the cookout. Veggies-R-Us sells veggie burgers in 4-lb packages. Betabel Burgers sells veggie burgers in 3-lb packages.

   a. Write an equation to represent the relationship between the number of packages $x$ Jada can buy from Veggies-R-Us, the number of packages $y$ she can buy from Betabel Burgers, and the total amount of veggie burgers she needs.
   $4x + 3y = 20$

   b. Is $(1.25, 5)$ a solution to both the equation and the problem? Explain your thinking.
   **Sample response: This ordered pair is a solution to the equation, but not to the problem because Jada can only buy whole number packages. This means the $x$-values must be whole numbers.**




Name: Date: Period:

Practice

3. Match each equation with the set of ordered pairs that are solutions to the equation. Some sets of ordered pairs may have no matching equation or more than one matching equation.

| Equation | Sets of ordered pairs |
|---|---|
| a. $y = 1.5x$ | **a, d** $(14, 21), (2, 3), (8, 12)$ |
| b. $2x + 3y = 7$ | **c** $(-3, -7), (0, -4), (-1, -5)$ |
| c. $x - y = 4$ | **e** $\left(\frac{1}{8}, \frac{7}{8}\right), \left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{4}, \frac{3}{4}\right)$ |
| d. $3x = 2y$ | **b** $\left(1, 1\frac{2}{3}\right), (-1, 3), \left(0, 2\frac{1}{3}\right)$ |
| e. $y = -x + 1$ | $(0.5, 3), (1, 6), (1.2, 7.2)$ |

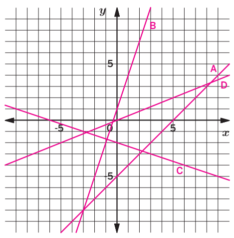
4. Use the coordinate plane to graph each equation.

   a. **Equation A:** $y = x - 5$

   b. **Equation B:** $y = 3x + 1$

   c. **Equation C:** $y = -\frac{1}{3}x - 2$

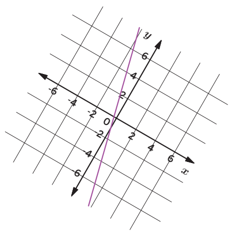
   d. **Equation D:** $y = 0.4x$




5. Consider the graph shown.

   a. What do you notice?
   **Sample response: I notice the coordinate plane has been rotated.**

   b. What do you wonder?
   **Sample response: Does the slope of the line change if the plane is rotated?**







## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 1 | 2 |
| Spiral | 3 | Unit 3 Lesson 16 | 1 |
| | 4 | Unit 3 Lesson 12 | 1 |
| Formative | 5 | Unit 3 Lesson 19 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.


# Rogue Planes

Let's see what happens when the coordinate plane acts in unusual ways.

## Focus

### Goals

1. **Language Goal:** Describe how the values of $m$ and $b$ in the equation $y = mx + b$ affect the line on the coordinate plane. **(Speaking and Listening)**

## Coherence

### Today

Students are presented with coordinate planes oriented in unusual ways. They apply what they have learned about the equation $y = mx + b$ to organize the values of $m$ and $b$ to fit these unusual coordinate planes.

### Previously

Over the course of Unit 3, students developed their understanding of proportional and linear relationships. They learned different ways of representing these relationships and gained experience using graphs, equations, and tables to represent real-world examples of linear and proportional relationships.

### Coming Soon

In Unit 4, students will continue their study of linear equations, first by looking at methods for solving algebraically, and later by exploring systems of linear equations.

## Rigor

- Students **apply** their understanding of linear equations to match graphs with coordinate planes oriented in unusual ways.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 20 min | 10 min | 5 min | 5 min |
| Pairs | Small Groups | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

## Materials

- Exit Ticket
- Additional Practice
- geometry toolkits: protractors, rulers, tracing paper

## Math Language Development

**Review words**

- *linear relationship*
- *slope*
- *$y$-intercept*

**Amps** | **Featured Activity**

### Activity 1
**Digital** Rogue Planes

Students match coordinate planes to lines using digital rotation tools.

**Amps** POWERED BY desmos

## Building Math Identity and Community
Connecting to Mathematical Practices

In Activity 2, students may feel overwhelmed with the process of just starting. Remind students that they have a large mathematical tool box and that the first decision that they probably need to make for this activity is which tool(s) to use. In order to make the best choice of tool, however, students will need to identify the problem and analyze the situation.

## Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- **Activity 2** may be omitted.

Pairs | 5 min

## Warm-up True or False?

Students critique a statement about the slope of a line on a rotated coordinate plane to notice a relationship between the line and the coordinate plane.

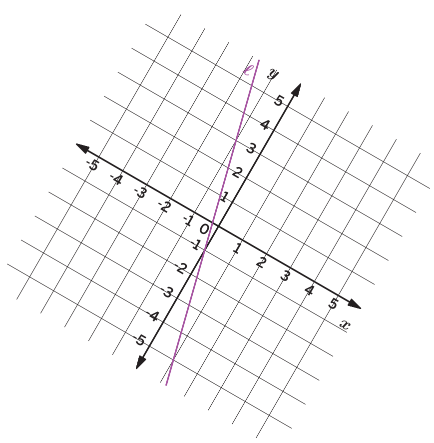
Unit 3 | Lesson 19 – Capstone

# Rogue Planes

Let's see what happens when the coordinate plane acts in unusual ways.


### Warm-up True or False?

Is it true that the slope of line $\ell$ is $-4$? Explain your thinking.



True; Sample response: The slope is $-4$ even if it looks like it is increasing because of the way the coordinate plane has been rotated. The line for the equation is still $y = -4x - 1$.

Log in to Amplify Math to complete this lesson online.

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### 1 Launch

Have students use the *Think-Pair-Share* routine. Provide them 1 minute to think independently. Then have them complete the Warm-up with a partner.

### 2 Monitor

**Help students get started** by asking them what they notice about the coordinate plane and generating ideas for how they can find the slope of the line

**Look for points of confusion:**

- **Thinking that the statement is false.** Acknowledge that the slope does appear to be positive. Ask students what they notice about the coordinate plane and have students rotate the page to position the coordinate plane in the regular way.

**Look for productive strategies:**

- Rotating their papers to orient the $y$-axis vertically.
- Finding two points on the line to find the slope of the line.

### 3 Connect

**Display** the Warm-up.

**Have students share** if they think the statement is true or false.

**Ask:**

- "What made this problem challenging?" Answers may vary, but students may struggle with the unusual orientation of the graph.
- "What is the equation of the line?" $y = -4x - 1$

**Highlight** that students will find that the coordinate plane they are used to seeing will be acting in unusual ways in this lesson. Even with a coordinate plane oriented in multiple directions, students can still find the equation of the line by using strategies discussed in this unit.

## Power-up

**To power up students' ability to generate ideas about a rotated coordinate plane, have students complete:**

Examine the rotated coordinate plane. Determine which statements are true. Select *all* that apply.

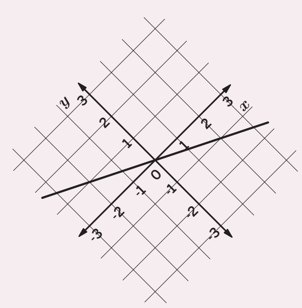
**A.** The slope of the line is positive.

**B.** The slope of the line is negative.

**C.** The slope of the line is $\frac{1}{2}$.

**D.** The slope of the line is $-\frac{1}{2}$.

**E.** The slope of the line cannot be determined.



**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 18, Practice Problem 5

Small Groups | 20 min

# Activity 1 Something Weird Is Happening . . .

Students manipulate coordinate planes and match lines to the given equations, deepening their understanding about relationships between $m$ and $b$ and the position of the line.

**Amps Featured Activity** Digital Rogue Planes

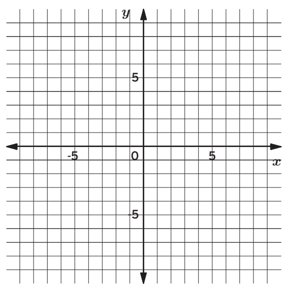
Name: ______________ Date: ________ Period: ________

**Activity 1** Something Weird Is Happening . . .

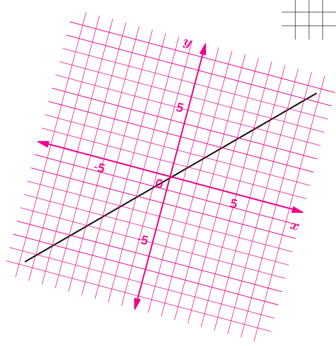
The coordinate plane we know and love has gone rogue!

Trace the coordinate plane on a piece of tracing paper. Orient the coordinate plane on each line such that the line matches the equation on the rotated, rogue coordinate plane.

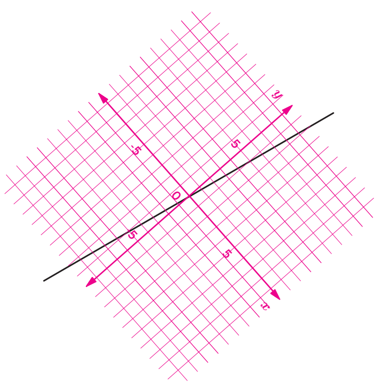
Sketch your graph on top of each line.



1. $y = x$



2. $y = 5x$







## 1 Launch

Distribute geometry toolkits including tracing paper and rulers. Help students create their coordinate plane on tracing paper. Assign students to groups of 2–4.

## 2 Monitor

**Help students get started** by showing them how to place their coordinate plane sketch paper on the line. Place it incorrectly and ask students what the slope of the line appears to be.

**Look for points of confusion:**

- **Creating a graph of the equation $y = \frac{1}{5}x$ for Problem 2.** Ask students to draw a slope triangle and have them restate the definition of slope.
- **Creating graphs of the equations $y = -\frac{3}{2}x - 2$ or $y = -\frac{3}{2}x + 3$ for Problem 3.** Confirm for students that the slope is correct and ask students to check their $y$-intercept by circling it on the graph and in the equation.

**Look for productive strategies:**

- Using the $m$ or the $b$ from $y = mx + b$ to help place a line in its correct place in the plane.
- Using a point that should be on the line to position the coordinate plane.
- Creating a line that matches the equation on the coordinate plane, and then rotating the plane so the line is matching.

Activity 1 continued >

## Differentiated Support

**Accessibility:** ***Optimize Access to Technology***

Have students use the Amps slides for this activity, in which they can use digital rotation tools to match coordinate planes to lines. This will help them visualize how the coordinate plane has rotated.

**Accessibility:** ***Guide Processing and Visualization***

Demonstrate how to use the Activity 1 PDF, *Coordinate Plane Template* and tracing paper to create the coordinate plane for Problem 1.

**Extension:** ***Math Enrichment***

Have students complete the activity without access to the Activity 1 PDF. This will challenge students to find strategies and tools for creating their own coordinate plane and scale on tracing paper.

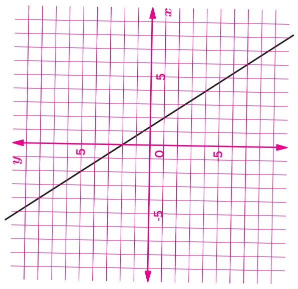
Small Groups | 20 min

# Activity 1 Something Weird Is Happening . . . (continued)

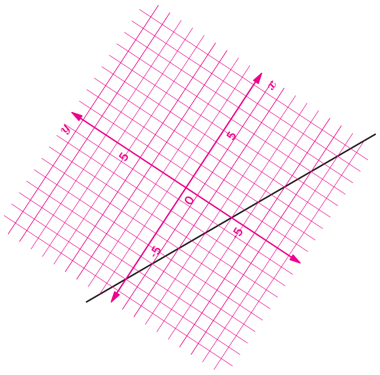
Students manipulate coordinate planes and match lines to the given equations, deepening their understanding about relationships between $m$ and $b$ and the position of the line.

**Activity 1** Something Weird Is Happening . . . (continued)

3. $y = -\frac{3}{2}x + 2$



4. $y = -\frac{1}{2}x - 4$



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## 3 Connect

**Display** student work showing correct responses for Problems 1–4.

**Have students share** what strategies they used to place the coordinate plane on the lines.

**Ask:**

- "What strategies did you find effective?" Answers may vary.
- "What strategies did you find ineffective?" Answers may vary.

**Highlight** different strategies students used and support discussion by clarifying or providing vocabulary as needed. Knowing the values of $m$ and the $b$ from the equation $y = mx + b$ means students draw a line $y = mx$, parallel to the line $y = mx + b$, that passes through the point $(1, m)$. Then students can translate this line to make it pass through the point $(0, b)$ so that it matches the equation $y = mx + b$.

Pairs | 10 min

# Activity 2 Partner Planes

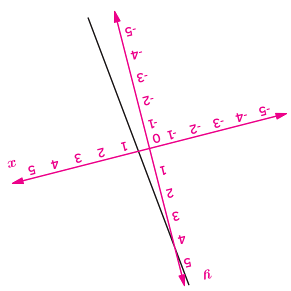
Students write equations for a line not on a coordinate plane, challenging their partner to match the position of the plane to the equation.

Name: ______ Date: ______ Period: ______

## Activity 2 Partner Planes

**Up to this point, you have seen how lines can change on the coordinate plane. Today, you also encountered planes that were being transformed. Understanding changes to the coordinate plane, or "frame of reference," for moving objects was key to the development of relativity by scientists and mathematicians like Albert Einstein and Emmy Noether.**

Write an equation for the line on the space provided. Trade with a partner to see if they can place the coordinate plane on the line to correctly match your equation. Check their sketch to confirm they are correct.



**Equation:** Sample response: $y = -10x + 5$

**Featured Mathematician**

**Emmy Noether**

Born in Bavaria, Germany in 1882, Noether was a pioneer in abstract algebra. After completing her doctorate, she taught at a German university for seven years without pay due to sexism in academia. As the Nazis rose to power in the 1930s, Noether moved to the United States, teaching at Bryn Mawr and Princeton.

In 1915, she worked with David Hilbert and Felix Klein to further develop Albert Einstein's theory of general relativity — a geometric interpretation of gravity. Noether proved that energy and momentum are indeed conserved in different physical systems, no matter how they are oriented — that is, whether or not their planes have gone rogue!

Public Domain

STOP





### 1 Launch

Remind students that they should be able to solve the problem they prepare for their partner.

### 2 Monitor

Help students get started by asking them to identify the values of $m$ and $b$ of their partner's equation, and then ask whether they need to adjust their scales accordingly.

**Look for points of confusion:**

- **Not being able to place the plane on the line because the equation uses numbers that exceed their scale.** Encourage students to come up with a new scale they can use.

**Look for productive strategies:**

- Creating a new scale that is appropriate for a line and a plane with a slope or $y$-intercept of magnitude greater than the scale from Activity 1.

### 3 Connect

**Display** examples of student work.

**Ask**, "What tools were most helpful?"
Answers may vary.

**Have students share** any questions they have about the activity.

**Highlight** examples where students made interesting choices about tools that helped them complete the activity. Showcase examples of where students persevered in problem solving.

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge***

Restrict students to using positive or negative integers between −5 to 5 (inclusive) when writing their equations.

**Extension: *Math Enrichment***

Have students write an equation in the form of $Ax + By = C$.

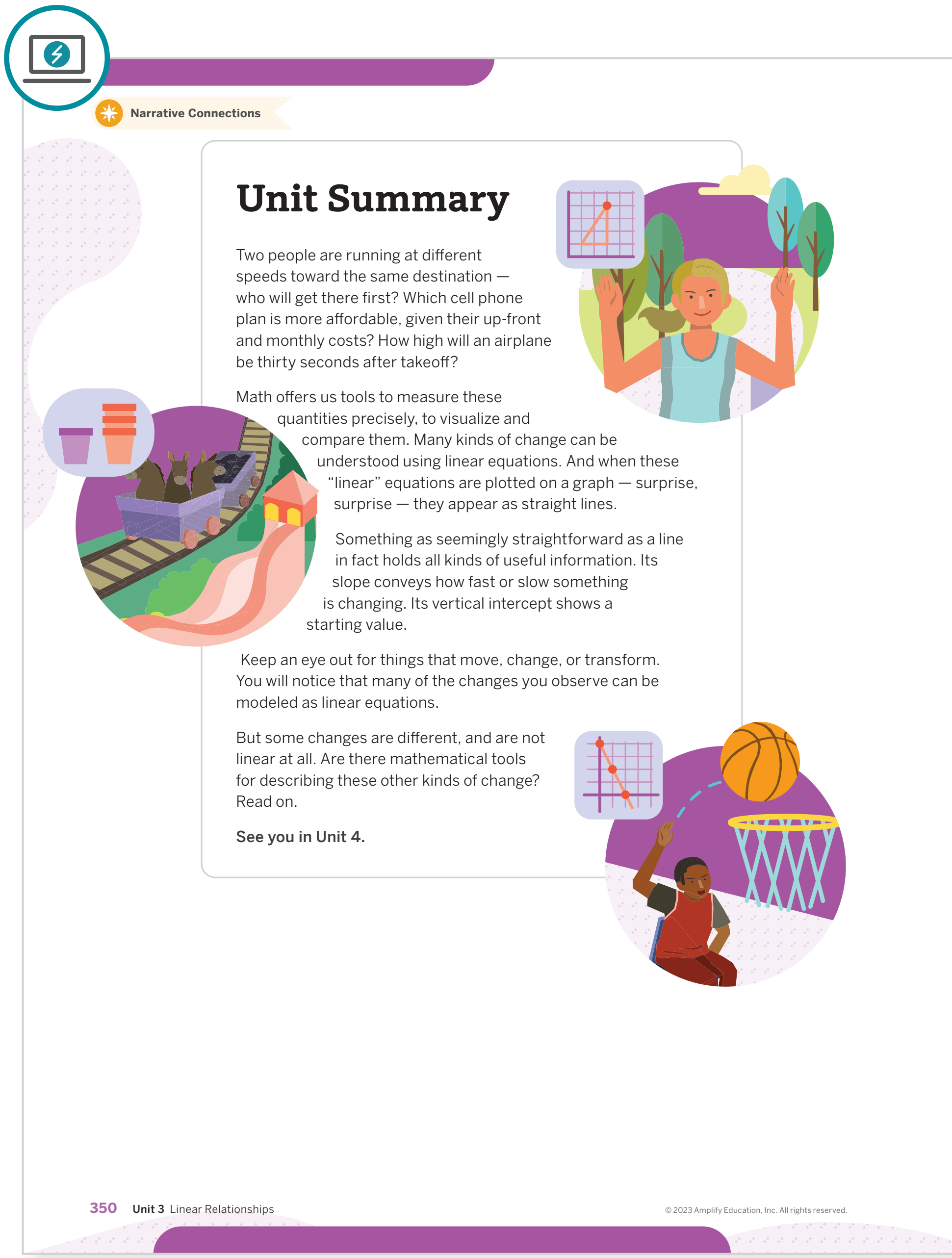
## Featured Mathematician

**Emmy Noether**

Have students read about Emmy Noether, who worked to further develop Albert Einstein's theory of general relativity.

# Unit Summary

Review and synthesize takeaways and questions students have about Unit 3.


Narrative Connections

## Unit Summary

Two people are running at different speeds toward the same destination — who will get there first? Which cell phone plan is more affordable, given their up-front and monthly costs? How high will an airplane be thirty seconds after takeoff?

Math offers us tools to measure these quantities precisely, to visualize and compare them. Many kinds of change can be understood using linear equations. And when these "linear" equations are plotted on a graph — surprise, surprise — they appear as straight lines.

Something as seemingly straightforward as a line in fact holds all kinds of useful information. Its slope conveys how fast or slow something is changing. Its vertical intercept shows a starting value.

Keep an eye out for things that move, change, or transform. You will notice that many of the changes you observe can be modeled as linear equations.

But some changes are different, and are not linear at all. Are there mathematical tools for describing these other kinds of change? Read on.

**See you in Unit 4.**

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### Narrative Connections

Read the narrative aloud as a class or have students read it individually.

### Synthesize

**Display** the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

**Have students share** their reflections from their work in this unit.

**Ask:**

- "What are your biggest takeaways from this unit?" Answers may vary.
- "What are your biggest questions about this unit?" Answers may vary.

**Highlight** that students will continue to study linear relationships in Unit 4. Consider posting any questions that can be revisited in Unit 4.

### Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

- Did anything surprise you while reading the narratives of this unit?
- Is there anything you would like to learn more about these topics? What are some steps you can take to learn more?

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by reflecting on how the values of $m$ and $b$ in the equation $y = mx + b$ affect the position of the line on the coordinate plane.

Printable

Name: ____________ Date: ________ Period: ________

## Exit Ticket 3.19

**Reflect on what you have learned in this unit.**

1. Three things I learned:
   **Answers may vary.**

2. Two things I found interesting or surprising:
   **Answers may vary.**

3. One question I still have:
   **Answers may vary.**

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I know how the values of $m$ and $b$ in the equation $y = mx + b$ affect the graph of the line on the coordinate plane.

1 2 3





## Success looks like . . .

- **Language Goal:** Describing how the values of $m$ and $b$ in the equation $y = mx + b$ affect the line on the coordinate plane. **(Speaking and Listening)**

## Suggested next steps

**If students are unsure what to write, consider:**

- Activating students' prior knowledge by having students reference the Unit 3 Anchor Charts. Consider displaying them, if they are not currently displayed.
- Encourage students to write any remaining questions if they cannot think of how to respond to Problems 1 and 2.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- What was especially satisfying about seeing how students approached Activity 1?
- What, if anything, did students find frustrating or challenging about Activity 1? What helped them work through this frustration?

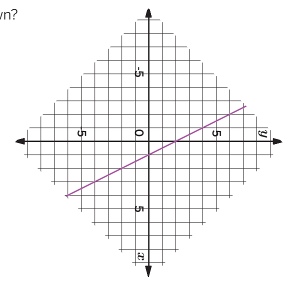
# Practice

Independent

Name: ______ Date: ______ Period: ______

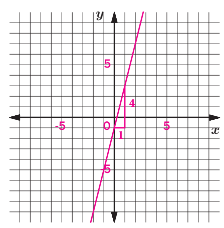
Practice

1. What is the equation of the line shown?
$y = -2x + 2$




2. For each pair of points, find the equation of the line that passes through both points.
   a. $(0, 1)$ and $(2, 5)$ $y = 2x + 1$
   b. $(1, 1)$ and $(7, 5)$ $y = \frac{2}{3}x + \frac{1}{3}$
   c. $(-2, 11)$ and $(1, -1)$ $y = -4x + 3$
   d. $\left(-5, \frac{3}{2}\right)$ and $\left(4, \frac{3}{2}\right)$ $y = \frac{3}{2}$

3. Graph the linear relationship with a slope of $4$ and a negative $y$-intercept. Show how you know the slope is $4$ and then write an equation for the line.
**Sample response: The slope of the line is $4$ because I can draw a slope triangle on the line with a vertical distance of $4$ and a horizontal distance of $1$. The equation is $y = 4x - 1$.**







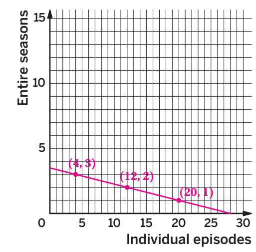

Name: ______ Date: ______ Period: ______

Practice

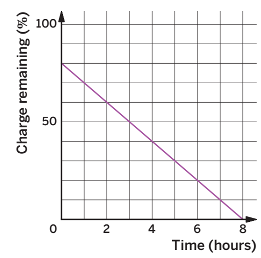
4. It costs \$1.50 to download a television episode to watch and \$12 to download the entire season of episodes. Jada has \$42 to spend downloading television shows.
   a. Complete the table showing three possible ways Jada can spend \$42 downloading.

| Individual episodes | Entire seasons |
|---|---|
| 20 | 1 |
| 12 | 2 |
| 4 | 3 |

   b. Write an equation relating the number of individual episodes $x$ and the number of entire seasons $y$ Jada can download.
   $1.5x + 12y = 42$
   c. Graph the relationship on the coordinate plane.




5. The graph shows the percentage of charge $c$ remaining on Clare's cell phone after it has been in use for $h$ hours.
   a. Write an equation that gives the percentage of charge $c$ remaining on her phone after $h$ hours of use.
   $c = 80 - 10h$
   b. Explain what each number in your equation represents in this situation.
   **The number 80 represents the initial value of the charge of Clare's phone. The number $-10$ means the phone loses 10% of charge every hour.**




   c. When will Clare's phone run out of charge? Where do you see this on the graph?
   **Her phone runs out of charge after 8 hours. It is the $x$-intercept of the line.**





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| Spiral | 2 | Unit 3 Lesson 14 | 1 |
| | 3 | Unit 3 Lesson 12 | 1 |
| | 4 | Unit 3 Lesson 15 | 2 |
| | 5 | Unit 3 Lesson 13 | 2 |

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

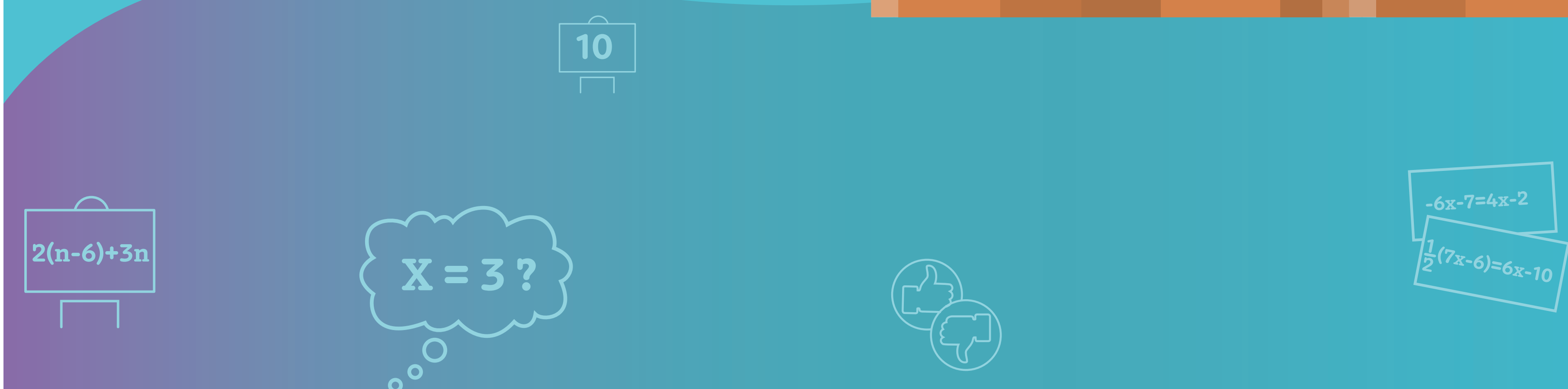
UNIT 4

# Linear Equations and Systems of Linear Equations

Students begin this unit by developing algebraic methods for solving linear equations with variables on both sides of the equation. They then use these algebraic methods, along with graphs and tables, to solve systems of linear equations.

**Essential Questions**

- How can you determine the solution to an equation with variables on both sides?
- What does the number of solutions (none, one, or infinite) to a system of linear equations represent?
- How can systems of equations be used to represent situations and solve problems?
- *(By the way, does a female plumber earn the same amount of money as a male plumber?)*

# Key Shifts in Mathematics

## Focus

### In this unit . . .

The unit begins with lessons on number puzzles and hanger diagrams, which help students develop the algebraic thinking they will use to write expressions and balance equations. Students will then study algebraic methods for solving linear equations in one variable. They analyze groups of linear equations, noting that they fall into three categories: no solution, exactly one solution, and infinitely many solution. The second Sub-Unit focuses on systems of linear equations in two variables.

## Coherence

### Previously . . .

In Grades 6 and 7, students worked with different representations, including hanger diagrams, to solve linear equations with a variable on one side. In Unit 3, students identified and drew graphs for proportional and linear relationships, which helps them reason about graphs for systems of linear equations.

### Coming soon . . .

In Unit 5, students learn the definition of a function, linear or nonlinear. In high school, students will continue their exploration of systems of linear equations by studying more complex ways of solving systems using algebraic methods.

## Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

### Conceptual Understanding

Students learn they can set two expressions equal to find when two situations are the same (Lessons 10 and 11). They learn that the *solution to the system of equations* can be seen as a point of intersection of lines that represent the equations (Lesson 13).

### Procedural Fluency

Equipped with skills for keeping equations balanced (Lessons 2–8), students practice strategic solving of linear equations (Lesson 9). To solve a system of equations, students practice graphing the lines of two equations (Lesson 14).

### Application

Students consider how a system of equations can be used to describe real-world scenarios (Lesson 13). In the final lesson, students look at median earnings for men and women and graph a system of equations to project the gender pay gap over time (Lesson 17).

# The Path the Mind Takes

SUB-UNIT

1

Lessons 2–9

### Linear Equations in One Variable

This Sub-Unit is devoted to solving linear equations in one variable. Students build fluency with a variety of strategies and reason about the processes they use to solve linear equations, which prepares them for solving systems of linear equations in the next Sub-Unit.


**Narrative:** Without the work of mathematicians like Al-Khwarizmi, math might not be the universal language you know today.

SUB-UNIT

2

Lessons 10–16

### Systems of Linear Equations

Students discover how ***systems of linear equations*** can be used to model and solve everyday problems. Using graphs, tables, and equations, they determine and interpret the meaning of a ***solution to a system***, including systems with no solution or infinitely many solutions.


**Narrative:** Discover how more than one equation can help you solve problems with more than one constraint.

## Launch

Lesson 1

### Number Puzzles

Students solve puzzles with number machines, building skills and concepts that mirror what they will do when solving linear equations.

## Capstone

Lesson 17

### Pay Gaps

Supplied with U.S. Census data, students conduct an analysis of data describing the gender pay gap. They consider the implications of this gender pay gap over time using systems of linear equations.

# Unit at a Glance

**Spoiler Alert:** To determine when two equations — each written in the form $y = mx + b$ — have the same solution(s), you can set the two expressions equal to one another, creating one linear equation.

**Assessment**

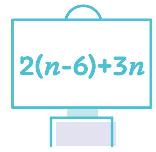
**A** **Pre-Unit Readiness Assessment**

**Launch Lesson**


**1** **Number Puzzles**

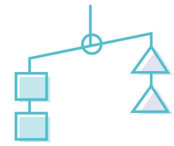
Calculate a missing value for a number puzzle that can be represented by a linear equation with one variable.

**Sub-Unit 1: Linear Equations in One Variable**



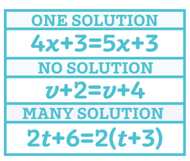

**2** **Writing Expressions and Equations**

Apply the Properties of Equality to solve equations.



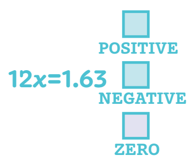
**3** **Keeping the Balance** •

Determine unknown weights on balanced hanger diagrams.



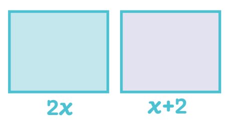

**8** **How Many Solutions?** (Part 2) •

Consider what the structure of an equation says about its solutions.




**9** **Strategic Solving**

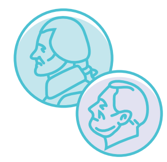
Apply strategies for solving linear equations by participating in a scavenger hunt.




**10** **When Are They the Same?** (optional) •

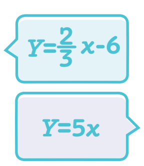
Determine when two conditions are the same by setting expressions equal.

**Sub-Unit 2:**



**11** **On or Off the Line?**

Determine and interpret a point that satisfies two relationships simultaneously.




**16** **Writing Systems of Linear Equations**

Write systems of linear equations representing different contexts and interpret the solution for those systems.

**Capstone Lesson**

**17** **Pay Gaps**

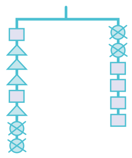
Examine the gender pay gap by graphing the disparity between the median earnings for men and women.

**Assessment**

**A** **End-of-Unit Assessment**

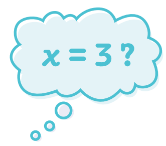
**Key Concepts**

**Lesson 5:** The structure of an equation can be used to determine possible next steps when solving linear equations with variables on both sides.
**Lesson 12:** A graph of two intersecting lines has one solution, while one of lines that never intersect has no solution, and a graph of two lines directly on top of one another has infinitely many solutions.
**Lesson 13:** A solution to a system of linear equations is the ordered pair, $(x, y)$, that makes all equations in the system true.



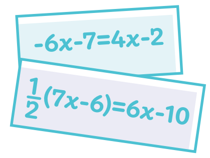
### 4 Balanced Moves
(Part 1) ●

Explore solution paths to solve an equation in one variable.



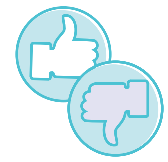

### 5 Balanced Moves
(Part 2)

Rewrite equations while keeping the same solutions.




### 6 Solving Linear Equations

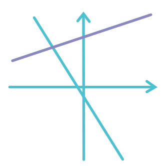
Explain and critique steps for solving linear equations.



### 7 How Many Solutions?
(Part 1) ●

Explore how many solutions equations can have.

## Systems of Linear Equations



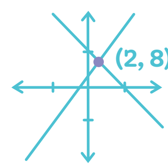
### 12 On Both of the Lines

Interpret graphs with one solution, no solution, and infinitely many solutions.

$$\begin{cases} x+y=-2 \\ x-y=12 \end{cases}$$

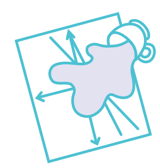
### 13 Systems of Linear Equations

Comprehend that solving a system of equations means determining what ordered pair makes both equations true.




### 14 Solving Systems of Linear Equations
(Part 1)

Solve systems of linear equations by graphing.



### 15 Solving Systems of Linear Equations
(Part 2)

Generalize a process for solving systems of linear equations algebraically.

**Pacing**

**17 Lessons:** 45 min each
**2 Assessments:** 45 min each
**Full Unit:** 19 days
● **Modified Unit:** 16 days

Assumes 45-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

**● Modifications to Pacing**

**Lessons 3–4:** If students are demonstrating proficiency with hanger diagrams and balancing equations, Lessons 3 and 4 can be combined.

**Lessons 7–8:** Lessons 7 and 8 can be combined if students are demonstrating fluency with solving equations.

**Lesson 10:** Lesson 10 helps students to better see why they can set expressions equal to each other to determine that two situations are the same. Because this concept is also covered in Lessons 11 and 12, Lesson 10 is considered optional and can be omitted.

# Unit Supports

## Math Language Development

| Lesson | New Vocabulary |
|---|---|
| 13 | solution to a system of equations<br>system of equations |

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

| Lesson(s) | Mathematical Language Routines |
|---|---|
| 3, 17 | **MLR1:** Stronger and Clearer Each Time |
| 1, 2, 10, 12, 13 | **MLR2:** Collect and Display |
| 4–6, 8, 16 | **MLR3:** Critique, Correct, Clarify |
| 16 | **MLR4:** Information Gap |
| 11, 17 | **MLR5:** Co-craft Questions |
| 2, 6, 10–13 | **MLR6:** Three Reads |
| 1, 4, 8, 13–15 | **MLR7:** Compare and Connect |
| 1–3, 7, 9, 15, 16 | **MLR8:** Discussion Supports |

## Materials

Every lesson includes:

Exit Ticket

Additional Practice

Additional required materials include:

| Lesson(s) | Materials |
|---|---|
| 10, 17 | calculators |
| 2 | glue or tape (optional) |
| 16 | graph paper |
| 13, 14, 16 | graphing technology |
| 8 | index cards |
| 1–7, 9, 10, 12–17 | PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs. |
| 8 | plain sheets of paper |
| 11–14 | rulers |
| 1 | sticky notes |

## Instructional Routines

Activities throughout this unit include the following instructional routines:

| Lesson(s) | Instructional Routines |
|---|---|
| 16 | Algebra Talk |
| 3, 13 | Card Sort |
| 6 | Find and Fix |
| 16 | Info Gap |
| 17 | Notice and Wonder |
| 15 | Partner Problems |
| 10 | Poll the Class |
| 7, 14 | True or False |
| 3, 6, 9, 10, 11, 13, 15, 16 | Think-Pair-Share |
| 12 | Which One Doesn't Belong? |

# Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

| Assessments | When to Administer |
|---|---|
| **Pre-Unit Readiness Assessment**<br>This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit. | Prior to Lesson 1 |
| **Exit Tickets**<br>Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned. | End of each lesson |
| **End-of-Unit Assessment**<br>This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide. | After Lesson 17 |

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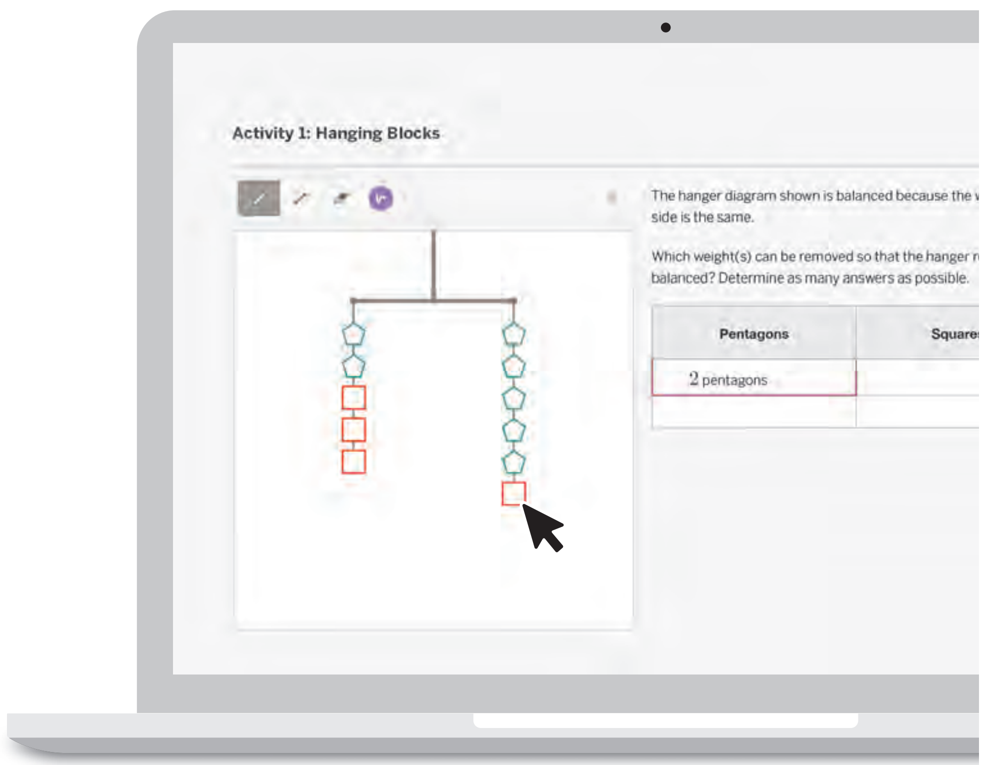
## Social & Collaborative Digital Moments

### Featured Activity

**Hanging Blocks**

Put on your student hat and work through **Lesson 3, Activity 1:**




Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

**Other Featured Activities:**

- Number Machines **(Lesson 1)**
- Trading Equations, Revisited **(Lesson 8)**
- A New Way of Solving **(Lesson 11)**
- How Many Solutions? **(Lesson 14)**
- Mind the Gap **(Lesson 17)**

# Unit Study
## Professional Learning

This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

## Anticipating the Student Experience With Fawn Nguyen

**Sub-Unit 2** introduces students to systems of linear equations. Students work with hanger diagrams as visual models for equations. They learn to solve linear equations in multiple steps and explain each step in their work, such as using the Distributive Property and combining like terms. Students see how graphing can help solve a system of linear equations that arise from everyday problems. They learn to identify if a system of linear equations has one solution, no solution, or infinitely many solutions. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

### Do the Math

Put on your student hat and tackle these problems from Lesson 16, Activity 1:

**Write a system of equations to model each scenario. Define the variables you choose to use. Without solving the system, interpret what the solution to the system would tell you about the scenario.**

1. Elena plans a kayaking trip. Kayak Rental A charges a base fee of \$15 plus \$4.50 per hour. Kayak Rental B charges a base fee of \$12.50 plus \$5 per hour.
2. Diego works at a smoothie stand and prepares a batch of smoothies. The recipe calls for 3 cups of sliced strawberries for every cup of sliced apples. Diego uses a total of 5 cups of sliced strawberries and apples.
3. Andre orders some posters. At Store A, he can order 6 large posters and 4 small posters for \$70. At Store B, he can order 5 large posters and 9 small posters for \$81.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

**Points to Ponder . . .**

- What was it like to engage in this problem as a learner?
- The scenario in question 1 might lend itself to equations in slope-intercept form, while those of question 3 are in standard form. How might you help students recognize the different types in real-world contexts?
- What implications might this have for your teaching in this unit?

## Focus on Instructional Routines

### Info Gap

**Rehearse . . .**

How you'll facilitate the *Info Gap* instructional routine in Lesson 16, Activity 2:

**You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.**

| If you are given the *problem card*: | If you are given the *data card*: |
|---|---|
| 1. Silently read your card and think about what information you need to be able to solve the problem. | 1. Silently read your card. |
| 2. Ask your partner for the specific information that you need. | 2. Ask your partner "What specific information do you need?" and wait for them to ask for information. |
| 3. Explain how you will use the information to solve the problem. | 3. If your partner asks for information that is not on the card, do not perform the calculations for them. Tell them you don't have that information. |
| 4. Continue to ask questions until you have enough information to solve the problem. | 4. Before sharing the information, ask "Why do you need that information?" Listen to your partner's reasoning and ask clarifying questions. |
| 5. Share the problem card and solve the problem independently. | 5. Read the problem card and solve the problem independently. |
| 6. Read the data card and discuss your thinking. | 6. Share the data card and discuss your thinking. |

**Pause here so your teacher can review your work. You will be given a new set of cards. Repeat the activity, trading roles with your partner.**

**Points to Ponder . . .**

- It will be helpful for students to see a demonstration of this routine before they participate. What can you model in this activity that will help students understand the routine, without revealing anything about the math in the activity?

**This routine . . .**

- Strengthens the opportunities and supports for high-quality mathematical conversations.
- Helps students learn new mathematical language.
- Places an emphasis on communication in order to bridge information gaps.

**Anticipate . . .**

- Which part of the routine, posing or answering questions, will be most difficult for your students?
- How will you facilitate the multiple rounds of dialog such that students strengthen their discourse over time?
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

## Strengthening Your Effective Teaching Practices

**Facilitate meaningful mathematical discourse.**

### This effective teaching practice . . .

- Ensures that there is a shared understanding of the mathematical ideas students have explored and discovered during each lesson's activities.
- Allows students to listen to and critique the strategies and conclusions of others.

### Points to Ponder . . .

- How can you establish a classroom environment in which diverse approaches to solving problems are cultivated?
- Some students may not know how to dive deeper into discussions about mathematics. How can you model these discussions?

## Math Language Development

### MLR6: Three Reads

MLR6 appears in Lessons 2, 6, 10–13.

- Encourage students to read introductory text multiple times before jumping into a task. By doing so, they will have more opportunities to understand the task and the quantities and relationships presented. The ***Three Reads*** routine asks students to focus on the following for each read:
  - **Read 1:** Make sense of the overall information or scenario, without focusing on specific quantities.
  - **Read 2:** Look for specific quantities and relationships and make note of them.
  - **Read 3:** Brainstorm strategies for how to approach the task.
- **English Learners:** Annotate or highlight key words and phrases in the introductory text to help students understand the relationships between quantities, such as each, *twice*, etc.

### Point to Ponder . . .

- Some students may resist reading information multiple times. How will you help them see the benefits to doing so before jumping into the actual task?

## Fostering Diverse Thinking

Use these opportunities for students to connect mathematics to the world around them:

- In Lesson 12, students research Wilma Rudolph, one of the first athletes to advocate for civil rights in the U.S.
- In Lesson 17, students research and learn about National Equal Pay Day in the U.S., what it represents mathematically, how it is calculated, and how it compares to prior years.

### Point to Ponder . . .

- How can I help raise my students' awareness of the contributions of mathematicians around the world, and connect the math they are learning in this unit to conversations about equity?

## Unit Assessments

Use the results of the Pre-Unit Readiness Assessment to understand your students' prior knowledge and determine their prerequisite skills.

### Look Ahead . . .

- Review and unpack the End-of-Unit Assessments, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

### Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with solving one-step equations throughout the unit? Do you think your students will generally:
  - miss the underlying concept of balance and mathematical equality?
  - struggle using graphs to solve a system of equations?
  - have difficulty using a system of equations to describe a story?

## Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their social awareness and self-management.

### Points to Ponder . . .

- Are students able to motivate themselves to deepen their understanding of equations and the relationship they have to graphs? Do they use the tools available to explore new concepts and gain more knowledge on systems of equations?
- How do the students relate to each other? Are they able to communicate clearly? Do they work as a team? How do they build their relationships?


# Number Puzzles

Let's solve some puzzles!

## Focus

### Goals

1. **Language Goal:** Calculate a missing value for a number puzzle that can be represented by a linear equation with one variable, and explain the solution method. **(Speaking and Listening, Writing)**

2. Create a number puzzle that can be represented by a linear equation with one variable.

## Rigor

- Students build **conceptual understanding** for solving linear equations.

## Coherence

### Today

Students begin this unit by finding solutions to number puzzles, including considering inputs and outputs of a number machine with given steps. These puzzles are good preparation for solving linear equations, in which students have to perform operations on each side of the equation to isolate the variable. Students use representations of their choosing, such as line diagrams, tape diagrams, and equations.

### Previously

In Grade 7, students worked with different representations to solve equations, including hanger diagrams. In Unit 3, students were introduced to the term *linear relationship* by studying graphs, but did not yet get the opportunity to practice solving linear equations algebraically.

### Coming Soon

In Lesson 2, students will continue studying puzzles, this time by writing equations to help them find a solution.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 8 min | 15 min | 15 min | 5 min | 5 min |
| Pairs | Small Groups | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Graphic Organizer PDF *Guess and Check* (as needed)
- sticky notes

### Math Language Development

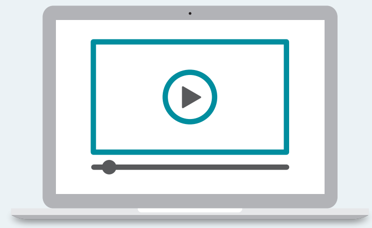
**Review words**

- *input*
- *order of operations*
- *output*

**Amps** | Featured Activity

### Activity 1
### Digital Number Machines

Students work with digital number machines to develop their equation-solving intuition.



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### Building Math Identity and Community

Connecting to Mathematical Practices

Students may lack confidence to represent each step as they work to solve the puzzle. Remind students to use their strengths as they compose their explanations. Assure them that they may not always be correct, but the attempt should be made with confidence.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 2 may be omitted.
- In **Activity 1**, have students work only with one partner, instead of three.
- **Activity 2** may be omitted.

Pairs | 8 min

# Warm-up Number Machine

Students explore solutions for given number machines to gain an understanding for the number puzzles they will soon create.

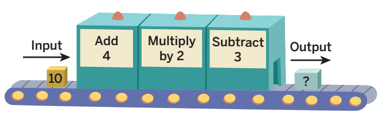
Unit 4 | Lesson 1 – Launch

## Number Puzzles

Let's solve some puzzles!

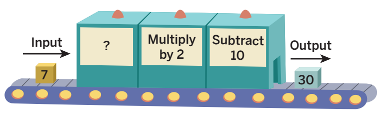

### Warm-up Number Machine

1. Math, Inc. built a number machine. If 10 is the input, what number will be the output? Show or explain your thinking.




Sample response: $(10 + 4) \cdot 2 - 3 = 25$

2. Advanced Algebra Corporation built their own number machine, but they lost the part that describes the first step. When 7 was the input, 30 was the output. Can you determine the missing part for the first step? Show or explain your thinking.




The first step is adding 13.

Sample response:

$$(7 \square x) \cdot 2 - 10 = 30$$
$$(7 \square x) \cdot 2 - 10 + 10 = 30 + 10$$
$$(7 \square x) \cdot 2 = 40$$
$$(7 \square x) = 20$$

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Tell students they will be working with different number puzzles throughout this lesson, and their goal for the Warm-up is to understand how number machines work.

## 2 Monitor

**Help students get started** by reviewing the terms *input* and *output* and asking, "What is the first thing that happens when a number is put into the number machine?"

**Look for points of confusion:**

- **Not being sure how the number machine works.** Ask students how many steps there are in the machine, and help students find the output after the first step.
- **Not being sure how to find the missing step in Problem 2.** Suggest that students try by starting with the output and reversing each step.

## 3 Connect

**Have students share** their reasoning for Problems 1 and 2. Sequence responses by starting with students who guessed the answer, and by ending with students who wrote an expression, if applicable.

**Highlight** that the number machine produces an output by performing a series of operations on an input, the same way we evaluate an expression. Identify different strategies or representations students may have used to make sense of the number machine.

**Ask:**

- "What was different about your process for solving Problem 1 and Problem 2?"
- "Do these number machines follow the order of operations? What does this suggest?"
- "What are some other ways you could represent the number machine?"

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

To scaffold students' thinking for Problem 1, omit the second and third step and have students work with a number machine that has one step. Continue adding additional steps until students are able to work with the three-step number machine.

## Math Language Development

### *MLR8: Discussion Supports — Restate It!*

During the Connect, as students share their reasoning, have students who guessed first share, followed by students who wrote an expression. After each student shares their strategy, pause and ask another student to restate their reasoning in their own words. Ask, "How does the number machine relate to expressions that you can evaluate?"

### English Learners

Annotate each number machine with a corresponding expression and write the term *expression* next to it.

# Activity 1 Think of a Number . . .

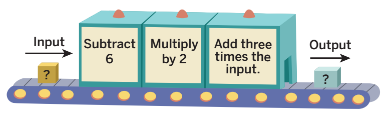
Students determine an input based on their partners' output to develop new representations for finding the solutions.

**Amps Featured Activity** Digital Number Machines

Name: ________ Date: ________ Period: ________

## Activity 1 Think of a Number . . .

You will be given some sticky notes. Consider the number machine shown.




1. Think of a number to represent the input of the number machine. Record your input on a sticky note, without showing anyone.
   Answers may vary. Sample response: 5

2. When you put your number into the machine, what is the output? Record your output on a different sticky note, and share your output with a partner.
   Answers may vary. Sample response: 13

3. Record your partner's output and guess their input. Show your thinking and any calculations needed. Switch partners in your group until you have worked with all three partners.
   a. If Partner 1's output is 38, Partner 1's input is 10.
   b. If Partner 2's output is 18, Partner 2's input is 6.
   c. If Partner 3's output is −12, Partner 3's input is 0.

**Collect and Display:** As you discuss your results, your teacher will collect words and phrases you use. This language will be added to a class display for your reference.

4. How did you determine the input, when given the output?
   Sample response:
   - I estimated different numbers for the input and plugged them into the machine until I found the input that gave me the correct output.
   - I wrote an equation for the machine and used it to solve for the input. For example, I wrote the equation $2(n-6) + 3n =$ [output] and solved for $n$, the input.




## 1 Launch

Assign students to groups of 4. Distribute sticky notes. **Note:** Students will represent this number machine with an expression in Lesson 2. Discuss here only if you see students comfortably using expressions on their own.

## 2 Monitor

**Help students get started** by asking them to describe what is happening to their input in each step of the number machine.

**Look for points of confusion:**

- **Multiplying their result by 3 or adding 3 in the third step of the number machine, instead of adding 3 times their input.** Ask students to identify their input, and then ask what 3 times that number would be.

## 3 Connect

**Display** student work showing different representations. Ask students to share any strategies they used to determine their partners' input.

**Ask:**

- "What made it challenging to work backwards with this number machine?"
- "How did your strategies evolve as you worked with other partners?"
- "What representations, if any, made determining the output more efficient? Were any representations or strategies ineffective?"

**Highlight** students who worked through a challenge by trying different strategies. Remind them that you will not be providing any hints or strategies for solving, but that they will need to rely on their own ideas, or their partners', to determine solutions.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Omit the second step of the number machine and have students work with a number machine that has two steps.

### Accessibility: *Guide Processing and Visualization*

Have students use the Graphic Organizer PDF, *Guess and Check* to help organize their thinking.

## Math Language Development

### *MLR2: Collect and Display*

During partner discussion, circulate and listen to students explain their representations of the problems to one another. Listen for the variety of ways students solve for their partners' input. Write student-generated words on the class display and continue adding to the display throughout the unit.

### English Learners

Highlight any visual representations students create and show how the visual representation connects to the number machine.

Pairs | 15 min

# Activity 2 Build Your Own Number Machine

Students create their own number puzzle to apply new strategies about how number puzzles work and can be solved.

**Activity 2** Build Your Own Number Machine

**The number machine shows three steps. Create your own descriptions for the three steps. Using a sticky note, choose an input for your number machine and record the output.**

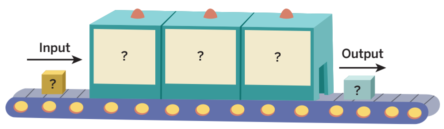
**My puzzle:**

Sample response:

Step 1: Add 3.

Step 2: Subtract 4.

Step 3: Multiply by 10.




1. Trade puzzles and outputs with your partner to see if you can determine each other's inputs. Show your thinking. Then check whether your partner used your number machine correctly by confirming their input is correct.
   Answers will vary.

2. With your partner, compare your solutions to each puzzle. Did each of you solve the puzzle the same way? If not, be prepared to share with the class which solution strategy you think is the most efficient.
   Sample response: My partner used an equation to solve, but I worked backward by undoing each step.

**Are you ready for more?**

Consider a number machine with the following steps:

- Think of a number.
- Double the number.
- Add 9.
- Subtract 3.
- Divide by 2.
- Subtract the original number.
- The output should be 3.

Why does this always work?

Sample response: Because when you double your number, then divide by 2 and subtract your original number, the result is zero. You are left with the expression $(9 - 3) \div 2$, which is 3.

STOP





## 1 Launch

Give five minutes for students to write their own puzzle before trading their puzzle with a partner to solve. Make sure students write their input on a different sticky note than their output so their partner cannot see the solution.

## 2 Monitor

**Help students get started** by asking them to describe the steps they see in their partner's number machine.

**Look for points of confusion:**

- **Creating a number machine that is challenging for students to solve on their own.** Remind students that they must be able to determine the output for a given input for their own puzzle to be able to confirm their partner solved it correctly. If students cannot, or are discouraged from solving their own puzzle, consider providing support or suggesting they rewrite their puzzle with simpler steps.

**Look for productive strategies:**

- Trying different strategies or representations based on their partner's work.

## 3 Connect

**Have pairs of students share** the puzzles they created with the class and any representations they created. If students do not mention this in their explanations, ask which of their representations was the most efficient one for solving the puzzle.

**Highlight** that there is no "best" representation for solving number puzzles. The best representation is the one that makes sense to each student and helps them solve the problem. However, as problems grow more complex, students are likely to find that certain representations are more useful for solving problems than others.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

To support students getting started creating their own number machines, provide them with a simple input, such as 2. Alternatively, suggest they create a two-step expression that would represent a two-step number machine.

## Math Language Development

MLR

### *MLR7: Compare and Connect*

During the Connect, as students share their puzzles and any representations or strategies they used to solve their partner's puzzle, ask the class to identify similarities and differences between their strategies and representations. Ask volunteers to share which solution strategy they think is the most efficient and why. Draw students' attention to the fact that, as long as the strategy makes sense mathematically, it is a valid strategy.

Whole Class | 5 min

# Summary The Path the Mind Takes

Review and synthesize strategies for working with number machines.

Narrative Connections

Unit 4 Linear Equations and Systems of Linear Equations

## The Path the Mind Takes

Riddles exist in almost every culture.

There is the ancient Greek tale of the Sphinx. The Sphinx was a fearsome creature with a human head, the body of a lion, and the wings of an eagle. It stood guard over the city of Thebes. Any traveler who wished to enter or leave the city had to answer the Sphinx's riddle. Those who failed were instantly devoured!

The Dagara people of West Africa tell *zupkai* to their children — a blending of folktales, riddles, and proverbs that provide valuable lessons through storytelling.

Meanwhile, in East Asia, the Zen Buddhist *kōans* present seemingly impossible scenarios — such as the sound of one hand clapping. These *kōans* were designed to inspire thought and reflection in the listener.

