## Reintroducing Inequalities

Let's work with inequalities.


## Focus

## Goals

1. Language Goal: Comprehend the terms less than or equal to and greater than or equal to and the symbols $\leq$ and $\geq$. (Speaking and Listening, Reading and Writing)
2. Represent solutions to an inequality on a number line.
3. Recognize that more than one value for a variable makes the same inequality true.

## Coherence

## - Today

Students write inequalities to represent scenarios, test values to determine whether they are solutions, and reason about solving one-step inequalities. The inequalities of less than or equal to and greater than or equal to are introduced.

## < Previously

In Grade 6, students reasoned about and represented solutions to inequalities of the forms $x>a$ and $x<a$.

## > Coming Soon

In Lessons 14-18, students will formalize the process to solve inequalities and notice similarities to solving equations.

## Rigor

- Students discuss real-world scenarios to build conceptual understanding of less than or equal to and greater than or equal to.


## Standards

## Addressing

## 7.EE.B. 4

Use variables to represent quantities in a realworld or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

| Building On | Building Toward |
| :--- | :--- |
| 6.EE.B. 5 | 7.EE.B.4.B |



Warm-up

## Activity 1

## Activity 2

## Activity 3

Summary
Exit Ticket

| (J) 5 min | © 10 min | (1) 10 min | ( $) 10 \mathrm{~min}$ | (1) 5 min | (1) 5 min |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\bigcirc$ Independent | กํำ Pairs | กํำ Pairs | คํำ Pairs | กำกำก Whole Class | $\bigcirc$ ○ Independent |
| MP2 | MP1 | MP1 |  |  |  |
| 7.EE.B. 4 | 7.EE.B. 4 | 7.EE.B. 4 | 7.EE.B. 4 | 7.EE.B. 4 | 7.EE. B. 4 |
| Amps powered by desmos | Activity and Presentation Slides |  |  |  |  |

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

## Practice $\bigcirc$ Independent

## Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, pre-cut cards, one set per pair
- Anchor Chart PDF, Solving Inequalities (for display)
- Anchor Chart PDF, Solving Inequalities (answers)


## Math Language Development

## New words

- greater than or equal to
- less than or equal to
- solution to an inequality.


## Review words

- greater than
- inequality
- less than
- solution to an equation


## Amps Featured Activity

## Warm-up <br> Dynamic Dog Walking Diagrams

Dog walking diagrams and measures of strength will automatically update as students change their input values.


## Building Math Identity and Community Connecting to Mathematical Practices

Students might not spend enough time analyzing the inequalities in Activity 3. Ask students to list some steps that they should take in their analysis in order to match inequalities to their graphed solutions (MP7). Ask them to explain which steps are absolutely necessary for solving the problem correctly and which steps could be beneficial, but might not be critical.

## - Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- Activity 3 may be omitted. Consider assigning the activity as Additional Practice using the Digital Card Sort.

Students make connections between a dog walking diagram and all possible solutions that satisfy a given criteria, as an introduction to writing and solving inequalities.

Amps Featured Activity Dynamic Dog Walking Diagrams


Warm-up Greater Than One
Study the diagram.


1. Select all the values that could represent $x$ so that the dog walker is pulled to the left.
(A.) 3
C. -3
(®.) 5
G. 0
B. 0.6
D. 1
©. 1.05
2. Plot the solutions from Problem 1 on the number line.

>3. Write an inequality to represent all the solutions from Problem 2.
$x>1$

Critique and Correct: Your teacher will provide an incorrect statement about
this scenario. Work with a partner to identify the erro and correct the statement.

## 1) Launch

Set an expectation for the amount of time students will have to work independently on the activity.

## 2 Monitor

Help students get started by asking, "What numbers listed will pull the dog walker to the left, i.e., are they less than or greater than 1?" (MP2)

Look for points of confusion:

- Difficulty understanding which values are greater than $\mathbf{1}$. Identify where the numbers are on the number line.
- Thinking 700 is not a solution because it is not visible on the number line. Continue the number line until students realize it will eventually reach 700.


## 3 Connect

Have students share responses to Problem 1 by conducting the Poll the Class routine. Ask students to explain their thinking.

## Highlight:

- Substituting the value in the inequality and reading it aloud can help students determine if the statement is true. Say, " $-3>1$ is not true; therefore, -3 is not a solution."
- Comparing this statement to the location of -3 on the number line.
- The boundary value of 1 is not included because 1 is not greater than 1 and is represented with an open circle.
- Write $1<x$, read it, and discuss if this inequality is the same as or different from the inequality $x>1$.
Define solution to an inequality: a value that will make an inequality a true statement when substituted into the inequality.


## Math Language Development

MLR3: Critique, Correct, Clarify
During the Connect, display an incorrect statement about this scenario, such as "The value of $x$ could be any number that is 1 or greater." Ask:

- Critique: "Do you agree or disagree with this statement? Explain your thinking." Listen for students who reason that the strength of the dog on the left cannot be equal to 1 .
- Correct: "How would you correct this statement?"
- Clarify: "How can you convince someone that your statement is true?"


## (7) <br> Power-up

To power up students' ability to determine which values make an inequality true, have students complete:
Apples from a certain orchard are always picked before they weigh 0.5 lbs . Select all of the values that are less than 0.5 .
A. 0.5
C. $\frac{1}{3}$
(E.) 0
(B.) 0.25
D. $\frac{2}{3}$
F. 0.9

Use: Before the Warm-up
Informed by: Performance on Lesson 12, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3

Students review the meanings of the symbols < and > and then read a scenario which facilitates the necessity for the new inequality symbols $\leq$ and $\geq$.

Activity 1 The Roller Coaster

A sign next to a roller coaster at an amusement park reads, "You must be at least 60 in. tall to ride." Noah is happy to know that he is tall enough to ride.

## 1 Launch

Activate background knowledge by asking students whether they have been on a roller coaster or a ride at the fair and whether they experienced being with someone who was not able to ride because of their height.

## 2 Monitor

Help students get started by asking, "If Noah is _ inches tall, can he ride?" Continue with similar examples until students can continue on their own (MP1).

## Look for points of confusion:

- Not knowing they can pick two options for Problem 1. Discuss why Noah can either be greater than 60 or equal to 60 .
- Struggling to represent their responses to Problems 1-3. Have them use the number line to help identify solutions.
- Not knowing how to show that 58 is included. Ask students how they would plot the point 58 on a number line. Discuss that a closed circle is used because it represents shading the values that make the inequality true.


## 3 Connect

Have students share possible heights for Noah's friend and mark the solutions on the number line. Ask students to share their responses to Problem 5. Students may write, " $y=58$ or $y>58$."

Highlight that there is a symbol meaning greater than or equal to and introduce the notation $\geq$. Since Noah's friend can be 58 in. tall, have students mark 58 with a closed circle on the number line.

## Define:

- greater than or equal to
- less than or equal to


## Differentiated Support

## Accessibility: Guide Processing and Visualization

Before students begin the activity, ask them to generate possible heights for Noah that would allow him to ride the roller coaster. Then ask them to generate 1 or 2 heights for Noah that would mean he would not be allowed to ride the roller coaster.

## Extension: Math Enrichment

Have students complete the following problem:
Suppose Noah's sister will be allowed to ride the roller coaster when her height increases by at least 8 in. Define a variable and write an inequality that represents Noah's sister's current height. Sample response: Let $z$ represent Noah's sister's current height; $z+8 \geq 60$.

## Math Language Development

## MLR7: Compare and Connect

During the Connect, as you introduce the $\geq$ and $\leq$ symbols, add them to the class display, along with common words and phrases that can indicate them. For example, consider displaying the following table.

|  |
| :--- | :--- | :--- |
| Greater than or equal to  <br> At least Less than or equal to <br> At most  |

## English Learners

Provide examples of phrases, such as "I own at least 4 baseball hats, which means the number of baseball hats I own is greater than or equal to 4 ."

Students solve one-step inequalities by reasoning about the solutions. They are not expected to formally
solve inequalities in this activity.

## Activity 2 Understanding Inequalities

Clare wants to buy her mother a gift which costs at least \$20. Let $x$ represent the amount of money Clare will need.

1. This scenario is represented by the inequality $x \geq 20$. Graph the solutions to $x \geq 20$ on the number line.

2. If Clare drives to the mall, she must spend $\$ 5$ on parking. The scenario is now represented by the inequality $x-5 \geq 20$. Graph the solutions on the number line. How did the solutions change from $x \geq 20$ ?

3. If Clare's father gives her $\$ 5$ and drops her off at the mall, the scenario is now represented by $x+5 \geq 20$. Graph the solutions for the inequality. How did the solutions change from $x \geq 20$ ?

4. Clare's siblings want to help by dividing the cost of the gift among themselves. Assume her father does not give her $\$ 5$. There are 5 siblings altogether, so the inequality $5 x \geq 20$ can be used to represent this scenario. Graph the solutions for the inequality. Scale your own number line. How did the inequality and the solutions change from $x \geq 20$ ?


The solutions are now greater than or equal to 4 .

## 1 Launch

Be sure students understand what the symbols $<,>, \leq$, and $\geq$ represent.

## Monitor

Help students get started by asking, "What does $\geq$ mean?" or "Is $\qquad$ greater than or equal to 20 ?" (MP1)

## Look for points of confusion:

- Wanting to shade the direction in which the inequality points (i.e., < shades left because the arrow points in that direction). Encourage students to test values in the inequality to determine in which direction to shade. To further emphasize this point, show an inequality such as $3<x$, and discuss the solutions.


## Look for productive strategies:

- Wanting to solve the inequalities algebraically. Although this is not the goal of this lesson, encourage students to test values before shading the number line. This process will be formalized in the next lesson.


## 3 Connect

Highlight that students can test values to determine whether the inequality is true. Students are not expected to solve inequalities during this lesson.

Ask, "Why is the boundary point shaded?" It makes the inequality true and is a solution.

## Differentiated Support

## Accessibility: Vary Demands to Optimize Challenge

If students need more processing time, have them focus on completing Problems 1-3. As time allows, they can work on Problem 4.

## Accessibility: Guide Processing and Visualization

Test possible solutions from Problem 1 into the inequality in Problem 2, starting with values that are true for both inequalities, such as 25 and 30 . Then test a value that is only a solution to Problem 1, such as 20 or 24 . Ask students why these values are not solutions to the inequality in Problem 2.

## Math Language Development

## MLRT: Compare and Connect

During the Connect, display the four inequalities and ask students how the inequalities represent each scenario. Ask them to look for the key words and phrases that indicate why the inequality symbol is always $\geq$, and what operation(s) are performed on the variable. Ask:

- "What does $x$ represent in these scenarios? What does the phrase 'at least' \$20 tell you?"
- "If Clare spends $\$ 5$ on parking, what does this mean for the amount of money she will need? Why is 5 subtracted from $x$ ?"
Ask similar questions for the two remaining scenarios.


## Activity 3 Card Sort: Inequalities

Students sort cards to match inequalities with solutions on number lines. This will help further their understanding of testing values to determine whether inequalities are true.

Activity 3 Card Sort: Inequalities

The symbols $>,<, \geq$, and $\leq$ help to represent fundamental ideas about comparing amounts, but even these symbols had to come from somewhere. When mathematicians, such as Thomas Harriot or Giuseppe Peano, have brand new mathematical ideas, they create symbols that hold the special meaning of the new idea within them.

You will be given a set of cards. Match each inequality with a solution on the number line. Write the letter of the matches in the table. Have your teacher check your answers when you are finished.


Explain your strategies for matching.
Answers may vary. Some students may guess and check, while others may think about solving inequalities similarly to how they solve equations. Som students may also test values to determine which set shows the solutions.


Giuseppe Peano
How would you explain the concept of numbers without using the words number, math, or even any words related to math? This was a challenge in mathematics until 1889, when Giuseppe Peano wrote a set of axioms that did just that. For Peano's other work, he even had to create new symbols to represent his brand new ideas in mathematical logic. You will likely encounter the symbols $\cup, n$, $\ni, \mathbb{N}$, or $\mathbb{Q}$ a bit later in your math studies, but you will already know there are some big ideas contained inside of them.

## Differentiated Support

## Accessibility: Vary Demands to Optimize Challenge

Distribute the cards with addition or subtraction inequalities and their solutions first. After these are matched, distribute the cards with multiplication inequalities and their solutions.

## Extension: Interdisciplinary Connections

Ask students, "Have you ever wondered where the symbols for =, >, or < came from?" Tell them that the first use of the equal sign, $=$, is attributed to Robert Recorde who was a Welsh physician and mathematician. He intentionally used two parallel lines to represent equality because they are always the same distance apart. The works of British mathematician Thomas Harriot included the first inequality symbols. These were actually introduced by his book's editor, who altered the original triangular symbols Harriot used. Ask students to think of and create their own symbols they would use to indicate equality, greater than, or less than, and explain why they chose the symbols they did. (History)

Featured Mathematician

## Giuseppe Peano

Have students read about Featured Mathematician Giuseppe Peano, the creator of several symbols for mathematical logic.

Summary
7.EE.B. 4

Review and synthesize how inequalities can be used to represent real-world situations, and how testing values can help determine whether inequalities are true.
(3)

> Reflect:

## Synthesize

Display the Anchor Chart PDF, Solving Inequalities and complete the top section as a class.

## Ask:

- "Which inequality symbol should be place in each box?"
- "How are the two inequalities for each number lines similar? How are they different?"
- "What clue from the number line lets you know when to use $>$ versus $\geq$ ?"
- "Are inequalities with $\leq$ and $<$ always shaded to the left? Why or why not?"

Highlight that the location of the variable in an inequality, on the left or the right, impacts the side of the number line which is shaded. Bring attention to the fact that when the value and the variable swap sides the direction of the inequality symbol also changes direction (e.g. $x>2$ is the same as $2<x$ ).

## Formalize vocabulary:

- less than or equal to
- greater than or equal to
- solutionto an inequality.


## (1) Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the Reflect space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does it mean to be a solution to an inequality?"
- "Does it make sense that an inequality can have more than one solution? Why or why not?"


## Math Language Development

MLR2: Collect and Display
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms solution to an inequality, less than, greater than, less than or equal to, and greater than or equal to that were added to the display during the lesson.

## Exit Ticket

Students demonstrate their understanding by giving solutions to an inequality and determining how solutions change when the term equal to is added to an inequality.


## Success looks like . . .

- Language Goal: Comprehending the terms less than or equal to and greater than or equal to, and the symbols $\leq$ and $\geq$. (Speaking and Listening, Reading and Writing)
» Explaining how the solutions to the inequality $2 x<10$ are different from the solutions to $2 x \leq 10$ in Problem 2.
- Goal: Representing solutions to an inequality on a number line.
- Goal: Recognizing that more than one value for a variable makes the same inequality true.


## - Suggested next steps

If students determine solutions satisfying $x \geq 5$, they most likely understand the relationship with $\mathbf{2}$ and $\mathbf{1 0}$. If students possibly did not test their points to determine whether they made the inequality true, consider:

- Assigning Practice Problem 3.

If students need more practice with understanding the inequality symbols, consider:

- Assigning Practice Problem 1.
- Assigning Practice Problem 2.


## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.
$\mathrm{O}_{0}$ Points to Ponder ...

- What worked and didn't work today? How did the Card Sort set up students to develop understanding of solutions of inequalities?
- In what ways did the Warm-up go as planned? What might you change for the next time you teach this lesson?



## Additional Practice Available



For students that need additional practice in this lesson, assign the Grade 7 Additional Practice.

## Solving Inequalities

Let's solve more complicated inequalities.

## Focus

## Goals

1. Use substitution to determine whether a given value for a variable makes an inequality true.
2. Generalize that it is possible to solve an inequality of the form $x+q>r$ or $x+q<r$ by solving the equation $x+q=r$ and then testing a value to determine the direction of the inequality in the solution.
3. Generalize that it is possible to solve an inequality of the form $q x>r$ or $q x<r$ by solving the equation $q x=r$ and then testing a value to determine the direction of the inequality in the solution.

## Coherence

## - Today

Students write and solve equations and use those solutions to help them determine the solutions of corresponding one-step inequalities that may include negative values (MP6, MP7).

## < Previously

In Lesson 13, students wrote one-step inequalities to represent realworld scenarios and tested values to determine if they were solutions.

## Rigor

- Students use substitution to build conceptual understanding of what is meant by a solution to an inequality.
- Students use tables of values to build their conceptual understanding of solution sets of one-step inequalities.


## Standards

## Addressing

## 7.EE.B. 4

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

| Building On | Building Toward |
| :--- | :--- |
| 6.EE.B.5 | 7.EE.B.4.B |
| 6.EE.B.8 |  |

## >Coming Soon

In Lesson 15, students will write and solve two-step inequalities.

## ()

Warm-up

Activity 2

Activity 3 (Optional)

Summary

Exit Ticket

| (1) 5 min | (1) 5 min |
| :---: | :---: |
| กัํากำก Whole Class | $\bigcirc$ ค Independent |
| 7.EE.B.4.B | 7.EE.B. 4 |

## Amps powered by desmos : Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

## Practice $\bigcirc$ Independent

## Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- two colors of colored pencils, markers, or highlighters for each group


## Math Language <br> Development

## Review words

- greater than
- greater than or equal to
- inequality
- less than
- less than or equal to
- solution to an equation
- solutions to an inequality


## Amps : Featured Activity

Activity 1
Overlay Student Work
Students individually plot points on a number line and can then see all their classmates' data shown together on one number line.


## Building Math Identity and Community Connecting to Mathematical Practices

In Activity 3, students write possible inequalities that have given solutions and then check their work with a partner. Remind them to communicate clearly and precisely (MP6) as they share the inequalities they wrote, and why they believe they are correct.

## - Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- During the Warm-up, choose only one row for the class to complete.
- In Activity 1, Problem 3 may be omitted.
- Optional Activity 3 may be omitted, or students may choose one problem to complete.


## Warm-up True and False Inequalities

Students complete a table by determining which values make an inequality true or false to practice identifying solutions to inequalities.


## 1 Launch

Conduct the Think-Pair-Share routine. Give students 2 minutes of silent work time. Then give them 2 minutes to compare their work with a partner.

## (2) Monitor

Help students get started by modeling how to reason about one of the inequalities in the table. Choose an inequality and model for students how to substitute each value into the inequality to determine if the inequality is true or false.

## Look for points of confusion:

- Misinterpreting inequalities with the variable on the right (e.g., assuming $100<4 x$ is equivalent to $x<25$ ). Suggest substituting values for $x$ to see the difference between " $x$ is less than 25 " and " 25 is less than $x$."
- Forgetting that inequalities with $\geq$ and $\leq$ signs are true when both sides are equal. Ask students to explain the difference between $</>$ and $\leq / \geq$.


## 3 Connect

Have students share their entries for each row of the table and the strategies they used.

Highlight the differences between substituting 25 for $x$ in $100<4 x$ and $10 \geq 35-x$. Note that inequalities using the symbols $\leq$ and $\geq$ are considered true when both sides are equal, whereas < and > inequalities are considered false when both sides are equal. Emphasize that substituting a value for $x$ and checking if the resulting inequality is true is the most direct way to check whether the value is a solution.

## (7) Power-up

To power up students' ability to determine whether a given value makes an equation true, have students complete:

Match each equation with the value that makes it true.
a. $x+0=25$
a, c $\quad x=25$
b. $x+25=25$
d $. x=-25$
c. $100=4 x$.
b $\quad x=0$
d. $10=x+35$

Use: Before the Warm-up.
Informed by: Performance on Lesson 13, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4.

## Activity 1 Inequalities with Tables (Part 1)

7.EE.B. 4

Students complete and analyze a table of values in order to better understand the relationship between inequalities and their solutions.


Amps Featured Activity Overlay Student Work

Activity 1 Inequalities With Tables (Part 1)

1. Complete the table.

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x-3$ | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | 1 |

2. Refer to the number line and the values of $x$ in the table from Problem 1 . Sample responses shown.

a Which value of $x$ makes $x-3=-2$ true? Mark it on the number line. $x=1$
b Which values of $x$ make $x-3$ greater than -2 ? Mark them in one color on the number line
$x=2,3,4$
c Which values of $x$ make $x-3$ less than -2 ? Mark them in another color on the number line.
$x=0,-1,-2,-3,-4$
d Find a value of $x$ between -4 and 4 not listed in the table that makes $x-3$ greater than -2 . Plot and label it on the number line. Mark it in the same color you used for part b.
Answers may vary, but must be non-whole numbers between 1 and 4 .

Find a value of $x$ between -4 and 4 not listed in the table that makes $x-3$ less than -2 . Plot and label it on the number line. Mark it in the same color you used for part c.
Answers may vary, but must be non-whole numbers between -4 and 1 .

## 1. Launch

Display the Activity 1 PDF. Ask, "How are the numbers in the top row and bottom row related? Think about the equation $x+2=-2$. What value of $x$ makes this true? Where do you see that in the table? How about the inequality $x+2>3$ ?" Distribute two different colored pencils, markers, or highlighters to each group.

## 2 Monitor

Look for points of confusion:

- Struggling to identify values not listed in the table for Problem 2, parts d and e. Suggest students consider non-integer values.
- Saying the solution of $x-3>-2$ is $x>2$, based on the integer values. Suggest they check whether any values between 1 and 2 make the inequality true.
- Making mistakes in Problem 3 when deciding if the circle on each graph is open or closed. Ask them to consider if each inequality is true when $x=1$.

Activity 1 continued >

## Differentiated Support

## Accessibility: Optimize Access to Technology

Have students use the Amps slides for this activity, in which they can individually plot points on a digital number line and then see all of their classmate's data shown together on one number line.

## Extension: Math Enrichment

Ask students to explain how the solutions to the following inequalities are similar to and different from the solutions to the inequalities in Problem 3
$x+1>-2$
$x+1 \geq-2$
$x+1<-2$
$x+1 \leq-2$
Sample response: The direction in which the inequality is shaded will remain the same. Whether the boundary value is an open or closed circle will remain the same. The boundary values will change to be 4 less than they were in Problem 3; the boundary values will now be -3 .