Across the globe, people have been fascinated by riddles in one form or another. But the most exciting part of any riddle is not the answer itself, but the path the mind takes to find the answer.

Math has plenty of riddles too, but they are not typically presented through stories, poems, or rhymes. Instead, they are posed using numbers and symbols, in a discipline called algebra. And much like the sphinx's riddle, the *zupkai*, or the *kōan*, the joy should not be in the answer itself, but the journey of finding the answer.

**Welcome to Unit 4.**





## Narrative Connections

Read the narrative aloud as a class or have students read it individually.

## Synthesize

**Display** the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

**Have students share** if they felt like they had strategies that helped them solve number machines.

**Ask:**

- "What strategies or techniques do you have for when you are working on a problem and feel stuck or frustrated?"
- "In what ways is the equation $x + 5 - 2 = 10$ like a number machine?"

**Highlight** that it may feel like students are missing a skill or strategy for solving number machines. Pique curiosity by previewing that students will learn a powerful strategy for representing and solving number puzzles, such as number machines in Lesson 2.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

- "Which number puzzles did you find most challenging to solve today?"
- "Were any strategies or tools not helpful? Why?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by solving a number puzzle similar to a number machine.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 4.01

Clare asked Diego to consider a number machine with the following steps.

- Choose an input.
- Multiply by 3.
- Add 2.
- Subtract 7.
- Add your input.

Diego's output is 27. What was his input?
Show or explain your thinking.

8; Sample response: I used guess and check. Because the first step is to multiply by 3, and I know the final result is 27, I wanted a number that would multiply by 3 and get close to 30, based on how the following steps seemed to not increase or decrease the value by much; $8 \cdot 3 + 2 - 7 + 8 = 27$.

**Self-Assess**

| ? 1 I don't really get it | 2 I'm starting to get it | 3 I got it |
|---|---|---|

a I can solve number puzzles using various strategies, such as guess and check or by working backward.

1 2 3





## Success looks like . . .

- **Language Goal:** Calculating a missing value for a number puzzle that can be represented by a linear equation with one variable, and explaining the solution method. **(Speaking and Listening, Writing)**
  - » Solving for Diego's input and explaining how they determined the value.
- **Goal:** Creating a number puzzle that can be represented by a linear equation with one variable.

## Suggested next steps

**If students work in the opposite order, consider:**

- Reminding students that they must work backward from the output to determine the input.
- Reviewing how to work backward from an output by looking at Problem 2 of the Warm-up.

**If students give up or feel like they are unable to solve the number puzzle, consider:**

- Offering words of encouragement and suggesting that they will learn new strategies in Lesson 2 that may be helpful.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- What trends do you see in participation?
- What did students find frustrating about writing or solving number machines? What helped them work through this frustration?

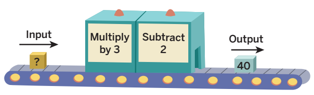
# Practice

Independent

Name: ____________ Date: ________ Period: ________

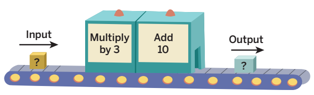
Practice

1. Diego chose a number to represent the input of the machine shown and 40 was the output. What was his input? Explain your thinking.



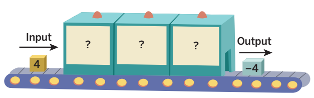

**14; Sample response: $14 \cdot 3 - 2 = 40$**

2. Clare wants to solve this puzzle represented by the number machine shown. She says, "If you take the output, then divide by 3 and subtract 10, the result is the same as the input." Do you agree? Explain your thinking.




**No; I do not agree. Sample response: The order of the operations matters. You would subtract 10 first, and then divide by 3.**

3. The number machine shown has three steps. Create your own descriptions of these three steps so that an input of 4 gives an output of $-4$.




**Sample response shown.**

Step 1: **Add 5.**

Step 2: **Subtract 10.**

Step 3: **Multiply by 4.**




Name: ____________ Date: ________ Period: ________

Practice

4. Solve each equation. Show your thinking.

a. $4x + 9 = 11$

$$4x + 9 - 9 = 11 - 9$$
$$4x = 2$$
$$4x \div 4 = 2 \div 4$$
$$x = \frac{1}{2}$$

b. $-3(x + 7) = -15$

$$-3(x + 7) \div (-3) = -15 \div (-3)$$
$$x + 7 = 5$$
$$x + 7 - 7 = 5 - 7$$
$$x = -2$$

5. Select *all* of the given points that lie on the graph of the linear equation $4x - y = 3$.

A. $(-1, -7)$

B. $(0, 3)$

C. $\left(\frac{3}{4}, 0\right)$

D. $(1, 1)$

E. $(5, 2)$

F. $(4, -1)$

6. Write each verbal description as a mathematical expression.

a. 5 more than $x$

**$5 + x$**

b. $k$ less than $\frac{1}{2}$

**$\frac{1}{2} - k$**

c. Half of $r$

**$\frac{r}{2}$**

d. The product of 12 and $p$

**$12p$**




## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 1 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Grade 7 | 1 |
| | 5 | Unit 3 Lesson 16 | 1 |
| Formative | 6 | Unit 4 Lesson 2 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Whole Class

**Sub-Unit 1**

# Linear Equations in One Variable

In this Sub-Unit, students solve linear equations to build fluency with the strategies they will need to solve systems of linear equations in the second Sub-Unit.

SUB-UNIT

**1** **Linear Equations in One Variable**

**Narrative Connections**

## Who was the Father of Algebra?

Situated along the banks of the Tigris river, the city of Baghdad was a bustling hub of business and commerce in the ninth century. But as economic activity grew, so too did disputes. Laborers needed wages. Inheritances needed to be split. Land had to be divided.

But resolving each dispute individually was time-consuming. But thanks to the Persian mathematician Muhammad ibn Mūsā al-Khwārizmī, a system was developed to help settle these disputes more efficiently.

Very little is known about Al-Khwārizmī's early life. By the age of 40, Al-Khwārizmī was invited by Caliph al-Ma'mun to Baghdad's House of Wisdom. This academic center hosted leading scholars and was considered the center of knowledge in the world at the time. There, Al-Khwārizmī was appointed as an astronomer and later as the head of the library.

Al-Khwārizmī's methods for settling disputes made up a great portion of his book, *The Compendious Book on Calculation by Completion and Balancing*, or *Hisab al-jabr w'al-muqabala*. The "al-jabr" in the title is where the word *algebra* is derived. Al-Khwārizmī's book brought together the geometry of Greeks and the algorithmic methods of Indian, Mesopotamian, and Chinese scholars.

It might seem like math is a universal language, but it wasn't always so. It took the work of mathematicians like Al-Khwārizmī to create the mathematical language and balancing methods we still use today to solve for unknowns.





## Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore mathematical language and balancing methods related to linear equations in one variable in the following places:

- **Lesson 2, Activity 1:** Think of a Number, Revisited
- **Lesson 3, Activities 1-3:** Hanging Blocks, Card Sort: Hanger Diagrams, More Hanging Blocks
- **Lesson 4, Activities 1-2:** Matching Hangers, Matching Equations Moves
- **Lesson 5, Activities 1-2:** Step by Step, Create Your Own Steps
- **Lesson 6, Activities 1-2:** Trading Equations, Find and Fix

UNIT 4 | **LESSON 2**

# Writing Expressions and Equations

Let's write expressions and equations.

## Focus

### Goals

1. Write expressions and equations to represent real-world scenarios.
2. Generate an equivalent expression with fewer terms, including using the Distributive Property.
3. Use the Properties of Equality to solve equations.

## Coherence

### Today

Students model the number machines from Lesson 1 and new verbal descriptions with expressions and equations. Students apply the Distributive Property, combining like terms, and the Properties of Equality to solve the equations.

### Previously

In Grade 7, students solved equations of the form $px + q = r$ and $p(x + q) = r$ and began to write expressions with fewer terms. In Lesson 1, students used representations of their choosing to determine solutions to number puzzles to begin the conversation about solving linear equations.

### Coming Soon

In Lessons 3 and 4, students will use hanger diagrams to show balancing equations to lead toward solving linear equations with variables on both sides of the equal sign.

## Rigor

- Students grow their **conceptual understanding** of expressions and equations by creating them to represent scenarios.
- Students **apply** their knowledge of solving equations to new situations and scenarios.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 10 min | 20 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

## Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards (optional)
- Anchor Chart PDF, *Properties of Operations*
- Anchor Chart PDF, *Properties of Equality*
- glue or tape (optional)

## Math Language Development

**Review words**

- *coefficient*
- *constant*
- *Distributive Property*
- *equation*
- *equivalent equations*
- *expression*
- *input*
- *like terms*
- *output*
- *Properties of Equality*
- *solution*
- *substitution*
- *term*
- *variable*

**Amps** | **Featured Activity**

### Exit Ticket
### Real-Time Exit Ticket

Check in real time if your students can write and solve an equation by using a digital Exit Ticket.

## Building Math Identity and Community

Connecting to Mathematical Practices

Students may be confused and think of mathematics as a foreign language throughout the activities. Encourage students to resist their impulses to quickly write down the first thing that comes to mind. Have them first identify what they do know about writing and solving equations, and then ask them to develop a solution plan rather than just a solution attempt.

## Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 2**, have students only complete Problems 1–3.

Pairs | 5 min

# **Warm-up** Think of a Number, Revisited

Students use a number machine from Lesson 1 to create an expression representing the operations of the machine.

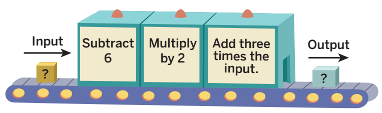
Unit 4 | Lesson 2

## Writing Expressions and Equations

Let's write expressions and equations.

### **Warm-up** Think of a Number, Revisited

**In Lesson 1, you worked with the number machine shown. Now, you will build an expression to represent the number machine.**




1. The table shows the number machine steps. Let $n$ represent any chosen input. Complete the table with a possible expression to represent the resulting value for each step.

| Description from number machine | Expression |
|---|---|
| Let $n$ represent the input. | $n$ |
| Subtract 6. | $n - 6$ |
| Multiply by 2. | $2(n - 6)$ |
| Add three times the input. | $2(n - 6) + 3n$ |

2. How can you use the final expression you wrote in Problem 1 to:
   a. Determine the output if you use any input?
   Sample response: To find the output, the input can be substituted for $n$ in the expression, and then the expression can be evaluated following the order of operations.
   b. Determine the input if you know the output?
   Sample response: Write an equation using the expression on one side of the equal sign and the output on the other side of the equal sign. Then solve the equation for $n$, the input.

Log in to Amplify Math to complete this lesson online.

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## 1 Launch

Activate prior knowledge, and review the definition of *expression*. Consider having students provide examples and counterexamples of expressions.

## 2 Monitor

**Help students get started** by asking how they can write an expression showing subtracting 6 from the variable $n$.

**Look for points of confusion:**

- **Forgetting the parentheses in the expression.** Remind students they want to multiply the entire result from the first step and ask them what symbols allow them to do the subtraction first, and then the multiplication. If further support is needed, review the order of operations and show that, without the parentheses, the multiplication step will happen first.
- **Adding 3 in the last step instead of $3n$.** Have students read the first row of the table and describe what variable was used to represent the input and how they can represent three times that number.

## 3 Connect

**Display** the number machine and table.

**Have students share** their expressions and responses to Problem 2.

**Highlight** the connection between the description and the expression showing how the words model the mathematical expression. Review any necessary vocabulary, including, *expression*, *constant*, *coefficient*, and/or *order of operations*.

**Ask**, "How would you write the following phrases as mathematical expressions?

- 7 less than a number $x - 7$
- Twice a number $2x$
- Twice more than 7 less than a number $2(x - 7)$

## Math Language Development

### *MLR8: Discussion Supports*

Provide sentence frames, such as the following, while students work with their partner to write the expressions representing the number machine descriptions.

- "I think the expression is ___ because . . ."
- "I agree/disagree because . . ."

Show how the phrase "Multiply by 2" is represented by the expression $2(n - 6)$ instead of $2n - 6$. Ask students to explain why. Sample response: The entire expression from the previous line, $n - 6$ is multiplied by 2, not just the input $n$.

## Power-up

**To power up students' ability to write verbal phrases as mathematical expressions, have students complete:**

Match each verbal phrase with its corresponding mathematical expression.

a. 5 more than a number. c $7x + 5$
b. 7 times a number. b $7x$
c. 5 more than 7 times a number. d $7(x + 5)$
d. 7 times 5 more than a number. a $x + 5$

**Use:** Before the Warm-up
**Informed by:** Performance on Lesson 1, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 6

Pairs | 10 min

# Activity 1 Think of a Number, Revisited

Students provide an explanation for each step in a student's process for solving an equation to prepare for solving similar equations in the future activities.

Name: ______ Date: ______ Period: ______

## Activity 1 Think of a Number, Revisited

**Kiran used the following steps to find the input for the number machine from the Warm-up if the output is 17. Describe what Kiran did in each step.**

| Equation | Description |
|---|---|
| $2(n-6)+3n=17$ | Set the expression from the Warm-up equal to the output, 17. |
| $2n-12+3n=17$ | Apply the Distributive Property. |
| $2n+3n-12=17$ | Apply the Commutative Property of Addition. |
| $5n-12=17$ | Combine like terms. |
| $5n-12+12=17+12$ | Add 12 to each side. |
| $5n=29$ | Combine like terms. |
| $5n \div 5 = 29 \div 5$ | Divide each side by the coefficient of $n$, 5. |
| $n=\frac{29}{5}$ | This is the input, the solution to the equation. |

**Are you ready for more?**

Consider a number machine that processes the following steps. Write an expression that represents the output, for any input. Define the variable you choose to use.

- Think of a number.
- Double the number.
- Add 9.
- Subtract 3.
- Divide by 2.
- Subtract the original number.

Sample response: Let $n$ represent the chosen input. The expression that represents the output is $\frac{2n+9-3}{2}-n$.




### 1 Launch

Activate prior knowledge and review examples of the associative, commutative, and Distributive Properties.

### 2 Monitor

**Help students get started** by asking, "How has the equation changed from the first row to the second equation?"

**Look for points of confusion:**

- **Not remembering the names of the properties.** Consider displaying the Anchor Chart PDF, *Properties of Operations* for students to reference throughout the unit.

**Look for productive strategies:**

- Using mathematically precise language such as "applying the Distributive Property," instead of "multiplying by 2," or saying, "dividing by the coefficient," instead of "getting rid of the 5."

### 3 Connect

**Display** the table.

**Have students share** their responses starting with the least sophisticated and finishing with the most mathematically precise.

**Highlight** and encourage mathematically precise language. Remind students that the terms $2n$ and $3n$ are called *like terms* because they have the same variable component and can be combined together using the operations present in the problem.

**Ask:**

- "Did Kiran perform any steps that surprised you? Would you have done anything differently?"
- "How can you check whether the input was actually $\frac{29}{5}$?"

## Differentiated Support

**Accessibility: *Guide Processing and Visualization, Clarify Vocabulary and Symbols***

Provide pre-cut slips from the Activity 1 PDF and glue or tape for students to attach the slips on their Student Edition page. This will support students who would benefit from matching the vocabulary and explanations to the appropriate steps in the table.

## Math Language Development

***MLR2: Collect and Display***

During the Connect, as you highlight mathematically precise language, add these terms and phrases to the class display, such as *like terms, coefficient, constant*, and the *Properties of Equality*.

**English Learners**

Allow students to create a reference sheet with the help of a partner showing the mathematical terms and phrases in their primary language and in English. Have them include examples and illustrations or diagrams, as applicable.

Pairs | 20 min

# Activity 2 How Much Did Each Give?

Students use an ancient problem to practice writing and solving an equation, specifically one where like terms must be combined.

### Activity 2 How Much Did Each Give?

**In 1881, a local farmer from a village called Bakhshali, a region in modern-day Pakistan, noticed a piece of birch bark buried in their field. Turned out, this was not some ordinary piece of bark. The bark was actually an ancient Indian mathematical text, the *oldest* known Indian mathematical text, now known as the Bakhshali manuscript. The manuscript is so old, researchers cannot say for certain when it was written. Some estimates suggest it was written as early as 224 CE.**

**Here is a similar problem to one written in the Bakhshali manuscript:**

Of four coin donors, the second donor gave twice the first donor. The third donor gave three times more than the first donor and the fourth donor gave four more than the first. Together, all four donors gave 32 coins. How much did each give?

1. Choose a variable to represent the number of coins the first donor gave.
   **Sample response: Let $x$ be the number of coins the first donor gave.**

2. Write an expression that represents the number of coins each donor gave, based on the number of coins the first donor gave.
   a. First donor: $x$
   b. Second donor: $2x$
   c. Third donor: $3x$
   d. Fourth donor: $x + 4$

3. Write an expression that represents how much the donors gave altogether.
   $x + 2x + 3x + x + 4$

4. Recall that together they donated 32. Write an equation that represents this statement.
   $x + 2x + 3x + x + 4 = 32$

5. Solve the equation you wrote in Problem 4. Show your thinking.
   $$\begin{aligned} x + 2x + 3x + x + 4 &= 32 \\ 7x + 4 &= 32 \\ 7x + 4 - 4 &= 32 - 4 \\ 7x &= 28 \\ 7x \div 7 &= 28 \div 7 \\ x &= 4 \end{aligned}$$

6. How many coins did each of the donors give? Explain your thinking.
   **Sample response: The first donor gave 4 coins, and the second donor gave twice that, which is 8 coins. The third donor gave $3(4) = 12$ coins, and the fourth donor gave 4 more than the first, which is also 8 coins.**

**Reflect:** How can identifying and defining the variable help you to be more successful in solving this problem?

STOP

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## 1 Launch

Have students read the information regarding the Bakhshali manuscript and the problem. Activate students' background knowledge regarding the meaning of *donors*.

## 2 Monitor

**Help students get started** by asking how they can represent the second donor's expression. Consider providing numerical examples. Ask, "If the first donor gave 8 coins, how many did the second donor give?"

**Look for points of confusion:**

- **Thinking the fourth donor's expression is $4x$.** Have students explain the difference between 4 more and 4 times more.
- **Not knowing how to write the expression for Problem 3.** Ask students what it means by *altogether*. Continue to prompt them until they give the answer of addition.

**Look for productive strategies:**

- Recognizing they can combine like terms to solve the equation.

## 3 Connect

**Have students share** their responses to Problem 3 and 4.

**Highlight** that this equation, such as the one from Activity 1, has multiple variable terms. These can be combined together to create an equivalent equation with fewer terms. After this step, students can reason abstractly as they use the Properties of Equality to solve the equation. Consider displaying the Anchor Chart PDF, *Properties of Equality* to support this discussion.

**Ask:**

- "How did you use the answer from Problem 5 to find the number of coins the other donors gave?"
- "How do the equations from today differ from the equations from previous units or grades? How are they similar?"

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Chunk this task into more manageable parts. After students complete Problem 2, provide feedback before they continue to Problems 3 and 4. For example, ask, "What do you notice about the relationship between the expressions for the first donor and the other donors?"

### Accessibility: *Guide Processing and Visualization*

Have students color-code information in the text and subsequent expressions by using a different color to represent each donor. For example, have students color-code the text and subsequent expressions for the second donor blue. This will help students keep track of the structure of the expressions and what each term represents.

## Math Language Development

### *MLR6: Three Reads*

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand the historical context for the problem. Define the term *donor* as this term may be unfamiliar.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as "The second donor gave twice the first donor."
- **Read 3:** Have students brainstorm strategies for choosing and defining a variable to represent the number of coins given by the first donor.

### English Learners

Have students highlight key words and phrases, such as *gave twice, gave three times more than, gave four more than the first,* and *together.*

Whole Class | 5 min

# Summary

Review and synthesize the process of solving equations.

Name: ______ Date: ______ Period: ______

## Summary

**In today's lesson . . .**

You wrote expressions and equations to represent scenarios. You then worked to write the expressions into fewer terms by using the Distributive Property and combining like terms. To solve the equations, you reviewed the properties of equality to ensure your equations were equivalent.

You will continue to practice solving equations for the remainder of this unit and develop strategies which will be useful for the rest of your mathematical career.

**Reflect:**



**Lesson 2** Writing Expressions and Equations 367

## Synthesize

**Have students share** when they know to use which steps in solving. For example, ask, "When do you know to use the Distributive Property?"

**Highlight** that they will continue to practice solving equations for the rest of the unit.

**Ask**, "What are some strategies or steps you used when solving the equations?"

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What is helpful about working step by step when solving equations? What is challenging?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by writing and solving an equation based on a verbal description.

**Amps Featured Activity** Real-Time Exit Ticket

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 4.02

Bard has two cousins. One cousin is twice Bard's age and the other cousin is 2 years older than Bard. The sum of Bard's age and both cousins' ages is 22 years. Define a variable. Then write and solve an equation to determine Bard's age.

Sample response: Let $b$ represent Bard's age.

The expression $2b$ represents one cousin's age and the expression $b + 2$ represents the other cousin's age.

The sum of their ages can be represented by the equation $b + 2b + b + 2 = 22$.

$$b + 2b + b + 2 = 22$$
$$4b + 2 = 22$$
$$4b + 2 - 2 = 22 - 2$$
$$4b = 20$$
$$4b \div 4 = 20 \div 4$$
$$b = 5$$

Bard is 5 years old.

**Self-Assess**

1 I don't really get it · 2 I'm starting to get it · 3 I got it

a. I can write expressions and equations representing scenarios. 1 2 3

b. I can combine like terms to write expressions and equations in fewer terms. 1 2 3



Lesson 2 Writing Expressions and Equations

## Success looks like . . .

- **Goal:** Writing expressions and equations to represent real-world scenarios.
  - » Writing an equation to determine Bard's age.
- **Goal:** Generating an equivalent expression with fewer terms, including using the Distributive Property.
- **Goal:** Using the Properties of Equality to solve equations.
  - » Solving the equation to determine Bard's age by using the Properties of Equality.

## Suggested next steps

**If students have trouble writing the correct equation, consider:**

- Reviewing Activities 1 and 2.
- Assigning Practice Problem 3.

**If students write the correct equation but incorrectly solve the equation, consider:**

- Reviewing Activities 1 and 2.
- Assigning Practice Problems 1 and 2.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- What worked and didn't work today? The instructional goal for this lesson was to generate equivalent expressions with fewer terms. How well did students accomplish this? What did you specifically do to help students accomplish it?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

# Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. For each expression, combine like terms and write an equivalent expression with fewer terms. Show your thinking. **Sample responses shown.**

   a. $4(x-9)+6$
   $$=4(x)-4(9)+6$$
   $$=4x-36+6$$
   $$=4x-30$$

   b. $\frac{2}{3}(6x-9)-(4x+1)$
   $$=\frac{2}{3}(6x)-\frac{2}{3}(9)-4x-1$$
   $$=4x-6-4x-1$$
   $$=-7$$

   c. $3-2(x-9)+7$
   $$=3-2(x)-2(-9)+7$$
   $$=3-2x+18+7$$
   $$=28-2x$$

2. Solve each equation. Show your thinking.

   a. $-3y-9y+5y=2.1$
   $$-7y=2.1$$
   $$-7y\div-7=2.1\div-7$$
   $$y=-0.3$$

   b. $0.5x-\frac{1}{2}(4x-6)=7.5$
   $$0.5x-2x+3=7.5$$
   $$-1.5x+3=7.5$$
   $$-1.5x+3-3=7.5-3$$
   $$-1.5x=4.5$$
   $$-1.5x\div-1.5=4.5\div-1.5$$
   $$x=-3$$

3. In a basketball game, Elena scored twice as many points as Tyler. Tyler scored 4 points fewer than Noah, and Noah scored three times as many points as Mai. If Mai scored 5 points, how many points did Elena score? Explain your thinking.

   **Sample response: Let $E$, $T$, $N$, and $M$ represent the number of points Elena, Tyler, Noah, and Mai scored respectively. $E=2T$, $T=N-4$, $N=3M$, $M=5$.**

   $M=5$

   $N=3\cdot5=15$

   $T=15-4=11$

   $E=2\cdot11=22$

4. Triangle A is an isosceles triangle. One angle measures $x$ degrees and another angle measures $y$ degrees.

   a. What values could $x$ and $y$ represent? Determine three pairs of values for $x$ and $y$ that could be the angle measures of the triangle.

   **Sample response: 40°, 40°, 100°; 45°, 45°, 90°; 50°, 50°, 80°**

   b. Write an equation relating $x$ and $y$.

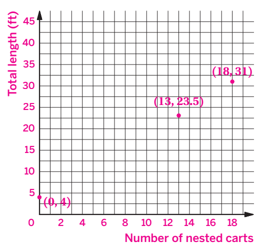
   **Sample response: $2x+y=180$**




Name: ______ Date: ______ Period: ______

Practice

5. A store is designing the space for rows of nested shopping carts. Each row has a starting cart that is 4 ft long, followed by the nested carts (so 0 nested carts means there is only the starting cart). The store measured a row of 13 nested carts to be 23.5 ft long, and a row of 18 nested carts to be 31 ft long.




   a. Create a graph of the situation. Remember to scale and label your axes.

   **Sample response shown.**

   b. How many feet does each additional nested cart add to the length of the row? Explain your thinking.

   **1.5 ft is added by each cart.**

   **Sample response: $\frac{31-23.5}{18-13}=\frac{7.5}{5}=1.5$**

   c. If the store design allows for each row of nested carts to have a maximum length of 43 ft, how many total carts can fit in a row?

   **Sample response: Let $x$ represent the number of nested carts. Let $y$ represent the length of a row of nested carts. The equation $y=4+1.5x$ represents this situation. Substitute 43 for $y$ and solve the equation for $x$.**
   $$43=4+1.5x$$
   $$43-4=4+1.5x-4$$
   $$39=1.5x$$
   $$39\div1.5=1.5x\div1.5$$
   $$26=x$$

   **A total of 27 carts (26 nested carts plus the starting cart) can fit in each row.**

6. Match each expression with an equivalent expression.

   a. $5(x-7)$

   b. $-5(x-7)$

   c. $2x-30-5-7x$

   d. $3x+2x-5(-7)$

   ...**d**... $5x+35$

   ...**a**... $5x-35$

   ...**b**... $-5x+35$

   ...**c**... $-5x-35$




## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activities 1 and 2 | 2 |
| | 2 | Activities 1 and 2 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 3 Lesson 16 | 2 |
| | 5 | Unit 3 Lesson 7 | 2 |
| Formative | 6 | Unit 4 Lesson 3 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Keeping the Balance

Let's determine unknown weights on balanced hanger diagrams.

## Focus

### Goals

1. **Language Goal:** Calculate the weight of an unknown object using a hanger diagram, and explain the solution method. **(Speaking and Listening)**

2. Comprehend that adding and removing equal items from each side of a hanger diagram or multiplying and dividing items on each side of the hanger diagram by the same amount are moves that keep the hanger balanced.

## Rigor

- Students strengthen their **conceptual understanding** of maintaining balance as one of the key strategies in solving equations.

## Coherence

### Today

Students recall a representation that they have seen in prior grades: the hanger diagram. They learn to work with hanger diagrams with variables on each side. In Activity 3, they are introduced to concepts related to infinite and no solutions, discussed formally in Lesson 7. Students use concrete quantities to develop their power of abstract reasoning about equations.

### Previously

In Lessons 1 and 2, students worked with number machines by representing them with equations and solving the equations.

### Coming Soon

Students will build on their conceptual understanding by solving equations while keeping the hanger diagram balanced.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Activity 3 | Summary | Exit Ticket |
|---|---|---|---|---|---|
| 5 min | 10 min | 10 min | 10 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

## Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair

## Math Language Development

**Review words**

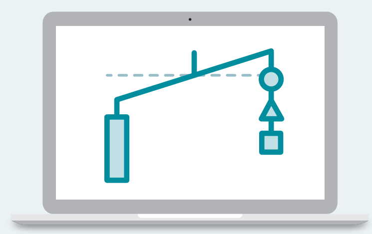
- *equivalent*
- *hanger diagram*
- *like terms*
- *Properties of Equality*
- *substitution*
- *term*

**Amps** | **Featured Activity**

**Activities 1 and 3**

### Digital Hanger Diagrams

Students manipulate digital hanger diagrams and check whether they are balanced in real time.



Amps POWERED BY desmos

## Building Math Identity and Community

Connecting to Mathematical Practices

At first, students may not immediately be able to identify the number of solutions and may want to quit before really getting started with Activity 3. Encourage students to set a goal of identifying what they do know about the diagram and then work toward a solution, one step at a time. By looking only one step ahead, a task can seem much more manageable.

## Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- Consider giving students the matches for **Activity 2** and having them focus on completing the *Possible Move* column.

Pairs | 5 min

# Warm-up What's True?

Students reason about hanger diagrams to determine what a balanced or unbalanced hanger represents.

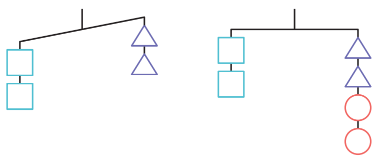
Unit 4 | Lesson 3

## Keeping the Balance

Let's determine unknown weights on balanced hanger diagrams.

### Warm-up What's True?

In the two hanger diagrams shown, all the triangles weigh the same as one another, all the squares weigh the same as one another, and all the circles weigh the same as one another.



Based on the diagrams, what is . . .

1. One thing that *must* be true?
   Sample responses:
   - A square must weigh more than a triangle.
   - A triangle and a circle must weigh the same as a square.

2. One thing that *could* be true?
   Sample response: A circle could weigh the same as a triangle.

3. One thing that *cannot* be true?
   Sample response:
   - A circle weighs more than a square.
   - A triangle weighs more than a square.

Log in to Amplify Math to complete this lesson online.

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## 1 Launch

Activate prior knowledge by asking what students remember about balance and hanger diagrams.

## 2 Monitor

**Help students get started** by asking what they notice about the first hanger diagram.

**Look for points of confusion:**

- **Thinking that a triangle and circle must be equal.** Explain that we only know the two squares are balanced with two triangles and two circles but not that a triangle is balanced with a circle.
- **Thinking that a triangle and a circle *could* equal a square.** Discuss with students that removing one square, one triangle, and one circle is removing half of the objects, which would still maintain balance and *must* be true.

**Look for productive strategies:**

- Assigning possible weights to the shapes to support their statements.

## 3 Connect

**Display** the hanger diagrams.

**Have students share** their statements of what must be, could be, or cannot be true and explain why.

**Ask:**

- "What shapes can you remove from the second diagram and still maintain the balance?" one square, one triangle, and one circle
- "If you remove one square from the first diagram, will the hanger become balanced?" That is unknown because we do not know the weight of each shape or how they compare to each other.

## Differentiated Support

**Accessibility: *Activate Prior Knowledge, Guide Processing and Visualization***

Display one of the hanger diagrams and ask students what they recall about balancing a hanger diagram. Suggest that students assign possible numerical values to each shape to help their thinking.

**Extension: *Math Enrichment***

Have students assign $x$, $y$, and $z$ to represent the weight of each square, triangle, and circle, respectively. Challenge them to write algebraic statements that represent the hanger diagrams.

Sample response: $x > y$ and $x = y + z$

## Power-up

**To power up students' ability to determine equivalent expressions, have students complete:**

Which of the following expressions are equivalent to $-3(x - 7) + 4x$? Select *all* that apply.

A. $-3x - 21 + 4x$
B. $x - 21$
C. $-3x + 21 + 4x$
D. $x + 21$
E. $7x + 21$

**Use:** Before Activity 1
**Informed by:** Performance on Lesson 2, Practice Problem 6 and Pre-Unit Readiness Assessment, Problems 1, 2, and 3

Pairs | 10 min

# Activity 1 Hanging Blocks

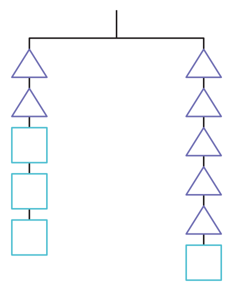
Students explain why adding or subtracting blocks from both sides of the hanger diagram will maintain balance to begin their work with solving equations.

**Amps Featured Activity** Digital Hanger Diagrams

Name: ______________________ Date: ____________ Period: ____________

## Activity 1 Hanging Blocks

The hanger diagram shown is balanced because the weight on each side is the same.



1. Which weight(s) can be removed so that the hanger remains balanced? Determine as many answers as possible.
   Sample responses:
   - Remove one or two triangles from each side.
   - Remove one square from each side.
   - Remove two triangles and one square from each side.

2. If a triangle weighs $1$ g, how much does a square weigh? Explain your thinking.
   $1.5$ g; Sample response: After removing two triangles and one square from each side, there are two squares remaining on the left side and three triangles on the right side. Each triangle weighs $1$ g, making the balance show that two squares are equal in weight to $3$ g. Dividing this weight equally among the two squares makes each square weigh $1.5$ g.

3. Determine another pair of measurements that keep the hanger diagram balanced.
   Answers may vary, but should result in $1$ square weighing the same as $1.5$ triangles.

**Are you ready for more?**

If the weight of a square is $x$ grams and the weight of a triangle is $1$ g, what equation could represent the hanger diagram?
Sample response: $3x + 2 = x + 5$





## 1 Launch

Let students know the hanger diagram is balanced, and they will perform moves to maintain that balance.

## 2 Monitor

**Help students get started** by asking, "If you remove a triangle from the left side, what do you need to do on the right side to maintain balance?" Consider giving an incorrect answer to help students' process.

**Look for points of confusion:**

- **Thinking they can only remove weights from the bottom.** Let students know that if a weight is removed, then the others are rehung on the hanger diagram.
- **Thinking that removing one shape from the right side will be balanced because there are 5 shapes on each side.** Remind students each shape has a different weight.

**Look for productive strategies:**

- Drawing on the diagram or redrawing the diagram to show removed pieces or known weights.
- Representing the hanger diagram with an equation.

## 3 Connect

**Highlight** that it is acceptable to add or remove blocks of the same "size" from both sides of the hanger diagram. The sides will still be balanced and the resulting hanger diagram is equivalent to the starting diagram.

**Have students share** their responses and reasoning while displaying the hanger diagram. Start with students who first assigned a value to the triangles and end with students who removed equivalent weights and then assigned a value to the triangles in Problem 2. For Problem 3, discuss any proportional reasoning they used.

**Ask**, "How do you know if your move will keep the hanger diagram balanced?"

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Represent the same information through different modalities by using concrete representations. For example, create a physical model of the hanger diagrams by using a clothes hanger and weighted objects. Highlight how the weights of objects on either side impact whether the hanger is balanced or unbalanced.

### Accessibility: *Vary Demands to Optimize Challenge*

For Problem 3, consider providing a weight for one square and having the students determine the corresponding weight of the triangle. For example, provide the weight of a square as 6 grams. Students would then determine the weight of the triangle to be 4 grams.

## Math Language Development

### *MLR8: Discussion Supports — Press for Reasoning*

During the Connect, amplify mathematical language that explains how to balance the hanger diagram. Press for reasoning by asking students "How do you know the hanger diagram is balanced?" Highlight any proportional reasoning students use and emphasize words and phrases, such as *balance, same size*, and *equal weights*.

### English Learners

As students discuss which weights can be removed, mark up the diagram to show connections between student descriptions of maintaining balance and the visual diagram.

Pairs | 10 min

# Activity 2 Card Sort: Hanger Diagrams

Students match two equivalent hanger diagrams and describe the possible move to turn one diagram into the other.

## Activity 2 Card Sort: Hanger Diagrams

**You will be given a set of cards. Each card contains a hanger diagram.**

1. For each card listed in the "Hanger 1" column of the table, match its hanger diagram with an equivalent hanger diagram. Record the card number in the "Hanger 2" column of the table.
2. Describe a possible move or moves that can be applied to Hanger 1 so that it will look like Hanger 2.

| Hanger 1 | Hanger 2 | Possible Move |
|---|---|---|
| Card 1 | Card 10 | Remove one triangle from each side. |
| Card 2 | Card 6 | Create two equivalent groups where two triangles on the left side balances one square and one circle on the right. Then remove one set from both sides. |
| Card 3 | Card 8 | Remove one pentagon and one square from each side. |
| Card 4 | Card 7 | Add one triangle, one square, and one pentagon to each side. |
| Card 5 | Card 9 | Triple the number of shapes on each side. |

**Are you ready for more?**

Bard has 24 pencils and 3 cups. The second cup holds one more pencil than the first cup. The third cup holds one more pencil than the second cup. How many pencils does each cup hold? Explain your thinking.

Sample response: Let $x$ represent the number of pencils in the first cup. Then $x+1$ represents the number of pencils in the second cup, and $x+2$ represents the number of pencils in the third cup; therefore, $x+x+1+x+2=24$.

$3x+3=24$; So, $x=7$; the first cup holds 7 pencils, the second cup holds 8 pencils, and the third cup holds 9 pencils.

### 1 Launch

Provide the pre-cut cards from the Activity 2 PDF for each pair of students.

### 2 Monitor

**Help students get started** by suggesting they match the cards first before writing the possible move.

**Look for points of confusion:**

- **Explaining Card 2 to Card 6 as doubling (or switching any card pair).** Remind students they are finding the possible moves from the card listed under Hanger 1 to make the card listed under Hanger 2.

**Look for productive strategies:**

- Using mathematically precise language, such as "subtract one triangle from each side" or "multiply both sides by 3."

### 3 Connect

**Display** any necessary cards to help with the discussion.

**Have students share** their matches and possible moves to turn Hanger 1 into Hanger 2.

**Highlight** mathematically precise language for the possible moves. Review the possible moves used: adding blocks, subtracting blocks, increasing by the same multiple, and grouping the blocks to remove redundancy.

**Ask**, "Are there any additional moves which could be made and which still keep the hanger diagrams balanced?"

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge***

Provide students with the card matches for each card. This will allow students to focus on writing their descriptions for the possible moves.

**Accessibility: *Guide Processing and Visualization***

Suggest that students color code each shape to help them determine the matches. For example, color all of the squares blue, the triangles red, and the circles yellow.

**Extension: *Math Enrichment***

Have students create their own pair of equivalent hanger diagrams according to their own descriptions of possible moves. Challenge them to incorporate two or three different types of possible moves, instead of just one.

Pairs | 10 min

# Activity 3 More Hanging Blocks

Students reason about hanger diagrams and find unknown weights to begin the discussion on one, infinitely many, and no solutions.

**Amps Featured Activity** Digital Hanger Diagrams

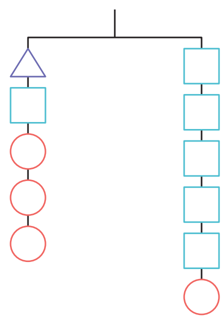
Name: ______ Date: ______ Period: ______

## Activity 3 More Hanging Blocks

Consider the following hanger diagrams. Each triangle weighs 3 g, and each circle weighs 6 g.

1. Determine the weight of 1 square. Show or explain your thinking.

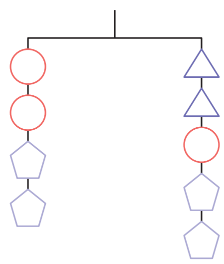
   3.75 g; Sample response: By removing like shapes from each side of the diagram, I can determine the weight of a square based on the remaining shapes. First, remove 1 square and 1 circle from each side. This leaves the left side with 1 triangle and 2 circles for a total of 15 g, and the right side with 4 squares. Dividing 15 g evenly among the 4 squares means each square must weigh 3.75 g.



2. Determine the weight of 1 pentagon. Show or explain your thinking.

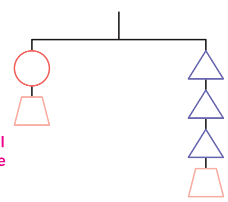
   Any amount; Sample response: First I would remove 1 circle from each side to reduce the number of shapes. That leaves 1 circle and 2 pentagons on the left side, and 2 triangles and 2 pentagons on the right side. Because 1 circle weighing 6 g equals 2 triangles weighing 3 g each, the circle and triangles can also be removed.

   The remaining 2 pentagons will always be equal in weight to two identical pentagons; therefore, 1 pentagon can weigh any amount.



3. Determine the weight of 1 trapezoid. Show or explain your thinking.

   Sample response: The circle on the left side weighs 6 g, so I can remove it, along with 2 triangles weighing a total of 6 g from the right side. This leaves 1 trapezoid on the left side, and 1 triangle and 1 trapezoid on the right side. The identical trapezoids should weigh the same amount, but the right side has an additional 3 g from the triangle, which means this hanger diagram is not actually balanced.



STOP





### 1 Launch

Use the *Think-Pair-Share* routine as students work through each problem.

### 2 Monitor

**Help students get started** by asking if there are any shapes that can be removed but still maintain balance.

**Look for points of confusion:**

- **Thinking that triangles weigh 1 gram as in the previous activity.** Have students read and highlight the weights in the directions.

**Look for productive strategies:**

- Removing shapes first before substituting numerical values for the known shapes.
- Using an equation to represent the hanger diagrams.

### 3 Connect

**Display** the hanger diagrams.

**Have students share** strategies for finding the unknown weight without using equations. Ask students to be clear how they are changing each side of the hanger diagram.

**Highlight** how removing shapes before substituting in numerical values can help make the process more efficient. Explain the square in Problem 1 has only one value for the hanger diagram to stay balanced, the pentagon in Problem 2 can be any weight and the hanger diagram will always be balanced, and the diagram in Problem 3 will never be balanced. **Note:** The number of possible solutions will be formalized in Lesson 7.

**Ask**, "How can you change the known weights in Problem 3 so that you can determine the weight of the trapezoid?" You will not be able to determine the weight of the trapezoids because they balance with each other and can be any value regardless of the weight of the other shapes.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Have students focus on Problem 1. Consider providing a simplified hanger diagram, with fewer shapes. Encourage students to label the values they know, and then determine the weight of 1 square.

### Extension: *Math Enrichment*

Have students complete the following as a follow-up to Problem 3: If the weight of a square is $a$ grams, the weight of a pentagon is $b$ grams, and the weight of a trapezoid is $c$ grams, write an equation that could represent each hanger diagram.

1. $1a + 21 = 5a + 6$
2. $2b + 12 = 2b + 12$
3. $1c + 6 = 1c + 9$

## Math Language Development

### *MLR1: Stronger and Clearer Each Time*

After students complete Problem 1, use this routine to support students in crafting a well written explanation. Say, "Explain how you determined the weight of 1 square." Give students time to individually write an initial draft of their response. Have them meet with 2–3 partners to both give and receive feedback. Encourage partners to ask clarifying questions and invite students to write a final draft based on the feedback.

### English Learners

Allow students to partner with at least one peer who speaks the same primary language. This will give students an opportunity to clarify feedback in their primary language as they work to improve their draft response.

Whole Class | 5 min

# Summary

Review and synthesize how to maintain balance in the hanger diagrams.

## Summary

**In today's lesson . . .**

You balanced hanger diagrams and saw that adding or removing the same amount from each side kept the diagram balanced. You also saw that multiplying or dividing each side by the same amount kept the resulting hanger diagram balanced.

In the next lessons, you will connect this idea of balance to solving equations.

**Reflect:**

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## Synthesize

**Highlight** that, if a possible move is done on the left side of the hanger diagram, it must also be done on the right side so that the hanger diagram maintains its balance.

**Ask**, "How can you determine the value of an unknown weight in a hanger diagram?"

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do you make sure the hanger diagram maintains balance?"

Independent | 5 min

# Exit Ticket

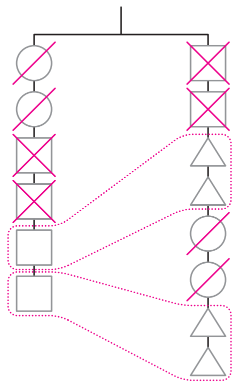
Students demonstrate their understanding by finding an unknown weight on a balanced hanger diagram.

Printable

Name: ______________ Date: ________ Period: ________

## Exit Ticket 4.03

A balanced hanger diagram is shown. The weight of the square is 8 g. What could the weights of the triangle and circle be? Explain your thinking.



Sample response: Removing two circles and two squares from each side leaves two squares on the left side and four triangles on the right side. Therefore, one square could balance with two triangles. This means that 8 grams balances with two triangles, so each triangle must weigh 4 grams. The circle could weigh any amount because the same number of circles are on each side, canceling each other out.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can add or remove like shapes from a hanger diagram to keep it balanced.
1 2 3

b I can multiply or divide each side of a hanger diagram by the same amount to keep it balanced.
1 2 3





## Success looks like . . .

- **Language Goal:** Calculating the weight of an unknown object using a hanger diagram, and explaining the solution method. **(Speaking and Listening)**
  - » Determining the weights of the triangle and circle in the balanced hanger diagram.
- **Goal:** Comprehending that adding and removing equal items from each side of a hanger diagram or multiplying and dividing items on each side of the hanger diagram by the same amount are moves that keep the hanger balanced.
  - » Explaining how to determine the weights of the triangle and circle by first removing equal items from each side.

## Suggested next steps

**If students find that one square balances with two triangles but do not determine the weight of the triangle, consider:**

- Highlighting that a square weighs 8 grams.
- Reviewing Activity 1.
- Reviewing Problem 1 from Activity 3.
- Assigning Practice Problems 1 and 3.

**If students determine the weight of the triangle but not the weight of the circle, consider:**

- Reminding them the question asked for the weight of both the triangle and circle.
- Asking if the weight of the circle could be 5 grams, 3 grams, or any other amount.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- What worked and didn't work today? What did students find frustrating about Problem 3 of Activity 2? What helped them work through this frustration?
- In this lesson, students balanced hanger diagrams. How will that support their work of solving equations? What might you change for the next time you teach this lesson?

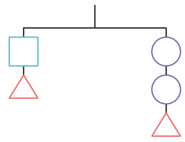
# Practice

Independent

Name: ______ Date: ______ Period: ______

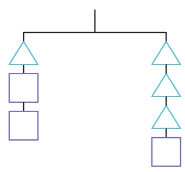
Practice

1. Which of these moves would keep the hanger diagram in balance? Select *all* that apply.



   - A. Add 2 circles to the left side and 1 square to the right side.
   - B. Add 2 triangles to each side.
   - C. Add 1 triangle to the left side and 1 square to the right side.
   - D. Remove 2 circles from the right side and 1 square from the left side.
   - E. Add 1 circle to the left side and 1 square to the right side.

2. What is the weight of 1 square if a triangle weighs 4 g? Explain your thinking.



   **8 g; Sample response: Removing 1 triangle and 1 square from each side leaves the left side with 1 square and the right side with 2 triangles. If each triangle weighs 4 g, the right side weighs 8 g, making one square's weight equivalent to 8 g.**

3. Solve each equation. Show your thinking.

   a. $5(2x-3)=-10$

   $$10x-15=-10$$
   $$10x-15+15=-10+15$$
   $$10x=5$$
   $$x=\frac{5}{10}$$
   $$x=\frac{1}{2}$$

   b. $4x-5(3x-2)=10$

   $$4x-15x+10=10$$
   $$-11x+10=10$$
   $$-11x+10-10=10-10$$
   $$-11x=0$$
   $$-11x\div -11=0\div -11$$
   $$x=0$$





Name: ______ Date: ______ Period: ______

Practice

4. Andre created this puzzle: "I am three years younger than my brother, and I am two years older than my sister. My mom's age is one less than three times my brother's age. When you add all of our ages, you get 87. What are our ages?"

   a. To solve the puzzle, Jada writes the equation: $(x)+(x+3)+(x-2)+3(x+3)-1=87$. What does the variable $x$ represent in this scenario? Explain which parts of the equation represent Andre's brother's, sister's, and mother's ages.

   **$x$ represents Andre's age. $(x+3)$ represents his brother's age, $(x-2)$ represents his sister's age, and $3(x+3)-1$ represents his mother's age.**

   b. Write an equivalent equation in fewer terms.

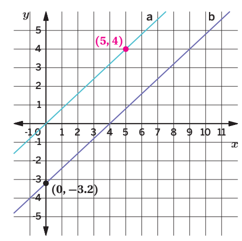
   **Sample response: $6x+9=87$**

   c. Solve the equation. How old is each member of Andre's family?

   $$6x+9=87$$
   $$6x+9-9=87-9$$
   $$6x=78$$
   $$6x\div 6=78\div 6$$
   $$x=13$$

   **Andre is 13 years old, his brother is 16 years old, his sister is 11 years old, and his mother is 47 years old.**

5. These two lines are parallel. Write an equation for each line.




   **The slope of each line is $\frac{4}{5}=0.8$. Line a passes through the origin, while line b has a $y$-intercept of $-3.2$.**

   **Line a: $y=0.8x$**

   **Line b: $y=0.8x-3.2$**

6. Consider each statement.

   a. If the expression $6x+9$ is divided by 3, which expression would be the result?

   A. $6x+3$ B. $2x+9$ C. $2x+3$ D. $x+3$

   b. If the expression $3(6x+9)$ is divided by 3, which expression would be the result?

   A. $2x+3$ B. $6x+9$ C. $2x+9$ D. $6x+3$





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 2 |
| Spiral | 3 | Unit 4 Lesson 2 | 1 |
| | 4 | Unit 4 Lesson 2 | 2 |
| | 5 | Unit 3 Lesson 12 | 2 |
| Formative | 6 | Unit 4 Lesson 4 | 2 |

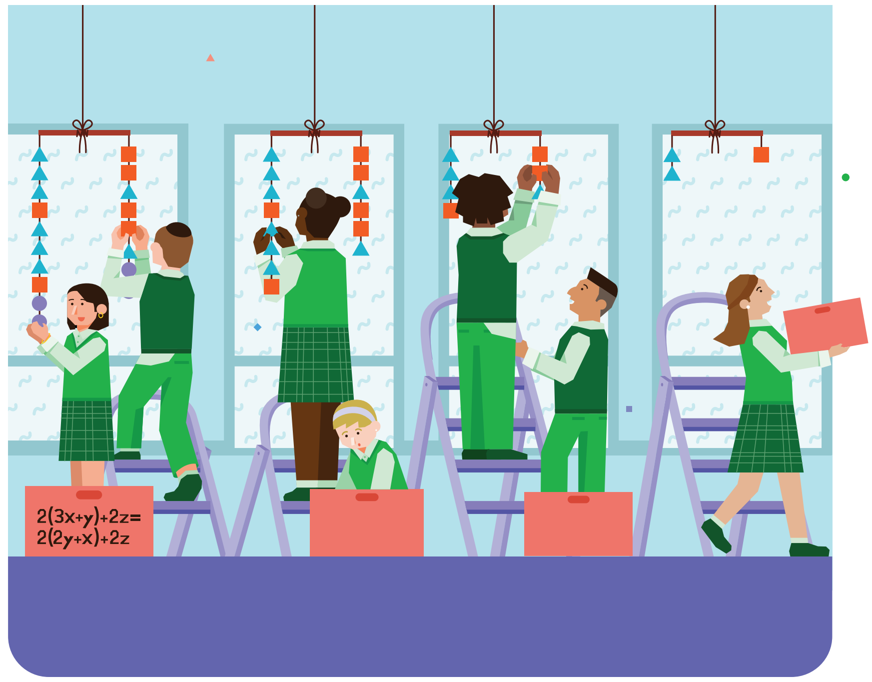
**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Balanced Moves (Part 1)

Let's rewrite equations, while keeping the same solutions.




## Focus

### Goals

1. **Language Goal:** Compare and contrast solution paths to solve an equation in one variable by performing the same operation on each side. **(Speaking and Listening, Writing)**
2. **Language Goal:** Correlate changes on hanger diagrams with steps that create equivalent equations. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students move from using hanger diagrams to using equations to represent a problem. They see how moves that maintain the balance of a hanger diagram correspond to steps that maintain the equality of an equation, such as halving the value of each side or subtracting the same unknown value from each side. Students reason about the equation which represents the hanger diagram and about the steps in solving an equation.

### Previously

In Lesson 3, students made possible moves to keep hanger diagrams balanced and found unknown quantities of weights.

### Coming Soon

In Lesson 5, students will focus on solving equations with variables on both sides.

## Rigor

- Students build **conceptual understanding** of solving equations by relating it to keeping the hanger diagram balanced.
- Students **apply** the work they did with hanger diagrams to the process of solving equations.

## Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 20 min | 10 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

**Practice** Independent

### Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Properties of Operations*
- Anchor Chart PDF, *Properties of Equality*

### Math Language Development

**Review words**

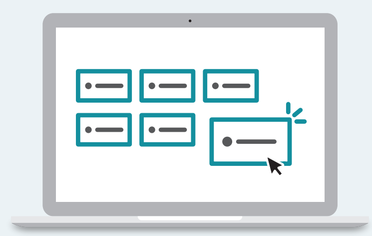
- *coefficient*
- *constant*
- *Distributive Property*
- *equation*
- *equivalent equations*
- *expression*
- *like terms*
- *Properties of Equality*
- *solution*
- *substitution*
- *term*
- *variable*

**Amps** | **Featured Activity**

**Activity 2**

### Digital Card Sort

Students match pairs of equations with the corresponding step that produces the second equation from the first equation.



Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Without a hanger diagram, students might feel that the task in Activity 2 is too difficult or even impossible. Encourage students to manage their stress levels by decontextualizing the processes used with the hanger diagram to those used with the equation. To stay organized and to visualize what they are doing, students might want to actually create and use a hanger diagram.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, consider writing the equations in Problem 2 together as a whole class.

Independent | 5 min

## **Warm-up** What's Being Represented?

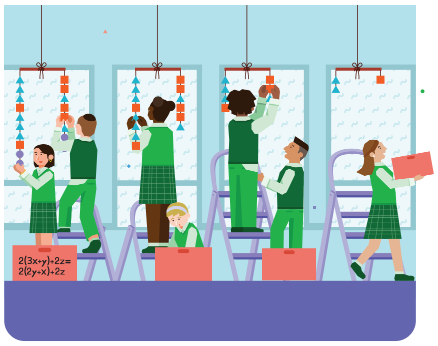
Students represent each side of the hanger diagram with expressions to begin their work of solving equations.

Name: ______ Date: ______ Period: ______

Unit 4 | Lesson 4

### Balanced Moves (Part 1)

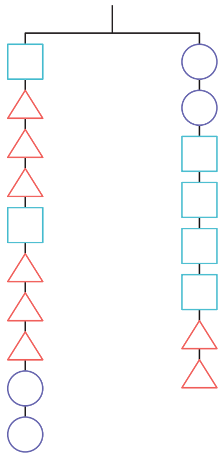
Let's rewrite equations, while keeping the same solutions.




#### **Warm-up** What's Being Represented?

**Refer to the hanger diagram.**

Let $x$ represent the weight of $1$ square, $y$ represent the weight of $1$ triangle, and $z$ represent the weight of $1$ circle. Describe the hanger diagram using these variables.



**Sample responses (listed in order of a suggested way of how to display them during the class discussion during the Connect):**

- **On the left side, there are 2 $x$s, 6 $y$s, and 2 $z$s. On the right side, there are 2 $z$s, 4 $x$s, and 2 $y$s.**
- **On the left side, the expressions are $2x$, $6y$, and $2z$. On the right side, the expressions are $2z$, $4x$, and $2y$.**
- **The left is represented by the expression $x+y+y+y+x+y+y+y+z+z$, and the right side is represented by the expression $z+z+x+x+x+x+y+y$.**
- **The left is represented by the expression $2x+6y+2z$, and the right is represented by the expression $2z+4x+2y$.**
- **An equation that represents this hanger diagram is $2x+6y+2z=2z+4x+2y$.**
- **An equation that represents this hanger diagram is $2(x+3y)+2z=2z+4x+2y$.**

Log in to Amplify Math to complete this lesson online.



Lesson 4 Balanced Moves (Part 1) 377

### 1 Launch

Set an expectation for the amount of time that students will have to work individually on the activity.

### 2 Monitor

**Help students get started** by asking them to describe the shapes on one side of the hanger diagram. Write down what they say, and then have them replace the word *square* with the variable $x$.

**Look for points of confusion:**

- **Wanting to remove shapes from both sides of the hanger diagram.** Let students know they will be able to do this in the next activity, but right now they should describe each side separately.

**Look for productive strategies:**

- Representing the balanced hanger diagram with an equation. Have these students share their responses last during the discussion.

### 3 Connect

**Display** the hanger diagram.

**Have students share** their responses. Start with students who used phrases, followed by students who wrote expressions, and end with students who used equations to represent the balanced hanger diagram.

**Highlight** the many representations for the hanger diagram and how like shapes can be combined similarly to the like terms in Lesson 2. The many equations that the students created are considered *equivalent equations*.

**Ask**, "What could be done to both sides of this hanger diagram so it would still be balanced?"

**Note:** If a student writes the equation $2(x+3y)+2z=2z+4x+2y$, have them share last to segue to Problem 1 of Activity 1.

### Math Language Development

#### *MLR7: Compare and Connect*

During the Connect, have pairs of students share and compare their responses. As students discuss, highlight those who used phrases compared to those who used expressions or equations. Ask:

- "How does the expression $x+x$ compare to the combined term $2x$?"
- "How does the verbal phrase 'there are 2 $x$s, 6 $y$s, and 2 $z$s on the left' compare with the expression $2x+6y+2z$? Why are these terms added?"

#### English Learners

Annotate the shapes on the hanger diagram with their corresponding variables to make connections between the phrases and expressions or equations.

### Power-up

**To power up students' ability to write equivalent expressions involving division, have students complete:**

Andre solves the equation $5(x-10)=35$ by dividing both sides by 5.

**a.** Show Andre's next step.

$x-10=7$

**b.** Finish solving Andre's equation.

$x-10+10=7+10$

$x=17$

**Use:** Before Activity 1

**Informed by:** Performance on Lesson 3, Practice Problem 6

Pairs | 20 min

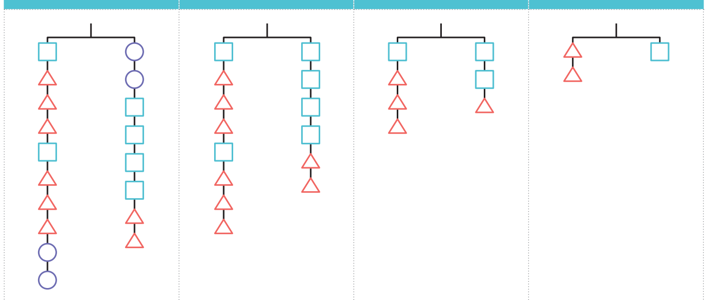
# Activity 1 Matching Hangers

Students revisit the hanger diagram from the Warm-up to connect possible moves with hanger diagrams to possible next steps with equivalent equations.

### Activity 1 Matching Hangers

Hanger Diagrams 2, 3, and 4 show the result of simplifying the previous hanger diagram by removing equal weights from each side.

| Diagram 1 | Diagram 2 | Diagram 3 | Diagram 4 |
|---|---|---|---|



| Equation 1 | Equation 2 | Equation 3 | Equation 4 |
|---|---|---|---|
| $2(x + 3y) + 2z = 2z + 4x + 2y$ | $2(x + 3y) = 4x + 2y$ | $x + 3y = 2x + y$ | $2y = x$ |

1. How does Equation 1 represent Diagram 1? Recall that $x$, $y$, and $z$ represent the weight of a square, triangle, and circle, respectively.
   **Sample response: The left side has 2 groups of 1 square and 3 triangles, which is represented by the expression $2(x + 3y)$, and there are 2 circles, which is represented by the term $2z$. The right side has 2 circles, 4 squares, and 2 triangles, which is represented by the expression $2z + 4x + 2y$.**

2. Write an equation for the remaining hanger diagrams in the table.
   **Sample responses shown in the table.**

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## 1 Launch

Give pairs a few minutes to think about Problem 1. Discuss it as a class to ensure everyone understands Equation 1 is a possible representation of Diagram 1.

## 2 Monitor

**Help students get started** by asking what changed from Diagram 1 to Diagram 2.

**Look for points of confusion:**

- **Writing Equation 2 as $2x + 6y = 4x + 2y$.** Although this is correct, students may have trouble determining the possible move to Equation 3. Review factoring by finding the greatest common factor.

**Look for productive strategies:**

- Using mathematically precise language to describe the moves, specifically in terms of describing variable terms instead of shapes.

**Activity 1 continued >**

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge***

Provide students with a list of possible equations for Hanger Diagrams 2, 3, and 4. Have students match the equations with the appropriate diagrams. This will allow students to focus on making connections between the symbols and the structure of the equivalent equations.

**Extension: *Math Enrichment***

If students complete the *Are you ready for more?* activity, challenge them to create their own cryptarithmetic puzzle. While not all of the digits $0–9$ must be used, each digit can only represent one letter. Have students create their cryptarithmetic puzzles and trade them with a partner to try to solve.

Pairs | 20 min

# Activity 1 Matching Hangers (continued)

Students revisit the hanger diagram from the Warm-up to connect possible moves with hanger diagrams to possible next steps with equivalent equations.

Name: ______________ Date: ________ Period: ________

## Activity 1 Matching Hangers (continued)

3. Explain what operation(s) were performed on each equation to create the next equation. Consider referencing the changes in the hanger diagrams to help with your thinking.

| Equation 1 to 2 | Equation 2 to 3 | Equation 3 to 4 |
|---|---|---|
| Sample response: $2z$ was subtracted from each side of the equation. | Sample response: Each side was divided by 2. | Sample response: A $y$-term and an $x$-term were subtracted from each side. |

4. If the weight of 1 square is 6 g, what is the weight of 1 triangle? Which equation or diagram did you use to find this value?
Sample response: Using the equation $2y = x$ and substituting 6 for $x$ gives the equation $2y = 6$. This equation can be solved by dividing each side by 2, giving the solution $y = 3$. Therefore, the weight of 1 triangle is 3 g.

**Are you ready for more?**

In a cryptarithmetic puzzle, the digits 0–9 are represented with letters of the alphabet. Use your understanding of addition to find which digits represent the letters A, B, E, G, H, L, N, and R if the following statement is true.

HANGER + HANGER + HANGER = ALGEBRA
920614 + 920614 + 920614 = 2761842





## 3 Connect

**Have students share** their possible moves. Start with students who described moving shapes (i.e., "remove two circles"), and end with students who described the moves in terms of variable expressions (i.e., "subtract $2z$").

**Highlight** how the moves in the hanger diagrams relate to the steps in the equations. Display the Anchor Chart PDF, *Properties of Equality* to show adding or subtracting the same terms, or multiplying or dividing by the same value, on each side keeps the hanger diagram and the equation in balance.

**Ask:**

- "Why is it acceptable to halve both sides when moving from Equation 2 to 3 when that step removes different objects from each side?"
- "If you substitute 6 for every $x$ in Equation 2 or 3, will you get the same answer as when you substitute that same value in Equation 4? Why?"

Pairs | 10 min

# Activity 2 Matching Equation Moves

Students reason about pairs of equations to identify possible next steps in the solving process.

**Amps Featured Activity** Digital Card Sort

**Activity 2** Matching Equation Steps

The following shows a series of equations and possible moves or steps.

1. Match each set of equations with a possible step that turns the first equation into the second equation.
**Note:** You may not have a matching equation for every possible step listed.

| Equations | | Possible Steps |
|---|---|---|
| a | $3x + 7 = 5x$<br>$7 = 2x$ | c Divide each side by $-3$. |
| b | $-\frac{5x}{3} = 12$<br>$5x = -36$ | a Subtract $3x$ from each side. |
| c | $-3(4x - 3) = -15$<br>$4x - 3 = 5$ | d Add $3x$ to each side. |
| d | $4 - 3x = 12x$<br>$4 = 15x$ | e Subtract 3 from each side. |
| e | $10 - 6x = 4 + 5x$<br>$7 - 6x = 1 + 5x$ | ____ Multiply each side by 3. |
| f | $12x + 3 = 6$<br>$4x + 1 = 2$ | f Divide each term by 3. |
| | | b Multiply each side by $-3$. |

**Critique and Correct:** Your teacher will display an incorrect statement. Work with your partner to critique the statement, write a corrected statement, and clarify how and why you corrected it.

STOP

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## 1 Launch

Let students know this activity does not use hanger diagrams, but if they need help, they can create a hanger diagram to represent some of the equations.

## 2 Monitor

**Help students get started** by asking what is missing from the first pair of equations and how that step could be done.

**Look for points of confusion:**

- **Switching answer choices for part a and part e.** Have students carefully look at which terms are changing from the first equation to the second equation.
- **Switching answers for part b and part c.** Have students perform the selected operations on the equation and determine whether they get the second equation.

**Look for productive strategies:**

- Focusing on the step to turn 12 into $-36$ for the equations in part b.

## 3 Connect

**Display** any equation pairs needed to help with the discussion, and have students share their matches and reasoning. Consider displaying the Anchor Chart PDF, *Properties of Equality*.

**Ask**, "Were there any steps that left you wondering why that step was taken?" Some students may wonder why, in part e, 3 was subtracted or, in part f, why it was divided by 3 because they do not work towards isolating the variable.

**Highlight** that the Properties of Equality are used in every set of equations. The operation performed on the left side is also done on the right side to maintain equality. Although some steps are possible, such as the ones in part e and part f, they may not be helpful in solving equations.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, limit the number of pairs of equations so students focus on the sets of equations in parts a, b, c, and d first. If time permits, encourage students to complete the other sets of equations.

### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can match pairs of equations with the corresponding step that produces the second equation from the first equation.

## Math Language Development

### *MLR3: Critique, Correct, Clarify*

During the Connect, display a partially incorrect statement, such as "When you add to both sides, it is the same." Ask:

- ***Critique:*** "Do you agree with this statement? Why or why not?" Prompt students to consider cases with positive and negative numbers, as well as fractions.
- ***Correct:*** "Write a revised statement that is correct and clearer."
- ***Clarify:*** "How did you revise the statement? How can you verify that your statement is correct?"

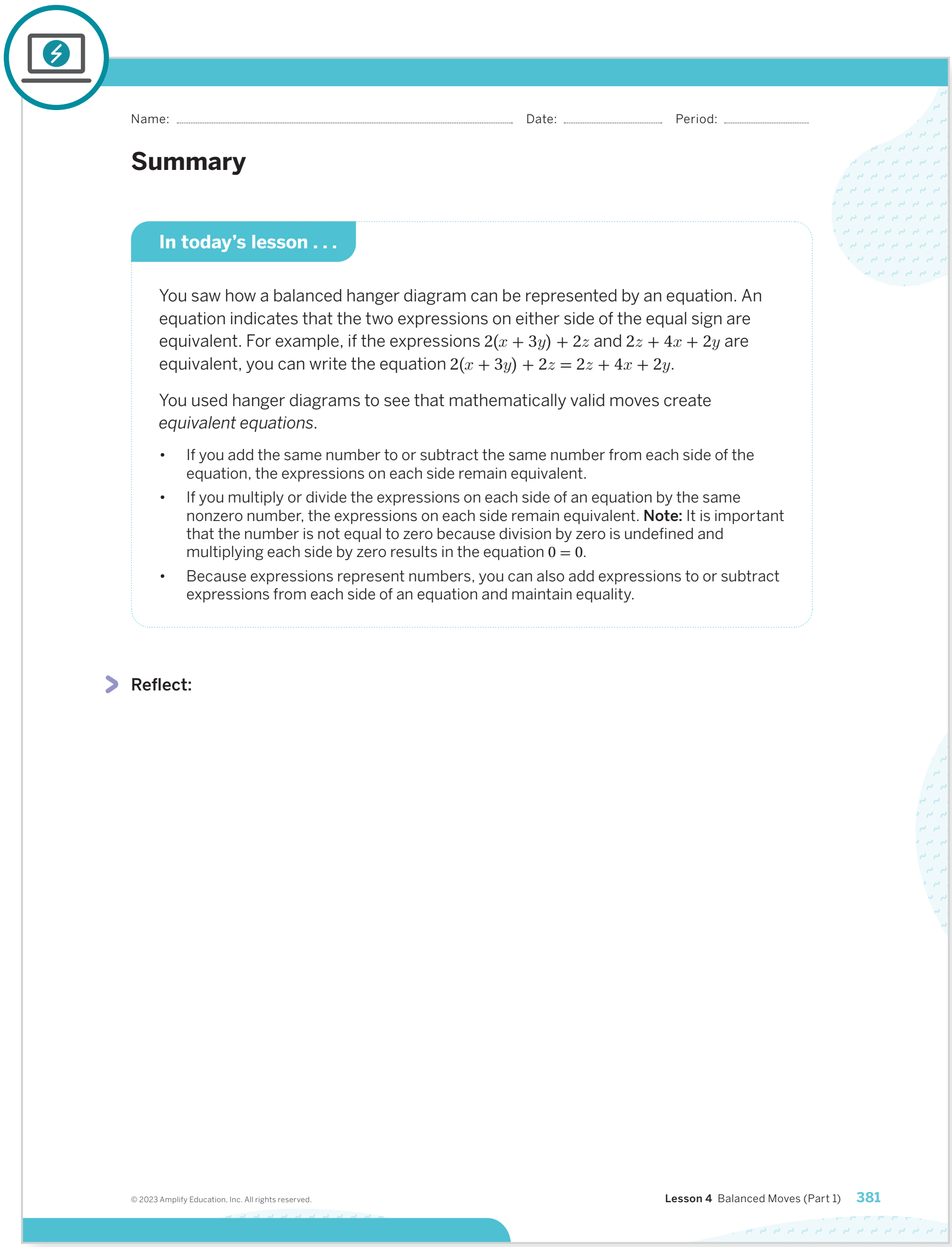
### English Learners

Allow students to share their revised statements with a partner before sharing with the whole class.

Whole Class | 5 min

# Summary

Review and synthesize the possible next steps in solving an equation.



Name: Date: Period:

**Summary**

**In today's lesson . . .**

You saw how a balanced hanger diagram can be represented by an equation. An equation indicates that the two expressions on either side of the equal sign are equivalent. For example, if the expressions $2(x + 3y) + 2z$ and $2z + 4x + 2y$ are equivalent, you can write the equation $2(x + 3y) + 2z = 2z + 4x + 2y$.

You used hanger diagrams to see that mathematically valid moves create *equivalent equations*.

- If you add the same number to or subtract the same number from each side of the equation, the expressions on each side remain equivalent.
- If you multiply or divide the expressions on each side of an equation by the same nonzero number, the expressions on each side remain equivalent. **Note:** It is important that the number is not equal to zero because division by zero is undefined and multiplying each side by zero results in the equation $0 = 0$.
- Because expressions represent numbers, you can also add expressions to or subtract expressions from each side of an equation and maintain equality.

**Reflect:**

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## Synthesize

**Display** the equation $6x + 12 = 10x - 4$.

**Have students share** possible steps they could make to maintain equality.

**Ask:**

- "How do you know when a possible move is a mathematically valid step?"
- "Is multiplying both sides by $0$ a valid step?" It will maintain equality but will cause all terms to become $0$, which is unhelpful.
- "What is the goal when solving an equation? How do you choose your steps based on that goal?" The goal is to isolate the variable and to find the solution of the equation, which is the value that makes the equation true. Steps can be chosen to isolate the variable.

**Highlight** that there are many possible steps, such as subtracting $6x$ from both sides, adding $4$ to both sides, dividing both sides by $2$, to name a few. The Properties of Equality are used to maintain equality (or balance). To help determine a possible step, students can use the terms presented in the problem. For instance, the equation adds $12$ on the left side. If the goal is to remove this term, it makes sense to subtract $12$ from both sides. Students should try to decide on steps which will isolate the variable. A more formal algorithm will be defined in Lesson 5.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Thinking about your work with hanger diagrams, how can you manipulate an equation with variables on both sides?"

# Exit Ticket

Students demonstrate their understanding by analyzing the structure of the equation pairs to determine the possible step.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 4.04

**Match each set of equations with a possible step that turns the first equation into the second equation.**

**Equations**

a. $2(6x - 3) = 4x$
$6x - 3 = 2x$

b. $6x - 3 = 2x$
$4x - 3 = 0$

c. $4x - 3 = 0$
$4x = 3$

d. $4x = 3$
$x = \frac{3}{4}$

**Possible Steps**

b Subtract $2x$ from each side.

d Divide each side by 4.

a Divide each side by 2.

c Add 3 to each side.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can represent balanced hanger diagrams with equations.
1 2 3

b. I can perform operations on each side of an equation that results in an equivalent equation with fewer terms.
1 2 3




## Success looks like . . .

- **Language Goal:** Comparing and contrasting solution paths to solve an equation in one variable by performing the same operation on each side. **(Speaking and Listening, Writing)**
  - » Matching each set of equations with the step that turns the first equation into the second equation.
- **Language Goal:** Correlating changes on hanger diagrams with steps that create equivalent equations. **(Speaking and Listening, Writing)**

## Suggested next steps

**If students mismatch the possible steps with the equations, consider:**

- Reviewing Activity 2.
- Having them draw a hanger diagram to represent the equations and discuss the possible moves.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- What worked and didn't work today? What challenges did students encounter as they worked on Problem 2 from Activity 1? How did they work through them?
- How did matching possible moves set students up to develop strategies for solving equations, particularly with variables on both sides? What might you change for the next time you teach this lesson?

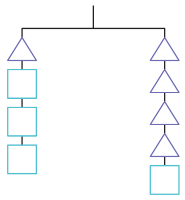
# Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

**1.** In this balanced hanger diagram, the weight of the triangle is $x$ and the weight of the square is $y$.



**a** Write an equation using $x$ and $y$ to represent the hanger diagram.

**Sample response: $x + 3y = 4x + y$**

**b** If $x = 6$, what is the value of $y$? Explain your thinking.

**$y = 9$; Sample response: Remove one triangle and one square from each side, leaving 2 squares on the left side and 3 triangles on the right side. Because each triangle weighs 6, the remaining weight on the right side is 18. Dividing the 18 equally among the two squares makes each square have a weight of 9.**

**2.** Andre and Diego were each trying to solve $2x + 6 = 3x - 8$. Describe the first step they each made to the equation.

**a** The result of Andre's first step was $-x + 6 = -8$.

**Sample response: Andre subtracted $3x$ from each side.**

**b** The result of Diego's first step was $6 = x - 8$.

**Sample response: Diego subtracted $2x$ from each side.**

**3.** Match each set of equations with a possible step that turns the first equation into the second equation.

| Equations | Possible Steps |
|---|---|
| **a** $6x + 9 = 4x - 3$, $2x + 9 = -3$ | **b** Divide each side by $-4$. |
| **b** $-4(5x - 7) = -18$, $5x - 7 = 4.5$ | **d** Multiply each side by $-4$. |
| **c** $8 - 10x = 7 + 5x$, $4 - 10x = 3 + 5x$ | **e** Divide each side by $4$. |
| **d** $-\frac{5}{4}x = 4$, $5x = -16$ | **a** Subtract $4x$ from each side. |
| **e** $12x + 4 = 20x + 24$, $3x + 1 = 5x + 6$ | **c** Subtract $4$ from each side. |





Name: ______ Date: ______ Period: ______

Practice

**4.** Consider the equation $3x + y = 15$.

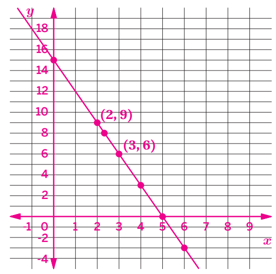
**a** Complete the table with pairs of values for $x$ and $y$ which make the equation true.

| $x$ | $y$ |
|---|---|
| 2 | **9** |
| **4** | 3 |
| 6 | **−3** |
| 0 | **15** |
| 3 | **6** |
| **5** | 0 |
| $\frac{7}{3}$ | 8 |

**b** Create a graph and plot the points in the table. Find the slope of the line that passes through the points.

**The slope is −3; Sample response:**

$$\frac{9-6}{2-3} = \frac{3}{-1} = -3$$




**5.** Solve each equation. Show your thinking.

**a** $5(x + 2) = 30$

$$5(x + 2) \div 5 = 30 \div 5$$
$$x + 2 = 6$$
$$x + 2 - 2 = 6 - 2$$
$$x = 4$$

**b** $5x + 2 = 30$

$$5x + 2 - 2 = 30 - 2$$
$$5x = 28$$
$$5x \div 5 = 28 \div 5$$
$$x = \frac{28}{5}$$





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 2 | 1 |
| Spiral | 4 | Unit 3 Lesson 16 | 2 |
| Formative | 5 | Unit 4 Lesson 5 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available


For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Balanced Moves
(Part 2)

Let's rewrite some more equations, while keeping the same solutions.


## Focus

### Goals

1. **Language Goal:** Calculate a value that is a solution for a linear equation in one variable, and compare and contrast solution strategies with others. **(Speaking and Listening)**
2. **Language Goal:** Critique the reasoning of others in solving a linear equation in one variable. **(Writing)**

## Coherence

### Today

Students continue to reinforce the connection between three fundamental ideas: a solution to an equation is a value that makes the equation true, performing the same operation on each side of an equation maintains the equality in the equation, and, therefore, two equations related by such a step have the same solution. They use the structure of the equation to determine the possible next steps as they practice solving linear equations with variables on both sides.

### Previously

In Lessons 3 and 4, students used hanger diagrams as tools to solve linear equations.

### Coming Soon

In Lesson 6, students will continue to practice solving linear equations with variables on both sides, and, in Lessons 7 and 8, they will solve equations with no solution or infinitely many solutions.

## Rigor

- Students practice **procedural skills** as they develop an algorithm to solve equations with variables on both sides.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 8 min | 5 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Solving Linear Equations*

### Math Language Development

**Review words**

- *coefficient*
- *constant*
- *Distributive Property*
- *equation*
- *equivalent equations*
- *expression*
- *like terms*
- *Properties of Equality*
- *solution*
- *substitution*
- *term*
- *variable*

**Amps** | **Featured Activity**

**Exit Ticket**
**Real-Time Exit Ticket**

Check in real time to see whether your students can find errors in a solution and can correctly solve an equation with variables on both sides using a digital Exit Ticket.

Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students might get frustrated if their algorithms are not the same as the given algorithm in Activity 1. Remind students that it is *an* algorithm, not *the* algorithm, and therefore, there could be multiple correct ways to solve a problem. It might also help for them to think of an algorithm as simply a list of steps to take to solve an equation. As they work toward developing an algorithm for solving equations with variables on both sides, they can evaluate their process and makes changes.

### Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- Have students choose one problem from **Activity 2** and assign the remaining problems as additional practice.

Pairs | 5 min

# Warm-up Is It a Solution?

Students substitute a value into an equation to determine whether it is the solution.

Unit 4 | Lesson 5

## Balanced Moves (Part 2)

Let's rewrite some more equations, while keeping the same solutions.


### Warm-up Is It a Solution?

Consider the equation $10x - 2x + 9 = 3(2x + 9)$. Is $x = 3$ a solution to the equation? Show or explain your thinking.

Sample response:

$$10x - 2x + 9 = 3(2x + 9)$$
$$10(3) - 2(3) + 9 = 3(2(3) + 9)$$ Substitute 3 for $x$ and evaluate the expression on each side.
$$30 - 6 + 9 = 3(6 + 9)$$
$$24 + 9 = 3(15)$$
$$33 = 45$$

This is not a true statement; therefore, $x = 3$ is not a solution.

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Activate prior knowledge and ask students to define the *solution* of an equation.

## 2 Monitor

**Help students get started** by asking how they can check whether 3 is the solution.

**Look for points of confusion:**

- **Not substituting 3 in for every $x$-variable.** Remind students they are checking if 3 makes the equation true, so they must replace every $x$ with 3.

**Look for productive strategies:**

- Solving the equation correctly.

## 3 Connect

**Have students share** their work and reasoning.

**Highlight** that, when checking a solution, it is a best practice to substitute the value into the original equation. When 3 was substituted into the $x$-variables, the left part and the right part of the equation were not equal. This means that 3 is not the solution to the equation.

**Ask**, "What steps could you take to solve this equation?" If time permits, use the suggestions from the students and attempt to solve the equation. If they determine a solution, check to ensure it makes the equation true.

## Differentiated Support

**Accessibility: *Guide Processing and Visualization***

Demonstrate or suggest that students substitute $x = 3$ for each $x$-value in the equation. Consider chunking the problem into the following steps.

- Substitute $x = 3$ into the left side of the equation.
  $10(3) - 2(3) + 9 = ?$
- Substitute $x = 3$ into the right side of the equation.
  $10(2 \cdot 3 + 9) = ?$
- Compare these two values. Are they the same?

## Power-up

**To power up students' ability to solve equations containing only one variable term, have students complete:**

Solve each equation and check your answer.

1. $\frac{1}{3}x = 8$
   $\frac{1}{3}x \div \frac{1}{3} = 8 \div \frac{1}{3}$
   $x = 24$

2. $3x - 4 = 11$
   $3x - 4 + 4 = 11 + 4$
   $3x = 15$
   $x = 5$

**Use:** Before Activity 1
**Informed by:** Performance on Lesson 4, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 4

Pairs | 15 min

# Activity 1 Step by Step by Step by Step

Students solve an equation to build an algorithm to use when solving linear equations with variables on both sides.

Name: ______ Date: ______ Period: ______

## Activity 1 Step by Step by Step by Step

**Bob Moses, a civil rights icon and algebra teacher, has dedicated his career to improving how algebra is taught and learned, especially for students who have not benefited from high-quality instruction. Being able to manipulate an equation using an algorithm is just one example of what Moses and others would consider to be important to studying algebra. An *algorithm* is a list of steps to follow and is particularly useful in solving equations.**

1. The following table shows the description of steps for one method of solving the equation shown. Complete the table, using the steps shown, to solve the equation.

| Description | Example |
|---|---|
| Original equation | $\frac{1}{2}(4x + 7) + \frac{3}{2} = 3(2x + 5) + x$ |
| Use the Distributive Property. | $2x + \frac{7}{2} + \frac{3}{2} = 6x + 15 + x$ |
| Multiply each term by the least common denominator to eliminate the fractions. | $4x + 7 + 3 = 12x + 30 + 2x$ |
| Combine like terms on each side. | $4x + 10 = 14x + 30$ |
| Add or subtract expressions so that the variable terms are on one side. | $4x + 10 - 4x = 14x + 30 - 4x$ <br> $10 = 10x + 30$ |
| Add or subtract expressions so that the constant terms are on the other side. | $10 - 30 = 10x + 30 - 30$ <br> $-20 = 10x$ |
| Divide by the coefficient to isolate the variable. | $-20 \div 10 = 10x \div 10$ |
| Solution | $x = -2$ |




## 1 Launch

Display the equation from Problem 1 and have students compare this equation with ones they have solved previously. Consider completing Problem 1 as a class to ensure understanding.

## 2 Monitor

**Help students get started** by asking whether they notice any familiar steps they could take.

**Look for points of confusion:**

- **Being intimidated by the fractions.** Let students know these are just numbers and they know how to perform operations with fractions.
- **Not understanding the Distributive Property.** Rewrite distribution as multiplication of the term on the outside with every term, such as $3(2x + 9)$ as $3(2x) + 3(9)$.
- **Incorrectly distributing the negative sign in Problem 3.** Have students rewrite the right side as $-1(x - 2)$ before proceeding with the Distributive Property.
- **Miscalculating the second term on the right side of Problem 3.** Consider rewriting the terms inside the parentheses as $(x + (-2))$ to remind students that the 2 is negative.

**Look for productive strategies:**

- Rearranging the order of the steps in the algorithm, especially if it allows for a more efficient method.
- Wanting to subtract $14x$ in Problem 1 instead of $4x$. If time permits, consider showing that this step is acceptable.
- Performing addition and subtraction with the rational values without multiplying by the LCD first.
- Multiplying both sides by 3 before using the Distributive Property in Problem 3.

Activity 1 continued ›

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Display or provide students with a copy of the Anchor Chart PDF, *Solving Linear Equations* in a sheet protector so they can mark completed steps throughout their solving process.

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problem 1 and choosing to complete either Problem 2 or Problem 3.

# Activity 1 Step by Step by Step by Step (continued)

Students solve an equation to build an algorithm to use when solving linear equations with variables on both sides.

## Activity 1 Step by Step by Step by Step (continued)

**2.** Solve the equation $10x - 2x + 9 = 3(2x + 9)$ from the Warm-up. Show your thinking and check your solution.

$$\begin{aligned}10x - 2x + 9 &= 3(2x + 9)\\ 10x - 2x + 9 &= 6x + 27\\ 8x + 9 &= 6x + 27\\ 8x + 9 - 8x &= 6x + 27 - 8x\\ 9 &= -2x + 27\\ 9 - 27 &= -2x + 27 - 27\\ -18 &= -2x\\ -18 \div (-3) &= -2x \div (-3)\\ 9 &= x\end{aligned}$$

**Check your solution:**

$$\begin{aligned}10(9) - 2(9) + 9 &= 3(2(9) + 9)\\ 90 - 18 + 9 &= 3(18 + 9)\\ 72 + 9 &= 3(27)\\ 81 &= 81\end{aligned}$$

**This is a true statement; therefore, $x = 9$ is a solution.**

**3.** Solve the equation $\frac{2}{3}(6x - 1) = -(x - 2)$. Show your thinking and check your solution.

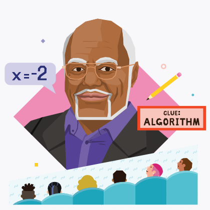
$$\begin{aligned}\tfrac{2}{3}(6x - 1) &= -(x - 2)\\ 4x - \tfrac{2}{3} &= -x + 2\\ 12x - 2 &= -3x + 6\\ 15x - 2 &= 6\\ 15x &= 8\\ x &= \tfrac{8}{15}\end{aligned}$$

**Check your solution:**

$$\begin{aligned}\tfrac{2}{3}\left(6\left(\tfrac{8}{15}\right) - 1\right) &= -\left(\left(\tfrac{8}{15}\right) - 2\right)\\ \tfrac{2}{3}\left(\tfrac{16}{5} - 1\right) &= -\left(-\tfrac{22}{15}\right)\\ \tfrac{2}{3}\left(\tfrac{11}{5}\right) &= \tfrac{22}{15}\\ \tfrac{22}{15} &= \tfrac{22}{15}\end{aligned}$$

**This is a true statement; therefore, $x = \frac{8}{15}$ is a solution.**

**Featured Mathematician**




**Bob Moses**

Bob Moses was an educator and was an activist in the Civil Rights Movement in the 1960s. He served as a leader of the Student Nonviolent Coordinating Committee, facing violence and intimidation as he helped register Black voters in Mississippi. In the 1980s, Moses founded the Algebra Project, which makes algebra more accessible to students, because "every child has a right to a quality education, to succeed in this technology-based society, and to exercise full citizenship." Bob Moses passed away in 2021.




## 3 Connect

**Display** any problem necessary to help with the discussion.

**Have students share** their solution process for Problems 2 and 3. Have students who used multiplication by the LCD in Problem 3 share their solution, or show this solution method if it was not used. Encourage the use of mathematically precise language.

**Highlight** how the structure of the equation determines which steps of the algorithm they need to take. For instance, Problem 2 does not contain fractions, so multiplying by the LCD is not necessary. Also, Problem 3 does not have like terms to combine, so that step of the algorithm can be omitted.

**Ask:**

- "How can you tell when distribution might be a helpful step?"
- "How can you tell when multiplying by the LCD might be a helpful step?"

**Note:** There are multiple ways to solve equations. The algorithm presented in this activity is just one way. Have students use the solving method of their choice.

## Featured Mathematician

### Bob Moses

Have students read about featured mathematician Bob Moses, a civil rights activist and algebra teacher. In the 1960s, Moses was a leader of the Student Nonviolent Coordinating Committee, facing violence and intimidation as he helped register Black voters in Mississippi. In the 1980s, he became an algebra teacher and received a MacArthur Fellowship grant to found the Algebra Project. Starting with one high school in Mississippi, Mosses worked to transform math education for students who had been historically underserved due to Jim Crow and racial discrimination. He enlisted community support, doubled up on math instructional time, and made the curriculum more student-centered and culturally aware. The Algebra Project expanded to serve students in more than 200 schools, and now partners with schools and organizations across the country to improve math literacy for students from kindergarten to high school.

Pairs | 15 min

# Activity 2 Create Your Own Steps

Students practice solving linear equations with variables on both sides and with rational coefficients.

Name: ______ Date: ______ Period: ______

## Activity 2 Create Your Own Steps

Solve each equation. Show your thinking and check your solution. Correct any mistakes you may have made.

1. $8x + 7 = 6x - 13$

Sample response:

$$\begin{aligned} 2x + 7 &= -13 \\ 2x &= -20 \\ x &= -10 \end{aligned}$$

Check your solution:

$$\begin{aligned} 8(-10) + 7 &= 6(-10) - 13 \\ -80 + 7 &= -60 - 13 \\ -73 &= -73 \end{aligned}$$

This is a true statement; therefore, $x = -10$ is a solution.

2. $-3(a - 4) = 9a - 4$

Sample response:

$$\begin{aligned} -3a + 12 &= 9a - 4 \\ 12 &= 12a - 4 \\ 16 &= 12a \\ \frac{4}{3} &= a \end{aligned}$$

Check your solution:

$$\begin{aligned} -3\left(\frac{4}{3} - 4\right) &= 9\left(\frac{4}{3}\right) - 4 \\ -4 + 12 &= 12 - 4 \\ 8 &= 8 \end{aligned}$$

This is a true statement; therefore, $a = \frac{4}{3}$ is a solution.





## 1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

## 2 Monitor

**Help students get started** by referencing the algorithm from Activity 1 and asking whether the Distributive Property, multiplying by the LCD, or collecting like terms on the left side or the right side is needed. All of these steps can be omitted, so students should begin with moving the variables to one side by adding or subtracting a variable term from both sides.

**Look for points of confusion:**

- **Distributing the –3 incorrectly in Problem 2.** Have students rewrite the equation as $-3(x + (-4)) = 9x - 4$ to show the $-3$ is multiplied by $-4$ to make $12$.
- **Not knowing how to get started with Problem 3.** Have students refer to the algorithm in Activity 1 and start with the first step of using the Distributive Property.
- **Not knowing how to get started with Problem 4.** If following the algorithm from Activity 1, have students skip the step with the Distributive Property and multiply by the LCD of 6. Or, they can rewrite the equation as $\frac{1}{3}(12 + 6x) = \frac{1}{2}(5x - 9)$ and proceed with the algorithm.

**Look for productive strategies:**

- Choosing to multiply both sides by 3 in Problem 3. This possible step yields the equation $3m - 12 = 6m - 54$.
- In Problem 4, starting by multiplying both sides by 6. This possible strategy yields the equation $2(12 + 6x) = 3(5x - 9)$.

Activity 2 continued ›

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them complete Problem 1 and then choose to complete either Problem 2 or Problem 3. These problems will give them opportunities to solve equations with variables on both sides, with and without parentheses.

### Extension: *Math Enrichment*

Have students think of as many different strategies as they can to solve the equation in Problem 4. For example, they could begin by:

- Rewrite the division as multiplication by a fraction.
- Multiply both sides by 6 to eliminate the fractions.
- Divide each numerator by its denominator.

## Math Language Development

### *MLR3: Critique, Correct, Clarify*

During the Connect, display an incorrect solution pathway for Problem 3, such as, adding 4 to both sides first, but then distributing the value $\frac{1}{3}$ to the 4 that is now on the right side. Ask:

- ***Critique:*** "Do you agree with this solution pathway? Why or why not?" Listen for students who correctly realize that the $\frac{1}{3}$ should only be distributive to the terms inside the parentheses.
- ***Correct:*** "What should have been the correct next step?"
- ***Clarify:*** "How would you use words to explain to someone who made this error why it is incorrect and what they should have done instead?"

Pairs | 15 min

# Activity 2 Create Your Own Steps (continued)

Students practice solving linear equations with variables on both sides and with rational coefficients.

## Activity 2 Create Your Own Steps (continued)

3. $m - 4 = \frac{1}{3}(6m - 54)$

Sample response:

$m - 4 = 2m - 18$
$-4 = m - 18$
$14 = m$

Check your solution:

$14 - 4 = \frac{1}{3}(6(14) - 54)$
$10 = \frac{1}{3}(84 - 54)$
$10 = \frac{1}{3}(30)$
$10 = 10$

This is a true statement; therefore, $m = 14$ is a solution.

4. $\frac{12 + 6x}{3} = \frac{5x - 9}{2}$

Sample response:

$\frac{1}{3}(12 + 6x) = \frac{1}{2}(5x - 9)$
$4 + 2x = \frac{5}{2}x - \frac{9}{2}$
$8 + 4x = 5x - 9$
$8 = x - 9$
$17 = x$

Check your solution:

$\frac{12 + 6(17)}{3} = \frac{5(17) - 9}{2}$
$\frac{12 + 102}{3} = \frac{85 - 9}{2}$
$\frac{114}{3} = \frac{76}{2}$
$38 = 38$

This is a true statement; therefore, $x = 17$ is a solution.

**Are you ready for more?**

A gaggle — or group — of geese are flying north after their summer migration. Half of the geese stop to rest on a lake while the other half continue the trip. When they pass another lake, half of the remaining geese stop at the lake, while the rest continue to fly. This continues until the geese are spread out over 7 lakes. What is the fewest number of geese in the gaggle? Show or explain your thinking.

Sample response: $1 + 1 + 2 + 4 + 8 + 16 + 32 = 64$ geese

STOP

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## 3 Connect

**Display** any necessary problems to help with the discussion.

**Have students share** their solution methods for each problem. If students show different methods, embrace this diversity in problem solving and have a discussion about how the steps are different but still accurate.

**Highlight** that the algorithm provides a list of steps to follow, but only if they pertain to the equation. Not every equation involves the Distributive Property, but, if one does, using the Distributive Property first helps start the process of solving. The structure of the equation should determine the steps that are taken.

**Ask**, "In Problem 1, which step do you prefer: subtracting $8x$ or $6x$ from both sides? Why?"

Whole Class | 8 min

# Summary

Review and synthesize the steps for solving linear equations with variables on both sides.

Name: ______________________ Date: __________ Period: __________

## Summary

**In today's lesson . . .**

You solved equations with variables on each side of the equal sign. How do you make sure your solution is correct? Accidentally adding when you meant to subtract, missing a negative sign when you distribute, forgetting to write an $x$ from one line to the next — there are many possible mistakes to watch out for!

Fortunately, each step you take solving an equation results in a new equation with the same solution as the original. This means you can check your work by substituting the value of your solution into the original equation. If the resulting equation is true, you found the correct solution.

> **Reflect:**





## Synthesize

**Display** and complete the Anchor Chart PDF, *Solving Linear Equations*. Consider displaying the necessary steps for the problem to make sense for your class. The steps shown on the answer key represent possible steps which are not always necessary.

**Ask**, "Do the following steps maintain the equality of the equation?"

- Subtracting a number from each side. Maintains equality
- Adding $4x$ to each side. Maintains equality
- Dividing each side by $7$. Maintains equality
- Adding $5x$ to one side and $10x$ to the other. Does not maintain equality (unless $x = 0$)
- Adding $4$ to the left side and subtracting $4$ from the right side. Does not maintain equality
- Multiplying both sides by $-3$. Maintains equality
- Multiplying both sides by $0$. Maintains equality but causes everything to become $0$, so it is not useful when solving.

**Highlight** that the algorithm is useful in knowing how to get started, but it is not the only way to solve equations. Students can expect to become more fluent with different methods for solving as they practice and become more familiar with the process.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you determine the solution to an equation with variables on both sides?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by analyzing an incorrect solution and determining the correct solution for a linear equation with variables on both sides.

Amps Featured Activity | Real-Time Exit Ticket | Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 4.05

Lin solved the equation $8(x - 3) + 7 = 2(3x - 7)$ incorrectly. Circle the step(s) in which she made an error. Then find the correct solution and check your answer. Show or explain your thinking.

Sample response:

$8(x - 3) + 7 = 2(3x - 7)$

$8x - 24 + 7 = 6x - 14$

$8x - 17 = 6x - 14$

$2x - 17 = -14$

$2x = 3$

$x = \frac{3}{2}$

Solution check:

$8\left(\frac{3}{2} - 3\right) + 7 = 2\left(3\left(\frac{3}{2}\right) - 7\right)$

$12 - 24 + 7 = 2\left(\frac{9}{2} - 7\right)$

$-5 = 9 - 14$

$-5 = -5$ This equation is true.

Therefore, $x = \frac{3}{2}$.

Lin's work:

$8(x - 3) + 7 = 2(3x - 7)$

$8x - 24 + 7 = 6x - 14$

$8x - 17 = 6x - 14$

$8x - 17 + 6x = 6x - 14 + 6x$

$14x - 17 = -14$

$14x - 17 + 17 = -14 + 17$

$14x = 3$

$14x \div 14 = 3 \div 14$

$x = \frac{3}{14}$

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can solve linear equations with variables on each side.

1 2 3





## Success looks like . . .

- **Language Goal:** Calculating a value that is a solution for a linear equation in one variable, and comparing and contrasting solution strategies with others. **(Speaking and Listening)**
- **Language Goal:** Critiquing the reasoning of others in solving a linear equation in one variable. **(Writing)**
  - » Explaining the error Lin made when solving an equation.

## Suggested next steps

**If students identify the error in Lin's solution but do not solve it correctly, consider:**

- Reviewing the algorithm in Activity 1.
- Assigning Practice Problems 2 and 3.

**If students solve the equation correctly but do not find Lin's error, consider:**

- Reviewing the Warm-up.
- Assigning Practice Problems 1 and 3.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- What worked and didn't work today? Knowing where students need to be by the end of this unit, how did the algorithm in Activity 1 influence that future goal?
- What surprised you as your students worked on solving linear equations? What might you change the next time you teach this lesson?

# Practice

Independent

Practice

Name: ________ Date: ________ Period: ________

1. Mai and Tyler are each solving the equation $\frac{2}{5}b - 10 = 3b$. Mai's solution is $b = \frac{50}{17}$, and Tyler's solution is $b = -\frac{10}{13}$. Their work is shown. Do you agree with either of their solutions? Show or explain your thinking.

**Mai's work:**

$$\frac{2}{5}b - 10 = 3b$$
$$\frac{2}{5}b + 3b - 10 = 0$$
$$\frac{17}{5}b - 10 = 0$$
$$\frac{17}{5}b = 10$$
$$17b = 50$$
$$b = \frac{50}{17}$$

**Tyler's work:**

$$\frac{2}{5}b - 10 = 3b$$
$$2b - 10 = 15b$$
$$-10 = 13b$$
$$-\frac{10}{13} = b$$

**Sample response: Both Mai and Tyler made errors. Mai's mistake is adding $3b$; she should have subtracted $3b$ on the left side instead. Her second line should be the equation $\frac{2}{5}b - 3b - 10 = 0$. Tyler did not multiply by 5 by each term. His second line should be the equation $2b - 50 = 15b$.**

2. Solve the equation $3(x - 4) = 12x$. Show or explain your thinking. Remember to check your solution.

**Sample response:**

$$3(x - 4) = 12x$$
$$x - 4 = 4x$$
$$-4 = 3x$$
$$-\frac{4}{3} = x$$

**Solution check:**

$$3\left(-\frac{4}{3} - 4\right) = 12\left(-\frac{4}{3}\right)$$
$$-4 - 12 = -16$$
$$-16 = -16$$

**This is a true statement; therefore, $x = -\frac{4}{3}$ is a solution.**




Practice

Name: ________ Date: ________ Period: ________

3. Andre solved the equation shown, but when he checked his solution, he realized it was incorrect. He knows he made a mistake, but he cannot find it. Find Andre's mistake, and then correctly solve the equation.

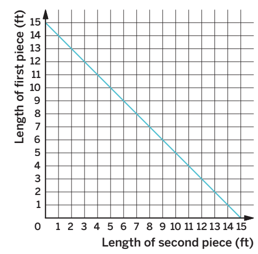
**Andre's work:**

$$-2(3x - 5) = 4(x + 3) + 8$$
$$-6x + 10 = 4x + 12 + 8$$
$$-6x + 10 = 4x + 20$$
$$10 = -2x + 20$$
$$-10 = -2x$$
$$5 = x$$

**Andre made a mistake in the fourth line. He subtracted $6x$ from $4x$ when he should have added.**

$$-2(3x - 5) = 4(x + 3) + 8$$
$$-6x + 10 = 4x + 12 + 8$$
$$-6x + 10 = 4x + 20$$
$$-10x + 10 = 20$$
$$-10x = 10$$
$$x = -1$$

4. A length of ribbon is cut into two pieces to use in a craft project. The graph shows the length $x$ of the second piece for each length $y$ of the first piece.




a. How long is the ribbon? Explain your thinking.

**15 ft because this is represented by the vertical intercept of the graph.**

b. What is the slope of the line?

**$-1$**

c. Explain what the slope of the line represents in context of the scenario.

**For every 1 ft increase in the length of the second piece, the length of the first piece will decrease by 1 ft.**

5. For each equation, determine whether $x = -3$ is a solution. Show or explain your thinking.

a. $\frac{2}{3}x = -2$

$$\frac{2}{3}(-3) = -2$$
$$-2 = -2$$

**True; therefore, $x = -3$ is a solution.**

b. $4(x + 7) - 9 = 7$

$$4(-3 + 7) - 9 = 7$$
$$4(4) - 9 = 7$$
$$16 - 9 = 7$$
$$7 = 7$$

**True; therefore, $x = -3$ is a solution.**

c. $-2(x + 2) = -10$

$$-2(-3 + 2) = -10$$
$$-2(-1) = -10$$
$$2 = -10$$

**False; therefore, $x = -3$ is not a solution.**



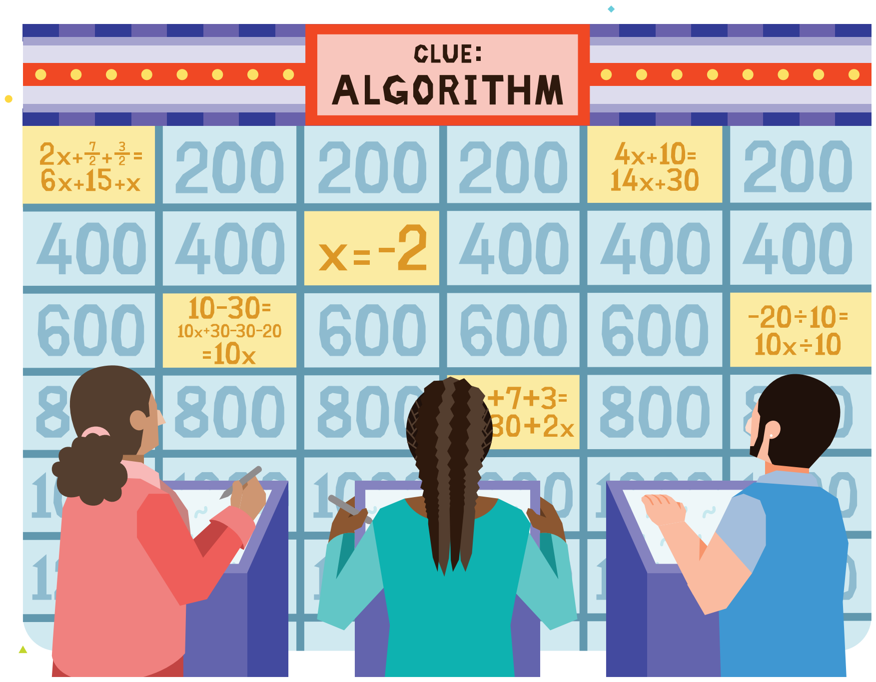

## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 1 | 2 |
| Spiral | 4 | Unit 3 Lesson 16 | 2 |
| Formative | 5 | Unit 4 Lesson 6 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.




# Solving Linear Equations

Let's solve linear equations.

## Focus

### Goals

1. **Language Goal:** Calculate a value that is a solution to a linear equation in one variable, and explain the steps used to solve. **(Speaking and Listening)**
2. **Language Goal:** Justify that each step used in solving a linear equation maintains equality. **(Speaking and Listening)**

## Coherence

### Today

Students encounter several different structures of equations and suggest steps for solving them. They explain their reasoning for choosing a particular step while solving equations. Students also critique their partner's choice.

### Previously

In Lesson 5, students developed an algorithm for solving linear equations with variables on both sides.

### Coming Soon

In Lessons 7 and 8, students will solve linear equations with no solution or infinitely many solutions.

## Rigor

- Students work toward **fluency** with solving linear equations with variables on both sides.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per pair
- Anchor Chart PDF, *Solving Linear Equations*

### Math Language Development

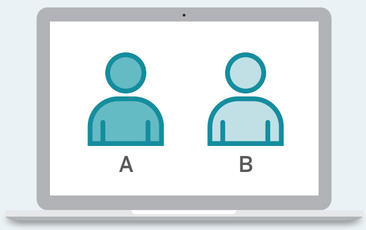
**Review words**

- *coefficient*
- *constant*
- *Distributive Property*
- *equation*
- *equivalent equations*
- *expression*
- *like terms*
- *Properties of Equality*
- *solution*
- *substitution*
- *term*
- *variable*

**Amps** | Featured Activity

### Activity 1
### Digital Collaboration

Students work together to solve linear equations.

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### Building Math Identity and Community

Connecting to Mathematical Practices

Students might not see a benefit to identifying common errors in the solutions to equations. Explain that to find a mistake, they must look closely at the solution's structure. They must have confidence in their understanding of equations, but also possess a growth mindset if they do not have the confidence yet.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, have students only complete the two cards.
- Have students choose one or two problems to complete in **Activity 2**.

Pairs | 5 min

# Warm-up Is It Equivalent?

Students analyze several equations to find equivalent equations and identify possible next steps in solving for $x$.

Unit 4 | Lesson 6

## Solving Linear Equations

Let's solve linear equations.


### Warm-up Is It Equivalent?

**Analyze each of the following equations.**

- Place a checkmark next to each equation that is equivalent to the original equation.
- Circle the equation that you think represents the best next step for determining the value of $b$ in the original equation.
- Solve the equation for $b$.

Original equation: $2b - 3 = \frac{1}{2}(2b - 3)$

- [x] $2b - 3 = b - \frac{3}{2}$
- [ ] $20b - 3 = b - 3$
- [x] $4b - 6 = 2b - 3$
- [ ] $2b = b$
- [x] $20b - 30 = 10b - 15$

$b = \frac{3}{2}$

**Students' responses for which equation they circle as the best next step may vary. Students may prefer the equation $4b - 6 = 2b - 3$ because it no longer contains fractions. Or students may prefer the equation $2b - 3 = b - \frac{3}{2}$ because they want to distribute first.**

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Display the Anchor Chart PDF, *Solving Linear Equations* for the remainder of the lesson. Use the *Think-Pair-Share* routine.

### 2 Monitor

**Help students get started** by asking what step they would use to start the process of solving the equation and if there is an equation which matches their answer.

**Look for points of confusion:**

- **Not being able to determine the equivalent equations that do not match their process for solving.** For instance, if a student used the Distributive Property, they may not notice $4b - 6 = 2b - 3$ is equivalent. Have students work with their partner to determine if there are additional equivalent equations.

**Look for productive strategies:**

- Solving the original equation first and then substituting the solution into the remaining equations to determine whether they are equivalent.
- Solving each equation to determine whether they reach the same solution as with the original equation.

### 3 Connect

**Display** the equations.

**Have students share** their selections and reasoning. Ask which equation they think represents the best next step and why.

**Highlight** that some problems might have several good next steps. Regardless of the possible next step taken, the equation should be equivalent whether it is a result of using the Properties of Equality, the Distributive Property, or combining like terms, and will have the same solution.

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization***

If students need more processing time, have them focus on identifying and solving one equivalent equation. Provide students with a copy of the Anchor Chart PDF, *Solving Linear Equations* to support them in organizing their solution pathways.

## Power-up

**To power up students' ability to determine whether a value is a solution to an equation, have students complete:**

Recall that in order to determine whether a value is a solution to an equation, you can solve the equation or substitute the value into the equation and evaluate it.

Determine whether $x = 4$ is a solution of the equation $(x + 5) + 6 = 9$.

It is not a solution; Sample response:
$$3(4 + 5) + 6 = 9$$
$$3(9) + 6 = 9$$
$$27 + 6 = 9$$
$$33 = 9 \text{ not true}$$

**Use:** Before Activity 1
**Informed by:** Performance on Lesson 5, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 5

Pairs | 15 min

# Activity 1 Trading Equations

Students work together to find the next step for solving equations.

**Amps Featured Activity** Digital Collaboration

Name: ____________ Date: ________ Period: ________

## Activity 1 Trading Equations

**Plan ahead:** As you trade and solve these equations, how will you use purposeful and precise communication?

**You will be given a set of cards with equations on them. Follow these instructions.**

1. Choose one card and have your partner choose a different card.
2. On your card, complete the first step in solving the equation. Fold your card so that only your step is visible and the original equation is hidden.
3. Trade cards with your partner and complete the next step for solving the equation on the card you received. Then fold the paper so only your step is visible.
4. Trade cards again. Complete the next step in solving the equation, fold the paper, and trade again.
5. Continue trading the cards back and forth after each step until the equations are solved.
6. Complete this process again with the remaining two equations and check your solutions.

**Student responses will be on the cards. Sample responses shown:**

**Card 1:**

$$-6x - 7 = 4x - 2$$
$$-7 = 10x - 2$$
$$-5 = 10x$$
$$-\frac{5}{10} = x$$
$$x = -\frac{1}{2}$$

**Card 2:**

$$\frac{1}{2}(7x - 6) = 6x - 10$$
$$7x - 6 = (6x - 10) \cdot 2$$
$$7x - 6 = 12x - 20$$
$$-6 = 5x - 20$$
$$14 = 5x$$
$$\frac{14}{5} = x$$

**Card 3:**

$$\frac{1}{2}x + 7 = x + 13$$
$$\frac{1}{2}x \cdot 2 + 7(2) = x(2) + 13(2)$$
$$x + 14 = 2x + 26$$
$$14 = x + 26$$
$$-12 = x$$

**Card 4:**

$$2(x + 7) = -4x + 14$$
$$2x + 14 = -4x + 14$$
$$6x + 14 = 14$$
$$6x = 0$$
$$x = 0$$




## 1 Launch

Distribute one set of pre-cut cards from the Activity 1 PDF to each pair of students. Read and discuss the procedure for this activity.

## 2 Monitor

**Help students get started** by referencing the Anchor Chart PDF, *Solving Linear Equations* and asking which step is needed for their equation.

**Look for points of confusion:**

- **Not understanding that 0 is a solution for Card 4.** Have students check the solution by substituting 0 in for $x$ and evaluating the equation.

**Look for productive strategies:**

- Looking for an efficient way to solve the equations. For instance, multiply by 2 on both sides for Card 2, or divide both sides by 2 on Card 4.

## 3 Connect

**Have students share** any interesting steps made by their partners while solving the equations and then display their solutions.

**Highlight** that while students might think about solving equations as an entire process, this activity shows each intermediate step produces an equivalent equation which can be solved independently and will yield the same solution. This is true if the possible steps involve using the Properties of Equality, the Distributive Property, or combining like terms until the variable is isolated.

**Ask**, "How can you check your solution?"

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization***

Chunk this task into smaller, more manageable parts by having partners focus on solving the equations on one pair of cards. Continue having students refer to the Anchor Chart PDF, *Solving Linear Equations* to support them in organizing their solution pathways.

**Extension: *Math Enrichment***

Have students complete a similar activity, but have them start with the solution first and perform operations to create equivalent equations with each trade.

Sample response:

$-12 = x$

$-2 = x + 10$

$x - 2 = 2x + 10$

## Math Language Development

***MLR6: Three Reads***

Use this routine to help students make sense of the activity's directions.

- **Read 1:** Students should read the directions individually, noting any questions they may have.
- **Read 2:** Ask students to read the directions aloud in pairs and clarify what the directions are asking them to do.
- **Read 3:** Ask students to read the directions again and this time, perform the actions described in each step.

Pairs | 15 min

# Activity 2 Find and Fix

Students critique solutions to equations to analyze common mistakes and how to fix them.

### Activity 2 Find and Fix

Four equations are shown, with an attempt to solve each one. In each solution attempt, there may be one or more errors. If there are any errors, circle them, explain why they are errors, and then correct them.
If there are no errors, state whether you would solve the equation in the same way or take a different approach. Sample responses shown.