## Activity 1 Inequalities with Tables (Part 1) (continued)

Students complete and analyze a table of values in order to better understand the relationship between inequalities and their solutions.

Name: $\rightarrow$ Date: $\square$ Period
Activity 1 Inequalities With Tables (Part 1) (continued)
3. Use the number line from Problem 2 to help you think about which values of $x$ will make each of the following inequalities true. Graph the solution to each inequality on the number line, and write an inequality to represent the solution that has $x$ by itself on one side

| Inequality | Graph | Solution |
| :---: | :---: | :---: |
| (a) $x-3>-2$ |  | $x>1$ |
| b $x-3 \geq-2$ |  | $x \geq 1$ |
| c $x-3<-2$ |  | $x<1$ |
| d $x-3 \leq-2$ |  | $x \leq 1$ |

## Activity 2 Inequalities With Tables (Part 2)

## Students use tables and numbers lines to reason about the solutions of inequalities, including those with negative coefficients, in preparation for solving two-step inequalities.

Activity 2 Inequalities With Tables (Part 2)
$>1$. Consider the inequality $2 x<6$.
a Predict which values of $x$ will make the inequality $2 x<6$ true, and show them on the number line.

b Complete the table. Compare the values of $x$ in the table with your graph to check your prediction.

(c) Write an inequality to represent the solutions to the inequality $2 x<6$ $x<3$
2. Consider the inequality $-2 x<6$.
a Predict which values of $x$ will make the inequality $-2 x<6$ true, and show them on the number line.
Sample response shown. This sample response is inaccurate but reflects the anticipated prediction that students will make.

b Complete the table. Compare the values of $x$ in the table with your graph to check your prediction.

>3. How are the solutions to $2 x<6$ different from the solutions to $-2 x<6$ ? The solutions to $2 x<6$ are numbers less than 3 . The solutions to $-2 x<6$ are numbers greater than -3 .

## 1 Launch

Explain that students will be making and checking predictions about the solutions to inequalities. Suggest that they use strategies discussed in the last activity, such as solving a related equation, to help them predict.

## 2 Monitor

## Look for points of confusion:

- Predicting the arrows point to the left on both graphs because they have the same inequality sign. Encourage them to check their predictions with the table.

Look for productive strategies:

- Writing and solving a related equation for each inequality. Note students who use this method.


## 3 Connect

Have students share their solutions for each inequality and explain how they are different.

Highlight that solving the equation related to the inequality gives the boundary value between solutions and non-solutions. Demonstrate that a table isn't necessary to check values on either side of the boundary value. Instead, test one number greater than the boundary value and one number less than the boundary value. Whichever number makes the inequality true is on the same side of the boundary value as all the points that make the inequality true (MP7).

Ask, "How can you use a related equation to help you solve an inequality? How would the solutions to these inequalities change if the sign was $\leq$ instead of <?"

## Differentiated Support

## Accessibility: Vary Demands to Optimize Challenge

Provide pre-completed tables for students to use for Problems 1 b and 2 b to check their predictions. This will allow students to spend more time comparing the two inequalities.

## Math Language Development

## MLR7: Compare and Connect

During the Connect, display the two inequalities and their solutions. Draw students' attention to how the inequalities and their solutions are similar and different. Ask:

- "How are the inequalities $2 x<6$ and $-2 x<6$ similar? How are they different?"
- "How are the solutions $x<3$ and $x>-3$ similar? How are they different?"
- "Why do you think the solutions to $-2 x<6$ aren't represented by the inequality $x<-3$ ?"


## English Learners

Annotate and/or color code the inequalities with their similarities and differences.

## Activity 3 Inequality Jeopardy

Students write inequalities that have a given solution and trade with a partner to check their work to practice reasoning about and solving inequalities (MP6).


## 1 Launch

Have a student read the directions to the class. Explain that students should swap with their partner after Problem 1 and again after Problem 2.
Each graph is the solution to an inequality. Fill in the boxes with positive or negative numbers to write three inequalities that each have the solution shown. Then trade books with your partner to check each other's work.


Sample responses shown.
$2 x>20$
$+x>11.5$
$-5 x<\boxed{-50}$


Sample responses shown.

| $-\frac{1}{2}$ | $x \leq-1$ |
| :--- | :--- |
| -4 | $x+2$ |

## Monitor

Help students get started by suggesting they write an inequality to represent the solution. Then tell them to consider how they could use what they know about creating equivalent equations to create a one-step inequality.

Look for points of confusion:

- Writing incorrect inequalities. Monitor pairs to make sure they are really checking each other's work.
- Disagreeing with each other about whether an inequality has the given solution. Urge students to use mathematical reasoning and precise language to defend their positions (MP3, MP6).


## 3 Connect

Display the two number lines.
Have students share an inequality they wrote for each number line. Select a few students to share how they checked that their partner's inequality was correct.

Highlight how to check that the solution of an inequality is correct.

## Differentiated Support

## Accessibility: Vary Demands to Optimize Challenge

If students need more processing time, have them write just one inequality for each number line. This will provide each student with additional time to reason about the solution before checking their partner's work.

## Extension: Math Enrichment

Have students explain why the third inequality in each problem must include a negative coefficient on the variable. Sample response: The inequality sign changed directions. In Problem 1, for example, in order for the product of the variable and a number to be less than another number, the number multiplied by the variable must be negative.

Review and synthesize how solving an inequality can be thought of as solving a related equation and then checking values greater and less than the solution.

## Summary

## In today's lesson ..

You tested values to determine what values make an inequality true. You used tables to organize your work and to help you write and graph the solutions to inequalities that involve addition, subtraction, or multiplication.
You noticed that in an inequality involving multiplication, the sign of the coefficient affected the direction of the solution. For example, consider the solutions to $3 x \geq 9$ and $-3 x \geq 9$ :


Solution: $x \geq 3$


Solution: $x \leq-3$


[^0]
## Synthesize

Display the inequality $5 x<-15$ and a blank number line.

Highlight the process for solving an inequality. Write the related equation $5 x=-15$. Solve for $x$. Discuss whether the solution makes the equation true. Then have students choose one value greater than the solution and one value less than the solution. Check which of the values makes the inequality true. Then write the solution and review how to graph it.

Ask, "Why is there an open circle on the graph instead of a closed circle? How do you know that the solution is less than -3 , instead of greater than?"

## (1) Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the Reflect space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Which strategies that worked for solving simple equations or inequalities can be put to use when solving more complex ones?"


## Exit Ticket

Students demonstrate their understanding by matching solutions to inequalities and explaining their choice.


## Success looks like . . .

- Goal: Using substitution to determine whether a given value for a variable makes an inequality true.
» Substituting values on either side of the boundary value to determine which side makes the inequality true in Problems 1 and 2.
- Goal: Generalizing that it is possible to solve an inequality of the form $x+q>r$ or $x+q<$ $r$ by solving the equation $x+q=r$ and then testing a value to determine the direction of the inequality in the solution.
- Goal: Generalizing that it is possible to solve an inequality of the form $q x>r$ or $q x<r$ by solving the equation $q x=r$ and then testing a value to determine the direction of the inequality in the solution.
» Solving the inequalities $4 x>10$ and $-4 x>-10$ in Problems 1 and 2.


## - Suggested next steps

If students say $x>2.5$ is the solution for both inequalities because they both use the greater than sign, consider:

- Suggesting they check a value greater than 2.5 to see whether it makes both inequalities true.
- Assigning Practice Problem 2.

If students choose solutions for each inequality without providing any explanation for their choice, consider:

- Reviewing the process of checking values greater than or less than the boundary value to find the solution to an inequality.
- Assigning Practice Problem 3.


## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

## O. Points to Ponder . .

- What worked and didn't work today? Which students' ideas were you able to highlight during Activity 1 ?
- Have you changed any ideas you used to have about solving and understanding inequalities as a result of today's lesson? What might you change for the next time you teach this lesson?

| Practice Problem Analysis |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Type | Problem | Refer to | Standard(s) | DOK |
| On-lesson | 1 | Activity 1 | 7.EE.B. 4 | 1 |
|  | 2 | Activity 2 | 7.EE.B. 4 | 2 |
|  | 3 | Activity 2 | 7.EE.B. 4 | 2 |
| Spiral | 4 | Unit 5 Lesson 18 | 7.NS.A. 3 | 1 |
|  | 5 | Unit 4 Lesson 5 | 7.RPA. 3 | 1 |
| Formative 0 | 6 | Unit 6 Lesson 15 | 7.EE.B.4.B | 1 |

(1) Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available



For students that need additional practice in this lesson, assign the Grade 7 Additional Practice.

# Finding Solutions to Inequalities in Context 

Let's solve more complicated inequalities.


## Focus

## Goals

1. Interpret inequalities representing situations with a constraint.
2. Solve an equation of the form $p x+q=r$ to determine the boundary value for an inequality of the form $p x+q>r$ or $p x+q<r$.
3. Language Goal: Use substitution or reasoning about the context to justify whether the values making an inequality true are greater than or less than the boundary value. (Speaking and Listening)

## Coherence

## - Today

Students solve contextual problems involving inequalities using the strategies from previous lessons. After solving for the boundary value, students determine the direction of the inequality. Students reason about the context, substitute values on either side of the boundary value, or reason about the number lines. These techniques exemplify making the problem more concrete and visual by asking, "Does this make sense?" (MP1, MP2).

## < Previously

Students wrote and solved equations from scenarios in Lessons 9-11. In Lesson 14, students wrote related equations and solved them to help find the solutions to the inequality.

## > Coming Soon

Students will continue to solve problems involving inequalities in Lessons 16-18.

## Rigor

- Students analyze real-world scenarios to develop procedural fluency in determining boundary values and direction of inequalities.
- Students apply their understanding of writing equations of the form $p x+q=y$ to write inequalities of the form $p x+q<y$ and $p x+q>y$.


## Standards

## Addressing

## 7.EE.B.4.B

Solve word problems leading to inequalities of the form $p x+q>r$ or $p x+q<r$, where $p, q$, and $r$ are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

| Building On | Building Toward |
| :--- | :--- |
| 6.EE.B.5 | 7.EE.B.4.B |
| 6.EE.B.8 |  |
| 7.EE.B.4.A |  |
|  |  |


| Warm-up | Activity 1 | Activity 2 | Summary | Exit Ticket |
| :---: | :---: | :---: | :---: | :---: |
| (J) 10 min | (J) 12 min | (J) 12 min | () 5 min | (J) 5 min |
| $\bigcirc \bigcirc \bigcirc{ }^{\circ}$ Independent | $\bigcirc \bigcirc \bigcirc\left({ }^{\circ}\right.$ Pairs | $\stackrel{\circ}{\circ} \mathrm{O}$ Pairs | คํํํํ คำำ Whole Class | $\bigcirc \bigcirc \bigcirc{ }^{\circ}$ Independent |
|  | MP1, MP2 | MP1, MP2 | MP1, MP2 | MP1 |
| 7.EE.B.4.B | 7.EE.B.4.B | 7.EE.B.4.B | 7.EE.B.4.B | 7.EE.B.4.B |
| AmpS powered by desmos | Activity and Presentation Slides |  |  |  |
| For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com. |  |  |  |  |

## Practice $\bigcirc$ Independent

## Materials

- Exit Ticket
- Additional Practice
- number lines (optional)


## Math Language Development

## Review words

- at least
- at most
- inequality
- greater than or equal to
- less than or equal to
- solution to an inequality


## Amps ! Featured Activity

## Activities 1 and 2 <br> See Student Thinking

As students solve equations step by step, see their thinking in real time.


## Building Math Identity and Community <br> Connecting to Mathematical Practices

Students tend to get concerned when new skills are being applied in realworld situations, but, to alleviate that concern, remind them that they have all of the skills they need to make sense of the problem (MP1). Ask students to give examples of self-talk that they use to build their self confidence. Ask students to choose one new way that they will encourage themselves during an internal dialog.

- Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- The Warm-up may be omitted


## Warm-up One Solution or Many Solutions?

Students see the link between an inequality and its related equation, while recognizing equations with the same solution do not imply the inequalities have the same solutions.


## 1 Launch

Set an expectation for the amount of time students will have to work independently on the activity.

## 2 Monitor

Help students get started by reminding students $-x$ can be written as $-1 x$.

## Look for points of confusion:

- Needing a number line to help visualize the boundary value. Print several copies of a line with unlabeled, evenly-spaced tick marks, and place these in sheet protectors. Students can write on these with dry erase markers and wipe them off.
- Having difficulty substituting negative values into the inequality. Remind students that $-x$ can be written as $-(x)$ or $-1 \cdot x$.


## 3 Connect

Display two number lines centered at -10 .
Have students share their responses to Problems 1 and 2, marking them on the number line while gauging the class for agreement. Repeat this with Problems 3 and 4.

Highlight that the equations in Problem 1 have the same solution. The inequalities in Problem 3 have the same structure as the equations, yet they do not have the same solutions. Reinforce that it is important to test values to know the direction of the solutions to an inequality.

## (7) Power-up

To power up students' ability to determine whether a value makes an inequality which has a negative coefficient true, have students complete:

Recall that $-x$ is equivalent to $-1 x$ or $-1 \cdot x$.
Select all values that make the inequality $-x<6$ true.
(A.) 6
C. 5
(E.) 0
B. -6
(D.) -5
(F. 12
Use: Before the Warm-up

Informed by: Performance on Lesson 14, Practice Problem 6

MP1, MP2
7.EE.B.4.B

## Activity 1 Earning Money for Soccer Apparel

Students solve an inequality (whole-number solutions only) by writing a related equation first to answer questions about a real-world scenario.

Amps Featured Activity See Student Thinking

Activity 1 Earning Money for Soccer Apparel

Han was hired for a summer job selling magazine subscriptions. He will earn $\$ 25$ per week, plus $\$ 3$ for every subscription he sells. Han hopes to make enough money this week to buy a new pair of soccer cleats.

1. Let $n$ represent the number of magazine subscriptions Han will sell this week. Write an expression for the amount of money he will make $25+3 n$
2. The most affordable cleats in the store will cost Han \$67. Write and solve an equation to determine how many magazine subscriptions he will need to sell to earn $\$ 67$. Show your thinking
$25+3 n=67$
$25+3 n-25=67-25$
$3 n=42$
$3 n \div 3=42 \div 3$
$n=14$
Han would need to sell 14 subscriptions to earn $\$ 67$.
>3. If Han sells 16 subscriptions this week, will he reach his goal and be able to buy the new cleats? Explain your thinking.
Yes, because $25+3 \cdot 16=73$. If he sold 16 subscriptions, he would earn $\$ 73$
3. What are some other numbers of subscriptions Han could sell to reach his goal?
Answers may vary, but must be whole numbers greater than or equal to 14
4. Write an inequality expressing how much Han will have to earn to afford at least $\$ 67$ for the cleats
$25+3 n \geq 67$
5. Write an inequality describing the number of subscriptions Han must sell to reach his goal.
$n \geq 14$

## 1 Launch

Set an expectation for the amount of time students have to work in pairs, or small groups, on the activity.

## 2 Monitor

Help students get started by asking, "If Han sells one subscription, how much money will he have? If he sells two subscriptions, how much money will he have?" Asking questions like these helps students develop the expression $3 n+25$ (MP1).

## Look for points of confusion:

- Thinking at least means "less than or equal to." Give examples of possible amounts Han needs. Ask, "Would Han be able to afford his soccer cleats with $\$ 45$ or with $\$ 70$ ?"


## 3 Connect

Have students share their solutions and strategies on how to determine which inequality to use.

Highlight that solving the related equation helps find the boundary value, but to determine the solutions to the inequality, students should test values and/or use the context of the scenario to help.

Ask:

- "How does solving the related equation help you solve the inequality?"
- "Are there restrictions to the types of numbers that are solutions?" Han can only sell whole-number subscriptions.
- "Is this always the case or just with some scenarios?" Only some scenarios are restricted to specific values. A common occurence of this is when the scenario requires the counting of a certain item.


## Differentiated Support

## Accessibility: Optimize Access to Technology

Have students use the Amps slides for this activity, in which they can see their classmate's responses after they submit their own response.

## Extension: Math Enrichment

Have students complete the following problem:
If Han can sell subscriptions for two weeks, how would the inequality and solution change? The inequality would become $50+3 n \geq 67$ and the solution would be $n \geq 5 \frac{2}{3}$, which means that Han needs to sell at least 6 subscriptions.

## Math Language Development

## MLR5: Co-craft Questions

During the Launch, reveal the introductory text and ask students to work with their partner and to write $2-3$ mathematical questions they could ask about this situation. Have volunteers share their questions with the class. Listen for and amplify questions students write that use the phrase at least.
Sample questions shown.

- How much do the soccer cleats cost?
- If Han sells 10 subscriptions this week, how much will he earn?
- Does Han need to earn exactly the same amount as the cost of the soccer cleats, at most this amount, or at least this amount?


## English Learners

Consider showing an image of soccer cleats to help students understand what this term means.

## Activity 2 Earning More Money for Soccer Apparel

## Students solve an inequality (rational solutions) by writing a related equation first to answer questions about a real-world scenario.



## 1 Launch

Set an expectation for the amount of time students will have to work in pairs, or small groups, on the activity.

## 2 Monitor

Help students get started by asking, "What do you know about Elena?" and "What do you need to know?"

## Look for points of confusion:

- Thinking at most means greater than or equal to. Ask, "Would Elena be within budget if she spent \$4 per pair or \$2 per pair?"


## 3 Connect

Have students share their solutions and strategies on which inequality to use.

Highlight similarities and differences among Han's and Elena's scenarios. Testing values will always help determine what the solutions are, but students can also reason about the scenario (MP1, MP2). If Elena wants to spend less money, she should spend less on each pair of socks.

## Ask:

- "Can Elena spend exactly $\$ 3.01$ on a pair of socks?"
- "Are there restrictions to these values like there were with Han's subscriptions?" No. In this situation, the variable represents money, which can be decimals to the hundredths place. In Han's situation, the variable represented the number of magazine subscriptions, which are restricted to whole numbers.


## Differentiated Support

## Accessibility: Optimize Access to Technology

Have students use the Amps slides for this activity, in which they can see their classmate's responses after they submit their own response.

## Extension: Math Enrichment

Have students complete the following problem:
If Elena selects socks that cost $\$ 4$ per pair, how much can she spend on the pair of shorts, if her budget remains the same? At most $\$ 15$; Let $s$ represent the cost of the pair of shorts; $5(4)+s \leq 35 ; s \leq 15$.

## Math Language Development

## MLR5: Co-craft Questions

During the Launch, reveal the introductory text and ask students to work with their partner and to write 2-3 mathematical questions they could ask about this situation. Have volunteers share their questions with the class. Listen for and amplify questions students write that use the phrase at most. Sample questions shown.

- How much do the pairs of socks cost?
- Can Elena spend exactly $\$ 35$, at least $\$ 35$, or at most $\$ 35$ ?
- How much can Elean spend on each pair of socks?


## English Learners

Be sure students understand that a "pair of shorts" represents one quantity, not two.

Review and synthesize how to interpret and solve inequalities that represent real-world situations (MP2).


## Synthesize

Display the inequality symbols on the board and write common phrases used for each.

Have students share strategies they use to determine which inequality symbol to use.

Highlight that substituting values into the inequality will always tell students the direction of the solutions to the inequality, and that reasoning through the language of the problem is a way to ensure that the solutions in context make sense (MP1).

Ask, "Which phrases do you find most challenging to understand?" Address any concerns presented by the students.

Reflect
After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the Reflect space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Which strategies that worked for solving simple equations or inequalities can be put to use when solving more complex ones?"

Students demonstrate their understanding of solving an inequality by first solving a related equation, and then testing values on either side of the boundary value.


Name:
Date: $\qquad$ Period:

## Exit Ticket

It is currently 20 degrees outside, and the temperature is dropping
3 degrees every hour. The temperature after $h$ hours is $20-3 h$.

1. Explain what the equation $20-3 h=0$ represents.

It represents when the temperature will be $\mathbf{0}$ degrees.
2. What value of $h$ makes the equation true?
$20-3 h-20=0-20$
$-3 h \div(-3)=-20 \div(-3)$
$h=6 \frac{2}{3}$
3. Explain what the inequality $20-3 h<0$ represents. It represents when the temperature will be below 0 degrees.
4. What values of $h$ make the inequality true?

Answers may vary, but must be greater than $6 \frac{2}{3}$.


## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

## $\mathrm{O}_{0}$ Points to Ponder . .

- What worked and didn't work today? In this lesson, students wrote and solved inequalities of the form $p x+q>r$ and $p x+q<r$. How did that build on the earlier work students did with writing and solving equations of the form $p x+q=r$ ?
- 7.EE.B.4.b asks students to interpret the solution set of an inequality in the context of a problem. Where in your students' work today did you see or hear evidence of them doing this? What might you change for the next time you teach this lesson?


## Success looks like . . .

- Goal: Interpreting inequalities representing situations with a constraint.
» Interpreting the inequality $20-3 h<0$ in the context of temperature in Problem 3.
- Goal: Solving an equation of the form $p x+q=r$ to determine the boundary value for an inequality of the form $p x+q>r$ or $p x+q<r$.
» Determining the boundary value $h$ that makes the equation true in Problem 2.
- Language Goal: Using substitution or reasoning about the context to justify whether the values making an inequality true are greater than or less than the boundary value. (Speaking and Listening)


## - Suggested next steps

If students solve the equation correctly but solve the inequality incorrectly, consider:

- Reminding them to test values on either side of the boundary value. The side where the values make the inequality true is the solution.
- Assigning Practice Problems 1 and 2.