1. Equation 1:

$$\begin{aligned} 4-2(3x-2) &= 14-x \\ 2(3x-2) &= 14-x \\ 6x-4 &= 14-x \\ 5x-4 &= 14 \\ 5x &= 18 \\ x &= \frac{18}{5} \end{aligned}$$

The subtraction of 2 from 4 should not be performed first because of the order of operations. The value −2 should be distributed first.

$$\begin{aligned} 4-2(3x-2) &= 14-x \\ 4-6x+4 &= 14-x \\ 8-6x &= 14-x \\ 8 &= 14+5x \\ -6 &= 5x \\ -\frac{6}{5} &= x \end{aligned}$$

2. Equation 2:

$$\begin{aligned} \frac{1}{3}(12x-5) &= 10x-9x-6 \\ 4x-5 &= 10x-9x-6 \\ 4x-5 &= x-6 \\ 3x-5 &= -6 \\ 3x &= -1 \\ x &= -\frac{1}{3} \end{aligned}$$

The value $\frac{1}{3}$ should be distributed to the value 5 also.

$$\begin{aligned} \frac{1}{3}(12x-5) &= 10x-9x-6 \\ 4x-\frac{5}{3} &= x-6 \\ 12x-5 &= 3x-18 \\ 9x &= -13 \\ x &= -\frac{13}{9} \end{aligned}$$




## 1 Launch

Let students know they will be analyzing the solutions for linear equations. They should look closely and determine whether there are errors and then correct the errors.

## 2 Monitor

**Help students get started** by asking how they want to approach the problem. Do they want to check the solution first, solve the problems first, or analyze the work shown? Once students decide which process they want to take, ask them what their first step would be. Clarify any procedural issues.

**Look for points of confusion:**

- **Not finding the errors in Problems 1 or 2.** Have students substitute their answer into the equation to determine if it is the solution. Once they see it is not, have them work through the problem using the algorithm from Lesson 5.
- **Multiplying only one side of the equation in Problem 2 by the LCD.** Students may think the resulting step is $12x - 5 = x - 6$, instead of $12x - 5 = 3x - 18$. If students choose to multiply by the LCD to eliminate fractions, remind them to maintain balance and multiply both sides by 3. Refer to Cards 5 and 9 from Lesson 3, Activity 2, if needed.
- **Not realizing the first step in Problem 4 as multiplying by 10.** Ask students to determine which parts changed from the first equation to the second.

**Look for productive strategies:**

- Checking the solution before deciding if there is an error.

Activity 2 continued ›

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

To help students get started and remain organized throughout the activity, provide students with the following checklist to keep track of their work:

- Check the solution to the equation to determine accuracy.
- Analyze the work shown to identify potential errors.
- Solve the equation using correct mathematical reasoning.
- Explain why the equation was incorrect and how it was corrected.

### Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Equation 1. Provide students with a copy of the Anchor Chart PDF, *Solving Linear Equations* to support them in organizing their solution pathways.

## Math Language Development

### *MLR3: Critique, Correct, Clarify*

During the Connect, display Problem 2's incorrect solution pathway. Ask:

- ***Critique:*** "Where do you see any mathematical errors in this solution attempt?"
- ***Correct:*** "How would you correct any errors? What is a correct solution strategy?"
- ***Clarify:*** "How can you verify that your solution strategy is correct?"

### English Learners

Allow students to share their correct solution strategy with a partner before sharing with the whole class.

Pairs | 15 min

# Activity 2 Find and Fix (continued)

Students critique solutions to equations to analyze common mistakes and how to fix them.

Name: ______ Date: ______ Period: ______

## Activity 2 Find and Fix (continued)

3. Equation 3:

$$\begin{aligned} 3x - 6 + 4\left(x - \frac{1}{2}\right) &= \frac{1}{4}(2x - 6) \\ 3x - 6 + 4x - 2 &= \frac{1}{2}x - \frac{6}{4} \\ 7x - 8 &= \frac{1}{2}x - \frac{6}{4} \\ 28x - 32 &= 2x - 6 \\ 26x &= 26 \\ x &= 1 \end{aligned}$$

There are no errors present. Other ways to solve the equation:

- Students may may wait to combine like terms.
- They may simplify $\frac{6}{4}$ to $\frac{3}{2}$, which will cause them to only have to multiply by the LCD of 2.
- They may want to move the variables and constant terms in separate steps.

4. Equation 4:

$$\begin{aligned} 1.1(x - 3) &= 0.1(2x - 6) \\ 11(x - 3) &= 1(2x - 6) \\ 11x - 33 &= 2x - 6 \\ 9x - 33 &= -6 \\ 9x &= 27 \\ x &= 3 \end{aligned}$$

There are no errors present. Other ways to solve the equation:

- Students may want to use the Distributive Property first and then multiply by 10 on each side.
- Or students may not want to multiply by 10 at all and instead solve the equation using the decimal values.

STOP





## 3 Connect

**Display** any necessary problems to help with the discussion.

**Have students share** the errors they uncovered, why they are errors, and how they corrected them.

**Highlight** strategies for solving the equations and reference the Anchor Chart PDF, *Solving Linear Equations* for ways to determine the errors. For instance, in Problem 1, the solution started with combining like terms, which is not appropriate, considering the multiplication from the Distributive Property should be done first.

**Ask,** "Let's look at Problem 2. If you check the solution of $x = -\frac{1}{3}$ into the equation $4x - 5 = x - 6$, it is true. But it is not the solution. What happened? How should you fix it?" The error in solving started in the first step. You should always substitute the solution into the original equation to avoid potential errors in solving.

Whole Class | 5 min

# Summary

Review and synthesize strategies for solving linear equations with variables on both sides.

## Summary

**In today's lesson . . .**

You solved equations in one variable, and there are many ways to solve these types of equations. Generally, you want to perform steps which will get you closer to an equation where the variable is isolated, such as *variable = some number.*

Using the algorithm from Lesson 5 can help ensure the correct steps are taken to find the solution. However, the steps can be switched if it makes the process more efficient for you. Just remember to always maintain equality by using the properties of equality when moving terms across the equal sign.

Every time you solve an equation, remember you can always check your solution by substituting the value into the original equation and evaluating to see whether the resulting equation is true.

> **Reflect:**

396 Unit 4 Linear Equations and Systems of Linear Equations



## Synthesize

**Display** the Anchor Chart PDF, *Solving Linear Equations*.

**Have students share** their approaches to solving equations with different structures.

**Highlight** that the algorithm from Lesson 5 is a strategy to use if students are unsure where to start; it will help them get through the solving process. However, if they notice the structure of the equation lends itself to a different approach, they can use a more efficient path as long as equality is maintained throughout the process.

**Ask**, "After solving an equation, how can you check whether you found the correct solution?"

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you determine the solution to an equation with variables on both sides?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by reasoning about a solved equation to identify the steps taken.

Printable

Name: Date: Period:

## Exit Ticket 4.06

Noah solved the equation $\frac{1}{2}(7x - 6) = 6x - 10$. Review his work and describe the steps he used.

| Noah's work | Description |
|---|---|
| $\frac{1}{2}(7x - 6) = 6x - 10$ | Original equation |
| $\frac{7}{2}x - 3 = 6x - 10$ | Distribute $\frac{1}{2}$ on the left side. |
| $7x - 6 = 12x - 20$ | Multiply each term by 2 to eliminate the fraction. |
| $-5x - 6 = -20$ | Subtract $12x$ from each side. |
| $-5x = -14$ | Add 6 to each side. |
| $x = \frac{14}{5}$ | Divide each side by $-5$. |

**Self-Assess**

1 I don't really get it 2 I'm starting to get it 3 I got it

a I can solve an equation where the variable terms appear on each side.
1 2 3





## Success looks like . . .

- **Language Goal:** Calculating a value that is a solution to a linear equation in one variable, and explaining the steps used to solve. **(Speaking and Listening)**
- **Language Goal:** Justifying that each step used in solving a linear equation maintains equality. **(Speaking and Listening)**
  - » Explaining each step of Noah's work in the table.

## Suggested next steps

**If students incorrectly identify Noah's steps, consider:**

- Reviewing the Anchor Chart PDF, *Solving Linear Equations*.
- Reviewing Activity 2.
- Assigning Practice Problems 1, 2, and 3.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- What worked and didn't work today? What different ways did students approach solving the equations in Activity 1? What does that tell you about similarities and differences among your students?
- Have you changed any ideas you used to have about teaching how to solve linear equations as a result of today's lesson? What might you change for the next time you teach this lesson?

## Math Language Development

**Language Goal: Justifying that each step used in solving a linear equation maintains equality.**

Reflect on students' language development toward this goal.

- How have students progressed in their precision of describing the steps or reasoning behind the steps when solving a linear equation?
- Do students' descriptions provided for each step of the Exit Ticket problem demonstrate that they understand that equality is preserved?

# Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

**1.** Solve each equation. Show or explain your thinking.

a. $2(x+5) = 3x + 1$

$2x + 10 = 3x + 1$

$10 = x + 1$

$9 = x$

b. $3y - 4 = 6 - 2y$

$5y - 4 = 6$

$5y = 10$

$y = 2$

c. $3(n+2) = 9(6-n)$

$3n + 6 = 54 - 9n$

$12n + 6 = 54$

$12n = 48$

$n = 4$

**2.** Clare solved the equation shown, but when she checked her solution, she realized it was incorrect. She knows she made at least one mistake, but she cannot find it. Find Clare's mistake(s) and then correctly solve the equation.

**Clare's work:**

$$12(5 + 2y) = 4y - (5 - 9y)$$
$$72 + 24y = 4y - 5 - 9y$$
$$72 + 24y = -5y - 5$$
$$24y = -5y - 77$$
$$29y = -77$$
$$y = -\frac{77}{29}$$

**Clare made two mistakes in the second line. She incorrectly multiplied 12 by 5, which should be 60, not 72. She did not distribute the negative sign correctly on the right side. It should be $4y - 5 + 9y$.**

$12(5 + 2y) = 4y - (5 - 9y)$

$60 + 24y = 4y - 5 + 9y$

$60 + 24y = 13y - 5$

$60 + 11y = -5$

$11y = -65$

$y = -\frac{65}{11}$

**3.** Solve each equation. Show your thinking.

a. $\frac{1}{9}(2m - 16) = \frac{1}{3}(2m + 4)$

$\frac{1}{9}(2m - 16) \cdot 9 = \frac{1}{3}(2m + 4) \cdot 9$

$2m - 16 = 3(2m + 4)$

$2m - 16 = 6m + 12$

$-16 = 4m + 12$

$-28 = 4m$

$-7 = m$

b. $1.5(5 + 0.2y) = 0.4y - (0.6 - 0.9y)$

$7.5 + 0.3y = 0.4y - 0.6 + 0.9y$

$75 + 3y = 4y - 6 + 9y$

$75 + 3y = 13y - 6$

$75 = 10y - 6$

$81 = 10y$

$8.1 = y$

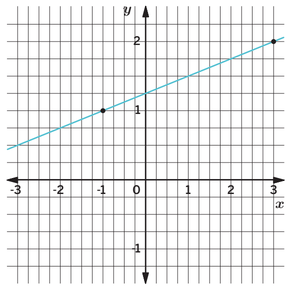




Practice

Name: ______ Date: ______ Period: ______

**4.** The graph of a linear equation is shown. Select *all* the true statements about this line and its equation.




A. One solution to the equation is the ordered pair $(3, 2)$.

B. One solution to the equation is the ordered pair $(-1, 1)$.

C. One solution to the equation is the ordered pair $\left(\frac{3}{2}, 1\right)$.

D. There are only 2 solutions.

E. There are infinitely many solutions.

F. The equation of the line is $y = \frac{1}{4}x + \frac{5}{4}$.

G. The equation of the line is $y = \frac{5}{4}x + \frac{1}{4}$.

**5.** Tyler invented a number puzzle. He asks Clare to choose any number, and then complete the steps shown.

**Steps:**

- Triple the number.
- Subtract 7.
- Double the result.
- Subtract 22.
- Divide by 6.

Clare says the output is $-3$. Tyler says the input must have been 3. How did Tyler know that? Explain or show your thinking.

**Sample response:**
**Let $x$ be the input; then the expression $\frac{2(3x-7)-22}{6}$ represents the number puzzle.**

$\frac{2(3x-7)-22}{6} = -3$

$2(3x - 7) - 22 = -18$

$6x - 14 - 22 = -18$

$6x - 36 = -18$

$6x = 18$

$x = 3$

**6.** Solve the equation $2(x - 10) = -2(x + 8) - 4$. Show your thinking.

$2x - 20 = -2x - 16 - 4$

$2x - 20 = -2x - 20$

$4x = 0$

$x = 0$





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 1 | 2 |
| Spiral | 4 | Unit 3 Lesson 16 | 2 |
| | 5 | Unit 4 Lesson 2 | 2 |
| Formative | 6 | Unit 4 Lesson 7 | 1 |

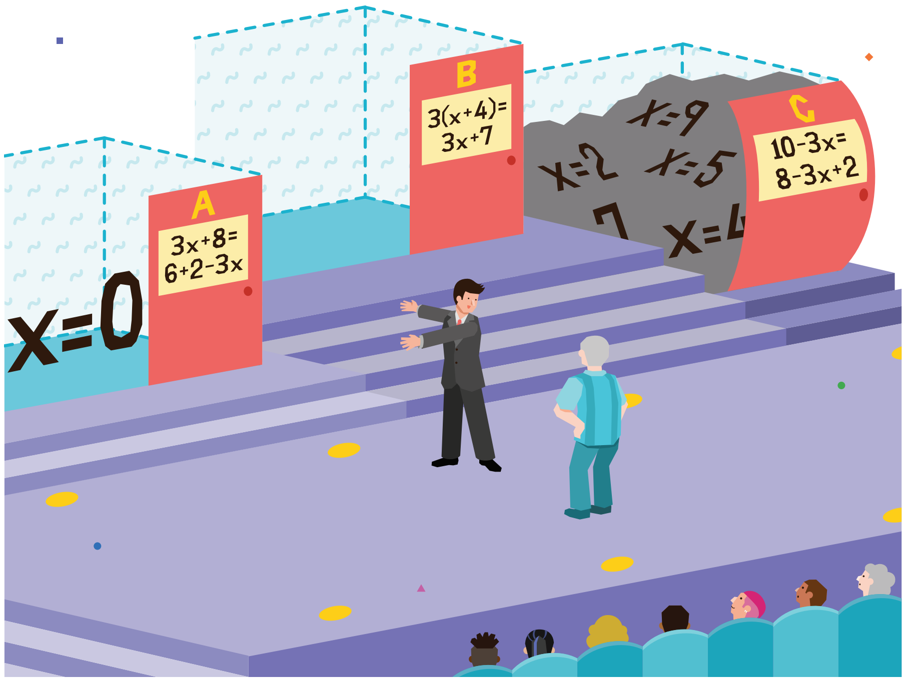
**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# How Many Solutions? (Part 1)

Let's think about how many solutions an equation can have.




## Focus

### Goals

1. Correlate equations that are *never true* as equations with *no solution* and equations that are *always true* as equations with *infinitely many solutions*.

2. **Language Goal:** Describe a linear equation as having one solution, no solution, or an infinite number of solutions, and solve equations in one variable with one solution. **(Speaking and Listening)**

## Coherence

### Today

Students explore the idea of one solution, no solution, and infinitely many solutions of an equation, but without hanger diagrams. They substitute numbers, where there is one number, no numbers, or infinitely many numbers that make the equation true. Students then solve the equation, resulting with false statements, such as $27 = 22$, or true statements, such as $5 = 5$, and relate the statement to the number of solutions for the equation.

### Previously

In Lessons 1–6, students balanced equations and explored the steps to solving equations with one solution. In Lesson 3, students explored the idea of one, none, and infinitely many solutions using hanger diagrams.

### Coming Soon

In Lesson 9, students will determine the number of solutions for a linear equation by using the structure of the equation, instead of solving it.

## Rigor

- Students strengthen their **fluency** in solving linear equations and identifying the number of solutions a linear equation might have.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 10 min | 10 min | 20 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Anchor Chart PDF, *Solving Linear Equations* (optional)
- Anchor Chart PDF, *Properties of Equality*

### Math Language Development

**Review words**

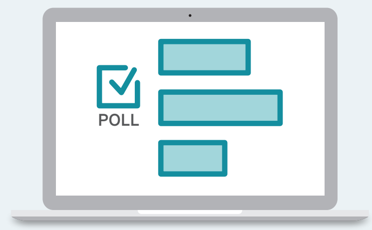
- *coefficient*
- *constant*
- *Distributive Property*
- *equation*
- *equivalent equations*
- *expression*
- *like terms*
- *Properties of Equality*
- *solution*
- *substitution*
- *term*
- *variable*

**Amps** | Featured Activity

**Warm-up**

**Take a Poll**

Digitally poll the class so that students can see which of their classmates' chosen number, if any, makes the equations true or false.




### Building Math Identity and Community

Connecting to Mathematical Practices

Students might show disinterest in the work of others as they share reasoning and strategies with one another in Activity 2. Prior to the sharing, review guidelines for social engagement. Emphasize how to show interest when others are speaking. Healthy communication in both directions will lead towards establishing healthy relationships.

### Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- Replace the Warm-up with **Activity 1**.
- In **Activity 2**, have students only complete Problems 1–4.

Independent | 10 min

# Warm-up True or False?

Students test different values to discover that equations can be always true or be always false.

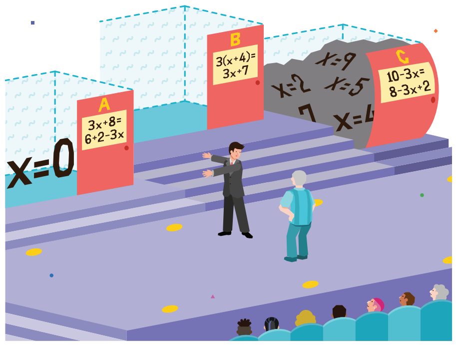
**Amps Featured Activity** Take a Poll

Name: ______ Date: ______ Period: ______

Unit 4 | Lesson 7

## How Many Solutions? (Part 1)

Let's think about how many solutions an equation can have.




### Warm-up True or False?

Choose any integer, decimal, or fraction to substitute for $x$ in each equation. Determine whether your number makes the equation true or false by placing a check mark in each box. Sample responses shown.

My number: 5

| | True | False |
|---|---|---|
| 1. $3x - 10 = -3x + 5 + 15$<br>$3(5) - 10 = -3(5) + 20$<br>$5 = 5$ | ✓ | |
| 2. $3(x + 4) = 3x + 7$<br>$3(5 + 4) = 3(5) + 7$<br>$3(9) = 15 + 7$<br>$27 = 22$ | | ✓ |
| 3. $10 - 3x = 8 - 3x + 2$<br>$10 - 3(5) = 8 - 3(5) + 2$<br>$10 - 15 = 8 - 15 + 2$<br>$-5 = -5$ | ✓ | |

For any value substituted into the equation in Problem 2, the equation will be false. For any value substituted into the equation in Problem 3, the equation will be true. Depending on the value substituted into the equation in Problem 1, the equation may be true or false. Here is an example of a value that gives a false statement: $x = 4$; false.

$3(4) - 10 = -3(4) + 5 + 15$

$2 = 8$

Log in to Amplify Math to complete this lesson online.





## 1 Launch

Have students choose a number to substitute in each equation. Encourage them to choose an integer, decimal, or fraction that they think will be unique. Then conduct the *True or False* routine.

## 2 Monitor

**Help students get started** by activating their prior knowledge and asking them how to determine whether a number makes an equation true or false.

**Look for productive strategies:**

- Responding *true* or *false* for Problem 1.
- Using the order of operations to evaluate the left and right sides of the equation.

## 3 Connect

**Have students share** their responses. For each equation, invite students to share their number and whether the equation was true or false. Record for all to see.

**Ask**, "Do you think there is a number that will make Equation 1/Equation 2/Equation 3 true?" yes/no/yes

**Highlight:**

- For Problem 1, there is only one value for $x$ that will make the equation true. That value is 5. This means there is only *one solution* for the equation.
- For Problem 2, there is no value of $x$ that will ever make the equation true. When there is no value of $x$ that makes an equation true, the equation has *no solution*.
- For Problem 3, any value of $x$ will always make the equation true. When any value of $x$ makes an equation true, the equation has *infinitely many solutions*.

## Math Language Development

### *MLR8: Discussion Supports — Revoicing*

During the Connect, as students share their responses, ask them to revise what their classmates shared using mathematical language. Ask the original speaker whether their peer accurately restated their thinking. For example, if a classmate says, "Problem 2 never works," a student could revoice this statement using mathematical language by saying, "The equation in Problem 2 is a false equation because there is no value for $x$ that makes both sides of the equation equal."

### English Learners

Provide wait time for students to formulate a response. Encourage students to rehearse with a partner before sharing with the class.

## Power-up

**To power up students' ability to solve equations containing multiple variable terms:**

Provide students with a copy of the Power-up PDF.

**Use:** Before Activity 1
**Informed by:** Performance on Lesson 6, Practice Problem 6

Pairs | 10 min

# Activity 1 Thinking About Solutions

Students solve equations to compare and contrast linear equations that have one solution, no solution, and infinitely many solutions.

**Activity 1** Thinking About Solutions

The three equations from the Warm-up are shown. Solve each equation. Show or explain your thinking.

1. $3x - 10 = -3x + 5 + 15$

   **Sample response:**

   $3x - 10 = -3x + 20$

   $6x - 10 = 20$

   $6x = 30$

   $x = 5$

2. $3(x + 4) = 3x + 7$

   **Sample response:**

   $3x + 12 = 3x + 7$

   $12 = 7$

   This equation will never be true for any value of $x$.

3. $10 - 3x = 8 - 3x + 2$

   **Sample response:**

   $10 - 3x = 10 - 3x$

   $10 = 10$

   This equation will always be true for any value of $x$.

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## 1 Launch

Ask students if they can think of another way to check for the number of solutions other than substituting different values of $x$. If no student suggests to solve the equation, tell them that solving is one way to determine the number of solutions for a linear equation.

## 2 Monitor

**Help students get started** by having them rewrite each side of the equation with fewer terms.

**Look for points of confusion:**

- **Struggling to solve for $x$ in Problem 2 or 3.** After writing the equation with fewer terms, suggest that students collect variables on one side. This will eliminate the variable and leave students with a false equation for Problem 2 and a true equation for Problem 3. Tell students to leave the equation as is, and revisit these equations during the whole-class discussion.

## 3 Connect

**Have students share** their solutions and their strategies for solving each equation.

**Highlight** that when students solve an equation, they rewrite the equation using equivalent equations. If the equivalent equation is of the form $x = a$, then the equation is true only for one number, so there is only *one solution*. If the equivalent equation is of the form $a = b$, where $a$ and $b$ are different values, then there are no values that make the equation true, so there is *no solution*. If the equivalent equation is of the form $a = a$, then any value makes this equation true, and so, there are *infinitely many solutions*.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the Anchor Chart PDF, *Solving Linear Equations* to support them in organizing their solution pathways. Consider displaying a table similar to the following for students to reference.

**After solving the equation, if the end result is . . .**

| An equation of the form $x = a$ | An equation of the form $a = a$ | An equation of the form $a = b$ |
|---|---|---|
| One solution | Infinitely many solutions | No solution |

**where $a$ and $b$ are numbers and $a$ does not equal $b$.**

## Math Language Development

### *MLR8: Discussion Supports — Press for Details*

During the Connect, press for details in students' reasoning. For example, if a student merely says, "Problem 2 has no solution," ask these questions:

- "How do you know the equation has no solution? Show me."
- "Did you try values for $x$? Did you try *all* the possible values for $x$? How else can you verify the equation has no solution?"
- "When you arrive at a statement that is always false, what does that tell you?"

### English Learners

Provide wait time for students to formulate a response. Encourage students to rehearse with a partner before sharing with the class.

Pairs | 20 min

# Activity 2 Looking for Solutions

Students solve linear equations to build fluency in determining the number of solutions for an equation.

Name: ______ Date: ______ Period: ______

**Activity 2** Looking for Solutions

Determine whether each equation has *one solution, no solution,* or *infinitely many solutions*. Show or explain your thinking.

1. $v + 2 = v + 4$
   No solution.
   Sample response:
   $2 = 4$
   This equation is never true for any value of $v$.

2. $-4 + 3x = -4 + 3x$
   Infinitely many solutions.
   Sample response:
   $-4 = -4$
   This equation is always true for any value of $x$.

3. $2t + 6 = 2(t + 3)$
   Infinitely many solutions.
   Sample response:
   $2t + 6 = 2t + 6$
   $6 = 6$
   This equation is alway true for any value of $t$.

4. $4x + 3 = -5x + 3$
   One solution.
   Sample response:
   $9x = 0$
   $x = 0$

5. $\frac{1}{2} + 5x = \frac{1}{3} + 5x$
   No solution.
   Sample response:
   $\frac{1}{2} = \frac{1}{3}$
   This equation is never true for any value of $x$.

6. $2(n - 1) = 10n + 6$
   One solution.
   Sample response:
   $2n - 2 = 10n + 6$
   $-2 = 8n + 6$
   $-8 = 8n$
   $n = -1$

STOP





## 1 Launch

Have students work individually to complete each problem, and then have them share responses with a partner. If there is a disagreement, have students work together to come to an agreement.

## 2 Monitor

**Help students get started** by having them use the Properties of Equality to solve each equation.

**Look for points of confusion:**

- **Thinking that their solution must be written as $x =$ ___.** Remind students that equations with no solution and infinitely many solutions may not be written in this form.
- **Confusing an equation with the solution $x = 0$ with an equation with no solution.** Use Problem 4 and have students substitute $x = 0$ to see if the equation is true or false. Remind students that $x = 0$ means that $0$ is the only value that will make the equation true, so there is one solution.

**Look for productive strategies:**

- Using the structure of the equation, instead of solving the equation, to determine the number of solutions. **Note:** Students will explore this concept further in Lesson 8.

## 3 Connect

**Highlight** that students can rewrite an equation until it is written with the fewest terms to determine the number of solutions for an equation.

**Ask** students to give examples of numbers that will make each equation true to develop further understanding of what it means for an equation to have one solution, no solution, or infinitely many solutions.

## Differentiated Support

**Accessibility:** ***Clarify Vocabulary and Visualization***

Display or provide copies of the Anchor Chart PDF, *Properties of Equality* for students to use as a reference during this activity.

**Accessibility:** ***Vary Demands to Optimize Challenge, Guide Processing and Visualization***

Before solving, invite students to create a flow chart diagram that describes what to look for to determine whether a linear equation has one solution, no solutions, or infinitely many solutions. If students need more processing time, have them focus on Problems 1–4 only.

**Extension:** ***Math Enrichment***

Have students complete the following equation three different ways so that one equation has one solution, one equation has no solution, and one equation has infinitely many solutions.

$3x + 8 =$ ___

Sample response:

- One solution: $3x + 8 = x - 5$
- No solution: $3x + 8 = 3x + 5$
- Infinitely many solutions: $3x + 8 = 3\left(x + \frac{8}{3}\right)$

Whole Class | 5 min

# Summary

Review and synthesize how to determine the number of solutions for any linear equation.

### Summary

**In today's lesson . . .**

You discovered that some equations have one solution, no solution, or infinitely many solutions.

Here are some examples.

| One solution: | No solution: | Infinitely many solutions: |
|---|---|---|
| $3x + 8 = 6 + 2 - 3x$<br>$3x + 8 = 8 - 3x$<br>$6x + 8 = 8$<br>$6x = 0$<br>$x = 0$ | $3(x + 4) = 3x + 7$<br>$3x + 12 = 3x + 7$<br>$12 = 7$ | $10 - 3x = 8 - 3x + 2$<br>$10 - 3x = 10 - 3x$<br>$10 = 10$ |
| This equation is only true when $x = 0$. | This equation is never true for any value of $x$. | This equation is always true for any value of $x$. |

Reflect:




## Synthesize

**Ask:**

- "What does it mean for an equation to have one/no/infinitely many solution(s)?" If an equation has one solution, only one number will make the equation true. If an equation has no solution, there is no number that will make the equation true. If there are infinitely many solutions, any number will make the equation true.
- "Do you think there is a linear equation with another type of solution other than one solution, no solution, or infinitely many solutions?" No; Sample response: When a linear equation is rewritten with the fewest terms, it could only be represented as $x = a\ number$ or $a\ number = a\ number$. Therefore, there are three different types of solutions possible.

**Have students share** how they know the number of solutions for an equation in their own words.

**Highlight** that linear equations could have one solution, no solution, or infinitely many solutions.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies did you find helpful today when determining the number of solutions for an equation?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by explaining how they know when an equation has one solution, no solution, or infinitely many solutions.

Printable

Name: ______________ Date: ________ Period: ________

## Exit Ticket 4.07

Explain how you know when a linear equation has . . .

a. One solution.

Sample response: The equation with the fewest number of terms will have a variable on one side and a constant on the other side. For example, $x = 5$.

b. No solution.

Sample response: The equation with the fewest number of terms will have different values on the left and right side. For example, $7 = 10$.

c. Infinitely many solutions.

Sample response: The equation with the fewest number of terms will have the same values on the left and right side. For example, $9 = 9$.

**Self-Assess**

| 1 | 2 | 3 |
|---|---|---|
| I don't really get it | I'm starting to get it | I got it |

a. I can determine whether a linear equation has one solution, no solution, or infinitely many solutions.

1 2 3





## Success looks like . . .

- **Goal:** Correlating equations that are *never true* as equations with *no solution* and equations that are *always true* as equations with *infinitely many solutions*.
  - » Describing the conditions for a linear equation to have one solution, no solution, and infinitely many solutions.
- **Language Goal:** Describing a linear equation as having one solution, no solution, or an infinite number of solutions, and solving equations in one variable with one solution. **(Speaking and Listening)**

## Suggested next steps

**If students do not correctly describe when a linear equation has one solution, no solution, or infinitely many solutions, consider:**

- Giving them three solved equations, each with a different type of solution, and asking them to identify which equation has one solution, no solution, or infinitely many solutions.
- Reassessing after Lesson 8.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- Which students' ideas were you able to highlight during Activity 1?
- In earlier lessons, students learned how to balance equations. How did that support their understanding of equations with no solution and infinitely many solutions?

# Practice

Independent

Practice

Name: ____________ Date: ________ Period: ________

1. For each equation, decide if it has one solution, no solution, or infinitely many solutions. Show or explain your thinking.

   a. $x - 13 = x + 1$
   No solution.
   Sample response:
   $-13 = 1$
   This equation is never true for any value of $x$.

   b. $x + \frac{1}{2} = x - \frac{1}{2}$
   No solution.
   Sample response:
   $\frac{1}{2} = -\frac{1}{2}$
   This equation is never true for any value of $x$.

   c. $2(x + 3) = 5x + 6 - 3x$
   Infinitely many solutions.
   Sample response:
   $2x + 6 = 2x + 6$
   $6 = 6$
   This equation is always true for any value of $x$.

   d. $-(7 - 5x) = 6x - 3$
   One solution.
   Sample response:
   $-7 + 5x = 6x - 3$
   $-7 + 3 = x$
   $x = -4$

2. Elena says the equation $3x + 10 = 5x - 4$ has no solution because when she substitutes each of the following numbers, all of the equations are false. Her work is shown.

   Elena's work:

| | | |
|---|---|---|
| $x = 2$ | $x = -6$ | $x = 0.5$ |
| $3(2) + 10 = 5(2) - 4$ | $3(-6) + 10 = 5(-6) - 4$ | $3(0.5) + 10 = 5(0.5) - 4$ |
| $6 + 10 = 10 - 4$ | $-18 + 10 = -30 - 4$ | $1.5 + 10 = 2.5 - 4$ |
| $16 = 6$ | $-8 = -34$ | $11.5 = -1.5$ |
| False | False | False |

   Do you agree with Elena? Explain your thinking.
   Sample response: I do not agree with Elena. I solved the equation:
   $3x + 10 = 5x - 4$
   $10 = 2x - 4$
   $14 = 2x$
   $x = 7$
   Because the solution is $x = 7$, I know that 7 is the only number that will make the equation true, so the equation has one solution. Elena substituted three different numbers that each happened to not be a solution to the equation.




Practice

Name: ____________ Date: ________ Period: ________

3. Kiran solved the equation $2(x - 3) = 8x - 6$ and got the answer "$x = 0$, no solution." Do you agree with Kiran's answer? Explain your thinking.
   Sample response: I do not agree with his answer. Although the answer $x = 0$ is correct, this means there is only one solution, 0, that will make the equation true. Writing "no solution" means that there is no number, not even 0, that makes the equation true.

4. Solve each equation. Show or explain your thinking. Sample strategies shown.

   a. $3x - 6 = 4(2 - 3x) - 8x$
   $3x - 6 = 8 - 12x - 8x$
   $3x - 6 = 8 - 20x$
   $23x - 6 = 8$
   $23x = 14$
   $x = \frac{14}{23}$

   b. $\frac{1}{2}z + 6 = \frac{3}{2}(z + 6)$
   $\frac{1}{2}z \cdot 2 + 6 \cdot 2 = 3(z + 6)$
   $z + 12 = 3z + 18$
   $-2z + 12 = 18$
   $-2z = 6$
   $z = -3$

5. The point $(-2, 3)$ is on a line that has a slope of 2.

   a. Write an equation for the line. Show or explain your thinking.
   I need to write the equation in the form $y = mx + b$ using the slope, 2. I substitute the slope and the coordinates of the points into the equation $y = mx + b$ and solve the equation for $b$:
   $3 = 2(-2) + b$
   $3 = -4 + b$
   $7 = b$ So, the equation is $y = 2x + 7$.

   b. Determine two more points on the line.
   Sample response: $(-1, 5)$ and $(0, 7)$

6. Several students are asked to identify the coefficient and constant in the expression $-5 + 3x + 12$. Select the statement that correctly identifies the coefficient and constant term.

   A. Jada: "The coefficient is $-5$, and the constant is 12."
   B. Shawn: "The coefficient is 3, and the constant is 7."
   C. Diego: "The coefficient is 12, and the constant is $-5$.
   D. Priya: "The coefficient is $3x$, and the constant is 7."
   E. Clare: "The coefficient is 7, and the constant is 3."




## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 1 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 1 | 2 |
| Spiral | 4 | Unit 4 Lesson 6 | 1 |
| | 5 | Unit 3 Lesson 11 | 2 |
| Formative | 6 | Unit 4 Lesson 8 | 1 |

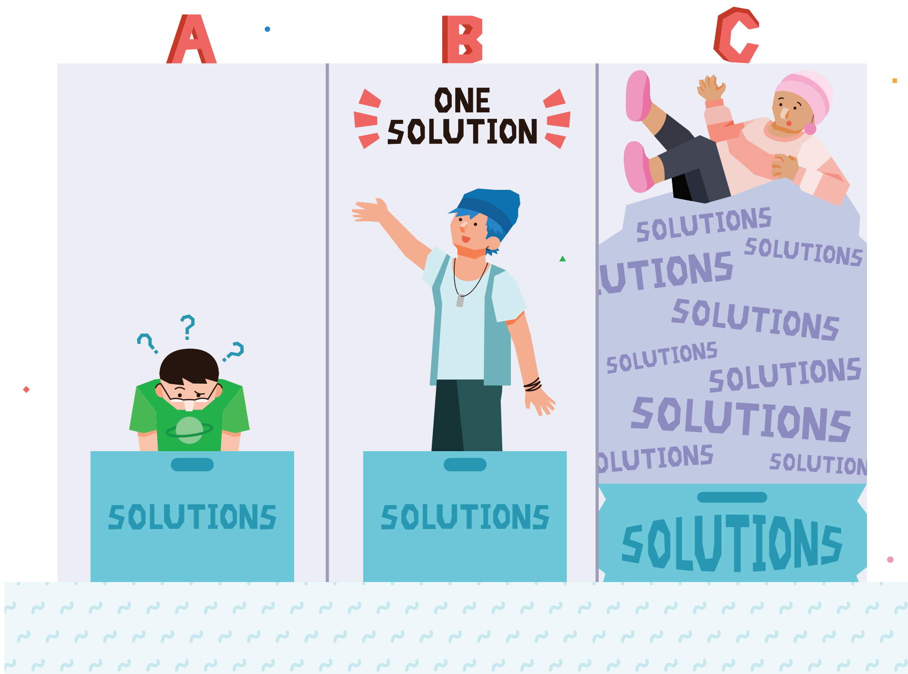
**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# How Many Solutions? (Part 2)

Let's solve equations with different numbers of solutions.




## Focus

### Goals

1. **Language Goal:** Compare and contrast the structure of linear equations that have no solution or infinitely many solutions. **(Speaking and Listening, Writing)**
2. Create linear equations in one variable that have either no solution or infinitely many solutions.

## Rigor

- Students use the structure of linear equations to identify whether an equation has one solution, no solution, or infinitely many solutions to develop **procedural fluency**.

## Coherence

### Today

Students compare linear equations to see that the structure of equations could be used to identify the number of solutions. Students create their own equations to think strategically about which coefficients and constants in a linear equation will result in one solution, no solution, or infinitely many solutions.

### Previously

In Lesson 7, students solved linear equations to determine whether an equation had one solution, no solution, or infinitely many solutions.

### Coming Soon

In Lesson 9, students will practice solving all types of linear equations to build fluency in solving equations.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 7 min | 12 min | 15 min | 5 min | 5 min |
| Independent | Small Groups | Small Groups | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- index cards
- plain sheets of paper

### Math Language Development

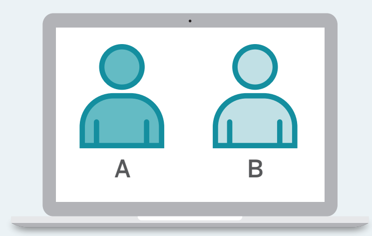
**Review words**

- *coefficient*
- *constant*
- *Distributive Property*
- *equation*
- *equivalent equations*
- *expression*
- *like terms*
- *Properties of Equality*
- *solution*
- *substitution*
- *term*
- *variable*

**Amps** | Featured Activity

**Activity 2**
**Digital Collaboration**

Students create three equations and challenge their classmates to match each equation with its number of solutions.




Amps powered by desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Working in groups during Activity 1 might be intimidating for some students, especially as they are identifying each other's mistakes and correcting them. Throughout the activity, students should show each other respect. More importantly, if a member of the group is nervous, the other members should empathize with the discomfort and behave in a way that comforts that member through encouragement.

### Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- **Activity 2** may be omitted.

Independent | 7 min

# Warm-up Making Use of Structure

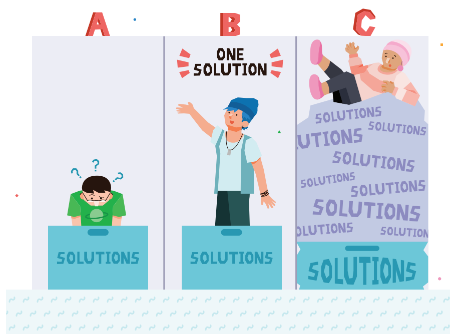
Students compare and contrast linear equations to see that the structure of equations could be used to identify the number of solutions for the equation.

Name: ______ Date: ______ Period: ______

Unit 4 | Lesson 8

## How Many Solutions? (Part 2)

Let's solve equations with different numbers of solutions.




### Warm-up Making Use of Structure

Refer to these equations from a previous lesson.

| One solution | No solution | Infinitely many solutions |
|---|---|---|
| $4x + 3 = -5x + 3$ | $v + 2 = v + 4$ | $-4 + 3x = -4 + 3x$ |
| $2(n - 1) = 10n + 6$ | $\frac{1}{2} + 5x = \frac{1}{3} + 5x$ | $2t + 6 = 2(t + 3)$ |
| $3x + 8 = 6 + 2 - 3x$ | $3(x + 4) = 3x + 7$ | $10 - 3x = 8 - 3x + 2$ |

What patterns do you notice when a linear equation has:

a. One solution?

**Sample response: There are a different amount of variables on the left side and right side.**

b. No solution?

**Sample response: There are the same amount of variables on the left side and right side.**

c. Infinitely many solutions?

**Sample response: There are the same amount of variables and constants on the left side and right side.**

Log in to Amplify Math to complete this lesson online.



Lesson 8 How Many Solutions? (Part 2) 405

## 1 Launch

Tell students that in this lesson, they will look more closely at the equations they solved in Lesson 7. Say, "Mathematics does not always require solving for an answer. It involves analyzing problems to look for patterns."

## 2 Monitor

**Help students get started** by having them circle the coefficients and underline the constants on both sides.

**Look for points of confusion:**

- **Thinking that the coefficients and constants are different when the equation has more than two terms on each side.** Have students write each side of the equation in fewer terms and then compare the left and right side of the equation.

## 3 Connect

**Highlight** that students do not need to solve a linear equation to determine the number of solutions it will have. Instead, they can look at the structure of the equation to determine the number of solutions.

- If a linear equation has the same coefficients and constants on both sides, it will have *infinitely many solutions*.
- If a linear equation has the same coefficients, but different constants on both sides, it will have *no solution*.
- If a linear equation has different coefficients and the same or different constants on both sides, it will have *one solution*.

**Ask** students for which equations they had difficulty in identifying patterns. Then, ask what students could do to identify a pattern more easily. I could write an equivalent equation with fewer terms.

## Power-up

**To power up students' ability to identify the constant and the coefficient from an equation, have students complete:**

Recall that a constant is a value that does not change, such as 2 or $-\frac{2}{3}$. A *coefficient* is a constant by which a variable is multiplied. For example, in the expression $3x$, 3 is a *coefficient*. For each expression, identify the constant and the coefficient.

| | $-2x + 1$ | $7 + g$ | $4 - 2h$ |
|---|---|---|---|
| **Constant** | 1 | 7 | 4 |
| **Coefficient** | $-2$ | 1 | $-2$ |

**Use:** Before Activity 1

**Informed by:** Performance on Lesson 7, Practice Problem 6

Small Groups | 12 min

# Activity 1 Three Responses!

Students use the structure of linear equations to identify if the equation has one solution, no solution, or infinitely many solutions.

**Activity 1** Three Responses

With your group, decide who will complete Column A, who will complete Column B, and who will complete Column C.

For each problem, without solving each equation, determine whether there will be *one solution*, *no solution*, or *infinitely many solutions*. After each row, share your responses with your group. For each row, your group should have three *different* responses. If there is an error, work together to solve the equation and correct your responses.

| | Column A | Column B | Column C |
|---|---|---|---|
| 1. | $6x + 8 = 8 + 6x$ Infinitely many solutions | $6x + 8 = 6x + 13$ No solution | $6x + 8 = 7x + 13$ One solution |
| 2. | $5x + 3x + 12 = 8x - 4$ No solution | $5x - 4x - 2 = -6x + 12$ One solution | $-5x + 2 - 3x = -8x + 2$ Infinitely many solutions |
| 3. | $12r - 6 + 12r = -6$ One solution | $-3(4r - 2) = -12r + 6$ Infinitely many solutions | $\frac{1}{4}(12 - 4r) = 6 - r$ No solution |
| 4. | $4n + 4n - 6 = 8n - 8$ No solution | $4n + 2(2n - 3) = 2(4n - 3)$ Infinitely many solutions | $4(2x - 2) = -8(x - 2)$ One solution |
| 5. | $-6 + 9c + c = 10c - 6$ Infinitely many solutions | $c + 3(2 + 3c) = 4(c - 6)$ One solution | $c - 3(2 - 3c) = 2(5c + 3)$ No solution |

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## 1 Launch

In groups of three, have each student choose a different column to complete. For each row, each group should have three *different* responses. If there is an error, have students solve their equations to identify their mistake and to correct their responses.

## 2 Monitor

**Help students get started** by having them look at the coefficients and constants to identify whether their equation has one solution, no solution, or infinitely many solutions.

**Look for points of confusion:**

- **Thinking that the equation for Problem 3 in Column A has infinitely many solutions.** Students may think that, if an equation has the same number of coefficients and constants, then there are infinitely many solutions. Circle the equal sign and remind students that they should be comparing the left side with the right side of the equation.
- **Not realizing that the equation for Problem 4 in Column C has one solution.** Students may think that $8x$ on the left side and $-8x$ on the right side will result in an equation with no solution. Point out that the signs of the coefficients are different, and then ask how the different coefficients will affect the number of solutions.

## 3 Connect

**Ask** students what they did to identify the number of solutions.

**Highlight** that students can rewrite each equation by using fewer terms and then compare the coefficients and constants on each side of the equation to determine the number of solutions of the equation.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Provide students with a checklist to keep track of the expectations of the task. For example, display the following for students:

- ☐ Highlight or circle your assigned *column*.
- ☐ When determining the number of solutions, be sure to compare both sides of the equal sign. Circle the equal sign as a reminder.
- ☐ Compare the solutions of each *row* with your group. If there is disagreement, solve the equation using inverse operations and revise your responses.

### Extension: *Math Enrichment*

Have students choose one of the equations they identified as having no solution and alter it so that it has one solution. Then have them alter it a different way so that it has infinitely many solutions.

## Math Language Development

### *MLR7: Compare and Connect*

Ask group members to describe how they determined the number of solutions by studying the equations. Ask:

- "If the coefficients are the same and the constants are different, what do you notice?"
- "What do you notice when the coefficients and constants are both the same?"

### English Learners

Consider providing a graphic organizer for students to keep track of the different scenarios, such as, when both the coefficients and constants are the same, or when the coefficients are the same, but the constants are different.

Small Groups | 15 min

# Activity 2 Trading Equations, Revisted

Students use the structure of equations to create and identify linear equations with one solution, no solutions, or infinitely many solutions.

**Amps Featured Activity** Digital Collaboration

Name: ______ Date: ______ Period: ______

## Activity 2 Trading Equations, Revisited

**You will be given an index card and a plain sheet of paper.**

1. On the index card, complete each equation so that one equation has one solution, one equation has no solution, and one equation has infinitely many solutions. On the back of the index card, solve the equation that has one solution.

   a. $12x + \square = \square(4x + \square)$ Sample responses shown for parts a–c.

   $12x + 15 = 3(4x + 5)$; Infinitely many solutions

   b. $2x - \square = \square x + \square$

   $2x - 10 = 2x + 15$; No solution

   c. $\square x + 5 = \square x - \square x + 5$

   $6x + 5 = 4x - 3x + 5$; One solution

   $6x + 5 = x + 5$

   $5x + 5 = 5$

   $5x = 0$

   $x = 0$

2. Trade index cards with a partner, without telling them the number of solutions.

3. Using your partner's equations, decide which equation has *one solution*, *no solution*, and *infinitely many solutions*. For the equation that has one solution, solve the equation to determine the value of $x$ that makes the equation true. Once you have made your decisions, check with your partner to see whether you are correct.

**Are you ready for more?**

**Use whole numbers 1–10 to replace the boxes so that you create three different equations: one equation with one solution, one equation with no solution, and one equation with infinitely many solutions. Use each number only once for each equation.**

$\square x + \square x + \square = \square(x + \square)$

Sample response:

One solution: $1x + 2x + 3 = 5(x + 4)$

No solution: $3x + 1x + 2 = 4(x + 5)$

Infinitely many solutions: $1x + 3x + 8 = 4(x + 2)$

STOP





## 1 Launch

Distribute an index card and a plain sheet of paper to each student. Have students write their three equations on the front of the index card and solve the equation with one solution on the back of their index card. Once students write their equations, have them switch cards with a partner. Tell students to use the extra paper to show their thinking. Allow students to switch index cards with additional partners, as time allows.

## 2 Monitor

**Help students get started** by telling them that they should have three equations, each with a different number of solutions.

**Look for points of confusion:**

- **Struggling to create equations with different numbers of solutions.** Have students look at the first row of the Warm-up. Using two different colors, highlight the coefficients and constants in an equation to emphasize when an equation has one solution, no solution, and infinitely many solutions. Allow students to use this as a reference when finishing the three equations.

## 3 Connect

**Have students share** their strategies for identifying each type of solution.

**Ask** students if anyone was able to create an equation with no solution using the equation in Problem 1c. Discuss reasons why the equation could have only one solution or infinitely many solutions. The constants are the same.

**Highlight** that students can use the structure of an equation to identify the number of solutions.

## Differentiated Support

### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create three equations and challenge their classmates to match each equation with its correct number of solutions.

### Accessibility: *Vary Demands to Optimize Challenge*

Consider one of these alternative approaches to this activity:

- Provide 6 equations from which students can choose, two equations from each category.
- Remove the restrictions on the other values and allow students to write any equation, as long as they meet the given criteria.

## Math Language Development

### *MLR3: Critique, Correct, Clarify*

During the Connect, display an incorrectly identified equation, such as "The equation $10x + 6 = 2(5x + 3)$ has no solution." Ask:

- ***Critique:*** "Do you agree with this statement? Why or why not?"
- ***Correct:*** "Write a revised statement that is correct."
- ***Clarify:*** "How did you revise the statement? Did you choose to alter the equation or did you choose to alter the number of solutions the equation has? How can you verify that your statement is correct?"

### English Learners

Allow students to share their revised statements with a partner before sharing with the whole class.

Whole Class | 5 min

# Summary

Review and synthesize the features of linear equations with one solution, no solution, or infinitely many solutions.

### Summary

**In today's lesson . . .**

You discovered that the structure of an equation can tell you if the equation has one solution, no solution, or infinitely many solutions.

**Infinitely many solutions:** A linear equation has *infinitely many solutions* when the coefficients and constants are the same on each side. For example, the equation $2x + 5 = 2x + 5$ has infinitely many solutions.

**No solution:** A linear equation has *no solution* when the coefficients are the same, but the constants are not the same on each side. For example, the equation $2x + 5 = 2x + 10$ has no solution.

**One solution:** A linear equation has *one solution* when the coefficients are different on each side. The constants may or may not be the same on each side. For example, the equation $2x + 5 = 3x + 10$ has one solution.

> Reflect:

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## Synthesize

**Ask:**

- "What are two different ways in which you can determine the number of solutions for a linear equation?" Sample response: Solve the equation or look at the structure of the equation.
- "How many solutions does $5x - 8 = 5x + 8$ have?" no solution Point out that it is important to pay attention to the signs when looking at the structures of equations. In this example, the signs before the constants are different.

**Have students share** whether they prefer solving the equation or looking at the structure of the equation to identify the number of solutions it has and why.

**Highlight** that before students solve an equation, they could look at the structure of an equation to determine how many solutions it has.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you determine the solution to an equation with variables on both sides?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by using the structure of equations to identify the number of solutions to three different equations.

Printable

Name: ______________ Date: ________ Period: ________

## Exit Ticket 4.08

Without solving, determine whether each equation has *one solution*, *no solution*, or *infinitely many solutions*. Explain your thinking.

a. $3(x-6)+2x=3x-18+2x$

Infinitely many solutions. Sample response: When I rewrote the left and right sides with fewer terms, $5x-18=5x-18$, I noticed that the coefficients and constants are the same on each side, so any number will make the equation true.

b. $3(x-6)+2x=3x+18+2x$

No solution. Sample response: When I rewrote the left and right sides with fewer terms, $5x-18=5x+18$, I noticed that the coefficients are the same on each side, but the constants are different, so there is no number that will make the equation true.

c. $3(x-6)+2x=3x-18+6x$

One solution. Sample response: When I rewrote the left and right sides with fewer terms, $5x-18=9x-18$, I noticed that the coefficients are different and the constants are the same, so only one solution, $x=0$, will make the equation true.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can determine whether an equation has one solution, no solution, or infinitely many solutions, without solving the equation.

1 2 3



**Lesson 8** How Many Solutions? (Part 2)

## Success looks like . . .

- **Language Goal:** Comparing and contrasting the structure of linear equations that have no solution or infinitely many solutions. **(Speaking and Listening, Writing)**
  - » Explaining why a given linear equation has one solution, no solution, or infinitely many solutions.
- **Goal:** Creating linear equations in one variable that have either no solution or infinitely many solutions.

## Suggested next steps

**If students incorrectly identify the number of solutions for the equations, consider:**

- Rewriting each equation using the fewest terms and reassessing.
- Reassessing after Lesson 15.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is?
- Which groups of students did and didn't have their ideas seen and heard today?

# Practice

Independent

Name: ______ Date: ______ Period: ______

**1.** Consider the unfinished equation $12(x-3)+18=$ ______.
Complete the equation so that it has:

**a** One solution.
**Sample response: $12(x-3)+18=5x+10$**

**b** No solution.
**Sample response: $12(x-3)+18=12x+10$**

**c** Infinitely many solutions.
**Sample response: $12(x-3)+18=12x-18$**

**2.** Decide whether each equation has *one solution*, *no solution*, or *infinitely many solutions*. If an equation has one solution, solve the equation to determine the value of $x$ that makes the equation true.

**a** $6x-4=-4+6x$
**Infinitely many solutions**

**b** $4x-6=4x+3$
**No solution**

**c** $-2x+4=-3x+4$
**One solution**
**Sample response:**
$x+4=4$
$x=0$

**d** $10x+\frac{1}{4}=10x+\frac{2}{3}$
**No solution**

**e** $3(2x-5)=4x+2x-15$
**Infinitely many solutions**

**f** $-4\,(x-2)=-2\left(x-\frac{17}{2}\right)$
**One solution**
$-4x+8=-2x+17$
$-2x+8=17$
$-2x=9$
$x=-\frac{9}{2}$




Name: ______ Date: ______ Period: ______

**3.** For each problem, determine whether you agree with the statement made by each person. Explain your thinking.

**a** Lin studied the equation $2x-32+4(3x-25)=14x$. She said, "I can tell right away there is no solution because, on the left side, you will have $2x+12x$ and a bunch of constants, but you have just $14x$ on the right side."
**Sample response: Yes, I agree with Lin. On the left side of the equation, I know that $2x$ and $4(3x)$ will give a total of $14x$. Because all of the constants on the left side are negative, they will not be equivalent to the constant of 0 on the right side.**

**b** Han studied the equation $6x-4+2(5x+2)=16x$. He said, "I can tell right away there is no solution because, on the left side, you will have $6x+10x$ and a bunch of constants, but you have just $16x$ on the right side."
**Sample response: No, I do not agree with Han. On the left side of the equation, I know that $6x$ and $2(5x)$ will give a total of $16x$. The constants on the left side will give $-4+2(2)$, which is 0. Because the $x$s and constants are the same on the left side and right side, there will be infinitely many solutions.**

**4.** The points $(-2, 2)$ and $(0, -6)$ lie on the graph of the same linear equation. Does the point $(2, 6)$ also lie on the graph of this linear equation? Explain your thinking.
**No. Sample response: The $y$-intercept is $(0,-6)$ and the slope is $-4$ because $\frac{-6-2}{0-(-2)}=\frac{-8}{2}=-4$. So, the equation of the line is $y=-4x-6$.**
**When I substitute $(2, 6)$ into the equation, the equation is not true:**
$6=-4(2)-6$
$6=-14$
**Therefore, I know that the point $(2, 6)$ is not a point on the line.**

**5.** What strategies can you use to determine whether $x=5$ is a solution to the equation $2(x-3)=2x+6$?
**Sample response:**
- **Substitute the value 5 for $x$ and see if the equation is true.**
- **Rewrite the equation as $2x-6=2x+6$ and look at the structure of the equation. I know there is no solution because the coefficients are the same, but the constants are different on each side.**




## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 3 Lesson 11 | 2 |
| Formative | 5 | Unit 4 Lesson 9 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Strategic Solving

Let's practice solving linear equations.


## Focus

### Goal

1. **Language Goal:** Describe strategies for solving linear equations in one variable with different features or structures. **(Speaking and Listening)**

## Coherence

### Today

Students complete a scavenger hunt to determine solutions of linear equations. As they solve equations, students make use of structure and strengthen their fluency in solving equations.

### Previously

In Lessons 7 and 8, students learned that linear equations are not limited to one solution, but could have no solution or infinitely many solutions.

### Coming Soon

In Lesson 10, students will discover that they can set two expressions equal to each other to find when two amounts are equal in context.

## Rigor

- Students strengthen their **fluency** in solving equations.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Summary | Exit Ticket |
|---|---|---|---|
| 10 min | 30 min | 5 min | 5 min |
| Pairs | Independent | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards (for display)
- Anchor Chart PDF, *Solving Linear Equations*

### Math Language Development

**Review words**

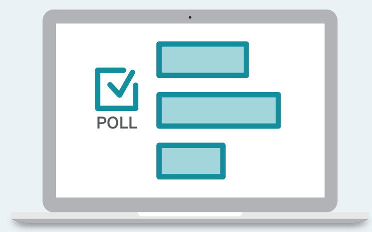
- *coefficient*
- *constant*
- *Distributive Property*
- *equation*
- *equivalent equations*
- *expression*
- *hanger diagram*
- *like terms*
- *Properties of Equality*
- *solution*
- *substitution*
- *term*
- *variable*

**Amps** | **Featured Activity**

**Warm-up**

**Take a Poll**

See what your students are thinking in real time by digitally polling the class.




Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students might become so excited about the scavenger hunt that they disregard those around them. Before they begin the activity, have students work together to determine ways they can analyze the structure of each equation — before solving it — to determine whether there is one, none, or infinitely many solutions. Remind students that checking their work is a way to identify whether they have solved a problem correctly, and encourage them to help each other check their equations.

### Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be omitted.

Pairs | 10 min

## Warm-up Predicting Solutions

Students shift the focus from solving equations to thinking about how the structure of an equation changes the solution.

**Amps Featured Activity** Take a Poll

Name: ________ Date: ________ Period: ________

**Unit 4 | Lesson 9**

# Strategic Solving

Let's practice solving linear equations.


**Warm-up** Predicting Solutions

Without solving, identify whether each equation has a solution that is *positive*, *negative*, or *zero*. Be prepared to explain your thinking.

1. $12x = 1.63$
   Positive. Sample response: To produce a positive value, I must multiply 12 by a positive number.

2. $-2 + 11x = -24$
   Negative. Sample response: If I multiply 11 by a positive number or 0, I will get a number greater than −2, but −24 is less than −2.

3. $\frac{x}{6} = \frac{3x}{4}$
   Zero. Sample response: The equation asks for a number so that $\frac{1}{6}$ of that number is equal to $\frac{3}{4}$ of the number. The only number for which this can be true is 0.

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Instruct students to inspect each equation carefully and use reasoning to answer the question rather than trying to solve each equation for a specific value. Then conduct the *Think-Pair-Share* routine.

### 2 Monitor

**Help students get started** by activating prior knowledge of operations with integers. For example, a negative number divided by a positive number will result in a negative number.

**Look for points of confusion:**

- **Not understanding how to determine the sign of the answer without solving the equation.** Provide different strategies that students can use. For example, ask students to substitute a positive or negative value for $x$ and think about the sign of the outcome.

### 3 Connect

**Have pairs of students share** the strategies they used to determine whether the solution was positive, negative, or zero. After each pair shares their strategy, ask if anyone else thought about the problem in a different way, and invite them to share their thinking.

**Highlight** that students can look at the structure of an equation and the properties of integers to help them think about the solution to an equation without solving it.

**Ask**, "How can you apply your strategies to help you when solving equations?" Sample response: After I solve an equation, I can look at the sign of my solution and reason whether it makes the equation true.

## Differentiated Support

**Accessibility: *Activate Prior Knowledge***

Before revealing the Warm-up, ask, "What happens when you multiply or divide a negative number by a positive number? What about multiplying or dividing a negative number by a negative number?" Once students come to a consensus, reveal the Warm-up.

## Power-up

**To power up students' ability to determine whether a value is a solution to an equation with more than one variable term, have students complete:**

Recall that when checking if a value is a solution to an equation, you can substitute the given value for each variable in the equation.

Determine whether $x = 3$ is the solution to the equation $-2(x - 4) + 1 = 2x - 3$.

Yes, it is a solution; Sample response:

$$-2((3) - 4) + 1 = 2(3) - 3$$
$$-2(-1) + 1 = 6 - 3$$
$$2 + 1 = 3$$
$$3 = 3 \text{ true}$$

**Use:** Before the Warm-up

**Informed by:** Performance on Lesson 8, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 5

Independent | 30 min

# Activity 1 Equations Scavenger Hunt

Students practice solving all types of linear equations to develop procedural fluency.

**Activity 1** Equations Scavenger Hunt

**Begin with any of the scavenger hunt cards and solve the problem, using the space provided here. Then look for your answer at the top of another scavenger hunt card and solve the problem on that card. Note: Use the hanger diagrams in the last box, if it helps your thinking.** Sample responses shown.

**Problem C**

How many solutions does this equation have?

$3x + 6 - 5x = 5x - 3x + 6$

$-2x + 6 = 2x + 6$

Because the coefficients are different on each side, there will be only one solution.

Answer: One solution

**Problem E**

Solve the equation.

$\frac{2}{3}x - 13 = 2\left(\frac{1}{3}x - 5\right) - 3$

$\frac{2}{3}x - 13 = \frac{2}{3}x - 10 - 3$

$\frac{2}{3}x - 13 = \frac{2}{3}x - 13$

Because the coefficients and constants are the same on each side, the equation will always be true for any value of $x$.

Answer: Infinitely many solutions

**Problem G**

Solve the equation.

$3(4 - 2x) + 6 = 4 - 2x$

$12 - 6x + 6 = 4 - 2x$

$18 - 6x = 4 - 2x$

$-4x = -14$

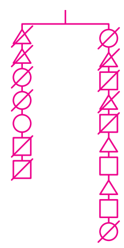
$x = \frac{14}{4}$

Answer: $\frac{7}{2}$

**Problem B**

A balanced hanger diagram is shown. A circle weighs 10 g, and a square weighs 2 g.

How many grams does each triangle weigh?



$10 = 2x + 4$

Answer: 3

**Problem I**

Solve the equation.

$x - 2(x + 5) = x - 10$

$x - 2x - 10 = x - 10$

$-x - 10 = x - 10$

$-2x = 0$

$x = 0$

Answer: 0

**Problem A**

Solve the equation.

$5(x - 7) = 3x + 7$

$5x - 35 = 3x + 7$

$2x = 42$

$x = 21$

Answer: 21

(continued)

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## 1 Launch

Shuffle the cards from the Activity 1 PDF, and post them around the classroom. If possible, laminate or use sheet protectors to facilitate reuse with other classes. Have students start with any of the *Scavenger Hunt* cards. Invite students to solve the problem and record their thinking, and then look for their answer at the top of a different *Scavenger Hunt* card and solve the problem on that card. This process continues until the students have solved all 10 problems and are back to their starting problem.

**Note:** Problem B and Problem D require students to solve for a missing weight in a hanger diagram. Tell students that they can write on the hanger diagrams provided at the end of their recording sheet if it helps them with their thinking.

## 2 Monitor

**Help students get started** by assigning a few students to different *Scavenger Hunt* cards and having them solve the problem on that card. Model how to find the next problem. Tell students if they do not see their answer, they should check their answer for any mistakes.

**Look for points of confusion:**

- **Not knowing how to solve a certain equation.** Encourage students to refer to the Anchor Chart PDF, *Solving Linear Equations*.
- **Struggling to find their answer or getting the wrong solution.** Remind students how to check their answers by substituting the value for $x$ in the original equation and evaluating it.

**Look for productive strategies:**

- Analyzing the structure of the equation, instead of solving it, to determine whether there is one, none, or infinitely many solutions.
- For the hanger diagrams, writing an equation and rewriting it with fewer terms before substituting the weight of the object.

Activity 1 continued ›

## Differentiated Support

### Accessibility: *Activate Prior Knowledge*

Display the hanger diagrams from the end of the activity. Ask them what they recall from working with hanger diagrams, paying attention to explanations about balancing the diagrams and assigning weights. Encourage students to use the diagrams as they complete the activity.

### Extension: *Math Enrichment*

Display the equation for Problem H. Ask, "Will multiplying both sides of the equation by 3 eliminate both fractions? Why or why not?" No; It will only eliminate the fraction with the denominator of 3. To eliminate the other fraction, I then need to multiply both sides by 2, or I could multiply both sides of the original equation by 6.

## Math Language Development

### *MLR8: Discussion Supports*

Assign pairs of students to different Scavenger Hunt sheets from the Activity 1 PDF. Before solving any of the equations, ask pairs to reason about the number of solutions (one, infinitely many, or none). After students have determined and agreed upon the number of solutions, invite them to solve their respective equations.

### English Learners

Provide students independent think time to formulate a response before discussing with their partner.

# Activity 1 Equations Scavenger Hunt (continued)

Students practice solving all types of linear equations to develop procedural fluency.

Name: Date: Period:

## Activity 1 Equations Scavenger Hunt (continued)

**Problem J**

What value will make this equation always true?

$12 - 8x = 2(6 + \boxed{\phantom{0}}\, x)$

Answer: $-4$

**Problem F**

Solve the equation.

$-5(x + 9) = -3(x - 8) - 2x$

$-5x - 45 = -3x + 24 - 2x$

$-5x - 45 = -5x + 24$

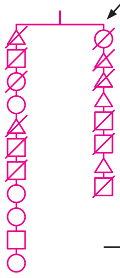
I noticed the coefficients are the same and the constants are different, so this equation will never be true for any value of $x$.

Answer: No solution

**Problem D**

A balanced hanger diagram is shown. A circle weighs 3 g, and a square weighs 2 g.

How many grams does each triangle weigh?



$12 + 2 = 2x$

Answer: 7

**Problem H**

Solve the equation.

$$\frac{2}{3}x - 1 = \frac{1}{2}x + 1$$

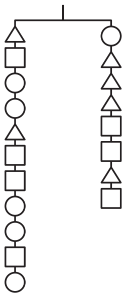
$$\frac{2}{3}x \cdot 6 - 1 \cdot 6 = \frac{1}{2}x \cdot 6 + 1 \cdot 6$$

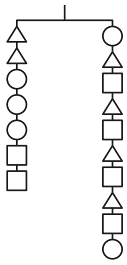
$$4x - 6 = 3x + 6$$

$$x = 12$$

Answer: 12

Use these diagrams if they help you with your thinking.





Reflect: During the scavenger hunt, how well did you view this challenge as an opportunity to stretch your knowledge?

STOP





## 3 Connect

**Ask** students whether there were any equations they had trouble solving and what strategies or tools helped them work through the problem.

**Have students share** the strategies they used to solve an equation with fractions and parentheses, such as Problem E.

**Highlight** the different strategies that students can use to solve equations. For fractions, they can determine a common denominator and multiply each side of the equation by the denominator to eliminate the fractions. For equations with parentheses and multiple terms, they could apply the Distributive Property and combine like terms to rewrite an equivalent expression with fewer terms.

Whole Class | 5 min

# Summary

Review and synthesize how students can apply different strategies for solving equations.

## Summary

**In today's lesson . . .**

You showed how to apply strategies for solving linear equations that include fractions, decimals, negative values, and equations written with many terms.

For example:

| | |
|---|---|
| $3(4 - 2x) + 6 = 4 - 2x$ | |
| $12 - 6x + 6 = 4 - 2x$ | Use the Distributive Property. |
| $18 - 6x = 4 - 2x$ | Combine like terms on each side. |
| $18 = 4 + 4x$ | Add $6x$ to each side. |
| $14 = 4x$ | Subtract 4 from each side. |
| $\frac{14}{4} = x$ | Divide each side by the coefficient. |

So, $x = \frac{7}{2}$.

> **Reflect:**




## Synthesize

**Have students share** their strategies for solving linear equations with different structures.

**Highlight** that there are different strategies students can use to solve an equation. Students could check whether their solution is correct by substituting their answer into the original equation and checking if the equation is true.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How has your understanding of equations changed since earlier lessons? What helped you develop your understanding of equations?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by solving an equation with variables on both sides.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 4.09

Solve the equation $\frac{2}{3}(4x+3)=\frac{1}{4}(3x-15)$. Show or explain your thinking.

$$\frac{2}{3}(4x+3)\cdot 12=\frac{1}{4}(3x-15)\cdot 12$$
$$8(4x+3)=3(3x-15)$$
$$32x+24=9x-45$$
$$23x+24=-45$$
$$23x=-69$$
$$x=-3$$

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can solve a variety of linear equations in one variable, including equations with fractions, decimals, negative values, grouping symbols, and multiple terms.

1 2 3



**Lesson 9** Strategic Solving

## Success looks like . . .

- **Language Goal:** Describing strategies for solving linear equations in one variable with different features or structures. **(Speaking and Listening)**
  - » Showing how to solve the equation using the Distributive Property.

## Suggested next steps

**If students do not correctly solve the equation, consider:**

- Providing the equivalent equation without fractions or parentheses, $32x + 24 = 9x - 45$, and having them solve it. If students still struggle solving this equation, consider reviewing Lessons 5 and 6.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- In what ways did Activity 1 go as planned?
- During the discussion about solving equations, how did you encourage each student to listen to one another's strategies?

# Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

**1.** Without solving, identify whether each equation has a solution that is *positive*, *negative*, or *zero*.

- **a** $7x = 3.25$ **Positive**
- **b** $-7x = 32.5$ **Negative**
- **c** $3x + 11 = 11$ **Zero**
- **d** $9 - 4x = 4$ **Positive**
- **e** $-8 + 5x = -20$ **Negative**

**2.** Solve each equation. Show or explain your thinking. **Sample strategies shown.**

**a** $2b + 8 - 5b + 3 = -13 + 8b - 5$

$-3b + 11 = -18 + 8b$

$-11b + 11 = -18$

$-11b = -29$

$b = \frac{29}{11}$

**b** $2x + 7 - 5x + 8 = 3(5 + 6x) - 12x$

$-3x + 15 = 15 + 18x - 12x$

$-3x + 15 = 15 + 6x$

$-9x + 15 = 15$

$-9x = 0$

$x = 0$

**c** $5c + 3 = 2(6 - c) + 7c$

$5c + 3 = 12 - 2c + 7c$

$5c + 3 = 12 + 5c$

$3 = 12$

**This equation is never true for any value of $x$; therefore, it has no solution.**

**d** $1.3 + 6d = 2.7 - 8d$

$1.3 + 14d = 2.7$

$14d = 1.4$

$d = 0.1$

**3.** Priya said the equation $9x + 15 = 3x + 15$ has no solution because $9x$ is greater than $3x$. Do you agree with Priya? Explain your thinking.

**Priya is incorrect. Sample response: $9x$ is greater than $3x$ only when $x > 0$. The solution is $x = 0$ because it is the only value that will make the equation true: $9(0) + 15 = 3(0) + 15$.**

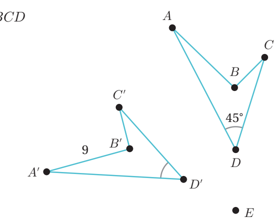




Name: ______ Date: ______ Period: ______

Practice

**4.** Polygon $A'B'C'D'$ is the image of Polygon $ABCD$ after a rotation about point $E$.




**a** What is the length of side $AB$? Explain your thinking.

**9 units. Sample response: The corresponding side lengths of a rotated figure are congruent.**

**b** What is the measure of $\angle D'$? Explain your thinking.

**45°. Sample response: The corresponding angles of a rotated figure are congruent.**

**5.** Select *all* of the situations for which only zero or negative slopes make sense.

- **A.** The height of a candle as it burns over an hour.
- **B.** A bank account balance over a year.
- **C.** The elevation above sea level of a hiker descending into a canyon.
- **D.** The number of students remaining in school after 6:00 p.m.
- **E.** The temperature in degrees Fahrenheit of an oven used on a hot summer day.

**6.** If you were babysitting, which option would you rather choose?

**Option A:** Charge \$5 for the first hour and \$8 for each additional hour.

**Option B:** Charge \$15 for the first hour and \$6 for each additional hour.

Explain your thinking.

**Sample responses:**

- **I would choose Option B because if I babysit for 4 hours, I would earn more.**
- **I would choose Option A because if I babysit for 6 hours, I would earn more.**
- **It doesn't matter which choice I choose because I would earn the same amount if I babysit for 5 hours.**





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Warm-up | 2 |
| | 2 | Activity 1 | 1 |
| | 3 | Activity 1 | 2 |
| Spiral | 4 | Unit 1 Lesson 9 | 1 |
| | 5 | Unit 3 Lesson 13 | 2 |
| Formative | 6 | Unit 4 Lesson 10 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

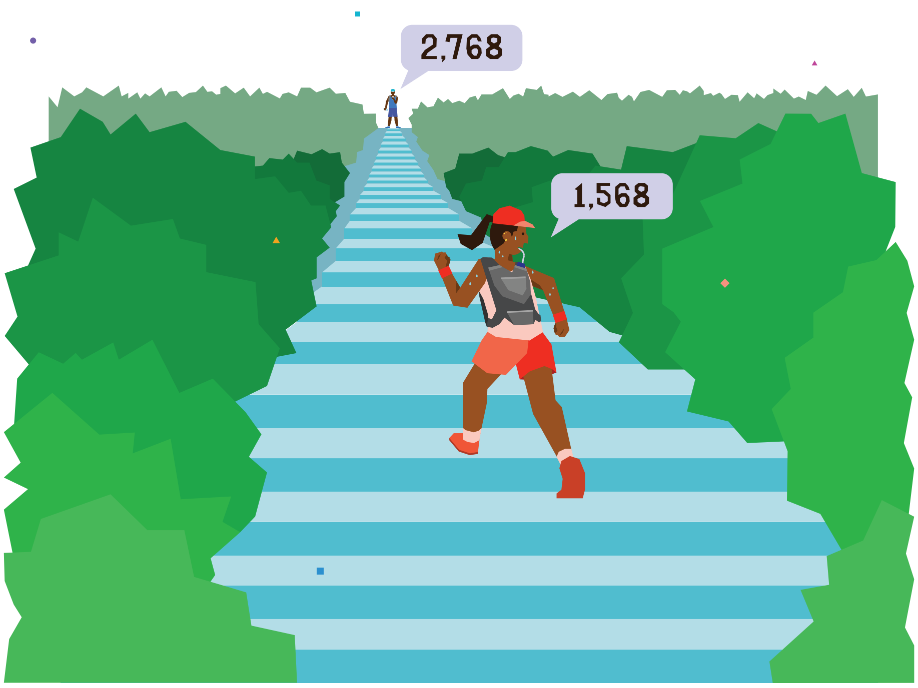
For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

Optional

UNIT 4 | LESSON 10

# When Are They the Same?

Let's use equations to think about situations.




## Focus

### Goals

1. Create an equation in one variable to represent a situation in which two quantities are equal.
2. **Language Goal:** Interpret the solution of an equation in one variable in context. **(Speaking and Listening, Writing)**

## Coherence

### Today

In this lesson, students apply their knowledge of solving equations by considering two real-world situations. Students are asked to determine when amounts in context will be the same. It is the work of the student to recognize that they can set the two expressions equal and solve the equation for the unknown. This work sets up the concept of substitution for the coming Sub-Unit on systems of linear equations.

### Previously

In Lesson 9, students strengthened their fluency for solving linear equations.

### Coming Soon

Starting in Lesson 11, students will begin exploring systems of linear equations by considering graphs of equations in context and the meaning of the solution. In Lesson 13, students will be formally introduced to the term *system of linear equations*.

## Rigor

- Students develop **conceptual understanding** for finding which value makes two expressions by setting two expressions equal to each other and solving the linear equation.
- Students **apply** strategies for solving linear equations by setting two expressions equal to each other and solving for $x$.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 8 min | 15 min | 15 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- calculators

### Math Language Development

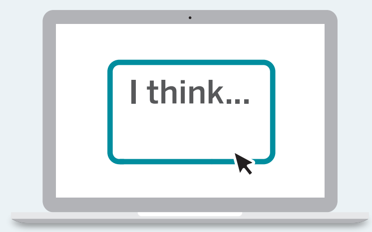
**Review words**

- *slope*
- *solution*
- *variable*

**Amps** | Featured Activity

### Exit Ticket
### See Student Thinking

Students are asked to explain what they think an equation represents related to a context, and these explanations are available to you digitally, in real time.




Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

As students begin to use equations as models for real-world scenarios, they might experience a sense of doubt. Ask them to explain the scenario in their own terms, making sure that they understand it and then have them draw connections between the scenario and the equation. These connections will help them complete the activity and will help them build confidence in their ability to work with equations.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, omit Problems 1 and 2.
- In **Activity 2**, omit Problems 1–4. Instead, provide students with expressions that represent Han and Priya's locations on the stairs.

Pairs | 8 min

# Warm-up Perimeter Puzzle

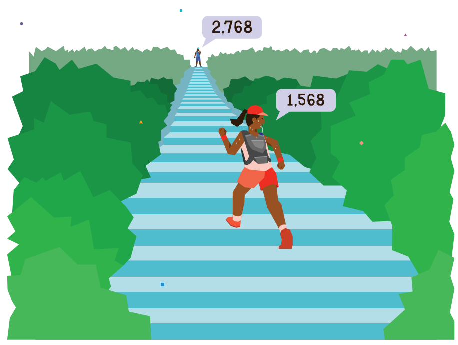
Students solve a problem by setting two expressions equal to each other to prepare their thinking for the lesson.

Name: ______________ Date: ________ Period: ________

Unit 4 | Lesson 10

## When Are They the Same?

Let's use equations to think about situations.




### Warm-up Perimeter Puzzle

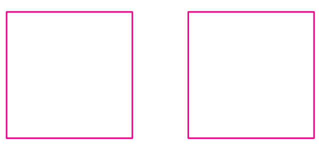
Bard drew a square with side lengths of $2x$ units. Lin drew a square with side lengths of $x + 2$ units.

Bard says to Lin, "The two perimeters can never be equal for any number $x$ because my square's side length multiplies $x$ by 2, while your square's side length just adds 2 to $x$."

Lin responds, "I disagree. There must be some number $x$ that makes these two perimeters equal."

Who do you think is correct? Explain your thinking

Sample response: Lin is correct because if you set the two expressions for the perimeters equal to each other, $4(2x) = 4(x + 2)$, you can see that $8x = 4x + 8$, which means $4x = 8$ and therefore, $x = 2$. When $x = 2$, both perimeters will equal 16.




Log in to Amplify Math to complete this lesson online.





## 1 Launch

Conduct the ***Think-Pair-Share*** routine.

## 2 Monitor

**Help students get started** by having them write in the lengths for all four sides and asking what expression they could use to find the perimeter.

**Look for points of confusion:**

- **Thinking Bard is correct.** Have students substitute the value 2 for $x$. Ask students what this means about Bard's statement and have students determine another way to show it.

**Look for productive strategies:**

- Finding out that 2 makes the perimeters equal and substituting 2 for $x$ to confirm.
- Setting expressions for the perimeters equal to each other and solving for $x$.

## 3 Connect

**Have students share** who they think is correct by conducting the ***Poll the Class*** routine.

**Display** student work that shows a sequence of strategies, such as students who:

- Used substitution or guess-and-check.
- Set the perimeters equal: $4(2x) = 4(x + 2)$.
- Set the side lengths equal, $2x = x + 2$.

**Ask:**

- "After you set the expressions equal to each other, what is represented by the solution $x = 2$?"
- "Will the perimeters be the same when $x = 3$? Why or why not?"
- "What is the value of the perimeter when both are equal? How do you know?"

**Highlight** that, by setting two expressions in the same variable equal to each other, students can solve for the value that makes both expressions true.

## Power-up

**To power up students' ability to compare rates to determine the best option when an initial value is given, have students complete:**

Two gyms open in your town. Gym A charges a starting fee of \$50, then \$10 per month. Gym B charges a starting fee of \$30, then \$15 per month. Complete each table to determine the costs for 6 months at each gym.

Gym A:

| Months | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Cost (\$) | 50 | 60 | 70 | 80 | 90 | 100 | 110 |

Gym B:

| Months | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Cost (\$) | 30 | 45 | 60 | 75 | 90 | 105 | 120 |

**Use:** Before Activity 1

**Informed by:** Performance on Lesson 9, Practice Problem 6

Pairs | 15 min

# Activity 1 Education Gap?

Students work within a real-world context to see that setting two separate expressions equal to one another is one way to determine more information about the context.

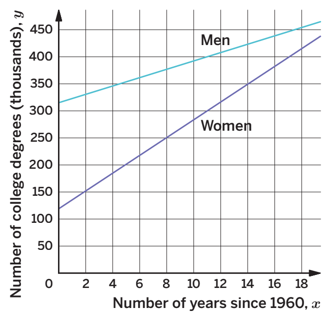
## Activity 1 Education Gap?

Now, more than at any point in history, women are earning college degrees comparably to men. However, that wasn't always the case.

The graph shown can be used to model the trend in college degrees earned by men and women starting in 1960.

The line represented by the equation $y = 16.4x + 118.3$ shows the number of women that earned a college degree.

The line represented by the equation $y = 7.7x + 314.3$ shows the number of men that earned a college degree. In both equations, $x$ represents the years since 1960 and $y$ represents the number of people in thousands.




1. According to the trend line, how many women earned a degree in the year 1970? How many men?

   Because 1970 is 10 years after 1960, I can substitute 10 into both equations.

   Women: $16.4(10) + 118.3 = 282.3$; Men: $7.7(10) + 314.3 = 391.3$

   About 282,300 women and about 391,300 men earned a degree in 1970.

2. During what year, if ever, do you think the number of women receiving a college degree will be the same as the number of men? Be as precise as possible.

   Sample response: If I set the expressions equal to each other, $16.4x + 118.3 = 7.7x + 314.3$, I can find the number of years after 1960, $x$, when the numbers of degrees earned by men and women are equal.

   $16.4x + 118.3 = 7.7x + 314.3$

   $8.7x = 196$

   $x \approx 22.5$

   During the year 1982, because $1960 + 22.5 = 1982.5$.

3. Make a prediction based on the trend lines: will there be more men or more women earning a college degree in the year 2025? How many more?

   I predict more women will receive degrees in 2025.

   Women: $16.4(65) + 118.3 = 1{,}184.3$; Men: $7.7(65) + 314.3 = 814.8$

   $1{,}184.3 - 814.8 = 369.5$, which means I predict there will be about 369,500 more women earning college degrees in 2025 than men.

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## 1 Launch

Read the text aloud with students. Activate students' background knowledge by asking them if they want to share whether their grandparents earned a college degree. Ask, "What year does the value 10 on the $x$-axis represent?" Provide access to calculators for the remainder of this lesson.

## 2 Monitor

**Help students get started** by asking them to use the graph to estimate the number of degrees for women and men in the year 1970.

**Look for points of confusion:**

- **Substituting 1970 for the year in Problem 1 or not interpreting $x = 22.5$ as 22.5 years after 1960 in Problem 2.** Have students underline the text that describes what $x$ represents. Ask students what $x = 1$ represents. Ask students what the value of $x$ would be for the year 1965 in this situation.

## 3 Connect

**Have students share** their responses to Problems 1, 2, and 3.

**Ask** students to explain why they knew to set the expressions equal.

**Display** the Activity 1 PDF. Have students compare their predictions for Problems 2 and 3 with the actual data.

**Highlight** that the trend lines do not represent the exact number of degrees in a given year, but the expected trend over time. Ask students if they think it is reasonable to expect these trend lines to continue in the same direction for the next 10 years or the next 100 years. Preview that students will look more at a different gap in the Capstone Lesson — the gender disparity in wages.

## Differentiated Support

### Accessibility: *Activate Background Knowledge*

Ask students if they would like to share if they know whether or not their grandparents earned a college degree. After students have shared, ask the class if they noticed any patterns about what their peers have said about their grandparents' earning of college degrees.

### Extension: *Math Enrichment*

Have students determine the percentage of all degrees earned in 1970 that were earned by women, according to the graph in this activity.

$$\frac{282.3}{282.3 + 391.3} \approx 0.42$$

## Math Language Development

### *MLR6: Three Reads*

Use this routine to help students make sense of the graph and introductory text.

- **Read 1:** Students should understand that the graph represents the trend in college degrees earned by men and women starting in 1960. Ask students to explain what the scale along the $x$-axis represents.
- **Read 2:** Ask students to state or highlight the given equations that model the number of men and women that earned a college degree.
- **Read 3:** Ask students to plan their solution strategy as to how they will complete Problem 1.

Pairs | 15 min

# Activity 2 Staircase to the Sky

Students determine when two hikers meet to elicit reasoning about why setting two expressions equal to one another is a way to solve the problem.

Name: ____________ Date: ________ Period: ________

## Activity 2 Staircase to the Sky

Han and Priya organize a week-long trip to hike the Rocky Mountains in Colorado. On the trip, they decide to visit the Manitou Incline, a 2,768-step staircase, nearly 1 mile long, which serves as a popular destination for anyone looking for a good workout.

Han reaches the top before Priya and texts Priya from the top to say he is out of water. Han hopes to meet Priya so he can have a drink from Priya's water bottle. Han walks down as Priya continues walking up, both at constant rates.

The table shows how many stairs Han and Priya are from the start of the Manitou Incline, and the time, in minutes, after Han texted Priya.

Yobab/Shutterstock.com

| Time (minutes after Priya's text) | Han (number of stairs from the start) | Priya (number of stairs from the start) |
|---|---|---|
| 0 | 2,768 | 1,568 |
| 5 | 2,368 | 1,768 |
| 10 | 1,968 | 1,968 |
| 15 | 1,568 | 2,168 |
| 20 | 1,168 | 2,368 |
| 25 | 768 | 2,568 |

1. How many stairs per minute does Han walk down? Show your thinking.
   $\frac{2768-2368}{0-5} = \frac{400}{-5} = -80$
   Han walks down at a rate of 80 stairs per minute.

2. How many stairs per minute does Priya walk up? Show your thinking.
   $\frac{1768-1568}{5-0} = \frac{200}{5} = 40$
   Priya walks up at a rate of 40 stairs per minute.





### 1 Launch

Have students read the text. Ask, "Where is each hiker when the time is zero?" Then ask students to predict when they think the hikers will meet. After taking some guesses, ask students how they can use algebra to be more precise.

### 2 Monitor

**Help students get started** by asking what it means when each hiker is hiking at a constant rate and reminding them of the formula for determining the rate of change.

**Look for points of confusion:**

- **Getting a positive rate of change for Han in Problem 1.** Ask students to consider whether Han is climbing up or down.
- **Not knowing what to do in Problem 5.** Ask them how to write on which step each hiker will be at $t$ minutes. Then ask them what does it mean for the hikers to meet.

**Look for productive strategies:**

- Using the table to estimate the time when the two hikers are at the same spot.
- Setting expressions equal in Problem 5.

**Activity 2 continued >**

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

After students complete Problems 1 and 2, provide students with expressions representing the number of stairs for Han and Priya and instruct students to focus on completing Problem 5.

### Extension: *Math Enrichment*

Have students find the number of stairs from the start when both hikers meet and explain their thinking. 1968; Sample response: Substitute 10 (the number of minutes) into either expression to determine the number of stairs; $1568 + 40(10) = 1968$.

## Math Language Development

### *MLR2: Collect and Display*

During the Connect, provide students with an opportunity to discuss their solutions to Problems 1–4 in groups of 3–4. Circulate through the groups and record language students use to describe what is happening with each hiker. Listen for language related to *rate of change, differences between rates, initial height of each hiker,* etc. Display the collected language so that students can refer to it throughout the rest of the activity and lesson.

Pairs | 15 min

# Activity 2 Staircase to the Sky (continued)

Students determine when two hikers meet to elicit reasoning about why setting two expressions equal to one another is a way to solve the problem.

## Activity 2 Staircase to the Sky (continued)

3. Write an expression that represents the number of stairs Han is from the start $x$ minutes after Priya's text.
   $2768 - 80x$

4. Write an expression that represents the number of stairs Priya is from the start after $x$ minutes after she texts Han.
   $1568 + 40x$

5. When will they meet? Be as precise as you can. Show or explain your thinking.
   They will meet in 10 minutes, 1868 stairs from the start. Sample response: Let $x$ be the time, in minutes, when they meet.
   $$2768 - 80x = 1568 + 40x$$
   $$2768 - 1568 = 40x + 80x$$
   $$1200 = 120x$$
   $$x = 10$$
   This means they will meet in 10 minutes.

STOP

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## 3 Connect

**Display** student work showing a correct expression for Han and Priya set equal to each other.

**Have students share** why they decided to set the expressions equal to each other. If no student set the expressions equal, stop the class and present an equation with the expressions set equal. Have students share with a partner what the equation represents and then solve for the time.

**Ask:**

- "What does the variable represent in the first expression? the second expression?"
- "How did you find the time when the two hikers met using the expressions?"

**Highlight** that, by writing and setting expressions equal, students can find a precise solution. Remind students to pay attention to what that variable represents in context when considering their solution.

Whole Class | 5 min

# Summary

Review and synthesize how to represent a situation by setting two expressions equal.

Name: ______ Date: ______ Period: ______

## Summary

**In today's lesson . . .**

You saw how multiple expressions can be used to represent a scenario. You saw that you can set these expressions equal to each other to solve for an unknown variable.

For example, imagine two hikers walking towards each other on a mountain, one hiking down from the top and one on their way to the top.

Now imagine when the hikers meet each other on the mountain, when they are at the same altitude at the same time. To find out when this time is, you can write an expression representing the altitude of each hiker and set those expressions equal to each other.

**Reflect:**



**Lesson 10** When Are They the Same? 421

## Synthesize

**Ask:**

- "If you cannot guess exactly when two values are the same, what can you do to find a precise answer?"
- "What other real-world examples can you think of when you might set two expressions equal to each other?"

**Have students share** examples with the class.

**Highlight** that students can sometimes guess when two values will be equal, either by looking at a table, estimating on a graph, or by doing some quick mental math. Another option can be to write two expressions that can be set equal to each other. This is a precise way to find when two expressions are the same.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies did you find helpful today when setting two amounts equal in a context? How were they helpful?"
- "Were any strategies not helpful? Why?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by explaining what is represented when two expressions are set equal.

**Amps Featured Activity** See Student Thinking

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 4.10

**To own and operate a home printer, it costs \$100 for the printer and an additional \$0.05 per page for the ink. To print pages at an office store, it costs \$0.25 per page. Let $p$ represent the number of pages printed.**

1. What does the equation $100 + 0.05p = 0.25p$ represent?

   It represents when the costs are the same to print at home or at an office store.

2. What is the solution to the equation, and what does it mean?

   $$100 + 0.05p = 0.25p$$
   $$100 = 0.25p - 0.05p$$
   $$100 = 0.2p$$
   $$p = 500$$

   This means that the costs are equal for printing 500 pages at home or at an office store.

**Self-Assess**

| 1 | 2 | 3 |
|---|---|---|
| I don't really get it | I'm starting to get it | I got it |

a. I can write two expressions and set them equal to each other to determine when two quantities, such as height, are the same in a real-world situation.

1 2 3



**Lesson 10** When Are They the Same?

## Success looks like . . .

- **Goal:** Creating an equation in one variable to represent a situation in which two quantities are equal.
- **Language Goal:** Interpreting the solution of an equation in one variable in context. **(Speaking and Listening, Writing)**
  - » Explaining that the costs of printing at home and at an office store are equal for 500 pages.

## Suggested next steps

**If students use general, imprecise language to describe the meaning of the equation in Problem 1, consider:**

- Asking students to keep in mind the units that are represented by the variable $p$ and have them revise their answers to include more precise language.

**If students do not solve for $p$ correctly in Problem 2, consider:**

- Reviewing how to solve equations with a variable on each side of the equal sign in Activity 1 or by referencing the Anchor Chart PDF, *Solving Linear Equations*.
- Assigning Practice Problem 2.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- The focus of this lesson was for students to see that they can set expressions equal to each other based on a context. How did this go?
- Which teacher actions made the Connect in Activities 1 and 2 strong?

# Practice

Independent

Practice

Name: Date: Period:

**1.** Which story could be represented by the equation $-6 + 3x = 2 + 4x$?

**A.** At 5 p.m., the temperatures recorded at two weather stations in Antarctica were $-6$ degrees at the first station and 2 degrees at the second station. The temperature changes at the same constant rate, $x$ degrees per hour, throughout the night at both locations. The temperature at the first station, 3 hours after this recording, was the same as the temperature at the second station, 4 hours after this recording.

**B.** Elena and Kiran play a card game. Every time they collect a pair of matching cards, they earn $x$ points. At one point in the game, Kiran has $-6$ points and Elena has 2 points. After Elena collects 3 pairs and Kiran collects 4 pairs, they have the same number of points.

**2.** Priya and Han are biking in the same direction on the same path.

a. Han rides at a constant speed of 16 miles per hour. Write an expression that represents the number of miles Han has traveled after $t$ hours.

$16t$

b. Priya started riding a half hour before Han. If Han has been riding for $t$ hours, how long has Priya been riding?

$t + \frac{1}{2}$ hours

c. Priya rides at a constant speed of 12 miles per hour. Write an expression that represents the number of miles Priya has traveled after Han has been riding for $t$ hours.

$12\left(t + \frac{1}{2}\right)$

d. Use your expressions to determine when Han and Priya meet. Show your thinking.

$16t = 12\left(t + \frac{1}{2}\right)$

$16t = 12t + 6$

$4t = 6$

$t = \frac{3}{2}$ They meet after 1.5 hours.

**3.** Cell phone Plan A costs $70 per month and comes with a free phone that is worth $500. Cell phone Plan B costs $50 per month, but does not come with a phone. Suppose you buy the $500 phone and choose Plan B. After how many months is your total cost the same as it would have been had you chosen Plan A?

Let $x$ be the number of months. Using Plan A, I spend $70x$ after $x$ months and using Plan B, I spend $50x + 500$ after $x$ months.

$70x = 50x + 500$

$20x = 500$

$x = 25$

The plans will cost the same after 25 months.





Name: Date: Period:

Practice

**4.** Solve each equation. Show or explain your thinking.

a. $3d + 16 = -2(5-3d)$

$3d + 16 = -10 + 6d$

$16 = -10 + 3d$

$26 = 3d$

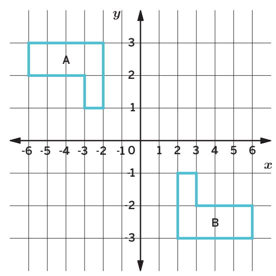
$d = \frac{26}{3}$

b. $2k - 3(4 - k) = 5k + 4$

$2k - 12 + 3k = 5k + 4$

No solution

**5.** Describe a rigid transformation that takes Polygon A to Polygon B.




Sample response: Rotate Polygon A 180° clockwise about the origin.

**6.** Choose the equation for which the ordered pairs $(5, 7)$ and $(8, 13)$ are each solutions.

**A.** $3x - y = 8$

**B.** $y = x + 2$

**C.** $y - x = 5$

**D.** $y = 2x - 3$





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 1 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 4 Lesson 5 | 1 |
| | 5 | Unit 1 Lesson 4 | 1 |
| Formative | 6 | Unit 4 Lesson 11 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

## Sub-Unit 2
# Systems of Linear Equations

Whole Class

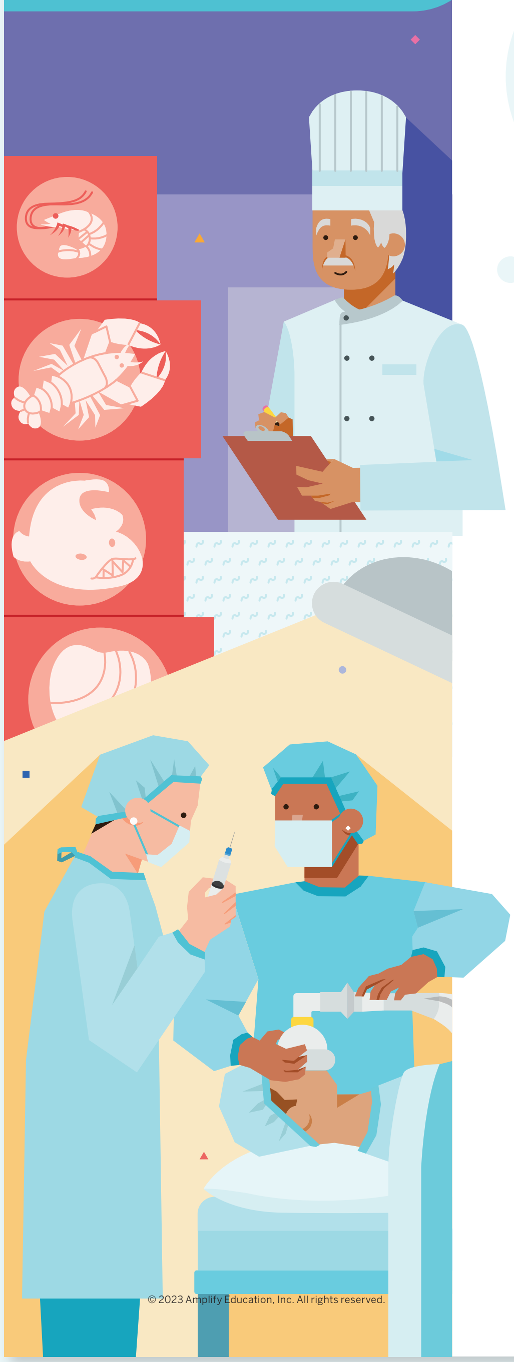
In this Sub-Unit, students discover how systems of linear equations can be used to solve everyday problems. Using graphs, tables, and equations, students find and interpret the meaning of a solution to a system, including systems with no solution or infinitely many solutions.

SUB-UNIT

2 Systems of Linear Equations

Narrative Connections

### How is anesthesia like buying live lobsters?



A patient lies on an operating table. Their anesthesiologist must take into account the patient's body mass before delivering just the right amount of anesthesia without endangering their life.

Elsewhere, a chef opens a restaurant in a beach town. As the weather gets warmer, crowds start to gather. Working from recipes that feed just ten people, the chef must now figure out how many clams, shrimp, and pounds of monkfish to order each week.

What do these scenarios have in common? They are two of the many everyday problems that can be solved with linear equations. Linear equations describe relationships where one value changes with another at a constant rate. Think about the way the amount of medicine changes with the weight of the patient. Or the number of clams for a recipe changes with the number of hungry diners.

But what happens when we need to use more than one linear equation at the same time? In these next lessons, you'll see how you can translate real-world problems into mathematical terms and solve them with a few handy methods. All with the help of a few linear equations!





### Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how systems of linear equations can be used to model and solve everyday problems in the following places:

- **Lesson 11, Activities 1-2:** Pocket Full of Change, A New Way of Solving
- **Lesson 12, Activities 1-2:** Can a Computer Science Teacher Run as Fast as Grete Waitz?, A Different Pace
- **Lesson 13, Activity 1:** Time to Refuel
- **Lesson 16, Activities 1-2:** Situations and Systems, Info Gap: Walking, Jogging, Running
- **Lesson 17, Activities 1-2:** Mind the Gap, Gender Pay Gap

# On or Off the Line?

Let's interpret the meaning of points on the coordinate plane.


## Focus

### Goals

1. Determine a point that satisfies two relationships simultaneously, using tables or graphs.
2. **Language Goal:** Interpret points that lie on one, both, or neither line on a graph of two simultaneous equations in context. **(Speaking and Listening, Writing)**

## Rigor

- Students build **conceptual understanding** for the meaning of a solution to two simultaneous linear equations that can be used to model a real-world scenario.

## Coherence

### Today

In this lesson, students consider pairs of linear equations, of the form $Ax + By = C$, in context and interpret the meaning of points on the graphs of the equations. Students build upon earlier work with linear equations in two variables where there is an equation constraining the possible combinations of two quantities. **Note:** The goal of this lesson is not for students to write equations or learn the language *system of equations*, but rather to investigate the mathematical structure with two stated facts by using familiar representations and to develop the need for new solving strategies.

### Previously

In Lesson 10, students set two expressions equal to one another to determine a common value where both expressions are true (if it exists).

### Coming Soon

In Lesson 12, students will continue exploring the meaning of a solution for linear equations graphed on the coordinate plane, focusing on equations of the form $y = mx + b$. In Lesson 13, students will be formally introduced to the term *system of equations*.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 20 min | 15 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

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For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- rulers

### Math Language Development

**Review word**

- *solution*

**Amps** | Featured Activity

### Activity 2
### Using Work From Previous Slides

In later slides, students can build on their work from previous slides. It's their work, so they get to hold onto it!

Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students may resist thinking deeply when they try to make sense of the coin problem. Have them engage in metacognitive functions by asking them to think about their own thinking process. For example, have them conduct their own ***Notice and Wonder*** routine, which will help them record their thought processes.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted and discussed briefly in the launch of Activity 1.
- In **Activity 1**, omit the last row of the table.
- In **Activity 2**, have students focus on Problems 1 and 3.

Pairs 5 min

# **Warm-up** Counting Coins

Students consider combinations of coins that could equal to \$2 to develop the need for using other methods for solving for two unknowns.

Unit 4 | Lesson 11

## On or Off the Line?

Let's interpret the meaning of points on the coordinate plane.


### **Warm-up** Counting Coins

**I have \$2 in my pocket. If I have only nickels and dimes, how many nickels and dimes do I have?**

**Sample response: 15 dimes and 10 nickels**



Log in to Amplify Math to complete this lesson online.



### 1 Launch

Conduct the *Think-Pair-Share* routine.

### 2 Monitor

**Help students get started** by asking, "What if I only had nickels in my pocket?"

**Look for points of confusion:**

- **Thinking that there is only one correct answer.** Ask students to reread the problem to see if other coin combinations could be considered.

**Look for productive strategies:**

- Creating a table or drawing to strategically determine multiple combinations of nickels and dimes that total to \$2.

### 3 Connect

**Display** multiple correct combinations for all to see.

**Ask**, "If I said there were exactly 10 nickels and dimes worth \$2 in my pocket, what strategies could you use to figure out how many of each coin I have? What if I had 20 coins that were nickels and dimes? 30 coins?"

**Highlight** that without knowing the number of coins in the pocket, it is not possible to definitively say how many nickels and dimes there are. If the number of coins are known, students can use guess-and-check strategies to find the amount of each coin. However, when the quantities for the combinations become too great, students need a different, more efficient strategy.

## Differentiated Support

**Accessibility: *Guide Processing and Visualization***

Provide the partially completed table below for possible coin combinations and ask students to continue completing the table with other possibilities.

| Nickels | Dimes | Solution |
|---|---|---|
| 10 | 15 | $10(0.05) + 15(0.1) = 2$ |
| | | |

**Accessibility: *Optimize Access to Tools***

Consider bringing in nickels and dimes and allow students to physically manipulate the coins to determine possibilities that represent \$2.

## Power-up

**To power up students' ability to determine whether a point is a solution to a two-variable equation, have students complete:**

Recall that in order to determine if a point is a solution to a two-variable equation, you can substitute the $x$-coordinate for the value of $x$ in the equation and the $y$-coordinate for the value of $y$ in the equation.

Determine which ordered pairs are solutions to the equation $y = 2x - 4$. Select *all* that apply.

A. $(0, 4)$
B. $(0, -4)$
C. $(-4, -12)$
D. $(2, 0)$

**Use:** Before Activity 1

**Informed by:** Performance on Lesson 10, Practice Problem 6

Pairs | 20 min

# Activity 1 Pocket Full of Change

Students focus on a context involving coins represented with a table to represent the context algebraically.

Name: __________ Date: __________ Period: __________

## Activity 1 Pocket Full of Change

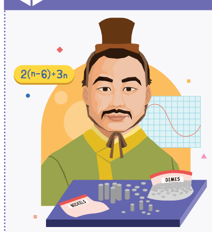
**Mathematicians like the renowned Zhang Quijian have long considered strategies for solving problems that lead to the Diophantine equations you explored in Unit 3. In this activity, you will work to find a strategy to help Jada and Noah solve the following problem:**

**Jada told Noah that she has $2 worth of nickels and dimes in her pocket and 31 coins altogether. She asked him to guess how many of each type of coin she has.**

Use the table to find combinations of nickels and dimes that have a total value of $2. Use the number of nickels given to determine the number of dimes, and then complete the rest of the first two columns. Then complete the third column to find the total number of coins for each combination. Can you find a combination of nickels and dimes that uses a total of 31 coins?

| Number of nickels | Number of dimes | Number of coins |
|---|---|---|
| 0 | 20 | 20 |
| 2 | 19 | 21 |
| 4 | 18 | 22 |
| 6 | 17 | 23 |
| 8 | 16 | 24 |
| 40 | 0 | 40 |

**Featured Mathematician**




**Zhang Qiujian**

Little is known about the Chinese mathematician Zhang Quijian. His 5th century book, *Zhang Qiujian Suanjing*, is considered one of the most important mathematical texts in history. In this book, Zhang explores different mathematical methods and problems. Perhaps the most famous one is the "Hundred Fowls Problem." Can you solve it?

*"Roosters cost 5 qian each, hens cost 3 qian each, and three chicks cost 1 qian. If 100 fowls are bought for 100 qian, how many roosters, hens and chicks are there?"*





### 1 Launch

Read the problem context together. Ensure students understand that Jada has exactly $2 in her pocket, that she only has nickels and dimes, and that she has exactly 31 coins.

### 2 Monitor

**Help students get started** by asking for the value of a one nickel and one dime, written as a decimal.

**Look for points of confusion:**

- **Not being able to find the number of dimes given the number of nickels.** Ask students to find the value of the nickels and subtract that from $2. Ask them how many dimes would they need to make equal to the remaining dollars value.
- **Being unsure how to find new combinations of coins.** Ask students what they notice about the nickel amounts already in the table and what value they could try based on what has worked in previous rows.

**Look for productive strategies:**

- Noticing when students increase by 2 nickels, students must subtract 1 dime.

### 3 Connect

**Have students share** how they determined the number of dimes and what patterns they notice in the table.

**Ask:**

- "Is it possible to have a combination with 1 nickel?"
- "What patterns do you see when you add 2 nickels?"

**Highlight** that the solution must make both facts true. To solve this problem more efficiently, students will learn how graphs can be used to help them find the exact solution.

## Math Language Development

### *MLR6: Three Reads*

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that Jada has a certain amount of coins and is asking Noah to guess how many of each type of coin she has.
- **Read 2:** Ask students to name or highlight the given quantities and their relationships, such as, "$2 worth of nickels and dimes in her pocket and 31 coins altogether".
- **Read 3:** Ask students to plan their solution strategy as to how they will complete the table to find combinations of nickels and dimes that have a total value of $2.

### English Learners

Annotate the term *altogether* with the term *total* to help students make the connection that these words mean the same thing.

## Featured Mathematician

### Zhang Qiujian

Have students read about the featured mathematician Zhang Qiujian, who wrote an influential book in the fifth century.

Possible solutions to the Hundred Fowls Problem:

- 0 roosters, 25 hens, 75 chicks
- 4 roosters, 18 hens, 78 chicks
- 8 roosters, 11 hens, 81 chicks
- 12 roosters, 4 hens, 84 chicks

Pairs | 15 min

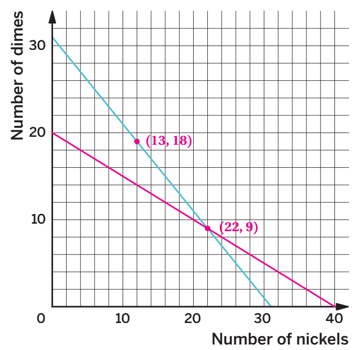
# Activity 2 A New Way of Solving

Students graph simultaneous equations representing the number and value of the coins from Activity 1 to discover new strategies for finding and interpreting the solution to simultaneous equations.

**Amps Featured Activity** Using Work From Previous Slides

## Activity 2 A New Way of Solving

**Refer to the scenario from Activity 1. The graph shows the relationship between the number of nickels and the number of dimes if the total number of coins is 31.**




1. Choose a point on the graph and explain what it means in context.
   Sample response: The point $(13, 18)$ means that 13 nickels and 18 dimes represent a possible combination of 31 coins.

2. Using values from your table in Activity 1, sketch the graph of the line that shows combinations of nickels and dimes that have a total value of \$2.

3. Label the point where the two lines intersect. What does this point represent?
   $(22, 9)$; 22 nickels and 9 dimes represent a total of 31 coins that have a total value of \$2.00 because, $0.05(22) + 0.1(9) = 2$.

4. Let $x$ represent the number of nickels and $y$ represent the number of dimes.
   a. Write an equation that shows that there are 31 total coins.
   $x + y = 31$ or $y = -x + 31$
   b. Write an equation that shows that the total value of the coins is \$2.
   $0.05x + 0.1y = 2$ or $y = -\frac{1}{2}x + 20$

STOP




## 1 Launch

Ensure students understand the graph representing coin combinations is already given, but that they will add a graph of the line that represents the values of the coin combinations found in Activity 1. Provide access to rulers.

## 2 Monitor

**Help students get started** by having them identify the labels for the $x$- and $y$-axes.

**Look for points of confusion:**

- **Having difficulty graphing a second line on the plane.** Ask students to identify their variables and to confirm they match the $x$- and $y$-axes. Then direct students to their tables.

## 3 Connect

**Display** student work showing a correct graph, and have students compare and contrast using a graph to using a table for solving in Activity 1.

**Ask:**

- "What does the point of intersection represent? Does it make sense in context?"
- "Is $(22, 9)$ the only combination that works for both lines? Why or why not?"
- "What is an example of a combination of coins that is a solution for one line but not the other? A combination that is not on either line?"
- "How can you use your equations to confirm the ordered pair $(22, 9)$ is on each line?"

**Highlight** that the solution $(22, 9)$ represents the number of nickels and dimes that makes both requirements true. Then highlight that a point of intersection on a graph is a solution for both equations because it lies on both lines of the equations. This strategy can be more efficient than using a table to find combinations that work.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Provide a sample point for Problem 1 instead of having students generate one on their own. Have students focus on completing Problems 1–3 and provide equations for Problem 4 so that they can refer to them during the Connect.

### Extension: *Math Enrichment*

Ask students to solve each equation they wrote in Problem 4 so that it is written in the form $y = mx + b$. Then have them set each of the expressions written in the form $mx + b$ equal to each other and solve for $x$. Ask them what they notice about this value of $x$. The value of $x$ is 22, which is also the $x$-coordinate of the point of intersection.

## Math Language Development

### *MLR5: Co-craft Questions*

During the Launch, display the graph before revealing the introductory text or any of the problems. Ask students to work with their partner to generate 1–2 mathematical questions they have about the graph. Ask student volunteers to share their questions with the class.

### English Learners

Provide sample questions, such as:

- "Can there be non-whole number inputs for nickels or dimes?"
- "What does the horizontal intercept mean in terms of the number of nickels or dimes?"

Displaying sample questions will help support students in developing metalinguistic awareness as they learn different types of mathematical questions that can be asked.

Whole Class | 5 min

# Summary

Review and synthesize how two simultaneous linear equations can be used to represent information from the same scenario.

Name: ______ Date: ______ Period: ______

## Summary

**In today's lesson . . .**

You saw an example of how you can use linear relationships to represent real-world scenarios. You saw that two equations can be used simultaneously to represent the same scenario, both graphed on the same coordinate plane. In some cases, this can be a more efficient way of finding a solution.

**Reflect:**





## Synthesize

**Display** the Student Edition Summary page.

**Highlight** what a solution to two simultaneous linear equations means.

**Ask:**

- "What are some advantages of tables? If you used two tables to describe the two relationships, how would you know whether a common point exists? If it did exist, how would you find it?" Tables are good for knowing the exact values for individual points. If the common point is listed in each table, it might be easy to notice, but it may be missing from at least one table or may be difficult to find if the tables are large and unordered. If the common point is listed in each table, one row of the table should match in both columns.
- "What are some advantages of graphs?" Graphs give a better overall picture of the relationships and usually makes estimating (if not finding exactly) the common point easier.
- "When using graphs, where are the points whose coordinates do not make a given relationship true?"

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What did you find challenging about math today? What did you do to overcome any challenges?"

Independent | 5 min

# Exit Ticket

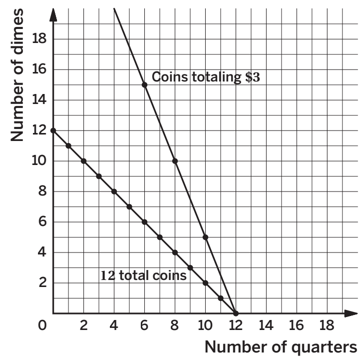
Students demonstrate their understanding by graphing the lines of two simultaneous linear equations and examining the solution in context.

Printable

Name: ______________ Date: ________ Period: ________

## Exit Ticket 4.11

**On the coordinate plane shown, one line represents the possible combinations of dimes and quarters that have a total value of $3. The other line represents the possible combinations of dimes and quarters when the total number of coins is 12.**




1. Name one combination of 12 coins shown on the graph.
   Sample response: 6 quarters and 6 dimes.

2. Name one combination of coins shown on the graph that have a total value of $3.
   Sample response: 6 quarters and 15 dimes.