If students have difficulty with the process of solving a related equation, consider:

- Assigning Practice Problem 3.


| Practice Problem Analysis |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Type | Problem | Refer to | Standard(s) | DOK |
| On-lesson | 1 | Activities 1 and 2 | 7.EE.B.4.B | 2 |
|  | 2 | Activities 1 and 2 | 7.EE.B.4.B | 2 |
|  | 3 | Activities 1 and 2 | 7.EE.B.4.B | 2 |
| Spiral | 4 | Unit 6 Lesson 14 | 7.EE.B.4.B | 1 |
|  | 5 | Unit 4 <br> Lesson 5 | 7.RP.A. 3 | 1 |
| Formative 0 | 6 | Unit 6 Lesson 16 | 7.EE.B.4.B | 1 |

© Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Efficiently Solving Inequalities

## Let's solve more complicated inequalities.



## Focus

## Goals

1. Language Goal: Compare and contrast solutions to equations and solutions to inequalities. (Speaking and Listening)
2. Draw and label a graph on the number line that represents all the solutions to an inequality.
3. Language Goal: Generalize the solutions of an inequality of the form $p x+q>r$ or $p x+q<r$ by solving the equation $p x+q=r$ and then testing a value to determine the direction of the inequality in the solution. (Speaking and Listening)

## Coherence

## - Today

Students solve inequalities of the forms $p x+q<r$ and $p(x+q)<r$ by first writing and solving a related equation. Then they test values to determine the direction of the inequality in the solution (MP1).

## < Previously

In Lesson 14, students solved inequalities of the forms $p x<q$ and $x+p<q$ by writing and solving a related equation and testing values to determine the direction of the inequality in the solution.

## Coming Soon

In Lesson 17, students will solve word problems by writing inequalities of the forms $p x+q<r$ and $p(x+q)<r$ and solving them using the methods addressed in today's lesson.

## Rigor

- Students solve inequalities and test solutions to develop their conceptual understanding of graphing the solutions of an inequality on a number line.
- Students develop procedural fluency in solving and graphing the solutions of an inequality.


## Standards

## Addressing

## 7.EE.B.4.B

Solve word problems leading to inequalities of the form $p x+q>r$ or $p x+q<r$, where $p, q$, and $r$ are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

## Building On

## Building Toward

6.EE.B. 5

HSA.REI.B. 3
6.EE.B. 8
7.EE.B.4.A


For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

## Practice $\bigcirc$ Independent

## Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, Inequalities (for display, as needed)
- Anchor Chart PDF Inequalities (answers)


## Math Language <br> Development

## Review words

- inequality
- solution to an equation
- solutions to an inequality


## Building Math Identity and Community <br> Connecting to Mathematical Practices

Students might impulsively solve an inequality just like they would solve an equation but without considering the special cases required of inequalities with signed numbers (MP1). Encourage students to write anything extra that they need to remember when solving an inequality at the top of the page. After they have solved all of the inequalities, they need to persevere and go back to look at each case making sure that they did not forget to apply the additional steps.

## Amps Featured Activity

## Activity 1 <br> Dynamic Number Lines

Students can represent solutions to inequalities on digital numbers lines. You can view their responses in real time.


## - Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In Activity 1, Problem 2 may be omitted.
- In Activity 2, Problem 1 may be omitted.

Students make a prediction about an inequality with a negative coefficient and then test values to check their prediction. This helps prepare them for thinking about solving inequalities with negative coefficients.


## 1 Launch

Set an expectation for the amount of time students have to work independently on the activity.
(2) Monitor

Help students get started by asking, "How do you represent solutions to inequalities on the number line? and How can you test if a value is a solution to an inequality?"

Look for points of confusion:

- Ignoring the negative sign on the variable when selecting values that are solutions. Remind students that $-x$ is the same as $-1 \cdot x$ or $-1 x$. Have students substitute the given values of $x$ into the inequality and check if it is still true (MP6).


## 3 Connect

Have students share which of the given values they predicted were solutions. Select some students to explain their thinking for each value and others to share how their predictions differed from their final solutions.

Highlight how negative values in the inequality can make it difficult to predict the solutions. Review the strategy of first determining the value for which both sides are equal, and then testing values to determine the direction of the solutions to the inequality.

Ask, "How are the solutions to $-x \geq-4$ different from the solutions to $x \geq 4$ ?"

## (1) <br> Power-up

To power up students' ability to determine what value make more complex inequalities true, have students complete:

Select all values that make the inequality $2 x+8>6$ true .
(A. 1
C. 1.5
E. -1
B. 0
(D.) 2
(F.) 12

Use: Before Activity 1
Informed by: Performance on Lesson 15, Practice Problem 6

15 min
Activity 1 Which Side Has the Solutions?
Students solve inequalities by solving related equations and testing solutions and by formalizing a process for solving inequalities.

## Amps Featured Activity Dynamic Number Lines

Activity 1 Which Side Has the Solutions?
$>1$. Let's investigate the inequality $-4 x+5 \geq 25$.
a Solve the equation $-4 x+5=25$, and place an open circle at the solution on the number line below.

$$
\begin{aligned}
-4 x+5-5 & =25-5 \\
-4 x & =20 \\
-4 x \div(-4) & =20 \div \div(-4) \\
x & =-5
\end{aligned}
$$

b Is the inequality $-4 x+5 \geq 25$ true when:

- $x$ equals the solution to the equation $-4 x+5=25$ ? Explain your thinking Yes; The solution is $x=-5$, and $x=-5$ makes the inequality true.
- $x$ is greater than the solution to the equation? Explain your thinking. No; $\mathbf{0}$ is greater than the solution, and $x=\mathbf{0}$ doesn't make the inequality true.
- $x$ is less than the solution to the equation? Explain your thinking. Yes; -10 is less than the solution, and $x=-10$ makes the inequality true.

Complete the graph to show the solutions to the inequality $-4 x+5 \geq 25$ on the number line. Then write an inequality to represent the solution.


Solution: $x \leq-5$

## 1 Launch

Conduct the Think-Pair-Share routine. Give students 5 minutes of independent work time. Then give pairs of students time to share their responses and reasoning with each other.

## 2 Monitor

Help students get started solving the related equations by reviewing the process of subtracting the constant from both sides and dividing by the coefficient.

## Look for points of confusion:

- Struggling to select values greater than or less than the solutions in Problem 1b. Rephrase the problem and ask students to select a number to the left and to the right on the number line (MP1).
- Struggling to use their responses to part b to help them complete part c. Remind students of their work in Lesson 14 and have them consider which values will make the inequality true.
- Making errors in labeling the number line when graphing the solution. Suggest they start in the middle with the boundary value and then label the tick marks to the left and right of this value.


## Activity 1 continued >

## Differentiated Support

## Accessibility: Optimize Access to Technology

Have students use the Amps slides for this activity, in which they can represent solutions to inequalities on digital numbers lines. You can view their responses in real time.

## Accessibility: Vary Demands to Optimize Challenge

Have students focus on Problem 1 and then review their responses together before they move on to Problem 2.

## Math Language Development

## MLR8: Discussion Supports-Press for Details

During the Connect, as you discuss how students chose the values to test for each inequality, press them for details in their reasoning. For example:

## If a student says . . .

"I chose to test the values 1.7 and 1.9." (Problem 2)

## Press for detail by asking .

"Why did you choose these values? Are there different values you could have chosen? Are some values less challenging to use than others?"

## English Learners

Annotate the number lines with how they illustrate whether the boundary value is/is not a solution and whether values on each side of the boundary value are/are not solutions.

Students solve inequalities by solving related equations and testing solutions and by formalizing a process for solving inequalities.


## 3 Connect

Have students share how they graphed the solution to each inequality and how they wrote the solution in the form of an inequality, based on the graph.

Highlight how students determined the boundary value, their process for testing numbers on either side to determine which side has the values that make the inequality true, and how to represent the solution on the number line. Discuss how students chose the values to test (part b), emphasizing that since they can select any value, they should choose convenient ones.

Ask, If someone asked for your help with how to solve an inequality, what would you tell them?"

## Students practice solving inequalities by solving related equations and testing solutions.

Activity 2 Solving Inequalities

1. Consider the inequality $\frac{23}{3}<\frac{4}{3} x+3$.

Solve the related equation and test values less than and greater than the solution. Then graph the solution on the number line and write an inequality to represent the solution.
$\frac{23}{3}=\frac{4}{3} x+3$
$\frac{23}{3}-3=\frac{4}{3} x+3-3 \quad$ Sample response:
$\frac{14}{3}=\frac{4}{3} x$
$\frac{14}{3} \div \frac{4}{3}=\frac{4}{3} x \div \frac{4}{3}$
$\frac{7}{2}=x$ Check values less than and greater than $\frac{7}{2}$ $\begin{array}{ll}x=0 & x=6 \\ \frac{23}{3}<\frac{4}{3} \cdot 0+3 & \frac{23}{3}<\frac{4}{3} \cdot 6+3 \\ \frac{23}{3}<3 \text { is not true } & \frac{23}{3}<11 \text { is true }\end{array}$


Solution: $x>\frac{7}{2}$
2. Consider the inequality $-3\left(x-\frac{4}{3}\right) \leq 6$.

Solve the related equation and test values less than and greater than the solution. Then graph the solution on the number line and write an inequality to represent the solution.


Solution: $x \geq-\frac{2}{3}$

## Differentiated Support

## Accessibility: Guide Processing and Visualization

Help students create a checklist that documents the steps for solving an inequality. A sample checklist is shown. Alternatively, provide students with a copy of the Anchor Chart PDF, Inequalities.
$\square$ Write a related equation and solve the equation. The solution is the boundary value.
$\square$ Determine if the boundary value is a solution to the inequality. This will tell you whether to use the $>/<$ or $\geq / \leq$.
$\square$ Test values on either side of the inequality to determine if they are solutions. This will tell you whether to use the symbols $>/ \geq$ or $</ \leq$.Write and graph the solution.

## Math Language Development

## MLR2: Collect and Display

During the Connect, as students respond to the Ask question, "How would you describe to someone how to solve any inequality?," ask them to consider how multiplying or dividing by a negative coefficient affects the solution to an inequality. Collect and display language students use in their response and connect their language to number line diagrams.

## English Learners

Provide students time to record their ideas individually and then share with a partner before sharing with the whole class.

## Review and synthesize how to solve a more complicated inequality using the same reasoning used for solving simpler inequalities.



## Synthesize

Display the Anchor Chart PDF, Solving Inequalities and complete the bottom section as a class.

Highlight the steps for solving an inequality by writing and solving a related equation and then checking whether values less than or greater than the equation's solution make the inequality true. Demonstrate how to graph the solution and write an inequality to represent the solution.

## Ask:

- "How does the equation relate to the inequality?"
- "How do you use the solution to the equation to help you solve the inequality?
- "What are you looking for when you test values less than and greater than the solution to the equation?"
- "What will the graph of the solution look like?"
- "How do you know whether 7 is included in the solution?"
- "How do you determine the inequality you write for the solution?"


## D. Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the Reflect space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Which strategies that worked for solving simple equations or inequalities can be put to use when solving more complex ones?"


## Exit Ticket

Students demonstrate their understanding by determining which of two similar solutions solves each of two inequalities.

## Printable

Name: Date: $\qquad$ Period:

## Exit Ticket

 03Lin solved both inequalities below, but she mixed up her solutions. Help her by deciding whether the solution to each inequality is represented by $x<2.5$ or $x>2.5$. Explain your thinking.

1. $-4 x+5>-5$

Solution: $x<2.5$

Explanation: Answers may vary, but should mention testing values greater and less than 2.5 to determine which makes the inequality true.
2. $-25>-5(x+2.5)$

Solution: $x>2.5$
Explanation: Answers may vary, but should include some mention of testing values greater and less than 2.5 to determine which makes the inequality true.


## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

0 Points to Ponder . .

- What worked and didn't work today? Who participated and who not participate in Activity 1 today? What trends do you see in participation?
What did students find frustrating about Activity 1 ? What helped them work through this frustration? What might you change for the next time you teach this lesson?


## Practice

| Practice Problem | Analysis |  |  |  |
| :--- | :---: | :--- | :--- | :--- | :--- |
| Type | Problem | Refer to | Standard(s) | DOK |
| On-lesson | $\mathbf{1}$ | Warm-up | 7.EE.B.4 | 2 |
| Spiral | $\mathbf{2}$ | Activity 1 | 7.EE.B.4.B | 1 |
| Formative 0 | 6 | Activity 2 | 7.EE.B.4.B | 2 |
|  | $\mathbf{4}$ | Unit 6 <br> Lesson 12 | 7.RP.A.3 | 2 |
| Unit 5 <br> Lesson 4 <br> Unit 6 <br> Lesson 17 | 7.NS.A.1 | 2 |  |  |

(6) Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.
Name: $\square+$ Date: + Period $\square$ _

1. Consider the inequality $-4(x-6)>16$.
a Which numbers are solutions to the inequality?

| A. 2 | (巨.) 1.9 |
| :--- | :--- |
| (B.) -2 | F. 2.1 |
| C. 0 | (.. $\frac{2}{3}$ |
| (D.) -2.1 |  |

b Write two more solutions to the inequality.
Answers may vary, but should be less than 2 .
2. Diego is solving the inequality $100-3 x \geq-50$. He first solves the equation $100-3 x=-50$ and obtains $x=50$. What is the solution to the inequality?
A. $x<50$
(B.) $x \leq 50$
C. $x>50$
D. $x \geq 50$
. Solve each inequality. Show your solution as a graph on the number

42 Unit 6 Expressions. Equations. and Inequalities
A. $2 x-10<48$
B. $2 x+10<48$
C. $2 x-10 \leq 48$
(D.) $2 x+10 \leq 48$
4. The price of a pair of earrings is $\$ 22$, but Priya buys them on sale for $\$ 13.20$.
a How much, in dollars, was the price discounted?
The price was discounted $\$ 8.80$; Sample response: $22-13.20=8.80$
b What was the percent of the discount? Show or explain your thinking
5. Complete the magic square so that the sums of each row, column, and diagonal in the grid are equal.

| 1 | 2 | 6 |
| :---: | :---: | :---: |
| 8 | 3 | -2 |
| 0 | 4 | 5 |

6. You are building a tower with 2 -in. tall blocks on top of a 10 -in. tall base Your tower will topple when it is taller than 48 in. Which inequality represents the number of blocks you can use to build your tower? The percent of the discount was $\mathbf{4 0 \%}$; Sample response: $\frac{8.80}{22} \cdot 100=\mathbf{0 . 4 0}$


# Interpreting Inequalities 

Let's write some inequalities.


## Focus

## Goals

1. Language Goal: Identify the inequality that represents a situation, and justify the choice. (Writing)
2. Language Goal: Present (using multiple representations) the solution method for a problem involving an inequality and interpret the solution. (Speaking and Listening, Writing)

## Coherence

## - Today

Students interpret and solve inequalities that represent real-world situations, making sense of quantities and their relationships in the problem (MP2).

## < Previously

Students wrote and solved equations from scenarios in Lesson 9-11. In Lesson 14, students wrote related equations and solved them to help find the solutions to the inequality.

## > Coming Soon

In Lesson 18, students will begin to focus on the modeling process, starting with a question they want to answer and then independently deciding how they will represent the situation mathematically.

## Rigor

- Students build their procedural fluency in solving and graphing the solutions of inequalities.


## Standards

## Addressing

7.EE.B.4.B

Solve word problems leading to inequalities of the form $p x+q>r$ or $p x+q<r$, where $p, q$, and $r$ are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

| Building On | Building Toward |
| :--- | :--- |
| 6.EE.B.6 | HSA.REI.B. 3 |
| 6.EE.B.8 |  |



For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

## Practice $\bigcirc$ Independent | Slides $x-x$

## Materials

- Exit Ticket
- Additional Practice


## Math Language <br> Development

## Review words

- at least
- at most
- inequality
- solution to an inequality


## Amps Featured Activity

## Warm-up <br> Interactive Inequalities

Students drag and drop values to test whether their inequality works and receive instant feedback.


## Building Math Identity and Community <br> Connecting to Mathematical Practices

Students might become distracted as they try to match inequalities and scenarios in Activity 1. They might not put forth the needed focus to approach the problem with both abstract and quantitative reasoning (MP2). While working in pairs, have students help each other stay focused. Encourage them to explain their thinking to their partner so that they can make sure their reasoning is sound.

## - Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- The Warm-up may be omitted.

Students create their own inequality following certain rules. This helps students reason about the components of an inequality in an abstract way.

Amps Featured Activity Interactive Inequalities

Unit 6 | Lesson 17

Interpreting Inequalities Let's write some inequalities.


Warm-up Mystery Inequalities
Using each of the numbers from 1 to 9 at most once, fill in the boxes so that the solution to the inequality is $x \leq 2$. Repeat as many times as possible to determine different sets of numbers.

| Sample responses: |  |  |  |  |  |  |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- |
| $2,1,3$ | $3,1,5$ | $4,1,7$ | $5,1,9$ | $6,3,9$ | $7,5,9$ | $8,7,9$ |
| $2,3,1$ | $3,2,4$ | $4,2,6$ | $5,2,8$ | $6,4,8$ | $7,6,8$ | $8,9,7$ |
|  | $3,4,2$ | $4,3,5$ | $5,3,7$ | $6,5,7$ | $7,8,6$ |  |
|  | $3,5,1$ | $4,5,3$ | $5,4,6$ | $6,7,5$ | $7,9,5$ |  |
|  |  | $4,6,2$ | $5,6,4$ | $6,8,4$ |  |  |
|  |  | $4,7,1$ | $5,7,3$ | $6,9,3$ |  |  |
|  |  |  | $5,8,2$ |  |  |  |
|  |  |  | $5,9,1$ |  |  |  |

## 1. Launch

Read the directions aloud, to ensure that students understand the expectations for this problem. It may be helpful to give one example of an inequality that satisfies the prompt, such as $2 x-1 \leq 3$.

## 2 Monitor

Help students get started by suggesting they try different numbers and check which work.

## Look for points of confusion:

- Using a number more than once. Ask, "Have you noticed that you've broken one of the rules?"
- Trying to place the 2 to the right of the $\leq$ symbol. Say, "The solution needs to only have an $x$ on the left side. How can you do that?"


## Look for productive strategies:

- Finding a pattern in the solutions. Say, "Explain to me what you are noticing. How has it helped you?" (MP8)


## 3 Connect

Display the problem and a blank chart to be filled in with possible answers.
Have students share their process for selecting numbers with a partner.

Highlight a student that substituted 2 for $x$ in order to make sense of the problem.

Ask, "Once you found one solution, was there a pattern you could use to find other solutions?"

## Differentiated Support

## Accessibility: Optimize Access to Technology

Have students use the Amps slides for this activity, in which they drag and drop values to test whether their inequality works and receive instant feedback.

## Power-up

To power up students' ability to write expressions to represent real-world scenarios, have students complete:

1. It costs $\$ 0.25$ to play each game at a fair. Admission is $\$ 4$. Write an expression to show the total cost of playing $x$ games at the fair. $0.25 x+4$
2. It costs $\$ 0.25$ to play each game at a new arcade. Today, in honor of opening day, the arcade is giving away $\$ 4$ of free tokens. Write an expression to show the total cost of playing $x$ games at the arcade today. $0.25 x-4$
Use: Before Activity 1
Informed by: Performance on Lesson 16, Practice Problem 6

## Students interpret a scenario that leads to an inequality. This activity helps students make sense of the quantities and their relationships.

Activity 1 Matching an Inequality to a Scenario

The Science Club is investigating the effect of a liquid's density on the height of an object floating within that liquid. They place an egg in a $25-\mathrm{cm}$ tall beaker filled with salt water. It floats 5 cm above the bottom. They notice that each time they add a spoonful of salt, the egg rises $\frac{1}{2} \mathrm{~cm}$. How many spoonfuls of salt can be added without the egg reaching the top of the cup?

1. Choose the inequality that best matches the scenario.
A. $25 x+5<\frac{1}{2}$
(B.) $\frac{1}{2} x+5<25$
C. $\frac{1}{2} x+25<5$
D. $5 x+\frac{1}{2}<25$
2. Explain what each part of the inequality represents.
$x$ represents the number of spoonfuls of salt, $\frac{1}{2}$ is the height in centimeters that the egg rises for each additional spoonful of salt, the egg started at 5 cm from the bottom of the beaker, and 25 is the maximum height in centimeters.
3. Solve for $x$, graph the solution, and write an inequality to represent the solution. Show your work.

4. Explain what the solution means in terms of the scenario. The solution of $x<40$ means that as long as less than 40 spoonfuls of salt are added, the egg will not reach the top of the cup.

## 1) Launch

Activate students' background knowledge by asking, "Has anyone noticed that it is easier to float in the ocean than in a pool? Why do you think that is?" The salt in the ocean makes it easier for objects (and people) to float.

## 2 Monitor

Help students get started by asking, "What quantity could be represented by the variable in this scenario?"

## Look for points of confusion:

- Assuming one quantity will always be on the opposite side of the variable. Allow for this conjecture and ask students to re-evaluate their thinking at the end of the lesson.


## Look for productive strategies:

- Expressing the solution in words or by graphing on a number line. Applaud student use of these representations while encouraging them to express the solution using the efficient algebraic notation.


## 3 Connect

Display one student's solution to Problem 3.
Have students share what each quantity and variable represent in the original inequality. Annotate the inequality as the student explains.
Highlight what the solution to the inequality represents in the scenario (MP2).

Ask, "What does it mean for $x$ to be less than 40 ?"

## Differentiated Support

## Accessibility: Vary Demands to Optimize Challenge, Guide Processing and Visualization

Instead of asking students to select the correct inequality for Problem 1, provide them with the correct inequality and ask them to explain how it matches the scenario. Provide access to colored pencils and suggest students color code key words and phrases from the text and how they are represented in the inequality.

## Math Language Development

## MLR6: Three Reads

Use this routine to help students make sense of the introductory text

- Read 1: Students should understand that an egg is floating in a beaker of salt water.
- Read 2: Ask students to name or highlight the given quantities and relationships, such as each time a spoonful of salt is added, the height of the egg in the water increases by $\frac{1}{2} \mathrm{~cm}$.
- Read 3: Ask students to identify what the unknown amount should represent in this context.


## English Learners

Draw a picture representing this context showing an egg floating in a beaker. Then draw a new picture showing as salt is added, the egg rises.

Students are now asked to write and solve their own inequality to match a scenario. This is a gradual release of support from Activity 1 to prepare students for the Exit Ticket.