3. What combination of quarters and dimes would equal both 12 coins and total $3?
   12 quarters and 0 dimes.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can identify and interpret ordered pairs that are solutions to an equation.
1 2 3

b. When two linear equations are graphed on the same coordinate plane and intersect, I can identify the ordered pair that is a solution to both equations.
1 2 3



**Lesson 11** On or Off the Line?

## Success looks like . . .

- **Goal:** Determining a point that satisfies two relationships simultaneously, using tables or graphs.
  - » Determining the combination of quarters and dimes that equals both 12 coins and gives a total fo $3 In Problem 3.
- **Language Goal:** Interpreting points that lie on one, both, or neither line on a graph of two simultaneous equations in context. **(Speaking and Listening, Writing)**
  - » Using the graph to give a combination of 12 coins from the graph in Problem 1 and a combination of coins that total to $3 in Problem 2.

## Suggested next steps

**If students have difficulty explaining Problem 1 or 2 in context, consider:**

- Reviewing Problem 1 from Activity 2.
- Asking students to identify which line represents which fact from the text.
- Assigning Practice Problem 2.

**If students have difficulty identifying the point of intersection in Problem 3, consider:**

- Reviewing Problem 3 from Activity 2.
- Assigning Practice Problem 1.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- In what different ways did students approach the process of finding a solution to simultaneous linear equations? What does that tell you about similarities and differences among your students?
- What might you change for the next time you teach this lesson?

# Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. Refer to the coordinate plane.

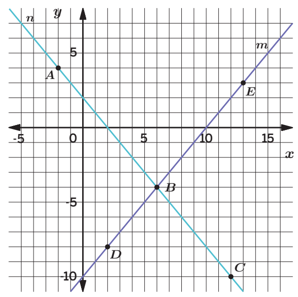
  a. Which line, line $m$ or line $n$, represents each statement?

  - A set of points where the coordinates of each point have a sum of 2.

    Line $n$

  - A set of points where the $y$-coordinate of each point is 10 less than its $x$-coordinate.

    Line $m$




  b. Which labeled point(s) represent each of the following statements?

  - The coordinates are two numbers with a sum of 2.

    Points $A$, $B$, and $C$

  - The coordinates are two numbers, where the $y$-coordinate of each point is 10 less than the $x$-coordinate.

    Points $B$, $D$, and $E$

  - The coordinates are two numbers with a sum of 2 and where the $y$-coordinate is 10 less than the $x$-coordinate.

    Point $B$

2. Mai earns $7 per hour mowing her neighbors' lawns. She also earned $14 for hauling away bags of recyclables from some neighbors.

  Priya babysits her neighbor's children. The table shows the amount of money $m$ she earns in $h$ hours.

| $h$ | $m$ |
|---|---|
| 1 | $8.40 |
| 2 | $16.80 |
| 4 | $33.60 |

  Priya and Mai have agreed to go to the movies the weekend after they have earned the *same* amount of money for the *same* number of hours worked.

  a. How many hours do they each have to work before they go to the movies?

    10 hours

  b. How much will each of them have earned?

    $84




Name: ______ Date: ______ Period: ______

Practice

3. Diego has $11 and begins saving $5 each week toward buying a new phone. At the same time that Diego begins saving, Lin has $60 and begins spending $2 per week on supplies for her art class. Is there a week when they will have the same amount of money? How much do they have at that time? Show or explain your thinking.

| Week | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Diego | 11 | 16 | 21 | 26 | 31 | 36 | 41 | 46 |
| Lin | 60 | 58 | 56 | 54 | 52 | 50 | 48 | 46 |

  After 7 weeks, they both will have $46.

4. Consider the equation $4x - 4 = 4x +$ ______. What value or expression could you write in the blank so that the equation would be true for:

  a. No values of $x$? Sample response: 3

  b. All values of $x$? $-4$

  c. One value of $x$? Sample response: $2x$

5. Solve each equation. Show or explain your thinking.

  a. $\frac{3y-6}{9} = \frac{4-2y}{-3}$

  $\frac{3y-6}{9} \cdot 9 = \frac{4-2y}{-3} \cdot 9$

  $3y - 6 = -3(4 - 2y)$

  $3y - 6 = -12 + 6y$

  $6 = 3y$

  $2 = y$

  b. $0.3(x - 10) - 1.8 = 2.7x$

  $0.3x - 3 - 1.8 = 2.7x$

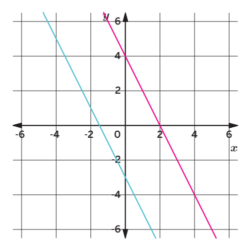
  $3x - 30 - 18 = 27x$

  $3x - 48 = 27x$

  $-48 = 24x$

  $-2 = x$

6. Draw a line with the same slope as the line given and a $y$-intercept of $(0, 4)$.







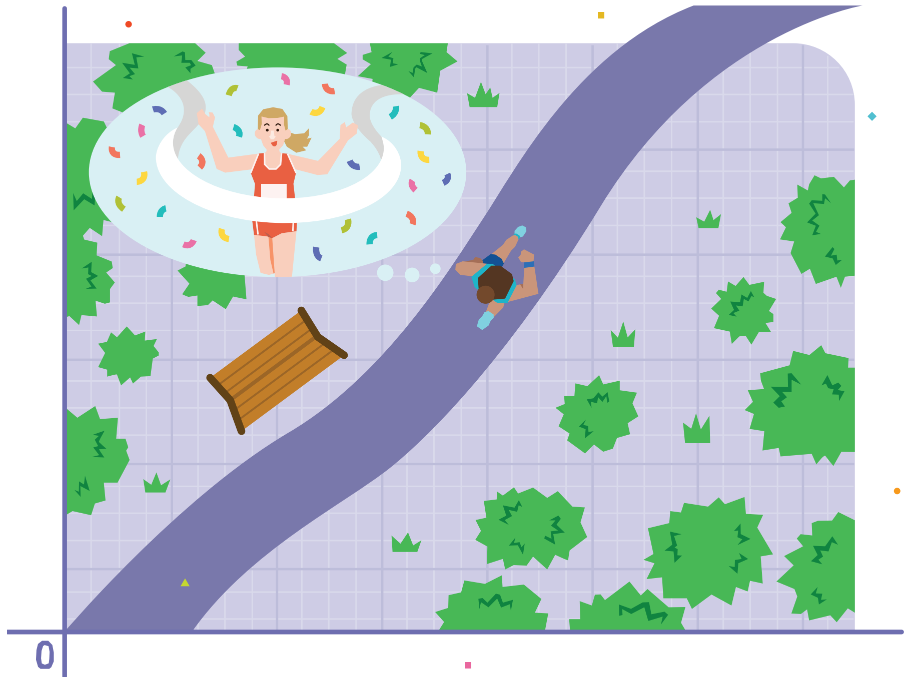
## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| | 2 | Activity 2 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 4 Lesson 8 | 1 |
| | 5 | Unit 4 Lesson 5 | 1 |
| Formative | 6 | Unit 4 Lesson 12 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available


For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.




# On Both of the Lines

Let's use lines to analyze real-world situations.

## Focus

### Goals

1. **Language Goal:** Create a graph that represents two linear relationships in context, and interpret the point of intersection. **(Speaking and Listening, Writing)**

2. **Language Goal:** Interpret a graph of two equivalent lines and a graph of two parallel lines in context. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students study simultaneous equations in context, where the equations are in the form $y = mx + b$. The purpose of this lesson is to introduce students to the graphical interpretation of simultaneous equations that have one point of intersection, no points of intersection, or infinitely many points of intersection. Keeping the graphs in mind will be useful as students navigate algebraic techniques for solving systems in the lessons to come.

### Previously

In Lesson 11, students studied graphs of simultaneous equations and considered the meaning of the point of intersection of the lines in context.

### Coming Soon

In Lesson 13, students will be formally introduced to the term *system of equations*. In Lesson 14, students will begin learning strategies for solving systems of linear equations algebraically.

## Rigor

- Students build **procedural skills** for graphing simultaneous equations and for finding and interpreting the solution in context.
- Students deepen their **conceptual understanding** of a solution to simultaneous equations by looking at scenarios where there are no solution or infinite solutions.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Representations of Linear Relationships*
- rulers

### Math Language Development

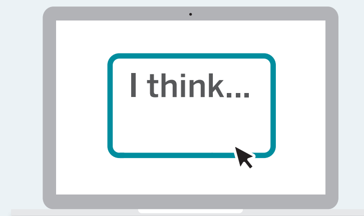
**Review words**

- *slope*
- *solution*

**Amps** | Featured Activity

### Activity 2
### See Student Thinking

Students are asked to explain their thinking behind the meaning of different simultaneous graphs. These explanations are digitally available to you in real time.




**Amps** POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

Students may be able to complete the background mathematics without being able to interpret the results in context. Because these interpretations are all similar in pattern, have students reflect on the work that they did. Ask them to write a summary of how to interpret the graph in a context, focusing on decisions that need to be made to get to the solution.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up**, may be omitted.
- In **Activity 2**, provide the line students are asked to create in Problem 1.

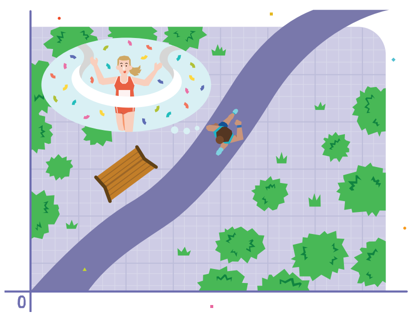
# Warm-up Which One Doesn't Belong?

Students review graphed equations to elicit ways they can describe different characteristics that arise when more than one line is graphed on a coordinate plane.

Unit 4 | Lesson 12

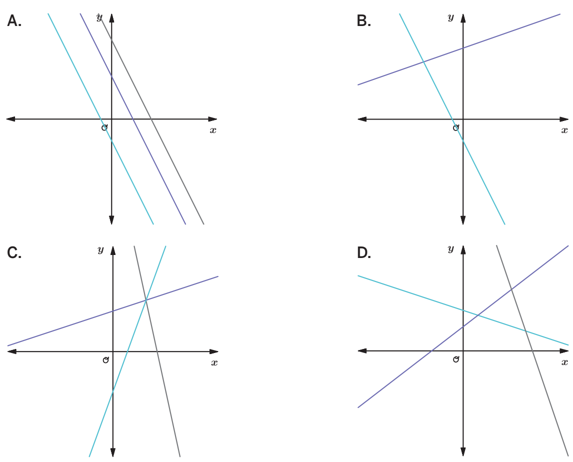
## On Both of the Lines

Let's use lines to analyze real-world situations.




**Warm-up** Which One Doesn't Belong?

Consider the lines graphed. Which graph doesn't belong?




Sample responses:

- Graph A is the only one with no intersection points.
- Graph B is the only one with two lines.
- Graph C is the only one with three lines that intersect at a single point.
- Graph D is the only one with multiple intersection points.

Log in to Amplify Math to complete this lesson online.

432 Unit 4 Linear Equations and Systems of Linear Equations



### 1 Launch

Conduct the *Which One Doesn't Belong?* routine. Distribute rulers for the duration of the lesson.

### 2 Monitor

**Help students get started** by saying, "Try looking for differences in the points of intersection."

**Look for points of confusion:**

- **Thinking there is only one correct answer.** Ask students to consider why a different graph does not belong.
- **Thinking Graphs C and D are the same because all three lines touch.** Ask students which graph shares one point of intersection with all three lines (Graph C) and then have students describe how the lines are intersecting in Graph D.

**Look for productive strategies:**

- Describing the points of intersection in terms of "solutions."
- Determining more than one solution for Graph D.

### 3 Connect

**Display** the Warm-up page from the Student Edition.

**Have students share** one reason why a particular graph might not belong. Record and display the responses for all to see. Have students share if they agree or disagree with each response.

**Ask**, "Why do you think the lines do not intersect in Graph A?"

**Highlight** students' use of concepts and language introduced in previous lessons about lines, such as *slope* and *intercepts*. Discuss that, for two distinct lines, the lines will either be parallel or they will intersect.

## Math Language Development

### *MLR2: Collect and Display*

Collect and display language as students describe which graph doesn't belong. Highlight and add specific terms, such as *slope*, *parallel*, and *intersect*. Continue adding to the display in Activity 2.

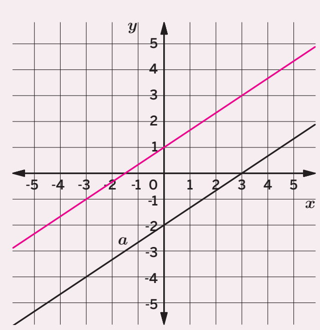
### English Learners

As students describe the various features of the graphs, annotate the graphs to indicate which lines are parallel and intersecting, amplifying that parallel lines have the same slope.

## Power-up

**To power up students' ability to graph a line given the slope and $y$-intercept, have students complete:**

1. Determine the slope of line $a$. $\frac{2}{3}$
2. Draw a line with the same slope as line $a$ that has a $y$-intercept of $(0, 1)$.




**Use:** Before Activity 2
**Informed by:** Performance on Lesson 11, Practice Problem 6

# Activity 1 Can a Computer Science Teacher Run as Fast as Grete Waitz?

Pairs | 15 min

Students create a graph from a table to compare two simultaneous linear equations on the same plane and interpret their point of intersection.

Name: ________ Date: ________ Period: ________

## Activity 1 Can a Computer Science Teacher Run as Fast as Grete Waitz?

**Ms. Hernández, a computer science teacher, wants to run a marathon as fast as Grete Waitz, the geography teacher who became a marathon legend.**

Ms. Hernández starts training by running shorter races with her trainer. The graph shows the distance and time run by her trainer. Ms. Hernández hopes to beat his time. To add a challenge, Ms. Hernández starts the race 20 seconds after her trainer.

1. Ms. Hernández records her distance and time ran, from when her trainer starts, in the given table. Use the table to sketch the graph for Ms. Hernández.

| Time (seconds) | Distance (m) |
|---|---|
| 20 | 0 |
| 40 | 160 |
| 60 | 320 |
| 120 | 800 |


2. At what speed, in meters per second, is Ms. Hernández running? Show or explain your thinking.
   $\frac{160-0}{40-20} = 8$, which means 8 m per second.

3. At what speed, in meters per second, is Ms. Hernández's trainer running? Show or explain your thinking.
   $\frac{600-0}{100-0} = 6$, which means 6 m per second.

4. Estimate the coordinates of the point where the two lines intersect. Explain what the point means in context.
   (80, 480). This means at 80 seconds, Ms. Hernández catches up to the same distance, 480 m, as her trainer.





### 1 Launch

Activate students' background knowledge about Grete Waitz, whom they studied in Unit 3. Ask students to explain what it would look like on the graph if Ms. Hernández catches up to her trainer. Provide access to rulers for the remainder of the lesson.

### 2 Monitor

**Help students get started** by asking, "What ordered pair is represented by the first row of the table?"

**Look for points of confusion:**

- **Not being able to find each runner's speed.** Refer students to the Anchor Chart PDF, *Representations of Linear Relationships* to remind them of the formula for finding the slope of a line.
- **Being unclear what the point represents in Problem 4.** Have students label the point of intersection and then ask what the $x$- and $y$-values represent.

**Look for productive strategies:**

- Identifying that, because Ms. Hernández is running at a faster speed, her graph has a steeper slope and it will catch up to, or intersect, the graph of her trainer.

### 3 Connect

**Display** student work showing the correct graph.

**Ask:**

- "How did you graph the line for Ms. Hernández?"
- "What can the graph tell you about what is happening at 50 seconds? At 200 seconds?"
- "How many points of intersection are there? What does that mean?"

**Highlight** that the point of intersection represents the one and only point when the two lines will meet. This is why we can say there is one solution that satisfies the equations of the lines of both runners.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Display or provide students with a copy of the Anchor Chart PDF, *Representations of Linear Relationships* to remind them of the formula for determining the slope of a line.

### Extension: *Math Enrichment*

Have students write the equation for each line in the form $y = mx + b$ and describe what they notice about the values for $m$ and $b$ in this context. Each value of $m$ represents each person's rate. The values for $b$ represent the trainer starting at time 0 and Ms. Hernández starting at time 20 seconds.

Ms. Hernández: $y = 8x + 20$ Trainer: $y = 6x$

## Math Language Development

### *MLR6: Three Reads*

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that Ms. Hernández and her trainer are both running races together.
- **Read 2:** Ask students to name or highlight given quantities and relationships, such as "Ms. Hernández starts the race 20 seconds after her trainer."
- **Read 3:** Ask students to plan their solution strategy in Problem 1 as to how they will complete the graph using the information given in the table.

### English Learners

Annotate the first row of the table with the phrase *starts race 20 seconds after trainer.*

Pairs | 15 min

# Activity 2 A Different Pace

Students create a graph for two simultaneous situations to see how different positions of the lines can be interpreted in the context.

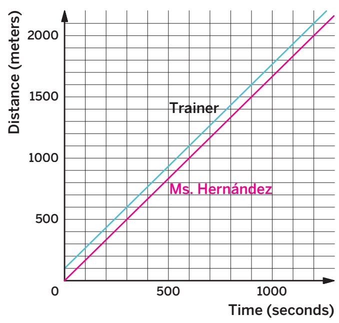
Amps Featured Activity | See Student Thinking

### Activity 2 A Different Pace

To run a longer race, Ms. Hernández's trainer reminds her she will need to slow down to conserve energy. Ms. Hernández is now preparing to run a 5K, or 5,000 m race. Her trainer starts 100 m ahead of the start line so that Ms. Hernández can run a comfortable distance behind him, still within eyesight. The graph shows Ms. Hernández's trainer's distance $y$, related to the time, in seconds, $x$.

Eduard Moldoveanu/Shutterstock.com

1. Graph a line representing Ms. Hernández's distance if she runs at the same speed as her trainer, but starts at the starting line.




2. Write an equation to represent each line.

   Trainer: $y = \frac{5}{3}x + 100$

   Ms. Hernández: $y = \frac{5}{3}x$

3. What do you notice about the two lines?

   Sample response: They are parallel.

4. Ms. Hernández says that she will never catch up to her trainer at the pace they are both running. Does your graph support this? Explain your thinking.

   Sample response: Yes. Because the two lines are parallel, they will never intersect.

5. Mr. Patel, an art teacher, who ran the same race, said that his graph looks exactly the same as Ms Hernández's graph. What do you think this could mean?

   Sample response: He ran alongside Ms. Hernández at the same speed and from the same starting point.

STOP




## 1 Launch

Set an expectation for time to work in pairs on the activity.

## 2 Monitor

**Help students get started** by asking, "What does it mean that Ms. Hernández runs at the same speed as her trainer? How can you represent this on the graph?"

**Look for points of confusion:**

- **Not being able to write a correct description in Problem 5.** Ask students to draw a line on top of the line representing Ms. Hernández. Ask students where each runner is at 0, 5, and 10 minutes.

## 3 Connect

**Ask:**

- "How many solutions to the equations of the lines are there for Ms.Hernández and her trainer? What does this mean in context?"
- "How many solutions to the equations of the lines are there for Ms.Hernández and Mr. Patel? What does this mean in context?"
- "What would be the equation for Mr. Patel's line?"

**Have pairs of students share** examples of points that show there is no solution for Problem 3 and infinitely many solutions for the equations of the lines for Ms. Hernández and Mr. Patel.

**Highlight** that parallel lines will never intersect, and, therefore, there will be no point that is a solution for both lines. In this context, that means that there will be no time when the trainer and Ms. Hernández are at the same distance at the same time. One line that is completely on top of another shares infinitely many points, and, therefore, the equations share infinitely many solutions.

## Fostering Diverse Thinking

### Running for Change

Have students research Wilma Rudolph, who earned three Olympic gold medals and was one of the first athletes to advocate for civil rights. She was the first American woman in track and field to win three gold medals at one Olympics, setting a world record for each. She refused to attend her hometown's parade and banquet unless it was nonsegregated, and so it became the first nonsegregated event in the town's history. Rudolph has been quoted as saying, "I would be very sad if I was only remembered as Wilma Rudolph, the great sprinter."

Ask:

- "In 1960, Rudolph ran 200 m in 23.2 seconds, setting a world record at the time. How did Rudolph's speed compare to Ms. Hernandez's speed from Activity 1?"
- "How are today's athletes using their platforms to show their support for different causes?"

Whole Class | 5 min

# Summary

Review and synthesize different situations with simultaneous equations.

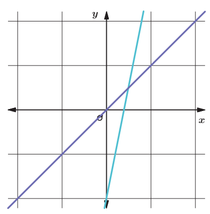
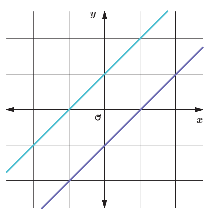
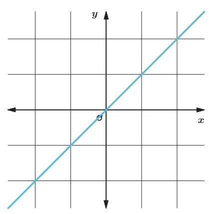
Name: ______________ Date: ________ Period: ________

## Summary

**In today's lesson . . .**

You continued to explore linear relationships. You saw that if you have two simultaneous equations, you can find if there is a solution to both equations by studying the lines of the equations on the same plane.

You saw examples of types of solutions for simultaneous equations:

| One solution | No solution | Infinitely many solutions |
| --- | --- | --- |
|  There is one point of intersection, which is the solution. |  There are no points of intersection. The lines are parallel. |  There are infinitely many points of intersection. The lines are directly on top of each other. |

**Reflect:**





### Synthesize

**Display** the Summary from the Student Edition.

**Have students share** what it means for simultaneous equations to have one solution, no solution, or infinitely many solutions.

**Ask:**

- "What do you notice about the slopes and the $y$-intercepts of the lines when there is one solution? No solution? Infinitely many solutions?"
- What must be true about the $m$ and the $b$ for the equations of the lines, in $y = mx + b$ form, that have infinitely many solutions?"

**Highlight** the three types of two simultaneous equations explored today:

1. **One solution:** When simultaneous lines intersect at one point.
2. **No solution:** When simultaneous lines will never intersect because they are parallel.
3. **Infinitely many solutions:** When simultaneous lines are on top of each other and share an infinite number of points of intersection.

### Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What can graphs tell you about the number of solutions for simultaneous linear equations?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by writing and graphing an equation from a scenario and interpreting the solution in context.

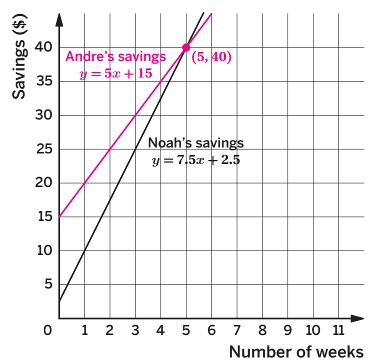
Printable

Name: ____________ Date: ________ Period: ________

## Exit Ticket 4.12

**Andre and Noah started tracking their savings at the same time. Andre started with \$15 and deposits \$5 per week. Noah started with \$2.50 and deposits \$7.50 per week. The graph of Noah's savings is represented by the equation $y = 7.5x + 2.5$, where $x$ represents the number of weeks and $y$ represents Noah's savings.**

1. Write the equation for Andre's savings and graph the line on the same coordinate plane.
   $y = 5x + 15$




2. Identify the point of intersection. What does the intersection point mean in this context?
   In this context, the intersection point located at (5, 40) means that after 5 weeks, Noah and Andre each have \$40 in savings.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use graphs that represent real-world scenarios to identify the intersection point of two equations and explain what it means within context.
1 2 3



**Lesson 12** On Both of the Lines

## Success looks like . . .

- **Language Goal:** Creating a graph that represents two linear relationships in context, and interpreting the point of intersection. **(Speaking and Listening, Writing)**
  - » Graphing the equation for Andre's savings on the same coordinate plane as the equation for Noah's savings in Problem 1.
- **Language Goal:** Interpreting a graph of two equivalent lines and a graph with two parallel lines in context. **(Speaking and Listening, Writing)**

## Suggested next steps

**If students have difficulty writing or graphing Andre's line, consider:**

- Reviewing graphing strategies from Activity 1.
- Assigning Practice Problem 2.

**If students cannot explain the point of intersection in context, consider:**

- Reviewing Problem 4 from Activity 1.
- Assigning Practice Problem 3.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- Knowing where students need to be by the end of this unit, how did Activities 1 and 2 influence that future goal?

# Practice

Independent

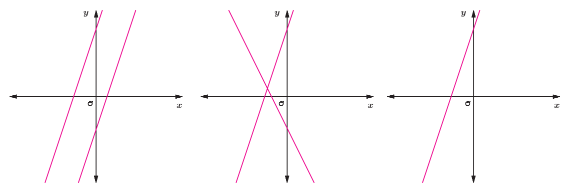
Name: ______ Date: ______ Period: ______

1. Sketch two lines that match each description. Then describe the number of solutions for each pair of lines.

   a. Two lines with the same slope and different $y$-intercepts.

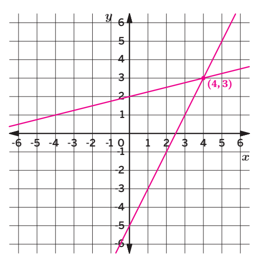
   b. Two lines with different slopes.

   c. Two lines with the same slope and same $y$-intercept.



There is no solution. There is one solution. There are infinitely many solutions.

2. Draw a graph to find $x$- and $y$-values that make both of the equations $y = \frac{1}{4}x + 2$ and $y = 2x - 5$ true.




The ordered pair that is a solution to both equations is $(4, 3)$.

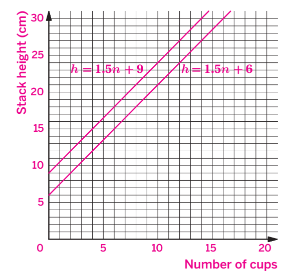



Name: ______ Date: ______ Period: ______

3. A stack of $n$ small cups has a height $h$, in centimeters, that is represented by the equation $h = 1.5n + 6$. A stack of $n$ large cups has a height $h$, in centimeters, that is represented by the equation $h = 1.5n + 9$.

   a. Graph the equations for each stack of cups on the same coordinate plane. Make sure to label the axes and decide on an appropriate scale.

   **Sample response shown.**




   b. For what number of cups will the two stacks have the same height? Explain your thinking.

   **The stacks will never have the same height because the lines are parallel, meaning they will never intersect.**

4. For what value of $x$ do the expressions $\frac{2}{3}x + 2$ and $\frac{4}{3}x - 6$ have the same value?

$$\frac{2}{3}x + 2 = \frac{4}{3}x - 6$$
$$2x + 6 = 4x - 18$$
$$6 = 2x - 18$$
$$24 = 2x$$
$$x = 12$$

**For $x = 12$, both expressions have the same value.**

5. Write two equations that represent the following scenario. Be sure to define your variables.

   A hiker descends a mountain at a rate of 3.5 miles per hour from a height of 0.5 miles above sea level. Another hiker ascends the mountain from the trailhead — which is located 0.1 miles above sea level — hiking at a rate of 2 miles per hour.

   **Let $x$ be the time in hours, and let $y$ be the hikers' height above sea level in miles.**

$$y = -3.5x + 0.5$$
$$y = 2x + 0.1$$



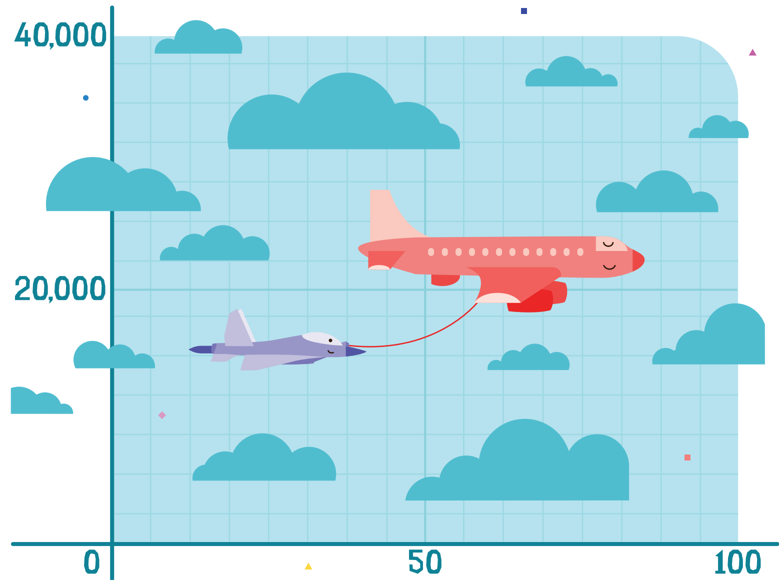

## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 1 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 4 Lesson 10 | 1 |
| Formative | 5 | Unit 4 Lesson 13 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.




# Systems of Linear Equations

Let's understand how a system of equations can be used to model a real-world context.

## Focus

### Goals

1. **Language Goal:** Comprehend that solving a system of equations means determining values of the variables that makes both equations true at the same time. **(Speaking and Listening)**
2. Create a graph of two lines that represents a system of linear equations in context.

## Coherence

### Today

Students are formally introduced to the concept of a *system of linear equations* with different contexts.

### Previously

In Lessons 11 and 12, students explored concepts of systems of equations without being formally introduced. In Lesson 12, they studied graphs of systems of linear equations that had one solution, no solution, or infinitely many solutions.

### Coming Soon

Starting in Lesson 14, students will spend the final lessons of the unit developing and applying strategies for solving systems of linear equations.

## Rigor

- Students build **conceptual understanding** for how to graph and solve systems of linear equations.
- Students develop **fluency** for writing systems of equations to match different contexts.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 20 min | 15 min | 5 min | 8 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair
- graphing technology (optional)
- rulers

### Math Language Development

**New words**

- ***solution to a system of equations***
- ***system of equations****

**Review words**

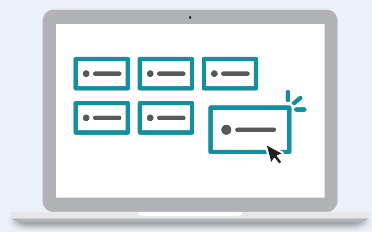
- *variable*

*Students may be familiar with the term *system* as it relates to an organizational structure, such as the public school system or a gaming system. Point out how a system of equations can be considered an organizational structure.

**Amps** | Featured Activity

### Activity 2
## Digital Card Sort

Students match systems of equations with their contexts by dragging and connecting them on-screen.



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### Building Math Identity and Community

Connecting to Mathematical Practices

At first, students may not immediately be able to connect the graph and the equations and might want to quit before really getting started. Encourage students to set a goal of identifying what they do know about them and build on that goal by using what they know to see how they are related. Ask them how they will motivate themselves to achieve this goal.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, have students use graphing technology to create their graphs.

Pairs | 5 min

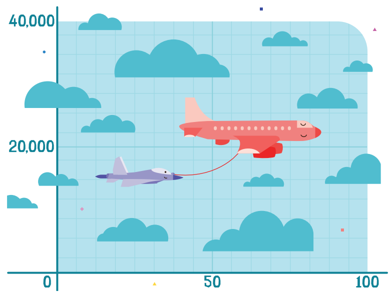
# **Warm-up** Midair Meetup?

Students study two lines on a plane and consider what it means for lines to be intersecting, even if not in view.

Unit 4 | Lesson 13

## Systems of Linear Equations

Let's understand how a system of equations can be used to model a real-world context.

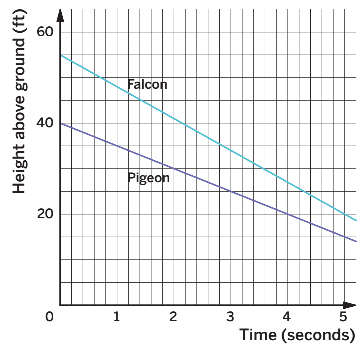



### **Warm-up** Midair Meetup?

A falcon chases a pigeon in midair. The graph shows the height of each bird as the time passes. Will the falcon catch the pigeon? Explain your thinking.

Sample responses:

- No, the falcon will run out of time. The lines do not intersect during this time interval.
- Yes, the falcon will catch the pigeon. When I used my ruler to extend the lines, they appeared to intersect just before the falcon and the pigeon reached ground level.




Log in to Amplify Math to complete this lesson online.





## 1 Launch

Conduct the *Think-Pair-Share* routine. Provide access to rulers for the duration of this lesson.

## 2 Monitor

**Help students get started** by asking them what the graph will look like if the falcon catches the pigeon.

**Look for points of confusion:**

- **Thinking that because the two lines do not intersect on the axes provided, the lines will never intersect.** Ask students to consider what the graph would look like if the $x$-axis continued further in the positive direction.

**Look for productive strategies:**

- Using a ruler to extend the lines of the falcon and of the pigeon.
- Extending the $x$-axis in the positive direction.

## 3 Connect

**Have students share** whether they think the falcon will catch the pigeon.

**Ask**, "How can you determine whether two lines will have a point of intersection, even if not shown on the plane?"

**Highlight** that if two lines have different slopes then they must intersect at some point. Based on the context, that point may or may not be a solution. Reveal that the line of the falcon and the line of the pigeon will intersect at the point (7.5, 2.5) by using graphing technology or a ruler.

## Power-up

**To power up students' ability to write a two-variable equation to represent a scenario, have students complete:**

Determine which equation matches the following scenario:

Noah's car has 3,000 mile on the odometer when he gets on the highway and travels at a constant speed of 60 mph. Write an equation to represent the number of miles on his odometer after traveling for a certain number of hours.

**A.** $y = 3000x + 60$, where $x$ represents hours and $y$ represents miles.
**B.** $y = 3000x + 60$, where $x$ represents miles and $y$ represents hours.
**C.** $y = 60x + 3000$, where $x$ represents hours and $y$ represents miles.
**D.** $y = 60x + 3000$, where $x$ represents miles and $y$ represents hours.

**Use:** Before Activity 1
**Informed by:** Performance on Lesson 12, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 8

Pairs | 20 min

# Activity 1 Time to Refuel

Students write a system of equations and solve for the point of intersection to strengthen their understanding of the connection between graphs and equations.

Name: ______________ Date: ________ Period: ________

## Activity 1 Time to Refuel

Occasionally, jet pilots need to refuel in the air. This complicated procedure is called *aerial refueling*. It requires mathematical precision and expert timing to be done safely.

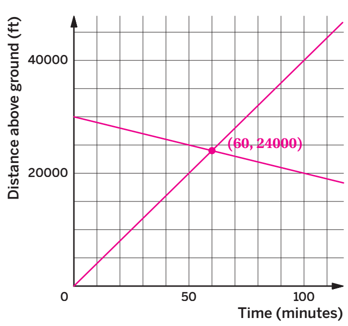
dmvphotos/Shutterstock.com

Suppose a pilot is flying a speed jet at an altitude of 30,000 ft when she recognizes her jet is soon going to run out of fuel.

- She requests an aerial refueling, and begins to descend at a constant rate of 100 vertical feet per minute to refuel at a safer altitude.
- The refueling plane takes off from ground level at the same time as the speed jet begins its descent, ascending at a speed of 400 vertical feet per minute.
- The refueling plane begins refueling the speed jet when the two planes reach the same altitude, at which point the refueling plane will position itself below the speed jet and connect to the jet's fuel tank.

1. Write two equations in the form $y = mx + b$ to represent the situation. Be sure to define your variables.
   Let $y$ represent the distance above the ground, in feet. Let $x$ represent the time in minutes.
   The equation $y = 30000 - 100x$ represents the altitude of the speed jet.
   The equation $y = 400x$ represents the altitude of the refueling plane.

2. Graph the lines representing each equation.







## 1 Launch

Read the context of the problem with the students to help them understand the situation. Ask, "When time is zero, where is the speed jet? Where is the refueling plane?"

Give 2–3 minutes of quiet work time and ask students to pause after they have completed the first problem to discuss their equations with a partner before starting to graph the equations. Give 5–7 minutes for students to complete the remaining problems with their partners followed by a whole-class discussion.

## 2 Monitor

**Help students get started** by helping them define their variables.

**Look for points of confusion:**

- **Thinking both lines have a positive slope.** Ask students to show you with their hands what it looks like for a plane to descend. Then ask whether that means the plane is adding or subtracting vertical feet for each unit of time.
- **Not being sure how to write the equations for each plane.** Ask students to pick a plane to start with and have them point to the value that represents the initial height. Then have them identify the value that represents the slope.
- **Not being able to correctly graph the line of each equation.** Have students point to the $y$-intercept for each equation and plot a point. Ask them which values of $x$ they could substitute to find values of $y$ for ordered pairs. Offer $x = 50$ and $x = 100$, if needed.

**Look for productive strategies:**

- Using the graph to identify the point of intersection.
- Precisely defining what the point of intersection represents in context.

Activity 1 continued >

## Differentiated Support

### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, or other graphing technology, to graph the lines representing each equation in Problem 2.

### Accessibility: *Guide Processing and Visualization*

Guide students to define the variables by asking the following questions:

- "What two quantities are being compared?" Distance and time.
- "Which quantity depends on the other? This is the dependent variable." Distance depends on the time.
- "Which quantity is the independent variable?" Time.
- "Which quantity will you use $x$ to represent it?" Time, because it is the independent variable.
- "Which quantity will you use $y$ to represent it?" Distance, because it is the dependent variable.

## Math Language Development

### *MLR6: Three Reads*

Use this routine to help students make sense of the introductory text.

- **Read 1:** Students should understand that a pilot needs to refuel her jet while she is flying. She descends at the same time while the refueling plane takes off from the ground.
- **Read 2:** Ask students to name the given quantities, such as the pilot descends at a constant rate of 100 vertical ft per minute.
- **Read 3:** Ask students to brainstorm strategies for representing this situation with a system of two equations.

### English Learners

Draw a quick sketch of the two planes with lines indicating their descent and ascent.

Pairs | 20 min

# Activity 1 Time to Refuel (continued)

Students write a system of equations and solve for the point of intersection to strengthen their understanding of the connection between graphs and equations.

**Activity 1** Time to Refuel (continued)

3. Find the point where the two graphs intersect each other. Estimate the coordinates of this point.
   Students' responses should be close to the point $(60, 24000)$.

4. What do the coordinates represent in this situation?
   $(60, 24000)$ means that at 60 minutes, the planes will be at the same altitude, 24,000 ft. This means that at 60 minutes, the refueling plane can attempt to connect to the speed jet's fuel tank.

**Are you ready for more?**

The Voyager, a refueling aircraft in the United Kingdom, can hold a little over 100 tons of fuel. The Voyager uses this fuel for its own engine *and* for aerial refuelling of other jets. Suppose the Voyager wanted to help out a fleet of American F-16 jets in need of aerial refueling. Each F-16 jet can hold 9.5 tons of fuel. If the Voyager burns about 6 tons of fuel per hour, and each refueling takes 0.5 hours, what is the greatest number of F-16 jets the Voyager could refuel completely and have fuel remaining to safely land at an airbase? Assume the Voyager needs 1 hour to land at an airbase.

The Voyager could refuel 7 F-16 jets and have 12.5 tons of fuel remaining to land, because $100 - 9.5(7) - (7)(0.5)(6) = 12.5$. If the Voyager refuelled 8 jets, $100 - 9.5(8) - (8)(0.5)(6) = 0$, means there would be 0 tons of fuel remaining for the Voyager to land.




## 3 Connect

**Display** the correct set of equations alongside the correct graph.

**Have students share** how they wrote their equations and how they graphed the lines of their equations.

**Define** the term ***system of equations*** and illustrate how the two equations can be written using a brace. Each equation has many solutions, represented by all of the points on the line, but a solution to a system of equations is the point that makes both equations true and that is a point on both lines. Thus a brace is used to show that students consider the equations in the system together.

**Define** the term ***solution to a system of equations*** as an ordered pair that makes all equations true. Tell students that "solving a system of equations" means to find this ordered pair. The solution to a system of equations is the point where the line representing the equation of the speed jet and the line representing the equation of the refueling plane intersect because it is the only point which satisfies both equations.

**Ask:**

- "What is true about the relationship between the coordinates of the point $(60, 24000)$ and the equation for the speed jet? And for the equation of the refueling plane?"
- "How can you confirm that $(60, 24000)$ is the solution to the system of equations?"
- "Would it still be the same system of equations if you used different variables, such as $t$ for time and $h$ for altitude?"

**Highlight** how the coordinates of the point $(60, 24000)$ can be substituted as an ordered pair for $x$ and $y$ into each equation of the system of equations. Discuss how using the graph to determine a solution provides only a good estimate for the solution.

Pairs | 15 min

# Activity 2 Card Sort: System Sort

Students match systems of equations to their context to develop fluency for writing systems that can be used to solve problems.

**Amps Featured Activity** Digital Card Sort

Name: ________________ Date: ________ Period: ________

## Activity 2 Card Sort: System Sort

You will be provided with a set of cards, some of which have a system of linear equations written on them, and others that have a real-world scenario described. For each scenario, define your variables. Then match the scenarios with the systems. Some cards may not have a match.

1. List the card pairs you found that matched. Explain how you defined your variables.

| Matching card pairs | Define your variables: |
|---|---|
| Cards 7 and 2 | Sample response: Let $x$ be the time spent hiking, let $y$ be the distance hiked. |
| Cards 3 and 10 | Sample response: Let $x$ be the number of silver dollar coins, let $y$ be the number of half-dollar coins. |
| Cards 5 and 6 | Sample response: Let $x$ be the minutes spent filling the pool, let $y$ be the gallons of water in the pool. |
| Cards 9 and 4, | Sample response: Let $x$ be the number of long wood boards, let $y$ be the number of short wood boards. |

2. For which cards did you not find a match?
   Card 1 and Card 8

**Are you ready for more?**

For a card that did not have a match, describe a scenario that could be represented by the equations or write a system of linear equations that could represent the scenario.

Sample response: Card 1 could be represented by the following scenario. Two runners are running in a race at a rate of $10$ ft per second. One runner starts running $1$ ft behind the starting line and one runner starts 5 ft ahead of the starting line. Card 8 does not have a match, but it cannot be written as a system of linear equations.

STOP





## 1 Launch

Distribute the cards from the Activity 2 PDF to the sets of partners. Tell students that if they use letters for variables that end up not matching the systems, they can still find the matching pairs.

## 2 Monitor

**Help students get started** by making sure students are precisely defining their variables.

**Look for points of confusion:**

- **Not being sure what strategy to take for Card 2.** Have students reference Activity 2 in Lesson 12 where Mr. Patel and Ms. Hernández run side by side and ask about their equations.
- **Missing that there are different units in the Card 4 scenario.** Ask students to check the units in the story.

**Look for productive strategies:**

- Seeing that Card 8 has one constraint, so there is no match.

## 3 Connect

**Have pairs of students share** their matches for all the even-numbered cards (story cards), starting with Card 4. Discuss their reasoning and any difficulties they might have had finding a match.

**Ask:**

- "What did you notice about the units given in Card 4? What implications does that have for writing systems of equations?" When writing an equation, it is important to be mindful of units. Adding or subtracting variables need to be in the same unit as in the equation.
- "Why was it not possible to find a match for Card 8?"
- "How can you quickly match a story to an equation?" Sample response: Checking the slope.

**Highlight** how to think of each story as being represented by two equations. Consider using color coding to help students make connections.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

The first pair of cards is already matched for students — Cards 7 and 2. Demonstrate how these cards make a matching pair by using a think-aloud, similar to the following.

- "Card 2 states that two friends hike at the same rate. This means the slopes should be the same. I need to find two cards for which the coefficients of $x$ are the same in each equation. This narrows the choices to Cards 1 and 7."
- "Card 2 also states that they start from the same distance from the parking lot. This means the initial values, the $y$-intercepts, should be the same. Only Card 7 has the same $y$-intercepts."

## Math Language Development

### *MLR7: Compare and Connect*

During the Connect, as students share their matches, draw their attention to the equations written in $y = mx + b$ form and how the slopes and $y$-intercepts are represented in the words of the story problem. Then highlight the equations that are written in $\_\_x + \_\_y = \_\_$ form and how their corresponding story problems refer to a total or a measure that is "combined."

### English Learners

Annotate key words and phrases in the story problems, such as *same rate, same distance, combined length, already contains*, and *total coins*.

Whole Class | 5 min

# Summary

Review and synthesize how a system of equations represents two equations that occur simultaneously and can represent real-world problems.

## Summary

**In today's lesson . . .**

You saw that a ***system of equations*** is a set of two equations with two variables where the variables represent the same unknown values. (In a later course, you will encounter systems with more than two equations and variables.)

A ***solution to a system of equations*** is an ordered pair that makes all equations in the system true.

For example, these equations make up a system of equations:

$$\begin{cases} x + y = -2 \\ x - y = 12 \end{cases}$$

One way to determine a solution to a system of equations is to graph both lines and locate the intersection point.

- If there is one point of intersection, you can conclude that the system of equations has one solution.
- If there is no point of intersection, you can conclude that the system of equations does not have a solution.
- If there are infinitely many points of intersection, you can conclude that the system has infinitely many solutions.

> Reflect:




## Synthesize

**Have students share** how systems of linear equations can be used to represent real-world scenarios.

**Formalize vocabulary:**
***system of equations***
***solution to a system of equations***

**Ask:**

- "What is a system of linear equations?"
- "What does the solution to a system of equations represent?"
- What does it look like when a system of equations has no solution? Infinite solutions?"

**Highlight** that writing and solving a system of equations can be an efficient way to find solutions to real-world problems. Remind students that the solution to the system of equations is not one number, but a pair of numbers — the ordered pair that makes all equations true. Tell students they will learn algebraic methods for solving systems of linear equations in upcoming lessons.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does the number of solutions (none, one, or infinite) to a system of equations represent?"

## Math Language Development

### *MLR2: Collect and Display*

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *system of equations* and *solution to a system of equations* that were added to the display during the lesson.

Independent | 8 min

# Exit Ticket

Students demonstrate their understanding by graphing a system of equations to estimate its solution.

Printable

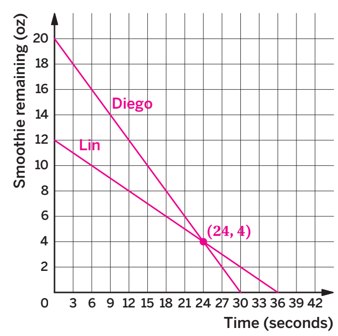
Name: ______ Date: ______ Period: ______

## Exit Ticket 4.13

Lin drinks a 12-oz smoothie at a rate of $\frac{1}{3}$ oz per second. Diego drinks a 20-oz smoothie and at a rate of $\frac{2}{3}$ oz per second.

1. Write a system of equations, each in the form $y = mx + b$, that represents this context. Be sure to define your variables.
   Let $x$ represent the number of seconds, and let $y$ represent the number of ounces of smoothie remaining.
   $\begin{cases} y = 12 - \frac{1}{3}x \\ y = 20 - \frac{2}{3}x \end{cases}$

2. Graph the equations you wrote on the same coordinate plane.
   Sample response shown.




3. Estimate the point of intersection. What does it tell you about the context?
   The point of intersection is estimated to be located at (24, 4), which means there is one time when both Lin and Diego have the same number of ounces of smoothie remaining. The point (24, 4) means that after 24 seconds, both of them had 4 oz of smoothie remaining.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can interpret the solution to a system of linear equations in a real-world context.
1 2 3

b. I can explain what a system of linear equations represents.
1 2 3

c. I can create graphs to identify an ordered pair that is a solution to a system of linear equations.
1 2 3



Lesson 13 Systems of Linear Equations

## Success looks like . . .

- **Language Goal:** Comprehending that solving a system of equations means determining values of the variables that makes both equations true at the same time. **(Speaking and Listening)**
  » Estimating the solution of a system of equations and explaining its meaning in the given context in Problem 3.
- **Goal:** Creating a graph of two lines that represents a system of linear equations in context.
  » Graphing the system of equations of the given context in Problem 2.

## Suggested next steps

**If students cannot correctly write a system of equations from context and define their variables, consider:**
- Reviewing strategies from Activity 2.
- Asking students to identify the initial value and rate of change by annotating the text.

**If students are unable to correctly graph the system of equations, consider:**
- Reviewing graphing strategies from Activity 1.
- Assigning Practice Problem 1.

**If students do not precisely describe the solution in context, consider:**
- Reviewing the solution in context from Activity 1.
- Asking, "What does the $x$-value represent? What does the $y$-value represent?"

**If students do not find the correct point of intersection, consider:**
- Having them substitute the $x$- and $y$-values into the system of equations to check whether it is a solution for both equations.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- How did activities in this lesson, and Lessons 11 and 12, set students up to develop a conceptual understanding of the term *system of linear equations*?
- Who participated and who didn't participate in the Card Sort activity today? What trends do you see in participation?

## Math Language Development

**Language Goal: Comprehending that solving a system of equations means determining values of the variables that makes both equations true at the same time.**

Reflect on students' language development toward this goal.

- Do students' responses to Problem 3 of the Exit Ticket demonstrate that they understand the point of intersection represents the solution to the system where both Lin and Diego have the same number of ounces of smoothie remaining?
- How can you help students be more precise in their description of what the point of intersection represents in this context?

# Practice

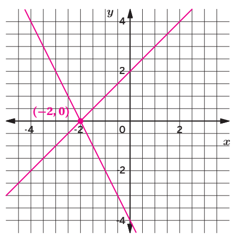
Independent

Name: Date: Period:

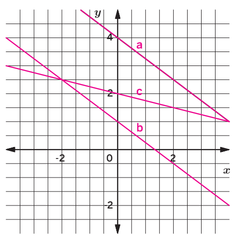
**1.** Graph the system of equations. Then estimate the coordinates of the ordered pair $(x, y)$ that makes both equations true.

$$\begin{cases} y = x + 2 \\ y = -2x - 4 \end{cases}$$

**Students' responses should be close to the point $(-2, 0)$.**




**2.** Consider the graph shown. Suppose it represents one equation in a system of two equations. Write a second equation for the system that would satisfy each of the following conditions.




a. The system has infinitely many solutions.

$y = -\frac{3}{4}x + 4$

b. The graph of the second equation passes through the point $(0, 1)$, and the system has no solution.

$y = -\frac{3}{4}x + 1$

c. The graph of the second equation passes through the point $(0, 2)$, and the system has one solution located at the point $(4, 1)$.

$y = -\frac{1}{4}x + 2$





Practice

Name: Date: Period:

**3.** The temperature in degrees Fahrenheit $F$ is related to the temperature in degrees Celsius $C$ by the equation $F = \frac{9}{5}C + 32$. There is one temperature for which the degrees Fahrenheit and degrees Celsius are the same, so that $C = F$. Use the expression from the equation, where $F$ is expressed in terms of $C$, to find this temperature. Show or explain your thinking.

$$C = \frac{9}{5}C + 32$$
$$-\frac{4}{5}C = 32$$
$$C = -40$$

**At −40 degrees Celsius, the temperature is also −40 degrees Fahrenheit.**

**4.** Decide whether each equation is true for *all*, *one*, or *no* values of $x$.

a. $2x + 8 = -3.5x + 19$

**True for one value of $x$.**

b. $9(x - 2) = 7x + 5$

**True for one value of $x$.**

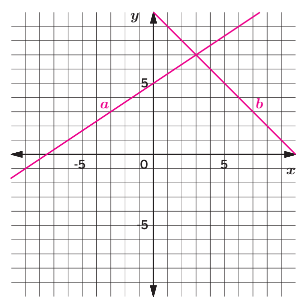
c. $3(3x + 2) - 2x = 7x + 6$

**True for all values of $x$.**

**5.** Think about the equations $y = \frac{2}{3}x + 5$ and $x + y = 10$.

a. Graph the equation $y = \frac{2}{3}x + 5$ and label its line as line $a$.

b. Graph the equation $x + y = 10$ and label its line as $b$.








## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 1 |
| | 2 | Activity 1 | 2 |
| Spiral | 3 | Unit 4 Lesson 10 | 1 |
| | 4 | Unit 4 Lesson 8 | 1 |
| Formative | 5 | Unit 4 Lesson 14 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

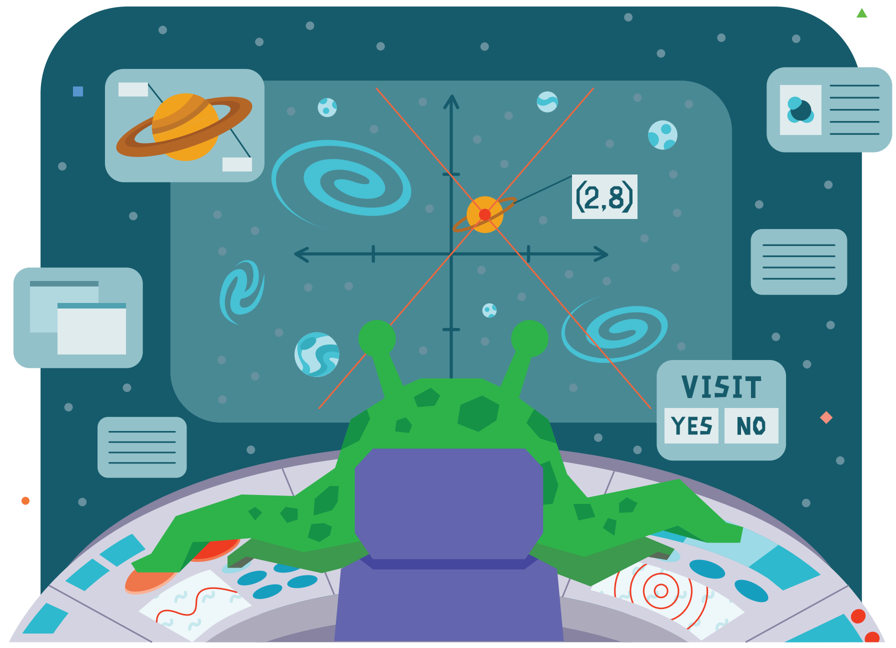
## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 4 | **LESSON 14**

# Solving Systems of Linear Equations (Part 1)

Let's solve systems of linear equations.




## Focus

### Goals

1. Create a graph of a system of linear equations, and identify the solution to the system of equations.
2. **Language Goal:** Justify that a particular system of equations has one solution, no solution, or infinitely many solutions by using the structure of the equations. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students continue to explore systems of linear equations. They connect algebraic and graphical representations of systems by drawing their own graphs and identifying the solution. Students use graphing technology to analyze the structure of equations and the number of solutions for the system of linear equations.

### Previously

In Lessons 7 and 8, students explored equations with no solution and infinitely many solutions. In Lesson 13, students were formally introduced to a system of equations based on a context.

### Coming Soon

In Lesson 15, students will continue to explore systems of linear equations and solve the system algebraically to determine its solution.

## Rigor

- Students graph a system of linear equations and identify the solution to develop **procedural fluency**.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 10 min | 20 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 2 PDF (optional)
- Activity 2 PDF (answers)
- graphing technology
- rulers

### Math Language Development

**Review words**

- *coefficient*
- *constant*
- *solution to a system of equations*
- *slope*
- *$y$-intercept*
- *system of equations*

**Amps** | Featured Activity

### Activity 2
### Graphing Systems of Equations

Students enter a system of equations and use the graph to identify the number of solutions for the system.

Amps POWERED BY desmos

### Building Math Identity and Community

Connecting to Mathematical Practices

When handed technology, sometimes students will turn their brains off. Encourage students to have a growth mindset instead. They need to think of technology as a tool to help them understand systems of equations, even when they might not be able to solve them on their own yet. Students see the benefit of graphing technology as they use the structure of the graphs of equations to identify the number of solutions to a systems of equations.

### Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 2**, display matching equations, graphs, and the number of solutions for Problem 1. Have students answer Problems 2 and 3 by using the provided information.

Pairs | 5 min

## Warm-up True or False?

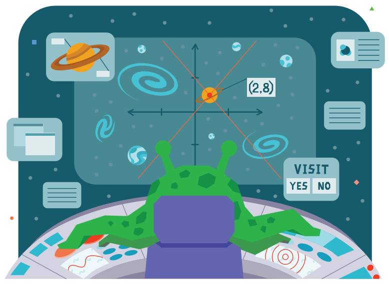
Students analyze equations and graphs to connect algebraic and graphical representations of equations.

Name: ______ Date: ______ Period: ______

Unit 4 | Lesson 14

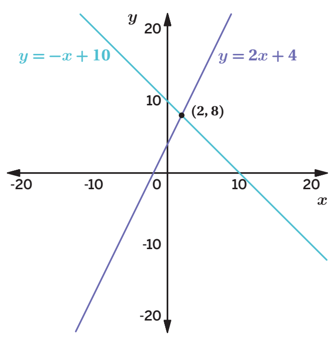
# Solving Systems of Linear Equations (Part 1)

Let's solve systems of linear equations.



### Warm-up True or False?

**Consider the two lines represented by the equations $y = -x + 10$ and $y = 2x + 4$. Use the lines to decide whether each statement is true or false. Be prepared to explain your thinking.**




a. The ordered pair $(2, 8)$ is a solution to the equation $y = -x + 10$.
True

b. The ordered pair $(8, 2)$ is a solution to the equation $y = 2x + 4$.
False

c. The ordered pair $(8, 2)$ is a solution to both the equations $y = -x + 10$ and $y = 2x + 4$.
False

d. The ordered pair $(2, 8)$ is a solution to both the equations $y = -x + 10$ and $y = 2x + 4$.
True

e. There are no values of $x$ and $y$ that make the equations $y = -x + 10$ and $y = 2x + 4$ true at the same time.
False

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Conduct the ***True or False*** routine. Record class responses. While some students may solve each equation to find whether it is true or false without relating it to the graphs, encourage all students to show why their answer is correct based on the graphs of the equations.

### 2 Monitor

**Help students get started** by having them identify the point and line given in statement a.

**Look for points of confusion:**

- **Thinking that statements b and c are true.** Remind students that ordered pairs are written as $(x, y)$, which is different from $(y, x)$.

**Look for productive strategies:**

- Remembering that a solution represents a point on the line.

### 3 Connect

**Display** the graph from the Warm-up.

**Have pairs of students share** their reasoning based on the lines.

**Highlight** that the ordered pair $(2, 8)$ is a solution to both $y = -x + 10$ and $y = 2x + 4$ because it is the only point on both lines. This point can be determined by looking at where the two lines intersect. Remind students that $(2, 8)$ is the solution to the system of equations $\begin{cases} y = 2x + 4 \\ y = -x + 10 \end{cases}$.

**Ask**, "Can there be another point of intersection for the two lines? Why or why not?" No. Because the slopes of the lines are different, and the lines continue straight in both directions, they will never intersect again.

## Differentiated Support

**Accessibility: *Activate Prior Knowledge***

Before launching the activity, ask students, "How do you know whether a point on a graph is a solution to an equation?" Listen for and highlight student ideas that describe that if a point lies on the line, then it is a solution to the equation of the line.

## Power-up

**To power up students' ability to graph a line given the equation:**

Provide students with a copy of the Power-up PDF.

**Use:** Before the Warm-up
**Informed by:** Performance on Lesson 13, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 7

Pairs | 10 min

# Activity 1 Graphing a System of Linear Equations

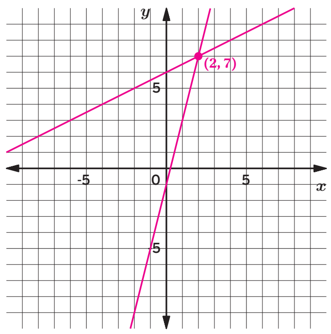
Students graph a system of linear equations to develop procedural fluency in estimating the solution from a graphical representation.

**Activity 1** Graphing a System of Equations

Graph each system of equations. Then estimate the coordinates of the ordered pair $(x, y)$ that makes both equations true.

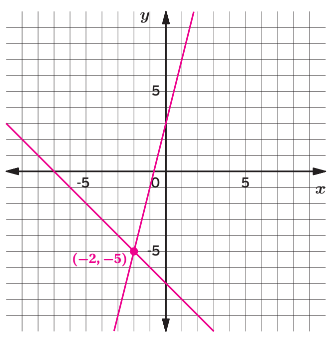
1. $\begin{cases} y = \frac{1}{2}x + 6 \\ y = 4x - 1 \end{cases}$

Students' responses should be close the point (2, 7).




2. $\begin{cases} y = 4x + 3 \\ x + y = -7 \end{cases}$

Students' responses should be close the point ( −2, −5).




446 Unit 4 Linear Equations and Systems of Linear Equations



## 1 Launch

Activate prior knowledge by asking students different ways they could graph a line, such as using the slope and $y$-intercept or substituting values to determine ordered pairs. Provide access to rulers.

## 2 Monitor

**Help students get started** by having them graph one line at a time and then having them look for the point of intersection.

**Look for points of confusion:**

- **Not knowing how to graph Problem 2 because the second equation is not written in the form $y = mx + b$.** Remind students that they can substitute values for $x$ and $y$ in the equation $x + y = -7$ to determine ordered pairs on the line and then graph the points.

**Look for productive strategies:**

- Drawing slope triangles.
- Using the $x$ and $y$-intercepts to graph $x + y = -7$.

## 3 Connect

**Display** student work showing the completed graphs.

**Have students share** which strategies they used to graph each system of equations.

**Ask**, "Can the ordered pair (7, 2) be a solution to the system of equations for Problem 1? Why or why not?" Sample response: No, an $x$-value of 7 will not produce an output of 2 for either equation. In addition, the lines do not intersect at (7, 2).

**Highlight** that, for systems of linear equations that intersect, there is only one ordered pair that is a solution for both equations.

## Differentiated Support

**Accessibility: *Vary Demands to Optimize Challenge, Optimize Access to Technology***

If students need more processing time, have them focus on completing Problem 1. Have students use the Amps slides, or provide access to other graphing technology, to help them estimate solutions to the systems of equations from a graphical representation.

**Extension: *Math Enrichment***

Have students write a system of equations in which the ordered pair (5, −3) is the solution to the system.

Sample response:

$y = 5x - 13$

$2y - x = -11$

# Activity 2 How Many Solutions?

Students graph systems of linear equations to connect the structure of equations and the number of solutions for the system of equations.

**Amps Featured Activity** Graphing Systems of Equations

Name: __________ Date: ______ Period: ______

## Activity 2 How Many Solutions?

You will need graphing technology to complete this activity.

1. Graph each system of equations. Determine whether the system of equations has *one solution*, *no solution*, or *infinitely many solutions* by placing a check mark in the appropriate box. If the system of equations has one solution, estimate the ordered pair that makes both equations true.
For the systems with one solution, estimates should be close to the points shown.

| System of equations | One solution | No solution | Infinitely many solutions |
|---|---|---|---|
| $\begin{cases} y = 5(x-3) \\ y = 2x - 15 \end{cases}$ | ✓ (0, – 15) | | |
| $\begin{cases} y = 2x + 3 \\ y = 2x - 5 \end{cases}$ | | ✓ | |
| $\begin{cases} y = -6x \\ y = -5x + 10 - x \end{cases}$ | | ✓ | |
| $\begin{cases} y = -4x + 6 \\ y = -4x + 6 \end{cases}$ | | | ✓ |
| $\begin{cases} y = -4x + 8 \\ y = -2x + 5 \end{cases}$ | ✓ (1.5, 2) | | |
| $\begin{cases} y = 6x + 3 - 4x \\ y = 2x + 3 \end{cases}$ | | | ✓ |

2. What do you notice about the coefficient and constants in each system of linear equations when it has . . . Sample responses shown.
   a. One solution?
   The coefficients are different, the constants may or may not be the same.
   b. No solution?
   The coefficients are the same, and the constants are different.
   c. Infinitely many solutions?
   The coefficients and constants are the same.




## 1 Launch

Have students use graphing technology to graph each system of equations to determine the number of solutions. Have students complete Problems 1 and 2 in pairs, and then discuss the problems as a whole class before moving to Problem 3.

Depending on time and resources, you may wish to use the Activity 2 PDF and have students match each system of equations with its graph before determining the number of solutions. Have students record their results in the table in the Student Edition.

## 2 Monitor

**Help students get started** by activating their prior knowledge and asking what the graph looks like when a system has one solution, no solutions, and infinitely many solutions.

**Look for points of confusion:**

- **Not recognizing any patterns in Problem 2.** Have students rewrite the equations with fewer terms, then compare the coefficients and constants for each equation. Consider having students reference the Summary from Lesson 8.
- **Thinking the coefficients in Problem 3d are the same.** After students use the Distributive Property in the second equation, point out that $x$ and $-x$ do not have the same coefficient.

**Look for productive strategies:**

- Rewriting the equations with fewer terms and noticing that they could compare the coefficient and constants, similar to Lesson 8, to determine the number of solutions.

Activity 2 continued >

## Differentiated Support

### Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can enter a system of equations and use the graph to identify the number of solutions for the system.

### Accessibility: *Vary Demands to Optimize Challenge*

For Problems 1 and 3, provide students with equivalent expressions with fewer terms. This will help facilitate the connection between the structure of the equations and the number of solutions to a system of equations.

## Math Language Development

### *MLR8: Discussion Support*

During the Connect, as students share their responses to the Ask question, add the following to the class display to help students make the connection between the mathematical terminology used and the structure of the equations.

**When both equations are written in the form $y = mx + b$**

| One solution | No solution | Infinitely many solutions |
|---|---|---|
| Different slopes | Same slopes | Same slopes |
| Same or different $y$-intercepts | Different $y$-intercepts | Same $y$-intercepts |

Pairs | 20 min

# Activity 2 How Many Solutions? (continued)

Students graph systems of linear equations to connect the structure of equations and the number of solutions for the system of equations.

### Activity 2 How Many Solutions? (continued)

3. Without graphing, determine whether each system of equations has *one solution*, *no solution*, or *infinitely many solutions*. Be prepared to explain your thinking.

a. $\begin{cases} y = -\frac{4}{3}x + 4 \\ y = -1 - \frac{4}{3}x \end{cases}$

No solution

b. $\begin{cases} y = -2x - 5 + 6x \\ y = -2x + 7 \end{cases}$

One solution

c. $\begin{cases} y = 5x - 15 \\ y = 5(x - 3) \end{cases}$

Infinitely many solutions

d. $\begin{cases} y = x + 6 \\ y = -(x + 6) \end{cases}$

One solution

**Discussion Support:** What math terms can you use during the discussion to explain how you determined the number of solutions?

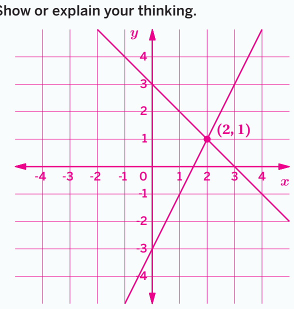
**Are you ready for more?**

The graphs of the equations $x + y = \square$ and $y = \square x - 3$ intersect at the point $(2, 1)$. Determine the missing values in the equations. Show or explain your thinking.

$x + y = 3$ and $y = 2x - 3$.

Sample response:

- For the first equation, I used the point $(2, 1)$ and substituted the $x$- and $y$-values into the equation to determine the missing value of 3.
- For the second equation, I graphed the $y$-intercept $(0, -3)$ and point $(2, 1)$ to determine the slope of 2. This means the missing value is 2.




STOP




## 3 Connect

**Have pairs of students share** their responses for Problem 2. Record responses for all to see.

**Ask**, "How do you know the number of solutions to a system of equations by looking at the slopes and $y$-intercepts of the two lines?" If the slopes and $y$-intercepts are the same, there will be infinitely many solutions. If the slopes are the same, but the $y$-intercepts are different, there will be no solution. If the slopes are different, there will be one solution.

### Highlight

- A system of linear equations with one solution has different coefficients. The constants may or may not be the same.
- A system of linear equations with no solution has the same coefficients and different constants.
- A system of linear equations with infinitely many solutions has the same coefficients and the same constants.

Whole Class | 5 min

# Summary

Review and synthesize how coefficients and slopes in a system of linear equations can help students determine the solution to a system of equations.

Name: ______ Date: ______ Period: ______

## Summary

**In today's lesson . . .**

You graphed a system of linear equations to determine the solution to the system of equations. You found that you can identify the number of solutions for a system of equations by studying the coefficients and constants of the equations.

Here are some examples:

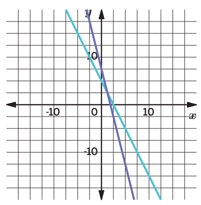
**One solution**

$$\begin{cases} y = -4x + 8 \\ y = -2x + 5 \end{cases}$$

Equations:

- Different coefficients
- Same or different constants

Graph:



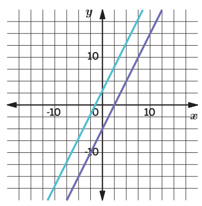

**No solution**

$$\begin{cases} y = 2x + 3 \\ y = 2x - 5 \end{cases}$$

Equations:

- Same coefficients
- Different constants

Graph:



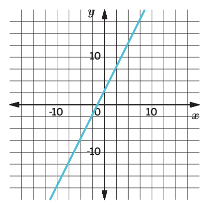

**Infinitely many solutions**

$$\begin{cases} y = 2x + 3 \\ y = 2x + 3 \end{cases}$$

Equations:

- Same coefficients
- Same constants

Graph:




**Reflect:**





## Synthesize

**Display** the Summary from the Student Edition.

**Have students share** how they can determine the number of solutions to a system of equations using the equations and the graph.

**Ask**, "How is determining the number of solutions to a system of equations similar to and different from determining the number of solutions to a single equation?" Sample response: They are similar because you can look at the coefficient and constants in a system of equations and in a single equation. They are different because a system of equations has more than one equation.

**Highlight** that students could rewrite the equations in a system with fewer terms and then compare the coefficients and constant terms to determine the number of solutions that the system has.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does the number of solutions (none, one, or infinite) to a system of linear equations represent?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding of identifying the number of solutions graphically and algebraically.

Printable

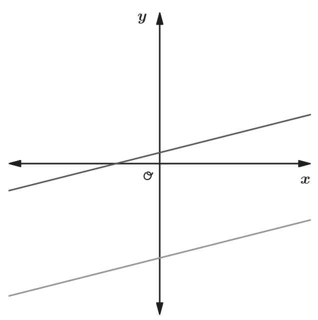
Name: ______________ Date: ________ Period: ________

## Exit Ticket 4.14

**Consider the parallel lines shown.**

1. Select two equations that could represent the system.

   A. $y = \frac{1}{4}x - 10$
   
   B. $y = 5(x - 2)$
   
   C. $y = \frac{1}{4} + 10x$
   
   D. $y = \frac{1}{4}x + 1$
   
   E. $y = x - 10$




2. How many solutions does this system of equations have? Explain your thinking.

   None. Sample response: The equations of the lines have the same coefficients and represent parallel lines. Because the lines are parallel and never intersect, there is no solution to the system of equations.

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can determine the solution to a system of linear equations by graphing. 1 2 3

b. I can use the structure of equations to determine the number of solutions for a system of linear equations, without graphing. 1 2 3





## Success looks like . . .

- **Goal:** Creating a graph of a system of linear equations, and identifying the solution to the system of equations.
  - » Selecting two equations for a system of equations that could represent the graph in Problem 1.
- **Language Goal:** Justifying that a particular system of equations has one solution, no solution, or infinitely many solutions by using the structure of the equations. **(Speaking and Listening, Writing)**
  - » Explaining the number of solutions of the system of equations in Problem 2.

## Suggested next steps

**If students do not choose A and D for Problem 1, consider:**

- Having them rewrite equations with fewer numbers of terms, if applicable.
- Asking them to circle the coefficient or the slope in each equation.
- Reviewing Activity 2.

**If students incorrectly answer Problem 2, consider:**

- Asking them where the point of intersection would be on the graph. Point out that, because the lines do not intersect, there will be no solution.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

### Points to Ponder . . .

- In what ways have your students improved in using the structure of equations to determine the number of solutions?
- During the discussion about determining the number of solutions by using the structure of equations, how did you encourage each student to share their understanding?

# Practice

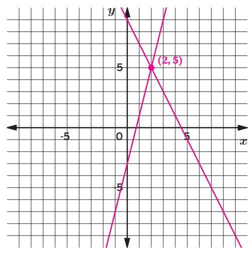
Independent

Name: ______ Date: ______ Period: ______

1. Graph each system of equations. Then estimate the coordinates of the ordered pair $(x, y)$ that makes both equations true.

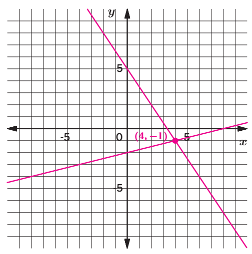
   a. $\begin{cases} y = 4x - 3 \\ y = -2x + 9 \end{cases}$

   Students' responses should be close to the point (2, 5).




   b. $\begin{cases} y = \frac{1}{4}x - 2 \\ 3x + 2y = 10 \end{cases}$

   Students' responses should be close the point $(4, -1)$.



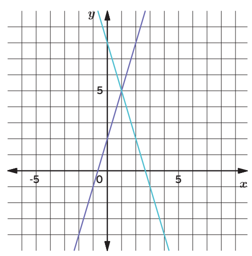

2. Consider the two lines shown.

   a. Write an equation that can represent each line.

   $y = 3x + 2$ and $y = -3x + 8$

   b. Describe how to estimate the solution to the corresponding system by studying the graph. Then estimate the ordered pair that makes both equations true.

   Sample response: Estimate the point where the two lines intersect: (1, 5).







Name: ______ Date: ______ Period: ______

3. Which equation, together with the equation $y = -5x + 10$, creates a system with infinitely many solutions?

   A. $y = 5 - 10x$

   B. $y = 2x + -6 - 3x$

   C. $y = 5(x - 2)$

   D. $y = -5(x + 3) + 25$

   E. $y = 10x + x - 5$

4. Solve each equation. Show or explain your thinking.

   a. $\frac{15(x-3)}{5} = 3(2x - 3)$

   $3(x - 3) = 3(2x - 3)$
   $x - 3 = 2x - 3$
   $-3 = x - 3$
   $x = 0$

   b. $0.4(x + 7) = 0.2(x + 40) - 5.2 + 0.2x$

   $0.4x + 2.8 = 0.2x + 8 - 5.2 + 0.2x$
   $0.4x + 2.8 = 0.4x + 2.8$

   There are infinitely many solutions because the coefficients and constants are the same on each side of the equal sign.

5. Determine the $y$-value for each equation if $x = 3$. What do you notice?

   **Equation 1:**
   $y = 3x + 9$
   $y = 3(3) + 9$
   $y = 18$

   **Equation 2:**
   $y = 7x - 3$
   $y = 7(3) - 3$
   $y = 18$

   Sample response: The $y$-values are both 18.




## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 1 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 2 | 2 |
| Spiral | 4 | Unit 4 Lesson 6 | 1 |
| Formative | 5 | Unit 4 Lesson 15 | 1 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Solving Systems of Linear Equations (Part 2)

Let's solve systems of linear equations.

## Focus

### Goals

1. **Language Goal:** Correlate the solution to an equation with variables on both sides to the solution to a system of two linear equations. **(Speaking and Listening)**

2. **Language Goal:** Generalize a process for solving systems of equations and calculate the values that are a solution to a system of linear equations. **(Speaking and Listening, Writing)**

## Coherence

### Today

Students solve a system of linear equations, where the equations are of the form $y = mx + b$. Students associate solving a system of linear equations with solving an equation when they set two $y$-values equal to each other to solve for $x$. They build fluency in solving systems of equations, and critique the reasoning of others as they complete *Partner Problems*.

### Previously

In Lesson 10, students determined when two amounts, given a context, would be the same and started to develop a process for solving a system of linear equations. In Lesson 14, students solved systems of equations by graphing.

### Coming Soon

In Lesson 16, students will apply their understanding of systems of equations to interpret and solve linear equations in a context.

## Rigor

- Students solve systems of linear equations to build **fluency**.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 12 min | 18 min | 5 min | 5 min |
| Pairs | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice
Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- Anchor Chart PDF, *Solving Linear Equations*

### Math Language Development

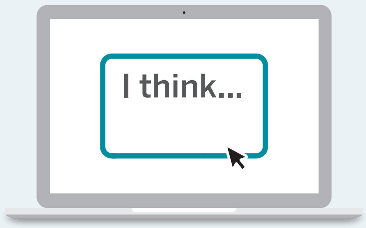
**Review words**

- *slope*
- *solution to a system of equations*
- *system of equations*

**Amps** | Featured Activity

### Activity 1
### See Student Thinking

Students are asked to explain their thinking when describing how to solve a system of equations.These explanations are digitally available to you, in real time.




### Building Math Identity and Community

Connecting to Mathematical Practices

As partners work to agree on a solution, they might get excited about their own work and forget to listen well to their partner's responses. Remind students that, by listening well, each person can determine whether they need to seek or offer help. Review signals that indicate whether a person is actively listening and encourage students to practice them.

### Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In **Activity 2**, consider having students complete the first row and assigning the remaining problems as additional practice.

Pairs | 5 min

## Warm-up Clean up on Quadrant Four

Students study the graph of a system of equations as a reminder that they do not need to graph lines to solve the system and to generate ideas for solving a system algebraically.

**Unit 4 | Lesson 15**

# Solving Systems of Linear Equations (Part 2)

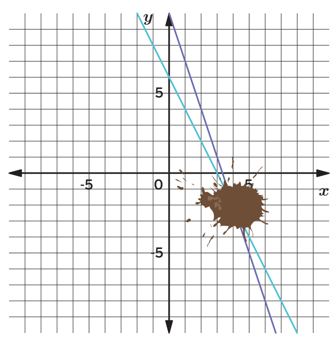
Let's solve systems of linear equations.

**Warm-up** Clean up on Quadrant Four

**While Priya was doing her homework, her mom accidently spilled coffee on it! Priya was graphing a system of equations to determine the solution, but can no longer see where the lines intersect. Her work is shown.**

Determine the ordered pair that makes both equations true.

$$\begin{cases} y = -3x + 10 \\ y = -2x + 6 \end{cases}$$




Show or explain another method Priya could use to determine the solution.

- **From the graph, it looks like the $x$-value is between 2 and 5, so she can substitute different $x$-values until she gets the same $y$-value for both equations.**
- **She can write one equation $-3x + 10 = -2x + 6$ and solve for the $x$-value before solving for the $y$-value.**

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Conduct the ***Think-Pair-Share*** routine.

### 2 Monitor

**Help students get started** by asking them how they can use the graph or equations to determine another method Priya could use.

**Look for productive strategies:**

- Using the visible points on the lines and the slope to estimate the ordered pair.
- Substituting values to guess the ordered pair.

### 3 Connect

**Display** the equation and graph from the Warm-up.

**Have pairs of students share** their response. Select pairs of students with varying responses. Record responses for all to see.

**Ask**:

- "How do you know there is a solution to the system of linear equations?" Sample response: The lines are not parallel, so they will intersect at one point. The coefficients are different, so I know there is one solution.
- "Which method could you use to solve the system if you were given only the equations, and not the graph?" Sample response: Write one equation by setting the $y$-values equal to each other.

**Highlight** that, to determine the point of intersection, students need to determine the value of $x$ so that both equations have the same $y$-value. One way to do this is to write one equation by setting the $y$-values equal to each other.

## Power-up

**To power up students' ability to determine the corresponding $y$-value after substituting a given $x$-value into a two-variable equation, have students complete:**

Which of the following demonstrate the correct work when determining the value of $y$ when $x = 2$ in the equation $y = -3x + 1$.

**A.** $y = -3(2) + 1$
$y = -3(3)$
$y = -9$

**B.** $2 = -3x + 1$
$2 = -3x$
$-\frac{1}{3} = x$

**C.** $y = -3(2) + 1$
$y = -6 + 1$
$y = -5$

**Use:** Before Activity 1
**Informed by:** Performance on Lesson 14, Practice Problem 5

Pairs | 12 min

# Activity 1 What's the Solution?

Students develop a method to solve a system of linear equations algebraically.

**Amps Featured Activity** See Student Thinking

Name: ______ Date: ______ Period: ______

**Activity 1** What's The Solution?

Elena solves the system of equations from the Warm-up. Some of her work is shown.

$$\begin{cases} y = -3x + 10 \\ y = -2x + 6 \end{cases}$$

Elena's work:

$-3x + 10 = -2x + 6$
$-x + 10 = 6$
$-x = -4$
$x = 4$

1. Describe Elena's method for calculating the value of $x$.
Sample response: Elena set both of the expressions $-3x + 10$ and $-2x + 6$ equal to each other by writing one equation, and then solved the equation.

2. Describe a method that Elena could use to calculate the value of $y$. Then use this method to determine the value of $y$.
Sample responses:
- Elena could substitute $x = 4$ into the first equation in the system of equations. $y = -3(4) + 10$, $y = -2$
- Elena could substitute $x = 4$ into the second equation in the system of equations. $-2(4) + 6 = -8 + 6 = -2$, $y = -2$

3. What is the ordered pair that is a solution to the system?
$(4, -2)$





## 1 Launch

Have students complete Problem 1 individually. Then have them share responses with a partner before completing Problems 2 and 3.

## 2 Monitor

**Help students get started** by having them review Elena's work step by step.

**Look for points of confusion:**

- **Not understanding Elena's work.** Use the graph from the Warm-up to point out that the point of intersection is where the $y$-values are equal. Then circle $-3x + 10$ and $-2x + 6$ and tell students that these represent the $y$-values algebraically.
- **Having trouble describing a method to calculate $y$.** Ask students to refer to the suggestions made in the Warm-up. For students who need more support, give them explicit instructions on how to substitute $x = 4$ in one of the equations in the system.

## 3 Connect

**Have students share** their responses for Problem 2.

**Highlight** that students can substitute the $x$-value into either equation from the original system to determine the $y$-value, but should check their answer using both equations to determine whether their solution is correct. Also highlight that when students solve a system with two linear equations, the final response should have two variables written as an ordered pair $(x, y)$.

**Display** the Activity 1 PDF. Ask, "Why are the $y$-values the same when the $x$-value is substituted into either equation?" Sample response: Because there is one point of intersection, no matter which line we look at, the coordinates $(x, y)$ are the same.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Consider allowing students to verbally describe Elena's method for Problem 1, instead of writing a full explanation at first. Scribe their thinking onto a display, creating a complete sentence for them to see.

### Extension: *Math Enrichment*

Ask students if they could still use Elena's method to solve the system of equations if both equations were not written in the form $y = mx + b$, such as the following system.

$\{y = -3x + 10 \;\; 2x + y = 6$

Yes; I could solve the second equation for $y$ and then set the two expressions equal to each other.

## Math Language Development

### *MLR8: Discussion Supports*

During the Connect, as students share their responses to Problem 2, display these sentence frames to help them organize their thinking:

- "First, I_____ because . . ."
- "I noticed_____, so I . . ."
- "I chose to use the first/second equation because . . ."

Ask, "Does it matter which equation you use to substitute the $x$-value to check the solution? Consider asking these follow-up questions:

- "Is the solution $(4, -2)$ a solution to one or both equations? How do you know?"
- "If you determined a solution and it only worked in one of the equations, would this be a solution to the system? Why or why not?"

Pairs | 18 min

# Activity 2 Partner Problems

Students solve systems of linear equations to build procedural fluency.

### Activity 2 Partner Problems

With your partner, decide who will solve the systems of equations in Column A and who will solve the systems of equations in Column B. After each row, share your responses with your partner. Although the problems in each row are different, your responses should be the same. If they are not the same, work together to correct any errors or resolve any disagreements

| | Column A | Column B |
|---|---|---|
| 1. | $\begin{cases} y = -3x + 9 \\ y = 2x + 4 \end{cases}$<br>$-3x + 9 = 2x + 4$<br>$-3x - 2x = 4 - 9$<br>$-5x = -5$<br>$x = 1$<br>$y = 2(1) + 4$<br>$y = 6$<br>Solution: $(1, 6)$ | $\begin{cases} y = -4x + 10 \\ y = 8x - 2 \end{cases}$<br>$-4x + 10 = 8x - 2$<br>$-4x - 8x = -2 - 10$<br>$-12x = -12$<br>$x = 1$<br>$y = -4(1) + 10$<br>$y = 6$<br>Solution: $(1, 6)$ |
| 2. | $\begin{cases} y = 5x + 7 \\ y = 6x + 4 \end{cases}$<br>$5x + 7 = 6x + 4$<br>$5x - 6x = 4 - 7$<br>$-x = -3$<br>$x = 3$<br>$y = 5(3) + 7$<br>$y = 22$<br>Solution: $(3, 22)$ | $\begin{cases} y = -2x + 28 \\ y = -x + 25 \end{cases}$<br>$-2x + 28 = -x + 25$<br>$-2x + x = 25 - 28$<br>$-x = -3$<br>$x = 3$<br>$y = -2(3) + 28$<br>$y = 22$<br>Solution: $(3, 22)$ |

454 Unit 4 Linear Equations and Systems of Linear Equations



## 1 Launch

Conduct the ***Partner Problems*** routine. Remind students that solving a system of equations means that they should have two variables written as an ordered pair for their final response. Consider providing students with additional paper to thoroughly show their thinking.

## 2 Monitor

**Help students get started** by asking them to inspect each system of equations to determine whether each system will have a solution.

**Look for points of confusion:**

- **Writing one value for their solution.** Tell students that they are looking for the same $x$- and $y$-values that will make both equations true.
- **Having trouble solving Problem 3**. Ask students whether they can identify an $x$- or $y$-value from either equation. Students should recognize the value of $x$. Then have them substitute $x = 4$ in the other equation to solve for $y$.

**Look for productive strategies:**

- For the first two rows, substituting their $x$-value in both equations to check whether the $y$-values will produce the same value.

**Activity 2 continued >**

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

To support students' organizational thinking, provide the following checklist for them to refer to while solving:

- Determine a strategy you can use to solve the system, by asking yourself these questions:
  - » Are both equations already solved for $x$ or $y$?
  - » Is the value of one coordinate of the solution already given?
  - » Can you determine if there is no solution or infinitely many solutions by just inspecting the structure of the equations?
- Write your solution as an ordered pair, $(x, y)$.

## Math Language Development

### *MLR7: Compare and Connect*

During the Connect, as students share how Problem 3 is different from Problems 1 and 2, listen for students who recognize that one of the equations gives the $x$-value, so the $x$-coordinate of the solution is 4. Draw students' attention to the connections between the structure of the equations and the strategies that are most efficient to use. For example, for Problem 4, ask:

- "Why do you not need to solve either equation for $x$ or $y$ to determine that there is no solution to these systems?"
- "How can visual inspection be an efficient method to use when solving a system of equations?"

Pairs | 18 min

# Activity 2 Partner Problems (continued)

Students solve systems of linear equations to build procedural fluency.

Name: ______ Date: ______ Period: ______

## Activity 2 Partner Problems (continued)

| Column A | Column B |
|---|---|
| 3. $\begin{cases} y = -3x + 12 \\ x = 4 \end{cases}$<br>$y = -3(4) + 12$<br>$y = 0$<br>Solution: $(4, 0)$ | $\begin{cases} x = 4 \\ y = -3(x - 4) \end{cases}$<br>$y = -3(4 - 4)$<br>$y = 0$<br>Solution: $(4, 0)$ |
| 4. $\begin{cases} 2x + y = 7 \\ 2x + y = 9 \end{cases}$<br>No solution.<br>Sample response: Because the left sides of both equations are the same and the right sides of both equations are different, I know there is no solution. | $\begin{cases} 3x - 2y = 11 \\ 3x - 2y = 7 \end{cases}$<br>No solution.<br>Sample response: Because the left sides of both equations are the same and the right sides of both equations are different, I know there is no solution. |

STOP





## 3 Connect

**Have pairs of students share** any problems in which they did not have the same solution as their partner, and how they came to an agreement of their final solution.

**Ask** students how Problem 3 is different from Problems 1 and 2, and what strategies they used to determine the solution.

**Highlight** that one way students can solve a system of equations is to write a single equation to solve for one variable and then substitute that value into one of the original equations to solve for the other variable.

Whole Class | 5 min

# Summary

Review and synthesize how to solve a system of linear equations algebraically.

## Summary

**In today's lesson . . .**

You discovered that for an ordered pair to be a solution to a system of equations, the $x$- and $y$-values of the ordered pair must make both of the equations true.

For example, consider the following system of equations:

$$\begin{cases} y = 4x - 5 \\ y = -2x + 7 \end{cases}$$

To determine the solution to the system, you can write a single equation that sets the two expressions — for which $y$ is equal to — equal to each other:

$$\begin{aligned} 4x - 5 &= -2x + 7 \\ 6x - 5 &= 7 \\ 6x &= 12 \\ x &= 2 \end{aligned}$$

Then you can use the solution for $x$ and either of the original equations in the system to determine the value of $y$:

If $x = 2$, then $y = 4(2) - 5$, $y = 3$.

The ordered pair (2, 3) is the solution to the system of equations.

**Reflect:**





## Synthesize

**Have students share** how to solve a system of equations algebraically in their own words.

**Highlight** that the solution to a system of equations is the ordered pair that makes all the equations true.

**Ask**, "After you solve a system of equations, how could you check whether the solution is correct?" Substitute the $x$- and $y$-values into all the equations in the system and check whether all the equations are true.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is solving a system of linear equations similar to solving an equation with variables on both sides? How is it different?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by solving a system of linear equations algebraically.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 4.15

Solve the following system of equations. Show or explain your thinking.

$$\begin{cases} y = -3x - 5 \\ y = 4x + 30 \end{cases}$$

Sample response:

$-3x - 5 = 4x + 30$

$-5 - 30 = 4x + 3x$

$-35 = 7x$

$x = -5$

Substitute the $x$-value into the second equation and solve for $y$:

$y = 4(-5) + 30$

$y = 10$

Solution: $(-5, 10)$

**Self-Assess**

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can solve a system of linear equations using algebraic thinking. 1 2 3





## Success looks like . . .

- **Language Goal:** Correlating the solution to an equation with variables on both sides to the solution to a system of two linear equations. **(Speaking and Listening)**
- **Language Goal:** Generalizing a process for solving systems of equations and calculating the values that are a solution to a system of linear equations. **(Speaking and Listening, Writing)**
  - » Explaining the process of solving the system of equations.

## Suggested next steps

**If students do not solve for $x$ correctly, consider:**

- Reviewing Lesson 10 and having students refer to the Anchor Chart PDF, *Solving Linear Equations*.

**If students do not correctly solve for $y$, consider:**

- Highlighting $x$ in either equation in the system and having them rewrite the equation by substituting $-5$ for $x$.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- During the discussion in the Warm-up, how did you encourage each student to listen to one another's strategies?
- What challenges did students encounter as they worked on Activity 1? How did they work through them?

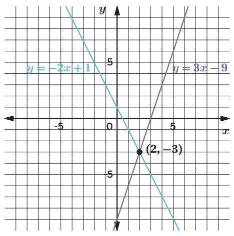
# Practice

Independent

Name: ________ Date: ________ Period: ________

Practice

1. Use the lines shown to decide whether each statement is true or false.




   a. The solution to the equation $-2x + 1 = 3x - 9$ is $x = 2$.
   True

   b. The point $(2, -3)$ is a solution to the following system of equations:
   $\begin{cases} y = -2x + 1 \\ x = 2 \end{cases}$
   True

   c. The point $(0, 1)$ is a solution to the equation $y = -2x + 1$.
   True

2. Solve each system of equations. Show or explain your thinking.

   a. $\begin{cases} y = 3x - 2 \\ y = -2x + 8 \end{cases}$

   $3x - 2 = -2x + 8$
   $3x + 2x = 8 + 2$
   $5x = 10$
   $x = 2$

   Substitute the $x$-value into the first equation and solve for $y$:
   $y = 3(2) - 2$
   $y = 4$

   Solution: (2, 4)

   b. $\begin{cases} y = -3x - 5 \\ y = 4x + 30 \end{cases}$

   $-3x - 5 = 4x + 30$
   $-3x - 4x = 30 + 5$
   $-7x = 35$
   $x = -5$

   Substitute the $x$-value into the second equation and solve for $y$:
   $y = 4(-5) + 30$
   $y = 10$

   Solution:( −5, 10)

   c. $\begin{cases} y = 2x - 9 \\ y = 4 + 2x \end{cases}$

   $2x - 9 = 4 + 2x$
   $2x - 2x = 4 + 9$
   $0 = 13$

   No solution

   d. $\begin{cases} x = 2 \\ y = 3x - 1 \end{cases}$

   $y = 3(2) - 1$
   $y = 5$

   The $x$-value is provided in the first equation, $x = 2$.

   Solution: (2, 5)





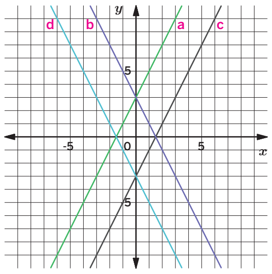
Name: ________ Date: ________ Period: ________

Practice

3. The solution to a system of equations is $(1, 5)$. Select two equations that might make up the system.

   A. $y = -3x + 6$
   B. $y = 2x + 3$
   C. $y = -7x + 1$
   D. $y = x + 4$
   E. $y = -2x + 9$

4. Label each line with its corresponding equation.




   a. $y = 2x + 3$
   b. $y = -2x + 3$
   c. $y = 2x - 3$
   d. $y = -2x - 3$

5. What is the slope of the line that passes through the points $(-3, 4)$ and $(1, 7)$?

   $\frac{7 - 4}{1 - (-3)} = \frac{3}{4}$

6. Write an equation that represents each situation.

   a. A gym charges a $50 one-time membership fee and then $15 each month. Write an equation to represent the total cost $c$ for $m$ months of membership.
   Sample response: $c = 15m + 50$

   b. Han purchases avocados and tomatoes. Each avocado costs $2 and each tomato costs $1.50. Write an equation to represent the number of avocados $a$ and tomatoes $t$ Han could buy with $15.
   Sample response: $2a + 1.50t = 15$





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 2 | 2 |
| | 2 | Activity 2 | 1 |
| | 3 | Activity 1 | 2 |
| Spiral | 4 | Unit 3 Lesson 12 | 1 |
| | 5 | Unit 3 Lesson 11 | 2 |
| Formative | 6 | Unit 4 Lesson 16 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

UNIT 4 | **LESSON 16**

# Writing Systems of Linear Equations

Let's write systems of equations to model real-world contexts.

## Focus

### Goals

1. Construct a system of linear equations that models a real-world context.
2. **Language Goal:** Determine the solution to a system of linear equations that represents a context and interpret its solution in context. **(Speaking and Listening)**

## Coherence

### Today

Students write systems of linear equations representing different contexts and interpret the solution to those systems. They use the ***Info Gap*** routine to request information from their partner and use appropriate tools to determine the solution to a system of linear equations.

### Previously

In Lessons 14 and 15, students developed procedural fluency in solving systems of linear equations graphically and algebraically.

### Coming Soon

In Lesson 17, students will apply what they have learned to solve problems about gender earning differences.

## Rigor

- Students **apply** their understanding of systems of linear equations to interpret solutions in context.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 10 min | 20 min | 5 min | 5 min |
| Independent | Pairs | Pairs | Whole Class | Independent |

**Amps** powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair
- Activity 2 PDF (answers)
- *Info Gap Routine PDF*
- Anchor Chart PDF, *Solving Linear Equations* (optional)
- graphing technology or graph paper

### Math Language Development

**Review words**

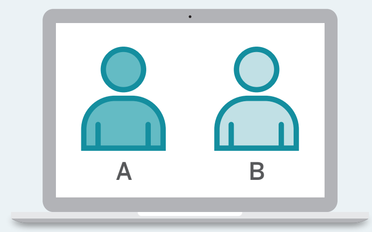
- *systems of equations*
- *solution to a system of equations*

**Amps** | Featured Activity

**Activity 2**

**Digital Collaboration**

Students work together to communicate and to determine a solution to a problem.




### Building Math Identity and Community

Connecting to Mathematical Practices

Students may use paper and pencil, or a graphing calculator, when they are working with systems of equations. As students solve each problem algebraically, they might identify limitations to their methods. As they transition to solve by graphing, have students reflect on the effectiveness of graphing technology and digital tools to solve problems.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, omit Problems 2 and 3.

Independent | 5 min

# Warm-up Algebra Talk

Students write an equation that is part of a system to remind them that they could look at the structure of equations to determine the number of solutions for a system of equations.

Name: ______ Date: ______ Period: ______

Unit 4 | Lesson 16

## Writing Systems of Linear Equations

Let's write systems of equations to model real-world contexts.

### Warm-up Algebra Talk

Consider the incomplete system of equations shown.

$$\begin{cases} y = \frac{2}{3}x - 6 \\ \boxed{\ ?\ } \end{cases}$$

Write a second equation so that the system of equations has:

a. Exactly one solution.
Sample response: $y = 3x - 7$

b. No solution.
Sample response: $y = \frac{2}{3}x - 10$

c. Infinitely many solutions.
Sample response: $y = \frac{2}{3}x - 6$

Log in to Amplify Math to complete this lesson online.





### 1 Launch

Conduct the *Algebra Talk* routine.

### 2 Monitor

**Help students get started** by activating their prior knowledge by asking, "How can you find the number of solutions for a system based on the coefficients and constants?"

**Look for productive strategies:**

- Using the coefficients and constants to write their equations.
- Solving the system of equations to determine the number of solutions.
- Writing an equation in a form other than $y = mx + b$.
- Making a general statement for each part. For example, for part b, students may write "$y = \frac{2}{3}x$ plus any number."

### 3 Connect

**Have students share** their equations. Record equations for all to see. Have students share their strategies for writing each equation. Start with students who solved the systems of equations to determine the number of solutions, and then with students who used the coefficient and constants.

**Ask** students to describe the possible different responses for each problem. For example, the equation that is part of a system with exactly one solution could have many different responses, while the equation that is part of a system with infinitely many solutions would not have many different responses because the coefficient and constants would remain the same.

**Highlight** that, before students solve a system of equations, they could check the number of solutions the system should have.

## Math Language Development

*MLR8: Discussion Supports*

During the Connect, as students share their strategies for determining the second equation for each part, listen for and amplify the mathematical vocabulary they use, such as *coefficient*, *slope*, *constant*, $y$*-intercept*, etc. Ask:

- "What values must be the same for there to be infinitely many solutions? No solution?"
- "What values must be different for there to be no solution?"

**English Learners**

Annotate the values that are the same or different for parts b and c.

## Power-up

**To power up students' ability to write an equation from a context, have students complete:**

A video game rental company charges an annual fee of $38 plus an additional $12 per month. Match each part of the equation $y = 12x + 38$ with what it represents in context.

| | | | |
|---|---|---|---|
| **a.** | $y$ | a | The total cost. |
| **b.** | 12 | c | The number of months. |
| **c.** | $x$ | b | The cost per month. |
| **d.** | 38 | d | The annual fee. |

**Use:** Before Activity 1
**Informed by:** Performance on Lesson 15, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8

Pairs | 10 min

# Activity 1 Situations and Systems

Students write systems of linear equations and interpret the solution in context to determine that different contexts can lead to systems in different forms.

**Activity 1** Situations and Systems

**Write a system of equations to model each scenario. Define the variables you choose to use. Without solving the system, interpret what the solution to the system would tell you about the scenario.**

1. Elena plans a kayaking trip. Kayak Rental A charges a base fee of $15 plus $4.50 per hour. Kayak Rental B charges a base fee of $12.50 plus $5 per hour.
   Sample response: $\begin{cases} c = 15 + 4.5h \\ c = 12.5 + 5h \end{cases}$
   The solution to the system would represent the time, in hours $h$, spent kayaking when the total cost $c$ would be the same from each rental company.

2. Diego works at a smoothie stand and prepares a batch of smoothies. The recipe calls for 3 cups of sliced strawberries for every cup of sliced apples. Diego uses a total of 5 cups of sliced strawberries and apples.
   Sample response: $\begin{cases} s = 3a \\ a + s = 5 \end{cases}$
   The solution to the system would represent the number of cups $a$ of sliced apples and the number of cups $s$ of strawberries used in the 5 cups.

3. Andre orders some posters. At Store A, he can order 6 large posters and 4 small posters for $70. At Store B, he can order 5 large posters and 9 small posters for $81.
   Sample response: $\begin{cases} 6\ell + 4s = 70 \\ 5\ell + 9s = 81 \end{cases}$
   The solution to the system would represent the price of one large poster $\ell$ and one small poster $s$.

**Are you ready for more?**

For Problem 3, determine the price of one large poster and one small poster.

One large poster costs $9 and one small poster costs $4.





## 1 Launch

Conduct the *Think-Pair-Share* routine.

## 2 Monitor

**Help students get started** by having them define the variables they use for the problem.

**Look for points of confusion:**

- **Having trouble writing a system of equations.** Underline key words or phrases. For students who might need more help, consider using the suggested activity provided under Differentiated Support.
- **Thinking that the equations in Problems 2 and 3 must be written in $y = mx + b$ form.** Remind them that linear equations could be written in different forms.
- **Not knowing how to interpret the solution.** Remind students to define their variables and that a solution to the system makes *both* equations true.

## 3 Connect

**Have students share** their systems of equations and interpretation of the solution for each problem. As students share, record their systems of equations for all to see. When necessary, ask students to explain the meaning of the variables they used.

**Highlight** that different contexts could lead to systems written in different forms. Also highlight that, although one student may write a system of equations that is different from another student, as long as the equations are equivalent the solution will be the same.

**Ask**, "What information was helpful in writing each system of equations?"

## Differentiated Support

### Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1 and 2. For each problem, provide an incomplete system of equations and have students complete it. For example:

$\{c = \square h + \square c = \square h + \square$

### Accessibility: *Clarify Vocabulary and Symbols*

Highlight the phrase *base fee* in Problem 1 and explain that this represents the amount of money charged regardless of the number of hours a kayak is rented.

## Math Language Development

### *MLR3: Critique, Correct, Clarify*

During the Connect, display an incorrect or ambiguous statement, such as "This solution represents the same amount of money for both rental places" for Problem 1.

- **Critique:** "Do you agree with this statement? Why or why not? Why might it be challenging to interpret what this statement really means?"
- **Correct:** "Write a revised statement that is clearer."
- **Clarify:** "How did you revise the statement? Is your revised statement completely clear or should you add more detail?"

### English Learners

Allow students to share their revised statements with a partner and rehearse what they will say before sharing with the whole class.

Pairs | 20 min

# Activity 2 Info Gap: Walking, Jogging, Running

Students use mathematical language to communicate with each other in order to determine a solution about a problem in context.

**Amps Featured Activity** Digital Collaboration

Name: ______________________ Date: ______________ Period: ______________

## Activity 2 Info Gap: Walking, Jogging, Running

**You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.**

| If you are given the *problem card*: | If you are given the *data card*: |
|---|---|
| **1.** Silently read your card and think about what information you need to be able to solve the problem. | **1.** Silently read your card. |
| **2.** Ask your partner for the specific information that you need. | **2.** Ask your partner "What specific information do you need?" and wait for them to ask for information. |
| **3.** Explain how you will use the information to solve the problem. | **3.** If your partner asks for information that is not on the card, do not perform the calculations for them. Tell them you don't have that information. |
| **4.** Continue to ask questions until you have enough information to solve the problem. | **4.** Before sharing the information, ask "Why do you need that information?" Listen to your partner's reasoning and ask clarifying questions. |
| **5.** Share the problem card and solve the problem independently. | **5.** Read the problem card and solve the problem independently. |
| **6.** Read the data card and discuss your thinking. | **6.** Share the data card and discuss your thinking. |

**Pause here so your teacher can review your work. You will be given a new set of cards. Repeat the activity, trading roles with your partner.**

**Info Gap:** To help you get started, ask yourself these questions:
- Do you know what the variables represent?
- Do you know each person's rate?

What else might you need to know?

STOP



Lesson 16 Writing Systems of Linear Equations 461

## 1 Launch

Give one partner Problem Card 1 and the other partner Data Card 1 from the Activity 2 PDF. Display the *Info Gap* Routine PDF and model the ***Info Gap*** routine. Tell students that they may solve each problem by creating a system of equations and solving it algebraically or by graphing. Once the first set of cards have been successfully solved, provide the second set of cards, and have students switch roles. Provide access to graphing technology or graph paper.

## 2 Monitor

**Help students get started** by encouraging them to refine their language and ask more precise questions until they get the information they need.

**Look for points of confusion:**

- **Having trouble asking for appropriate information from their partner.** Activate background knowledge by asking students what they know about walking, jogging, and running.
- **For Problem Card 1, not knowing which number from the ordered pair is the answer.** Have students define each variable and ask them which variable answers the question in the problem card.

## 3 Connect

**Ask**, "What information was most helpful in determining the solution for each problem?" Sample response: For Problem 1: Clare's running rate and her head start. For Problem 2: Defining $x$ and $y$ and Tyler's walking and jogging rates.

**Have pairs of students share** their strategies for solving each problem. Select students who used different strategies to show that systems of equations can be solved in different ways.

**Highlight** that there are different strategies to solving a problem that could be represented by a system of equations, such as writing and solving an equation or by using graphing technology.

## Differentiated Support

### Accessibility: *Guide Processing and Visualization*

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. Consider using the following questions during the think-aloud.

- "I am given the equation that represents one person's progress, but I don't know who that is. I will ask which person's rate is represented by this equation."
- "I know one person has a head start, but I don't know who has the head start. I will ask which person has the head start."
- "I don't know the rate for the other person. I will ask at what rate the other person is running."
- "I don't know how much of a head start one person has. I will ask for this information."

## Math Language Development

### *MLR4: Information Gap*

Display prompts for students who benefit from a starting point, such as:

- "Can you tell me . . . (specific piece of information)?"
- "Why do you need to know . . . (that piece of information)?"

### English Learners

Consider providing sample questions students could ask, such as the following, for Problem Card 1:

- "Which person has a head start?"
- "At what speed does Clare run?"

Whole Class | 5 min

# Summary

Review and synthesize writing systems of equations from a context.

### Summary

**In today's lesson . . .**

You discovered that writing and solving systems of equations can help solve everyday problems. When writing a system of equations to model a given real-world problem, it is important to define your variables. After you have solved the system, you will know what the solution represents if you have clearly defined your variables.

> **Reflect:**




## Synthesize

**Have students share** their strategies for writing and solving a system of equations.

**Highlight** that systems of equations could be used to solve real-world problems.

**Ask** students to think of a situation where a system of equations could be used to solve a problem in their life. Sample response: I can use systems of equations to calculate when I will get paid the same amount as my friend.

## Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can systems of equations be used to represent situations and solve problems?"

# Exit Ticket

Students demonstrate their understanding by writing and solving a system of linear equations.

Printable

Name: ______ Date: ______ Period: ______

## Exit Ticket 4.16

Han and Shawn are each saving money to buy the newest video game. Han already saved \$20 and plans to save \$5 per week. Shawn has not saved any money yet, but plans to save \$10 per week.

Write and solve a system of equations that describes the amount of money each person is saving, based on the number of weeks. Be sure to define your variables. What does the solution represent within the context of the problem?

Sample response:

Let $w$ represent the number of weeks and $a$ represent the amount each person is saving, in dollars.

$$\begin{cases} a = 20 + 5w \\ a = 10w \end{cases}$$

$20 + 5w = 10w$

$20 = 5w$

$w = 4$

Substitute $w = 4$ into the second equation and solve for $a$.

$a = 10(4)$

$a = 40$

Solution: $w = 4$ and $a = 40$

It will take Shawn 4 weeks to save the same amount as Han. That amount is \$40.

**Self-Assess**

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can write a system of linear equations to model a real-world context.

1 2 3

b. I can solve a system of linear equations that models a real-world context, and interpret the solution within the context.

1 2 3



Lesson 16 Writing Systems of Linear Equations

## Success looks like . . .

- **Goal:** Constructing a system of linear equations that models a real-world context.
  - » Writing a system of equations to represent the amount of money saved based on the number of weeks.
- **Language Goal:** Determining the solution to a system of linear equations that represents a context and interpret its solution in context. **(Speaking and Listening)**
  - » Solving the system of equations to determine how many weeks it will take Shawn to save the same amount as Han and the amount Shawn will save.

## Suggested next steps

**If students do not write the correct systems of equations, consider:**

- Reviewing Activity 1.
- Asking students to write one equation that represents Han's savings and another equation that represents Shawn's savings.

**If students do not solve the system correctly, consider:**

- Reviewing Lessons 10 and 15.
- Reviewing the Anchor Chart PDF, *Solving Linear Equations*.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- What resources did students use as they worked on Activity 2? Which resources were especially helpful?
- How did the *Info Gap* routine support students in solving systems of equations in context?

## Math Language Development

**Language Goal: Determining the solution to a system of linear equations that represents a context and interpret its solution in context.**

Reflect on students' language development toward this goal.

- How did using the *Critique, Correct, Clarify* routine in Activity 1 help students interpret the solution to a system of linear equations within context? Would you change anything the next time you use this routine?
- Do students' responses to the Exit Ticket problem include a correct interpretation of the solution to the system, including which variable represents which quantity?

# Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. Which story can be represented by the following system of equations?
$\begin{cases} y = x + 6 \\ x + y = 100 \end{cases}$
   - A. Diego's teacher creates a test worth 100 points. There are 6 more multiple choice questions than short answer questions.
   - B. Priya and her younger cousin each measure their heights. They notice that Priya is 6 in. taller, and their heights add up to exactly 100 in.
   - C. Kiran receives a \$6 allowance per week. At the end of the month, he saves \$100.

2. Clare and her brother play a game in which they earn the same number of points for each goal and lose the same number of points for each penalty. Clare earns 6 goals and has 3 penalties, ending the game with 6 points. Her brother earns 8 goals and has 9 penalties and ends the game with −22 points.

   Write a system of equations to model this scenario. Define the variables you choose to use. Without solving the system, interpret what the solution to the system would tell you about the scenario.

   **Sample response:**

   $\begin{cases} 6x + 3y = 6 \\ 8x + 9y = -22 \end{cases}$

   **The solution to the system would represent the amount of points $x$ earned by a goal and the amount of points $y$ lost by a penalty.**

3. Noah and his cousin each work during the summer for a landscaping company. Noah's cousin has been working for the company longer, so his pay is 30% more than Noah. Last week, his cousin worked 1 hour and Noah worked 3 hours. Together, they earned \$36.55. What is Noah's hourly pay? Show or explain your thinking.

   **Noah earns \$8.50 per hour. Sample response:**

   **Let $n$ represent Noah's hourly wage and $c$ represent Noah's cousin's hourly wage.**

   $\begin{cases} c = 1.3n \\ 1c + 3n = 36.55 \end{cases}$

   $$\begin{aligned} 1.3n + 3n &= 36.55 \\ 4.3n &= 36.55 \\ n &= 8.5 \end{aligned}$$





Practice

Name: ______ Date: ______ Period: ______

4. Solve each system of equations. Show or explain your thinking.

   a. $\begin{cases} y = 6x - 8 \\ y = -3x + 10 \end{cases}$

   $$\begin{aligned} 6x - 8 &= -3x + 10 \\ 6x + 3x &= 10 + 18 \\ 9x &= 18 \\ x &= 2 \end{aligned}$$

   **Substitute the $x$-value into the second equation and solve for $y$:**

   $y = 6(2) - 8$
   $y = 4$

   **Solution: (2, 4)**

   b. $\begin{cases} y = -8x + 2x - 4 \\ y = -6x - 4 \end{cases}$

   $$\begin{aligned} -8x + 2x - 4 &= -6x - 4 \\ -6x - 4 &= -6x - 4 \end{aligned}$$

   **There will be infinitely many solutions because the coefficients and constants are the same on each side of the equation.**

5. Consider the incomplete system of equations shown. Create a second equation so that the system has no solution.

   $\begin{cases} y = \frac{3}{4}x - 4 \\ \boxed{\quad ? \quad} \end{cases}$

   **Answers may vary. Any equation in the form $y = \frac{3}{4}x + b$, where $b$ is not equivalent to −4 will result in the system having no solution.**

6. Kiran read 182 pages of his new 232-page book. What percent of his new book has Kiran read? Round to the nearest tenth of a percent.

   $\frac{182}{232} \cdot 100 = 78.4$

   **Kiran has read 78.4% of his new book.**





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| On-lesson | 1 | Activity 1 | 2 |
| | 2 | Activity 1 | 2 |
| | 3 | Activity 1 | 3 |
| Spiral | 4 | Unit 4 Lesson 14 | 1 |
| | 5 | Unit 4 Lesson 14 | 2 |
| Formative | 6 | Unit 4 Lesson 17 | 2 |

**Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

# Pay Gaps

Let's learn about earning differences by gender.


## Focus

### Goals

1. **Language Goal:** Identify the gender pay gap by calculating and graphing the disparity between men and women's earnings. **(Speaking and Listening)**

## Coherence

### Today

Students look at data showing the median earnings for men and for women in different occupations. Students will discover that there is a pay gap — the gender pay gap — where men outearn women. Students will examine the impact of this pay gap over the course of a lifetime of earnings, if nothing were to change. **Note:** The purpose of this lesson is for students to see the data and to make observations about the data. Students may also have questions about why the gap exists and what can or should be done about it.

### Previously

In Lesson 16, students wrote systems of linear equations representing different contexts and interpreted the solution for those systems.

### Coming Soon

This is the final lesson of Unit 4. In Unit 5, students will study functions. In high school, students will continue working with systems of linear equations in deeper and more complex ways.

## Rigor

- Students **apply** concepts of systems of linear equations to examine the size and scope of the gender pay gap.

# Pacing Guide

Suggested Total Lesson Time ~**45 min**

| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 5 min | 5 min |
| Independent | Small Groups | Pairs | Whole Class | Independent |

**Amps** powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

## Practice

Independent

### Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one page per group
- Activity 1 PDF (answers)
- calculators

### Math Language Development

**Review word**

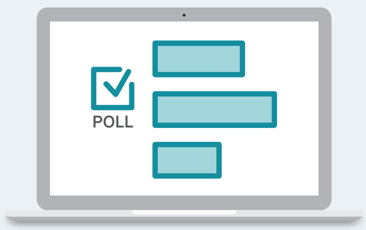
- *system of equations*

**Amps** | **Featured Activity**

**Activity 1**

### Take a Poll

See what your students are thinking in real time by digitally polling the class to see student estimates for the size of the gender gap.




### Building Math Identity and Community

Connecting to Mathematical Practices

Students will study and explore the gender pay gap in this lesson and use mathematics to model the data. They may feel overwhelmed in looking at all of the data and thinking of how to organize, represent, and display the data using the mathematics they know. Encourage them to pause when they feel overwhelmed, and try to tackle one thing at a time.

### Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 1**, have students only complete the first five rows of the table.
- **Activity 2** may be omitted.

Independent | 5 min

# Warm-up Notice and Wonder

Students study a chart that shows majors associated with the highest earnings have low percentages of female students to make observations about the gender pay gap.

Name: ______________ Date: ________ Period: ________

Unit 4 | Lesson 17 – Capstone

## Pay Gaps

Let's learn about earning differences by gender


### Warm-up Notice and Wonder

**Women have been earning college degrees at a higher rate than men since 1981. Let's take a closer look at the types of majors men and women have been pursuing. The following data, taken from the National Center for Education Statistics and the U.S. Census Bureau in 2014, show some college majors with the highest and lowest earnings. What do you notice? What do you wonder?**

1. I notice . . .

   Sample responses:
   - The median earnings range from \$41K to \$136K.
   - Many of the highest earning majors are more popular with men, while many of the lowest earning majors are more popular with women.
   - Engineering majors typically earn more than education or community service majors.

2. I wonder . . .

   Sample responses:
   - I wonder why a greater percent of women pursue majors with lower median earnings. Are women being discouraged from pursuing higher-earning majors?
   - Why is there such a discrepancy between the earnings for engineering majors versus education or community service majors?

**College majors with the highest/lowest earnings**

| Majors with highest earnings | Median earning | Percent female |
|---|---|---|
| Petroleum engineering | \$136K | 14% |
| Pharmacy, pharmaceutical sciences | \$113K | 59% |
| Metallurgical engineering | \$98K | 23% |
| Mining and mineral engineering | \$97K | 13% |
| Chemical engineering | \$96K | 32% |
| Electrical engineering | \$93K | 12% |
| Studio arts | \$42K | 69% |
| Social work | \$42K | 88% |
| Human/community services | \$41K | 85% |

Log in to Amplify Math to complete this lesson online.



Lesson 17 Pay Gaps 465

## 1 Launch

Activate students' background knowledge about the rates at which women and men have earned a college degree from Lesson 11. Conduct the *Notice and Wonder* routine. Ask students how they might make the survey more inclusive or otherwise improve it in future years.

## 2 Monitor

**Help students get started** by asking them to describe the labels they see in the chart.

**Look for points of confusion:**

- **Drawing inaccurate conclusions based on the chart such as certain majors on the chart are better suited for men or women.** Remind students that the goal is to focus on the data.

## 3 Connect

**Display** the chart from the Student Edition.

**Have students share** what they notice and wonder about the chart.

**Ask:**

- "Can you conclude that women earn less than men based on this chart?" No. "So, let's dive deeper into earnings to see what different occupations pay to men and women."
- "What other data would you want to see to better understand male and female earning outcomes?"

**Highlight** that there may be many reasons to explain why students see this trend, such as sexism, societal pressure, traditional family patterns, among others. However, explain that these are complicated reasons and deserve a greater study at a later time. Emphasize that, today, students will be focused on looking at what the data says about the earnings for male and female workers in the United States.

## Power-up

**To power up students' ability to determine percentages, have students complete:**

Recall that *percent* means out of 100.

Bard finished 318 minutes of the required 600 minutes of reading. What percent of reading minutes did Bard complete?

$53\%; \frac{318}{600} \cdot 100 = 53$

**Use:** Before the Warm-up
**Informed by:** Performance on Lesson 16, Practice Problem 6

Small Groups | 15 min

# **Activity 1** Mind the Gap

Students analyze salaries for ten different occupations to uncover that there is a gap in earnings called the *gender pay gap*.

**Amps Featured Activity** Take a Poll

### **Activity 1** Mind the Gap

The table shows the median annual earnings for veterinarians in the year 2018, according to data from the U.S. Census Bureau.

| Men's median earnings ($) | Women's median earnings ($) | Women's median earnings as a percentage of men's |
|---|---|---|
| 111,080 | 93,065 | 83.8% |

1. Complete the table to calculate the women's median annual earnings as a percentage of men's for veterinarians. Round to the nearest tenth of a percent. Explain your thinking.
   Sample response: To calculate the percentage, I divided the women's median earnings by the men's median earnings, and then multiplied by 100.
   $\frac{93,065}{111,080} \approx 0.838$
   $0.838 \cdot 100 = 83.8$ or about 83.8%

2. Your group will be given a sheet with data showing the median earnings for men and for women for ten different occupations. Calculate the women's median annual earnings as a percentage of the men's median annual earnings, for each occupation. Round to the nearest tenth of a percent. Record your responses in the table.

3. What conclusions can you draw from the data? What questions do you have?
   Sample responses:
   - Women earn about 80%, on average, what men do, for the same occupation.
   - Occupations in math and science tend to pay higher.
   - There is a wide range of salaries.
   - There were some professions where women earned more than men, but these instances were rare, and when they did occur, the difference was not as much as when the men earned a greater amount.
   - Some occupations had much greater gender pay gaps than others.

466 **Unit 4** Linear Equations and Systems of Linear Equations



## 1 Launch

Have students complete Problem 1 in pairs. Discuss the solution with the class. Then, assign students to groups of 4, and distribute one page of the Activity 1 PDF to each group. Provide access to calculators.

## 2 Monitor

**Help students get started** by asking them to recall the steps for determining the percent women earned compared to the percent men earned in Problem 1.

**Look for points of confusion:**

- **Drawing inaccurate or premature conclusions based on the data.** Remind students that they cannot draw many conclusions, at this point, with limited data and limited context. Have students focus on what they notice about the percentages they are finding and what they notice about the types of occupations with the greatest and least earnings.

## 3 Connect

**Have groups of students share** their completed table with another group. Have students look for patterns or trends they see in the data.

**Display** student observations for all to see. Ask students to guess the percentage representing the gender earnings gap for all occupations, and display these estimates on the board.

**Highlight** that this gap is called the *gender pay gap*. Reveal that, according to the U.S. Census Bureau 2018 data, the gap for all occupations averaged to be about 81.1%.

**Ask**, "Looking back to the Warm-up, can you conclude that women make less than men because they are choosing certain majors in college that lead to lower paying jobs?" No, that is only part of the story. You can see a trend across occupations that women are earning less than men.

## Differentiated Support

### Accessibility: *Vary Demands to Optimize Challenge*

Have students complete the table on the Activity 1 PDF for the first five occupations. Then provide the remaining percentages and have students record them in their tables.

### Accessibility: *Guide Processing and Visualization*

Consider demonstrating and displaying how to determine the women's median earnings as a percentage of the men's median earnings. For example, display the following and keep it displayed throughout the activity:
$\frac{\text{women's earnings}}{\text{men's earnings}} \cdot 100.$

## Math Language Development

### *MLR1: Stronger and Clearer Each Time*

Use this routine to provide students an opportunity to revise and refine the conclusions they stated in Problem 3. Encourage students to focus on (1) making a claim and (2) adding a reason to support their claim. After students have had time to write their conclusions, ask them to meet with 1–2 partners to share their responses and receive feedback. After receiving feedback, give students time to improve their responses.

Pairs | 15 min

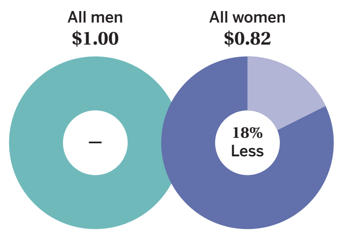
# Activity 2 Gender Pay Gap

Students graph a system of linear equations to explore the impact of the gender pay gap over the course of a lifetime of earnings.

Name: ________________ Date: ________ Period: ________

## Activity 2 Gender Pay Gap

The graphic shows what is commonly called the *gender pay gap*.



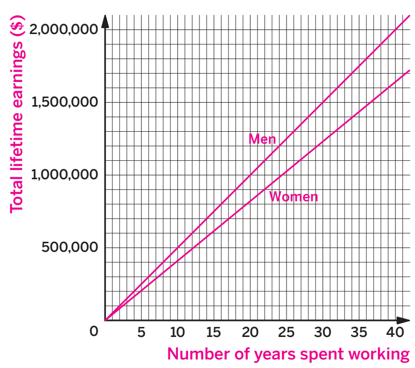

1. Describe the graphic in your own words.

   Sample response: If men earn $1.00, women earn $0.82. This seems to suggest women earn 82% of what men earn.

2. On the coordinate plane shown, sketch two graphs, one labeled "Men" and one labled "Women," that show the impact of the gender pay gap over the course of a lifetime of earnings, assuming no changes in the gender pay gap. Be sure to label the axes.

   Here are some figures you may find helpful:

   - The median full-time annual salary for men in 2018 was just over $51,000. **Note:** You can use $50,000 for the purposes of this activity.
   - Assume the typical worker can expect to work full time for about 40 years.




3. For typical women to earn the same amount of lifetime earnings as typical men, how much more would they need to earn by the end of their 40-year careers?

   About $360,000

STOP





## 1 Launch

Give students one minute to independently complete Problem 1 before discussing the problem with the class. Then have students work in pairs on Problems 2 and 3.

## 2 Monitor

**Help students get started** by helping them label their axes.

**Look for points of confusion:**

- **Having difficulty starting a graph.** Ask students what the earnings would be in Year 0 and help them see that both graphs will start at the origin.
- **Not being able to determine the slope of either graph.** Make sure students are using $50,000. Ask to identify the earnings after 1, 2, and 3 years. Suggest students use a table to determine 2 more values for men, and then plot their points before doing the same for women.
- **Being unsure how to solve Problem 3.** Ask students to identify the earnings for men and women after 40 years.

## 3 Connect

**Display** student work showing a correct graph.

**Ask**, "Using the graph, what is the gap in earnings after 10 years? 20 years? 30 years?" Demonstrate how vertical distance between the two lines for any year $x$ represents the difference in the earnings $y$.

**Highlight** that the gender pay gap is narrower, but still present, when students compare women and men with the same qualifications and experience. This is called the *controlled gender pay gap*. However, the gap is greater when students consider the occupation as a whole without controls. Ask students why both measures are important to consider.

## Differentiated Support

### Accessibility: *Activate Prior Knowledge, Clarify Vocabulary and Symbols*

Have students complete the table on the Activity 1 PDF for the first five occupations. Then provide the remaining percentages and have students record them in their tables.

### Extension: *Math Enrichment*

To confirm their answer to Problem 3, have students write and solve a system of equations in which the ordered pair (40, 2000000) is the solution to the system.

Sample response:

$$\begin{cases} y = 50000x \\ y = 41000x + 360000 \end{cases}$$

## Math Language Development

### *MLR5: Co-craft Questions*

After students have independently described the graphic in Problem 1, pause and give them an opportunity to work with a partner to process the information and write 1–2 questions they may have about the graphic.

### English Learners

Provide students examples of questions they can ask to make sense of the graphic, such as, "Do qualifications and experience account for this difference?"

Whole Class | 5 min

# Unit Summary

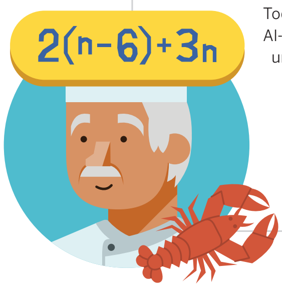
Review and synthesize how the concepts of this unit, particularly systems of linear equations, can be used to study the gender pay gap.

Narrative Connections

## Unit Summary


Algebra is a powerful tool. It gives us ways to talk about unknowns in ways that are concrete. You saw it in this very lesson with the gender pay gap. While it's already widely known that sexism has enabled men to earn more than women, algebra allows us to uncover exactly how wide the wage gap actually is.

Through a process of elimination and deduction that involves constants (numbers that stay the same) and variables (the numbers that change), you worked your way down to a solution. This algebraic process was first recorded by the Persian mathematician, Al-Khwārizmī, over a thousand years ago. By bringing together the work of ancient Greek, Chinese, Mesopotamian and Indian mathematicians, Al-Khwārizmī simplified the lives of Baghdad's merchants and traders. His detailed calculation methods allowed them to conduct business more efficiently.




Today algebra is just as useful as it was in Al-Khwārizmī's day. It offers a way to solve for unknowns through a process that's orderly and logical. Whether it's finding where two hikers will meet on a particular trail or describing the inequities in our society, algebra provides a path to uncovering the truth.

**See you in Unit 5.**




### Narrative Connections

Read the narrative aloud as a class or have students read it individually.

### Synthesize

**Display** the Summary from the Student Edition. Have students read the Summary or have a student volunteer read it aloud.

**Have students share** their reflections from their work in this unit.

**Ask:**

- "What are your biggest takeaways from this unit?"
- "What are your biggest questions about this unit?"

**Highlight** that students will continue to study linear relationships and systems of equations in high school.

### Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

- "Did anything surprise you while reading the narratives of this unit?"
- "Is there anything you would like to learn more about? What are some steps you can take to learn more?"

## Fostering Diverse Thinking

### Equal Pay Day

Have students research National Equal Pay Day in the U.S. Ask students what this day represents mathematically, and how it can be calculated. Based on what they saw in the lesson, do they think another day might be more representative?

Highlight also that this day is for women in general and is calculated using averages. Note that disparities are different for Black, Native American, Asian American, and Hispanic women.

Ask these questions to facilitate class discussion:

- "For the current year, which day has been designated as Equal Pay Day? How do you think this day was determined mathematically, and how else could it be determined?"
- "How does this day compare to prior years? What does this tell you?"
- "What barriers do you think women face when it comes to earning equal pay?"
- "What would it mean for growing the economy if Equal Pay Day occurred on January 1?"

Independent | 5 min

# Exit Ticket

Students demonstrate their understanding by reflecting on their work in this unit.

Printable

Name: ______________________ Date: __________ Period: __________

## Exit Ticket 4.17

**Reflect on what you have learned in this unit.**

**1.** Three things I learned:
**Answers may vary.**

**2.** Two things I found interesting or surprising:
**Answers may vary.**

**3.** One question I still have:
**Answers may vary.**

**Self-Assess**

| 1 | 2 | 3 |
|---|---|---|
| I don't really get it | I'm starting to get it | I got it |

**a** I can use algebraic representations to represent, analyze, and help describe real-world problems.
**1 2 3**





## Success looks like . . .

- **Language Goal:** Identifying the gender pay gap by calculating and graphing the disparity between men and women's earnings. **(Speaking and Listening)**
  - » Writing a response that includes what they learned about the gender pay gap.

## Suggested next steps

**If students are unsure what to write, consider:**

- Activating students' prior knowledge by pointing to the Unit 4 Anchor Charts posted in the room.
- Encourage students to write any remaining questions if they cannot think of how to respond to Problems 1 and 2.

## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

**Points to Ponder . . .**

- What worked and what didn't work today?
- What might you change for the next time you teach this lesson?

# Practice

Independent

Name: ______ Date: ______ Period: ______

Practice

1. Select the situation that could be represented by this system of equations:
$\begin{cases} y = x + 6 \\ x + y = 100 \end{cases}$

   A. Han and his younger cousin measure their heights. They notice that Han is 6 in. taller, and their heights add up to exactly 100 in.

   B. Andre's teacher writes a test worth 100 points. There are 6 more multiple choice questions than short answer questions.

2. Solve each system of equations. Show your thinking.

   a. $\begin{cases} y = 6x - 8 \\ y = -3x + 10 \end{cases}$

   $6x - 8 = -3x + 10$
   $9x - 8 = 10$
   $9x = 18$
   $x = 2$
   $y = 6(2) - 8$
   $y = 4$
   Solution: $(2, 4)$

   b. $\begin{cases} x = \frac{1}{2} \\ y = 3 - 4x \end{cases}$

   $y = 3 - 4\left(\frac{1}{2}\right)$
   $y = 3 - 2$
   $y = 1$
   Solution: $\left(\frac{1}{2}, 1\right)$

   c. $\begin{cases} y = 0.5x - 1 \\ y = 5 - 2.5x \end{cases}$

   $0.5x - 1 = 5 - 2.5x$
   $3x - 1 = 5$
   $3x = 6$
   $x = 2$
   $y = 0.5(2) - 1$
   $y = 1 - 1$
   $y = 0$
   Solution: $(2, 0)$

   d. $\begin{cases} y = 2x + 1 \\ y = \frac{1}{2} + 2x \end{cases}$

   $2x + 1 = \frac{1}{2} + 2x$
   No solution

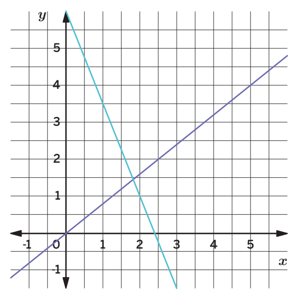




Name: ______ Date: ______ Period: ______

Practice

3. Refer to the two lines graphed on the coordinate plane.




   a. Estimate the coordinates of the point where the two lines meet.

   Sample response: $(1.75, 1.4)$

   b. Choose two equations that make up the system represented by the graph.

   A. $y = \frac{5}{4}x$

   B. $y = 6 - 2.5x$

   C. $y = 2.5x + 6$

   D. $y = 6 - x$

   E. $y = 0.8x$

   c. Solve the system of equations. Round the coordinates of the solution to the nearest hundredth. Then confirm the accuracy of your estimate you made in part a.

   $\begin{cases} y = 6 - 2.5x \\ y = 0.8x \end{cases}$

   $6 - 2.5x = 0.8x$
   $6 = 3.3x$ $\quad$ $y = 0.8(1.82)$
   $x = 1.82$ $\quad$ $y = 1.46$
   Solution: $(1.82, 1.46)$

4. A full 1,500-liter water tank springs a leak, losing 2 liters per minute. At the same time, a second tank contains 300 liters and is being filled at a rate of 6 liters per minute. When will the two water tanks have the same amount of water? Show or explain your thinking.

   Let $x$ be the number of minutes. Then the expression $1500 - 2x$ represents the number of liters in the first tank after $x$ minutes and the expression $300 + 6x$ represents the number of liters in the second tank after $x$ minutes.

   $\begin{cases} y = 1500 - 2x \\ y = 300 + 6x \end{cases}$

   $1500 - 2x = 300 + 6x$
   $1500 = 300 + 8x$
   $1{,}200 = 8x$
   $x = 150$

   The two water tanks will have the same amount of water after 150 minutes.





## Practice Problem Analysis

| Type | Problem | Refer to | DOK |
|---|---|---|---|
| Spiral | 1 | Unit 4 Lesson 13 | 1 |
| | 2 | Unit 4 Lesson 15 | 2 |
| | 3 | Unit 4 Lesson 15 | 2 |
| | 4 | Unit 4 Lesson 16 | 2 |

## Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

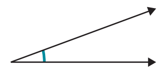
# Glossary/Glosario

## English

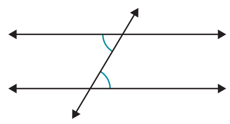
### A

**absolute value** The value that represents the distance between a number and zero. For example, because the distance between $-3$ and $0$ is $3$, the absolute value of $-3$ is $3$, or $|-3| = 3$.

**acute angle** An angle whose measure is less than $90$ degrees.



**alternate interior angles** Alternate interior angles are created when a pair of parallel lines are intersected by a transversal. These angles lie inside the parallel lines and on opposite (alternate) sides of the transversal.

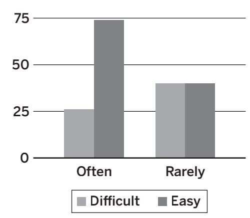


**angle of rotation** See the definition for *rotation*.

**area** The number of unit squares needed to fill a two-dimensional shape without gaps or overlaps.

### B

**bar graph** A graph that presents data using rectangular bars that have heights proportional to the values that they represent.




**bar notation** Notation that indicates the repeated part of a repeating decimal. For example, $0.\overline{6} = 0.66666\ldots$.

**base** The number that is raised to a power. The power indicates the number of times the base is multiplied by itself.

### C

**center of dilation** See the definition for *dilation*.

**center of rotation** See the definition for *rotation*.

**circle** A shape that is made up of all of the points that are the same distance from a given point.

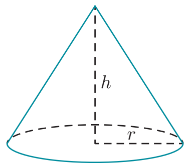
**circumference** The distance around a circle.

**clockwise** A rotation in the same direction as the way hands on a clock move is called a *clockwise* rotation.

**cluster** A cluster represents data values that are grouped closely together.

**coefficient** A constant by which a variable is multiplied, written in front of the variable. For example, in the expression $3x + 2y$, $3$ is the coefficient of $x$.

**cone** A three-dimensional solid that consists of a circular base connected by a curved surface to a single point.



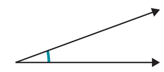

**congruent** Two figures are "congruent" to each other if one figure can be mapped onto the other by a sequence of rigid transformations.

## Español

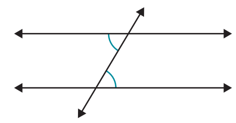
### A

**valor absoluto** Valor que representa la distancia entre un número y cero. Por ejemplo, dado que la distancia entre $-3$ y $0$ es $3$, el valor absoluto de $-3$ es $3$, o $|-3| = 3$.

**ángulo agudo** Ángulo cuya medida es menor que $90$ grados.



**ángulos interiores alternos** Se crean ángulos interiores alternos cuando un par de líneas paralelas son intersecadas por una transversal. Estos ángulos están dentro de las líneas paralelas y en lados opuestos (alternos) de la transversal.

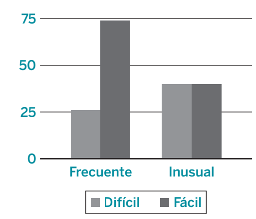


**ángulo de rotación** Ver *rotación*.

**área** Número de unidades cuadradas necesario para llenar una forma bidimensional sin dejar espacios vacíos ni superposiciones.

### B

**gráfica de barras** Gráfica que presenta datos por medio de barras con alturas proporcionales a los valores que representan.




**notación de barras** Notación que indica la parte repetida de un número decimal periódico. Por ejemplo, $0.\overline{6} = 0.66666\ldots$.

**base** Número que se eleva a una potencia. La potencia indica el número de veces que la base debe multiplicarse por sí mismo.

### C

**centro de dilatación** Ver *dilatación*.

**centro de rotación** Ver *rotación*.

**círculo** Forma constituida por todos los puntos que están a la misma distancia de un punto dado.

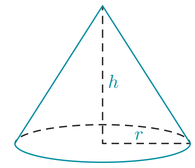
**circunferencia** Distancia alrededor de un círculo.

**en el sentido de las agujas del reloj** Una rotación en la misma dirección en que se mueven las agujas de un reloj es llamada una rotación *en el sentido de las agujas del reloj*.

**agrupación** Una agrupación representa valores de datos que se agrupan de manera cercana entre ellos.

**coeficiente** Constante por la cual una variable es multiplicada, escrita frente a la variable. Por ejemplo, en la expresión $3x + 2y$, $3$ es el coeficiente de $x$.

**cono** Sólido tridimensional compuesto de una base circular conectada a un solo punto por medio de una superficie curva.

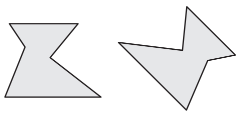



**congruente** Dos figuras son "congruentes" si una de las figuras puede mapearse con la otra mediante una secuencia de transformaciones rígidas.

# Glossary/Glosario

## English

**congruent** Two figures are *congruent* to each other if one figure can be mapped onto the other by a sequence of rigid transformations.



**constant** A value that does not change, meaning it is not a variable.

**constant of proportionality** The number in a proportional relationship that the value of one quantity is multiplied by to get the value of the other quantity.

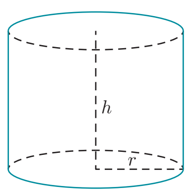
**coordinate plane** A two-dimensional plane that represents all the ordered pairs $(x, y)$, where $x$ and $y$ can both represent on values that are positive, negative, or zero.

**corresponding parts** Parts of two scaled copies that match up, or "correspond" with each other. These corresponding parts could be points, segments, angles, or lengths.

**counterclockwise** A rotation in the opposite direction as the way hands on a clock move is called a *counterclockwise* rotation.

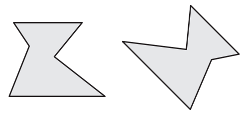
**cube root** The cube root of a positive number $p$ is a positive solution to equations of the form $x^3 = p$. Write the cube root of $p$ as $\sqrt[3]{p}$.

**cylinder** A three-dimensional solid that consists of two parallel, circular bases joined by a curved surface.




## Español

**congruente** Dos figuras son *congruentes* entre sí, si una figura puede adquirir la forma de la otra figura mediante una secuencia de transformaciones rígidas.



**constante** Valor que no cambia, lo que significa que no es una variable.

**constante de proporcionalidad** En una relación proporcional, el número por el cual el valor de una cantidad es multiplicado para obtener el valor de otra cantidad.

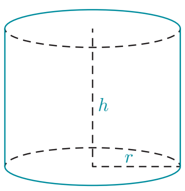
**plano de coordenadas** Plano bidimensional que representa todos los pares ordenados $(x, y)$, donde tanto $x$ como $y$ pueden representar valores positivos, negativos o cero.

**partes correspondientes** Partes de dos copias a escala que coinciden, o "se corresponden", entre sí. Estas partes correspondientes pueden ser puntos, segmentos, ángulos o longitudes.

**en el sentido contrario a las agujas del reloj** Una rotación en la dirección opuesta a la forma en que las agujas de un reloj se mueven es llamada una rotación *en el sentido contrario a las agujas del reloj*.

**raíz cúbica** La raíz cúbica de un número positivo $p$ es una solución positiva a las ecuaciones de la forma $x^3 = p$. Escribimos la raíz cúbica $p$ como $\sqrt[3]{p}$ .

**cilindro** Sólido tridimensional compuesto por dos bases paralelas y circulares unidas por una superficie curva.




## D

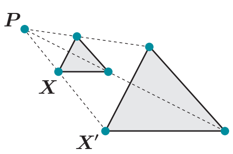
### English

**dependent variable** The dependent variable represents the output of a function.

**diagonal** A line segment connecting two vertices on different sides of a polygon or polyhedra.

**diameter** The distance across a circle through its center. The line segment with endpoints on the circle, that passes through its center.

**dilation** A transformation defined by a fixed point $P$ (called the *center of dilation*) and a scale factor $k$. The dilation moves each point $X$ to a point $X'$ along ray $PX$, such that its distance from $P$ changes by the scale factor.




**Distributive Property** A property relating addition and multiplication: $a(b + c) = ab + ac$.

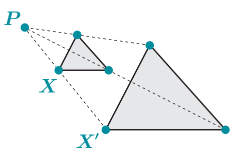
### Español

**variable dependiente** La variable dependiente representa el resultado, o salida, de una función.

**diagonal** Segmento de línea que conecta dos vértices que están en lados diferentes de un polígono o de un poliedro.

**diámetro** Distancia que atraviesa un círculo por su centro. El segmento de línea cuyos extremos se ubican en el círculo y que pasa a través de su centro.

**dilatación** Transformación definida por un punto fijo $P$ (llamado *centro de dilatación*) y un factor de escala $k$. La dilatación mueve cada punto $X$ a un punto $X'$ a lo largo del rayo $PX$, de manera tal que su distancia con respecto a $P$ es cambiada por el factor de escala.




**Propiedad distributiva** Propiedad que relaciona la suma con la multiplicación: $a(b + c) = ab + ac$.

## English

### E

**equation** A mathematical statement that two expressions are equal.

**equivalent** If two mathematical objects (especially fractions, ratios, or expressions) are equal in any form, then they are equivalent.

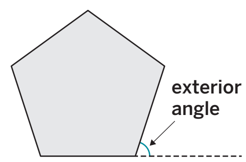
**equivalent equations** Equations that have the same solution or solutions.

**equivalent expressions** Two expressions whose values are equal when the same value is substituted into the variable for each expression.

**exponent** The number of times a factor is multiplied by itself.

**expression** A quantity that can include constants, variables, and operations.

**exterior angle** An angle between a side of a polygon and an extended adjacent side.



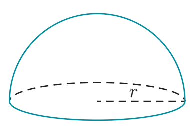

### F

**function** A function is a rule that assigns exactly one output to each possible input.

### H

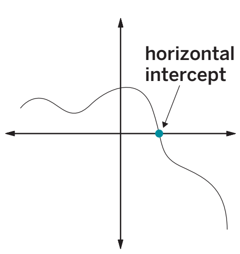
**hanger diagram** A model in which quantities are represented as weights attached to either side of a hanger. When the hanger is balanced, the sum of the quantities on either side must be equal.

**hemisphere** Half of a sphere.

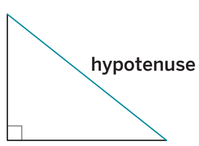



**horizontal** Running straight from left to right (or right to left).

**horizontal intercept** A point where a graph intersects the horizontal axis. Also known as the $x$-intercept, it is the value of $x$ when $y$ is $0$.




**hypotenuse** In a right triangle, the side opposite the right angle is called the hypotenuse.




## Español

### E

**ecuación** Declaración matemática de que dos expresiones son iguales.

**equivalente** Si dos objetos matemáticos (especialmente fracciones, razones o expresiones) son iguales de cualquier manera, entonces son equivalentes.

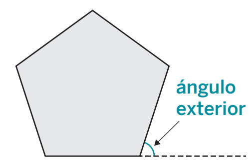
**ecuaciones equivalentes** Ecuaciones que tienen la misma solución o soluciones.

**expresiones equivalentes** Dos expresiones cuyos valores son iguales cuando se sustituye el mismo valor en la variable de cada expresión.

**exponente** Número de veces que un factor es multiplicado por sí mismo.

**expresión** Cantidad que puede incluir constantes, variables y operaciones.

**ángulo exterior** Ángulo que se encuentra entre un lado de un polígono y un lado extendido adyacente.



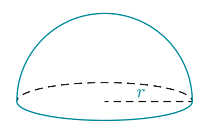

### F

**función** Una función es una regla que asigna exactamente un resultado, o salida, a cada posible entrada.

### H

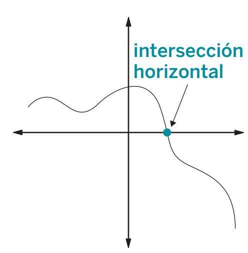
**diagrama de colgador** Modelo en el cual ciertas cantidades son representadas como pesos sujetos a cada lado de un colgador. Cuando el colgador está en equilibrio, la suma de las cantidades en cualquiera de los lados debe ser igual.

**hemisferio** La mitad de una esfera.

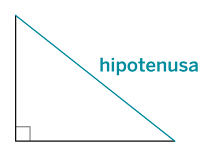



**horizontal** Que corre en línea recta de izquierda a derecha (o de derecha a izquierda).

**intersección horizontal** Punto en que una gráfica se interseca con el eje horizontal. Conocida también como intersección $x$, se trata del valor de $x$, cuando $y$ es $0$.




**hipotenusa** En un triangulo rectángulo, el lado opuesto al ángulo recto se llama la hipotenusa.



# Glossary/Glosario

## English

## I

**image** A new figure that is created from an original figure (called the *preimage*) by a transformation.

**independent variable** The independent variable represents the input of a function.

**initial value** The starting amount in a context.

**input** The independent variable of a function.

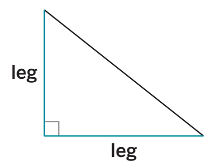
**integers** Whole numbers and their opposites. For example, $-4$, $0$, and $15$ are whole numbers.

**interior angle** An angle between two adjacent sides of a polygon.

**irrational number** A number that is not rational. That is, an irrational number cannot be written as a fraction.

## L

**legs** The two sides of a right triangle that form the right angle.




**like terms** Parts of an expression that have the same variables and exponents. *Like terms* can be added or subtracted into a single term.

**line of reflection** See the definition for *reflection*.

**linear association** If a straight line can model the data, the data have a linear association.

**linear function** A linear relationship which assigns exactly one output to each possible input.

**linear model** A linear equation that models a relationship between two quantities.

**linear relationship** A relationship between two quantities in which there is a constant rate of change. When one quantity increases by a certain amount, the other quantity increases or decreases by a proportional amount.

**long division** A way to show the steps for dividing base ten whole numbers and decimals, dividing one digit at a time, from left to right.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\;4\phantom{00} \\ \hline 60\phantom{0} \\ -56\phantom{0} \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

## Español

## I

**imagen** Nueva figura que se crea a partir de una figura original (llamada la *preimagen*) por medio de una transformación.

**variable independiente** La variable independiente representa la entrada de una función.

**valor inicial** Monto inicial en un contexto.

**entrada** La variable independiente de una función.

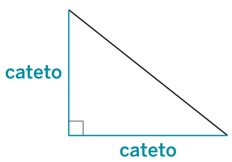
**enteros** Números completos y sus opuestos. Por ejemplo, $-4$, $0$ y $15$ son números enteros.

**ángulo interior** Ángulo que se encuentra entre dos lados adyacentes de un polígono.

**número irracional** Número que no es racional. Es decir, un número irracional no puede ser escrito como fracción.

## L

**catetos** Los dos lados de un triángulo rectángulo que componen el ángulo recto.




**términos similares** Partes de una expresión que tienen las mismas variables y exponentes. Los *términos similares* pueden ser reducidos a un solo término mediante su suma o resta.

**línea de reflexión** Ver *reflexión*.

**asociación lineal** Si una línea recta puede modelar los datos, los datos tienen una asociación lineal.

**función lineal** Relación lineal que asigna exactamente un resultado, o salida, a cada entrada posible.

**modelo lineal** Ecuación lineal que modela una relación entre dos cantidades.

**relación lineal** Relación entre dos cantidades en la cual existe una tasa de cambio constante. Cuando una cantidad aumenta un cierto monto, la otra cantidad aumenta o disminuye en un monto proporcional.

**división larga** Forma de mostrar los pasos necesarios para dividir números enteros en base diez y decimales, por medio de la división de un dígito a la vez, de izquierda a derecha.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -2\;4\phantom{00} \\ \hline 60\phantom{0} \\ -56\phantom{0} \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

## English

### N

**negative association** A negative association is a relationship between two quantities where one tends to decrease as the other increases.

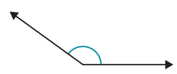
**nonlinear association** If a straight line cannot model the data, the data have a nonlinear association.

**nonlinear function** A function that does not have a constant rate of change. Its graph is not a straight line.

**nonproportional relationship** A relationship between two quantities in which the corresponding values do not have a constant ratio. (In other words, a relationship that is not a proportional relationship.)

### O

**obtuse angle** An angle that measures more than $90$ degrees.



**order of operations** When an expression has multiple operations, they are applied in a consistent order (the *order of operations*) so that the expression is evaluated the same way by everyone.

**ordered pair** Two values $x$ and $y$, written as $(x, y)$, that represent a point on the coordinate plane.

**orientation** The arrangement of the vertices of a figure before and after a transformation. A figure's orientation changes when it is reflected across a line.

**origin** The point represented by the ordered pair $(0, 0)$ on the coordinate plane. The *origin* is where the $x$- and $y$-axes intersect.

**outlier** Outliers are points that are far away from their predicted values.

**output** The dependent variable of a function.

## Español

### N

**asociación negativa** Una asociación negativa es una relación entre dos cantidades, en la cual una tiende a disminuir a medida que la otra aumenta.

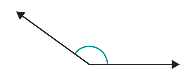
**asociación no lineal** Si una línea recta no puede modelar los datos, los datos tienen una asociación no lineal.

**función no lineal** Función que no tiene un índice constante de cambio. Su gráfica no es una línea recta.

**relación no proporcional** Relación entre dos cantidades, en la cual los valores correspondientes no tienen una razón constante. (En otras palabras, una relación que no es una relación proporcional.)

### O

**ángulo obtuso** Ángulo que mide más de $90$ grados.



**orden de las operaciones** Cuando una expresión tiene múltiples operaciones, estas se aplican en cierto orden consistente (el *orden de las operaciones*) de manera que la expresión sea evaluada de la misma manera por todas las personas.

**par ordenado** Dos valores $x$ y $y$, escritos como $(x, y)$, que representan un punto en el plano de coordenadas.

**orientación** El arreglo de los vertices de una figura antes y después de una transformación. La orientación de una figura cambia cuando esta es reflejada con respecto de una línea.

**origen** Punto representado por el par ordenado $(0, 0)$ en el plano de coordenadas. El *origen* es donde los ejes $x$ y $y$ se intersecan.

**valor atípico** Los valores atípicos son puntos que están muy lejos de sus valores predichos.

**resultado o salida** Variable dependiente de una función.

# Glossary/Glosario

| English | Español |
|---|---|

## P

**perfect cube** A number that is the cube of an integer. For example, $8$ is a perfect cube because $2^3 = 8$.

**cubo perfecto** Número que es el cubo de un número entero. Por ejemplo, $8$ es un cubo perfecto porque $2^3 = 8$.

**perfect square** A number that is the square of an integer. For example, $16$ is a perfect square because $4^2 = 16$.

**cuadrado perfecto** Número que es el cuadrado de un número entero. Por ejemplo, $16$ es un cuadrado perfecto porque $4^2 = 16$.

**pi** The ratio of the circumference of a circle to its diameter. It is usually represented by $\pi$.

**pi** Razón entre la circunferencia y el diámetro de un círculo. Usualmente se representa como $\pi$.

**piecewise function** A function that is defined by two or more equations. Each equation is valid for some interval.

**función por partes** Función definida por dos o más ecuaciones. Cada ecuación es válida para alguno de los intervalos.

**polygon** A closed, two-dimensional shape with straight sides that do not cross each other.

**polígono** Forma cerrada y bidimensional de lados rectos que no se entrecruzan.

**positive association** A positive association is a relationship between two quantities where one tends to increase as the other increases.

**asociación positiva** Una asociación positiva es una relación entre dos cantidades, en la cual una tiende a aumentar a medida que la otra disminuye.

**preimage** See the definition of *image*.

**preimagen** Ver *imagen*.

**prime notation** A labeling notation that uses a tick mark. *Prime notation* is typically applied to an image, to tell it apart from its preimage.

**notación prima** Notación para etiquetar que usa un signo de prima. Una *notación prima* usualmente se aplica a una imagen, para distinguirla de su preimagen.

**Properties of Equality** Rules that apply to all equations. These include properties of addition, subtraction, multiplication, and division, which state that if an equation is true, then applying the same operation to both sides will give a new equation that is also true.

**Propiedades de igualdad** Reglas que son aplicables a todas las ecuaciones. Incluyen las propiedades de suma, resta, multiplicación y división, las cuales señalan que si una ecuación es verdadera, al aplicar la misma operación a ambos lados se obtendrá una nueva ecuación que también es verdadera.

**proportional relationship** A relationship in which the values for one quantity are each multiplied by the same number (the *constant of proprtionality*) to get the values for the other quantity.

**relación proporcional** Relación en la que los valores de una cantidad se multiplican cada uno por el mismo número (la *constante de proporcionalidad*) para obtener los valores de la otra cantidad.

**Pythagorean Theorem** The Pythagorean Theorem states that, for any right triangle, $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$. Sometimes this can be presented as $a^2 + b^2 = c^2$, where $a$ and $b$ represent the length of the legs and $c$ represents the length of the hypotenuse.

**Teorema de Pitágoras** El Teorema de Pitágoras establece que para todo triángulo rectángulo: $\text{cateto}^2 + \text{cateto}^2 = \text{hipotenusa}^2$. A veces puede ser también presentado como $a^2 + b^2 = c^2$, donde $a$ y $b$ representan las longitudes de los catetos y $c$ representa la longitud de la hipotenusa.

**Pythagorean triple** Three positive integers $a$, $b$, and $c$, such that $a^2 + b^2 = c^2$.

**Triplete pitagórico** Tres enteros positivos $a$, $b$ y $c$, tales como $a^2 + b^2 = c^2$.

## Q

**quadrilateral** A polygon with exactly four sides.

**cuadrilátero** Polígono de exactamente cuatro lados.

## English

# R

**radius** A line segment that connects the center of a circle with a point on the circle. The term can also refer to the length of this segment.

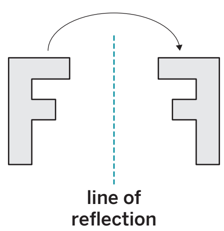
**rate of change** The amount one quantity (often $y$) changes when the value of another quantity (often $x$) increases by $1$. The *rate of change* in a linear relationship is also the slope of its graph.

**ratio** A comparison of two quantities by multiplication or division.

**rational numbers** The set of all the numbers that can be written as positive or negative fractions.

**rectangular prism** A polyhedron with two congruent and parallel bases, whose faces are all rectangles.

**reflection** A transformation that flips each point on a preimage across a *line of reflection* to a point on the opposite side of the line.

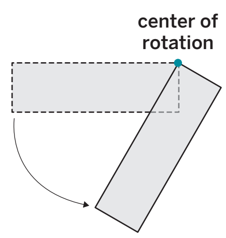



**relative frequency** The relative frequency is the ratio of the number of times an outcome occurs in a set of data. It can be written as a fraction, a decimal, or a percentage.

**repeating decimal** A decimal in which there is a sequence of non-zero digits that repeat indefinitely.

**rigid transformation** A move that does not change any of the measurements of a figure. Translations, rotations, and reflections are all examples of *rigid transformations* (as well as any sequence of these).

**rotation** A transformation that turns a figure a certain angle (called the *angle of rotation*) about a point (called the *center of rotation*).




## Español

**radio** Segmento de línea que conecta el centro de un círculo con cualquier punto del círculo. El término puede también referirse a la longitud de este segmento.

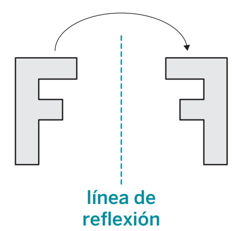
**tasa de cambio** Monto en que una cantidad (usualmente $y$) cambia cuando el valor de otra cantidad (usualmente $x$) aumenta en un factor de $1$. La *tasa de cambio* en una relación lineal es también la pendiente de su gráfica.

**razón** Comparación de dos cantidades a través de una multiplicación o una división.

**números racionales** Conjunto de todos los números que pueden ser escritos como fracciones positivas o negativas.

**prisma rectangular** Poliedro con dos bases congruentes y paralelas, cuyas caras son todas rectángulos.

**reflexión** Transformación que hace girar cada punto de una preimagen a lo largo de una *línea de reflexión* hacia un punto en el lado opuesto de la línea.

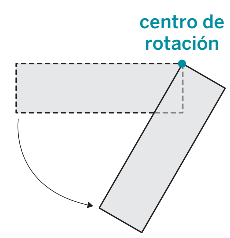



**frecuencia relativa** La frecuencia relativa es la razón del número de veces que ocurre un resultado en un conjunto de datos. Se puede escribir como una fracción, un decimal o un porcentaje.

**número decimal periódico** Decimal que tiene una secuencia de dígitos diferentes de cero que se repite de manera indefinida.

**transformación rígida** Movimiento que no cambia medida alguna de una figura. Traslaciones, rotaciones y reflexiones son ejemplos de *transformaciones rígidas* (como también cualquier secuencia de estas transformaciones).

**rotación** Transformación que hace girar una figura en cierto ángulo (llamado *ángulo de rotación*) alrededor de un punto (llamado *centro de rotación*).



# Glossary/Glosario

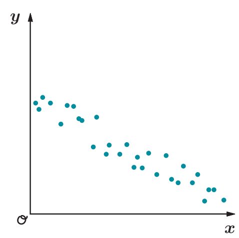
## English

**S**

**scale factor** The value that side lengths are multiplied by to produce a certain scaled copy.

**scaled copy** A copy of a figure where every length in the original figure is multiplied by the same value to produce corresponding lengths in the copy.

**scatter plot** A scatter plot is a graph that shows the values of two variables on a coordinate plane. It allows us to investigate connections between the two variables.

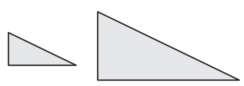



**scientific notation** A way of writing very large or very small numbers. When a number is written in scientific notation, the first factor is a number greater than or equal to one, but less than ten. The second factor is an integer power of ten. For example, $23000 = 2.3 \times 10^4$ and $0.00023 = 2.3 \times 10^{-4}$.

**segmented bar graph** A segmented bar graph compares two categories within a data set. The whole bar represents all the data within one category. Then, each bar is separated into parts (segments) that show the percentage of each part in the second category.

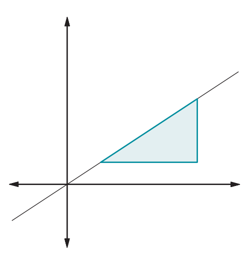
**sequence of transformations** Two or more transformations that are performed in a particular order.

**similar** Two figures are *similar* if they can be mapped onto each other by a sequence of transformations, including dilations.



**slope** The numerical value that represents the ratio of the vertical side length to the horizontal side length in a slope triangle. The rate of change in a linear relationship is also the slope of its graph.

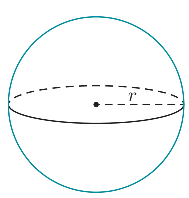
**slope triangle** A right triangle whose longest side is part of a line, and whose other sides are horizontal and vertical. *Slope triangles* can be used to calculate the slope of a line.



**solution** A value that makes an equation true.

**solution to a system of equations** An ordered pair that makes every equation in a system of equations true.

**sphere** A three-dimensional figure that consists of the set of points, in space, that are the same distance from a given point called the center.



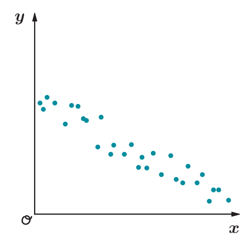

**square root** The square root of a positive number $p$ is a positive solution to equations of the form $x^2 = p$. Write the square root of $p$ as $\sqrt{p}$.

## Español

**factor de escala** Valor por el cual las longitudes de cada lado son multiplicadas para producir una cierta copia a escala.

**copia a escala** Copia de una figura donde cada longitud de la figura original es multiplicada por el mismo valor, para producir longitudes correspondientes en la copia.

**diagrama de dispersión** Un diagrama de dispersión es una gráfica que muestra los valores de dos variables en un plano de coordenadas. Nos ayuda a investigar relaciones entre las dos variables.

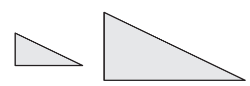



**notación científica** Manera de escribir números muy grandes o números muy pequeños. Cuando un número es escrito en notación científica, el primer factor es un número mayor o igual a uno, pero menor que diez. El segundo factor es un número entero que es potencia de diez. Por ejemplo, $23000 = 2.3 \times 10^4$ y $0.00023 = 2.3 \times 10^{-4}$.

**gráfica de barras segmentada** Una gráfica de barras segmentada compara dos categorías dentro de una serie de datos. La barra completa representa la totalidad de los datos dentro de una categoría. Entonces, cada barra es separada en partes (llamadas segmentos) que muestran el porcentaje de cada parte en la segunda categoría.

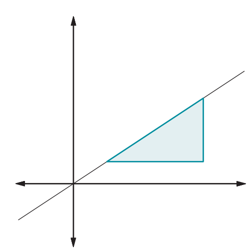
**secuencia de transformaciones** Dos o más transformaciones que se llevan a cabo en un orden particular.

**similar** Dos figuras son *similares* si pueden ser imagen la una de la otra, mediante una secuencia de transformaciones que incluyen las dilataciones.



**pendiente** El valor numérico que representa la razón entre la longitud del lado vertical y la longitud del lado horizontal en un triángulo de pendiente. Dada una línea, todo triángulo de pendiente tiene la misma pendiente.

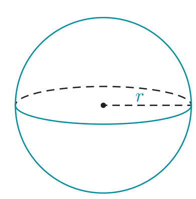
**triángulo de pendiente** Triángulo rectángulo cuyo lado más largo es parte de una línea, y cuyos otros lados son horizontales y verticales. Los *triángulos de pendiente* pueden ser usados para calcular la pendiente de una línea.



**solución** Valor que hace verdadera a una ecuación.

**solución al sistema de ecuaciones** Par ordenado que hace verdadera cada ecuación de un sistema de ecuaciones.

**esfera** Figura tridimensional que consiste en una serie de puntos en el espacio que están a la misma distancia de un punto específico, llamado centro.




**raíz cuadrada** La raíz cuadrada de un número positivo $p$ es una solución positiva a las ecuaciones de la forma $x^2 = p$. Escribimos la raíz cuadrada de $p$ como $\sqrt{p}$.

## English

**straight angle** An angle that forms a straight line. A straight angle measures $180$ degrees.

**substitution** Replacing an expression with another expression that is known to be equal.

**supplementary angles** Two angles whose measures add up to $180$ degrees.

**symmetry** When a figure can be transformed in a certain way so that it returns to its original position, it is said to have *symmetry*, or be *symmetric*.

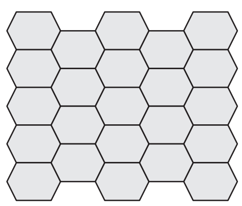
**system of equations** A set of two equations with two variables. (In a later course, you will see systems with more than two equations and variables.)

### T

**term** An expression with constants or variables that are multiplied or divided.

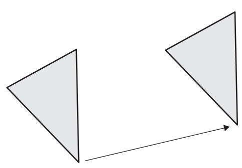
**terminating decimal** A decimal that ends in $0$s.

**tessellation** A pattern made of repeating shapes that completely covers a plane, without any gaps or overlaps.

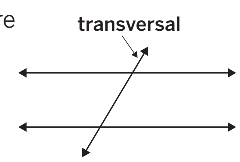


**transformation** A rule for moving or changing figures on the plane. Transformations include translations, reflections, and rotations.

**translation** A transformation that slides a figure without turning it. In a *translation*, each point of the figure moves the same distance in the same direction.



**transversal** A line that intersects two or more other lines.




**Triangle Sum Theorem** A theorem that states the sum of of the three interior angles of any triangle is $180$ degrees.

**two-way table** A two-way table provides a way to compare two categorical variables. It shows one of the variables across the top and the other down one side. Each entry in the table is the frequency or relative frequency of the category shown by the column and row headings.

### U

**unit rate** How much one quantity changes when the other changes by $1$.

## Español

**ángulo llano** Ángulo que forma una línea recta. Un ángulo llano mide $180$ grados.

**sustitución** Reemplazo de una expresión por otra expresión que se sabe es equivalente.

**ángulos suplementarios** Dos ángulos cuyas medidas suman $180$ grados.

**simetría** Cuando una figura puede ser transformada de manera tal que regrese a su posición original, se dice que tiene *simetría* o que es *simétrica*.

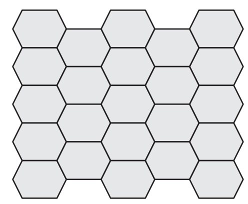
**sistema de ecuaciones** Conjunto de dos ecuaciones con dos variables. (En un curso posterior verán sistemas con más de dos ecuaciones y variables.)

### T

**término** Expresión con constantes o variables que son multiplicadas o divididas.

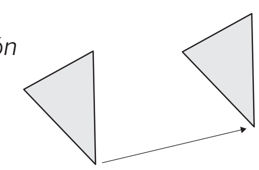
**decimal exacto** Un decimal que termina en ceros.

**teselado** Patrón compuesto por formas repetidas que cubren por completo un plano, sin dejar espacios vacíos ni superposiciones.

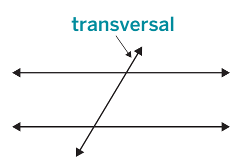


**transformación** Regla que se aplica al movimiento o al cambio de figuras en el plano. Traslaciones, rotaciones y reflexiones son ejemplos de transformaciones.

**traslación** Transformación que desliza una figura sin hacerla girar. En una *traslación* cada punto de la figura se mueve la misma distancia en la misma dirección.



**transversal** Línea que se interseca con dos o más líneas distintas.




**Teorema de la suma del triángulo** Teorema que afirma que la suma de los tres ángulos interiores de cualquier triángulo es $180$ grados.

**tabla de dos entradas** Una tabla de dos entradas provee una forma de comparar dos variables categóricas. Muestra una de las variables de forma horizontal y la otra de forma vertical. Cada entrada en la tabla es la frecuencia o frecuencia relativa de la categoría mostrada en los encabezados de la columna y la fila.

### U

**tasa unitaria** Cuánto cambia cierta cantidad cuando la otra cambia por un factor de $1$.

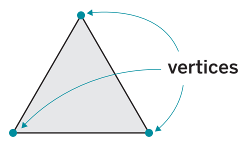
# Glossary/Glosario

## English

## V

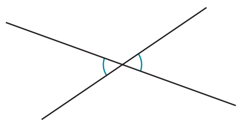
**variable** A quantity that can take on different values, or that has a single unknown value. Variables are typically represented using letters.

**vertex** A point where two sides of a two-dimensional shape or two or more edges of a three-dimensional figure intersect. (The plural of *vertex* is *vertices*.)

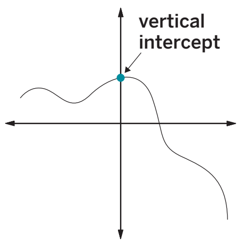



**vertical** Running straight up or down.

**vertical angles** Opposite angles that share the same vertex, formed by two intersecting lines. Vertical angles have equal measures.



**vertical intercept** A point where a graph intersects the vertical axis. Also known as the $y$-intercept, it is the value of $y$ when $x$ is $0$.




**volume** The number of unit cubes needed to fill a three-dimensional figure without gaps or overlaps.

## X

**$x$-intercept** See the definition for *horizontal intercept*.

## Y

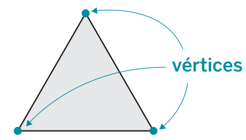
**$y$-intercept** See the definition for *vertical intercept*.

## Español

## V

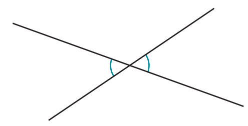
**variable** Cantidad que puede asumir diferentes valores o que tiene un solo valor desconocido. Las variables usualmente son representadas por letras.

**vértice** Punto donde se intersecan dos lados de una forma bidimensional, o dos o más aristas de una figura tridimensional.

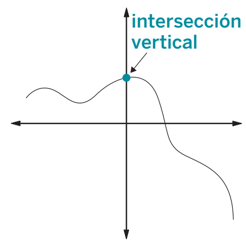



**vertical** Que corre en línea recta hacia arriba o hacia abajo.

**ángulos verticales** Ángulos opuestos que comparten el mismo vértice, conformado por dos líneas que se intersecan. Los ángulos verticales tienen las mismas medidas.



**intersección vertical** Punto en que una gráfica se interseca con el eje vertical. También conocida como intersección $y$, se trata del valor de $y$ cuando $x$ es $0$.




**volumen** Número de unidades cúbicas necesario para llenar una figura tridimensional sin dejar espacios vacíos ni superposiciones.

## X

**intersección $x$** Ver *intersección horizontal*.

## Y

**intersección $y$** Ver *intersección vertical*.

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## L



## M



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## T