## Activity 2 Writing an Inequality

 for a ScenarioThe Chemistry Club is experimenting with different mixtures of water and a chemical called sodium polyacrylate to make fake snow.

Each mixture starts with some amount of water, measured in grams. The amount of the chemical used in the mixture is $\frac{1}{7}$ of the amount of water used, plus 9 more grams of the chemical. The chemical is expensive, so there must be less than $\mathbf{5 0 g}$ of the chemical in any one mixture. How much water can the students use in the experiment?
>1. Describe the unknown amount that the variable $x$ will represent. $x$ represents the amount of water, measured in grams.
2. Write an inequality that represents the scenario, graph the solution, and write an inequality to represent the solution.
${ }_{7}^{1} x+9<50$ 1

Check values less than and greater than 287 .
$\frac{1}{7} x+9-9=50-9$
$x=0$
$=0 \quad x=700$
$\frac{1}{7} x=41$
$\frac{1}{7} x \div \frac{1}{7}=41 \div \frac{1}{7}$
$\frac{1}{7} \cdot 700+9<50$
$x=287$
Solution: $x<287$
> 3. Explain what the solution means in terms of the scenario. The solution $x<287$ means that the students can use any amount of water that is less than 287 g in the experiment.


Three Reads: Read the introductory information three times.
. Make sense of the
scenario.
What mathematical
quantities are given?
to solve the problem

646 Unit 6 Expressions. Equations, and Inequarites

## (1) Launch

Have the students in each pair take turns reading the scenario to each other. Then ask students to each write their inequality independently before comparing with their partner.

## 2 Monitor

Help students get started by suggesting they read the scenario backwards, starting with the last sentence and finishing with the first.

Look for points of confusion:

- Representing the scenario with $\frac{1}{7}+\mathbf{9 x}<\mathbf{5 0}$. Ask, "What does it mean to have $\frac{1}{7}$ of an amount? Do we know what that amount is?"
- Using " $\leq$ ". Ask, "Can the Chemistry Club use exactly 50 grams of the chemical? How do you know?"
- Thinking that the solution represents the amount of chemical in the mixture. Ask, "What did you say the variable represented when you read the scenario?"


## 3 Connect

Display one student's solution to Problem 2.
Have students share what each quantity and variable represent in their original inequality (MP2). Annotate the inequality as the student explains.

Highlight how the inequality and solution relate to the scenario.

## Ask:

- "How did you determine what the $\frac{1}{7}$ term represents?"
- "How did you decide on the direction of the inequality for the solution?"
- "What does it mean that $x$ is less than 287 ?"


## Differentiated Support

## Accessibility: Guide Processing and Visualization, Activate Prior Knowledge

Before students begin, ask them to explain in their own words what it means that the amount of the chemical is $\frac{1}{7}$ the amount of the water. Connect this relationship to their prior understanding of ratios. Have them complete the following statements.

- For every 1 gram of water, there are $\qquad$ grams of the chemical.
- For 7 grams of water, there are __ grams of the chemical.
- For 14 grams of water, there are __ grams of the chemical.
- For $x$ grams of water, there are __ grams of the chemical.


## Math Language Development

## MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

- Read 1: Students should understand that water is mixed with a chemical to make fake snow. Tell them they do not need to worry about how to pronounce the chemical name.
- Read 2: Ask students to name or highlight the given quantities and relationships, such as the amount of the chemical used is $\frac{1}{7}$ of the amount water used plus 9 more grams of the chemical.
- Read 3: Ask students to identify what the unknown amount should represent in this context.

Review and synthesize how inequalities can represent and help solve real-world problems.

## Synthesize

Display the following, "Suppose your friend asks you to write some practice problems for solving inequalities. You want to write an inequality that has a solution of $x \leq-8 \frac{2}{3}$. Describe how to write such an inequality."

Have students share with a partner how they would write such an inequality. Circulate and note the different strategies students use.

Highlight that there are many approaches to writing such an inequality. As students share different approaches, pause the class and highlight each one.

Ask, "How many different inequalities can be written with this solution?" An infinite number.

## (1) Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the Reflect space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is writing and solving inequalities the same or different from writing and solving equations?"
- "What strategies did you use when determining how to graph the solutions of an inequality?

Students demonstrate their understanding by explaining how each of the each of the terms, and the solution to, a given inequality relate to a scenario (MP2).


Name:
Date:
Period:

## Exit Ticket

Andre is making paper cranes to decorate for a party. He plans to make one larger paper crane as a centerpiece and several smaller ones to place around the table. It takes Andre 10 minutes to make the centerpiece and 3 minutes to make each small crane. He will only have 30 minutes to make them once he gets home.

1. Andre wrote the inequality $3 x+10 \leq 30$ to plan his time. Describe what $x, 3 x, 10$, and 30 represent in this inequality. $x$ represents the number of small cranes Andre can make and $3 x$ represents the total time it takes to make the small cranes. 10 represents the number of minutes it takes to make the larger crane. $\mathbf{3 0}$ represents the maximum amount of time Andre has to make his cranes.
2. Solve Andre's inequality, graph the solution, and explain what the


Explanation: Andre has enough time to make at most $6 \frac{2}{3}$ small cranes. Because $\frac{2}{3}$ of a crane would not be useful, Andrew really only has enough time to make 6 small cranes.


## Professional Learning

## Math Language Development

Language Goal: Presenting (using multiple representations) the solution method for a problem involving an inequality, and interpreting the solution.
Reflect on students' language development toward this goal.

- How have students progressed in this unit toward
- Making sense of real-world problems that involve equations or inequalities?
- Defining variables to represent the unknown quantities?
- Interpreting the solutions to their equations or inequalities within the context of the problem?


## Practice



| Practice Problem | Analysis |  |  |  |
| :--- | :---: | :--- | :--- | :---: | :---: |
| Type | Problem | Refer to | Standard(s) | DOK |
| On-lesson | $\mathbf{1}$ | Activity 1 | 7.EE.B.4.B | 2 |
| Spiral | $\mathbf{2}$ | Activity 2 | 7.EE.B.4.B | 1 |
| Formative $\mathbf{0}$ | $\mathbf{6}$ | Activity 2 | 7.EE.B.4.B | 2 |
|  | $\mathbf{4}$ | Unit 4 <br> Lesson 9 <br> Unit 6 | 7.RP.3 | 1 |

(1) Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## (2)

4. You are at a skateboard shop browsing items. Solve each problem.
(a) The price tag on a shirt reads $\$ 12.58$. Sales tax is $7.5 \%$ of the price.
How much will you pay for the shirt? Show or explain your thinking $\$ 13.52 ; 12.58 \cdot 1.075=13.5235$
b The shop manager buys a helmet for $\$ 19.00$ and sells it for $\$ 31.50$ What percent was the markup? Show or explain your thinking About $66 \% ; \frac{31.50-19.00}{19.00} \cdot 100 \approx 66$
c A shop pays workers $\$ 14.25$ per hour, plus $5.5 \%$ commission. If someone works 18 hours and sells $\$ 250$ worth of merchandise, what is the total amount of their paycheck for this pay period? Show or explain your thinking. 270.25: 14.25 $\cdot 18+0.055 \cdot 250=270.25$
5. Match each scenario with the inequality that could represent it. Scenario Inequality
a Han got $\$ 2$ from Clare, but still has
(b) Mai spent $\$ 2$ and now has less than $\$ 20 . \quad$ a $x+2<20$
c If Tyler had twice the amount of money
he currently has, he would still have less than $\$ 20$. b $\quad x-2<20$
d If Priya had half the money she currently has, she would have less than $\$ 20$.
( 6. Diego is buying juice packs for his party this weekend. Each pack of juice comes with 6 cartons, and he knows he wants enough to serve at least 16 people, so he writes the inequality $6 j \geq 16$. After solving, he determines the solution is $j \geq 2 \frac{2}{3}$. Is it possible for Diego to purchase $2 \frac{2}{3}$ juice packs? How many would you tell him to buy?
Diego cannot buy $2 \frac{2}{3}$ packs of juice. He should instead purchase a whole number of packs of juice greater than or equal to 3 .

## Additional Practice Available



For students that need additional practice in this lesson, assign the Grade 7 Additional Practice.

## Modeling With Inequalities



## Focus

## Goals

1. Language Goal: Critique the solution to an inequality, including whether fractional or negative values are reasonable. (Speaking and Listening)
2. Determine what information is needed to solve a problem involving a quantity constrained by a maximum or minimum acceptable value.
3. Write and solve an inequality of the form $p x+q>r$ or $p x+q<r$ to solve a problem about a situation with a constraint.

## Coherence

## - Today

In this lesson, students determine if their solutions are reasonable within context of the scenarios they represent. This lesson focuses on the modeling process, in which students start with a question they want to answer and independently decide how they will represent the situation mathematically (MP2, MP4).

## < Previously

In Lesson 16 and 17, students wrote and solved inequalities of the form $p x+q>r$ and $p(x+q)<r$.

## > Coming Soon

Students will continue their work with inequalities in Grade 8 when they solve linear inequalities.

## Rigor

- Students continue to build conceptual understanding of solutions to inequalities by analyzing real-world scenarios.
- Students develop procedural fluency in solving and graphing solutions to inequalities through an Info Gap routine.


## Standards

## Addressing

## 7.EE.B.4.B

Solve word problems leading to inequalities of the form $p x+q>r$ or $p x+q<r$, where $p, q$, and $r$ are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

| Building On | Building Toward |
| :--- | :--- |
| 6.EE.B. 5 | HSA.REI.B. 3 |
| 6.EE.B.8 |  |
| 7.EE.B.4.A |  |




Activity 2


Summary


Exit Ticket

| (1) 5 min | (1) 10 min | (1) 20 min | (1) 5 min | (1) 5 min |
| :---: | :---: | :---: | :---: | :---: |
| ํํํ Pairs |  | $\bigcirc \cap ํ ำ$ Pairs | กำกำก Whole Class | $\bigcirc$ ค Independent |
| MP2 | MP2, MP4 | MP4 | MP4 |  |
| 7.EE.B.4.B | 7.EE.B.4.B | 7.EE.B.4.B | 7.EE.B.4.B | 7.EE.B.4.B |

Amps powered by desmos ! Activity and Presentation Slides
For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

## Practice $\bigcirc$ Independent

## Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- Activity 1 PDF (answers)
- Activity 2 PDF, pre-cut cards, one set per pair


## Math Language <br> Development

## Review words

- inequality
- solution to an inequality


## Building Math Identity and Community Connecting to Mathematical Practices

When working with mathematical models, students must make sure that they are appropriate for the scenario, otherwise, the model is completely ineffective (MP4). The effectiveness of the model is evaluated after it has been applied by considering whether the solution is discrete or continuous and whether the answer needs to be rounded. Discuss how students evaluate their life decisions and why the reflection process is important.

## Amps Featured Activity

## Exit Ticket <br> Real-time Exit Ticket

Check in real time if your students can correct errors in an inequality using a digital Exit Ticket.


## - Modifications to Pacing

You may want to consider this additional modification if you are short on time.

- In Activity 2, have students only complete Problem 1 from the Info Gap.

Students read a real-world scenario and determine which solutions are possible based on the context. This will begin a discussion about realistic solutions.


## (1) Launch

Set an expectation for the amount of time students have to work in pairs on the activity.

## 2 Monitor

Help students get started by asking, "Can a fraction of a sandwich be ordered? Can the stage manager order a negative number of sandwiches?"

Look for points of confusion:

- Thinking that the value 14 is a solution because 13.86 rounds up. Model how to substitute 14 into the inequality showing that it is not a solution.


## (7) Power-up

To power up students' ability to determine appropriate solutions based on the context of a scenario, have students complete:
Noah needs to buy at least 3 packs of pencils at the store to have enough to last the school year.

Select all of the possible numbers of packs of pencils he could buy at the store.
(A.) 3 packs
B. 3.5 packs
C. 4 packs
D. $5 \frac{1}{3}$ packs

Use: Before the Warm-up
Informed by: Performance on Lesson 17, Practice Problem 6

10 min
Activity 1 Loading an Elevator
MP2, MP4
7.EE.B.4.B

Students write and solve an inequality to represent a real-world problem, and consider what solutions are realistic in context (MP2).

## Activity 1 Loading an Elevator

A mover is loading an elevator with many identical $48-\mathrm{lb}$ boxes.
The mover weighs 185 lb . The elevator can carry at most $2,000 \mathrm{lb}$.

1. Write an inequality that shows the mover would not overload the elevator on a particular ride.
$48 x+185 \leq 2000$, where $x$ is the number of boxes.
2. Solve your inequality and graph the solution on a number line.

|  |  | Sample response: |  |
| ---: | :--- | ---: | :--- |
| $48 x+185$ | $\leq 2000$ |  |  |
| $48 x+185$ | $=2000$ |  |  |
| $48 x+185-185$ | $=2000-185$ |  | $x=0$ |
| $48 x$ | $=1815$ |  | $48 \cdot 0+185 \leq 2000$ |

Solution: $x \leq 37.8125$

3. The mover asks, "How many boxes can I load on this elevator at a time?" What do you tell them?
Sample response: The mover can load at most 37.8125 boxes. However, it is unrealistic to have a fraction of a box; therefore, the mover only can load a whole number of boxes between 0 and 37 .

## 1) Launch

Ask students to close their books and display the Activity 1 PDF. Give pairs of students a few moments to brainstorm what information they need in order to answer the question (MP4). After students share what missing information is needed, have them open their books and read the scenario for the Activity.

## 2 Monitor

Help students get started by asking, "Can the mover put one box on the elevator? Would that be efficient?"

## Look for points of confusion:

- Thinking it is possible to have 38 boxes in the elevator. Have students substitute the value into the inequality and determine it is not a solution.


## 3 Connect

Have students share strategies for solving the inequality.
Highlight modeling the scenario with the inequality and how the related equation helps solve the inequality (MP2).

## Ask:

- "How can you represent the solution on a number line? Is 5.5 a solution?" Sample response: It is not a solution in the context of this problem because it doesn't make sense to have half a box.
- "Do you want to change the number line somehow to show this?" Sample response: I could plot points or I could simply leave it as is, but just know that for a problem with this context, I will only use integer solutions.
- "How did you know which way to round?" Sample response: I should round down, otherwise the mover has gone over the weight limit.
- "What other limitations do the contexts place on the solutions?" Sample response: There must be a positive number of boxes.


## Differentiated Support

## Accessibility: Guide Processing and Visualization

To help students make sense of the introductory text, ask these questions before they begin the activity. Then distribute the Activity 1 PDF for students to record all of the possibilities.

- "Can the mover take all 48 boxes in one load? Why or why not?"
- "Can the mover take 10 boxes in one load? More than 10 ?"
- "Can the mover take 24 boxes in one load? More than 10 ?"


## Extension: Math Enrichment

Have students complete the following problem:
If there were 140 boxes to move, how many trips would it take? 4 trips; 140 divided by 37 is about 3.7 , which means 4 trips are needed.

## Math Language Development

## MLR7: Compare and Connect

During the Connect, as you highlight how the inequality models the scenario, display the scenario and its related inequality. Ask the following questions. As students respond, annotate or color code the key words and phrases in the text with how they are represented in the inequality.
Ask, "Where do you see ...

- "The unknown? What does it represent?"
- "The weight constraint of the elevator in the text and in the inequality? Why was this particular inequality symbol used?"
- "The weight of the mover in each representation? Why is it added?"
- "The weight of each box? Why is it multiplied by the unknown?"

Activity 2 Info Gap: Giving Advice
Students set up and solve inequalities representing real-world scenarios. They use the context of the
scenario to interpret the solutions.

Activity 2 Info Gap: Giving Advice

You will be given either a problem card or a data card. Do not show or read your card to your partner.
If you are given a problem card:

If you are given a data card: 1. Silently read your card and think about
what information you need to be able to solve the problem
2. Ask your partner for the specific information that you need
3. Explain how you are using the information to solve the problem.
Continue to ask questions until you have enough information to solve the problem.
4. Share the problem card and solve the problem independently in the space below.
5. Read the data card and discuss your reasoning.

1. Silently read your card.
2. Ask your partner "What specific information do you need?" and wait for them to ask for information
3. Before sharing the information, ask "Why do you need that information?" Listen to your partner's reasoning and ask clarifying questions
4. Read the problem card and solve the problem independently in the space below.
5. Share the data card and discuss your reasoning.

Pause here so your teacher can review your work. You will be given a new set of cards and repeat the activity, trading roles with your partner.

| Problem 1 | Problem 2 |
| :---: | :---: |
| Let $x$ be the number of loads. <br> Then $-1.65 x+50 \geq 15$. $\begin{aligned} -1.65 x+50 & =15 \\ -1.65 x+50-50 & =15-50 \\ -1.65 x & =-35 \\ -1.65 x \div(-1.65) & =-35 \div(-1.65) \\ x & \approx 21.21 \end{aligned}$ <br> For $x=0$, the inequality $-\mathbf{1 . 6 5 \cdot 0}+\mathbf{5 0} \geq 15$ is true. <br> Solution: $x \leq 21.21$ <br> They can do at most 21.21 loads; however, it is unrealistic to do a fraction of a load. Therefore, they can do a whole number of loads between 0 and 21. | Let $w$ be the width. Then $2 w+14 \leq 65$. $\begin{aligned} 2 w+14 & =65 \\ 2 w+14-14 & =65-14 \\ 2 w & =51 \\ 2 w \div 2 & =51 \div 2 \\ w & =25.5 \end{aligned}$ <br> For $x=\mathbf{0}$, the inequality is true. <br> Solution: $x \leq 25.5$ <br> The width can be no longer than 25.5 cm ; however, it is unrealistic to have a negative width. Therefore, the width can be between 0 and 25.5 cm . |

## 1 Launch

Distribute a set of cards from Activity 2 PDF to each pair of students. Conduct the Info Gap routine.

## 2 Monitor

Help students get started by reminding students they can represent their situation using words, an inequality, and a graph. They also need to determine what the variable represents (MP4).

## Look for points of confusion:

- Calculating the area instead of perimeter for Problem Card 2. Remind students that the term border implies a distance (length) around the outside.
- Not remembering how to determine the perimeter or not remembering there are two lengths. Have them draw a picture of a rectangle and label the length as 7 cm and the width as the unknown quantity.


## 3 Connect

Highlight that some scenarios can only have discrete solutions. For instance, Noah cannot do 2.5 loads of laundry; he can only do whole numbers of loads. Some scenarios will have continuous solutions. For instance, Elena can make the width any amount between 0 and 25.5 cm .

## Ask:

- "In Noah's problem, should you round up or down? Why?" Down; Noah does not have enough money to do 3 loads.
- "Do you need to round for Elena's problem? Why or why not?" No; the width does not have to be a whole-number value.


## Differentiated Support

## Accessibility: Guide Processing and Visualization

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. Consider using the following during the think-aloud.

- "I know the family wants to keep a minimum balance on the card, but I don't know what that is. I will ask for this amount."
- "I need to determine how many loads of laundry Noah's family can do before needing to add money to the card, but I don't know how much money is already on the card. I will ask for this amount."


## Math Language Development

## MLR4: Information Gap

Display prompts for students who benefit from a starting point, such as:

- "Can you tell me ... (specific piece of information)?"
- "Why do you need to know . . . (that piece of information)?"


## English Learners

Consider providing sample questions students could ask, such as the following for Problem Card 1:

- "How much does a load of laundry cost?"
- "How much money is currently on the card?"


## Review and synthesize how to model real-world situations with inequalities (MP4).

## Synthesize

Display the scenario, "Andre is saving money to purchase something and needs at least \$100. He already has $\$ 20$ in his piggy bank and earns $\$ 7$ each week in allowance."

Ask students what information needs to be decided or what steps need to be completed. For example, students need to define a variable, write an inequality, solve the inequality, and interpret the solution within the context of the scenario.

Highlight that possible solutions to a scenario are different than the mathematical solutions. For instance, some solutions may only be positive whole-number values (number of people). Other scenarios may have continuous solutions (length of a rope).

## (1) Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the Reflect space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "When it was your turn for the "Problem" card, how did you decide which questions to ask?"
- "When it was your turn for the "Data" card, how did you decide what information you should share with your partner?"

Students demonstrate their understanding by critiquing Elena's inequality and solution to a real-world problem.


## Success looks like

- Language Goal: Critiquing the solution to an inequality, including whether fractional or negative values are reasonable. (Speaking and Listening)
» Explaining why Elena's solution is not correct in Problem 1.
- Goal: Determining what information is needed to solve a problem involving a quantity constrained by a maximum or minimum acceptable value.
- Goal: Writing and solving an inequality of the form $p x+q>r$ or $p x+q<r$ to solve a problem about a situation with a constraint.
» Correcting Elena's solution to the inequality in Problem 2.


## Suggested next steps

## If students do not find Elena's mistake with the inequality, consider:

- Reviewing the Summary for Lesson 15.
- Discussing examples of inequality phrases.

If students cannot explain the meaning of the parts of the inequality, consider:

- Having students identifying the meaning of the parts of the inequalities in Practice Problems 1, 2, and 3.


## Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.
O. Points to Ponder ...

- What worked and didn't work today? How did students model with mathematics today (MP4)? How are you helping students become aware of how they are progressing in this area?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?


## Practice



| Practice Problem | Analysis |  |  |  |
| :--- | :---: | :--- | :--- | :--- |
| Type | Problem | Refer to | Standard(s) | DOK |
| On-lesson | $\mathbf{1}$ | Activity 1 | 7.EE.B.4.B | 2 |
| Spiral | $\mathbf{2}$ | Activity 2 | 7.EE.B.4.B | 2 |
| Formative | 6 | Activity 2 | 7.EE.B.4.B | 2 |
|  | 5 | Unit 6 <br> Lesson 6 | 7.EE.B.4.A | 1 |
| Unit 6 <br> Lesson 14 <br> Unit 6 <br> Lesson 19 | 7.EE.B.4 | 1 |  |  |

(1) Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

## Additional Practice Available



For students that need additional practice in this lesson, assign the Grade 7 Additional Practice.


[^0]:    Reflect:

