Amplify Math
TENNESSEE

Teacher Edition

Algebra 1 | Volume 1

FEATURING Amps POWERED BY desmos

Amplify Math

Algebra 1

Volume 1: Units 1–3

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify Math is based on the Illustrative Mathematics (IM) curriculum. IM 6–8 Math™ was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is © 2017–2019 Open Up Resources. Additional adaptations and updates to IM 6–8 Math™ are © 2019 Illustrative Mathematics. IM 9–12 Math™ is © 2019 Illustrative Mathematics. IM 6–8 Math™ and IM 9–12 Math™ are licensed under the Creative Commons Attribution 4.0 International license (CC BY 4.0). Additional modifications contained in Amplify Math are © 2020 Amplify Education, Inc. and its licensors. Amplify is not affiliated with the Illustrative Mathematics organization.

Desmos® is a registered trademark of Desmos, Inc.

English Learners Success Forum is a fiscally sponsored project of the New Venture Fund (NVF), a 501(c)(3) public charity.

Universal Design for Learning Guidelines and framework are developed by the Center for Applied Special Technology. © 2018 CAST.

The Effective Mathematics Teaching Practices are developed by NCTM in Principles to Actions: Ensuring mathematical success for all. © 2014 NCTM.

Notice and Wonder and I Notice/I Wonder are trademarks of NCTM and the Math Forum.

No part of this publication may be reproduced or distributed in its original form, or stored in a database or retrieval system, without the prior written consent of Amplify Education, Inc., except for the classroom use of the worksheets included for students in some lessons.

Cover illustration by Caroline Hadilaksono.

© 2023 by Amplify Education, Inc.
55 Washington Street, Suite 800,
Brooklyn, NY 11201
www.amplify.com

ISBN: 978-1-63643-491-9
Printed in [e.g., the United States of America]
[# of print run] [print vendor] [year of printing]

Dear Teacher,

Thank you for choosing Amplify Math. Our team has worked hard to ensure you have strong, easy-to-follow instructional supports that get all students talking and thinking together about grade-level math.

You can learn more about the program design in the pages that follow, but we wanted to call out a few things that really drive our point home about the student experience:

Make math social

The student experience is **social and collaborative**. Your students will have plenty of opportunities to talk through their reasoning and work with their peers to come to new understandings. Thanks to our **partnership with Desmos**, you can kick off these social math experiences both offline and while logged in.

Power-ups

Your priority is teaching grade-level math to each and every student in your classroom, every day. To help with that, we've developed **Power-ups** to provide just-in-time support for your students.

Narrative

We kick off each sub-unit with a short, **engaging narrative** about historical or current-day events or phenomena. That way, your students can see the relevance of math in their everyday lives.

Featured Mathematicians

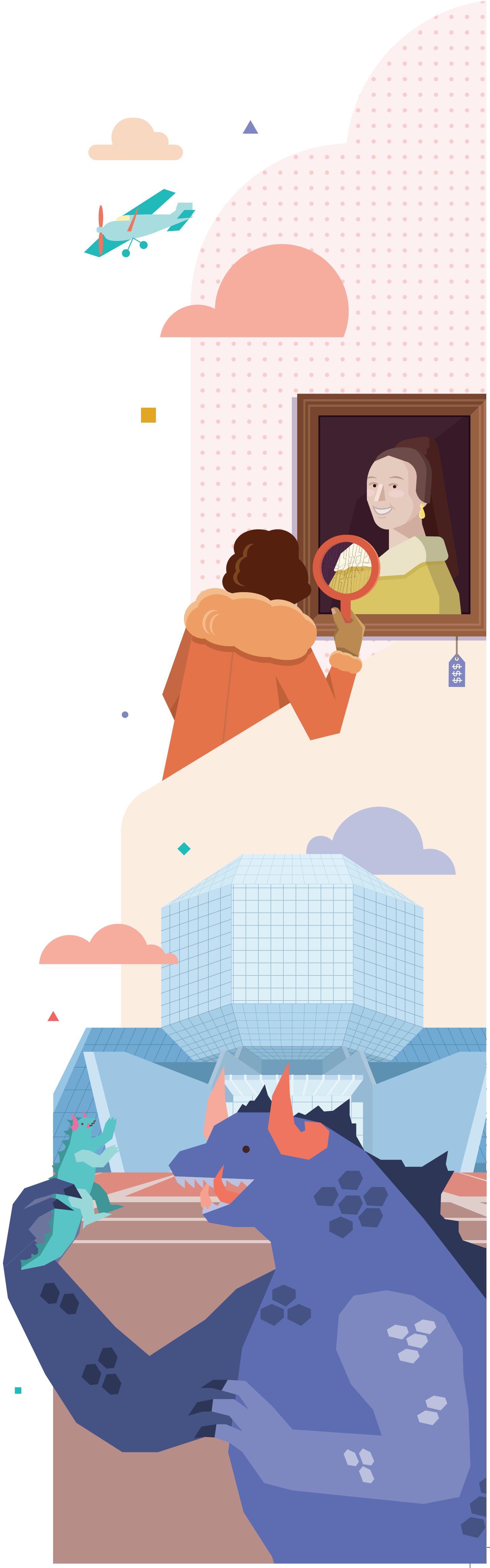
It's important to us that students see themselves in our materials. To that end, we've woven in **the work of innovative mathematical thinkers**. We've also included some of their personal stories, so that students can see themselves mirrored in the living history of mathematics.

Data

We provide plenty of **data to help you drive your instruction** and talk about student performance with your colleagues, as well as with caregivers who may not have had the best experiences with math when they were in school.

We hope you see the quality work of our team of editors and advisors in the program.

Sincerely,
The Amplify Math Team



Acknowledgments

Program Advisors

Amplify gratefully acknowledges the outstanding contributions and work of distinguished program advisors who have been integral to the development of Amplify Math. This product is testimony to their expertise, understanding of student learning needs, and dedication to rigorous and equitable mathematics instruction.

Phil Daro
Board member: Strategic Education Research Partnership (SERP)
Area of focus: Content strategy

Fawn Nguyen
Rio School District, California
Area of focus: Problem solving

Sunil Singh
Educator, author, storyteller
Area of focus: Narrative and storytelling

Paulo Tan, Ph.D.
Johns Hopkins University, School of Education
Area of focus: Meeting the needs of all students

Educator Advisory Board

Amplify gratefully acknowledges the outstanding contributions and work of esteemed members of our Educator Advisory Board. This product exhibits their respected observations and reflections.

Melvin Burnett
Alamance Burlington Schools, North Carolina

Jessica Childers
Putnam County Schools, Tennessee

Brent Christensen
Spokane Public Schools, Washington

Rhonda Creed-Harry
New York City Schools, New York

Jenny Croitoru
Chicago Public Schools, Illinois

Tara DeVaughn
Fairbanks Northstar Borough School District, Alaska

Elizabeth Hailey
Springfield R-XII School District, Missouri

Howie Hua
California State University at Fresno, California

Rachael Jones
Durham Public Schools, North Carolina

Rita Leskovec
Cleveland Metropolitan School District, Ohio

Corey Levin
New York City Schools, New York

Sandhya Raman
Berryessa Union School District, California

Jerry Schmidt
Brentwood School District, Missouri

Deloris Scott
Yazoo County School District, Mississippi

Noah Sharrow
Clarkston Community Schools, Michigan

Myla Simmons
Plainfield Public Schools, New Jersey

Michele Stassfurth
North Plainfield School District, New Jersey

Field Trials

Amplify gratefully acknowledges the time and efforts of educators from the following districts and schools whose participation in field trials provided constructive critiques and resulting improvements. This product reflects their valuable feedback.

Berryessa Union School District, California

Chicago Jesuit Academy, Illinois

Irvine Unified School District, California

Lake Tahoe Unified School District, California

Leadership Learning Academy, Utah

Lusher Charter School, Louisiana

Memphis Grizzlies Preparatory Charter School, Tennessee

Saddleback Valley Unified School District, California

San Juan Unified School District, California

Santa Paula Unified School District, California

Silver Summit Academy, Utah

Streetsboro City Schools, Ohio

West Contra Costa Unified School District, California

Wyoming City Schools, Ohio

Young Women's Leadership School of Brooklyn, New York

Amplify Math Product Development

Product

Molly Batchik
Candy Bratton
Stephanie Cheng
Rebecca Copley
Marsheela Evans
Christina Lee
Brad Shank
Kathleen Sheehy
Jennifer Skelley
Rey Vargas
Allen von Pallandt
Steven Zavari

Curriculum and Editorial

Toni Brokaw
Anna Buchina
Nora Castiglione
Jaclyn Claiborne
Kristina Clayton
Drew Corley
Karen Douglass
Karen Everly
Chris Ignaciuk
Justine Jackson
Louise Jarvis
Brian Kam
Rachel King
Suzanne Magargee
Mark Marzen
Nana Nam
Kim Petersen
Molly Pooler
Elizabeth Re
Allison Shatzman
Kristen Shebek
Ben Simon
Evan Spellman
Amy Sroka
Shelby Strong
Rajan Vedula
Zach Wissner-Gross

Digital Curriculum

Ian Cross
Phil DeOrsey
Ryan de la Garza
Sheila Jaung
Nokware Knight
Michelle Palker
Vincent Panetta
Aaron Robson
Sam Rodriguez
Eileen Rutherford
Elliot Shields
Gabe Turow

Design and Illustration

Amanda Behm
Irene Chan
Tim Chi Ly
Cindy Chung
Caroline Hadilaksono
Justin Moore
Christina Ogbotiti
Renée Park
Eddie Peña
Todd Rawson
Jordan Stine
Veronica Tolentino
J Yang

Narrative Design

Bill Cheng
Gala Mukomolova
Raj Parameswaran

Marketing

Megan Hunter
Zach Slack
Heath Williams

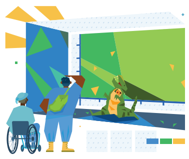
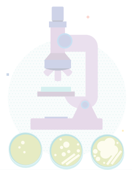
Engineering

Jessica Graham
Matt Hayes
Bardh Jahjaga
Eduard Korolchuk
Nick Maddalena
Syed Rizvi
Jon Tully

Digital Production

Andrew Avery
Ryan Cooper
Jessica Yin Gerena
Edward Johnson
Charvi Magdaong
Julie Palomba
Heather Ruiz
Ana Zapata

Program Scope and Sequence

	Unit 1	Unit 2	Unit 3	Unit 4
Grade 6 160 days total	Area and Surface Area 20 Instructional Days 3 Assessment Days 23 days total	Introducing Ratios 20 Instructional Days 2 Assessment Days 22 days total	Rates and Percentages 15 Instructional Days 2 Assessment Days 17 days total	Dividing Fractions 17 Instructional Days 3 Assessment Days 20 days total
Grade 7 153 days total	Scale Drawings 13 Instructional Days 2 Assessment Days 15 days total	Introducing Proportional Relationships 17 Instructional Days 2 Assessment Days 19 days total	Measuring Circles 12 Instructional Days 2 Assessment Days 14 days total	Percentages 13 Instructional Days 2 Assessment Days 15 days total
Grade 8 145 days total	Rigid Transformation and Congruence 18 Instructional Days 3 Assessment Days 21 days total	Dilations and Similarity 12 Instructional Days 2 Assessment Days 14 days total	Linear Relationships 19 Instructional Days 2 Assessment Days 21 days total	Linear Equations and Systems of Linear Equations 17 Instructional Days 2 Assessment Days 19 days total
Algebra 1 157 days total	 **Linear Equations, Inequalities, and Systems** 26 Instructional Days 3 Assessment Days 29 days total	 **Data Analysis and Statistics** 22 Instructional Days 3 Assessment Days 25 days total	 **Functions and Their Graphs** 22 Instructional Days 3 Assessment Days 25 days total	 Introducing Exponential Functions 22 Instructional Days 3 Assessment Days 25 days total

Volume 1

Unit 5	Unit 6	Unit 7	Unit 8
Arithmetic in Base Ten 14 Instructional Days 2 Assessment Days 16 days total	Expressions and Equations 19 Instructional Days 2 Assessment Days 21 days total	Rational Numbers 19 Instructional Days 2 Assessment Days 21 days total	Data Sets and Distributions 17 Instructional Days 3 Assessment Days 20 days total
Rational Number Arithmetic 20 Instructional Days 3 Assessment Days 23 days total	Expressions, Equations, and Inequalities 23 Instructional Days 3 Assessment Days 26 days total	Angles, Triangles, and Prisms 18 Instructional Days 3 Assessment Days 21 days total	Probability and Sampling 17 Instructional Days 3 Assessment Days 20 days total
Functions and Volume 21 Instructional Days 3 Assessment Days 24 days total	Exponents and Scientific Notation 15 Instructional Days 2 Assessment Days 17 days total	Irrationals and the Pythagorean Theorem 16 Instructional Days 2 Assessment Days 18 days total	Associations in Data 9 Instructional Days 2 Assessment Days 11 days total
Introducing Quadratic Functions 23 Instructional Days 3 Assessment Days 26 days total	Quadratic Equations 24 Instructional Days 3 Assessment Days 27 days total		

Volume 1

Unit 1 Linear Equations, Inequalities, and Systems

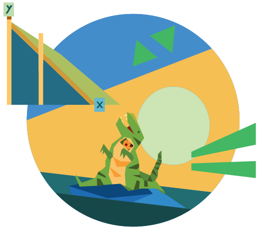
In this unit, students write and solve linear equations and inequalities to model the relationship between different quantities, recalling what is meant by a solution to an equation and interpreting solutions in context. They also solve systems of linear equations by graphing and using substitution and elimination methods.

Unit Narrative:
Adulting (Making Life Decisions)

Sub-Unit Narrative:
How did a tragic accident end a three-month strike?
Revisit how equations and inequalities can be used to model real-world situations, and how they can help you make decisions.

Sub-Unit Narrative:
How do first-gen Americans vault the hurdles of college?
"Solving" an equation doesn't always mean finding an unknown value — sometimes it can mean changing the equation's very structure.

Sub-Unit Narrative:
What's after high school?
Whether you work, intern, attend college, or do something else, inequalities can be used to model your time and money, helping you manage both of them.



Sub-Unit Narrative:
Are you a "boomerang-er"?
For better or for worse, life is full of constraints. Discover new strategies for solving problems with multiple constraints, which you will see time and again.

Sub-Unit Narrative:
Is there such a thing as too much choice?
What happens when the decisions become more complicated? Look at the big picture and then fine-tune where the decision overlaps.

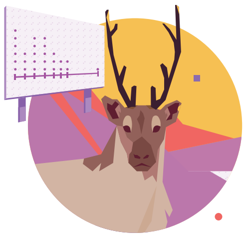
Unit 2 Data Analysis and Statistics

Students will explore univariate and bivariate data sets, many of which relate to climate change. Along the way, they will investigate, calculate, and interpret descriptive statistics, including measures of center, variability, and association.

Unit Narrative:
Analyzing Climate Change

PRE-UNIT READINESS ASSESSMENT

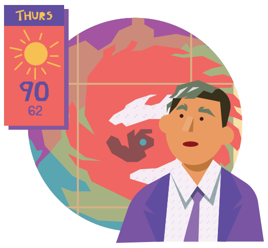
LAUNCH



Sub-Unit 1 Data Distributions 211

Sub-Unit Narrative:
How can we protect ourselves from a zombie virus?

Remember dot plots, histograms, and box plots? Revisit them through temperature data, while describing the data's center and variability.



Sub-Unit 2 Standard Deviation 251

Sub-Unit Narrative:
Is Sandy the new normal?

Meet the most commonly used measure of variability: standard deviation.

MID-UNIT ASSESSMENT

Sub-Unit 3 Bivariate Data 285

Sub-Unit Narrative:
What is "Day Zero"?

You have seen linear models before, but now you will (finally!) see how to identify the "best" model, by looking carefully at what are called residuals.

Sub-Unit Narrative:
What makes storms worse and has nothing to do with weather?

Use two-way tables to see how the changing climate has affected marginalized people around the world.

Sub-Unit Narrative:
Who is the "water warrior"?

Finally, you can say just how strong or weak an association is using a new statistical measure: the correlation coefficient.

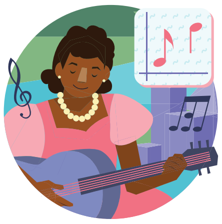
Unit 3 Functions and Their Graphs

Unit Narrative:
Artscapes

Students will expand on their understanding of the key features and multiple representations of functions. Along the way, they will write, graph, and interpret linear functions, piecewise functions, absolute functions, and the inverse of functions.

PRE-UNIT READINESS ASSESSMENT

LAUNCH



Sub-Unit 1 Functions and Their Representations

Sub-Unit Narrative:
How did the blues find a home in Memphis?

Remember representing functions with description, tables, graphs, and equations? Take another look as you visit Memphis, and meet another common tool to represent and interpret functions: *function notation*.

Sub-Unit 2 Analyzing and Creating Graphs of Functions

Sub-Unit Narrative:
What's the function of a jazz solo?

The way you describe a graph helps you gain insight on the relationship it represents. Average rate of change, domain, and range help to construct and interpret graphs more precisely.

MID-UNIT ASSESSMENT

Sub-Unit Narrative:
Where did the world meet soul?

Piecewise functions allow you to represent a relationship between two quantities as a set of rules. The absolute value function is a specific piecewise function that represents the distance from zero.

Sub-Unit Narrative:
How do you get Sunday shoppers to hear your song?

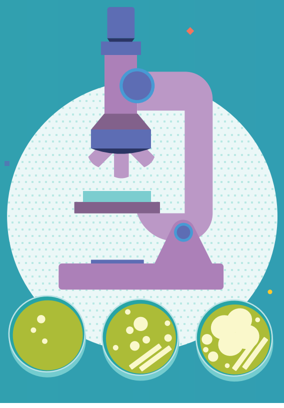
What happens if you reverse the process used in a relationship between two quantities? What changes? What stays the same? You can explore these questions using inverses of functions.

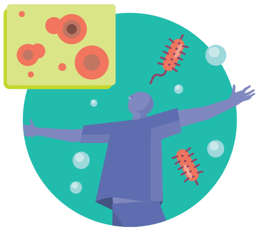
Volume 2

Unit 4 Introducing Exponential Functions

This is a unit of mathematical discovery, where the relationship between quantities is unlike any function students will have seen up to this point. Students encounter the explosiveness of exponential growth and and lingering of exponential decay through applications of infectious disease, vaccination, and prescription drug costs.

Unit Narrative:
Infectious Diseases, Vaccines, and Costs





Sub-Unit Narrative:
Where do baby bacteria come from?

Examine nonlinear functions using tables and graphs, before defining an exponential relationship. You'll represent exponential growth using an equation and explore it in context.

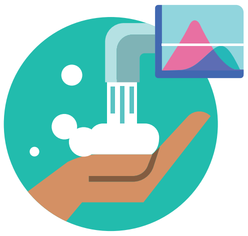
Sub-Unit Narrative:
How did an enslaved person save the city of Boston?

Examine growth factors between 0 and 1 as you develop an understanding of exponential decay.

Sub-Unit Narrative:
What does growing and shrinking look like on a graph?

Identify exponential relationships as exponential functions, and determine whether a graph is discrete.

Sub-Unit Narrative:
Want to be CEO for a day?
Make sense of repeated percent increase and see how it relates to compound interest.

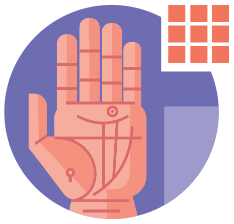


Sub-Unit Narrative:
Does distance make the curve grow flatter?
Compare the growth of different kinds of functions and finish with an exploration of how social distancing can combat the dangers of an epidemic.

Unit 5 Introducing Quadratic Functions

Students study quadratic functions. By analyzing and comparing patterns, tables, graphs, and equations, they gain an appreciation for the special features of quadratic functions and the situations they represent.

Unit Narrative:
Squares in Motion



Sub-Unit Narrative:
What's the best shape for a crystal ball?
Dive into quadratic expression by examining patterns of growth and change.

Sub-Unit Narrative:
What would sports be like without quadratics?
Use quadratic functions to model objects flying through the air or revenues earned by companies.

Sub-Unit Narrative:
How do you put the "quad-" in quadratics?
Use area diagrams and algebra tiles to factor quadratic expressions as you explore equivalent ways to write them.

Sub-Unit Narrative:
Mirror, mirror on the wall, what's the fairest function of them all?

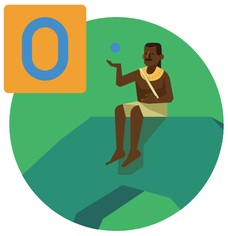
Quadratics have their own beauty, and different forms help you identify features of their graphs.

Unit 6 Quadratic Equations

In this unit, students write and solve quadratic equations and make sense of the solutions. They investigate the structure of quadratic equations and determine efficient strategies for solving them.

Unit Narrative:
The Evolution of Solving Quadratic Equations

Sub-Unit Narrative:
How did the Nile River spur on Egyptian mathematics?
Revisit projectile motion and maximizing revenue as you discover new meanings for the zeros of a quadratic function.



Sub-Unit Narrative:
When is zero more than nothing?
Understand the importance of zero when solving quadratic equations. Then, efficiently factor quadratics using a variety of strategies.

Sub-Unit Narrative:
How many ways can you crack an egg?
Discover the ancient art of taking a quadratic expression and completing the square. It's all about that missing piece.

Sub-Unit Narrative:
Where does a number call its home?

Subtraction and division took you from whole numbers to rationals. Now you must look beyond them as you operate with irrational numbers.

Sub-Unit Narrative:
What was the House of Wisdom?

Discover strategies for solving any quadratic equation. You will also determine which strategies are more efficient.

Get all students talking and thinking about grade-level math.

Amplify Math was designed around the idea that core math needs to serve 100% of students in accessing grade-level math every day. To that end, the program delivers:

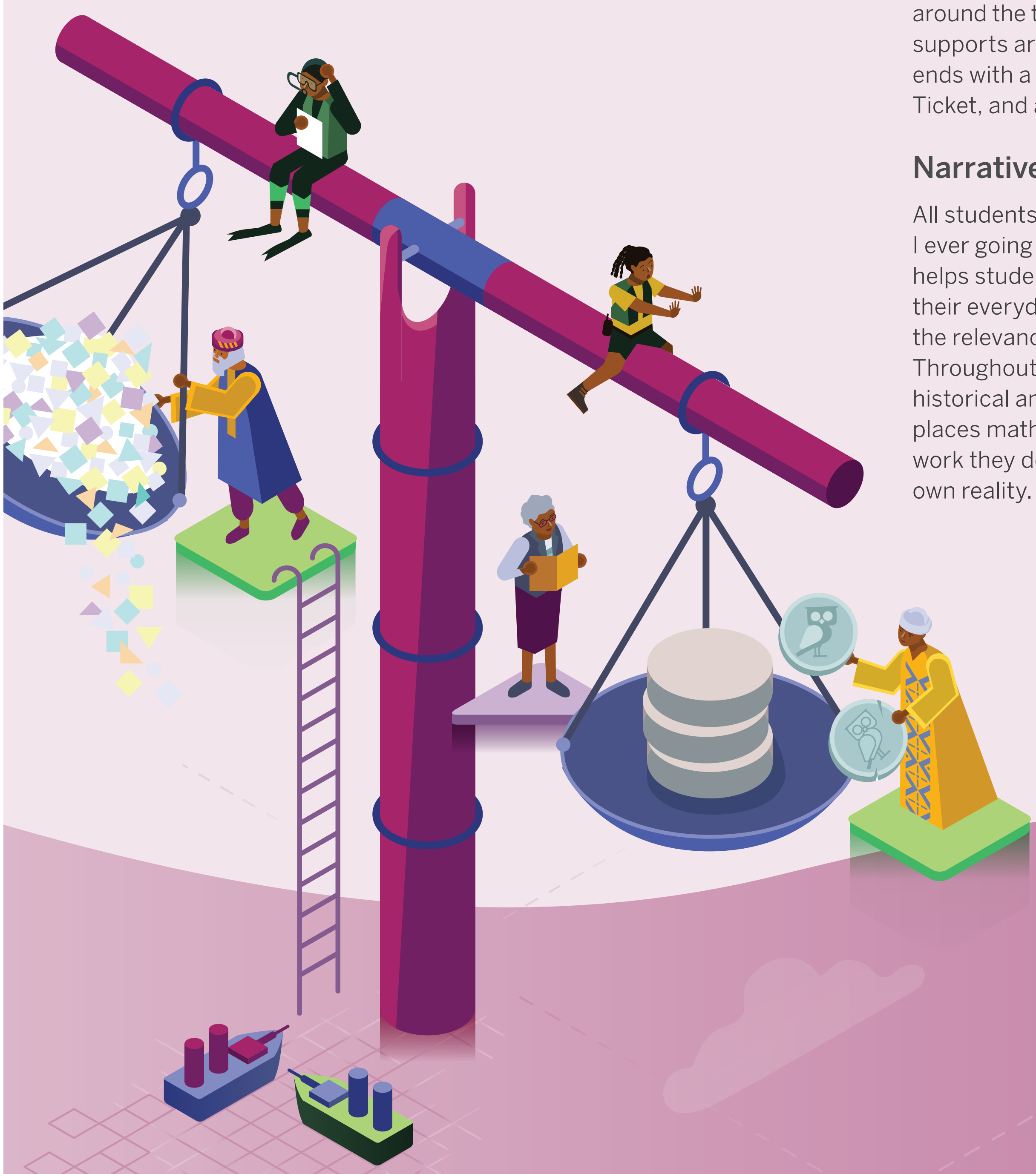
1 Productive discourse made easier to facilitate and more accessible for students

Clean and clear lesson design

The lessons all include straightforward "1, 2, 3 step" guidance for launching and facilitating discussions around the tasks. Thoughtful and specific differentiation supports are included for every activity. Every lesson ends with a summary and reflection moment, an Exit Ticket, and a practice problem set.

Narrative and storytelling

All students ask "Why do I need to know this? When am I ever going to use this in the real world?" Amplify Math helps students make the connections with math and their everyday lives to help them see and appreciate the relevance of the math they're figuring out in class. Throughout the units, students will be introduced to historical and current narratives that show the many places mathematics inhabits in our world and how the work they do in class connects to our history and their own reality.



2 Flexible, social problem-solving experiences online

Social learning experiences online

By partnering with Desmos, we've been able to deliver digital lessons, which we call **Amps**, that get students thinking, talking, revising, and celebrating their ideas. As students work in the interactive slides, new functionality may appear and they will often be asked to justify their actions and thinking. All of this is made visible to the teacher in real time.

Automatically differentiated activities

Our **Power-ups** automatically provide differentiated activities to students who need pre-requisite support, based on performance on past problems and assessments. They're available in this Teacher Edition, too. Phil Daro partnered with us on this feature to ensure we were giving all students, even the ones who might be three years behind in math, but only 15 minutes behind the day's lesson, the chance to experience success in math.

3 Real-time insights, data, and reporting that inform instruction

Teacher orchestration tools

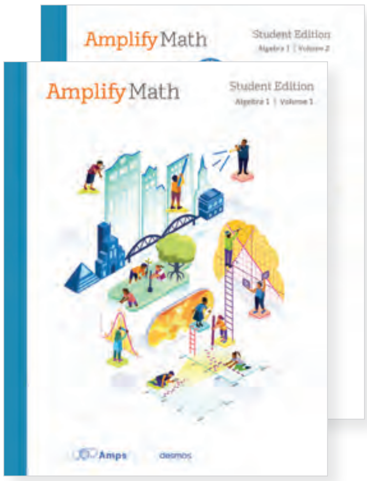
Once a teacher launches an Amp, students will be automatically moved to the lesson of the day and will see the interactive screens. Teachers will have the ability not only to pace the lesson the way they want to, but also to see student work in real time. The orchestration tools offer teachers ways to overlay student work to spot misconceptions and also the ability to spotlight student work anonymously to discuss with the class.

Embedded and standalone assessments

Amplify Math includes both a suite of standalone assessments and embedded assessments that allow teachers and leaders insights into where students are and how they might best be supported. The full reporting suite covers student and class performance based on work done in lessons, Exit Tickets, and practice sets, performance by standards, and performance on Interim assessments.

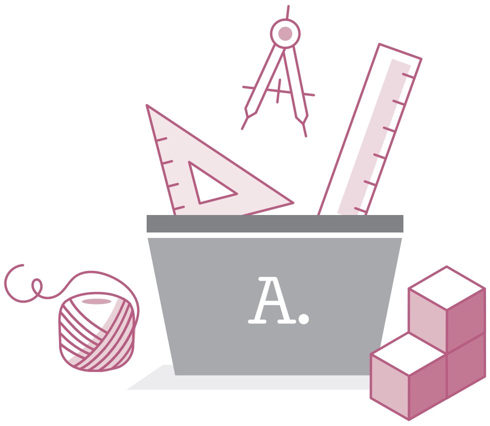
Amplify Math resources

Student Materials



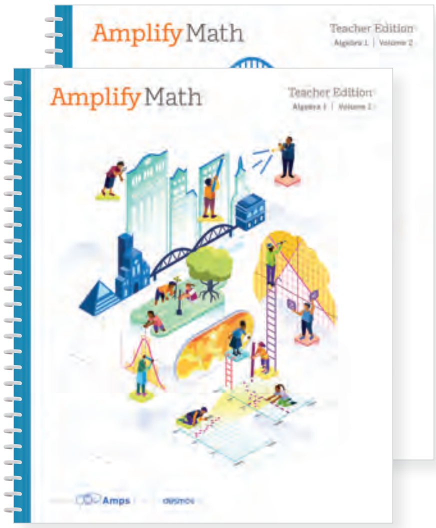
Student workbooks, 2 volumes

Amps, our exclusive collection of digital lessons powered by desmos



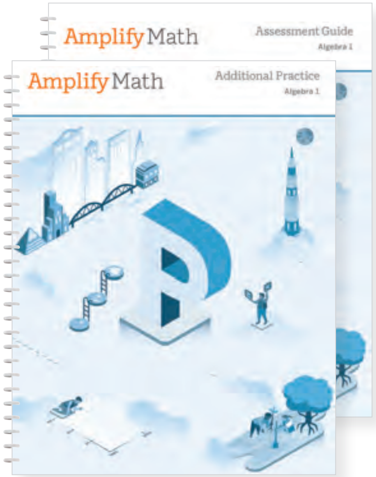
Hands-on manipulatives (optional)

Teacher Materials



Teacher Edition, 2 volumes

Digital Teacher Edition and classroom monitoring tools



Additional Practice and Assessment Guide blackline masters

Program architecture

Course

I Interim Assesments

UNIT 1	I	UNIT 2	I	UNIT 3	UNIT 4	I	UNIT 5	I	UNIT 6
26 lessons		22 lessons		22 lessons	22 lessons		23 lessons		22 lessons

Note: Interim assessments may be administered according to your district/school's timeline; this depiction is just one of many possible administrations.

Unit

A Pre-Unit Readiness Assessment

A Mid-Unit Assessment

End-of-Unit Assessment A

LAUNCH	Sub-Unit 1	Sub-Unit 2	A	Sub-Unit 3	CAPSTONE
1	2 3 4 5 6 7	8 9 10 11 12		13 14 15 16 17 18 19	20

Note: The number of sub-units and lessons vary from unit to unit; this depiction shows the general structure of a unit.

Lesson

Warm-up	>	Activity 1	>	Activity 2	>	Summary	>	Exit Ticket	>	Practice
5 min		15 min		15 min		5 min		5 min		timing varies
Independent Pairs Small Groups Whole Class		Independent Pairs Small Groups Whole Class		Independent Pairs Small Groups Whole Class		Whole Class		Independent		Independent

Note: The number of activities vary from lesson to lesson; this depiction shows the general structure of a lesson.

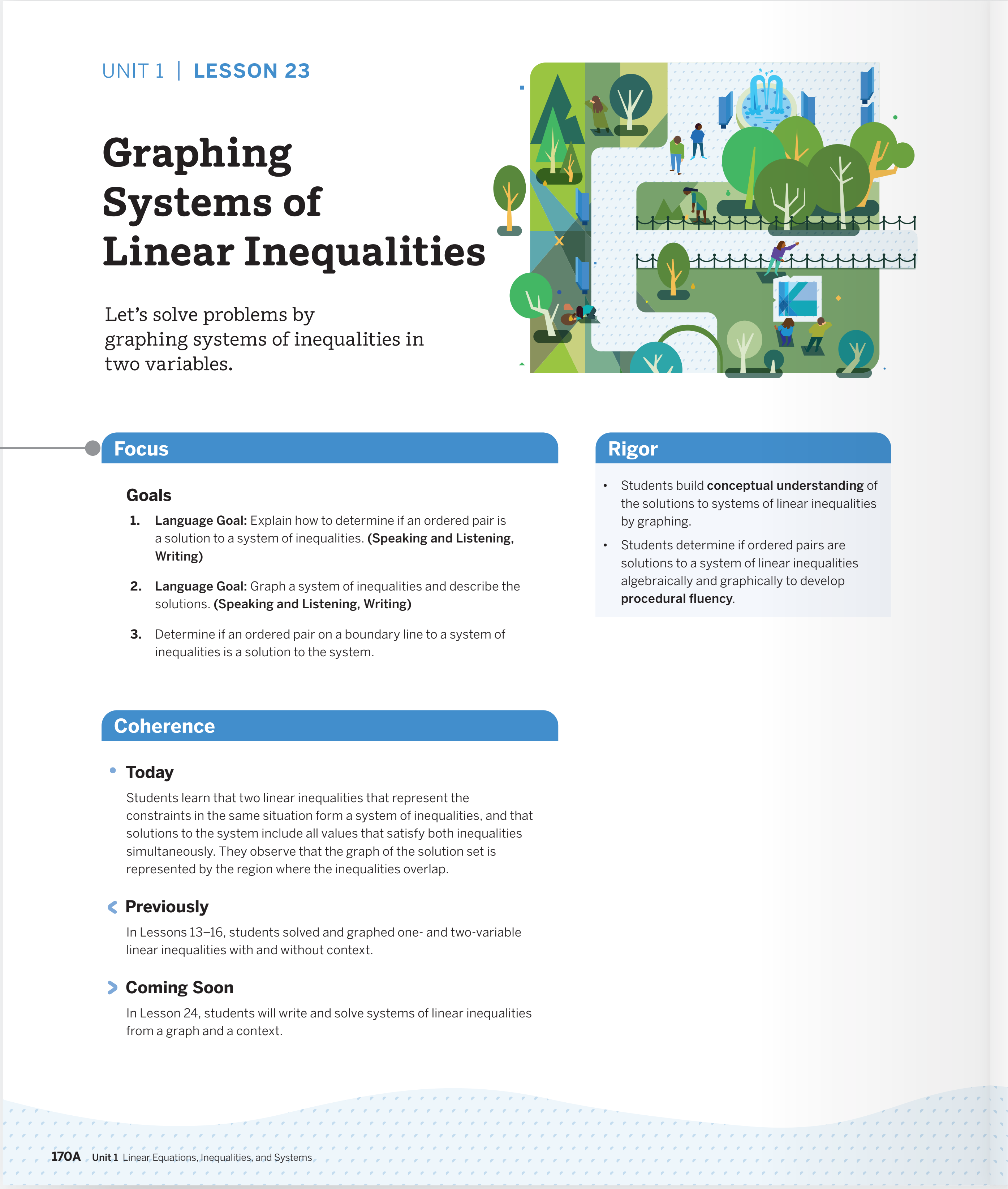
Key:

Independent
Small Groups
Pairs
Whole Class

Navigating This Program

Lesson Brief

Lesson goals, **coherence mapping**, and a breakdown for how **conceptual understanding**, **procedural fluency**, and **application** are addressed are included for each lesson.



UNIT 1 | **LESSON 23**

Graphing Systems of Linear Inequalities

Let's solve problems by graphing systems of inequalities in two variables.

Focus

Goals

1. **Language Goal:** Explain how to determine if an ordered pair is a solution to a system of inequalities. **(Speaking and Listening, Writing)**
2. **Language Goal:** Graph a system of inequalities and describe the solutions. **(Speaking and Listening, Writing)**
3. Determine if an ordered pair on a boundary line to a system of inequalities is a solution to the system.

Coherence

Today

Students learn that two linear inequalities that represent the constraints in the same situation form a system of inequalities, and that solutions to the system include all values that satisfy both inequalities simultaneously. They observe that the graph of the solution set is represented by the region where the inequalities overlap.

Previously

In Lessons 13–16, students solved and graphed one- and two-variable linear inequalities with and without context.

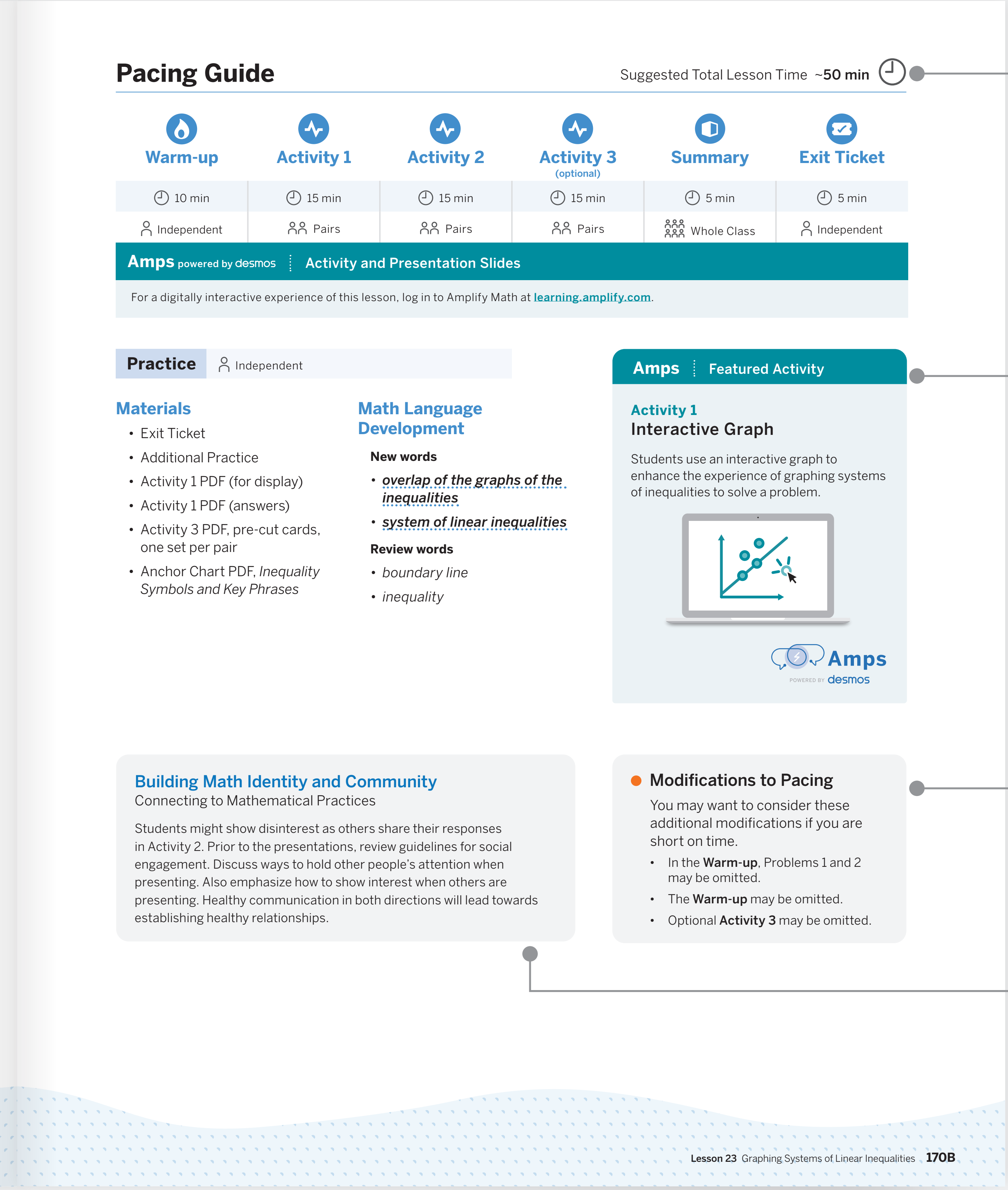
Coming Soon

In Lesson 24, students will write and solve systems of linear inequalities from a graph and a context.

Rigor

- Students build **conceptual understanding** of the solutions to systems of linear inequalities by graphing.
- Students determine if ordered pairs are solutions to a system of linear inequalities algebraically and graphically to develop **procedural fluency**.

170A Unit 1 Linear Equations, Inequalities, and Systems



Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
10 min	15 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- Activity 1 PDF (answers)
- Activity 3 PDF, pre-cut cards, one set per pair
- Anchor Chart PDF, *Inequality Symbols and Key Phrases*

Math Language Development

New words

- *overlap of the graphs of the inequalities*
- *system of linear inequalities*

Review words

- *boundary line*
- *inequality*

Amps Featured Activity

Activity 1
Interactive Graph

Students use an interactive graph to enhance the experience of graphing systems of inequalities to solve a problem.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might show disinterest as others share their responses in Activity 2. Prior to the presentations, review guidelines for social engagement. Discuss ways to hold other people's attention when presenting. Also emphasize how to show interest when others are presenting. Healthy communication in both directions will lead towards establishing healthy relationships.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problems 1 and 2 may be omitted.
- The **Warm-up** may be omitted.
- Optional **Activity 3** may be omitted.

Lesson 23 Graphing Systems of Linear Inequalities **170B**

Suggested timing for the lesson and each activity is included for quick reference.

The benefits of teaching one or more of the activities **online** are outlined for each lesson.

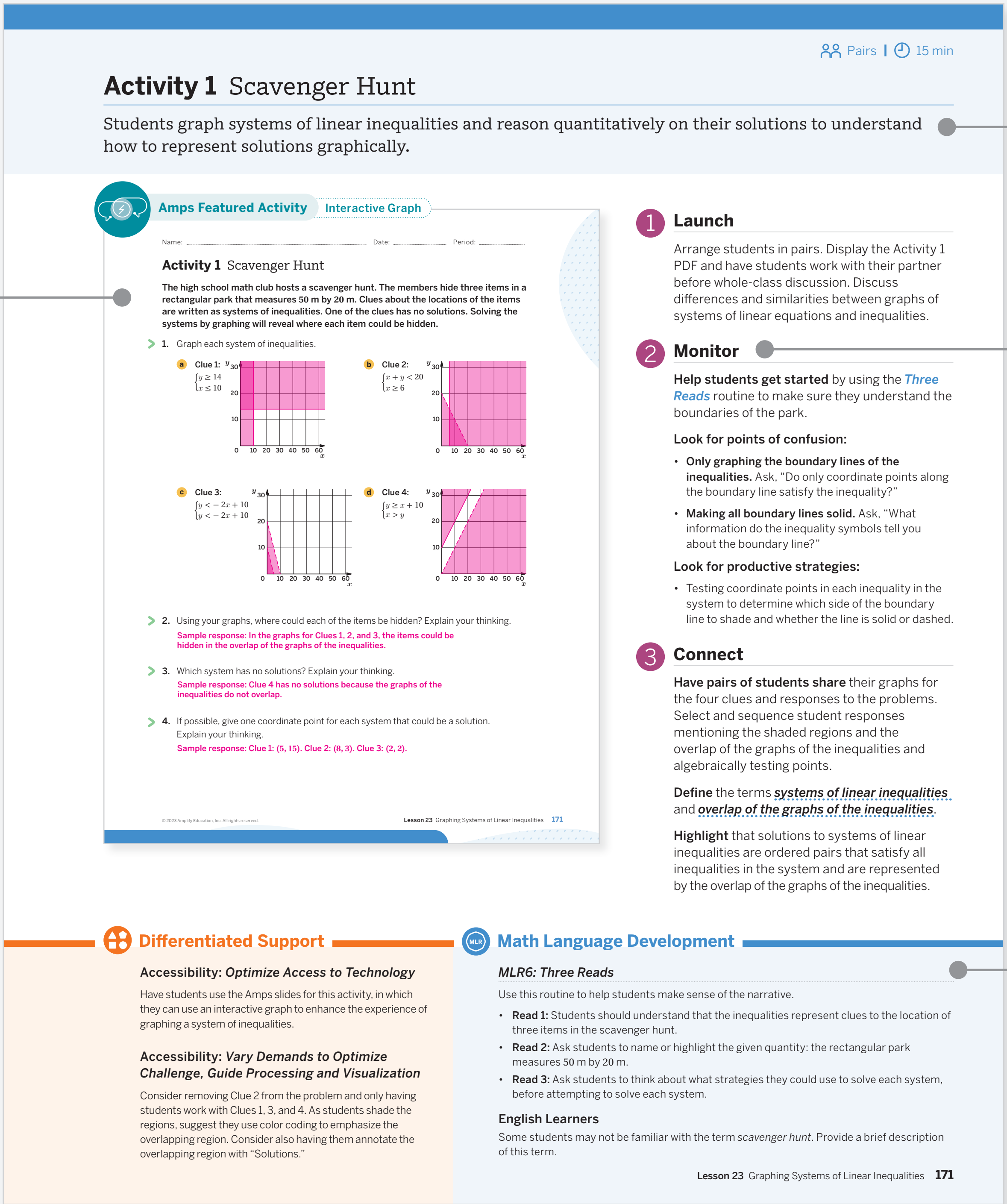
Every lesson pacing guide includes **modification** suggestions.

Building Math Identity and Community supports for teachers are included in the Lesson Brief. Student supports appear online and in the printed Student Edition.

Navigating This Program

Lesson

The **student-facing** content is presented to the left.



Pairs | 15 min

Activity 1 Scavenger Hunt

Students graph systems of linear inequalities and reason quantitatively on their solutions to understand how to represent solutions graphically.

Amps Featured Activity Interactive Graph

Name: ________ Date: ________ Period: ________

Activity 1 Scavenger Hunt

The high school math club hosts a scavenger hunt. The members hide three items in a rectangular park that measures 50 m by 20 m. Clues about the locations of the items are written as systems of inequalities. One of the clues has no solutions. Solving the systems by graphing will reveal where each item could be hidden.

1. Graph each system of inequalities.

a. **Clue 1:** $\begin{cases} y \geq 14 \\ x \leq 10 \end{cases}$

b. **Clue 2:** $\begin{cases} x + y < 20 \\ x \geq 6 \end{cases}$

c. **Clue 3:** $\begin{cases} y < -2x + 10 \\ y < -2x + 10 \end{cases}$

d. **Clue 4:** $\begin{cases} y \geq x + 10 \\ x > y \end{cases}$

2. Using your graphs, where could each of the items be hidden? Explain your thinking.

Sample response: In the graphs for Clues 1, 2, and 3, the items could be hidden in the overlap of the graphs of the inequalities.

3. Which system has no solutions? Explain your thinking.

Sample response: Clue 4 has no solutions because the graphs of the inequalities do not overlap.

4. If possible, give one coordinate point for each system that could be a solution. Explain your thinking.

Sample response: Clue 1: (5, 15). Clue 2: (8, 3). Clue 3: (2, 2).

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 23 Graphing Systems of Linear Inequalities 171

1 Launch

Arrange students in pairs. Display the Activity 1 PDF and have students work with their partner before whole-class discussion. Discuss differences and similarities between graphs of systems of linear equations and inequalities.

2 Monitor

Help students get started by using the *Three Reads* routine to make sure they understand the boundaries of the park.

Look for points of confusion:

- **Only graphing the boundary lines of the inequalities.** Ask, "Do only coordinate points along the boundary line satisfy the inequality?"
- **Making all boundary lines solid.** Ask, "What information do the inequality symbols tell you about the boundary line?"

Look for productive strategies:

- Testing coordinate points in each inequality in the system to determine which side of the boundary line to shade and whether the line is solid or dashed.

3 Connect

Have pairs of students share their graphs for the four clues and responses to the problems. Select and sequence student responses mentioning the shaded regions and the overlap of the graphs of the inequalities and algebraically testing points.

Define the terms ***systems of linear inequalities*** and ***overlap of the graphs of the inequalities***.

Highlight that solutions to systems of linear inequalities are ordered pairs that satisfy all inequalities in the system and are represented by the overlap of the graphs of the inequalities.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use an interactive graph to enhance the experience of graphing a system of inequalities.

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Consider removing Clue 2 from the problem and only having students work with Clues 1, 3, and 4. As students shade the regions, suggest they use color coding to emphasize the overlapping region. Consider also having them annotate the overlapping region with "Solutions."

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

- **Read 1:** Students should understand that the inequalities represent clues to the location of three items in the scavenger hunt.
- **Read 2:** Ask students to name or highlight the given quantity: the rectangular park measures 50 m by 20 m.
- **Read 3:** Ask students to think about what strategies they could use to solve each system, before attempting to solve each system.

English Learners

Some students may not be familiar with the term *scavenger hunt*. Provide a brief description of this term.

Lesson 23 Graphing Systems of Linear Inequalities **171**

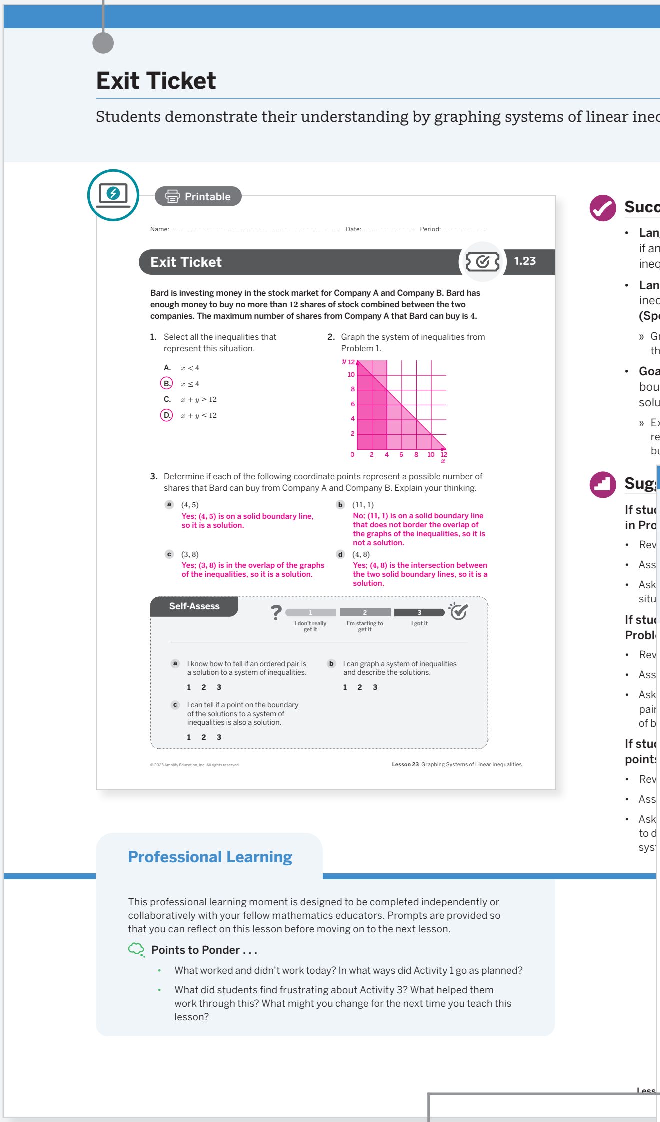
A short **description of the activity and its targeted goal** is outlined at the top.

Easy 1-2-3 guidance for teachers shortens the amount of time required to plan. The "look for" prompts are helpful to scan while teaching.

Differentiation supports, including our alternative warm-ups called Power-ups, provide practical guidance for scaffolding or extending the learning for all students. Differentiation supports, including our just-in-time supports called Power-ups, provide practical guidance for scaffolding or extending the learning for all students.

LESSON BRIEF | WARM-UP | ACTIVITIES | SUMMARY | EXIT TICKET | PRACTICE

Each lesson ends with an **Exit Ticket** which includes a self assessment for students.



Independent | 5 min

Exit Ticket

Students demonstrate their understanding by graphing systems of linear inequalities and their solutions.

Printable

Name: ________ Date: ________ Period: ________

Exit Ticket 1.23

Bard is investing money in the stock market for Company A and Company B. Bard has enough money to buy no more than 12 shares of stock combined between the two companies. The maximum number of shares from Company A that Bard can buy is 4.

1. Select all the inequalities that represent this situation.
   - A. $x < 4$
   - [✓ B.] $x \leq 4$
   - C. $x + y \geq 12$
   - [✓ D.] $x + y \leq 12$

2. Graph the system of inequalities from Problem 1.

3. Determine if each of the following coordinate points represent a possible number of shares that Bard can buy from Company A and Company B. Explain your thinking.
   - a $(4, 5)$ Yes; $(4, 5)$ is on a solid boundary line, so it is a solution.
   - b $(11, 1)$ No; $(11, 1)$ is on a solid boundary line that does not border the overlap of the graphs of the inequalities, so it is not a solution.
   - c $(3, 8)$ Yes; $(3, 8)$ is in the overlap of the graphs of the inequalities, so it is a solution.
   - d $(4, 8)$ Yes; $(4, 8)$ is the intersection between the two solid boundary lines, so it is a solution.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

- a I know how to tell if an ordered pair is a solution to a system of inequalities. **1 2 3**
- b I can graph a system of inequalities and describe the solutions. **1 2 3**
- c I can tell if a point on the boundary of the solutions to a system of inequalities is also a solution. **1 2 3**

© 2023 Amplify Education, Inc. All rights reserved. **Lesson 23** Graphing Systems of Linear Inequalities

Success looks like . . .

- **Language Goal:** Explaining how to determine if an ordered pair is a solution to a system of inequalities. **(Speaking and Listening, Writing)**
- **Language Goal:** Graphing a system of inequalities and describing the solutions. **(Speaking and Listening, Writing)**
  - » Graphing the system of inequalities representing the number of shares in Problem 2.
- **Goal:** Determining if a coordinate point on a boundary line to a system of inequalities is a solution to the system.
  - » Explaining whether each coordinate point could represent a number of shares that Bard could buy in Problem 3.

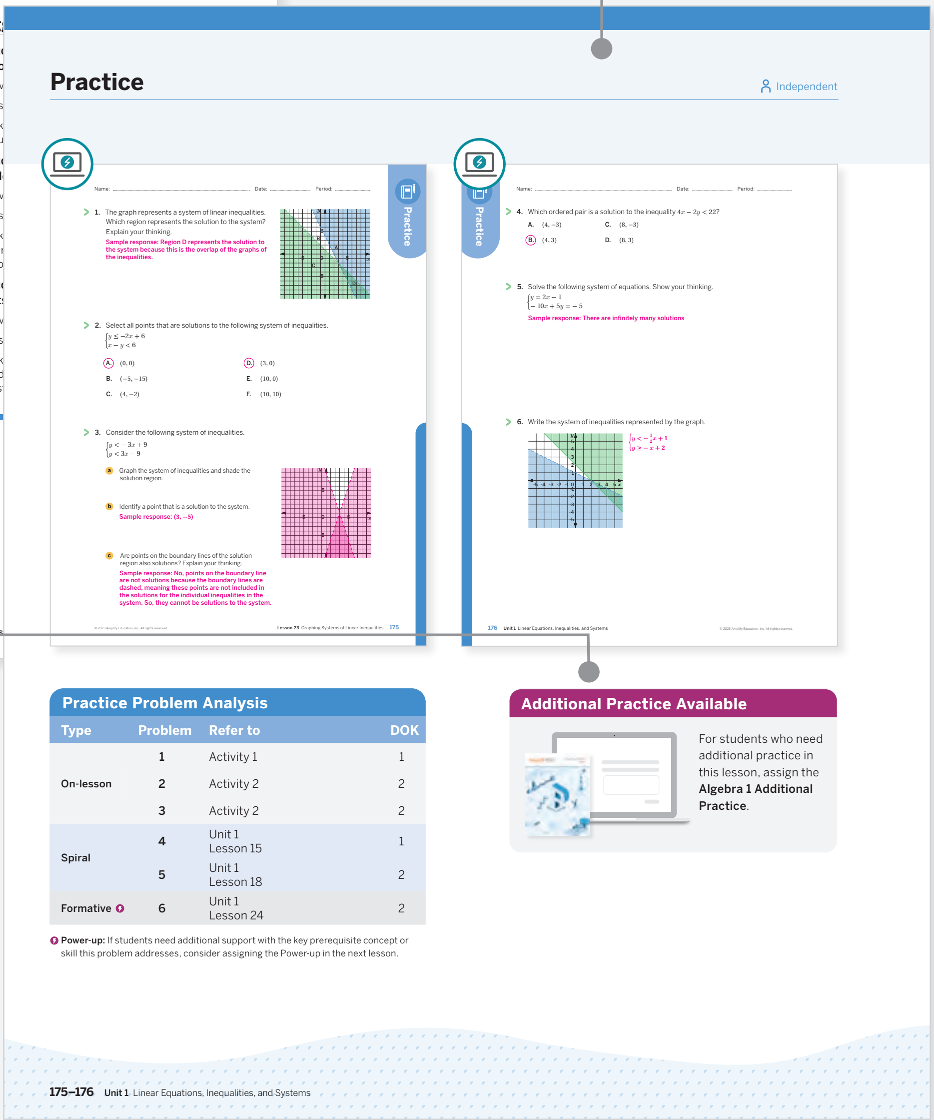
Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways did Activity 1 go as planned?
- What did students find frustrating about Activity 3? What helped them work through this? What might you change for the next time you teach this lesson?

A targeted set of 4-6 **practice problems** are included online and in the print Student Edition. Each set includes at least one spiral review problem and one formative problem as a prerequisite check for the next lesson.



Practice

Independent

Name: ________ Date: ________ Period: ________

1. The graph represents a system of linear inequalities. Which region represents the solution to the system? Explain your thinking.
   Sample response: Region D represents the solution to the system because this is the overlap of the graphs of the inequalities.

2. Select all points that are solutions to the following system of inequalities.
   $\begin{cases} y \leq -2x + 6 \\ x - y < 6 \end{cases}$
   - [✓ A.] $(0, 0)$
   - B. $(-5, -15)$
   - C. $(4, -2)$
   - [✓ D.] $(3, 0)$
   - E. $(10, 0)$
   - F. $(10, 10)$

3. Consider the following system of inequalities.
   $\begin{cases} y < -3x + 9 \\ y < 3x - 9 \end{cases}$
   - a Graph the system of inequalities and shade the solution region.
   - b Identify a point that is a solution to the system.
     Sample response: $(3, -5)$
   - c Are points on the boundary lines of the solution region also solutions? Explain your thinking.
     Sample response: No, points on the boundary line are not solutions because the boundary lines are dashed, meaning these points are not included in the solutions for the individual inequalities in the system. So, they cannot be solutions to the system.

© 2023 Amplify Education, Inc. All rights reserved. **Lesson 23** Graphing Systems of Linear Inequalities 175

Name: ________ Date: ________ Period: ________

4. Which ordered pair is a solution to the inequality $4x - 2y < 22$?
   - A. $(4, -3)$
   - [✓ B.] $(4, 3)$
   - C. $(8, -3)$
   - D. $(8, 3)$

5. Solve the following system of equations. Show your thinking.
   $\begin{cases} y = 2x - 1 \\ -10x + 5y = -5 \end{cases}$
   Sample response: There are infinitely many solutions

6. Write the system of inequalities represented by the graph.
   $\begin{cases} y < -\frac{1}{2}x + 1 \\ y \geq -x + 2 \end{cases}$

176 **Unit 1** Linear Equations, Inequalities, and Systems © 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 15	1
	5	Unit 1 Lesson 18	2
Formative ⚡	6	Unit 1 Lesson 24	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

175–176 **Unit 1** Linear Equations, Inequalities, and Systems

In the **Additional Practice book**, students will find a worked out example and four to eight practice problems per lesson.

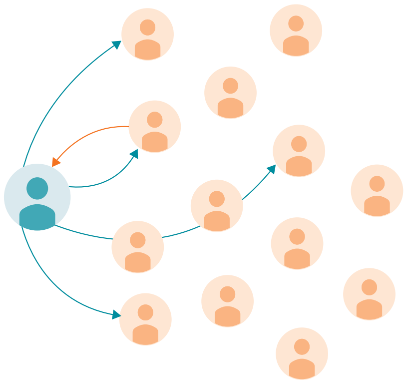
Social, collaborative digital experiences

Digital lessons, when designed the right way, can be powerful in their ability to surface student thinking and spark interesting and productive discussions. To bring our vision of what digital lessons can and should be to life, we've partnered with Desmos to create our complete library of **Amps**—social, collaborative lessons powered by Desmos technology that recast technology from simply mirroring what can be done in a workbook to presenting captivating scenarios where students work together and interact with the mathematics in real time.

Amps
POWERED BY desmos

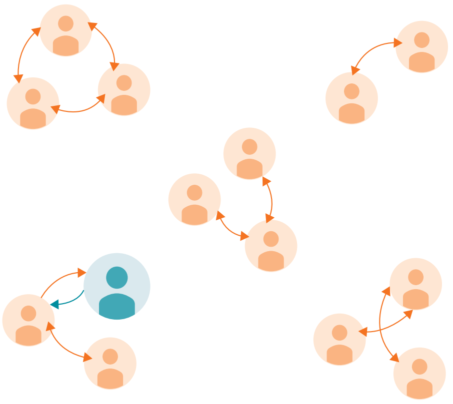
1 Launch

Teachers launch an activity and ensure students understand what's being asked.

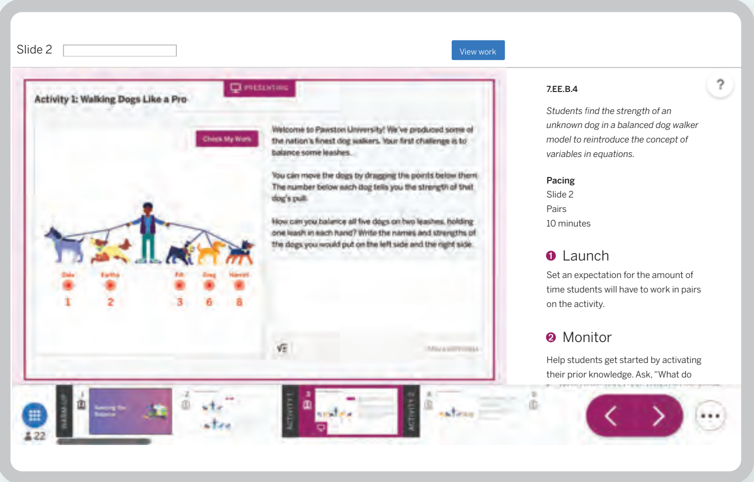


2 Monitor

Students interact with each other to discuss and work out strategies for solving a problem.



Teacher experience



Activity 1: Walking Dogs Like a Pro

Welcome to Pawston University! We've produced some of the nation's finest dog walkers. Your first challenge is to balance some leashes.

You can move the dogs by dragging the points below them. The number below each dog tells you the strength of that dog's pull.

How can you balance all five dogs on two leashes, holding one leash in each hand? Write the names and strengths of the dogs you would put on the left side and the right side.

7.EE.B.4

Students find the strength of an unknown dog in a balanced dog walker model to reintroduce the concept of variables in equations.

Pacing
Slide 2
Pairs
10 minutes

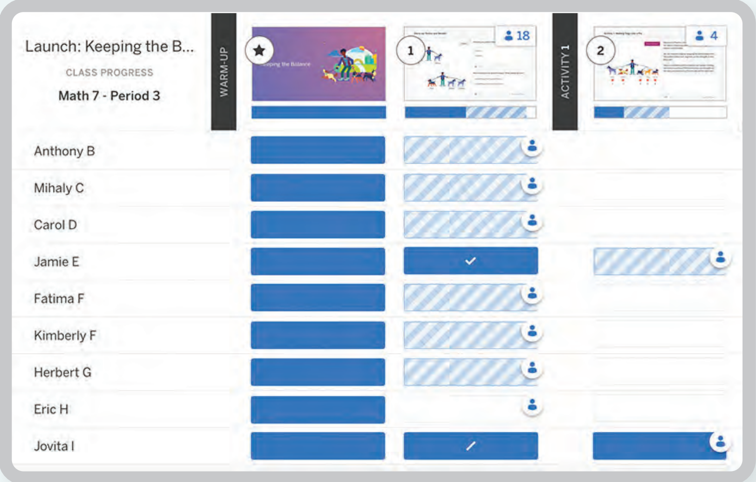
❶ Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

❷ Monitor

Help students get started by activating their prior knowledge. Ask, "What do

When you launch a lesson, you'll have access to **easy-to-skim teacher notes and all of the controls necessary** to manage the lesson.



Launch: Keeping the B...
CLASS PROGRESS
Math 7 - Period 3

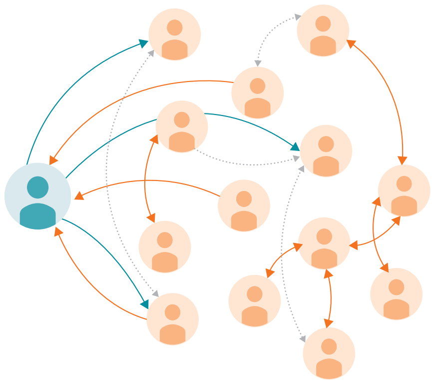
Anthony B
Mihaly C
Carol D
Jamie E
Fatima F
Kimberly F
Herbert G
Eric H
Jovita I

After students have started working you can access the Class Progress screen to **see where students are in the lesson and even control which problems they have access to.**

When you launch an **Amp**, you will be kickstarting small group and whole class discussions where students can see how their thinking can impact a situation and learn how their peers are justifying their actions and decisions.

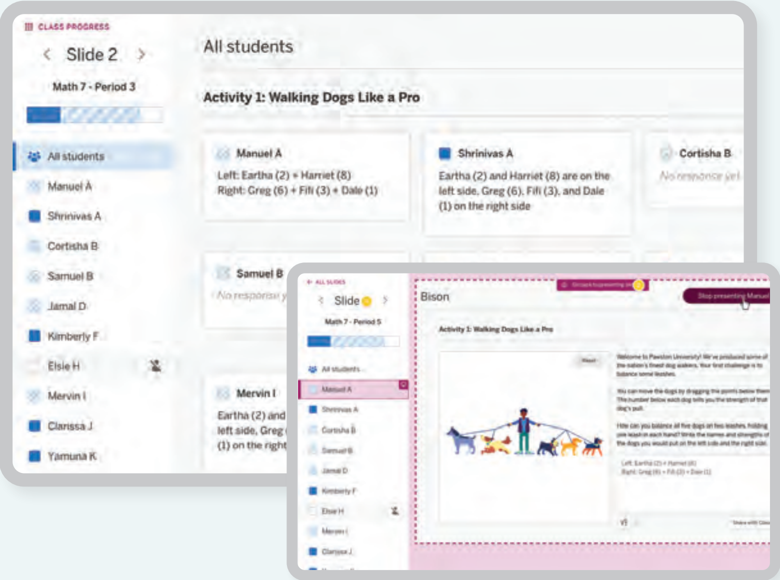
3 Connect

Students construct viable arguments and critique each other's reasoning, then synthesize with the teacher at the end.



4 Review

After class, teachers can provide feedback on submitted student work and run reports.



All students

Activity 1: Walking Dogs Like a Pro

Manuel A
Left: Eartha (2) + Harriet (8)
Right: Greg (6) + Fifi (3) + Dale (1)

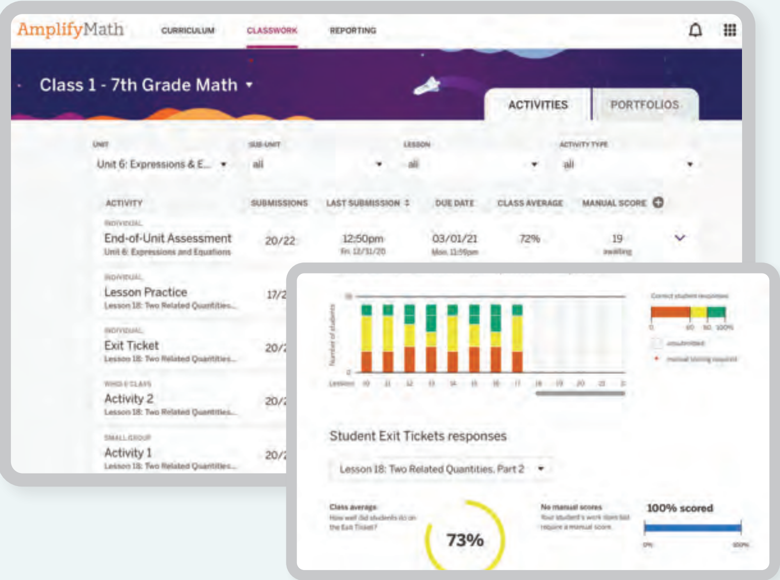
Shrinivas A
Eartha (2) and Harriet (8) are on the left side, Greg (6), Fifi (3), and Dale (1) on the right side

Cortisha B
No response yet

Samuel B
No response y

Mervin I
Eartha (2) and
left side, Greg
(1) on the right

All student responses can be viewed easily on the All Students screen. You can often view a composite view of responses and spotlight student work anonymously.



Unit 6: Expressions & E... all all all

ACTIVITY	SUBMISSIONS	LAST SUBMISSION	DUE DATE	CLASS AVERAGE	MANUAL SCORE
INDIVIDUAL End-of-Unit Assessment Unit 6: Expressions and Equations	20/22	12:50pm Fri. 12/31/20	03/01/21 Mon. 11:59pm	72%	19 awaiting
INDIVIDUAL Lesson Practice Lesson 18: Two Related Quantities...	17/2				
INDIVIDUAL Exit Ticket Lesson 18: Two Related Quantities...	20/2				
WHOLE CLASS Activity 2 Lesson 18: Two Related Quantities...	20/2				
SMALL GROUP Activity 1 Lesson 18: Two Related Quantities...	20/2				

Student Exit Tickets responses

Lesson 18: Two Related Quantities, Part 2

After students complete work that's ready for grading, you can head to Classwork to **quickly provide feedback.**

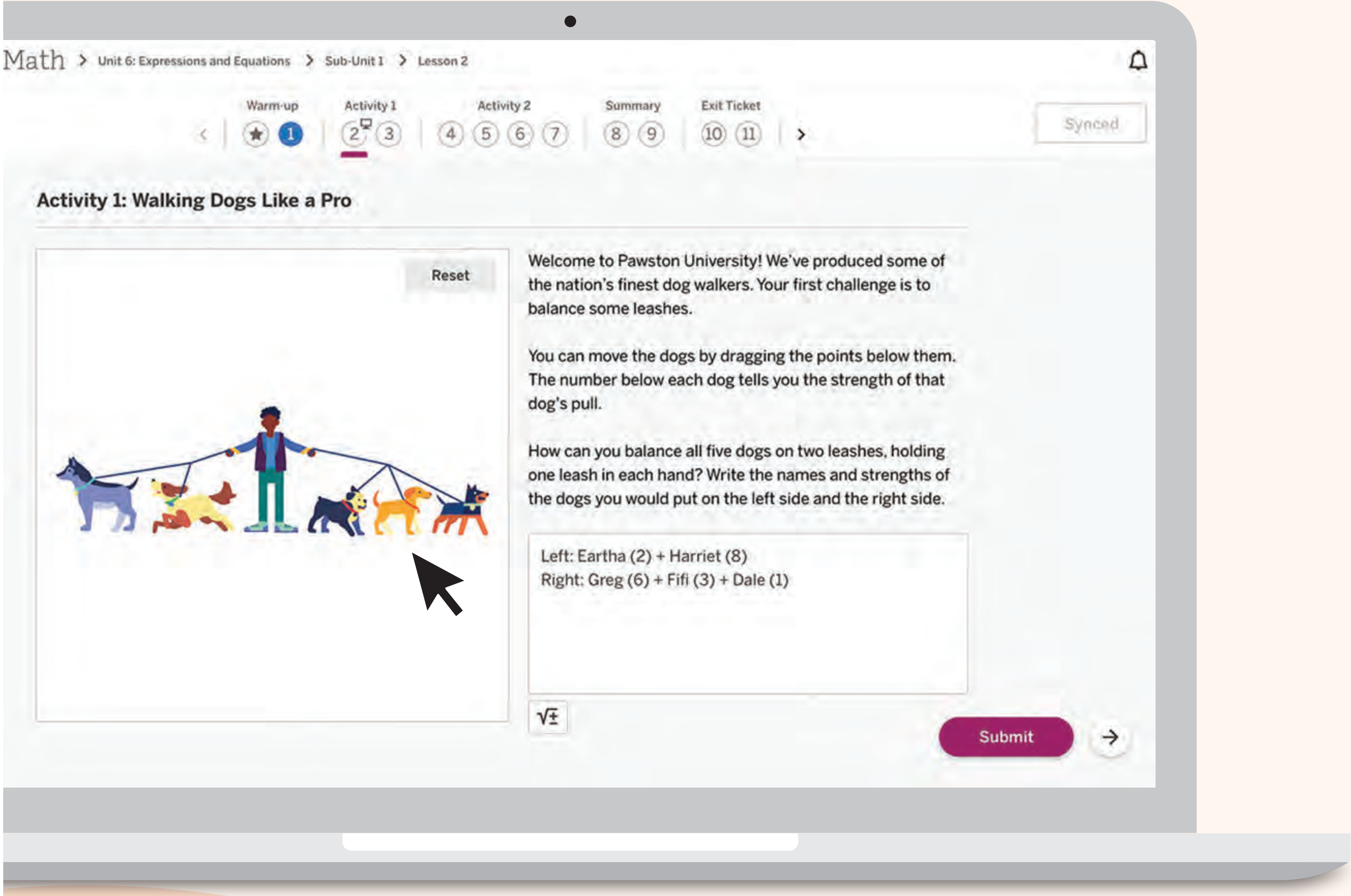
Once students have completed an Exit Ticket, a practice problem set, or an assessment, you can **run reports at the class, student, and standards levels to check in on student progress.**

Connecting everyone in the classroom

The student experience is interactive and responsive. As students manipulate the interactive elements of the Amps, they will be asked to justify their thinking, and often they will get to see how their peers are thinking, too. All along the way, teachers can monitor work in real time.

Student experience

The student experience is intuitive and engaging, offering students **low floors and high ceilings** as they engage with the lesson content.



Activity 1: Walking Dogs Like a Pro

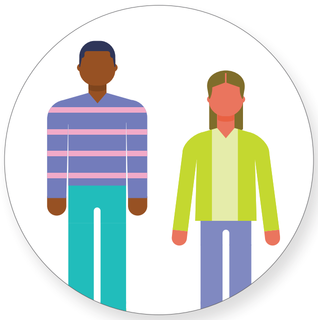
Welcome to Pawston University! We've produced some of the nation's finest dog walkers. Your first challenge is to balance some leashes.

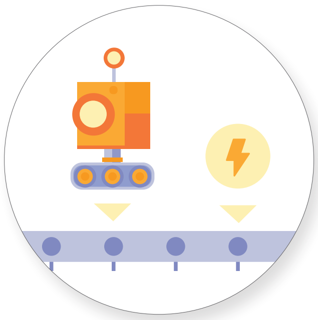
You can move the dogs by dragging the points below them. The number below each dog tells you the strength of that dog's pull.

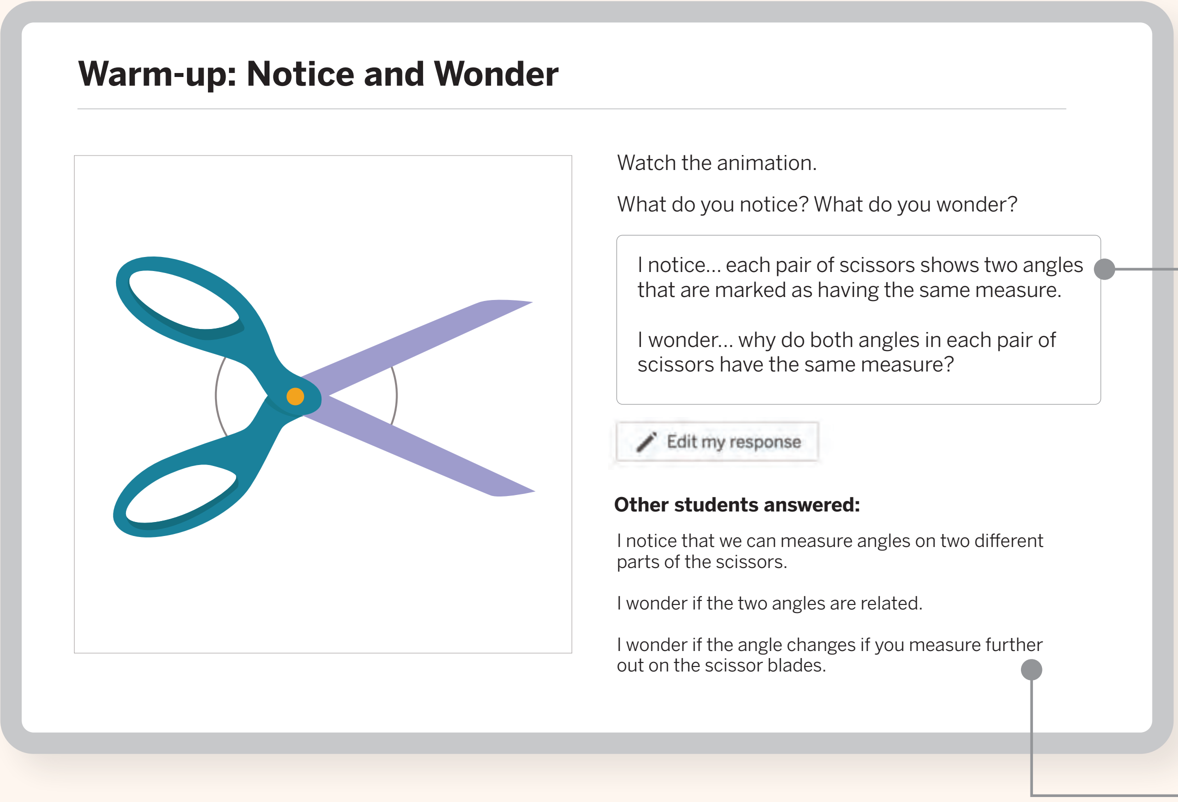
How can you balance all five dogs on two leashes, holding one leash in each hand? Write the names and strengths of the dogs you would put on the left side and the right side.

Left: Eartha (2) + Harriet (8)
Right: Greg (6) + Fifi (3) + Dale (1)

Many of the lessons feature rich, visual moments that students can interact with. Amps are meant to feel intuitive, immersive, and sometimes even playful.







Warm-up: Notice and Wonder

Watch the animation.

What do you notice? What do you wonder?

I notice... each pair of scissors shows two angles that are marked as having the same measure.

I wonder... why do both angles in each pair of scissors have the same measure?

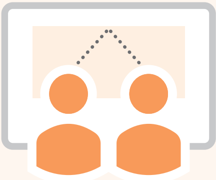
Other students answered:

I notice that we can measure angles on two different parts of the scissors.

I wonder if the two angles are related.

I wonder if the angle changes if you measure further out on the scissor blades.

As students work, the slides change, prompting students to **describe their strategies**. Teachers can see student work in real time and spotlight responses anonymously to support in-class discussion.



When working online, students will sometimes **see their peers' thinking** on their own screens. These connection points are great for sparking discussions ahead of the synthesis moment in the lesson.

Routines in Amplify Math

Routines help you and your students to maintain a sense of familiarity and structure throughout the school year. As the year progresses, routines free up time you would otherwise spend giving new directions for each activity.

Routine	What is it?	Where is it?
Turn and Talk	Turn and Talk can be done anytime, needs little preparation, and only takes a minute or two. Students turn and talk to their partner about another student's thinking, eventually deciding whether they agree or not and why. The students then switch roles. Eventually you will call on a few students to say what they think, or report what their partner said.	Use anytime students are working
Ask Three Before Me	Do you find yourself responding to all of your students' questions? This routine saves you time and empowers your students' agency and voice, while allowing them to view each other as knowledgeable resources. The routine is simple: When a student has a question, let them know they should ask three other students before they ask you.	
Go Find a Good Idea	When students are stuck and productive struggle has stalled, direct them to get up and walk around the room to find a good idea in the written work of other students. Encourage them to ask questions of other students and to explain their work. Then, they should bring a good idea back and continue working with it, citing from whom they got it.	
Notice and Wonder	Students are shown some media or a mathematical representation. They are prompted with "What do you notice? What do you wonder?," and are given a few minutes to think and share with a partner. Then you then ask several students to share what they noticed and wondered, recording responses for all to see. ***Note:*** Notice and Wonder and I Notice/I Wonder are trademarks of NCTM and the Math Forum and used in these materials with permission.	Warm-ups, Activity launches
Math Talks and Strings	Typically, one problem is displayed at a time. Problems are purposefully chosen to elicit different approaches, often in a way that builds from one problem to the next. These problems could be numerical or algebraic in nature. Give students a few moments to quietly think and give you a signal when they have an answer and a strategy, then select some to share different strategies for each problem. Ask questions like "Who thought about it a different way?"	Warm-ups
Which One Doesn't Belong?	Students are asked "Which one doesn't belong?" when presented with several figures, diagrams, graphs, or expressions. The best part is that every answer choice is correct! It's the similarities and differences among the options that are mathematically meaningful here. Prompt students to explain their rationale for deciding which option doesn't belong, and give them opportunities to make their rationale more precise.	Warm-ups
Card Sort	A card sort uses cards or slips of paper that can be manipulated and moved around. Individually, in pairs, or in small groups students put things into categories or groups based on shared characteristics or connections.	Activities
Find and Fix	Students are presented with the work of another hypothetical student whose work contains a flaw or error. They first identify any errors, then show or explain how to correct these errors, and explain why the person may have made the error.	Activities
Group Presentations and Gallery Tours	Instruct students—typically in groups—to create a visual display of their work, such as how they solved a problem with mathematical modeling, invented a new problem, designed a simulation or experiment, or organized and displayed data. In the Gallery Tour version of this routine, student work is captured on a piece of paper, a poster, or on an assigned portion of the board. Students then move around the room to observe, record notes or questions on their own paper, or write on each other's work (posing clarifying questions, giving kudos, or identifying portions they may disagree with). You lead a discussion, allowing students to respond to questions or critiques of their work.	Activities
Info Gap	One partner receives a problem card with a math question that doesn't have enough given information, and the other partner receives a data card with information relevant to the problem card. Students ask each other questions, such as "What information do you need?" and "Why do you need this information?" and are expected to explain what they will do with the information.	Activities

Math language development

All students, including English Learners, benefit from math lessons that are designed with strong math language development supports. Working in close partnership with the **English Learners Success Forum** (ELSF), the Amplify curriculum writers ensured the math language development strategies and supports in the program were clear, useful, and appropriate for all students. This was achieved by infusing the instruction with research-based Math Language Routines (MLRs) and by providing sentence frames where appropriate, both in the teacher language provided for each task and in the differentiation supports section. ELSF has helped review all student-facing content to ensure it's developmentally appropriate and reflects the fact that math is a new language students are mastering.

The math language development story starts at the unit level where teachers will see new vocabulary and a correlation of MLRs to lessons listed in the unit planning materials. In the Unit Study materials, one of the MLRs is highlighted under the Focus on Differentiated support section.

Embedded language development support

- **Course level:** The course design centers the development of communication skills.
- **Unit level:** Teachers will understand how language development progresses throughout the unit.
- **Lesson level:** Each lesson includes definitions of new vocabulary and language goals.
- **Activities:** Math Language Routines support the development of new vocabulary and activities often include additional supports based on language demands.
- **Assessments:** Suggested next steps offer guidance for teachers to support students' mathematical language development, based on their performance on assessment items.

Sentence frames

Sentence frames can give a student a sensible jumping off point for verbalizing or writing out an idea. Amplify Math leans on sentence frames that all serve one or more language functions, including describing a topic, explaining it, justifying thinking, generalizing, critiquing, representing, and interpreting ideas.

Math Language Routines

The Math Language Routines deployed throughout the Teacher Edition:

MLR1: Stronger and Clearer Each Time

MLR2: Collect and Display

MLR3: Critique, Correct, Clarify

MLR4: Information Gap

MLR5: Co-craft Questions

MLR6: Three Reads

MLR7: Compare and Connect

MLR8: Discussion Supports

Some routines adapted from Zwiers, J. (2014). Building academic language: Meeting Common Core Standards across disciplines, grades 5–12 (2nd ed.). San Francisco, CA: Jossey-Bass.

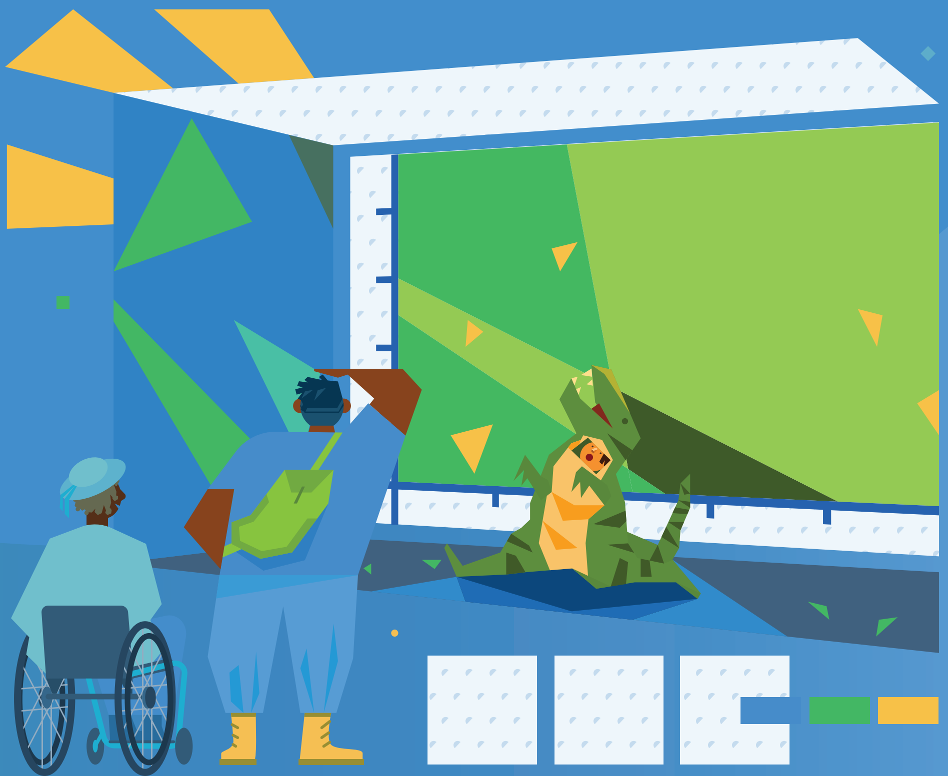
UNIT 1

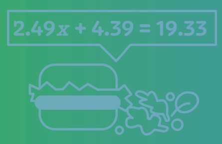
Linear Equations, Inequalities, and Systems

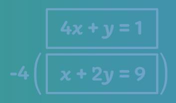
In this unit, students write and solve linear equations and inequalities to model the relationship between different quantities, recalling what is meant by a solution to an equation and interpreting them in context. They also solve systems of linear equations by graphing, substitution, and elimination.

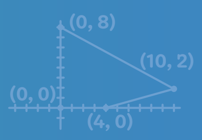
Essential Questions

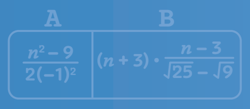
- How can equations and inequalities help you solve problems?
- Why is it useful to have different forms of linear equations?
- How can you use systems of equations or inequalities to model situations and solve problems?
- *(By the way, how can you make a decision if there are infinitely many possibilities?)*











Key Shifts in Mathematics

Focus

In this unit . . .

Students revisit solving one- and two-variable equations and inequalities. They further their understanding of solving equations by solving for a variable or variable expression. They learn new strategies to solve systems of equations. Students graph and solve a system of inequalities.

Coherence

Previously . . .

In Grade 7, students solved two-variable equations and inequalities. In Grade 8, students solved multi-step equations and systems of equations using tables, graphs, and substitution.

Coming soon . . .

In Unit 2, students will explore how the climate has changed over time in different regions using one- and two-dimensional statistics. They will represent and analyze data to draw their own conclusions.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students build their conceptual understanding of equivalent equations and their relationship to systems of linear equations across several lessons. They work toward understanding the solutions of two-variable inequalities by reasoning, graphing, and testing.

Procedural Fluency

Students build on their procedural fluency by solving one- and two variable equations and inequalities. They practice graphing equations of lines and inequalities. They develop procedural fluency solving systems of linear equations and inequalities.

Application

Throughout the unit, students apply their knowledge in multiple contexts. In the launch, they engage in decision making using the knapsack problem. They use spreadsheet technology to determine the number of employees who should be hired to work on a project.

Adulting
(Making Life Decisions)

SUB-UNIT

1

Lessons 2–6

Writing and Modeling With Equations and Inequalities

Students revisit how equations and inequalities can model and solve real-world problems, including summer jobs, transportation, and entertainment. They discover how equations and inequalities can help make decisions within the context of teenage money matters that involve ***constraints***.

Narrative: Making life decisions involves understanding unknowns and limitations — equations can help!

SUB-UNIT

2

Lessons 7–12

Manipulating Equations and Understanding Their Structure

Students begin to explore the structure of equations by understanding that multiple ***equivalent equations*** can represent the same relationship. They maintain equality when rearranging equations to isolate a variable of interest, including equations that contain two or more variables. Students rearrange linear equations from standard form to slope-intercept form and connect these equations to their graphs.

Narrative: Understanding equations can help you navigate decisions beyond high school.

SUB-UNIT

3

Lessons 13–16

Solving Inequalities and Graphing Their Solutions

Revisiting inequalities from middle school, students solve one-variable inequalities by reasoning about related equations. They move on to understand that a constraint on two variables can be represented by a linear inequality and the solution can be represented graphically as a ***half-plane*** constrained by a ***boundary line***.

Narrative: Your old friend — the inequality — is here to help you decide what's next after senior year.

Launch

Lesson 1

Homecoming In Style

Students wrangle with preparing for the homecoming dance through the eyes of other students. They consider transportation, clothing, hair, etc, while not only budgeting, but also considering what makes them happy. Students use the knapsack problem to explore their choices.

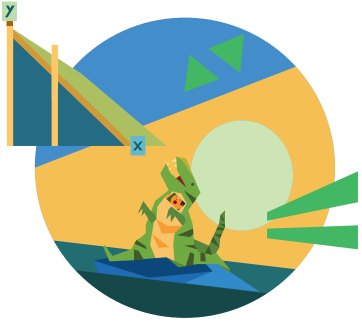
SUB-UNIT

4

Lessons 17–22

Systems of Linear Equations in Two Variables

Students use a variety of strategies to solve systems of linear equations, including graphing, substitution, and ***elimination*** methods. They analyze the structure of the equations of a linear system to determine whether the system will have one solution, no solution, or infinitely many solutions.



Narrative: Where to live after high school — systems of linear equations to the rescue!

SUB-UNIT

5

Lessons 23–25

Systems of Linear Inequalities in Two Variables

Students realize that the solution set of a ***system of linear inequalities*** in two variables consists of any pair of values that make both inequalities true. This solution set is represented graphically by the region where the graphs overlap.

Narrative: Discover how algebra can help you avoid choice overload.

Lesson 26

Capstone

Linear Programming

Students examine the fundraising efforts of two familiar characters. They analyze the maximum revenue they can raise by selling fundraising designs given specific constraints.

Unit at a Glance

Spoiler Alert: Solving systems of equations by elimination can actually be generalized into addition and multiplication.

Assessment

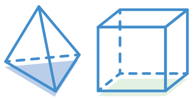
A Pre-Unit Readiness Assessment

Launch

1 Homecoming in Style

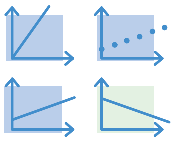
Use a variation of the knapsack problem to help weigh decisions about homecoming attire and accessories against costs and happiness.

Sub-Unit 1: Writing and Modeling With



2 Writing Equations to Model Relationships ●

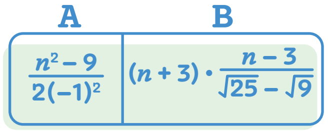
Given a description of a situation or an equation, identify varying quantities. Write equations with variables and numbers.



6 Equations and Their Graphs

Comprehend that the graph of a linear equation represents all pairs of values that are solutions to the equation.

Sub-Unit 2: Manipulating Equations and Understanding Their Structure

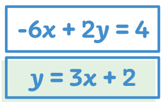


7 Equivalent Equations

Comprehend “equivalent equations” are equations that have exactly the same solutions, and multiple equivalent equations can represent the same relationship.

8 Explaining Steps for Rewriting Equations (optional) ●

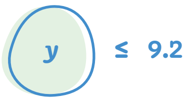
Explain why performing certain operations on an equation may result in equivalent equations, but performing other operations may not.



12 Connecting Equations in Slope-Intercept Form to Their Graphs

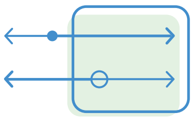
Given an equation of the form $ax + by = c$, write an equivalent equation of the form $y = mx + b$. Determine the slope and y-intercept in either form.

Sub-Unit 3: Solving Inequalities and Graphing Their Solutions



13 Inequalities and Their Solutions

Write and solve inequalities in one variable to represent the constraints in situations, understand solutions are a range of values.



14 Solving Two-Variable Inequalities

Determine the solution to two-variable inequalities by reasoning and by solving a related equation and testing values greater than and less than that solution.

Key Concepts

Lessons 9–10: Solving for variables or variable expressions.
Lesson 18–20: Solving systems of linear equations using substitution and elimination.
Lessons 23–25: Solving systems of linear equations and interpreting their graphs.

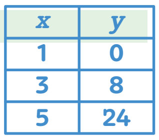
Pacing

26 Lessons: 50 min each
3 Assessments: 45 min each

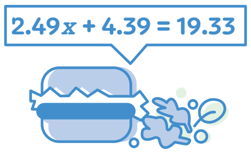
Full Unit: 29 days
● **Modified Unit:** 26 days

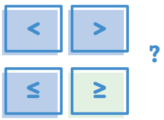
Assumes 50 minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

Equations and Inequalities



x	y
1	0
3	8
5	24





3 Strategies for Determining Relationships ●

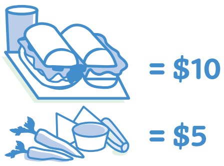
Identify and describe patterns in tables. Use patterns to generalize relationships and write equations.

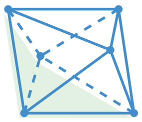
4 Equations and Their Solutions

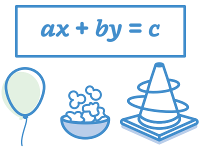
Explain, interpret, and determine solutions for one and two-variable equations given a context.

5 Writing Inequalities to Model Relationships

Interpret inequalities given a context. Write inequalities to represent the constraints of a context.







9 Rearranging Equations (Part 1)

Comprehend that to solve for a variable is to rearrange an equation to isolate a variable of interest.

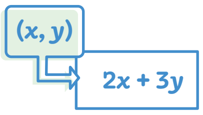
10 Rearranging Equations (Part 2)

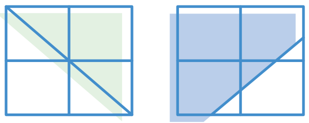
Write equations in two or more variables and solve for a specific variable.

11 Connecting Equations in Standard Form to Their Graphs

Analyze and graph equations of the form $ax + by = c$. Explain how a, b, and c are reflected on its graph.

Assessment





15 Graphing Two-Variable Linear Inequalities (Part 1)

Understand that the solutions to a linear inequality in two variables are represented graphically as a half-plane bounded by a line.

16 Graphing Two-Variable Linear Inequalities (Part 2)

Understand that a constraint on two variables can be represented by an inequality, a graph (a half-plane), and a verbal description.

A Mid-Unit Assessment

Unit at a Glance

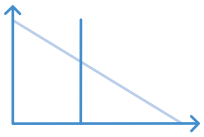
Spoiler Alert: Solving systems of equations by elimination can actually be generalized into addition and multiplication.

‹ continued

Sub-Unit 4: Systems of Linear Equations in Two Variables

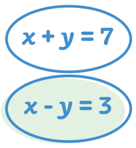
17 Writing and Graphing Systems of Linear Equations (optional) ●

Solve systems of linear equations by reasoning with tables and by graphing, and explain the solution method.



18 Solving Systems by Substitution

Recognize that there are multiple ways to perform substitution to solve a system of equations.



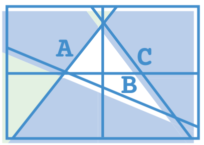
19 Solving Systems by Elimination: Adding and Subtracting (Part 1)

Recognize that adding or subtracting equations in a system creates a new equation with a solution that coincides with that of the original system.

Sub-Unit 5: Systems of Inequalities in Two Variables

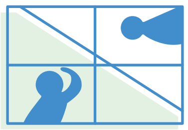
23 Graphing Systems of Inequalities

Explain how to tell whether an ordered pair is a solution to a system of inequalities. Graph systems of inequalities.



24 Solving and Writing Systems of Linear Inequalities

Understand that the solution set of a system of inequalities in two variables consists of any pair of values that make both inequalities true, and that it is represented graphically by the region where the graphs overlap.



25 Modeling With Systems of Linear Inequalities

Define the constraints in a situation and create a mathematical model to represent them.

Key Concepts

Lessons 9–10: Solving for variables or variable expressions.
Lesson 18–20: Solving systems of linear equations using substitution and elimination.
Lessons 23–25: Solving systems of linear equations and interpreting their graphs.

Pacing

26 Lessons: 50 min each
3 Assessments: 45 min each

Full Unit: 29 days
● **Modified Unit:** 26 days

Assumes 50 minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

$$\boxed{\begin{array}{l}4x + 3y = 10\\ -4x + 5y = 6\end{array}}$$

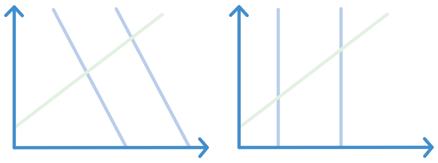
20 Solving Systems by Elimination: Adding and Subtracting (Part 2)

Explain why adding or subtracting two equations that share a solution results in a new equation that also shares the same solution.

$$\boxed{4x + y = 1}$$
$$-4\left(\boxed{x + 2y = 9}\right)$$

21 Solving Systems by Elimination: Multiplying

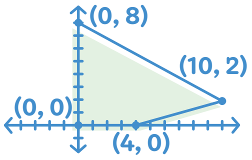
Solve systems of linear equations by multiplying one or both equations by a factor and then adding or subtracting the equations to eliminate a variable.



22 Systems of Linear Equations and Their Solutions

Determine whether a system of linear equations will have one solution, no solution, or infinitely many solutions by analyzing the structure of its equations.

Capstone



26 Linear Programming ●

Analyze given information about a scenario involving multiple constraints and write a mathematical model to represent it.

Assessment

A End-of-Unit Assessment

● Modifications to Pacing

Lessons 2, 3, and 17: These lessons revisit concepts first encountered in middle school, depending on the readiness check, you may choose to omit.

Lessons 8 and 17: These lessons are optional.

Lesson 26: The capstone is about linear programming which is outside the scope of Algebra 1.

Unit Supports

Math Language Development

Lesson	New vocabulary
1	constraint
7	equivalent equations
13	solution set
15	boundary line half-plane
19	elimination
21	equivalent systems
23	overlap of the graphs of the inequalities system of linear inequalities

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
1–3, 7, 13, 14	**MLR1:** Stronger and Clearer Each Time
1–3, 5, 7, 8, 12, 13, 15, 17–19, 21, 23, 24	**MLR2:** Collect and Display
15, 20, 21, 23	**MLR3:** Critique, Correct, Clarify
12, 23	**MLR4:** Information Gap
4, 11, 13, 16	**MLR5:** Co-craft Questions
5, 6, 10, 13, 15, 16, 20, 22, 26	**MLR6:** Three Reads
4, 6, 9, 11–14, 16–23, 25	**MLR7:** Compare and Connect
2–4, 6–10, 12, 15, 17, 19, 21, 22, 25, 26	**MLR8:** Discussion Supports

Materials

Every lesson includes:

Exit Ticket

Additional Practice

Additional required materials include:

Lesson(s)	Materials
17	counters
3, 12, 18	graph paper
6, 12, 16, 17, 19–22, 24	graphing technology
4, 7	music
1–13, 15–26	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
24	rulers
4, 6, 9, 10	scientific calculators
10	spreadsheet technology

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routines
11, 15	Algebra Talk
16, 21, 22	Card Sort
9, 12, 17	Gallery Tour
12, 23	Info Gap
7, 11, 19	Jigsaw
2, 3, 18	Math Talk
5, 7	Mix and Mingle
15, 16, 19, 22	Notice and Wonder
6, 12	Which One Doesn't Belong?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 16
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 25

Amps

POWERED BY desmos

Social & Collaborative Digital Moments

Featured Activity

Customer Receipts

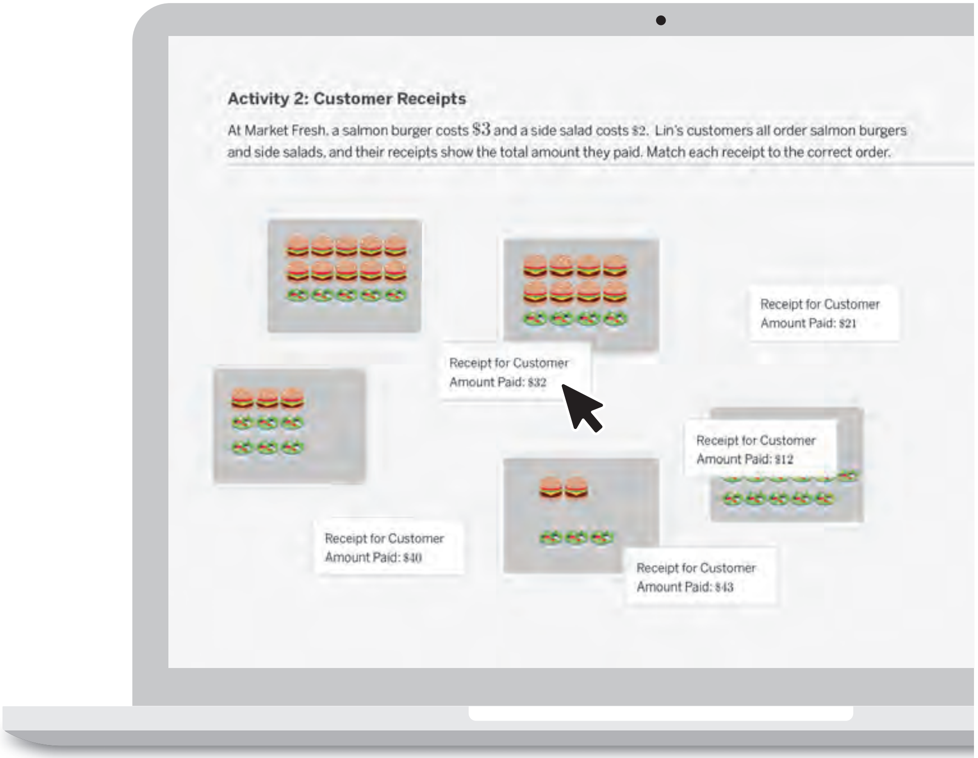
Put on your student hat and work through **Lesson 4, Activity 2:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Draining Gas Tank **(Lesson 9)**
- Digital Coin Jar **(Lesson 11)**
- Elevator Constraints **(Lesson 5)**
- Scavenger Hunt **(Lesson 23)**



Activity 2: Customer Receipts

At Market Fresh, a salmon burger costs $3 and a side salad costs $2. Lin's customers all order salmon burgers and side salads, and their receipts show the total amount they paid. Match each receipt to the correct order.

Receipt for Customer
Amount Paid: $21

Receipt for Customer
Amount Paid: $32

Receipt for Customer
Amount Paid: $12

Receipt for Customer
Amount Paid: $40

Receipt for Customer
Amount Paid: $43

Unit Study
Professional Learning

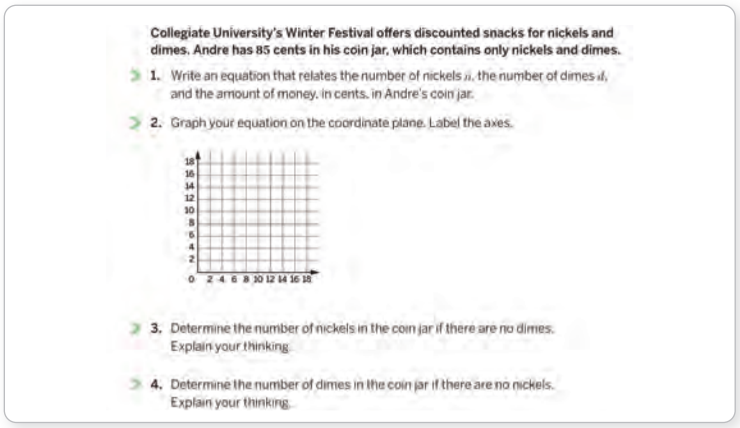
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Unit 1 begins with introducing students to constraints that they'd be familiar with and interested in. Students learn strategies to determine relationships that can be expressed as equations. They examine equations and their graphs and learn to solve equations by isolating one variable. They expand on their understanding of linear equations to linear inequalities and systems of linear equations. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from **Lesson 11, Activity 2:**



Collegiate University's Winter Festival offers discounted snacks for nickels and dimes. Andre has 85 cents in his coin jar, which contains only nickels and dimes.

1. Write an equation that relates the number of nickels n, the number of dimes d, and the amount of money, in cents, in Andre's coin jar.
2. Graph your equation on the coordinate plane. Label the axes.

3. Determine the number of nickels in the coin jar if there are no dimes. Explain your thinking.
4. Determine the number of dimes in the coin jar if there are no nickels. Explain your thinking.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

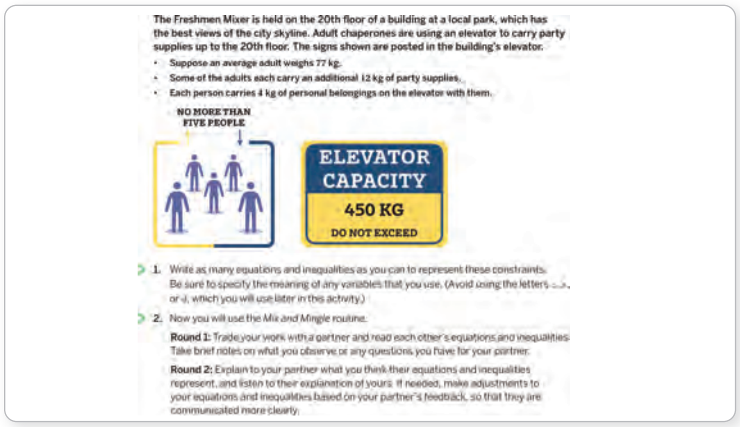
- What was it like to engage in this problem as a learner?
- This particular lesson addresses the equation in standard form. How might students differentiate this form from another, such as slope-intercept?
- Problem 2 asks students to graph the equation that they'd written. Do you anticipate students connecting the points on the graph? Why or why not?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Mix and Mingle

Rehearse . . .

How you'll facilitate the ***Mix and Mingle*** instructional routine in **Lesson 5, Activity 2:**



The Freshmen Mixer is held on the 20th floor of a building at a local park, which has the best views of the city skyline. Adult chaperones are using an elevator to carry party supplies up to the 20th floor. The signs shown are posted in the building's elevator.

- **Suppose an average adult weighs 77 kg.**
- **Some of the adults each carry an additional 12 kg of party supplies.**
- **Each person carries 4 kg of personal belongings on the elevator with them.**

1. Write as many equations and inequalities as you can to represent these constraints. Be sure to specify the meaning of any variables that you use. (Avoid using the letters [illegible], or [illegible], which you will use later in this activity.)
2. Now you will use the *Mix and Mingle* routine.

   Round 1: Trade your work with a partner and read each other's equations and inequalities. Take brief notes on what you observe or any questions you have for your partner.

   Round 2: Explain to your partner what you think their equations and inequalities represent, and listen to their explanation of yours. If needed, make adjustments to your equations and inequalities based on your partner's feedback, so that they are communicated more clearly.

Points to Ponder . . .

- Am I a model for giving good feedback? Do I only give 'cool' feedback on ways students can improve or strengthen their responses? Or do I also offer 'warm' feedback when students do good work? Do I allow opportunities for students to pause and make revisions based on my feedback or that of their peers? How can I be more intentional about using feedback to guide students to new understandings?

This routine . . .

- Encompasses MLR8 Discussion Supports.
- Encompasses the need for graphic organizers and sentence stem anchor charts for English Learners and any students who would benefit from them.
- Includes elements of MP3, where students construct viable arguments and critique the reasoning of their partners.
- Requires selection of appropriate music to signal the start/stop of partner discussions. Consider using instrumental music on a device or technology that can easily play and pause.

Anticipate . . .

- Preparing questions or providing those listed on the *Mix and Mingle* PDF to support student discussion.
- Providing a graphic organizer to help students organize their solutions or findings.
- If you *haven't* used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you *have* used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Establish mathematics goals to focus learning. Implement tasks that promote reasoning and problem solving.

These effective teaching practices . . .

- Establishes clear goals for both you and your students so that you both know what the lesson is targeting. If you know the target, you know whether or not you hit it.
- Provides opportunities for students to engage in low floor, high ceiling tasks that allow for multiple entry points and a variety of solution strategies.

Points to Ponder . . .

- How can you use the lesson goals to know if you need to redirect instruction or provide additional support?
- Where do you see opportunities to highlight different strategies that students used to approach the tasks in this unit?
- How can you encourage students to consider the alternative approaches that other students may have used?

Math Language Development

MLR7: Compare and Connect

MLR7 appears in Lessons 4, 6, 9, 11–14, 16–23, and 25.

- Students explore and learn about constraints in this unit, and connect constraints that are described verbally within a context to how they are represented algebraically and graphically.
- Throughout this unit, students connect verbal descriptions to graphical and algebraic representations for equations and inequalities in two variables. They connect the structure of the algebraic representations, such as coefficients and constants, to how they are represented graphically and look for key words and phrases in the verbal descriptions that indicate these quantities and relationships.
- **English Learners:** Annotate or highlight key phrases in the text that indicate constraints, such as *no more than* or *at least*.

Point to Ponder . . .

- How can you help your students understand the concept of solving problems within given constraints? What connections can you make to their daily lives?

Differentiated Support

Accessibility: Guide Processing and Visualization

Opportunities to provide visual support and guidance for information processing appear in Lessons 1, 2, 6, 8–10, 13, 15–17, 19–22, 24, and 26.

- Throughout the unit, anchor charts are provided for you to display or distribute to students, such as *Graphing Linear Inequalities*, *Writing a System of Equations From a Context*, and *Forms of Linear Equations*.
- Suggestions are provided in several lessons to display or distribute partially-completed tables or graphic organizers as a guide to support student thinking and organization.
- Use color coding or annotation to illustrate student thinking, such as:
  - Color coding given quantities and relationships in a narrative with how they are represented in a system of linear equations.
  - Color coding inequalities in a system of inequalities, where each inequality represents a given constraint.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to provide tables or graphic organizers, or when to suggest students use color coding to help them visualize and process information?

Unit Assessments

- Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **Mid-** and **End-of-Unit Assessments**, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with solving one-step equations with a variable throughout the unit? Do you think your students will generally:
  - Miss the underlying concept of balance and mathematical equality?
  - Simply struggle with the concept of variables and unknowns?
  - Be fully ready to solve procedurally and efficiently, but misapply the properties of equality?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their self awareness and self-management skills.

Points to Ponder . . .

- How do students perceive themselves? How does that perception affect their performance? Do students recognize their strengths and take confidence when a task fosters the ability to use their strengths? Can students remind themselves to have a growth mindset?
- Do students make constructive choices about their behavior and interactions with other people? Upon what do they base those choices? Can they evaluate a situation and determine what responsibility they have in solving the problem? How do they evaluate the consequences of their actions?

Homecoming in Style

Let's see how constraints can affect preparation for homecoming.

Focus

Goals

1. Use constraints in real-life contexts to maximize a value.
2. **Language Goal:** Determine which options meet the constraints in real-life context. **(Speaking and Listening, Writing)**
3. **Language Goal:** Comprehend the term *constraint* to mean a limitation on the possible or reasonable values a quantity could have. **(Speaking and Listening, Writing)**

Coherence

Today

Students brainstorm the constraints that restrict the decisions made in preparing for a school dance. In a version of the knapsack problem, with cost as the constraint, students choose from a bank of choices to maximize their "style points" when attending the homecoming dance. They explain how constraints affect their method of problem-solving.

Previously

Students investigated and interpreted how conditions constrain and affect the probability of events in Grade 7.

Coming Soon

Students will write linear equations to model various scenarios in Lessons 2 and 3.

Rigor

- Students build **conceptual understanding** of constraints.
- Students **apply** constraints in the context of decision making in context.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos ⋮ **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per pair
- Activity 1 PDF, one per student (as needed)
- Activity 2 PDF, pre-cut cards, one set per pair
- Activity 2 PDF, one per student (as needed)

Math Language Development

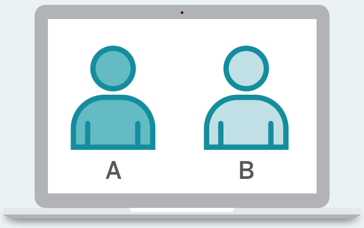
New words

- ***constraint***

Amps ⋮ **Featured Activity**

Activity 1
Digital Collaboration

Students work in pairs to determine which choices meet the constraints of attending the homecoming dance.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel disorganized in Activities 1 and 2 while attempting to determine a combination of options that meet the constraints. Discuss organizational strategies, including how to organize their choices in a table. By creating a structured display of the information, students will more easily be able to identify which combinations they have attempted, and to make calculations of total cost and total style points.

Modifications to Pacing

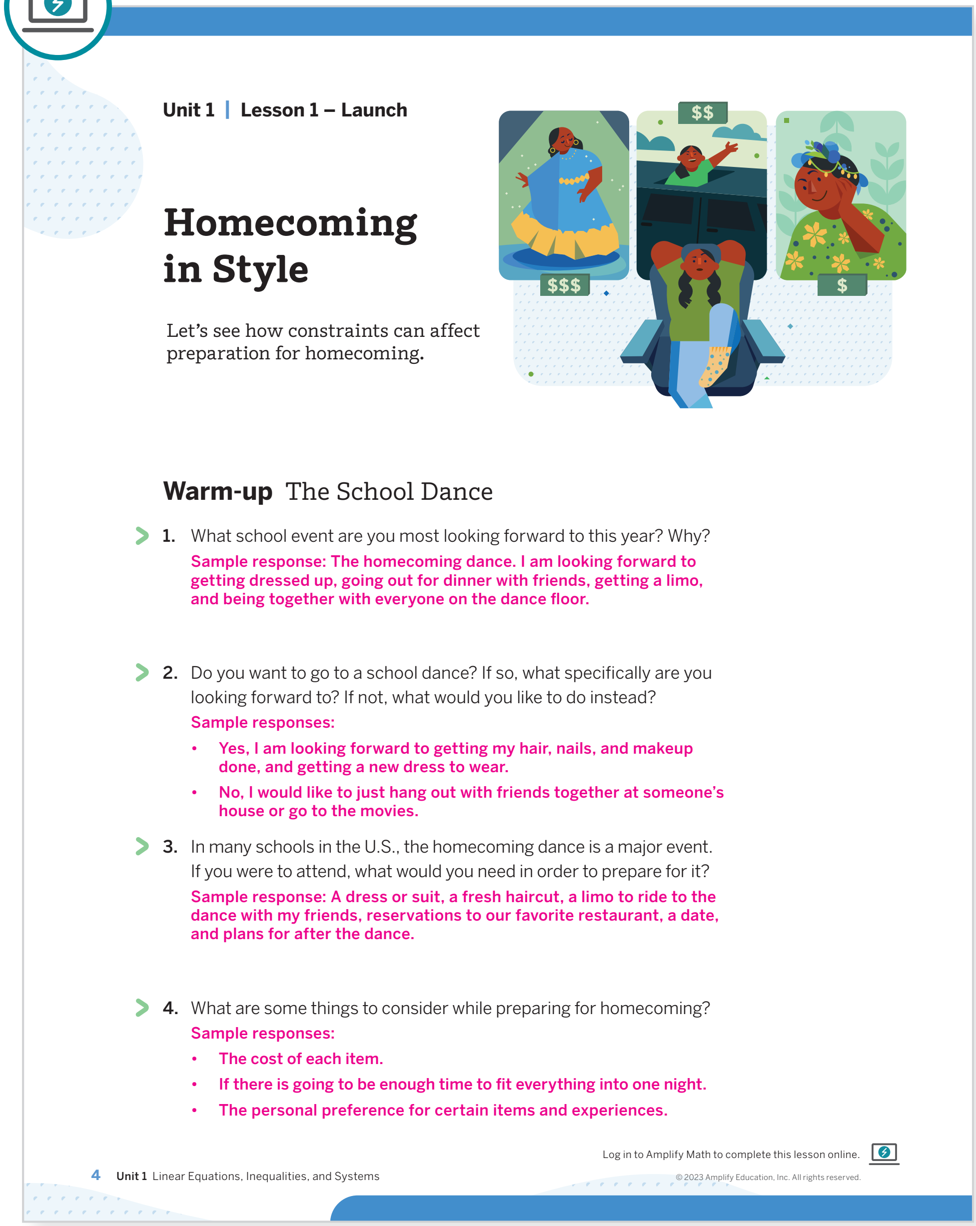
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 2 may be omitted.
- In **Activity 1** and **Activity 2**, have students only complete Problem 1.

Independent | 10 min

Warm-up The School Dance

Students explain how constraints impact choices in preparation for attending a high school dance.



Unit 1 | **Lesson 1 – Launch**

Homecoming in Style

Let's see how constraints can affect preparation for homecoming.

Warm-up The School Dance

1. What school event are you most looking forward to this year? Why?
   Sample response: The homecoming dance. I am looking forward to getting dressed up, going out for dinner with friends, getting a limo, and being together with everyone on the dance floor.

2. Do you want to go to a school dance? If so, what specifically are you looking forward to? If not, what would you like to do instead?
   Sample responses:
   - **Yes, I am looking forward to getting my hair, nails, and makeup done, and getting a new dress to wear.**
   - **No, I would like to just hang out with friends together at someone's house or go to the movies.**

3. In many schools in the U.S., the homecoming dance is a major event. If you were to attend, what would you need in order to prepare for it?
   Sample response: A dress or suit, a fresh haircut, a limo to ride to the dance with my friends, reservations to our favorite restaurant, a date, and plans for after the dance.

4. What are some things to consider while preparing for homecoming?
   Sample responses:
   - **The cost of each item.**
   - **If there is going to be enough time to fit everything into one night.**
   - **The personal preference for certain items and experiences.**

Log in to Amplify Math to complete this lesson online.

4 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Ask, "What are some school events that will occur this school year?" If "homecoming" was not a response, ask students if they know what it is. After a brief discussion, have students work individually to complete the problems, then have them share their responses with a partner.

2 Monitor

Help students get started by asking, "What are some things that could restrict your choices for homecoming?"

Look for points of confusion:

- **Listing only considerations that are not constraints.** Have students think about what other circumstances or limits restrict their choices for homecoming.

Look for productive strategies:

- Using their list from Problem 3 to determine considerations in Problem 4.

3 Connect

Have individual students share their responses to Problem 3.

Define the term ***constraint***.

Highlight that constraints are everywhere in the world, and limit the possible values or options that are available.

Ask, "How could constraints affect a student's enjoyment of going to homecoming?" Sample response: A student may not be able to afford their first choice for clothing, transportation, and other costs that go along with homecoming.

Math Language Development

MLR1: Stronger and Clearer Each Time

Have students write an initial draft of their responses for Problems 1–4. Then have them share their responses with 2–3 partners to give and receive feedback. After receiving feedback, provide students time to revise their response by incorporating or addressing new ideas they might not have previously considered.

English Learners

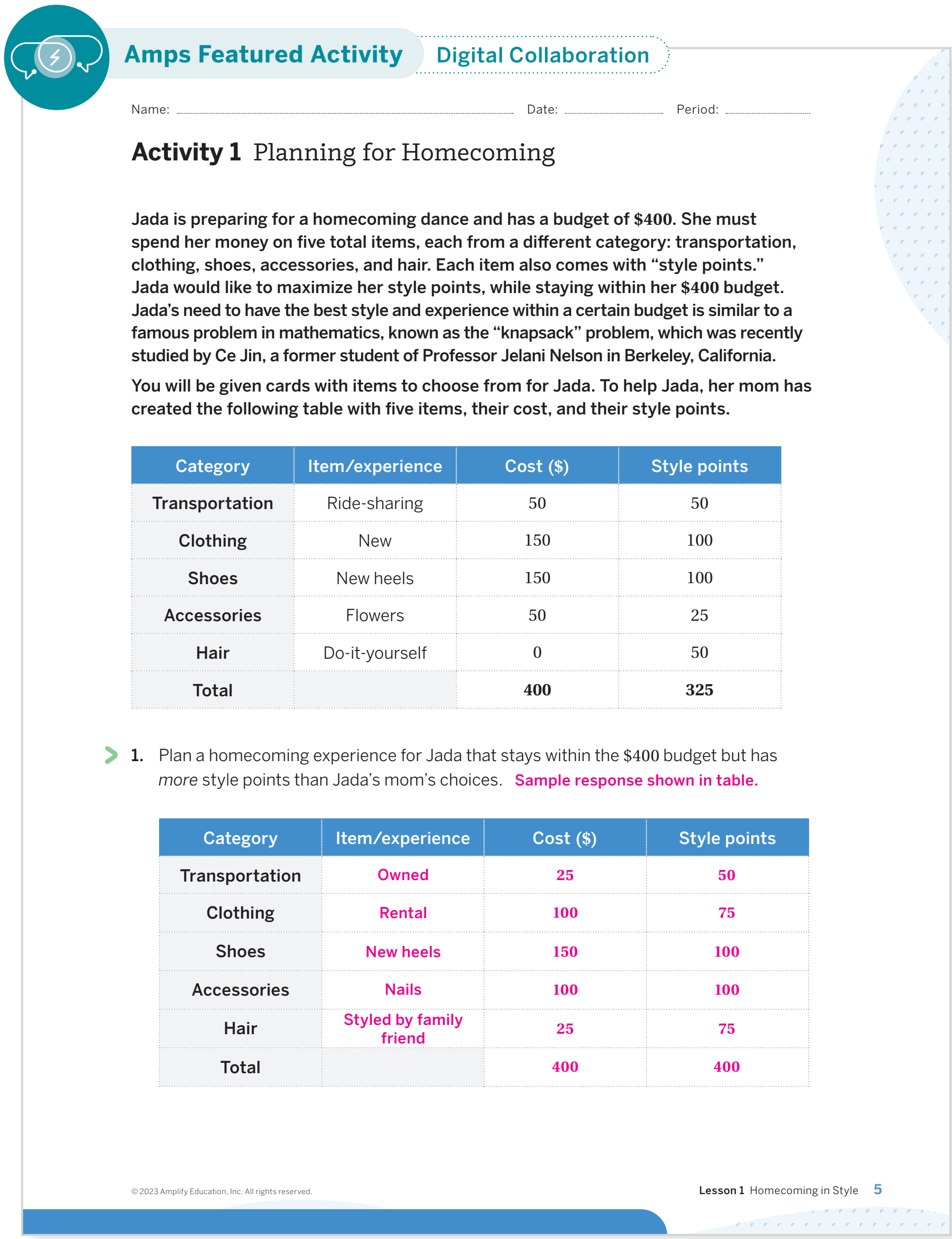
Allow pairs of students who speak the same primary language to draft their initial responses and provide feedback. Then have them write their revised responses in English.

Pairs | 15 min

Activity 1 Planning for Homecoming

Students reason quantitatively to prepare for homecoming by choosing from options with associated costs and styles points to make sense of budget constraints.

Amps Featured Activity Digital Collaboration



Name: ______________ Date: ______ Period: ______

Activity 1 Planning for Homecoming

Jada is preparing for a homecoming dance and has a budget of $400. She must spend her money on five total items, each from a different category: transportation, clothing, shoes, accessories, and hair. Each item also comes with "style points." Jada would like to maximize her style points, while staying within her $400 budget. Jada's need to have the best style and experience within a certain budget is similar to a famous problem in mathematics, known as the "knapsack" problem, which was recently studied by Ce Jin, a former student of Professor Jelani Nelson in Berkeley, California.

You will be given cards with items to choose from for Jada. To help Jada, her mom has created the following table with five items, their cost, and their style points.

Category	Item/experience	Cost ($)	Style points
Transportation	Ride-sharing	50	50
Clothing	New	150	100
Shoes	New heels	150	100
Accessories	Flowers	50	25
Hair	Do-it-yourself	0	50
Total		**400**	**325**

1. Plan a homecoming experience for Jada that stays within the $400 budget but has *more* style points than Jada's mom's choices. **Sample response shown in table.**

Category	Item/experience	Cost ($)	Style points
Transportation	Owned	25	50
Clothing	Rental	100	75
Shoes	New heels	150	100
Accessories	Nails	100	100
Hair	Styled by family friend	25	75
Total		400	400

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 1 Homecoming in Style 5

1 Launch

Arrange students in pairs. Read the scenario as a class. Provide each student with a set of the Activity 1 PDF, pre-cut cards.

2 Monitor

Help students get started by saying, "Keep track of the total cost and how much money is left in the budget to help you make the next choice."

Look for points of confusion:

- **Thinking that there is only one correct combination.** Have students attempt to determine other combinations after they have determined their first.
- **Making choices all at once leading to being over budget.** Have students make choices in one or two categories first, and then make choices in the remaining categories based on the money left in budget.

Look for productive strategies:

- Making choices in each category sequentially, so that subsequent choices are restricted by previous choices.
- Dividing up work in pairs where one student looks for a choice that meets the constraint and the other student calculates the total cost and money left in the budget.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with copies of the blank tables from the Activity 1 PDF. Suggest that they use these tables to help organize their thinking as they determine which items or experiences to choose, their cost, and style points.

Math Language Development

MLR2: Collect and Display

During the Connect, listen for the words and phrases students use as they share their choices and strategies. Write these words and phrases on a visual display, numbering each strategy, so students can refer back to these strategies in the next activity. If students do not use the term *constraint*, add it to the display and guide students towards using it. Encourage students to refer to this display throughout this unit.

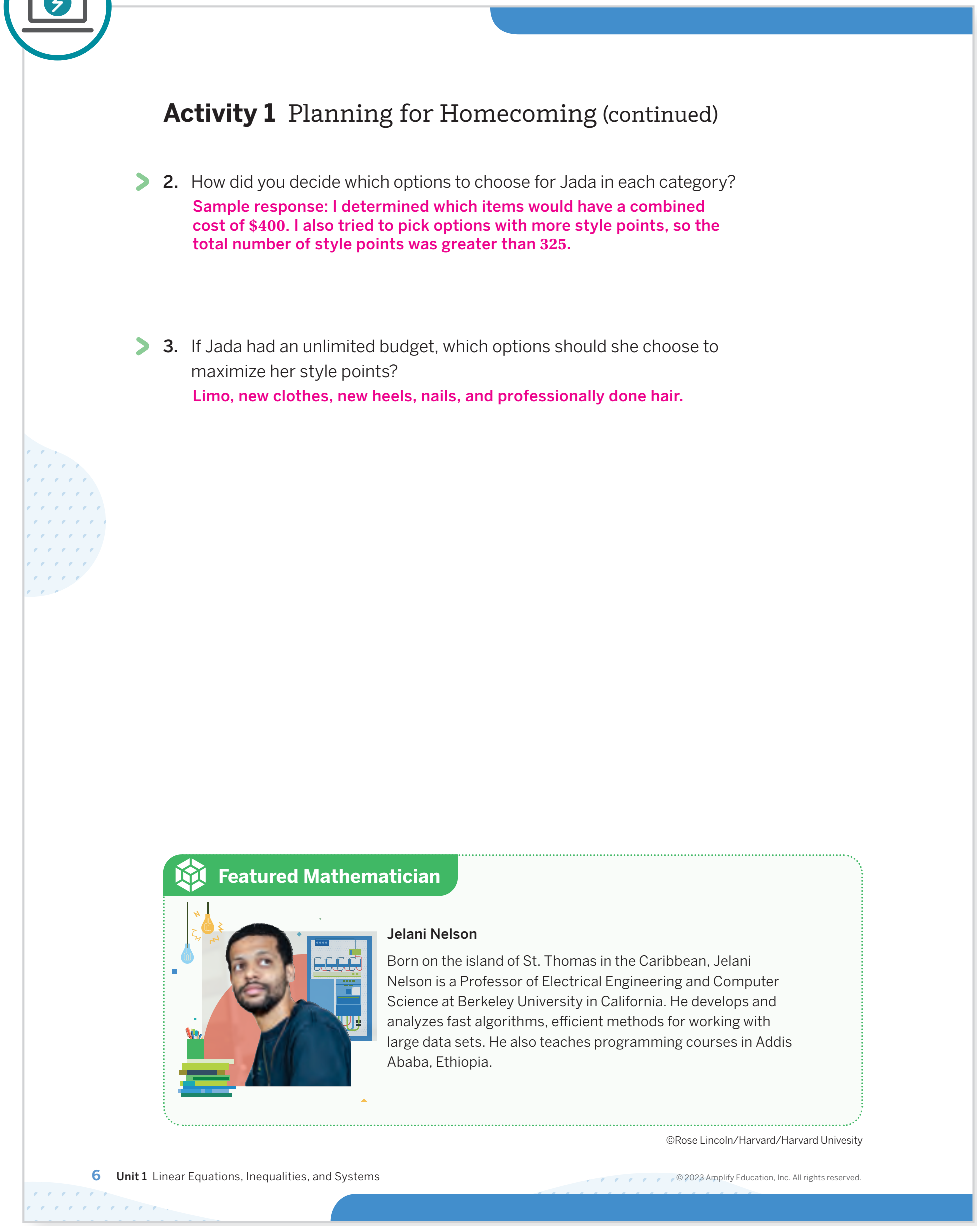
English Learners

Highlight student strategies that used a graphic organizer, table, or a similar visual method for keeping track of the total cost and budget.

Pairs | 15 min

Activity 1 Planning for Homecoming (continued)

Students reason quantitatively to prepare for homecoming by choosing from options with associated costs and style points to make sense of budget constraints.



Activity 1 Planning for Homecoming (continued)

2. How did you decide which options to choose for Jada in each category?
Sample response: I determined which items would have a combined cost of $400. I also tried to pick options with more style points, so the total number of style points was greater than 325.

3. If Jada had an unlimited budget, which options should she choose to maximize her style points?
Limo, new clothes, new heels, nails, and professionally done hair.

Featured Mathematician

Jelani Nelson

Born on the island of St. Thomas in the Caribbean, Jelani Nelson is a Professor of Electrical Engineering and Computer Science at Berkeley University in California. He develops and analyzes fast algorithms, efficient methods for working with large data sets. He also teaches programming courses in Addis Ababa, Ethiopia.

©Rose Lincoln/Harvard/Harvard Univesity

6 Unit 1 Linear Equations, Inequalities, and Systems

©2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have pairs of students share their combinations of choices and strategy they used to make these choices.

Highlight that the style points are not constrained. A choice made in one category will not limit the possible choices in other categories.

Ask:

- "What is the constraint in this problem? How do you know?" The budget, which limits the choices I can make.
- "How would this problem change if Jada and her date choose two options from each category, but within a higher budget?" Sample response: There would be many more combinations to check. Depending on the budget, there may be more, fewer, or the same number of combinations that would fit within the budget constraints.

Differentiated Support

Extension: *Math Enrichment*

Provide more context surrounding the "knapsack" problem mentioned in the activity. The idea is simple — you want to pack a knapsack given a set of items with assigned weights and values. The goal is to maximize the *value* of the items, while not going past a certain *weight limit*. The problem in this activity is a type of knapsack problem, which is really about resource allocation. Mention that variations of knapsack problems exist all around us in our everyday lives:

- How will you spend your paycheck to get the most value?
- How can you pack your suitcase for a flight to get the most needed items in, and so that the weight does not exceed 50 lb?

Ask students to generate their own "knapsack" problems they may face in their daily lives.

Featured Mathematician

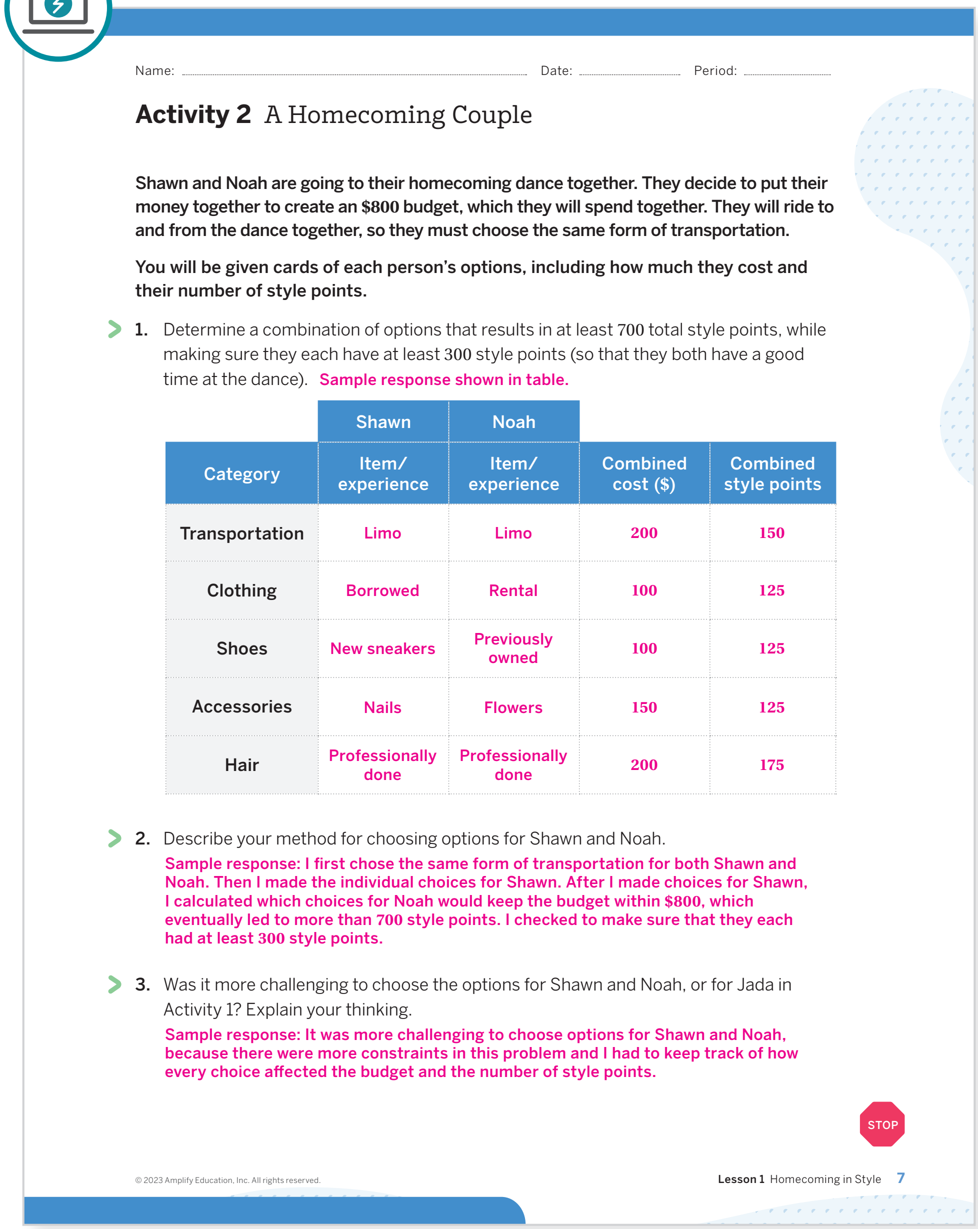
Jelani Nelson

Have students read about featured mathematician Jelani Nelson, who develops and analyzes fast algorithms and efficient methods for working with large data sets.

Pairs | 15 min

Activity 2 A Homecoming Couple

Students reason abstractly and quantitatively to choose homecoming options within the constraints of budget and style points.



Name: ______________ Date: ________ Period: ________

Activity 2 A Homecoming Couple

Shawn and Noah are going to their homecoming dance together. They decide to put their money together to create an $800 budget, which they will spend together. They will ride to and from the dance together, so they must choose the same form of transportation.

You will be given cards of each person's options, including how much they cost and their number of style points.

1. Determine a combination of options that results in at least 700 total style points, while making sure they each have at least 300 style points (so that they both have a good time at the dance). **Sample response shown in table.**

	Shawn	Noah		
Category	**Item/ experience**	**Item/ experience**	**Combined cost ($)**	**Combined style points**
Transportation	Limo	Limo	200	150
Clothing	Borrowed	Rental	100	125
Shoes	New sneakers	Previously owned	100	125
Accessories	Nails	Flowers	150	125
Hair	Professionally done	Professionally done	200	175

2. Describe your method for choosing options for Shawn and Noah.
Sample response: I first chose the same form of transportation for both Shawn and Noah. Then I made the individual choices for Shawn. After I made choices for Shawn, I calculated which choices for Noah would keep the budget within $800, which eventually led to more than 700 style points. I checked to make sure that they each had at least 300 style points.

3. Was it more challenging to choose the options for Shawn and Noah, or for Jada in Activity 1? Explain your thinking.
Sample response: It was more challenging to choose options for Shawn and Noah, because there were more constraints in this problem and I had to keep track of how every choice affected the budget and the number of style points.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Students remain in pairs. Read the scenario as a class. Provide each student with a set of the Activity 2 PDF, pre-cut cards. Have student pairs brainstorm a strategy before starting.

2 Monitor

Help students get started by asking, "How could you expand your method from Activity 1 to solve this problem?"

Look for points of confusion:

- **Meeting only one of the style constraints.** Have students check both the individual and combined style points after every choice to see if they meet the constraints.

Look for productive strategies:

- Persevering in problem-solving by replicating their Activity 1 strategy and organization by including a set of calculations to check the combined cost and style points.

3 Connect

Have pairs of students share their combinations of choices and strategy.

Display the choices student pairs share, and have the class check to see if these choices meet the constraints.

Highlight that the number of combinations that meet the constraints do not necessarily change from Activity 1.

Ask, "How do more constraints change a problem?" More constraints often limit the number of values or choices that are solutions, but sometimes more constraints do not affect the solution set.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with copies of the blank tables from the Activity 2 PDF. Suggest that they use these tables to help organize their thinking as they determine which items or experiences to choose, their combined cost, and combined style points.

Extension: *Math Enrichment*

Ask students to write equations or inequalities to represent the constraints in this activity. Have them define the variables they use.
Sample response: $T + C + S + A + H \le 800$ and $T_s + C_s + S_s + A_s + H_s \ge 700$. $T, C, S,$ A and H represent the total transportation, clothing, shoe, accessories, and hair cost, respectively. $T_s, C_s, S_s, A_s,$ and H_s represent the total transportation, clothing, shoe, accessories, and hair style points, respectively.

Math Language Development

MLR7: Compare and Connect

During the Connect, after students share their combinations and strategies, ask them how the number of combinations that meet the constraints in this activity compare to Activity 1. Then ask them how their strategy changed in this activity. Have each pair of students turn to another pair and discuss. Ask one pair of students to share their responses with the class.

English Learners

Connect the term *constraint* to the English word *restriction* and its Spanish equivalent, *restricción*, for students whose primary language is Spanish.

Whole Class | 5 min

Summary Adulting (Making Life Decisions)

Review and synthesize how constraints inform decision making.

Narrative Connections

Unit 1 Linear Equations, Inequalities, and Systems

Adulting (Making Life Decisions)

Pep rallies, bouquets, the crowning of a student court — homecoming can be a watershed moment in many teenagers' lives. It celebrates a school football team's first home game of a season. Students past and present "come home" to the school to celebrate, with the week's festivities culminating in a homecoming dance. Whether you choose to participate in homecoming or other activities, high school can be an exciting time. For many, it marks a step into the world of adulthood.

But becoming an adult can be a mixed bag of increased responsibility and freedom. Part of that process includes learning how to make choices.

In the years to come, you'll face all kinds of decisions — some as small as deciding what to have for lunch; and some big enough to impact the rest of your life (and even the lives of others). You'll have to make decisions about where you will live; what kind of work you will do; and what issues you will stand up for. And no matter what those choices are, there will be trade-offs to consider. It won't always be clear which option is best.

Constraints, like the budgets you saw in this lesson, can help frame those options. They can guide you in determining what your priorities are and establish what you *are* and *aren't* willing to compromise on.

In these next lessons, you will explore ways constraints can interact, as you build systems of linear equations and inequalities. These equations and systems will be powerful tools, allowing you to make choices that are thoughtful and consistent with the values that matter to you.

Welcome to Algebra 1.

8 Unit 1 Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Have students share what other constraints besides budget could limit choices for homecoming that could be included in the table.

Highlight that some constraints limit choices more than others. With enough constraints there may only be one choice or no choices for a scenario available.

Formalize vocabulary: ***constraint***

Display a completed table for Activity 2.

Ask:

- "What are the possible constraints for this table?" Sample response: The constraint could be a minimum amount of money spent or a minimum number of style points needed. There could also be a maximum budget, or the number of style points may only be under a certain amount.
- "How do the number of constraints affect the number of combinations that meet the constraints?" More constraints limit the number of combinations of choices that work, or it may not change the number of combinations that work.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

- "How do constraints affect decision-making?"
- "What strategies or tools did you find helpful today when considering constraints in your decision making? How were they helpful?"

Math Language Development

MLR2: Collect and Display

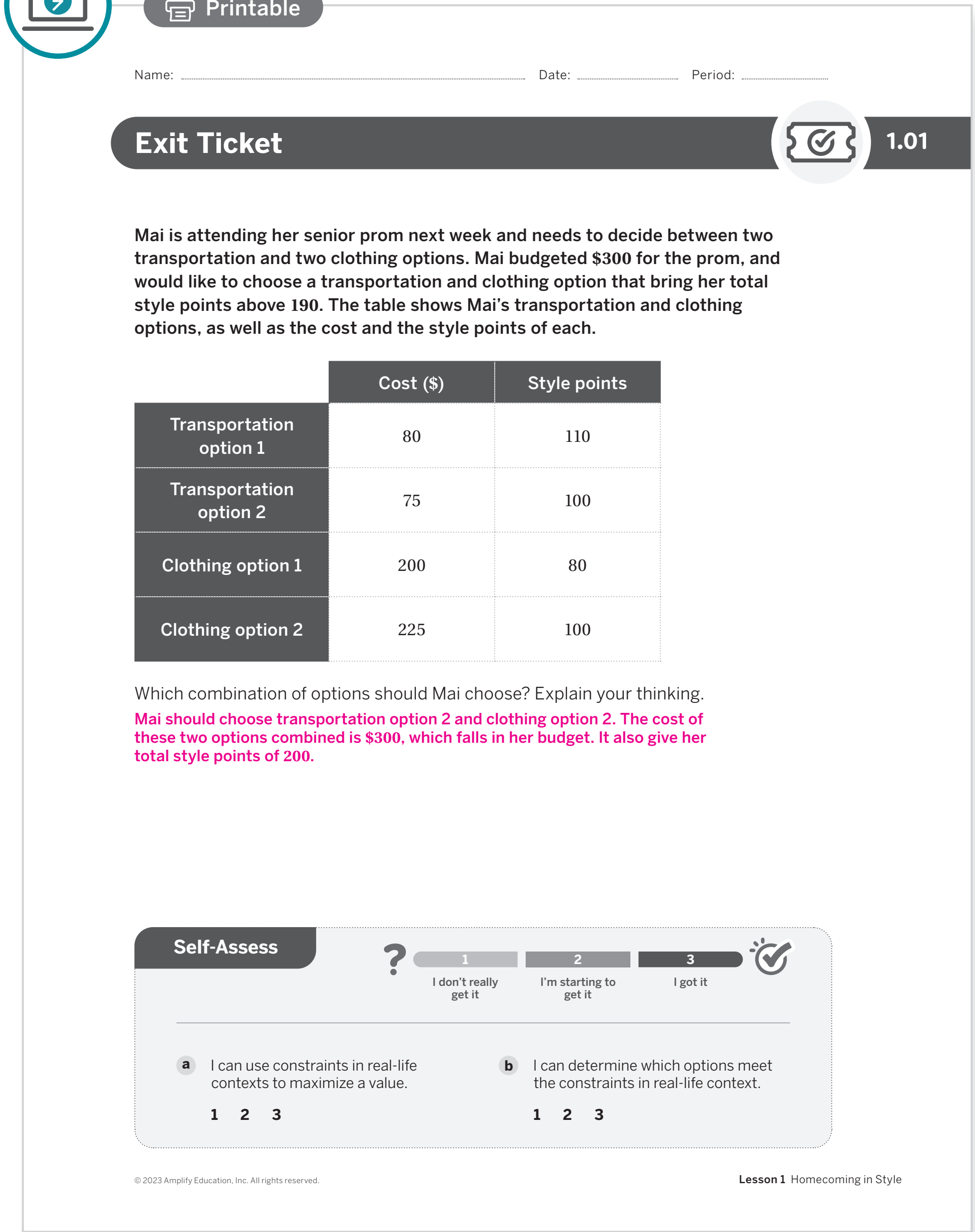
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in this lesson. Ask them to review and reflect on any terms and phrases related to the term *constraint* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students reason quantitatively by making choices for prom within a budget, demonstrating their understanding of the effects of constraints.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.01

Mai is attending her senior prom next week and needs to decide between two transportation and two clothing options. Mai budgeted $300 for the prom, and would like to choose a transportation and clothing option that bring her total style points above 190. The table shows Mai's transportation and clothing options, as well as the cost and the style points of each.

	Cost ($)	Style points
Transportation option 1	80	110
Transportation option 2	75	100
Clothing option 1	200	80
Clothing option 2	225	100

Which combination of options should Mai choose? Explain your thinking.

Mai should choose transportation option 2 and clothing option 2. The cost of these two options combined is $300, which falls in her budget. It also give her total style points of 200.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can use constraints in real-life contexts to maximize a value.
1 2 3

b I can determine which options meet the constraints in real-life context.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 1 Homecoming in Style

Success looks like . . .

- **Goal:** Using constraints in real-life contexts to maximize a value.
  - » Selecting the combination of options that fit the budget and desired total style points.
- **Language Goal:** Determining which options meet the constraints in real-life context. **(Speaking and Listening, Writing)**
  - » Choosing the option that costs less than $300 and gives style points above 190.
- **Language Goal:** Comprehending the term *constraint* to mean a limitation on the possible or reasonable values a quantity could have. **(Speaking and Listening, Writing)**

Suggested next step

If students cannot determine constraints that are within budget and above the style points goal, consider:

- Reviewing strategies of keeping track of the combinations of choices in Activity 1.
- Assigning Practice Problem 1.
- Asking, "How could you keep track of the combination of choices you have already checked against the constraints?"

Professional Learning

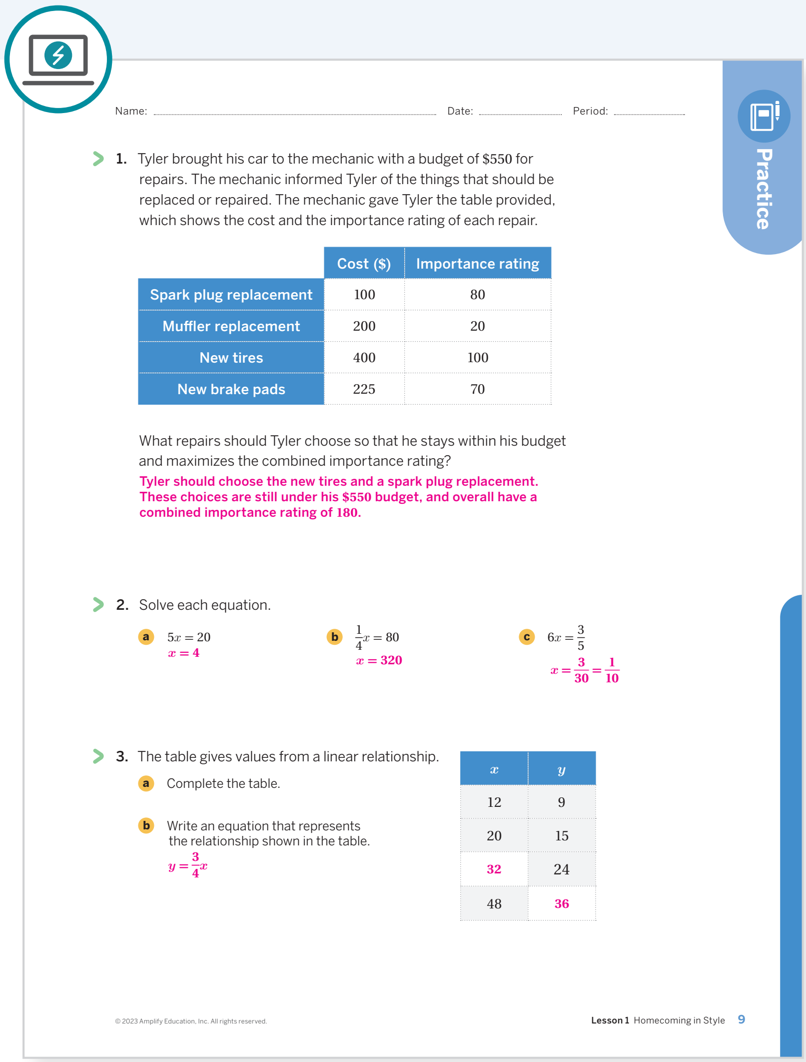
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students explored how constraints restrict decisions. How will that support their work with systems of linear equations and inequalities?
- What different ways did students approach organizing their choices? What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Tyler brought his car to the mechanic with a budget of $550 for repairs. The mechanic informed Tyler of the things that should be replaced or repaired. The mechanic gave Tyler the table provided, which shows the cost and the importance rating of each repair.

	Cost ($)	Importance rating
Spark plug replacement	100	80
Muffler replacement	200	20
New tires	400	100
New brake pads	225	70

What repairs should Tyler choose so that he stays within his budget and maximizes the combined importance rating?

Tyler should choose the new tires and a spark plug replacement. These choices are still under his $550 budget, and overall have a combined importance rating of 180.

2. Solve each equation.

a $5x = 20$
$x = 4$

b $\frac{1}{4}x = 80$
$x = 320$

c $6x = \frac{3}{5}$
$x = \frac{3}{30} = \frac{1}{10}$

3. The table gives values from a linear relationship.

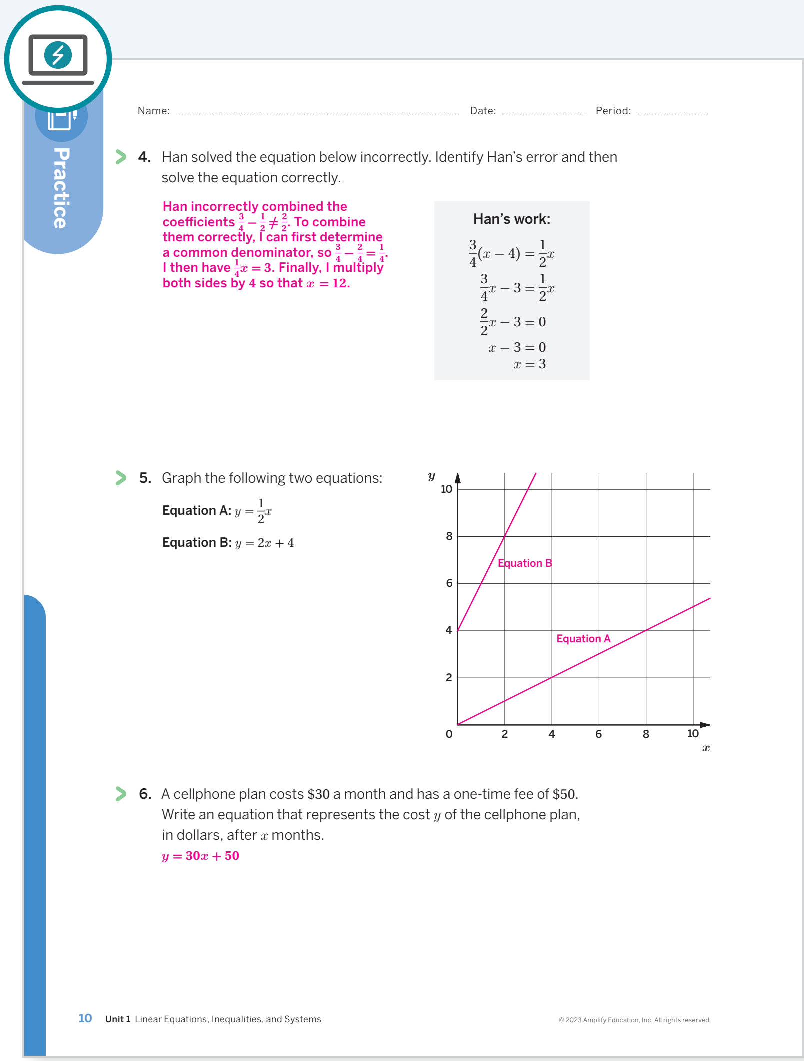
a Complete the table.

b Write an equation that represents the relationship shown in the table.
$y = \frac{3}{4}x$

x	y
12	9
20	15
32	24
48	36

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 1 Homecoming in Style 9



Name: Date: Period:

Practice

4. Han solved the equation below incorrectly. Identify Han's error and then solve the equation correctly.

Han incorrectly combined the coefficients $\frac{3}{4} - \frac{1}{2} \neq \frac{2}{2}$. To combine them correctly, I can first determine a common denominator, so $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$. I then have $\frac{1}{4}x = 3$. Finally, I multiply both sides by 4 so that $x = 12$.

Han's work:

$$\frac{3}{4}(x-4) = \frac{1}{2}x$$
$$\frac{3}{4}x - 3 = \frac{1}{2}x$$
$$\frac{2}{2}x - 3 = 0$$
$$x - 3 = 0$$
$$x = 3$$

5. Graph the following two equations:

Equation A: $y = \frac{1}{2}x$

Equation B: $y = 2x + 4$

6. A cellphone plan costs $30 a month and has a one-time fee of $50. Write an equation that represents the cost y of the cellphone plan, in dollars, after x months.
$y = 30x + 50$

10 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
Spiral	2	Grade 8	1
	3	Grade 8	2
	4	Grade 8	2
	5	Grade 8	2
Formative ⚡	6	Unit 1 Lesson 2	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice.**

Mathematical Modeling Prompt

At any point during the course and after completing this lesson, have students work in pairs or small groups on this unit's Mathematical Modeling Prompt, *Evaluating a Sample Response to a Modeling Prompt* (optional), which is available in the **Algebra 1 Additional Practice**.

Sub-Unit 1

Writing and Modeling With Equations and Inequalities

Whole Class

In this Sub-Unit, students revisit how equations and inequalities are used to model real-world situations. They discover how using equations and inequalities can help them make decisions.

SUB-UNIT

1 Writing and Modeling With Equations and Inequalities

Narrative Connections

How did a tragic accident end a three-month strike?

In the early 20th century, American factories employed children and young teenagers, expecting them to work long hours on dangerous machinery. Carmella Teoli was one such teenager. At 13, her hair got caught in the machinery at the mill where she worked. The incident left her hospitalized with a six-inch scar on her head.

At the same time, discontent was growing among the mill workers of New England — the majority of whom were immigrant women. Their hours were being cut and their wages lowered. By the time Teoli was released from the hospital, the Lawrence Textile Strike of 1912 was underway.

Teoli joined the striking workers. And when a Congressional hearing was called, she testified. Her account was so moving that President Taft launched a national investigation into factory working conditions. Three months after the strike began, the mill owners bowed to the strikers' demands.

As Teoli's story shows, working teenagers didn't always have options when it came to how they were treated. But thanks to the reforms spurred on by Teoli and the other strikers, teenagers working today have better choices when it comes to where and how they work. But the best choices aren't always obvious.

Will a summer job let you save enough money for a new car? Should you work somewhere closer to home for less money, or take a job farther away that you would have to commute to?

Making decisions like these can involve understanding what your unknowns are, taking into account the relevant facts, and being able to express your situation mathematically.

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students will see how constraints and algebraic thinking can help them analyze situations involving school, jobs, and even summer fun, in the following places:

- **Lesson 2, Activity 2:** Bus Fares and Summer Earnings
- **Lesson 5, Activity 1:** Planning the Freshman Mixer
- **Lesson 6, Activity 1:** Movie Night Snacks

UNIT 1 | **LESSON 2**

Writing Equations to Model Relationships

Let's write equations or inequalities that help us to model quantities and constraints.

Focus

Goals

1. **Language Goal:** Given a description of a situation, identify quantities that vary and quantities that do not. **(Reading and Writing)**
2. **Language Goal:** Given an equation, identify quantities that do and do not vary. **(Reading and Writing)**
3. **Language Goal:** Understand that variables can be used to represent both quantities that vary and those that are constant. **(Reading and Writing)**
4. **Language Goal:** Write equations with numbers and variables to describe relationships and constraints. **(Speaking and Listening, Writing)**

Coherence

Today

Students examine different situations which can be modeled with numeric and algebraic representations. They first create models for known quantities and move toward models in which the quantities are unknown or vary. Students interpret verbal descriptions and write equations and consider the featured mathematician, Leonhard Euler in Activity 1.

Previously

In the previous lesson, students brainstormed about the constraints in a specific situation and how it affected their decisions as well as their method of problem-solving.

Coming Soon

In Lesson 3, students will continue to build on their understanding of describing and writing equations to model relationships, by analyzing tables, looking for patterns, and interpreting the tables in context.

Rigor

- Students build **conceptual understanding** of using variables to model quantities given in a context.
- Students **apply** their equation-writing skills to several real-world contexts that are relevant to teenagers.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	10 min	5 min	5 min
Whole Class	Independent	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (as needed)
- Anchor Chart PDF, *Sentence Stems, Stronger and Clearer Each Time*
- Anchor Chart PDF, *Sentence Stems, Math Talk*

Math Language Development

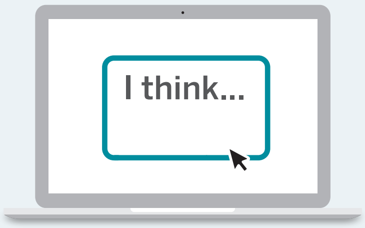
Review words

- *constraint*
- *variable*

Amps | Featured Activity

Activity 1
See Student Thinking

Students will explore the relationships between the faces, vertices, and edges of Platonic solids and write an equation to model the specific relationship that exists between all three.



Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel lost during Activity 2 as they are asked to write equations given progressively less information. Encourage students to develop a problem-solving plan where they look for what's the same in each subsequent problem and what changes. Suggest that they use their peers as a resource by asking them to explain the connections observed between each set of statements or equations.

Modifications to Pacing

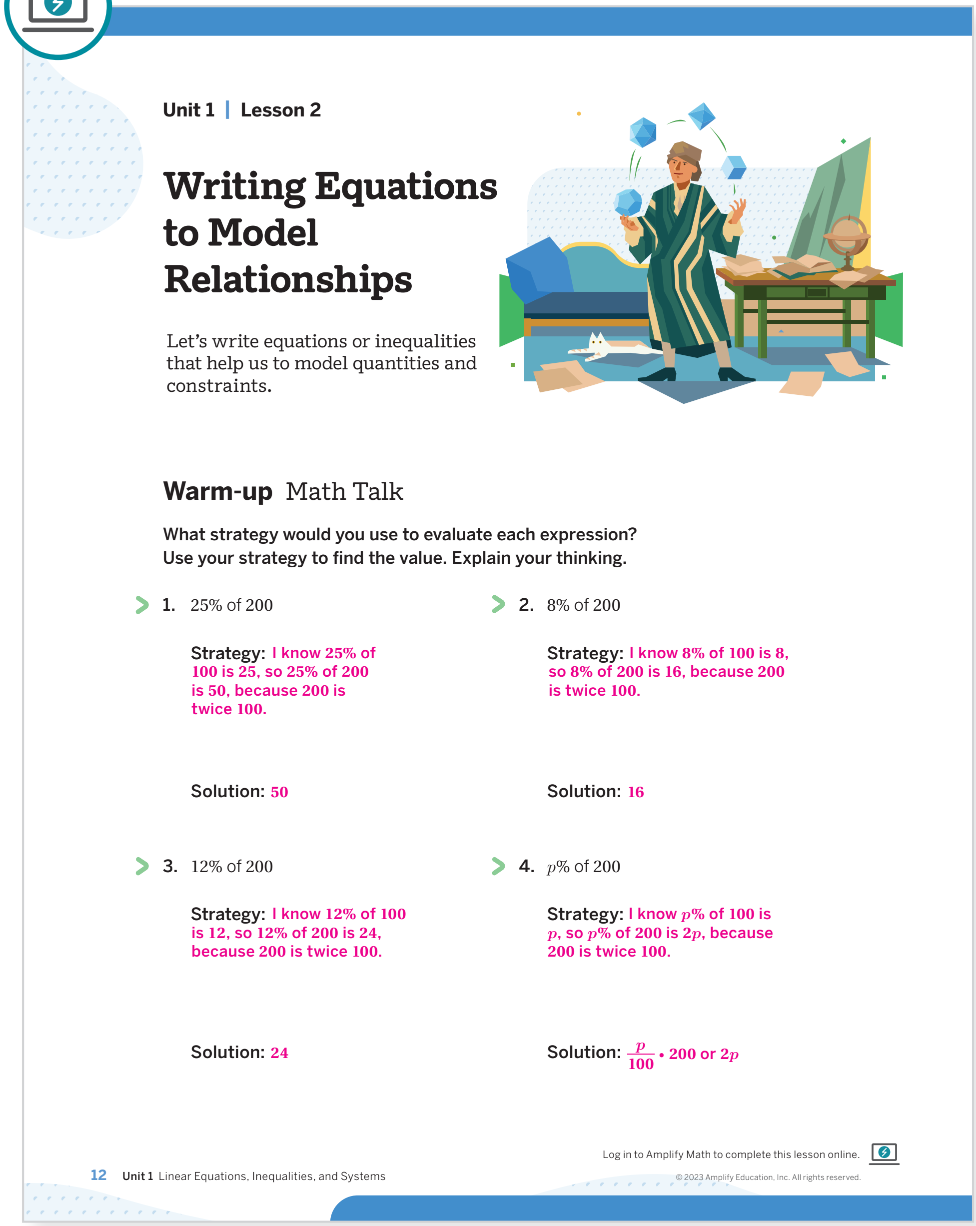
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problems 2a and 2b may be omitted.
- In **Activity 2**, have students only complete Problem 1 and incorporate Problem 2 into the discussion during the *Connect*.
- In **Activity 3**, Problems 3 and 4 may be omitted.

Whole Class | 5 min

Warm-up Math Talk

Students look for repeated reasoning by determining different percentages of 200 to elicit strategies and activate prior knowledge about the relationships between fractions, decimals, and percents.



Unit 1 | **Lesson 2**

Writing Equations to Model Relationships

Let's write equations or inequalities that help us to model quantities and constraints.

Warm-up Math Talk

What strategy would you use to evaluate each expression? Use your strategy to find the value. Explain your thinking.

1. 25% of 200

Strategy: I know 25% of 100 is 25, so 25% of 200 is 50, because 200 is twice 100.

Solution: 50

2. 8% of 200

Strategy: I know 8% of 100 is 8, so 8% of 200 is 16, because 200 is twice 100.

Solution: 16

3. 12% of 200

Strategy: I know 12% of 100 is 12, so 12% of 200 is 24, because 200 is twice 100.

Solution: 24

4. p% of 200

Strategy: I know p% of 100 is p, so p% of 200 is $2p$, because 200 is twice 100.

Solution: $\frac{p}{100} \cdot 200$ or $2p$

Log in to Amplify Math to complete this lesson online.

12 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Conduct the *Math Talk* routine. Display each problem one at a time. Allow students individual work time. Then place students in pairs to share their work with their partner. Keep all problems displayed throughout the discussion.

2 Monitor

Help students get started by asking, "What are some other ways you can represent 25%?"

Look for points of confusion:

- **Determining an operation to calculate percent.** Ask, "What operation is indicated by the word 'of?'"
- **Converting 8% to a decimal incorrectly in Problem 3.** Ask, "How many places must the decimal move when dividing by 100?"

Look for productive strategies:

- Converting each percentage into a fraction and multiplying the fraction by 200.
- Recognizing that 200 is $2 \cdot 100$, so any percentage of 200 can be found by multiplying any percentage of 100 by 2.

3 Connect

Have pairs of students share their strategies for each problem. Record and display their responses.

Ask:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone solve the problem in a different way?"

Highlight that a percentage is a fraction of 100, so a percentage of 200 would be the numerator of an equivalent fraction when 200 is the denominator.

Math Language Development

MLR8: Discussion Supports

During the Connect, display or provide students with a copy of the Anchor Chart PDF, *Sentence Stems*, *Math Talk* to support them as they explain the strategy they used for each problem. Consider providing students the opportunity to rehearse what they will say with a partner before they share with the whole class.

English Learners

Pair students together who speak the same primary language as they rehearse how they will explain their strategies.

Power-up

To power up students' ability to write an equation to model the relationship between two quantities, have students complete:

There is a 10% discount on cell phone plans.

1. If a cell phone plan costs $200, determine the discount, in dollars. Explain your thinking.
   $20; Sample response: I multiplied 200 by 0.10 to determine the discount.
2. Write an equation to calculate the discount y of a cell phone plan that costs x dollars, if the discount remains 10%.
   $y = 0.10x$

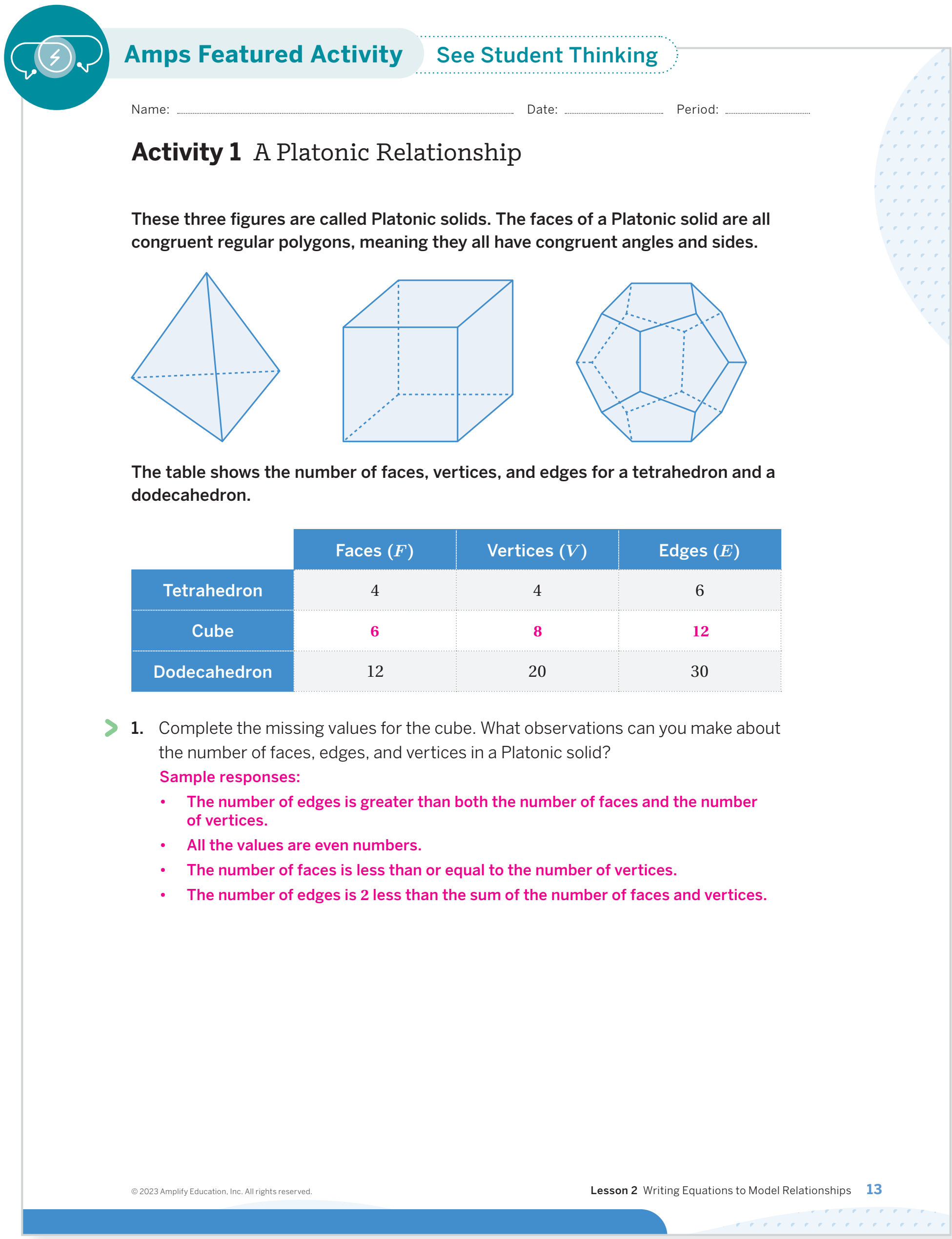
Use: Before the Warm-up
Informed by: Performance on Lesson 1, Practice Problem 6

Independent | 15 min

Activity 1 A Platonic Relationship

Students analyze the relationships between the faces, vertices, and edges of Platonic solids to look for and express regularity in repeated reasoning using an equation.

Amps Featured Activity See Student Thinking



Name: ______ Date: ______ Period: ______

Activity 1 A Platonic Relationship

These three figures are called Platonic solids. The faces of a Platonic solid are all congruent regular polygons, meaning they all have congruent angles and sides.

The table shows the number of faces, vertices, and edges for a tetrahedron and a dodecahedron.

	Faces (F)	Vertices (V)	Edges (E)
Tetrahedron	4	4	6
Cube	6	8	12
Dodecahedron	12	20	30

1. Complete the missing values for the cube. What observations can you make about the number of faces, edges, and vertices in a Platonic solid?

   Sample responses:
   - The number of edges is greater than both the number of faces and the number of vertices.
   - All the values are even numbers.
   - The number of faces is less than or equal to the number of vertices.
   - The number of edges is 2 less than the sum of the number of faces and vertices.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display the images of the three Platonic solids. If physical polyhedra are available, consider displaying those instead, or construct them using the Activity 1 PDF.

2 Monitor

Help students get started by asking how the solids are the same and how they are different, prompting them to use precise language to refer to the parts of the solid.

Look for points of confusion:

- **Having difficulty expressing a relationship they have found as an equation.** Have students write a statement to express the relationship and then to translate it into an equation, using the given variables.
- **Struggling to determine an equation that relates the parts of the Platonic solids in Problem 2c.** Prompt students to add the vertices and faces of each polyhedra.

Look for productive strategies:

- Substituting the values of F, V, and E into the given inequalities.
- Looking for patterns or performing operations within the table of values to determine a relationship.

Activity 1 continued ›

Differentiated Support

Accessibility: ***Optimize Access to Tools, Guide Processing and Visualization***

Consider bringing in physical polyhedra for students to feel and handle the solids. Alternatively, construct them using the Activity 1 PDF and make them available for students.

Accessibility: ***Activate Background Knowledge***

Remind students they learned about faces, vertices, and edges in middle school. Provide students with colored pencils or highlighters and ask them to color code a sample face, vertex, and edge of each solid to use as a reference during the activity.

Math Language Development

MLR2: Collect and Display

As students work, listen for and collect the language they use to describe how the Platonic solids are the same and how they are different. Record informal student language alongside precise mathematical terms on a visual display of the three solids and update the display throughout the lesson.

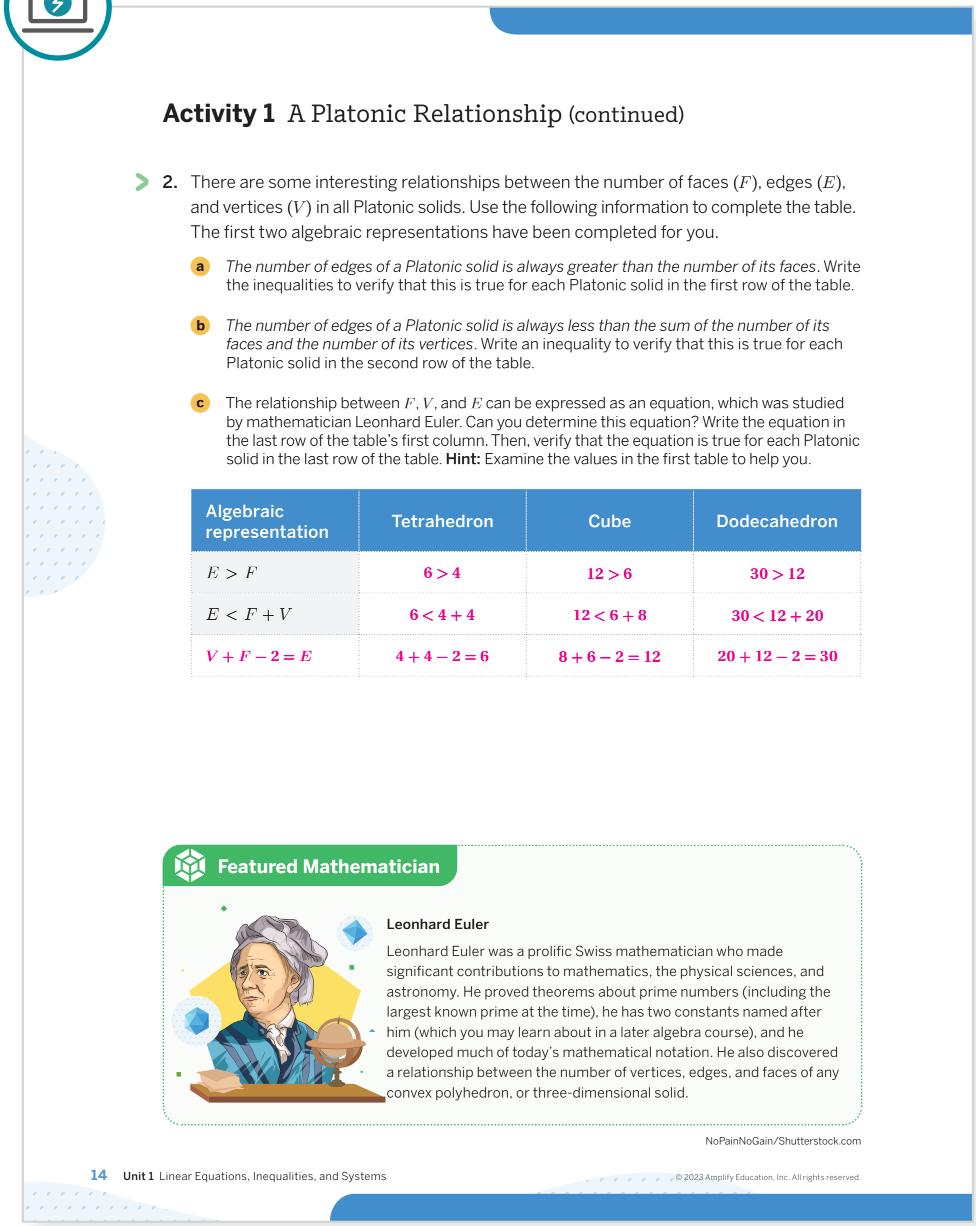
English Learners

Add visual images of a *face*, a *vertex*, and an *edge* with the terms labeled to help students make connections. Be sure to also include the term *vertices* as the plural of *vertex*.

Independent | 15 min

Activity 1 A Platonic Relationship (continued)

Students analyze the relationships between the faces, vertices, and edges of Platonic solids to look for and express regularity in repeated reasoning using an equation.



Activity 1 A Platonic Relationship (continued)

2. There are some interesting relationships between the number of faces (F), edges (E), and vertices (V) in all Platonic solids. Use the following information to complete the table. The first two algebraic representations have been completed for you.

   a *The number of edges of a Platonic solid is always greater than the number of its faces*. Write the inequalities to verify that this is true for each Platonic solid in the first row of the table.

   b *The number of edges of a Platonic solid is always less than the sum of the number of its faces and the number of its vertices*. Write an inequality to verify that this is true for each Platonic solid in the second row of the table.

   c The relationship between F, V, and E can be expressed as an equation, which was studied by mathematician Leonhard Euler. Can you determine this equation? Write the equation in the last row of the table's first column. Then, verify that the equation is true for each Platonic solid in the last row of the table. **Hint:** Examine the values in the first table to help you.

Algebraic representation	Tetrahedron	Cube	Dodecahedron
$E > F$	$6 > 4$	$12 > 6$	$30 > 12$
$E < F + V$	$6 < 4 + 4$	$12 < 6 + 8$	$30 < 12 + 20$
$V + F - 2 = E$	$4 + 4 - 2 = 6$	$8 + 6 - 2 = 12$	$20 + 12 - 2 = 30$

Featured Mathematician

Leonhard Euler

Leonhard Euler was a prolific Swiss mathematician who made significant contributions to mathematics, the physical sciences, and astronomy. He proved theorems about prime numbers (including the largest known prime at the time), he has two constants named after him (which you may learn about in a later algebra course), and he developed much of today's mathematical notation. He also discovered a relationship between the number of vertices, edges, and faces of any convex polyhedron, or three-dimensional solid.

NoPainNoGain/Shutterstock.com

14 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have students share their observations about the relationships between the quantities in the table. Have them support their observations using specific values from the table. Then select students who were able to determine a correct equation for Problem 2c and record and display them. (If students produce only one correct equation, introduce a variant such as $V + F - E = 2$ or $V + F - E - 2 = 0$.)

Display the *Featured Mathematician* box, if time permits, showing the equivalent equation $V - E + F = 2$ discovered by Euler, a Swiss mathematician.

Ask, "Do these equations all represent the same relationship? How do you know?"

Highlight that the equations displayed are equivalent. Show their equivalence by substituting the values in the table into each of them.

Differentiated Support

Extension: *Math Enrichment*

Tell students that there are actually 5 Platonic solids: Tetrahedron, Cube (the only regular Hexahedron), Octahedron, Dodecahedron, and Icosahedron. Consider showing a visual of each Platonic solid or have students research them. Provide students with the number of faces for each of the remaining solids and have them use the formula from this activity to determine the number of edges and vertices for the two remaining Platonic solids: Octahedron and Icosahedron.

Octahedron	Icosahedron
8 triangular faces	20 triangular faces

Octahedron: 6 vertices, 12 edges
Icosahedran: 12 vertices, 30 edges

Featured Mathematician

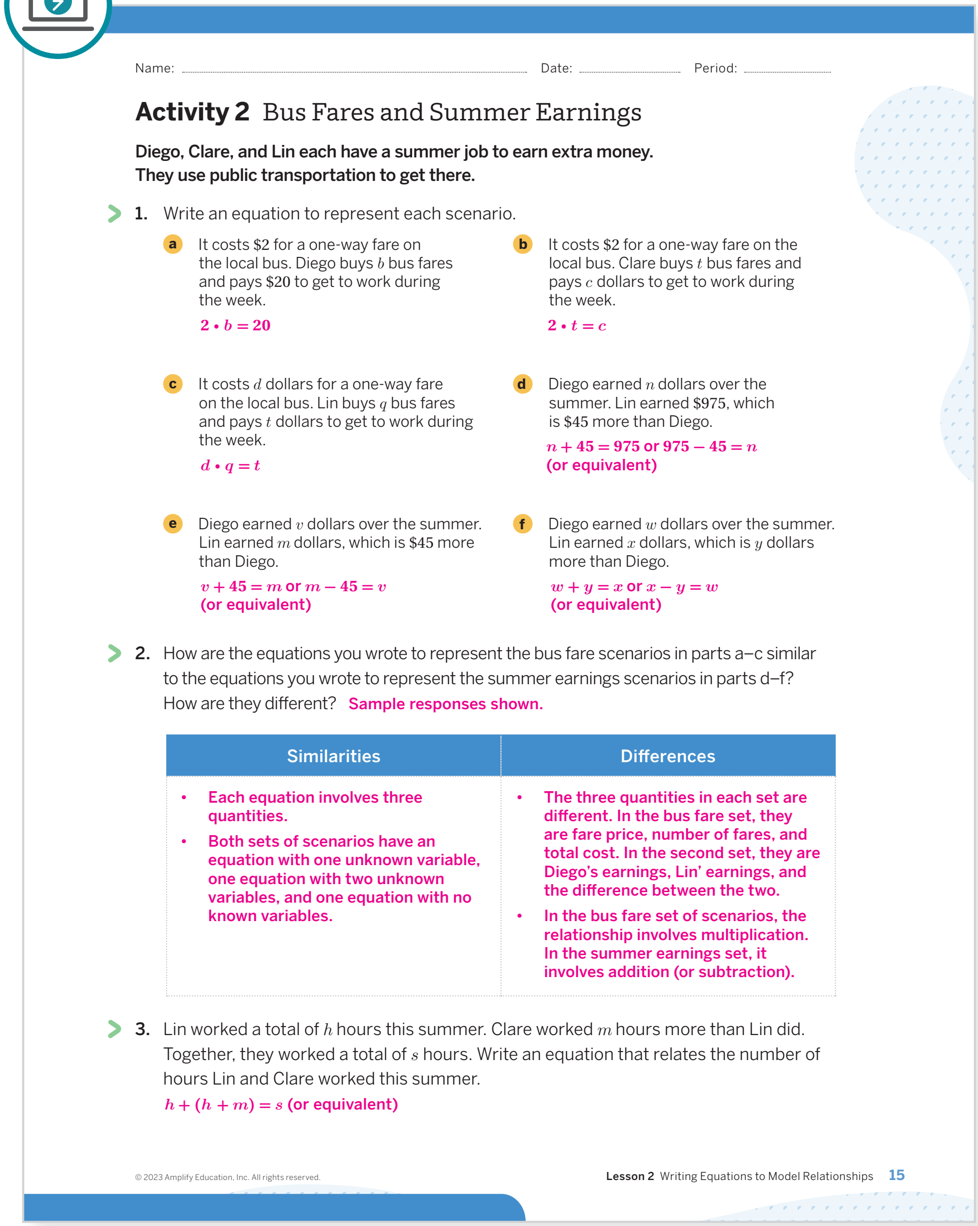
Leonhard Euler

Have students read about featured mathematician Leonhard Euler, who made significant contributions to mathematics, physical sciences, and astronomy, and discovered a relationship between the number of vertices, edges, and faces of any convex polyhedron, or three-dimensional solid.

Pairs | 10 min

Activity 2 Bus Fares and Summer Earnings

Students write equations to represent quantities and constraints to demonstrate how variables are used to represent unknown or varying quantities.



Name: ______ Date: ______ Period: ______

Activity 2 Bus Fares and Summer Earnings

Diego, Clare, and Lin each have a summer job to earn extra money. They use public transportation to get there.

1. Write an equation to represent each scenario.

   a. It costs $2 for a one-way fare on the local bus. Diego buys b bus fares and pays $20 to get to work during the week.

   $2 \cdot b = 20$

   b. It costs $2 for a one-way fare on the local bus. Clare buys t bus fares and pays c dollars to get to work during the week.

   $2 \cdot t = c$

   c. It costs d dollars for a one-way fare on the local bus. Lin buys q bus fares and pays t dollars to get to work during the week.

   $d \cdot q = t$

   d. Diego earned n dollars over the summer. Lin earned $975, which is $45 more than Diego.

   $n + 45 = 975$ or $975 - 45 = n$ (or equivalent)

   e. Diego earned v dollars over the summer. Lin earned m dollars, which is $45 more than Diego.

   $v + 45 = m$ or $m - 45 = v$ (or equivalent)

   f. Diego earned w dollars over the summer. Lin earned x dollars, which is y dollars more than Diego.

   $w + y = x$ or $x - y = w$ (or equivalent)

2. How are the equations you wrote to represent the bus fare scenarios in parts a–c similar to the equations you wrote to represent the summer earnings scenarios in parts d–f? How are they different? **Sample responses shown.**

Similarities	Differences
• Each equation involves three quantities. • Both sets of scenarios have an equation with one unknown variable, one equation with two unknown variables, and one equation with no known variables.	• The three quantities in each set are different. In the bus fare set, they are fare price, number of fares, and total cost. In the second set, they are Diego's earnings, Lin' earnings, and the difference between the two. • In the bus fare set of scenarios, the relationship involves multiplication. In the summer earnings set, it involves addition (or subtraction).

3. Lin worked a total of h hours this summer. Clare worked m hours more than Lin did. Together, they worked a total of s hours. Write an equation that relates the number of hours Lin and Clare worked this summer.

   $h + (h + m) = s$ (or equivalent)

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 2 Writing Equations to Model Relationships 15

1 Launch

Set an expectation for the amount of time students will have to work on the activity in pairs.

2 Monitor

Help students get started by asking, "How much does it cost for 2 bus fares? 3? 4?" Have students articulate what they are doing to determine the cost.

Look for points of confusion:

- **Writing an equation that does not model the given scenario.** Have students substitute values into their equation to see if it represents the given constraints.

Look for productive strategies:

- Articulating that the set of equations for each scenario are essentially the same, except for the number of known and unknown quantities.
- Recognizing the usage progression of numbers and variables to only variables for each scenario.

3 Connect

Have pairs of students share their observations about how the equations for parts a–c are similar to those for parts d–f. Then ask how the equations within each set are different. Select students who used productive strategies to explain what they noticed about the quantities or the use of numbers and variables within each specific set.

Highlight that sometimes students know how quantities are related, but their values may be unknown or may vary (change). When writing equations, they often use variables or symbols to represent unknown or varying quantities.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have student pairs focus on writing equations only for Problems 1a–1c, or only for Problems 1d–1f. Consider allowing them to choose which set of three parts they would like to complete. Save Problem 2 for a whole-class discussion.

Extension: *Math Enrichment*

Have students complete the following as a follow-up to Problem 3: Diego worked half as many hours as Lin and Clare worked combined. Write an equation that represents the total number of hours t the three friends worked. $t = h + (h + m) + \frac{h + (h + m)}{2}$ (or equivalent)

Math Language Development

MLR1: Stronger and Clearer Each Time

After students have responded to Problem 2 and before the Connect discussion, display the Anchor Chart PDF, *Sentence Stems, Stronger and Clearer Each Time*. Have each pair of students meet with another pair and use the anchor chart questions to ask clarifying questions to help partners improve their responses. Provide time for partners to revise their response, before moving into the Connect discussion.

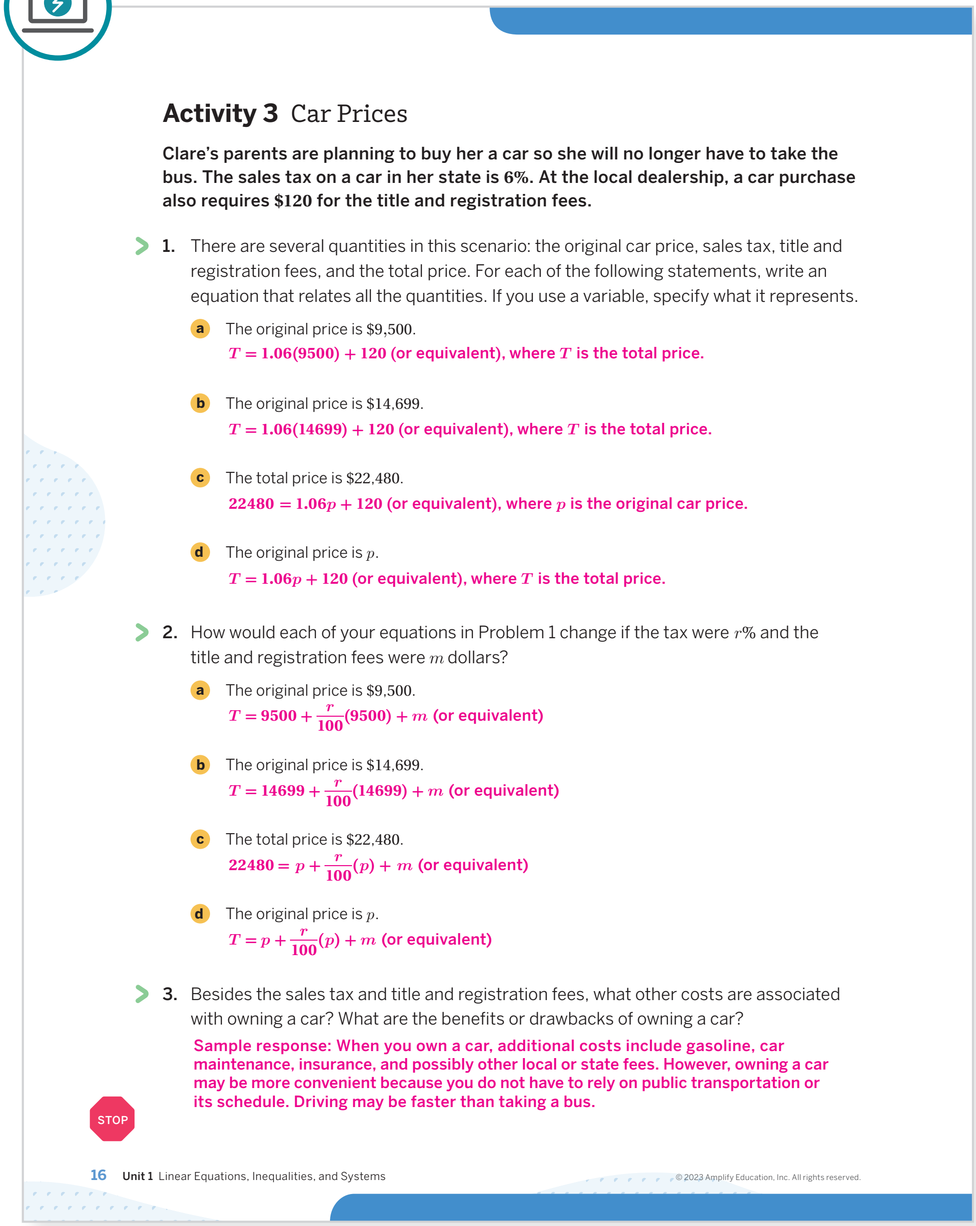
English Learners

Keep the Anchor Chart displayed and refer students to use it during future discussions. Model how to complete 1 or 2 of the prompts before students begin sharing.

Independent | 10 min

Activity 3 Car Prices

Students write equations to make sense of the relationships between the quantities in a problem.



Activity 3 Car Prices

Clare's parents are planning to buy her a car so she will no longer have to take the bus. The sales tax on a car in her state is 6%. At the local dealership, a car purchase also requires $120 for the title and registration fees.

1. There are several quantities in this scenario: the original car price, sales tax, title and registration fees, and the total price. For each of the following statements, write an equation that relates all the quantities. If you use a variable, specify what it represents.
   - a The original price is $9,500.
     $T = 1.06(9500) + 120$ (or equivalent), where T is the total price.
   - b The original price is $14,699.
     $T = 1.06(14699) + 120$ (or equivalent), where T is the total price.
   - c The total price is $22,480.
     $22480 = 1.06p + 120$ (or equivalent), where p is the original car price.
   - d The original price is p.
     $T = 1.06p + 120$ (or equivalent), where T is the total price.
2. How would each of your equations in Problem 1 change if the tax were r% and the title and registration fees were m dollars?
   - a The original price is $9,500.
     $T = 9500 + \frac{r}{100}(9500) + m$ (or equivalent)
   - b The original price is $14,699.
     $T = 14699 + \frac{r}{100}(14699) + m$ (or equivalent)
   - c The total price is $22,480.
     $22480 = p + \frac{r}{100}(p) + m$ (or equivalent)
   - d The original price is p.
     $T = p + \frac{r}{100}(p) + m$ (or equivalent)
3. Besides the sales tax and title and registration fees, what other costs are associated with owning a car? What are the benefits or drawbacks of owning a car?
   Sample response: When you own a car, additional costs include gasoline, car maintenance, insurance, and possibly other local or state fees. However, owning a car may be more convenient because you do not have to rely on public transportation or its schedule. Driving may be faster than taking a bus.

STOP

16 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Ask students if they have had to pay sales tax when making a purchase and to explain how sales tax works.

2 Monitor

Help students get started by having them calculate the cost of buying a $9,500 car.

Look for points of confusion:

- **Struggling to represent r algebraically in Problem 2.** Tell students to look at their work in Problem 1. Ask, "By what number did you multiply the car price? How do you represent a percentage as a fraction?"

Look for productive strategies:

- Converting 6% to a decimal or fraction and multiplying it by the original price.
- Using variables to represent unknown values.
- Writing r% as $\frac{r}{100}$.

3 Connect

Have pairs of students share their equations for Problems 1 and 2. Have students identify the quantities and constraints represented in each equation and explain what it means in context.

Ask these questions about Problem 1, and then again about Problem 2:

- "Which quantities are known? Which are unknown and how do you represent them?"
- "Which quantities vary? Which ones are fixed?"

Highlight that there are times when students might choose to use variables to represent quantities that vary or those that are constant. When they use variables to represent quantities that are known or are constant, it can help them to focus on the relationship between the quantities rather than the specific values.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a checklist, such as the following, to support them in making sure they have accounted for all of the given quantities in the equations they write in Problem 1.

☐ The original car price.

☐ Sales tax.

☐ Title and registration fees.

☐ Total price after all taxes and fees.

Accessibility: *Guide Processing and Visualization*

After students complete Problems 1a and 1b, ask, "How did you represent the total price in parts a and b? How might you represent the original price in part c, since you are not given its value?"

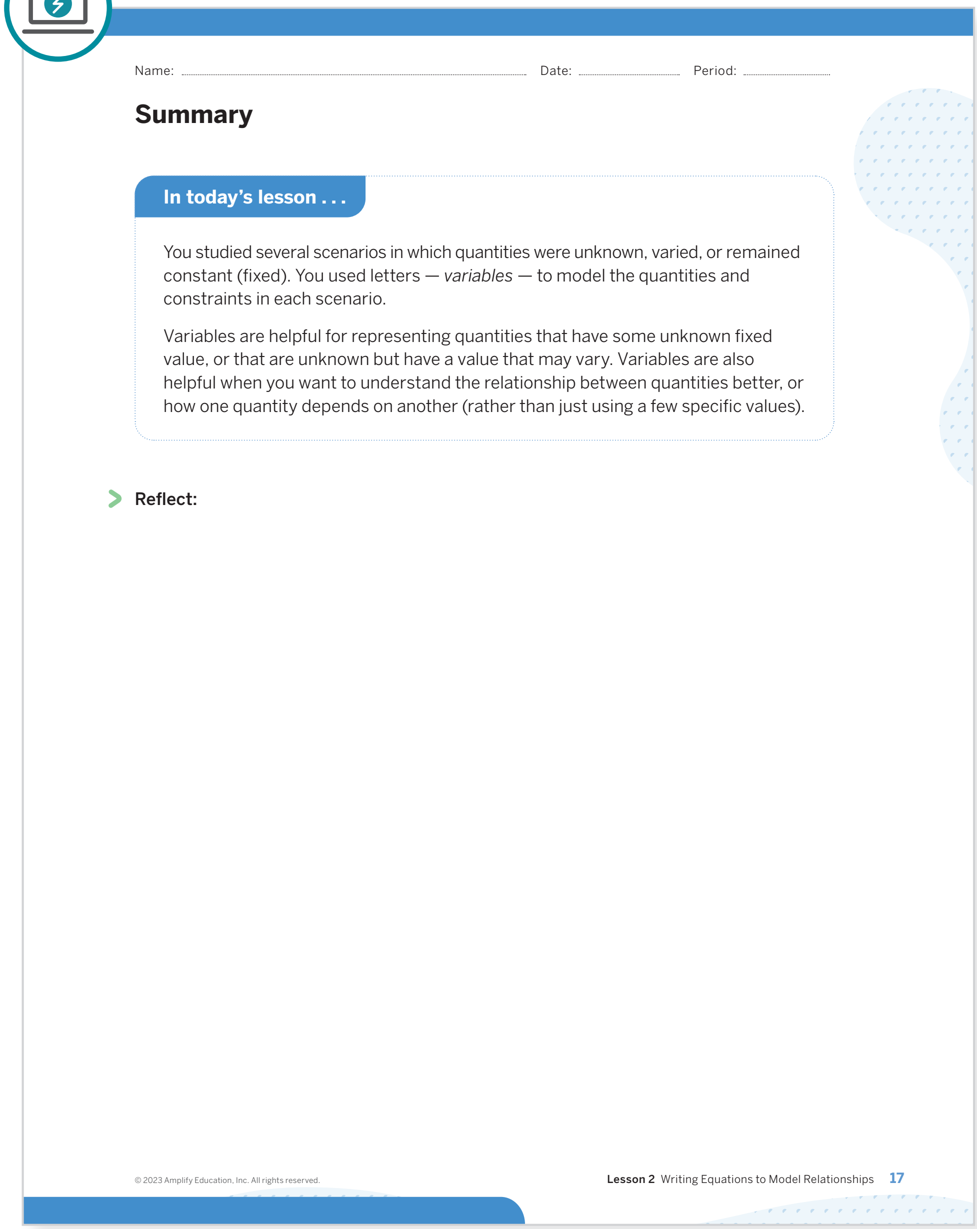
Extension: *Math Enrichment*

Provide the current minimum wage for hours worked. Have students determine their gross weekly pay if they work 20 hours a week at minimum wage. Have them subtract 25% of their pay to account for taxes and subtract estimated weekly costs associated with owning a car that they wrote about in Problem 3. Answers may vary.

Whole Class | 5 min

Summary

Review and synthesize when it is useful to use letters or numbers to write equations that model the quantities and constraints in a given scenario.



Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You studied several scenarios in which quantities were unknown, varied, or remained constant (fixed). You used letters — *variables* — to model the quantities and constraints in each scenario.

Variables are helpful for representing quantities that have some unknown fixed value, or that are unknown but have a value that may vary. Variables are also helpful when you want to understand the relationship between quantities better, or how one quantity depends on another (rather than just using a few specific values).

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 2 Writing Equations to Model Relationships 17

Synthesize

Ask students to write a response to one or both of the following prompts:

- "You could use numbers or variables to represent the quantities in a situation. When might it make sense to use only numbers? When might it make sense to use variables?"
- "You've heard the phrases 'a quantity that varies' and a 'quantity that stays constant' in this lesson. Describe what they mean in your own words. If possible, give an example of a situation that has a quantity that varies and a quantity that stays constant."

Highlight that variables are useful for representing unknown quantities or quantities that vary. Variables can also be used to represent quantities that are known or that are fixed when their goal is to understand the relationship between the quantities better.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

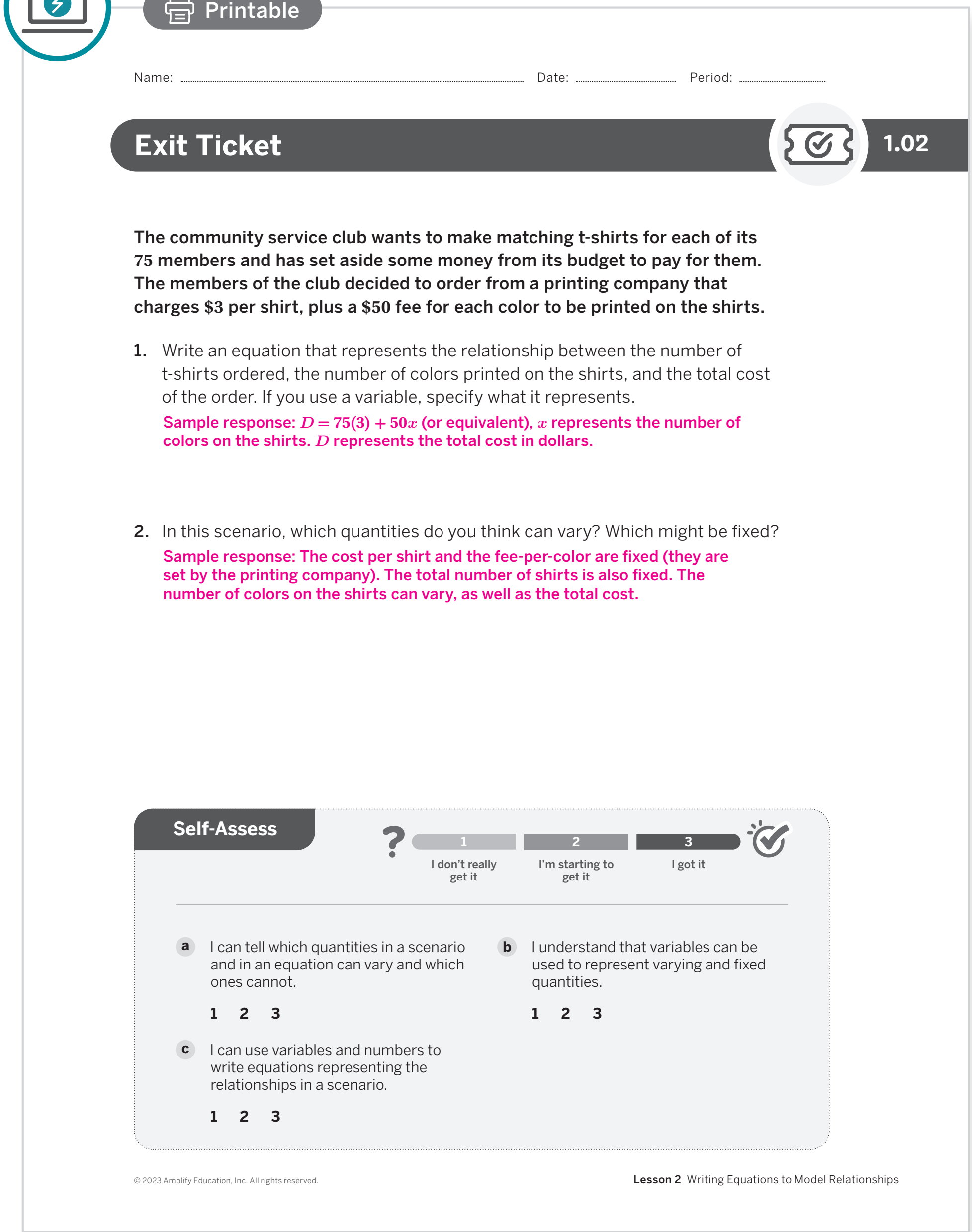
- "How does writing an equation help you to see the constraints in a scenario?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing equations that model relationships of fixed and varying quantities.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.02

The community service club wants to make matching t-shirts for each of its 75 members and has set aside some money from its budget to pay for them. The members of the club decided to order from a printing company that charges \$3 per shirt, plus a \$50 fee for each color to be printed on the shirts.

1. Write an equation that represents the relationship between the number of t-shirts ordered, the number of colors printed on the shirts, and the total cost of the order. If you use a variable, specify what it represents.
   Sample response: $D = 75(3) + 50x$ (or equivalent), x represents the number of colors on the shirts. D represents the total cost in dollars.

2. In this scenario, which quantities do you think can vary? Which might be fixed?
   Sample response: The cost per shirt and the fee-per-color are fixed (they are set by the printing company). The total number of shirts is also fixed. The number of colors on the shirts can vary, as well as the total cost.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can tell which quantities in a scenario and in an equation can vary and which ones cannot.
1 2 3

b. I understand that variables can be used to represent varying and fixed quantities.
1 2 3

c. I can use variables and numbers to write equations representing the relationships in a scenario.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 2 Writing Equations to Model Relationships

Success looks like . . .

- **Language Goal:** Given a description of a situation, identifying quantities that vary and quantities that do not. **(Reading and Writing)**
- **Language Goal:** Given an equation, identifying quantities that do and do not vary. **(Reading and Writing)**
  - » Identifying the quantities that can vary and those that can be fixed in Problem 2.
- **Language Goal:** Understanding that variables can be used to represent both quantities that vary and those that are constant. **(Reading and Writing)**
- **Language Goal:** Writing equations with numbers and variables to describe relationships and constraints. **(Speaking and Listening, Writing)**

Suggested next steps

If students are unable to write an equation to represent the given relationship in Problem 1, consider:

- Having students identify and write down the specific quantities given in the scenario and determine which quantities need to be assigned a variable.
- Reviewing Activity 2, Problem 1.
- Assigning Practice Problems 1 and 2.

If students are unable to determine which quantities vary and which are fixed in Problem 2, consider:

- Asking, "Which quantities are fixed by the company and which are determined by the community service club?"
- Reviewing Activity 3, Problems 1 and 2.

Professional Learning

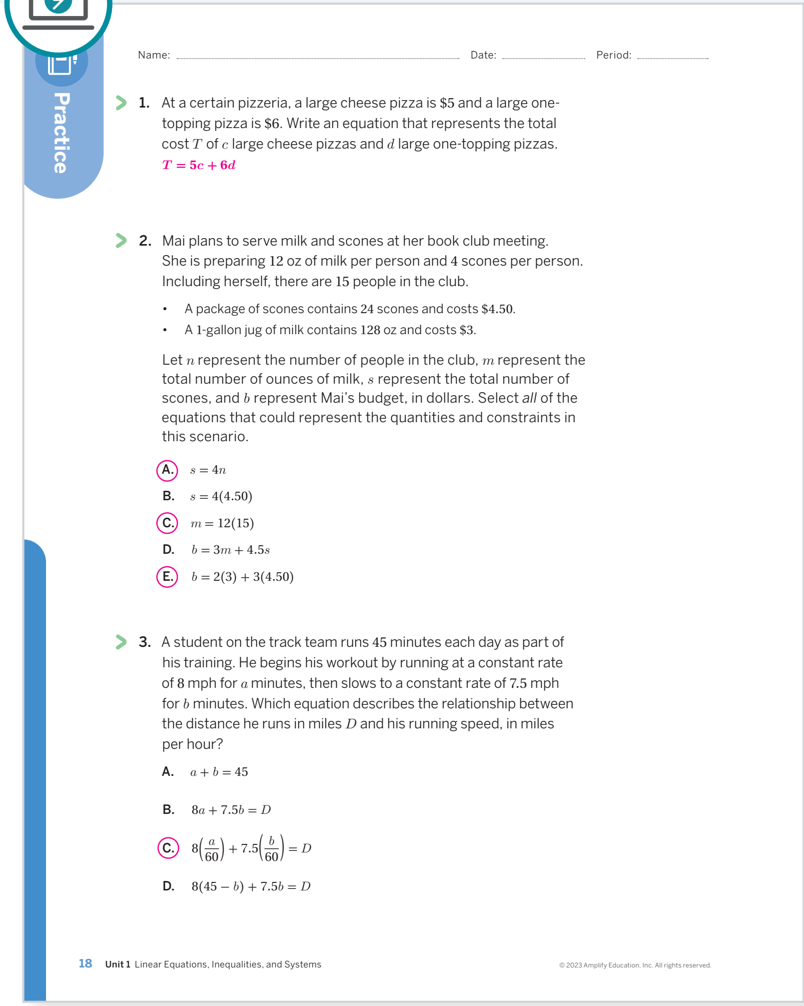
The instructional goal for this lesson was understanding that variables can be used to represent different quantities and writing equations to model the constraints on them. How well did students accomplish this? What did you specifically do to help students accomplish it?

Points to Ponder . . .

- In Activity 2, you used structured pairing with **MLR7** to group students who utilized different levels of precision in mathematical language. What effect did this grouping strategy have on their conversations? Would you change anything the next time you use **MLR7**?
- How did students look for and express regularity in repeated reasoning today? How are you helping students become aware of how they are progressing in this area? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. At a certain pizzeria, a large cheese pizza is $5 and a large one-topping pizza is $6. Write an equation that represents the total cost T of c large cheese pizzas and d large one-topping pizzas.

   $\mathbf{T = 5c + 6d}$

2. Mai plans to serve milk and scones at her book club meeting. She is preparing 12 oz of milk per person and 4 scones per person. Including herself, there are 15 people in the club.

   - A package of scones contains 24 scones and costs $4.50.
   - A 1-gallon jug of milk contains 128 oz and costs $3.

   Let n represent the number of people in the club, m represent the total number of ounces of milk, s represent the total number of scones, and b represent Mai's budget, in dollars. Select *all* of the equations that could represent the quantities and constraints in this scenario.

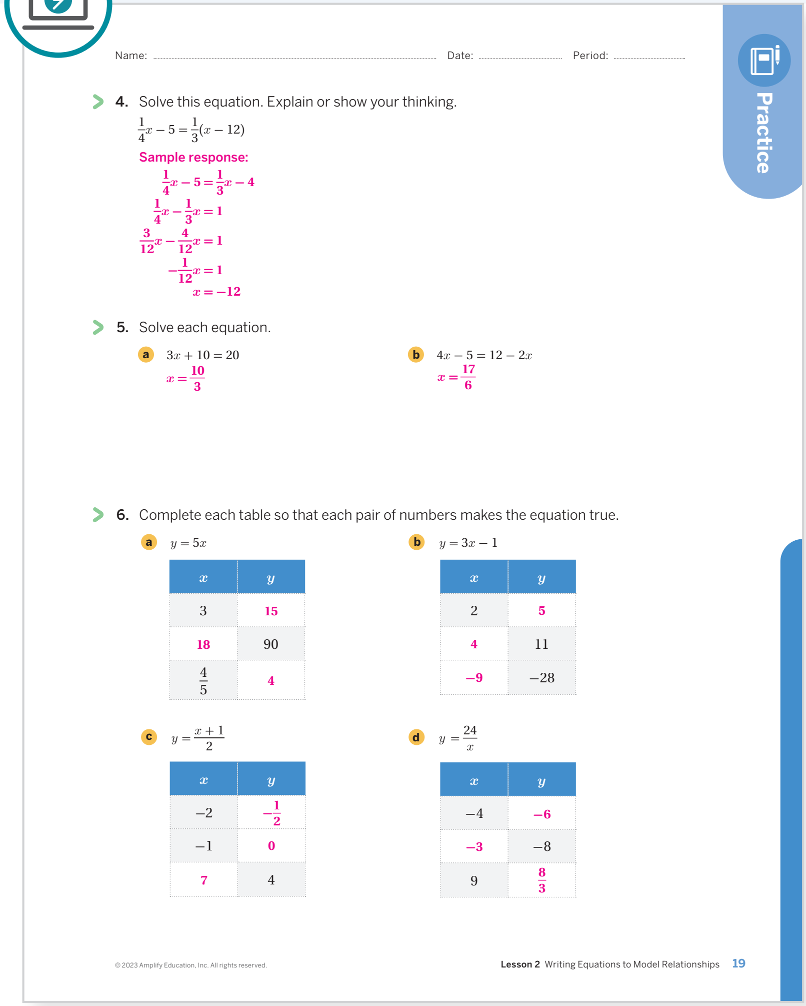
   [✓ A.] $s = 4n$
   B. $s = 4(4.50)$
   [✓ C.] $m = 12(15)$
   D. $b = 3m + 4.5s$
   [✓ E.] $b = 2(3) + 3(4.50)$

3. A student on the track team runs 45 minutes each day as part of his training. He begins his workout by running at a constant rate of 8 mph for a minutes, then slows to a constant rate of 7.5 mph for b minutes. Which equation describes the relationship between the distance he runs in miles D and his running speed, in miles per hour?

   A. $a + b = 45$
   B. $8a + 7.5b = D$
   [✓ C.] $8\left(\frac{a}{60}\right) + 7.5\left(\frac{b}{60}\right) = D$
   D. $8(45 - b) + 7.5b = D$

18 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ________ Date: ________ Period: ________

4. Solve this equation. Explain or show your thinking.

   $\frac{1}{4}x - 5 = \frac{1}{3}(x - 12)$

   Sample response:

   $$\frac{1}{4}x - 5 = \frac{1}{3}x - 4$$
   $$\frac{1}{4}x - \frac{1}{3}x = 1$$
   $$\frac{3}{12}x - \frac{4}{12}x = 1$$
   $$-\frac{1}{12}x = 1$$
   $$x = -12$$

5. Solve each equation.

   a. $3x + 10 = 20$

   $x = \frac{10}{3}$

   b. $4x - 5 = 12 - 2x$

   $x = \frac{17}{6}$

6. Complete each table so that each pair of numbers makes the equation true.

   a. $y = 5x$

x	y
3	15
18	90
$\frac{4}{5}$	4

   b. $y = 3x - 1$

x	y
2	5
4	11
−9	−28

   c. $y = \frac{x+1}{2}$

x	y
−2	$-\frac{1}{2}$
−1	0
7	4

   d. $y = \frac{24}{x}$

x	y
−4	−6
−3	−8
9	$\frac{8}{3}$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 2 Writing Equations to Model Relationships 19

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 3	2
Spiral	4	Grade 8	2
	5	Grade 8	2
Formative	6	Unit 1 Lesson 3	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 3**

Strategies for Determining Relationships

Let’s use patterns to help us write equations.

Focus

Goals

1. **Language Goal:** Identify and describe patterns in tables of values and in calculations. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Use observed patterns to generalize the relationships between quantities and to write equations. **(Reading and Writing)**

Coherence

Today

In this lesson, students continue to develop their ability to identify, describe, and model relationships using equations. Students are given tables of values and must generalize the relationship between pairs of quantities by studying the values and looking for patterns, and by interpreting them in context. They also determine effective strategies, including creating tables, that enable them to discover the relationship between pairs of quantities.

Previously

In the previous lesson, students wrote equations to model the quantities and constraints in a given scenario, identified the quantities as varying or fixed, and reflected on the usefulness of using equations to model relationships.

Coming Soon

In Lesson 4, students will examine the solutions of equations in one and two variables and make sense of the solutions in context.

Rigor

- Students build a **conceptual understanding** of equations in two variables.
- Students develop **fluency** in interpreting tables of values that model relationships.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice | Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Are you ready for more?* (answers)
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps*
- graph paper

Math Language Development

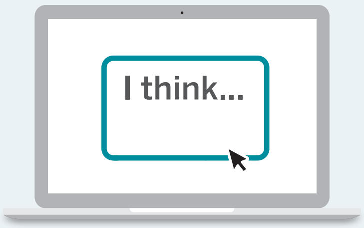
Review words

- *constraint*
- *model*
- *variable*

Amps | Featured Activity

Activity 2
See Student Thinking

Students describe a strategy they deem most appropriate to determine the relationship between two quantities in several scenarios.



Building Math Identity and Community

Connecting to Mathematical Practices

Students work with a partner in each activity and may become defensive if their partners disagree with or challenge their strategy or way of thinking, particularly if it is a correct strategy. Lead a discussion after the Warm-up about taking different approaches to problem-solving and multiple perspectives, identifying feelings and thoughts of others who adopt these strategies. Emphasize that there may be more than one correct way of finding a solution and the importance of valuing other people's perspectives.

Modifications to Pacing

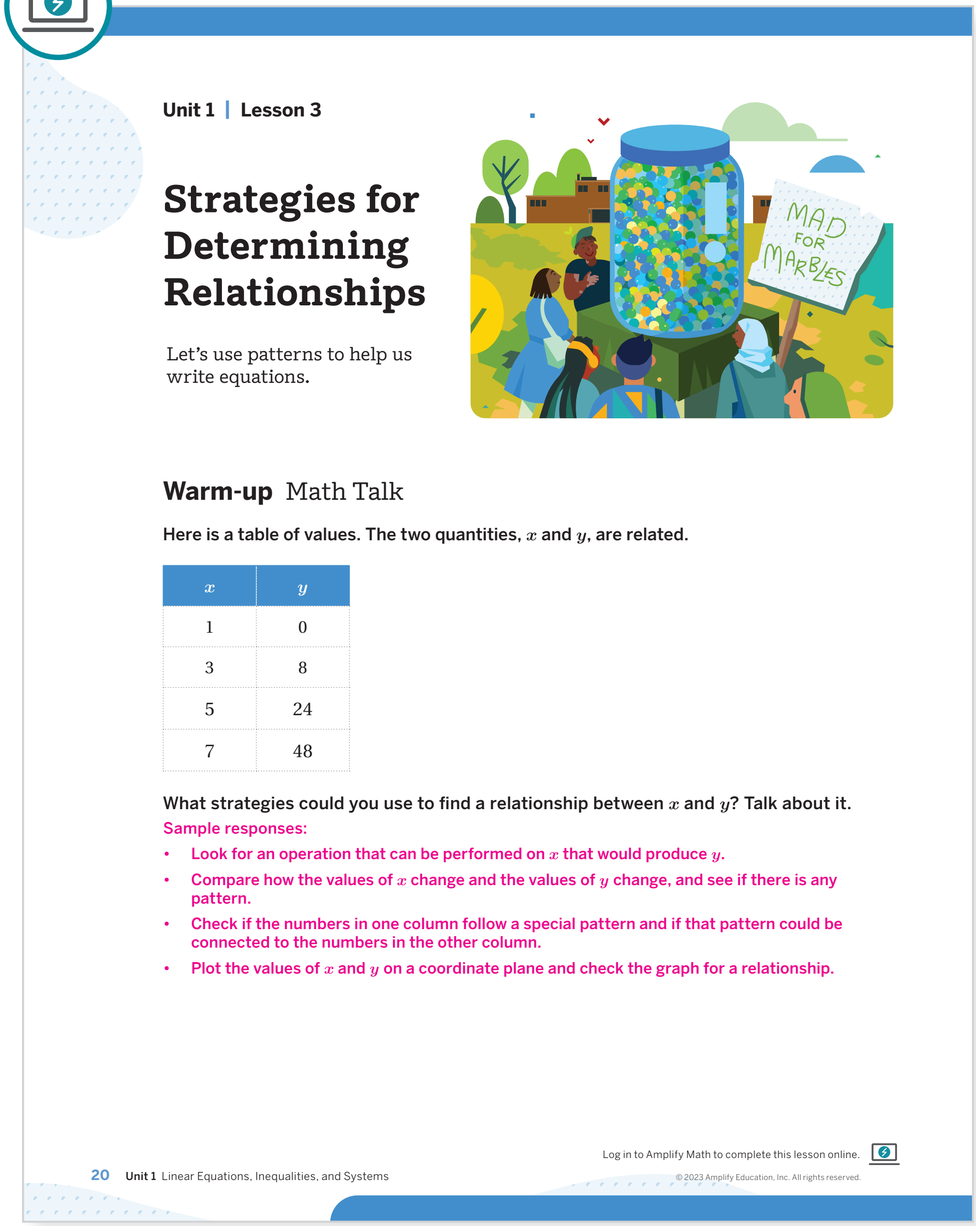
You may want to consider these additional modifications if you are short on time.

- Reduce allotted time for the **Warm-up** to 5 minutes.
- In **Activity 1**, assign only two tables for each student pair to analyze.
- In **Activity 2**, Problem 3 may be omitted.

Pairs | 5 min

Warm-up Math Talk

Students look for and make use of the structure of a table to determine strategies for determining the relationship between two quantities, in preparation for Activities 1 and 2.



Unit 1 | Lesson 3

Strategies for Determining Relationships

Let's use patterns to help us write equations.

Warm-up Math Talk

Here is a table of values. The two quantities, x and y, are related.

x	y
1	0
3	8
5	24
7	48

What strategies could you use to find a relationship between x and y? Talk about it.

Sample responses:

- Look for an operation that can be performed on x that would produce y.
- Compare how the values of x change and the values of y change, and see if there is any pattern.
- Check if the numbers in one column follow a special pattern and if that pattern could be connected to the numbers in the other column.
- Plot the values of x and y on a coordinate plane and check the graph for a relationship.

Log in to Amplify Math to complete this lesson online.

20 Unit 1 Linear Equations, Inequalities, and Systems

©2023 Amplify Education, Inc. All rights reserved.

1 Launch

Conduct the ***Math Talk*** routine. Explain that the quantities in each column are related. Emphasize the goal is to brainstorm strategies and not determine a solution. Provide graph paper to students who ask.

2 Monitor

Help students get started by asking, "Is there an operation that can be performed on x to produce y?"

Look for productive strategies:

- Trying to perform operations on the values of x to yield values of y.
- Looking for a pattern in each column of the table.
- Noticing that each time x increases by 2, y increases by a multiple of 8.
- Noticing that the values of y are 1 less than the square of the values of x.
- Plotting the values of x and y from the table on a graph.

3 Connect

Have pairs of students share their strategies, selecting and sequencing those using productive strategies. Have them explicitly show how they applied their strategy using the values from the table. Record the strategies and display for the remainder of the lesson.

Ask, "Did anyone use a different strategy that was not mentioned?"

Highlight that each specific strategy used is a way of determining a relationship between x and y. Model graphing as a strategy if it was not mentioned.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their strategies, collect and display informal language and diagrams students use. Highlight specific strategies as efficient and ask students why these strategies might be more efficient than others. Students can adopt these efficient strategies for future use.

English Learners

Amplify strategies that use a graph. Connect the table to the graph by annotating where students can see the tabular relationships on the graph.

Power-up

To power up students' ability to use an equation to complete a table of values, have students complete:

Recall that to use an equation to determine the missing values in a table, you substitute each value for the given variable and solve for the unknown variable.

Complete the table for $y = 2x - 1$.

x	y
2	**3**
5	**9**
3	5

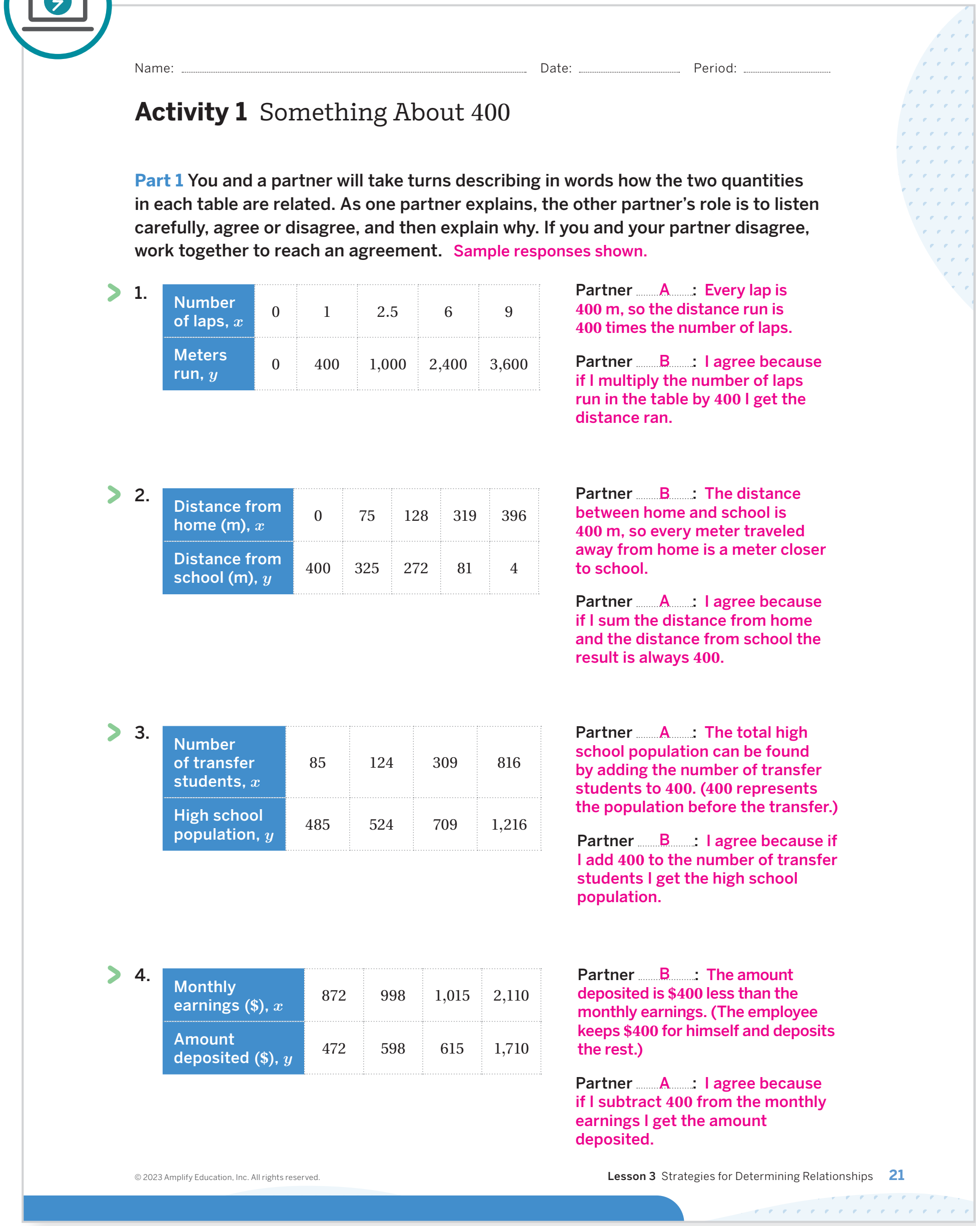
Use: Before the Warm-up

Informed by: Performance on Lesson 2, Practice Problem 6

Pairs | 20 min

Activity 1 Something About 400

Students determine the relationship between two quantities given a table of values to determine the linear equation that best matches it.



Name: ________ Date: ________ Period: ________

Activity 1 Something About 400

Part 1 You and a partner will take turns describing in words how the two quantities in each table are related. As one partner explains, the other partner's role is to listen carefully, agree or disagree, and then explain why. If you and your partner disagree, work together to reach an agreement. Sample responses shown.

1.

Number of laps, x	0	1	2.5	6	9
Meters run, y	0	400	1,000	2,400	3,600

Partner A: Every lap is 400 m, so the distance run is 400 times the number of laps.

Partner B: I agree because if I multiply the number of laps run in the table by 400 I get the distance ran.

2.

Distance from home (m), x	0	75	128	319	396
Distance from school (m), y	400	325	272	81	4

Partner B: The distance between home and school is 400 m, so every meter traveled away from home is a meter closer to school.

Partner A: I agree because if I sum the distance from home and the distance from school the result is always 400.

3.

Number of transfer students, x	85	124	309	816
High school population, y	485	524	709	1,216

Partner A: The total high school population can be found by adding the number of transfer students to 400. (400 represents the population before the transfer.)

Partner B: I agree because if I add 400 to the number of transfer students I get the high school population.

4.

Monthly earnings ($), x	872	998	1,015	2,110
Amount deposited ($), y	472	598	615	1,710

Partner B: The amount deposited is $400 less than the monthly earnings. (The employee keeps $400 for himself and deposits the rest.)

Partner A: I agree because if I subtract 400 from the monthly earnings I get the amount deposited.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Give students time to study the tables in Part 1. Have partners take turns describing the relationships in the tables. Have partners take turns describing the relationships in the tables and critiquing one another's reasoning.

2 Monitor

Help students get started by prompting them to use a strategy from the Warm-up to determine the relationship between x and y.

Look for points of confusion:

- **Describing incorrect relationships between x and y for the given quantities.** Ask questions that relate a specific x to its corresponding y to support students thinking for the context. For example, for the table in Problem 1, ask, "If you run one lap, how many meters have you run?"
- **Having difficulty matching the equations in Part 2 with the tables.** Prompt students to substitute values from the table into the equations to verify their matches.

Look for productive strategies:

- Using effective strategies discussed in the Warm-up.
- Using the given quantities to contextualize the values of x and y.
- Using values from the table to explain the relationship they see between the quantities.
- Performing operations on the values of x to yield values of y as a solution.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on describing the tables in Problems 1 and 2 in Part 1, and then move to the first two equations in Problem 5 in Part 2. Alternatively, allow them to choose two of the four tables to complete in Part 1 and have them match the same tables in Part 2 with two of the four equations.

Extension: *Math Enrichment*

If students complete the *Are you ready for more?* problem, consider introducing factorial notation to support them in generating additional ways to express the numbers between 1 and 20.

Math Language Development

MLR8: Discussion Supports

As students take turns describing how the quantities in each table are related in Part 1, display or provide the Anchor Chart PDF, *Sentence Stems*, *Explaining My Steps* to support student discussions. Encourage students to challenge each other when they disagree so that they have opportunities to clarify their reasoning.

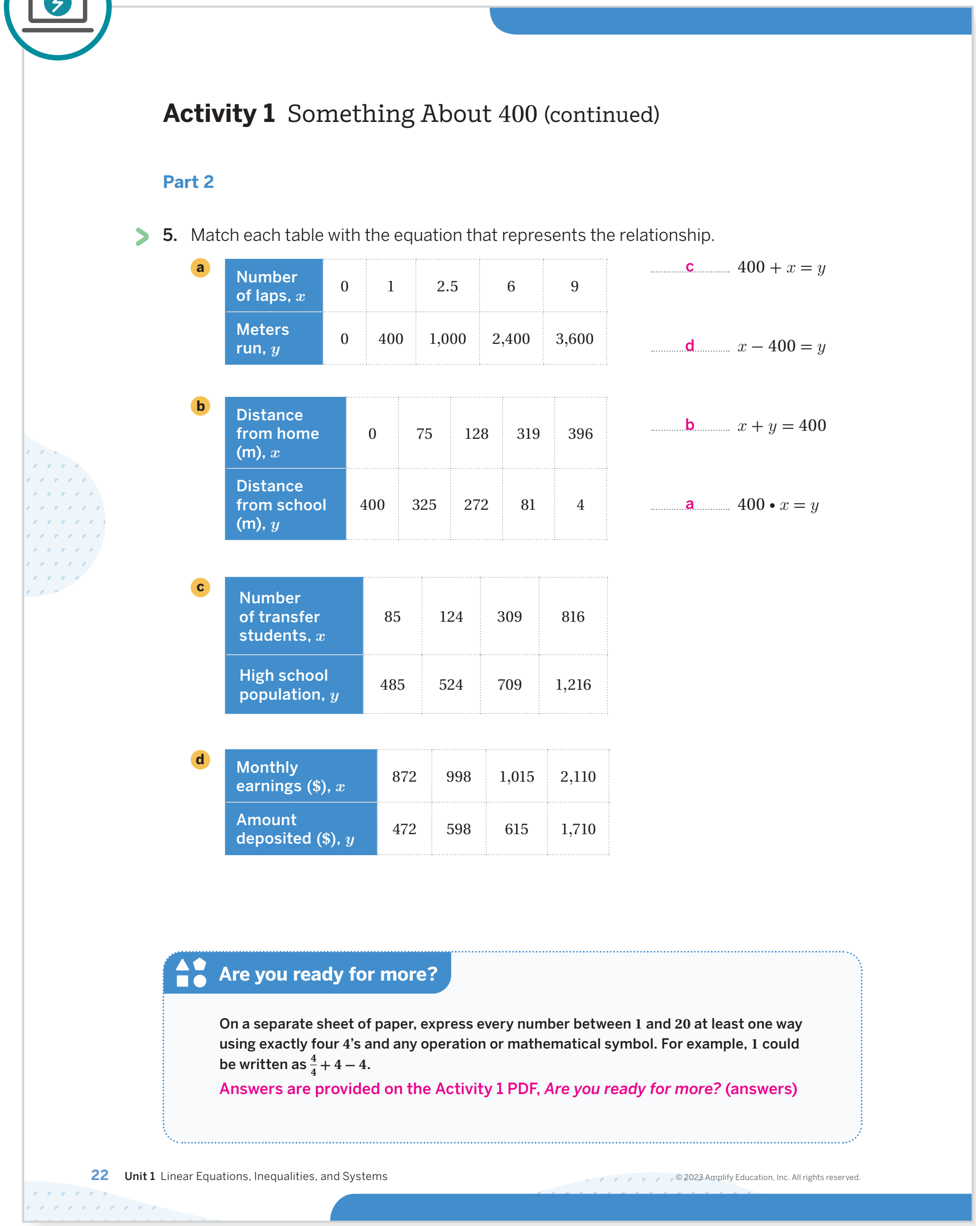
English Learners

Encourage students to visually annotate the tables with the operations they use to describe the relationships.

Pairs | 20 min

Activity 1 Something About 400 (continued)

Students determine the relationship between two quantities given a table of values to determine the linear equation that best matches it.



Activity 1 Something About 400 (continued)

Part 2

5. Match each table with the equation that represents the relationship.

a

Number of laps, x	0	1	2.5	6	9
Meters run, y	0	400	1,000	2,400	3,600

b

Distance from home (m), x	0	75	128	319	396
Distance from school (m), y	400	325	272	81	4

c

Number of transfer students, x	85	124	309	816
High school population, y	485	524	709	1,216

d

Monthly earnings ($), x	872	998	1,015	2,110
Amount deposited ($), y	472	598	615	1,710

c .... $400 + x = y$

d .... $x - 400 = y$

b .... $x + y = 400$

a .... $400 \cdot x = y$

Are you ready for more?

On a separate sheet of paper, express every number between 1 and 20 at least one way using exactly four 4's and any operation or mathematical symbol. For example, 1 could be written as $\frac{4}{4} + 4 - 4$.

Answers are provided on the Activity 1 PDF, *Are you ready for more?* (answers)

22 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

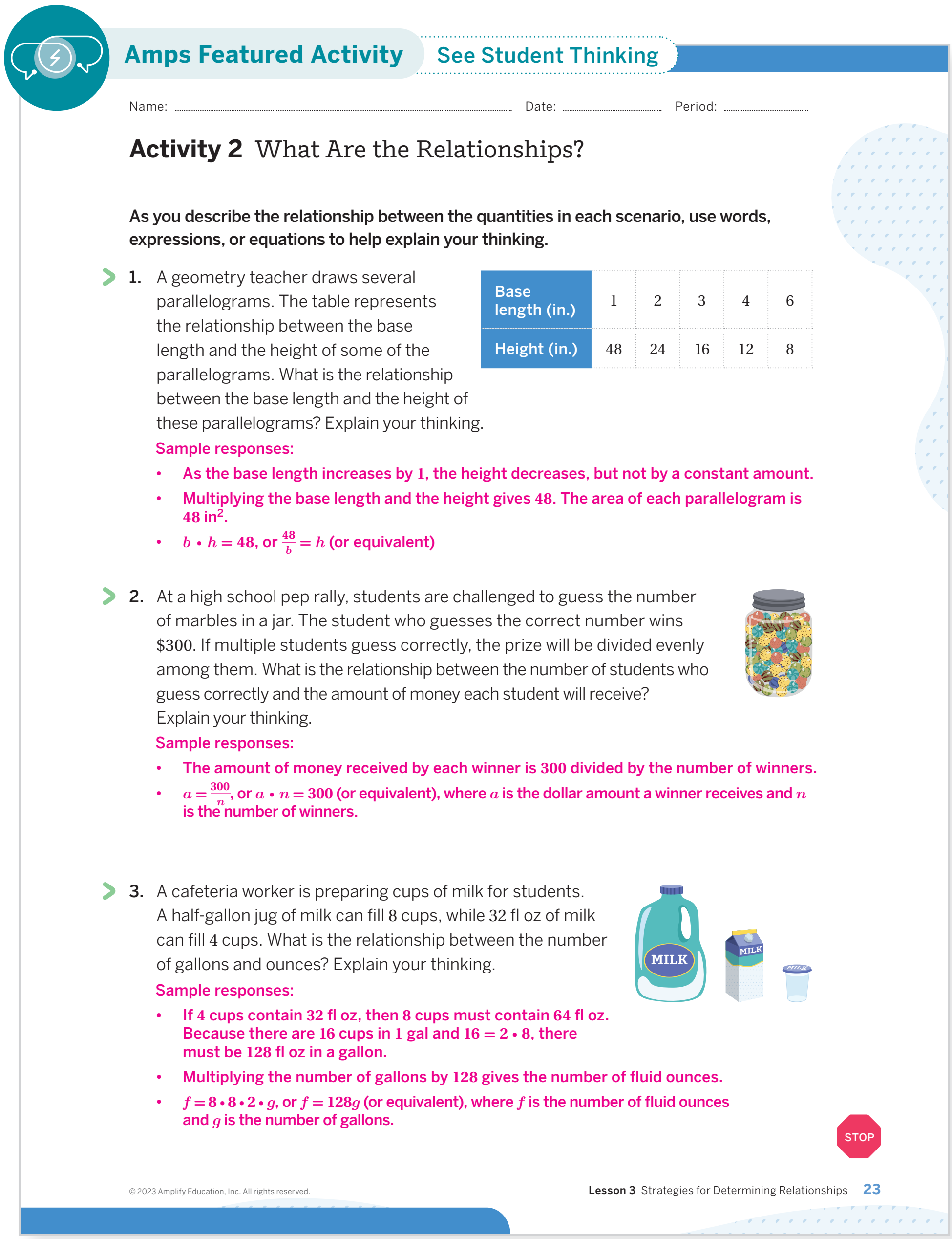
Have pairs of students share their thinking about each of the relationships in Part 1. Select and sequence student pairs who reasoned only in terms of numerical operations or who used strategies discussed in the Warm-up, and move toward those who interpreted the quantities in context. Record and display their descriptions, highlighting the connections between the different responses to help students look for and make use of structure.

Highlight that how they interpret the relationship between quantities can help them write equations that represent it. Use students' responses to model how their interpretation might have helped (or hindered) them to match the equations in Part 2. For example, describing the relationship in the table in Problem 2 as "The distance from home and the distance from school always add up to 400" matches the third equation, but describing the relationship in the table as "The distance from school is always 400 minus the distance from home" might yield a different (albeit equivalent) equation.

Pairs | 15 min

Activity 2 What Are the Relationships?

Students reason abstractly and quantitatively to determine relationships given a table or verbal description and express them in multiple ways.



Amps Featured Activity See Student Thinking

Name: ________ Date: ________ Period: ________

Activity 2 What Are the Relationships?

As you describe the relationship between the quantities in each scenario, use words, expressions, or equations to help explain your thinking.

1. A geometry teacher draws several parallelograms. The table represents the relationship between the base length and the height of some of the parallelograms. What is the relationship between the base length and the height of these parallelograms? Explain your thinking.

Base length (in.)	1	2	3	4	6
Height (in.)	48	24	16	12	8

Sample responses:

- **As the base length increases by 1, the height decreases, but not by a constant amount.**
- **Multiplying the base length and the height gives 48. The area of each parallelogram is 48 in^2.**
- **$b \bullet h = 48$, or $\frac{48}{b} = h$ (or equivalent)**

2. At a high school pep rally, students are challenged to guess the number of marbles in a jar. The student who guesses the correct number wins $300. If multiple students guess correctly, the prize will be divided evenly among them. What is the relationship between the number of students who guess correctly and the amount of money each student will receive? Explain your thinking.

Sample responses:

- **The amount of money received by each winner is 300 divided by the number of winners.**
- **$a = \frac{300}{n}$, or $a \bullet n = 300$ (or equivalent), where a is the dollar amount a winner receives and n is the number of winners.**

3. A cafeteria worker is preparing cups of milk for students. A half-gallon jug of milk can fill 8 cups, while 32 fl oz of milk can fill 4 cups. What is the relationship between the number of gallons and ounces? Explain your thinking.

Sample responses:

- **If 4 cups contain 32 fl oz, then 8 cups must contain 64 fl oz. Because there are 16 cups in 1 gal and $16 = 2 \bullet 8$, there must be 128 fl oz in a gallon.**
- **Multiplying the number of gallons by 128 gives the number of fluid ounces.**
- **$f = 8 \bullet 8 \bullet 2 \bullet g$, or $f = 128g$ (or equivalent), where f is the number of fluid ounces and g is the number of gallons.**

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Say, "You will now study some new situations and describe the relationship between two quantities."

2 Monitor

Help students get started by prompting students to draw a diagram of the parallelograms represented in the table.

Look for points of confusion:

- **Trying to express the relationships as linear equations.** Encourage students to use verbal descriptions, tables, and other representations if they cannot determine an equation.
- **Having difficulty relating the three quantities in Problem 3.** Prompt them to create a table with the headings "Gallons," "Cups," and "Fluid ounces."

Look for productive strategies:

- Recognizing that the base length multiplied by the height of a parallelogram yields its area.
- Using the given descriptions to construct tables of values.
- Writing non-linear equations to describe the relationships.

3 Connect

Have pairs of students share their responses and thinking. Select and sequence student pairs, beginning with those that described the relationships using words, then those that created tables, then those that determined equations for each relationship.

Display the equations that can represent the three situations.

Highlight that writing equations is an efficient way to capture the constraints in a situation. Point out that these equations contain two variables, such as the ones students saw in Activity 1, but they are not all linear.

Differentiated Support

Accessibility: *Optimize Access to Technology, Vary Demands to Optimize Challenge*

Have students use the Amps slides for this activity, in which they are given a choice of which strategy they deem most appropriate to determine the relationship between the quantities in the scenarios.

Extension: *Math Enrichment*

During the Connect, after you mention that not all the equations are linear, ask students to identify which equation(s) are not linear and explain their thinking. Problem 2's equation, $a = \frac{300}{n}$, is not linear because there is no constant rate of change. It cannot be written in slope-intercept form.

Math Language Development

MLR1: Stronger and Clearer Each Time

As students complete each problem, have them first write individual responses and then share them with their partner to help refine and clarify their responses. Have them revise their original written responses based on their partner's feedback.

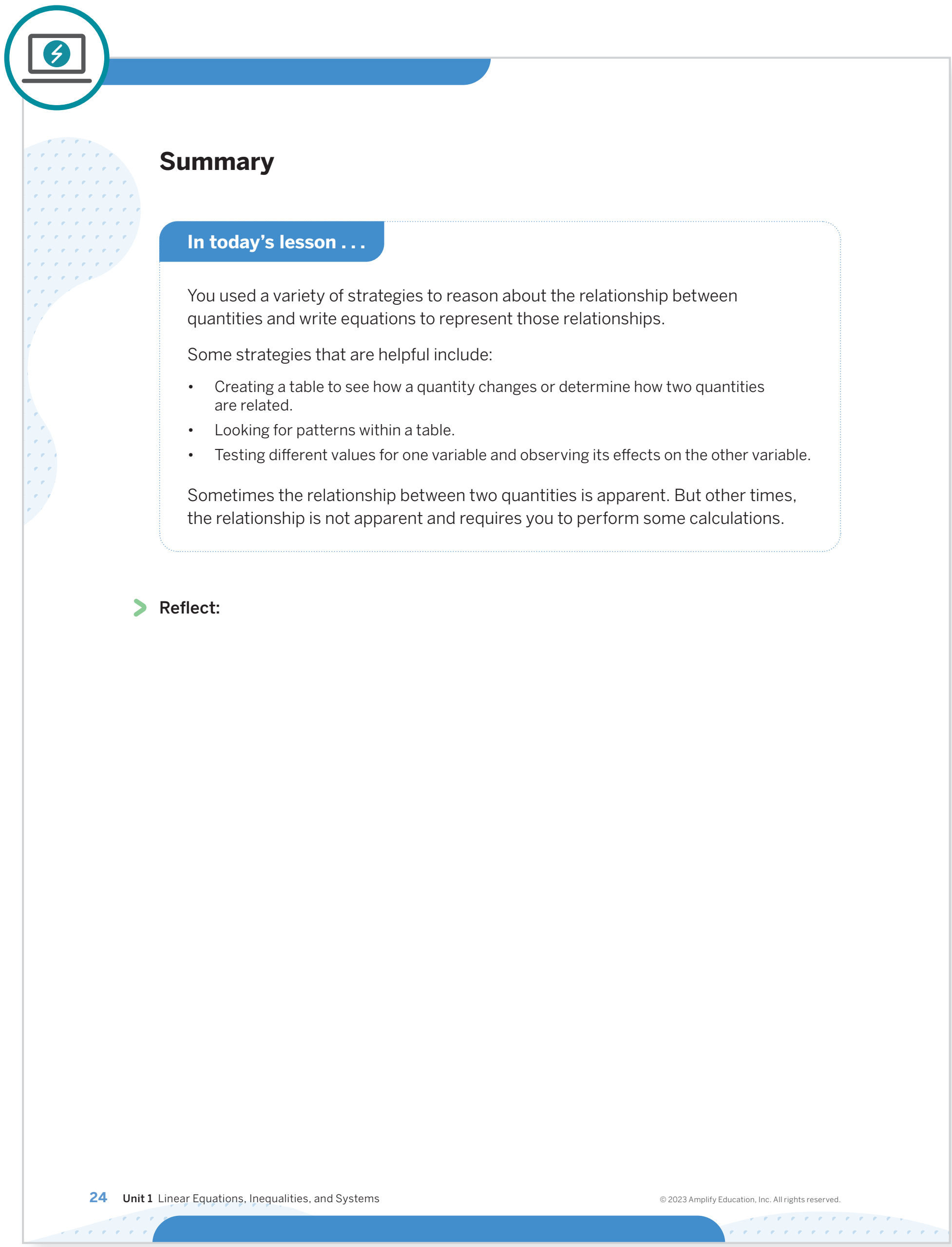
English Learners

In Problem 1, have students annotate the table with the relationship they see. In Problems 2 and 3, ask them to circle or color code the words and phrases that indicate the relationship in the text.

Whole Class | 5 min

Summary

Review and synthesize how to determine the relationship between two quantities.



Summary

In today's lesson . . .

You used a variety of strategies to reason about the relationship between quantities and write equations to represent those relationships.

Some strategies that are helpful include:

- Creating a table to see how a quantity changes or determine how two quantities are related.
- Looking for patterns within a table.
- Testing different values for one variable and observing its effects on the other variable.

Sometimes the relationship between two quantities is apparent. But other times, the relationship is not apparent and requires you to perform some calculations.

Reflect:

24 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Ask, "What are some strategies you used today to determine the relationship between two quantities?" Select a couple of tables and descriptions of scenarios from the lesson to elicit students' reflections.

Have students share their reasoning strategies, which might include creating a table, looking for a pattern in a table, or trying different numbers for one variable to see how it affects other variables.

Highlight that sometimes the relationship between two quantities is easy to determine. Other times, they have to use different strategies to do so. Creating a table or trying different numbers for one variable to see its effect on others can be effective strategies for determining what the relationship is, and for writing an equation to represent the relationship.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

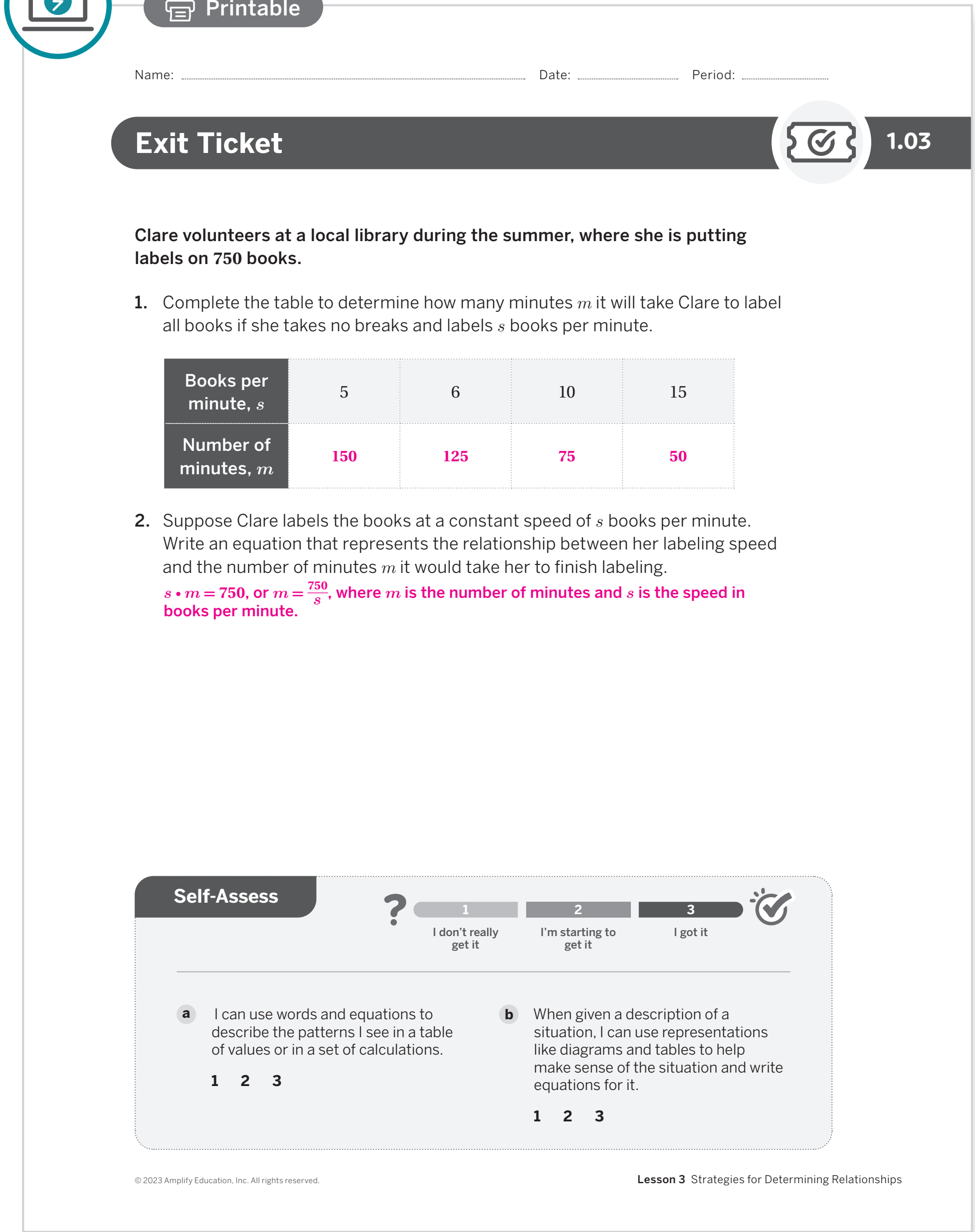
- "What are some strategies for determining the relationship between two quanitites?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the relationship between two quantities given a verbal description and writing an equation that represents it.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.03

Clare volunteers at a local library during the summer, where she is putting labels on 750 books.

1. Complete the table to determine how many minutes m it will take Clare to label all books if she takes no breaks and labels s books per minute.

Books per minute, s	5	6	10	15
Number of minutes, m	**150**	**125**	**75**	**50**

2. Suppose Clare labels the books at a constant speed of s books per minute. Write an equation that represents the relationship between her labeling speed and the number of minutes m it would take her to finish labeling.
   $s \cdot m = 750$, or $m = \frac{750}{s}$, where m is the number of minutes and s is the speed in books per minute.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a. I can use words and equations to describe the patterns I see in a table of values or in a set of calculations.

1 2 3

b. When given a description of a situation, I can use representations like diagrams and tables to help make sense of the situation and write equations for it.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 3 Strategies for Determining Relationships

Success looks like . . .

- **Language Goal:** Identifying and describing patterns in tables of values and in calculations. **(Speaking and Listening, Reading and Writing)**
  - » Completing the table to show the pattern of the number of minutes it takes Clare to label each amount of books.
- **Language Goal:** Using observed patterns to generalize the relationships between quantities and to write equations. **(Reading and Writing)**
  - » Writing an equation that relates Clare's labeling speed and the number of minutes she takes to finish labeling.

Suggested next steps

If students cannot complete the table in Problem 1, consider:

- Assigning Practice Problems 2 and 3.

If students cannot write an equation in Problem 2, consider:

- Reviewing Activity 2, Problems 1 and 2.
- Assigning Practice Problem 1.

Professional Learning

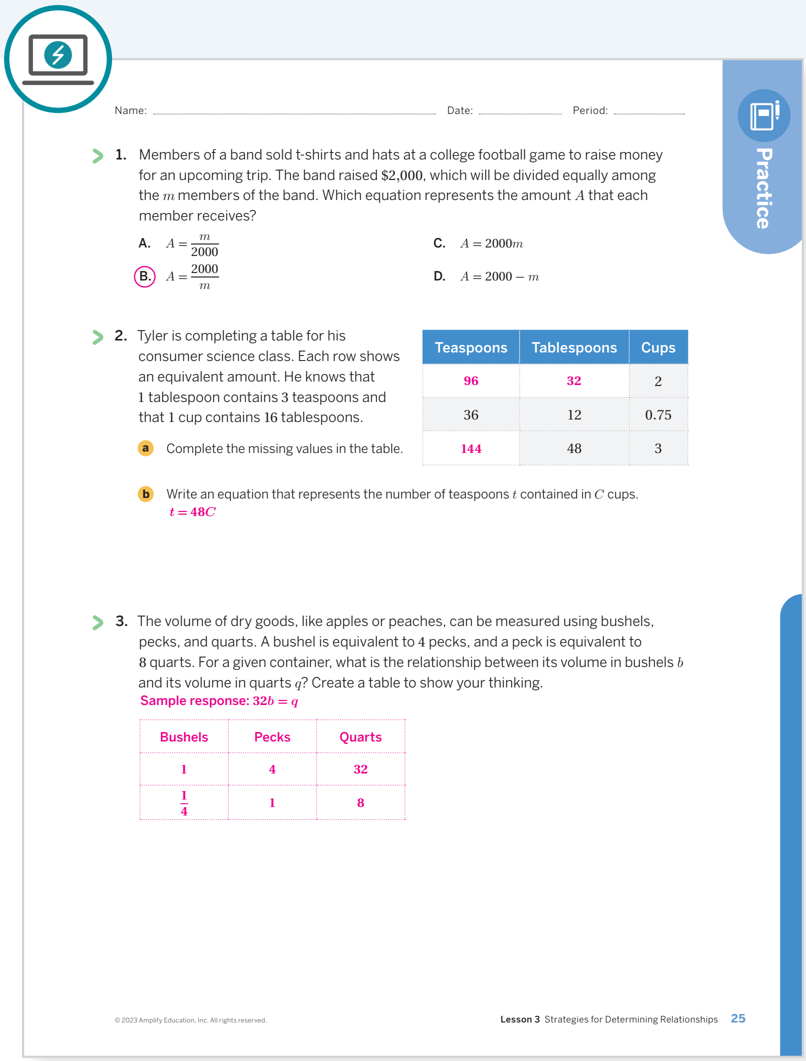
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did students' work in Activity 1 reveal about your students as learners?
- Which students' ideas were you able to highlight during Activity 2 when discussing the relationships between the quantities in each of the given scenarios? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Members of a band sold t-shirts and hats at a college football game to raise money for an upcoming trip. The band raised $\$2,000$, which will be divided equally among the m members of the band. Which equation represents the amount A that each member receives?

A. $A = \frac{m}{2000}$

[✓ B.] $A = \frac{2000}{m}$

C. $A = 2000m$

D. $A = 2000 - m$

2. Tyler is completing a table for his consumer science class. Each row shows an equivalent amount. He knows that 1 tablespoon contains 3 teaspoons and that 1 cup contains 16 tablespoons.

Teaspoons	Tablespoons	Cups
96	**32**	2
36	12	0.75
144	48	3

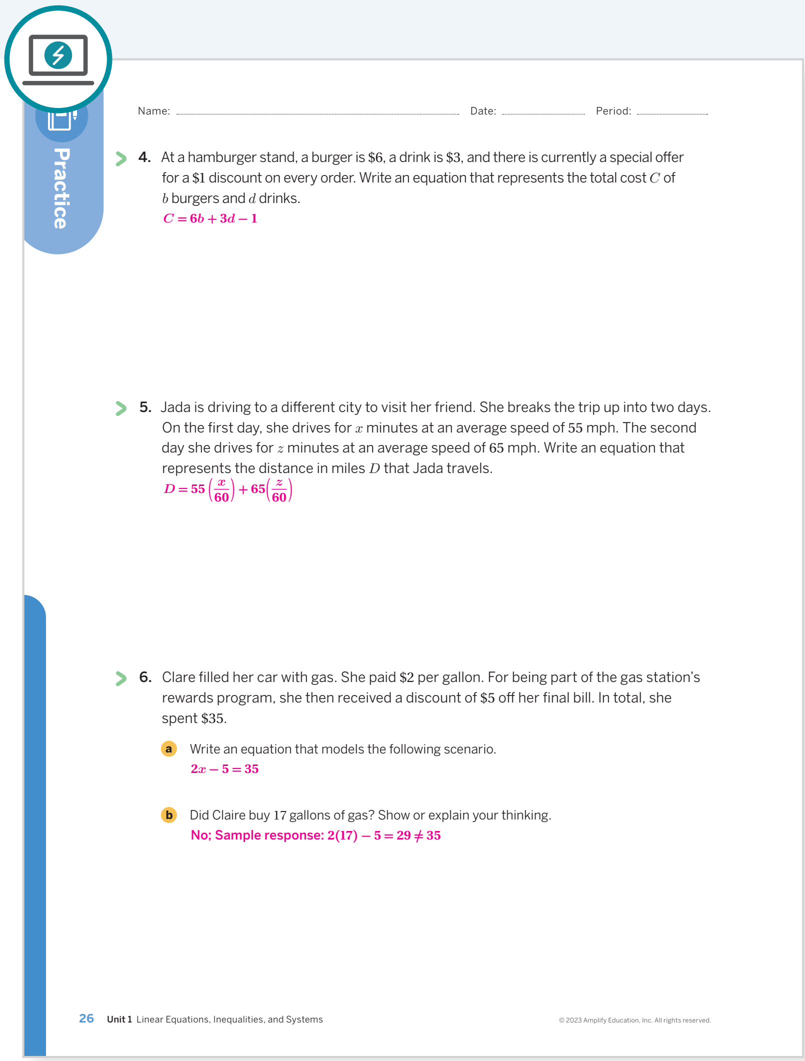
a. Complete the missing values in the table.

b. Write an equation that represents the number of teaspoons t contained in C cups.
$t = 48C$

3. The volume of dry goods, like apples or peaches, can be measured using bushels, pecks, and quarts. A bushel is equivalent to 4 pecks, and a peck is equivalent to 8 quarts. For a given container, what is the relationship between its volume in bushels b and its volume in quarts q? Create a table to show your thinking.
Sample response: $32b = q$

Bushels	Pecks	Quarts
1	4	32
$\frac{1}{4}$	1	8

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ______ Date: ______ Period: ______

4. At a hamburger stand, a burger is $\$6$, a drink is $\$3$, and there is currently a special offer for a $\$1$ discount on every order. Write an equation that represents the total cost C of b burgers and d drinks.
$C = 6b + 3d - 1$

5. Jada is driving to a different city to visit her friend. She breaks the trip up into two days. On the first day, she drives for x minutes at an average speed of 55 mph. The second day she drives for z minutes at an average speed of 65 mph. Write an equation that represents the distance in miles D that Jada travels.
$D = 55\left(\frac{x}{60}\right) + 65\left(\frac{z}{60}\right)$

6. Clare filled her car with gas. She paid $\$2$ per gallon. For being part of the gas station's rewards program, she then received a discount of $\$5$ off her final bill. In total, she spent $\$35$.

a. Write an equation that models the following scenario.
$2x - 5 = 35$

b. Did Claire buy 17 gallons of gas? Show or explain your thinking.
No; Sample response: $2(17) - 5 = 29 \neq 35$

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 2	3
Spiral	4	Unit 1 Lesson 2	2
	5	Unit 1 Lesson 2	2
Formative ⚡	6	Unit 1 Lesson 4	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 4**

Equations and Their Solutions

Let's recall what we know about solutions to equations.

Focus

Goals

1. Determine solutions to equations in one variable and two variables by reasoning about the relationships in context.
2. **Language Goal:** Interpret solutions to equations in one variable and in two variables. **(Speaking and Listening, Writing)**

Coherence

Today

Students continue using equations to model the quantities and constraints in context, focusing on what makes a value a solution. They verify and determine solutions to equations in one and two variables by evaluating given values and determining if they make the equation true. Students may choose to solve algebraically but they are not expected to rely on this strategy in this lesson.

Previously

In the previous lessons, students worked with equations in one variable and used them to model the relationships between quantities for given contexts.

Coming Soon

In the next lesson, students will explore scenarios that can be modeled with inequalities, and interpret and write inequalities to represent the constraints in those scenarios.

Rigor

- Students build on their **conceptual understanding** of what constitutes a solution to a linear equation.
- Students strengthen their **fluency** in determining solutions to one- and two-variable equations.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2 (optional)	Activity 3	Summary	Exit Ticket
5 min	20 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning*
- scientific calculators

Math Language Development

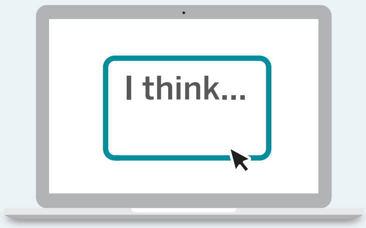
Review words

- *constraint*
- *variable*

Amps | Featured Activity

Activity 2
Can I Take Your Order?

Students play the role of a cashier, pairing customers with their orders by interpreting solutions to equations in two variables.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may resist thinking deeply about the relationships between the quantities in the given scenarios in Activities 1 and 2 and the equations that model them. Have students engage in metacognitive functions by asking them to think about their own thinking process. For example, have them conduct their own ***Notice and Wonder*** routine after reading a task statement that describes a scenario, which will help them record their thought processes.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- Optional **Activity 2** may be omitted.
- In **Activity 3**, Problem 5 may be omitted.

Independent | 5 min

Warm-up What Is a Solution?

Students write one-variable equations and reason abstractly and quantittively about their solutions to determine values that make the equation true and satisfy the constraints of the context.

Name: ______ Date: ______ Period: ______

Unit 1 | Lesson 4

Equations and Their Solutions

Let's recall what we know about solutions to equations.

Warm-up What is a Solution?

Bard orders veggie toast v for the table while dining with friends. Veggie toast is \$2.49. The total of the order is \$19.33. The equation $2.49v + 4.39 = 19.33$ represents the relationship between the quantities.

1. What could the 4.39 represent in the equation?
   Sample response: It could represent the cost of something else that Bard ordered.
2. If Bard orders 7 orders of veggie toast, is the total \$19.33? Explain or show your thinking.
   No; $2.49(7) + 4.39 \neq 19.33$
3. If Bard orders 2 orders of veggie toast, is the total \$19.33? Explain or show your thinking.
   No; $2.49(2) + 4.39 \neq 19.33$
4. How many orders of veggie toast does Bard order?
   6

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Equations and Their Solutions 27

1 Launch

Ask, "What is a solution?" Have students discuss their ideas before completing the activity independently.

2 Monitor

Help students get started by having them identify the items that Bard orders and drawing connections to the equation.

Look for points of confusion:

- **Having difficulty explaining why 7 or 2 is not a solution.** Ask, "What is the total when you substitute 7 (or 2) into the equation? What should it be?"
- **Not knowing which value to choose for Problem 4.** Ask, "Will the solution be greater than or less than 7? 2?"

Look for productive strategies:

- Making a table relating the number of orders of veggie toast to the order total.
- Estimating total cost by rounding the price of an order of veggie toast.
- Substituting 7 or 2 for v into the equation.

3 Connect

Display the equation $2.49v + 4.39 = 19.33$.

Have students share how they interpreted each term in the equation. Select and sequence students that used productive strategies.

Ask, "What is a *solution* to an equation?" A value for the variable that makes the equation true.

Highlight that the equation $2.49v + 4.39 = 19.33$ is an equation in one variable and has one solution that makes it true. If a value is substituted into the equation that does not make it true, it can be used to help approximate what value would make it true.

Math Language Development

MLR8: Discussion Supports

During the Connect, focus on how students interpreted each term in the equation. Have students share their definitions of what a *solution* is and ask them to give an example of a solution to a simple equation, focusing on what the solution means. For example, ask them what the equation $2.49s = 7.47$ represents and what its solution means.

English Learners

Use concrete objects or visuals to model the equation $2.49s = 7.47$ and its solution. For example, use 3 index cards and write 2.49 on each of them. Label each index card "veggie toast."

Power-up

To power up students' ability to identify solutions to equations in one variable, have students complete:

Recall that a value is a *solution* to an equation if, after substituting the value into the equation, it makes the equation true.

Which of the following values represents a solution to the equation $3c + 5 = 17$? Be prepared to explain your thinking.

A. 2
B. 3
[✓ C.] 4
D. 7

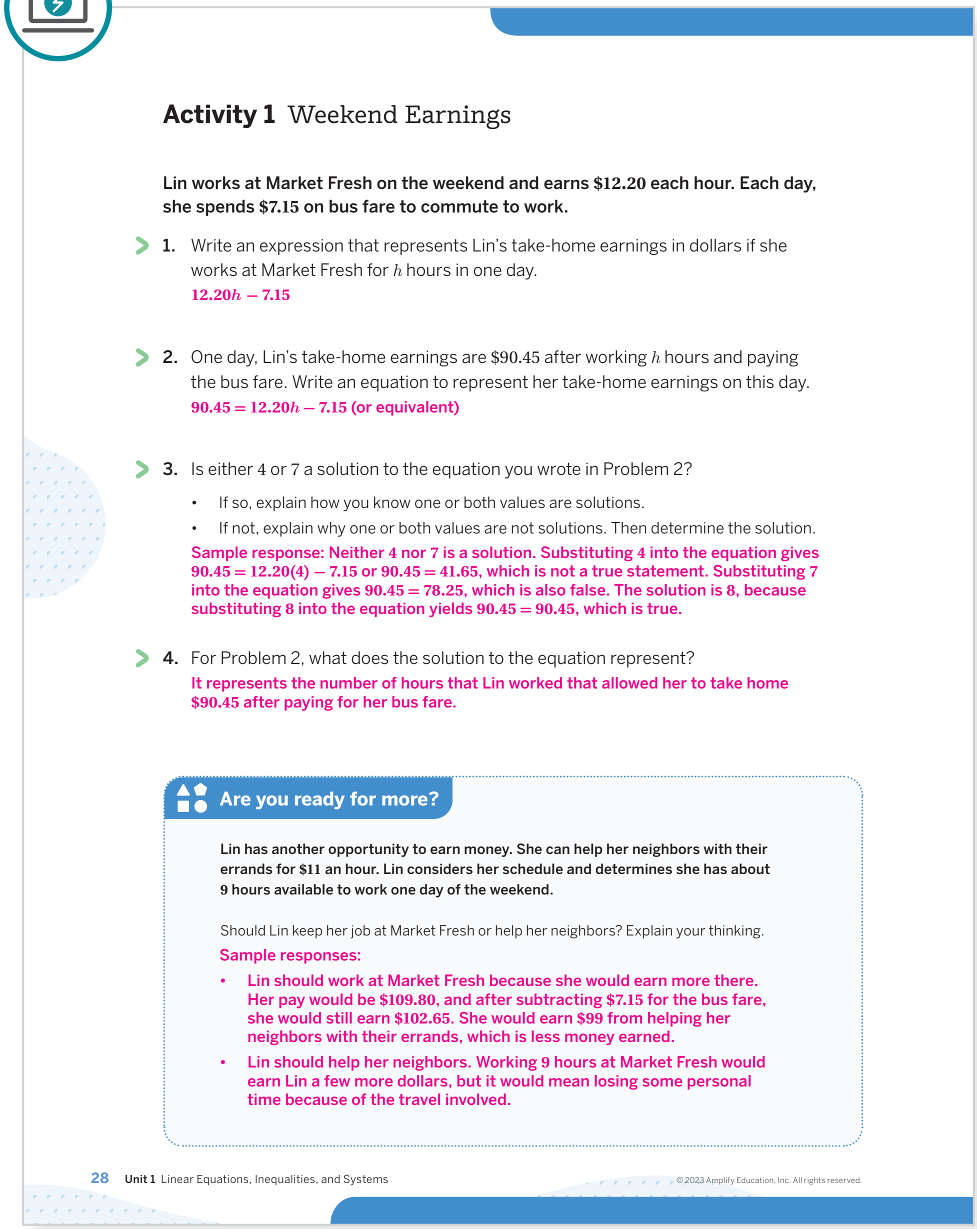
Use: Before the Warm-up

Informed by: Performance on Lesson 3, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 7

Pairs | 20 min

Activity 1 Weekend Earnings

Students write one-variable equations and reason abstractly and quantittively about their solutions to determine values that make the equation true and satisfy the constraints of the context.



Activity 1 Weekend Earnings

Lin works at Market Fresh on the weekend and earns $12.20 each hour. Each day, she spends $7.15 on bus fare to commute to work.

1. Write an expression that represents Lin's take-home earnings in dollars if she works at Market Fresh for h hours in one day.
   $12.20h - 7.15$
2. One day, Lin's take-home earnings are $90.45 after working h hours and paying the bus fare. Write an equation to represent her take-home earnings on this day.
   $90.45 = 12.20h - 7.15$ (or equivalent)
3. Is either 4 or 7 a solution to the equation you wrote in Problem 2?
   - If so, explain how you know one or both values are solutions.
   - If not, explain why one or both values are not solutions. Then determine the solution.

   Sample response: Neither 4 nor 7 is a solution. Substituting 4 into the equation gives $90.45 = 12.20(4) - 7.15$ or $90.45 = 41.65$, which is not a true statement. Substituting 7 into the equation gives $90.45 = 78.25$, which is also false. The solution is 8, because substituting 8 into the equation yields $90.45 = 90.45$, which is true.
4. For Problem 2, what does the solution to the equation represent?
   It represents the number of hours that Lin worked that allowed her to take home $90.45 after paying for her bus fare.

Are you ready for more?

Lin has another opportunity to earn money. She can help her neighbors with their errands for $11 an hour. Lin considers her schedule and determines she has about 9 hours available to work one day of the weekend.

Should Lin keep her job at Market Fresh or help her neighbors? Explain your thinking.

Sample responses:

- **Lin should work at Market Fresh because she would earn more there. Her pay would be $109.80, and after subtracting $7.15 for the bus fare, she would still earn $102.65. She would earn $99 from helping her neighbors with their errands, which is less money earned.**
- **Lin should help her neighbors. Working 9 hours at Market Fresh would earn Lin a few more dollars, but it would mean losing some personal time because of the travel involved.**

28 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in pairs and provide access to scientific calculators. Read the opening passage. You may want to clarify terms such as "commute" and "take-home earnings."

2 Monitor

Help students get started by asking, "Which quantities vary? Which are fixed?"

Look for points of confusion:

- **Struggling to write an equation in Problem 2.** Ask, "What does the expression you wrote in Problem 1 represent?"
- **Substituting 90.45 for h in Problem 2.** Have students articulate what is represented by each variable.

Look for productive strategies:

- Recognizing $90.45 as Lin's total take-home earnings.
- Approximating values above 7.
- Using properties of equality.

3 Connect

Have pairs of students share their strategies for writing the equations for Problems 1 and 2. Have them explain how it describes the constraints for Lin's situation. Select and sequence students using strategies from *guess-and-check* to algebraic manipulation for determining the solution of their equation.

Ask, "What do the results represent when substituting different values into the equation? How can you use those results to help reason about the solution to the equation?"

Highlight that if substituting a value into an equation leads to a false equation, it is not a solution. If the resulting equation is true, then it is a solution.

Differentiated Support

Accessibility: *Activate Background Knowledge, Clarify Vocabulary and Symbols*

Clarify the meanings of the phrases "bus fare" and "commute to work." Some students may be familiar with bus fare if they have used city transportation. Remind students that the phrase "each hour" indicates a constant rate of change.

Accessibility: *Optimize Access to Tools*

Provide a partially completed process table with the first column labeled "Hours, h" and the last column labeled "Take-home earnings, E." Include values for h and have students write the corresponding values for E based on the given constraints. Prompt students to use their table to determine an equation relating h and E.

Math Language Development

MLR5: Co-craft Questions

During the Launch, as you read the opening passage, ask students to think of 1 – 2 mathematical questions that could be asked about the scenario. Here are some sample questions.

- How many hours does she work at Market Fresh each day?
- After paying for bus fare, how much money does Lin get to take home each weekend?
- What expression or equation could I write to represent this scenario?

English Learners

Consider acting out the phrases "bus fare" and "commute to work" to illustrate what these mean.

Optional

Pairs | 15 min

Activity 2 Customer Receipts

Students match customers' receipts to their corresponding orders to practice reasoning quantitatively and abstractly in two variables.

Amps Featured Activity Can I Take Your Order?

Name: __________ Date: ________ Period: ________

Activity 2 Customer Receipts

At Market Fresh, a salmon burger costs \$3 and a side salad costs \$2. Lin's next five customers all order salmon burgers and side salads. Match each customer receipt with the correct customer order.

Receipt:

Receipt for Customer 121
Amount Paid: \$21

Receipt for Customer 122
Amount Paid: \$40

Receipt for Customer 124
Amount Paid: \$32

Receipt for Customer 120
Amount Paid: \$12

Receipt for Customer 123
Amount Paid: \$43

Customer order:

Customer 120:
2 salmon burgers + 3 side salads

Customer 121:
3 salmon burgers + 6 side salads

Customer 122:
10 salmon burgers + 5 side salads

Customer 123:
7 salmon burgers + 11 side salads

Customer 124:
8 salmon burgers + 4 side salads

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Equations and Their Solutions 29

1 Launch

Have students determine matches independently, then compare their responses with a partner. Prompt them to come to a consensus on responses they disagree on.

2 Monitor

Help students get started by prompting them to write an expression to model the total cost of salmon burgers and side salads.

Look for points of confusion:

- **Not knowing what to do with the receipt.** Ask, "How is the amount paid on the receipt determined?" Prompt students to assign a variable to the amount paid and write an equation that models an order of salmon burgers and side salads.

Look for productive strategies:

- Annotating each customer order with the total cost for each item.
- Writing an equation equal to the amount paid for each receipt and substituting pairs of values from the customer orders.
- Writing an expression to represent each customer order and evaluating it.

3 Connect

Have pairs of students share their matches and the strategy they used to determine them. Prompt students to write an equation that represents the relationship between items ordered and amount paid.

Ask, "Are the combinations in each customer order the only pairs of values that would result in the amounts paid on each receipt?"

Highlight that each receipt can be represented by the equation $3b + 2s = t$, where t is the amount paid, and that each customer order only represents one solution for each receipt.

Differentiated Support

Accessibility: ***Guide Processing and Visualization, Optimize Access to Tools***

Display or provide a table, such as the following, that students can use to organize their thinking.

Number of salmon burgers	Cost of salmon burgers (\$)	Number of side salads	Cost of side salads (\$)	Total cost of order (\$)

Extension: ***Math Enrichment***

Have students select one receipt and list all of the possible combinations of salmon burgers and side salads that would result in the amount paid.

Math Language Development

MLR8: Discussion Supports

Display the Anchor Chart PDF, *Sentence Stems*, *Partner and Group Questioning* while students work. Encourage students to use these prompts and ask each other to clarify their thinking. For example, they could ask, "How do you know that Customer 121 paid \$21?"

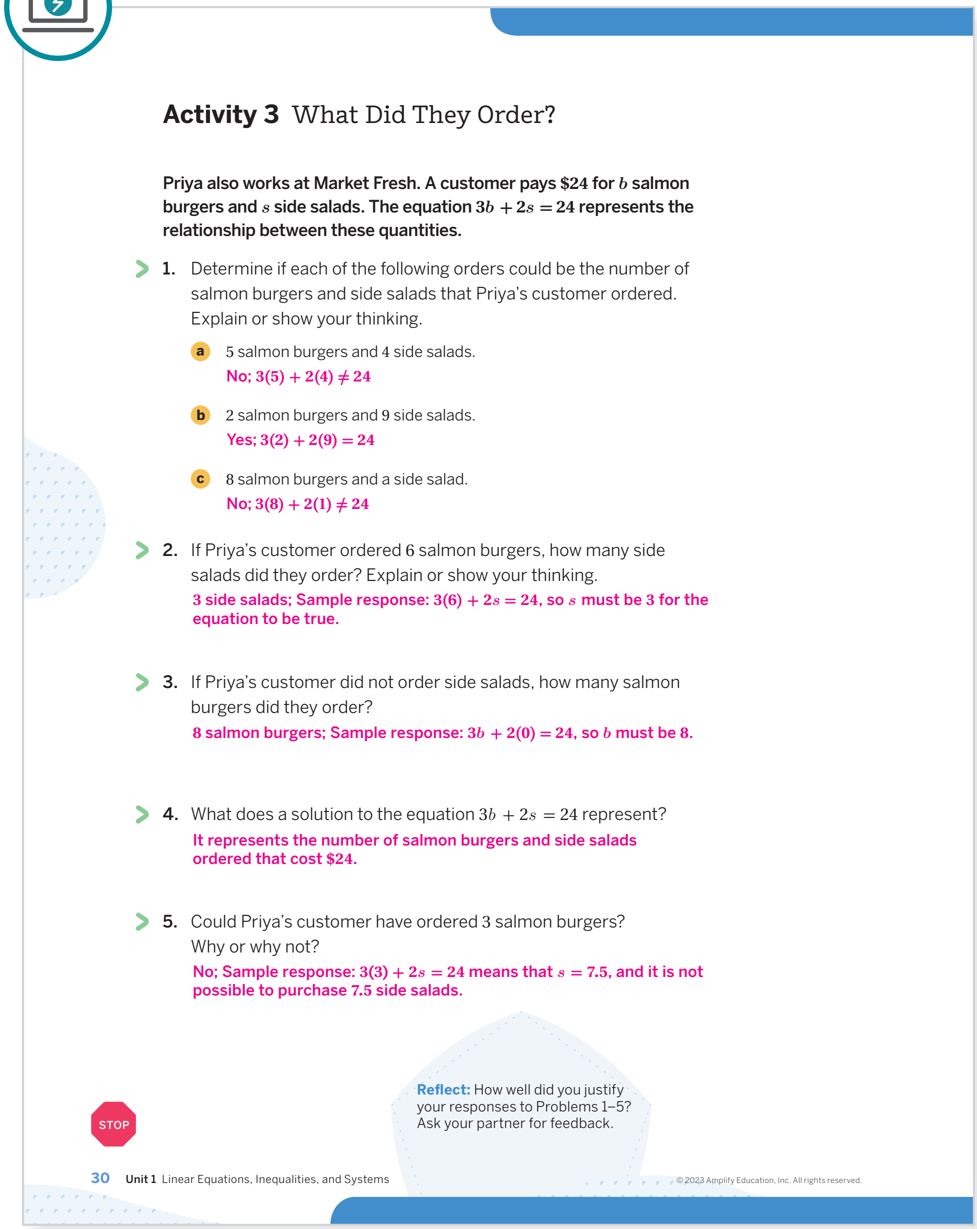
English Learners

During the Connect, annotate each customer with their corresponding equation.

Pairs | 15 min

Activity 3 What Did They Order?

Students reason abstractly and quantitatively on the solutions of an equation given a context to recall that a solution to a two-variable equation is a pair of values that make it true.



Activity 3 What Did They Order?

Priya also works at Market Fresh. A customer pays $24 for b salmon burgers and s side salads. The equation $3b + 2s = 24$ represents the relationship between these quantities.

1. Determine if each of the following orders could be the number of salmon burgers and side salads that Priya's customer ordered. Explain or show your thinking.
   - a 5 salmon burgers and 4 side salads.
     No; $3(5) + 2(4) \neq 24$
   - b 2 salmon burgers and 9 side salads.
     Yes; $3(2) + 2(9) = 24$
   - c 8 salmon burgers and a side salad.
     No; $3(8) + 2(1) \neq 24$
2. If Priya's customer ordered 6 salmon burgers, how many side salads did they order? Explain or show your thinking.
   3 side salads; Sample response: $3(6) + 2s = 24$, so s must be 3 for the equation to be true.
3. If Priya's customer did not order side salads, how many salmon burgers did they order?
   8 salmon burgers; Sample response: $3b + 2(0) = 24$, so b must be 8.
4. What does a solution to the equation $3b + 2s = 24$ represent?
   It represents the number of salmon burgers and side salads ordered that cost $24.
5. Could Priya's customer have ordered 3 salmon burgers? Why or why not?
   No; Sample response: $3(3) + 2s = 24$ means that $s = 7.5$, and it is not possible to purchase 7.5 side salads.

STOP

Reflect: How well did you justify your responses to Problems 1–5? Ask your partner for feedback.

30 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Students remain in pairs. Allow continued access to scientific calculators. Give students quiet time to work independently before sharing with their partners.

2 Monitor

Help students get started by prompting them to organize values of b and s for each pair of values with a table.

Look for points of confusion:

- **Having difficulty explaining how to determine the value of s given b in Problem 2.** Ask, "How much does the customer have left to spend on side salads after purchasing 6 salmon burgers?"

Look for productive strategies:

- Guessing and checking values for b and s that yield 24.
- Translating each order into algebraic expressions and evaluating them.

3 Connect

Have students share their thinking of what constitutes a solution and their strategies for determining the value of one variable, given the other. Select and sequence students with different responses for Problem 4.

Ask:

- "What does it mean when you say $b = 5$ and $s = 4$ are not solutions?"
- "How many possible solutions are there to the equation? How many possible combinations of salmon burgers and side salads would give a total of 24? Are these the same number?"

Highlight that a solution to an equation in two variables is a pair of values that make the equation true. It is possible to have more than one solution.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

Allow continued access to the table mentioned in the previous activity. Suggest that students color code the variables in the equation and what they represent in Problems 1, 2, 3, and 5.

Extension: *Math Enrichment*

Show students a graph of the equation $3b + 2s = 24$ (horizontal axis: b; vertical axis: s). Have them use the graph to determine how many salmon burgers and side salads the customer could purchase to have about the same number of each. 4 salmon burgers and 6 side salads is the closest to purchasing the same number of each.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you highlight what a solution to an equation in two variables means, display an equation in one variable and an equation in two variables, such as $3b = 24$ and $3b + 2s = 24$.

Ask students to compare and contrast these two equations. Ask:

- "How are these equations similar? How are they different?"
- "What does a solution mean to the first equation? To the second?"

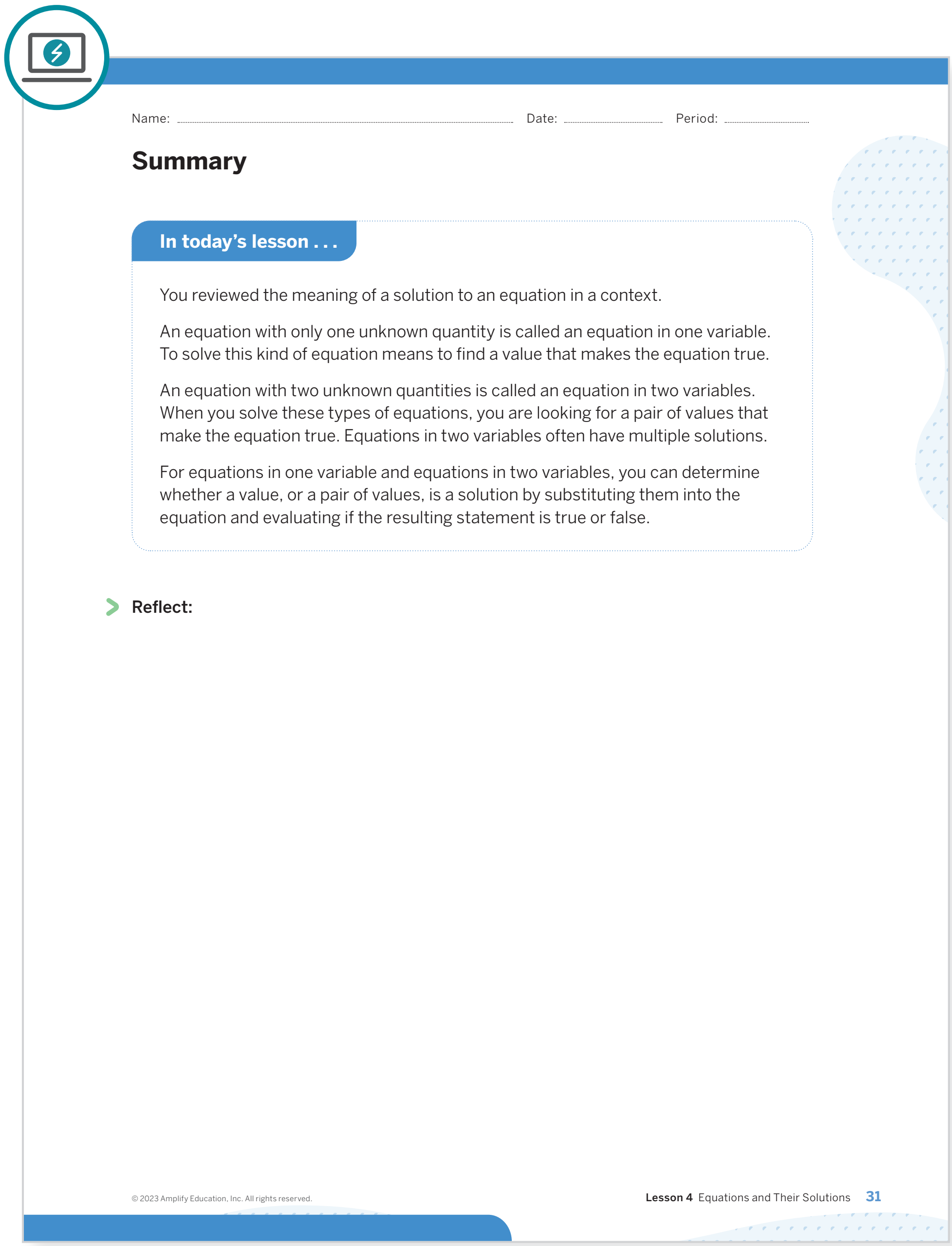
English Learners

Annotate the first equation by writing "1 variable" and the second equation by writing "2 variables."

Whole Class | 5 min

Summary

Review and synthesize the meanings of solutions of equations in one- and two-variables in context.



Name: Date: Period:

Summary

In today's lesson . . .

You reviewed the meaning of a solution to an equation in a context.

An equation with only one unknown quantity is called an equation in one variable. To solve this kind of equation means to find a value that makes the equation true.

An equation with two unknown quantities is called an equation in two variables. When you solve these types of equations, you are looking for a pair of values that make the equation true. Equations in two variables often have multiple solutions.

For equations in one variable and equations in two variables, you can determine whether a value, or a pair of values, is a solution by substituting them into the equation and evaluating if the resulting statement is true or false.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Equations and Their Solutions 31

Synthesize

Display the prices of a salmon burger and side salad at Market Fresh.

Ask:

- "What does the equation $3b + 2s = 75$ represent in this context?" The number of salmon burgers and side salads ordered, totaling \$75.
- "What does it mean to solve this equation?" To determine the number of salmon burgers and side salads ordered to get a total of \$75.
- "Is the combination of 15 salmon burgers and 16 side salads a solution? Why or why not?" No, because substituting 15 for b and 16 for s leads to a false equation.
- "What is a solution to this equation?" Sample response: $b = 15$ and $s = 15$
- "In this context, what does a solution to the equation $3(20) + 2s = 75$ represent?" The number of side salads that were ordered given 20 salmon burgers were ordered, for a total cost of \$75.

Highlight that a solution to an equation in one variable is a single value that, when substituted, makes the equation true. The solution to an equation in two variables requires a pair of values that make the equation true, and there can be many solutions.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

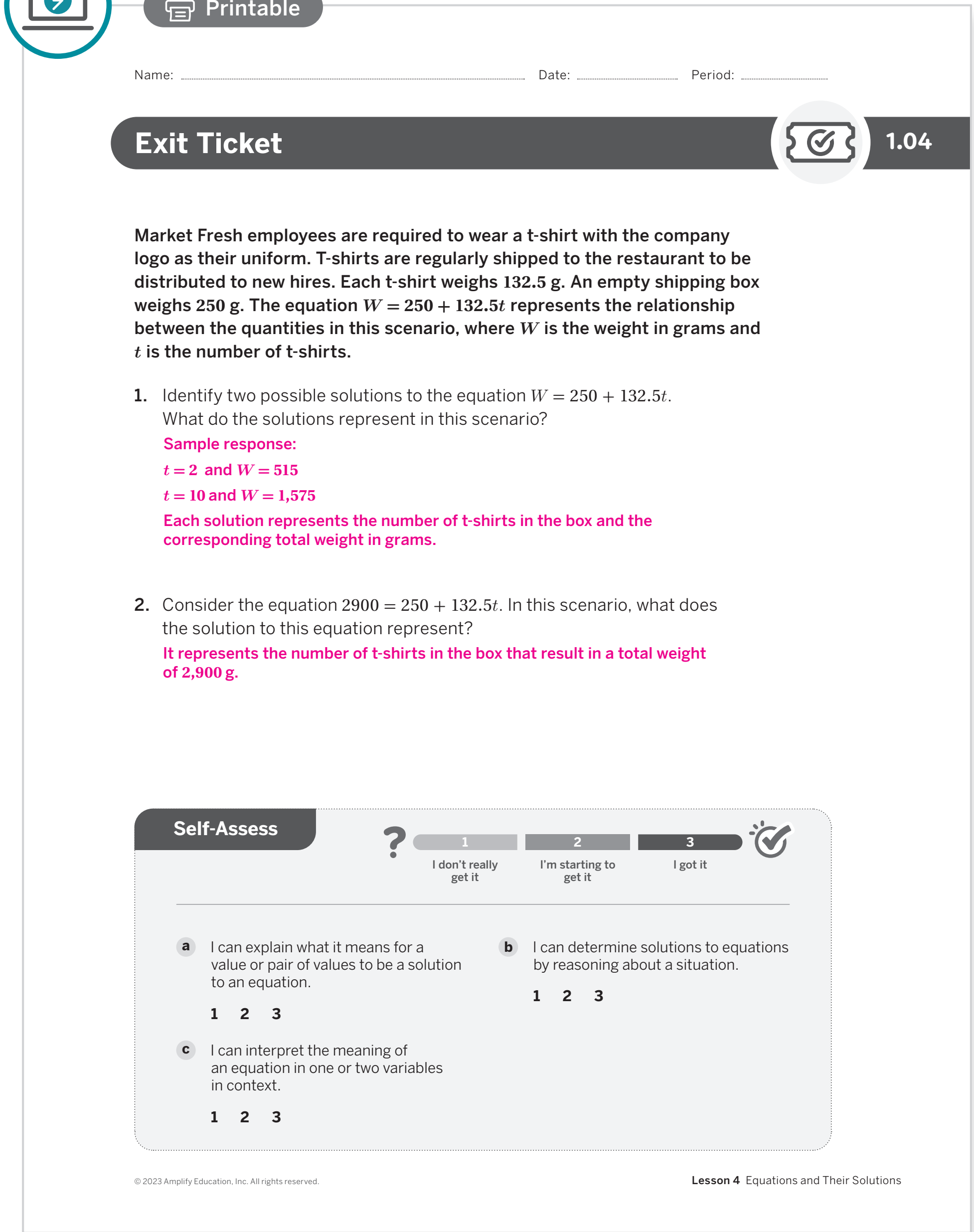
- "Explain what is meant by a solution to an equation."

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the solution of a linear equation and what it represents in a context.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 1.04

Market Fresh employees are required to wear a t-shirt with the company logo as their uniform. T-shirts are regularly shipped to the restaurant to be distributed to new hires. Each t-shirt weighs 132.5 g. An empty shipping box weighs 250 g. The equation $W = 250 + 132.5t$ represents the relationship between the quantities in this scenario, where W is the weight in grams and t is the number of t-shirts.

1. Identify two possible solutions to the equation $W = 250 + 132.5t$. What do the solutions represent in this scenario?
   Sample response:
   $t = 2$ **and** $W = 515$
   $t = 10$ **and** $W = 1{,}575$
   Each solution represents the number of t-shirts in the box and the corresponding total weight in grams.

2. Consider the equation $2900 = 250 + 132.5t$. In this scenario, what does the solution to this equation represent?
   It represents the number of t-shirts in the box that result in a total weight of 2,900 g.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can explain what it means for a value or pair of values to be a solution to an equation.
1 2 3

b. I can determine solutions to equations by reasoning about a situation.
1 2 3

c. I can interpret the meaning of an equation in one or two variables in context.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Equations and Their Solutions

Success looks like . . .

- **Goal:** Determining solutions to equations in one variable and in two variables by reasoning about the relationships in context.
  - » Determining two possible solutions to the equation for the number of t-shirts in the box and the weight of the box in Problem 1.
- **Language Goal:** Interpreting solutions to equations in one variable and in two variables. **(Speaking and Listening, Writing)**
  - » Interpreting the solution of the equation in Problem 2.

Suggested next steps

If students are unable to determine or interpret the solutions in Problem 1, consider:

- Asking, "How much does the box weigh if it has 2 t-shirts in it? 5? 10?"
- Reviewing Activity 2.
- Assigning Practice Problems 1 and 2.

If students are unable to interpret the solution in Problem 2, consider:

- Asking, "How is this equation different from the one in Problem 1?"
- Reviewing Activity 1.
- Assigning Practice Problem 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The focus of this lesson was understanding and being able to articulate what is meant by a solution to an equation. How did this go?
- What did you see in the way some students approached matching receipts to their orders in Activity 2 that you would like other students to try? What might you change for the next time you teach this lesson?

Math Language Development

Language Goal: Interpreting solutions to equations in one variable and in two variables.

Reflect on students' language development toward this goal.

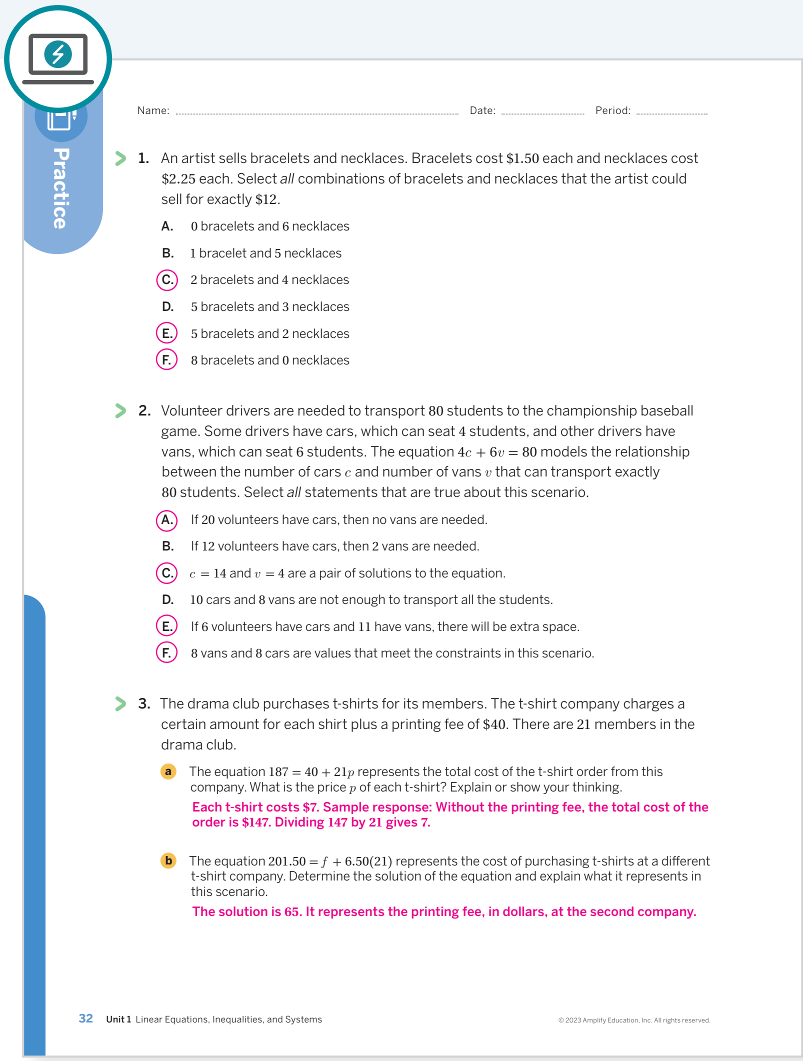
- Do students' responses to Problem 2 of the Exit Ticket demonstrate understanding of the meaning of the solution? Are their descriptions accurate and precise? How can you help them be more precise?

Sample descriptions:

Emerging	Expanding
Number of t-shirts.	The solution represents the number of t-shirts in the box that will result in a total weight of 2,900 g.

Practice

Independent



Name: ______ Date: ______ Period: ______

1. An artist sells bracelets and necklaces. Bracelets cost $1.50 each and necklaces cost $2.25 each. Select *all* combinations of bracelets and necklaces that the artist could sell for exactly $12.
   - A. 0 bracelets and 6 necklaces
   - B. 1 bracelet and 5 necklaces
   - [✓ C.] 2 bracelets and 4 necklaces
   - D. 5 bracelets and 3 necklaces
   - [✓ E.] 5 bracelets and 2 necklaces
   - [✓ F.] 8 bracelets and 0 necklaces

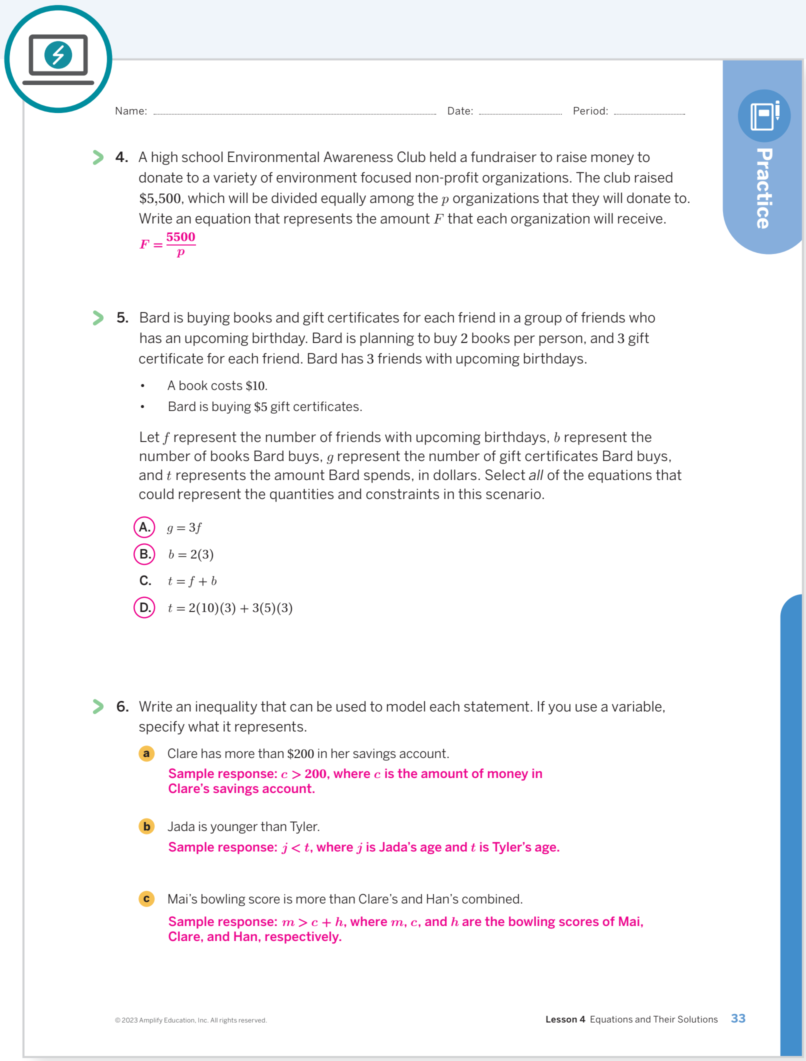
2. Volunteer drivers are needed to transport 80 students to the championship baseball game. Some drivers have cars, which can seat 4 students, and other drivers have vans, which can seat 6 students. The equation $4c + 6v = 80$ models the relationship between the number of cars c and number of vans v that can transport exactly 80 students. Select *all* statements that are true about this scenario.
   - [✓ A.] If 20 volunteers have cars, then no vans are needed.
   - B. If 12 volunteers have cars, then 2 vans are needed.
   - [✓ C.] $c = 14$ and $v = 4$ are a pair of solutions to the equation.
   - D. 10 cars and 8 vans are not enough to transport all the students.
   - [✓ E.] If 6 volunteers have cars and 11 have vans, there will be extra space.
   - [✓ F.] 8 vans and 8 cars are values that meet the constraints in this scenario.

3. The drama club purchases t-shirts for its members. The t-shirt company charges a certain amount for each shirt plus a printing fee of $40. There are 21 members in the drama club.
   - a. The equation $187 = 40 + 21p$ represents the total cost of the t-shirt order from this company. What is the price p of each t-shirt? Explain or show your thinking.

     Each t-shirt costs $7. Sample response: Without the printing fee, the total cost of the order is $147. Dividing 147 by 21 gives 7.
   - b. The equation $201.50 = f + 6.50(21)$ represents the cost of purchasing t-shirts at a different t-shirt company. Determine the solution of the equation and explain what it represents in this scenario.

     The solution is 65. It represents the printing fee, in dollars, at the second company.

© 2023 Amplify Education, Inc. All rights reserved.



Name: ______ Date: ______ Period: ______

4. A high school Environmental Awareness Club held a fundraiser to raise money to donate to a variety of environment focused non-profit organizations. The club raised $5,500, which will be divided equally among the p organizations that they will donate to. Write an equation that represents the amount F that each organization will receive.

   $F = \frac{5500}{p}$

5. Bard is buying books and gift certificates for each friend in a group of friends who has an upcoming birthday. Bard is planning to buy 2 books per person, and 3 gift certificate for each friend. Bard has 3 friends with upcoming birthdays.
   - A book costs $10.
   - Bard is buying $5 gift certificates.

   Let f represent the number of friends with upcoming birthdays, b represent the number of books Bard buys, g represent the number of gift certificates Bard buys, and t represents the amount Bard spends, in dollars. Select *all* of the equations that could represent the quantities and constraints in this scenario.
   - [✓ A.] $g = 3f$
   - [✓ B.] $b = 2(3)$
   - C. $t = f + b$
   - [✓ D.] $t = 2(10)(3) + 3(5)(3)$

6. Write an inequality that can be used to model each statement. If you use a variable, specify what it represents.
   - a. Clare has more than $200 in her savings account.

     Sample response: $c > 200$, where c is the amount of money in Clare's savings account.
   - b. Jada is younger than Tyler.

     Sample response: $j < t$, where j is Jada's age and t is Tyler's age.
   - c. Mai's bowling score is more than Clare's and Han's combined.

     Sample response: $m > c + h$, where m, c, and h are the bowling scores of Mai, Clare, and Han, respectively.

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity 2	3
	3	Activity 1	3
Spiral	4	Unit 1 Lesson 3	2
	5	Unit 1 Lesson 2	2
Formative ϟ	6	Unit 1 Lesson 5	2

ϟ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 5**

Writing Inequalities to Model Relationships

Let's use inequalities to represent constraints in a context.

Focus

Goals

1. **Language Goal:** Interpret inequalities in a given context. **(Speaking and Listening, Writing)**
2. Write inequalities to represent constraints in a given context.

Coherence

Today

Students work with inequalities by reviewing inequality symbols and recalling their meanings both with and without context. They examine key quantities in a context, define variables for the context, and write inequalities to model the constraints of the context.

Previously

Students determined what makes a value or pair of values a solution to an equation in one- or two-variables by verifying if it made the equation true.

Coming Soon

In the next lesson, students are introduced to graphing technology and use graphs as a way to represent the relationships between quantities in a context.

Rigor

- Students write inequalities to model relationships to build on **fluency** of skills from prior grades.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Review of Inequality Symbols*
- Instructional Routine PDF, *Mix and Mingle: Instructions*
- music

Math Language Development

Review words

- *boundary value*
- *inequality*
- *variable*

Amps | Featured Activity

Activity 2

Formative Feedback for Students

Students write equations/inequalities and see whether or not they satisfy the given elevator constraints, and if necessary, make adjustments to their work until they do.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may become frustrated when they are asked to write as many equations and inequalities as they can in Activity 2, particularly if they do not have a clear strategy for doing so. Encourage students to reflect and write down or highlight the quantities and constraints in the scenario before attempting the problems, and check in with their peers for strategies they are using after their own individual quiet think-time.

Modifications to Pacing

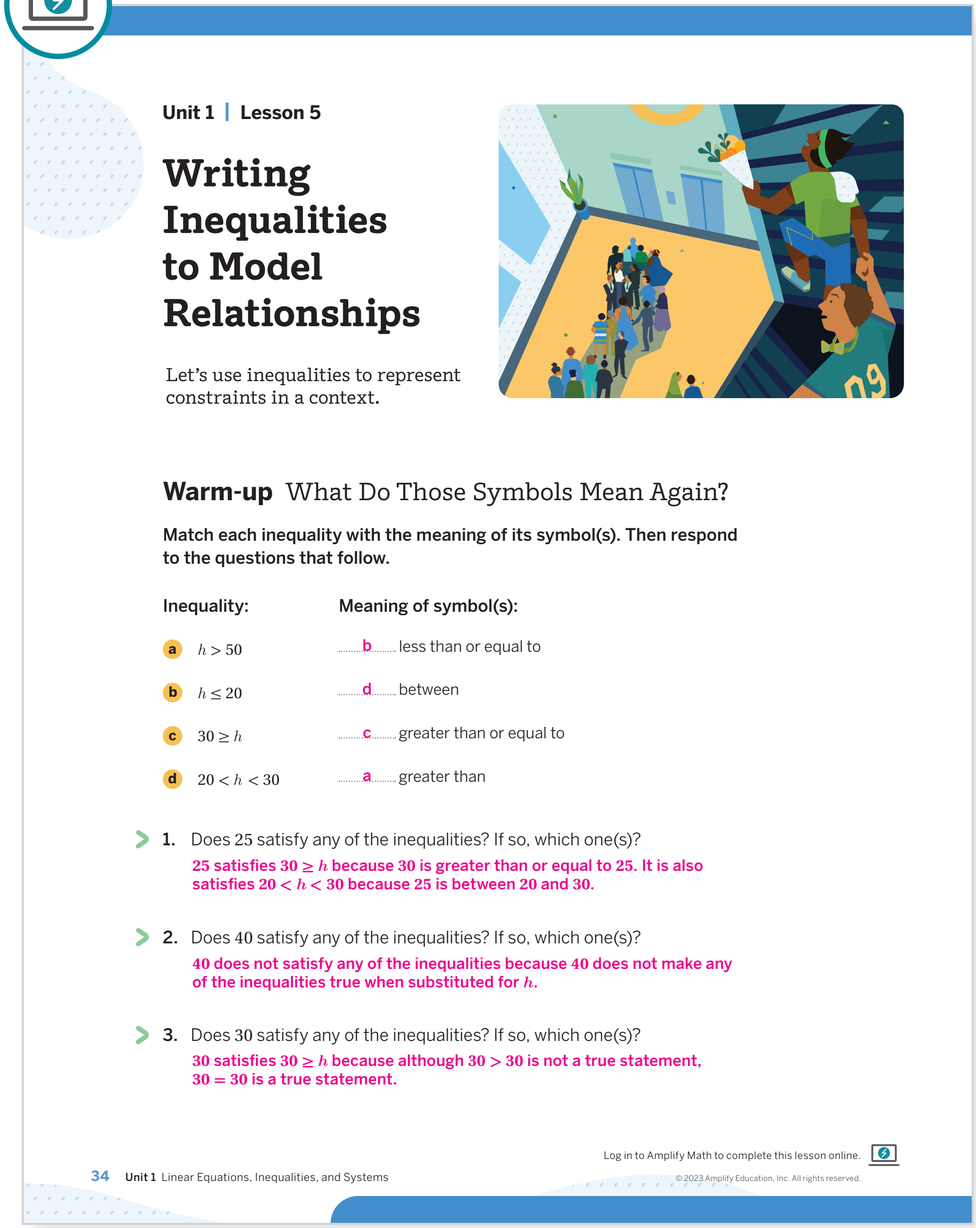
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, have students complete Problems 1 and 2 only, and reduce the time they have to work with a partner. Then consider Problem 3 during the *Connect*.

Independent | 5 min

Warm-up What Do Those Symbols Mean Again?

Students match inequalities with their verbal descriptions to recall representing and determining solutions of inequalities.



Unit 1 | Lesson 5

Writing Inequalities to Model Relationships

Let's use inequalities to represent constraints in a context.

Warm-up What Do Those Symbols Mean Again?

Match each inequality with the meaning of its symbol(s). Then respond to the questions that follow.

Inequality:	**Meaning of symbol(s):**
a $h > 50$	b less than or equal to
b $h \leq 20$	d between
c $30 \geq h$	c greater than or equal to
d $20 < h < 30$	a greater than

1. Does 25 satisfy any of the inequalities? If so, which one(s)?
   25 satisfies $30 \geq h$ because 30 is greater than or equal to 25. It is also satisfies $20 < h < 30$ because 25 is between 20 and 30.

2. Does 40 satisfy any of the inequalities? If so, which one(s)?
   40 does not satisfy any of the inequalities because 40 does not make any of the inequalities true when substituted for h.

3. Does 30 satisfy any of the inequalities? If so, which one(s)?
   30 satisfies $30 \geq h$ because although $30 > 30$ is not a true statement, $30 = 30$ is a true statement.

Log in to Amplify Math to complete this lesson online.

34 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Give students 1 minute of think-time and 2 minutes of work time.

2 Monitor

Help students get started by displaying the inequality $a < b$ and asking for possible values of a and b that make the inequality true.

Look for points of confusion:

- **Including the boundary value for the first inequality.** Ask, "If you substitute 50 for h, is the inequality true?"
- **Writing an inequality to model the relationship between the quantities in each statement, and then matching them to the given inequalities.**

Look for productive strategies:

- Using the boundary values to test solutions for each inequality.
- Using a number line to determine solutions for each inequality.

3 Connect

Have individual students share their strategies for reading or annotating each inequality. Ask students what the numbers in each inequality mean.

Ask, "How do you know whether a number is a solution to an inequality?"

Display the equations $h = 50$, $h = 20$, and $30 = h$.

Highlight that in these equations, there is only one value that could make the equation true, but in an inequality, there is a range of values that make it true.

Math Language Development

MLR2: *Collect and Display*

As students share their strategies for reading or annotating each inequality, display the Anchor Chart PDF, *Review of Inequality Symbols*. Add the language students use to the display, connecting words and phrases to the inequality symbols. Encourage students to refer to the display during class discussions.

English Learners

The term *satisfy* might be unfamiliar to students in this context. Clarify for students what it means to *satisfy* an inequality using simple examples. Add this to the class display.

Power-up

To power up students' ability to compare values using inequality symbols, have students complete:

Recall that < means *less than* and > means *greater than*. Place a < or > to correctly complete each inequality if $g = 5$ and $h = 0.1$.

a $g \underline{<} 10$

b $\frac{1}{5} \underline{>} h$

c $g \underline{>} -10$

d $-\frac{1}{5} \underline{<} h$

e $-g \underline{>} -10$

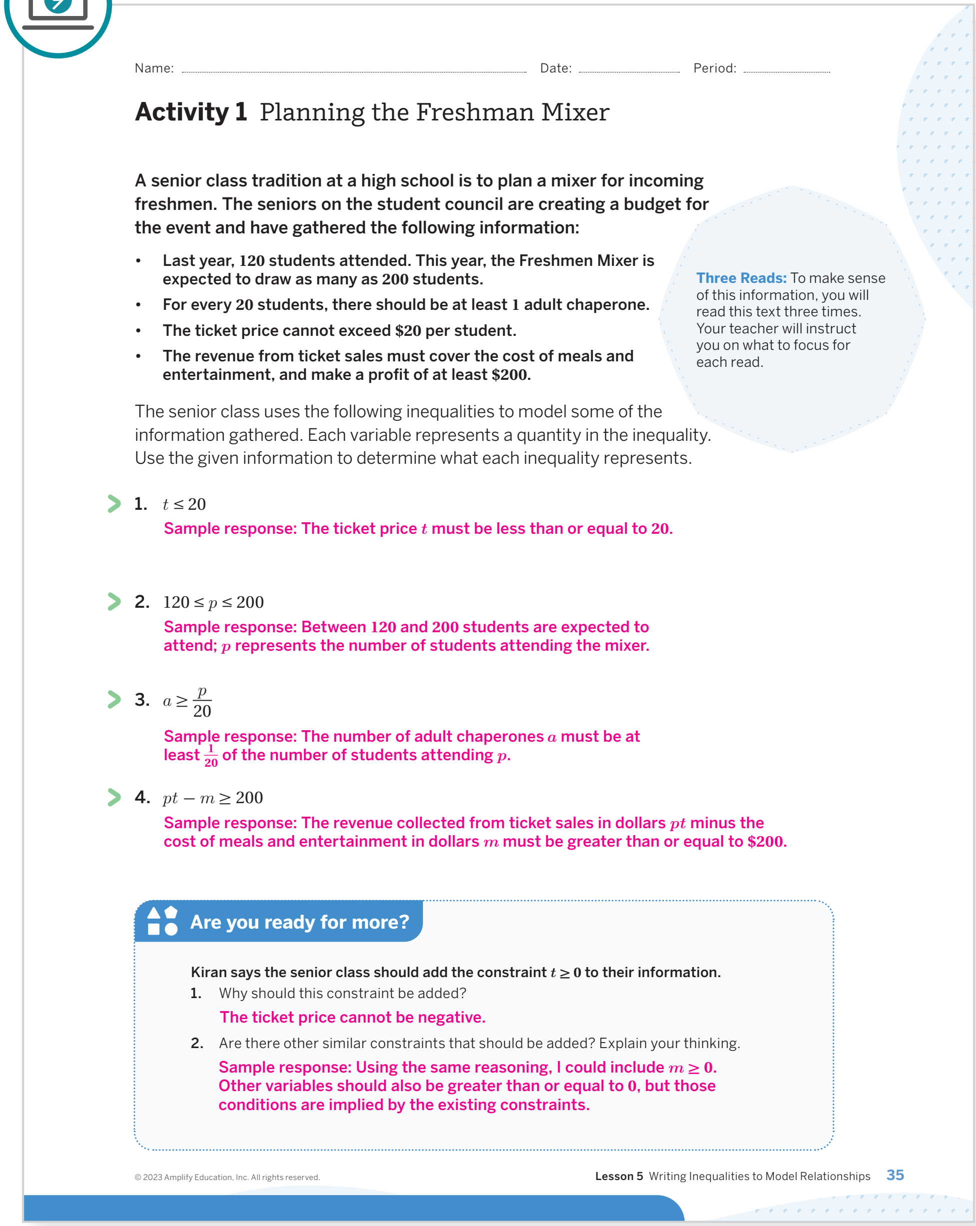
Use: Before the Warm-up

Informed by: Performance on Lesson 4, Practice Problem 6

Pairs | 15 min

Activity 1 Planning the Freshman Mixer

Students interpret inequalities in a context to reason quantitatively and abstractly.



Name: ______ Date: ______ Period: ______

Activity 1 Planning the Freshman Mixer

A senior class tradition at a high school is to plan a mixer for incoming freshmen. The seniors on the student council are creating a budget for the event and have gathered the following information:

- **Last year, 120 students attended. This year, the Freshmen Mixer is expected to draw as many as 200 students.**
- **For every 20 students, there should be at least 1 adult chaperone.**
- **The ticket price cannot exceed $20 per student.**
- **The revenue from ticket sales must cover the cost of meals and entertainment, and make a profit of at least $200.**

Three Reads: To make sense of this information, you will read this text three times. Your teacher will instruct you on what to focus for each read.

The senior class uses the following inequalities to model some of the information gathered. Each variable represents a quantity in the inequality. Use the given information to determine what each inequality represents.

1. $t \le 20$

   Sample response: The ticket price t must be less than or equal to 20.

2. $120 \le p \le 200$

   Sample response: Between 120 and 200 students are expected to attend; p represents the number of students attending the mixer.

3. $a \ge \frac{p}{20}$

   Sample response: The number of adult chaperones a must be at least $\frac{1}{20}$ of the number of students attending p.

4. $pt - m \ge 200$

   Sample response: The revenue collected from ticket sales in dollars pt minus the cost of meals and entertainment in dollars m must be greater than or equal to $200.

Are you ready for more?

Kiran says the senior class should add the constraint $t \ge 0$ to their information.

1. Why should this constraint be added?

   The ticket price cannot be negative.

2. Are there other similar constraints that should be added? Explain your thinking.

   Sample response: Using the same reasoning, I could include $m \ge 0$. Other variables should also be greater than or equal to 0, but those conditions are implied by the existing constraints.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 5 Writing Inequalities to Model Relationships 35

1 Launch

Allow students individual work time before having them share their work with a partner.

2 Monitor

Help students get started by prompting them to identify keywords and boundary values in the statements.

Look for points of confusion:

- **Having difficulty representing the second bullet.** Prompt students to create a table that relates the number of students to the minimum number of chaperones needed to satisfy the constraint.
- **Not understanding what pt represents.** Ask, "How would the senior class determine the revenue from ticket sales?"

Look for productive strategies:

- Associating boundary values in each statement with their corresponding inequality.
- Writing an inequality for each statement, then matching them to the given inequalities.

3 Connect

Have pairs of students share their strategies for interpreting the statements and matching them to each inequality. Select and sequence students to share in order of the problems.

Ask:

- "How many chaperones are needed if there are 120 students? 180 students?" At least $\frac{120}{20}$, or 6 chaperones. At least $\frac{180}{20}$, or 9 chaperones.
- "What term is used to represent the revenue from ticket sales?" pt

Highlight that the last two inequalities represent the constraints on profit and on the number of chaperones.

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Ask students to color code the variables in Problems 1–4 with what they represent in the text. Consider demonstrating the first one by color coding t in Problem 1 and the phrase "ticket price" in the second bullet.

Extension: *Math Enrichment*

Ask students to alter the inequalities to represent these changes:

- For every 10 students, there must now be at least 1 adult chaperone.
  $a \ge \frac{p}{10}$
- The ticket price will be lowered to not exceed $15 per student.
  $t \le 15$

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

- **Read 1:** Students should understand that the seniors are creating a budget for the freshman mixer and there are several constraints.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as the profit must be at least $200.
- **Read 3:** Ask students to think about how inequalities can represent this information.

English Learners

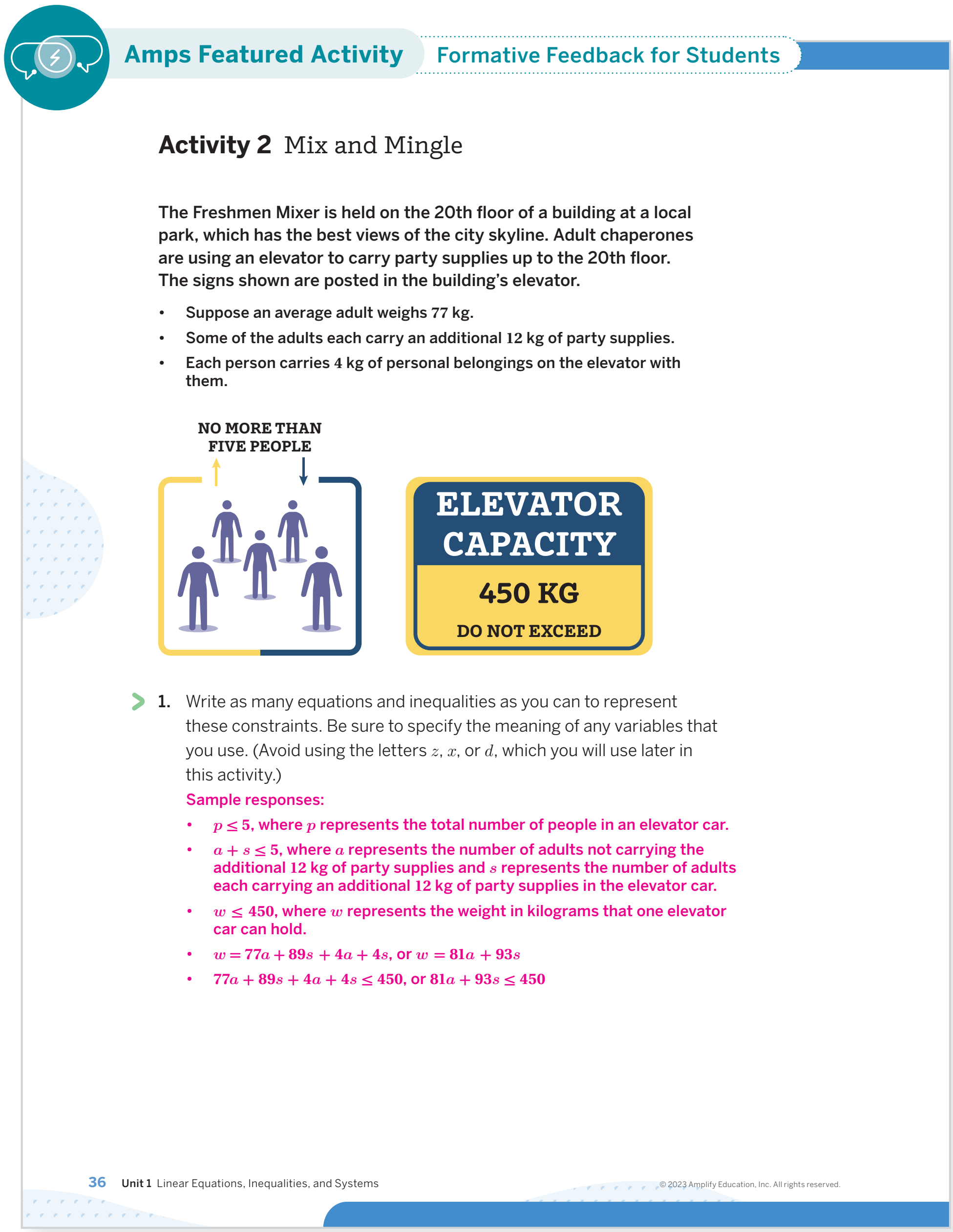
Have students highlight key phrases, such as *for every 20 students, there should be at least 1 adult chaperone.*

Pairs | 20 min

Activity 2 Mix and Mingle

Students write inequalities to model the relationships between them.

Amps Featured Activity Formative Feedback for Students



Activity 2 Mix and Mingle

The Freshmen Mixer is held on the 20th floor of a building at a local park, which has the best views of the city skyline. Adult chaperones are using an elevator to carry party supplies up to the 20th floor. The signs shown are posted in the building's elevator.

- Suppose an average adult weighs 77 kg.
- Some of the adults each carry an additional 12 kg of party supplies.
- Each person carries 4 kg of personal belongings on the elevator with them.

NO MORE THAN FIVE PEOPLE

ELEVATOR CAPACITY
450 KG
DO NOT EXCEED

1. Write as many equations and inequalities as you can to represent these constraints. Be sure to specify the meaning of any variables that you use. (Avoid using the letters z, x, or d, which you will use later in this activity.)
   Sample responses:
   - $p \leq 5$, where p represents the total number of people in an elevator car.
   - $a + s \leq 5$, where a represents the number of adults not carrying the additional 12 kg of party supplies and s represents the number of adults each carrying an additional 12 kg of party supplies in the elevator car.
   - $w \leq 450$, where w represents the weight in kilograms that one elevator car can hold.
   - $w = 77a + 89s + 4a + 4s$, or $w = 81a + 93s$
   - $77a + 89s + 4a + 4s \leq 450$, or $81a + 93s \leq 450$

36 Unit 1 Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in pairs to complete Problems 1 and 2. Use the Instructional Routine PDF, *Mix and Mingle: Instructions* to introduce the instructional routine. Provide students with sticky notes for note-taking. **Note:** This routine is designed to be used with clips of music.

2 Monitor

Help students get started by prompting them to list all the information they know from the context.

Look for points of confusion:

- **Having difficulty representing the total number of people.** Ask, "If you know the number of adults either carrying supplies or not carrying supplies, how would you determine the total number of adults?" Then prompt them to express this algebraically, using the variables they have already defined.
- **Not accounting for the additional weight of personal belongings.** Point to the third bullet and ask, "Where is this constraint represented in your equation/inequality?"

Look for productive strategies:

- Writing inequalities to model the constraints in the elevator signage.
- Using the inequalities from Problem 1 to determine inequalities for Problem 3.

Activity 2 continued ›

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Provide access to colored pencils or highlighters and suggest students color code the quantities in the text and diagrams with the variables they choose in Problem 1.

Extension: *Math Enrichment*

During the Connect, as students share their equations and inequalities, challenge students to rewrite as many of the inequalities as they can using fewer terms.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their responses, display any words and phrases that describe the meaning of each equation/inequality. Call students' attention to the different ways the constraints are represented by language and equations/inequalities of different forms.

English Learners

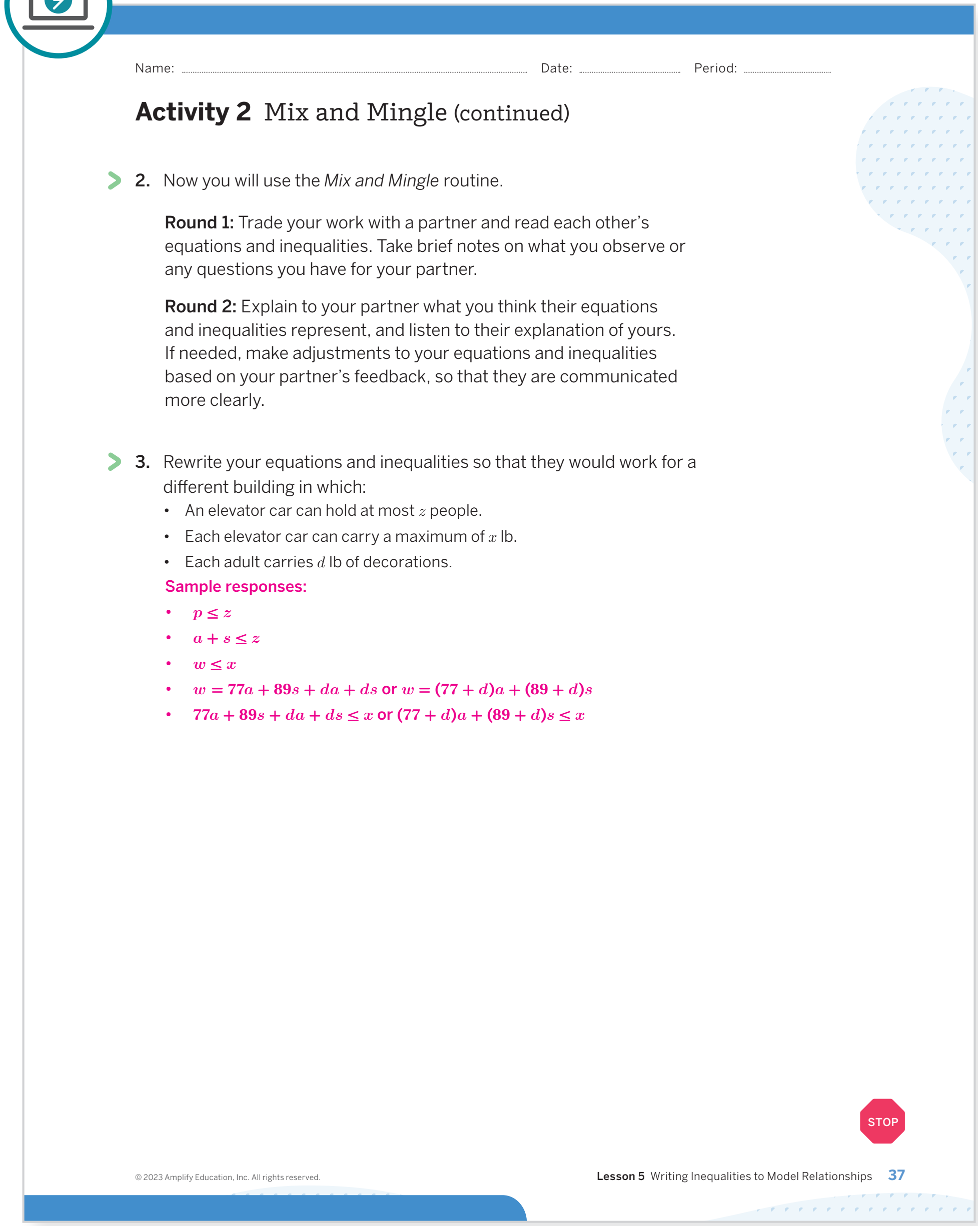
Use color coding or diagrams to show how different phrases and symbols represent the same constraint. For example:

- An elevator car can hold *at most* 5 people.
- An elevator car can hold *no more than* 5 people.
- An elevator car can hold *a maximum of* 5 people.
- $p \leq 5$

Pairs | 20 min

Activity 2 Mix and Mingle (continued)

Students write inequalities to model the relationships between them.



Name: Date: Period:

Activity 2 Mix and Mingle (continued)

2. Now you will use the *Mix and Mingle* routine.

   Round 1: Trade your work with a partner and read each other's equations and inequalities. Take brief notes on what you observe or any questions you have for your partner.

   Round 2: Explain to your partner what you think their equations and inequalities represent, and listen to their explanation of yours. If needed, make adjustments to your equations and inequalities based on your partner's feedback, so that they are communicated more clearly.

3. Rewrite your equations and inequalities so that they would work for a different building in which:
   - An elevator car can hold at most z people.
   - Each elevator car can carry a maximum of x lb.
   - Each adult carries d lb of decorations.

   Sample responses:
   - $p \leq z$
   - $a + s \leq z$
   - $w \leq x$
   - $w = 77a + 89s + da + ds$ or $w = (77 + d)a + (89 + d)s$
   - $77a + 89s + da + ds \leq x$ or $(77 + d)a + (89 + d)s \leq x$

STOP

© 2023 Amplify Education, Inc. All rights reserved.

③ Connect

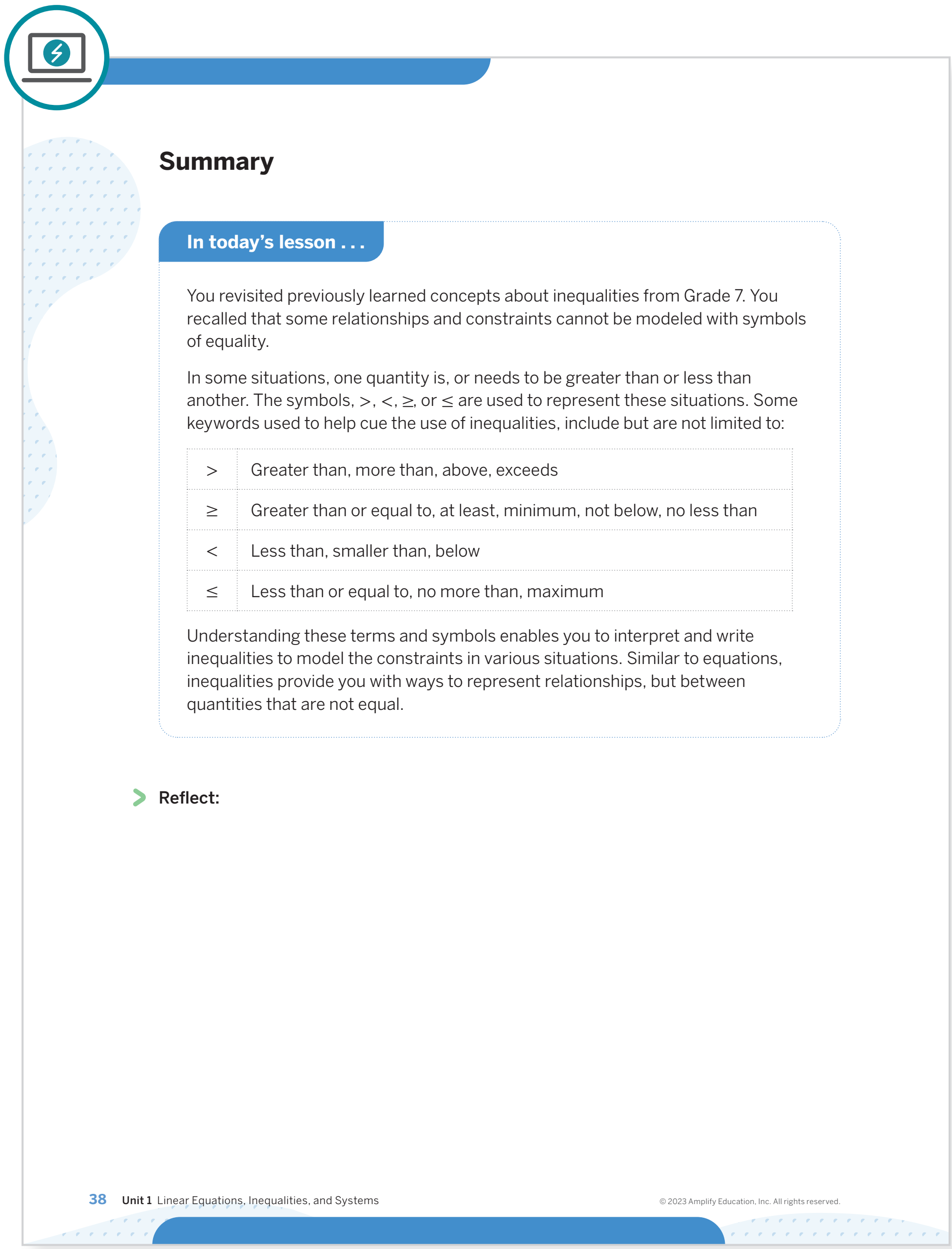
Have pairs of students share their equations and inequalities, selecting and sequencing those reasoning more concretely (Problem 1) to more abstractly (Problem 3). Record and display their responses. Have the class identify equivalent statements and explain how they model the same constraints.

Highlight that the same constraints may be accurately represented by statements of different forms. Consider reading the inequalities using different keywords, e.g. "The maximum weight is 450 kg" or "the weight is at most 450 kg."

Whole Class | 5 min

Summary

Review and synthesize how to write and interpret inequalities that represent the constraints in a given context.



Summary

In today's lesson . . .

You revisited previously learned concepts about inequalities from Grade 7. You recalled that some relationships and constraints cannot be modeled with symbols of equality.

In some situations, one quantity is, or needs to be greater than or less than another. The symbols, >, <, ≥, or ≤ are used to represent these situations. Some keywords used to help cue the use of inequalities, include but are not limited to:

>	Greater than, more than, above, exceeds
≥	Greater than or equal to, at least, minimum, not below, no less than
<	Less than, smaller than, below
≤	Less than or equal to, no more than, maximum

Understanding these terms and symbols enables you to interpret and write inequalities to model the constraints in various situations. Similar to equations, inequalities provide you with ways to represent relationships, but between quantities that are not equal.

Reflect:

38 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the following examples and have students write inequalities to model the given constraints:

- The area of a rectangle A, with a length of 4 m and a width of w m is no more than 100 m^2. $4 \times w \leq 100$
- To cover all the expenses of a musical production each week, the number of weekday tickets sold d, and the number of weekend tickets sold e, must be greater than 4,000. $d + e > 4000$
- Elena would like the number of hours she works in a week h, to be more than 5 but no more than 20. $5 < h \leq 20$
- The total cost T of buying a adult shirts and c child shirts must be less than \$150. Adult shirts are \$12 and children shirts are \$7 each. $12a + 7c < 150$

Highlight that inequalities, like equations, can also express relationships between quantities in a specific scenario and that the solution to an inequality indicates a range of values that make it true.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

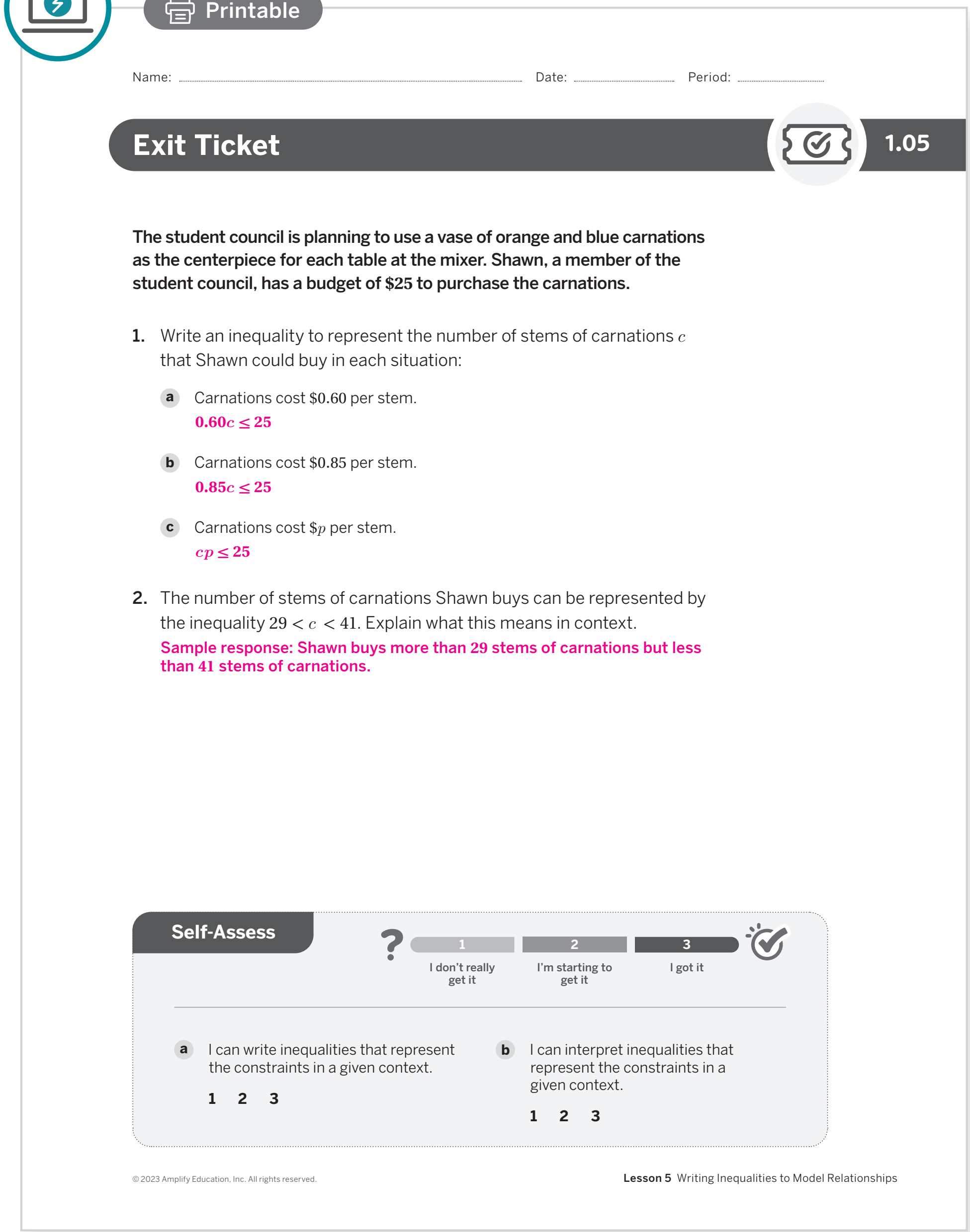
- "What strategies did you use to interpret inequalities that model constraints in a context?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by modeling the quantities and constraints as an inequality with a given scenario.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.05

The student council is planning to use a vase of orange and blue carnations as the centerpiece for each table at the mixer. Shawn, a member of the student council, has a budget of $25 to purchase the carnations.

1. Write an inequality to represent the number of stems of carnations c that Shawn could buy in each situation:
   - **a** Carnations cost $0.60 per stem.
     $0.60c \leq 25$
   - **b** Carnations cost $0.85 per stem.
     $0.85c \leq 25$
   - **c** Carnations cost $$p$ per stem.
     $cp \leq 25$
2. The number of stems of carnations Shawn buys can be represented by the inequality $29 < c < 41$. Explain what this means in context.
   Sample response: Shawn buys more than 29 stems of carnations but less than 41 stems of carnations.

Self-Assess

?	1	2	3	✓
	I don't really get it	I'm starting to get it	I got it	

a I can write inequalities that represent the constraints in a given context.

1 2 3

b I can interpret inequalities that represent the constraints in a given context.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 5 Writing Inequalities to Model Relationships

Success looks like . . .

- **Language Goal:** Interpreting inequalities in a given context. **(Speaking and Listening, Writing)**
  - » Explaining the meaning of an inequality that represents the number of stems of carnations that Shawn buys in Problem 2.
- **Goal:** Writing inequalities to represent constraints in a given context.
  - » Writing inequalities that represent the number of carnations that Shawn could buy in Problem 1.

Suggested next steps

If students are unable to write an inequality for Problem 1 parts a and b, consider:

- Asking, "What is Shawn's budget? What does that mean in this context?"
- Having students evaluate specific values of c and determining whether Shawn has enough to purchase it.
- Assigning Practice Problem 1.

If students are unable to write an inequality for Problem 1 part c, consider:

- Asking, "What quantity does c represent in this scenario?"
- Having students articulate what operation they used to write the inequalities for Problem 1 parts a and b.
- Assigning Practice Problem 3.

Professional Learning

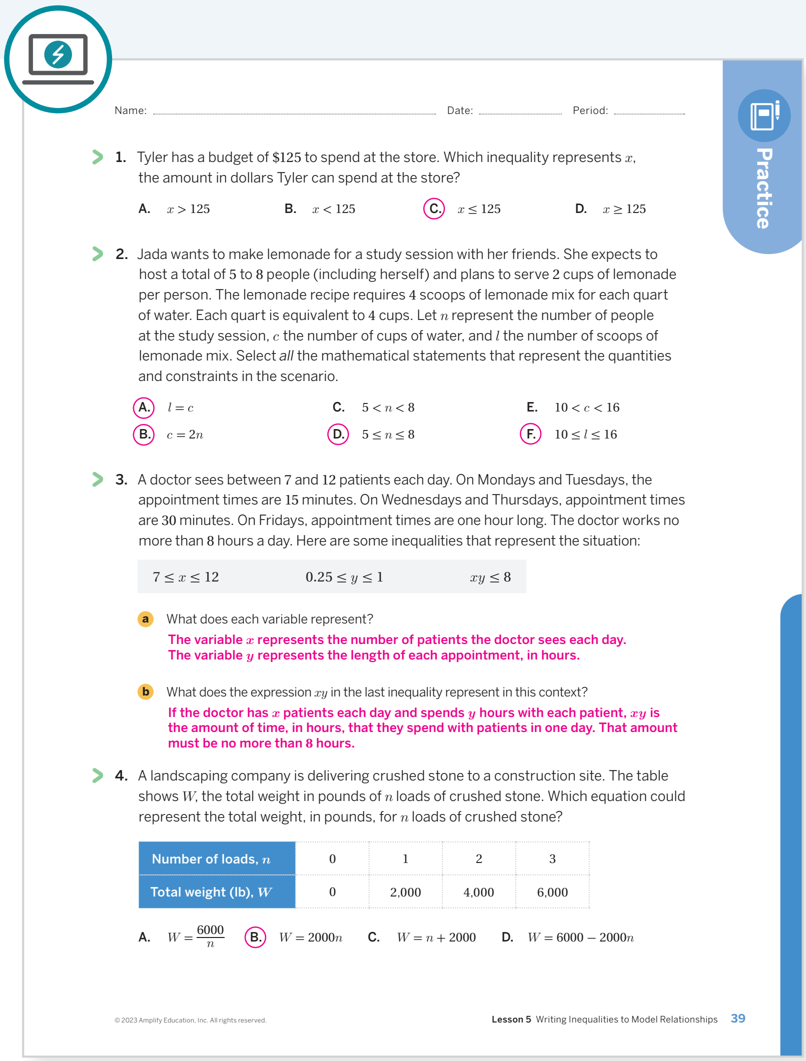
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students write inequalities to represent the constraints in a given context. How will this support them in solving and graphing linear inequalities?
- How did the *Mix and Mingle* routine in Activity 2 support students in writing and interpreting inequalities that represented the elevator constraints? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Tyler has a budget of $125 to spend at the store. Which inequality represents x, the amount in dollars Tyler can spend at the store?

   A. $x > 125$ B. $x < 125$ [✓ C.] $x \leq 125$ D. $x \geq 125$

2. Jada wants to make lemonade for a study session with her friends. She expects to host a total of 5 to 8 people (including herself) and plans to serve 2 cups of lemonade per person. The lemonade recipe requires 4 scoops of lemonade mix for each quart of water. Each quart is equivalent to 4 cups. Let n represent the number of people at the study session, c the number of cups of water, and l the number of scoops of lemonade mix. Select *all* the mathematical statements that represent the quantities and constraints in the scenario.

   [✓ A.] $l = c$ C. $5 < n < 8$ E. $10 < c < 16$
   [✓ B.] $c = 2n$ [✓ D.] $5 \leq n \leq 8$ [✓ F.] $10 \leq l \leq 16$

3. A doctor sees between 7 and 12 patients each day. On Mondays and Tuesdays, the appointment times are 15 minutes. On Wednesdays and Thursdays, appointment times are 30 minutes. On Fridays, appointment times are one hour long. The doctor works no more than 8 hours a day. Here are some inequalities that represent the situation:

   $7 \leq x \leq 12$ $0.25 \leq y \leq 1$ $xy \leq 8$

   a. What does each variable represent?
   The variable x represents the number of patients the doctor sees each day. The variable y represents the length of each appointment, in hours.

   b. What does the expression xy in the last inequality represent in this context?
   If the doctor has x patients each day and spends y hours with each patient, xy is the amount of time, in hours, that they spend with patients in one day. That amount must be no more than 8 hours.

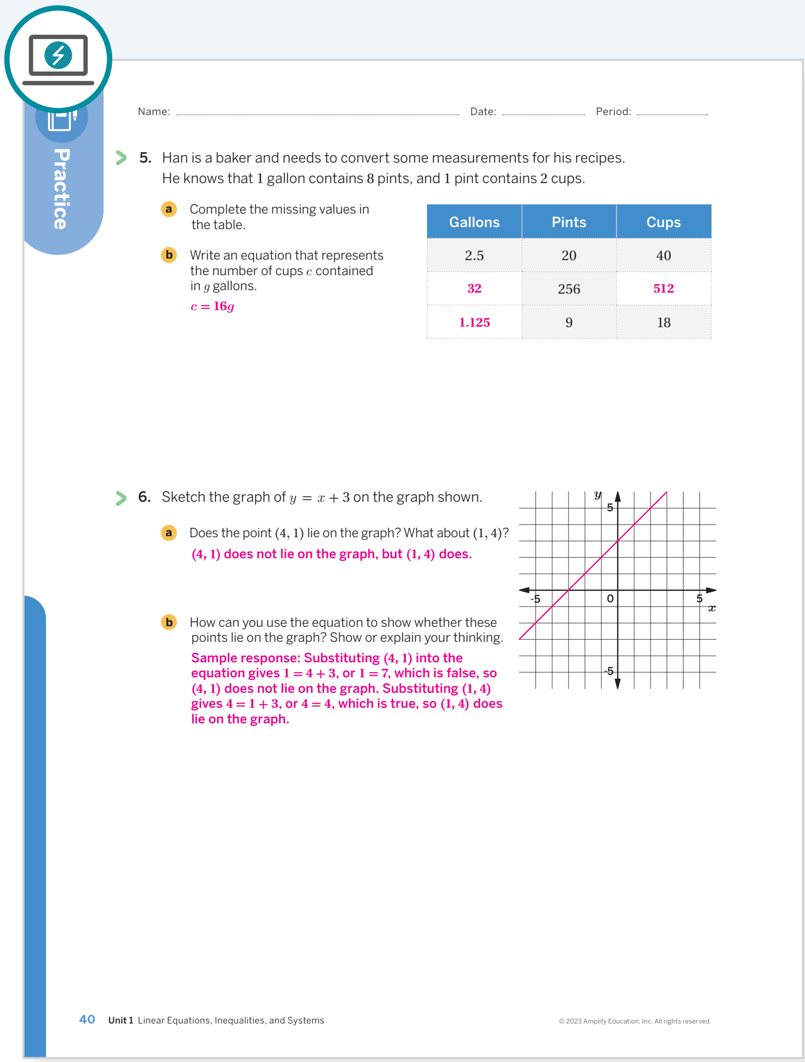
4. A landscaping company is delivering crushed stone to a construction site. The table shows W, the total weight in pounds of n loads of crushed stone. Which equation could represent the total weight, in pounds, for n loads of crushed stone?

Number of loads, n	0	1	2	3
Total weight (lb), W	0	2,000	4,000	6,000

   A. $W = \frac{6000}{n}$ [✓ B.] $W = 2000n$ C. $W = n + 2000$ D. $W = 6000 - 2000n$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 5 Writing Inequalities to Model Relationships 39



Practice

Name: ________ Date: ________ Period: ________

5. Han is a baker and needs to convert some measurements for his recipes. He knows that 1 gallon contains 8 pints, and 1 pint contains 2 cups.

   a. Complete the missing values in the table.

   b. Write an equation that represents the number of cups c contained in g gallons.
   $c = 16g$

Gallons	Pints	Cups
2.5	20	40
32	256	**512**
1.125	9	18

6. Sketch the graph of $y = x + 3$ on the graph shown.

   a. Does the point $(4, 1)$ lie on the graph? What about $(1, 4)$?
   $(4, 1)$ does not lie on the graph, but $(1, 4)$ does.

   b. How can you use the equation to show whether these points lie on the graph? Show or explain your thinking.
   Sample response: Substituting $(4, 1)$ into the equation gives $1 = 4 + 3$, or $1 = 7$, which is false, so $(4, 1)$ does not lie on the graph. Substituting $(1, 4)$ gives $4 = 1 + 3$, or $4 = 4$, which is true, so $(1, 4)$ does lie on the graph.

40 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
	4	Grade 8	2
Spiral	5	Unit 1 Lesson 3	2
Formative ⚡	6	Unit 1 Lesson 6	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Equations and Their Graphs

Let's graph equations in two variables.

Focus

Goals

1. **Language Goal:** Comprehend that the graph of a linear equation in two variables represents all pairs of values that are solutions to the equation. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Interpret points on a graph of a linear equation to respond to problems about the quantities in context. **(Reading and Writing)**
3. Use graphing technology to graph linear equations and identify solutions to the equations.

Coherence

Today

Students continue writing equations to represent relationships and constraints and determining their solutions. They are introduced to graphing technology as they revisit graphs that model relationships. Students analyze points on and off the graph and interpret them in context, making sense of problems as they draw connections between equations, verbal descriptions, and graphs.

Previously

Students wrote and interpreted two-variable equations, and wrote and found solutions for inequalities in one-variable as it related to a context.

Coming Soon

In the next lesson, students will revisit properties of equality to manipulate and write equivalent equations.

Rigor

- Students revisit graphs as a useful way to represent relationships to further their **conceptual understanding** of points on a graph being a solution to the equations they represent.
- Students build **procedural fluency** in writing equations in two-variables.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2 (optional)	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	20 min	5 min	5 min
Pairs	Pairs	Independent	Small Groups		Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Which One Doesn't Belong?*
- Anchor Chart PDF, *Slope-Intercept Form*
- graphing technology
- scientific calculators

Math Language Development

Review words

- *constraint*
- *initial value*
- *rate of change*
- *slope-intercept form*
- *variable*
- x*-intercept*
- y*-intercept*

Amps | Featured Activity

Activity 2

Interactive Graphs

Students are introduced to graphing technology and learn how to graph equations in two-variables and identify their solutions, along with some other helpful features.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students who are familiar with using graphing technology may be more confident with this work and may be able to assist more inexperienced students in Activity 2, as well as model how to use graphing technology strategically in Activity 3. Remind students to "step up" if they have something to add that could benefit the group, but also to "step back" to give other voices a chance to share.

Modifications to Pacing

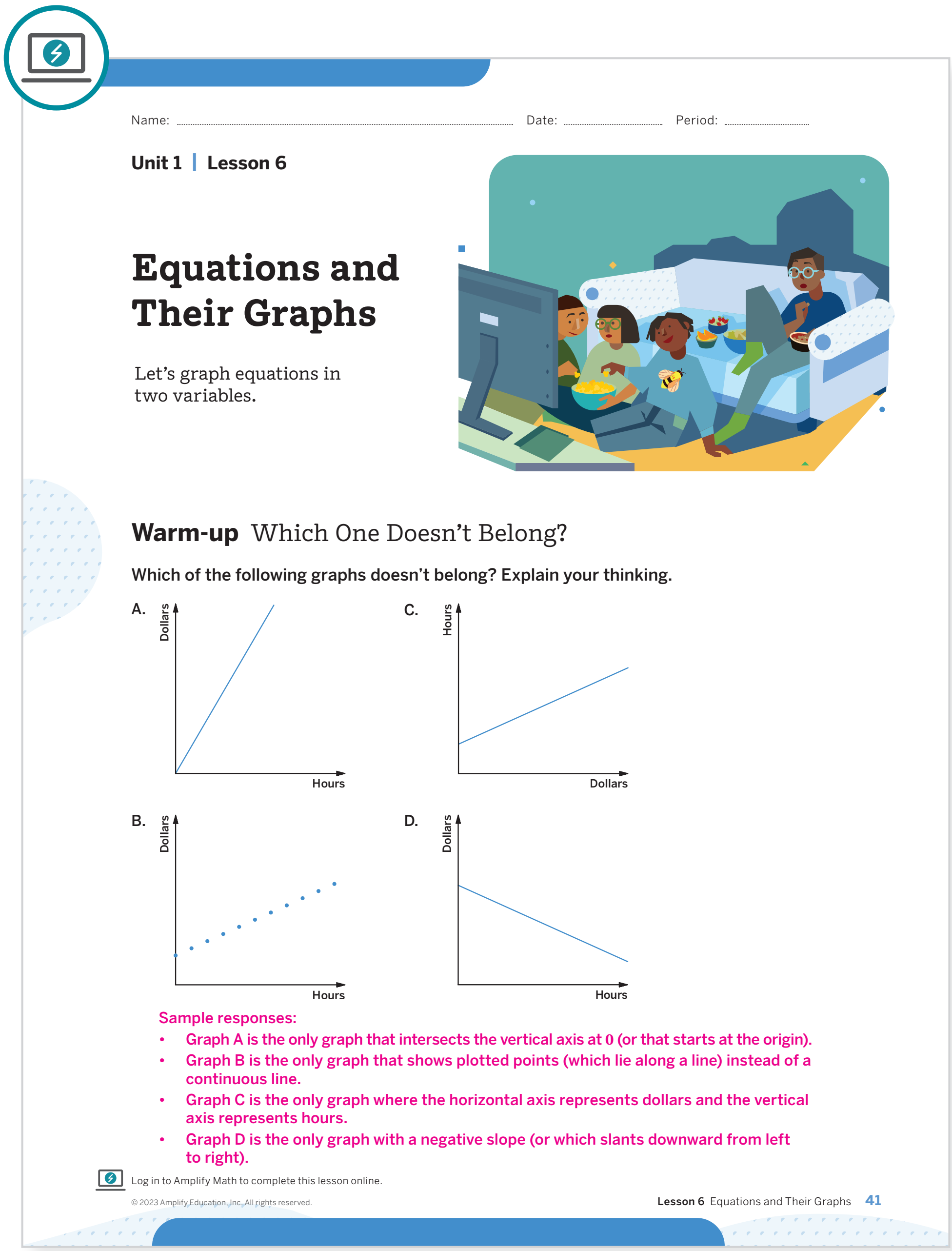
You may want to consider these additional modifications if you are short on time.

- **Activity 2** may be omitted. Consider having students complete the activity in advance of this lesson and review the questions in the *Connect*.
- In **Activity 3**, have students complete either Problem 1 or Problem 2.

Pairs | 5 min

Warm-up Which One Doesn't Belong?

Students analyze and compare the graphs of linear equations to practice using mathematical language precisely.


Name: ______ Date: ______ Period: ______

Unit 1 | Lesson 6

Equations and Their Graphs

Let's graph equations in two variables.

Warm-up Which One Doesn't Belong?

Which of the following graphs doesn't belong? Explain your thinking.

Sample responses:

- Graph A is the only graph that intersects the vertical axis at 0 (or that starts at the origin).
- Graph B is the only graph that shows plotted points (which lie along a line) instead of a continuous line.
- Graph C is the only graph where the horizontal axis represents dollars and the vertical axis represents hours.
- Graph D is the only graph with a negative slope (or which slants downward from left to right).

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 6 Equations and Their Graphs 41

1 Launch

Conduct the *Which One Doesn't Belong?* routine. Give students independent work time, before having them share their thinking with a partner. Have the pairs work together to determine one reason why each graph doesn't belong.

2 Monitor

Help students get started by having them notice similarities or differences in two of the graphs. Listen for students using key vocabulary.

Look for productive strategies:

- Noticing continuous graphs versus the discrete graphs.
- Noticing the axes labels and identifying the quantities compared.
- Referring to the slope or rate-of-change and associating it with line direction.
- Referring to the y-intercept or initial value.
- Describing the relationship between hours and dollars.

3 Connect

Display the Warm-up. Be prepared to record and capture student responses as they share.

Have pairs of students share one reason why each graph doesn't belong. Ask the class to agree or disagree with their reasoning. Amplify students using key terms, e.g. y-intercept, slope, rate of change, and prompt them to explain their meanings.

Highlight that different graphs represent specific types of relationships. Graph A represents a proportional relationship, $y = mx$, Graphs C and D non-proportional, $y = mx + b$, and Graph B is a discrete relationship because the points are not connected (not continuous).

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share why each graph does not belong, press for details in their reasoning. For example, if they say "Graph A is the only proportional relationship," ask them how they know this is true.

English Learners

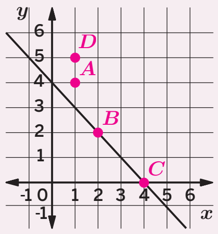
Annotate each graph with their characteristics. For example, annotate Graph C with the term *nonproportional* and highlight how the axes labels are different.

Power-up

To power up students' ability to determine whether a point lies on a graph, have students complete:

The graph of $y = -x + 4$ is shown. Graph each point to determine whether it lies on the line. Write *yes* or *no*.

1. A: $(1, 4)$ No
2. B: $(2, 2)$ Yes
3. C: $(4, 0)$ Yes
4. D: $(1, 5)$ No

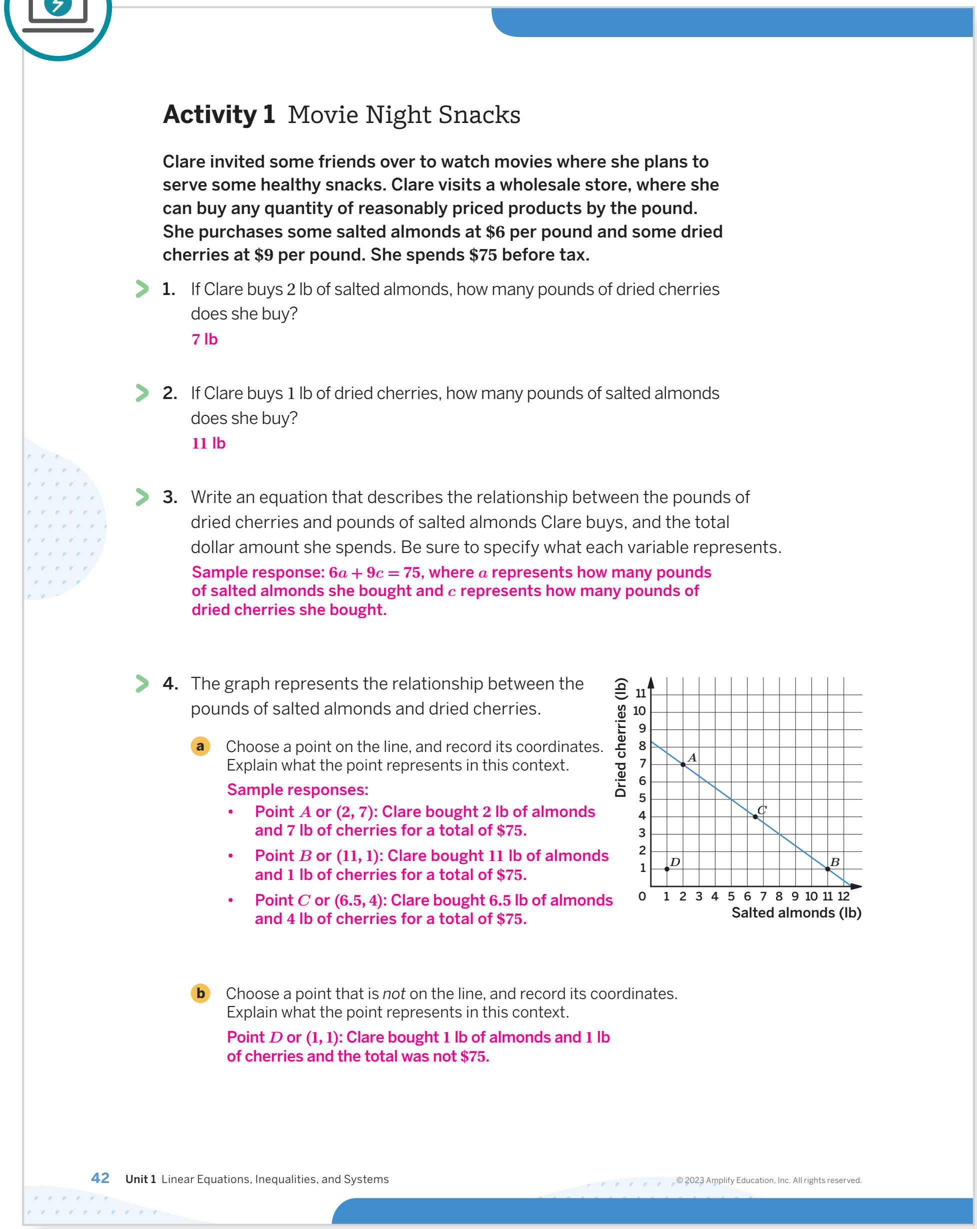


Use: Before Activity 1
Informed by: Performance on Lesson 5, Practice Problem 6

Pairs | 15 min

Activity 1 Movie Night Snacks

Students write an equation to model the relationship between two quantities, analyzing and interpreting its graph to determine solutions and non-solutions in context.



Activity 1 Movie Night Snacks

Clare invited some friends over to watch movies where she plans to serve some healthy snacks. Clare visits a wholesale store, where she can buy any quantity of reasonably priced products by the pound. She purchases some salted almonds at $6 per pound and some dried cherries at $9 per pound. She spends $75 before tax.

1. If Clare buys 2 lb of salted almonds, how many pounds of dried cherries does she buy?

   7 lb

2. If Clare buys 1 lb of dried cherries, how many pounds of salted almonds does she buy?

   11 lb

3. Write an equation that describes the relationship between the pounds of dried cherries and pounds of salted almonds Clare buys, and the total dollar amount she spends. Be sure to specify what each variable represents.

   Sample response: $6a + 9c = 75$, where a represents how many pounds of salted almonds she bought and c represents how many pounds of dried cherries she bought.

4. The graph represents the relationship between the pounds of salted almonds and dried cherries.

   a. Choose a point on the line, and record its coordinates. Explain what the point represents in this context.

   Sample responses:
   - **Point A or $(2, 7)$: Clare bought 2 lb of almonds and 7 lb of cherries for a total of $75.**
   - **Point B or $(11, 1)$: Clare bought 11 lb of almonds and 1 lb of cherries for a total of $75.**
   - **Point C or $(6.5, 4)$: Clare bought 6.5 lb of almonds and 4 lb of cherries for a total of $75.**

   b. Choose a point that is *not* on the line, and record its coordinates. Explain what the point represents in this context.

   Point D or $(1, 1)$: Clare bought 1 lb of almonds and 1 lb of cherries and the total was not $75.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Provide access to scientific calculators. Have students work independently before having them share and discuss their work with their partner.

2 Monitor

Help students get started by prompting them to create a table relating pounds of salted almonds to pounds of dried cherries.

Look for points of confusion:

- **Having difficulty making sense of points not on the line.** Ask, "What would the coordinate represent if Clare's total amount spent is unknown?"
- **Not understanding how to interpret $75 on the given graph.** Have students write the number of almonds and cherries as an ordered pair and relate the coordinates to the equation.

Look for productive strategies:

- Labeling points with their coordinates and relating them to the context.
- Using the equation to verify or explain the points on or off the graph.

3 Connect

Display the graph.

Ask:

- "What does the point $(10, 3)$ represent? Is this a solution?"
  10 lb of almonds and 3 lb of cherries. It is not a solution because it does not lie on the graph.
- "The point $(7, 3.5)$ appears to be a solution. How could you verify this?" Substituting the values into the equation to determine if it makes a true statement.

Highlight that a solution to an equation in two variables lies on its graph and is represented by an ordered pair.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*
Provide a partially-completed table with the headers "Almonds (lb)" and "Cherries (lb)." Include the values from Problems 1 and 2. Ask students to determine two more pairs of values that satisfy the constraints before writing an equation.

Extension: *Math Enrichment*
Ask students if the point $(-1, 9)$ is a solution to their equation, if the graph is extended beyond Quadrant 1. Have them explain their thinking. Yes, it is a solution to the equation because $6(-1) + 9(9) = 75$ is a true statement. However, it is not a reasonable solution, because it is not possible to buy a negative amount of almonds.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

- **Read 1:** Students should understand that Clare purchases an unspecified amount of almonds and cherries.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as the almonds cost $6 per pound.
- **Read 3:** Ask students to think about how an equation in two variables can represent this information.

English Learners

Have students highlight key phrases, such as $6 *per pound* and $9 *per pound*.

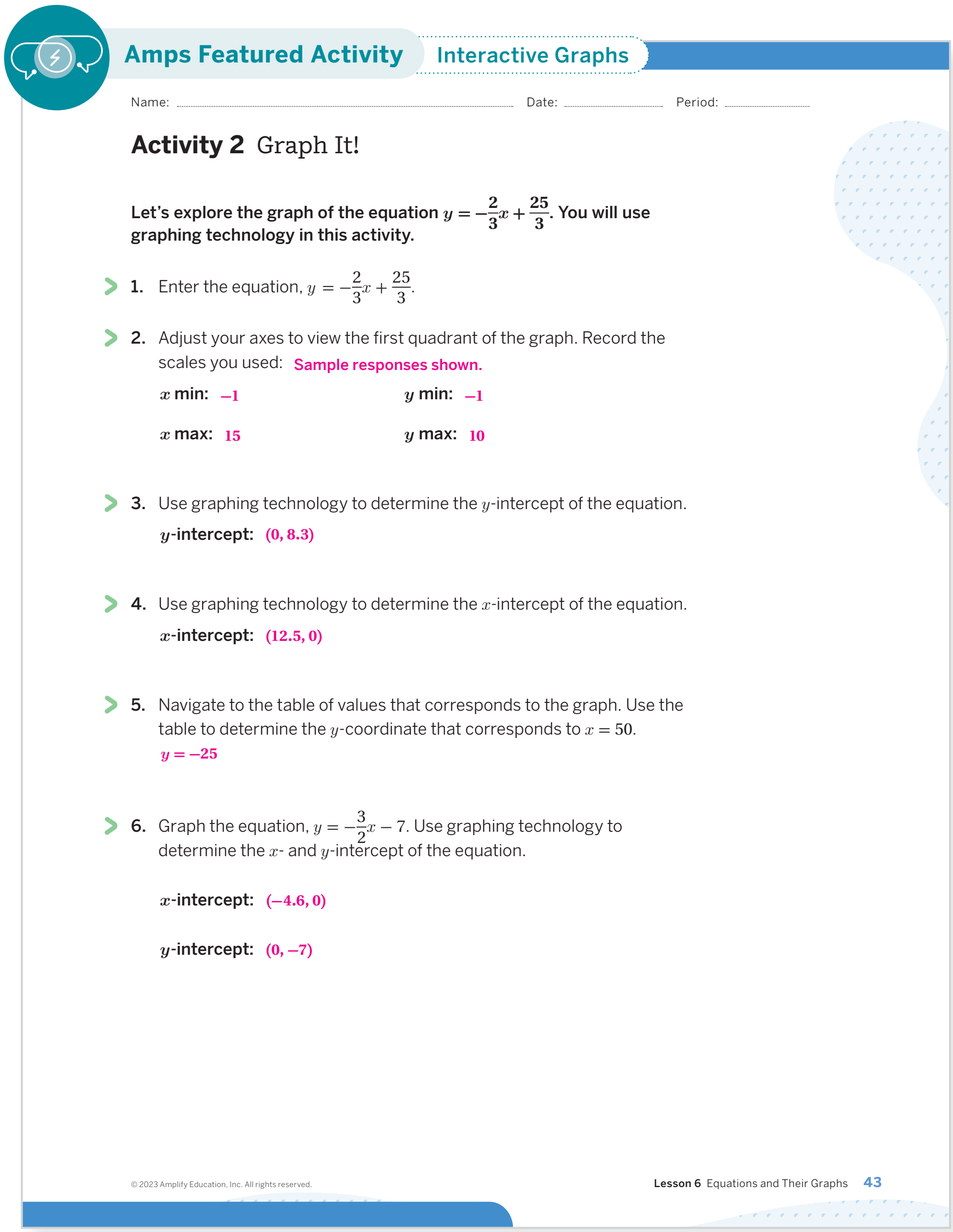
Optional

Independent | 10 min

Activity 2 Graph It!

Students explore graphing technology and perform several tasks to become familiar with using the technological tools.

Amps Featured Activity Interactive Graphs



Name: ______ Date: ______ Period: ______

Activity 2 Graph It!

Let's explore the graph of the equation $y = -\frac{2}{3}x + \frac{25}{3}$. You will use graphing technology in this activity.

1. Enter the equation, $y = -\frac{2}{3}x + \frac{25}{3}$.
2. Adjust your axes to view the first quadrant of the graph. Record the scales you used: **Sample responses shown.**
   x **min:** -1 y **min:** -1
   x **max:** 15 y **max:** 10
3. Use graphing technology to determine the y-intercept of the equation.
   y**-intercept:** $(0, 8.3)$
4. Use graphing technology to determine the x-intercept of the equation.
   x**-intercept:** $(12.5, 0)$
5. Navigate to the table of values that corresponds to the graph. Use the table to determine the y-coordinate that corresponds to $x = 50$.
   $y = -25$
6. Graph the equation, $y = -\frac{3}{2}x - 7$. Use graphing technology to determine the x- and y-intercept of the equation.
   x**-intercept:** $(-4.6, 0)$
   y**-intercept:** $(0, -7)$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 6 Equations and Their Graphs 43

1 Launch

Provide access to graphing technology. Use graphing technology to show the graph of $y = -2x + 6$. Ask students to identify the x- and y-intercepts. Ask, "Can you identify any other characteristics of the graph?"

2 Monitor

Help students get started by providing a tutorial or tour of the graphing technology being used.

Look for points of confusion:

- **Having difficulty adjusting the axes.** Provide written instructions with images appropriate for the graphing technology being used.
- **Having difficulty navigating between the equation, graph, and table.** Provide written instructions with images illustrating appropriate keystrokes.

Look for productive strategies:

- Clicking on points on the graph to view their coordinates.
- Zooming in or out to better view the graph.
- Revising the scales as needed to better view the graph.

3 Connect

Have students share their thinking or feeling about using graphing technology. Ask, "Did you find any shortcuts? Do you prefer graphing technology or graphing by hand?" Gauge students' comfort with graphing technology by eliciting a thumbs up or thumbs down.

Highlight that students will frequently be using graphing technology throughout the course to create graphs and analyze equations.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools and Technology*

Prepare a graphing technology cheat sheet, depending on the type of graphing technology your students use. Include images, keystrokes, and step-by-step directions for entering equations, adjusting the axes, navigating to the table of values, and plotting points.

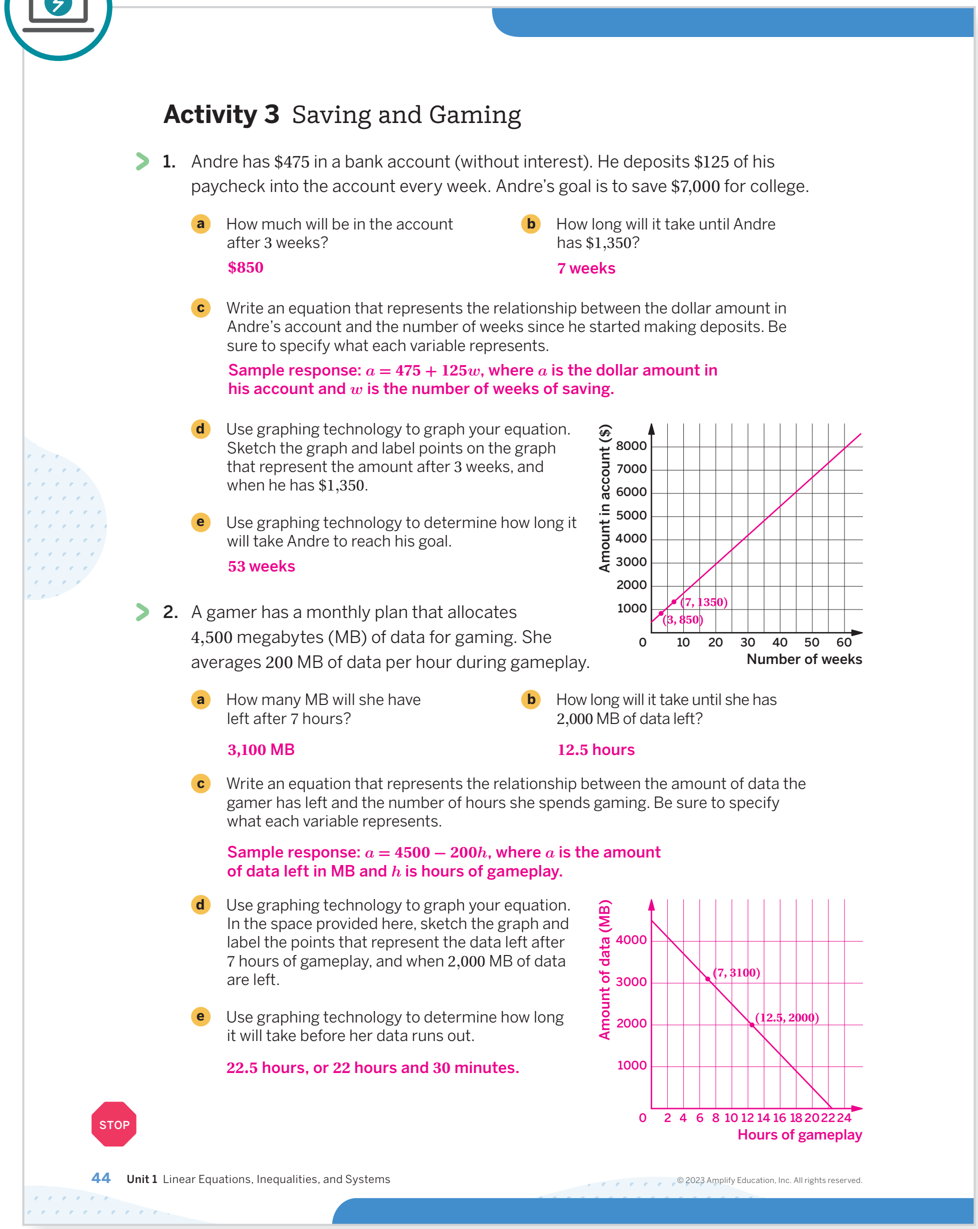
Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can explore how to use the graphing technology to graph equations in two variables, identify solutions, and explore other helpful features of the technology.

Small Groups | 20 min

Activity 3 Saving and Gaming

Students write equations modeling two different contexts and use their graphs to solve problems.



Activity 3 Saving and Gaming

1. Andre has $475 in a bank account (without interest). He deposits $125 of his paycheck into the account every week. Andre's goal is to save $7,000 for college.

   a. How much will be in the account after 3 weeks?

   $850

   b. How long will it take until Andre has $1,350?

   7 weeks

   c. Write an equation that represents the relationship between the dollar amount in Andre's account and the number of weeks since he started making deposits. Be sure to specify what each variable represents.

   Sample response: $a = 475 + 125w$, where a is the dollar amount in his account and w is the number of weeks of saving.

   d. Use graphing technology to graph your equation. Sketch the graph and label points on the graph that represent the amount after 3 weeks, and when he has $1,350.

   e. Use graphing technology to determine how long it will take Andre to reach his goal.

   53 weeks

2. A gamer has a monthly plan that allocates 4,500 megabytes (MB) of data for gaming. She averages 200 MB of data per hour during gameplay.

   a. How many MB will she have left after 7 hours?

   3,100 MB

   b. How long will it take until she has 2,000 MB of data left?

   12.5 hours

   c. Write an equation that represents the relationship between the amount of data the gamer has left and the number of hours she spends gaming. Be sure to specify what each variable represents.

   Sample response: $a = 4500 - 200h$, where a is the amount of data left in MB and h is hours of gameplay.

   d. Use graphing technology to graph your equation. In the space provided here, sketch the graph and label the points that represent the data left after 7 hours of gameplay, and when 2,000 MB of data are left.

   e. Use graphing technology to determine how long it will take before her data runs out.

   22.5 hours, or 22 hours and 30 minutes.

STOP

44 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in groups and assign one problem to each group. Assign Problem 1 to students who require more support analyzing and interpreting a graph, which is given, and Problem 2 to students who are ready to be challenged to create their own graph.

2 Monitor

Help students get started by displaying $y = 100x + 200$. Ask, "Which value represents the initial value? The rate of change?" Review these terms.

Look for points of confusion:

- **Having difficulty writing an equation for part c.** Ask, "What is the initial value? What is the rate of change?"

Look for productive strategies:

- Creating a table.
- Writing equations using the initial value and the rate of change.
- Substituting values into their equation.
- Using graphing technology to view the table of the equation.

3 Connect

Have groups of students share their strategies for writing equations and using their graphs to solve each problem. Select groups assigned to Problem 1 to share first before groups assigned to Problem 2 share. Compare each groups' strategies.

Display students' graphs.

Ask, "What do the points on each graph represent? How can you use the graphs to answer part e in both problems?"

Highlight that a graph can be used to answer questions about the quantities in a context, and the points on a graph represent solutions of an equation.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools and Technology*

Display the Anchor Chart PDF, *Slope-Intercept Form* for students to use during this activity. Assign Problem 1 to groups who would benefit from engaging with a pre-created graph. Provide access to the graphing technology cheat sheet, if you created one for the previous activity.

Extension: *Math Enrichment*

Have students complete this problem: If the gamer wants her data to last 30 hours, how should she adjust the amount of data she uses? She should reduce the data she uses, on average per hour, to 150 MB.

Math Language Development

MLR7: Compare and Connect

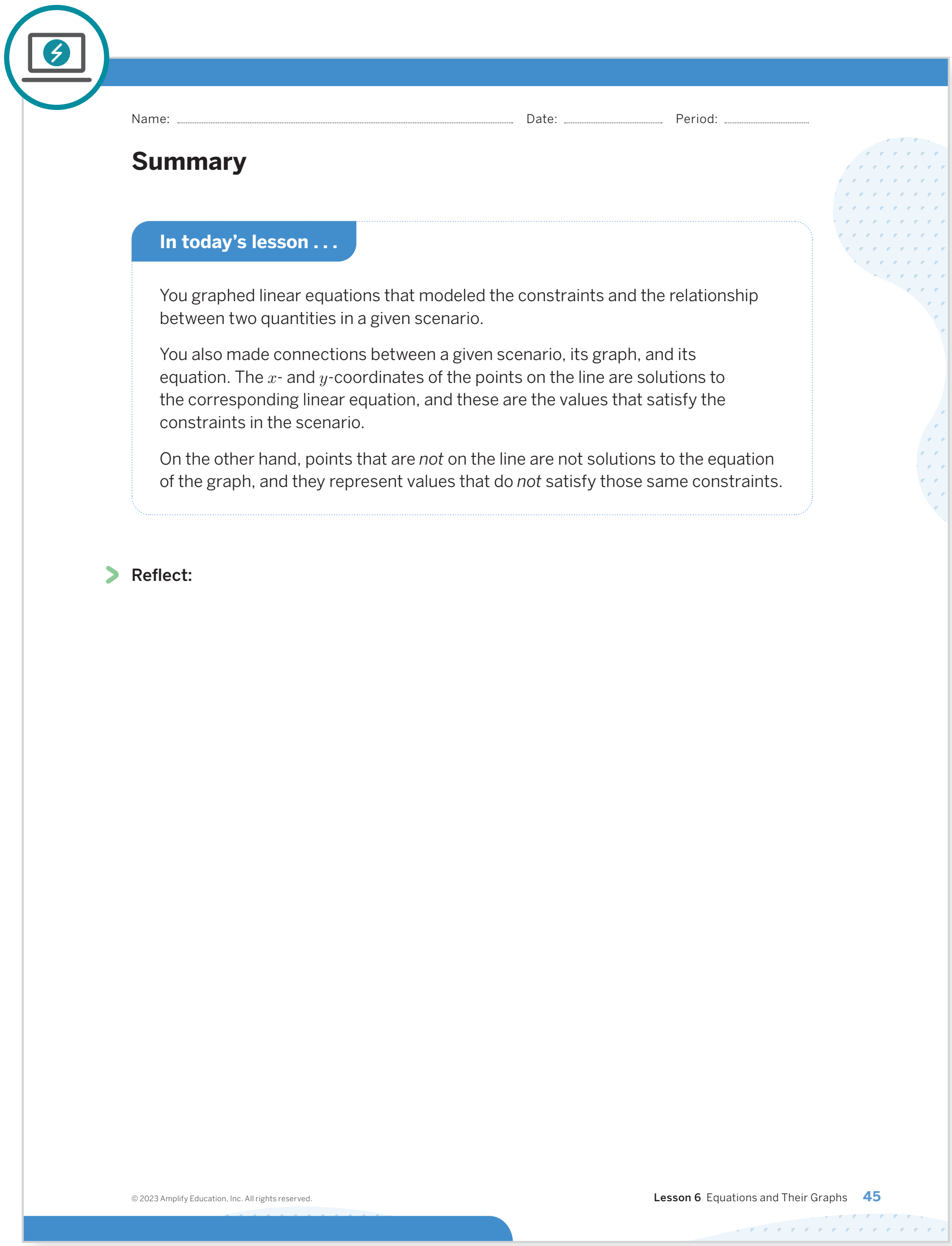
During the Connect, call attention to the different ways the quantities are represented graphically and within the context of each situation. Amplify student words and actions that describe the connections between a specific feature of one mathematical representation and a specific feature of another representation. For example, in part e of Problem 1, annotate Andre's goal of $7,000 on the graph. Then show where his goal is represented in the equation $a = 475 + 125w$ (when $a = 7000$).

English Learners

Use color coding or gestures, such as pointing, to highlight the connections between various representations.

Summary

Review and synthesize how to use the graph of an equation in two variables to determine its solutions.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You graphed linear equations that modeled the constraints and the relationship between two quantities in a given scenario.

You also made connections between a given scenario, its graph, and its equation. The x- and y-coordinates of the points on the line are solutions to the corresponding linear equation, and these are the values that satisfy the constraints in the scenario.

On the other hand, points that are *not* on the line are not solutions to the equation of the graph, and they represent values that do *not* satisfy those same constraints.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 6 Equations and Their Graphs 45

Synthesize

Display the graph of $y = 450 - 20x$. Say, "The graph represents a 450-gallon tank full of water draining at a rate of 20 gallons per minute." Prompt students to use the graph to respond to the problems shown.

Ask:

- "What is the initial value? The rate of change?" 450 gallons; 20 gallons per minute
- "Write an equation to model the water draining from the tank." $y = 450 - 20x$
- "After how many minutes should you stop the draining if you want to leave 150 gallons of water in the tank?" 15 minutes
- "Is $(25, -50)$ a solution to the equation? Why or why not?" Yes. The point is on the graph. But this solution does not make sense in this context because there will never be -50 gallons of water in the tank.

Highlight that like an equation, a graph provides information about a relationship between two quantities and the constraints on them. Points on the line represent solutions to the equation and points not on the line are not solutions, but still have meaning in the situation (whether it makes sense or not).

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

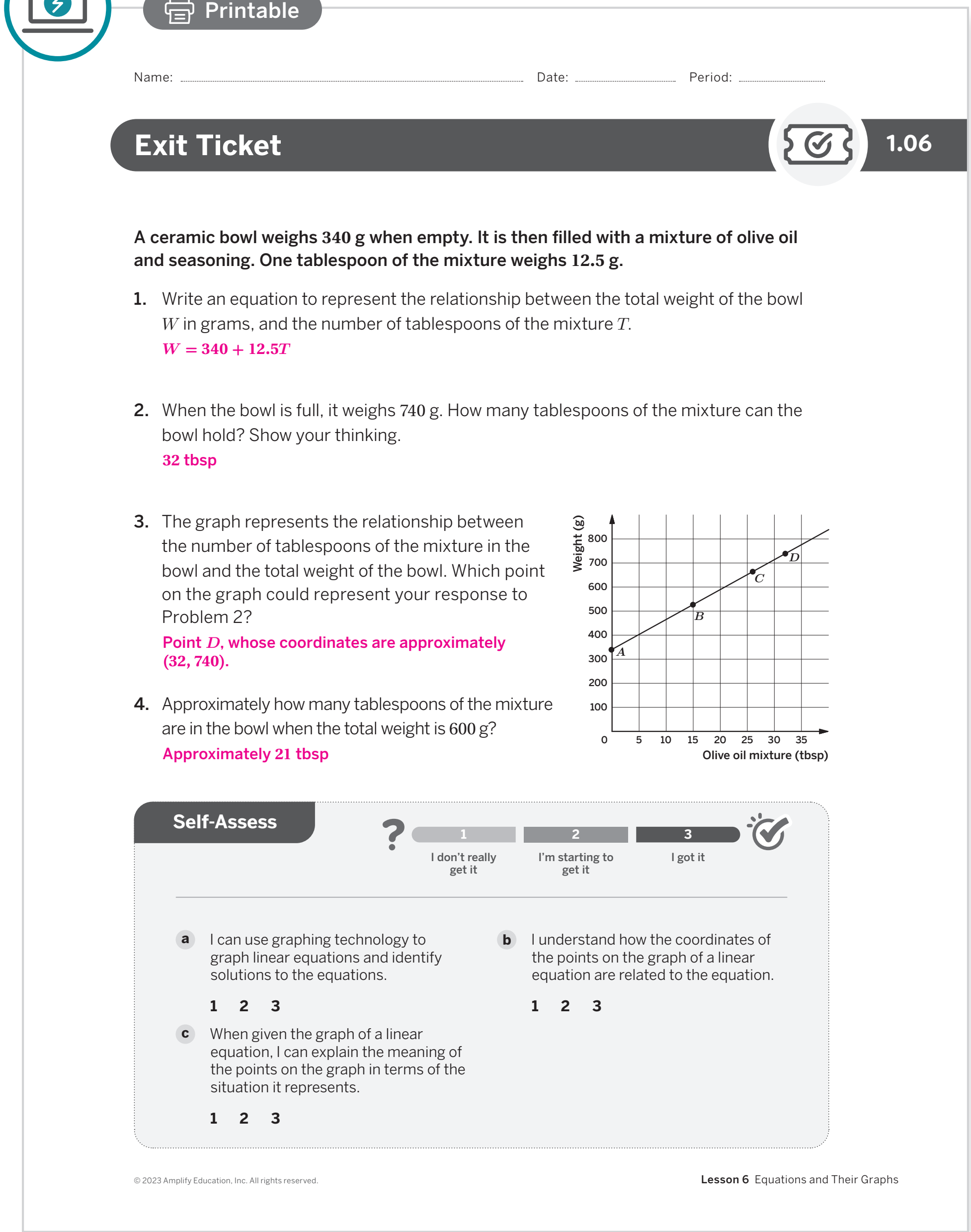
- "How can you use a graph of an equation to determine its solutions?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by analyzing the graph of an equation and interpreting its points in terms of the situation it represents.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket

1.06

A ceramic bowl weighs 340 g when empty. It is then filled with a mixture of olive oil and seasoning. One tablespoon of the mixture weighs 12.5 g.

1. Write an equation to represent the relationship between the total weight of the bowl W in grams, and the number of tablespoons of the mixture T.
   $W = 340 + 12.5T$
2. When the bowl is full, it weighs 740 g. How many tablespoons of the mixture can the bowl hold? Show your thinking.
   32 tbsp
3. The graph represents the relationship between the number of tablespoons of the mixture in the bowl and the total weight of the bowl. Which point on the graph could represent your response to Problem 2?
   Point D, whose coordinates are approximately (32, 740).
4. Approximately how many tablespoons of the mixture are in the bowl when the total weight is 600 g?
   Approximately 21 tbsp

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can use graphing technology to graph linear equations and identify solutions to the equations.
1 2 3

b. I understand how the coordinates of the points on the graph of a linear equation are related to the equation.
1 2 3

c. When given the graph of a linear equation, I can explain the meaning of the points on the graph in terms of the situation it represents.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 6 Equations and Their Graphs

Success looks like . . .

- **Language Goal:** Comprehending that the graph of a linear equation in two variables represents all pairs of values that are solutions to the equation. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Interpreting points on a graph of a linear equation to respond to problems about the quantities in context. **(Reading and Writing)**
  - » Identifying the point that corresponds to 32 tbsp in Problem 3.
- **Goal:** Using graphing technology to graph linear equations and identify solutions to the equations.

Suggested next steps

If students cannot write an equation to represent the scenario in Problem 1, consider:

- Having students create a table of values relating T to W.
- Reviewing Activity 1, Problems 1–3.

If students cannot determine the number of tablespoons needed to fill the ceramic bowl in Problem 2, consider:

- Prompting students to substitute the given weight into their equation.
- Reviewing Activity 3, Problems 1 and 2, parts a and b.

If students cannot identify the coordinate point in Problem 3, consider:

- Reviewing Activity 1, Problem 4.
- Assigning Practice Problems 1–2.

If students cannot determine the number of tablespoons of the mixture that corresponds to the given total weight, consider:

- Rreviewing Activity 3, Problems 1–2, parts d and e.

Professional Learning

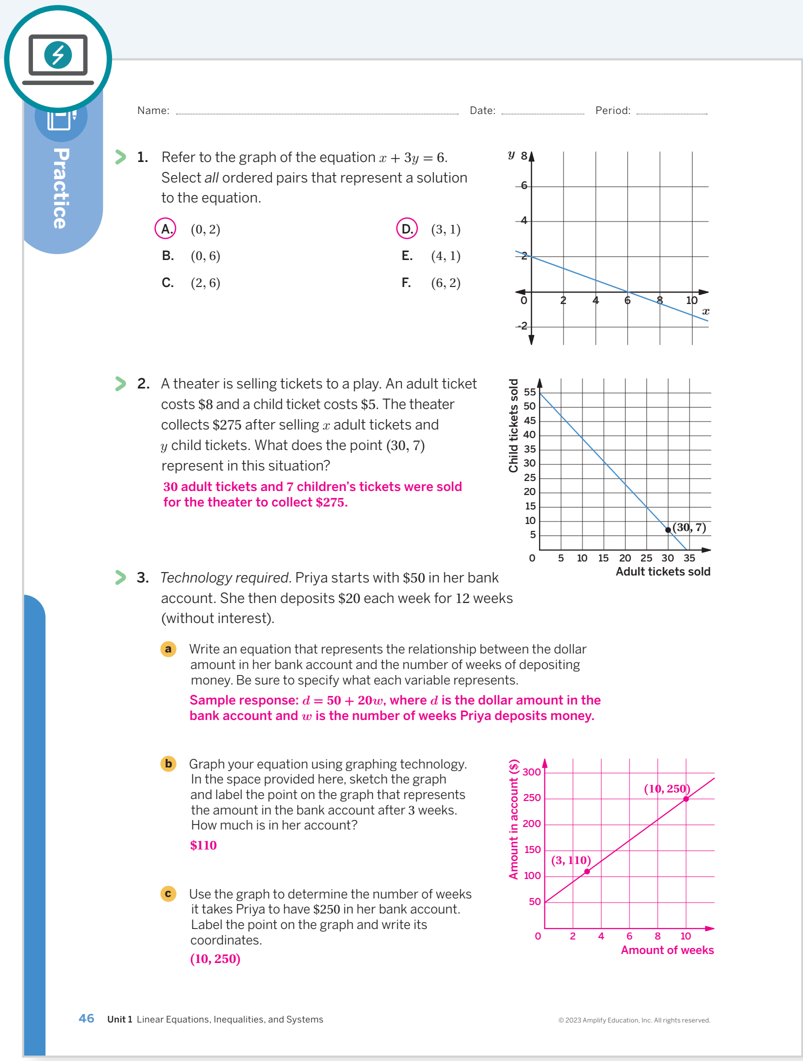
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways have your students gotten better at using appropriate tools strategically?
- During the discussion about Problems 1 and 2 in Activity 3, how did you encourage each student to listen to one another's strategies? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Refer to the graph of the equation $x + 3y = 6$. Select *all* ordered pairs that represent a solution to the equation.

   [✓ A.] $(0, 2)$
   B. $(0, 6)$
   C. $(2, 6)$
   [✓ D.] $(3, 1)$
   E. $(4, 1)$
   F. $(6, 2)$

2. A theater is selling tickets to a play. An adult ticket costs \$8 and a child ticket costs \$5. The theater collects \$275 after selling x adult tickets and y child tickets. What does the point $(30, 7)$ represent in this situation?

   30 adult tickets and 7 children's tickets were sold for the theater to collect \$275.

3. *Technology required.* Priya starts with \$50 in her bank account. She then deposits \$20 each week for 12 weeks (without interest).

   a. Write an equation that represents the relationship between the dollar amount in her bank account and the number of weeks of depositing money. Be sure to specify what each variable represents.

   Sample response: $d = 50 + 20w$, where d is the dollar amount in the bank account and w is the number of weeks Priya deposits money.

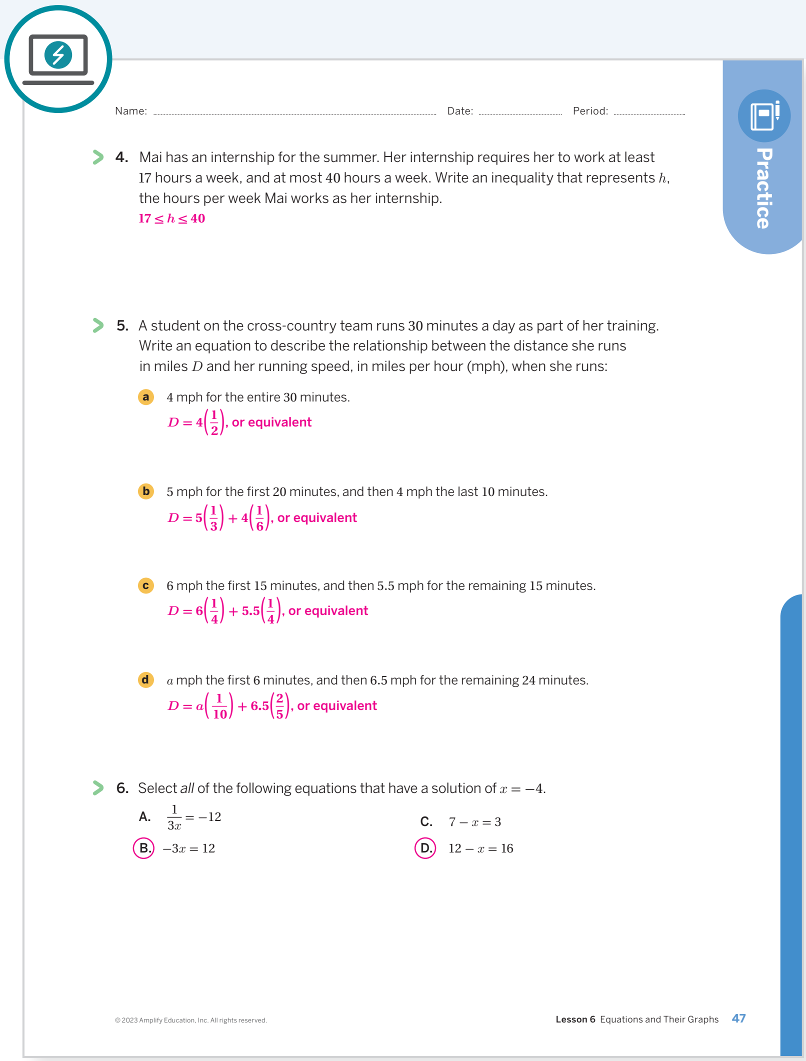
   b. Graph your equation using graphing technology. In the space provided here, sketch the graph and label the point on the graph that represents the amount in the bank account after 3 weeks. How much is in her account?

   \$110

   c. Use the graph to determine the number of weeks it takes Priya to have \$250 in her bank account. Label the point on the graph and write its coordinates.

   $(10, 250)$

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ________ Date: ________ Period: ________

4. Mai has an internship for the summer. Her internship requires her to work at least 17 hours a week, and at most 40 hours a week. Write an inequality that represents h, the hours per week Mai works as her internship.

   $17 \leq h \leq 40$

5. A student on the cross-country team runs 30 minutes a day as part of her training. Write an equation to describe the relationship between the distance she runs in miles D and her running speed, in miles per hour (mph), when she runs:

   a. 4 mph for the entire 30 minutes.

   $D = 4\left(\frac{1}{2}\right)$, or equivalent

   b. 5 mph for the first 20 minutes, and then 4 mph the last 10 minutes.

   $D = 5\left(\frac{1}{3}\right) + 4\left(\frac{1}{6}\right)$, or equivalent

   c. 6 mph the first 15 minutes, and then 5.5 mph for the remaining 15 minutes.

   $D = 6\left(\frac{1}{4}\right) + 5.5\left(\frac{1}{4}\right)$, or equivalent

   d. a mph the first 6 minutes, and then 6.5 mph for the remaining 24 minutes.

   $D = a\left(\frac{1}{10}\right) + 6.5\left(\frac{2}{5}\right)$, or equivalent

6. Select *all* of the following equations that have a solution of $x = -4$.

   A. $\frac{1}{3x} = -12$
   [✓ B.] $-3x = 12$
   C. $7 - x = 3$
   [✓ D.] $12 - x = 16$

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 3	2
Spiral	4	Unit 1 Lesson 2	2
	5	Unit 1 Lesson 3	2
Formative	6	Unit 1 Lesson 7	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 2

Manipulating Equations and Understanding Their Structure

Whole Class

In this Sub-unit, students further their understanding of solving equations and realize that solving an equation doesn't always lead to a value. Students make connections between multiple variables and real-life situations that have multiple possibilities.

SUB-UNIT
2 Manipulating Equations and Understanding Their Structure

Narrative Connections

How do first-gen Americans vault the hurdles of college?

College can be a time for finding yourself, making new friends, trying new experiences, and developing new intellectual passions. But for the children of immigrants, applying to college can come with its own unique challenges. For many immigrant homes, these students are the first in their family to attend college. Many parents and guardians are new to things like financial aid, scholarships, tuition fees, and student housing.

But these challenges are far from the whole story. First-generation students who thrive in college often have support in their schools and communities. Guidance counselors can help navigate the application process and steer students toward financial aid and scholarships. Many colleges also have departments whose goal is to support first-generation students. These departments offer mentoring services, textbook exchange programs, and even emergency funds when students encounter sudden, unexpected costs. There are even government funded programs — such as TRIO and the College Assistance Migrant Program (CAMP) — designed to provide services, counseling, and aid to first-generation students.

Above all, students have reported that simply having someone to talk to who looked like them helps with the challenges of college life.

All college-bound students will face decisions and trade-offs around budget, access to resources, and proximity to home. Equations help you model and weigh these sorts of decisions. But to efficiently solve equations, you have to be able to work with them: manipulating them, rearranging them, and graphing them. And that's exactly what you'll be doing over the next few lessons.

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students will encounter equations that model choices they might make in college or beyond high school in the following places:

- **Lesson 7, Activity 3:** Jigsaw: Buying College Textbooks
-
- **Lesson 9, Activity 1:** College Housing Decisions
- **Lesson 11, Activity 2:** Nickels and Dimes

UNIT 1 | **LESSON 7**

Equivalent Equations

Let's investigate what makes equations equivalent.

Focus

Goals

1. **Language Goal:** Comprehend that *equivalent equations* are equations that have exactly the same solutions, and that multiple equivalent equations can represent the same relationship. **(Writing)**
2. **Language Goal:** Determine and explain whether two equations are equivalent. **(Speaking and Listening, Writing)**
3. Identify operations that can be performed on an equation to create equivalent equations.

Coherence

Today

Students build on their Grade 8 understanding of equivalent expressions by defining equivalent equations. By making use of properties of equality, they examine, generate, and apply the steps to writing equivalent equations. Students interpret equivalent equations and their solutions in a context, reasoning abstractly and concretely.

Previously

In Lesson 6, students analyzed points on and off the graphs of linear equations and interpreted points and solutions in a given context.

Coming Soon

In optional Lesson 8, students will explain why certain steps produce equivalent equations and revisit equations with no solutions.

Rigor

- Students solve, manipulate, and interpret equations to build **conceptual understanding** of equivalent equations.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, pre-cut cards, one per pair
- Activity 3 PDF (answers)
- Anchor Chart PDF, *Sentence Stems, Describing My Thinking*
- Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning*
- Anchor Chart PDF, *Order of Operations*
- Instructional Routine PDF, *Jigsaw: Instructions*
- Instructional Routine PDF, *Mix and Mingle: Instructions*
- music

Math Language Development

New words

- ***equivalent equations***

Review words

- *Addition Property of Equality*
- *Division Property of Equality*
- *equivalent expressions*
- *Multiplication Property of Equality*
- *Subtraction Property of Equality*

Amps | **Featured Activity**

Activity 1
Multiple Equations

Students can enter multiple, equivalent equations on the lines of table. This makes it simpler for you to review their work.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may not have the self-discipline to read scenarios closely enough to accurately write and interpret equations that model them. Point out that they need to apply critical reading skills to these problems. Provide some strategies for careful reading, including how to identify and mark the important information. Ask students to share strategies that have helped them in the past.

● Modifications to Pacing

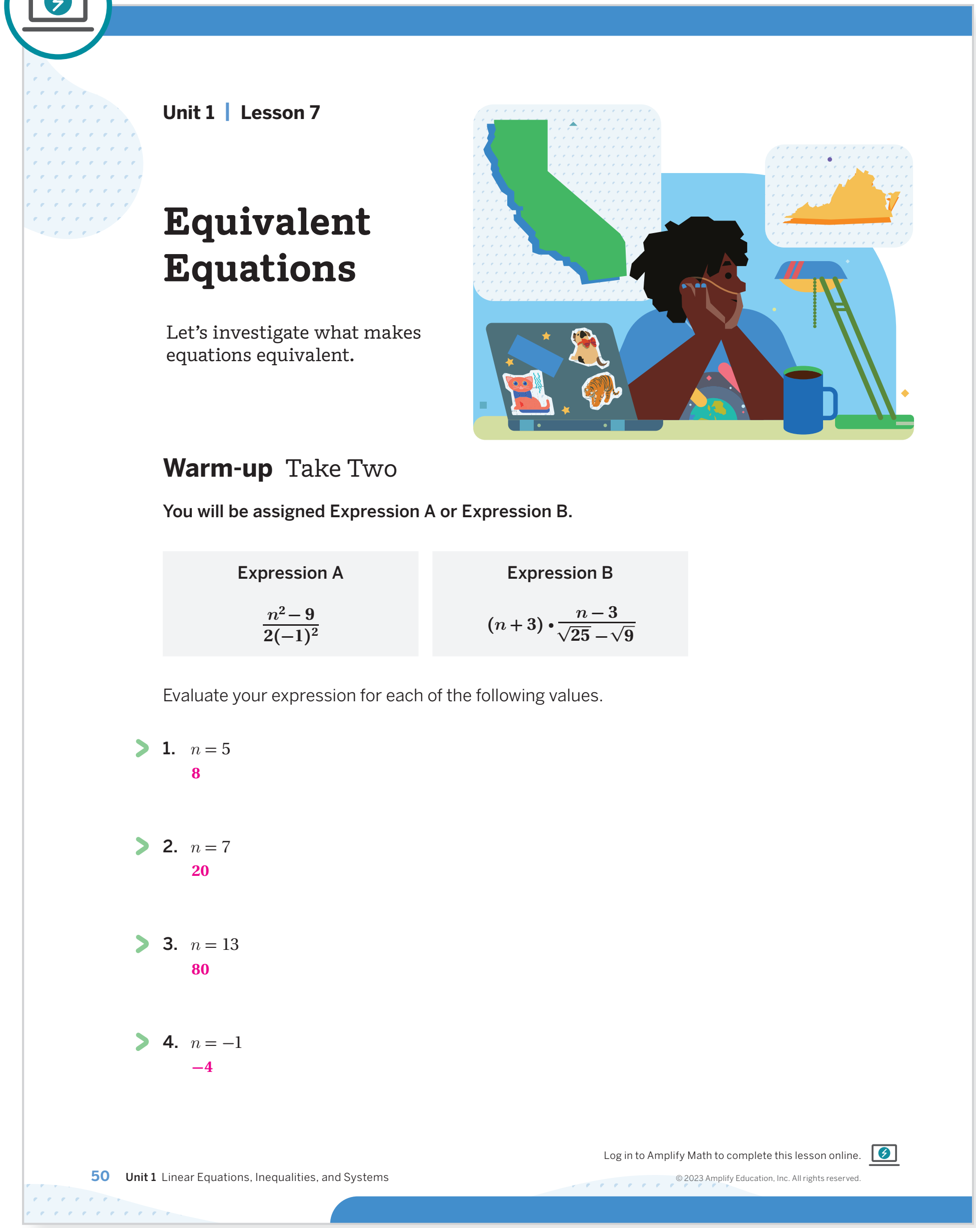
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, Problem 3 may be omitted.
- In **Activity 3**, Card 4 may be omitted.

Independent | 5 min

Warm-up Take Two

Students apply prior knowledge of equivalent expressions to prepare them to analyze, write, and interpret equivalent equations.



Unit 1 | Lesson 7

Equivalent Equations

Let's investigate what makes equations equivalent.

Warm-up Take Two

You will be assigned Expression A or Expression B.

Expression A	Expression B
$\frac{n^2 - 9}{2(-1)^2}$	$(n+3) \cdot \frac{n-3}{\sqrt{25} - \sqrt{9}}$

Evaluate your expression for each of the following values.

1. $n = 5$
   8
2. $n = 7$
   20
3. $n = 13$
   80
4. $n = -1$
   -4

Log in to Amplify Math to complete this lesson online.

50 Unit 1 Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Assign half of the class Expression A and the other half Expression B. Have students complete the problems independently.

2 Monitor

Help students get started by displaying the Anchor Chart PDF, *Order of Operations*.

Look for points of confusion:

- **Having difficulty applying the exponent in Expression A.** Prompt students to evaluate the exponent by using repeated multiplication.
- **Having difficulty evaluating the square roots or applying the exponents in Expression B.** Ask students what operation should be performed first according to the order of operations.

Look for productive strategies:

- Simplifying the expression before evaluating for each value of n.

3 Connect

Display the expressions and record student-provided solutions.

Have students share what they notice or wonder about the results.

Ask:

- "Evaluate the expressions with different values of n. What do you notice?"
- "Were you surprised that these expressions have the same result for different values of n? Why?"

Highlight that equivalent expressions are expressions that are equal no matter the value of the variable.

Ask, "Is there a better way to determine if expressions are equivalent than checking every value of n?"

Math Language Development

MLR8: Discussion Supports

Before the Connect, display or provide the Anchor Chart PDF, *Sentence Stems, Describing My Thinking* to support students as they share their thinking. Allow students to rehearse with a partner what they will say before sharing with the whole class.

English Learners

Pair students together who speak the same primary language to rehearse before the whole-class discussion.

Power-up

To power up students' ability to solve one-step equations, have students complete:

Match each equation with the operation that can be applied to each side to solve the equation. Then determine the solution to each equation.

a. $4x = 20$ $x = 5$	a Divide by 4.
b. $-4x = 12$ $x = -3$	c Add 4.
c. $x - 4 = 5$ $x = 9$	d Multiply by 4.
d. $\frac{1}{4}x = -2$ $x = -8$	b Divide by -4.

Use: Before Activity 1

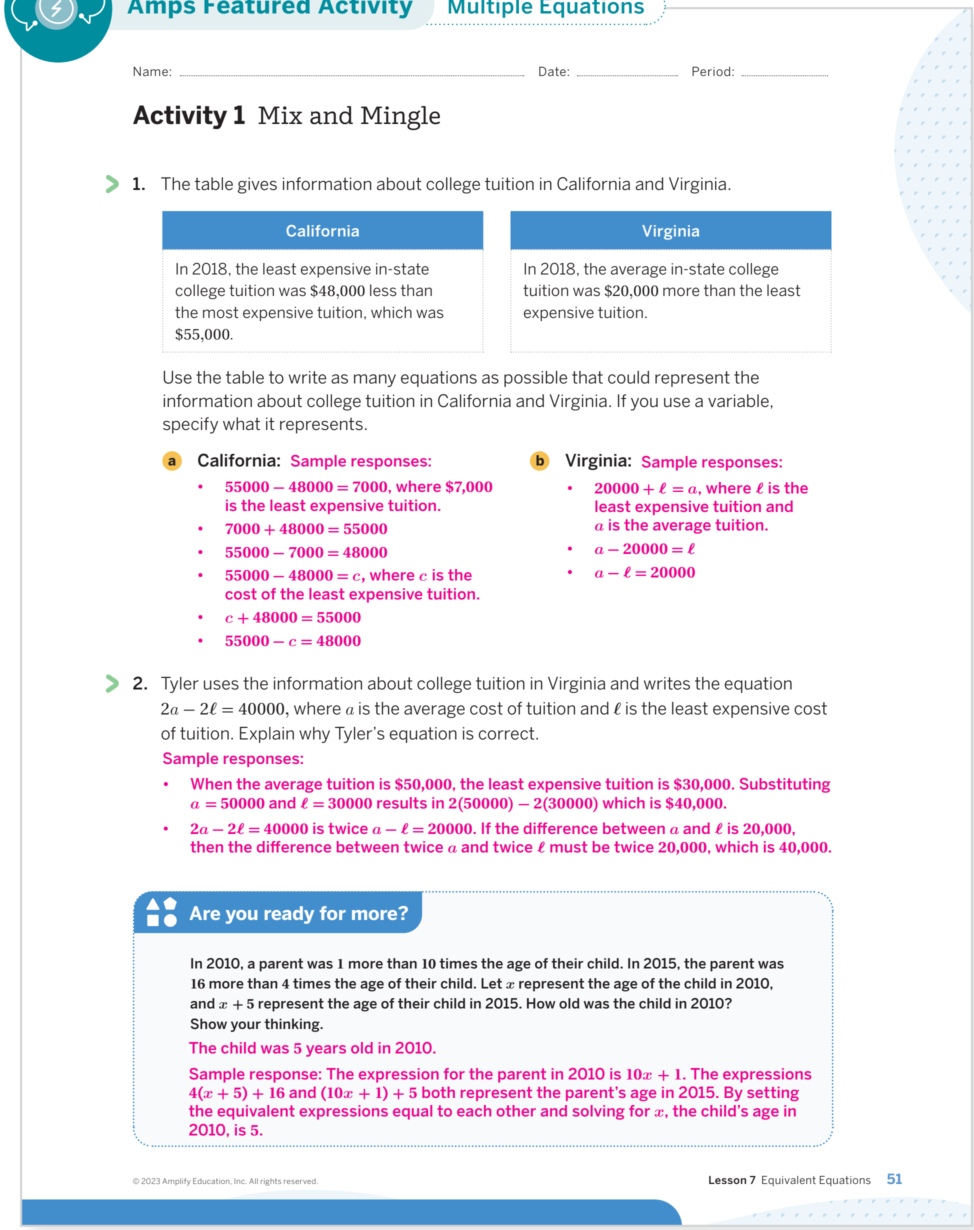
Informed by: Performance on Lesson 6 Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 1

Pairs | 10 min

Activity 1 Mix and Mingle

Students write multiple equations to represent relationships in a given context to conceptually understand what makes equations equivalent.

Amps Featured Activity Multiple Equations



Name: ______ Date: ______ Period: ______

Activity 1 Mix and Mingle

1. The table gives information about college tuition in California and Virginia.

California	Virginia
In 2018, the least expensive in-state college tuition was \$48,000 less than the most expensive tuition, which was \$55,000.	In 2018, the average in-state college tuition was \$20,000 more than the least expensive tuition.

Use the table to write as many equations as possible that could represent the information about college tuition in California and Virginia. If you use a variable, specify what it represents.

a. California: **Sample responses:**
- $55000 - 48000 = 7000$, where \$7,000 is the least expensive tuition.
- $7000 + 48000 = 55000$
- $55000 - 7000 = 48000$
- $55000 - 48000 = c$, where c is the cost of the least expensive tuition.
- $c + 48000 = 55000$
- $55000 - c = 48000$

b. Virginia: **Sample responses:**
- $20000 + \ell = a$, where ℓ is the least expensive tuition and a is the average tuition.
- $a - 20000 = \ell$
- $a - \ell = 20000$

2. Tyler uses the information about college tuition in Virginia and writes the equation $2a - 2\ell = 40000$, where a is the average cost of tuition and ℓ is the least expensive cost of tuition. Explain why Tyler's equation is correct.

Sample responses:
- When the average tuition is \$50,000, the least expensive tuition is \$30,000. Substituting $a = 50000$ and $\ell = 30000$ results in $2(50000) - 2(30000)$ which is \$40,000.
- $2a - 2\ell = 40000$ is twice $a - \ell = 20000$. If the difference between a and ℓ is 20,000, then the difference between twice a and twice ℓ must be twice 20,000, which is 40,000.

Are you ready for more?

In 2010, a parent was 1 more than 10 times the age of their child. In 2015, the parent was 16 more than 4 times the age of their child. Let x represent the age of the child in 2010, and $x + 5$ represent the age of their child in 2015. How old was the child in 2010? Show your thinking.

The child was 5 years old in 2010.

Sample response: The expression for the parent in 2010 is $10x + 1$. The expressions $4(x + 5) + 16$ and $(10x + 1) + 5$ both represent the parent's age in 2015. By setting the equivalent expressions equal to each other and solving for x, the child's age in 2010, is 5.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 7 Equivalent Equations 51

1 Launch

Arrange students in pairs. Have student-pairs discuss the following question: "Would you like to attend a college nearby, in-state, or out-of-state?"

Display the Instructional Routine PDF, *Mix and Mingle: Instructions*. Consider demonstrating if students are unfamiliar with it. **Note:** This routine is designed to be used with clips of music.

2 Monitor

Help students get started by using a graphic organizer to list all the information they know and do not know.

Look for points of confusion:

- **Thinking the equation in Problem 2 is incorrect.** Have students try determining values of a and ℓ that would make the equation true. Ask, "What do you notice about the values of the variables?"

Look for productive strategies:

- Noticing and applying the structure of one equation to write new equivalent equations.
- Substituting values to make sense of the scenarios.

3 Connect

Have students share their equations.

Display student equations in categories — numerical, one variable, and two variables.

Ask:

- "Consider the equations for each scenario. Are they all equivalent? Why or why not?"
- "What do you think it means for two equations to be equivalent?"

Define the term ***equivalent equations***.

Highlight that equivalent equations may look different, but are equivalent if they have the same solution.

Ask, "What strategies could you use to determine if two equations are equivalent?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can see the immediate consequences of their responses, and then have opportunities to correct any errors.

Accessibility: *Guide Processing and Visualization*

Display the following equation visual support for each state to assist students in visualizing the quantities and their relationships. Color code the variables with what they represent.

California	[least expensive] = [most expensive] − 48000
Virginia	[average] = 20000 + [least expensive]

Math Language Development

MLR1: Stronger and Clearer Each Time

As students work, display the Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning*. Encourage students to use these prompts to assist them as they discuss what equations to write or what approach to take to verify Tyler's equation in Problem 2.

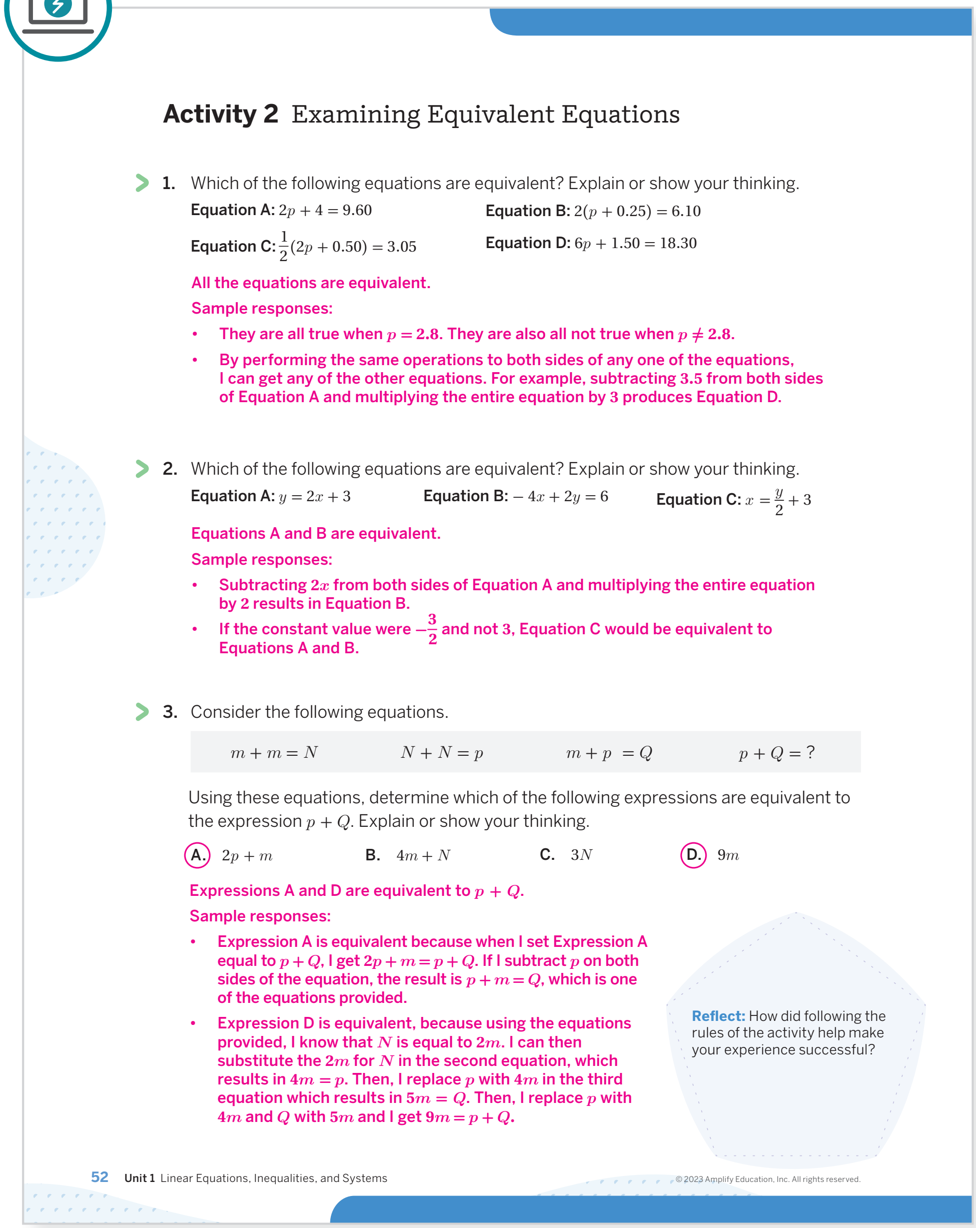
English Learners

Allow students to work with a partner who speaks the same primary language to support their use of developing mathematical language in their explanations.

Pairs | 10 min

Activity 2 Examining Equivalent Equations

Students examine and manipulate equations in multiple variables to determine their equivalency and solidify their understanding of equivalent equations.



Activity 2 Examining Equivalent Equations

1. Which of the following equations are equivalent? Explain or show your thinking.

Equation A: $2p + 4 = 9.60$
Equation B: $2(p + 0.25) = 6.10$
Equation C: $\frac{1}{2}(2p + 0.50) = 3.05$
Equation D: $6p + 1.50 = 18.30$

All the equations are equivalent.

Sample responses:

- They are all true when $p = 2.8$. They are also all not true when $p \neq 2.8$.
- By performing the same operations to both sides of any one of the equations, I can get any of the other equations. For example, subtracting 3.5 from both sides of Equation A and multiplying the entire equation by 3 produces Equation D.

2. Which of the following equations are equivalent? Explain or show your thinking.

Equation A: $y = 2x + 3$
Equation B: $-4x + 2y = 6$
Equation C: $x = \frac{y}{2} + 3$

Equations A and B are equivalent.

Sample responses:

- Subtracting $2x$ from both sides of Equation A and multiplying the entire equation by 2 results in Equation B.
- If the constant value were $-\frac{3}{2}$ and not 3, Equation C would be equivalent to Equations A and B.

3. Consider the following equations.

$m + m = N$ $N + N = p$ $m + p = Q$ $p + Q = ?$

Using these equations, determine which of the following expressions are equivalent to the expression $p + Q$. Explain or show your thinking.

[✓ A.] $2p + m$
B. $4m + N$
C. $3N$
[✓ D.] $9m$

Expressions A and D are equivalent to $p + Q$.

Sample responses:

- Expression A is equivalent because when I set Expression A equal to $p + Q$, I get $2p + m = p + Q$. If I subtract p on both sides of the equation, the result is $p + m = Q$, which is one of the equations provided.
- Expression D is equivalent, because using the equations provided, I know that N is equal to $2m$. I can then substitute the $2m$ for N in the second equation, which results in $4m = p$. Then, I replace p with $4m$ in the third equation which results in $5m = Q$. Then, I replace p with $4m$ and Q with $5m$ and I get $9m = p + Q$.

Reflect: How did following the rules of the activity help make your experience successful?

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have student pairs examine and discuss how to approach Problem 1 together. Then complete independently, before comparing solutions, strategies, and patterns.

2 Monitor

Help students get started by asking them to list strategies discussed in Activity 1.

Look for points of confusion:

- **Having difficulty performing multiple operations in Problem 1.** Ask, "How can you use the solution to Equation A to determine if the remaining equations are equivalent?"
- **Having difficulty manipulating multiple variables in Problems 2 and 3.** Use color or annotations to help students make connections and apply structure.

Look for productive strategies:

- Substituting values to compare equivalence.
- Comparing structure after one or two operations have been performed.
- Rewriting equations using different variables in Problem 3.

3 Connect

Have pairs of students share their strategies or processes for determining equivalence.

Highlight that solving an equation involves writing a series of equivalent equations that eventually isolates the variable on one side. To determine equivalency with multiple variables, perform a series of operations to both sides of the equation.

Ask, "Do you think the context changes when you write equivalent equations that model real-world scenarios?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider changing Equations B and C in Problem 1 to the following:
Equation B: $2p = 5.6$
Equation C: $6p = 16.8$

Accessibility: *Guide Processing and Visualization*

Provide students with a table or two-column graphic organizer in which they can record the steps they use, along with notes or explanations, to determine which equations are equivalent in Problems 1 and 2.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their strategies for determining equivalence, write the words and phrases they use on a visual display and update it throughout the remainder of the lesson. Continue adding to the display in Activity 3. Some phrases students may use are: *isolate the variable, perform operations on both sides, multiplying by* ____, etc.

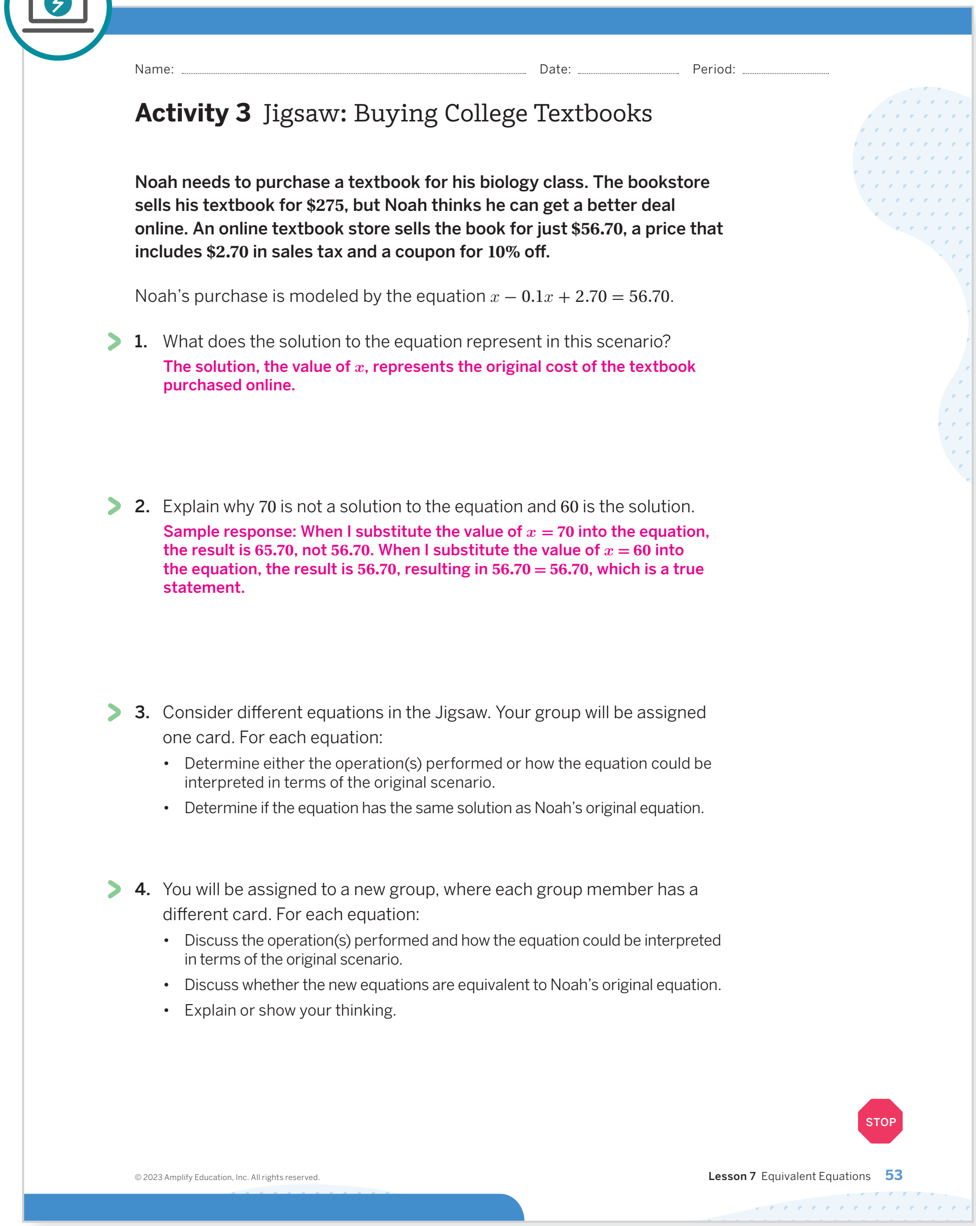
English Learners

Show visual examples next to the words and phrases. For example, next to *isolate the variable*, show an equation in which the variable is not isolated next to an equation in which the variable is isolated. Circle the equation in which the variable is isolated.

Small Groups | 15 min

Activity 3 Jigsaw: Buying College Textbooks

Students reason concretely and abstractly to interpret the structure of equivalent and non-equivalent equations in a given context.



Name: ______ Date: ______ Period: ______

Activity 3 Jigsaw: Buying College Textbooks

Noah needs to purchase a textbook for his biology class. The bookstore sells his textbook for $275, but Noah thinks he can get a better deal online. An online textbook store sells the book for just $56.70, a price that includes $2.70 in sales tax and a coupon for 10% off.

Noah's purchase is modeled by the equation $x - 0.1x + 2.70 = 56.70$.

1. What does the solution to the equation represent in this scenario?
   The solution, the value of x, represents the original cost of the textbook purchased online.

2. Explain why 70 is not a solution to the equation and 60 is the solution.
   Sample response: When I substitute the value of $x = 70$ into the equation, the result is 65.70, not 56.70. When I substitute the value of $x = 60$ into the equation, the result is 56.70, resulting in $56.70 = 56.70$, which is a true statement.

3. Consider different equations in the Jigsaw. Your group will be assigned one card. For each equation:
   - Determine either the operation(s) performed or how the equation could be interpreted in terms of the original scenario.
   - Determine if the equation has the same solution as Noah's original equation.

4. You will be assigned to a new group, where each group member has a different card. For each equation:
   - Discuss the operation(s) performed and how the equation could be interpreted in terms of the original scenario.
   - Discuss whether the new equations are equivalent to Noah's original equation.
   - Explain or show your thinking.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 7 Equivalent Equations 53

1 Launch

Arrange students in small groups to complete Problems 1 and 2, discuss as a whole class. Display the Instructional Routine PDF, *Jigsaw: Instructions*. Model the ***Jigsaw*** routine process and distribute the cards from the Activity 3 PDF.

Allow each group 5 minutes to work before re-grouping students with others who have different cards.

2 Monitor

Help students get started by having them annotate the prompt to connect the scenario with the equation.

Look for points of confusion:

- **Having difficulty interpreting the equation on their card.** Have students label the equation with original cost of the textbook, sales tax, total cost, and discount.

Look for productive strategies:

- Evaluating $x = 60$ for each equation.
- Solving each equation.
- Annotating, highlighting, or color coding to make sense of the equation.

3 Connect

Have students share the steps that lead to the same solution, different solutions, and their interpretations.

Display student steps in two lists to categorize steps that result in the same solution and different solutions.

Ask, "How does each equation model the same scenario? Explain your thinking."

Highlight that when the equations are equivalent, the equations model the same scenario.

Ask, "What steps will result in equivalent equations?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils or highlighters. Ask students to color code the equation presented at the beginning of the activity, $x - 0.1x + 2.70 = 5.670$, with what each term represents. For example:

x: the original cost of the textbook
$0.1x$: 10% of the original cost of the textbook
2.70: sales tax
56.70: the sale price of the online textbook
Suggest that students similarly color code and label each term in the equations of the card(s) they will receive.

Extension: *Math Enrichment*

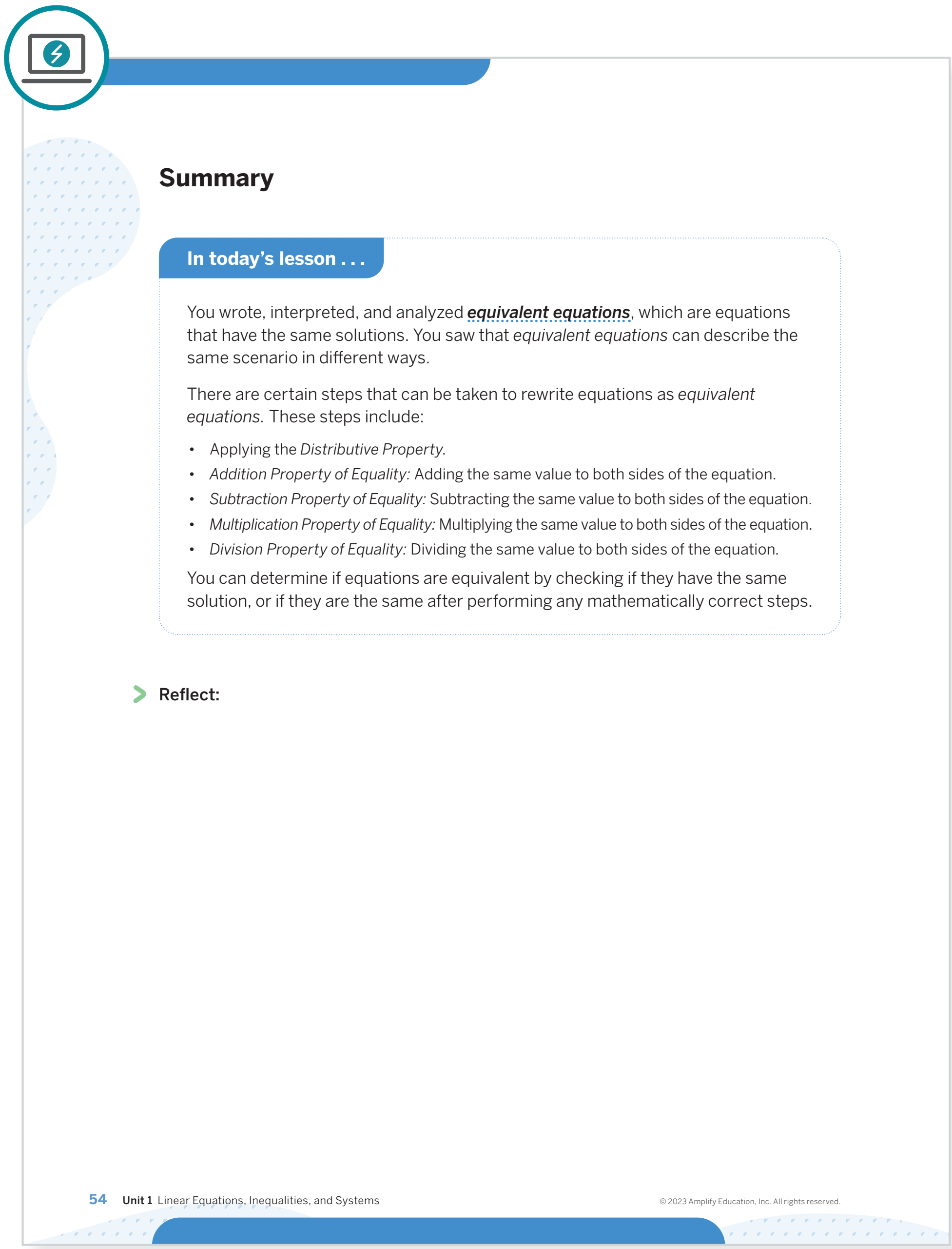
Have students write as many equivalent equations as possible to model the following scenario:

Han purchases a textbook online for $$x$, with a shipping fee of $4.99. He applies a 25% off coupon for this textbook. The total cost of his purchase is $46.24.
$x - 0.25x + 4.99 = 46.24$ or $0.75x + 4.99 = 46.24$ (or equivalent)

Whole Class | 5 min

Summary

Review and synthesize writing, analyzing, and making sense of equivalent equations.



Summary

In today's lesson . . .

You wrote, interpreted, and analyzed ***equivalent equations***, which are equations that have the same solutions. You saw that *equivalent equations* can describe the same scenario in different ways.

There are certain steps that can be taken to rewrite equations as *equivalent equations*. These steps include:

- Applying the *Distributive Property*.
- *Addition Property of Equality:* Adding the same value to both sides of the equation.
- *Subtraction Property of Equality:* Subtracting the same value to both sides of the equation.
- *Multiplication Property of Equality:* Multiplying the same value to both sides of the equation.
- *Division Property of Equality:* Dividing the same value to both sides of the equation.

You can determine if equations are equivalent by checking if they have the same solution, or if they are the same after performing any mathematically correct steps.

Reflect:

54 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the prompt, "The equation $5y = 6$ represents purchasing 5 tubs of yogurt for $\$6$."

Have students share what the solution represents and at least three equivalent equations.

Ask:

- "What steps did you take to write your equivalent equations?"
- "For your equivalent equations, what do they represent in the context of the yogurt purchase?"

Formalize vocabulary: *equivalent equations*

Highlight the strategies and properties of equalities that result in equivalent equations.

Ask, "Can you think of any operations that might not result in an equivalent equation?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies did you find helpful to determine if equations are equivalent? How were they helpful?"

Math Language Development

MLR2: Collect and Display

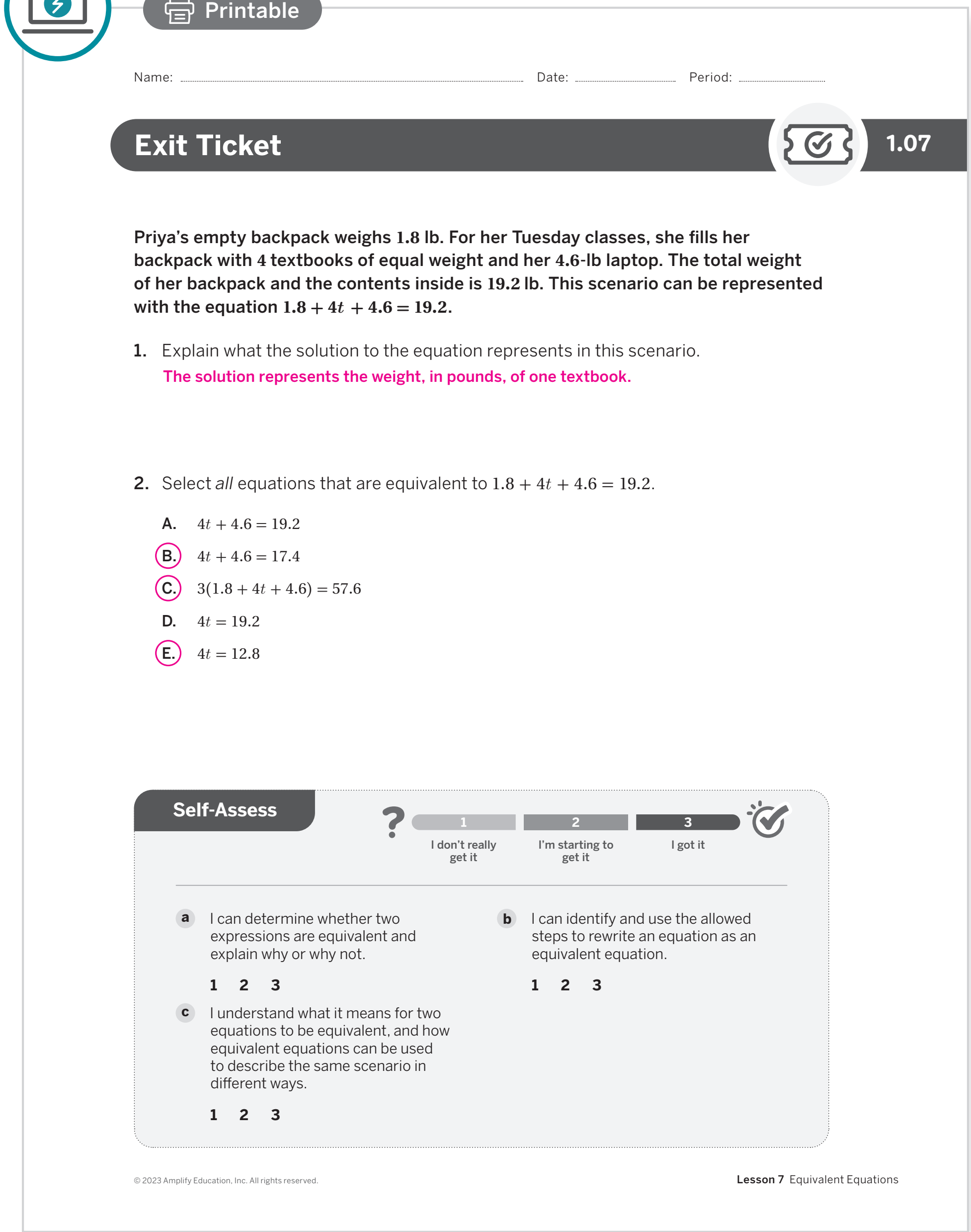
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *equivalent equations* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by selecting the equivalent equations for the equation provided.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 1.07

Priya's empty backpack weighs 1.8 lb. For her Tuesday classes, she fills her backpack with 4 textbooks of equal weight and her 4.6-lb laptop. The total weight of her backpack and the contents inside is 19.2 lb. This scenario can be represented with the equation $1.8 + 4t + 4.6 = 19.2$.

1. Explain what the solution to the equation represents in this scenario.

   The solution represents the weight, in pounds, of one textbook.

2. Select *all* equations that are equivalent to $1.8 + 4t + 4.6 = 19.2$.

   A. $4t + 4.6 = 19.2$
   [✓ B.] $4t + 4.6 = 17.4$
   [✓ C.] $3(1.8 + 4t + 4.6) = 57.6$
   D. $4t = 19.2$
   [✓ E.] $4t = 12.8$

Self-Assess

a I can determine whether two expressions are equivalent and explain why or why not.
1 2 3

b I can identify and use the allowed steps to rewrite an equation as an equivalent equation.
1 2 3

c I understand what it means for two equations to be equivalent, and how equivalent equations can be used to describe the same scenario in different ways.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 7 Equivalent Equations

Success looks like . . .

- **Language Goal:** Comprehending that *equivalent equations* are equations that have exactly the same solutions, and that multiple equivalent equations can represent the same relationship. **(Writing)**
- **Language Goal:** Determining and explaining whether two equations are equivalent. **(Speaking and Listening, Writing)**
  - » Selecting all equations equivalent to the given equation in Problem 2.
- **Goal:** Identifying operations that can be performed on an equation to create equivalent equations.

Suggested next steps

If students incorrectly explain the solution in context for Problem 1, consider:

- Reviewing Lesson 5.

If students do not select all the equivalent equations in Problem 2, consider:

- Reviewing strategies from Activities 2 and 3.
- Assigning Practice Problems 1 and 2.
- Asking, "What strategies can you use to determine if two equations are equivalent?"

Professional Learning

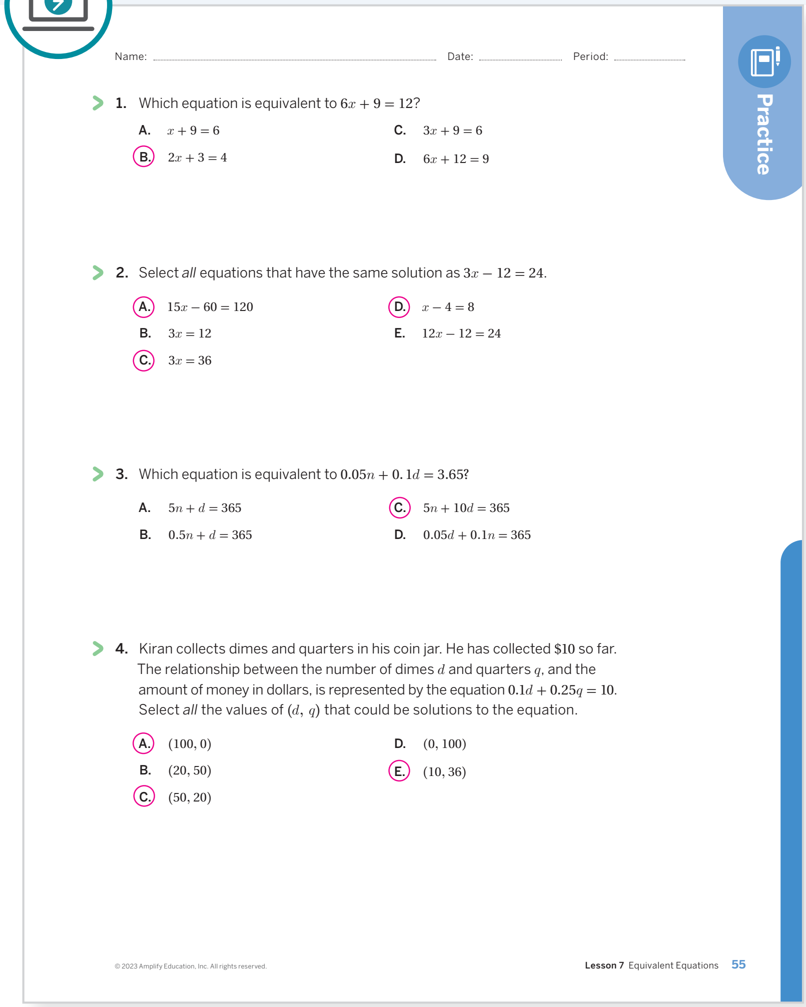
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In Activity 1, you used structured pairing with MLR1 to group students who spoke the same primary language. What effect did this grouping strategy have on their revisions? Would you change anything the next time you use MLR1?
- How did students self-manage today? How are you helping students become aware of how they are progressing in this area? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Which equation is equivalent to $6x + 9 = 12$?
   - A. $x + 9 = 6$
   - [✓ B.] $2x + 3 = 4$
   - C. $3x + 9 = 6$
   - D. $6x + 12 = 9$

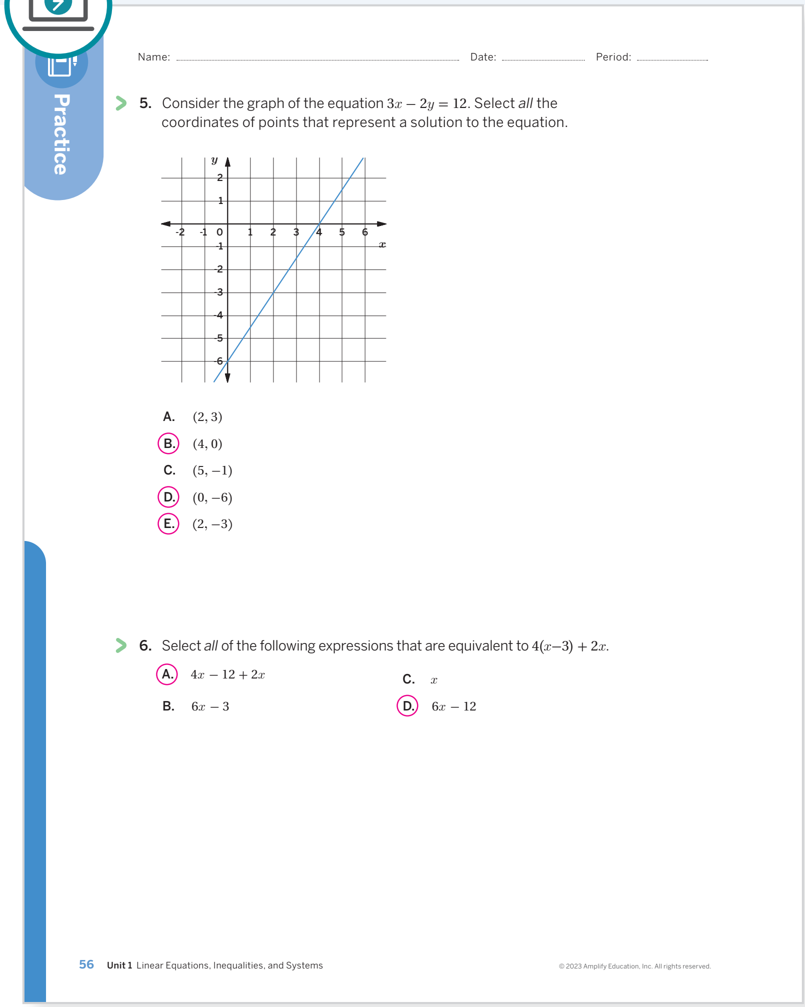
2. Select *all* equations that have the same solution as $3x - 12 = 24$.
   - [✓ A.] $15x - 60 = 120$
   - B. $3x = 12$
   - [✓ C.] $3x = 36$
   - [✓ D.] $x - 4 = 8$
   - E. $12x - 12 = 24$

3. Which equation is equivalent to $0.05n + 0.1d = 3.65$?
   - A. $5n + d = 365$
   - B. $0.5n + d = 365$
   - [✓ C.] $5n + 10d = 365$
   - D. $0.05d + 0.1n = 365$

4. Kiran collects dimes and quarters in his coin jar. He has collected $10 so far. The relationship between the number of dimes d and quarters q, and the amount of money in dollars, is represented by the equation $0.1d + 0.25q = 10$. Select *all* the values of (d, q) that could be solutions to the equation.
   - [✓ A.] $(100, 0)$
   - B. $(20, 50)$
   - [✓ C.] $(50, 20)$
   - D. $(0, 100)$
   - [✓ E.] $(10, 36)$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 7 Equivalent Equations 55



Name: Date: Period:

Practice

5. Consider the graph of the equation $3x - 2y = 12$. Select *all* the coordinates of points that represent a solution to the equation.

   - A. $(2, 3)$
   - [✓ B.] $(4, 0)$
   - C. $(5, -1)$
   - [✓ D.] $(0, -6)$
   - [✓ E.] $(2, -3)$

6. Select *all* of the following expressions that are equivalent to $4(x–3) + 2x$.
   - [✓ A.] $4x - 12 + 2x$
   - B. $6x - 3$
   - C. x
   - [✓ D.] $6x - 12$

56 Unit 1 Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	1
	3	Activity 3	2
Spiral	4	Unit 1 Lesson 4	2
	5	Unit 1 Lesson 6	2
Formative ⚡	6	Unit 1 Lesson 8	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Optional

UNIT 1 | LESSON 8

Explaining Steps for Rewriting Equations

Let's investigate why some steps for rewriting equations work but other steps do not.

Focus

Goals

1. **Language Goal:** Explain why performing certain operations on an equation may result in equivalent equations, but performing other operations may not. **(Speaking and Listening, Writing)**
2. Understand that dividing by a variable can lead to equations with fewer solutions than the original equation.
3. Understand that equations that are not true for any value of the variable(s) do not have solutions.

Coherence

Today

Students continue developing their understanding of equivalent equations by explaining which operations produce equivalent equations. They build on their Grade 8 understanding of equations with no solutions and division by zero, seeing that dividing each side of an equation by the same variable will not lead to an equivalent equation.

Previously

In Lesson 7, students defined, identified, and wrote equivalent equations quantitatively, in a context, and abstractly.

Coming Soon

In Lesson 9, students will use properties of equality to rearrange equations to solve for a desired variable.

Rigor

- Students further their **conceptual understanding** of why certain algebraic steps result in equivalent equations.
- Students strengthen their **fluency** in solving one-variable equations.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps*
- Anchor Chart PDF, *Properties of Operations*
- Instructional Routine PDF, *Take Turns: Instructions*

Math Language Development

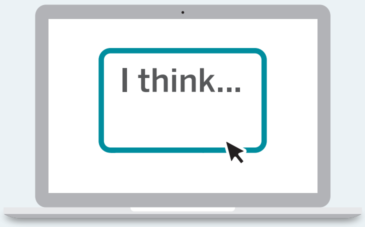
Review words

- *equivalent equations*

Amps | Featured Activity

Activity 2
See Student Thinking

Students are asked to explain their thinking about allowed operations on equivalent equations, and these explanations are available to you digitally, in real time.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may forget to actively listen, and they might not be able to communicate clearly when they disagree with or misunderstand their partner. Brainstorm with students how to respectfully disagree and engage in questioning to gain more clarity into their partner's thinking.

Modifications to Pacing

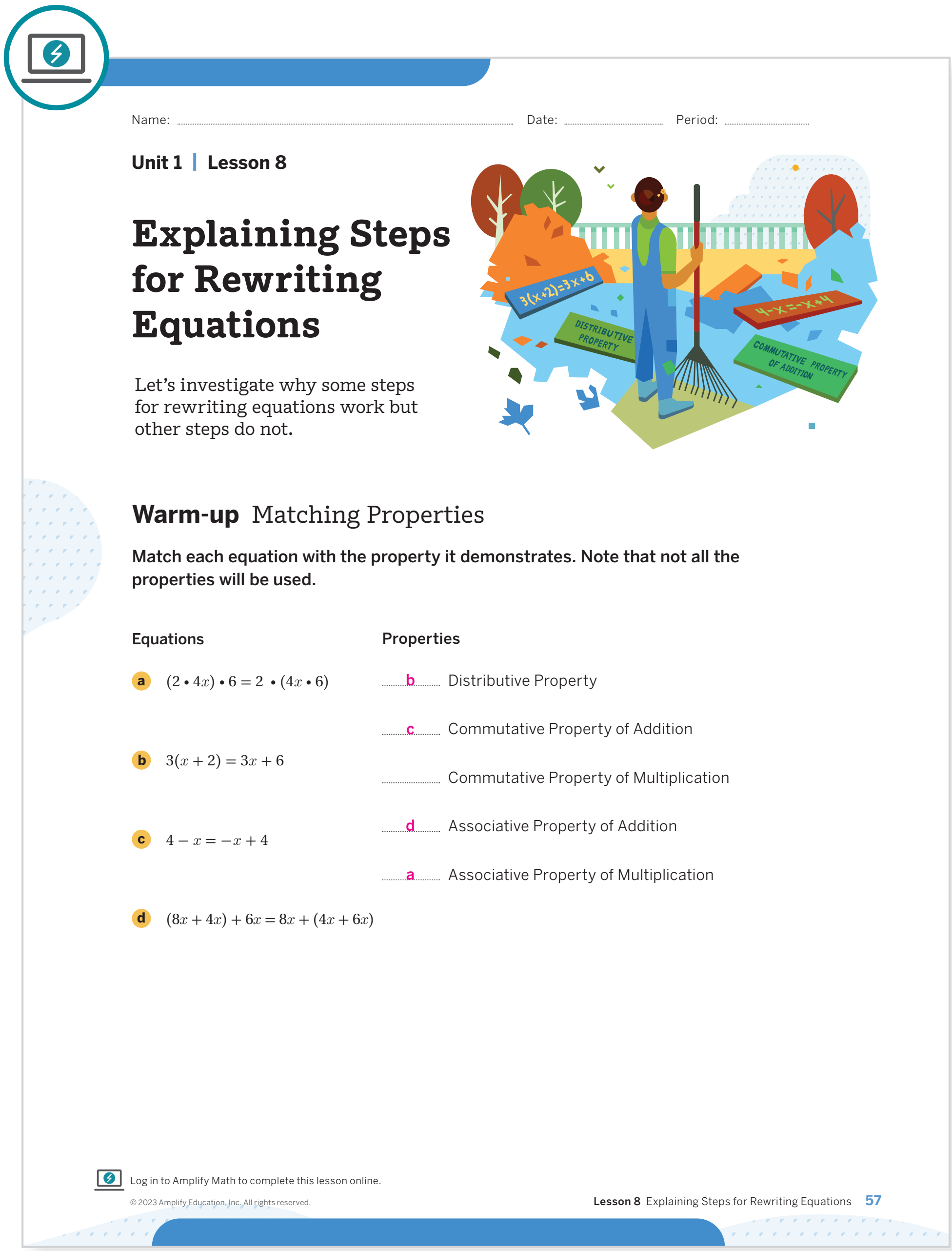
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, assign one partner Problems 1 and 2 and the other partner Problems 3 and 4.
- In **Activity 2**, have students complete at least two sets of cards.

Independent | 5 min

Warm-up Matching Properties

Students apply prior knowledge of the properties of operations to prepare them to explain allowed operations for equivalent equations in Activity 1.



Name: ______ Date: ______ Period: ______

Unit 1 | Lesson 8

Explaining Steps for Rewriting Equations

Let's investigate why some steps for rewriting equations work but other steps do not.

Warm-up Matching Properties

Match each equation with the property it demonstrates. Note that not all the properties will be used.

Equations	Properties
a $(2 \cdot 4x) \cdot 6 = 2 \cdot (4x \cdot 6)$	b Distributive Property
	c Commutative Property of Addition
b $3(x + 2) = 3x + 6$	____ Commutative Property of Multiplication
	d Associative Property of Addition
c $4 - x = -x + 4$	a Associative Property of Multiplication
d $(8x + 4x) + 6x = 8x + (4x + 6x)$	

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Set an expectation for the amount of time students will have to work independently on the Warm-up.

2 Monitor

Help students get started by displaying the Anchor Chart PDF, *Properties of Operations*.

Look for points of confusion:

- **Thinking there is no property for equation c.** Ask, "How could you rewrite the expression using a different operation?"

Look for productive strategies:

- Annotating or highlighting the parts of the equation that indicate the property.

3 Connect

Have students share their strategies for matching each property.

Highlight that the properties of operations can be used to write equivalent equations.

Ask, "Explain the properties of operations in your own words."

Math Language Development

MLR8: Discussion Supports

While students work, display the Anchor Chart PDF, *Sentence Stems, Explaining My Steps*. Encourage students to think about how they will explain their strategies as they work, ahead of the Connect discussion. Suggest they record some of the steps they use while they work so that they will have them ready to share during the Connect.

English Learners

Use color coding or gestures to draw connections between the two examples of the associative properties. Illustrate how $8x$ and $3x$ were added first in part d.

Power-up

To power up students' ability to simplify multi-step expressions, have students complete:

Diego simplified the expression $4(x + 1) - 3x + 2$, but his work is not in the correct order. Order his steps using 1, 2, 3, 4.

Step 4 $x + 6$ Step 3 $(4x - 3x) + (4 + 2)$

Step 2 $4x + 4 - 3x + 2$ Step 1 $4(x) + 4(1) - 3x + 2$

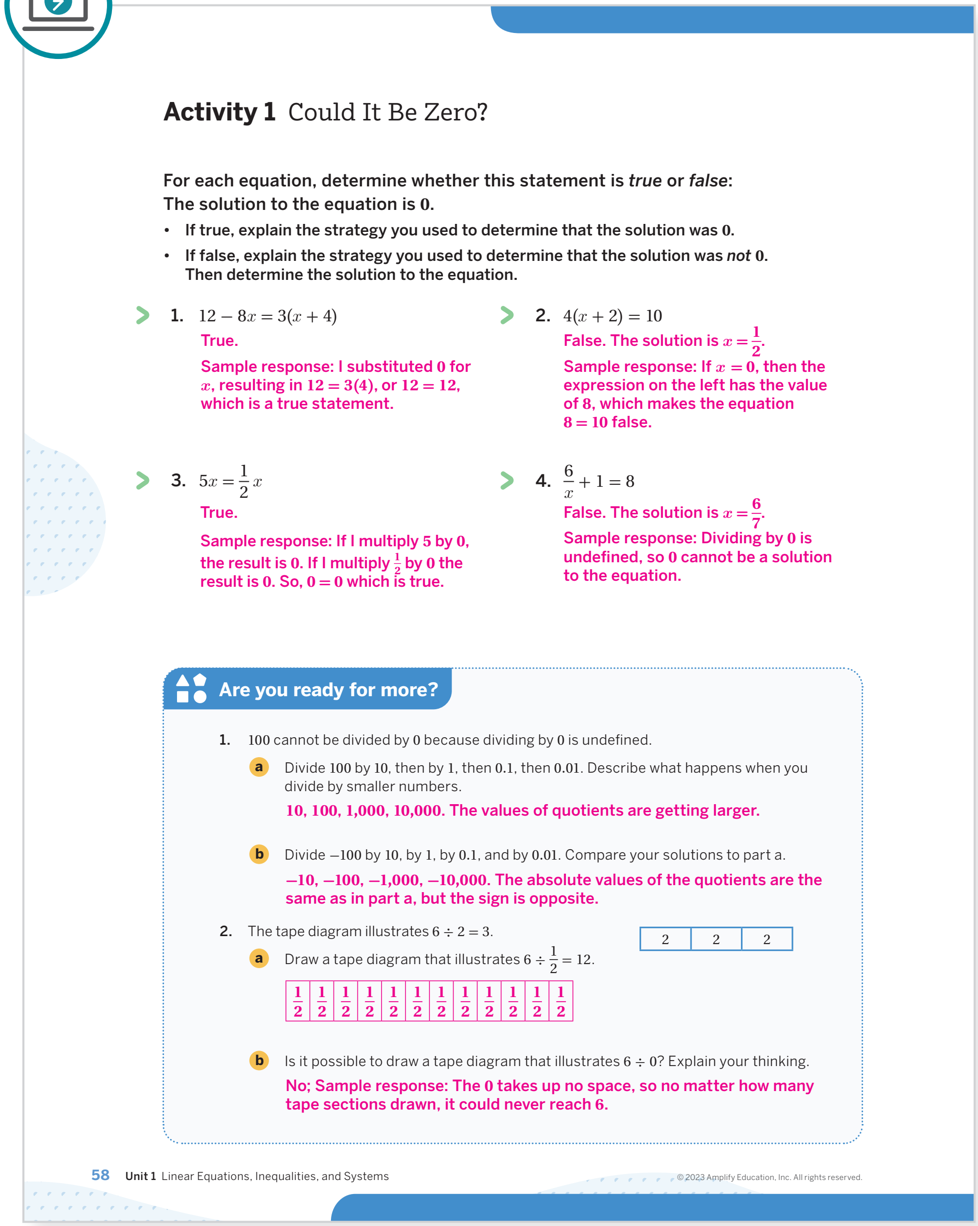
Use: Before the Warm-up

Informed by: Performance on Lesson 7, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 6

Pairs | 10 min

Activity 1 Could It Be Zero?

Students analyze the structure of linear equations and determine if 0 is a possible solution to recall invalid mathematical operations.



Activity 1 Could It Be Zero?

For each equation, determine whether this statement is *true* or *false*:
The solution to the equation is 0.

- **If true, explain the strategy you used to determine that the solution was 0.**
- **If false, explain the strategy you used to determine that the solution was *not* 0. Then determine the solution to the equation.**

1. $12 - 8x = 3(x + 4)$

   True.

   Sample response: I substituted 0 for x, resulting in $12 = 3(4)$, or $12 = 12$, which is a true statement.

2. $4(x + 2) = 10$

   False. The solution is $x = \frac{1}{2}$.

   Sample response: If $x = 0$, then the expression on the left has the value of 8, which makes the equation $8 = 10$ false.

3. $5x = \frac{1}{2}x$

   True.

   Sample response: If I multiply 5 by 0, the result is 0. If I multiply $\frac{1}{2}$ by 0 the result is 0. So, $0 = 0$ which is true.

4. $\frac{6}{x} + 1 = 8$

   False. The solution is $x = \frac{6}{7}$.

   Sample response: Dividing by 0 is undefined, so 0 cannot be a solution to the equation.

Are you ready for more?

1. 100 cannot be divided by 0 because dividing by 0 is undefined.
   - **a** Divide 100 by 10, then by 1, then 0.1, then 0.01. Describe what happens when you divide by smaller numbers.

     10, 100, 1,000, 10,000. The values of quotients are getting larger.
   - **b** Divide -100 by 10, by 1, by 0.1, and by 0.01. Compare your solutions to part a.

     -10, -100, $-1{,}000$, $-10{,}000$. The absolute values of the quotients are the same as in part a, but the sign is opposite.
2. The tape diagram illustrates $6 \div 2 = 3$.

2	2	2

   - **a** Draw a tape diagram that illustrates $6 \div \frac{1}{2} = 12$.

$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

   - **b** Is it possible to draw a tape diagram that illustrates $6 \div 0$? Explain your thinking.

     No; Sample response: The 0 takes up no space, so no matter how many tape sections drawn, it could never reach 6.

58 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have student-pairs examine and discuss how to approach each problem together, complete individually, then compare solutions and strategies.

2 Monitor

Help students get started by asking, "If the solution is $x = 0$, what terms would become 0?"

Look for points of confusion:

- **Having difficulty manipulating the rational expression in Problem 4.** Have students explain what is different about this problem, then describe the steps they took to solve Problem 2.

Look for productive strategies:

- Substituting $x = 0$ and evaluating.
- Examining the structure of the equation to determine if $x = 0$ is the solution.
- Solving the equation for x.
- Using precise language when explaining their strategies.

3 Connect

Have individual students share their strategies for each problem.

Ask:

- "Did anyone solve the problem in a different way?"
- "Can anyone add on to ___'s strategy?"
- "What properties did you use when examining or solving the equations?"

Display the problems and student responses.

Highlight that dividing by 0 results in an undefined solution. Discuss possible strategies for solving or manipulating Problem 4.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Chunk this task into smaller, more manageable parts by first having students determine if the statement is true for each equation.

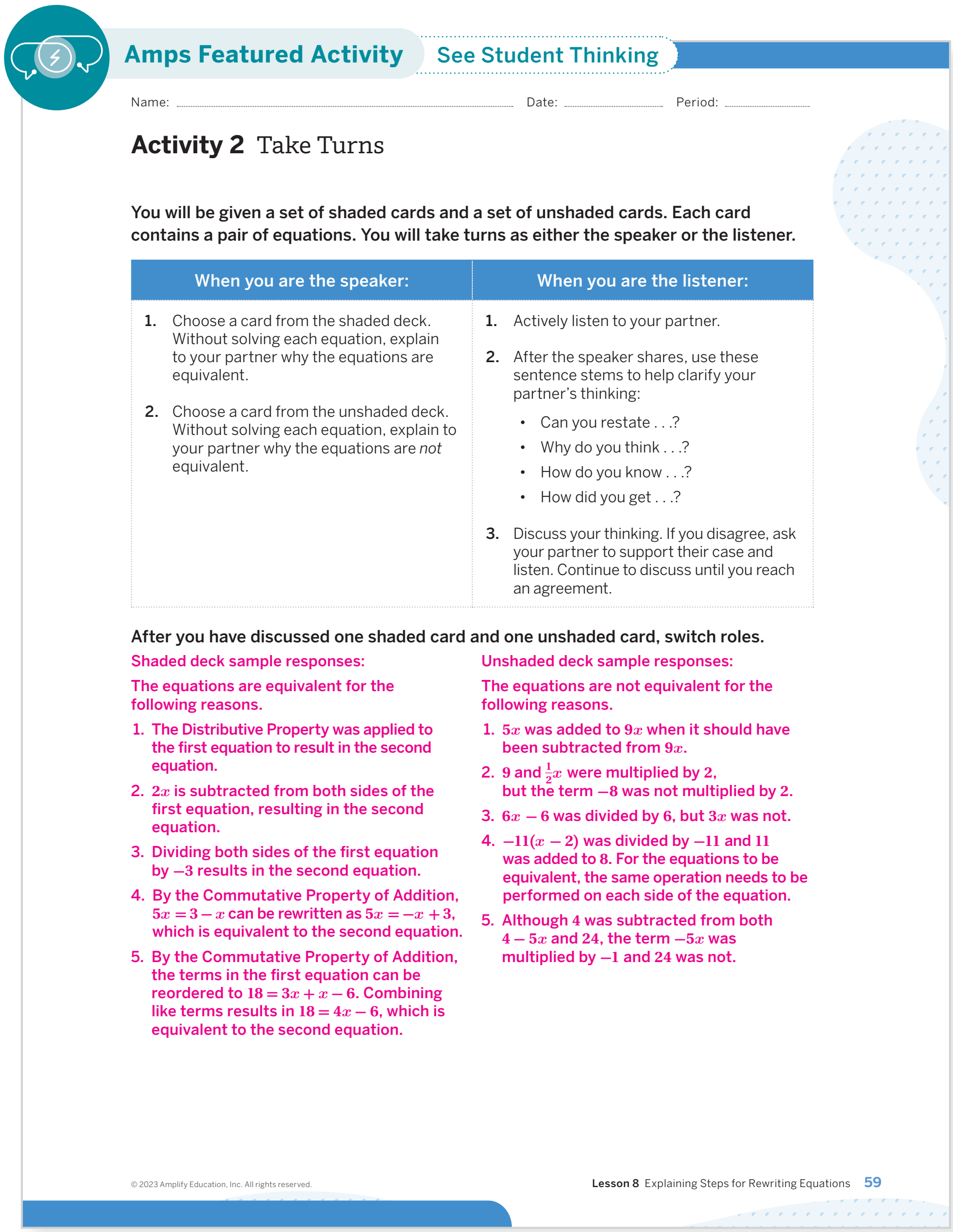
- Write the equations on slips of paper and distribute them. Have students sort the equations into two piles: whether the statement is true, or whether it is false.
- Instead of having them write their explanations, check for verbal reasoning and/or allow them to show the mathematical steps.
- Lastly, for the equations they determined the solution was not 0, have them determine the solution.

Pairs | 15 min

Activity 2 Take Turns

Students construct arguments about whether equations are equivalent without solving.

Amps Featured Activity See Student Thinking



Name: Date: Period:

Activity 2 Take Turns

You will be given a set of shaded cards and a set of unshaded cards. Each card contains a pair of equations. You will take turns as either the speaker or the listener.

When you are the speaker:	When you are the listener:
1. Choose a card from the shaded deck. Without solving each equation, explain to your partner why the equations are equivalent. 2. Choose a card from the unshaded deck. Without solving each equation, explain to your partner why the equations are *not* equivalent.	1. Actively listen to your partner. 2. After the speaker shares, use these sentence stems to help clarify your partner's thinking: • Can you restate . . .? • Why do you think . . .? • How do you know . . .? • How did you get . . .? 3. Discuss your thinking. If you disagree, ask your partner to support their case and listen. Continue to discuss until you reach an agreement.

After you have discussed one shaded card and one unshaded card, switch roles.

Shaded deck sample responses:

The equations are equivalent for the following reasons.

1. **The Distributive Property was applied to the first equation to result in the second equation.**
2. **$2x$ is subtracted from both sides of the first equation, resulting in the second equation.**
3. **Dividing both sides of the first equation by -3 results in the second equation.**
4. **By the Commutative Property of Addition, $5x = 3 - x$ can be rewritten as $5x = -x + 3$, which is equivalent to the second equation.**
5. **By the Commutative Property of Addition, the terms in the first equation can be reordered to $18 = 3x + x - 6$. Combining like terms results in $18 = 4x - 6$, which is equivalent to the second equation.**

Unshaded deck sample responses:

The equations are not equivalent for the following reasons.

1. **$5x$ was added to $9x$ when it should have been subtracted from $9x$.**
2. **9 and $\frac{1}{2}x$ were multiplied by 2, but the term -8 was not multiplied by 2.**
3. **$6x - 6$ was divided by 6, but $3x$ was not.**
4. **$-11(x - 2)$ was divided by -11 and 11 was added to 8. For the equations to be equivalent, the same operation needs to be performed on each side of the equation.**
5. **Although 4 was subtracted from both $4 - 5x$ and 24, the term $-5x$ was multiplied by -1 and 24 was not.**

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 8 Explaining Steps for Rewriting Equations 59

1 Launch

Display the Instructional Routine PDF, *Take Turns: Instructions*. Distribute the pre-cut cards from the Activity 2 PDF to each student-pair.

2 Monitor

Help students get started by asking, "What operation could have been performed resulting in the second equation?"

Look for points of confusion:

- **Thinking the equations in the unshaded deck are equivalent.** Prompt students to solve the first equation and substitute the solution into the second equation to see what happens.

Look for productive strategies:

- Solving the unshaded deck equation on their own, comparing their equation with the provided incorrect equation.
- Using evidence and precise mathematical language to justify their thinking to determine the operations and properties that result in equivalent equations.

3 Connect

Have pairs of students share their explanations for each deck.

Display and record lists of "acceptable" and "unacceptable" operations that lead to equivalent equations.

Highlight that if two expressions are equal, then performing arithmetic operations to both expressions, or applying the Distributive, commutative, or associative properties maintains equality.

Ask, "Do you think it is possible to perform all the acceptable operations and get a false statement?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Cards 1, 2, or 5 from the unshaded deck and Cards 1 or 3 from the shaded deck.

Extension: *Math Enrichment*

Challenge students to write their own equation that uses at least two operations. Then have them write two additional equations: one that is equivalent to their original equation, and one that is not equivalent because an "unacceptable" operation was performed. Have students exchange equations with a partner and each partner should determine which equation is equivalent to the original.

Math Language Development

MLR2: Collect and Display

During the Connect, as you display the "acceptable" and "unacceptable" operations, press for details in students' reasoning. Encourage the use of precise mathematical language, such as:

- The same operation(s) must be applied to both sides of the equation to maintain equivalence.
- Properties of equality, such as commutative and associative properties, maintain equivalence.

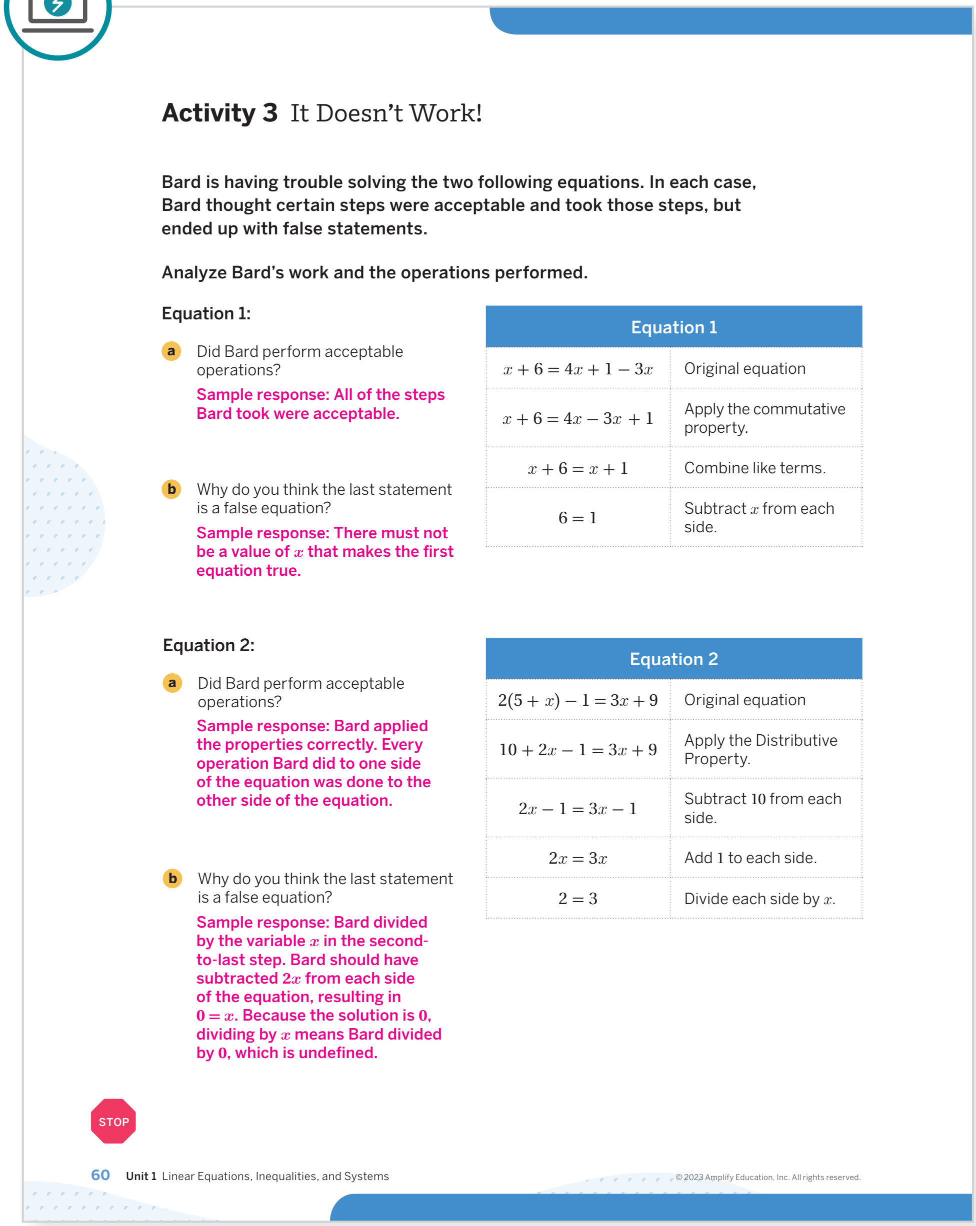
English Learners

Whenever possible, add visual examples that illustrate equivalence, such as adding the same number to each side.

Small Groups | 10 min

Activity 3 It Doesn’t Work!

Students analyze and critique the errors in a student’s work to make sense of equations with no solutions and why dividing by a variable can lead to a false statement.



Activity 3 It Doesn’t Work!

Bard is having trouble solving the two following equations. In each case, Bard thought certain steps were acceptable and took those steps, but ended up with false statements.

Analyze Bard’s work and the operations performed.

Equation 1:

a. Did Bard perform acceptable operations?

Sample response: All of the steps Bard took were acceptable.

b. Why do you think the last statement is a false equation?

Sample response: There must not be a value of x that makes the first equation true.

Equation 1	
$x + 6 = 4x + 1 - 3x$	Original equation
$x + 6 = 4x - 3x + 1$	Apply the commutative property.
$x + 6 = x + 1$	Combine like terms.
$6 = 1$	Subtract x from each side.

Equation 2:

a. Did Bard perform acceptable operations?

Sample response: Bard applied the properties correctly. Every operation Bard did to one side of the equation was done to the other side of the equation.

b. Why do you think the last statement is a false equation?

Sample response: Bard divided by the variable x in the second-to-last step. Bard should have subtracted $2x$ from each side of the equation, resulting in $0 = x$. Because the solution is 0, dividing by x means Bard divided by 0, which is undefined.

Equation 2	
$2(5 + x) - 1 = 3x + 9$	Original equation
$10 + 2x - 1 = 3x + 9$	Apply the Distributive Property.
$2x - 1 = 3x - 1$	Subtract 10 from each side.
$2x = 3x$	Add 1 to each side.
$2 = 3$	Divide each side by x.

STOP

60 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students into groups of four. Have group members analyze and discuss the student work together, complete the problems individually, then compare solutions and strategies.

2 Monitor

Help students get started by prompting them to use the list of acceptable steps created in Activity 2.

Look for points of confusion:

- **Finding no errors.** Have students determine possible values for x and substitute the value into each line to determine any errors.
- **Identifying correct steps as errors.** Provide simpler numerical examples to demonstrate the property used.

Look for productive strategies:

- Attempting to solve each equation and looking for structure from line to line in the table.

3 Connect

Have groups of students share their explanations, strategies, and possible errors for both equations.

Ask:

- “Can you determine a value of x that is true for $x + 6 = x + 1$? What can you conclude about the equation?”
- “For $2x = 3x$, what would happen if you subtracted $2x$ from both sides?”
- “For $2x = 3x$, when you divide by x, what value are you really dividing by?”

Highlight that, when dividing by a variable, students are assuming the variable does not equal 0. Otherwise, they will produce statements that are not equivalent, or even false. A solution of 0 must be checked separately.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils or highlighters and suggest that students color code the progression of Bard’s steps. Consider demonstrating this for the first step of Equation 1 as follows:

$x + 6 = 4x + 1 - 3x$

$x + 6 = 4x - 3x + 1$

The order in which the terms were added changed, which illustrates the commutative property.

Math Language Development

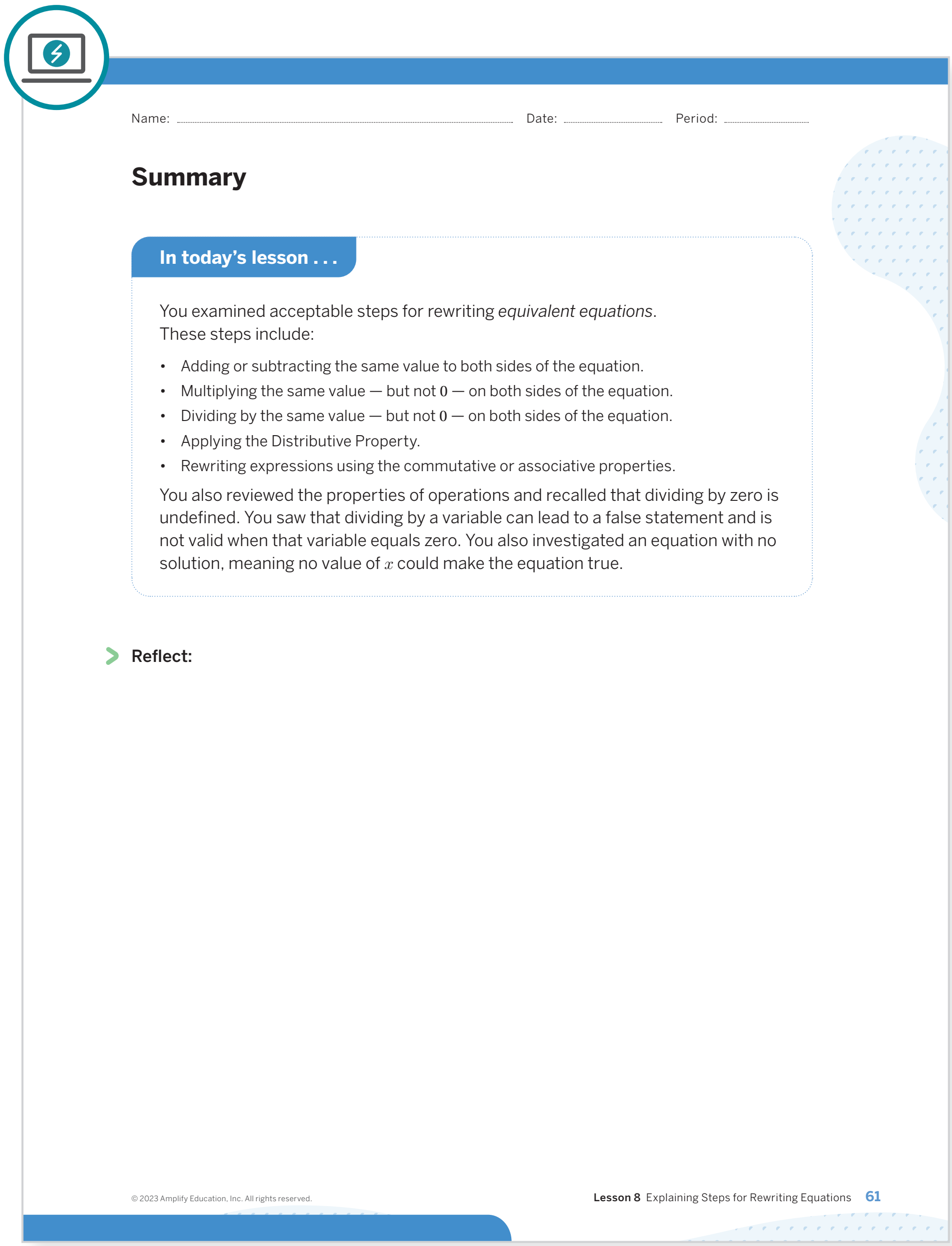
MLR8: Discussion Supports

Before the Connect, provide students time to ensure all group members can explain their group’s analysis of Bard’s work. Invite groups to rehearse before they share in the whole-class discussion. Emphasize to them that rehearsing provides additional opportunities to clarify their thinking. Consider displaying the last Ask question in the Connect before the discussion so that students can think about how they will respond ahead of the discussion. During the discussion ask, “Is $x = 0$ a solution to Equation 2? Why or why not?”

Whole Class | 5 min

Summary

Review and synthesize that performing certain operations on an equation will create equivalent equations while other operations will not.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You examined acceptable steps for rewriting *equivalent equations*. These steps include:

- Adding or subtracting the same value to both sides of the equation.
- Multiplying the same value — but not 0 — on both sides of the equation.
- Dividing by the same value — but not 0 — on both sides of the equation.
- Applying the Distributive Property.
- Rewriting expressions using the commutative or associative properties.

You also reviewed the properties of operations and recalled that dividing by zero is undefined. You saw that dividing by a variable can lead to a false statement and is not valid when that variable equals zero. You also investigated an equation with no solution, meaning no value of x could make the equation true.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 8 Explaining Steps for Rewriting Equations 61

Synthesize

Display the four equation sets.

Set A: $5(x-3)=5$
$x-3=1$

Set B: $5x+3=5$
$5x=2$

Set C: $5x-3=5x$
$x-3=x$

Set D: $(5-3)x=5x$
$5-3=5$

Have students share what they notice and wonder about the equations.

Ask:

- "What operation was performed to the original equation to obtain the second equation?"
- "Is the solution to the second equation the same as the solution to the original equation? Why does it stay the same or why does it change?"

Highlight the properties of operation that result in equivalent equations, e.g. subtraction and dividing by a constant, and the operations that may not result in equivalent equations, e.g. dividing by a variable.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

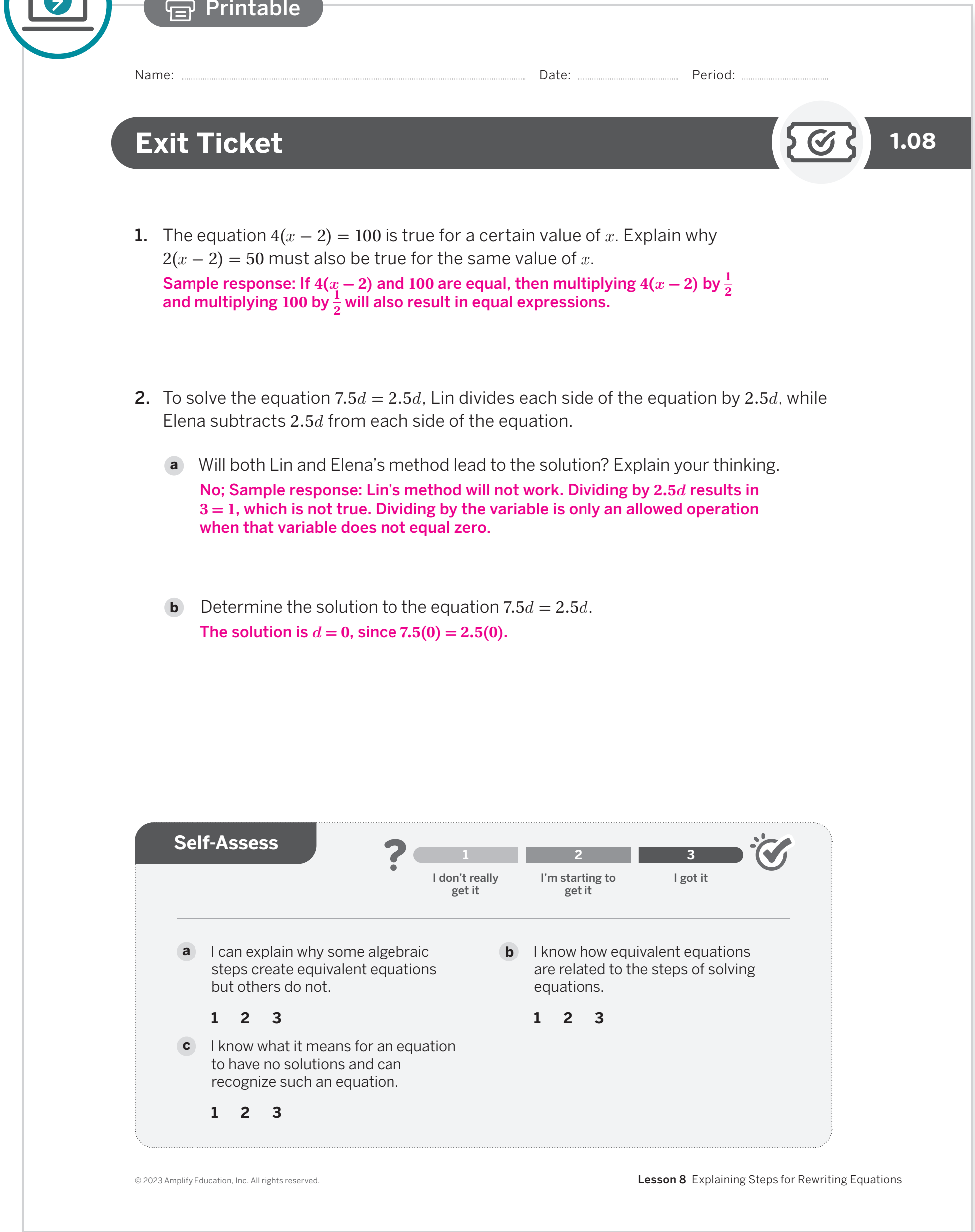
- "What new information did you learn about solving strategies today? What previously learned information did you recall? Did anything surprise you?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by examining equations and explaining why certain algebraic steps result in equivalent equations.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

1.08

1. The equation $4(x-2)=100$ is true for a certain value of x. Explain why $2(x-2)=50$ must also be true for the same value of x.
   Sample response: If $4(x-2)$ and 100 are equal, then multiplying $4(x-2)$ by $\frac{1}{2}$ and multiplying 100 by $\frac{1}{2}$ will also result in equal expressions.

2. To solve the equation $7.5d=2.5d$, Lin divides each side of the equation by $2.5d$, while Elena subtracts $2.5d$ from each side of the equation.

   a. Will both Lin and Elena's method lead to the solution? Explain your thinking.
   No; Sample response: Lin's method will not work. Dividing by $2.5d$ results in $3=1$, which is not true. Dividing by the variable is only an allowed operation when that variable does not equal zero.

   b. Determine the solution to the equation $7.5d=2.5d$.
   The solution is $d=0$, since $7.5(0)=2.5(0)$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can explain why some algebraic steps create equivalent equations but others do not.
1 2 3

b. I know how equivalent equations are related to the steps of solving equations.
1 2 3

c. I know what it means for an equation to have no solutions and can recognize such an equation.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Success looks like . . .

- **Language Goal:** Explaining why performing certain operations on an equation may create equivalent equations but performing other operations may not. **(Speaking and Listening, Writing)**
  - » Explaining why Lin's method of solving does not work in Problem 2a.
- **Goal:** Understanding that dividing by a variable is not used in solving equations because it can lead to equations that have fewer solutions than the original equation.
- **Goal:** Understanding that equations that are not true for any value of the variable(s) do not have solutions.

Suggested next steps

If students inadequately explain their thinking in Problem 1, consider:

- Reviewing strategies from Activity 2.
- Assigning Practice Problem 2.
- Asking, "What is the solution to the first equation? What happens when you substitute that value into the second equation?"

If students think the steps lead to the solution in Problem 2, consider:

- Reviewing the discussion from Activity 3.
- Asking, "What value of d makes the equation true? What happened when they divided by the variable?"

If students incorrectly solve the equation in Problem 2b, consider:

- Reviewing Lesson 5.

Professional Learning

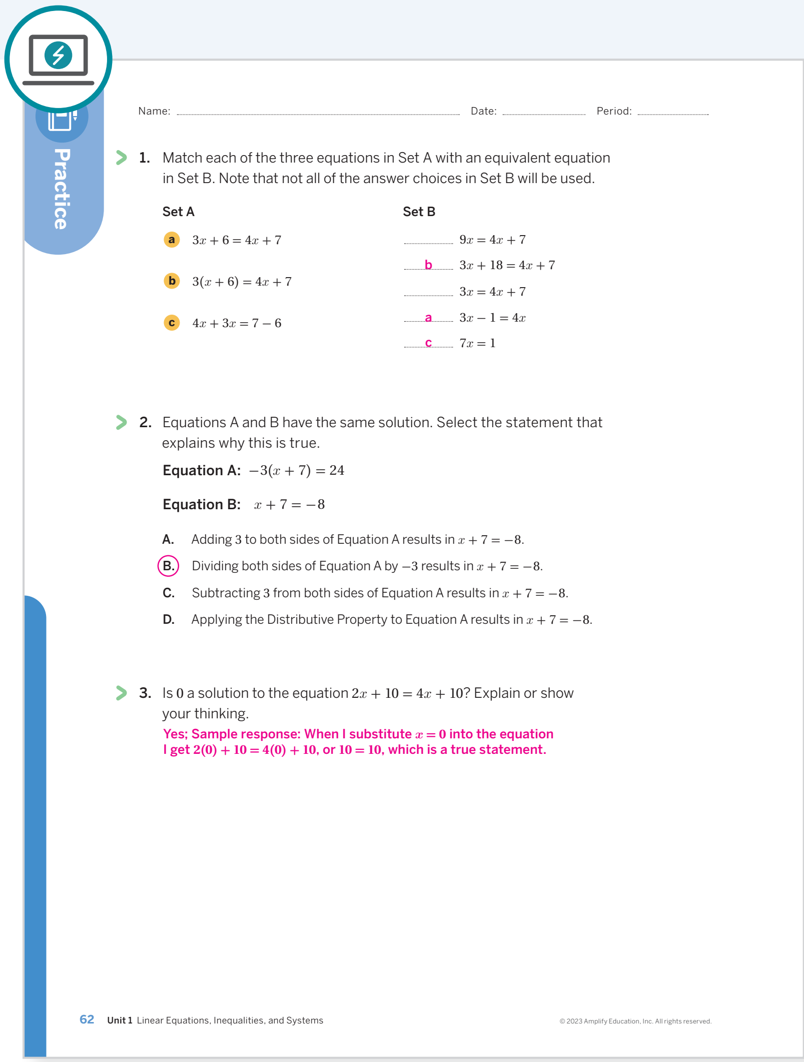
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students are asked to explain each step in solving a simple equation. Where in your students' work today did you see or hear evidence of them doing this?
- During the discussion in Activity 3 how did you encourage each student to listen to one another's strategies? What might you change for the next time you teach this lesson?

Practice

Independent



Name: .... Date: .... Period: ....

1. Match each of the three equations in Set A with an equivalent equation in Set B. Note that not all of the answer choices in Set B will be used.

Set A	Set B
a $3x + 6 = 4x + 7$	.... $9x = 4x + 7$
	b $3x + 18 = 4x + 7$
b $3(x + 6) = 4x + 7$	.... $3x = 4x + 7$
	a $3x - 1 = 4x$
c $4x + 3x = 7 - 6$	c $7x = 1$

2. Equations A and B have the same solution. Select the statement that explains why this is true.

Equation A: $-3(x + 7) = 24$

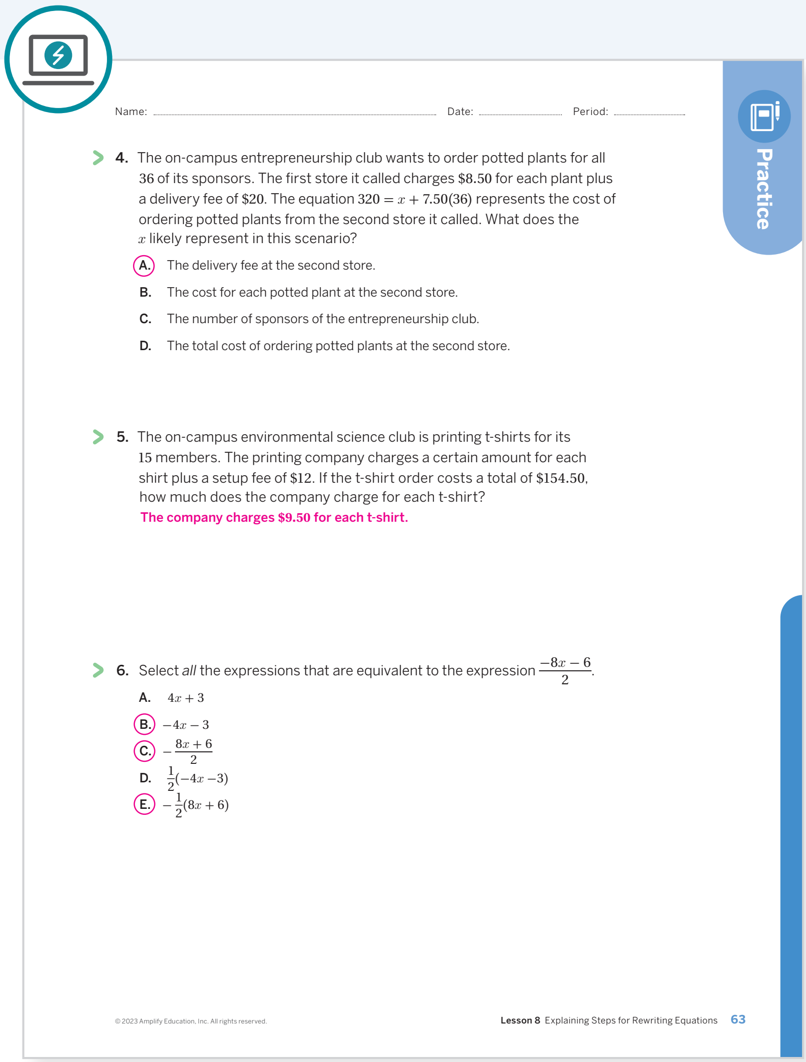
Equation B: $x + 7 = -8$

A. Adding 3 to both sides of Equation A results in $x + 7 = -8$.
[✓ B.] Dividing both sides of Equation A by -3 results in $x + 7 = -8$.
C. Subtracting 3 from both sides of Equation A results in $x + 7 = -8$.
D. Applying the Distributive Property to Equation A results in $x + 7 = -8$.

3. Is 0 a solution to the equation $2x + 10 = 4x + 10$? Explain or show your thinking.

Yes; Sample response: When I substitute $x = 0$ into the equation I get $2(0) + 10 = 4(0) + 10$, or $10 = 10$, which is a true statement.

© 2023 Amplify Education, Inc. All rights reserved.



Name: .... Date: .... Period: ....

4. The on-campus entrepreneurship club wants to order potted plants for all 36 of its sponsors. The first store it called charges $\$8.50$ for each plant plus a delivery fee of $\$20$. The equation $320 = x + 7.50(36)$ represents the cost of ordering potted plants from the second store it called. What does the x likely represent in this scenario?

[✓ A.] The delivery fee at the second store.
B. The cost for each potted plant at the second store.
C. The number of sponsors of the entrepreneurship club.
D. The total cost of ordering potted plants at the second store.

5. The on-campus environmental science club is printing t-shirts for its 15 members. The printing company charges a certain amount for each shirt plus a setup fee of $\$12$. If the t-shirt order costs a total of $\$154.50$, how much does the company charge for each t-shirt?

The company charges $\$9.50$ for each t-shirt.

6. Select *all* the expressions that are equivalent to the expression $\frac{-8x - 6}{2}$.

A. $4x + 3$
[✓ B.] $-4x - 3$
[✓ C.] $-\frac{8x + 6}{2}$
D. $\frac{1}{2}(-4x - 3)$
[✓ E.] $-\frac{1}{2}(8x + 6)$

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	1
	3	Activity 1	1
Spiral	4	Unit 1 Lesson 4	2
	5	Unit 1 Lesson 3	2
Formative ⚡	6	Unit 1 Lesson 9	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 9**

Rearranging Equations (Part 1)

Let's rearrange equations to determine certain quantities.

Focus

Goals

1. **Language Goal:** Comprehend that to "solve for a variable" is to rearrange an equation to isolate a variable of interest. **(Writing)**
2. Rearrange two-variable equations in slope-intercept form to highlight a particular quantity.

Rigor

- Students build a **conceptual understanding** of rearranging expressions and solving for a variable.
- Students strengthen their **fluency** in writing two-variable equations to model a scenario in slope-intercept form.

Coherence

Today

Students examine scenarios where one form of an equation may be more useful than others. They reason, rearrange, or solve for a variable based on the quantity of interest. Students calculate repeatedly and look for regularity as they manipulate equations to solve for a variable. They notice that solving for a variable is an efficient way to solve problems.

Previously

In optional Lesson 8, students solidified their understanding of equivalent equations by explaining operations that produce equivalent equations.

Coming Soon

In Lesson 10, students will solve for a variable in an equation in standard form to determine unknown quantities more efficiently.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps*
- scientific calculators

Math Language Development

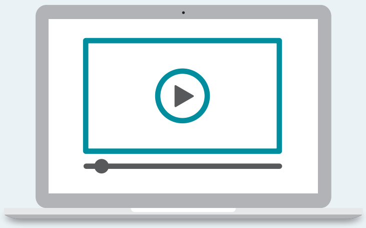
Review words

- *equivalent equations*
- *slope-intercept form*

Amps | **Featured Activity**

Activity 2
Pacing the Work

Students first complete tables that represent linear relationships. Only when they have filled out the table are they asked to write an equation for each relationship.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may have difficulty managing stress and self-motivating when writing and rewriting equations in Activities 1 and 2. Lead a discussion on challenges students may encounter and ways they could overcome them. Have students consider who might be able to help or what other resources might be available.

Modifications to Pacing

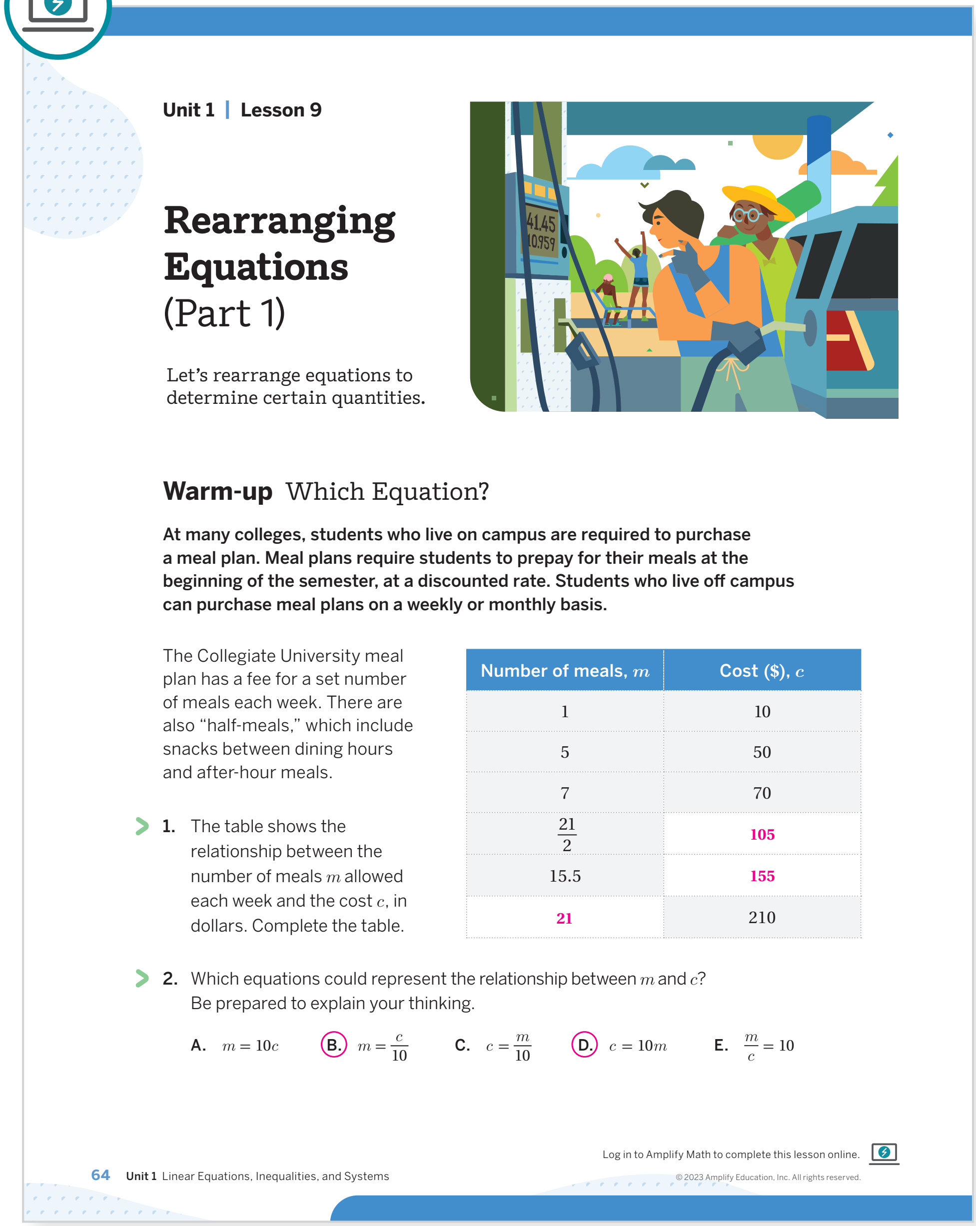
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, provide the completed table and focus on Problem 2.
- In **Activity 1**, have students only complete Problems 1–4.
- In **Activity 2**, have one student complete Problems 1–4 and the other student complete Problems 5–8.

Independent | 10 min

Warm-up Which Equation?

Students interpret a context written and presented in a table to determine equivalent equations representing the relationship.



Unit 1 | Lesson 9

Rearranging Equations (Part 1)

Let's rearrange equations to determine certain quantities.

Warm-up Which Equation?

At many colleges, students who live on campus are required to purchase a meal plan. Meal plans require students to prepay for their meals at the beginning of the semester, at a discounted rate. Students who live off campus can purchase meal plans on a weekly or monthly basis.

The Collegiate University meal plan has a fee for a set number of meals each week. There are also "half-meals," which include snacks between dining hours and after-hour meals.

1. The table shows the relationship between the number of meals m allowed each week and the cost c, in dollars. Complete the table.

Number of meals, m	Cost ($), c
1	10
5	50
7	70
$\frac{21}{2}$	**105**
15.5	**155**
21	210

2. Which equations could represent the relationship between m and c? Be prepared to explain your thinking.

A. $m = 10c$ [✓ B.] $m = \frac{c}{10}$ C. $c = \frac{m}{10}$ [✓ D.] $c = 10m$ E. $\frac{m}{c} = 10$

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Provide students with background knowledge about college meal plans. Consider sharing a story about your experience choosing a college meal plan. Set an expectation for the amount of time students will have to work independently on the Warm-up. Provide access to scientific calculators.

2 Monitor

Help students get started by prompting them to annotate the table with any patterns noticed.

Look for points of confusion:

- **Having difficulty selecting an equation.** Have students substitute values from the table into each equation and repeat to check remaining options.

Look for productive strategies:

- Substituting values of m and c into the given equations.
- Generalizing the pattern from the table into possible equations.

3 Connect

Have students share their thinking for choosing the equations in Problem 2.

Display Equations B and D.

Ask:

- "If the number of meals is known, which equation would most efficiently determine the cost? Why?"
- "If the cost is known, which equation would most efficiently determine the number of meals? Why?"

Highlight that the relationship between two quantities can be expressed in more than one way, but sometimes one form is more helpful depending on the context.

Math Language Development

MLR8: Discussion Supports

While students work, display the Anchor Chart PDF, *Sentence Stems, Explaining My Steps*. Encourage students to think about how they will explain how they chose the equations in Problem 2, ahead of the Connect discussion. Suggest they record some notes by each equation so that they will be ready to share during the discussion.

English Learners

Have students annotate the table to show where they see the value 10 in the table, other than in the first row.

Power-up

To power up students' ability to identify equivalent expressions, have students complete:

Recall that a quotient can be rewritten as the product of $\frac{1}{\text{divisor}}$ and the dividend. For example, $\frac{6x}{3} = \frac{1}{3} \bullet 6x$.

Select *all* expressions that are equivalent to $\frac{p-124}{9}$.

[✓ A.] $\frac{1}{9}(p - 124)$ [✓ C.] $\frac{p}{9} - \frac{124}{9}$

B. $\frac{p}{9} - 124$ D. $p - \frac{124}{9}$

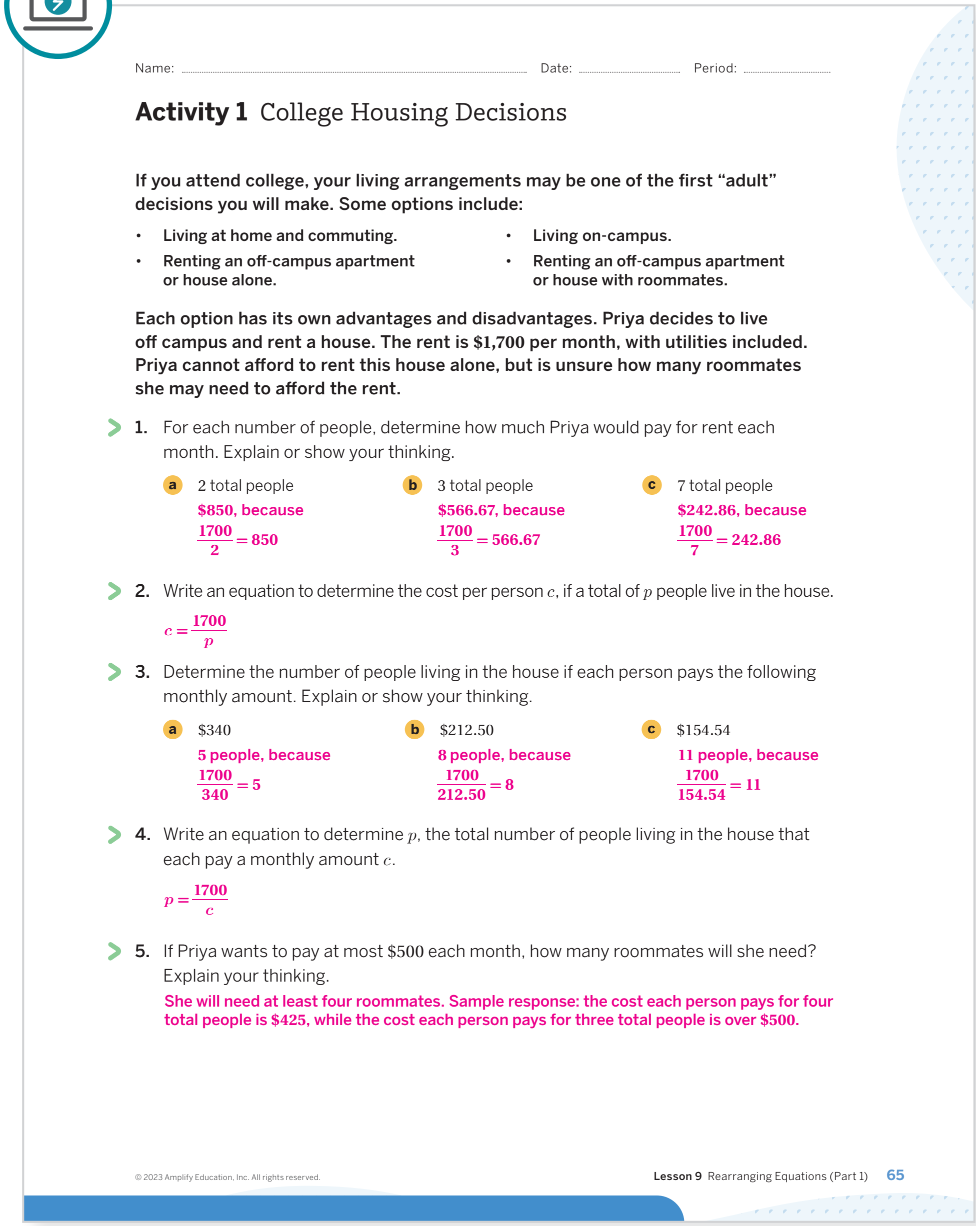
Use: Before the Warm-up

Informed by: Performance on Lesson 8, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 2

Pairs | 15 min

Activity 1 College Housing Decisions

Students use repeated calculations to write equivalent equations to recognize that one form of an equation can be more useful than the other.



Name: ____________ Date: ________ Period: ________

Activity 1 College Housing Decisions

If you attend college, your living arrangements may be one of the first "adult" decisions you will make. Some options include:

- **Living at home and commuting.**
- **Renting an off-campus apartment or house alone.**
- **Living on-campus.**
- **Renting an off-campus apartment or house with roommates.**

Each option has its own advantages and disadvantages. Priya decides to live off campus and rent a house. The rent is $1,700 per month, with utilities included. Priya cannot afford to rent this house alone, but is unsure how many roommates she may need to afford the rent.

1. For each number of people, determine how much Priya would pay for rent each month. Explain or show your thinking.

   a. 2 total people
   $850, because $\frac{1700}{2} = 850$

   b. 3 total people
   $566.67, because $\frac{1700}{3} = 566.67$

   c. 7 total people
   $242.86, because $\frac{1700}{7} = 242.86$

2. Write an equation to determine the cost per person c, if a total of p people live in the house.

   $c = \frac{1700}{p}$

3. Determine the number of people living in the house if each person pays the following monthly amount. Explain or show your thinking.

   a. $340
   5 people, because $\frac{1700}{340} = 5$

   b. $212.50
   8 people, because $\frac{1700}{212.50} = 8$

   c. $154.54
   11 people, because $\frac{1700}{154.54} = 11$

4. Write an equation to determine p, the total number of people living in the house that each pay a monthly amount c.

   $p = \frac{1700}{c}$

5. If Priya wants to pay at most $500 each month, how many roommates will she need? Explain your thinking.

   She will need at least four roommates. Sample response: the cost each person pays for four total people is $425, while the cost each person pays for three total people is over $500.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Rearranging Equations (Part 1) 65

1 Launch

Arrange students in pairs. Read the prompt aloud. Ask, "What do you know about college housing options?" Discuss each problem in pairs before completing individually. Provide access to scientific calculators.

2 Monitor

Help students get started by prompting them to draw a tape diagram to represent the total cost for two people.

Look for points of confusion:

- **Having difficulty writing equations for Problems 2 and 4.** Prompt students to describe the steps they took to calculate their previous answers.
- **Not understanding how to determine an approach for Problem 5.** Ask, "Which equation is most helpful for determining the number of people when you know the monthly cost?"

Look for productive strategies:

- Using proportional reasoning.
- Changing the form of the equation $p \cdot c = 1700$ to solve each problem.
- Generalizing repeated calculations to write and solve the equation $p \cdot c = 1700$.

3 Connect

Display student responses and strategies. Consider conducting a ***Gallery Tour***.

Have pairs of students share their strategies for Problems 2, 4, and 5. Selecting and sequencing students to share, beginning with the least straightforward approach, and ending with those using an equation.

Ask:

- "Are the equations equivalent? Explain your thinking."
- "Do you need both equations? What do they tell you about the context?"

Highlight that isolating a variable is called solving for a variable, and can be an efficient way for determining different unknown quantities.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Model how to approach Problems 1a and 3a. Ask students to use similar reasoning for parts b and c. Provide colored pencils and ask students to color code the quantities in Problems 1 and 3 with the variables they use to write the equations in Problems 2 and 4. Consider omitting Problem 5.

Extension: *Math Enrichment*

Have students decide between these two options and explain their choice:

Option A: Rent an apartment alone for $600 per month.

Option B: Rent an apartment with 3 others for $1,700 per month.

Sample response: Option B; It only costs $425 per person, per month.

Math Language Development

MLR7: Compare and Connect

Invite students to create a visual display of their strategies and equations prior to the Connect discussion. During the Connect, conduct a ***Gallery Tour*** and allow students time to quietly circulate and compare the strategies and equations displayed in at least two other visual displays. Listen for and amplify the connections between different strategies or equations. For example, ask:

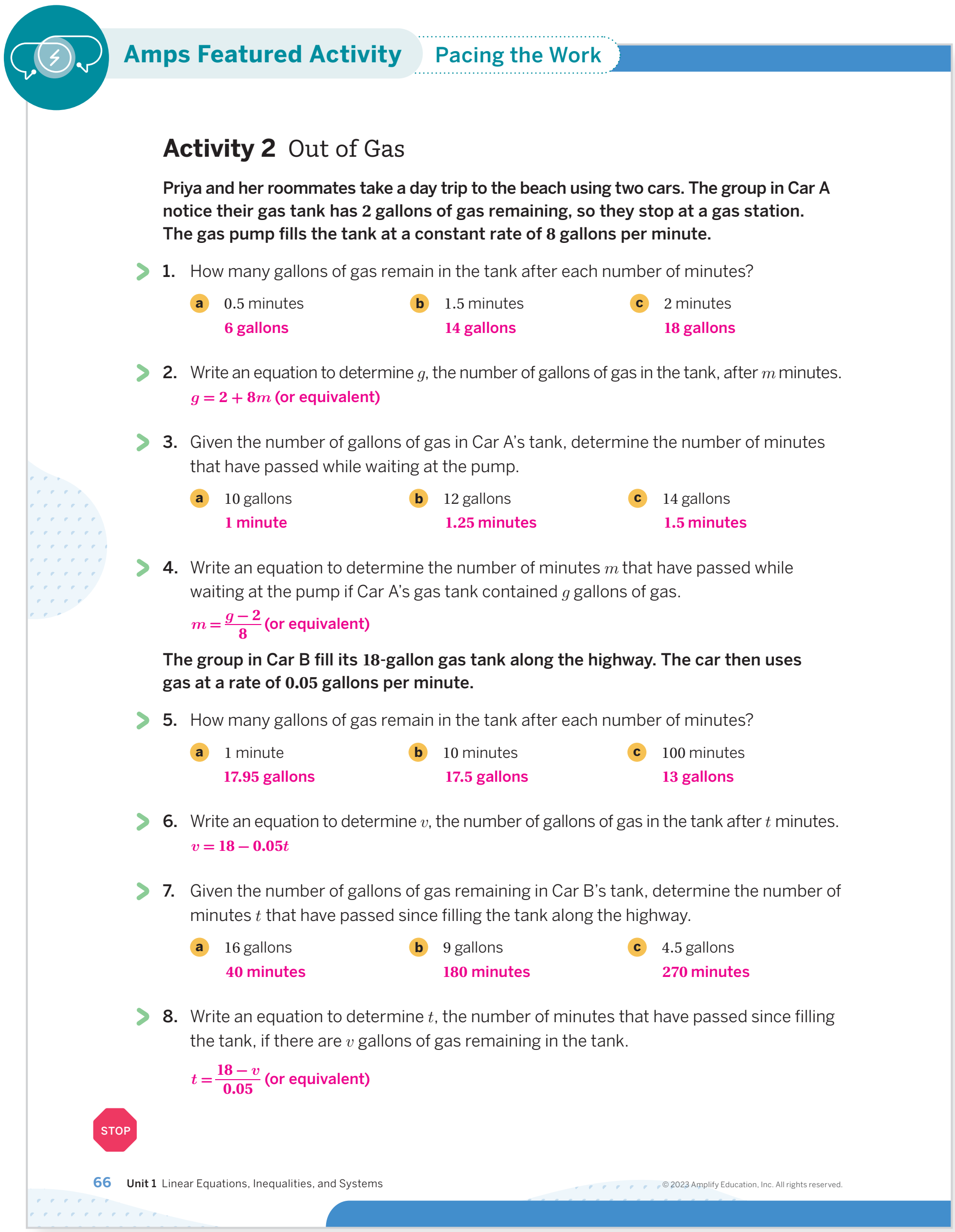
- "Where do you see the cost per person in each strategy or equation? The total number of people?"
- "Compare the equations in Problems 2 and 3. Why did the variables appear to switch places? Can you explain this mathematically?"

Pairs | 15 min

Activity 2 Out of Gas

Students write and solve non-proportional linear equations to build on their understanding of solving for a variable.

Amps Featured Activity Pacing the Work



Activity 2 Out of Gas

Priya and her roommates take a day trip to the beach using two cars. The group in Car A notice their gas tank has 2 gallons of gas remaining, so they stop at a gas station. The gas pump fills the tank at a constant rate of 8 gallons per minute.

1. How many gallons of gas remain in the tank after each number of minutes?

   a 0.5 minutes
   6 gallons

   b 1.5 minutes
   14 gallons

   c 2 minutes
   18 gallons

2. Write an equation to determine g, the number of gallons of gas in the tank, after m minutes.
   $g = 2 + 8m$ **(or equivalent)**

3. Given the number of gallons of gas in Car A's tank, determine the number of minutes that have passed while waiting at the pump.

   a 10 gallons
   1 minute

   b 12 gallons
   1.25 minutes

   c 14 gallons
   1.5 minutes

4. Write an equation to determine the number of minutes m that have passed while waiting at the pump if Car A's gas tank contained g gallons of gas.
   $m = \frac{g-2}{8}$ **(or equivalent)**

The group in Car B fill its 18-gallon gas tank along the highway. The car then uses gas at a rate of 0.05 gallons per minute.

5. How many gallons of gas remain in the tank after each number of minutes?

   a 1 minute
   17.95 gallons

   b 10 minutes
   17.5 gallons

   c 100 minutes
   13 gallons

6. Write an equation to determine v, the number of gallons of gas in the tank after t minutes.
   $v = 18 - 0.05t$

7. Given the number of gallons of gas remaining in Car B's tank, determine the number of minutes t that have passed since filling the tank along the highway.

   a 16 gallons
   40 minutes

   b 9 gallons
   180 minutes

   c 4.5 gallons
   270 minutes

8. Write an equation to determine t, the number of minutes that have passed since filling the tank, if there are v gallons of gas remaining in the tank.
   $t = \frac{18-v}{0.05}$ **(or equivalent)**

STOP

66 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have student-pairs discuss how to approach each problem together, then complete independently before comparing solutions and strategies. Provide access to scientific calculators.

2 Monitor

Help students get started by having them highlight or circle the known information about Car A.

Look for points of confusion:

- **Rewriting the equation in terms of t in Problem 6.** Prompt students to describe the steps they took to calculate the minutes in Problem 4.

Look for productive strategies:

- Creating a table of values to determine the slope and y-intercept.
- Noticing through repeated calculations that m and g can replace numbers.
- Solving for the variable of interest in an equation.

3 Connect

Have pairs of students share their strategies for determining the equations for Problems 2, 4, 6, and 8. Select and sequence students reasoning informally first, before those using an equation to solve for a variable.

Ask:

- "How do you know that your equation represents the number of gallons of gas in Car A after m minutes?"
- "How do you know that your equation represents the number of minutes after g gallons in Car A have been used?"

Highlight the steps for writing Problem 2 as an equation, $g = 2 + 8m$, and solving for m, $m = \frac{g-2}{8}$ in Problem 4.

Ask, "When would solving for the variable m be helpful?"

Differentiated Support

Accessibility: *Optimize Access To Technology*

Have students use the Amps slides for this activity, in which they can compare their models to a gas tank animation. This visual support will help them know if they need to revise or refine their thinking.

Accessibility: *Guide Processing and Visualization*

Provide a table for students to use which will help them see the patterns for writing the equations. Here is a sample table for Problem 3.

Number of gallons, g	Substitute g into your equation from Problem 2.	Solve for m.
10	$10 = 2 + 8m$	$m = \frac{10-2}{8}$

Math Language Development

MLR8: Discussion Supports – Press for Details

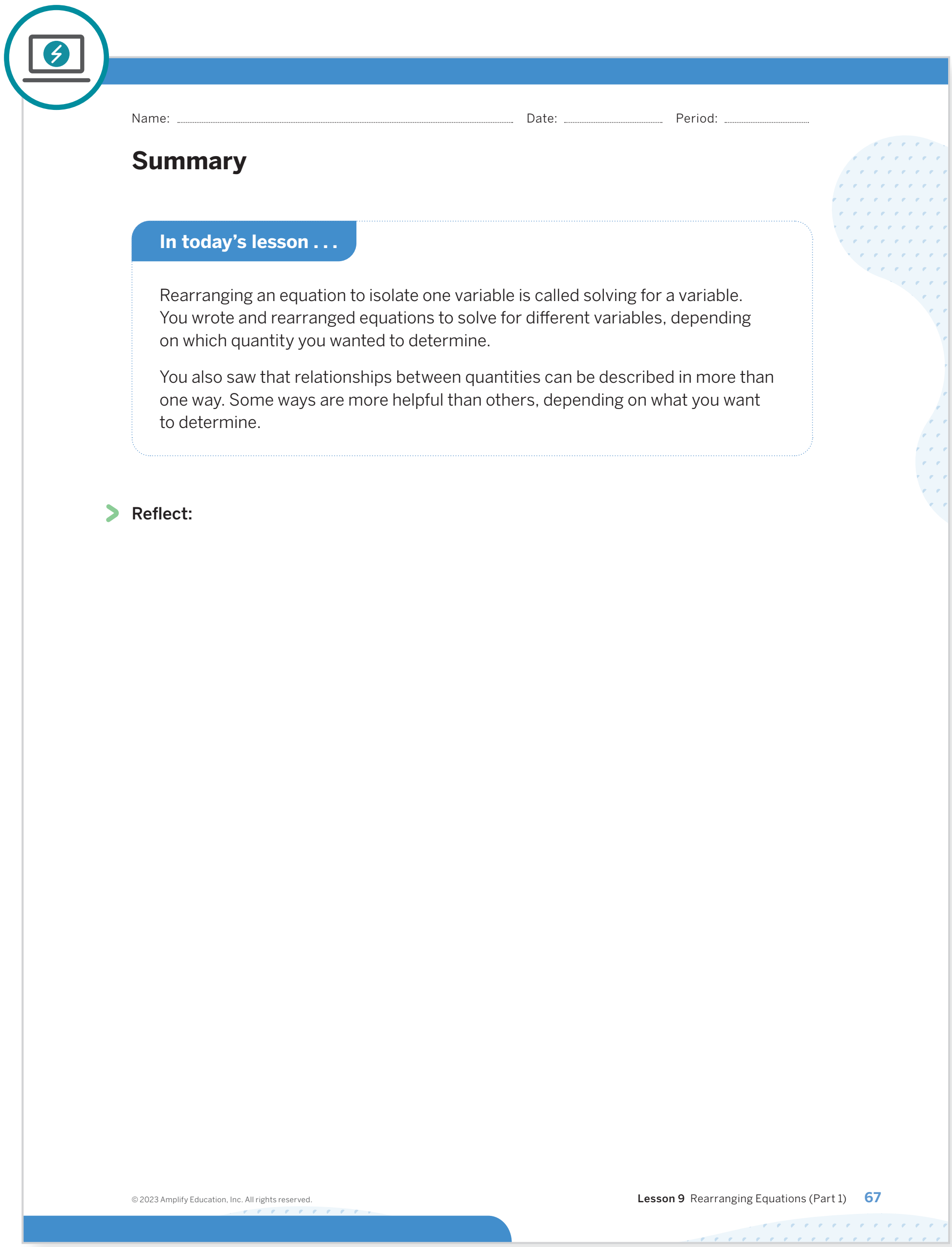
During the Connect, as students respond to the Ask questions, press for details in their reasoning. For example, if a student says, "My equation represents the number of gallons because it begins with $g =$, ask:

- "How do you know that the right side of the equation gives the number of gallons?"
- "Where do you see the 2 gallons of gas in your equation? Where do you see the constant rate of 8 gallons per minute?"

Whole Class | 5 min

Summary

Review and synthesize the value of rearranging equations to solve for a variable.



Name: Date: Period:

Summary

In today's lesson . . .

Rearranging an equation to isolate one variable is called solving for a variable. You wrote and rearranged equations to solve for different variables, depending on which quantity you wanted to determine.

You also saw that relationships between quantities can be described in more than one way. Some ways are more helpful than others, depending on what you want to determine.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Rearranging Equations (Part 1) 67

Synthesize

Display the four equations:

$c = 10m$

$c \cdot p = 1700$

$2 + 8m = g$

$18 - 0.05t = v$

Have students share the steps they would take to solve each equation for a different variable.

Ask, "How would you describe each of the following to a student who was absent today:"

- "What does it mean to 'solve for a variable'?"
- "Why should you solve for a variable?"
- "How do you solve for a variable?"

Highlight that rearranging an equation to isolate one variable is called solving for a variable. This can be useful to determine a certain quantity that you are interested in.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

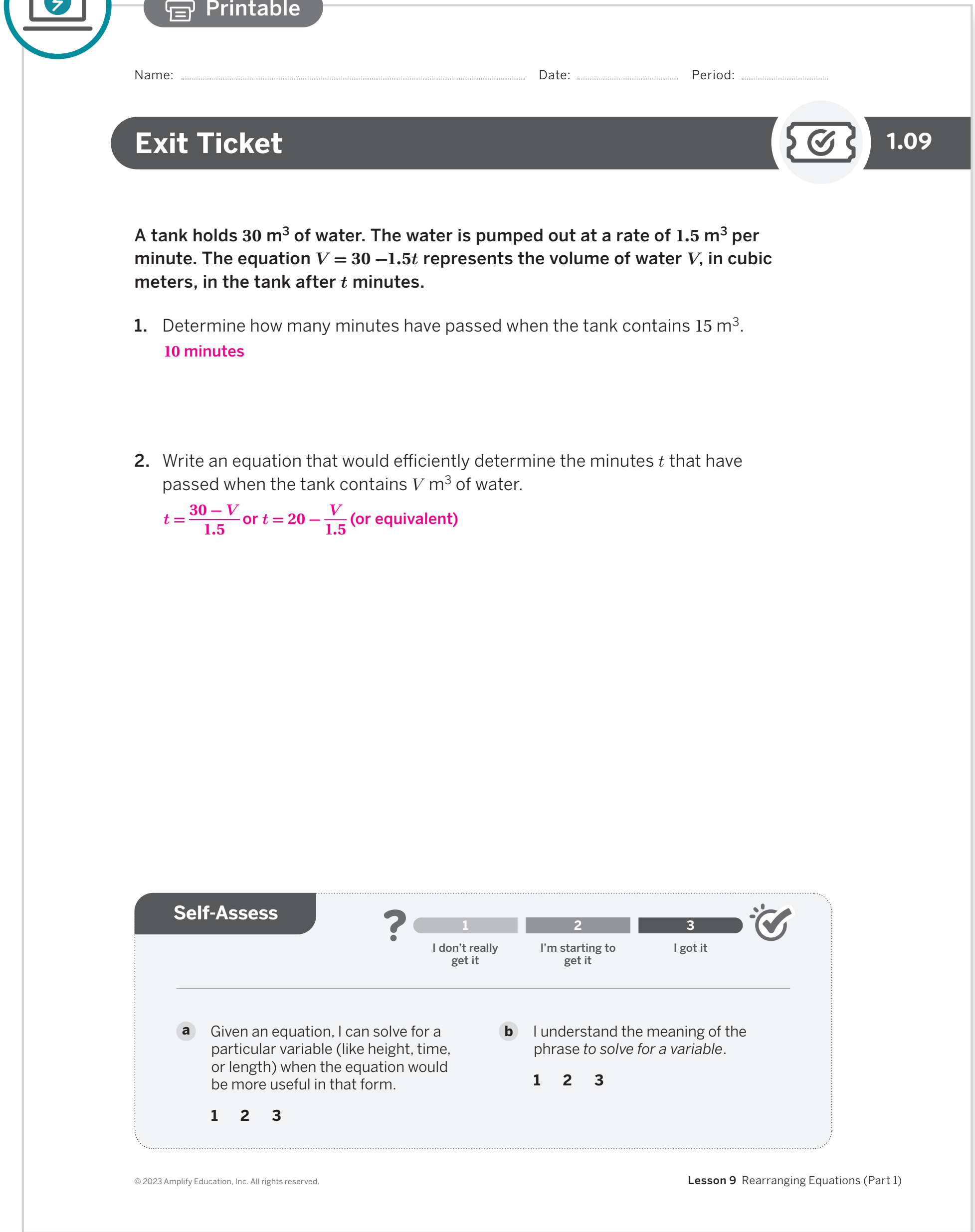
- "What strategies did you find helpful for rearranging the equations? How were they helpful?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of solving for a variable by rearranging a given equation.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.09

A tank holds 30 m^3 of water. The water is pumped out at a rate of 1.5 m^3 per minute. The equation $V = 30 - 1.5t$ represents the volume of water V, in cubic meters, in the tank after t minutes.

1. Determine how many minutes have passed when the tank contains 15 m^3.

   10 minutes

2. Write an equation that would efficiently determine the minutes t that have passed when the tank contains V m^3 of water.

   $t = \frac{30 - V}{1.5}$ **or** $t = 20 - \frac{V}{1.5}$ **(or equivalent)**

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a Given an equation, I can solve for a particular variable (like height, time, or length) when the equation would be more useful in that form.

1 2 3

b I understand the meaning of the phrase *to solve for a variable*.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Rearranging Equations (Part 1)

Success looks like . . .

- **Language Goal:** Comprehending that "to solve for a variable" is to rearrange an equation to isolate a variable of interest. **(Writing)**
- **Goal:** Rearranging two-variable equations in slope-intercept form to highlight a particular quantity.
  - » Rearranging the equation to isolate the variable t in Problem 2.

Suggested next steps

If students incorrectly determine the solution in Problem 1, consider:

- Reviewing Lesson 5.

If students write an incorrect equation in Problem 2, consider:

- Reviewing generalizing and rearranging strategies from Activity 2.
- Assigning Practice Problem 1.
- Asking, "Describe your steps for Problems 1. How could you generalize that process for Problem 2? How could you solve the given equation for t?"

Professional Learning

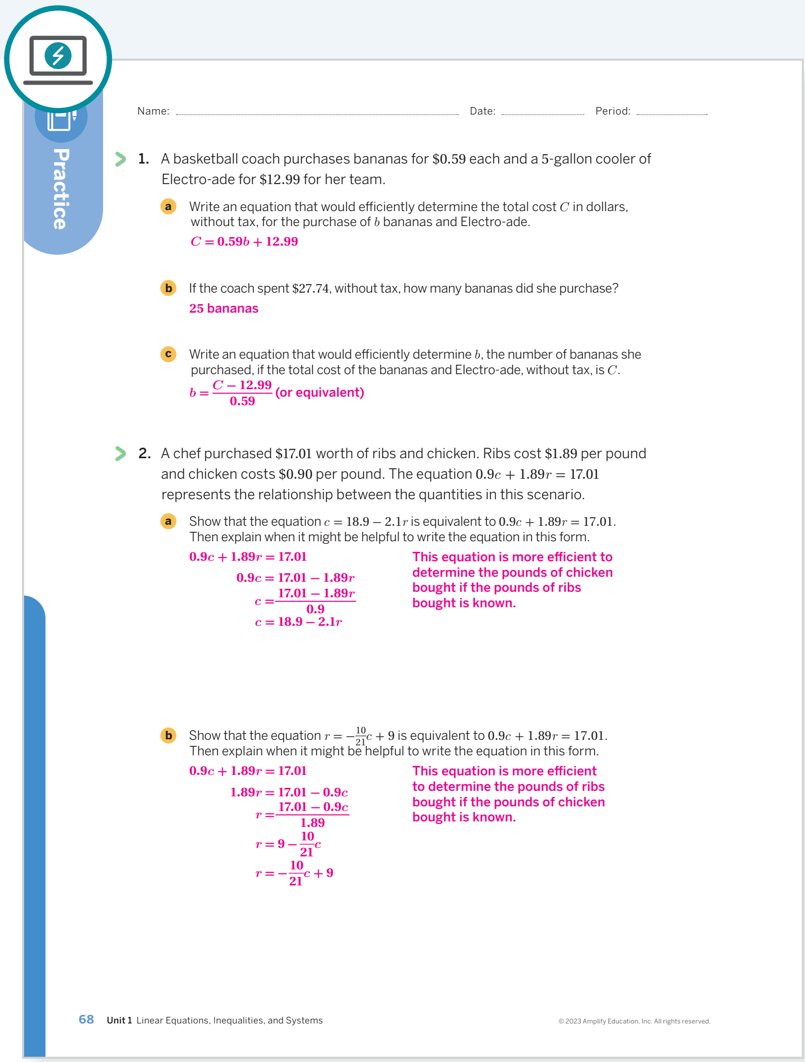
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The instructional goal for this lesson was comprehending that to solve for a variable is to rearrange an equation to isolate a variable of interest. How well did students accomplish this? What did you specifically do to help students accomplish it?
- During the discussion about isolating the variable in Activity 2, how did you encourage each student to share their understandings? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. A basketball coach purchases bananas for $\$0.59$ each and a 5-gallon cooler of Electro-ade for $\$12.99$ for her team.
   - a Write an equation that would efficiently determine the total cost C in dollars, without tax, for the purchase of b bananas and Electro-ade.
     $C = 0.59b + 12.99$
   - b If the coach spent $\$27.74$, without tax, how many bananas did she purchase?
     25 bananas
   - c Write an equation that would efficiently determine b, the number of bananas she purchased, if the total cost of the bananas and Electro-ade, without tax, is C.
     $b = \frac{C - 12.99}{0.59}$ (or equivalent)

2. A chef purchased $\$17.01$ worth of ribs and chicken. Ribs cost $\$1.89$ per pound and chicken costs $\$0.90$ per pound. The equation $0.9c + 1.89r = 17.01$ represents the relationship between the quantities in this scenario.
   - a Show that the equation $c = 18.9 - 2.1r$ is equivalent to $0.9c + 1.89r = 17.01$. Then explain when it might be helpful to write the equation in this form.

$$\begin{aligned} 0.9c + 1.89r &= 17.01 \\ 0.9c &= 17.01 - 1.89r \\ c &= \frac{17.01 - 1.89r}{0.9} \\ c &= 18.9 - 2.1r \end{aligned}$$

This equation is more efficient to determine the pounds of chicken bought if the pounds of ribs bought is known.

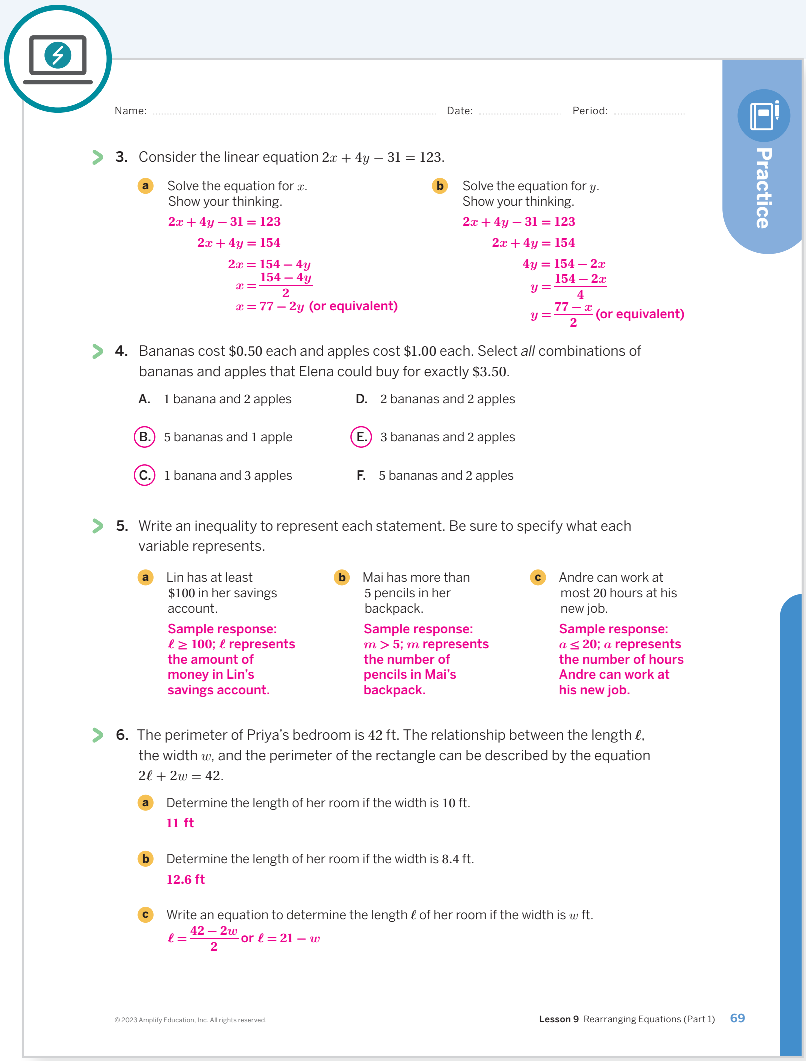
   - b Show that the equation $r = -\frac{10}{21}c + 9$ is equivalent to $0.9c + 1.89r = 17.01$. Then explain when it might be helpful to write the equation in this form.

$$\begin{aligned} 0.9c + 1.89r &= 17.01 \\ 1.89r &= 17.01 - 0.9c \\ r &= \frac{17.01 - 0.9c}{1.89} \\ r &= 9 - \frac{10}{21}c \\ r &= -\frac{10}{21}c + 9 \end{aligned}$$

This equation is more efficient to determine the pounds of ribs bought if the pounds of chicken bought is known.

68 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: Date: Period:

3. Consider the linear equation $2x + 4y - 31 = 123$.
   - a Solve the equation for x. Show your thinking.

$$\begin{aligned} 2x + 4y - 31 &= 123 \\ 2x + 4y &= 154 \\ 2x &= 154 - 4y \\ x &= \frac{154 - 4y}{2} \\ x &= 77 - 2y \text{ (or equivalent)} \end{aligned}$$

   - b Solve the equation for y. Show your thinking.

$$\begin{aligned} 2x + 4y - 31 &= 123 \\ 2x + 4y &= 154 \\ 4y &= 154 - 2x \\ y &= \frac{154 - 2x}{4} \\ y &= \frac{77 - x}{2} \text{ (or equivalent)} \end{aligned}$$

4. Bananas cost $\$0.50$ each and apples cost $\$1.00$ each. Select *all* combinations of bananas and apples that Elena could buy for exactly $\$3.50$.

A. 1 banana and 2 apples
D. 2 bananas and 2 apples
[✓ B.] 5 bananas and 1 apple
[✓ E.] 3 bananas and 2 apples
[✓ C.] 1 banana and 3 apples
F. 5 bananas and 2 apples

5. Write an inequality to represent each statement. Be sure to specify what each variable represents.
   - a Lin has at least $\$100$ in her savings account.
     Sample response: $\ell \geq 100$; ℓ represents the amount of money in Lin's savings account.
   - b Mai has more than 5 pencils in her backpack.
     Sample response: $m > 5$; m represents the number of pencils in Mai's backpack.
   - c Andre can work at most 20 hours at his new job.
     Sample response: $a \leq 20$; a represents the number of hours Andre can work at his new job.

6. The perimeter of Priya's bedroom is 42 ft. The relationship between the length ℓ, the width w, and the perimeter of the rectangle can be described by the equation $2\ell + 2w = 42$.
   - a Determine the length of her room if the width is 10 ft.
     11 ft
   - b Determine the length of her room if the width is 8.4 ft.
     12.6 ft
   - c Write an equation to determine the length ℓ of her room if the width is w ft.
     $\ell = \frac{42 - 2w}{2}$ or $\ell = 21 - w$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Rearranging Equations (Part 1) 69

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 4	2
	5	Unit 1 Lesson 5	2
Formative	6	Unit 1 Lesson 10	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 10**

Rearranging Equations (Part 2)

Let's rearrange equations to solve for one of its variables.

Focus

Goals

1. Write equations in two or more variables and solve for a particular variable.
2. Solve for a variable by performing acceptable operations, including when the values of other quantities in a multi-variable equation are not known.

Rigor

- Students develop **procedural skills** in solving equations for a variable of interest.
- Students strengthen their **fluency** in writing two-variable equations to model a scenario in standard form.

Coherence

Today

Students write standard form equations and solve for a particular variable. They practice solving for a variable first, before performing any calculations or substituting known values. Students observing this process allows them to determine information more efficiently. Students model different budgeting scenarios and interpret their equations in the given context.

Previously

In Lesson 9, students used repeated reasoning to write and rearrange equations to solve for a variable given the values of the other variables.

Coming Soon

In Lesson 11, students consider how standard form linear equations relate to features on the graphs and equations of different forms.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	20 min	15 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Pairs	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF (as needed)
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps*
- scientific calculators
- spreadsheet technology

Math Language Development

Review words

- *equivalent equations*
- *slope-intercept form*
- *standard form*

Amps | **Featured Activity**

Activity 2
What Would You Do?

Students input their own grocery choices to make the problem interactive and personal to their budgeting plans.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may become frustrated or lost when they attempt to model the different scenarios in Activities 1 and 2. Lead a class discussion about the ways you motivate yourself and have students brainstorm ways to motivate themselves. Have students set one small motivating goal that they can work toward throughout the lesson.

Modifications to Pacing

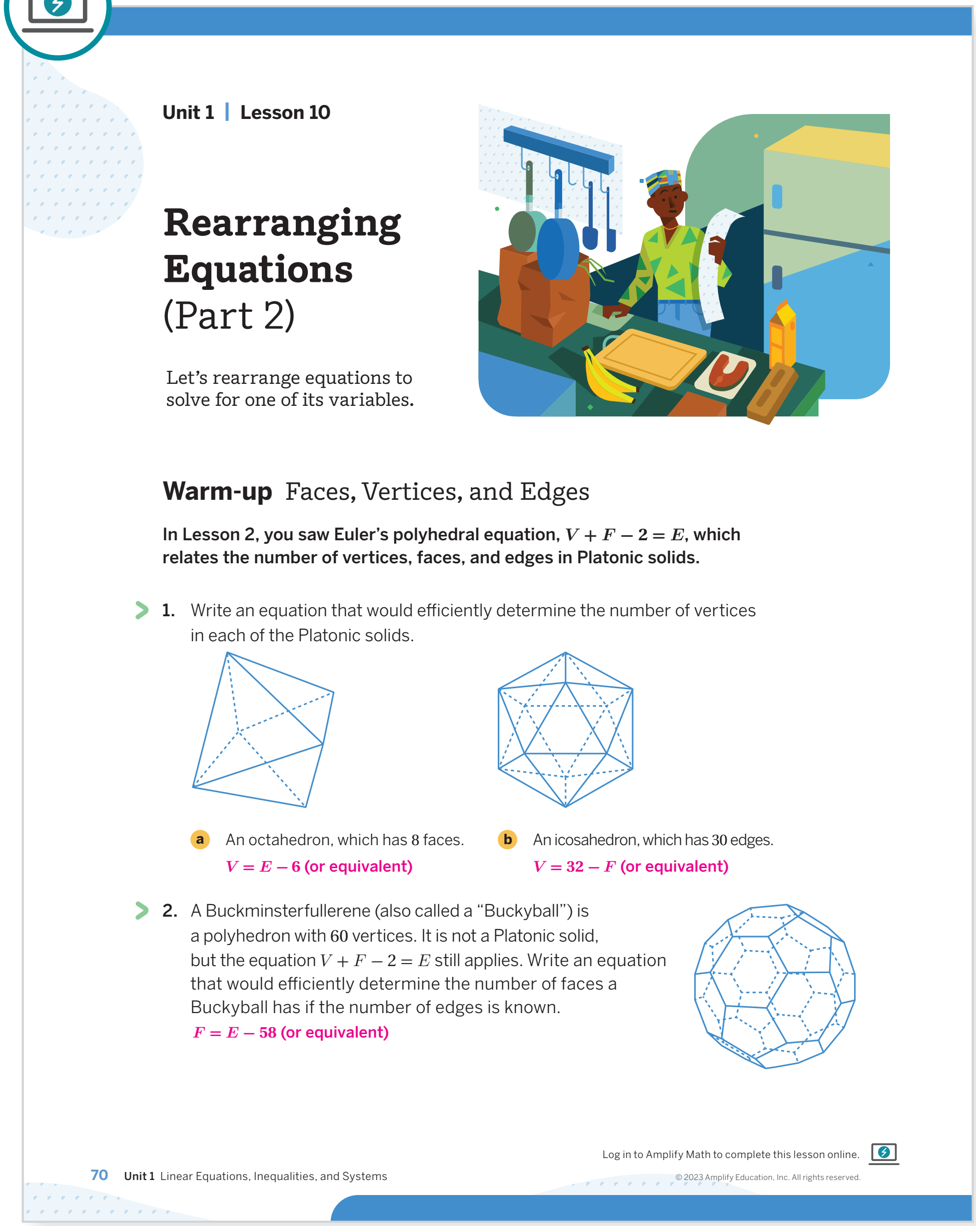
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 2 may be omitted.
- In **Activity 1**, Problem 4 may be omitted.
- Optional digital **Activity 3** may be omitted.

Independent | 5 min

Warm-up Faces, Vertices, and Edges

Students rearrange Euler's equation to prepare them to solve for a variable.



Unit 1 | **Lesson 10**

Rearranging Equations (Part 2)

Let's rearrange equations to solve for one of its variables.

Warm-up Faces, Vertices, and Edges

In Lesson 2, you saw Euler's polyhedral equation, $V + F - 2 = E$, which relates the number of vertices, faces, and edges in Platonic solids.

1. Write an equation that would efficiently determine the number of vertices in each of the Platonic solids.

   a. An octahedron, which has 8 faces.
   $V = E - 6$ (or equivalent)

   b. An icosahedron, which has 30 edges.
   $V = 32 - F$ (or equivalent)

2. A Buckminsterfullerene (also called a "Buckyball") is a polyhedron with 60 vertices. It is not a Platonic solid, but the equation $V + F - 2 = E$ still applies. Write an equation that would efficiently determine the number of faces a Buckyball has if the number of edges is known.
   $F = E - 58$ (or equivalent)

Log in to Amplify Math to complete this lesson online.

70 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Activate prior knowledge from Lesson 2. Provide a minute of think-time, then have students complete the problems independently.

2 Monitor

Help students get started by having them circle the variable they want to solve for.

Look for points of confusion:

- **Having difficulty manipulating an equation with three variables.** Have students identify the variable of interest. Ask, "What information is known? How can you use this information to reduce the number of variables?"

Look for productive strategies:

- Substituting the given value first.
- Isolating the variable of interest first.

3 Connect

Have students share their strategies or processes for determining their equations. Select and sequence students using the productive strategies.

Display student steps taken to rearrange the equations. Emphasizing that each step constitutes an acceptable move, keeping the equation true.

Highlight that they can either substitute known values into the given equation first or rearrange the equation first.

Ask:

- "When might it be more helpful to substitute a value first? Why?"
- "When might it be more helpful to rearrange the equation before substituting? Why?"

Math Language Development

MLR8: Discussion Support

While students work, display the Anchor Chart PDF, *Sentence Stems, Explaining My Steps*. Encourage students to think about how they will explain how they wrote the equations in Problems 1 and 2, ahead of the Connect discussion. Suggest they record some notes by each equation so that they will be ready to share during the discussion.

Power-up

To power up students' ability to solve an equation with more than one variable, have students complete:

A puzzle box says the rectangular puzzle covers 36 in^2 when assembled. The area can be modeled using the formula $\ell \cdot w = 36$.

a. What is the length of the puzzle if the width is 9 in.? 4 in.

b. What is the width of the puzzle if the length is 12 in.? 3 in.

c. Write an equation to determine the width w for any length ℓ. $w = \frac{36}{\ell}$

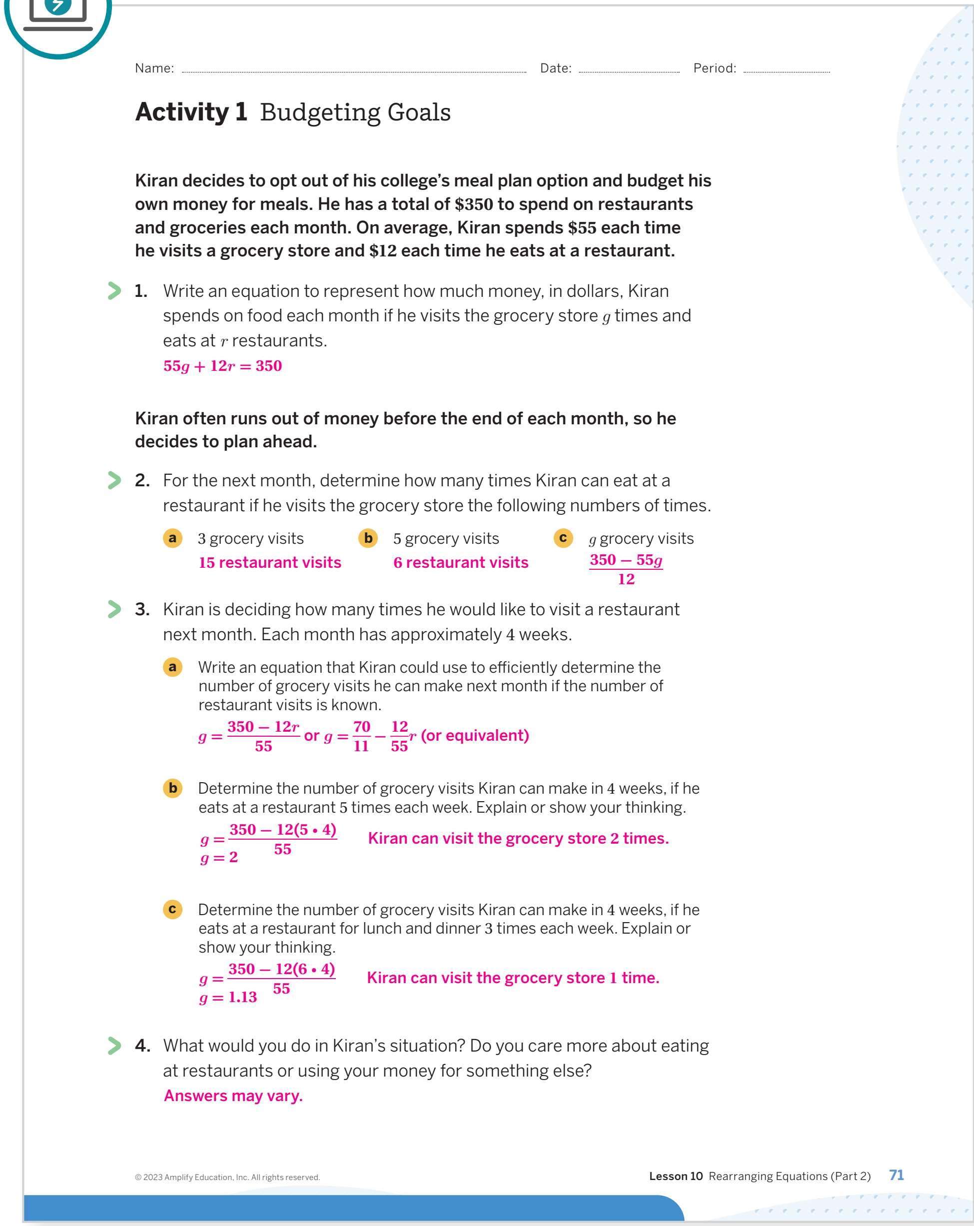
Use: Before the Warm-up

Informed by: Performance on Lesson 9, Practice Problem 6

Pairs | 20 min

Activity 1 Budgeting Goals

Students write and solve an equation in two variables in a given context to practice solving for a variable.



Name: ______ Date: ______ Period: ______

Activity 1 Budgeting Goals

Kiran decides to opt out of his college's meal plan option and budget his own money for meals. He has a total of $350 to spend on restaurants and groceries each month. On average, Kiran spends $55 each time he visits a grocery store and $12 each time he eats at a restaurant.

1. Write an equation to represent how much money, in dollars, Kiran spends on food each month if he visits the grocery store g times and eats at r restaurants.
   $55g + 12r = 350$

Kiran often runs out of money before the end of each month, so he decides to plan ahead.

2. For the next month, determine how many times Kiran can eat at a restaurant if he visits the grocery store the following numbers of times.
   - a 3 grocery visits
     15 restaurant visits
   - b 5 grocery visits
     6 restaurant visits
   - c g grocery visits
     $\frac{350 - 55g}{12}$
3. Kiran is deciding how many times he would like to visit a restaurant next month. Each month has approximately 4 weeks.
   - a Write an equation that Kiran could use to efficiently determine the number of grocery visits he can make next month if the number of restaurant visits is known.
     $g = \frac{350 - 12r}{55}$ **or** $g = \frac{70}{11} - \frac{12}{55}r$ **(or equivalent)**
   - b Determine the number of grocery visits Kiran can make in 4 weeks, if he eats at a restaurant 5 times each week. Explain or show your thinking.
     $g = \frac{350 - 12(5 \cdot 4)}{55}$
     $g = 2$
     Kiran can visit the grocery store 2 times.
   - c Determine the number of grocery visits Kiran can make in 4 weeks, if he eats at a restaurant for lunch and dinner 3 times each week. Explain or show your thinking.
     $g = \frac{350 - 12(6 \cdot 4)}{55}$
     $g = 1.13$
     Kiran can visit the grocery store 1 time.
4. What would you do in Kiran's situation? Do you care more about eating at restaurants or using your money for something else?
   Answers may vary.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Rearranging Equations (Part 2) 71

1 Launch

Arrange students in pairs. Ask, "Do you ever make decisions about where to eat based on how much items cost or how much money you have?"

Discuss and solve Problem 1 together. Then have student-pairs discuss each problem before completing independently, comparing solutions and equations upon completion.

2 Monitor

Help students get started by prompting them to use a graphic organizer to list all known and unknown information from the prompt.

Look for points of confusion:

- **Having difficulty writing the equation in Problem 3a.** Prompt students to consider their original equation from Problem 1 and circle the variable that represents the number of grocery visits.

Look for productive strategies:

- Rounding the solutions to make sense in the given context, e.g. number of visits must be a whole number.
- Rearranging the equation first to solve for the desired unknown variable.

3 Connect

Have pairs of students share their strategies for determining the equations for Problems 2c and 3a.

Highlight the steps for rearranging the original equation to solve for each variable. Discuss the equivalent simplified equations and what they mean in context.

Ask, "What helps you decide which variable to solve for in a given word problem?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide a table for students to use which will help them see the patterns for writing the equations. Here is a sample table for Problem 2a.

g	Substitute g into Problem 1's equation.	Solve for r.
3	$55(3) + 12r = 350$	$r = \frac{350 - 55(3)}{12}$

Extension: *Math Enrichment*

Ask students to determine all of the possible combinations of grocery store and restaurant visits. For (g, r): $(0, 29)$, $(1, 24)$, $(2, 20)$, $(3, 15)$, $(4, 10)$, $(5, 6)$, and $(6, 1)$.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

- **Read 1:** Students should understand that Kiran has a total budget for food and already spends a certain amount on each category.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as Kiran spends $12 each time at a restaurant.
- **Read 3:** Ask students to think about how an equation in two variables can represent this information.

English Learners

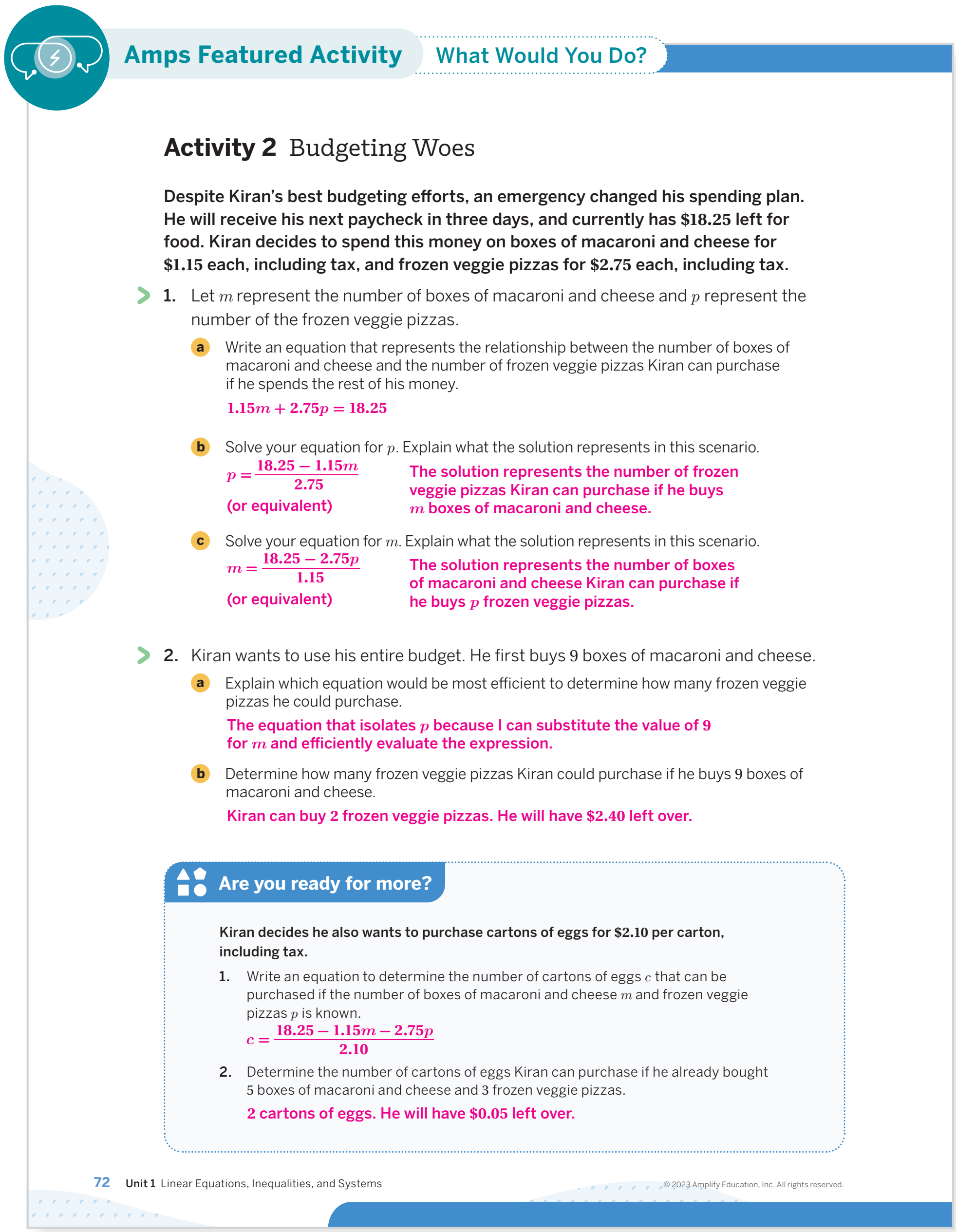
Have students highlight key phrases, such as $55 *each visit* (grocery store) and $12 *each time* (restaurant).

Pairs | 15 min

Activity 2 Budgeting Woes

Students write and rearrange equations in two variables modeling a given context to practice solving for a variable without first calculating numerically.

Amps Featured Activity What Would You Do?



Activity 2 Budgeting Woes

Despite Kiran's best budgeting efforts, an emergency changed his spending plan. He will receive his next paycheck in three days, and currently has $18.25 left for food. Kiran decides to spend this money on boxes of macaroni and cheese for $1.15 each, including tax, and frozen veggie pizzas for $2.75 each, including tax.

1. Let m represent the number of boxes of macaroni and cheese and p represent the number of the frozen veggie pizzas.
   - a. Write an equation that represents the relationship between the number of boxes of macaroni and cheese and the number of frozen veggie pizzas Kiran can purchase if he spends the rest of his money.

     $1.15m + 2.75p = 18.25$
   - b. Solve your equation for p. Explain what the solution represents in this scenario.

     $p = \frac{18.25 - 1.15m}{2.75}$ (or equivalent)

     The solution represents the number of frozen veggie pizzas Kiran can purchase if he buys m boxes of macaroni and cheese.
   - c. Solve your equation for m. Explain what the solution represents in this scenario.

     $m = \frac{18.25 - 2.75p}{1.15}$ (or equivalent)

     The solution represents the number of boxes of macaroni and cheese Kiran can purchase if he buys p frozen veggie pizzas.

2. Kiran wants to use his entire budget. He first buys 9 boxes of macaroni and cheese.
   - a. Explain which equation would be most efficient to determine how many frozen veggie pizzas he could purchase.

     The equation that isolates p because I can substitute the value of 9 for m and efficiently evaluate the expression.
   - b. Determine how many frozen veggie pizzas Kiran could purchase if he buys 9 boxes of macaroni and cheese.

     Kiran can buy 2 frozen veggie pizzas. He will have $2.40 left over.

Are you ready for more?

Kiran decides he also wants to purchase cartons of eggs for $2.10 per carton, including tax.

1. Write an equation to determine the number of cartons of eggs c that can be purchased if the number of boxes of macaroni and cheese m and frozen veggie pizzas p is known.

   $c = \frac{18.25 - 1.15m - 2.75p}{2.10}$
2. Determine the number of cartons of eggs Kiran can purchase if he already bought 5 boxes of macaroni and cheese and 3 frozen veggie pizzas.

   2 cartons of eggs. He will have $0.05 left over.

72 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the prompt aloud, continue the discussion on budgeting. Then, have student-pairs discuss each problem before completing independently, and comparing solutions and equations upon completion. Provide access to scientific calculators.

2 Monitor

Help students get started by asking, "How would you calculate the cost for 1 of each? 2 of each? How could you model that using an equation?"

Look for points of confusion:

- **Having difficulty explaining each solution in context in Problem 1b and 1c.** Prompt students to evaluate the equation at different values, explain the results in context, then generalize.

Look for productive strategies:

- Annotating or color-coding the prompt to help represent the scenario symbolically.
- Using the context to determine the best equation for each scenario.

3 Connect

Have pairs of students share their thinking for writing equations for Problem 1. Select students who wrote different equivalent forms of the equation.

Display student equations.

Ask:

- "Is each equation equivalent?"
- "How much of each item would you suggest Kiran buy? Why?"

Highlight that solving for p or m makes it possible to efficiently determine the number of veggie pizzas or boxes of macaroni to buy while staying within the budget. Discuss what information an equation in standard form provides.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can input their own grocery choices. This will allow them to make the problem more personal to their own budgeting plans.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and have students color code the information in the narrative and the variables described in Problem 1. For example, have them use one color to color code 1.15 and m. Have them use another color to color code 2.75 and p.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

- **Read 1:** Students should understand that Kiran needs to determine how best to spend his paycheck, given several constraints.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as Kiran currently has $18.25 left to spend.
- **Read 3:** Ask students to think about how an equation in two variables can represent this information.

English Learners

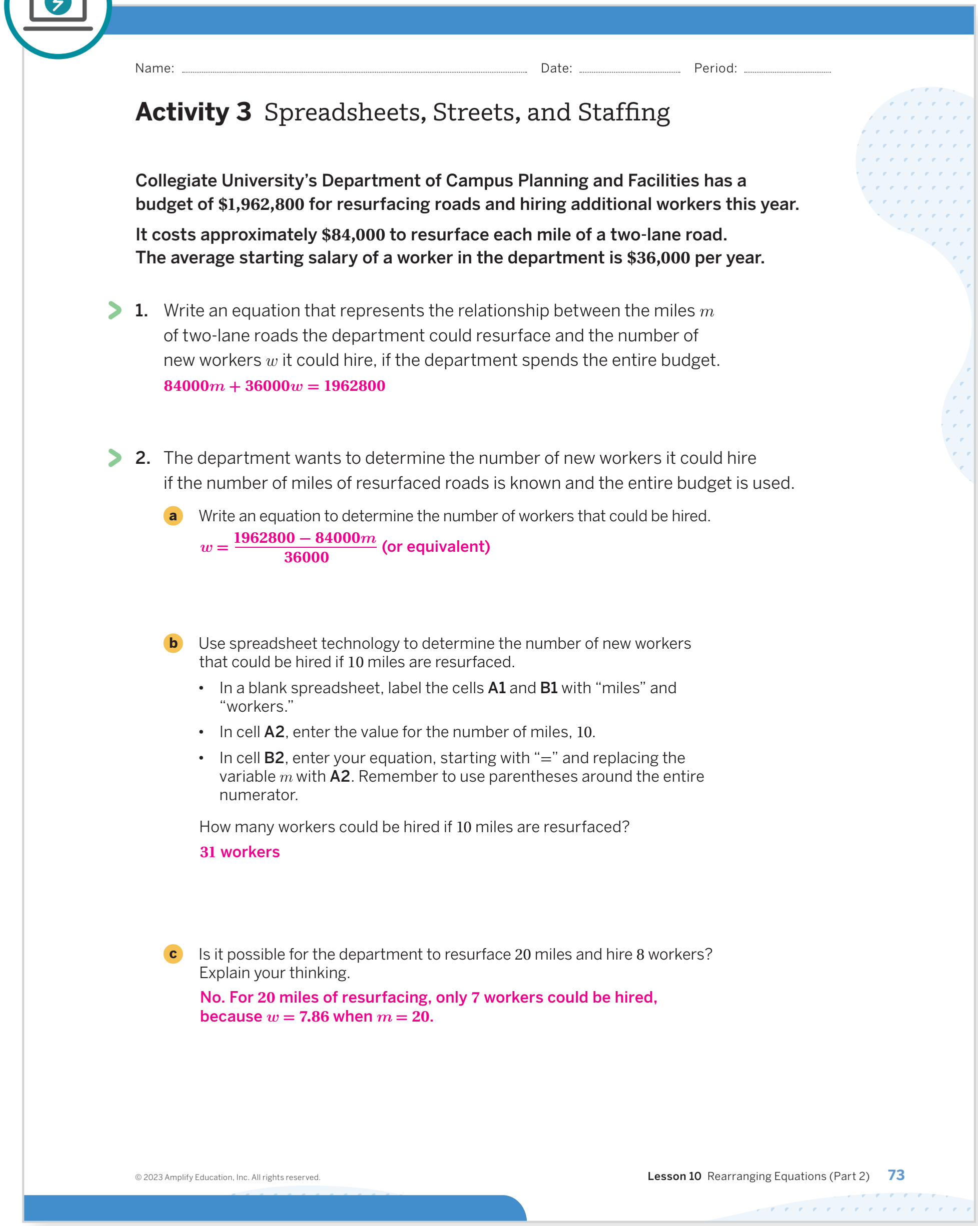
Have students highlight key phrases, such as $1.15 each (macaroni and cheese) and $2.75 each (frozen veggie pizza).

Optional

Pairs | 20 min

Activity 3 Spreadsheets, Streets, and Staffing

Students use spreadsheet technology to see how to efficiently calculate the value of one quantity when given the value of the other.



Name: ______ Date: ______ Period: ______

Activity 3 Spreadsheets, Streets, and Staffing

Collegiate University's Department of Campus Planning and Facilities has a budget of $1,962,800 for resurfacing roads and hiring additional workers this year.

It costs approximately $84,000 to resurface each mile of a two-lane road. The average starting salary of a worker in the department is $36,000 per year.

1. Write an equation that represents the relationship between the miles m of two-lane roads the department could resurface and the number of new workers w it could hire, if the department spends the entire budget.
   $84000m + 36000w = 1962800$
2. The department wants to determine the number of new workers it could hire if the number of miles of resurfaced roads is known and the entire budget is used.
   - a Write an equation to determine the number of workers that could be hired.
     $w = \frac{1962800 - 84000m}{36000}$ (or equivalent)
   - b Use spreadsheet technology to determine the number of new workers that could be hired if 10 miles are resurfaced.
     - In a blank spreadsheet, label the cells **A1** and **B1** with "miles" and "workers."
     - In cell **A2**, enter the value for the number of miles, 10.
     - In cell **B2**, enter your equation, starting with "=" and replacing the variable m with **A2**. Remember to use parentheses around the entire numerator.

     How many workers could be hired if 10 miles are resurfaced?
     31 workers
   - c Is it possible for the department to resurface 20 miles and hire 8 workers? Explain your thinking.
     No. For 20 miles of resurfacing, only 7 workers could be hired, because $w = 7.86$ when $m = 20$.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Rearranging Equations (Part 2) 73

1 Launch

Revisit the meaning of the term "budget." Discuss large-scale, organizational budgets. Ask, "What are some expenses that a college might have?" Allow student-pairs to work together on each problem. Display or provide copies of the Activity 3 PDF, as needed.

2 Monitor

Help students get started by listing each variable with what it represents and any associated quantities.

Look for points of confusion:

- **Having difficulty rewriting the equation.** Have students use a two-column graphic organizer to keep track of their work and thinking.
- **Not considering the constraints in Problem 4.** Prompt students to test different values for each equation using spreadsheet technology and explain what the results represent.

Look for productive strategies:

- Using spreadsheet technology to:
  - » Test their equation.
  - » Determine which equation to use for each scenario.
  - » Efficiently calculate and make sense of possible solutions.

Activity 3 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider providing students a copy of the Activity 3 PDF, which contains written directions and visual examples for using spreadsheet technology with this activity.

Extension: *Math Enrichment*

Have students refer back to Activity 2 and use spreadsheet technology to determine the most amount of food Kiran can buy within his budget. 13 boxes of macaroni and cheese and 1 veggie pizza or just 15 boxes of macaroni and cheese.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

- **Read 1:** Students should understand that the university wants to hire workers to help resurface roads, given several budget constraints.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as the budget is $1,962,800.
- **Read 3:** Ask students to think about how an equation in two variables can represent this information.

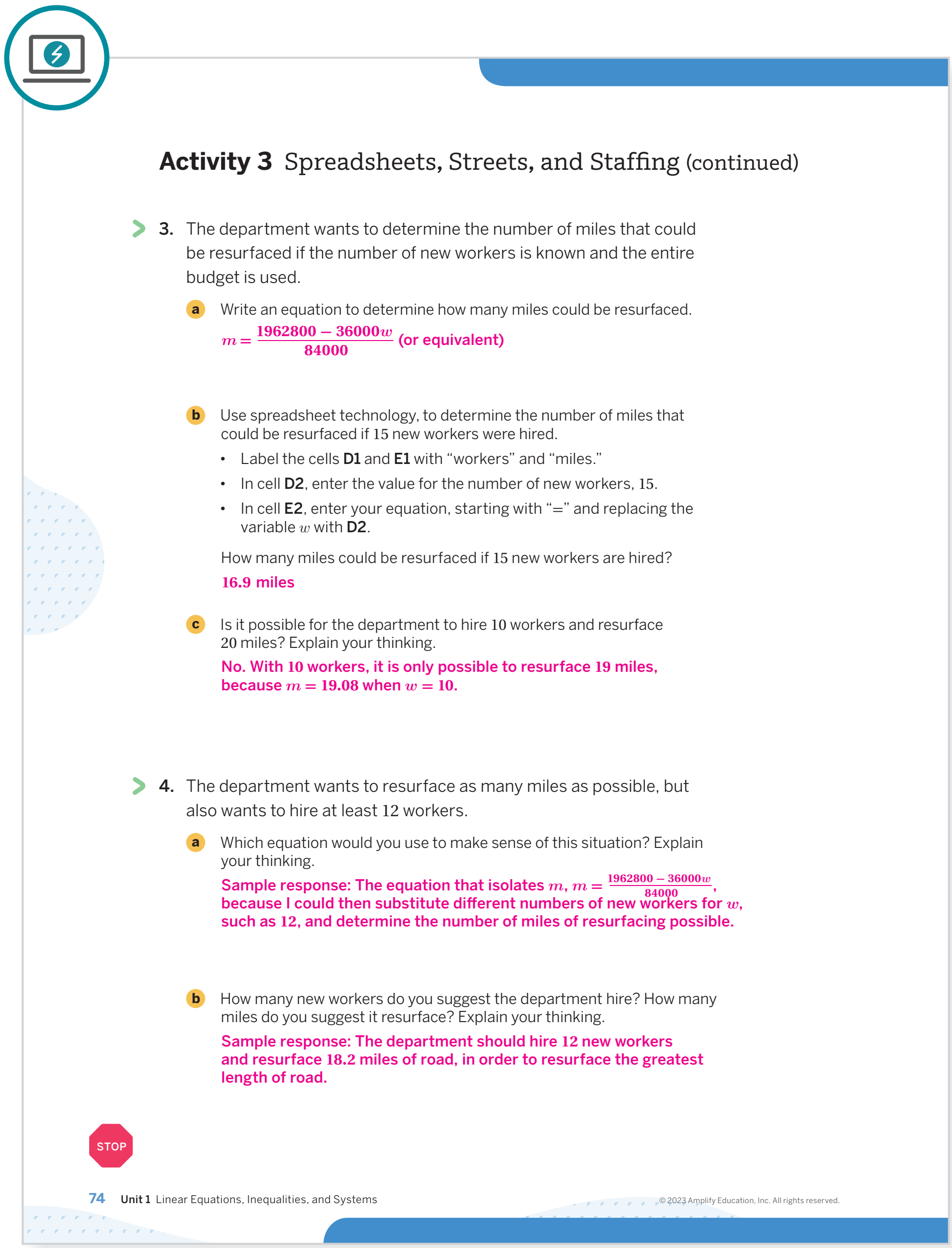
English Learners

Have students highlight key phrases, such as the cost to resurface each *mile* or the average worker's salary *per year*.

Pairs | 20 min

Activity 3 Spreadsheets, Streets, and Staffing (continued)

Students use spreadsheet technology to see how to efficiently calculate the value of one quantity when given the value of the other.



Activity 3 Spreadsheets, Streets, and Staffing (continued)

3. The department wants to determine the number of miles that could be resurfaced if the number of new workers is known and the entire budget is used.

a Write an equation to determine how many miles could be resurfaced.

$m = \frac{1962800 - 36000w}{84000}$ **(or equivalent)**

b Use spreadsheet technology, to determine the number of miles that could be resurfaced if 15 new workers were hired.

- Label the cells **D1** and **E1** with "workers" and "miles."
- In cell **D2**, enter the value for the number of new workers, 15.
- In cell **E2**, enter your equation, starting with "=" and replacing the variable w with **D2**.

How many miles could be resurfaced if 15 new workers are hired?

16.9 miles

c Is it possible for the department to hire 10 workers and resurface 20 miles? Explain your thinking.

No. With 10 workers, it is only possible to resurface 19 miles, because $m = 19.08$ when $w = 10$.

4. The department wants to resurface as many miles as possible, but also wants to hire at least 12 workers.

a Which equation would you use to make sense of this situation? Explain your thinking.

Sample response: The equation that isolates m, $m = \frac{1962800 - 36000w}{84000}$, because I could then substitute different numbers of new workers for w, such as 12, and determine the number of miles of resurfacing possible.

b How many new workers do you suggest the department hire? How many miles do you suggest it resurface? Explain your thinking.

Sample response: The department should hire 12 new workers and resurface 18.2 miles of road, in order to resurface the greatest length of road.

STOP

74 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have pairs of students share how using spreadsheet technology was helpful or challenging, and their solutions for Problem 4b.

Ask:

- "How many people could be hired to resurface 16 miles of road? 3 miles?" 17 people, 47 people
- "How many miles of road could be resurfaced by hiring 2 new workers? No new workers?" 22.5 miles, About 23.37 miles

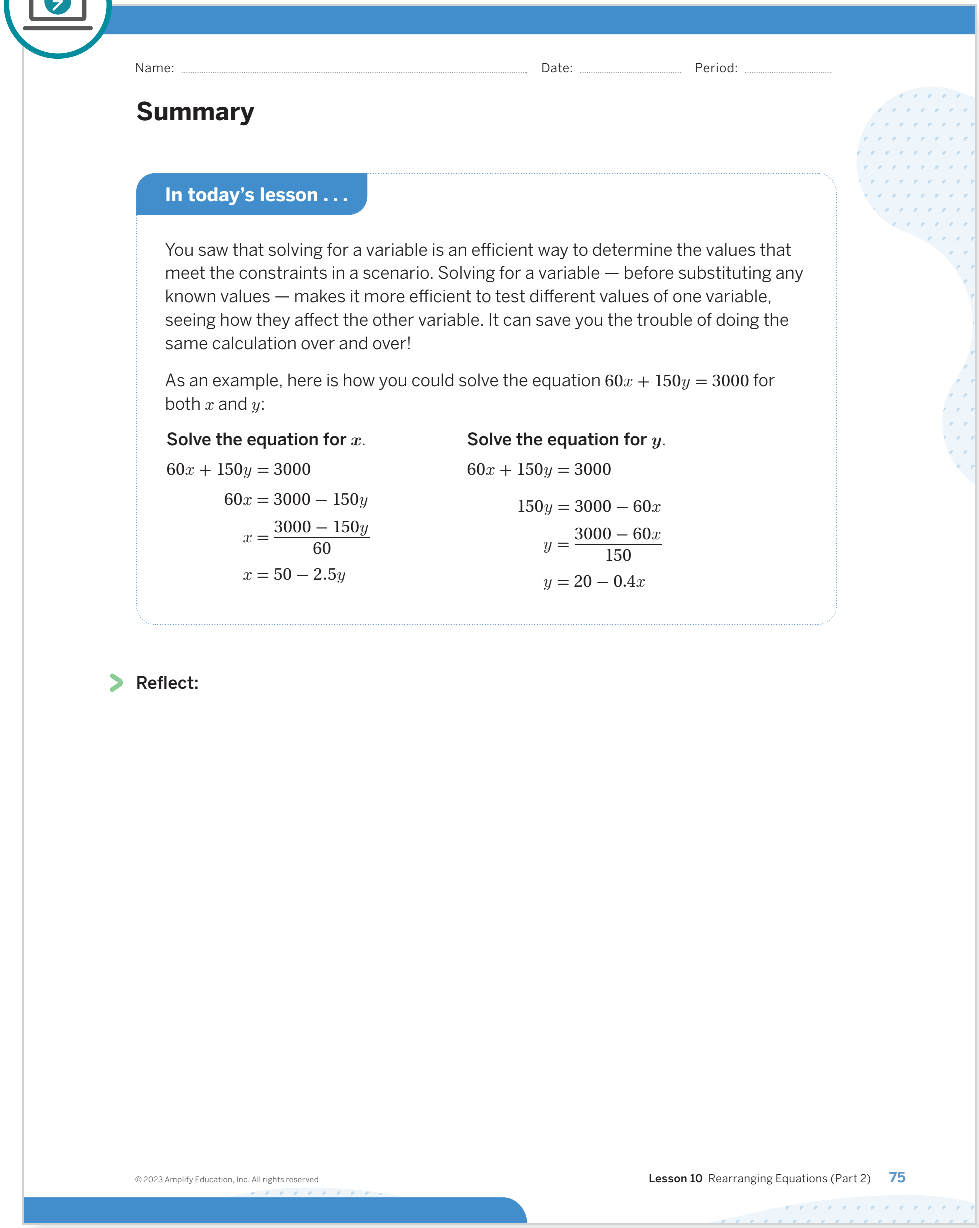
Highlight that solving for w allows efficient determination of the number of workers that could be hired given the miles to be resurfaced, while staying within the budget. Similarly, solving for m allows efficient determination of the miles that could be resurfaced given any number of new hires.

Ask, "How could you use spreadsheet technology to determine the total cost for each combination and what remains in the budget?" In any empty cell type the formula $= 84000m + 36000w$, use different cells to represent and manipulate the variables. To determine how much remains in the budget with each combination, subtract the displayed total cost from 1,962,800.

Pairs | 5 min

Summary

Review and synthesize solving for a variable as an efficient strategy to determine unknown values that meet the constraints in a scenario.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that solving for a variable is an efficient way to determine the values that meet the constraints in a scenario. Solving for a variable — before substituting any known values — makes it more efficient to test different values of one variable, seeing how they affect the other variable. It can save you the trouble of doing the same calculation over and over!

As an example, here is how you could solve the equation $60x + 150y = 3000$ for both x and y:

Solve the equation for x.

$$\begin{aligned} 60x + 150y &= 3000 \\ 60x &= 3000 - 150y \\ x &= \frac{3000 - 150y}{60} \\ x &= 50 - 2.5y \end{aligned}$$

Solve the equation for y.

$$\begin{aligned} 60x + 150y &= 3000 \\ 150y &= 3000 - 60x \\ y &= \frac{3000 - 60x}{150} \\ y &= 20 - 0.4x \end{aligned}$$

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the prompt: "Suppose you are organizing a party and have a budget of b dollars for the appetizers. You plan to order v vegetarian spring rolls at $\$0.75$ each and s shrimp rolls at $\$0.95$ each. The equation $0.75v + 0.95s = b$ represents this constraint." Have half of the class solve for v and the other half of the class solve for s.

Have students share their equations, what each equation represents, and when they might use each equation.

Highlight that solving for a variable is an efficient way to determine the values that meet the constraints in a scenario.

Ask, "Why is solving for a variable a more efficient strategy than substituting known values?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

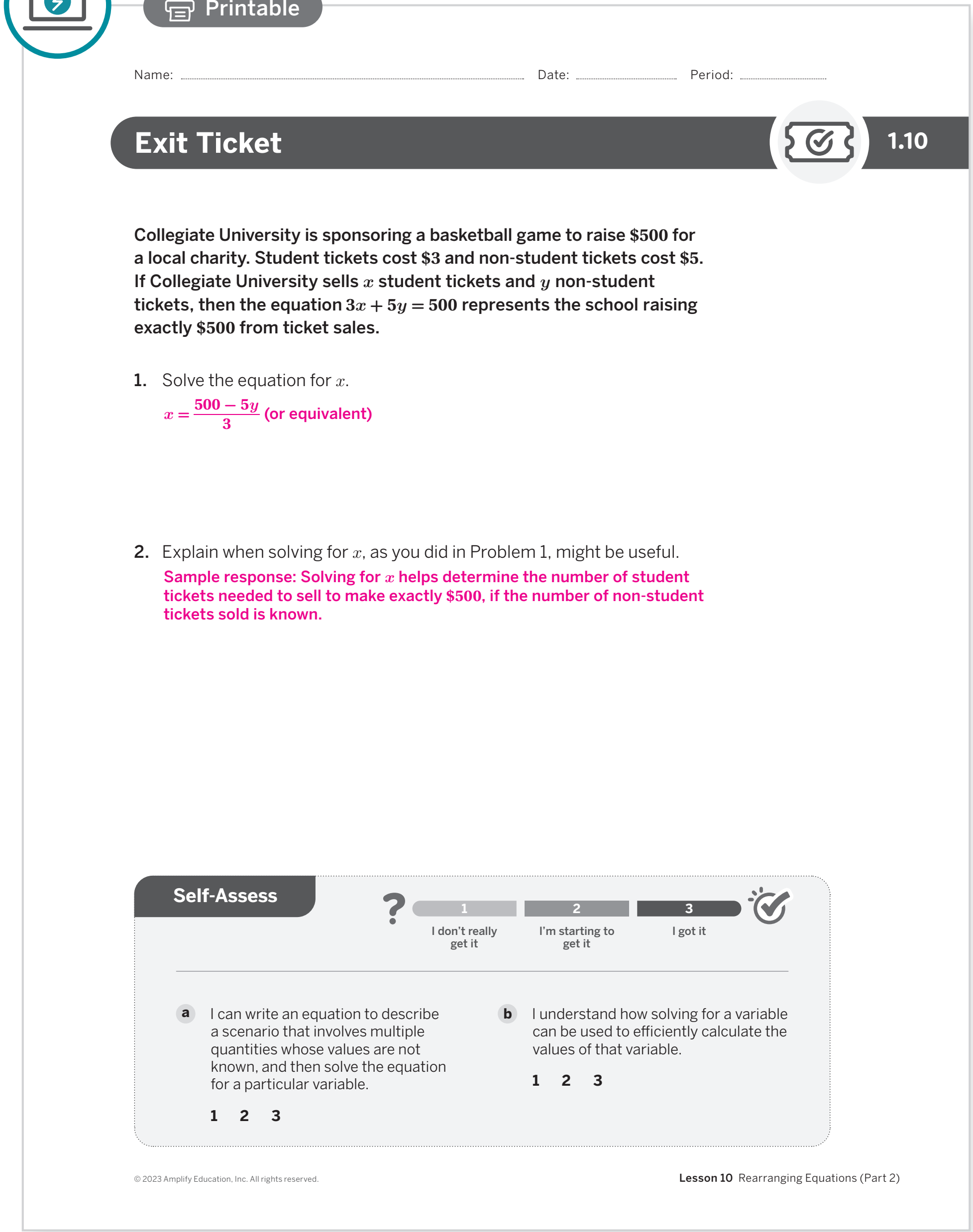
- "What strategies are helpful for solving an equation for a certain variable? How are they helpful?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by solving a two-variable equation for a given variable and explaining how it might be helpful in the context.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 1.10

Collegiate University is sponsoring a basketball game to raise \$500 for a local charity. Student tickets cost \$3 and non-student tickets cost \$5. If Collegiate University sells x student tickets and y non-student tickets, then the equation $3x + 5y = 500$ represents the school raising exactly \$500 from ticket sales.

1. Solve the equation for x.

   $x = \frac{500 - 5y}{3}$ (or equivalent)

2. Explain when solving for x, as you did in Problem 1, might be useful.

   Sample response: Solving for x helps determine the number of student tickets needed to sell to make exactly \$500, if the number of non-student tickets sold is known.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can write an equation to describe a scenario that involves multiple quantities whose values are not known, and then solve the equation for a particular variable.

1 2 3

b. I understand how solving for a variable can be used to efficiently calculate the values of that variable.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Rearranging Equations (Part 2)

Success looks like . . .

- Writing equations in two or more variables and solving for a particular variable.
- Solving for a variable by performing acceptable operations, including when the values of other quantities in a multi-variable equation are not known.
  - » Solving the equation $3x + 5y = 500$ for x in Problem 1.

Suggested next steps

If students incorrectly solve for x in Problem 1, consider:

- Reviewing rearranging strategies from Activity 2.
- Assigning Practice Problem 3.

If students explain incorrectly or incompletely in Problem 2, consider:

- Reviewing Activity 2.
- Assigning Practice Problem 1.

Professional Learning

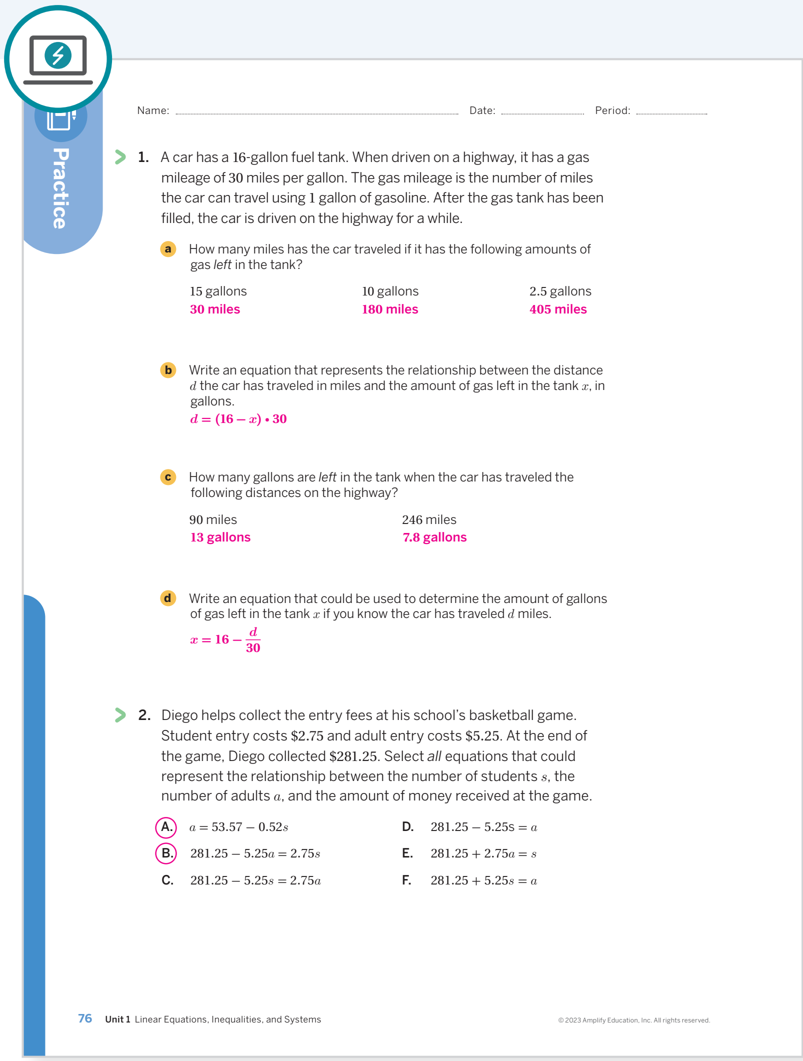
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is?
- Who participated and who didn't participate in Activity 2 today? What trends do you see in participation? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ____ Date: ____ Period: ____

1. A car has a 16-gallon fuel tank. When driven on a highway, it has a gas mileage of 30 miles per gallon. The gas mileage is the number of miles the car can travel using 1 gallon of gasoline. After the gas tank has been filled, the car is driven on the highway for a while.

a How many miles has the car traveled if it has the following amounts of gas *left* in the tank?

15 gallons	10 gallons	2.5 gallons
30 miles	**180 miles**	**405 miles**

b Write an equation that represents the relationship between the distance d the car has traveled in miles and the amount of gas left in the tank x, in gallons.

$d = (16 - x) \cdot 30$

c How many gallons are *left* in the tank when the car has traveled the following distances on the highway?

90 miles	246 miles
13 gallons	**7.8 gallons**

d Write an equation that could be used to determine the amount of gallons of gas left in the tank x if you know the car has traveled d miles.

$x = 16 - \frac{d}{30}$

2. Diego helps collect the entry fees at his school's basketball game. Student entry costs \$2.75 and adult entry costs \$5.25. At the end of the game, Diego collected \$281.25. Select *all* equations that could represent the relationship between the number of students s, the number of adults a, and the amount of money received at the game.

[✓ A.] $a = 53.57 - 0.52s$

[✓ B.] $281.25 - 5.25a = 2.75s$

C. $281.25 - 5.25s = 2.75a$

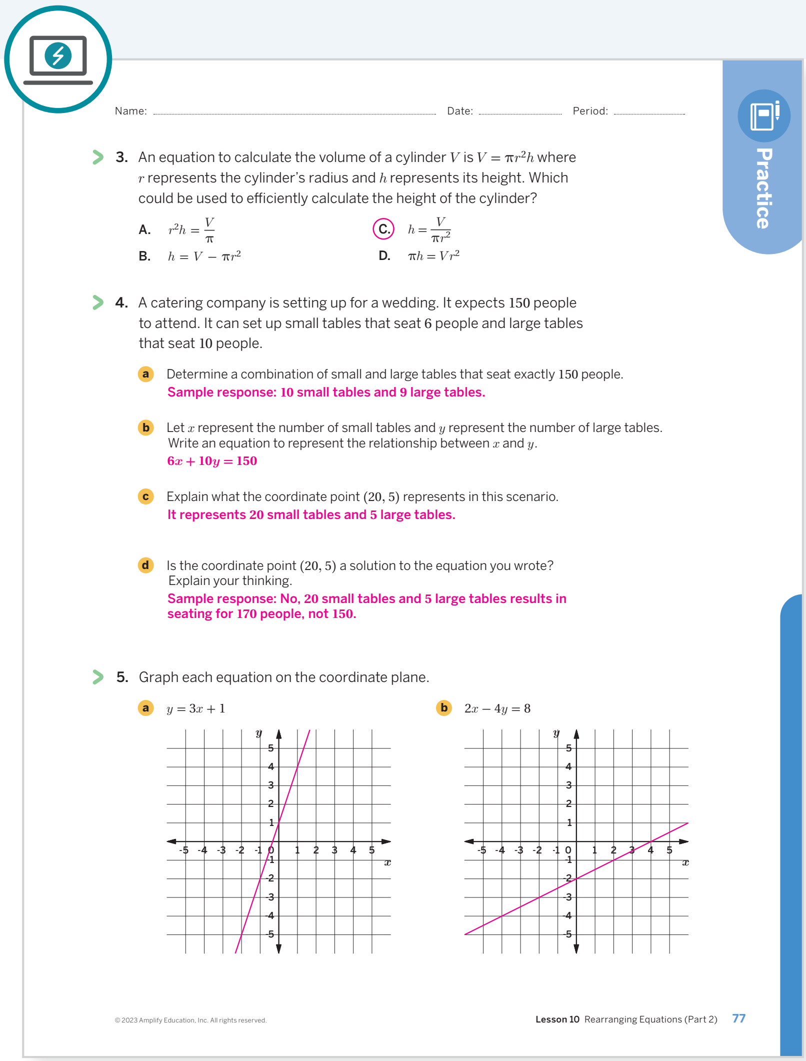
D. $281.25 - 5.25s = a$

E. $281.25 + 2.75a = s$

F. $281.25 + 5.25s = a$

76 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ____ Date: ____ Period: ____

3. An equation to calculate the volume of a cylinder V is $V = \pi r^2 h$ where r represents the cylinder's radius and h represents its height. Which could be used to efficiently calculate the height of the cylinder?

A. $r^2 h = \frac{V}{\pi}$

B. $h = V - \pi r^2$

[✓ C.] $h = \frac{V}{\pi r^2}$

D. $\pi h = V r^2$

4. A catering company is setting up for a wedding. It expects 150 people to attend. It can set up small tables that seat 6 people and large tables that seat 10 people.

a Determine a combination of small and large tables that seat exactly 150 people.

Sample response: 10 small tables and 9 large tables.

b Let x represent the number of small tables and y represent the number of large tables. Write an equation to represent the relationship between x and y.

$6x + 10y = 150$

c Explain what the coordinate point $(20, 5)$ represents in this scenario.

It represents 20 small tables and 5 large tables.

d Is the coordinate point $(20, 5)$ a solution to the equation you wrote? Explain your thinking.

Sample response: No, 20 small tables and 5 large tables results in seating for 170 people, not 150.

5. Graph each equation on the coordinate plane.

a $y = 3x + 1$

b $2x - 4y = 8$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Rearranging Equations (Part 2) 77

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	3
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 6	2
Formative ⚡	5	Unit 1 Lesson 11	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 11**

Connecting Equations in Standard Form to Their Graphs

Let's investigate what graphs can tell us about the equations and relationships they represent.

Focus

Goals

1. Analyze how a, b, and c of equations in standard form $ax + by = c$ are reflected on its graph.
2. **Language Goal:** Explain how a, b, and c of an equation in standard form are related to the rate of change in a relationship. **(Speaking and Listening, Writing)**
3. Graph linear equations in standard form and interpret points on the graph in context.
4. **Language Goal:** Understand that different forms of a linear equation can provide different insights about the relationship it represents and about the graph. **(Speaking and Listening, Writing)**

Coherence

Today

Students relate the terms of two-variable standard form linear equations to their graphs. They analyze different forms of two-variable linear equations to determine certain features of its graph or relationships between its quantities. Students reason quantitatively and abstractly as they interpret equations and graphs in context.

Previously

In Lesson 10, students rearranged and solved equations by isolating one of the variables.

Coming Soon

In Lesson 12, students will continue to practice relating the structure of equations to contexts, corresponding graphs, and features of graphs.

Rigor

- Students build **conceptual understanding** of the relationship between different forms of linear equations and their graphs.
- Students **apply** linear equations and their graphs to determine all of its solutions.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Individual	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (for display)
- Anchor Chart PDF, *Forms of Linear Equations*
- Anchor Chart PDF, *Sentence Stems, Math Talk*
- Instructional Routine PDF, *Jigsaw: Instructions*
- manipulatives for nickels and dimes

Math Language Development

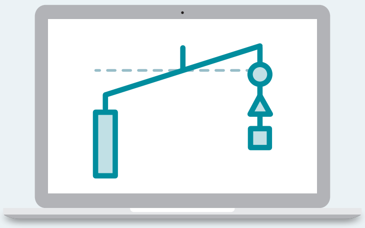
Review words

- *equivalent equations*
- *slope-intercept form*
- *slope*
- *standard form*
- x*-intercept*
- y*-intercept*

Amps | **Featured Activity**

Activity 2
Digital Coin Jar

Students interact with different representations of a coin jar to make connections between the scenario, equation, and graph.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may choose a preference of which form of the linear relationship they prefer as they alternate between written scenarios, equations, and graphs in each activity. Discuss why there are different ways to express the same relationship. Encourage students to think about why each could be helpful at different times and set goals to use each at an appropriate time.

Modifications to Pacing

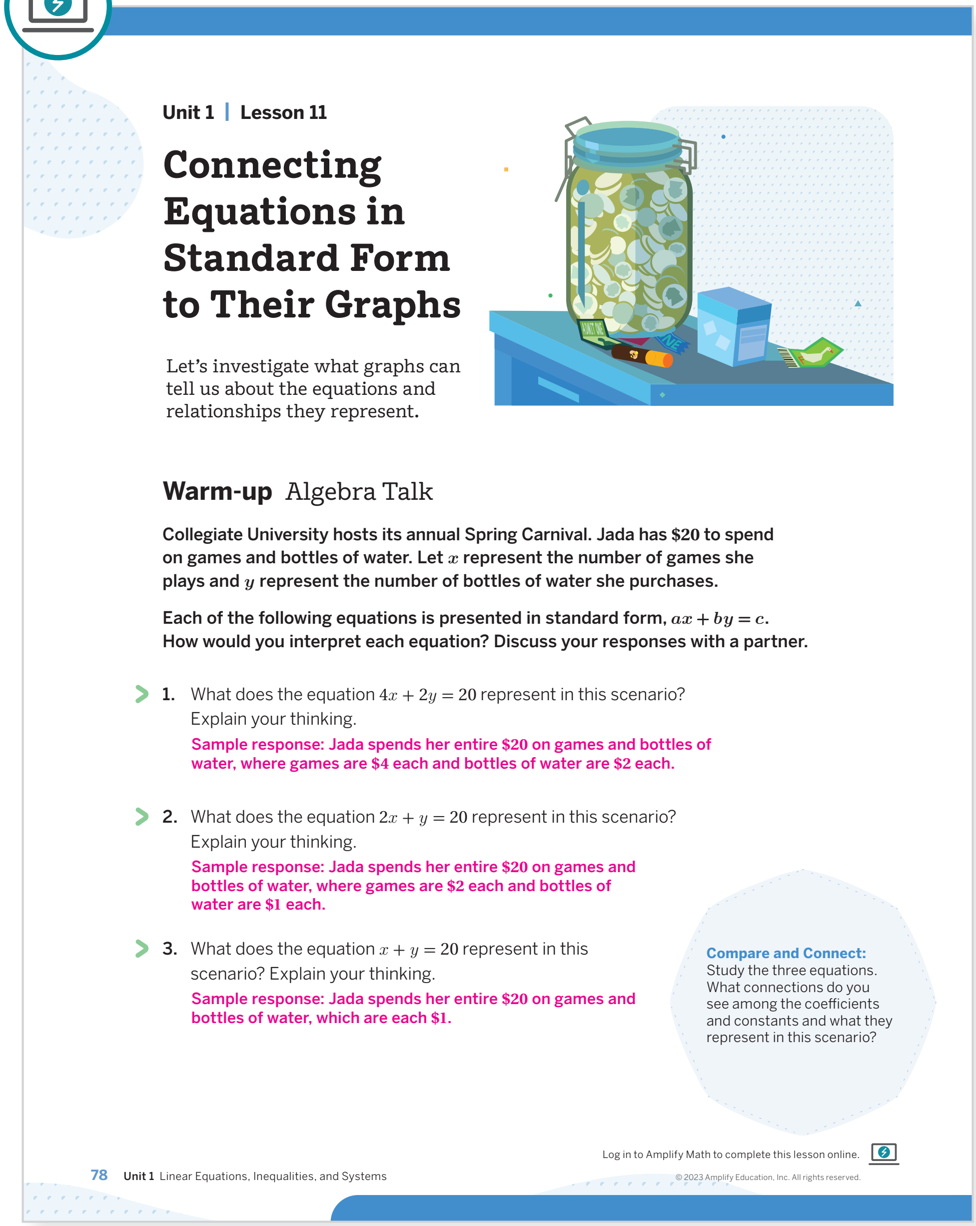
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 1 may be omitted.
- In **Activity 1**, instead of grouping students into a new group, discuss observations as a whole class.

Individual | 5 min

Warm-up Algebra Talk

Students interpret and make sense of equations in standard form in a context to prepare for the next activity.



Unit 1 | Lesson 11

Connecting Equations in Standard Form to Their Graphs

Let's investigate what graphs can tell us about the equations and relationships they represent.

Warm-up Algebra Talk

Collegiate University hosts its annual Spring Carnival. Jada has $20 to spend on games and bottles of water. Let x represent the number of games she plays and y represent the number of bottles of water she purchases.

Each of the following equations is presented in standard form, $ax + by = c$. How would you interpret each equation? Discuss your responses with a partner.

1. What does the equation $4x + 2y = 20$ represent in this scenario? Explain your thinking.
   Sample response: Jada spends her entire $20 on games and bottles of water, where games are $4 each and bottles of water are $2 each.

2. What does the equation $2x + y = 20$ represent in this scenario? Explain your thinking.
   Sample response: Jada spends her entire $20 on games and bottles of water, where games are $2 each and bottles of water are $1 each.

3. What does the equation $x + y = 20$ represent in this scenario? Explain your thinking.
   Sample response: Jada spends her entire $20 on games and bottles of water, which are each $1.

Compare and Connect: Study the three equations. What connections do you see among the coefficients and constants and what they represent in this scenario?

Log in to Amplify Math to complete this lesson online.

78 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the prompt aloud. Conduct the ***Algebra Talk*** routine giving students think-time and provide a signal to use after they have interpreted the equations.

2 Monitor

Help students get started by having them list everything they know and want to know about the problem.

Look for points of confusion:

- **Thinking 20 represents the combined number of games and bottles of water purchased in Problems 1 and 2.** Have students evaluate each equation for the purchase of 2 bottles of water. Ask, "What do the results represent? How many items were purchased?"

Look for productive strategies:

- Creating a table to test different values to make sense of the quantities and relationship.
- Using precise language to explain their thinking.

3 Connect

Display the prompt.

Have individual students share their thinking for interpreting each equation.

Ask:

- "Who can restate ____'s thinking differently?"
- "Did anyone think about the problem differently?"
- "Can anyone add on to ____'s thinking?"

Highlight that relationships between quantities can be modeled with standard form linear equations and that the coefficients change the context.

Ask, "In Problem 3, why does the 20 represent both the total cost and the total combined purchases?"

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share how they interpreted each equation, display or provide the Anchor Chart PDF, *Sentence Stems, Math Talk*. Ask them to borrow prompts from this display during the discussion. Draw connections between the quantities in the scenario and how they appear in the equations. Ask these questions:

- "What does the coefficient of x represent in this scenario?" The cost of each game.
- "What does the coefficient of y represent in this scenario?" The cost of each bottle of water.

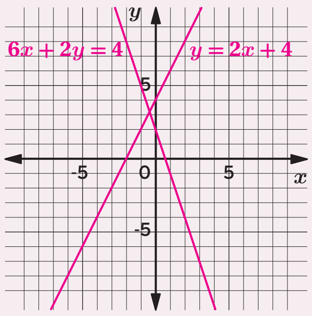
Power-up

To power up students' ability to graph a two-variable linear equation, have students complete:

Graph the equations $y = 2x + 4$ and $6x + 2y = 4$.

Use: Before Activity 1

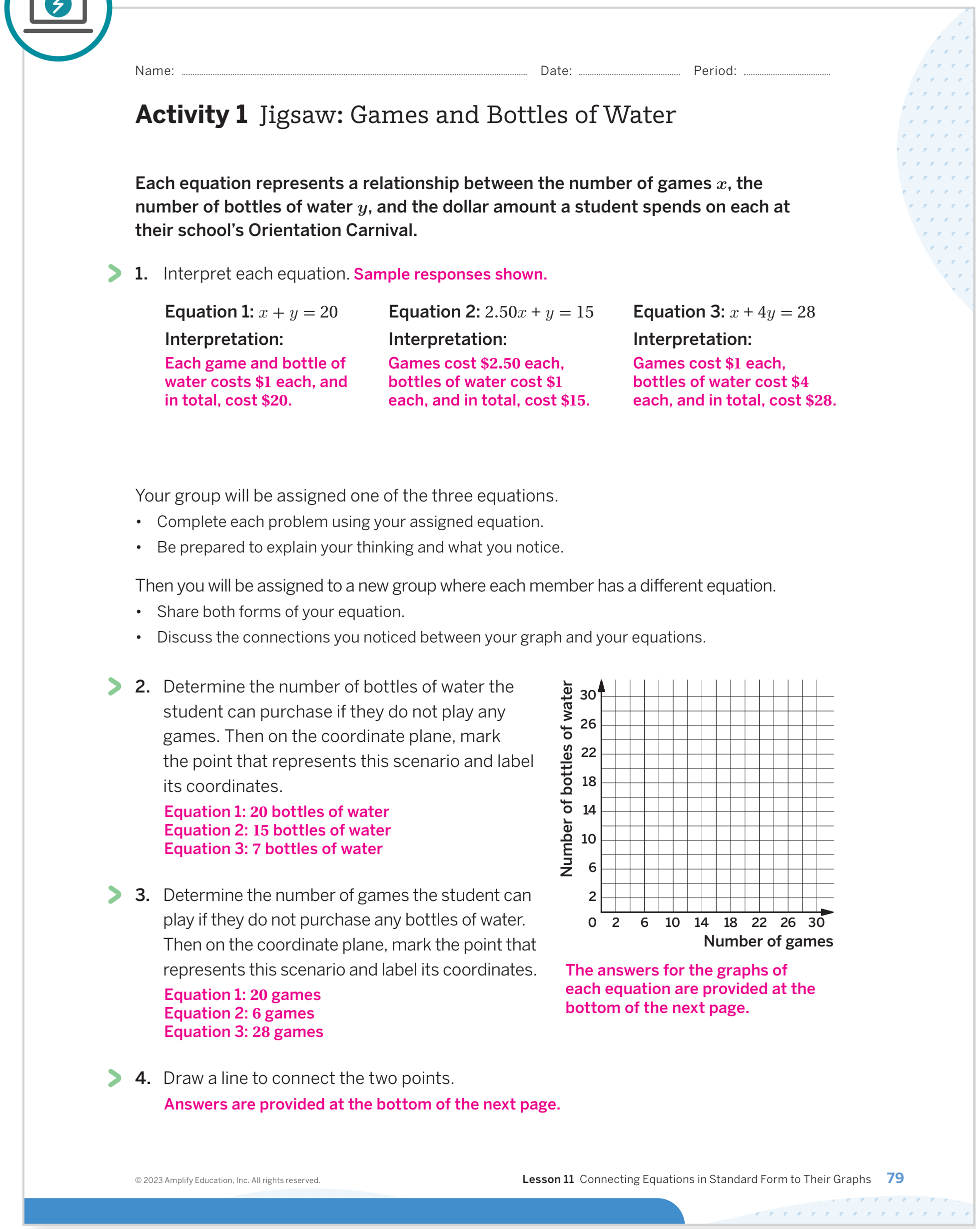
Informed by: Performance on Lesson 10, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 3



Small Groups | 20 min

Activity 1 Jigsaw: Games and Bottles of Water

Students interpret and rearrange linear equations in standard form to slope-intercept form to graph and make sense of the context.



Name: Date: Period:

Activity 1 Jigsaw: Games and Bottles of Water

Each equation represents a relationship between the number of games x, the number of bottles of water y, and the dollar amount a student spends on each at their school's Orientation Carnival.

1. Interpret each equation. **Sample responses shown.**

Equation 1: $x + y = 20$	Equation 2: $2.50x + y = 15$	Equation 3: $x + 4y = 28$
Interpretation:	Interpretation:	Interpretation:
Each game and bottle of water costs \$1 each, and in total, cost \$20.	**Games cost \$2.50 each, bottles of water cost \$1 each, and in total, cost \$15.**	**Games cost \$1 each, bottles of water cost \$4 each, and in total, cost \$28.**

Your group will be assigned one of the three equations.
- Complete each problem using your assigned equation.
- Be prepared to explain your thinking and what you notice.

Then you will be assigned to a new group where each member has a different equation.
- Share both forms of your equation.
- Discuss the connections you noticed between your graph and your equations.

2. Determine the number of bottles of water the student can purchase if they do not play any games. Then on the coordinate plane, mark the point that represents this scenario and label its coordinates.
Equation 1: 20 bottles of water
Equation 2: 15 bottles of water
Equation 3: 7 bottles of water

The answers for the graphs of each equation are provided at the bottom of the next page.

3. Determine the number of games the student can play if they do not purchase any bottles of water. Then on the coordinate plane, mark the point that represents this scenario and label its coordinates.
Equation 1: 20 games
Equation 2: 6 games
Equation 3: 28 games

4. Draw a line to connect the two points.
Answers are provided at the bottom of the next page.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 11 Connecting Equations in Standard Form to Their Graphs 79

1 Launch

Read the prompt aloud. Discuss the interpretation for each equation together. Assign each group an equation and use the *Jigsaw* routine. Remind students to focus on thinking and not on answers during group discussions. If needed, display the Instructional Routine PDF, *Jigsaw: Instructions*.

Provide 10 minutes to complete the first equation. Then assign each student to a new group so that each new group member has a different equation. Have students discuss their thinking and what they noticed for 5 minutes.

2 Monitor

Help students get started by annotating the problem, writing x for the "number of games" and y for the "number of bottles of water."

Look for points of confusion:

- **Having difficulty interpreting the phrase "for every additional game" in Problem 5b.** Have students determine the number of bottles of water they could purchase if they played 3, 4, or 5 games. Ask, "How do the number of bottles of water change when one more game is played?"

Look for productive strategies:

- Annotating the prompt, graph, or equation to make sense of the quantities and relationship.
- Calculating the slope by counting or using the slope formula.
- Determining the slope or y-intercept by rewriting the equation in slope-intercept form.
- Using the graph to interpret the slope.
- Noticing the relationship between a, b, and c of each equation form and the intercepts and the slope of the graph.

Activity 1 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a two-column graphic organizer to record an explanation for each step of their work. Display or provide the Anchor Chart PDF, *Forms of Linear Equations* for students to reference during this activity.

Extension: *Math Enrichment*

Ask students to write a fourth equation that represents a different student's purchases. Have them trade their equation with a group member and each group member should determine what the equation represents within the scenario, including the slope and intercepts (and their meaning).

Math Language Development

MLR7: Compare and Connect

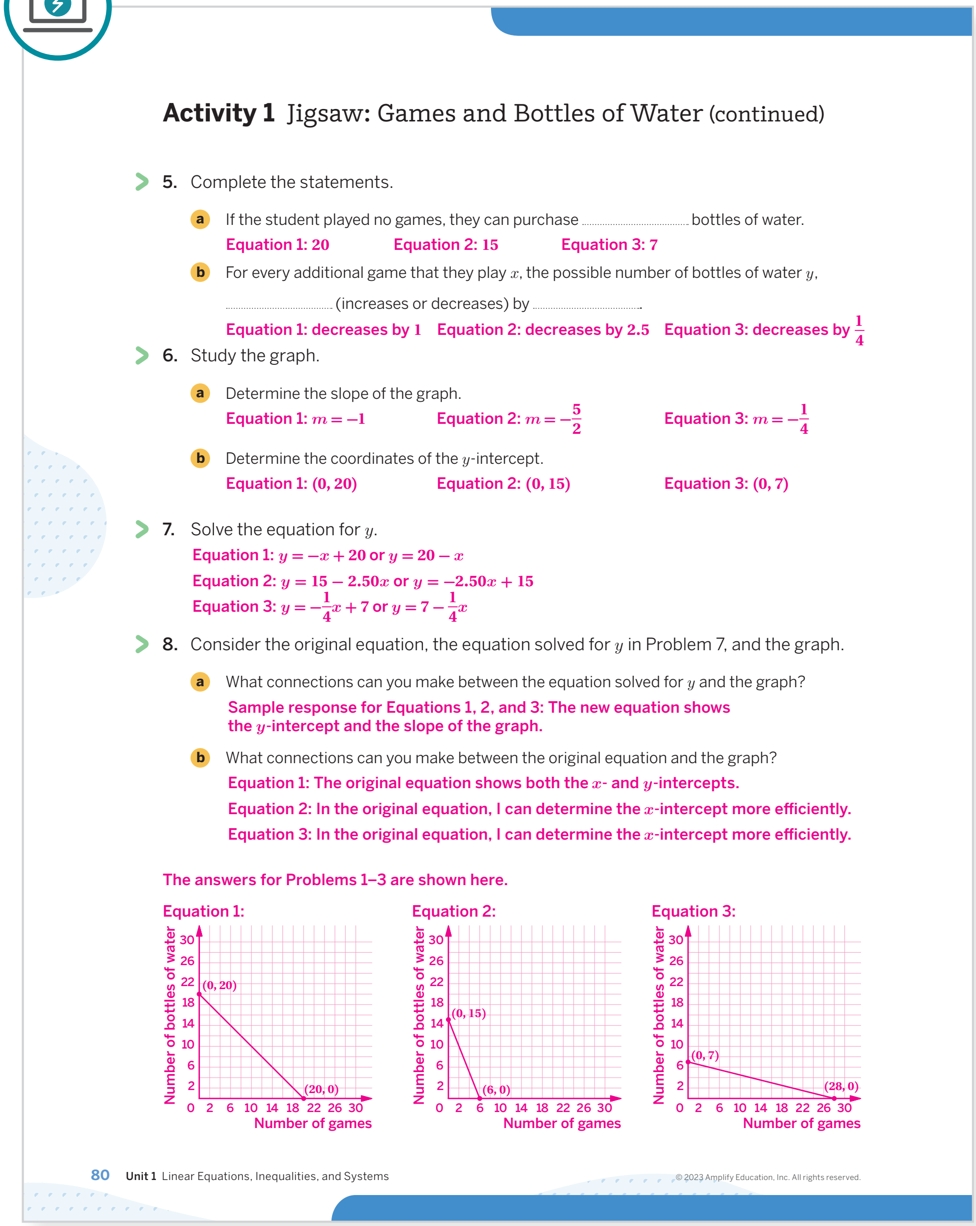
During the Connect, as students share the connections they noticed between the forms of the equations and their graphs, press for details in their reasoning. For example, if a student says, "Equation 1 shows both intercepts," follow-up with these questions:

- "What do you mean? Where specifically in the equation do you see the x-intercept? y-intercept?"
- "What is it about the structure of Equation 1 that allows you to see both intercepts?"

Small Groups | 20 min

Activity 1 Jigsaw: Games and Bottles of Water (continued)

Students interpret and rearrange linear equations in standard form to slope-intercept form to graph and make sense of the context.



Activity 1 Jigsaw: Games and Bottles of Water (continued)

5. Complete the statements.
   - a If the student played no games, they can purchase .... bottles of water.
     Equation 1: 20 Equation 2: 15 Equation 3: 7
   - b For every additional game that they play x, the possible number of bottles of water y, ... (increases or decreases) by ....
     Equation 1: decreases by 1 Equation 2: decreases by 2.5 Equation 3: decreases by $\frac{1}{4}$

6. Study the graph.
   - a Determine the slope of the graph.
     Equation 1: $m = -1$ Equation 2: $m = -\frac{5}{2}$ Equation 3: $m = -\frac{1}{4}$
   - b Determine the coordinates of the y-intercept.
     Equation 1: $(0, 20)$ Equation 2: $(0, 15)$ Equation 3: $(0, 7)$

7. Solve the equation for y.
   Equation 1: $y = -x + 20$ or $y = 20 - x$
   Equation 2: $y = 15 - 2.50x$ or $y = -2.50x + 15$
   Equation 3: $y = -\frac{1}{4}x + 7$ or $y = 7 - \frac{1}{4}x$

8. Consider the original equation, the equation solved for y in Problem 7, and the graph.
   - a What connections can you make between the equation solved for y and the graph?
     Sample response for Equations 1, 2, and 3: The new equation shows the y-intercept and the slope of the graph.
   - b What connections can you make between the original equation and the graph?
     Equation 1: The original equation shows both the x- and y-intercepts.
     Equation 2: In the original equation, I can determine the x-intercept more efficiently.
     Equation 3: In the original equation, I can determine the x-intercept more efficiently.

The answers for Problems 1–3 are shown here.

Equation 1:

Equation 2:

Equation 3:

80 Unit 1 Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have groups of students share their rearranged equations, graphs and any connections noticed between the forms of the equations and their graphs.

Display the original equations and student responses.

Ask:

- "How did you determine your answers for Problems 2 and 3? Where did you mark the points on the coordinate plane?"
- "How can the graph help you complete the sentences in Problem 5?"
- "What strategies did you use to interpret the slope in the context of this problem?"

Highlight that each equation form provides some insights about the relationship between the quantities. Solving for y results in the slope and y-intercept, which are helpful for creating or visualizing a graph. Even without a graph, the slope and y-intercept can tell them about the relationship between the quantities.

Ask:

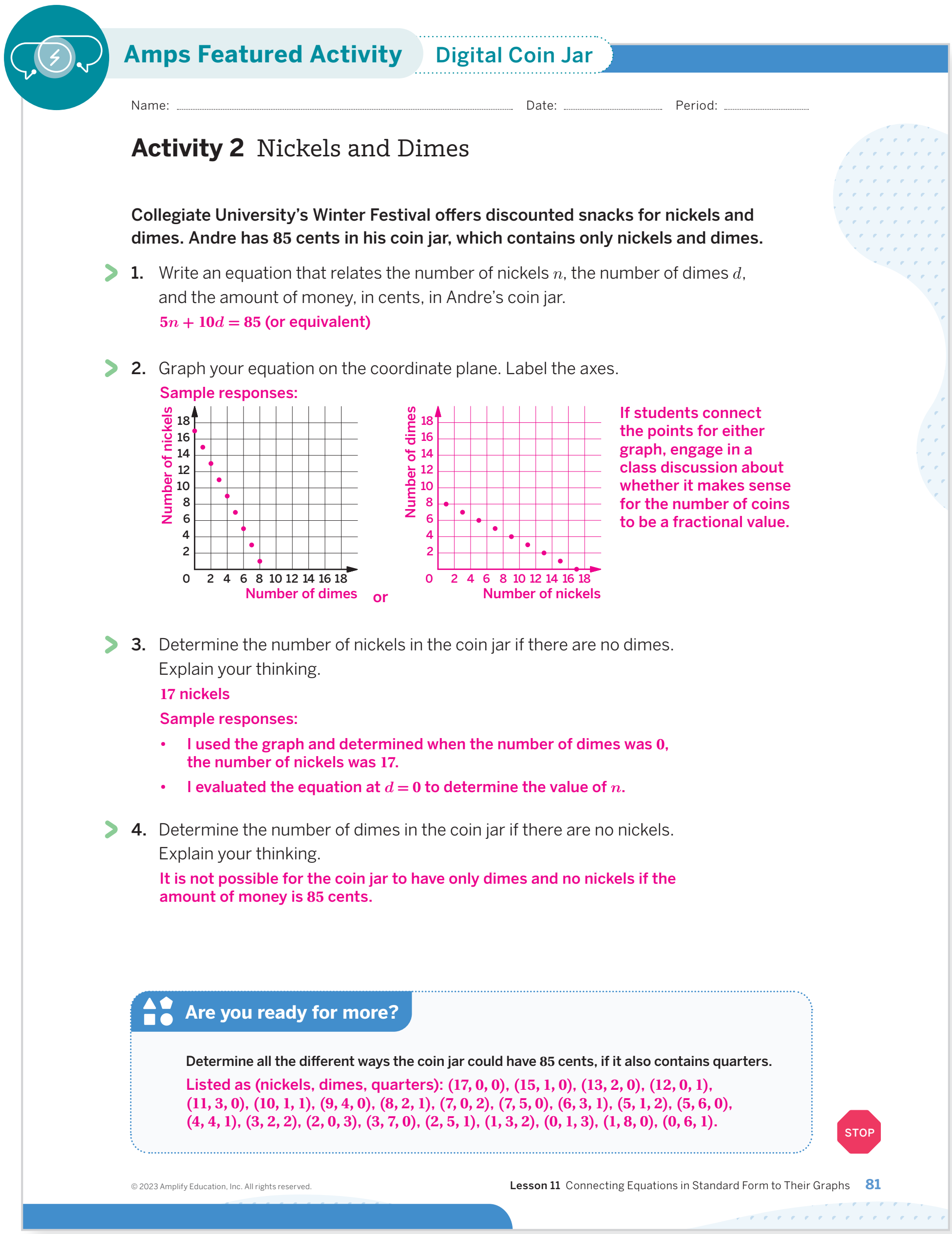
- "What information can you determine about the graph from an equation in standard form?"
- "What information can you determine about the graph from an equation in slope-intercept form?"

Small Groups | 15 min

Activity 2 Nickels and Dimes

Students write, graph, and interpret an equation in standard form in a context to consider reasonableness in their solutions.

Amps Featured Activity Digital Coin Jar



Name: ____________ Date: ____________ Period: ____________

Activity 2 Nickels and Dimes

Collegiate University's Winter Festival offers discounted snacks for nickels and dimes. Andre has 85 cents in his coin jar, which contains only nickels and dimes.

1. Write an equation that relates the number of nickels n, the number of dimes d, and the amount of money, in cents, in Andre's coin jar.
   $5n + 10d = 85$ (or equivalent)

2. Graph your equation on the coordinate plane. Label the axes.
   Sample responses:

If students connect the points for either graph, engage in a class discussion about whether it makes sense for the number of coins to be a fractional value.

3. Determine the number of nickels in the coin jar if there are no dimes. Explain your thinking.
   17 nickels
   Sample responses:
   - I used the graph and determined when the number of dimes was 0, the number of nickels was 17.
   - I evaluated the equation at $d = 0$ to determine the value of n.

4. Determine the number of dimes in the coin jar if there are no nickels. Explain your thinking.
   It is not possible for the coin jar to have only dimes and no nickels if the amount of money is 85 cents.

Are you ready for more?

Determine all the different ways the coin jar could have 85 cents, if it also contains quarters.

Listed as (nickels, dimes, quarters): (17, 0, 0), (15, 1, 0), (13, 2, 0), (12, 0, 1), (11, 3, 0), (10, 1, 1), (9, 4, 0), (8, 2, 1), (7, 0, 2), (7, 5, 0), (6, 3, 1), (5, 1, 2), (5, 6, 0), (4, 4, 1), (3, 2, 2), (2, 0, 3), (3, 7, 0), (2, 5, 1), (1, 3, 2), (0, 1, 3), (1, 8, 0), (0, 6, 1).

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 11 Connecting Equations in Standard Form to Their Graphs 81

1 Launch

Display the prompt utilizing the ***Co-craft Questions*** routine. Allow think-time before having small group discussions. Discuss whole-class, then release students to work together in small groups.

For each problem, students discuss strategies as a group, complete individually, then compare solutions before moving on to the next problem.

2 Monitor

Help students get started by asking, "What is the value of a dime? a nickel? How can that be used to write an equation to represent the scenario?" $0.05n + 0.1d = 0.85$ (or equivalent)

Look for points of confusion:

- **Having difficulty determining which variable represents y.** Ask, "How does the graph change if the y-axis is the number of dimes? Number of nickels?"

Look for productive strategies:

- Creating a table to determine points and graph.
- Solving the equation for one variable to graph.
- Modeling using a discrete graph.

3 Connect

Have groups of students share their graphs and thinking for Problems 3 and 4.

Ask, "What strategies did you use to graph? In Problems 3 and 4?"

Display the Activity 2 PDF.

Highlight that each graph models the scenario, but only whole-number coordinate values represent the solutions to the scenario. If not mentioned, the standard form of an equation is helpful for identifying the x- and y-intercepts. While slope-intercept form is best for identifying the rate of change or slope.

Differentiated Support

Accessibility: *Optimize Access to Tools, Guide Processing and Visualization*

Provide students with manipulatives to represent nickels and dimes (or actual nickels and dimes) to support sense making of the problem. Before students write an equation in Problem 1, ask them to think of one possible combination of nickels and dimes that could equal 85 cents.

Extension: *Math Enrichment*

Have students complete one or both of the following problems:

- Write an equation in Problem 1 so that the units are in dollars, not cents. $0.05n + 0.10d = 0.85$ (or equivalent)
- Write an equation in three variables that represents the *Are you ready for more*? problem. Sample response: $5n + 10d + 25q = 85$

Math Language Development

MLR5: Co-craft Questions

Display the introductory text. Ask students to individually write 1–2 mathematical questions that could be asked about the scenario. Have students compare their questions with their group members before they complete the activity. Some sample questions are:

- "How many nickels does Andre have? How many dimes?"
- "How much do the snacks cost? How many snacks can Andre buy?"

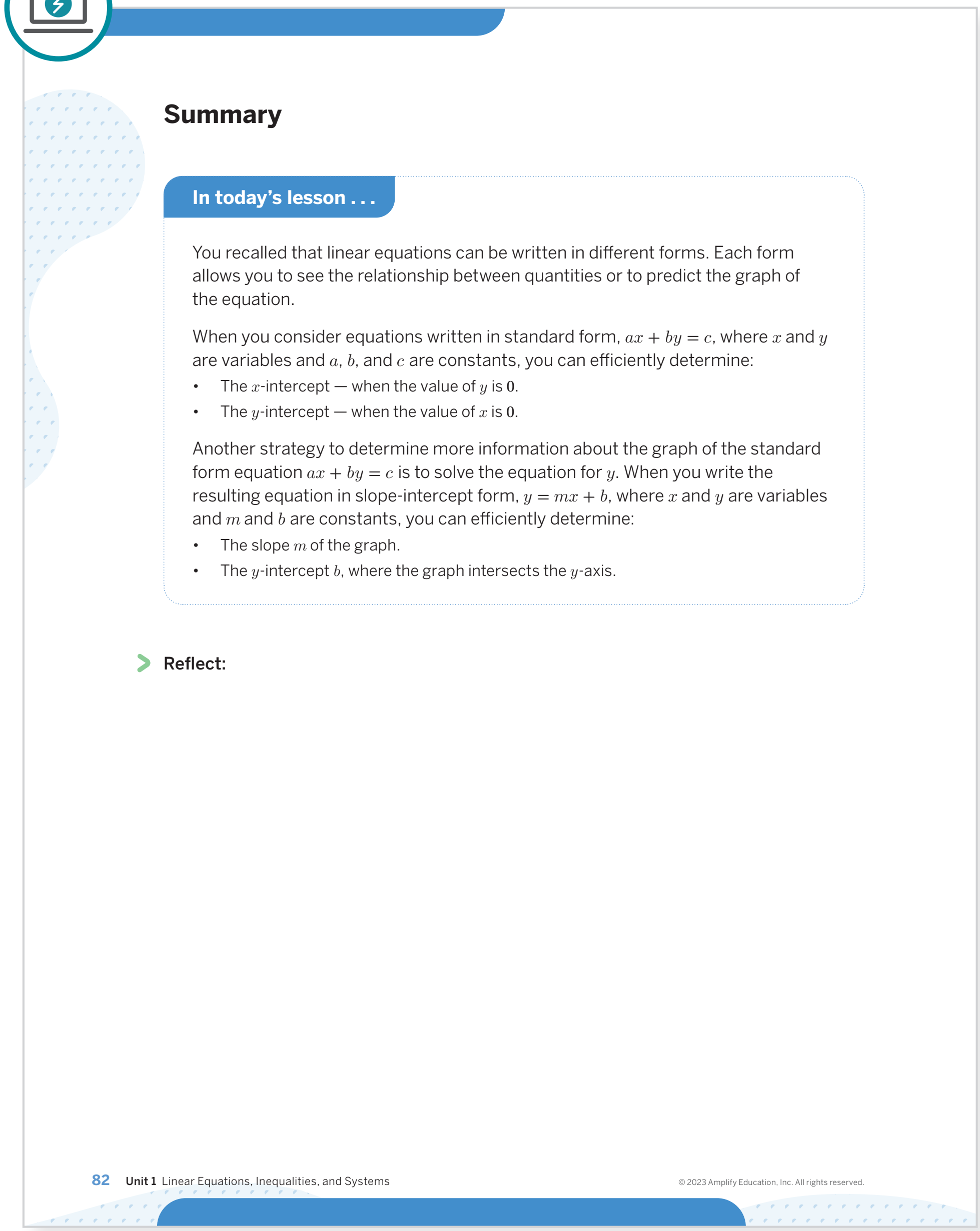
English Learners

Bring in some nickels and dimes and explain how much each type of coin is worth in U.S. currency.

Whole Class | 5 min

Summary

Review and synthesize that different forms of linear equations highlight different relationships between the quantities and graph.



Summary

In today's lesson . . .

You recalled that linear equations can be written in different forms. Each form allows you to see the relationship between quantities or to predict the graph of the equation.

When you consider equations written in standard form, $ax + by = c$, where x and y are variables and a, b, and c are constants, you can efficiently determine:

- The x-intercept — when the value of y is 0.
- The y-intercept — when the value of x is 0.

Another strategy to determine more information about the graph of the standard form equation $ax + by = c$ is to solve the equation for y. When you write the resulting equation in slope-intercept form, $y = mx + b$, where x and y are variables and m and b are constants, you can efficiently determine:

- The slope m of the graph.
- The y-intercept b, where the graph intersects the y-axis.

Reflect:

82 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the equations $x + 4y = 28$ and $y = -\frac{1}{4}x + 7$, representing the number of games and bottles of water purchased within a fixed budget.

Ask:

- "What information does the standard form equation provide about the context? The graph?"
- "What information does the equation in slope-intercept form provide about the context? The graph?"
- "What strategy is most efficient for graphing an equation in standard form?"

Highlight that each form of an equation provides different information. The slope-intercept form shows the slope and y-intercept, while the x- and y-intercepts can be efficiently calculated from the standard form equation.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

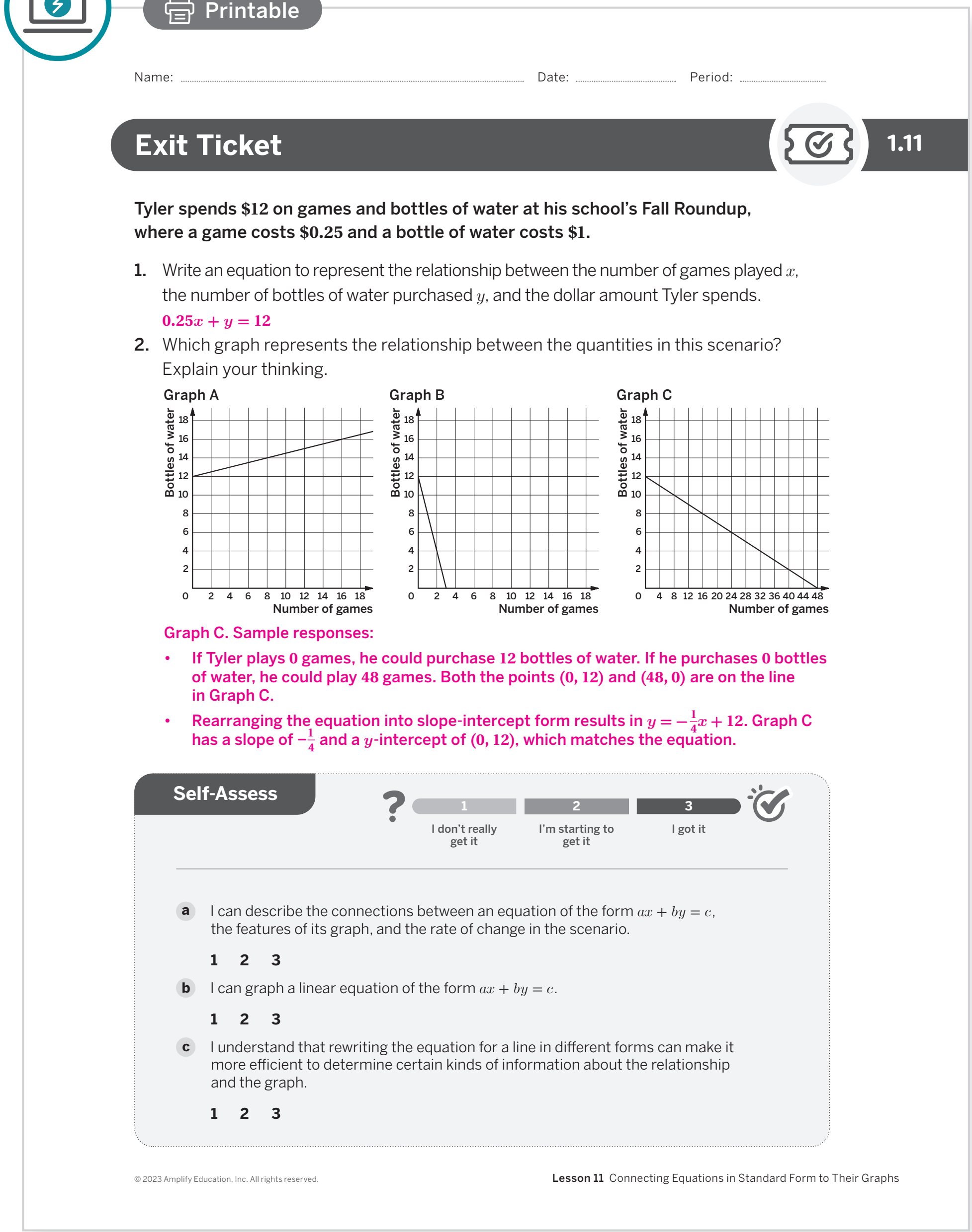
- "Why is it useful to have different forms of linear equations?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing a standard form equation and interpreting its graph.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.11

Tyler spends $\$12$ on games and bottles of water at his school's Fall Roundup, where a game costs $\$0.25$ and a bottle of water costs $\$1$.

1. Write an equation to represent the relationship between the number of games played x, the number of bottles of water purchased y, and the dollar amount Tyler spends.

   $0.25x + y = 12$

2. Which graph represents the relationship between the quantities in this scenario? Explain your thinking.

Graph C. Sample responses:

- **If Tyler plays 0 games, he could purchase 12 bottles of water. If he purchases 0 bottles of water, he could play 48 games. Both the points $(0, 12)$ and $(48, 0)$ are on the line in Graph C.**
- **Rearranging the equation into slope-intercept form results in $y = -\frac{1}{4}x + 12$. Graph C has a slope of $-\frac{1}{4}$ and a y-intercept of $(0, 12)$, which matches the equation.**

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can describe the connections between an equation of the form $ax + by = c$, the features of its graph, and the rate of change in the scenario.

1 2 3

b I can graph a linear equation of the form $ax + by = c$.

1 2 3

c I understand that rewriting the equation for a line in different forms can make it more efficient to determine certain kinds of information about the relationship and the graph.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 11 Connecting Equations in Standard Form to Their Graphs

Success looks like . . .

- **Goal:** Analyzing how a, b, and c of equations in standard form $ax + by = c$ are reflected on its graph.
  - » Relating the equation relating the number of games played, the number of water bottles purchased, and the amount spent by Tyler to the correct graph in Problem 2.
- **Language Goal:** Explaining how a, b, and c of an equation in standard form are related to the rate of change in a relationship. **(Speaking and Listening, Writing)**
- **Goal:** Graphing linear equations in standard form and interpreting points on the graph in context.
- **Language Goal:** Understanding that different forms of a linear equation can provide different insights about the relationship it represents and about the graph. **(Speaking and Listening, Writing)**

Suggested next steps

If students incorrectly write the equation in Problem 1, consider:

- Reviewing Lesson 6.

If students choose the incorrect graph or incompletely explain their choice in Problem 2, consider:

- Reviewing graphing strategies from Activity 2.
- Assigning Practice Problem 2.
- Asking, "How could you determine the x-intercept or the y-intercept?"

Professional Learning

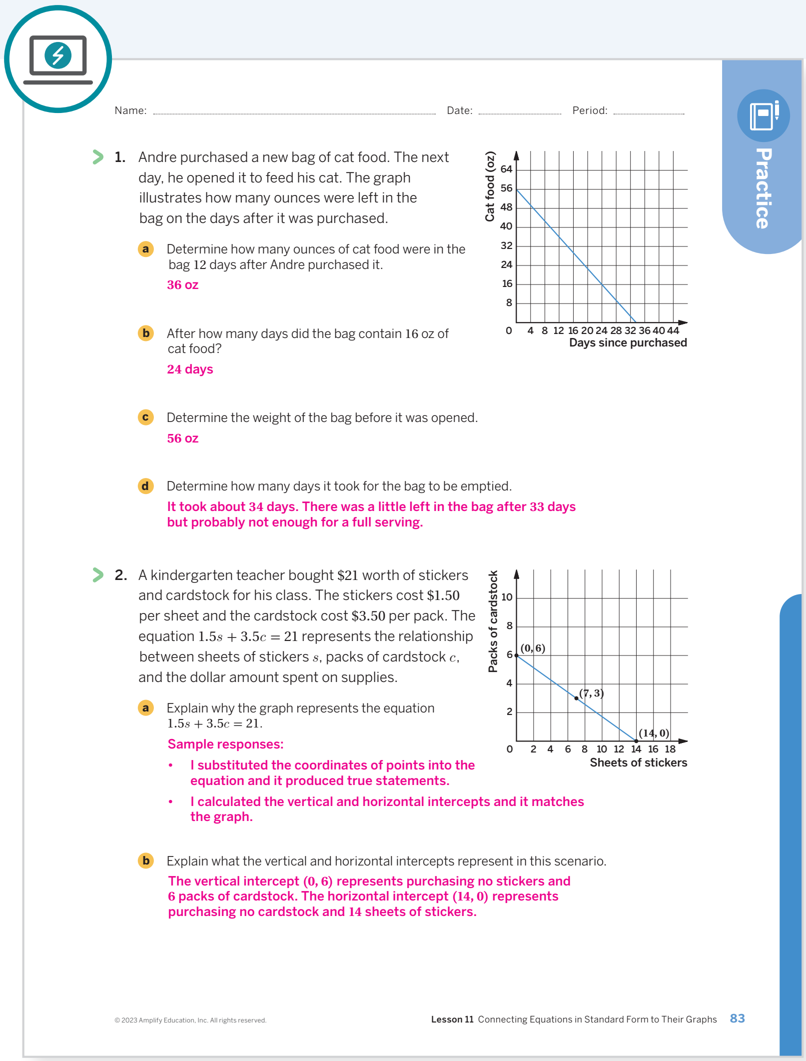
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did students self-manage today? How are you helping students become aware of how they are progressing in this area?
- What different ways did students approach graphing in Activity 2? What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Andre purchased a new bag of cat food. The next day, he opened it to feed his cat. The graph illustrates how many ounces were left in the bag on the days after it was purchased.

   a. Determine how many ounces of cat food were in the bag 12 days after Andre purchased it.

   36 oz

   b. After how many days did the bag contain 16 oz of cat food?

   24 days

   c. Determine the weight of the bag before it was opened.

   56 oz

   d. Determine how many days it took for the bag to be emptied.

   It took about 34 days. There was a little left in the bag after 33 days but probably not enough for a full serving.

2. A kindergarten teacher bought $21 worth of stickers and cardstock for his class. The stickers cost $1.50 per sheet and the cardstock cost $3.50 per pack. The equation $1.5s + 3.5c = 21$ represents the relationship between sheets of stickers s, packs of cardstock c, and the dollar amount spent on supplies.

   a. Explain why the graph represents the equation $1.5s + 3.5c = 21$.

   Sample responses:

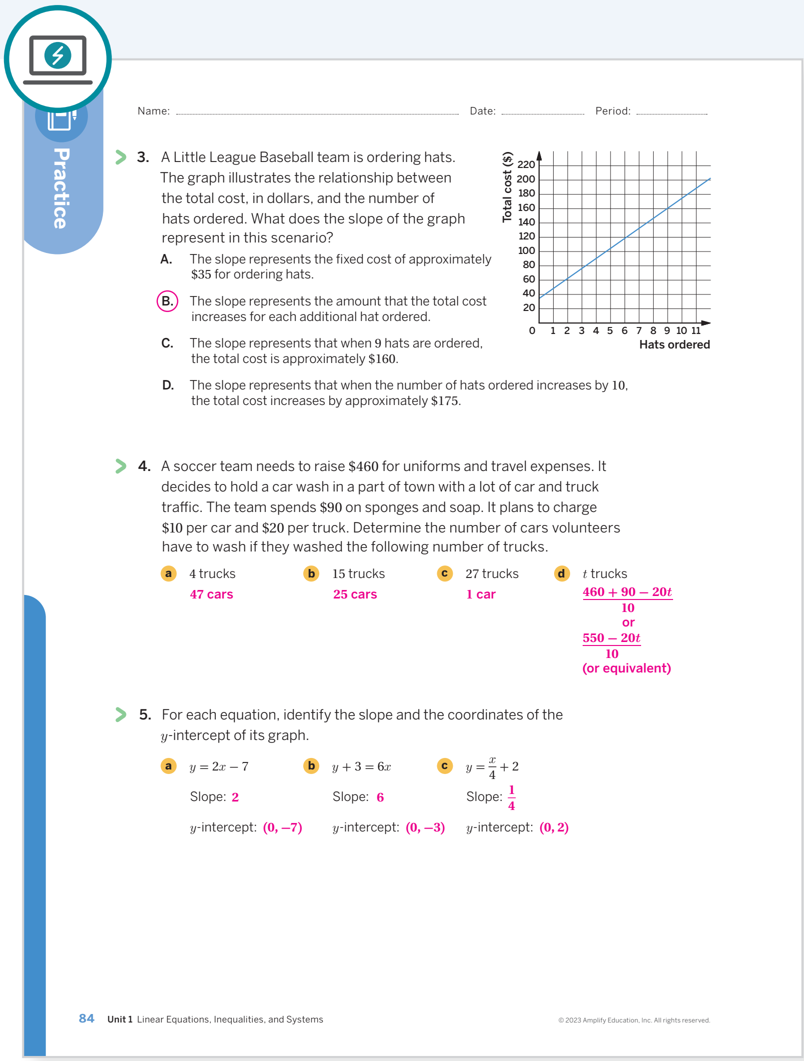
   - **I substituted the coordinates of points into the equation and it produced true statements.**
   - **I calculated the vertical and horizontal intercepts and it matches the graph.**

   b. Explain what the vertical and horizontal intercepts represent in this scenario.

   The vertical intercept (0, 6) represents purchasing no stickers and 6 packs of cardstock. The horizontal intercept (14, 0) represents purchasing no cardstock and 14 sheets of stickers.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 11 Connecting Equations in Standard Form to Their Graphs 83



Name: Date: Period:

3. A Little League Baseball team is ordering hats. The graph illustrates the relationship between the total cost, in dollars, and the number of hats ordered. What does the slope of the graph represent in this scenario?

   A. The slope represents the fixed cost of approximately $35 for ordering hats.

   [✓ B.] The slope represents the amount that the total cost increases for each additional hat ordered.

   C. The slope represents that when 9 hats are ordered, the total cost is approximately $160.

   D. The slope represents that when the number of hats ordered increases by 10, the total cost increases by approximately $175.

4. A soccer team needs to raise $460 for uniforms and travel expenses. It decides to hold a car wash in a part of town with a lot of car and truck traffic. The team spends $90 on sponges and soap. It plans to charge $10 per car and $20 per truck. Determine the number of cars volunteers have to wash if they washed the following number of trucks.

   a. 4 trucks **47 cars**

   b. 15 trucks **25 cars**

   c. 27 trucks **1 car**

   d. t trucks $\frac{460 + 90 - 20t}{10}$ **or** $\frac{550 - 20t}{10}$ **(or equivalent)**

5. For each equation, identify the slope and the coordinates of the y-intercept of its graph.

   a. $y = 2x - 7$
   Slope: **2**
   y-intercept: **(0, –7)**

   b. $y + 3 = 6x$
   Slope: **6**
   y-intercept: **(0, –3)**

   c. $y = \frac{x}{4} + 2$
   Slope: $\frac{1}{4}$
   y-intercept: **(0, 2)**

84 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 9	2
Formative	5	Unit 1 Lesson 12	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 12**

Connecting Equations in Slope-Intercept Form to Their Graphs

Let's analyze different forms of linear equations and how the forms relate to their graphs.

Focus

Goals

1. Determine the slope and vertical intercept of the graphs of linear equations by making use of structure or by rearranging the equations.
2. Given an equation of the form $ax + by = c$, write an equivalent equation of the form $y = mx + b$.

Coherence

- ### Today

Students continue relating the structure of equations in standard form to a context and corresponding graphs. They analyze constraints, equations, and points on a graph to determine whether they match an equation or context. Students reason abstractly to determine the slope and y-intercept of equations in standard form without context.

Previously

In Lesson 11, students made connections between equations in standard form, the features of its graph, and interpreted the rate of change for a context.

Coming Soon

In Lesson 13, students will write inequalities in one variable, reason about solutions, and represent solutions on a number line.

Rigor

- Students build **fluency** analyzing the structure and rearranging linear equations to determine the slope and y-intercept.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	15 min	15 min	10 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, pre-cut cards, one per pair
- Anchor Chart PDF, *Sentence Stems, Matching Prompts*
- Anchor Chart PDF, *Sentence Stems, Types of Questioning*
- Anchor Chart PDF, *Sentence Stems, Which One Doesn't Belong?*
- Instructional Routine PDF, *Info Gap: Instructions*
- graph paper
- graphing technology

Math Language Development

Review words

- *equivalent equations*
- *slope-intercept form*
- *slope*
- *standard form*
- x*-intercept*
- y*-intercept*

Amps | **Featured Activity**

Activity 1

Interactive Graphs

Students engage with interactive graphs to help analyze the graphs, interpret the points, and relate them to the structure of the equations.

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel overwhelmed making sense of the equations, scenarios, and graphs in Activity 1. Ask students how they are feeling and listen deeply and reflect what you heard about their feelings. For example, "It sounds like you're feeling very frustrated right now . . ." Then have students describe other challenging lessons or concepts they have persevered and succeeded in.

Modifications to Pacing

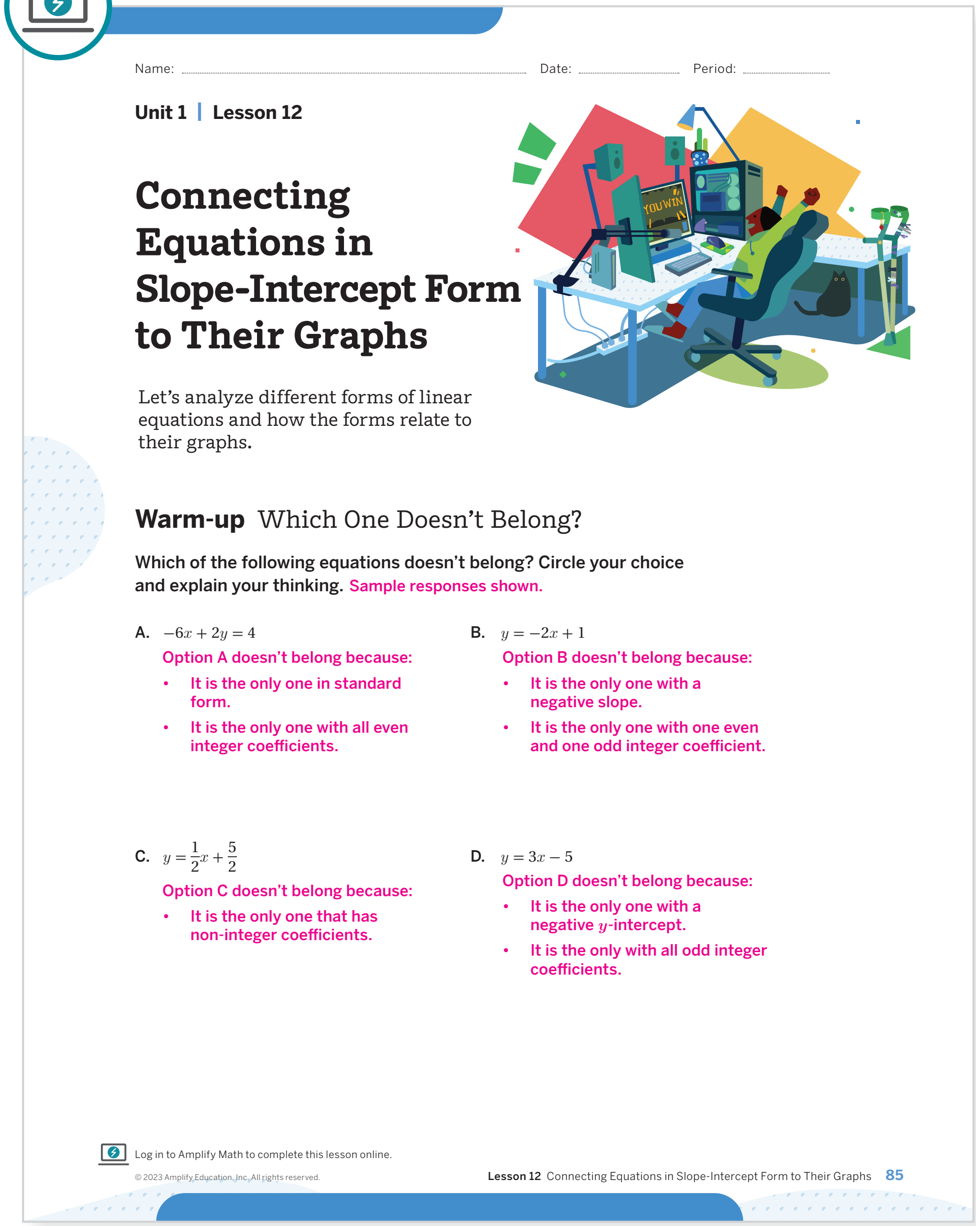
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, Problems 1d, 1e, and 2 may be omitted.
- In **Activity 3**, have students only complete 2 cards.

Independent | 5 min

Warm-up Which One Doesn't Belong?

Students consider linear equations of different forms to notice and analyze their structure.



Name: ______ Date: ______ Period: ______

Unit 1 | Lesson 12

Connecting Equations in Slope-Intercept Form to Their Graphs

Let's analyze different forms of linear equations and how the forms relate to their graphs.

Warm-up Which One Doesn't Belong?

Which of the following equations doesn't belong? Circle your choice and explain your thinking. Sample responses shown.

A. $-6x + 2y = 4$

Option A doesn't belong because:

- It is the only one in standard form.
- It is the only one with all even integer coefficients.

B. $y = -2x + 1$

Option B doesn't belong because:

- It is the only one with a negative slope.
- It is the only one with one even and one odd integer coefficient.

C. $y = \frac{1}{2}x + \frac{5}{2}$

Option C doesn't belong because:

- It is the only one that has non-integer coefficients.

D. $y = 3x - 5$

Option D doesn't belong because:

- It is the only one with a negative y-intercept.
- It is the only with all odd integer coefficients.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 12 Connecting Equations in Slope-Intercept Form to Their Graphs 85

1 Launch

Conduct the *Which One Doesn't Belong?* routine. Have students consider each option. Provide independent think-time before students share their thinking with a partner.

2 Monitor

Help students get started by having them compare two options at a time.

Look for points of confusion:

- **Having difficulty comparing Option A.** Ask students what strategies they have learned to graph, rewrite, or make sense of standard form equations.

Look for productive strategies:

- Comparing coefficients.
- Comparing forms of equations.
- Rearranging all equations to slope-intercept form.
- Comparing slope and y-intercepts.

3 Connect

Have students share one reason why an option does not belong. After each response, ask the class if they agree or disagree.

Highlight that different strategies can be used to compare and determine information about equations in different forms.

Ask:

- "What information can be determined from each equation without performing any calculations?"
- "Which form of an equation do you prefer to use in any given problem? Why?"

Math Language Development

MLR8: Discussion Supports

During the Connect, consider displaying the Anchor Chart PDF, *Sentence Stems, Which One Doesn't Belong?* for students to use as a reference as they share what they noticed and wondered. Encourage the use of mathematical language in their responses, such as *standard form*, *integer*, *slope*, *coefficient*, *even*, or *odd*.

English Learners

As each mathematical term is mentioned, point or highlight how that term is represented in the equation.

Power-up

To power up students' ability to identify the slope and y-intercept from linear equations, have students complete:

Recall that, for equations of the form $y = mx + b$, m represents the slope and b represents the y-coordinate of the vertical intercept. Complete the missing information in the table.

Equation	$y = 2x + 1$	$y = -3x + 4$	$2x + y = 8$
Slope	2	-3	-2
Coordinates of the y-intercept	$(0, 1)$	$(0, 4)$	$(0, 8)$

Use: Before Activity 1

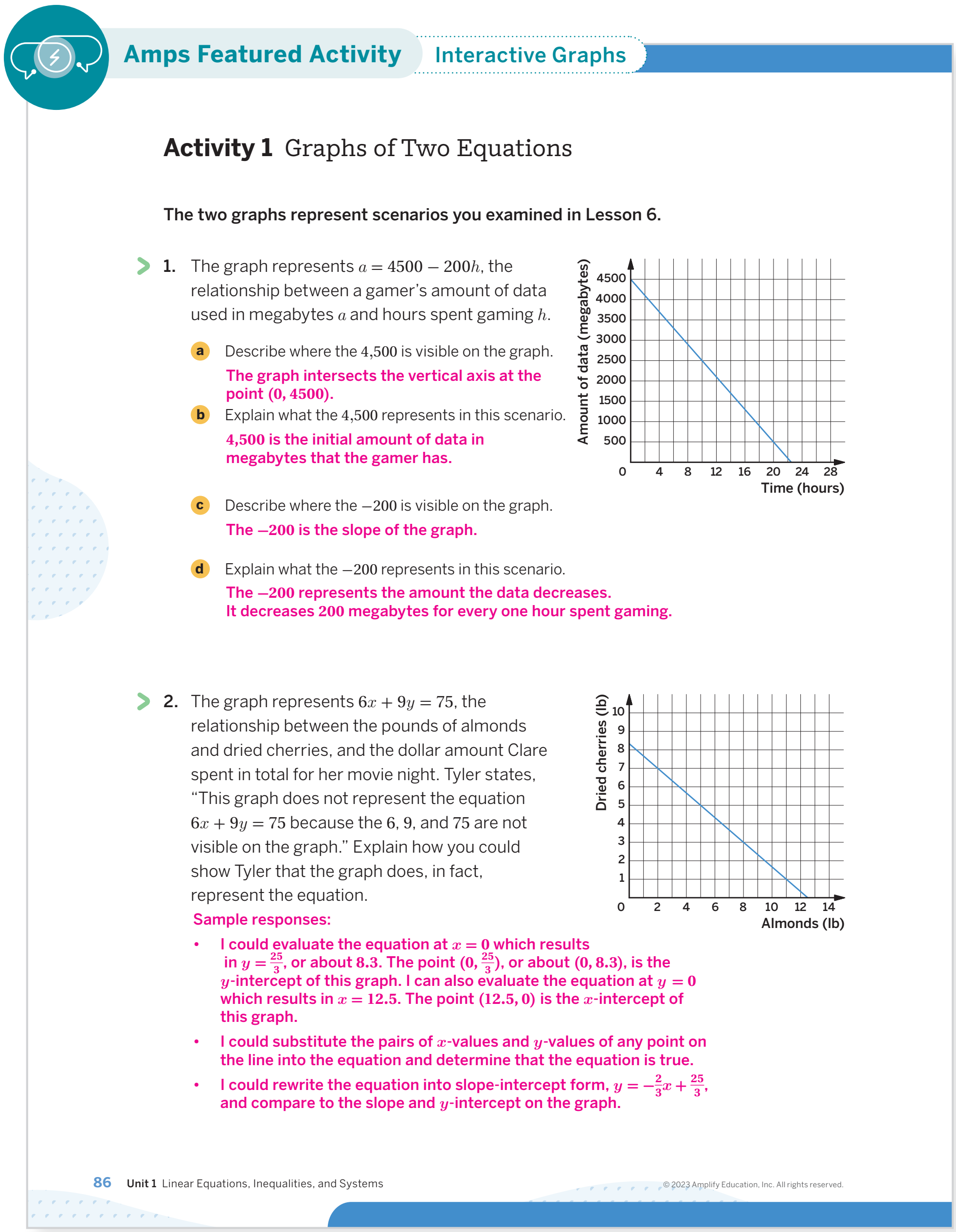
Informed by: Performance on Lesson 11, Practice Problem 5

Pairs | 15 min

Activity 1 Graphs of Two Equations

Students reason quantitatively and abstractly about an equation and graph to construct a logical explanation for a graph of an equation in standard form.

Amps Featured Activity Interactive Graphs



Activity 1 Graphs of Two Equations

The two graphs represent scenarios you examined in Lesson 6.

1. The graph represents $a = 4500 - 200h$, the relationship between a gamer's amount of data used in megabytes a and hours spent gaming h.

- a Describe where the 4,500 is visible on the graph.
  The graph intersects the vertical axis at the point (0, 4500).
- b Explain what the 4,500 represents in this scenario.
  4,500 is the initial amount of data in megabytes that the gamer has.
- c Describe where the −200 is visible on the graph.
  The −200 is the slope of the graph.
- d Explain what the −200 represents in this scenario.
  The −200 represents the amount the data decreases. It decreases 200 megabytes for every one hour spent gaming.

2. The graph represents $6x + 9y = 75$, the relationship between the pounds of almonds and dried cherries, and the dollar amount Clare spent in total for her movie night. Tyler states, "This graph does not represent the equation $6x + 9y = 75$ because the 6, 9, and 75 are not visible on the graph." Explain how you could show Tyler that the graph does, in fact, represent the equation.

Sample responses:

- **I could evaluate the equation at $x = 0$ which results in $y = \frac{25}{3}$, or about 8.3. The point $(0, \frac{25}{3})$, or about $(0, 8.3)$, is the y-intercept of this graph. I can also evaluate the equation at $y = 0$ which results in $x = 12.5$. The point $(12.5, 0)$ is the x-intercept of this graph.**
- **I could substitute the pairs of x-values and y-values of any point on the line into the equation and determine that the equation is true.**
- **I could rewrite the equation into slope-intercept form, $y = -\frac{2}{3}x + \frac{25}{3}$, and compare to the slope and y-intercept on the graph.**

86 **Unit 1** Linear Equations, Inequalities, and Systems © 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have the students discuss each problem in pairs, complete individually, and then compare solutions and patterns.

2 Monitor

Help students get started by having them organize what they know and notice about the context, equation, and graph.

Look for points of confusion:

- **Not considering fractional x- and y-intercepts in Problem 2.** Prompt students to explain other possible strategies to determine if the equation matches the graph.

Look for productive strategies:

- Determining and comparing the x- and y-intercepts.
- Rearranging the equation into slope-intercept form to use the slope and y-intercept.

3 Connect

Display the graph from Problem 2.

Have student-pairs share their thinking and strategies for Problem 2. Select and sequence students interpreting the points on the graph using the context first, ending with those rearranging the equation into slope-intercept form.

Ask:

- "Where can you see the values $\frac{25}{3}$ and $-\frac{2}{3}$ on the graph or in the equations?"
- "What does each of those values tell you about the almonds and the cherries?"

Highlight that the structure of an equation can provide insights about the properties of a graph. Solving for y can be an efficient strategy.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can engage with interactive graphs to help them interpret points and draw connections to the equations.

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils or highlighters and have students color code where they see the quantities in the graphs and the equations. For example, in Problem 1, have them color code 4,500 on the graph and in the equation with one color and −200 in another color.

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their strategies, listen for and collect vocabulary, gestures, patterns, and diagrams they use to explain their thinking. Record these on a visual display and update it throughout the remainder of the lesson. For example, students may use terms such as *slope, coefficient, initial value, vertical axis,* etc.

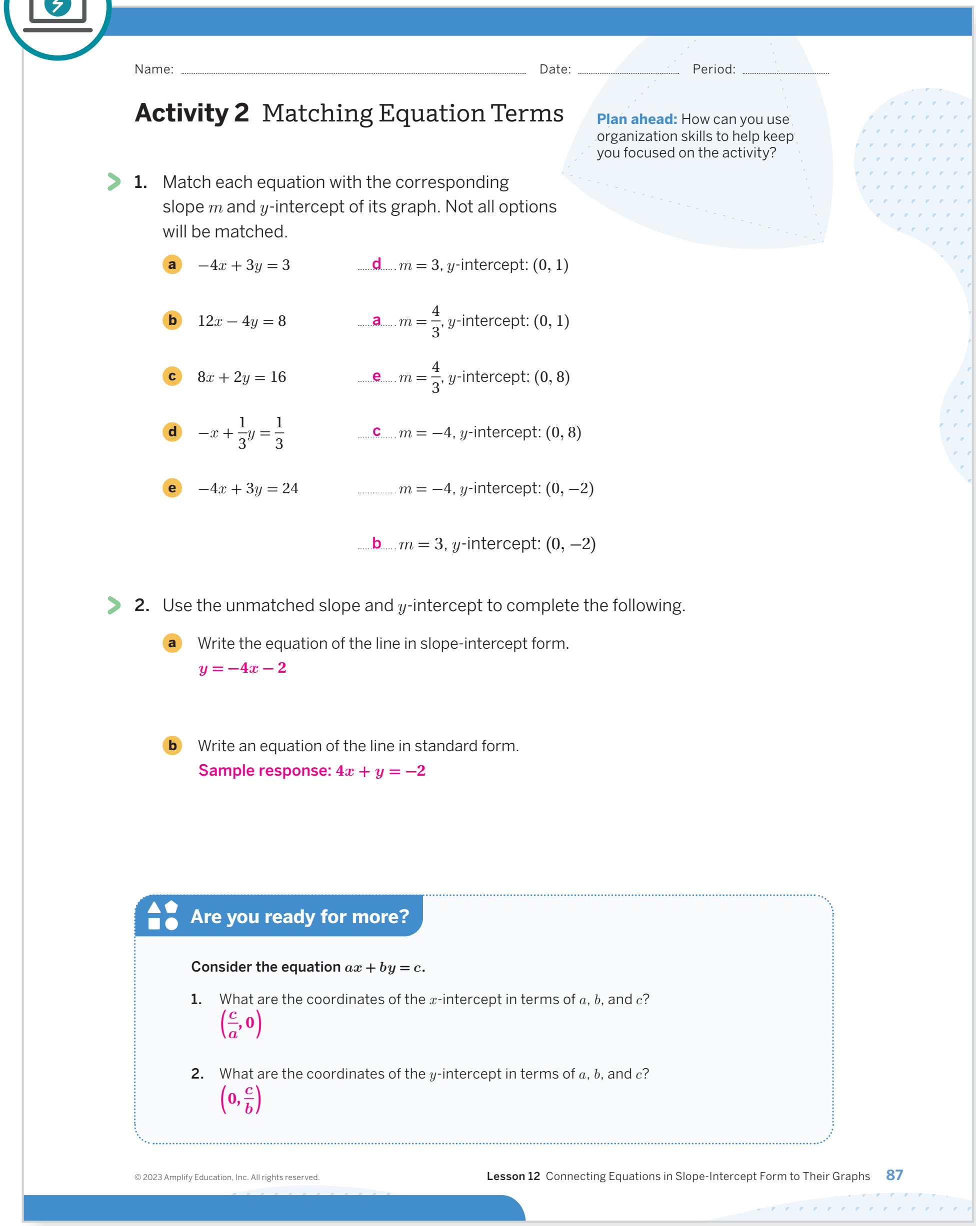
English Learners

Include a visual example of a graph and sample equation and annotate them with the terms and phrases students use to describe their strategies.

Pairs | 15 min

Activity 2 Matching Equation Terms

Students look for and make use of structure to determine the slope and y-intercept of equations in standard form without context to shift to reasoning symbolically and abstractly.



Name: ………… Date: ………… Period: …………

Activity 2 Matching Equation Terms

Plan ahead: How can you use organization skills to help keep you focused on the activity?

1. Match each equation with the corresponding slope m and y-intercept of its graph. Not all options will be matched.

- a $-4x + 3y = 3$
- b $12x - 4y = 8$
- c $8x + 2y = 16$
- d $-x + \frac{1}{3}y = \frac{1}{3}$
- e $-4x + 3y = 24$

- d $m = 3$, y-intercept: $(0, 1)$
- a $m = \frac{4}{3}$, y-intercept: $(0, 1)$
- e $m = \frac{4}{3}$, y-intercept: $(0, 8)$
- c $m = -4$, y-intercept: $(0, 8)$
- ……… $m = -4$, y-intercept: $(0, -2)$
- b $m = 3$, y-intercept: $(0, -2)$

2. Use the unmatched slope and y-intercept to complete the following.
   - a Write the equation of the line in slope-intercept form.
     $y = -4x - 2$
   - b Write an equation of the line in standard form.
     Sample response: $4x + y = -2$

Are you ready for more?

Consider the equation $ax + by = c$.

1. What are the coordinates of the x-intercept in terms of a, b, and c?
   $\left(\frac{c}{a}, 0\right)$
2. What are the coordinates of the y-intercept in terms of a, b, and c?
   $\left(0, \frac{c}{b}\right)$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 12 Connecting Equations in Slope-Intercept Form to Their Graphs 87

1 Launch

Have student-pairs take turns determining each match and explaining their strategy to their partner. Provide access to graphing technology or graph paper.

2 Monitor

Help students get started by prompting them to list and explain the strategies discussed in Activity 1 and selecting one to try.

Look for points of confusion:

- **Isolating x rather than y.** Ask, "What form of a linear equation helps identify the slope and y-intercept? What variable is isolated in this form?"
- **Dividing one of two terms by the coefficient of y.** Ask, "Will this result in equivalent equations?"

Look for productive strategies:

- Substituting the coordinates of the given y-intercepts.
- Rearranging the equation into slope-intercept form.
- Recognizing $-\frac{a}{b} = m$ and $\frac{c}{b}$ is the y-intercept.

3 Connect

Have pairs of students share their strategies and any patterns noticed. Consider conducting a ***Gallery Tour***, selecting and sequencing common points of confusion first, followed by productive strategies in the order listed.

Ask, "What do you notice about the coefficients of the standard form equation and the corresponding slope and y-intercept?"

Highlight that in standard form, the slope is the opposite of the ratio between the coefficients of x and y.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, consider omitting Problems 1d, 1e, and 2. Consider removing the option $m = 4$, y-intercept: $(0, -2)$.

Accessibility: *Guide Processing and Visualization*

Consider displaying the Anchor Chart PDF, *Sentence Stems, Matching Prompts* for students to refer to as they take turns determining matches and explaining their thinking.

Math Language Development

MLR7: Compare and Connect

As students respond to the Ask question, draw connections between the coefficients of the terms in standard form and slope-intercept form. Ask students to solve the equation $ax + by = c$ for y, write it in slope-intercept form, and describe what they notice. Encourage them to use mathematical language, such as *ratio* and *coefficient*.

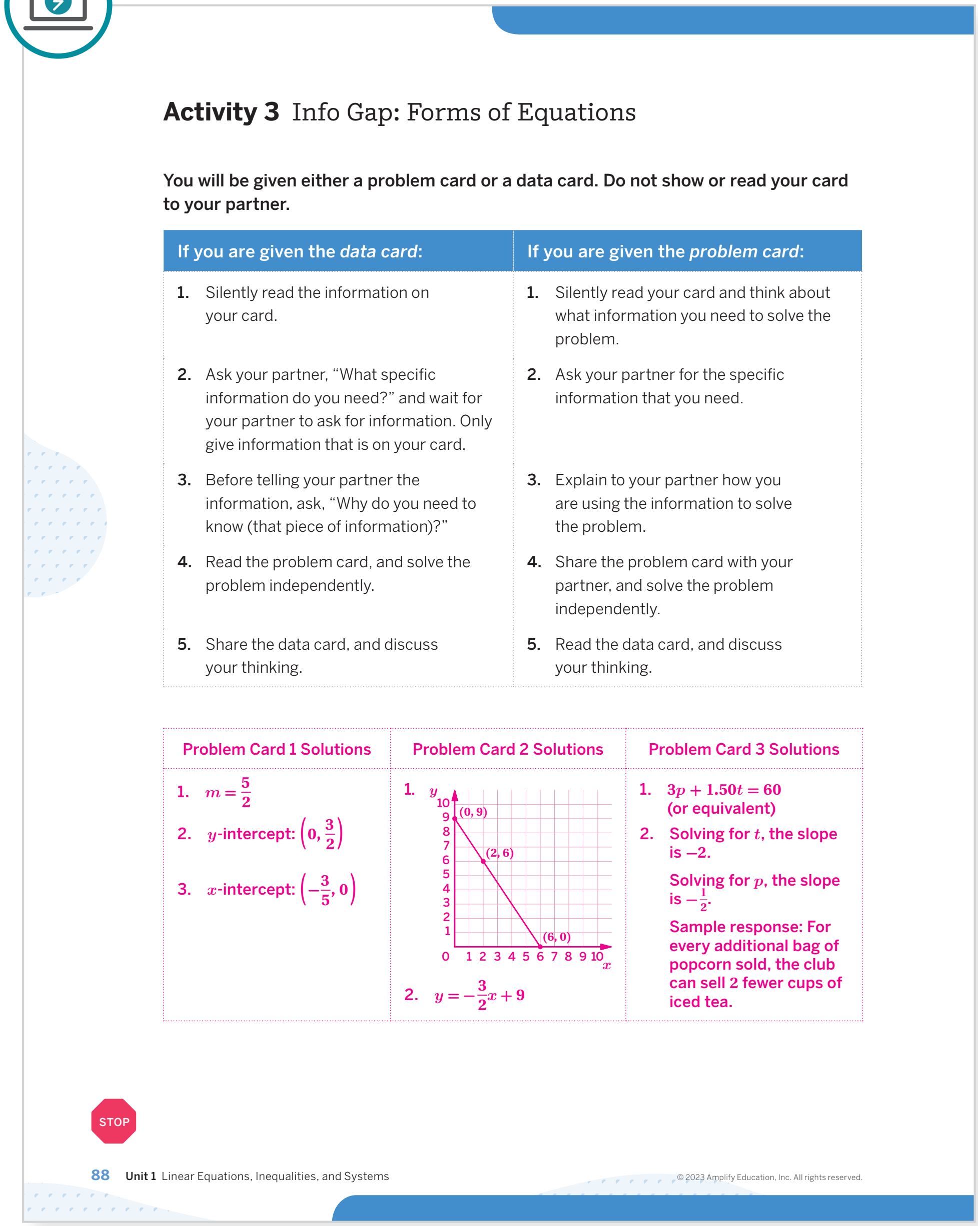
$y = \frac{c - ax}{b}$ or $y = -\frac{a}{b}x + \frac{c}{b}$; Sample response: The slope m is the opposite of the ratio of the coefficients a and b. The y-intercept is the ratio of the constant c to the coefficient of y, which is b.

Optional

Pairs | 15 min

Activity 3 Info Gap: Forms of Equations

Students determine and request the information needed to write, graph, and interpret linear equations written in different forms.



Activity 3 Info Gap: Forms of Equations

You will be given either a problem card or a data card. Do not show or read your card to your partner.

If you are given the *data card*:	If you are given the *problem card*:
1. Silently read the information on your card.	**1.** Silently read your card and think about what information you need to solve the problem.
2. Ask your partner, "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card.	**2.** Ask your partner for the specific information that you need.
3. Before telling your partner the information, ask, "Why do you need to know (that piece of information)?"	**3.** Explain to your partner how you are using the information to solve the problem.
4. Read the problem card, and solve the problem independently.	**4.** Share the problem card with your partner, and solve the problem independently.
5. Share the data card, and discuss your thinking.	**5.** Read the data card, and discuss your thinking.

Problem Card 1 Solutions	Problem Card 2 Solutions	Problem Card 3 Solutions
1. $m = \frac{5}{2}$ 2. y-intercept: $\left(0, \frac{3}{2}\right)$ 3. x-intercept: $\left(-\frac{3}{5}, 0\right)$	1. (graph) 2. $y = -\frac{3}{2}x + 9$	1. $3p + 1.50t = 60$ (or equivalent) 2. Solving for t, the slope is -2. Solving for p, the slope is $-\frac{1}{2}$. Sample response: For every additional bag of popcorn sold, the club can sell 2 fewer cups of iced tea.

STOP

88 Unit 1 Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Explain the ***Info Gap*** instructional routine and display the Instructional Routine PDF, *Info Gap: Instructions*. Consider demonstrating the routine if students are unfamiliar with it. Provide a problem and data card to each pair of students from the Activity 3 PDF. **Note:** Graphing technology should not be used.

2 Monitor

Help students get started by modeling the questioning process.

Look for points of confusion:

- **Having difficulty discerning which information to use in Data Card 1.** Have students list the different strategies discussed in Activity 2.

Look for productive strategies:

- Creating a table to evaluate the equations for different values.
- Calculating the slope from given points.
- Graphing the given points and analyzing the graph.
- Rearranging equations into different forms to get the information necessary.

3 Connect

Have pairs of students share their strategies or any challenges they experienced.

Ask:

- "What strategies do you use to determine the slope and y-intercept of an equation in standard form?"
- "When should you rearrange a standard form equation into a slope-intercept form equation? Why?"

Highlight the different possible strategies to determine the slope and y-intercept of a line given an equation in standard form.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. Consider saying, "It looks like I need to determine the features of a particular graph, but I do not know anything about the graph. I think I could determine the slope if I knew two points that the line passes through. I will ask for two points that the line passes through."

Accessibility: *Vary Demands to Optimize Challenge*

Have students complete Problem Cards 1 and 2. Consider omitting Problem Card 3, or display the first sentence of Data Card 3 before students start working with Problem Card 3.

Math Language Development

MLR4: Information Gap

Consider displaying the Anchor Chart PDF, *Sentence Stems, Types of Questioning* for students who need a starting point to form questions.

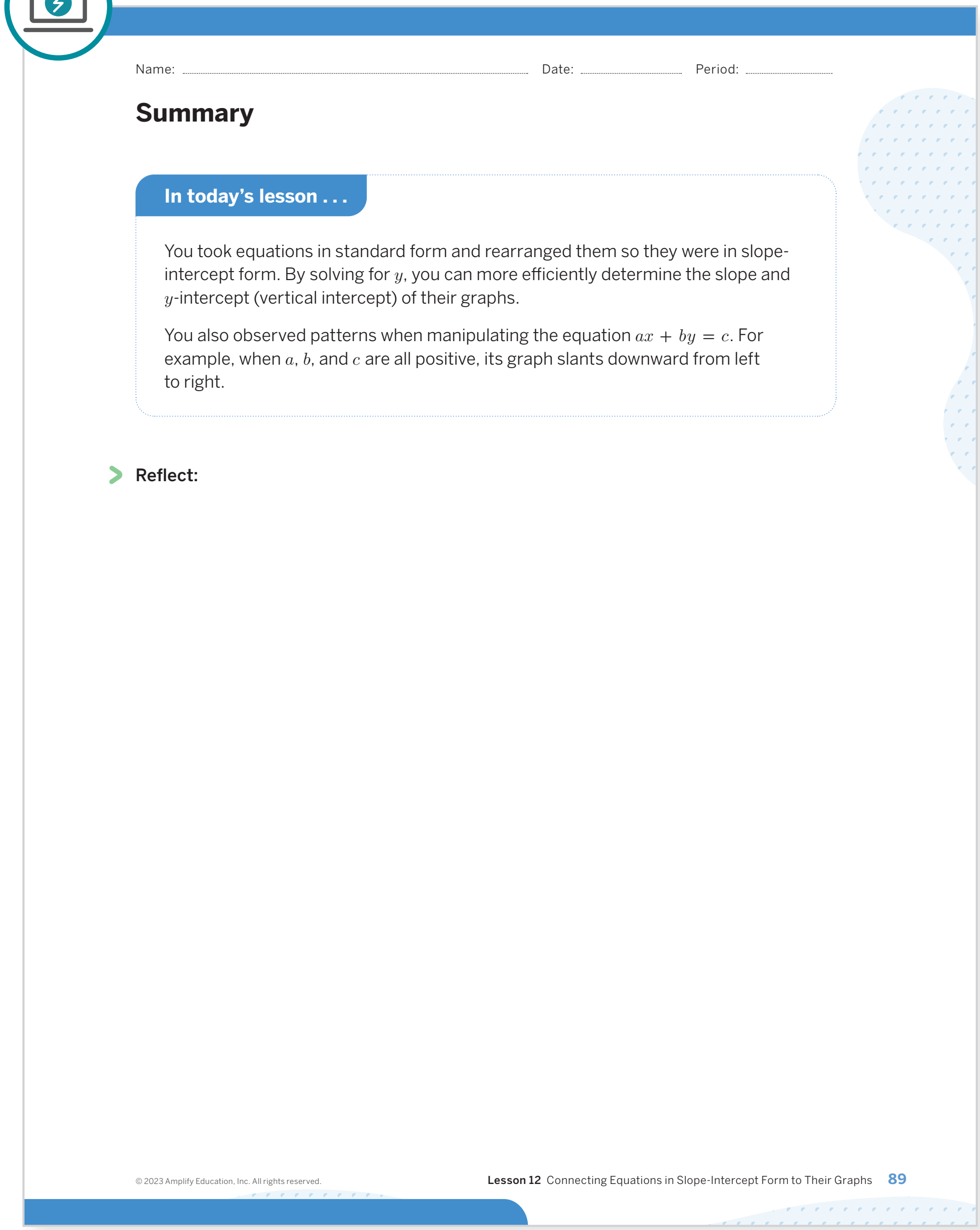
English Learners

Consider providing sample questions students could ask, such as:

- What is the equation of the line? (Problem Cards 1 and 2)
- What is the scenario about? What are the two variables? (Problem Card 3)
- What constraints are given? (Problem Card 3)

Summary

Review and synthesize relating the structure of equations to the scenario and corresponding graphs.



Name: Date: Period:

Summary

In today's lesson . . .

You took equations in standard form and rearranged them so they were in slope-intercept form. By solving for y, you can more efficiently determine the slope and y-intercept (vertical intercept) of their graphs.

You also observed patterns when manipulating the equation $ax + by = c$. For example, when a, b, and c are all positive, its graph slants downward from left to right.

Reflect:

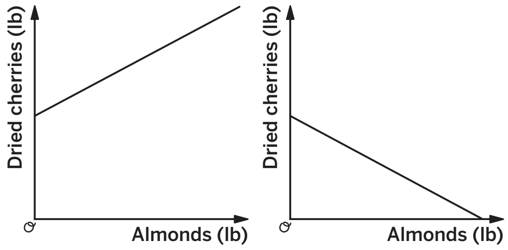
© 2023 Amplify Education, Inc. All rights reserved.

Lesson 12 Connecting Equations in Slope-Intercept Form to Their Graphs 89

Synthesize

Display the following scenario:

Suppose Clare went back to the store to get more almonds and dried cherries but spent \$108 this time. Almonds cost \$6 a pound and dried cherries cost \$9 a pound. Clare's purchase can be represented by the equation $6x + 9y = 108$. Consider the two graphs that represent the relationship between pounds of almonds x, and pounds of dried cherries y.



Ask:

- "Without performing any calculations, can you determine which graph represents the equation $6x + 9y = 108$? Explain your thinking."
- "What does the vertical intercept represent in this scenario?"
- "What is the slope of the graph? Explain your strategy for determining it."
- "What information does the slope provide about the almonds and cherries? Explain your thinking."

Highlight that students can determine the slope and y-intercept of a line in a standard form equation using different strategies, including rearranging to the equivalent form in slope-intercept form.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

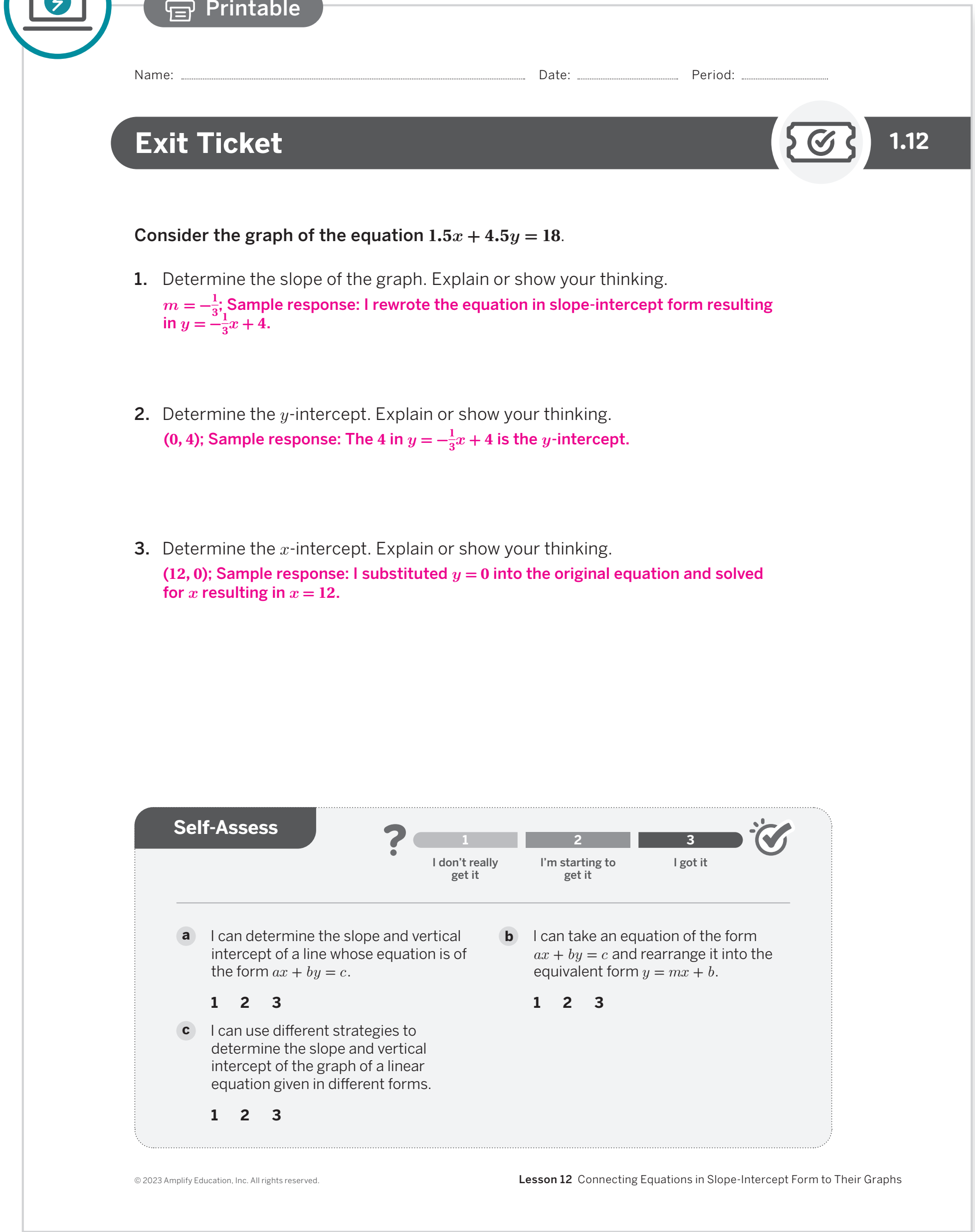
- "What information do equations in standard form and slope-intercept form tell you?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the slope and x- and y-intercepts of an equation in standard form.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.12

Consider the graph of the equation $1.5x + 4.5y = 18$.

1. Determine the slope of the graph. Explain or show your thinking.
   $m = -\frac{1}{3}$; Sample response: I rewrote the equation in slope-intercept form resulting in $y = -\frac{1}{3}x + 4$.
2. Determine the y-intercept. Explain or show your thinking.
   $(0, 4)$; Sample response: The 4 in $y = -\frac{1}{3}x + 4$ is the y-intercept.
3. Determine the x-intercept. Explain or show your thinking.
   $(12, 0)$; Sample response: I substituted $y = 0$ into the original equation and solved for x resulting in $x = 12$.

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can determine the slope and vertical intercept of a line whose equation is of the form $ax + by = c$.

1 2 3

b I can take an equation of the form $ax + by = c$ and rearrange it into the equivalent form $y = mx + b$.

1 2 3

c I can use different strategies to determine the slope and vertical intercept of the graph of a linear equation given in different forms.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 12 Connecting Equations in Slope-Intercept Form to Their Graphs

Success looks like . . .

- **Goal:** Determining the slope and vertical intercept of the graphs of linear equations by making use of structure or by rearranging the equations.
  - » Determining the slope and y-intercept of the graph by rewriting the equation in slope-intercept form in Problems 1 and 2.
- **Goal:** Given an equation of the form $ax + by = c$, writing an equivalent equation of the form $y = mx + b$.

Suggested next steps

If students incorrectly calculate the slope in Problem 1, consider:

- Reviewing strategies from Activity 2.
- Assigning Practice Problem 1.

If students incorrectly calculate the y-intercept in Problem 2, consider:

- Reviewing strategies from Activity 2.
- Assigning Practice Problem 2.

If students incorrectly calculate the x-intercept in Problem 3, consider:

- Assigning Practice Problem 3.

Professional Learning

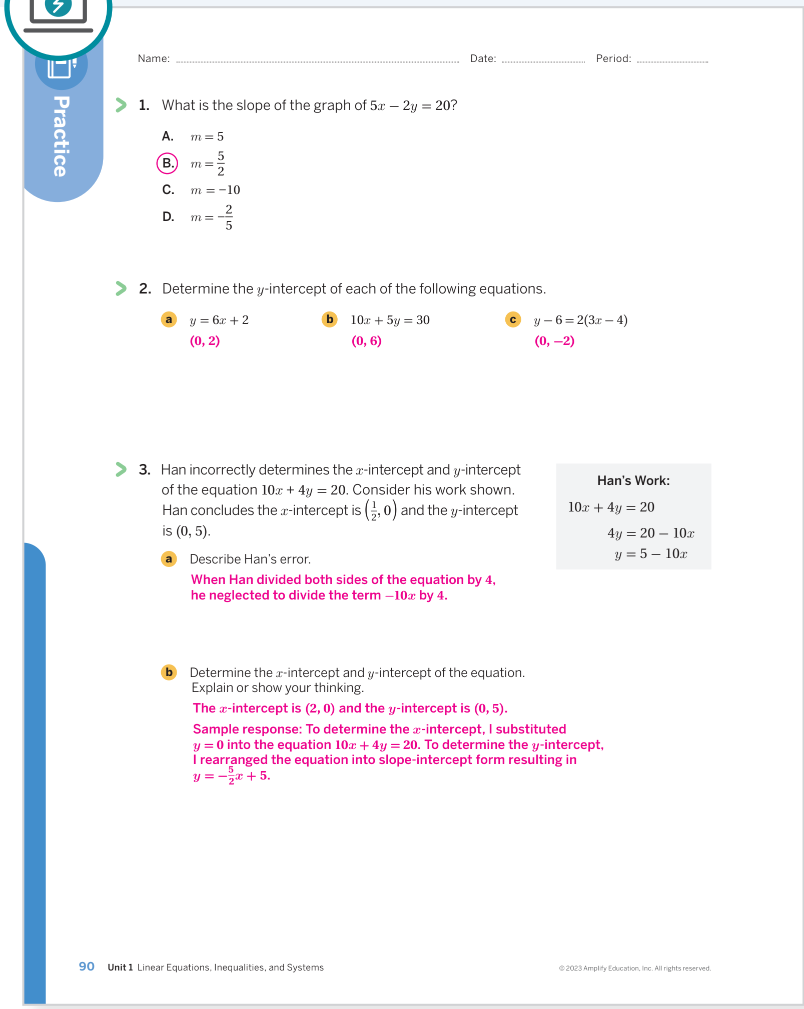
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which teacher actions made Activity 1 strong?
- What trends do you see in participation? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. What is the slope of the graph of $5x - 2y = 20$?

   A. $m = 5$

   [✓ B.] $m = \frac{5}{2}$

   C. $m = -10$

   D. $m = -\frac{2}{5}$

2. Determine the y-intercept of each of the following equations.

   a $y = 6x + 2$ **(0, 2)**

   b $10x + 5y = 30$ **(0, 6)**

   c $y - 6 = 2(3x - 4)$ **(0, –2)**

3. Han incorrectly determines the x-intercept and y-intercept of the equation $10x + 4y = 20$. Consider his work shown. Han concludes the x-intercept is $\left(\frac{1}{2}, 0\right)$ and the y-intercept is $(0, 5)$.

   Han's Work:

   $$10x + 4y = 20$$
   $$4y = 20 - 10x$$
   $$y = 5 - 10x$$

   a Describe Han's error.

   When Han divided both sides of the equation by 4, he neglected to divide the term $-10x$ by 4.

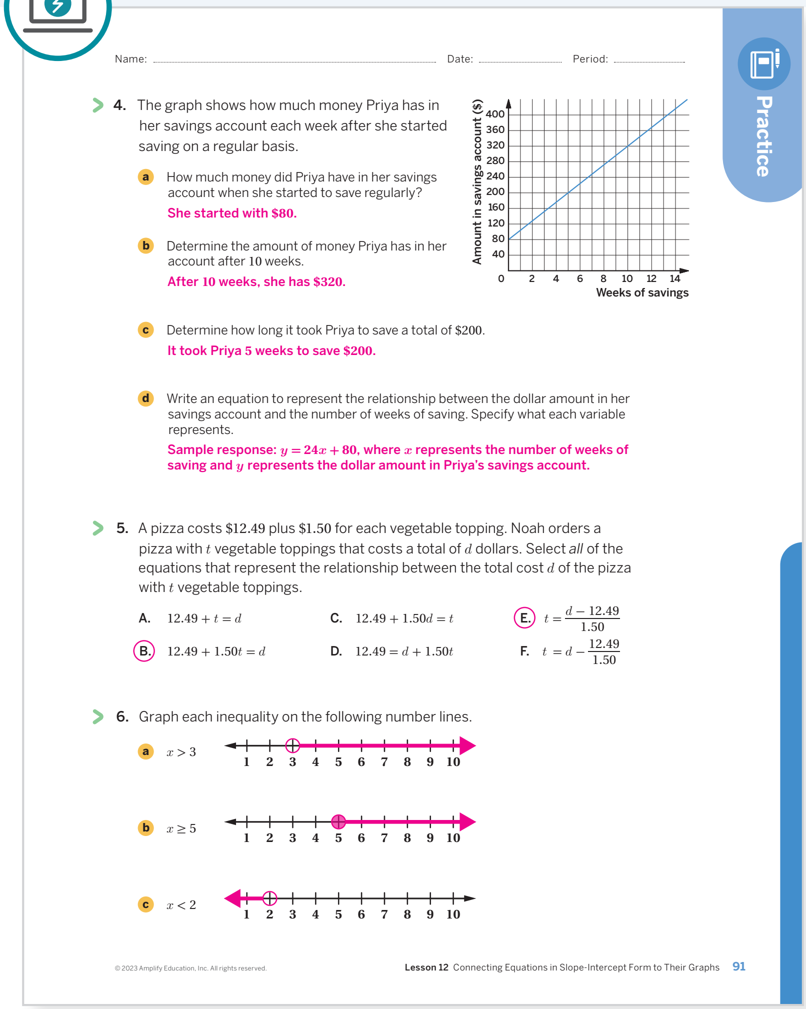
   b Determine the x-intercept and y-intercept of the equation. Explain or show your thinking.

   The x-intercept is (2, 0) and the y-intercept is (0, 5).

   Sample response: To determine the x-intercept, I substituted $y = 0$ into the equation $10x + 4y = 20$. To determine the y-intercept, I rearranged the equation into slope-intercept form resulting in $y = -\frac{5}{2}x + 5$.

90 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.



Name: Date: Period:

4. The graph shows how much money Priya has in her savings account each week after she started saving on a regular basis.

   a How much money did Priya have in her savings account when she started to save regularly?

   She started with $80.

   b Determine the amount of money Priya has in her account after 10 weeks.

   After 10 weeks, she has $320.

   c Determine how long it took Priya to save a total of $200.

   It took Priya 5 weeks to save $200.

   d Write an equation to represent the relationship between the dollar amount in her savings account and the number of weeks of saving. Specify what each variable represents.

   Sample response: $y = 24x + 80$, where x represents the number of weeks of saving and y represents the dollar amount in Priya's savings account.

5. A pizza costs $12.49 plus $1.50 for each vegetable topping. Noah orders a pizza with t vegetable toppings that costs a total of d dollars. Select *all* of the equations that represent the relationship between the total cost d of the pizza with t vegetable toppings.

   A. $12.49 + t = d$

   [✓ B.] $12.49 + 1.50t = d$

   C. $12.49 + 1.50d = t$

   D. $12.49 = d + 1.50t$

   [✓ E.] $t = \frac{d - 12.49}{1.50}$

   F. $t = d - \frac{12.49}{1.50}$

6. Graph each inequality on the following number lines.

   a $x > 3$

   b $x \geq 5$

   c $x < 2$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 12 Connecting Equations in Slope-Intercept Form to Their Graphs 91

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	1
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 6	2
	5	Unit 1 Lesson 10	2
Formative	6	Unit 1 Lesson 13	1

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 3

Solving Inequalities and Graphing Their Solutions

Whole Class

In this Sub-Unit, students learn that regardless of whether they work, intern, or attend college, time and money are decision drivers. They discover that using inequalities can help them to manage their time and money.

SUB-UNIT

3 Solving Inequalities and Graphing Their Solutions

Narrative Connections

What's after high school?

Imagine it's senior year. In a few short months, you'll finally be out of high school. Most of your classmates will likely go straight to college or enter the workforce. For many, those might be the right choices. But those aren't the *only* choices.

Some graduates take what's called a "gap year." These students take a year off from their studies or employment to travel, work on community projects, or develop a skill. That way, when they continue their education or start a new job, they're more experienced, mature, and have a stronger sense of purpose.

What life after high school looks like will come down to any constraints that might be in place, as well as the choices you make — whether it's taking a gap year, attending a college, or pursing a career. You will have to weigh your options carefully, scrutinizing what each path offers and comparing their pros and cons.

In this next set of lessons, You will learn how to make these kinds of comparisons mathematically. That way, you can see the impact of those options more clearly and choose the one that's best for you. But in order to do that, we must first pay a visit to our old friend: inequalities.

© 2023 Amplify Education, Inc. All rights reserved.

Sub-Unit 3 Solving Inequalities and Graphing Their Solutions 93

Narrative Connections

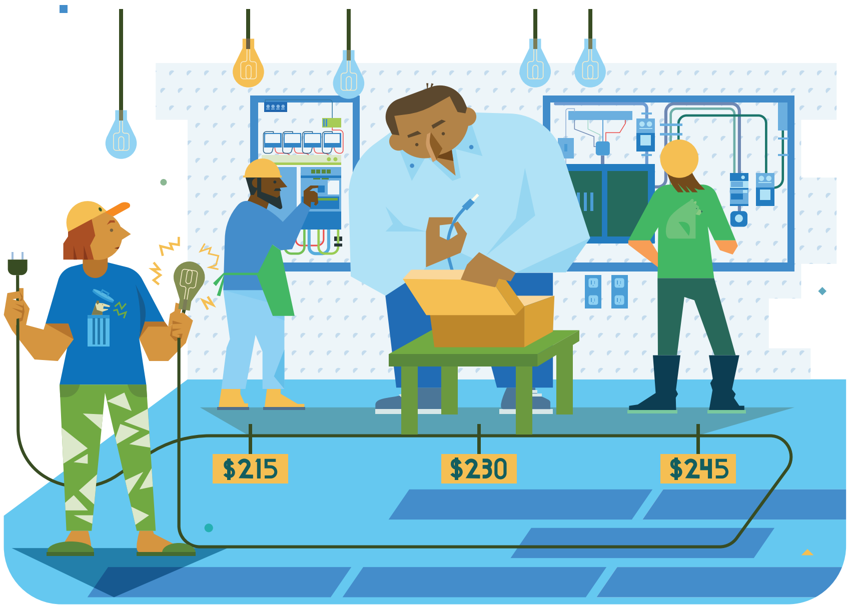
Read the narrative aloud as a class or have students read it individually. Students will continue exploring inequalities across various post-secondary settings (in the workplace, in education, and within a gap year) in the following places:

- **Lesson 13, Activity 3:** Union Dues
- **Lesson 15, Activity 3:** Continuing Education
- **Lesson 16, Activity 1:** Gap Year Options

UNIT 1 | **LESSON 13**

Inequalities and Their Solutions

Let’s solve problems by writing and solving inequalities in one variable.



Focus

Goals

1. **Language Goal:** Understand that the solution to an inequality is a range of values that make the inequality true. **(Speaking and Listening, Reading and Writing)**
2. Analyze and use the structure in inequalities to determine whether the solution is greater or less than the solution to a related equation.
3. **Language Goal:** Write and solve inequalities in one variable to represent the constraints in situations and solve problems. **(Reading and Writing)**

Coherence

Today

Students build on their Grade 7 understanding of inequality solutions, and are introduced to the term “solution set.” They write an inequality in one variable to model scenarios. They investigate different strategies for determining the solution set to an inequality — by reasoning about the quantities and by substituting a value into the inequality and checking to see it makes the inequality true.

Previously

In Grade 7, students solved word problems by solving inequalities in one variable.

Coming Soon

Students will solve inequalities in two variables by graphing in Lesson 14.

Rigor

- Students build on their **conceptual understanding** of solutions of inequalities in one variable with and without context.
- Students practice graphing solutions of inequalities to build **procedural fluency**.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	12 min	12 min	12 min	5 min	5 min
Independent	Independent	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)

Math Language Development

New words

- ***solution set****

Review words

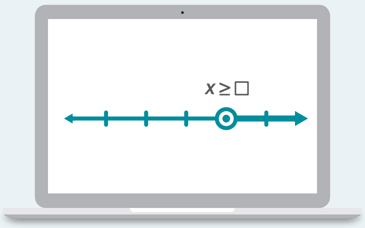
- *boundary value*
- *constraint*
- *inequality*
- *line segment*
- *ray*

*Students may confuse the mathematical term *solution* with the scientific term that refers to a liquid mixture. Be ready to address the differences between them.

Amps | **Featured Activity**

Activity 1
Digital Card Sort

Students solve inequalities in one variable, then match the inequalities with their graphs.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated or overwhelmed in Activity 3 as they make attempts and fail to write inequalities to model the scenario and make sense of the problem solving method. Encourage students to note what information they obtained from the text and give authentic feedback when students demonstrate perseverance (e.g. "I noticed you asked a peer for help and tried their suggestion.")

Modifications to Pacing

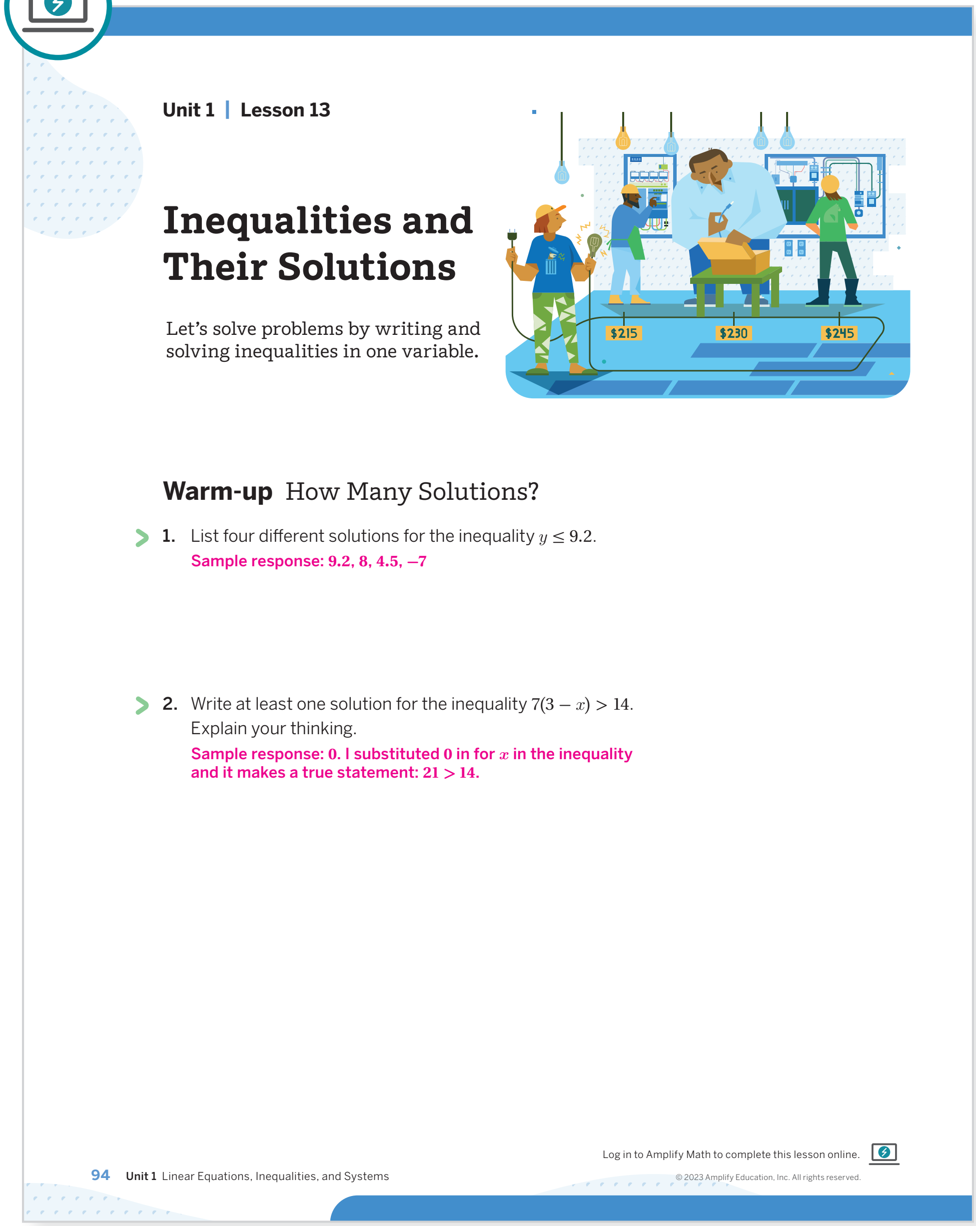
You may want to consider this additional modification if you are short on time.

- **Activity 1** may be omitted, if students demonstrate proficiency in the Warm-up.

Independent | 5 min

Warm-up How Many Solutions?

Students determine solutions of inequalities to activate prior knowledge of inequality solutions.



Unit 1 | **Lesson 13**

Inequalities and Their Solutions

Let's solve problems by writing and solving inequalities in one variable.

Warm-up How Many Solutions?

1. List four different solutions for the inequality $y \leq 9.2$.
   Sample response: 9.2, 8, 4.5, –7

2. Write at least one solution for the inequality $7(3 - x) > 14$. Explain your thinking.
   Sample response: 0. I substituted 0 in for x in the inequality and it makes a true statement: $21 > 14$.

Log in to Amplify Math to complete this lesson online.

94 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Activate students' prior knowledge by asking them the difference between solutions to an equation and an inequality. Set an expectation for the amount of time students will have to work individually on the Warm-up.

2 Monitor

Help students get started by having them substitute values into the inequality and determine if they make a true statement.

Look for points of confusion:

- **Choosing only positive values or only one solution for x.** Have students check multiple values, including negative values.
- **Not changing the inequality sign for Problem 2 if they solved for x.** Have students check solutions in the original inequality and determine what should change in their final inequality.

Look for productive strategies:

- Using a number line.
- Substituting different values in Problem 2.
- Isolating x to solve the inequality in Problem 2.

3 Connect

Have individual students share one solution to each inequality.

Define the term ***solution set*** as a set of values which satisfy a given inequality.

Highlight that students can use a number line to concisely show the solution set to an inequality, but they can also write another inequality that shows the same information.

Ask, "How does the solution to the equation $7(3 - x) = 14$ relate to the solution set to the inequality?" The solution to the equation, $x = 1$, is the boundary value used in the inequality representing the solution set.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students respond to the Ask question, display both the inequality $7(3 - x) > 14$ and the equation $7(3 - x) = 14$. Ask them to substitute $x = 1$ into each and describe what they notice. Draw students' attention to the fact that the inequality is a false statement, $14 \not> 14$, and the equation is a true statement, $14 = 14$. Ask students to explain why this is the case. The value 14 is the boundary value for the inequality and the inequality symbol did not include the boundary value.

If students do not use the term *boundary value*, display this term and activate prior knowledge as to what this term means.

Power-up

To power up students' ability to graph the solution set to an inequality on a number line:

Provide students with a copy of the Power-up PDF.

Use: Before the Warm-up

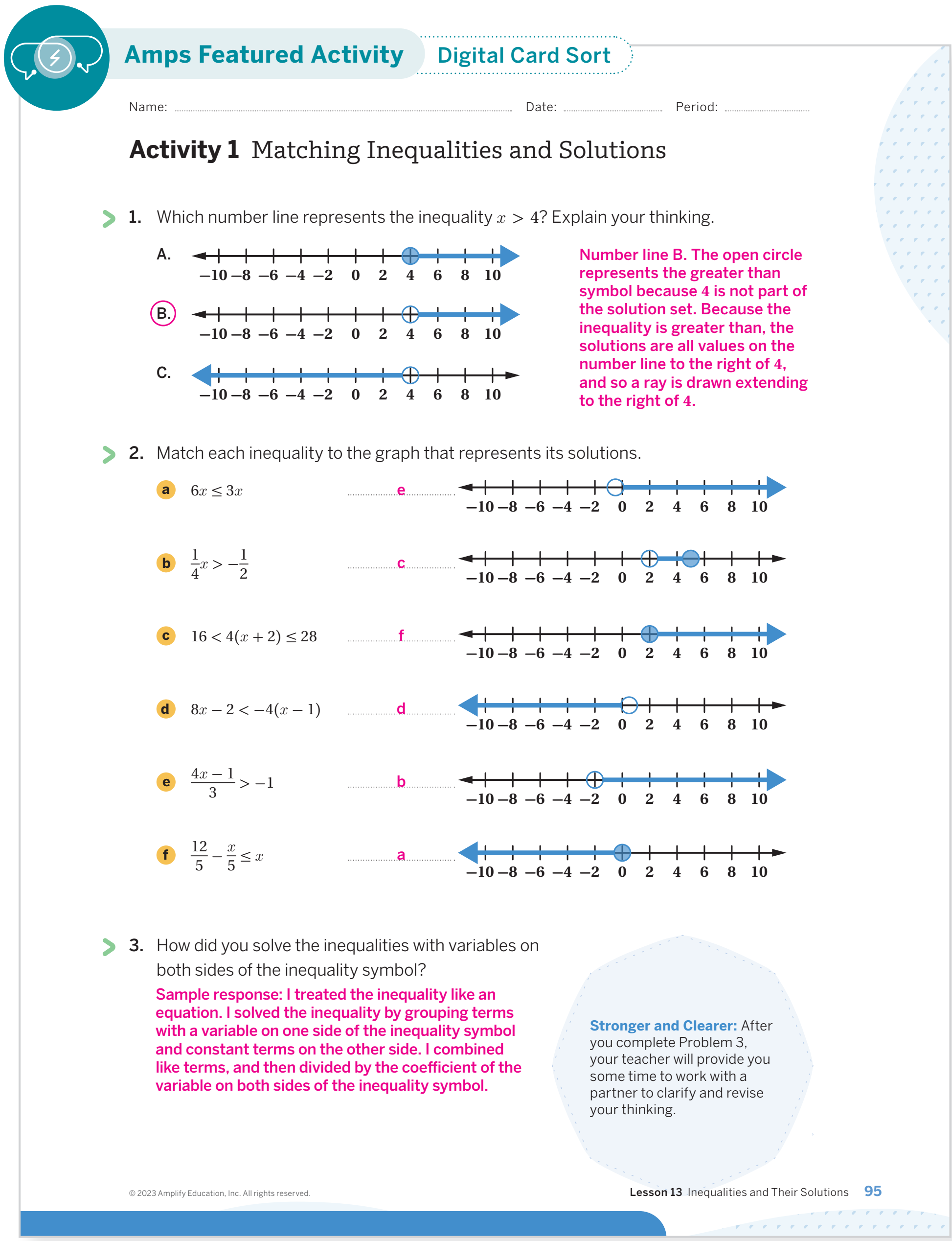
Informed by: Performance on Lesson 12, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8

Independent | 12 min

Activity 1 Matching Inequalities and Solutions

Students match inequalities in one variable with solutions represented on number lines to activate procedural skills from Grade 7.

Amps Featured Activity Digital Card Sort



Name: Date: Period:

Activity 1 Matching Inequalities and Solutions

1. Which number line represents the inequality $x > 4$? Explain your thinking.

A.

B.

C.

Number line B. The open circle represents the greater than symbol because 4 is not part of the solution set. Because the inequality is greater than, the solutions are all values on the number line to the right of 4, and so a ray is drawn extending to the right of 4.

2. Match each inequality to the graph that represents its solutions.

a $6x \leq 3x$ e

b $\frac{1}{4}x > -\frac{1}{2}$ c

c $16 < 4(x + 2) \leq 28$ f

d $8x - 2 < -4(x - 1)$ d

e $\frac{4x - 1}{3} > -1$ b

f $\frac{12}{5} - \frac{x}{5} \leq x$ a

3. How did you solve the inequalities with variables on both sides of the inequality symbol?

Sample response: I treated the inequality like an equation. I solved the inequality by grouping terms with a variable on one side of the inequality symbol and constant terms on the other side. I combined like terms, and then divided by the coefficient of the variable on both sides of the inequality symbol.

Stronger and Clearer: After you complete Problem 3, your teacher will provide you some time to work with a partner to clarify and revise your thinking.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 13 Inequalities and Their Solutions 95

1 Launch

Have students complete Problem 1 in pairs, then discuss as a class before having students work independently.

2 Monitor

Help students get started by labeling the inequality symbol for each problem.

Look for points of confusion:

Dividing by x for Problem 2. Remind students of valid operations used when solving equations from Lesson 8.

Look for productive strategies:

- Substituting the value of the circle's location to determine if the value is an inequality's boundary value, which helps narrow choices.
- Noticing the type of inequality symbol used to eliminate number lines with open or filled circles.

3 Connect

Highlight that a boundary value is the open or filled in point on the number line. The ray graphed from the boundary value represents the infinite solutions that are in the solution set.

Have individual students share their strategy for determining if the point is open or filled.

Ask, "Why did you use a line segment for Problem 2c?" The solution set has two boundary values, and does not extend infinitely in either direction.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display a table like the one shown for students to record their takeaways from the Connect discussion.

Inequality symbol	Open or closed circle?	Why?
> <		
≥ ≤		

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 3, provide them time to share their responses with a partner. Have partners provide feedback to help improve the clarity of responses, such as including an example or using mathematical terms (*variable terms, constant terms, combine like terms, coefficient,* etc). Then provide time for students to revise their responses.

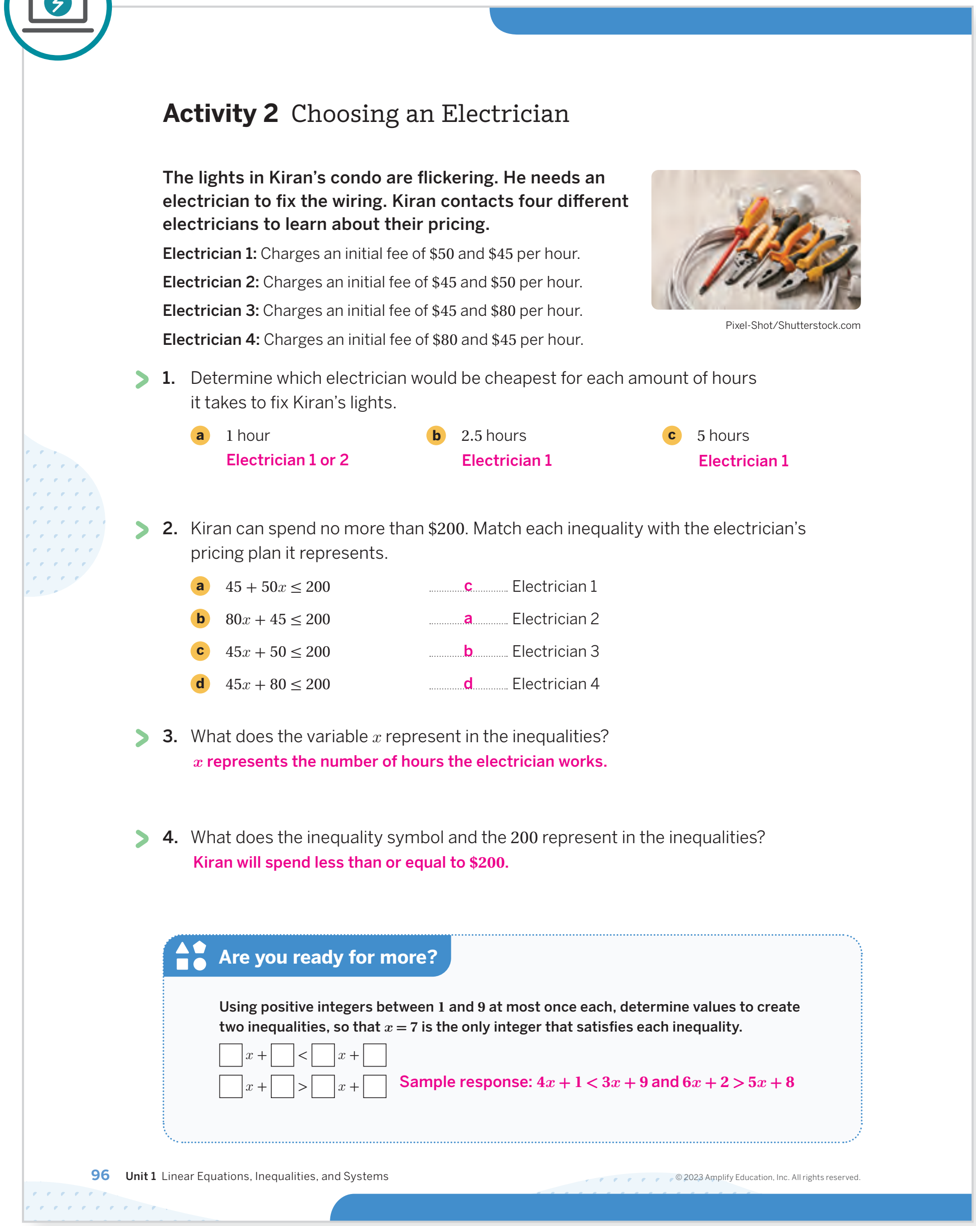
English Learners

Consider pairing a stronger English speaker with a student who is developing their English proficiency.

Pairs | 12 min

Activity 2 Choosing an Electrician

Students match inequalities that model a constraint to reason quantitatively and abstractly about their solutions.



Activity 2 Choosing an Electrician

The lights in Kiran's condo are flickering. He needs an electrician to fix the wiring. Kiran contacts four different electricians to learn about their pricing.

Electrician 1: Charges an initial fee of $50 and $45 per hour.

Electrician 2: Charges an initial fee of $45 and $50 per hour.

Electrician 3: Charges an initial fee of $45 and $80 per hour.

Electrician 4: Charges an initial fee of $80 and $45 per hour.

Pixel-Shot/Shutterstock.com

1. Determine which electrician would be cheapest for each amount of hours it takes to fix Kiran's lights.

   a. 1 hour
   Electrician 1 or 2

   b. 2.5 hours
   Electrician 1

   c. 5 hours
   Electrician 1

2. Kiran can spend no more than $200. Match each inequality with the electrician's pricing plan it represents.

   a. $45 + 50x \leq 200$
   b. $80x + 45 \leq 200$
   c. $45x + 50 \leq 200$
   d. $45x + 80 \leq 200$

   c Electrician 1
   a Electrician 2
   b Electrician 3
   d Electrician 4

3. What does the variable x represent in the inequalities?
   x represents the number of hours the electrician works.

4. What does the inequality symbol and the 200 represent in the inequalities?
   Kiran will spend less than or equal to $200.

Are you ready for more?

Using positive integers between 1 and 9 at most once each, determine values to create two inequalities, so that $x = 7$ is the only integer that satisfies each inequality.

$\square x + \square < \square x + \square$

$\square x + \square > \square x + \square$

Sample response: $4x + 1 < 3x + 9$ and $6x + 2 > 5x + 8$

96 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the narrative aloud. Arrange students in pairs. Have them discuss the electricians' descriptions before working independently.

2 Monitor

Help students get started by prompting them to write an expression for each electrician.

Look for points of confusion:

- **Having difficulty matching the expression for Electrician 3.** Ask, "How is Electrician 3's pricing different from all the others? Which inequality reflects Electrician 3's cost for up to two hours of work?"

Look for productive strategies:

- Eliminating answer choices based on hourly rate, and then initial fee.
- Writing an algebraic expression for each electrician.
- Creating a table to compare the electricians' fees.
- Graphing the solutions to compare the fees.

3 Connect

Have pairs of students share their strategies for matching the inequalities and thinking for Problem 4.

Ask, "Why do all the electricians use the same inequality symbol?" Because the budget is $200, the amount charged for the job has to be less than or equal to $200.

Highlight that the initial charge is the constant in the expression on the left, and the hourly rate is the coefficient of x.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils or highlighters and have students color code the initial fee for each electrician in one color and the hourly rate in another color. Consider displaying a template of an inequality in slope-intercept form: $y \leq$ hourly rate • x + initial fee.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

- **Read 1:** Students should understand that Kiran is choosing from 4 electricians who charge varying amounts.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as the initial fee each electrician charges.
- **Read 3:** Ask students to think about how they could use variables to represent each electrician's fees.

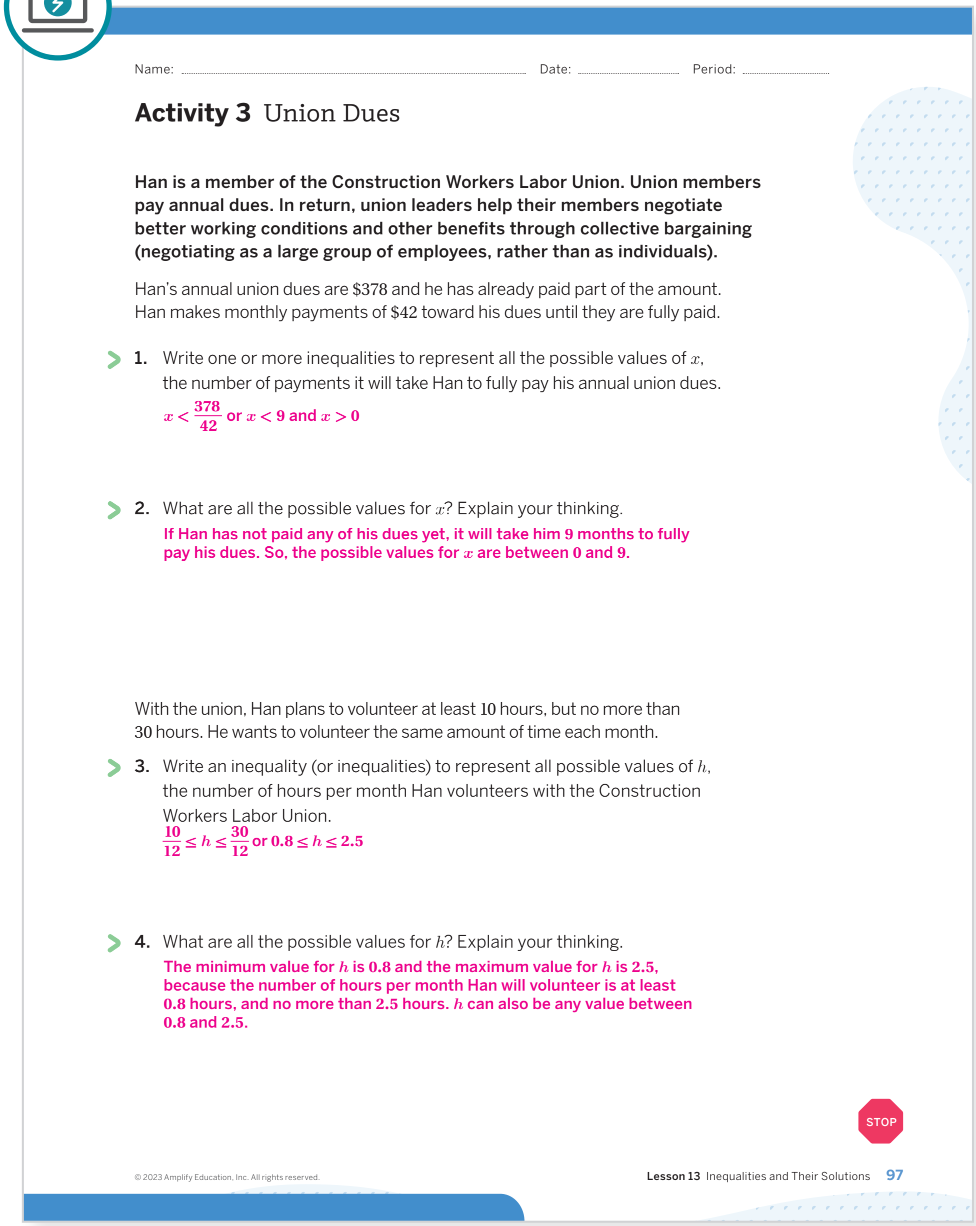
English Learners

Have students highlight key phrases, such as *initial fee* and *per hour*.

Independent | 12 min

Activity 3 Union Dues

Students write inequalities in one variable modeling a context to make sense of solution sets of inequalities in context.



Name: ______ Date: ______ Period: ______

Activity 3 Union Dues

Han is a member of the Construction Workers Labor Union. Union members pay annual dues. In return, union leaders help their members negotiate better working conditions and other benefits through collective bargaining (negotiating as a large group of employees, rather than as individuals).

Han's annual union dues are $378 and he has already paid part of the amount. Han makes monthly payments of $42 toward his dues until they are fully paid.

1. Write one or more inequalities to represent all the possible values of x, the number of payments it will take Han to fully pay his annual union dues.
   $x < \frac{378}{42}$ **or** $x < 9$ **and** $x > 0$

2. What are all the possible values for x? Explain your thinking.
   If Han has not paid any of his dues yet, it will take him 9 months to fully pay his dues. So, the possible values for x are between 0 and 9.

With the union, Han plans to volunteer at least 10 hours, but no more than 30 hours. He wants to volunteer the same amount of time each month.

3. Write an inequality (or inequalities) to represent all possible values of h, the number of hours per month Han volunteers with the Construction Workers Labor Union.
   $\frac{10}{12} \leq h \leq \frac{30}{12}$ **or** $0.8 \leq h \leq 2.5$

4. What are all the possible values for h? Explain your thinking.
   The minimum value for h is 0.8 and the maximum value for h is 2.5, because the number of hours per month Han will volunteer is at least 0.8 hours, and no more than 2.5 hours. h can also be any value between 0.8 and 2.5.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the narrative aloud. Discuss unions and their purpose, provide examples of other professions that have unions (police, fireman, teachers, etc.) Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by asking, "What is the minimum or maximum value of each variable?"

Look for points of confusion:

- **Struggling to write an inequality to represent the scenario.** Have students write the constraints of the scenario in their own words incorporating 12 months and x.

Look for productive strategies:

- Using the minimum and maximum value of each variable to write their inequalities.
- Using key words and phrases from the scenario to describe and make sense of the solution set.

3 Connect

Have individual students share their strategies for writing their inequalities.

Ask, "How did you determine the minimum number of months in Problem 1?" Han cannot pay a negative number of months, and has already paid some amount.

Highlight that sometimes students need to account for other real world considerations when interpreting solutions of inequalities in a context. For example, Han could not make negative payments here.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Some students may or may not be familiar with union dues. Provide some examples of occupations in the U.S. in which workers typically join a union and pay an amount – union dues – so that the union advocates on their behalf for fair pay and satisfactory working conditions.

Accessibility: *Guide Processing and Visualization*

For Problems 1 and 2, display the following prompt to help students make sense of the situation and consider the possible values of x.

If Han has not paid any of his dues yet, it will take him _____ months.

Math Language Development

MLR5: Co-craft Questions

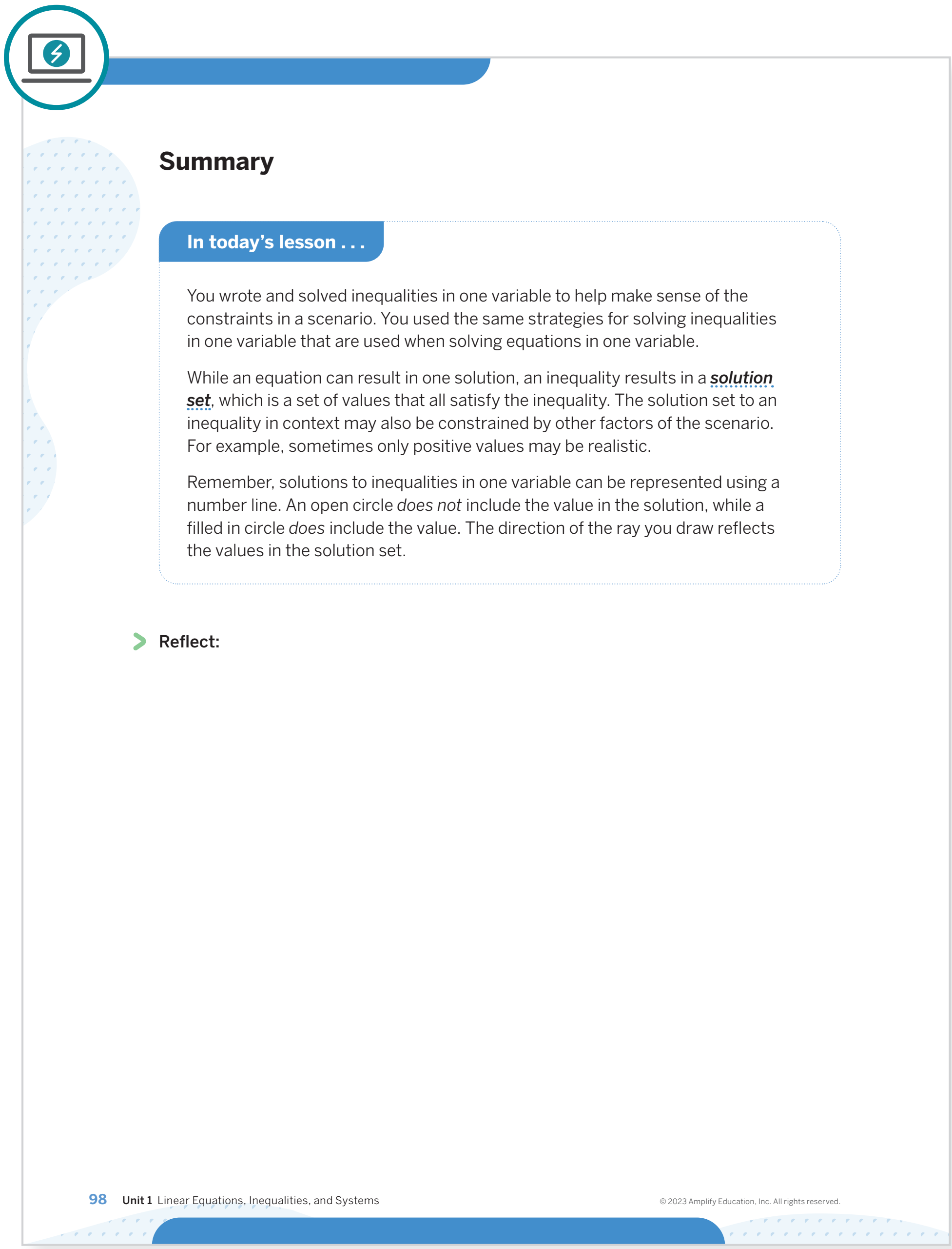
Display only the introductory text of this problem and have students write 1–2 mathematical questions they have about the scenario. Invite them to share their questions with a partner. Amplify any questions that involve possible constraints, such as the following:

- What would be the maximum number of months, or monthly payments, Han would need to pay?
- What could be the minimum possible amount Han has already paid?
- What could be the maximum possible amount Han has already paid?

Whole Class | 5 min

Summary

Review and synthesize writing, solving, and understanding the solution sets of inequalities in one variable.



Summary

In today's lesson . . .

You wrote and solved inequalities in one variable to help make sense of the constraints in a scenario. You used the same strategies for solving inequalities in one variable that are used when solving equations in one variable.

While an equation can result in one solution, an inequality results in a ***solution set***, which is a set of values that all satisfy the inequality. The solution set to an inequality in context may also be constrained by other factors of the scenario. For example, sometimes only positive values may be realistic.

Remember, solutions to inequalities in one variable can be represented using a number line. An open circle *does not* include the value in the solution, while a filled in circle *does* include the value. The direction of the ray you draw reflects the values in the solution set.

Reflect:

98 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display $x = 3$ and $x > 3$, as well as the solution set to the inequality graphed on a number line.

Formalize vocabulary: ***solution set***

Highlight that an equation can result in one solution and an inequality results in a solution set. The solution set may also be constrained by other factors of the scenario.

Have students share their strategies for determining the solutions of an inequality in one variable.

Ask, "How does replacing the inequality symbol with an equal sign help you solve the inequality?" Sample response: Once the inequality symbol is replaced with an equal sign, I can solve the equation to determine a boundary value. Then I can substitute a value either greater than or less than this value back into the inequality to determine if the solutions are values less than or greater than the boundary value.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful today when determining and representing the solution set to an inequality in one variable? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

Math Language Development

MLR2: Collect and Display

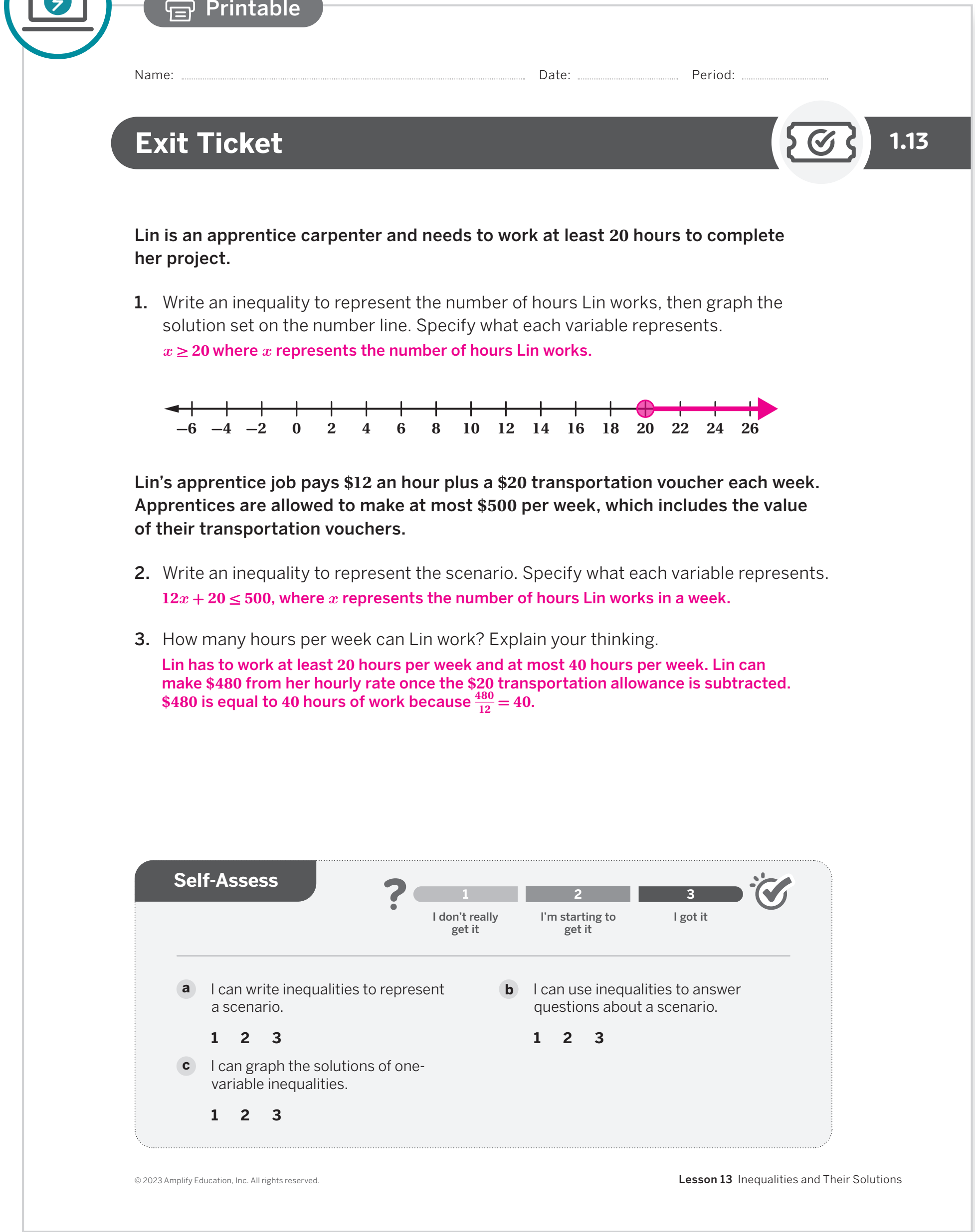
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term solution set that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing, solving, and interpreting the solutions of inequalities in one variable that model the constraints of a scenario.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.13

Lin is an apprentice carpenter and needs to work at least 20 hours to complete her project.

1. Write an inequality to represent the number of hours Lin works, then graph the solution set on the number line. Specify what each variable represents.
   $x \geq 20$ where x represents the number of hours Lin works.

Lin's apprentice job pays $12 an hour plus a $20 transportation voucher each week. Apprentices are allowed to make at most $500 per week, which includes the value of their transportation vouchers.

2. Write an inequality to represent the scenario. Specify what each variable represents.
   $12x + 20 \leq 500$, where x represents the number of hours Lin works in a week.
3. How many hours per week can Lin work? Explain your thinking.
   Lin has to work at least 20 hours per week and at most 40 hours per week. Lin can make $480 from her hourly rate once the $20 transportation allowance is subtracted. $480 is equal to 40 hours of work because $\frac{480}{12} = 40$.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can write inequalities to represent a scenario.
1 2 3

b I can use inequalities to answer questions about a scenario.
1 2 3

c I can graph the solutions of one-variable inequalities.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Success looks like . . .

- **Language Goal:** Understanding that the solution to an inequality is a range of values that make the inequality true. **(Speaking and Listening, Reading and Writing)**
- **Goal:** Analyzing and using the structure in inequalities to determine whether the solution is greater or less than the solution to a related equation.
- **Language Goal:** Writing and solving inequalities in one variable to represent the constraints in situations and solve problems. **(Reading and Writing)**
  - » Writing and solving an inequality to represent the number of hours Lin can work per week in Problems 2 and 3.

Suggested next steps

If students incorrectly graph the solution set on the number line in Problem 1, consider:

- Reviewing strategies for graphing solutions on a number line from Activity 1.
- Assigning Practice Problem 1.
- Asking, "How could you use the inequality symbol to determine which direction the arrow faces and if the point is open or filled in?"

If students incorrectly create the inequality in Problem 2, consider:

- Reviewing strategies for defining a variable and creating an inequality in Activity 3.
- Assigning Practice Problem 3.

If students incorrectly calculate one or both constraints in Problem 3, consider:

- Reviewing calculating constraints in Activity 3.
- Assigning Practice Problems 2 and 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did writing inequalities in one variable from a scenario reveal about your students as learners?
- In earlier lessons, students wrote equations to model relationships. How did that support writing inequalities from scenarios? What might you change for the next time you teach this lesson?

MLR Math Language Development

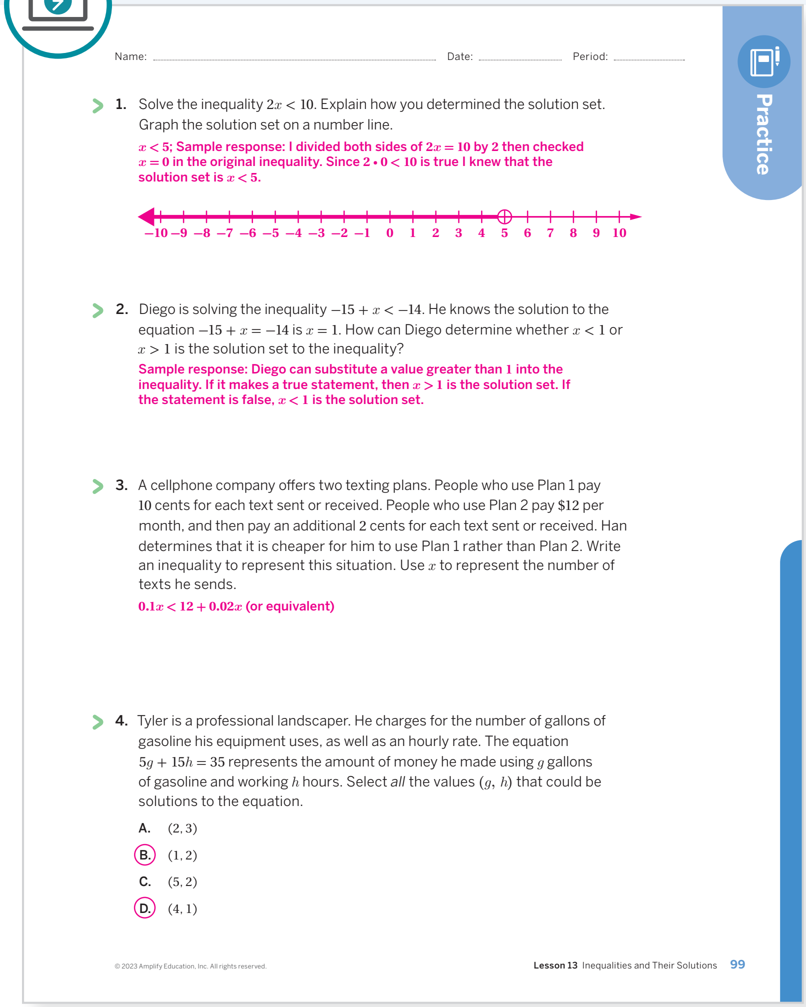
Language Goal: Writing and solving inequalities in one variable to represent the constraints in situations and solve problems.

Reflect on students' language development toward this goal.

- Are students progressing in their interpretation of real-world problems that involve constraints in order to determine key phrases that will help them write an inequality to represent the situation?
- How did using the *Three Reads* routine in Activity 1 help students look for key words and phrases to begin to write the inequality? Are students choosing to use this routine on their own as they read real-world situations?

Practice

Independent



Name: Date: Period:

Practice

1. Solve the inequality $2x < 10$. Explain how you determined the solution set. Graph the solution set on a number line.

   $x < 5$; Sample response: I divided both sides of $2x = 10$ by 2 then checked $x = 0$ in the original inequality. Since $2 \cdot 0 < 10$ is true I knew that the solution set is $x < 5$.

2. Diego is solving the inequality $-15 + x < -14$. He knows the solution to the equation $-15 + x = -14$ is $x = 1$. How can Diego determine whether $x < 1$ or $x > 1$ is the solution set to the inequality?

   Sample response: Diego can substitute a value greater than 1 into the inequality. If it makes a true statement, then $x > 1$ is the solution set. If the statement is false, $x < 1$ is the solution set.

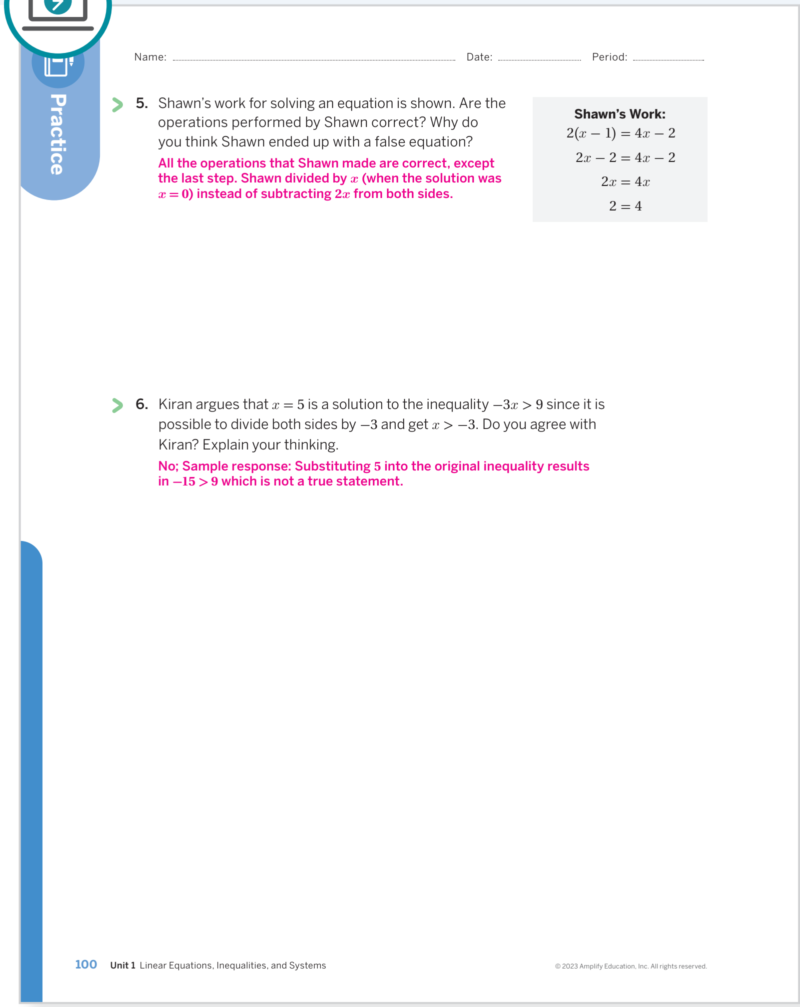
3. A cellphone company offers two texting plans. People who use Plan 1 pay 10 cents for each text sent or received. People who use Plan 2 pay $12 per month, and then pay an additional 2 cents for each text sent or received. Han determines that it is cheaper for him to use Plan 1 rather than Plan 2. Write an inequality to represent this situation. Use x to represent the number of texts he sends.

   $0.1x < 12 + 0.02x$ (or equivalent)

4. Tyler is a professional landscaper. He charges for the number of gallons of gasoline his equipment uses, as well as an hourly rate. The equation $5g + 15h = 35$ represents the amount of money he made using g gallons of gasoline and working h hours. Select *all* the values (g, h) that could be solutions to the equation.

   A. $(2, 3)$
   [✓ B.] $(1, 2)$
   C. $(5, 2)$
   [✓ D.] $(4, 1)$

© 2023 Amplify Education, Inc. All rights reserved.



Name: Date: Period:

Practice

5. Shawn's work for solving an equation is shown. Are the operations performed by Shawn correct? Why do you think Shawn ended up with a false equation?

   All the operations that Shawn made are correct, except the last step. Shawn divided by x (when the solution was $x = 0$) instead of subtracting $2x$ from both sides.

Shawn's Work:

$$2(x-1) = 4x - 2$$
$$2x - 2 = 4x - 2$$
$$2x = 4x$$
$$2 = 4$$

6. Kiran argues that $x = 5$ is a solution to the inequality $-3x > 9$ since it is possible to divide both sides by -3 and get $x > -3$. Do you agree with Kiran? Explain your thinking.

   No; Sample response: Substituting 5 into the original inequality results in $-15 > 9$ which is not a true statement.

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 6	2
	5	Unit 1 Lesson 8	2
Formative ⚡	6	Unit 1 Lesson 14	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 14**

Solving Two-Variable Linear Inequalities

Let's practice writing and solving linear inequalities in two variables.

Focus

Goals

1. **Language Goal:** Determine the solution to a one-variable inequality by reasoning and by solving a related equation and testing values greater than and less than that solution. **(Speaking and Listening, Reading and Writing)**
2. Graph the solution to an inequality as a ray on a number line.
3. **Language Goal:** Understand that the solution to an inequality is a range of values that make the inequality true. **(Reading and Writing)**

Coherence

Today

Students revisit changing the direction of an inequality symbol when dividing or multiplying an inequality by a negative value. They explore strategies for solving multi-step one- and two-variable linear inequalities.

Previously

Students wrote and graphed linear inequalities in one variable to represent the constraints of a situation in Lesson 13.

Coming Soon

Students will graph linear inequalities in two variables in Lesson 15.

Rigor

- Students build **conceptual understanding** of solving linear inequalities in two variables.
- Students strengthen their **procedural skills** in solving and graphing linear inequalities in one variable.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1 (optional)	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	20 min	5 min	5 min
Independent	Independent	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

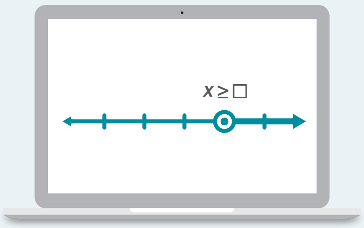
Review words

- *boundary value*
- *linear inequality*
- *slope-intercept form*
- *solution set*

Amps | Featured Activity

Activity 1
Interactive Number Line

Students investigate the reflection of the solution set on a number to understand the need to reverse the inequality symbol when multiplying or dividing an inequality by a negative value.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may not have the self-discipline to look for every instance of multiplying or dividing by a negative number to be sure that they reverse the direction of the inequality symbol. Have students develop strategies for how to avoid this error, including what they can do before they start the assignment and after they complete it.

Modifications to Pacing

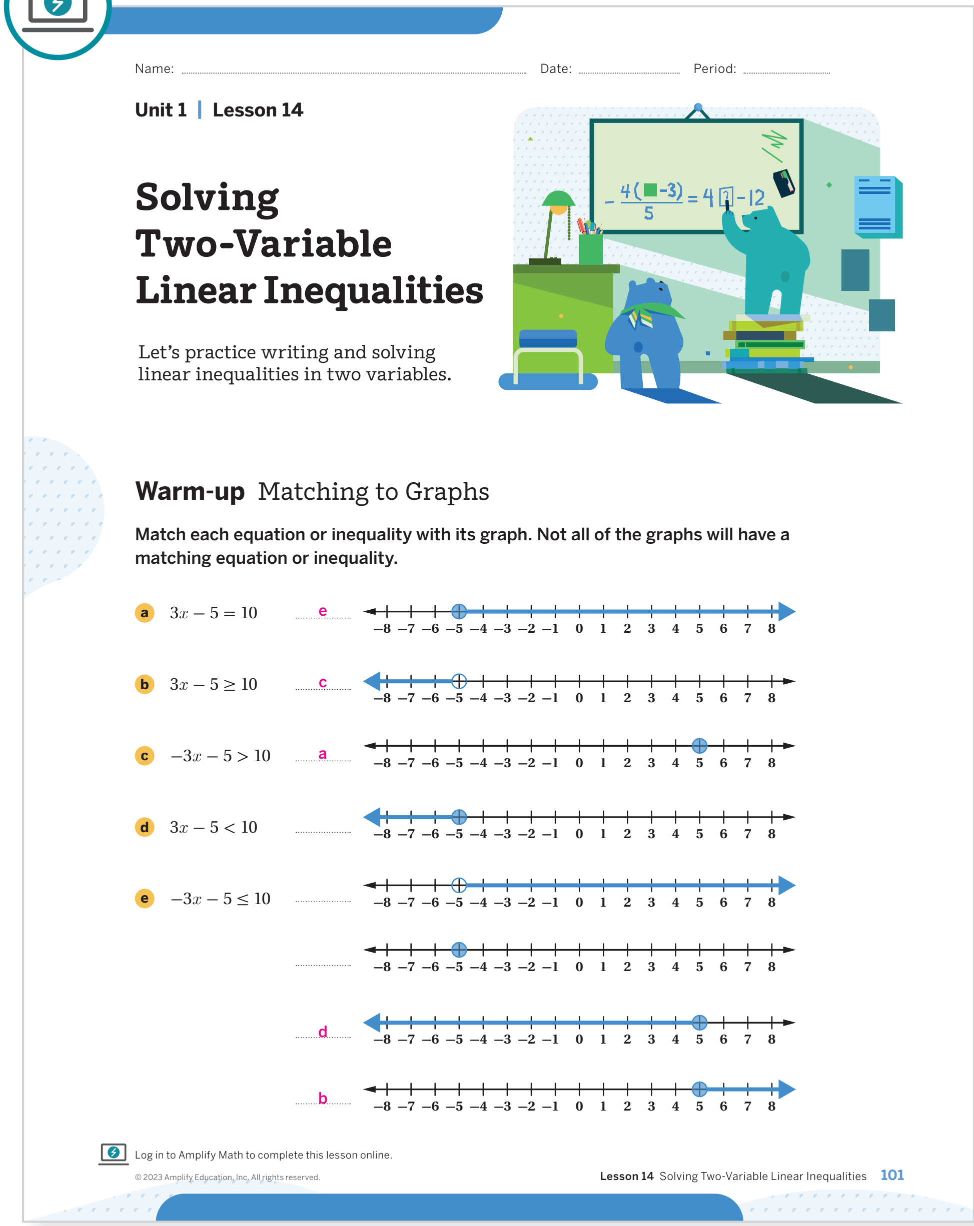
You may want to consider these additional modifications if you are short on time.

- **Activity 1** may be omitted if the majority of your students understand and can explain the Warm-up.
- In **Activity 3**, Problems 5 and 6 may be omitted.

Independent | 5 min

Warm-up Matching to Graphs

Students match the solutions to inequalities and equations to graphs on number lines to review the difference between solutions of equations and inequalities.



Name: Date: Period:

Unit 1 | Lesson 14

Solving Two-Variable Linear Inequalities

Let's practice writing and solving linear inequalities in two variables.

Warm-up Matching to Graphs

Match each equation or inequality with its graph. Not all of the graphs will have a matching equation or inequality.

a. $3x - 5 = 10$ e
b. $3x - 5 \geq 10$ c
c. $-3x - 5 > 10$ a
d. $3x - 5 < 10$
e. $-3x - 5 \leq 10$

d

b

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 14 Solving Two-Variable Linear Inequalities 101

1 Launch

Set an expectation for the amount of time students will have to work individually on the Warm-up. Highlight that each equation or inequality has exactly one graph that represents its solution.

2 Monitor

Help students get started by asking, "How could you use the strategies for solving equations to help you solve the inequalities?"

Look for points of confusion:

- **Choosing graphs that do not match.** Have students test a value from the number line to see if it makes the inequality or equation true.

Look for productive strategies:

- Making use of structure to eliminate graphs that do not represent an equation or inequality depending on which is given for each problem.

3 Connect

Have student pairs share what they noticed or wondered. Record some of the responses.

Ask, "What is the difference between the solutions to inequalities and equations?" Equations may have one solution, while inequalities have infinitely many solutions, a solution set.

Highlight that they need to account for the negative coefficient in Problems 3 and 5 somehow; otherwise, the solution set will make the inequality false if they solve the inequality just like Problems 2 and 4.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share what they noticed and wondered about the equations or inequalities and their graphs, draw connections between the graphs of the solutions to equations and the graphs of the solutions to inequalities. Guide students towards using mathematical phrases, such as *infinitely many solutions*, when describing the solutions to inequalities.

English Learners

Use gestures when highlighting the direction of the graphs of the inequalities.

Power-up

To power up students' ability to solve inequalities in one variable, have students complete:

Match the equivalent inequalities.

a. $3x > 12$	d	$x \leq 4$
b. $-3x > 12$	b	$x < -4$
c. $3x \leq -12$	a	$x > 4$
d. $-3x -12$	c	$x \leq -4$

Use: Before the Warm-up

Informed by: Performance on Lesson 5, Practice Problem 6

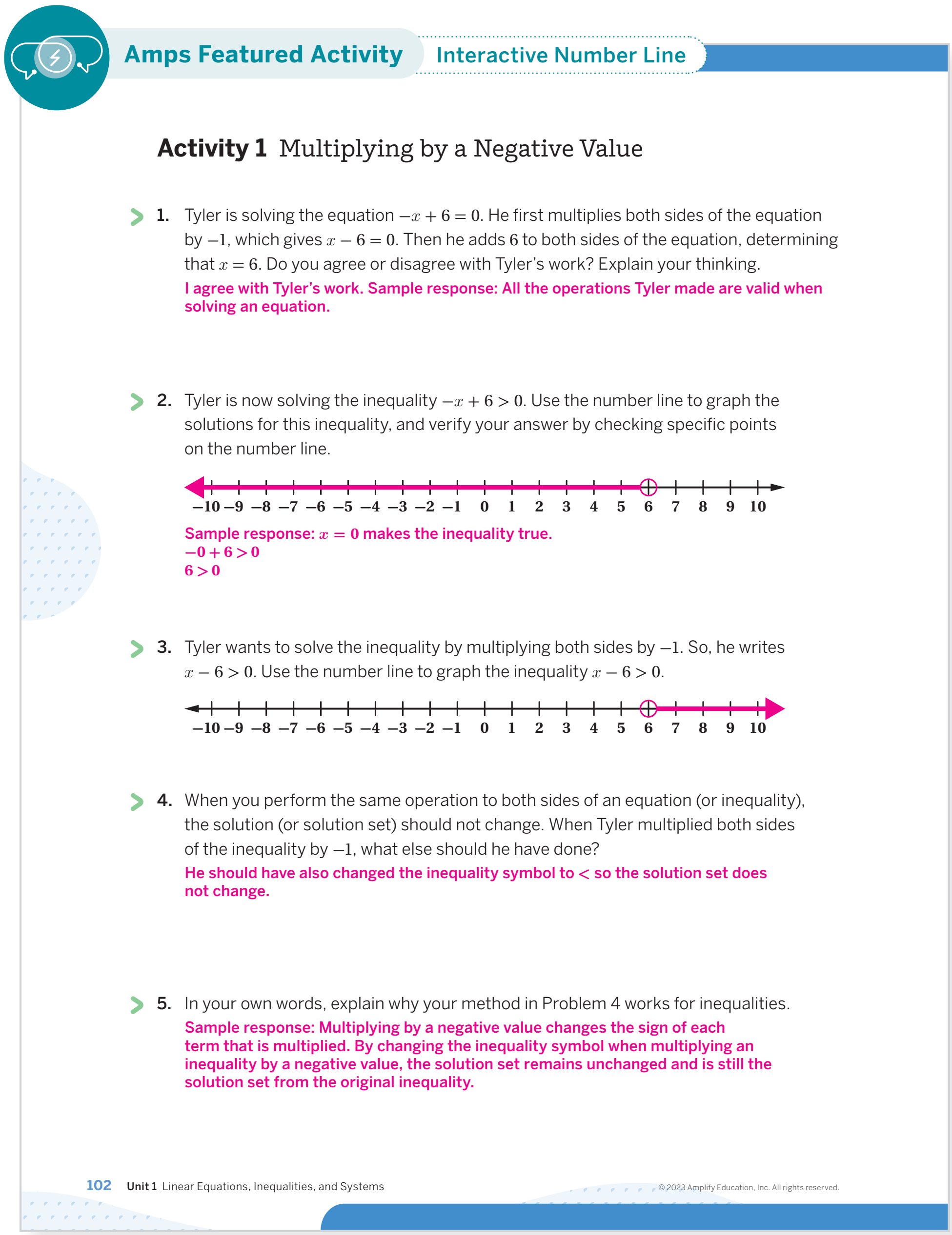
Optional

Independent | 10 min

Activity 1 Multiplying by a Negative Value

Students investigate multiplying an inequality by a negative value to revisit why the direction of an inequality symbol changes.

Amps Featured Activity Interactive Number Line



Activity 1 Multiplying by a Negative Value

1. Tyler is solving the equation $-x + 6 = 0$. He first multiplies both sides of the equation by -1, which gives $x - 6 = 0$. Then he adds 6 to both sides of the equation, determining that $x = 6$. Do you agree or disagree with Tyler's work? Explain your thinking.
   I agree with Tyler's work. Sample response: All the operations Tyler made are valid when solving an equation.

2. Tyler is now solving the inequality $-x + 6 > 0$. Use the number line to graph the solutions for this inequality, and verify your answer by checking specific points on the number line.

   Sample response: $x = 0$ makes the inequality true.
   $-0 + 6 > 0$
   $6 > 0$

3. Tyler wants to solve the inequality by multiplying both sides by -1. So, he writes $x - 6 > 0$. Use the number line to graph the inequality $x - 6 > 0$.

4. When you perform the same operation to both sides of an equation (or inequality), the solution (or solution set) should not change. When Tyler multiplied both sides of the inequality by -1, what else should he have done?
   He should have also changed the inequality symbol to $<$ so the solution set does not change.

5. In your own words, explain why your method in Problem 4 works for inequalities.
   Sample response: Multiplying by a negative value changes the sign of each term that is multiplied. By changing the inequality symbol when multiplying an inequality by a negative value, the solution set remains unchanged and is still the solution set from the original inequality.

102 Unit 1 Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Students work independently before sharing the method and explanation they wrote in Problem 5. Say, "Lets see why the inequality symbol changed when dividing by a negative value in the Warm-up."

2 Monitor

Help students get started by having them perform the operations independently that are described in Problem 1.

Look for points of confusion:

- **Isolating x without changing the inequality symbol and checking values in the final inequality in Problem 2.** Have students substitute values of x into the original inequality.

Look for productive strategies:

- Testing their method by substituting values in both the final and original inequality.

3 Connect

Have individual students share their method and explanation as to why this method is true.

Highlight that this method holds true whenever they divide or multiply any inequality by a negative value. This method also applies to a linear inequality in two variables.

Ask, "Why does this method not hold true for multiplying or dividing by a positive value?" Multiplying by a positive value does not change the sign of a value.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they use interactive number lines to investigate the reflection of the solution set on a number line. This visually reinforces the need to reverse the inequality symbol when multiplying or dividing an inequality by a negative value.

Accessibility: *Guide Processing and Visualization*

Provide a visual display of Tyler's work in Problem 1, as opposed to the text description. For example, display the work shown here.

Tyler's work:

$$\begin{aligned} -x + 6 &= 0 \\ -1(-x + 6) &= -1(0) \\ x - 6 &= 0 \\ x &= 6 \end{aligned}$$

Math Language Development

MLR1: Stronger and Clearer Each Time

After students write a first draft of their responses to Problem 5, have them share their responses with a partner to receive and give feedback. Provide them with time to revise their original written responses by incorporating or addressing new ideas or language.

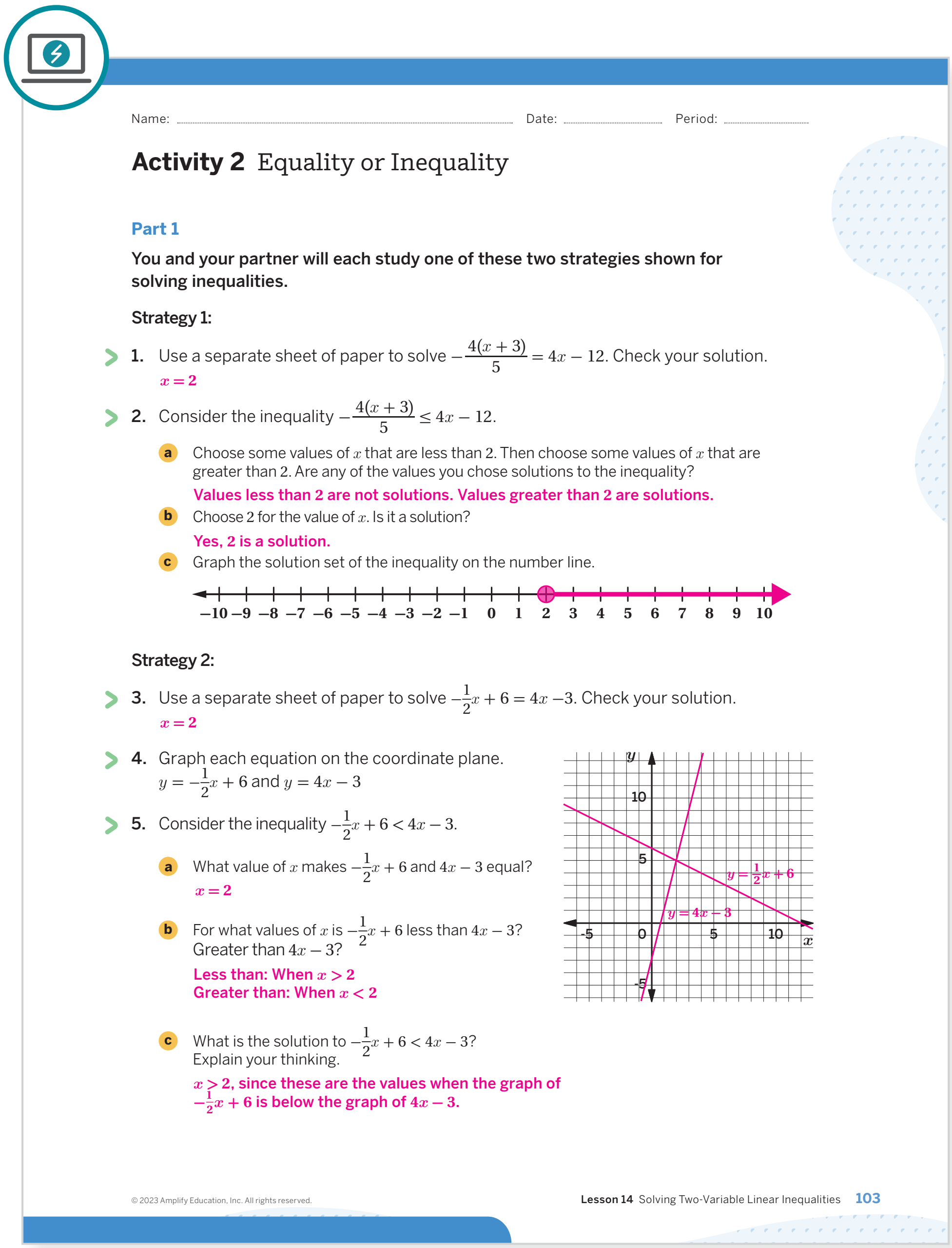
English Learners

Use gestures to show how the direction of the shading changed in Problems 2 and 3.

Pairs | 15 min

Activity 2 Equality or Inequality

Students look for and express regularity in repeaated reasoning to further develop the idea that inequalities can be solved by first solving a related equation.



Name: ______ Date: ______ Period: ______

Activity 2 Equality or Inequality

Part 1

You and your partner will each study one of these two strategies shown for solving inequalities.

Strategy 1:

1. Use a separate sheet of paper to solve $-\frac{4(x+3)}{5} = 4x - 12$. Check your solution.
   $x = 2$

2. Consider the inequality $-\frac{4(x+3)}{5} \leq 4x - 12$.
   - a Choose some values of x that are less than 2. Then choose some values of x that are greater than 2. Are any of the values you chose solutions to the inequality?
     Values less than 2 are not solutions. Values greater than 2 are solutions.
   - b Choose 2 for the value of x. Is it a solution?
     Yes, 2 is a solution.
   - c Graph the solution set of the inequality on the number line.

Strategy 2:

3. Use a separate sheet of paper to solve $-\frac{1}{2}x + 6 = 4x - 3$. Check your solution.
   $x = 2$

4. Graph each equation on the coordinate plane.
   $y = -\frac{1}{2}x + 6$ and $y = 4x - 3$

5. Consider the inequality $-\frac{1}{2}x + 6 < 4x - 3$.
   - a What value of x makes $-\frac{1}{2}x + 6$ and $4x - 3$ equal?
     $x = 2$
   - b For what values of x is $-\frac{1}{2}x + 6$ less than $4x - 3$? Greater than $4x - 3$?
     Less than: When $x > 2$
     Greater than: When $x < 2$
   - c What is the solution to $-\frac{1}{2}x + 6 < 4x - 3$? Explain your thinking.
     $x > 2$, since these are the values when the graph of $-\frac{1}{2}x + 6$ is below the graph of $4x - 3$.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 14 Solving Two-Variable Linear Inequalities 103

1 Launch

Arrange students in pairs. Say, "You and your partner will each choose a strategy to review and answer questions related to each strategy. You will then explain the strategy and your responses to your partner." Give students 5 minutes of independent work time before sharing with their partner. Repeat for Strategies 3 and 4.

2 Monitor

Help students get started by assigning students Strategy 1 and having them refer to the Warm-up for a reminder of how to graph solutions of an inequality.

Look for points of confusion:

- **Incorrectly using the graphs to compare values in Strategy 2.** Ask, "How can you determine which equation is greater for a specific value of x by using the graph?"
- **Vaguely explaining why m must include negative values in Strategy 4.** Have students test negative values in the original inequality. Ask, "If negative values make the inequality true, why does this imply the solution set is $m < 3$?"

Look for productive strategies:

- Plotting points on the graph for Strategy 2.
- Checking multiple values in their inequalities.
- Checking solutions in the final inequality and original inequality.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider using one of these alternative approaches to this activity.

- Assign students Strategies 1 and 3, and ask them to focus on how the strategies are similar.
- Allow students to choose one of the four strategies to investigate.

Extension: *Math Enrichment*

Ask, "Does multiplying or dividing both sides of a linear inequality in two variables by a negative value change which ordered pairs are solutions? Explain your thinking." No. The solution set does not change; only the inequality symbol changes.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Part 3, have them write a description of the strategy they chose to use on a separate sheet of paper. Have them share their descriptions with a partner to help refine and clarify them. Provide students with the opportunity to revise their descriptions by incorporating or addressing new ideas or language.

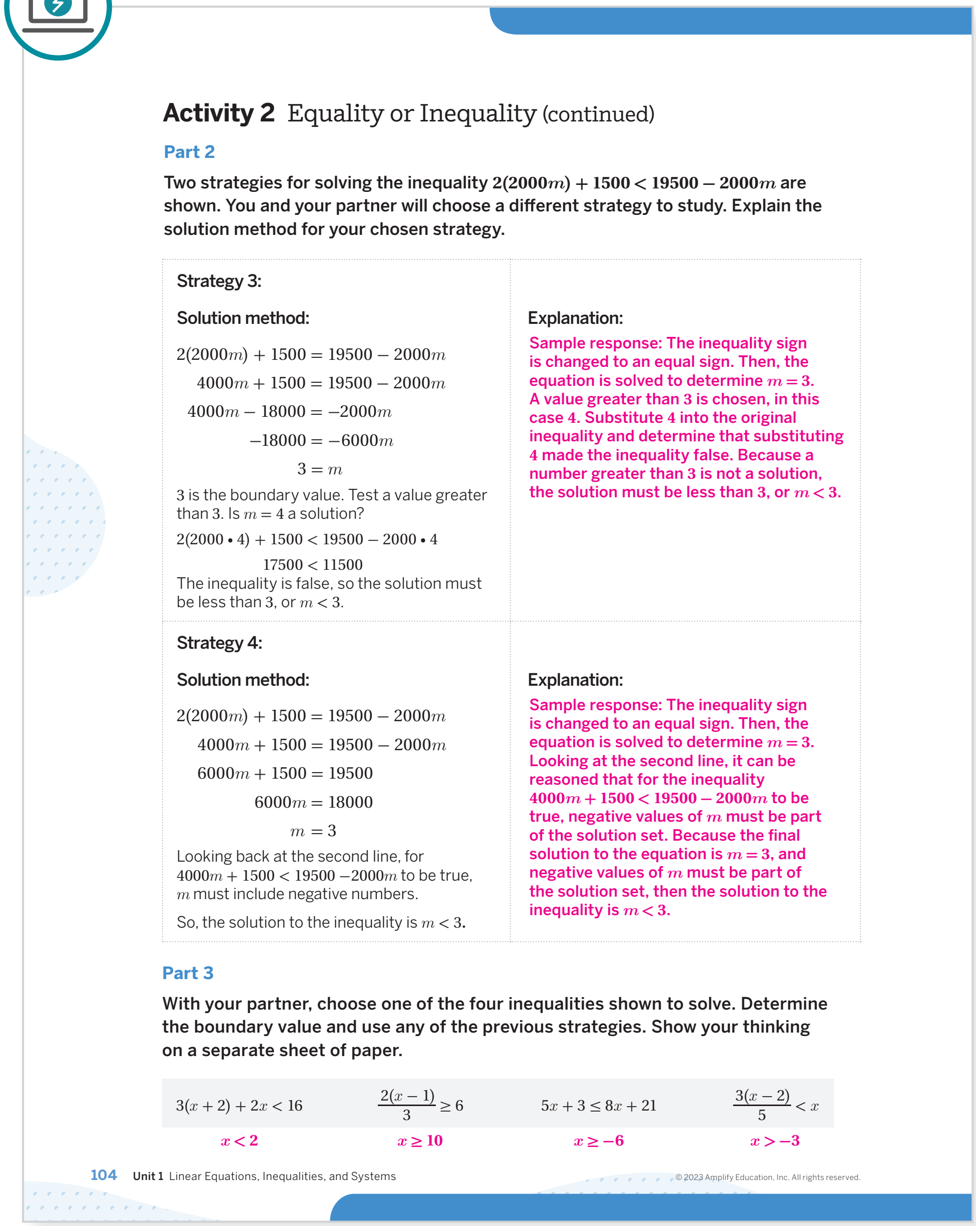
English Learners

Pair students with different language proficiencies together to provide English Learners an opportunity to hear the language of a native English speaker.

Pairs | 15 min

Activity 2 Equality or Inequality (continued)

Students look for and express regularity in repeaated reasoning to further develop the idea that inequalities can be solved by first solving a related equation.



Activity 2 Equality or Inequality (continued)

Part 2

Two strategies for solving the inequality $2(2000m) + 1500 < 19500 - 2000m$ are shown. You and your partner will choose a different strategy to study. Explain the solution method for your chosen strategy.

Strategy 3: Solution method: $2(2000m) + 1500 = 19500 - 2000m$ $4000m + 1500 = 19500 - 2000m$ $4000m - 18000 = -2000m$ $-18000 = -6000m$ $3 = m$ 3 is the boundary value. Test a value greater than 3. Is $m = 4$ a solution? $2(2000 \cdot 4) + 1500 < 19500 - 2000 \cdot 4$ $17500 < 11500$ The inequality is false, so the solution must be less than 3, or $m < 3$.	Explanation: Sample response: The inequality sign is changed to an equal sign. Then, the equation is solved to determine $m = 3$. A value greater than 3 is chosen, in this case 4. Substitute 4 into the original inequality and determine that substituting 4 made the inequality false. Because a number greater than 3 is not a solution, the solution must be less than 3, or $m < 3$.
Strategy 4: Solution method: $2(2000m) + 1500 = 19500 - 2000m$ $4000m + 1500 = 19500 - 2000m$ $6000m + 1500 = 19500$ $6000m = 18000$ $m = 3$ Looking back at the second line, for $4000m + 1500 < 19500 - 2000m$ to be true, m must include negative numbers. So, the solution to the inequality is $m < 3$.	Explanation: Sample response: The inequality sign is changed to an equal sign. Then, the equation is solved to determine $m = 3$. Looking at the second line, it can be reasoned that for the inequality $4000m + 1500 < 19500 - 2000m$ to be true, negative values of m must be part of the solution set. Because the final solution to the equation is $m = 3$, and negative values of m must be part of the solution set, then the solution to the inequality is $m < 3$.

Part 3

With your partner, choose one of the four inequalities shown to solve. Determine the boundary value and use any of the previous strategies. Show your thinking on a separate sheet of paper.

$3(x + 2) + 2x < 16$	$\frac{2(x - 1)}{3} \geq 6$	$5x + 3 \leq 8x + 21$	$\frac{3(x - 2)}{5} < x$
$x < 2$	$x \geq 10$	$x \geq -6$	$x > -3$

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have individual students share which strategy they chose to use to solve their inequality and their reasoning.

Highlight that if they solve an inequality by using a related equation, it is important to make sure that the solution to the equation is correct because that solution gives a boundary from which they could check the solutions to the inequality. If the boundary value is incorrect, they may not be able to correctly find the solution set to the inequality.

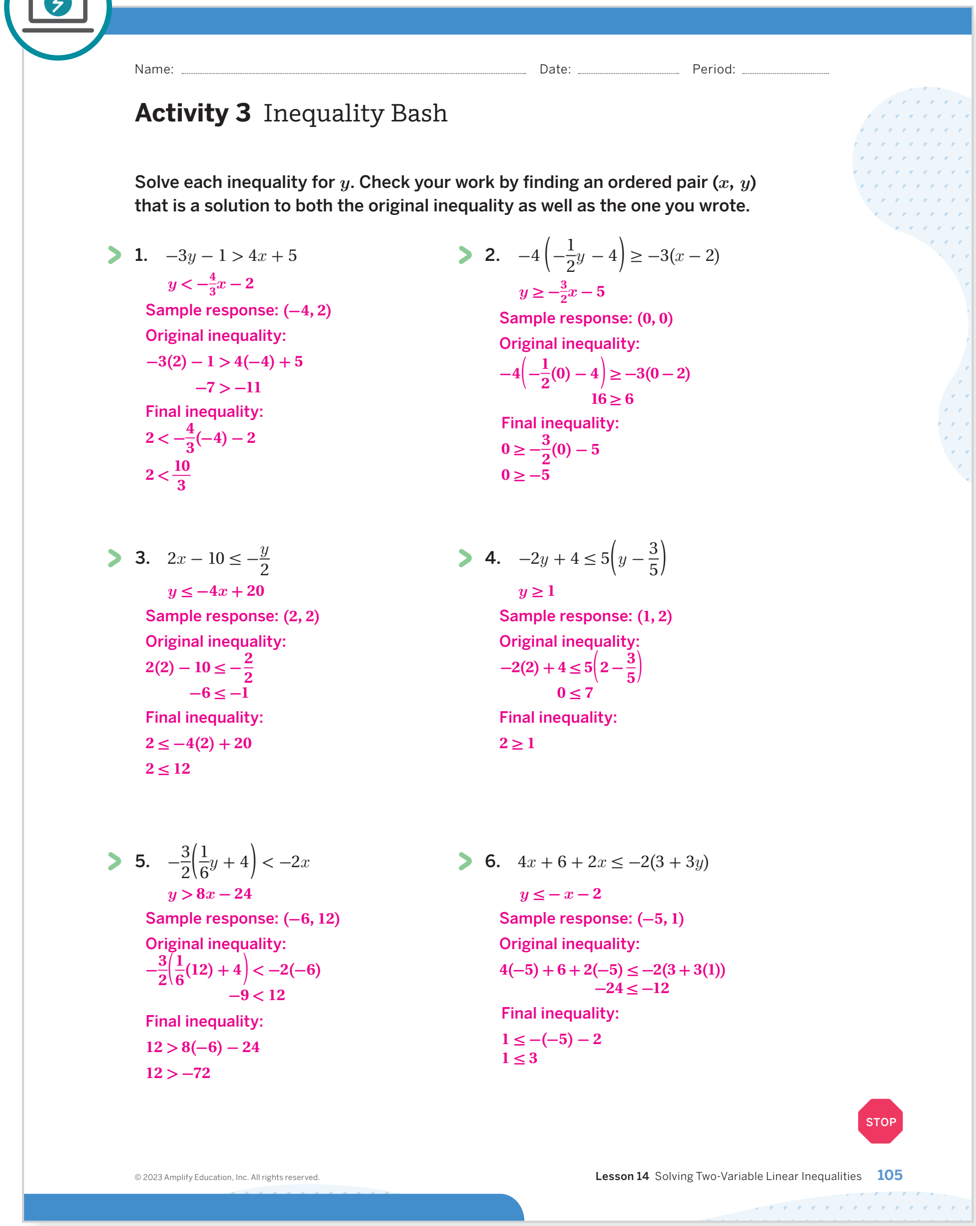
Ask:

- "If you isolated x on the right side of the inequality symbol for $3(x + 2) + 2x < 16$, how do you interpret the solution set $2 > x$?" This inequality would be read as "2 is greater than x," which means that the solution set contains any value less than 2.
- "How could you make sure you determined the correct solutions to the inequality?" I could substitute a value of x that is a solution into the final inequality and original inequality to make sure they are both true.

Independent | 20 min

Activity 3 Inequality Bash

Students rewrite linear inequalities in two variables in slope-intercept form to reason about solutions as ordered pairs.



Name: ______________ Date: ________ Period: ________

Activity 3 Inequality Bash

Solve each inequality for y. Check your work by finding an ordered pair (x, y) that is a solution to both the original inequality as well as the one you wrote.

1. $-3y - 1 > 4x + 5$

$y < -\frac{4}{3}x - 2$

Sample response: $(-4, 2)$

Original inequality:

$-3(2) - 1 > 4(-4) + 5$

$-7 > -11$

Final inequality:

$2 < -\frac{4}{3}(-4) - 2$

$2 < \frac{10}{3}$

2. $-4\left(-\frac{1}{2}y - 4\right) \geq -3(x - 2)$

$y \geq -\frac{3}{2}x - 5$

Sample response: $(0, 0)$

Original inequality:

$-4\left(-\frac{1}{2}(0) - 4\right) \geq -3(0 - 2)$

$16 \geq 6$

Final inequality:

$0 \geq -\frac{3}{2}(0) - 5$

$0 \geq -5$

3. $2x - 10 \leq -\frac{y}{2}$

$y \leq -4x + 20$

Sample response: $(2, 2)$

Original inequality:

$2(2) - 10 \leq -\frac{2}{2}$

$-6 \leq -1$

Final inequality:

$2 \leq -4(2) + 20$

$2 \leq 12$

4. $-2y + 4 \leq 5\left(y - \frac{3}{5}\right)$

$y \geq 1$

Sample response: $(1, 2)$

Original inequality:

$-2(2) + 4 \leq 5\left(2 - \frac{3}{5}\right)$

$0 \leq 7$

Final inequality:

$2 \geq 1$

5. $-\frac{3}{2}\left(\frac{1}{6}y + 4\right) < -2x$

$y > 8x - 24$

Sample response: $(-6, 12)$

Original inequality:

$-\frac{3}{2}\left(\frac{1}{6}(12) + 4\right) < -2(-6)$

$-9 < 12$

Final inequality:

$12 > 8(-6) - 24$

$12 > -72$

6. $4x + 6 + 2x \leq -2(3 + 3y)$

$y \leq -x - 2$

Sample response: $(-5, 1)$

Original inequality:

$4(-5) + 6 + 2(-5) \leq -2(3 + 3(1))$

$-24 \leq -12$

Final inequality:

$1 \leq -(-5) - 2$

$1 \leq 3$

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 14 Solving Two-Variable Linear Inequalities 105

1 Launch

Set an expectation for the amount of time students will have to work individually on the activity. Emphasize that they should show their work so that they can explain their procedure or analyze their work for any errors.

2 Monitor

Help students get started by saying, "Isolate y on the left side of the inequality symbol, and group all the other terms on the right side."

Look for points of confusion:

- **Changing the inequality symbol when subtracting or adding a negative value.** Have students revisit the method they wrote in Activity 1.

Look for productive strategies:

- Reasoning quantitatively by comparing their inequality to the original and testing specific values of x and y.

3 Connect

Display the inequalities with y isolated.

Have individual students share any inequalities that look different, but still are an equivalent inequality.

Highlight that distributing by a negative value does not change the inequality symbol because this is an operation that is just used to simplify one side of the inequality.

Ask, "If you changed the inequalities to equations, how could you use the graph of the equation to determine an ordered pair that is a solution to the inequality?" If the inequality includes "or equal to," I can choose a point on the line as a solution, if not, I can test points on either side of the line.

Differentiated Support

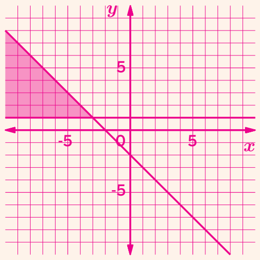
Accessibility: *Vary Demands to Optimize Challenge*

Consider using one of these alternative approaches to this activity.

- Allow students to choose four of the inequalities to solve.
- Provide students with the goal, the final inequality solved for y, and have them work toward showing valid mathematical steps to arrive at that final inequality.

Extension: *Math Enrichment*

Have students change the inequalities to equation for Problems 4 and 6, and graph these equations on the same coordinate plane. Then have students shade in the region which represents the ordered pairs that are solutions to both original inequalities.



Math Language Development

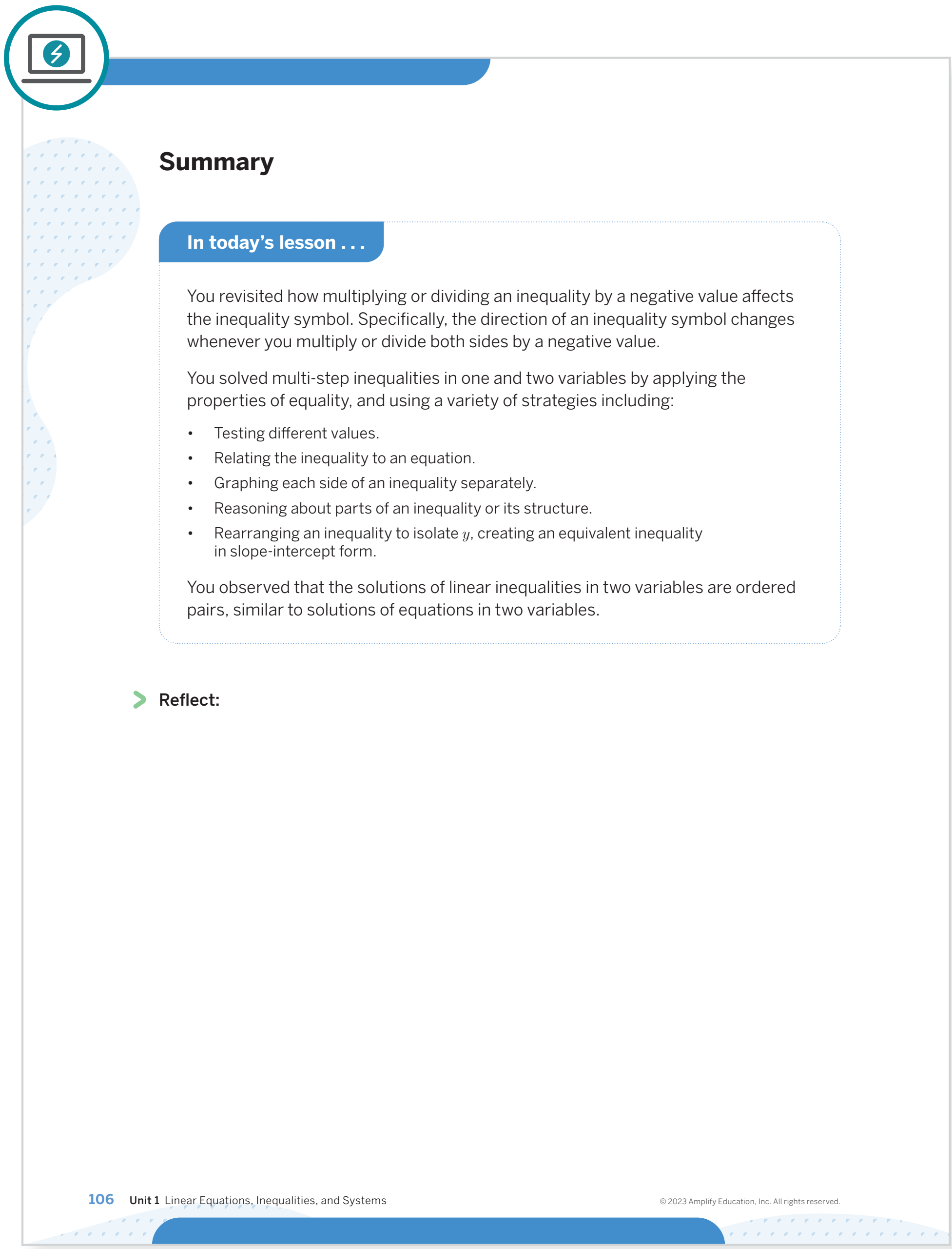
MLR7: Compare and Connect

During the Connect, as students share any inequalities that look different, but are still equivalent, press for details in their reasoning. For example, for Problem 1, if a student says, "The inequalities $y < \frac{3}{4}x - 2$ and $y < -2 + 0.75x$ are equivalent," consider asking, "How do you know that they are equivalent? What properties or strategies can you use to say they are equivalent?"

Whole Class | 5 min

Summary

Review and synthesize solving linear inequalities in one variable.



Summary

In today's lesson . . .

You revisited how multiplying or dividing an inequality by a negative value affects the inequality symbol. Specifically, the direction of an inequality symbol changes whenever you multiply or divide both sides by a negative value.

You solved multi-step inequalities in one and two variables by applying the properties of equality, and using a variety of strategies including:

- Testing different values.
- Relating the inequality to an equation.
- Graphing each side of an inequality separately.
- Reasoning about parts of an inequality or its structure.
- Rearranging an inequality to isolate y, creating an equivalent inequality in slope-intercept form.

You observed that the solutions of linear inequalities in two variables are ordered pairs, similar to solutions of equations in two variables.

Reflect:

106 Unit 1 Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the inequalities $-5x \geq 5$ and $5x > 5$.

Have students share how the inequalities are alike and different.

Highlight that the inequality symbols both include the "greater than" symbol, but only one of the inequalities will include the value that makes both sides of the inequality equal.

Ask:

- "Which inequality contains solutions that makes both slides of the inequality equal? How do you know?" The first inequality because the inequality symbol is "greater than or equal to."
- "How would you graph the solutions to each inequality?" Sample response: I would isolate x by dividing by –5 and 5. The first inequality would require the inequality symbol to be switched because I divided by a negative value. Then I would graph the solution set on a number line.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

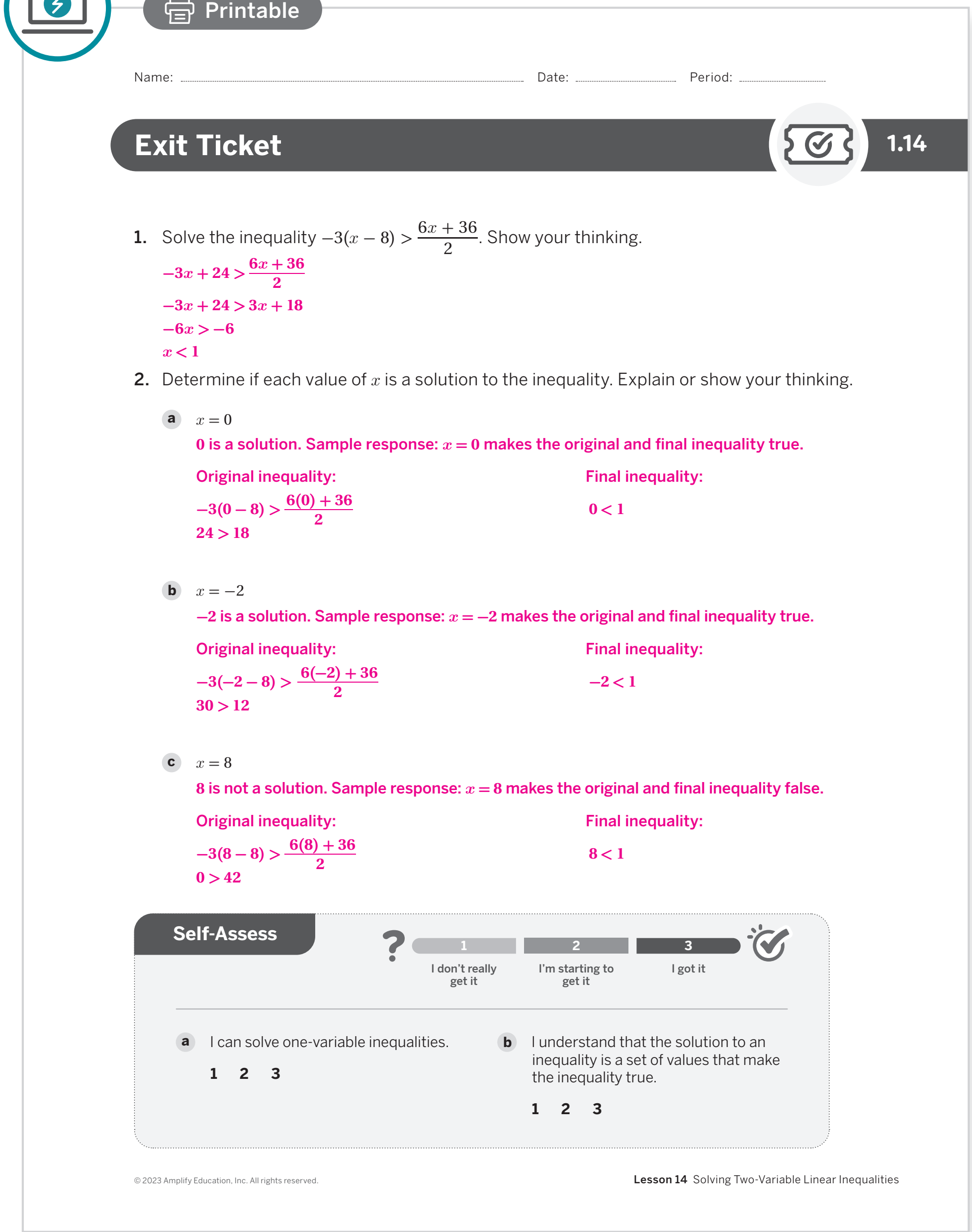
- "Why do you change the inequality symbol when dividing or multiplying by a negative value?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by solving an inequality in one variable and determining if values of x are solutions.

Printable



Name: Date: Period:

Exit Ticket 1.14

1. Solve the inequality $-3(x-8) > \frac{6x+36}{2}$. Show your thinking.

$-3x + 24 > \frac{6x+36}{2}$
$-3x + 24 > 3x + 18$
$-6x > -6$
$x < 1$

2. Determine if each value of x is a solution to the inequality. Explain or show your thinking.

a $x = 0$

0 is a solution. Sample response: $x = 0$ makes the original and final inequality true.

Original inequality:	Final inequality:
$-3(0-8) > \frac{6(0)+36}{2}$ $24 > 18$	$0 < 1$

b $x = -2$

−2 is a solution. Sample response: $x = -2$ makes the original and final inequality true.

Original inequality:	Final inequality:
$-3(-2-8) > \frac{6(-2)+36}{2}$ $30 > 12$	$-2 < 1$

c $x = 8$

8 is not a solution. Sample response: $x = 8$ makes the original and final inequality false.

Original inequality:	Final inequality:
$-3(8-8) > \frac{6(8)+36}{2}$ $0 > 42$	$8 < 1$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can solve one-variable inequalities.

1 2 3

b I understand that the solution to an inequality is a set of values that make the inequality true.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 14 Solving Two-Variable Linear Inequalities

Success looks like . . .

- **Language Goal:** Determining the solution to a one-variable inequality by reasoning and by solving a related equation and testing values greater than and less than that solution. **(Speaking and Listening, Reading and Writing)**
  - » Determining whether given values are solutions to the inequality in Problem 2.
- **Goal:** Graphing the solution to an inequality as a ray on a number line.
- **Language Goal:** Understanding that the solution to an inequality is a range of values that make the inequality true. **(Reading and Writing)**

Suggested next steps

If students incorrectly isolate x in Problem 1, consider:

- Reviewing strategies from Activity 2.
- Assigning Practice Problems 1 and 2.
- Asking, "What change occurs when dividing or multiplying both sides of an inequality by a negative value?"

If students make incorrect calculations when checking values in Problem 2, consider:

- Reviewing checking values in the original and final inequality in Activity 3.
- Assigning Practice Problem 1.
- Asking, "Where else could you check the value besides just the final inequality that you found?"

Professional Learning

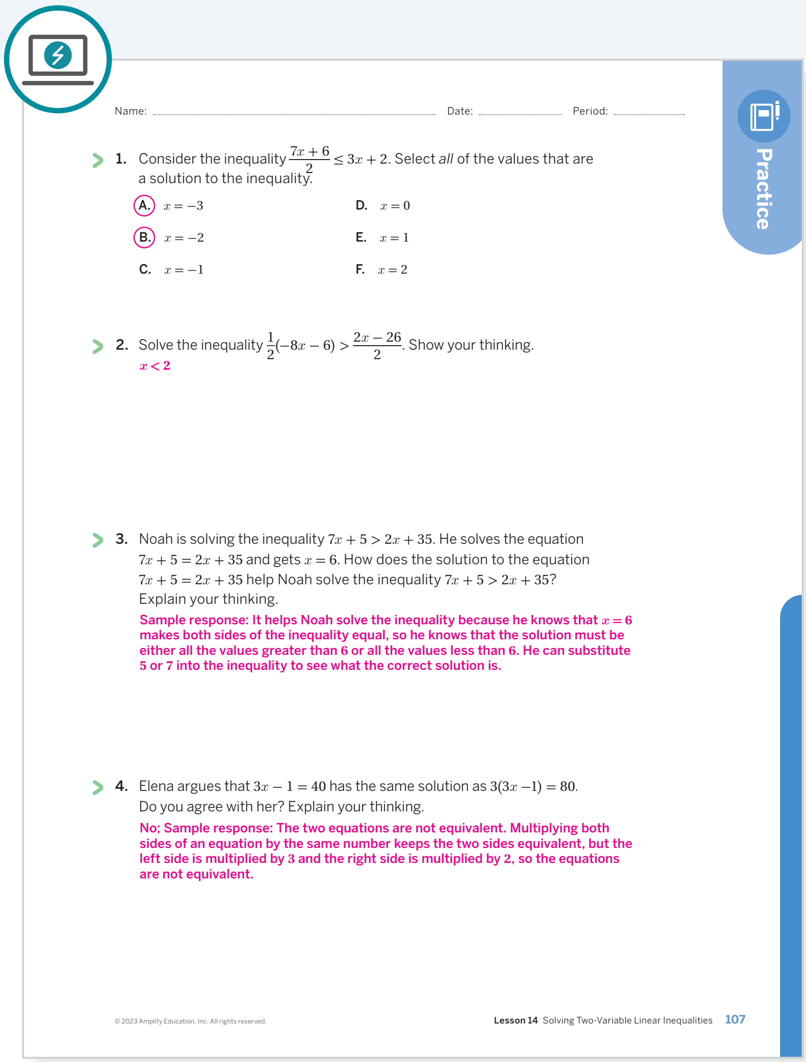
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In Activity 2, you used structured pairing with MLR1 to group students who spoke at different levels of language proficiency. What effect did this grouping strategy have on their revisions? Would you change anything the next time you use MLR1?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Consider the inequality $\frac{7x+6}{2} \leq 3x + 2$. Select *all* of the values that are a solution to the inequality.

[✓ A.] $x = -3$

[✓ B.] $x = -2$

C. $x = -1$

D. $x = 0$

E. $x = 1$

F. $x = 2$

2. Solve the inequality $\frac{1}{2}(-8x - 6) > \frac{2x - 26}{2}$. Show your thinking.

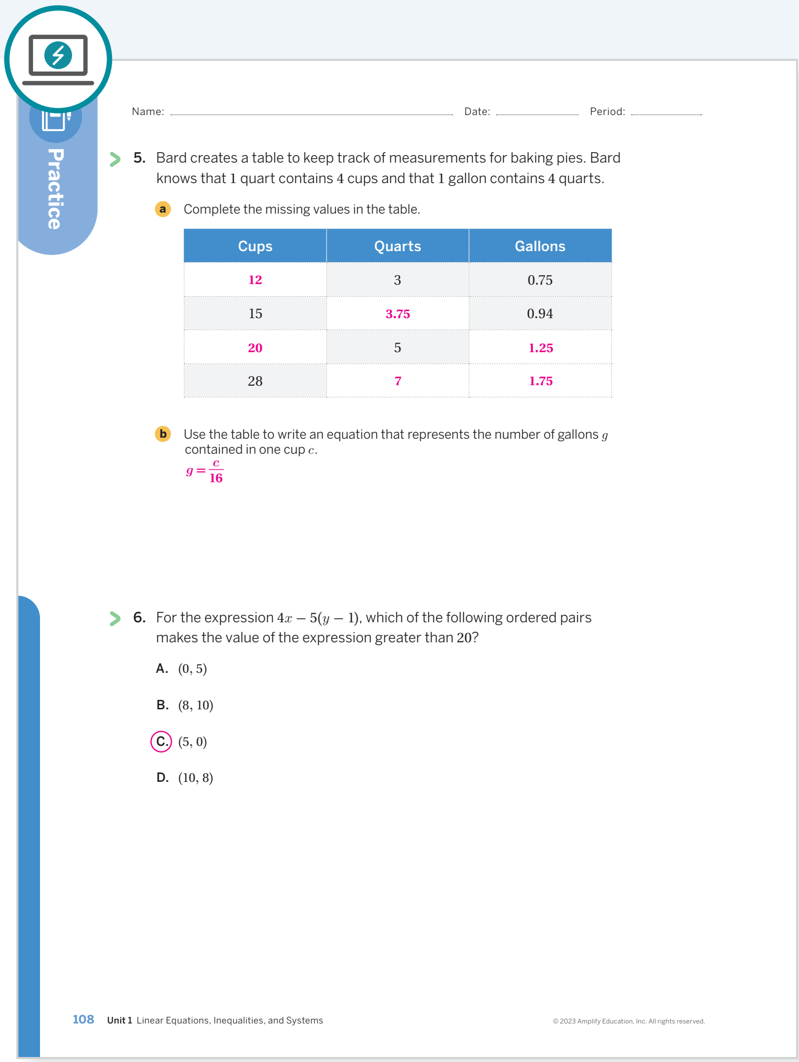
$x < 2$

3. Noah is solving the inequality $7x + 5 > 2x + 35$. He solves the equation $7x + 5 = 2x + 35$ and gets $x = 6$. How does the solution to the equation $7x + 5 = 2x + 35$ help Noah solve the inequality $7x + 5 > 2x + 35$? Explain your thinking.

Sample response: It helps Noah solve the inequality because he knows that $x = 6$ makes both sides of the inequality equal, so he knows that the solution must be either all the values greater than 6 or all the values less than 6. He can substitute 5 or 7 into the inequality to see what the correct solution is.

4. Elena argues that $3x - 1 = 40$ has the same solution as $3(3x - 1) = 80$. Do you agree with her? Explain your thinking.

No; Sample response: The two equations are not equivalent. Multiplying both sides of an equation by the same number keeps the two sides equivalent, but the left side is multiplied by 3 and the right side is multiplied by 2, so the equations are not equivalent.



Name: Date: Period:

5. Bard creates a table to keep track of measurements for baking pies. Bard knows that 1 quart contains 4 cups and that 1 gallon contains 4 quarts.

a. Complete the missing values in the table.

Cups	Quarts	Gallons
12	3	0.75
15	3.75	0.94
20	5	1.25
28	7	1.75

b. Use the table to write an equation that represents the number of gallons g contained in one cup c.

$g = \frac{c}{16}$

6. For the expression $4x - 5(y - 1)$, which of the following ordered pairs makes the value of the expression greater than 20?

A. (0, 5)

B. (8, 10)

[✓ C.] (5, 0)

D. (10, 8)

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 7	2
	5	Unit 1 Lesson 3	2
Formative ⚡	6	Unit 1 Lesson 15	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 15**

Graphing Two-Variable Linear Inequalities (Part 1)

Let's use graphs to represent solutions of two-variable linear inequalities.

Focus

Goals

1. **Language Goal:** Given the graph of a related equation, determine the solution region to an inequality in two variables by testing the points on the line and on either side of the line. **(Speaking and Listening, Writing)**
2. **Language Goal:** Understand that the solutions to a linear inequality in two variables are represented graphically as a half-plane bounded by a line. **(Speaking and Listening, Writing)**

Coherence

Today

Students learn that solutions of two-variable inequalities involve pairs of values similarly to two-variable equations. They graph solutions of inequalities, observing that solutions are not single points on a line but are comprised of a region bounded by a line. Students determine if the boundary line is included in the solution set. They write inequalities given graphs that represent solution regions.

Previously

In Lesson 14, students wrote and solved linear inequalities in one and two variables.

Coming Soon

In Lesson 16, students will graph inequalities in two variables to solve problems in context.

Rigor

- Students develop **conceptual understanding** of graphical representations of solution sets of linear inequalities in two variables by making connections to graphs of two-variable linear equations.
- Students graph linear inequalities in two variables to build **procedural skills**.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	10 min	5 min	5 min
Pairs	Small Groups	Independent	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (as needed)
- Anchor Chart PDF, *Graphing Linear Inequalities*
- Anchor Chart PDF, *Inequality Symbols and Key Phrases*
- Anchor Chart PDF, *Sentence Stems, Math Talk*
- colored pencils

Math Language Development

New words

- ***boundary line***
- ***half-plane***

Review words

- *inequality*
- *solution*
- *solution set*

Amps | Featured Activity

Activity 1
Interactive Graphs

Students test ordered pairs in inequalities. The solutions and non-solutions are represented by different symbols on the graph. Students' points are then generated on a graph to reveal the boundary line and solution set to the inequality.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated with their difficulty in looking for and making use of structure as they attempt to identify a clear boundary between the region of solutions and non-solutions. Encourage students to persevere and continue plotting more points until the boundary becomes clearer. Encourage students to ask others to explain their strategy of the points they chose to plot.

Modifications to Pacing

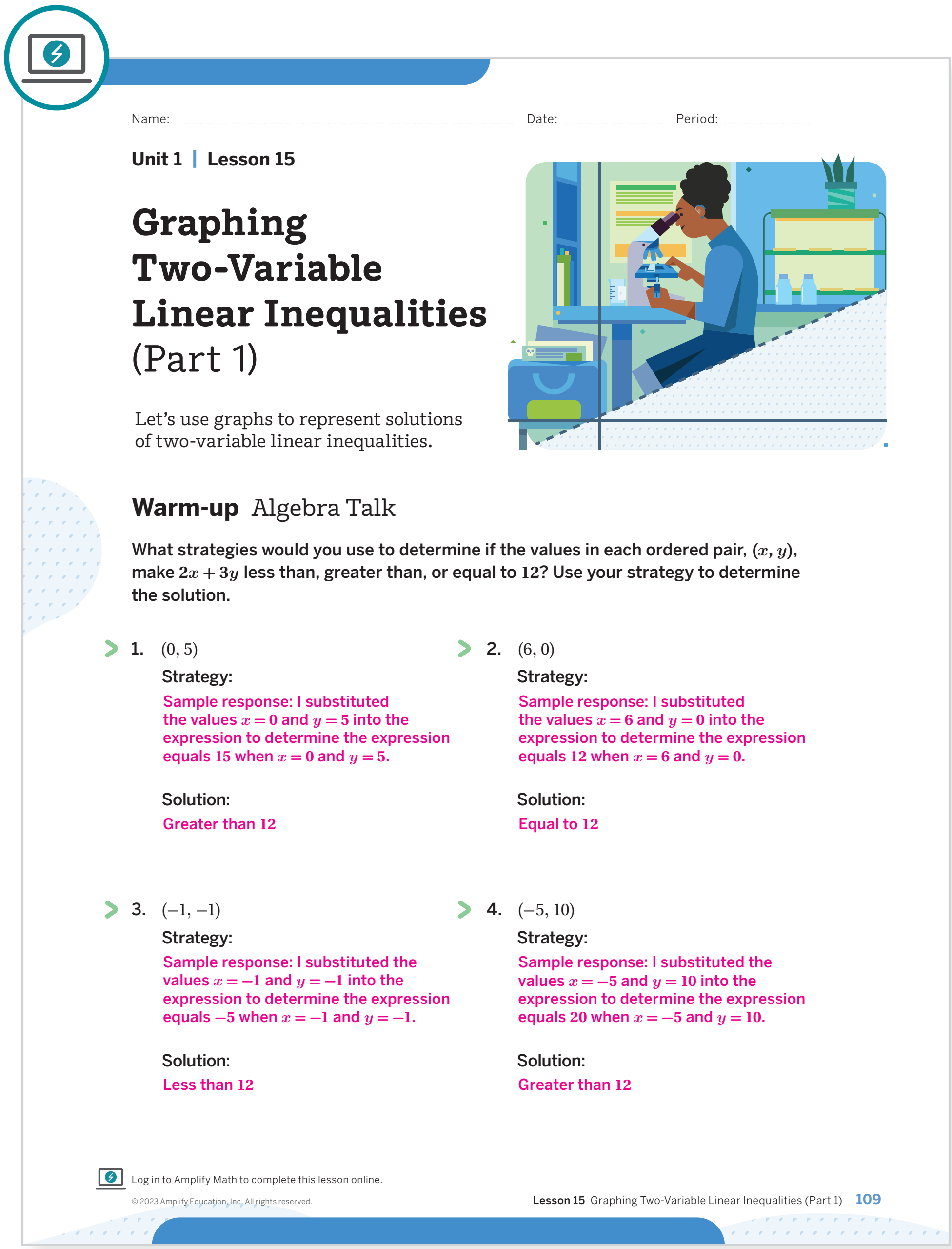
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 4 may be omitted.
- In **Activity 1**, Problem 1 may be omitted, and students can be held to 5 minutes of plotting points.
- In **Activity 3**, have students complete only Problems 1–3, or omit the Activity and assign Problems 1–3 as additional practice.

Pairs | 5 min

Warm-up Algebra Talk

Students substitute ordered pairs into an expression to determine whether an expression is greater than, less than, or equal to a value.



Name: ________ Date: ________ Period: ________

Unit 1 | Lesson 15

Graphing Two-Variable Linear Inequalities (Part 1)

Let's use graphs to represent solutions of two-variable linear inequalities.

Warm-up Algebra Talk

What strategies would you use to determine if the values in each ordered pair, (x, y), make $2x + 3y$ less than, greater than, or equal to 12? Use your strategy to determine the solution.

1. $(0, 5)$

Strategy:

Sample response: I substituted the values $x = 0$ and $y = 5$ into the expression to determine the expression equals 15 when $x = 0$ and $y = 5$.

Solution:

Greater than 12

2. $(6, 0)$

Strategy:

Sample response: I substituted the values $x = 6$ and $y = 0$ into the expression to determine the expression equals 12 when $x = 6$ and $y = 0$.

Solution:

Equal to 12

3. $(-1, -1)$

Strategy:

Sample response: I substituted the values $x = -1$ and $y = -1$ into the expression to determine the expression equals -5 when $x = -1$ and $y = -1$.

Solution:

Less than 12

4. $(-5, 10)$

Strategy:

Sample response: I substituted the values $x = -5$ and $y = 10$ into the expression to determine the expression equals 20 when $x = -5$ and $y = 10$.

Solution:

Greater than 12

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 15 Graphing Two-Variable Linear Inequalities (Part 1) 109

1 Launch

Display each problem one at a time. Allow students individual work time. Then place students in pairs and conduct the ***Algebra Talk*** routine. Keep all problems displayed throughout the discussion.

2 Monitor

Help students get started by asking, "How could you efficiently compare the value of the expression to 12?"

Look for points of confusion:

- **Misusing the equation $2x + 3y = 12$ to determine if the expression is less than or greater than 12.** Have students substitute values only into $2x + 3y$ and then compare.

Look for productive strategies:

- Substituting the coordinate pairs in the expression.
- Using the signs of the values of x and y to reason about the answers.
- Writing the expression as an inequality.

3 Connect

Have individual students share their strategies for comparing the values from the expression to 12. Select and sequence students in order of their productive strategies.

Highlight that if students graphed $2x + 3y = 12$, $(6, 0)$ would be a point on the line.

Ask:

- "Would the points other than $(6, 0)$ be graphed on the line? How do you know?" No, because these values make the expressions less than or greater than 12.
- "Which pairs, if any, are solutions to the inequality $2x + 3y \leq 12$?" $(6, 0)$ and $(-1, -1)$

Math Language Development

MLR2: Collect and Display

During the Connect, provide students with language models, such as "I substituted the values $x = 6$ and $y = 0$ into the expression to determine the expression equals 12 when $x = 6$ and $y = 0$." Collect the math terms and phrases students use and add them to the class display.

English Learners

Display or provide copies of the Anchor Chart PDF, *Sentence Stems, Math Talk* to support them as they share their strategies.

Power-up

To power up students' ability to evaluate expressions with two variables, have students complete:

Evaluate the expression $2x - 4(y + 3)$ for $x = 2$ and $y = -1$. Show your thinking.

-4; $2(2) - 4(-1 + 3) = 4 - 4(2) = 4 - 8 = -4$

Use: Before the Warm-up

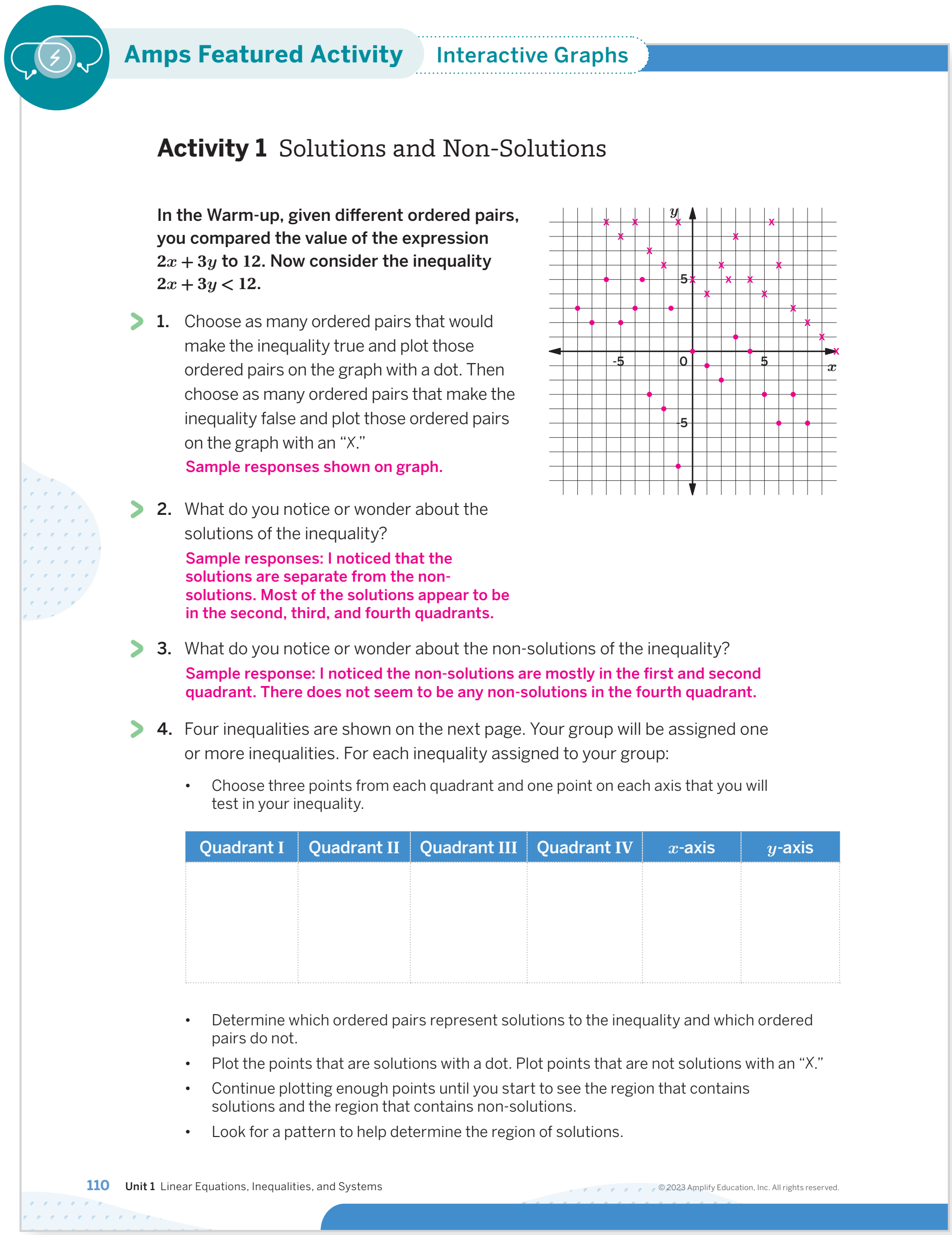
Informed by: Performance on Lesson 14, Practice Problem 6

Small Groups | 15 min

Activity 1 Solutions and Non-Solutions

Students reason quantitatively and abstractly on the solutions and non-solutions of a linear inequality and explore the graphical representations of their solution sets.

Amps Featured Activity Interactive Graphs



Activity 1 Solutions and Non-Solutions

In the Warm-up, given different ordered pairs, you compared the value of the expression $2x + 3y$ to 12. Now consider the inequality $2x + 3y < 12$.

1. Choose as many ordered pairs that would make the inequality true and plot those ordered pairs on the graph with a dot. Then choose as many ordered pairs that make the inequality false and plot those ordered pairs on the graph with an "X."

   Sample responses shown on graph.

2. What do you notice or wonder about the solutions of the inequality?

   Sample responses: I noticed that the solutions are separate from the non-solutions. Most of the solutions appear to be in the second, third, and fourth quadrants.

3. What do you notice or wonder about the non-solutions of the inequality?

   Sample response: I noticed the non-solutions are mostly in the first and second quadrant. There does not seem to be any non-solutions in the fourth quadrant.

4. Four inequalities are shown on the next page. Your group will be assigned one or more inequalities. For each inequality assigned to your group:

   - Choose three points from each quadrant and one point on each axis that you will test in your inequality.

Quadrant I	Quadrant II	Quadrant III	Quadrant IV	x-axis	y-axis

   - Determine which ordered pairs represent solutions to the inequality and which ordered pairs do not.
   - Plot the points that are solutions with a dot. Plot points that are not solutions with an "X."
   - Continue plotting enough points until you start to see the region that contains solutions and the region that contains non-solutions.
   - Look for a pattern to help determine the region of solutions.

110 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display the activity opener. Give each group time to determine and graph points. Collect ordered pairs from each group, using differing symbols to plot their points. Conduct the ***Notice and Wonder*** routine before releasing groups to work.

2 Monitor

Help students get started by providing the general form of an ordered pair for each quadrant and axis, such as $(-x, y)$ and $(x, 0)$. Prompt students to choose ordered pairs using these forms.

Look for points of confusion:

- **Having difficulty distinguishing between solutions and non-solutions.** Provide students with two colored pencils, assigning different colors for solutions and non-solutions.
- **Choosing a limited number of points and not seeing the boundary line.** Have students try to plot non-solution and solution points that are closer and closer together.

Look for productive strategies:

- Continuing to choose points from each quadrant and each axes until a boundary line becomes clearer.
- Testing more points close to the apparent boundary line to confirm its location.
- Changing the inequality symbol to an equal sign and graphing the equation as the boundary line.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with the Activity 1 PDF table. Have them use the given ordered pairs for each graph as a starting point. Then ask them to generate 6 more of their own ordered pairs to test, recording their ordered pairs in the table.

Extension: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which the solutions and non-solutions are represented by different symbols on the graph. Students' points are generated on the graph to reveal the boundary line and solution set to the inequality.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students respond to the Ask questions, highlight the mathematical terms and phrases they use, such as *half-plane*, *boundary line*, *solution*, *solid line*, *dashed line*, *inequality symbol*, etc.

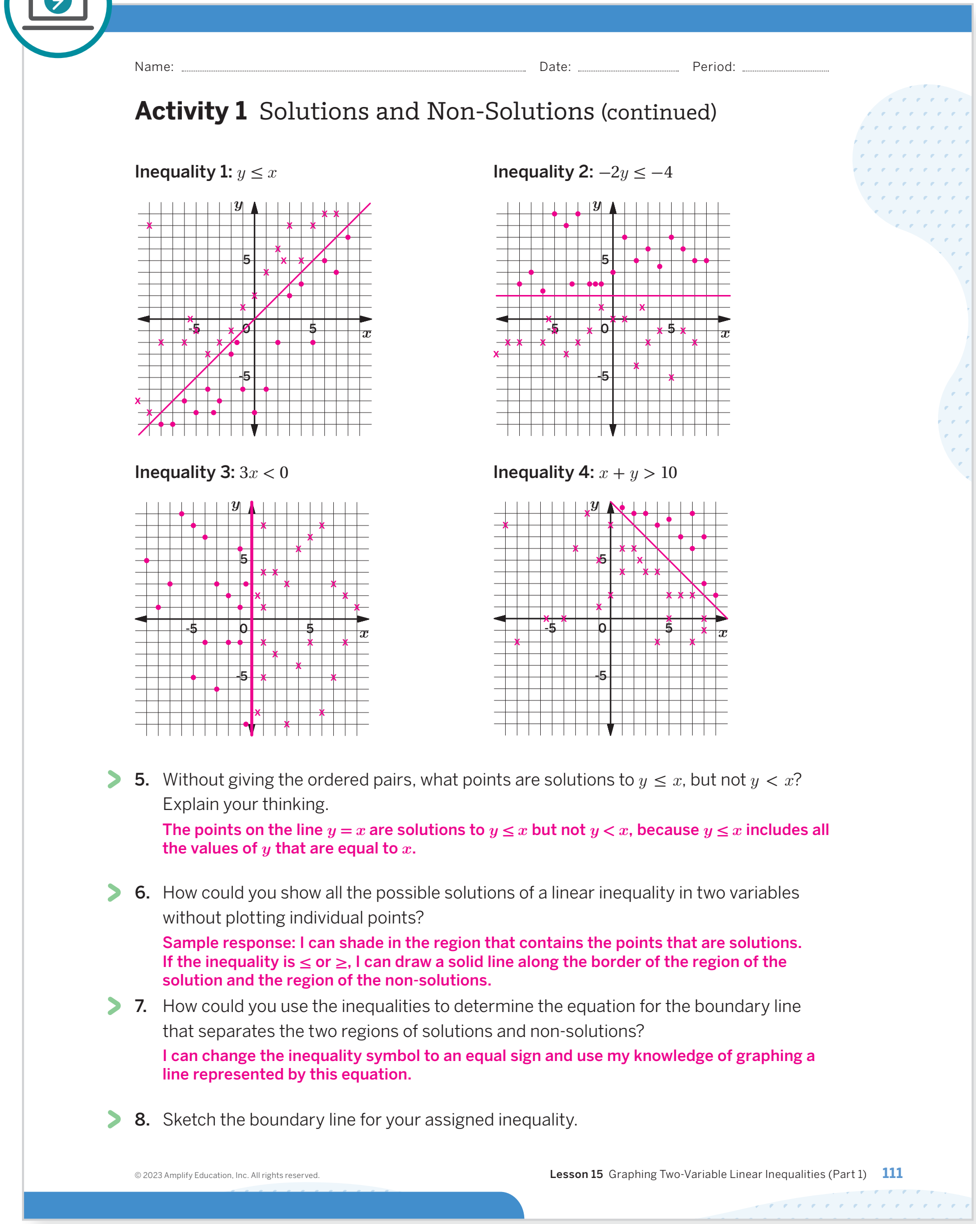
English Learners

Annotate the graph of Inequality 1 to illustrate the boundary line and the two half-planes.

Small Groups | 15 min

Activity 1 Solutions and Non-Solutions (continued)

Students reason quantitatively and abstractly on the solutions and non-solutions of a linear inequality and explore the graphical representations of their solution sets.



Name: ______ Date: ______ Period: ______

Activity 1 Solutions and Non-Solutions (continued)

Inequality 1: $y \le x$

Inequality 2: $-2y \le -4$

Inequality 3: $3x < 0$

Inequality 4: $x + y > 10$

5. Without giving the ordered pairs, what points are solutions to $y \le x$, but not $y < x$? Explain your thinking.
   The points on the line $y = x$ are solutions to $y \le x$ but not $y < x$, because $y \le x$ includes all the values of y that are equal to x.

6. How could you show all the possible solutions of a linear inequality in two variables without plotting individual points?
   Sample response: I can shade in the region that contains the points that are solutions. If the inequality is ≤ or ≥, I can draw a solid line along the border of the region of the solution and the region of the non-solutions.

7. How could you use the inequalities to determine the equation for the boundary line that separates the two regions of solutions and non-solutions?
   I can change the inequality symbol to an equal sign and use my knowledge of graphing a line represented by this equation.

8. Sketch the boundary line for your assigned inequality.

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have pairs of students share their graphs where a clear boundary can be seen between two regions.

Highlight that the solution and non-solution regions are separated by a clear boundary, called a boundary line. The boundary line separates the coordinate plane into two half-planes.

Define the terms ***boundary line*** and ***half-plane***.

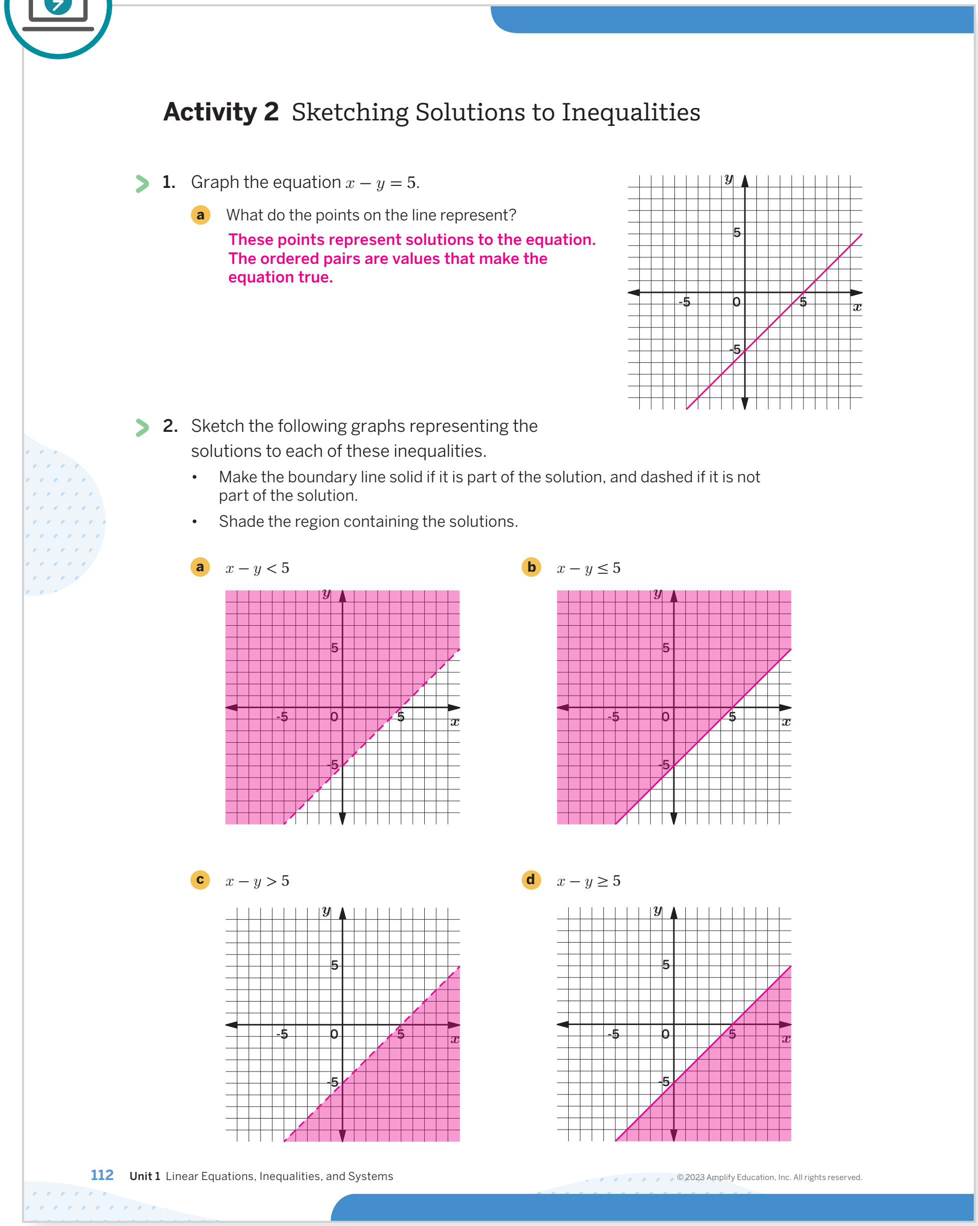
Ask:

- "How could you account for all solutions on these graphs?" I can shade the half-plane that contains the points that are solutions. (Point out that if the boundary line is part of the solution, a solid line should be used. If it is not, a dashed line should be used.)
- "How does the inequality symbol affect this boundary?" There are only solutions along this boundary line if the inequality symbol is ≥ or ≤.

Independent | 10 min

Activity 2 Sketching Solutions to Inequalities

Students look for and express regularity in repeated reasoning by graphing the solutions of linear inequalities and writing inequalities whose solutions could be represented by given graphs.



Activity 2 Sketching Solutions to Inequalities

1. Graph the equation $x - y = 5$.

   a. What do the points on the line represent?

   These points represent solutions to the equation. The ordered pairs are values that make the equation true.

2. Sketch the following graphs representing the solutions to each of these inequalities.

   - Make the boundary line solid if it is part of the solution, and dashed if it is not part of the solution.
   - Shade the region containing the solutions.

   a. $x - y < 5$

   b. $x - y \leq 5$

   c. $x - y > 5$

   d. $x - y \geq 5$

112 **Unit 1** Linear Equations, Inequalities, and Systems

©2023 Amplify Education, Inc. All rights reserved.

1 Launch

Ask, “How do you know if the boundary line is included in the solutions?” If the inequality symbol includes “equal to,” then the points along the line are included.

Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by saying, “Use ordered pairs of points on either side of the boundary line to determine which region contains the solutions.”

Look for points of confusion:

- **Assuming the original inequality symbol can be used to determine whether to shade “above” (for $>$ or $\geq$) or “below” (for $<$ or $\leq$) the boundary line.** Have students check this reasoning by testing ordered pairs on either side of the boundary line.
- **Struggling to write an equation for a vertical line or a horizontal line.** Have students determine and plot the coordinates of several points on the line and look for a pattern.

Look for productive strategies:

- Determining intercepts and the slope of the boundary line in Problem 3 to determine the equation of the boundary line.
- Testing ordered pairs on either side of the boundary line to determine the inequality symbol.

Activity 2 continued ›

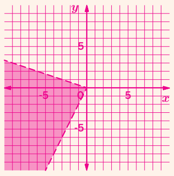
Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display or provide copies of the Anchor Chart PDF, *Graphing Linear Inequalities* for students to reference as they complete Problem 2. For Problem 3, encourage students to first think of the graph as a straight line, without an inequality or any shading. Then have them determine the inequality symbol.

Extension: *Math Enrichment*

Have students graph the inequality $2x < y < -\frac{1}{3}x$.



Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display an incorrect inequality and incorrect reasoning, such as “The inequality in Problem 3a is $y > 3$ because all the points to the right of 3 are shaded.” Ask these questions:

- **Critique:** “Why is this statement incorrect?”
- **Correct:** “How would you correct this statement?”
- **Clarify:** “How do you know your statement is correct?”

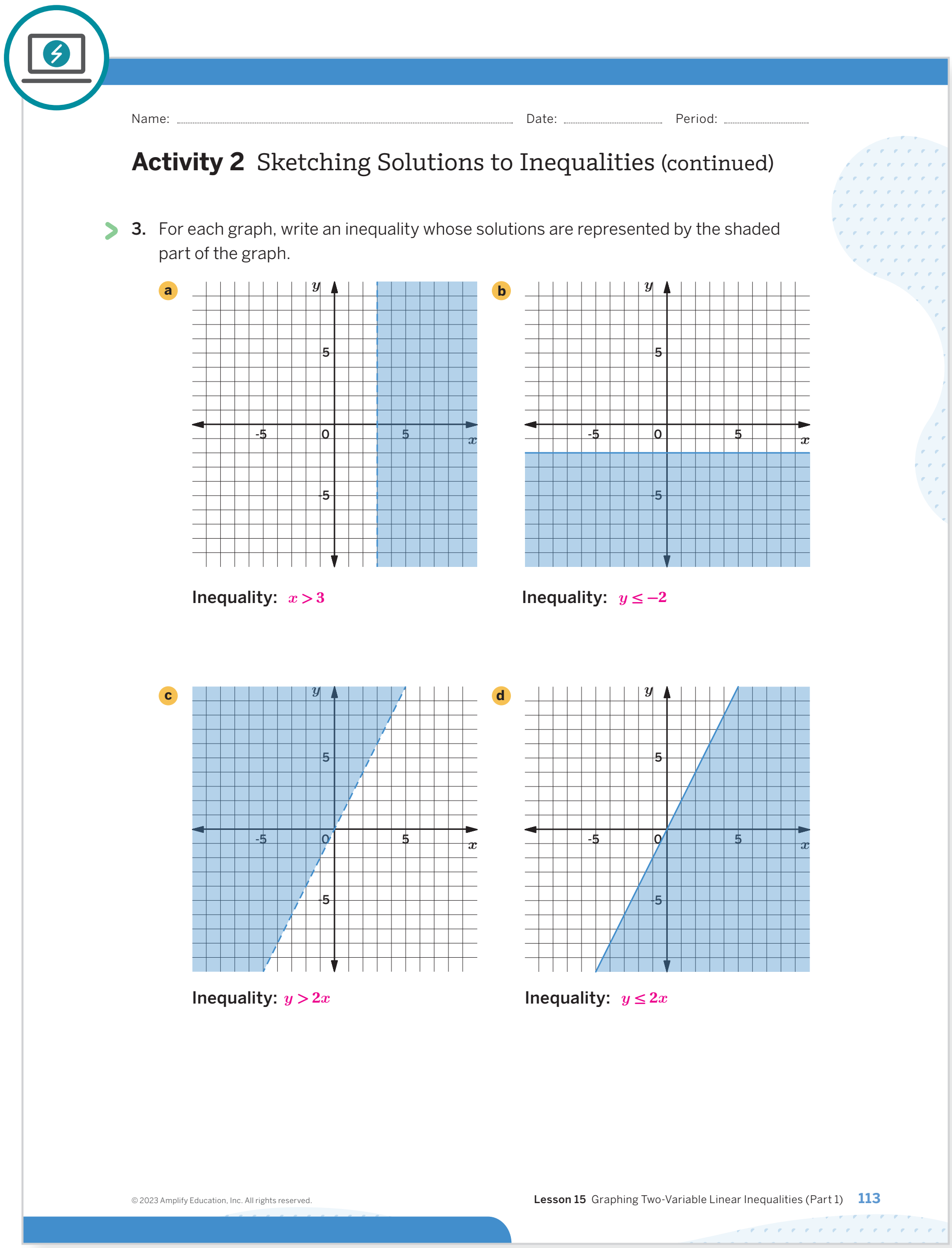
English Learners

After the discussion, clearly annotate the incorrect part of the statement.

Independent | 10 min

Activity 2 Sketching Solutions to Inequalities (continued)

Students look for and express regularity in repeated reasoning by graphing the solutions of linear inequalities and writing inequalities whose solutions could be represented by given graphs.



Name: ______ Date: ______ Period: ______

Activity 2 Sketching Solutions to Inequalities (continued)

3. For each graph, write an inequality whose solutions are represented by the shaded part of the graph.

a Inequality: $x > 3$

b Inequality: $y \le -2$

c Inequality: $y > 2x$

d Inequality: $y \le 2x$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 15 Graphing Two-Variable Linear Inequalities (Part 1) 113

3 Connect

Have individual students share their graphs for Problem 1 and their strategies for writing their inequalities in Problem 2.

Highlight that graphing an accurate boundary line is critical in clearly separating the solutions and non-solutions regions. Once they graph the boundary line, they can change it to a dashed or solid line depending on the inequality symbol.

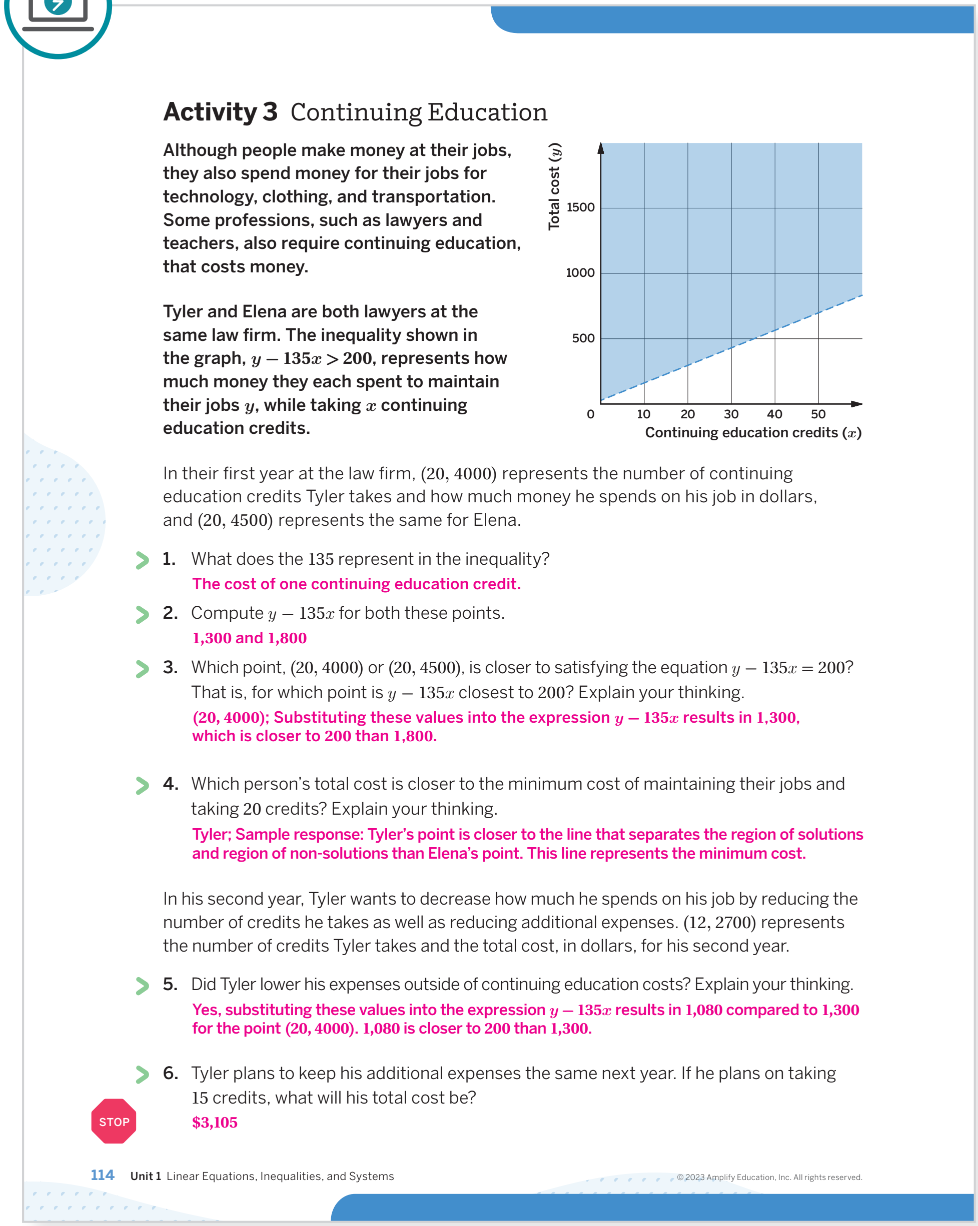
Ask:

- "What methods did you use to graph the boundary line?" Sample response: I determined the x- and y-intercepts of the equation that go along with each inequality, and then connected these two points with a line.
- "Does checking one ordered pair in the solution region confirm that you graphed the inequality accurately?" Not necessarily. The point chosen in the solution region could make the inequality true, but if the boundary line was graphed incorrectly, part of the shaded region will not actually contain the solutions.

Independent | 10 min

Activity 3 Continuing Education

Students use the graph of a linear inequality in two variables modeling a context to explore the cost of maintaining a job.



Activity 3 Continuing Education

Although people make money at their jobs, they also spend money for their jobs for technology, clothing, and transportation. Some professions, such as lawyers and teachers, also require continuing education, that costs money.

Tyler and Elena are both lawyers at the same law firm. The inequality shown in the graph, $y - 135x > 200$, represents how much money they each spent to maintain their jobs y, while taking x continuing education credits.

In their first year at the law firm, $(20, 4000)$ represents the number of continuing education credits Tyler takes and how much money he spends on his job in dollars, and $(20, 4500)$ represents the same for Elena.

1. What does the 135 represent in the inequality?
   The cost of one continuing education credit.
2. Compute $y - 135x$ for both these points.
   1,300 and 1,800
3. Which point, $(20, 4000)$ or $(20, 4500)$, is closer to satisfying the equation $y - 135x = 200$? That is, for which point is $y - 135x$ closest to 200? Explain your thinking.
   $(20, 4000)$; Substituting these values into the expression $y - 135x$ results in 1,300, which is closer to 200 than 1,800.
4. Which person's total cost is closer to the minimum cost of maintaining their jobs and taking 20 credits? Explain your thinking.
   Tyler; Sample response: Tyler's point is closer to the line that separates the region of solutions and region of non-solutions than Elena's point. This line represents the minimum cost.

In his second year, Tyler wants to decrease how much he spends on his job by reducing the number of credits he takes as well as reducing additional expenses. $(12, 2700)$ represents the number of credits Tyler takes and the total cost, in dollars, for his second year.

5. Did Tyler lower his expenses outside of continuing education costs? Explain your thinking.
   Yes, substituting these values into the expression $y - 135x$ results in 1,080 compared to 1,300 for the point $(20, 4000)$. 1,080 is closer to 200 than 1,300.
6. Tyler plans to keep his additional expenses the same next year. If he plans on taking 15 credits, what will his total cost be?
   \$3,105

STOP

114 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Use the *Three Reads* routine to read and help students make sense of the narrative. Ask students, "What words or phrases do you associate with each inequality symbol?"

2 Monitor

Help students get started by having them plot points to visualize how close each point is to the boundary line.

Look for points of confusion:

- **Using only the total costs to compare Tyler's expenses in Problem 5.** Have students first calculate the total cost for Tyler's second year and the cost of the continuing education separately.

Look for productive strategies:

- Using original points to determine which student is closest to the minimum cost of 20 continuing education credits.

3 Connect

Display the graph and plot the points of each problem on the graph.

Have individual students share their strategy for comparing Tyler's expenses from each year.

Highlight that the minimum cost of just the continuing education credits is represented by the boundary line.

Ask, "Why is the boundary line dashed in this scenario?" There are other costs to maintaining their jobs in addition to the continuing education credits, so the total cost will always be greater than just the continuing education costs.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with the alternative form of the inequality, $y > 200 + 135x$, and ask them to determine the y-coordinate of the point on the boundary line when $x = 12$. Ask them how this value will help them complete Problem 5.

Extension: *Math Enrichment*

Have students complete the following problem:

Tyler decided that he wanted to make sure his additional job-related expenses were less than twice those expenses in the second year at this job. What inequality would represent this scenario? $y - 135x < 2160$ (or equivalent)

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

- **Read 1:** Students should understand that Tyler and Elena need to pay for education classes as part of their job requirements.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as the given inequality $y - 135x > 200$.
- **Read 3:** Ask students to think about what this inequality represents.

Display the Anchor Chart PDF, *Inequality Symbols* and *Key Phrases*.

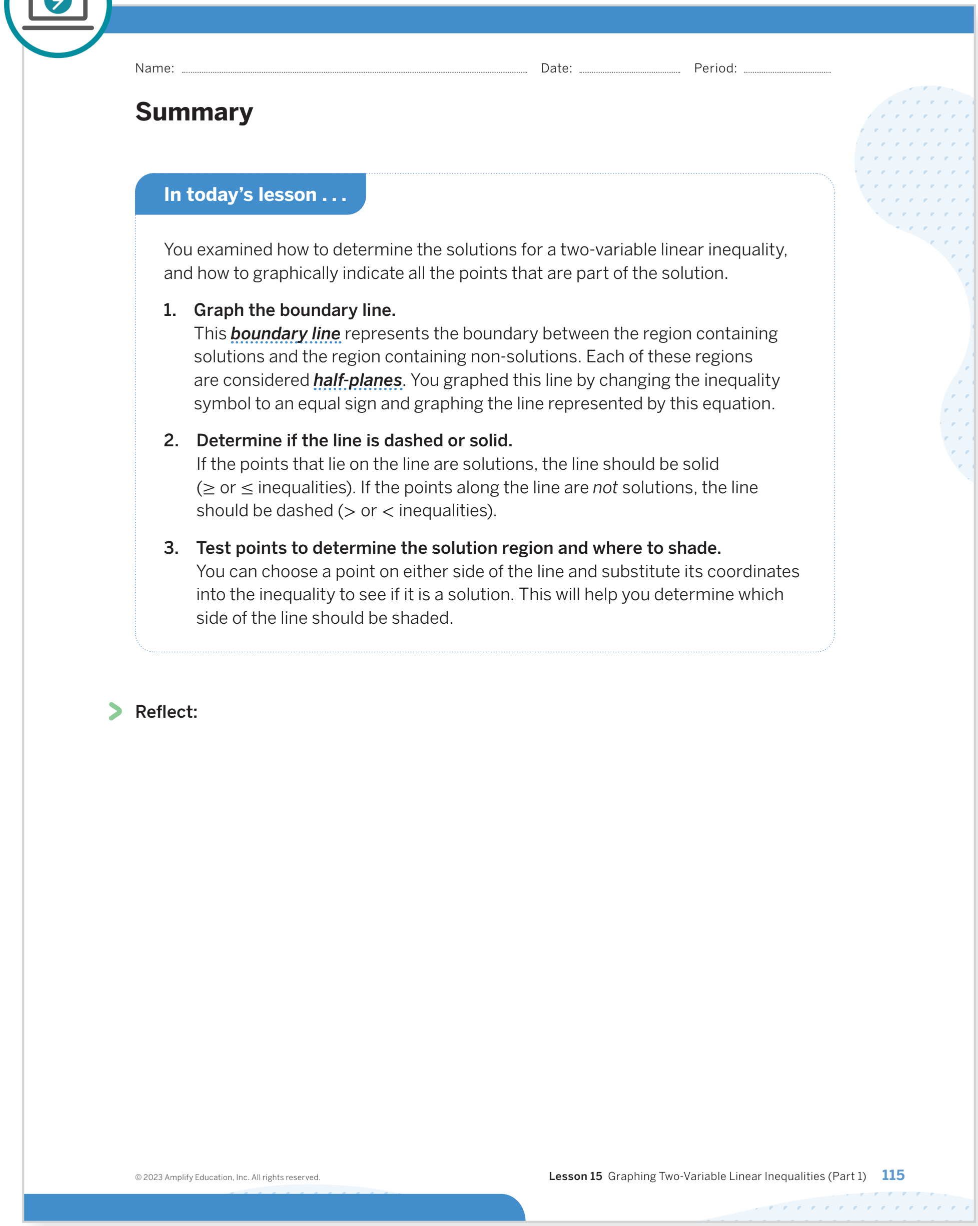
English Learners

Have students highlight what each of the variables x and y represent in the problem.

Whole Class | 5 min

Summary

Review and synthesize graphing the solution to a linear inequality in two variables.



Name: ______________________ Date: __________ Period: __________

Summary

In today's lesson . . .

You examined how to determine the solutions for a two-variable linear inequality, and how to graphically indicate all the points that are part of the solution.

1. **Graph the boundary line.**
   This ***boundary line*** represents the boundary between the region containing solutions and the region containing non-solutions. Each of these regions are considered ***half-planes***. You graphed this line by changing the inequality symbol to an equal sign and graphing the line represented by this equation.
2. **Determine if the line is dashed or solid.**
   If the points that lie on the line are solutions, the line should be solid ($\geq$ or $\leq$ inequalities). If the points along the line are *not* solutions, the line should be dashed ($>$ or $<$ inequalities).
3. **Test points to determine the solution region and where to shade.**
   You can choose a point on either side of the line and substitute its coordinates into the inequality to see if it is a solution. This will help you determine which side of the line should be shaded.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the graph of and the inequality $y - x > 0$.

Have students share their method for graphing the boundary line.

Highlight that graphing the boundary line is a critical first step in determining where to shade to represent the solutions of linear inequalities in two variables.

Formalize vocabulary: *boundary line*, *half-plane*

Ask:

- "What would you do next after graphing the boundary line?" Sample response: I would look at the inequality symbol to determine if the line is solid or dashed, pick a point on either side of the boundary, and then substitute the values of the ordered pair into the inequality to determine where to shade.
- "What methods can you use to check your graph?" Sample response: I could test points from both sides of the boundary line, as well as on the boundary line, to confirm the shading and whether the line should be dashed or solid.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies did you find helpful today when graphing a boundary line and determining which region contains the solutions?"
- "Were any strategies or tools not helpful? Why?"

Math Language Development

MLR2: Collect and Display

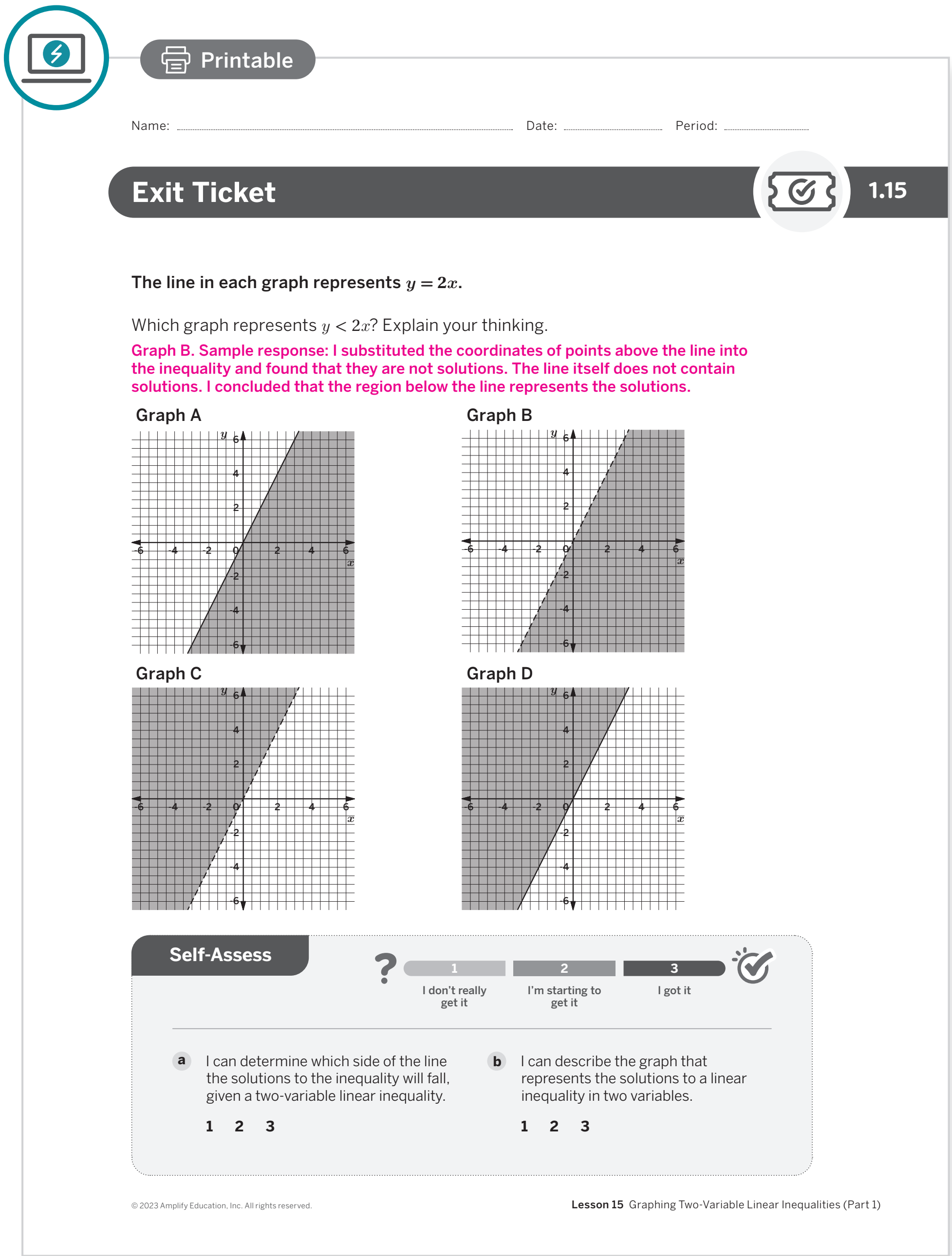
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *boundary line* and *half plane* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying and explaining the graph of the solutions to a linear inequality in two variables.

Printable



Name: ______________ Date: __________ Period: __________

Exit Ticket 1.15

The line in each graph represents $y = 2x$.

Which graph represents $y < 2x$? Explain your thinking.

Graph B. Sample response: I substituted the coordinates of points above the line into the inequality and found that they are not solutions. The line itself does not contain solutions. I concluded that the region below the line represents the solutions.

Graph A

Graph B

Graph C

Graph D

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can determine which side of the line the solutions to the inequality will fall, given a two-variable linear inequality.

1 2 3

b I can describe the graph that represents the solutions to a linear inequality in two variables.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 15 Graphing Two-Variable Linear Inequalities (Part 1)

Success looks like . . .

- **Language Goal:** Given the graph of a related equation, determining the solution region to an inequality in two variables by testing the points on the line and on either side of the line. **(Speaking and Listening, Writing)**
  - » Substituting points above or below the line to determine the correct graph for the inequality $y < 2x$.
- **Language Goal:** Understanding that the solutions to a linear inequality in two variables are represented graphically as a half-plane bounded by a line. **(Speaking and Listening, Writing)**
  - » Selecting the graph for the inequality $y < 2x$ by choosing the correct half plane and graphed line.

Suggested next steps

If students select Graph A, consider:

- Reviewing how to determine whether the boundary line is included in the solution, from Activity 2.
- Assigning Practice Problems 1–3.
- Asking, "What does a solid boundary line represent?"

If students select the Graphs B or D, consider:

- Reviewing how to determine which region to shade from Activity 2.
- Assigning Practice Problems 1–3.
- Asking, "How can you use a point from either side of the boundary line to determine where to shade?"

Professional Learning

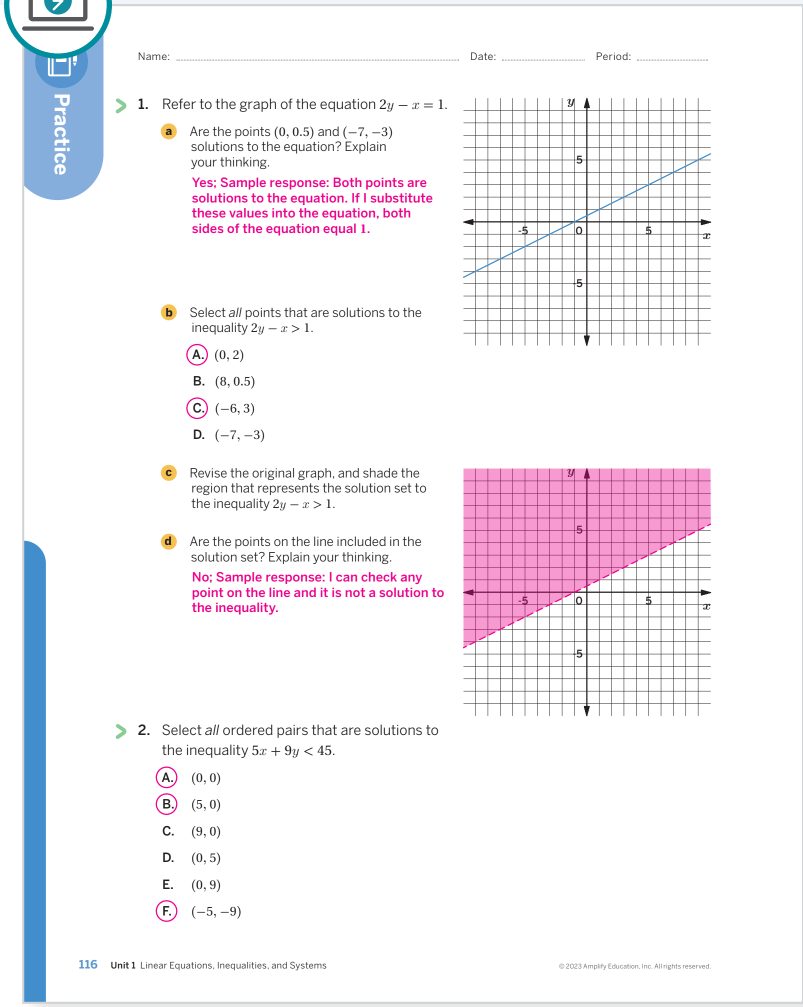
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students graphed linear inequalities in two variables. How did that build on the earlier work students did with graphing linear equations?
- What different ways did students approach graphing the boundary line? What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: .... Date: .... Period: ....

1. Refer to the graph of the equation $2y - x = 1$.

   a. Are the points $(0, 0.5)$ and $(-7, -3)$ solutions to the equation? Explain your thinking.

      Yes; Sample response: Both points are solutions to the equation. If I substitute these values into the equation, both sides of the equation equal 1.

   b. Select *all* points that are solutions to the inequality $2y - x > 1$.

      [✓ A.] $(0, 2)$
      B. $(8, 0.5)$
      [✓ C.] $(-6, 3)$
      D. $(-7, -3)$

   c. Revise the original graph, and shade the region that represents the solution set to the inequality $2y - x > 1$.

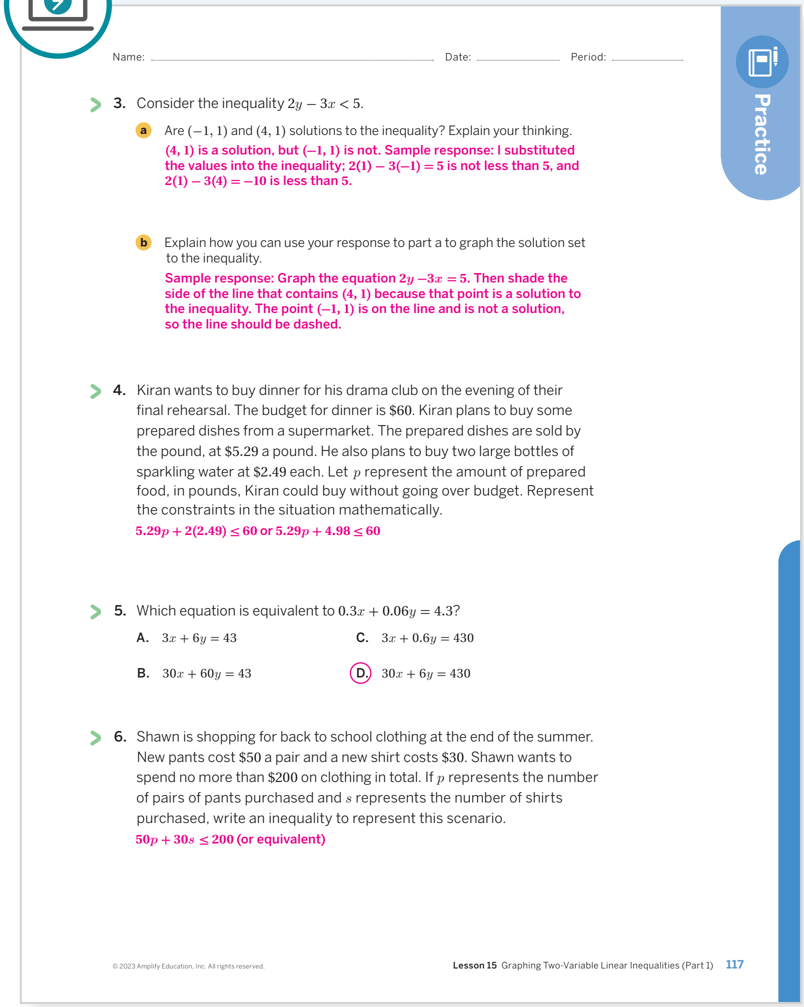
   d. Are the points on the line included in the solution set? Explain your thinking.

      No; Sample response: I can check any point on the line and it is not a solution to the inequality.

2. Select *all* ordered pairs that are solutions to the inequality $5x + 9y < 45$.

   [✓ A.] $(0, 0)$
   [✓ B.] $(5, 0)$
   C. $(9, 0)$
   D. $(0, 5)$
   E. $(0, 9)$
   [✓ F.] $(-5, -9)$

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: .... Date: .... Period: ....

3. Consider the inequality $2y - 3x < 5$.

   a. Are $(-1, 1)$ and $(4, 1)$ solutions to the inequality? Explain your thinking.

      $(4, 1)$ is a solution, but $(-1, 1)$ is not. Sample response: I substituted the values into the inequality; $2(1) - 3(-1) = 5$ is not less than 5, and $2(1) - 3(4) = -10$ is less than 5.

   b. Explain how you can use your response to part a to graph the solution set to the inequality.

      Sample response: Graph the equation $2y - 3x = 5$. Then shade the side of the line that contains $(4, 1)$ because that point is a solution to the inequality. The point $(-1, 1)$ is on the line and is not a solution, so the line should be dashed.

4. Kiran wants to buy dinner for his drama club on the evening of their final rehearsal. The budget for dinner is \$60. Kiran plans to buy some prepared dishes from a supermarket. The prepared dishes are sold by the pound, at \$5.29 a pound. He also plans to buy two large bottles of sparkling water at \$2.49 each. Let p represent the amount of prepared food, in pounds, Kiran could buy without going over budget. Represent the constraints in the situation mathematically.

   $5.29p + 2(2.49) \leq 60$ or $5.29p + 4.98 \leq 60$

5. Which equation is equivalent to $0.3x + 0.06y = 4.3$?

   A. $3x + 6y = 43$
   C. $3x + 0.6y = 430$
   B. $30x + 60y = 43$
   [✓ D.] $30x + 6y = 430$

6. Shawn is shopping for back to school clothing at the end of the summer. New pants cost \$50 a pair and a new shirt costs \$30. Shawn wants to spend no more than \$200 on clothing in total. If p represents the number of pairs of pants purchased and s represents the number of shirts purchased, write an inequality to represent this scenario.

   $50p + 30s \leq 200$ (or equivalent)

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 13	2
	5	Unit 1 Lesson 7	2
Formative ⚡	6	Unit 1 Lesson 16	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 16**

Graphing Two-Variable Linear Inequalities (Part 2)

Let's practice writing, interpreting, and graphing solutions to linear inequalities in two variables.

Focus

Goals

1. **Language Goal:** Identify an inequality, a graph, an ordered pair, and a description that represent the constraints and possible solutions in a situation. **(Speaking and Listening, Writing)**
2. **Language Goal:** Understand that a constraint on two variables can be represented by an inequality, a graph (a half-plane), and a verbal description. **(Speaking and Listening, Writing)**
3. Write inequalities in two variables to represent the constraints in a situation and use technology to graph the solution set to answer questions about the situation.

Coherence

Today

Students write linear inequalities to represent the constraints in situations and then use the representations (including the graphs of the solutions) to answer questions about the situations. As they write inequalities from descriptions, decide on the solution sets, and interpret points in a solution region, students engage in quantitative and abstract reasoning.

Previously

In Lesson 14, students graphed linear inequalities in two variables and determined the solution region by testing points.

Coming Soon

In Lessons 23 and 24, students will graph systems of linear inequalities.

Rigor

- Students strengthen their **fluency** in writing and graphing linear inequalities.
- Students **apply** linear inequalities in two variables in the context of gap year pathways.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per student
- Activity 1 PDF (answers)
- Activity 1 PDF, *Are you ready for more?*
- Activity 1 PDF, *Are you ready for more?* (answers)
- Activity 2 PDF (as needed)
- Activity 2 PDF, pre-cut cards, one set per student
- Anchor Chart PDF, *Inequality Symbols and Key Phrases*
- Anchor Chart PDF, *Graphing Linear Inequalities*
- Anchor Chart PDF, *Sentence Stems, Notice and Wonder*
- graphing technology

Math Language Development

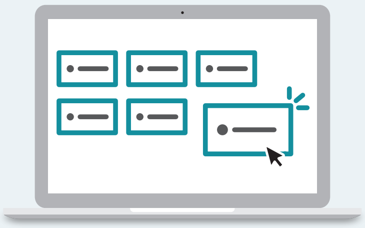
Review words

- *boundary line*
- *half-plane*
- *inequality*
- *solution*

Amps | **Featured Activity**

Activity 2
Digital Card Sort

Students match scenarios, inequalities, solutions, and graphs by dragging and connecting them on screen.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may become distracted while creating and moving between different representations of linear inequalities in Activity 1. Inspire students to set immediate goals to help them maintain focus. Encourage students to ask other pairs of students for other avenues of thinking when moving between each representation and record strategies that they find helpful.

Modifications to Pacing

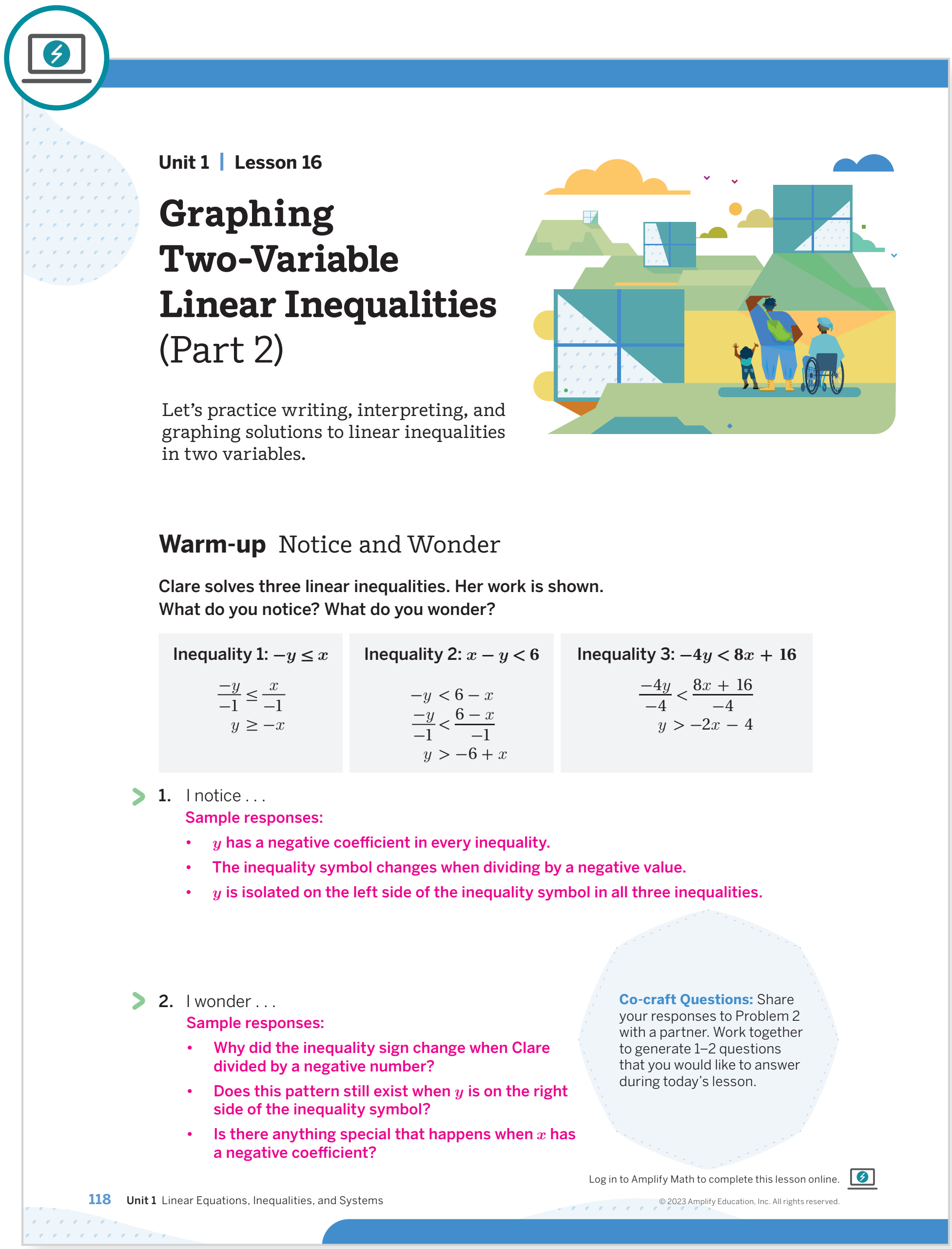
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, provide pairs of students one or two of the cards in a set and have them determine the other cards that go with the set.
- Optional **Activity 2** may be omitted or shortened.

Independent | 5 min

Warm-up Notice and Wonder

Students notice the effects of dividing an inequality by a negative value and ask questions to generalize these effects.



Unit 1 | Lesson 16

Graphing Two-Variable Linear Inequalities (Part 2)

Let's practice writing, interpreting, and graphing solutions to linear inequalities in two variables.

Warm-up Notice and Wonder

Clare solves three linear inequalities. Her work is shown. What do you notice? What do you wonder?

Inequality 1: $-y \leq x$	Inequality 2: $x - y < 6$	Inequality 3: $-4y < 8x + 16$
$\frac{-y}{-1} \leq \frac{x}{-1}$ $y \geq -x$	$-y < 6 - x$ $\frac{-y}{-1} < \frac{6 - x}{-1}$ $y > -6 + x$	$\frac{-4y}{-4} < \frac{8x + 16}{-4}$ $y > -2x - 4$

1. I notice . . .

   Sample responses:
   - y has a negative coefficient in every inequality.
   - The inequality symbol changes when dividing by a negative value.
   - y is isolated on the left side of the inequality symbol in all three inequalities.

2. I wonder . . .

   Sample responses:
   - Why did the inequality sign change when Clare divided by a negative number?
   - Does this pattern still exist when y is on the right side of the inequality symbol?
   - Is there anything special that happens when x has a negative coefficient?

Co-craft Questions: Share your responses to Problem 2 with a partner. Work together to generate 1–2 questions that you would like to answer during today's lesson.

Log in to Amplify Math to complete this lesson online.

118 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Conduct the *Notice and Wonder* routine. Prompt students to examine each inequality individually first. Then have them compare the inequalities before they write what they notice and wonder.

2 Monitor

Help students get started by asking, "What is common in the work of all three problems?"

Look for points of confusion:

- **Thinking that reversing the inequality symbol only occurs when dividing.** Ask, "What operation is equivalent to dividing by −1? How would this affect any generalization you made?"

Look for productive strategies:

- Testing an ordered pair in the original inequality and the final inequality to help determine if the final inequality still has the same solution set.

3 Connect

Have student pairs share what they noticed or wondered. Record some of the responses.

Highlight that dividing by a value is equivalent to multiplying by the reciprocal of the value, so any pattern they notice would hold true for multiplying by a negative value as well.

Ask, "Would this pattern be true when multiplying or dividing one side of the inequality by a negative value?" No. Multiplying or dividing just one side of an inequality symbol by a negative value does not make an equivalent expression.

Math Language Development

MLR5: Co-craft Questions

After students complete Problem 2, have them share what they wondered with a partner and work together to generate 1–2 questions that they would like to answer during today's lesson. Consider posting these questions and returning to them at the end of the lesson to see if students have been able to answer them after completing the activities in this lesson.

English Learners

Display or provide the Anchor Chart PDF, *Sentence Stems, Notice and Wonder* to support students as they complete the Warm-up.

Power-up

To power up students' ability to write an inequality from context, have students complete:

Mai needs to buy some supplies for school. Pens cost $\$0.50$ each and notebooks cost $\$1.25$ each. She can spend up to $\$25$. Which inequality represents the number of pens p and notebooks n she can purchase?

A. $0.50p + 1.25n < 25$

B. $0.50p + 1.25n > 25$

C. $0.50p + 1.25n \geq 25$

[✓ D.] $0.50p + 1.25n \leq 25$

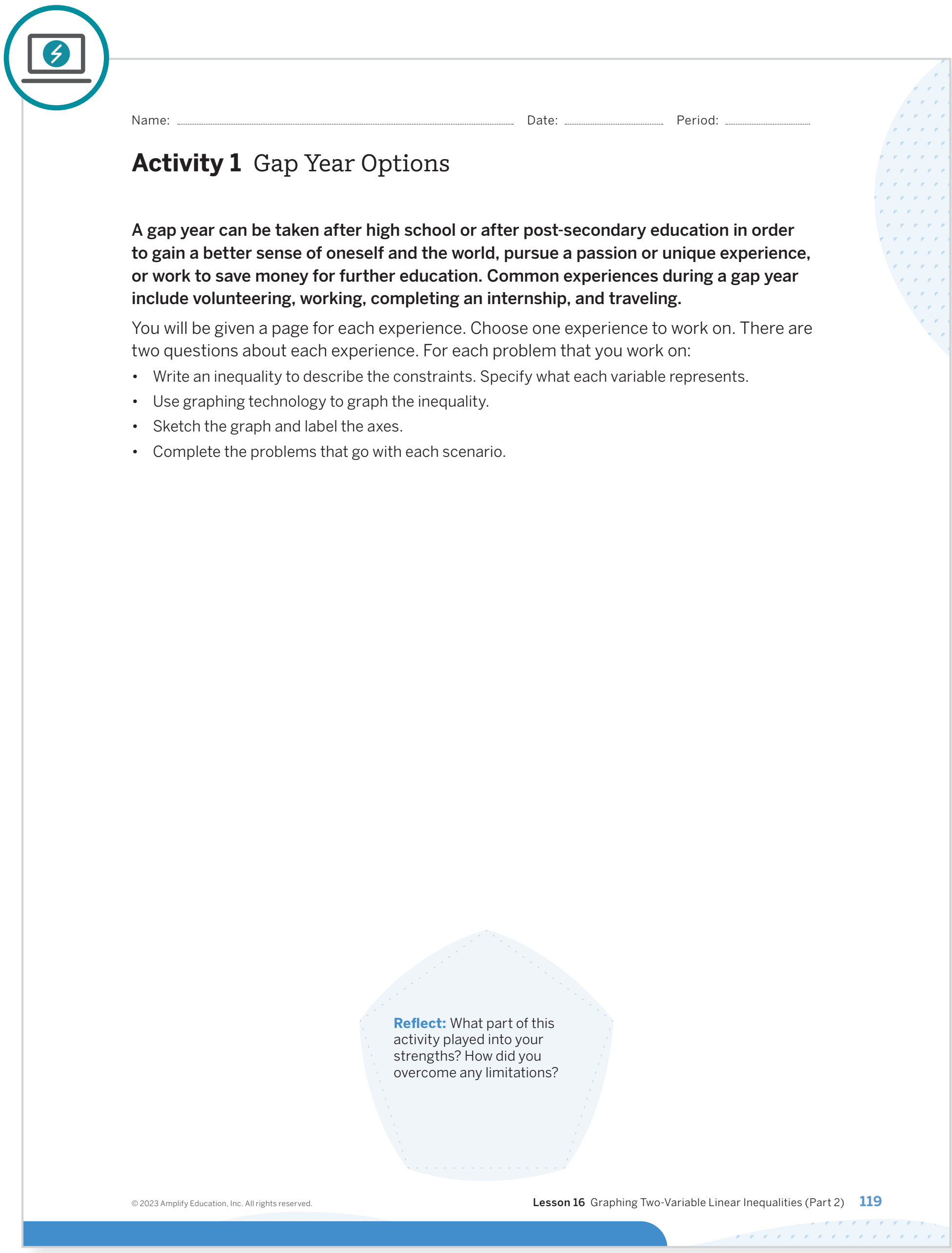
Use: Before Activity 1

Informed by: Performance on Lesson 15, Practice Problem 6

Pairs | 15 min

Activity 1 Gap Year Options

Students write, graph, and interpret inequalities in two variables to represent constraints in situations to answer contextual questions.



Name: ____________ Date: ________ Period: ________

Activity 1 Gap Year Options

A gap year can be taken after high school or after post-secondary education in order to gain a better sense of oneself and the world, pursue a passion or unique experience, or work to save money for further education. Common experiences during a gap year include volunteering, working, completing an internship, and traveling.

You will be given a page for each experience. Choose one experience to work on. There are two questions about each experience. For each problem that you work on:

- Write an inequality to describe the constraints. Specify what each variable represents.
- Use graphing technology to graph the inequality.
- Sketch the graph and label the axes.
- Complete the problems that go with each scenario.

Reflect: What part of this activity played into your strengths? How did you overcome any limitations?

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 16 Graphing Two-Variable Linear Inequalities (Part 2) 119

1 Launch

Arrange students in pairs. Distribute the Activity 1 PDF and provide access to graphing technology to each student. Read through the directions together as a class.

2 Monitor

Help students get started by saying, “Define each constraint by using a variable first.”

Look for points of confusion:

- **Using incorrect symbols for their inequalities.** Help students identify key words or phrases that indicate which symbol to use.

Look for productive strategies:

- Testing points in the solutions region and on the boundary line to check the accuracy of their inequality.
- Changing the window settings in their graphing technology to match the axes on the blank graphs.

3 Connect

Have pairs of students share their variables, inequality, graph, responses, and the meaning of a solution to the inequality in the context of the scenario.

Highlight that sometimes a scenario will be represented by a linear inequality in one variable if there is only one constraint.

Ask, “Which gap year experience would you choose if you were trying to save money for college? Which would you choose if money was not an issue?”

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Provide the variables and their meanings for Problems 1 and 5 from the Activity 1 PDF. Provide the inequalities for Problems 2 and 4 and the graphs for Problems 3 and 6. Have students determine the other missing representations and answer the questions for each problem. Display or provide the Anchor Chart PDF, *Graphing Linear Inequalities*.

Extension: *Math Enrichment*

Have students complete the *Are you ready for more?* problems, from the Activity 1 PDF, *Are you ready for more?*

Math Language Development

MLR7: Compare and Connect

Have pairs create a display of one experience. During the Connect, select and arrange a display of each experience. Provide students with 2–3 minutes of think time to interpret the displays before students present their work. Draw connections and comparisons between Problems 1 and 2, Problems 3 and 4, and Problems 5 and 6.

English Learners

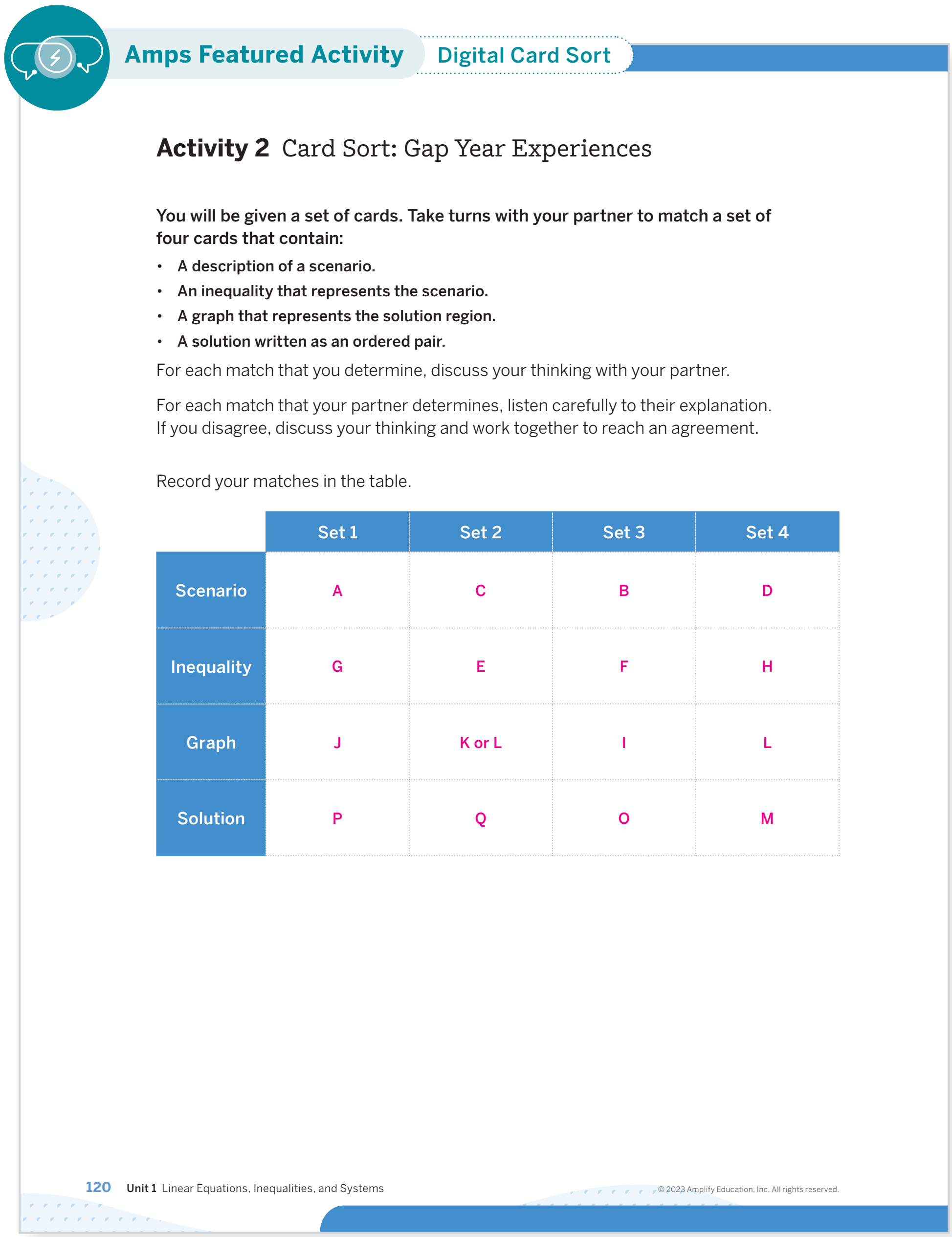
Annotate the graphs on the displays to highlight the initial value, the slope, and the inequality symbol and how they are represented in each graph and inequality.

Pairs | 10 min

Activity 2 Card Sort: Gap Year Experiences

Students analyze representations, statements, and structures to interpret linear inequalities in context.

Amps Featured Activity Digital Card Sort



Activity 2 Card Sort: Gap Year Experiences

You will be given a set of cards. Take turns with your partner to match a set of four cards that contain:

- **A description of a scenario.**
- **An inequality that represents the scenario.**
- **A graph that represents the solution region.**
- **A solution written as an ordered pair.**

For each match that you determine, discuss your thinking with your partner.

For each match that your partner determines, listen carefully to their explanation. If you disagree, discuss your thinking and work together to reach an agreement.

Record your matches in the table.

	Set 1	Set 2	Set 3	Set 4
Scenario	A	C	B	D
Inequality	G	E	F	H
Graph	J	K or L	I	L
Solution	P	Q	O	M

120 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have students remain in pairs. Distribute the pre-cut cards from the Activity 2 PDF to each student. Conduct the ***Card Sort*** routine, having students work independently for 5 minutes before discussing their thinking with their partner.

2 Monitor

Help students get started by having them underline key words and phrases that could help determine the inequality symbol.

Look for points of confusion:

- **Using only 200 to choose an inequality.** Have students determine the variables and inequalities of each scenario first.

Look for productive strategies:

- Defining each variable to help match to an inequality.
- Determining the intercepts and/or slope of the boundary line of an inequality to match it with a graph.

3 Connect

Display the correct groupings.

Have pairs of students share their results and the strategy they used to match the cards.

Highlight that they can use key phrases like “at least,” “no more than,” and “less than” to determine the inequality symbol that represents the scenario.

Ask, “Why are the graphs restricted to the first quadrant?” We are only concerned about positive values for the constraints in these scenarios.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with only the cards that match with Scenarios A and B. Have students match these cards first, and then provide the cards that match with Scenarios C and D.

Accessibility: *Guide Processing and Visualization*

Display or provide the Anchor Chart PDF, *Graphing Linear Inequalities*. Ask students to first match each scenario with its corresponding inequality and then use the inequality to determine the graph and solution.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you display the correct groupings, ask students to compare the quantities and constraints provided in each scenario with its corresponding inequality and graph. Display or provide copies of the Anchor Chart PDF, *Inequality Symbols and Key Phrases* and consider annotating the chart with whether a corresponding graph would have a solid line or a dashed line.

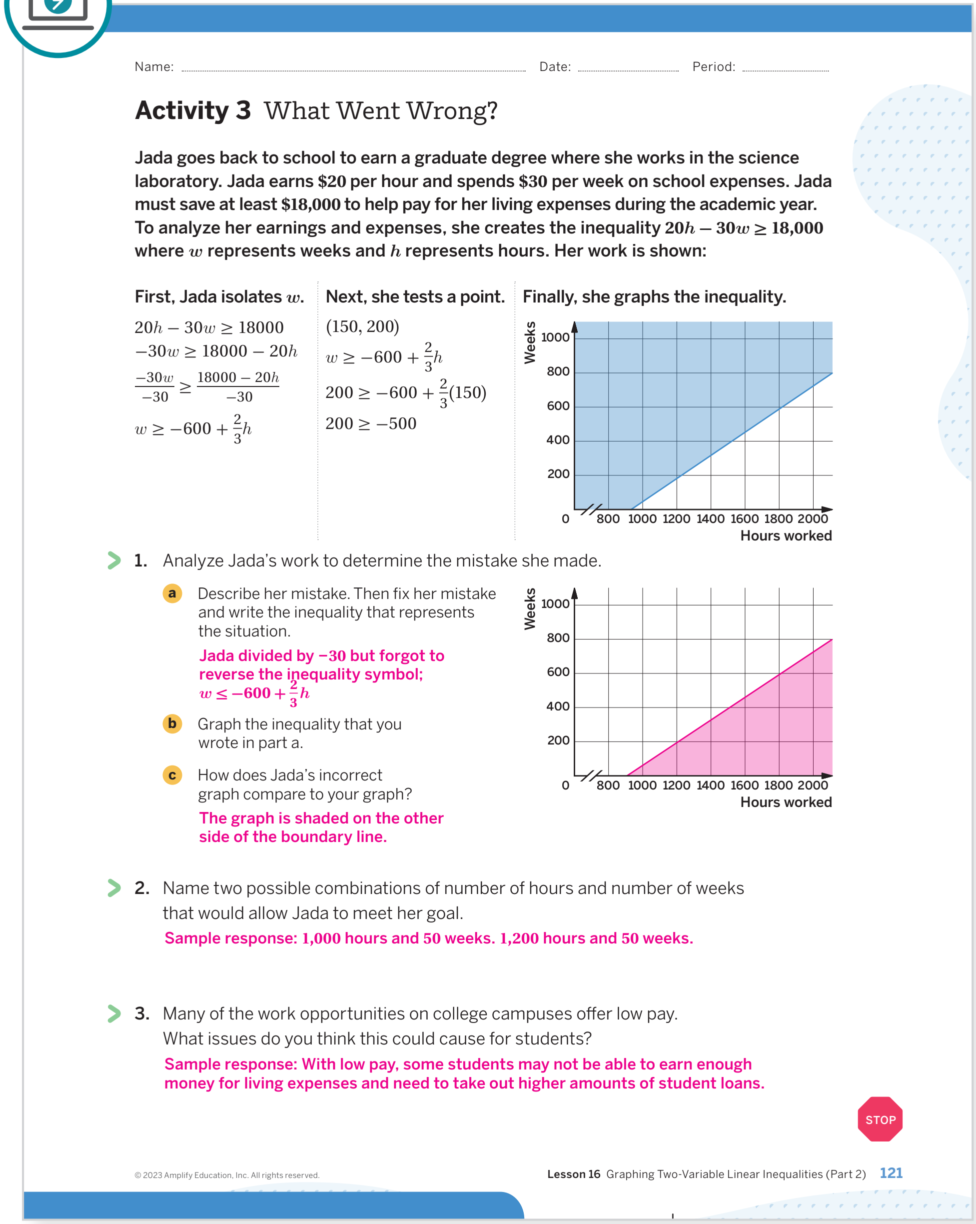
English Learners

Annotate or highlight the key phrases and words in each scenario and how those quantities are represented in the inequality and graph.

Independent | 10 min

Activity 3 What Went Wrong?

Students analyze an error in presented work and graph the solutions to a linear inequality to represent constraints of internships.



Name: ____ Date: ____ Period: ____

Activity 3 What Went Wrong?

Jada goes back to school to earn a graduate degree where she works in the science laboratory. Jada earns $20 per hour and spends $30 per week on school expenses. Jada must save at least $18,000 to help pay for her living expenses during the academic year. To analyze her earnings and expenses, she creates the inequality $20h - 30w \geq 18{,}000$ where w represents weeks and h represents hours. Her work is shown:

First, Jada isolates w.

$$20h - 30w \geq 18000$$
$$-30w \geq 18000 - 20h$$
$$\frac{-30w}{-30} \geq \frac{18000 - 20h}{-30}$$
$$w \geq -600 + \frac{2}{3}h$$

Next, she tests a point.

$$(150, 200)$$
$$w \geq -600 + \frac{2}{3}h$$
$$200 \geq -600 + \frac{2}{3}(150)$$
$$200 \geq -500$$

Finally, she graphs the inequality.

1. Analyze Jada's work to determine the mistake she made.
   - a. Describe her mistake. Then fix her mistake and write the inequality that represents the situation.
     Jada divided by −30 but forgot to reverse the inequality symbol; $w \leq -600 + \frac{2}{3}h$
   - b. Graph the inequality that you wrote in part a.
   - c. How does Jada's incorrect graph compare to your graph?
     The graph is shaded on the other side of the boundary line.

2. Name two possible combinations of number of hours and number of weeks that would allow Jada to meet her goal.
   Sample response: 1,000 hours and 50 weeks. 1,200 hours and 50 weeks.

3. Many of the work opportunities on college campuses offer low pay. What issues do you think this could cause for students?
   Sample response: With low pay, some students may not be able to earn enough money for living expenses and need to take out higher amounts of student loans.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 16 Graphing Two-Variable Linear Inequalities (Part 2) 121

1 Launch

Arrange students in pairs. Use the ***Three Reads*** strategy to review the narrative. Then have students discuss with their partner how the inequality represents the scenario.

2 Monitor

Help students get started by having them write an explanation of the operation that took place next to each line.

Look for points of confusion:

- **Thinking the inequality should be the sum of the two terms.** Have students choose a number of hours and weeks and determine the amount in savings. Connect their work to the inequality.

Look for productive strategies:

- Completing their own work of isolating w and comparing it to Jada's work.

3 Connect

Have individual students share the error they identified and their strategy used for graphing.

Highlight that testing points in the original inequality can help determine if the inequality is graphed correctly.

Ask, "If you forgot to change the inequality symbol, how could you catch your error by using the graph?" I could substitute the values of an ordered pair in the solution regions into the original inequality and determine if these values make a false statement.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display each of Jada's steps one at a time, or have students use an index card to cover up the other steps. This will allow them to focus on analyzing each step, without being distracted by the other steps.

Extension: *Math Enrichment*

Ask students to solve the inequality they wrote in Problem 1a for h. Then ask them how they can use this new inequality to graph the relationship on the same coordinate plane. $h \geq \frac{3}{2}w + 900$; The horizontal intercept is 900. To plot additional points, for every increase of 2 in the number of weeks, the number of hours increases by 3.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

- **Read 1:** Students should understand that Jada needs to save a certain amount from her job to pay for living expenses.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as Jada spends $30 per week on school expenses.
- **Read 3:** Ask students to think about how the given inequality represents this situation.

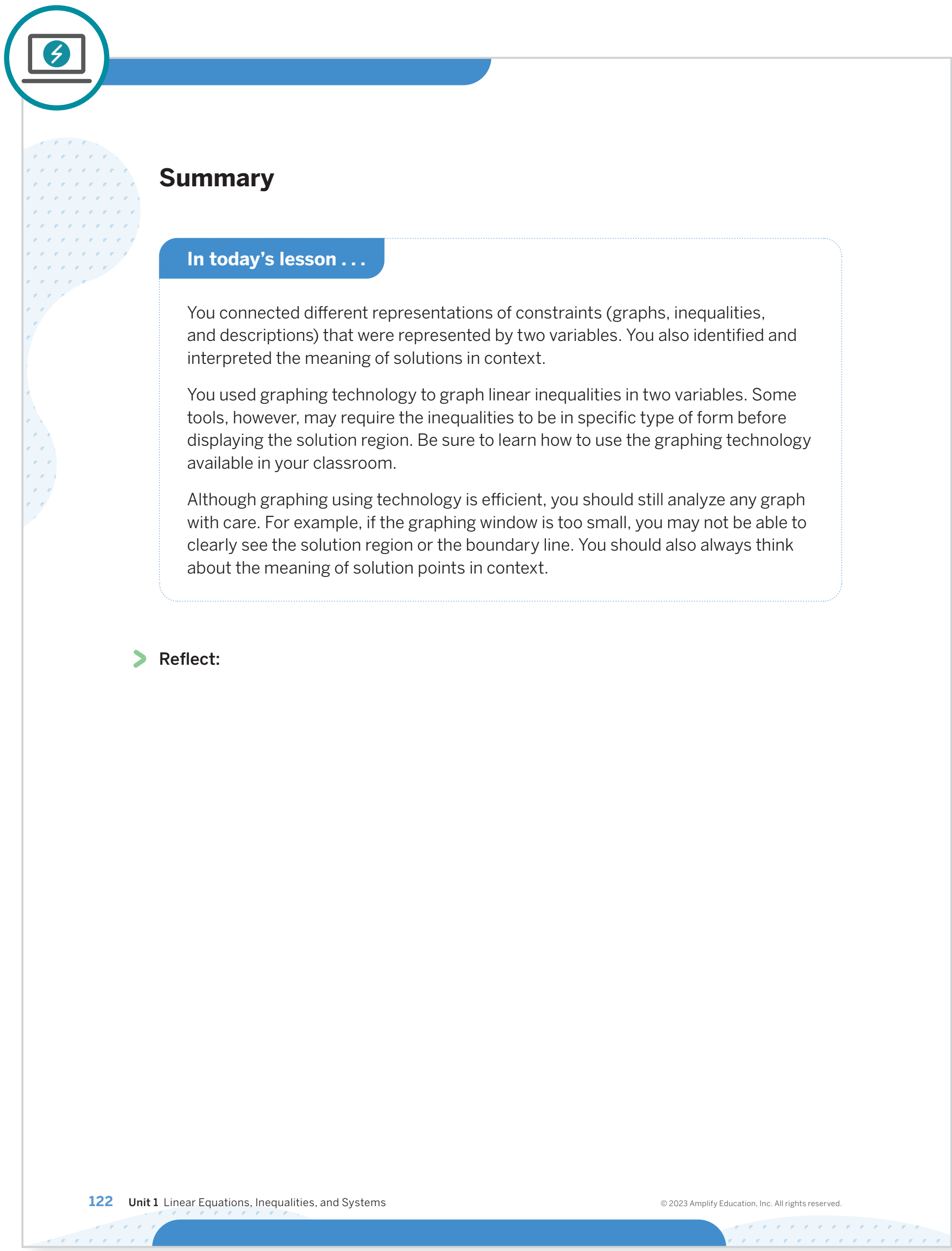
English Learners

Have students highlight key phrases in the text, such as $20 *per hour*, $30 *per week*, and *at least* $18,000.

Whole Class | 5 min

Summary

Review and synthesize connecting and interpreting different representations of a scenario modeled by a linear inequality.



Summary

In today's lesson . . .

You connected different representations of constraints (graphs, inequalities, and descriptions) that were represented by two variables. You also identified and interpreted the meaning of solutions in context.

You used graphing technology to graph linear inequalities in two variables. Some tools, however, may require the inequalities to be in specific type of form before displaying the solution region. Be sure to learn how to use the graphing technology available in your classroom.

Although graphing using technology is efficient, you should still analyze any graph with care. For example, if the graphing window is too small, you may not be able to clearly see the solution region or the boundary line. You should also always think about the meaning of solution points in context.

Reflect:

122 Unit 1 Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display a set of matching representations from Activity 2.

Highlight that it's important to determine the meaning of each variable in context when exploring a graph to help interpret its key features like slope and intercepts.

Ask:

- "Of the four things you were asked to do in the last activity — writing an inequality, graphing the solutions, identifying and interpreting a particular solution, and answering the question about the situation — which one did you find most challenging or prone to error?" Sample response: Writing an inequality. I have to determine each term and the inequality symbol, which can be difficult from the scenario description.
- "How is graphing linear inequalities using technology similar to graphing them by hand? How is it different?" Sample response: I have to resize the window when using graphing technology, just like I have to when determining the axes by hand. With graphing technology, I do not have to determine the slope or the intercepts myself to graph.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

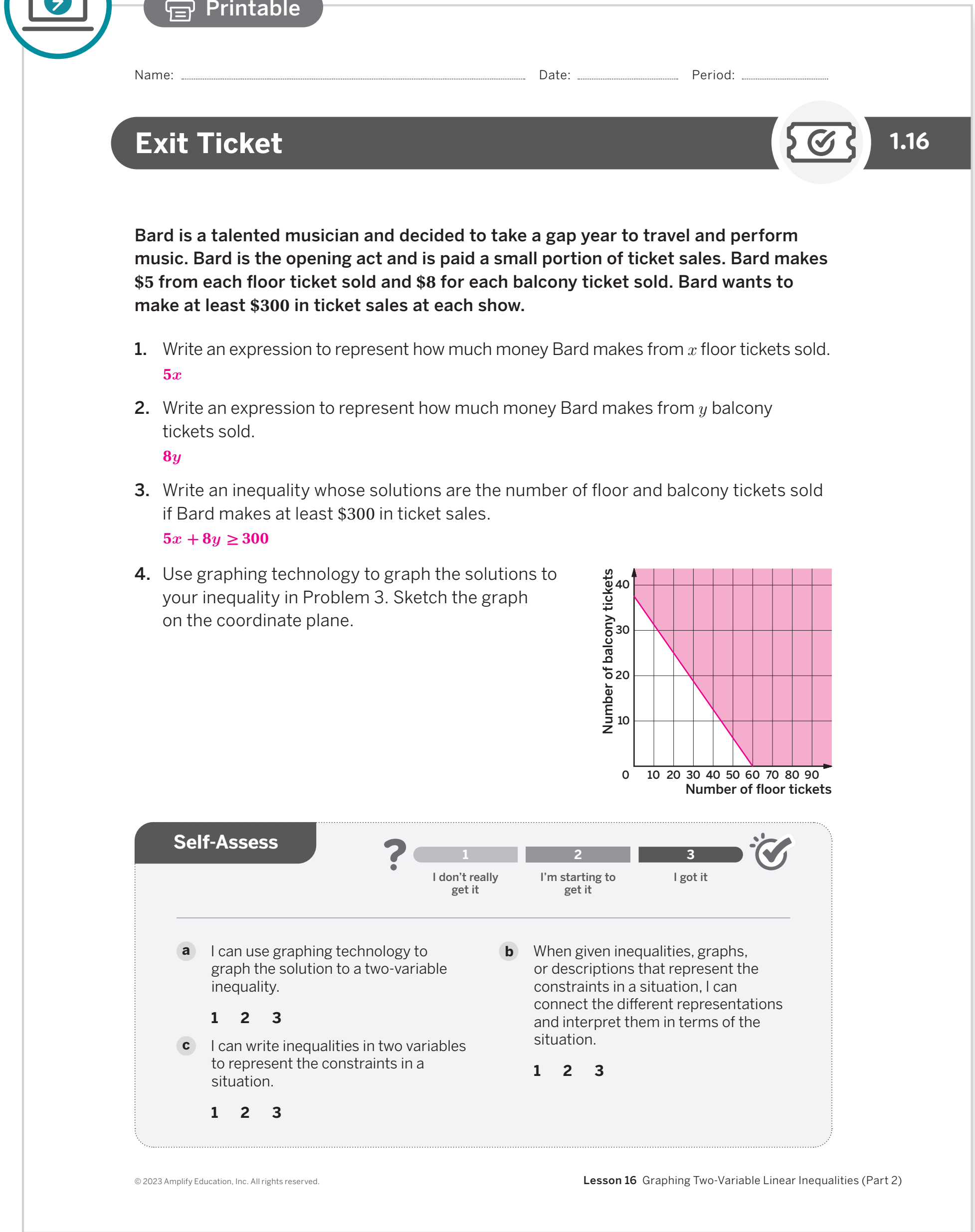
- "What strategies or tools did you find helpful today when connecting the different representations? How were they helpful?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by writing and graphing a linear inequality in two variables that represents a scenario.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

1.16

Bard is a talented musician and decided to take a gap year to travel and perform music. Bard is the opening act and is paid a small portion of ticket sales. Bard makes \$5 from each floor ticket sold and \$8 for each balcony ticket sold. Bard wants to make at least \$300 in ticket sales at each show.

1. Write an expression to represent how much money Bard makes from x floor tickets sold.
   $\mathbf{5x}$
2. Write an expression to represent how much money Bard makes from y balcony tickets sold.
   $\mathbf{8y}$
3. Write an inequality whose solutions are the number of floor and balcony tickets sold if Bard makes at least \$300 in ticket sales.
   $\mathbf{5x + 8y \geq 300}$
4. Use graphing technology to graph the solutions to your inequality in Problem 3. Sketch the graph on the coordinate plane.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can use graphing technology to graph the solution to a two-variable inequality.

1 2 3

c. I can write inequalities in two variables to represent the constraints in a situation.

1 2 3

b. When given inequalities, graphs, or descriptions that represent the constraints in a situation, I can connect the different representations and interpret them in terms of the situation.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 16 Graphing Two-Variable Linear Inequalities (Part 2)

Success looks like . . .

- **Language Goal:** Identifying an inequality, a graph, an ordered pair, and a description that represent the constraints and possible solutions in a situation. **(Speaking and Listening, Writing)**
  - » Writing an inequality that represents ticket sales in Problem 3.
- **Language Goal:** Understanding that a constraint on two variables can be represented by an inequality, a graph (a half-plane), and a verbal description. **(Speaking and Listening, Writing)**
- **Goal:** Writing inequalities in two variables to represent the constraints in a situation and using technology to graph the solution set to answer questions about the situation.
  - » Using technology to graph the solutions to the inequality in Problem 4.

Suggested next steps

If students write incorrect expressions for Problems 1 and 2, consider:

- Reviewing Activity 1.
- Assigning Practice Problems 1–3.

If students write an incorrect inequality for Problem 3, consider:

- Reviewing Activity 1.
- Asking, "What key words or phrases are in the description of the scenario that can tell you what inequality symbol to use?"

If students incorrectly graph the inequality in Problem 4, consider:

- Reviewing the Warm-up.
- Assigning Practice Problems 2 and 3.

Professional Learning

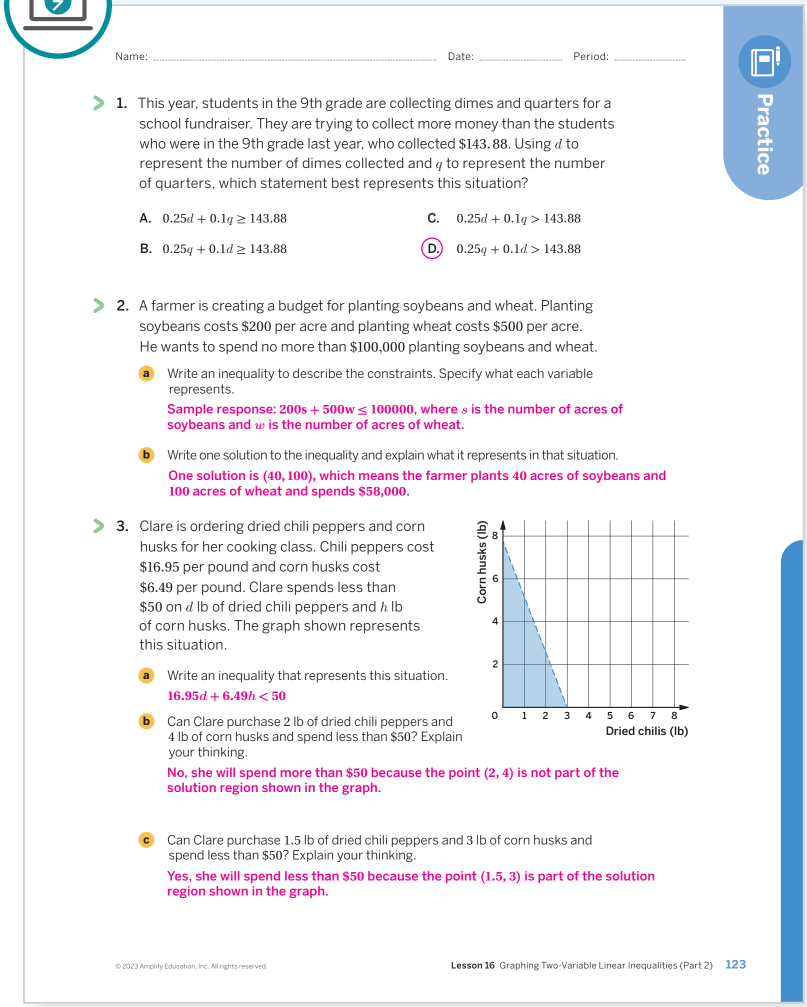
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion about Activity 1 how did you encourage each student to share their understandings?
- Have you changed any ideas you used to have about graphing linear inequalities as a result of today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. This year, students in the 9th grade are collecting dimes and quarters for a school fundraiser. They are trying to collect more money than the students who were in the 9th grade last year, who collected $\$143.88$. Using d to represent the number of dimes collected and q to represent the number of quarters, which statement best represents this situation?

   A. $0.25d + 0.1q \geq 143.88$
   C. $0.25d + 0.1q > 143.88$
   B. $0.25q + 0.1d \geq 143.88$
   [✓ D.] $0.25q + 0.1d > 143.88$

2. A farmer is creating a budget for planting soybeans and wheat. Planting soybeans costs $\$200$ per acre and planting wheat costs $\$500$ per acre. He wants to spend no more than $\$100{,}000$ planting soybeans and wheat.

   a. Write an inequality to describe the constraints. Specify what each variable represents.

   Sample response: $200s + 500w \leq 100000$, where s is the number of acres of soybeans and w is the number of acres of wheat.

   b. Write one solution to the inequality and explain what it represents in that situation.

   One solution is $(40, 100)$, which means the farmer plants 40 acres of soybeans and 100 acres of wheat and spends $\$58{,}000$.

3. Clare is ordering dried chili peppers and corn husks for her cooking class. Chili peppers cost $\$16.95$ per pound and corn husks cost $\$6.49$ per pound. Clare spends less than $\$50$ on d lb of dried chili peppers and h lb of corn husks. The graph shown represents this situation.

   a. Write an inequality that represents this situation.

   $16.95d + 6.49h < 50$

   b. Can Clare purchase 2 lb of dried chili peppers and 4 lb of corn husks and spend less than $\$50$? Explain your thinking.

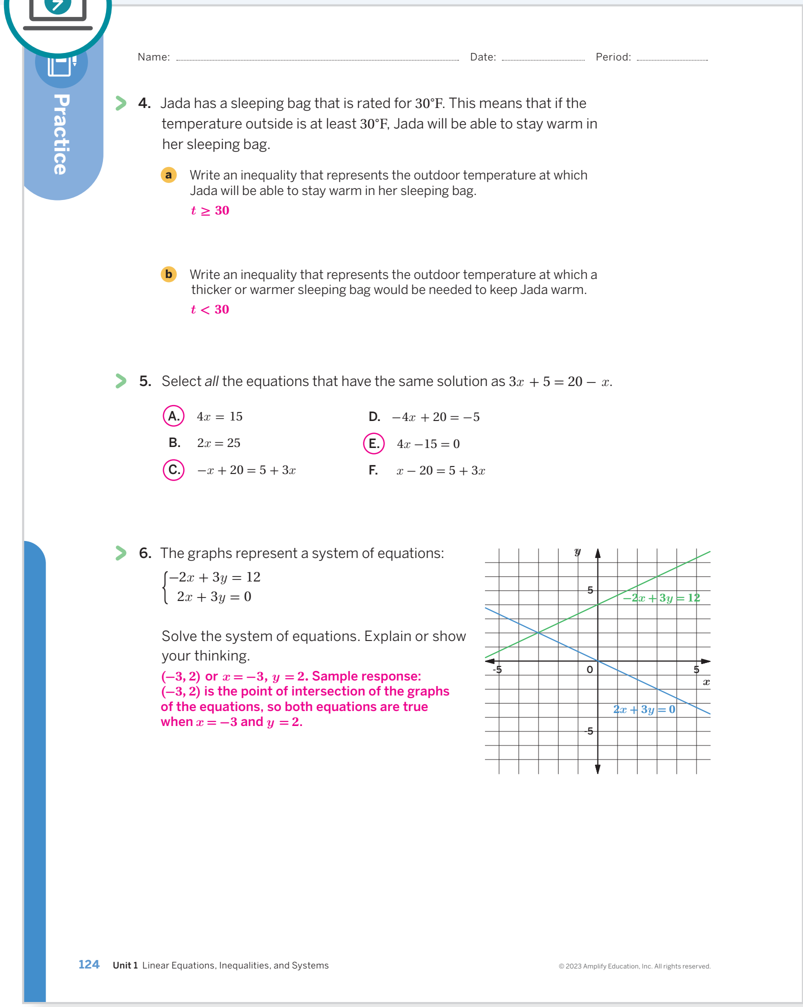
   No, she will spend more than $\$50$ because the point $(2, 4)$ is not part of the solution region shown in the graph.

   c. Can Clare purchase 1.5 lb of dried chili peppers and 3 lb of corn husks and spend less than $\$50$? Explain your thinking.

   Yes, she will spend less than $\$50$ because the point $(1.5, 3)$ is part of the solution region shown in the graph.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 16 Graphing Two-Variable Linear Inequalities (Part 2) 123



Name: ______ Date: ______ Period: ______

4. Jada has a sleeping bag that is rated for $30°F$. This means that if the temperature outside is at least $30°F$, Jada will be able to stay warm in her sleeping bag.

   a. Write an inequality that represents the outdoor temperature at which Jada will be able to stay warm in her sleeping bag.

   $t \geq 30$

   b. Write an inequality that represents the outdoor temperature at which a thicker or warmer sleeping bag would be needed to keep Jada warm.

   $t < 30$

5. Select *all* the equations that have the same solution as $3x + 5 = 20 - x$.

   [✓ A.] $4x = 15$
   D. $-4x + 20 = -5$
   B. $2x = 25$
   [✓ E.] $4x - 15 = 0$
   [✓ C.] $-x + 20 = 5 + 3x$
   F. $x - 20 = 5 + 3x$

6. The graphs represent a system of equations:

$$\begin{cases} -2x + 3y = 12 \\ 2x + 3y = 0 \end{cases}$$

   Solve the system of equations. Explain or show your thinking.

   $(-3, 2)$ or $x = -3$, $y = 2$. Sample response: $(-3, 2)$ is the point of intersection of the graphs of the equations, so both equations are true when $x = -3$ and $y = 2$.

124 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 5	2
	5	Unit 1 Lesson 7	2
Formative ⚡	6	Unit 1 Lesson 17	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Mathematical Modeling Prompt

At any point during the course and after completing this lesson, have students work in pairs or small groups on this unit's Mathematical Modeling Prompt, *A New Heating System*, which is available in the **Algebra 1 Additional Practice**.

Sub-Unit 4

Systems of Linear Equations in Two Variables

Whole Class

In this Sub-Unit, students explore how life is full of constraints. They discover new strategies for approaching and solving real-world problems involving multiple decision points and constraints.

SUB-UNIT

4 Systems of Linear Equations in Two Variables

Narrative Connections

Are you a ‘Boomerang-er’?

When we think about being an adult, we think of being independent and living on one’s own. But recently, more and more young adults move back home after college.

According to the U.S. Census, over the last 20 years there has been an increase of roughly 2 million people between the ages of 25 and 34 who move in with their parents. Dubbed the “Boomerang Generation,” these individuals often found it difficult to afford living on their own, given a scarcity of high-paying jobs and rising levels of student debt.

This trend might not seem so strange for many cultures outside the U.S., where multiple generations can be found living under the same roof. Moving back home is not only more affordable — it allows you to be in a system of mutual support and care with the rest of your family. And for city dwellers returning to the suburbs, access to a washer and dryer is a notable perk!

But there is much to consider when choosing the right living arrangement. Does the price of gas commuting from your family’s home cost more than public transit over time? Does splitting the electric bill with several people cost less than paying only for yourself?

Earlier in this unit, you learned to solve problems with one variable. But the complexities of life can often involve multiple variables. This next set of lessons will show how to solve equations with more than one variable, so that you can make a confident decision, no matter where you hang your hat!

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students will examine systems of linear equations in different settings (and living arrangements!) in the following places:

- **Lesson 17, Warm-up:** Grocery Shopping With Roomies
- **Lesson 17, Activity 1:** Working Two Jobs
- **Lesson 22, Activity 1:** Gym Membership and Personal Training

Optional

UNIT 1 | **LESSON 17**

Writing and Graphing Systems of Linear Equations

Let's recall what it means to solve a system of linear equations and represent the solution graphically.

Focus

Goals

1. **Language Goal:** Solve systems of linear equations by reasoning with tables and by graphing, and explain the solution method. **(Speaking and Listening, Writing)**
2. **Language Goal:** Understand that the solution to a system of equations in two variables is a pair of values that simultaneously make both equations true, and that it is represented by the intersection point of the graphs of the equations. **(Speaking and Listening, Writing)**
3. **Language Goal:** Understand that two (or more) equations that represent the constraints on the same quantities in the same situation form a system. **(Speaking and Listening, Writing)**

Coherence

Today

Students build on their Grade 8 understanding of solutions of systems of equations using tables and graphs to determine a solution. They recall that a solution of a system of equations is the point of intersection on its graph. Students write systems of equations to model different constraints in a situation and choose appropriate tools for solving. For this lesson, students are not required to solve algebraically.

Previously

Students graphed linear inequalities in two variables and considered when it was necessary to change the inequality symbol when solving them.

Coming Soon

Students will consider algebraic strategies for solving a system of equations. They revisit how to solve by substitution in the next lesson.

Rigor

- Students further their **conceptual understanding** of systems of linear equations by considering their solutions in a context.
- Students build **fluency** writing systems of equations and solving them by graphing.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	20 min	10 min	5 min	5 min
Small Groups	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Describing My Thinking*
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps*
- Graphic Organizer PDF, *Writing a System of Equations From a Context*
- counters
- graphing technology

Math Language Development

Review words

- *constraint*
- *slope-intercept form*
- *solution to a system*
- *system of equations*

Amps | **Featured Activity**

Activity 1

Interactive Graph

Students graph two linear equations on the same set of axes and interpret their intersection in a context to recall what is meant by a solution to a system of equations.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Some students may be more proficient at using the different functions on their graphing tool to determine solutions to systems and use this technology more strategically than their peers in Activity 3. Encourage these students to take on the role of "expert" and share their expertise so that their peers can lean on them as a resource as they familiarize themselves with how to use graphing technology more strategically.

Modifications to Pacing

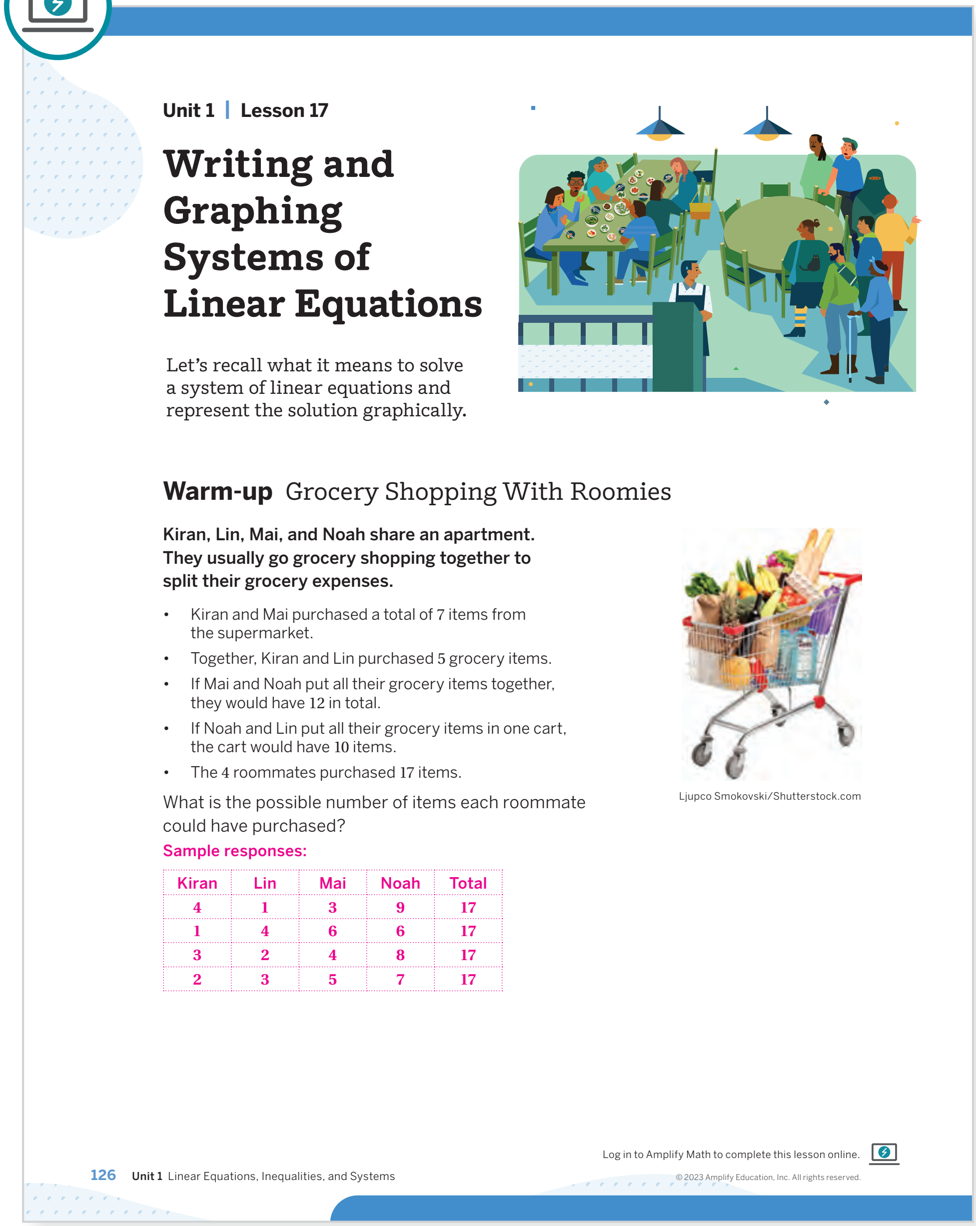
You may want to consider this additional modification if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, assign student pairs Problem 2 or Problem 3.

Small Groups | 10 min

Warm-up Grocery Shopping With Roomies

Students determine possible values satisfying multiple constraints in a context to recall solving simultaneous linear equations.



Unit 1 | **Lesson 17**

Writing and Graphing Systems of Linear Equations

Let's recall what it means to solve a system of linear equations and represent the solution graphically.

Warm-up Grocery Shopping With Roomies

Kiran, Lin, Mai, and Noah share an apartment. They usually go grocery shopping together to split their grocery expenses.

- Kiran and Mai purchased a total of 7 items from the supermarket.
- Together, Kiran and Lin purchased 5 grocery items.
- If Mai and Noah put all their grocery items together, they would have 12 in total.
- If Noah and Lin put all their grocery items in one cart, the cart would have 10 items.
- The 4 roommates purchased 17 items.

What is the possible number of items each roommate could have purchased?

Ljupco Smokovski/Shutterstock.com

Sample responses:

Kiran	Lin	Mai	Noah	Total
4	1	3	9	17
1	4	6	6	17
3	2	4	8	17
2	3	5	7	17

Log in to Amplify Math to complete this lesson online.

126 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students into small groups. Read the narrative and discuss the pros and cons of sharing grocery expenses. Make counters available to each group.

2 Monitor

Help students get started by prompting them to focus on one clue and determine possible values which make it true.

Look for points of confusion:

- **Having difficulty determining the number of items needed to simultaneously satisfy the constraints.** Ask, "How many total items did the roommates purchase? What are some values that satisfy this constraint?"
- **Struggling to compare the constraints for each clue.** Prompt students to write an equation representing each constraint.

Look for productive strategies:

- Substituting numeric values for one quantity to determine the values of other quantities.
- Rearranging equations to isolate specific variables.
- Comparing equations that have a variable in common.
- Solving simultaneous equations.

3 Connect

Have groups of students share their strategies for determining the solutions. Consider using a ***Gallery Tour*** to select and sequence groups reasoning concretely, quantitatively, and abstractly. Record and display each group's responses.

Ask, "Do your responses satisfy all the constraints simultaneously?"

Highlight that each set of values is a solution because they satisfy the constraints for all clues simultaneously. These constraints can be represented with a system of equations.

Math Language Development

MLR8: Discussion Supports

After students read the clues from the Warm-up, have them take turns describing what steps they think should be taken to determine the possible number of items each roommate could have purchased. Have them explain their thinking for each step. Provide access to the Anchor Chart PDF, *Sentence Stems, Describing My Thinking*.

English Learners

Have students highlight key phrases, such as *a total of 7 items* and *put their grocery items together*.

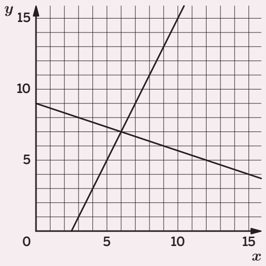
Power-up

To power up students' ability to determine the solution to a system of equations from a graph, have students complete:

Recall that the solution to a system of equations is the point of intersection of the graphs of the equations.

The graph represents the system:

$$\begin{cases} y = 2x - 5 \\ y = -\frac{1}{3}x + 9 \end{cases}$$



Determine the solution to the system.

(6, 7)

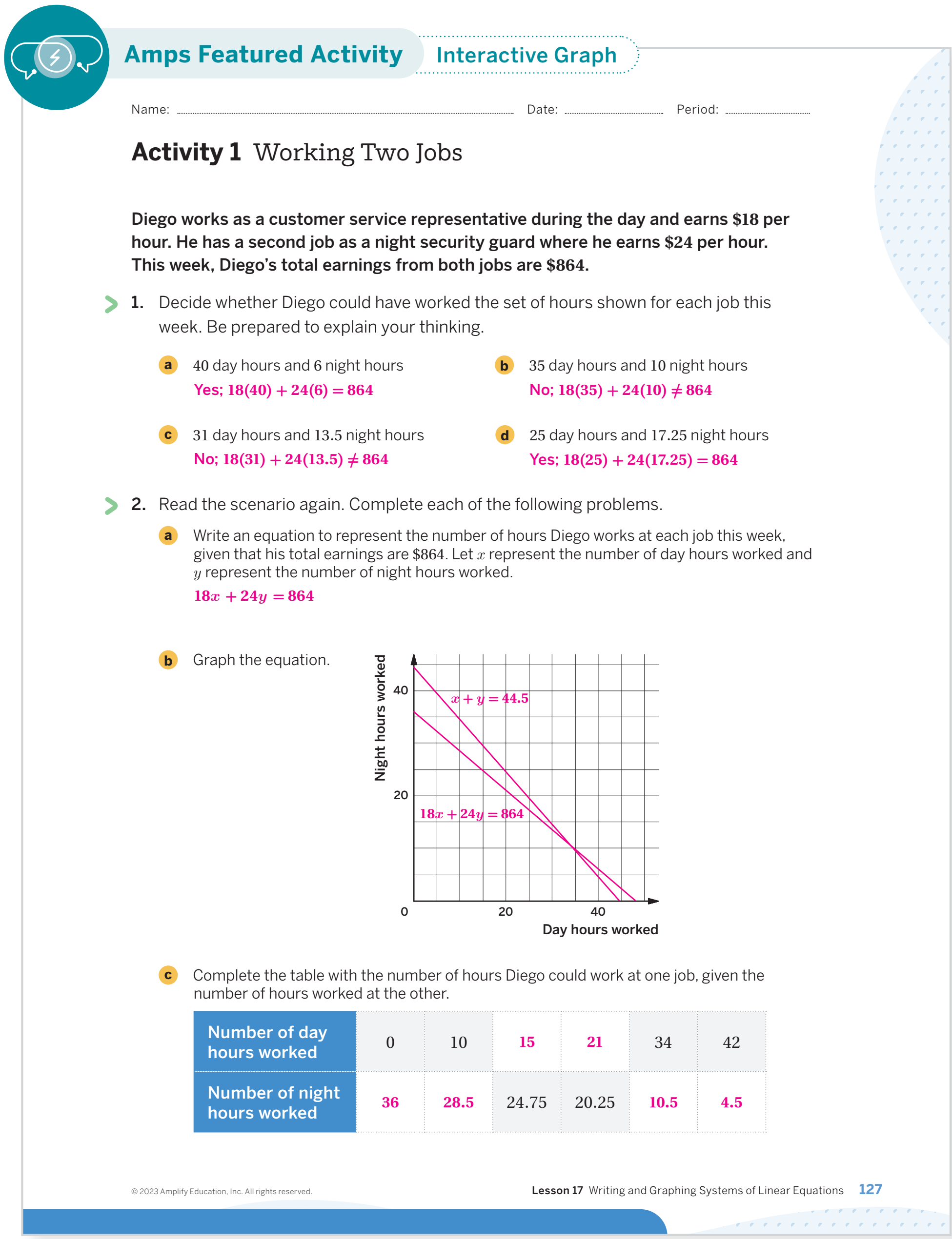
Use: Before the Warm-up

Informed by: Performance on Lesson 16, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4

Pairs | 20 min

Activity 1 Working Two Jobs

Students reason abstractly and quantitatively to determine possible values satisfying multiple constraints in a context and recall solving simultaneous linear equations.



Amps Featured Activity Interactive Graph

Name: ____________________ Date: __________ Period: __________

Activity 1 Working Two Jobs

Diego works as a customer service representative during the day and earns $18 per hour. He has a second job as a night security guard where he earns $24 per hour. This week, Diego's total earnings from both jobs are $864.

1. Decide whether Diego could have worked the set of hours shown for each job this week. Be prepared to explain your thinking.

   a. 40 day hours and 6 night hours
   Yes; $18(40) + 24(6) = 864$

   b. 35 day hours and 10 night hours
   No; $18(35) + 24(10) \neq 864$

   c. 31 day hours and 13.5 night hours
   No; $18(31) + 24(13.5) \neq 864$

   d. 25 day hours and 17.25 night hours
   Yes; $18(25) + 24(17.25) = 864$

2. Read the scenario again. Complete each of the following problems.

   a. Write an equation to represent the number of hours Diego works at each job this week, given that his total earnings are $864. Let x represent the number of day hours worked and y represent the number of night hours worked.
   $18x + 24y = 864$

   b. Graph the equation.

   c. Complete the table with the number of hours Diego could work at one job, given the number of hours worked at the other.

Number of day hours worked	0	10	15	21	34	42
Number of night hours worked	36	28.5	24.75	20.25	10.5	4.5

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 17 Writing and Graphing Systems of Linear Equations 127

1 Launch

Read the narrative aloud. Have students complete Problem 1 independently and then discuss their responses with the whole class. Arrange students in pairs to resume the activity. Provide access to graphing technology.

2 Monitor

Help students get started by modeling a combination in Problem 1 with a numeric equation. Ask, "What is the constraint in this scenario?"

Look for points of confusion:

- **Having difficulty completing the table.** Ask, "How could you use your equation or graph?"
- **Determining a pair of values that do not meet both constraints in Problem 4.** Ask, "What information does your graph of your equation in Problem 3a provide?"

Look for productive strategies:

- Guessing and checking.
- Using graphing technology to determine the missing values in the table.
- Substituting the given value of one variable into the equation and solving for the other variable.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1a and 1b, 2, and 3. Consider providing the equation and graph in Problem 2 and have students write the equation and create the graph in Problem 3.

Math Language Development

MLR2: Collect and Display

During the Connect, look for and amplify productive strategies that focus on multiple representations, such as guess and check, making a table, writing equations, and graphing. Add these productive strategies to the class display.

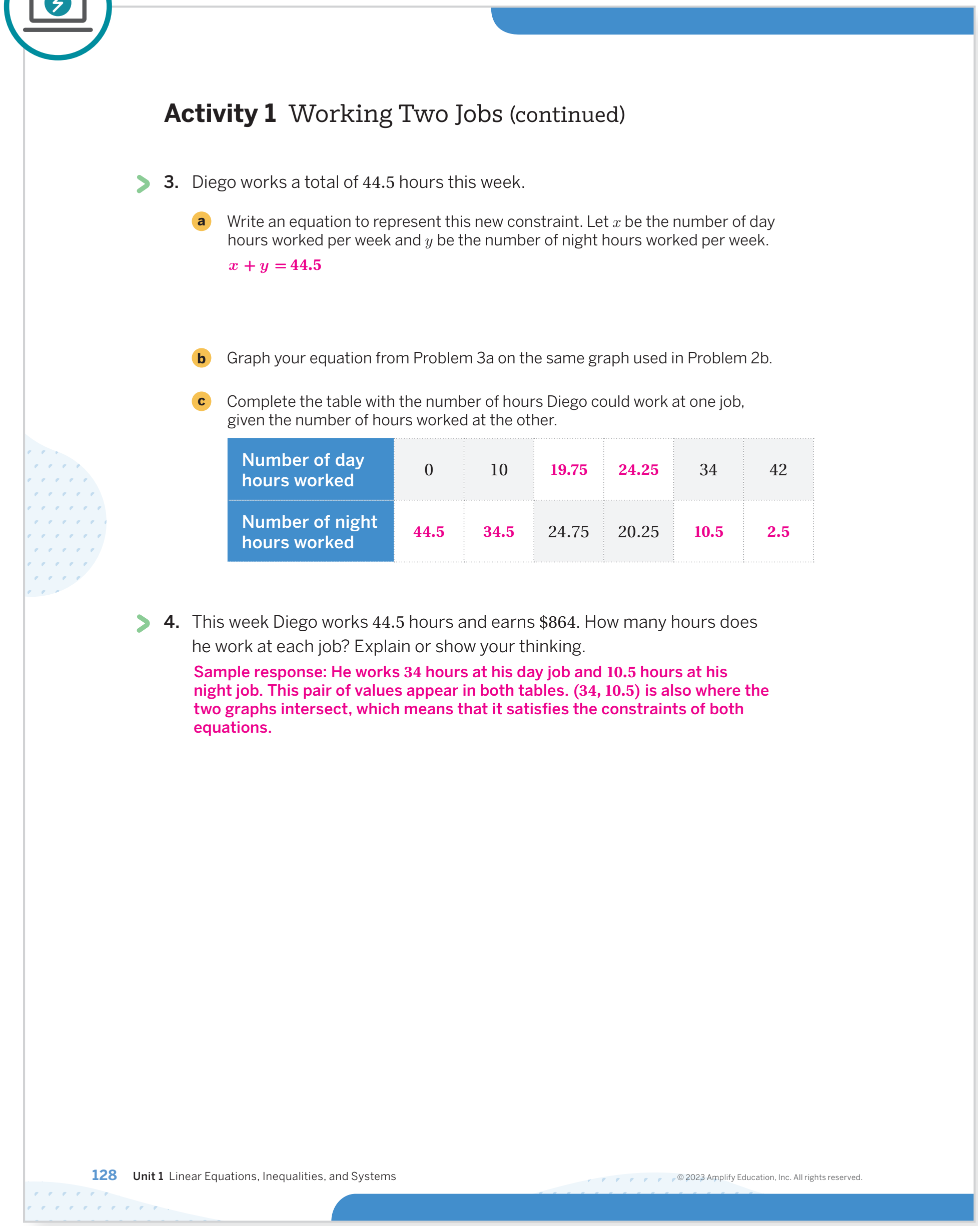
English Learners

Make connections on the display by drawing arrows between the representations and annotating how the different representations connect to one another.

Pairs | 20 min

Activity 1 Working Two Jobs (continued)

Students reason abstractly and quantitatively to determine possible values satisfying multiple constraints in a context and recall solving simultaneous linear equations.



Activity 1 Working Two Jobs (continued)

3. Diego works a total of 44.5 hours this week.

a. Write an equation to represent this new constraint. Let x be the number of day hours worked per week and y be the number of night hours worked per week.

$x + y = 44.5$

b. Graph your equation from Problem 3a on the same graph used in Problem 2b.

c. Complete the table with the number of hours Diego could work at one job, given the number of hours worked at the other.

Number of day hours worked	0	10	19.75	24.25	34	42
Number of night hours worked	44.5	34.5	24.75	20.25	10.5	2.5

4. This week Diego works 44.5 hours and earns $864. How many hours does he work at each job? Explain or show your thinking.

Sample response: He works 34 hours at his day job and 10.5 hours at his night job. This pair of values appear in both tables. (34, 10.5) is also where the two graphs intersect, which means that it satisfies the constraints of both equations.

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have pairs of students share their strategies for completing each table and answering Problem 4. Select and sequence pairs using the productive strategies in the order listed.

Ask:

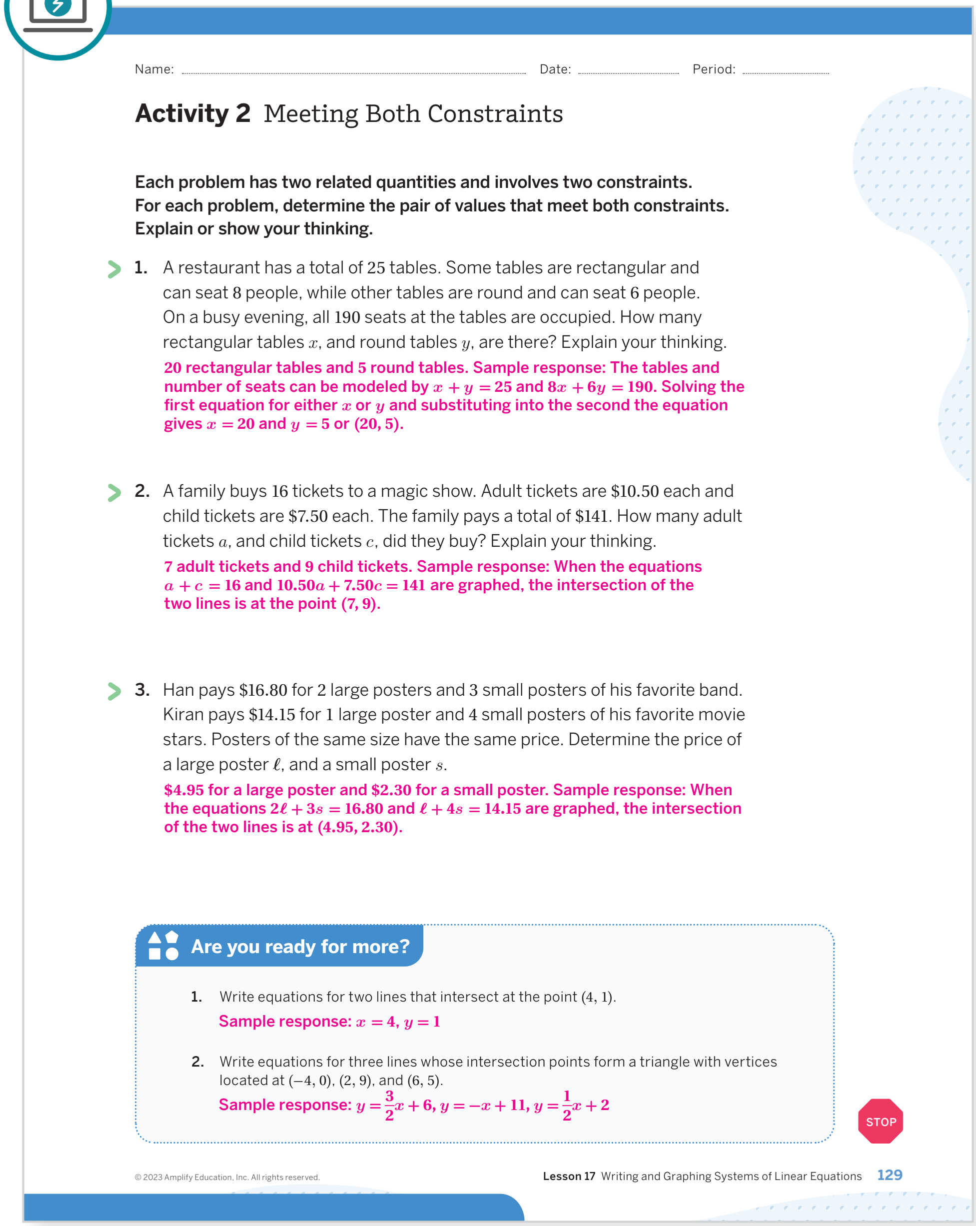
- "What constraint does the first equation represent in this context? The second?"
- "How many possible combinations of day and night hours meet both constraints? How do you know?"

Highlight that the two equations form a system of equations. Model writing the system using curly brackets. The solution to the system is the pair of values that meet the constraints of both equations. Graphing is an effective way to see a solution of a system, if one exists.

Pairs | 10 min

Activity 2 Meeting Both Constraints

Students reason quantitatively and abstractly about two related quantities to write and solve systems of linear equations in a context.



Name: ______ Date: ______ Period: ______

Activity 2 Meeting Both Constraints

Each problem has two related quantities and involves two constraints. For each problem, determine the pair of values that meet both constraints. Explain or show your thinking.

1. A restaurant has a total of 25 tables. Some tables are rectangular and can seat 8 people, while other tables are round and can seat 6 people. On a busy evening, all 190 seats at the tables are occupied. How many rectangular tables x, and round tables y, are there? Explain your thinking.

   20 rectangular tables and 5 round tables. Sample response: The tables and number of seats can be modeled by $x + y = 25$ and $8x + 6y = 190$. Solving the first equation for either x or y and substituting into the second the equation gives $x = 20$ and $y = 5$ or $(20, 5)$.

2. A family buys 16 tickets to a magic show. Adult tickets are \$10.50 each and child tickets are \$7.50 each. The family pays a total of \$141. How many adult tickets a, and child tickets c, did they buy? Explain your thinking.

   7 adult tickets and 9 child tickets. Sample response: When the equations $a + c = 16$ and $10.50a + 7.50c = 141$ are graphed, the intersection of the two lines is at the point $(7, 9)$.

3. Han pays \$16.80 for 2 large posters and 3 small posters of his favorite band. Kiran pays \$14.15 for 1 large poster and 4 small posters of his favorite movie stars. Posters of the same size have the same price. Determine the price of a large poster ℓ, and a small poster s.

   \$4.95 for a large poster and \$2.30 for a small poster. Sample response: When the equations $2\ell + 3s = 16.80$ and $\ell + 4s = 14.15$ are graphed, the intersection of the two lines is at $(4.95, 2.30)$.

Are you ready for more?

1. Write equations for two lines that intersect at the point $(4, 1)$.

   Sample response: $x = 4$, $y = 1$

2. Write equations for three lines whose intersection points form a triangle with vertices located at $(-4, 0)$, $(2, 9)$, and $(6, 5)$.

   Sample response: $y = \frac{3}{2}x + 6$, $y = -x + 11$, $y = \frac{1}{2}x + 2$

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 17 Writing and Graphing Systems of Linear Equations 129

1 Launch

Give students think-time to consider the first problem. Have them discuss possible strategies with their partner before completing the rest of the activity. Provide continued access to graphing technology.

2 Monitor

Help students get started by having them identify the two constraints for each problem. Prompt them to write equations for each constraint.

Look for points of confusion:

- **Struggling to make sense of the information in each problem.** Prompt students to draw a picture or diagram to represent the situation or to make a list or table to reason quantitatively.

Look for productive strategies:

- Using the *guess-and-check* strategy.
- Drawing a picture or diagram.
- Creating tables to list possible combinations of values.
- Graphing the systems.
- Solving by substitution.

3 Connect

Have pairs of students share their responses and thinking. Select and sequence less systematic productive strategies moving towards those more systematic.

Display students' work for each new strategy presented.

Ask:

- "Why is it helpful to write a pair of equations for each problem?"
- "Are there situations when writing equations may not be useful?"

Highlight graphing as an efficient strategy for solving systems. To solve by graphing, it is necessary to first write a pair of equations that model the situation.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Allow students to choose two of the three problems to complete.

Accessibility: *Guide Processing and Visualization*

Display or provide access to the Graphic Organizer PDF, *Writing a System of Equations From a Context* to help students make sense of each problem and write a system that models the problem. Provide access to colored pencils or highlighters and have them annotate the quantities and constraints in each context that help them write an appropriate system of equations.

Math Language Development

MLR7: Compare and Connect

Have students choose one problem and create a visual display of their work. Begin the Connect by selecting and arranging 2–4 displays. Invite other students to analyze and interpret each display. Provide students with time to interpret the displays before inviting the students who created each display to share their strategies.

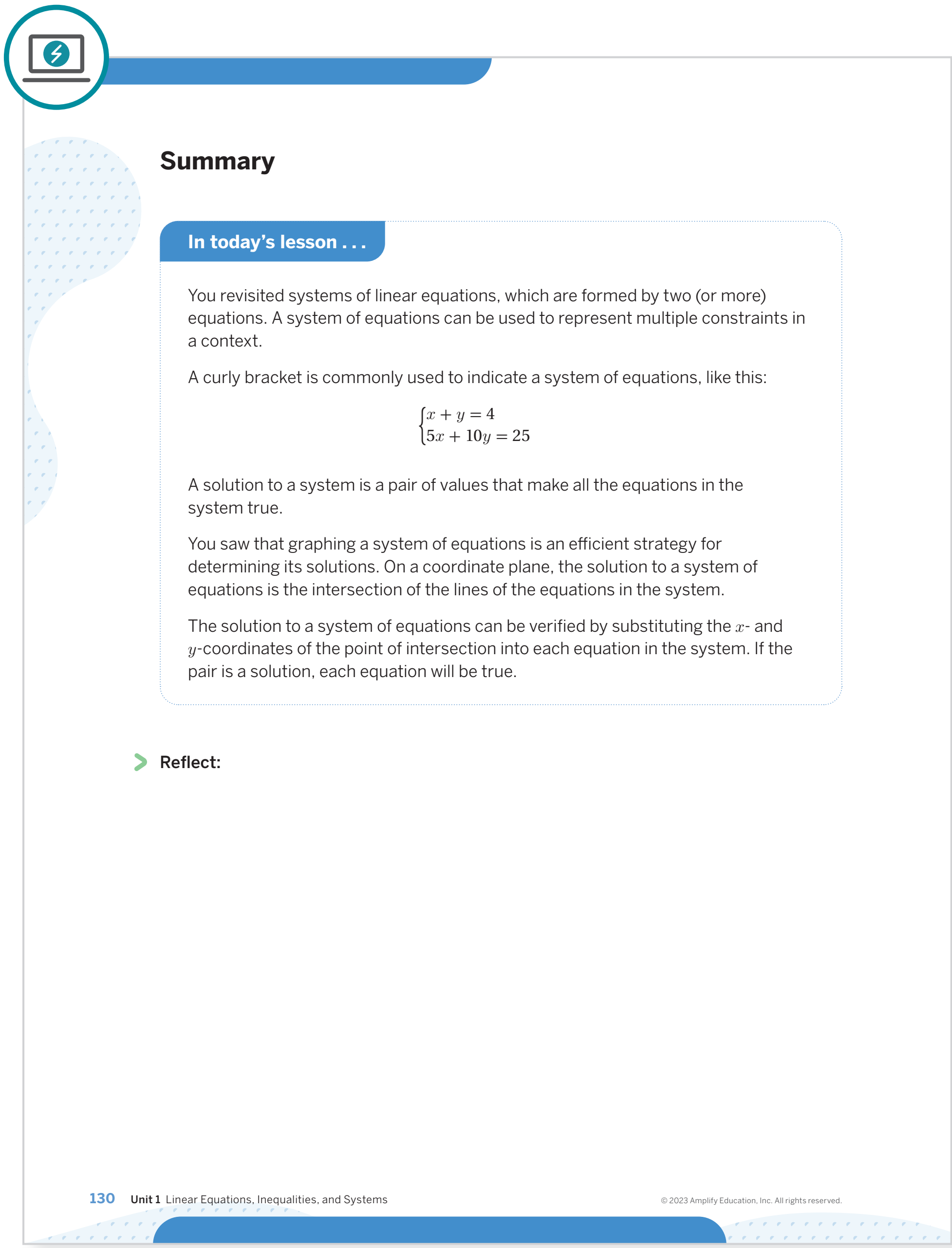
English Learners

Annotate the displays with key words and phrases that help to write an appropriate system of equations. For example, highlight *a total of* 25 *tables*, *rectangular* (8 *people*), *round* (6 *people*), and 190 *seats* in Problem 1.

Whole Class | 5 min

Summary

Review and synthesize systems of linear equations and their solutions and efficient strategies for solving them.



Summary

In today's lesson . . .

You revisited systems of linear equations, which are formed by two (or more) equations. A system of equations can be used to represent multiple constraints in a context.

A curly bracket is commonly used to indicate a system of equations, like this:

$$\begin{cases} x + y = 4 \\ 5x + 10y = 25 \end{cases}$$

A solution to a system is a pair of values that make all the equations in the system true.

You saw that graphing a system of equations is an efficient strategy for determining its solutions. On a coordinate plane, the solution to a system of equations is the intersection of the lines of the equations in the system.

The solution to a system of equations can be verified by substituting the x- and y-coordinates of the point of intersection into each equation in the system. If the pair is a solution, each equation will be true.

Reflect:

130 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display a graph of the system: $\begin{cases} x + y = 4 \\ 5x + 10y = 25 \end{cases}$

Ask, "What are the coordinates of a point that is:"

- "A solution to the first equation?"
- "A solution to the second equation?"
- "A solution to the system?"

Have students share what is meant by a solution to a system and how the solution is represented graphically.

Highlight that solving a system means to find a pair of values that simultaneously make both equations in the system true or meet both constraints in a situation. A two-variable equation has many solutions (represented by the graph of its line), but the solution to a system of linear equations is the point of intersection of the two lines.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

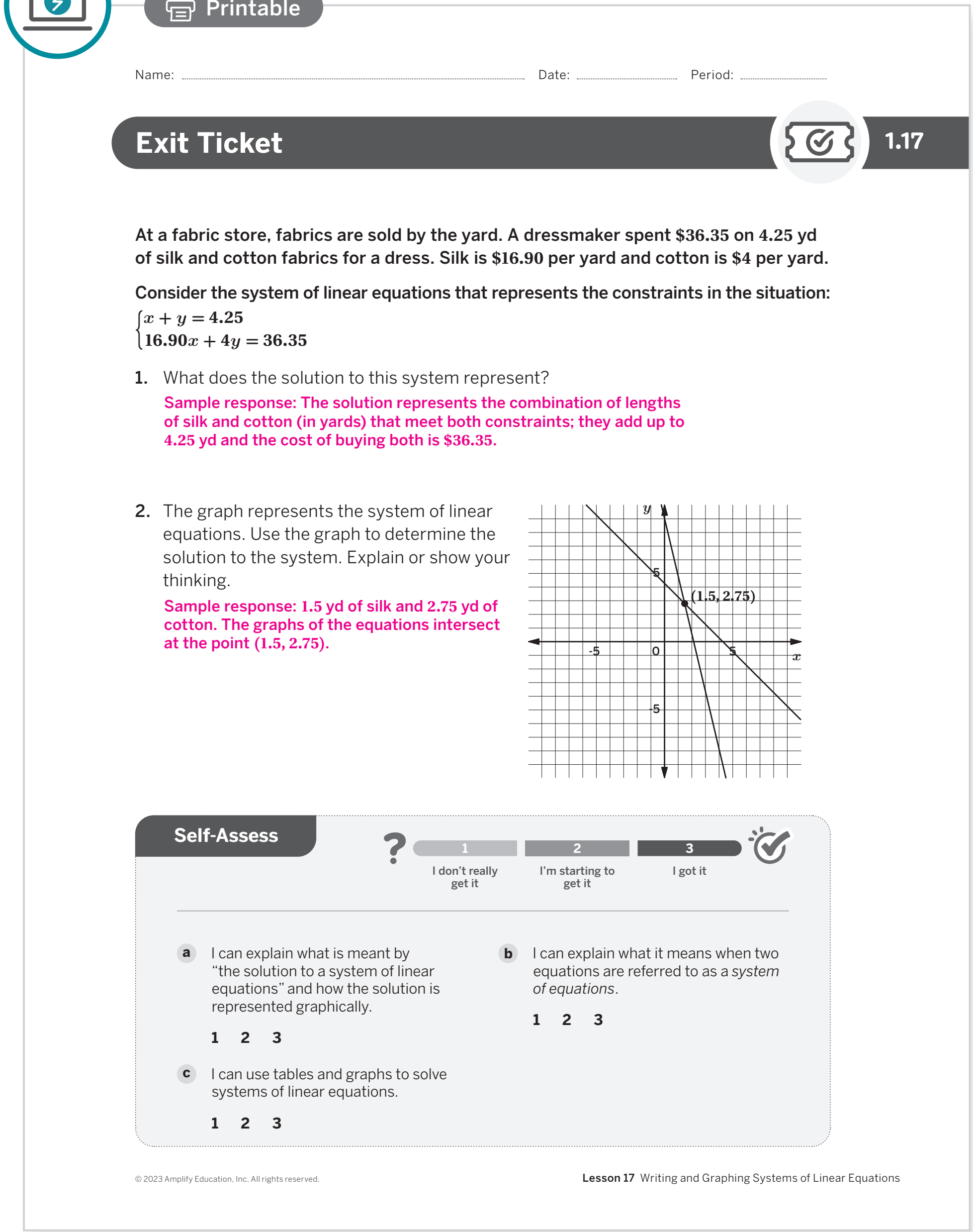
- "How would you explain a 'system of linear equations' to a classmate who is absent today? What does it mean to solve a system of linear equations?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining a solution to a system of linear equations and explaining what it represents in context.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.17

At a fabric store, fabrics are sold by the yard. A dressmaker spent $36.35 on 4.25 yd of silk and cotton fabrics for a dress. Silk is $16.90 per yard and cotton is $4 per yard.

Consider the system of linear equations that represents the constraints in the situation:

$$\begin{cases} x + y = 4.25 \\ 16.90x + 4y = 36.35 \end{cases}$$

1. What does the solution to this system represent?

   Sample response: The solution represents the combination of lengths of silk and cotton (in yards) that meet both constraints; they add up to 4.25 yd and the cost of buying both is $36.35.

2. The graph represents the system of linear equations. Use the graph to determine the solution to the system. Explain or show your thinking.

   Sample response: 1.5 yd of silk and 2.75 yd of cotton. The graphs of the equations intersect at the point (1.5, 2.75).

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can explain what is meant by "the solution to a system of linear equations" and how the solution is represented graphically.

1 2 3

b I can explain what it means when two equations are referred to as a *system of equations*.

1 2 3

c I can use tables and graphs to solve systems of linear equations.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Success looks like . . .

- **Language Goal:** Solving systems of linear equations by reasoning with tables and by graphing, and explaining the solution method. **(Speaking and Listening, Writing)**
- **Language Goal:** Understanding that the solution to a system of equations in two variables is a pair of values that simultaneously make both equations true, and that it is represented by the intersection point of the graphs of the equations. **(Speaking and Listening, Writing)**
  - » Determining the solution to the system of equations in Problem 2.
- **Language Goal:** Understanding that two (or more) equations that represent the constraints on the same quantities in the same situation form a system. **(Speaking and Listening, Writing)**

Suggested next steps

If students cannot articulate what is meant by the solution to the given system in Problem 1, consider:

- Asking, "What are the quantities in the given situation? What is the relationship between them?"
- Reviewing the quantities given in each problem in Activity 2.
- Assigning Practice Problem 1.

If students cannot determine a solution to the system in Problem 2, consider:

- Prompting them to use graphing technology to graph each equation in the system on the same set of axes.
- Assigning Practice Problems 2 and 3.

Professional Learning

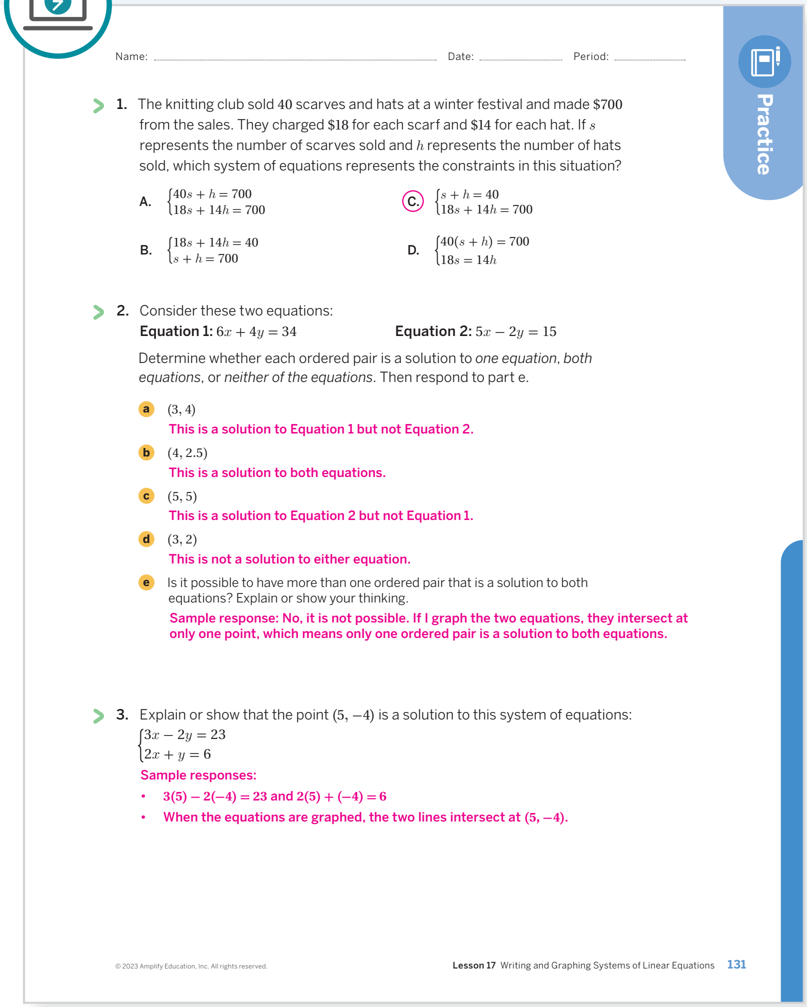
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In Activity 1, students discuss whether different pairs of values are possible combinations of hours that Diego could have worked. How did this discussion build on students' earlier work explaining a solution to an equation?
- What strategies did students use to approach Activity 2? How might this inform your instruction for the next lesson? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. The knitting club sold 40 scarves and hats at a winter festival and made $700 from the sales. They charged $18 for each scarf and $14 for each hat. If s represents the number of scarves sold and h represents the number of hats sold, which system of equations represents the constraints in this situation?

A. $\begin{cases} 40s + h = 700 \\ 18s + 14h = 700 \end{cases}$

[✓ C.] $\begin{cases} s + h = 40 \\ 18s + 14h = 700 \end{cases}$

B. $\begin{cases} 18s + 14h = 40 \\ s + h = 700 \end{cases}$

D. $\begin{cases} 40(s + h) = 700 \\ 18s = 14h \end{cases}$

2. Consider these two equations:

Equation 1: $6x + 4y = 34$ **Equation 2:** $5x - 2y = 15$

Determine whether each ordered pair is a solution to *one equation*, *both equations*, or *neither of the equations*. Then respond to part e.

a. $(3, 4)$

This is a solution to Equation 1 but not Equation 2.

b. $(4, 2.5)$

This is a solution to both equations.

c. $(5, 5)$

This is a solution to Equation 2 but not Equation 1.

d. $(3, 2)$

This is not a solution to either equation.

e. Is it possible to have more than one ordered pair that is a solution to both equations? Explain or show your thinking.

Sample response: No, it is not possible. If I graph the two equations, they intersect at only one point, which means only one ordered pair is a solution to both equations.

3. Explain or show that the point $(5, -4)$ is a solution to this system of equations:

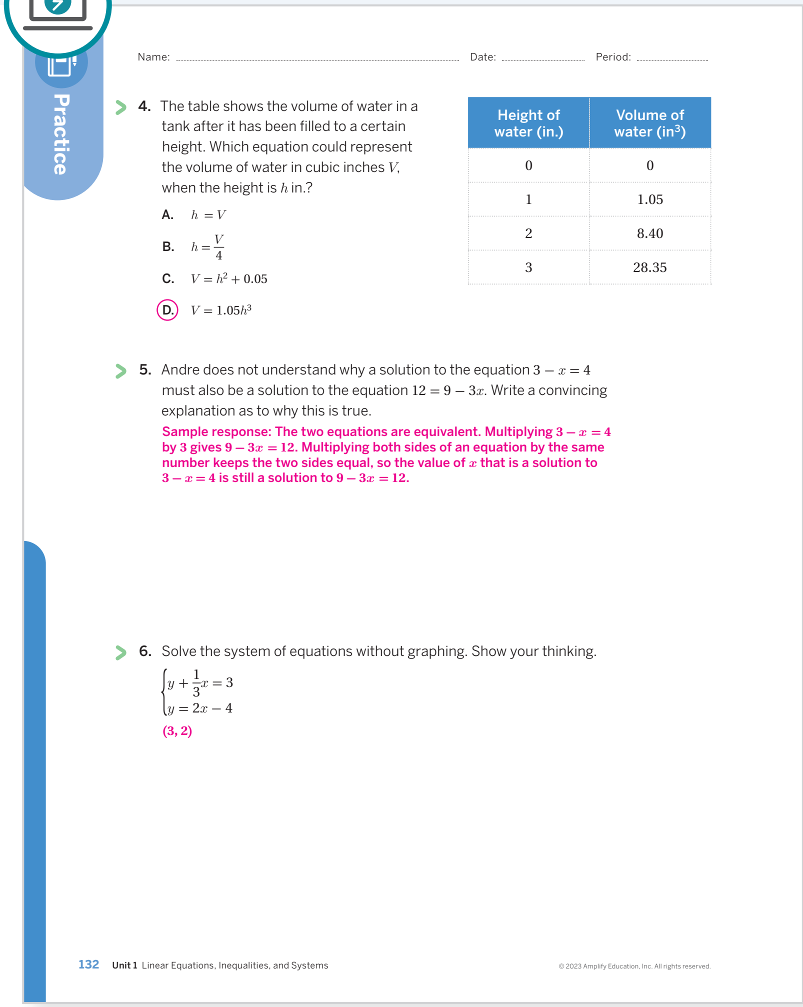
$\begin{cases} 3x - 2y = 23 \\ 2x + y = 6 \end{cases}$

Sample responses:

- **$3(5) - 2(-4) = 23$ and $2(5) + (-4) = 6$**
- **When the equations are graphed, the two lines intersect at $(5, -4)$.**

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 17 Writing and Graphing Systems of Linear Equations 131



Practice

Name: Date: Period:

4. The table shows the volume of water in a tank after it has been filled to a certain height. Which equation could represent the volume of water in cubic inches V, when the height is h in.?

A. $h = V$

B. $h = \frac{V}{4}$

C. $V = h^2 + 0.05$

[✓ D.] $V = 1.05h^3$

Height of water (in.)	Volume of water (in^3)
0	0
1	1.05
2	8.40
3	28.35

5. Andre does not understand why a solution to the equation $3 - x = 4$ must also be a solution to the equation $12 = 9 - 3x$. Write a convincing explanation as to why this is true.

Sample response: The two equations are equivalent. Multiplying $3 - x = 4$ by 3 gives $9 - 3x = 12$. Multiplying both sides of an equation by the same number keeps the two sides equal, so the value of x that is a solution to $3 - x = 4$ is still a solution to $9 - 3x = 12$.

6. Solve the system of equations without graphing. Show your thinking.

$\begin{cases} y + \frac{1}{3}x = 3 \\ y = 2x - 4 \end{cases}$

$(3, 2)$

132 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 3	2
	5	Unit 1 Lesson 8	3
Formative ⚡	6	Unit 1 Lesson 18	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

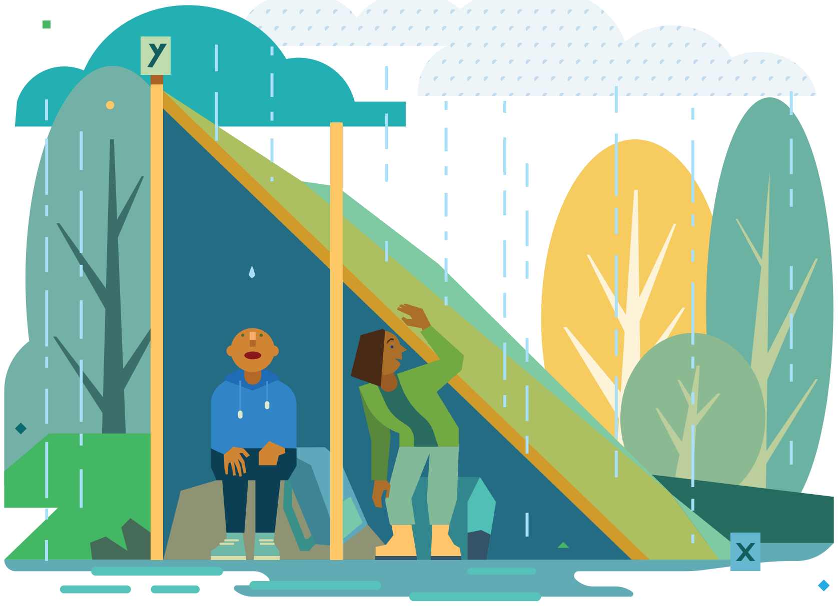
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 18**

Solving Systems by Substitution

Let's use substitution to solve systems of linear equations.



Focus

Goals

1. **Language Goal:** Recognize that a system can be efficiently solved by substitution if one variable is already isolated or can readily be isolated. **(Speaking and Listening, Writing)**
2. **Language Goal:** Recognize that there are multiple ways to perform substitution to solve a system of equations. **(Speaking and Listening, Writing)**
3. Solve systems of linear equations by substituting a variable with a number or an expression, and check solutions by substituting them back into the equations.

Coherence

Today

Students build on their Grade 8 understanding of solving a system of equations in slope-intercept form by substitution. Now, they perform substitution on systems written in different forms to solve the same system. Students examine the structure of the linear equations in a system and determine the most efficient variable to substitute to solve the system.

Previously

In the previous lesson, students wrote and solved systems of linear equations and determined the solutions using a graph.

Coming Soon

In the next lesson, students will be introduced to solving systems of equations by elimination when substitution is not efficient.

Rigor

- Students build **procedural fluency** solving systems of linear equations by substitution.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
10 min	15 min	15 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps*
- graph paper

Math Language Development

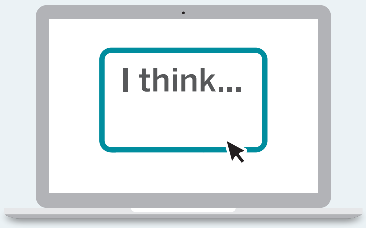
Review words

- *slope-intercept form*
- *solution to a system*
- *standard form*
- *substitution*
- *system of equations*

Amps | Featured Activity

Activity 1
Equation-Solving Organizer

Students can track their steps in solving an equation line by line.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated if they substitute correctly but still yield an incorrect solution in Activity 3. Offer positive feedback for the correct steps they have already taken and encourage them to perform this first step for all problems. Then, motivate them to finish the problem by having them solve the equations they wrote on a clean sheet of paper, reminding them of the properties of equality that they have already learned.

Modifications to Pacing

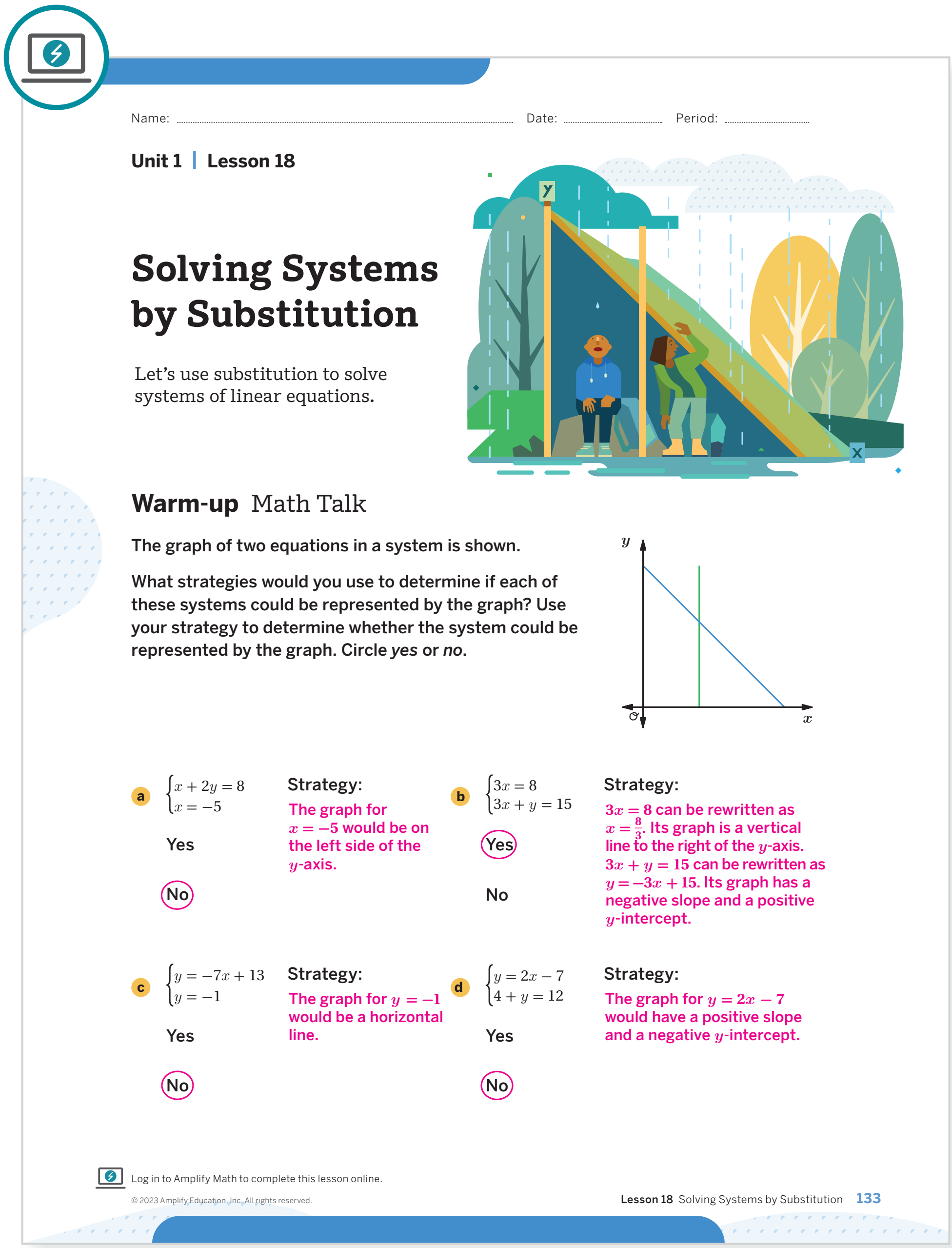
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, part d may be omitted.
- In **Activity 1**, Problem 4 may be omitted.
- In **Activity 2**, have each partner solve only two systems.

Independent | 10 min

Warm-up Math Talk

Students determine if systems of linear equations represent an unlabeled graph on a coordinate plane to make connections between features of graphs and equations.



Name: ____________ Date: ________ Period: ________

Unit 1 | Lesson 18

Solving Systems by Substitution

Let's use substitution to solve systems of linear equations.

Warm-up Math Talk

The graph of two equations in a system is shown.

What strategies would you use to determine if each of these systems could be represented by the graph? Use your strategy to determine whether the system could be represented by the graph. Circle *yes* or *no*.

a $\begin{cases} x + 2y = 8 \\ x = -5 \end{cases}$

Yes [✓ No]

Strategy: The graph for $x = -5$ would be on the left side of the y-axis.

b $\begin{cases} 3x = 8 \\ 3x + y = 15 \end{cases}$

[✓ Yes] No

Strategy: $3x = 8$ can be rewritten as $x = \frac{8}{3}$. Its graph is a vertical line to the right of the y-axis. $3x + y = 15$ can be rewritten as $y = -3x + 15$. Its graph has a negative slope and a positive y-intercept.

c $\begin{cases} y = -7x + 13 \\ y = -1 \end{cases}$

Yes [✓ No]

Strategy: The graph for $y = -1$ would be a horizontal line.

d $\begin{cases} y = 2x - 7 \\ 4 + y = 12 \end{cases}$

Yes [✓ No]

Strategy: The graph for $y = 2x - 7$ would have a positive slope and a negative y-intercept.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 18 Solving Systems by Substitution 133

1 Launch

Conduct the *Math Talk* routine. Give students think-time for each problem. Have them signal when they have an answer and a strategy for each problem. **Note:** Graphing technology should not be used.

2 Monitor

Help students get started by providing access to graph paper.

Look for points of confusion:

- **Interchanging equations of horizontal lines with vertical lines.** Prompt students to make a table or list possible coordinates of each line and plot the points.

Look for productive strategies:

- Sketching the graphs of each system.
- Rewriting equations in slope-intercept form.
- Using the structure of the graph to eliminate systems with horizontal lines and positive slopes.
- Solving each system algebraically.

3 Connect

Have students share their responses and strategies for each problem. Listen for students using precise language. Display their responses.

Highlight why parts a, c, and d cannot represent the graph by identifying the horizontal lines in parts c and d and identifying the positive slope in part d, and the vertical line on the negative x-axis in part a.

Ask, "How can you determine the solution for part b without graphing?"

Math Language Development

MLR2: Collect and Display

During the Connect, as students share their responses, listen for and encourage the use of precise mathematical language. For example, for part a, ask students why the graph of $x = -5$ would be on the left side of the y-axis. The constant term is negative. Display this language on the class display for students to refer to during class discussions.

English Learners

Display or provide access to the Anchor Chart PDF, *Sentence Stems, Explaining My Steps*.

Power-up

To power up students' ability to solve a system of equations without graphing, have students complete:

Recall that, in order to solve a system of equations without graphing, you can begin by setting the equations equal to each other to solve for x. Then substitute the value of x to determine the corresponding value of y.

Solve the system. Show or explain your thinking.

$\begin{cases} y = 2x - 4 \\ y = x + 1 \end{cases}$

$2x - 4 = x + 1$
$2x - x = 1 + 4$
$x = 5$

$y = 5 + 1$
$y = 6$

$(5, 6)$

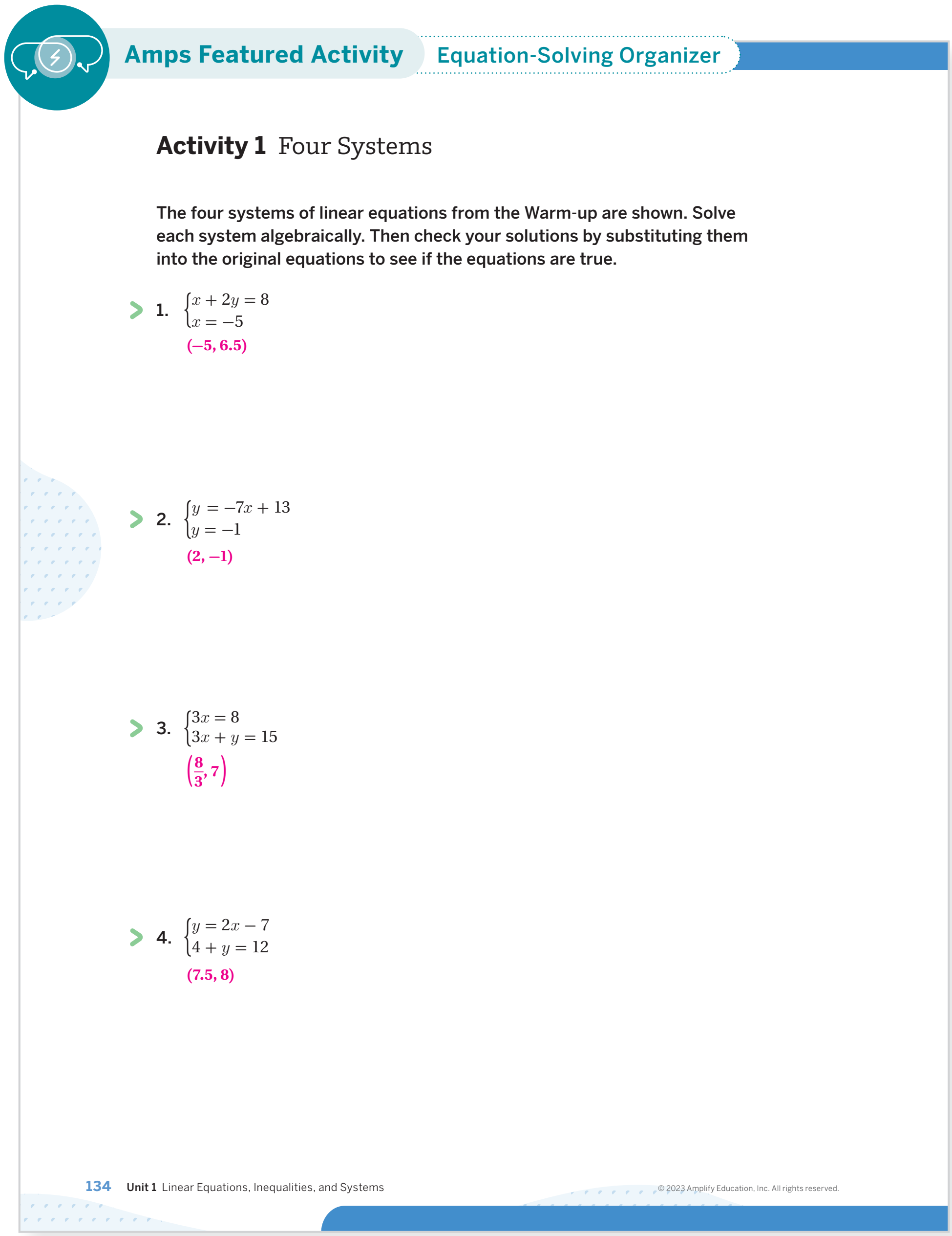
Use: Before the Warm-up
Informed by: Performance on Lesson 17, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 5

Pairs | 15 min

Activity 1 Four Systems

Students look for structure in systems of linear equations to determine a strategy for solving them algebraically.

Amps Featured Activity Equation-Solving Organizer



Activity 1 Four Systems

The four systems of linear equations from the Warm-up are shown. Solve each system algebraically. Then check your solutions by substituting them into the original equations to see if the equations are true.

1. $\begin{cases} x + 2y = 8 \\ x = -5 \end{cases}$

   (−5, 6.5)

2. $\begin{cases} y = -7x + 13 \\ y = -1 \end{cases}$

   (2, −1)

3. $\begin{cases} 3x = 8 \\ 3x + y = 15 \end{cases}$

   $\left(\frac{8}{3}, 7\right)$

4. $\begin{cases} y = 2x - 7 \\ 4 + y = 12 \end{cases}$

   (7.5, 8)

134 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in pairs. Have them discuss each problem together, complete the problems independently, then share their strategies as they finish.

2 Monitor

Help students get started by displaying the work from the Warm-up.

Look for points of confusion:

- **Forgetting to solve for the second variable.** Remind students that the solution to a system is a pair of values.
- **Struggling to identify which equation to substitute in Problem 4.** Ask, "Which variable can be solved in one-step?"

Look for productive strategies:

- Graphing each system.
- Substituting a variable or expression in one equation with an equivalent value or expression into the other.
- Verifying solutions by substituting the values into the original equations.

3 Connect

Have pairs of students share their strategies for solving the systems. Select students using different substitutions for Problems 3 and 4. Display each substitution strategy.

Highlight that there may be multiple ways to solve by substitution and that one may be more efficient than the others. When a variable is already isolated or can readily be isolated, substituting the value (or expression) of that variable into the other equation can be an efficient way to solve the system.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use an interactive substitution tool to visually see what it means to replace one value with another.

Extension: *Math Enrichment*

Challenge students to solve for the other variable in one of the equations in each system and then use substitution. Ask them to compare both methods and articulate which method they thought was more efficient. Ask them to explain how the structure of each equation in the system indicates which variable might be the most efficient to choose.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies, ask students who used different substitution methods to compare their work and connect how the different methods still yield the same solution. Ask, "Was one substitution method more efficient than the other? Why?" Highlight for students that it is important to be strategic in choosing which substitution method to choose.

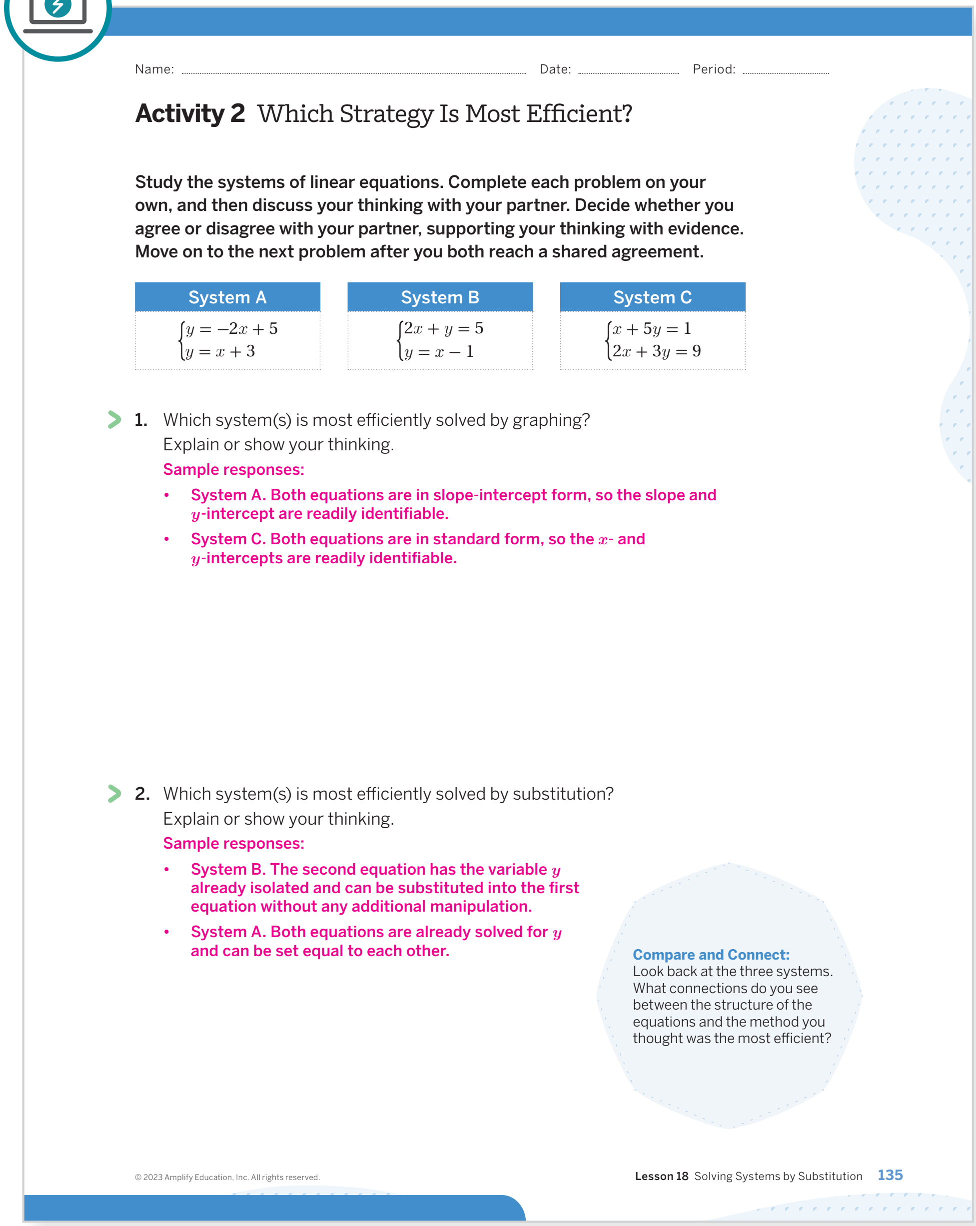
English Learners

Display or provide access to the Anchor Chart PDF, *Sentence Stems, Explaining My Steps.*

Pairs | 15 min

Activity 2 Which Strategy Is Most Efficient?

Students examine three different systems of linear equations to determine which strategy is most efficient for solving.



Name: ______ Date: ______ Period: ______

Activity 2 Which Strategy Is Most Efficient?

Study the systems of linear equations. Complete each problem on your own, and then discuss your thinking with your partner. Decide whether you agree or disagree with your partner, supporting your thinking with evidence. Move on to the next problem after you both reach a shared agreement.

System A	System B	System C
$\begin{cases} y = -2x + 5 \\ y = x + 3 \end{cases}$	$\begin{cases} 2x + y = 5 \\ y = x - 1 \end{cases}$	$\begin{cases} x + 5y = 1 \\ 2x + 3y = 9 \end{cases}$

1. Which system(s) is most efficiently solved by graphing? Explain or show your thinking.
   Sample responses:
   - **System A. Both equations are in slope-intercept form, so the slope and y-intercept are readily identifiable.**
   - **System C. Both equations are in standard form, so the x- and y-intercepts are readily identifiable.**

2. Which system(s) is most efficiently solved by substitution? Explain or show your thinking.
   Sample responses:
   - **System B. The second equation has the variable y already isolated and can be substituted into the first equation without any additional manipulation.**
   - **System A. Both equations are already solved for y and can be set equal to each other.**

Compare and Connect: Look back at the three systems. What connections do you see between the structure of the equations and the method you thought was the most efficient?

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Give students one minute to study the systems. Ask students which strategy they think is most efficient and record their responses. Then, have students work independently before sharing their thinking with their partner. Provide graph paper to students who request it.

2 Monitor

Help students get started by asking what "most efficient" means to them. Sample response: Requiring the least amount of manipulation.

Look for points of confusion:

- **Rearranging equations.** Prompt students to use the current structure of the given equations.

Look for productive strategies:

- Looking for equations with isolated variables.
- Looking for equations in standard or slope-intercept form.
- Recognizing all systems could be solved using either strategy.

3 Connect

Display the systems of equations and ask students if they still agree with their original responses.

Have students share their thinking about which strategies would be most efficient for solving the systems. Select and sequence students that used the structure of the equations.

Highlight that graphing is an efficient way to solve systems with equations given in slope-intercept or standard form since no manipulation of the equations is needed. Substitution is an efficient way to solve systems that contain equations where a variable is already isolated or can be isolated without much manipulation.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Tell students that System B could be solved efficiently by using substitution. Ask students to explain why. Repeat for other systems and solution methods.

Extension: *Math Enrichment*

Have students write a few sentences about how the structure of a system of linear equations can indicate which method might be the most efficient to use to solve the system.

Math Language Development

MLR7: Compare and Connect

During the Connect, capture the words and phrases students use as they share their solution strategies. Amplify phrases students use such as "written in slope-intercept form," "x- and y-intercepts are easily identified," "the variable _____ is already isolated," etc. Connect these words to the solution method that might be more efficient.

English Learners

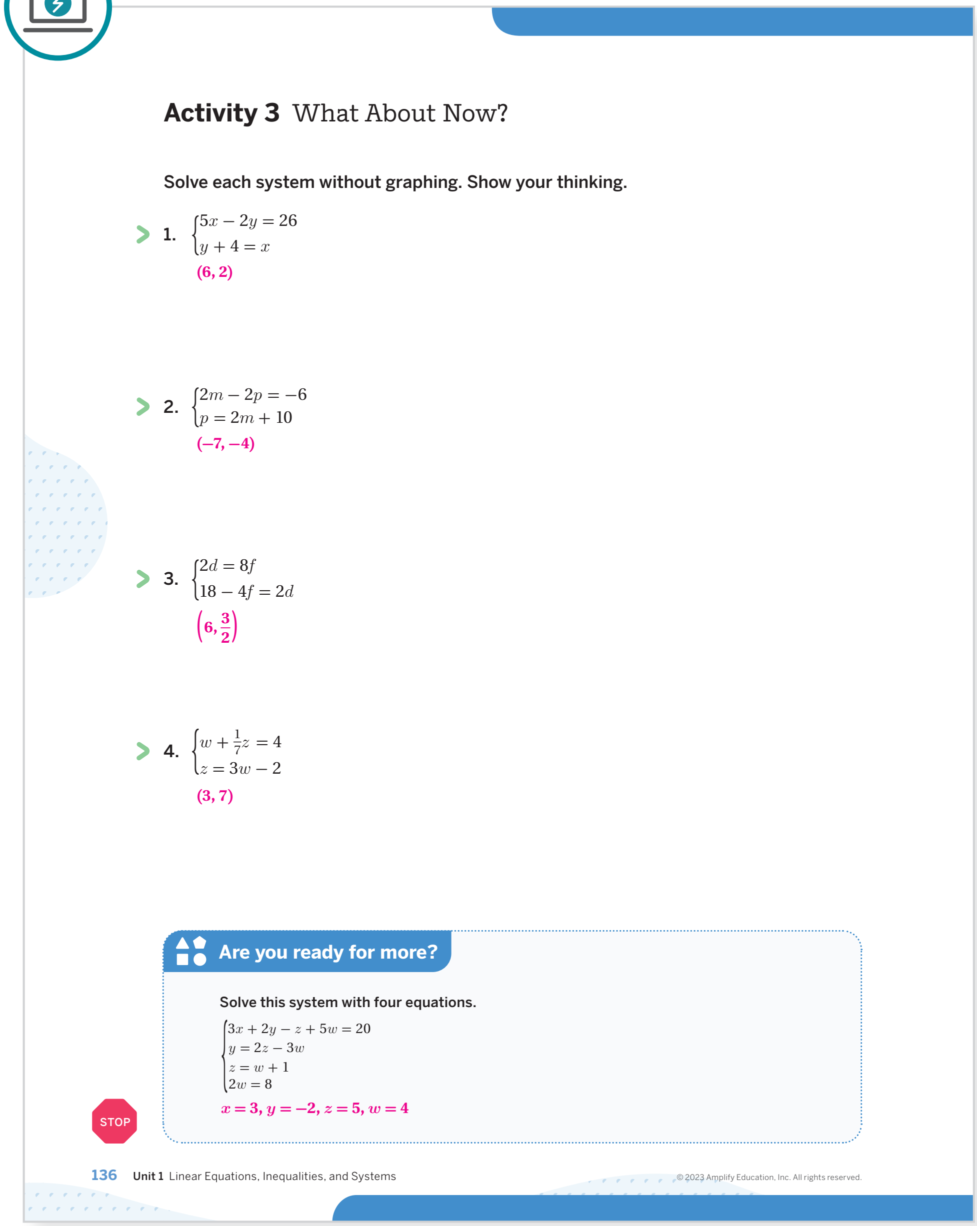
Use gestures or annotations to highlight each of the three systems as students share these words and phrases. For example, point to $y = x - 1$ in System B as you say "the variable _____ is already isolated."

Optional

Independent | 10 min

Activity 3 What About Now?

Students practice solving systems without graphing to reinforce that there are multiple ways to perform substitution.



Activity 3 What About Now?

Solve each system without graphing. Show your thinking.

1. $\begin{cases} 5x - 2y = 26 \\ y + 4 = x \end{cases}$

   (6, 2)

2. $\begin{cases} 2m - 2p = -6 \\ p = 2m + 10 \end{cases}$

   (−7, −4)

3. $\begin{cases} 2d = 8f \\ 18 - 4f = 2d \end{cases}$

   $\left(6, \frac{3}{2}\right)$

4. $\begin{cases} w + \frac{1}{7}z = 4 \\ z = 3w - 2 \end{cases}$

   (3, 7)

Are you ready for more?

Solve this system with four equations.

$$\begin{cases} 3x + 2y - z + 5w = 20 \\ y = 2z - 3w \\ z = w + 1 \\ 2w = 8 \end{cases}$$

$x = 3, y = -2, z = 5, w = 4$

STOP

136 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Keep students in pairs and have them work independently before sharing their thinking with their partner.

2 Monitor

Help students get started by prompting them to use parentheses around any expression(s) they substitute into another equation.

Look for points of confusion:

- **Substituting an expression without parentheses in Problems 1, 2, or 4.** Remind students that a coefficient multiplies the entire expression that is equivalent to the variable.
- **Not distributing the negative in Problem 2.** Cover the term $2m$ in the expression and ask, "What is being distributed?" −2

Look for productive strategies:

- Rearranging an equation to isolate a variable.
- Using the distributive property after replacing a variable with an equivalent expression.
- Verifying solutions by substituting the values into the original equations.

3 Connect

Have pairs of students share their responses and thinking. Select students solving for different variables or using different substitutions and display their work.

Highlight different ways of performing substitution to solve the same system using students' displayed work. For example, in Problem 2, they can replace p with $2m + 10$, or $2m$ with $p - 10$ in the first equation. In Problem 3, they can replace $2d$ with $8f$, or f with $\frac{1}{4}d$ in the second equation.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students focus on completing Problems 1–3. Allow students to complete the problems in any order. Suggest they first look for any system in which one or both equations are already solved for one variable.

Accessibility: *Guide Processing and Visualization*

Provide colored pencils and suggest that students color code variables with their equivalent expressions. For example, in Problem 1, have students color code x and $y + 4$ with one color in the second equation. In the first equation, have them color code x with the same color to visually see the needed substitution.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses, call their attention to the different ways their classmates have chosen to make substitutions. Wherever possible, amplify student words and actions that involve the language of substitution, such as *replace, substitute, solve for,* and *isolate*.

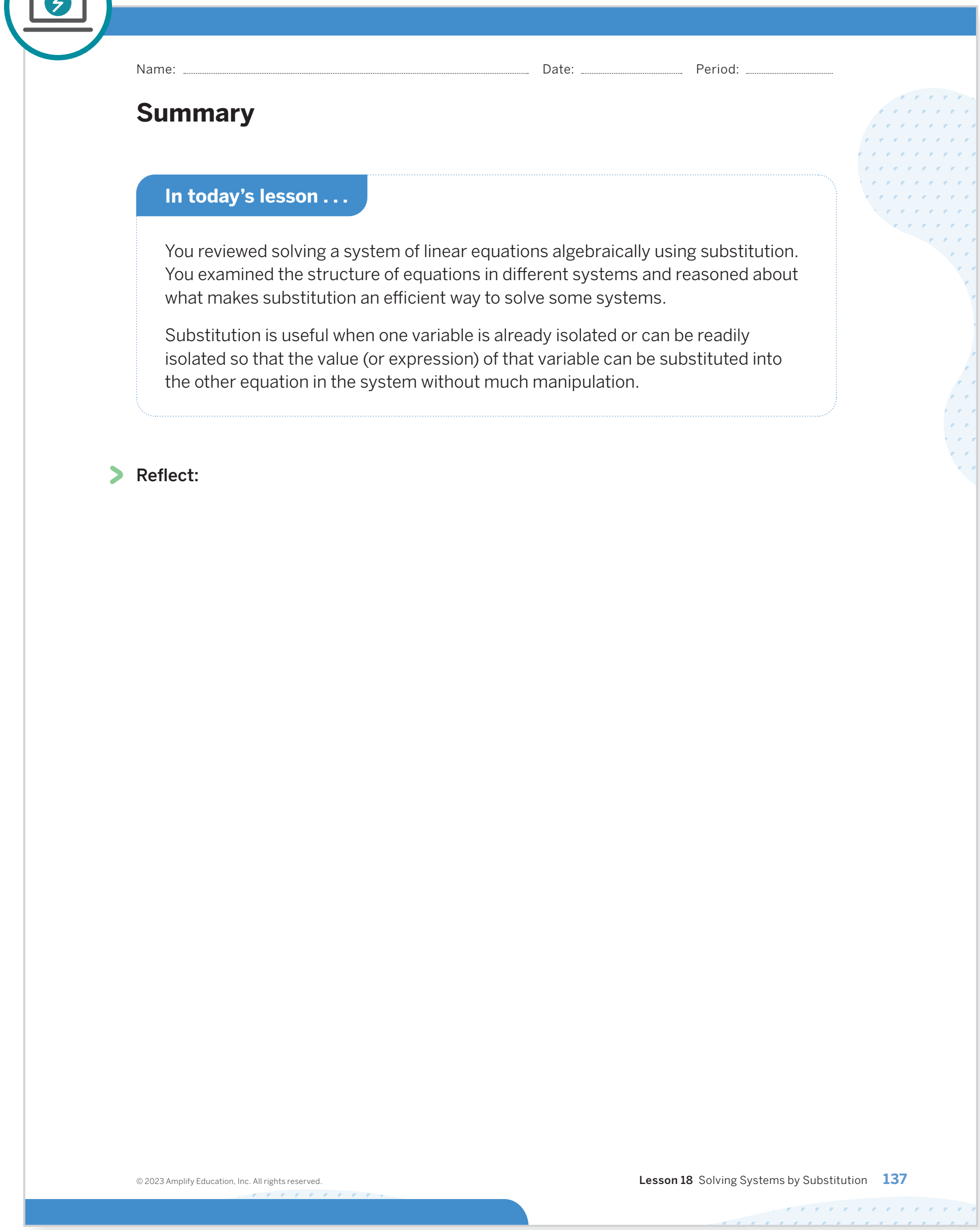
English Learners

Display the words *replace* and *substitute* and highlight how they mean the same thing when replacing/substituting a value or expression for a variable. Similarly, show how the phrases *solve for* _____ and *isolate* _____ mean the same thing.

Whole Class | 5 min

Summary

Review and synthesize how to solve a system by substituting a variable or an expression.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You reviewed solving a system of linear equations algebraically using substitution. You examined the structure of equations in different systems and reasoned about what makes substitution an efficient way to solve some systems.

Substitution is useful when one variable is already isolated or can be readily isolated so that the value (or expression) of that variable can be substituted into the other equation in the system without much manipulation.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the following three systems.

System 1	System 2	System 3
$\begin{cases} 3m + n = 71 \\ 2m - n = 30 \end{cases}$	$\begin{cases} 4x + y = 1 \\ y = -2x + 9 \end{cases}$	$\begin{cases} 5x + 4y = 15 \\ 5x + 11y = 22 \end{cases}$

Ask, "Which system can readily be solved by substitution? Which system might require more effort to solve using substitution?"

Have students share why they would or would not choose to solve one of the given systems by substitution.

Highlight that System 2 is the most conducive to being solved by substitution because it is already solved for the variable, y. System 1 can be rearranged so that n is isolated in the second equation. System 3 is the least conducive to solving by substitution, but it can be done!

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

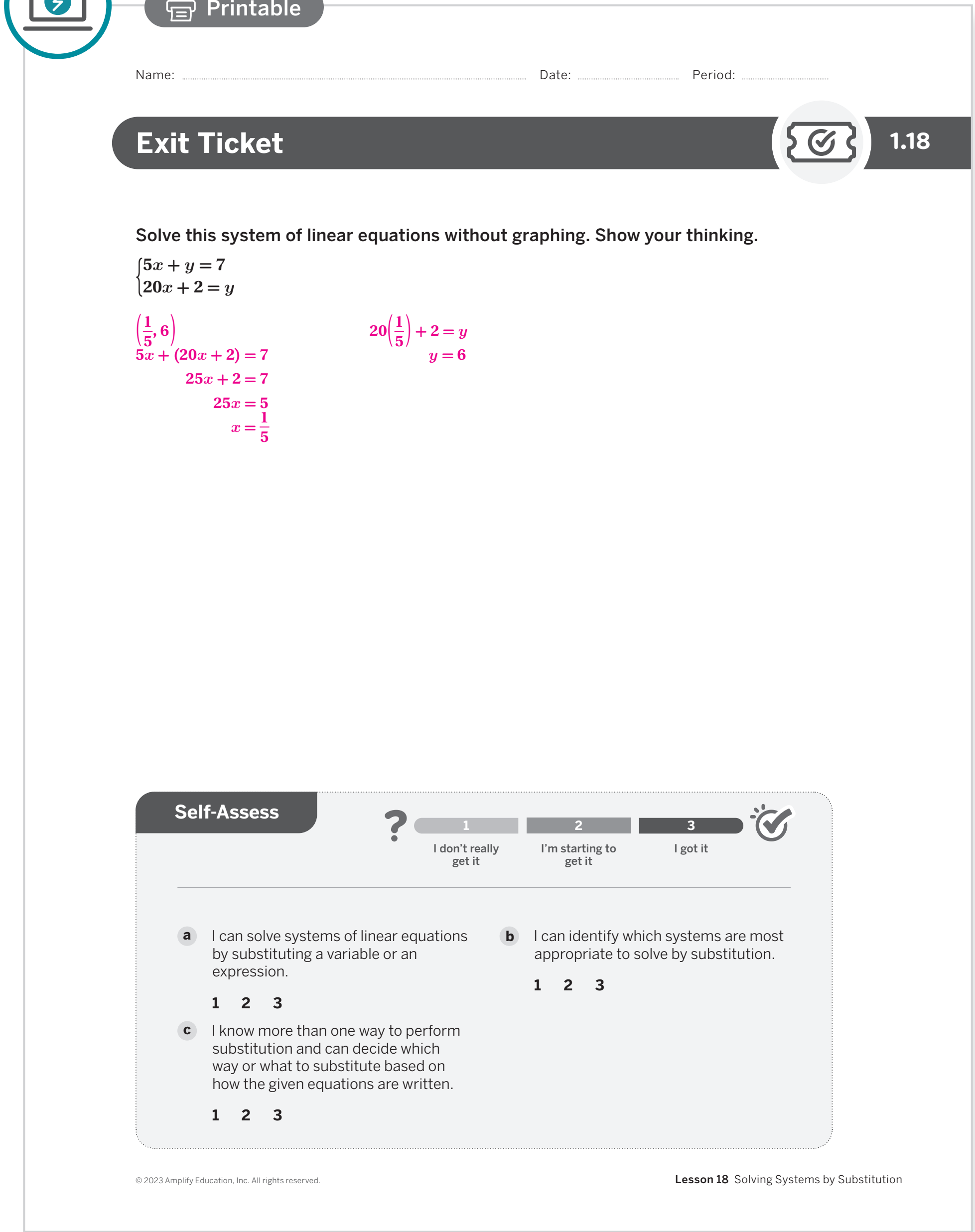
- "How can you determine by inspection if a system of equations is conducive to being solved by substitution?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by solving a system of linear equations using substitution.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket

1.18

Solve this system of linear equations without graphing. Show your thinking.

$$\begin{cases} 5x + y = 7 \\ 20x + 2 = y \end{cases}$$

$\left(\frac{1}{5}, 6\right)$

$5x + (20x + 2) = 7$

$25x + 2 = 7$

$25x = 5$

$x = \frac{1}{5}$

$20\left(\frac{1}{5}\right) + 2 = y$

$y = 6$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can solve systems of linear equations by substituting a variable or an expression.

1 2 3

b I can identify which systems are most appropriate to solve by substitution.

1 2 3

c I know more than one way to perform substitution and can decide which way or what to substitute based on how the given equations are written.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 18 Solving Systems by Substitution

Success looks like . . .

- **Language Goal:** Recognizing that a system can be efficiently solved by substitution if one variable is already isolated or can be easily isolated. **(Speaking and Listening, Writing)**
- **Language Goal:** Recognizing that there are multiple ways to perform substitution to solve a system of equations. **(Speaking and Listening, Writing)**
- **Goal:** Solving systems of linear equations by substituting a variable with a number or an expression, and checking solutions by substituting them back into the equations.
  - » Solving the system of equations by substituting the expression $20x + 2$ for y in the first equation.

Suggested next steps

If students are unable to solve the given system by substitution, consider:

- Reviewing Problems 1 and 2 in Activity 2.
- Assigning Practice Problems 2 and 3.

Professional Learning

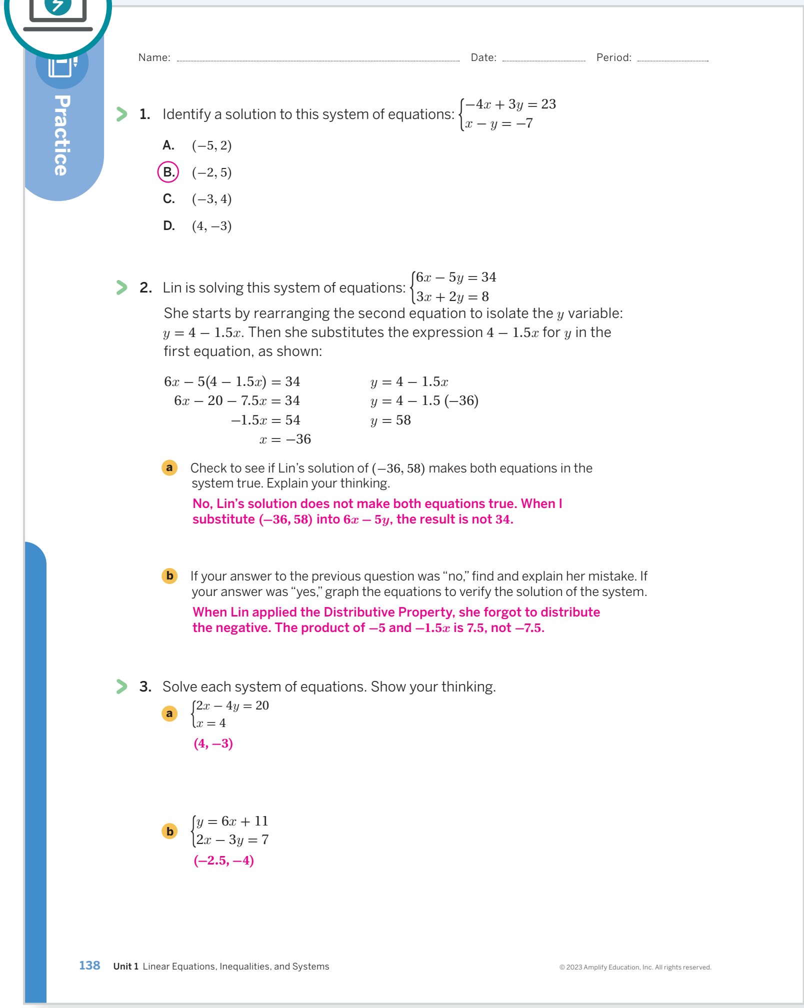
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Think about the types of questions you asked students today. Were they assessing or advancing? What did students say or do in response? What question was most effective in helping students use substitution efficiently?
- What did students find frustrating about Activity 2? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Identify a solution to this system of equations: $\begin{cases} -4x + 3y = 23 \\ x - y = -7 \end{cases}$
   - A. $(-5, 2)$
   - [✓ B.] $(-2, 5)$
   - C. $(-3, 4)$
   - D. $(4, -3)$

2. Lin is solving this system of equations: $\begin{cases} 6x - 5y = 34 \\ 3x + 2y = 8 \end{cases}$
   She starts by rearranging the second equation to isolate the y variable: $y = 4 - 1.5x$. Then she substitutes the expression $4 - 1.5x$ for y in the first equation, as shown:

$$\begin{aligned} 6x - 5(4 - 1.5x) &= 34 \\ 6x - 20 - 7.5x &= 34 \\ -1.5x &= 54 \\ x &= -36 \end{aligned} \qquad \begin{aligned} y &= 4 - 1.5x \\ y &= 4 - 1.5\,(-36) \\ y &= 58 \end{aligned}$$

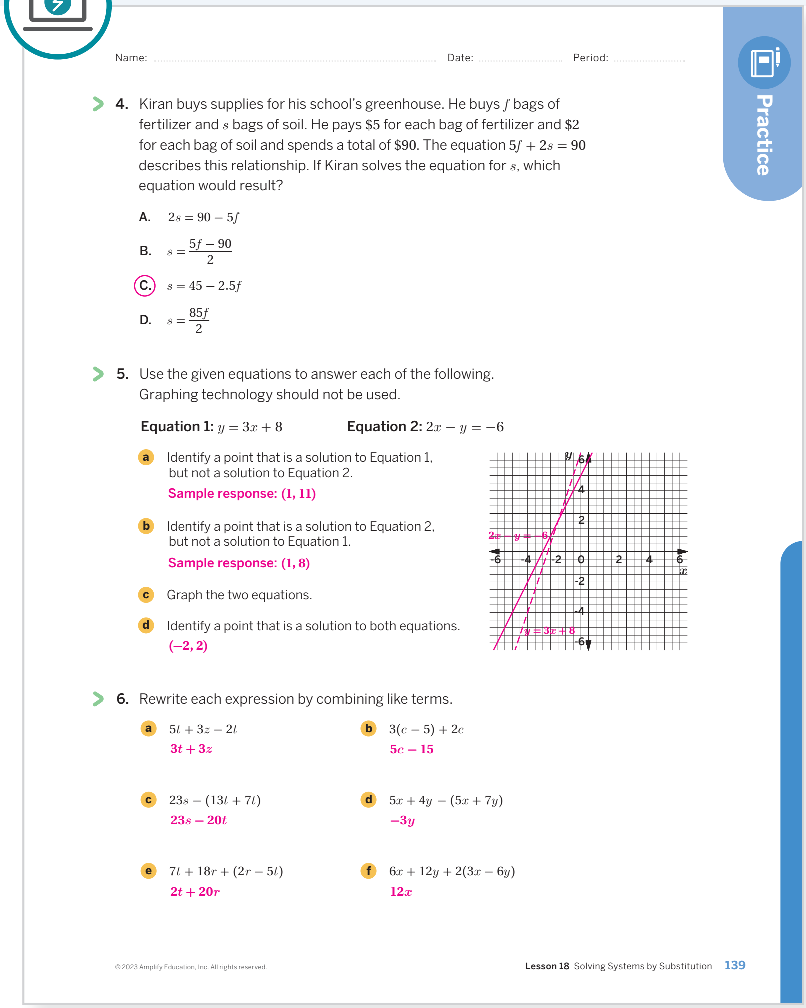
   a. Check to see if Lin's solution of $(-36, 58)$ makes both equations in the system true. Explain your thinking.
   No, Lin's solution does not make both equations true. When I substitute $(-36, 58)$ into $6x - 5y$, the result is not 34.

   b. If your answer to the previous question was "no," find and explain her mistake. If your answer was "yes," graph the equations to verify the solution of the system.
   When Lin applied the Distributive Property, she forgot to distribute the negative. The product of -5 and $-1.5x$ is 7.5, not -7.5.

3. Solve each system of equations. Show your thinking.
   a. $\begin{cases} 2x - 4y = 20 \\ x = 4 \end{cases}$
   $(4, -3)$

   b. $\begin{cases} y = 6x + 11 \\ 2x - 3y = 7 \end{cases}$
   $(-2.5, -4)$

138 **Unit 1** Linear Equations, Inequalities, and Systems © 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ________ Date: ________ Period: ________

4. Kiran buys supplies for his school's greenhouse. He buys f bags of fertilizer and s bags of soil. He pays \$5 for each bag of fertilizer and \$2 for each bag of soil and spends a total of \$90. The equation $5f + 2s = 90$ describes this relationship. If Kiran solves the equation for s, which equation would result?
   - A. $2s = 90 - 5f$
   - B. $s = \frac{5f - 90}{2}$
   - [✓ C.] $s = 45 - 2.5f$
   - D. $s = \frac{85f}{2}$

5. Use the given equations to answer each of the following. Graphing technology should not be used.

   Equation 1: $y = 3x + 8$ **Equation 2:** $2x - y = -6$

   a. Identify a point that is a solution to Equation 1, but not a solution to Equation 2.
   Sample response: $(1, 11)$

   b. Identify a point that is a solution to Equation 2, but not a solution to Equation 1.
   Sample response: $(1, 8)$

   c. Graph the two equations.

   d. Identify a point that is a solution to both equations.
   $(-2, 2)$

6. Rewrite each expression by combining like terms.
   a. $5t + 3z - 2t$
   $3t + 3z$
   b. $3(c - 5) + 2c$
   $5c - 15$
   c. $23s - (13t + 7t)$
   $23s - 20t$
   d. $5x + 4y - (5x + 7y)$
   $-3y$
   e. $7t + 18r + (2r - 5t)$
   $2t + 20r$
   f. $6x + 12y + 2(3x - 6y)$
   $12x$

© 2023 Amplify Education, Inc. All rights reserved. **Lesson 18** Solving Systems by Substitution 139

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	3
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 9	2
	5	Unit 1 Lesson 17	2
Formative ⚡	6	Unit 1 Lesson 19	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 19**

Solving Systems by Elimination: Adding and Subtracting (Part 1)

Let's investigate how adding or subtracting equations can help us solve systems of linear equations.

Focus

Goals

1. Recognize that adding or subtracting equations in a system creates a new equation with a solution that coincides with that of the original system, so the new equation can be used to solve the original system.
2. Solve systems of equations by adding or subtracting the equations strategically to eliminate a variable.
3. **Language Goal:** Use graphing technology to graph the sums and differences of the equations in a system, and analyze and describe the behaviors of the graphs. **(Speaking and Listening, Writing)**

Coherence

Today

Students are introduced to elimination, a new strategy for solving a system of equations. They add and subtract equations in systems of equations, creating a new equation used to solve the system. Students connect the common point of intersection on the graphs of all three equations to the solution of the system. This is the first of three lessons for solving by elimination.

Previously

Students solved systems of linear equations using substitution.

Coming Soon

In Lessons 20 and 21, students will further their understanding of using elimination to solve systems of inear equations.

Rigor

- Students build a **conceptual understanding** of solving systems of linear equations by elimination using addition and subtraction.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	20 min	10 min	5 min	5 min
Independent	Pairs	Small Groups	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Note: Activity 2 is recommended to be completed digitally.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (answers)
- graphing technology

Math Language Development

New words

- ***elimination***

Review words

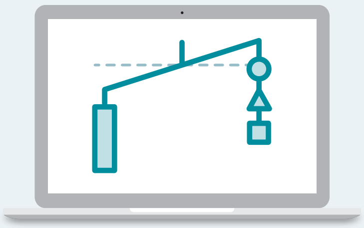
- *standard form*
- *solution to a system*
- *substitution*
- *system of equations*

Amps | Featured Activity

Activity 1

Equation-Solving Organizer

Students can track their steps in solving an equation line by line.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may have conflicting ideas when critiquing the arguments posed in Activity 3 or with each other's arguments. Establish protocols for students to use when they disagree, where they take turns speaking and actively listening to one another. Give students authentic feedback anytime they work well with others and thank them whenever they listen well and interact respectfully, particularly when they have opposing arguments with their peers.

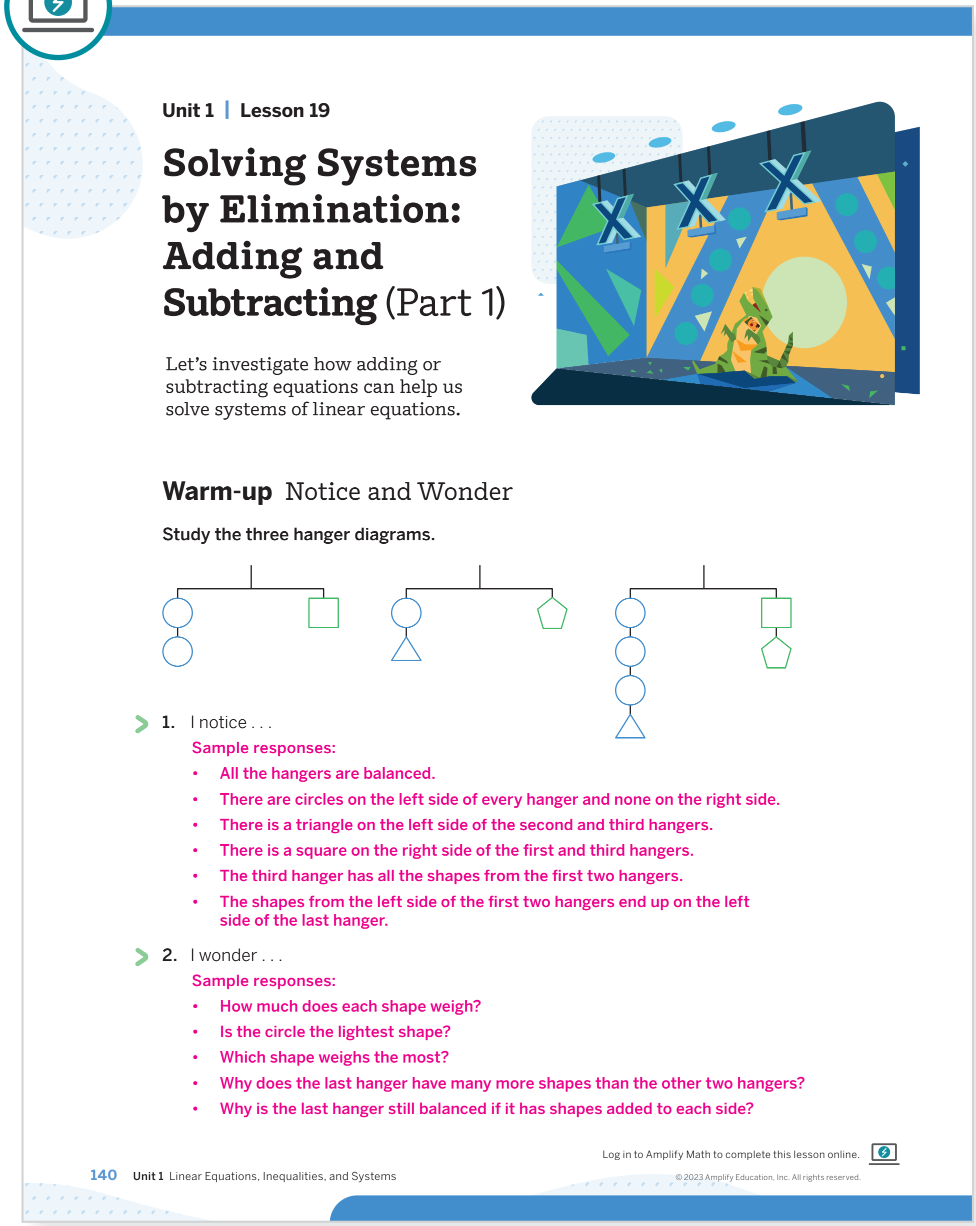
● Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, have students identify one item they notice and one question for what they wonder.
- In **Activity 1**, omit Problem 2, part a.
- Optional **Activity 3** can be omitted.

Warm-up Notice and Wonder

Students study a hanger diagram of three equations, making use of its structure to think concretely about combining two true equations.



Unit 1 | **Lesson 19**

Solving Systems by Elimination: Adding and Subtracting (Part 1)

Let's investigate how adding or subtracting equations can help us solve systems of linear equations.

Warm-up Notice and Wonder

Study the three hanger diagrams.

1. I notice . . .
   Sample responses:
   - All the hangers are balanced.
   - There are circles on the left side of every hanger and none on the right side.
   - There is a triangle on the left side of the second and third hangers.
   - There is a square on the right side of the first and third hangers.
   - The third hanger has all the shapes from the first two hangers.
   - The shapes from the left side of the first two hangers end up on the left side of the last hanger.
2. I wonder . . .
   Sample responses:
   - How much does each shape weigh?
   - Is the circle the lightest shape?
   - Which shape weighs the most?
   - Why does the last hanger have many more shapes than the other two hangers?
   - Why is the last hanger still balanced if it has shapes added to each side?

Log in to Amplify Math to complete this lesson online.

140 Unit 1 Linear Equations, Inequalities, and Systems

©2023 Amplify Education, Inc. All rights reserved.

① Launch

Give students a minute of think-time to study the hanger diagrams. Conduct the ***Notice and Wonder*** routine. Tell them there are no wrong answers.

② Monitor

Help students get started by asking, "What does it mean to be balanced?"

Look for points of confusion:

- **Not recognizing that each hanger represents a balanced equation.** Suggest students think of each shape as a different weight, with the left and right sides balancing each other.

Look for productive strategies:

- Recognizing the left side of the third hanger is the combination of the left sides of the first and second hangers and the right side of the third hanger is the combination of the right sides of the first and second hangers.

③ Connect

Have students share what they notice and wonder about the diagrams. Record and display their thinking. Ask students if they have questions about what is on the list.

Highlight that the weights on each side of the third hanger are the combination of the weights from the corresponding sides of the first two balanced hangers.

Ask, "If you only saw the first two hangers, but you know the third hanger has the combined weights of the corresponding sides of the two hangers, could you predict whether the third hanger would balance?" Yes. I can think of it as adding the first and second hangers. If the same amount is added to each side of a balanced hanger, the third hanger would still be balanced.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students respond to the Ask question, consider displaying just the first two hanger diagrams and cover up the third. Ask students to visually describe the third hanger diagram and explain *why* it would still be balanced. Look for language that calls attention to the first two hanger diagrams being balanced and adding the shapes from the two hanger diagrams together.

English Learners

Visually annotate how the third hanger diagram is composed of the shapes from the first two hanger diagrams combined.

Power-up

To power up students' ability to simplify expressions by combining like terms, have students complete:

Recall that *like terms* are parts of an expression that have the same variables and exponents. Like terms can be added or subtracted into a single term.

Simplify each expression by combining like terms.

a. $2z - 3w + 4 + 3w + 2z$
$= 4z + 4$

b. $2(x + 3) - 3(x + y)$
$= -x - 3y + 6$

Use: Before Activity 1

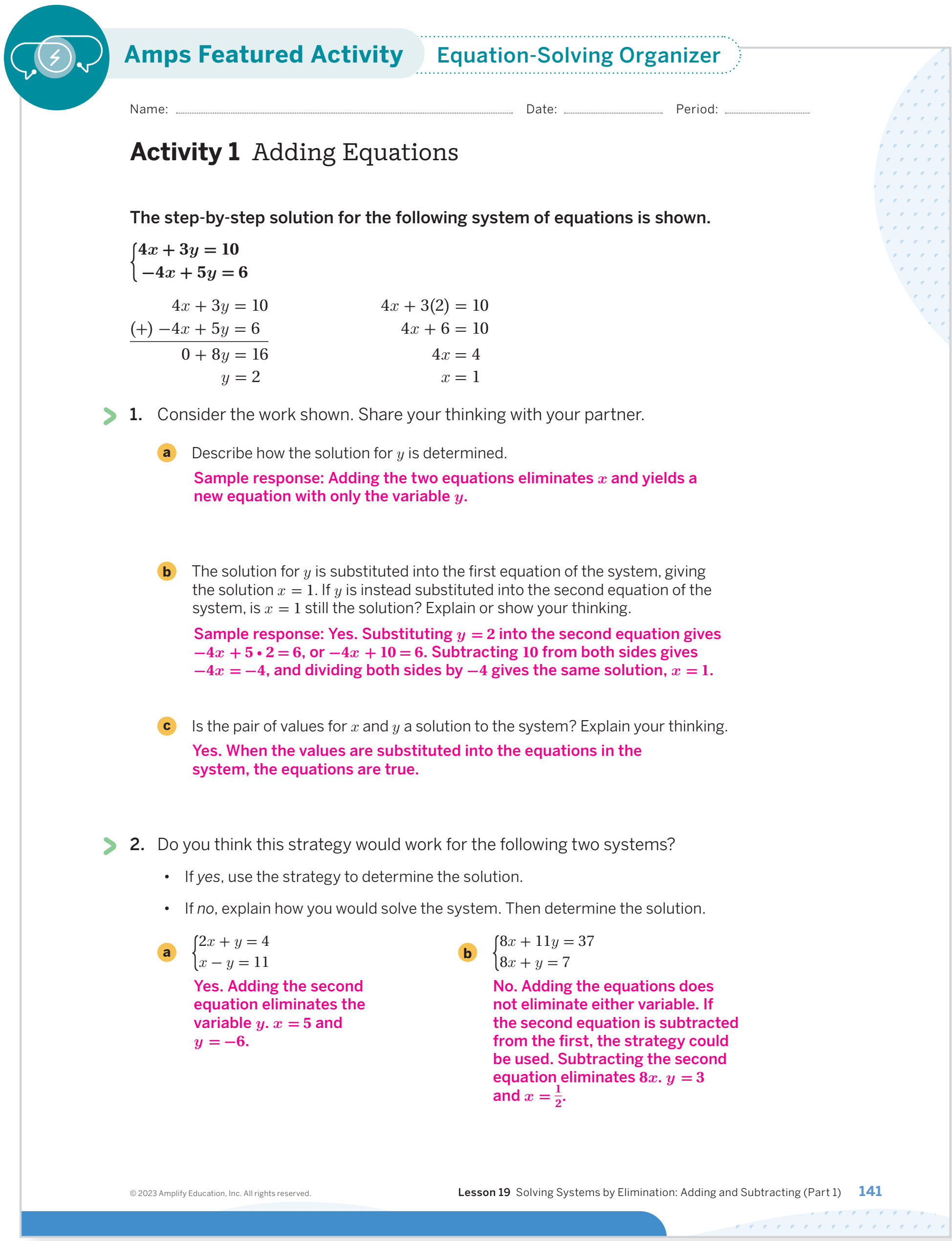
Informed by: Performance on Lesson 18, Practice Problem 6

Pairs | 15 min

Activity 1 Adding Equations

Students analyze a system of equations solved by elimination to develop their understanding.

Amps Featured Activity Equation-Solving Organizer



Name: ______ Date: ______ Period: ______

Activity 1 Adding Equations

The step-by-step solution for the following system of equations is shown.

$$\begin{cases} 4x + 3y = 10 \\ -4x + 5y = 6 \end{cases}$$

$$\begin{array}{rl} & 4x + 3y = 10 \\ (+) & -4x + 5y = 6 \\ \hline & 0 + 8y = 16 \\ & y = 2 \end{array} \qquad \begin{array}{r} 4x + 3(2) = 10 \\ 4x + 6 = 10 \\ 4x = 4 \\ x = 1 \end{array}$$

1. Consider the work shown. Share your thinking with your partner.

   a. Describe how the solution for y is determined.

   Sample response: Adding the two equations eliminates x and yields a new equation with only the variable y.

   b. The solution for y is substituted into the first equation of the system, giving the solution $x = 1$. If y is instead substituted into the second equation of the system, is $x = 1$ still the solution? Explain or show your thinking.

   Sample response: Yes. Substituting $y = 2$ into the second equation gives $-4x + 5 \cdot 2 = 6$, or $-4x + 10 = 6$. Subtracting 10 from both sides gives $-4x = -4$, and dividing both sides by -4 gives the same solution, $x = 1$.

   c. Is the pair of values for x and y a solution to the system? Explain your thinking.

   Yes. When the values are substituted into the equations in the system, the equations are true.

2. Do you think this strategy would work for the following two systems?
   - If *yes*, use the strategy to determine the solution.
   - If *no*, explain how you would solve the system. Then determine the solution.

   a. $\begin{cases} 2x + y = 4 \\ x - y = 11 \end{cases}$

   Yes. Adding the second equation eliminates the variable y. $x = 5$ and $y = -6$.

   b. $\begin{cases} 8x + 11y = 37 \\ 8x + y = 7 \end{cases}$

   No. Adding the equations does not eliminate either variable. If the second equation is subtracted from the first, the strategy could be used. Subtracting the second equation eliminates $8x$. $y = 3$ and $x = \frac{1}{2}$.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 19 Solving Systems by Elimination: Adding and Subtracting (Part 1) 141

1 Launch

Arrange students in pairs, giving them time to analyze and discuss Problem 1. Then discuss as a class. Ask, "What do you notice about the structure of the equations? What happens when you add them?" Have students complete the remaining problem.

2 Monitor

Help students get started by asking "What should happen when adding two equations in a system?"

Look for points of confusion:

- **Having difficulty recognizing the eliminated terms.** Ask, "What is true of the terms that add up to 0?"

Look for productive strategies:

- Recognizing the equations in Problem 2b must be subtracted in order to eliminate a variable.

3 Connect

Display the systems in Problem 2.

Have pairs of students share their strategies for solving each system. Record and display their work.

Ask, "Why does adding the equations work for Problem 2a and not 2b?"

Highlight that the two equations in a system can be added or subtracted to form a third equivalent equation, eliminating a variable to obtain the other. This strategy is known as elimination, and is often used with equations in standard form.

Define the term ***elimination***.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

For Problem 2, provide a T-chart with the sum or difference of equations in one column and space in the other column for students to describe what is happening in each step. Consider including some pre-filled descriptions for some of the steps.

Accessibility: *Activate Prior Knowledge*

Connect understanding of balance using simpler, numerical statements. Ask, "If $2 + 2 = 4$ and $3 + 1 = 4$ are balanced, true equations, what happens when you add the equations to get $5 + 3 = 8$?" $5 + 3 = 8$ is a balanced, true equation.

Math Language Development

MLR8: Discussion Supports

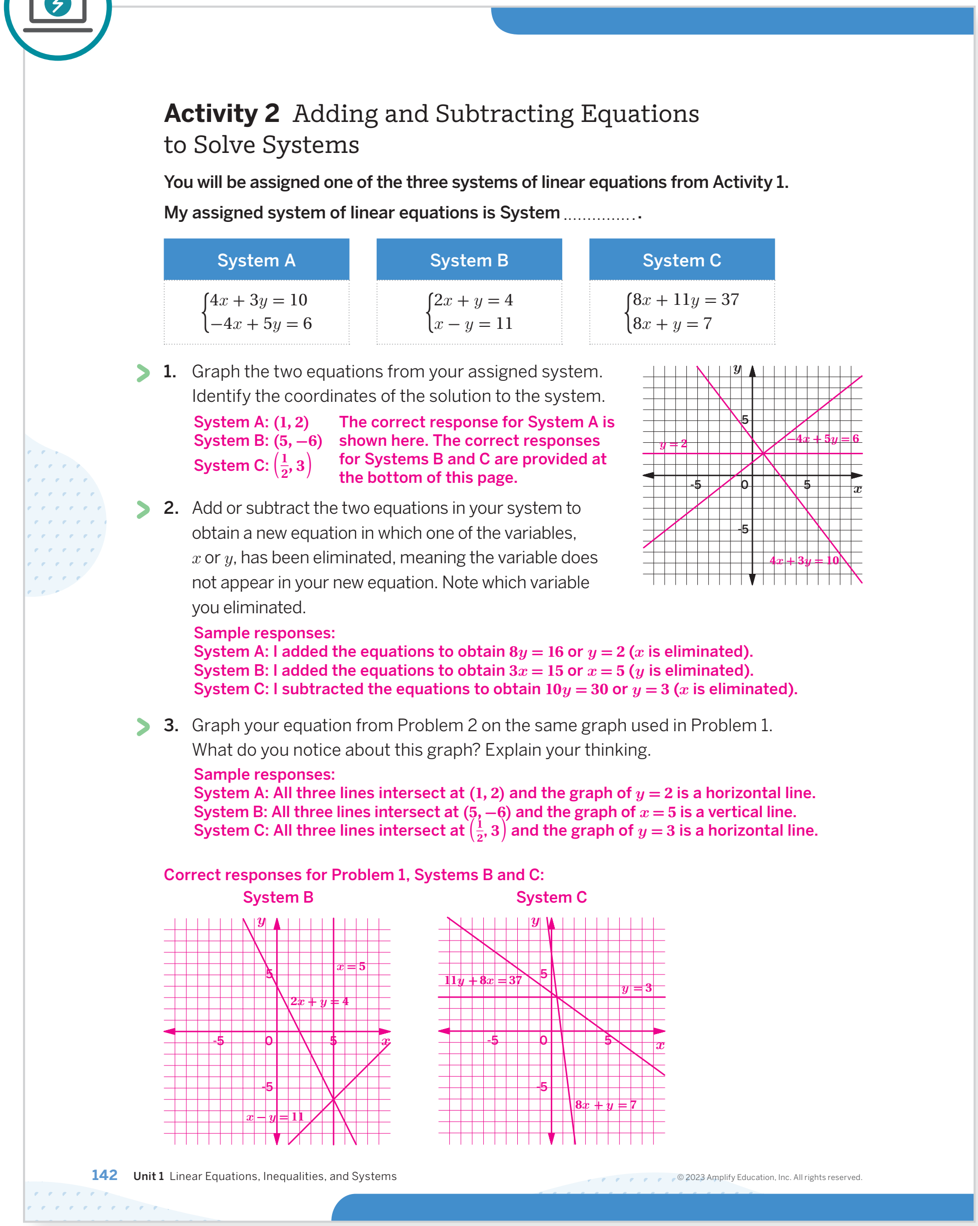
During the Connect, as students respond to the Ask question, ask these follow-up questions to clarify student reasoning:

- "When you add the equations in Problem 2a, do you get a resulting true, balanced equation? What about in Problem 2b?" Yes, in both Problems 2a and 2b, the resulting equations are true and balanced.
- "If the equation in Problem 2b is true and balanced, why does this strategy not work?" While it is a true equation, it does not help me isolate one of the variables.

Small Groups | 20 min

Activity 2 Adding and Subtracting Equations to Solve Systems

Students use graphs of systems of linear equations to understand the connection of the third equation determined by elimination.



Activity 2 Adding and Subtracting Equations to Solve Systems

You will be assigned one of the three systems of linear equations from Activity 1.

My assigned system of linear equations is System ....

System A	System B	System C
$\begin{cases}4x+3y=10\\-4x+5y=6\end{cases}$	$\begin{cases}2x+y=4\\x-y=11\end{cases}$	$\begin{cases}8x+11y=37\\8x+y=7\end{cases}$

1. Graph the two equations from your assigned system. Identify the coordinates of the solution to the system.

   System A: $(1, 2)$
   System B: $(5, -6)$
   System C: $\left(\frac{1}{2}, 3\right)$

   The correct response for System A is shown here. The correct responses for Systems B and C are provided at the bottom of this page.

2. Add or subtract the two equations in your system to obtain a new equation in which one of the variables, x or y, has been eliminated, meaning the variable does not appear in your new equation. Note which variable you eliminated.

   Sample responses:
   System A: I added the equations to obtain $8y=16$ or $y=2$ (x is eliminated).
   System B: I added the equations to obtain $3x=15$ or $x=5$ (y is eliminated).
   System C: I subtracted the equations to obtain $10y=30$ or $y=3$ (x is eliminated).

3. Graph your equation from Problem 2 on the same graph used in Problem 1. What do you notice about this graph? Explain your thinking.

   Sample responses:
   System A: All three lines intersect at $(1, 2)$ and the graph of $y=2$ is a horizontal line.
   System B: All three lines intersect at $(5, -6)$ and the graph of $x=5$ is a vertical line.
   System C: All three lines intersect at $\left(\frac{1}{2}, 3\right)$ and the graph of $y=3$ is a horizontal line.

Correct responses for Problem 1, Systems B and C:

System B

System C

142 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

① Launch

Use the ***Jigsaw*** routine. Arrange students into groups of three, assigning each group a system. Provide access to graphing technology. Give groups enough time to complete the problems for their system before rearranging the groups. Remind students to focus on strategies rather than answers during group discussions.

② Monitor

Help students get started by reviewing which form of an equation is more appropriate when using graphing technology.

Look for points of confusion:

- **Having difficulty rearranging equations from standard form to slope-intercept form to use graphing technology.** Prompt students to graph the equations in standard form by determining their intercepts.

Look for productive strategies:

- Adding or subtracting equations in the system.
- Graphing a horizontal or vertical third line.
- Determining the intersection point of all three lines.

③ Connect

Have groups of students share their graphs from Problem 3 and any observations they noticed during the ***Jigsaw*** routine. Display the graphs for each system.

Highlight that the new equation intersects the original equations at the same point.

Ask students to subtract the equations they previously added in their assigned system (or vice versa). Graph the resulting equation on the same coordinate plane. Then ask, "What do you notice about the graphs of these new equations?" The new line intersects the others at the same point.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Instead of having students draw the graphs of each line, provide pre-created graphs of each system in Problem 1 for students to analyze. Have them begin the activity with Problem 2 and ask them to graph their equation from Problem 3 on the pre-created graph they were given.

Extension: *Math Enrichment*

Ask students to create two systems of linear equations, one in which the equations could be added together to eliminate one variable and the other one in which the equations could be subtracted to eliminate one variable.

Math Language Development

MLR7: Compare and Connect

During the Connect, press for details as students compare the graphs of the three equations (the two from the original system and the third new equation from Problem 3). Ask these follow-up questions:

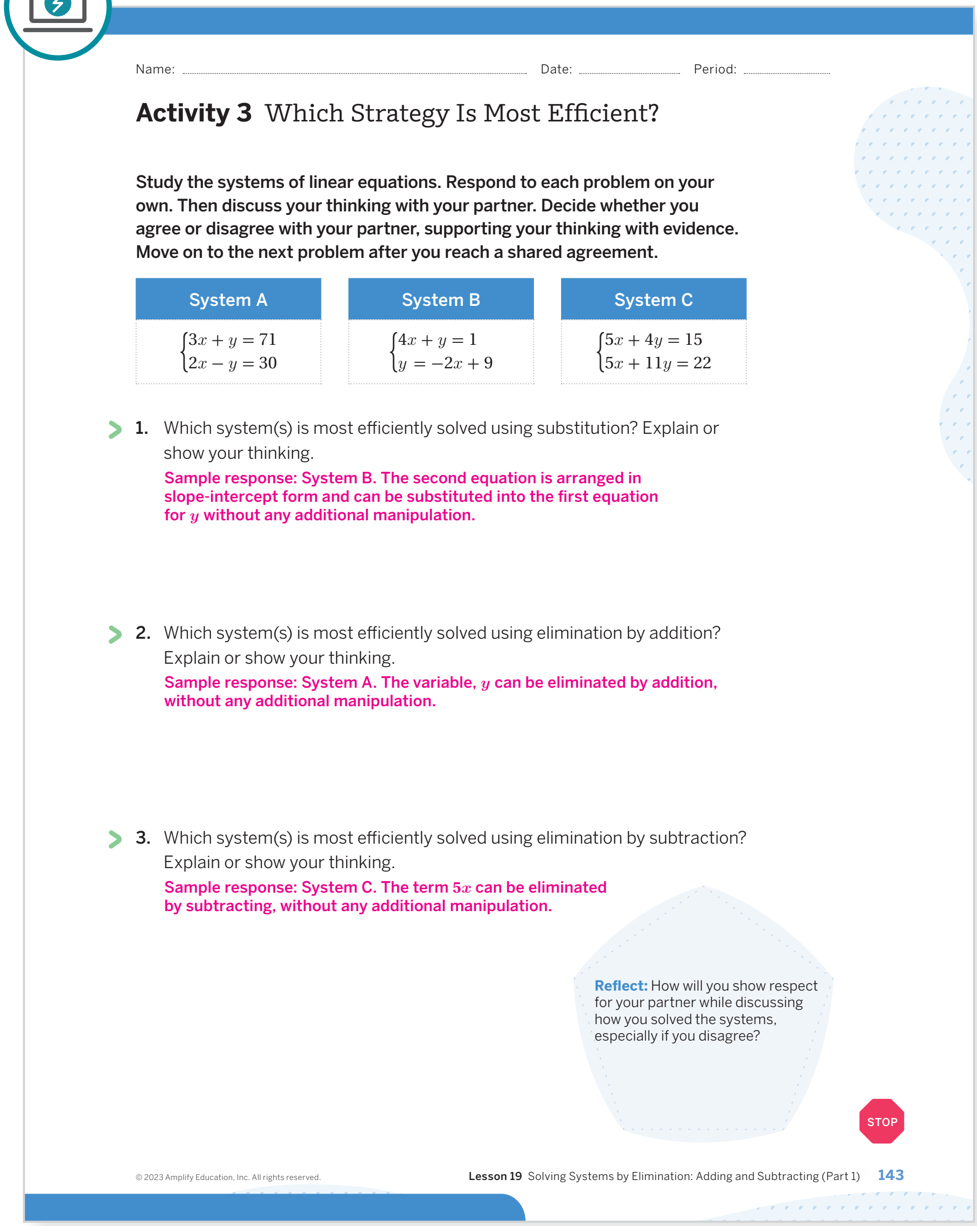
- "Why are the lines that represent the new equation either horizontal or vertical?" When adding or subtracting the equations, one variable was eliminated. The new equation is either in the form $x =$ _____ or $y =$ _____.
- "Why does the new equation intersect in the same point as the original system?" The new equation shares the same x- or y-value as the solution to the system.

Optional

Independent | 10 min

Activity 3 Which Strategy Is Most Efficient?

Students examine three different systems of linear equations to determine which strategy is most efficient for solving.



Name: ______________________ Date: __________ Period: __________

Activity 3 Which Strategy Is Most Efficient?

Study the systems of linear equations. Respond to each problem on your own. Then discuss your thinking with your partner. Decide whether you agree or disagree with your partner, supporting your thinking with evidence. Move on to the next problem after you reach a shared agreement.

System A	System B	System C
$\begin{cases} 3x + y = 71 \\ 2x - y = 30 \end{cases}$	$\begin{cases} 4x + y = 1 \\ y = -2x + 9 \end{cases}$	$\begin{cases} 5x + 4y = 15 \\ 5x + 11y = 22 \end{cases}$

1. Which system(s) is most efficiently solved using substitution? Explain or show your thinking.
Sample response: System B. The second equation is arranged in slope-intercept form and can be substituted into the first equation for y without any additional manipulation.

2. Which system(s) is most efficiently solved using elimination by addition? Explain or show your thinking.
Sample response: System A. The variable, y can be eliminated by addition, without any additional manipulation.

3. Which system(s) is most efficiently solved using elimination by subtraction? Explain or show your thinking.
Sample response: System C. The term $5x$ can be eliminated by subtracting, without any additional manipulation.

Reflect: How will you show respect for your partner while discussing how you solved the systems, especially if you disagree?

STOP

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Give students one minute to study the systems. Using the ***Poll the Class*** routine, ask students which strategy they think is most efficient. Record their responses. Then have students work independently before sharing their thinking with their partner.

2 Monitor

Help students get started by reminding them what "most efficient" means.

Look for points of confusion:

- **Having difficulty identifying terms that can be eliminated.** Have students circle terms with coefficients that are the same or opposites.

Look for productive strategies:

- Using the structure of the equations in each system to determine which strategy is most efficient.
- Looking for terms with coefficients that are opposites.
- Noticing if equations are in standard or slope-intercept form.
- Recognizing all systems could be solved using either strategy.

3 Connect

Display the systems of equations and the results from the ***Poll the Class*** routine. Ask students if they still agree with their responses.

Have students share their thinking for determining which strategies would be most efficient for solving the systems. Select and sequence students utilizing the structure of the equations.

Highlight that by rearranging equations, the systems can be solved using either substitution or elimination. However, choosing the "most efficient" strategy involves taking advantage of an equation's structure and performing as few steps as possible.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students determine which system(s) are most efficiently solved using each method, provide a sample statement and ask students to critique it. For example: System A can be most efficiently solved using substitution because I can solve the first equation for y. While it is true that substitution can be used, elimination by addition will eliminate the variable y, which can be a more efficient method.

Extension: *Math Enrichment*

Ask students to create three systems of linear equations, one in which each method — substitution, elimination (addition), elimination (subtraction) — is the most efficient, and explain their thinking.

Math Language Development

MLR7: Compare and Connect

During the Connect, call attention to how the structure of the equations in each system help indicate which strategies will be more efficient than others. Ask these follow-up questions:

- "What is it about the structure of System A that indicates which strategy might be the most efficient? System B? System C?"
- "What do you look for to determine if substitution would be an efficient strategy? Elimination using addition or subtraction?"

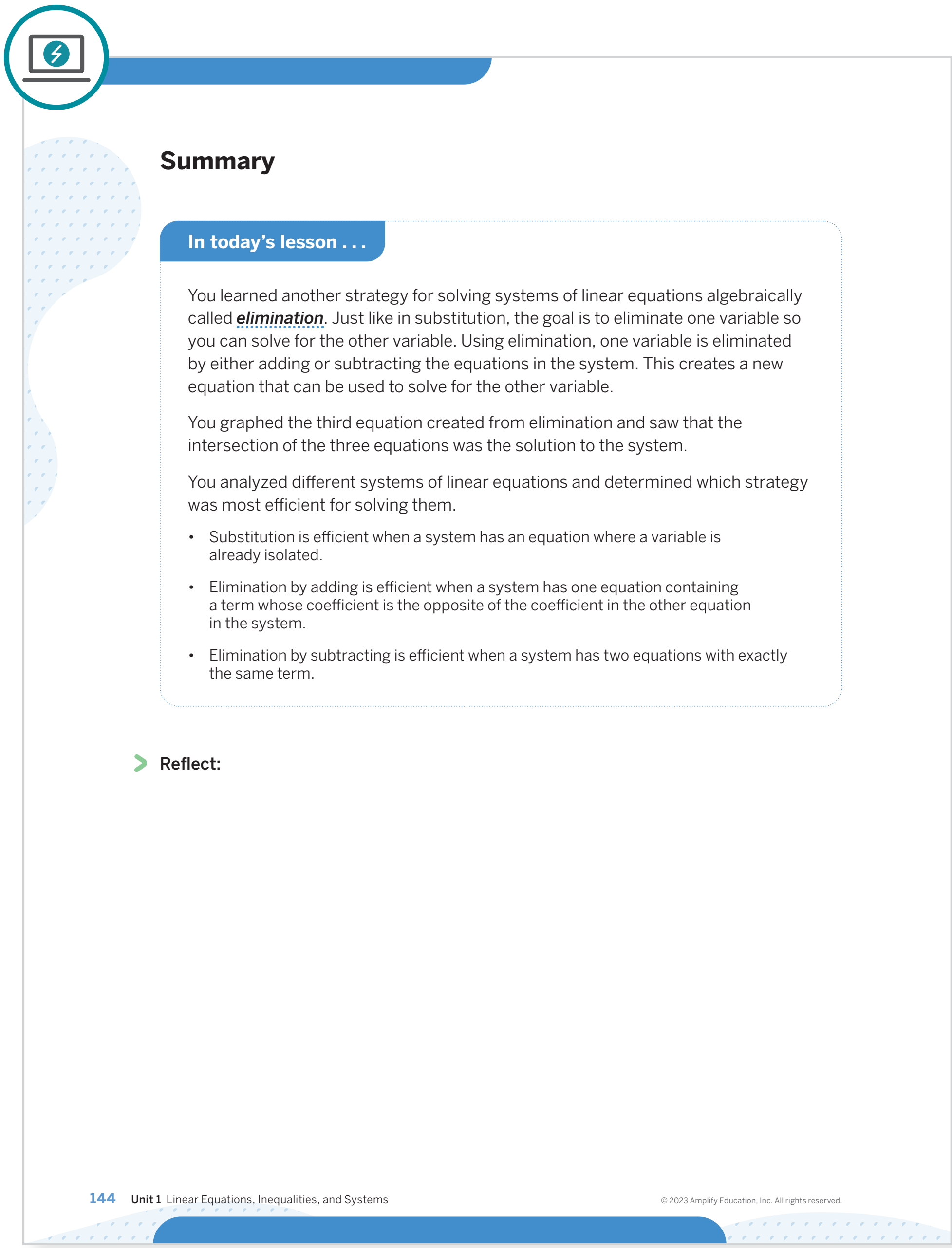
English Learners

Use gestures, such as pointing, and annotations to draw attention to the structure of the equations.

Whole Class | 5 min

Summary

Review and synthesize that adding or subtracting equations in a system creates a new equation that shares the same solution.



Summary

In today's lesson . . .

You learned another strategy for solving systems of linear equations algebraically called ***elimination***. Just like in substitution, the goal is to eliminate one variable so you can solve for the other variable. Using elimination, one variable is eliminated by either adding or subtracting the equations in the system. This creates a new equation that can be used to solve for the other variable.

You graphed the third equation created from elimination and saw that the intersection of the three equations was the solution to the system.

You analyzed different systems of linear equations and determined which strategy was most efficient for solving them.

- Substitution is efficient when a system has an equation where a variable is already isolated.
- Elimination by adding is efficient when a system has one equation containing a term whose coefficient is the opposite of the coefficient in the other equation in the system.
- Elimination by subtracting is efficient when a system has two equations with exactly the same term.

Reflect:

144 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display student language gathered throughout the lesson alongside the systems from Activity 3.

Have students share how they determined the third equation for each system and how that equation is related to the system.

Highlight that adding or subtracting two equations in a system results in a new equation that has the same solution as the system. Explain that addition is used when a system has two equations with opposite terms, and subtraction is used when a system has two equations with exactly the same term.

Formalize vocabulary: *elimination*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What is the goal of adding or subtracting equations in a system? How is this similar to substitution?"

Math Language Development

MLR2: Collect and Display

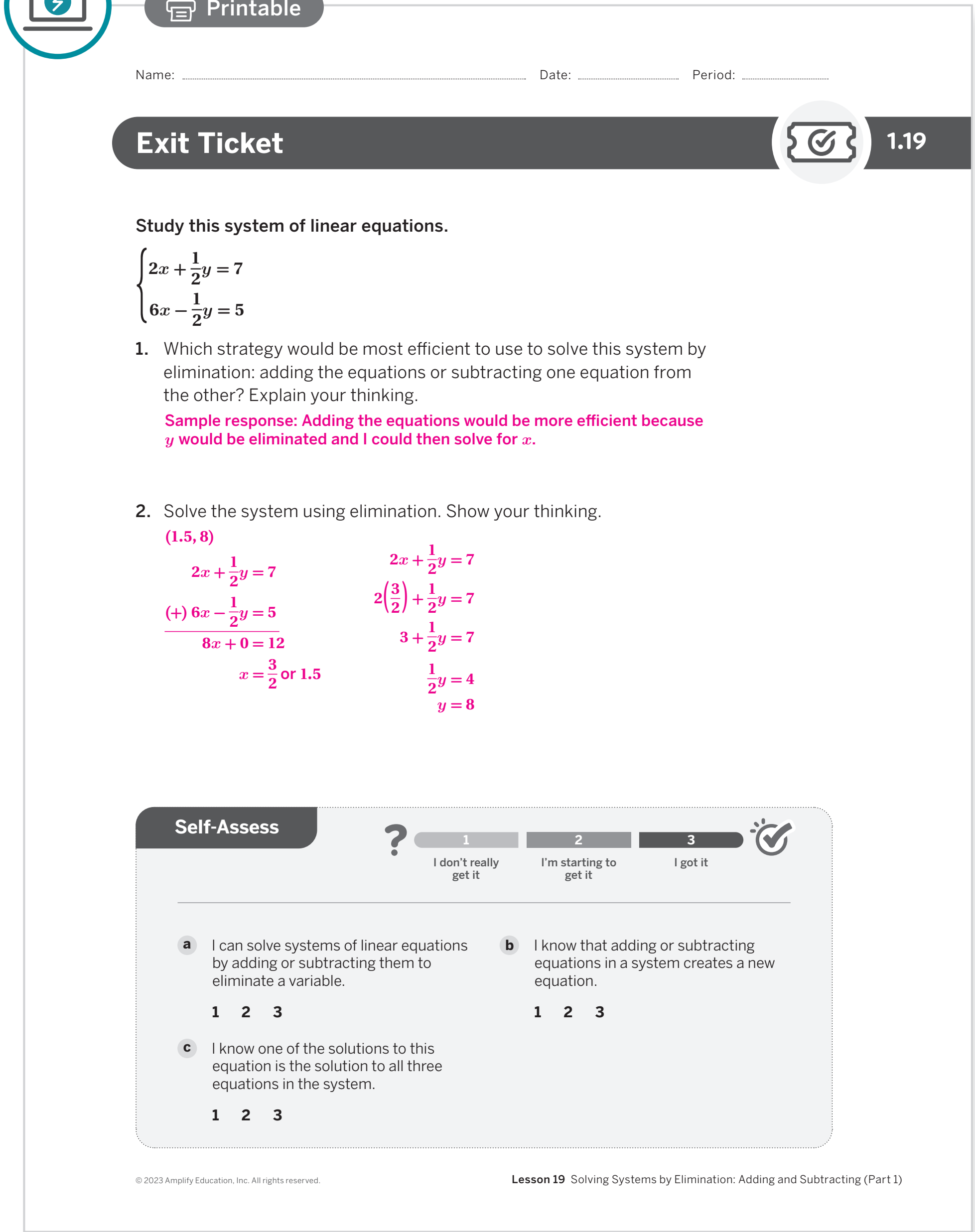
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *elimination* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using the structure of linear equations in a system to determine the most efficient strategy for solving and applying the elimination method.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.19

Study this system of linear equations.

$$\begin{cases} 2x + \frac{1}{2}y = 7 \\ 6x - \frac{1}{2}y = 5 \end{cases}$$

1. Which strategy would be most efficient to use to solve this system by elimination: adding the equations or subtracting one equation from the other? Explain your thinking.
   Sample response: Adding the equations would be more efficient because y would be eliminated and I could then solve for x.

2. Solve the system using elimination. Show your thinking.
   $(1.5, 8)$

$$\begin{array}{r} 2x + \frac{1}{2}y = 7 \\ (+)\ 6x - \frac{1}{2}y = 5 \\ \hline 8x + 0 = 12 \\ x = \frac{3}{2} \text{ or } 1.5 \end{array}$$

$$\begin{aligned} 2x + \frac{1}{2}y &= 7 \\ 2\left(\frac{3}{2}\right) + \frac{1}{2}y &= 7 \\ 3 + \frac{1}{2}y &= 7 \\ \frac{1}{2}y &= 4 \\ y &= 8 \end{aligned}$$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can solve systems of linear equations by adding or subtracting them to eliminate a variable.
1 2 3

b I know that adding or subtracting equations in a system creates a new equation.
1 2 3

c I know one of the solutions to this equation is the solution to all three equations in the system.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Success looks like . . .

- **Goal:** Recognizing that adding or subtracting equations in a system creates a new equation with a solution that coincides with that of the original system, so the new equation can be used to solve the original system.
- **Goal:** Solving systems of equations by adding or subtracting the equations strategically to eliminate a variable.
  - » Adding the two equations so that the y-variable is eliminated.
- **Language Goal:** Using graphing technology to graph the sums and differences of the equations in a system, and analyzing and describing the behaviors of the graphs. **(Speaking and Listening, Writing)**

Suggested next steps

If students struggle to answer Problem 1, consider:

- Reviewing substitution and elimination strategies from Activities 2 and 3.
- Assigning Practice Problem 3.
- Asking, "Could this system be solved by adding or subtracting without having to first rearrange equations?"

If students use substitution to solve Problem 2, consider:

- Assigning Practice Problem 2.

Professional Learning

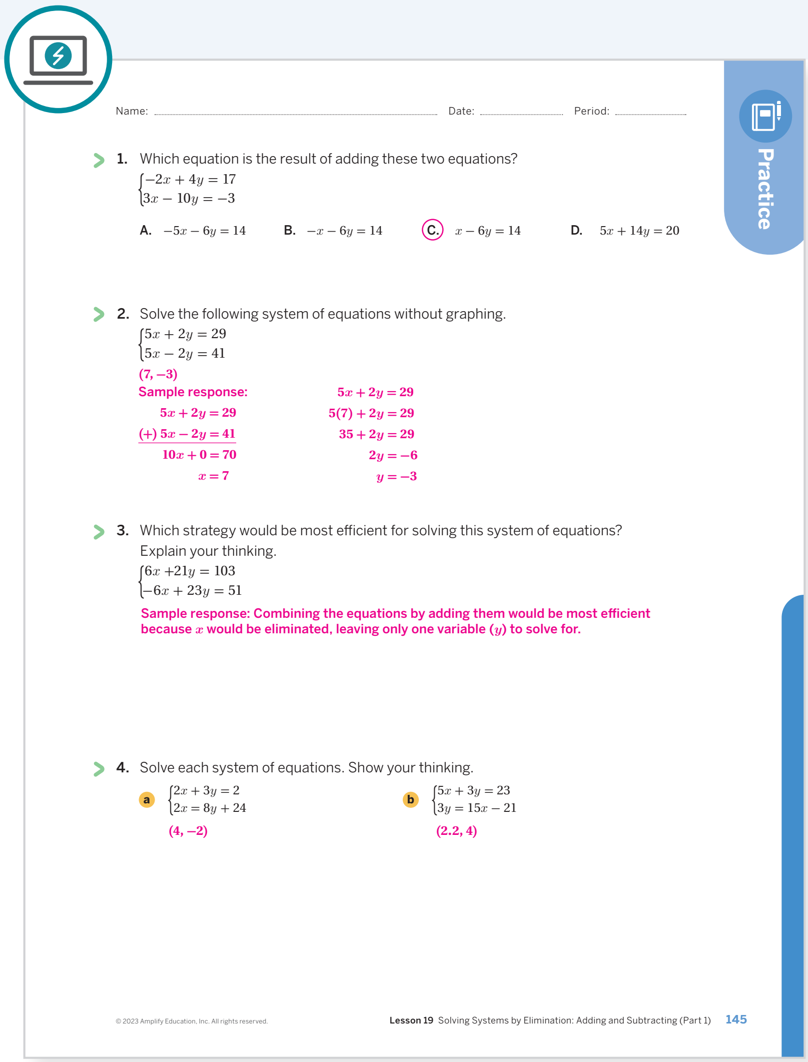
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students solved systems of equations by summing or finding the difference. How will that support writing equivalent systems by writing multiples of equations?
- What did partner discussions during the activities reveal about your students as learners? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Which equation is the result of adding these two equations?

$$\begin{cases}-2x + 4y = 17\\3x - 10y = -3\end{cases}$$

A. $-5x - 6y = 14$ B. $-x - 6y = 14$ [✓ C.] $x - 6y = 14$ D. $5x + 14y = 20$

2. Solve the following system of equations without graphing.

$$\begin{cases}5x + 2y = 29\\5x - 2y = 41\end{cases}$$

(7, –3)

Sample response:

$$\begin{array}{rr} 5x + 2y = 29 \\ \underline{(+)\ 5x - 2y = 41} \\ 10x + 0 = 70 \\ x = 7 \end{array} \qquad \begin{array}{r} 5x + 2y = 29 \\ 5(7) + 2y = 29 \\ 35 + 2y = 29 \\ 2y = -6 \\ y = -3 \end{array}$$

3. Which strategy would be most efficient for solving this system of equations? Explain your thinking.

$$\begin{cases}6x + 21y = 103\\-6x + 23y = 51\end{cases}$$

Sample response: Combining the equations by adding them would be most efficient because x would be eliminated, leaving only one variable (y) to solve for.

4. Solve each system of equations. Show your thinking.

a $\begin{cases}2x + 3y = 2\\2x = 8y + 24\end{cases}$

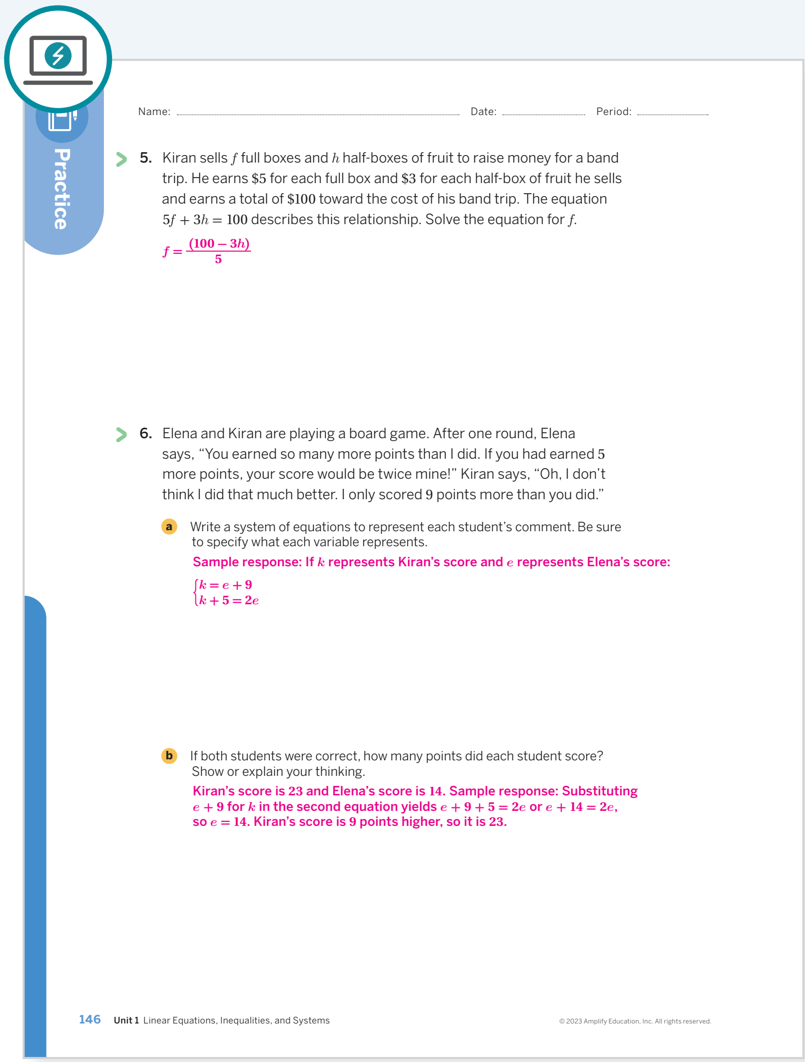
(4, –2)

b $\begin{cases}5x + 3y = 23\\3y = 15x - 21\end{cases}$

(2.2, 4)

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 19 Solving Systems by Elimination: Adding and Subtracting (Part 1) 145



Name: Date: Period:

5. Kiran sells f full boxes and h half-boxes of fruit to raise money for a band trip. He earns \$5 for each full box and \$3 for each half-box of fruit he sells and earns a total of \$100 toward the cost of his band trip. The equation $5f + 3h = 100$ describes this relationship. Solve the equation for f.

$$f = \frac{(100 - 3h)}{5}$$

6. Elena and Kiran are playing a board game. After one round, Elena says, “You earned so many more points than I did. If you had earned 5 more points, your score would be twice mine!” Kiran says, “Oh, I don’t think I did that much better. I only scored 9 points more than you did.”

a Write a system of equations to represent each student’s comment. Be sure to specify what each variable represents.

Sample response: If k represents Kiran’s score and e represents Elena’s score:

$$\begin{cases}k = e + 9\\k + 5 = 2e\end{cases}$$

b If both students were correct, how many points did each student score? Show or explain your thinking.

Kiran’s score is 23 and Elena’s score is 14. Sample response: Substituting $e + 9$ for k in the second equation yields $e + 9 + 5 = 2e$ or $e + 14 = 2e$, so $e = 14$. Kiran’s score is 9 points higher, so it is 23.

146 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	3
	3	Activity 3	2
Spiral	4	Unit 1 Lesson 18	2
	5	Unit 1 Lesson 10	2
Formative ⚡	6	Unit 1 Lesson 20	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 20**

Solving Systems by Elimination: Adding and Subtracting (Part 2)

Let's think about why adding and subtracting equations works for solving systems of linear equations.

Focus

Goals

1. **Language Goal:** Explain why adding or subtracting two equations that share a solution results in a new equation that shares the same solution. **(Speaking and Listening, Writing)**
2. Solve systems of linear equations by adding or subtracting equations to eliminate a variable.
3. **Language Goal:** Use a context to make sense of an equation that is the sum of two equations in a system, and reason about why this equation shares a solution with the system. **(Speaking and Listening, Writing)**

Coherence

Today

Students build on their understanding of solving systems of linear equations by elimination. Given a grocery shopping context, students interpret the solutions for each individual equation. Then, using the context, they make sense of the sum of two equations and understand why the third equation shares a solution with the system. This second lesson on elimination allows students to formulate a logical argument explaining why the process works.

Previously

Students were introduced to solving systems of linear equations by elimination using addition or subtraction in Lesson 19.

Coming Soon

Students will solve systems of linear equations by elimination using multiplication or division in Lesson 21.

Rigor

- Students continue to build their **conceptual understanding** of solving systems of linear equations by elimination using addition or subtraction.
- Students develop **procedural skills** by adding or subtracting equations in a system and using the resultant equation to solve the system.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Anchor Chart PDF, *Solving Systems of Linear Equations by Elimination*
- Graphic Organizer PDF, *Balance Scale* (as needed)
- graphing technology

Math Language Development

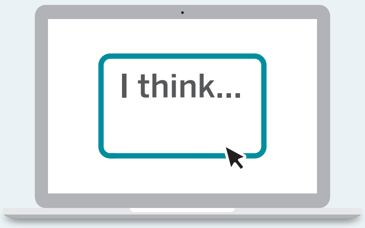
Review words

- *elimination*
- *equivalent equations*
- *solution to a system of equations*
- *substitution*
- *system of equations*

Amps | **Featured Activity**

Activity 1
See Student Thinking

Students are asked to explain their solution to a system of linear equations in context, and these explanations are available to you digitally, in real time.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may lack motivation to focus while working independently when they know they will get to discuss their thinking with their partner. Explain that everyone's thinking is valuable because we can learn from each other. Discuss the responsibility each person has to do their best at interpreting the models. Remind them that their partner might need their help, so they should be prepared to provide it.

Modifications to Pacing

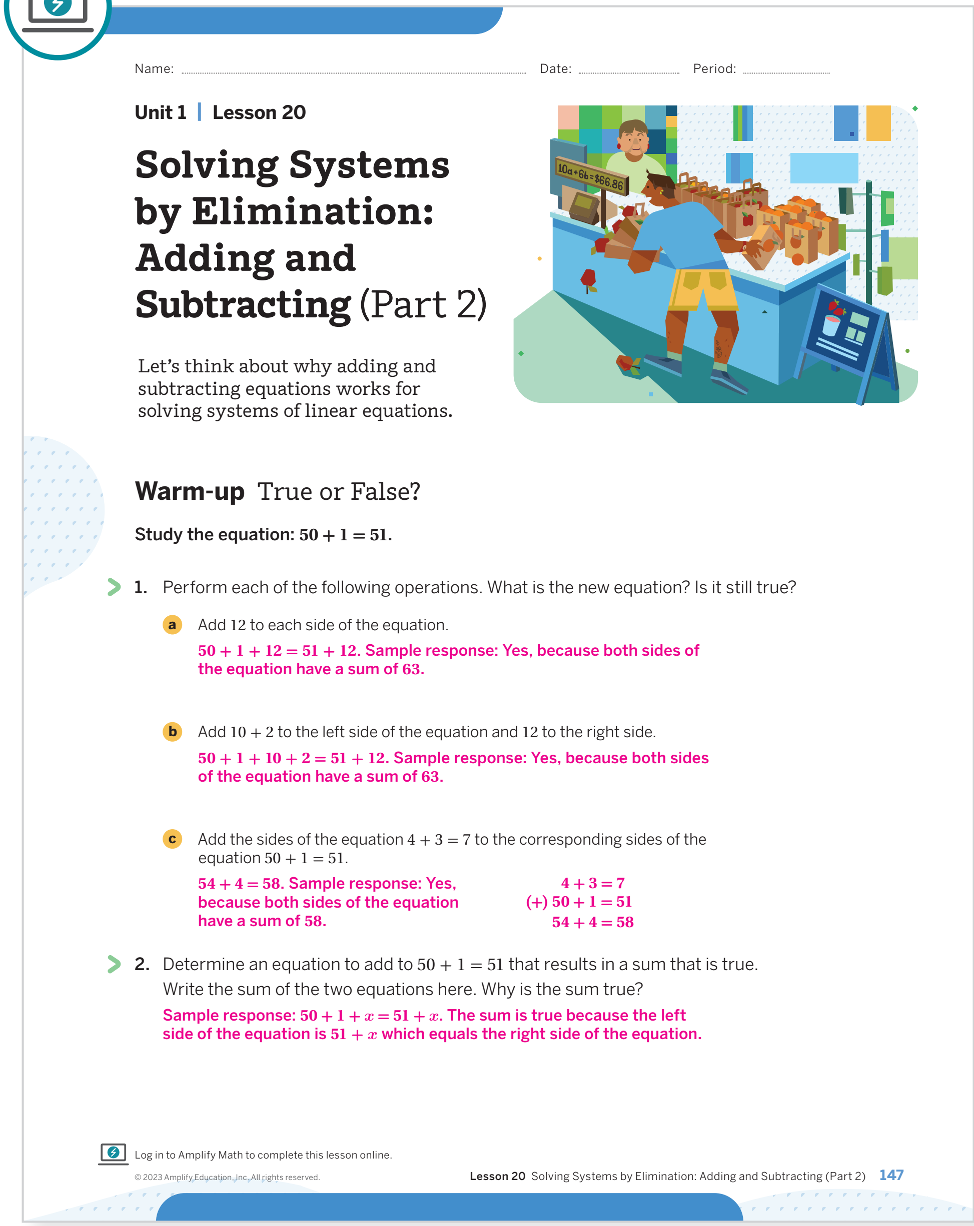
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, Problem 3 may be omitted.

Independent | 10 min

Warm-up True or False?

Students make sense of the sums of numeric equations to determine why values simultaneously satisfy two equations in a system and their sum.



Name: .... Date: .... Period: ....

Unit 1 | Lesson 20

Solving Systems by Elimination: Adding and Subtracting (Part 2)

Let's think about why adding and subtracting equations works for solving systems of linear equations.

Warm-up True or False?

Study the equation: $50 + 1 = 51$.

1. Perform each of the following operations. What is the new equation? Is it still true?

   a. Add 12 to each side of the equation.

   $50 + 1 + 12 = 51 + 12$. Sample response: Yes, because both sides of the equation have a sum of 63.

   b. Add $10 + 2$ to the left side of the equation and 12 to the right side.

   $50 + 1 + 10 + 2 = 51 + 12$. Sample response: Yes, because both sides of the equation have a sum of 63.

   c. Add the sides of the equation $4 + 3 = 7$ to the corresponding sides of the equation $50 + 1 = 51$.

   $54 + 4 = 58$. Sample response: Yes, because both sides of the equation have a sum of 58.

$$\begin{array}{r} 4 + 3 = 7 \\ (+)\ 50 + 1 = 51 \\ \hline 54 + 4 = 58 \end{array}$$

2. Determine an equation to add to $50 + 1 = 51$ that results in a sum that is true. Write the sum of the two equations here. Why is the sum true?

   Sample response: $50 + 1 + x = 51 + x$. The sum is true because the left side of the equation is $51 + x$ which equals the right side of the equation.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Solving Systems by Elimination: Adding and Subtracting (Part 2) 147

1 Launch

Give students a minute to study the equation. Set an expectation for the amount of time students will have to work individually on the Warm-up.

2 Monitor

Help students get started by asking them to explain how adding a quantity to both sides of the equal sign affects the equation.

Look for points of confusion:

- **Thinking that because the values added to both sides of the equal sign look different, the resulting equation is no longer true.** Have students verify that the amounts added to each side are equal.

Look for productive strategies:

- Writing an equation using a variable for Problem 2.

3 Connect

Display examples of student work.

Have individual students share strategies for determining if the resulting equations are still true.

Highlight that when the same amount is added to both sides of the equal sign, the resulting equation is also true. If no student used a variable for Problem 2, provide an example.

Ask, "Why are the resulting equations true even after you add numbers or expressions that look different to each side?" The numbers being added to both sides are always equal amounts, even though they are written in a different form.

Math Language Development

MLR7: Compare and Connect

During the whole-class discussion, draw attention to the language students use to determine if the resulting equations are still true. For example, "the same value was added to each side" or "each side of the resulting equation has the same value."

English Learners

Use color coding or annotations to point out that $10 + 2$ and 12 are equivalent values, and $4 + 3$ and 7 are equivalent values.

Power-up

To power up students' ability to understand what the solution of a system of equations represents in context:

Provide students with a copy of the Power-up PDF.

Use: Before the Warm-up

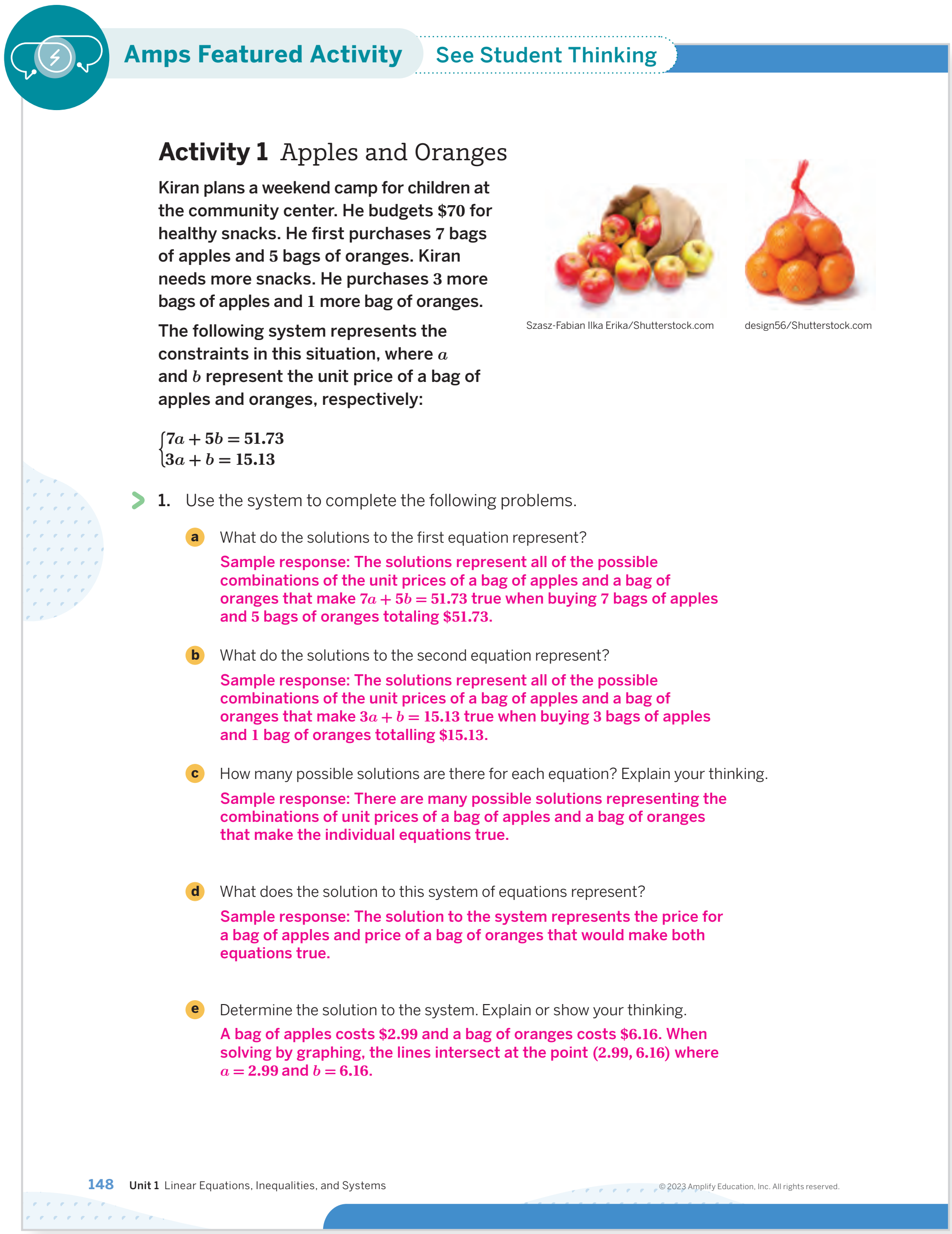
Informed by: Performance on Lesson 19, Practice Problem 6

Pairs | 15 min

Activity 1 Apples and Oranges

Students explore the meaning of the linear equations formed by adding two equations in a system to make sense of their shared solution.

Amps Featured Activity See Student Thinking



Activity 1 Apples and Oranges

Kiran plans a weekend camp for children at the community center. He budgets $70 for healthy snacks. He first purchases 7 bags of apples and 5 bags of oranges. Kiran needs more snacks. He purchases 3 more bags of apples and 1 more bag of oranges.

The following system represents the constraints in this situation, where a and b represent the unit price of a bag of apples and oranges, respectively:

$$\begin{cases} 7a + 5b = 51.73 \\ 3a + b = 15.13 \end{cases}$$

Szasz-Fabian Ilka Erika/Shutterstock.com

design56/Shutterstock.com

1. Use the system to complete the following problems.

   a. What do the solutions to the first equation represent?

   Sample response: The solutions represent all of the possible combinations of the unit prices of a bag of apples and a bag of oranges that make $7a + 5b = 51.73$ true when buying 7 bags of apples and 5 bags of oranges totaling $51.73.

   b. What do the solutions to the second equation represent?

   Sample response: The solutions represent all of the possible combinations of the unit prices of a bag of apples and a bag of oranges that make $3a + b = 15.13$ true when buying 3 bags of apples and 1 bag of oranges totalling $15.13.

   c. How many possible solutions are there for each equation? Explain your thinking.

   Sample response: There are many possible solutions representing the combinations of unit prices of a bag of apples and a bag of oranges that make the individual equations true.

   d. What does the solution to this system of equations represent?

   Sample response: The solution to the system represents the price for a bag of apples and price of a bag of oranges that would make both equations true.

   e. Determine the solution to the system. Explain or show your thinking.

   A bag of apples costs $2.99 and a bag of oranges costs $6.16. When solving by graphing, the lines intersect at the point (2.99, 6.16) where $a = 2.99$ and $b = 6.16$.

148 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the narrative together. Have students discuss Problem 1 with their partner. Then, have students work independently on Problem 2 before sharing their thinking with their partner.

2 Monitor

Help students get started by making sure they understand the first set of problems do not require solving.

Look for points of confusion:

- **Not understanding they are solving for the unit price of a bag of apples and a bag of oranges.** Have them reason about one equation at a time.
- **Attempting to solve Problem 1e using elimination.** Ask, “What other methods do you have for solving a system of linear equations? Which do you prefer and why?”
- **Having difficulty making sense of the text in Problem 2.** Define reimbursement as a sum paid to cover the costs a person spends. Have students calculate the total amount Kiran spent after both trips.

Look for productive strategies:

- Understanding the variables represent the unit price of apples and oranges and the dollar amount represents the total cost.
- Using substitution to solve the system by first isolating b in the second equation.
- Checking the solution to the system in the equation of Problem 2a.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider one of these alternative approaches to this activity:

- If students need more processing time, omit Problems 1a, 1b, and 1c. Prompt students to use substitution to solve the system in Problem 1e and then return to this problem to use elimination as time allows.
- Omit Problem 1 entirely and provide the equation, showing the sum of the two equations in the original system, in Problem 2. Have students proceed with the activity by completing Problems 2b–2d.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

- **Read 1:** Students should understand that Kiran has a budget for purchasing bags of apples and oranges.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as Kiran first purchases 7 bags of apples.
- **Read 3:** Ask students to think about how the given system represents this information.

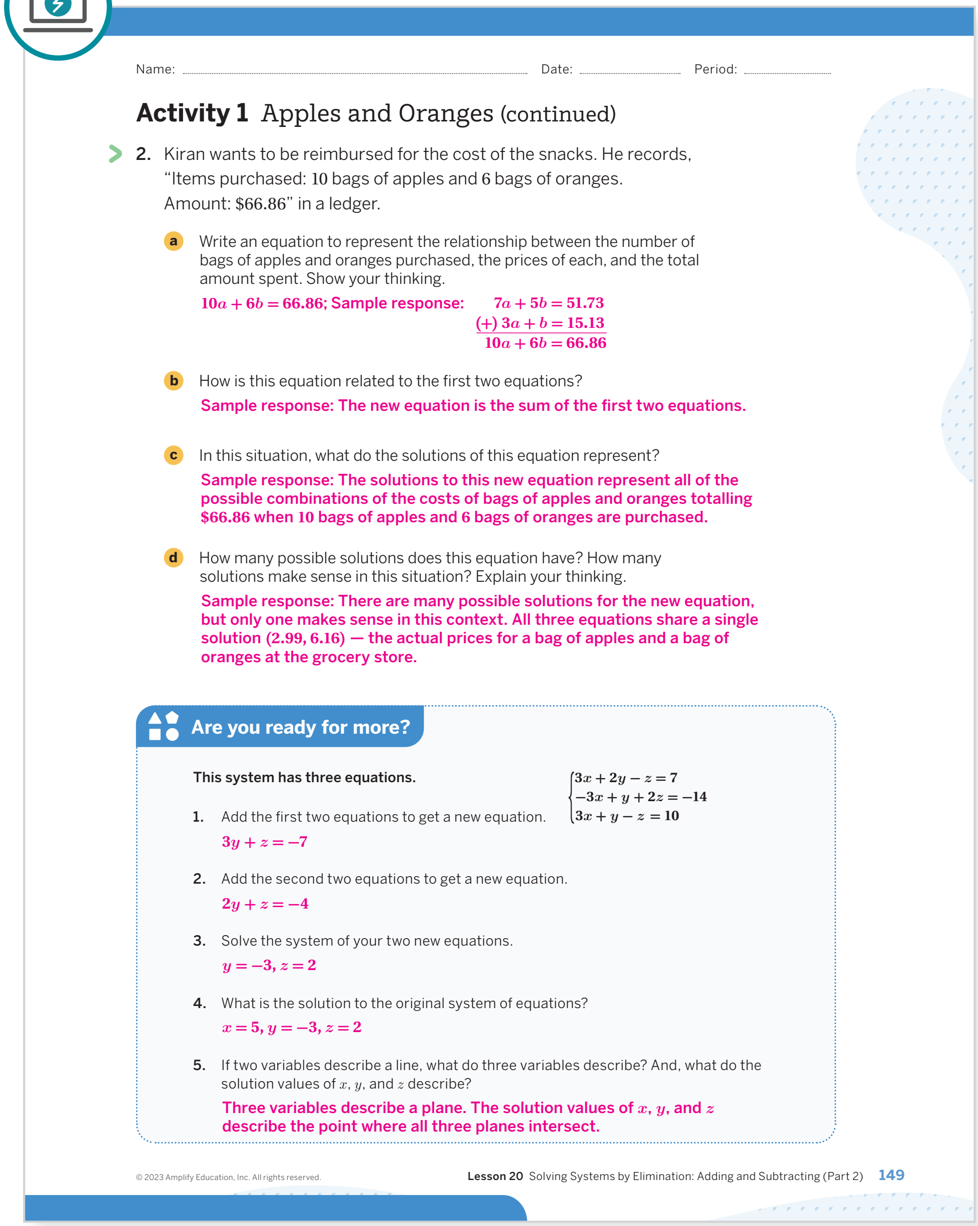
English Learners

Have students highlight key phrases in the text, such as 3 *more bags of apples and* 1 *more bag of oranges*.

Pairs | 15 min

Activity 1 Apples and Oranges (continued)

Students explore the meaning of the linear equations formed by adding two equations in a system to make sense of their shared solution.



Name: ______ Date: ______ Period: ______

Activity 1 Apples and Oranges (continued)

2. Kiran wants to be reimbursed for the cost of the snacks. He records, "Items purchased: 10 bags of apples and 6 bags of oranges. Amount: $66.86" in a ledger.

   a. Write an equation to represent the relationship between the number of bags of apples and oranges purchased, the prices of each, and the total amount spent. Show your thinking.

   $10a + 6b = 66.86$; Sample response:

   $$\begin{array}{r} 7a + 5b = 51.73 \\ \underline{(+)\ 3a + b = 15.13} \\ 10a + 6b = 66.86 \end{array}$$

   b. How is this equation related to the first two equations?

   Sample response: The new equation is the sum of the first two equations.

   c. In this situation, what do the solutions of this equation represent?

   Sample response: The solutions to this new equation represent all of the possible combinations of the costs of bags of apples and oranges totalling $66.86 when 10 bags of apples and 6 bags of oranges are purchased.

   d. How many possible solutions does this equation have? How many solutions make sense in this situation? Explain your thinking.

   Sample response: There are many possible solutions for the new equation, but only one makes sense in this context. All three equations share a single solution (2.99, 6.16) — the actual prices for a bag of apples and a bag of oranges at the grocery store.

Are you ready for more?

This system has three equations.

$$\begin{cases} 3x + 2y - z = 7 \\ -3x + y + 2z = -14 \\ 3x + y - z = 10 \end{cases}$$

1. Add the first two equations to get a new equation.

   $3y + z = -7$

2. Add the second two equations to get a new equation.

   $2y + z = -4$

3. Solve the system of your two new equations.

   $y = -3, z = 2$

4. What is the solution to the original system of equations?

   $x = 5, y = -3, z = 2$

5. If two variables describe a line, what do three variables describe? And, what do the solution values of x, y, and z describe?

   Three variables describe a plane. The solution values of x, y, and z describe the point where all three planes intersect.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Solving Systems by Elimination: Adding and Subtracting (Part 2) 149

3 Connect

Have pairs of students share how they interpreted each equation and reasoned about their solutions. Select and sequence students using a graph for Problem 1e last.

Display the graph of the system of equations if no students used this strategy.

Ask:

- "What do you think the graph of all three equations would look like?" The graphs would intersect at the same point.
- "Why does it make sense for the graphs to intersect at the same point? What does that point represent?" The graphs intersect at the same point since this is the solution to the system, which is a solution for all three equations.

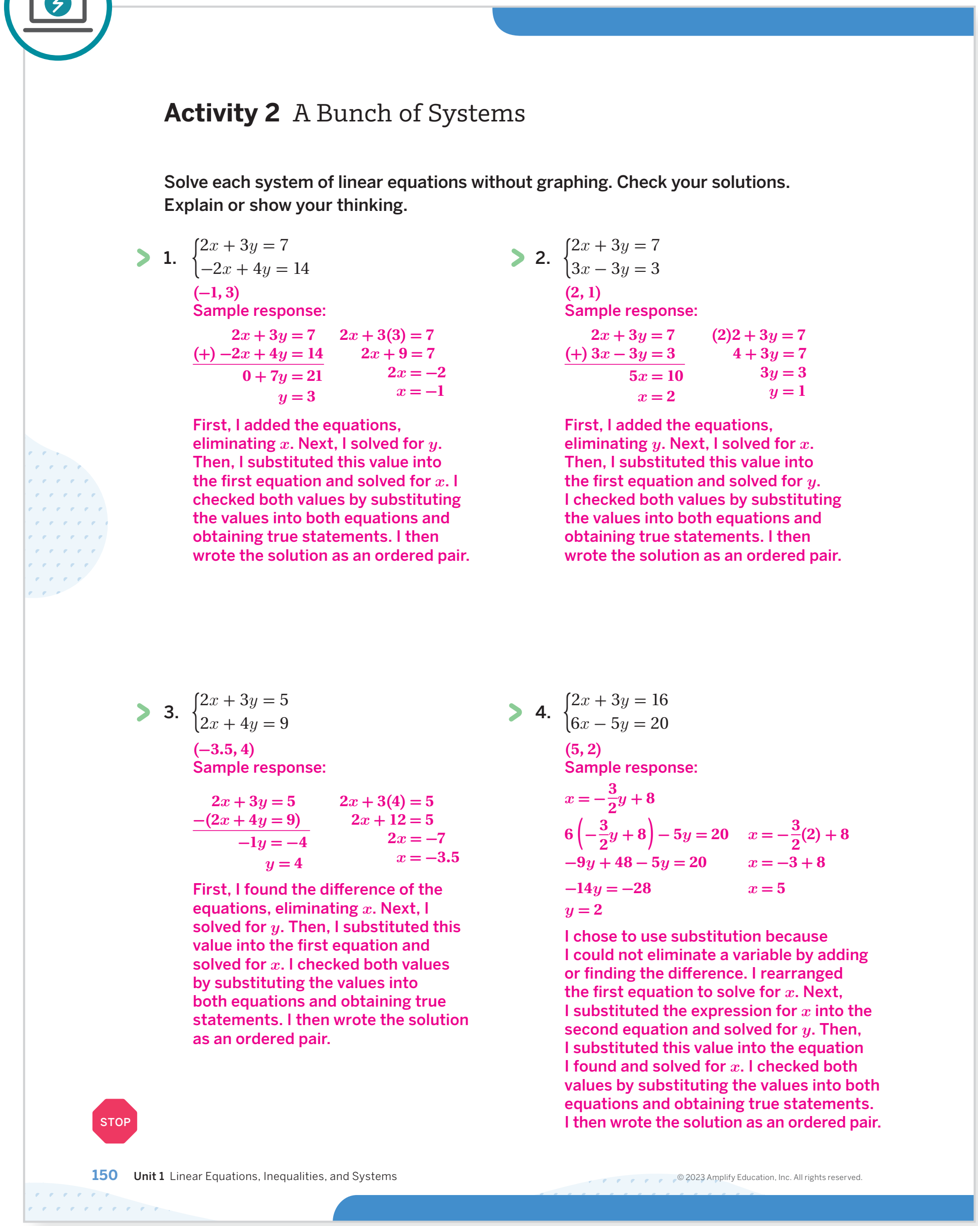
Highlight that the solutions are pairs of values of a and b that make each equation true. In Problem 1, there are many possible combinations that make each equation true, but only one possible solution for the system.

Ask, "What difficulties were there when using elimination to solve the system?" Because the coefficients on each variable are not the same or opposite, adding or subtracting the equations will not eliminate a variable.

Pairs | 15 min

Activity 2 A Bunch of Systems

Students solve systems of linear equations algebraically and encounter a system requiring a new strategy in order to anticipate the next lesson.



Activity 2 A Bunch of Systems

Solve each system of linear equations without graphing. Check your solutions. Explain or show your thinking.

1. $\begin{cases} 2x + 3y = 7 \\ -2x + 4y = 14 \end{cases}$

(−1, 3)

Sample response:

$$\begin{array}{r} 2x + 3y = 7 \\ (+)\ -2x + 4y = 14 \\ \hline 0 + 7y = 21 \\ y = 3 \end{array} \qquad \begin{array}{r} 2x + 3(3) = 7 \\ 2x + 9 = 7 \\ 2x = -2 \\ x = -1 \end{array}$$

First, I added the equations, eliminating x. Next, I solved for y. Then, I substituted this value into the first equation and solved for x. I checked both values by substituting the values into both equations and obtaining true statements. I then wrote the solution as an ordered pair.

2. $\begin{cases} 2x + 3y = 7 \\ 3x - 3y = 3 \end{cases}$

(2, 1)

Sample response:

$$\begin{array}{r} 2x + 3y = 7 \\ (+)\ 3x - 3y = 3 \\ \hline 5x = 10 \\ x = 2 \end{array} \qquad \begin{array}{r} (2)2 + 3y = 7 \\ 4 + 3y = 7 \\ 3y = 3 \\ y = 1 \end{array}$$

First, I added the equations, eliminating y. Next, I solved for x. Then, I substituted this value into the first equation and solved for y. I checked both values by substituting the values into both equations and obtaining true statements. I then wrote the solution as an ordered pair.

3. $\begin{cases} 2x + 3y = 5 \\ 2x + 4y = 9 \end{cases}$

(−3.5, 4)

Sample response:

$$\begin{array}{r} 2x + 3y = 5 \\ -(2x + 4y = 9) \\ \hline -1y = -4 \\ y = 4 \end{array} \qquad \begin{array}{r} 2x + 3(4) = 5 \\ 2x + 12 = 5 \\ 2x = -7 \\ x = -3.5 \end{array}$$

First, I found the difference of the equations, eliminating x. Next, I solved for y. Then, I substituted this value into the first equation and solved for x. I checked both values by substituting the values into both equations and obtaining true statements. I then wrote the solution as an ordered pair.

4. $\begin{cases} 2x + 3y = 16 \\ 6x - 5y = 20 \end{cases}$

(5, 2)

Sample response:

$$\begin{array}{ll} x = -\frac{3}{2}y + 8 & \\ 6\left(-\frac{3}{2}y + 8\right) - 5y = 20 & x = -\frac{3}{2}(2) + 8 \\ -9y + 48 - 5y = 20 & x = -3 + 8 \\ -14y = -28 & x = 5 \\ y = 2 & \end{array}$$

I chose to use substitution because I could not eliminate a variable by adding or finding the difference. I rearranged the first equation to solve for x. Next, I substituted the expression for x into the second equation and solved for y. Then, I substituted this value into the equation I found and solved for x. I checked both values by substituting the values into both equations and obtaining true statements. I then wrote the solution as an ordered pair.

STOP

150 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have student pairs study each system and discuss possible strategies for solving with their partners. Provide access to graphing technology to check solutions.

2 Monitor

Help students get started by having them record their proposed strategy before attempting each system.

Look for points of confusion:

- **Thinking that the equations must be added in order to eliminate a variable.** Remind students that a variable can also be eliminated by subtracting the equations.

Look for productive strategies:

- Using elimination by addition or subtraction for Problems 1–3.
- Using substitution or graphing for Problem 4.
- Using graphing technology to verify their solutions.

3 Connect

Have pairs of students share their strategies for solving Problems 1–3. Display student work for Problem 4, selecting and sequencing graphs first, and then substitution.

Ask, "Why were you able to solve the systems in Problems 1–3 using elimination, but not the system in Problem 4?" In the first three systems, at least one variable in each pair of equations have the same or opposite coefficients, so when the terms were added or subtracted, the result is 0. This allowed a variable to be eliminated.

Highlight that sometimes an extra step may be needed to eliminate the variable in a system.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Display or provide the Anchor Chart PDF, *Solving Systems of Linear Equations by Elimination*. Consider having students choose two of the four problems to complete, including Problem 4.

Extension: *Math Enrichment*

Ask, "If a system of equations has one solution, how many equations must be in the system to determine the solution? Explain your thinking." At least the same number of equations as there are variables. Otherwise, there would be infinitely many solutions.

Math Language Development

MLR3: Critique, Correct, Clarify

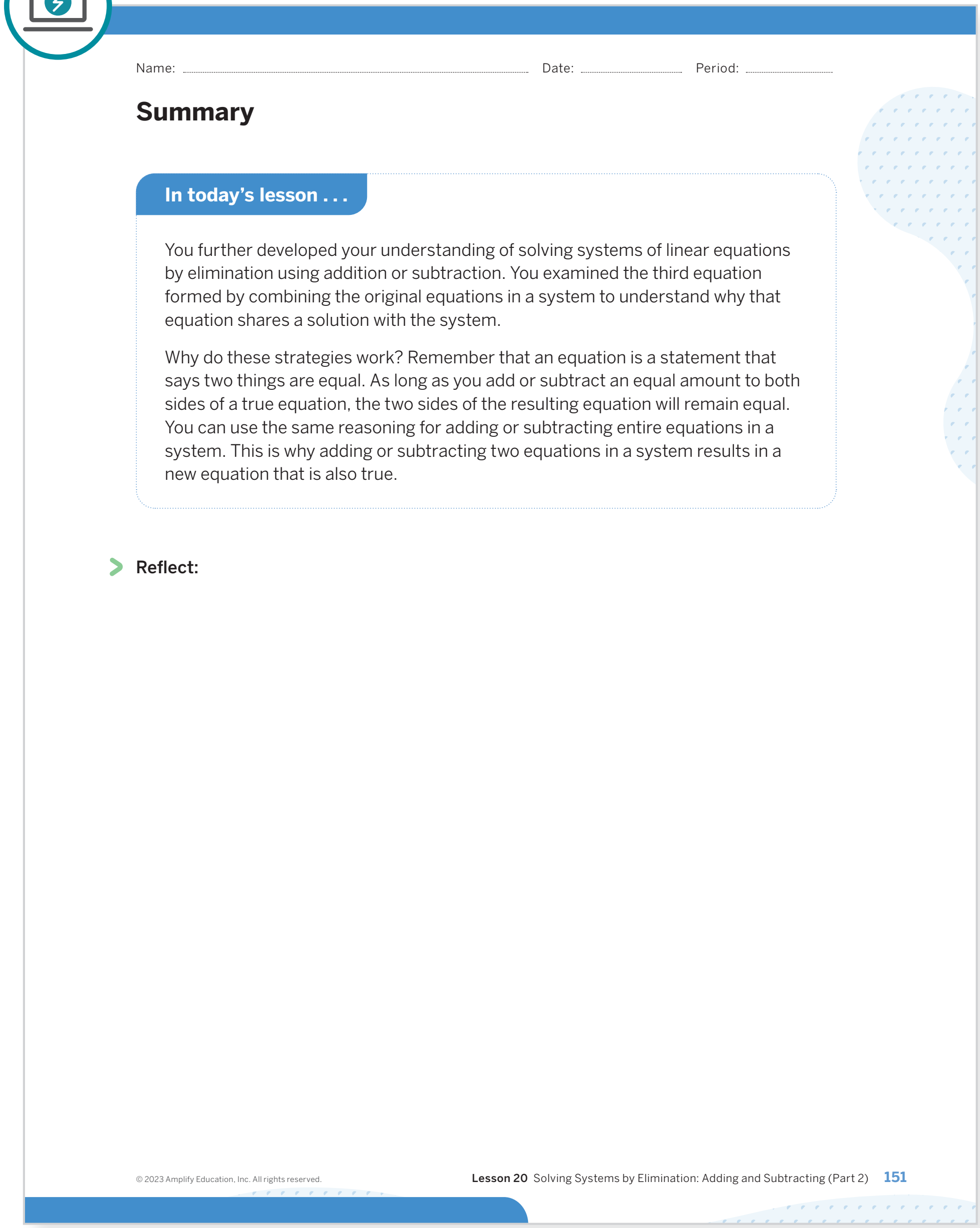
Before students share their strategies during the Connect, display reasoning for solving the system in Problem 1. For example: "I added the equations to get $7y = 14$, so I know $y = 2$. Then I substituted 2 for y into the first equation and solved that equation for x."

- **Critique:** Ask students to critique the reasoning.
- **Correct:** Ask students to write a corrected statement.
- **Clarify:** Ask students to explain how they know their statement is correct.

Whole Class | 5 min

Summary

Review and synthesize that the sum or difference of equations in a system results in an equation that shares the same solution as the system.



Name: Date: Period:

Summary

In today's lesson . . .

You further developed your understanding of solving systems of linear equations by elimination using addition or subtraction. You examined the third equation formed by combining the original equations in a system to understand why that equation shares a solution with the system.

Why do these strategies work? Remember that an equation is a statement that says two things are equal. As long as you add or subtract an equal amount to both sides of a true equation, the two sides of the resulting equation will remain equal. You can use the same reasoning for adding or subtracting entire equations in a system. This is why adding or subtracting two equations in a system results in a new equation that is also true.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Solving Systems by Elimination: Adding and Subtracting (Part 2) 151

Synthesize

Display the system: $\begin{cases} -2x + 8y = 20 \\ 2x + y = 7 \end{cases}$

Ask, "Is there a pair of x and y values that make both equations true?" (2, 3)

Have students share their strategies for determining the solution. Display the third equation, $9y = 27$. Graph all three equations.

Highlight that the original two equations and the third new equation, created by adding or subtracting, all share the same solution. This can be confirmed by graphing all three equations and observing that they intersect at one point.

Ask, "When solving a system with two equations, why is it acceptable to add the two equations or to subtract one equation from the other?" As long as I an equal amount is added to or subtracted from each side of a true equation, the two sides of the resulting equation will remain equal. I can reason the same to be true about variable equations.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

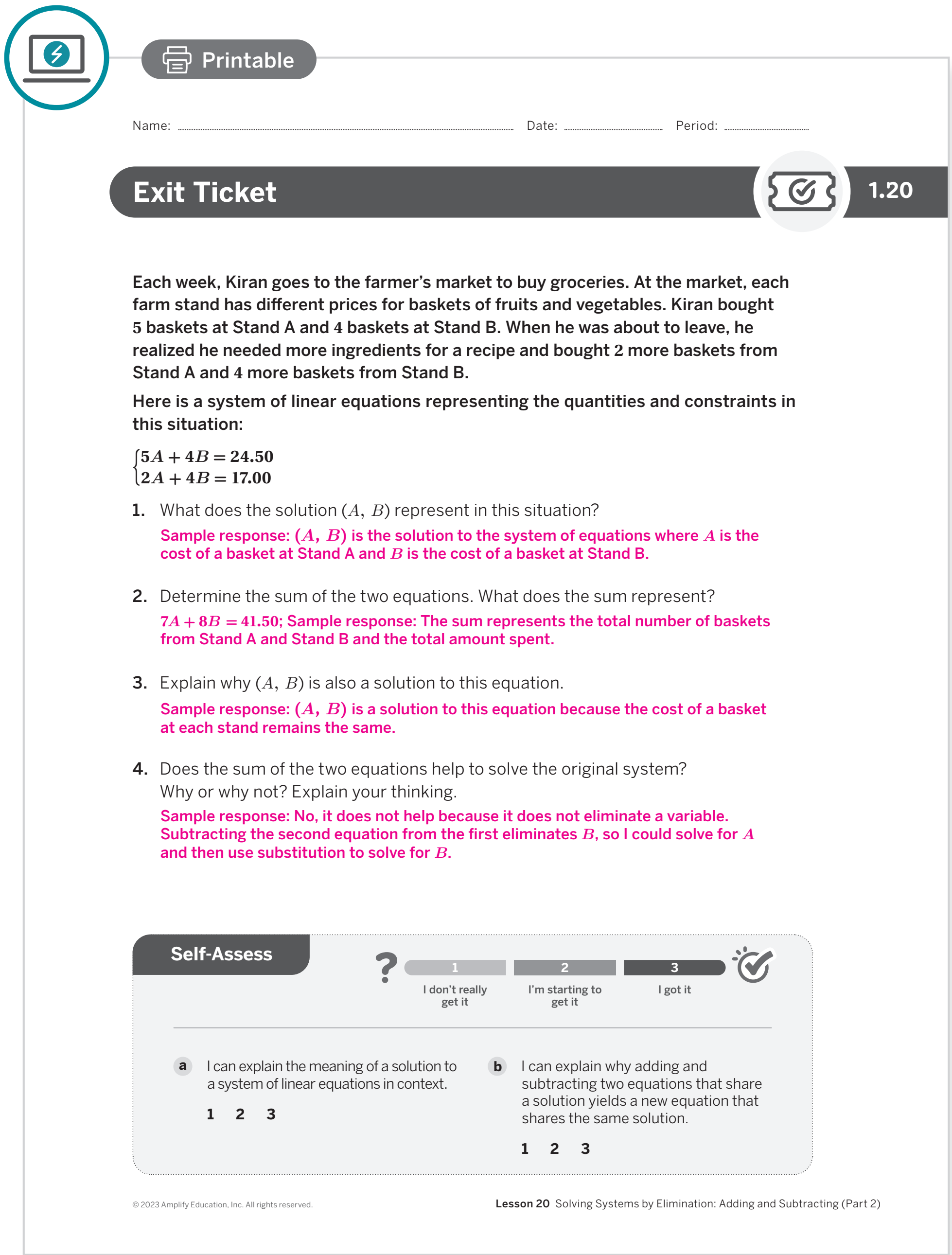
- "When you model a situation using a system of equations, how do you determine the solution and the meaning of the solution?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of elimination by explaining why adding or subtracting two equations creates a new equation with the same solutions.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket

1.20

Each week, Kiran goes to the farmer's market to buy groceries. At the market, each farm stand has different prices for baskets of fruits and vegetables. Kiran bought 5 baskets at Stand A and 4 baskets at Stand B. When he was about to leave, he realized he needed more ingredients for a recipe and bought 2 more baskets from Stand A and 4 more baskets from Stand B.

Here is a system of linear equations representing the quantities and constraints in this situation:

$$\begin{cases} 5A + 4B = 24.50 \\ 2A + 4B = 17.00 \end{cases}$$

1. What does the solution $(A,\ B)$ represent in this situation?

   Sample response: $(A,\ B)$ is the solution to the system of equations where A is the cost of a basket at Stand A and B is the cost of a basket at Stand B.

2. Determine the sum of the two equations. What does the sum represent?

   $7A + 8B = 41.50$; Sample response: The sum represents the total number of baskets from Stand A and Stand B and the total amount spent.

3. Explain why $(A,\ B)$ is also a solution to this equation.

   Sample response: $(A,\ B)$ is a solution to this equation because the cost of a basket at each stand remains the same.

4. Does the sum of the two equations help to solve the original system? Why or why not? Explain your thinking.

   Sample response: No, it does not help because it does not eliminate a variable. Subtracting the second equation from the first eliminates B, so I could solve for A and then use substitution to solve for B.

Self-Assess

?	1	2	3
	I don't really get it	I'm starting to get it	I got it

a I can explain the meaning of a solution to a system of linear equations in context.

1 2 3

b I can explain why adding and subtracting two equations that share a solution yields a new equation that shares the same solution.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Solving Systems by Elimination: Adding and Subtracting (Part 2)

Success looks like . . .

- **Language Goal:** Explaining why adding or subtracting two equations that share a solution results in a new equation that shares the same solution. **(Speaking and Listening, Writing)**
- **Goal:** Solving systems of linear equations by adding or subtracting equations to eliminate a variable.
- **Language Goal:** Using a context to make sense of an equation that is the sum of two equations in a system, and reasoning about why this equation shares a solution with the system. **(Speaking and Listening, Writing)**
  - » Explaining what the sum of the two equations represents in relation to number of baskets and total amount spent in Problem 2.

Suggested next steps

If students cannot describe the significance of (A, B) in Problem 1, consider:

- Reviewing the meaning of the solutions from Activity 1.
- Assigning Practice Problem 3.
- Asking, "What values of A and B make all three equations true? What do A and B represent?"

If students do not accurately determine the sum or interpret the equation in Problem 2, consider:

- Reviewing determining the sum of equations and interpreting the sum from Activity 1.
- Assigning Practice Problems 2 and 3.

If students give a vague or inaccurate explanation in Problems 3 and 4, consider:

- Reviewing how the equation is related to the other two equations in the system from Activity 1.
- Assigning Practice Problems 2 and 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In earlier lessons, students solved systems of linear equations by substitution and elimination. How did that support determining and interpreting the sum of equations in linear systems from Activity 1?
- What different ways did students approach solving systems in Activity 2? What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

Math Language Development

Language Goal: Using a context to make sense of an equation that is the sum of two equations in a system, and reasoning about why this equation shares a solution with the system.

Reflect on students' language development toward this goal.

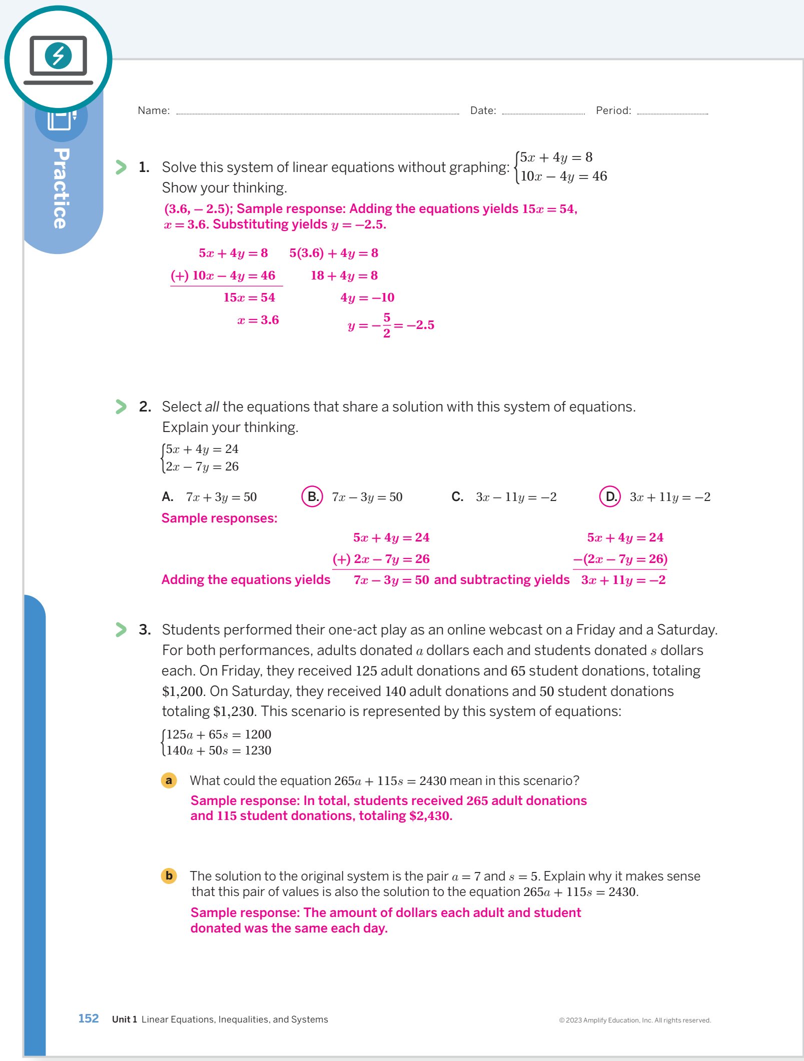
- Do students' responses to Problems 2 and 3 of the Exit Ticket demonstrate they can interpret the sum in context and why the solution representing the sum is also a solution to the system?
- How can you help them be more precise in their descriptions?

Sample descriptions for Problem 3.

Emerging	Expanding
It represents the total.	The equation represents the total number of baskets from each stand and the total spent.

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. Solve this system of linear equations without graphing: $\begin{cases}5x + 4y = 8\\10x - 4y = 46\end{cases}$ Show your thinking.

(3.6, – 2.5); Sample response: Adding the equations yields $15x = 54$, $x = 3.6$. Substituting yields $y = -2.5$.

$$\begin{array}{rr} 5x + 4y = 8 \\ (+)\ 10x - 4y = 46 \\ \hline 15x = 54 \\ x = 3.6 \end{array} \qquad \begin{array}{r} 5(3.6) + 4y = 8 \\ 18 + 4y = 8 \\ 4y = -10 \\ y = -\frac{5}{2} = -2.5 \end{array}$$

2. Select *all* the equations that share a solution with this system of equations. Explain your thinking.

$\begin{cases}5x + 4y = 24\\2x - 7y = 26\end{cases}$

A. $7x + 3y = 50$ [✓ B.] $7x - 3y = 50$ C. $3x - 11y = -2$ [✓ D.] $3x + 11y = -2$

Sample responses:

Adding the equations yields $\begin{array}{r} 5x + 4y = 24 \\ (+)\ 2x - 7y = 26 \\ \hline 7x - 3y = 50 \end{array}$ **and subtracting yields** $\begin{array}{r} 5x + 4y = 24 \\ -(2x - 7y = 26) \\ \hline 3x + 11y = -2 \end{array}$

3. Students performed their one-act play as an online webcast on a Friday and a Saturday. For both performances, adults donated a dollars each and students donated s dollars each. On Friday, they received 125 adult donations and 65 student donations, totaling $1,200. On Saturday, they received 140 adult donations and 50 student donations totaling $1,230. This scenario is represented by this system of equations:

$\begin{cases}125a + 65s = 1200\\140a + 50s = 1230\end{cases}$

a. What could the equation $265a + 115s = 2430$ mean in this scenario?

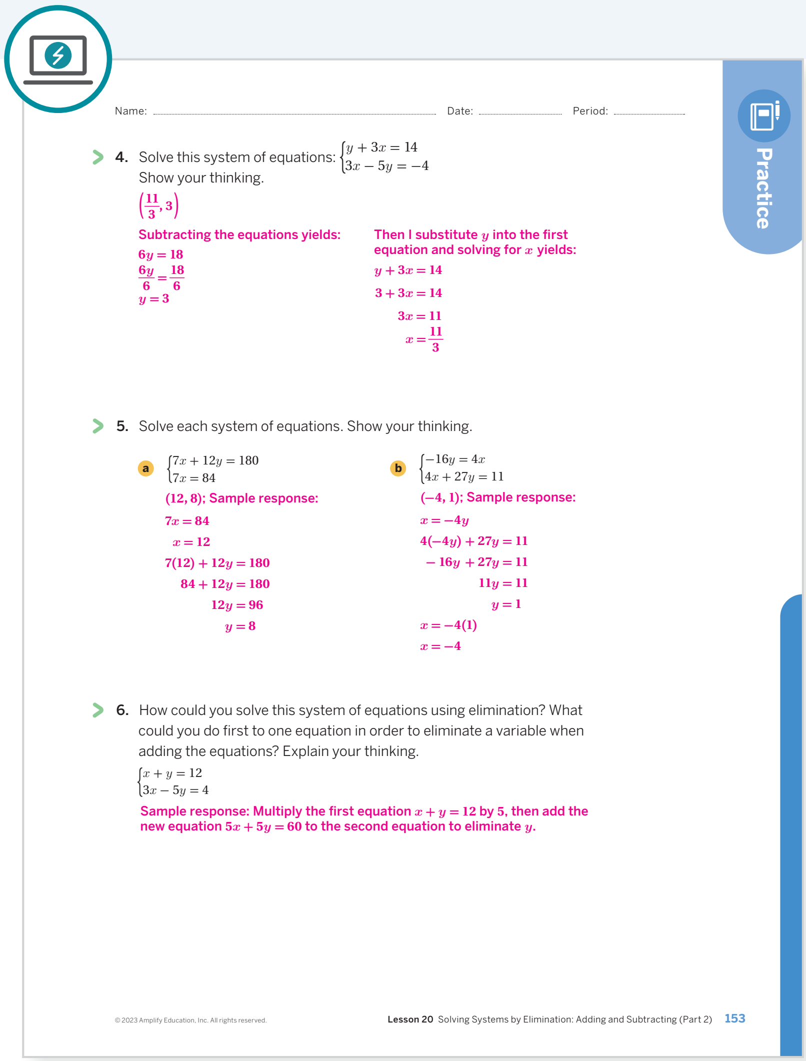
Sample response: In total, students received 265 adult donations and 115 student donations, totaling $2,430.

b. The solution to the original system is the pair $a = 7$ and $s = 5$. Explain why it makes sense that this pair of values is also the solution to the equation $265a + 115s = 2430$.

Sample response: The amount of dollars each adult and student donated was the same each day.

152 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ________ Date: ________ Period: ________

4. Solve this system of equations: $\begin{cases}y + 3x = 14\\3x - 5y = -4\end{cases}$ Show your thinking.

$\left(\frac{11}{3}, 3\right)$

Subtracting the equations yields:

$6y = 18$

$\frac{6y}{6} = \frac{18}{6}$

$y = 3$

Then I substitute y into the first equation and solving for x yields:

$y + 3x = 14$

$3 + 3x = 14$

$3x = 11$

$x = \frac{11}{3}$

5. Solve each system of equations. Show your thinking.

a. $\begin{cases}7x + 12y = 180\\7x = 84\end{cases}$

(12, 8); Sample response:

$7x = 84$

$x = 12$

$7(12) + 12y = 180$

$84 + 12y = 180$

$12y = 96$

$y = 8$

b. $\begin{cases}-16y = 4x\\4x + 27y = 11\end{cases}$

(–4, 1); Sample response:

$x = -4y$

$4(-4y) + 27y = 11$

$-16y + 27y = 11$

$11y = 11$

$y = 1$

$x = -4(1)$

$x = -4$

6. How could you solve this system of equations using elimination? What could you do first to one equation in order to eliminate a variable when adding the equations? Explain your thinking.

$\begin{cases}x + y = 12\\3x - 5y = 4\end{cases}$

Sample response: Multiply the first equation $x + y = 12$ by 5, then add the new equation $5x + 5y = 60$ to the second equation to eliminate y.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Solving Systems by Elimination: Adding and Subtracting (Part 2) 153

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	3
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 19	2
	5	Unit 1 Lesson 18	2
Formative	6	Unit 1 Lesson 21	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice.**

UNIT 1 | **LESSON 21**

Solving Systems by Elimination: Multiplying

Let's investigate how multiplying equations by a factor can help us solve systems of linear equations.

Focus

Goals

1. Recognize that multiplying an equation by a factor creates an equivalent equation whose graph is the same as that of the original equation.
2. Solve systems of equations by multiplying one or both equations by a factor and then adding or subtracting the equations to eliminate a variable.
3. **Language Goal:** Understand that solving a system by elimination entails creating one or more equivalent systems that allows students to solve the original one. **(Speaking and Listening, Writing)**

Coherence

Today

This is the third lesson students are solving systems by elimination. They are introduced to multiplying systems of linear equations by a factor to eliminate a variable. They see that multiplying, adding, or subtracting in a system of equations creates equivalent systems. Students construct logical arguments to support their thinking.

Previously

In Lesson 7, students wrote equivalent equations in two variables. In Lesson 20, students made sense of why the sum or difference of a system share a solution.

Coming Soon

In the next lesson, students will explore systems of linear equations with infinitely many, one, or no solutions.

Rigor

- Students further their **conceptual understanding** of solving systems of equations by elimination using multiplication.
- Students build **procedural fluency** by continuing to solve systems of linear equations by elimination using addition or subtraction.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
10 min	10 min	10 min	10 min	5 min	5 min
Pairs	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 3 PDF, pre-cut cards, one set per pair
- Activity 3 PDF (answers)
- Anchor Chart PDF, *Sentence Stems, Critiquing*
- Anchor Chart PDF, *Solving Systems of Linear Equations by Elimination*
- Anchor Chart PDF, *Sentence Stems, Generalizing*
- graphing technology

Math Language Development

New words

- ***equivalent systems***

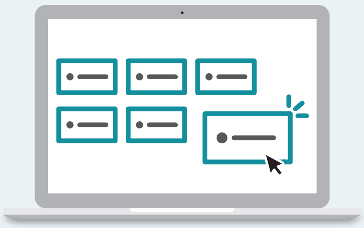
Review words

- *elimination*
- *equivalent equation*
- *solution to a system of equations*
- *substitution*
- *system of equations*

Amps | **Featured Activity**

Activity 3
Digital Card Sort

Students order the steps to solving a system of linear equations by dragging and connecting them on screen.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 2, students might think that one good strategy for creating an equivalent system is enough. Remind students that they can learn from each other. They should listen to others' arguments, too, as they build their sets of evidence. They also can help each other by looking for errors in the thinking so that a correction can be made. This requires engagement by all students.

Modifications to Pacing

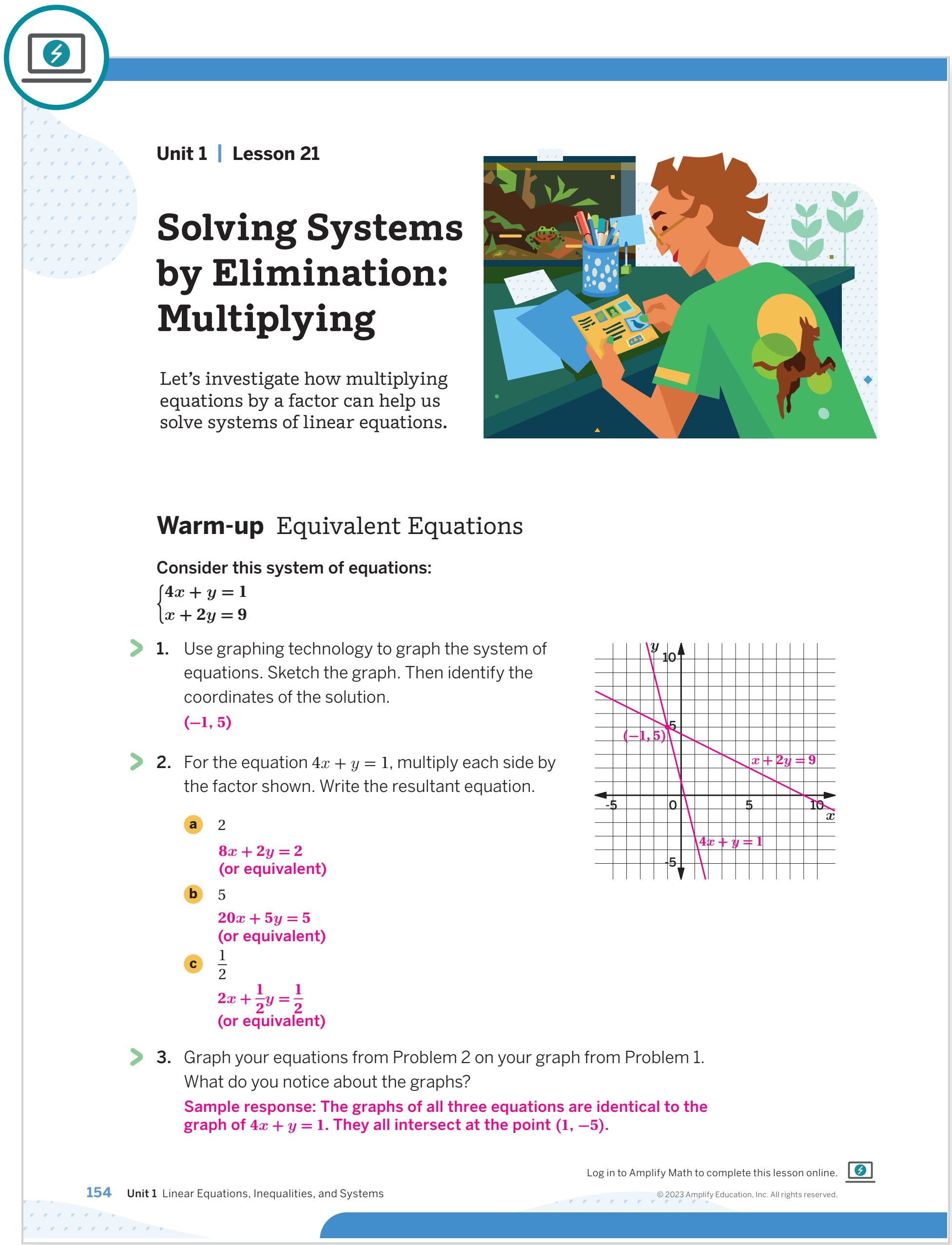
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 2, have students write only one equivalent equation.
- In **Activity 1**, have students complete one work sample and graph two out of the three systems.
- In **Activity 2**, Problem 1 may be omitted.

Pairs | 10 min

Warm-up Equivalent Equations

Students create and graph equivalent equations in a linear system to recall that they have the same solutions and identical graphs.



Unit 1 | Lesson 21

Solving Systems by Elimination: Multiplying

Let's investigate how multiplying equations by a factor can help us solve systems of linear equations.

Warm-up Equivalent Equations

Consider this system of equations:

$$\begin{cases} 4x + y = 1 \\ x + 2y = 9 \end{cases}$$

1. Use graphing technology to graph the system of equations. Sketch the graph. Then identify the coordinates of the solution.

   $(-1, 5)$

2. For the equation $4x + y = 1$, multiply each side by the factor shown. Write the resultant equation.

   a. 2

   $8x + 2y = 2$
   (or equivalent)

   b. 5

   $20x + 5y = 5$
   (or equivalent)

   c. $\frac{1}{2}$

   $2x + \frac{1}{2}y = \frac{1}{2}$
   (or equivalent)

3. Graph your equations from Problem 2 on your graph from Problem 1. What do you notice about the graphs?

   Sample response: The graphs of all three equations are identical to the graph of $4x + y = 1$. They all intersect at the point $(1, -5)$.

Log in to Amplify Math to complete this lesson online.

154 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

In pairs, provide access to graphing technology. To save time, allow group members to divide up the tasks. Ask, "Where can you identify the solution to the system on the graph?" The solution to the system is the point of intersection of the lines.

2 Monitor

Help students get started by having them identify the slope and x- and y-intercepts of the original equation.

Look for points of confusion:

- **Having difficulty understanding why the graphs of the new equations are identical to the original equations.** Have them verify that each new equation has the same slope and intercepts as the original.

Look for productive strategies:

- Verifying that the new equations are equivalent to the original and the new system has the same solution as the original.

3 Connect

Display the graph from Problem 3.

Have pairs of students share their observations about the three new equations and their graphs.

Highlight that when students multiply both sides of the equal sign by the same number, they create an equation that is equivalent to the original.

Ask, "What if you multiply both sides of a two-variable equation by the same factor? Would the resulting equation have the same solutions as the original equation?"

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their observations about the graphs, draw connections between the structure of the equations in Problem 2 and how their graphs are the same line. If students are unsure why they are the same line, have them write each equation in Problem 2 in slope-intercept form to show that the slope and the y-intercept are the same.

English Learners

Use color coding or annotations to highlight how each coefficient and constant term in each equation in Problem 2 is related to the factors given.

Power-up

To power up students' ability to solve systems of equations using elimination, have students complete:

Select *all* of the systems of equations that would eliminate one variable through addition or subtraction.

[✓ A.] $\begin{cases} 2x + 4y = 8 \\ -2x + 3y = 7 \end{cases}$

C. $\begin{cases} 2x + 4y = -5 \\ 4x + 2y = 6 \end{cases}$

[✓ B.] $\begin{cases} 5x - 3y = 9 \\ -2x + 3y = 10 \end{cases}$

[✓ D.] $\begin{cases} 6x + 3y = 12 \\ 6x - y = 9 \end{cases}$

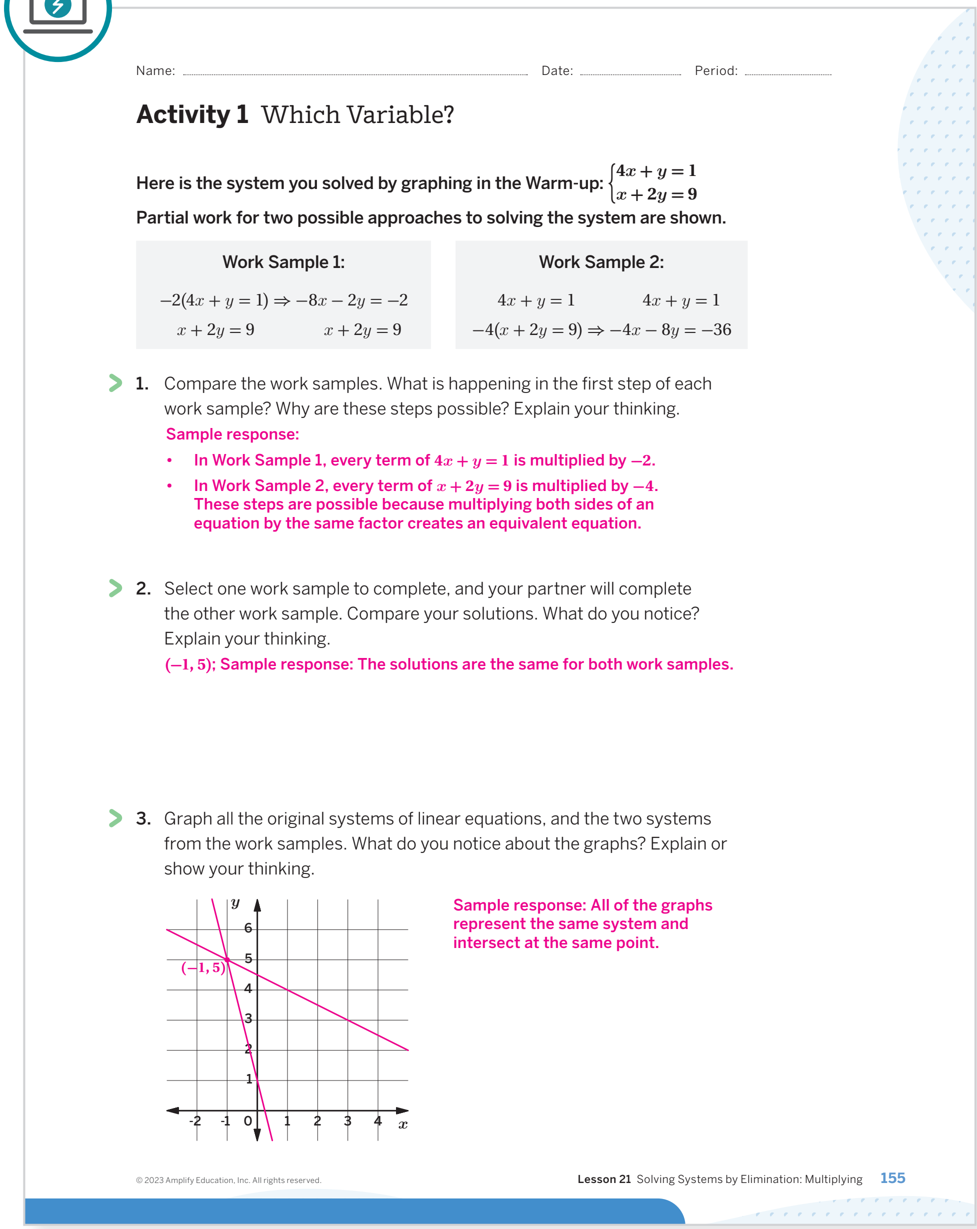
Use: Before the Warm-up

Informed by: Performance on Lesson 20, Practice Problem 6

Pairs | 10 min

Activity 1 Which Variable?

Students reason quantitatively to create an equivalent system of equations with the same solution set by multiplying by a factor.



Name: Date: Period:

Activity 1 Which Variable?

Here is the system you solved by graphing in the Warm-up: $\begin{cases} 4x + y = 1 \\ x + 2y = 9 \end{cases}$

Partial work for two possible approaches to solving the system are shown.

Work Sample 1:	Work Sample 2:
$-2(4x + y = 1) \Rightarrow -8x - 2y = -2$ $x + 2y = 9 \qquad x + 2y = 9$	$4x + y = 1 \qquad 4x + y = 1$ $-4(x + 2y = 9) \Rightarrow -4x - 8y = -36$

1. Compare the work samples. What is happening in the first step of each work sample? Why are these steps possible? Explain your thinking.
   Sample response:
   - **In Work Sample 1, every term of $4x + y = 1$ is multiplied by -2.**
   - **In Work Sample 2, every term of $x + 2y = 9$ is multiplied by -4. These steps are possible because multiplying both sides of an equation by the same factor creates an equivalent equation.**

2. Select one work sample to complete, and your partner will complete the other work sample. Compare your solutions. What do you notice? Explain your thinking.
   $(-1, 5)$; Sample response: The solutions are the same for both work samples.

3. Graph all the original systems of linear equations, and the two systems from the work samples. What do you notice about the graphs? Explain or show your thinking.

Sample response: All of the graphs represent the same system and intersect at the same point.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Provide pairs with access to graphing technology. Students work on Problem 1 together, then work on the two work samples in Problem 2 independently, and before moving on to Problem 3, compare their solutions and discuss. Follow with a whole-class discussion.

2 Monitor

Help students get started by asking "What is the difference between the two work samples?"

Look for points of confusion:

- **Forgetting to solve for the second variable.** Remind students that solutions to systems of linear equations are written as ordered pairs.

Look for productive strategies:

- Substituting the solution into the equations in both work samples to verify their work.
- Writing the equations in slope-intercept form to graph.

3 Connect

Display the Anchor Chart PDF, *Solving Systems of Linear Equations by Elimination*.

Have students share what they noticed about the solution to each work sample and the graphs of the systems. Listen for student responses that mention equivalent equations.

Define the term ***equivalent systems***.

Highlight that students can create an equivalent system by multiplying an equation by a factor to create a new equation that is a multiple of the original.

Ask, "Is it possible to multiply both equations in a linear system by a factor to eliminate a variable?" Yes, once I pick a variable to eliminate, I can multiply both equations by different factors to create opposite leading coefficients.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display or provide the Anchor Chart PDF, *Solving Systems of Linear Equations by Elimination* for students to use as a reference throughout the activity. Provide access to colored pencils or highlighters and have students color code the factor -2 in Work Sample 1 and -4 in Work Sample 2. Ask these questions:

- "In Work Sample 1, look at the equation $-8x - 2y = -2$ and the equation below it. Can this system now be solved by elimination?"
- "In Work Sample 2, look at the equation $-4x - 8y = -36$ and the equation above it. Can this system now be solved by elimination?"

Math Language Development

MLR3: Critique, Correct, Clarify

Before or during the Connect, consider presenting an incorrect statement for Problem 1, such as "In Work Sample 1, each term in the top equation was doubled."

- **Critique:** Ask students to critique the reasoning. Listen for students who notice that the signs of the terms also changed.
- **Correct and Clarify:** Ask students to write a corrected statement and explain how they know their statement is correct.

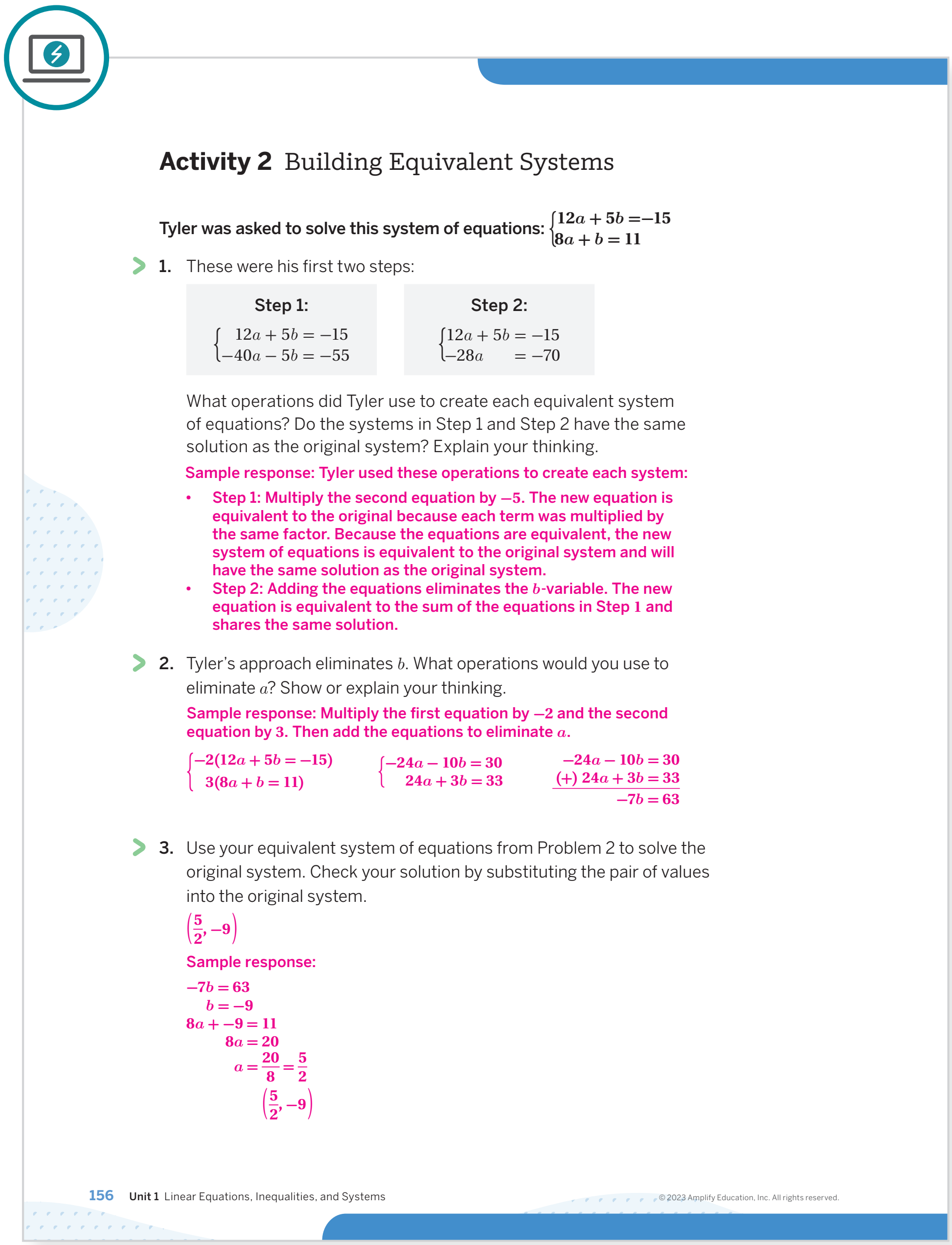
English Learners

Display the Anchor Chart PDF, *Sentence Stems, Critiquing* for students to use as a reference.

Pairs | 10 min

Activity 2 Building Equivalent Systems

Students multiply both equations in a system by different factors to understand different entry points for solving linear systems by elimination.



Activity 2 Building Equivalent Systems

Tyler was asked to solve this system of equations: $\begin{cases} 12a + 5b =-15 \\ 8a + b = 11 \end{cases}$

1. These were his first two steps:

Step 1:	**Step 2:**
$\begin{cases} 12a + 5b = -15 \\ -40a - 5b = -55 \end{cases}$	$\begin{cases} 12a + 5b = -15 \\ -28a \quad = -70 \end{cases}$

What operations did Tyler use to create each equivalent system of equations? Do the systems in Step 1 and Step 2 have the same solution as the original system? Explain your thinking.

Sample response: Tyler used these operations to create each system:

- **Step 1: Multiply the second equation by –5. The new equation is equivalent to the original because each term was multiplied by the same factor. Because the equations are equivalent, the new system of equations is equivalent to the original system and will have the same solution as the original system.**
- **Step 2: Adding the equations eliminates the b-variable. The new equation is equivalent to the sum of the equations in Step 1 and shares the same solution.**

2. Tyler's approach eliminates b. What operations would you use to eliminate a? Show or explain your thinking.

Sample response: Multiply the first equation by –2 and the second equation by 3. Then add the equations to eliminate a.

$$\begin{cases} -2(12a + 5b = -15) \\ 3(8a + b = 11) \end{cases} \qquad \begin{cases} -24a - 10b = 30 \\ 24a + 3b = 33 \end{cases} \qquad \begin{array}{r} -24a - 10b = 30 \\ \underline{(+)\ 24a + 3b = 33} \\ -7b = 63 \end{array}$$

3. Use your equivalent system of equations from Problem 2 to solve the original system. Check your solution by substituting the pair of values into the original system.

$\left(\frac{5}{2}, -9\right)$

Sample response:

$$\begin{aligned} -7b &= 63 \\ b &= -9 \\ 8a + -9 &= 11 \\ 8a &= 20 \\ a &= \frac{20}{8} = \frac{5}{2} \\ &\left(\frac{5}{2}, -9\right) \end{aligned}$$

156 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Provide students with 5 minutes of independent think-time before discussing with their partner. Allow time for pairs to revise thier thinking based on their partner's critiques.

2 Monitor

Help students get started by asking, "How do you know which factor to use to eliminate a variable? Can it be a fraction or a negative number?"

Look for points of confusion:

- **Not understanding which variable to eliminate and what factor to use to create a pair with opposite coefficients.** Ask them which variable should be eliminated. Then have them identify a factor that will allow them to eliminate the chosen variable when adding or subtracting the equations.

Look for productive strategies:

- Identifying a variable to eliminate and the factor necessary to eliminate it when adding or subtracting the equations.

3 Connect

Display the Anchor Chart PDF, *Solving Systems of Linear Equations by Elimination*.

Have students share strategies for creating the equivalent system and solving it in Problems 2 and 3.

Highlight that there are many ways to determine the solution, either variable may be eliminated, and one or both equations may be multiplied by a factor or factors.

Ask, "Why does each system generate the same solution as the original system or the system before it?" The systems are equivalent systems.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display the system of equations and guide students towards understanding Tyler's first step by asking these questions:

- "If you just added these equations together, would you eliminate a or b? Why not?"
- "Could you multiply the bottom equation by some number so that the b-values could eliminate?"

Extension: *Math Enrichment*

Have students explore using different factors to eliminate the variables and then explain how creating multiples of the original equations creates equivalent systems of linear equations.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share the strategies they used for Problems 2 and 3, display the Anchor Chart PDF, *Sentence Stems, Critiquing*. Ask students to borrow phrases from the anchor chart as they respond to their classmate's strategies. Highlight that the equations may be multiplied by different factors as long as one of the variables is eliminated. Ask, "Which variable do you think is more efficiently eliminated? Based on that, what factor would you use to multiply each equation by?"

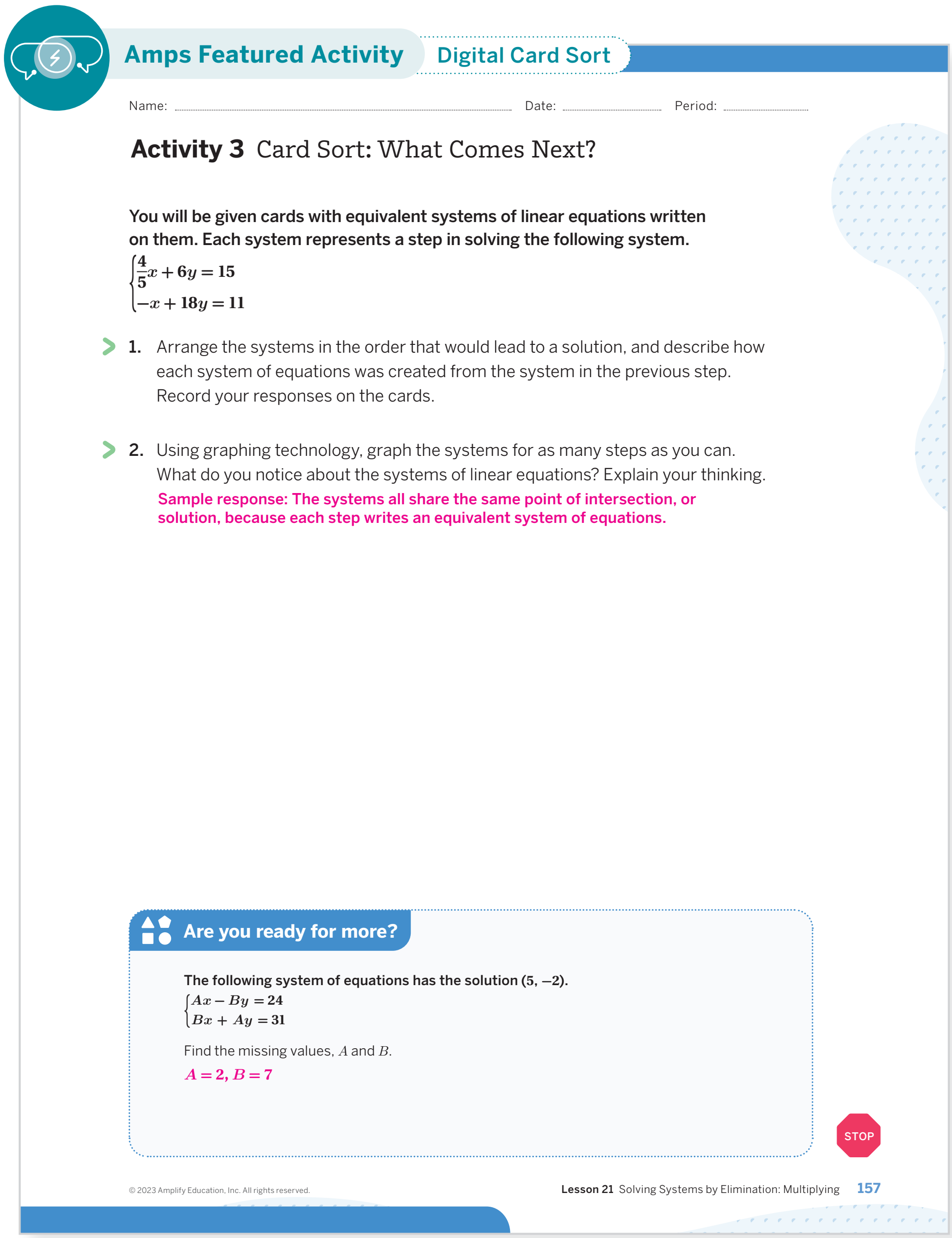
English Learners

Display different equivalent systems of equations students created. Annotate them as *equivalent systems*.

Pairs | 10 min

Activity 3 Card Sort: What Comes Next?

Students construct an argument that explains why each new system of equations is equivalent to the previous system.



Amps Featured Activity Digital Card Sort

Name: Date: Period:

Activity 3 Card Sort: What Comes Next?

You will be given cards with equivalent systems of linear equations written on them. Each system represents a step in solving the following system.

$$\begin{cases}\frac{4}{5}x + 6y = 15\\ -x + 18y = 11\end{cases}$$

1. Arrange the systems in the order that would lead to a solution, and describe how each system of equations was created from the system in the previous step. Record your responses on the cards.

2. Using graphing technology, graph the systems for as many steps as you can. What do you notice about the systems of linear equations? Explain your thinking.
   Sample response: The systems all share the same point of intersection, or solution, because each step writes an equivalent system of equations.

Are you ready for more?

The following system of equations has the solution (5, –2).

$$\begin{cases}Ax - By = 24\\ Bx + Ay = 31\end{cases}$$

Find the missing values, A and B.

$A = 2, B = 7$

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 21 Solving Systems by Elimination: Multiplying 157

1 Launch

Provide the pre-cut cards from the Activity 3 PDF to each student pair and conduct the ***Card Sort*** routine. Allow students three minutes of work time to sort the systems. Then, allow three minutes to discuss with their partner and record their explanations for why they chose each step in the cards.

2 Monitor

Help students get started by asking, “Which variable would you choose to eliminate?”

Look for points of confusion:

- **Thinking that only one equation can be multiplied by a factor when using elimination.** Remind students that in Activity 2, they found they can multiply both equations by different factors to create opposite leading coefficients.

Look for productive strategies:

- Multiplying the first equation by 5 to eliminate the fraction and then multiplying the second equation by 4 to eliminate the x-variable.
- Working backwards from solution to determine prior steps.

3 Connect

Have pairs of students share strategies for determining the sequence and explanations of why each system is equivalent to the previous system.

Display the correct order of the cards in each step.

Highlight that each step represents an equivalent system of equations and Steps 2 and 3 can be interchanged.

Ask, “Was it necessary to multiply each equation by different factors?” No, I could multiply the first equation by $\frac{5}{4}$ to eliminate x or by 3 to eliminate y. Or I could multiply the second equation by $\frac{4}{5}$ to eliminate x or by $\frac{1}{3}$ to eliminate y.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider displaying or providing the Anchor Chart PDF, *Solving Systems of Linear Equations by Elimination* for students to use as a reference. Ask students to first find the card that shows the original system of equations and then find the card that shows the final solution. Then ask them to find the card(s) where the value for one of the variables first appears.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their strategies, display the Anchor Chart PDF, *Sentence Stems, Critiquing*. Ask students to borrow phrases from the anchor chart as they respond to their classmate's strategies. For example, students may disagree with whether Step 2 or Step 3 comes first. Either step can actually be performed first; encourage students to explain why.

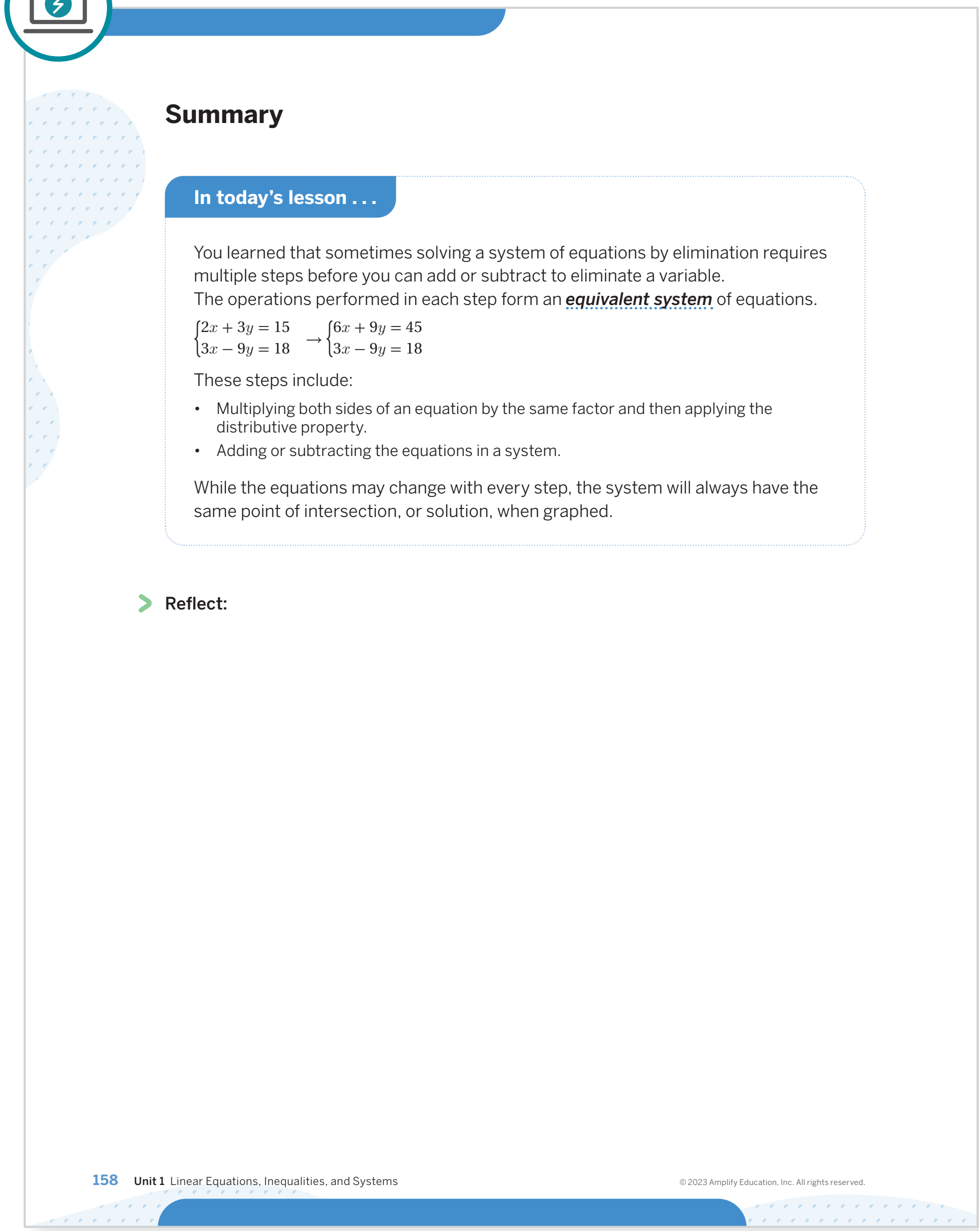
English Learners

Consider using color coding to annotate what changed in the equations from each step to the next.

Whole Class | 5 min

Summary

Review and synthesize that systems of linear equations can be solved using elimination by first creating equivalent systems of linear equations.



Summary

In today's lesson . . .

You learned that sometimes solving a system of equations by elimination requires multiple steps before you can add or subtract to eliminate a variable. The operations performed in each step form an ***equivalent system*** of equations.

$$\begin{cases} 2x + 3y = 15 \\ 3x - 9y = 18 \end{cases} \rightarrow \begin{cases} 6x + 9y = 45 \\ 3x - 9y = 18 \end{cases}$$

These steps include:

- Multiplying both sides of an equation by the same factor and then applying the distributive property.
- Adding or subtracting the equations in a system.

While the equations may change with every step, the system will always have the same point of intersection, or solution, when graphed.

Reflect:

158 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the Anchor Chart PDF, *Solving Systems of linear equations by Elimination*.

Have students share their strategies for creating equivalent systems and explain why multiplying by different factors leads to the same solution.

Highlight that adding equal amounts to the two sides of an equation keeps the two sides equal, so the same x- and y-values that make the first equation true also makes this new equation true. This means that the same pair of values is also a solution to the new system.

Formalize vocabulary: *equivalent systems*

Ask, "How do you know the new system is equivalent to the one before it even though one equation has been transformed?" Multiplying an equation by a factor creates an equivalent equation. If the equations are equivalent, the systems are equivalent.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How does creating an equivalent system of linear equations help solve the system using elimination?"

Math Language Development

MLR2: Collect and Display

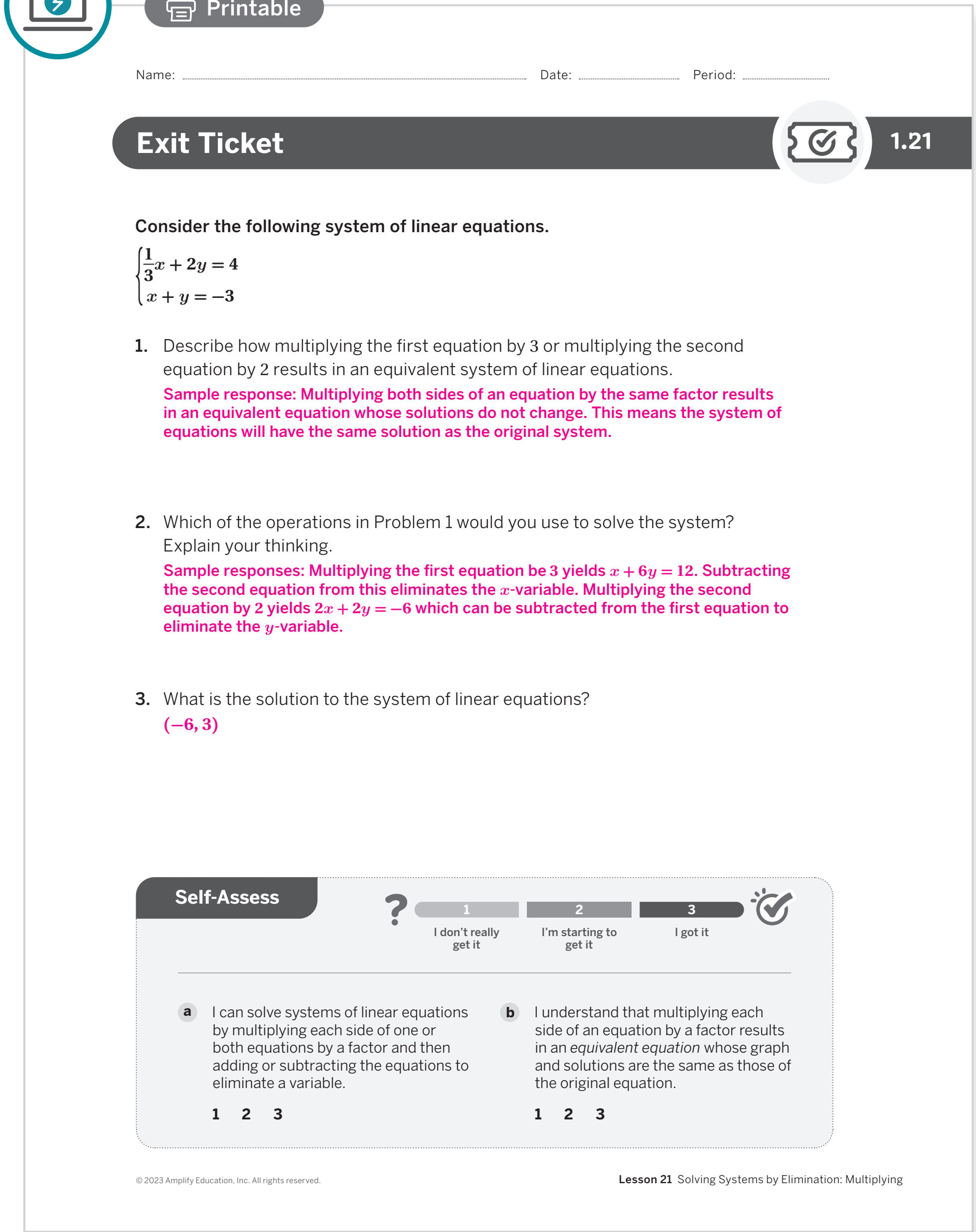
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *equivalent systems* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of solving systems of linear equations using elimination by creating equivalent systems of linear equations.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

1.21

Consider the following system of linear equations.

$$\begin{cases} \frac{1}{3}x + 2y = 4 \\ x + y = -3 \end{cases}$$

1. Describe how multiplying the first equation by 3 or multiplying the second equation by 2 results in an equivalent system of linear equations.

   Sample response: Multiplying both sides of an equation by the same factor results in an equivalent equation whose solutions do not change. This means the system of equations will have the same solution as the original system.

2. Which of the operations in Problem 1 would you use to solve the system? Explain your thinking.

   Sample responses: Multiplying the first equation be 3 yields $x + 6y = 12$. Subtracting the second equation from this eliminates the x-variable. Multiplying the second equation by 2 yields $2x + 2y = -6$ which can be subtracted from the first equation to eliminate the y-variable.

3. What is the solution to the system of linear equations?

   $(-6, 3)$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can solve systems of linear equations by multiplying each side of one or both equations by a factor and then adding or subtracting the equations to eliminate a variable.

1 2 3

b I understand that multiplying each side of an equation by a factor results in an *equivalent equation* whose graph and solutions are the same as those of the original equation.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Success looks like . . .

- **Goal:** Recognizing that multiplying an equation by a factor creates an equivalent equation whose graph is the same as that of the original equation.
  - » Explaining why multiplying the first equation by 3 or the second equation by 2 gives an equivalent system of equations in Problem 1.
- **Goal:** Solving systems of equations by multiplying one or both equations by a factor and then adding or subtracting the equations to eliminate a variable.
- **Goal:** Understanding that solving a system by elimination entails creating one or more equivalent systems that allows students to solve the original one. **(Speaking and Listening, Writing)**
  - » Explaining how to eliminate a variable in order to solve the system of equations in Problem 2.

Suggested next steps

If students struggle to explain how multiplying an equation by a factor creates an equivalent system of equations in Problem 1, consider:

- Reviewing multiplicative strategies from Activities 1, 2, and 3.
- Assigning Practice Problem 2.
- Asking, "How would you solve this if, instead of multiplying the second equation by 2, you were multiplying by −2?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The instructional goal for this lesson was to solve systems of linear equations using elimination by first creating an equivalent system of equations. How well did students accomplish this? What did you specifically do to help students accomplish it?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ________ Date: ________ Period: ________

1. Solve each system of equations. Show your thinking.

a $\begin{cases} 2x - 4y = 10 \\ x + 5y = 40 \end{cases}$

(15, 5)

Sample response:

$$\begin{aligned} 2x - 4y &= 10 \\ -2(x + 5y &= 40) \end{aligned}$$

$$\begin{array}{rr} 2x - 4y = 10 & x + 5(5) = 40 \\ \underline{(+)\ -2x - 10y = -80} & x + 25 = 40 \\ -14y = -70 & x = 15 \\ y = 5 & \end{array}$$

b $\begin{cases} 3x - 5y = 4 \\ -2x + 6y = 18 \end{cases}$

(14.25, 7.75)

Sample response:

$$\begin{aligned} 2(3x - 5y &= 4) \\ 3(-2x + 6y &= 18) \end{aligned}$$

$$\begin{array}{rr} 6x - 10y = 8 & 3x - 5(7.75) = 4 \\ \underline{(+)\ -6x + 18y = 54} & 3x - 38.75 = 4 \\ 8y = 62 & 3x = 42.75 \\ y = 7.75 & x = 14.25 \end{array}$$

2. Consider these potential strategies for solving the following system.

$\begin{cases} 4p + 2q = 62 \\ 8p - q = 59 \end{cases}$

- Multiply $4p + 2q = 62$ by 2. Then subtract $8p - q = 59$ from the result.
- Multiply $8p - q = 59$ by 2. Then add the result to $4p + 2q = 62$.

Do both strategies work for solving the system? Explain or show your thinking.

Yes, both strategies work. Sample response: The first strategy eliminates p. The second eliminates q. The solution is (9, 13), regardless of the strategy used.

3. Select all systems that are equivalent to this system.

$\begin{cases} 6d + 4.5e = 16.5 \\ 5d + 0.5e = 4 \end{cases}$

A. $\begin{cases} 6d + 4.5e = 16.5 \\ 45d + 4.5e = 4 \end{cases}$

[✓ B.] $\begin{cases} 6d + 4.5e = 16.5 \\ 6d + 0.6e = 4.8 \end{cases}$

[✓ C.] $\begin{cases} 30d + 22.5e = 82.5 \\ 30d + 3e = 24 \end{cases}$

[✓ D.] $\begin{cases} 30d + 22.5e = 82.5 \\ 5d + 0.5e = 4 \end{cases}$

E. $\begin{cases} 12d + 9e = 33 \\ 10d + 0.5e = 8 \end{cases}$

F. $\begin{cases} 6d + 4.5e = 16.5 \\ 11d + 5e = 20.5 \end{cases}$

© 2023 Amplify Education, Inc. All rights reserved.

Name: ________ Date: ________ Period: ________

4. The cost to mail a package is \$5. Noah has postcard stamps that are worth \$0.34 each and First Class stamps that are worth \$0.49 each.

a Write an equation that relates the number of postcard stamps p, the number of First Class stamps f, and the cost of mailing the package.

$0.34p + 0.49f = 5$

b Solve the equation for f.

$f = \frac{5 - 0.34p}{0.49}$

c Solve the equation for p.

$p = \frac{5 - 0.49f}{0.34}$

d If Noah puts 7 First Class stamps on the package, how many postcard stamps will he need?

$p \approx 4.6$. Sample response: Noah will need 5 postcard stamps.

5. Priya buys 2.4 lb of bananas and 3.6 lb of grapes for \$9.38 at a grocery store. At the same grocery store, Andre buys 1.2 lb of bananas and 1.8 lb of grapes for \$4.69. This information can be represented by the following system of equations:

$\begin{cases} 2.4b + 3.6g = 9.38 \\ 1.2b + 1.8g = 4.69 \end{cases}$

What happens when you try to solve the system of equations using elimination? Explain or show your thinking.

Sample response: When $1.2b + 1.8g = 4.69$ is multiplied by –2, all terms are eliminated when the equations are added.

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	3
	2	Activity 3	3
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 10	2
Formative ⚡	5	Unit 1 Lesson 22	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

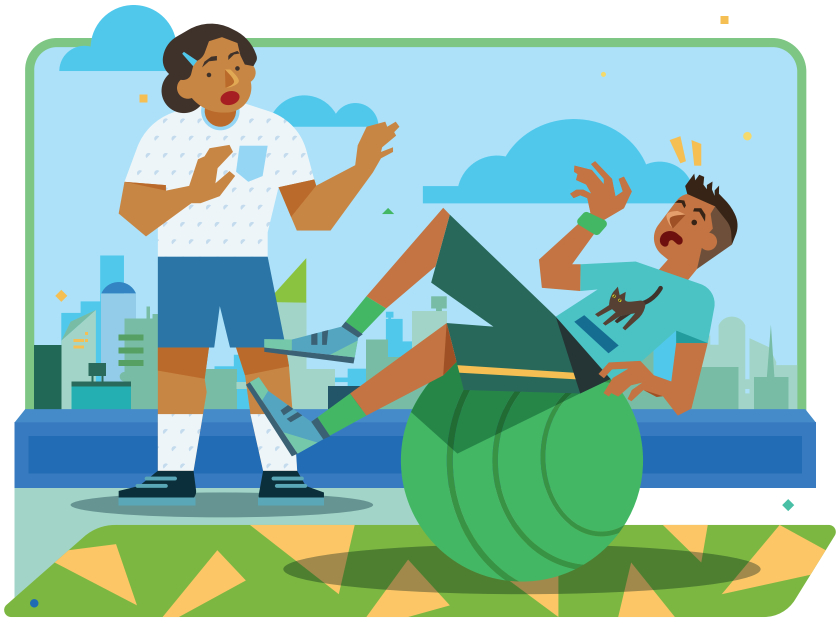
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 22**

Systems of Linear Equations and Their Solutions

Let's determine how many solutions there are to a system of linear equations.



Focus

Goals

1. **Language Goal:** Determine whether a system of equations will have one solution, no solution, or infinitely many solutions by analyzing the structure of its equations. **(Reading and Writing)**
2. Determine how many solutions a system has by graphing.
3. Explain how the slope and y-intercept of a system of equations affect the features of its graphs.

Coherence

Today

Students revisit the Grade 8 concept that all systems of linear equations do not have a single solution. They investigate and use the structure of the equations in a system to understand the number of solutions and effects on the graph.

Previously

In Lessons 17–21, students solved systems of linear equations by graphing, substitution, and elimination.

Coming Soon

In Lesson 23, students will solve systems of linear inequalities by graphing.

Rigor

- Students build on their **conceptual understanding** of the different types of systems of equations by studying their structure to determine the number of solutions it has.
- Students solve systems of linear equations using any strategy to develop **procedural fluency**.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
10 min	10 min	20 min	10 min	5 min	5 min
Small Groups	Small Groups	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (as needed)
- Activity 1 PDF (answers)
- Activity 2 PDF, pre-cut cards, one set per group
- Anchor Chart PDF, *Types of Linear Systems*
- graphing technology

Math Language Development

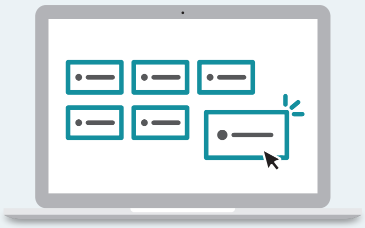
Review words

- *elimination*
- *equivalent systems*
- *parallel*
- *substitution*
- *system of equations*

Amps | **Featured Activity**

Activity 2
Digital Card Sort

Students match systems of linear equations with their number of solutions by dragging and connecting them on screen. Instead of walking from student to student, work can be seen digitally in real-time.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated as they sort the cards in Activity 2. Encourage students to persist as they make sense of the structure of the given systems. For instance, have them think back to equivalent systems and how they can apply this knowledge to sorting the cards.

Modifications to Pacing

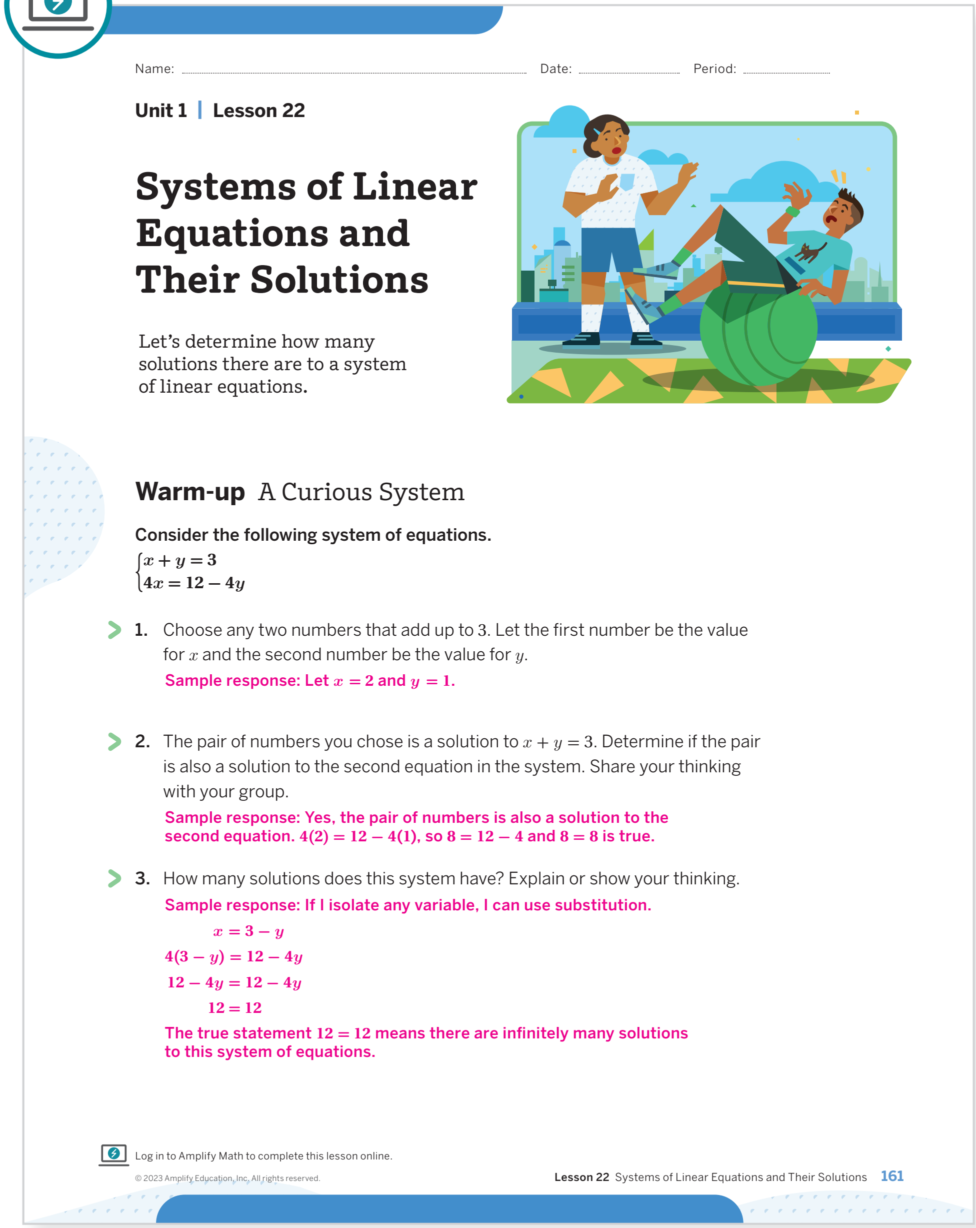
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, have students omit Cards 1 and/or 9.
- Optional **Activity 3** may be omitted.

Small Groups | 10 min

Warm-up A Curious System

Students reason quantitatively on the solutions of a linear system to prepare for a discussion about the number of solutions a system can have.



Name: Date: Period:

Unit 1 | Lesson 22

Systems of Linear Equations and Their Solutions

Let's determine how many solutions there are to a system of linear equations.

Warm-up A Curious System

Consider the following system of equations.

$$\begin{cases} x + y = 3 \\ 4x = 12 - 4y \end{cases}$$

1. Choose any two numbers that add up to 3. Let the first number be the value for x and the second number be the value for y.
   Sample response: Let $x = 2$ and $y = 1$.

2. The pair of numbers you chose is a solution to $x + y = 3$. Determine if the pair is also a solution to the second equation in the system. Share your thinking with your group.
   Sample response: Yes, the pair of numbers is also a solution to the second equation. $4(2) = 12 - 4(1)$, so $8 = 12 - 4$ and $8 = 8$ is true.

3. How many solutions does this system have? Explain or show your thinking.
   Sample response: If I isolate any variable, I can use substitution.

$$\begin{aligned} x &= 3 - y \\ 4(3 - y) &= 12 - 4y \\ 12 - 4y &= 12 - 4y \\ 12 &= 12 \end{aligned}$$

   The true statement $12 = 12$ means there are infinitely many solutions to this system of equations.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 Systems of Linear Equations and Their Solutions 161

1 Launch

Arrange students in groups of three or four. Have students work independently on Problems 1 and 2, pause for a group discussion, and then work on Problem 3 individually.

2 Monitor

Help students get started by asking, "What pairs of numbers sum to 3?"

Look for points of confusion:

- **Interchanging chosen values from Problem 1 into Problem 2.** Have students label which number represents x and y.
- **Choosing only whole numbers.** Suggest they try -1 and 4.

Look for productive strategies:

- Using a table, list, or chart to substitute multiple values for x and y.

3 Connect

Display students' solutions. Record their responses as they share.

Have students share their solutions and strategies for verifying the solution. Select and sequence responses using tables, substitution, graphing, elimination, and reasoning about equivalence.

Ask, "How could you have shown the system has infinite solutions without graphing or using calculations?" Multiply the first equation in the system by 4 to show the linear system are equivalent equations.

Highlight that the linear system has equivalent equations. All pairs of values for x and y that make one equation true will make the other equation true, yielding infinitely many solutions.

MLR Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses, listen for language they use to determine the number of solutions to the system, such as "*every pair of numbers that added to 3* was also a solution to the second equation." Draw connections between the structure of the equations and the fact that there are infinitely many solutions to the system. Consider asking these follow-up questions:

- "What would happen if you added $4y$ in the second equation to either side?"
- "What do you now notice?"

Power-up

To power up students' ability to solve a system using elimination, have students complete:

Determine which sequence of operations would eliminate one variable in the system. Select *all* that apply.

$$\begin{cases} 2x + 4y = -5 \\ 4x + 2y = 6 \end{cases}$$

[✓ A.] Multiply the top equation by -2 and then add the equations to eliminate x.
B. Multiply the top equation by 2 and then add the equations to eliminate x.
[✓ C.] Multiply the top equation by 2 and then subtract the equations to eliminate x.
[✓ D.] Multiply the bottom equation by -2 and then add the equations to eliminate y.

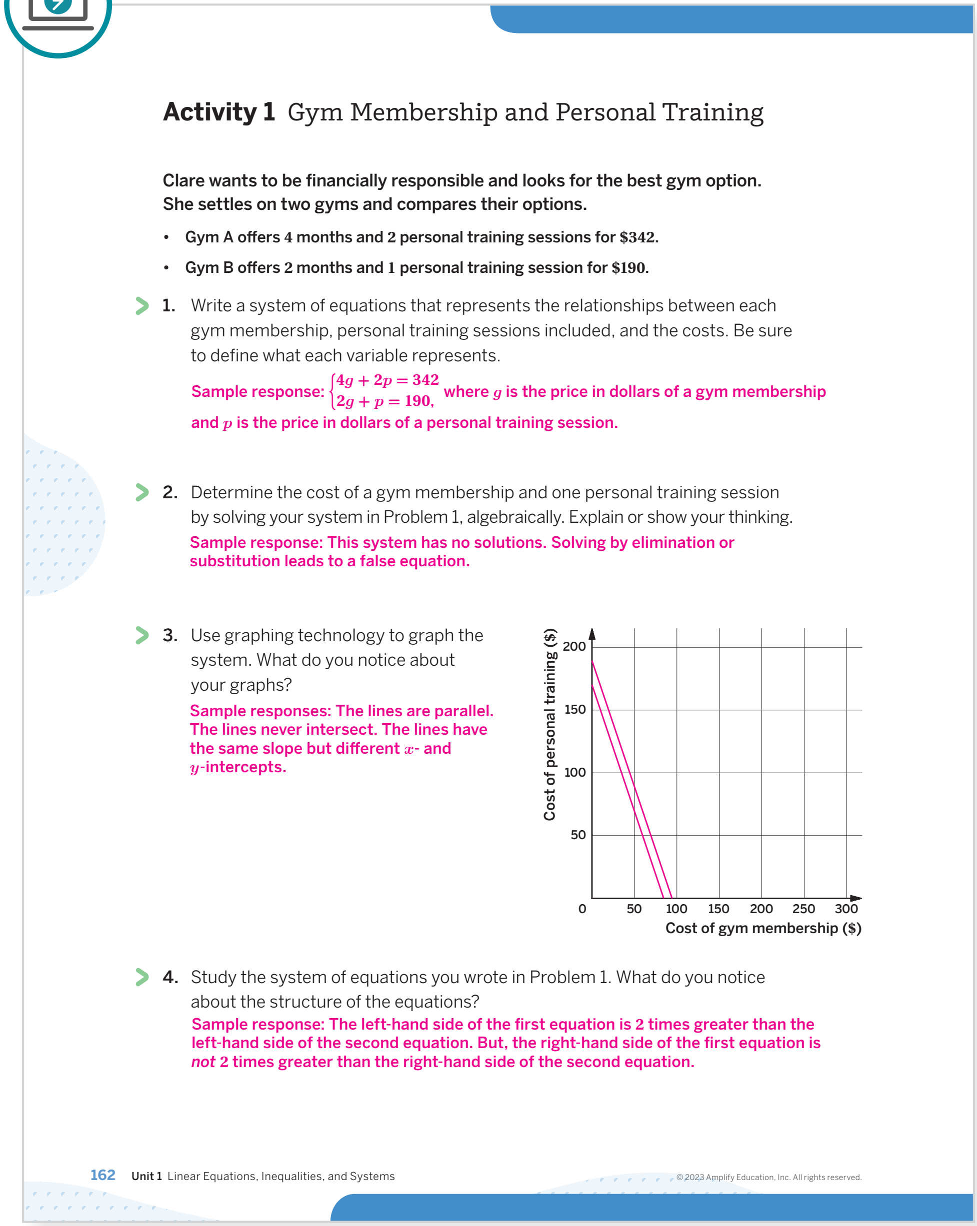
Use: Before the Warm-up

Informed by: Performance on Lesson 21, Practice Problem 5

Small Groups | 10 min

Activity 1 Gym Membership and Personal Training

Students write linear equations to represent a scenario to further analyze the structure of linear systems.



Activity 1 Gym Membership and Personal Training

Clare wants to be financially responsible and looks for the best gym option. She settles on two gyms and compares their options.

- **Gym A offers 4 months and 2 personal training sessions for $342.**
- **Gym B offers 2 months and 1 personal training session for $190.**

1. Write a system of equations that represents the relationships between each gym membership, personal training sessions included, and the costs. Be sure to define what each variable represents.

Sample response: $\begin{cases} 4g + 2p = 342 \\ 2g + p = 190, \end{cases}$ where g is the price in dollars of a gym membership and p is the price in dollars of a personal training session.

2. Determine the cost of a gym membership and one personal training session by solving your system in Problem 1, algebraically. Explain or show your thinking.

Sample response: This system has no solutions. Solving by elimination or substitution leads to a false equation.

3. Use graphing technology to graph the system. What do you notice about your graphs?

Sample responses: The lines are parallel. The lines never intersect. The lines have the same slope but different x- and y-intercepts.

4. Study the system of equations you wrote in Problem 1. What do you notice about the structure of the equations?

Sample response: The left-hand side of the first equation is 2 times greater than the left-hand side of the second equation. But, the right-hand side of the first equation is *not* 2 times greater than the right-hand side of the second equation.

162 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the narrative aloud and discuss gym memberships to help familiarize students with the context. Students should work independently before sharing in their groups. **Note:** Graphing technology is needed for Problem 3.

2 Monitor

Help students get started by having them identify quantities and define variables for the constraints.

Look for points of confusion:

- **Writing an equation in slope-intercept form in Problem 1.** Have students identify other forms of linear equations they know and how it could apply to this context.

Look for productive strategies:

- Recognizing that because both equations have the same slope and different y-intercepts, the lines will be parallel and the system will have no solutions.

3 Connect

Have students share their systems from Problem 1, and then reach a consensus. Select and sequence strategies for Problem 2. Conduct a ***Notice and Wonder*** routine using the graph before discussing Problem 4.

Highlight that the graph of the system is two parallel lines which have the same slope and different y-intercepts.

Display the Anchor Chart PDF, *Types of Linear Systems.*

Ask, "What can you determine about the price of a gym membership and personal training session?" The prices of a gym membership and personal training session differ between Gym A and Gym B.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with the Activity 1 PDF which they can use to guide them towards the correct system of equations to write for Problem 1.

Extension: *Math Enrichment*

Have students describe a third gym, Gym C that, along with Gyms A and B, creates a system of three equations with no solutions.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

- **Read 1:** Students should understand that Clare is comparing two different options for gym membership.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as the number of months for each option.
- **Read 3:** Ask students to think about how they could represent this information with a system of equations.

English Learners

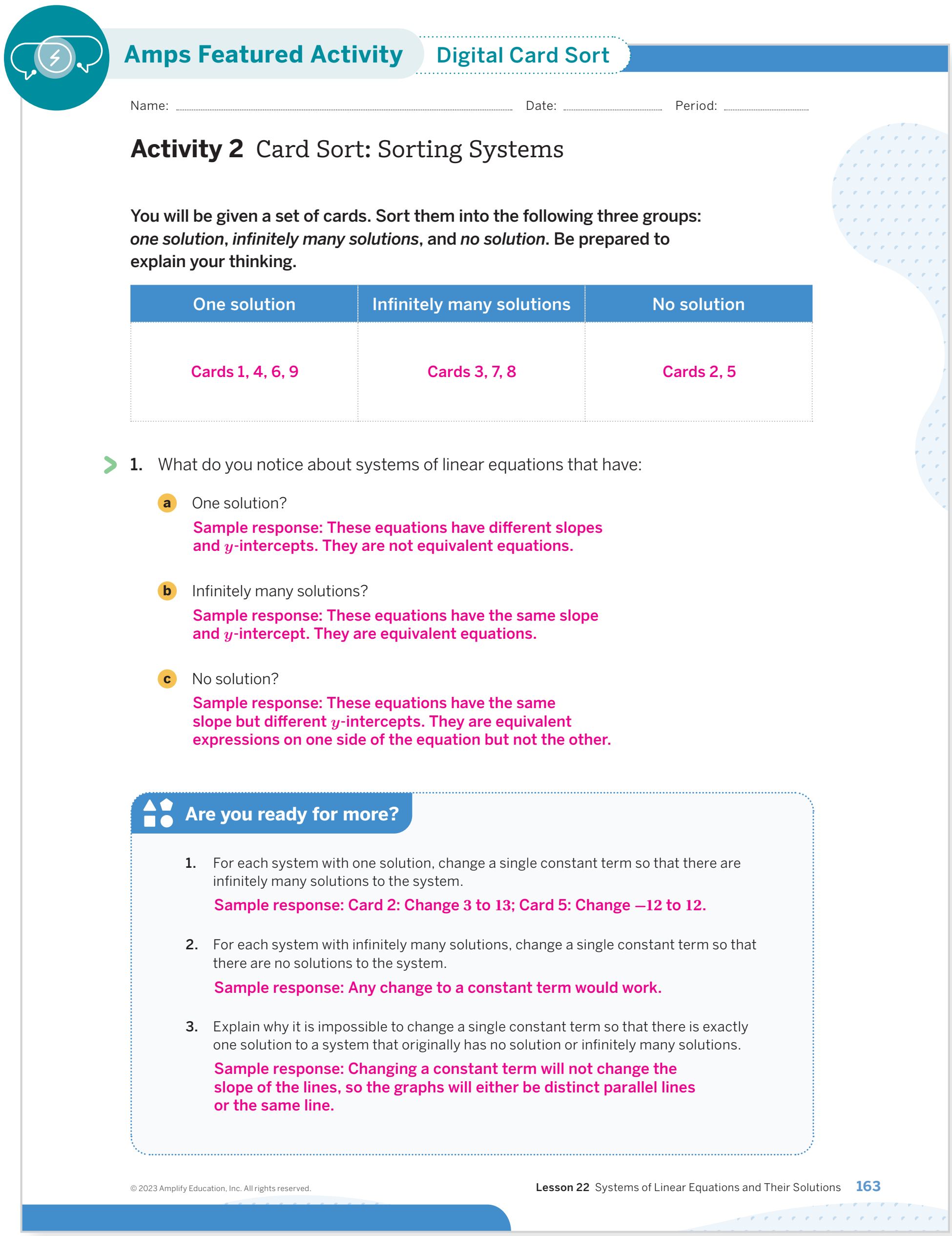
Have students color code the number of months in one color and the number of training sessions in another color.

Pairs | 20 min

Activity 2 Card Sort: Sorting Systems

Students analyze the structure of linear systems as a way to categorize them by the number of solutions.

Amps Featured Activity Digital Card Sort



Name: ______ Date: ______ Period: ______

Activity 2 Card Sort: Sorting Systems

You will be given a set of cards. Sort them into the following three groups: *one solution*, *infinitely many solutions*, and *no solution*. Be prepared to explain your thinking.

One solution	Infinitely many solutions	No solution
Cards 1, 4, 6, 9	Cards 3, 7, 8	Cards 2, 5

1. What do you notice about systems of linear equations that have:

   a. One solution?

   Sample response: These equations have different slopes and y-intercepts. They are not equivalent equations.

   b. Infinitely many solutions?

   Sample response: These equations have the same slope and y-intercept. They are equivalent equations.

   c. No solution?

   Sample response: These equations have the same slope but different y-intercepts. They are equivalent expressions on one side of the equation but not the other.

Are you ready for more?

1. For each system with one solution, change a single constant term so that there are infinitely many solutions to the system.

   Sample response: Card 2: Change 3 to 13; Card 5: Change –12 to 12.

2. For each system with infinitely many solutions, change a single constant term so that there are no solutions to the system.

   Sample response: Any change to a constant term would work.

3. Explain why it is impossible to change a single constant term so that there is exactly one solution to a system that originally has no solution or infinitely many solutions.

   Sample response: Changing a constant term will not change the slope of the lines, so the graphs will either be distinct parallel lines or the same line.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 Systems of Linear Equations and Their Solutions 163

1 Launch

Students remain in pairs. Distribute the pre-cut cards from the Activity 2 PDF to each pair. Conduct the ***Card Sort*** routine, with students working together to sort the cards into the three groups. **Note:** Graphing technology should not be used.

2 Monitor

Help students get started by having them make observations about the structure of the equations in each system, for example, the same slopes, different intercepts, etc.

Look for points of confusion:

- **Attempting to solve each system algebraically.** Ask, "How could you analyze the slopes or y-intercepts instead?"
- **Noticing equations with the same slope but not the y-intercept.** Ask, "Besides slope, what are other important characteristics of linear equations?"

Look for productive strategies:

- Examining the slope and y-intercept or rewriting equations in standard form to look for equivalent equations.
- Looking for equations with the same slope but different y-intercepts.

3 Connect

Have pairs of students share their strategies for sorting the equations. Select and sequence students, beginning with those who solved algebraically, then those who re-wrote equations, and then those who analyzed the different parts of the equations.

Highlight that solving each system algebraically is not always the most efficient strategy. Here, analyzing by looking for equivalent equations works more efficiently.

Ask, "Why is analyzing the structure of the equations in a linear system beneficial?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider using one of these alternative approaches to this activity:

- Remove all cards that have one solution and have students only sort the cards with no solution or infinitely many solutions.
- Provide students a subset of the cards, no more than five cards at a time, and introduce the remaining cards once the initial set has been sorted.

Math Language Development

MLR8: Discussion Supports

While pairs of students work, encourage them to take turns selecting and sorting cards. Display the sentence frame: "This system has _____ solutions because . . ." Encourage students to challenge each other if they disagree and to challenge each other to use their developing mathematical language.

English Learners

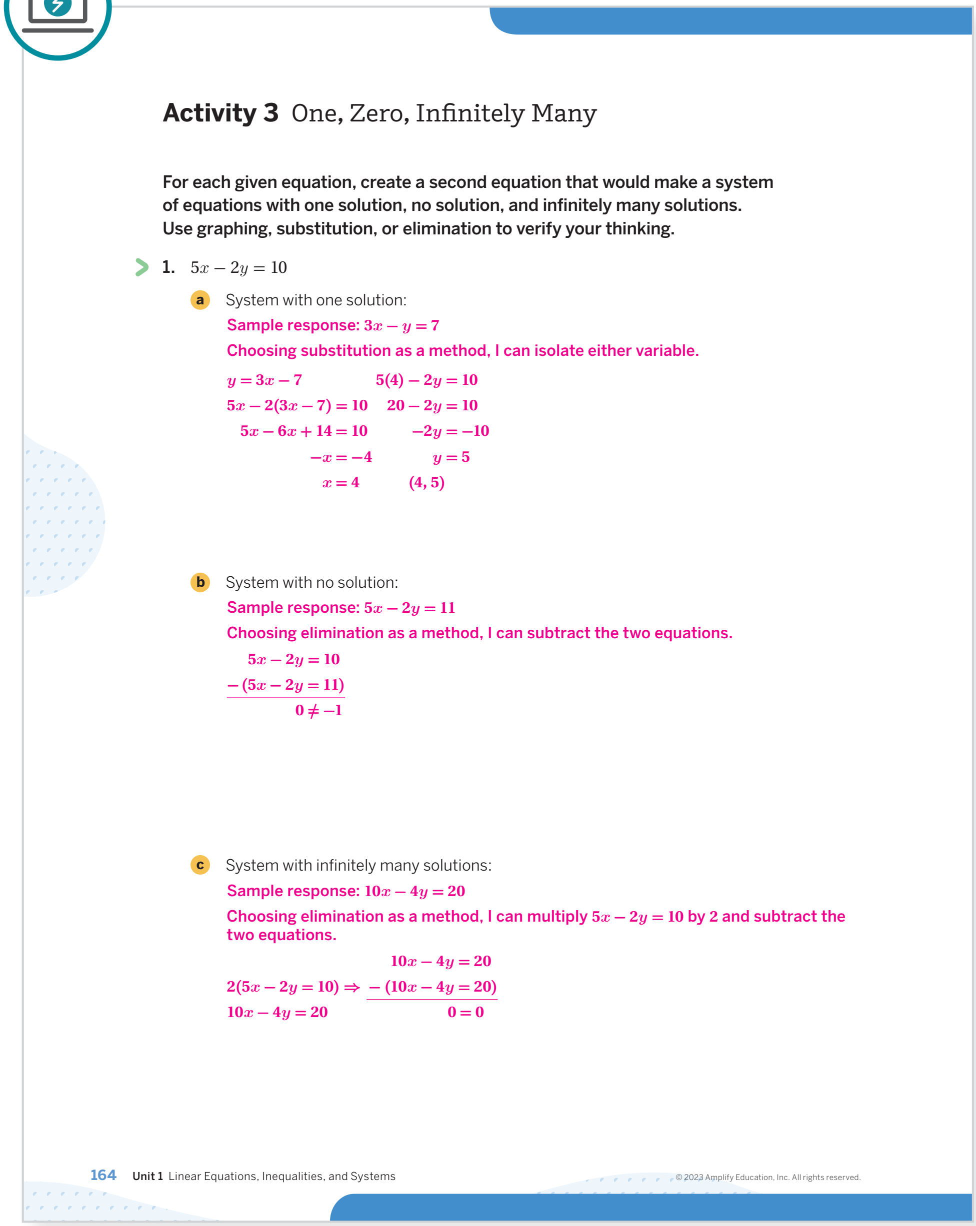
Provide a sample completed sentence frame, such as, "Card 1 has one solution because the equations have different slopes, 2 and –7, and different y-intercepts, –7 and 2."

Optional

Pairs | 10 min

Activity 3 One, Zero, Infinitely Many

Students create linear systems that have one, zero, or infinitely many solutions to make use of the structure of linear equations.



Activity 3 One, Zero, Infinitely Many

For each given equation, create a second equation that would make a system of equations with one solution, no solution, and infinitely many solutions. Use graphing, substitution, or elimination to verify your thinking.

1. $5x - 2y = 10$

   a. System with one solution:

   Sample response: $3x - y = 7$

   Choosing substitution as a method, I can isolate either variable.

   $$\begin{aligned} y &= 3x - 7 & 5(4) - 2y &= 10 \\ 5x - 2(3x - 7) &= 10 & 20 - 2y &= 10 \\ 5x - 6x + 14 &= 10 & -2y &= -10 \\ -x &= -4 & y &= 5 \\ x &= 4 & &(4, 5) \end{aligned}$$

   b. System with no solution:

   Sample response: $5x - 2y = 11$

   Choosing elimination as a method, I can subtract the two equations.

   $$\begin{array}{r} 5x - 2y = 10 \\ \underline{-\,(5x - 2y = 11)} \\ 0 \neq -1 \end{array}$$

   c. System with infinitely many solutions:

   Sample response: $10x - 4y = 20$

   Choosing elimination as a method, I can multiply $5x - 2y = 10$ by 2 and subtract the two equations.

   $$\begin{array}{ll} & 10x - 4y = 20 \\ 2(5x - 2y = 10) \Rightarrow & \underline{-\,(10x - 4y = 20)} \\ 10x - 4y = 20 & \qquad\qquad 0 = 0 \end{array}$$

164 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Keep students in pairs. Students restate directions in their own words to verify understanding. **Note:** Graphing technology is optional.

2 Monitor

Help students get started by asking, "How are the equations of linear systems with one solution different from other systems?"

Look for points of confusion:

- **Creating second equations with illogical reasoning.** Refer to the similarities and differences highlighted in Activity 2.

Look for productive strategies:

- Changing the slope and y-intercept of the given equation for one solution.
- Changing the value of the constant for no solutions.
- Multiplying any constant to the given equation for infinitely many solutions.

Activity 3 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with the following template for writing their equations.

$\_\_\_x + \_\_\_y = \_\_\_$

Extension: *Math Enrichment*

Have students use the given equation in Problem 1 to write a system of three equations that has no solution, and then to write a system of three equations that has infinitely many solutions. Sample responses shown.

No solution:
$$\begin{cases} 5x - 2y = 10 \\ 5x - 2y = 11 \\ 5x - 2y = 12 \end{cases}$$

Infinitely many solutions:
$$\begin{cases} 5x - 2y = 10 \\ 10x - 4y = 20 \\ 15x - 6y = 30 \end{cases}$$

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their strategies for writing the equations, draw connections to the coefficients and constant terms of each equation and how they knew the system would have no solution, one solution, or infinitely many solutions. Encourage language use such as *coefficient*, *constant*, *slope*, y-*intercept*, etc.

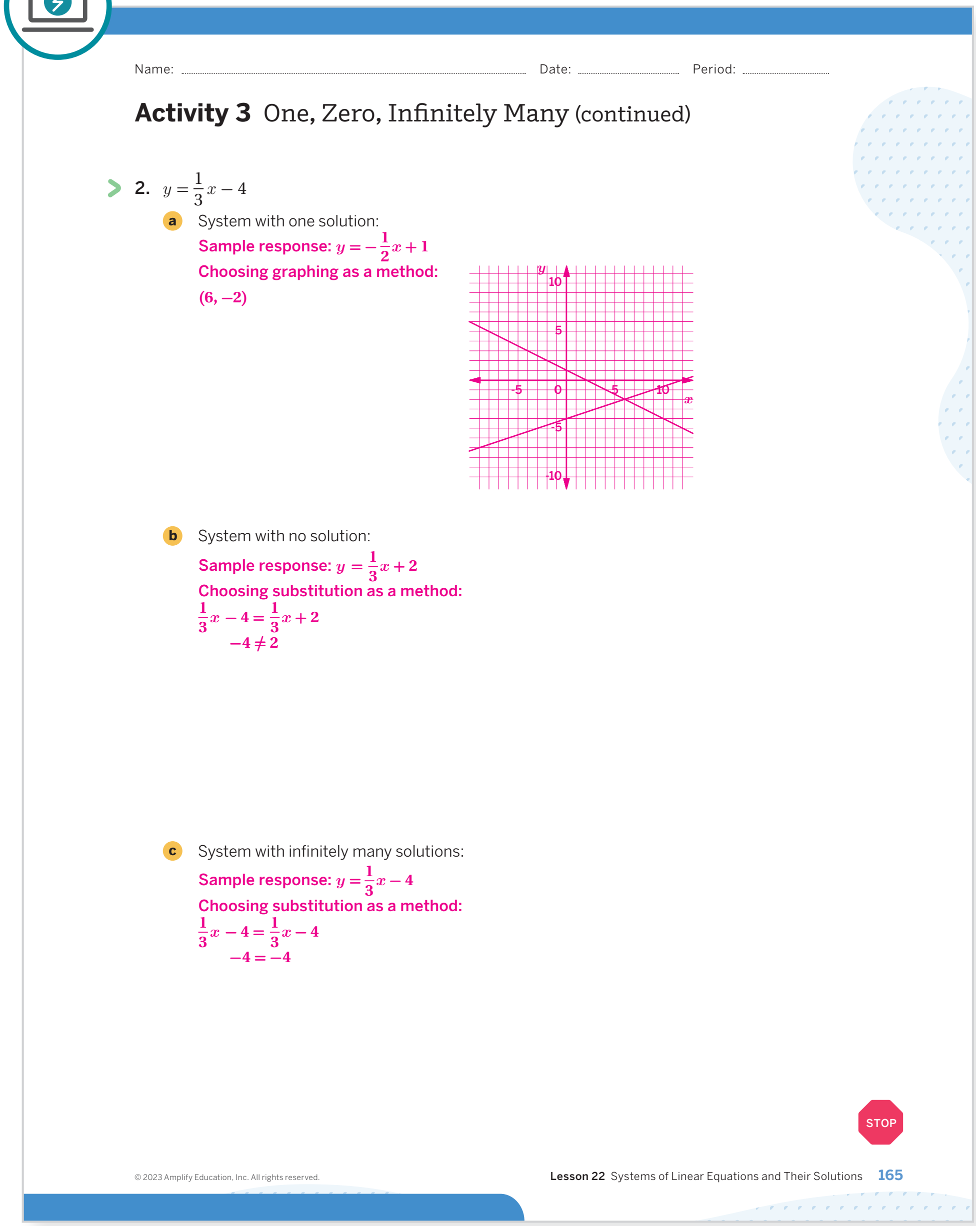
English Learners

Display three systems for Problem 1 and annotate them as no solution, one solution, or infinitely many solutions. Color code the connections between the coefficients and constants of the terms.

Pairs | 10 min

Activity 3 One, Zero, Infinitely Many (continued)

Students create linear systems that have one, zero, or infinitely many solutions to make use of the structure of linear equations.



Name: Date: Period:

Activity 3 One, Zero, Infinitely Many (continued)

2. $y = \frac{1}{3}x - 4$

a System with one solution:

Sample response: $y = -\frac{1}{2}x + 1$

Choosing graphing as a method:

(6, –2)

b System with no solution:

Sample response: $y = \frac{1}{3}x + 2$

Choosing substitution as a method:

$$\frac{1}{3}x - 4 = \frac{1}{3}x + 2$$
$$-4 \neq 2$$

c System with infinitely many solutions:

Sample response: $y = \frac{1}{3}x - 4$

Choosing substitution as a method:

$$\frac{1}{3}x - 4 = \frac{1}{3}x - 4$$
$$-4 = -4$$

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 Systems of Linear Equations and Their Solutions 165

3 Connect

Have pairs of students share strategies for writing the second equation. Select and sequence students using graphing technology, algebraic manipulation, and the structure of the equations to reason about the second equation.

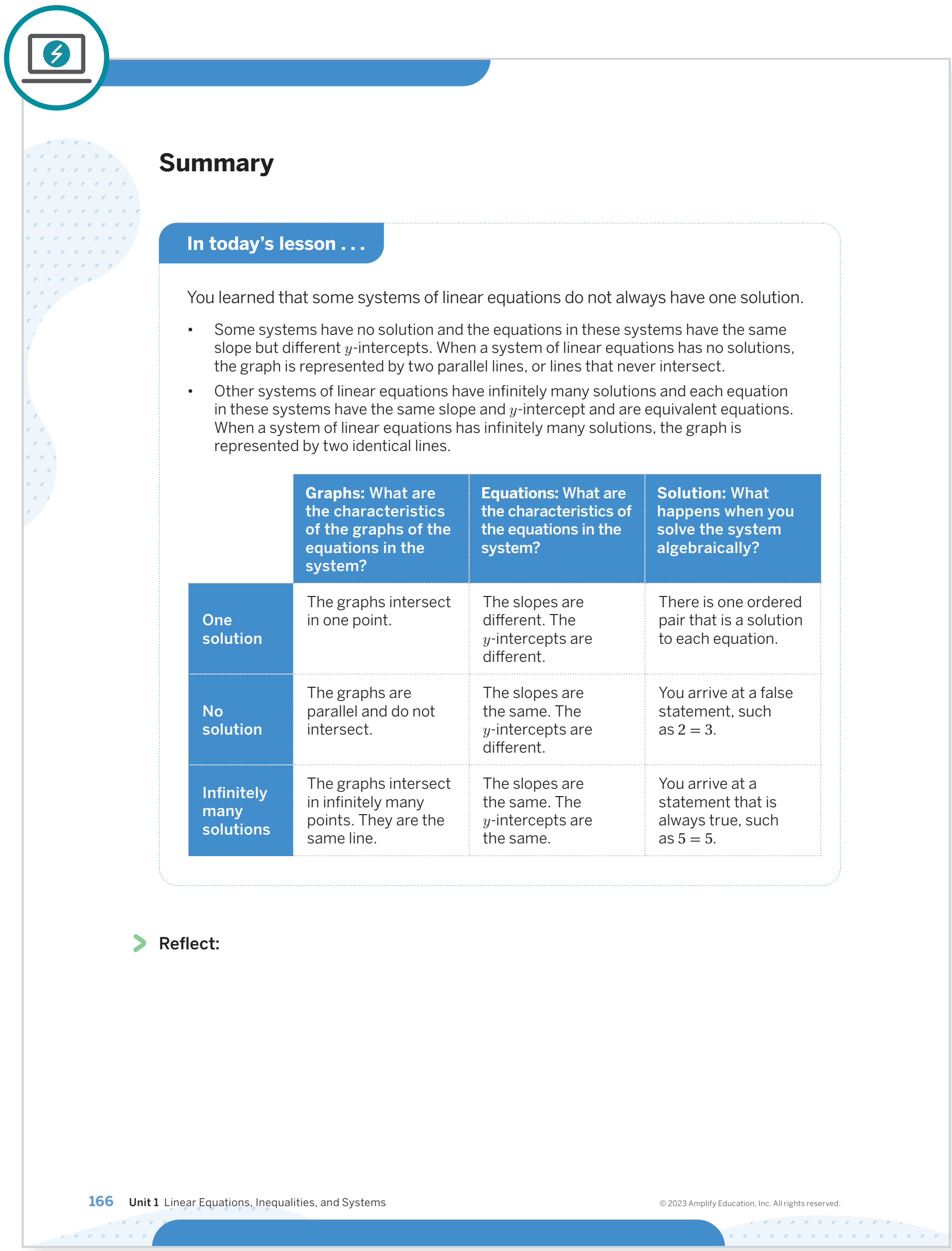
Highlight that changing the coefficients and constants create one-solution systems. Having the same slope but different y-intercepts creates no solutions, and having equivalent equations creates infinite solutions.

Display a graph of the given equation. Demonstrate strategies outlined in the Highlight to provide students a visual representation.

Whole Class | 5 min

Summary

Review and synthesize the different possibilities for the number solutions to a system of linear equations.



Summary

In today's lesson . . .

You learned that some systems of linear equations do not always have one solution.

- Some systems have no solution and the equations in these systems have the same slope but different y-intercepts. When a system of linear equations has no solutions, the graph is represented by two parallel lines, or lines that never intersect.
- Other systems of linear equations have infinitely many solutions and each equation in these systems have the same slope and y-intercept and are equivalent equations. When a system of linear equations has infinitely many solutions, the graph is represented by two identical lines.

	Graphs: What are the characteristics of the graphs of the equations in the system?	**Equations:** What are the characteristics of the equations in the system?	**Solution:** What happens when you solve the system algebraically?
One solution	The graphs intersect in one point.	The slopes are different. The y-intercepts are different.	There is one ordered pair that is a solution to each equation.
No solution	The graphs are parallel and do not intersect.	The slopes are the same. The y-intercepts are different.	You arrive at a false statement, such as $2 = 3$.
Infinitely many solutions	The graphs intersect in infinitely many points. They are the same line.	The slopes are the same. The y-intercepts are the same.	You arrive at a statement that is always true, such as $5 = 5$.

Reflect:

166 Unit 1 Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the following equations. Record students' responses as they share.

- $\begin{cases} 3x + 4y = 8 \\ 3x - 4y = 8 \end{cases}$
- $\begin{cases} 3x + 4y = 8 \\ 6x + 8y = 16 \end{cases}$
- $\begin{cases} 3x + 4y = 8 \\ 3x + 4y = -4 \end{cases}$

Have students share the characteristics for each number of solutions as it relates to graphs, equations, and solutions.

Highlight that equivalent equations give a linear system with infinite solutions with identical lines. Two linear equations with the same slope but a different y-intercept gives a system with no solutions, or two parallel lines. Two linear equations with a different slope gives a system with one solution with lines that intersect at one point.

Ask, "Why is examining the structure of equations in a system useful?"

If I examine the slope or y-intercept, or look for equivalent equations, I can determine the number of solutions to the system without solving, which is efficient.

Reflect

After synthesizing the concepts of the lesson, allow a few moments for students to reflect on determining the number of solutions to a system of linear equations. Provide students with the Anchor Chart PDF, *Types of Linear Systems*. Encourage them to record any notes in the *Reflect* space provided in the Student Edition.

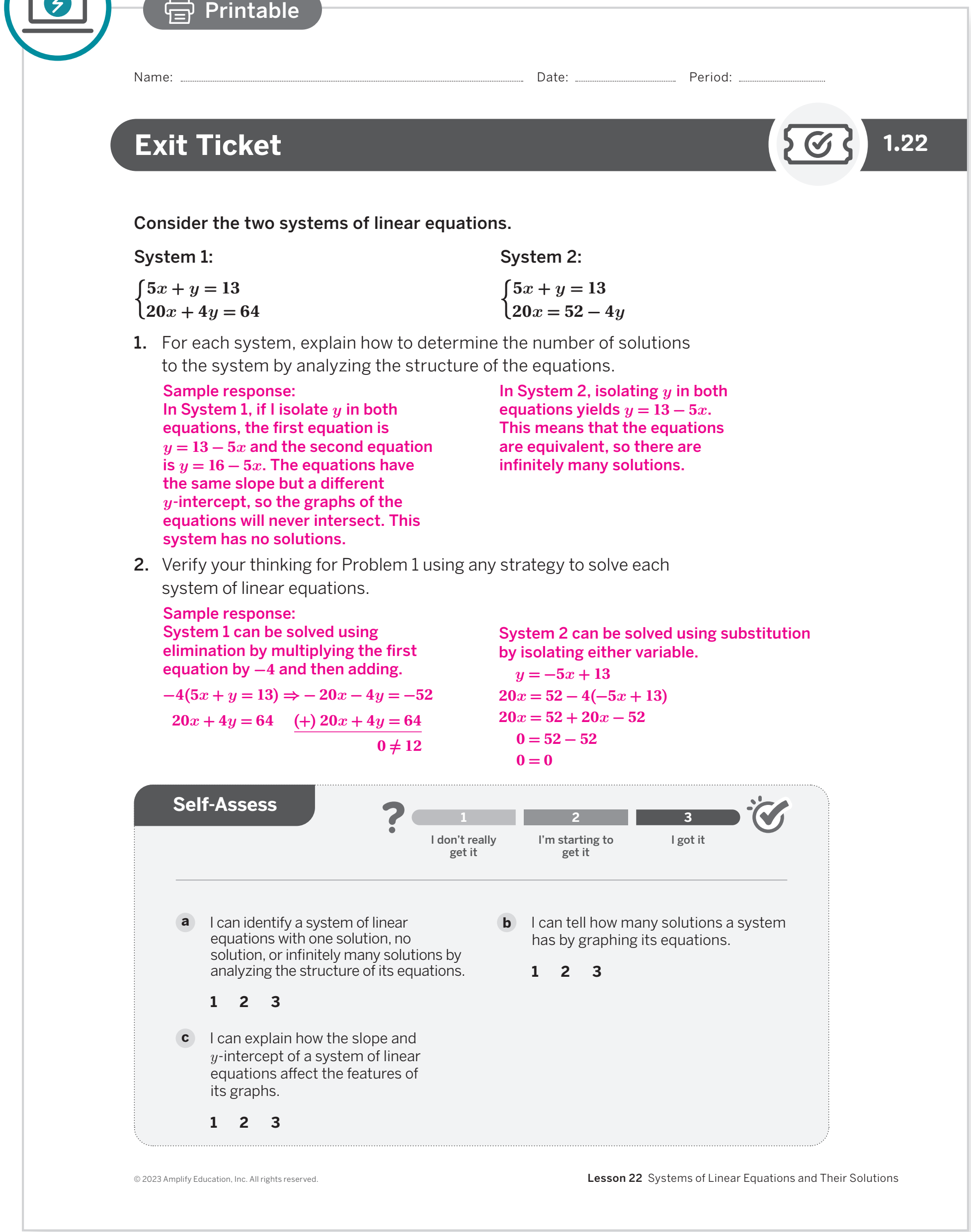
- "What are some different ways to determine the number of solutions to a system of linear equations?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the number of solutions to systems of linear equations by making use of structure.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 1.22

Consider the two systems of linear equations.

System 1:

$$\begin{cases} 5x + y = 13 \\ 20x + 4y = 64 \end{cases}$$

System 2:

$$\begin{cases} 5x + y = 13 \\ 20x = 52 - 4y \end{cases}$$

1. For each system, explain how to determine the number of solutions to the system by analyzing the structure of the equations.

Sample response:
In System 1, if I isolate y in both equations, the first equation is $y = 13 - 5x$ and the second equation is $y = 16 - 5x$. The equations have the same slope but a different y-intercept, so the graphs of the equations will never intersect. This system has no solutions.

In System 2, isolating y in both equations yields $y = 13 - 5x$. This means that the equations are equivalent, so there are infinitely many solutions.

2. Verify your thinking for Problem 1 using any strategy to solve each system of linear equations.

Sample response:
System 1 can be solved using elimination by multiplying the first equation by -4 and then adding.

$$-4(5x + y = 13) \Rightarrow -20x - 4y = -52$$

$$20x + 4y = 64 \quad \underline{(+)\ 20x + 4y = 64}$$

$$0 \neq 12$$

System 2 can be solved using substitution by isolating either variable.

$$y = -5x + 13$$
$$20x = 52 - 4(-5x + 13)$$
$$20x = 52 + 20x - 52$$
$$0 = 52 - 52$$
$$0 = 0$$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can identify a system of linear equations with one solution, no solution, or infinitely many solutions by analyzing the structure of its equations.
1 2 3

b. I can tell how many solutions a system has by graphing its equations.
1 2 3

c. I can explain how the slope and y-intercept of a system of linear equations affect the features of its graphs.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 Systems of Linear Equations and Their Solutions

Success looks like . . .

- **Goal:** Determining whether a system of equations will have one solution, no solution, or infinitely many solutions by analyzing the structure of its equations.
  - » Analyzing Systems 1 and 2 and explaining how many solutions each system has in Problem 1.
- **Goal:** Determining how many solutions a system has by graphing.
- **Language Goal:** Explaining how the slope and y-intercept of a system of equations affect the features of its graphs. **(Speaking and Listening, Writing)**

Suggested next steps

If students incorrectly determine the number of solutions to each system in Problem 1, consider:

- Reviewing strategies for determining the number of solutions to a system from Activity 2.
- Assigning Practice Problem 1.
- Asking, "How would you use equivalent equations, the slope, or the y-intercept to analyze the system?"

If students incorrectly solve the systems in Problem 2, consider:

- Reviewing strategies for solving a linear system of equations from Activity 1.
- Assigning Practice Problem 4.
- Asking, "How would you use the form of the linear equations to determine what strategy to use?"

Professional Learning

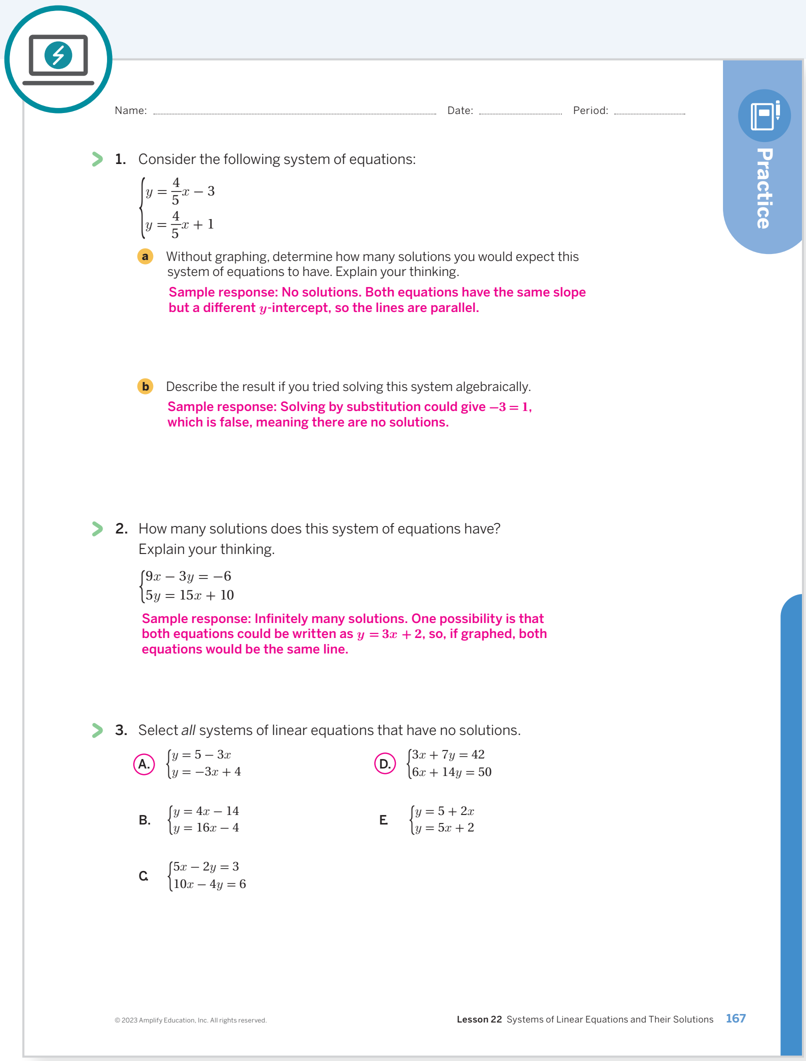
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did sorting systems of linear equations by number of solutions reveal about your students as learners?
- What surprised you about how students reasoned when sorting the systems in Activity 2? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Consider the following system of equations:

$$\begin{cases} y = \frac{4}{5}x - 3 \\ y = \frac{4}{5}x + 1 \end{cases}$$

a. Without graphing, determine how many solutions you would expect this system of equations to have. Explain your thinking.

Sample response: No solutions. Both equations have the same slope but a different y-intercept, so the lines are parallel.

b. Describe the result if you tried solving this system algebraically.

Sample response: Solving by substitution could give $-3 = 1$, which is false, meaning there are no solutions.

2. How many solutions does this system of equations have? Explain your thinking.

$$\begin{cases} 9x - 3y = -6 \\ 5y = 15x + 10 \end{cases}$$

Sample response: Infinitely many solutions. One possibility is that both equations could be written as $y = 3x + 2$, so, if graphed, both equations would be the same line.

3. Select *all* systems of linear equations that have no solutions.

[✓ A.] $\begin{cases} y = 5 - 3x \\ y = -3x + 4 \end{cases}$

B. $\begin{cases} y = 4x - 14 \\ y = 16x - 4 \end{cases}$

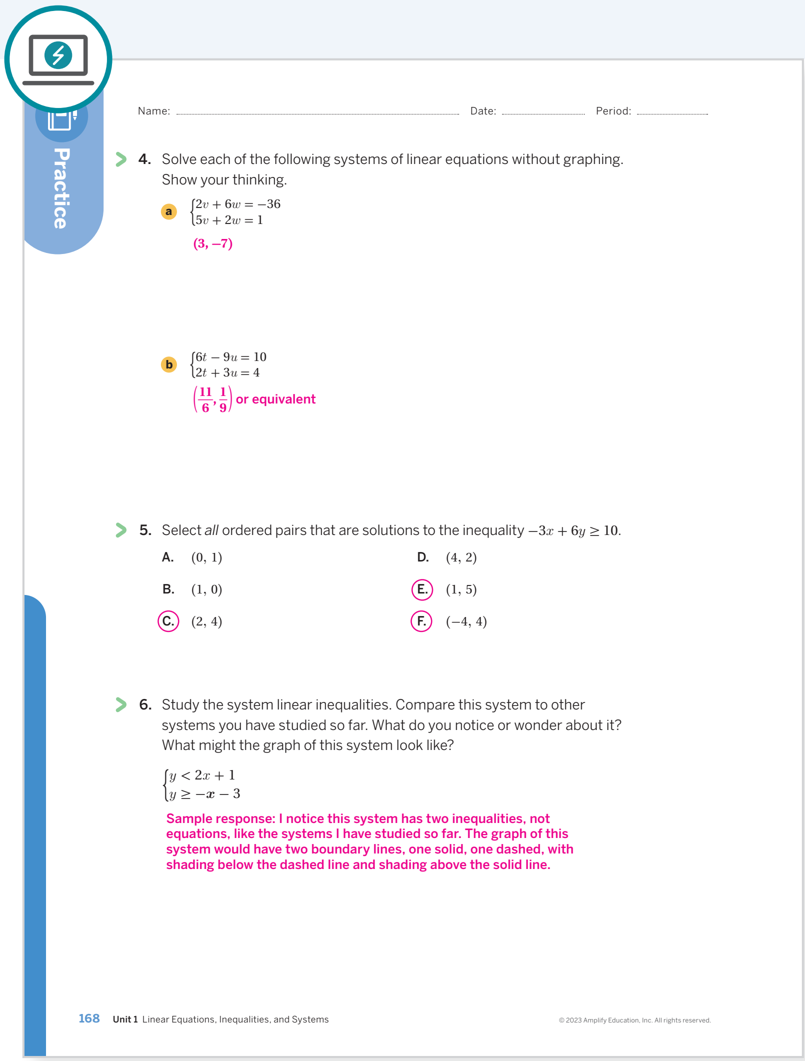
C. $\begin{cases} 5x - 2y = 3 \\ 10x - 4y = 6 \end{cases}$

[✓ D.] $\begin{cases} 3x + 7y = 42 \\ 6x + 14y = 50 \end{cases}$

E. $\begin{cases} y = 5 + 2x \\ y = 5x + 2 \end{cases}$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 Systems of Linear Equations and Their Solutions 167



Practice

Name: Date: Period:

4. Solve each of the following systems of linear equations without graphing. Show your thinking.

a. $\begin{cases} 2v + 6w = -36 \\ 5v + 2w = 1 \end{cases}$

$(3, -7)$

b. $\begin{cases} 6t - 9u = 10 \\ 2t + 3u = 4 \end{cases}$

$\left(\frac{11}{6}, \frac{1}{9}\right)$ or equivalent

5. Select *all* ordered pairs that are solutions to the inequality $-3x + 6y \geq 10$.

A. $(0, 1)$

B. $(1, 0)$

[✓ C.] $(2, 4)$

D. $(4, 2)$

[✓ E.] $(1, 5)$

[✓ F.] $(-4, 4)$

6. Study the system linear inequalities. Compare this system to other systems you have studied so far. What do you notice or wonder about it? What might the graph of this system look like?

$$\begin{cases} y < 2x + 1 \\ y \geq -x - 3 \end{cases}$$

Sample response: I notice this system has two inequalities, not equations, like the systems I have studied so far. The graph of this system would have two boundary lines, one solid, one dashed, with shading below the dashed line and shading above the solid line.

168 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	1
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 21	2
	5	Unit 1 Lesson 15	2
Formative	6	Unit 1 Lesson 23	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 5

Systems of Linear Inequalities in Two Variables

Whole Class

In this Sub-Unit, students are faced with real-world situations in which the decision becomes more complicated. They step back to take a look at the larger picture and then fine tune the details where their decisions overlap.

SUB-UNIT

5 Systems of Linear Inequalities in Two Variables

Narrative Connections

Is there such a thing as *too much* choice?

Next time you're at the supermarket, take a walk down the cereal aisle.

There, you will find dozens of boxes in a wide assortment of colors, flavors, and mascots. Some will have marshmallows, others whole-grain oats, puffed corn kernels, or almond clusters. There are cereals shaped like animals, hearts, O's, or squares. With the vast number of choices at your disposal, a shopper could be excused for feeling downright exhausted.

Some psychologists call this phenomenon "overchoice," or "choice overload." They believe that whenever a person is confronted with too many options, their mind can become overwhelmed by the risks and outcomes should they make the wrong choice.

As far as cereals go, the stakes aren't exactly high. But it's a different story altogether when it comes to things like choosing a career or picking a college major. These decisions can lead to an individual to agonize, procrastinate, or be unmotivated, to the point of not making any choice at all.

The key is being able to whittle down your choices, taking into account constraints. By putting boundaries around what might feel like a sea of options, you can reduce them to a size that is manageable. One way to do this is to express your constraints mathematically, such as with a system of inequalities. The more constraints you have, the more you can narrow your options and avoid the trap of overchoice.

© 2023 Amplify Education, Inc. All rights reserved.

Sub-Unit 5 Systems of Linear Inequalities in Two Variables 169

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students will graph systems of inequalities and see how each new inequality constrains the solution further (and hopefully prevents overchoice) in the following places:

- **Lesson 24, Activity 2:** Help Design the Graphic
- **Lesson 25, Activity 1:** Custom Food Bars
- **Lesson 26, Activity 3:** Optimizing Revenue

UNIT 1 | **LESSON 23**

Graphing Systems of Linear Inequalities

Let's solve problems by graphing systems of inequalities in two variables.

Focus

Goals

1. **Language Goal:** Explain how to determine if an ordered pair is a solution to a system of inequalities. **(Speaking and Listening, Writing)**
2. **Language Goal:** Graph a system of inequalities and describe the solutions. **(Speaking and Listening, Writing)**
3. Determine if an ordered pair on a boundary line to a system of inequalities is a solution to the system.

Rigor

- Students build **conceptual understanding** of the solutions to systems of linear inequalities by graphing.
- Students determine if ordered pairs are solutions to a system of linear inequalities algebraically and graphically to develop **procedural fluency**.

Coherence

Today

Students learn that two linear inequalities that represent the constraints in the same situation form a system of inequalities, and that solutions to the system include all values that satisfy both inequalities simultaneously. They observe that the graph of the solution set is represented by the region where the inequalities overlap.

Previously

In Lessons 13–16, students solved and graphed one- and two-variable linear inequalities with and without context.

Coming Soon

In Lesson 24, students will write and solve systems of linear inequalities from a graph and a context.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
10 min	15 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- Activity 1 PDF (answers)
- Activity 3 PDF, pre-cut cards, one set per pair
- Anchor Chart PDF, *Inequality Symbols and Key Phrases*

Math Language Development

New words

- ***overlap of the graphs of the inequalities***
- ***system of linear inequalities***

Review words

- *boundary line*
- *inequality*

Amps | Featured Activity

Activity 1
Interactive Graph

Students use an interactive graph to enhance the experience of graphing systems of inequalities to solve a problem.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might show disinterest as others share their responses in Activity 2. Prior to the presentations, review guidelines for social engagement. Discuss ways to hold other people's attention when presenting. Also emphasize how to show interest when others are presenting. Healthy communication in both directions will lead towards establishing healthy relationships.

Modifications to Pacing

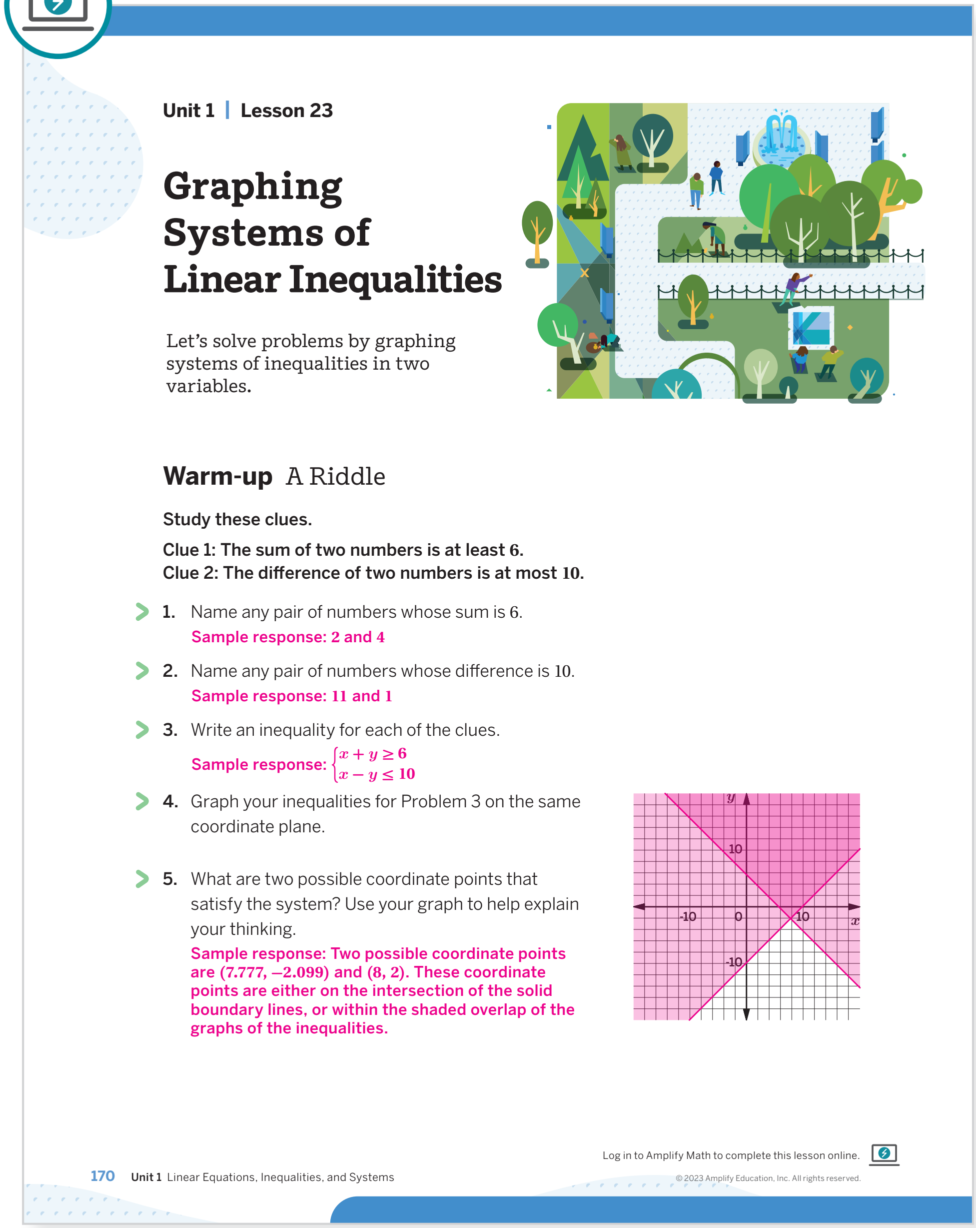
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problems 1 and 2 may be omitted.
- The **Warm-up** may be omitted.
- Optional **Activity 3** may be omitted.

Independent | 10 min

Warm-up A Riddle

Students write, solve, and graph a system of linear inequalities to prepare for identifying solutions of these systems.



Unit 1 | Lesson 23

Graphing Systems of Linear Inequalities

Let's solve problems by graphing systems of inequalities in two variables.

Warm-up A Riddle

Study these clues.

Clue 1: The sum of two numbers is at least 6.
Clue 2: The difference of two numbers is at most 10.

1. Name any pair of numbers whose sum is 6.
   Sample response: 2 and 4
2. Name any pair of numbers whose difference is 10.
   Sample response: 11 and 1
3. Write an inequality for each of the clues.
   Sample response: $\begin{cases} x + y \geq 6 \\ x - y \leq 10 \end{cases}$
4. Graph your inequalities for Problem 3 on the same coordinate plane.
5. What are two possible coordinate points that satisfy the system? Use your graph to help explain your thinking.
   Sample response: Two possible coordinate points are (7.777, –2.099) and (8, 2). These coordinate points are either on the intersection of the solid boundary lines, or within the shaded overlap of the graphs of the inequalities.

Log in to Amplify Math to complete this lesson online.

170 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have students work on the problems independently and then share their responses with a partner.

2 Monitor

Help students get started by asking, "What are two numbers that, when added, give 5.678 and, when subtracted, give 9.876?"

Look for points of confusion:

- **Writing the system as $x + y > 9.876$ and $x - y < 5.678$.** Ask students to read the problem to themselves and pay attention to the wording "at least" and "at most."

Look for productive strategies:

- Using a table or chart to organize pairs of numbers whose sum is 5.678 or whose difference is 9.876.
- Writing a linear system of equations to determine a solution.

3 Connect

Display students' responses for Problems 1 and 2. Record their responses as they share.

Have students share their systems and graphs for Problems 3 and 4. Refer to the responses from Problems 1 and 2, and ask which of the responses are solutions to the system of inequalities.

Ask, "How many solutions are there to the riddle?"

Highlight the differences between the solutions to systems of linear equations and systems of linear inequalities.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you highlight the differences between the solutions to systems of linear equations and systems of linear inequalities, consider asking students how the clues would be altered if this situation was represented by a system of equations. Then ask them how the graph would be altered. Instead of "is at least" in the clues, the clues would say "is." The graph would not have any shading.

English Learners

Display the clues and graphs of the system of equations side-by-side with the system of inequalities. Point to the differences as you highlight them.

Power-up

To power up students' ability to identify the difference between a system of equations and a system of inequalities, have students complete:

Determine which systems are a system of inequalities. Select *all* that apply.

A. $\begin{cases} x + y = 6 \\ x - y = 10 \end{cases}$

[✓ C.] $\begin{cases} x + y \geq 6 \\ x - y \leq 10 \end{cases}$

B. $\begin{cases} y = 6 - x \\ y = x - 10 \end{cases}$

[✓ D.] $\begin{cases} y \geq 6 - x \\ y \geq x - 10 \end{cases}$

Use: Before the Warm-up

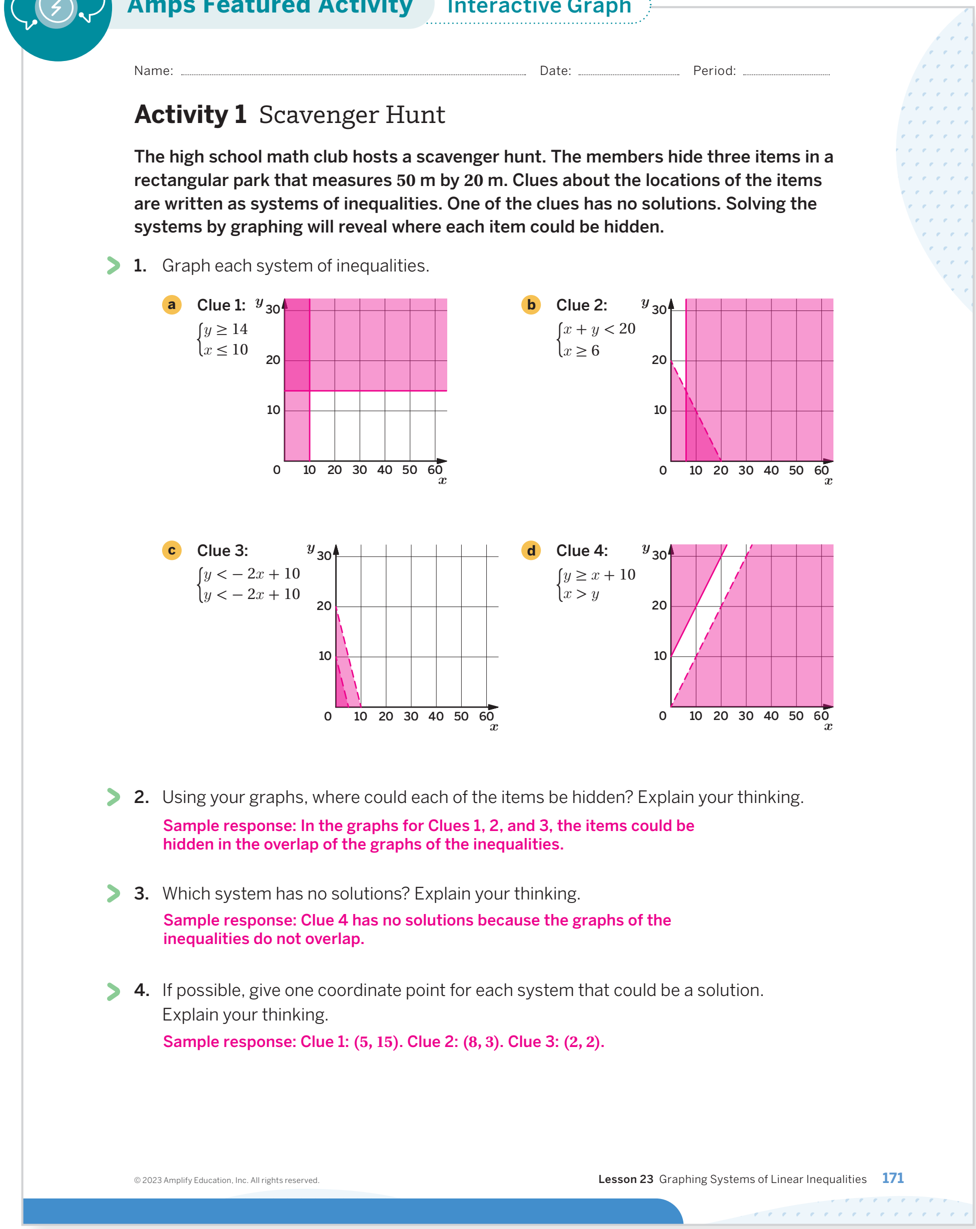
Informed by: Performance on Lesson 22, Practice Problem 6

Pairs | 15 min

Activity 1 Scavenger Hunt

Students graph systems of linear inequalities and reason quantitatively on their solutions to understand how to represent solutions graphically.

Amps Featured Activity Interactive Graph



Name: ______ Date: ______ Period: ______

Activity 1 Scavenger Hunt

The high school math club hosts a scavenger hunt. The members hide three items in a rectangular park that measures 50 m by 20 m. Clues about the locations of the items are written as systems of inequalities. One of the clues has no solutions. Solving the systems by graphing will reveal where each item could be hidden.

1. Graph each system of inequalities.

   a. Clue 1: $\begin{cases} y \geq 14 \\ x \leq 10 \end{cases}$

   b. Clue 2: $\begin{cases} x + y < 20 \\ x \geq 6 \end{cases}$

   c. Clue 3: $\begin{cases} y < -2x + 10 \\ y < -2x + 10 \end{cases}$

   d. Clue 4: $\begin{cases} y \geq x + 10 \\ x > y \end{cases}$

2. Using your graphs, where could each of the items be hidden? Explain your thinking.

   Sample response: In the graphs for Clues 1, 2, and 3, the items could be hidden in the overlap of the graphs of the inequalities.

3. Which system has no solutions? Explain your thinking.

   Sample response: Clue 4 has no solutions because the graphs of the inequalities do not overlap.

4. If possible, give one coordinate point for each system that could be a solution. Explain your thinking.

   Sample response: Clue 1: (5, 15). Clue 2: (8, 3). Clue 3: (2, 2).

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in pairs. Display the Activity 1 PDF and have students work with their partner before whole-class discussion. Discuss differences and similarities between graphs of systems of linear equations and inequalities.

2 Monitor

Help students get started by using the ***Three Reads*** routine to make sure they understand the boundaries of the park.

Look for points of confusion:

- **Only graphing the boundary lines of the inequalities.** Ask, "Do only coordinate points along the boundary line satisfy the inequality?"
- **Making all boundary lines solid.** Ask, "What information do the inequality symbols tell you about the boundary line?"

Look for productive strategies:

- Testing coordinate points in each inequality in the system to determine which side of the boundary line to shade and whether the line is solid or dashed.

3 Connect

Have pairs of students share their graphs for the four clues and responses to the problems. Select and sequence student responses mentioning the shaded regions and the overlap of the graphs of the inequalities and algebraically testing points.

Define the terms ***systems of linear inequalities*** and ***overlap of the graphs of the inequalities***.

Highlight that solutions to systems of linear inequalities are ordered pairs that satisfy all inequalities in the system and are represented by the overlap of the graphs of the inequalities.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use an interactive graph to enhance the experience of graphing a system of inequalities.

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Consider removing Clue 2 from the problem and only having students work with Clues 1, 3, and 4. As students shade the regions, suggest they use color coding to emphasize the overlapping region. Consider also having them annotate the overlapping region with "Solutions."

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

- **Read 1:** Students should understand that the inequalities represent clues to the location of three items in the scavenger hunt.
- **Read 2:** Ask students to name or highlight the given quantity: the rectangular park measures 50 m by 20 m.
- **Read 3:** Ask students to think about what strategies they could use to solve each system, before attempting to solve each system.

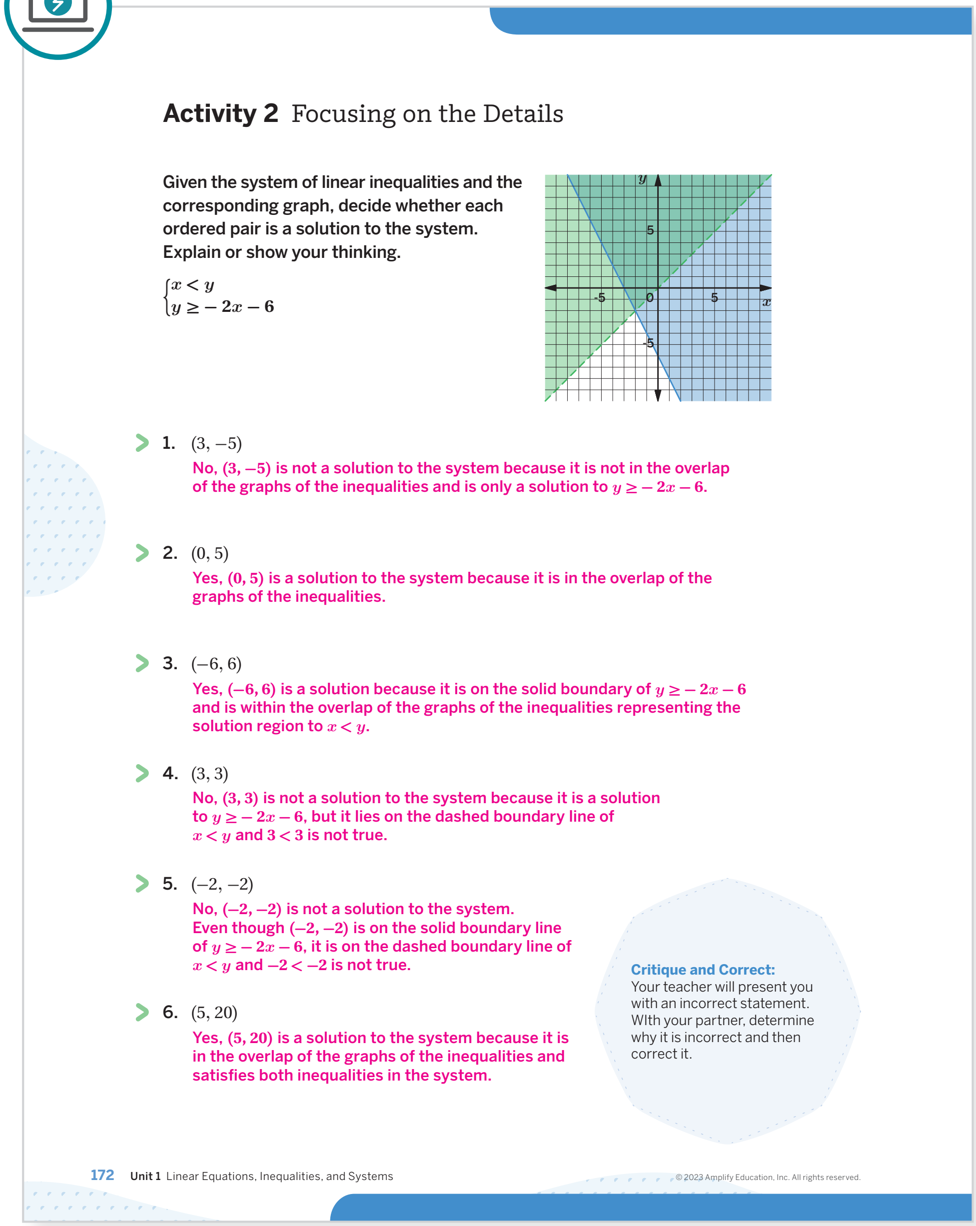
English Learners

Some students may not be familiar with the term *scavenger hunt*. Provide a brief description of this term.

Pairs | 15 min

Activity 2 Focusing on the Details

Students analyze ordered pairs on boundary lines to determine when they are also solutions.



Activity 2 Focusing on the Details

Given the system of linear inequalities and the corresponding graph, decide whether each ordered pair is a solution to the system. Explain or show your thinking.

$$\begin{cases} x < y \\ y \geq -2x - 6 \end{cases}$$

1. $(3, -5)$
   No, $(3, -5)$ is not a solution to the system because it is not in the overlap of the graphs of the inequalities and is only a solution to $y \geq -2x - 6$.

2. $(0, 5)$
   Yes, $(0, 5)$ is a solution to the system because it is in the overlap of the graphs of the inequalities.

3. $(-6, 6)$
   Yes, $(-6, 6)$ is a solution because it is on the solid boundary of $y \geq -2x - 6$ and is within the overlap of the graphs of the inequalities representing the solution region to $x < y$.

4. $(3, 3)$
   No, $(3, 3)$ is not a solution to the system because it is a solution to $y \geq -2x - 6$, but it lies on the dashed boundary line of $x < y$ and $3 < 3$ is not true.

5. $(-2, -2)$
   No, $(-2, -2)$ is not a solution to the system. Even though $(-2, -2)$ is on the solid boundary line of $y \geq -2x - 6$, it is on the dashed boundary line of $x < y$ and $-2 < -2$ is not true.

6. $(5, 20)$
   Yes, $(5, 20)$ is a solution to the system because it is in the overlap of the graphs of the inequalities and satisfies both inequalities in the system.

Critique and Correct:
Your teacher will present you with an incorrect statement. WIth your partner, determine why it is incorrect and then correct it.

172 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Students remain in pairs. Have them restate directions in their own words to verify understanding. Ask, "Are ordered pairs on the boundary lines of the overlap of the graphs of the inequalities also solutions?" It depends on whether these ordered pairs algebraically satisfy the system of inequalities.

2 Monitor

Help students get started by asking, "How could plotting the given ordered pairs help?"

Look for points of confusion:

- **Thinking a ordered pair in any shaded region, boundary line, or intersection is a solution.** Ask, "How can you test to see if these ordered pairs really are solutions?"

Look for productive strategies:

- Plotting ordered pairs on the given graph.
- Substituting the ordered pairs in the system of inequalities to determine if they are solutions.

3 Connect

Have pairs of students share their responses to the problems. Select and sequence students using the graph, types of boundary lines, and algebraic testing.

Highlight that not all boundary or intersection points are a solution. The x-and y-values must satisfy both inequalities in the system.

Ask, "How does knowing the type of boundary help you determine if ordered pairs on the boundary line are solutions?" Ordered pairs on solid boundary lines of the solution region will also be solutions to the system. Ordered pairs on dashed boundary lines of the solution region will not be solutions to the system.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Problems 1–4. Encourage them to plot each point to help assess whether it is a solution to the system.

Extension: *Math Enrichment*

Have students complete the following problem:
Suppose the inequality $y < 4$ is added to the system. How would this affect your responses to Problems 1–5? The points $(0, 5)$ and $(-6, 6)$ would no longer be solutions. The other responses would remain the same.

Math Language Development

MLR3: Critique, Correct, Clarify

Before or during the Connect, present the following incorrect statement: "The point $(3, -5)$ is a solution to the system because it lies in a shaded region and not in an unshaded region."

- **Critique:** Ask students to critique the statement and reasoning.
- **Correct and Clarify:** Ask students to write a corrected statement and explain how they know their statement is correct.

English Learners

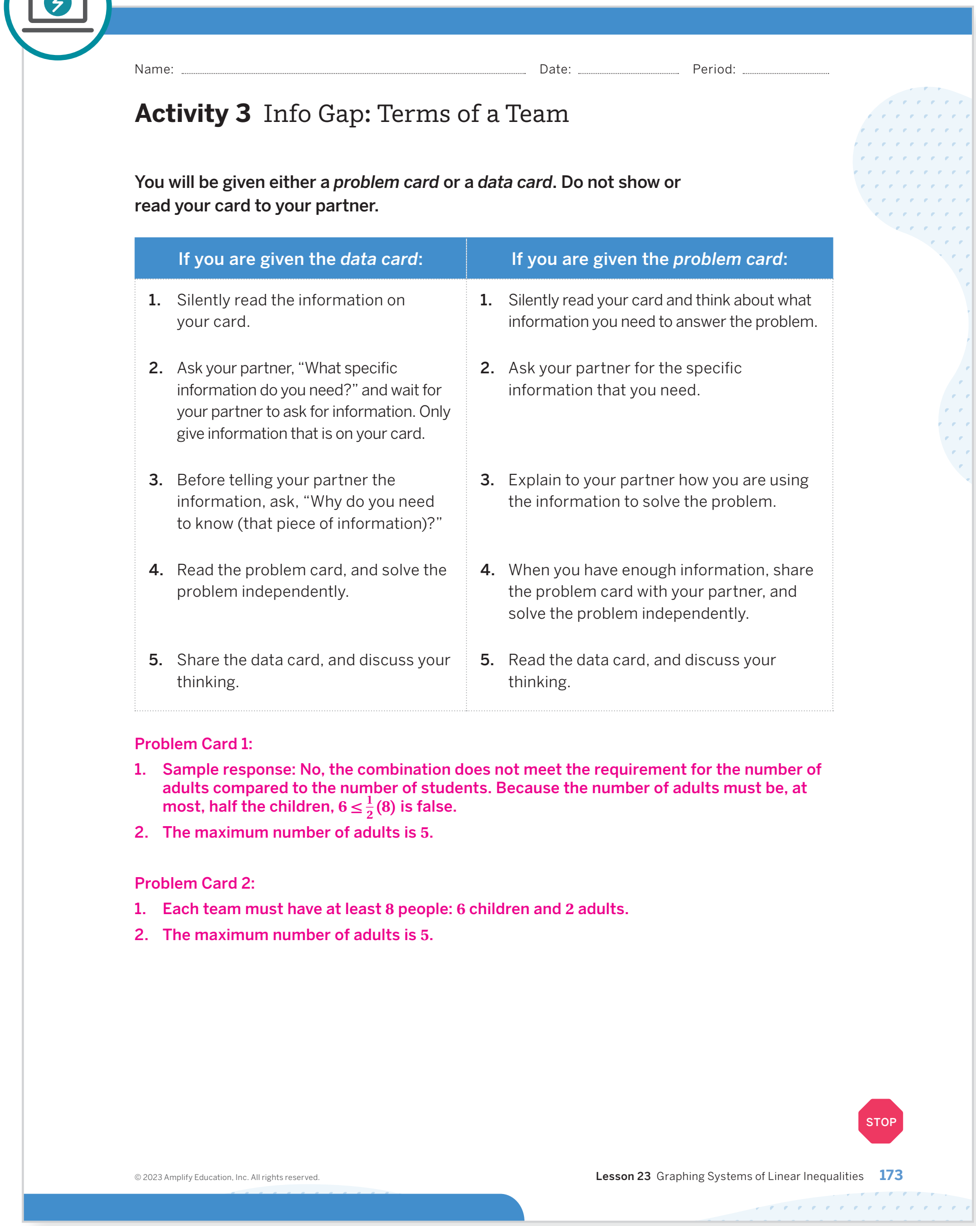
Draw an outline around the overlapping region on the graph and annotate it with the phrase "Solutions."

Optional

Pairs | 15 min

Activity 3 Info Gap: Terms of a Team

Students demonstrate their understanding by graphing systems of linear inequalities and reasoning quantitatively about their solutions.



Name: ______ Date: ______ Period: ______

Activity 3 Info Gap: Terms of a Team

You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given the *data card*:	If you are given the *problem card*:
1. Silently read the information on your card.	1. Silently read your card and think about what information you need to answer the problem.
2. Ask your partner, "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card.	2. Ask your partner for the specific information that you need.
3. Before telling your partner the information, ask, "Why do you need to know (that piece of information)?"	3. Explain to your partner how you are using the information to solve the problem.
4. Read the problem card, and solve the problem independently.	4. When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Share the data card, and discuss your thinking.	5. Read the data card, and discuss your thinking.

Problem Card 1:

1. Sample response: No, the combination does not meet the requirement for the number of adults compared to the number of students. Because the number of adults must be, at most, half the children, $6 \leq \frac{1}{2}(8)$ is false.
2. The maximum number of adults is 5.

Problem Card 2:

1. Each team must have at least 8 people: 6 children and 2 adults.
2. The maximum number of adults is 5.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 23 Graphing Systems of Linear Inequalities 173

1 Launch

Explain the ***Info Gap*** instructional routine, and consider demonstrating the routine if students are unfamiliar with it. Distribute the pre-cut cards from the Activity 3 PDF to each student pair.

2 Monitor

Help students get started by modeling the questioning process.

Look for points of confusion:

- **Guessing random values for the number of adults and children.** Suggest identifying quantities and organizing data in a chart or table.

Look for productive strategies:

- Defining variables and writing inequalities to represent data.
- Translating phrases like "at most" or "at least" into inequalities.
- Representing constraints graphically.

3 Connect

Have pairs of students share their strategies for communicating to answer the problem cards. Select and sequence students who used tables or charts, wrote inequalities, and graphed constraints.

Highlight that writing and graphing inequalities to represent the data can reveal a solution region of possibilities.

Display the data and problem cards. Demonstrate the strategy outlined in the Highlight to provide students a visual representation.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. For example:

"I wonder what the membership rules are.

- I think I should ask some questions about the relationship between adults and children for each team. I'll ask how many members can be on the team altogether.
- Then I'll ask if there is a minimum number of adults or children allowed. For example, can a team have 0 adults or 0 children?
- I wonder what else I can ask about."

Math Language Development

MLR4: Information Gap

Display prompts for students who benefit from a starting point, such as:

- "Can you tell me . . . (specific piece of information)?"
- "Why do you need to know . . . (that piece of information)?"

English Learners

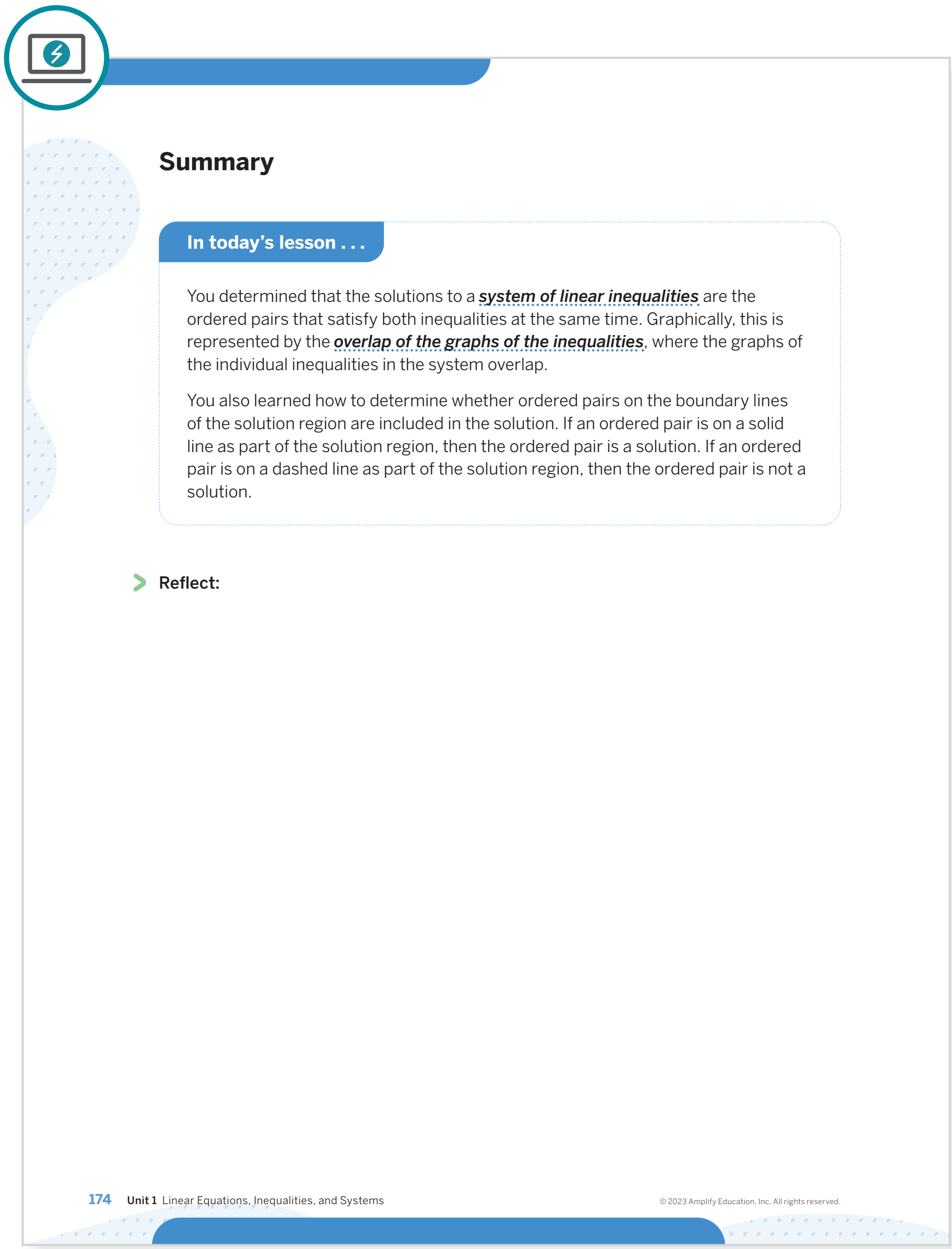
Consider providing sample questions students could ask for Problem Card 1, such as the following:

- "How many members are allowed on the team altogether?"
- "Is there a ratio of adults to children that must be satisfied?"

Whole Class | 5 min

Summary

Review and synthesize representing solutions to systems of linear inequalities graphically and numerically.



Summary

In today's lesson . . .

You determined that the solutions to a ***system of linear inequalities*** are the ordered pairs that satisfy both inequalities at the same time. Graphically, this is represented by the ***overlap of the graphs of the inequalities***, where the graphs of the individual inequalities in the system overlap.

You also learned how to determine whether ordered pairs on the boundary lines of the solution region are included in the solution. If an ordered pair is on a solid line as part of the solution region, then the ordered pair is a solution. If an ordered pair is on a dashed line as part of the solution region, then the ordered pair is not a solution.

Reflect:

174 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

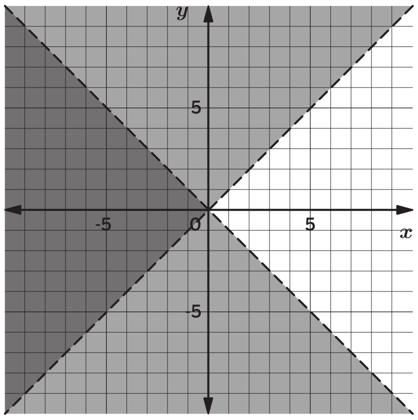
Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit, that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *overlap of the graphs of the inequalities* and *system of linear inequalities* that were added to the display during the lesson.

Synthesize

Display the following graph to the class.



Have students share how to determine where the solutions to a system of linear inequalities are.

Highlight that the overlap of the graphs of the inequalities represents the solutions. Ordered pairs on the boundary line are included in the solution if these points satisfy both inequalities in the system, indicated by solid boundary lines.

Formalize vocabulary:

- *overlap of the graphs of the inequalities*
- *system of linear inequalities*

Ask, "How do you know if an ordered pair is a solution to a system of inequalities?"

An ordered pair is a solution if the point lies in the overlapping shaded solution region of the system, the point is on a solid boundary line, or it algebraically satisfies all inequalities in the system.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

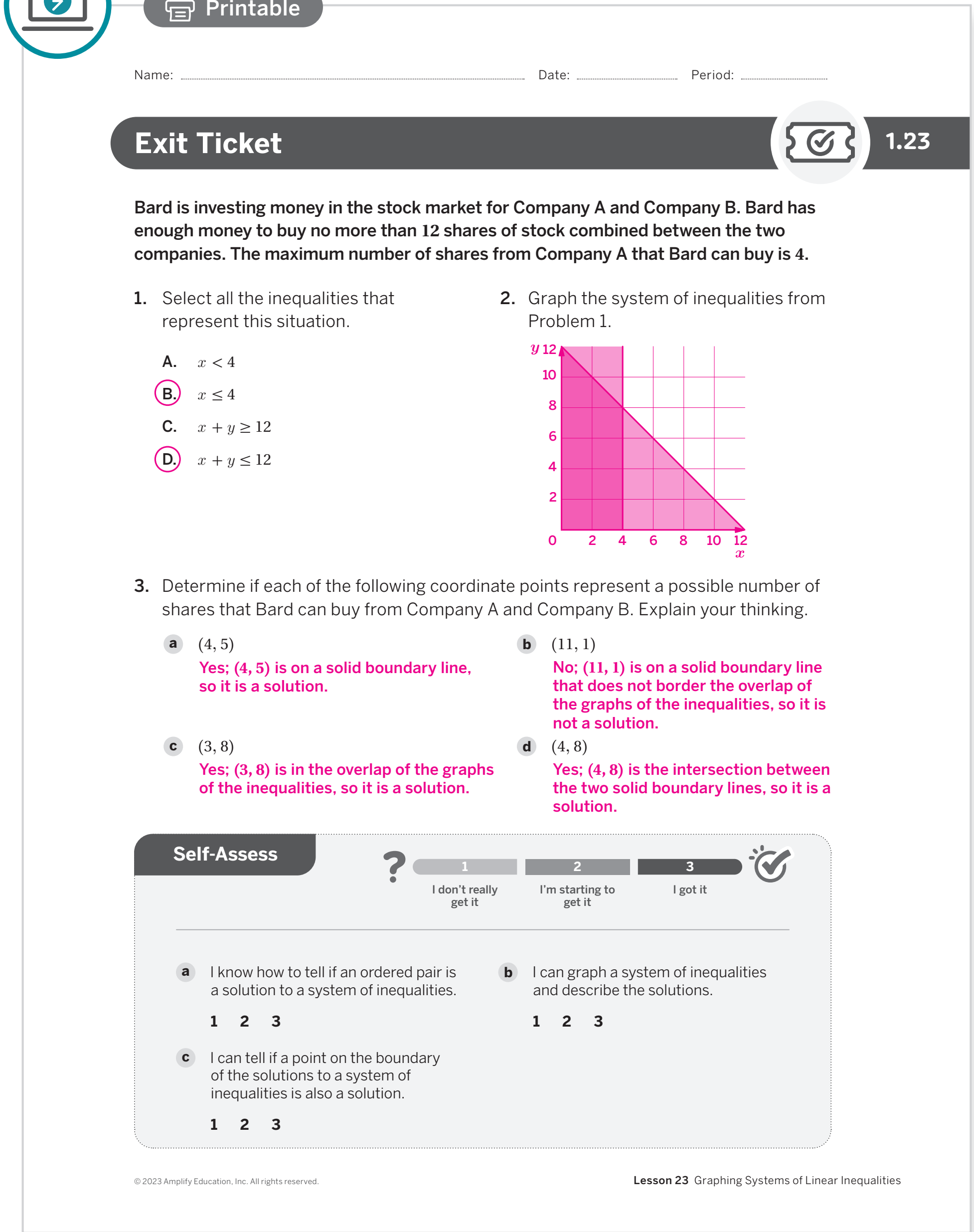
- "How did graphing help you visualize the solutions to a linear system of inequalities?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by graphing systems of linear inequalities and their solutions.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 1.23

Bard is investing money in the stock market for Company A and Company B. Bard has enough money to buy no more than 12 shares of stock combined between the two companies. The maximum number of shares from Company A that Bard can buy is 4.

1. Select all the inequalities that represent this situation.

A. $x < 4$

[✓ B.] $x \le 4$

C. $x + y \ge 12$

[✓ D.] $x + y \le 12$

2. Graph the system of inequalities from Problem 1.

3. Determine if each of the following coordinate points represent a possible number of shares that Bard can buy from Company A and Company B. Explain your thinking.

a $(4, 5)$

Yes; $(4, 5)$ is on a solid boundary line, so it is a solution.

b $(11, 1)$

No; $(11, 1)$ is on a solid boundary line that does not border the overlap of the graphs of the inequalities, so it is not a solution.

c $(3, 8)$

Yes; $(3, 8)$ is in the overlap of the graphs of the inequalities, so it is a solution.

d $(4, 8)$

Yes; $(4, 8)$ is the intersection between the two solid boundary lines, so it is a solution.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I know how to tell if an ordered pair is a solution to a system of inequalities.

1 2 3

b I can graph a system of inequalities and describe the solutions.

1 2 3

c I can tell if a point on the boundary of the solutions to a system of inequalities is also a solution.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 23 Graphing Systems of Linear Inequalities

Success looks like . . .

- **Language Goal:** Explaining how to determine if an ordered pair is a solution to a system of inequalities. **(Speaking and Listening, Writing)**
- **Language Goal:** Graphing a system of inequalities and describing the solutions. **(Speaking and Listening, Writing)**
  - » Graphing the system of inequalities representing the number of shares in Problem 2.
- **Goal:** Determining if a coordinate point on a boundary line to a system of inequalities is a solution to the system.
  - » Explaining whether each coordinate point could represent a number of shares that Bard could buy in Problem 3.

Suggested next steps

If students incorrectly select the inequalities in Problem 1, consider:

- Reviewing strategies from Activity 3.
- Assigning Practice Problem 6.
- Asking, "How could you translate phrases from the situation into inequality symbols?"

If students incorrectly graph the system in Problem 2, consider:

- Reviewing strategies from Activity 1.
- Assigning Practice Problem 1.
- Asking, "How could you algebraically test ordered pairs to determine where to shade and what type of boundary line you will use?"

If students incorrectly determine which points are solutions in Problem 3, consider:

- Reviewing strategies from Activity 2.
- Assigning Practice Problem 2.
- Asking, "How could you test the ordered pairs to determine if they satisfy all inequalities in the system?"

Professional Learning

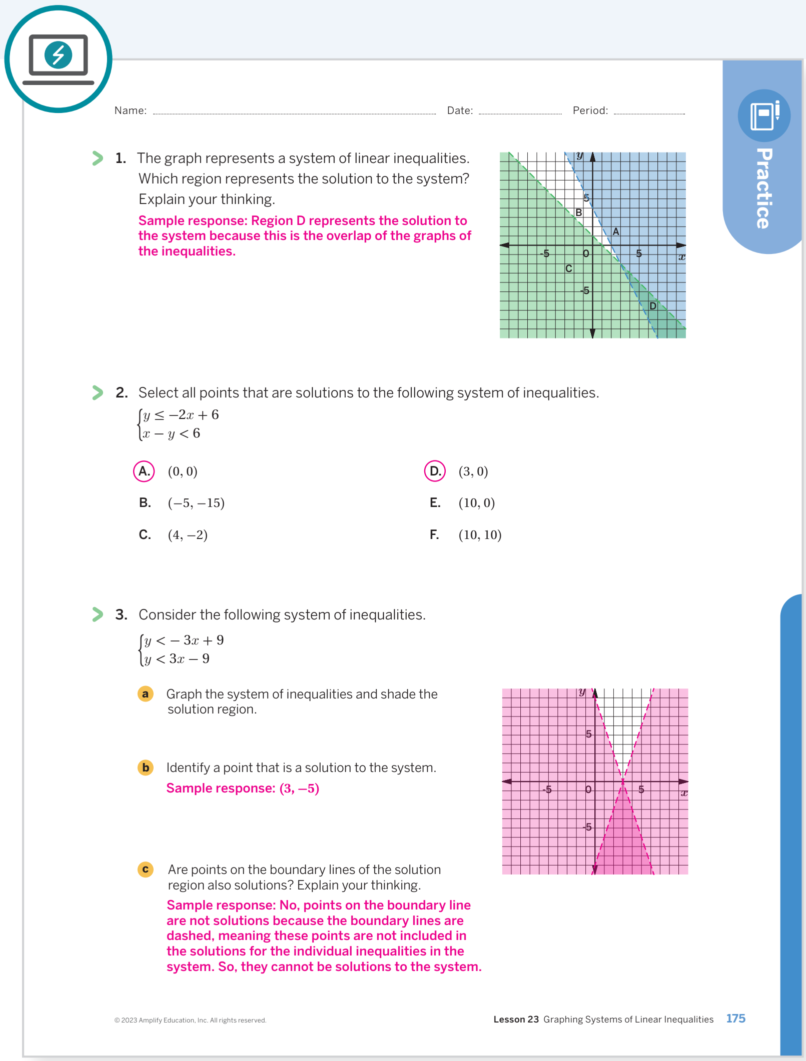
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways did Activity 1 go as planned?
- What did students find frustrating about Activity 3? What helped them work through this? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. The graph represents a system of linear inequalities. Which region represents the solution to the system? Explain your thinking.

Sample response: Region D represents the solution to the system because this is the overlap of the graphs of the inequalities.

2. Select all points that are solutions to the following system of inequalities.

$$\begin{cases} y \leq -2x + 6 \\ x - y < 6 \end{cases}$$

[✓ A.] $(0, 0)$
B. $(-5, -15)$
C. $(4, -2)$
[✓ D.] $(3, 0)$
E. $(10, 0)$
F. $(10, 10)$

3. Consider the following system of inequalities.

$$\begin{cases} y < -3x + 9 \\ y < 3x - 9 \end{cases}$$

a. Graph the system of inequalities and shade the solution region.

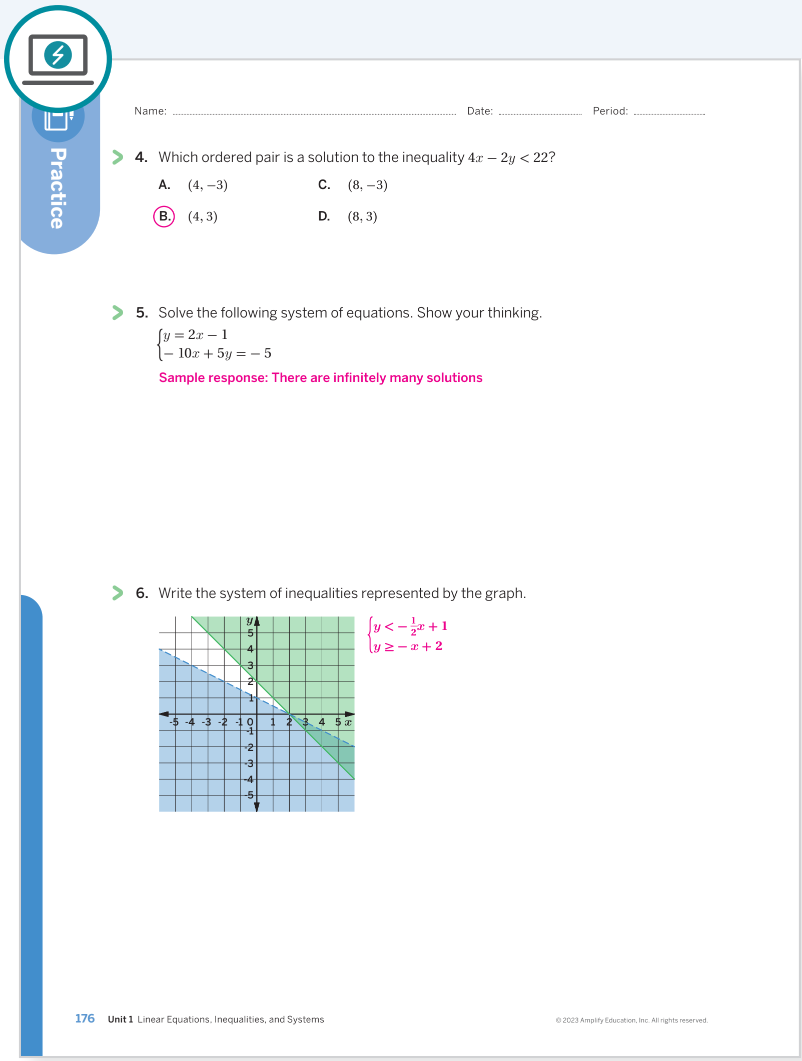
b. Identify a point that is a solution to the system.

Sample response: (3, –5)

c. Are points on the boundary lines of the solution region also solutions? Explain your thinking.

Sample response: No, points on the boundary line are not solutions because the boundary lines are dashed, meaning these points are not included in the solutions for the individual inequalities in the system. So, they cannot be solutions to the system.

© 2023 Amplify Education, Inc. All rights reserved.



Name: ______ Date: ______ Period: ______

4. Which ordered pair is a solution to the inequality $4x - 2y < 22$?

A. $(4, -3)$
[✓ B.] $(4, 3)$
C. $(8, -3)$
D. $(8, 3)$

5. Solve the following system of equations. Show your thinking.

$$\begin{cases} y = 2x - 1 \\ -10x + 5y = -5 \end{cases}$$

Sample response: There are infinitely many solutions

6. Write the system of inequalities represented by the graph.

$$\begin{cases} y < -\frac{1}{2}x + 1 \\ y \geq -x + 2 \end{cases}$$

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 15	1
	5	Unit 1 Lesson 18	2
Formative ⚡	6	Unit 1 Lesson 24	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

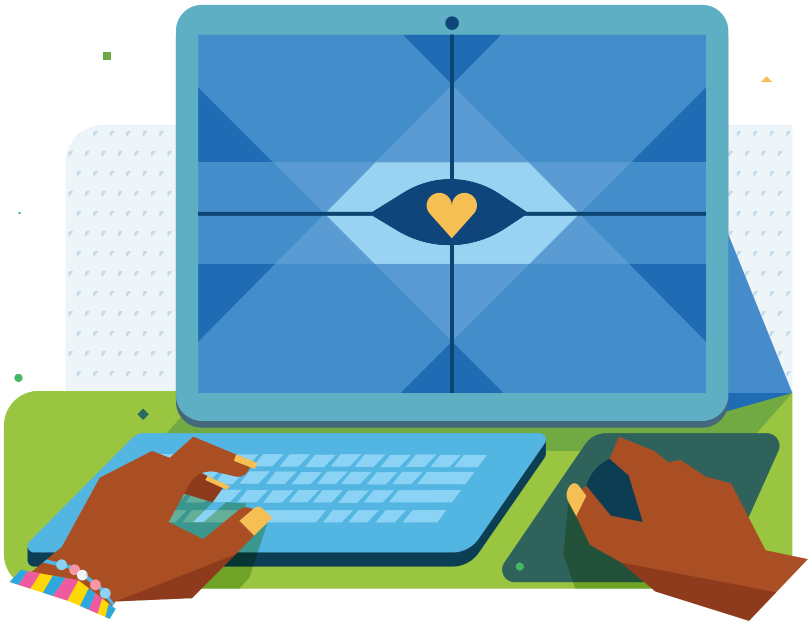
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 24**

Solving and Writing Systems of Linear Inequalities

Let's explore the use of systems of linear inequalities in design.



Focus

Goals

1. **Language Goal:** Write systems of inequalities in two variables to represent the solution of a given graph, and interpret the solutions in context. **(Reading and Writing)**
2. Understand that the solution set of a system of inequalities in two variables consists of any pair of values that make both inequalities true and can be represented graphically by the region where the graphs overlap.

Coherence

Today

Students graph a system of three or more linear inequalities. Students are given a graph of a system of linear inequalities and use boundary lines and a shaded region to write the system that represents the graph.

Previously

In Lesson 23, students graphed systems of linear inequalities to understand that the solution to the system is represented graphically by the area where the graphs overlap.

Coming Soon

In Lesson 25, students will create a system of linear inequalities to model the constraints and conditions in a situation.

Rigor

- Students build **procedural fluency** of writing a system of linear inequalities to represent the solution of a graph.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
10 min	15 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Anchor Chart PDF, *Forms of Linear Equations*
- Anchor Chart PDF, *Graphing Linear Inequalities*
- blank coordinate planes
- rulers

Math Language Development

Review words

- *boundary line*
- *linear inequality*
- *system of linear inequalities*

Amps | Featured Activity

Activity 2
Interactive Graph

Students choose from several boundary lines to design their own company logo. Students use their chosen boundary lines to write a system of inequalities whose graph represents their design.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel overwhelmed with all that they have to keep track of while designing the logo. Before starting, have students discuss ways that they are going to keep their work organized. By staying organized, their stress levels will decrease, and ultimately they reduce the workload because they track what they are doing in real time as they work towards their goal.

Modifications to Pacing

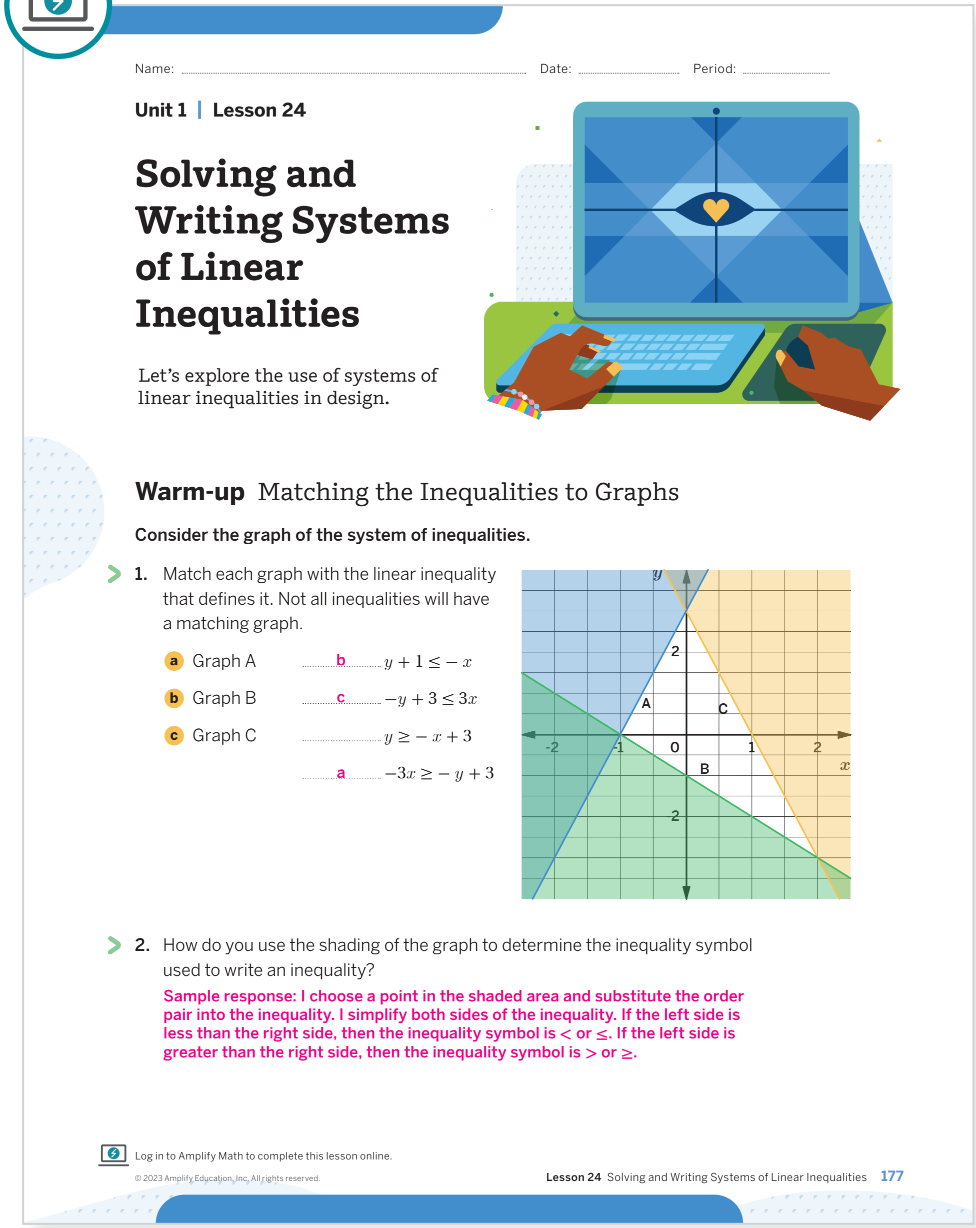
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, have students only match Graphs A and B.
- In **Activity 2**, Problems 4 and 5 may be omitted.
- Optional **Activity 3** may be omitted.

Independent | 10 min

Warm-up Matching the Inequalities to Graphs

Students determine the solution set of a system of linear inequalities to reason quantitatively on whether boundary lines are present on its graph.



Name: ______________________ Date: ____________ Period: ____________

Unit 1 | Lesson 24

Solving and Writing Systems of Linear Inequalities

Let's explore the use of systems of linear inequalities in design.

Warm-up Matching the Inequalities to Graphs

Consider the graph of the system of inequalities.

1. Match each graph with the linear inequality that defines it. Not all inequalities will have a matching graph.

   a Graph A
   b Graph B
   c Graph C

   b $y + 1 \leq -x$
   c $-y + 3 \leq 3x$
   ______ $y \geq -x + 3$
   a $-3x \geq -y + 3$

2. How do you use the shading of the graph to determine the inequality symbol used to write an inequality?

   Sample response: I choose a point in the shaded area and substitute the order pair into the inequality. I simplify both sides of the inequality. If the left side is less than the right side, then the inequality symbol is < or ≤. If the left side is greater than the right side, then the inequality symbol is > or ≥.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have students work independently to match each inequality with the corresponding region of the graph. Then have them compare their solutions with a partner.

2 Monitor

Help students get started by asking, "How could you manipulate an inequality to help match it to a graph?"

Look for points of confusion:

- **Leaving the inequality symbol unchanged when dividing by a negative value.** Have students test a point that is a solution. Highlight that the inequality symbol changes when dividing or multiplying by a negative value.

Look for productive strategies:

- Determining the slope of the boundary lines.
- Isolating y in each inequality.

3 Connect

Have individual students share their strategies for matching.

Highlight that they can test a point from where the shading overlaps and from the boundary line, if it is solid, to check their matching.

Ask, "If you were not provided with the inequalities, how would you write the inequality yourself?" Sample response: I would determine the slope and the y-intercept of each boundary line, and then test a point from where the shading overlaps to determine the inequality symbol. I would then test a point on the boundary line to determine whether the boundary line is solid or dashed.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Activate Prior Knowledge*

Before the Warm-up, or while students work, display or provide copies of the Anchor Chart PDFs, *Graphing Linear Inequalities* and *Forms of Linear Equations* for students to reference. These anchor charts will also help activate prior knowledge of slope-intercept form and graphing linear inequalities.

Power-up

To power up students' ability to write an inequality represented by a graph:

Provide students with a copy of the Power-up PDF.

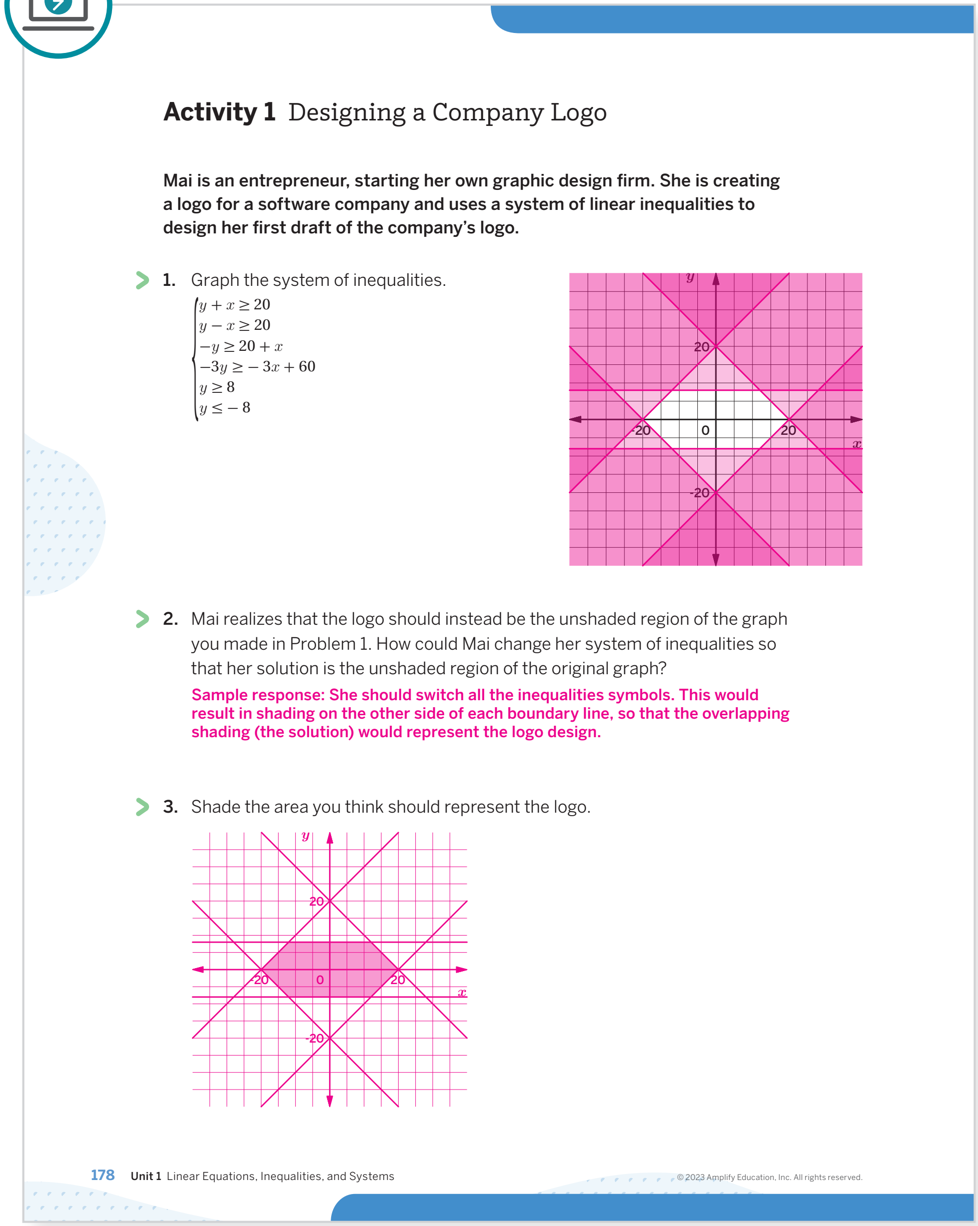
Use: Before the Warm-up

Informed by: Performance on Lesson 23, Practice Problem 6

Pairs | 20 min

Activity 1 Designing a Company Logo

Students graph a system of three or more linear inequalities to reveal a logo design.



Activity 1 Designing a Company Logo

Mai is an entrepreneur, starting her own graphic design firm. She is creating a logo for a software company and uses a system of linear inequalities to design her first draft of the company's logo.

1. Graph the system of inequalities.

$$\begin{cases} y + x \geq 20 \\ y - x \geq 20 \\ -y \geq 20 + x \\ -3y \geq -3x + 60 \\ y \geq 8 \\ y \leq -8 \end{cases}$$

2. Mai realizes that the logo should instead be the unshaded region of the graph you made in Problem 1. How could Mai change her system of inequalities so that her solution is the unshaded region of the original graph?
Sample response: She should switch all the inequalities symbols. This would result in shading on the other side of each boundary line, so that the overlapping shading (the solution) would represent the logo design.

3. Shade the area you think should represent the logo.

178 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the narrative as a class. Provide access to rulers. Have students work individually and then check work with a partner periodically while graphing.

2 Monitor

Help students get started by saying, "Rewrite the inequalities in slope-intercept or standard form to help determine the intercepts and/or slope."

Look for points of confusion:

- **Losing track of the shading that corresponds to each inequality.** Have students use different colors for each boundary line and shading.

Look for productive strategies:

- Determining the x- and y-intercept to graph the boundary lines.

3 Connect

Have individual students share their graphs.

Highlight that no matter the number of inequalities in a system of inequalities, the solution for the system is still the region where the shading from the inequalities overlap.

Ask, "Which region contains the solutions to this system of inequalities?" There is no solution to this system because there is no region where the shading from all the inequalities overlap.

Differentiated Support

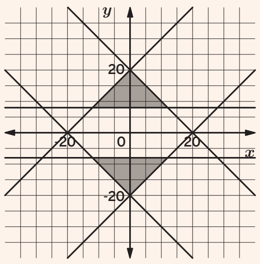
Accessibility: *Vary Demands to Optimize Challenge*

Consider using this alternative approachs to this activity. Provide students with the inequalities where y is isolated.

$$\begin{cases} y \geq 20 - x \\ y \geq 20 + x \\ y \leq -20 - x \\ y \leq x - 20 \\ y \geq 8 \\ y \leq -8 \end{cases}$$

Extension: *Math Enrichment*

Rewrite the system of inequalities to reflect the logo design.



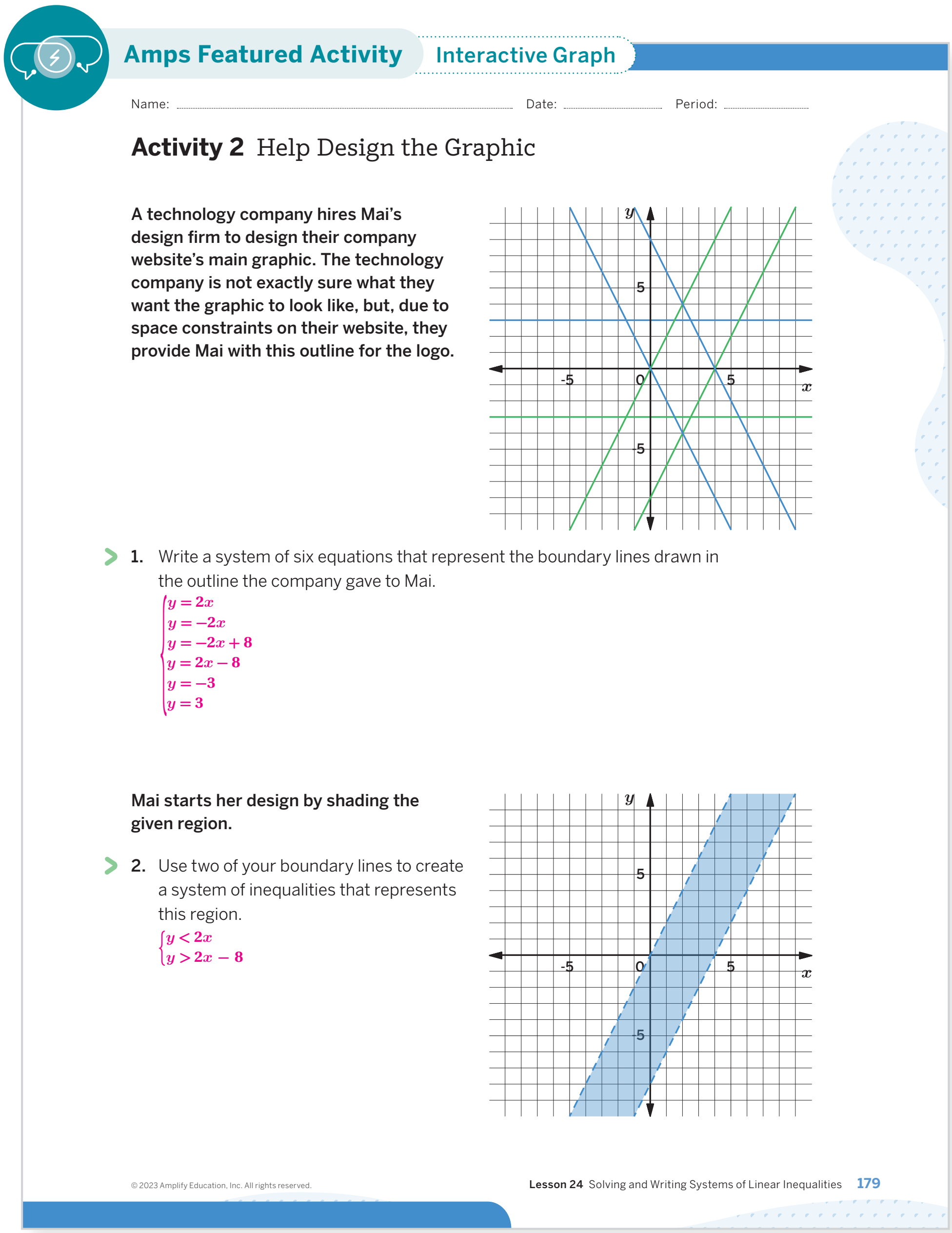
$$\begin{cases} y + x \leq 20 \\ y - x \leq 20 \\ -y \leq 20 + x \\ -3y \leq -3x + 60 \\ y \geq 8 \\ y \leq -8 \end{cases}$$

Pairs | 15 min

Activity 2 Help Design the Graphic

Students use a bounded region and boundary lines to write a system of linear inequalities to represent a graphic design.

Amps Featured Activity Interactive Graph



Name: ______ Date: ______ Period: ______

Activity 2 Help Design the Graphic

A technology company hires Mai's design firm to design their company website's main graphic. The technology company is not exactly sure what they want the graphic to look like, but, due to space constraints on their website, they provide Mai with this outline for the logo.

1. Write a system of six equations that represent the boundary lines drawn in the outline the company gave to Mai.

$$\begin{cases} y = 2x \\ y = -2x \\ y = -2x + 8 \\ y = 2x - 8 \\ y = -3 \\ y = 3 \end{cases}$$

Mai starts her design by shading the given region.

2. Use two of your boundary lines to create a system of inequalities that represents this region.

$$\begin{cases} y < 2x \\ y > 2x - 8 \end{cases}$$

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the narrative as a class. Have students work individually and then check their equations of the boundary lines with a partner.

2 Monitor

Help students get started by saying, "Determine the intercepts and slope of each line to help you write the equations."

Look for points of confusion:

- **Using ≥ and ≤ for Problems 2 and 3.** Ask, "How is a solid and dashed line reflected in the inequality?"
- **Estimating the y-intercept for the boundary line in Problem 3.** Have students extend each line using a ruler.

Look for productive strategies:

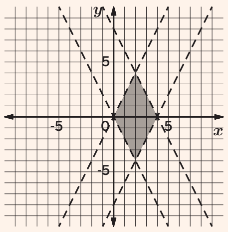
- Identifying and using intercepts to write the boundary line equations in standard form.
- Identifying the slope and y-intercept to write the boundary line equations in slope-intercept form.
- Testing a point from the shaded region in each inequality to check their inequalities.
- Writing their system of inequalities for their design first and then using the system to graph their design.

Activity 2 continued >

Differentiated Support

Accessibility: ***Guide Processing and Visualization***

Provide students with extended colored boundary lines for Problem 3.



Accessibility: ***Guide Processing and Visualization, Activate Prior Knowledge***

While students work, display or provide copies of the Anchor Chart PDFs, *Graphing Linear Inequalities* and *Forms of Linear Equations* for students to reference. These anchor charts will also help activate prior knowledge of slope-intercept form and graphing linear inequalities.

Math Language Development

MLR2: Collect and Display

During the whole-class discussion, listen for and collect language students use to generalize the process of writing an inequality from a graph. Write students' words and phrases on a visual display, keeping track of each step from start to finish.

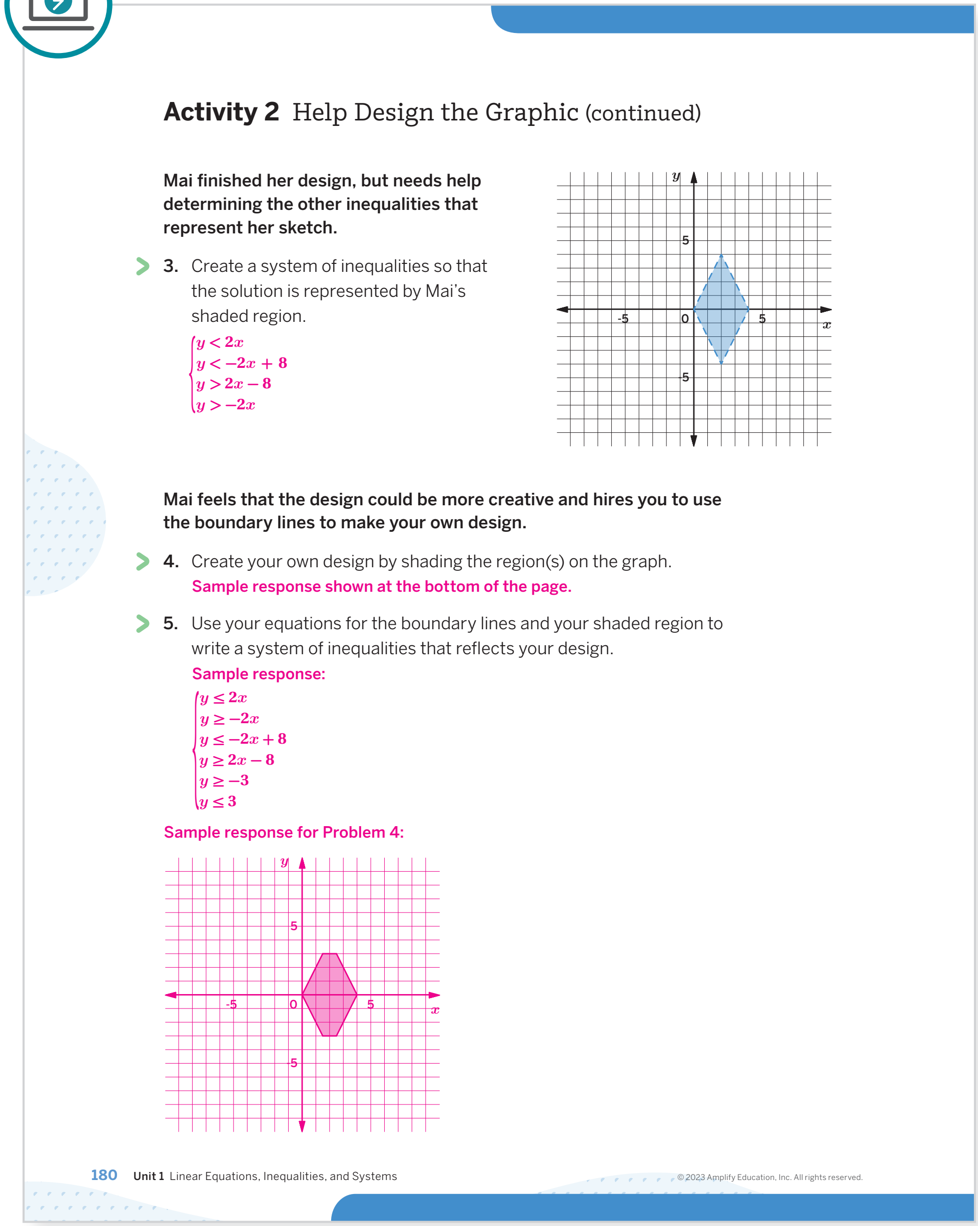
English Learners

Label the boundary lines on the graph with the inequalities that students write to make the connection between graph and inequality clear.

Pairs | 15 min

Activity 2 Help Design the Graphic (continued)

Students use a bounded region and boundary lines to write a system of linear inequalities to represent a graphic design.



Activity 2 Help Design the Graphic (continued)

Mai finished her design, but needs help determining the other inequalities that represent her sketch.

3. Create a system of inequalities so that the solution is represented by Mai's shaded region.

$$\begin{cases} y < 2x \\ y < -2x + 8 \\ y > 2x - 8 \\ y > -2x \end{cases}$$

Mai feels that the design could be more creative and hires you to use the boundary lines to make your own design.

4. Create your own design by shading the region(s) on the graph.
Sample response shown at the bottom of the page.

5. Use your equations for the boundary lines and your shaded region to write a system of inequalities that reflects your design.
Sample response:

$$\begin{cases} y \leq 2x \\ y \geq -2x \\ y \leq -2x + 8 \\ y \geq 2x - 8 \\ y \geq -3 \\ y \leq 3 \end{cases}$$

Sample response for Problem 4:

180 Unit 1 Linear Equations, Inequalities, and Systems

©2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have individual students share their designs and the system of inequalities that represent those designs.

Highlight that to represent a shaded polygon with a system of inequalities, it is helpful to extend the boundary line.

Ask:

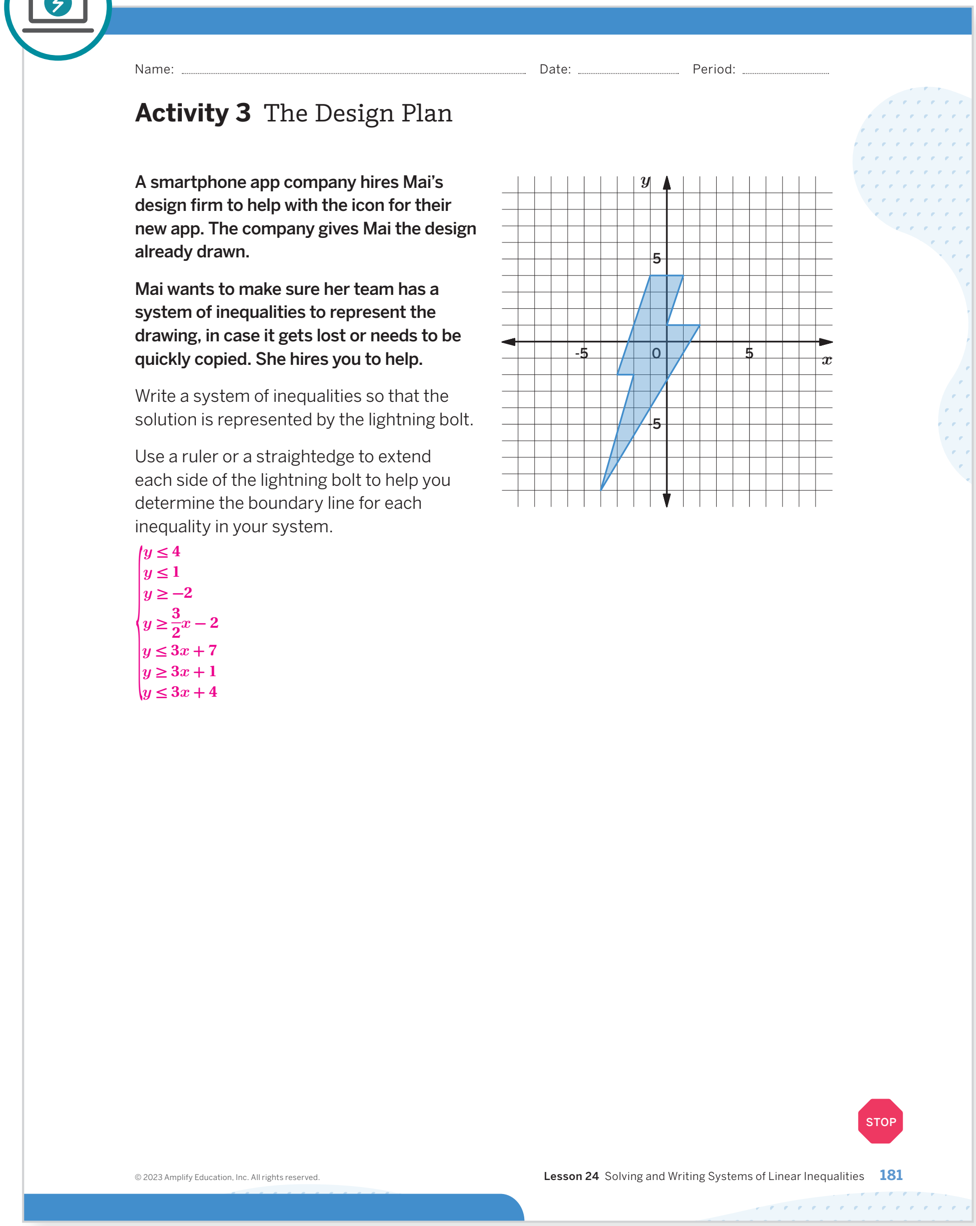
- "To create a graph representing only the solutions of the system of inequalities, do you extend the boundary line indefinitely? Explain your thinking." If the boundary line is dashed, I can always extend it since a dashed boundary line does not contain solutions. If the boundary line is solid, I restrict the boundary line to just the segment that borders the solutions because a solid line represents solutions to the system.
- "To represent just the outline of the design, what changes would you make to the boundary lines?" The domain and range of the boundary line would be restricted.

Optional

Independent | 15 min

Activity 3 The Design Plan

Students write a system of linear inequalities to represent a graphic design.



Name: ____________ Date: ____________ Period: ____________

Activity 3 The Design Plan

A smartphone app company hires Mai's design firm to help with the icon for their new app. The company gives Mai the design already drawn.

Mai wants to make sure her team has a system of inequalities to represent the drawing, in case it gets lost or needs to be quickly copied. She hires you to help.

Write a system of inequalities so that the solution is represented by the lightning bolt.

Use a ruler or a straightedge to extend each side of the lightning bolt to help you determine the boundary line for each inequality in your system.

$$\begin{cases} y \leq 4 \\ y \leq 1 \\ y \geq -2 \\ y \geq \frac{3}{2}x - 2 \\ y \leq 3x + 7 \\ y \geq 3x + 1 \\ y \leq 3x + 4 \end{cases}$$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 24 Solving and Writing Systems of Linear Inequalities 181

1 Launch

Use the *Three Reads* routine to review the activity before independent work.

2 Monitor

Help students get started by saying, "Write the equation of the boundary line that represents each side first, and then determine the inequality symbol."

Look for points of confusion:

- **Using graphing technology to check their inequalities.** Many graphing technologies will extend shading indefinitely for each inequality. Tell students to graph by hand on a blank coordinate plane.

Look for productive strategies:

- Graphing their inequalities over the design to check that the inequalities represent the sides and shaded region.

3 Connect

Have individual students share an inequality in their system and which side it corresponds to.

Highlight that if y is isolated on the left side of an inequality, they can determine the inequality symbol if their shading is clearly above or below the boundary line.

Ask, "How could you use a point outside the lightning bolt to check your inequalities?" I could substitute the ordered pair into each inequality to determine if it makes a false statement.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate how to determine one of the inequalities by using a ruler or straightedge to extend one of the sides of the lightning bolt. Consider displaying the following as a checklist for writing each inequality. Suggest that students extend each side of the lightning bolt using a different color and then writing the inequality using that color.

- Extend one side of the image. Use a ruler or straightedge.
- Determine the slope and y-intercept of the line.
- Write the equation of the boundary line in slope-intercept form.
- Use the shaded region to determine the inequality symbol.

Extension: *Math Enrichment*

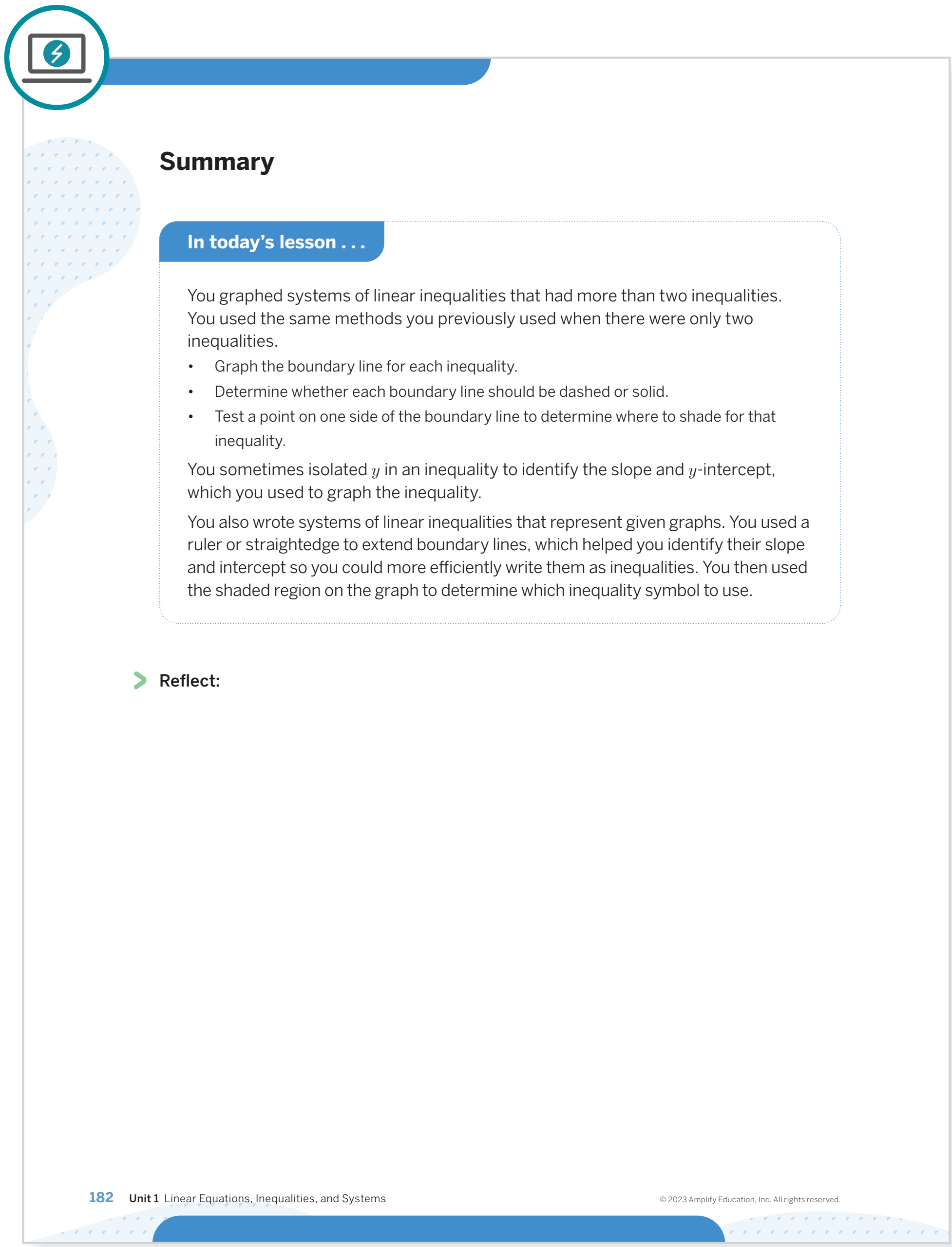
Tell students, "The general form of the equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$ where (h, k) is the center of the circle and r is the radius of the circle. Write an equation of a circle that encloses the lightning bolt."

Sample response: $(x + 1)^2 + (y + 2)^2 = 49$

Whole Class | 5 min

Summary

Review and synthesize graphing systems of three or more inequalities and writing inequalities from graphs.



Summary

In today's lesson . . .

You graphed systems of linear inequalities that had more than two inequalities. You used the same methods you previously used when there were only two inequalities.

- Graph the boundary line for each inequality.
- Determine whether each boundary line should be dashed or solid.
- Test a point on one side of the boundary line to determine where to shade for that inequality.

You sometimes isolated y in an inequality to identify the slope and y-intercept, which you used to graph the inequality.

You also wrote systems of linear inequalities that represent given graphs. You used a ruler or straightedge to extend boundary lines, which helped you identify their slope and intercept so you could more efficiently write them as inequalities. You then used the shaded region on the graph to determine which inequality symbol to use.

> **Reflect:**

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the lightning bolt design from Activity 3.

Have students share their procedure for writing a system of inequalities when there are several sides to an enclosed region.

Highlight that if they are graphing the design from a system of several inequalities, they would need to go back and either erase the shading that is not part of the solution or use darker shading or color to highlight the solution.

Ask:

- "What other situations could three or more inequalities be used to model or represent constraints?" Sample response: The shape of a sports field.
- "What about graphing or writing a system of three or more inequalities is most difficult for you? What detail(s) do you need to pay attention to more closely?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

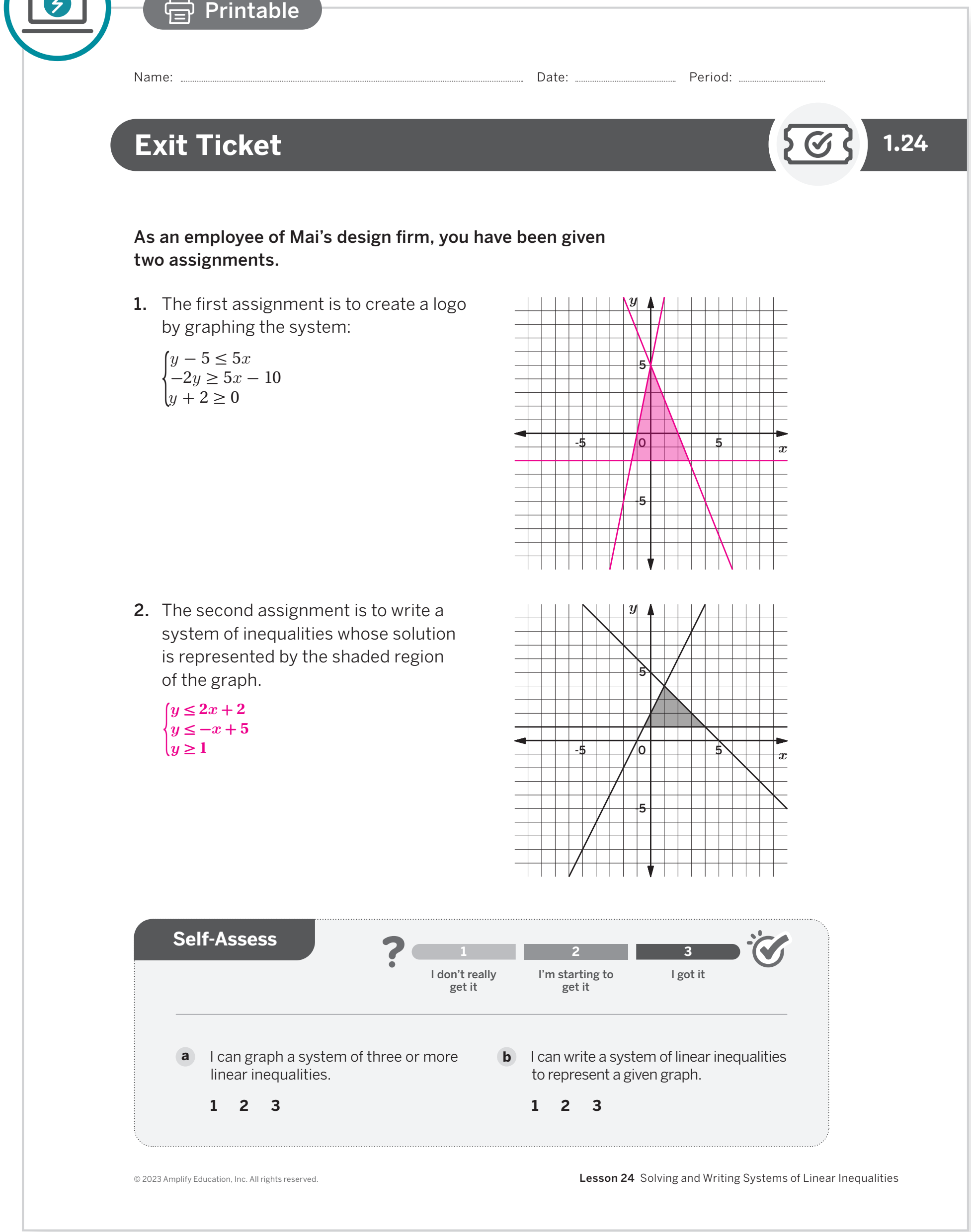
- "What strategies or tools did you find helpful today when writing inequalities from a graph? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by graphing and writing a system of three inequalities to represent a bounded region.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 1.24

As an employee of Mai's design firm, you have been given two assignments.

1. The first assignment is to create a logo by graphing the system:

$$\begin{cases} y - 5 \le 5x \\ -2y \ge 5x - 10 \\ y + 2 \ge 0 \end{cases}$$

2. The second assignment is to write a system of inequalities whose solution is represented by the shaded region of the graph.

$$\begin{cases} y \le 2x + 2 \\ y \le -x + 5 \\ y \ge 1 \end{cases}$$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can graph a system of three or more linear inequalities.

1 2 3

b I can write a system of linear inequalities to represent a given graph.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 24 Solving and Writing Systems of Linear Inequalities

Success looks like . . .

- **Language Goal:** Writing systems of inequalities in two variables to represent the solution of a given graph, and interpreting the solutions in context. **(Reading and Writing)**
  - » Writing a system of inequalities for the given graph.
- **Goal:** Understanding that the solution set of a system of inequalities in two variables consists of any pair of values that make both inequalities true and can be represented graphically by the region where the graphs overlap.

Suggested next steps

If students incorrectly graph a linear inequality Problem 1, consider:

- Reviewing strategies of isolating y and determining the slope and y-intercept of the boundary line to graph from Activity 1.
- Assigning Practice Problem 1.
- Asking, "How could you determine the slope and intercepts of the boundary line for each inequality?"

If students incorrectly create a linear inequality from the graph in Problem 2, consider:

- Reviewing strategies of determining the equation of the boundary line and then using the shaded region to determine the inequality symbol in Activity 2.
- Assigning Practice Problems 2 and 3.
- Asking, "What do you need to know to create the equation of the boundary line?"

Professional Learning

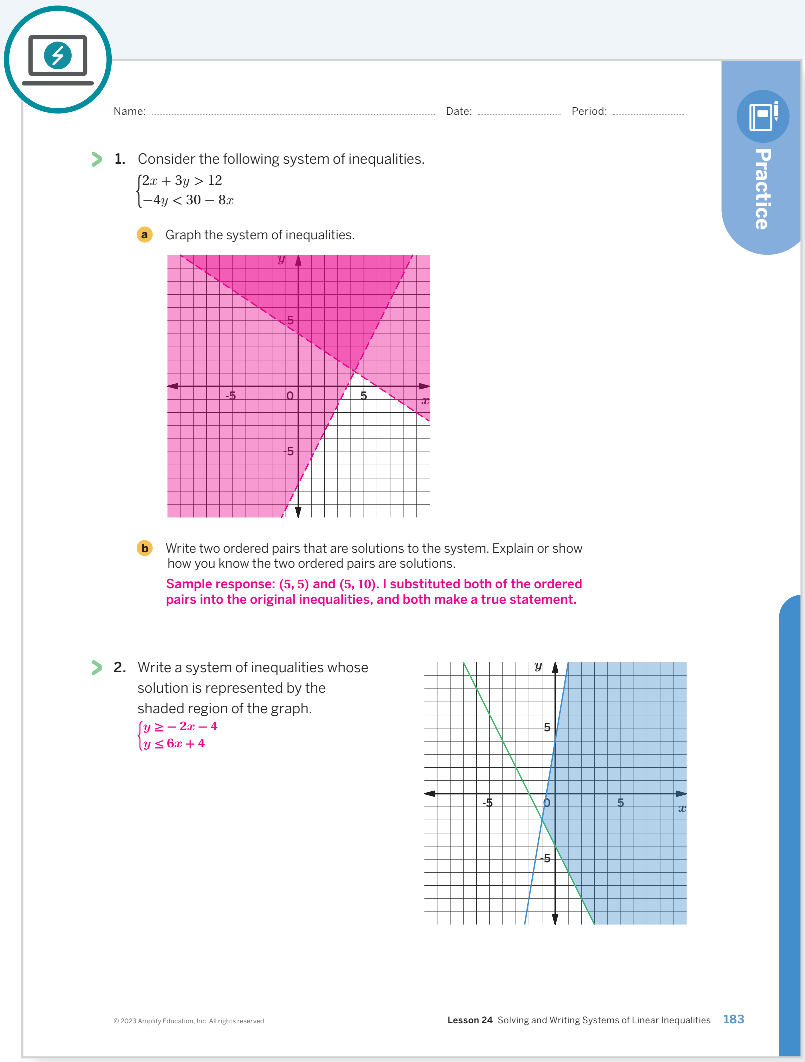
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students wrote systems of linear equations from a given graph. How did that build on the earlier work students did with graphing linear equations?
- What did you see in the way some students approached creating the equation for the boundary lines that you would like other students to try? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. Consider the following system of inequalities.

$$\begin{cases} 2x + 3y > 12 \\ -4y < 30 - 8x \end{cases}$$

a Graph the system of inequalities.

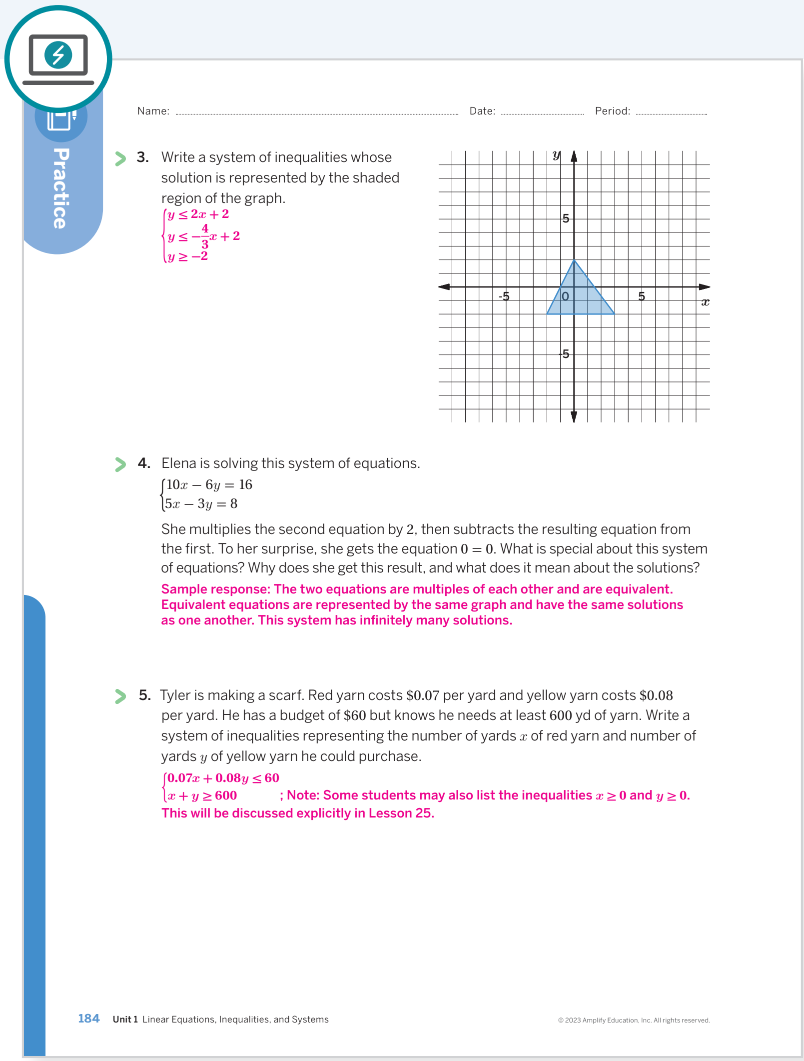
b Write two ordered pairs that are solutions to the system. Explain or show how you know the two ordered pairs are solutions.

Sample response: $(5, 5)$ and $(5, 10)$. I substituted both of the ordered pairs into the original inequalities, and both make a true statement.

2. Write a system of inequalities whose solution is represented by the shaded region of the graph.

$$\begin{cases} y \geq -2x - 4 \\ y \leq 6x + 4 \end{cases}$$

© 2023 Amplify Education, Inc. All rights reserved.



Name: Date: Period:

Practice

3. Write a system of inequalities whose solution is represented by the shaded region of the graph.

$$\begin{cases} y \leq 2x + 2 \\ y \leq -\frac{4}{3}x + 2 \\ y \geq -2 \end{cases}$$

4. Elena is solving this system of equations.

$$\begin{cases} 10x - 6y = 16 \\ 5x - 3y = 8 \end{cases}$$

She multiplies the second equation by 2, then subtracts the resulting equation from the first. To her surprise, she gets the equation $0 = 0$. What is special about this system of equations? Why does she get this result, and what does it mean about the solutions?

Sample response: The two equations are multiples of each other and are equivalent. Equivalent equations are represented by the same graph and have the same solutions as one another. This system has infinitely many solutions.

5. Tyler is making a scarf. Red yarn costs $\$0.07$ per yard and yellow yarn costs $\$0.08$ per yard. He has a budget of $\$60$ but knows he needs at least 600 yd of yarn. Write a system of inequalities representing the number of yards x of red yarn and number of yards y of yellow yarn he could purchase.

$$\begin{cases} 0.07x + 0.08y \leq 60 \\ x + y \geq 600 \end{cases}$$

; Note: Some students may also list the inequalities $x \geq 0$ and $y \geq 0$. This will be discussed explicitly in Lesson 25.

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 22	2
Formative ⚡	5	Unit 1 Lesson 25	2

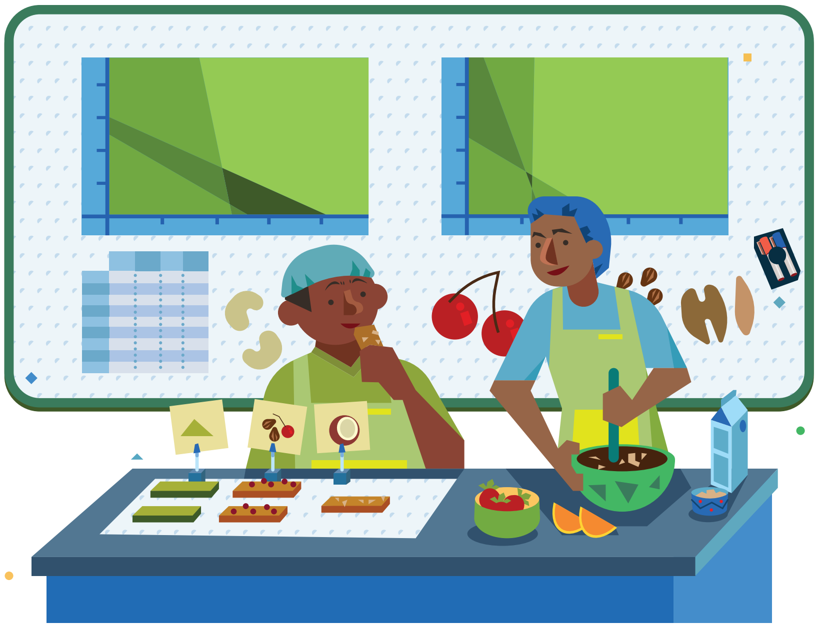
⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 25**

Modeling With Systems of Linear Inequalities



Let’s create mathematical models using systems of linear inequalities.

Focus

Goals

1. Define the constraints in a situation and create a mathematical model to represent them.
2. **Language Goal:** Interpret a mathematical model, presented as inequalities and graphs, that represents a situation. **(Speaking and Listening, Writing)**

Coherence

Today

Students interpret and analyze given models that represent the constraints and conditions in a situation. Then they create their own models after specifying quantities of interest, identifying relevant information and setting the constraints.

Previously

In Lessons 23 and 24, students graphed systems of linear inequalities and wrote systems of inequalities to represent a given graph of a system of linear inequalities.

Coming Soon

In Lesson 26, students will analyze a bounded region and determine maximum values for different two-variable expressions.

Rigor

- Students **apply** systems of linear inequalities to create mathematical models.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Independent	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per student
- Activity 2 PDF (for display)

Math Language Development

Review words

- *boundary line*
- *inequality*
- *system of linear inequalities*

Amps | Featured Activity

Activity 2
Interactive Graph

Students create their own food bar by choosing ingredients and nutritional constraints. They represent their constraints with a graph of a system of inequalities.

Building Math Identity and Community

Connecting to Mathematical Practices

At first, students may not immediately be able to determine the constraints and might want to quit before really getting started. Encourage students to set a goal of identifying what they do know about the situation and build on that goal by using what they know, one part at a time. By looking only one step ahead, a task can seem much more manageable.

Modifications to Pacing

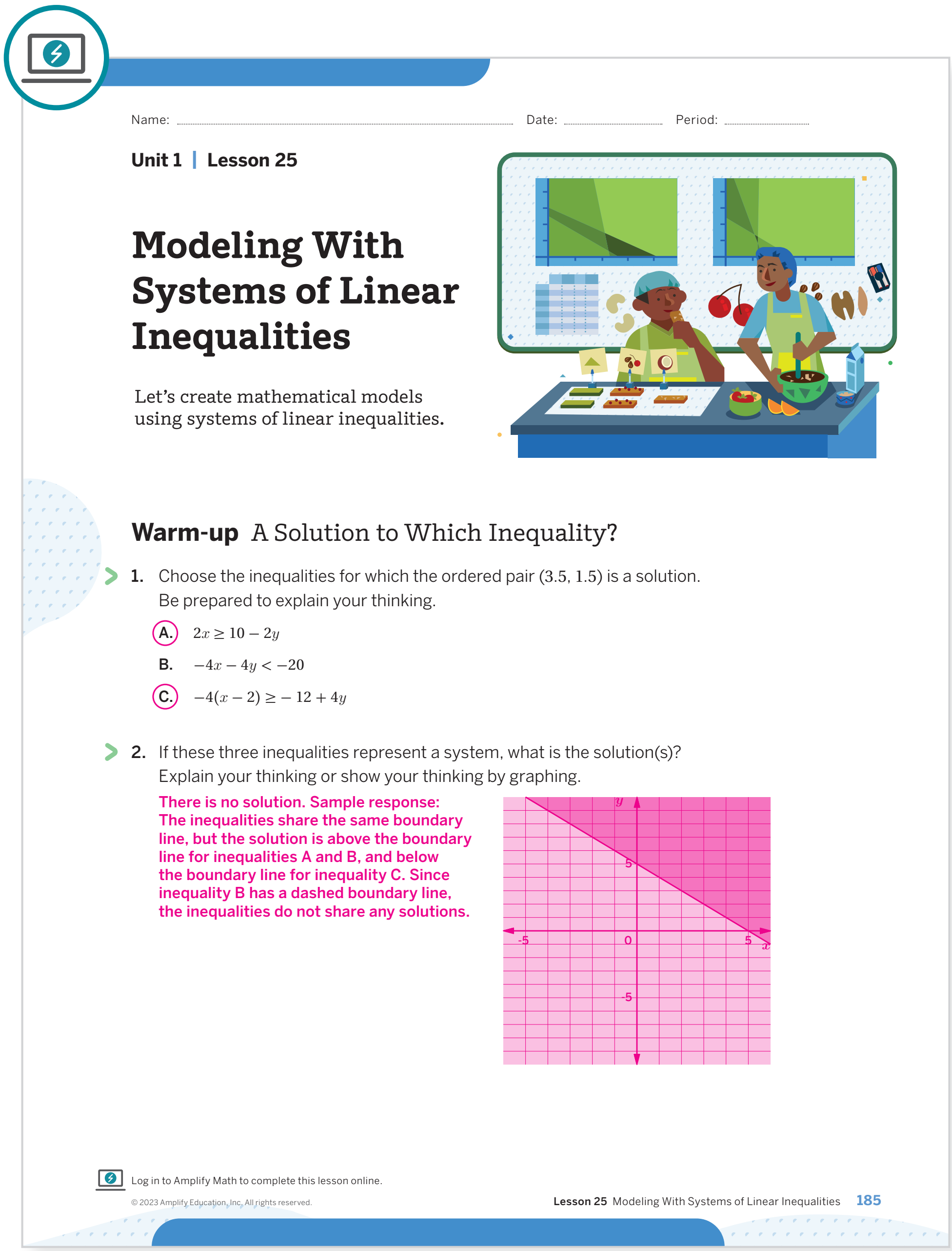
You may want to consider these additional modifications if you are short on time.

- In Problem 2 of the **Warm-up**, have students determine the solution set of the systems of linear inequalities consisting of only the first two inequalities.
- In **Activity 2**, have students analyze either Tyler's or Elena's system of inequalities and graph.

Independent | 5 min

Warm-up A Solution to Which Inequality?

Students determine the solution set of a system of linear inequalities to think carefully about whether boundary lines are included.



Name: Date: Period:

Unit 1 | Lesson 25

Modeling With Systems of Linear Inequalities

Let's create mathematical models using systems of linear inequalities.

Warm-up A Solution to Which Inequality?

1. Choose the inequalities for which the ordered pair (3.5, 1.5) is a solution. Be prepared to explain your thinking.

   [✓ A.] $2x \geq 10 - 2y$

   B. $-4x - 4y < -20$

   [✓ C.] $-4(x - 2) \geq -12 + 4y$

2. If these three inequalities represent a system, what is the solution(s)? Explain your thinking or show your thinking by graphing.

   There is no solution. Sample response: The inequalities share the same boundary line, but the solution is above the boundary line for inequalities A and B, and below the boundary line for inequality C. Since inequality B has a dashed boundary line, the inequalities do not share any solutions.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 25 Modeling With Systems of Linear Inequalities 185

① Launch

To activate prior knowledge, ask students to define and explain a "boundary line." Have students work independently.

② Monitor

Help students get started by saying, "To help graph each inequality, isolate y to identify the slope and y-intercept of the boundary line."

Look for points of confusion:

- **Thinking there are infinite solutions because the inequalities have the same boundary line.** Have students test points on both sides of and on the boundary line to verify their answer.

Look for productive strategies:

- Using the expressions in Problem 1 after isolating y to explain that the system has no solutions.

③ Connect

Display a graph of the system of linear inequalities.

Have individual students share their thinking for each problem.

Highlight that when inequalities in a system share a boundary line, it is difficult to determine the solution set just from the graph. For example, the graphs of inequality A and B shared the same boundary line, but the graph of inequality A used a solid line. The graph does not indicate that inequality B used a dashed line.

Ask, "Is it possible for a system of linear inequalities to have one solution?" No. If the boundary lines intersect, the shading will overlap and the solution set will have infinitely many solutions.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students respond to the Ask question, listen for and amplify the language they use, such as *boundary line, intersect, overlapping region, solution set, infinitely many*, etc. Show two visual examples of graphs of a system of inequalities: one in which the system has no solution and one in which the system has infinitely many solutions. Ask students to explain why these are the only possibilities.

Power-up

To power up students' ability to write a system of inequalities to represent a scenario, have students complete:

Which system of linear inequalities could represent the following riddle: "The sum of two numbers is less then 10. If I subtract the second number from the first, the difference is greater then 3."

[✓ A.] $\begin{cases} x + y < 10 \\ x - y > 3 \end{cases}$

B. $\begin{cases} x + y = 10 \\ x - y = 3 \end{cases}$

C. $\begin{cases} x + y \leq 10 \\ x - y \geq 3 \end{cases}$

D. $\begin{cases} x + y > 10 \\ x - y < 3 \end{cases}$

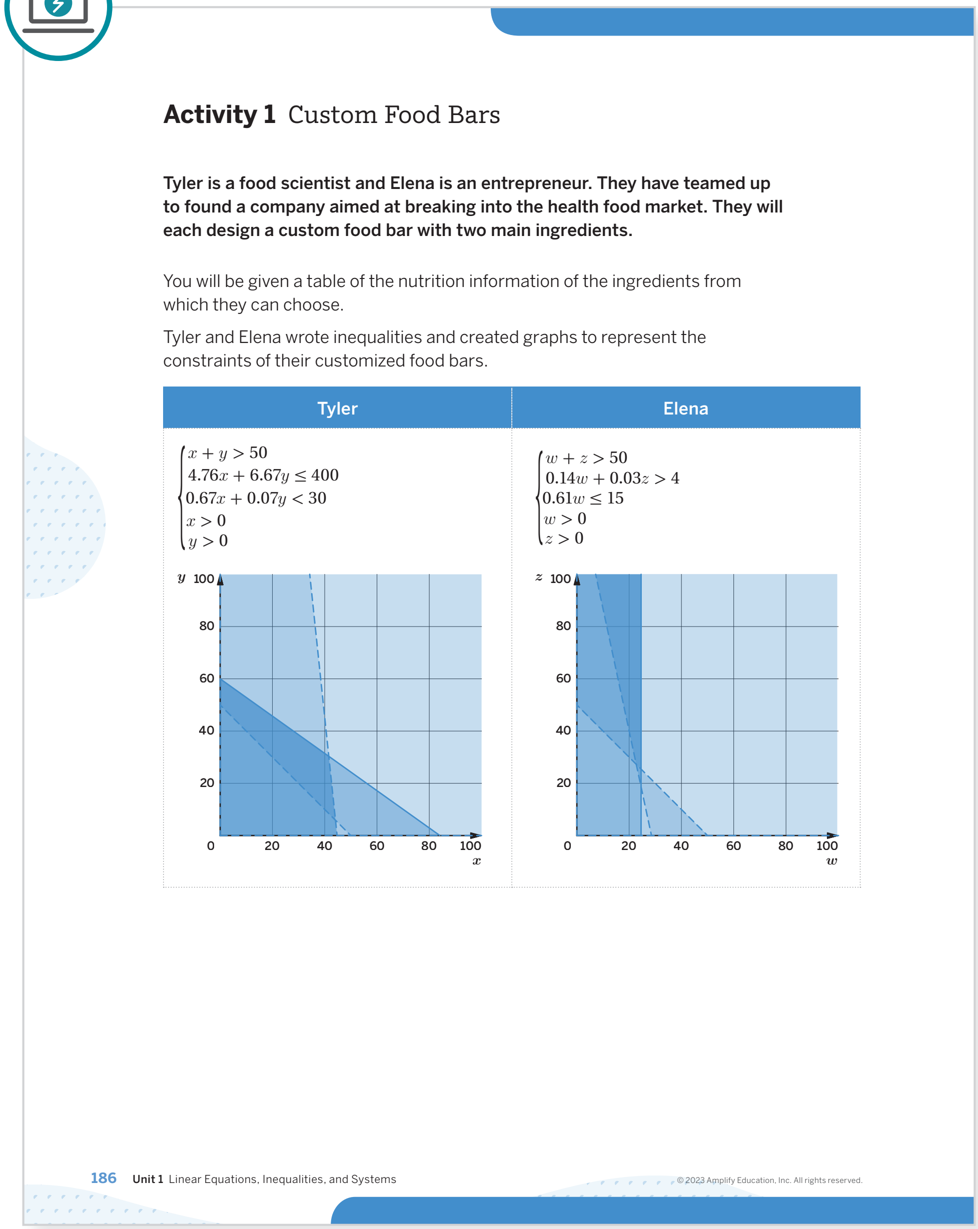
Use: Before Activity 1

Informed by: Performance on Lesson 24, Practice Problem 5

Independent | 15 min

Activity 1 Custom Food Bars

Students interpret multiple representations of mathematical models to make connections between the inequalities, the graphs, and the data set.



Activity 1 Custom Food Bars

Tyler is a food scientist and Elena is an entrepreneur. They have teamed up to found a company aimed at breaking into the health food market. They will each design a custom food bar with two main ingredients.

You will be given a table of the nutrition information of the ingredients from which they can choose.

Tyler and Elena wrote inequalities and created graphs to represent the constraints of their customized food bars.

Tyler	Elena
$\begin{cases} x+y>50 \\ 4.76x+6.67y \le 400 \\ 0.67x+0.07y<30 \\ x>0 \\ y>0 \end{cases}$	$\begin{cases} w+z>50 \\ 0.14w+0.03z>4 \\ 0.61w \le 15 \\ w>0 \\ z>0 \end{cases}$

186 Unit 1 Linear Equations, Inequalities, and Systems

©2023 Amplify Education, Inc. All rights reserved.

1 Launch

Distribute the Activity 1 PDF. Have students read through the task independently. Have them share one thing they notice and wonder.

2 Monitor

Help students get started by saying, “Use the coefficients from the second and third inequalities in each system to identify the ingredients in the table.”

Look for points of confusion:

- **Missing one of the combinations of ingredients for Elena.** Have students circle the values in the table that reflect the coefficients used in the inequalities
- **Thinking that $0.61w \le 15$ does not reflect constraints for both ingredients.** Have students identify the nutritional information this inequality represents. Ask, “What is the coefficient of z for this inequality for this constraint?”

Look for productive strategies:

- Highlighting the columns and rows values are used from the table.
- Writing a description for each inequality.
- Testing the values of the combination of ingredients in each inequality.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students choose one person, Tyler or Elena, to analyze their system of inequalities. Pair students with a partner who chose the other person and have them share their thinking and responses.

Extension: *Math Enrichment*

Tell students that Tyler also wants his bar to have no more than 20 g of protein and at least 5 g of fiber. Have them add inequalities to his current system and name one possible combination of ingredients. 5 g of chocolate pieces and 51 g of shredded coconut. $0.05x + 0.07y \le 20$ and $0.02x + 0.13y \ge 5$

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share what constraint each inequality represents, draw attention to the similarities between the inequalities in each system. For example, each person wrote an inequality of the form ___ + ___ > 50. Each person also wrote the inequalities of the form ___ > 0. Ask students how these similarities and the coefficients in the second and third inequalities can help them determine the variables and their constraints.

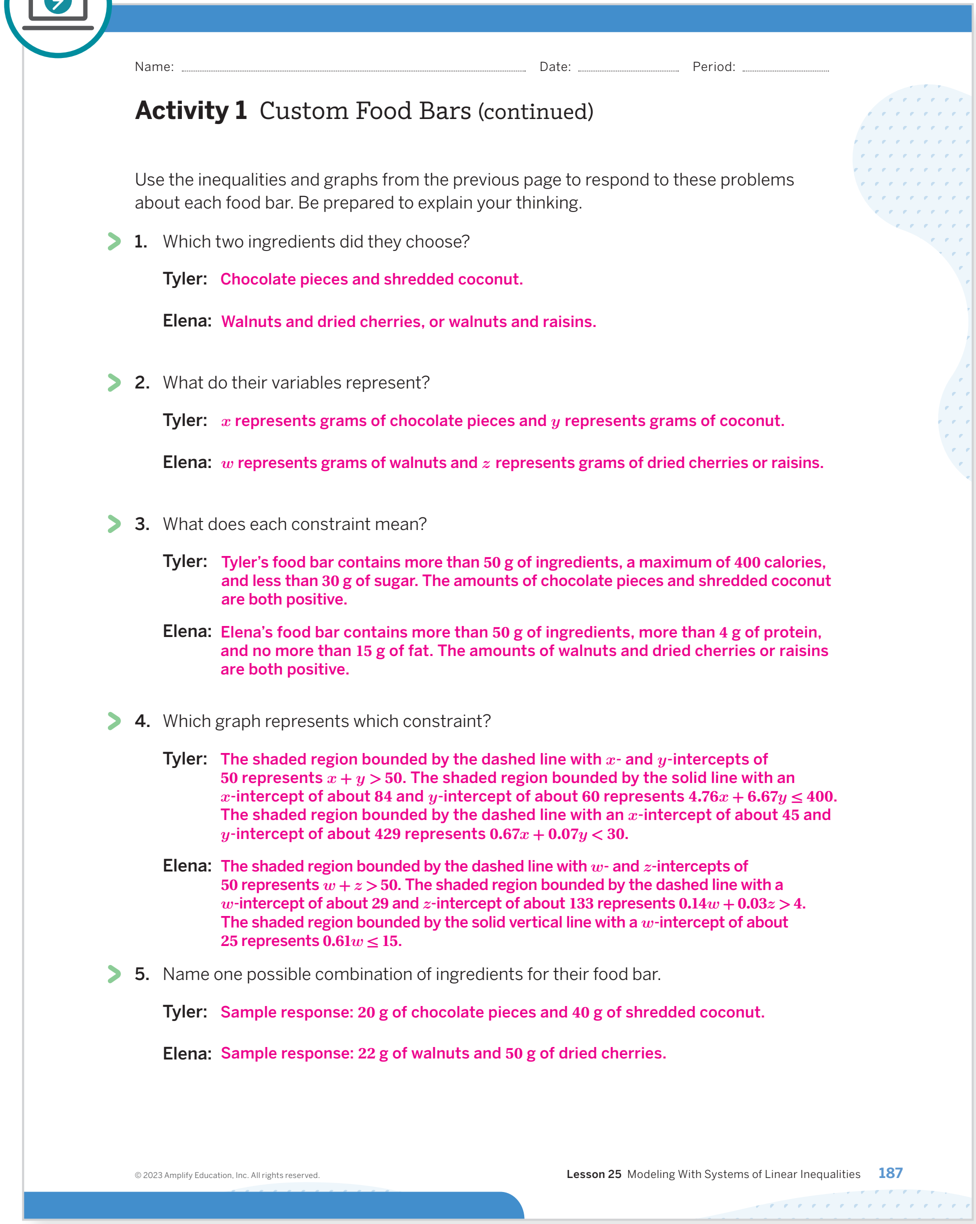
English Learners

Use color coding to code the information from the Activity 1 PDF and the inequalities to help make connections.

Independent | 15 min

Activity 1 Custom Food Bars (continued)

Students interpret multiple representations of mathematical models to make connections between the inequalities, the graphs, and the data set.



Name: ______ Date: ______ Period: ______

Activity 1 Custom Food Bars (continued)

Use the inequalities and graphs from the previous page to respond to these problems about each food bar. Be prepared to explain your thinking.

1. Which two ingredients did they choose?

   Tyler: Chocolate pieces and shredded coconut.

   Elena: Walnuts and dried cherries, or walnuts and raisins.

2. What do their variables represent?

   Tyler: x represents grams of chocolate pieces and y represents grams of coconut.

   Elena: w represents grams of walnuts and z represents grams of dried cherries or raisins.

3. What does each constraint mean?

   Tyler: Tyler's food bar contains more than 50 g of ingredients, a maximum of 400 calories, and less than 30 g of sugar. The amounts of chocolate pieces and shredded coconut are both positive.

   Elena: Elena's food bar contains more than 50 g of ingredients, more than 4 g of protein, and no more than 15 g of fat. The amounts of walnuts and dried cherries or raisins are both positive.

4. Which graph represents which constraint?

   Tyler: The shaded region bounded by the dashed line with x- and y-intercepts of 50 represents $x + y > 50$. The shaded region bounded by the solid line with an x-intercept of about 84 and y-intercept of about 60 represents $4.76x + 6.67y \leq 400$. The shaded region bounded by the dashed line with an x-intercept of about 45 and y-intercept of about 429 represents $0.67x + 0.07y < 30$.

   Elena: The shaded region bounded by the dashed line with w- and z-intercepts of 50 represents $w + z > 50$. The shaded region bounded by the dashed line with a w-intercept of about 29 and z-intercept of about 133 represents $0.14w + 0.03z > 4$. The shaded region bounded by the solid vertical line with a w-intercept of about 25 represents $0.61w \leq 15$.

5. Name one possible combination of ingredients for their food bar.

   Tyler: Sample response: 20 g of chocolate pieces and 40 g of shredded coconut.

   Elena: Sample response: 22 g of walnuts and 50 g of dried cherries.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 25 Modeling With Systems of Linear Inequalities 187

3 Connect

Display each system and matching graph one at a time.

Have pairs of students share what constraint each inequality represents.

Highlight that clarifying the meaning of each variable and each inequality symbol in your own words can help you then describe each constraint.

Ask:

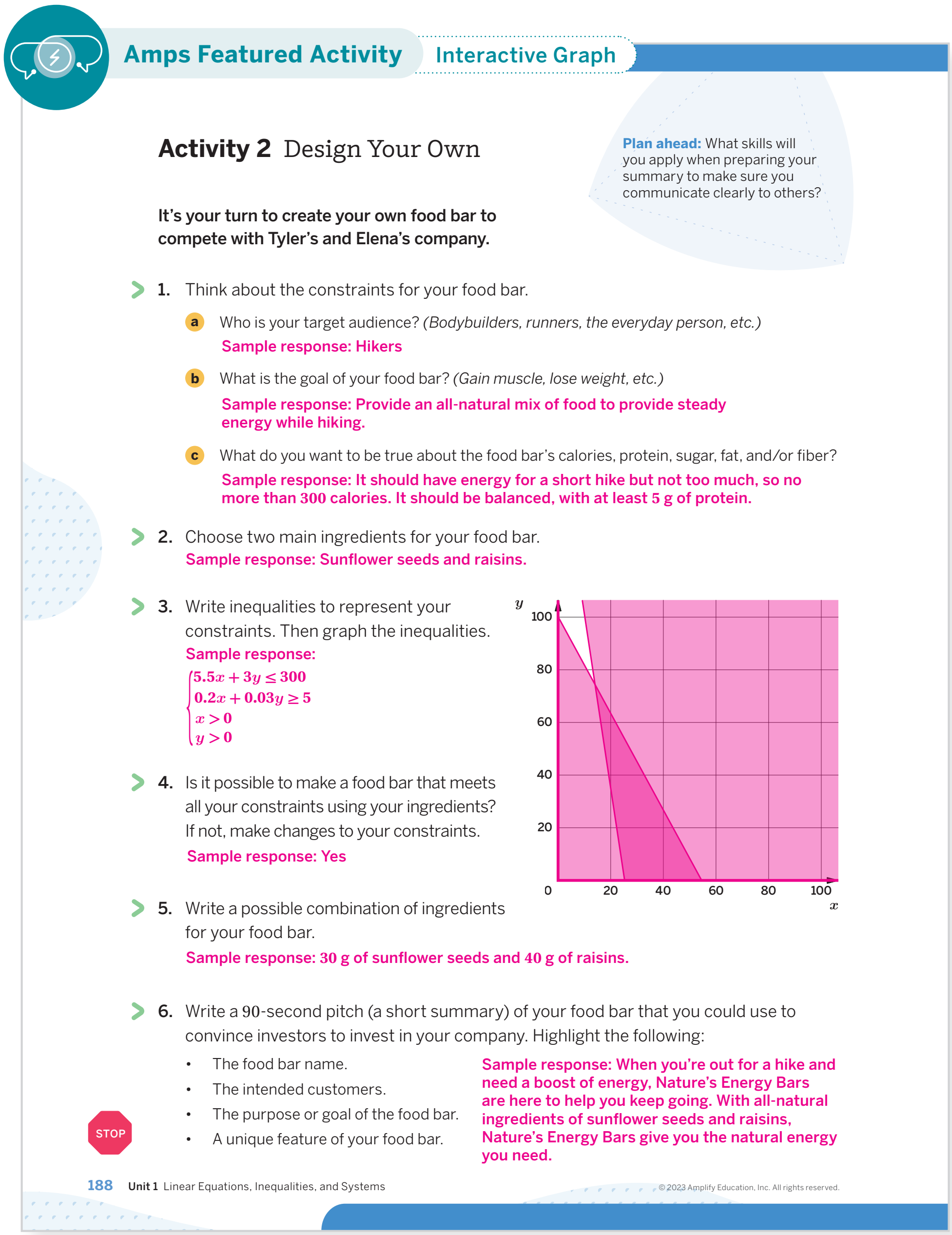
- "Why do you think Elena and Tyler both included the inequalities $x > 0$ and $y > 0$, and $w > 0$ and $z > 0$?" The number of grams of each ingredient cannot be negative, so both variables must be greater than 0.
- "How do those inequalities affect the graph of the solution?" They limit the region to the first quadrant.

Pairs | 20 min

Activity 2 Design Your Own

Students use their understanding of systems of linear inequalities to perform mathematical modeling of their own constraints.

Amps Featured Activity Interactive Graph



Activity 2 Design Your Own

Plan ahead: What skills will you apply when preparing your summary to make sure you communicate clearly to others?

It's your turn to create your own food bar to compete with Tyler's and Elena's company.

1. Think about the constraints for your food bar.
   - a. Who is your target audience? *(Bodybuilders, runners, the everyday person, etc.)*
     Sample response: Hikers
   - b. What is the goal of your food bar? *(Gain muscle, lose weight, etc.)*
     Sample response: Provide an all-natural mix of food to provide steady energy while hiking.
   - c. What do you want to be true about the food bar's calories, protein, sugar, fat, and/or fiber?
     Sample response: It should have energy for a short hike but not too much, so no more than 300 calories. It should be balanced, with at least 5 g of protein.
2. Choose two main ingredients for your food bar.
   Sample response: Sunflower seeds and raisins.
3. Write inequalities to represent your constraints. Then graph the inequalities.
   Sample response:
   $$\begin{cases} 5.5x + 3y \leq 300 \\ 0.2x + 0.03y \geq 5 \\ x > 0 \\ y > 0 \end{cases}$$
4. Is it possible to make a food bar that meets all your constraints using your ingredients? If not, make changes to your constraints.
   Sample response: Yes
5. Write a possible combination of ingredients for your food bar.
   Sample response: 30 g of sunflower seeds and 40 g of raisins.
6. Write a 90-second pitch (a short summary) of your food bar that you could use to convince investors to invest in your company. Highlight the following:
   - The food bar name.
   - The intended customers.
   - The purpose or goal of the food bar.
   - A unique feature of your food bar.

   Sample response: When you're out for a hike and need a boost of energy, Nature's Energy Bars are here to help you keep going. With all-natural ingredients of sunflower seeds and raisins, Nature's Energy Bars give you the natural energy you need.

STOP

188 Unit 1 Linear Equations, Inequalities, and Systems

©2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in pairs. Display the Activity 2 PDF. Say, "With your partner, use these range of nutritional values to help determine your constraints."

2 Monitor

Help students get started by asking, "The values displayed are for an entire day. What constraints would be reasonable for just one food bar?"

Look for points of confusion:

- **Excluding inequalities for their variables to be greater than zero.** Have students compare their graph to those in Activity 1. Ask, "What inequalities can you add to your system so that only positive values are considered?"

Look for productive strategies:

- Rewriting their inequalities in slope-intercept form to graph the system of linear inequalities.

3 Connect

Have pairs of students share their thinking behind their chosen constraints.

Highlight that there could be more than five inequalities in the system if several of the nutritional categories were constrained as well as other constraints, such as total number of grams.

Ask:

- "Did anyone have to revise or change their model in order to come up with a solution they could use? How did you revise your model?"
- "How did you use the graph to choose a recipe for your food bar?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students sample constraints to use, such as the following:

- There should be no more than 300 calories per bar.
- Each bar should contain at least 5 g of protein, and be made from sunflower seeds and raisins.

Then have students complete Problems 1a, 1b, and 3–5.

Math Language Development

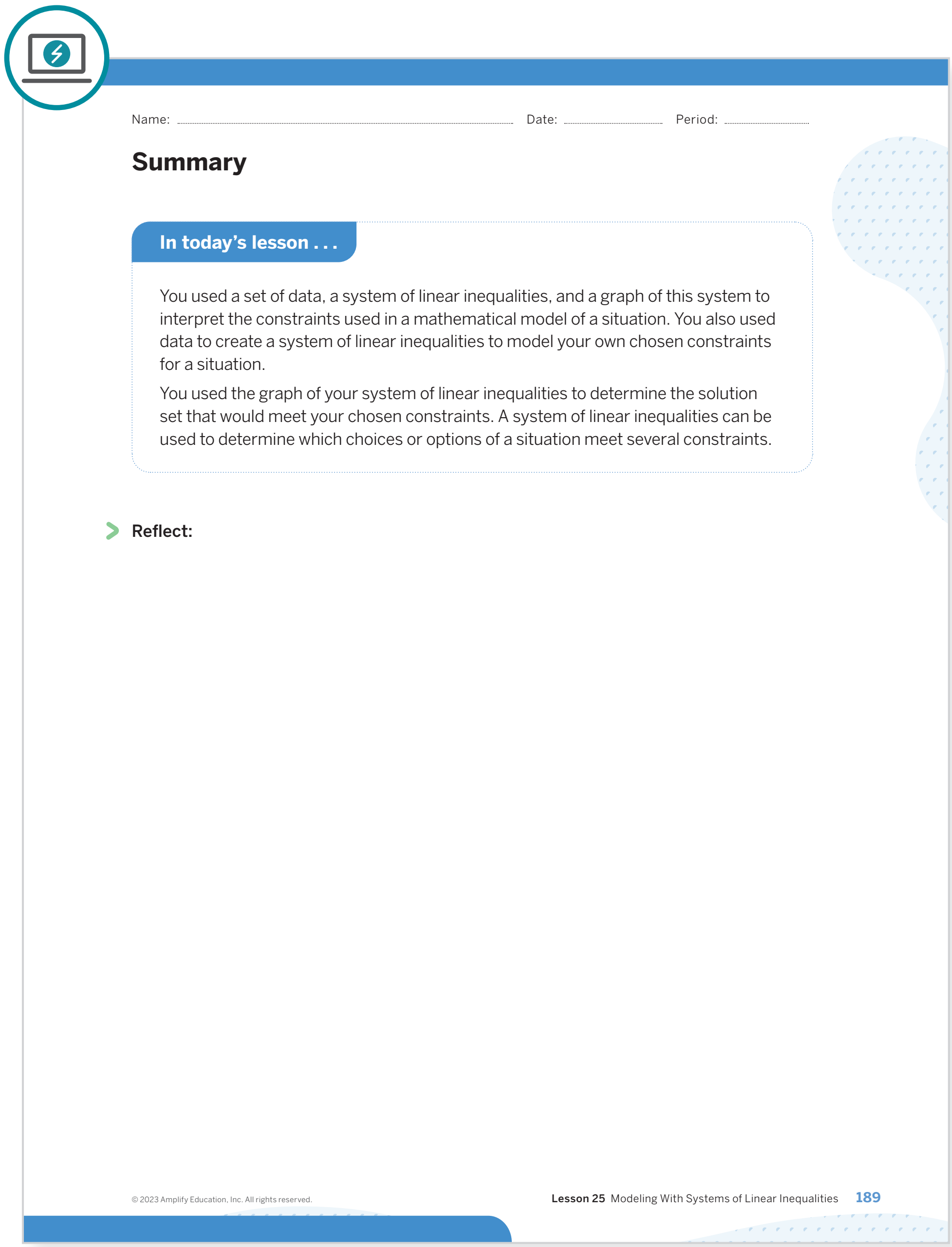
MLR7: Compare and Connect

Have groups create a display of their work with annotations, notes, diagrams, arrows, etc. Begin the Connect by selecting and arranging 2–4 displays for all to see. Give students time to analyze and interpret the displays before having the students who created them present their work.

Whole Class | 5 min

Summary

Review and synthesize interpreting and creating mathematical models that represent constraints in context.



Name: Date: Period:

Summary

In today's lesson . . .

You used a set of data, a system of linear inequalities, and a graph of this system to interpret the constraints used in a mathematical model of a situation. You also used data to create a system of linear inequalities to model your own chosen constraints for a situation.

You used the graph of your system of linear inequalities to determine the solution set that would meet your chosen constraints. A system of linear inequalities can be used to determine which choices or options of a situation meet several constraints.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display sample response of the inequalities and graph from Activity 2.

Have students share possible combinations of ingredients in the food bar represented by the system of inequalities.

Highlight that systems of linear inequalities that are used for mathematical models are not always restricted to the first quadrant.

Ask:

- "Can you give a real life example where a system of linear inequalities would include solutions in other quadrants?" Sample response: Earning profit or going into debt.
- "Could you add constraints relating to a third ingredient to your graph?" No; The coordinate plane represents two-dimensions or two variables, so I could not add a third constraint.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

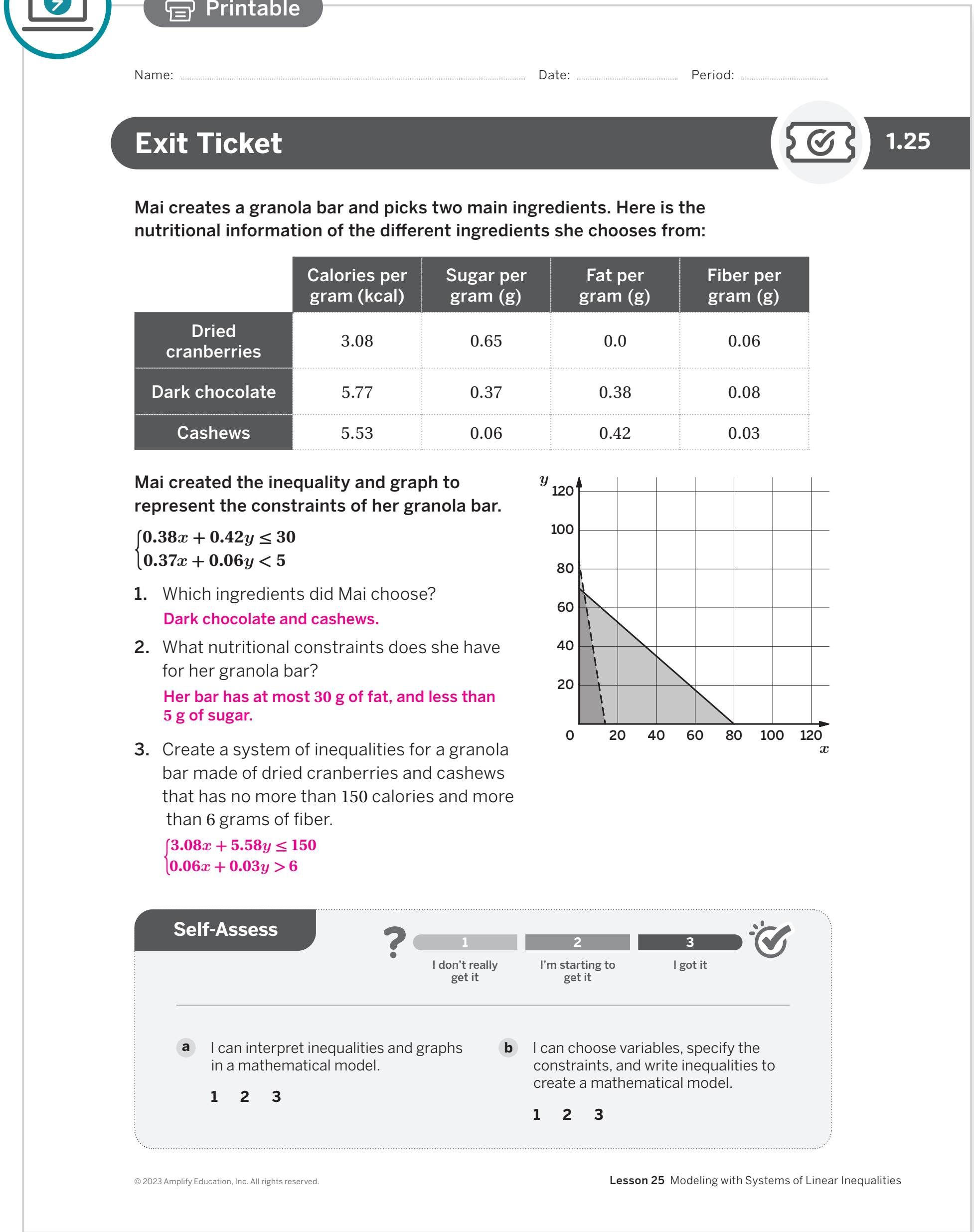
- "What strategies did you find helpful today when creating your own mathematical model?"
- "Were any strategies not helpful? Why?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by interpreting a system of inequalities and write inequalities to create a mathematical model.

Printable



Name: Date: Period:

Exit Ticket 1.25

Mai creates a granola bar and picks two main ingredients. Here is the nutritional information of the different ingredients she chooses from:

	Calories per gram (kcal)	Sugar per gram (g)	Fat per gram (g)	Fiber per gram (g)
Dried cranberries	3.08	0.65	0.0	0.06
Dark chocolate	5.77	0.37	0.38	0.08
Cashews	5.53	0.06	0.42	0.03

Mai created the inequality and graph to represent the constraints of her granola bar.

$$\begin{cases} 0.38x + 0.42y \le 30 \\ 0.37x + 0.06y < 5 \end{cases}$$

1. Which ingredients did Mai choose?
   Dark chocolate and cashews.
2. What nutritional constraints does she have for her granola bar?
   Her bar has at most 30 g of fat, and less than 5 g of sugar.
3. Create a system of inequalities for a granola bar made of dried cranberries and cashews that has no more than 150 calories and more than 6 grams of fiber.

$$\begin{cases} 3.08x + 5.58y \le 150 \\ 0.06x + 0.03y > 6 \end{cases}$$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can interpret inequalities and graphs in a mathematical model.
1 2 3

b I can choose variables, specify the constraints, and write inequalities to create a mathematical model.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 25 Modeling with Systems of Linear Inequalities

Success looks like . . .

- **Goal:** Defining the constraints in a situation and creating a mathematical model to represent them.
- **Language Goal:** Interpreting a mathematical model, presented as inequalities and graphs, that represents a situation. **(Speaking and Listening, Writing)**
  - » Explaining the ingredients and nutritional constraints represented by Mai's system of inequalities In Problems 1 and 2.

Suggested next steps

If students misidentify the ingredients in Problem 1, consider:

- Reviewing using values from the table to identify ingredients from Activity 1.
- Assigning Practice Problems 1 and 2.
- Asking, "How could you use the coefficients to identify the ingredient from the table?"

If students misidentify the constraints in Problem 2, consider:

- Reviewing using values from the table to identify the constraints of the granola bar from Activity 1.
- Assigning Practice Problems 1 and 2.
- Asking, "Which columns are used for each inequality?"

If students incorrectly write the inequalities in Problem 3, consider:

- Reviewing using their own constraints and the table of nutritional information in Activity 2.
- Assigning Practice Problems 1 and 2.
- Asking, "Which values from the table will you use to write each inequality?"

Professional Learning

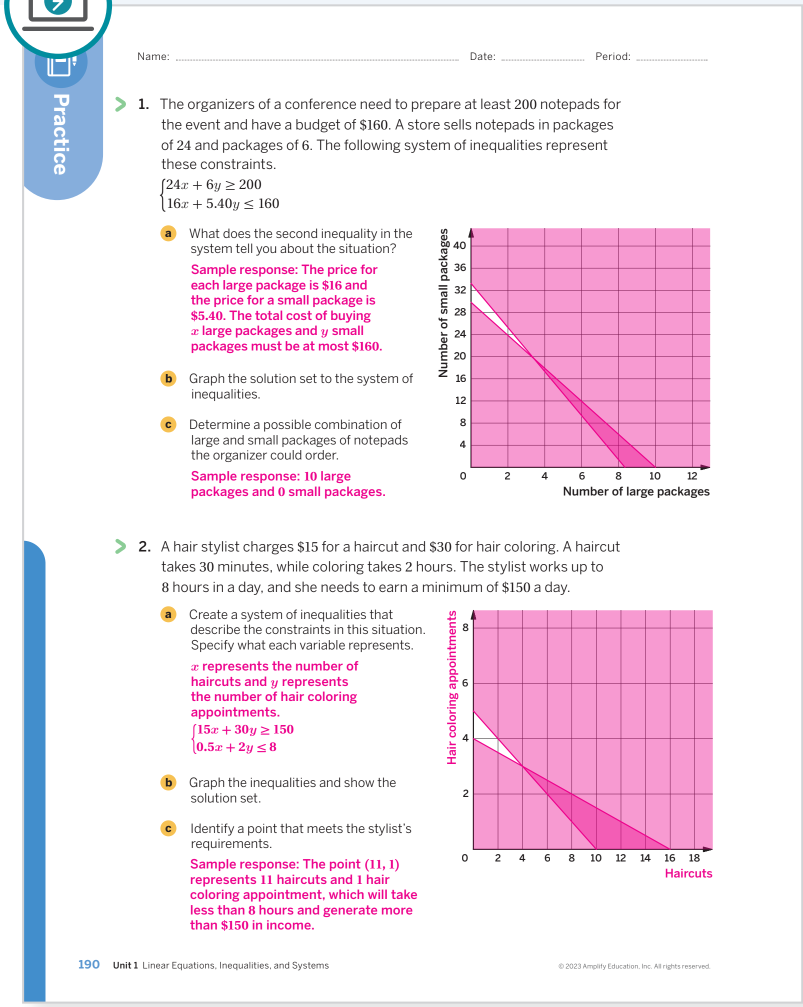
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In earlier lessons, students wrote systems of linear equations. How did that support writing linear inequalities to represent constraints?
- What did students find frustrating about modeling their chosen constraints with a system of linear inequalities? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

1. The organizers of a conference need to prepare at least 200 notepads for the event and have a budget of $160. A store sells notepads in packages of 24 and packages of 6. The following system of inequalities represent these constraints.

$$\begin{cases}24x + 6y \geq 200\\16x + 5.40y \leq 160\end{cases}$$

a. What does the second inequality in the system tell you about the situation?

Sample response: The price for each large package is $16 and the price for a small package is $5.40. The total cost of buying x large packages and y small packages must be at most $160.

b. Graph the solution set to the system of inequalities.

c. Determine a possible combination of large and small packages of notepads the organizer could order.

Sample response: 10 large packages and 0 small packages.

2. A hair stylist charges $15 for a haircut and $30 for hair coloring. A haircut takes 30 minutes, while coloring takes 2 hours. The stylist works up to 8 hours in a day, and she needs to earn a minimum of $150 a day.

a. Create a system of inequalities that describe the constraints in this situation. Specify what each variable represents.

x represents the number of haircuts and y represents the number of hair coloring appointments.

$$\begin{cases}15x + 30y \geq 150\\0.5x + 2y \leq 8\end{cases}$$

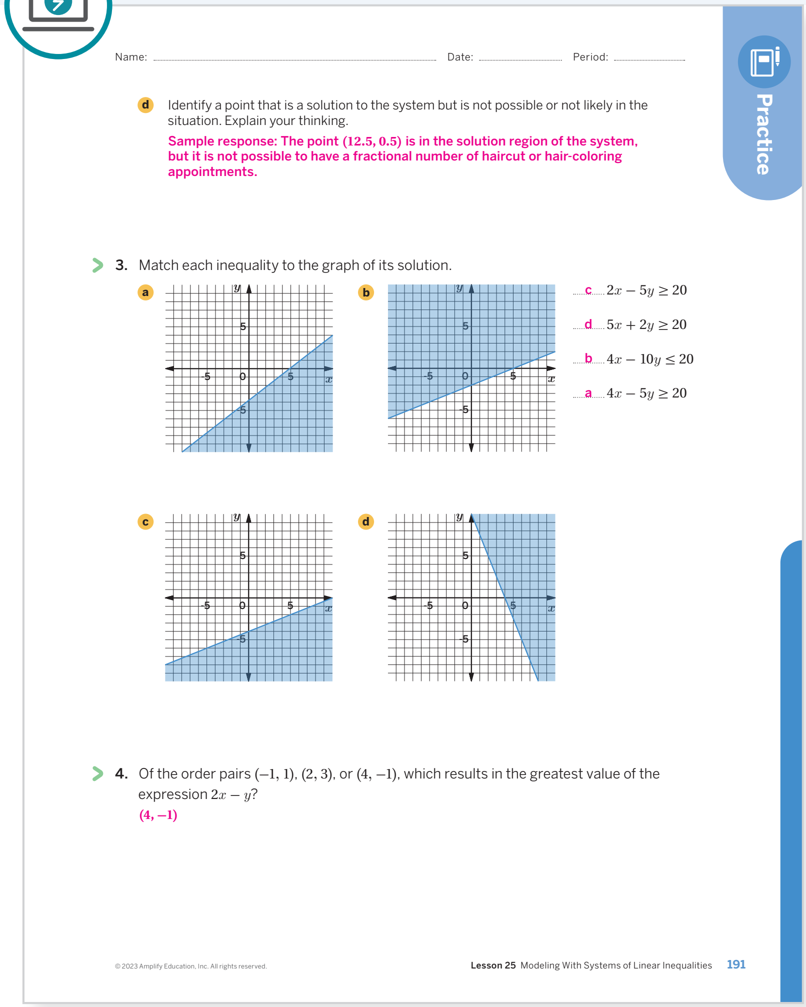
b. Graph the inequalities and show the solution set.

c. Identify a point that meets the stylist's requirements.

Sample response: The point (11, 1) represents 11 haircuts and 1 hair coloring appointment, which will take less than 8 hours and generate more than $150 in income.

190 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.



Name: ________ Date: ________ Period: ________

d. Identify a point that is a solution to the system but is not possible or not likely in the situation. Explain your thinking.

Sample response: The point (12.5, 0.5) is in the solution region of the system, but it is not possible to have a fractional number of haircut or hair-coloring appointments.

3. Match each inequality to the graph of its solution.

c $2x - 5y \geq 20$

d $5x + 2y \geq 20$

b $4x - 10y \leq 20$

a $4x - 5y \geq 20$

4. Of the order pairs (–1, 1), (2, 3), or (4, –1), which results in the greatest value of the expression $2x - y$?

(4, –1)

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 25 Modeling With Systems of Linear Inequalities 191

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
Spiral	3	Unit 2 Lesson 15	2
Formative	4	Unit 2 Lesson 26	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 1 | **LESSON 26 – CAPSTONE**

Linear Programming

Let's investigate how to maximize time and revenue within given constraints.

Focus

Goals

1. Given a system of linear inequalities and its graph, determine the maximum value in the solution region for a given expression.
2. **Language Goal:** Analyze given information about a scenario involving multiple constraints and write a mathematical model to represent it. **(Reading and Writing)**

Coherence

Today

Students apply their understanding of writing and graphing systems of linear inequalities. They analyze a bounded region and determine maximum values for different two-variable expressions. Then, students model a real-world scenario to determine the maximum revenue within the provided time constraints.

Previously

In Lesson 25, students interpreted, analyzed, and created their own systems of linear inequalities based on given constraints.

Coming Soon

In Unit 3, students will study linear models and how to analyze and represent data.

Rigor

- Students **apply** their understanding of systems of inequalities to linear programming.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2 (optional)	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	20 min	5 min	5 min
Independent	Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF (for display)
- Activity 2 PDF, one per group
- Anchor Chart PDF, *Forms of Linear Equations*
- Anchor Chart PDF, *Graphing Linear Inequalities*

Math Language Development

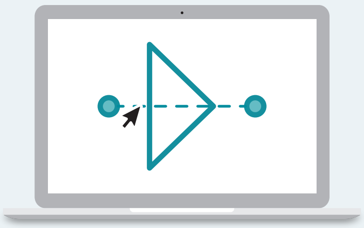
Review words

- *constraint*
- *system of linear inequalities*

Amps | Featured Activity

Activity 2
Digital Designs

Students are able to digitally trace and cut their designs using interactive tools in order for them to collect the data necessary to complete a linear programming problem.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may become frustrated or lost when they are asked to determine the maximum value for different expressions in Activity 1 and to maximize fundraising profits in Activity 2 if they do not have a clear strategy to accomplish these tasks. Encourage students to reflect and write down strategies they can use before attempting the problems, and check in with their peers for strategies they are using after their own individual quiet think-time.

Modifications to Pacing

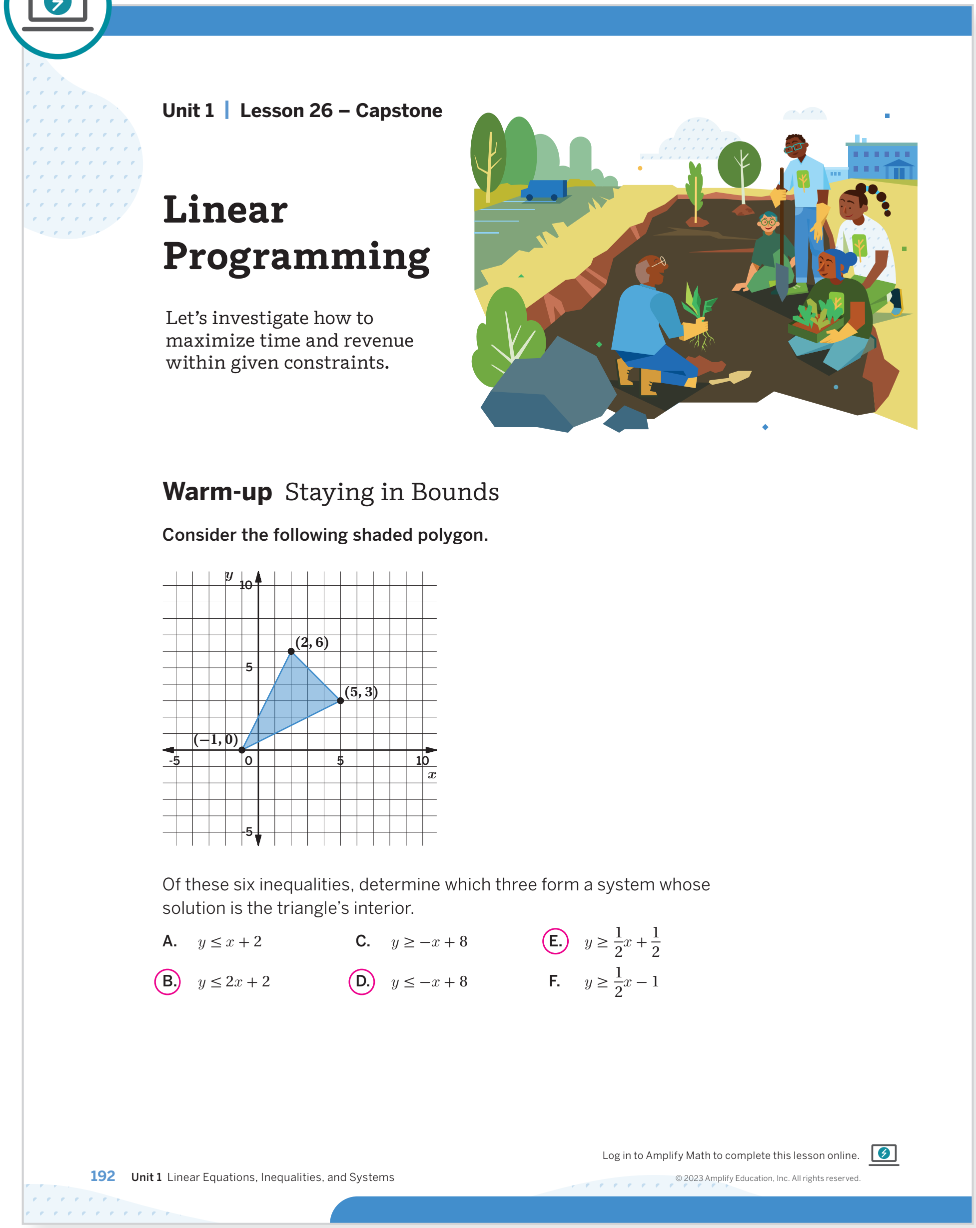
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Options A and F may be omitted.
- In **Activity 1**, have each group member complete the problems for a different point.
- In **Activity 2**, have groups complete two to four tracings and find the average time for the completed amount.

Independent | 5 min

Warm-up Staying in Bounds

Students match the linear inequalities with the polygon graphed to practice writing linear inequalities and to consider a bounded region.



Unit 1 | Lesson 26 – Capstone

Linear Programming

Let's investigate how to maximize time and revenue within given constraints.

Warm-up Staying in Bounds

Consider the following shaded polygon.

Of these six inequalities, determine which three form a system whose solution is the triangle's interior.

A. $y \leq x + 2$

[✓ B.] $y \leq 2x + 2$

C. $y \geq -x + 8$

[✓ D.] $y \leq -x + 8$

[✓ E.] $y \geq \frac{1}{2}x + \frac{1}{2}$

F. $y \geq \frac{1}{2}x - 1$

Log in to Amplify Math to complete this lesson online.

192 Unit 1 Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

① Launch

Have students work independently to match each inequality with the corresponding side of the polygon. Then have them compare their solutions with a partner.

② Monitor

Help students get started by activating their prior knowledge. Ask, "How could you determine the equation of each line?"

Look for productive strategies:

- Graphing each provided option on the coordinate plane.
- Evaluating the inequalities at different points from the solution region.
- Calculating the slope from the two points given.
- Counting the slope.
- Extending each line to determine the y-intercept.

③ Connect

Have individual students share their strategies and solutions.

Highlight that the bounded region formed by the three inequalities is the solution region for all points that make the system true.

Ask:

- "If you were not provided options to match, what strategies would you use to determine the inequalities that form the bounded triangle?"
- "What is the greatest value of x possible for the system of linear inequalities? y? How did you determine these?"

Differentiated Support

Accessibility: *Guide Processing and Visualization, Activate Prior Knowledge*

Before the Warm-up, or while students work, display or provide copies of the Anchor Chart PDFs, *Graphing Linear Inequalities* and *Forms of Linear Equations* for students to reference. These anchor charts will also help activate prior knowledge of slope-intercept form and graphing linear inequalities.

Power-up

To power up students' ability to compare the value of an expressions for multiple ordered pairs, have students complete:

Recall that, when given an ordered pair, the first value is the x-value and the second value is the y-value.

For the expression $3x + 2y$, determine which ordered pair results in the greatest value.

A. $(0, 2)$

B. $(-2, 4)$

[✓ C.] $(6, -4)$

D. $(1, 1)$

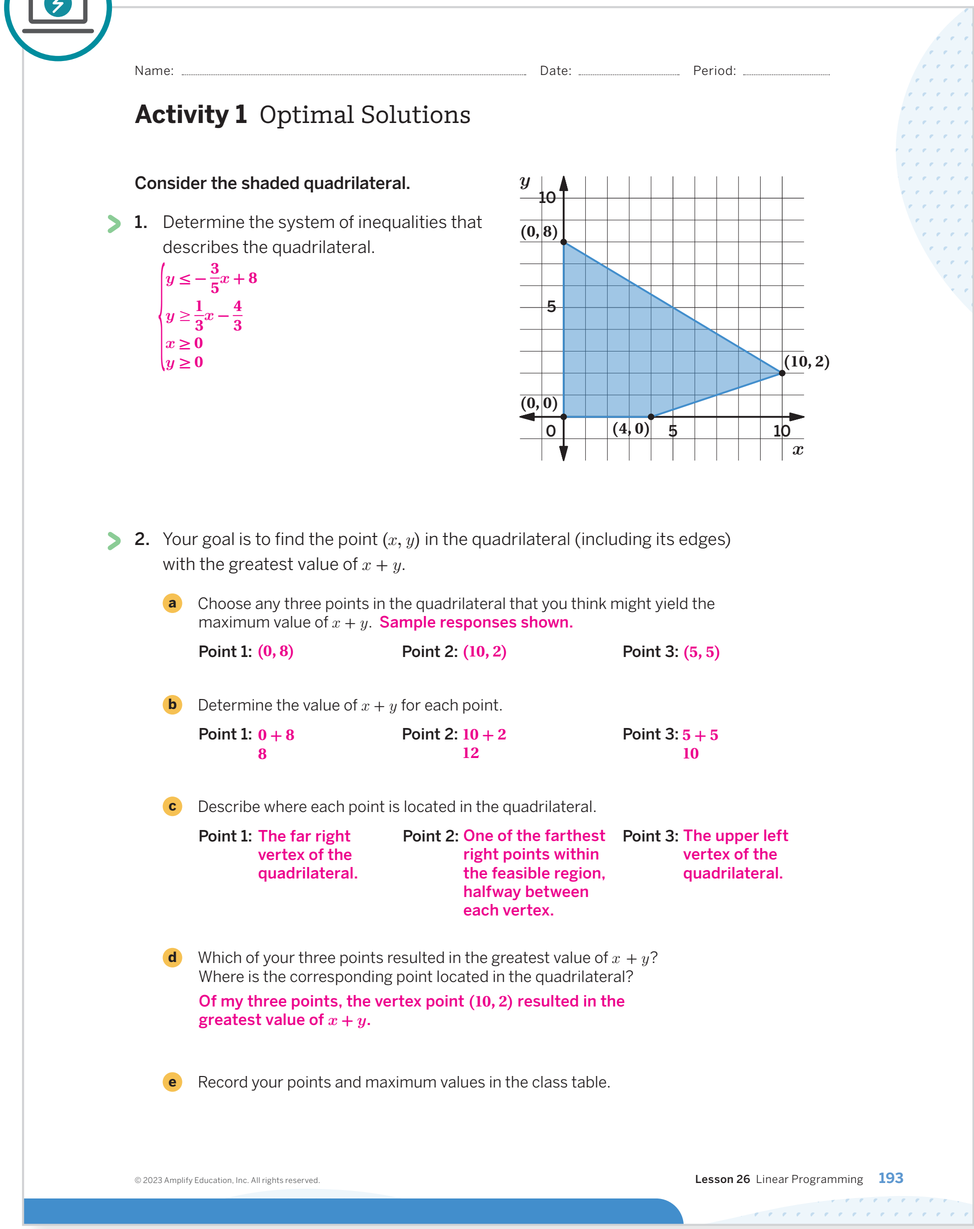
Use: Before Activity 1

Informed by: Performance on Lesson 25, Practice Problem 4

Small Groups | 15 min

Activity 1 Optimal Solutions

Students investigate maximizing the value for different expressions within a constrained region to discover that the optimal solutions occur at the vertices.



Name: Date: Period:

Activity 1 Optimal Solutions

Consider the shaded quadrilateral.

1. Determine the system of inequalities that describes the quadrilateral.

$$\begin{cases} y \leq -\frac{3}{5}x + 8 \\ y \geq \frac{1}{3}x - \frac{4}{3} \\ x \geq 0 \\ y \geq 0 \end{cases}$$

2. Your goal is to find the point (x, y) in the quadrilateral (including its edges) with the greatest value of $x + y$.

a. Choose any three points in the quadrilateral that you think might yield the maximum value of $x + y$. Sample responses shown.

Point 1: $(0, 8)$ Point 2: $(10, 2)$ Point 3: $(5, 5)$

b. Determine the value of $x + y$ for each point.

Point 1: $0 + 8$
8

Point 2: $10 + 2$
12

Point 3: $5 + 5$
10

c. Describe where each point is located in the quadrilateral.

Point 1: The far right vertex of the quadrilateral.

Point 2: One of the farthest right points within the feasible region, halfway between each vertex.

Point 3: The upper left vertex of the quadrilateral.

d. Which of your three points resulted in the greatest value of $x + y$? Where is the corresponding point located in the quadrilateral?

Of my three points, the vertex point $(10, 2)$ resulted in the greatest value of $x + y$.

e. Record your points and maximum values in the class table.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Define and explain optimal solutions and maximum values. Display the Activity 1 PDF. Arrange students in groups and have them complete Problems 1 and 2.

Discuss the table as a whole-class. Focus the discussion on the location of the point that produces the greatest value. Next, have students complete Problems 3 and 4 with group members.

2 Monitor

Help students get started by prompting them to recall the strategies discussed in the Warm-up.

Look for points of confusion:

- **Not evaluating the points in the expression.** Have students evaluate the expression for any values of x and y. Then, ask them if those ordered pairs are within the solution region.
- **Overlooking the location of the point when choosing possible points that yield a maximum value.** Have students plot their chosen points on the graph provided to help bring their attention to the location of each point.

Look for productive strategies:

- Creating a table for the values of x and y and the result of the expression.
- Considering different types of maximums — highest, most right — when making their choice.
- Testing different possible values in the expression to determine the best guesses.
- Creating different expressions to determine if their conjecture fits.
- Noticing the relationship between the vertices and the maximum value.

Activity 1 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with the system of inequalities in Problem 1 and have them begin the activity with Problem 2. This will allow them to focus on the goal of this activity, which is to determine optimal solutions. Consider demonstrating how to check the sum of one point in Problem 2a. Ask students if they can find a point with a greater sum.

Extension: *Math Enrichment*

Have students create their own triangle or quadrilateral plotted on the coordinate plane to verify that the solution in a constrained region that maximizes the value of an expression is one of the region's vertices.

Math Language Development

MLR8: Discussion Supports

During the Connect, as you highlight that the solution in a given constrained region that maximizes the value an expression will be one of the region's vertices, consider asking students to explain this, using their own words. For example, a student may say "If I want to find the greatest value of an expression relating x and y that is restricted by a region, one of the vertices will give me that greatest value."

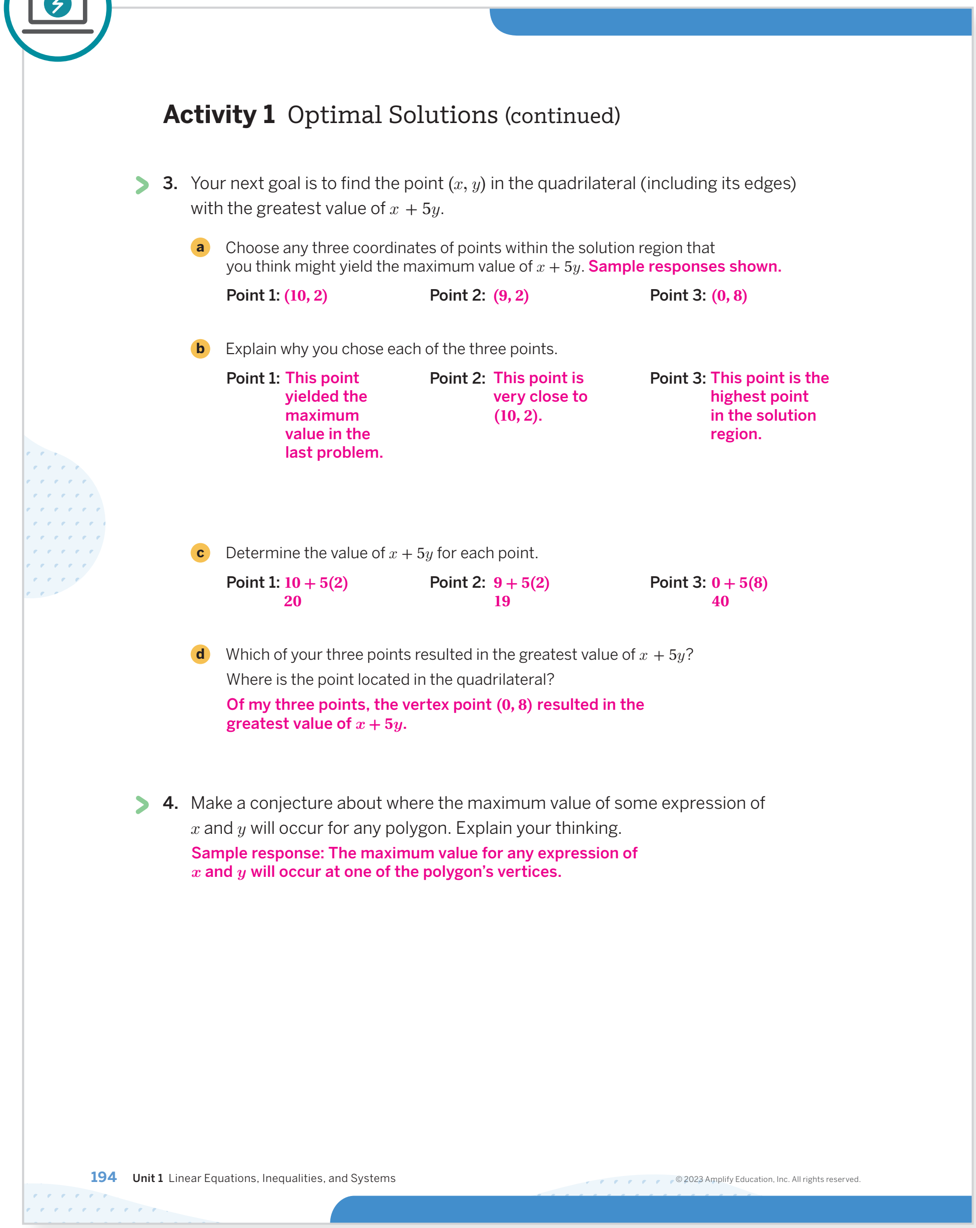
English Learners

Point out that the term maximum value of $x + y$ means the same as the greatest value of $x + y$.

Small Groups | 15 min

Activity 1 Optimal Solutions (continued)

Students investigate maximizing the value for different expressions within a constrained region to discover that the optimal solutions occur at the vertices.



Activity 1 Optimal Solutions (continued)

3. Your next goal is to find the point (x, y) in the quadrilateral (including its edges) with the greatest value of $x + 5y$.

   a. Choose any three coordinates of points within the solution region that you think might yield the maximum value of $x + 5y$. **Sample responses shown.**

Point 1: $(10, 2)$	Point 2: $(9, 2)$	Point 3: $(0, 8)$

   b. Explain why you chose each of the three points.

Point 1: **This point yielded the maximum value in the last problem.**	Point 2: **This point is very close to $(10, 2)$.**	Point 3: **This point is the highest point in the solution region.**

   c. Determine the value of $x + 5y$ for each point.

Point 1: $10 + 5(2)$ 20	Point 2: $9 + 5(2)$ 19	Point 3: $0 + 5(8)$ 40

   d. Which of your three points resulted in the greatest value of $x + 5y$? Where is the point located in the quadrilateral?

   Of my three points, the vertex point $(0, 8)$ resulted in the greatest value of $x + 5y$.

4. Make a conjecture about where the maximum value of some expression of x and y will occur for any polygon. Explain your thinking.

   Sample response: The maximum value for any expression of x and y will occur at one of the polygon's vertices.

194 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the graph of the constrained region.

Have groups of students share their strategies for Problem 3 and their thinking for Problem 4.

Highlight that the solution in a given constrained region that maximizes the value of an expression will be one of the region's vertices.

Ask:

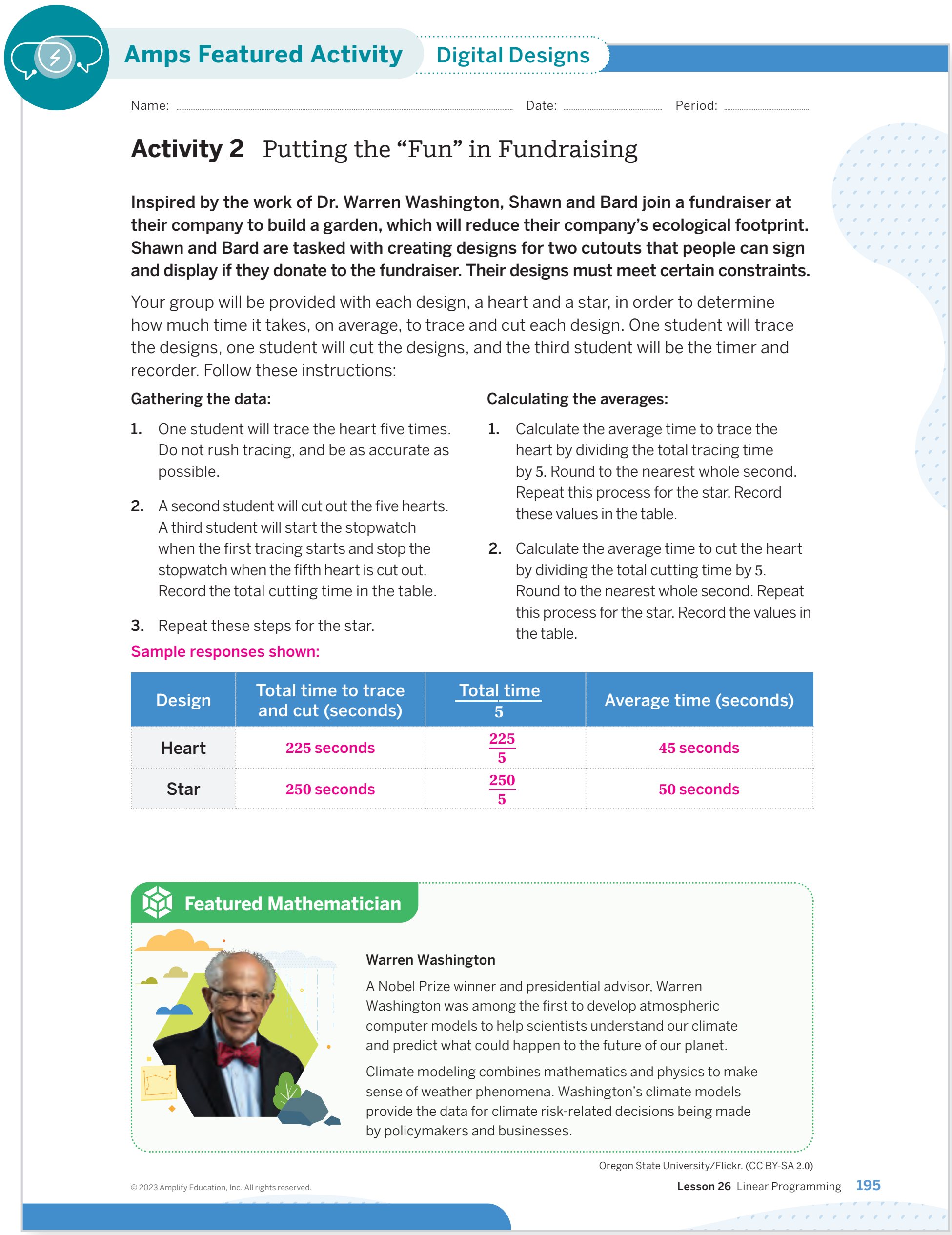
- "Do you think the minimum solutions for an expression in the bounded region will also occur at the vertices? Why or why not?"
- "How do you think this could be useful in real-life scenarios?" Sample responses: Maximizing profit or time, minimizing cost or time.

Optional

Small Groups | 10 min

Activity 2 Putting the “Fun” in Fundraising

Students collect data from tracing and cutting designs to use in creating inequalities that represent optimizing revenue.



Amps Featured Activity Digital Designs

Name: ______ Date: ______ Period: ______

Activity 2 Putting the “Fun” in Fundraising

Inspired by the work of Dr. Warren Washington, Shawn and Bard join a fundraiser at their company to build a garden, which will reduce their company’s ecological footprint. Shawn and Bard are tasked with creating designs for two cutouts that people can sign and display if they donate to the fundraiser. Their designs must meet certain constraints.

Your group will be provided with each design, a heart and a star, in order to determine how much time it takes, on average, to trace and cut each design. One student will trace the designs, one student will cut the designs, and the third student will be the timer and recorder. Follow these instructions:

Gathering the data:

1. One student will trace the heart five times. Do not rush tracing, and be as accurate as possible.
2. A second student will cut out the five hearts. A third student will start the stopwatch when the first tracing starts and stop the stopwatch when the fifth heart is cut out. Record the total cutting time in the table.
3. Repeat these steps for the star.

Calculating the averages:

1. Calculate the average time to trace the heart by dividing the total tracing time by 5. Round to the nearest whole second. Repeat this process for the star. Record these values in the table.
2. Calculate the average time to cut the heart by dividing the total cutting time by 5. Round to the nearest whole second. Repeat this process for the star. Record the values in the table.

Sample responses shown:

Design	Total time to trace and cut (seconds)	$\frac{\text{Total time}}{5}$	Average time (seconds)
Heart	225 seconds	$\frac{225}{5}$	45 seconds
Star	250 seconds	$\frac{250}{5}$	50 seconds

Featured Mathematician

Warren Washington

A Nobel Prize winner and presidential advisor, Warren Washington was among the first to develop atmospheric computer models to help scientists understand our climate and predict what could happen to the future of our planet.

Climate modeling combines mathematics and physics to make sense of weather phenomena. Washington’s climate models provide the data for climate risk-related decisions being made by policymakers and businesses.

Oregon State University/Flickr. (CC BY-SA 2.0)

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 26 Linear Programming 195

1 Launch

Arrange students in small groups. Have students take two minutes to read the instructions to themselves, then assign roles. Distribute the Activity 2 PDF.

2 Monitor

Help students get started by modeling how to time the tracing and cutting of each design.

Look for points of confusion:

- **Not converting the total tracing or cutting time from minutes to seconds.** Remind students to read the table and ask “What units should time be recorded in?”

Look for productive strategies:

- Accurately tracing and cutting the designs.
- Accurately calculating the average tracing and cutting time and rounding.

3 Connect

Have groups of students share their average tracing and cutting time with the class.

Highlight that most groups will have different average times for tracing and cutting each design, and that this will effect Activity 3. In Activity 3, students will need to know the maximum number of total designs that can be made. For each group, this is found by the following, where 1,600 seconds, or 30 minutes, is the maximum amount of time allowed.

$$\frac{1800}{\text{Longer average time}} < \text{Maximum number of designs} < \frac{1800}{\text{Faster average time}}$$

Ask, “Why do you think you took the average tracing and cutting time of five attempts instead of just timing ourselves once?”

Math Language Development

MLR8: Discussion Supports

During the Connect, display the inequality presented in the Highlight section. Mention that students will use this inequality in Activity 3, where 1,800 is the maximum time allowed, in seconds (30 minutes). To help students understand this inequality, ask:

- “Suppose the average times were 45 seconds and 60 seconds. What inequality represents the maximum number of designs?” $\frac{1800}{60} < d < \frac{1800}{45}$ or $30 < d < 40$
- “Describe the meaning of this inequality in words.” The maximum number of designs that can be made is between 30 and 40.

English Learners

Emphasize that a “longer average time” means that the number of seconds was greater, while a “faster average time” means that the number of seconds was less.

Featured Mathematician

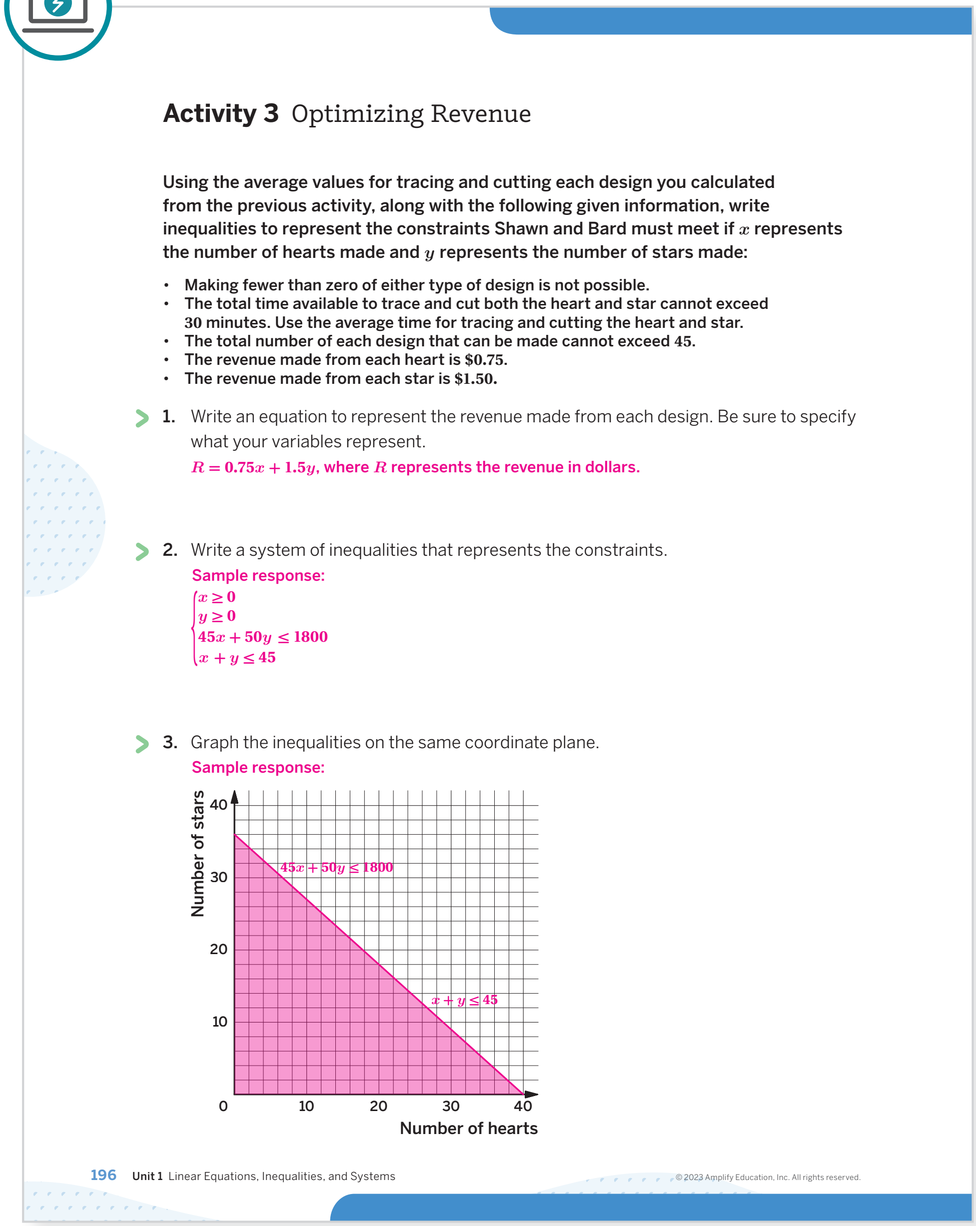
Warren Washington

Have students read about featured mathematician Warren Washington who helped pioneer the development of atmospheric computer models. These models help scientists and meteorologists understand Earth’s climate patterns.

Small Groups | 20 min

Activity 3 Optimizing Revenue

Students create and use inequalities that represent a viable solution region to determine maximum revenue.



Activity 3 Optimizing Revenue

Using the average values for tracing and cutting each design you calculated from the previous activity, along with the following given information, write inequalities to represent the constraints Shawn and Bard must meet if x represents the number of hearts made and y represents the number of stars made:

- **Making fewer than zero of either type of design is not possible.**
- **The total time available to trace and cut both the heart and star cannot exceed 30 minutes. Use the average time for tracing and cutting the heart and star.**
- **The total number of each design that can be made cannot exceed 45.**
- **The revenue made from each heart is $0.75.**
- **The revenue made from each star is $1.50.**

1. Write an equation to represent the revenue made from each design. Be sure to specify what your variables represent.
   $R = 0.75x + 1.5y$, where R represents the revenue in dollars.

2. Write a system of inequalities that represents the constraints.
   Sample response:
   $$\begin{cases} x \geq 0 \\ y \geq 0 \\ 45x + 50y \leq 1800 \\ x + y \leq 45 \end{cases}$$

3. Graph the inequalities on the same coordinate plane.
   Sample response:

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Note: If you omitted optional Activity 2, please provide the maximum number of total design that can be made, given by the inequality provided in the Connect section of Activity 2.

Allow students 10 minutes to complete Problems 1-2, pausing for a class discussion for two minutes, and then having students work on Problems 3–5.

2 Monitor

Help students get started by asking, "What key phrases or words can be translated into math symbols or operations?"

Look for points of confusion:

- **Thinking the revenue equation must also be graphed.** Ask, "How many unknowns are in the equation? Is it possible to graph this?"
- **Rounding the coordinates of the intersection points of the boundary lines.** Ask, "What are some possible issues that could happen when you round these values? Will they always be solutions? Why or why not?"

Look for productive strategies:

- Simplifying and rewriting inequalities before graphing them.
- Using a test point to determine what side of each inequality to shade.
- If the coordinates of the intersection points are not whole numbers, making sure that when rounding these values, they still fall in the solution region of all inequalities in the system.

Activity 3 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display the inequality from the Connect section from Activity 2. Consider demonstrating how to write either the revenue equation in Problem 1 or the inequalities $x \geq 0$ and $y \geq 0$ in Problem 2, using a think-aloud approach.

Extension: *Math Enrichment*

Have students complete the following problem:
If the total number of each design that could be made cannot exceed 70, what is the maximum revenue and how much of each design can be made? Sample response: The maximum revenue will now be $30 for 40 hearts and 0 stars.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

- **Read 1:** Students should understand that Shawn and Bard are creating designs and need to meet several constraints.
- **Read 2:** Ask students to name or highlight the given quantities and relationships, such as the time cannot exceed 30 minutes.
- **Read 3:** Ask students to think about how they could write inequalities to represent the given constraints.

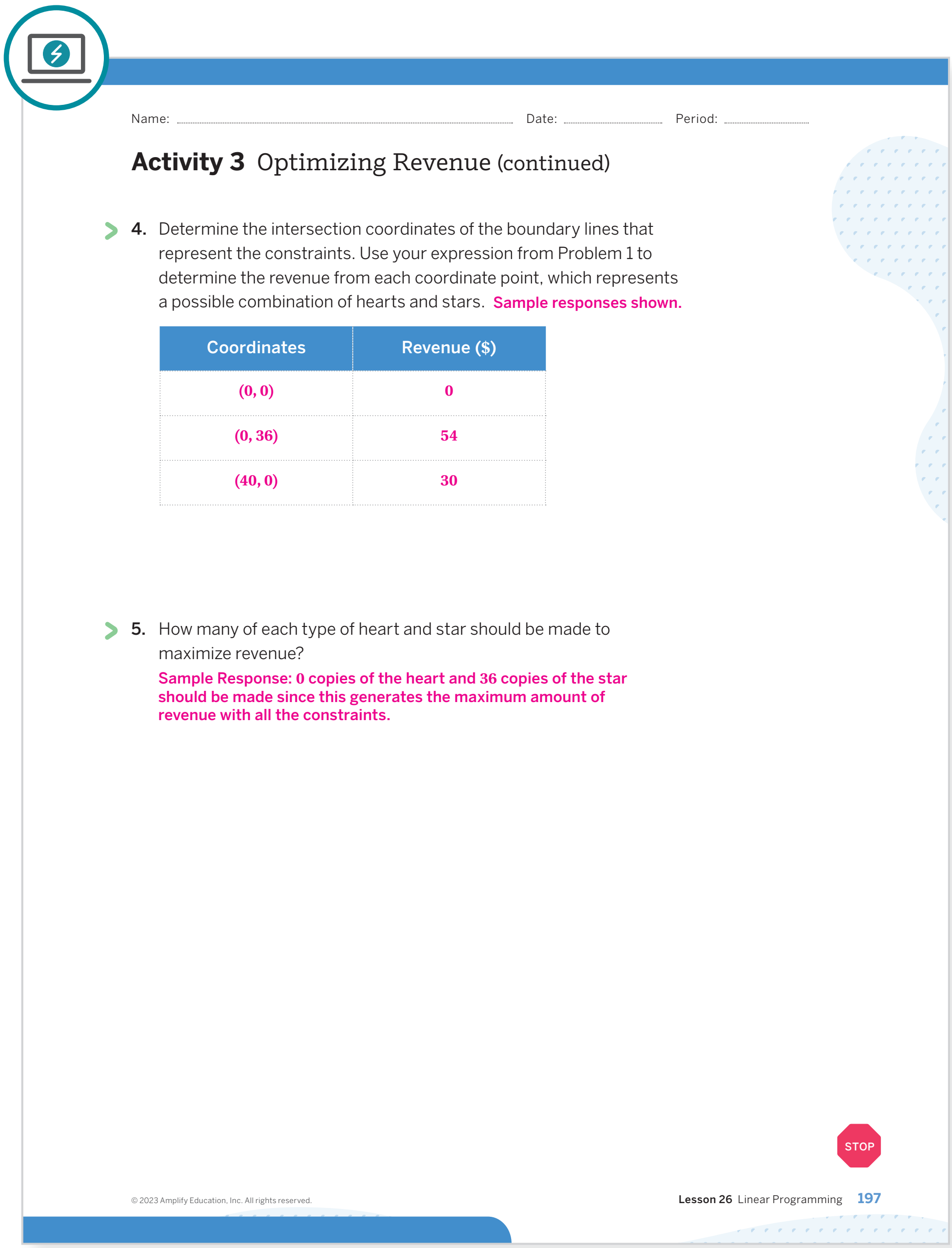
English Learners

Have students highlight key phrases in the text, such as *cannot exceed, total number, each heart, each star*, etc.

Small Groups | 20 min

Activity 3 Optimizing Revenue (continued)

Students create and use inequalities that represent a viable solution region to determine maximum revenue.



Name: ____________ Date: ________ Period: ________

Activity 3 Optimizing Revenue (continued)

4. Determine the intersection coordinates of the boundary lines that represent the constraints. Use your expression from Problem 1 to determine the revenue from each coordinate point, which represents a possible combination of hearts and stars. Sample responses shown.

Coordinates	Revenue ($)
(0, 0)	0
(0, 36)	54
(40, 0)	30

5. How many of each type of heart and star should be made to maximize revenue?
Sample Response: 0 copies of the heart and 36 copies of the star should be made since this generates the maximum amount of revenue with all the constraints.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have groups of students share their system of inequalities, graphs, and which combination of each design will generate the maximum amount of revenue.

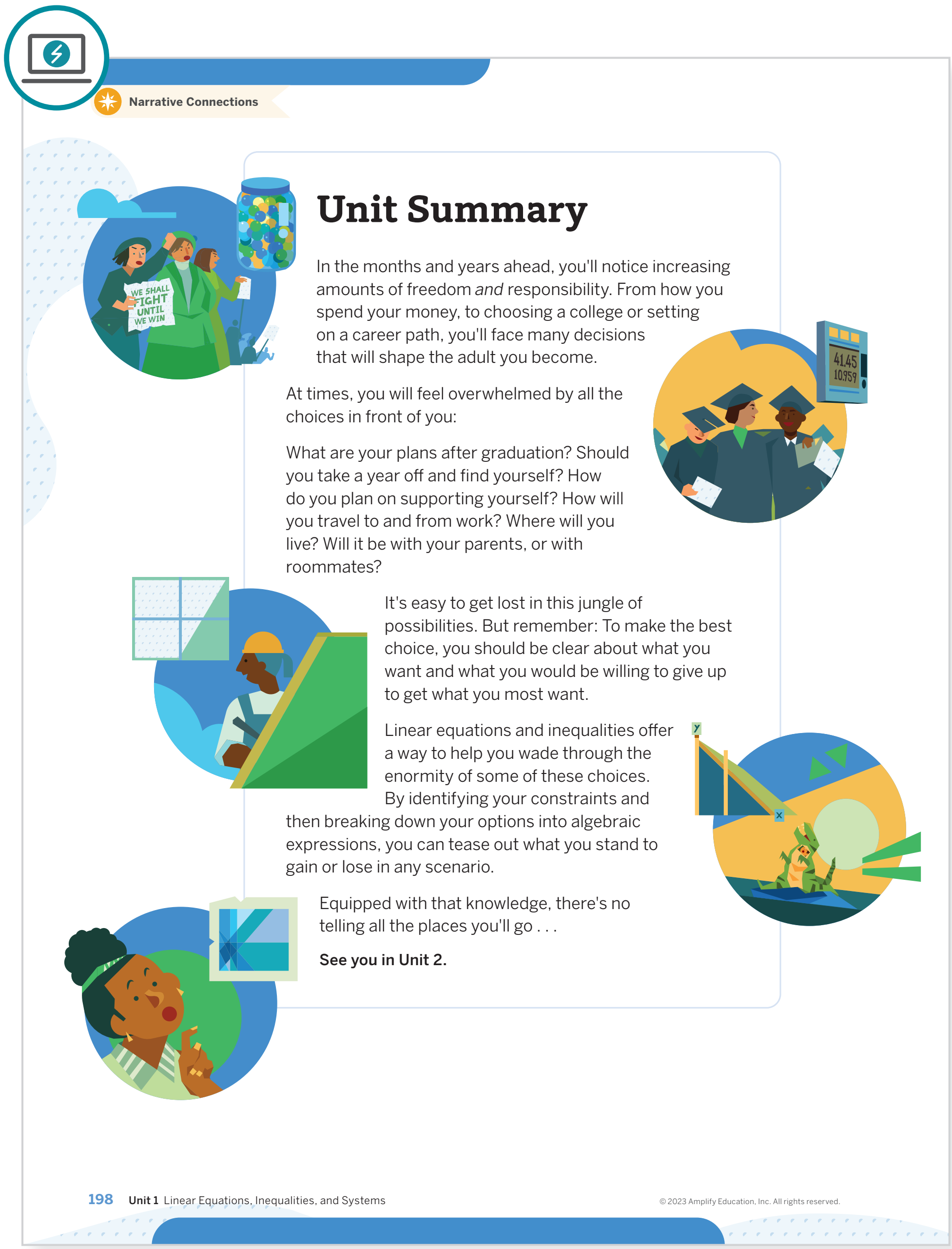
Display student work and explanations.

Highlight that linear programming is a strategy to achieve the best solution or outcome in a given scenario. It is applicable to several real-world applications including production, routes, and business. This strategy can help maximize or minimize desired outcomes within constraints. The optimal solutions for a linear programming problem will be one of the vertices of the constrained region.

Ask, “Where might you use linear programming in your own life? What kind of information would you need?”

Unit Summary

Review and synthesize Unit 1, writing, graphing, and modeling linear equations, one-variable inequalities, systems of linear equations, and systems of linear inequalities.



Narrative Connections

Unit Summary

In the months and years ahead, you'll notice increasing amounts of freedom *and* responsibility. From how you spend your money, to choosing a college or setting on a career path, you'll face many decisions that will shape the adult you become.

At times, you will feel overwhelmed by all the choices in front of you:

What are your plans after graduation? Should you take a year off and find yourself? How do you plan on supporting yourself? How will you travel to and from work? Where will you live? Will it be with your parents, or with roommates?

It's easy to get lost in this jungle of possibilities. But remember: To make the best choice, you should be clear about what you want and what you would be willing to give up to get what you most want.

Linear equations and inequalities offer a way to help you wade through the enormity of some of these choices. By identifying your constraints and then breaking down your options into algebraic expressions, you can tease out what you stand to gain or lose in any scenario.

Equipped with that knowledge, there's no telling all the places you'll go . . .

See you in Unit 2.

198 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Have students share what concepts from the unit surprised them, interested them, or they enjoyed and give reasons for their answers.

Ask:

- "What types of real-world applications can you model with linear equations? With systems of linear equations?"
- "How are linear inequalities and systems of linear inequalities applicable in the real-world?"
- "How could you use linear equations and inequalities to help you make decisions in your life?"

Highlight that in this unit students expanded on their understanding of linear equations, inequalities, and systems of linear equations and inequalities. They also modeled relationships and constraints to help make decisions about real-world scenarios.

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

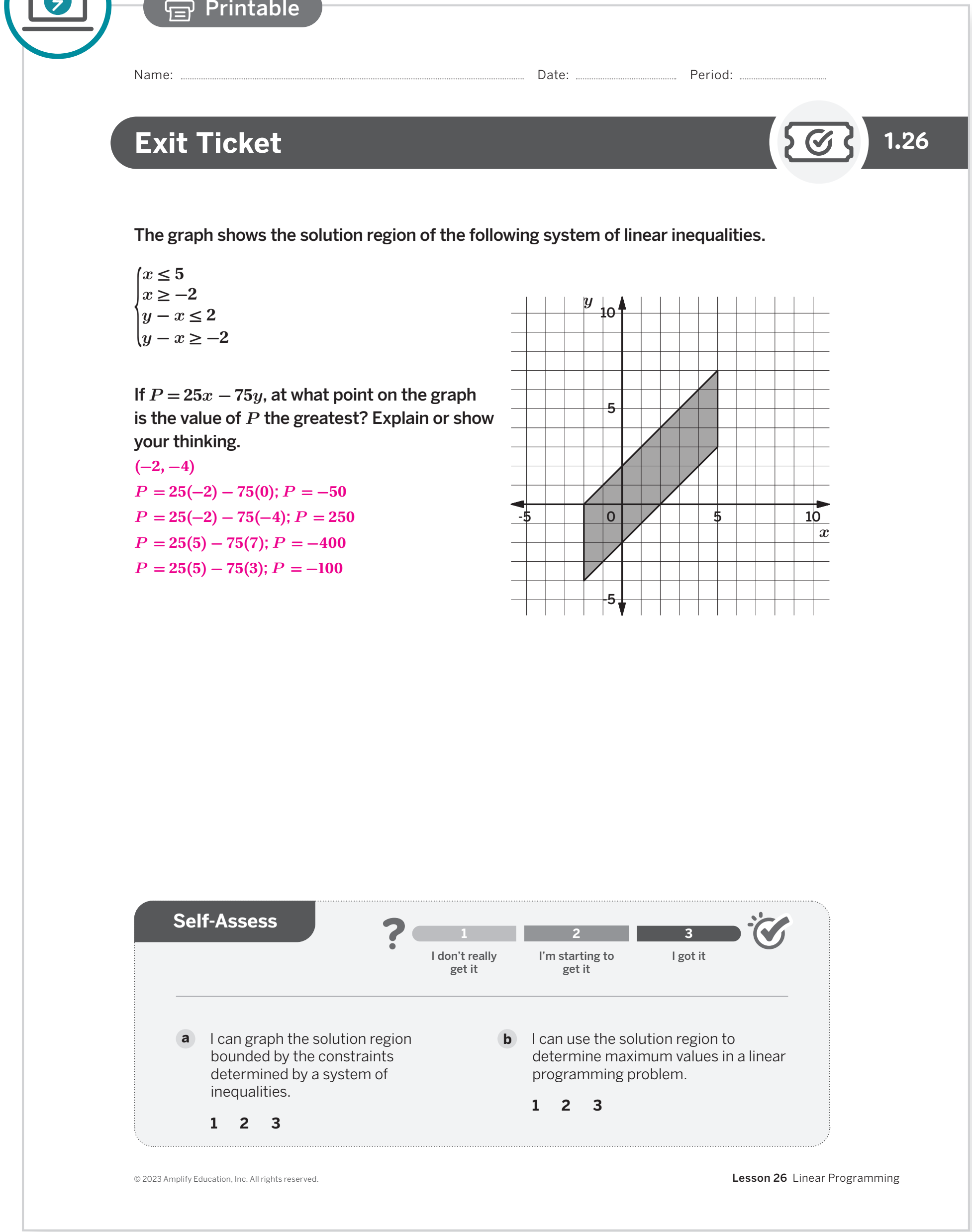
- "Did anything surprise you while reading the narratives of this unit?"
- "Is there anything you would like to learn more about these topics? What are some steps you can take to learn more?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of linear programming by determining the maximum profit given a written scenario and a graph of the solution region.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 1.26

The graph shows the solution region of the following system of linear inequalities.

$$\begin{cases} x \leq 5 \\ x \geq -2 \\ y - x \leq 2 \\ y - x \geq -2 \end{cases}$$

If $P = 25x - 75y$, at what point on the graph is the value of P the greatest? Explain or show your thinking.

$(-2, -4)$

$P = 25(-2) - 75(0); P = -50$

$P = 25(-2) - 75(-4); P = 250$

$P = 25(5) - 75(7); P = -400$

$P = 25(5) - 75(3); P = -100$

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can graph the solution region bounded by the constraints determined by a system of inequalities.

1 2 3

b I can use the solution region to determine maximum values in a linear programming problem.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Success looks like . . .

- **Goal:** Given a system of linear inequalities and its graph, determining the maximum value in the solution region for a given expression.
  - » Determining the point that makes the value of P the greatest.
- **Language Goal:** Analyzing given information about a scenario involving multiple constraints and writing a mathematical model to represent it. **(Reading and Writing)**

Suggested next steps

If students test several points besides the vertices, consider:

- Reviewing strategies from Activity 2.
- Assigning Practice Problem 1.

Professional Learning

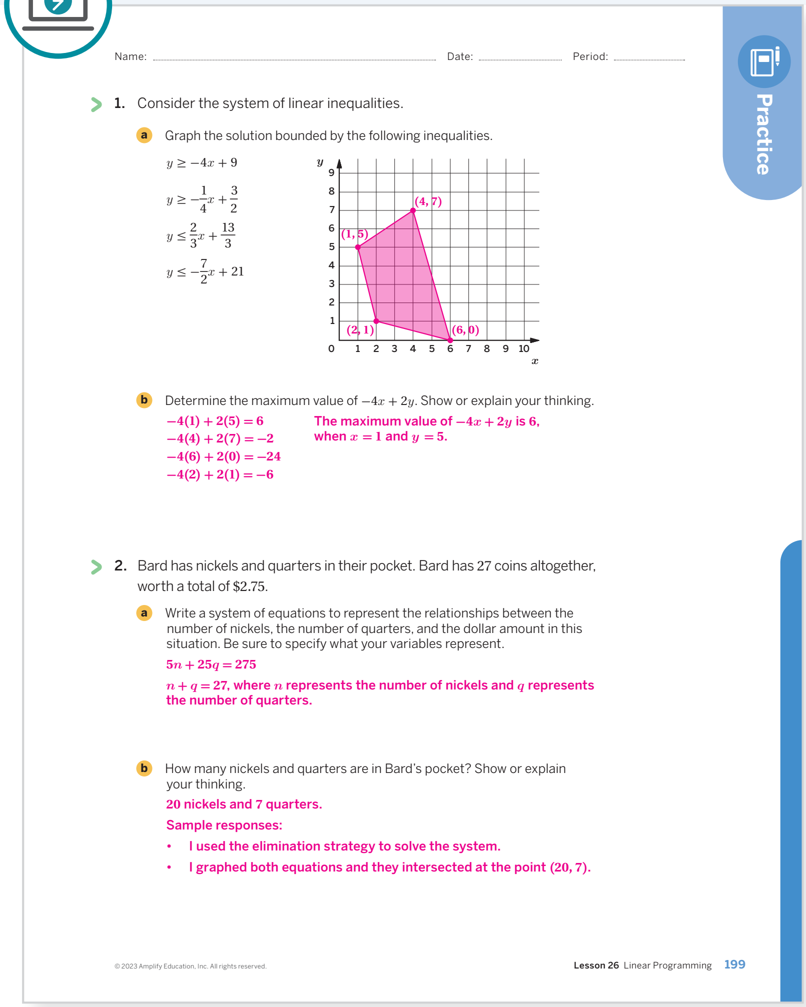
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work in this unit? Have you changed any ideas you used to have about equations and inequalities as a result of teaching this unit?
- In this unit, students expanded their understanding of two-variable linear inequalities and equations. How will that support the next unit on functions? What might you change for the next time you teach this unit?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Consider the system of linear inequalities.

   a. Graph the solution bounded by the following inequalities.

   $y \geq -4x + 9$

   $y \geq -\frac{1}{4}x + \frac{3}{2}$

   $y \leq \frac{2}{3}x + \frac{13}{3}$

   $y \leq -\frac{7}{2}x + 21$

   b. Determine the maximum value of $-4x + 2y$. Show or explain your thinking.

   $-4(1) + 2(5) = 6$

   $-4(4) + 2(7) = -2$

   $-4(6) + 2(0) = -24$

   $-4(2) + 2(1) = -6$

   The maximum value of $-4x + 2y$ is 6, when $x = 1$ and $y = 5$.

2. Bard has nickels and quarters in their pocket. Bard has 27 coins altogether, worth a total of $2.75.

   a. Write a system of equations to represent the relationships between the number of nickels, the number of quarters, and the dollar amount in this situation. Be sure to specify what your variables represent.

   $5n + 25q = 275$

   $n + q = 27$, where n represents the number of nickels and q represents the number of quarters.

   b. How many nickels and quarters are in Bard's pocket? Show or explain your thinking.

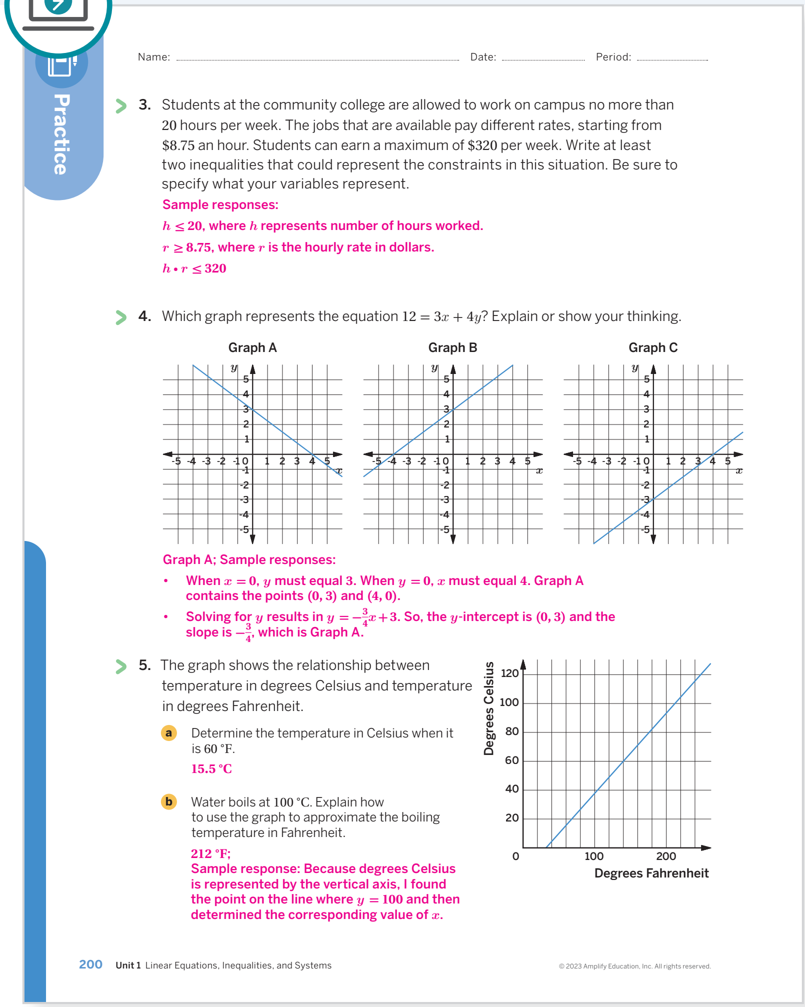
   20 nickels and 7 quarters.

   Sample responses:

   - I used the elimination strategy to solve the system.
   - I graphed both equations and they intersected at the point (20, 7).

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 26 Linear Programming 199



Practice

Name: ______ Date: ______ Period: ______

3. Students at the community college are allowed to work on campus no more than 20 hours per week. The jobs that are available pay different rates, starting from $8.75 an hour. Students can earn a maximum of $320 per week. Write at least two inequalities that could represent the constraints in this situation. Be sure to specify what your variables represent.

   Sample responses:

   $h \leq 20$, where h represents number of hours worked.

   $r \geq 8.75$, where r is the hourly rate in dollars.

   $h \cdot r \leq 320$

4. Which graph represents the equation $12 = 3x + 4y$? Explain or show your thinking.

   Graph A; Sample responses:

   - When $x = 0$, y must equal 3. When $y = 0$, x must equal 4. Graph A contains the points (0, 3) and (4, 0).
   - Solving for y results in $y = -\frac{3}{4}x + 3$. So, the y-intercept is (0, 3) and the slope is $-\frac{3}{4}$, which is Graph A.

5. The graph shows the relationship between temperature in degrees Celsius and temperature in degrees Fahrenheit.

   a. Determine the temperature in Celsius when it is 60 °F.

   15.5 °C

   b. Water boils at 100 °C. Explain how to use the graph to approximate the boiling temperature in Fahrenheit.

   212 °F;

   Sample response: Because degrees Celsius is represented by the vertical axis, I found the point on the line where $y = 100$ and then determined the corresponding value of x.

200 **Unit 1** Linear Equations, Inequalities, and Systems

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
Spiral	2	Unit 1 Lesson 22	2
	3	Unit 1 Lesson 13	2
	4	Unit 1 Lesson 12	1
	5	Unit 1 Lesson 6	1

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 2

Data Analysis and Statistics

Students will explore univariate and bivariate data sets, many of which relate to climate change. Along the way, they will investigate, calculate, and interpret descriptive statistics, including measures of center, variability, and association.

Essential Questions

- What is data and who uses it?
- How do people understand and communicate data?
- How can graphical displays be manipulated to present misleading information?
- *(By the way, when making decisions, do you think there is too much data or not enough data?)*

Key Shifts in Mathematics

Focus

In this unit . . .

Students build on their understanding of statistics and data distributions from middle school. They now encounter a new measure of variability, the standard deviation, and use it to compare distributions. They develop a mathematical method for determining outliers. Topics in univariate and bivariate data are bridged and students use mathematical strategies to fit lines to data and explore the relationship between correlation and causation.

Coherence

Previously . . .

In middle school, students were introduced to data distributions — such as dot plots, histograms, and box plots. They described and compared distributions using measures of center and variation. In Grade 8, students informally fit a line to bivariate data to interpret trends for linear models.

Coming soon . . .

In Algebra 2, students will use sampling to estimate population characteristics and explore the normal distribution to provide estimates with a margin of error. Using experimental studies, they develop a strategy for analyzing the data using a randomization along with normal distributions.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students build a conceptual understanding of standard deviation in Lesson 7, and least squares and correlation coefficient using a geometrical representation in Lessons 14 and 19.

Procedural Fluency

Students build procedural fluency of creating and describing histograms, dot plots, and box plots in Lessons 2–6 and determining values of two-way and relative frequency tables in Lessons 15–17.

Application

Students apply their understanding of the correlation coefficient In Lessons 20 and 22 and relative frequencies in Lessons 16 and 17 to analyze the effects of changes in the environment.

Analyzing Climate Change

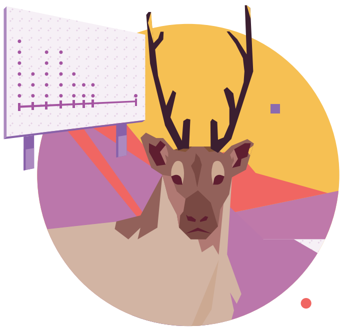
SUB-UNIT

1

Lessons 2–6

Data Distributions

Students extend their understanding of data from middle school, this time to expand their vocabulary as they describe the shape of distributions, using terms such as ***bell-shaped***, ***bimodal***, ***skewed***, and ***uniform***. They revisit measures of center and variability and use spreadsheet technology to create distributions.



Narrative: Understanding statistics can help protect the world from a "zombie virus".

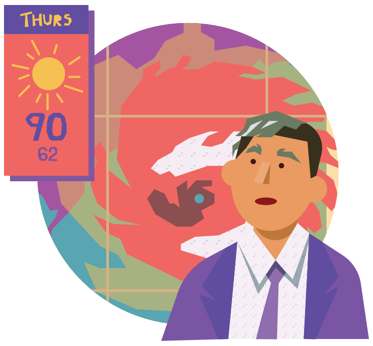
SUB-UNIT

2

Lessons 7–10

Standard Deviation

Students develop a visual understanding of the ***standard deviation*** of a data set and calculate it to summarize the variability of a data set. They determine which measure(s) of center and variability are appropriate for data sets that contain extreme values.



Narrative: A *new measure* can help determine whether the current weather is the *new normal*.

SUB-UNIT

3

Lessons 11–14

Bivariate Data

Students use linear models to estimate values in data sets. They calculate the ***residuals*** of a data set to judge whether a linear model is a good fit and use the sum of the squared residuals to determine the ***line of best fit***.

Narrative: Linear models can help predict when — or if — a shortage of natural resources may occur.

Launch

Lesson 1

What Is a Statistical Question?

Students describe data representations by using the titles, scales, and values of the data. They explain how these features can influence how data is displayed and recall the concept of variability to determine if a question is a statistical question.

SUB-UNIT

4

Lessons 15–17

Categorical Data

Students use ***two-way tables*** and ***relative frequency tables*** to examine bivariate ***categorical*** data. They look for ***association*** by comparing relative frequency tables by row and column.

Narrative: Mathematical tables can help you understand the effects of climate change.

SUB-UNIT

5

Lessons 18–21

Correlation

Students describe the strength of association between categorical and quantitative data and gain mathematical precision as they are introduced to, calculate, and use the ***correlation coefficient***. They distinguish between correlation and ***causation*** as they study bivariate data.

Narrative: Analyze possible associations related to climate change and global sustainability.

Lesson 22

Capstone

Cutting Through Misleading Statistical Claims

Students encounter multiple statistical fallacies and use what they have learned over the course of this unit to avoid falling into these traps. They will play the role of skeptic to better understand correlation versus causation, cherrypicking of data, and linear models.

Unit at a Glance

Spoiler Alert: What's the connection between statistics and geometry? Understanding statistics from a geometric perspective helps.

Assessment

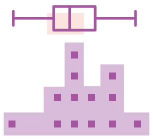
A **Pre-Unit Readiness Assessment**

Launch Lesson

1 **What Is a Statistical Question?**

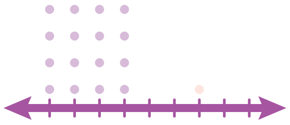
Explain how features of data representations influence how data is displayed, and determine if a question is a statistical question.

Sub-Unit 1: Data Distributions



2 **Data Representations** ●

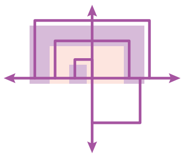
Represent data using histograms and box plots, and calculate values for a five-number summary to create box plots.



6 **Data With Spreadsheets**

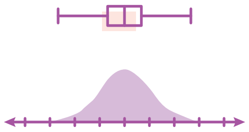
Use spreadsheet technology to create data distributions to explore how features of data sets impact possible conclusions.

Sub-Unit 2: Standard Deviation



7 **Standard Deviation**

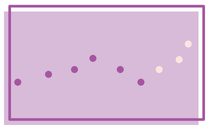
Calculate and conceptualize standard deviation, comparing it to the MAD of a data set.



8 **Choosing Appropriate Measures** (Part 1)

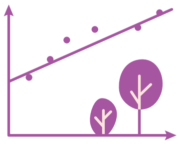
Investigate how the mean and standard deviation are more influenced by extreme values than the median and IQR.

Sub-Unit 3: Bivariate Data



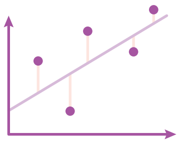
11 **Representing Data With Two Variables** ●

Revisit representing data sets with two variables with a scatter plot, and determining if the data is linear or non-linear.



12 **Linear Models** ●

Describe how well a line fits a scatter plot, and use the line to predict values not represented.



13 **Residuals**

Compute and plot residuals to quantify how well a linear model fits the data.

Key Concepts

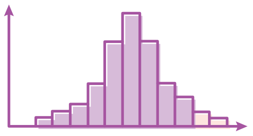
Lesson 7: Understand standard deviation as another measure of variability.
Lesson 15–17: Data can be organized in two-way and relative frequency tables.
Lessons 19–20: The correlation coefficient is defined and used to describe associations of bivariate data.

Pacing

22 Lessons: 50 min each
3 Assessments: 45 min each

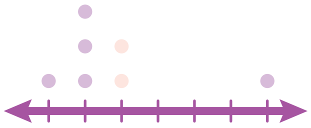
Full Unit: 25 days
● **Modified Unit:** 23 days

Assumes 50-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



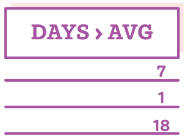
3 The Shape of Distributions ●

Connect different data representations of the same data set by using and describing the shape of the distributions.



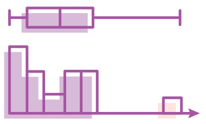
4 Deviation From the Center

Calculate the mean, MAD, median, and IQR and use these values to describe the center and variability of data.



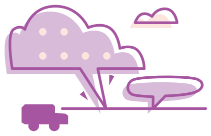
5 Measuring Outliers

Use the IQR to identify outliers, and investigate how the mean and median are affected by outliers.



9 Choosing Appropriate Measures (Part 2)

Compare measures of center and variability in a real world context, using the shape of the distribution to select the more appropriate measures.



10 Outliers and Standard Deviation

Use standard deviation and mean to identify outliers and how they affect the statistics of the data set.

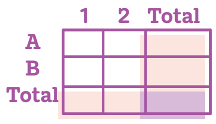
Assessment

A Mid-Unit Assessment

14 Line of Best Fit

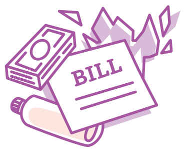
Use the absolute values of the residuals and least squares method to determine the line of best fit for a data set.

Sub-Unit 4: Categorical Data



15 Two-Way Tables

Creating and interpreting categorical data on the social impacts of climate change using two-way tables.



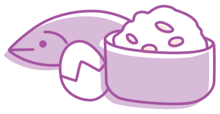
16 Relative Frequency Tables

Create and interpret relative frequency tables by row and column in the context of the effects of climate change on people.

Unit at a Glance

Spoiler Alert: What's the connection between statistics and geometry? Understanding statistics from a geometric perspective helps.

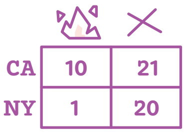
continued



17 **Associations in Categorical Data**

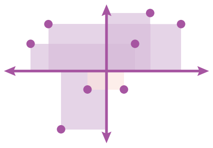
Analyze data from two-way tables to look for patterns and possible association.

Sub-Unit 5: Correlation



18 **"Strength" of Association**

Examining and quantifying associations of data, ordering them from weakest to strongest.



19 **Correlation Coefficient** (Part 1)

Use the correlation coefficient to describe how well a line fits data.

Assessment

A **End-of-Unit Assessment**

Key Concepts

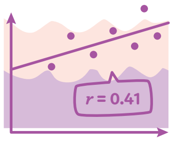
Lesson 7: Understand standard deviation as another measure of variability.
Lesson 15–17: Data can be organized in two-way and relative frequency tables.
Lessons 19–20: The correlation coefficient is defined and used to describe associations of bivariate data.

Pacing

22 Lessons: 50 min each
3 Assessments: 45 min each

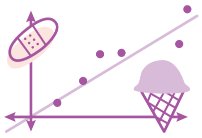
Full Unit: 25 days
● **Modified Unit:** 23 days

Assumes 50-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



20 Correlation Coefficient (Part 2)

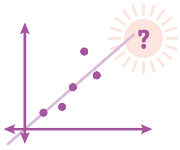
Explore sea level change by using spreadsheet technology to create the line of best fit and determine the correlation coefficient.



21 Correlation vs. Causation

Examine scenarios of correlation to see how experiments can show causation.

Capstone Lesson



22 Cutting Through Misleading Statistical Claims

Perform an experiment and analyze data to draw conclusions about causation.

● Modifications to Pacing

Lessons 2–3: These two lessons can be combined. Lesson 2 revisits creating data representations from Grade 6. A review of the five-number summary and creating dot plots, histograms, and box plots can be incorporated into Lesson 3.

Lessons 11–12: These lessons can be combined. Lesson 11 revisits modeling data with scatterplots and Lesson 12 uses scatter plots to determine a line of fit for a data set.

Lessons 18–19: These lessons can be combined. Lesson 18 revisits describing associations between data and Lesson 19 quantifies associations between data using the correlation coefficient.

Unit Supports

Math Language Development

Lesson	New Vocabulary
3	bell-shaped bimodal skewed left skewed right uniform
7	standard deviation
8	discrete
13	residuals residual plot
14	line of best fit
15	categorical variable two-way table
16	relative frequency table
17	association
19	correlation coefficient
21	causation

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
3–6, 8, 13	**MLR1:** Stronger and Clearer Each Time
1, 3, 5–7, 10, 13, 14–19, 21, 22	**MLR2:** Collect and Display
11, 19	**MLR3:** Critique, Correct, Clarify
15	**MLR4:** Information Gap
7, 9, 16	**MLR5:** Co-craft Questions
12, 13, 15, 15, 20–22	**MLR6:** Three Reads
2–4, 9, 11–14, 16–19, 20, 22	**MLR7:** Compare and Connect
1, 2, 4, 5, 7, 8, 10–13, 17–21	**MLR8:** Discussion Supports

Materials

Every lesson includes:

Exit Ticket

Additional Practice

Additional required materials include:

Lesson(s)	Materials
1, 2	colored pencils/markers
21	dice
5, 7	four-function calculators
7, 11	graph paper
1–22	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
4	pennies
2	poster paper
10, 13	scientific calculators
5–8, 10, 20–22	spreadsheet technology
4, 11–13, 18	rulers
4, 12	yardsticks

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routines
1, 2, 10, 12, 16, 19	Notice and Wonder
5	Math Talk
15	Info Gap
3, 14, 18	Which One Doesn't Belong?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 10
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 21

Amps

POWERED BY desmos

Social & Collaborative Digital Moments

Featured Activity

r We There Yet?

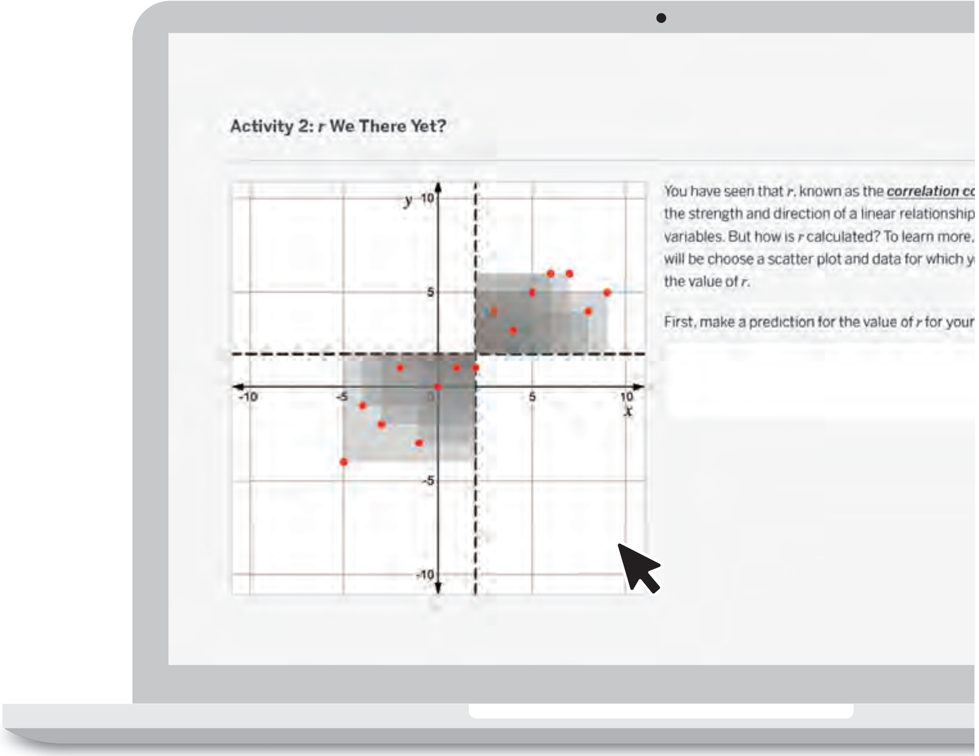
Put on your student hat and work through **Lesson 19, Activity 2:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Temperature on a Global Scale **(Lesson 1)**
- Outliers' Effect on Measures of Center **(Lesson 5)**
- Interpreting Data Distributions **(Lesson 8)**
- Least Squares Method **(Lesson 14)**



Activity 2: r We There Yet?

You have seen that r, known as the ***correlation c***
the strength and direction of a linear relationship
variables. But how is r calculated? To learn more,
will be choose a scatter plot and data for which y
the value of r.

First, make a prediction for the value of r for your

Unit Study
Professional Learning

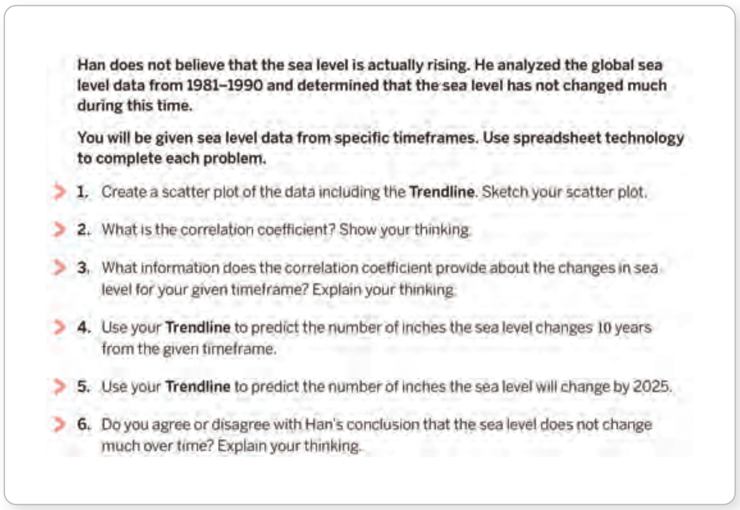
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 5 introduces methods to describe the correlation between data. Students investigate the strength of association between categorical and quantitative data. Students understand that determining the strength of correlation between data is useful in real-world context, such as investigating the sustainability of the changing climate. They calculate and use the correlation coefficient to describe the strength of correlation and use their knowledge of statistical experiments to determine if causation may exist. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from **Lesson 20, Activity 2:**



Han does not believe that the sea level is actually rising. He analyzed the global sea level data from 1981–1990 and determined that the sea level has not changed much during this time.

You will be given sea level data from specific timeframes. Use spreadsheet technology to complete each problem.

1. Create a scatter plot of the data including the **Trendline**. Sketch your scatter plot.
2. What is the correlation coefficient? Show your thinking.
3. What information does the correlation coefficient provide about the changes in sea level for your given timeframe? Explain your thinking.
4. Use your **Trendline** to predict the number of inches the sea level changes 10 years from the given timeframe.
5. Use your **Trendline** to predict the number of inches the sea level will change by 2025.
6. Do you agree or disagree with Han's conclusion that the sea level does not change much over time? Explain your thinking.

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

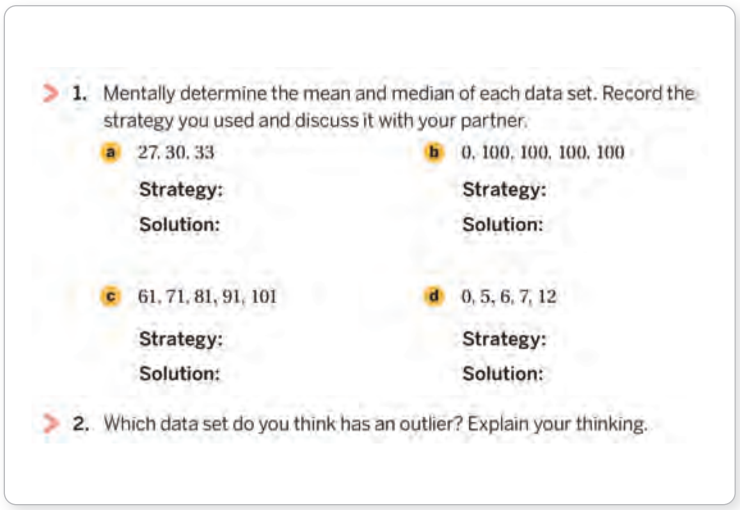
- What was it like to engage in this problem as a learner?
- It's not uncommon for students to be unsure of the accuracy of their trendline if a majority of the data is far away from the trendline. How might you help students think this through and build their confidence?
- What approaches might your students take?
- Do any approaches reveal a misconception that might arise for students?
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Math Talk

Rehearse . . .

How you'll facilitate the ***Math Talk*** instructional routine in **Lesson 5, Warm-up:**



1. Mentally determine the mean and median of each data set. Record the strategy you used and discuss it with your partner.

   a. 27, 30, 33
   Strategy:
   Solution:

   b. 0, 100, 100, 100, 100
   Strategy:
   Solution:

   c. 61, 71, 81, 91, 101
   Strategy:
   Solution:

   d. 0, 5, 6, 7, 12
   Strategy:
   Solution:

2. Which data set do you think has an outlier? Explain your thinking.

Points to Ponder . . .

- How am I supporting students to develop fluent use of precise mathematical language?
- How should I emphasize or have my class identify phrases and language that are most relevant to our learning goal?

This routine . . .

- Facilitates the use of sentence frames and other discussion supports that benefit not only English Learners, but all students.
- Provides opportunities for students to engage in mathematical discourse and explain their thinking to others.
- Provides opportunities to foster a safe space for students to respond freely as they share their unique ways of thinking.

Anticipate . . .

- Intentional grouping of students to best support dialogue and focus.
- Preparing scaffolds or questions to help students get started.
- If you haven't used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you have used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Support productive struggle in learning mathematics.

This effective teaching practice . . .

- Provides students with the opportunity to wrestle with mathematical concepts and relationships before you intervene, which builds student confidence and perseverance.
- Allows you greater opportunities to monitor student progress, probe for student understanding, and offer differentiation support.

Points to Ponder . . .

- How comfortable are you with allowing students the time to wrestle with mathematical ideas, before you intervene?
- When is the right moment to intervene? What can you look for as you monitor student work and student conversations to know when they are engaging in productive struggle or unproductive struggle?

Math Language Development

MLR8: Discussion Supports

MLR8 appears in Lessons 1, 2, 4, 5, 7, 8, 10–13, 17–21.

- Throughout the unit, sentence frames are provided for you to display to your students. Students can use these prompts to help frame their responses and add structure and organization to their thinking. Anchor charts, such as the Anchor Chart PDFs, *Sentence Stems* are also provided that you can use to display sentence frames for students to use as a guide.
- In Lessons 4, 5, 20, and 21, further probing questions are provided so that you can ask your students for more clarification or to press for details in their reasoning.
- **English Learners:** Provide wait time to allow students to formulate a response before sharing with others.

Point to Ponder . . .

- During class discussions, how will you know when to probe further to assess student understanding, provide sentence frames, and encourage your students to use their developing mathematical vocabulary?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students' prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **Mid**- and **End-of-Unit Assessments**, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students' understanding of the use of measures of center and variability to describe and compare data sets throughout the unit? Do you think your students will generally:
  - » Miss the underlying concept of center and spread?
  - » Struggle with the concept of summarizing data using the shape of a distribution?
  - » Miss the underlying concept of "best" line of fit?
  - » Be fully ready to make connections from univariate data to bivariate data in order to understand correlation?

Fostering Diverse Thinking

Use these opportunities for students to connect mathematics to the world around them:

- In Lesson 16, students research more information about the Flint water crisis and learn about the Safe Water Drinking Act, which was enacted in 1974 and provides regulated standards for drinking water.
- In the Sub-Unit 5 narrative, students see how 14-year old Autumn Peltier advocated for clean water for the Indigenous communities of Canada.

Point to Ponder . . .

- How can I help raise my students' awareness of equity concerns and encourage them to think about how they can use data analysis and statistics to help advocate for change?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students' capacity to build strong mathematical habits of mind.

In this unit, pay particular attention to supporting students in building their self awareness and self-management skills.

Points to Ponder . . .

- Are students able to view a situation from another person's perspective? How do they show empathy? In what ways do they show respect for others?
- Are students able to make constructive decisions about their choices? How do their decisions lead to solutions to problems? Do they consider the well-being of others as well as themselves? How do they show that they accept the responsibility for their choices?

What Is a Statistical Question?

Let's investigate the information data representations can reveal and the statistical questions those representations can answer.

Focus

Goals

1. **Language Goal:** Describe the features of data representations that are important to examine when making conclusions about data sets. **(Speaking and Listening, Writing)**
2. **Language Goal:** Describe the difference between statistical and non-statistical questions. **(Speaking and Listening, Writing)**

Coherence

Today

Students describe data representations using their titles, scales, and trends. They explain how these features affect how the data may be interpreted. Students also recall the concept of variability to determine if a question is a statistical question.

Previously

In Grade 6, students created dot plots, histograms, and box plots to display data, and recognized a statistical question as one that anticipates variability.

Coming Soon

In Lesson 2, students will review creating dot plots and histograms, and calculate a five-number summary to create box plots.

Rigor

- Students build on their Grade 6 **conceptual understanding** of statistical questions.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (for display)
- Activity 1 PDF (for display)
- Anchor Chart PDF, *Sentence Stems, Notice and Wonder*
- colored pencils/markers

Math Language Development

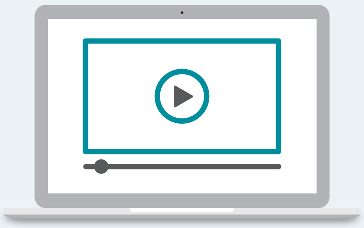
Review words

- *categorical data*
- *numerical data*
- *statistical question*
- *variability*

Amps | Featured Activity

Activity 1
Global Temperature Maps

Students examine global temperatures at various years, represented by changing colors on a global map. They use this to begin examining details of data representations that are important to pay close attention to when drawing conclusions about the data sets represented.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may get excited about the quantitative topics covered in these activities and lose focus on their task. Have students set goals for the lesson at the beginning, including being able to analyze questions. Remind students that while their questions might be quantitative in nature, they will have to reason about whether the questions are statistical.

Modifications to Pacing

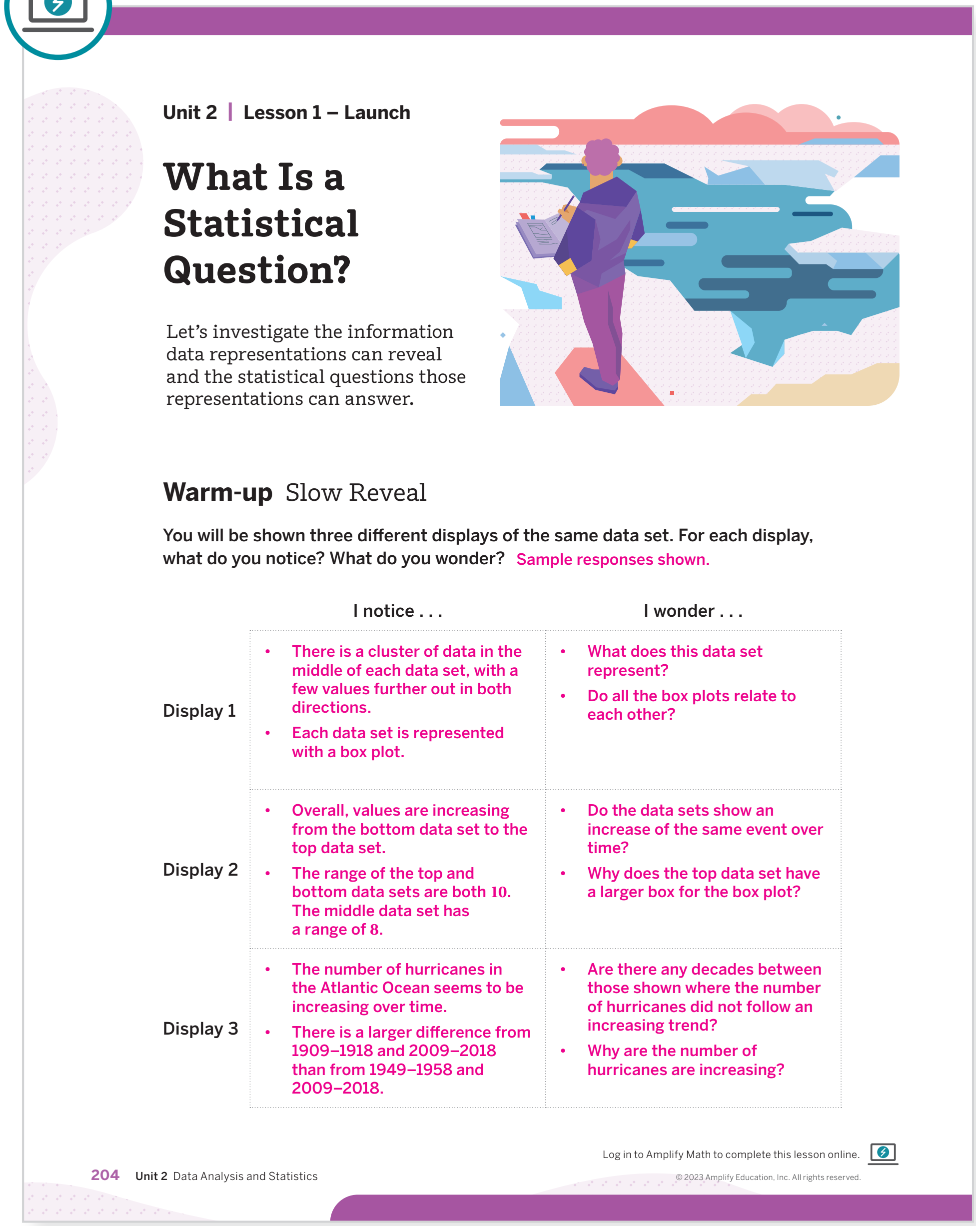
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, the first display may be omitted.
- In **Activity 1**, the displays and Problem 2 may be omitted.
- In **Activity 2**, Problems 1 and 2 may be omitted.

Pairs | 10 min

Warm-up Slow Reveal

Students notice and wonder about the features of three box plots to make sense of what labels, scales, and range reveal about data.



Unit 2 | Lesson 1 – Launch

What Is a Statistical Question?

Let's investigate the information data representations can reveal and the statistical questions those representations can answer.

Warm-up Slow Reveal

You will be shown three different displays of the same data set. For each display, what do you notice? What do you wonder? Sample responses shown.

	I notice . . .	I wonder . . .
Display 1	There is a cluster of data in the middle of each data set, with a few values further out in both directions. Each data set is represented with a box plot.	What does this data set represent? Do all the box plots relate to each other?
Display 2	Overall, values are increasing from the bottom data set to the top data set. The range of the top and bottom data sets are both 10. The middle data set has a range of 8.	Do the data sets show an increase of the same event over time? Why does the top data set have a larger box for the box plot?
Display 3	The number of hurricanes in the Atlantic Ocean seems to be increasing over time. There is a larger difference from 1909–1918 and 2009–2018 than from 1949–1958 and 2009–2018.	Are there any decades between those shown where the number of hurricanes did not follow an increasing trend? Why are the number of hurricanes are increasing?

Log in to Amplify Math to complete this lesson online.

204 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in pairs. Using the Warm-up PDF, display one graph at a time, giving students time to record their thinking. Between each display, have students share with their partner before moving to the next display.

2 Monitor

Help students get started by suggesting that they look for differences among the displays.

Look for points of confusion:

- **Thinking the center line in the box plot evenly divides the maximum and minimum values.** Remind students that the center line represents the data set's median.

Look for productive strategies:

- Adding mathematical details by using the range and data set values to describe the box plots with precision.

3 Connect

Have pairs of students share what they notice and wonder about. Record and display their observations.

Highlight that without the labels and scales of the box plots, the trend of the data set is vague. More key features shown allow for more detailed conjectures and conclusions.

Ask:

- "What benefit is there to displaying all three box plots on the same axis?" When they all use the same axis, I can compare them more easily.
- "What other information could be helpful to have in the display?" Sample response: More decades.

Math Language Development

MLR8: Discussion Supports

Before the Connect, display or provide the Anchor Chart PDF, *Sentence Stems, Notice and Wonder* to support students as they record what they notice and wonder and think about how they will share these responses with the class.

English Learners

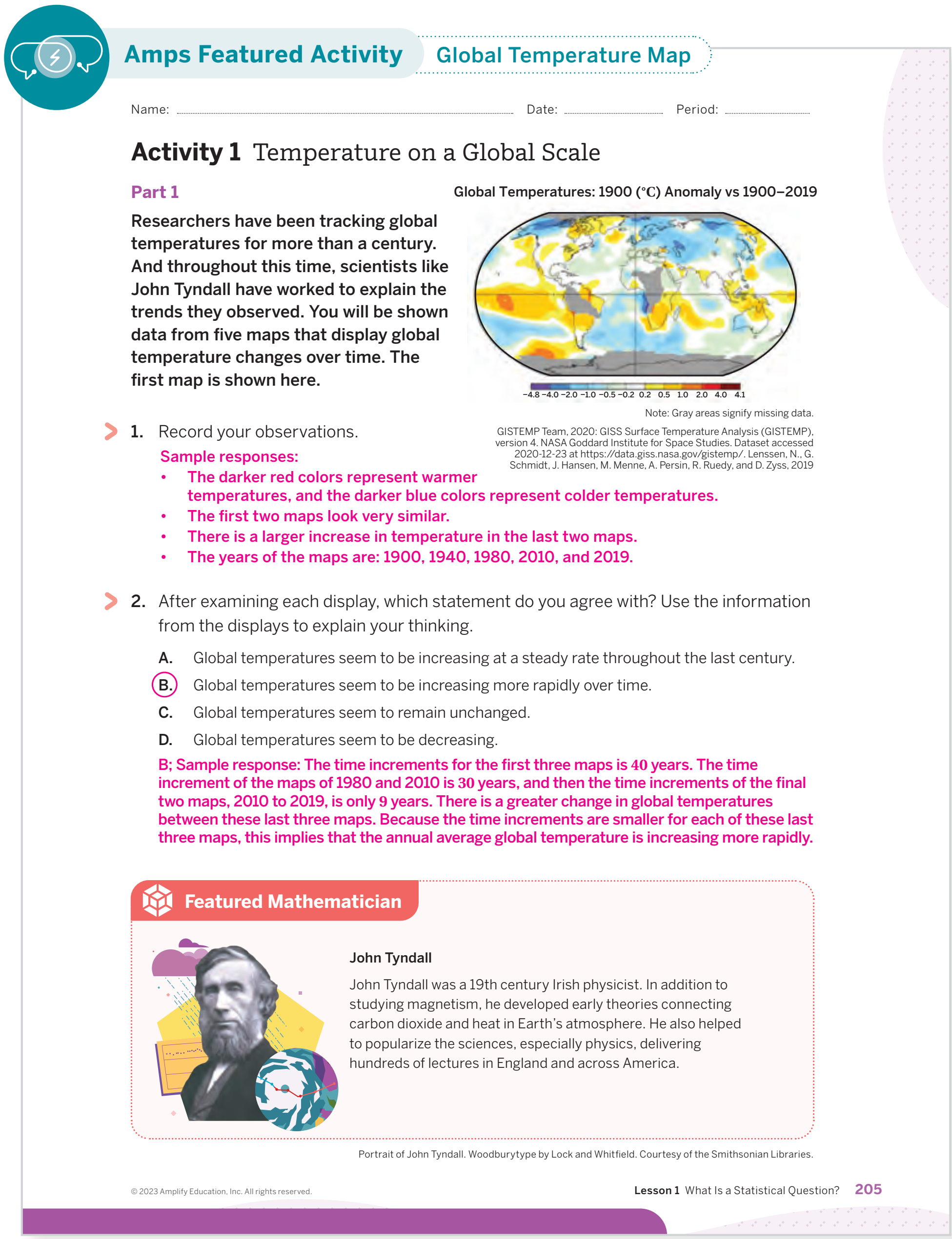
Allow students to rehearse and formulate what they will say with their partner before sharing with the class.

Pairs | 15 min

Activity 1 Temperature on a Global Scale

Students examine global and U.S. temperature averages to understand how key features in data representations influence perception.

Amps Featured Activity Global Temperature Map



Name: Date: Period:

Activity 1 Temperature on a Global Scale

Part 1

Researchers have been tracking global temperatures for more than a century. And throughout this time, scientists like John Tyndall have worked to explain the trends they observed. You will be shown data from five maps that display global temperature changes over time. The first map is shown here.

Global Temperatures: 1900 (°C) Anomaly vs 1900–2019

Note: Gray areas signify missing data.

GISTEMP Team, 2020: GISS Surface Temperature Analysis (GISTEMP), version 4. NASA Goddard Institute for Space Studies. Dataset accessed 2020-12-23 at https://data.giss.nasa.gov/gistemp/. Lenssen, N., G. Schmidt, J. Hansen, M. Menne, A. Persin, R. Ruedy, and D. Zyss, 2019

1. Record your observations.

   Sample responses:
   - **The darker red colors represent warmer temperatures, and the darker blue colors represent colder temperatures.**
   - **The first two maps look very similar.**
   - **There is a larger increase in temperature in the last two maps.**
   - **The years of the maps are: 1900, 1940, 1980, 2010, and 2019.**

2. After examining each display, which statement do you agree with? Use the information from the displays to explain your thinking.

   A. Global temperatures seem to be increasing at a steady rate throughout the last century.
   [✓ B.] Global temperatures seem to be increasing more rapidly over time.
   C. Global temperatures seem to remain unchanged.
   D. Global temperatures seem to be decreasing.

   B; Sample response: The time increments for the first three maps is 40 years. The time increment of the maps of 1980 and 2010 is 30 years, and then the time increments of the final two maps, 2010 to 2019, is only 9 years. There is a greater change in global temperatures between these last three maps. Because the time increments are smaller for each of these last three maps, this implies that the annual average global temperature is increasing more rapidly.

Featured Mathematician

John Tyndall

John Tyndall was a 19th century Irish physicist. In addition to studying magnetism, he developed early theories connecting carbon dioxide and heat in Earth's atmosphere. He also helped to popularize the sciences, especially physics, delivering hundreds of lectures in England and across America.

Portrait of John Tyndall. Woodburytype by Lock and Whitfield. Courtesy of the Smithsonian Libraries.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Students remain in pairs. Display the Activity 1 PDF showing infographics for global temperatures during various years. Have students complete Problems 1 and 2 independently, then share with their partner before completing Problems 3–5 in pairs.

2 Monitor

Help students get started by modeling with the first U.S. graph (Graph A).

Look for points of confusion:

- **Using only a few values to make claims about temperatures in the U.S.** Have students look for an overall trend in the data, rather than focusing on specific values.
- **Using Graphs A and C to make claims about the temperatures in the U.S.** Ask, "What is similar about Graphs A and C? What is similar about Graphs B and D? Which pair is more helpful when determining differences among data points?"

Look for productive strategies:

- Utilizing other possible key features of the maps to examine the data, such as legends, data sources, and notes on the display.
- Using the scale of the axes to describe the differences between graphs.
- Noticing that the data from Graphs A and B are subsets of Graphs C and D.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students highlight corresponding axes' scales and titles of each set of graphs with different colored pencils or highlighters to help them draw attention to the similarities and differences between the graphs.

Math Language Development

MLR2: Collect and Display

As students discuss the data representations, record important phrases you hear them say onto a visual display. Throughout the remainder of the lesson and unit, update the display and remind students to use language from the display, as needed. Highlight key features of the maps useful for examining the data by pointing to items, such as the legend and scale of the axes.

Featured Mathematician

John Tyndall

John Tyndall was a 19th century Irish physicist. In addition to studying magnetism, he developed early theories connecting carbon dioxide and heat in Earth's atmosphere. He also helped to popularize the sciences, especially physics, delivering hundreds of lectures in England and across America.

Pairs | 15 min

Activity 1 Temperature on a Global Scale (continued)

Students examine global and U.S. temperature averages to understand how key features in data representations influence perception.

Activity 1 Temperature on a Global Scale (continued)

Collect and Display: Your teacher will highlight the math language you use to discuss these graphs and add it to a class display. Refer to the display during future discussions.

Part 2

The following four graphs all show the annual average U.S. temperature, but with different scales on the axes.

3. While all four graphs show the same data, how do they appear different?

   Sample response: Graph A and Graph C's vertical axes each goes from 0 to about 60, so these graphs appear flat. Graph B and Graph D's vertical axes each goes from 49 to 56, so more change is visible. Graph D is the only graph where a nonlinear pattern is clear.

4. Which graph(s) support the statement, "The U.S. average temperature is rising"? Explain your thinking.

   Graph D; Sample response: The time period is 1900 to 2020, and the trend of temperatures is increasing after 1960. It is challenging to tell the trend in Graph A because of the vertical axis scale, and Graph B also does not show a clear trend. The vertical scale of Graph C makes it challenging to determine if there is an increase in temperature.

5. If you wanted to convince someone the average U.S. temperature was *not* rising, which graph would you show them? Explain your thinking.

   Sample response: I would show them Graph C, because it shows all the years from 1900 to 2020, but the vertical scaling makes it challenging to see any change.

206 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display all four graphs.

Have pairs of students share the differences between Graphs A, B, C, and D.

Highlight that each graph of U.S. temperature uses numerical data. The data from Graphs A and B are a subset of the data from Graphs C and D. Cherry-picking a subset of data and adjusting scales of a graph can contribute to vague or misleading conclusions.

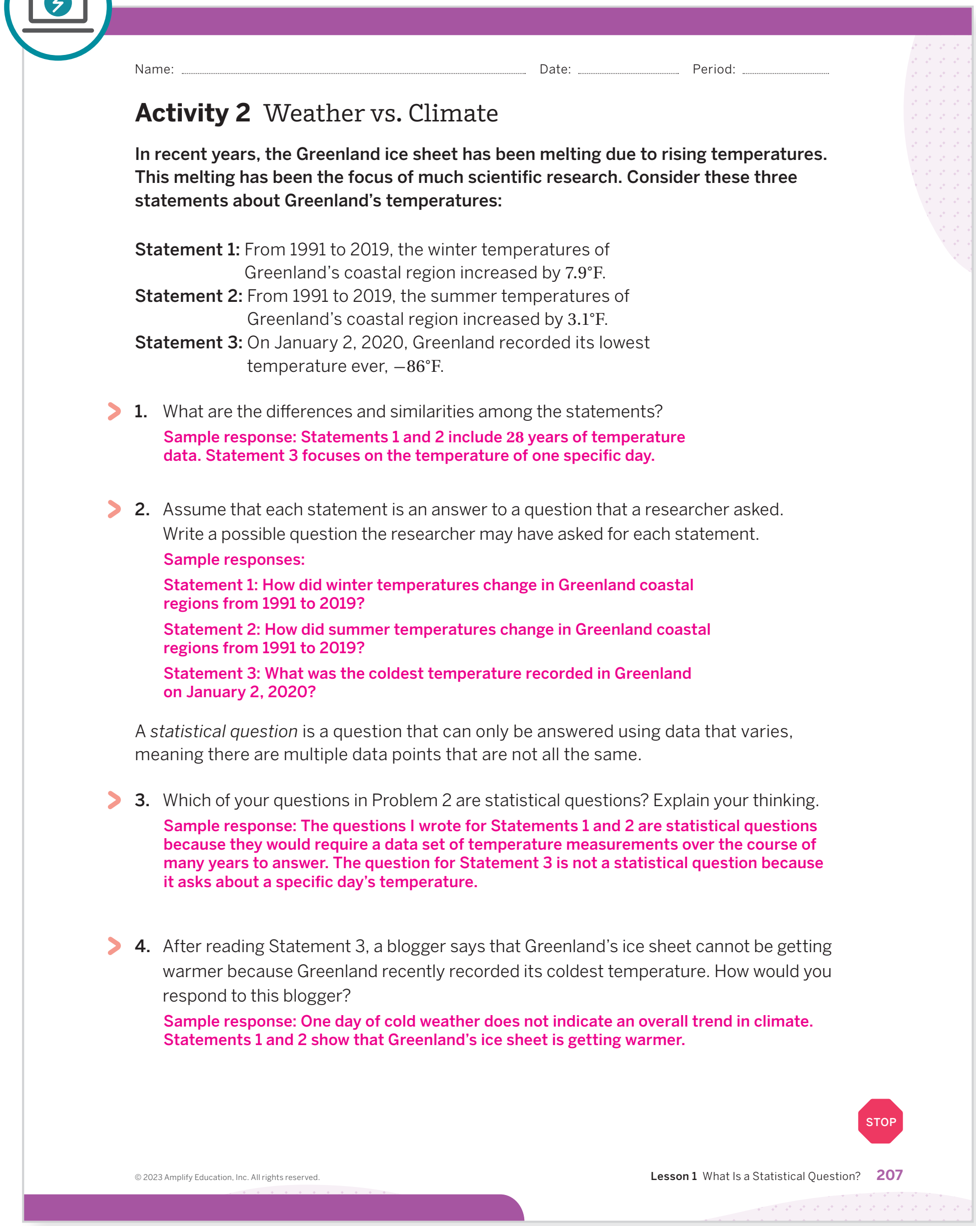
Ask:

- "How might someone use this data set to argue that the average U.S. temperature is showing little to no change?" Someone may use Graph C because it is difficult to determine the change amongst values, or argue that the change in the temperatures are indeed present, but the changes do not appear significant.
- "How might someone cherry-pick the global maps to argue that temperatures are decreasing?" Someone may use a subset of data that shows a decreasing trend. For example, the values of 2000 to 2010 seem to show a slightly decreasing trend in average temperatures.

Independent | 15 min

Activity 2 Weather vs. Climate

Students examine Greenland's weather and climate data to explain differences between statistical and non-statistical questions.



Name: ______ Date: ______ Period: ______

Activity 2 Weather vs. Climate

In recent years, the Greenland ice sheet has been melting due to rising temperatures. This melting has been the focus of much scientific research. Consider these three statements about Greenland's temperatures:

Statement 1: From 1991 to 2019, the winter temperatures of Greenland's coastal region increased by 7.9°F.
Statement 2: From 1991 to 2019, the summer temperatures of Greenland's coastal region increased by 3.1°F.
Statement 3: On January 2, 2020, Greenland recorded its lowest temperature ever, −86°F.

1. What are the differences and similarities among the statements?
   Sample response: Statements 1 and 2 include 28 years of temperature data. Statement 3 focuses on the temperature of one specific day.

2. Assume that each statement is an answer to a question that a researcher asked. Write a possible question the researcher may have asked for each statement.
   Sample responses:
   Statement 1: How did winter temperatures change in Greenland coastal regions from 1991 to 2019?
   Statement 2: How did summer temperatures change in Greenland coastal regions from 1991 to 2019?
   Statement 3: What was the coldest temperature recorded in Greenland on January 2, 2020?

A *statistical question* is a question that can only be answered using data that varies, meaning there are multiple data points that are not all the same.

3. Which of your questions in Problem 2 are statistical questions? Explain your thinking.
   Sample response: The questions I wrote for Statements 1 and 2 are statistical questions because they would require a data set of temperature measurements over the course of many years to answer. The question for Statement 3 is not a statistical question because it asks about a specific day's temperature.

4. After reading Statement 3, a blogger says that Greenland's ice sheet cannot be getting warmer because Greenland recently recorded its coldest temperature. How would you respond to this blogger?
   Sample response: One day of cold weather does not indicate an overall trend in climate. Statements 1 and 2 show that Greenland's ice sheet is getting warmer.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 1 What Is a Statistical Question? 207

1 Launch

Say, "Greenland is a large island with massive ice sheets between the Arctic and Atlantic Ocean." Explain that weather refers to the conditions in the atmosphere at a specific point in time, whereas climate focuses on the long-term conditions of the atmosphere and involves large data sets.

2 Monitor

Help students get started by asking, "What data should researchers use to make the conclusions in Statements 1 and 2?"

Look for points of confusion:

- **Thinking that the change in temperatures is the difference between the 1991 and 2019 averages.** Explain that data throughout the entire time span is being used to make this conclusion.

Look for productive strategies:

- Annotating words, phrases, and values to compare and contrast the statements.

3 Connect

Display the three statements.

Have individual students share the questions they wrote in Problem 2. Record and display their questions. Use the ***Poll the Class*** routine to determine the statistical and non-statistical questions.

Highlight that statistical questions require a collection of data and anticipate variability in data. This data can be numerical or categorical, such as "hot, cold, warm, etc."

Ask, "Does weather or climate focus on statistical questions? Why?" Climate. The study of climate involves patterns, trends, and changes over longer periods of time. It involves larger data sets and variability.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they have heard of the Greenland ice sheet. Let students know the Greenland ice sheet is a body of ice that covers about 79% of the surface of Greenland. It is the second largest body of ice in the world, second to the Antarctic ice sheet.

Extension: *Interdisciplinary Connections*

Preview the online "Greenland Surface Melt Extent Interactive Chart" from NASA. Decide if you would like your students to explore the interactive chart, in which they can click on various years to compare the melt area in square kilometers for selected months of the year. Ask them what they notice and what questions they have about the data. **(Science)**

Math Language Development

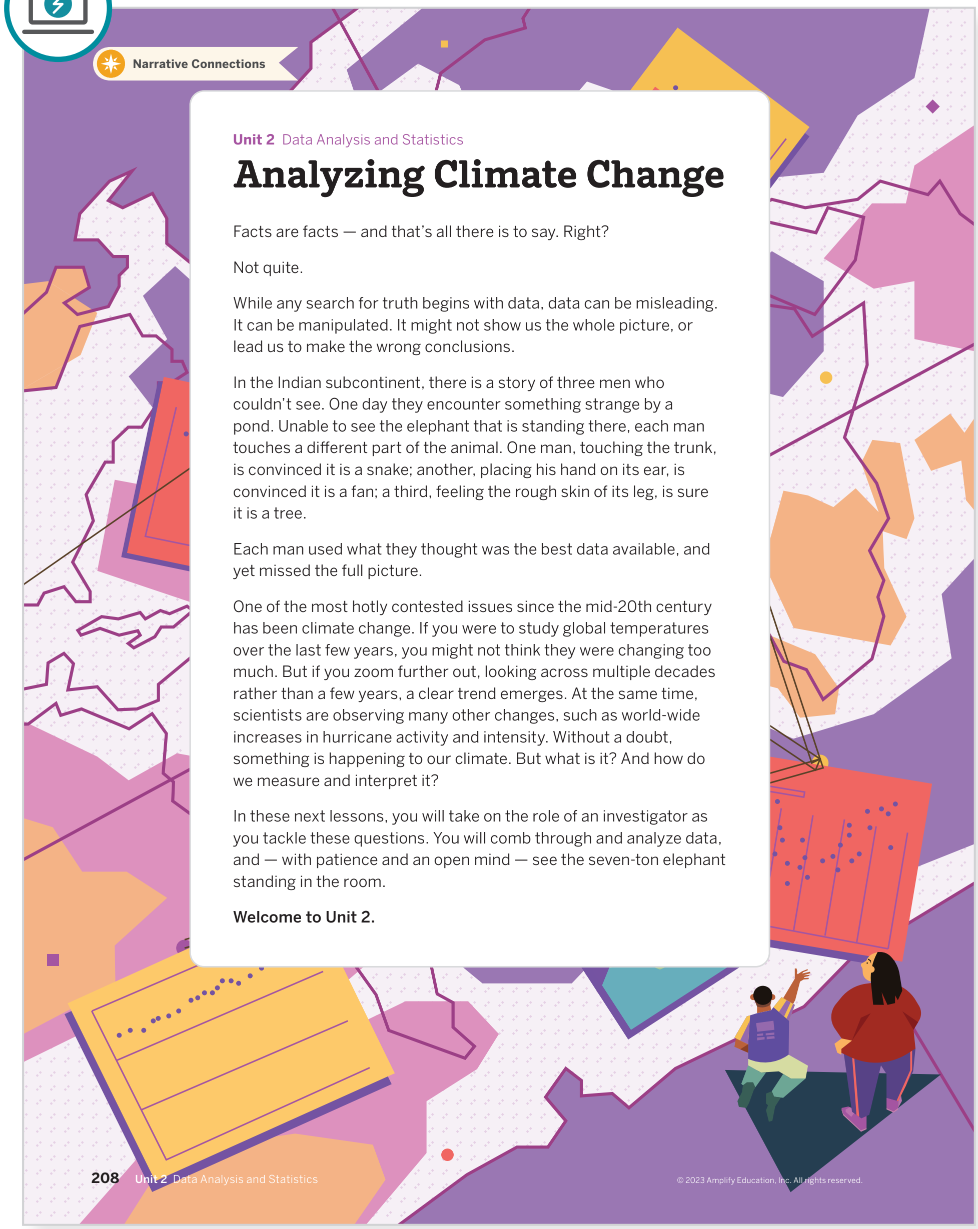
MLR8: Discussion Supports — Restate It!

As students complete Problems 3 and 4, have them meet with a partner to share their responses. Ask them to restate their partner's thinking for these problems using their developing mathematical language. Have the original author share whether their partner accurately restated their thinking. Call students' attention to mathematical language that helped to clarify the original statement, such as *statistical question, variability in responses, overall trend*, etc.

Whole Class | 5 min

Summary Analyzing Climate Change

Review and synthesize how key features of data representations can be exploited to mislead their audience, and how to determine if a question is a statistical question.



Narrative Connections

Unit 2 Data Analysis and Statistics

Analyzing Climate Change

Facts are facts — and that's all there is to say. Right?

Not quite.

While any search for truth begins with data, data can be misleading. It can be manipulated. It might not show us the whole picture, or lead us to make the wrong conclusions.

In the Indian subcontinent, there is a story of three men who couldn't see. One day they encounter something strange by a pond. Unable to see the elephant that is standing there, each man touches a different part of the animal. One man, touching the trunk, is convinced it is a snake; another, placing his hand on its ear, is convinced it is a fan; a third, feeling the rough skin of its leg, is sure it is a tree.

Each man used what they thought was the best data available, and yet missed the full picture.

One of the most hotly contested issues since the mid-20th century has been climate change. If you were to study global temperatures over the last few years, you might not think they were changing too much. But if you zoom further out, looking across multiple decades rather than a few years, a clear trend emerges. At the same time, scientists are observing many other changes, such as world-wide increases in hurricane activity and intensity. Without a doubt, something is happening to our climate. But what is it? And how do we measure and interpret it?

In these next lessons, you will take on the role of an investigator as you tackle these questions. You will comb through and analyze data, and — with patience and an open mind — see the seven-ton elephant standing in the room.

Welcome to Unit 2.

208 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display Graph D from Activity 1.

Have students share what else they would add to the data representation to clarify the information presented.

Ask:

- "Assume this data is given to a researcher to answer a question she asked. What is a possible question she may have asked?" Sample response: What has been the average temperature in the U.S. within the past century?
- "Is this a statistical question?" Yes; The answer to this question involves using data that is expected to have variability.

Highlight that data representations are helpful in answering statistical questions because variability, trends, and summary of data sets can be readily observed.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

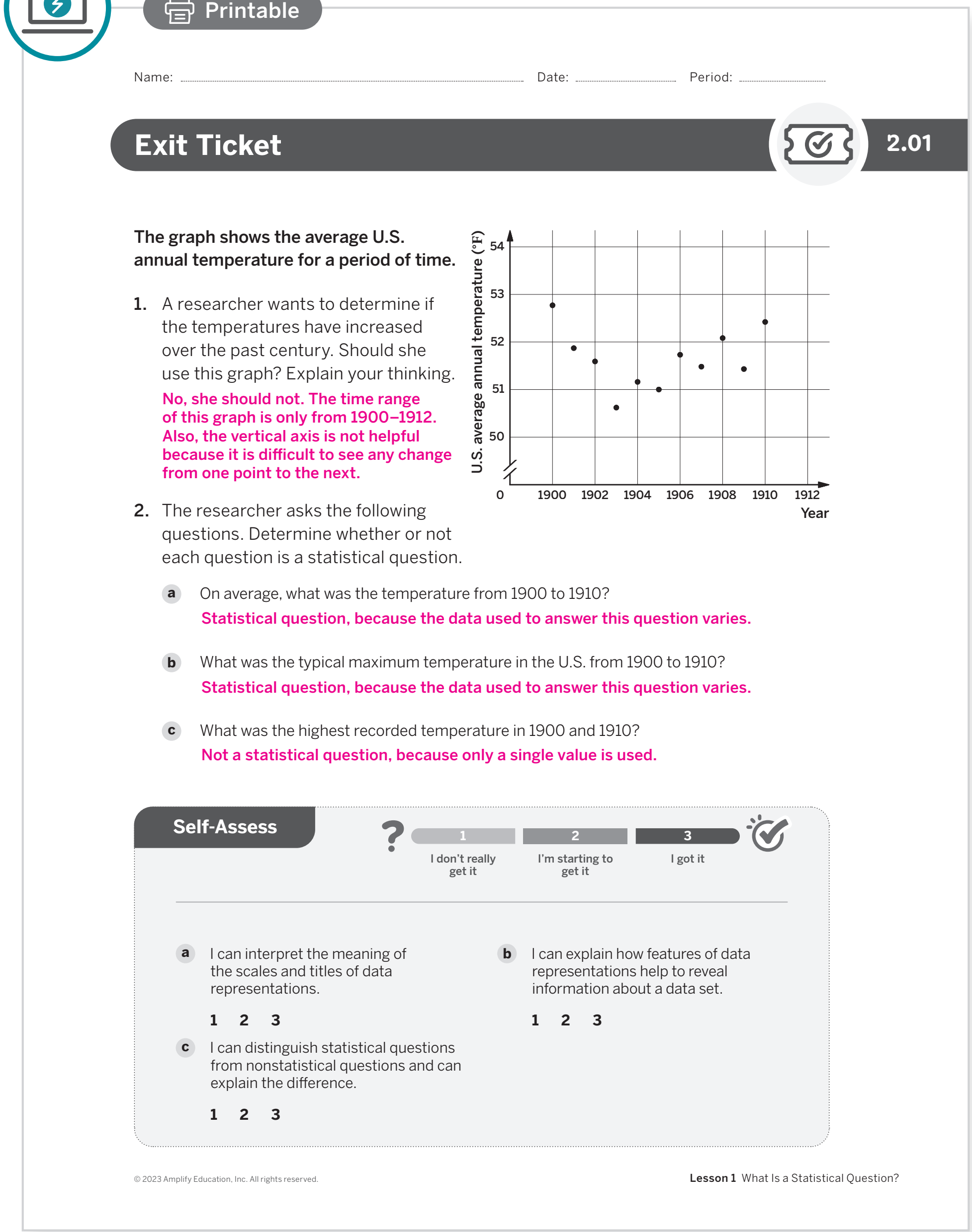
- "What makes a question a statistical question?"
- "How can key features of data representations be used to display trends of a data set?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by describing key features of a data representation and determining if a question is a statistical question.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 2.01

The graph shows the average U.S. annual temperature for a period of time.

1. A researcher wants to determine if the temperatures have increased over the past century. Should she use this graph? Explain your thinking.
   No, she should not. The time range of this graph is only from 1900–1912. Also, the vertical axis is not helpful because it is difficult to see any change from one point to the next.
2. The researcher asks the following questions. Determine whether or not each question is a statistical question.
   - a. On average, what was the temperature from 1900 to 1910?
     Statistical question, because the data used to answer this question varies.
   - b. What was the typical maximum temperature in the U.S. from 1900 to 1910?
     Statistical question, because the data used to answer this question varies.
   - c. What was the highest recorded temperature in 1900 and 1910?
     Not a statistical question, because only a single value is used.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can interpret the meaning of the scales and titles of data representations.
1 2 3

b. I can explain how features of data representations help to reveal information about a data set.
1 2 3

c. I can distinguish statistical questions from nonstatistical questions and can explain the difference.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Success looks like . . .

- **Language Goal:** Describing the features of data representations that are important to examine when making conclusions about data sets. **(Speaking and Listening, Writing)**
  - » Explaining whether the researcher should use the graph of annual temperatures in Problem 1.
- **Language Goal:** Describing the difference between statistical and non-statistical questions. **(Speaking and Listening, Writing)**
  - » Determining whether each question is statistical in Problem 2.

Suggested next steps

If students give an inaccurate or vague explanation in Problem 1, consider:

- Reviewing how specific graphs may or may not be helpful when making conclusions about a data set in Activity 1.
- Assigning Practice Problem 3.
- Asking, "Can you determine the differences between values in the data set? Why or why not?"

If students incorrectly categorize questions in Problem 2, consider:

- Reviewing the definition of a statistical question from Activity 2.
- Assigning Practice Problems 1 and 2.
- Asking, "Which type of measures involve statistical questions: weather or climate? What is the difference between a statistical and non-statistical question?"

Professional Learning

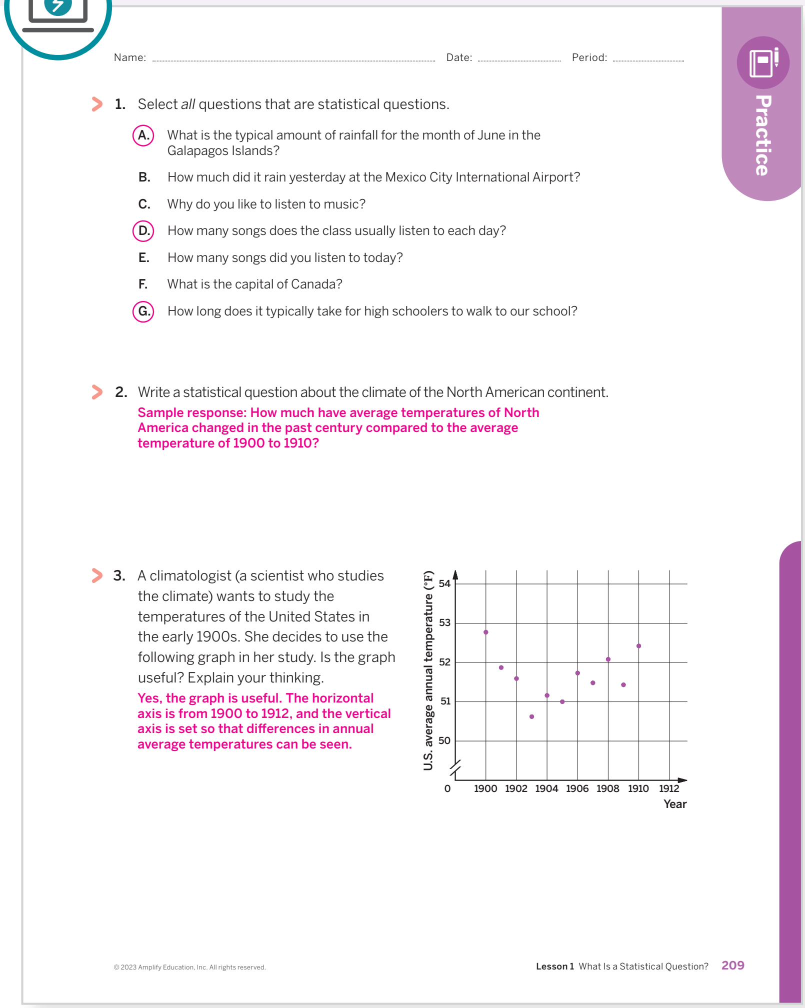
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?
- How did the slow reveal support students in describing the importance and use of key features of data representations? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

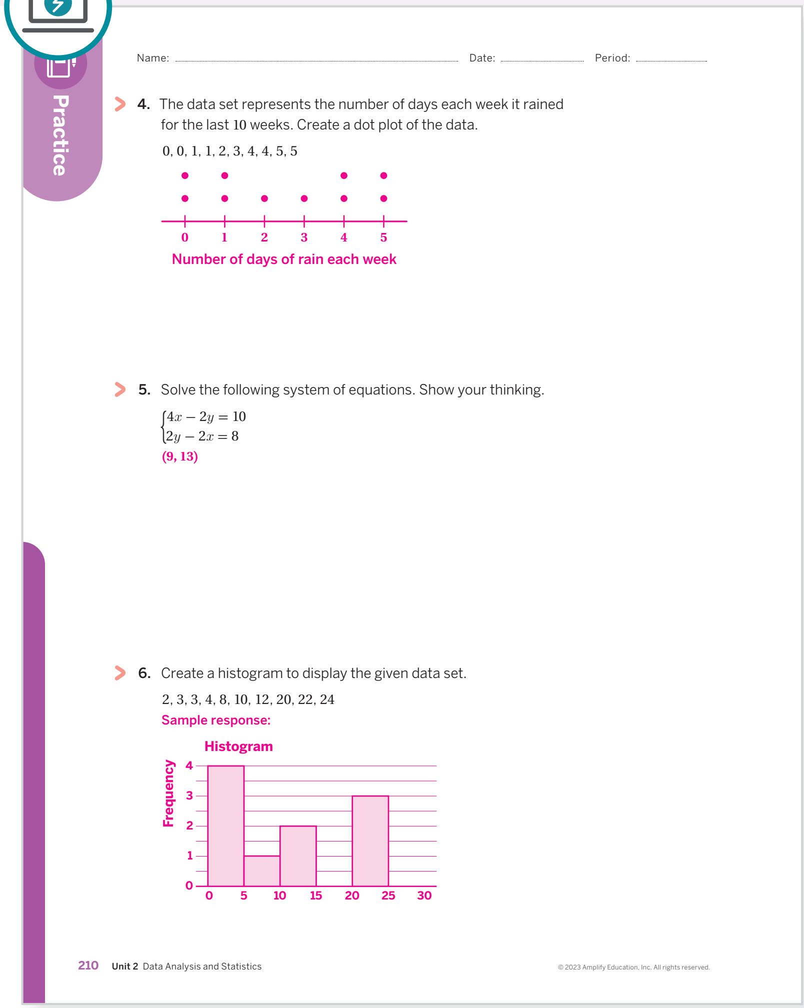
Practice

1. Select *all* questions that are statistical questions.
   - [✓ A.] What is the typical amount of rainfall for the month of June in the Galapagos Islands?
   - B. How much did it rain yesterday at the Mexico City International Airport?
   - C. Why do you like to listen to music?
   - [✓ D.] How many songs does the class usually listen to each day?
   - E. How many songs did you listen to today?
   - F. What is the capital of Canada?
   - [✓ G.] How long does it typically take for high schoolers to walk to our school?

2. Write a statistical question about the climate of the North American continent.
   Sample response: How much have average temperatures of North America changed in the past century compared to the average temperature of 1900 to 1910?

3. A climatologist (a scientist who studies the climate) wants to study the temperatures of the United States in the early 1900s. She decides to use the following graph in her study. Is the graph useful? Explain your thinking.
   Yes, the graph is useful. The horizontal axis is from 1900 to 1912, and the vertical axis is set so that differences in annual average temperatures can be seen.

© 2023 Amplify Education, Inc. All rights reserved.



Name: Date: Period:

Practice

4. The data set represents the number of days each week it rained for the last 10 weeks. Create a dot plot of the data.

   0, 0, 1, 1, 2, 3, 4, 4, 5, 5

5. Solve the following system of equations. Show your thinking.

$$\begin{cases} 4x - 2y = 10 \\ 2y - 2x = 8 \end{cases}$$

   (9, 13)

6. Create a histogram to display the given data set.

   2, 3, 3, 4, 8, 10, 12, 20, 22, 24

   Sample response:

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Grade 6	1
	5	Unit 1 Lesson 20	2
Formative ⚡	6	Unit 2 Lesson 2	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 1

Data Distributions

Whole Class

In this Sub-Unit, students create and interpret one-variable data distributions using measures of center and variability, and also learn a method for determining outliers.

SUB-UNIT 1 Data Distributions

Narrative Connections

How can we protect ourselves from a zombie virus?

When you read “zombie virus,” you might imagine a virus that turns humans into undead monsters. In the world of science, however, it refers to a past disease that has been revived. For example, consider the strange case of a reindeer carcass found in the Siberian Arctic permafrost. The reindeer, which had died from an anthrax outbreak in 1941, was buried and frozen in the Arctic’s permafrost. But in 2016, a record-setting heat wave caused that permafrost to thaw.

While the deer itself remained dead, the anthrax bacteria had other plans. It made its way from the carcass to the topsoil to the water. From there, it went into the bodies of 2,000 living reindeer and the people who lived alongside them. In total, 115 people were hospitalized and one 12-year-old boy died.

This might seem like an isolated case. But as Arctic temperatures rise, more and more viruses and bacteria once frozen could be resurrected.

Some scientists have downplayed the risk, noting that bacteria like anthrax are commonly found in warmer climates. But others are less quick to dismiss the threat, warning that an outbreak could be disastrous when coupled with higher rates of exposure.

In order to assess the risk these viruses might pose, we must first make sense of the temperature data. What is a typical temperature in the Arctic? What is an extreme temperature? And how much does it vary? To help us answer these questions, we turn to data distributions.

© 2023 Amplify Education, Inc. All rights reserved.

Sub-Unit 1 Data Distributions 211

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how measures of center and variation can help determine typical and extreme values — within the context of temperature and weather — in the following places:

- **Lesson 2, Activity 1:** Revisiting Dot Plots and Histograms
- **Lesson 3, Activities 1–2:** Matching Distributions, Examining the Pairs
- **Lesson 4, Activities 2–3:** Global Temperatures From the Early 1900s, Deviation in Global Temperature
- **Lesson 6, Activities 2–3:** Using Spreadsheets to Create Box Plots, Comparing Representations

UNIT 2 | **LESSON 2**

Data Representations

Let's make, compare, and interpret representations of data.

Focus

Goals

1. Create a dot plot, histogram, and box plot to represent numerical data.
2. Identify the five-number summary that describes given statistical data.
3. **Language Goal:** Interpret a box plot that represents a data set. **(Speaking and Listening, Writing)**

Coherence

Today

Students revisit representing data using dot plots, histograms, and box plots. They compare these types of representations for the same data set. Students learn to calculate a five-number summary for creating box plots. They represent, interpret, and present temperature data using their representation of choice.

Previously

In Grade 6, students displayed numerical data in dot plots, histograms, and box plots.

Coming Soon

In Lesson 3, students will use mathematical language to describe the shape of data distributions and interpret its meaning in context.

Rigor

- Students build on their **conceptual understanding** of histograms and box plots.
- Students strengthen their **procedural fluency** in representing data using dot plots, histograms, and box plots.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1 (optional)	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	25 min	5 min	5 min
Independent	Pairs	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 1 PDF, one per pair (also used in Activity 2)
- Activity 2 PDF (as needed)
- Activity 3 PDF, one data set per group
- Activity 3 PDF (answers)
- Anchor Chart PDF, *Sentence Stems, Notice and Wonder*
- colored pencils/markers
- poster paper

Math Language Development

New words

- *five-number summary*

Review words

- *box plot*
- *categorical data*
- *distribution*
- *dot plot*
- *histogram*
- *median*
- *numerical data*
- *quartile*
- *statistic*

Amps | Featured Activity

Activity 3
Interactive Graphs

Students create a dot plot, box plot, and/or histogram to represent temperature data, and interpret their representations.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may lack confidence as others look at their data displays during Activity 3. Help build students' confidence by reviewing critical parts of each display before having students show others their work. Provide time for any last-minute corrections so that their displays show their best work of which they can be proud.

Modifications to Pacing

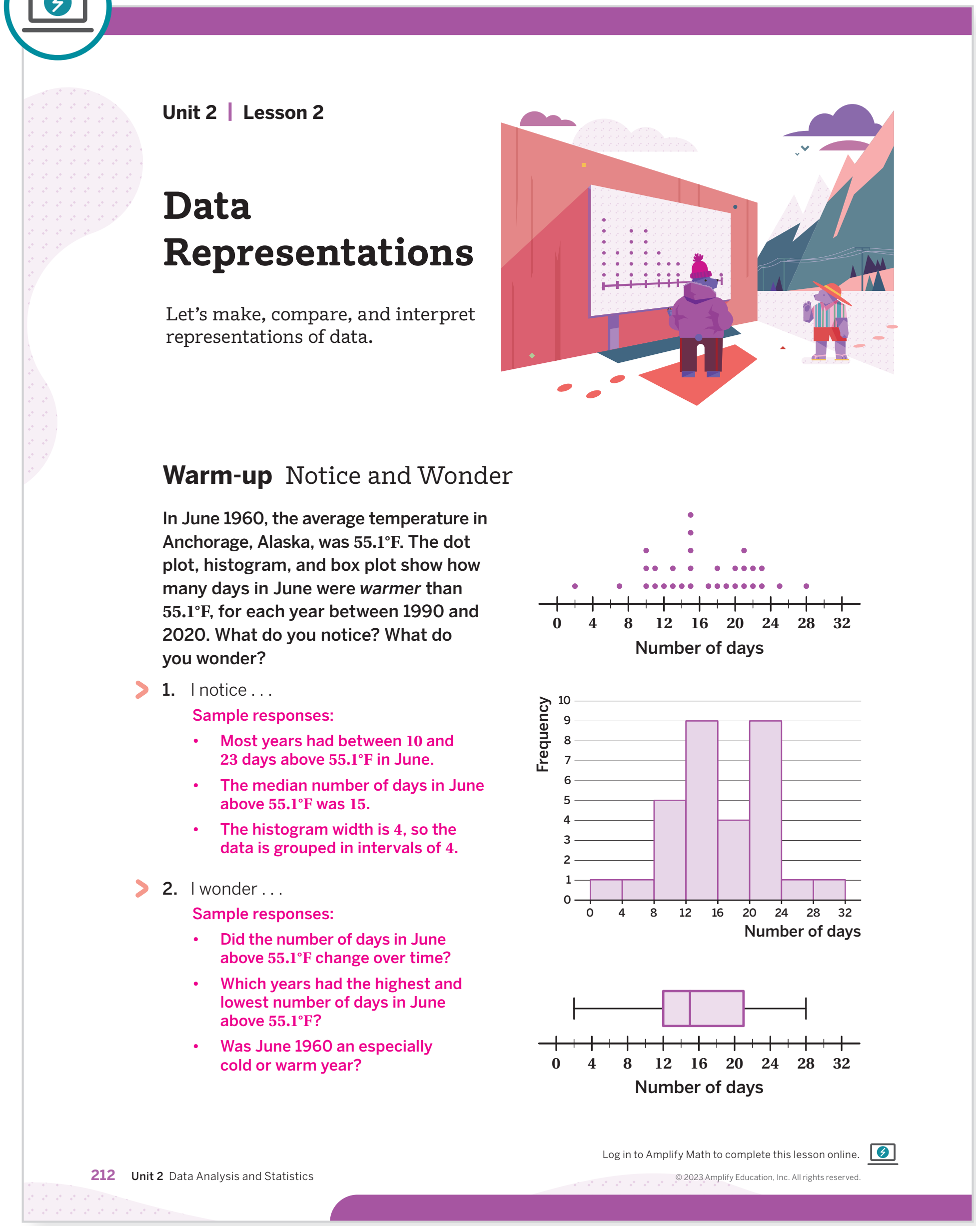
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, omit the dot plot.
- Depending on pre-assessment data, Optional **Activity 1** may be omitted.
- In **Activity 3**, have students create their representation in the activity and omit the gallery tour.

Independent | 5 min

Warm-up Notice and Wonder

Students examine different representations of the same data to consider what mathematical observations are observable for each different display.



Unit 2 | Lesson 2

Data Representations

Let's make, compare, and interpret representations of data.

Warm-up Notice and Wonder

In June 1960, the average temperature in Anchorage, Alaska, was 55.1°F. The dot plot, histogram, and box plot show how many days in June were *warmer* than 55.1°F, for each year between 1990 and 2020. What do you notice? What do you wonder?

1. I notice . . .

   Sample responses:
   - Most years had between 10 and 23 days above 55.1°F in June.
   - The median number of days in June above 55.1°F was 15.
   - The histogram width is 4, so the data is grouped in intervals of 4.

2. I wonder . . .

   Sample responses:
   - Did the number of days in June above 55.1°F change over time?
   - Which years had the highest and lowest number of days in June above 55.1°F?
   - Was June 1960 an especially cold or warm year?

Log in to Amplify Math to complete this lesson online.

212 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Say, "The data displays represent the same data recorded in Anchorage, Alaska, from June 1990 to 2020." Discuss the axes labels, before conducting the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by interpreting the first data value on the dot plot.

Look for points of confusion:

- **Thinking each bar of the histogram represents one measurement.** Have students examine the axis label and compare the dot plot intervals to the histograms.

Look for productive strategies:

- Identifying values from the five-number summary to describe the data.
- Making use of the x-axis to make mathematical observations.

3 Connect

Have individual students share what they noticed or wondered. Record and display their observations.

Highlight that the shape of the dot plot is similar to the shape of the histogram, but the histogram groups categorical data by intervals.

Ask:

- "Which representation(s) shows all the data values?" The dot plot.
- "How would you describe the distribution of the data?" Sample response: Most of the data is between 12 and 20.

Math Language Development

MLR8: Discussion Supports

Before the Connect, display or provide the Anchor Chart PDF, *Sentence Stems, Notice and Wonder* to support students as they record what they notice and wonder and think about how they will share these responses with the class. During the Connect, as students share their responses, listen for and amplify the language students use to describe the graphs, such as *median, intervals, cluster*, etc. Display and review the definitions of review vocabulary.

Power-up

To power up students' ability to understand how data is modeled by a histogram:

Provide students with a copy of the Power-up PDF.

Use: Before the Warm-up

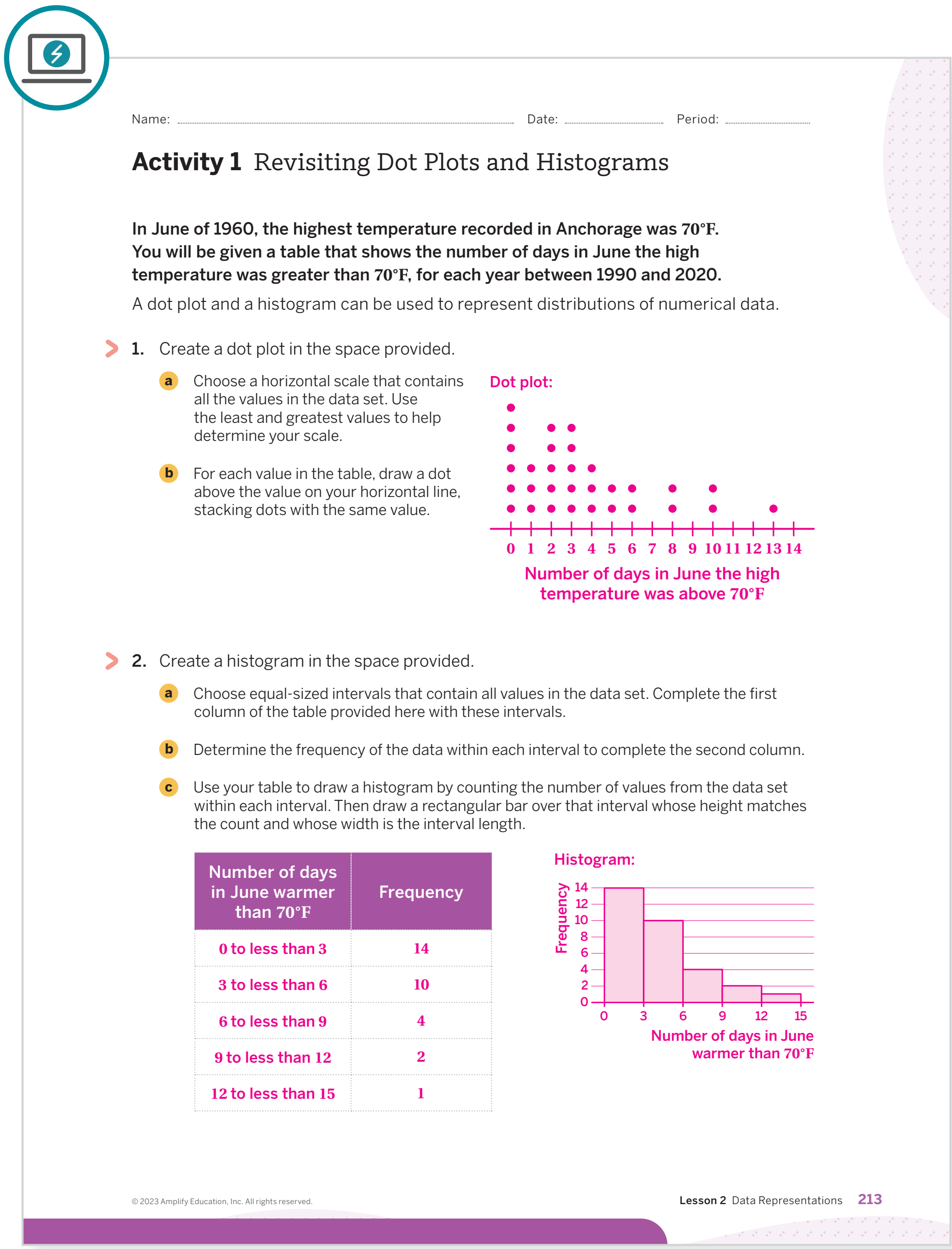
Informed by: Performance on Lesson 1, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 2

Optional

Pairs | 15 min

Activity 1 Revisiting Dot Plots and Histograms

Students revisit creating dot plots and histograms to build procedural fluency.



Name: ______________________ Date: __________ Period: __________

Activity 1 Revisiting Dot Plots and Histograms

In June of 1960, the highest temperature recorded in Anchorage was 70°F. You will be given a table that shows the number of days in June the high temperature was greater than 70°F, for each year between 1990 and 2020.

A dot plot and a histogram can be used to represent distributions of numerical data.

1. Create a dot plot in the space provided.
   - a Choose a horizontal scale that contains all the values in the data set. Use the least and greatest values to help determine your scale.
   - b For each value in the table, draw a dot above the value on your horizontal line, stacking dots with the same value.

2. Create a histogram in the space provided.
   - a Choose equal-sized intervals that contain all values in the data set. Complete the first column of the table provided here with these intervals.
   - b Determine the frequency of the data within each interval to complete the second column.
   - c Use your table to draw a histogram by counting the number of values from the data set within each interval. Then draw a rectangular bar over that interval whose height matches the count and whose width is the interval length.

Number of days in June warmer than 70°F	Frequency
0 to less than 3	14
3 to less than 6	10
6 to less than 9	4
9 to less than 12	2
12 to less than 15	1

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 2 Data Representations 213

1 Launch

Distribute the Activity 1 PDF to each student pair. Review the data as a whole class before completing the problems in pairs.

2 Monitor

Help students get started by having them create the dot plot. Then ask, "How are dot plots and histograms similar?"

Look for points of confusion:

- **Using histogram intervals that are too small or too large.** Remind students that a histogram should have a shape similar to a dot plot, typically with 5–10 intervals.

Look for productive strategies:

- Creating the x-axis using the minimum and maximum values from the data.
- Comparing the shape of the histogram to the dot plot and revising interval lengths to make adjustments.

3 Connect

Display the dot plot from Problem 1.

Have pairs of students share their histograms. Select and sequence those with very small or very large interval lengths first. Explain how the interval lengths were chosen.

Ask, "What do you notice if the interval lengths are too small? Too large?"

Highlight that histograms are accurate regardless of the interval length, but very large or small intervals may obscure the distribution of data.

Ask, "What information can be observed in both representations?" The shape of the distribution and the number of data in a set.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Consider providing students with a pre-labeled number line for which they can use to create their dot plot and a partially-completed histogram with axes scales and intervals given. Suggest they cross off data values on the Activity 1 PDF as they add them to their dot plot.

Extension: *Math Enrichment*

Ask, "What might be a problem with using interval lengths of one?" Sample response: In situations where each data value occurs only once or twice, every bar will have the same height, not allowing us to get a good sense of the shape of the distribution.

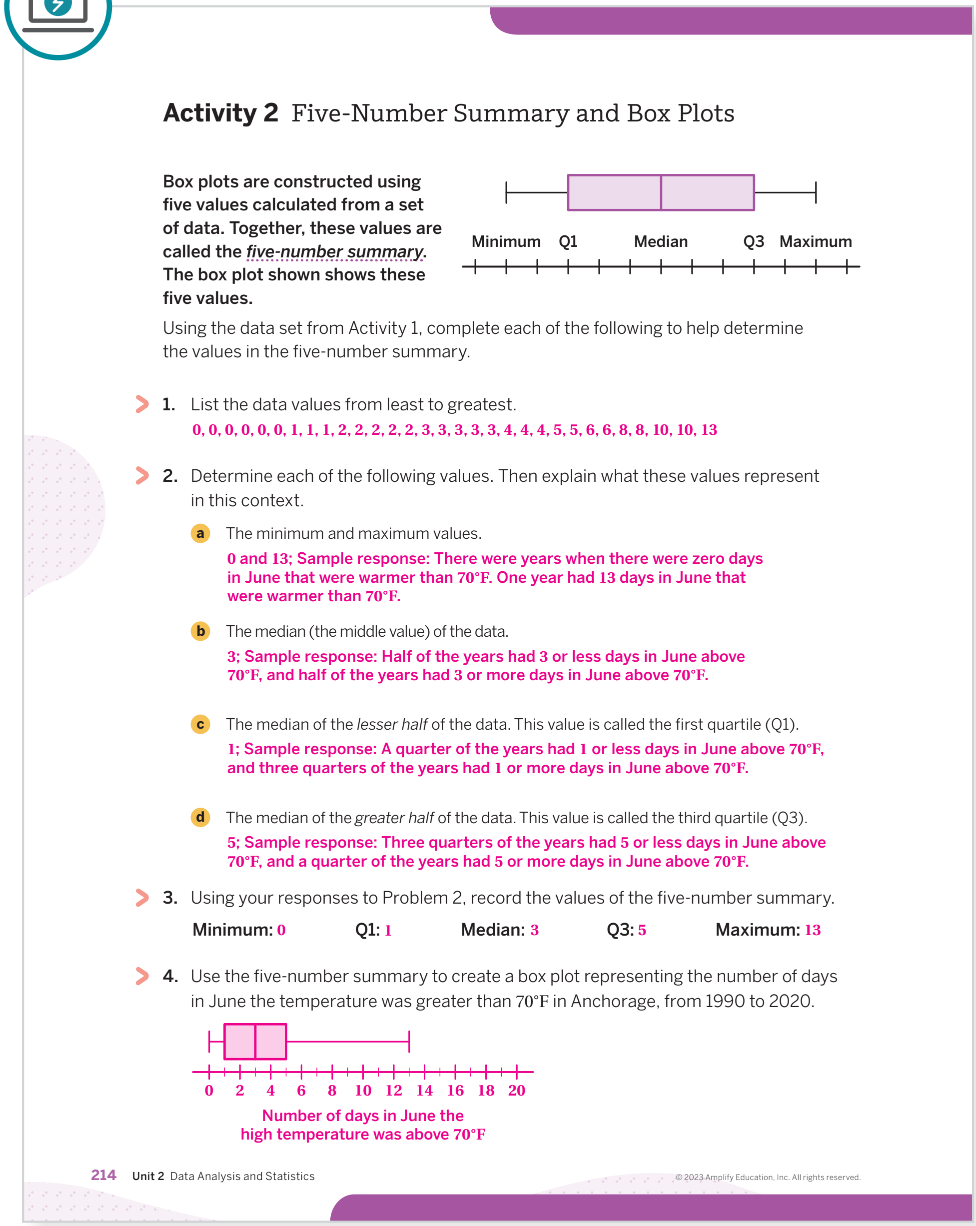
Extension: *Math Enrichment*

Ask, "If you created two histograms of the same data, but one histogram used intervals 0–2, 3–5, 6–8, etc., and the other histogram used intervals 0–4, 5–9, and 10–14, how would the bar heights compare?" Sample response: The distribution would look similar, but the second histogram would have bar heights of 22, 8, and 3.

Pairs | 10 min

Activity 2 Five-Number Summary and Box Plots

Students create a box plot from a five-number summary to begin informally thinking about measures of center and spread.



Activity 2 Five-Number Summary and Box Plots

Box plots are constructed using five values calculated from a set of data. Together, these values are called the *five-number summary*. The box plot shown shows these five values.

Using the data set from Activity 1, complete each of the following to help determine the values in the five-number summary.

1. List the data values from least to greatest.

   0, 0, 0, 0, 0, 0, 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 5, 5, 6, 6, 8, 8, 10, 10, 13

2. Determine each of the following values. Then explain what these values represent in this context.

   a. The minimum and maximum values.

   0 and 13; Sample response: There were years when there were zero days in June that were warmer than 70°F. One year had 13 days in June that were warmer than 70°F.

   b. The median (the middle value) of the data.

   3; Sample response: Half of the years had 3 or less days in June above 70°F, and half of the years had 3 or more days in June above 70°F.

   c. The median of the *lesser half* of the data. This value is called the first quartile (Q1).

   1; Sample response: A quarter of the years had 1 or less days in June above 70°F, and three quarters of the years had 1 or more days in June above 70°F.

   d. The median of the *greater half* of the data. This value is called the third quartile (Q3).

   5; Sample response: Three quarters of the years had 5 or less days in June above 70°F, and a quarter of the years had 5 or more days in June above 70°F.

3. Using your responses to Problem 2, record the values of the five-number summary.

   Minimum: **0** **Q1:** **1** **Median:** **3** **Q3:** **5** **Maximum:** **13**

4. Use the five-number summary to create a box plot representing the number of days in June the temperature was greater than 70°F in Anchorage, from 1990 to 2020.

214 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Students remain in pairs and continue using the Activity 1 PDF.

2 Monitor

Help students get started by reviewing the term "quartile" and addressing common language misunderstandings associated with "quarters".

Look for points of confusion:

- **Thinking that quartiles and medians must be values in a data set.** Review using adjacent values to calculate the median and quartile when these fall in between the values in the data.

Look for productive strategies:

- Grouping data below and above the median to determine the quartiles.

3 Connect

Have pairs of students share their five-number summary and box plots.

Highlight that the median and quartiles are useful for describing the data's center and observing its spread.

Ask:

- "What percent of data falls above and below Q1?" 25% of the data is below Q1, and 75% of the data is above Q1.
- "What does Q3 represent in this scenario?" About 25% of the years had 5 or more days in June with a high temperature above 70°F.

Differentiated Support

Accessibility: ***Guide Processing and Visualization***

Provide students with the Activity 2 PDF to help them organize the data to determine the five-number summary. After they have determined these values, have them annotate the box plot in their Student Edition with where these values are located on the box plot. Guide students to see how the box plot clearly shows the five-number summary, which is why those five numbers are calculated when creating a box plot.

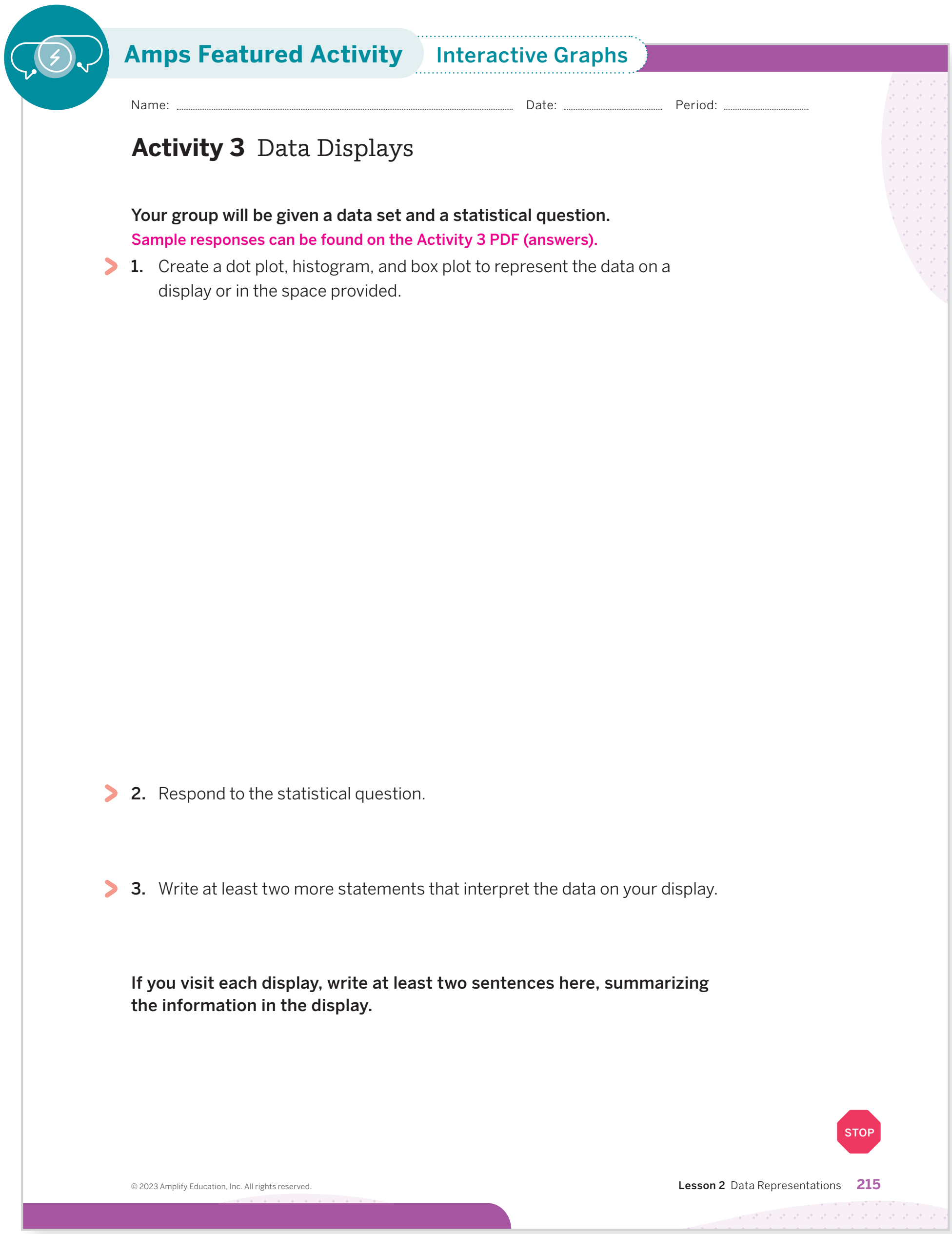
Extension: ***Math Enrichment***

Ask, "How will a box plot change if a value is added to the data set that is equivalent to the maximum or minimum?"
The quartiles and median will move closer to the value because there are now more values in the greater or lesser half.

Small Groups | 25 min

Activity 3 Data Displays

Students create data displays and answer statistical questions to determine how each representation is most helpful for interpreting data.



Amps Featured Activity Interactive Graphs

Name: ______ Date: ______ Period: ______

Activity 3 Data Displays

Your group will be given a data set and a statistical question.

Sample responses can be found on the Activity 3 PDF (answers).

1. Create a dot plot, histogram, and box plot to represent the data on a display or in the space provided.

2. Respond to the statistical question.

3. Write at least two more statements that interpret the data on your display.

If you visit each display, write at least two sentences here, summarizing the information in the display.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 2 Data Representations 215

1 Launch

Distribute one data set from the Activity 3 PDF and materials for creating displays to each group. Review the activity instructions together. Consider providing poster paper and colored pencils/markers to each group, then using the ***Gallery Tour*** routine to display student work during the Connect.

2 Monitor

Help students get started by having them reference the problems from Activities 1 and 2 to help in creating data representations.

Look for points of confusion:

- **Overlooking the two different time periods used in Data Set 3.** Have students group the data into two groups (1960–1961 and 2018–2019), and create representations for each group.

Look for productive strategies:

- Creating two of the same types of representations along one number line to compare subgroups of data from a data set.

3 Connect

Have groups of students share their data display.

Highlight that a dot plot and histogram share a similar shape. Students can see where data is clustered and how it is distributed across a range of values.

Ask:

- "What information is displayed on the box plot that is not displayed on the dot plot?" The box plot displays the quartiles and the median.
- "Which representation do you find most helpful to display the data?"
- "How would you describe the distribution of your data?"

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Consider providing students with blank number lines and blank displays they can use to create their dot plots, histograms, and box plots. Consider providing options of intervals students can choose from to create their histograms. Provide the Activity 2 PDF from the prior activity to help students determine the five-number summary.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital dot plots, histograms, and box plots.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their data displays, ask them to consider what is the same and what is different about each display. Draw their attention to the different ways the data are represented and the benefit of each display for interpreting data.

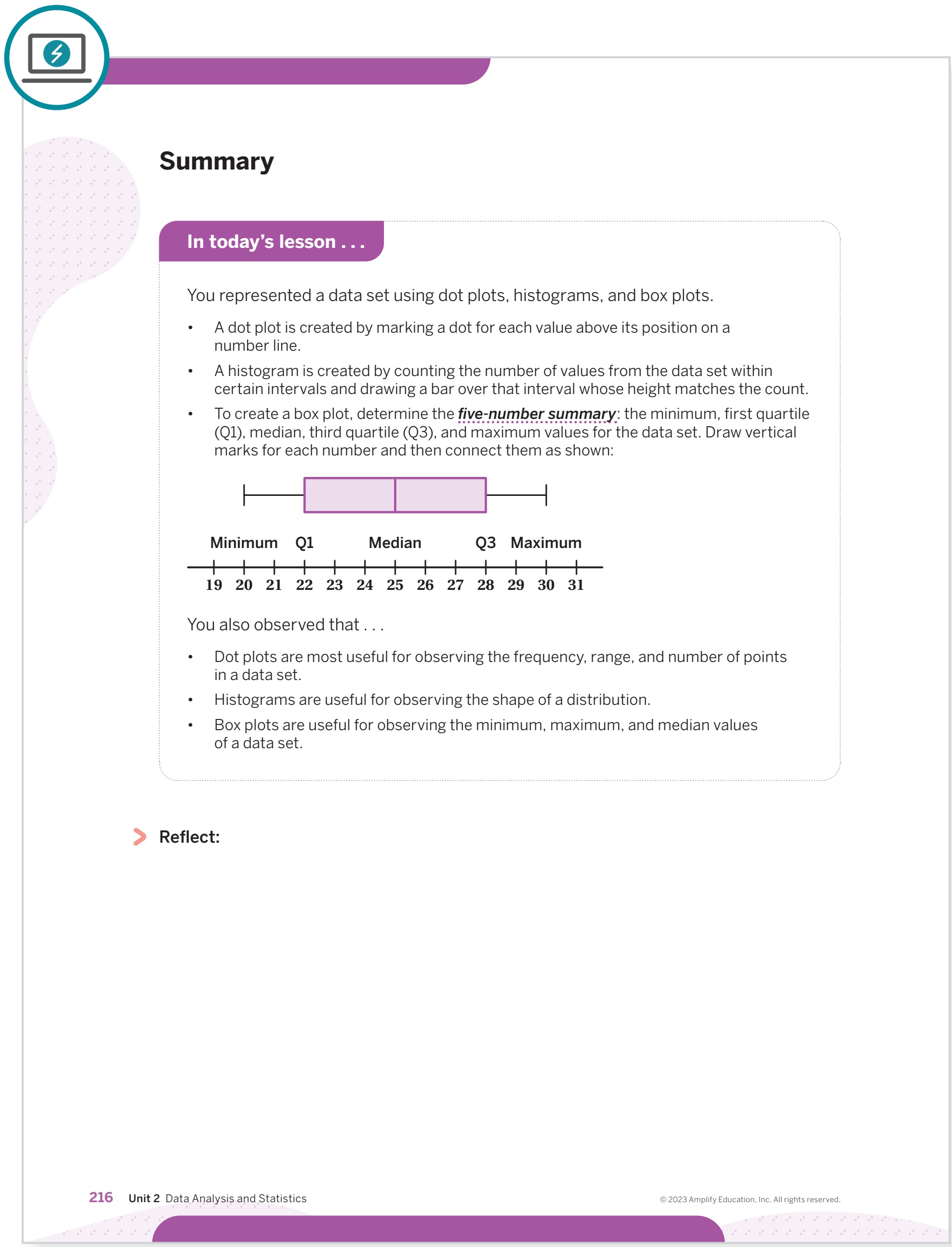
English Learners

Use annotations on the displays to highlight the different ways the data are represented.

Whole Class | 5 min

Summary

Review and synthesize each type of data representation and information that they provide and determining the five-number summary of a data set.



Summary

In today's lesson . . .

You represented a data set using dot plots, histograms, and box plots.

- A dot plot is created by marking a dot for each value above its position on a number line.
- A histogram is created by counting the number of values from the data set within certain intervals and drawing a bar over that interval whose height matches the count.
- To create a box plot, determine the ***five-number summary***: the minimum, first quartile (Q1), median, third quartile (Q3), and maximum values for the data set. Draw vertical marks for each number and then connect them as shown:

You also observed that . . .

- Dot plots are most useful for observing the frequency, range, and number of points in a data set.
- Histograms are useful for observing the shape of a distribution.
- Box plots are useful for observing the minimum, maximum, and median values of a data set.

Reflect:

216 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the five-number summary and box plot.

Have students share how each value is determined in the five-number summary and used to create a box plot.

Highlight that the quartiles and median of box plots allow students to observe how data is clustered, but they are unable to observe the specific shape and distribution of data over smaller intervals. Histograms and dot plots are more useful for those types of observations.

Ask, "What are the advantages and disadvantages of each representation?"

Reflect

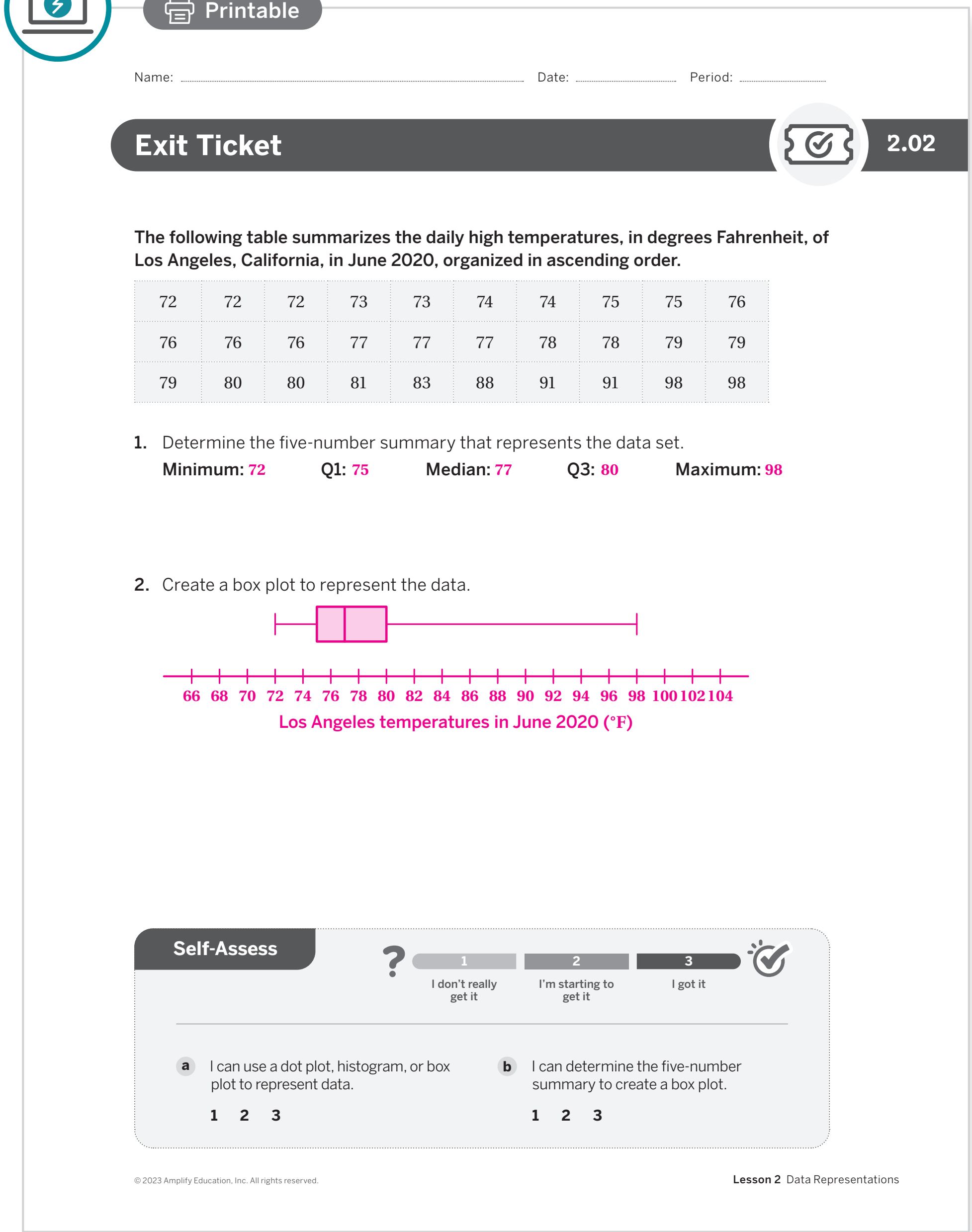
After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do people understand and communicate data?"

Exit Ticket

Students demonstrate their understanding by determining the five-number summary and creating a box plot of temperature data.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 2.02

The following table summarizes the daily high temperatures, in degrees Fahrenheit, of Los Angeles, California, in June 2020, organized in ascending order.

72	72	72	73	73	74	74	75	75	76
76	76	76	77	77	77	78	78	79	79
79	80	80	81	83	88	91	91	98	98

1. Determine the five-number summary that represents the data set.

Minimum: 72 **Q1:** 75 **Median:** 77 **Q3:** 80 **Maximum:** 98

2. Create a box plot to represent the data.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can use a dot plot, histogram, or box plot to represent data.

1 2 3

b. I can determine the five-number summary to create a box plot.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 2 Data Representations

Success looks like . . .

- **Goal:** Creating a dot plot, histogram, and box plot to represent numerical data.
  - » Creating a box plot to represent the daily high temperatures in Problem 2.
- **Goal:** Identifying the five-number summary that describes given statistical data.
  - » Determining the five-number summary for the daily high temperatures in Problem 1.
- **Language Goal:** Interpreting a box plot that represents a data set. **(Speaking and Listening, Writing)**

Suggested next steps

If students incorrectly determine values in the five-number summary in Problem 1, consider:

- Reviewing the process for determining the five-number summary of data from Activity 2.
- Assigning Practice Problems 1 and 2.
- Asking, "How could you rearrange the data to help determine the values in the five-number summary?"

If students inaccurately create the box plot from their five-number summary in Problem 2, consider:

- Reviewing how to use the five-number summary to create a box plot from Activity 2.
- Assigning Practice Problems 1 and 2.
- Asking, "What does each vertical line in a box plot represent?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

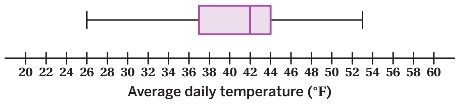
- What worked and didn't work today? What different ways did students approach creating a histogram? What does that tell you about similarities and differences among your students?
- How did creating the data representations prepare students to develop an understanding of distribution and spread? What might you change for the next time you teach this lesson?

Practice

Independent

Name: Date: Period:

1. The box plot represents the distribution of average daily temperatures of a town during 20 days of winter. Determine the five-number summary that represents this data.



Minimum: 26 **Q1:** 37 **Median:** 42 **Q3:** 44 **Maximum:** 53

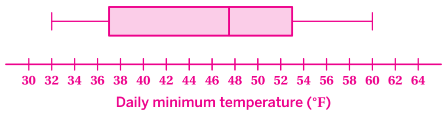
2. The table summarizes the daily minimum temperature of the first 16 days of 2020 in Atlanta, Georgia.

36	46	50	37	32	38	43	36
37	50	60	49	59	60	56	50

a) Determine the five-number summary that represents this data.

Minimum: 32 **Q1:** 37 **Median:** 47.5 **Q3:** 53 **Maximum:** 60

b) Create a box plot that represents the data.



© 2023 Amplify Education, Inc. All rights reserved.

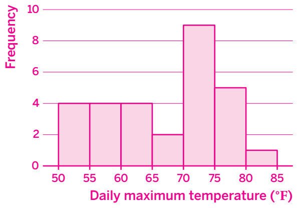
Name: Date: Period:

3. The table summarizes the daily maximum temperatures, in degrees Fahrenheit, of San Antonio, Texas, in February 2020.

72	76	73	78	50	59	74	70	78	73
51	68	61	59	64	72	81	75	53	50
57	64	71	73	68	56	62	73	76	

Use the table to create a frequency table for the data. Then use the table to create a histogram. **Sample response shown.**

Daily maximum temperature (°F)	Frequency
50 to less than 55	4
55 to less than 60	4
60 to less than 65	4
65 to less than 70	2
70 to less than 75	9
75 to less than 80	5
80 to less than 85	1



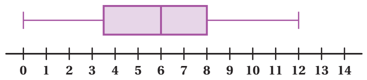
4. Solve the following system of equations. Show your thinking.

$$\begin{cases} y = 4x + 10 \\ -5x + y = 3 \end{cases}$$

(7, 38)

5. Determine which statements about the box plot are true. Select *all* that apply.

[✓ A.] The range of the data is 12.
B. The first quartile is 4.
[✓ C.] The IQR (interquartile range) is 4.5.
D. The mean is 6.
[✓ E.] The median is 6.



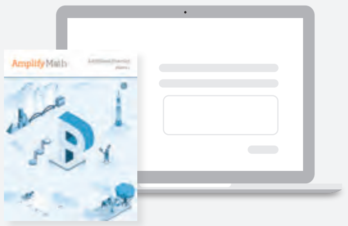
© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 1 Lesson 18	2
Formative ⚡	5	Unit 2 Lesson 3	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available



For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 2 | **LESSON 3**

The Shape of Distributions

Let's describe data distributions.

Focus

Goals

1. **Language Goal:** Describe the shape of a distribution using words, such as *symmetric*, *skewed*, *uniform*, *bimodal*, and *bell-shaped*. **(Speaking and Listening, Writing)**
2. **Language Goal:** Interpret a distribution to suggest a possible context for the data. **(Speaking and Listening, Writing)**

Coherence

Today

Students are introduced to several terms for precisely describing the shape of data distributions: symmetric, skewed, uniform, bimodal, and bell-shaped. They match dot plots and box plots to corresponding histograms based on their shape. Students interpret data representations of snow coverage and temperature.

Previously

In Lesson 2, students created box plots, dot plots, and histograms, and determined the five-number summary of a data set.

Coming Soon

In Lesson 4, students will calculate the mean, mean absolute deviation, median, and IQR and use them to measure the center and variability of data sets.

Rigor

- Students build on their **conceptual understanding** of different types of data distributions and their shape in relation to the data.
- Students build **fluency** describing data distributions and connecting dot plots and histograms.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 1 PDF, pre-cut cards, one per pair
- Anchor Chart PDF, *Sentence Stems, Describing My Thinking*
- Anchor Chart PDF, *Shapes of Distributions* (as needed)

Math Language Development

New words

- ***bell-shaped***
- ***bimodal***
- ***shape***
- ***skewed left***
- ***skewed right***
- ***uniform****

Review words

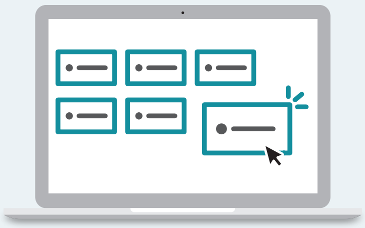
- *box plot*
- *distribution*
- *histogram*
- *mean*
- *median*
- *statistic*
- *symmetric*

*Students may confuse the statistical term uniform with the everyday use describing an athletic or school uniform. Be ready to address the differences between them.

Amps | Featured Activity

Activity 1
Digital Card Sort

Students match dot plots to their corresponding histogram and describe the distribution of the data.



Building Math Identity and Community

Connecting to Mathematical Practices

During Activity 1, students might struggle to get along with a partner. As a class, review ways partners show respect and build each other up when working together. Remind students that they have each other as resources and that they can learn from each other.

Modifications to Pacing

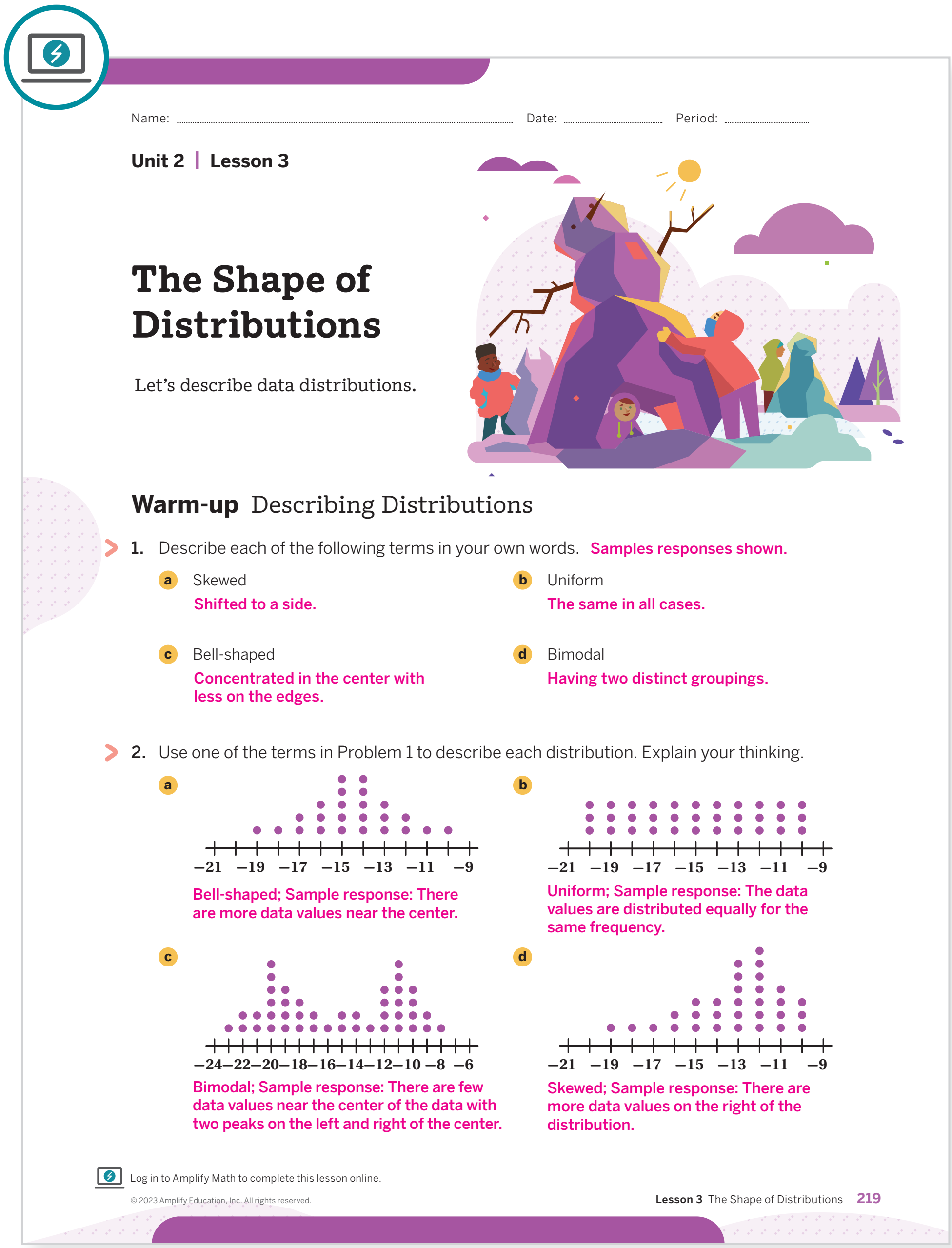
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Cards 13 and 10 can be omitted and reviewed together as a class.
- In **Activity 2**, Problem 1 can be omitted.

Pairs | 10 min

Warm-up Describing Distributions

Students connect mathematical terminology to data distributions to describe the shapes of dot plots.



Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 3

The Shape of Distributions

Let's describe data distributions.

Warm-up Describing Distributions

1. Describe each of the following terms in your own words. **Samples responses shown.**

   a. Skewed
   Shifted to a side.

   b. Uniform
   The same in all cases.

   c. Bell-shaped
   Concentrated in the center with less on the edges.

   d. Bimodal
   Having two distinct groupings.

2. Use one of the terms in Problem 1 to describe each distribution. Explain your thinking.

   a.

   Bell-shaped; Sample response: There are more data values near the center.

   b.

   Uniform; Sample response: The data values are distributed equally for the same frequency.

   c.

   Bimodal; Sample response: There are few data values near the center of the data with two peaks on the left and right of the center.

   d.

   Skewed; Sample response: There are more data values on the right of the distribution.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Give students time to work independently before sharing their thinking with their partner.

2 Monitor

Help students get started by developing non-mathematical definitions for each term.

Look for points of confusion:

- **Struggling to define the terms *skewed* and *bimodal*.** Use each term in a sentence in a non-mathematical context or ask what the prefix "bi" means.

Look for productive strategies:

- Using the symmetric or asymmetric properties of the dot plots to describe their matching term.
- Drawing lines or curves to make sense of a distribution's shape.

3 Connect

Display the Warm-up.

Have pairs of students share their definitions and how they used them to describe each dot plot.

Ask, "Which distributions appear symmetrical?" Uniform and bell-shaped. Bimodal can also be symmetrical.

Define the terms:

- *bell-shaped*
- *bimodal*
- *shape*
- *skewed left*
- *skewed right*
- *uniform*

Highlight that skewed data can be skewed left (a tail extending to the left) or skewed right (a tail extending to the right), and a bimodal distribution has two separate clusters of data.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students write their responses to Problem 2, have them share their responses with another pair of students to give and receive feedback. Encourage reviewers to ask clarifying questions such as:

- "How did you decide which distribution was bell-shaped versus bimodal? Bell-shaped versus uniform?"
- "Were the scales on the axes part of your decision? Why or why not?"

Have students write a final response, based on the feedback they received.

English Learners

Consider using intentional grouping with this routine to pair students with developing English proficiency with students who are more proficient with the English language.

Power-up

To power up students' ability to read information from box plots:

Provide students with a copy of the Power-up PDF.

Use: Before Activity 1

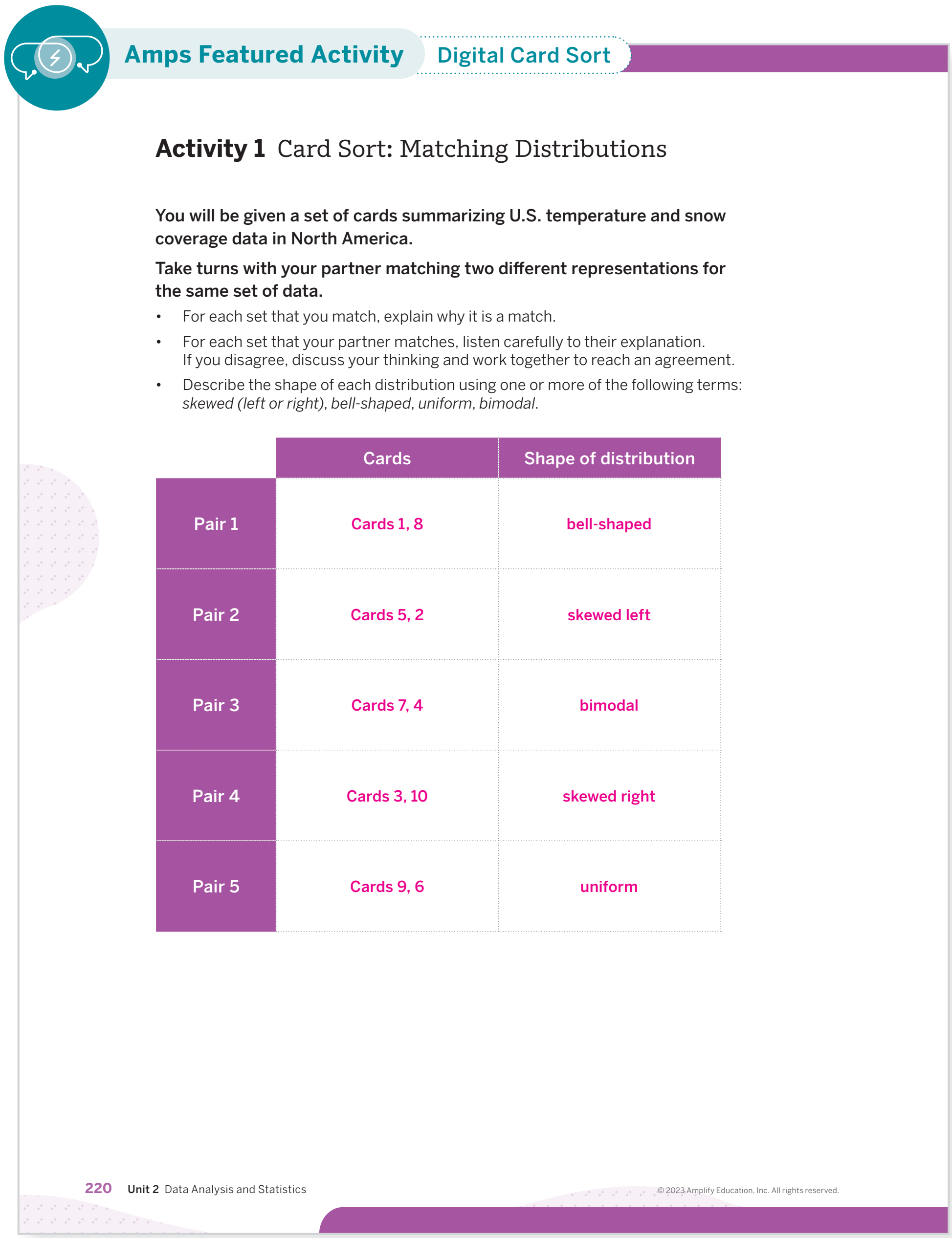
Informed by: Performance on Lesson 2, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 1

Pairs | 15 min

Activity 1 Matching Distributions

Students match data representations showing the same data to practice describing the shape of the distributions.

Amps Featured Activity Digital Card Sort



Activity 1 Card Sort: Matching Distributions

You will be given a set of cards summarizing U.S. temperature and snow coverage data in North America.

Take turns with your partner matching two different representations for the same set of data.

- For each set that you match, explain why it is a match.
- For each set that your partner matches, listen carefully to their explanation. If you disagree, discuss your thinking and work together to reach an agreement.
- Describe the shape of each distribution using one or more of the following terms: *skewed (left or right), bell-shaped, uniform, bimodal.*

	Cards	Shape of distribution
Pair 1	Cards 1, 8	bell-shaped
Pair 2	Cards 5, 2	skewed left
Pair 3	Cards 7, 4	bimodal
Pair 4	Cards 3, 10	skewed right
Pair 5	Cards 9, 6	uniform

220 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Distribute the pre-cut cards from the Activity 1 PDF to each student pair. Read the directions as a whole class, and then model a discussion that could occur between partners.

2 Monitor

Help students get started by having them examine the horizontal axis of each data representation.

Look for points of confusion:

- **Mismatching dot plots with corresponding histograms.** Have students identify histogram intervals, then determine the number of values on the dot plot in each interval.
- **Having difficulty determining a corresponding match for box plots.** Have students determine a five-number summary and ask, "How can you use these values to determine where data is clustered?"

Look for productive strategies:

- Comparing the overall shapes of the distributions for each display type.
- Comparing the horizontal axis for each display type.

3 Connect

Have pairs of students share their strategy for matching the cards. Select and sequence those using productive strategies.

Highlight that students can use the height of the bars in the histograms, and the box of the box plot to determine how data is clustered and the shape of the distribution.

Ask, "Which distribution pair(s) is symmetric?" Pair 5, the uniform distribution.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with a subset of the cards, such as Cards 1, 2, 5, 8, 9, 11, 12, and 14. Have them describe the shape of the distributions in their own words first, before determining which terms match their descriptions.

Accessibility: *Guide Processing and Visualization*

Display or provide students with a copy of the Anchor Chart PDF, *Shapes of Distributions*, which shows a visual example of a data distribution for each of these terms: *symmetric, uniform, bimodal, bell-shaped, skewed left*, and *skewed right*.

Math Language Development

MLR7: Compare and Connect

Before the Connect, ask students to prepare a visual display that shows their matches. Post the displays. During the Connect, as students share their strategies, have them refer to their visual displays. Ask them to point to locations on the distributions that helped them decide how to sort the cards.

English Learners

Annotate specific locations on the distributions that illustrate whether the distribution is *skewed left, skewed right, bell-shaped, uniform*, or *bimodal*.

Pairs | 15 min

Activity 2 Examining the Pairs

Students analyze distributions from Activity 1 to interpret possible applications for the data.

Name: ______ Date: ______ Period: ______

Activity 2 Examining the Pairs

Snow is often associated with cold weather, but snow influences temperature as well. Snow can reflect heat from the Sun back into space, cooling the planet. The presence or absence of snow contributes to patterns of warming and cooling.

1. A journalist uses the data cards from Activity 1 to investigate snowfall coverage in North America and U.S. temperatures from recent decades. Which pair of cards does not belong? Explain your thinking.

   Pair 5 (Cards 9 and 6); Sample response: The other sets of cards focus on the same range of years and times of year, whereas Cards 9 and 6 focus on the monthly temperatures of one year.

2. The journalist writes an article about snow coverage and temperature during summer in recent decades. Which pair(s) of cards do you think should be used for the article? Explain your thinking.

   Pair 2 (Cards 5 and 2) and Pair 4 (Cards 3 and 10). These pairs summarize the August temperatures and the summer snow coverage.

3. What statistical questions might the journalist have wanted answers to, based on the data used in the article? Be prepared to explain your thinking.

   Sample responses:
   - **How have summer temperatures and snow coverage changed in recent decades?**
   - **What is the typical summer temperature and snow coverage?**

4. Examine the cards with a bimodal distribution.

   a. Why do you think this data set is bimodal?

   Sample response: In many places in North America, there is usually very little snowfall in the summer months, much higher snowfall in the winter months, and lower amounts in the fall and spring.

   b. Which months of the year do you think the smaller peak of data values are from? The larger peak of data values? Explain your thinking.

   I think the smaller peak of data values are from summer months because many parts of North America have little or no snowfall. The larger peak is from the winter months because many places in North America have large amounts of snowfall during this time of year.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 3 The Shape of Distributions 221

1 Launch

Read the prompt aloud and consider questions that may arise. Have student pairs compare their responses with other pairs after 5 minutes.

2 Monitor

Help students get started by displaying the Activity 1 solutions and conducting the ***Which One Doesn't Belong?*** routine for Problem 1.

Look for points of confusion:

- **Having difficulty distinguishing between periods of time and times of year.** Try grouping similar distributions together by time periods, asking what connections they notice. Then group by times of year. Ask, "What do you notice?"
- **Struggling to determine statistical questions.** Ask, "What is a question that requires a large data set to answer?"

Look for productive strategies:

- Grouping two pairs of cards by time periods.
- Grouping pairs of cards by types of data (temperature versus snow coverage).

3 Connect

Have pairs of students share how they used key features of the data representations to choose pairs of cards for Problems 1 and 3.

Highlight that using different time periods or different size intervals in the histogram may portray a different story or generalization of the data distribution.

Ask, "What information do the pairs of cards provide about average U.S. temperatures over the last century?"

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Have students first organize the cards into two categories to help them respond to Problem 1: displays that represent the same range of years and times of year, and displays that show different ranges of years or times of year.

Extension: *Math Enrichment*

Using some of the representations on the cards as a guide, have students create a histogram that shows the U.S. average August temperatures from 1980–2019 (or other years). Ask them to describe the shape of the distribution and what it tells them about the data.

Math Language Development

MLR2: Collect and Display

While students work, circulate and listen to their discourse as they share their thinking. Capture common or important phrases that you hear and call student attention to these terms during the Connect discussion. For example, listen for these terms: *line of symmetry, bell-shaped, skewed, bimodal*, and *uniform*.

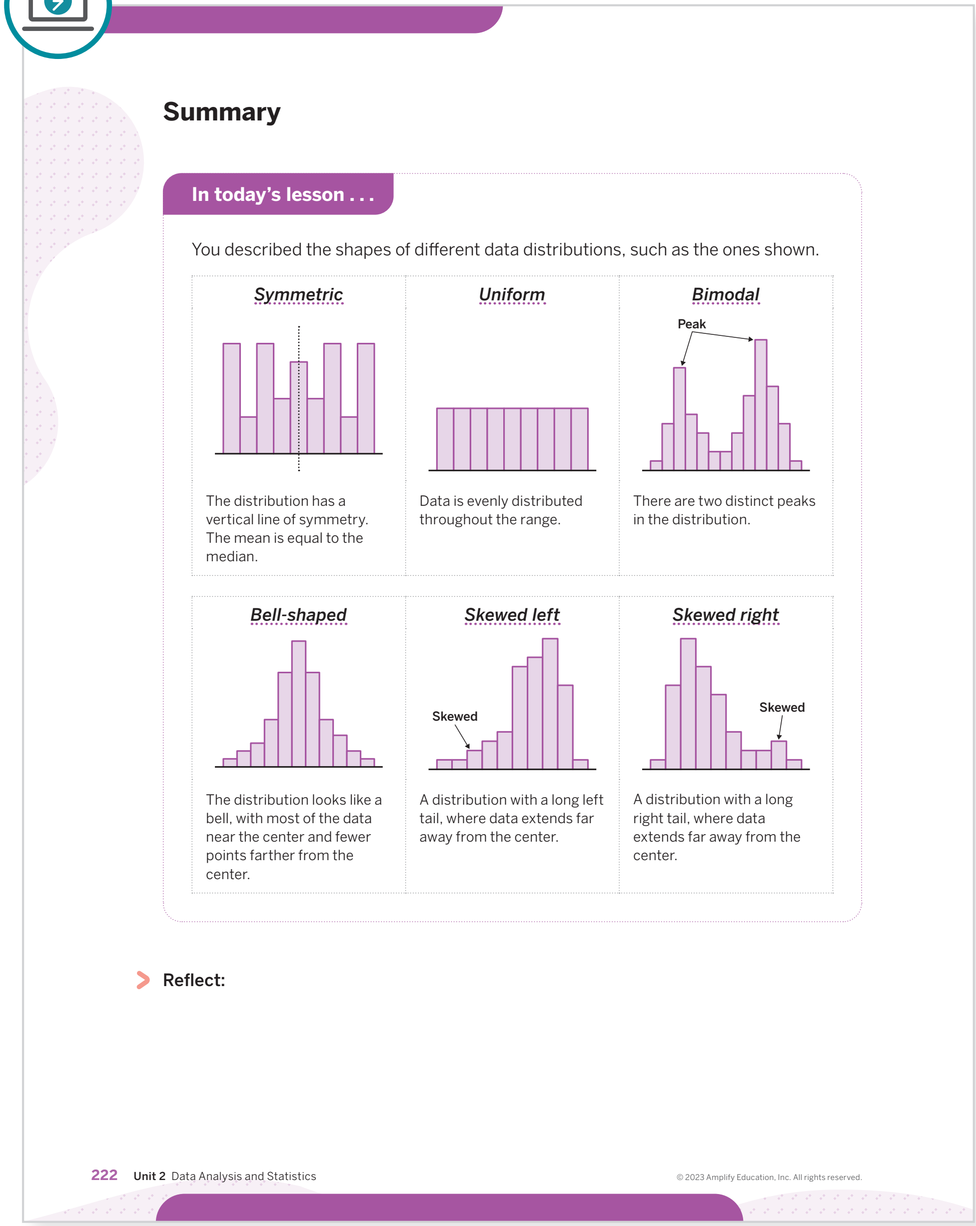
English Learners

Have students annotate the horizontal labels on the histograms to highlight key words, such as August temperature, *monthly* snow coverage, *monthly* temperature, *annual* temperature, and *summer monthly* snow coverage. Be ready to explain what the terms monthly and annual mean.

Whole Class | 5 min

Summary

Review and synthesize terminology and strategies for describing distributions of data.



Summary

In today's lesson . . .

You described the shapes of different data distributions, such as the ones shown.

Symmetric	*Uniform*	*Bimodal*
The distribution has a vertical line of symmetry. The mean is equal to the median.	Data is evenly distributed throughout the range.	There are two distinct peaks in the distribution.
Bell-shaped	***Skewed left***	***Skewed right***
The distribution looks like a bell, with most of the data near the center and fewer points farther from the center.	A distribution with a long left tail, where data extends far away from the center.	A distribution with a long right tail, where data extends far away from the center.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display an example of a skewed, bell-shaped, bimodal, and uniform histogram.

Have students share their description of the shape of the distribution for each histogram.

Highlight that the shape of a distribution can provide a rough visual of the data's center, how the data is spread, and the number of data close in value.

Formalize vocabulary:

- *bell-shaped*
- *bimodal*
- *skewed left*
- *skewed right*
- *uniform*

Ask:

- "Can a skewed distribution also be symmetric? Why or why not?" No, because skewed means that one side of the peak has more data values further away from it than the other side.
- "Which terms from today can be used with 'symmetric' to describe the same histogram?" The term "symmetric" can be used with every term besides skewed.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies or tools did you find helpful today when matching dot plots, box plots, and histograms? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"
- "What information can the shape of a distribution provide?"

Math Language Development

MLR2: Collect and Display

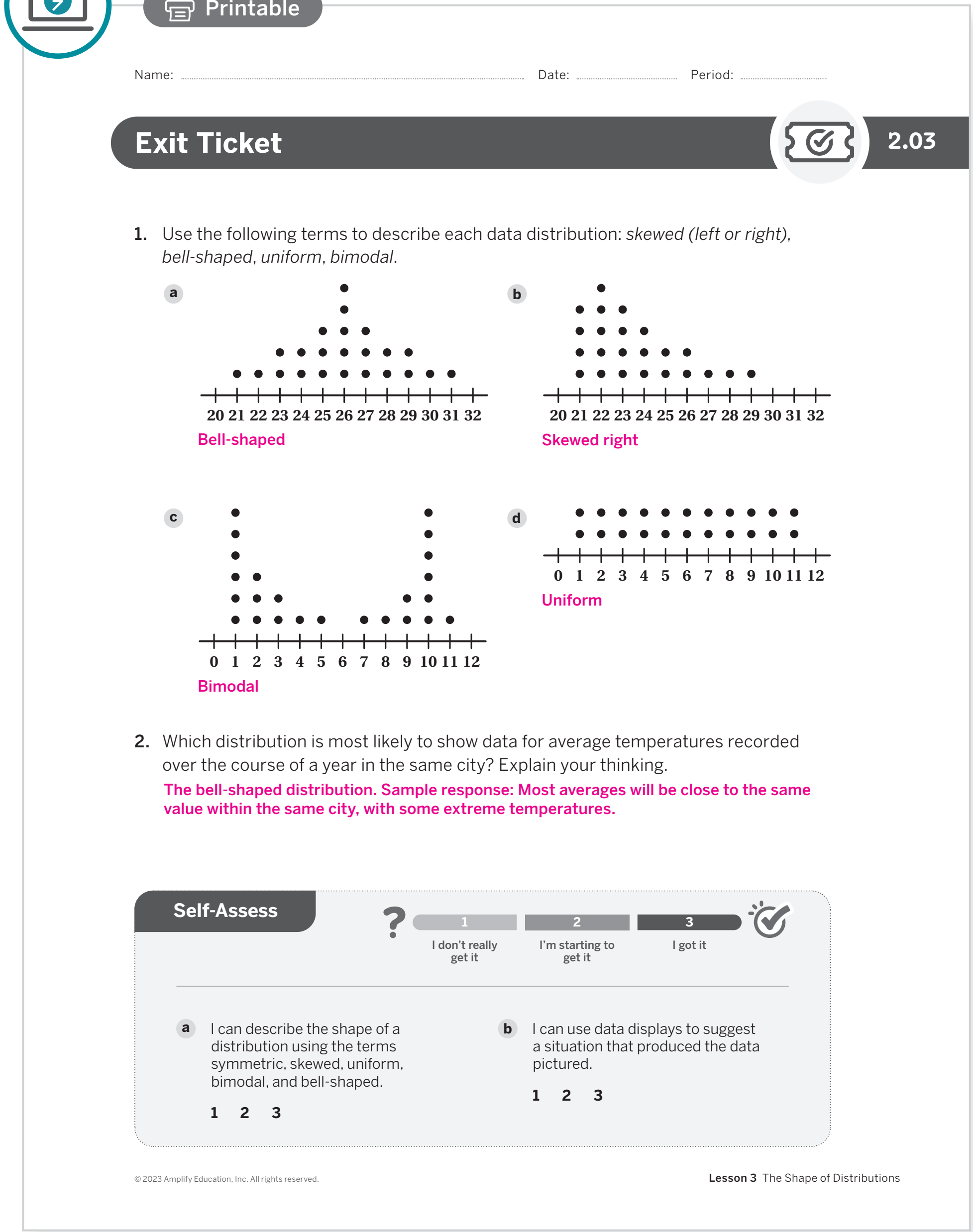
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *bell-shaped*, *bimodal*, *skewed left*, *skewed right*, and *uniform* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by describing the shapes of distributions and connecting them to a context.

Printable



Name: __________ Date: __________ Period: __________

Exit Ticket 2.03

1. Use the following terms to describe each data distribution: *skewed (left or right)*, *bell-shaped*, *uniform*, *bimodal*.

a. Bell-shaped

b. Skewed right

c. Bimodal

d. Uniform

2. Which distribution is most likely to show data for average temperatures recorded over the course of a year in the same city? Explain your thinking.
The bell-shaped distribution. Sample response: Most averages will be close to the same value within the same city, with some extreme temperatures.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can describe the shape of a distribution using the terms symmetric, skewed, uniform, bimodal, and bell-shaped.
1 2 3

b. I can use data displays to suggest a situation that produced the data pictured.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 3 The Shape of Distributions

Success looks like . . .

- **Language Goal:** Describing the shape of a distribution using words, such as *symmetric*, *skewed*, *uniform*, *bimodal*, and *bell-shaped*. **(Speaking and Listening, Writing)**
  - » Describing data distributions using these terms in Problem 1.
- **Language Goal:** Interpreting a distribution to suggest a possible context for the data. **(Speaking and Listening, Writing)**
  - » Selecting the distribution that could represent the data of average temperatures over a year in one city in Problem 2.

Suggested next steps

If students inaccurately described the distributions in Problem 1, consider:

- Reviewing the definitions of each term from the Warm-up.
- Assigning Practice Problems 1–3.
- Asking, "How would you describe the distribution in your own words?"

If students inaccurately choose a dot plot or provide a vague explanation of their choice in Problem 2, consider:

- Reviewing strategies for choosing cards for the article in Activity 2.
- Assigning Practice Problems 1–3.
- Asking, "How do temperatures change over the course of the year and in different seasons?"

Professional Learning

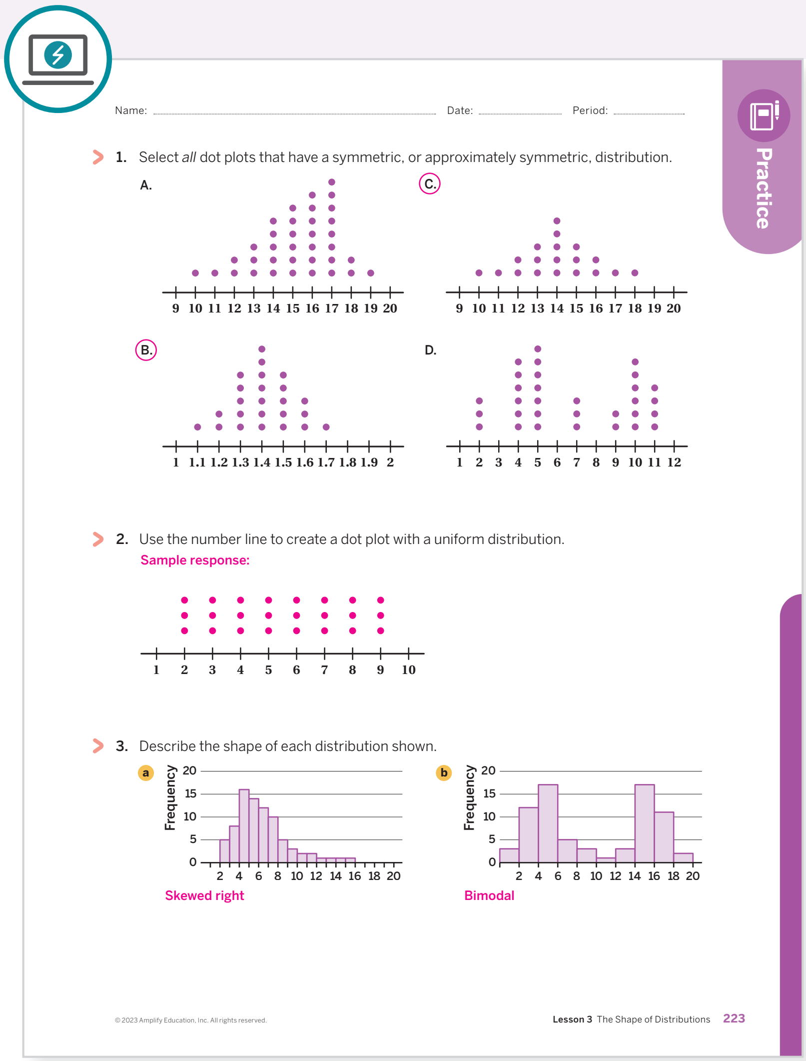
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What different ways did students approach matching data representations to titles? What does that tell you about similarities and differences among your students?
- Which groups of students did and did not have their ideas seen and heard today? What might you change for the next time you teach this lesson?
- In the Warm-up activity, you used intentional grouping with MLR1. What effect did this grouping strategy have on students' final responses? Would you change anything the next time you use this routine?

Practice

Independent



Practice

Name: Date: Period:

1. Select *all* dot plots that have a symmetric, or approximately symmetric, distribution.

2. Use the number line to create a dot plot with a uniform distribution.

Sample response:

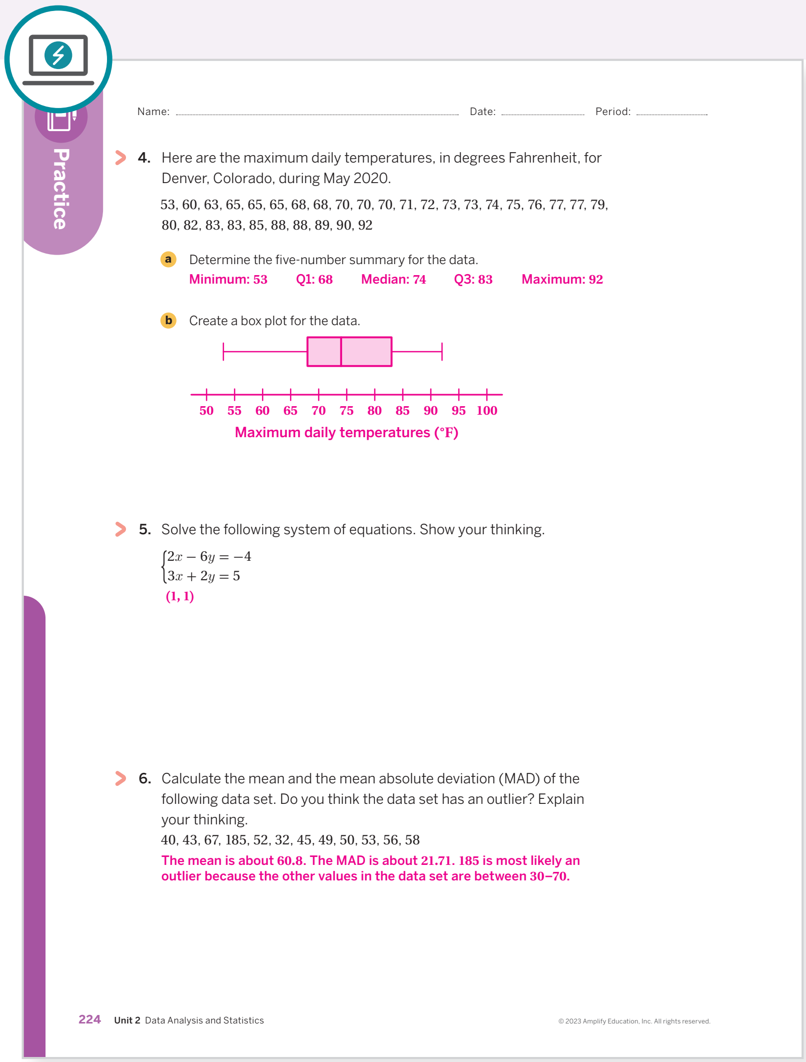
3. Describe the shape of each distribution shown.

a. **Skewed right**

b. **Bimodal**

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 3 The Shape of Distributions 223



Practice

Name: Date: Period:

4. Here are the maximum daily temperatures, in degrees Fahrenheit, for Denver, Colorado, during May 2020.

53, 60, 63, 65, 65, 65, 68, 68, 70, 70, 70, 71, 72, 73, 73, 74, 75, 76, 77, 77, 79, 80, 82, 83, 83, 85, 88, 88, 89, 90, 92

a. Determine the five-number summary for the data.

Minimum: 53 Q1: 68 Median: 74 Q3: 83 Maximum: 92

b. Create a box plot for the data.

5. Solve the following system of equations. Show your thinking.

$$\begin{cases} 2x - 6y = -4 \\ 3x + 2y = 5 \end{cases}$$

(1, 1)

6. Calculate the mean and the mean absolute deviation (MAD) of the following data set. Do you think the data set has an outlier? Explain your thinking.

40, 43, 67, 185, 52, 32, 45, 49, 50, 53, 56, 58

The mean is about 60.8. The MAD is about 21.71. 185 is most likely an outlier because the other values in the data set are between 30–70.

224 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 2 Lesson 2	2
	5	Unit 1 Lesson 21	2
Formative ⚡	6	Unit 2 Lesson 4	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 2 | **LESSON 4**

Deviation From the Center

Let's calculate measures of center and variability, and use them to describe distributions of data.

Focus

Goals

1. Calculate the MAD, IQR, mean, and median.
2. **Language Goal:** Describe how the MAD and IQR are measures of variability. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students revisit calculating the mean, MAD, median, and IQR, and use these values as measures of center and variability and to describe the distribution of global land-ocean temperatures. They also create data sets that reflect given measures of center and variability.

Previously

In Lesson 3, students described and interpreted the shape of the distribution of a data set.

Coming Soon

In Lessons 5 and 6, students will describe how an outlier affects different measures of center and variability.

Rigor

- Students build onto their Grade 6 **conceptual understanding** of the mean absolute deviation (MAD) of a data set and interquartile range (IQR).
- Students build **procedural fluency** calculating mean, median, mean absolute value deviation, and interquartile range.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1 (optional)	Activity 2	Activity 3	Summary	Exit Ticket
10 min	15 min	15 min	15 min	5 min	5 min
Independent	Small Groups	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 1 PDF, Determining the Median, Q1, and Q3, one per student (as needed)
- Anchor Chart PDF, *Sentence Stems, Critiquing* (for display)
- Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning* (as needed)
- pennies
- rulers
- yardsticks

Math Language Development

Review words

- *center*
- *five-number summary*
- *interquartile range*
- *mean*
- *mean absolute deviation*
- *median*
- *outlier*
- *quartile*
- *skewed distribution*
- *statistic*
- *variability*

Amps | Featured Activity

Activity 1
Interactive Graphs

Students place coins along a meter stick to investigate and create data sets that have given measures of center and variability.

Building Math Identity and Community

Connecting to Mathematical Practices

Throughout Activity 2, students may become disorganized, losing focus on the purpose of the activity. Have students make a graphic organizer for the process of using the five-number summary and IQR to describe spread. Encourage them to reference it and make notes about how the quantitative measures help them reason about the data.

Modifications to Pacing

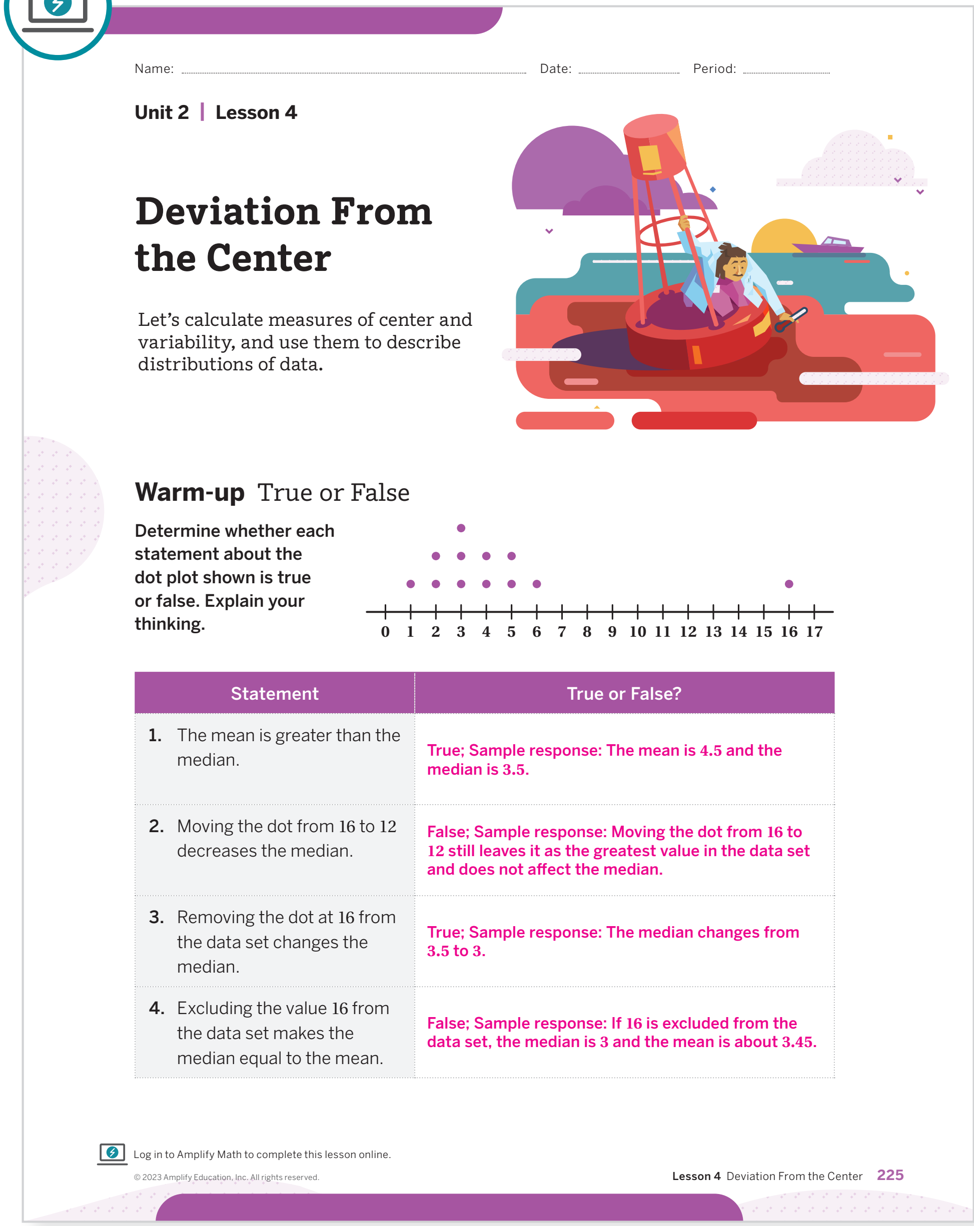
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 4 may be omitted.
- **Activity 1** is optional and may be omitted.
- In **Activity 3**, Problems 4 and 5 may be omitted.

Independent | 10 min

Warm-up True or False

Students determine the validity of statements about a data set to understand how statistics may change as specific values in the set change.



Name: ______________ Date: ________ Period: ________

Unit 2 | Lesson 4

Deviation From the Center

Let's calculate measures of center and variability, and use them to describe distributions of data.

Warm-up True or False

Determine whether each statement about the dot plot shown is true or false. Explain your thinking.

Statement	True or False?
1. The mean is greater than the median.	True; Sample response: The mean is 4.5 and the median is 3.5.
2. Moving the dot from 16 to 12 decreases the median.	False; Sample response: Moving the dot from 16 to 12 still leaves it as the greatest value in the data set and does not affect the median.
3. Removing the dot at 16 from the data set changes the median.	True; Sample response: The median changes from 3.5 to 3.
4. Excluding the value 16 from the data set makes the median equal to the mean.	False; Sample response: If 16 is excluded from the data set, the median is 3 and the mean is about 3.45.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Deviation From the Center 225

1 Launch

Display one problem at a time. Give students 1 minute of quiet think time followed by a whole-class discussion after each problem.

2 Monitor

Help students get started by reviewing the statistical terms in each statement.

Look for points of confusion:

- **Having difficulty calculating the median for an even number of data values.** Ask, " How could you determine a middle number between two values?"
- **Not recalculating the median after a change in the data set.** Ask, "Does this change affect which value will be in the middle of the data set?"

Look for productive strategies:

- Listing the values from least to greatest.
- Updating the dot plot to reflect changes of the data.

3 Connect

Have individual students share their thinking for verifying each statement including processes for determining the mean, median, and how they change by adding or removing data values.

Ask, "Does changing 16 to 12 affect the mean? Why or why not?" This change decreases the mean. 16 is greater than the original mean of 4.5, so decreasing this value decreases the mean.

Highlight that outliers can affect measures of centers. The median and mean describe typical values in a data set. Changing a data value may (or may not) change the median, but will always affect the mean because all values are used in its calculation.

Math Language Development

MLR8: Discussion Supports

During the Connect, display the Anchor Chart PDF, *Sentence Stems, Critiquing* to support students as they share their thinking for verifying whether each statement is true or false. After each student shares, ask other students if they agree, disagree, or if they would like to ask clarifying questions.

English Learners

As students share their thinking for each statement, annotate the dot plot with the words that are used to describe it. For example, annotate where the mean and median is on the dot plot for Statement 1.

Power-up

To power up students' ability to calculate the mean and the MAD:

Provide students with a copy of the Power-up PDF.

Use: Before Activity 1

Informed by: Performance on Lesson 3, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3

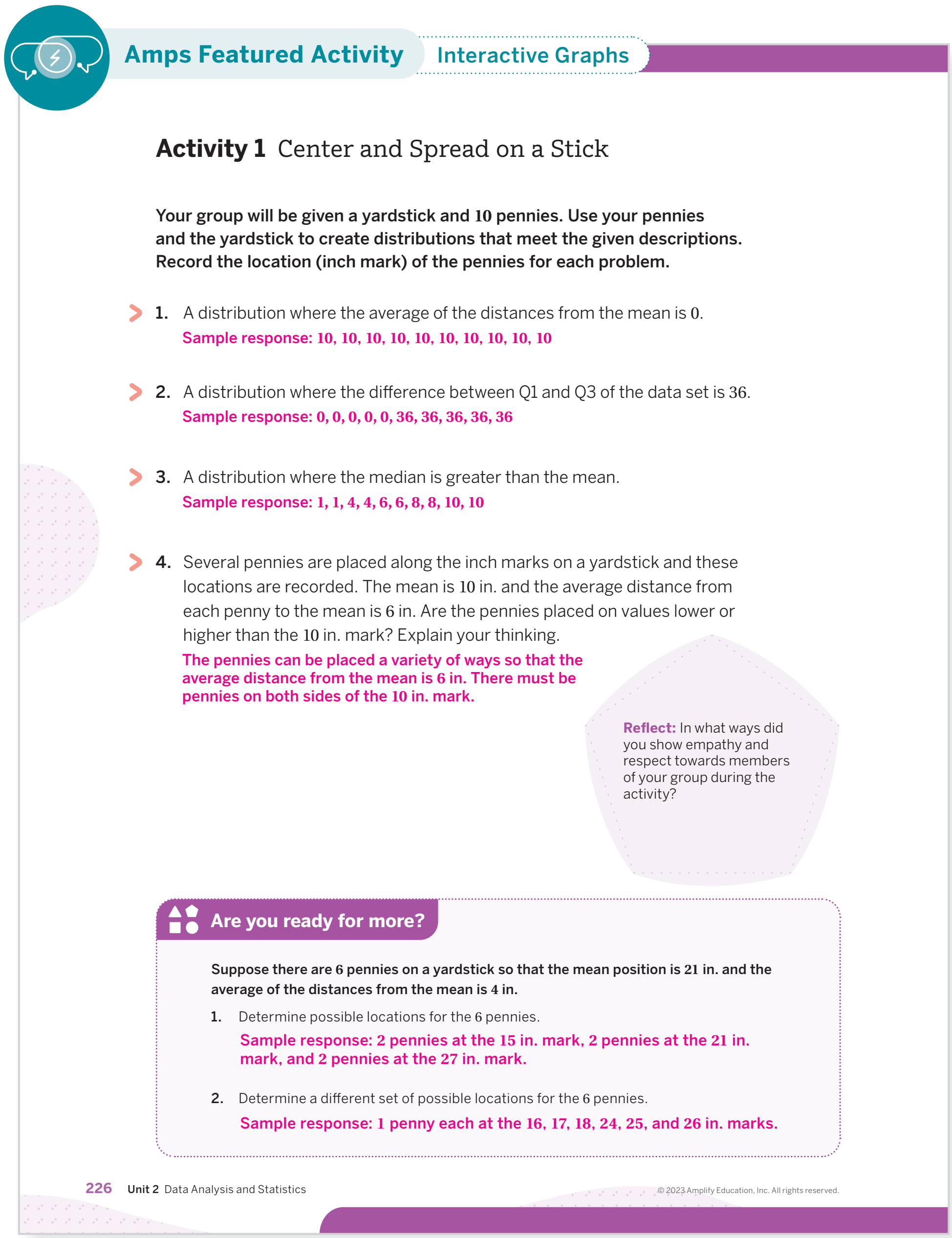
Optional

Small Groups | 15 min

Activity 1 Center and Spread on a Stick

Students create tactile data sets using pennies and a yardstick to conceptualize measures of center and variability.

Amps Featured Activity Interactive Graphs



Activity 1 Center and Spread on a Stick

Your group will be given a yardstick and 10 pennies. Use your pennies and the yardstick to create distributions that meet the given descriptions. Record the location (inch mark) of the pennies for each problem.

1. A distribution where the average of the distances from the mean is 0.
   Sample response: 10, 10, 10, 10, 10, 10, 10, 10, 10, 10
2. A distribution where the difference between Q1 and Q3 of the data set is 36.
   Sample response: 0, 0, 0, 0, 0, 36, 36, 36, 36, 36
3. A distribution where the median is greater than the mean.
   Sample response: 1, 1, 4, 4, 6, 6, 8, 8, 10, 10
4. Several pennies are placed along the inch marks on a yardstick and these locations are recorded. The mean is 10 in. and the average distance from each penny to the mean is 6 in. Are the pennies placed on values lower or higher than the 10 in. mark? Explain your thinking.
   The pennies can be placed a variety of ways so that the average distance from the mean is 6 in. There must be pennies on both sides of the 10 in. mark.

Reflect: In what ways did you show empathy and respect towards members of your group during the activity?

Are you ready for more?

Suppose there are 6 pennies on a yardstick so that the mean position is 21 in. and the average of the distances from the mean is 4 in.

1. Determine possible locations for the 6 pennies.
   Sample response: 2 pennies at the 15 in. mark, 2 pennies at the 21 in. mark, and 2 pennies at the 27 in. mark.
2. Determine a different set of possible locations for the 6 pennies.
   Sample response: 1 penny each at the 16, 17, 18, 24, 25, and 26 in. marks.

226 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Conduct a physical demonstration using pennies on a yardstick.

2 Monitor

Help students get started by having them calculate the mean and five-number summary of the data set.

Look for points of confusion:

- **Placing all pennies at separate locations.** Show students how multiple pennies can be at the same location, similar to a dot plot.

Look for productive strategies:

- Determining the location of the pennies one at a time while contemplating how each penny's location will affect the measure of variability.

3 Connect

Display a yardstick set up with 10 pennies, ready to move around as students share their responses.

Have groups of students share their strategies for Problems 1–3.

Highlight that the distance from the mean is always positive because distance cannot be negative. Therefore, two data sets can look very different, but still have the same average of the distances from the mean.

Ask, "If the yardstick is to be balanced at its center, 18 in., how could you use the distances from the mean to help determine approximate locations for the pennies?" The sum of the distances of the pennies on one side of 18 needs to equal the sum of the distances on the other side.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate how to place the 10 pennies on the yardstick, where each penny corresponds to a certain location, in inches, on the yardstick. Encourage students to use whole-number inch marks. Demonstrate how to place pennies on the yardstick to create a distribution that is symmetric, bimodal, or one in which the mean is 5.

Accessibility: *Clarify Vocabulary and Symbols*

Display review vocabulary terms and their definitions, such as *mean*, *"average distance from the mean"* (*mean absolute deviation*), *Q1 (first quartile)*, *Q3 (third quartile)*, and *median*. Consider demonstrating what the "average distance from the mean" means. Students learned the mean absolute deviation in middle school, but may benefit from a reminder.

Math Language Development

MLR8: Discussion Supports

During the Launch, after you demonstrate how to place the 10 pennies on the yardstick to create a distribution, ask these questions:

- "What do you think this demonstration has in common with measures of center and variability?"
- "How do you think I can balance this yardstick? What does it mean for the yardstick to be balanced?"
- "How is the balance point related to the mean or median?"

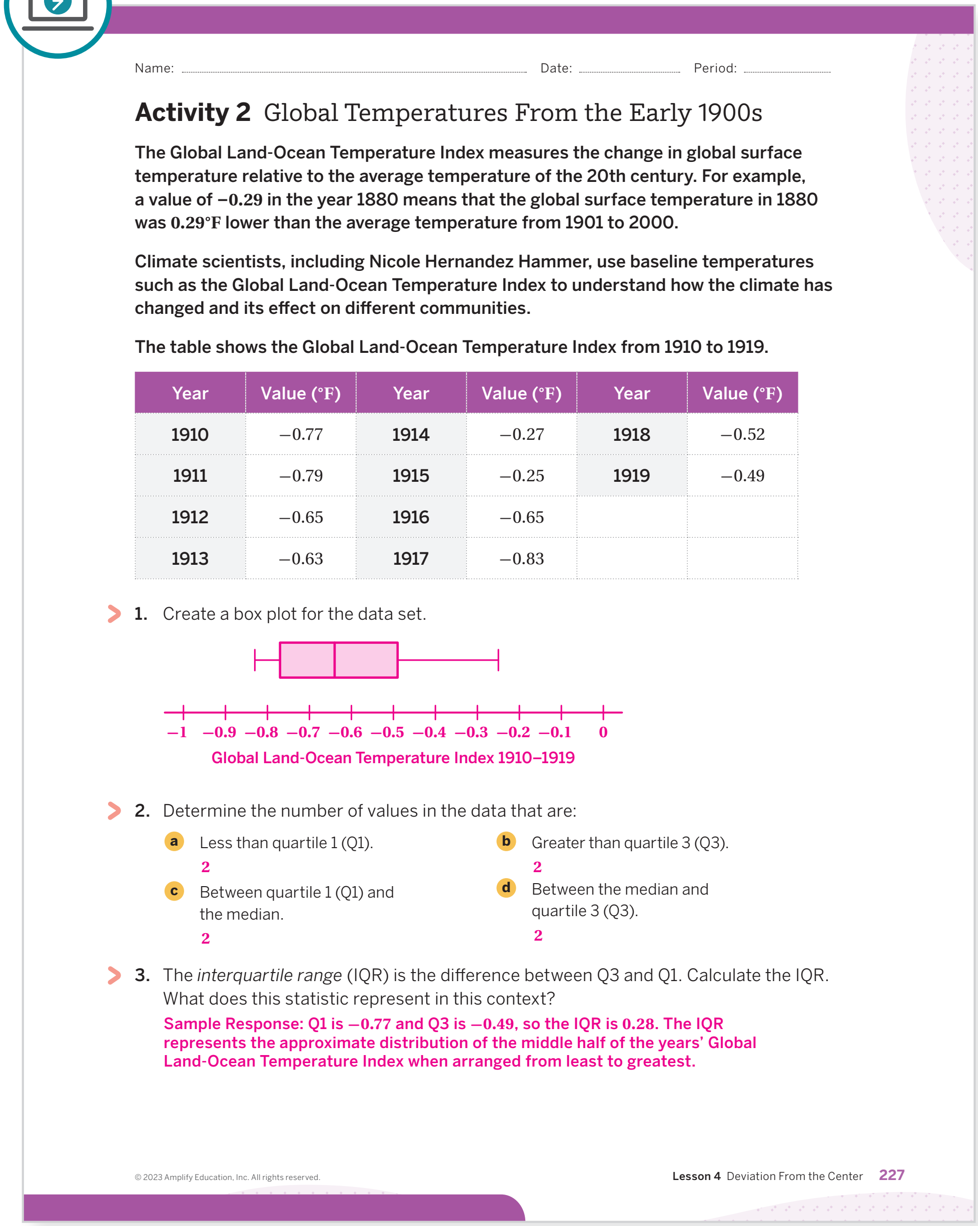
English Learners

Use hand gestures, such as pointing, to illustrate where the mean, median, Q1, and Q3 are located on the distribution.

Pairs | 15 min

Activity 2 Global Temperatures From the Early 1900s

Students calculate a five-number summary and IQR to describe the spread and interpret a data set.



Name: ______ Date: ______ Period: ______

Activity 2 Global Temperatures From the Early 1900s

The Global Land-Ocean Temperature Index measures the change in global surface temperature relative to the average temperature of the 20th century. For example, a value of −0.29 in the year 1880 means that the global surface temperature in 1880 was 0.29°F lower than the average temperature from 1901 to 2000.

Climate scientists, including Nicole Hernandez Hammer, use baseline temperatures such as the Global Land-Ocean Temperature Index to understand how the climate has changed and its effect on different communities.

The table shows the Global Land-Ocean Temperature Index from 1910 to 1919.

Year	Value (°F)	Year	Value (°F)	Year	Value (°F)
1910	−0.77	1914	−0.27	1918	−0.52
1911	−0.79	1915	−0.25	1919	−0.49
1912	−0.65	1916	−0.65		
1913	−0.63	1917	−0.83		

1. Create a box plot for the data set.

2. Determine the number of values in the data that are:

   a. Less than quartile 1 (Q1).
   2

   b. Greater than quartile 3 (Q3).
   2

   c. Between quartile 1 (Q1) and the median.
   2

   d. Between the median and quartile 3 (Q3).
   2

3. The *interquartile range* (IQR) is the difference between Q3 and Q1. Calculate the IQR. What does this statistic represent in this context?
   Sample Response: Q1 is −0.77 and Q3 is −0.49, so the IQR is 0.28. The IQR represents the approximate distribution of the middle half of the years' Global Land-Ocean Temperature Index when arranged from least to greatest.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read and discuss the prompt as a whole class. Have students complete Problem 1 independently, then pause to review their box plots as a whole class before resuming to work in pairs. Provide access to rulers to aid students in constructing accurate displays.

2 Monitor

Help students get started by reviewing parts of a five-number summary and box plot.

Look for points of confusion:

- **Struggling to use the median and IQR to describe the center and variability of the temperature data.** Have students identify the values of Q1, Q3, and the median from the box plot, as well as their responses to Problem 2. Ask, "What do these values all together tell you about typical land-ocean temperatures in this period, compared to the average temperature from 1901 to 2000?"

Look for productive strategies:

- Annotating the box plot with the values from the five-number summary, the IQR, the difference of Q1 and the median, and the difference of the median and Q3.
- Using the values from the five-number summary to describe how temperatures in this time period were lower than average from 1901 to 2000.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display or provide copies of the Activity 2 PDF, *Determining the Median, Q1, and Q3* to help students organize their thinking as they determine the median, first quartile, and third quartile of the data set. Consider providing a number line to students either pre-labeled or not pre-labeled for them to use as they create their box plots.

Math Language Development

MLR1: Stronger and Clearer Each Time

Have students share their responses to Problems 3–5 with another pair of students to give and receive feedback. Display these questions:

- "Does the response describe what each measure means within the context of the Global Land-Ocean Temperature Index?"
- "Is the response clear? What suggestions do you have for improvement?"

Have students write a final response, based on the feedback they received.

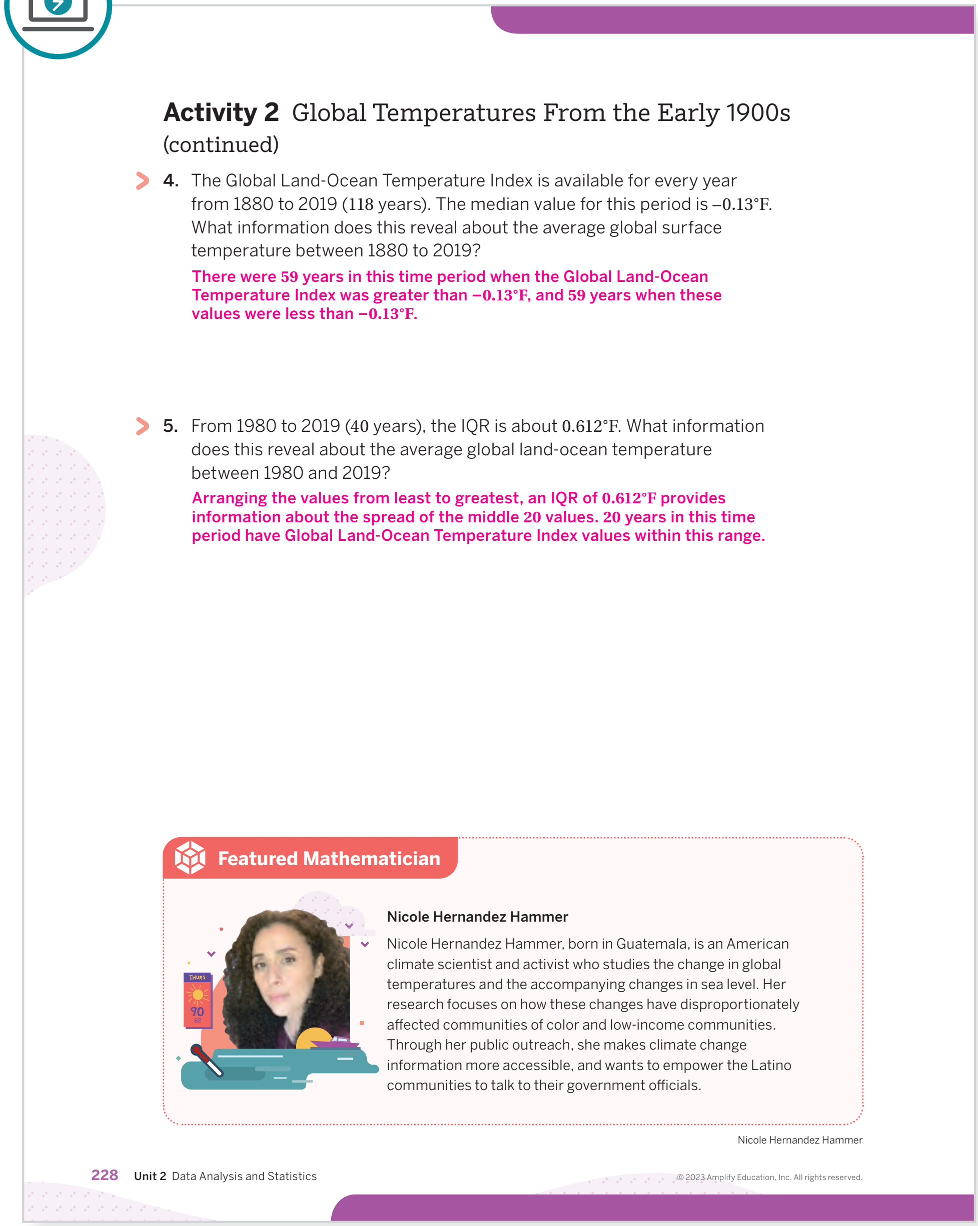
English Learners

Display the Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning* from which students can choose questions to ask as they give and receive feedback.

Pairs | 15 min

Activity 2 Global Temperatures From the Early 1900s (continued)

Students calculate a five-number summary and IQR to describe the spread and interpret a data set.



Activity 2 Global Temperatures From the Early 1900s (continued)

4. The Global Land-Ocean Temperature Index is available for every year from 1880 to 2019 (118 years). The median value for this period is –0.13°F. What information does this reveal about the average global surface temperature between 1880 to 2019?

   There were 59 years in this time period when the Global Land-Ocean Temperature Index was greater than –0.13°F, and 59 years when these values were less than –0.13°F.

5. From 1980 to 2019 (40 years), the IQR is about 0.612°F. What information does this reveal about the average global land-ocean temperature between 1980 and 2019?

   Arranging the values from least to greatest, an IQR of 0.612°F provides information about the spread of the middle 20 values. 20 years in this time period have Global Land-Ocean Temperature Index values within this range.

Featured Mathematician

Nicole Hernandez Hammer

Nicole Hernandez Hammer, born in Guatemala, is an American climate scientist and activist who studies the change in global temperatures and the accompanying changes in sea level. Her research focuses on how these changes have disproportionately affected communities of color and low-income communities. Through her public outreach, she makes climate change information more accessible, and wants to empower the Latino communities to talk to their government officials.

Nicole Hernandez Hammer

228 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the box plot in Problem 1.

Ask:

- "What does the IQR reveal about a data set?" The IQR is a measure of variability that describes the range of the middle half of the data.
- "What are the advantages and disadvantages of using the IQR to describe the spread of a data set?" Advantages: The IQR provides information about the middle half of the data, so I can get a sense of the spread of the middle values. Disadvantages: The IQR does not take into account all the values in a data set, and only reflects the middle half of values.

Have pairs of students share their responses and thinking for Problems 3–5.

Highlight that the IQR describes the spread of the data set. Using this in combination with the Q1, Q3, and the median, they can get a sense of how the data varies and typical values in the data set.

Featured Mathematician

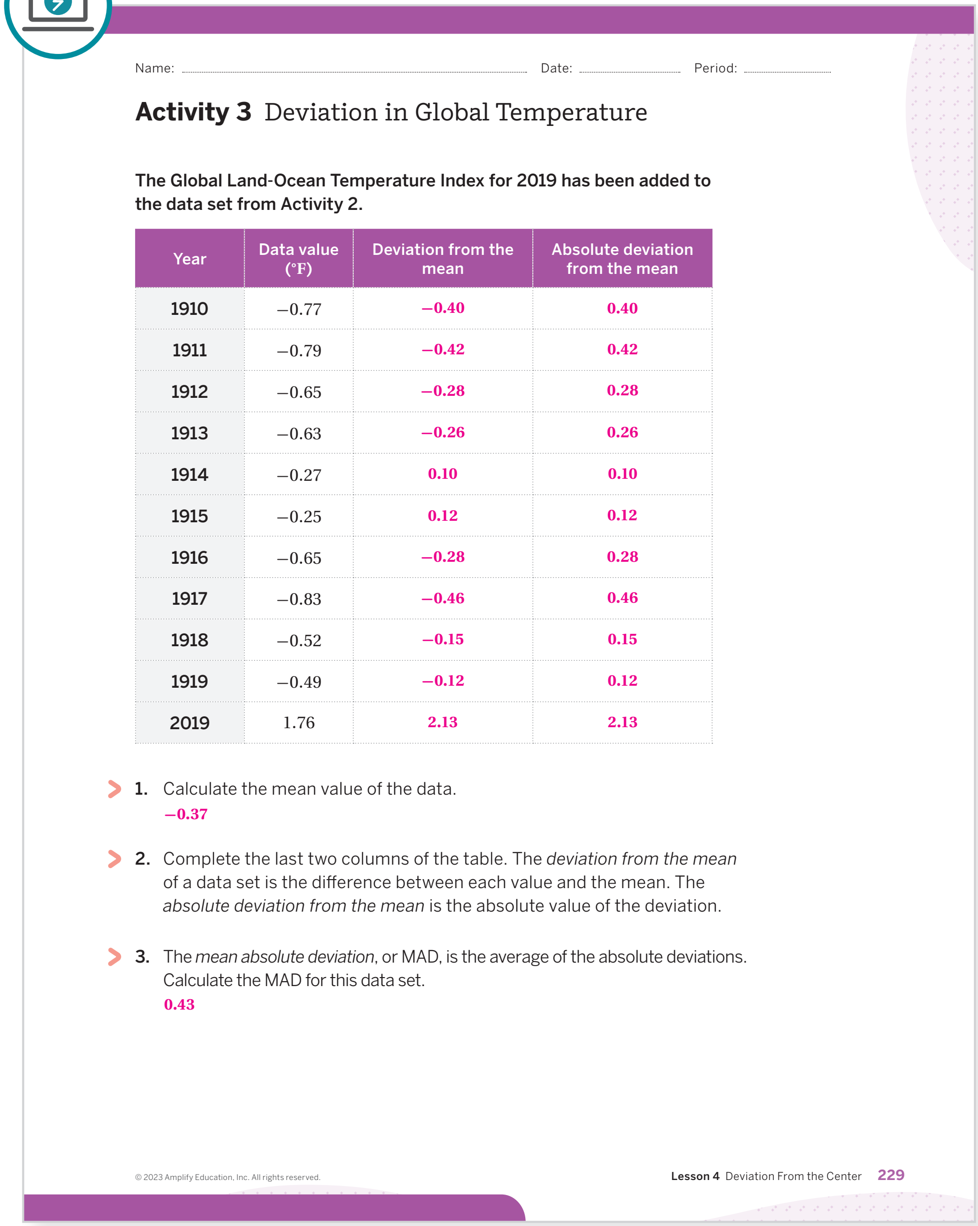
Nicole Hernandez Hammer

Have students read about Nicole Hernandez Hammer, a Guatemalan American climate scientist and activist, who studies how changes in climate disproportionately affect communities of color and low-income communities.

Pairs | 15 min

Activity 3 Deviation in Global Temperature

Students calculate the MAD for a data set with an outlier to see how these values affect this measure of spread.



Name: ______ Date: ______ Period: ______

Activity 3 Deviation in Global Temperature

The Global Land-Ocean Temperature Index for 2019 has been added to the data set from Activity 2.

Year	Data value (°F)	Deviation from the mean	Absolute deviation from the mean
1910	−0.77	**−0.40**	**0.40**
1911	−0.79	**−0.42**	**0.42**
1912	−0.65	**−0.28**	**0.28**
1913	−0.63	**−0.26**	**0.26**
1914	−0.27	**0.10**	**0.10**
1915	−0.25	**0.12**	**0.12**
1916	−0.65	**−0.28**	**0.28**
1917	−0.83	**−0.46**	**0.46**
1918	−0.52	**−0.15**	**0.15**
1919	−0.49	**−0.12**	**0.12**
2019	1.76	**2.13**	**2.13**

1. Calculate the mean value of the data.
   −0.37
2. Complete the last two columns of the table. The *deviation from the mean* of a data set is the difference between each value and the mean. The *absolute deviation from the mean* is the absolute value of the deviation.
3. The *mean absolute deviation*, or MAD, is the average of the absolute deviations. Calculate the MAD for this data set.
   0.43

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Explain that the data set is the same as the one in Activity 2 with an additional value added for 2019. Ask, "How might the 2019 value affect the data?"

2 Monitor

Help students get started by asking what they remember about "absolute value."

Look for points of confusion:

- **Misidentifying the effect of an outlier on the mean and MAD.** Ask, "Will the mean increase or decrease if a very large value is added to the data set? Do you think this value will be closer or further from the new mean?"
- **Needing more information for Problem 6.** Provide students with a sample data set: 1, 1, 2, 3, 3. Have them calculate the mean and MAD, and describe what they notice about the data set and these values.

Look for productive strategies:

- Estimating the median and the MAD of the data set in Activity 1.
- Creating a data set that reflects the variability described in Problem 6.

Activity 3 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Have students complete Problem 1 and then complete the first three rows of the table in Problem 2. Then provide them with the remaining values in the table. This will allow them to focus more on the activity's goals, rather than on all of the calculations. Have them proceed with Problem 3.

Extension: *Math Enrichment*

Have students complete the following problem:
Without seeing the data values, determine the MAD of a data set in which the following is true. The MAD is 2.

- One fourth of the values are located 1 unit above the mean.
- Half of the values are located 2 units below the mean.
- One fourth of the values are located 3 units above the mean.

Math Language Development

MLR1: Stronger and Clearer Each Time

Have students share their responses to Problems 6 and 7 with another pair of students to give and receive feedback. Display these questions for reviewers to use as they provide feedback:

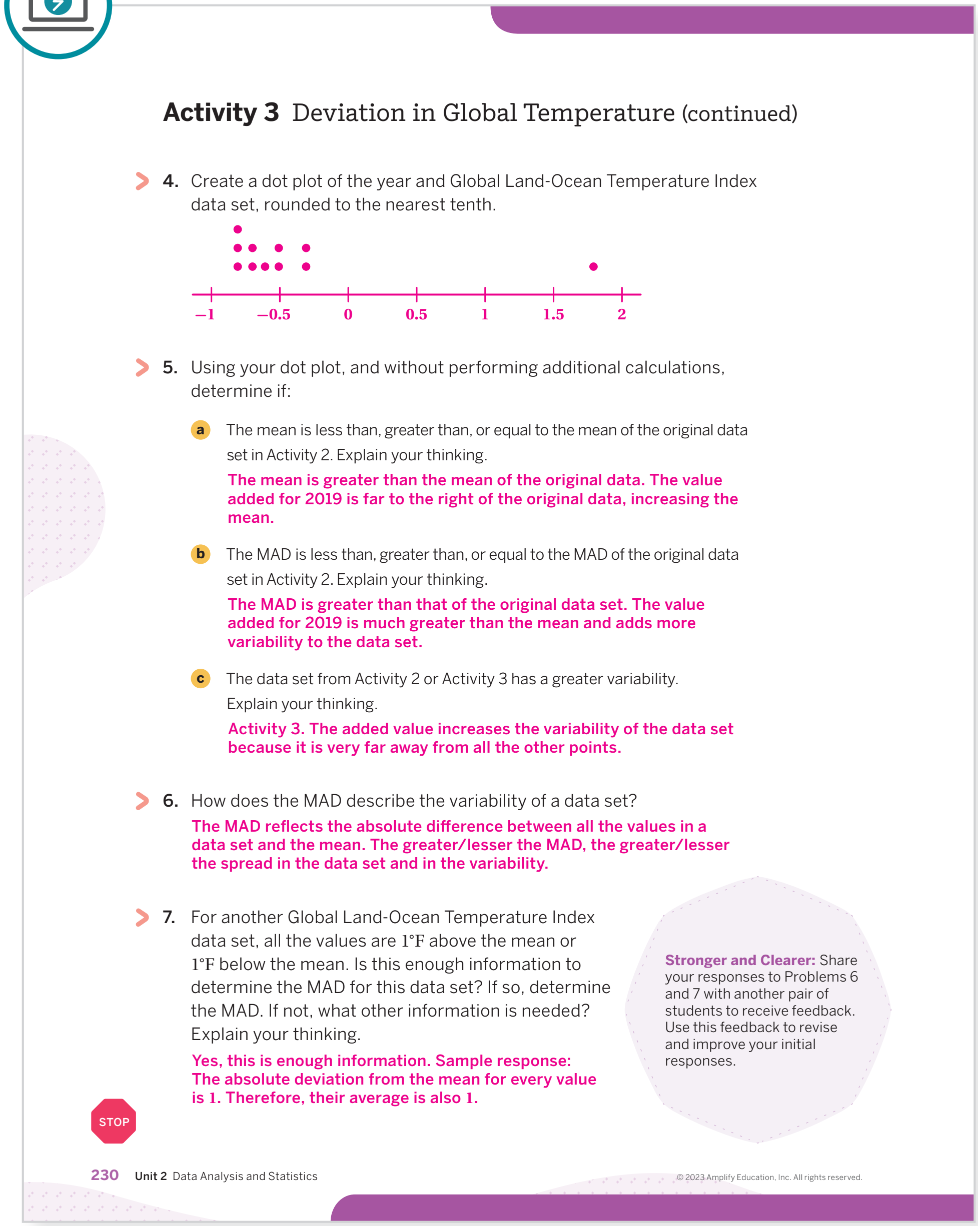
- "Does the response include more than just saying 'the MAD is a measure of variation'?"
- "Does the response include how a greater or lesser MAD describes the spread or variability? If you have a lesser MAD, what does that mean? If you have a greater MAD?"

Have students write a final response, based on the feedback they received.

Pairs | 15 min

Activity 3 Deviation in Global Temperature (continued)

Students calculate the MAD for a data set with an outlier to see how these values affect this measure of spread.



Activity 3 Deviation in Global Temperature (continued)

4. Create a dot plot of the year and Global Land-Ocean Temperature Index data set, rounded to the nearest tenth.

5. Using your dot plot, and without performing additional calculations, determine if:

a The mean is less than, greater than, or equal to the mean of the original data set in Activity 2. Explain your thinking.

The mean is greater than the mean of the original data. The value added for 2019 is far to the right of the original data, increasing the mean.

b The MAD is less than, greater than, or equal to the MAD of the original data set in Activity 2. Explain your thinking.

The MAD is greater than that of the original data set. The value added for 2019 is much greater than the mean and adds more variability to the data set.

c The data set from Activity 2 or Activity 3 has a greater variability. Explain your thinking.

Activity 3. The added value increases the variability of the data set because it is very far away from all the other points.

6. How does the MAD describe the variability of a data set?

The MAD reflects the absolute difference between all the values in a data set and the mean. The greater/lesser the MAD, the greater/lesser the spread in the data set and in the variability.

7. For another Global Land-Ocean Temperature Index data set, all the values are 1°F above the mean or 1°F below the mean. Is this enough information to determine the MAD for this data set? If so, determine the MAD. If not, what other information is needed? Explain your thinking.

Yes, this is enough information. Sample response: The absolute deviation from the mean for every value is 1. Therefore, their average is also 1.

Stronger and Clearer: Share your responses to Problems 6 and 7 with another pair of students to receive feedback. Use this feedback to revise and improve your initial responses.

STOP

230 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the dot plot of the data set.

Have pairs of students share their thinking for Problem 6.

Highlight that an extremely large or small value can greatly affect the MAD, particularly in smaller data sets. A value that does so may become misleading when attempting to interpret a data set.

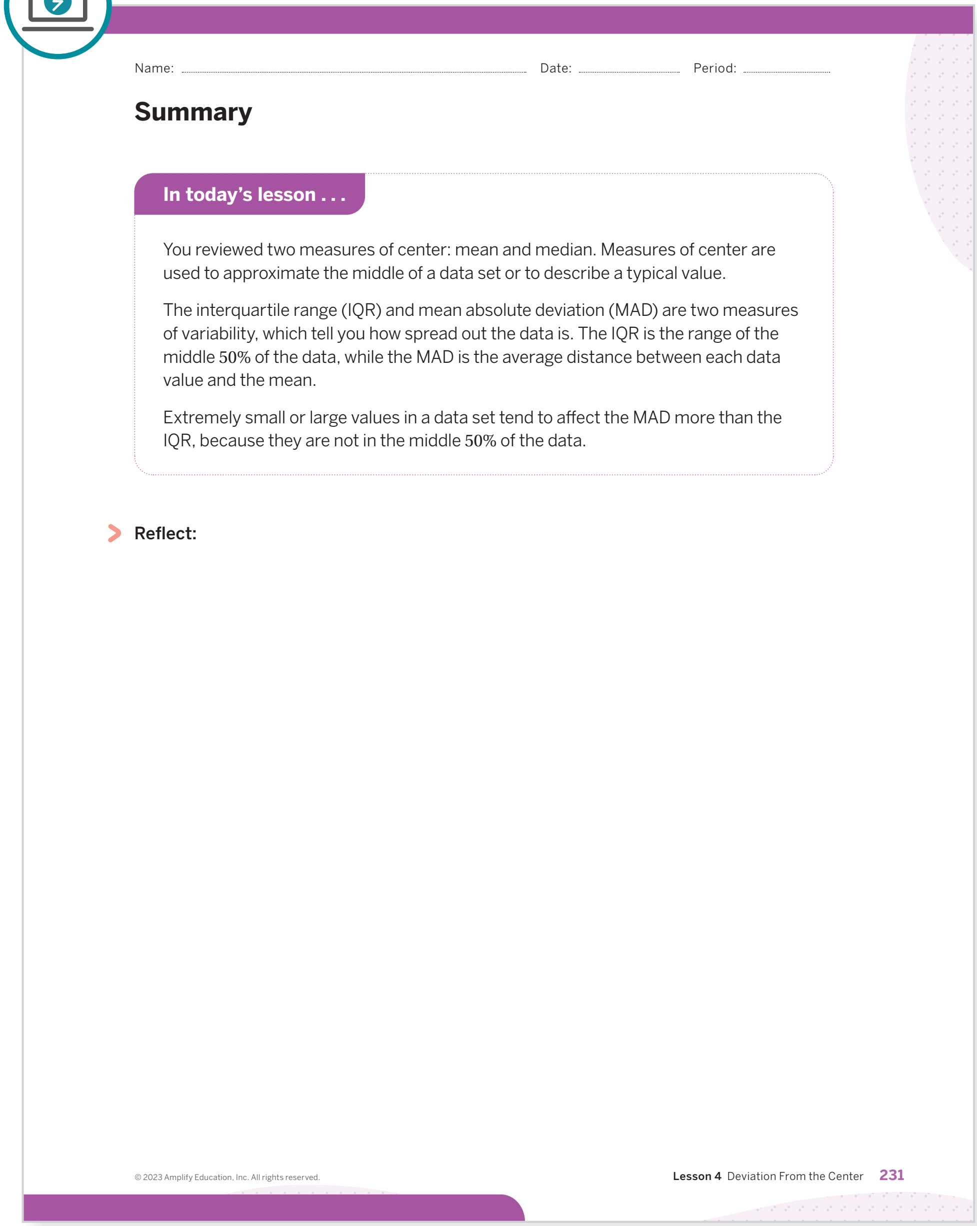
Ask:

- "Why do you use the absolute deviation (rather than just deviation) to help describe variability?" The absolute deviation helps me get a sense of the spread of the data for values greater than or less than the mean. (And the deviation will always average to zero!)
- "What are the advantages and disadvantages of using the MAD to describe the spread of a data set?" Advantages: It is a measure of variability that reflects all the values in the data set. Disadvantages: It can be greatly affected by extremely large or small values in the data set.

Whole Class | 5 min

Summary

Review and synthesize calculating and interpreting the IQR and MAD and understand why they are measures of variability.



Name: Date: Period:

Summary

In today's lesson . . .

You reviewed two measures of center: mean and median. Measures of center are used to approximate the middle of a data set or to describe a typical value.

The interquartile range (IQR) and mean absolute deviation (MAD) are two measures of variability, which tell you how spread out the data is. The IQR is the range of the middle 50% of the data, while the MAD is the average distance between each data value and the mean.

Extremely small or large values in a data set tend to affect the MAD more than the IQR, because they are not in the middle 50% of the data.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display a dot plot with the mean and MAD annotated, and a box plot with the median and IQR annotated.

Have students share why the IQR and the MAD are considered measures of variability.

Highlight that the MAD and IQR will always be positive, because both use the difference of values, with the MAD using the distance from the mean, which is a positive value. Usually an outlier has more effect on the MAD than the IQR, particularly in smaller data sets. An outlier that greatly affects the MAD may become misleading when attempting to interpret a data set.

Ask:

- "How do you calculate the IQR and the MAD?"
  For the IQR, determine the difference between the value of the Q3 and Q1. For the MAD, determine the distance of each value from the mean. Then take the mean of those values.
- "If Data set 1 has a greater IQR but a lower MAD than Data set 2, what does that tell you about Data set 1?"
  The middle half of Data set 1 is more spread out from the center than the middle half of the Data set 2. However, the points in Data set 2 are, on average, further from the mean. The lowest and highest quartiles of Data set 2 may be very far from the mean.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

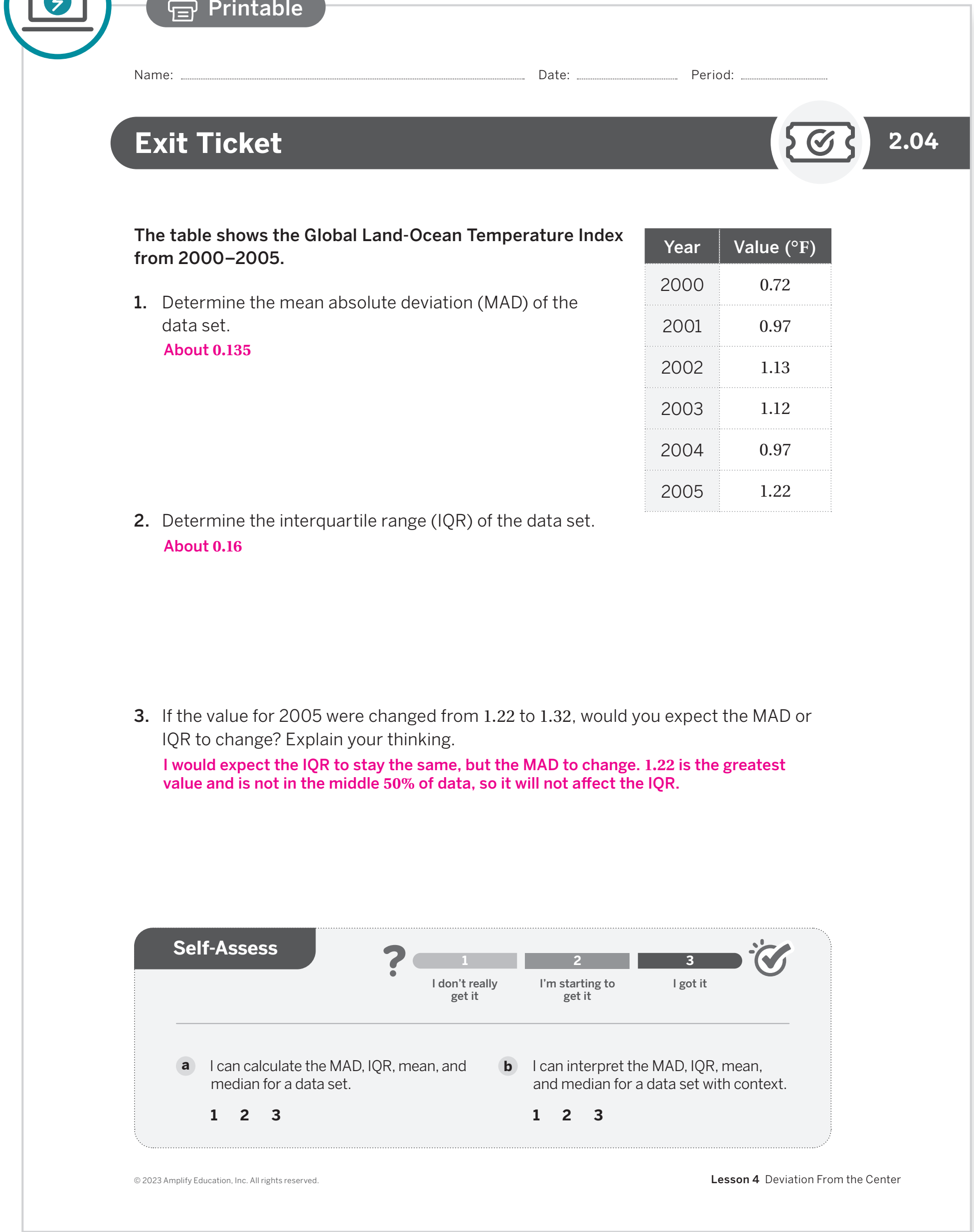
- "Why are the IQR and MAD considered measures of variability?"
- "Will the IQR and MAD both increase or decrease with additional values added to a data set? Why or why not?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by calculating the MAD and IQR of a data set and determining whether a value in the data set is an outlier.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 2.04

The table shows the Global Land-Ocean Temperature Index from 2000–2005.

Year	Value (°F)
2000	0.72
2001	0.97
2002	1.13
2003	1.12
2004	0.97
2005	1.22

1. Determine the mean absolute deviation (MAD) of the data set.
   About 0.135

2. Determine the interquartile range (IQR) of the data set.
   About 0.16

3. If the value for 2005 were changed from 1.22 to 1.32, would you expect the MAD or IQR to change? Explain your thinking.
   I would expect the IQR to stay the same, but the MAD to change. 1.22 is the greatest value and is not in the middle 50% of data, so it will not affect the IQR.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can calculate the MAD, IQR, mean, and median for a data set.
1 2 3

b I can interpret the MAD, IQR, mean, and median for a data set with context.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Deviation From the Center

Success looks like . . .

- **Goal:** Calculating the MAD, IQR, mean, and median.
  - » Calculating the MAD and IQR of the ocean temperature data in Problems 1 and 2.
- **Language Goal:** Describing how the MAD and IQR are measures of variability. **(Speaking and Listening, Reading and Writing)**

Suggested next steps

If students inaccurately calculate the MAD for Problem 1, consider:

- Reviewing the procedure for calculating the MAD of a data set from Activity 3.
- Assigning Practice Problem 2.
- Asking, "How do you calculate the mean of a data set? How do you determine the absolute deviation from the mean for a value?"

If students inaccurately calculate the IQR for Problem 2, consider:

- Reviewing the procedure for calculating the IQR of a data set from Activity 2.
- Assigning Practice Problem 1.
- Asking, "How do you determine the values of quartile 1 and 3? What does it mean to determine the 'range' of two values?"

If students inaccurately choose a measure of variability or provide a vague explanation of their choice in Problem 3, consider:

- Reviewing Activity 3, Problem 4.
- Assigning Practice Problems 1 and 2.
- Asking, "What do the MAD and IQR reveal about a data set? Will either of these change if the maximum value of the data set is replaced with a larger value?"

Professional Learning

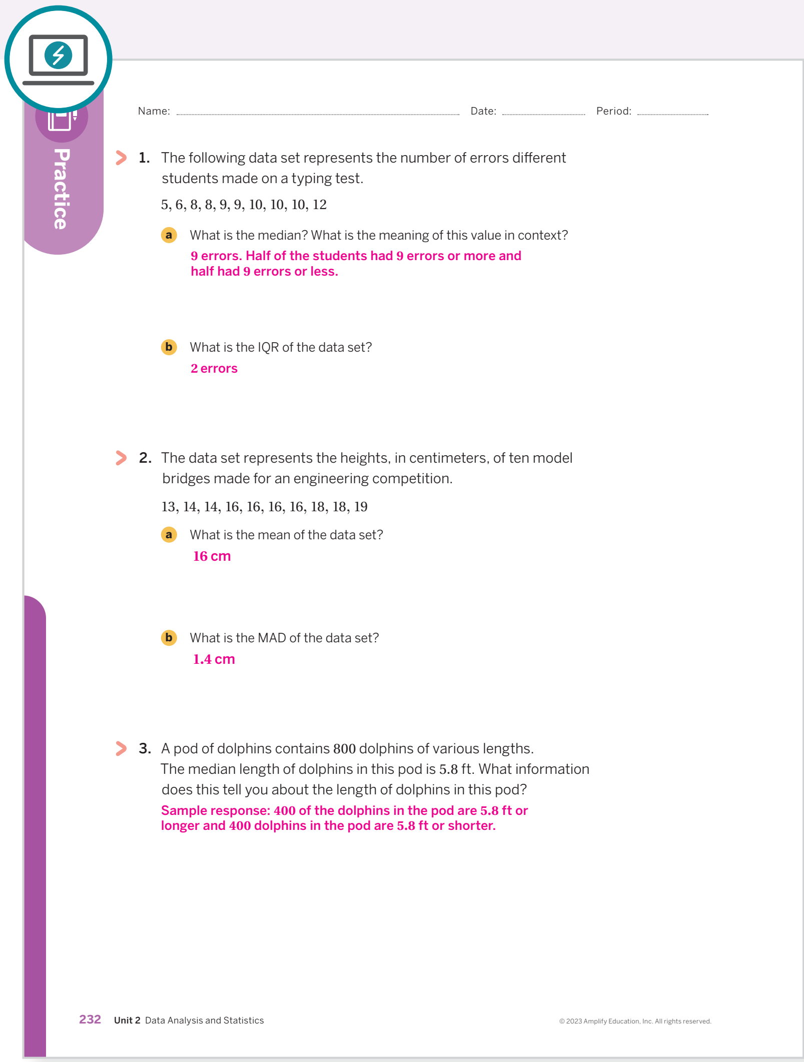
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The focus of this lesson was to calculate measures of center and variability and use them to describe the shape of the distribution. How did this go?
- How did interpreting MAD as a measure of variability set students up to develop an understanding of standard deviation in future lessons? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. The following data set represents the number of errors different students made on a typing test.

   5, 6, 8, 8, 9, 9, 10, 10, 10, 12

   a What is the median? What is the meaning of this value in context?

   9 errors. Half of the students had 9 errors or more and half had 9 errors or less.

   b What is the IQR of the data set?

   2 errors

2. The data set represents the heights, in centimeters, of ten model bridges made for an engineering competition.

   13, 14, 14, 16, 16, 16, 16, 18, 18, 19

   a What is the mean of the data set?

   16 cm

   b What is the MAD of the data set?

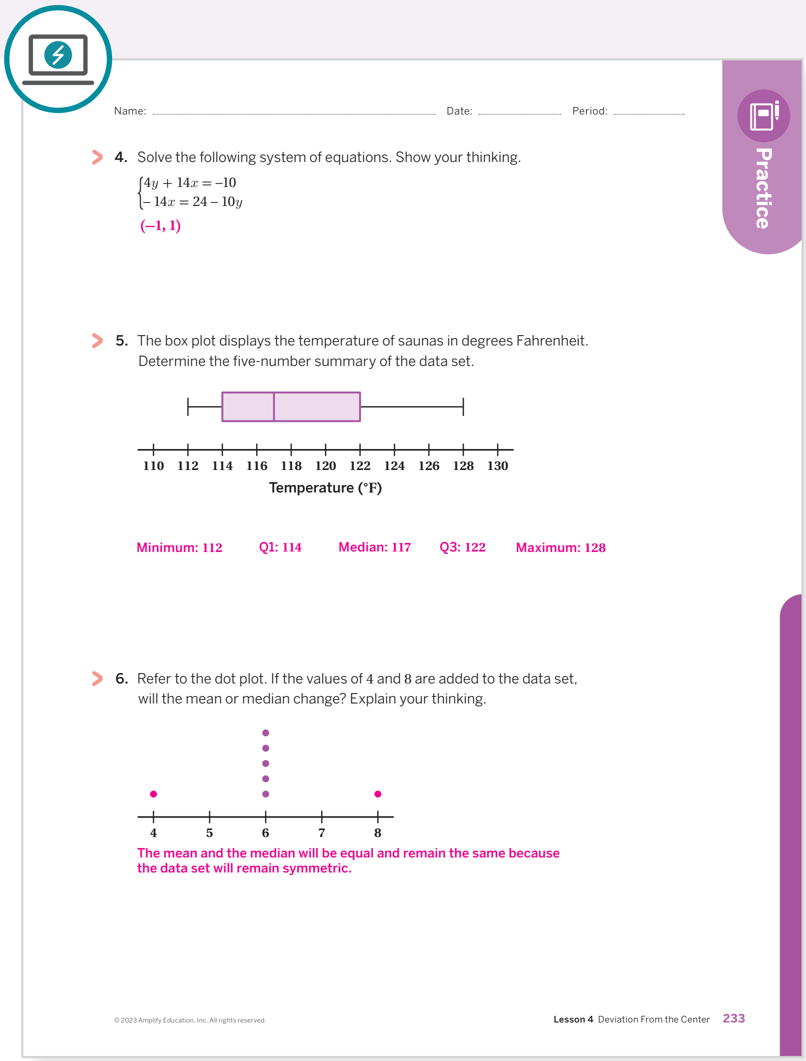
   1.4 cm

3. A pod of dolphins contains 800 dolphins of various lengths. The median length of dolphins in this pod is 5.8 ft. What information does this tell you about the length of dolphins in this pod?

   Sample response: 400 of the dolphins in the pod are 5.8 ft or longer and 400 dolphins in the pod are 5.8 ft or shorter.

232 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: Date: Period:

4. Solve the following system of equations. Show your thinking.

   $$\begin{cases} 4y + 14x = -10 \\ -14x = 24 - 10y \end{cases}$$

   (–1, 1)

5. The box plot displays the temperature of saunas in degrees Fahrenheit. Determine the five-number summary of the data set.

   Minimum: 112 **Q1: 114** **Median: 117** **Q3: 122** **Maximum: 128**

6. Refer to the dot plot. If the values of 4 and 8 are added to the data set, will the mean or median change? Explain your thinking.

   The mean and the median will be equal and remain the same because the data set will remain symmetric.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Deviation From the Center 233

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 1 Lesson 20	2
	5	Unit 2 Lesson 2	2
Formative ⚡	6	Unit 2 Lesson 6	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Measuring Outliers

Let's look at a definition for outliers and apply it.

Focus

Goals

1. **Language Goal:** Recognize the relationship between the mean and median based on the shape of the distribution. **(Reading and Writing)**
2. **Language Goal:** Understand the effects of extreme values on measures of center. **(Reading and Writing)**

Coherence

Today

Students determine if a value in a data set is an outlier by using calculations involving the IQR. They investigate the effects of outliers on the mean and median. Students determine preferred measures of center for symmetric and skewed distributions based on their investigations.

Previously

In Lesson 4, students informally identified outliers and began investigating their effect on measures of variability.

Coming Soon

In Lesson 7, students will be introduced to standard deviation as a measure of variability.

Rigor

- Students build **conceptual understanding** of the mathematical definition of outliers and investigate their effect on measures of center.
- Students develop **procedural fluency** determining if a value is an outlier by using a data set's IQR.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	15 min	5 min	5 min
Pairs	Pairs	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps* (as needed)
- four-function calculators
- spreadsheet technology

Math Language Development

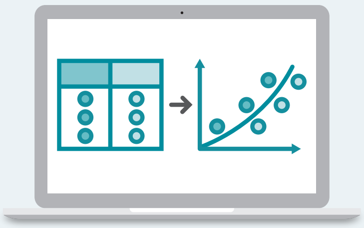
Review words

- *center*
- *interquartile range*
- *histogram*
- *dot plot*
- *mean*
- *median*
- *outlier*
- *quartile*
- *variability*

Amps | Featured Activity

Activity 2
Using Work From Previous Slides

Students see the outliers they and their classmates chose on a dot plot, along with the values of the data set's mean and median, to reinforce the effect that outliers have on these measures of center.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

While working with spreadsheets in Activities 1 and 2, students might be tempted to use the technology for purposes other than it is intended. Prior to giving students access to the technology, set guidelines that all are expected to follow. Also, ask them to help each other stay focused and on-task with reminders, if needed.

Modifications to Pacing

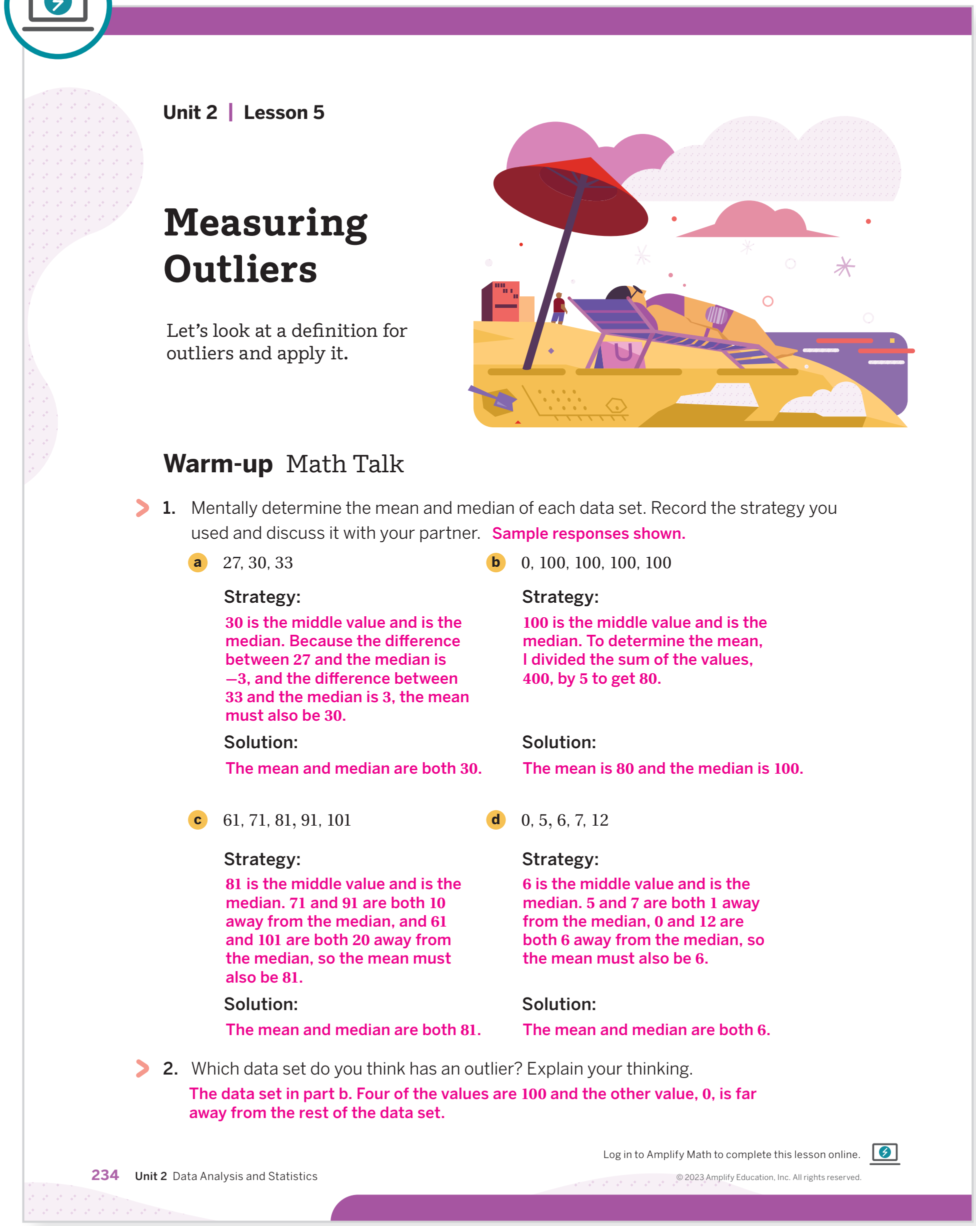
You may want to consider these additional modifications if you are short on time:

- In **Activity 1,** have students complete Problem 3 only for Data sets C and D.
- In **Activity 2,** Problem 1 may be omitted.

Pairs | 5 min

Warm-up Math Talk

Students mentally determine the mean and median of data sets to examine how the symmetry of data sets can be used to determine measures of center.



Unit 2 | Lesson 5

Measuring Outliers

Let's look at a definition for outliers and apply it.

Warm-up Math Talk

1. Mentally determine the mean and median of each data set. Record the strategy you used and discuss it with your partner. **Sample responses shown.**

   a 27, 30, 33

   Strategy:

   30 is the middle value and is the median. Because the difference between 27 and the median is —3, and the difference between 33 and the median is 3, the mean must also be 30.

   Solution:

   The mean and median are both 30.

   b 0, 100, 100, 100, 100

   Strategy:

   100 is the middle value and is the median. To determine the mean, I divided the sum of the values, 400, by 5 to get 80.

   Solution:

   The mean is 80 and the median is 100.

   c 61, 71, 81, 91, 101

   Strategy:

   81 is the middle value and is the median. 71 and 91 are both 10 away from the median, and 61 and 101 are both 20 away from the median, so the mean must also be 81.

   Solution:

   The mean and median are both 81.

   d 0, 5, 6, 7, 12

   Strategy:

   6 is the middle value and is the median. 5 and 7 are both 1 away from the median, 0 and 12 are both 6 away from the median, so the mean must also be 6.

   Solution:

   The mean and median are both 6.

2. Which data set do you think has an outlier? Explain your thinking.

   The data set in part b. Four of the values are 100 and the other value, 0, is far away from the rest of the data set.

Log in to Amplify Math to complete this lesson online.

234 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Conduct the ***Math Talk*** routine, displaying each problem one at a time. Have students complete the problems independently then discuss their work with their partners. Keep all problems displayed throughout the discussion.

2 Monitor

Help students get started by reminding them how medians are determined for even and odd numbers of values in a data set.

Look for points of confusion:

- **Struggling to articulate their strategy for the symmetrical data sets.** Ask, "What would the dot plots of these data sets look like? How are these data sets different from Data set B?"

Look for productive strategies:

- Using the symmetry of the data set to determine the mean.

3 Connect

Have pairs of students share their strategies. Select and sequence students using the standard algorithm before those using symmetry.

Ask:

- "How is the mean of Data set B different from the mean of the other data sets?" Data set B has an outlier, so the mean is not equal to the median. The data set is not symmetric like the other data sets.
- "How does the outlier affect the median and mean of Data set B?"
  The outlier does not affect the median, but has a greater effect on the mean.

Highlight that students can only use symmetrical data sets to determine that the mean and median are the same value.

Math Language Development

MLR7: Compare and Connect

During the Connect, as pairs of students share their strategies, draw connections between using the standard algorithm to determine the mean and using symmetry. Depending on the data set, one method may be more efficient or accurate than another. Ask students to share what the structure of a data set would look like if they could use symmetry to determine the mean. For example, in Data set C, there are 5 values and the differences between consecutive values are always the same. This means that the data are symmetric and the mean (and median) is 81.

Power-up

To power up students' ability to use the symmetry of a data set to make conclusions about its mean and median:

Provide students with a copy of the Power-up PDF.

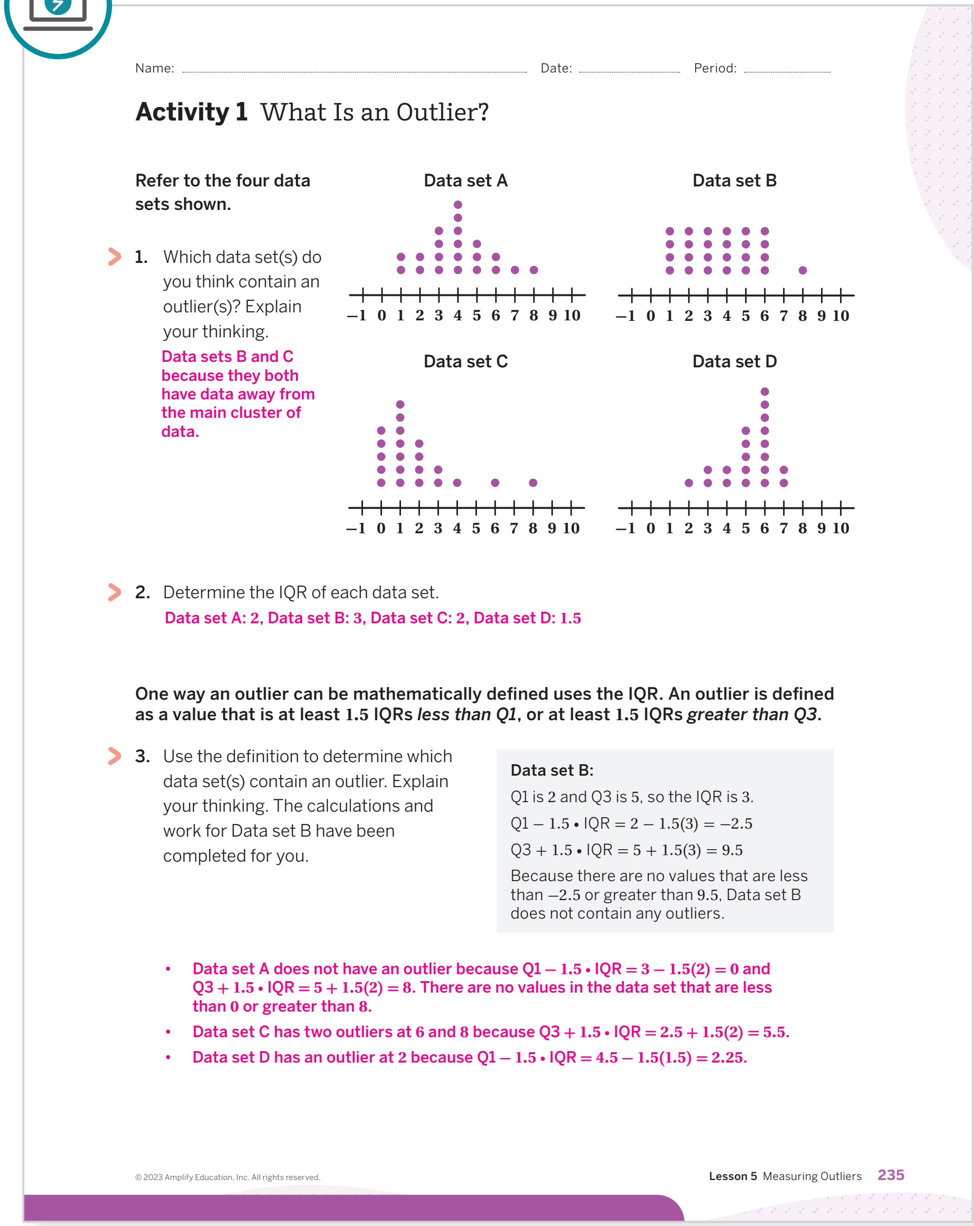
Use: Before the Warm-up

Informed by: Performance on Lesson 4, Practice Problem 6

Pairs | 10 min

Activity 1 What Is an Outlier?

Students use the mathematical definition of an outlier to identify outliers in data sets.



Name: ______ Date: ______ Period: ______

Activity 1 What Is an Outlier?

Refer to the four data sets shown.

Data set A

Data set B

Data set C

Data set D

1. Which data set(s) do you think contain an outlier(s)? Explain your thinking.
   Data sets B and C because they both have data away from the main cluster of data.

2. Determine the IQR of each data set.
   Data set A: 2, Data set B: 3, Data set C: 2, Data set D: 1.5

One way an outlier can be mathematically defined uses the IQR. An outlier is defined as a value that is at least 1.5 IQRs *less than Q1*, or at least 1.5 IQRs *greater than Q3*.

3. Use the definition to determine which data set(s) contain an outlier. Explain your thinking. The calculations and work for Data set B have been completed for you.

Data set B:

Q1 is 2 and Q3 is 5, so the IQR is 3.

$Q1 - 1.5 \cdot IQR = 2 - 1.5(3) = -2.5$

$Q3 + 1.5 \cdot IQR = 5 + 1.5(3) = 9.5$

Because there are no values that are less than –2.5 or greater than 9.5, Data set B does not contain any outliers.

- **Data set A does not have an outlier because $Q1 - 1.5 \cdot IQR = 3 - 1.5(2) = 0$ and $Q3 + 1.5 \cdot IQR = 5 + 1.5(2) = 8$. There are no values in the data set that are less than 0 or greater than 8.**
- **Data set C has two outliers at 6 and 8 because $Q3 + 1.5 \cdot IQR = 2.5 + 1.5(2) = 5.5$.**
- **Data set D has an outlier at 2 because $Q1 - 1.5 \cdot IQR = 4.5 - 1.5(1.5) = 2.25$.**

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have student pairs complete Problems 1 and 2, then pause and discuss as a whole class before releasing them to work on the remaining problems. **Note:** Provide students access to four-function calculators.

2 Monitor

Help students get started by reminding them that the IQR is the difference between Q3 and Q1.

Look for points of confusion:

- **Thinking that outliers have to be far away from the rest of the data.** Refer to the example work for Data set B to highlight that only these calculations are used to determine outliers.

Look for productive strategies:

- Counting dots to determine the IQR of each dot plot.
- Marking the cut-off for outliers on the dot plots.

3 Connect

Have individual students share their thinking for Problem 1.

Ask, "What shapes of distributions tend to have outliers more often? Why?" Skewed distributions, because there is often a cluster of data with at least a few extreme values.

Highlight that by using this calculation, students can not necessarily rely on just the look of a data set to determine if there are outliers. There may be a value separate from a cluster of values that is not an outlier, or a value close to a cluster that is an outlier.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide Q1 and Q3 for Data sets A, C, and D, so that students can focus on using these values to determine outliers, instead of calculating them.

	Data set A	Data set C	Data set D
Q1	3	0.5	4.5
Q3	5	2.5	6

Extension: *Math Enrichment*

Have students complete the following problem:
Create a data set that contains an outlier, but has no gaps between intervals when a histogram is created of the data set. Sample response: 8, 9, 9, 9, 10, 10, 10, 10, 10, 11, 12, 13

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 2, have them pause to read the definition of an outlier that is given before Problem 3. After reading this definition, have students individually write a procedure they could use for determining whether a value is an outlier. Have students share their procedure with their partner to both give and receive feedback. Have them revise their procedure based on the feedback they received. Select 1 or 2 pairs of students to share their procedure with the class.

English Learners

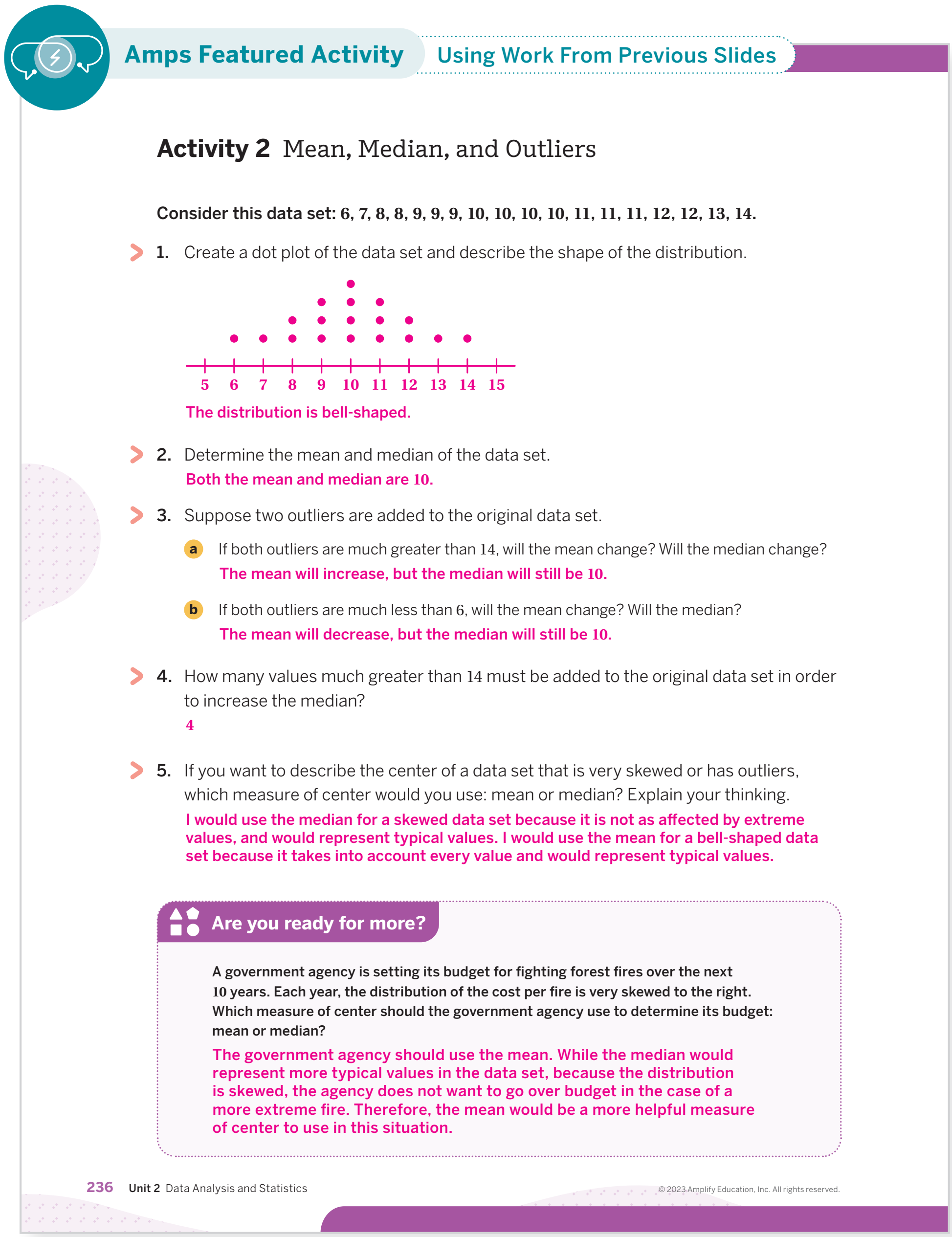
After sharing with the class, post an agreed-upon procedure that students can use to determine whether a data value is an outlier.

Independent | 10 min

Activity 2 Mean, Median, and Outliers

Students add outliers to a data set and determine the preferred measure of center for both a skewed versus a symmetric distribution, and when extreme values are present.

Amps Featured Activity Using Work From Previous Slides



Activity 2 Mean, Median, and Outliers

Consider this data set: 6, 7, 8, 8, 9, 9, 9, 10, 10, 10, 10, 11, 11, 11, 12, 12, 13, 14.

1. Create a dot plot of the data set and describe the shape of the distribution.

The distribution is bell-shaped.

2. Determine the mean and median of the data set.
   Both the mean and median are 10.

3. Suppose two outliers are added to the original data set.
   - a. If both outliers are much greater than 14, will the mean change? Will the median change?
     The mean will increase, but the median will still be 10.
   - b. If both outliers are much less than 6, will the mean change? Will the median?
     The mean will decrease, but the median will still be 10.

4. How many values much greater than 14 must be added to the original data set in order to increase the median?
   4

5. If you want to describe the center of a data set that is very skewed or has outliers, which measure of center would you use: mean or median? Explain your thinking.
   I would use the median for a skewed data set because it is not as affected by extreme values, and would represent typical values. I would use the mean for a bell-shaped data set because it takes into account every value and would represent typical values.

Are you ready for more?

A government agency is setting its budget for fighting forest fires over the next 10 years. Each year, the distribution of the cost per fire is very skewed to the right. Which measure of center should the government agency use to determine its budget: mean or median?

The government agency should use the mean. While the median would represent more typical values in the data set, because the distribution is skewed, the agency does not want to go over budget in the case of a more extreme fire. Therefore, the mean would be a more helpful measure of center to use in this situation.

236 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have students complete Problem 1, and pause to discuss the distribution before moving on.

2 Monitor

Help students get started by highlighting that the data set is symmetric, so they can use the strategy from the Warm-up to determine the mean and median.

Look for points of confusion:

- **Not choosing the median in Problem 5.** Ask, "Which measure of center, the mean or median, would change the least if an outlier is added to the data set?"

Look for productive strategies:

- Using the quartiles and IQR to determine outliers for the data set.

3 Connect

Have individual students share their responses for Problems 4 and 5.

Highlight that the addition of extreme values tends to have a greater effect on the mean than the median.

Ask, "When is the median a better statistic to describe typical values? When is the mean a better statistic?" The median is a better statistic to describe typical values when the distribution is skewed, and the mean is better to use when the distribution is symmetric.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with a pre-made dot plot, along with the mean and median of the data set for Problems 1–2. This will allow them to focus on analyzing how adding outliers to the data set affect the mean and median.

Accessibility: *Guide Processing and Visualization*

Consider providing students with three copies of the pre-made dot plot for Problem 1, extending the number line on both ends. They can refer to one of these copies as the original dot plot. On the second copy, have them add two outliers that are much greater than 14 so they can visually see how the distribution is affected. On the third copy, have them add two outliers that are much less than 6.

Math Language Development

MLR2: Collect and Display

During the Connect, listen for and collect language students use to describe how outliers affect the mean and median, and when each measure is more appropriate to use. Write students' words and phrases on a visual display and have students use this as a reference throughout the lesson. For example, listen for words and phrases, such as *skewed, symmetric, affected, not affected, bell-shaped, represent typical values,* etc.

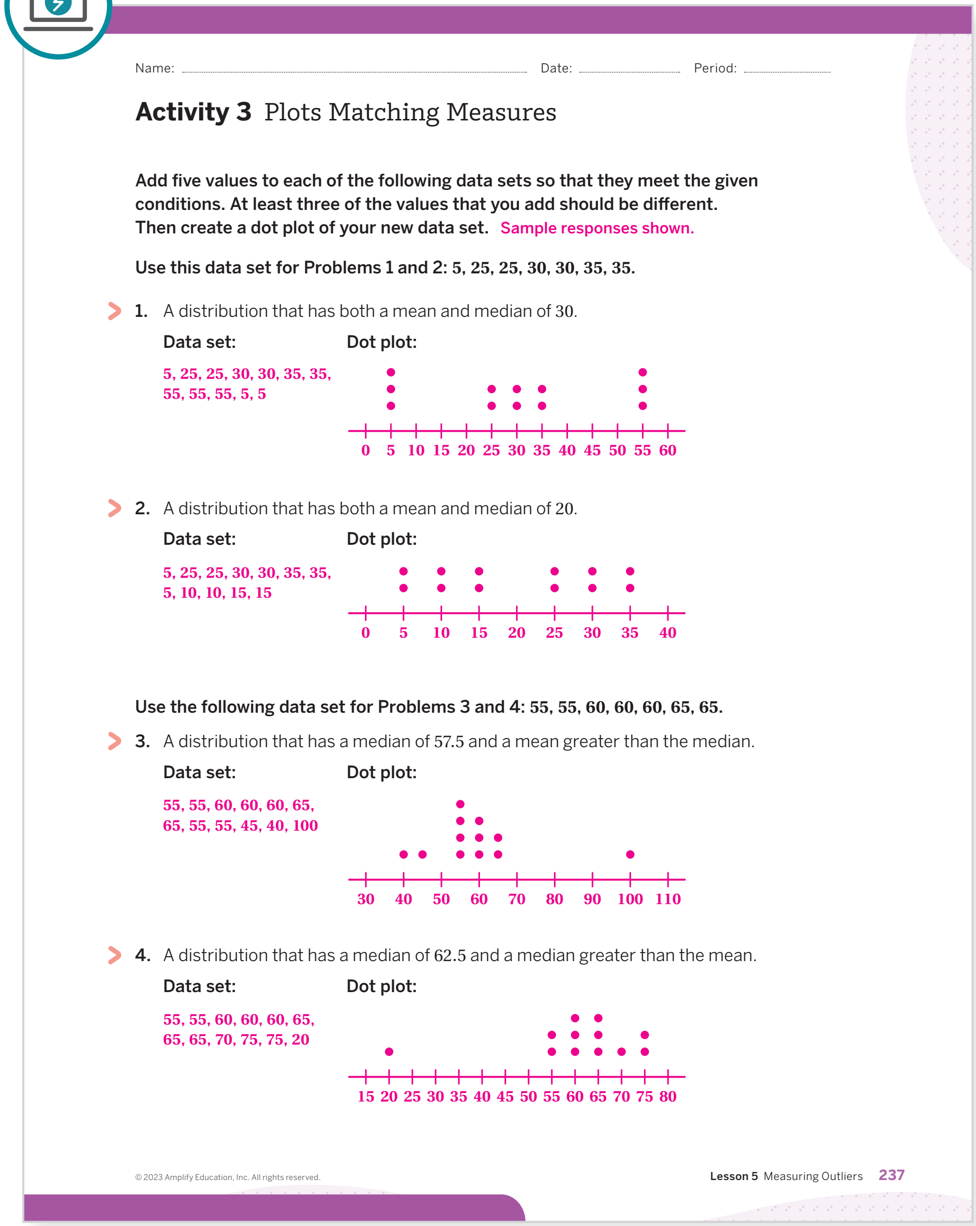
English Learners

Include diagrams or representations on the class display to connect the words and phrases to the different representations.

Pairs | 15 min

Activity 3 Plots Matching Measures

Students recognize the relationship between measures of center and outliers by creating distributions with given measures of center.



Name: ________ Date: ________ Period: ________

Activity 3 Plots Matching Measures

Add five values to each of the following data sets so that they meet the given conditions. At least three of the values that you add should be different. Then create a dot plot of your new data set. Sample responses shown.

Use this data set for Problems 1 and 2: 5, 25, 25, 30, 30, 35, 35.

1. A distribution that has both a mean and median of 30.

   Data set: 5, 25, 25, 30, 30, 35, 35, 55, 55, 55, 5, 5

   Dot plot:

2. A distribution that has both a mean and median of 20.

   Data set: 5, 25, 25, 30, 30, 35, 35, 5, 10, 10, 15, 15

   Dot plot:

Use the following data set for Problems 3 and 4: 55, 55, 60, 60, 60, 65, 65.

3. A distribution that has a median of 57.5 and a mean greater than the median.

   Data set: 55, 55, 60, 60, 60, 65, 65, 55, 55, 45, 40, 100

   Dot plot:

4. A distribution that has a median of 62.5 and a median greater than the mean.

   Data set: 55, 55, 60, 60, 60, 65, 65, 65, 70, 75, 75, 20

   Dot plot:

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Say, "Consider thinking about the shape of the distribution of each data set before you attempt to add values." **Note:** Consider making spreadsheet technology available.

2 Monitor

Help students get started by having them calculate the mean and median of the original data sets first.

Look for points of confusion:

- **Using only the *guess-and-check* strategy to add values.** Have students determine how the mean and median changed, and which side(s) of these values to add data to, to achieve this change.

Look for productive strategies:

- Marking the original mean and median on the box plot and using distances from these measures to determine values to add.
- Using *symmetric, uniform,* and *skew* to describe distributions and make connections to the mean and median.
- Understanding the structure of the distribution to adjust individual data values to change the measures of center.

Activity 3 continued >

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing students with pre-made dot plots of the data sets that they can add the values to, which will help them save time and focus on analyzing how the distributions are affected.

Extension: *Math Enrichment*

Some mathematicians use the phrase "negative skew" when data are skewed left and "positive skew" when data are skewed right. Ask students to use these phrases to explain how the mean compares to the median for these types of distributions. Sample response: When the data are negatively skewed, the mean is less than the median. When the data are positively skewed, the mean is greater than the median.

Math Language Development

MLR8: Discussion Supports — Revoicing

During the Connect, as students respond to the Ask questions, amplify the use of mathematical language by asking these clarifying questions:

- "Why does it make sense that the distributions would be skewed right when the mean is greater than the median?"
- "Why does it make sense that the distributions would be skewed left when the mean is less than the median?"

Press for details in students' explanations by having other students elaborate on and revoice their peers' responses.

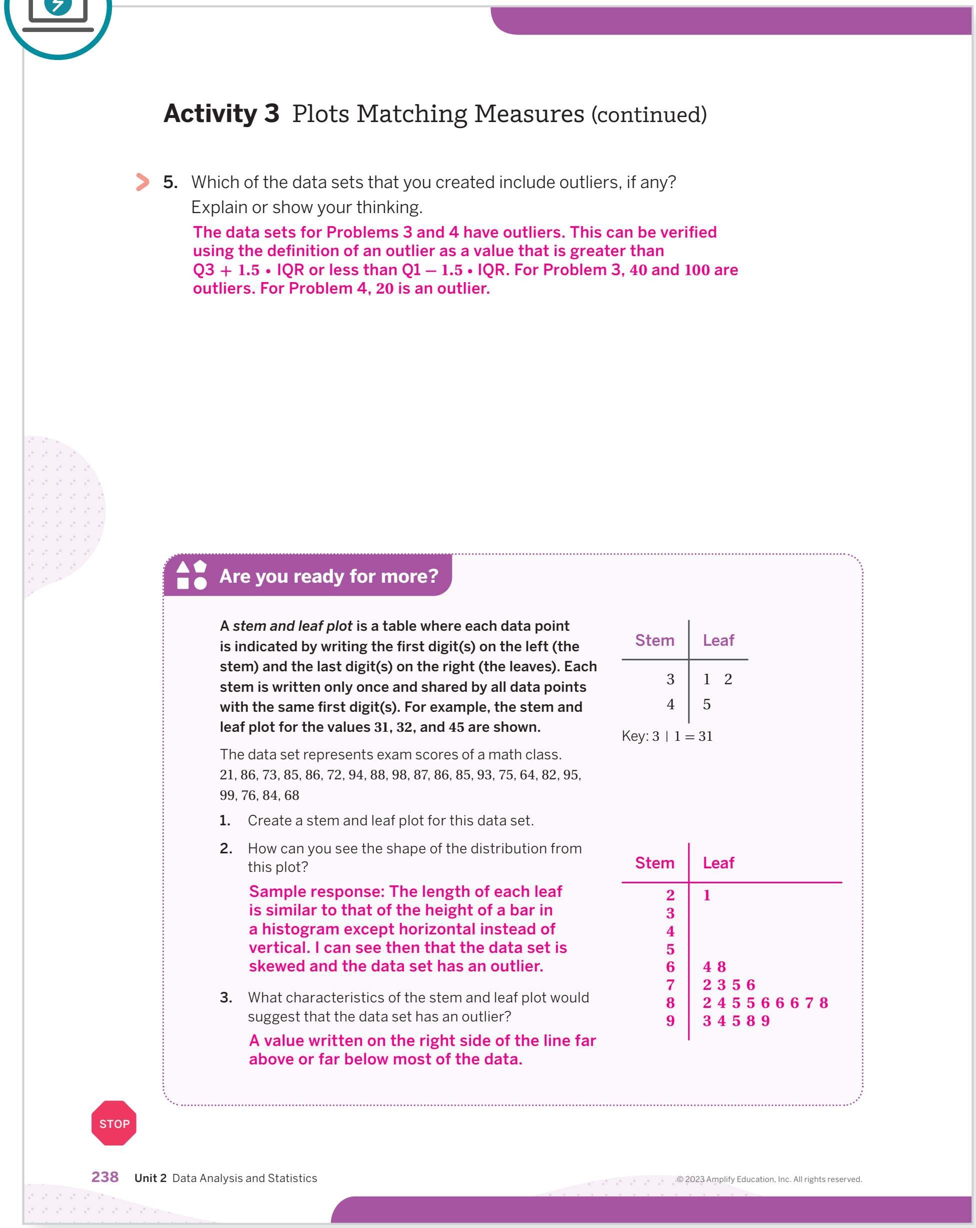
English Learners

Give students time to formulate a response with a partner before sharing with the whole class.

Pairs | 15 min

Activity 3 Plots Matching Measures (continued)

Students recognize the relationship between measures of center and outliers by creating distributions with given measures of center.



Activity 3 Plots Matching Measures (continued)

5. Which of the data sets that you created include outliers, if any? Explain or show your thinking.
   The data sets for Problems 3 and 4 have outliers. This can be verified using the definition of an outlier as a value that is greater than Q3 + 1.5 • IQR or less than Q1 − 1.5 • IQR. For Problem 3, 40 and 100 are outliers. For Problem 4, 20 is an outlier.

Are you ready for more?

A *stem and leaf plot* is a table where each data point is indicated by writing the first digit(s) on the left (the stem) and the last digit(s) on the right (the leaves). Each stem is written only once and shared by all data points with the same first digit(s). For example, the stem and leaf plot for the values 31, 32, and 45 are shown.

Stem	Leaf
3	1 2
4	5

Key: 3 | 1 = 31

The data set represents exam scores of a math class.
21, 86, 73, 85, 86, 72, 94, 88, 98, 87, 86, 85, 93, 75, 64, 82, 95, 99, 76, 84, 68

1. Create a stem and leaf plot for this data set.

Stem	Leaf
2	1
3	
4	
5	
6	4 8
7	2 3 5 6
8	2 4 5 5 6 6 6 7 8
9	3 4 5 8 9

2. How can you see the shape of the distribution from this plot?
   Sample response: The length of each leaf is similar to that of the height of a bar in a histogram except horizontal instead of vertical. I can see then that the data set is skewed and the data set has an outlier.
3. What characteristics of the stem and leaf plot would suggest that the data set has an outlier?
   A value written on the right side of the line far above or far below most of the data.

STOP

238 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have pairs of students share their dot plots.

Highlight that the median is the preferred measure of center when a distribution is skewed or if there are extreme values, because the median is usually not influenced greatly by extreme values and would still reflect typical values in the data set. The mean is the preferred measure of center when a distribution is symmetric and there are no extreme values because it accounts for all values in the data set and would reflect typical values.

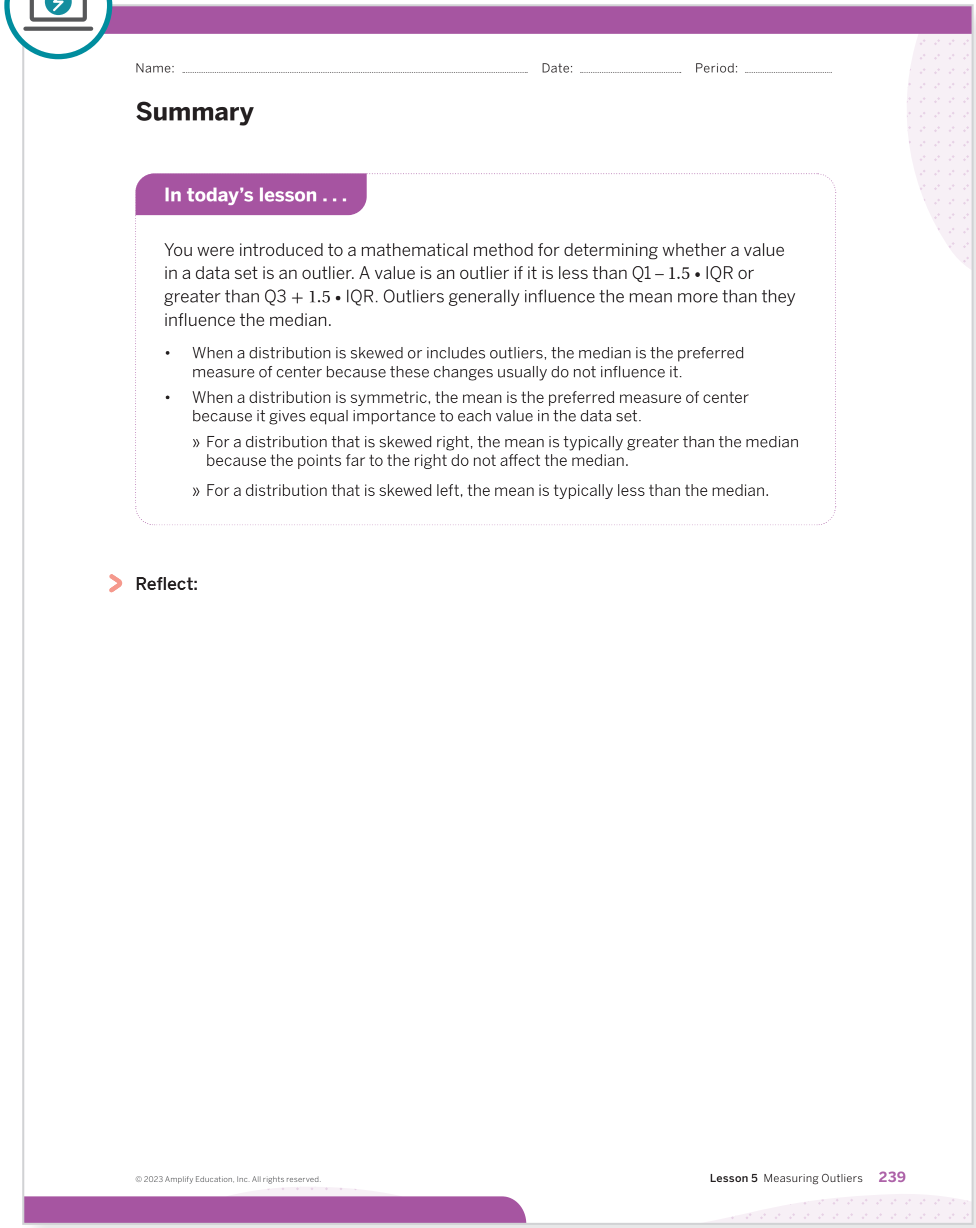
Ask:

- "What do the shapes of the dot plots have in common when the mean is greater than the median?" They are both skewed right.
- "What information does the shape of the skewed distributions tell you about the median and mean?" Whether the distribution is skewed left/right tells me whether the mean will likely be greater than/less than the median.

Whole Class | 5 min

Summary

Review and synthesize how outliers affect the mean, MAD, median, and IQR, and why outliers affect these measures differently.



Name: Date: Period:

Summary

In today's lesson . . .

You were introduced to a mathematical method for determining whether a value in a data set is an outlier. A value is an outlier if it is less than Q1 – 1.5 • IQR or greater than Q3 + 1.5 • IQR. Outliers generally influence the mean more than they influence the median.

- When a distribution is skewed or includes outliers, the median is the preferred measure of center because these changes usually do not influence it.
- When a distribution is symmetric, the mean is the preferred measure of center because it gives equal importance to each value in the data set.
  - For a distribution that is skewed right, the mean is typically greater than the median because the points far to the right do not affect the median.
  - For a distribution that is skewed left, the mean is typically less than the median.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the dot plots from Activity 1.

Have students share which measure of center they would use to describe each data set.

Highlight that while there are preferred measures of center for skewed distribution and symmetric distributions, there may be cases when these measures are still close or are affected similarly by outliers.

Ask, "Why is the median often preferred for skewed distributions and the mean often preferred for symmetric distributions?" The extreme values in skewed data have a greater effect on the mean, so the median tends to better reflect typical values. The mean takes into account every data value, so it is the preferred measure when it is representative of the data.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

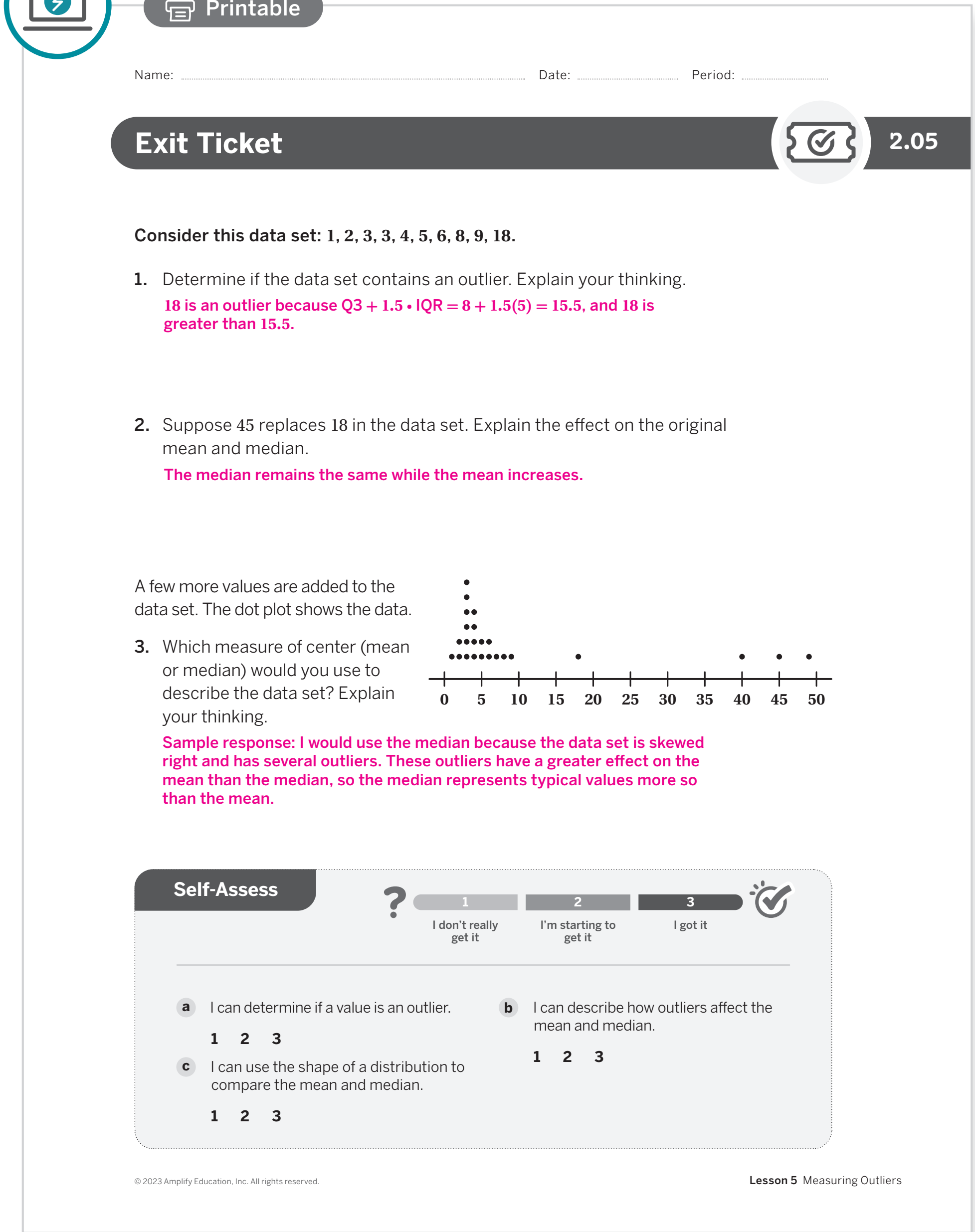
- "Do other extreme values that are not outliers have similar effects on these measures of center and variability?"
- "Why are Q1 and Q3 used in the calculations to determine if a value is an outlier?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining outliers and selecting an appropriate measure of center.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 2.05

Consider this data set: 1, 2, 3, 3, 4, 5, 6, 8, 9, 18.

1. Determine if the data set contains an outlier. Explain your thinking.
   18 is an outlier because Q3 + 1.5 • IQR = 8 + 1.5(5) = 15.5, and 18 is greater than 15.5.

2. Suppose 45 replaces 18 in the data set. Explain the effect on the original mean and median.
   The median remains the same while the mean increases.

A few more values are added to the data set. The dot plot shows the data.

3. Which measure of center (mean or median) would you use to describe the data set? Explain your thinking.
   Sample response: I would use the median because the data set is skewed right and has several outliers. These outliers have a greater effect on the mean than the median, so the median represents typical values more so than the mean.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can determine if a value is an outlier.
1 2 3

b. I can describe how outliers affect the mean and median.
1 2 3

c. I can use the shape of a distribution to compare the mean and median.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 5 Measuring Outliers

Success looks like . . .

- **Language Goal:** Recognizing the relationship between the mean and median based on the shape of the distribution. **(Reading and Writing)**
- **Language Goal:** Understanding the effects of extreme values on measures of center. **(Reading and Writing)**
  - » Explaining how replacing 18 with 45 changes the original mean and median in Problem 2.

Suggested next steps

If students make inaccurate calculations in Problem 1, consider:

- Reviewing the procedure for determining whether a value is an outlier in Activity 1.
- Assigning Practice Problem 3.
- Asking, "What measures do you need to determine first to use in your calculations for determining outliers?"

If students provide an inaccurate value or explanation in Problem 2, consider:

- Reviewing how outliers affect measures of center from Activity 2.
- Assigning Practice Problems 2 and 3.
- Asking, "Which measures give equal importance to each value in the data set and are more affected by extreme values?"

If students provide an inaccurate value or explanation in Problem 3, consider:

- Reviewing Problem 6 from Activity 2.
- Assigning Practice Problems 1 and 2.
- Asking, "How would you describe the shape of the distribution? Which shapes of distributions tend to have outliers more often?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The focus of this lesson was to examine how outliers affect measures of center and variability.
- How did students' conclusions about the effect of outliers go?
- How will the work from this lesson help students understand how outliers affect measures of center? What might you change for the next time you teach this lesson?

MLR Math Language Development

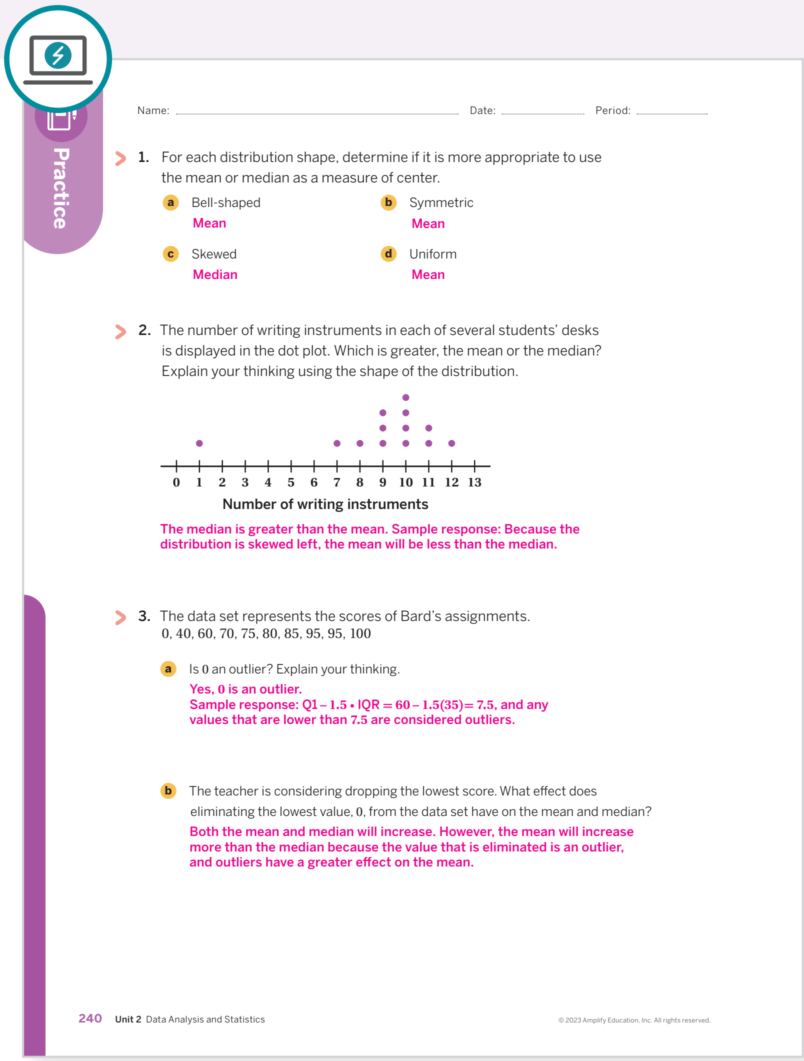
Language Goal: Recognizing the relationship between the mean and median based on the shape of the distribution.

Reflect on students' language development toward this goal.

- Do students' responses to Problem 3 of the Exit Ticket demonstrate they understand how the shape of a data distribution indicates whether the mean or median is an appropriate measure?
- Are their explanations accurate and precise? For example, do they use the terms and phrases *skewed right* and *outlier(s)*, and do their explanations include mention of which measure of center is more affected by outliers?

Practice

Independent



Practice

Name: Date: Period:

1. For each distribution shape, determine if it is more appropriate to use the mean or median as a measure of center.

   a. Bell-shaped
   Mean

   b. Symmetric
   Mean

   c. Skewed
   Median

   d. Uniform
   Mean

2. The number of writing instruments in each of several students' desks is displayed in the dot plot. Which is greater, the mean or the median? Explain your thinking using the shape of the distribution.

The median is greater than the mean. Sample response: Because the distribution is skewed left, the mean will be less than the median.

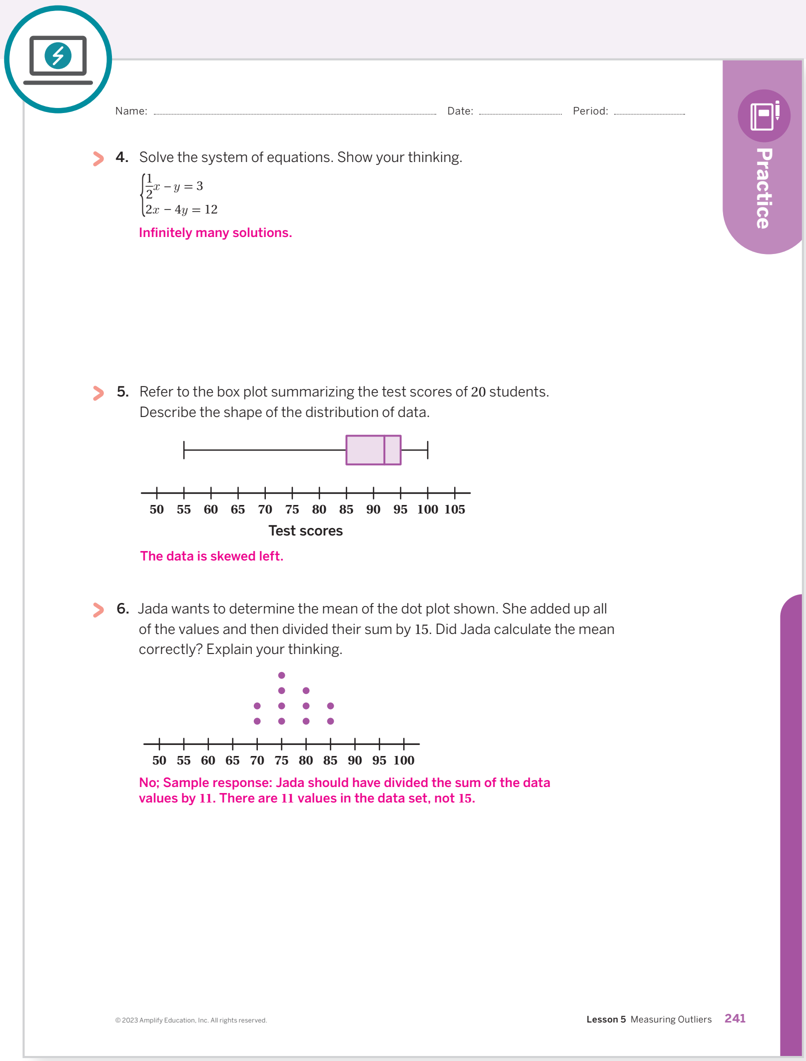
3. The data set represents the scores of Bard's assignments.
   $0, 40, 60, 70, 75, 80, 85, 95, 95, 100$

   a. Is 0 an outlier? Explain your thinking.
   Yes, 0 is an outlier.
   Sample response: Q1 – 1.5 • IQR $= 60 - 1.5(35) = 7.5$, and any values that are lower than 7.5 are considered outliers.

   b. The teacher is considering dropping the lowest score. What effect does eliminating the lowest value, 0, from the data set have on the mean and median?
   Both the mean and median will increase. However, the mean will increase more than the median because the value that is eliminated is an outlier, and outliers have a greater effect on the mean.

240 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: Date: Period:

4. Solve the system of equations. Show your thinking.

$$\begin{cases} \frac{1}{2}x - y = 3 \\ 2x - 4y = 12 \end{cases}$$

Infinitely many solutions.

5. Refer to the box plot summarizing the test scores of 20 students. Describe the shape of the distribution of data.

The data is skewed left.

6. Jada wants to determine the mean of the dot plot shown. She added up all of the values and then divided their sum by 15. Did Jada calculate the mean correctly? Explain your thinking.

No; Sample response: Jada should have divided the sum of the data values by 11. There are 11 values in the data set, not 15.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 5 Measuring Outliers 241

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 3	2
Spiral	4	Unit 1 Lesson 22	2
	5	Unit 2 Lesson 3	2
Formative	6	Unit 2 Lesson 6	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 2 | **LESSON 6**

Data With Spreadsheets

Let's use technology to organize and visualize data.

Focus

Goal

1. Use spreadsheet technology to graphically represent data and calculate useful statistics.

Coherence

Today

Students use spreadsheet technology to create and analyze histograms, dot plots, and box plots. They determine appropriate intervals for their histograms, as well as appropriate axes scales and titles. They explain how subsets from the same data set can reflect vague or conflicting conclusions.

Previously

In Lessons 2–5, students created histograms, dot plots, and box plots by hand to represent data sets.

Coming Soon

In Lesson 7, students will be introduced to standard deviation, a new measure of variability.

Rigor

- Students build **procedural fluency** creating and analyzing histograms, dot plots, and box plots using spreadsheet technology.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (as needed)
- Activity 1 PDF (as needed)
- Activities 2 & 3 PDF
- Activity 2 PDF (as needed)
- spreadsheet technology

Math Language Development

Review words

- *box plot*
- *dot plot*
- *histogram*
- *median*
- *quartile*

Amps | **Featured Activity**

Activity 3
Interactive Graphs

Students create digital box plots and histograms to represent two different time periods of temperature data. They then explain how the time period of each data set influences the conclusions that can be made about changes in global ocean temperatures.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might forget to use precise language when sharing their thinking during Activity 3. Create a word wall with new vocabulary so that students can see specific mathematical vocabulary that will help them express their thoughts in a way that their partner can understand. Encourage them to help each other be as specific with their words as possible.

Modifications to Pacing

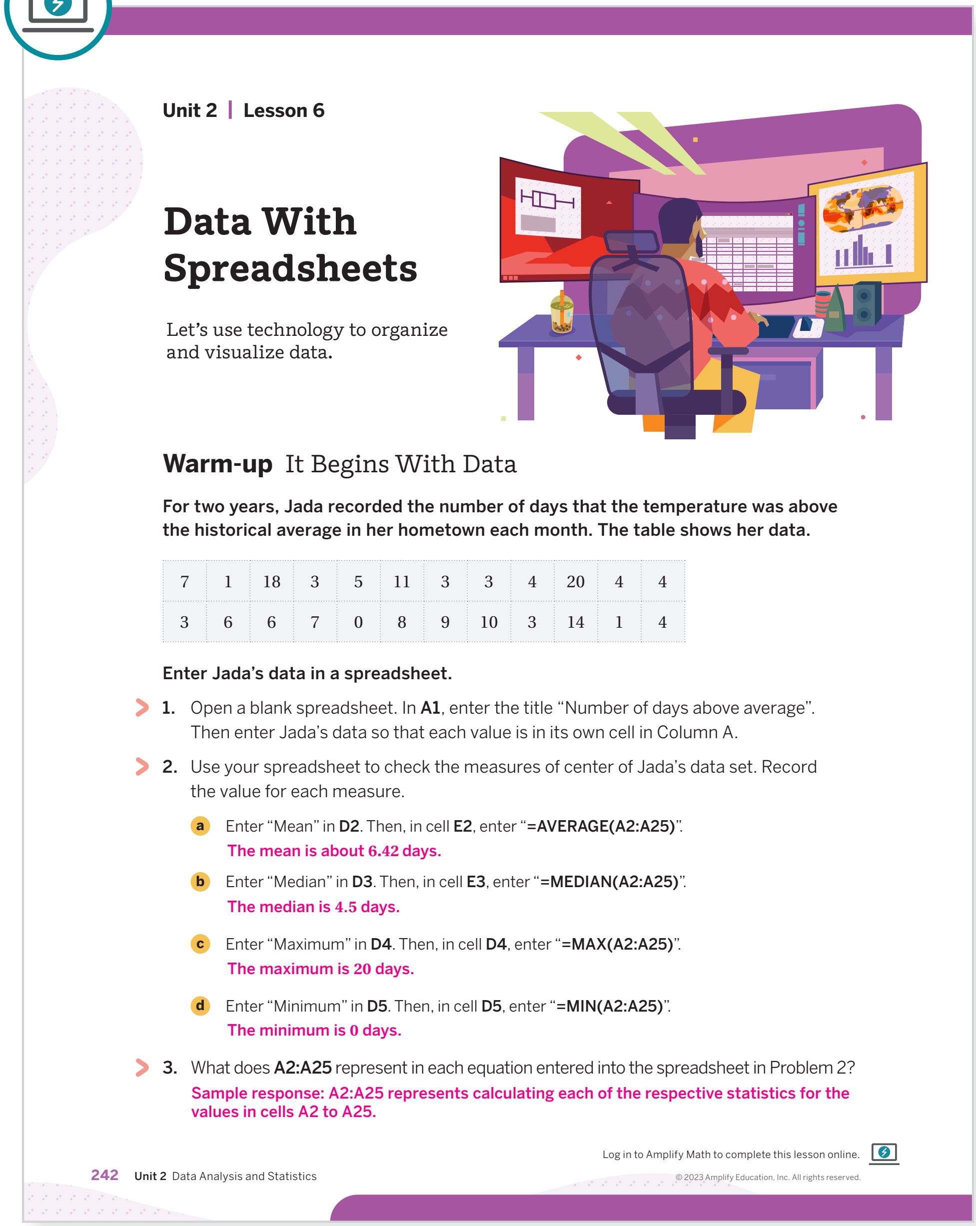
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 3 may be omitted.
- In **Activity 1**, Problems 6–12 may be omitted.
- In **Activity 3**, have students only create data representations for one time period and complete either Problem 2 or 3, and Problem 4.

Independent | 5 min

Warm-up It Begins With Data

Students enter data and determine statistics using spreadsheet functions to prepare them to create data representations.



Unit 2 | Lesson 6

Data With Spreadsheets

Let's use technology to organize and visualize data.

Warm-up It Begins With Data

For two years, Jada recorded the number of days that the temperature was above the historical average in her hometown each month. The table shows her data.

7	1	18	3	5	11	3	3	4	20	4	4
3	6	6	7	0	8	9	10	3	14	1	4

Enter Jada's data in a spreadsheet.

1. Open a blank spreadsheet. In **A1**, enter the title "Number of days above average". Then enter Jada's data so that each value is in its own cell in Column A.
2. Use your spreadsheet to check the measures of center of Jada's data set. Record the value for each measure.
   - a Enter "Mean" in **D2**. Then, in cell **E2**, enter "**=AVERAGE(A2:A25)**".
     The mean is about 6.42 days.
   - b Enter "Median" in **D3**. Then, in cell **E3**, enter "**=MEDIAN(A2:A25)**".
     The median is 4.5 days.
   - c Enter "Maximum" in **D4**. Then, in cell **D4**, enter "**=MAX(A2:A25)**".
     The maximum is 20 days.
   - d Enter "Minimum" in **D5**. Then, in cell **D5**, enter "**=MIN(A2:A25)**".
     The minimum is 0 days.
3. What does **A2:A25** represent in each equation entered into the spreadsheet in Problem 2?
   Sample response: A2:A25 represents calculating each of the respective statistics for the values in cells A2 to A25.

Log in to Amplify Math to complete this lesson online.

242 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have students open a spreadsheet using spreadsheet technology. Make sure students are familiar with frequently used terminology for spreadsheets, i.e., cells, column names, and row numbers.

2 Monitor

Help students get started by modeling how to enter the first few values in Column A.

Look for points of confusion:

- **Thinking that A2:A25 only includes A2 and A25.** Have students determine if the minimum and maximum of just these two cells would give those calculated in the spreadsheet.

Look for productive strategies:

- Selecting cells in Column A as an alternative method to enter A2:A25.

3 Connect

Display the data set entered into Column A.

Highlight that students can accurately and quickly calculate a variety of statistics of large data sets using formulas in spreadsheets.

Ask:

- "What else would you need to do if you calculated these statistics by hand?" Order the values from least to greatest.
- "If a value in the data set changed, would you prefer to calculate these statistics by hand or using a spreadsheet?" Sample response: I would prefer to use a spreadsheet because these statistics would automatically update.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with the Warm-up PDF that they can use to help organize their thinking and use as a guide to enter the data into their spreadsheet.

Accessibility: *Bridge Knowledge Gaps*

If students are unfamiliar with spreadsheets, spend some time demonstrating to them what a spreadsheet is, what one looks like, and how they can be used. Be sure they understand how rows and columns are named and how formulas can be used and entered. Spending some time upfront will pay off in later lessons as students will be using spreadsheets to enter and analyze data.

Power-up

To power up students' ability to relate the amount of data in a set to determine its measures of center, have students complete:

Identify which description explains how to determine the *mean* and which describes how to determine the *median*.

1. List the values in order from least to greatest and identify the value in the middle. If there are two values, calculate their average.
   Median
2. Determine the sum of all of the values in the data set and then divide by the total number of values.
   Mean

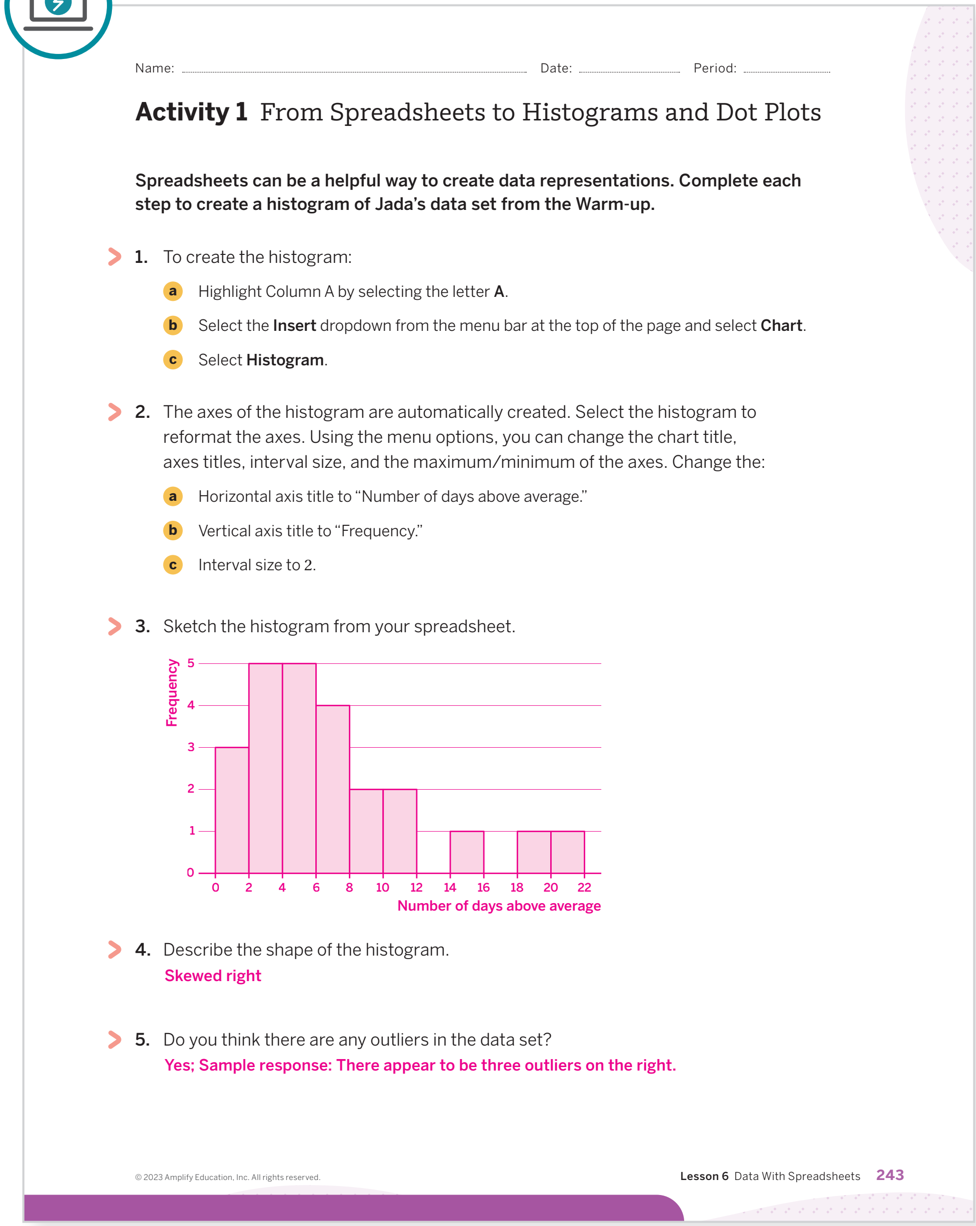
Use: Before the Warm-up

Informed by: Performance on Lesson 5, Practice Problem 6

Pairs | 15 min

Activity 1 From Spreadsheets to Histograms and Dot Plots

Students create histograms and dot plots using spreadsheet technology to examine the shape of the distribution of a data set.



Name: ______ Date: ______ Period: ______

Activity 1 From Spreadsheets to Histograms and Dot Plots

Spreadsheets can be a helpful way to create data representations. Complete each step to create a histogram of Jada's data set from the Warm-up.

1. To create the histogram:
   - a Highlight Column A by selecting the letter **A**.
   - b Select the **Insert** dropdown from the menu bar at the top of the page and select **Chart**.
   - c Select **Histogram**.

2. The axes of the histogram are automatically created. Select the histogram to reformat the axes. Using the menu options, you can change the chart title, axes titles, interval size, and the maximum/minimum of the axes. Change the:
   - a Horizontal axis title to "Number of days above average."
   - b Vertical axis title to "Frequency."
   - c Interval size to 2.

3. Sketch the histogram from your spreadsheet.

4. Describe the shape of the histogram.
   Skewed right

5. Do you think there are any outliers in the data set?
   Yes; Sample response: There appear to be three outliers on the right.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 6 Data With Spreadsheets 243

1 Launch

Have students work independently and then discuss their work with a partner.

2 Monitor

Help students get started by showing them where to find the chart options and the various charts to choose from.

Look for points of confusion:

- **Selecting *Chart* and having a default chart type appear.** Have students look through the chart options to find the option titled "Histogram."
- **Not entering the frequency in the count column.** Review the example in Problem 8.

Look for productive strategies:

- Changing the interval size for the histogram to observe how the shape of the distribution is affected.
- Adjusting the interval size until the shape of the distribution is clear.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Visualization and Processing*

Display or provide students with a copy of the Activity 1 PDF, which they can use to check to see if they created their histogram using the spreadsheet correctly.

If students need more processing time, consider omitting the dot plot from the activity and have them focus on creating and analyzing the histogram.

Extension: *Math Enrichment*

Ask, "How could you change the key features on your histogram so that the shape of the distribution appears to be bell-shaped?" I could restrict the horizontal axis so that the histogram does not show any outliers on the right.

Math Language Development

MLR1: Stronger and Clearer Each Time

Have students share their histogram in Problem 3, and their responses to Problems 4 and 5 with their partner to give and receive feedback. Display these questions for reviewers to consider:

- "Does the response to Problem 4 use mathematical language from this unit? Does it correctly describe the shape of the histogram?"
- "Does the response to Problem 5 provide any more information other than a yes/no response?"

Have students revise their histogram and responses, based on any feedback they received.

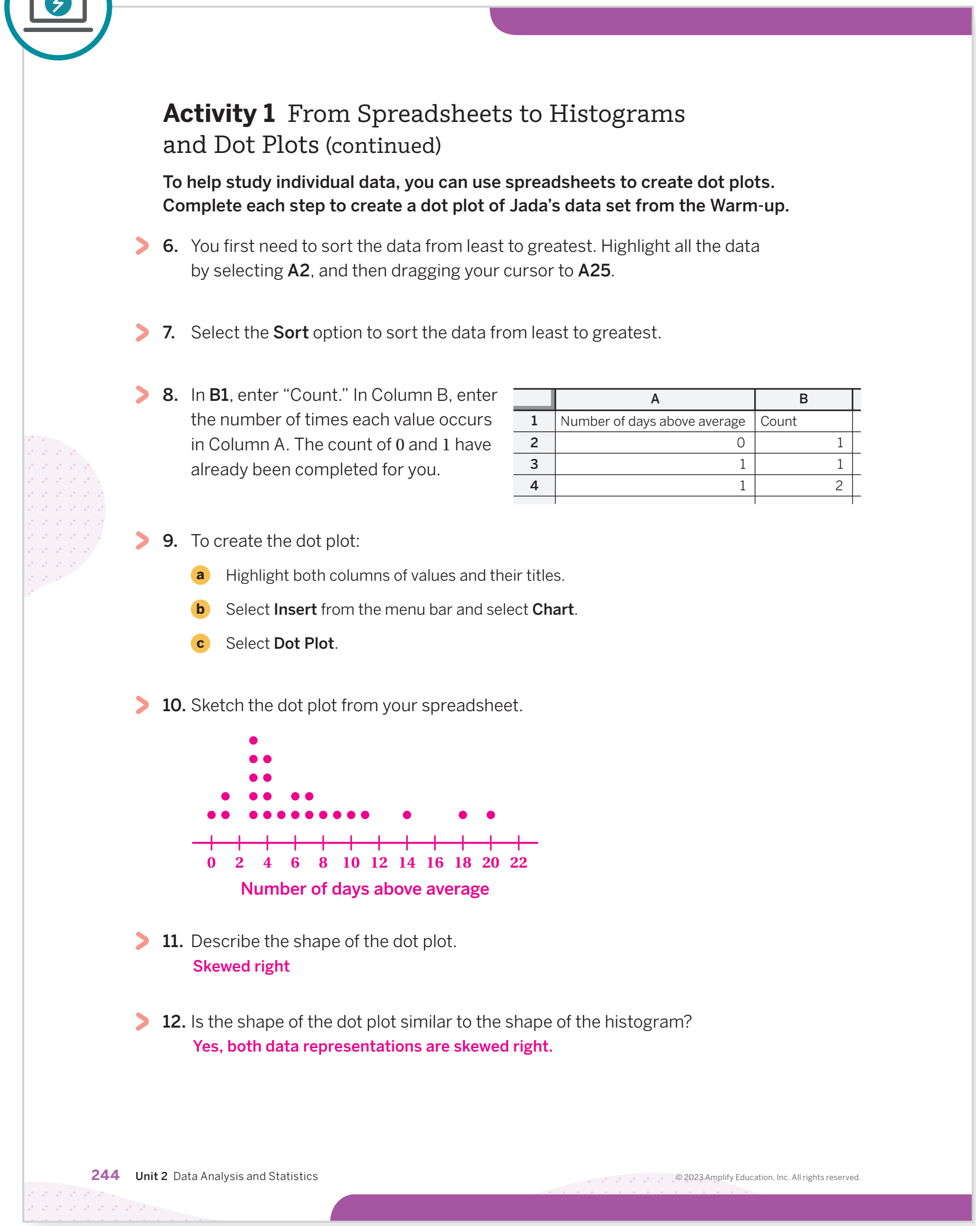
English Learners

Provide access to the Anchor Chart PDF, *Sentence Stems, Stronger and Clearer Each Time* and encourage students to borrow phrases from this chart during discussions.

Activity 1 From Spreadsheets to Histograms and Dot Plots (continued)

Pairs | 15 min

Students create histograms and dot plots using spreadsheet technology to examine the shape of the distribution of a data set.



Activity 1 From Spreadsheets to Histograms and Dot Plots (continued)

To help study individual data, you can use spreadsheets to create dot plots. Complete each step to create a dot plot of Jada's data set from the Warm-up.

6. You first need to sort the data from least to greatest. Highlight all the data by selecting **A2**, and then dragging your cursor to **A25**.

7. Select the **Sort** option to sort the data from least to greatest.

8. In **B1**, enter "Count." In Column B, enter the number of times each value occurs in Column A. The count of 0 and 1 have already been completed for you.

	A	B
1	Number of days above average	Count
2	0	1
3	1	1
4	1	2

9. To create the dot plot:

- a Highlight both columns of values and their titles.
- b Select **Insert** from the menu bar and select **Chart**.
- c Select **Dot Plot**.

10. Sketch the dot plot from your spreadsheet.

11. Describe the shape of the dot plot.
Skewed right

12. Is the shape of the dot plot similar to the shape of the histogram?
Yes, both data representations are skewed right.

244 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the histogram and dot plot of the data set.

Highlight that the count column is important for the spreadsheet to plot an accurate number of dots for each value in the dot plot. The spreadsheet uses the this column to count how many times a value is repeated to determine the frequency for the dot plot, just as when students were creating dot plots by hand.

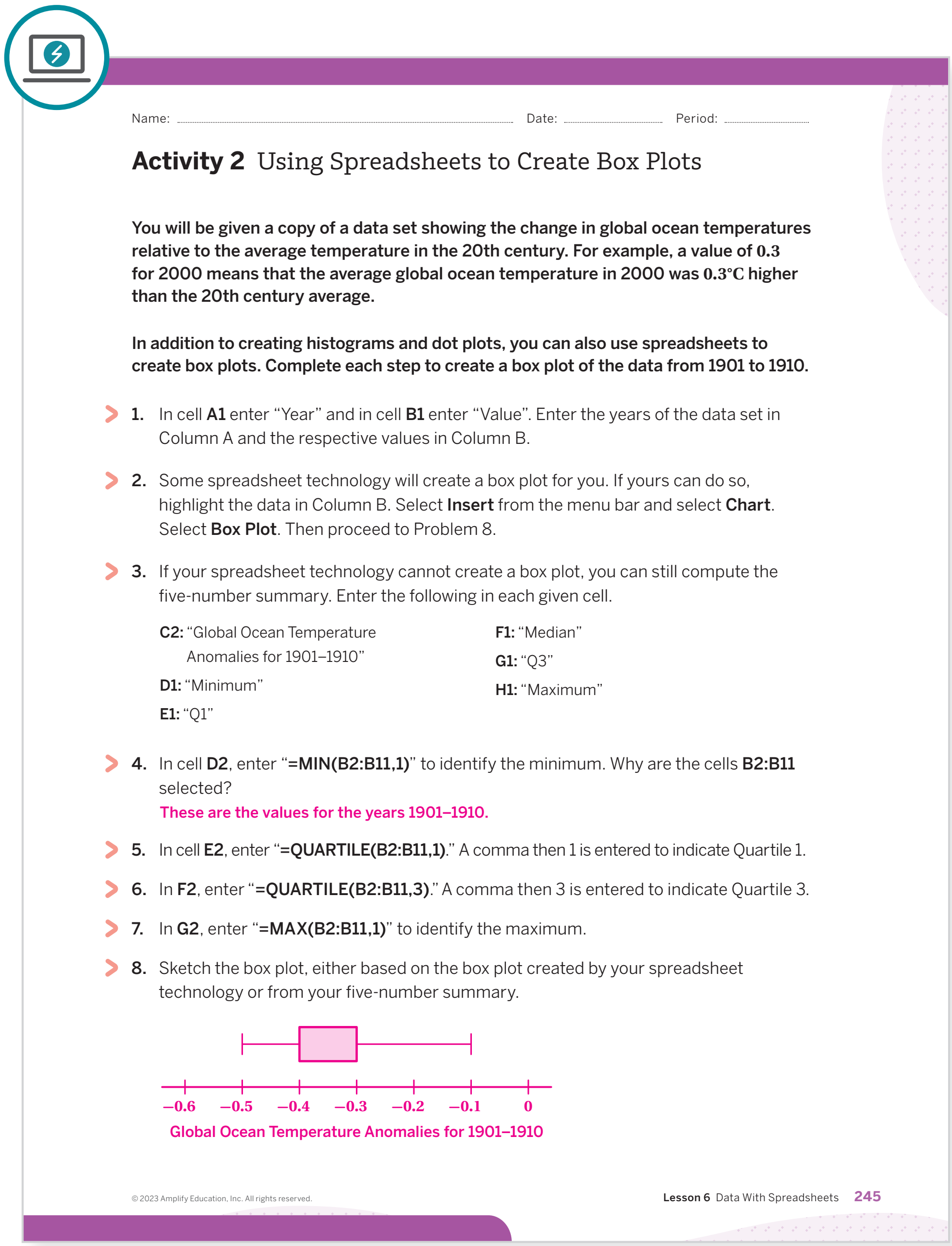
Have pairs of students share their responses and thinking to Problems 4 and 5.

Ask:

- "How does the interval size of the histogram affect how someone may describe the shape of the distribution?" If the interval size is too large or small, it may be difficult to determine the shape of the distribution.
- "Why might the shape of the histogram and the dot plot look slightly different?" The interval size of the histogram may affect the shape of the distribution so that it does not clearly resemble the shape of the dot plot.
- "If the values that appear to be outliers are removed from the data set, how would this affect the shape of the distribution?" With the removal of the outliers, the shape of the distribution would be bell-shaped.

Activity 2 Using Spreadsheets To Create Box Plots

Students create a box plot using spreadsheet technology to examine the shape of the center and spread of the data set.



Name: ______ Date: ______ Period: ______

Activity 2 Using Spreadsheets to Create Box Plots

You will be given a copy of a data set showing the change in global ocean temperatures relative to the average temperature in the 20th century. For example, a value of 0.3 for 2000 means that the average global ocean temperature in 2000 was 0.3°C higher than the 20th century average.

In addition to creating histograms and dot plots, you can also use spreadsheets to create box plots. Complete each step to create a box plot of the data from 1901 to 1910.

1. In cell **A1** enter "Year" and in cell **B1** enter "Value". Enter the years of the data set in Column A and the respective values in Column B.
2. Some spreadsheet technology will create a box plot for you. If yours can do so, highlight the data in Column B. Select **Insert** from the menu bar and select **Chart**. Select **Box Plot**. Then proceed to Problem 8.
3. If your spreadsheet technology cannot create a box plot, you can still compute the five-number summary. Enter the following in each given cell.

   C2: "Global Ocean Temperature Anomalies for 1901–1910"
   D1: "Minimum"
   E1: "Q1"
   F1: "Median"
   G1: "Q3"
   H1: "Maximum"
4. In cell **D2**, enter "**=MIN(B2:B11,1)**" to identify the minimum. Why are the cells **B2:B11** selected?
   These are the values for the years 1901–1910.
5. In cell **E2**, enter "**=QUARTILE(B2:B11,1)**." A comma then 1 is entered to indicate Quartile 1.
6. In **F2**, enter "**=QUARTILE(B2:B11,3)**." A comma then 3 is entered to indicate Quartile 3.
7. In **G2**, enter "**=MAX(B2:B11,1)**" to identify the maximum.
8. Sketch the box plot, either based on the box plot created by your spreadsheet technology or from your five-number summary.

Global Ocean Temperature Anomalies for 1901–1910

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 6 Data With Spreadsheets 245

① Launch

Distribute the Activities 2 & 3 PDF to each pair. Display the PDF, highlighting the data that should be entered for the activity.

② Monitor

Help students get started by showing them which chart type is the box plot.

Look for points of confusion:

- **Thinking that the spreadsheet can be used to calculate the 2nd and 4th quartile.** Ask, "What values in the five-number summary already represent these values?"
  The median and maximum.

Look for productive strategies:

- Copying and pasting formulas, but changing the function or the number for the quartile.

③ Connect

Display the box plot.

Have pairs of students share any strategies they found helpful when creating the box plots.

Highlight that it may be helpful to construct all three data representations to gain clarity about the center and spread of the data set, while keeping the axis scale the same so the representations can be easily compared.

Ask, "How is constructing a box plot in a spreadsheet different from constructing one by hand?" Answers may vary. Sample response: I do not need to calculate all the values in the five-number summary to construct the box plot.

Differentiated Support

Accessibility: *Guide Visualization and Processing*

Display or provide students with a copy of the Activity 2 PDF, which they can use to check their spreadsheets to see if they have entered values and formulas correctly.
Consider creating a pre-filled spreadsheet and displaying or making copies of the spreadsheet to distribute to students.

Extension: *Math Enrichment*

Have students complete the following problem:
Change one value in the data set so that the shape of the distribution is bell-shaped. Sample response: Change -0.5 to -0.1.

Math Language Development

MLR2: Collect and Display

As pairs work on the activity, circulate and collect key words, phrases, and methods students use while creating the box plot. Display phrases and spreadsheet formulas, depending upon the specific type of spreadsheet technology used. Continue to update collected student language throughout the entire lesson.

English Learners

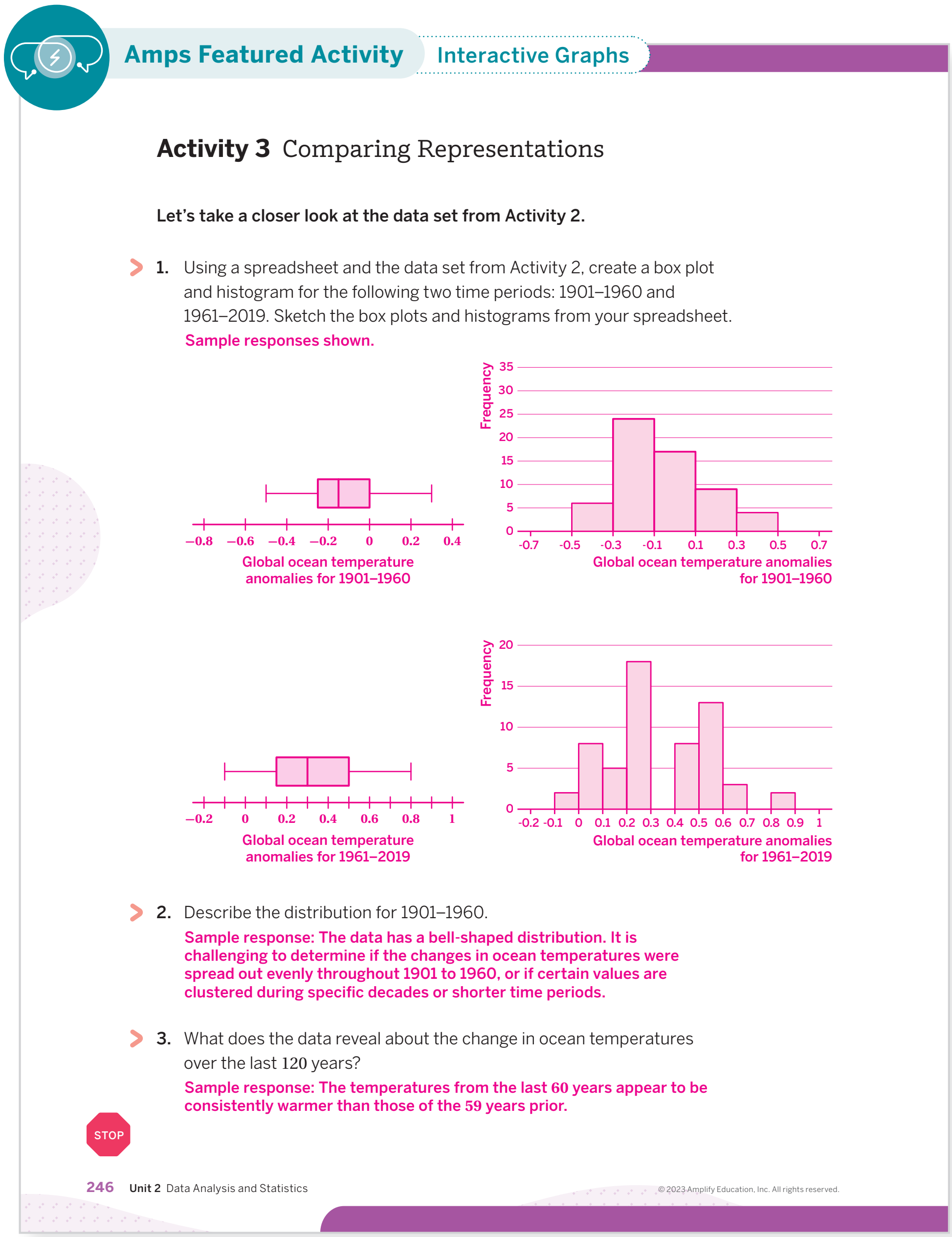
Annotate a box plot on the class display to highlight how the key words and phrases connect to the box plot.

Independent | 10 min

Activity 3 Comparing Representations

Students use technology to construct box plots and histograms for two large data sets to compare them.

Amps Featured Activity Interactive Graphs



Activity 3 Comparing Representations

Let's take a closer look at the data set from Activity 2.

1. Using a spreadsheet and the data set from Activity 2, create a box plot and histogram for the following two time periods: 1901–1960 and 1961–2019. Sketch the box plots and histograms from your spreadsheet.
   Sample responses shown.

2. Describe the distribution for 1901–1960.
   Sample response: The data has a bell-shaped distribution. It is challenging to determine if the changes in ocean temperatures were spread out evenly throughout 1901 to 1960, or if certain values are clustered during specific decades or shorter time periods.

3. What does the data reveal about the change in ocean temperatures over the last 120 years?
   Sample response: The temperatures from the last 60 years appear to be consistently warmer than those of the 59 years prior.

STOP

246 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Highlight the two time periods that will be used for the activity.

2 Monitor

Help students get started by helping them identify which values for the data set they will use that correspond to the two time periods.

Look for points of confusion:

- **Using only the shape of the distribution to draw conclusions.** Have students examine and use the horizontal and vertical scales of their data representations.

Look for productive strategies:

- Using the same horizontal and vertical scale for all data representations so that they can be easily compared.

3 Connect

Have individual students share their data representations.

Highlight that the shape of the distribution for both data sets is bell-shaped, but that the scale of the axes show that the ocean temperatures are increasing.

Ask, "Which representations, the box plots or histograms, did you use to respond to Problem 3? Explain your thinking."

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, assign different students to create either a histogram or a box plot for one of the two different time periods. Consider allowing students to choose which representation and time period they will use, but make sure that all four displays are created by various students in the class. After the displays are created, display all four representations and facilitate a class discussion for Problems 2–3.

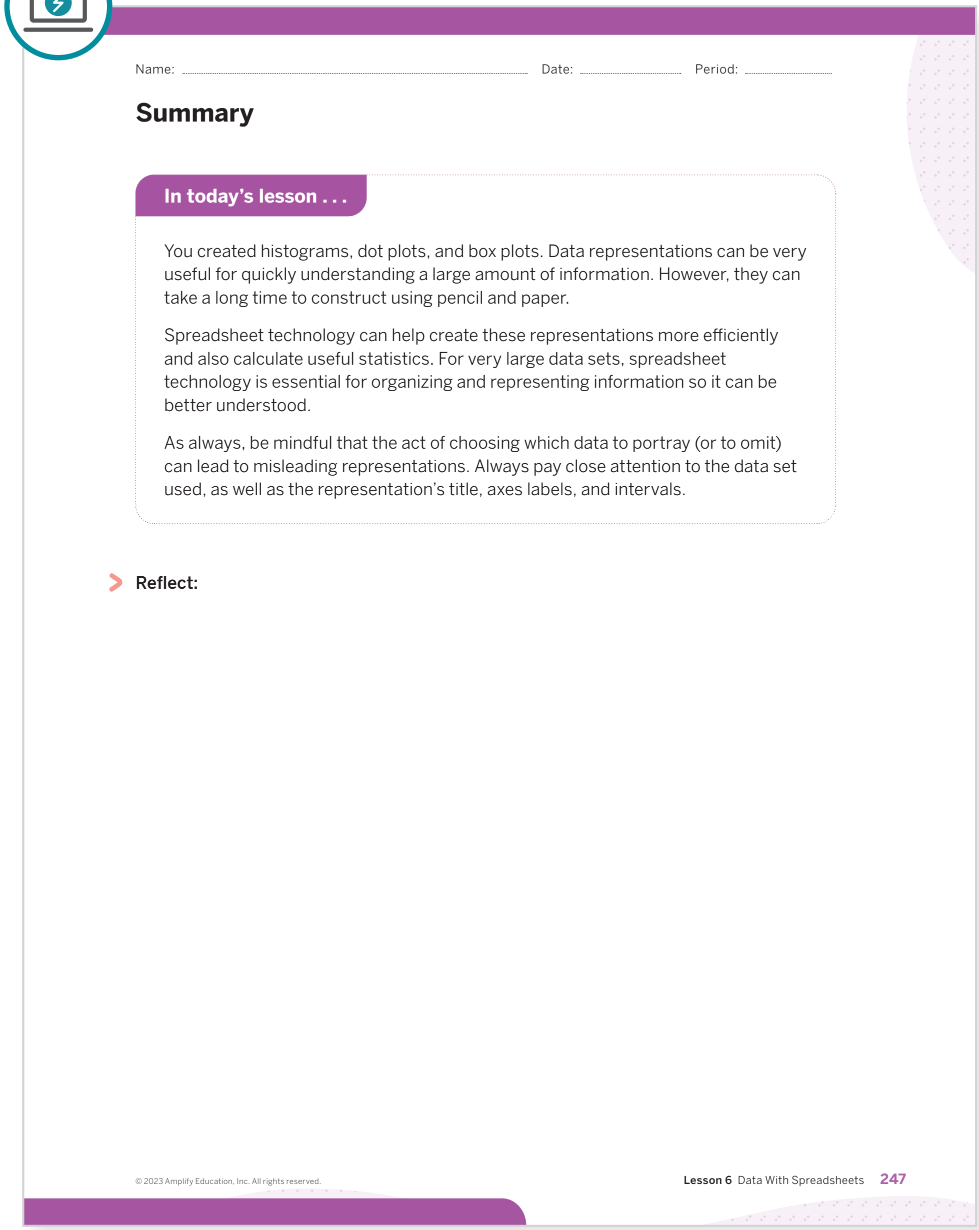
Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital box plots and histograms to represent the two different time periods.

Whole Class | 5 min

Summary

Review and synthesize creating data representations and calculating statistics of a data set using spreadsheet technology.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You created histograms, dot plots, and box plots. Data representations can be very useful for quickly understanding a large amount of information. However, they can take a long time to construct using pencil and paper.

Spreadsheet technology can help create these representations more efficiently and also calculate useful statistics. For very large data sets, spreadsheet technology is essential for organizing and representing information so it can be better understood.

As always, be mindful that the act of choosing which data to portray (or to omit) can lead to misleading representations. Always pay close attention to the data set used, as well as the representation's title, axes labels, and intervals.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display a student's data representations from Activity 3.

Have students share which data representations they found most efficient or most challenging to create using spreadsheets.

Highlight that using spreadsheet technology allows students to compute statistics, create data representations, and analyze large amounts of data quickly.

Ask:

- "When do you think it is helpful to use technology to construct data representations or to calculate statistics?" Sample response: With large data sets, using technology for data representations and to calculate statistics helps to limit the possibility of making incorrect calculations.
- "What are the benefits in using multiple data representations for a data set?" Multiple data representations provide further clarity of the spread, center, and shape of the distribution of a data set.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

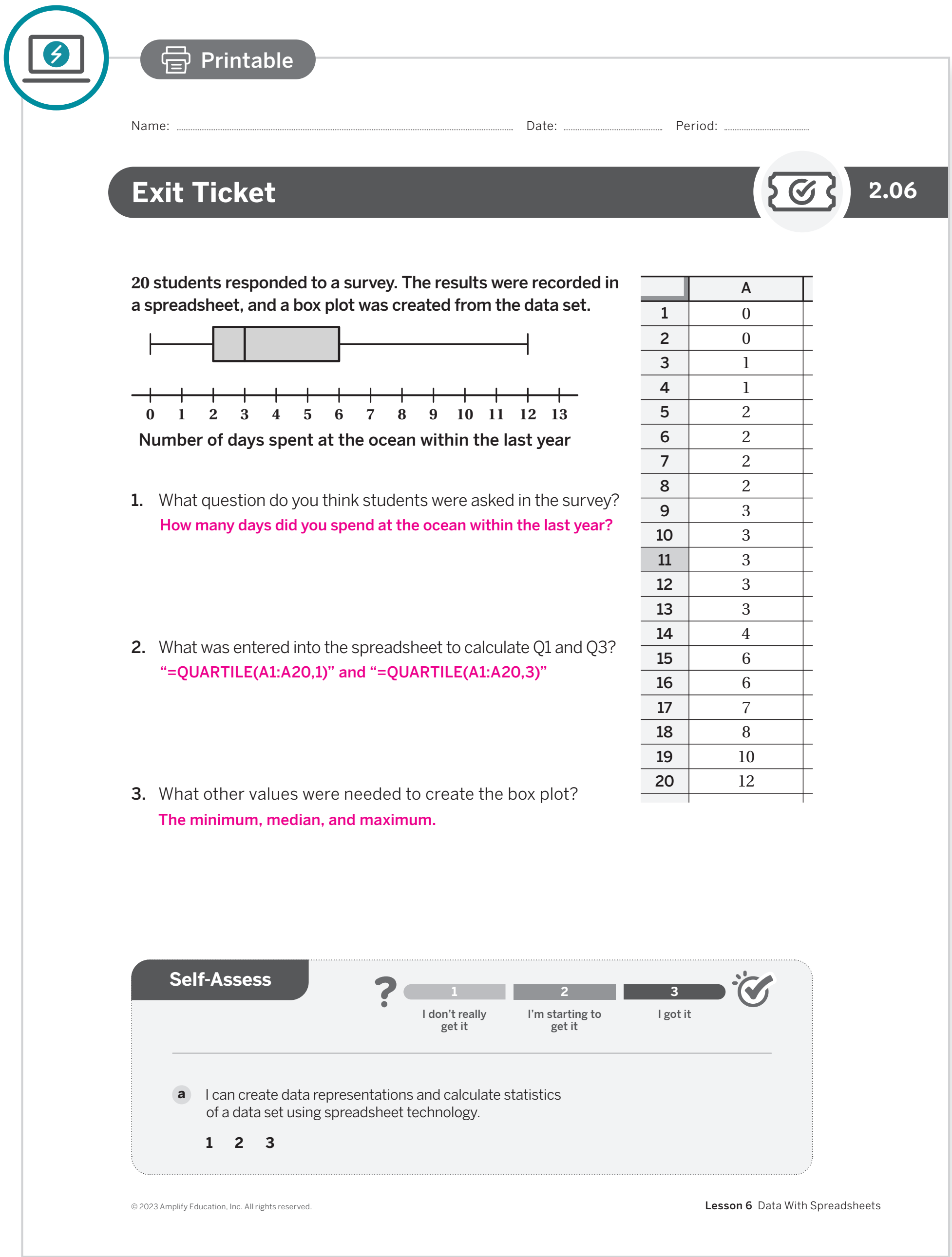
- "Why are spreadsheets useful in creating data representations?"
- "Is there a data representation or statistic you would prefer to create or calculate by hand?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by describing how to use spreadsheet technology to create the given data representations.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 2.06

20 students responded to a survey. The results were recorded in a spreadsheet, and a box plot was created from the data set.

	A
1	0
2	0
3	1
4	1
5	2
6	2
7	2
8	2
9	3
10	3
11	3
12	3
13	3
14	4
15	6
16	6
17	7
18	8
19	10
20	12

1. What question do you think students were asked in the survey?
   How many days did you spend at the ocean within the last year?

2. What was entered into the spreadsheet to calculate Q1 and Q3?
   "=QUARTILE(A1:A20,1)" and "=QUARTILE(A1:A20,3)"

3. What other values were needed to create the box plot?
   The minimum, median, and maximum.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can create data representations and calculate statistics of a data set using spreadsheet technology.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Success looks like . . .

- **Goal:** Using technology tools to graphically represent data and calculate useful statistics.
  - » Explaining the formula used in the spreadsheet to determine the quartiles in Problem 2.

Suggested next steps

If students provide a vague or inaccurate response for Problem 1, consider:

- Reviewing creating axes titles from Activity 1.
- Assigning Practice Problems 2 and 3.
- Asking, "How could you use the title of the horizontal axis to write a possible question?"

If students provide an incorrect response for Problems 2 and 3, consider:

- Reviewing the values needed for a box plot and how to determine them from Activity 3.
- Assigning Practice Problem 2.
- Asking, "What are the values of the five-number summary?"

Professional Learning

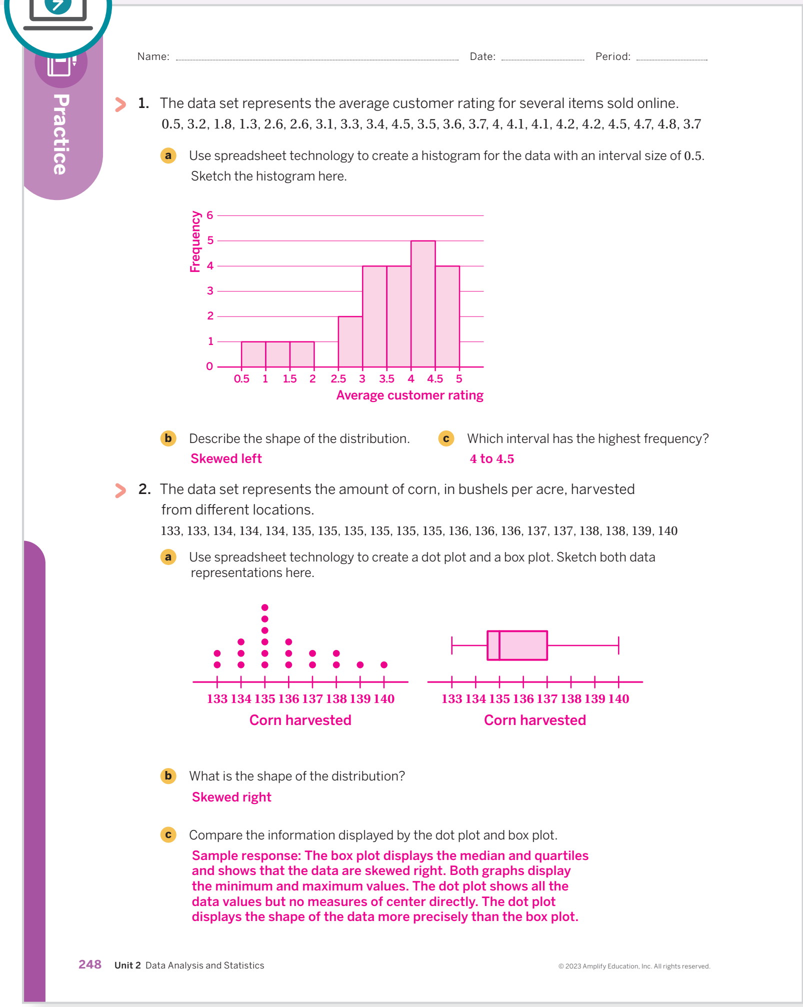
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students used spreadsheets to create histograms, dot plots, and box plots. How will that support using spreadsheet technology to investigate standard deviation?
- What different ways did students approach describing outliers in Activity 1? What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. The data set represents the average customer rating for several items sold online.
   0.5, 3.2, 1.8, 1.3, 2.6, 2.6, 3.1, 3.3, 3.4, 4.5, 3.5, 3.6, 3.7, 4, 4.1, 4.1, 4.2, 4.2, 4.5, 4.7, 4.8, 3.7

   a. Use spreadsheet technology to create a histogram for the data with an interval size of 0.5. Sketch the histogram here.

   b. Describe the shape of the distribution.
   Skewed left

   c. Which interval has the highest frequency?
   4 to 4.5

2. The data set represents the amount of corn, in bushels per acre, harvested from different locations.
   133, 133, 134, 134, 134, 135, 135, 135, 135, 135, 135, 136, 136, 136, 137, 137, 138, 138, 139, 140

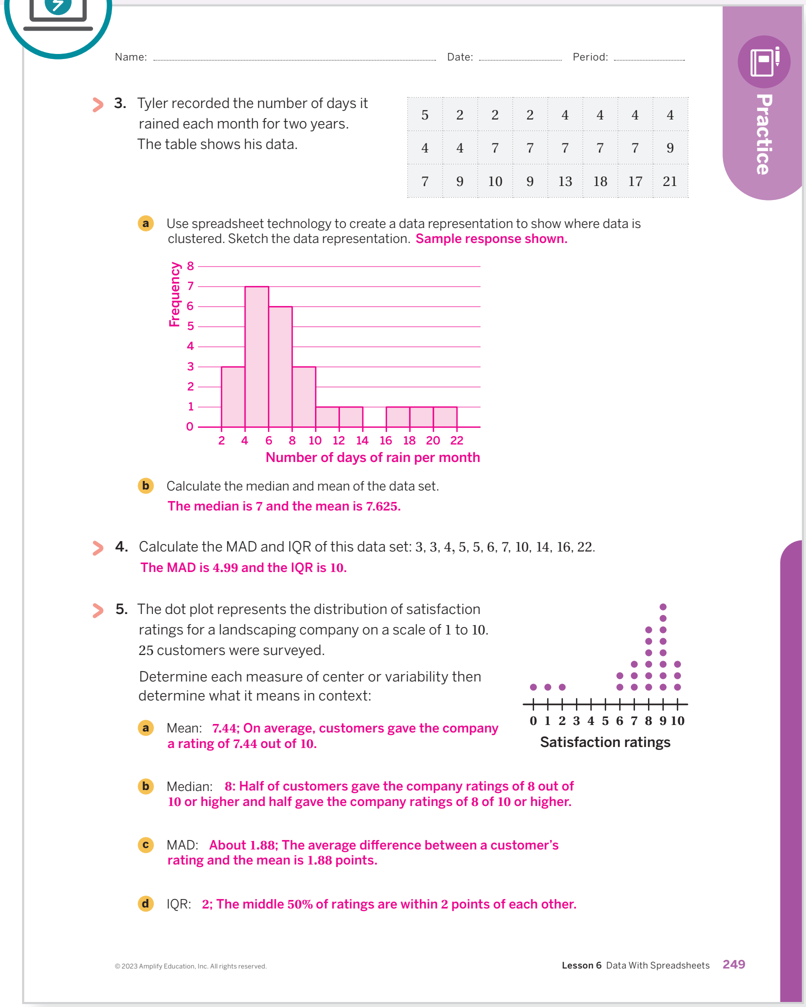
   a. Use spreadsheet technology to create a dot plot and a box plot. Sketch both data representations here.

   b. What is the shape of the distribution?
   Skewed right

   c. Compare the information displayed by the dot plot and box plot.
   Sample response: The box plot displays the median and quartiles and shows that the data are skewed right. Both graphs display the minimum and maximum values. The dot plot shows all the data values but no measures of center directly. The dot plot displays the shape of the data more precisely than the box plot.

248 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: Date: Period:

3. Tyler recorded the number of days it rained each month for two years. The table shows his data.

5	2	2	2	4	4	4	4
4	4	7	7	7	7	7	9
7	9	10	9	13	18	17	21

   a. Use spreadsheet technology to create a data representation to show where data is clustered. Sketch the data representation. **Sample response shown.**

   b. Calculate the median and mean of the data set.
   The median is 7 and the mean is 7.625.

4. Calculate the MAD and IQR of this data set: 3, 3, 4, 5, 5, 6, 7, 10, 14, 16, 22.
   The MAD is 4.99 and the IQR is 10.

5. The dot plot represents the distribution of satisfaction ratings for a landscaping company on a scale of 1 to 10. 25 customers were surveyed.

   Determine each measure of center or variability then determine what it means in context:

   a. Mean: **7.44; On average, customers gave the company a rating of 7.44 out of 10.**

   b. Median: **8: Half of customers gave the company ratings of 8 out of 10 or higher and half gave the company ratings of 8 of 10 or higher.**

   c. MAD: **About 1.88; The average difference between a customer's rating and the mean is 1.88 points.**

   d. IQR: **2; The middle 50% of ratings are within 2 points of each other.**

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 6 Data With Spreadsheets 249

Practice Problem Analysis

Type	Problem	Refer to	DOK
	1	Activity 1	2
On-lesson	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 2 Lesson 4	2
Formative ⚡	5	Unit 2 Lesson 7	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 2

Standard Deviation

Whole Class

In this Sub-Unit, students encounter standard deviation, the most commonly used measure of variability, and learn how it is computed. They also choose measures of center and variability based on a distribution's shape.

SUB-UNIT

2 Standard Deviation

Narrative Connections

Is Sandy the new normal?

New York City is known as the city that never sleeps. But when Hurricane Sandy slammed into the east coast of the U.S. in 2012, it brought much of the city to a standstill. The hurricane created a massive storm surge — a 14-foot wall of water — that poured into streets and subways. It paralyzed the city and devastated the surrounding areas. By the time the storm had passed, it had killed over 200 people and caused $70 billion worth of damage to the Caribbean and the eastern U.S.

Usually, storms like Sandy are pushed away into the North Atlantic by air currents known as the jet stream. But an unusual bend in the jet stream caused by a mass of high-pressure cold air redirected Sandy toward the northeastern U.S. Combined with hurricane winds, this created a rare post-tropical cyclone called a "superstorm." Through warming oceans, rising sea levels, and increased atmospheric moisture, researchers believe that superstorm events could become more frequent. Rather than occuring once every 400 years, they believe events like Sandy will occur about once every 23 years by the end of the century.

Researchers reached these conclusions by investigating historical data and analyzing climate models. By observing how distributions of storm data change over time, they can determine how likely extreme events (or outliers) like Sandy occur. You already encountered different ways of measuring how spread out a distribution is to identify outliers. However, there is one measure that is used more frequently than the others. In the next few lessons, you will learn about this measure. Then, you will be able to determine for yourself whether superstorms like Sandy are the new normal.

© 2023 Amplify Education, Inc. All rights reserved.

Sub-Unit 2 Standard Deviation 251

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how the standard deviation can help them understand weather anomalies in the following places:

- **Lesson 7, Activity 3:** Comparing Statistics
- **Lesson 8, Activity 1:** Extreme Heat and Extreme Cold
- **Lesson 9, Activity 1:** Hurricane Frequency
- **Lesson 10, Activities 1–2:** Investigating Outliers, Where Do Outliers Come From?

UNIT 2 | **LESSON 7**

Standard Deviation

Let's explore another measure of variability.

Focus

Goals

1. **Language Goal:** Comprehend standard deviation as a measure of variability. **(Speaking and Listening, Reading and Writing)**
2. Calculate an approximate value for the standard deviation of a data set.
3. Use technology to compute standard deviation.

Coherence

Today

Students draw squares to calculate and conceptualize the standard deviation of a data set, comparing this to the calculation of the MAD. Students also use spreadsheet technology to calculate standard deviation and compare the variability of data sets.

Previously

Students have calculated MAD and IQR as measures for variability. In Lesson 6, they used spreadsheet technology to compute MAD and IQR.

Coming Soon

Students will compare standard deviation and IQR as they determine which is a more appropriate measure of variability for a data set (based on the distribution).

Rigor

- Students build a **conceptual understanding** of standard deviation using a geometric interpretation of squares and square roots.
- Students compare data sets to develop **fluency** in using standard deviation as a measure of variability.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	20 min	15 min	10 min	5 min	5 min
Independent	Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- colored pens/pencils
- four-function calculators
- graph paper
- rulers (or straightedges)
- spreadsheet technology

Math Language Development

New words

- ***standard deviation***

Review words

- *MAD*
- *statistic*
- *variability*

Amps | Featured Activity

Activity 1

Dynamic Standard Deviation

Students calculate standard deviation using a digitally interactive geometric approach and are able to visualize what it means and how it compares to the MAD.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might struggle to figure out how to use a spreadsheet to calculate the standard deviation in Activity 2. In order to achieve this goal, remind students that they need to understand the process they follow each time. Encourage them to write down their own code for what steps they need to take to calculate with the spreadsheet.

Modifications to Pacing

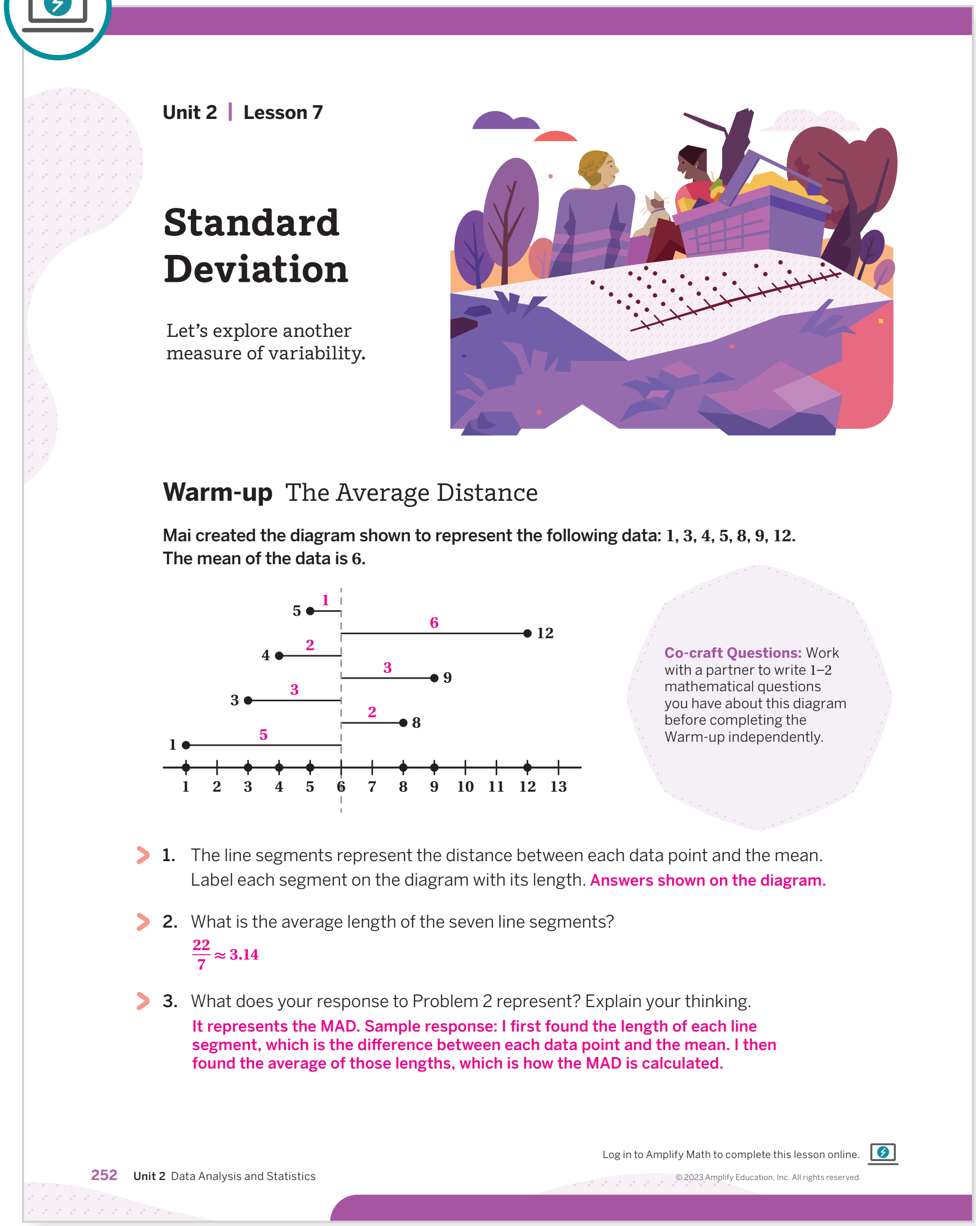
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, Problem 3 may be omitted.
- **Activity 3** is optional and may be omitted.

Independent | 5 min

Warm-up The Average Distance

Students display data on a number line to visualize calculating the mean absolute deviation (MAD).



Unit 2 | Lesson 7

Standard Deviation

Let's explore another measure of variability.

Warm-up The Average Distance

Mai created the diagram shown to represent the following data: 1, 3, 4, 5, 8, 9, 12. The mean of the data is 6.

Co-craft Questions: Work with a partner to write 1–2 mathematical questions you have about this diagram before completing the Warm-up independently.

1. The line segments represent the distance between each data point and the mean. Label each segment on the diagram with its length. **Answers shown on the diagram.**

2. What is the average length of the seven line segments?

   $\frac{22}{7} \approx 3.14$

3. What does your response to Problem 2 represent? Explain your thinking.

   It represents the MAD. Sample response: I first found the length of each line segment, which is the difference between each data point and the mean. I then found the average of those lengths, which is how the MAD is calculated.

Log in to Amplify Math to complete this lesson online.

252 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display Mai's diagram, and make sure students recognize that the endpoint of each line segment is a value from the data set.
Make four-function calculators available.

2 Monitor

Help students get started by asking, "What does the dotted line represent?" The mean.

Look for points of confusion:

- **Calculating negative lengths.** Ask, "Can a distance be negative?" Prompt students to count the distance between the endpoints or use absolute value.
- **Adding the data set values when determining the average length of the line segments.** Ask, "Does it make sense for the data to represent the lengths of the segments?"

Look for productive strategies:

- Annotating the diagram with the length of each segment.
- Using absolute value when determining the distance for each line segment.
- Recognizing MAD in Problem 4.

3 Connect

Display the diagram.

Have students share their process for determining the average length using the diagram and which statistic it represents.

Ask students to sketch the average length of the line segment from Problem 2 on their graphs. "What does this line represent in terms of the data?" It represents the MAD because it is the mean distance between the data and the mean.

Power-up

To power up students' ability to interpret the measures of center and variability in context, have students complete:

Mai collected data on the number of wi-fi enabled devices in seven randomly selected households. Her results were:

Mean: 6 Median: 5 MAD: 3.14 IQR: 6

Determine which of the following statements are true based on her results. Select *all* that apply.

A. The mean distance between the data values and the mean is 6.

[✓ **B.**] Half of the households had 5 or fewer wi-fi-enabled devices.

[✓ **C.**] On average, the households had 6 wifi-enabled devices.

D. The difference between the least number of wi-fi-enabled devices and greatest number of wifi-enabled devices is 6.

Use: Before the Warm-up

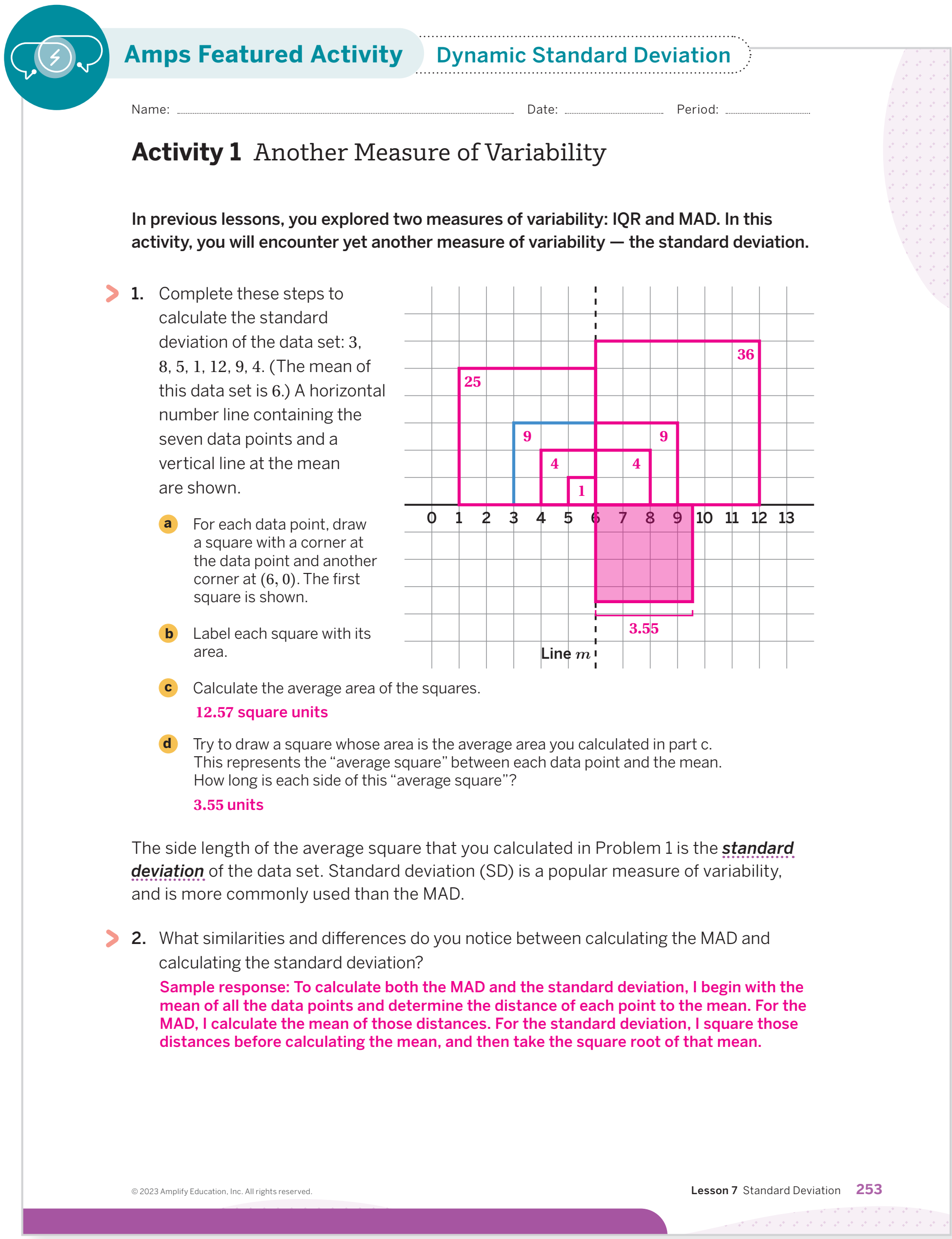
Informed by: Performance on Lesson 6, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4

Independent | 20 min

Activity 1 Another Measure of Variability

Students draw squares to geometrically calculate and interpret a data set's standard deviation.

Amps Featured Activity Dynamic Standard Deviation



Name: ________ Date: ________ Period: ________

Activity 1 Another Measure of Variability

In previous lessons, you explored two measures of variability: IQR and MAD. In this activity, you will encounter yet another measure of variability — the standard deviation.

1. Complete these steps to calculate the standard deviation of the data set: $3, 8, 5, 1, 12, 9, 4$. (The mean of this data set is 6.) A horizontal number line containing the seven data points and a vertical line at the mean are shown.

   a. For each data point, draw a square with a corner at the data point and another corner at $(6, 0)$. The first square is shown.

   b. Label each square with its area.

   c. Calculate the average area of the squares.

   12.57 square units

   d. Try to draw a square whose area is the average area you calculated in part c. This represents the "average square" between each data point and the mean. How long is each side of this "average square"?

   3.55 units

The side length of the average square that you calculated in Problem 1 is the ***standard deviation*** of the data set. Standard deviation (SD) is a popular measure of variability, and is more commonly used than the MAD.

2. What similarities and differences do you notice between calculating the MAD and calculating the standard deviation?

   Sample response: To calculate both the MAD and the standard deviation, I begin with the mean of all the data points and determine the distance of each point to the mean. For the MAD, I calculate the mean of those distances. For the standard deviation, I square those distances before calculating the mean, and then take the square root of that mean.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 7 Standard Deviation 253

1 Launch

Distribute graph paper, rulers, and colored pens/pencils to each student. Have students complete Problems 1 and 2 independently, then share their work with a partner. If differing responses arise, have them work together to reach a consensus. Then have students complete Problem 3 with their partner.

2 Monitor

Help students get started by modeling the instructions using the provided example.

Look for points of confusion:

- **Excluding the area of squares with double values in Problems 3 and 4.** Have students compare the number of data values to the number of squares they sketched.
- **Squaring the values of the data set to determine the areas of the squares.** Refer to the Warm-up, reminding students that they are working with distances rather than data values.

Look for productive strategies:

- Using a ruler to sketch the side lengths of the "average square".
- Understanding properties of a square to utilize the line segment lengths.
- Generalizing a process to calculate the standard deviation in Problem 3.

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology, Guide Processing and Visualization*

Have students use the Amps slides for this activity, in which they can use a geometric approach to visualize and calculate the standard deviation and how this measure of variability compares to the MAD.

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with a graph of several squares pre-populated for the data set. Have them determine the areas of the pre-populated squares and then sketch the remaining squares and determine those areas.

Accessibility: *Activate Prior Knowledge, Clarify Vocabulary and Symbols*

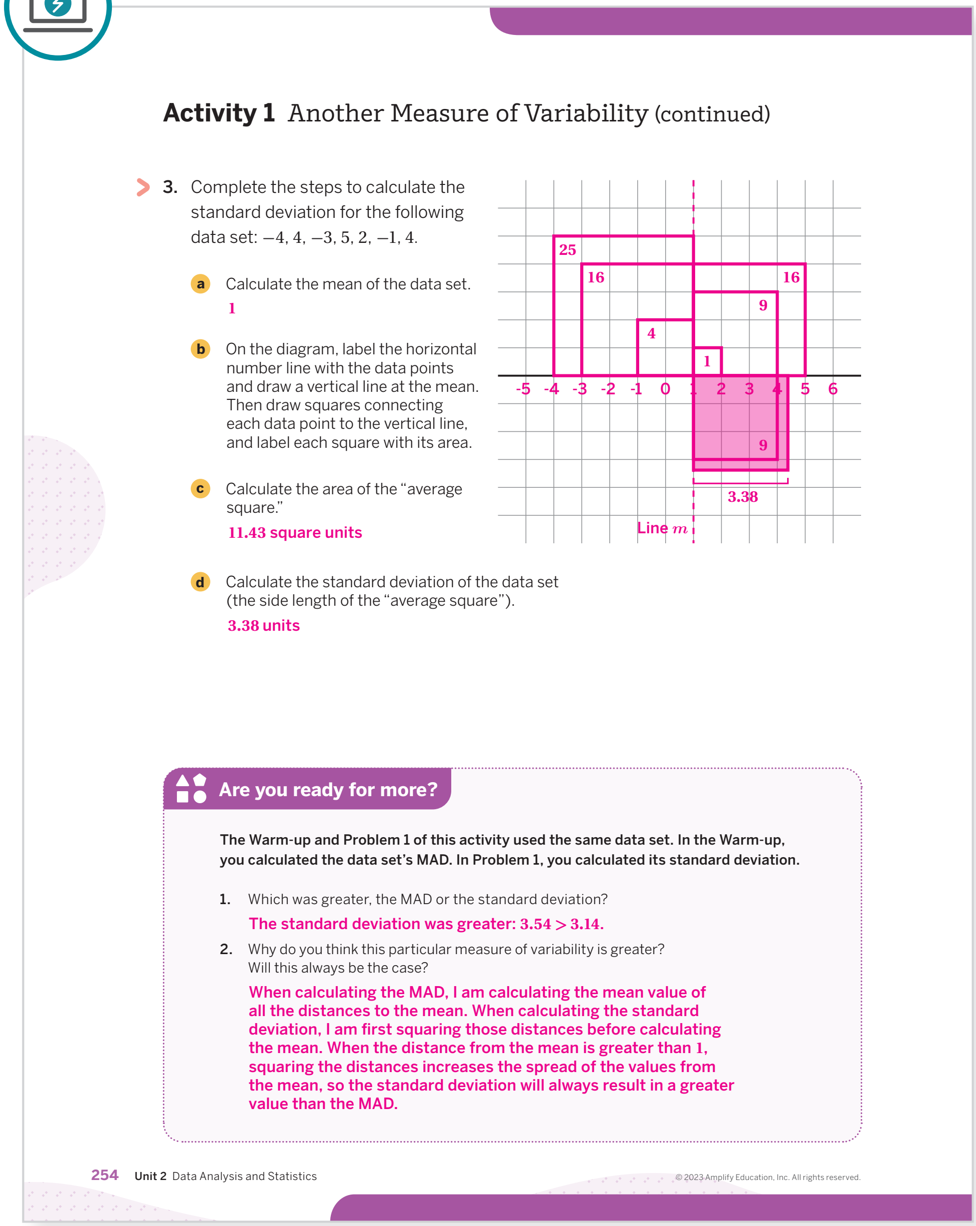
Before students begin the activity, remind them of the IQR (interquartile range) and MAD (mean absolute deviation) and what they describe about a data set. At some point in the activity, or during the Connect, emphasize the acronym they will use for the standard deviation is SD. Consider creating and displaying a graphic organizer that compares and contrasts these three measures of variability:

- Interquartile range (IQR)
- Mean absolute deviation (MAD)
- Standard deviation (SD)

Independent | 20 min

Activity 1 Another Measure of Variability (continued)

Students draw squares to geometrically calculate and interpret a data set's standard deviation.



Activity 1 Another Measure of Variability (continued)

3. Complete the steps to calculate the standard deviation for the following data set: $-4, 4, -3, 5, 2, -1, 4$.

   a. Calculate the mean of the data set.

   1

   b. On the diagram, label the horizontal number line with the data points and draw a vertical line at the mean. Then draw squares connecting each data point to the vertical line, and label each square with its area.

   c. Calculate the area of the "average square."

   11.43 square units

   d. Calculate the standard deviation of the data set (the side length of the "average square").

   3.38 units

Are you ready for more?

The Warm-up and Problem 1 of this activity used the same data set. In the Warm-up, you calculated the data set's MAD. In Problem 1, you calculated its standard deviation.

1. Which was greater, the MAD or the standard deviation?

   The standard deviation was greater: $3.54 > 3.14$.

2. Why do you think this particular measure of variability is greater? Will this always be the case?

   When calculating the MAD, I am calculating the mean value of all the distances to the mean. When calculating the standard deviation, I am first squaring those distances before calculating the mean. When the distance from the mean is greater than 1, squaring the distances increases the spread of the values from the mean, so the standard deviation will always result in a greater value than the MAD.

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have pairs of students share their resulting graphs for Problems 1 and 3, modeling their strategies for creating their graphs. Select and sequence students using productive strategies, highlighting anyone generalizing the process. Discuss the process for calculating the side length of the "average square."

Define the term ***standard deviation***.

Ask, "How are the processes for calculating the standard deviation and MAD similar? How are they different?"

Highlight that the side length of the "average square" represents the standard deviation of a data set, similar to the average distance representing the MAD. This measure of variability is used most commonly in applications. It is also typically calculated using technology.

Note: There are *two* formulas for standard deviation. In this activity, students computed the average before taking the square root, dividing the sum of the squares by the number of data points. Standard deviation is more commonly computed by dividing the sum by *one less* than the number of data points. Students should be made aware that their hand calculations of standard deviation with "average squares" may be slightly different from the standard deviation calculated using technology. They can learn about this distinction in an advanced statistics course.

Differentiated Support

Extension: *Math Enrichment*

Have students write a procedure they can use to determine the MAD and SD for a data set, and then explain how they are similar and how they are different. Sample response: When calculating either the MAD or the SD, I need to determine the distance each data value is from the mean. However, with the MAD, I then determine the absolute value of these distances and then determine the average distance. With the SD, I determine the squares of these distances, determine the average square, and then take the square root.

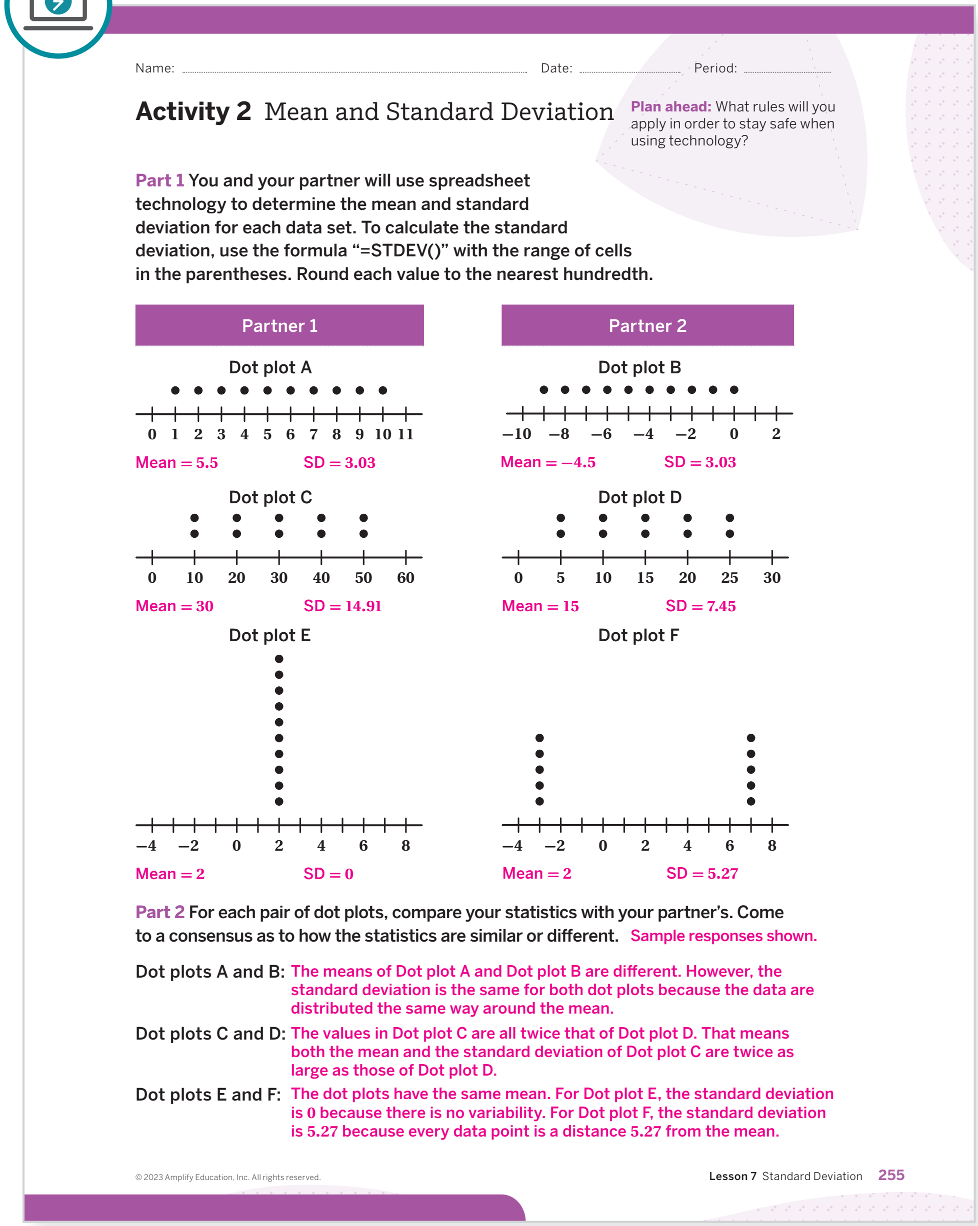
Ask students to explain why using either the MAD or the SD ensures that the distances each data value is from the mean is a positive value. Sample response: When using the MAD, I determine the absolute value of the distances and absolute value is always positive. When using the SD, I determine the squares of these distances and squaring a value always results in a positive value.

Tell students that most mathematicians and statisticians use the SD, as opposed to the MAD. This is because the SD has some nice mathematical properties that students can study further in advanced statistics or science courses.

Pairs | 15 min

Activity 2 Mean and Standard Deviation

Students use technology to calculate the mean and standard deviation of different data sets to understand how they are affected by a distribution's shape and scale.



Name: Date: Period:

Activity 2 Mean and Standard Deviation

Plan ahead: What rules will you apply in order to stay safe when using technology?

Part 1 You and your partner will use spreadsheet technology to determine the mean and standard deviation for each data set. To calculate the standard deviation, use the formula "=STDEV()" with the range of cells in the parentheses. Round each value to the nearest hundredth.

Partner 1

Dot plot A
0 1 2 3 4 5 6 7 8 9 10 11
Mean = 5.5 SD = 3.03

Dot plot C
0 10 20 30 40 50 60
Mean = 30 SD = 14.91

Dot plot E
−4 −2 0 2 4 6 8
Mean = 2 SD = 0

Partner 2

Dot plot B
−10 −8 −6 −4 −2 0 2
Mean = −4.5 SD = 3.03

Dot plot D
0 5 10 15 20 25 30
Mean = 15 SD = 7.45

Dot plot F
−4 −2 0 2 4 6 8
Mean = 2 SD = 5.27

Part 2 For each pair of dot plots, compare your statistics with your partner's. Come to a consensus as to how the statistics are similar or different. Sample responses shown.

Dot plots A and B: The means of Dot plot A and Dot plot B are different. However, the standard deviation is the same for both dot plots because the data are distributed the same way around the mean.

Dot plots C and D: The values in Dot plot C are all twice that of Dot plot D. That means both the mean and the standard deviation of Dot plot C are twice as large as those of Dot plot D.

Dot plots E and F: The dot plots have the same mean. For Dot plot E, the standard deviation is 0 because there is no variability. For Dot plot F, the standard deviation is 5.27 because every data point is a distance 5.27 from the mean.

© 2023 Amplify Education, Inc. All rights reserved. Lesson 7 Standard Deviation 255

1 Launch

Spreadsheet technology is required. Introduce the spreadsheet function "=STDEV()" for calculating standard deviation.

2 Monitor

Help students get started by demonstrating how to translate data from a dot plot onto a spreadsheet.

Look for points of confusion:

- **Attempting to calculate the mean and standard deviation by hand.** Encourage students to use their spreadsheet technology, perhaps by asking which is more efficient.
- **Struggling to articulate similarities and differences between the dot plots.** Have students focus on the horizontal axis, noting similarities and differences.

Look for productive strategies:

- Creating numeric lists of data before entering the data into a spreadsheet.
- Sketching the mean on the dot plots.

3 Connect

Display each pair of dot plots.

Have pairs of students share the statistics they calculated for the dot plots.

Ask:

- "For Dot plots A and B, why is the standard deviation the same?" The data is distributed the same way around the mean.
- "The data in Dot plot C are double the data in Dot plot D. What happened to the mean and standard deviation?" The mean and standard deviation both doubled as well.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

Provide more structure to Part 2 by asking these questions:

- "For which dot plots are the means different, but the standard deviations are the same? Why do you think that is the case?"
- "For which dot plots are the means the same? How do the standard deviations compare? Why are they different?"
- "Which dot plots 'look the same' without considering the scale on the number line? What do you notice about their statistics?"

Extension: ***Math Enrichment***

Challenge students to create the following data sets:

- 10 data values whose mean is 6 and whose standard deviation is equal to that of Dot plot A.
- 10 data values whose standard deviation is 3 times greater than that of Dot plot A.

Math Language Development

MLR5: Co-craft Questions

After completing Part 2 of the activity, ask pairs of students to write 1–2 mathematical questions they may have about the distributions. Invite students to share their questions with the class. This will help students process the relationships between the mean and standard deviation in this task.

English Learners

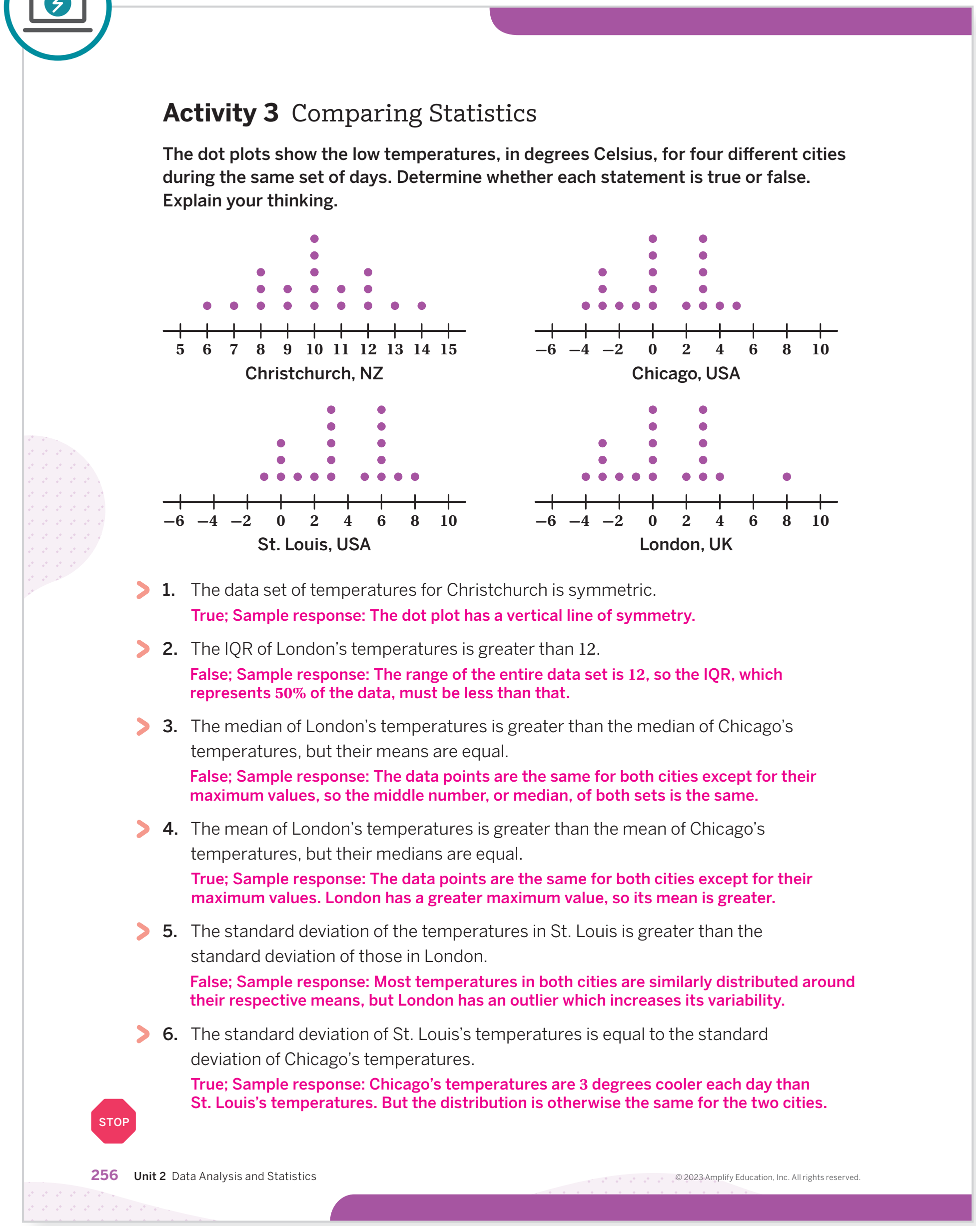
Model a mathematical question for students such as, "How does the changing of the scale affect the data points?" to help students build metalinguistic awareness.

Optional

Pairs | 10 min

Activity 3 Comparing Statistics

Students interpret and compare dot plots to determine if statements about their distribution, measures of center, and measures of variability are true.



Activity 3 Comparing Statistics

The dot plots show the low temperatures, in degrees Celsius, for four different cities during the same set of days. Determine whether each statement is true or false. Explain your thinking.

1. The data set of temperatures for Christchurch is symmetric.
   True; Sample response: The dot plot has a vertical line of symmetry.
2. The IQR of London's temperatures is greater than 12.
   False; Sample response: The range of the entire data set is 12, so the IQR, which represents 50% of the data, must be less than that.
3. The median of London's temperatures is greater than the median of Chicago's temperatures, but their means are equal.
   False; Sample response: The data points are the same for both cities except for their maximum values, so the middle number, or median, of both sets is the same.
4. The mean of London's temperatures is greater than the mean of Chicago's temperatures, but their medians are equal.
   True; Sample response: The data points are the same for both cities except for their maximum values. London has a greater maximum value, so its mean is greater.
5. The standard deviation of the temperatures in St. Louis is greater than the standard deviation of those in London.
   False; Sample response: Most temperatures in both cities are similarly distributed around their respective means, but London has an outlier which increases its variability.
6. The standard deviation of St. Louis's temperatures is equal to the standard deviation of Chicago's temperatures.
   True; Sample response: Chicago's temperatures are 3 degrees cooler each day than St. Louis's temperatures. But the distribution is otherwise the same for the two cities.

256 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Allow students individual work time. Then have them share their responses with their partner, coming to a consensus if their responses differ.

2 Monitor

Help students get started by prompting them to estimate the measures of center for each dot plot.

Look for points of confusion:

- **Determining the IQR of London's temperatures in Problem 2.** Ask, "How does the range compare to the IQR?"
- **Calculating exact values for the measures of center of both data sets in Problem 3 (or 4).** Prompt students to determine where the measures of center are for each data set in comparison to one another.

Look for productive strategies:

- Annotating different statistical values in the dot plot.
- Comparing the scales of each dot plot.
- Identifying gaps or extreme values.

3 Connect

Display the dot plots from the activity. Use the *Poll the Class* routine for each statement and record the answers based on the responses of the majority.

Have pairs of students share how they determined their responses, using the data representations to support their responses, where appropriate.

Highlight that students can compare the measures of centers of different data sets if they are distributed similarly. Standard deviation, like IQR, is used to measure variability and gets larger as the data points spread out farther from the mean.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students use annotations to help guide their thinking. For example, in Problem 3, have them study the two dot plots and annotate the data value of 5 in the Chicago dot plot being removed from the London dot plot and the data value of 8 that was added to the London dot plot.

Accessibility: *Guide Processing and Visualization*

Consider demonstrating how students do not necessarily have to calculate values to determine the validity of each statement. Use a think-aloud to model how the statement in Problem 2 must be false because the scale of the data set does not go past 8.

MLR Math Language Development

MLR8: Discussion Supports

During the Connect, as students share how they determined the validity of each statement, display or provide the Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning*. Encourage students to use sentence frames from this display to ask for clarification or to challenge an idea if they disagree.

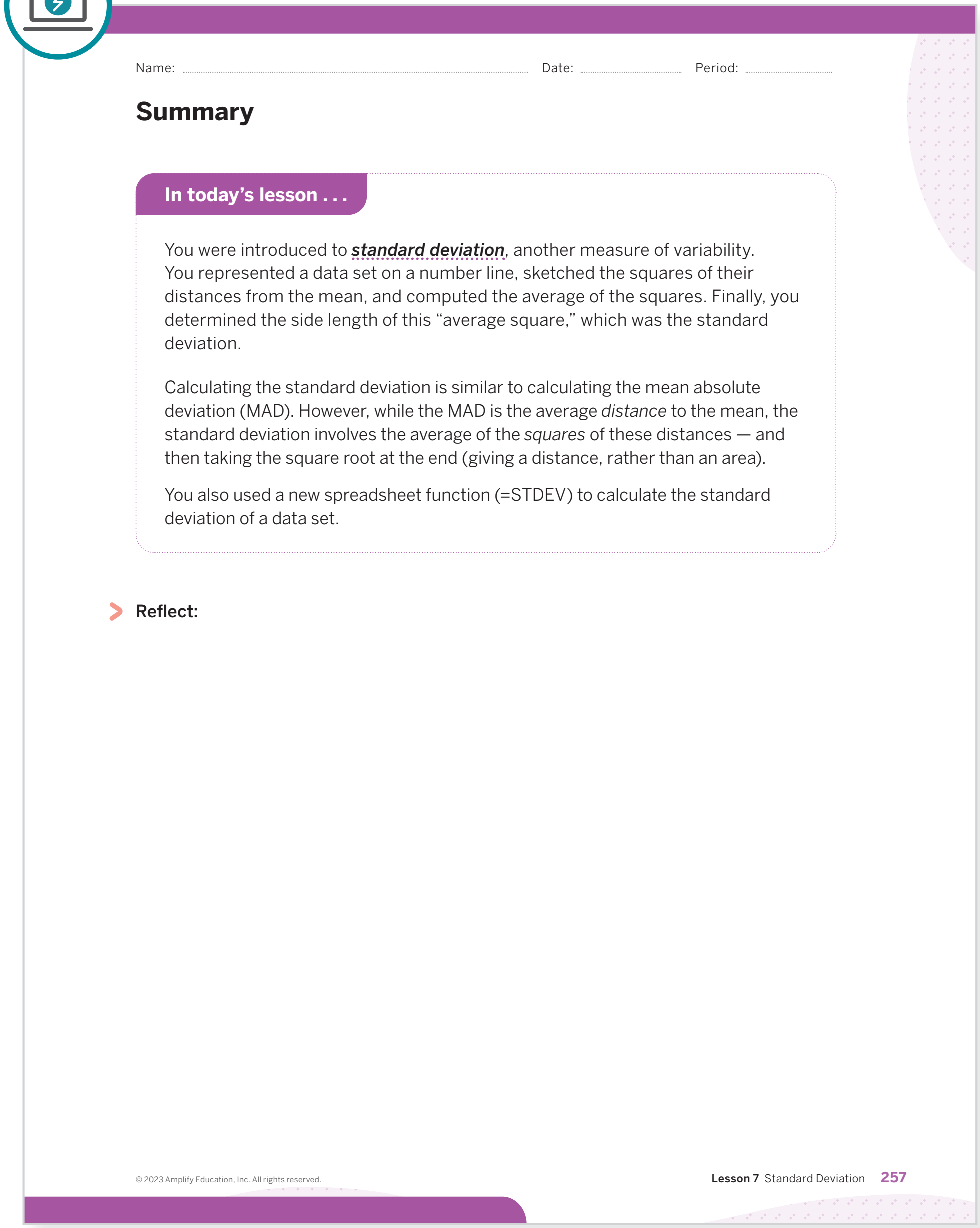
English Learners

As students share their responses, use gestures or annotations to show how the structure of the displays supports or does not support each statement.

Whole Class | 5 min

Summary

Review and synthesize how standard deviation measures variability and how it is calculated.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You were introduced to ***standard deviation***, another measure of variability. You represented a data set on a number line, sketched the squares of their distances from the mean, and computed the average of the squares. Finally, you determined the side length of this "average square," which was the standard deviation.

Calculating the standard deviation is similar to calculating the mean absolute deviation (MAD). However, while the MAD is the average *distance* to the mean, the standard deviation involves the average of the *squares* of these distances — and then taking the square root at the end (giving a distance, rather than an area).

You also used a new spreadsheet function (=STDEV) to calculate the standard deviation of a data set.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 7 Standard Deviation 257

Synthesize

Have students share the steps for calculating the standard deviation using a number line, in their own words.

Ask:

- "One data set has a standard deviation of 5 and another data set has a standard deviation of 10. What does this tell you?" The second data set shows greater variability than the first data set.
- "How does the standard deviation compare to MAD?" Both are measures of variability calculated from the mean. But for standard deviation, I square the lengths first before determining the average. I then take the square root to get a length rather than an area.

Formalize vocabulary: *standard deviation*

Highlight that the standard deviation is more commonly used to measure variability than MAD. Moving forward, students will typically use standard deviation to measure variability of a data set rather than MAD.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Describe the steps for calculating the standard deviation using a number line."

Math Language Development

MLR2: Collect and Display

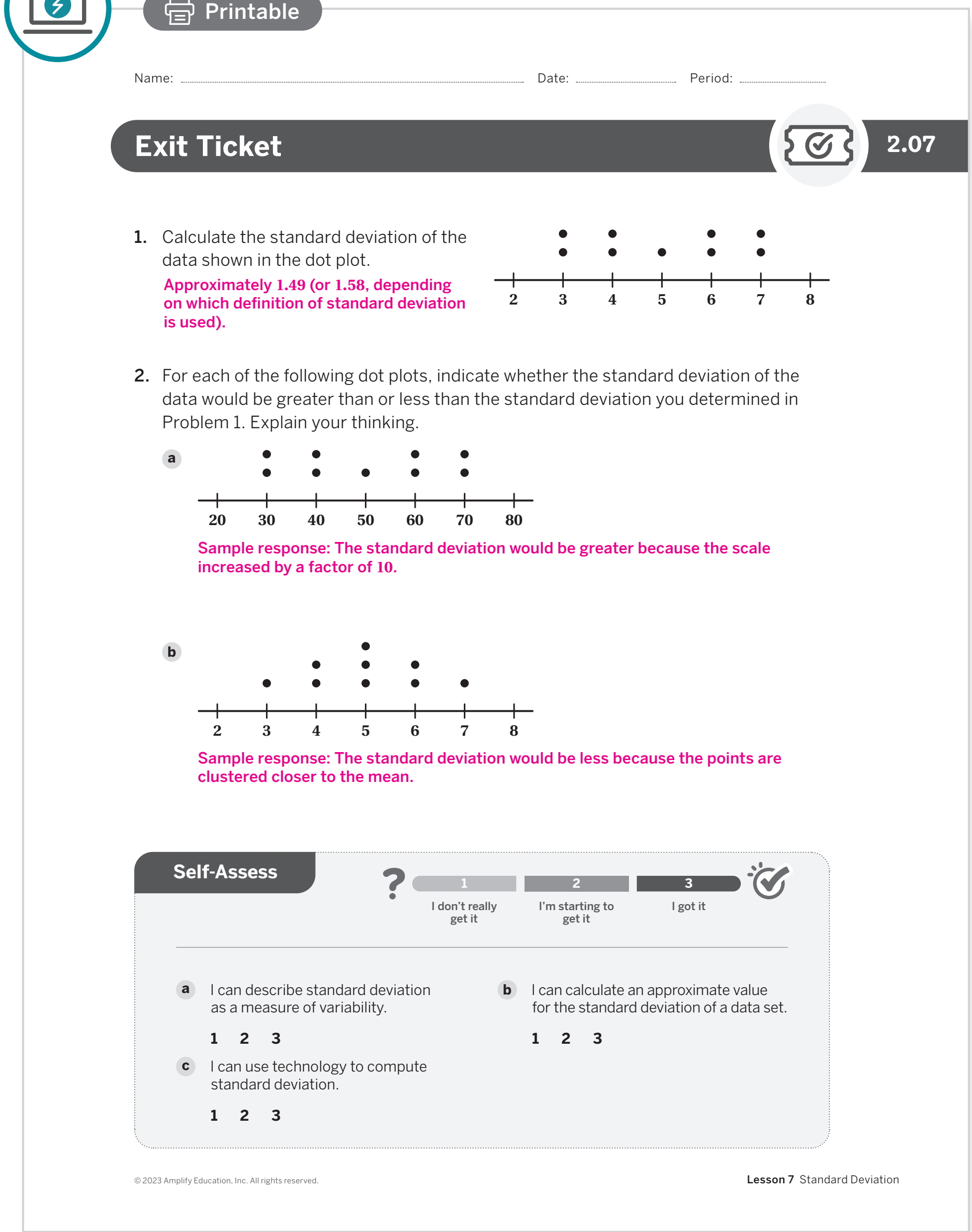
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *standard deviation* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by calculating the standard deviation of a data set and using it to measure variability.

Printable



Name: Date: Period:

Exit Ticket 2.07

1. Calculate the standard deviation of the data shown in the dot plot.
   Approximately 1.49 (or 1.58, depending on which definition of standard deviation is used).

2. For each of the following dot plots, indicate whether the standard deviation of the data would be greater than or less than the standard deviation you determined in Problem 1. Explain your thinking.

   a. Sample response: The standard deviation would be greater because the scale increased by a factor of 10.

   b. Sample response: The standard deviation would be less because the points are clustered closer to the mean.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can describe standard deviation as a measure of variability.
1 2 3

b. I can calculate an approximate value for the standard deviation of a data set.
1 2 3

c. I can use technology to compute standard deviation.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 7 Standard Deviation

Success looks like . . .

- **Language Goal:** Comprehending standard deviation as a measure of variability. **(Speaking and Listening, Reading, and Writing)**
  - » Comparing standard deviations of data sets based on the distribution of each set in Problem 2.
- **Goal:** Calculating an approximate value for the standard deviation of a data set.
  - » Calculating the standard deviation of the data in the dot plot in Problem 1.
- **Goal:** Using technology to compute standard deviation.

Suggested next steps

If students cannot calculate the standard deviation in Problem 1, consider:

- Reviewing Activity 1.
- Providing spreadsheet technology.

If students cannot describe the standard deviation of both sets in Problem 2, consider:

- Reviewing Activity 2.
- Assigning Practice Problem 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What challenges did students encounter as they worked on calculating the average length in Activity 1? How did they work through them?
- How did Activity 2 support students in describing standard deviation as a measure of variability?

Math Language Development

Language Goal: Comprehending standard deviation as a measure of variability.

Reflect on students' language development toward this goal.

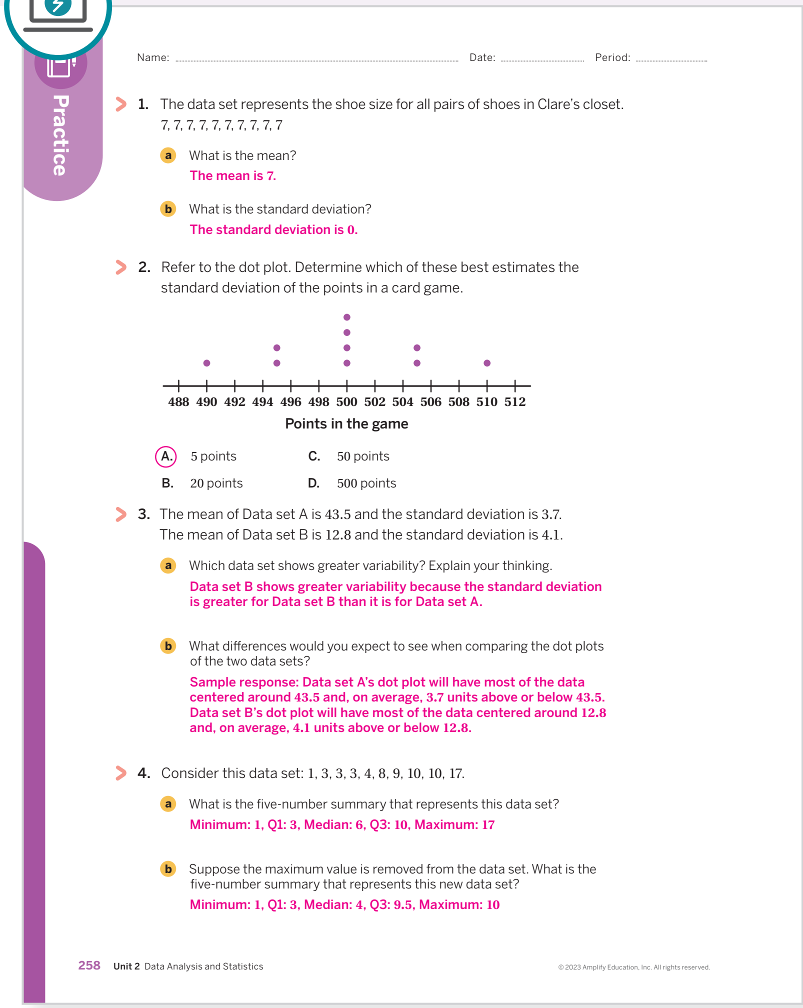
- Do students' responses to Problem 2 of the Exit Ticket demonstrate they understand that the standard deviation is a measure of variability? Are their explanations accurate and precise? How can you help them be more precise?

Sample explanations for Problem 2b:

Emerging	Expanding
Less because the points are closer.	The standard deviation would be less because the points are closer to the mean.

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. The data set represents the shoe size for all pairs of shoes in Clare's closet.
   7, 7, 7, 7, 7, 7, 7, 7, 7, 7

   a. What is the mean?
   The mean is 7.

   b. What is the standard deviation?
   The standard deviation is 0.

2. Refer to the dot plot. Determine which of these best estimates the standard deviation of the points in a card game.

   [✓ A.] 5 points
   B. 20 points
   C. 50 points
   D. 500 points

3. The mean of Data set A is 43.5 and the standard deviation is 3.7.
   The mean of Data set B is 12.8 and the standard deviation is 4.1.

   a. Which data set shows greater variability? Explain your thinking.
   Data set B shows greater variability because the standard deviation is greater for Data set B than it is for Data set A.

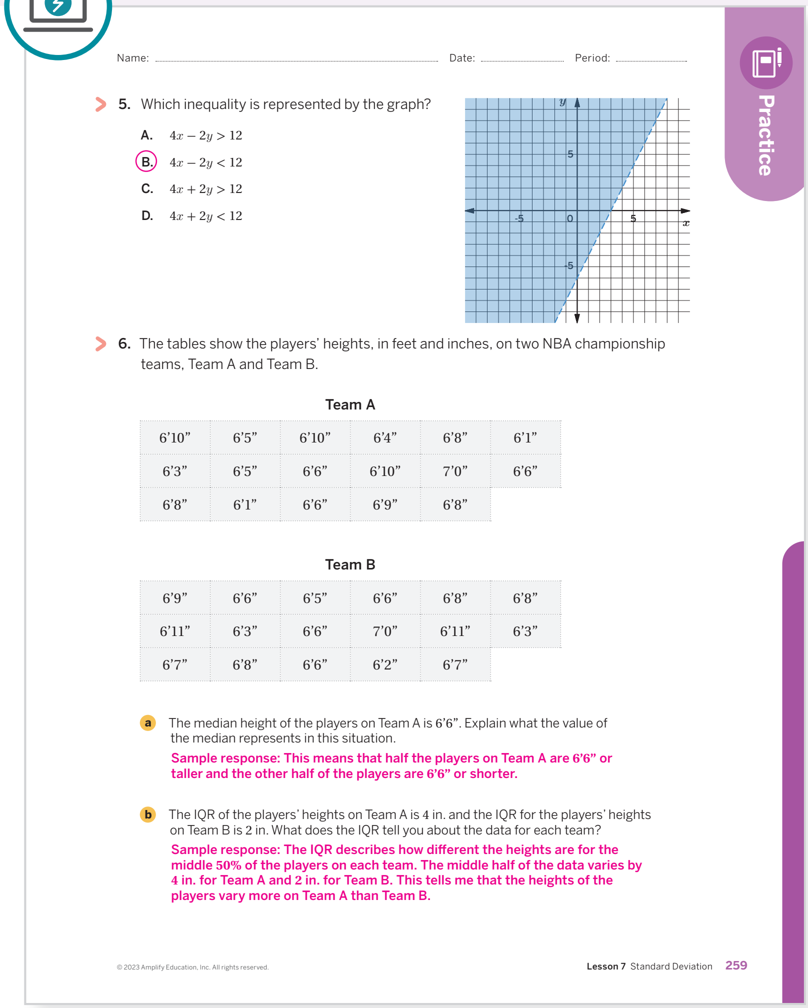
   b. What differences would you expect to see when comparing the dot plots of the two data sets?
   Sample response: Data set A's dot plot will have most of the data centered around 43.5 and, on average, 3.7 units above or below 43.5. Data set B's dot plot will have most of the data centered around 12.8 and, on average, 4.1 units above or below 12.8.

4. Consider this data set: 1, 3, 3, 3, 4, 8, 9, 10, 10, 17.

   a. What is the five-number summary that represents this data set?
   Minimum: 1, Q1: 3, Median: 6, Q3: 10, Maximum: 17

   b. Suppose the maximum value is removed from the data set. What is the five-number summary that represents this new data set?
   Minimum: 1, Q1: 3, Median: 4, Q3: 9.5, Maximum: 10

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ________ Date: ________ Period: ________

5. Which inequality is represented by the graph?

   A. $4x - 2y > 12$
   [✓ B.] $4x - 2y < 12$
   C. $4x + 2y > 12$
   D. $4x + 2y < 12$

6. The tables show the players' heights, in feet and inches, on two NBA championship teams, Team A and Team B.

Team A

6'10"	6'5"	6'10"	6'4"	6'8"	6'1"
6'3"	6'5"	6'6"	6'10"	7'0"	6'6"
6'8"	6'1"	6'6"	6'9"	6'8"	

Team B

6'9"	6'6"	6'5"	6'6"	6'8"	6'8"
6'11"	6'3"	6'6"	7'0"	6'11"	6'3"
6'7"	6'8"	6'6"	6'2"	6'7"	

   a. The median height of the players on Team A is 6'6". Explain what the value of the median represents in this situation.
   Sample response: This means that half the players on Team A are 6'6" or taller and the other half of the players are 6'6" or shorter.

   b. The IQR of the players' heights on Team A is 4 in. and the IQR for the players' heights on Team B is 2 in. What does the IQR tell you about the data for each team?
   Sample response: The IQR describes how different the heights are for the middle 50% of the players on each team. The middle half of the data varies by 4 in. for Team A and 2 in. for Team B. This tells me that the heights of the players vary more on Team A than Team B.

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 2	2
	5	Unit 1 Lesson 16	2
Formative ⚡	6	Unit 2 Lesson 8	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

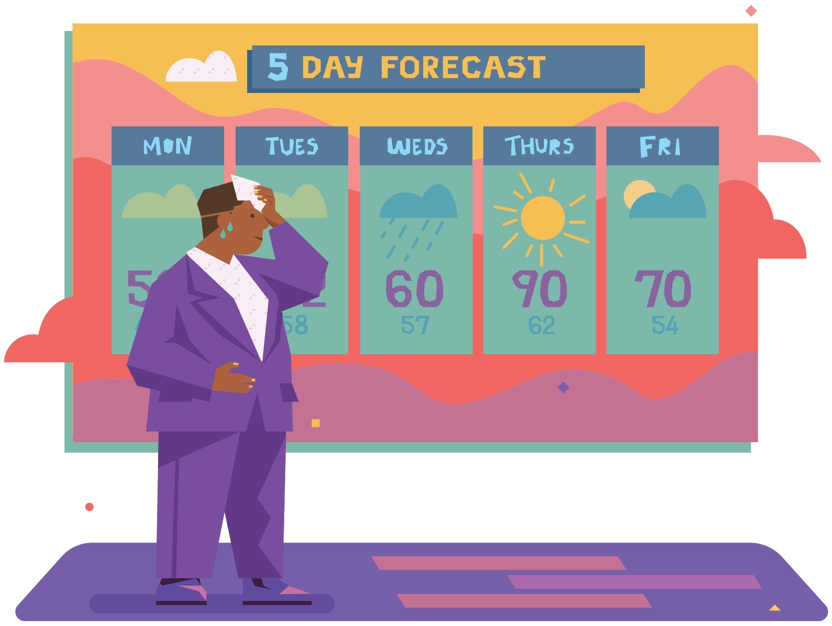
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 2 | **LESSON 8**

Choosing Appropriate Measures (Part 1)

Let's investigate relationships between shapes of distributions and measures of center and variability.



Focus

Goals

1. **Language Goal:** Explain the effect of an extreme value on the statistics of a data set. **(Reading and Writing)**
2. **Language Goal:** Use the shape of a distribution to determine which measure of center and variability is most appropriate for a data set. **(Speaking and Listening)**

Coherence

Today

Students investigate the effect of extreme values on measures of center and variability. They determine which measures best summarize symmetric and skewed data sets.

Previously

In Lesson 6, students were introduced to a mathematical method of determining outliers and observed its effects on the interquartile range (IQR) and mean absolute deviation (MAD) of a data set.

Coming Soon

In Lesson 9, students will continue determining which measures of center and variability are most appropriate, applied to the context of hurricane activity.

Rigor

- Students continue to build a **conceptual understanding** of standard deviation as a measure of variability.
- Students develop **fluency** in determining appropriate measures of center given a distribution's shape.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Independent	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, one per student
- Warm-up PDF, *Exact Statistics of the Distributions* (for display)
- Activity 2 PDF, one per student
- Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning*
- Anchor Chart PDF, *Shapes of Distributions*
- spreadsheet technology

Math Language Development

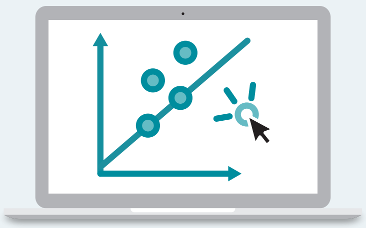
Review words

- *IQR*
- *mean*
- *median*
- *standard deviation*
- *variability*

Amps | **Featured Activity**

Activity 2
Dynamic Data Distributions

Students manipulate a histogram to visually see how changing the shape of the distribution affects the measures of center and variability and determine which is more appropriate to measure for the data set.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might not understand appropriate ways to resolve conflict with their partners. Help students establish healthy ways of communicating, listening, and coming to a consensus when they have different answers to any of the problems. Remind them that they should both seek and offer help when needed so that both partners can interpret the data sets correctly.

Modifications to Pacing

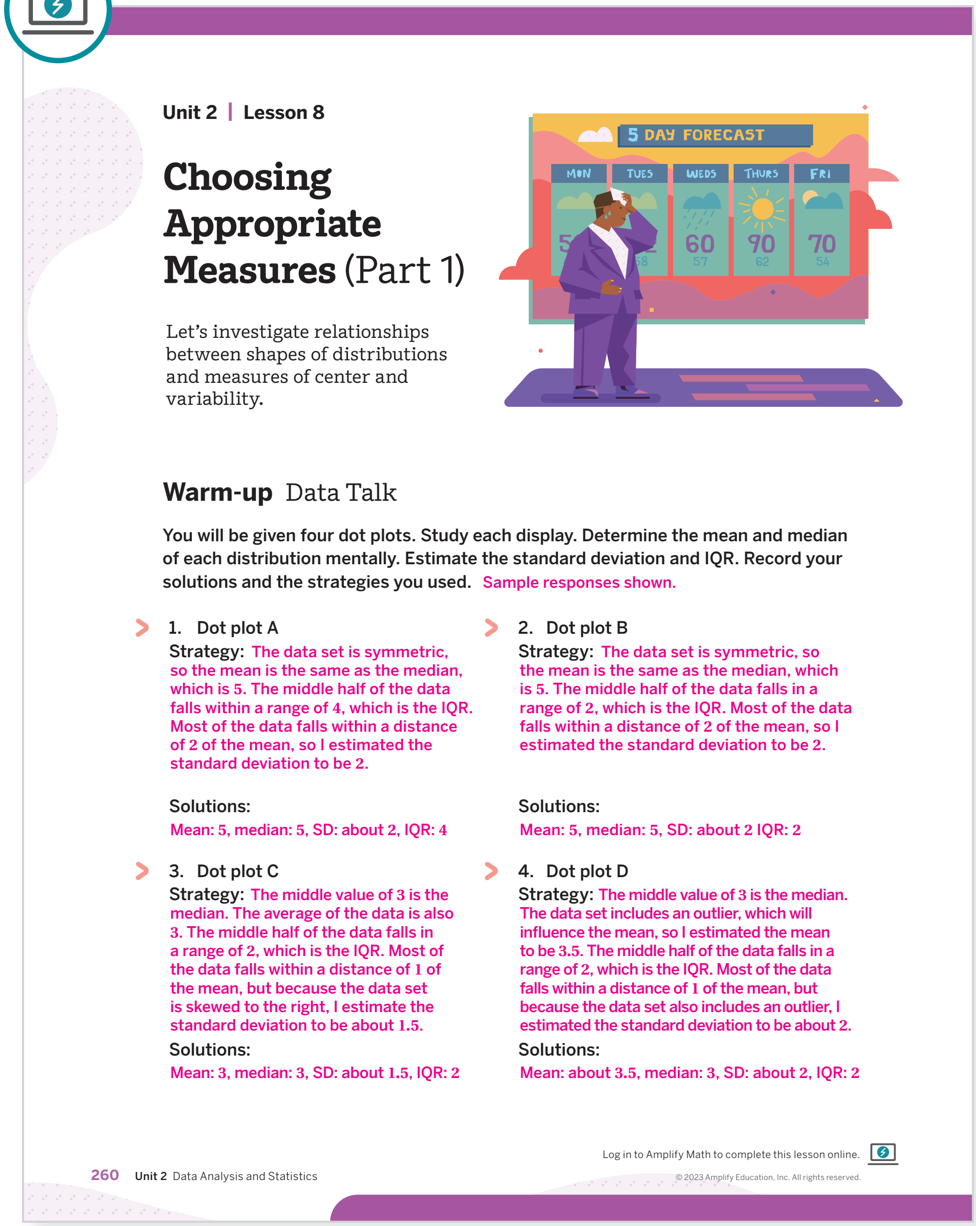
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, consider assigning Problems 3 and 4 to student pairs, and having them compare results in the Connect.

Independent | 5 min

Warm-up Data Talk

Students estimate the SD and IQR of data sets to understand the relationships.



Unit 2 | Lesson 8

Choosing Appropriate Measures (Part 1)

Let's investigate relationships between shapes of distributions and measures of center and variability.

Warm-up Data Talk

You will be given four dot plots. Study each display. Determine the mean and median of each distribution mentally. Estimate the standard deviation and IQR. Record your solutions and the strategies you used. Sample responses shown.

1. Dot plot A
   Strategy: The data set is symmetric, so the mean is the same as the median, which is 5. The middle half of the data falls within a range of 4, which is the IQR. Most of the data falls within a distance of 2 of the mean, so I estimated the standard deviation to be 2.
   Solutions:
   Mean: 5, median: 5, SD: about 2, IQR: 4

2. Dot plot B
   Strategy: The data set is symmetric, so the mean is the same as the median, which is 5. The middle half of the data falls in a range of 2, which is the IQR. Most of the data falls within a distance of 2 of the mean, so I estimated the standard deviation to be 2.
   Solutions:
   Mean: 5, median: 5, SD: about 2 IQR: 2

3. Dot plot C
   Strategy: The middle value of 3 is the median. The average of the data is also 3. The middle half of the data falls in a range of 2, which is the IQR. Most of the data falls within a distance of 1 of the mean, but because the data set is skewed to the right, I estimate the standard deviation to be about 1.5.
   Solutions:
   Mean: 3, median: 3, SD: about 1.5, IQR: 2

4. Dot plot D
   Strategy: The middle value of 3 is the median. The data set includes an outlier, which will influence the mean, so I estimated the mean to be 3.5. The middle half of the data falls in a range of 2, which is the IQR. Most of the data falls within a distance of 1 of the mean, but because the data set also includes an outlier, I estimated the standard deviation to be about 2.
   Solutions:
   Mean: about 3.5, median: 3, SD: about 2, IQR: 2

Log in to Amplify Math to complete this lesson online.

260 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Distribute the Warm-up PDF to each student. Have students complete each problem independently then share their work with a partner.

2 Monitor

Help students get started by suggesting they look for symmetry or skew in each distribution.

Look for points of confusion:

- **Having difficulty estimating the standard deviation.** Have students review the definition of standard deviation. Ask, "Approximately how far from the mean is a typical data point in the data set?"

Look for productive strategies:

- Using the shape of the distribution to determine which measure of center to use.
- Estimating or calculating the values of the mean, median, standard deviation, or IQR.

3 Connect

Have students share their strategies for estimating the measures of center and variability for each distribution.

Display the Warm-up PDF, *Exact Statistics of the Distributions*.

Ask:

- "How did your estimates compare to the actual statistics?"
- "Which statistics were more straightforward to estimate?"

Highlight that the shape of a data distribution should be considered when calculating and interpreting the measures of variability and summarizing its statistics.

Math Language Development

MLR2: Collect and Display

During the Connect, listen for and collect students' language connecting the shape of the distributions with measures and estimates of center and variability, capturing words and phrases such as *symmetric, skewed,* and *uniform*. Add these to the class display.

English Learners

Annotate the distributions from the Warm-up PDF with these words and phrases and keep the display up for students to refer in future discussions.

Power-up

To power up students' ability to calculate and interpret the IQR, have students complete:

Recall that the *interquartile range* (IQR) is the range of the middle 50% of a set of data. Two teams of volleyball players — A and B — have a median height of 60 in. Team A has an IQR of 3 in. and Team B has an IQR of 5 in. Which statement correctly describes the heights of each team?

A. The teams have the same variability.

B. The heights of the middle 50% of players in Team A are more spread out than in Team B.

[✓ **C.**] The heights of the middle 50% of players in Team B are more spread out than in Team A.

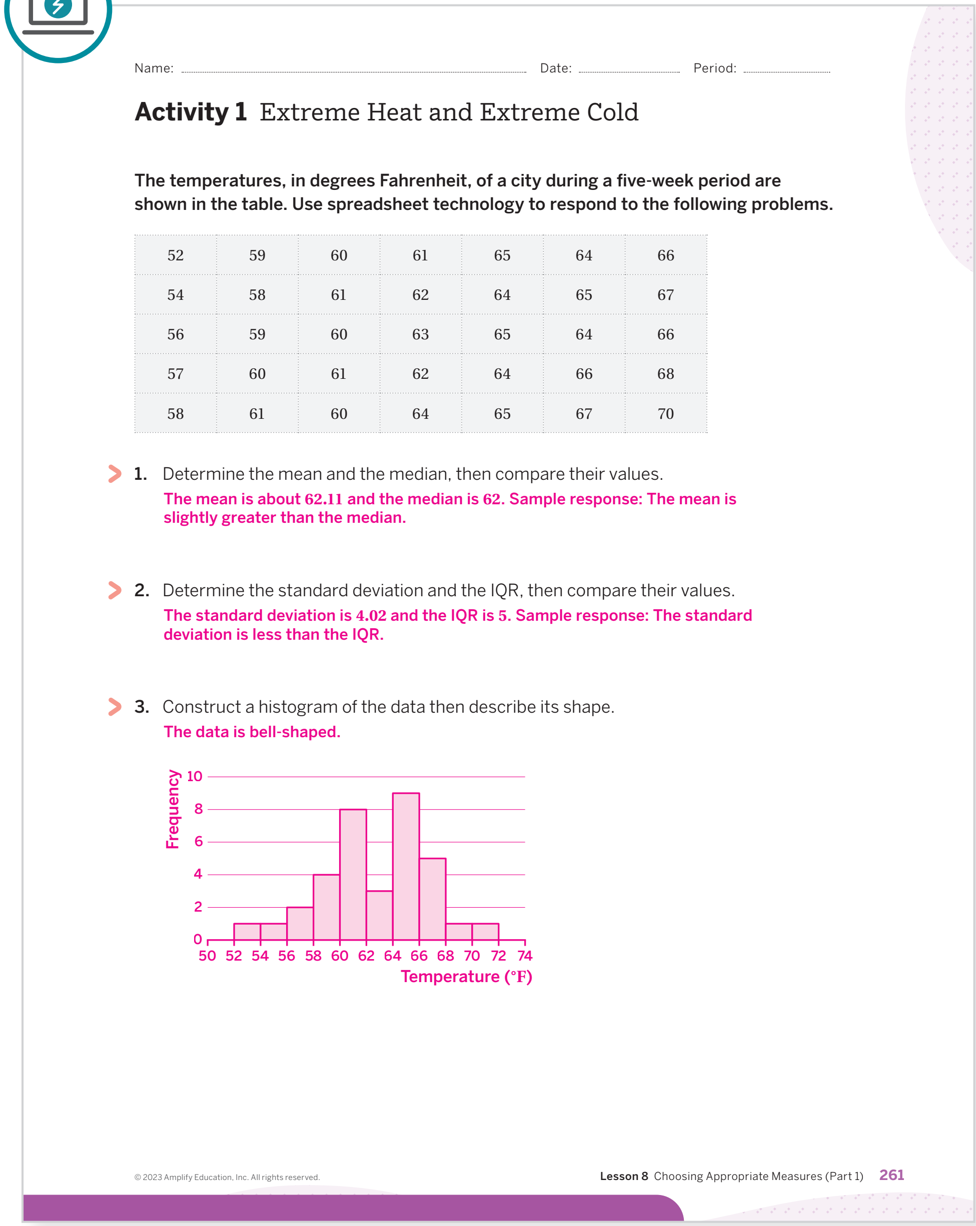
D. The range of heights on Team B is greater than the range of heights on Team A.

Use: Before the Warm-up
Informed by: Performance on Lesson 7, Practice Problem 6

Independent | 20 min

Activity 1 Extreme Heat and Extreme Cold

Students investigate how extreme data affects measures of center and variability to build understanding for when each measure is appropriate.



Name: ______ Date: ______ Period: ______

Activity 1 Extreme Heat and Extreme Cold

The temperatures, in degrees Fahrenheit, of a city during a five-week period are shown in the table. Use spreadsheet technology to respond to the following problems.

52	59	60	61	65	64	66
54	58	61	62	64	65	67
56	59	60	63	65	64	66
57	60	61	62	64	66	68
58	61	60	64	65	67	70

1. Determine the mean and the median, then compare their values.
   The mean is about 62.11 and the median is 62. Sample response: The mean is slightly greater than the median.

2. Determine the standard deviation and the IQR, then compare their values.
   The standard deviation is 4.02 and the IQR is 5. Sample response: The standard deviation is less than the IQR.

3. Construct a histogram of the data then describe its shape.
   The data is bell-shaped.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Spreadsheet technology is required. Give students time to calculate the statistics for Problems 1 and 2, then pause to ensure the class agrees on these values before resuming the activity.

2 Monitor

Help students get started by displaying the spreadsheet functions students will use (such as =AVERAGE, =MEDIAN, =STDEV, and =QUARTILE) as a reminder.

Look for points of confusion:

- **Replacing the minimum or maximum value in their original spreadsheet.** Prompt students to create separate data sets so that they can compare them with one another.

Look for productive strategies:

- Comparing three separate sets of statistics.
- Mentioning any tails or gaps in the data.
- Changing the bucket size of a histogram to discern the shape of the data distribution.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display or provide copies of the Anchor Chart PDF, *Shapes of Distributions* for students to reference as they describe the shape of their histogram in Problem 3.

Extension: *Math Enrichment*

Have students add values or remove values from the original data set, so that the median and IQR change. Ask them to respond to the following question: "What changes to the data set must occur in order for the median and IQR values to change?" Student responses may vary.

Math Language Development

MLR1: Stronger and Clearer Each Time

Provide students time to write a draft explanation for Problem 4b. Have them meet with 1–2 partners to give and receive feedback. Display these questions for reviewers to consider:

- "Does the response use mathematical language from this unit?"
- "Does the response describe *how* the statistics were affected?"

Have students revise their histogram and responses, based on any feedback they received.

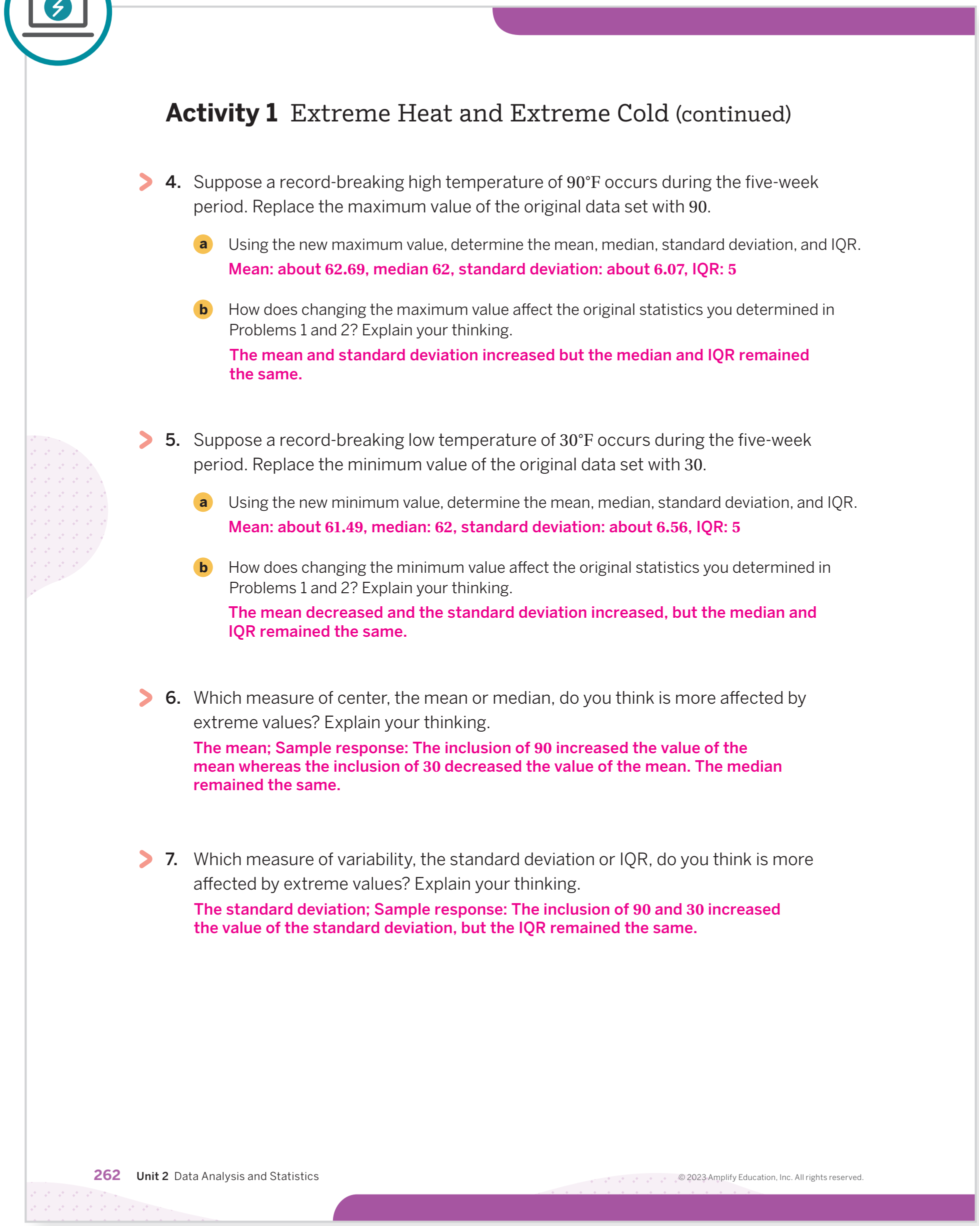
English Learners

Provide access to the Anchor Chart PDF, *Sentence Stems, Stronger and Clearer Each Time* and encourage students to borrow phrases from this chart during discussions.

Independent | 20 min

Activity 1 Extreme Heat and Extreme Cold (continued)

Students investigate how extreme data affects measures of center and variability to build understanding for when each measure is appropriate.



Activity 1 Extreme Heat and Extreme Cold (continued)

4. Suppose a record-breaking high temperature of 90°F occurs during the five-week period. Replace the maximum value of the original data set with 90.

   a. Using the new maximum value, determine the mean, median, standard deviation, and IQR.

   Mean: about 62.69, median 62, standard deviation: about 6.07, IQR: 5

   b. How does changing the maximum value affect the original statistics you determined in Problems 1 and 2? Explain your thinking.

   The mean and standard deviation increased but the median and IQR remained the same.

5. Suppose a record-breaking low temperature of 30°F occurs during the five-week period. Replace the minimum value of the original data set with 30.

   a. Using the new minimum value, determine the mean, median, standard deviation, and IQR.

   Mean: about 61.49, median: 62, standard deviation: about 6.56, IQR: 5

   b. How does changing the minimum value affect the original statistics you determined in Problems 1 and 2? Explain your thinking.

   The mean decreased and the standard deviation increased, but the median and IQR remained the same.

6. Which measure of center, the mean or median, do you think is more affected by extreme values? Explain your thinking.

   The mean; Sample response: The inclusion of 90 increased the value of the mean whereas the inclusion of 30 decreased the value of the mean. The median remained the same.

7. Which measure of variability, the standard deviation or IQR, do you think is more affected by extreme values? Explain your thinking.

   The standard deviation; Sample response: The inclusion of 90 and 30 increased the value of the standard deviation, but the IQR remained the same.

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have groups of students share the shape of the distribution for the original data and how replacing the maximum value affected (or did not affect) the mean, standard deviation, median, and IQR. Then discuss these changes when the minimum value is replaced.

Ask:

- "Which measure of center seems to be more influenced by an extreme value?" Mean
- "Which measure of variability seems to be more influenced by an extreme value?" Standard deviation
- "If data is skewed, do you think the skew has a greater effect on the mean or the median? On the standard deviation or the IQR?" Skew has a greater effect on the mean than the median, and a greater effect on the standard deviation than the IQR.

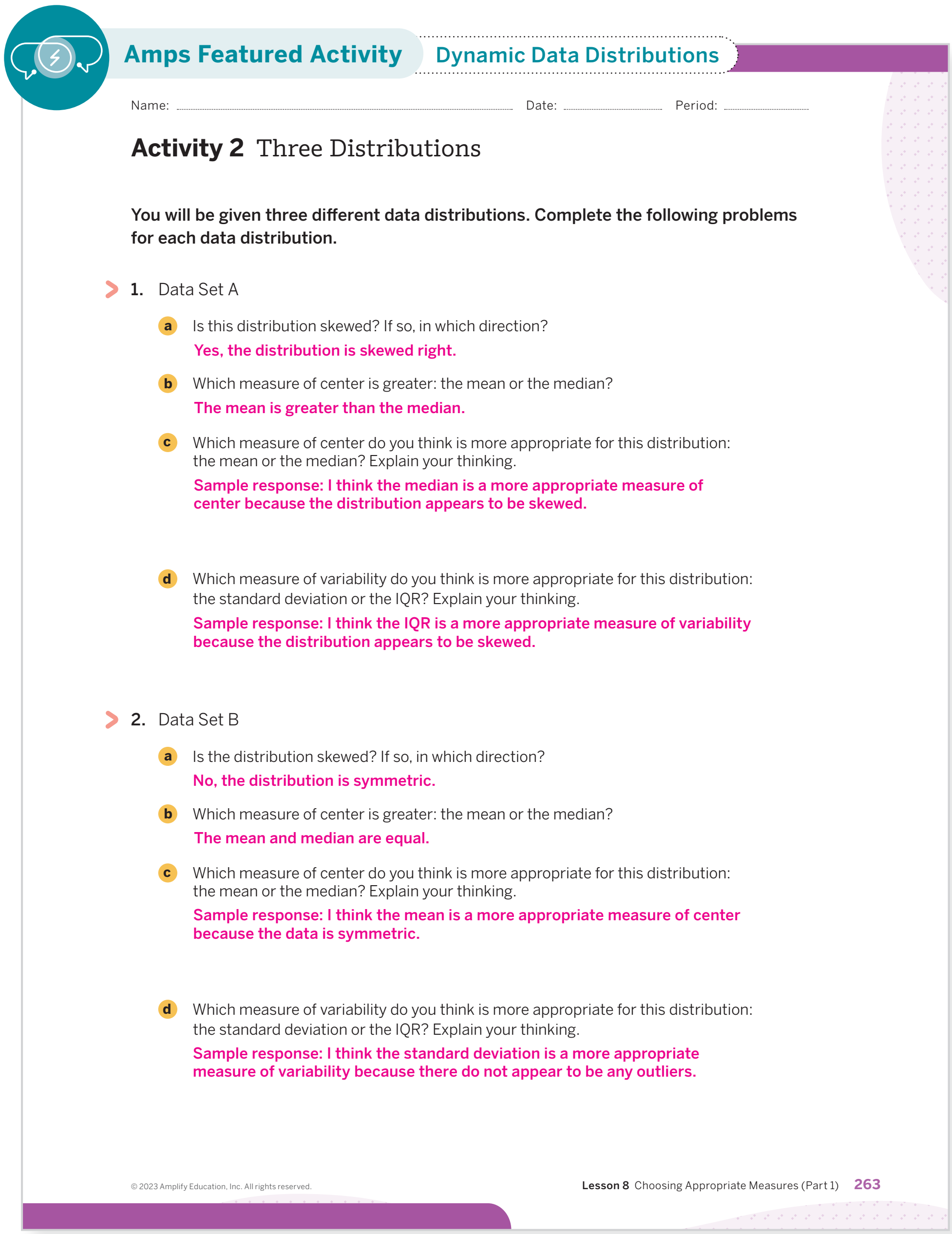
Highlight that the mean and standard deviation are more sensitive to extreme values in a data set while the median and IQR resist the effects of extreme values.

Pairs | 15 min

Activity 2 Three Distributions

Students analyze symmetric and skewed distributions to connect appropriate measures of center and variability with a distribution's shape.

Amps Featured Activity Dynamic Data Distributions



Name: ________ Date: ________ Period: ________

Activity 2 Three Distributions

You will be given three different data distributions. Complete the following problems for each data distribution.

1. Data Set A
   - a Is this distribution skewed? If so, in which direction?
     Yes, the distribution is skewed right.
   - b Which measure of center is greater: the mean or the median?
     The mean is greater than the median.
   - c Which measure of center do you think is more appropriate for this distribution: the mean or the median? Explain your thinking.
     Sample response: I think the median is a more appropriate measure of center because the distribution appears to be skewed.
   - d Which measure of variability do you think is more appropriate for this distribution: the standard deviation or the IQR? Explain your thinking.
     Sample response: I think the IQR is a more appropriate measure of variability because the distribution appears to be skewed.
2. Data Set B
   - a Is the distribution skewed? If so, in which direction?
     No, the distribution is symmetric.
   - b Which measure of center is greater: the mean or the median?
     The mean and median are equal.
   - c Which measure of center do you think is more appropriate for this distribution: the mean or the median? Explain your thinking.
     Sample response: I think the mean is a more appropriate measure of center because the data is symmetric.
   - d Which measure of variability do you think is more appropriate for this distribution: the standard deviation or the IQR? Explain your thinking.
     Sample response: I think the standard deviation is a more appropriate measure of variability because there do not appear to be any outliers.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 8 Choosing Appropriate Measures (Part 1) 263

1 Launch

Distribute the Activity 2 PDF to each student. Have students complete the problems independently, then share their work with a partner. Have student pairs work together to come to a consensus if they have different responses to any of the problems.

2 Monitor

Help students get started by explaining that an appropriate measure of center is one whose value is "typical" of the data.

Look for points of confusion:

- **Struggling to determine whether the mean or median is greater.** Ask, "How does the mean compare to the median when the distribution is symmetric? Which measure of center is more affected by a skew in the data?"
- **Struggling to determine an appropriate measure of variability.** Ask, "Which measure of variability is more affected by the skew?"

Look for productive strategies:

- Drawing vertical lines or otherwise annotating the values of the mean or median on the data representations.
- Writing arguments that use skew as a reason for choosing a measure.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can manipulate a histogram to visually see how changing the shape of the distribution affects the measures of center and variability. They can use this visual support to help them determine which measure is more appropriate.

Accessibility: *Guide Processing and Visualization*

Display or provide copies of the Anchor Chart PDF, *Shapes of Distributions* for students to reference as they determine whether the distributions are skewed in Problem 1a and Problem 2a.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their responses, display or provide the Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning.* Encourage students to use sentence frames from this display to ask for clarification or to challenge an idea if they disagree.

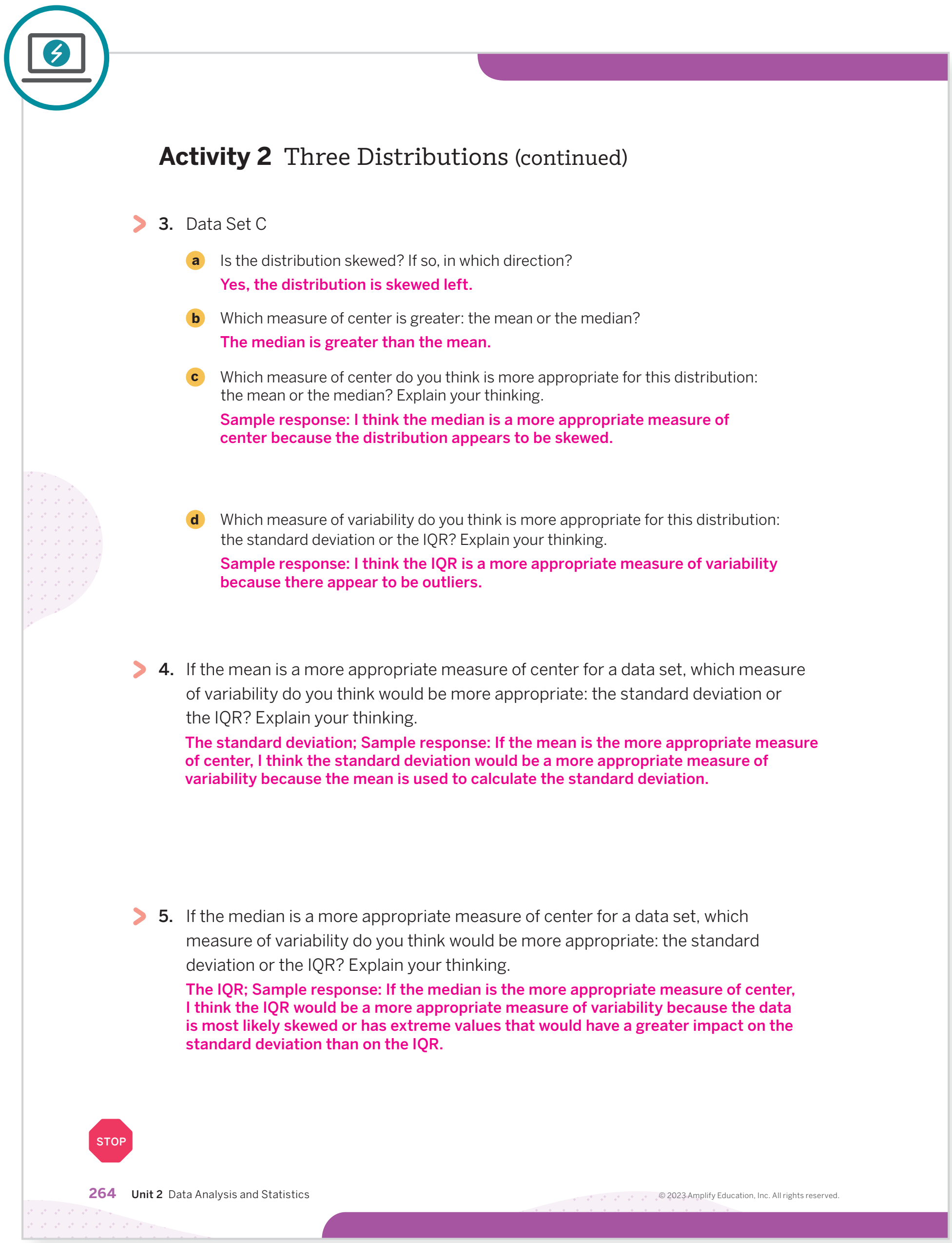
English Learners

Annotate the displays with whether the distribution is skewed left, skewed right, or symmetric, and which measure of center and variability is more appropriate to use. Keep these displays up for students to reference during future discussions.

Pairs | 15 min

Activity 2 Three Distributions (continued)

Students analyze symmetric and skewed distributions to connect appropriate measures of center and variability with a distribution's shape.



Activity 2 Three Distributions (continued)

3. Data Set C

a Is the distribution skewed? If so, in which direction?

Yes, the distribution is skewed left.

b Which measure of center is greater: the mean or the median?

The median is greater than the mean.

c Which measure of center do you think is more appropriate for this distribution: the mean or the median? Explain your thinking.

Sample response: I think the median is a more appropriate measure of center because the distribution appears to be skewed.

d Which measure of variability do you think is more appropriate for this distribution: the standard deviation or the IQR? Explain your thinking.

Sample response: I think the IQR is a more appropriate measure of variability because there appear to be outliers.

4. If the mean is a more appropriate measure of center for a data set, which measure of variability do you think would be more appropriate: the standard deviation or the IQR? Explain your thinking.

The standard deviation; Sample response: If the mean is the more appropriate measure of center, I think the standard deviation would be a more appropriate measure of variability because the mean is used to calculate the standard deviation.

5. If the median is a more appropriate measure of center for a data set, which measure of variability do you think would be more appropriate: the standard deviation or the IQR? Explain your thinking.

The IQR; Sample response: If the median is the more appropriate measure of center, I think the IQR would be a more appropriate measure of variability because the data is most likely skewed or has extreme values that would have a greater impact on the standard deviation than on the IQR.

STOP

264 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the data representations from Problems 1, 2, and 3, one at a time.

Have pairs of students share their responses for each problem and why they chose the measure of center and measure of variability that they did.

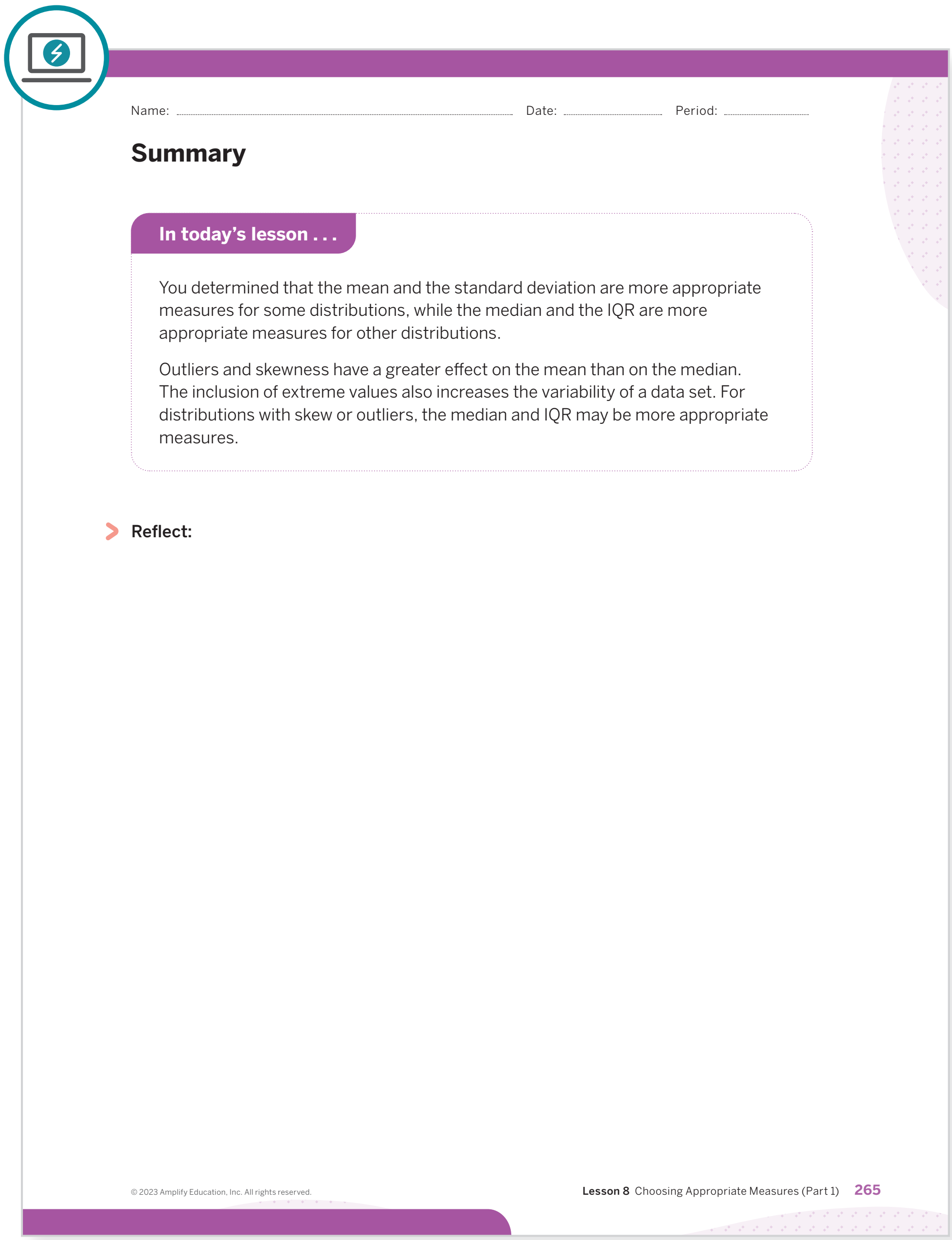
Ask, "How does the mean compare to the median when the distribution is skewed right? Skewed left?" The mean is greater than the median if the data is skewed right and less than the median if it is skewed left.

Highlight that the median and IQR are more appropriate measures of center and variability (respectively) when the data is skewed because they resist the effects of extreme values. Otherwise, we use the mean and standard deviation, both of which are calculated using every value in the data set.

Whole Class | 5 min

Summary

Review and synthesize how to compare data sets using appropriate measures of center and variability.



Name: Date: Period:

Summary

In today's lesson . . .

You determined that the mean and the standard deviation are more appropriate measures for some distributions, while the median and the IQR are more appropriate measures for other distributions.

Outliers and skewness have a greater effect on the mean than on the median. The inclusion of extreme values also increases the variability of a data set. For distributions with skew or outliers, the median and IQR may be more appropriate measures.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Have students share how they can use the shape of a distribution to determine the most appropriate measure of center and measure of variability for the data set.

Ask:

- "One data set's measure of center is best represented by a median of 7 and another data set by a median of 10. How would you determine which set has greater variability?" Whichever one has a larger IQR.
- "How do you determine which of two nearly symmetric distributions has less variability?" Whichever one has a lesser standard deviation.
- "Which measures of center and variability are more affected by an extreme value?" The mean and standard deviation.
- "What does it mean to say that one data set or distribution has more variability than the other?" That one distribution is more spread out than the other.

Highlight that just as the mean is often the more appropriate measure of center for a symmetric distribution, the standard deviation is the more appropriate measure of spread (because it is similarly calculated taking into account every point). Similarly, just as the median is often the more appropriate measure of center for a skewed distribution, the IQR is the more appropriate measure of variability because it similarly resists the effects of extreme values.

Reflect

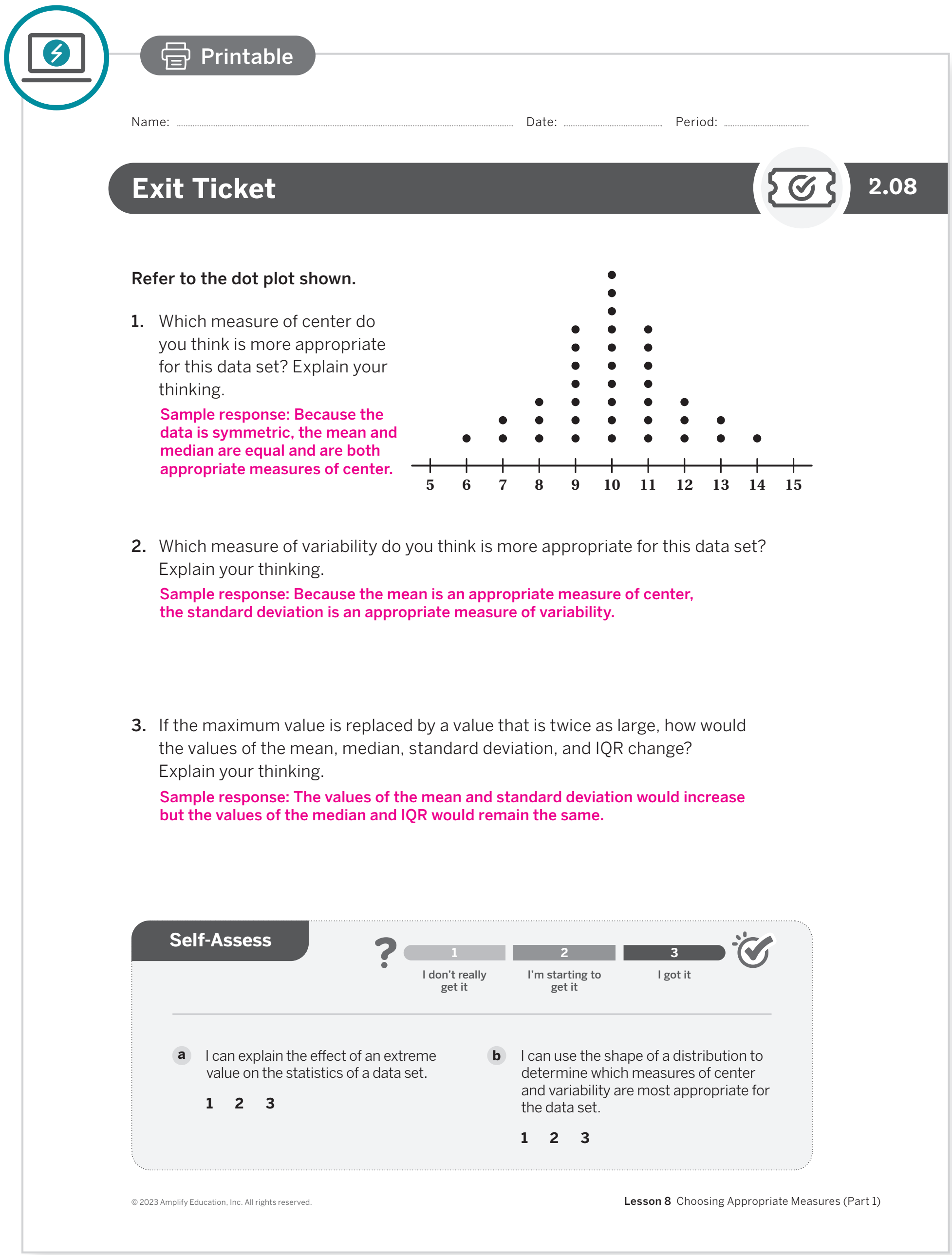
After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can you determine which measure of center and which measure of variability is most appropriate for a data set?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining which measures of center and variability are most appropriate for a given data set.



Printable

Name: ____________ Date: ________ Period: ________

Exit Ticket 2.08

Refer to the dot plot shown.

1. Which measure of center do you think is more appropriate for this data set? Explain your thinking.
   Sample response: Because the data is symmetric, the mean and median are equal and are both appropriate measures of center.

2. Which measure of variability do you think is more appropriate for this data set? Explain your thinking.
   Sample response: Because the mean is an appropriate measure of center, the standard deviation is an appropriate measure of variability.

3. If the maximum value is replaced by a value that is twice as large, how would the values of the mean, median, standard deviation, and IQR change? Explain your thinking.
   Sample response: The values of the mean and standard deviation would increase but the values of the median and IQR would remain the same.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can explain the effect of an extreme value on the statistics of a data set.
1 2 3

b I can use the shape of a distribution to determine which measures of center and variability are most appropriate for the data set.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 8 Choosing Appropriate Measures (Part 1)

Success looks like . . .

- **Language Goal:** Explaining the effect of an extreme value on the statistics of a data set. **(Reading and Writing)**
  - » Explaining how changing the maximum value of the data set affects the measures of center and variability in Problem 3.
- **Language Goal:** Using the shape of a distribution to determine which measures of center and variability are most appropriate for a data set. **(Speaking and Listening)**
  - » Selecting the appropriate measures of center and variability for the data shown in the dot plot in Problems 1 and 2.

Suggested next steps

If students cannot determine an appropriate measure of center in Problem 1, consider:

- Asking, "What shape is the distribution of the data?"
- Having students review the Anchor Chart PDF, *Shapes of Distribution*.

If students cannot determine an appropriate measure of variability in Problem 2, consider:

- Asking, "Which measure of variability can be determined using your response to Problem 1?"
- Reviewing Activity 2.

If students cannot determine how the statistics in Problem 3 would change, consider:

- Reviewing Activity 1.
- Assigning Practice Problem 2.

Professional Learning

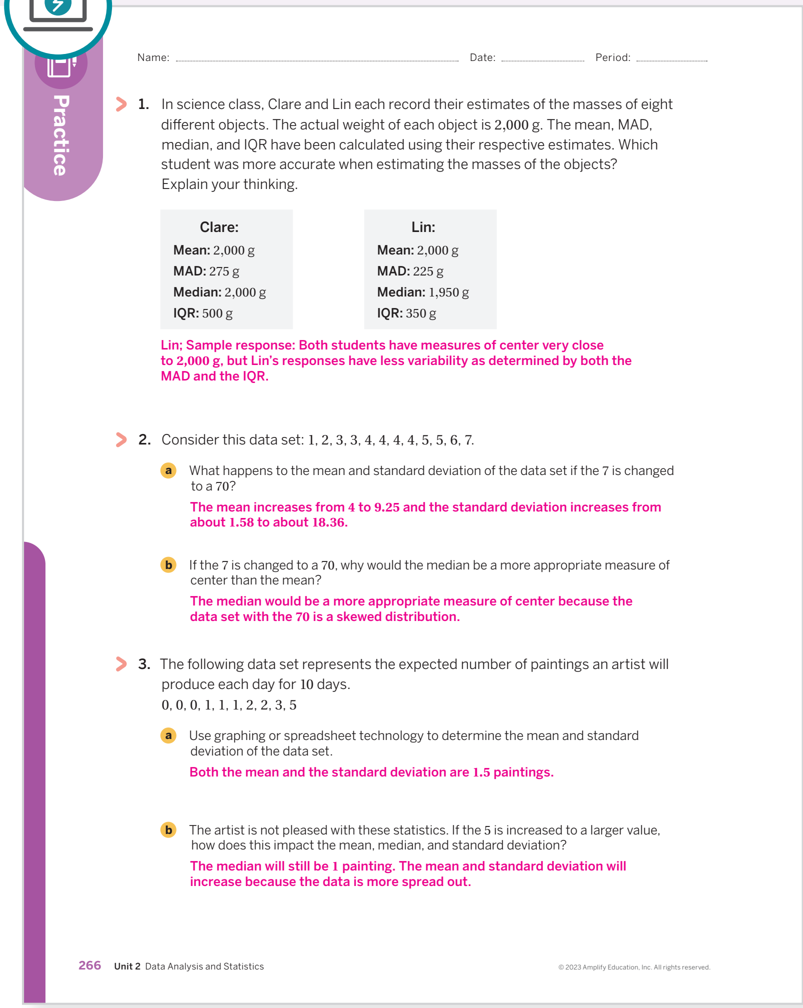
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In this lesson, students compare variability using standard deviation. How did that build on the earlier work students did with mean absolute deviation (MAD)?
- During the discussion about appropriate measures of center and variability in Activity 2, how did you encourage each student to share their understandings?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. In science class, Clare and Lin each record their estimates of the masses of eight different objects. The actual weight of each object is 2,000 g. The mean, MAD, median, and IQR have been calculated using their respective estimates. Which student was more accurate when estimating the masses of the objects? Explain your thinking.

Clare:	Lin:
Mean: 2,000 g	**Mean:** 2,000 g
MAD: 275 g	**MAD:** 225 g
Median: 2,000 g	**Median:** 1,950 g
IQR: 500 g	**IQR:** 350 g

Lin; Sample response: Both students have measures of center very close to 2,000 g, but Lin's responses have less variability as determined by both the MAD and the IQR.

2. Consider this data set: 1, 2, 3, 3, 4, 4, 4, 4, 5, 5, 6, 7.

   a. What happens to the mean and standard deviation of the data set if the 7 is changed to a 70?

   The mean increases from 4 to 9.25 and the standard deviation increases from about 1.58 to about 18.36.

   b. If the 7 is changed to a 70, why would the median be a more appropriate measure of center than the mean?

   The median would be a more appropriate measure of center because the data set with the 70 is a skewed distribution.

3. The following data set represents the expected number of paintings an artist will produce each day for 10 days.
   0, 0, 0, 1, 1, 1, 2, 2, 3, 5

   a. Use graphing or spreadsheet technology to determine the mean and standard deviation of the data set.

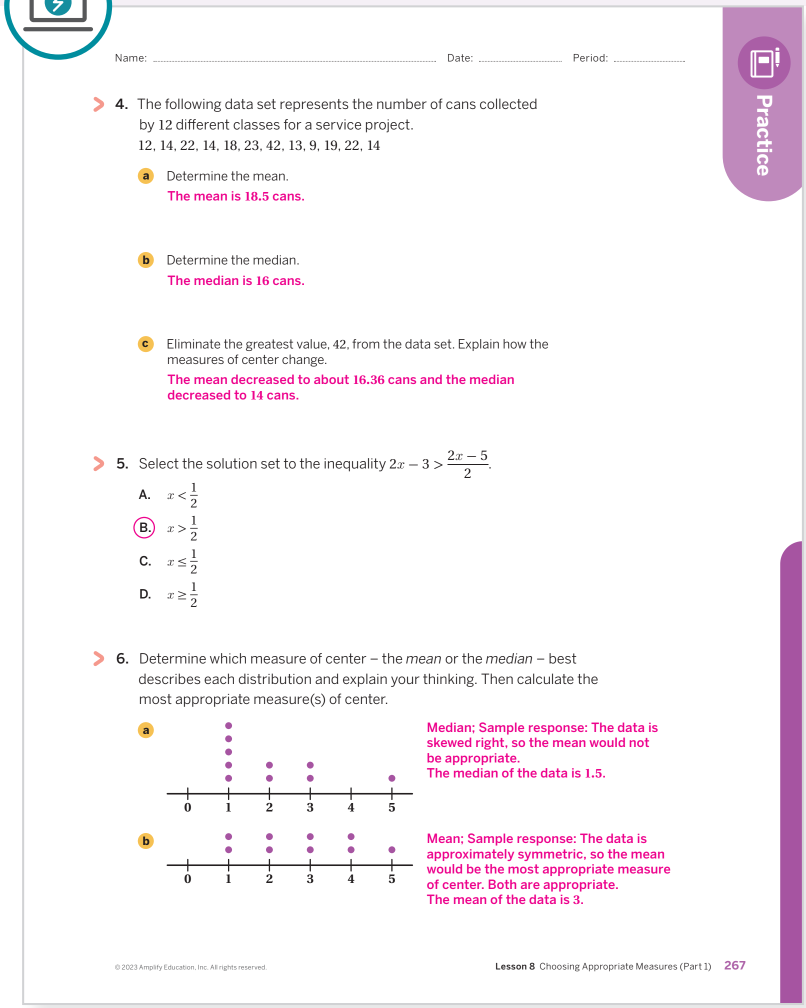
   Both the mean and the standard deviation are 1.5 paintings.

   b. The artist is not pleased with these statistics. If the 5 is increased to a larger value, how does this impact the mean, median, and standard deviation?

   The median will still be 1 painting. The mean and standard deviation will increase because the data is more spread out.

266 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.



Name: ______ Date: ______ Period: ______

4. The following data set represents the number of cans collected by 12 different classes for a service project.
   12, 14, 22, 14, 18, 23, 42, 13, 9, 19, 22, 14

   a. Determine the mean.

   The mean is 18.5 cans.

   b. Determine the median.

   The median is 16 cans.

   c. Eliminate the greatest value, 42, from the data set. Explain how the measures of center change.

   The mean decreased to about 16.36 cans and the median decreased to 14 cans.

5. Select the solution set to the inequality $2x - 3 > \frac{2x-5}{2}$.

   A. $x < \frac{1}{2}$

   [✓ B.] $x > \frac{1}{2}$

   C. $x \leq \frac{1}{2}$

   D. $x \geq \frac{1}{2}$

6. Determine which measure of center – the *mean* or the *median* – best describes each distribution and explain your thinking. Then calculate the most appropriate measure(s) of center.

   a. **Median; Sample response: The data is skewed right, so the mean would not be appropriate. The median of the data is 1.5.**

   b. **Mean; Sample response: The data is approximately symmetric, so the mean would be the most appropriate measure of center. Both are appropriate. The mean of the data is 3.**

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 8 Choosing Appropriate Measures (Part 1) 267

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 2 Lesson 5	2
	5	Unit 1 Lesson 14	2
Formative ⚡	6	Unit 2 Lesson 9	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 2 | **LESSON 9**

Choosing Appropriate Measures (Part 2)

Let's apply what you've learned about statistics to understand hurricanes.

Focus

Goals

1. **Language Goal:** Compare and contrast situations using measures of center and measures of variability. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students apply their statistical understanding to compare measures of center and variability in real-world and mathematical contexts. They summarize hurricane data and use it to make predictions regarding future hurricane seasons.

Previously

Students used the shapes of distributions to compare the statistics of different data sets and explored the effect of extreme values and outliers on data.

Coming Soon

In Lesson 10, students will be introduced to another mathematical method for determining outliers, this time using standard deviation.

Rigor

- Students **apply** their understanding of statistics to interpret measures of center and variability in a real-world context.
- Students strengthen their **fluency** in using statistics to compare different sets of data.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Explaining My Steps*
- Anchor Chart PDF, *Partner and Group Questioning*

Math Language Development

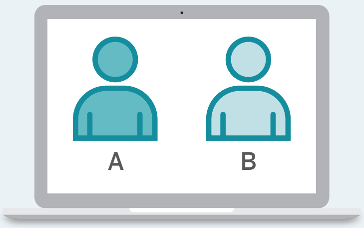
Review words

- *center*
- *IQR*
- *standard deviation*
- *variability*

Amps | **Featured Activity**

Activity 2
Digital Collaboration

Students work independently to choose the best measure of center and variability for a set of data representations. Then they compare their representations with their partner's to see who has the greater measure of center and the greater measure of variability.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might impulsively want to disregard any outlier for a data set. Ask students to discipline themselves to create a list of the ways outliers affect a data set. Encourage them to set a goal of understanding why outliers can be important.

Modifications to Pacing

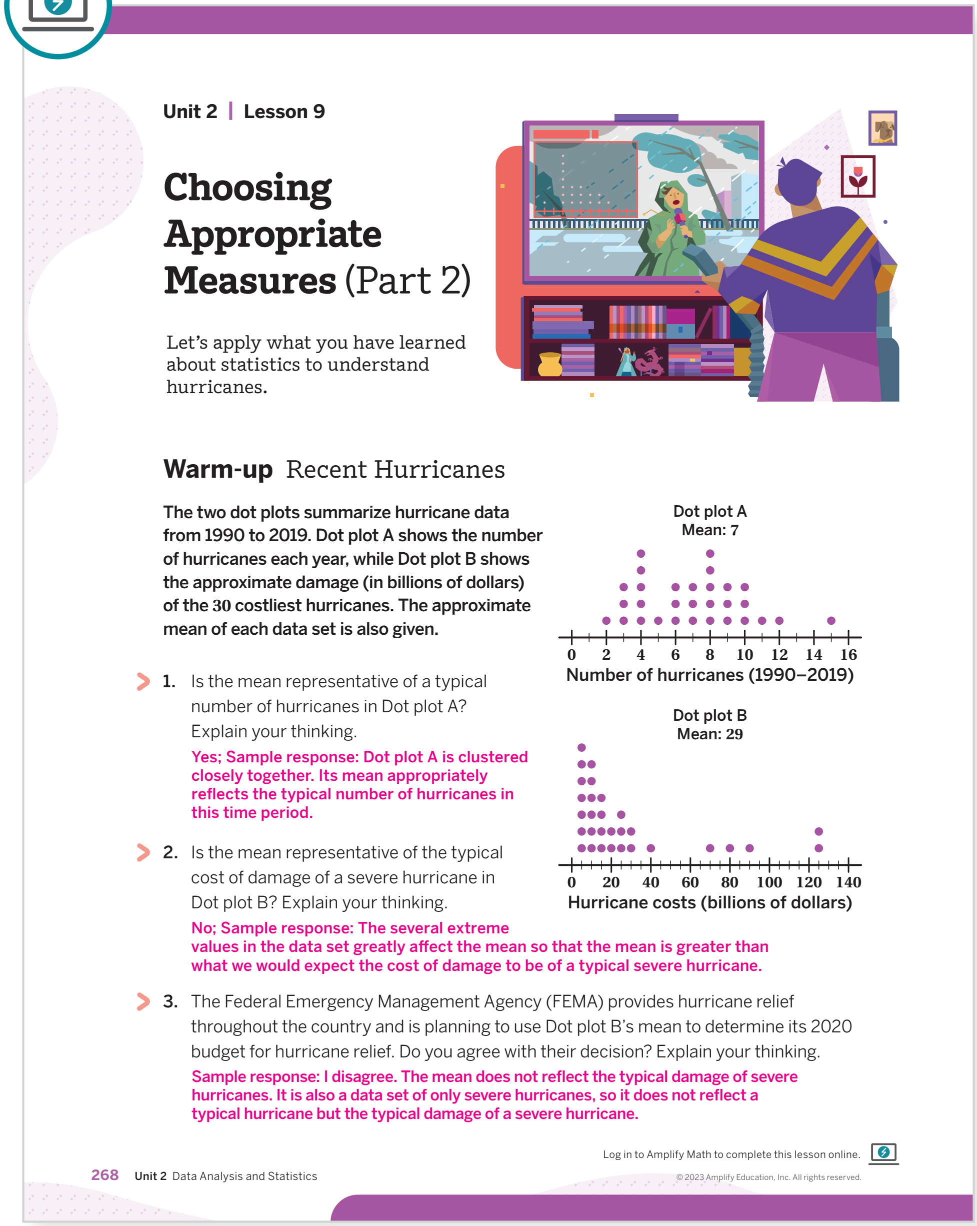
You may want to consider this additional modification if you are short on time:

- In **Activity 2**, reduce the number of data representations for students to consider.

Pairs | 10 min

Warm-up Recent Hurricanes

Students analyze two dotplots to determine if their mean represents typical values in the data set.



Unit 2 | Lesson 9

Choosing Appropriate Measures (Part 2)

Let's apply what you have learned about statistics to understand hurricanes.

Warm-up Recent Hurricanes

The two dot plots summarize hurricane data from 1990 to 2019. Dot plot A shows the number of hurricanes each year, while Dot plot B shows the approximate damage (in billions of dollars) of the 30 costliest hurricanes. The approximate mean of each data set is also given.

Dot plot A
Mean: 7

Number of hurricanes (1990–2019)

Dot plot B
Mean: 29

Hurricane costs (billions of dollars)

1. Is the mean representative of a typical number of hurricanes in Dot plot A? Explain your thinking.

   Yes; Sample response: Dot plot A is clustered closely together. Its mean appropriately reflects the typical number of hurricanes in this time period.

2. Is the mean representative of the typical cost of damage of a severe hurricane in Dot plot B? Explain your thinking.

   No; Sample response: The several extreme values in the data set greatly affect the mean so that the mean is greater than what we would expect the cost of damage to be of a typical severe hurricane.

3. The Federal Emergency Management Agency (FEMA) provides hurricane relief throughout the country and is planning to use Dot plot B's mean to determine its 2020 budget for hurricane relief. Do you agree with their decision? Explain your thinking.

   Sample response: I disagree. The mean does not reflect the typical damage of severe hurricanes. It is also a data set of only severe hurricanes, so it does not reflect a typical hurricane but the typical damage of a severe hurricane.

Log in to Amplify Math to complete this lesson online.

268 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display the graph of the two dot plots and read the prompt together as a whole class. Have students complete the problems independently before discussing their responses with a partner.

2 Monitor

Help students get started by prompting them to annotate the dot plots with descriptions of each.

Look for points of confusion:

- **Relating the graphs.** Prompt students to annotate and explain any differences they notice between the dot plots.

Look for productive strategies:

- Identifying and using the shape of the distribution to analyze the dot plots.
- Using measures of center or variability to explain why FEMA should not use the mean for its budget.

3 Connect

Display the dot plots and their means.

Have individual students share how they determined whether the mean represents typical values for each dot plot.

Highlight that Dot plot B is skewed right, while Dot plot A's distribution is close to bell-shaped. The mean for Dot plot B does not represent typical values because the extreme values in the data set skew the mean.

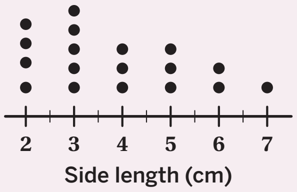
Ask, "What other statistics could you use to analyze Dot plot B?" I could use the median because this data set is skewed right. I could also examine the IQR and standard deviation of the dot plot and determine if these are appropriate measures of variability.

Power-up

To power up students' ability to determine the appropriateness of a measure of center from a dot plot, have students complete:

Recall that when data is symmetric, both the mean or median are appropriate measures of center. When data are skewed, the median is more appropriate than the mean.

Which measure of center is most appropriate to describe the distribution; *mean*, *median*, or *both*? Median

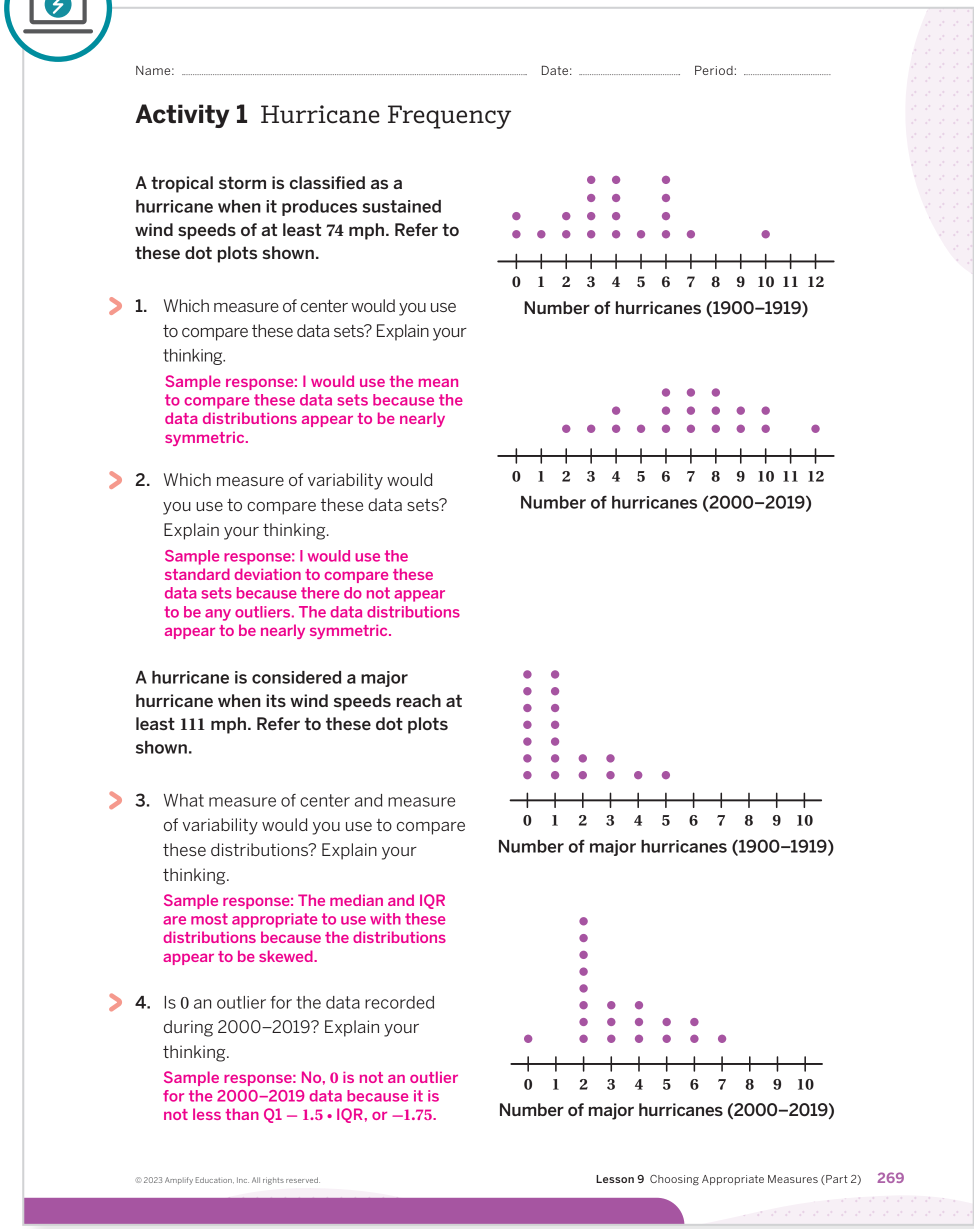


Use: Before the Warm-up

Informed by: Performance on Lesson 8, Practice Problem 6 and Exit Ticket

Activity 1 Hurricane Frequency

Students examine dot plots representing hurricane data to compare and interpret the measures of center and variability in context.



Name: ______ Date: ______ Period: ______

Activity 1 Hurricane Frequency

A tropical storm is classified as a hurricane when it produces sustained wind speeds of at least 74 mph. Refer to these dot plots shown.

Number of hurricanes (1900–1919)

Number of hurricanes (2000–2019)

1. Which measure of center would you use to compare these data sets? Explain your thinking.

   Sample response: I would use the mean to compare these data sets because the data distributions appear to be nearly symmetric.

2. Which measure of variability would you use to compare these data sets? Explain your thinking.

   Sample response: I would use the standard deviation to compare these data sets because there do not appear to be any outliers. The data distributions appear to be nearly symmetric.

A hurricane is considered a major hurricane when its wind speeds reach at least 111 mph. Refer to these dot plots shown.

Number of major hurricanes (1900–1919)

Number of major hurricanes (2000–2019)

3. What measure of center and measure of variability would you use to compare these distributions? Explain your thinking.

   Sample response: The median and IQR are most appropriate to use with these distributions because the distributions appear to be skewed.

4. Is 0 an outlier for the data recorded during 2000–2019? Explain your thinking.

   Sample response: No, 0 is not an outlier for the 2000–2019 data because it is not less than Q1 – 1.5 • IQR, or –1.75.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Choosing Appropriate Measures (Part 2) 269

1 Launch

Arrange students in pairs. Have student-pairs discuss each problem before completing individually, then compare solutions and strategies.

2 Monitor

Help students get started by asking, "What information can you determine from each dot plot?"

Look for points of confusion:

- **Having difficulty determining which measure of center and variability is more appropriate.** Prompt students to create two-column graphic organizers to compare each measure of center and variability.
- **Struggling to use statistical measures to compare the data.** Ask, "What does it mean when a measure of center is greater? What does it mean when a measure of variability is greater?"
- **Having difficulty distinguishing between relevant statistics for a meteorologist and climatologist.** Ask, "What kind of information does a meteorologist/climatologist need and want to know? Why?"

Look for productive strategies:

- Annotating the dot plots to identify any possible outliers and the general shape of the distribution.
- Using shapes of data distributions, measures of center, or measures of variability to justify responses.

Activity 1 continued ›

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they know the difference between a tropical storm, a hurricane, and a major hurricane. Mention that these are all storms and even a tropical storm can cause major wind or water damage. The categories of these storms are determined by wind speed.

Extension: *Interdisciplinary Connections, Math Enrichment*

Provide students with hurricane data for other current years. Have them add these to the dot plots in this activity and ask them to recalculate the statistics from Problem 4. Data for 2020 is shown.

Hurricanes: 13 hurricanes **Major hurricanes:** 6 hurricanes

Math Language Development

MLR5: Co-craft Questions

Ask students to read the introductory text and study the first two dot plots shown. Have them work with their partner to write 1–2 questions they have about the information presented before they begin the activity. Ask volunteers to share their questions with the class.

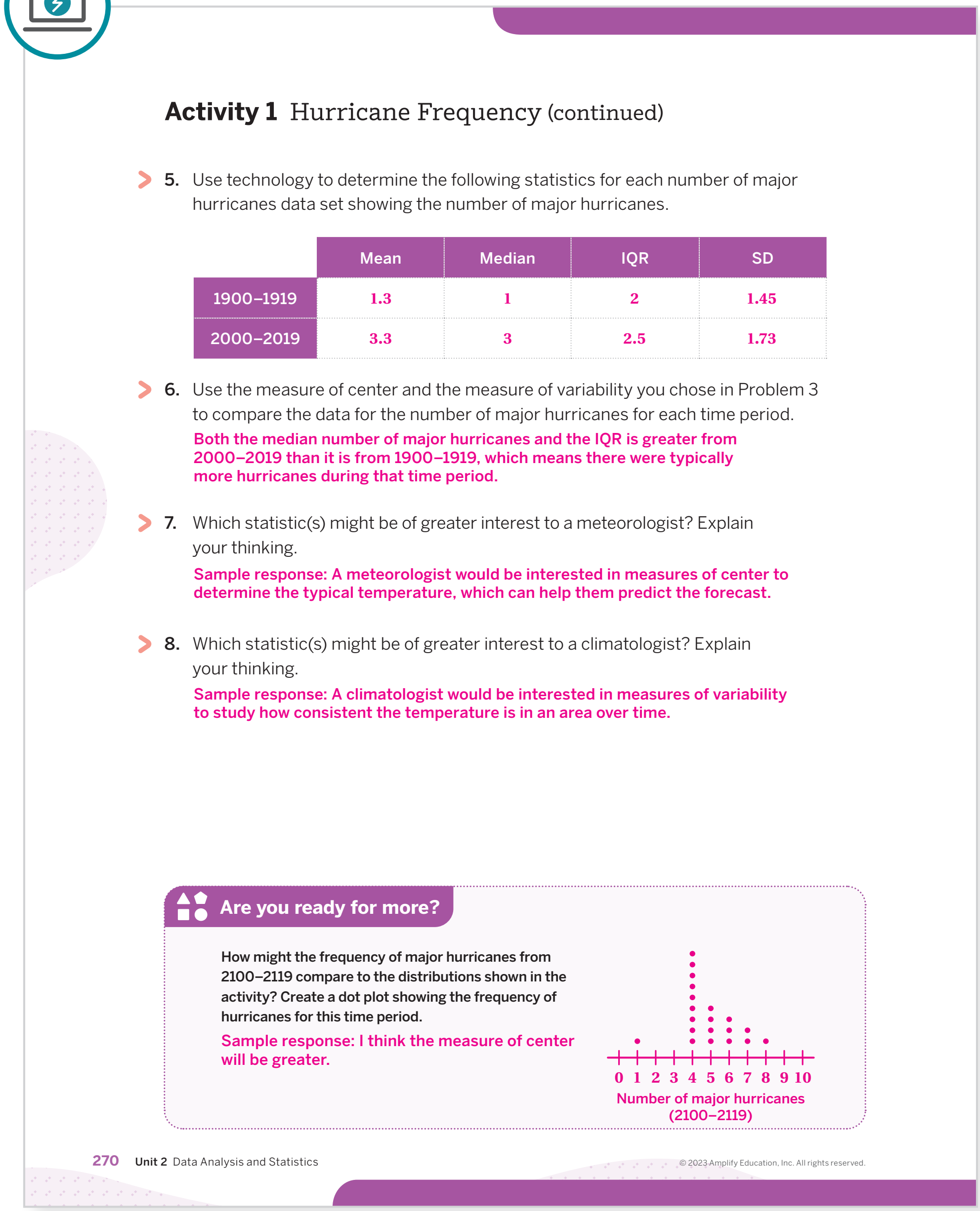
English Learners

Model a mathematical question for students such as, "What does each dot represent on the dot plot?"

Pairs | 15 min

Activity 1 Hurricane Frequency (continued)

Students examine dot plots representing hurricane data to compare and interpret the measures of center and variability in context.



Activity 1 Hurricane Frequency (continued)

5. Use technology to determine the following statistics for each number of major hurricanes data set showing the number of major hurricanes.

	Mean	Median	IQR	SD
1900–1919	1.3	1	2	1.45
2000–2019	3.3	3	2.5	1.73

6. Use the measure of center and the measure of variability you chose in Problem 3 to compare the data for the number of major hurricanes for each time period.
   Both the median number of major hurricanes and the IQR is greater from 2000–2019 than it is from 1900–1919, which means there were typically more hurricanes during that time period.

7. Which statistic(s) might be of greater interest to a meteorologist? Explain your thinking.
   Sample response: A meteorologist would be interested in measures of center to determine the typical temperature, which can help them predict the forecast.

8. Which statistic(s) might be of greater interest to a climatologist? Explain your thinking.
   Sample response: A climatologist would be interested in measures of variability to study how consistent the temperature is in an area over time.

Are you ready for more?

How might the frequency of major hurricanes from 2100–2119 compare to the distributions shown in the activity? Create a dot plot showing the frequency of hurricanes for this time period.

Sample response: I think the measure of center will be greater.

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the data representations.

Have students share how they determined which measure of center and variability they chose for each data representation.

Highlight that the first set of dot plots are approximately symmetric, so it is more appropriate to use the mean and standard deviation as measures of center and variability. The second set of dot plots are more skewed, so using the median and IQR is more appropriate.

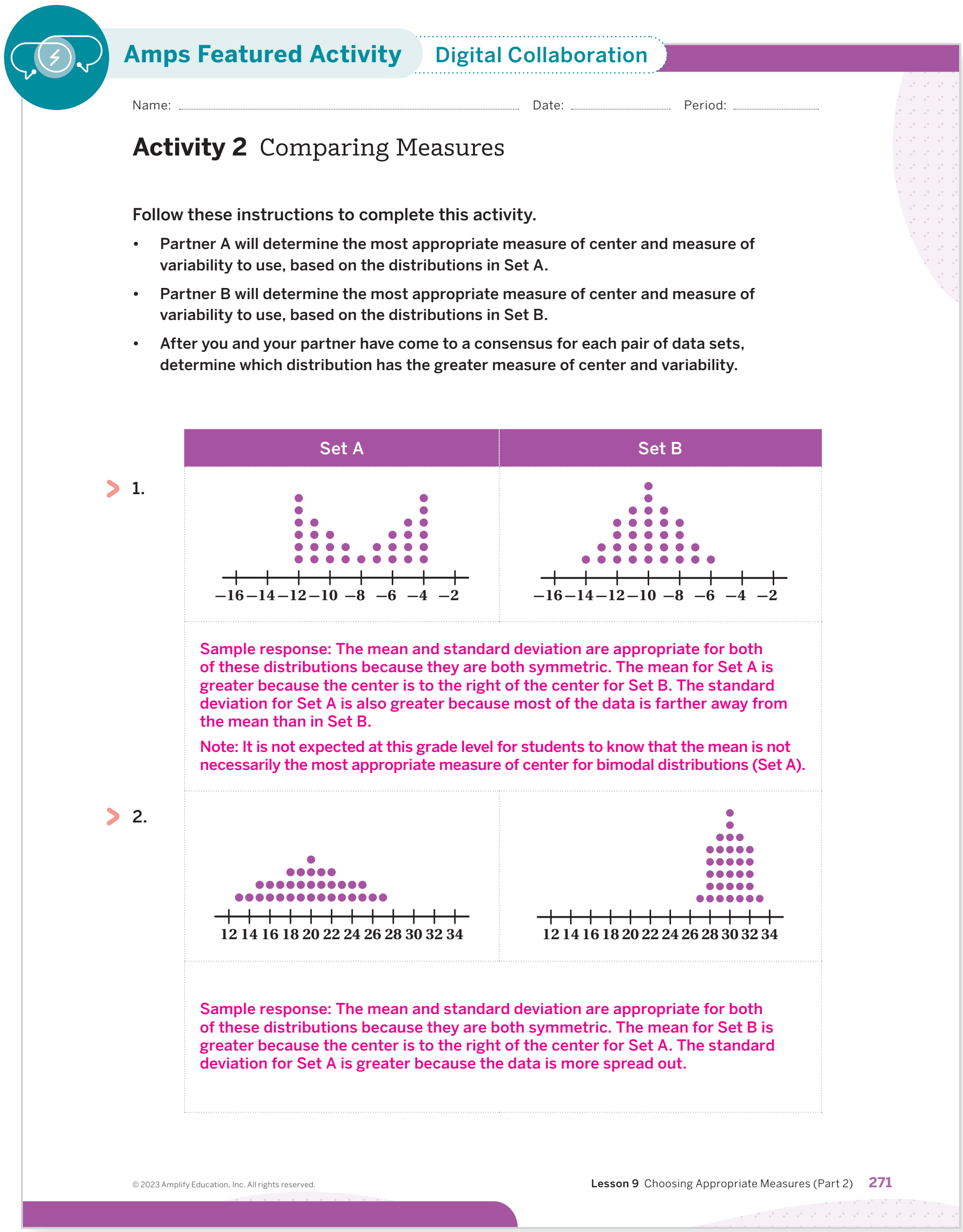
Ask:

- "Can you use these statistics to make any predictions about the future of hurricanes?"
- "What additional data might you want in order to make predictions? Why?"

Pairs | 15 min

Activity 2 Comparing Measures

Students determine the most appropriate measure of center and variability for a data set and then use those measures to compare data representations.



Amps Featured Activity Digital Collaboration

Name: ____________ Date: ________ Period: ________

Activity 2 Comparing Measures

Follow these instructions to complete this activity.

- Partner A will determine the most appropriate measure of center and measure of variability to use, based on the distributions in Set A.
- Partner B will determine the most appropriate measure of center and measure of variability to use, based on the distributions in Set B.
- After you and your partner have come to a consensus for each pair of data sets, determine which distribution has the greater measure of center and variability.

	Set A	Set B

1. Sample response: The mean and standard deviation are appropriate for both of these distributions because they are both symmetric. The mean for Set A is greater because the center is to the right of the center for Set B. The standard deviation for Set A is also greater because most of the data is farther away from the mean than in Set B.

   Note: It is not expected at this grade level for students to know that the mean is not necessarily the most appropriate measure of center for bimodal distributions (Set A).

2. Sample response: The mean and standard deviation are appropriate for both of these distributions because they are both symmetric. The mean for Set B is greater because the center is to the right of the center for Set A. The standard deviation for Set A is greater because the data is more spread out.

©2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Choosing Appropriate Measures (Part 2) 271

1 Launch

Arrange students in pairs and review instructions for the activity. Allow time for students to determine the appropriate measures for each representation in their assigned column before they begin discussing their selections and comparing representations.

2 Monitor

Help students get started by prompting them to write justifications for their choices to refer to during their partner discussions.

Look for points of confusion:

- **Struggling to determine which statistical measure is more appropriate in Problem 6.** Remind students that for this comparison, they can just consider which measure is greater.
- **Thinking there is not enough information to compare center and variability in Problem 6.** Ask, "What do you think the distribution would look like for each scenario?"

Look for productive strategies:

- Indicating whether the data is symmetric or skewed.
- Comparing the clusters or spreads of two data sets.
- Calculating IQR for the box plots.
- Sketching a representation for the word problems.
- Using symmetry or spread to justify which is the most appropriate measure of center and variability.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Chunk this task into smaller, more manageable parts by scaffolding the directions.

- Determine whether the distribution is symmetric or not symmetric.
- Determine the most appropriate measure of center and variability.
- Determine which distribution has a greater measure of center and variability.

Math Language Development

MLR7: Compare and Connect

As partners analyze their respective data sets, encourage them to identify similarities and differences among their distributions. As you circulate, consider asking these questions:

- "Which distributions are symmetric? How is it possible to look different from each other and still be symmetric?"
- "How can you tell just by looking at the symmetric distributions in Problem 2 which standard deviation is greater?"

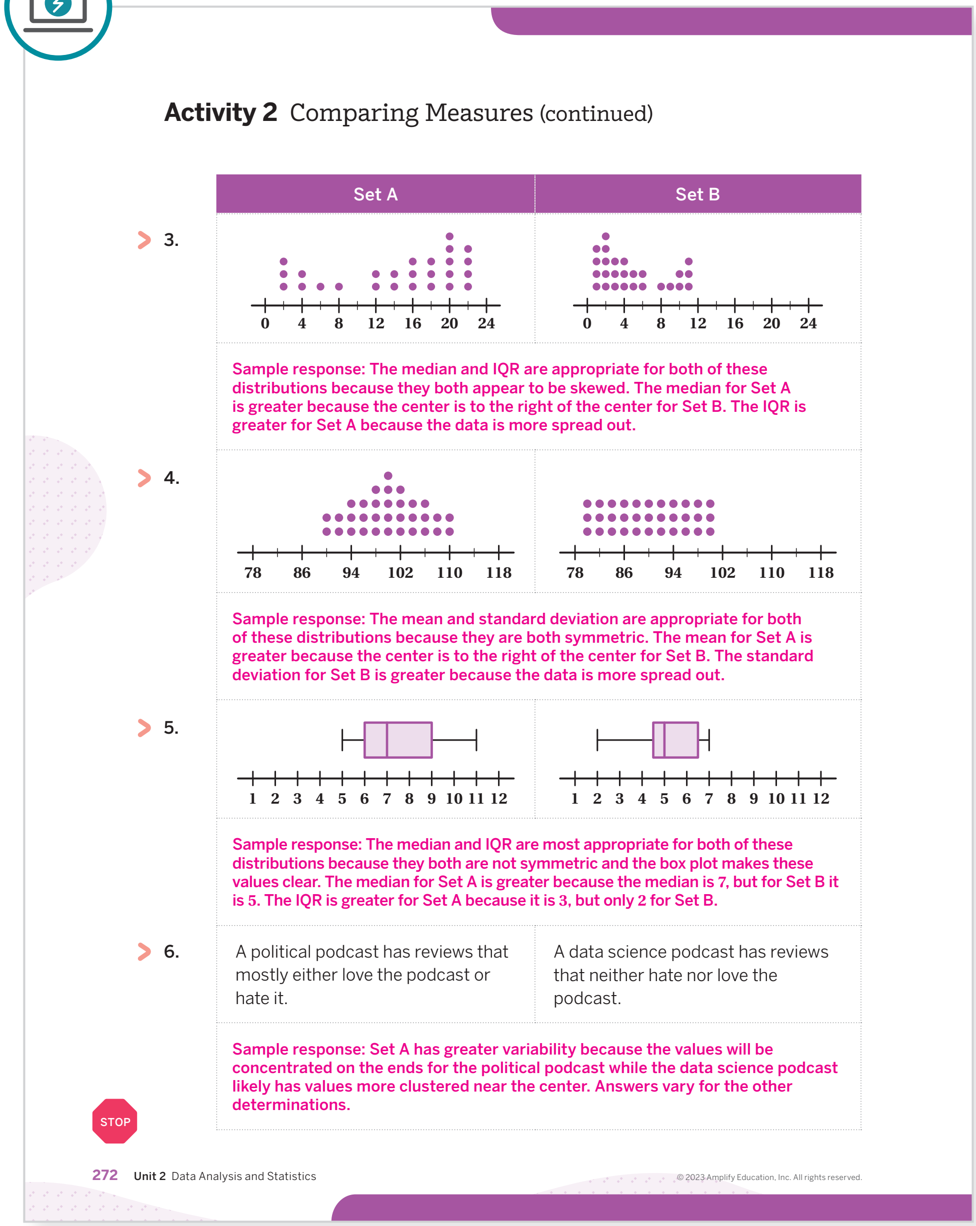
English Learners

Provide access to the Anchor Chart PDF, *Sentence Stems, Explaining My Steps* to support students in justifying their choices and access to the Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning* to support partner discussions when comparing data sets.

Pairs | 15 min

Activity 2 Comparing Measures (continued)

Students determine the most appropriate measure of center and variability for a data set and then use those measures to compare data representations.



Activity 2 Comparing Measures (continued)

	Set A	Set B
3.		
	Sample response: The median and IQR are appropriate for both of these distributions because they both appear to be skewed. The median for Set A is greater because the center is to the right of the center for Set B. The IQR is greater for Set A because the data is more spread out.	
4.		
	Sample response: The mean and standard deviation are appropriate for both of these distributions because they are both symmetric. The mean for Set A is greater because the center is to the right of the center for Set B. The standard deviation for Set B is greater because the data is more spread out.	
5.		
	Sample response: The median and IQR are most appropriate for both of these distributions because they both are not symmetric and the box plot makes these values clear. The median for Set A is greater because the median is 7, but for Set B it is 5. The IQR is greater for Set A because it is 3, but only 2 for Set B.	
6.	A political podcast has reviews that mostly either love the podcast or hate it.	A data science podcast has reviews that neither hate nor love the podcast.
	Sample response: Set A has greater variability because the values will be concentrated on the ends for the political podcast while the data science podcast likely has values more clustered near the center. Answers vary for the other determinations.	

STOP

272 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display each pair of representations one at a time.

Have pairs of students share how they determined whether to use the mean or median, and which data set showed greater variability.

Ask:

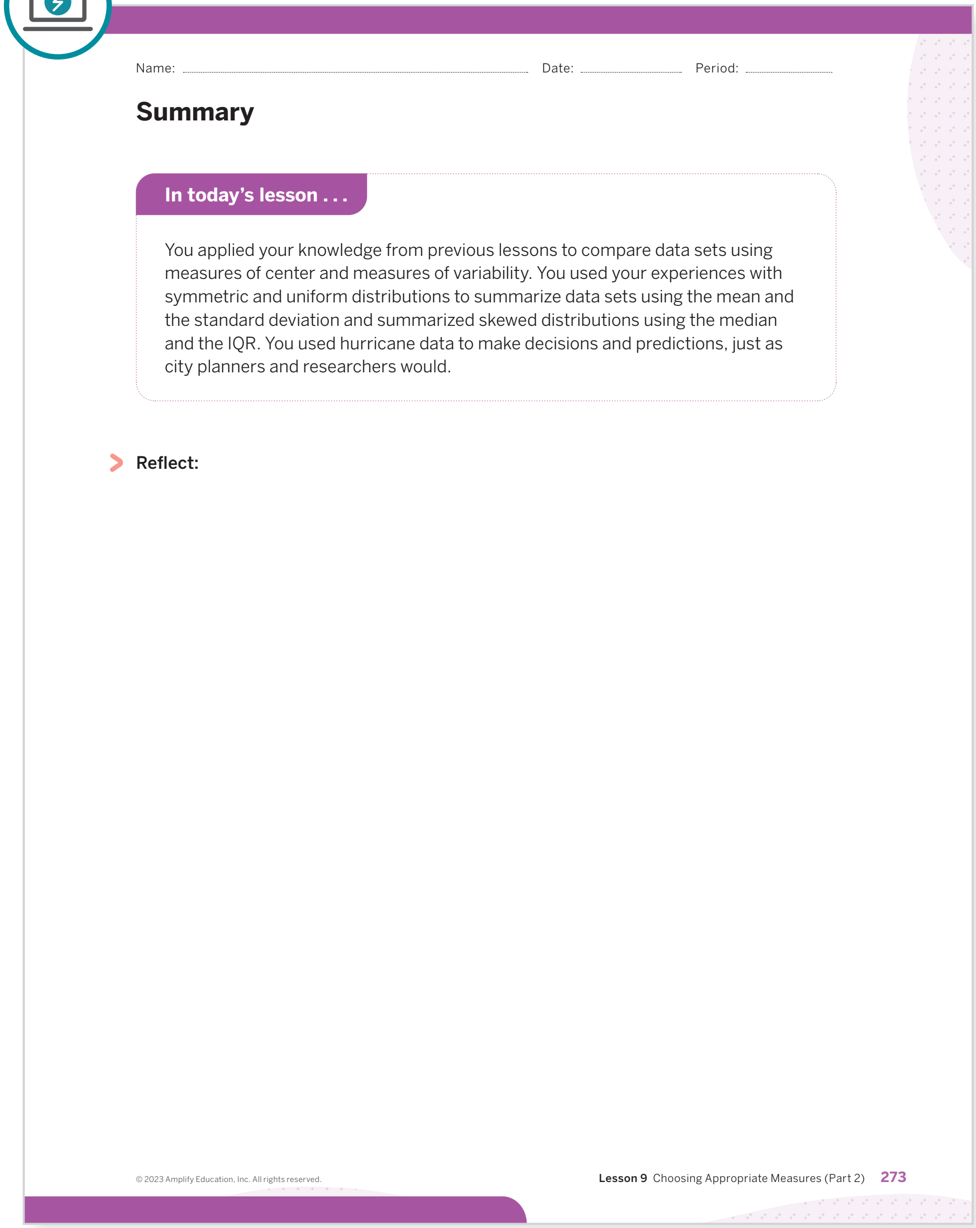
- "What strategies were useful when determining the most appropriate measure of center and variability?"
- "What type of data would you want to show a greater variability? What type of data would you want to show less variability?"

Highlight the data sets that are symmetric and those that are skewed. Model how to compare variability as each problem is displayed by estimating the center and how spread apart the data is relative to the center. Sketch a dot plot for the scenarios in Problem 6 to help students visualize the spread of each.

Whole Class | 5 min

Summary

Review and synthesize strategies for comparing data sets using the appropriate measures of center and variability.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You applied your knowledge from previous lessons to compare data sets using measures of center and measures of variability. You used your experiences with symmetric and uniform distributions to summarize data sets using the mean and the standard deviation and summarized skewed distributions using the median and the IQR. You used hurricane data to make decisions and predictions, just as city planners and researchers would.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Have students share the connections they saw in this lesson among shapes of distributions, measures of center, and measures of variability.

Ask:

- "How do you determine which measure of center to use for a data set?" Look at the shape and use the mean when it is symmetric, or really close to symmetric, and the median when it is skewed or if there are outliers.
- "How do you compare the measures of variability for a data set?" Either calculate them or estimate them from a data representation.
- "How do you estimate variability when looking at data representations?" Try to estimate the center and then estimate how spread apart the data is.

Highlight that selecting the appropriate measure of center (and variability) is the first step when comparing data sets. Once selected, these measures can be compared with data sets that have similar shapes.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

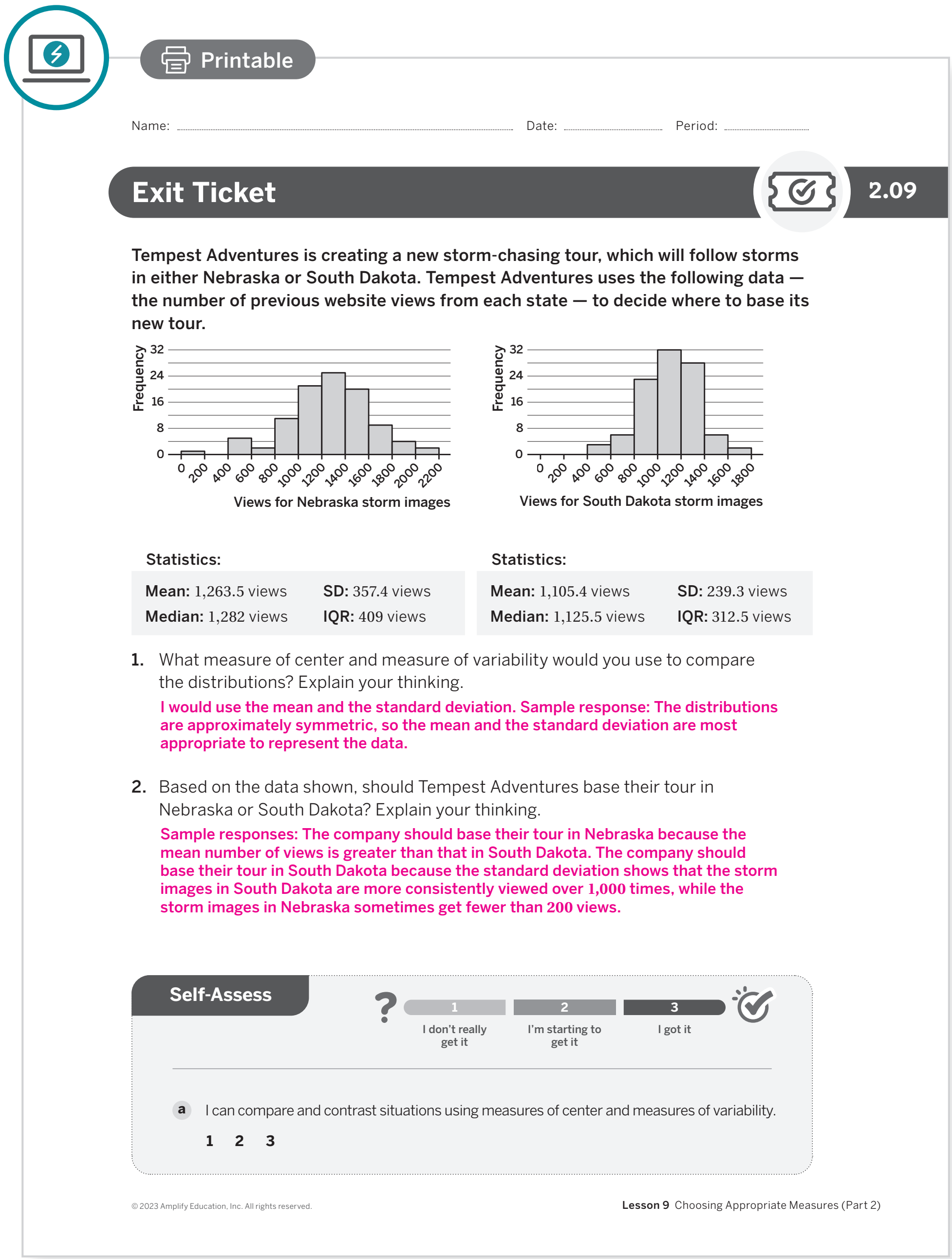
- "How do you compare the measures of center and measures of variability of two data sets?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using appropriate measures of center and variability to compare two data sets.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 2.09

Tempest Adventures is creating a new storm-chasing tour, which will follow storms in either Nebraska or South Dakota. Tempest Adventures uses the following data — the number of previous website views from each state — to decide where to base its new tour.

Statistics:

Mean: 1,263.5 views	**SD:** 357.4 views
Median: 1,282 views	**IQR:** 409 views

Statistics:

Mean: 1,105.4 views	**SD:** 239.3 views
Median: 1,125.5 views	**IQR:** 312.5 views

1. What measure of center and measure of variability would you use to compare the distributions? Explain your thinking.

   I would use the mean and the standard deviation. Sample response: The distributions are approximately symmetric, so the mean and the standard deviation are most appropriate to represent the data.

2. Based on the data shown, should Tempest Adventures base their tour in Nebraska or South Dakota? Explain your thinking.

   Sample responses: The company should base their tour in Nebraska because the mean number of views is greater than that in South Dakota. The company should base their tour in South Dakota because the standard deviation shows that the storm images in South Dakota are more consistently viewed over 1,000 times, while the storm images in Nebraska sometimes get fewer than 200 views.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can compare and contrast situations using measures of center and measures of variability.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Choosing Appropriate Measures (Part 2)

Success looks like . . .

- **Language Goal:** Comparing and contrasting situations using measures of center and measures of variability. **(Speaking and Listening, Reading and Writing)**
  - » Explaining which measures of center and variability to use to compare the two distributions in Problem 1.

Suggested next steps

If students cannot select the appropriate measure of center and variability for Problem 1, consider:

- Asking, "Is the data symmetric or skewed?"
- Assigning Practice Problem 1.

If students do not use specific statistics to justify their response in Problem 2, consider:

- Reviewing Activity 3.
- Assigning Practice Problem 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The instructional goal for this lesson was to compare and contrast situations using measures of center and measures of variability. How well did students accomplish this? What did you specifically do to help students accomplish this?
- What resources did students use as they worked on Activity 2? Which resources were especially helpful?

Math Language Development

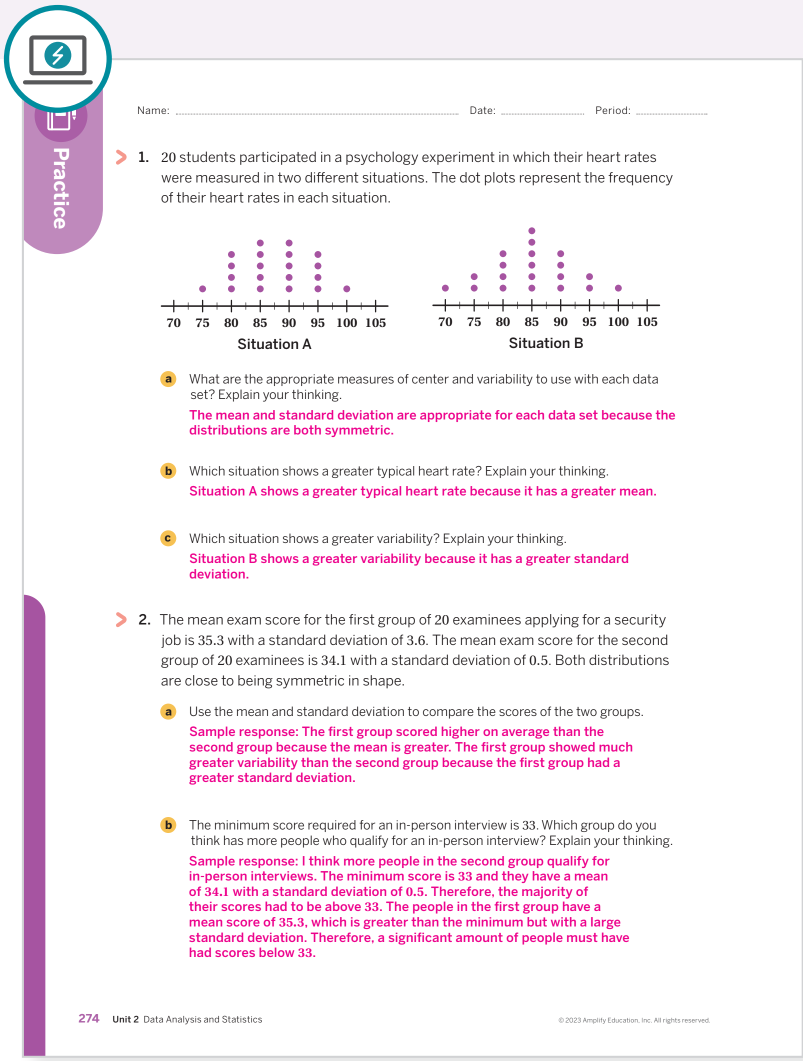
Language Goal: Comparing and contrasting situations using measures of center and measures of variability.

Reflect on students' language development toward this goal.

- How have students progressed in their descriptions that compare two data distributions using their measures of center and variability? Are they choosing appropriate measures and justifying their selection?
- Are they using terms and phrases such as *symmetric* or *not symmetric* and do their descriptions include a comparison of the mean or median and the standard deviation or interquartile range?

Practice

Independent



Practice

Name: Date: Period:

1. 20 students participated in a psychology experiment in which their heart rates were measured in two different situations. The dot plots represent the frequency of their heart rates in each situation.

a What are the appropriate measures of center and variability to use with each data set? Explain your thinking.

The mean and standard deviation are appropriate for each data set because the distributions are both symmetric.

b Which situation shows a greater typical heart rate? Explain your thinking.

Situation A shows a greater typical heart rate because it has a greater mean.

c Which situation shows a greater variability? Explain your thinking.

Situation B shows a greater variability because it has a greater standard deviation.

2. The mean exam score for the first group of 20 examinees applying for a security job is 35.3 with a standard deviation of 3.6. The mean exam score for the second group of 20 examinees is 34.1 with a standard deviation of 0.5. Both distributions are close to being symmetric in shape.

a Use the mean and standard deviation to compare the scores of the two groups.

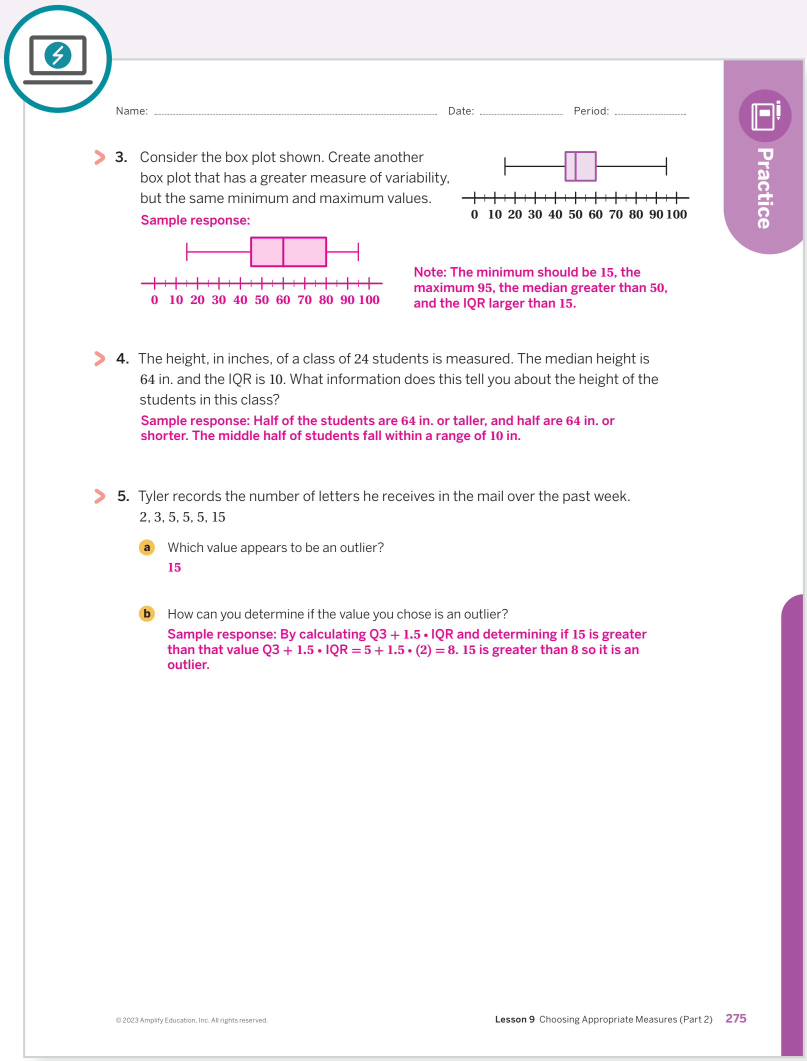
Sample response: The first group scored higher on average than the second group because the mean is greater. The first group showed much greater variability than the second group because the first group had a greater standard deviation.

b The minimum score required for an in-person interview is 33. Which group do you think has more people who qualify for an in-person interview? Explain your thinking.

Sample response: I think more people in the second group qualify for in-person interviews. The minimum score is 33 and they have a mean of 34.1 with a standard deviation of 0.5. Therefore, the majority of their scores had to be above 33. The people in the first group have a mean score of 35.3, which is greater than the minimum but with a large standard deviation. Therefore, a significant amount of people must have had scores below 33.

274 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: Date: Period:

3. Consider the box plot shown. Create another box plot that has a greater measure of variability, but the same minimum and maximum values.

Sample response:

Note: The minimum should be 15, the maximum 95, the median greater than 50, and the IQR larger than 15.

4. The height, in inches, of a class of 24 students is measured. The median height is 64 in. and the IQR is 10. What information does this tell you about the height of the students in this class?

Sample response: Half of the students are 64 in. or taller, and half are 64 in. or shorter. The middle half of students fall within a range of 10 in.

5. Tyler records the number of letters he receives in the mail over the past week.
2, 3, 5, 5, 5, 15

a Which value appears to be an outlier?

15

b How can you determine if the value you chose is an outlier?

Sample response: By calculating $Q3 + 1.5 \cdot IQR$ and determining if 15 is greater than that value $Q3 + 1.5 \cdot IQR = 5 + 1.5 \cdot (2) = 8$. 15 is greater than 8 so it is an outlier.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 Choosing Appropriate Measures (Part 2) 275

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 3	3
Spiral	4	Unit 2 Lesson 4	2
Formative ⚡	5	Unit 2 Lesson 10	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 2 | **LESSON 10**

Outliers and Standard Deviation

Let's take a closer look at outliers, now that we know more about standard deviation.

Focus

Goals

1. Use the standard deviation and mean to determine if a value in a data set is an outlier.
2. **Language Goal:** Investigate the source of an outlier and determine whether to include or exclude it from data analysis. **(Speaking and Listening)**
3. **Language Goal:** Explain how an outlier impacts the mean and standard deviation. **(Reading and Writing)**

Coherence

Today

Students consider how to determine the presence of outliers in a data set using standard deviation and mean, and how their inclusion can affect the statistics of the data set. They also consider the source of such outliers and reason abstractly and quantitatively about whether to exclude them, based on context and the data collection process.

Previously

Students were introduced to one method for determining if a value is an outlier using the IQR. They have also used standard deviation to measure a data set's typical distance to the mean.

Coming Soon

Students will study data representations involving two variables and learn how to interpret them.

Rigor

- Students further their **conceptual understanding** of outliers with respect to standard deviation.
- Students develop **fluency** in identifying outliers and deciding if they should be included in the analysis of a data set.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	12 min	12 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning*
- Anchor Chart PDF, *Shapes of Distributions*
- scientific calculators
- spreadsheet technology

Math Language Development

Review words

- *IQR*
- *outlier*
- *standard deviation*

Amps | Featured Activity

Activity 1
Visualizing Outliers and Standard Deviation

Students visualize what it means for data values to fall within a certain number of standard deviations of the mean and consider a new definition for outlier that involves this statistic.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Some students may feel pressured to just agree with their partners in order to avoid conflict. Encourage everyone to use their voice to speak their opinions throughout Activity 2. Discuss ways to internally deal with social pressures in order to gain the confidence to speak up. Thinking of the pair as a team, rather than two individuals will highlight the importance of working together, rather than having one person do all of the work.

● Modifications to Pacing

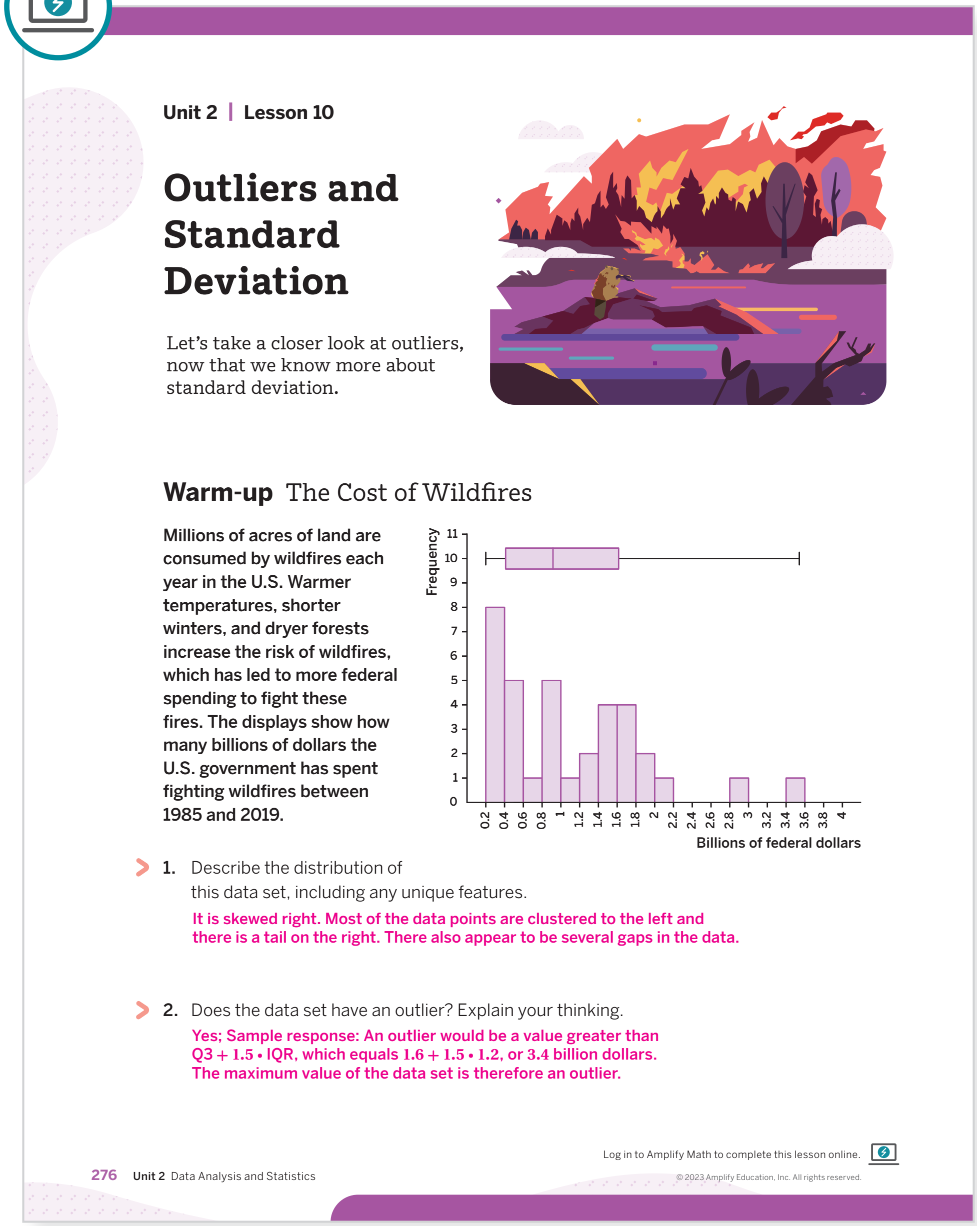
You may want to consider these additional modifications if you are short on time.

- In **Activity 2,** have student pairs consider problem 2a, 2b, or 2c only and discuss all 3 during the Connect.
- In **Activity 3**, Problems 1 and 3a, have students calculate the mean and standard deviation only.

Independent | 5 min

Warm-up The Cost of Wildfires

Students describe the distribution of a data set and determine if an outlier is present to activate prior knowledge about the mathematical definition of an outlier.



Unit 2 | Lesson 10

Outliers and Standard Deviation

Let's take a closer look at outliers, now that we know more about standard deviation.

Warm-up The Cost of Wildfires

Millions of acres of land are consumed by wildfires each year in the U.S. Warmer temperatures, shorter winters, and dryer forests increase the risk of wildfires, which has led to more federal spending to fight these fires. The displays show how many billions of dollars the U.S. government has spent fighting wildfires between 1985 and 2019.

1. Describe the distribution of this data set, including any unique features.

   It is skewed right. Most of the data points are clustered to the left and there is a tail on the right. There also appear to be several gaps in the data.

2. Does the data set have an outlier? Explain your thinking.

   Yes; Sample response: An outlier would be a value greater than Q3 + $1.5 \cdot$ IQR, which equals $1.6 + 1.5 \cdot 1.2$, or 3.4 billion dollars. The maximum value of the data set is therefore an outlier.

Log in to Amplify Math to complete this lesson online.

276 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

① Launch

Display the histogram and box plot and perform the ***Notice and Wonder*** routine with the class. Point out that the horizontal axis is in billions of dollars. Make sure students are able to express the intervals shown in the correct dollar amount.

② Monitor

Help students get started by asking, "What is one way to determine if a value is an outlier?"

Look for points of confusion:

- **Describing only the skew of the distribution of the data set.** Ask students if the representation has any other unique features worth mentioning.

Look for productive strategies:

- Describing the gaps and tails in the distribution.
- Measuring the width of the box in the box plot.
- Calculating Q3 + $1.5 \cdot$ IQR to determine outliers.

③ Connect

Have students share their descriptions of the distribution of the data set and how they determined if an outlier was present.

Ask, "Which measure of center is more appropriate for this distribution? Which measure of variability is more appropriate?" The median and IQR.

Highlight that there are two gaps in the data, and because it has a right tail, the distribution is skewed right. Using the definition Q3 + $1.5 \cdot$ IQR, it can be determined that the maximum value in the data set is an outlier. In this lesson, students will consider another definition for determining if a value is an outlier using the standard deviation.

Math Language Development

MLR2: Collect and Display

During the Connect, capture the language students use to describe the distribution and determine whether the visual display shows an outlier. Write helpful phrases on a display, such as *cluster*, *gap*, and the formula for determining an outlier, so that students can refer to it during subsequent activities in which they must identify outliers. Continue adding to the display during this lesson and throughout the unit.

English Learners

Provide access to the Anchor Chart PDF, *Shapes of Distributions* to support students as they describe the distribution of this data set.

Power-up

To power up students' ability to use the IQR to determine whether a value is an outlier, have students complete:

Recall that a value is considered an outlier if it is 1.5 IQRs less than Q1 or greater than Q3. A group of students recorded the distance in miles of the park nearest to their homes: $0.5, 0.75, 1, 1.25, 1.5, 2, 2, 2{,}5, 2.75, 8$.

a. Do any values appear to be an outlier? 8

b. Determine the values of Q1, Q3, and the IQR. Q1: 1, Q3: 2.5, IQR: 1.5

c. Use your values from part b to verify the existence of any outliers. Be prepared to explain your thinking. Sample response: 8 is an outlier because $8 > 1.5 \cdot 1.5 + 2.5$.

Use: Before the Warm-up

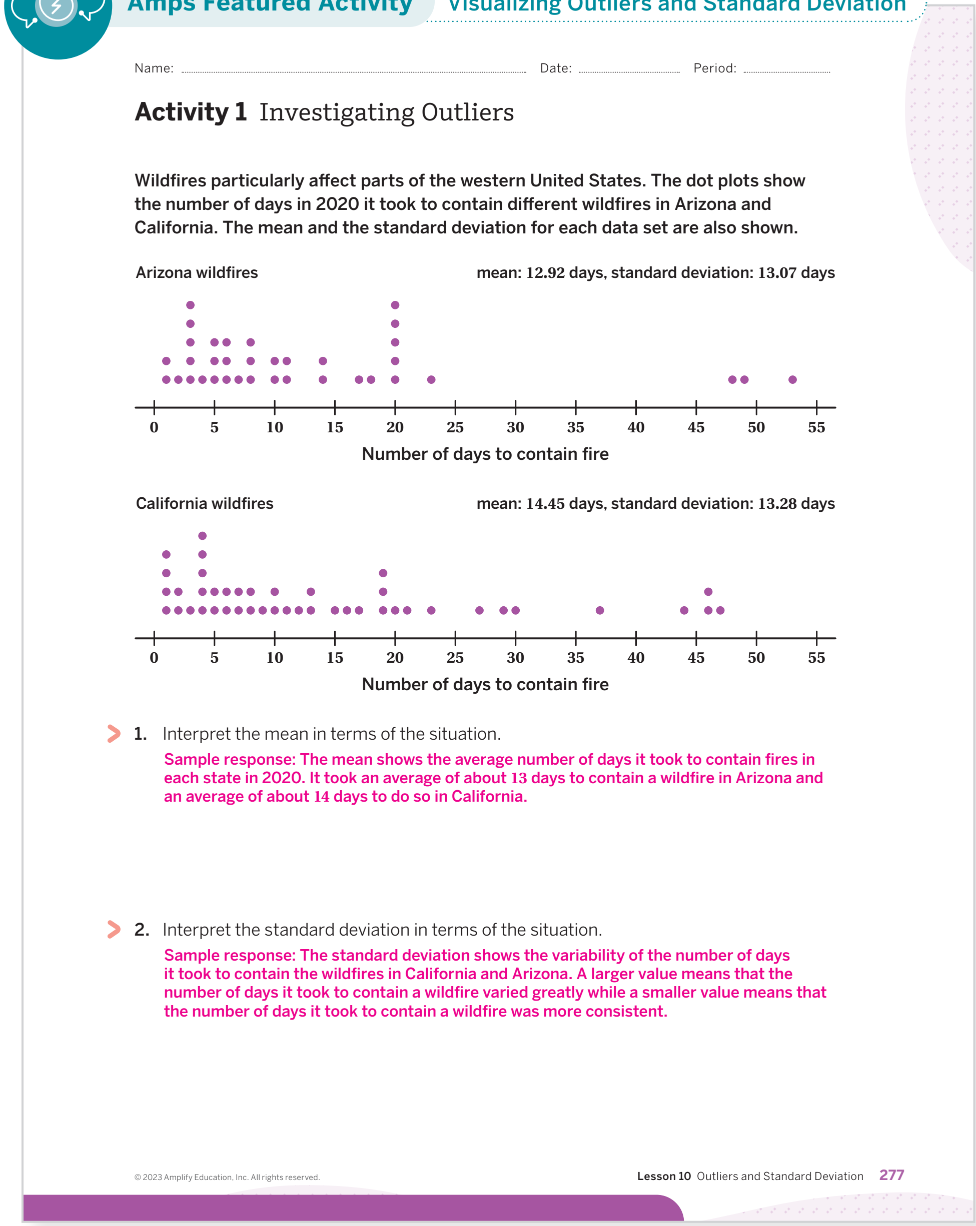
Informed by: Performance on Lesson 9, Practice Problem 5

Pairs | 12 min

Activity 1 Investigating Outliers

Students interpret mean and standard deviation in context and use these statistics to compare two data sets and determine whether either set contains an outlier.

Amps Featured Activity Visualizing Outliers and Standard Deviation



Name: ________ Date: ________ Period: ________

Activity 1 Investigating Outliers

Wildfires particularly affect parts of the western United States. The dot plots show the number of days in 2020 it took to contain different wildfires in Arizona and California. The mean and the standard deviation for each data set are also shown.

Arizona wildfires — mean: 12.92 days, standard deviation: 13.07 days

California wildfires — mean: 14.45 days, standard deviation: 13.28 days

1. Interpret the mean in terms of the situation.

   Sample response: The mean shows the average number of days it took to contain fires in each state in 2020. It took an average of about 13 days to contain a wildfire in Arizona and an average of about 14 days to do so in California.

2. Interpret the standard deviation in terms of the situation.

   Sample response: The standard deviation shows the variability of the number of days it took to contain the wildfires in California and Arizona. A larger value means that the number of days it took to contain a wildfire varied greatly while a smaller value means that the number of days it took to contain a wildfire was more consistent.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Outliers and Standard Deviation 277

1 Launch

Allow students individual work time. Then have them share their work with their partner and compare their definitions of an outlier.

2 Monitor

Help students get started by asking, "What does the measure of center tell you about each data set?"

Look for points of confusion:

- **Struggling to interpret the standard deviation in context.** Encourage students to focus less on the values of the standard deviations and more on which of the two distributions have a greater standard deviation.

Look for productive strategies:

- Annotating the mean of each data set on the dot plot.
- Multiplying the standard deviation by a number to indicate distance from the mean.

Activity 1 continued >

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can visualize what it means for data values to fall within a certain number of standard deviations from the mean. This will help them visually access this concept to support them as they explore a new definition for an outlier that relates to the standard deviation and the mean.

Accessibility: *Guide Processing and Visualization*

Provide annotated dot plots that show the locations of the mean and 1, 2, and 3 standard deviations from the mean. This will help students visualize what it means to be "_____ deviations from the mean."

Math Language Development

MLR8: Discussion Supports

During the Connect, display or provide access to the Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning* to support students as they compare their responses and definitions for an outlier that include the mean and standard deviation. Acknowledge for students that their previous definition of an outlier is being redefined here.

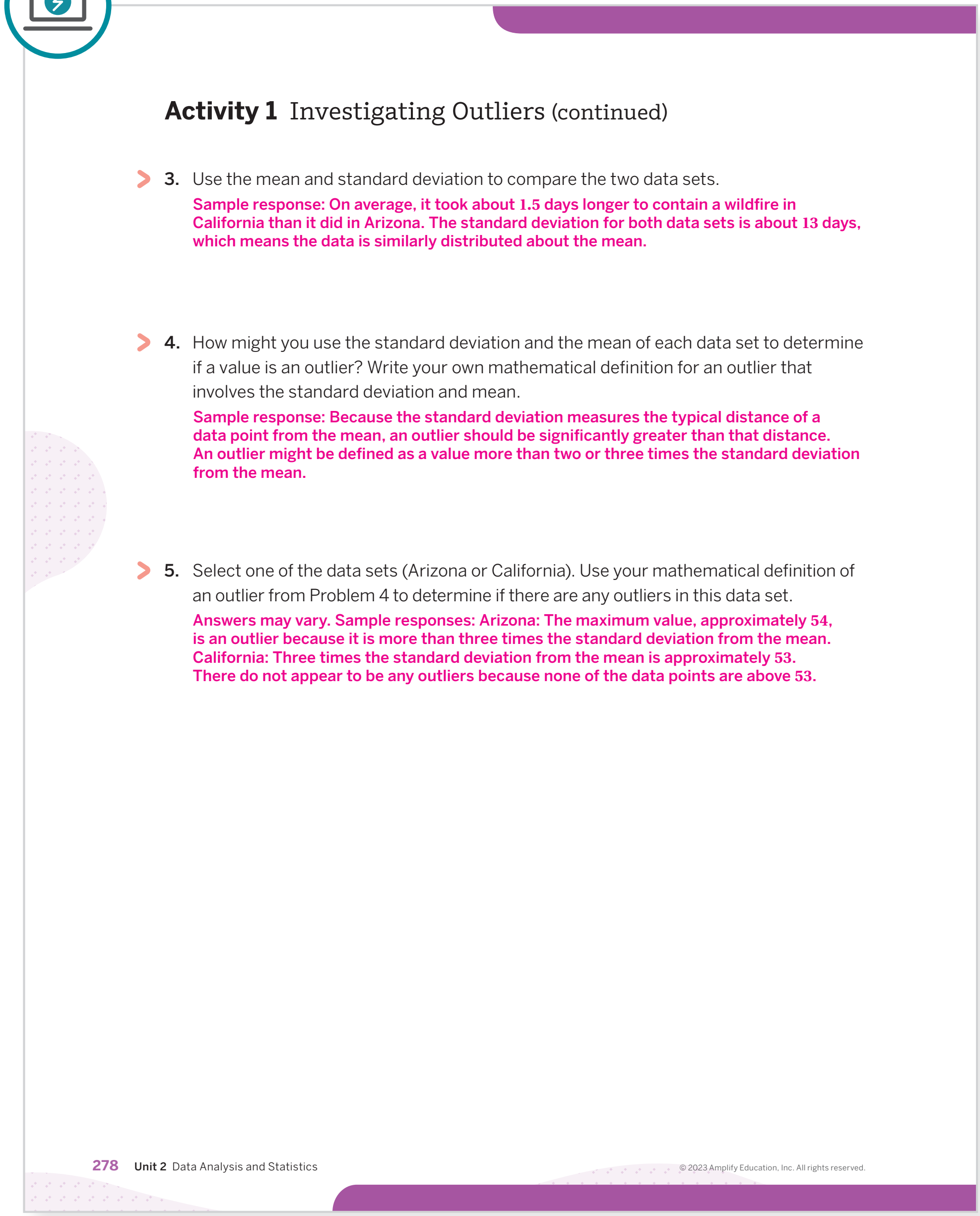
English Learners

Annotate the dot plots using dashed vertical lines to locate the mean and 3 standard deviations above or below the mean. Write the term *outlier* above these vertical lines.

Pairs | 12 min

Activity 1 Investigating Outliers (continued)

Students interpret mean and standard deviation in context and use these statistics to compare two data sets and determine whether either set contains an outlier.



Activity 1 Investigating Outliers (continued)

3. Use the mean and standard deviation to compare the two data sets.

Sample response: On average, it took about 1.5 days longer to contain a wildfire in California than it did in Arizona. The standard deviation for both data sets is about 13 days, which means the data is similarly distributed about the mean.

4. How might you use the standard deviation and the mean of each data set to determine if a value is an outlier? Write your own mathematical definition for an outlier that involves the standard deviation and mean.

Sample response: Because the standard deviation measures the typical distance of a data point from the mean, an outlier should be significantly greater than that distance. An outlier might be defined as a value more than two or three times the standard deviation from the mean.

5. Select one of the data sets (Arizona or California). Use your mathematical definition of an outlier from Problem 4 to determine if there are any outliers in this data set.

Answers may vary. Sample responses: Arizona: The maximum value, approximately 54, is an outlier because it is more than three times the standard deviation from the mean. California: Three times the standard deviation from the mean is approximately 53. There do not appear to be any outliers because none of the data points are above 53.

278 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display both dot plots.

Have pairs of students share their interpretation of what the mean and standard deviation indicate about the situation and how the data sets compare to each other. Select students to share their definitions of an outlier using standard deviation and mean.

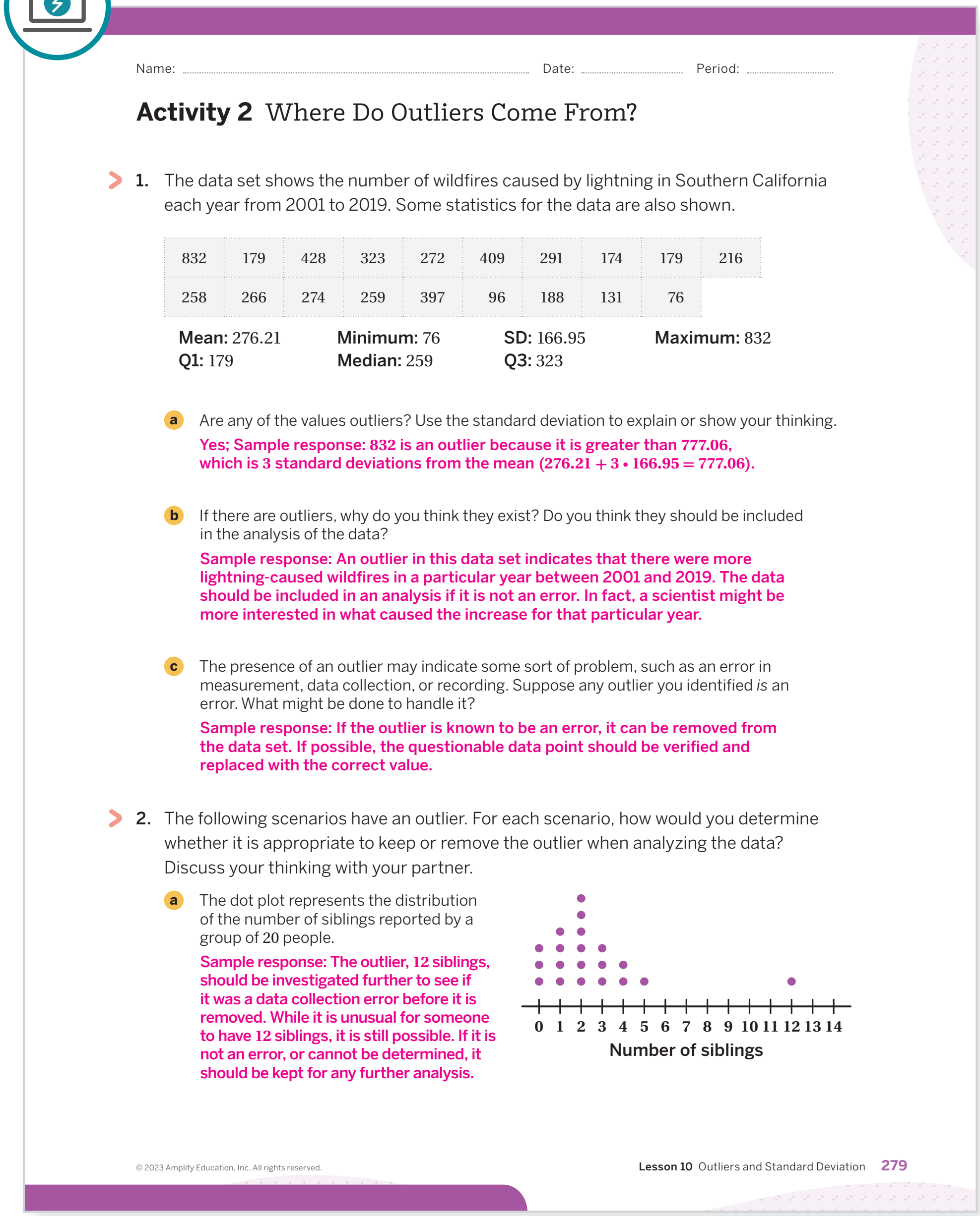
Highlight that if a value is 3 standard deviations above or below the mean, it is considered an outlier. Annotate the dot plots to indicate intervals that are 1, 2, and 3 standard deviations from the mean, in both directions.

Ask, “Are any of the values in either set outliers by this definition? What about the IQR definition?”

Pairs | 12 min

Activity 2 Where Do Outliers Come From?

Students reason on the source of different outliers and consider best practices for dealing with outliers when analyzing a data set.



Name: .... Date: .... Period: ....

Activity 2 Where Do Outliers Come From?

1. The data set shows the number of wildfires caused by lightning in Southern California each year from 2001 to 2019. Some statistics for the data are also shown.

832	179	428	323	272	409	291	174	179	216
258	266	274	259	397	96	188	131	76	

Mean: 276.21 **Minimum:** 76 **SD:** 166.95 **Maximum:** 832
Q1: 179 **Median:** 259 **Q3:** 323

a. Are any of the values outliers? Use the standard deviation to explain or show your thinking.

Yes; Sample response: 832 is an outlier because it is greater than 777.06, which is 3 standard deviations from the mean ($276.21 + 3 \cdot 166.95 = 777.06$).

b. If there are outliers, why do you think they exist? Do you think they should be included in the analysis of the data?

Sample response: An outlier in this data set indicates that there were more lightning-caused wildfires in a particular year between 2001 and 2019. The data should be included in an analysis if it is not an error. In fact, a scientist might be more interested in what caused the increase for that particular year.

c. The presence of an outlier may indicate some sort of problem, such as an error in measurement, data collection, or recording. Suppose any outlier you identified *is* an error. What might be done to handle it?

Sample response: If the outlier is known to be an error, it can be removed from the data set. If possible, the questionable data point should be verified and replaced with the correct value.

2. The following scenarios have an outlier. For each scenario, how would you determine whether it is appropriate to keep or remove the outlier when analyzing the data? Discuss your thinking with your partner.

a. The dot plot represents the distribution of the number of siblings reported by a group of 20 people.

Sample response: The outlier, 12 siblings, should be investigated further to see if it was a data collection error before it is removed. While it is unusual for someone to have 12 siblings, it is still possible. If it is not an error, or cannot be determined, it should be kept for any further analysis.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Outliers and Standard Deviation 279

1 Launch

Have students respond to Problem 1 independently, then pause to consider their responses. Discuss reasons a scientist might be interested in an outlier in this situation, as well as reasons that would cause him to exclude the outlier in his analysis. Then arrange students in pairs to respond to Problem 2. **Note:** Provide access to scientific calculators for the remainder of this lesson.

2 Monitor

Help students get started by asking, "Why might an outlier in this data set be of interest to a scientist or researcher examining lightning-caused wildfires?"

Look for points of confusion:

- **Thinking that an outlier should be removed from the data set without consideration.** Prompt students to justify their decision to remove an outlier from the data set.

Look for productive strategies:

- Reasoning on possible outcomes for each data set.
- Identifying possible errors or anomalies in the data.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Before students begin, have them make sense of the data set by asking these questions:

- "What is the minimum value? The maximum value?"
- "Do you see any values that look like they are much farther away from the other data values?"

Extension: *Math Enrichment*

Have students choose one of the data sets provided in Problem 2. Challenge them to determine what is the least possible data value that lies above the mean that would be considered an outlier. Then challenge them to determine what is the greatest possible data value that lies below the mean that would be considered an outlier.

Math Language Development

MLR8: Discussion Supports

During the Connect, display or provide access to the Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning* to support students as they share how they determined whether to keep or remove the outliers in the scenarios in Problem 2.

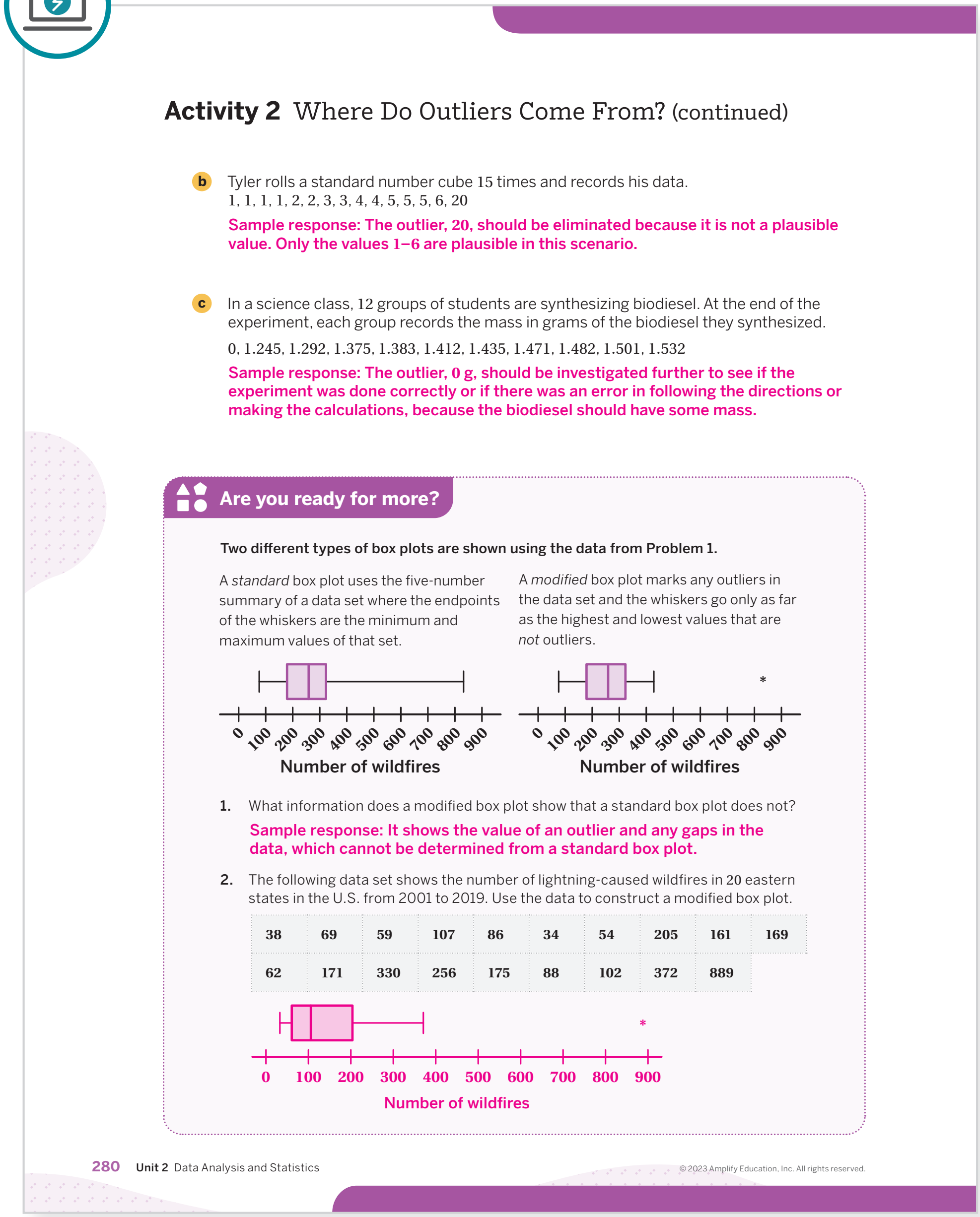
English Learners

In Problem 2b, display a number cube to illustrate why 20 is not a plausible value. In Problem 2c, consider changing the text to "In a science class, 12 groups of students record the mass, in grams, of a substance. At the end of the experiment, each group records the mass of the substance."

Pairs | 12 min

Activity 2 Where Do Outliers Come From? (continued)

Students reason on the source of different outliers and consider best practices for dealing with outliers when analyzing a data set.



Activity 2 Where Do Outliers Come From? (continued)

b Tyler rolls a standard number cube 15 times and records his data.
1, 1, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 20

Sample response: The outlier, 20, should be eliminated because it is not a plausible value. Only the values 1–6 are plausible in this scenario.

c In a science class, 12 groups of students are synthesizing biodiesel. At the end of the experiment, each group records the mass in grams of the biodiesel they synthesized.

0, 1.245, 1.292, 1.375, 1.383, 1.412, 1.435, 1.471, 1.482, 1.501, 1.532

Sample response: The outlier, 0 g, should be investigated further to see if the experiment was done correctly or if there was an error in following the directions or making the calculations, because the biodiesel should have some mass.

Are you ready for more?

Two different types of box plots are shown using the data from Problem 1.

A *standard* box plot uses the five-number summary of a data set where the endpoints of the whiskers are the minimum and maximum values of that set.

A *modified* box plot marks any outliers in the data set and the whiskers go only as far as the highest and lowest values that are *not* outliers.

1. What information does a modified box plot show that a standard box plot does not?

Sample response: It shows the value of an outlier and any gaps in the data, which cannot be determined from a standard box plot.

2. The following data set shows the number of lightning-caused wildfires in 20 eastern states in the U.S. from 2001 to 2019. Use the data to construct a modified box plot.

38	69	59	107	86	34	54	205	161	169
62	171	330	256	175	88	102	372	889	

280 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have pairs of students share how they determined whether it was appropriate to keep or remove an outlier from each data set in Problem 2.

Ask, “Is it possible:”

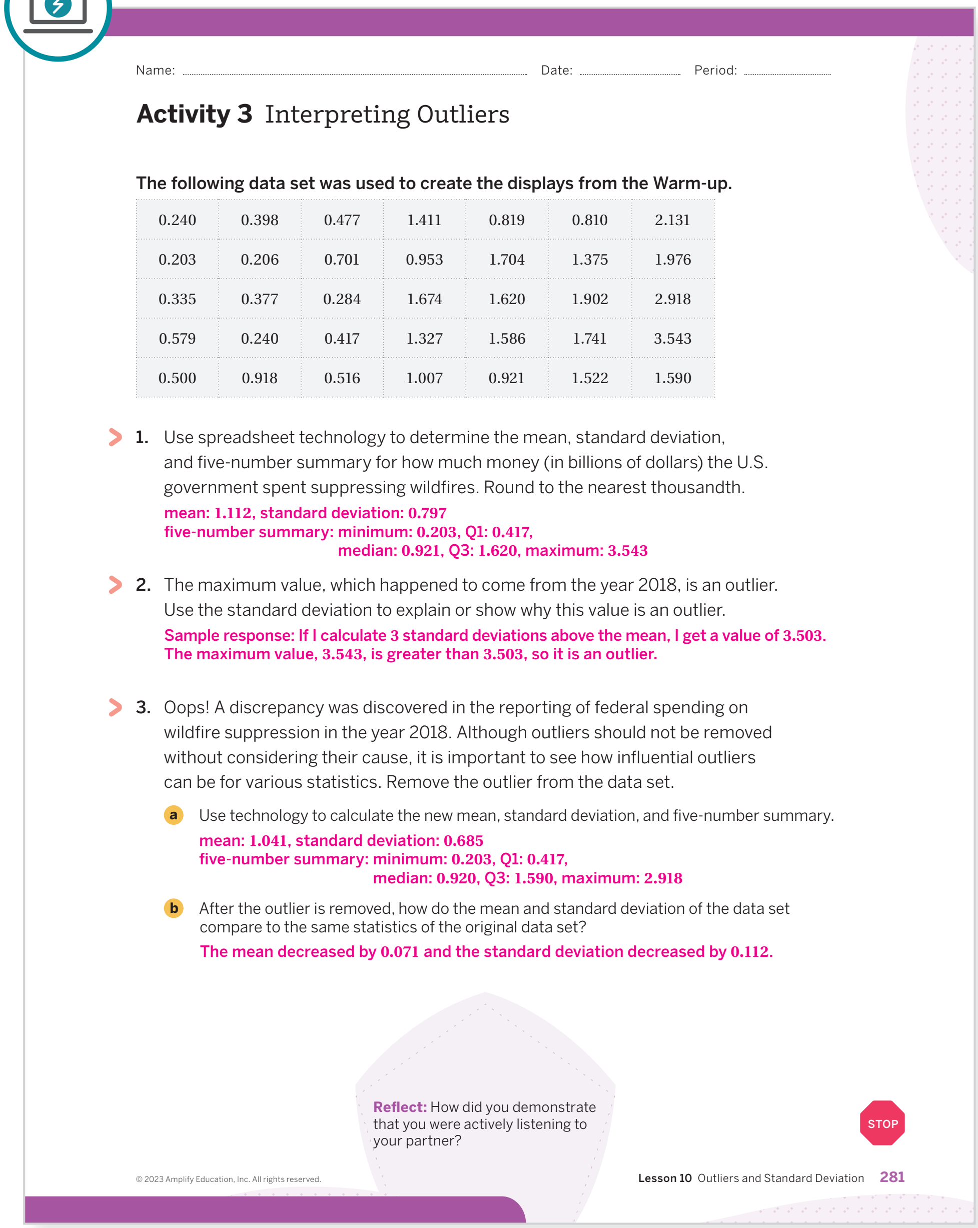
- “To roll a 20 on a 6-sided die?”
- “To have 12 siblings?”
- “For the mass of biodiesel (or any matter) to be 0 grams?”

Highlight the importance of investigating the source of an outlier when analyzing data. If the outlier is a valid part of the data set, it should be included in the analysis. But if it is the result of an error, it should be removed as an outlier tends to skew a data set. A data point should not be removed simply because it is an outlier.

Independent | 10 min

Activity 3 Interpreting Outliers

Students compare the statistics of a data set when an outlier is included with when the outlier is removed, to see how mean and standard deviation are affected.



Name: ______ Date: ______ Period: ______

Activity 3 Interpreting Outliers

The following data set was used to create the displays from the Warm-up.

0.240	0.398	0.477	1.411	0.819	0.810	2.131
0.203	0.206	0.701	0.953	1.704	1.375	1.976
0.335	0.377	0.284	1.674	1.620	1.902	2.918
0.579	0.240	0.417	1.327	1.586	1.741	3.543
0.500	0.918	0.516	1.007	0.921	1.522	1.590

1. Use spreadsheet technology to determine the mean, standard deviation, and five-number summary for how much money (in billions of dollars) the U.S. government spent suppressing wildfires. Round to the nearest thousandth.

   mean: 1.112, standard deviation: 0.797
   five-number summary: minimum: 0.203, Q1: 0.417,
   median: 0.921, Q3: 1.620, maximum: 3.543

2. The maximum value, which happened to come from the year 2018, is an outlier. Use the standard deviation to explain or show why this value is an outlier.

   Sample response: If I calculate 3 standard deviations above the mean, I get a value of 3.503. The maximum value, 3.543, is greater than 3.503, so it is an outlier.

3. Oops! A discrepancy was discovered in the reporting of federal spending on wildfire suppression in the year 2018. Although outliers should not be removed without considering their cause, it is important to see how influential outliers can be for various statistics. Remove the outlier from the data set.

   a. Use technology to calculate the new mean, standard deviation, and five-number summary.

      mean: 1.041, standard deviation: 0.685
      five-number summary: minimum: 0.203, Q1: 0.417,
      median: 0.920, Q3: 1.590, maximum: 2.918

   b. After the outlier is removed, how do the mean and standard deviation of the data set compare to the same statistics of the original data set?

      The mean decreased by 0.071 and the standard deviation decreased by 0.112.

Reflect: How did you demonstrate that you were actively listening to your partner?

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Outliers and Standard Deviation 281

1 Launch

Provide access to spreadsheet technology.

2 Monitor

Help students get started by reminding them of the spreadsheet functions they will need to determine the statistics of the data set (such as =average, =median, =stdev, and =quartile).

Look for points of confusion:

- **Removing the outlier from the data set in the spreadsheet.** Prompt students to create a separate data set so that they can compare them to one another.

Look for productive strategies:

- Calculating Q3 + 1.5 • IQR.
- Calculating the value that is 3 standard deviations above the mean.
- Calculating the differences between corresponding statistics in the data with the outlier included and removed.

3 Connect

Have students share how they determined whether the maximum value was an outlier. Then select students to explain how the statistics changed when the outlier was removed from the data, focusing on the mean and standard deviation.

Ask:

- "Why is it appropriate to remove the outlier from this data set?" Because there was a discrepancy in the reporting of this value.
- "How did removing the outlier affect the mean? The standard deviation?" Both values decreased.

Highlight that removing the outlier caused the mean to decrease by $71 million and the standard deviation to decrease by $112 million, which the U.S. federal government might regard as a significant difference. Removing an outlier can significantly influence the statistics of a data set, so It is important to use discretion.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Keep the box plot and histogram from the Warm-up displayed while students complete this activity. Display the values that are part of the five-number summary: minimum, first quartile (Q1), median, third quartile (Q3), and maximum for students to use as a reference. Consider displaying the new definition for an outlier as any value that is 3 standard deviations above or below the mean.

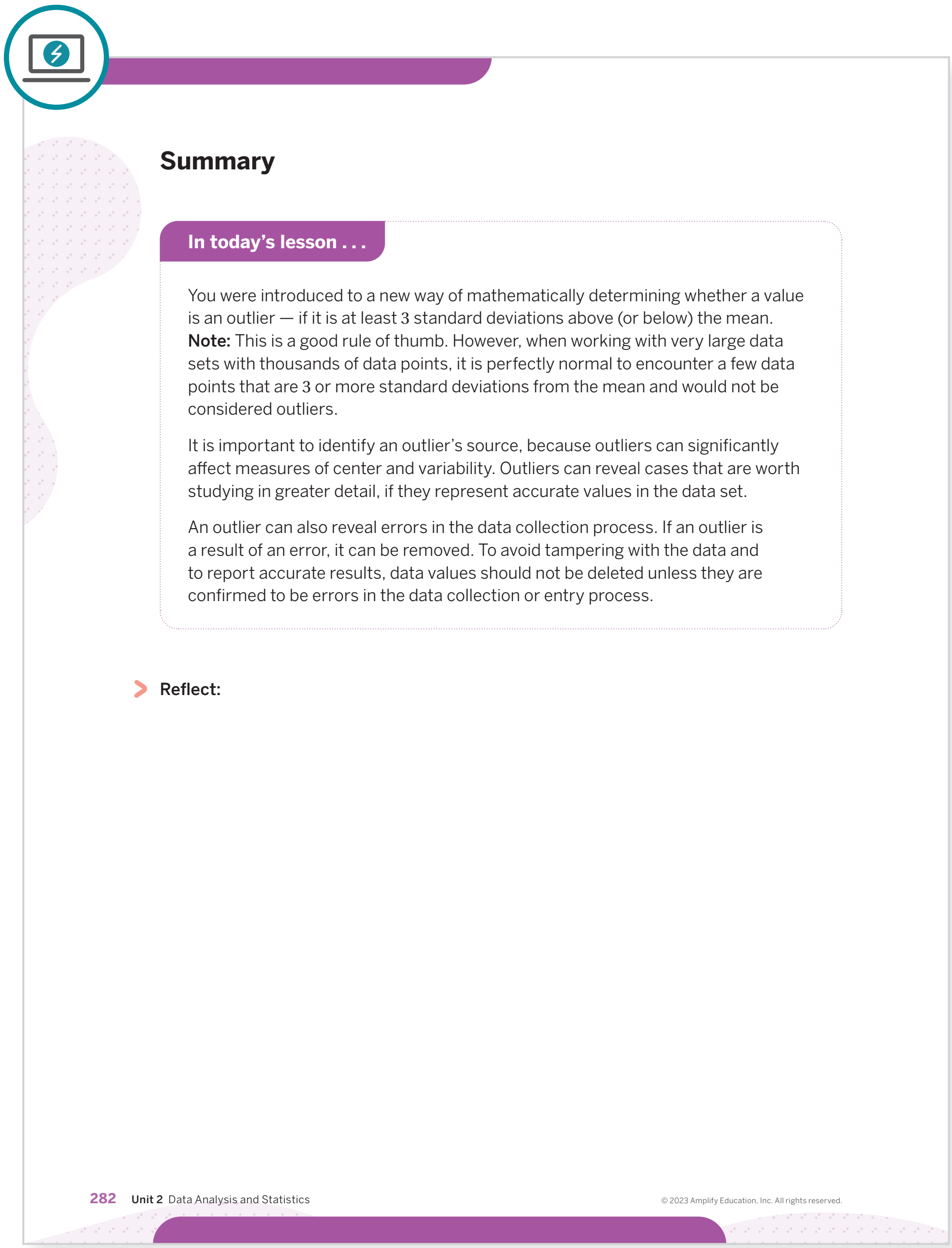
Accessibility: *Vary Demands to Optimize Challenge*

Consider providing a spreadsheet template with pre-populated data values to support students as they create their own spreadsheet. They can use this template to check their own work.

Whole Class | 5 min

Summary

Review and synthesize how to determine if a value is an outlier, using the standard deviation, and under what circumstances an outlier should be removed from a data set.



Summary

In today's lesson . . .

You were introduced to a new way of mathematically determining whether a value is an outlier — if it is at least 3 standard deviations above (or below) the mean. **Note:** This is a good rule of thumb. However, when working with very large data sets with thousands of data points, it is perfectly normal to encounter a few data points that are 3 or more standard deviations from the mean and would not be considered outliers.

It is important to identify an outlier's source, because outliers can significantly affect measures of center and variability. Outliers can reveal cases that are worth studying in greater detail, if they represent accurate values in the data set.

An outlier can also reveal errors in the data collection process. If an outlier is a result of an error, it can be removed. To avoid tampering with the data and to report accurate results, data values should not be deleted unless they are confirmed to be errors in the data collection or entry process.

Reflect:

282 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Have students share how to determine if a value is an outlier using standard deviation and mean and compare it to the IQR method they learned in a previous lesson.

Ask:

- "Why are outliers important to notice in a data set?" They can indicate an error in the data or a special case that could be studied more closely.
- "Why would you eliminate an outlier?" If it is an error or not representative of the sample as a whole. It depends on the context of the problem and the data collection process.
- "Which statistics are most impacted by the inclusion of an outlier in a data set?" The mean and standard deviation.

Highlight the need to determine the source of an outlier before deciding to include or remove it from a data set. Note the impact it can have on measures of center and variability, particularly the mean and standard deviation.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

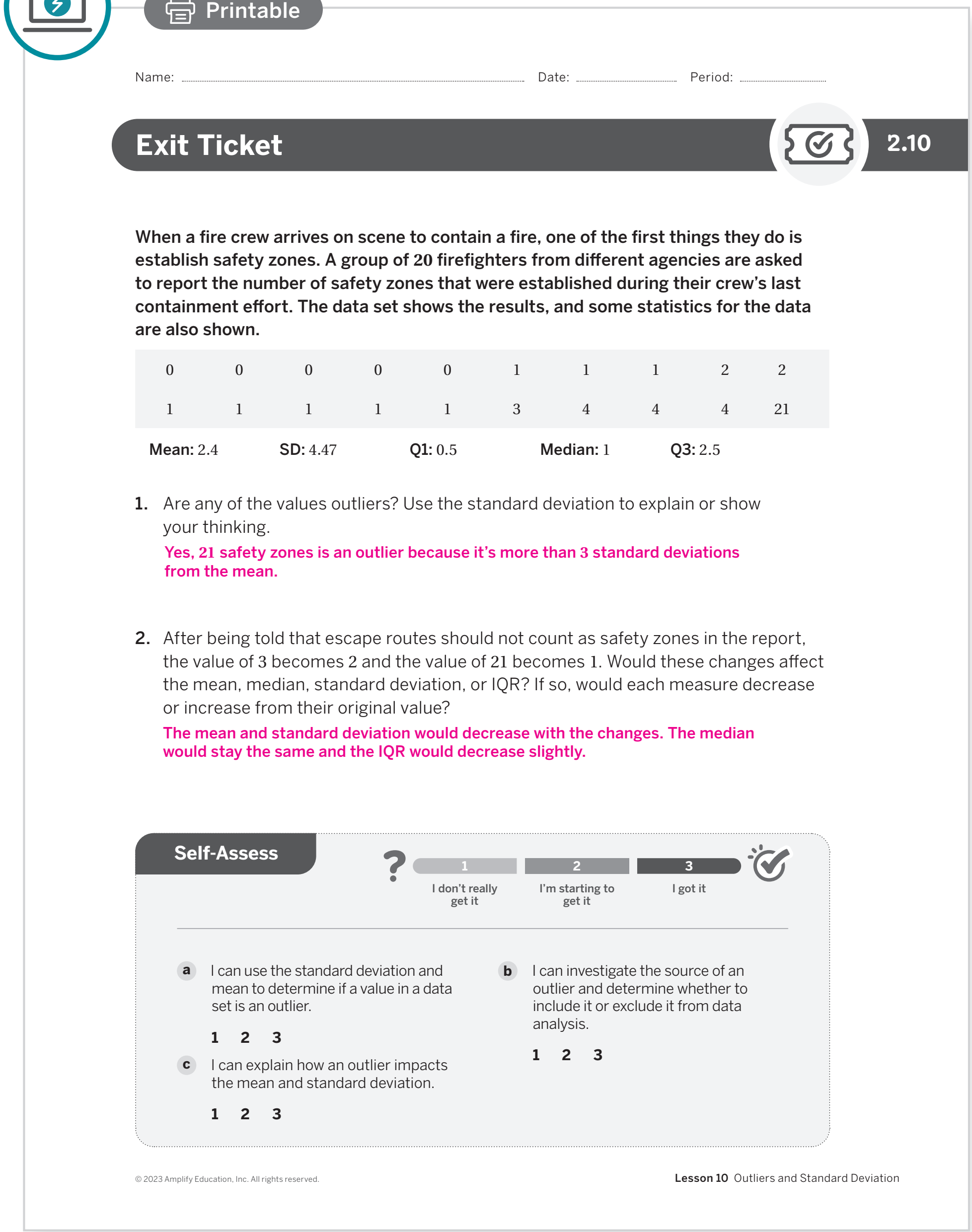
- "When should an outlier be removed from a data set? When should it be included?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the presence of an outlier and its effect on the mean and standard deviation of a data set.

Printable



Name: ____________________ Date: __________ Period: __________

Exit Ticket 2.10

When a fire crew arrives on scene to contain a fire, one of the first things they do is establish safety zones. A group of 20 firefighters from different agencies are asked to report the number of safety zones that were established during their crew's last containment effort. The data set shows the results, and some statistics for the data are also shown.

0	0	0	0	0	1	1	1	2	2
1	1	1	1	1	3	4	4	4	21

Mean: 2.4 **SD:** 4.47 **Q1:** 0.5 **Median:** 1 **Q3:** 2.5

1. Are any of the values outliers? Use the standard deviation to explain or show your thinking.
   Yes, 21 safety zones is an outlier because it's more than 3 standard deviations from the mean.

2. After being told that escape routes should not count as safety zones in the report, the value of 3 becomes 2 and the value of 21 becomes 1. Would these changes affect the mean, median, standard deviation, or IQR? If so, would each measure decrease or increase from their original value?
   The mean and standard deviation would decrease with the changes. The median would stay the same and the IQR would decrease slightly.

Self-Assess

? 1 I don't really get it 2 I'm starting to get it 3 I got it

a I can use the standard deviation and mean to determine if a value in a data set is an outlier.
1 2 3

b I can investigate the source of an outlier and determine whether to include it or exclude it from data analysis.
1 2 3

c I can explain how an outlier impacts the mean and standard deviation.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Outliers and Standard Deviation

Success looks like . . .

- **Goal:** Using the standard deviation and mean to determine if a value in a data set is an outlier.
  - » Determining whether any values are outliers in Problem 1.
- **Language Goal:** Investigating the source of an outlier and determining whether to include or exclude it from data analysis. **(Speaking and Listening)**
- **Language Goal:** Explaining how an outlier impacts the mean and standard deviation. **(Reading and Writing)**
  - » Explaining how the mean and standard deviation are affected after removing the escape routes in Problem 2.

Suggested next steps

If students are unable to determine the presence of outliers in the data, consider:

- Reviewing Problem 4 from Activity 1.
- Assigning Practice Problem 3.

If students are unable to explain how changes to the data affected the original statistics, consider:

- Reviewing Activity 3.
- Allowing students to use spreadsheet technology to compare both data sets.

Professional Learning

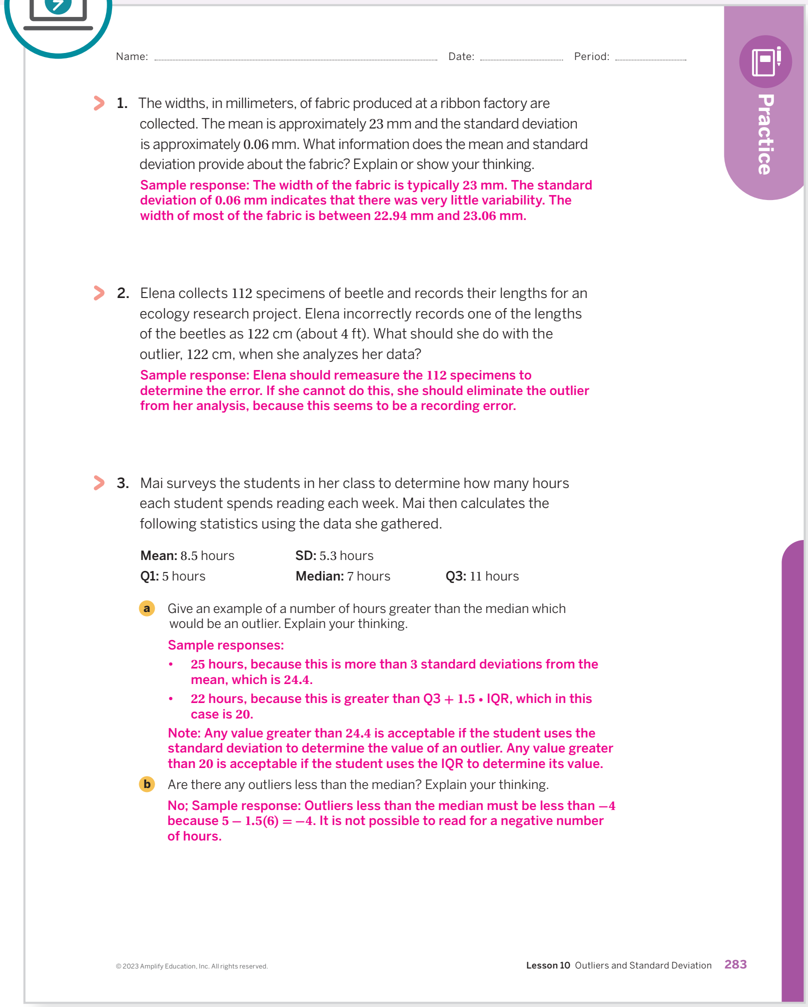
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- Which students' ideas were you able to highlight during Activity 1 when defining the term *outlier* using standard deviation?
- Think about the questions you asked students today during the discussion after Problem 1 in Activity 2. Which question(s) was most effective, based on what the students said or did as a result in Problem 2?

Practice

Independent



Name: ______________________ Date: __________ Period: __________

Practice

1. The widths, in millimeters, of fabric produced at a ribbon factory are collected. The mean is approximately 23 mm and the standard deviation is approximately 0.06 mm. What information does the mean and standard deviation provide about the fabric? Explain or show your thinking.

   Sample response: The width of the fabric is typically 23 mm. The standard deviation of 0.06 mm indicates that there was very little variability. The width of most of the fabric is between 22.94 mm and 23.06 mm.

2. Elena collects 112 specimens of beetle and records their lengths for an ecology research project. Elena incorrectly records one of the lengths of the beetles as 122 cm (about 4 ft). What should she do with the outlier, 122 cm, when she analyzes her data?

   Sample response: Elena should remeasure the 112 specimens to determine the error. If she cannot do this, she should eliminate the outlier from her analysis, because this seems to be a recording error.

3. Mai surveys the students in her class to determine how many hours each student spends reading each week. Mai then calculates the following statistics using the data she gathered.

   Mean: 8.5 hours **SD:** 5.3 hours
   Q1: 5 hours **Median:** 7 hours **Q3:** 11 hours

   a. Give an example of a number of hours greater than the median which would be an outlier. Explain your thinking.

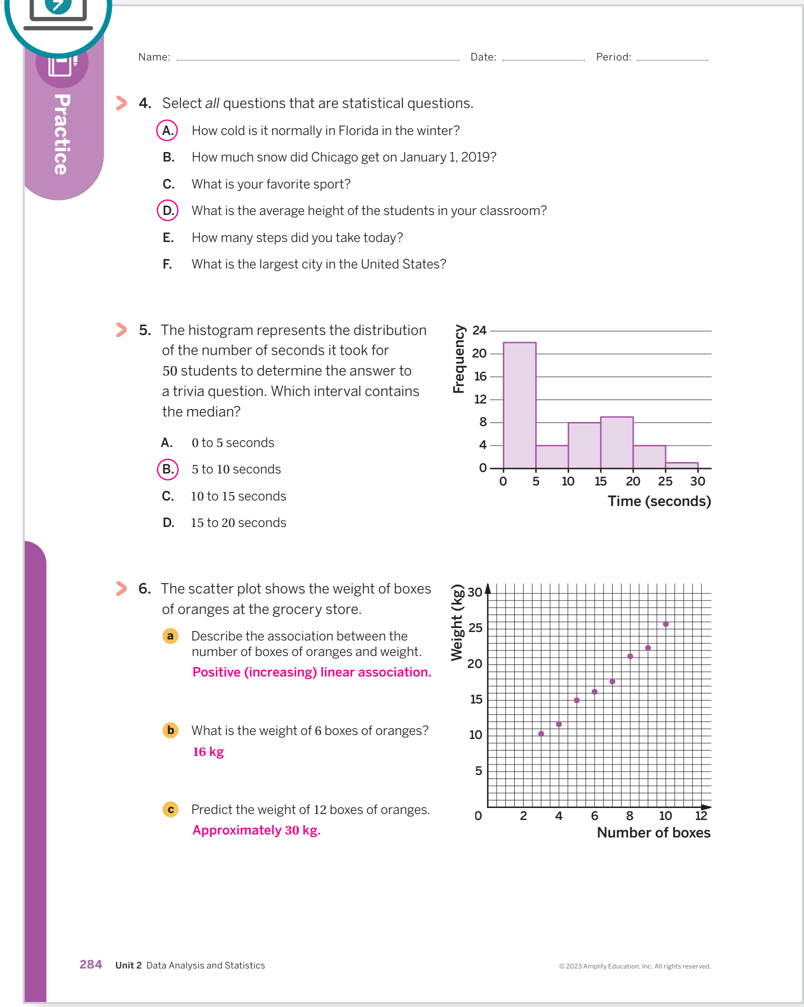
      Sample responses:
      - **25 hours, because this is more than 3 standard deviations from the mean, which is 24.4.**
      - **22 hours, because this is greater than Q3 + 1.5 • IQR, which in this case is 20.**

      Note: Any value greater than 24.4 is acceptable if the student uses the standard deviation to determine the value of an outlier. Any value greater than 20 is acceptable if the student uses the IQR to determine its value.

   b. Are there any outliers less than the median? Explain your thinking.

      No; Sample response: Outliers less than the median must be less than −4 because $5 - 1.5(6) = -4$. It is not possible to read for a negative number of hours.

© 2023 Amplify Education, Inc. All rights reserved. **Lesson 10** Outliers and Standard Deviation 283



Name: ______________________ Date: __________ Period: __________

Practice

4. Select *all* questions that are statistical questions.

   [✓ A.] How cold is it normally in Florida in the winter?
   B. How much snow did Chicago get on January 1, 2019?
   C. What is your favorite sport?
   [✓ D.] What is the average height of the students in your classroom?
   E. How many steps did you take today?
   F. What is the largest city in the United States?

5. The histogram represents the distribution of the number of seconds it took for 50 students to determine the answer to a trivia question. Which interval contains the median?

   A. 0 to 5 seconds
   [✓ B.] 5 to 10 seconds
   C. 10 to 15 seconds
   D. 15 to 20 seconds

6. The scatter plot shows the weight of boxes of oranges at the grocery store.

   a. Describe the association between the number of boxes of oranges and weight.

      Positive (increasing) linear association.

   b. What is the weight of 6 boxes of oranges?

      16 kg

   c. Predict the weight of 12 boxes of oranges.

      Approximately 30 kg.

284 **Unit 2** Data Analysis and Statistics © 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 3	1
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 1	2
	5	Unit 2 Lesson 2	2
Formative	6	Unit 2 Lesson 11	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Mathematical Modeling Prompt

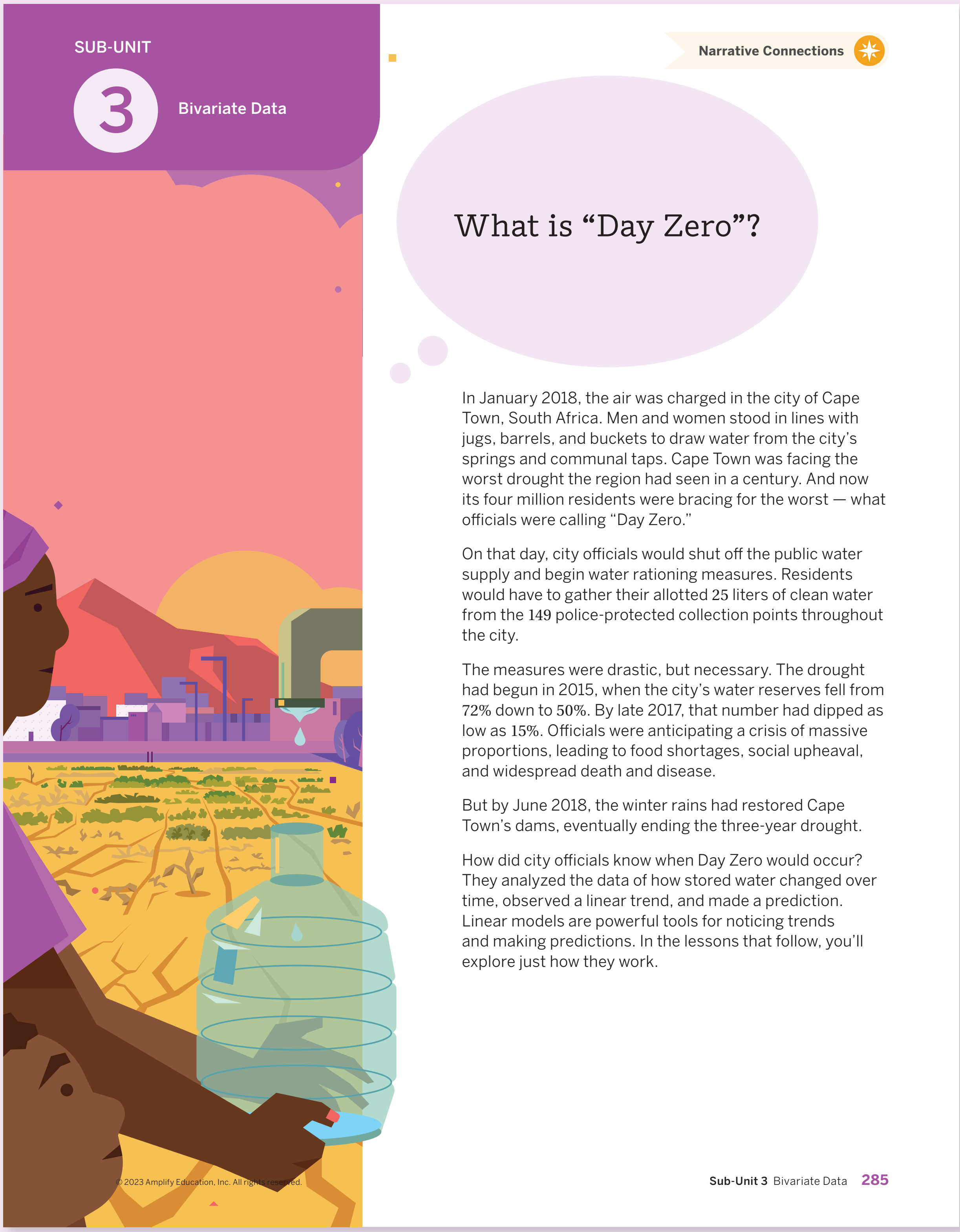
At any point during the course and after completing this lesson, have students work in pairs or small groups on this unit's **Mathematical Modeling Prompt**, *Display Your Data*, which is available in the **Algebra 1 Additional Practice**.

Sub-Unit 3

Bivariate Data

Whole Class

In this Sub-Unit, students graphically represent bivariate data and explore the mathematics of what makes a linear model the “best.”



SUB-UNIT

3 Bivariate Data

Narrative Connections

What is “Day Zero”?

In January 2018, the air was charged in the city of Cape Town, South Africa. Men and women stood in lines with jugs, barrels, and buckets to draw water from the city’s springs and communal taps. Cape Town was facing the worst drought the region had seen in a century. And now its four million residents were bracing for the worst — what officials were calling “Day Zero.”

On that day, city officials would shut off the public water supply and begin water rationing measures. Residents would have to gather their allotted 25 liters of clean water from the 149 police-protected collection points throughout the city.

The measures were drastic, but necessary. The drought had begun in 2015, when the city’s water reserves fell from 72% down to 50%. By late 2017, that number had dipped as low as 15%. Officials were anticipating a crisis of massive proportions, leading to food shortages, social upheaval, and widespread death and disease.

But by June 2018, the winter rains had restored Cape Town’s dams, eventually ending the three-year drought.

How did city officials know when Day Zero would occur? They analyzed the data of how stored water changed over time, observed a linear trend, and made a prediction. Linear models are powerful tools for noticing trends and making predictions. In the lessons that follow, you’ll explore just how they work.

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore how linear models can be used to notice trends and make predictions — within the context of natural resource usage — in the following places:

- **Lesson 11, Activity 2:** Defoliating Insects
- **Lesson 12, Activities 1 and 3:** Team Trees, Planting Mangroves
- **Lesson 13, Activities 2–3:** Which Line Fits Better?, Reducing Emissions
- **Lesson 14, Activity 3:** Hybrid Cars

UNIT 2 | **LESSON 11**

Representing Data With Two Variables

Let's model data with two variables.

Focus

Goals

1. Use a scatter plot to represent data in two variables.
2. **Language Goal:** Interpret the relationship between two variables from a scatter plot. **(Speaking and Listening, Writing)**
3. Use a scatter plot to distinguish between linear and nonlinear data.

Coherence

Today

Students recall how to create a scatter plot to represent bivariate data and how to describe the trend of that data from Grade 8. They reason abstractly and quantitatively to determine relationships between the variables in different scatter plots based on their trend and use more precise language to describe its direction (increasing or decreasing) and form (linear or nonlinear).

Previously

In this unit, students studied how to represent univariate data using dot plots, histograms, and box plots. In Grade 8, students also considered how to identify the trend in a data set from a scatter plot.

Coming Soon

Students will focus on data best fit by a linear model, which they will use to make predictions. They will also study strategies for fitting a line to data and judging its goodness of fit.

Rigor

- Students further their **conceptual understanding** of how to represent two-variable data with a scatter plot.
- Students develop **fluency** analyzing and interpreting data using a scatter plot.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	10 min	5 min	5 min
Independent	Independent	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 2 PDF
- Activity 2 PDF (answers)
- Activity 3 PDF, pre-cut cards, one set per pair
- Anchor Chart PDF, *Sentence Stems, Comparing and Contrasting*
- Anchor Chart PDF, *Sentence Stems, Explaining My Thinking*
- Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning*
- graph paper

Math Language Development

Review words

- *linear model*
- *scatter plot*
- *trend**

*Students may be familiar with the word *trend* as it relates to fashion trends or ideas that are trending on social media. Explain to students that in statistics a trend describes mathematical change over time that is statistically detectable.

Amps | Featured Activity

Activity 2

Interactive Graphs

Students analyze and interpret the data for four different species of defoliating insects and use the trends to make predictions. They also compare their scatter plots to determine which insect was the most or least destructive.

Building Math Identity and Community

Connecting to Mathematical Practices

In Activity 1, students might feel their stress levels rise as they consider a graph with large numbers and lots of points. Have students practice emotional regulation routines, such as deep breathing. Then have them draw similarities to tasks that they already feel confident in. By recognizing the skills that they already possess, the task will become less daunting.

Modifications to Pacing

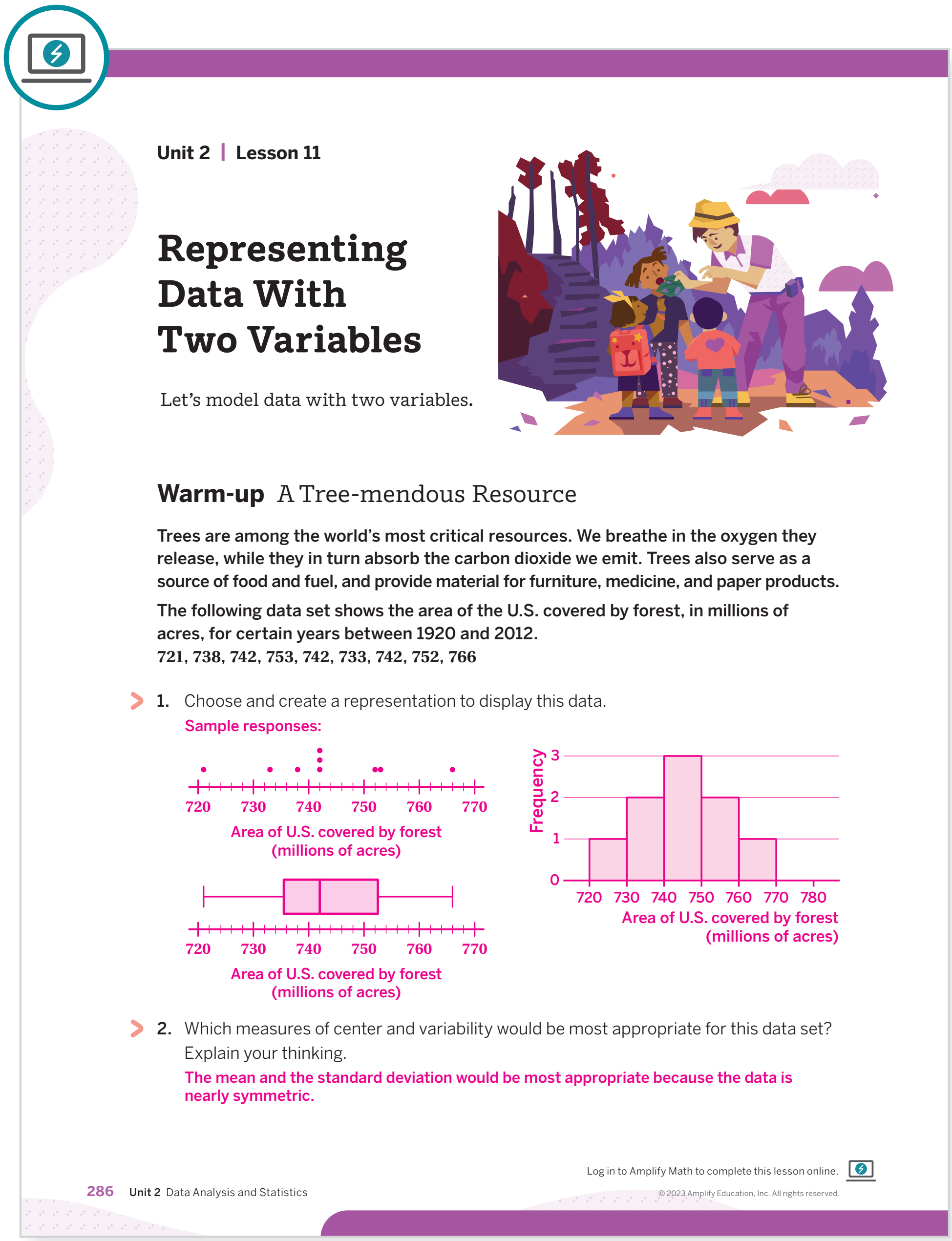
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, have students complete Part 1 and consider Part 2 as a whole class.
- In **Activity 3**, remove two of the nonlinear data cards from the set that students consider.

Independent | 5 min

Warm-up A Tree-mendous Resource

Students create a representation for a univariate data set and determine its appropriate measure of center and variability.



Unit 2 | Lesson 11

Representing Data With Two Variables

Let's model data with two variables.

Warm-up A Tree-mendous Resource

Trees are among the world's most critical resources. We breathe in the oxygen they release, while they in turn absorb the carbon dioxide we emit. Trees also serve as a source of food and fuel, and provide material for furniture, medicine, and paper products.

The following data set shows the area of the U.S. covered by forest, in millions of acres, for certain years between 1920 and 2012.
721, 738, 742, 753, 742, 733, 742, 752, 766

1. Choose and create a representation to display this data.

Sample responses:

2. Which measures of center and variability would be most appropriate for this data set? Explain your thinking.

The mean and the standard deviation would be most appropriate because the data is nearly symmetric.

Log in to Amplify Math to complete this lesson online.

286 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Ask, "What are some examples of natural resources?" Have students share their ideas, differentiating between resources that are renewable and nonrenewable. Then read the introduction and instructions to the Warm-up. Provide access to graph paper.

2 Monitor

Help students get started by asking, "What information would you need to know to determine the appropriate measures of center and variability?"

Look for points of confusion:

- **Having difficulty determining which representation to use.** Prompt students to create a representation where they can efficiently determine the shape of the distribution.

Look for productive strategies:

- Constructing a dot plot, histogram, or box plot.
- Identifying the data as symmetric.

3 Connect

Display a dot plot, histogram, and box plot of the data, using student samples, if available.

Ask, "How can you use each representation to determine which measure of center or variability is most appropriate?"

Have students share why they chose the mean and standard deviation as the better measures of center and variability.

Highlight that, so far, students have been using dot plots, histograms, and box plots as a way to represent univariate data, or data in one variable. Today, they will revisit what they learned in Grade 8 about how to represent bivariate data, or data in two variables.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they are familiar with the interdependent relationship humans have with trees, particularly in that humans breathe in the oxygen that trees release into the air. Reciprocally, trees absorb the carbon dioxide that humans release when breathing.

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing pre-created displays for students to analyze, instead of asking students to create them. Have students begin the Warm-up with Problem 2.

Power-up

To power up students' ability to describe the relationship between two quantities shown on a scatter plot:

Provide students with a copy of the Power-up PDF.

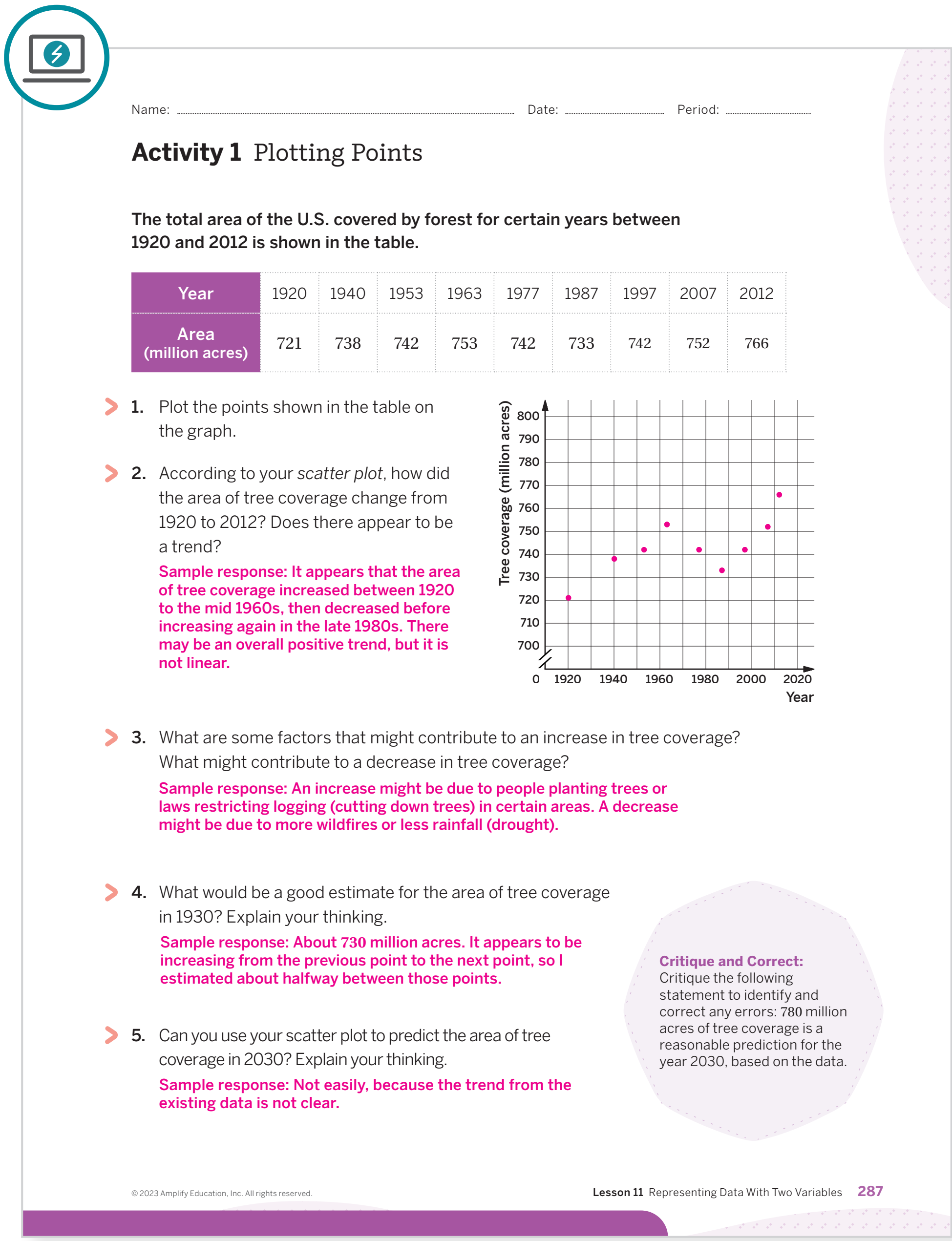
Use: Before Activity 1

Informed by: Performance on Lesson 10, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 5

Independent | 10 min

Activity 1 Plotting Points

Students activate prior knowledge, creating a scatter plot to model bivariate data, describe its trend, and make predictions.



Name: ______ Date: ______ Period: ______

Activity 1 Plotting Points

The total area of the U.S. covered by forest for certain years between 1920 and 2012 is shown in the table.

Year	1920	1940	1953	1963	1977	1987	1997	2007	2012
Area (million acres)	721	738	742	753	742	733	742	752	766

1. Plot the points shown in the table on the graph.

2. According to your *scatter plot*, how did the area of tree coverage change from 1920 to 2012? Does there appear to be a trend?

   Sample response: It appears that the area of tree coverage increased between 1920 to the mid 1960s, then decreased before increasing again in the late 1980s. There may be an overall positive trend, but it is not linear.

3. What are some factors that might contribute to an increase in tree coverage? What might contribute to a decrease in tree coverage?

   Sample response: An increase might be due to people planting trees or laws restricting logging (cutting down trees) in certain areas. A decrease might be due to more wildfires or less rainfall (drought).

4. What would be a good estimate for the area of tree coverage in 1930? Explain your thinking.

   Sample response: About 730 million acres. It appears to be increasing from the previous point to the next point, so I estimated about halfway between those points.

5. Can you use your scatter plot to predict the area of tree coverage in 2030? Explain your thinking.

   Sample response: Not easily, because the trend from the existing data is not clear.

Critique and Correct: Critique the following statement to identify and correct any errors: 780 million acres of tree coverage is a reasonable prediction for the year 2030, based on the data.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 11 Representing Data With Two Variables 287

1 Launch

Say, "You are going to use the data from the Warm-up about tree coverage in the U.S. and use a scatter plot to determine its relationship to time." Provide each student with a ruler.

2 Monitor

Help students get started by asking what is meant by the term "trend." A general pattern in the data.

Look for points of confusion:

- **Having difficulty explaining how they estimated tree coverage in 1930.** Ask, "What was your strategy for estimating? Why did you not estimate a greater or smaller value for y?"
- **Struggling to determine if it would make sense to predict tree coverage in 2020.** Ask students if they think it will be less than or greater than tree coverage in 2012.

Look for productive strategies:

- Describing the data as increasing or decreasing.
- Drawing a trendline.
- Describing the data or trend as nonlinear.

3 Connect

Display a scatter plot of the data.

Have students share their descriptions of how tree coverage changed over time using the scatter plot. Select students to describe how they estimated tree coverage and to explain why it does or does not make sense to make predictions.

Highlight that students' estimates are likely close to each other because they used a line to connect the data points at 1920 and 1940. We tend to think linearly because it is more efficient to make predictions when the data is linear. The scatter plot displayed is not linear, so they may not be as confident in predictions for values beyond those they are given.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students plot the points in Problem 1, provide them with a pre-populated graph to analyze. Have them begin the activity with Problem 2. This will still allow students to access the mathematical goal of this activity.

Extension: *Math Enrichment*

Have students use technology to plot the given data points. Prompt them to adjust the axes scales so that $y = 0$ is represented and to describe the trend of the data from this perspective. Ask students to determine what might be an appropriate range for the values of y and what values might be considered outliers.

Math Language Development

MLR3: Critique, Correct, Clarify

After students complete Problem 4, ask them to critique the statement shown in their Student Edition: 780 million acres of tree coverage is a reasonable prediction for the year 2030, based on the data. Ask these questions:

- ***Critique:*** "Is this statement true? Why or why not?"
- ***Correct:*** "Write a corrected statement that is now true."
- ***Clarify:*** "How did you correct the statement? How do you know that the statement is now true?"

English Learners

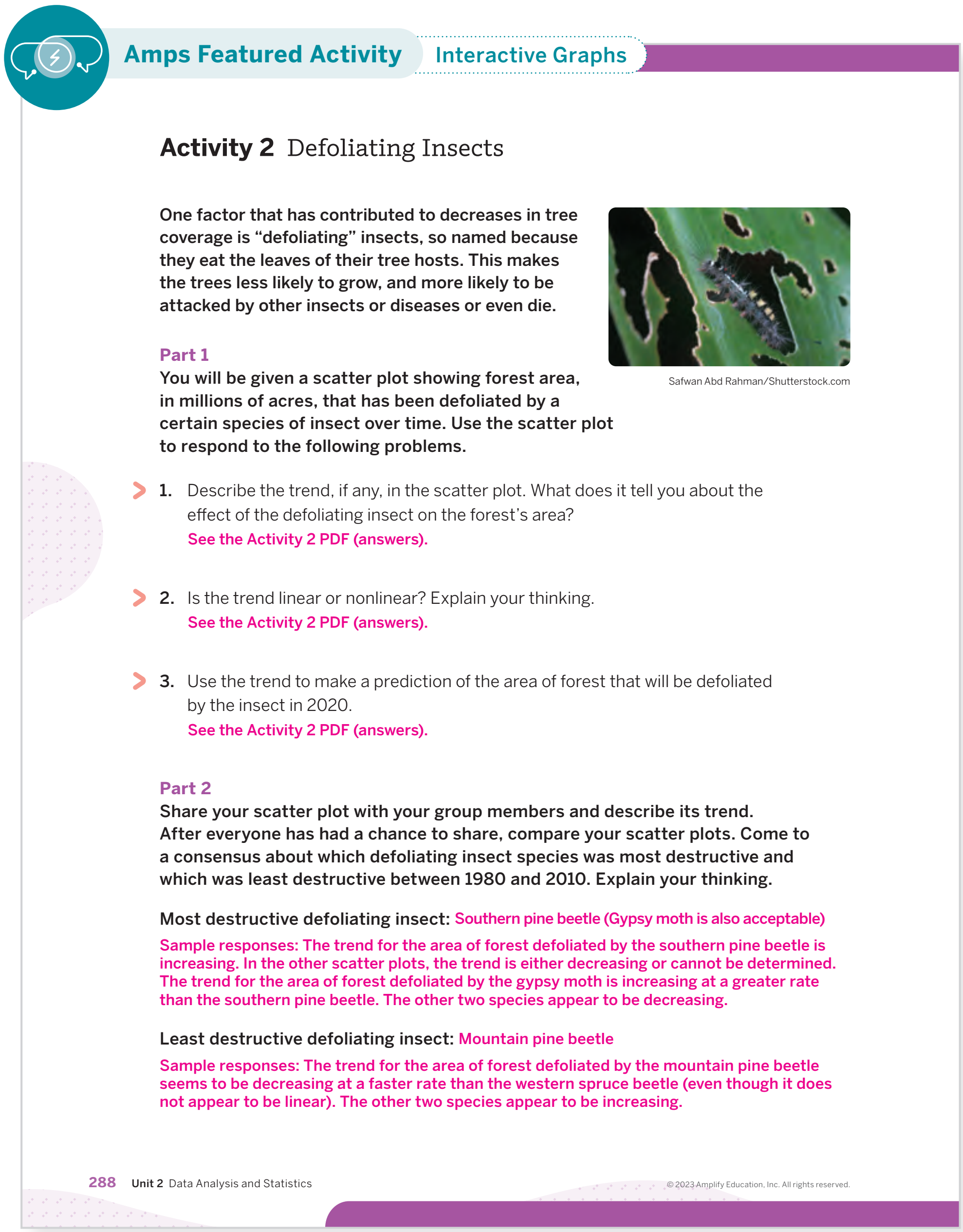
Provide students time to formulate a response before sharing with their small group.

Small Groups | 15 min

Activity 2 Defoliating Insects

Students describe how two variables are related on scatter plots with different trends, compare them, and determine if their trends are linear or nonlinear.

Amps Featured Activity Interactive Graphs



Activity 2 Defoliating Insects

One factor that has contributed to decreases in tree coverage is "defoliating" insects, so named because they eat the leaves of their tree hosts. This makes the trees less likely to grow, and more likely to be attacked by other insects or diseases or even die.

Safwan Abd Rahman/Shutterstock.com

Part 1

You will be given a scatter plot showing forest area, in millions of acres, that has been defoliated by a certain species of insect over time. Use the scatter plot to respond to the following problems.

1. Describe the trend, if any, in the scatter plot. What does it tell you about the effect of the defoliating insect on the forest's area?
   See the Activity 2 PDF (answers).
2. Is the trend linear or nonlinear? Explain your thinking.
   See the Activity 2 PDF (answers).
3. Use the trend to make a prediction of the area of forest that will be defoliated by the insect in 2020.
   See the Activity 2 PDF (answers).

Part 2

Share your scatter plot with your group members and describe its trend. After everyone has had a chance to share, compare your scatter plots. Come to a consensus about which defoliating insect species was most destructive and which was least destructive between 1980 and 2010. Explain your thinking.

Most destructive defoliating insect: Southern pine beetle (Gypsy moth is also acceptable)

Sample responses: The trend for the area of forest defoliated by the southern pine beetle is increasing. In the other scatter plots, the trend is either decreasing or cannot be determined. The trend for the area of forest defoliated by the gypsy moth is increasing at a greater rate than the southern pine beetle. The other two species appear to be decreasing.

Least destructive defoliating insect: Mountain pine beetle

Sample responses: The trend for the area of forest defoliated by the mountain pine beetle seems to be decreasing at a faster rate than the western spruce beetle (even though it does not appear to be linear). The other two species appear to be increasing.

288 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the prompt and the instructions for Part 1 together as a whole class. Arrange students into groups of four. Distribute the Activity 2 PDF so each member receives data for a different insect. Give students independent work time for Part 1.

2 Monitor

Help students get started on Part 2 by asking, "How can you analyze the trend in each scatter plot to determine which insect is more destructive?"

Look for points of confusion:

- **Having difficulty describing the trend for nonlinear data.** Ask, "Is the data increasing or decreasing?"

Look for productive strategies:

- Drawing a trendline, or a vertical line, from $x = 2020$.
- Comparing the rates of change of the scatter plots.

3 Connect

Display the four scatter plots from the Activity 2 PDF. Select groups that chose the southern pine beetle as the most destructive and the mountain pine beetle as the least destructive to share their thinking.

Highlight the two linear and nonlinear scatter plots and compare how quickly the quantities change. Ask students to predict the value of y when $x = 2020$ and whether those predictions are reasonable.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they have ever seen the leaves of trees after an insect has eaten parts of the leaves. Consider showing other images of leaves that have been eaten by insects, in addition to the one in the Student Edition. Consider showing images of the various insects: western spruce beetle, southern pine beetle, mountain pine beetle, and gypsy moth.

Accessibility: *Vary Demands to Optimize Challenge*

Instead of having each group member receive a different scatter plot, distribute 1 or 2 scatter plots to each group. Be sure that all four scatter plots are distributed to different groups. In Part 2, have groups share their scatter plots with another group who received different data.

Math Language Development

MLR7: Compare and Connect

Use this routine as students focus on Part 2 of the activity. Ask pairs of students to compare their scatter plots and identify what is the same and what is different between the displays. After giving them time to compare and share, ask them to come to a consensus about which defoliating insect species was the most destructive and which was the least destructive between 1980 and 2010.

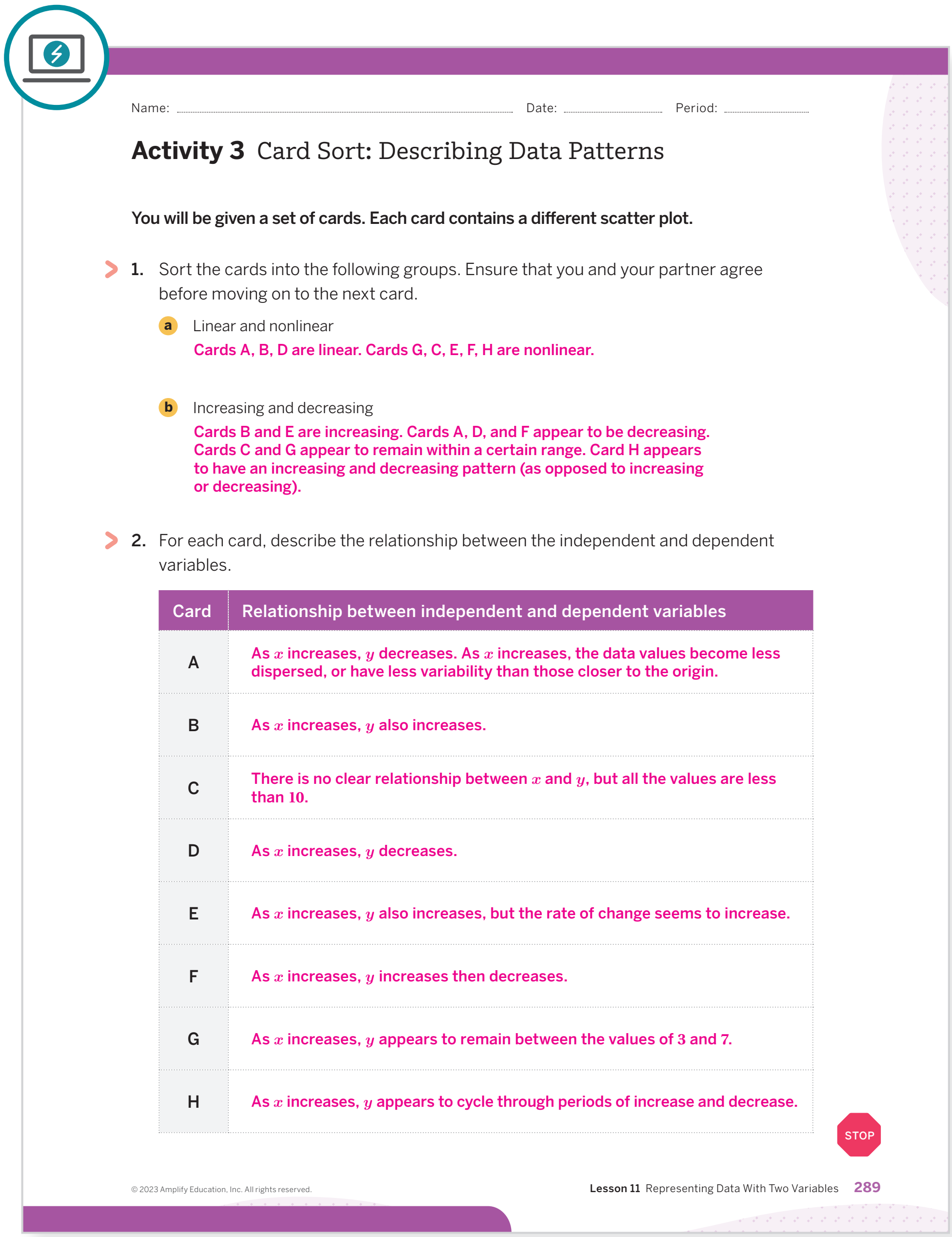
English Learners

Be sure students understand what the term *destructive* means. The most destructive insect that will be the one that *destroys* (in terms of defoliating) the greatest number of acres of forest.

Pairs | 10 min

Activity 3 Card Sort: Describing Data Patterns

Students sort linear and nonlinear scatter plots according to their trends and give quantitative descriptions of how the variables are related.



Name: ______ Date: ______ Period: ______

Activity 3 Card Sort: Describing Data Patterns

You will be given a set of cards. Each card contains a different scatter plot.

1. Sort the cards into the following groups. Ensure that you and your partner agree before moving on to the next card.
   - a Linear and nonlinear
     Cards A, B, D are linear. Cards G, C, E, F, H are nonlinear.
   - b Increasing and decreasing
     Cards B and E are increasing. Cards A, D, and F appear to be decreasing. Cards C and G appear to remain within a certain range. Card H appears to have an increasing and decreasing pattern (as opposed to increasing or decreasing).
2. For each card, describe the relationship between the independent and dependent variables.

Card	Relationship between independent and dependent variables
A	As x increases, y decreases. As x increases, the data values become less dispersed, or have less variability than those closer to the origin.
B	As x increases, y also increases.
C	There is no clear relationship between x and y, but all the values are less than 10.
D	As x increases, y decreases.
E	As x increases, y also increases, but the rate of change seems to increase.
F	As x increases, y increases then decreases.
G	As x increases, y appears to remain between the values of 3 and 7.
H	As x increases, y appears to cycle through periods of increase and decrease.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 11 Representing Data With Two Variables 289

1 Launch

Distribute the pre-cut cards from the Activity 3 PDF to each student pair. Encourage students to challenge each other and reach an agreement if they disagree with their partner.

2 Monitor

Help students get started by prompting them to begin with the cards they can efficiently identify as linear or nonlinear (or increasing and decreasing).

Look for points of confusion:

- **Having difficulty describing the trend on Card G.** Ask students to determine a range for the values of y.
- **Thinking Card A is linear.** Have students compare Card A to the other cards they determined were linear.

Look for productive strategies:

- Drawing trend lines or curves on scatter plots.
- Creating a third pile for cards that contain scatter plots that are neither increasing nor decreasing.

3 Connect

Display each card, one at a time.

Have pairs of students share whether each card is linear or nonlinear, increasing or decreasing, and how they described the relationship between the variables. If there are disagreements about how to classify a specific card, discuss until the class reaches a consensus.

Highlight that students can use a linear model to predict values on scatter plots showing linear data. Model how to do so for Cards B, D, and G. Note that the nonlinear scatter plots all appear to have a trend, or pattern, but students only need to be able to determine if the data is increasing or decreasing for now.

Ask, "Which of these cards shows no trend?" Card C

Differentiated Support

Accessibility: *Guide Processing and Visualization, Optimize Access to Tools*

Demonstrate, using Card A, how to determine that the data shows a linear and decreasing pattern. Provide access to rulers, index cards, or other straightedges so that students can use them to draw lines that follow the trend of the data for the linear relationships, to help with their thinking.

Extension: *Math Enrichment*

Challenge students to study the graphs on Cards A, B, D, and G to draw an approximate line of fit that models the data, write an equation for their line of fit, and use their equation to predict the y-value for an x-value not shown on the plot, such as $x = 20$.

Math Language Development

MLR8: Discussion Supports

During the Launch, display or provide access to the Anchor Chart PDF, *Sentence Stems, Describing My Thinking* to support students as they justify to their partners why they chose to sort certain cards in certain categories.

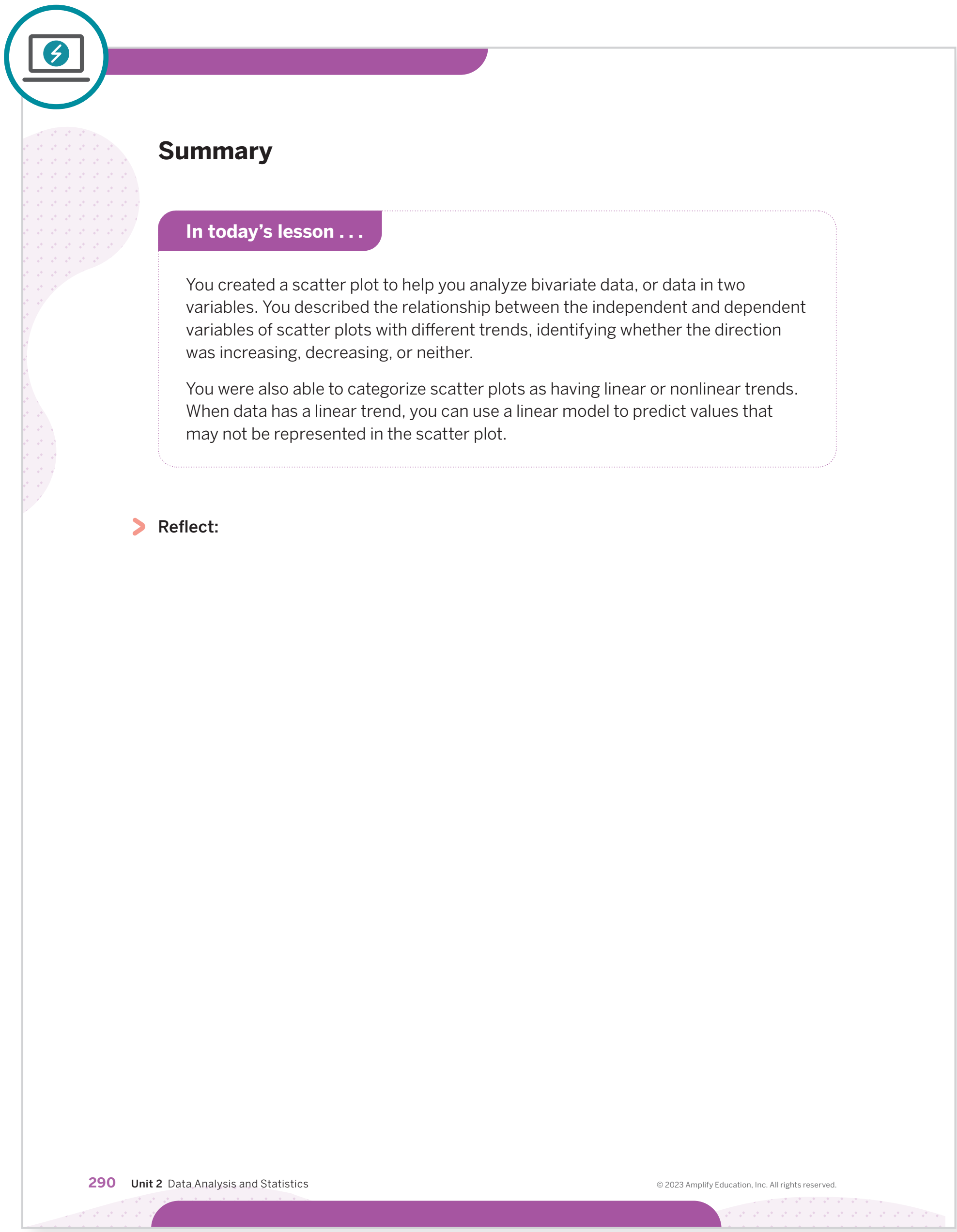
English Learners

During the Connect, annotate the graphs with *linear*, *nonlinear*, *increasing*, and/or *decreasing*. Consider also annotating the x-axis with *independent variable* and the y-axis with *dependent variable*.

Whole Class | 5 min

Summary

Review and synthesize how to represent bivariate data and describe its trend (linear vs. nonlinear, positive vs. negative) using a scatter plot.



Summary

In today's lesson . . .

You created a scatter plot to help you analyze bivariate data, or data in two variables. You described the relationship between the independent and dependent variables of scatter plots with different trends, identifying whether the direction was increasing, decreasing, or neither.

You were also able to categorize scatter plots as having linear or nonlinear trends. When data has a linear trend, you can use a linear model to predict values that may not be represented in the scatter plot.

> **Reflect:**

290 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Have students share why a scatter plot is a good way to represent two-variable data and their strategy for describing the trend in a scatter plot.

Display Cards B, D, and G from the Activity 3 PDF.

Ask:

- "How are these scatter plots similar?" They are all linear.
- "How are they different?" Card B is increasing, Card D is decreasing, and Card G is neither increasing nor decreasing.
- "Why is using a linear model for each of these scatter plots helpful?"

Highlight that a scatter plot helps students represent data in two variables. If there is a linear trend, they can use a linear model to help them see the trend more clearly or make predictions.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

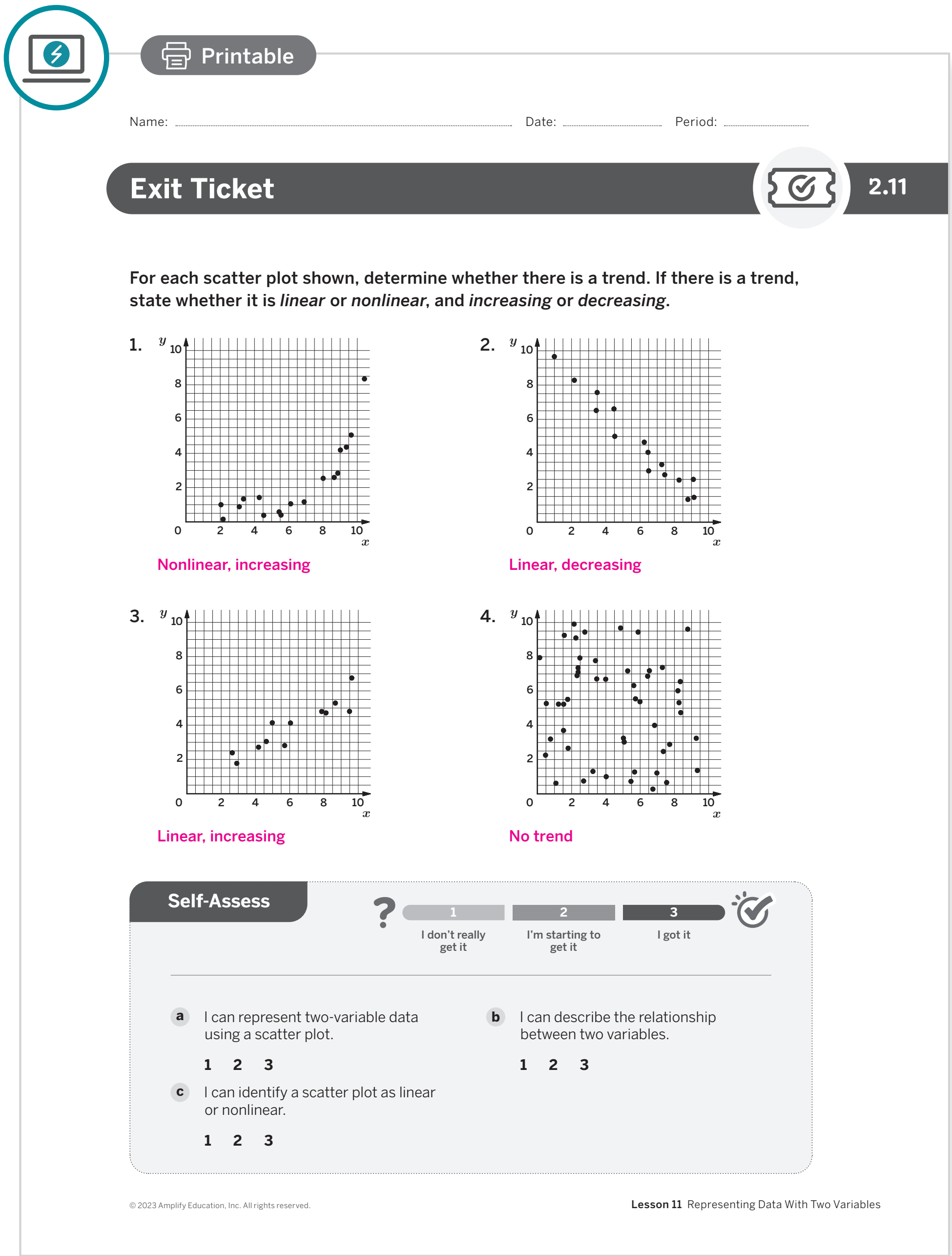
- "Why is a scatter plot a good representation for two-variable data?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by identifying whether a data set is linear or nonlinear and increasing or decreasing based on its scatter plot.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 2.11

For each scatter plot shown, determine whether there is a trend. If there is a trend, state whether it is *linear* or *nonlinear*, and *increasing* or *decreasing*.

1\. Nonlinear, increasing

2\. Linear, decreasing

3\. Linear, increasing

4\. No trend

Self-Assess

1 I don't really get it

2 I'm starting to get it

3 I got it

a I can represent two-variable data using a scatter plot. 1 2 3

b I can describe the relationship between two variables. 1 2 3

c I can identify a scatter plot as linear or nonlinear. 1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 11 Representing Data With Two Variables

Success looks like . . .

- **Goal:** Using a scatter plot to represent data in two variables.
- **Language Goal:** Interpreting the relationship between two variables from a scatter plot. **(Speaking and Listening, Writing)**
  - » Determining whether a scatter plot has a trend in Problem 1–4.
- **Goal:** Using a scatter plot to distinguish between linear and nonlinear data.
  - » Identifying a linear or nonlinear trend in the scatter plots of Problems 1–3.

Suggested next steps

If students are unable to correctly determine if the trend is linear or nonlinear, consider:

- Reviewing Activity 2.
- Assigning Practice Problem 1–2.

If students are unable to correctly determine if the trend is increasing or decreasing, consider:

- Reviewing each card from Activity 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In Grade 8, students are introduced to scatter plots and trends. How did that support students' ability to analyze and interpret scatter plots in today's activities?
- Which students' ideas were you able to highlight about the form and direction of scatter plots when comparing the data sets in Activity 2?

Math Language Development

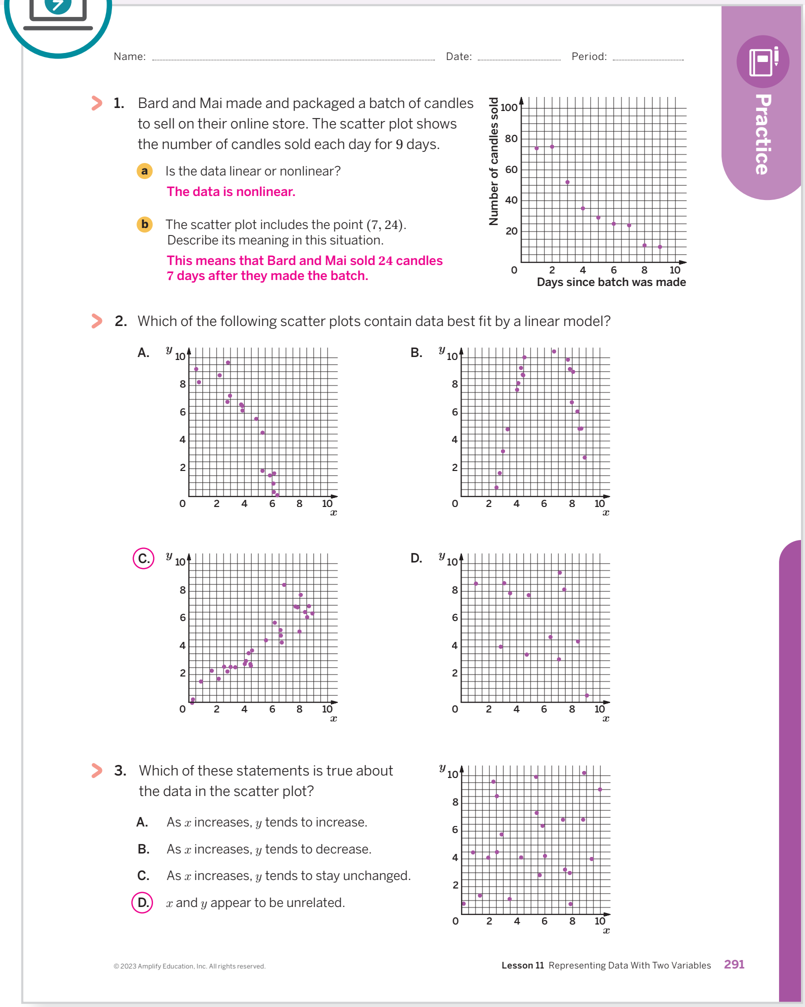
Language Goal: Interpreting the relationship between two variables from a scatter plot.

Reflect on students' language development toward this goal.

- How are students growing in comfort using the terms *linear*, *nonlinear*, *increasing*, and *decreasing* as they describe whether there is a trend shown on a given scatter plot? Are they using the terms accurately?
- How did using the *Discussion Supports* routine in Activity 3 help students practice using these terms? Would you change anything the next time you use this routine?

Practice

Independent


Name: Date: Period:

Practice

1. Bard and Mai made and packaged a batch of candles to sell on their online store. The scatter plot shows the number of candles sold each day for 9 days.
   - a Is the data linear or nonlinear?
     The data is nonlinear.
   - b The scatter plot includes the point (7, 24). Describe its meaning in this situation.
     This means that Bard and Mai sold 24 candles 7 days after they made the batch.

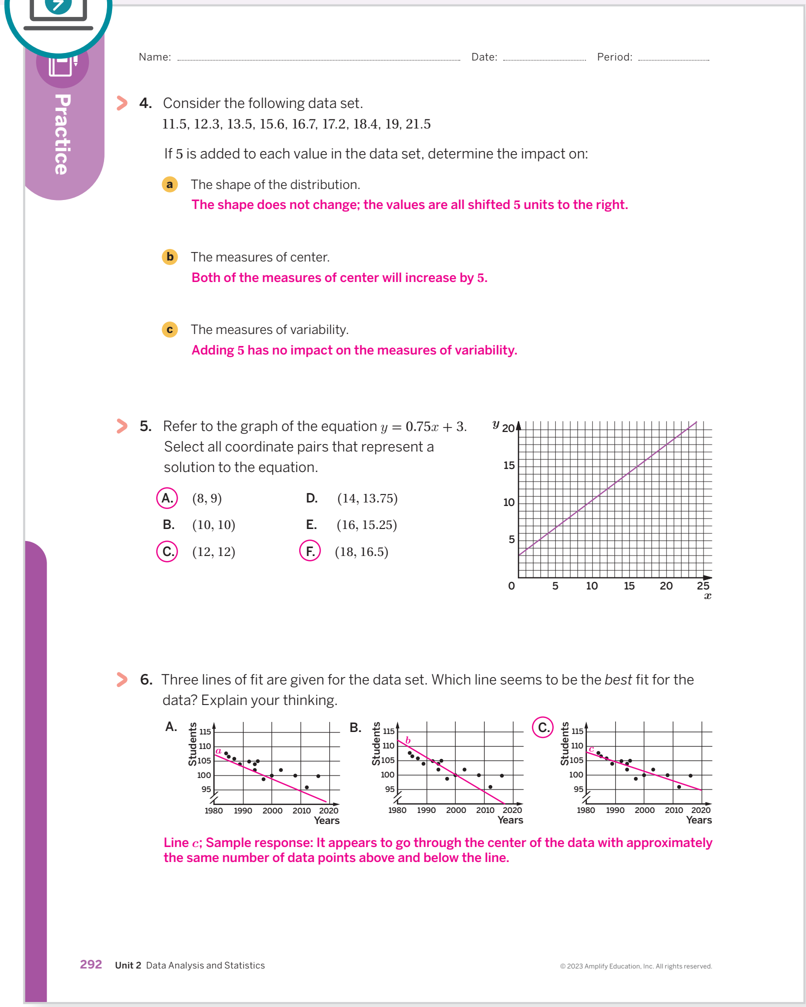
2. Which of the following scatter plots contain data best fit by a linear model?

[✓ C.]

3. Which of these statements is true about the data in the scatter plot?
   - A. As x increases, y tends to increase.
   - B. As x increases, y tends to decrease.
   - C. As x increases, y tends to stay unchanged.
   - [✓ D.] x and y appear to be unrelated.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 11 Representing Data With Two Variables 291


Name: Date: Period:

Practice

4. Consider the following data set.
   11.5, 12.3, 13.5, 15.6, 16.7, 17.2, 18.4, 19, 21.5

   If 5 is added to each value in the data set, determine the impact on:
   - a The shape of the distribution.
     The shape does not change; the values are all shifted 5 units to the right.
   - b The measures of center.
     Both of the measures of center will increase by 5.
   - c The measures of variability.
     Adding 5 has no impact on the measures of variability.

5. Refer to the graph of the equation $y = 0.75x + 3$. Select all coordinate pairs that represent a solution to the equation.

[✓ A.]	(8, 9)	D.	(14, 13.75)
B.	(10, 10)	E.	(16, 15.25)
[✓ C.]	(12, 12)	[✓ F.]	(18, 16.5)

6. Three lines of fit are given for the data set. Which line seems to be the *best* fit for the data? Explain your thinking.

Line c; Sample response: It appears to go through the center of the data with approximately the same number of data points above and below the line.

292 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity 3	1
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 10	2
	5	Unit 1 Lesson 11	2
Formative ⚡	6	Unit 2 Lesson 12	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 2 | **LESSON 12**

Linear Models

Let’s explore the relationships between two numerical variables.

Focus

Goals

1. **Language Goal:** Interpret the rate of change and vertical intercept for a linear model in everyday language. **(Speaking and Listening, Reading and Writing)**
2. Predict (extrapolate) and estimate (interpolate) values not given in the data set by using the linear model.
3. Fit a linear model to a scatter plot of data and informally judge its goodness of fit.

Coherence

Today

Students are reminded of how to informally draw a line to model data in a scatter plot and assess its goodness of fit. They use linear models to make predictions about values not represented on the scatter plot and reason abstractly by making sense of slope and y-intercept in different contexts. Students also investigate how an outlier might affect a linear model.

Previously

In Lesson 11, students created scatter plots to model bivariate data and describe its trend. In Grade 8, students informally fit a linear model to data. They evaluated the fit of the model by observing the closeness of the data points to the line.

Coming Soon

Students will determine the residuals of the values on a scatter plot and use a residual plot to make judgments about how well a linear model fits a data set.

Rigor

- Students build **conceptual understanding** of how to use a linear model to describe the relationship between two variables.
- Students develop **fluency** fitting a line to data on a scatter plot.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	10 min	10 min	5 min	5 min
Independent	Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (for display)
- Activity 3 PDF (for display)
- Anchor Chart PDF, *Sentence Stems, Comparing and Contrasting*
- Anchor Chart PDF, *Sentence Stems, Describing My Thinking*
- rulers
- yardsticks

Math Language Development

Review words

- *linear model*
- *scatter plot*
- *trend*

Amps | Featured Activity

Activity 3
Interactive Graphs

Students compare a line fit to data that includes an outlier to a line fit to data that does not include one to draw conclusions about the effect of an outlier and visualize how it changes the linear model.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may try to rush through the creation of their models in order to complete Activity 1. Have students discuss why they should discipline themselves to be careful when making their models. Create two sample models, one with precision and one that is much less accurate. Walk through the difference in results to show the importance of taking their time.

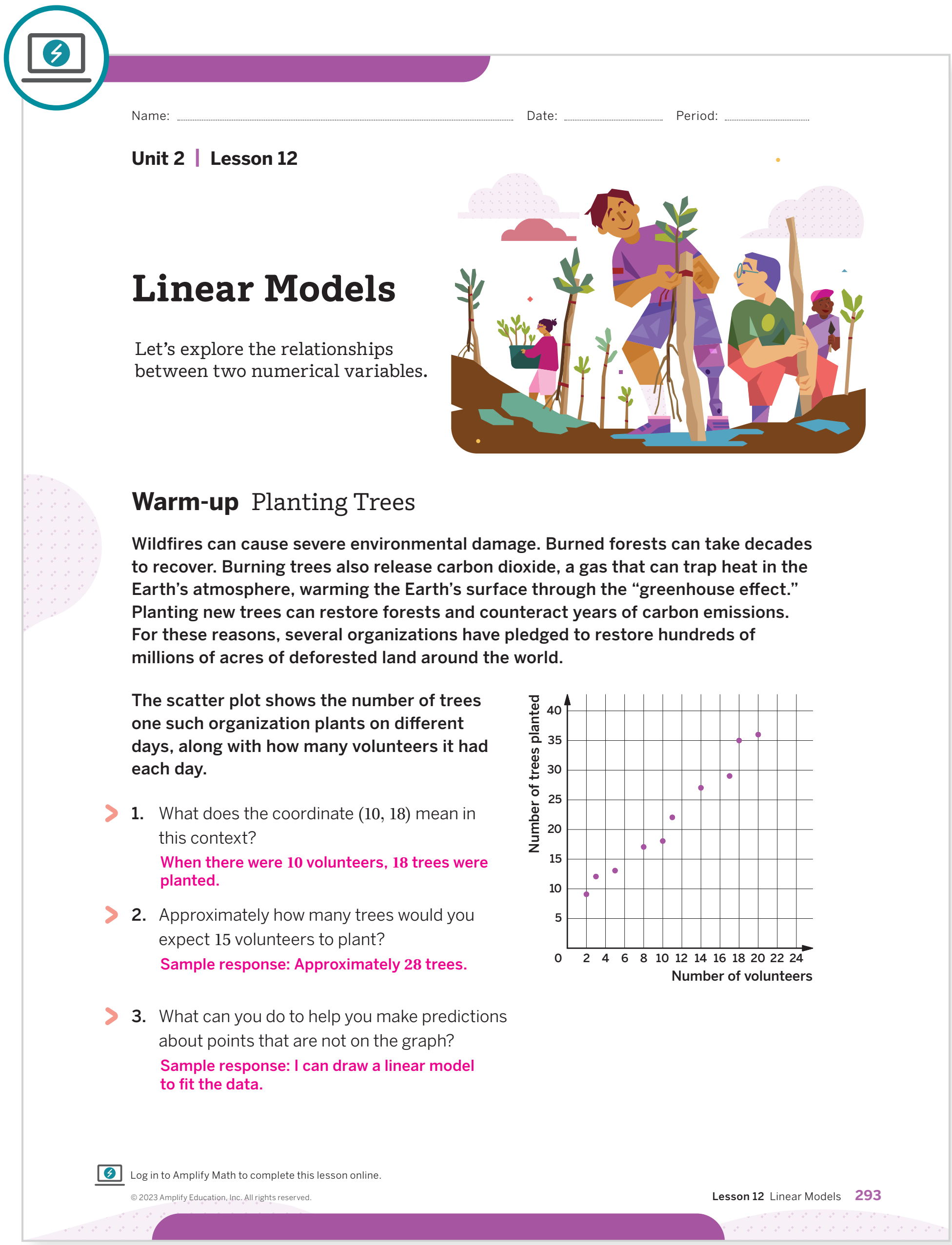
Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

- Omit the use of the Warm-up PDF in the **Warm-up**.
- In **Activity 1**, omit Problems 4 and 5 and address in the whole-class discussion during the Connect.
- Omit the ***Notice and Wonder*** routine from the **Activity 3** Launch.

Warm-up Planting Trees

Students examine a scatter plot to activate prior knowledge about trends in data.



Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 12

Linear Models

Let's explore the relationships between two numerical variables.

Warm-up Planting Trees

Wildfires can cause severe environmental damage. Burned forests can take decades to recover. Burning trees also release carbon dioxide, a gas that can trap heat in the Earth's atmosphere, warming the Earth's surface through the "greenhouse effect." Planting new trees can restore forests and counteract years of carbon emissions. For these reasons, several organizations have pledged to restore hundreds of millions of acres of deforested land around the world.

The scatter plot shows the number of trees one such organization plants on different days, along with how many volunteers it had each day.

1. What does the coordinate (10, 18) mean in this context?
   When there were 10 volunteers, 18 trees were planted.
2. Approximately how many trees would you expect 15 volunteers to plant?
   Sample response: Approximately 28 trees.
3. What can you do to help you make predictions about points that are not on the graph?
   Sample response: I can draw a linear model to fit the data.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 12 Linear Models 293

1 Launch

Read the prompt together as a whole class. Display the Warm-up PDF and prompt students to use it to explain how wildfires contribute to the greenhouse effect. Explain that planting trees, which absorb carbon dioxide, is one way to help combat this greenhouse effect.

2 Monitor

Help students get started by asking, "What is the trend of the data?" The trend is positive and linear.

Look for points of confusion:

- **Connecting the plotted points.** Ask, "What can you do instead to show how many more trees are planted for every additional volunteer that shows up?"

Look for productive strategies:

- Drawing a vertical line at $x = 15$ to estimate a corresponding y-value.
- Sketching a line to fit the data.
- Using key terms like "line of fit" or "trendline."

3 Connect

Display the scatter plot.

Ask:

- "What is the trend of the data?"
- "How is this data set modeled?"

Have students share what a coordinate in this context represents and how they estimated the number of trees planted by 15 volunteers. Select students that drew a linear model to fit the data and have them explain why they chose this as a strategy and how it helped, using precise mathematical language.

Highlight that a linear model can be drawn to fit the data to help students visualize the trend and make predictions about how many trees are planted based on the number of volunteers.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative. Consider reading aloud the first paragraph, or asking a volunteer to do so.

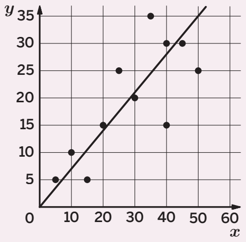
Read 1: Students should understand several organizations have pledged to plant trees to restore deforested land due to wildfire damage. The data for one organization is shown in the scatter plot.
Read 2: Ask students to name or highlight the given quantities and relationships, such as the scatter plot shows the relationship between the number of volunteers and the number of trees planted.
Read 3: Ask students to think about what patterns are shown in the data.

Power-up

To power up students' ability to analyze a line of fit for bivariate data, have students complete:

Does the line fit the data well? Be prepared to explain your thinking.

Yes; Sample response: There are 5 points that are on or very close to the line. There are 3 points above the line and 3 points below the line.



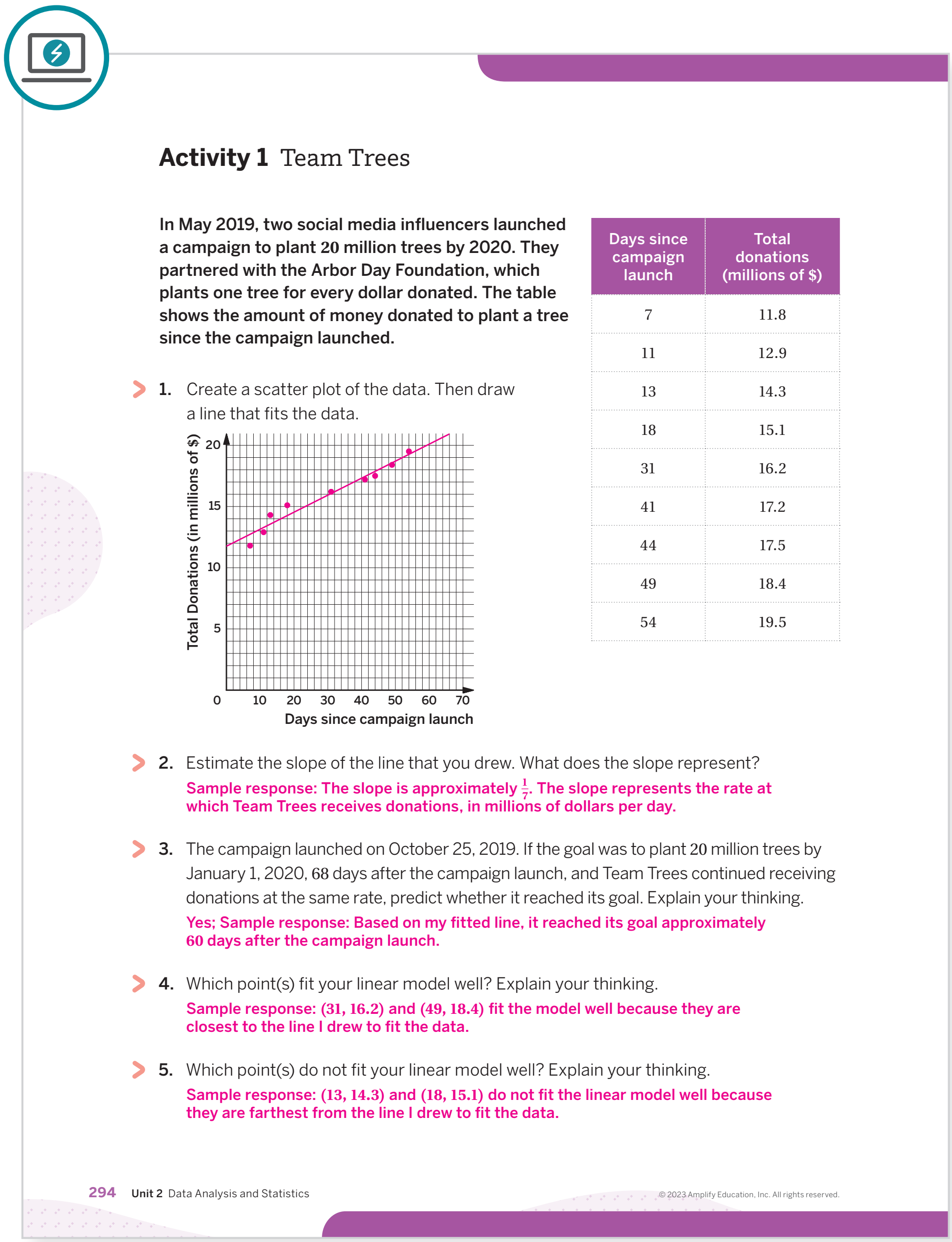
Use: Before Activity 1

Informed by: Performance on Lesson 11, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 7

Independent | 15 min

Activity 1 Team Trees

Students informally create a linear model, using it to make predictions and assess how well it fits the data.



Activity 1 Team Trees

In May 2019, two social media influencers launched a campaign to plant 20 million trees by 2020. They partnered with the Arbor Day Foundation, which plants one tree for every dollar donated. The table shows the amount of money donated to plant a tree since the campaign launched.

Days since campaign launch	Total donations (millions of $)
7	11.8
11	12.9
13	14.3
18	15.1
31	16.2
41	17.2
44	17.5
49	18.4
54	19.5

1. Create a scatter plot of the data. Then draw a line that fits the data.

2. Estimate the slope of the line that you drew. What does the slope represent?
   Sample response: The slope is approximately $\frac{1}{7}$. The slope represents the rate at which Team Trees receives donations, in millions of dollars per day.

3. The campaign launched on October 25, 2019. If the goal was to plant 20 million trees by January 1, 2020, 68 days after the campaign launch, and Team Trees continued receiving donations at the same rate, predict whether it reached its goal. Explain your thinking.
   Yes; Sample response: Based on my fitted line, it reached its goal approximately 60 days after the campaign launch.

4. Which point(s) fit your linear model well? Explain your thinking.
   Sample response: (31, 16.2) and (49, 18.4) fit the model well because they are closest to the line I drew to fit the data.

5. Which point(s) do not fit your linear model well? Explain your thinking.
   Sample response: (13, 14.3) and (18, 15.1) do not fit the linear model well because they are farthest from the line I drew to fit the data.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the prompt together as a whole class. Provide an expectation for the amount of time students will have to work on the activity independently.

2 Monitor

Help students get started by providing them with rulers to draw straight lines after creating their scatter plots.

Look for points of confusion:

- **Struggling to estimate the slope of their linear model**. Prompt students to estimate the values of two coordinates on their line and substitute them into the slope formula.

Look for productive strategies:

- Using a ruler to draw a straight line through the trend of the data so that there are points above and below the line.
- Judging the fit of their line by the proximity of the plotted points to the line.

3 Connect

Display the scatter plot of the data.

Have students share their strategy for drawing a line and how they determined its equation. Select a student with an appropriate linear model to demonstrate the process on the scatter plot displayed. Then select and sequence students to share their responses to Problems 3–5, prompting them to use the scatter plot in their explanations.

Highlight the connection between the positive slope of the linear model and the positive trend of the data. Also highlight that a good linear model should pass through the majority of the data points and that there will be points on either side of the line.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with a pre-populated scatter plot to use in Problem 1. This will allow them to still access the mathematical goal of the activity, without needing to spend additional time plotting the points. Consider allowing students to also work with a partner during this activity.

Extension: *Math Enrichment*

Challenge students to write an equation that represents their linear model. Display different equations that represent different lines of fit and ask students to compare the slopes and y-intercepts and describe any similarities they notice.

Math Language Development

MLR7: Compare and Connect

After students have completed Problems 1 and 2, have them circulate and examine the lines of fit of at least two other students in the room, making comparisons. Listen for any challenges students faced when drawing the lines and amplify the similarities between their models. Discuss students' observations during the Connect. Provide students with sticky notes so that they can record their comparisons as they circulate around the room.

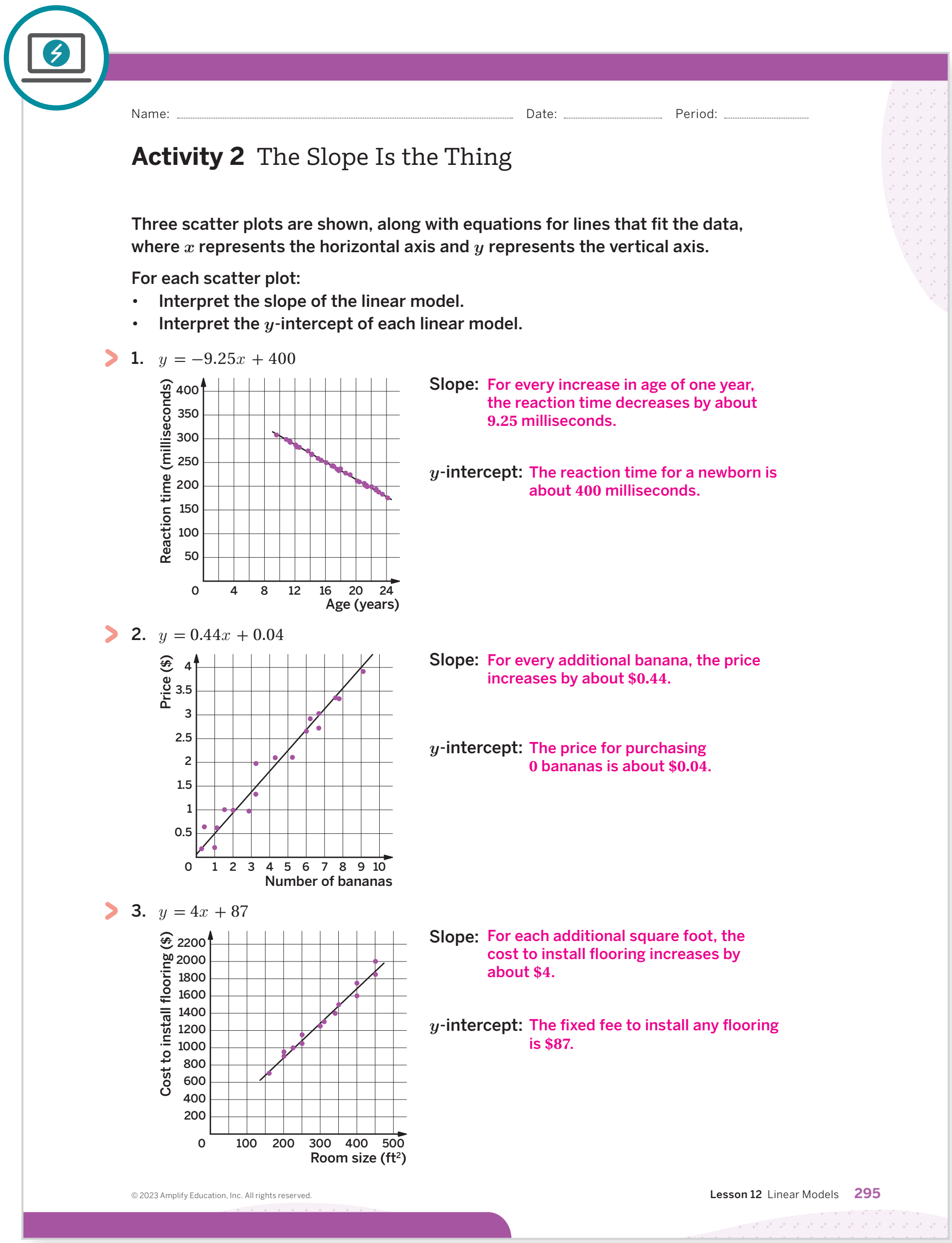
English Learners

Use color coding or annotations to illustrate which data points fit a linear model well and which do not.

Pairs | 10 min

Activity 2 The Slope Is the Thing

Students interpret the slope and y-intercept of linear models in context to strengthen their connections between statistical and algebraic representations.



Name: ________ Date: ________ Period: ________

Activity 2 The Slope Is the Thing

Three scatter plots are shown, along with equations for lines that fit the data, where x represents the horizontal axis and y represents the vertical axis.

For each scatter plot:

- Interpret the slope of the linear model.
- Interpret the y-intercept of each linear model.

1. $y = -9.25x + 400$

Slope: For every increase in age of one year, the reaction time decreases by about 9.25 milliseconds.

y-intercept: The reaction time for a newborn is about 400 milliseconds.

2. $y = 0.44x + 0.04$

Slope: For every additional banana, the price increases by about $0.44.

y-intercept: The price for purchasing 0 bananas is about $0.04.

3. $y = 4x + 87$

Slope: For each additional square foot, the cost to install flooring increases by about $4.

y-intercept: The fixed fee to install any flooring is $87.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have students complete the problems independently and then compare their responses with a partner.

2 Monitor

Help students get started by reminding them that slope is determined by $\frac{\text{change in } y}{\text{change in } x}$.

Look for points of confusion:

- **Having difficulty interpreting the slope in decimal form**. Explain that for each situation, the slope is $\frac{\text{change in } y}{\text{change in } x}$ where change in $x = 1$.

Look for productive strategies:

- Interpreting slope as a unit rate.
- Interpreting the y-intercept as the initial value, when $x = 0$.
- Interpreting slope as a unit rate in context.

3 Connect

Display each linear model, one at a time.

Have students share their interpretations of the slope and y-intercept for each linear model.

Ask:

- "For Problem 2, why is the intercept for the bananas not $(0, 0)$?" A linear model is not exact; it is an approximation based on the data.
- "Does the value of the y-intercept make sense in context for Problem 2?" No. "Problem 3?" Yes. "Problem 1?" Open to interpretation.

Highlight that the slope of a linear model tells students how much the dependent variable changes for every increase of one in the independent variable. Also, the y-intercept may not always make sense in context because the linear model is an approximation.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Instead of assigning each student all three problems, assign each student one of the three problems. Consider allowing them to choose which problem to complete. Then, before the Connect discussion, have students share their responses with a partner who completed a different problem.

Accessibility: *Activate Prior Knowledge*

Display the slope-intercept form of a linear equation $y = mx + b$ and elicit the meaning of m and b from students before beginning the activity. Remind students that slope is determined by $\frac{\text{change in } y}{\text{change in } x}$.

Math Language Development

MLR8: Discussion Supports

During the Launch, display or provide access to the Anchor Chart PDF, *Sentence Stems, Describing My Thinking* and encourage students to borrow from these phrases as they describe how they interpreted the slope and y-intercept.

English Learners

Display the following sentence frame to support students as they interpret the slope.

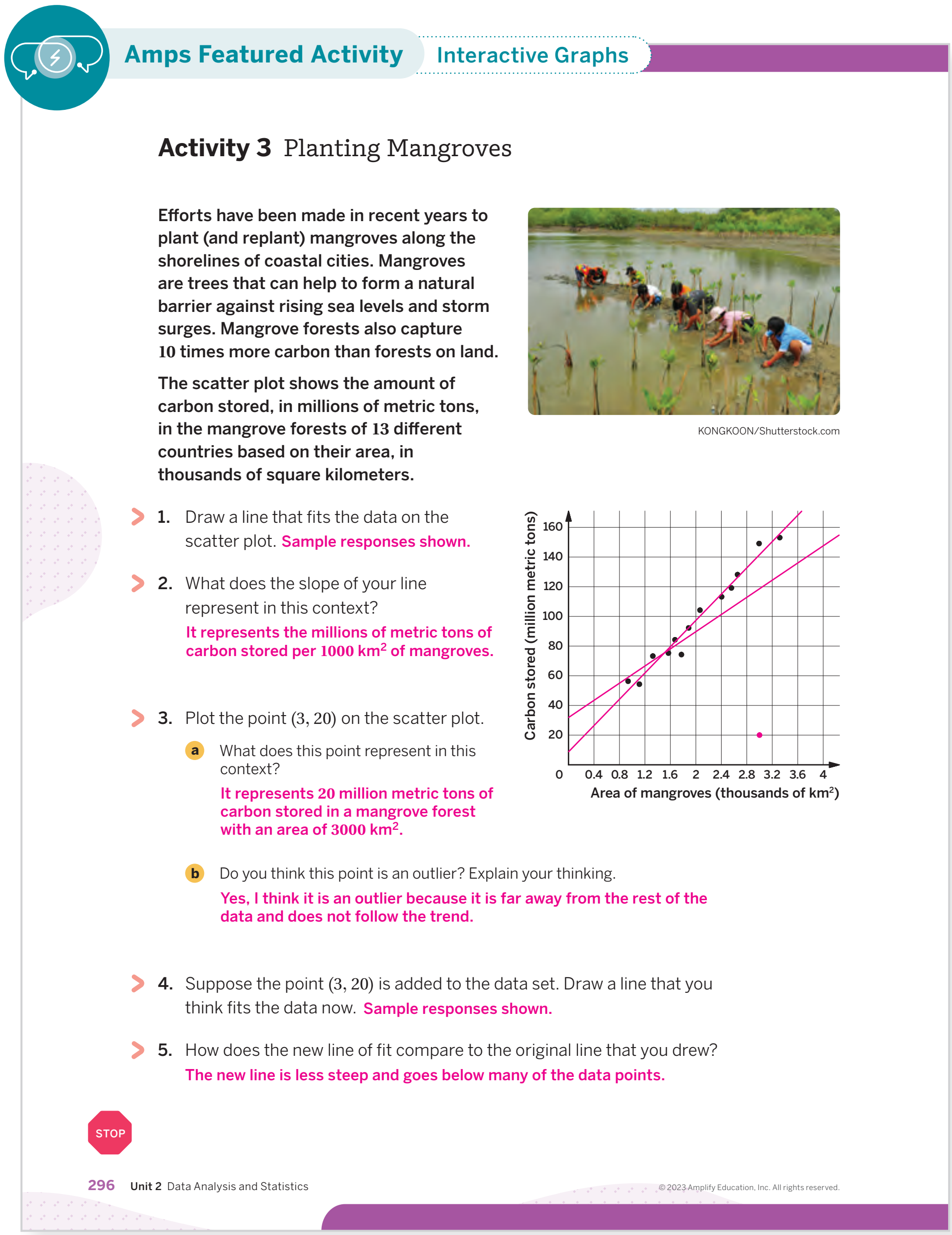
"For every _____, the _____ increases/decreases by _____."

Pairs | 10 min

Activity 3 Planting Mangroves

Students are reintroduced to outliers to see how they might affect linear models.

Amps Featured Activity Interactive Graphs



Activity 3 Planting Mangroves

Efforts have been made in recent years to plant (and replant) mangroves along the shorelines of coastal cities. Mangroves are trees that can help to form a natural barrier against rising sea levels and storm surges. Mangrove forests also capture 10 times more carbon than forests on land.

The scatter plot shows the amount of carbon stored, in millions of metric tons, in the mangrove forests of 13 different countries based on their area, in thousands of square kilometers.

KONGKOON/Shutterstock.com

1. Draw a line that fits the data on the scatter plot. **Sample responses shown.**

2. What does the slope of your line represent in this context?
   It represents the millions of metric tons of carbon stored per 1000 km^2 of mangroves.

3. Plot the point (3, 20) on the scatter plot.
   - a. What does this point represent in this context?
     It represents 20 million metric tons of carbon stored in a mangrove forest with an area of 3000 km^2.
   - b. Do you think this point is an outlier? Explain your thinking.
     Yes, I think it is an outlier because it is far away from the rest of the data and does not follow the trend.

4. Suppose the point (3, 20) is added to the data set. Draw a line that you think fits the data now. **Sample responses shown.**

5. How does the new line of fit compare to the original line that you drew?
   The new line is less steep and goes below many of the data points.

STOP

296 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display the Activity 3 PDF and facilitate the *Notice and Wonder* routine. Activate students' prior knowledge by asking, "What do you know about outliers?" Say that they will be looking at outliers again, but this time on a scatter plot (rather than in a distribution). Allow students individual work time, and then have them compare their work with their partner.

2 Monitor

Help students get started by asking, "What do you notice about the trend of the data?"
It is positive and linear.

Look for points of confusion:

- **Having difficulty determining if the point is an outlier in Problem 3b**. Explain that using an informal definition of an outlier is sufficient (without IQR and standard deviation, which were for distributions).
- **Struggling to draw a line of fit for the data with an outlier**. Ask, "How do you think the line will differ from your original line?"

Look for productive strategies:

- Recognizing that the unit change in x is 1000 km^2.
- Drawing a linear model whose slope is less steep when the data includes the outlier.

3 Connect

Display the scatter plot, including the point (3, 20) and both lines of best fit.

Ask, "Is (3, 20) an outlier? Use the scatter plot to explain your thinking."

Have pairs of students share their strategy for drawing a line of best fit when (3, 20) was included. Discuss how this line compares to the original line.

Highlight that an outlier does not follow the trend of the other data. When an outlier is present, the linear model may not fit as well, and the outlier will be far from the linear model.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students draw the lines of fit, provide a pre-completed graph with sample lines of fit shown for Problems 1 and 4. Conduct the *Notice and Wonder* routine by having students study the two lines of fit and record what they notice and wonder.

Extension: *Math Enrichment*

Ask students to name the coordinates of a point that would result in the line of fit becoming steeper. Ask them to interpret the coordinates within the context of the scenario. Sample response: (0.8, 140); For a mangrove area of 800 km^2, 140 million metric tons of carbon are stored.

Math Language Development

MLR8: Discussion Supports

While students work, display or provide access to the Anchor Chart PDF, *Sentence Stems, Comparing and Contrasting* to support student conversation as they compare their original lines of fit (Problem 1) and their new lines of fit (Problem 4) with their partner's lines of fit.

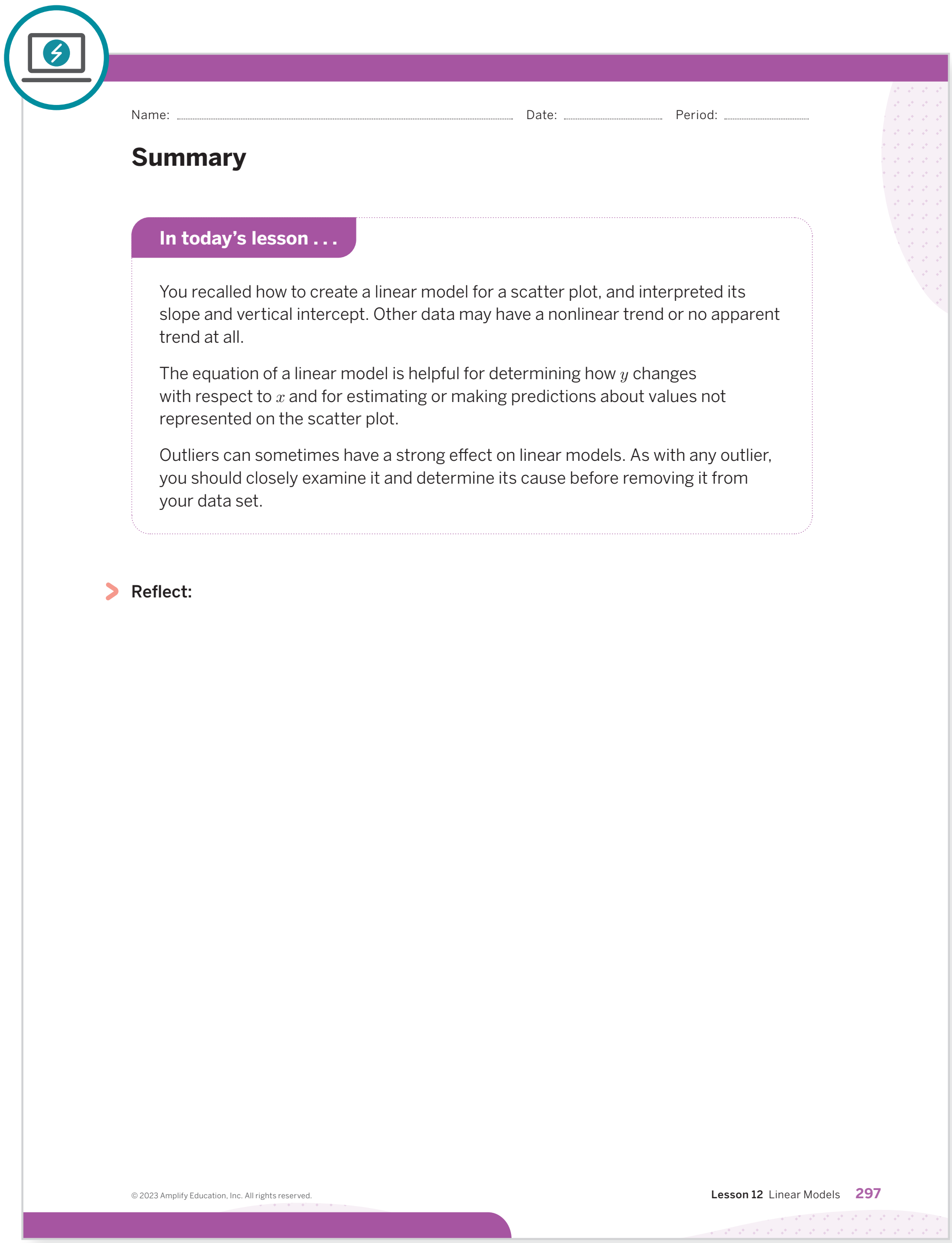
English Learners

During the Connect, display the original scatter plot and line of fit. Demonstrate by using a yardstick or other manipulative the movement of the line as the point (3, 20) is added.

Whole Class | 5 min

Summary

Review and synthesize how to make connections between bivariate data, a linear model, and the data's context.



Name: Date: Period:

Summary

In today's lesson . . .

You recalled how to create a linear model for a scatter plot, and interpreted its slope and vertical intercept. Other data may have a nonlinear trend or no apparent trend at all.

The equation of a linear model is helpful for determining how y changes with respect to x and for estimating or making predictions about values not represented on the scatter plot.

Outliers can sometimes have a strong effect on linear models. As with any outlier, you should closely examine it and determine its cause before removing it from your data set.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Have students share their strategy for drawing a line to fit a data set and what a good line of fit looks like.

Ask:

- "How can you tell which points in a data set fit a linear model well and which do not?" The closer they are to the line, the better they fit.
- "How might the presence of an outlier in a data set affect its linear model?" The linear model may not fit the data as well if outliers are included.

Highlight that a linear model allows students to make predictions for the data in a range near the given data. It also helps them to describe the relationship between two variables quantitatively.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

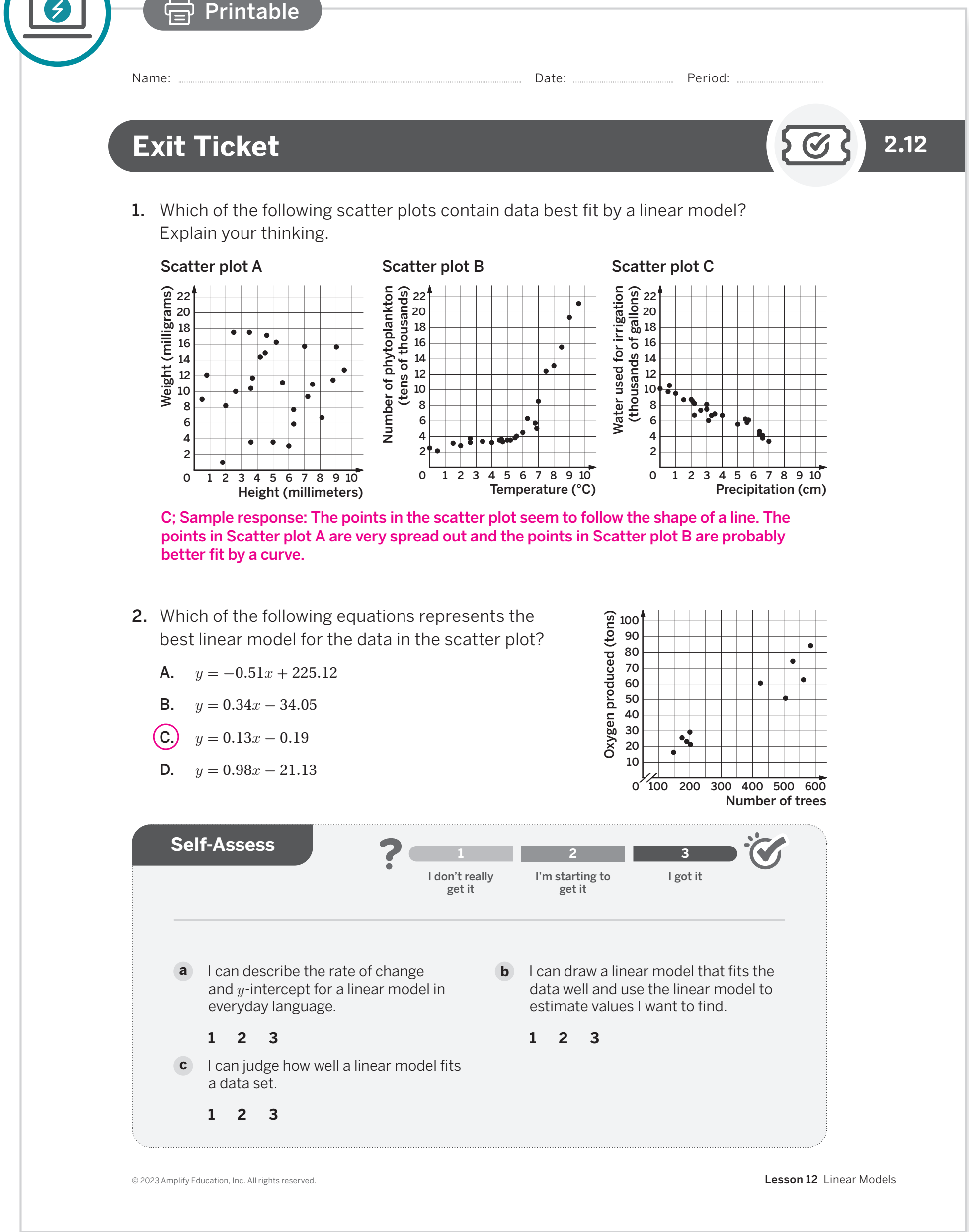
- "How can you tell if a linear model is a good fit for a set of data? Why is creating a linear model useful?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by fitting a line to data on a scatter plot and judging how well it models a trend.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 2.12

1. Which of the following scatter plots contain data best fit by a linear model? Explain your thinking.

C; Sample response: The points in the scatter plot seem to follow the shape of a line. The points in Scatter plot A are very spread out and the points in Scatter plot B are probably better fit by a curve.

2. Which of the following equations represents the best linear model for the data in the scatter plot?

A. $y = -0.51x + 225.12$

B. $y = 0.34x - 34.05$

[✓ C.] $y = 0.13x - 0.19$

D. $y = 0.98x - 21.13$

Self-Assess

1 I don't really get it
2 I'm starting to get it
3 I got it

a I can describe the rate of change and y-intercept for a linear model in everyday language.
1 2 3

b I can draw a linear model that fits the data well and use the linear model to estimate values I want to find.
1 2 3

c I can judge how well a linear model fits a data set.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 12 Linear Models

Success looks like . . .

- **Language Goal:** Interpreting the rate of change and vertical intercept for a linear model in everyday language. **(Speaking and Listening, Reading and Writing)**
- **Goal:** Predicting (extrapolate) and estimating (interpolate) values not given in the data set by using the linear model.
- **Goal:** Fitting a linear model to a scatter plot of data and informally judging its goodness of fit.
  - » Identifying the linear model that best fits the data shown in the scatter plot in Problem 2.

Suggested next steps

If students identify Scatter plot B in Problem 1, consider:

- Asking students to add a line of fit to each scatter plot.
- Asking, "What would fit Scatter plot B better, a line or a curve?"

If students have difficulty in determining the best line of fit for Problem 2, consider:

- Asking, "How could you determine which slope is most appropriate for a line that models the given data?"
- Assigning Practice Problem 2.

Professional Learning

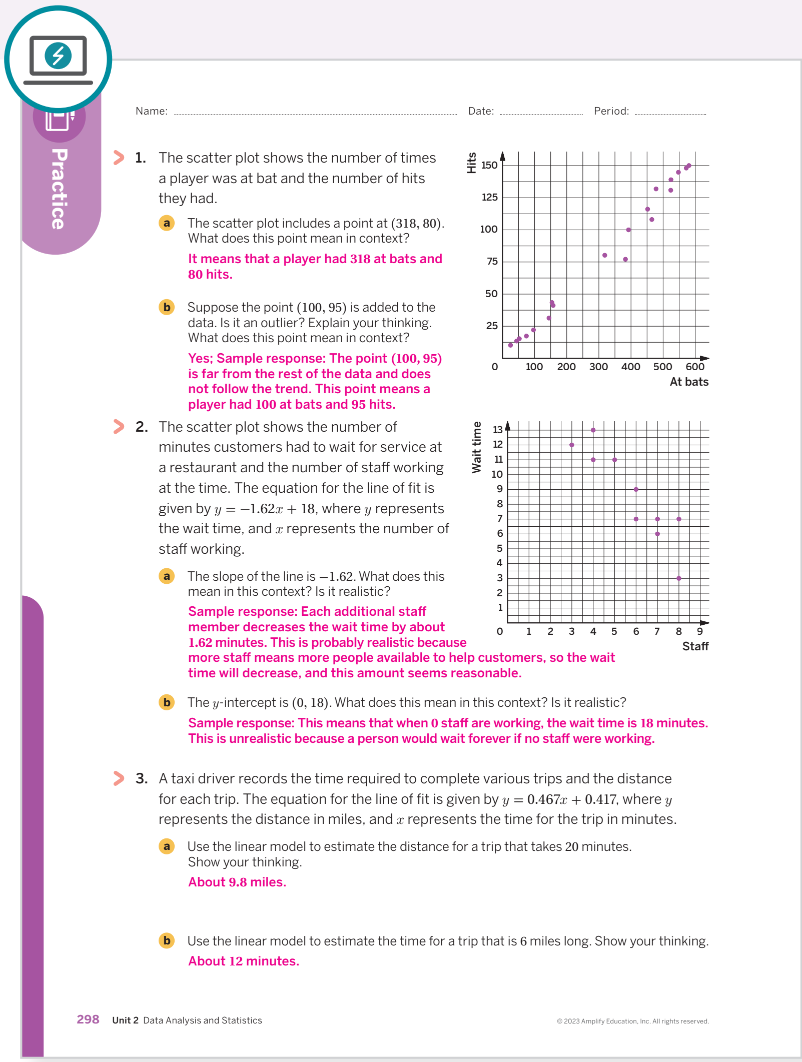
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What routines enabled all students to do math in today's lesson?
- During the discussion about the effect of an outlier on a linear model, how did you encourage each student to share their understandings?

Practice

Independent



Practice

Name: Date: Period:

1. The scatter plot shows the number of times a player was at bat and the number of hits they had.

   a. The scatter plot includes a point at $(318, 80)$. What does this point mean in context?

      It means that a player had 318 at bats and 80 hits.

   b. Suppose the point $(100, 95)$ is added to the data. Is it an outlier? Explain your thinking. What does this point mean in context?

      Yes; Sample response: The point (100, 95) is far from the rest of the data and does not follow the trend. This point means a player had 100 at bats and 95 hits.

2. The scatter plot shows the number of minutes customers had to wait for service at a restaurant and the number of staff working at the time. The equation for the line of fit is given by $y = -1.62x + 18$, where y represents the wait time, and x represents the number of staff working.

   a. The slope of the line is -1.62. What does this mean in this context? Is it realistic?

      Sample response: Each additional staff member decreases the wait time by about 1.62 minutes. This is probably realistic because more staff means more people available to help customers, so the wait time will decrease, and this amount seems reasonable.

   b. The y-intercept is $(0, 18)$. What does this mean in this context? Is it realistic?

      Sample response: This means that when 0 staff are working, the wait time is 18 minutes. This is unrealistic because a person would wait forever if no staff were working.

3. A taxi driver records the time required to complete various trips and the distance for each trip. The equation for the line of fit is given by $y = 0.467x + 0.417$, where y represents the distance in miles, and x represents the time for the trip in minutes.

   a. Use the linear model to estimate the distance for a trip that takes 20 minutes. Show your thinking.

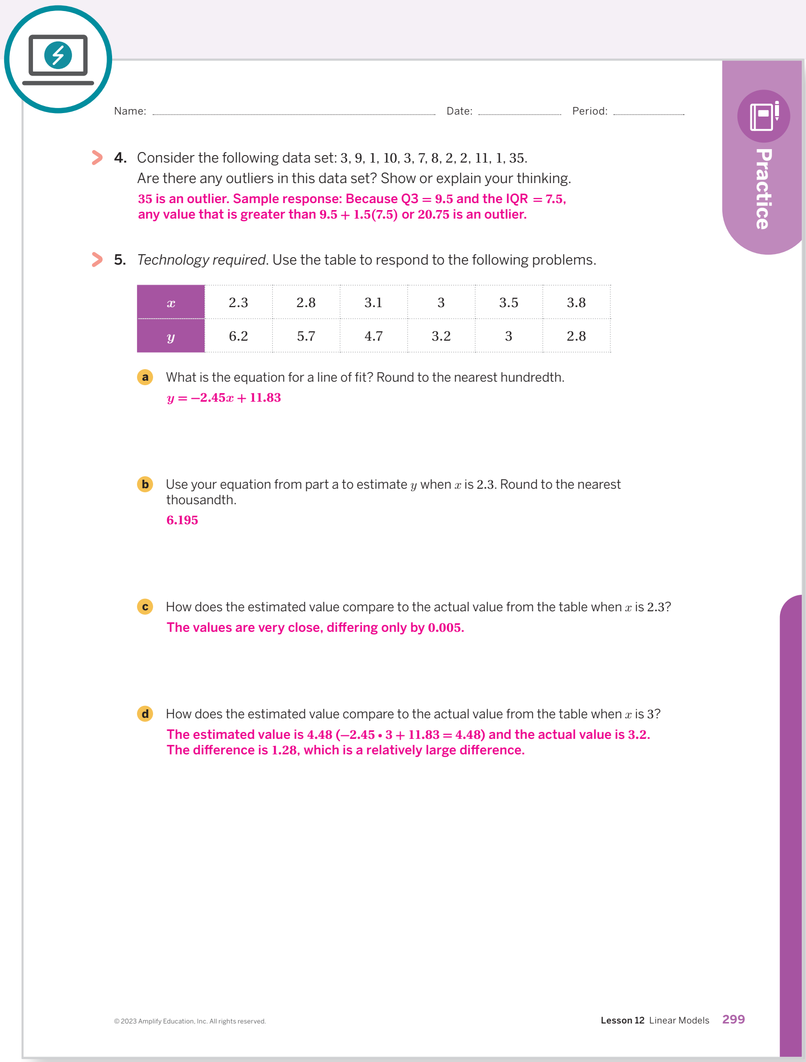
      About 9.8 miles.

   b. Use the linear model to estimate the time for a trip that is 6 miles long. Show your thinking.

      About 12 minutes.

298 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.



Name: Date: Period:

Practice

4. Consider the following data set: 3, 9, 1, 10, 3, 7, 8, 2, 2, 11, 1, 35. Are there any outliers in this data set? Show or explain your thinking.

   35 is an outlier. Sample response: Because Q3 = 9.5 and the IQR = 7.5, any value that is greater than 9.5 + 1.5(7.5) or 20.75 is an outlier.

5. *Technology required*. Use the table to respond to the following problems.

x	2.3	2.8	3.1	3	3.5	3.8
y	6.2	5.7	4.7	3.2	3	2.8

   a. What is the equation for a line of fit? Round to the nearest hundredth.

      $y = -2.45x + 11.83$

   b. Use your equation from part a to estimate y when x is 2.3. Round to the nearest thousandth.

      6.195

   c. How does the estimated value compare to the actual value from the table when x is 2.3?

      The values are very close, differing only by 0.005.

   d. How does the estimated value compare to the actual value from the table when x is 3?

      The estimated value is 4.48 ($-2.45 \cdot 3 + 11.83 = 4.48$) and the actual value is 3.2. The difference is 1.28, which is a relatively large difference.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 12 Linear Models 299

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 2 Lesson 5	2
Formative	5	Unit 2 Lesson 13	2

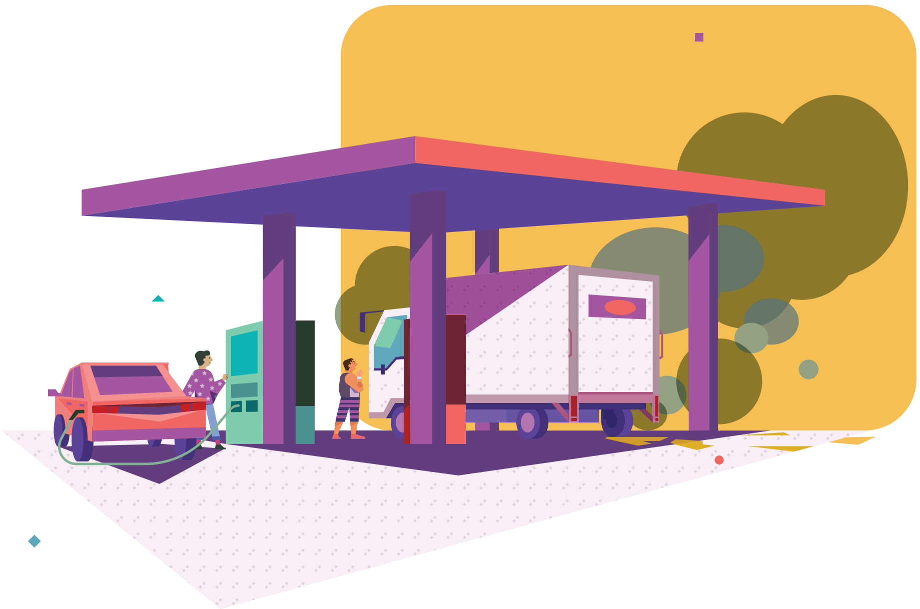
Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Residuals

Let's examine how close linear models are to the data they represent.



Focus

Goals

1. Calculate and plot the residuals for a given data set.
2. **Language Goal:** Use residuals to determine the goodness of fit for a linear model. **(Speaking and Listening)**

Coherence

Today

Students learn how to compute the residuals of a linear model and use those residuals to judge how well the linear model fits the data. They interpret what positive and negative residuals mean in context. Students compare residual plots of linear models to determine which line fits better, and are given the opportunity to explain their reasoning, listen to their peers, and critique the reasoning of others.

Previously

Students examined and created linear models to help them make predictions. They considered what makes a line a good fit for a data set.

Coming Soon

Students will learn a new strategy for precisely determining a line of best fit for a set of data that builds on their knowledge of residuals.

Rigor

- Students build **conceptual understanding** of residuals of linear models.
- Students develop **fluency** judging the goodness of fit of a linear model.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	10 min	5 min	5 min
Independent	Small Groups	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 1 PDF, one per student
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps*
- Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning*
- rulers
- scientific calculators

Math Language Development

New words

- ***residual***
- ***residual plot***

Review words

- *linear model*
- *scatter plot*

Amps | Featured Activity

Activity 2
Interactive Graphs

Students use interactive tools to draw a linear model fit to a data set and see how it compares to the linear models of their classmates. They have the opportunity to visualize how residuals are a measure of each data point from the line of fit.

Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might try to guess at which line fits better without any reasoning in Activity 2. Ask each person to pair up with someone else and prove which line is better. Encourage them to challenge each other to explain their answers more thoroughly by asking lots of questions, until they are satisfied that the answer is correct.

Modifications to Pacing

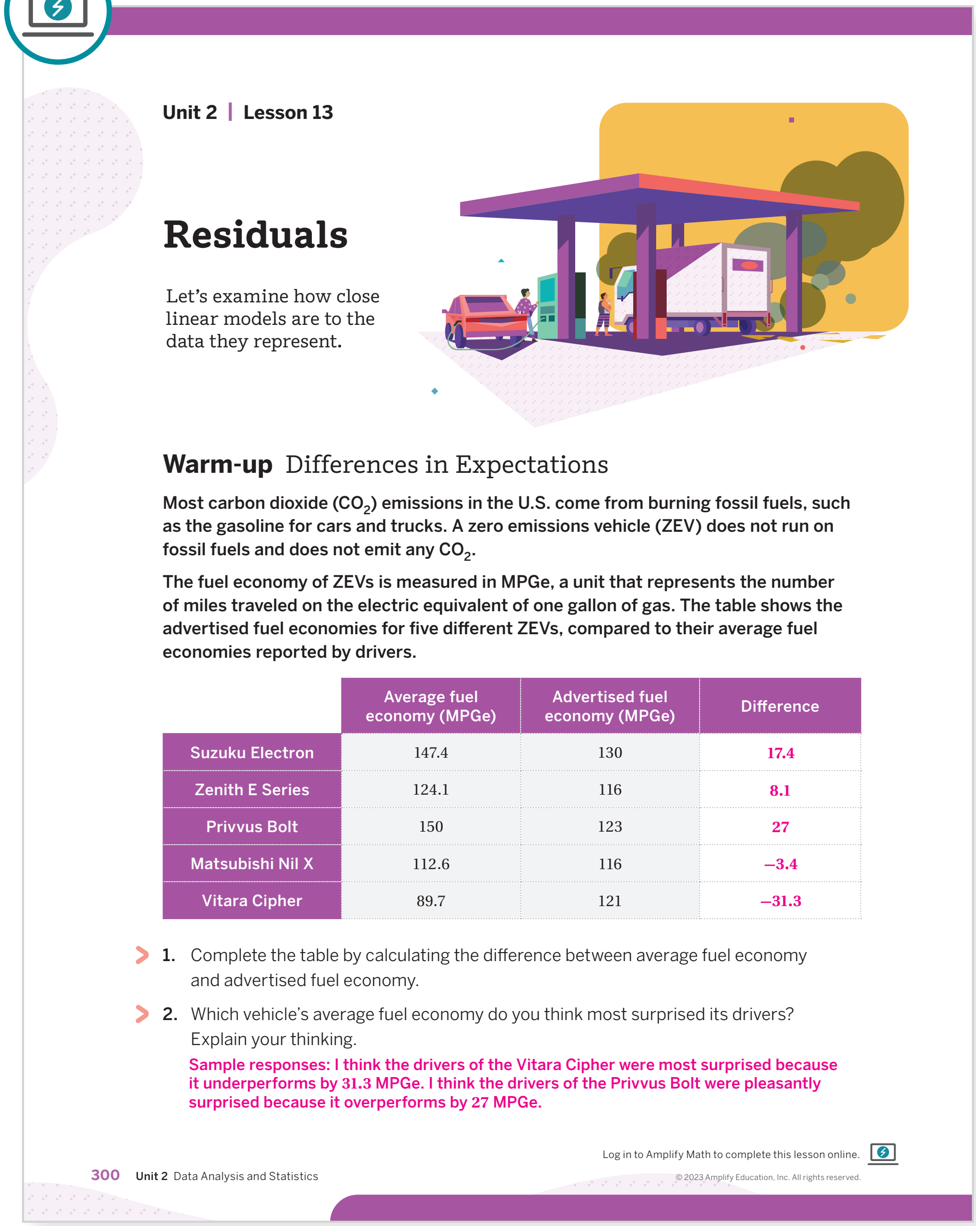
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, Problem 4 may be omitted.
- In **Activity 3**, Problem 1 may be omitted.

Independent | 5 min

Warm-up Differences in Expectations

Students calculate the difference between advertised values and actual values to elicit understandings that they will need to compute residuals from a linear model.



Unit 2 | Lesson 13

Residuals

Let's examine how close linear models are to the data they represent.

Warm-up Differences in Expectations

Most carbon dioxide (CO_2) emissions in the U.S. come from burning fossil fuels, such as the gasoline for cars and trucks. A zero emissions vehicle (ZEV) does not run on fossil fuels and does not emit any CO_2.

The fuel economy of ZEVs is measured in MPGe, a unit that represents the number of miles traveled on the electric equivalent of one gallon of gas. The table shows the advertised fuel economies for five different ZEVs, compared to their average fuel economies reported by drivers.

	Average fuel economy (MPGe)	Advertised fuel economy (MPGe)	Difference
Suzuku Electron	147.4	130	17.4
Zenith E Series	124.1	116	8.1
Privvus Bolt	150	123	27
Matsubishi Nil X	112.6	116	−3.4
Vitara Cipher	89.7	121	−31.3

1. Complete the table by calculating the difference between average fuel economy and advertised fuel economy.
2. Which vehicle's average fuel economy do you think most surprised its drivers? Explain your thinking.
   Sample responses: I think the drivers of the Vitara Cipher were most surprised because it underperforms by 31.3 MPGe. I think the drivers of the Privvus Bolt were pleasantly surprised because it overperforms by 27 MPGe.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Ask, "Have you ever been to a car dealership? What information about a car do you think is important to a potential buyer?" If not mentioned, explain that gas mileage, or fuel economy, is something buyers are typically interested in because it determines how much they will need to spend on gas.

2 Monitor

Help students get started by asking, "Which car is advertised as being the most fuel efficient?"

Look for points of confusion:

- **Having difficulty calculating the difference.** Remind students to subtract the advertised fuel economy from the average fuel economy.

Look for productive strategies:

- Annotating the table with a subtraction sign between the values in the second and third column.
- Identifying ZEVs with the greatest differences.

3 Connect

Display the names of the five ZEVs and have a student share the differences they calculated for each.

Have students share what the difference values represent in this context.

Ask:

- "What does a positive difference mean in this context? What about a negative difference?"
- "Which differences do you think most closely match drivers' expectations? Which are most surprising to drivers?"

Highlight that students will learn a new strategy for judging how well a linear model fits data that involves calculating differences.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

Read 1: Students should understand that there is a difference between the advertised fuel economy and the actual fuel economy that is reported by drivers.
Read 2: Ask students to name or highlight the given quantities and relationships, such as the Zenith E Series has a greater average fuel economy reported by drivers than the advertised fuel economy.
Read 3: Ask students to preview Problems 1 and 2 and think about how they might approach completing them.

Power-up

To power up students' ability to compare the predicted value from a linear model and the actual value:

Provide students with a copy of the Power-up PDF.

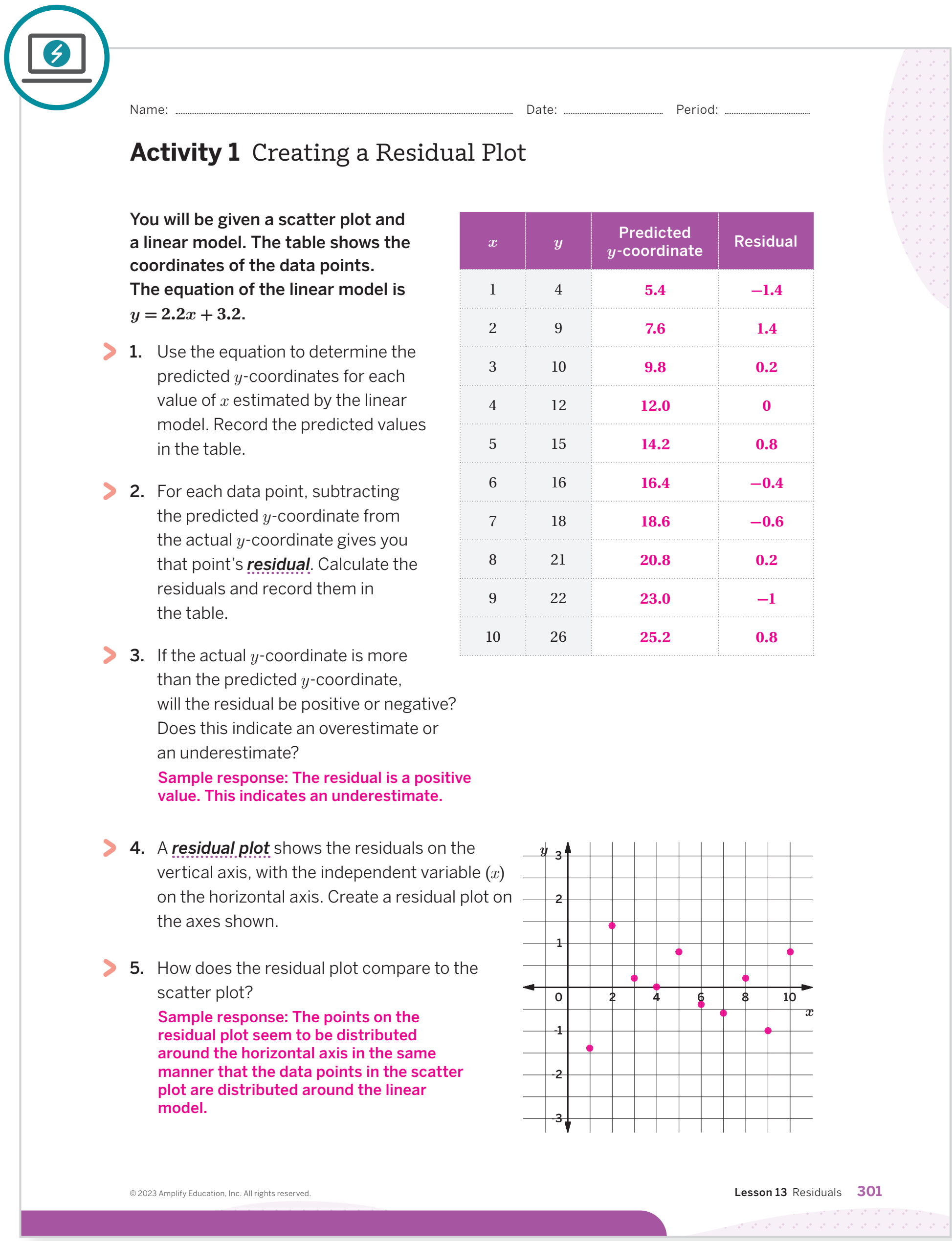
Use: Before Activity 1

Informed by: Performance on Lesson 12, Practice Problem 5

Small Groups | 10 min

Activity 1 Creating a Residual Plot

Students learn about residuals and create their very first residual plot, observing how it relates to the original scatter plot and linear model.



Name: ______________________ Date: __________ Period: __________

Activity 1 Creating a Residual Plot

You will be given a scatter plot and a linear model. The table shows the coordinates of the data points. The equation of the linear model is $y = 2.2x + 3.2$.

x	y	Predicted y-coordinate	Residual
1	4	5.4	–1.4
2	9	7.6	1.4
3	10	9.8	0.2
4	12	12.0	0
5	15	14.2	0.8
6	16	16.4	–0.4
7	18	18.6	–0.6
8	21	20.8	0.2
9	22	23.0	–1
10	26	25.2	0.8

1. Use the equation to determine the predicted y-coordinates for each value of x estimated by the linear model. Record the predicted values in the table.

2. For each data point, subtracting the predicted y-coordinate from the actual y-coordinate gives you that point's ***residual***. Calculate the residuals and record them in the table.

3. If the actual y-coordinate is more than the predicted y-coordinate, will the residual be positive or negative? Does this indicate an overestimate or an underestimate?
   Sample response: The residual is a positive value. This indicates an underestimate.

4. A ***residual plot*** shows the residuals on the vertical axis, with the independent variable (x) on the horizontal axis. Create a residual plot on the axes shown.

5. How does the residual plot compare to the scatter plot?
   Sample response: The points on the residual plot seem to be distributed around the horizontal axis in the same manner that the data points in the scatter plot are distributed around the linear model.

©2023 Amplify Education, Inc. All rights reserved.

1 Launch

Distribute the Activity 1 PDF to each student. Provide access to scientific calculators.

2 Monitor

Help students get started by modeling how to calculate the predicted value for the point $(1, 4)$.

Look for points of confusion:

- **Having difficulty determining the residual values.** Ask, "For $x = 1$, what is the 'actual value' on the scatter plot? The 'predicted value'? What is the distance between the two?"

Look for productive strategies:

- Using the equation of the linear model to calculate the predicted values for each value of x.
- Noticing a positive residual indicates an overestimate and a negative residual indicates an underestimate.

3 Connect

Display the Activity 1 PDF and its corresponding residual plot.

Define the terms ***residual*** and ***residual plot***.

Have groups of students share how their residual plots compare to the scatter plot.

Ask:

- "When is a residual positive? Negative? Zero?"
- "How are residuals represented on the scatter plot? On the residual plot?"

Highlight that the smaller the absolute value of the residuals, the closer they are to 0 (the horizontal axis), and the better the fit of the linear model. The residual plot shows some residuals scattered randomly above and below the horizontal axis as you go from left to right, indicating a mix of positive and negative values.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider providing an alternate table, such as the one shown here, for students to use that provides additional scaffolding to help students determine the predicted y-coordinate and the residual. The first row is shown.

x	y	$y = 2.2x + 3.2$	Predicted y-coordinate	Predicted minus actual y-coordinate	Residual
1	4	$y = 2.2(1) + 3.2$	5.4	5.4 – 4	1.4

Extension: *Math Enrichment*

Have students draw a sample residual plot for a scatter plot whose linear model is a good fit. Then have them draw a sample residual plot for a scatter plot whose linear model is not a good fit.

Math Language Development

MLR7: Compare and Connect

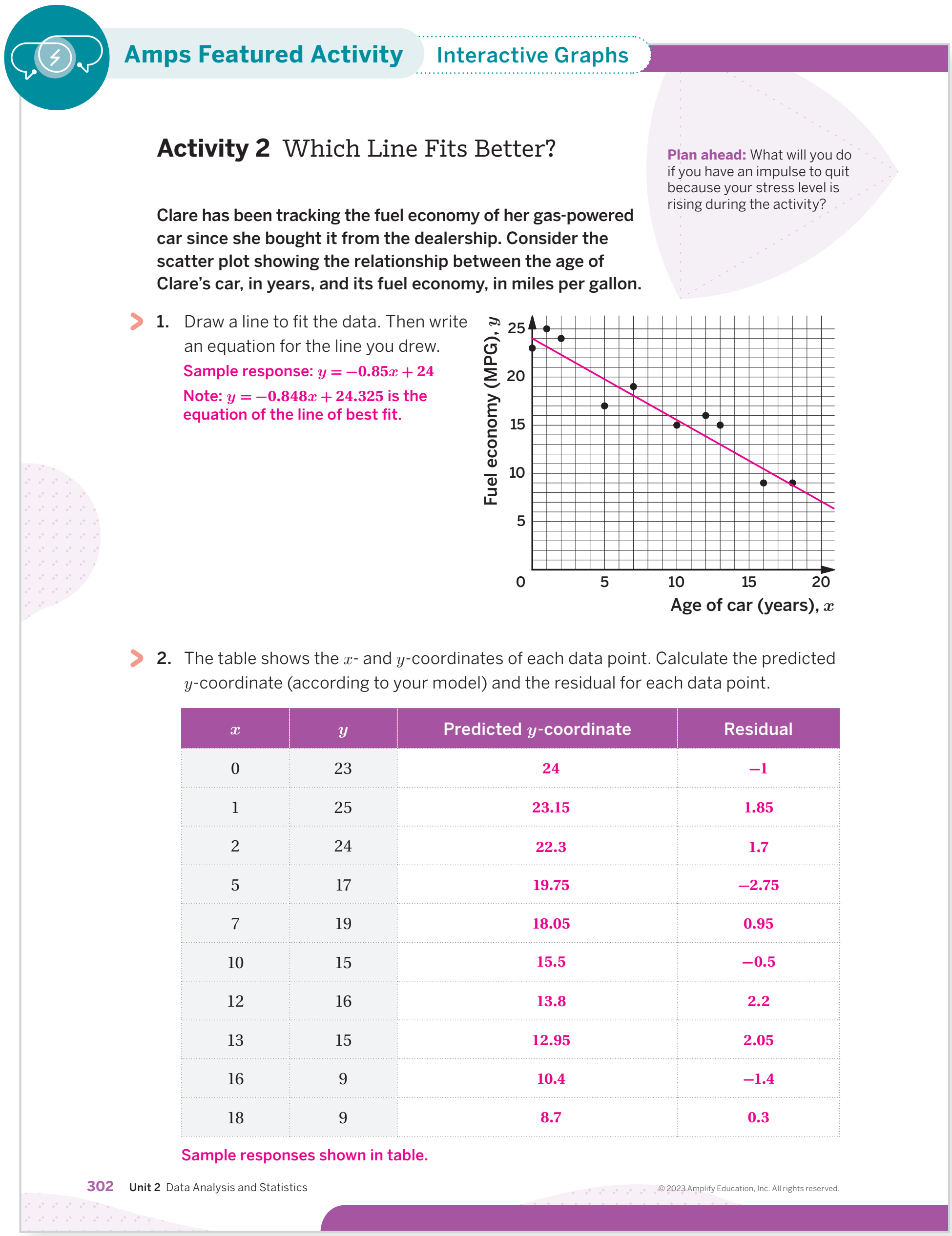
During the Connect, consider asking these follow-up questions as you point to each of the first four points on the scatter plot:

- "Does this point lie above or below the linear model on the scatter plot?"
- "Does its corresponding residual point lie above or below the horizontal axis?"
- "Look at one of the points that appears to lie on the line of the linear model of the scatter plot. Where is its corresponding residual located in comparison to the horizontal axis on the residual plot?"

Pairs | 15 min

Activity 2 Which Line Fits Better?

Students draw linear models to fit a data set and judge its goodness of fit by comparing its residuals to those of a different linear model.



Amps Featured Activity Interactive Graphs

Activity 2 Which Line Fits Better?

Plan ahead: What will you do if you have an impulse to quit because your stress level is rising during the activity?

Clare has been tracking the fuel economy of her gas-powered car since she bought it from the dealership. Consider the scatter plot showing the relationship between the age of Clare's car, in years, and its fuel economy, in miles per gallon.

1. Draw a line to fit the data. Then write an equation for the line you drew.
   Sample response: $y = -0.85x + 24$
   Note: $y = -0.848x + 24.325$ is the equation of the line of best fit.

2. The table shows the x- and y-coordinates of each data point. Calculate the predicted y-coordinate (according to your model) and the residual for each data point.

x	y	Predicted y-coordinate	Residual
0	23	24	−1
1	25	23.15	1.85
2	24	22.3	1.7
5	17	19.75	−2.75
7	19	18.05	0.95
10	15	15.5	−0.5
12	16	13.8	2.2
13	15	12.95	2.05
16	9	10.4	−1.4
18	9	8.7	0.3

Sample responses shown in table.

302 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display the scatter plot and ask students to interpret the slope of the linear model drawn to fit this data. (It shows the rate at which her car's fuel economy decreases each year.) Allow individual work time, then place students in pairs to compare their linear models and residuals. Provide access to scientific calculators.

2 Monitor

Help students get started by prompting students to round coordinate values on the scatter plot to the nearest whole number.

Look for points of confusion:

- **Having difficulty determining the slope of the line drawn to fit the data.** Encourage students to determine the coordinates of two points on their line and apply the slope formula.
- **Struggling to determine whose linear model is a better fit for the data.** Prompt students to compare the values of the residuals.

Look for productive strategies:

- Positioning the ruler so that it matches the negative trend of the data.
- Drawing a line that goes through the data points so some are above and some are below the line.
- Using the values of the residuals in the tables to judge goodness of fit.
- Comparing the residuals in both tables.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide a scatter plot with a sample line of fit already drawn, along with its equation. Have students begin with Problem 2 by calculating the residuals for the first 3–4 rows. Provide them with the rest of the table for Problem 2. This will still allow them to access the mathematical goal of the activity, which is to compare residuals of two different linear models.

Accessibility: *Guide Processing and Visualization*

Consider providing an alternate table, similar to the one shown in the previous activity under Accessibility, to help students organize their thinking as they determine the residuals.

Math Language Development

MLR8: Discussion Supports

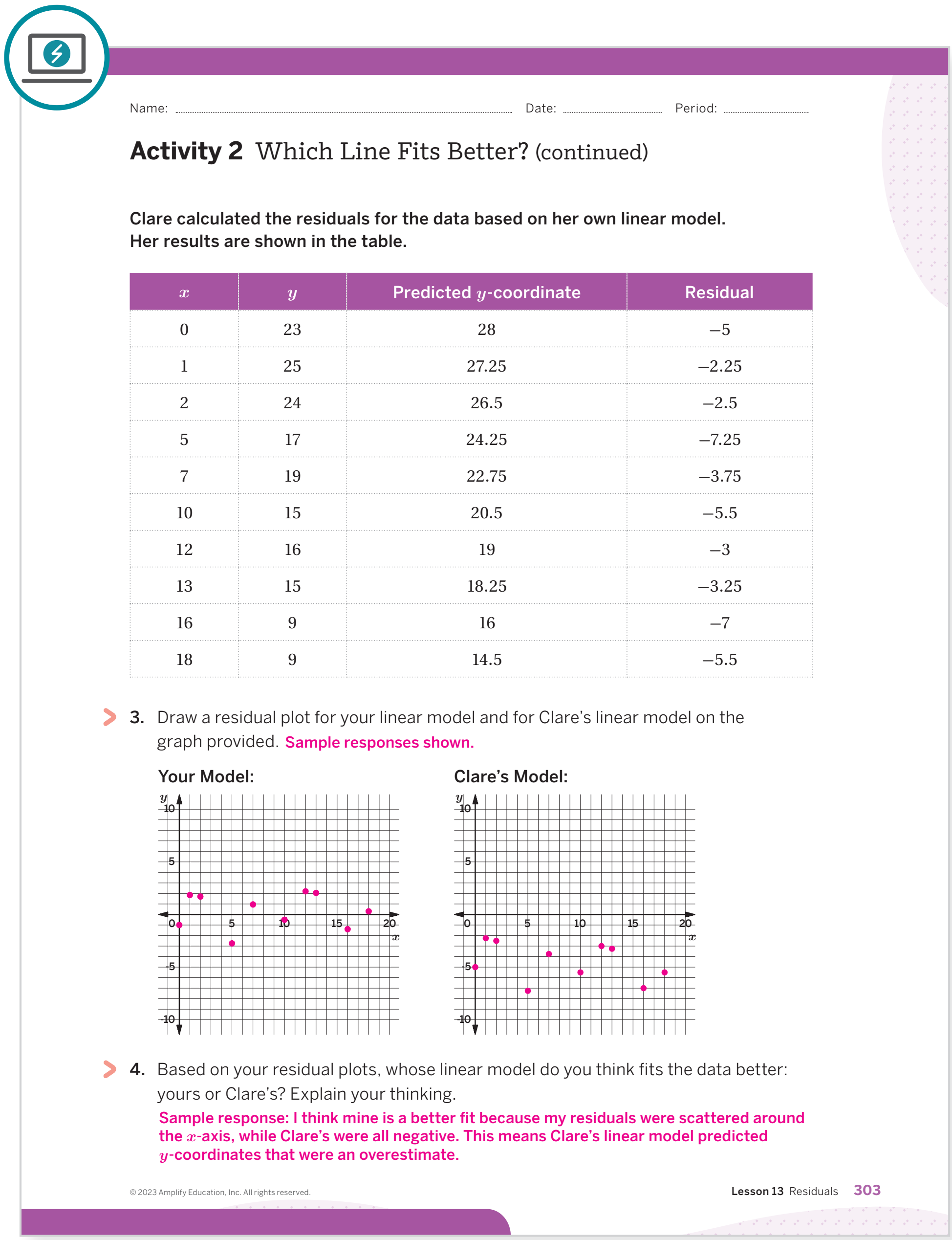
As students share their linear models and residuals with their partner, display or provide access to the Anchor Chart PDF, Sentence Stems, *Partner and Group Questioning* to support them in asking questions of each other.

English Learners

During the Connect discussion, as you display different residual plots, annotate the plots in which the points are located closer to the horizontal axis as plots that indicate linear models that are good fits for the data.

Activity 2 Which Line Fits Better? (continued)

Students draw linear models to fit a data set and judge its goodness of fit by comparing its residuals to those of a different linear model.



Name: ______________________ Date: __________ Period: __________

Activity 2 Which Line Fits Better? (continued)

Clare calculated the residuals for the data based on her own linear model. Her results are shown in the table.

x	y	Predicted y-coordinate	Residual
0	23	28	−5
1	25	27.25	−2.25
2	24	26.5	−2.5
5	17	24.25	−7.25
7	19	22.75	−3.75
10	15	20.5	−5.5
12	16	19	−3
13	15	18.25	−3.25
16	9	16	−7
18	9	14.5	−5.5

3. Draw a residual plot for your linear model and for Clare's linear model on the graph provided. Sample responses shown.

Your Model:

Clare's Model:

4. Based on your residual plots, whose linear model do you think fits the data better: yours or Clare's? Explain your thinking.
Sample response: I think mine is a better fit because my residuals were scattered around the x-axis, while Clare's were all negative. This means Clare's linear model predicted y-coordinates that were an overestimate.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 13 Residuals 303

3 Connect

Have pairs of students share whether their linear model was a good fit for the data and if it was better than Clare's or their partner's linear model. Have students explain how they judged which linear model was a better fit or any challenges they ran into while trying to decide.

Display the linear models and table of residual values of three students who claim to have a good fit.

Ask:

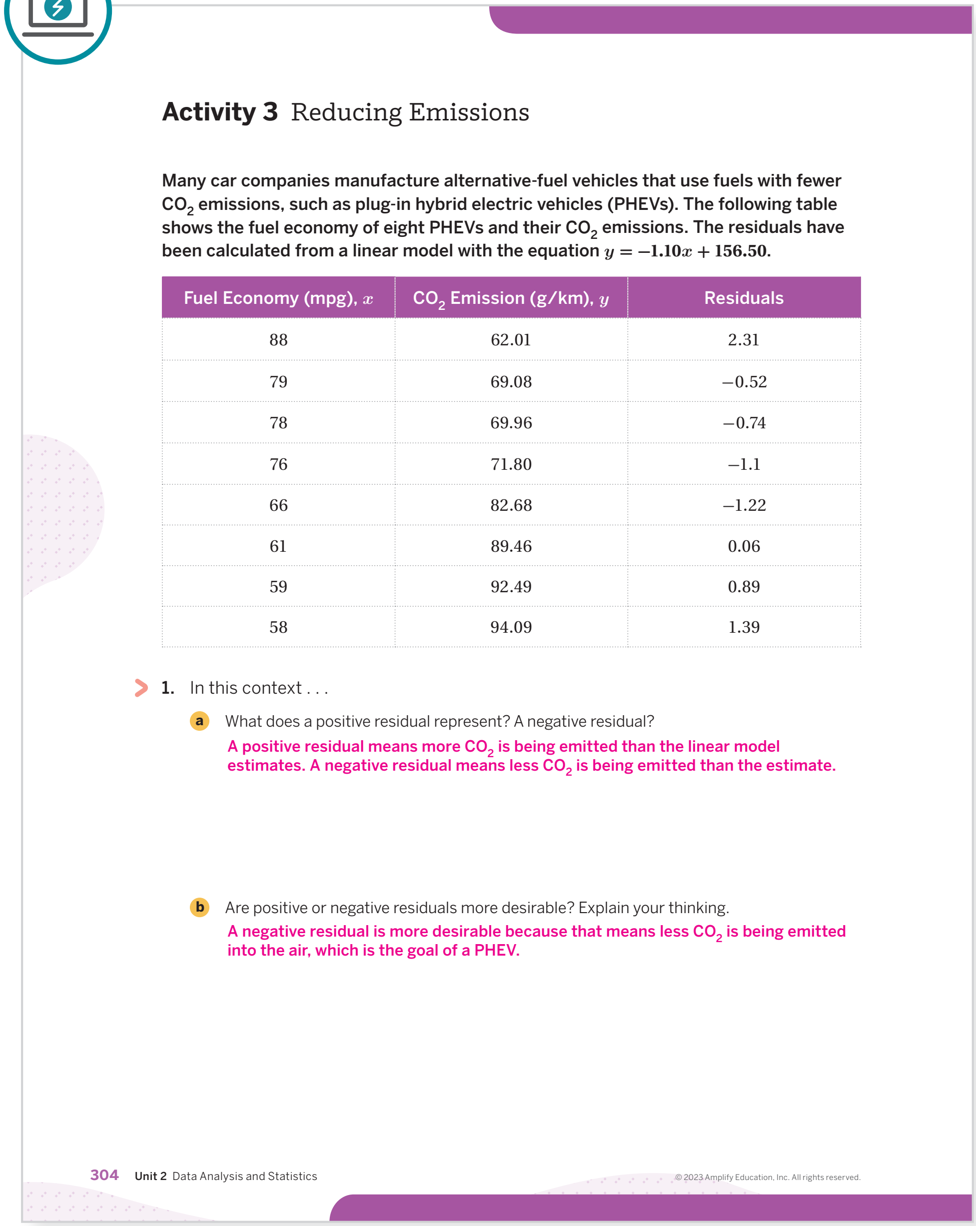
- "Can you tell from the residuals which linear model is a better fit ?"
- "How can you compare linear models if the residuals are similarly distributed about the horizontal axis?"

Highlight that students can compare residuals to judge which linear model fits a data set better. This can help them to eliminate linear models that fit the data poorly. However, if more than one linear model is a good fit for the data, comparing their residuals may not be sufficient to determine which fits best.

Independent | 10 min

Activity 3 Reducing Emissions

Students interpret residuals in a context and construct a residual plot, seeing that it has a pattern when data is nonlinear.



Activity 3 Reducing Emissions

Many car companies manufacture alternative-fuel vehicles that use fuels with fewer CO_2 emissions, such as plug-in hybrid electric vehicles (PHEVs). The following table shows the fuel economy of eight PHEVs and their CO_2 emissions. The residuals have been calculated from a linear model with the equation $y = -1.10x + 156.50$.

Fuel Economy (mpg), x	CO_2 Emission (g/km), y	Residuals
88	62.01	2.31
79	69.08	−0.52
78	69.96	−0.74
76	71.80	−1.1
66	82.68	−1.22
61	89.46	0.06
59	92.49	0.89
58	94.09	1.39

1. In this context . . .

   a. What does a positive residual represent? A negative residual?

   A positive residual means more CO_2 is being emitted than the linear model estimates. A negative residual means less CO_2 is being emitted than the estimate.

   b. Are positive or negative residuals more desirable? Explain your thinking.

   A negative residual is more desirable because that means less CO_2 is being emitted into the air, which is the goal of a PHEV.

304 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Ask, "How might cars contribute to the 'greenhouse effect' discussed in the previous lesson?" Elicit responses about how gas from the exhaust of a car contributes to air pollution. One way car companies are striving to help counteract this is by manufacturing alternative-fuel vehicles. Display the table and read the prompt together as a whole class.

2 Monitor

Help students get started by reminding them that the residuals are the dependent variable.

Look for points of confusion:

- **Having difficulty determining how the residuals were calculated.** Prompt students to verify the residuals using the equation of the linear model to determine the predicted values and comparing them to the values of the CO_2 emissions.

Look for productive strategies:

- Judging goodness of fit by looking at the residuals.
- Noticing that the residual plot has a U-shaped pattern.

Activity 3 continued ›

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they are familiar with PHEVs (plug-in hybrid electric vehicles) or if they have seen designated parking places for these types of vehicles to recharge. Mention that these types of cars typically have greater fuel economy than traditional vehicles.

Accessibility: *Guide Processing and Visualization*

Display or provide access to the scatter plot and its corresponding residual plot from Activity 1 for students to use as a comparison reference.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students write their responses to Problem 4, have them share their responses with another pair of students to give and receive feedback.

Encourage listeners to consider these questions:

- "Does the response indicate whether a linear model is a good fit?"
- "Does the response provide reasoning or justification for why a linear model would be a good fit or not?"

Have students use the feedback to revise their responses.

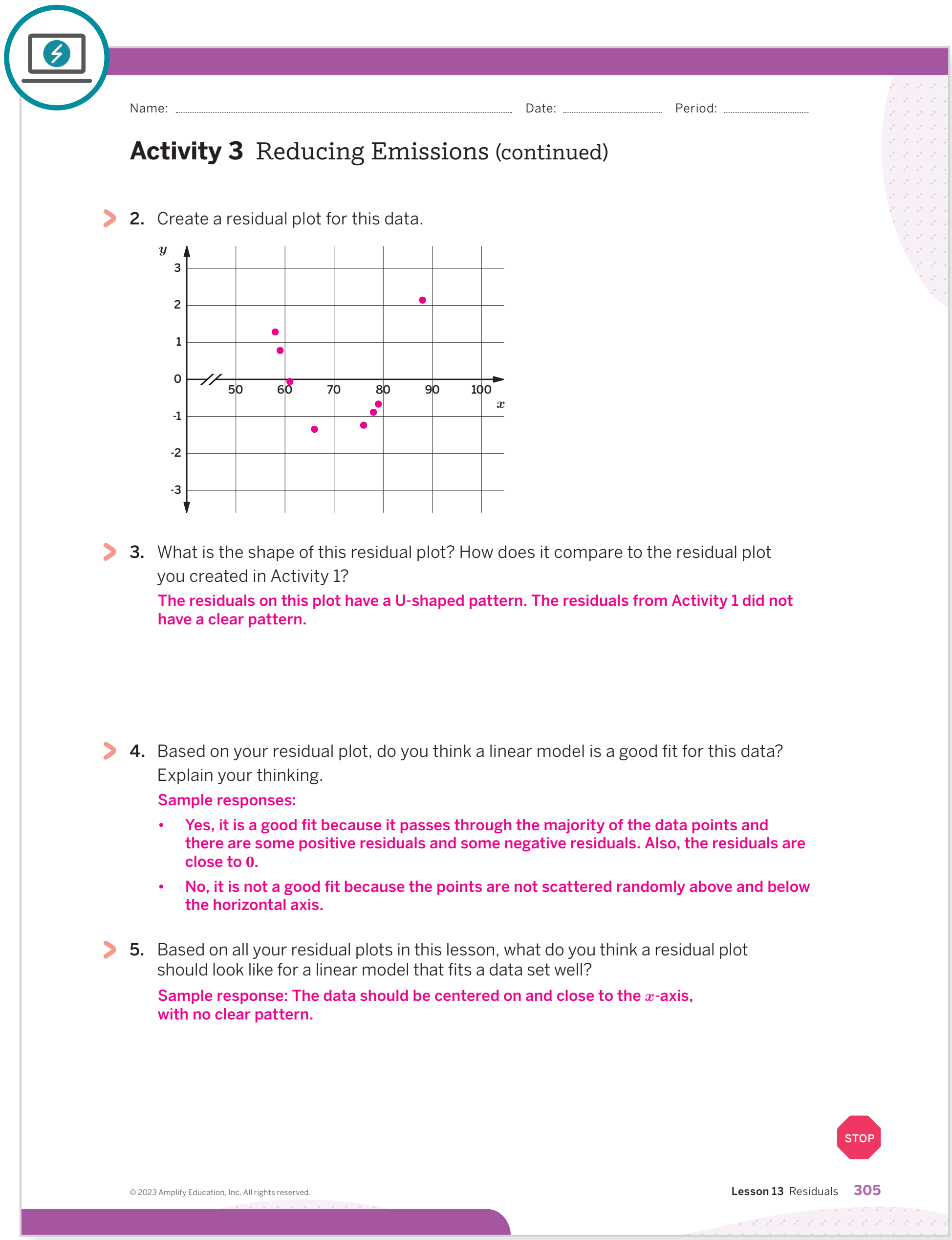
English Learners

Provide access to the Anchor Chart PDF, Sentence Stems, *Stronger and Clearer Each Time*.

Pairs | 15 min

Activity 3 Reducing Emissions (continued)

Students interpret residuals in a context and construct a residual plot, seeing that it has a pattern when data is nonlinear.



Name: Date: Period:

Activity 3 Reducing Emissions (continued)

2. Create a residual plot for this data.

3. What is the shape of this residual plot? How does it compare to the residual plot you created in Activity 1?

The residuals on this plot have a U-shaped pattern. The residuals from Activity 1 did not have a clear pattern.

4. Based on your residual plot, do you think a linear model is a good fit for this data? Explain your thinking.

Sample responses:

- **Yes, it is a good fit because it passes through the majority of the data points and there are some positive residuals and some negative residuals. Also, the residuals are close to 0.**
- **No, it is not a good fit because the points are not scattered randomly above and below the horizontal axis.**

5. Based on all your residual plots in this lesson, what do you think a residual plot should look like for a linear model that fits a data set well?

Sample response: The data should be centered on and close to the x-axis, with no clear pattern.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the residual plot of the data.

Have students share how they interpreted positive and negative residuals in this context and why negative residuals are more desirable. Then select students to describe the shape of the residual and how it compares to the ones we saw in Activity 1.

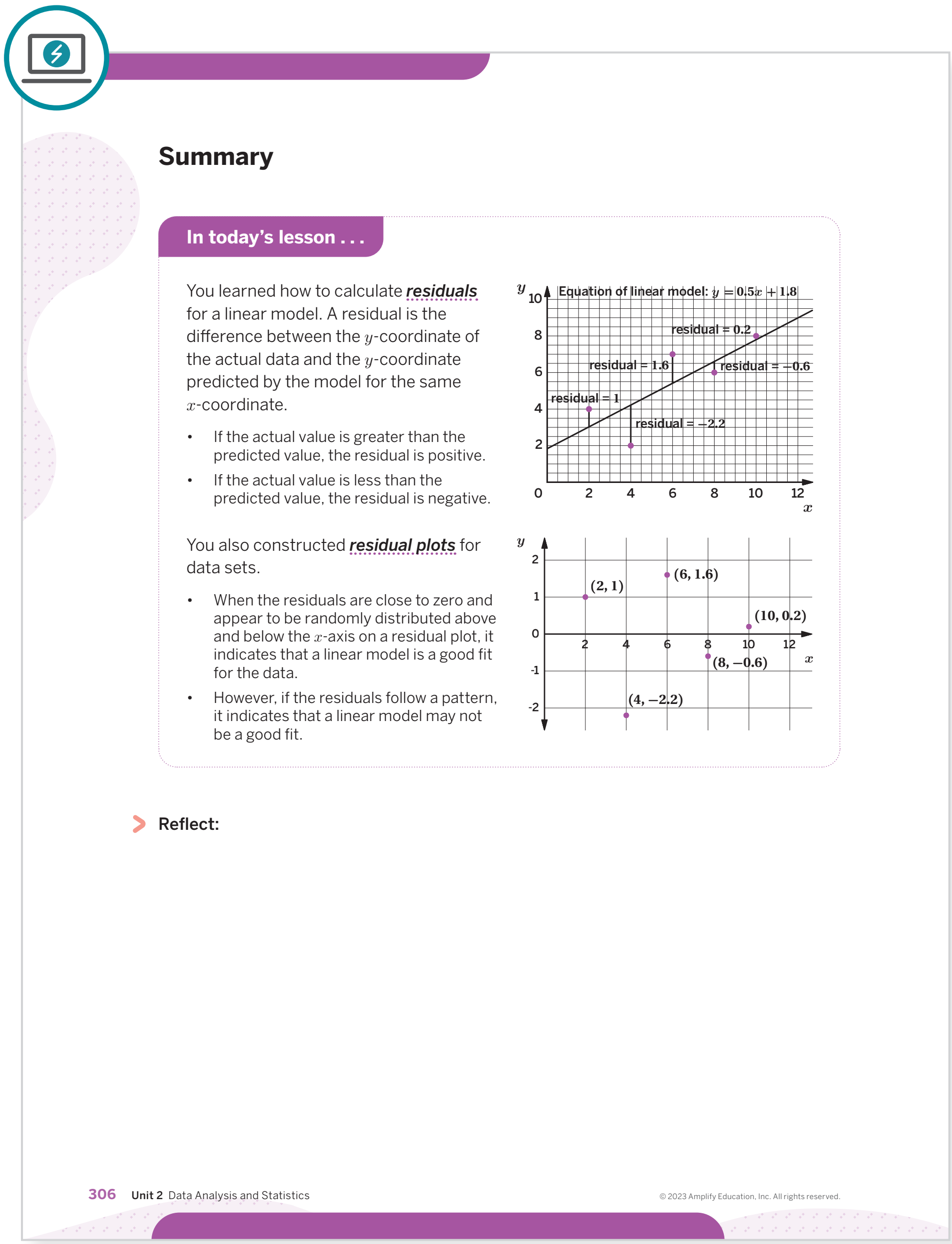
Ask, “Does the residual plot indicate that a linear model would be a good fit?”

Highlight that when a residual plot has a clear pattern (such as the U-shaped pattern displayed), that means that a linear model is not a good fit for the data; it would be better fit by a nonlinear model.

Whole Class | 5 min

Summary

Review and synthesize how to calculate residuals and use them to judge how well a linear model fits a data set.



Summary

In today's lesson . . .

You learned how to calculate ***residuals*** for a linear model. A residual is the difference between the y-coordinate of the actual data and the y-coordinate predicted by the model for the same x-coordinate.

- If the actual value is greater than the predicted value, the residual is positive.
- If the actual value is less than the predicted value, the residual is negative.

You also constructed ***residual plots*** for data sets.

- When the residuals are close to zero and appear to be randomly distributed above and below the x-axis on a residual plot, it indicates that a linear model is a good fit for the data.
- However, if the residuals follow a pattern, it indicates that a linear model may not be a good fit.

Reflect:

306 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the Activity 1 PDF and the residual plot of the linear model that corresponds to the Activity 1 PDF.

Have students share how they can use the residual plot to determine if the linear model is a good fit.

Ask, "How can you use the values of residuals to judge whether a linear model is a good fit for a data set?"

Highlight the connections between a linear model and its corresponding residual plot. A good linear model for the data will have residuals that are close to the horizontal axis and scattered on either side of the axis without a clear pattern.

Formalize vocabulary:

- *residual*
- *residual plot*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do residuals help to determine if a linear model is a good fit for a set of data?"

Math Language Development

MLR2: Collect and Display

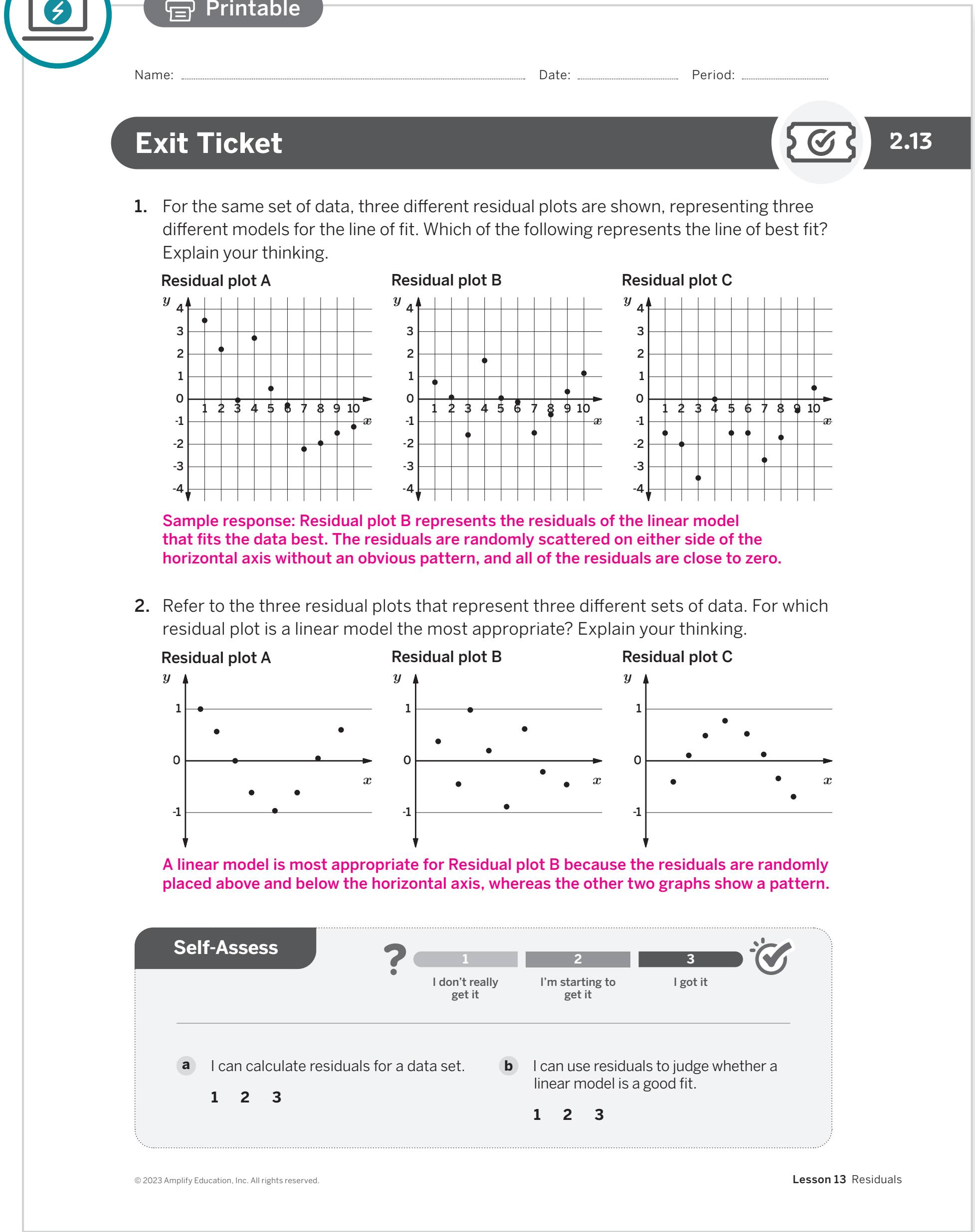
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *residual* and *residual plot* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using residual plots to judge whether a linear model is a good fit for a set of data.

Printable



Name: Date: Period:

Exit Ticket 2.13

1. For the same set of data, three different residual plots are shown, representing three different models for the line of fit. Which of the following represents the line of best fit? Explain your thinking.

Residual plot A Residual plot B Residual plot C

Sample response: Residual plot B represents the residuals of the linear model that fits the data best. The residuals are randomly scattered on either side of the horizontal axis without an obvious pattern, and all of the residuals are close to zero.

2. Refer to the three residual plots that represent three different sets of data. For which residual plot is a linear model the most appropriate? Explain your thinking.

Residual plot A Residual plot B Residual plot C

A linear model is most appropriate for Residual plot B because the residuals are randomly placed above and below the horizontal axis, whereas the other two graphs show a pattern.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can calculate residuals for a data set.
1 2 3

b I can use residuals to judge whether a linear model is a good fit.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved. Lesson 13 Residuals

Success looks like . . .

- **Goal:** Calculating and plotting the residuals for a given data set.
- **Language Goal:** Using residuals to determine the goodness of fit for a linear model. **(Speaking and Listening)**
  - » Determining which residual plot represents the line of best fit in Problem 1.

Suggested next steps

If students cannot determine which residual plot represents the best-fitting linear model, consider:

- Comparing the residual plots to those that correspond to the scatter plots in Activity 1.
- Asking, "Which residual plot matches some of the criteria for a good linear model?"

If students cannot determine which residual plot is most appropriate for a linear model, consider:

- Reviewing Activity 3.

Professional Learning

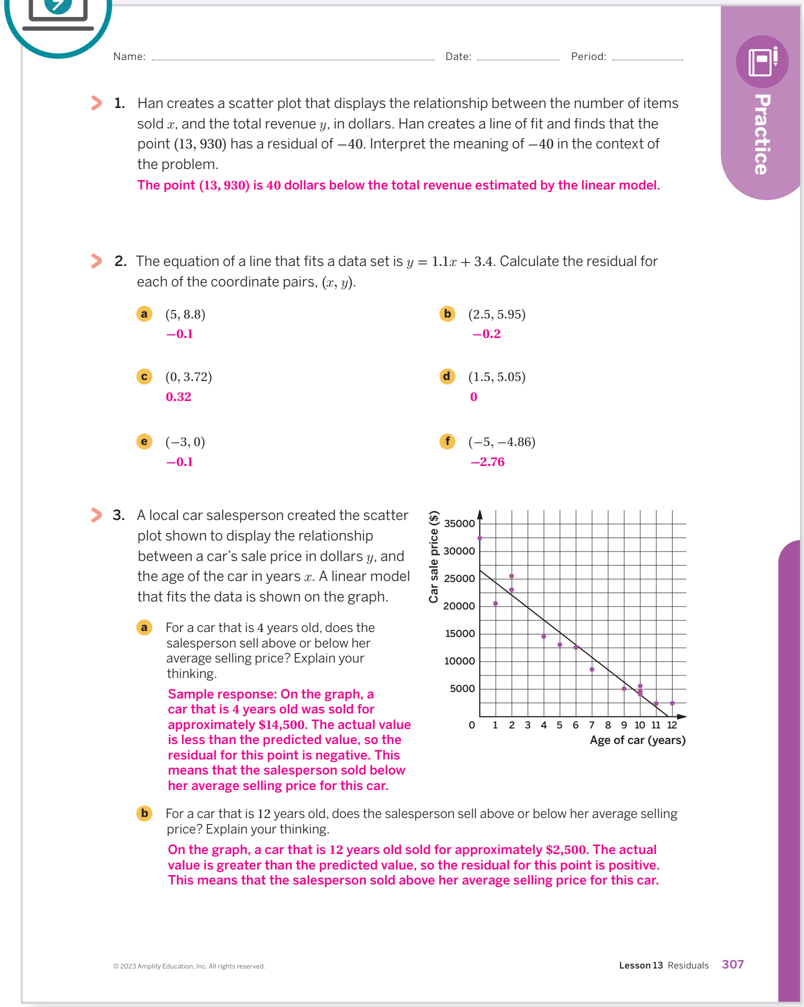
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- During the discussion about judging if a linear model is a good fit, how did you encourage each student to listen to one another's strategies?
- Which students' ideas were you able to highlight during Activity 3 about what the appearance is of a residual plot representing a good linear model?

Practice

Independent



Name: Date: Period:

1. Han creates a scatter plot that displays the relationship between the number of items sold x, and the total revenue y, in dollars. Han creates a line of fit and finds that the point $(13, 930)$ has a residual of -40. Interpret the meaning of -40 in the context of the problem.

   The point $(13, 930)$ is 40 dollars below the total revenue estimated by the linear model.

2. The equation of a line that fits a data set is $y = 1.1x + 3.4$. Calculate the residual for each of the coordinate pairs, (x, y).

   a. $(5, 8.8)$ **-0.1**

   b. $(2.5, 5.95)$ **-0.2**

   c. $(0, 3.72)$ **0.32**

   d. $(1.5, 5.05)$ **0**

   e. $(-3, 0)$ **-0.1**

   f. $(-5, -4.86)$ **-2.76**

3. A local car salesperson created the scatter plot shown to display the relationship between a car's sale price in dollars y, and the age of the car in years x. A linear model that fits the data is shown on the graph.

   a. For a car that is 4 years old, does the salesperson sell above or below her average selling price? Explain your thinking.

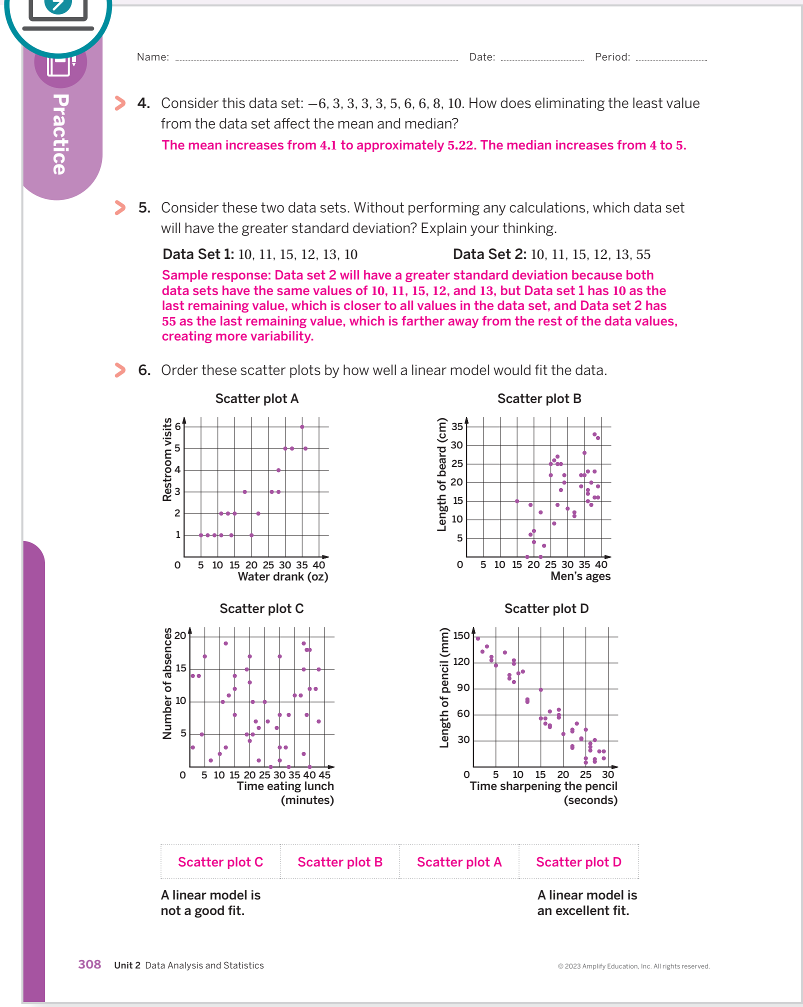
      Sample response: On the graph, a car that is 4 years old was sold for approximately $\$14,500$. The actual value is less than the predicted value, so the residual for this point is negative. This means that the salesperson sold below her average selling price for this car.

   b. For a car that is 12 years old, does the salesperson sell above or below her average selling price? Explain your thinking.

      On the graph, a car that is 12 years old sold for approximately $\$2,500$. The actual value is greater than the predicted value, so the residual for this point is positive. This means that the salesperson sold above her average selling price for this car.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 13 Residuals 307



Name: Date: Period:

4. Consider this data set: $-6, 3, 3, 3, 3, 5, 6, 6, 8, 10$. How does eliminating the least value from the data set affect the mean and median?

   The mean increases from 4.1 to approximately 5.22. The median increases from 4 to 5.

5. Consider these two data sets. Without performing any calculations, which data set will have the greater standard deviation? Explain your thinking.

   Data Set 1: $10, 11, 15, 12, 13, 10$ **Data Set 2:** $10, 11, 15, 12, 13, 55$

   Sample response: Data set 2 will have a greater standard deviation because both data sets have the same values of $10, 11, 15, 12$, and 13, but Data set 1 has 10 as the last remaining value, which is closer to all values in the data set, and Data set 2 has 55 as the last remaining value, which is farther away from the rest of the data values, creating more variability.

6. Order these scatter plots by how well a linear model would fit the data.

Scatter plot C	**Scatter plot B**	**Scatter plot A**	**Scatter plot D**

A linear model is not a good fit. **A linear model is an excellent fit.**

308 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	2
	2	Activity 1	1
	3	Activity 1	2
Spiral	4	Unit 2 Lesson 8	2
	5	Unit 2 Lesson 7	2
Formative ⚡	6	Unit 2 Lesson 14	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 2 | **LESSON 14**

Line of Best Fit

Let's figure out which linear model is the *best* linear model for a data set.

Focus

Goal

1. Evaluate how good a line of fit is using the sum of the squares of the residuals.

Coherence

Today

Students compare residual plots to determine when a line of fit models a data set well. Students evaluate which linear model is best by evaluating the sum of the squares of the residuals.

Previously

In Lesson 13, students calculated and plotted residuals for a given data set.

Coming Soon

In Lessons 19 and 20, students will calculate and interpret the correlation coefficient.

Rigor

- Students build **conceptual understanding** in how a line of best fit can be determined with mathematical precision.
- Students build **procedural fluency** in calculating and interpreting the sum of the squares of residuals.
- Students **apply** using the sum of the squares of residuals to evaluate how good a line of fit is.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	20 min	10 min	5 min	5 min
Independent	Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (for display)
- Anchor Chart PDF, *Sentence Stems, Which One Doesn't Belong?*
- graphing technology

Math Language Development

New words

- ***line of best fit***

Review words

- *residual*
- *scatter plot*

Amps | Featured Activity

Activity 3
Dynamic Squares and Residuals

Students are able to see how the sum of the squares of the residuals changes in real time as the line of fit is adjusted.

Building Math Identity and Community

Connecting to Mathematical Practices

When working in small groups, sometimes students might not not seem to "get" each other. Spend some time with a group that struggles trying to figure out what the real issue is. Encourage them to celebrate their differences, by not only showing respect, but also trying to understand where that person's background or culture comes into play. Have students identify ways that they can show appreciation for the diverse responses in their group.

Modifications to Pacing

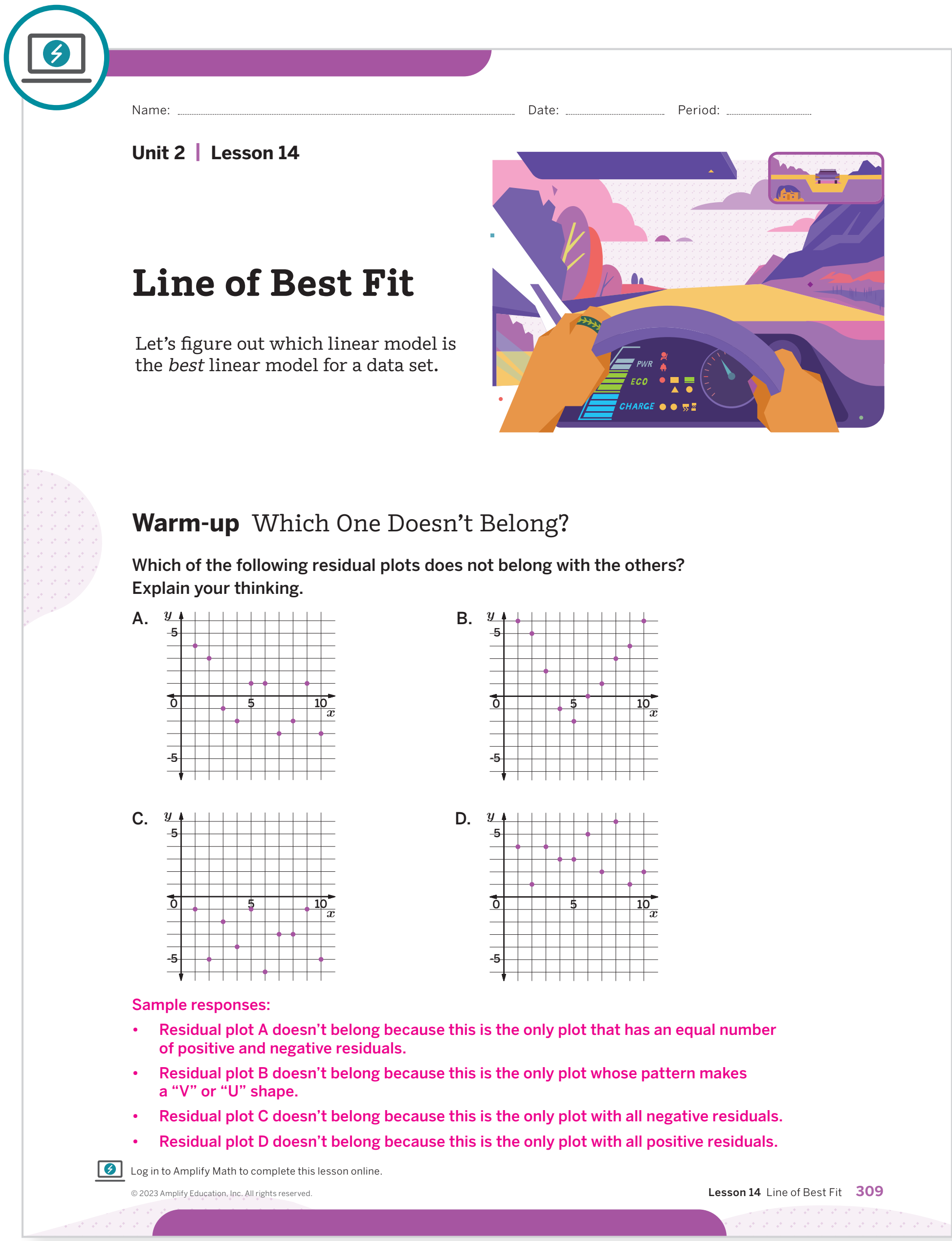
You may want to consider this additional modification if you are short on time.

- Optional **Activity 3** may be omitted.

Independent | 5 min

Warm-up Which One Doesn't Belong?

Students analyze residual plots to prepare for determining when a residual plot models a good line of fit.



Name: ______________ Date: ________ Period: ________

Unit 2 | Lesson 14

Line of Best Fit

Let's figure out which linear model is the *best* linear model for a data set.

Warm-up Which One Doesn't Belong?

Which of the following residual plots does not belong with the others? Explain your thinking.

Sample responses:

- Residual plot A doesn't belong because this is the only plot that has an equal number of positive and negative residuals.
- Residual plot B doesn't belong because this is the only plot whose pattern makes a "V" or "U" shape.
- Residual plot C doesn't belong because this is the only plot with all negative residuals.
- Residual plot D doesn't belong because this is the only plot with all positive residuals.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 14 Line of Best Fit 309

1 Launch

Conduct the ***Which One Doesn't Belong***? routine. Provide one minute of think time before students share with a partner. Then have students share their thinking during the whole class discussion.

2 Monitor

Help students get started by having them compare two residual plots at a time.

Look for points of confusion:

- **Misunderstanding what the given plots represent.** Ask students to re-read instructions. Ask, "What is the difference between a scatter plot and residual plot?"
- **Misreading the axes labels.** Ask students to identify the x and y axes on the graph.

Look for productive strategies:

- Looking for linear or nonlinear patterns in the residual plot.
- Describing the residual value relative to the line of fit.

3 Connect

Display the four residual plots.

Have individual students share their thinking about the residual plots with the class. Select and sequence responses that include positive/negative residuals, patterns in the residual plot, and location of residuals relative to the line of fit. Record student responses for display.

Highlight that the different graphs represent four possible lines of fit for the same set of data.

Ask, "Do all of these residual plots represent good lines of fit?"

Math Language Development

MLR2: Collect and Display

As students work, circulate and listen to the language students use to describe positive/negative residuals, patterns in the residual plots, and the location of the residuals relative to the horizontal axis. Add these terms, phrases, and diagrams to the class display. Continue adding to this display during Activity 1.

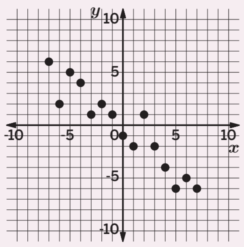
English Learners

Display or provide access to the Anchor Chart PDF, *Sentence Stems, Which One Doesn't Belong?* to support students as they discuss reasons why each of the given graphs might not not belong with the others.

Power-up

To power up students' ability to determine whether a linear model would fit data, have students complete:

For this scatter plot, does a linear model fit the data well? Be prepared to explain your thinking.



Yes; Sample response: The data points would be close to a line of fit, with some points above the line and some points below the line.

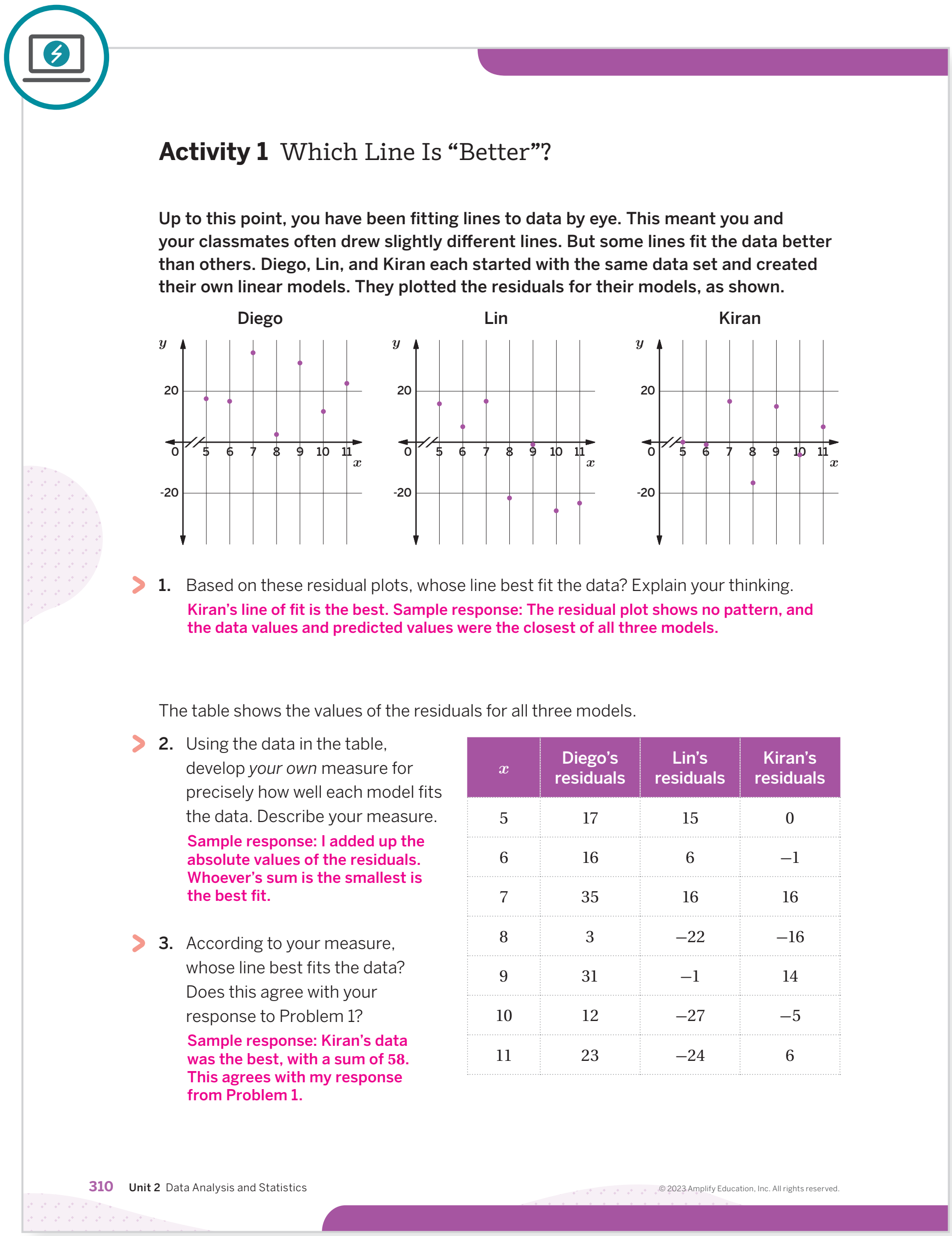
Use: Before the Warm-up

Informed by: Performance on Lesson 13, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 6

Small Groups | 15 min

Activity 1 Which Line is "Better"?

Students analyze residual plots to determine when a residual plot shows a line of fit models data well.



Activity 1 Which Line Is "Better"?

Up to this point, you have been fitting lines to data by eye. This meant you and your classmates often drew slightly different lines. But some lines fit the data better than others. Diego, Lin, and Kiran each started with the same data set and created their own linear models. They plotted the residuals for their models, as shown.

1. Based on these residual plots, whose line best fit the data? Explain your thinking.
   Kiran's line of fit is the best. Sample response: The residual plot shows no pattern, and the data values and predicted values were the closest of all three models.

The table shows the values of the residuals for all three models.

x	Diego's residuals	Lin's residuals	Kiran's residuals
5	17	15	0
6	16	6	−1
7	35	16	16
8	3	−22	−16
9	31	−1	14
10	12	−27	−5
11	23	−24	6

2. Using the data in the table, develop *your own* measure for precisely how well each model fits the data. Describe your measure.
   Sample response: I added up the absolute values of the residuals. Whoever's sum is the smallest is the best fit.

3. According to your measure, whose line best fits the data? Does this agree with your response to Problem 1?
   Sample response: Kiran's data was the best, with a sum of 58. This agrees with my response from Problem 1.

310 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in small groups. Provide time for students to complete Problem 1 in their groups. Pause to discuss Problem 1 together, and then have students complete the rest of the activity.

2 Monitor

Help students get started by asking, "What does absolute value mean?"

Look for points of confusion:

- **Determining the sum of the residuals and then taking the absolute value of the sum.** Remind students that order matters here. Ask, "What must first be found before determining the sum?"

Look for productive strategies:

- Describing residuals as being positive or negative to describe a good or bad line of fit.
- Describing the residual value relative to the line of fit.
- Sketching a scatter plot of the data and using the residual to estimate a line of fit.

3 Connect

Display the three data sets and residual plots.

Have groups of students share their responses to Problems 3 and 4. Select and sequence responses that mention what a possible line of fit would look like, where the residuals are relative to the line of fit, and drawing conclusions comparing the lines of fit to the sum of the absolute value of the residuals.

Highlight precise language used by students and that a line of fit that models data well should minimize the sum of the absolute value of the residuals.

Ask, "Why do you want the sum of the absolute value of the residuals to be minimized?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

After the class discussion about Problem 1 and before students begin Problem 2, guide students towards considering the absolute value by asking these questions:

- "Whose residual values were located entirely above the horizontal axis?" Diego's
- "Whose residual values were located both above and below the horizontal axis?" Lin's and Kiran's
- "How can you compare these distances when some are positive and some are negative?" Sample response: I can compare the absolute values.

Extension: *Math Enrichment*

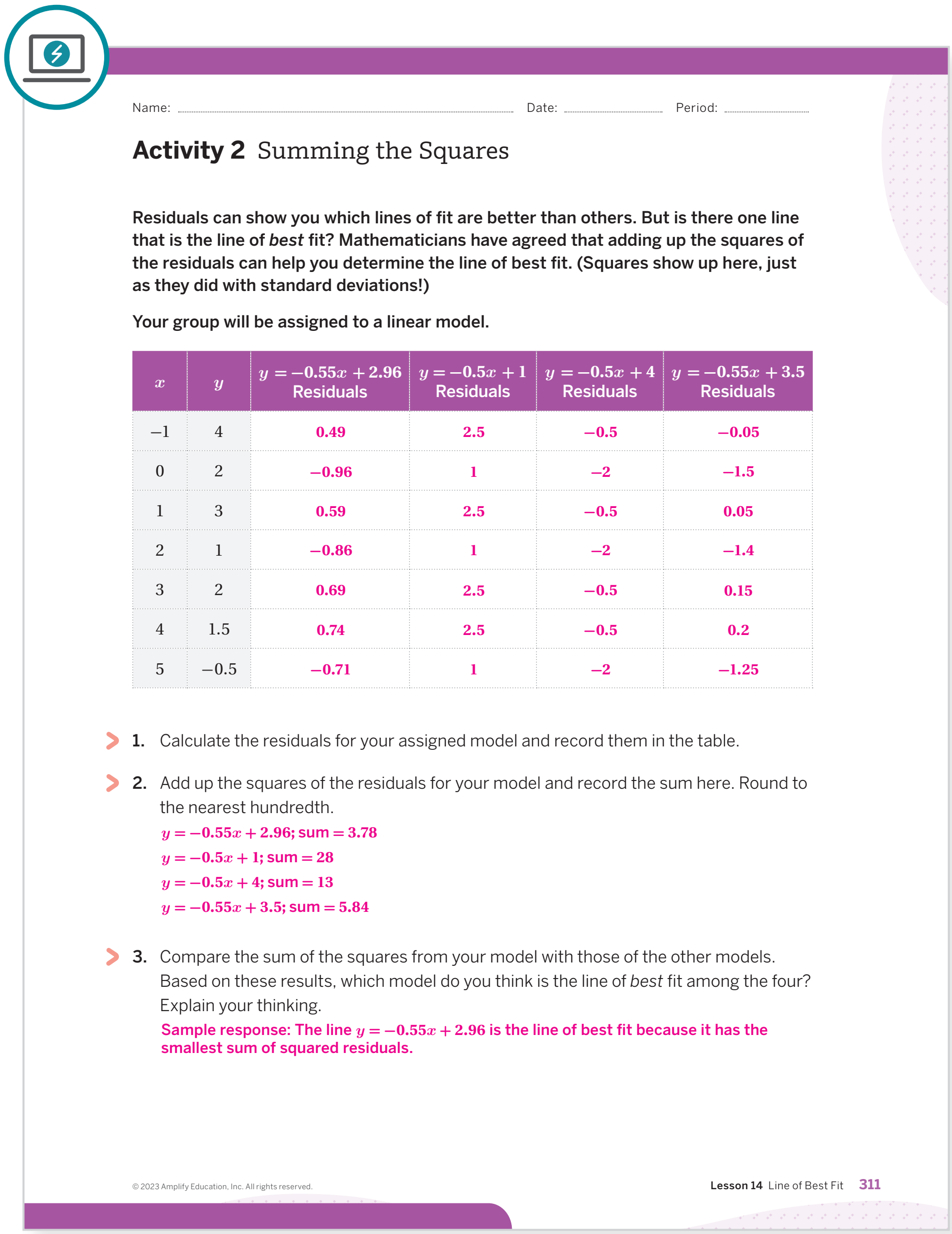
Have students complete the following problem:

A line of fit is drawn on a scatter plot where three data points lie above the line and three data points lie below the line. All of the data points have an equal distance to the line of fit. What is the sum of the absolute values of the residuals? Sample response: Let n and $-n$ represent the residual values. The sum of the absolute values of the residuals is $6n$.

Small Groups | 20 min

Activity 2 Summing the Squares

Students calculate and interpret the sum of the squares of the residuals to determine the line of best fit.



Name: ________ Date: ________ Period: ________

Activity 2 Summing the Squares

Residuals can show you which lines of fit are better than others. But is there one line that is the line of *best* fit? Mathematicians have agreed that adding up the squares of the residuals can help you determine the line of best fit. (Squares show up here, just as they did with standard deviations!)

Your group will be assigned to a linear model.

x	y	$y = -0.55x + 2.96$ Residuals	$y = -0.5x + 1$ Residuals	$y = -0.5x + 4$ Residuals	$y = -0.55x + 3.5$ Residuals
−1	4	0.49	2.5	−0.5	−0.05
0	2	−0.96	1	−2	−1.5
1	3	0.59	2.5	−0.5	0.05
2	1	−0.86	1	−2	−1.4
3	2	0.69	2.5	−0.5	0.15
4	1.5	0.74	2.5	−0.5	0.2
5	−0.5	−0.71	1	−2	−1.25

1. Calculate the residuals for your assigned model and record them in the table.

2. Add up the squares of the residuals for your model and record the sum here. Round to the nearest hundredth.

   $y = -0.55x + 2.96$; sum $= 3.78$
   $y = -0.5x + 1$; sum $= 28$
   $y = -0.5x + 4$; sum $= 13$
   $y = -0.55x + 3.5$; sum $= 5.84$

3. Compare the sum of the squares from your model with those of the other models. Based on these results, which model do you think is the line of *best* fit among the four? Explain your thinking.

   Sample response: The line $y = -0.55x + 2.96$ is the line of best fit because it has the smallest sum of squared residuals.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Assign one line of fit per group. After students determine their sum, pause for a discussion so students can record sums for the other lines of fit.

2 Monitor

Help students get started by asking, "How can you calculate the residual for a data point?"

Look for points of confusion:

- **Having difficulty determining the sum of the squares of the residuals.** Remind students that they first need to square each residual value before determining the sum.

Look for productive strategies:

- Squaring the residual values before determining the sum.
- Recognizing that the sum of the squared residuals gives similar information about the line of fit as the sum of the absolute value of the residuals.

3 Connect

Display each scatter plot on the Activity 2 PDF, one at a time, the completed table, and the sums.

Have groups of students share their responses to Problem 3.

Highlight that the line of best fit is the unique line of fit that minimizes the sum of the squared residuals, or the sum of the areas of the squares they found. This is the standard way the line of best fit is found.

Define the ***line of best fit*** as the linear model that minimizes the sum of the squares of the residuals.

Ask, "How does the comparison between the sum of the squared residuals and the sum of the absolute value of the residuals relate to standard deviation and MAD?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Consider providing a pre-populated table with the residuals already calculated. Have students begin the activity with Problem 2. This will still allow them to access the mathematical goal of the activity, which is to interpret the sum of the squares of the residuals.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses to Problem 3, draw connections between the numerical sum of the squares of the residuals for each scatter plot and the square shown on the scatter plot for each data. Ask these questions:

- "How does the size of the square relate to the sum of the squares of the residuals?"
- "Which equation was the least appropriate fit for the data? What was the sum of the squares of the residuals? Describe the size of the square shown on the scatter plot."

English Learners

Annotate scatter plots with the sum of the squares of the residuals. Consider placing the scatter plots in order from least appropriate line of fit to most appropriate line of fit.

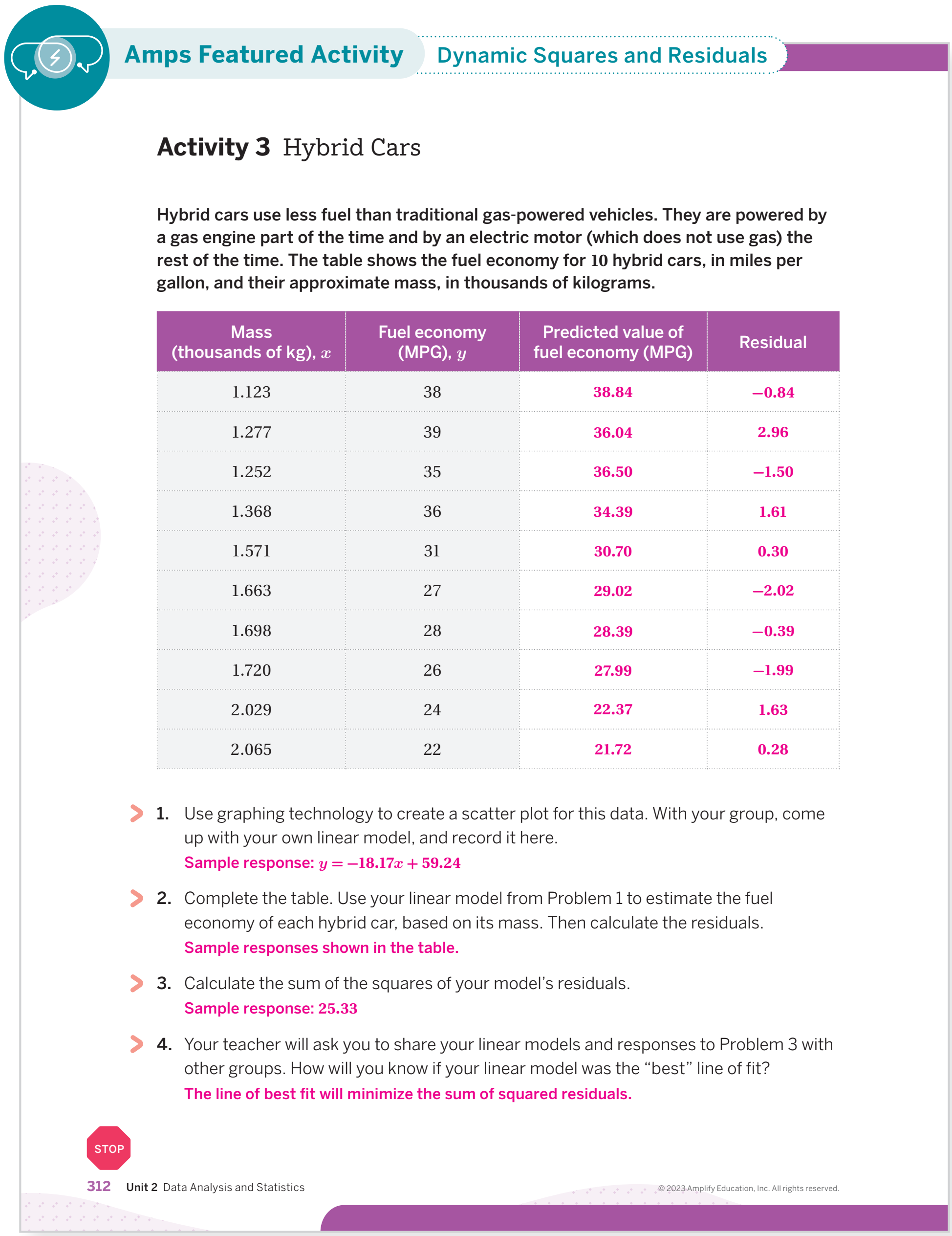
Optional

Small Groups | 10 min

Activity 3 Hybrid Cars

Students make their own linear models for a real-world scenario to practice evaluating models (and to see who did it best!).

Amps Featured Activity Dynamic Squares and Residuals



Activity 3 Hybrid Cars

Hybrid cars use less fuel than traditional gas-powered vehicles. They are powered by a gas engine part of the time and by an electric motor (which does not use gas) the rest of the time. The table shows the fuel economy for 10 hybrid cars, in miles per gallon, and their approximate mass, in thousands of kilograms.

Mass (thousands of kg), x	Fuel economy (MPG), y	Predicted value of fuel economy (MPG)	Residual
1.123	38	38.84	−0.84
1.277	39	36.04	2.96
1.252	35	36.50	−1.50
1.368	36	34.39	1.61
1.571	31	30.70	0.30
1.663	27	29.02	−2.02
1.698	28	28.39	−0.39
1.720	26	27.99	−1.99
2.029	24	22.37	1.63
2.065	22	21.72	0.28

1. Use graphing technology to create a scatter plot for this data. With your group, come up with your own linear model, and record it here.
   Sample response: $y = -18.17x + 59.24$
2. Complete the table. Use your linear model from Problem 1 to estimate the fuel economy of each hybrid car, based on its mass. Then calculate the residuals.
   Sample responses shown in the table.
3. Calculate the sum of the squares of your model's residuals.
   Sample response: 25.33
4. Your teacher will ask you to share your linear models and responses to Problem 3 with other groups. How will you know if your linear model was the "best" line of fit?
   The line of best fit will minimize the sum of squared residuals.

STOP

312 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have students complete Problems 1–3 with their small groups. Before completing Problem 4, have them share their linear models with at least 2 other groups.

2 Monitor

Help students get started by modeling how to input data and determine the line of best fit using graphing technology.

Look for points of confusion:

- **Making calculation errors when determining the residuals or being unsure how to determine them.** Have students look back to Lesson 13 on how to calculate residual values.

Look for productive strategies:

- Using the residual values to determine the best and worst estimates given by the line of best fit.
- Calculating the sum of squared residuals to compare the linear models.

3 Connect

Have groups of students share how they determined how their linear model compared with those from other groups. Select and sequence student responses that compare fuel economy values, use residuals, and calculate the sum of squared residuals.

Highlight that graphing technology and other types of technology will give the line of best fit as the default line of fit.

Ask, "How might a car company use residuals to determine whether a car should continue to be produced or not?" Sample response: A car could be discontinued if the residuals are negative, and does not meet expectations on fuel economy.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Students will use graphing technology in this optional activity. Consider providing a sheet of step-by-step directions that shows how to create a scatter plot of the data, with visual examples.

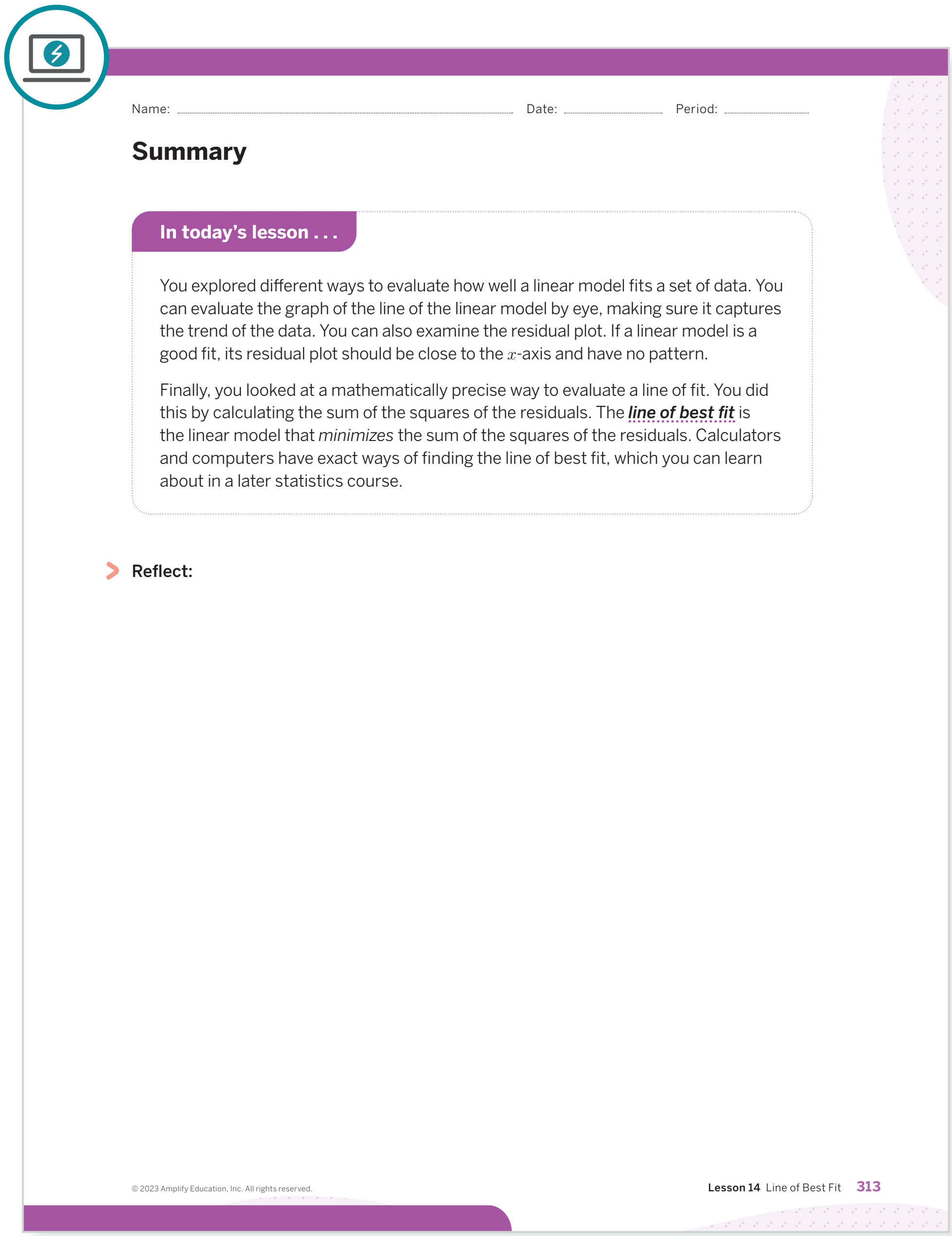
Math Language Development

MLR7: Compare and Connect

During the Connect, as students share how they compared their linear model with those from other groups, draw connections to how the sum of the squares of the residuals compare for each linear model. Consider asking, "Compare the sum of the squares of the residuals for the linear models you thought were better fits than others. What do you notice?"

Summary

Review and synthesize how to determine the line of best fit using the least squares method.



Name: ______________ Date: ________ Period: ________

Summary

In today's lesson . . .

You explored different ways to evaluate how well a linear model fits a set of data. You can evaluate the graph of the line of the linear model by eye, making sure it captures the trend of the data. You can also examine the residual plot. If a linear model is a good fit, its residual plot should be close to the x-axis and have no pattern.

Finally, you looked at a mathematically precise way to evaluate a line of fit. You did this by calculating the sum of the squares of the residuals. The ***line of best fit*** is the linear model that *minimizes* the sum of the squares of the residuals. Calculators and computers have exact ways of finding the line of best fit, which you can learn about in a later statistics course.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

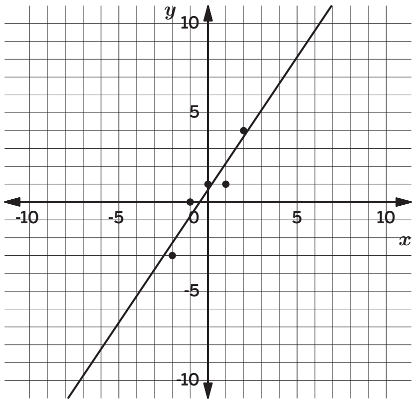
Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *line of best fit* that were added to the display during the lesson.

Synthesize

Display the following scatter plot and line of fit.



Have students share their thinking on how well the line of fit models the data and how they would calculate the sum of the squared residuals.

Highlight that the line of best fit is the unique line found through the least squares method, where the sum of the squares of the residuals is minimized.

Formalize vocabulary: ***line of best fit***

Ask, "Why might it be important to find the line of best fit for a set of data?" Sample response: The line of best fit gives the equation of the line that most closely models the set of data. This can be helpful when working with real-world data sets and wanting to make the more accurate estimates possible.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

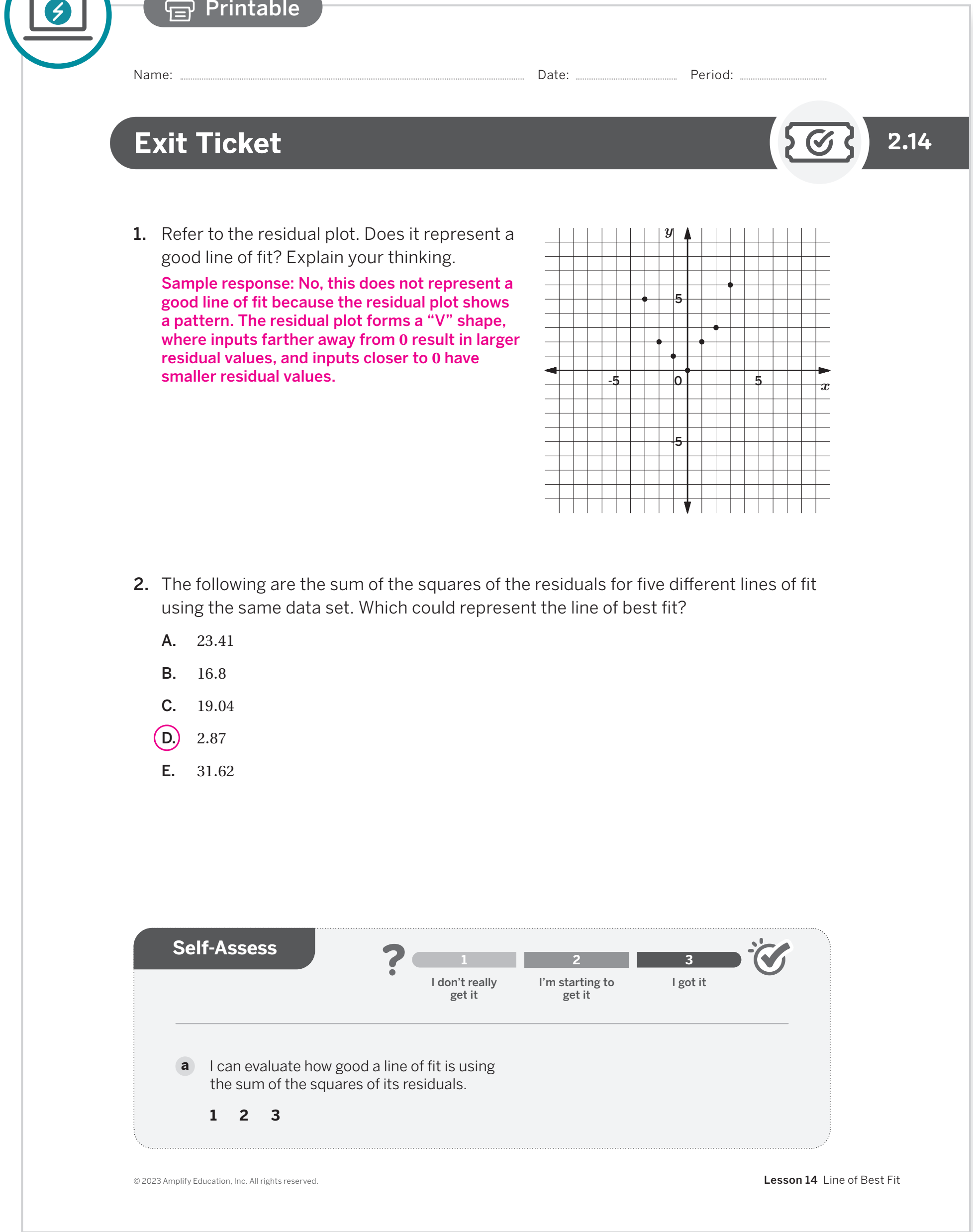
- "What strategies or tools did you find helpful today when finding the line of best fit? How were they helpful?"
- "Were any strategies or tools not helpful? Why?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of finding the line of best fit by analyzing residual plots and sum of the squared residuals.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 2.14

1. Refer to the residual plot. Does it represent a good line of fit? Explain your thinking.

Sample response: No, this does not represent a good line of fit because the residual plot shows a pattern. The residual plot forms a "V" shape, where inputs farther away from 0 result in larger residual values, and inputs closer to 0 have smaller residual values.

2. The following are the sum of the squares of the residuals for five different lines of fit using the same data set. Which could represent the line of best fit?

A. 23.41
B. 16.8
C. 19.04
[✓ D.] 2.87
E. 31.62

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I can evaluate how good a line of fit is using the sum of the squares of its residuals.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 14 Line of Best Fit

Success looks like . . .

- **Goal:** Evaluating how good a line of fit is using the sum of the squares of the residuals.
  - » Comparing the sums of squares of residuals of five different lines of fit in Problem 2.

Suggested next steps

If students incorrectly determine the residual plot shows a good line of fit in Problem 1, consider:

- Reviewing strategies for analyzing residual plots from Activity 1.
- Assigning Practice Problem 1.
- Asking, "What might the scatter plot of the data look like based on the residual plot?"

If students incorrectly select the sum of the squares residuals in Problem 2, consider:

- Reviewing interpretation of the sum of squared residuals from Activity 2.
- Assigning Practice Problem 2.
- Asking, "What should be the goal for the sum of squared residuals when finding the line of best fit?"

Professional Learning

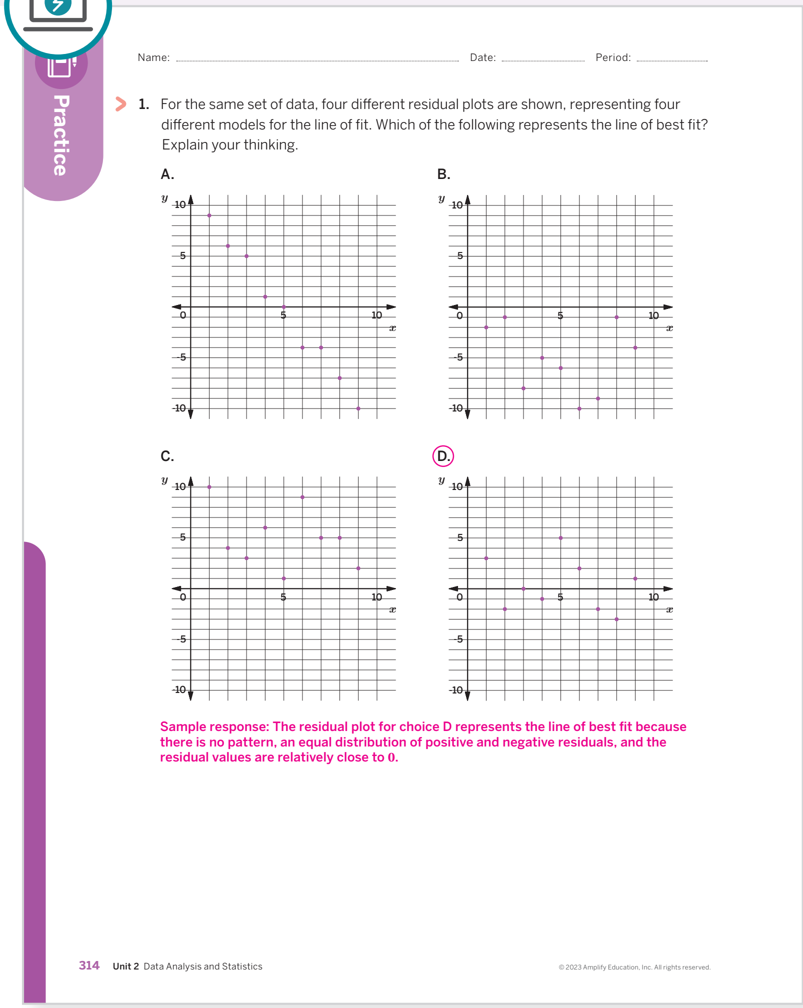
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In earlier lessons, students geometrically interpret standard deviation. How did that support geometrically interpreting the sum of the squared residuals?
- What challenges did students encounter as they worked on Activity 2? How did they work through them? What might you change for the next time you teach this lesson?

Practice

Independent



Name: .... Date: .... Period: ....

1. For the same set of data, four different residual plots are shown, representing four different models for the line of fit. Which of the following represents the line of best fit? Explain your thinking.

A.

B.

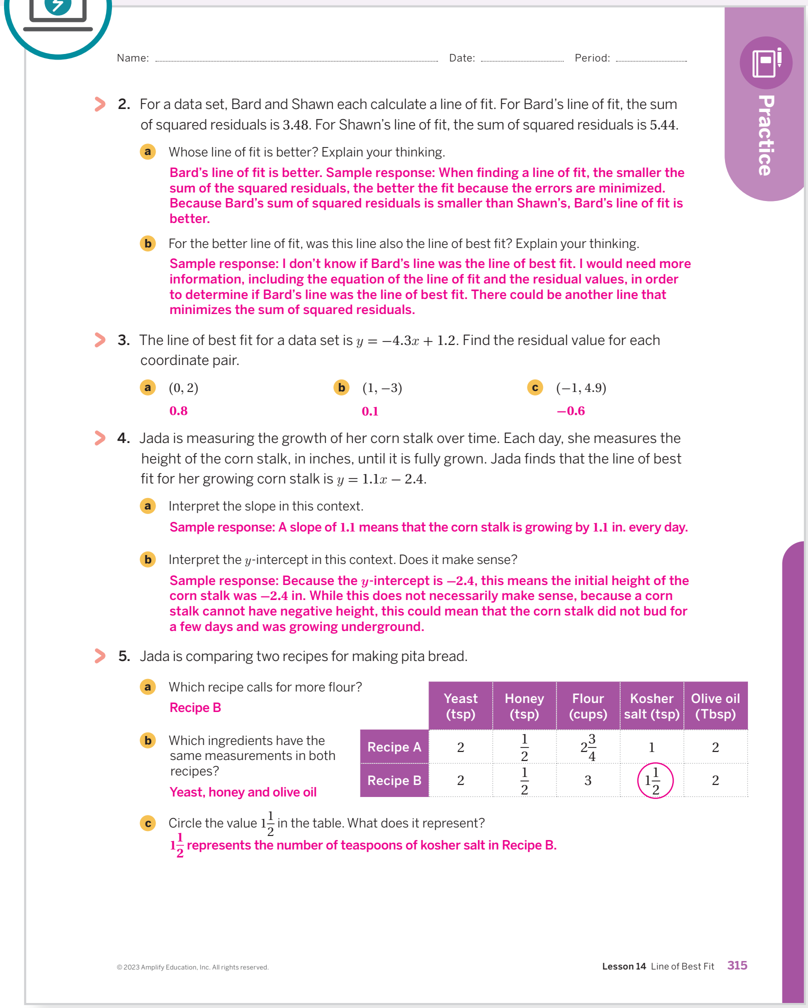
C.

[✓ D.]

Sample response: The residual plot for choice D represents the line of best fit because there is no pattern, an equal distribution of positive and negative residuals, and the residual values are relatively close to 0.

314 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.



Name: .... Date: .... Period: ....

2. For a data set, Bard and Shawn each calculate a line of fit. For Bard's line of fit, the sum of squared residuals is 3.48. For Shawn's line of fit, the sum of squared residuals is 5.44.

   a. Whose line of fit is better? Explain your thinking.

   Bard's line of fit is better. Sample response: When finding a line of fit, the smaller the sum of the squared residuals, the better the fit because the errors are minimized. Because Bard's sum of squared residuals is smaller than Shawn's, Bard's line of fit is better.

   b. For the better line of fit, was this line also the line of best fit? Explain your thinking.

   Sample response: I don't know if Bard's line was the line of best fit. I would need more information, including the equation of the line of fit and the residual values, in order to determine if Bard's line was the line of best fit. There could be another line that minimizes the sum of squared residuals.

3. The line of best fit for a data set is $y = -4.3x + 1.2$. Find the residual value for each coordinate pair.

a. $(0, 2)$	b. $(1, -3)$	c. $(-1, 4.9)$
0.8	**0.1**	**−0.6**

4. Jada is measuring the growth of her corn stalk over time. Each day, she measures the height of the corn stalk, in inches, until it is fully grown. Jada finds that the line of best fit for her growing corn stalk is $y = 1.1x - 2.4$.

   a. Interpret the slope in this context.

   Sample response: A slope of 1.1 means that the corn stalk is growing by 1.1 in. every day.

   b. Interpret the y-intercept in this context. Does it make sense?

   Sample response: Because the y-intercept is −2.4, this means the initial height of the corn stalk was −2.4 in. While this does not necessarily make sense, because a corn stalk cannot have negative height, this could mean that the corn stalk did not bud for a few days and was growing underground.

5. Jada is comparing two recipes for making pita bread.

   a. Which recipe calls for more flour?

   Recipe B

   b. Which ingredients have the same measurements in both recipes?

   Yeast, honey and olive oil

	Yeast (tsp)	Honey (tsp)	Flour (cups)	Kosher salt (tsp)	Olive oil (Tbsp)
Recipe A	2	$\frac{1}{2}$	$2\frac{3}{4}$	1	2
Recipe B	2	$\frac{1}{2}$	3	$1\frac{1}{2}$	2

   c. Circle the value $1\frac{1}{2}$ in the table. What does it represent?

   $1\frac{1}{2}$ represents the number of teaspoons of kosher salt in Recipe B.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 14 Line of Best Fit 315

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
Spiral	3	Unit 2 Lesson 13	1
	4	Unit 1 Lesson 12	2
Formative ⚡	5	Unit 2 Lesson 15	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 4

Categorical Data

In this Sub-Unit, students use two-way tables and frequency tables to examine associations within climate data, and see how climate change affects marginalized people around the world.

SUB-UNIT

4 Categorical Data

Narrative Connections

What makes storms worse and has nothing to do with the weather?

When Hurricane Harvey hit Louisiana and Texas in 2017, it arrived with shocking force. Making landfall with 130-mile-per-hour winds, the storm and flooding killed dozens, caused $125 billion worth of damage, and forced 32,000 people from their homes. It was the worst natural disaster the country had seen since Hurricane Katrina.

What you may not realize is that small differences in where people live within a town or city can make a big difference in how they are affected by a regional event. For example, when it came to Houston's embankments, some communities were better protected than others. It turned out that some of the city's low-income housing was located directly in a high-risk flood zone! Meanwhile, people who lived closer to chemical plants were at greater risk to sources of toxic spillage from the flooding.

While natural disasters affect everyone, they can inflict the worst damage on those who are least able to withstand it. Imagine if you were a city planner or in charge of infrastructure. You wouldn't want to leave anything, or anyone, to chance.

To do that, you have to break the numbers down. Mathematical tools such as two-way tables and relative frequency tables can help us understand data by seeing which categories might be associated. With these tools, we can parse the effects of environmental damage on different groups, and hopefully even fend off future disasters.

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore two-way tables and relative frequency tables — within the context of climate change data — in the following places:

- **Lesson 15, Activities 1–2:** Social Impacts of Climate Change, Info Gap: Droughts and Flooding
- **Lesson 16, Activities 1–2:** Age and Wildfires; Flint, Michigan
- **Lesson 17, Activity 1:** Droughts in Kenya

UNIT 2 | **LESSON 15**

Two-Way Tables

Let's create and interpret categorical data using two-way tables.

Focus

Goals

1. Calculate missing values in a two-way table.
2. Create two-way tables for categorical data, given information in everyday language.
3. **Language Goal:** Describe what the values in a two-way table mean in everyday language. **(Speaking and Listening, Writing)**

Coherence

Today

Students interpret and use two-way frequency tables to examine the social impacts of climate change. During Activity 2, students ask questions to determine missing information in a two-way table to help respond to the problems.

Previously

In Lessons 11–14, students determined how to use statistics to analyze data with two variables.

Coming Soon

In Lesson 16, students apply their knowledge of two-way tables to interpret and create relative frequency tables.

Rigor

- Students build **conceptual understanding** of two-way tables.
- Students use two-way tables to calculate values and solve problems to develop **procedural fluency**.
- Students **apply** two-way tables in the context of climate change.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one per pair
- Activity 2 PDF (answers)
- Instructional Routine PDF, *Info Gap: Instructions*
- Instructional Routine PDF, *Info Gap: Types of Questioning*

Math Language Development

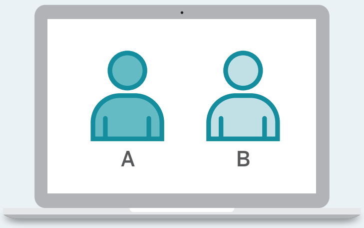
New words

- ***categorical variable***
- ***two-way table***

Amps | Featured Activity

Activity 2
Digital Collaboration

Students are paired to determine and request the information needed to understand the relationship of cells in a two-way table.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may struggle to communicate clearly and precisely in the Info Gaps activity. Explain that they first need to make sure they are listening well. Then discuss how they can encourage their partner to ask clarifying questions or to reword their request. Their partner can then repeat the request in their own words to make sure that both students have the same understanding of the request.

Modifications to Pacing

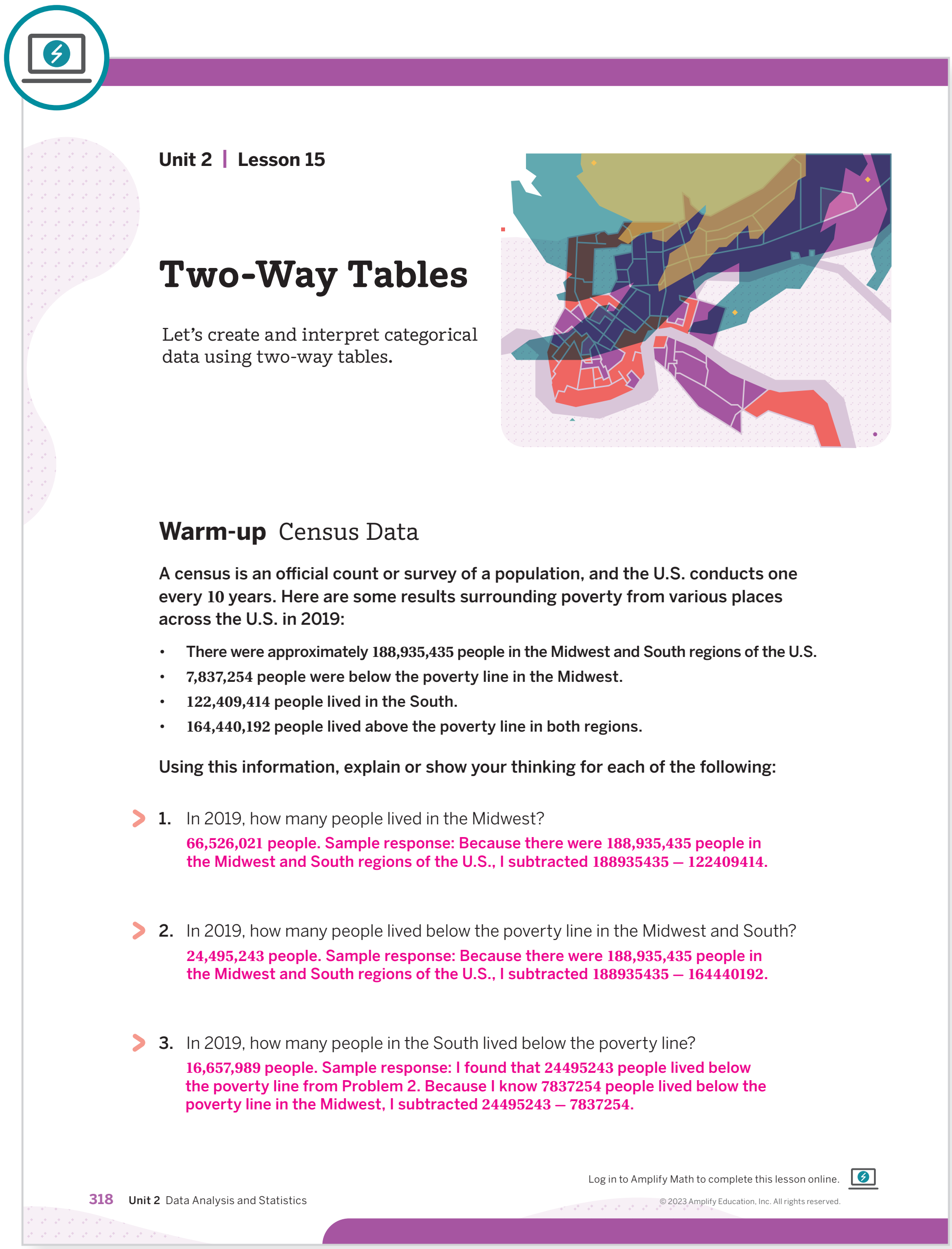
You may want to consider this additional modification if you are short on time.

- In the **Warm-up**, Problem 3 may be omitted.

Independent | 10 min

Warm-up Census Data

Students use and interpret census data to respond to problems in context to motivate the need for efficient representation of the data.



Unit 2 | Lesson 15

Two-Way Tables

Let's create and interpret categorical data using two-way tables.

Warm-up Census Data

A census is an official count or survey of a population, and the U.S. conducts one every 10 years. Here are some results surrounding poverty from various places across the U.S. in 2019:

- There were approximately 188,935,435 people in the Midwest and South regions of the U.S.
- 7,837,254 people were below the poverty line in the Midwest.
- 122,409,414 people lived in the South.
- 164,440,192 people lived above the poverty line in both regions.

Using this information, explain or show your thinking for each of the following:

1. In 2019, how many people lived in the Midwest?
   66,526,021 people. Sample response: Because there were 188,935,435 people in the Midwest and South regions of the U.S., I subtracted 188935435 – 122409414.

2. In 2019, how many people lived below the poverty line in the Midwest and South?
   24,495,243 people. Sample response: Because there were 188,935,435 people in the Midwest and South regions of the U.S., I subtracted 188935435 – 164440192.

3. In 2019, how many people in the South lived below the poverty line?
   16,657,989 people. Sample response: I found that 24495243 people lived below the poverty line from Problem 2. Because I know 7837254 people lived below the poverty line in the Midwest, I subtracted 24495243 – 7837254.

Log in to Amplify Math to complete this lesson online.

318 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the prompt together as a whole class. Have a brief discussion about what the "poverty line" is. Allow students five minutes to work independently.

2 Monitor

Help students get started by saying, "How could you organize the data to make sense of it?"

Look for points of confusion:

- **Using only the provided values to respond to each problem.** Ask, "How can you use the descriptions of each value to help determine the values described by each problem?"

Look for productive strategies:

- Calculating the difference between given values to determine the values described in each problem.
- Using an organizational strategy to represent the given values.

3 Connect

Have individual students share their strategies for determining the values described in each problem and organizing the data.

Define the term ***categorical variable***.

Highlight that while students can still respond to the given problems using the list of values and descriptions given, it is not necessarily the most efficient way to represent data.

Ask, "What are some ways you could represent the data given to you?"

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

Read 1: Students should understand that the U.S. government conducts a census — an official population count and survey — every 10 years.

Read 2: Ask students to name or highlight the given quantities and relationships, such as 122,409,414 people lived in the South.

Read 3: Ask students to brainstorm strategies for determining the number of people who lived in the Midwest in 2019.

Power-up

To power up students' ability to read information given in a table of values, have students complete:

Use the given table of values to determine whether each statement is *true* or *false*.

	Pencils	Erasers
Mai	8	3
Shawn	5	4

a. Mai has more pencils than Shawn. True
b. Mai has more erasers than Shawn. False
c. Shawn has 4 pencils. False
d. Mai has 3 erasers. True

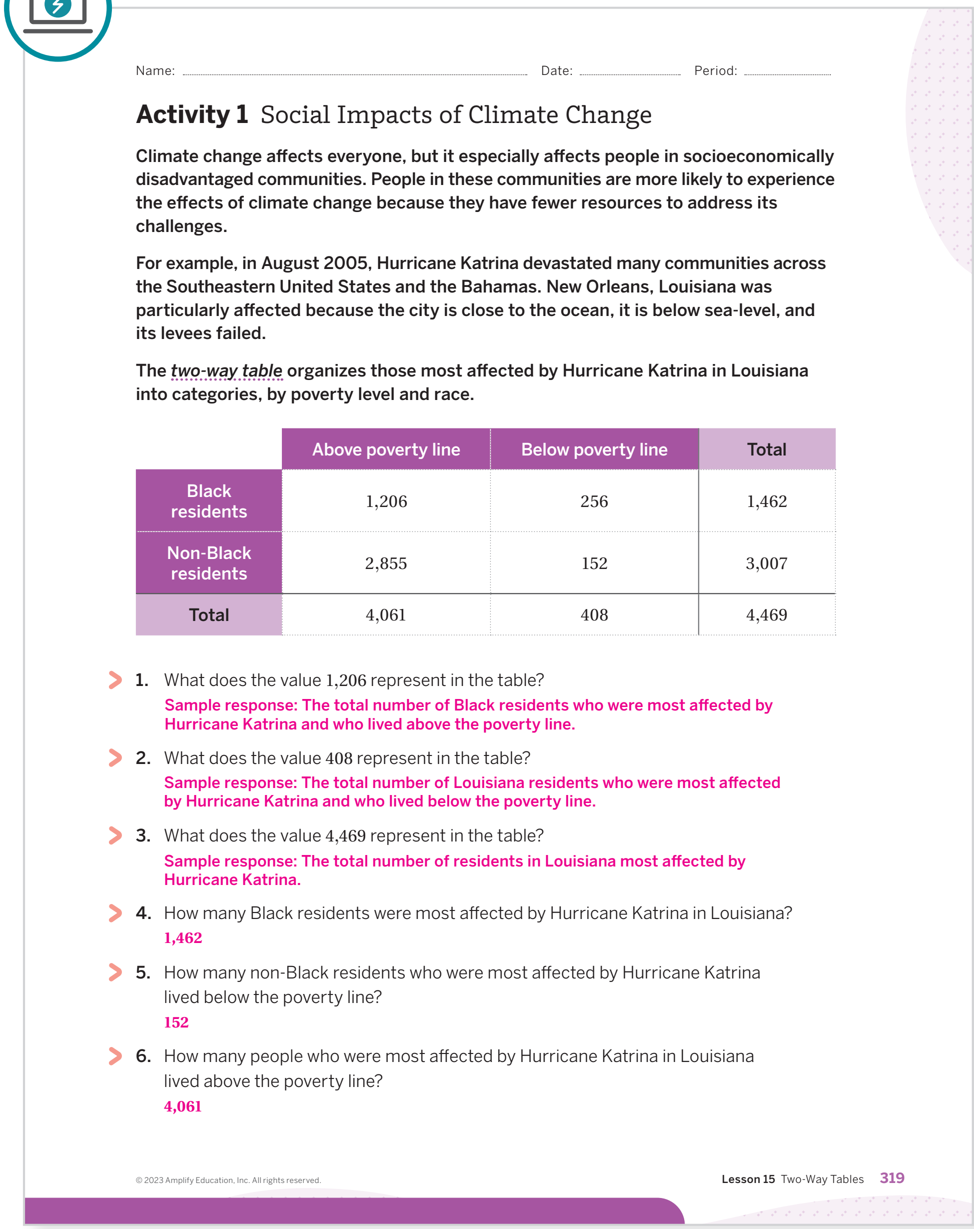
Use: Before Activity 1

Informed by: Performance on Lesson 14, Practice Problem 5

Pairs | 15 min

Activity 1 Social Impacts of Climate Change

Students interpret data in a two-way table to respond to problems investigating how Hurricane Katrina affected different residents.



Name: ________________ Date: ________ Period: ________

Activity 1 Social Impacts of Climate Change

Climate change affects everyone, but it especially affects people in socioeconomically disadvantaged communities. People in these communities are more likely to experience the effects of climate change because they have fewer resources to address its challenges.

For example, in August 2005, Hurricane Katrina devastated many communities across the Southeastern United States and the Bahamas. New Orleans, Louisiana was particularly affected because the city is close to the ocean, it is below sea-level, and its levees failed.

The *two-way table* organizes those most affected by Hurricane Katrina in Louisiana into categories, by poverty level and race.

	Above poverty line	Below poverty line	Total
Black residents	1,206	256	1,462
Non-Black residents	2,855	152	3,007
Total	4,061	408	4,469

1. What does the value 1,206 represent in the table?
   Sample response: The total number of Black residents who were most affected by Hurricane Katrina and who lived above the poverty line.
2. What does the value 408 represent in the table?
   Sample response: The total number of Louisiana residents who were most affected by Hurricane Katrina and who lived below the poverty line.
3. What does the value 4,469 represent in the table?
   Sample response: The total number of residents in Louisiana most affected by Hurricane Katrina.
4. How many Black residents were most affected by Hurricane Katrina in Louisiana?
   1,462
5. How many non-Black residents who were most affected by Hurricane Katrina lived below the poverty line?
   152
6. How many people who were most affected by Hurricane Katrina in Louisiana lived above the poverty line?
   4,061

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in pairs. Read the prompt as a whole class, discuss what a "levee" is. Have students share their thinking for the first problem with their partner, then the whole class.

2 Monitor

Help students get started by saying, "Use the titles for each column and row to help interpret the values in each problem."

Look for points of confusion:

- **Only using one descriptor to describe values in the cells of the table.** Ask, "Where does this value occur in the table? What does that mean for how it should be described?"

Look for productive strategies:

- Using the labels from the two-way table to describe values.
- Recognizing a single descriptor corresponds with a value from a column, row, or grand total.
- Recognizing two descriptors correspond with a value from a cell in the table.

3 Connect

Display the two-way table.

Define the term ***two-way table***.

Have pairs of students share how they interpreted values from the table and determined what values corresponded with a description.

Highlight that two-way tables are an efficient representation for two categorical variables. The frequency, or count, appears in each cell, and often the "totals" for rows and columns are displayed.

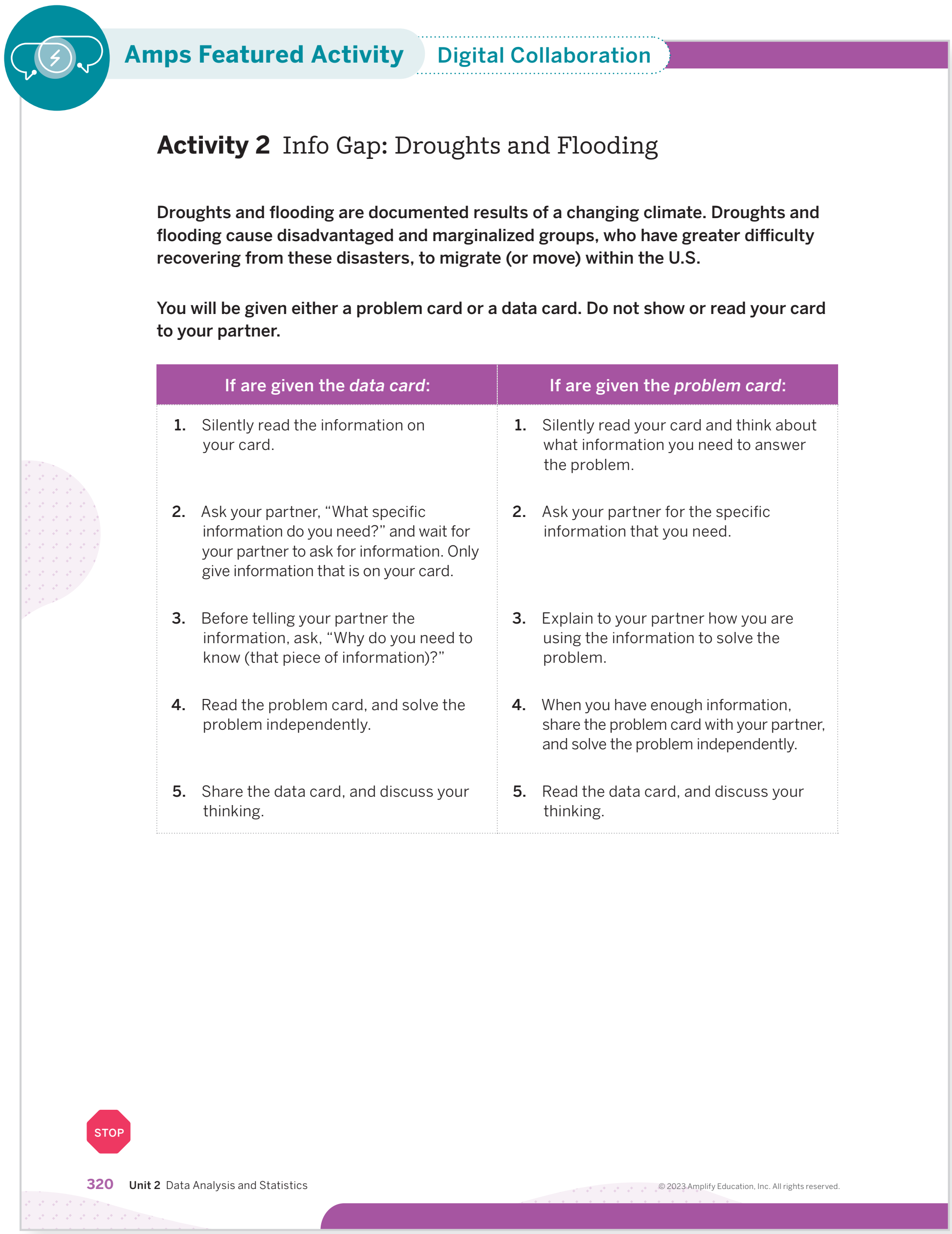
Ask, "If all totals from rows and columns were removed, would you be able to determine these values? How?"

Pairs | 15 min

Activity 2 Info Gap: Droughts and Flooding

Students determine and request the information needed to understand the relationships in a two-way table.

Amps Featured Activity Digital Collaboration



Activity 2 Info Gap: Droughts and Flooding

Droughts and flooding are documented results of a changing climate. Droughts and flooding cause disadvantaged and marginalized groups, who have greater difficulty recovering from these disasters, to migrate (or move) within the U.S.

You will be given either a problem card or a data card. Do not show or read your card to your partner.

If are given the *data card*:	If are given the *problem card*:
1. Silently read the information on your card.	1. Silently read your card and think about what information you need to answer the problem.
2. Ask your partner, "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card.	2. Ask your partner for the specific information that you need.
3. Before telling your partner the information, ask, "Why do you need to know (that piece of information)?"	3. Explain to your partner how you are using the information to solve the problem.
4. Read the problem card, and solve the problem independently.	4. When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Share the data card, and discuss your thinking.	5. Read the data card, and discuss your thinking.

STOP

320 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display the Instructional Routine PDF, *Info Gap: Instructions* and review the ***Info Gap*** routine. Consider demonstrating it if students are unfamiliar. Distribute the pre-cut cards from the Activity 2 PDF to each student pair.

2 Monitor

Help students get started by practicing the instructional routine with them.

Look for points of confusion:

- **Using only values given by their partner to respond to problems.** Ask, "How can you use the given values to work backwards and determine other missing values?"

Look for productive strategies:

- Using row and column header labels to formulate questions.
- Using given values to work backwards, using subtraction, to determine other missing table values.

3 Connect

Have pairs of students share what questions they asked and how they may have had to refine their questions to help complete the tables.

Display the completed two-way tables.

Highlight the meaning of the values in the cells and various ways to describe the intersection of two categories.

Ask, "How did you determine the values that were not provided on the data card?" Sample response: I subtracted the known value in a row from the total of the row to determine the missing value.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. Consider using these questions as part of the think-aloud:

- "I know the total number of states is 50. Once I know a few of these values, I can determine the other values."
- "If I look at the first row, I will ask for the number of states who experienced effects due to drought. This will give me the total for that row."
- "Once I know the total for that row, I can subtract that value from 50 to determine the total for the second row."

Math Language Development

MLR4: Information Gap

Consider displaying the Instructional Routine PDF, *Info Gap: Types of Questioning* for students who need a starting point to form questions.

English Learners

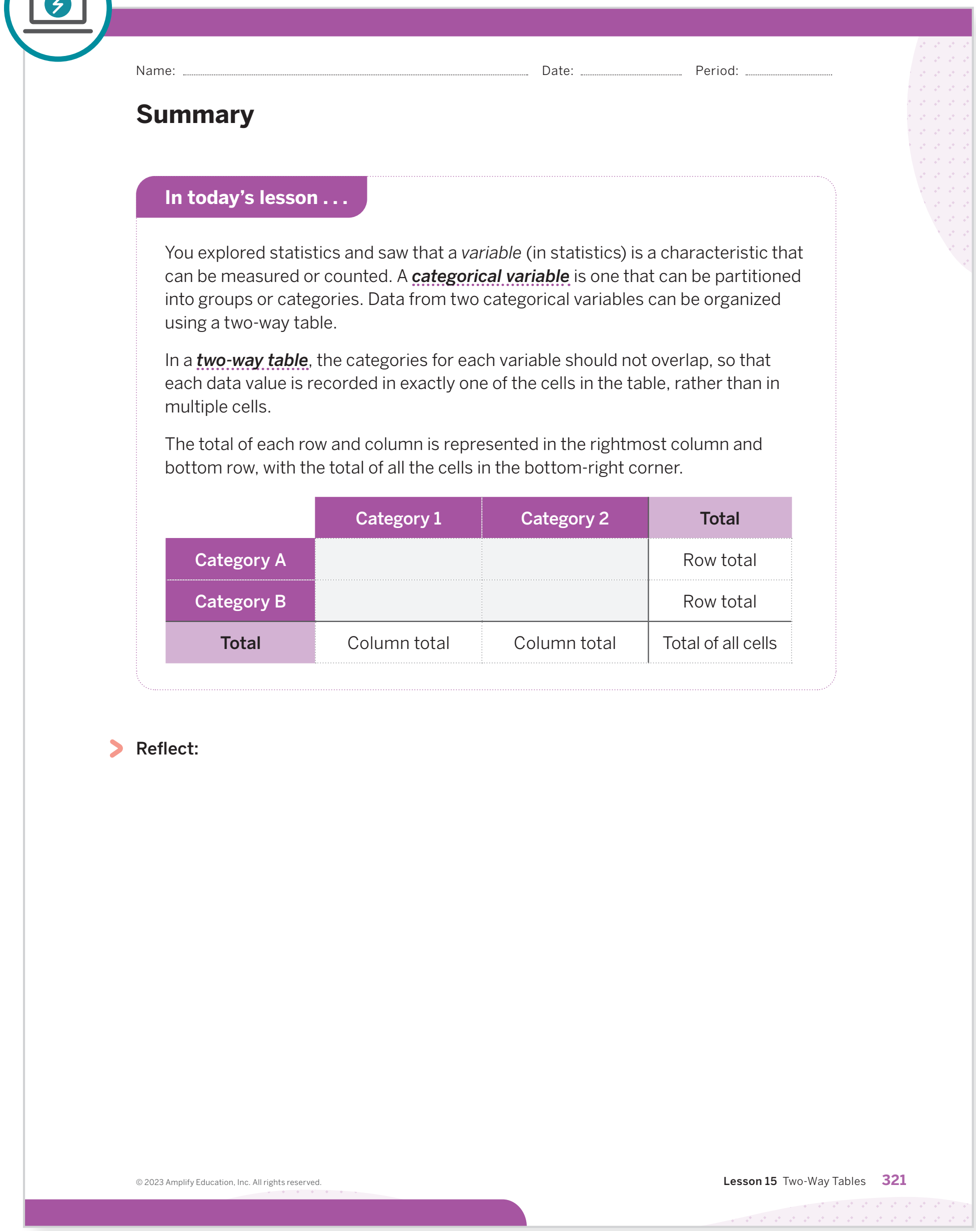
Consider providing sample questions students could ask their partner, or themselves, for Problem Card 1, such as:

- "How many states have experienced effects due to drought?"
- "How many states have not experienced effects due to drought?"

Whole Class | 5 min

Summary

Review and synthesize creating and interpreting the values of a two-way table.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You explored statistics and saw that a *variable* (in statistics) is a characteristic that can be measured or counted. A ***categorical variable*** is one that can be partitioned into groups or categories. Data from two categorical variables can be organized using a two-way table.

In a ***two-way table***, the categories for each variable should not overlap, so that each data value is recorded in exactly one of the cells in the table, rather than in multiple cells.

The total of each row and column is represented in the rightmost column and bottom row, with the total of all the cells in the bottom-right corner.

	Category 1	Category 2	Total
Category A			Row total
Category B			Row total
Total	Column total	Column total	Total of all cells

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the two way table.

Have students share an example of two categorical variables of climate and social variables that may relate to one another.

Highlight that categories for a single variable should not overlap. For instance, each individual in a survey should only be able to be included in one category, not both.

Formalize vocabulary:

- ***categorical variable***
- ***two-way table***

Ask, "What does the phrase *two-way* in *two-way* table mean?" Sample response: It means there are two different categories being examined.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How can information be displayed to communicate and justify data in real-life?"
- "How are two-way tables used to organize data?"

Math Language Development

MLR2: Collect and Display

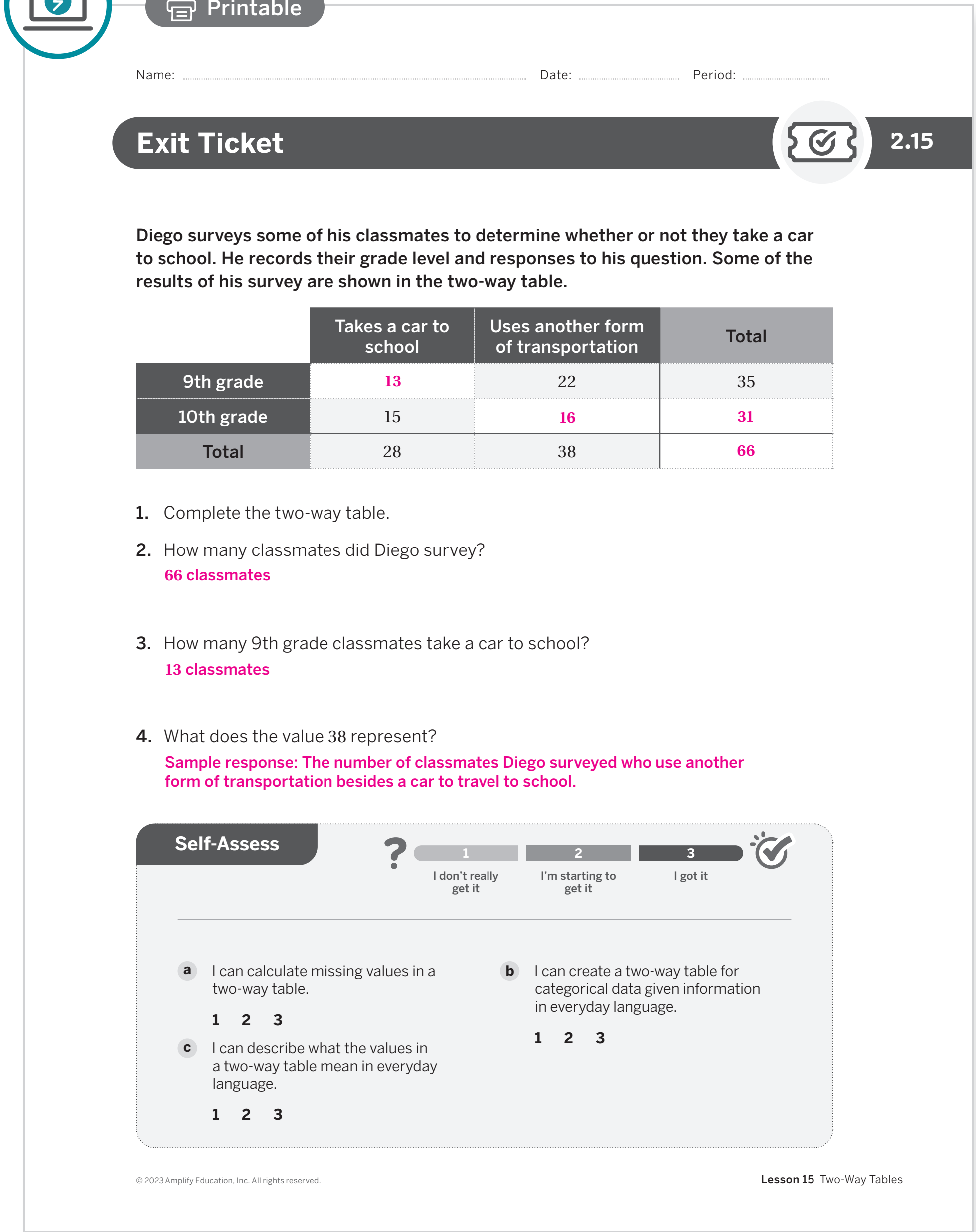
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the terms *categorical variable* and *two-way table* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by calculating missing values in a two-way table and describing their meaning.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket

2.15

Diego surveys some of his classmates to determine whether or not they take a car to school. He records their grade level and responses to his question. Some of the results of his survey are shown in the two-way table.

	Takes a car to school	Uses another form of transportation	Total
9th grade	13	22	35
10th grade	15	16	31
Total	28	38	66

1. Complete the two-way table.
2. How many classmates did Diego survey?
   66 classmates
3. How many 9th grade classmates take a car to school?
   13 classmates
4. What does the value 38 represent?
   Sample response: The number of classmates Diego surveyed who use another form of transportation besides a car to travel to school.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can calculate missing values in a two-way table.

1 2 3

b I can create a two-way table for categorical data given information in everyday language.

1 2 3

c I can describe what the values in a two-way table mean in everyday language.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 15 Two-Way Tables

Success looks like . . .

- **Goal:** Calculating missing values in a two-way table.
  - » Completing the two-way table about the car survey in Problem 1.
- **Goal:** Creating two-way tables for categorical data, given information in everyday language.
- **Language Goal:** Describing what the values in a two-way table mean in everyday language. **(Speaking and Listening, Writing)**
  - » Explaining the meaning of the value 38 in Problem 4.

Suggested next steps

If students incorrectly fill in the table in Problem 1, consider:

- Reviewing strategies on how to fill in a two-way table from Activity 2.
- Assigning Practice Problem 1.
- Asking, "How could you use the totals of each column and row to help determine missing values?"

If students incorrectly identify the value in Problems 2 or 3, consider:

- Reviewing the meaning of values from Activity 1.
- Assigning Practice Problem 2.
- Asking, "How could you use the row and column labels to identify these values?"

If students incorrectly interpret the value in Problem 4, consider:

- Reviewing how to interpret values from Activity 1.
- Assigning Practice Problem 1.
- Asking, "How could you use the row and column labels to help interpret these values?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The instructional goal for this lesson was to create and interpret two-way tables. How well did students accomplish this? What did you specifically do to help students accomplish it?
- What routines enabled all students to do math in today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent

Name: ______ Date: ______ Period: ______

1. Tyler surveys some of his classmates to determine whether or not they bring their lunch from home. He records their grade levels and responses to his question. Some of the results of his survey are shown in the table. Complete the two-way table.

	Brings lunch from home	Does not bring lunch from home	Total
10th Grade	29	**16**	**45**
12th Grade	**10**	**50**	60
Total	39	**66**	105

2. Mai conducts a survey in her community asking if the homeowners think they have safe drinking water and if they think more efforts should be made to provide clean water. The results of her survey are shown.

	More can be done to provide clean water	More cannot be done to provide clean water	Total
Have safe drinking water	32	23	55
Do not have safe drinking water	61	11	72
Total	93	34	127

a. How many members of the community participated in Mai's survey?
127 members

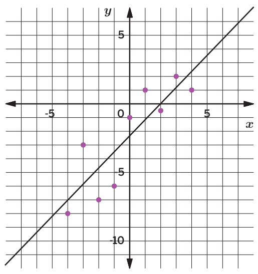
b. How many members of the community think more cannot be done to provide clean water?
34 members

c. Of the members of the community who do not have safe drinking water, how many think more can be done to provide clean water?
61 members

© 2023 Amplify Education, Inc. All rights reserved.

Name: ______ Date: ______ Period: ______

3. Which of the following could *not* be the equation for the line of best fit for the scatter plot shown? Explain your thinking.



A. $y = 1.18x - 2.39$
B. $y = 1.19x - 2.41$
C. $y = x - 2$
[✓ D.] $y = 1.18x + 2.39$

Sample response: The equation for the line of fit must have a positive slope and negative y-intercept, so choice D cannot represent the line of best fit because it has a positive y-intercept.

4. The same data from Mai's survey is shown in the two way table.

	More can be done to provide clean water	More cannot be done to provide clean water	Total
Have safe drinking water	32	23	55
Do not have safe drinking water	61	11	72
Total	93	34	127

a. What percent of community members surveyed do not have safe drinking water?
About 56.7%

b. What percent of community members think more can be done to provide clean water?
About 73.2%

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 1	2
Spiral	3	Unit 2 Lesson 12	2
Formative ⚡	4	Unit 2 Lesson 16	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 2 | **LESSON 16**

Relative Frequency Tables

Let's analyze two-way tables relative to their totals.

Focus

Goals

1. **Language Goal:** Describe the meaning of values in a relative frequency table. **(Speaking and Listening, Writing)**
2. Calculate the values in a relative frequency table.

Coherence

Today

Students are introduced to relative frequency tables, which are created by dividing each value in a two-way table by the total number of responses either in the entire table, in a row, or in a column. Students create and interpret relative frequency tables in context.

Previously

In Lesson 15, students created and used two-way tables to interpret relationships between two categorical variables.

Coming Soon

In Lesson 17, students will use information from two-way tables to look for associations in data.

Rigor

- Students build **conceptual understanding** of relative frequency.
- Students calculate the relative frequency of events to develop **procedural fluency**.
- Students **apply** relative frequency in the context of climate change and its effect on marginalized groups of people.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Anchor Chart PDF, *Choosing Two-Way Tables or Relative Frequency Tables*
- Anchor Chart PDF, *Sentence Stems, Calculating Relative Frequencies*
- Anchor Chart PDF, *Sentence Stems, Notice and Wonder*

Math Language Development

New words

- ***relative frequency table***

Review words

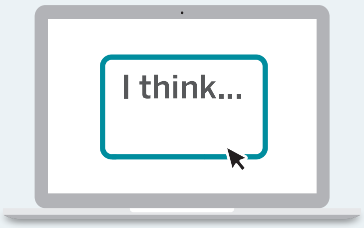
- *categorical variable*
- *two-way table*

Amps | Featured Activity

Activity 2

See Student Thinking

Students are asked to explain their thinking behind how values were calculated in different relative frequency tables, and these explanations are available to you digitally in real time.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated with their ability to look for and make use of structure as they develop a strategy to determine which type of relative frequency table to construct. Encourage students to seek clues from the problem itself and use any connections they notice between problems already completed as a class to help develop a strategy. Encourage students to ask others to explain their strategy.

Modifications to Pacing

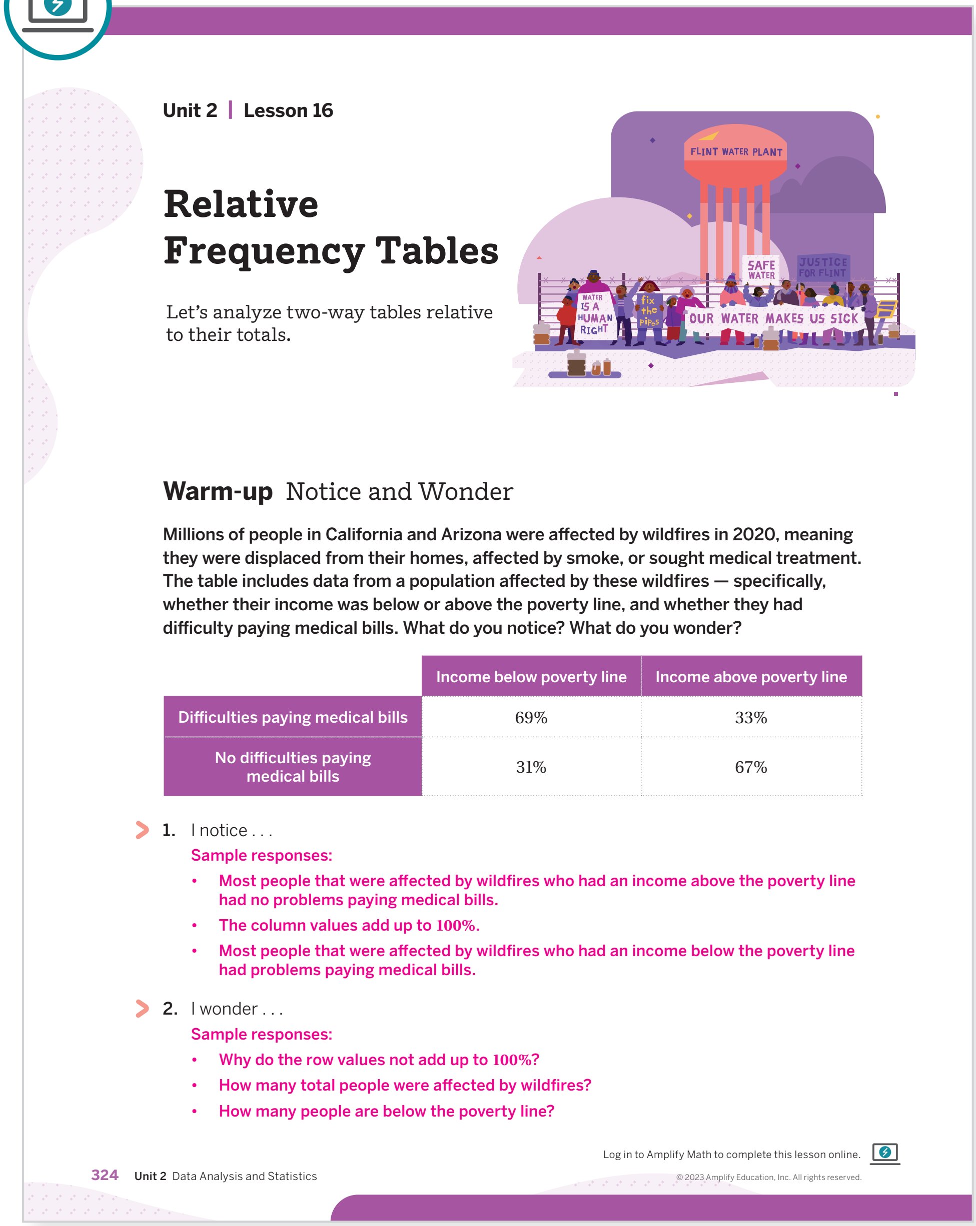
You may want to consider this additional modification if you are short on time.

- In **Activity 2**, provide students with the relative frequency value in the first cell for the tables in Problems 3 and 6.

Pairs | 10 min

Warm-up Notice and Wonder

Students notice relationships between two categorical variables and ask questions to help interpret relative frequency tables.



Unit 2 | Lesson 16

Relative Frequency Tables

Let's analyze two-way tables relative to their totals.

Warm-up Notice and Wonder

Millions of people in California and Arizona were affected by wildfires in 2020, meaning they were displaced from their homes, affected by smoke, or sought medical treatment. The table includes data from a population affected by these wildfires — specifically, whether their income was below or above the poverty line, and whether they had difficulty paying medical bills. What do you notice? What do you wonder?

	Income below poverty line	Income above poverty line
Difficulties paying medical bills	69%	33%
No difficulties paying medical bills	31%	67%

1. I notice . . .

Sample responses:

- Most people that were affected by wildfires who had an income above the poverty line had no problems paying medical bills.
- The column values add up to 100%.
- Most people that were affected by wildfires who had an income below the poverty line had problems paying medical bills.

2. I wonder . . .

Sample responses:

- Why do the row values not add up to 100%?
- How many total people were affected by wildfires?
- How many people are below the poverty line?

Log in to Amplify Math to complete this lesson online.

324 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the prompt as a whole class. Take a moment to elicit prior knowledge students have on wildfires. Conduce the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by asking, "What do you think each percentage represents? What questions do you have about the data?"

Look for points of confusion:

- **Using the sum of the rows to draw conclusions.** Ask students to compare the sum of the rows to the sum of the columns.

Look for productive strategies:

- Using the row and column titles to write conclusions.
- Making connections to two-way frequency tables.

3 Connect

Have pairs of students share what they noticed and wondered. Record some of the responses for display.

Highlight that the key difference between the two-way table from the previous lesson and the one here is that the values in this table are expressed as relative frequencies, or percentages.

Define the term ***relative frequency***.

Ask, "What are the categorical variables that are being represented here?" Sample response: Income (above vs. below the poverty line) and whether or not that person had difficulties paying medical bills.

Math Language Development

MLR5: Co-craft Questions

Display or provide the Anchor Chart PDF, *Sentence Stems, Notice and Wonder* to support students as they complete the Warm-up. After students complete Problem 2, have them share what they wondered with another pair of students and work together to generate 1–2 questions that they have about this table and scenario.

English Learners

Before the Connect, allow students to rehearse with a partner what they will say before sharing with the whole class.

Power-up

To power up students' ability to draw conclusions from a two-way table:

Provide students with a copy of the Power-up PDF.

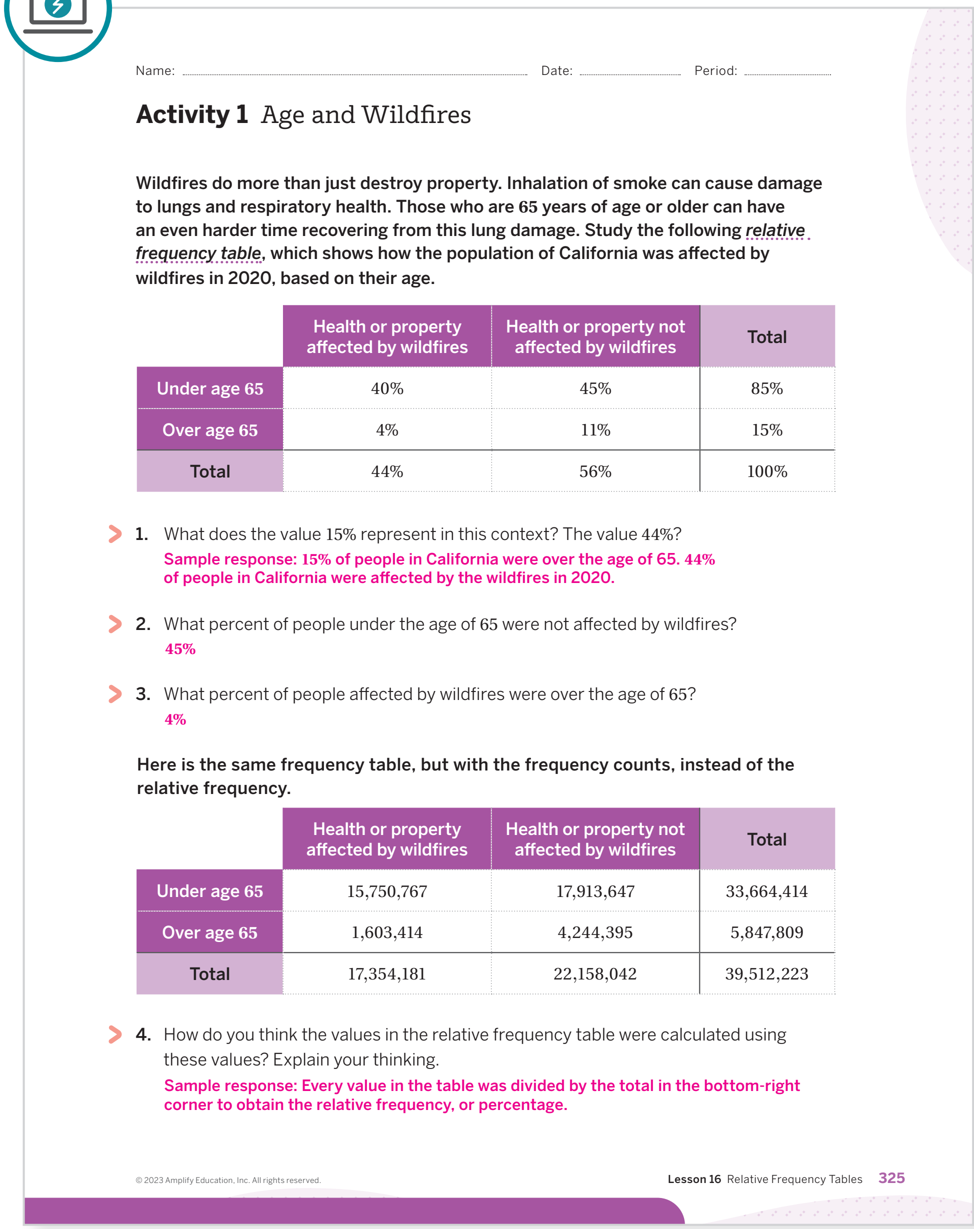
Use: Before the Warm-up

Informed by: Performance on Lesson 15, Practice Problem 4

Pairs | 15 min

Activity 1 Age and Wildfires

Students interpret values from a relative frequency table to make meaning of these values in context.



Name: ______ Date: ______ Period: ______

Activity 1 Age and Wildfires

Wildfires do more than just destroy property. Inhalation of smoke can cause damage to lungs and respiratory health. Those who are 65 years of age or older can have an even harder time recovering from this lung damage. Study the following *relative frequency table*, which shows how the population of California was affected by wildfires in 2020, based on their age.

	Health or property affected by wildfires	Health or property not affected by wildfires	Total
Under age 65	40%	45%	85%
Over age 65	4%	11%	15%
Total	44%	56%	100%

1. What does the value 15% represent in this context? The value 44%?
   Sample response: 15% of people in California were over the age of 65. 44% of people in California were affected by the wildfires in 2020.

2. What percent of people under the age of 65 were not affected by wildfires?
   45%

3. What percent of people affected by wildfires were over the age of 65?
   4%

Here is the same frequency table, but with the frequency counts, instead of the relative frequency.

	Health or property affected by wildfires	Health or property not affected by wildfires	Total
Under age 65	15,750,767	17,913,647	33,664,414
Over age 65	1,603,414	4,244,395	5,847,809
Total	17,354,181	22,158,042	39,512,223

4. How do you think the values in the relative frequency table were calculated using these values? Explain your thinking.
   Sample response: Every value in the table was divided by the total in the bottom-right corner to obtain the relative frequency, or percentage.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Students remain in pairs. Using the ***Three Reads*** strategy, read the passage and highlight the column and row titles.

2 Monitor

Help students get started by having them note the column and/or row where specific values are located.

Look for points of confusion:

- **Describing the percentages using the incorrect row and column labels.** Have students circle the column and/or row where the value is located.

Look for productive strategies:

- Using the context along with the row and column labels to both describe percentages and locate values.
- Recognizing all values in the columns and rows should add to 100%, so the table total was used to determine all values.

3 Connect

Display both two-way tables.

Have pairs of students share how they interpreted and located values in the two-way table.

Highlight that this two-way relative frequency table is one of a few different types of relative frequency tables. The values were found by dividing each cell, row, and column total by the total for the whole table.

Ask, "What other types of relative frequency tables could be found?" Sample response: Instead of dividing by the total for the entire table, every cell could be divided by either the row or column totals.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they learned about relative frequency tables in Grade 8. Consider reviewing how to interpret a relative frequency table by showing how the percentages add up to the total for each row and column. The total of all the percentages should be 100%, but there may be some differences due to rounding.

Extension: *Math Enrichment*

Have students respond to these questions:

- "If you are under the age of 65, what does it mean that 40% and 45% are relatively close to each other?"
- "If you are over the age of 65%, what does it mean that 11% is almost three times as great as 4%?"

Math Language Development

MLR7: Compare and Connect

During the Connect, as you display both two-way tables, draw students' attention to how they represent the same data, yet in different ways. Consider illustrating how students can verify the percentages given in the relative frequency table, using the frequency counts in the frequency table.

English Learners

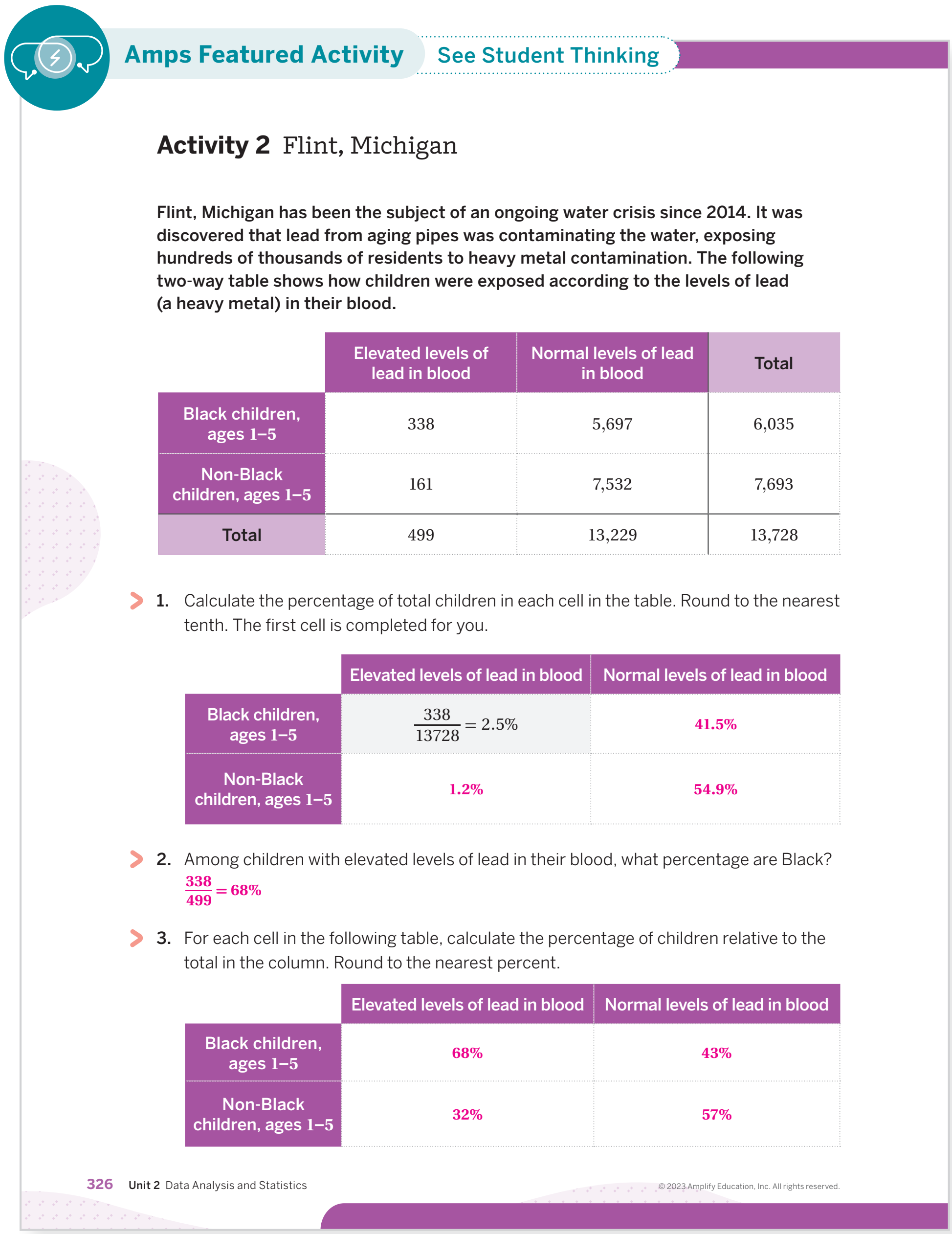
Consider displaying a relative frequency table that shows the expressions or calculations needed to determine the percentages, such as $15{,}750{,}767 \div 39{,}515{,}223 \approx 0.40$, which is about 40%.

Pairs | 15 min

Activity 2 Flint, Michigan

Students create and study relative frequency tables to understand the relationship between types of two-way tables.

Amps Featured Activity See Student Thinking



Activity 2 Flint, Michigan

Flint, Michigan has been the subject of an ongoing water crisis since 2014. It was discovered that lead from aging pipes was contaminating the water, exposing hundreds of thousands of residents to heavy metal contamination. The following two-way table shows how children were exposed according to the levels of lead (a heavy metal) in their blood.

	Elevated levels of lead in blood	Normal levels of lead in blood	Total
Black children, ages 1–5	338	5,697	6,035
Non-Black children, ages 1–5	161	7,532	7,693
Total	499	13,229	13,728

1. Calculate the percentage of total children in each cell in the table. Round to the nearest tenth. The first cell is completed for you.

	Elevated levels of lead in blood	Normal levels of lead in blood
Black children, ages 1–5	$\frac{338}{13728} = 2.5\%$	41.5%
Non-Black children, ages 1–5	1.2%	54.9%

2. Among children with elevated levels of lead in their blood, what percentage are Black?

$\frac{338}{499} = 68\%$

3. For each cell in the following table, calculate the percentage of children relative to the total in the column. Round to the nearest percent.

	Elevated levels of lead in blood	Normal levels of lead in blood
Black children, ages 1–5	68%	43%
Non-Black children, ages 1–5	32%	57%

326 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Using the *Three Reads* strategy, read the passage and highlight the column and row titles.

2 Monitor

Help students get started by asking, "What does the denominator of the work shown tell you about the types of relative frequency tables being used?"

Look for points of confusion:

- **Dividing all values by 499 in Problem 3 or 6035 in Problem 6.** Ask, "Does each column or row have the same total? How does this affect how you calculate the relative frequencies?"
- **Having difficulty choosing a row or column relative frequency to answer Problem 7.** Ask, "Because you want to see if race is an indicator for blood lead levels, what subgroups should you be looking at? Why?"

Look for productive strategies:

- Showing the calculations for determining each of the percentages in each type of relative frequency table.
- Recognizing what totals to use for relative frequencies by row or column.
- Using relative frequency by row to answer Problem 7.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display or provide copies of the Anchor Chart PDF, *Choosing Two-Way Tables or Relative Frequency Tables* as a reference for students to use during this activity. Consider also displaying or providing copies of the Anchor Chart PDF, Sentence Stems, *Calculating Relative Frequencies* to support students in explaining how they calculated the values.

Extension: *Math Enrichment*

Have students write their own question that they could answer using one of the types of frequency tables in this activity. Tell them that the question they ask must be able to be answered by this data alone, so it needs to be based on the data presented at the beginning of the activity.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative.

Read 1: Students should understand that the residents of Flint, Michigan were exposed to metal contamination in their water supply.

Read 2: Ask students to describe what the two-way table illustrates, such as the levels of lead in the blood of Black and non-Black children ages 1–5.

Read 3: Ask students to preview Problems 1 and 2 and think about how they might approach completing them.

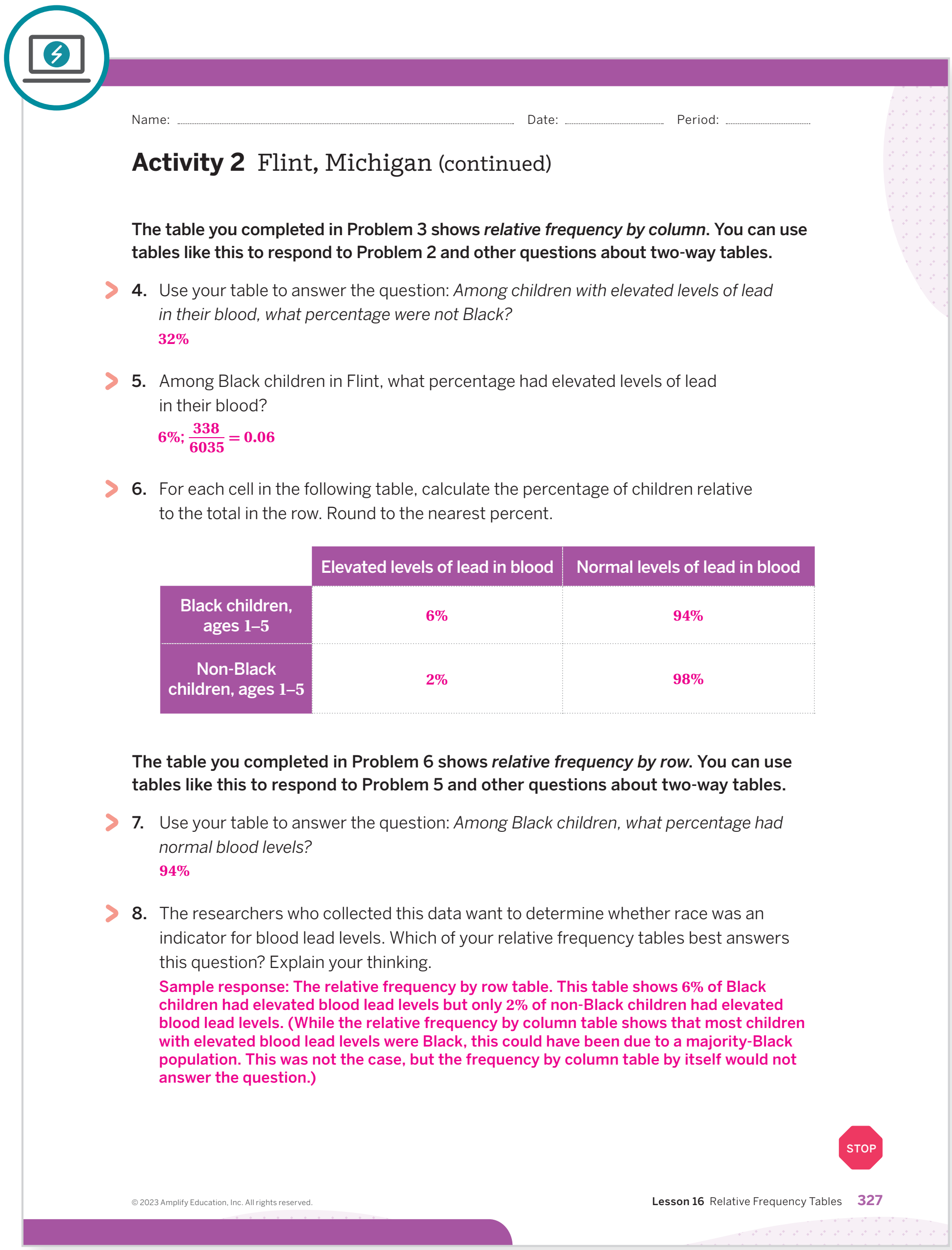
English Learners

Be sure students understand the meaning of "elevated," or consider changing the word to "increased."

Pairs | 15 min

Activity 2 Flint, Michigan (continued)

Students create and study relative frequency tables to understand the relationship between types of two-way tables.



Name: ______ Date: ______ Period: ______

Activity 2 Flint, Michigan (continued)

The table you completed in Problem 3 shows *relative frequency by column*. You can use tables like this to respond to Problem 2 and other questions about two-way tables.

4. Use your table to answer the question: *Among children with elevated levels of lead in their blood, what percentage were not Black?*
   32%

5. Among Black children in Flint, what percentage had elevated levels of lead in their blood?
   6%; $\frac{338}{6035} = 0.06$

6. For each cell in the following table, calculate the percentage of children relative to the total in the row. Round to the nearest percent.

	Elevated levels of lead in blood	Normal levels of lead in blood
Black children, ages 1–5	6%	94%
Non-Black children, ages 1–5	2%	98%

The table you completed in Problem 6 shows *relative frequency by row*. You can use tables like this to respond to Problem 5 and other questions about two-way tables.

7. Use your table to answer the question: *Among Black children, what percentage had normal blood levels?*
   94%

8. The researchers who collected this data want to determine whether race was an indicator for blood lead levels. Which of your relative frequency tables best answers this question? Explain your thinking.
   Sample response: The relative frequency by row table. This table shows 6% of Black children had elevated blood lead levels but only 2% of non-Black children had elevated blood lead levels. (While the relative frequency by column table shows that most children with elevated blood lead levels were Black, this could have been due to a majority-Black population. This was not the case, but the frequency by column table by itself would not answer the question.)

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 16 Relative Frequency Tables 327

3 Connect

Have pairs of students share how they calculated the different types of relative frequency tables and their strategies for determining which type of table to use.

Display the completed tables.

Highlight that first identifying a subgroup will help students determine whether to construct a row or column relative frequency table. Knowing the totals for the rows, columns, or entire table is necessary to construct a relative frequency table.

Ask, "Why do the column values not add up to 100% in Problem 6?" Sample response: This table was a row relative frequency table, so the row values add up to 100%.

Fostering Diverse Thinking

The Safe Water Drinking Act

Have students work in small groups to research the Flint water crisis. Highlight the following information:

- The Safe Water Drinking Act was enacted in 1974 and provides regulated standards for drinking water.
- A lawsuit was brought against the city of Flint and state officials, claiming that they had violated the Safe Water Drinking Act. A financial settlement was eventually reached.

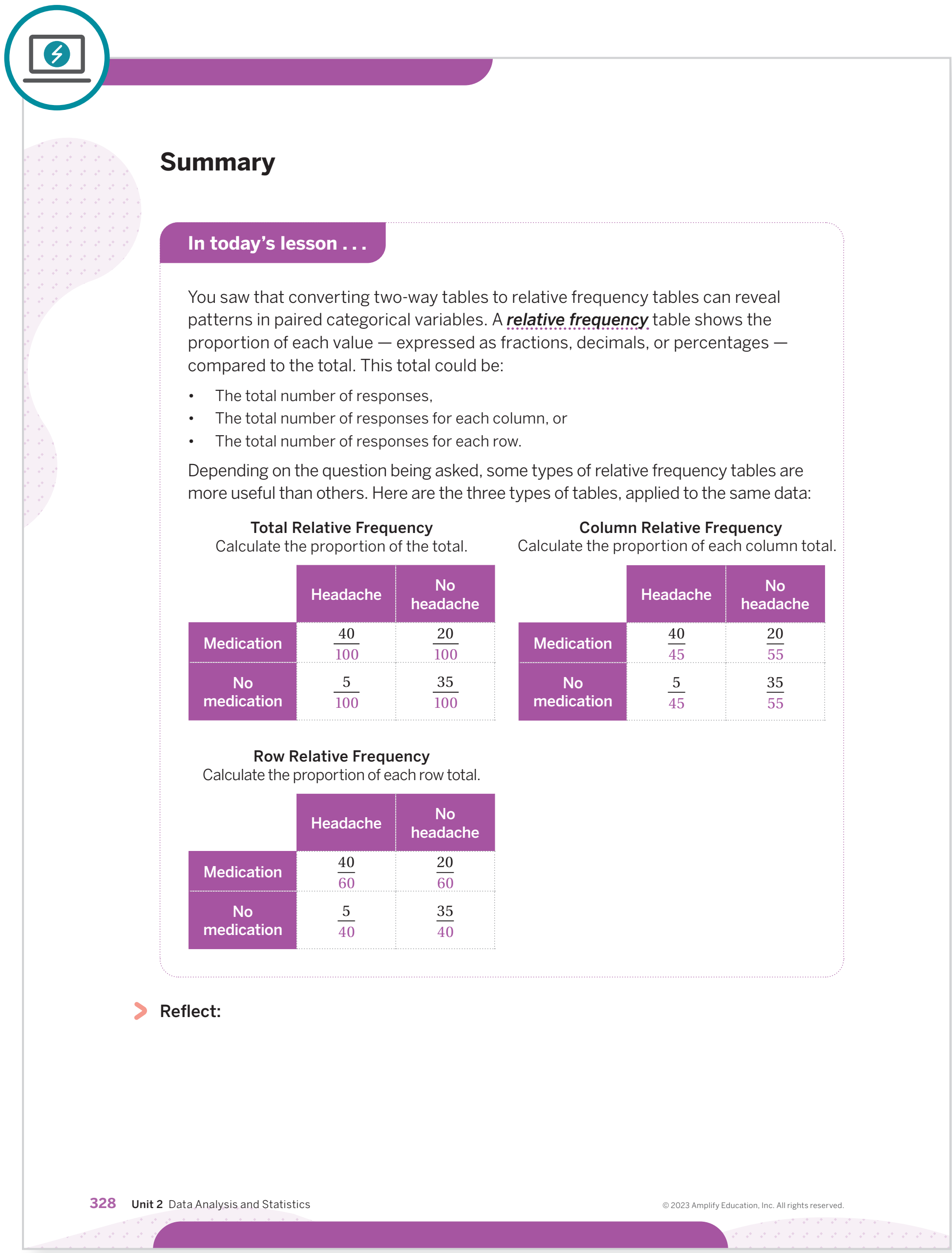
Facilitate a class discussion by asking these questions:

- "What new information did you learn about the Flint water crisis?"
- "What did you notice or wonder about the duration of the crisis? Explain your thinking."
- "What do you think would have been an appropriate settlement? How did you mathematically arrive at this figure? How does your thinking compare to the settlement that was reached?"

Whole Class | 5 min

Summary

Review and synthesize creating and interpreting relative frequency tables.



Summary

In today's lesson . . .

You saw that converting two-way tables to relative frequency tables can reveal patterns in paired categorical variables. A ***relative frequency*** table shows the proportion of each value — expressed as fractions, decimals, or percentages — compared to the total. This total could be:

- The total number of responses,
- The total number of responses for each column, or
- The total number of responses for each row.

Depending on the question being asked, some types of relative frequency tables are more useful than others. Here are the three types of tables, applied to the same data:

Total Relative Frequency
Calculate the proportion of the total.

	Headache	No headache
Medication	$\frac{40}{100}$	$\frac{20}{100}$
No medication	$\frac{5}{100}$	$\frac{35}{100}$

Column Relative Frequency
Calculate the proportion of each column total.

	Headache	No headache
Medication	$\frac{40}{45}$	$\frac{20}{55}$
No medication	$\frac{5}{45}$	$\frac{35}{55}$

Row Relative Frequency
Calculate the proportion of each row total.

	Headache	No headache
Medication	$\frac{40}{60}$	$\frac{20}{60}$
No medication	$\frac{5}{40}$	$\frac{35}{40}$

Reflect:

328 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the two way table.

	Headache	No headache
Medication	2%	98%
No medication	5%	95%

Have students share what type of relative frequency table is shown and how they know.

Highlight that column, row, or total relative frequency tables are found by dividing values by the respective column totals, row totals, or overall total.

Formalize vocabulary: *relative frequency table*

Ask, "Why are these tables called relative frequency tables? Why are they useful?" Sample response: Relative frequency means the proportion of time something occurs. These tables are helpful because they show the proportion of times something occurs within a subset or within the entire group.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How do you know when a two-way table or relative frequency table is more helpful when investigating data?"

Math Language Development

MLR2: Collect and Display

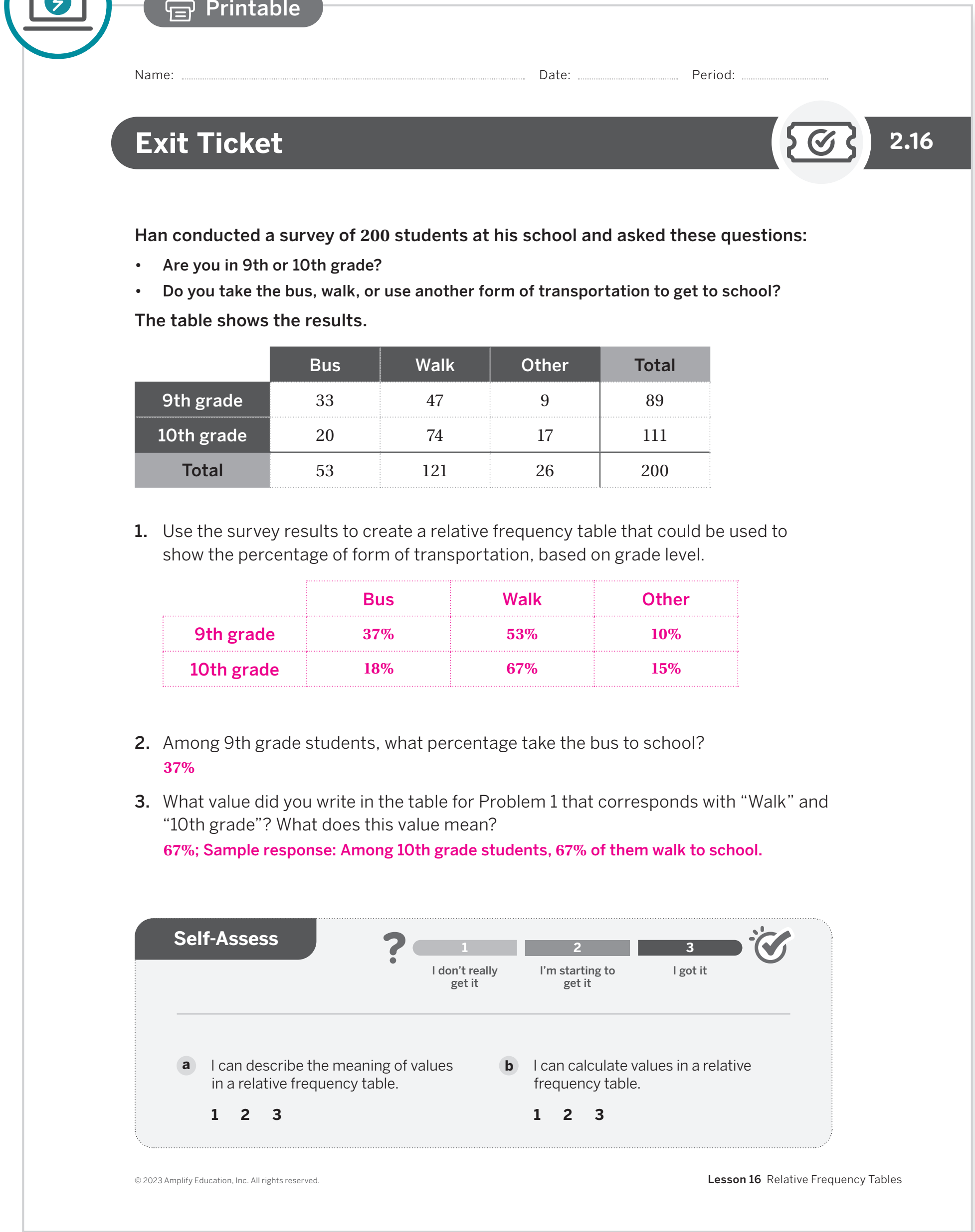
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *relative frequency table* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by completing and interpreting a relative frequency table by using a two-way table.

Printable



Name: Date: Period:

Exit Ticket 2.16

Han conducted a survey of 200 students at his school and asked these questions:

- **Are you in 9th or 10th grade?**
- **Do you take the bus, walk, or use another form of transportation to get to school?**

The table shows the results.

	Bus	Walk	Other	Total
9th grade	33	47	9	89
10th grade	20	74	17	111
Total	53	121	26	200

1. Use the survey results to create a relative frequency table that could be used to show the percentage of form of transportation, based on grade level.

	Bus	Walk	Other
9th grade	37%	53%	10%
10th grade	18%	67%	15%

2. Among 9th grade students, what percentage take the bus to school?
37%

3. What value did you write in the table for Problem 1 that corresponds with "Walk" and "10th grade"? What does this value mean?
67%; Sample response: Among 10th grade students, 67% of them walk to school.

Self-Assess

? 1	2	3 ✓
I don't really get it	I'm starting to get it	I got it

a I can describe the meaning of values in a relative frequency table.
1 2 3

b I can calculate values in a relative frequency table.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Success looks like . . .

- **Language Goal:** Describing the meaning of values in a relative frequency table. **(Speaking and Listening, Writing)**
  - » Explaining the meaning of the value for "Walk" and "10th grade" in Problem 3.
- **Goal:** Calculating the values in a relative frequency table.
  - » Determining the relative frequency table for the percentage of form of transportation for each grade in Problem 1.

Suggested next steps

If students incorrectly create the relative frequency table in Problem 1, consider:

- Reviewing how to create a relative frequency table from Activity 2.
- Assigning Practice Problem 2.
- Asking, "What subgroup are you focusing on? How does this help determine the type of relative frequency table to use?"

If students determine the incorrect percentage in Problem 2 or 3, consider:

- Reviewing interpreting relative frequency tables from Activity 1.
- Assigning Practice Problem 1.
- Asking, "How can you use labels from the table to help determine the percentage?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The focus of this lesson was relative frequency tables. How did it go?
- Which groups of students did and didn't have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Math Language Development

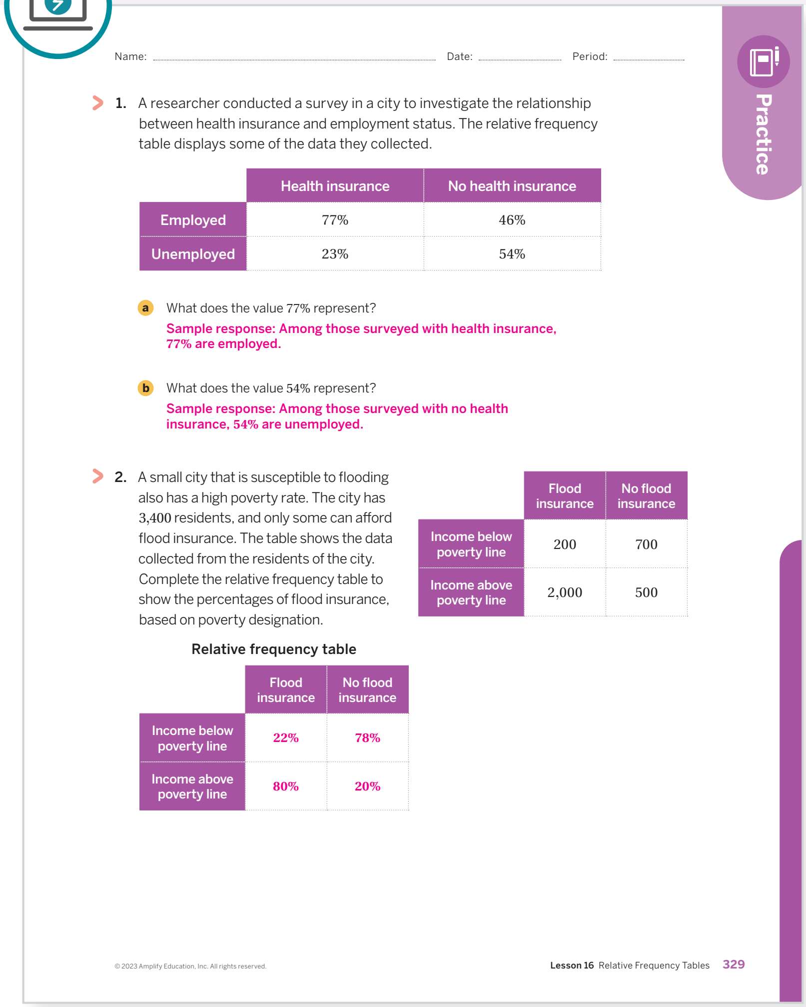
Language Goal: Describing the meaning of values in a relative frequency table.

Reflect on students' language development toward this goal.

- How have students progressed in their interpretations of values in a two-way table and relative frequency table in Lessons 15 and 16? Do their descriptions demonstrate they understand the differences between the two types of tables?
- Do students' responses to Problem 3 of the Exit Ticket include percentages? How can you help them be more precise in their responses?

Practice

Independent



Practice

Name: ____________ Date: ________ Period: ________

1. A researcher conducted a survey in a city to investigate the relationship between health insurance and employment status. The relative frequency table displays some of the data they collected.

	Health insurance	No health insurance
Employed	77%	46%
Unemployed	23%	54%

a. What does the value 77% represent?

Sample response: Among those surveyed with health insurance, 77% are employed.

b. What does the value 54% represent?

Sample response: Among those surveyed with no health insurance, 54% are unemployed.

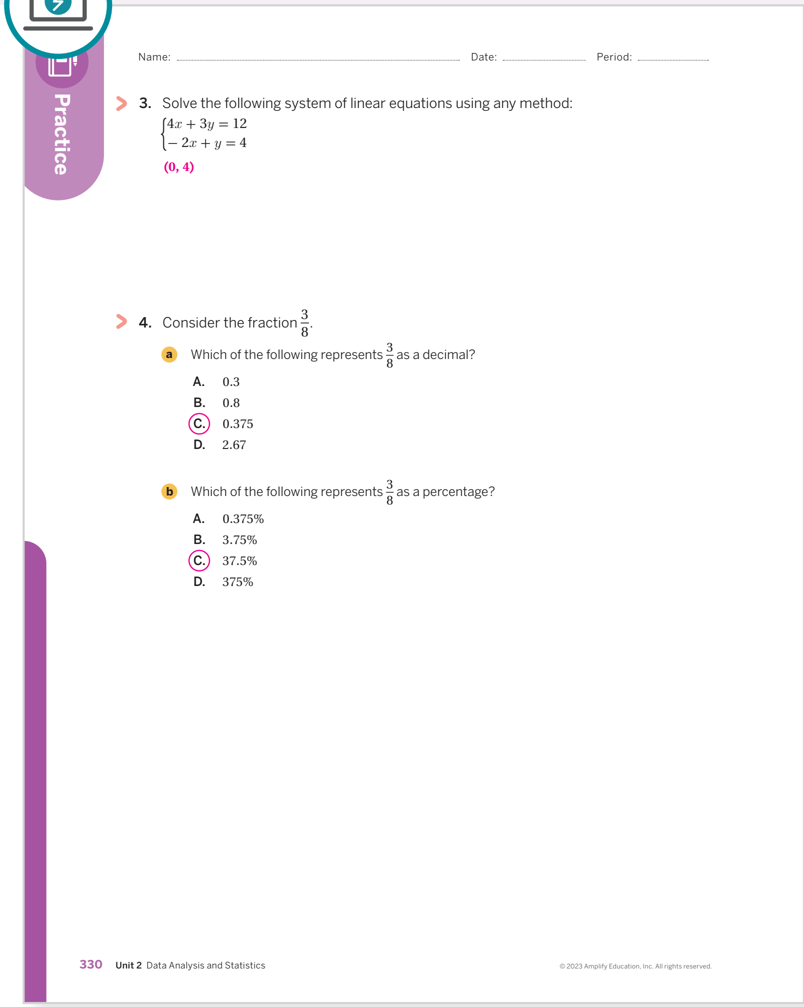
2. A small city that is susceptible to flooding also has a high poverty rate. The city has 3,400 residents, and only some can afford flood insurance. The table shows the data collected from the residents of the city. Complete the relative frequency table to show the percentages of flood insurance, based on poverty designation.

	Flood insurance	No flood insurance
Income below poverty line	200	700
Income above poverty line	2,000	500

Relative frequency table

	Flood insurance	No flood insurance
Income below poverty line	22%	78%
Income above poverty line	80%	20%

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ____________ Date: ________ Period: ________

3. Solve the following system of linear equations using any method:

$$\begin{cases} 4x + 3y = 12 \\ -2x + y = 4 \end{cases}$$

(0, 4)

4. Consider the fraction $\frac{3}{8}$.

a. Which of the following represents $\frac{3}{8}$ as a decimal?

A. 0.3
B. 0.8
[✓ C.] 0.375
D. 2.67

b. Which of the following represents $\frac{3}{8}$ as a percentage?

A. 0.375%
B. 3.75%
[✓ C.] 37.5%
D. 375%

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
Spiral	3	Unit 1 Lesson 21	2
Formative ⚡	4	Unit 2 Lesson 17	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 2 | **LESSON 17**

Associations in Categorical Data

Let's look for associations in categorical data.

Focus

Goal

1. Determine if there is a possible association between two variables from two-way and relative frequency tables.

Coherence

Today

Students analyze data presented to them in two-way tables to look for patterns and possible associations that could exist between two variables in the context of how climate change has affected people in other countries. Students create and analyze relative frequency tables in order to look for associations. In Activity 2, students sort cards based on whether the two-way tables on each card have a likely or unlikely association.

Previously

In Lessons 15 and 16, students created and interpreted two-way and relative frequency tables.

Coming Soon

In Lesson 18, students will analyze scatter plots to determine the strength of association in a data set.

Rigor

- Students build **conceptual understanding** of association in categorical data.
- Students compare differences in relative frequency percentages to develop **procedural fluency**.
- Students **apply** their ability to look for association in the context of climate change.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Are you ready for more?*
- Activity 1 PDF, *Are you ready for more?* (answers)
- Activity 2 PDF, pre-cut cards, one set per pair

Math Language Development

New words

- ***association***

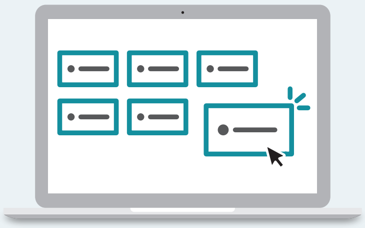
Review words

- *relative frequency*
- *two-way table*

Amps | Featured Activity

Activity 2
Digital Card Sort

Students match cards with different two-way tables that have an unlikely or likely association by dragging them on screen.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated with their ability to determine which value from the two-way tables to use as the total number of outcomes when calculating the probabilities. Encourage students to annotate each problem to help determine if the focus is on a subgroup or the total group. Have students ask classmates to explain their strategy in their own words.

Modifications to Pacing

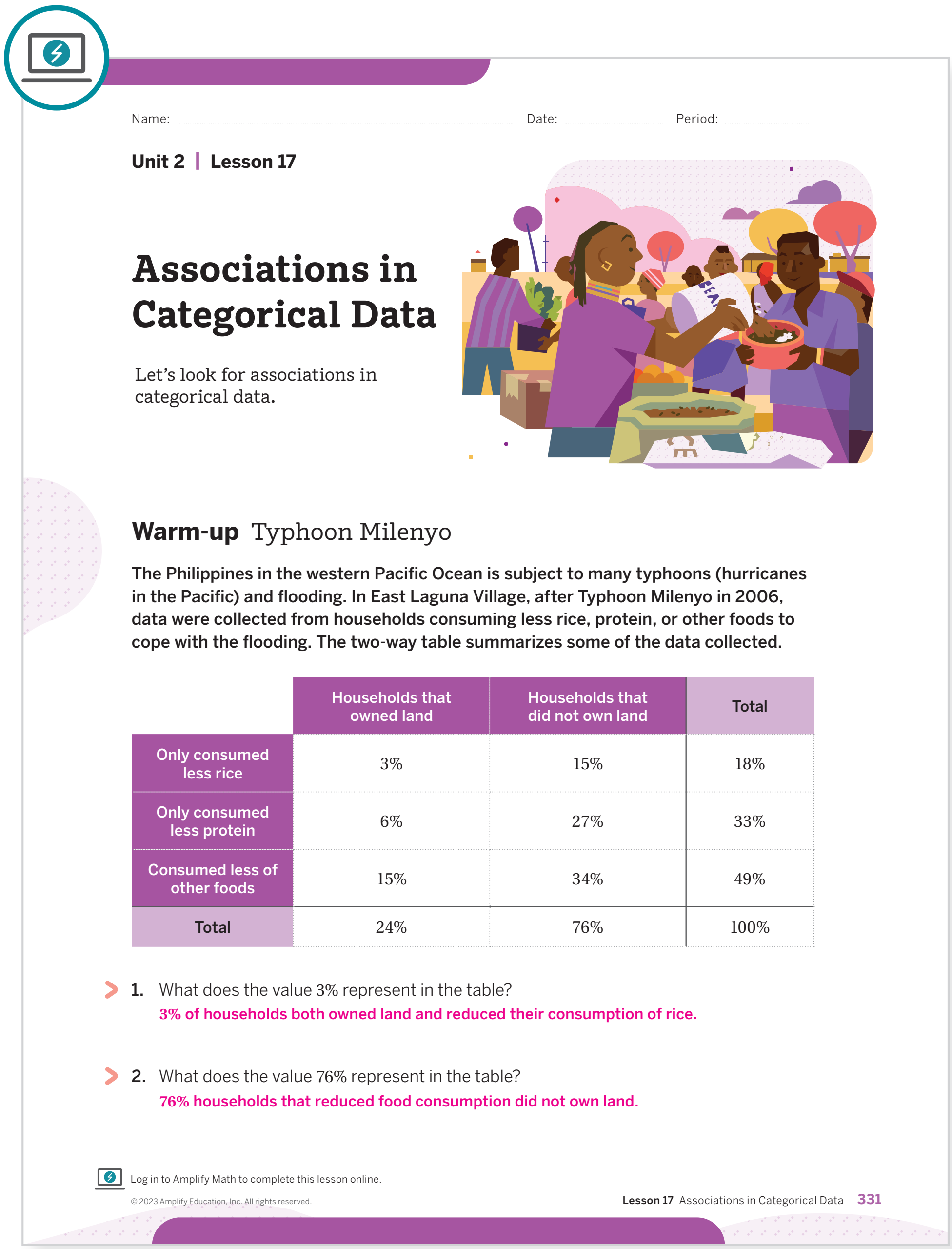
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 2 may be omitted.
- In **Activity 2**, either Cards 5–8, or 7 and 8 can be removed.

Independent | 5 min

Warm-up Typhoon Milenyo

Students recall how to interpret values in a relative frequency table to prepare for interpreting differences in percentages when looking for association.



Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 17

Associations in Categorical Data

Let's look for associations in categorical data.

Warm-up Typhoon Milenyo

The Philippines in the western Pacific Ocean is subject to many typhoons (hurricanes in the Pacific) and flooding. In East Laguna Village, after Typhoon Milenyo in 2006, data were collected from households consuming less rice, protein, or other foods to cope with the flooding. The two-way table summarizes some of the data collected.

	Households that owned land	Households that did not own land	Total
Only consumed less rice	3%	15%	18%
Only consumed less protein	6%	27%	33%
Consumed less of other foods	15%	34%	49%
Total	24%	76%	100%

1. What does the value 3% represent in the table?
   3% of households both owned land and reduced their consumption of rice.

2. What does the value 76% represent in the table?
   76% households that reduced food consumption did not own land.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 17 Associations in Categorical Data 331

1 Launch

Read the passage out loud. Activate background knowledge by asking students what they know about hurricanes, storms, and typhoons.

2 Monitor

Help students get started by having them locate where the percentages can be found in the table and circling the headings in the table.

Look for points of confusion:

- **Only using one descriptor when interpreting Problem 1.** Ask, "Where does 3% fall in the table? How many headings in the table describe this value?"

Look for productive strategies:

- Using table headings when interpreting the values.
- Recognizing the difference between how to interpret a cell in the table versus a row or column total.

3 Connect

Display the relative frequency table.

Have individual students share how they interpreted the relative frequency percentages.

Highlight that locating where each value falls in the table, using the headings, and determining if the value belongs to a "total" are all important for interpreting these percentages.

Ask, "Are there any patterns you notice in the table?" Sample response: Households without land had to reduce food consumption at a higher rate in all categories compared to households with land.

Math Language Development

MLR8: Discussion Supports

During the Connect, listen for the language students use to interpret each relative frequency percentage. Draw connections to the words and phrases shown in the headings of the two-way table. For example, to interpret 3%, draw students' attention to the headings "Households that owned land" and "Only consumed less rice."

English Learners

Use gestures, such as pointing to the column and row headings in the two-way table, as students share how they interpreted each percentage.

Power-up

To power up students' ability to write a fraction as a percentage, have students complete:

Recall that *percent* means out of 100. Which of the following percents represents $\frac{2}{5}$?

A. 20%
[✓ B.] 40%
C. 2%
D. 4%

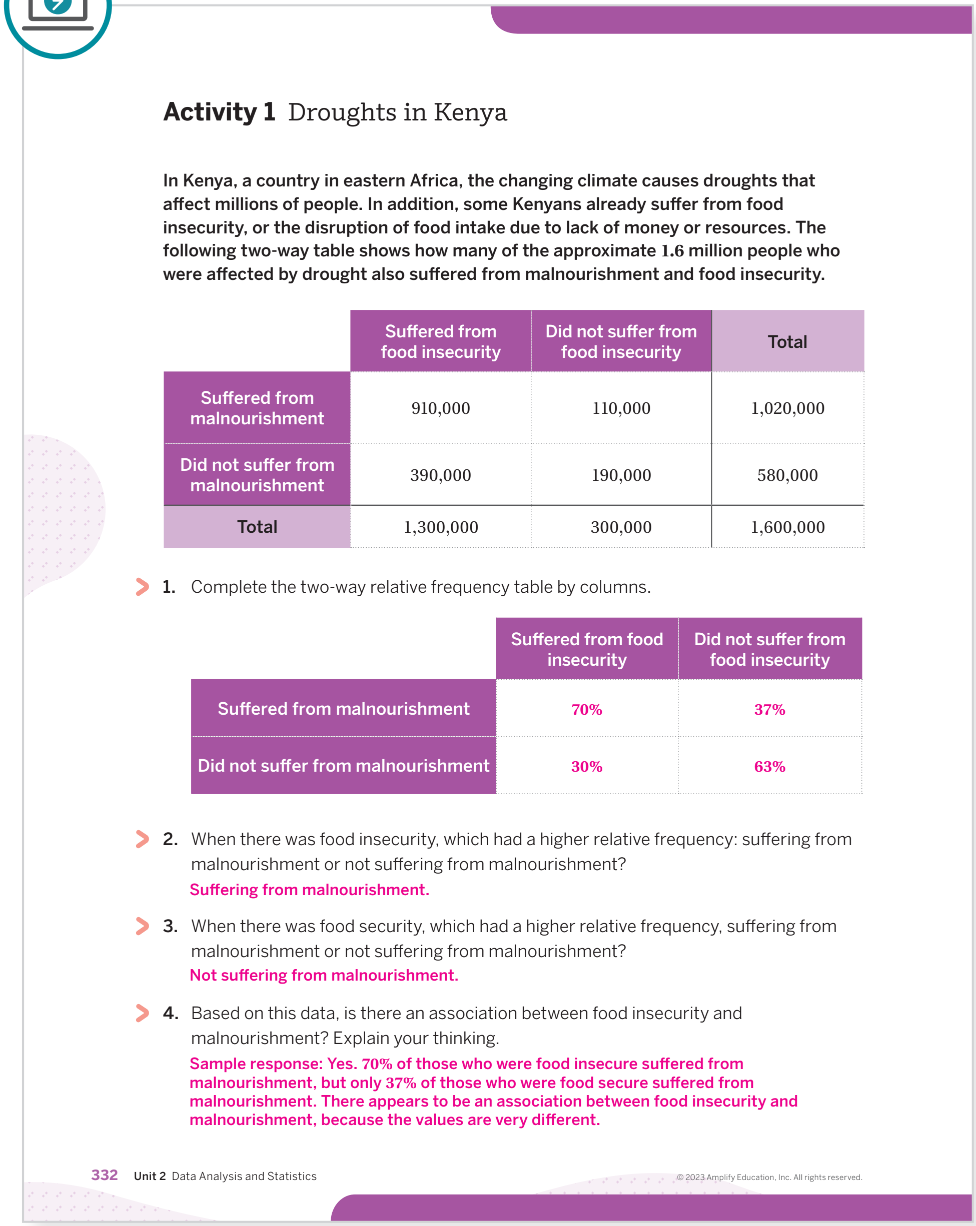
Use: Activity 1

Informed by: Performance on Lesson 16, Practice Problem 4

Pairs | 15 min

Activity 1 Droughts in Kenya

Students create and interpret data from a relative frequency table to look for associations in data on food insecurity and malnourishment.



Activity 1 Droughts in Kenya

In Kenya, a country in eastern Africa, the changing climate causes droughts that affect millions of people. In addition, some Kenyans already suffer from food insecurity, or the disruption of food intake due to lack of money or resources. The following two-way table shows how many of the approximate 1.6 million people who were affected by drought also suffered from malnourishment and food insecurity.

	Suffered from food insecurity	Did not suffer from food insecurity	Total
Suffered from malnourishment	910,000	110,000	1,020,000
Did not suffer from malnourishment	390,000	190,000	580,000
Total	1,300,000	300,000	1,600,000

1. Complete the two-way relative frequency table by columns.

	Suffered from food insecurity	Did not suffer from food insecurity
Suffered from malnourishment	70%	37%
Did not suffer from malnourishment	30%	63%

2. When there was food insecurity, which had a higher relative frequency: suffering from malnourishment or not suffering from malnourishment?
   Suffering from malnourishment.

3. When there was food security, which had a higher relative frequency, suffering from malnourishment or not suffering from malnourishment?
   Not suffering from malnourishment.

4. Based on this data, is there an association between food insecurity and malnourishment? Explain your thinking.
   Sample response: Yes. 70% of those who were food insecure suffered from malnourishment, but only 37% of those who were food secure suffered from malnourishment. There appears to be an association between food insecurity and malnourishment, because the values are very different.

332 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in pairs. Read the passage together as a class. Ask, “What does it mean to be malnourished?”

2 Monitor

Help students get started by asking, “What does it mean to create a two-way relative frequency table by columns?”

Look for points of confusion:

- **Creating a relative frequency table using the overall total or row total.** Have students read the problem again, then ask, “What values should be used when calculating the values for this table?”

Look for productive strategies:

- Showing all calculations for the relative frequency table.
- Recognizing that the significant difference between the percentages found in each column show a pattern, or association, in the data.

3 Connect

Have pairs of students share how they determined the values for the relative frequency table and if there was an association between food insecurity and malnourishment.

Display the completed two-way relative frequency table.

Define the term ***association***.

Highlight that association is possible, or likely, when relative frequencies by row or column are significantly different from the other row or column subgroup.

Ask, “What would no association have looked like for this problem?”
Sample response: If all the percentages calculated had been closer to each other in values, no association would have been more likely.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Consider displaying a map of Africa, showing the location of Kenya. Explain how droughts, especially ones that last for a long time, affect the quantity of food resources that are available for people.

Accessibility: *Guide Processing and Visualization*

Demonstrate how to complete the first cell of the table in Problem 1 to ensure students understand this relative frequency table is by *columns*, not totals or rows. Ask students to highlight or circle the phrase “by columns” in the directions for Problem 1. Consider displaying the calculations you demonstrate for the first cell, such as $910{,}000 \div 1{,}300{,}000 = 0.70$.

Extension: *Math Enrichment*

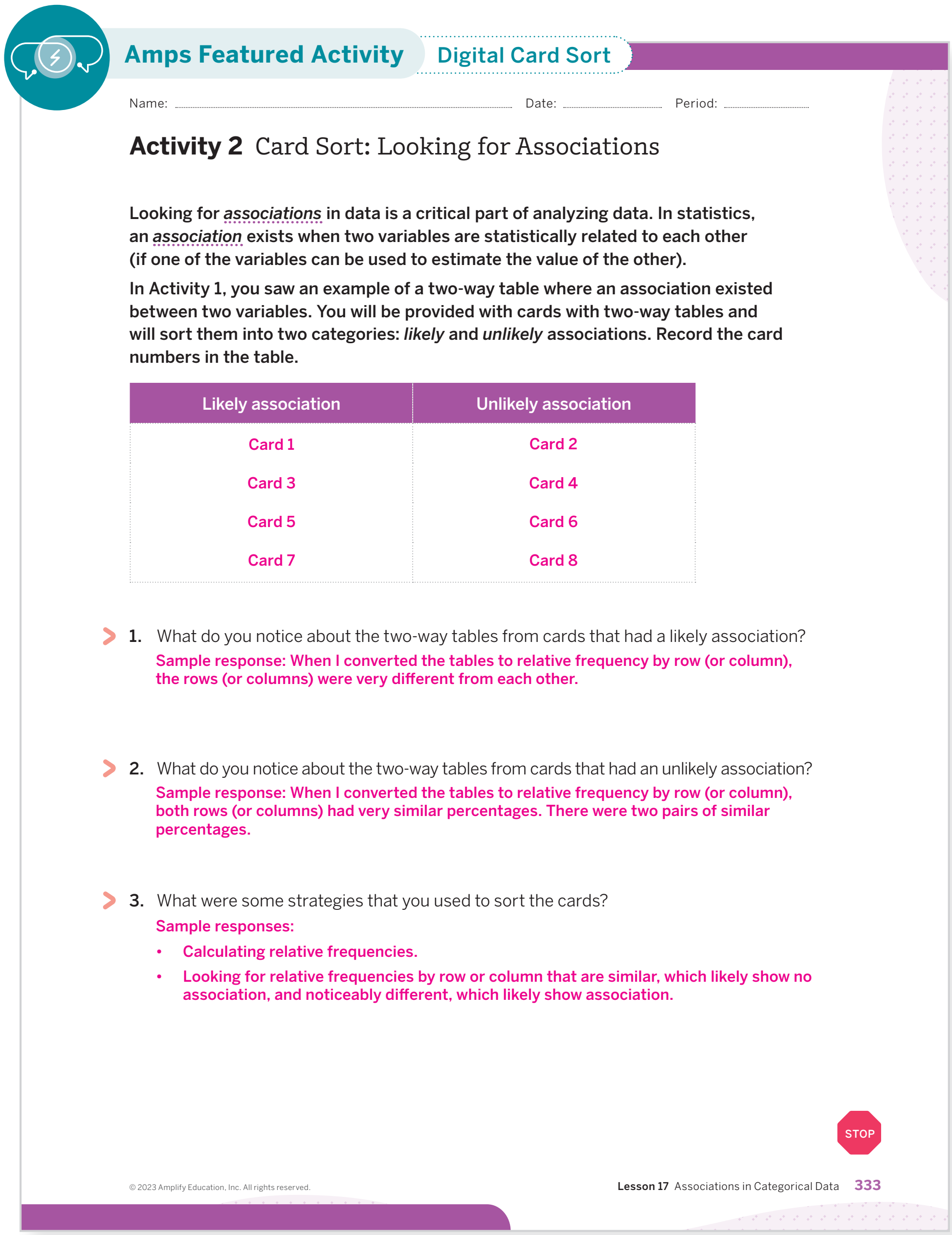
Display or provide students with a copy of the *Are you ready for more?* PDF and have them complete the problems. Students are presented with an incomplete two-way table and are asked to determine values that would complete the table to show that there is an association, as well as values that would show there is no association between the data.

Pairs | 20 min

Activity 2 Card Sort: Looking for Associations

Students sort cards with two-way tables into either likely or unlikely association to develop conceptual understanding of when association could exist.

Amps Featured Activity Digital Card Sort



Name: ________ Date: ________ Period: ________

Activity 2 Card Sort: Looking for Associations

Looking for *associations* in data is a critical part of analyzing data. In statistics, an *association* exists when two variables are statistically related to each other (if one of the variables can be used to estimate the value of the other).

In Activity 1, you saw an example of a two-way table where an association existed between two variables. You will be provided with cards with two-way tables and will sort them into two categories: *likely* and *unlikely* associations. Record the card numbers in the table.

Likely association	Unlikely association
Card 1	Card 2
Card 3	Card 4
Card 5	Card 6
Card 7	Card 8

1. What do you notice about the two-way tables from cards that had a likely association?
   Sample response: When I converted the tables to relative frequency by row (or column), the rows (or columns) were very different from each other.

2. What do you notice about the two-way tables from cards that had an unlikely association?
   Sample response: When I converted the tables to relative frequency by row (or column), both rows (or columns) had very similar percentages. There were two pairs of similar percentages.

3. What were some strategies that you used to sort the cards?
   Sample responses:
   - Calculating relative frequencies.
   - Looking for relative frequencies by row or column that are similar, which likely show no association, and noticeably different, which likely show association.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Distribute the cards from the Activity 2 PDF to each pair of students. Conduct the ***Card Sort*** routine, providing students with time to sort the cards and then complete Problems 1–3.

2 Monitor

Help students get started by asking, "How have you looked for association in prior activities?"

Look for points of confusion:

- **Computing a relative frequency table using the whole table total.** Ask, "What type of relative frequency tables do you use to look for association? Why?"

Look for productive strategies:

- Creating a row or column relative frequency table.
- Recognizing drastic differences in percentages show likely association and similar percentages show unlikely association.

3 Connect

Have pairs of students share what cards belong to each category and the strategies they used to sort the cards.

Display the category to which each card belongs.

Highlight that while sometimes looking at the two-way frequency table can give clues to a likely or unlikely association, it is only through relative frequency that these conclusions can be drawn. This is because different subgroups will not have an equal number of values.

Ask, "Why is it important to calculate relative frequencies when looking for association?" Sample response: Because not every subgroup will have the same frequency of values, I must find the proportion of one variable in relation to the other to look for likely or unlikely association.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest students create relative frequency tables below the existing two-way table on each card, to help with their thinking. Consider providing them with copies of a blank two-way table template that they can use to tape down on each card and calculate the relative frequencies.

Extension: *Math Enrichment*

Ask students to alter the values in one of the cards that did not show a likely association so that it now would show a likely association. Ask them to explain if they think it would make sense, given the labels, for there to be an unlikely or likely association.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share how they sorted each card, call their attention to the language they use in their explanations. For example, for two-way tables that show an unlikely association, the relative frequencies (by row or by column) are similar to each other. For two-way tables that show a likely association, the relative frequencies (by row or by column) are noticeably different from each other.

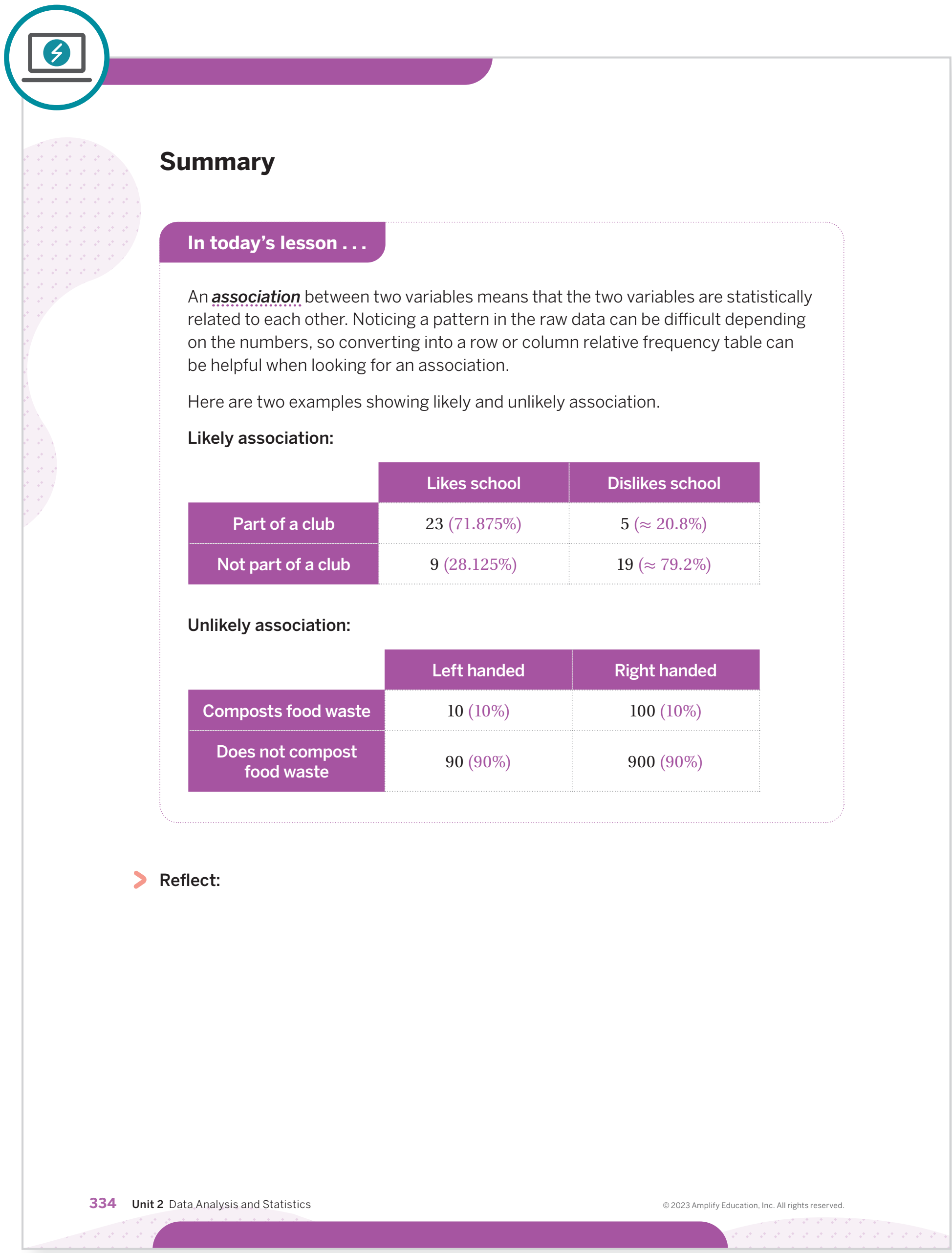
English Learners

Display a sample relative frequency table that shows a likely association and one that shows an unlikely association. Annotate each table as *likely* or *unlikely* and highlight the similarities or differences among the values.

Whole Class | 5 min

Summary

Review and synthesize what association is and how to determine if a two-way relative frequency table has a likely or unlikely association.



Summary

In today's lesson . . .

An ***association*** between two variables means that the two variables are statistically related to each other. Noticing a pattern in the raw data can be difficult depending on the numbers, so converting into a row or column relative frequency table can be helpful when looking for an association.

Here are two examples showing likely and unlikely association.

Likely association:

	Likes school	Dislikes school
Part of a club	23 (71.875%)	5 (≈ 20.8%)
Not part of a club	9 (28.125%)	19 (≈ 79.2%)

Unlikely association:

	Left handed	Right handed
Composts food waste	10 (10%)	100 (10%)
Does not compost food waste	90 (90%)	900 (90%)

Reflect:

334 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the two way table.

	Headache	No headache
Medication	45%	51%
No medication	55%	49%

Have students share their thinking on whether or not this relative frequency table shows a likely or unlikely association.

Highlight that an association between two variables means the variables are statistically related, and row and column relative frequencies are often used to look for association.

Formalize vocabulary: ***association***

Ask, "When is association likely or unlikely?"
Sample response: When row or column relative frequencies are vastly different, association is likely. When row or column relative frequencies are similar, association is unlikely.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does association mean?"
- "What is the difference between two realtive frequency tables that have a likely and unlikely association?"

Math Language Development

MLR2: Collect and Display

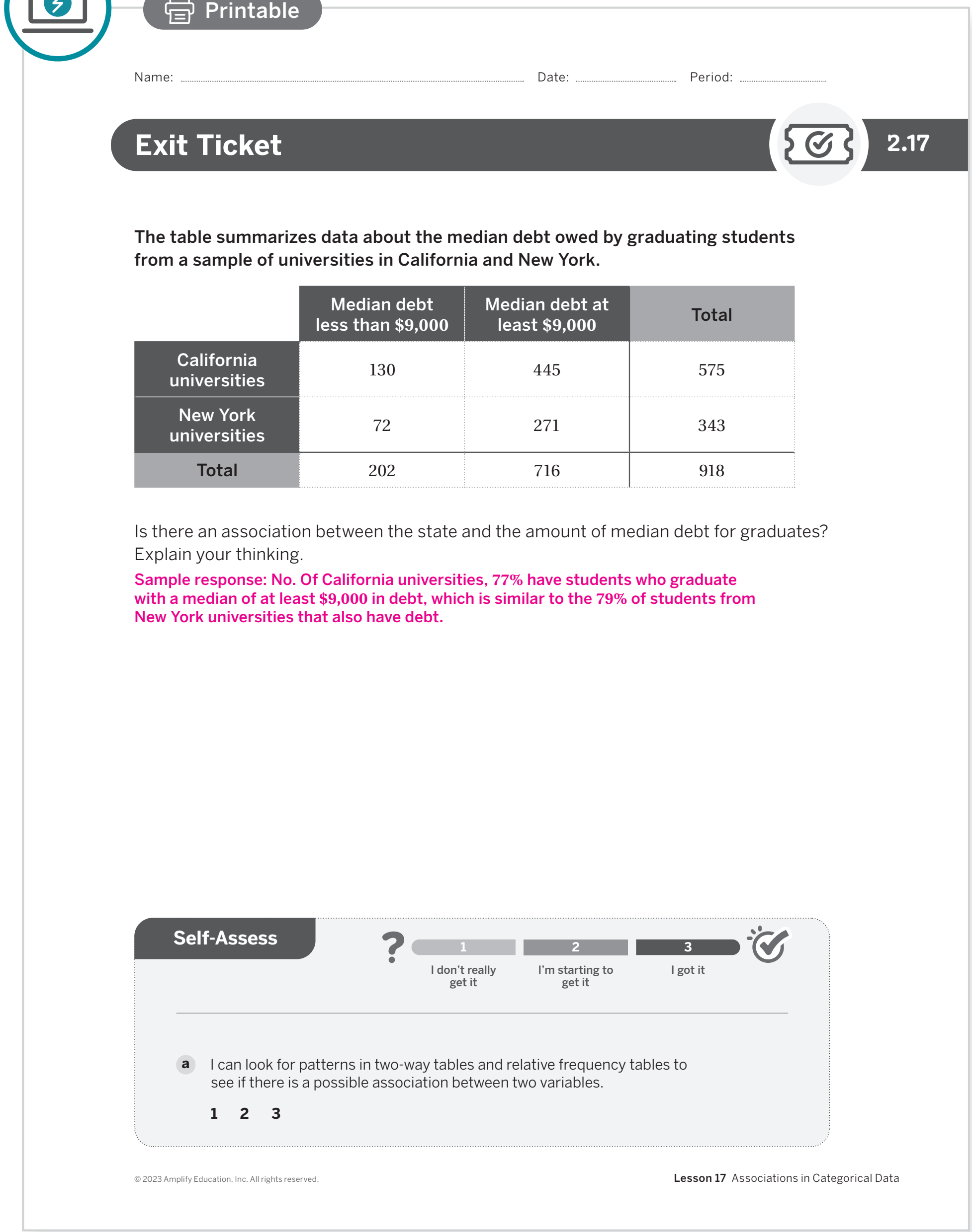
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask students to review and reflect on any terms and phrases related to the term *association* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining if association exists given a two-way table.

Printable



Name: ______________ Date: ______ Period: ______

Exit Ticket

2.17

The table summarizes data about the median debt owed by graduating students from a sample of universities in California and New York.

	Median debt less than $9,000	Median debt at least $9,000	Total
California universities	130	445	575
New York universities	72	271	343
Total	202	716	918

Is there an association between the state and the amount of median debt for graduates? Explain your thinking.

Sample response: No. Of California universities, 77% have students who graduate with a median of at least $9,000 in debt, which is similar to the 79% of students from New York universities that also have debt.

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can look for patterns in two-way tables and relative frequency tables to see if there is a possible association between two variables.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Success looks like . . .

- **Goal:** Determining if there is a possible association between two variables from two-way and relative frequency tables.
  - » Explaining whether there is an association between the state and the amount of median debt for graduates.

Suggested next steps

If students say an association exists, consider:

- Reviewing how to create a relative frequency table and look for association from Activity 1.
- Assigning Practice Problem 2.
- Asking, “What kind of table are we given? What might be helpful to determine using this given data?”

Professional Learning

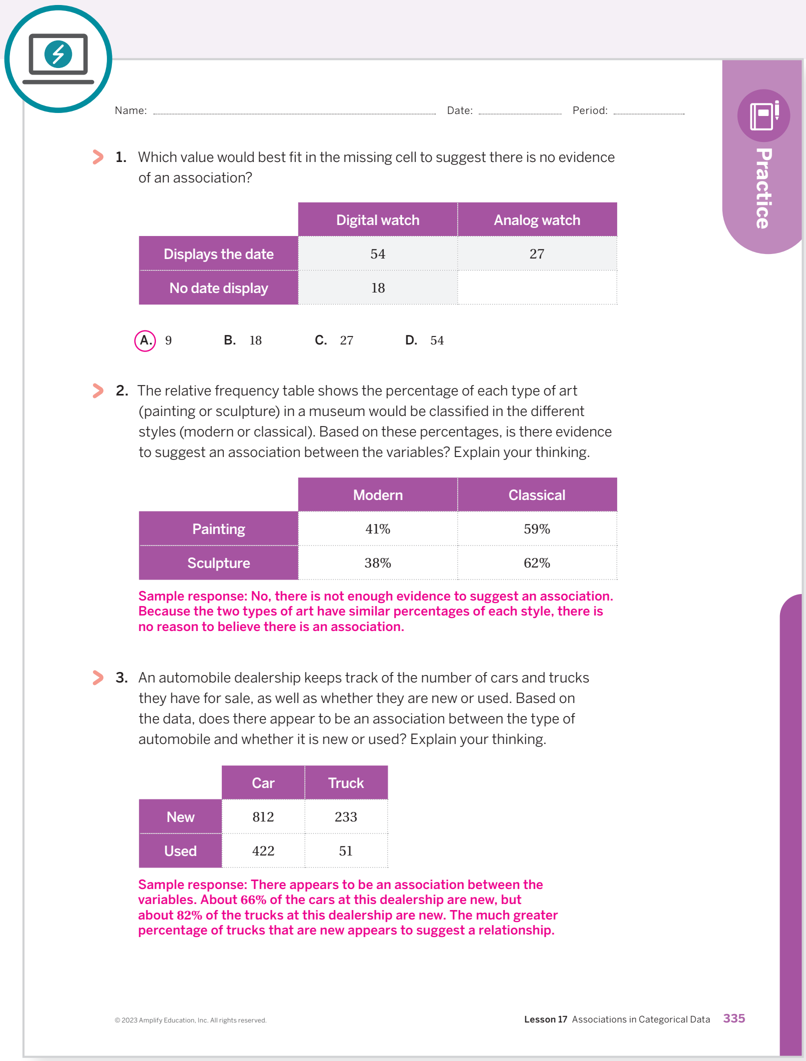
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn’t work today? In what ways did sorting cards in Activity 2 go as planned?
- What surprised you as your students worked on sorting cards in Activity 2? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______________ Date: ________ Period: ________

Practice

1. Which value would best fit in the missing cell to suggest there is no evidence of an association?

	Digital watch	Analog watch
Displays the date	54	27
No date display	18	

[✓ A.] 9 B. 18 C. 27 D. 54

2. The relative frequency table shows the percentage of each type of art (painting or sculpture) in a museum would be classified in the different styles (modern or classical). Based on these percentages, is there evidence to suggest an association between the variables? Explain your thinking.

	Modern	Classical
Painting	41%	59%
Sculpture	38%	62%

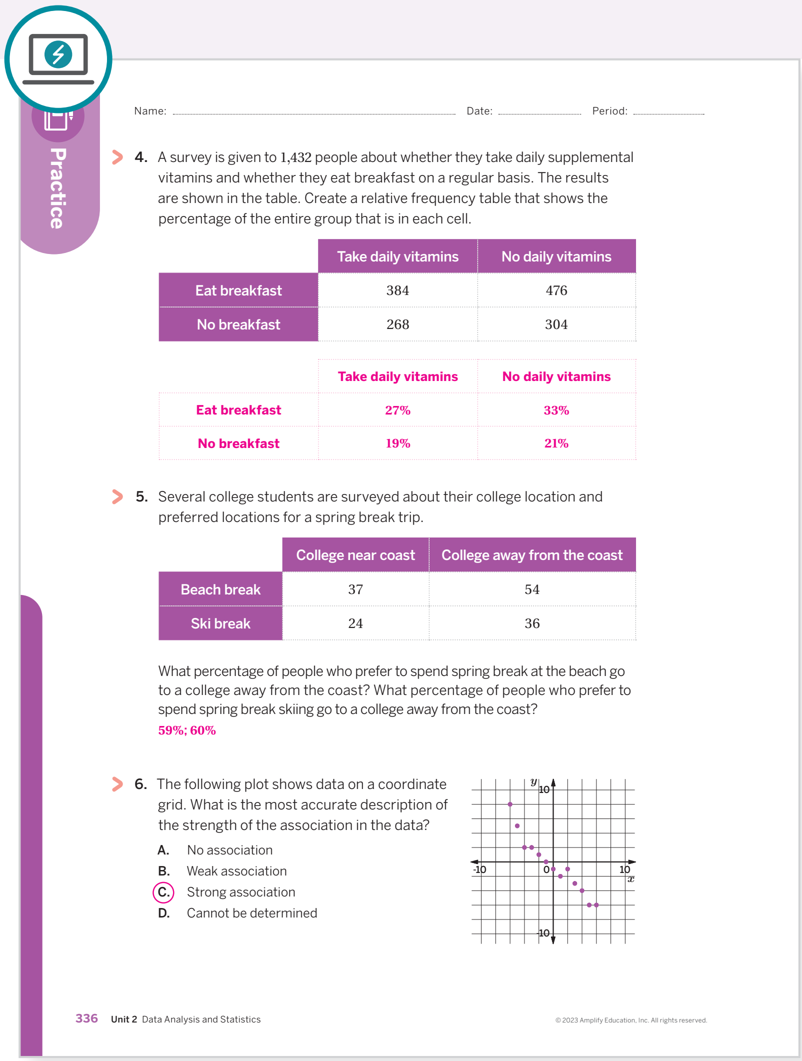
Sample response: No, there is not enough evidence to suggest an association. Because the two types of art have similar percentages of each style, there is no reason to believe there is an association.

3. An automobile dealership keeps track of the number of cars and trucks they have for sale, as well as whether they are new or used. Based on the data, does there appear to be an association between the type of automobile and whether it is new or used? Explain your thinking.

	Car	Truck
New	812	233
Used	422	51

Sample response: There appears to be an association between the variables. About 66% of the cars at this dealership are new, but about 82% of the trucks at this dealership are new. The much greater percentage of trucks that are new appears to suggest a relationship.

© 2023 Amplify Education, Inc. All rights reserved.



Name: ______________ Date: ________ Period: ________

Practice

4. A survey is given to 1,432 people about whether they take daily supplemental vitamins and whether they eat breakfast on a regular basis. The results are shown in the table. Create a relative frequency table that shows the percentage of the entire group that is in each cell.

	Take daily vitamins	No daily vitamins
Eat breakfast	384	476
No breakfast	268	304

	Take daily vitamins	No daily vitamins
Eat breakfast	27%	33%
No breakfast	19%	21%

5. Several college students are surveyed about their college location and preferred locations for a spring break trip.

	College near coast	College away from the coast
Beach break	37	54
Ski break	24	36

What percentage of people who prefer to spend spring break at the beach go to a college away from the coast? What percentage of people who prefer to spend spring break skiing go to a college away from the coast?
59%; 60%

6. The following plot shows data on a coordinate grid. What is the most accurate description of the strength of the association in the data?

A. No association
B. Weak association
[✓ C.] Strong association
D. Cannot be determined

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 16	2
	5	Unit 2 Lesson 16	2
Formative ⚡	6	Unit 2 Lesson 18	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

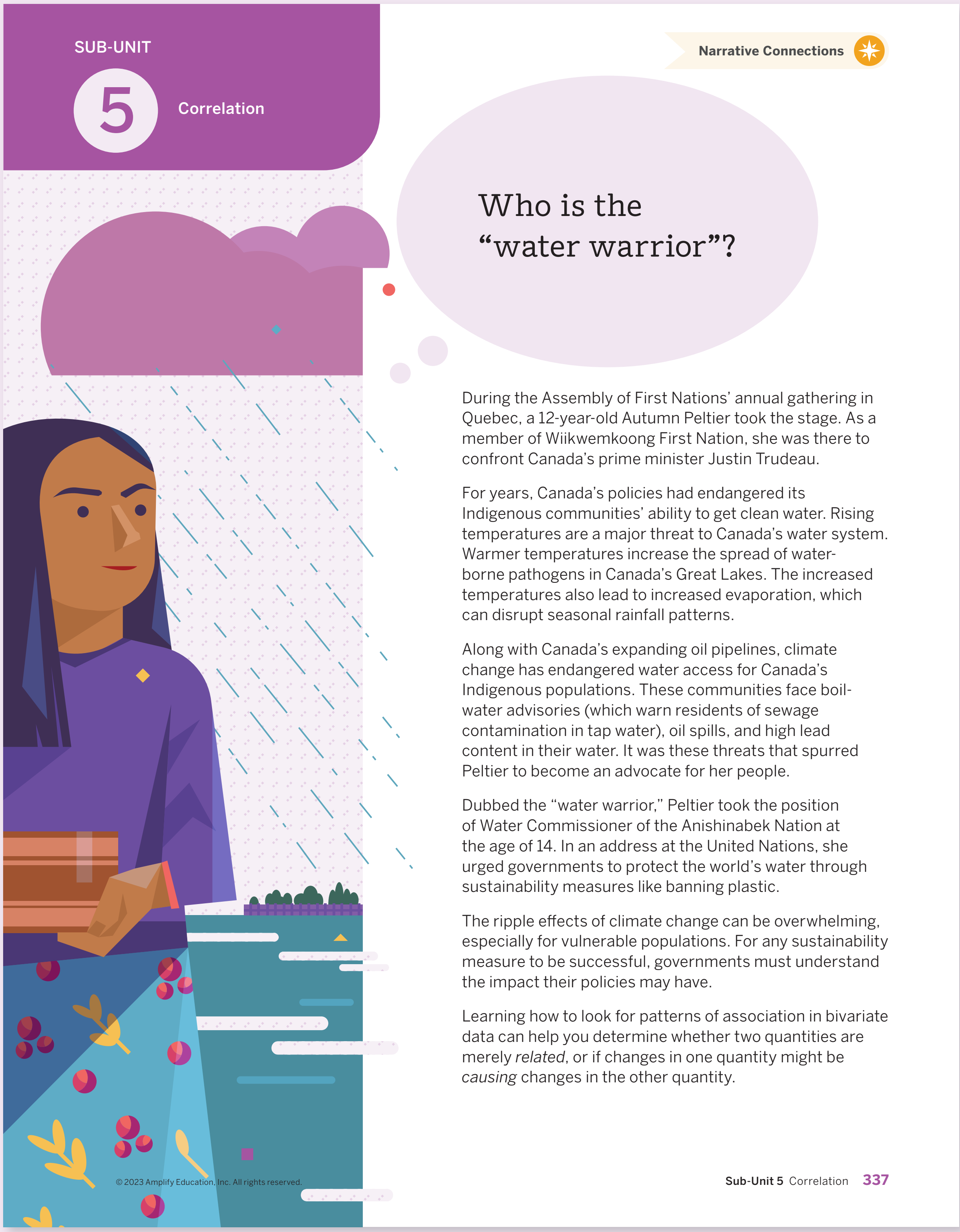
For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 5

Correlation

Whole Class

In this Sub-Unit, students encounter the correlation coefficient and learn how it is computed. They also explore statistical fallacies, like correlation versus causation.



SUB-UNIT

5 Correlation

Narrative Connections

Who is the “water warrior”?

During the Assembly of First Nations’ annual gathering in Quebec, a 12-year-old Autumn Peltier took the stage. As a member of Wiikwemkoong First Nation, she was there to confront Canada’s prime minister Justin Trudeau.

For years, Canada’s policies had endangered its Indigenous communities’ ability to get clean water. Rising temperatures are a major threat to Canada’s water system. Warmer temperatures increase the spread of water-borne pathogens in Canada’s Great Lakes. The increased temperatures also lead to increased evaporation, which can disrupt seasonal rainfall patterns.

Along with Canada’s expanding oil pipelines, climate change has endangered water access for Canada’s Indigenous populations. These communities face boil-water advisories (which warn residents of sewage contamination in tap water), oil spills, and high lead content in their water. It was these threats that spurred Peltier to become an advocate for her people.

Dubbed the “water warrior,” Peltier took the position of Water Commissioner of the Anishinabek Nation at the age of 14. In an address at the United Nations, she urged governments to protect the world’s water through sustainability measures like banning plastic.

The ripple effects of climate change can be overwhelming, especially for vulnerable populations. For any sustainability measure to be successful, governments must understand the impact their policies may have.

Learning how to look for patterns of association in bivariate data can help you determine whether two quantities are merely *related*, or if changes in one quantity might be *causing* changes in the other quantity.

© 2023 Amplify Education, Inc. All rights reserved.

Sub-Unit 5 Correlation 337

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore associations and correlation — within the context of sustainability and the changing climate — in the following places:

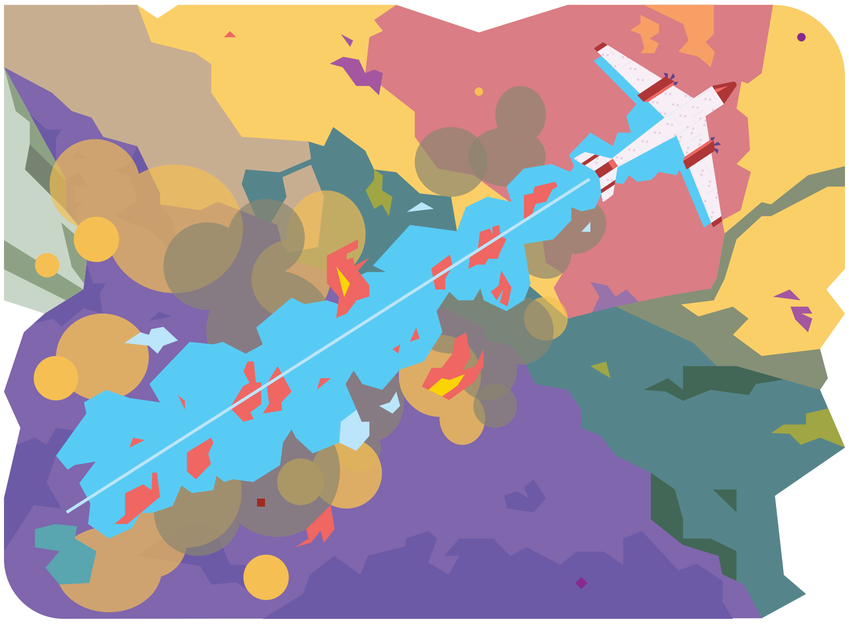
- **Lesson 18, Activity 2:** Associations in Two-Way Tables
- **Lesson 20, Activities 1–3:** Sea Level Change, An Incomplete Story, Two Truths and a Lie
- **Lesson 21, Activities 2–3:** What Comes Next?, Design Your Own Experiment
- **Lesson 22, Activities 1–2:** Increasing Number of Hurricanes, Global Average Temperature Change

Optional

UNIT 2 | **LESSON 18**

“Strength” of Association

Let’s measure associations in data.



Focus

Goals

1. **Language Goal:** Determine the strength of association between data values from a scatter plot visually. **(Reading)**
2. **Language Goal:** Determine how to measure association using statistics. **(Reading and Writing)**

Rigor

- Students build **conceptual understanding** of determining the strength of association between data sets.
- Students build **procedural fluency** by using and improving a method for quantifying association.

Coherence

Today

Students determine the strength of linear associations by visually ranking scatter plots from weakest to strongest. Because this leaves room for interpretation, students examine associations in two-way tables and a method for quantifying how strong some associations can be. They then suggest their own improvements to this method.

Previously

In Grade 8, students determined associations in two-way tables and from scatter plots.

Coming Soon

In Lesson 19, students will learn about the correlation coefficient, the most common way of quantifying association.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	15 min	5 min	5 min
Independent	Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 3 PDF
- rulers

Math Language Development

Review words

- *association*
- *mean*
- *scatter plot*

Amps | Featured Activity

Activity 1
Ordering the Plots

Students order scatter plots by the strength of their association. Meanwhile, you get to see these rankings in real time.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

After working through the strength of associations in Activity 1, students may feel deflated to learn that there is another way to determine associations in Activity 2. Ask students to label their emotions and explain how that emotion is influencing their behavior. Also, ask them to adapt a growth mindset, where they are not finished learning yet.

Modifications to Pacing

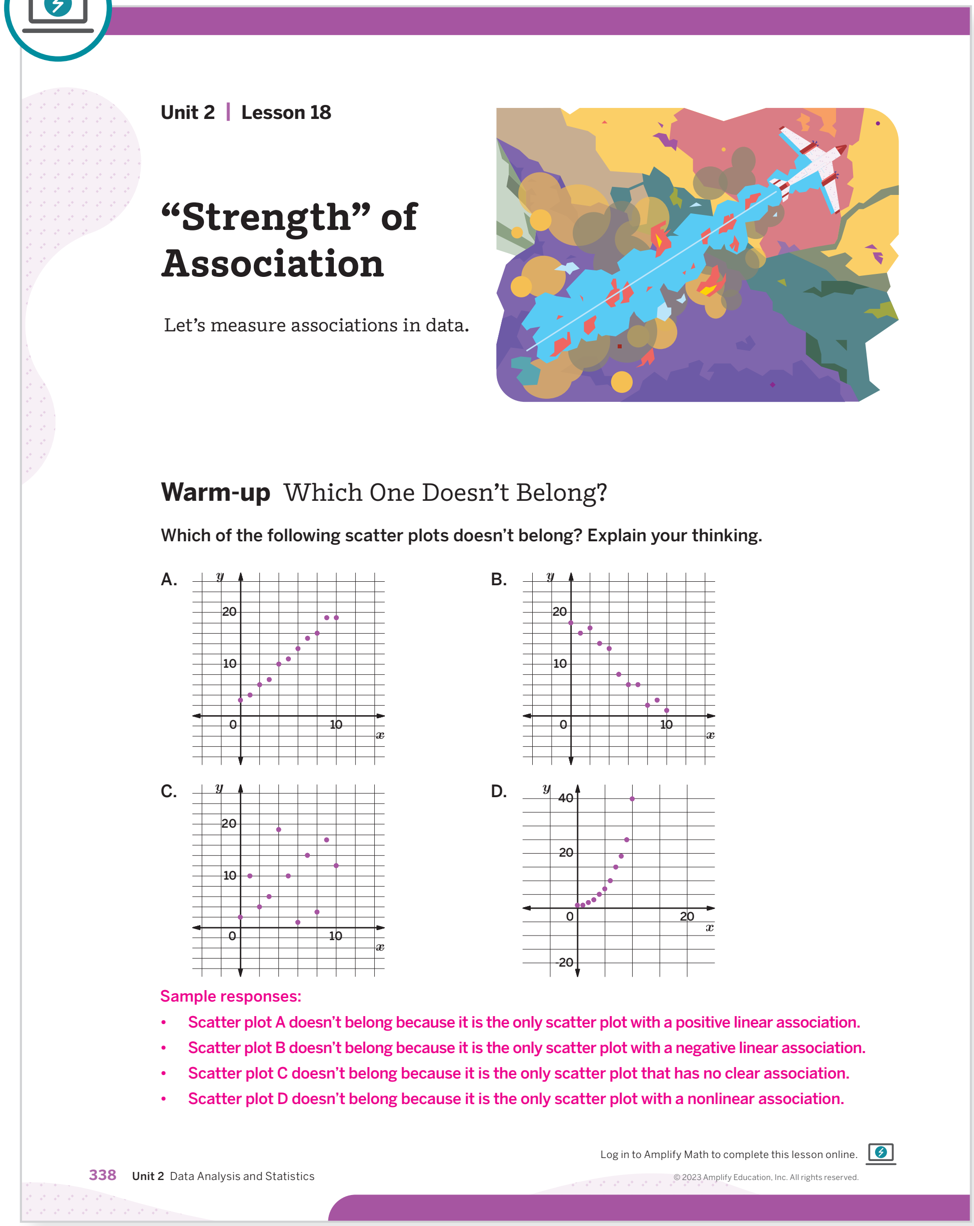
You may want to consider these additional modifications if you are short on time.

- In **Activity 2**, Problem 3 may be omitted.
- In **Activity 3**, Problems 7 and 8 may be omitted.

Independent | 5 min

Warm-up Which One Doesn't Belong?

Students determine which scatter plot doesn't belong to recall how to determine different types and strengths of association.



Unit 2 | Lesson 18

"Strength" of Association

Let's measure associations in data.

Warm-up Which One Doesn't Belong?

Which of the following scatter plots doesn't belong? Explain your thinking.

Sample responses:

- Scatter plot A doesn't belong because it is the only scatter plot with a positive linear association.
- Scatter plot B doesn't belong because it is the only scatter plot with a negative linear association.
- Scatter plot C doesn't belong because it is the only scatter plot that has no clear association.
- Scatter plot D doesn't belong because it is the only scatter plot with a nonlinear association.

Log in to Amplify Math to complete this lesson online.

338 Unit 2 Data Analysis and Statistics

©2023 Amplify Education, Inc. All rights reserved.

1 Launch

Conduct the *Which One Doesn't Belong* routine. Have students work independently for three minutes before participating in a whole class discussion.

2 Monitor

Help students get started by having them compare two pairs of scatter plots at a time and note similarities and differences.

Look for points of confusion:

- **Thinking there must be a pattern**. Remind students that there are some mathematical patterns that are different, and that not all are the same.

Look for productive strategies:

- Recognizing that Scatter plots A and B are the only distinct linear patterns.
- Describing Scatter plot D as being nonlinear.

3 Connect

Display the four scatter plots.

Have individual students share reasons why each scatter plot does not belong. Select and sequence responses involving linear versus nonlinear, and strong or weak patterns.

Highlight that even though Scatter plots A and B might both describe a linear association, there are differences between them, such as how close the points are and the direction of the trend.

Ask, "How would you describe the differences between Scatter plots A and B?"

Math Language Development

MLR2: Collect and Display

As students discuss which scatterplot doesn't belong, circulate and collect the language they use to describe the different associations shown. Add any key words and phrases students use to the class display, reminding them to refer to the display throughout the lesson.

English Learners

Provide sentence frames such as, "I think that Scatter plot ________ doesn't belong because . . ." Provide students time to share and formulate a response with a partner before sharing with the whole class.

Power-up

To power up students' ability to recognize linear associations in a scatter plot:

Provide students with a copy of the Power-up PDF.

Use: Before the Warm-up

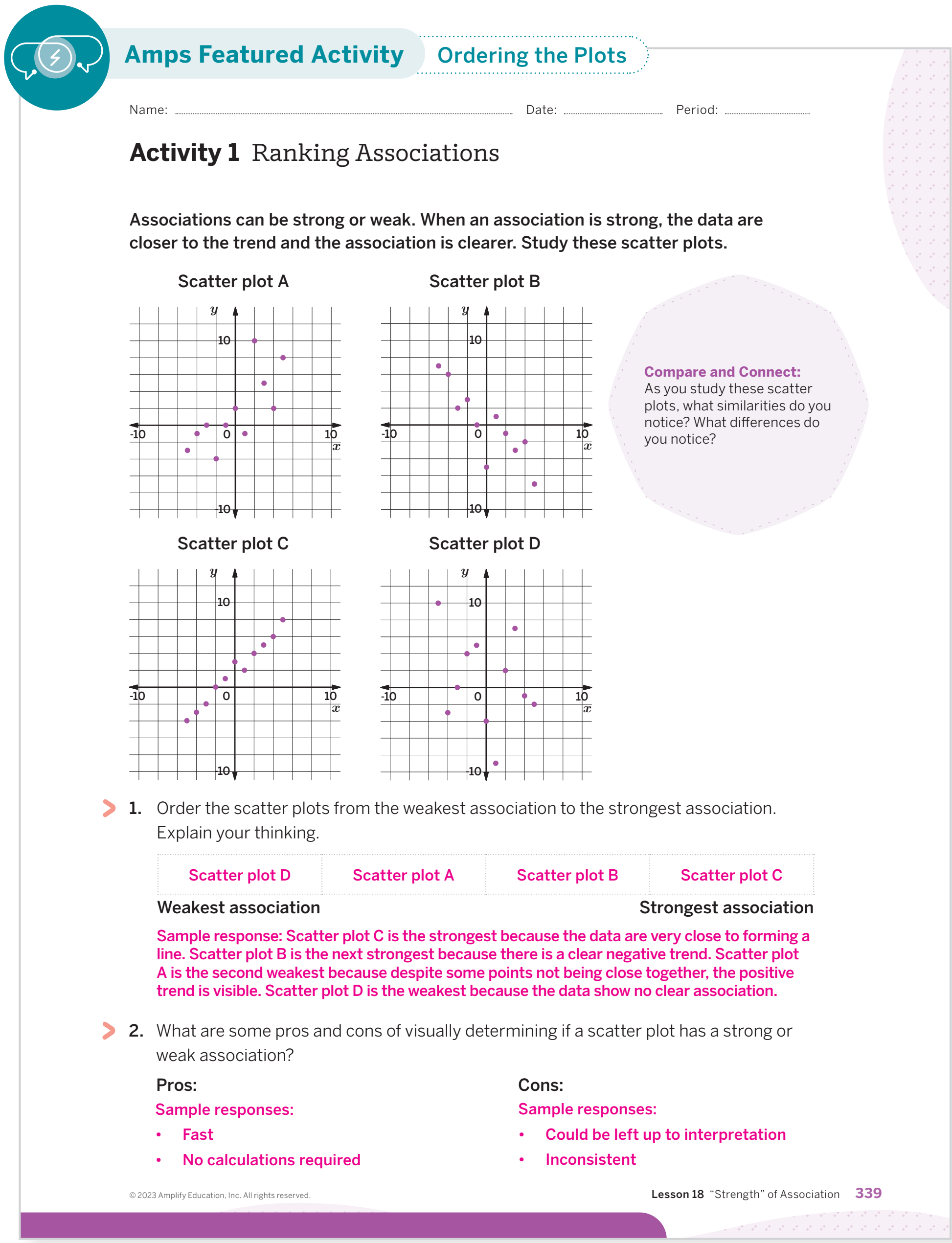
Informed by: Performance on Lesson 6, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 8

Independent | 10 min

Activity 1 Ranking Associations

Students visually determine how strong or weak the associations are in different scatter plots to explore the need for quantifying associations.

Amps Featured Activity Ordering the Plots



Name: ____________ Date: ________ Period: ________

Activity 1 Ranking Associations

Associations can be strong or weak. When an association is strong, the data are closer to the trend and the association is clearer. Study these scatter plots.

Scatter plot A

Scatter plot B

Scatter plot C

Scatter plot D

Compare and Connect: As you study these scatter plots, what similarities do you notice? What differences do you notice?

1. Order the scatter plots from the weakest association to the strongest association. Explain your thinking.

Scatter plot D	Scatter plot A	Scatter plot B	Scatter plot C

Weakest association — Strongest association

Sample response: Scatter plot C is the strongest because the data are very close to forming a line. Scatter plot B is the next strongest because there is a clear negative trend. Scatter plot A is the second weakest because despite some points not being close together, the positive trend is visible. Scatter plot D is the weakest because the data show no clear association.

2. What are some pros and cons of visually determining if a scatter plot has a strong or weak association?

Pros:	Cons:
Sample responses:	Sample responses:
Fast	Could be left up to interpretation
No calculations required	Inconsistent

©2023 Amplify Education, Inc. All rights reserved.

Lesson 18 "Strength" of Association 339

1 Launch

Students independently order the scatter plots from weakest to strongest before pausing for a 2-minute class discussion and then completing Problem 2.

2 Monitor

Help students get started by asking, "Do you see any clear trends? Which scatter plots have no clear trends?"

Look for points of confusion:

- **Thinking that a negative association means a weak association**. Ask, "How is a positive or negative association different from the strength of an association?"

Look for productive strategies:

- Describing how well the points on each scatter plot form a linear trend.
- Recognizing that visually determining the strength of association could be left to interpretation, especially when comparing the first two scatter plots.

3 Connect

Display the four scatter plots.

Have individual students share how they ordered each scatter plot by the strength of association. Have students who had different orderings of the strength of association share, and what the pros and cons of this visual approach are.

Highlight that when scatter plots have extreme differences in the strength of their association, it is more straightforward to determine visually which one has the stronger association. When two scatter plots seem to have a close strength in the association, visual methods of determining strength become challenging.

Ask, "How do you think mathematicians measure the strength of association from a scatter plot? Why would they measure it?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital scatter plots that have varying levels of association and receive feedback on their rankings in real time.

Accessibility: *Optimize Access to Tools*

Provide access to rulers or straightedges that students can use to informally fit a line to model the data. This will help them visualize the strength of each association.

Extension: *Math Enrichment*

Ask students why they think it might be important to know whether a data set showed a strong or weak linear association.

Math Language Development

MLR7: Compare and Connect

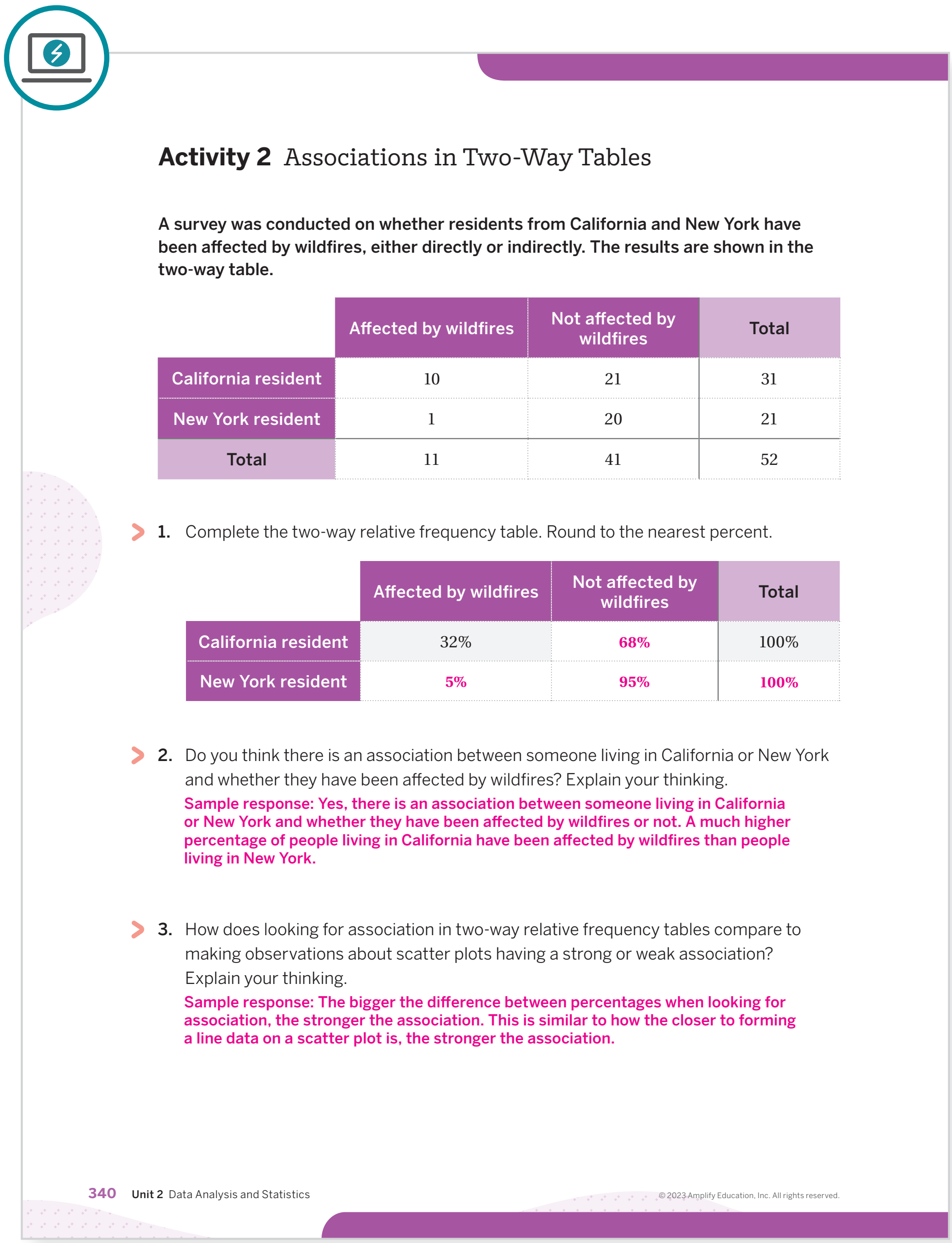
Before students begin Problem 1, have them study the four scatter plots shown to look for any similarities and differences that they notice. For example, they may notice that Scatter plots A and C both show a positive association, yet the points in Scatter plot C are closer to forming a line than in Scatter plot A.

English Learners

Provide students with words and phrases they could use to record similarities and differences, such as *positive association, negative association, no association,* and *linear association*.

Activity 2 Associations in Two-Way Tables

Students look for association in two-way tables to determine a way to measure the strength of association.



Activity 2 Associations in Two-Way Tables

A survey was conducted on whether residents from California and New York have been affected by wildfires, either directly or indirectly. The results are shown in the two-way table.

	Affected by wildfires	Not affected by wildfires	Total
California resident	10	21	31
New York resident	1	20	21
Total	11	41	52

1. Complete the two-way relative frequency table. Round to the nearest percent.

	Affected by wildfires	Not affected by wildfires	Total
California resident	32%	68%	100%
New York resident	5%	95%	100%

2. Do you think there is an association between someone living in California or New York and whether they have been affected by wildfires? Explain your thinking.
   Sample response: Yes, there is an association between someone living in California or New York and whether they have been affected by wildfires or not. A much higher percentage of people living in California have been affected by wildfires than people living in New York.

3. How does looking for association in two-way relative frequency tables compare to making observations about scatter plots having a strong or weak association? Explain your thinking.
   Sample response: The bigger the difference between percentages when looking for association, the stronger the association. This is similar to how the closer to forming a line data on a scatter plot is, the stronger the association.

340 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Say, "You will create a relative frequency table and use it to look for associations." Give students an expectation for the amount of time they will have to work on the activity in pairs.

2 Monitor

Help students get started by asking, "How were the values that are already in the table calculated?"

Look for points of confusion:

- **Thinking that the relative frequencies are calculated by dividing by the table total.** Ask, "Because the row relative frequency is 100%, how would the other relative frequencies be calculated?"

Look for productive strategies:

- Recognizing the difference in the relative frequencies between people who live in California versus New York.

3 Connect

Have pairs of students share their relative frequency tables and responses to Problems 2 and 3.

Highlight that, while large differences in percentages of a two-way frequency table can indicate association with close percentages, it can be challenging to tell if there is an association.

Ask, "How could you use two-way frequency tables to measure association in scatter plots in which the data points are located in one or more quadrants of the coordinate plane?" Sample response: The data could be split up into four categories by which quadrant the point falls into, and a relative frequency table could be created to look for association.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Demonstrate how the given percentage in the two-way table for Problem 1 was calculated; $10 \div 31 \approx 0.32$. Annotate the table as a row relative frequency table to distinguish it from a column relative frequency table.

Extension: *Math Enrichment*

After students complete Problem 2, have them answer the following: Would you have reached the same conclusion if you had calculated the *column* relative frequencies? Explain your thinking. Yes; Sample response: 91% of California residents were affected by wildfires, compared to only 9% of New York residents.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their responses, provide sentence frames such as:

- "There is/is not an association because . . ."
- "The column relative frequencies are similar/very different, which means that there is/is not an association."

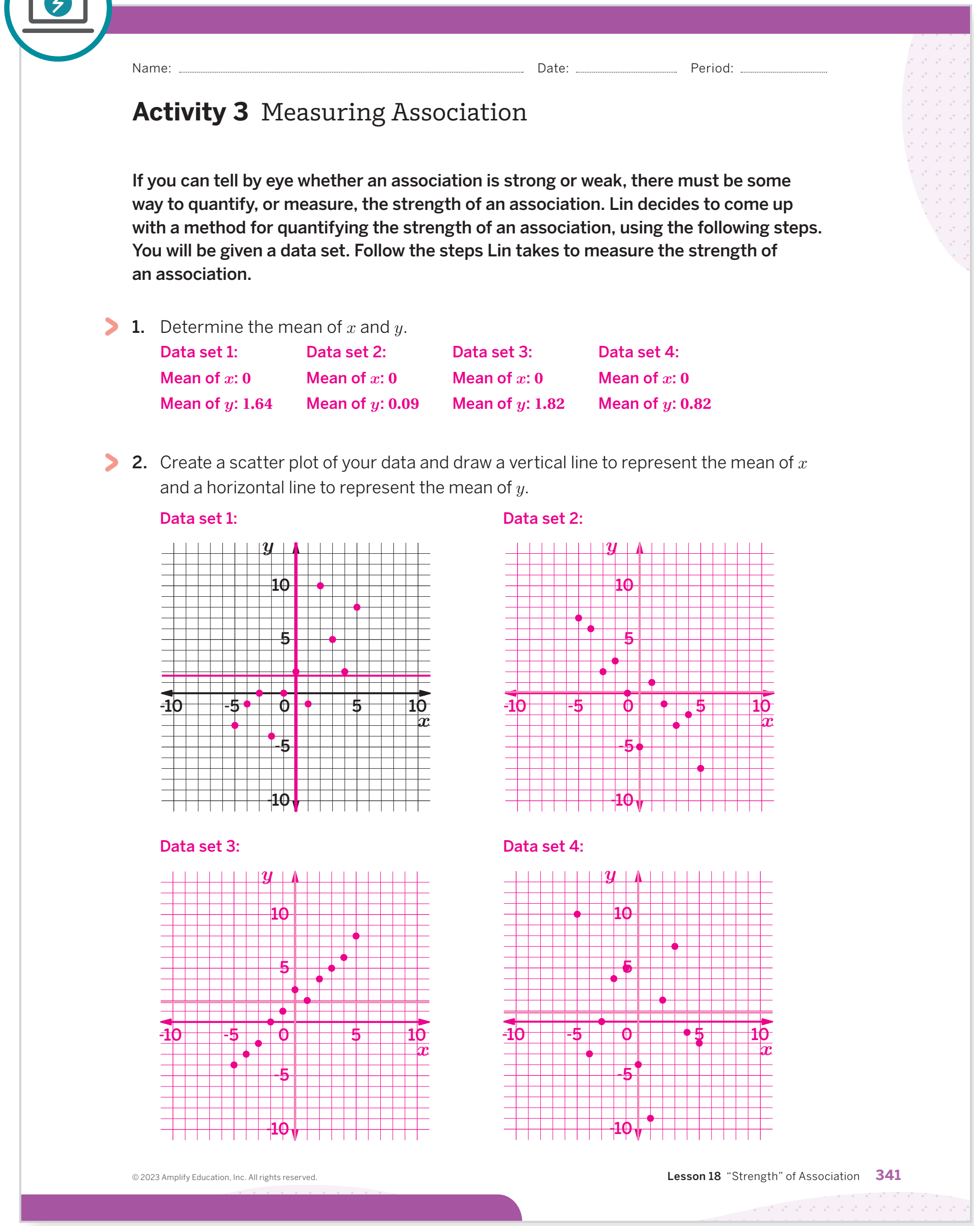
English Learners

Provide students time to formulate a response before sharing with the whole class.

Pairs | 15 min

Activity 3 Measuring Association

Students use a method to quantify association in scatter plots to see a numerical value in the strength of different associations.



Name: ______ Date: ______ Period: ______

Activity 3 Measuring Association

If you can tell by eye whether an association is strong or weak, there must be some way to quantify, or measure, the strength of an association. Lin decides to come up with a method for quantifying the strength of an association, using the following steps. You will be given a data set. Follow the steps Lin takes to measure the strength of an association.

1. Determine the mean of x and y.

Data set 1:	Data set 2:	Data set 3:	Data set 4:
Mean of x: 0	Mean of x: 0	Mean of x: 0	Mean of x: 0
Mean of y: 1.64	Mean of y: 0.09	Mean of y: 1.82	Mean of y: 0.82

2. Create a scatter plot of your data and draw a vertical line to represent the mean of x and a horizontal line to represent the mean of y.

Data set 1:

Data set 2:

Data set 3:

Data set 4:

©2023 Amplify Education, Inc. All rights reserved.

Lesson 18 "Strength" of Association 341

1 Launch

Have students pause after Problem 5, share their responses, and then complete the rest of the activity.

2 Monitor

Help students get started by modeling drawing a horizontal or vertical line for the mean of a different data set.

Look for points of confusion:

- **Placing points of the vertical or horizontal lines in the incorrect category.** Ask students to read the labels on the frequency table and explain what "greater than or equal to" means.
- **Calculating the incorrect value in Problem 5.** Using a different data set, model adding the percentages from the indicated cells and subtracting them.

Look for productive strategies:

- Recognizing that a value far away from 0% in Problem 5 indicates a strong association and close to 0% means a weak association.
- Recognizing that a negative value in Problem 5 indicates a negative trend in the association.

Activity 3 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Instead of students creating the scatter plots, provide pre-created scatter plots along with each data set. Have students determine the mean of x and y and then draw the vertical and horizontal lines as described in Problem 2. Have them continue the activity with Problem 3.

Accessibility: *Guide Processing and Visualization*

As students begin Problem 5, consider demonstrating or displaying Lin's method using sample calculations. For example, for Data set 1, display the following:

Top left cell + Bottom right cell: $45\% + 45\% = 90\%$
Top right cell + Bottom left cell: $0\% + 9\% = 9\%$
Subtract second value from the first: $90\% - 9\% = 81\%$

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses to Problems 5–6, draw connections between the associations shown on the scatter plots and the results students calculated for Problem 5. Ask these questions:

- "Look at the scatter plot for Data set 3, which has the strongest linear association shown. What do you notice about its corresponding calculation for Problem 5?" It was 100%.
- "What do you notice about the scatter plots that had negative calculations for Problem 5?" They have negative associations.

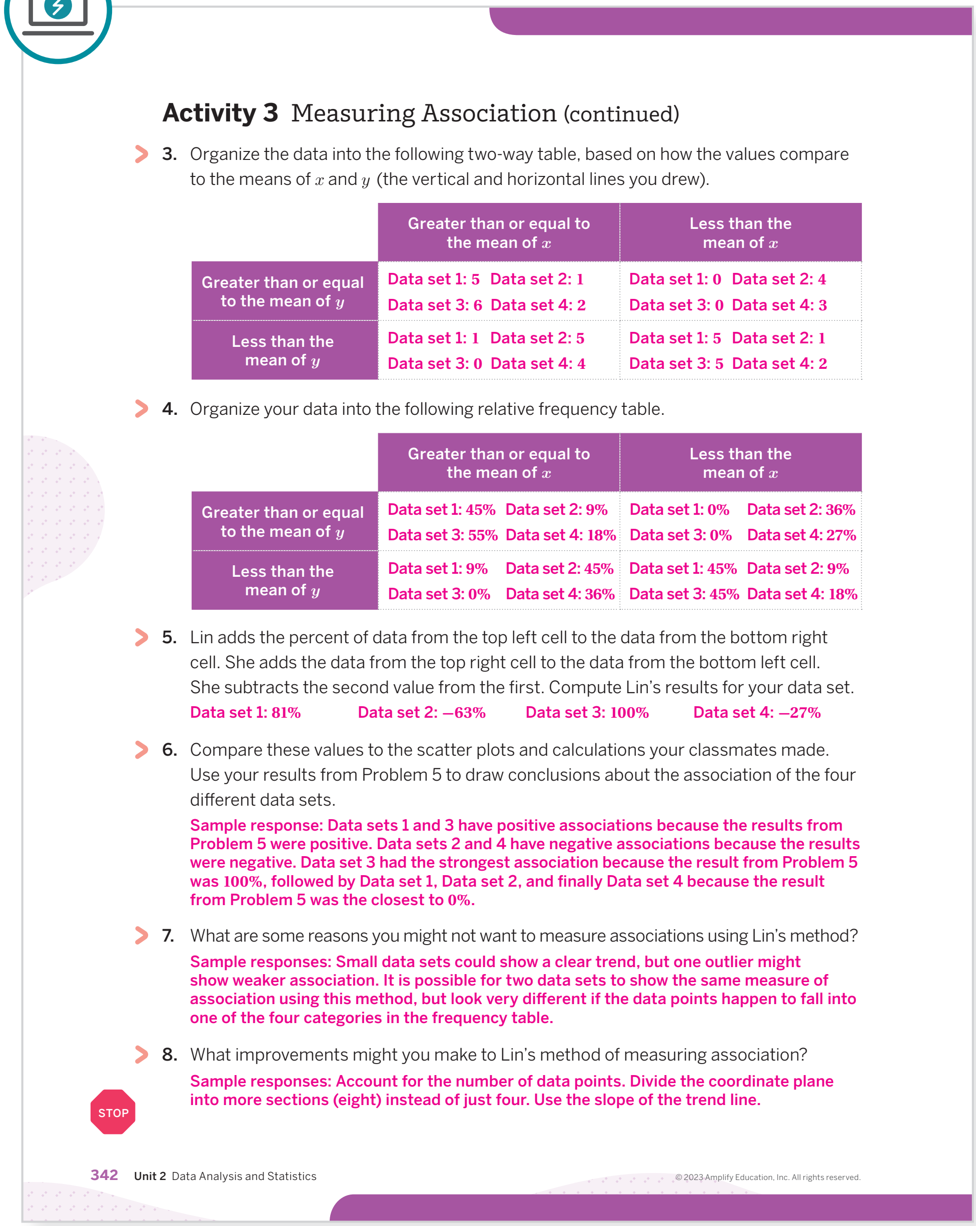
English Learners

Annotate the scatter plots with the calculations from Problem 5. Consider arranging the scatter plots in order from weakest association to strongest association.

Pairs | 15 min

Activity 3 Measuring Association (continued)

Students use a method to quantify association in scatter plots to see a numerical value in the strength of different associations.



Activity 3 Measuring Association (continued)

3. Organize the data into the following two-way table, based on how the values compare to the means of x and y (the vertical and horizontal lines you drew).

	Greater than or equal to the mean of x	Less than the mean of x
Greater than or equal to the mean of y	Data set 1: 5 Data set 2: 1 Data set 3: 6 Data set 4: 2	Data set 1: 0 Data set 2: 4 Data set 3: 0 Data set 4: 3
Less than the mean of y	Data set 1: 1 Data set 2: 5 Data set 3: 0 Data set 4: 4	Data set 1: 5 Data set 2: 1 Data set 3: 5 Data set 4: 2

4. Organize your data into the following relative frequency table.

	Greater than or equal to the mean of x	Less than the mean of x
Greater than or equal to the mean of y	Data set 1: 45% Data set 2: 9% Data set 3: 55% Data set 4: 18%	Data set 1: 0% Data set 2: 36% Data set 3: 0% Data set 4: 27%
Less than the mean of y	Data set 1: 9% Data set 2: 45% Data set 3: 0% Data set 4: 36%	Data set 1: 45% Data set 2: 9% Data set 3: 45% Data set 4: 18%

5. Lin adds the percent of data from the top left cell to the data from the bottom right cell. She adds the data from the top right cell to the data from the bottom left cell. She subtracts the second value from the first. Compute Lin's results for your data set.
   Data set 1: 81% Data set 2: –63% Data set 3: 100% Data set 4: –27%

6. Compare these values to the scatter plots and calculations your classmates made. Use your results from Problem 5 to draw conclusions about the association of the four different data sets.
   Sample response: Data sets 1 and 3 have positive associations because the results from Problem 5 were positive. Data sets 2 and 4 have negative associations because the results were negative. Data set 3 had the strongest association because the result from Problem 5 was 100%, followed by Data set 1, Data set 2, and finally Data set 4 because the result from Problem 5 was the closest to 0%.

7. What are some reasons you might not want to measure associations using Lin's method?
   Sample responses: Small data sets could show a clear trend, but one outlier might show weaker association. It is possible for two data sets to show the same measure of association using this method, but look very different if the data points happen to fall into one of the four categories in the frequency table.

8. What improvements might you make to Lin's method of measuring association?
   Sample responses: Account for the number of data points. Divide the coordinate plane into more sections (eight) instead of just four. Use the slope of the trend line.

STOP

342 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have pairs of students share their responses to Problems 5 and 6. Select and sequence student responses comparing each calculated value, comparing positive and negative values, and noting the distance from 0.

Display student samples of the four scatter plots.

Have pairs of students share possible issues and improvements they would make to this method.

Highlight that calculating the mean of x and y and drawing a vertical and horizontal line to represent each is part of the method used to measure association.

Ask, "Why does it make sense to have a maximum and minimum measurement using this method from Activity 3?" Sample response: Because the amount of data in any one category cannot exceed 100%, all measurements will range between –100% and 100%.

Differentiated Support

Extension: *Math Enrichment*

Have students explain why Data set 4 shows the weakest association, yet its calculation from Problem 5 was not the least value. (Data set 2 had the least value from Problem 5.) Sample response: The negative sign indicates negative association. Values that are closer to 0% indicate weaker associations than values closer to –100% or 100%.

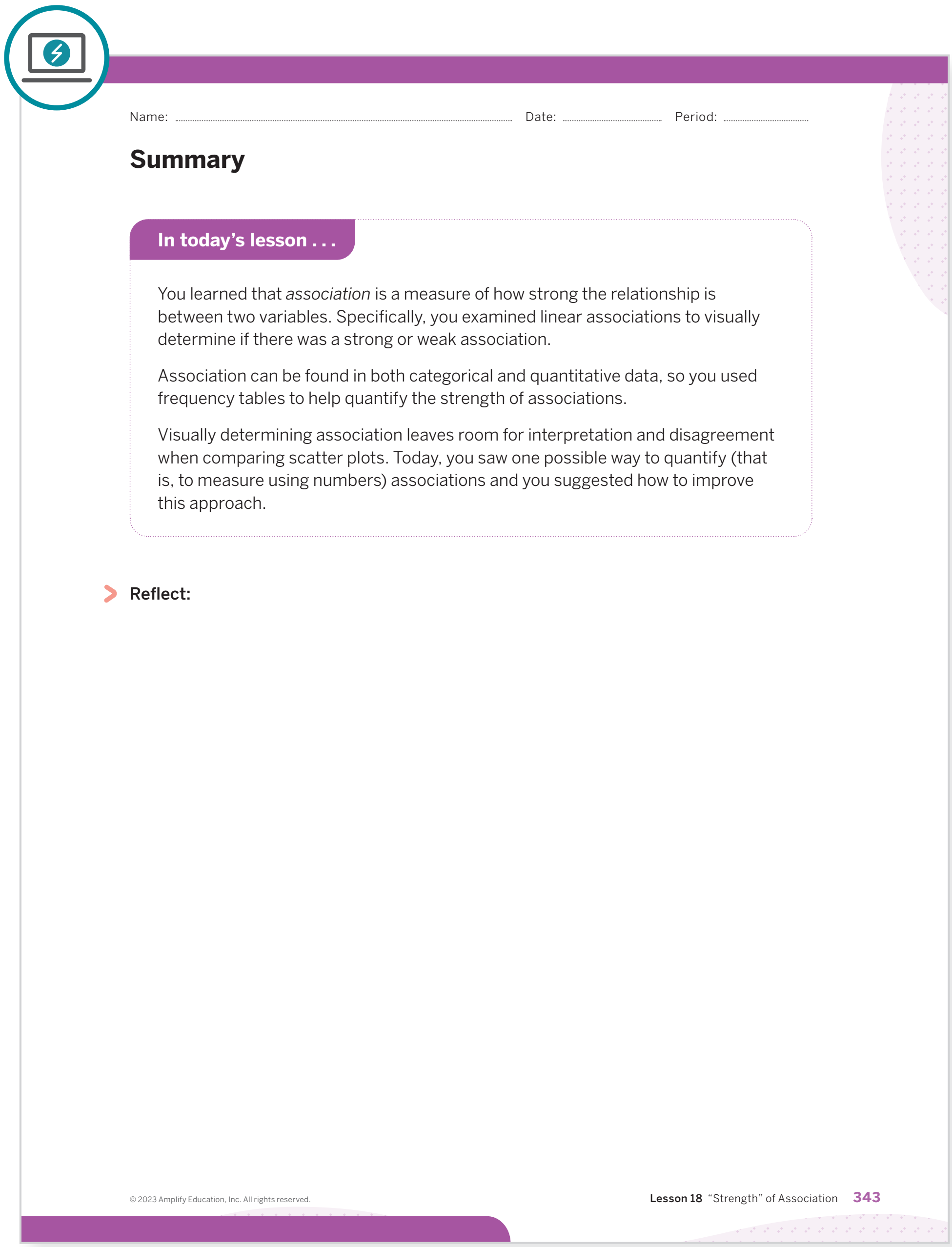
Then ask students to name a value that would indicate each of the following: Sample responses shown.

- A very weak positive association. 10%
- A very weak negative association. –10%
- A very strong positive association. 100%
- A positive association that was somewhat strong. 75%

Whole Class | 5 min

Summary

Review and synthesize how to quantify association visually and through measurements.



Name: Date: Period:

Summary

In today's lesson . . .

You learned that *association* is a measure of how strong the relationship is between two variables. Specifically, you examined linear associations to visually determine if there was a strong or weak association.

Association can be found in both categorical and quantitative data, so you used frequency tables to help quantify the strength of associations.

Visually determining association leaves room for interpretation and disagreement when comparing scatter plots. Today, you saw one possible way to quantify (that is, to measure using numbers) associations and you suggested how to improve this approach.

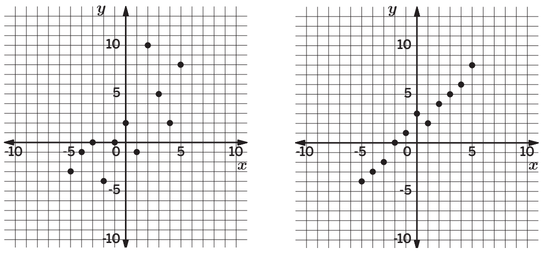
Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 18 "Strength" of Association 343

Synthesize

Display the following scatter plots.



Have students share which scatter plot they think has a stronger association and why.

Highlight that visually determining the strength of association between two scatter plots is quick, but leaves room for interpretation, so students must find ways to quantify, or measure, the strength of association.

Ask, "When is it beneficial to have a way to quantify the strength of association from a scatter plot?" Sample response: When you have a data set where a trend can be seen, but it is not very strong, there needs to be a consistent way to measure the association.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

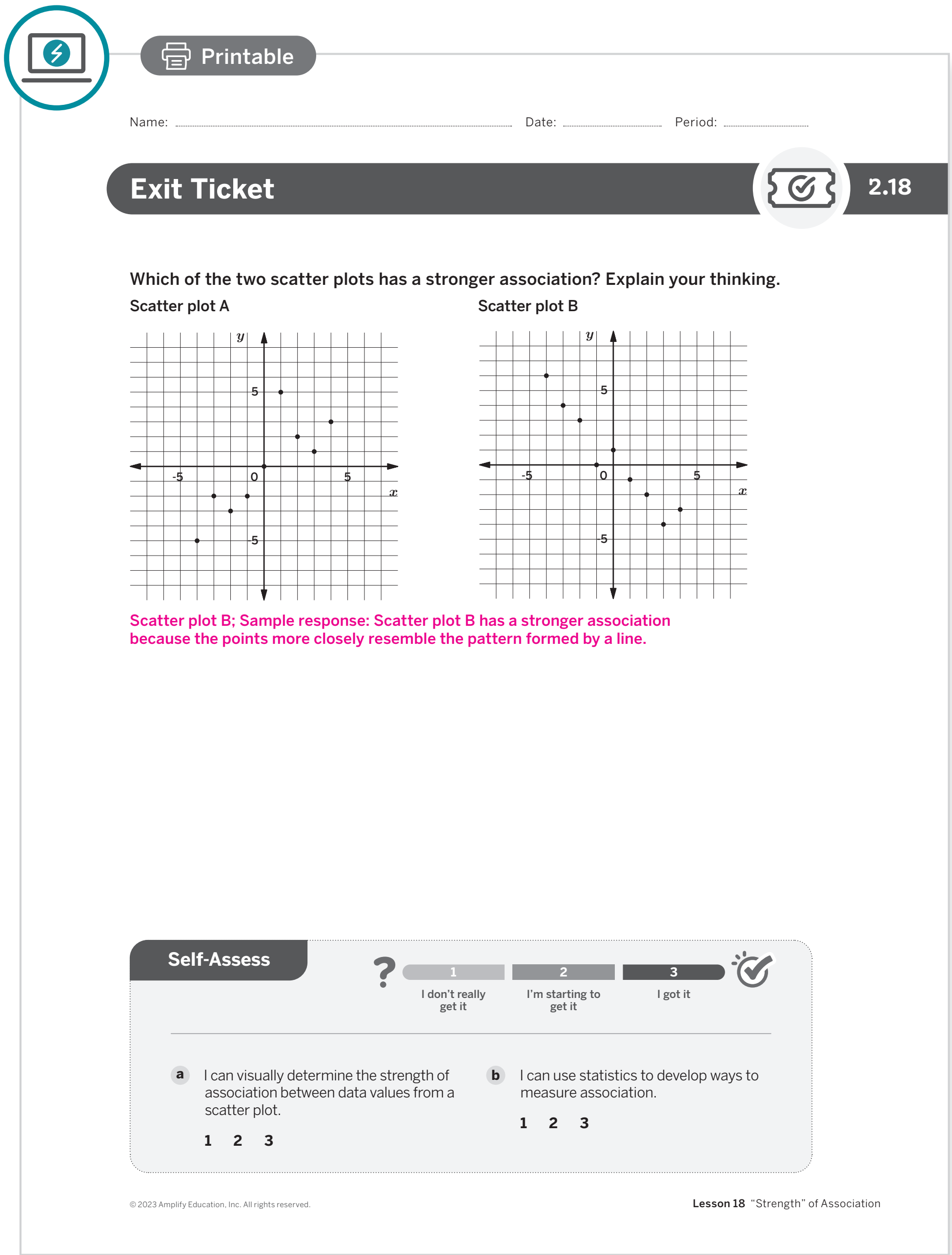
- "What were the different ways you measured association?"
- "How can you tell the strength of association from a scatter plot?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by comparing scatter plots and determining which has a stronger association.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 2.18

Which of the two scatter plots has a stronger association? Explain your thinking.

Scatter plot A

Scatter plot B

Scatter plot B; Sample response: Scatter plot B has a stronger association because the points more closely resemble the pattern formed by a line.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can visually determine the strength of association between data values from a scatter plot.

1 2 3

b I can use statistics to develop ways to measure association.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 18 "Strength" of Association

Success looks like . . .

- **Language Goal:** Determining the strength of association between data values from a scatter plot visually. **(Reading)**
  - » Selecting the scatter plot with the stronger association.
- **Language Goal:** Determining how to measure association using statistics. **(Reading and Writing)**

Suggested next steps

If students select the incorrect scatter plot in Problem 1, consider:

- Reviewing the visual method for determining the strength of association from Activity 1.
- Assigning Practice Problem 1.
- Asking, "Which scatter plot has points that more closely resemble a linear trend?"

Professional Learning

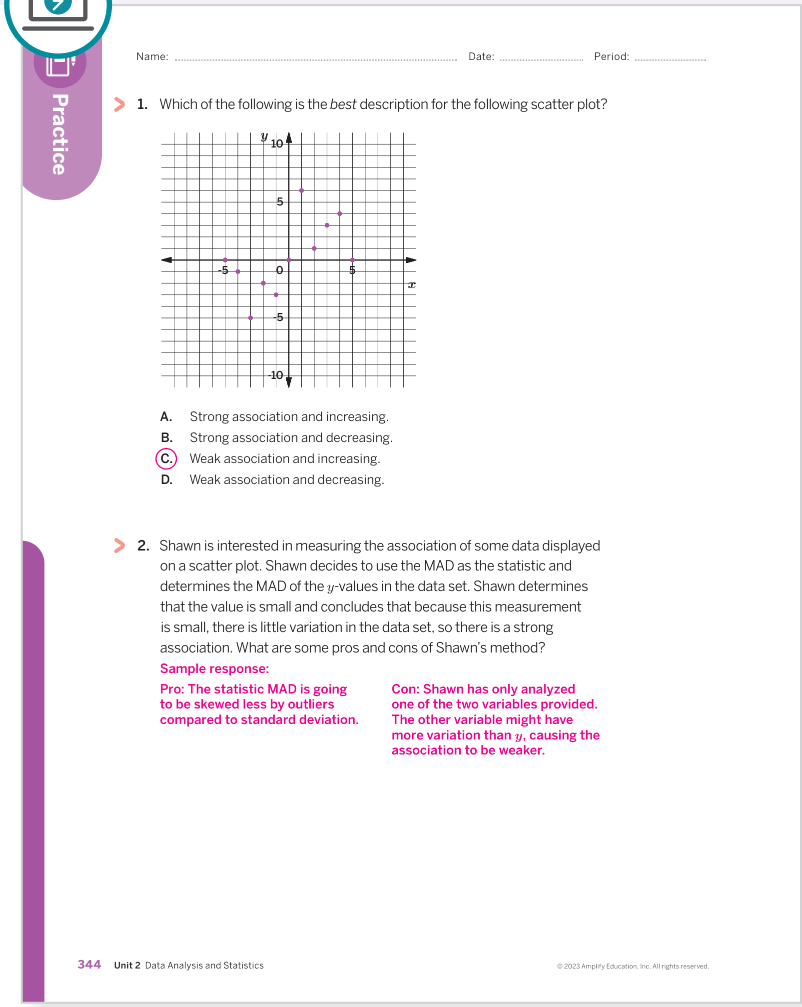
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did students find frustrating about Activity 3? What helped them work through this frustration?
- What did having students make improvements to the method for quantifying strength of association reveal about your students as learners? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. Which of the following is the *best* description for the following scatter plot?

A. Strong association and increasing.
B. Strong association and decreasing.
[✓ C.] Weak association and increasing.
D. Weak association and decreasing.

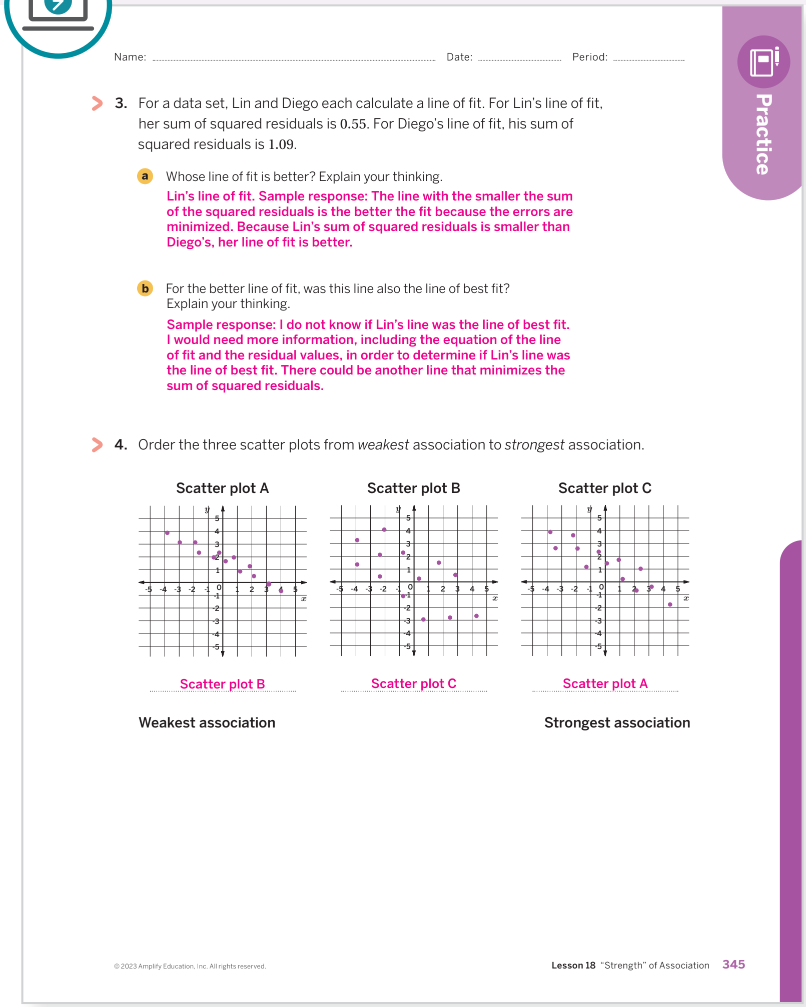
2. Shawn is interested in measuring the association of some data displayed on a scatter plot. Shawn decides to use the MAD as the statistic and determines the MAD of the y-values in the data set. Shawn determines that the value is small and concludes that because this measurement is small, there is little variation in the data set, so there is a strong association. What are some pros and cons of Shawn's method?

Sample response:

Pro: The statistic MAD is going to be skewed less by outliers compared to standard deviation.

Con: Shawn has only analyzed one of the two variables provided. The other variable might have more variation than y, causing the association to be weaker.

© 2023 Amplify Education, Inc. All rights reserved.



Name: ______ Date: ______ Period: ______

Practice

3. For a data set, Lin and Diego each calculate a line of fit. For Lin's line of fit, her sum of squared residuals is 0.55. For Diego's line of fit, his sum of squared residuals is 1.09.

   a. Whose line of fit is better? Explain your thinking.

   Lin's line of fit. Sample response: The line with the smaller the sum of the squared residuals is the better the fit because the errors are minimized. Because Lin's sum of squared residuals is smaller than Diego's, her line of fit is better.

   b. For the better line of fit, was this line also the line of best fit? Explain your thinking.

   Sample response: I do not know if Lin's line was the line of best fit. I would need more information, including the equation of the line of fit and the residual values, in order to determine if Lin's line was the line of best fit. There could be another line that minimizes the sum of squared residuals.

4. Order the three scatter plots from *weakest* association to *strongest* association.

Scatter plot A **Scatter plot B** **Scatter plot C**

Scatter plot B ______ **Scatter plot C** ______ **Scatter plot A**

Weakest association **Strongest association**

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
Spiral	3	Unit 2 Lesson 17	2
Formative	4	Unit 2 Lesson 19	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 2 | **LESSON 19**

Correlation Coefficient (Part 1)

Let's put a number to the strength of association in data.

Focus

Goals

1. **Language Goal:** Understand how the correlation coefficient describes associations in bivariate data. **(Reading and Writing)**
2. Understand that there is a precise method for calculating the correlation coefficient, given a scatter plot.

Coherence

Today

Students first inspect values of r (the correlation coefficient), given scatter plots with corresponding lines of fit. Students then further explore r and learn its meaning. Finally, students work through a method for geometrically calculating r and use it to describe how well a linear model fits data.

Previously

In Lesson 18, students reviewed association from Grade 8.

Coming Soon

In Lesson 20, students will apply their understanding of the correlation coefficient in the context of real-world situations.

Rigor

- Students build **conceptual understanding** of the correlation coefficient.
- Students build **procedural fluency** in calculating and interpreting the correlation coefficient.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, one per pair
- Activity 2 PDF (answers)
- Anchor Chart PDF, *Sentence Stems, Notice and Wonder*

Math Language Development

New words

- ***correlation coefficient***

Review words

- *correlation*
- *line of best fit*
- *residual*

Amps | Featured Activity

Activity 2
Real-Time Correlation Coefficient

Students are able to digitally represent the geometric interpretation of the correlation coefficient and see how it is calculated.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might think that the correlation coefficient is too complicated and unnecessary in Activity 2. Help them shift their thinking in a more positive direction by brainstorming as a class why the correlation coefficient might be useful.

Modifications to Pacing

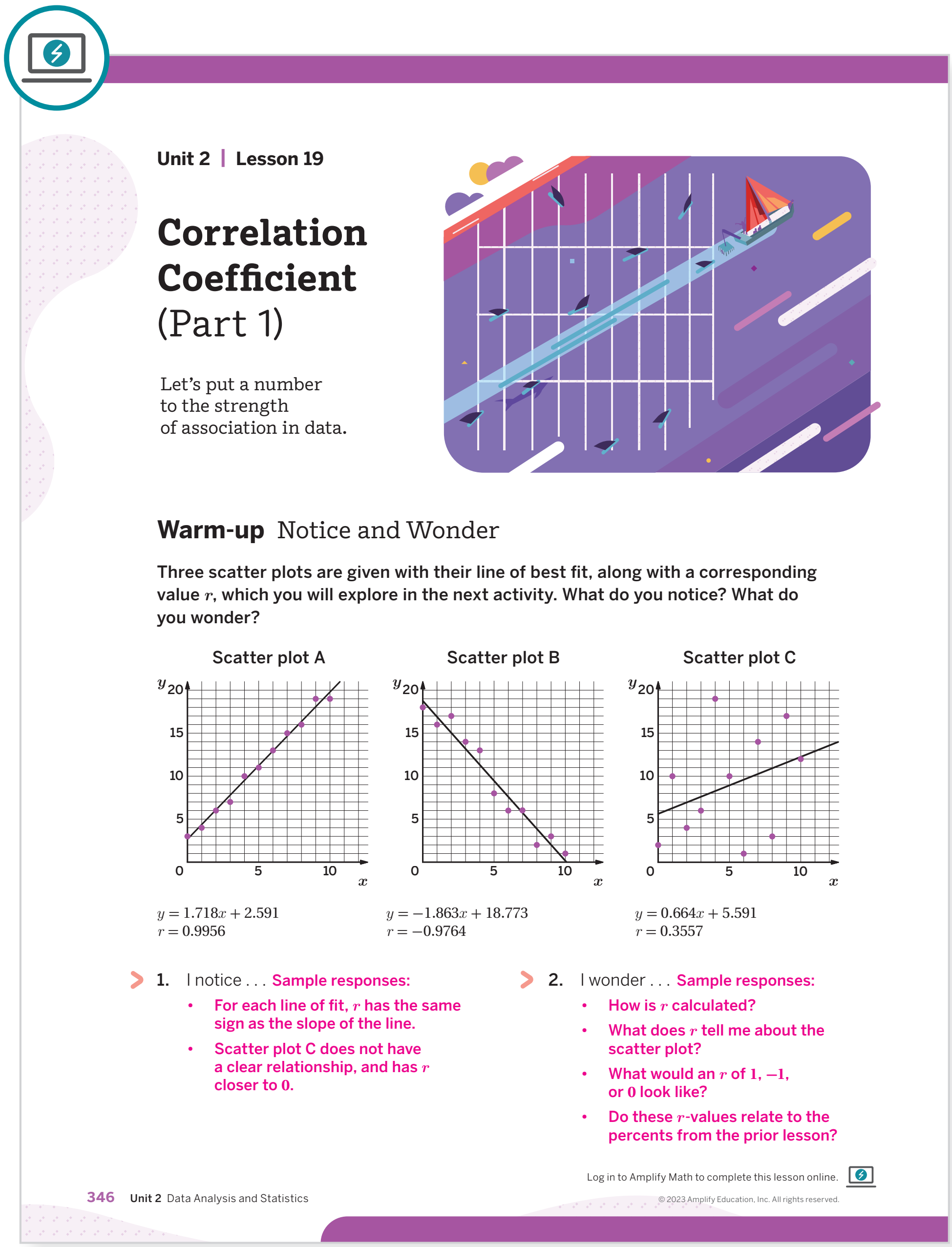
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, Problem 1 may be omitted.

Pairs | 5 min

Warm-up Notice and Wonder

Students notice how the value of r depends on the shape of a scatter plot to infer its meaning.



Unit 2 | Lesson 19

Correlation Coefficient (Part 1)

Let's put a number
to the strength
of association in data.

Warm-up Notice and Wonder

Three scatter plots are given with their line of best fit, along with a corresponding value r, which you will explore in the next activity. What do you notice? What do you wonder?

$y = 1.718x + 2.591$
$r = 0.9956$

$y = -1.863x + 18.773$
$r = -0.9764$

$y = 0.664x + 5.591$
$r = 0.3557$

1. I notice . . . **Sample responses:**
   - **For each line of fit, r has the same sign as the slope of the line.**
   - **Scatter plot C does not have a clear relationship, and has r closer to 0.**

2. I wonder . . . **Sample responses:**
   - **How is r calculated?**
   - **What does r tell me about the scatter plot?**
   - **What would an r of 1, -1, or 0 look like?**
   - **Do these r-values relate to the percents from the prior lesson?**

Log in to Amplify Math to complete this lesson online.

346 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Conduct the *Notice and Wonder* routine. Give students a minute of independent think time, and a minute to discuss what they notice and wonder about the four scatter plots with their partner before responding.

2 Monitor

Help students get started by asking, "Are there any comparisons you can make between the r-value and other values you see?"

Look for points of confusion:

- **Thinking that the least r-value implies there is no relationship**. Ask students, "What other relationships or patterns exist besides linear relationships? Can these relationships still be described mathematically?"

Look for productive strategies:

- Comparing the slope and r-value.
- Comparing how close the data points are to the line of best fit with the r-value.

3 Connect

Have pairs of students share what they noticed and wondered. Record these responses for display.

Highlight that the sign of the r-value is related to the slope and is also related to how well the data represents a linear pattern.

Ask, "What could be the purpose of having an r-value?" The r-value could be a way to more precisely determine how well a linear model represents a set of data.

Math Language Development

MLR8: Discussion Supports

Before the Connect, display or provide the Anchor Chart PDF, *Sentence Stems, Notice and Wonder* to support students as they record what they notice and wonder and think about how they will share these responses with the class.

English Learners

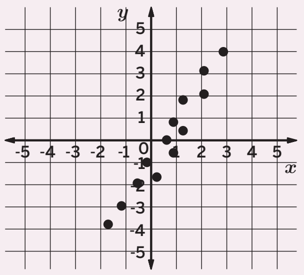
Allow students to rehearse and formulate what they will say with their partner before sharing with the class.

Power-up

To power up students' ability to determine whether a scatter plot models a linear association, have students complete:

Determine which statement about the given graph is true.

[✓ A.] It is a positive linear association.
B. It is a negative linear association.
C. It is a positive nonlinear association.
D. It is a negative nonlinear association.



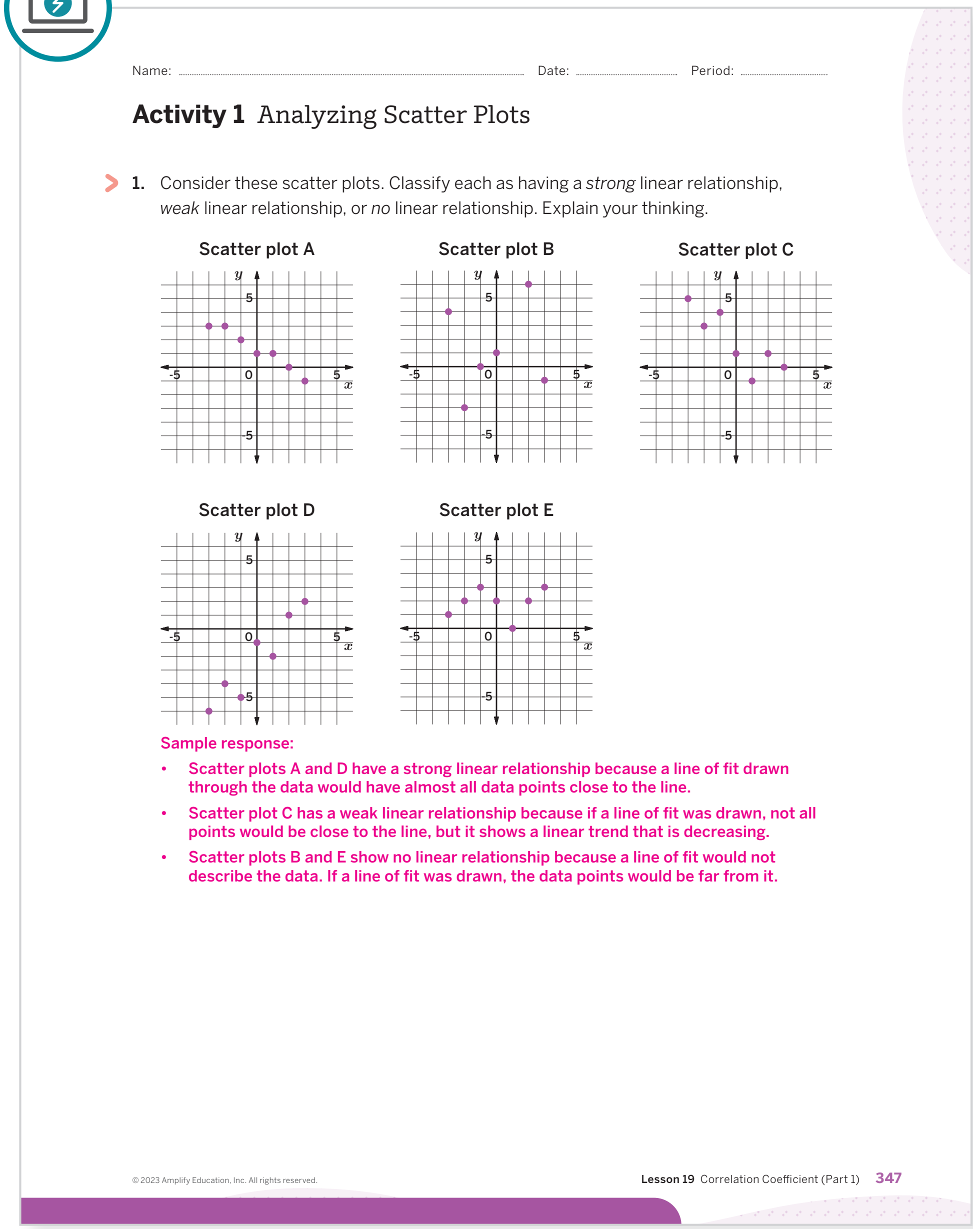
Use: Before Activity 1

Informed by: Performance on Lesson 18, Practice Problem 4

Pairs | 15 min

Activity 1 Analyzing Scatter Plots

Students now decide whether an association is strong or weak before seeing the corresponding value of r to further interpret its meaning.



Name: Date: Period:

Activity 1 Analyzing Scatter Plots

1. Consider these scatter plots. Classify each as having a *strong* linear relationship, *weak* linear relationship, or *no* linear relationship. Explain your thinking.

Sample response:

- Scatter plots A and D have a strong linear relationship because a line of fit drawn through the data would have almost all data points close to the line.
- Scatter plot C has a weak linear relationship because if a line of fit was drawn, not all points would be close to the line, but it shows a linear trend that is decreasing.
- Scatter plots B and E show no linear relationship because a line of fit would not describe the data. If a line of fit was drawn, the data points would be far from it.

©2023 Amplify Education, Inc. All rights reserved.

Lesson 19 Correlation Coefficient (Part 1) 347

1 Launch

Give students three minutes to work on each problem, pausing for a one minute class discussion after each.

2 Monitor

Help students get started by asking, "What makes a linear relationship strong or weak?"

Look for points of confusion:

- **Thinking an equal number of points above and below the line of fit represents a distinct linear relationship.** Ask, "Are all relationships linear? Do all scatter plots represent distinct relationships?"

Look for productive strategies:

- Recognizing that when points are close to the line of fit, the r-value is closer to -1 or 1.
- Noticing the sign of the slope corresponds with the sign of the r-value.

Activity 1 continued ›

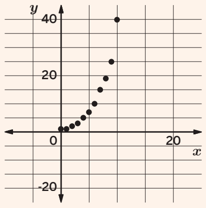
Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider making copies of the scatter plots shown in Problem 1. Cut out the scatter plots, and have students sort them into the categories described. For Problem 2, have students annotate the scatter plots shown with the categories they determined for Problem 1.

Extension: *Math Enrichment*

Ask students if all relationships are linear. Display or provide copies of the graph shown for students to consider. Ask them to draw a "curve of best fit" that they think would model the data.



Math Language Development

MLR7: Compare and Connect

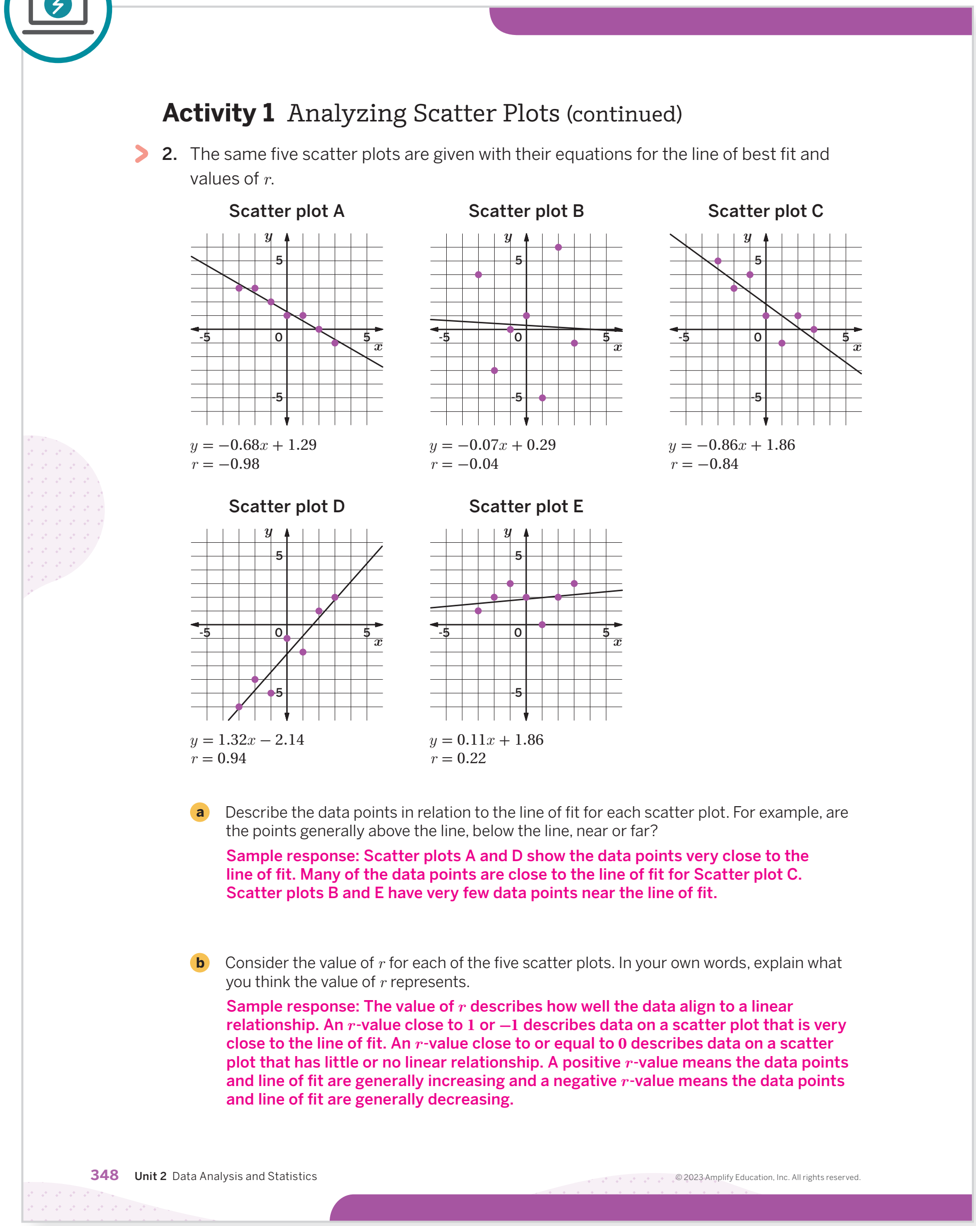
During the Connect, as students share how they interpreted the value of r, draw connections between the values of r shown in Problem 2 and the categories students determined in Problem 1. Ask these questions:

- "Look at Scatter plot D, which has the strongest linear association shown. What do you notice about its corresponding value of r, compared to the other values of r?" It is positive and the closest to 1 than the other values of r.
- "What do you notice about these values of r and the percentages you explored in the previous lesson?" The percentages were between -100% and 100%. The r-values are between -1 and 1.

Pairs | 15 min

Activity 1 Analyzing Scatter Plots (continued)

Students now decide whether an association is strong or weak before seeing the corresponding value of r to further interpret its meaning.



Activity 1 Analyzing Scatter Plots (continued)

2. The same five scatter plots are given with their equations for the line of best fit and values of r.

Scatter plot A

$y = -0.68x + 1.29$
$r = -0.98$

Scatter plot B

$y = -0.07x + 0.29$
$r = -0.04$

Scatter plot C

$y = -0.86x + 1.86$
$r = -0.84$

Scatter plot D

$y = 1.32x - 2.14$
$r = 0.94$

Scatter plot E

$y = 0.11x + 1.86$
$r = 0.22$

a. Describe the data points in relation to the line of fit for each scatter plot. For example, are the points generally above the line, below the line, near or far?

Sample response: Scatter plots A and D show the data points very close to the line of fit. Many of the data points are close to the line of fit for Scatter plot C. Scatter plots B and E have very few data points near the line of fit.

b. Consider the value of r for each of the five scatter plots. In your own words, explain what you think the value of r represents.

Sample response: The value of r describes how well the data align to a linear relationship. An r-value close to 1 or −1 describes data on a scatter plot that is very close to the line of fit. An r-value close to or equal to 0 describes data on a scatter plot that has little or no linear relationship. A positive r-value means the data points and line of fit are generally increasing and a negative r-value means the data points and line of fit are generally decreasing.

348 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the scatter plots with their line of fit and r-value.

Have pairs of students share what they think the r-value represents.

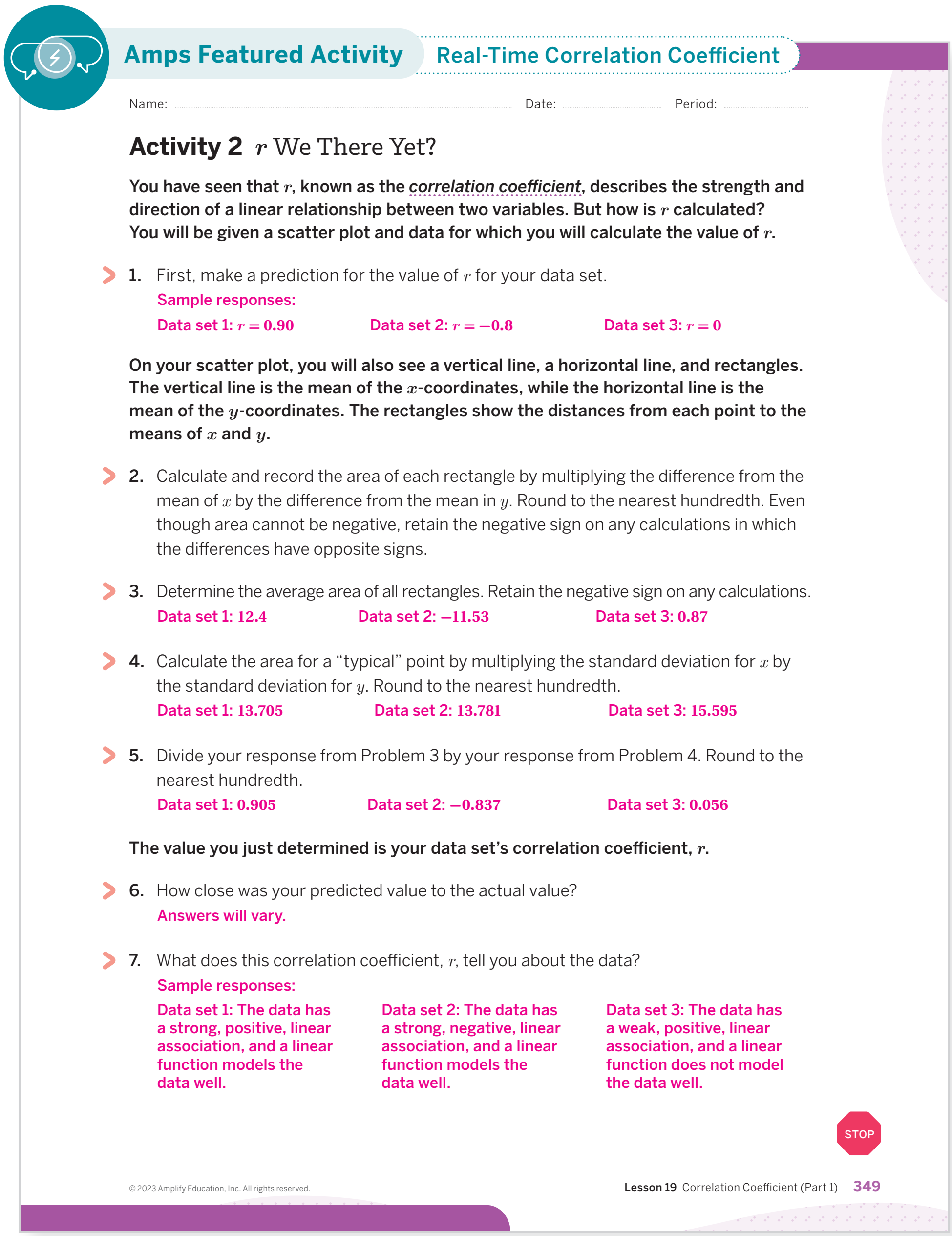
Highlight that the r-value can range from −1 to 1. An r-value close to 0 means little or no linear relationship exists, and values close to −1 or 1 mean a strong linear relationship exists.

Ask, "Why is this scale from −1 to 1 valuable when determining the strength of a linear relationship?" Sample response: It ensures a consistent way of measuring the strength of a linear relationship within certain bounds.

Pairs | 20 min

Activity 2 r We There Yet?

Students use geometric methods to calculate correlation coefficients for conceptual understanding.



Amps Featured Activity Real-Time Correlation Coefficient

Name: ______ Date: ______ Period: ______

Activity 2 r We There Yet?

You have seen that r, known as the *correlation coefficient*, describes the strength and direction of a linear relationship between two variables. But how is r calculated? You will be given a scatter plot and data for which you will calculate the value of r.

1. First, make a prediction for the value of r for your data set.
   Sample responses:
   Data set 1: $r = 0.90$ Data set 2: $r = -0.8$ Data set 3: $r = 0$

On your scatter plot, you will also see a vertical line, a horizontal line, and rectangles. The vertical line is the mean of the x-coordinates, while the horizontal line is the mean of the y-coordinates. The rectangles show the distances from each point to the means of x and y.

2. Calculate and record the area of each rectangle by multiplying the difference from the mean of x by the difference from the mean in y. Round to the nearest hundredth. Even though area cannot be negative, retain the negative sign on any calculations in which the differences have opposite signs.
3. Determine the average area of all rectangles. Retain the negative sign on any calculations.
   Data set 1: 12.4 Data set 2: –11.53 Data set 3: 0.87
4. Calculate the area for a "typical" point by multiplying the standard deviation for x by the standard deviation for y. Round to the nearest hundredth.
   Data set 1: 13.705 Data set 2: 13.781 Data set 3: 15.595
5. Divide your response from Problem 3 by your response from Problem 4. Round to the nearest hundredth.
   Data set 1: 0.905 Data set 2: –0.837 Data set 3: 0.056

The value you just determined is your data set's correlation coefficient, r.

6. How close was your predicted value to the actual value?
   Answers will vary.
7. What does this correlation coefficient, r, tell you about the data?
   Sample responses:
   Data set 1: The data has a strong, positive, linear association, and a linear function models the data well.
   Data set 2: The data has a strong, negative, linear association, and a linear function models the data well.
   Data set 3: The data has a weak, positive, linear association, and a linear function does not model the data well.

STOP

©2023 Amplify Education, Inc. All rights reserved.

Lesson 19 Correlation Coefficient (Part 1) 349

1 Launch

Distribute the Activity 2 PDF. Tell students they will work with their partner to determine how the value of r is calculated.

2 Monitor

Help students get started by labeling the side lengths of a rectangle and determining its area.

Look for points of confusion:

- **Thinking that the side lengths of each rectangle must be measured**. Remind students the side lengths are represented in the given table.

Look for productive strategies:

- Predicting a positive value for the correlation coefficient.
- Interpreting the correlation coefficient as meaning there is a strong, positive association.

3 Connect

Have pairs of students share the correlation coefficient they determined.

Display the three data sets and their correlation coefficient.

Define the term ***correlation coefficient***.

Highlight that the correlation coefficient is the measurement used to determine the strength of association. Be sure students understand that while the area of the rectangles they determined cannot be negative, they retained the negative sign on the calculations to know whether the association is positive or negative.

Ask, "What is the benefit to having a method for quantifying association?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can explore a geometric interpretation of the correlation coefficient r and watch how it is calculated in real time.

Accessibility: *Guide Processing and Visualization*

Demonstrate how to determine the area of one of the rectangles in one of the data sets. Consider demonstrating both a positive value and a negative value. Let students know to retain the negative sign on any calculations, even though the actual area of the rectangle cannot be negative. The negative sign will be important to indicate that the data shows a negative association.

Math Language Development

MLR3: Critique, Correct, and Clarify

During the Connect, display an incorrect statement and incorrect reasoning for Problem 7, such as "For Data set 2, the data has a strong, positive, linear association because the value of r is close to –1. A linear function would model the data well." Ask these questions:

- **Critique:** "Why is this statement incorrect?"
- **Correct:** "How would you correct this statement?"
- **Clarify:** "How do you know your statement is correct?"

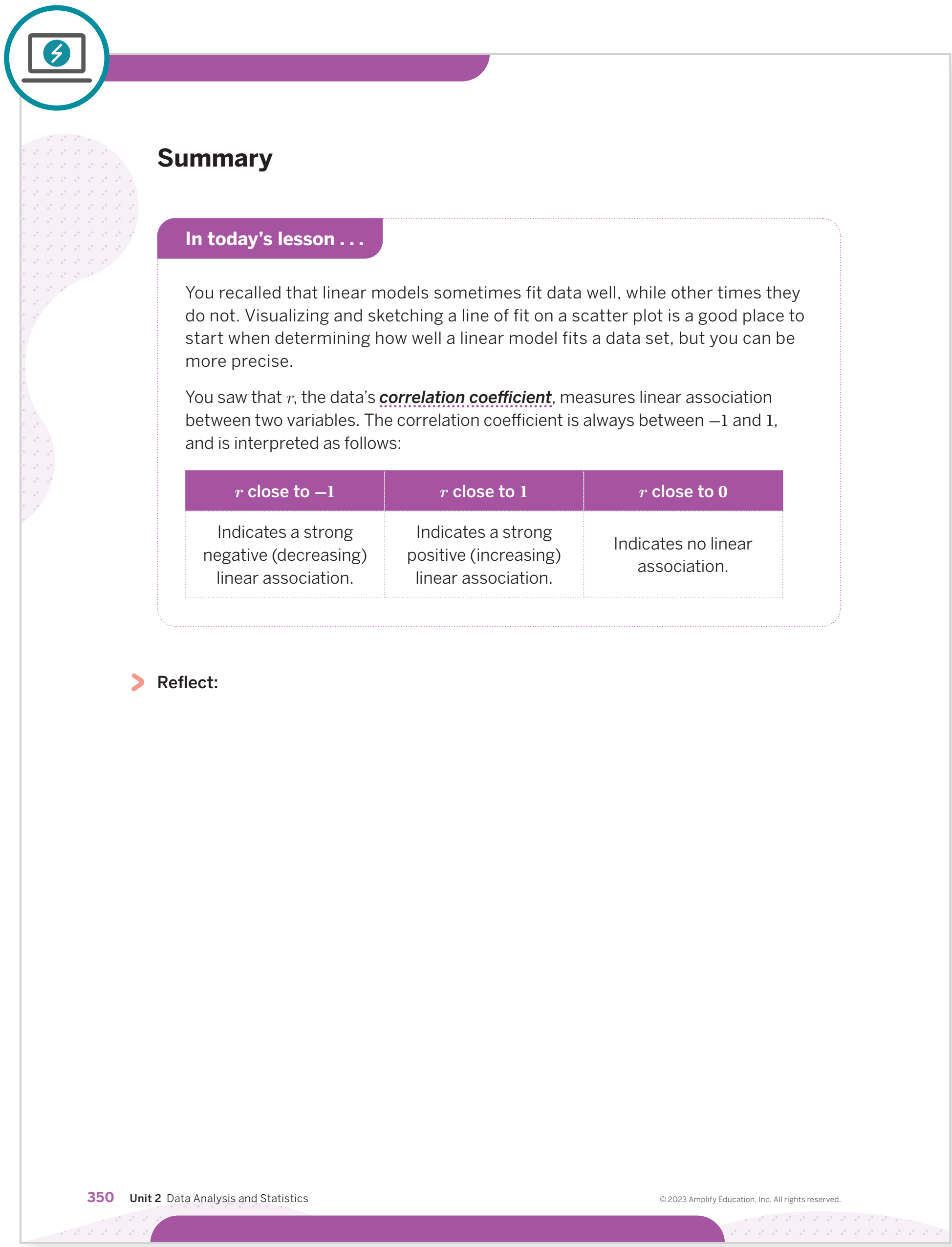
English Learners

After the discussion, clearly annotate the incorrect part(s) of the statement.

Whole Class | 5 min

Summary

Review and synthesize how to determine the correlation coefficient and what it represents.



Summary

In today's lesson . . .

You recalled that linear models sometimes fit data well, while other times they do not. Visualizing and sketching a line of fit on a scatter plot is a good place to start when determining how well a linear model fits a data set, but you can be more precise.

You saw that r, the data's ***correlation coefficient***, measures linear association between two variables. The correlation coefficient is always between -1 and 1, and is interpreted as follows:

r close to -1	r close to 1	r close to 0
Indicates a strong negative (decreasing) linear association.	Indicates a strong positive (increasing) linear association.	Indicates no linear association.

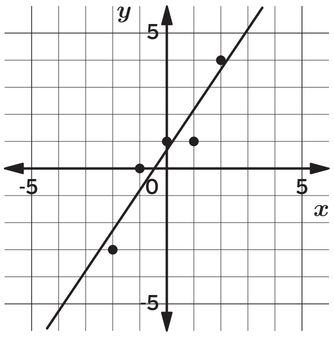
> **Reflect:**

350 Unit 2 Data Analysis and Statistics

©2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the following scatter plot, line of best fit, and correlation coefficient. **Note:** There is at least one error shown.



$y = 1.5x + 0.6$
$r = -0.94$

Have students share what they notice is incorrect about the given values that correspond with the scatter plot and line of best fit. After students share, reveal that the correct value for r is 0.94 because the line is increasing.

Highlight that the correlation coefficient always has a value between -1 to 1 (for a challenge, consider asking why), and it describes both the strength and direction of linear association within the data.

Formalize vocabulary: ***correlation coefficient***

Ask, "Is it always convenient to calculate the correlation coefficient by drawing many rectangles? How do you think mathematicians or others might calculate it?" Sample response: No, it can be time consuming. Mathematicians and others probably use technology to determine the correlation coefficient.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What did you find helpful in determining what the correlation coefficient represents? Why?"

Math Language Development

MLR2: Collect and Display

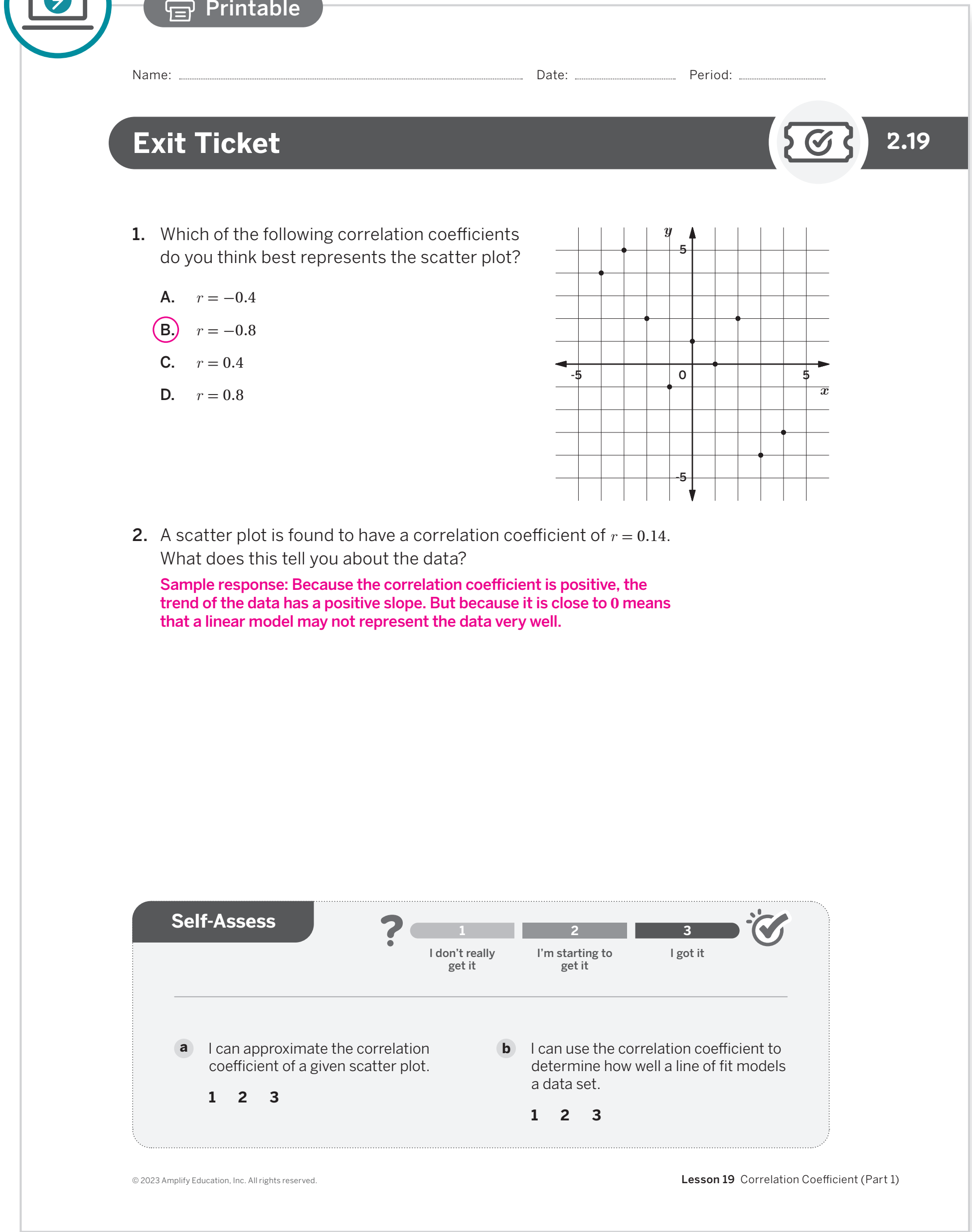
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *correlation coefficient* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining and interpreting the correlation coefficient.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 2.19

1. Which of the following correlation coefficients do you think best represents the scatter plot?
   - A. $r = -0.4$
   - [✓ B.] $r = -0.8$
   - C. $r = 0.4$
   - D. $r = 0.8$

2. A scatter plot is found to have a correlation coefficient of $r = 0.14$. What does this tell you about the data?

   Sample response: Because the correlation coefficient is positive, the trend of the data has a positive slope. But because it is close to 0 means that a linear model may not represent the data very well.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can approximate the correlation coefficient of a given scatter plot.
1 2 3

b I can use the correlation coefficient to determine how well a line of fit models a data set.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 19 Correlation Coefficient (Part 1)

Success looks like . . .

- **Language Goal:** Understanding how the correlation coefficient describes associations in bivariate data. **(Reading and Writing)**
  - » Explaining what the correlation coefficient of $r = 0.14$ means in Problem 2.
- **Goal:** Understanding that there is a precise method for calculating the correlation coefficient, given a scatter plot.

Suggested next steps

If students select the incorrect correlation coefficient in Problem 1, consider:

- Reviewing responses from Activity 1.
- Assigning Practice Problem 1.
- Asking, "How can determining whether there is a positive or negative association help you find the correlation coefficient?"

If students incorrectly interpret the correlation coefficient in Problem 2, consider:

- Reviewing responses from Activity 2.
- Assigning Practice Problem 2.
- Asking, "What does a correlation coefficient close to 0 mean?"

Professional Learning

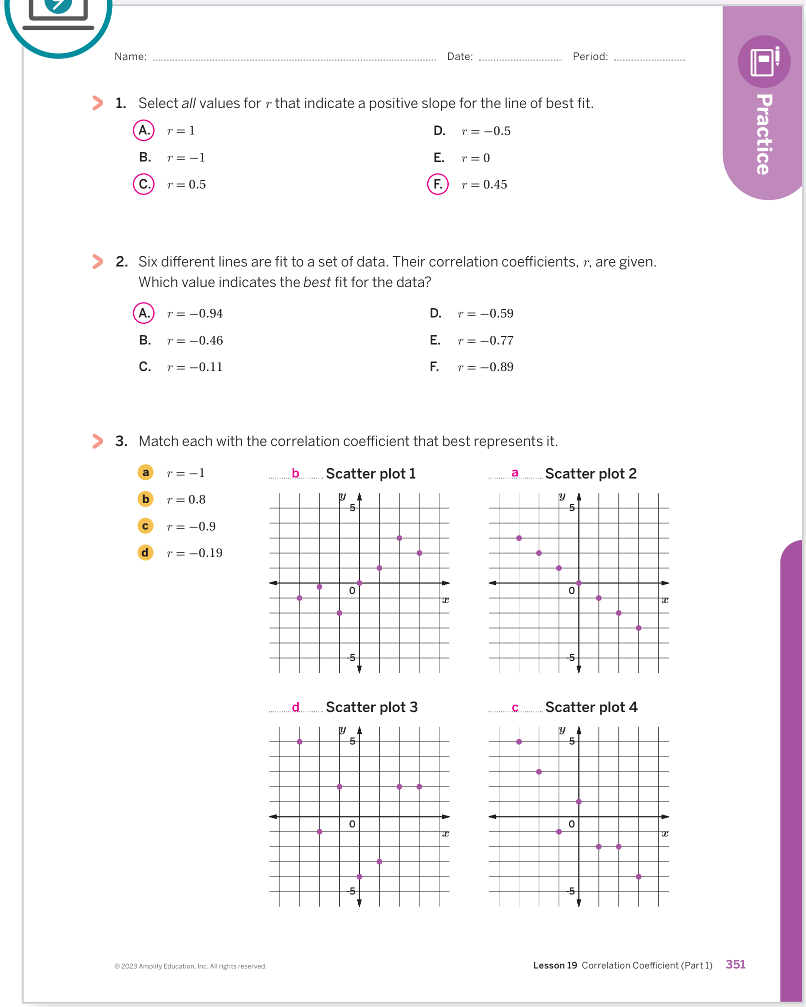
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What was especially satisfying about seeing students geometrically find the correlation coefficient?
- Have you changed any ideas you used to have about the correlation coefficient as a result of today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

Practice

1. Select *all* values for r that indicate a positive slope for the line of best fit.

[✓ A.] $r = 1$
B. $r = -1$
[✓ C.] $r = 0.5$
D. $r = -0.5$
E. $r = 0$
[✓ F.] $r = 0.45$

2. Six different lines are fit to a set of data. Their correlation coefficients, r, are given. Which value indicates the *best* fit for the data?

[✓ A.] $r = -0.94$
B. $r = -0.46$
C. $r = -0.11$
D. $r = -0.59$
E. $r = -0.77$
F. $r = -0.89$

3. Match each with the correlation coefficient that best represents it.

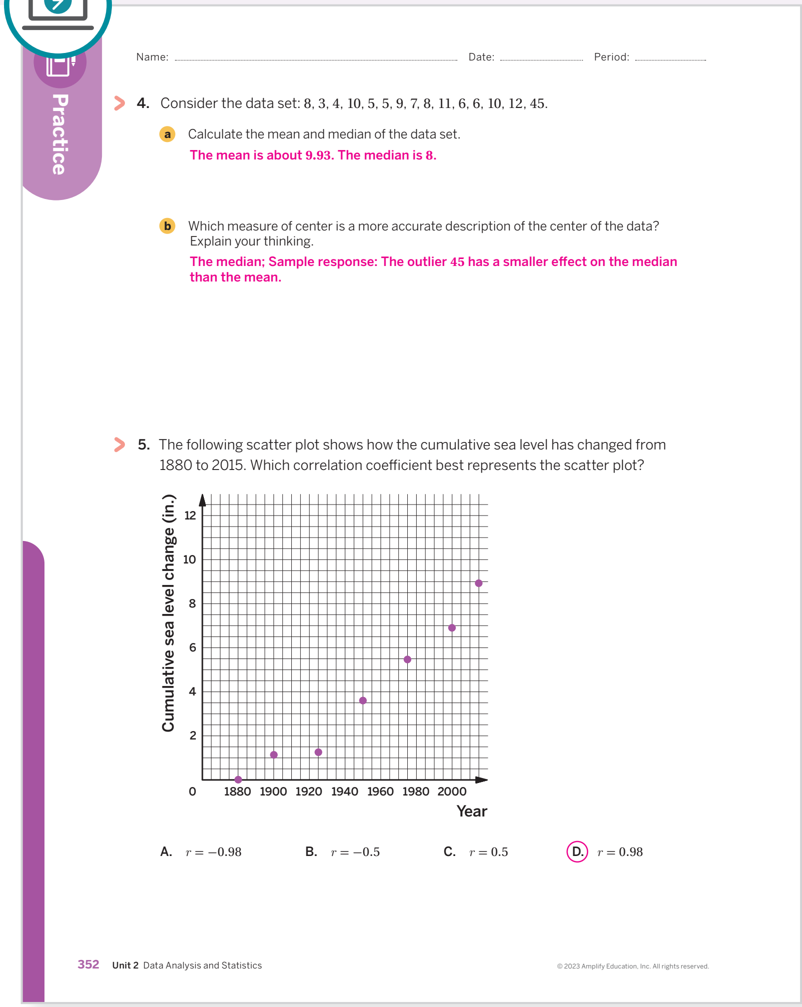
a $r = -1$
b $r = 0.8$
c $r = -0.9$
d $r = -0.19$

b Scatter plot 1

a Scatter plot 2

d Scatter plot 3

c Scatter plot 4



Name: ________ Date: ________ Period: ________

Practice

4. Consider the data set: 8, 3, 4, 10, 5, 5, 9, 7, 8, 11, 6, 6, 10, 12, 45.

a Calculate the mean and median of the data set.

The mean is about 9.93. The median is 8.

b Which measure of center is a more accurate description of the center of the data? Explain your thinking.

The median; Sample response: The outlier 45 has a smaller effect on the median than the mean.

5. The following scatter plot shows how the cumulative sea level has changed from 1880 to 2015. Which correlation coefficient best represents the scatter plot?

A. $r = -0.98$
B. $r = -0.5$
C. $r = 0.5$
[✓ D.] $r = 0.98$

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	1
	3	Activity 1	2
Spiral	4	Unit 2 Lesson 6	2
Formative ⚡	5	Unit 2 Lesson 20	1

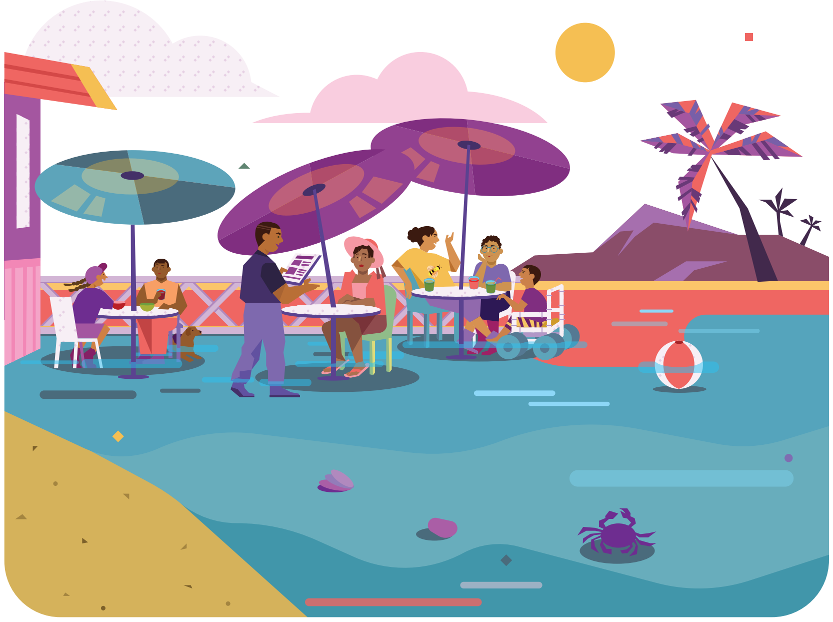
⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 2 | **LESSON 20**

Correlation Coefficient (Part 2)



Let's calculate and use the correlation coefficient to describe linear models in real-world scenarios.

Focus

Goals

1. Determine the correlation coefficient using graphing or spreadsheet technology.
2. **Language Goal:** Interpret the correlation coefficient in context. **(Speaking and Listening, Writing)**

Coherence

Today

Students use spreadsheet technology to create scatter plots, the line of best fit, and determine the correlation coefficient. They then determine the correlation coefficient in the context of real-world situations. Students calculate the correlation coefficient for different time frames from a large data set to see how selectively picking data can lead to different outcomes.

Previously

In Lesson 19, students geometrically found and interpreted the correlation coefficient.

Coming Soon

In Lesson 21, students will determine the difference between correlation and causation.

Rigor

- Students build **procedural fluency** in using spreadsheet technology to calculate the correlation coefficient.
- Students **apply** their understanding of the correlation coefficient and interpret it in context.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	20 min	15 min	10 min	5 min	5 min
Independent	Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 1 PDF, one per group
- Activity 2 PDF, one per group
- spreadsheet technology

Math Language Development

Review words

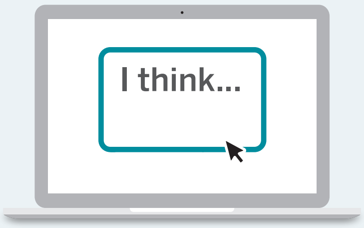
- *correlation coefficient*
- *line of best fit*

Amps | Featured Activity

Activity 2

See Student Thinking

Students are asked to explain their thinking behind interpreting the correlation coefficient, and these explanations are available to you digitally, in real time.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might not see a problem with choosing only part of the data set. Guide students to evaluate the consequences of doing so as they complete Activity 2. Afterwards, lead the class in a discussion about the integrity of statistics and how looking at only part of the data can be misleading and deceptive, and ultimately unethical.

Modifications to Pacing

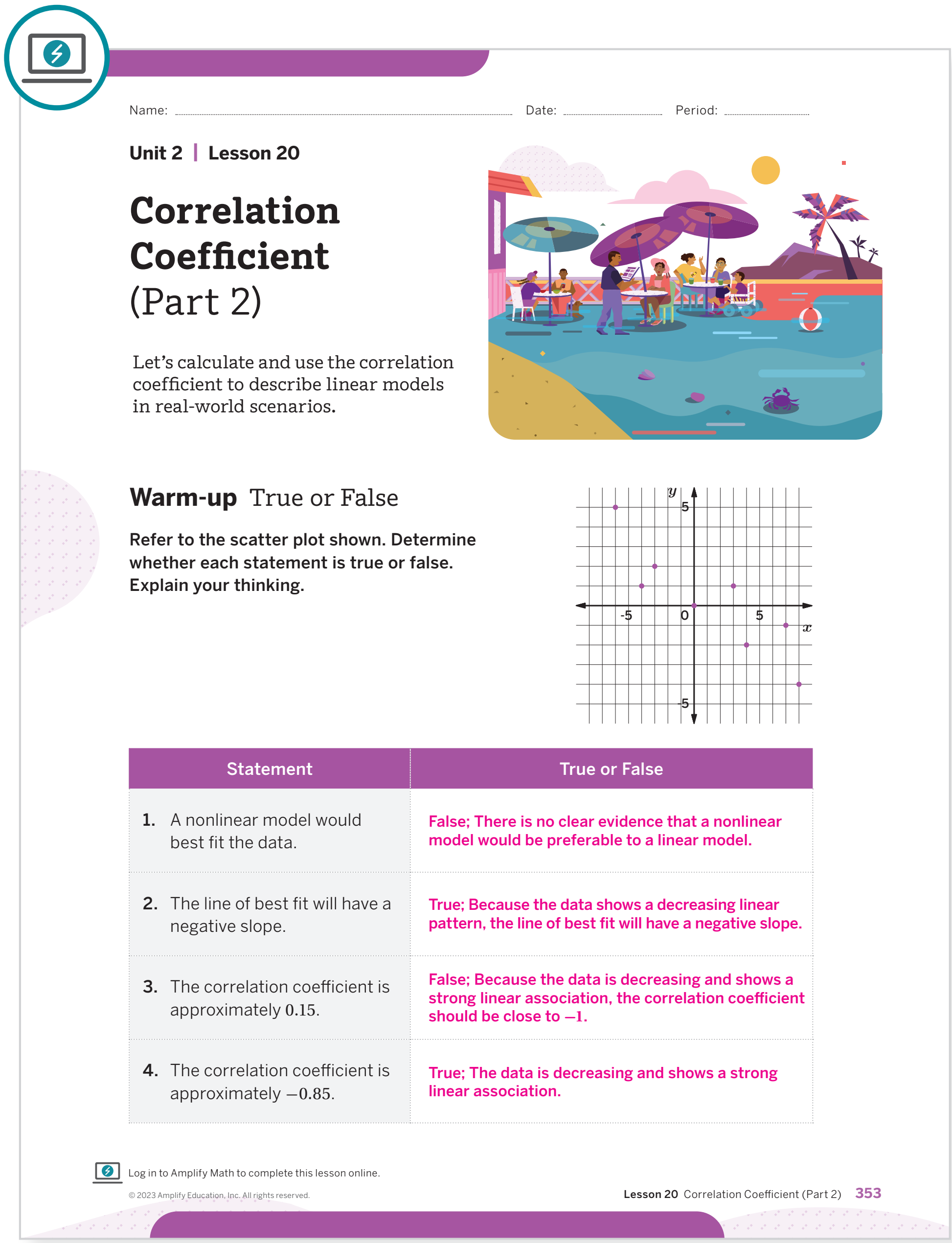
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problems 1 and 2 may be omitted.
- In **Activity 1**, Problem 4 may be omitted.
- In **Activity 3**, Problem 2 may be omitted.

Independent | 5 min

Warm-up True or False

Students determine if statements are true or false for a scatter plot to recall descriptions and analysis of a scatter plot.



Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 20

Correlation Coefficient (Part 2)

Let's calculate and use the correlation coefficient to describe linear models in real-world scenarios.

Warm-up True or False

Refer to the scatter plot shown. Determine whether each statement is true or false. Explain your thinking.

Statement	True or False
1. A nonlinear model would best fit the data.	**False; There is no clear evidence that a nonlinear model would be preferable to a linear model.**
2. The line of best fit will have a negative slope.	**True; Because the data shows a decreasing linear pattern, the line of best fit will have a negative slope.**
3. The correlation coefficient is approximately 0.15.	**False; Because the data is decreasing and shows a strong linear association, the correlation coefficient should be close to -1.**
4. The correlation coefficient is approximately -0.85.	**True; The data is decreasing and shows a strong linear association.**

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Correlation Coefficient (Part 2) 353

1 Launch

Have students work individually for 3 minutes to determine if the statements are true or false before participating in a whole-class discussion.

2 Monitor

Help students get started by asking, "What pattern or trend do you notice in the data?"

Look for points of confusion:

- **Thinking the slope and/or correlation coefficient will be positive.** Have students sketch an approximation for the line of best fit and make observations.

Look for productive strategies:

- Sketching a line of fit for the data.
- Using the slope to help determine which correlation coefficient is a better approximation.

3 Connect

Have individual students share their responses and thinking for each problem.

Display the scatter plot and statements from the table.

Highlight that the sign of the slope of the line of best fit should always match the sign of the correlation coefficient. This is because a negative correlation coefficient indicates a negative (decreasing) association, which is also indicated by a negative slope. In Problem 3, $r = 0.15$ is close to 0 and positive, so it cannot be the correlation coefficient.

Ask, "Why is the connection between slope and the correlation coefficient important?" Sample response: Knowing that the sign of both should be the same is a way to check to see if the slope for the line of best fit and correlation coefficient are correct.

Math Language Development

MLR7: Compare and Connect

During the Connect, draw connections between the sign of the correlation coefficient and the slope of the line of best fit. Ask these questions:

- "What does it mean for a line to have a positive slope? Negative slope?"
- "What does it mean for a correlation coefficient to be positive? Negative?"
- "Can a scatter plot with a negative linear association have a line of best fit with a positive correlation coefficient? Why or why not?"

English Learners

Highlight the negative sign in Statement 4 from the table with the negative trend of the data in the scatter plot shown.

Power-up

To power up students' ability to determine the most appropriate approximation for the correlation coefficient, have students complete:

Provide students with a copy of the Power-Up PDF.

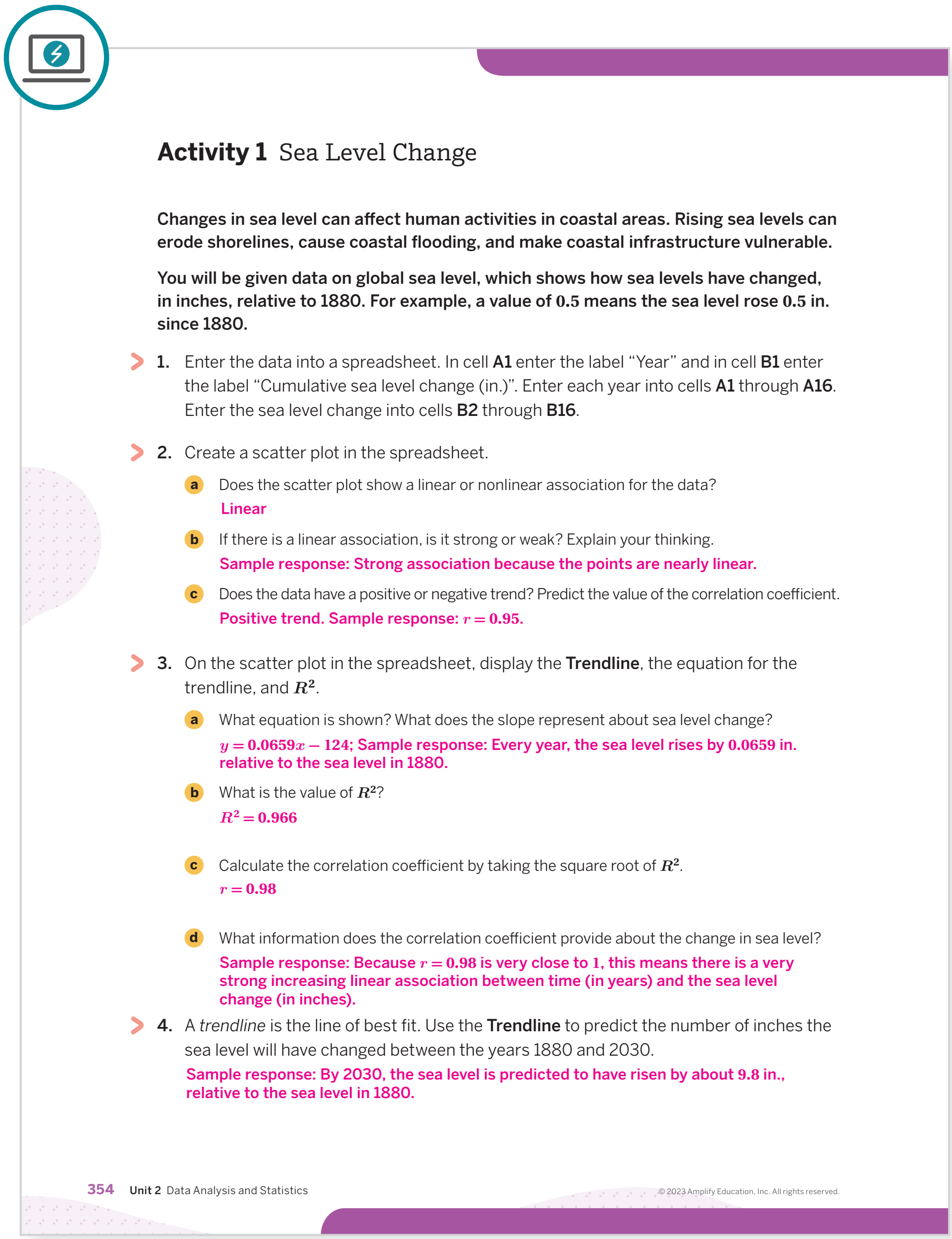
Use: Before the Warm-up

Informed by: Performance on Lesson 19, Practice Problem 5 and Exit Ticket

Small Groups | 20 min

Activity 1 Sea Level Change

Students use spreadsheet technology to calculate the correlation coefficient and interpret it in context.



Activity 1 Sea Level Change

Changes in sea level can affect human activities in coastal areas. Rising sea levels can erode shorelines, cause coastal flooding, and make coastal infrastructure vulnerable.

You will be given data on global sea level, which shows how sea levels have changed, in inches, relative to 1880. For example, a value of 0.5 means the sea level rose 0.5 in. since 1880.

1. Enter the data into a spreadsheet. In cell **A1** enter the label "Year" and in cell **B1** enter the label "Cumulative sea level change (in.)". Enter each year into cells **A1** through **A16**. Enter the sea level change into cells **B2** through **B16**.

2. Create a scatter plot in the spreadsheet.
   - a. Does the scatter plot show a linear or nonlinear association for the data?
     Linear
   - b. If there is a linear association, is it strong or weak? Explain your thinking.
     Sample response: Strong association because the points are nearly linear.
   - c. Does the data have a positive or negative trend? Predict the value of the correlation coefficient.
     Positive trend. Sample response: $r = 0.95$.

3. On the scatter plot in the spreadsheet, display the **Trendline**, the equation for the trendline, and R^2.
   - a. What equation is shown? What does the slope represent about sea level change?
     $y = 0.0659x - 124$; Sample response: Every year, the sea level rises by 0.0659 in. relative to the sea level in 1880.
   - b. What is the value of R^2?
     $R^2 = 0.966$
   - c. Calculate the correlation coefficient by taking the square root of R^2.
     $r = 0.98$
   - d. What information does the correlation coefficient provide about the change in sea level?
     Sample response: Because $r = 0.98$ is very close to 1, this means there is a very strong increasing linear association between time (in years) and the sea level change (in inches).

4. A *trendline* is the line of best fit. Use the **Trendline** to predict the number of inches the sea level will have changed between the years 1880 and 2030.
   Sample response: By 2030, the sea level is predicted to have risen by about 9.8 in., relative to the sea level in 1880.

354 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Provide access to spreadsheet technology. Tell students they will learn how to use spreadsheets to create scatter plots, the line of best fit, and calculate the correlation coefficient. Distribute the Activity 1 PDF to each group.

2 Monitor

Help students get started by showing them how to insert a scatter plot into their spreadsheet, once they have entered the data. Show them how to display the trendline.

Look for points of confusion:

- **Not having the data show up on the scatter plot.** Make sure students have the columns with the data highlighted before inserting the scatter plot.
- **Thinking the data represents measured sea level each year.** Remind students that the data shows the change in sea level, in inches, relative to 1880.

Look for productive strategies:

- Creating a scatter plot with appropriate labels, line of best fit, and R^2 value.
- Interpreting the correlation coefficient and predicted value in the context of sea level change relative to 1880.

3 Connect

Have groups of students share their scatter plots, lines of best fit, correlation coefficients, and sea level change predictions.

Display the scatter plot, line of best fit, and R^2 value.

Highlight that a properly labeled and scaled scatter plot can help with interpreting the slope and seeing overall association in the data. Taking the square root of R^2 only works for linear models.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Students may need a reminder of how to enter values into a spreadsheet. Consider demonstrating how to enter values, create a scatter plot, and display the trendline for a sample set of data.

Extension: *Math Enrichment*

Ask students to interpret the y-intercept within the context of this problem and ask them whether the value makes sense. Sample response: In Year 0, the sea level was 124 in. below the measured level in 1880. Because Year 0 is very far from 1880, when the data was collected, this is unlikely to have been true.

Math Language Development

MLR8: Discussion Supports

During the Connect, draw students' attention to the value of R^2 and how they needed to take the square root of this value to determine the value of the correlation coefficient r. Consider asking, "How do you know that you need to take the square root of R^2?" The value is given as the square of the correlation coefficient. I need to take the square root to determine the value of r.

English Learners

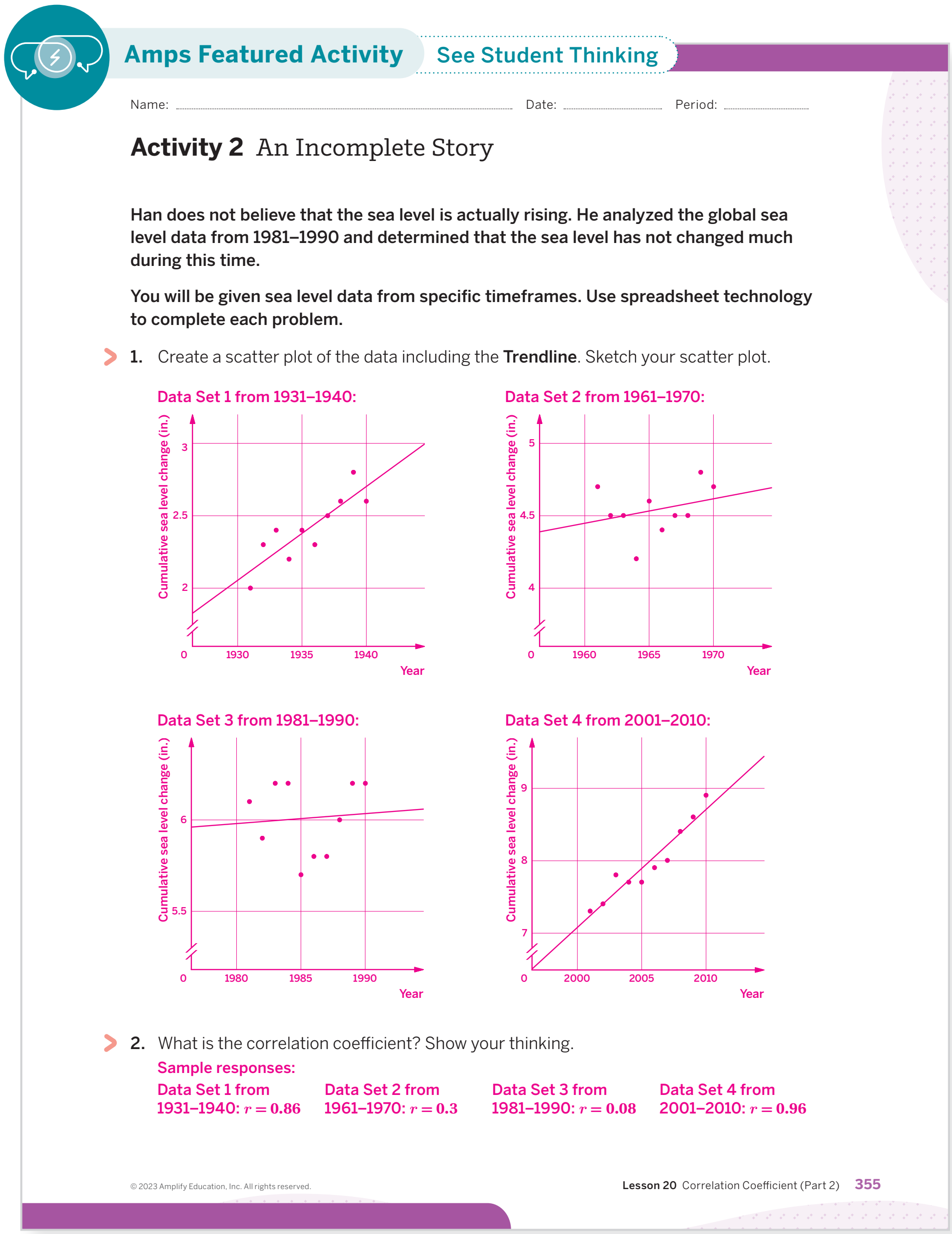
Use gestures to show how the term *trendline* corresponds to the term *line of best fit* because the trendline shows the *trend* of the data.

Small Groups | 15 min

Activity 2 An Incomplete Story

Students use spreadsheet technology to analyze data from specific time frames to understand how analyzing only subsets of data can be misleading.

Amps Featured Activity See Student Thinking



Name: ________ Date: ________ Period: ________

Activity 2 An Incomplete Story

Han does not believe that the sea level is actually rising. He analyzed the global sea level data from 1981–1990 and determined that the sea level has not changed much during this time.

You will be given sea level data from specific timeframes. Use spreadsheet technology to complete each problem.

1. Create a scatter plot of the data including the **Trendline**. Sketch your scatter plot.

2. What is the correlation coefficient? Show your thinking.

Sample responses:

Data Set 1 from 1931–1940: $r = 0.86$	Data Set 2 from 1961–1970: $r = 0.3$	Data Set 3 from 1981–1990: $r = 0.08$	Data Set 4 from 2001–2010: $r = 0.96$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Correlation Coefficient (Part 2) 355

1 Launch

Tell students they will use spreadsheet technology to analyze sea level change across different time frames. Distribute the Activity 2 PDF to each group.

2 Monitor

Help students get started by modeling how to find charts to insert into a spreadsheet once data has been entered.

Look for points of confusion:

- **Thinking the R^2 value displayed on the spreadsheet is the correlation coefficient.** Have students refer back to Activity 1, Problem 3. Ask, "What does the exponent tell you?"
- **Trying to estimate the predicted number of inches the sea level will rise using only the graph.** Ask, "How can you use the line of best fit to make predictions?"

Look for productive strategies:

- Taking the square root of the displayed value of R^2 to determine the correlation coefficient, and interpreting this value in context.
- Substituting values into the equation for the line of best fit to make predictions.

Activity 2 continued >

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Have students preview Problem 2. Mention they can determine the value of R^2 using their spreadsheet. Guide them to see that the value of R^2 will always be positive, so they must use the trend of the data to help them know if the value of the correlation coefficient r will be positive or negative. Ask these questions:

- "Will the value of R^2 ever be negative? Why or why not?" No, the square of any number, positive or negative, is always positive.
- "When taking the square root of R^2, how will you know whether the value of r should be positive or negative?" The square root can be either positive or negative. I need to use the trend of the scatter plot to tell me if the value of r should be positive or negative.

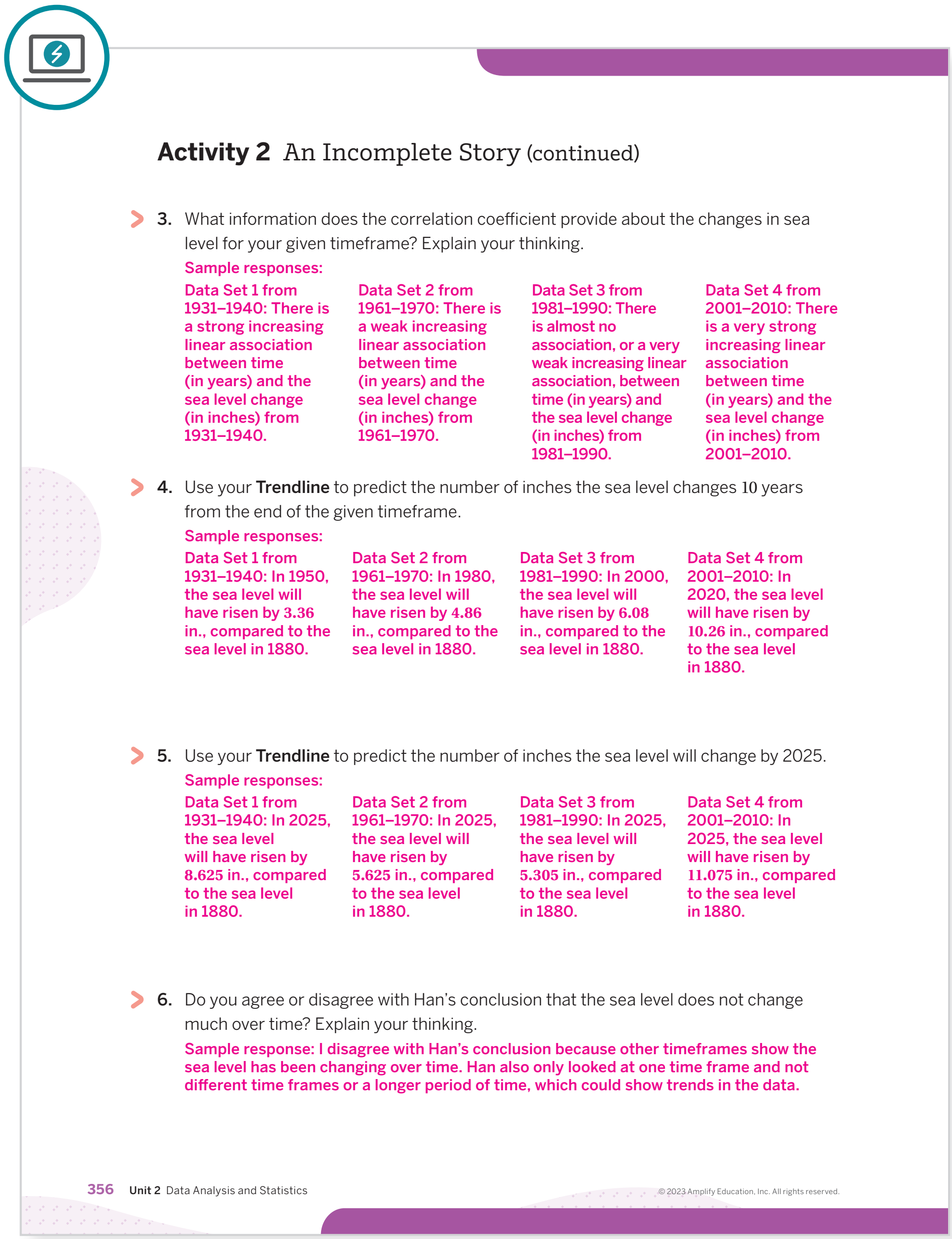
Extension: *Math Enrichment*

Have students respond to this question and explain their thinking:
"Using spreadsheet technology, a set of data is found to have a value of R^2 of 0.81. Does this mean the data shows a positive association?" Not necessarily. R^2 will always be positive, because it is a square value. The trend of the data should inform the sign of the correlation coefficient after taking the square root of R^2.

Small Groups | 15 min

Activity 2 An Incomplete Story (continued)

Students use spreadsheet technology to analyze data from specific time frames to understand how analyzing only subsets of data can be misleading.



Activity 2 An Incomplete Story (continued)

3. What information does the correlation coefficient provide about the changes in sea level for your given timeframe? Explain your thinking.

Sample responses:

Data Set 1 from 1931–1940: There is a strong increasing linear association between time (in years) and the sea level change (in inches) from 1931–1940.	Data Set 2 from 1961–1970: There is a weak increasing linear association between time (in years) and the sea level change (in inches) from 1961–1970.	Data Set 3 from 1981–1990: There is almost no association, or a very weak increasing linear association, between time (in years) and the sea level change (in inches) from 1981–1990.	Data Set 4 from 2001–2010: There is a very strong increasing linear association between time (in years) and the sea level change (in inches) from 2001–2010.

4. Use your **Trendline** to predict the number of inches the sea level changes 10 years from the end of the given timeframe.

Sample responses:

Data Set 1 from 1931–1940: In 1950, the sea level will have risen by 3.36 in., compared to the sea level in 1880.	Data Set 2 from 1961–1970: In 1980, the sea level will have risen by 4.86 in., compared to the sea level in 1880.	Data Set 3 from 1981–1990: In 2000, the sea level will have risen by 6.08 in., compared to the sea level in 1880.	Data Set 4 from 2001–2010: In 2020, the sea level will have risen by 10.26 in., compared to the sea level in 1880.

5. Use your **Trendline** to predict the number of inches the sea level will change by 2025.

Sample responses:

Data Set 1 from 1931–1940: In 2025, the sea level will have risen by 8.625 in., compared to the sea level in 1880.	Data Set 2 from 1961–1970: In 2025, the sea level will have risen by 5.625 in., compared to the sea level in 1880.	Data Set 3 from 1981–1990: In 2025, the sea level will have risen by 5.305 in., compared to the sea level in 1880.	Data Set 4 from 2001–2010: In 2025, the sea level will have risen by 11.075 in., compared to the sea level in 1880.

6. Do you agree or disagree with Han's conclusion that the sea level does not change much over time? Explain your thinking.

Sample response: I disagree with Han's conclusion because other timeframes show the sea level has been changing over time. Han also only looked at one time frame and not different time frames or a longer period of time, which could show trends in the data.

356 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the four data sets.

Have groups of students share their scatter plots, correlation coefficients, predictions, and interpretations with the class.

Ask, "Why were the predicted amounts that the sea level will rise very different from one data set to another?" Sample response: Each data set has a different line of best fit and correlation coefficient, so each will have a different prediction.

Display the four scatter plots with their corresponding lines of fit, correlation coefficients, and predictions.

Highlight that over short periods of time, data sets can look very different compared to another point in time. However, overall, large data sets can show trends over longer periods of time.

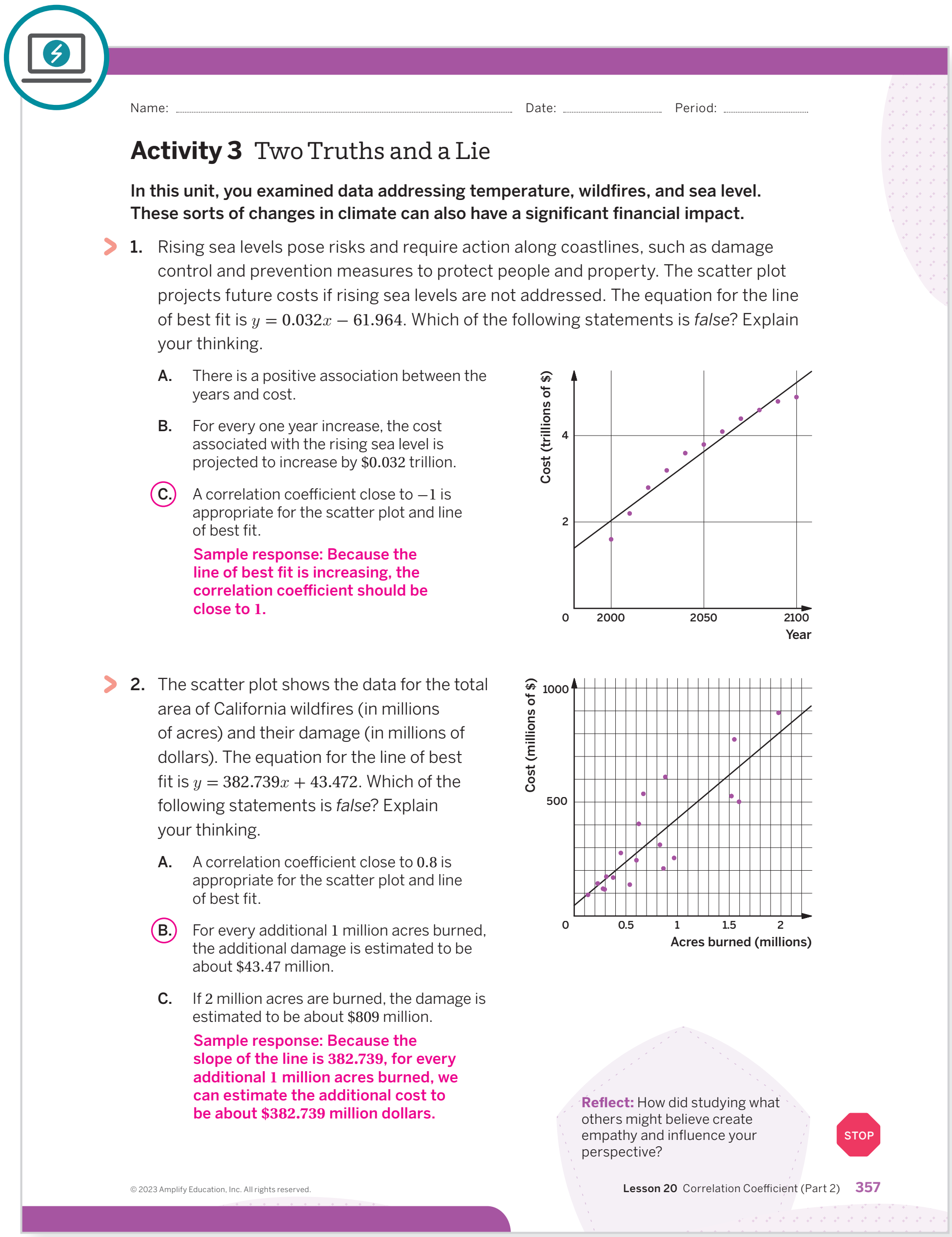
Ask, "How can analyzing portions of a data set be misleading?" Sample response: While large data sets might have a strong association, only selecting a small portion of the data might show no association when there really is one.

Optional

Small Groups | 10 min

Activity 3 Two Truths and a Lie

Students analyze data from sustainability topics to determine which statements accurately describe the data.



Name: ________ Date: ________ Period: ________

Activity 3 Two Truths and a Lie

In this unit, you examined data addressing temperature, wildfires, and sea level. These sorts of changes in climate can also have a significant financial impact.

1. Rising sea levels pose risks and require action along coastlines, such as damage control and prevention measures to protect people and property. The scatter plot projects future costs if rising sea levels are not addressed. The equation for the line of best fit is $y = 0.032x - 61.964$. Which of the following statements is *false*? Explain your thinking.
   - A. There is a positive association between the years and cost.
   - B. For every one year increase, the cost associated with the rising sea level is projected to increase by $\$0.032$ trillion.
   - [✓ C.] A correlation coefficient close to -1 is appropriate for the scatter plot and line of best fit.

   Sample response: Because the line of best fit is increasing, the correlation coefficient should be close to 1.

2. The scatter plot shows the data for the total area of California wildfires (in millions of acres) and their damage (in millions of dollars). The equation for the line of best fit is $y = 382.739x + 43.472$. Which of the following statements is *false*? Explain your thinking.
   - A. A correlation coefficient close to 0.8 is appropriate for the scatter plot and line of best fit.
   - [✓ B.] For every additional 1 million acres burned, the additional damage is estimated to be about $\$43.47$ million.
   - C. If 2 million acres are burned, the damage is estimated to be about $\$809$ million.

   Sample response: Because the slope of the line is 382.739, for every additional 1 million acres burned, we can estimate the additional cost to be about $\$382.739$ million dollars.

Reflect: How did studying what others might believe create empathy and influence your perspective?

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Correlation Coefficient (Part 2) 357

1 Launch

Say, "You will now look at two different scenarios in sustainability and determine which statements are true and false."

2 Monitor

Help students get started by asking, "What does the line of best fit tell you about the context of the problem?"

Look for points of confusion:

- **Thinking they must select the true statements.** Remind students they are looking for the "lie," or false statement.

Look for productive strategies:

- Comparing the slope of the line of best fit to the scatter plot and correlation coefficient.
- Using the labels of the scatter plot to interpret the slope in context.
- Substituting 2 million acres into the equation for the line of best fit to predict the cost.

3 Connect

Display the two scatter plots and statements.

Have groups of students share which statement they selected as false and why they selected that statement.

Highlight that even when the correlation coefficient cannot be computed using spreadsheet technology, using other information, such as a given scatter plot and slope, are good ways to estimate what it should be.

Ask, "Does it make sense that both of these sustainability situations show positive association?" Sample response: Yes, as the sea level rises, or more acres are burned, the cost to address these issues should increase as well.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest students use color coding to highlight the slope in each line of best fit equation and where they see that value in any of the statement choices.

Extension: *Math Enrichment*

Have students research sustainability topics from their state or community. Ask them to select one of these topics and use the data they found to create a scatter plot using spreadsheet technology. Have them use the spreadsheet technology to determine the line of best fit and correlation coefficient. Then ask them to write a few sentences about what the data tells them.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the prompt for each problem. Sample directions for each read are provided for Problem 1.

Read 1: Students should understand that the scatter plot shows the relationship between total cost to fix damage to coastlines, due to rising sea levels, and years.

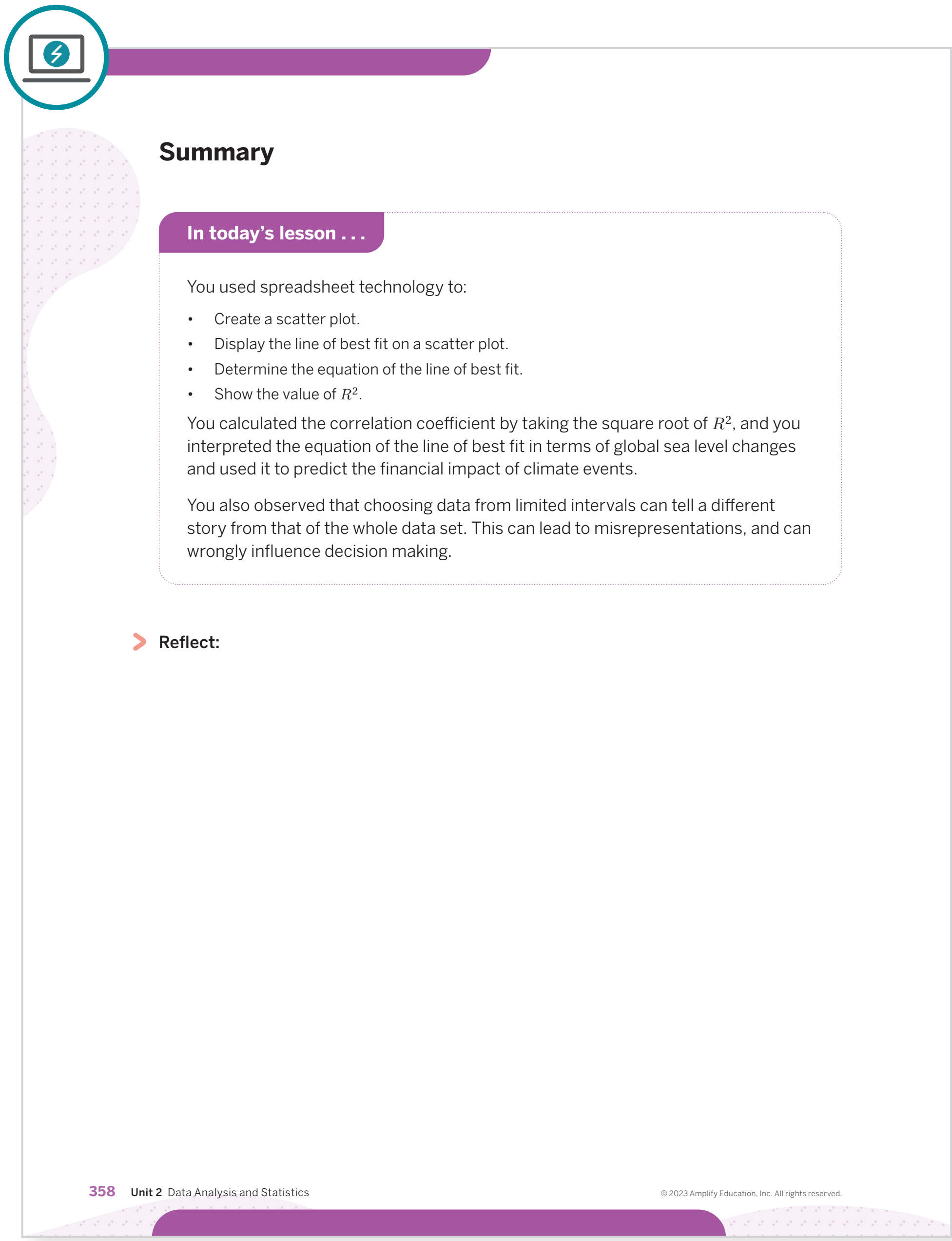
Read 2: Ask students to identify what the values in the line of best fit equation tell them. For example, there is a positive slope which means the cost is projected to increase.

Read 3: Ask students to brainstorm how the line of best fit equation can help them select which statement is false.

Whole Class | 5 min

Summary

Review and synthesize how to use spreadsheet technology to create scatter plots, draw lines of best fit, and compute correlation coefficients.



Summary

In today's lesson . . .

You used spreadsheet technology to:

- Create a scatter plot.
- Display the line of best fit on a scatter plot.
- Determine the equation of the line of best fit.
- Show the value of R^2.

You calculated the correlation coefficient by taking the square root of R^2, and you interpreted the equation of the line of best fit in terms of global sea level changes and used it to predict the financial impact of climate events.

You also observed that choosing data from limited intervals can tell a different story from that of the whole data set. This can lead to misrepresentations, and can wrongly influence decision making.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display a student's scatter plot, line of best fit, and correlation coefficient from Activity 2.

Have students share how they would use spreadsheet technology to find all this information.

Highlight that using spreadsheet technology allows students to find a scatter plot, line of best fit, and the correlation coefficient quickly.

Ask, "When do you think it is helpful to use spreadsheets to calculate the correlation coefficient?" Sample response: With large data sets, using spreadsheet technology helps to minimize the possibility of errors.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

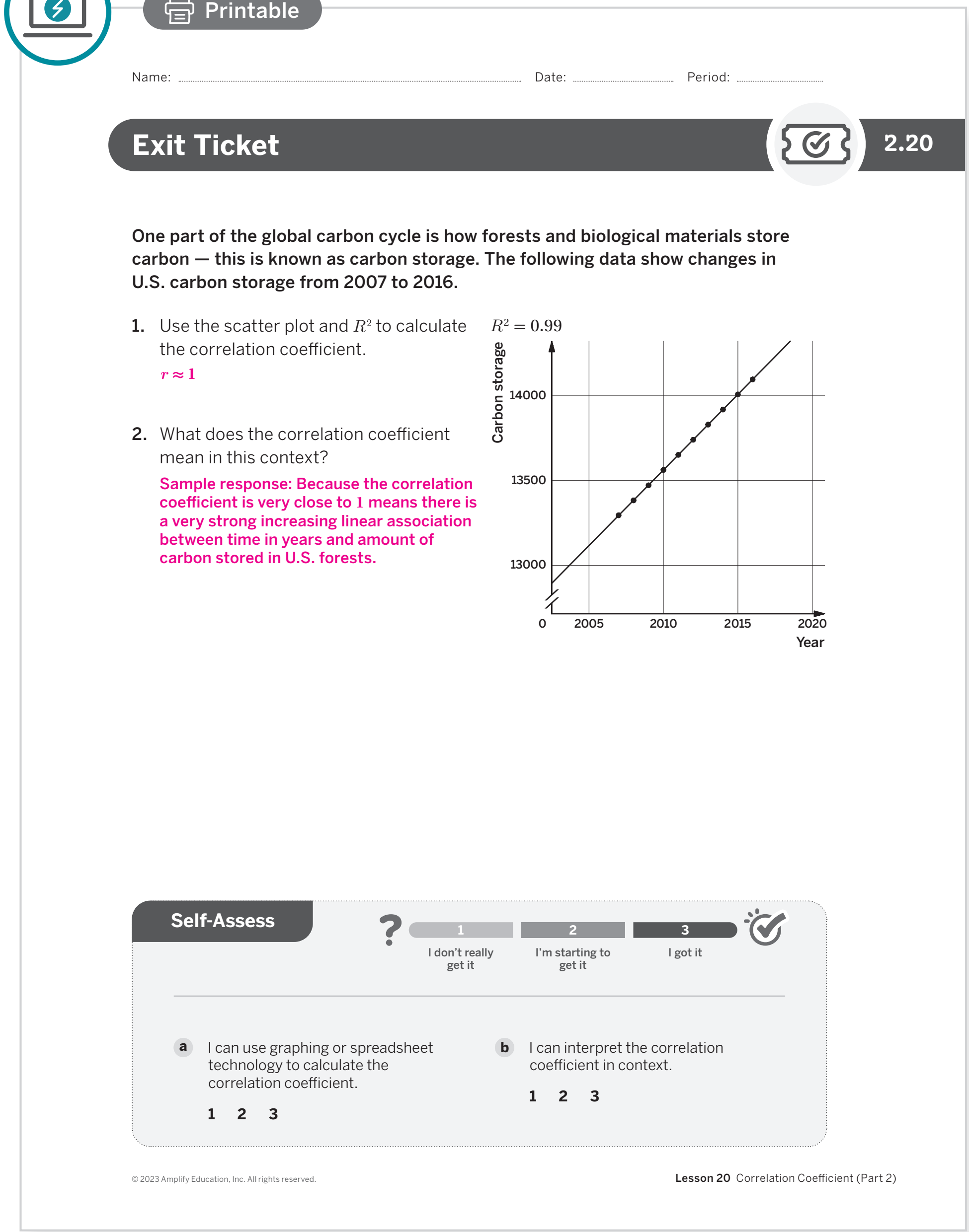
- "How is finding the correlation coefficient using spreadsheet technology helpful?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by using a scatter plot to calculate the correlation coefficient and interpret its meaning in context.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 2.20

One part of the global carbon cycle is how forests and biological materials store carbon — this is known as carbon storage. The following data show changes in U.S. carbon storage from 2007 to 2016.

1. Use the scatter plot and R^2 to calculate the correlation coefficient.
   $r \approx 1$
2. What does the correlation coefficient mean in this context?
   Sample response: Because the correlation coefficient is very close to 1 means there is a very strong increasing linear association between time in years and amount of carbon stored in U.S. forests.

Self-Assess

? 1 I don't really get it	2 I'm starting to get it	3 I got it

a. I can use graphing or spreadsheet technology to calculate the correlation coefficient.
1 2 3

b. I can interpret the correlation coefficient in context.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Correlation Coefficient (Part 2)

Success looks like . . .

- **Goal:** Determining the correlation coefficient using spreadsheet technology.
- **Language Goal:** Interpreting the correlation coefficient in context. **(Speaking and Listening, Writing)**
  - » Explaining the meaning of the correlation coefficient in the context of carbon storage.

Suggested next steps

If students incorrectly calculate the correlation coefficient in Problem 1, consider:

- Reviewing Problem 3, parts c and d from Activity 1.
- Assigning Practice Problem 1.
- Asking, "How do you determine the correlation coefficient if you are given R^2?"

If students incorrectly interpret the correlation coefficient in Problem 2, consider:

- Reviewing Problem 3d from Activity 1.
- Assigning Practice Problem 1.
- Asking, "What does a correlation coefficient close to –1 or 1 tell you about the data?"

Professional Learning

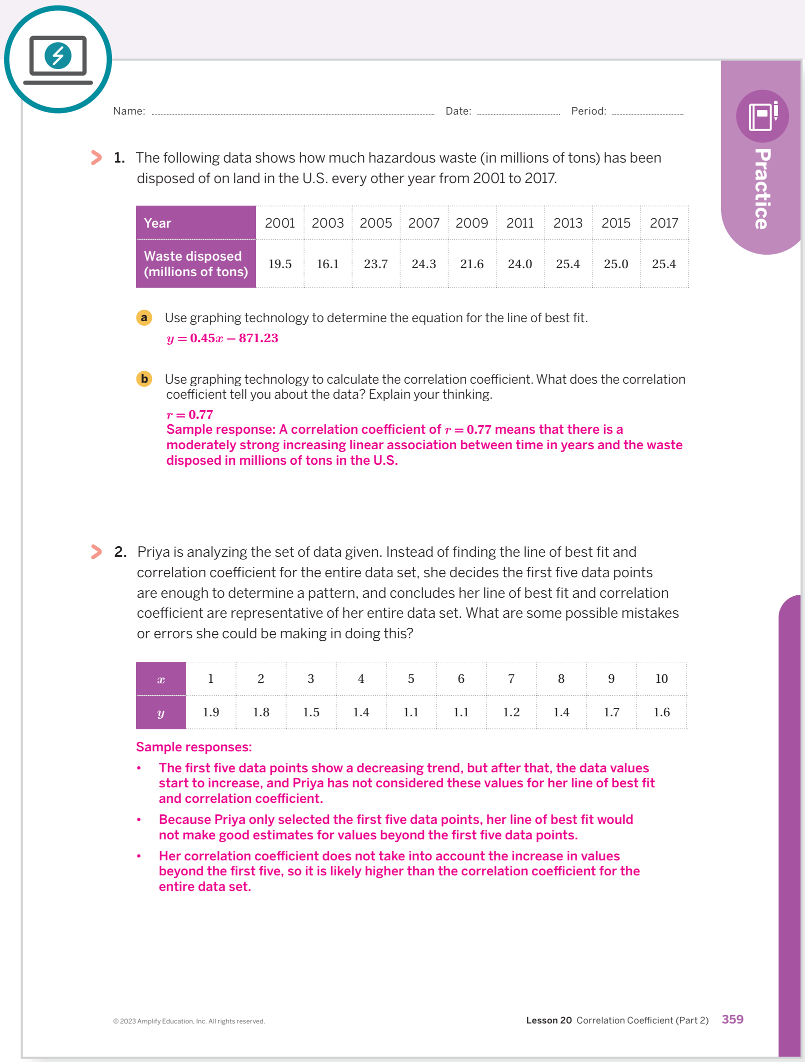
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways did using spreadsheet technology go as planned?
- What did students find frustrating about using spreadsheet technology? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

Practice

1. The following data shows how much hazardous waste (in millions of tons) has been disposed of on land in the U.S. every other year from 2001 to 2017.

Year	2001	2003	2005	2007	2009	2011	2013	2015	2017
Waste disposed (millions of tons)	19.5	16.1	23.7	24.3	21.6	24.0	25.4	25.0	25.4

a. Use graphing technology to determine the equation for the line of best fit.

$y = 0.45x - 871.23$

b. Use graphing technology to calculate the correlation coefficient. What does the correlation coefficient tell you about the data? Explain your thinking.

$r = 0.77$
Sample response: A correlation coefficient of $r = 0.77$ means that there is a moderately strong increasing linear association between time in years and the waste disposed in millions of tons in the U.S.

2. Priya is analyzing the set of data given. Instead of finding the line of best fit and correlation coefficient for the entire data set, she decides the first five data points are enough to determine a pattern, and concludes her line of best fit and correlation coefficient are representative of her entire data set. What are some possible mistakes or errors she could be making in doing this?

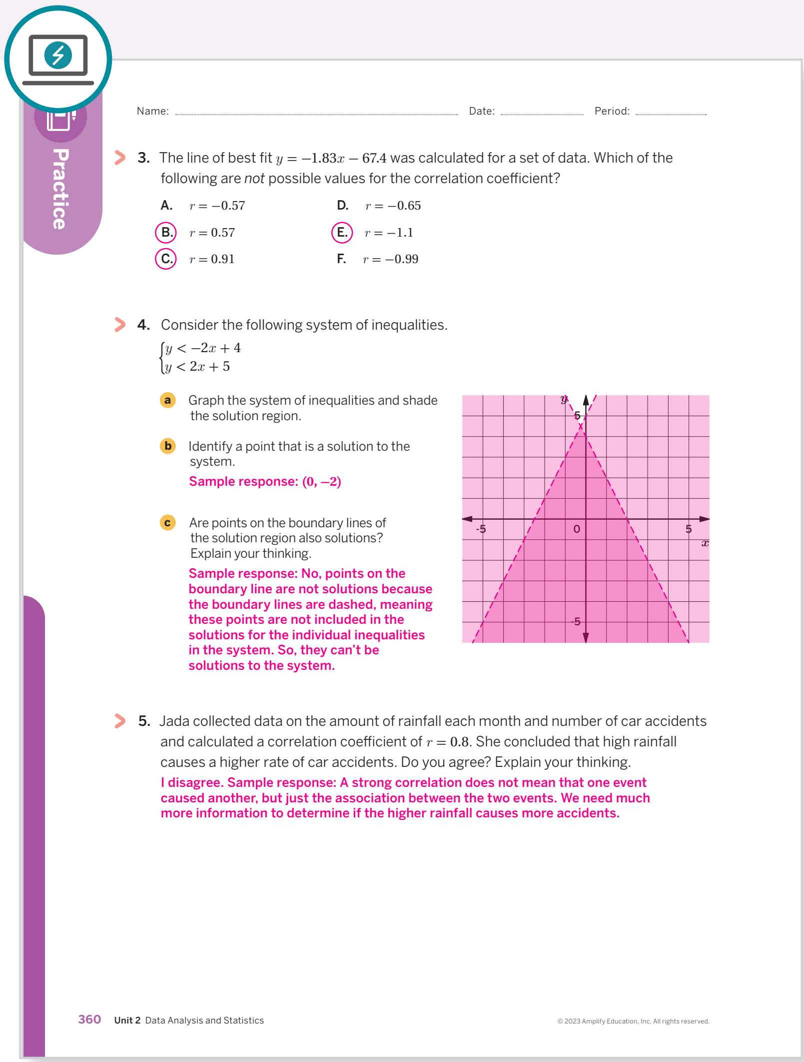
x	1	2	3	4	5	6	7	8	9	10
y	1.9	1.8	1.5	1.4	1.1	1.1	1.2	1.4	1.7	1.6

Sample responses:

- **The first five data points show a decreasing trend, but after that, the data values start to increase, and Priya has not considered these values for her line of best fit and correlation coefficient.**
- **Because Priya only selected the first five data points, her line of best fit would not make good estimates for values beyond the first five data points.**
- **Her correlation coefficient does not take into account the increase in values beyond the first five, so it is likely higher than the correlation coefficient for the entire data set.**

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Correlation Coefficient (Part 2) 359



Name: Date: Period:

Practice

3. The line of best fit $y = -1.83x - 67.4$ was calculated for a set of data. Which of the following are *not* possible values for the correlation coefficient?

A. $r = -0.57$
[✓ B.] $r = 0.57$
[✓ C.] $r = 0.91$
D. $r = -0.65$
[✓ E.] $r = -1.1$
F. $r = -0.99$

4. Consider the following system of inequalities.

$$\begin{cases} y < -2x + 4 \\ y < 2x + 5 \end{cases}$$

a. Graph the system of inequalities and shade the solution region.

b. Identify a point that is a solution to the system.

Sample response: $(0, -2)$

c. Are points on the boundary lines of the solution region also solutions? Explain your thinking.

Sample response: No, points on the boundary line are not solutions because the boundary lines are dashed, meaning these points are not included in the solutions for the individual inequalities in the system. So, they can't be solutions to the system.

5. Jada collected data on the amount of rainfall each month and number of car accidents and calculated a correlation coefficient of $r = 0.8$. She concluded that high rainfall causes a higher rate of car accidents. Do you agree? Explain your thinking.

I disagree. Sample response: A strong correlation does not mean that one event caused another, but just the association between the two events. We need much more information to determine if the higher rainfall causes more accidents.

360 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	**1**	Activity 1	2
	2	Activity 2	2
Spiral	**3**	Unit 2 Lesson 16	1
	4	Unit 1 Lesson 23	2
Formative ⚡	**5**	Unit 2 Lesson 21	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Correlation vs. Causation

Let’s determine the difference between correlation and causation.

Focus

Goals

1. **Language Goal:** Determine whether correlation or causation exists between two variables. **(Reading and Writing)**
2. **Language Goal:** Understand the steps of a statistical experiment. **(Speaking and Listening, Reading)**

Rigor

- Students build **conceptual understanding** of the difference between correlation and causation.
- Students build **procedural fluency** recognizing the steps of an experiment.

Coherence

Today

Students examine different scenarios where correlation exists between two variables, but the variables do not cause a change in each other. Students then learn that experiments can show causation. They order the steps of an experiment and design their own.

Previously

In Lesson 20, students used spreadsheet technology to determine and interpret the correlation coefficient.

Coming Soon

In Lesson 22, students will apply their understanding of how data can be misrepresented, interpreted, or analyzed.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	20 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair
- dice
- spreadsheet technology

Math Language Development

New words

- ***causation***

Review words

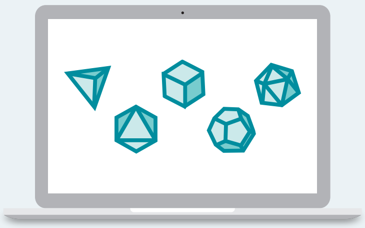
- *association*
- *correlation*
- *correlation coefficient*
- *line of best fit*

Amps | **Featured Activity**

Activity 1

Digital Simulation

Students have the opportunity to simulate 10 rolls of 2 dice. This way, students can see more outcomes of 10 rolls to see if any of them result in associated data.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might make assumptions about correlation and causation in Activity 3 as they make their own experiment. Remind students to stay focused on the interpretation of their mathematical results in the context of their experiment. They should keep asking themselves if it makes sense to say that the experiment proves causality.

Modifications to Pacing

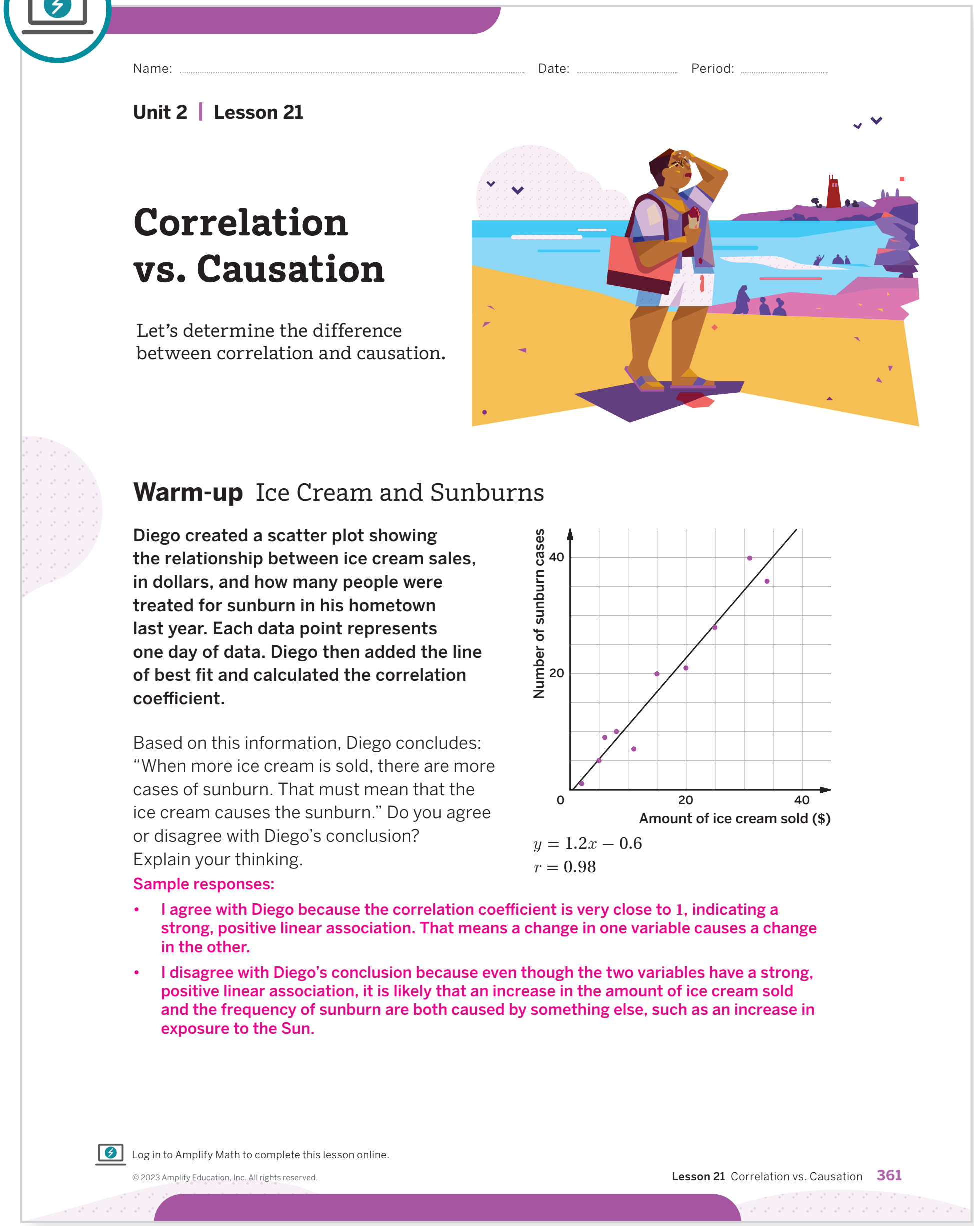
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problems 3–7 may be omitted.
- In **Activity 3**, Problems 5 and 6 may be omitted.

Independent | 5 min

Warm-up Ice Cream and Sunburns

Students evaluate a scatter plot to see how correlation does not imply causation when there are external factors.



Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 21

Correlation vs. Causation

Let's determine the difference between correlation and causation.

Warm-up Ice Cream and Sunburns

Diego created a scatter plot showing the relationship between ice cream sales, in dollars, and how many people were treated for sunburn in his hometown last year. Each data point represents one day of data. Diego then added the line of best fit and calculated the correlation coefficient.

Based on this information, Diego concludes: "When more ice cream is sold, there are more cases of sunburn. That must mean that the ice cream causes the sunburn." Do you agree or disagree with Diego's conclusion? Explain your thinking.

$y = 1.2x - 0.6$

$r = 0.98$

Sample responses:

- I agree with Diego because the correlation coefficient is very close to 1, indicating a strong, positive linear association. That means a change in one variable causes a change in the other.
- I disagree with Diego's conclusion because even though the two variables have a strong, positive linear association, it is likely that an increase in the amount of ice cream sold and the frequency of sunburn are both caused by something else, such as an increase in exposure to the Sun.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 21 Correlation vs. Causation 361

1 Launch

Have students work independently before participating in a class discussion.

2 Monitor

Help students get started by asking, "Based on the data, does this prove ice cream causes sunburn? Why or why not?"

Look for points of confusion:

- **Thinking the reason ice cream does not cause sunburns is because not all points are on the line.** Ask, "What does the line represent? Does real life data usually form an exact linear association?"

Look for productive strategies:

- Disagreeing because ice cream cannot cause sunburns, so the association could be random.
- Disagreeing because there is another reason causing both ice cream sales and sunburn to increase.

3 Connect

Display the scatter plot, line of fit, and correlation coefficient.

Have individual students share why they agree or disagree with Diego's conclusion.

Highlight that in real-world scenarios, there could be many reasons they see patterns or trends because more than one variable can cause a change in another variable.

Ask, "What could be some reasons why you see an association or correlation between two variables that is not due to causation?" Sample responses: A third variable, randomness, data manipulation.

Math Language Development

MLR2: Collect and Display

Listen for and collect the math language that students use to explain whether or not they agree with Diego's conclusion, such as *correlation coefficient, strong, positive, linear relationship*, etc. Write students' words on a visual display and update it throughout the remainder of the lesson.

English Learners

Annotate the scatter plot with these words and phrases.

Power-up

To power up students' ability to interpret the meaning of a correlation coefficient, have students complete:

Which of the following correlation coefficients describes a strong, increasing (positive), linear association?

[✓ A.] $r = 0.95$
B. $r = 0.15$
C. $r = -0.15$
D. $r = -0.95$

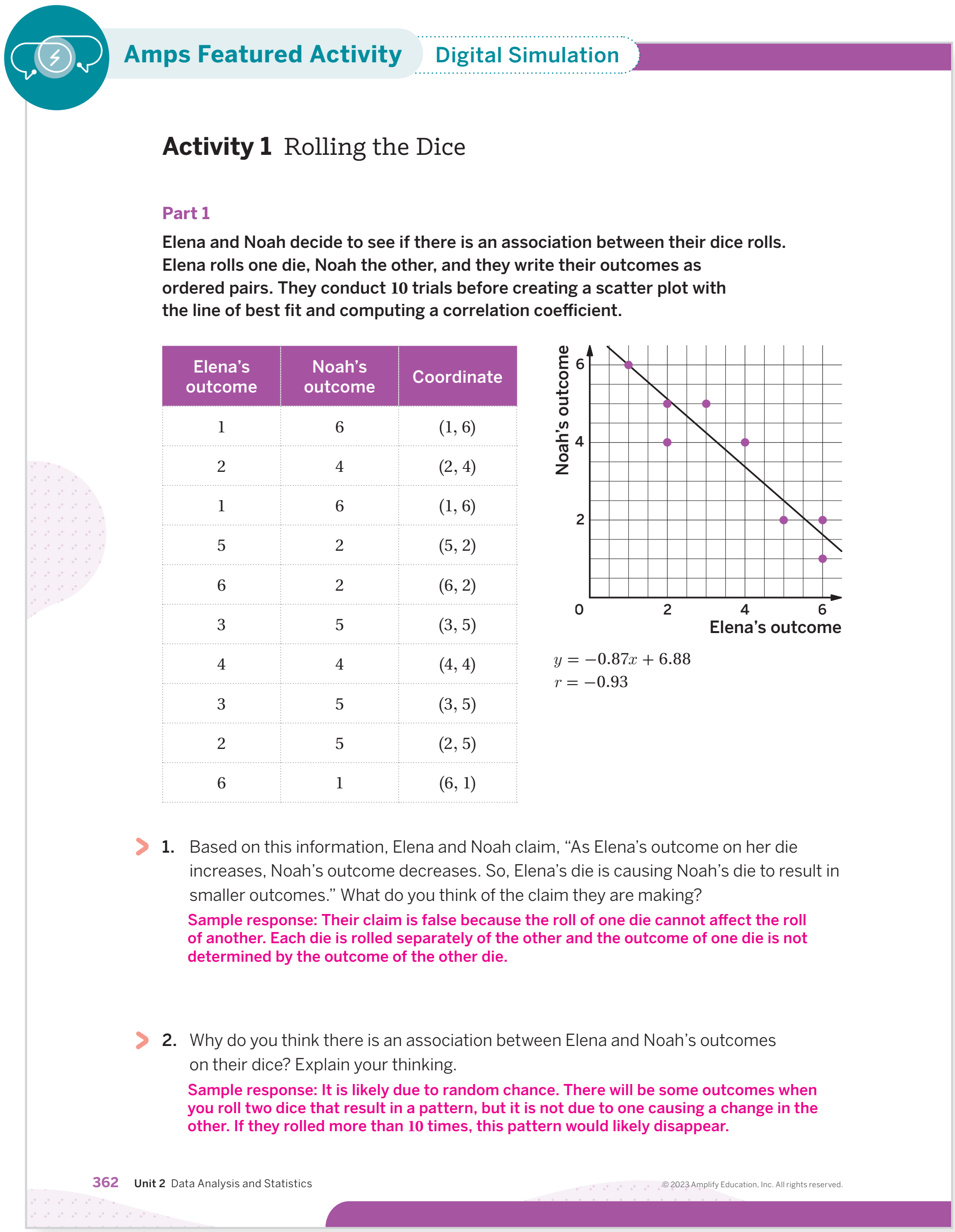
Use: Before the Warm-up
Informed by: Performance on Lesson 20, Practice Problem 5 and Exit Ticket

Pairs | 15 min

Activity 1 Rolling the Dice

Students roll dice to see how correlation does not imply causation due to random associations.

Amps Featured Activity Digital Simulation



Activity 1 Rolling the Dice

Part 1

Elena and Noah decide to see if there is an association between their dice rolls. Elena rolls one die, Noah the other, and they write their outcomes as ordered pairs. They conduct 10 trials before creating a scatter plot with the line of best fit and computing a correlation coefficient.

Elena's outcome	Noah's outcome	Coordinate
1	6	(1, 6)
2	4	(2, 4)
1	6	(1, 6)
5	2	(5, 2)
6	2	(6, 2)
3	5	(3, 5)
4	4	(4, 4)
3	5	(3, 5)
2	5	(2, 5)
6	1	(6, 1)

$y = -0.87x + 6.88$
$r = -0.93$

1. Based on this information, Elena and Noah claim, "As Elena's outcome on her die increases, Noah's outcome decreases. So, Elena's die is causing Noah's die to result in smaller outcomes." What do you think of the claim they are making?

Sample response: Their claim is false because the roll of one die cannot affect the roll of another. Each die is rolled separately of the other and the outcome of one die is not determined by the outcome of the other die.

2. Why do you think there is an association between Elena and Noah's outcomes on their dice? Explain your thinking.

Sample response: It is likely due to random chance. There will be some outcomes when you roll two dice that result in a pattern, but it is not due to one causing a change in the other. If they rolled more than 10 times, this pattern would likely disappear.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have students work on Problems 1 and 2 before a short class discussion. Distribute two dice to each pair of students and provided access to spreadsheet technology before they complete Problems 3–7.

2 Monitor

Help students get started by asking, "Do you agree or disagree with the claim Elena and Noah are making?"

Look for points of confusion:

- **Not remaining consistent with writing each coordinate from the outcomes of both partners during Part 2.** Remind students that once they choose whose outcome to list first and second as an ordered pair, they must write each ordered pair the same.

Look for productive strategies:

- Recognizing that the roll of two dice does not show causation because the outcomes of either dice do not affect each other.
- Recognizing that even though a correlation coefficient close to 1 or –1 may have been found, this is due to randomness from the outcome of rolling dice, not causation.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can simulate multiple rolls of 2 dice for Part 2. This will allow them to spend more time analyzing the simulated data using spreadsheet technology.

Math Language Development

MLR8: Discussion Supports

During the Connect, ask these questions to help students distinguish between terms *correlation* and *causation*:

- "Do you think there is a *correlation* between increased temperature and increased air conditioning costs?"
- "Would this mean that an increase in temperature *causes* an increase in air conditioning costs? Why or why not? What other variables might need to be considered?"

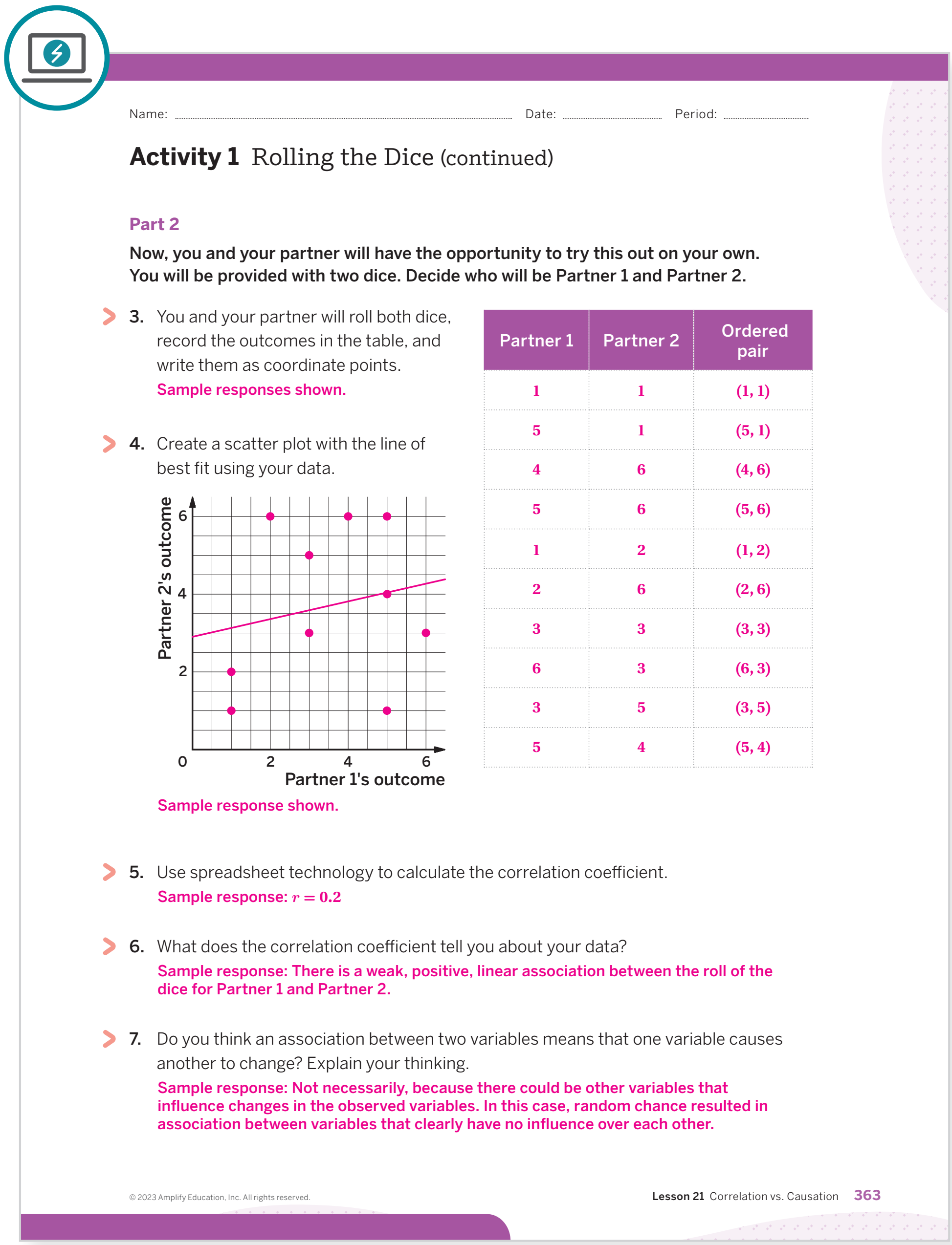
English Learners

Consider translating the terms *correlation* and *causation* into students' primary languages to help them understand the differences between these two terms. For example, in Spanish, the terms are *correlación* and *causalidad*.

Pairs | 15 min

Activity 1 Rolling the Dice (continued)

Students roll dice to see how correlation does not imply causation due to random associations.



Name: ______ Date: ______ Period: ______

Activity 1 Rolling the Dice (continued)

Part 2

Now, you and your partner will have the opportunity to try this out on your own. You will be provided with two dice. Decide who will be Partner 1 and Partner 2.

3. You and your partner will roll both dice, record the outcomes in the table, and write them as coordinate points.
Sample responses shown.

Partner 1	Partner 2	Ordered pair
1	1	(1, 1)
5	1	(5, 1)
4	6	(4, 6)
5	6	(5, 6)
1	2	(1, 2)
2	6	(2, 6)
3	3	(3, 3)
6	3	(6, 3)
3	5	(3, 5)
5	4	(5, 4)

4. Create a scatter plot with the line of best fit using your data.

Sample response shown.

5. Use spreadsheet technology to calculate the correlation coefficient.
Sample response: $r = 0.2$

6. What does the correlation coefficient tell you about your data?
Sample response: There is a weak, positive, linear association between the roll of the dice for Partner 1 and Partner 2.

7. Do you think an association between two variables means that one variable causes another to change? Explain your thinking.
Sample response: Not necessarily, because there could be other variables that influence changes in the observed variables. In this case, random chance resulted in association between variables that clearly have no influence over each other.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 21 Correlation vs. Causation 363

③ Connect

Display samples of student scatter plots from Problem 4.

Have pairs of students share their responses to Problems 5–7. Select and sequence student responses with no association, moderate association, and strong association.

Ask, "Why are there varying levels of association from all these scatter plots?"

Highlight that random events, such as rolling dice, can or cannot lead to trends and association in data. Even though there might be clear association, this is not due to one event *causing* another.

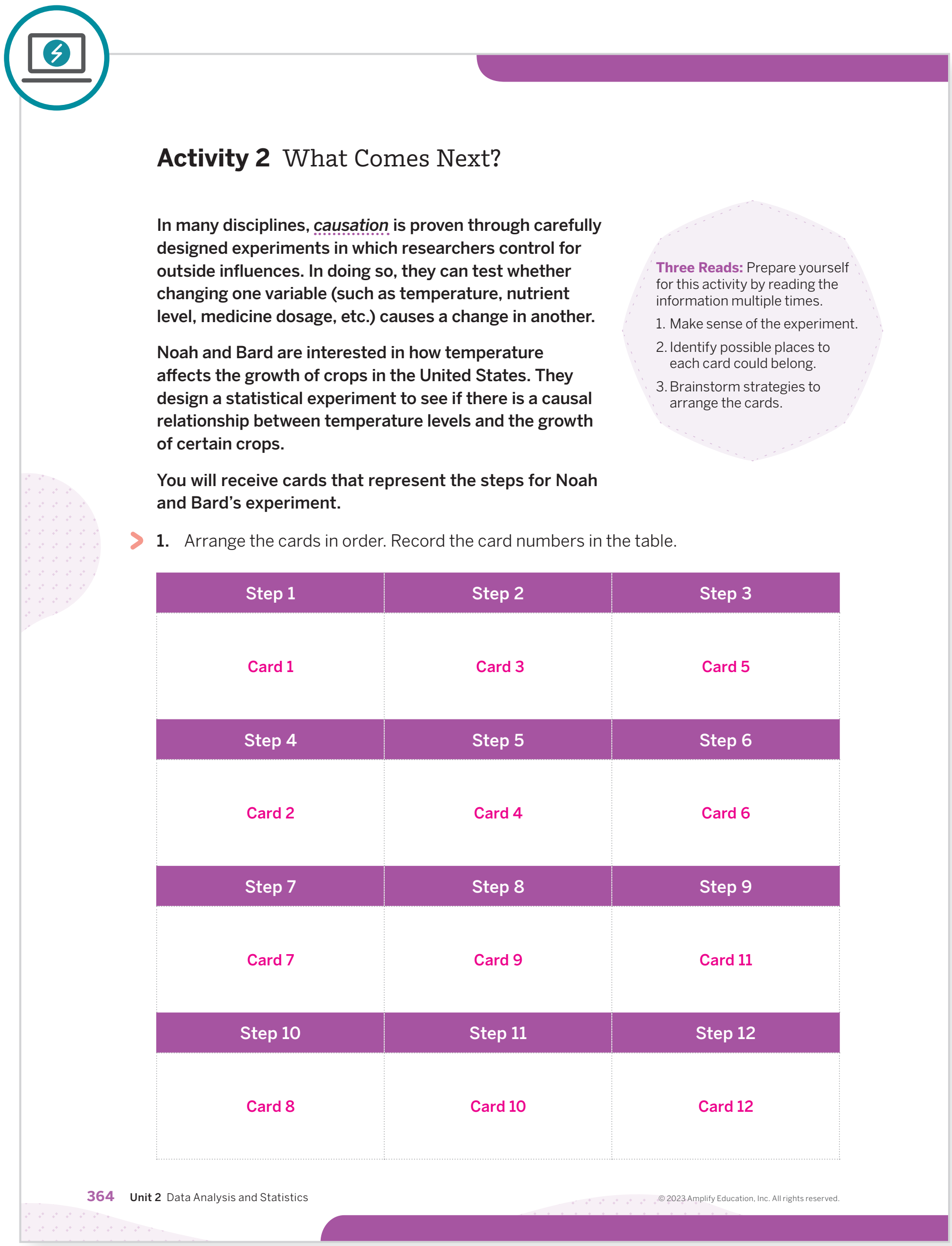
Define the term ***causation***.

Ask, "How do you think you could show causation between two variables?"

Pairs | 20 min

Activity 2 What Comes Next?

Students arrange the steps of a statistical experiment in order and analyze its data to see how an experiment might demonstrate causation.



Activity 2 What Comes Next?

In many disciplines, *causation* is proven through carefully designed experiments in which researchers control for outside influences. In doing so, they can test whether changing one variable (such as temperature, nutrient level, medicine dosage, etc.) causes a change in another.

Noah and Bard are interested in how temperature affects the growth of crops in the United States. They design a statistical experiment to see if there is a causal relationship between temperature levels and the growth of certain crops.

You will receive cards that represent the steps for Noah and Bard's experiment.

Three Reads: Prepare yourself for this activity by reading the information multiple times.

1. Make sense of the experiment.
2. Identify possible places to each card could belong.
3. Brainstorm strategies to arrange the cards.

1. Arrange the cards in order. Record the card numbers in the table.

Step 1	Step 2	Step 3
Card 1	Card 3	Card 5
Step 4	**Step 5**	**Step 6**
Card 2	Card 4	Card 6
Step 7	**Step 8**	**Step 9**
Card 7	Card 9	Card 11
Step 10	**Step 11**	**Step 12**
Card 8	Card 10	Card 12

364 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Shuffle the pre-cut cards from the Activity 2 PDF and distribute to each pair of students.

2 Monitor

Help students get started by asking, "Are there any steps you know must come at the beginning of the experiment or toward the end?"

Look for points of confusion:

- **Mixing up the first two steps of the experiment.** Ask, "In the description of what Noah and Bard are exploring, what is their area of interest?"
- **Thinking that because all scatter plots show strong positive associations, this means the experiment does not show anything.** Ask, "Even though all three scatter plots show corn growing over time, what is different about their slopes? What does this show?"

Look for productive strategies:

- Recognizing that some steps must logically come at the beginning or end of an experiment, and creating groups to represent the beginning, middle, and end of an experiment.
- Showing that the correlation coefficient, along with the slope found through statistical experimentation, show causation.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge, Guide Processing and Visualization*

Consider giving students three cards at a time and ask them to arrange them in order. For example, give them Cards 1, 3, and 5 and tell them that these represent Steps 1–3. Have them arrange them in the correct order, before giving them Cards 2, 4, and 6 and telling them these represent Steps 4–6.

Math Language Development

MLR6: Three Reads

During the Launch, after you have distributed the cards, have students use this routine to prepare themselves for the activity.

Read 1: Students should understand that Noah and Bard design an experiment to study temperature data and growth of certain crops.

Read 2: Ask students to identify possible places to which each card could belong. For example, summarizing one's findings (Card 12) is likely the last step.

Read 3: Ask students to brainstorm strategies for arranging the cards.

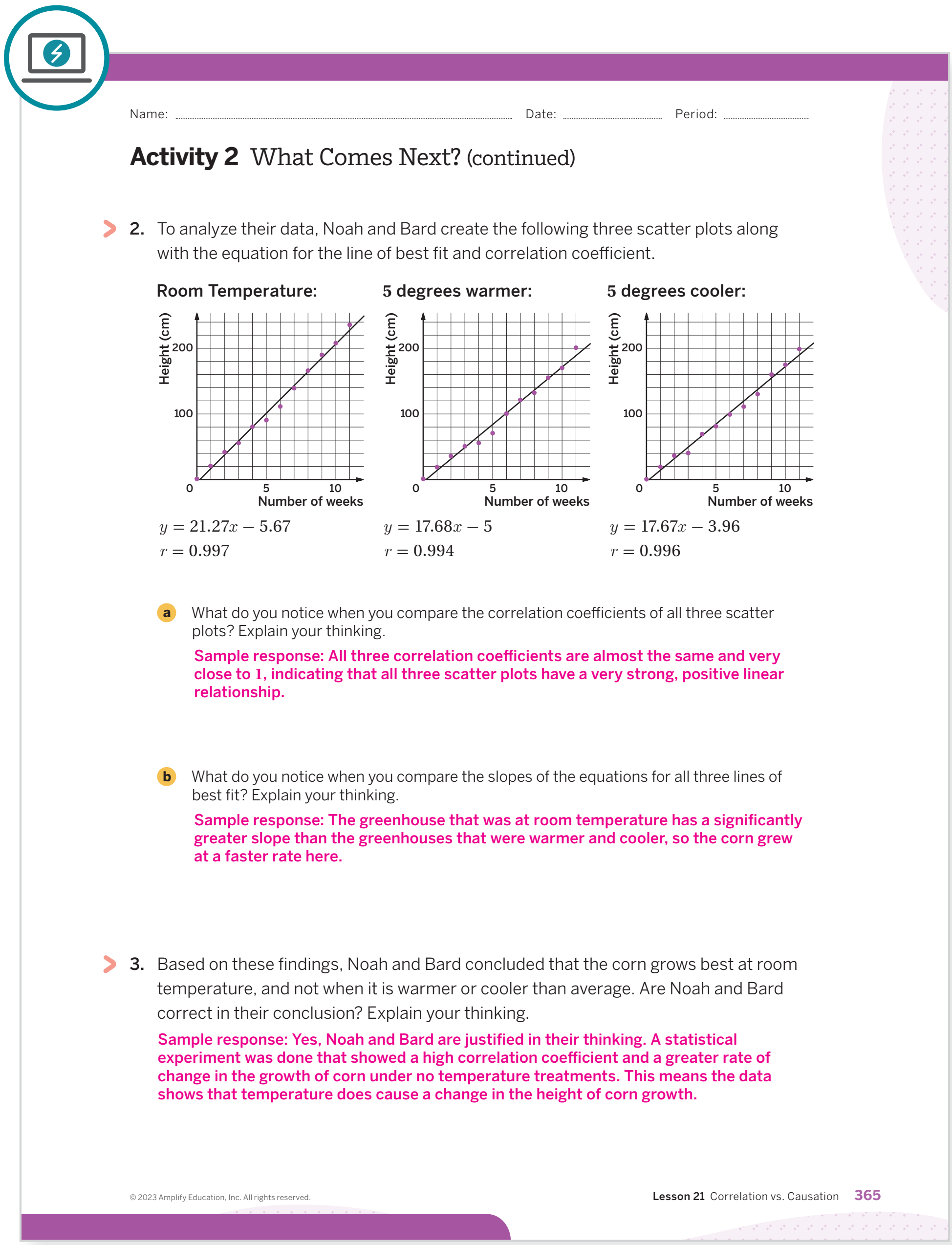
English Learners

Consider demonstrating, or asking any student to volunteer to do so, what some of the card descriptions mean by acting them out.

Pairs | 20 min

Activity 2 What Comes Next? (continued)

Students arrange the steps of a statistical experiment in order and analyze its data to see how an experiment might demonstrate causation.



Name: ______________ Date: ______________ Period: ______________

Activity 2 What Comes Next? (continued)

2. To analyze their data, Noah and Bard create the following three scatter plots along with the equation for the line of best fit and correlation coefficient.

Room Temperature: $y = 21.27x - 5.67$, $r = 0.997$

5 degrees warmer: $y = 17.68x - 5$, $r = 0.994$

5 degrees cooler: $y = 17.67x - 3.96$, $r = 0.996$

a What do you notice when you compare the correlation coefficients of all three scatter plots? Explain your thinking.

Sample response: All three correlation coefficients are almost the same and very close to 1, indicating that all three scatter plots have a very strong, positive linear relationship.

b What do you notice when you compare the slopes of the equations for all three lines of best fit? Explain your thinking.

Sample response: The greenhouse that was at room temperature has a significantly greater slope than the greenhouses that were warmer and cooler, so the corn grew at a faster rate here.

3. Based on these findings, Noah and Bard concluded that the corn grows best at room temperature, and not when it is warmer or cooler than average. Are Noah and Bard correct in their conclusion? Explain your thinking.

Sample response: Yes, Noah and Bard are justified in their thinking. A statistical experiment was done that showed a high correlation coefficient and a greater rate of change in the growth of corn under no temperature treatments. This means the data shows that temperature does cause a change in the height of corn growth.

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have pairs of students share the order of their cards. Invite students to share strategies they used to determine the order of the steps of the experiment.

Display the three scatter plots.

Have pairs of students share their responses to Problems 2 and 3. Select and sequence student responses comparing the values of the slope, interpreting the slope in context, and connecting the slope to show causation.

Highlight that a carefully designed and controlled statistical experiment can show causation because it controls many outside variables from affecting measurement. This they can then if show one variable can causes another to change.

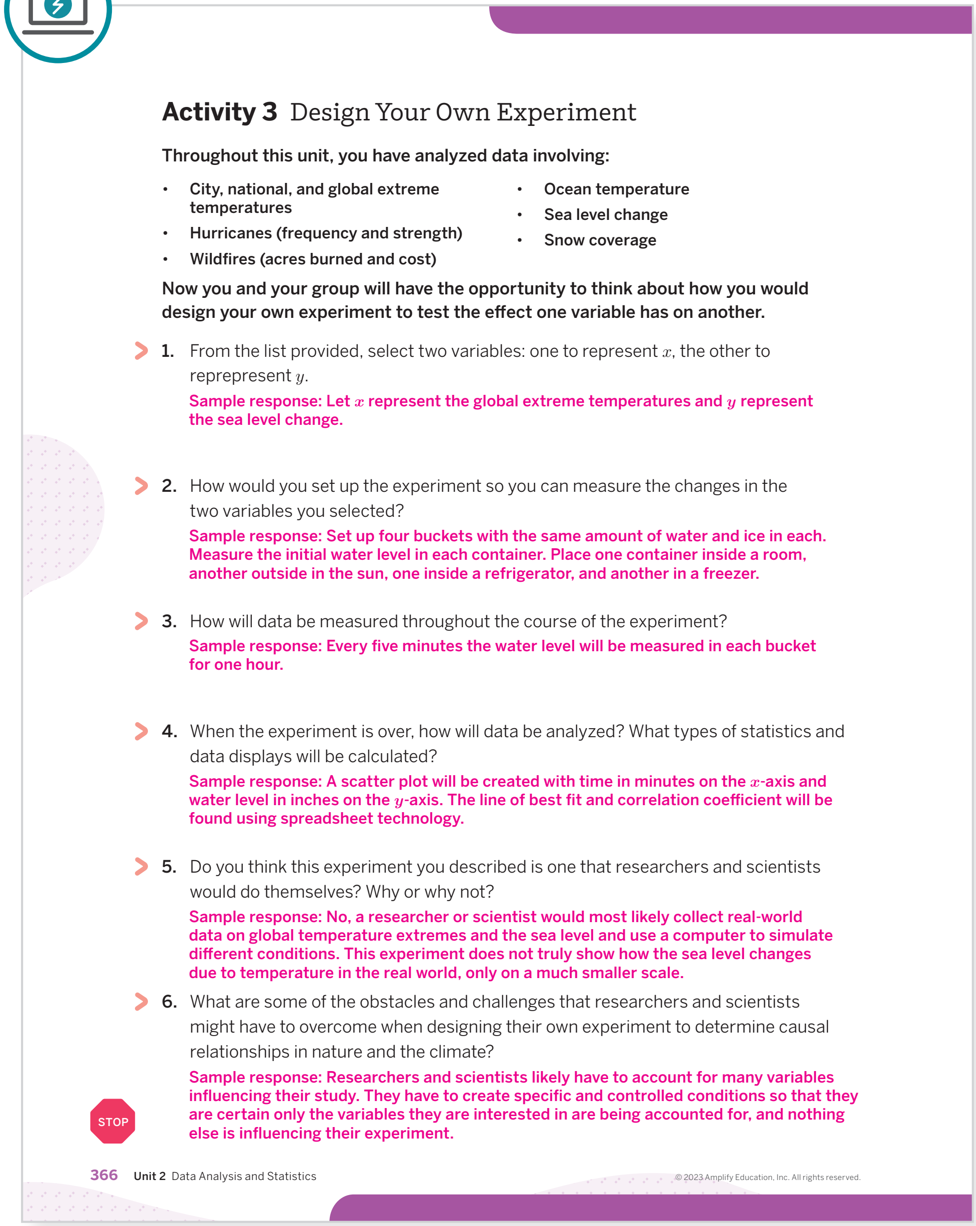
Ask, "What could be some improvements or oversights Noah and Bard made in their experiment?"

Optional

Pairs | 10 min

Activity 3 Design Your Own Experiment

Students design their own experiment using topics from the unit to apply their knowledge of statistical experiments.



Activity 3 Design Your Own Experiment

Throughout this unit, you have analyzed data involving:

- **City, national, and global extreme temperatures**
- **Hurricanes (frequency and strength)**
- **Wildfires (acres burned and cost)**
- **Ocean temperature**
- **Sea level change**
- **Snow coverage**

Now you and your group will have the opportunity to think about how you would design your own experiment to test the effect one variable has on another.

1. From the list provided, select two variables: one to represent x, the other to reprepresent y.

Sample response: Let x represent the global extreme temperatures and y represent the sea level change.

2. How would you set up the experiment so you can measure the changes in the two variables you selected?

Sample response: Set up four buckets with the same amount of water and ice in each. Measure the initial water level in each container. Place one container inside a room, another outside in the sun, one inside a refrigerator, and another in a freezer.

3. How will data be measured throughout the course of the experiment?

Sample response: Every five minutes the water level will be measured in each bucket for one hour.

4. When the experiment is over, how will data be analyzed? What types of statistics and data displays will be calculated?

Sample response: A scatter plot will be created with time in minutes on the x-axis and water level in inches on the y-axis. The line of best fit and correlation coefficient will be found using spreadsheet technology.

5. Do you think this experiment you described is one that researchers and scientists would do themselves? Why or why not?

Sample response: No, a researcher or scientist would most likely collect real-world data on global temperature extremes and the sea level and use a computer to simulate different conditions. This experiment does not truly show how the sea level changes due to temperature in the real world, only on a much smaller scale.

6. What are some of the obstacles and challenges that researchers and scientists might have to overcome when designing their own experiment to determine causal relationships in nature and the climate?

Sample response: Researchers and scientists likely have to account for many variables influencing their study. They have to create specific and controlled conditions so that they are certain only the variables they are interested in are being accounted for, and nothing else is influencing their experiment.

STOP

366 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Students remain in pairs. Say, "You will create an experiment to determine causality between sets of data you have seen throughout this unit."

2 Monitor

Help students get started by saying, "How can you use the previous activity to help create an experiment?"

Look for points of confusion:

- **Thinking the experiment must occur on a large scale in the real world.** Ask, "Is there a way to design a smaller version of the variables you are experimenting on?"

Look for productive strategies:

- Creating multiple treatment groups, one with no changes or treatments being applied.
- Recognizing that researchers most likely have a lot of resources to create more complex experiments.

3 Connect

Have pairs of students share the variables they selected for their experiments, the descriptions of the experiments, and how they believe researchers might carry out a similar experiment.

Highlight that when conducting research on the climate and the environment, researchers use computer models to simulate different situations to make predictions and determine causation.

Ask, "Why is it difficult to create an experiment that is not a computer simulation for some of these listed topics?"

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider providing a sample experiment design checklist for students to study and use as a guide as they create their own.

☐	Define the variables.	x represents: y represents:
☐	Set up the experiment.	Description:
☐	Decide how to measure the data.	Description:
☐	Decide how to analyze the data.	What statistical measures will you use? What statistical displays will you use?

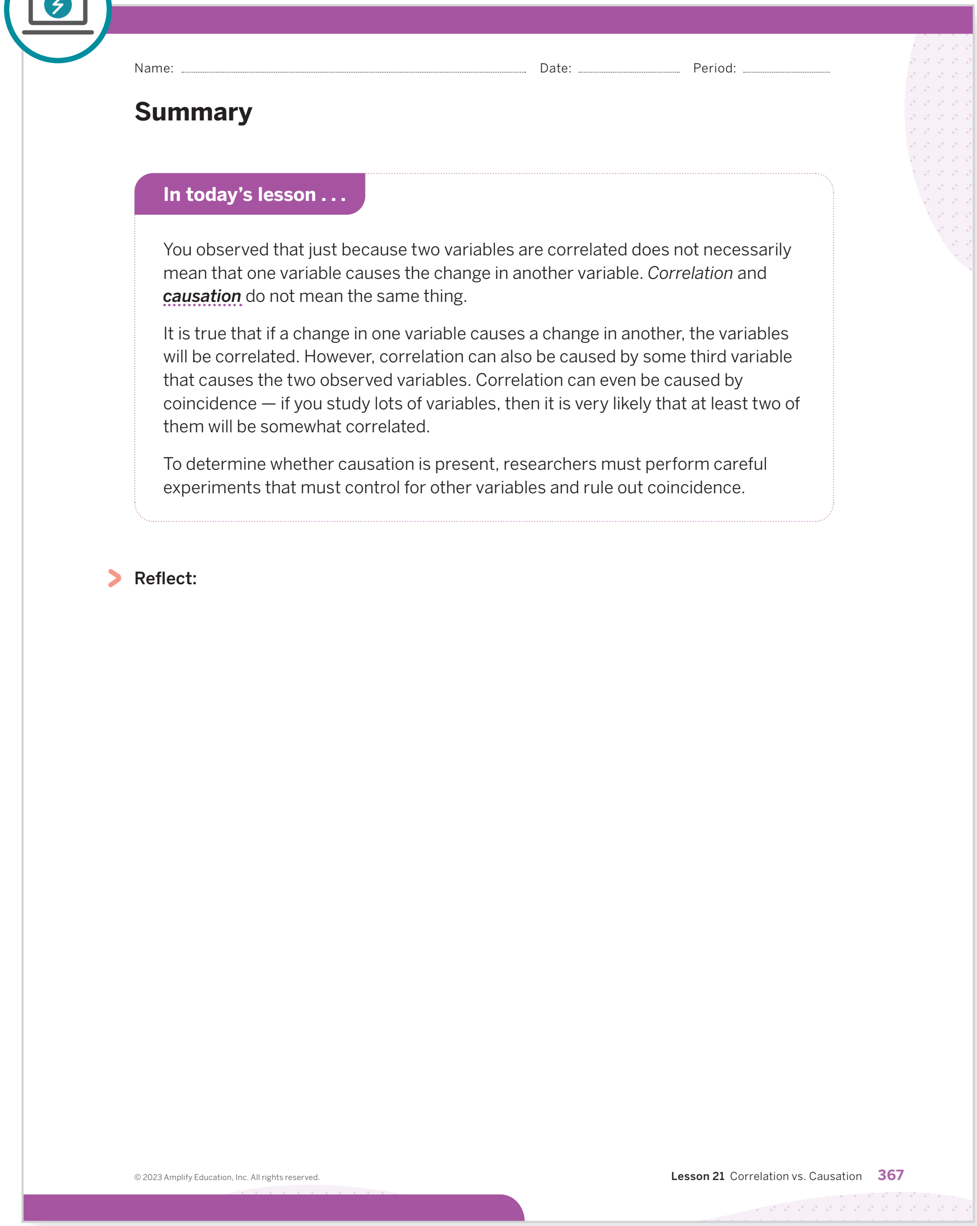
Extension: *Math Enrichment, Interdisciplinary Connections*

Ask students why it is important to have multiple treatment groups when designing an experiment in order to test the effect one variable has on another variable. **(Science)** Sample response: When you have multiple treatment groups, you can see how changes in the variable in which you are interested affect the other variable.

Whole Class | 5 min

Summary

Review and synthesize the difference between correlation and causation.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You observed that just because two variables are correlated does not necessarily mean that one variable causes the change in another variable. *Correlation* and ***causation*** do not mean the same thing.

It is true that if a change in one variable causes a change in another, the variables will be correlated. However, correlation can also be caused by some third variable that causes the two observed variables. Correlation can even be caused by coincidence — if you study lots of variables, then it is very likely that at least two of them will be somewhat correlated.

To determine whether causation is present, researchers must perform careful experiments that must control for other variables and rule out coincidence.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the scenario, "Lin creates a scatter plot showing a strong positive association between cellphone and organic food sales over time. She concludes that people buying more cellphones is causing organic food sales to increase."

Have students share if they agree or disagree with Lin's statement and why.

Highlight that just because two variables have a strong association and are correlated does not always mean a change in one variable is causing a change in another variable. Students show causation through statistical experiments.

Formalize vocabulary: *causation*

Ask, "Why are statistical experiments used to show causation?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is correlation different from causation?"

Math Language Development

MLR2: Collect and Display

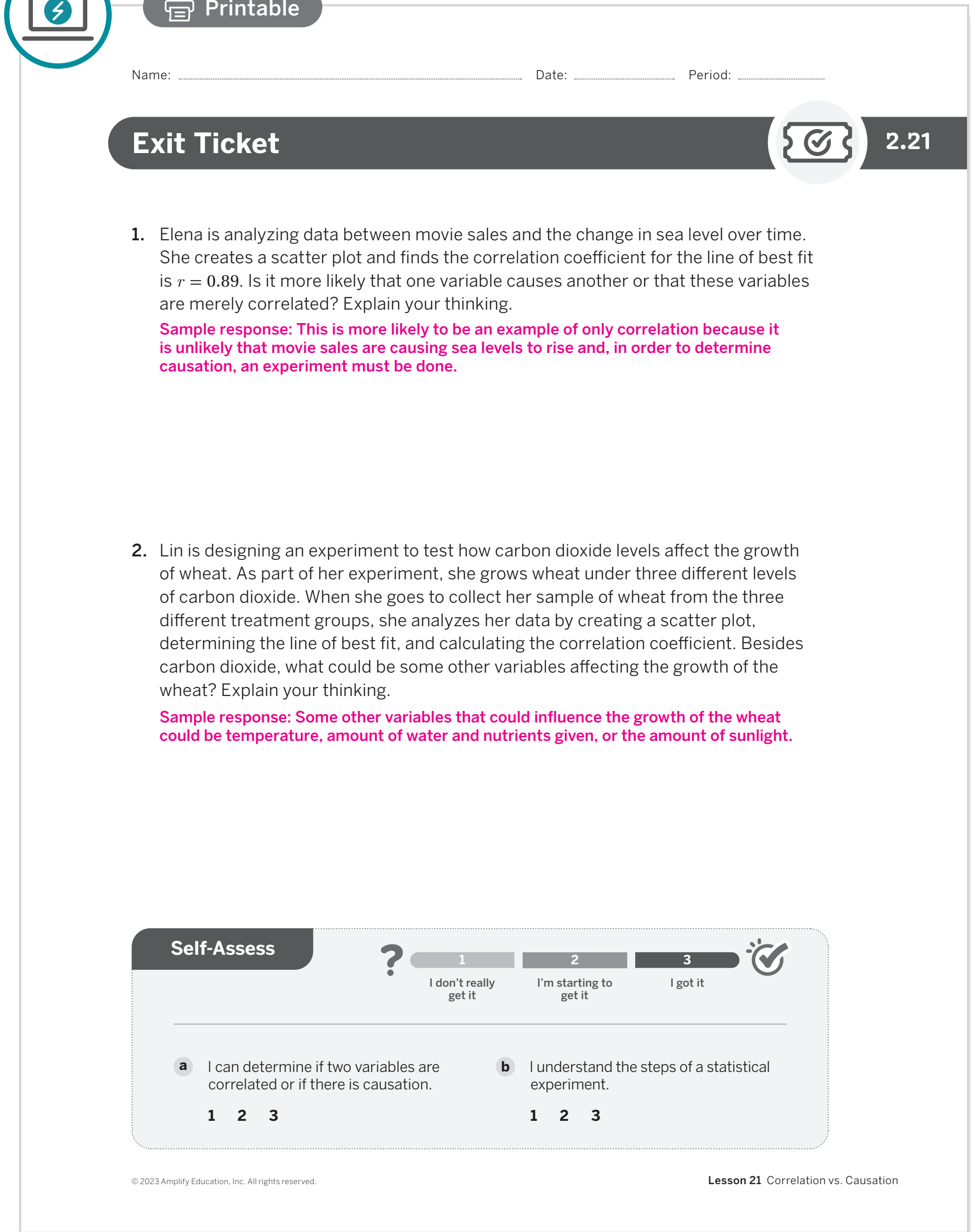
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 1. Ask them to review and reflect on any terms and phrases related to the term *causation* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by analyzing claims of causation and a statistical experiment.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 2.21

1. Elena is analyzing data between movie sales and the change in sea level over time. She creates a scatter plot and finds the correlation coefficient for the line of best fit is $r = 0.89$. Is it more likely that one variable causes another or that these variables are merely correlated? Explain your thinking.

   Sample response: This is more likely to be an example of only correlation because it is unlikely that movie sales are causing sea levels to rise and, in order to determine causation, an experiment must be done.

2. Lin is designing an experiment to test how carbon dioxide levels affect the growth of wheat. As part of her experiment, she grows wheat under three different levels of carbon dioxide. When she goes to collect her sample of wheat from the three different treatment groups, she analyzes her data by creating a scatter plot, determining the line of best fit, and calculating the correlation coefficient. Besides carbon dioxide, what could be some other variables affecting the growth of the wheat? Explain your thinking.

   Sample response: Some other variables that could influence the growth of the wheat could be temperature, amount of water and nutrients given, or the amount of sunlight.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can determine if two variables are correlated or if there is causation.

1 2 3

b I understand the steps of a statistical experiment.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 21 Correlation vs. Causation

Success looks like . . .

- **Language Goal:** Determining whether correlation or causation exists between two variables. **(Reading and Writing)**
  - » Explaining whether there is correlation or causation between movie sales and the change in sea level over time in Problem 1.
- **Language Goal:** Understanding the steps of a statistical experiment. **(Speaking and Listening, Reading)**
  - » Explaining other variables that Lin could consider for her experiment about carbon dioxide levels and the growth of wheat in Problem 2.

Suggested next steps

If students think the scenario shows causation in Problem 1, consider:

- Reviewing Problem 7 from Activity 1.
- Assigning Practice Problem 1.
- Asking, "What could be some other reasons why movie sales and the sea level have increased over time?"

If students can't think of outside variables in Problem 2, consider:

- Reviewing the Warm-up.
- Assigning Practice Problem 3.
- Asking, "What affects the growth of plants?"

Professional Learning

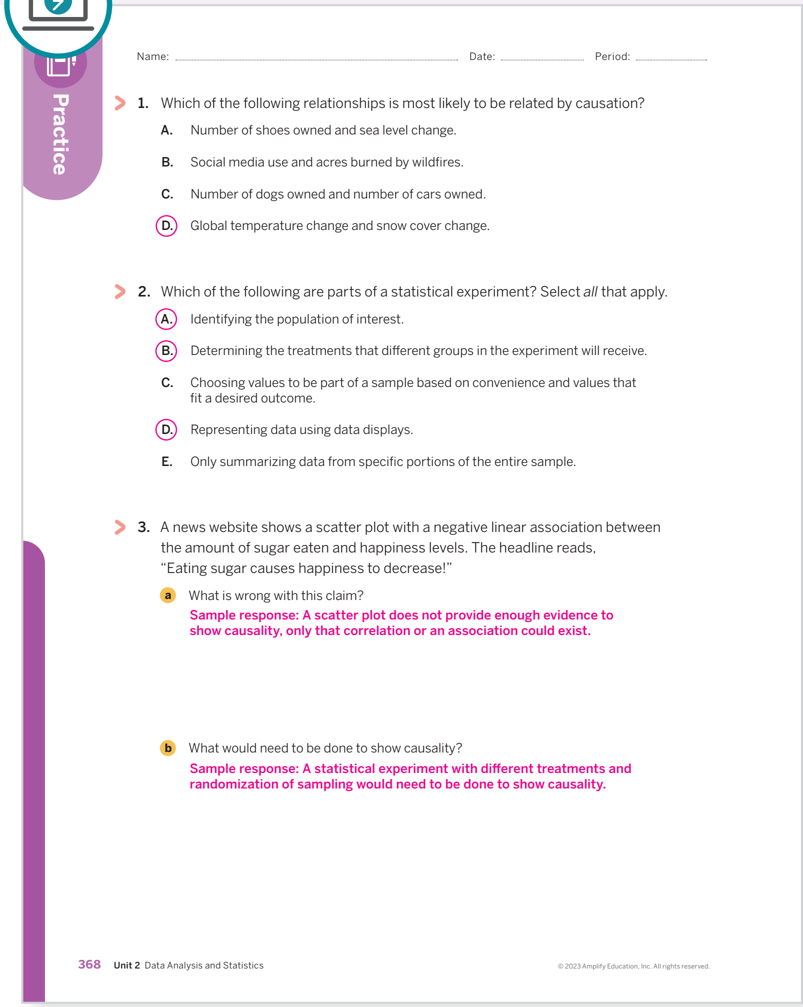
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Which students' ideas were you able to highlight during Activity 1?
- What did students find frustrating about Activity 2? What helped them work through this frustration? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

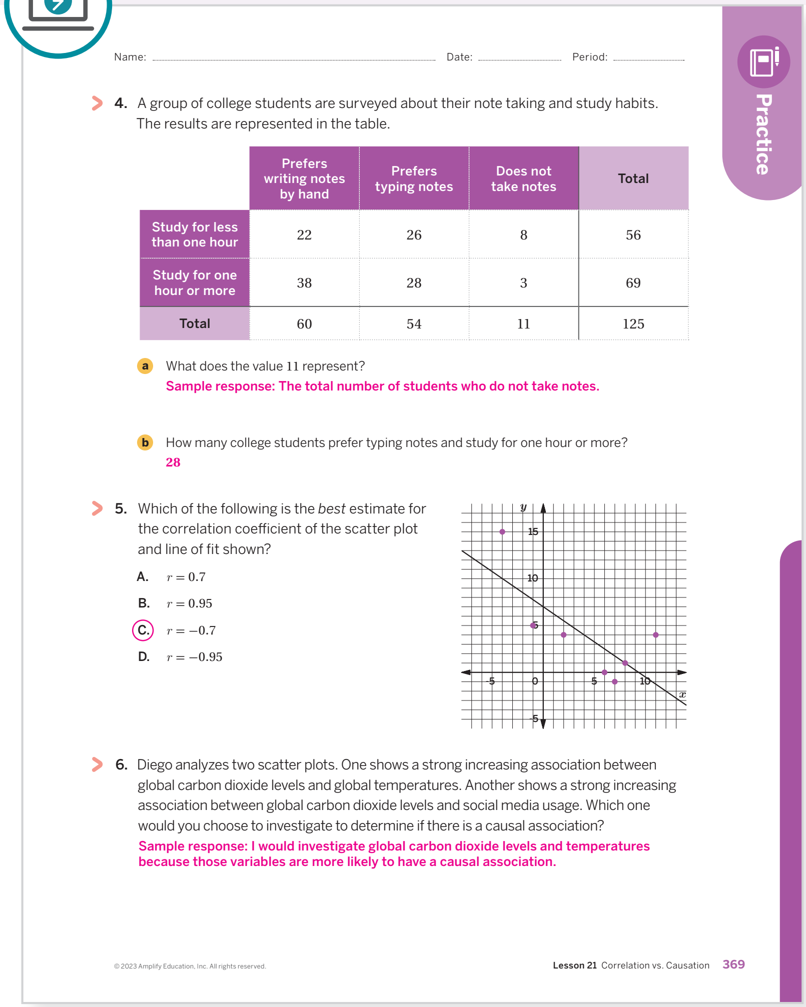
Practice

1. Which of the following relationships is most likely to be related by causation?
   - A. Number of shoes owned and sea level change.
   - B. Social media use and acres burned by wildfires.
   - C. Number of dogs owned and number of cars owned.
   - [✓ D.] Global temperature change and snow cover change.

2. Which of the following are parts of a statistical experiment? Select *all* that apply.
   - [✓ A.] Identifying the population of interest.
   - [✓ B.] Determining the treatments that different groups in the experiment will receive.
   - C. Choosing values to be part of a sample based on convenience and values that fit a desired outcome.
   - [✓ D.] Representing data using data displays.
   - E. Only summarizing data from specific portions of the entire sample.

3. A news website shows a scatter plot with a negative linear association between the amount of sugar eaten and happiness levels. The headline reads, "Eating sugar causes happiness to decrease!"
   - a What is wrong with this claim?
     Sample response: A scatter plot does not provide enough evidence to show causality, only that correlation or an association could exist.
   - b What would need to be done to show causality?
     Sample response: A statistical experiment with different treatments and randomization of sampling would need to be done to show causality.

© 2023 Amplify Education, Inc. All rights reserved.



Name: ______ Date: ______ Period: ______

Practice

4. A group of college students are surveyed about their note taking and study habits. The results are represented in the table.

	Prefers writing notes by hand	Prefers typing notes	Does not take notes	Total
Study for less than one hour	22	26	8	56
Study for one hour or more	38	28	3	69
Total	60	54	11	125

   - a What does the value 11 represent?
     Sample response: The total number of students who do not take notes.
   - b How many college students prefer typing notes and study for one hour or more?
     28

5. Which of the following is the *best* estimate for the correlation coefficient of the scatter plot and line of fit shown?
   - A. $r = 0.7$
   - B. $r = 0.95$
   - [✓ C.] $r = -0.7$
   - D. $r = -0.95$

6. Diego analyzes two scatter plots. One shows a strong increasing association between global carbon dioxide levels and global temperatures. Another shows a strong increasing association between global carbon dioxide levels and social media usage. Which one would you choose to investigate to determine if there is a causal association?
   Sample response: I would investigate global carbon dioxide levels and temperatures because those variables are more likely to have a causal association.

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 2 Lesson 16	2
	5	Unit 2 Lesson 19	1
Formative ⚡	6	Unit 2 Lesson 22	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Mathematical Modeling Prompt
At any point during the course and after completing this lesson, have students work in pairs or small groups on this unit's Mathematical Modeling Prompt, *College Characteristics*, which is available in the **Algebra 1 Additional Practice**.

UNIT 2 | **LESSON 22 – CAPSTONE**

Cutting Through Misleading Statistical Claims

Numbers might not lie, but what about their interpretation? Let's explore this further.

Focus

Goals

1. **Language Goal:** Understand how to evaluate claims about data to determine if there are fallacies or misrepresentations. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students apply their understanding of how data can be misrepresented, interpreted, or analyzed. They first examine newspaper headlines about data relating to climate change and determine errors made by each headline. They then examine the AMO index, which represents a cycle in the Atlantic Ocean temperature. Students then put themselves in the shoes of someone trying to show climate change is not real by selectively picking data, and then analyzing the entire data set.

Previously

In Lesson 21, students determined the difference between correlation and causation, and ordered the steps of an experiment.

Coming Soon

Students will further their study of statistics and data analysis in Algebra 2.

Rigor

- Students **apply** their understanding of how to analyze and interpret data properly.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	20 min	5 min	5 min
Independent	Pairs	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF
- spreadsheet technology

Math Language Development

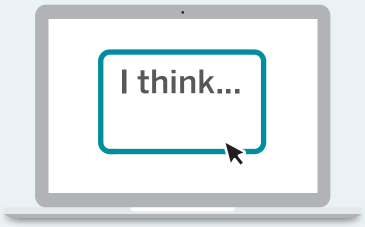
Review words

- *causation*
- *correlation coefficient*
- *line of best fit*
- *scatter plot*

Amps | Featured Activity

Activity 2
See Student Thinking

Students are asked to explain their thinking behind how selectively choosing data can be misleading, and these explanations are available to you digitally, in real time.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might think that because they are using technology, that they will not need to employ their brains in Activity 3. Point out that the technology can do some of the tedious calculations, but their brains have to do the deep dive into the interpretation of those results. Ask them how to identify ways they can keep themselves motivated to the very end of the lesson.

Modifications to Pacing

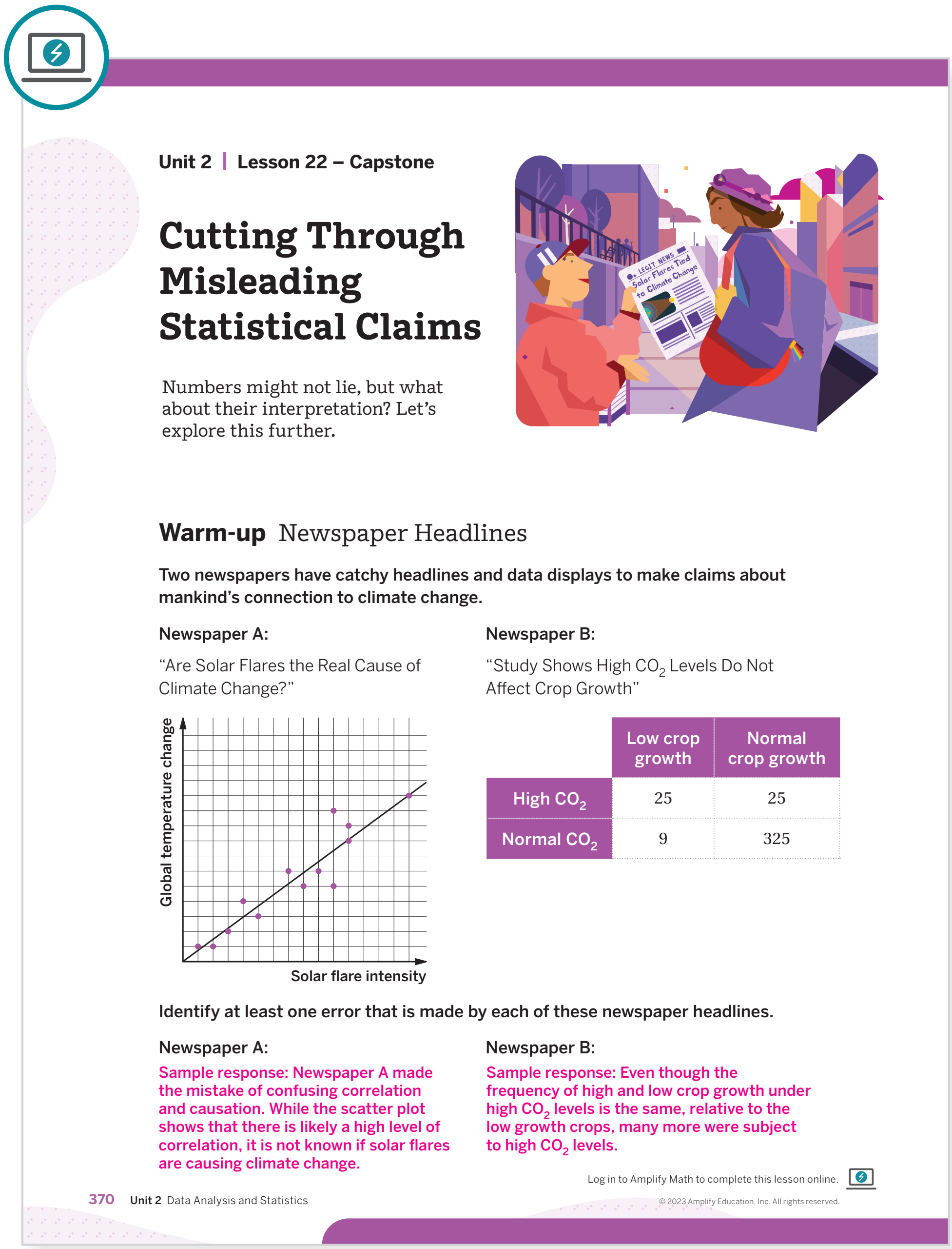
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Newspaper B may be omitted.
- In **Activity 2**, Problems 3 and 8 may be omitted.

Independent | 10 min

Warm-up Newspaper Headlines

Students evaluate two newspaper headlines about climate change data to determine errors made by their assertions.



Unit 2 | Lesson 22 – Capstone

Cutting Through Misleading Statistical Claims

Numbers might not lie, but what about their interpretation? Let's explore this further.

Warm-up Newspaper Headlines

Two newspapers have catchy headlines and data displays to make claims about mankind's connection to climate change.

Newspaper A:

"Are Solar Flares the Real Cause of Climate Change?"

Newspaper B:

"Study Shows High CO_2 Levels Do Not Affect Crop Growth"

	Low crop growth	Normal crop growth
High CO_2	25	25
Normal CO_2	9	325

Identify at least one error that is made by each of these newspaper headlines.

Newspaper A:

Sample response: Newspaper A made the mistake of confusing correlation and causation. While the scatter plot shows that there is likely a high level of correlation, it is not known if solar flares are causing climate change.

Newspaper B:

Sample response: Even though the frequency of high and low crop growth under high CO_2 levels is the same, relative to the low growth crops, many more were subject to high CO_2 levels.

Log in to Amplify Math to complete this lesson online.

370 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Activate students' prior knowledge by asking, "How many of you have read something in the news that seemed like it contained an error?"

2 Monitor

Help students get started by saying, "How do you determine causation or association?"

Look for points of confusion:

- **Asserting there is another cause to global temperature change**. Ask, "While there may be another cause, what is the error made by the headline?"

Look for productive strategies:

- Recognizing the difference between correlation and causation.
- Recognizing that a relative frequency table may have to be computed before looking for associations in two-way tables.

3 Connect

Display each newspaper headline and data representation to the class.

Have individual students share the errors they found and described with each newspaper headline.

Highlight that sometimes, newspaper headlines might be misleading because of common misconceptions or mistakes that are made when analyzing data.

Ask, "Why might newspapers or other sources of information create these types of headlines?" Sample response: Some newspapers may create these types of headlines to attract attention or push their own ideas.

Math Language Development

MLR2: Collect and Display

Listen for and collect the language students use to identify and explain any errors they see in the newspaper headlines, such as *confusing correlation with causation, strong, positive association, relative frequency,* etc. Record students' phrases on a display. Remind students to borrow language from the display as needed.

English Learners

Have students highlight key words or phrases in the newspaper headlines, such as *cause* and *do not affect.*

Power-up

To power up students' ability to understand the difference between correlation and causation, have students complete:

Recall that *causation* is when a change in one variable is shown, through careful experimentation, to cause a change in another variable. For each scenario, determine whether causation is *likely* or *unlikely*.

a. There is a strong decreasing association between the number of hours slept and the number of cups of coffee a person has to drink. Likely

b. There is a strong increasing association between the number of hours of sunlight and the number of hours spent indoors. Likely

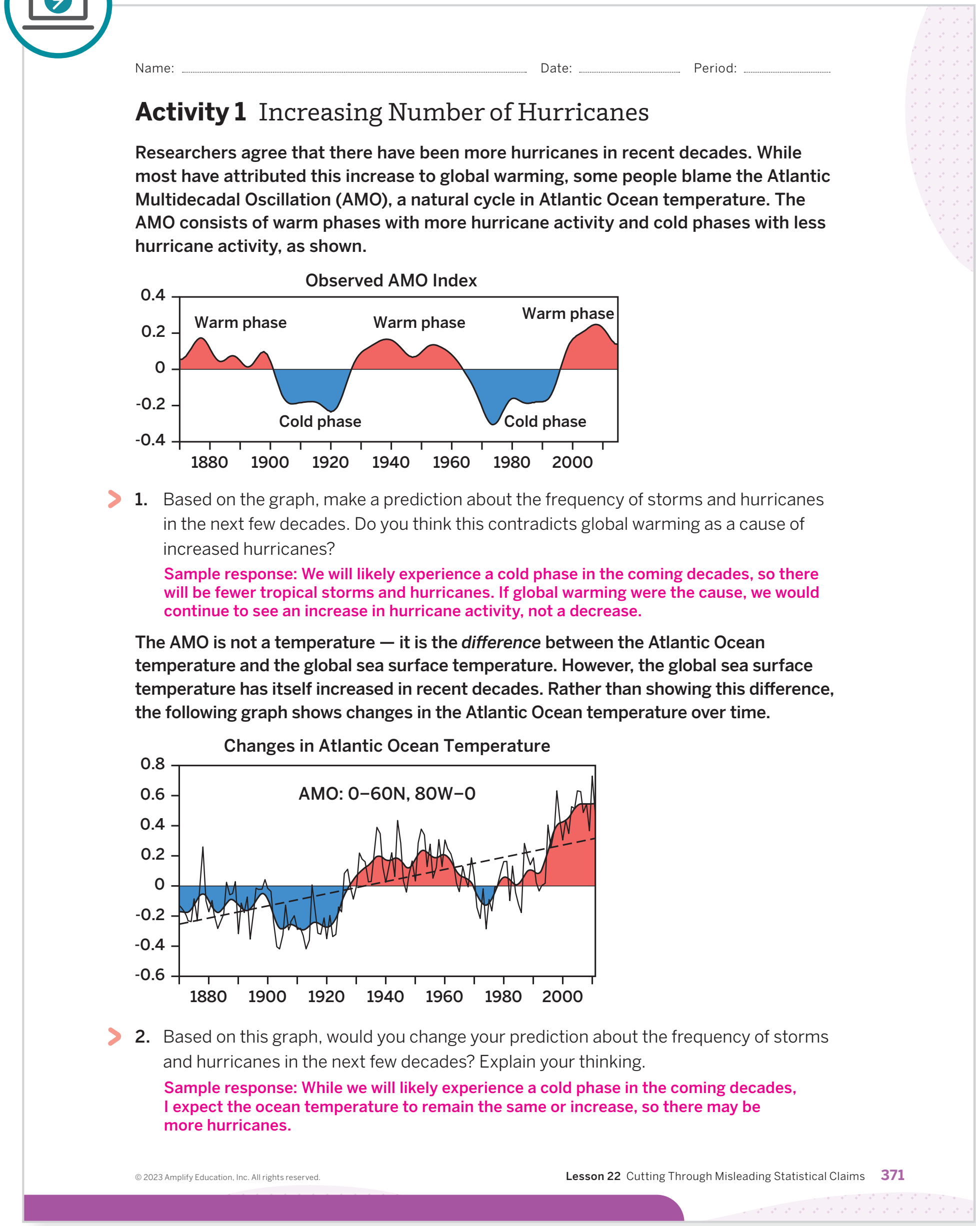
c. There is a strong increasing association between the average global temperature and the number of online videos posted. Unlikely

Use: Before the Warm-up

Informed by: Performance on Lesson 21, Practice Problem 6 and Exit Ticket

Activity 1 Increasing Number of Hurricanes

Students analyze infographics to evaluate two competing opinions regarding the increased frequency of hurricanes.



Name: ______ Date: ______ Period: ______

Activity 1 Increasing Number of Hurricanes

Researchers agree that there have been more hurricanes in recent decades. While most have attributed this increase to global warming, some people blame the Atlantic Multidecadal Oscillation (AMO), a natural cycle in Atlantic Ocean temperature. The AMO consists of warm phases with more hurricane activity and cold phases with less hurricane activity, as shown.

1. Based on the graph, make a prediction about the frequency of storms and hurricanes in the next few decades. Do you think this contradicts global warming as a cause of increased hurricanes?

   Sample response: We will likely experience a cold phase in the coming decades, so there will be fewer tropical storms and hurricanes. If global warming were the cause, we would continue to see an increase in hurricane activity, not a decrease.

The AMO is not a temperature — it is the *difference* between the Atlantic Ocean temperature and the global sea surface temperature. However, the global sea surface temperature has itself increased in recent decades. Rather than showing this difference, the following graph shows changes in the Atlantic Ocean temperature over time.

2. Based on this graph, would you change your prediction about the frequency of storms and hurricanes in the next few decades? Explain your thinking.

   Sample response: While we will likely experience a cold phase in the coming decades, I expect the ocean temperature to remain the same or increase, so there may be more hurricanes.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display the graph of the observed AMO index and read the prompt together as a whole class. Have students complete the problems independently before discussing their responses with a partner.

2 Monitor

Help students get started by prompting them to annotate the data representation.

Look for points of confusion:

- **Relating the graphs.** Prompt students to use the horizontal axis and annotate where the graph from Problem 1 appears in Problem 2.
- **Interpreting the graph in Problem 2.** Ask, "How is global sea surface temperature represented in the graph?"

Look for productive strategies:

- Applying background knowledge of positive linear associations.

3 Connect

Display the graphs.

Have students share how they determined which time period had a greater frequency of hurricanes and more variability in the number of hurricanes.

Ask:

- "What does the horizontal line represent in the first graph? How is it represented in the second graph?" Average temperature of the Atlantic Ocean; It is the diagonal dashed-line.
- "What does the first graph fail to consider?" That the average global sea surface temperature itself is increasing.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Ask students to use the pattern of warm and cold phases on the Observed AMO Index graph and extend it to sketch the next cycle. Help students process the information shown in the Changes in Atlantic Ocean Temperature graph, by asking:

- "For what years did the temperature of the Atlantic Ocean decrease? Increase?"
- "What does a change of 0 represent?"

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the narrative and Observed AMO Index graph.

Read 1: Students should understand that there is a disagreement in what is causing an increase of hurricanes, whether due to global warming or the Atlantic Multidecadal Oscillation (AMO).

Read 2: Ask students to name or highlight any quantities or relationships they see in the graph. For example, there are alternating warm phases and cold phases.

Read 3: Ask students to think about what the AMO display might indicate for the next cycle of years past 2010.

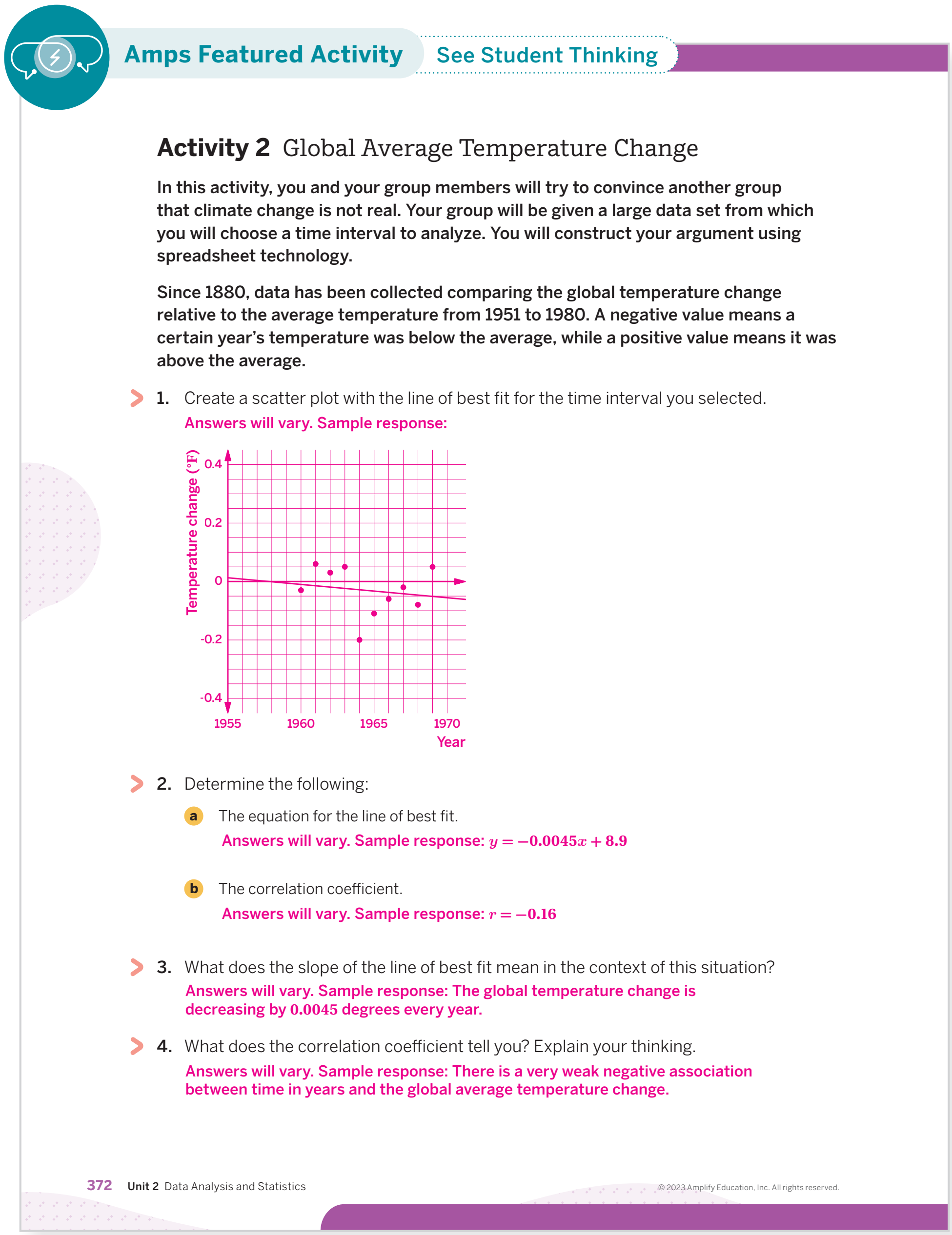
English Learners

Annotate the warm phases of the graph with "more hurricanes" and the cold phases of the graph with "less hurricanes."

Small Groups | 20 min

Activity 2 Global Average Temperature Change

Students choose data to try and show climate change is not real to see how selectively picking data is misleading.



Amps Featured Activity See Student Thinking

Activity 2 Global Average Temperature Change

In this activity, you and your group members will try to convince another group that climate change is not real. Your group will be given a large data set from which you will choose a time interval to analyze. You will construct your argument using spreadsheet technology.

Since 1880, data has been collected comparing the global temperature change relative to the average temperature from 1951 to 1980. A negative value means a certain year's temperature was below the average, while a positive value means it was above the average.

1. Create a scatter plot with the line of best fit for the time interval you selected.

   Answers will vary. Sample response:

2. Determine the following:

   a. The equation for the line of best fit.

   Answers will vary. Sample response: $y = -0.0045x + 8.9$

   b. The correlation coefficient.

   Answers will vary. Sample response: $r = -0.16$

3. What does the slope of the line of best fit mean in the context of this situation?

   Answers will vary. Sample response: The global temperature change is decreasing by 0.0045 degrees every year.

4. What does the correlation coefficient tell you? Explain your thinking.

   Answers will vary. Sample response: There is a very weak negative association between time in years and the global average temperature change.

372 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have students read the prompt, and then distribute the data from the Activity 2 PDF and provide access to spreadsheet technology.

2 Monitor

Help students get started by asking, "If you wanted to show that there is not a pattern or trend in the data, what values would you be looking for in the data set?"

Look for points of confusion:

- **Only selecting a few values for their data set.** Ask, "If you saw someone present data with only 3, 4, or 5 data points, would you be convinced by their conclusions? Why or why not?"
- **Thinking that because the slope is small for the entire data set, this means there is no convincing evidence of changing temperature over time.** Have students compare and interpret the correlation coefficients. Ask, "Even though the temperature change seems small, what is the time period of the data set? What does this mean for the future?"

Look for productive strategies:

- Selecting a portion of the data set with a sufficient number of data values (at least 10).
- Looking for a range of data values that show little to no trend over the given time period.
- Recognizing that making conclusions about the future using a small sample of a large data set is inaccurate and can lead to misrepresentations.

Activity 2 continued ›

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Suggest that students select a time interval of any 10 years that will help show that climate change is not real. Alternatively, consider providing them with a range of years to use.

Accessibility: *Optimize Access to Technology*

Have students use spreadsheet technology to construct the scatter plot for their chosen time period and determine the line of best fit and correlation coefficient. Consider displaying scaffolded steps they can use.

Math Language Development

MLR7: Compare and Connect

During the Connect, as you display samples of student work for different time periods and the scatter plot representing the entire data set, ask students to note any similarities and differences in how the axes were scaled and labeled. Ask, "For the entire data set, if we chose a vertical scale of −10 to 10 degrees instead of −2 to 2, how might the display be interpreted?" It may look like there is little change in the temperature.

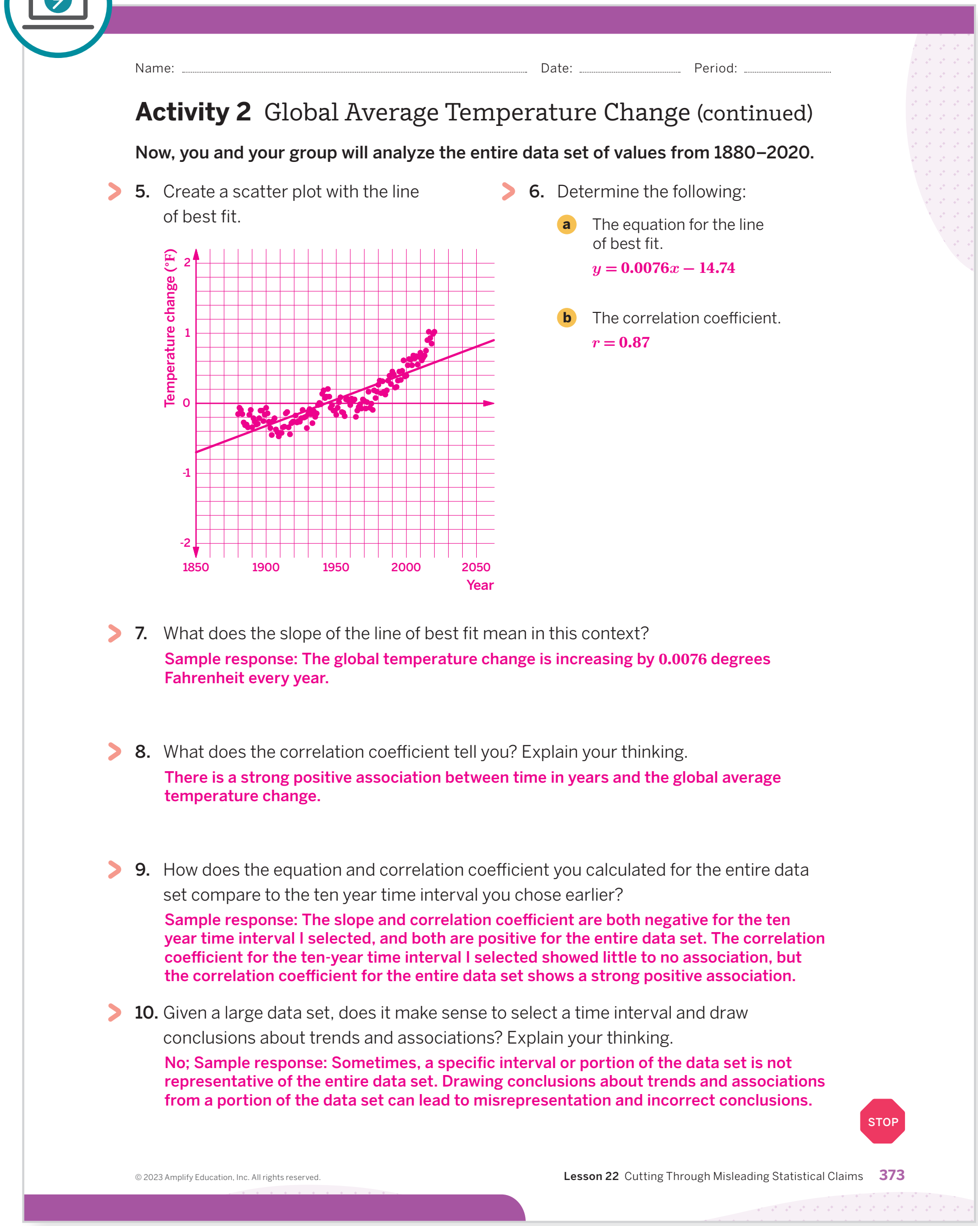
English Learners

Annotate the scatter plots to highlight the axes scales and labels.

Small Groups | 20 min

Activity 2 Global Average Temperature Change (continued)

Students choose data to try and show climate change is not real to see how selectively picking data is misleading.



Name: Date: Period:

Activity 2 Global Average Temperature Change (continued)

Now, you and your group will analyze the entire data set of values from 1880–2020.

5. Create a scatter plot with the line of best fit.

6. Determine the following:

a The equation for the line of best fit.

$y = 0.0076x - 14.74$

b The correlation coefficient.

$r = 0.87$

7. What does the slope of the line of best fit mean in this context?

Sample response: The global temperature change is increasing by 0.0076 degrees Fahrenheit every year.

8. What does the correlation coefficient tell you? Explain your thinking.

There is a strong positive association between time in years and the global average temperature change.

9. How does the equation and correlation coefficient you calculated for the entire data set compare to the ten year time interval you chose earlier?

Sample response: The slope and correlation coefficient are both negative for the ten year time interval I selected, and both are positive for the entire data set. The correlation coefficient for the ten-year time interval I selected showed little to no association, but the correlation coefficient for the entire data set shows a strong positive association.

10. Given a large data set, does it make sense to select a time interval and draw conclusions about trends and associations? Explain your thinking.

No; Sample response: Sometimes, a specific interval or portion of the data set is not representative of the entire data set. Drawing conclusions about trends and associations from a portion of the data set can lead to misrepresentation and incorrect conclusions.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 Cutting Through Misleading Statistical Claims 373

3 Connect

Display samples of student work showing the scatter plot representing the data they chose alongside the scatter plot for the entire data set.

Have groups of students share their conclusions they made based on their analysis comparing their data set to the entire data set. Select and sequence student responses comparing scatter plots, correlation coefficients, slopes, and recognizing the errors made in drawing conclusions from a specific time interval.

Highlight that sometimes, people misrepresent, or only report certain parts of a data set, in order to fit with what they are trying to show. It is important to take all data into consideration to have an accurate picture of the trends occurring in a data set.

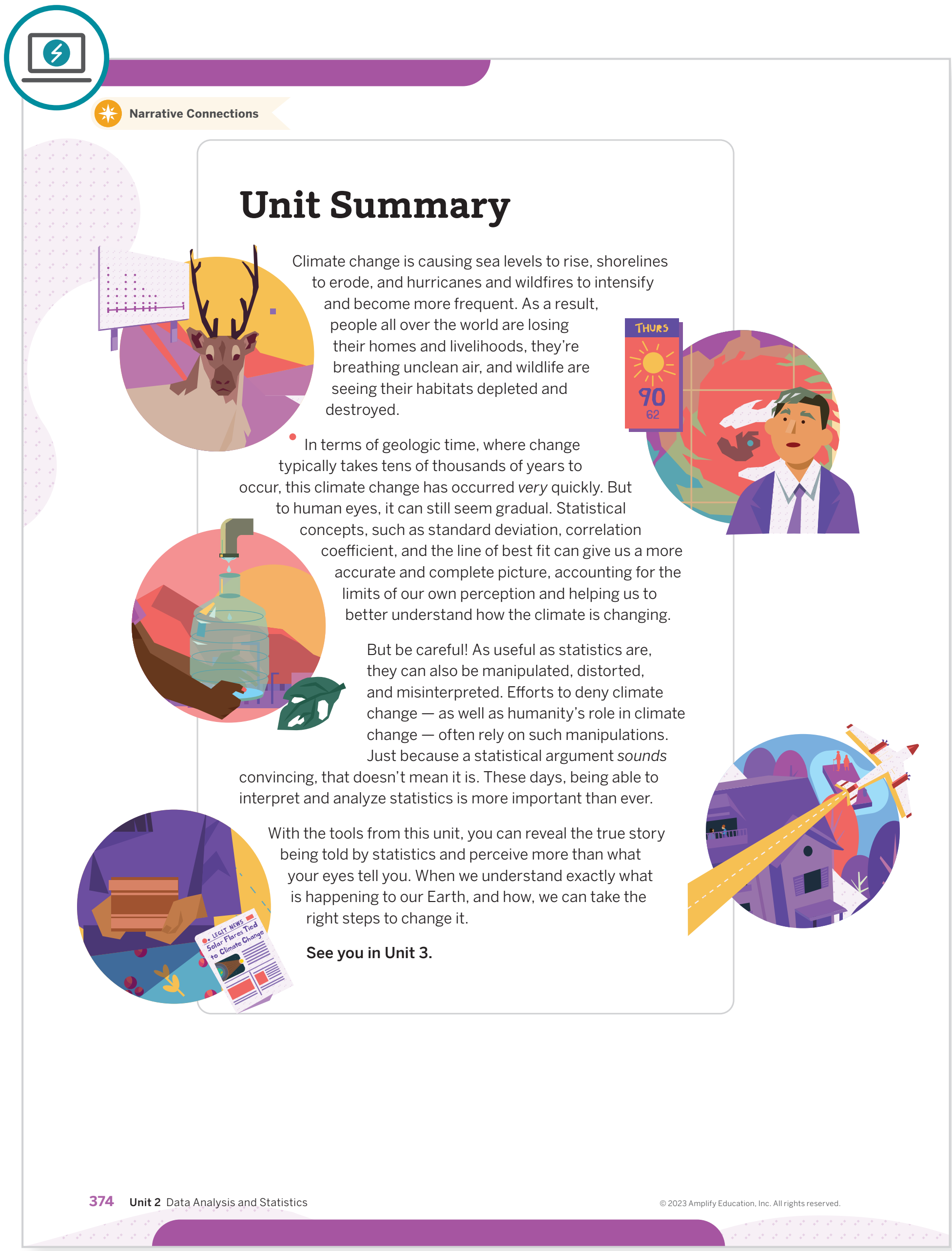
Ask:

- "Does the association in the data look to be linear or nonlinear? Why might that be? Explain your thinking."
  Sample response: Nonlinear. This might be because the rate of human emissions has not increased at a constant rate, causing an increase in the rate at which temperature has increased.
- "Outside of climate science, what are some other areas where data could be manipulated?"
  Sample responses:
  - Medical studies
  - Politics
  - Bias in surveys

Whole Class | 5 min

Unit Summary

Review and synthesize how data displays, statistics, and experiments can be used to study the climate.



Narrative Connections

Unit Summary

Climate change is causing sea levels to rise, shorelines to erode, and hurricanes and wildfires to intensify and become more frequent. As a result, people all over the world are losing their homes and livelihoods, they're breathing unclean air, and wildlife are seeing their habitats depleted and destroyed.

In terms of geologic time, where change typically takes tens of thousands of years to occur, this climate change has occurred *very* quickly. But to human eyes, it can still seem gradual. Statistical concepts, such as standard deviation, correlation coefficient, and the line of best fit can give us a more accurate and complete picture, accounting for the limits of our own perception and helping us to better understand how the climate is changing.

But be careful! As useful as statistics are, they can also be manipulated, distorted, and misinterpreted. Efforts to deny climate change — as well as humanity's role in climate change — often rely on such manipulations. Just because a statistical argument *sounds* convincing, that doesn't mean it is. These days, being able to interpret and analyze statistics is more important than ever.

With the tools from this unit, you can reveal the true story being told by statistics and perceive more than what your eyes tell you. When we understand exactly what is happening to our Earth, and how, we can take the right steps to change it.

See you in Unit 3.

374 Unit 2 Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the table.

x	1	2	5	7
y	-1	0	3	2

Have students share what data displays or statistics they could use to summarize this data set.

Highlight that students could use the mean, median, IQR, MAD, and standard deviation, to analyze one of the variables and create a dot plot, box plot, or histogram. To analyze both variables, they could find the line of best fit or correlation coefficient and create a scatter plot.

Ask, "How did you take the knowledge you have learned over the course of this unit and apply it to today's analysis of misrepresenting data?"

Reflect

After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

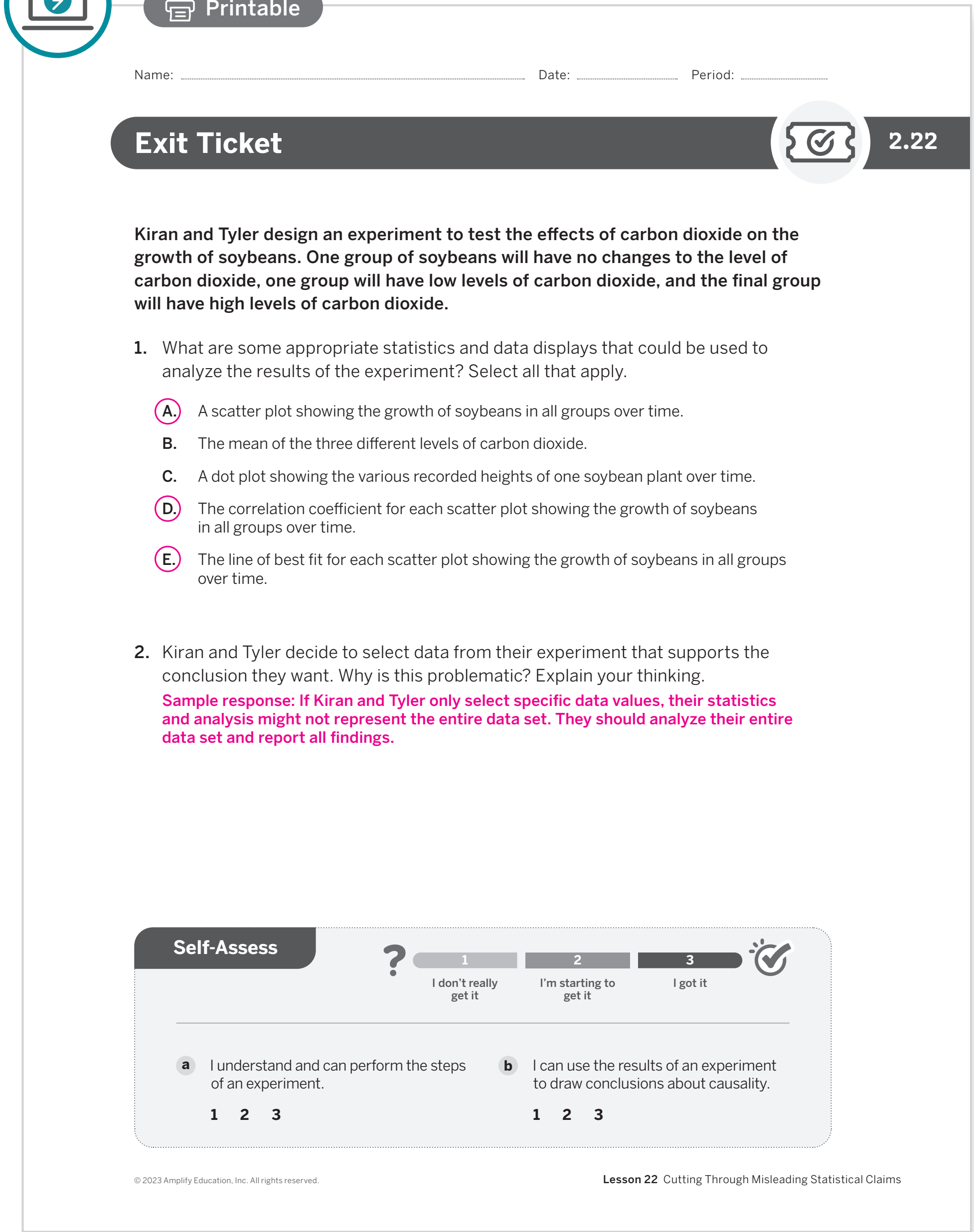
- "Did anything surprise you while reading the narratives of this unit?"
- "Is there anything more you would like to learn about these topics? What are some steps you can take to learn more?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining appropriate statistics and outside influences in an experiment.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 2.22

Kiran and Tyler design an experiment to test the effects of carbon dioxide on the growth of soybeans. One group of soybeans will have no changes to the level of carbon dioxide, one group will have low levels of carbon dioxide, and the final group will have high levels of carbon dioxide.

1. What are some appropriate statistics and data displays that could be used to analyze the results of the experiment? Select all that apply.
   - [✓ A.] A scatter plot showing the growth of soybeans in all groups over time.
   - B. The mean of the three different levels of carbon dioxide.
   - C. A dot plot showing the various recorded heights of one soybean plant over time.
   - [✓ D.] The correlation coefficient for each scatter plot showing the growth of soybeans in all groups over time.
   - [✓ E.] The line of best fit for each scatter plot showing the growth of soybeans in all groups over time.

2. Kiran and Tyler decide to select data from their experiment that supports the conclusion they want. Why is this problematic? Explain your thinking.
   Sample response: If Kiran and Tyler only select specific data values, their statistics and analysis might not represent the entire data set. They should analyze their entire data set and report all findings.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I understand and can perform the steps of an experiment.
1 2 3

b I can use the results of an experiment to draw conclusions about causality.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 Cutting Through Misleading Statistical Claims

Success looks like . . .

- **Language Goal:** Understanding how to evaluate claims about data to determine if there are fallacies or misrepresentations. **(Speaking and Listening, Reading and Writing)**
  - » Explaining why Kiran and Tyler should not select data that only supports the conclusion they want.

Suggested next steps

If students select the incorrect statistics or data displays in Problem 1, consider:

- Reviewing Activity 2.
- Assigning Practice Problem 4.
- Asking, "How many variables are represented when calculating the mean or making a dot plot?"

If students have trouble determining errors in Problem 2, consider:

- Reviewing Problems 9 and 10 from Activity 2.
- Assigning Practice Problem 2.
- Asking, "Are trends from portions of a data set always accurate representations of the entire data set?"

Professional Learning

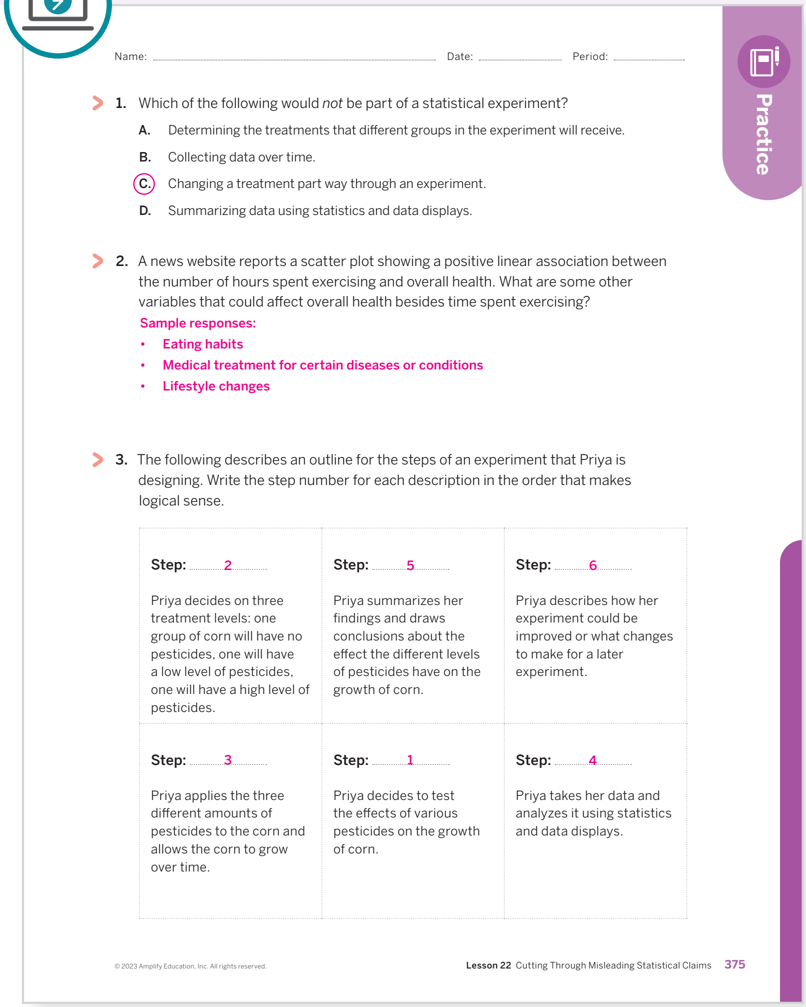
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students worked on Activity 2?
- What was especially satisfying about having your students analyze data in Activity 2? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

Practice

1. Which of the following would *not* be part of a statistical experiment?
   - A. Determining the treatments that different groups in the experiment will receive.
   - B. Collecting data over time.
   - [✓ C.] Changing a treatment part way through an experiment.
   - D. Summarizing data using statistics and data displays.

2. A news website reports a scatter plot showing a positive linear association between the number of hours spent exercising and overall health. What are some other variables that could affect overall health besides time spent exercising?

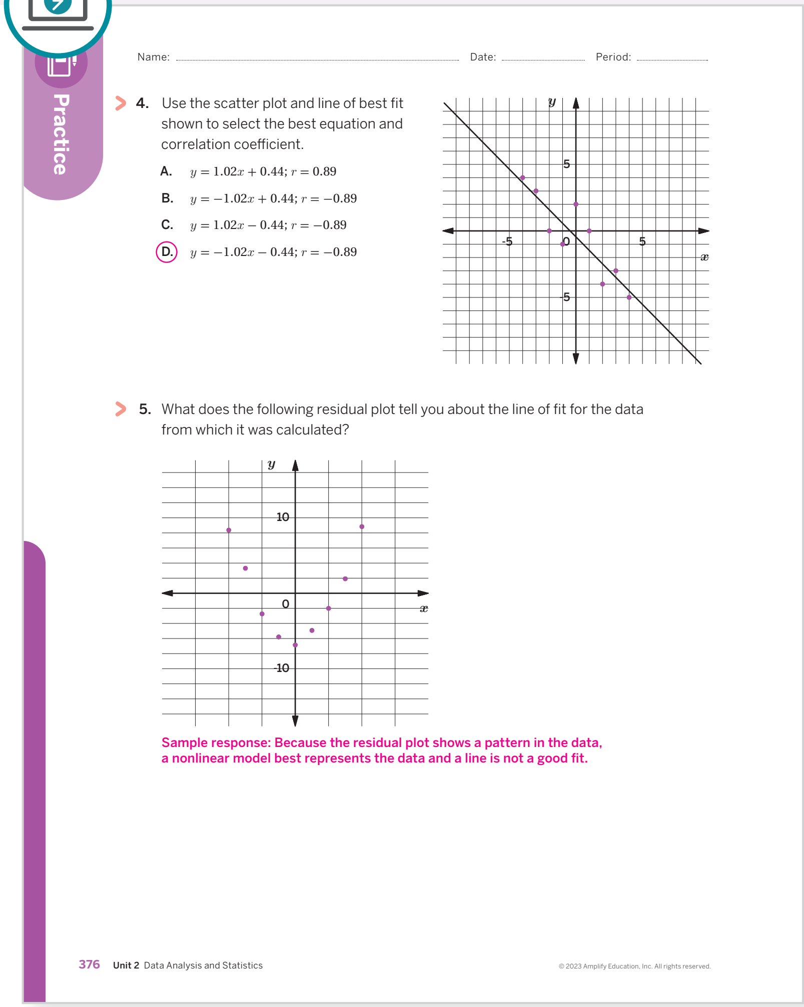
   Sample responses:
   - **Eating habits**
   - **Medical treatment for certain diseases or conditions**
   - **Lifestyle changes**

3. The following describes an outline for the steps of an experiment that Priya is designing. Write the step number for each description in the order that makes logical sense.

Step: 2 Priya decides on three treatment levels: one group of corn will have no pesticides, one will have a low level of pesticides, one will have a high level of pesticides.	**Step:** 5 Priya summarizes her findings and draws conclusions about the effect the different levels of pesticides have on the growth of corn.	**Step:** 6 Priya describes how her experiment could be improved or what changes to make for a later experiment.
Step: 3 Priya applies the three different amounts of pesticides to the corn and allows the corn to grow over time.	**Step:** 1 Priya decides to test the effects of various pesticides on the growth of corn.	**Step:** 4 Priya takes her data and analyzes it using statistics and data displays.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 Cutting Through Misleading Statistical Claims 375



Name: ______ Date: ______ Period: ______

Practice

4. Use the scatter plot and line of best fit shown to select the best equation and correlation coefficient.
   - A. $y = 1.02x + 0.44;\ r = 0.89$
   - B. $y = -1.02x + 0.44;\ r = -0.89$
   - C. $y = 1.02x - 0.44;\ r = -0.89$
   - [✓ D.] $y = -1.02x - 0.44;\ r = -0.89$

5. What does the following residual plot tell you about the line of fit for the data from which it was calculated?

Sample response: Because the residual plot shows a pattern in the data, a nonlinear model best represents the data and a line is not a good fit.

376 **Unit 2** Data Analysis and Statistics

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
Spiral	1	Unit 2 Lesson 18	1
	2	Unit 2 Lesson 18	2
	3	Unit 2 Lesson 21	2
	4	Unit 2 Lesson 19	1
	5	Unit 2 Lesson 13	2

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

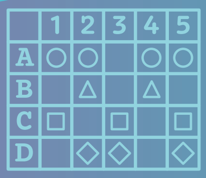
UNIT 3

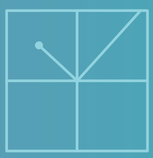
Functions and Their Graphs

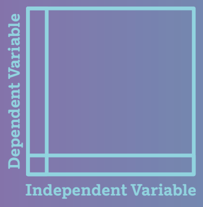
Students will expand on their understanding of the key features and multiple representations of functions. Along the way, they will write, graph, and interpret linear functions, piecewise functions, absolute value functions, and the inverses of functions.

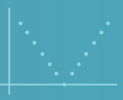
Essential Questions

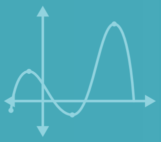
- How can you represent and describe functions?
- Why are relations and functions represented in multiple ways?
- Why should functions be analyzed graphically?
- *(By the way, what is the inverse of putting on your socks and shoes?)*

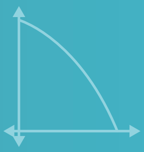












Key Shifts in Mathematics

Focus

In this unit . . .

Students expand their understanding of functions, building on what they learned in Grade 8. They are introduced to new tools for communicating about functions: function notation, domain and range, average rates of change, and mathematical terms for describing key features of graphs. Students connect features of the graph to features of the situation and other representations. They begin to distinguish categories of functions: linear functions, piecewise-defined functions (the absolute value function, in particular), and inverses of functions. Throughout the unit, students use, interpret, and connect the different representations of functions, both in and out of context.

Coherence

Previously . . .

In Grade 8, students were introduced to the concept of a function. They learned to understand and use the terms “input,” “output,” and “function,” e.g., “temperature is a function of time.” They described functions as increasing or decreasing between specific numerical inputs, and they considered the inputs of a function to be values of its independent variable and its outputs to be values of its dependent variable. Students used tables, equations, and graphs to represent functions, and described information presented in tables, equations, or graphs in terms of functions. In working with linear functions, they synthesized their understanding of “constant of proportionality” (Grade 7), “rate of change” and “slope” (Grade 8), and increasing and decreasing.

Coming soon . . .

Later in this course, students will formally define the non-linear functions introduced in this unit as exponential and quadratic. In Algebra 2, students continue their studies in functions by analyzing polynomial, rational, logarithm, square root, and trigonometric functions. They will analyze the graphs of these functions, furthering their understanding of key features of graphs such as multiplicity, end-behavior, and asymptotes. Students will formalize their understanding of inverse functions of non-linear functions and its notation. This work will lay the foundation for more advanced topics in mathematics.

Rigor

In this unit, students engage in rich tasks that address one or more of these aspects of rigor. The following are examples of how each pillar is addressed. See each Lesson Brief to learn more about how rigor is balanced throughout the unit.

Conceptual Understanding

Students build a conceptual understanding of functions in Lessons 1–5, piecewise and absolute value functions in Lessons 14 and 15, and the inverse of functions in Lesson 18.

Procedural Fluency

Students build procedural fluency of writing, graphing, and interpreting linear functions in Lessons 3–6 and determining and describing key features of functions in Lessons 8–12.

Application

Students apply their understanding of key features of functions in Lessons 13 and 22 to sketch graphs that represent real-world relationships, and of piecewise functions in Lesson 15 to model the pitch of a melody.

Artscapes

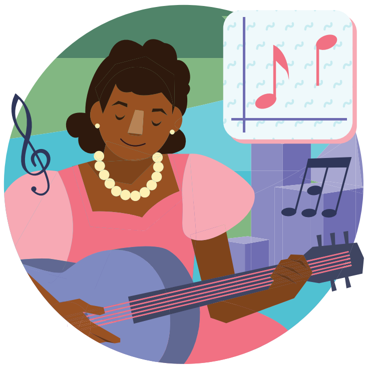
SUB-UNIT

1

Lessons 2–6

Functions and Their Representations

Students build on their understanding of functions from Grade 8 and represent them with verbal descriptions, tables, graphs, and equations. They are introduced to a new tool for communicating about functions — ***function notation*** — and use it to interpret functions within real-world contexts.



Narrative: Much like playing an instrument, you can "play" a mathematical function.

SUB-UNIT

2

Lessons 7–13

Analyzing and Creating Graphs of Functions

Students describe key features of functions, using the terms ***domain***, ***range***, and ***average range of change***. They explore functions that show ***discrete*** data and understand the importance of the scale of a function's graph. Using ***interval notation***, they represent a function's domain and range.

Narrative: Reading a sheet of music is similar to interpreting the graph of a function.

SUB-UNIT

3

Lessons 14–17

Piecewise Functions

Students examine ***piecewise functions*** as a set of rules defining a relationship, paying close attention to their domains. They understand the absolute value function as both a distance and a piecewise function, and examine how horizontal and vertical translations are represented in its equation and graph.

Narrative: Explore how a function can model the *pieces* of sound.

Launch

Lesson 1

Music to Our Ears

Students are introduced to the idea that there is a connection between sound and math through graphing. They take turns creating sounds and sketching graphs of what they think each sound could look like. Then, they examine graphical music notation in the context of a one-person band and make connections to what defines a function.

SUB-UNIT

4

Lessons 18–21

Inverses of Functions

Students are introduced to the ***inverse of a function*** as a reverse process, switching input and output values. They graph the inverse using a line of symmetry and a table of values and compare multiple representations of the inverse of a function.

Narrative: Functions and their inverses can help you go from acoustics to amplified sounds and back again.

Capstone

Lesson 22

Freerunning Functions

Students identify key features of a piecewise function, such as the domain and range, intervals of increasing, decreasing, and constant, local and global maximums and minimums, average rate of change over an interval, and vertical intercept, after completing the path of a freerunner. Students then are given the features to write a piecewise function that models a freerunner course.

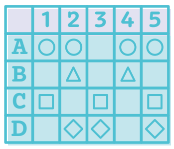
Unit at a Glance

Spoiler Alert: There's more to functions than meets the eye (or what students learned about in Grade 8). Function notation, interval notation, and inverses of functions are explored in this unit.

Assessment

A **Pre-Unit Readiness Assessment**

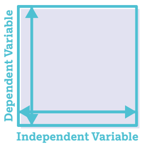
Launch Lesson



	1	2	3	4	5
A	○	○		○	○
B		△		△	
C	□		□		□
D		◇	◇		◇

1 **Music to Our Ears**

Sketch graphs to represent different scenarios and determine when a relationship is a function.



2 **Describing and Graphing Situations**

Review independent variable and dependent variable and represent functions with words, tables, and graphs.

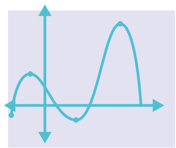
$$y = f(x) = ax + b$$

Linear Functions

6 **Using Function Notation to Describe Rules** (Part 2) ●

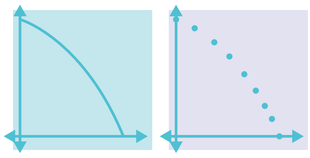
Use different ways to evaluate functions and solve equations written in function notation with a focus on linear relationships.

Sub-Unit 2: Analyzing and Creating Graphs of Functions



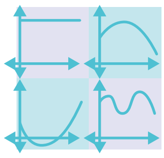
7 🔑 **Features of Graphs**

Understand and interpret important features of graphs such as *horizontal intercept*, *vertical intercept*, *maximum*, and *minimum*.



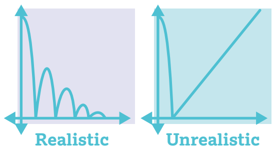
8 **Understanding Scale**

Understand how the scale of each axis affects a graph and determine whether a scenario is discrete or not.



12 **Interpreting Graphs**

Determine important features of graphs and the average rate of change in context.



13 **Creating Graphs of Functions**

Sketch graphs of functions that show important features in context.

Assessment

A **Mid-Unit Assessment**

Key Concepts

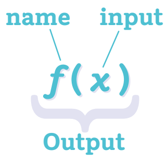
Lesson 3: Function notation is formally introduced.
Lesson 7: Mathematical terminology can be used to describe key features of graphs.
Lesson 11: Domain and range are formally defined and interval notation is introduced.
Lesson 18: The inverse of a function reverses its input and output values.

Pacing

22 Lessons: 50 min each
3 Assessments: 45 min each
Full Unit: 25 days
● **Modified Unit:** 22 days

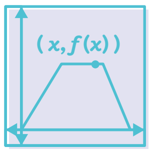
Assumes 50-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.

Sub-Unit 1: Functions and Their Representations



3 Function Notation

Use function notation to express functions that have specific input and output values in terms of a real-world situation.



4 Interpreting and Using Function Notation

Use function notation to represent a relationship between quantities and sketch a graph of the function.

$$f(s) = s^2$$

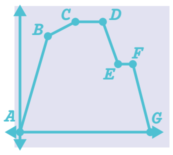
5 Using Function Notation to Describe Rules (Part 1) ●

Interpret and write equations when they are written in function notation and create graphs and tables to represent the functions.

$$\frac{f(b) - f(a)}{b - a}$$

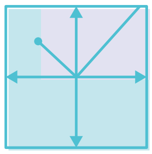
9 How Do Graphs Change?

Understand and interpret the average rate of change.



10 Where Are Functions Changing? ●

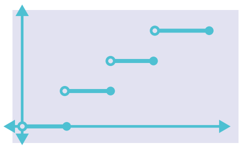
Determine reasonable input and output values for a function given a description of a situation.



11 Domain and Range ●

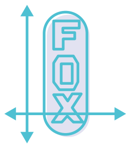
Understand the terms domain and range, how to represent values that continue forever, and use interval notation.

Sub-Unit 3: Piecewise Functions



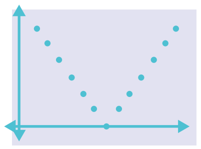
14 Piecewise Functions (Part 1) ●

Evaluate piecewise functions using the rules of the piecewise function, and graph and describe piecewise functions.



15 Piecewise Functions (Part 2) ●

Adjust the vertical intercept and domain of pieces of a piecewise function to eliminate breaks and jumps in a melody.



16 Another Function?

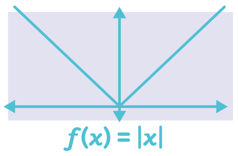
Calculate and plot absolute guessing errors to explore the absolute value function as a distance function.

Unit at a Glance

Spoiler Alert: There's more to functions than meets the eye (or what students learned about in Grade 8). Function notation, interval notation, and inverses of functions are explored in this unit.

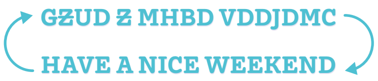
continued

Sub-Unit 4: Inverse of Functions



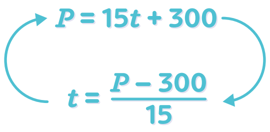
17 Absolute Value Functions

Define the absolute value function as a distance and piecewise function. Make sense of how translations of the function are represented.



18 Inverses of Functions

Use and create ciphers to encode and decode messages to define the inverse of a function.



19 Finding and Interpreting Inverses of Functions ●

Write and interpret the inverses of linear functions and their domain and range in terms of situations.

Assessment

A End-of-Unit Assessment

Key Concepts

Lesson 3: Function notation is formally introduced.

Lesson 7: Mathematical terminology can be used to describe key features of graphs.

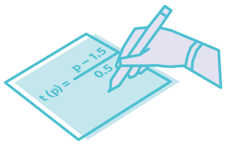
Lesson 11: Domain and range are formally defined and interval notation is introduced.

Lesson 18: The inverse of a function reverses its input and output values.

Pacing

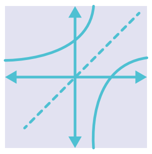
22 Lessons: 50 min each	**Full Unit:** 25 days
3 Assessments: 45 min each	● **Modified Unit:** 22 days

Assumes 50-minute class periods per day. For block scheduling or other durations, adjust the number of days accordingly.



20 Writing Inverses of Functions to Solve Problems ●

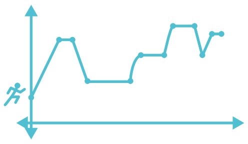
Find and interpret the inverses of functions in situations and determine which equation is more efficient to use when determining certain values.



21 Graphing Inverses of Functions

Examine and make sense of the relationship between the graph of a function and the graph of its inverse.

Capstone Lesson



22 Freerunning Functions

Create piecewise functions using descriptions of their key features and parts of their graphs.

● Modifications to Pacing

Lessons 5 and 6: These two lessons can be combined. Both lessons involve writing and solving equations in function notation. An activity in Lesson 6 requires the use of graphing technology to graph and interpret functions, which can be omitted.

Lessons 10 and 11: These two lessons may be combined. Lesson 10 develops the concept of domain and range in terms of input and output, whereas Lesson 11 formally defines domain and range and introduces interval notation.

Lessons 14 and 15: These two lessons can be combined. Lesson 14 focuses on the writing and graphing of piecewise functions. Lesson 15 applies these skills to creating melodies in music. One of the activities from Lesson 15 can be an addition on Lesson 14.

Lessons 19 and 20: These lessons can be combined. Lesson 20 continues a focus on writing and interpreting the inverse of linear functions from Lesson 19.

Unit Supports

Math Language Development

Lesson	New vocabulary
3	function notation
6	linear function
7	global maximum global minimum local maximum local minimum
8	discrete
9	average rate of change
11	domain infinite (infinity, ∞) interval notation range
14	piecewise function step function
17	absolute value function vertex
18	inverse of a function

Mathematical Language Routines (MLRs) support students' language development in the context of mathematical sense-making.

Lesson(s)	Mathematical Language Routines
14, 17, 22	**MLR1:** Stronger and Clearer Each Time
1–3, 7–9, 11, 12, 14, 16-18, 21	**MLR2:** Collect and Display
3, 10, 11, 13	**MLR3:** Critique, Correct, Clarify
19	**MLR4:** Information Gap
2, 5, 8, 20	**MLR5:** Co-craft Questions
3, 6–8, 10, 13, 18, 19	**MLR6:** Three Reads
1, 4, 7–12, 14, 15, 17, 19, 21, 22	**MLR7:** Compare and Connect
2–5, 10, 16, 19, 20, 22	**MLR8:** Discussion Supports

Materials

Every lesson includes:

Exit Ticket

Additional Practice

Additional required materials include:

Lesson(s)	Materials
18, 19	four-function calculator
6, 16, 17, 20, 21	graphing technology
1	music
1–5, 7, 10–12, 14–22	PDFs are required for these lessons. Refer to each lesson's overview to see which activities require PDFs.
4, 6, 15, 22	rulers
2	scissors straws tape
16	transparent jar with 30–50 small objects

Instructional Routines

Activities throughout this unit include the following instructional routines:

Lesson(s)	Instructional Routines
8, 10	Card Sort
19	Info Gap
5	Jigsaw
1, 2, 5, 8, 21	Notice and Wonder
4	Partner Problems
9, 11	Poll the Class
1	Take Turns
11	True or False
11	Two Truths and a Lie
12	Which One Doesn't Belong?

Unit Assessments

Each unit includes diagnostic, formative, and summative assessment opportunities to gauge student understanding of the concepts and skills taught in the unit. All assessments are available in print and digital formats.

Assessments	When to Administer
Pre-Unit Readiness Assessment This *diagnostic assessment* evaluates students' proficiency with prerequisite concepts and skills they need to feel successful in this unit.	Prior to Lesson 1
Exit Tickets Each lesson includes *formative assessments* to evaluate students' proficiency with the concepts and skills they learned.	End of each lesson
Mid-Unit Assessment This *summative assessment* provides students the opportunity to demonstrate their proficiency with the concepts and skills they learned in the first part of the unit.	After Lesson 13
End-of-Unit Assessment This *summative assessment* allows students to demonstrate their mastery of the concepts and skills they learned in the lessons preceding this assessment. Additionally, this unit's **Performance Task** is available in the Assessment Guide.	After Lesson 22

Amps
POWERED BY desmos

Social & Collaborative Digital Moments

Featured Activity

Mapping the Path of a Freerunner

Put on your student hat and work through **Lesson 22, Activity 2:**

Points to Ponder . . .

- What was it like to engage in this activity as a learner?
- How might you pace your students through this activity?
- What best practices might you employ for integrating technology in your classroom?

Other Featured Activities:

- Making Music **(Lesson 2)**
- The Flag of St. Louis **(Lesson 12)**
- Creating a Melody **(Lesson 15)**
- Plotting the Guesses **(Lesson 16)**

Activity 2: Creating a Freerunner Course

Sketch the route on the coordinate plane using a com
of linear, constant, and absolute value pieces. Start al
on the left.

Press play to test your route. Make changes to your sk
as needed.

Unit Study
Professional Learning

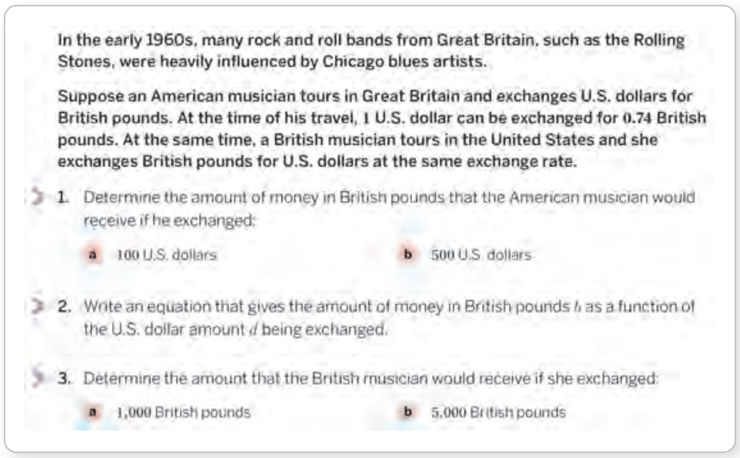
This unit study is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can engage in a meaningful professional learning experience to deepen your own understanding of this unit and employ effective pedagogical practices to support your students.

Anticipating the Student Experience With Fawn Nguyen

Sub-Unit 4 introduces students to inverses of functions. They learn to find the inverse of a function given an input/output table, an equation, and/or a graph. Students understand that inverses of functions are useful in real-world contexts, such as conversion rates and population changes over time. They notice that the graph of a function and its inverse are reflections across the line $y = x$. Students also determine that an inverse of a function is itself sometimes not a function. Equip yourself to support your students with this concept by engaging in a brief problem-solving exercise to deepen your own learning.

Do the Math

Put on your student hat and tackle these problems from Lesson 18, Activity 2:



In the early 1960s, many rock and roll bands from Great Britain, such as the Rolling Stones, were heavily influenced by Chicago blues artists.

Suppose an American musician tours in Great Britain and exchanges U.S. dollars for British pounds. At the time of his travel, 1 U.S. dollar can be exchanged for 0.74 British pounds. At the same time, a British musician tours in the United States and she exchanges British pounds for U.S. dollars at the same exchange rate.

1. Determine the amount of money in British pounds that the American musician would receive if he exchanged:
   - a 100 U.S. dollars
   - b 500 U.S. dollars
2. Write an equation that gives the amount of money in British pounds b as a function of the U.S. dollar amount d being exchanged.
3. Determine the amount that the British musician would receive if she exchanged:
   - a 1,000 British pounds
   - b 5,000 British pounds

Put your teacher hat back on to share your work with one or more colleagues and discuss your approaches.

Points to Ponder . . .

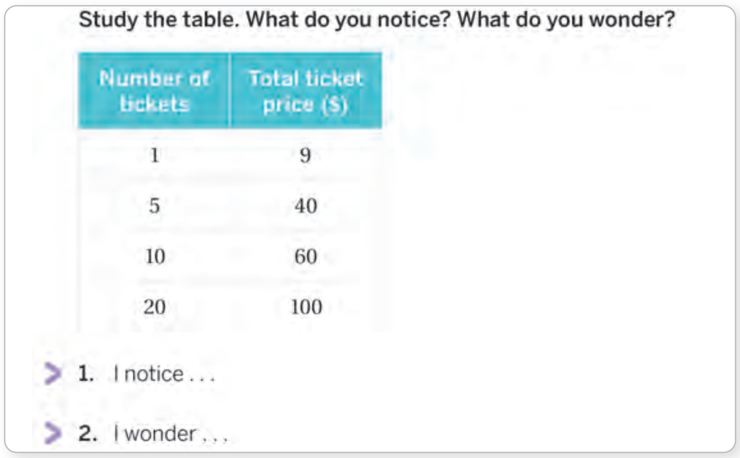
- What was it like to engage in this problem as a learner?
- It's not uncommon for students to interpret a scenario incorrectly by writing the equation for Problem 2 as $d = 0.74b$. How might you help students think this through and build their confidence?
- Students may enjoy selecting a currency of their choice and track its exchange rate with the U.S. dollar over time.
- What implications might this have for your teaching in this unit?

Focus on Instructional Routines

Notice and Wonder

Rehearse . . .

How you'll facilitate the *Notice and Wonder* instructional routine in Lesson 2, Warm-up:



Study the table. What do you notice? What do you wonder?

Number of tickets	Total ticket price ($)
1	9
5	40
10	60
20	100

1. I notice . . .
2. I wonder . . .

Points to Ponder . . .

- How am I capturing student responses as they share? Am I honoring all student responses?
- How should I emphasize or have my class identify responses that are most relevant to our learning goal?

This routine . . .

- Encompasses MLR8 Discussion Supports.
- Facilitates the use of graphic organizers, sentence frames, and other discussion supports that benefit not only English Learners, but all students.
- Provides opportunities for students to work with a partner to co-craft questions.
- Provides opportunities to foster a safe space for students to answer freely, because there are no right or wrong responses.

Anticipate . . .

- Intentional grouping of students to best support dialogue and focus.
- Preparing scaffolds or questions to help students get started.
- If you haven't used this routine before, what classroom management strategies might you need to put in place to help it run smoothly?
- If you have used this routine before, what went well? What would you want to refine?

Strengthening Your Effective Teaching Practices

Facilitate meaningful mathematical discourse. Pose purposeful questions.

These effective teaching practices . . .

- Ensures that there is a shared understanding of the mathematical concepts presented in each lesson.
- Allows students to listen to and critique the strategies and conclusions of others.
- Helps you assess the reasoning behind student responses, and advance their sense-making skills by asking deeper questions about mathematical ideas and relationships.

Points to Ponder . . .

- Some students may not not know how to dive deeper into discussions about mathematics. How can you model these discussions?
- How can you probe for student reasoning to ensure they understand a mathematical concept, beyond just providing a correct response?
- What kinds of questions can you ask that will help advance their understanding to a deeper level?

Math Language Development

MLR2: Collect and Display

MLR2 appears in Lessons 1–3, 7–9, 11, 12, 14, 16–18, 21.

- Students will be introduced to several new terms in this unit as they explore describing key features of functions, learn about piecewise and absolute value functions, and determine inverses of functions. Throughout the unit, as students formalize the new vocabulary they are learning, have them refer to the class display to continually review and reflect on new terms that are added.
- **English Learners:** Add diagrams or illustrations to the class display so that students can visualize the terms or phrases. For example, in Lesson 18, add illustrations that show what it means to “undo” or “reverse” the input-output pairs of a function.

Point to Ponder . . .

- How will you encourage or guide students toward using their developing math language to describe key features of functions?

Unit Assessments

Use the results of the **Pre-Unit Readiness Assessment** to understand your students’ prior knowledge and determine their prerequisite skills.

Look Ahead . . .

- Review and unpack the **Mid-** and **End-of-Unit Assessments**, noting the concepts and skills assessed in each.
- With your student hat on, complete each problem.

Points to Ponder . . .

- What concepts or skills in this unit might need more emphasis?
- Where might your students need additional scaffolding and support?
- How might you support your students with using key features of functions to describe and interpret a function throughout the unit? Do you think your students will generally:
  - Miss the underlying concept of function notation?
  - Simply struggle with the concept of average rate of change?

Differentiated Support

Accessibility: *Optimize Access to Technology*

Throughout this unit, have students use the Amps slides. Specific suggested opportunities to have students use technology to deepen their conceptual understanding appear in Lessons 2–6, 9, 11–17, 21, and 22.

- In Lesson 4, students view an animation of a panda climbing a tree, while the graph of the panda’s height and time are simultaneously displayed.
- In Lesson 10, students can digitally interact with the graphs of functions that have restricted domains and ranges. This will support their visualization of how the interval notation changes in real time to match restricted domain and range.
- In Lesson 15, students digitally change pieces of a piecewise function to eliminate breaks and jumps in a melody by adjusting the expression and domain representing that piece. They can check their accuracy by hearing the melody the graph represents.

Point to Ponder . . .

- As you preview or teach the unit, how will you decide when to use technology to optimize student understanding?

Building Math Identity and Community

Throughout this unit, each lesson includes opportunities to support your students’ capacity to build strong mathematical habits of mind.

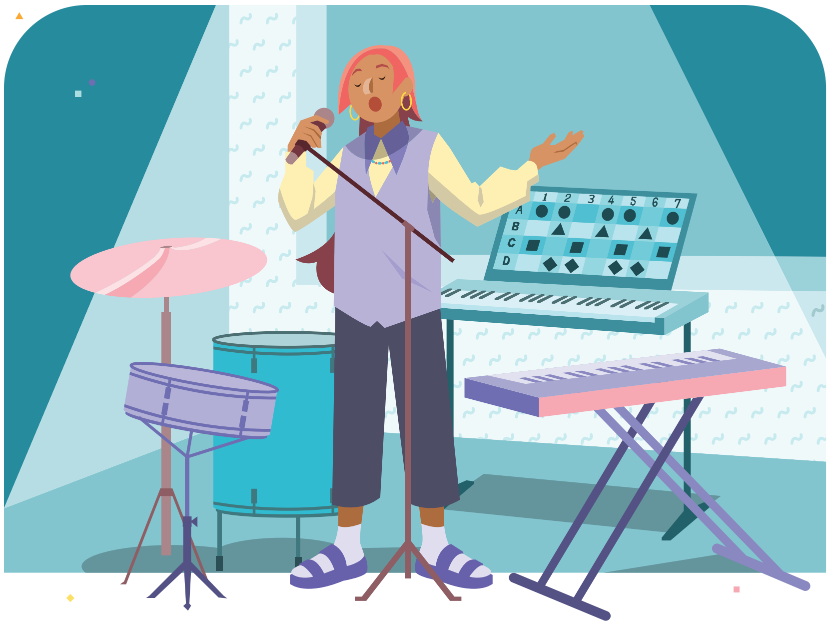
In this unit, pay particular attention to supporting students in building their self-management and self-awareness skills.

Points to Ponder . . .

- Can students regulate their emotions? When they approach a challenge, do they have skills that help them manage their stress? Are they able to overcome their impulses and focus on highly-detailed tasks? Do they set and accomplish their goals?
- Are students able to approach new tasks with confidence? Do they know how to use their strengths to their advantage? Do they have a growth mindset that expresses being ok with not knowing something yet? Are they optimistic in their work? Are they able to recognize their emotions and control how they affect their behavior?

Music to Our Ears

Let's determine how graphs and functions can be used to tell a story.



Focus

Goals

1. **Language Goal:** Sketch graphs to represent different scenarios. **(Writing)**
2. **Language Goal:** Determine whether a relationship is a function. **(Reading and Writing)**

Coherence

Today

Students are exposed to the connections between music and functions. They first get to see how sounds can be represented graphically. Then, students get to take turns creating and sketching graphs of sounds they and their partner make. Finally, students are introduced to graphic notation, a way to represent musical notation. In doing so, they get to draw connections between what is or is not a mathematical function.

Previously

In Grade 8, students learned when a relationship is or is not a function in context.

Coming Soon

In Lesson 2, students will sketch functions and determine independent and dependent variables.

Rigor

- Students build **conceptual understanding** of when a relationship is or is not a function.
- Students **apply** their understanding of sketching functions to represent different sounds.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (for display)
- Activity 1 PDF, pre-cut cards, one set per pair
- music

Math Language Development

Review words

- *function*

Amps | Featured Activity

Activity 2
Interactive Graphic Score

Students are able to move symbols that represent graphic notation in music. You can overlay student responses to provide immediate feedback.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Starting a new unit might raise some students' stress level because they are unsure of the change. Encourage students to use the creativity involved with modeling music with graphs to be a regulation mechanism for their stress. The models are a visual representation of something that most people find soothing, music. As their stress levels drop, they will be able to focus on the connection between the images and the new mathematics.

Modifications to Pacing

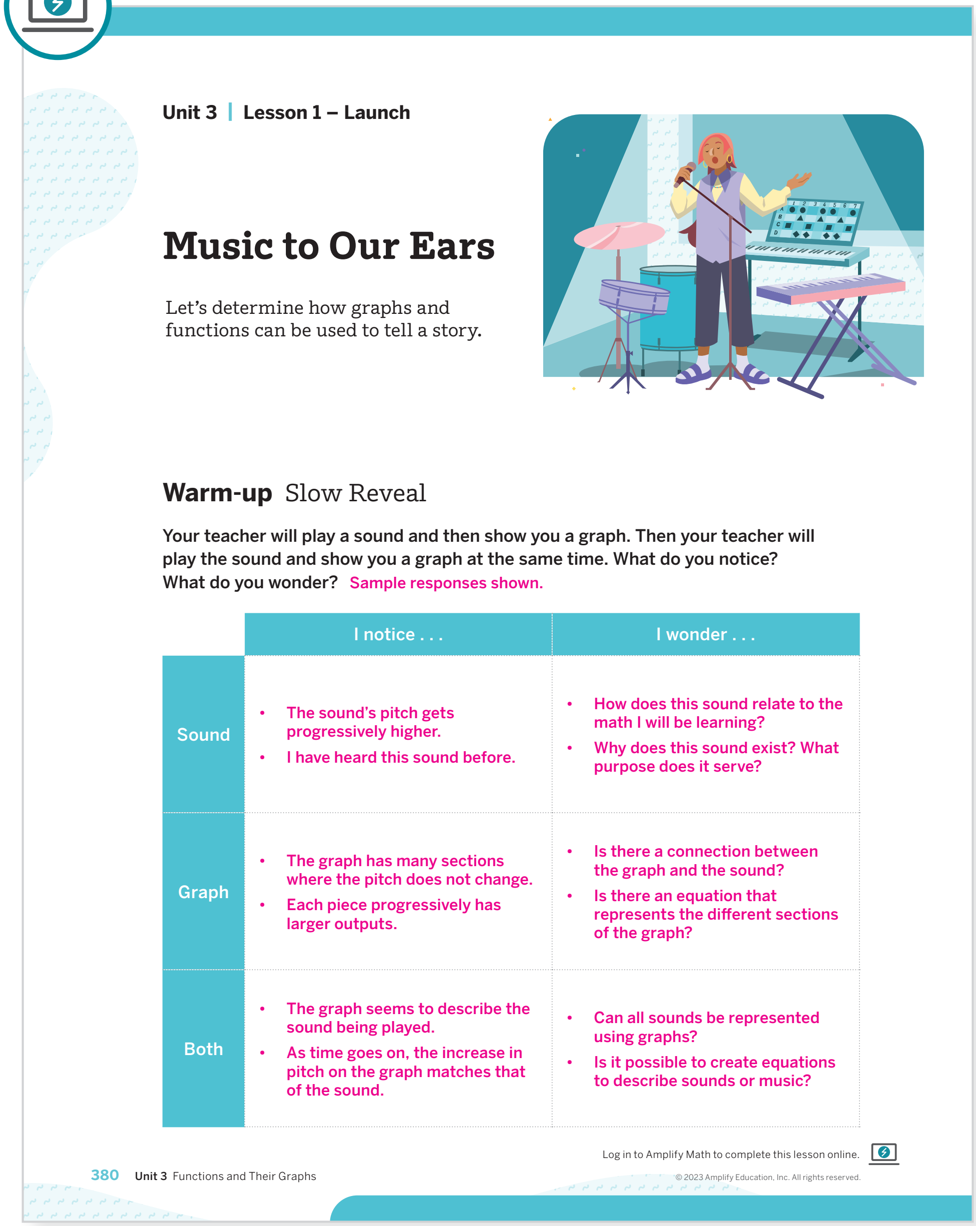
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, Cards 4 and 8 may be omitted.

Independent | 5 min

Warm-up Slow Reveal

Students describe what they notice and wonder from a sound and graph to observe how sound can be represented graphically.



Unit 3 | Lesson 1 – Launch

Music to Our Ears

Let's determine how graphs and functions can be used to tell a story.

Warm-up Slow Reveal

Your teacher will play a sound and then show you a graph. Then your teacher will play the sound and show you a graph at the same time. What do you notice? What do you wonder? Sample responses shown.

	I notice . . .	I wonder . . .
Sound	The sound's pitch gets progressively higher. I have heard this sound before.	How does this sound relate to the math I will be learning? Why does this sound exist? What purpose does it serve?
Graph	The graph has many sections where the pitch does not change. Each piece progressively has larger outputs.	Is there a connection between the graph and the sound? Is there an equation that represents the different sections of the graph?
Both	The graph seems to describe the sound being played. As time goes on, the increase in pitch on the graph matches that of the sound.	Can all sounds be represented using graphs? Is it possible to create equations to describe sounds or music?

Log in to Amplify Math to complete this lesson online.

380 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Play the "Do Re Mi Fa So La Ti Do" sound for students found on the internet, display Graph 1 from the Warm-up PDF, then play the sound while displaying Graph 1. Pause after each step to allow students to complete the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by asking, "How does the sound change over time?"

Look for points of confusion:

- **Thinking the graph describes an increase in volume.** Ask, "Is the loudness of what you are hearing changing? What other aspects of the voice you hear are changing?"

Look for productive strategies:

- Noticing the pitch in the sound is changing over time.
- Recognizing the graph is a representation of the sound they are hearing.

3 Connect

Display Graph 2 from the Warm-up PDF.

Have individual students share what they notice and wonder about the sound and graph.

Highlight that the graph represents how the pitch of the sound they are hearing is increasing over time, and each horizontal segment represents the constant pitch at which the sounds occur. Tell students that *pitch* generally refers to how high or low a note sounds.

Ask, "Why do you think there are gaps between each of the horizontal segments on the graph?" Sample response: Because each sound remains at the same pitch, gaps occur to represent a jump in the change of pitch.

Math Language Development

MLR2: Collect and Display

Listen for the vocabulary, images, and diagrams students use to describe what they notice and wonder about the sound and graph from the Warm-up. During the Connect, highlight terms such as *pitch, increasing, horizontal segment, constant, gaps, jumps,* etc. If students are unfamiliar with these terms, describe what they mean.

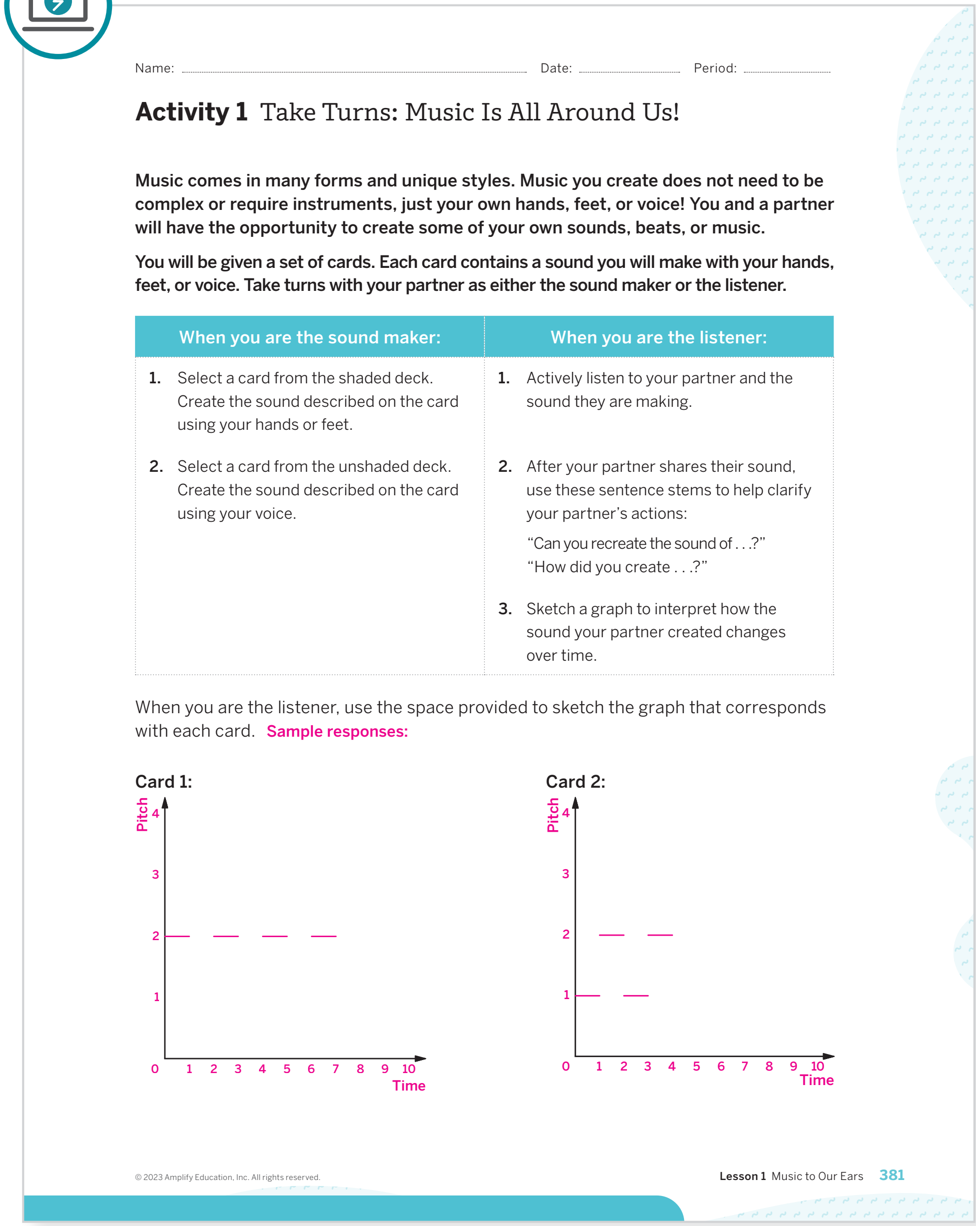
English Learners

As you play the sound, display the term *pitch* on a piece of paper and use gestures to illustrate how the pitch increases over time by raising the piece of paper as the sound progresses.

Pairs | 20 min

Activity 1 Take Turns: Music Is All Around Us!

Students take turns creating and sketching graphs of sounds they make to understand that graphs can model different scenarios.



Name: ______ Date: ______ Period: ______

Activity 1 Take Turns: Music Is All Around Us!

Music comes in many forms and unique styles. Music you create does not need to be complex or require instruments, just your own hands, feet, or voice! You and a partner will have the opportunity to create some of your own sounds, beats, or music.

You will be given a set of cards. Each card contains a sound you will make with your hands, feet, or voice. Take turns with your partner as either the sound maker or the listener.

When you are the sound maker:	When you are the listener:
1. Select a card from the shaded deck. Create the sound described on the card using your hands or feet.	**1.** Actively listen to your partner and the sound they are making.
2. Select a card from the unshaded deck. Create the sound described on the card using your voice.	**2.** After your partner shares their sound, use these sentence stems to help clarify your partner's actions: "Can you recreate the sound of . . .?" "How did you create . . .?"
	3. Sketch a graph to interpret how the sound your partner created changes over time.

When you are the listener, use the space provided to sketch the graph that corresponds with each card. **Sample responses:**

Card 1:

Card 2:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 1 Music to Our Ears 381

1 Launch

Distribute the pre-cut cards from the Activity 1 PDF to each student pair. Say, "You will take turns making sounds and sketching a graph of what you think represents the change in that sound."

2 Monitor

Help students get started by modeling creating a sound with your hands, feet, or voice.

Look for points of confusion:

- **Representing the sounds graphically with points.** Ask, "How do the sounds change over time? Even if a sound occurs for a short period of time, how can these changes be represented on a graph?"

Look for productive strategies:

- Using lines and segments to sketch sounds.
- Creating segments at varying outputs to represent changes in pitch of sound.
- Creating increasing or decreasing lines for gradual changes in sound.

Activity 1 continued ›

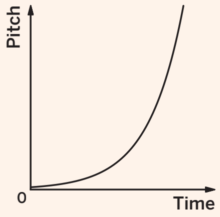
Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Instead of asking students to create graphs for each sound they hear, ask them to use hand gestures to show how the pitch changes over time. For example, for Card 1, students could hold their hand steady as time passes, but withdraw their hand every time they hear a gap in the sound.

Extension: *Math Enrichment*

Have students describe what the sound represented by this graph might sound like. Sample response: The pitch starts off low (slowly increasing) and then rapidly increases.



Math Language Development

MLR7: Compare and Connect

During the Connect, as students share how they created their graphs, listen for and amplify any mathematical language they use, such as *straight line segments, gaps, constant rate, increasing, decreasing, positive/negative slope, horizontal line,* etc. Highlight the connections across the different graphs.

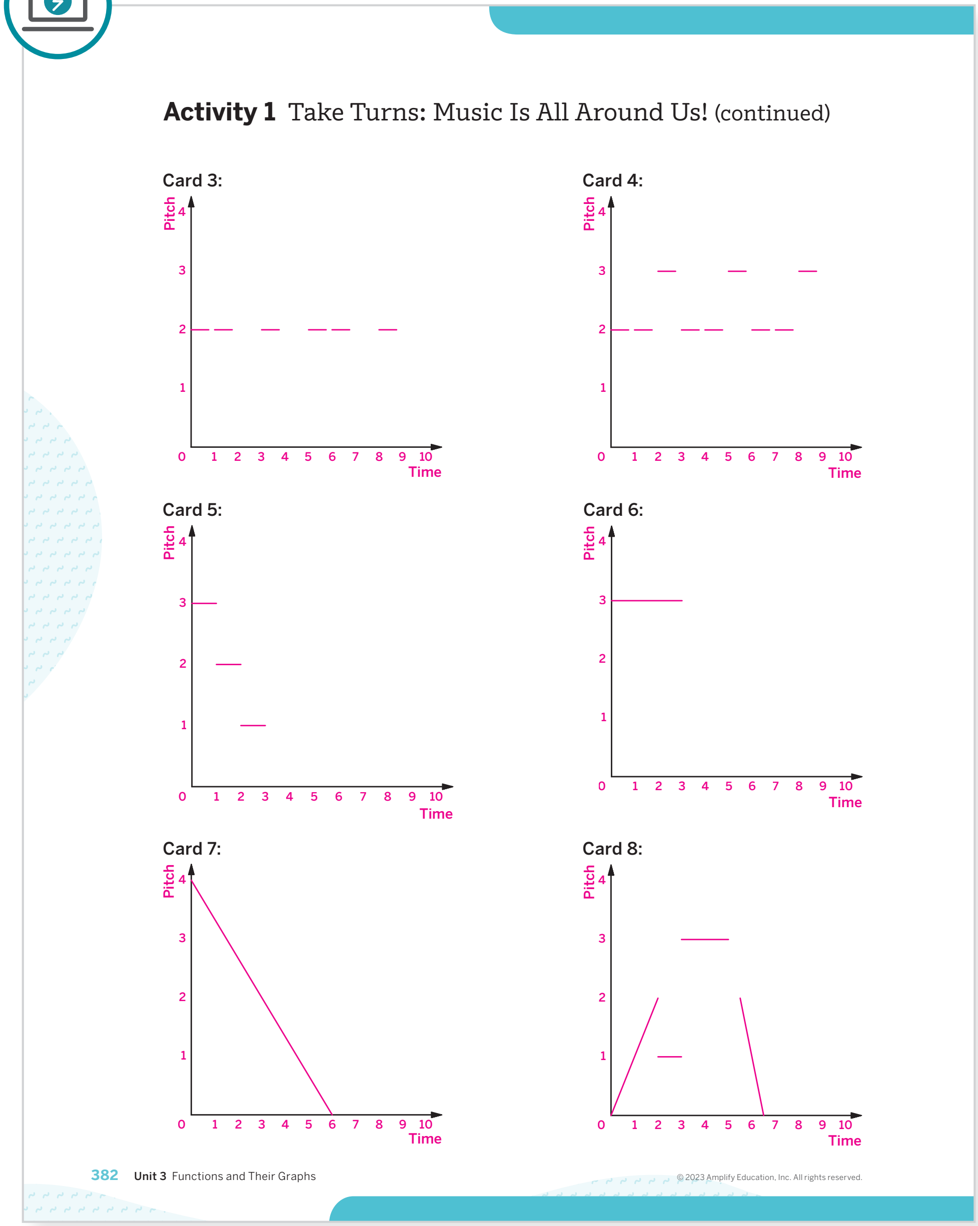
English Learners

Display an example of one of the graphs and annotate the graph using mathematical language.

Pairs | 20 min

Activity 1 Take Turns: Music is All Around Us! (continued)

Students take turns creating and sketching graphs of sounds they make to understand that graphs can model different scenarios.



Activity 1 Take Turns: Music Is All Around Us! (continued)

Card 3:

Card 4:

Card 5:

Card 6:

Card 7:

Card 8:

382 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display each card from the Activity 1 PDF.

Have pairs of students share their graphs for each card. Select and sequence student responses using points, line segments, and increasing or decreasing line segments.

Highlight that while it is not yet critical to be accurate with sketching graphs of scenarios, the changes the students hear everyday can be modeled mathematically and displayed graphically.

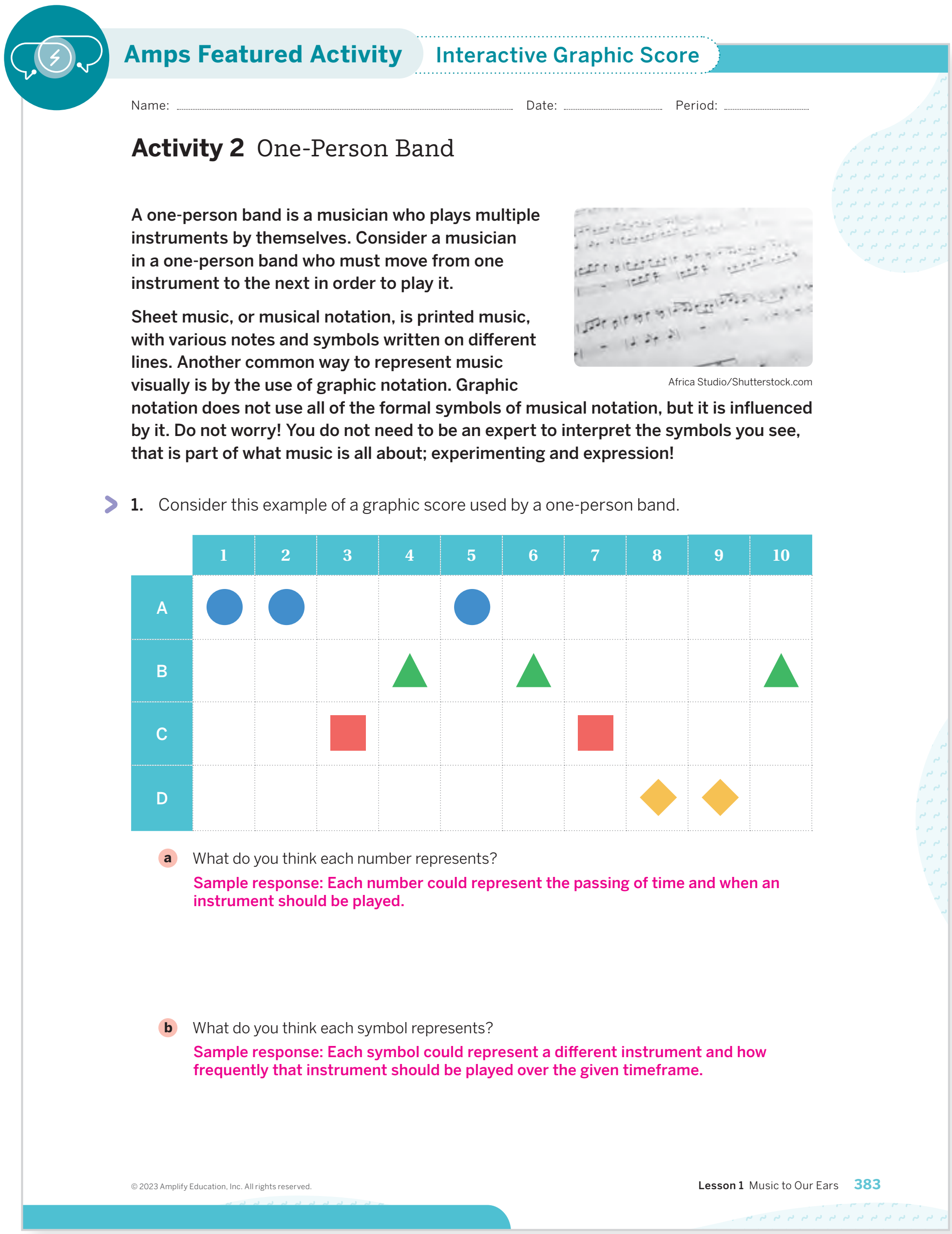
Ask, "What real-world uses might there be for creating mathematical representations of sound?"
Sample responses: Recreating music, digital music production, programming sounds.

Pairs | 15 min

Activity 2 One-Person Band

Students make observations about graphic scores in musical notation to make connections to functions.

Amps Featured Activity Interactive Graphic Score



Name: ______ Date: ______ Period: ______

Activity 2 One-Person Band

A one-person band is a musician who plays multiple instruments by themselves. Consider a musician in a one-person band who must move from one instrument to the next in order to play it.

Sheet music, or musical notation, is printed music, with various notes and symbols written on different lines. Another common way to represent music visually is by the use of graphic notation. Graphic notation does not use all of the formal symbols of musical notation, but it is influenced by it. Do not worry! You do not need to be an expert to interpret the symbols you see, that is part of what music is all about; experimenting and expression!

Africa Studio/Shutterstock.com

1. Consider this example of a graphic score used by a one-person band.

	1	2	3	4	5	6	7	8	9	10
A	●	●			●					
B				▲		▲				▲
C			■				■			
D								◆	◆	

a. What do you think each number represents?

Sample response: Each number could represent the passing of time and when an instrument should be played.

b. What do you think each symbol represents?

Sample response: Each symbol could represent a different instrument and how frequently that instrument should be played over the given timeframe.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Activate prior knowledge by asking, "Who has seen a one-person band? Is anyone familiar with sheet music?"

2 Monitor

Help students get started by asking "For the graphs of different sounds you have seen or created, what did each axis represent?"

Look for points of confusion:

- **Thinking each letter represents a different note.** Ask, "Given the context of a one-person band, what would that person be responsible for?"
- **Creating a graphic score where two different instruments are being played at the same time.** Ask, "Based on the video you saw, what are the challenges faced by a one-person band? What would be impossible for them to do?"

Look for productive strategies:

- Recognizing that each row could represent an instrument and each column a unit of time.
- Knowing that two instruments cannot be played at once.
- Creating a graphic score where there is only one instrument being played at any one moment in time.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Represent the given graphic scores in Problems 1 and 2 through different modalities. For example, provide students with four "instruments" they can use to represent each of the four symbols. The "instruments" could be as simple as the following:

- **Circle:** Clap your hands one time.
- **Triangle:** Snap your fingers one time.
- **Square:** Stomp your feet on time.
- **Rhombus:** Whistle one time (or say a word).

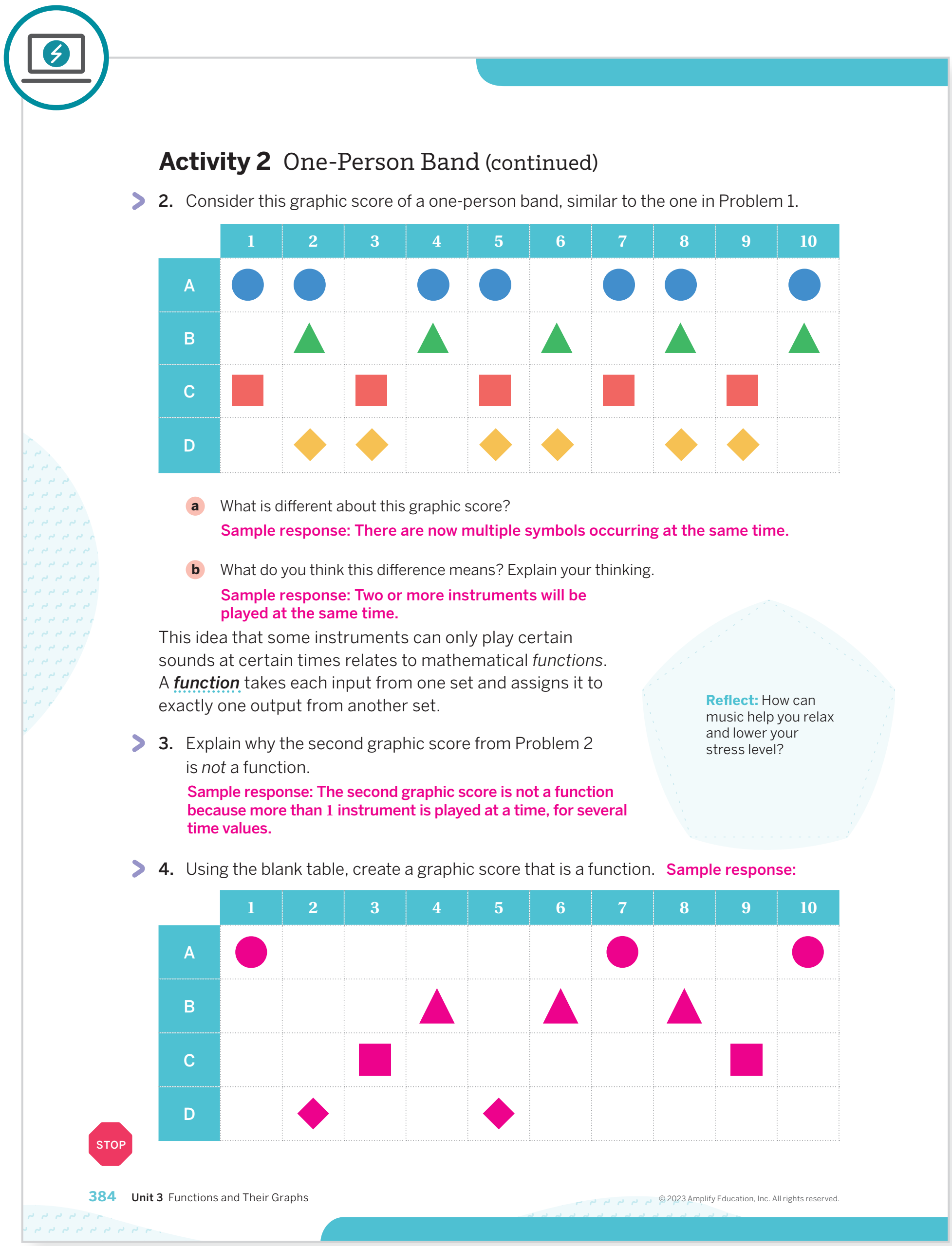
Accessibility: *Clarify Vocabulary and Symbols, Guide Processing and Visualization*

Annotate the graphic score in Problem 1 as "function" and the graphic score in Problem 2 as "not a function." Illustrate why the graphic score in Problem 1 is a function by having students note the number of shapes that are played at each time value (1 instrument is played at each time). Illustrate why the graphic score in Problem 2 is not a function by having students note that, for example, more than 1 instrument is played at a time, for several time values.

Pairs | 15 min

Activity 2 One-Person Band (continued)

Students make observations about graphic scores in musical notation to make connections to functions.



Activity 2 One-Person Band (continued)

2. Consider this graphic score of a one-person band, similar to the one in Problem 1.

	1	2	3	4	5	6	7	8	9	10
A	●	●		●	●		●	●		●
B		▲		▲		▲		▲		▲
C	■		■		■		■		■	
D		◆	◆		◆	◆		◆	◆	

a. What is different about this graphic score?
Sample response: There are now multiple symbols occurring at the same time.

b. What do you think this difference means? Explain your thinking.
Sample response: Two or more instruments will be played at the same time.

This idea that some instruments can only play certain sounds at certain times relates to mathematical *functions*. A ***function*** takes each input from one set and assigns it to exactly one output from another set.

Reflect: How can music help you relax and lower your stress level?

3. Explain why the second graphic score from Problem 2 is *not* a function.
Sample response: The second graphic score is not a function because more than 1 instrument is played at a time, for several time values.

4. Using the blank table, create a graphic score that is a function. **Sample response:**

	1	2	3	4	5	6	7	8	9	10
A	●						●			●
B				▲		▲		▲		
C			■						■	
D		◆			◆					

STOP

384 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the two graphic scores from Problems 1 and 2.

Have pairs of students share the graphic score they created and explain why it represents a function.

Highlight that the graphic scores students examined and created are one way to represent functions in this context. Functions play an important role in mathematics and will be explored further in this unit.

Ask, "What are some different ways you have seen functions represented?"
Sample response:
- Tables
- Graphs
- Verbal descriptions
- Equation

Whole Class | 5 min

Summary Artscapes

Review and synthesize how to define and represent functions of different scenarios.

Unit 3 Functions and Their Graphs

Artscapes

Music moves: from note-to-note, beat-to-beat. It moves through us, into us, out of us. There is the movement of air in a trumpeter's lungs before the blast, or the stamping of feet against a dance hall floor.

According to the philosopher Plato, "Music gives soul to the universe, wings to the mind, flight to the imagination, and life to everything." Music can lift us up when we are down, say the words we do not know how, and let us connect with a stadium full of people we have never met.

And yet, the music you know and love today did not just spring up fully formed. The jams on your playlist are the end result of hundreds of years and thousands upon thousands of miles of shifting cultural forces. To understand what gives our most beloved music its particular sound, we have to look at where that music came from.

There is no musical art form more quintessentially American than blues and jazz. Their legacy can still be found in rock, country, hip-hop, and R&B. But the story of blues music begins hundreds of years ago with the rhythms of the enslaved West African people, then to the Great Migration, where Black men, women, and children left the rural South to settle northward in America's growing cities. It was in these cities that their music gathered, mingled, and morphed.

In these next lessons, we will look at the way music such as blues and jazz were functions of the cities they have passed through, responding to the forces of place, time, people, and history.

Welcome to Unit 3.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 1 Music to Our Ears 385

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display a sketch of one of the graphs from Activity 1.

Have students share whether or not they think the graph is a function and why.

Highlight that music and sound can be represented graphically and mathematically. These representations can help students to visualize what they hear. Functions play an important role in math because they can help define rules for different relationships.

Ask, "How did you determine whether a graphic score was or was not a function?"
Sample response: If two instruments were being played at the same time by one person, this was not a function because it is not possible for one person to play two instruments at once.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. To help them engage in meaningful reflection, consider asking:

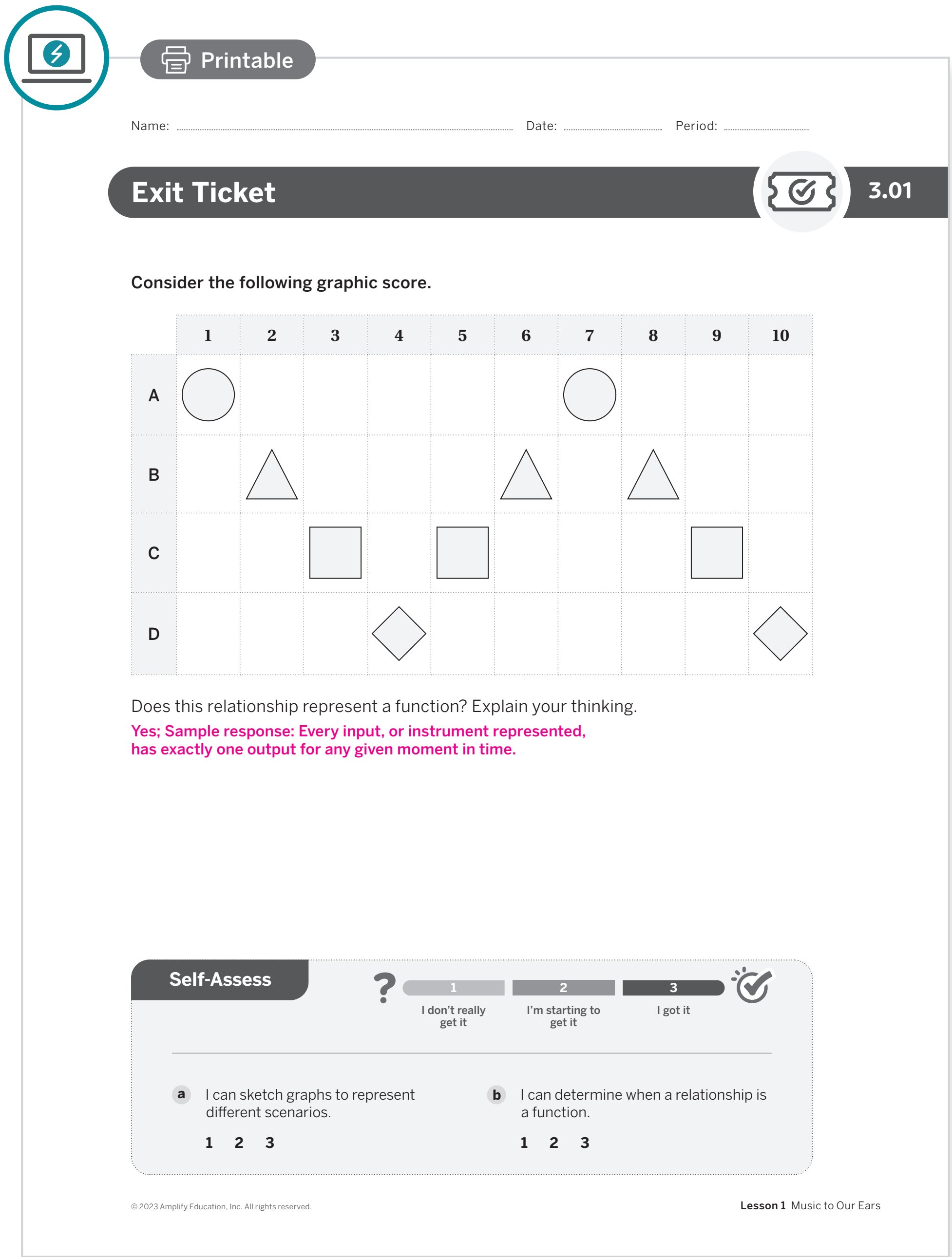
- "What is a function? How does this relate to music and sound?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining if a graphic score is a function.

Printable



Name: Date: Period:

Exit Ticket 3.01

Consider the following graphic score.

	1	2	3	4	5	6	7	8	9	10
A	○						○			
B		△				△		△		
C			□		□				□	
D				◇						◇

Does this relationship represent a function? Explain your thinking.

Yes; Sample response: Every input, or instrument represented, has exactly one output for any given moment in time.

Self-Assess

? 1 I don't really get it 2 I'm starting to get it 3 I got it

a I can sketch graphs to represent different scenarios.
1 2 3

b I can determine when a relationship is a function.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 1 Music to Our Ears

Success looks like . . .

- **Language Goal:** Sketching graphs to represent different scenarios. **(Writing)**
- **Language Goal:** Determining whether a relationship is a function. **(Reading and Writing)**
  - » Determining whether the graphic score represents a function.

Suggested next steps

If students determine the graphic score is not a function in Problem 1, consider:

- Reviewing graphic scores and functions from Activity 2.
- Assigning Practice Problem 2.
- Asking, "How did you determine if the graphic scores from Activity 2 were functions?"

Professional Learning

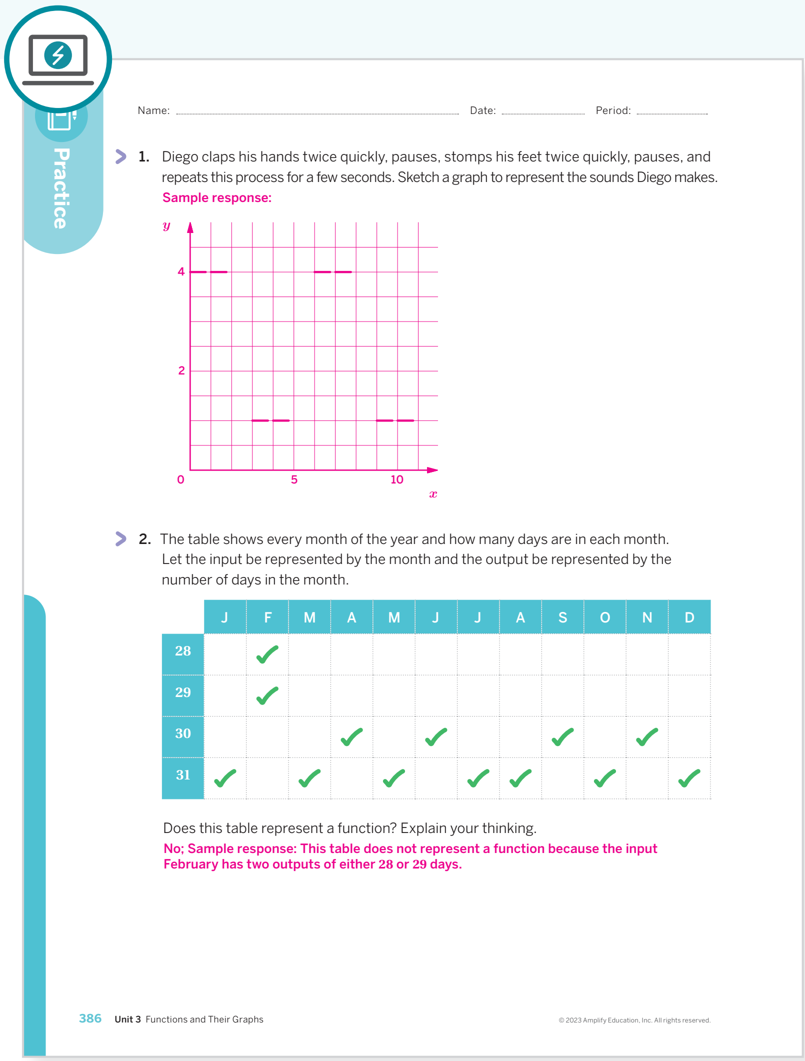
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did students find frustrating about Activity 2? What helped them work through this frustration?
- What surprised you as your students worked on Activity 1? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Diego claps his hands twice quickly, pauses, stomps his feet twice quickly, pauses, and repeats this process for a few seconds. Sketch a graph to represent the sounds Diego makes.
Sample response:

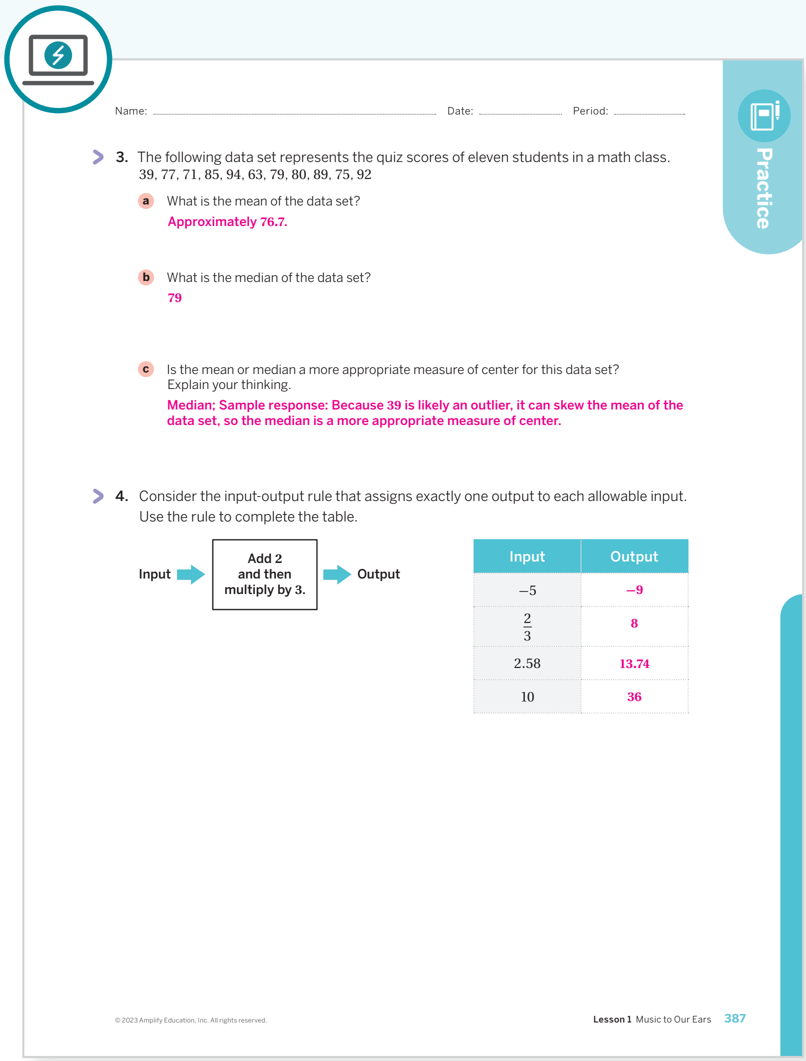
2. The table shows every month of the year and how many days are in each month. Let the input be represented by the month and the output be represented by the number of days in the month.

	J	F	M	A	M	J	J	A	S	O	N	D
28		✓										
29		✓										
30				✓		✓			✓		✓	
31	✓		✓		✓		✓	✓		✓		✓

Does this table represent a function? Explain your thinking.
No; Sample response: This table does not represent a function because the input February has two outputs of either 28 or 29 days.

386 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.



Name: Date: Period:

3. The following data set represents the quiz scores of eleven students in a math class.
39, 77, 71, 85, 94, 63, 79, 80, 89, 75, 92

a. What is the mean of the data set?
Approximately 76.7.

b. What is the median of the data set?
79

c. Is the mean or median a more appropriate measure of center for this data set? Explain your thinking.
Median; Sample response: Because 39 is likely an outlier, it can skew the mean of the data set, so the median is a more appropriate measure of center.

4. Consider the input-output rule that assigns exactly one output to each allowable input. Use the rule to complete the table.

Input	Output
-5	**-9**
$\frac{2}{3}$	**8**
2.58	**13.74**
10	**36**

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 1 Music to Our Ears 387

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	1
Spiral	3	Unit 2 Lesson 8	2
Formative	4	Unit 3 Lesson 2	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 1

Functions and Their Representations

Whole Class

In this Sub-Unit, students will use function notation to represent situations about the city of Memphis presented verbally, graphically, and tabularly.

SUB-UNIT

1 Functions and Their Representations

Narrative Connections

How did the blues find a home in Memphis?

Follow the Mississippi River long enough and you'll end up in the city of Memphis, Tennessee. For more than a hundred years, this city has been a waystation for countless blues legends.

This story begins after the Civil War. The city's cotton sellers had established a trade association, which allowed them to control the prices of cotton being sold in Memphis. This kept the city's economy strong, drawing in more workers from throughout the South. As more jobs came, so too did traveling performers. These performers exposed their Memphis audiences to the music that originated in rural Black communities.

By the early 1900s, vaudeville acts overtook the older performance acts. Places like Beale Street and Church Park blossomed, becoming centers for Black business and culture. Musicians brought the sounds of the country to the city. Work songs, ragtime, and country blues bursted in every Memphis theater, dance hall, and juke joint.

In the years to come, the city became home to performers like Memphis Minnie, Furry Lewis, Sleepy John Estes, and the "Father of the Blues" himself: W.C. Handy. In 1912, Handy composed "The Memphis Blues," a 12-bar musical composition set down in sheet music. It was this form and structure that would inspire blues players for generations to come.

It's not just music that has its own notation. As you'll see in the next few lessons, the same goes for functions. Just as a musician can write and play from sheet music, a mathematician can concisely write a function and even "play" it, by analyzing its structure and studying its graph.

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore functions — within the context of music and the city of Memphis — in the following places:

- **Lesson 2, Activities 1–2:** Going to the Museum, Making Music
- **Lesson 3, Activity 2:** Name That Song
- **Lesson 5, Activity 2:** The Memphis Pyramid
- **Lesson 6, Activity 1:** A Steady Pace

Describing and Graphing Situations

Let's explore different ways to represent a relationship between two quantities.

Focus

Goals

1. Understand that a relationship between two quantities is a function if there is only one possible output value for each input value.
2. **Language Goal:** Interpret descriptions and graphs of functions in context. **(Reading and Writing)**
3. **Language Goal:** Use words and graphs to represent relationships that are functions, including identifying the independent and dependent variables. **(Reading and Writing)**

Coherence

Today

Students identify independent and dependent variables to determine when a real-world scenario can be modeled with a function and when it cannot. They analyze and sketch functions in context as they look for and explain connections between verbal descriptions and graphs.

Previously

In Lesson 1, students sketched graphs of different relationships and determined when these relationships were functions.

Coming Soon

Students will talk about functions more formally in the upcoming Lessons 3 and 4 by learning to use and interpret function notation in different situations.

Rigor

- Students reason about the relationship between two variables in different scenarios and use verbal descriptions and graphs to build **conceptual understanding** of functions.

Pacing Guide

Suggested Total Lesson Time **~50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Notice and Wonder* (as needed)
- scissors
- straws
- tape

Math Language Development

Review words

- *dependent variable*
- *function*
- *independent variable*

Amps | Featured Activity

Activity 2
Interactive Graphs

Students will be able to move a slider to see how the pitch changes when a person plays an instrument. This allows them to experiment with changing the dependent variable to see how this affects the graph.

Building Math Identity and Community

Connecting to Mathematical Practices

Because the analysis of graphs is a new skill, students might have quite different interpretations or thoughts as they share with their partners. Encourage students to pre-plan how they will handle any conflict resolution and stay focused on the goal of helping each other learn. Discuss conflict negotiation skills and remind them that we can all learn from mistakes, too.

Modifications to Pacing

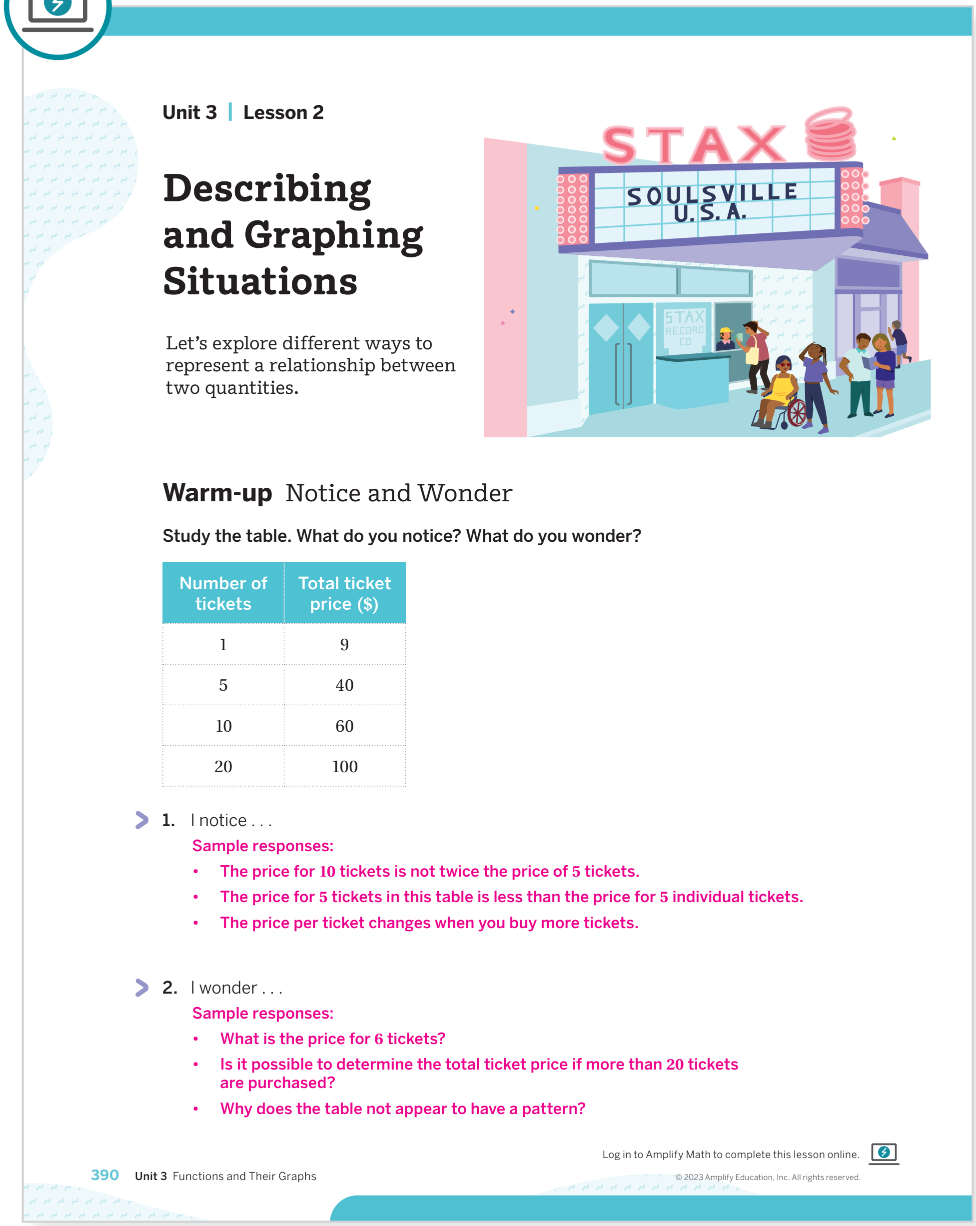
You may want to consider these additional modifications if you are short on time.

- The **Warm-up** may be omitted.
- In **Activity 2**, Description 3 may be omitted.

Independent | 5 min

Warm-up Notice and Wonder

Students examine a table of input and output values to notice the possible relationships between the two quantities.



Unit 3 | Lesson 2

Describing and Graphing Situations

Let's explore different ways to represent a relationship between two quantities.

Warm-up Notice and Wonder

Study the table. What do you notice? What do you wonder?

Number of tickets	Total ticket price ($)
1	9
5	40
10	60
20	100

1. I notice . . .
   Sample responses:
   - The price for 10 tickets is not twice the price of 5 tickets.
   - The price for 5 tickets in this table is less than the price for 5 individual tickets.
   - The price per ticket changes when you buy more tickets.

2. I wonder . . .
   Sample responses:
   - What is the price for 6 tickets?
   - Is it possible to determine the total ticket price if more than 20 tickets are purchased?
   - Why does the table not appear to have a pattern?

Log in to Amplify Math to complete this lesson online.

390 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Give the students a minute of think-time to study the table. Have students complete the problems independently and tell them there are no wrong answers. Conduct the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by asking, "What is the price per ticket for the different ticket packages?"

Look for points of confusion:

- **Thinking that there is exactly one price for each number of tickets.** Ask students if it is possible to calculate the price of 6 tickets in more than one way.

Look for productive strategies:

- Comparing the price using the single rate and the group rates.

3 Connect

Have students share what they noticed and wondered. Encourage students to point out contradicting information.

Highlight that the relationship between the number of tickets and their price does not form a function. Review the definition of a *function*. Say, "A function assigns exactly one output value for each input value."

Ask, "Is the relationship between the number of tickets and the ticket price considered a function?" Sample response: No, it is not a function because some input values can have more than one possible output value.

Math Language Development

MLR5: Co-craft Questions

After students complete Problems 1 and 2, have them meet with a partner to write 2–3 mathematical questions about the values shown in the table. The collaboration will help them consider other aspects of the relationships in the table they might not have considered on their own.

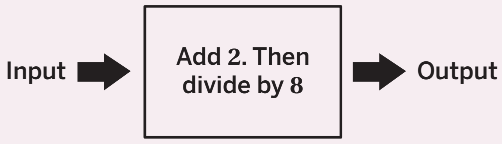
English Learners

Display or provide the Anchor Chart PDF, *Sentence Stems, Notice and Wonder* to support students in structuring their responses. Allow students to rehearse what they will say before sharing with the whole class.

Power-up

To power up students' ability to determine input-output values, have students complete:

For the given function machine, determine which expression represents how to calculate the output for an input of 6.



[✓ **A.**] $(6 + 2) \div 8$

B. $6 + 2 \div 8$

C. $2 + 6 \div 8$

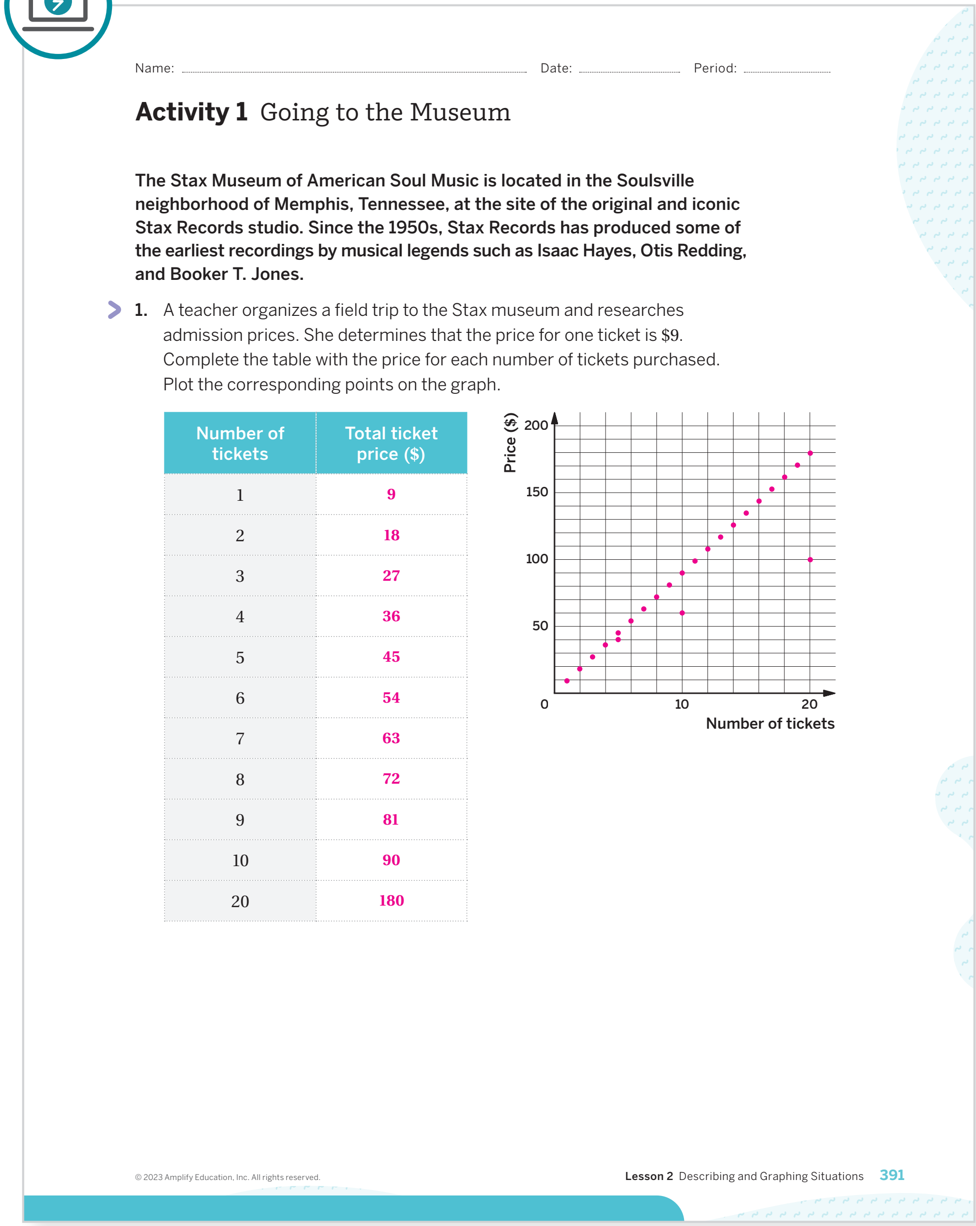
D. $6 \div 8 + 2$

Use: Before Activity 1
Informed by: Performance on Lesson 1, Practice Problem 4 and Pre-Unit Readiness Assessment, Problem 2

Pairs | 10 min

Activity 1 Going to the Museum

Students analyze a situation using a table, verbal description, and graph to determine whether a relationship is a function. They critique the reasoning of others to solve a problem.



Name: ______________________ Date: __________ Period: __________

Activity 1 Going to the Museum

The Stax Museum of American Soul Music is located in the Soulsville neighborhood of Memphis, Tennessee, at the site of the original and iconic Stax Records studio. Since the 1950s, Stax Records has produced some of the earliest recordings by musical legends such as Isaac Hayes, Otis Redding, and Booker T. Jones.

1. A teacher organizes a field trip to the Stax museum and researches admission prices. She determines that the price for one ticket is $9. Complete the table with the price for each number of tickets purchased. Plot the corresponding points on the graph.

Number of tickets	Total ticket price ($)
1	9
2	18
3	27
4	36
5	45
6	54
7	63
8	72
9	81
10	90
20	180

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the prompt as a class. Have students work independently before sharing their thinking with a partner.

2 Monitor

Help students get started by asking, "How much would it cost to purchase 5 tickets?"

Look for points of confusion:

- **Having difficulty determining the total ticket price.** Tell students to repeatedly add the value of a single ticket to determine the total ticket price.

Look for productive strategies:

- Extending the table by writing the prices for individual tickets using the group rates.
- Listing quantities in the table as points and relating them to the context.

Activity 1 continued ›

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they have ever received a discount on group tickets (or other bulk items) for buying a certain quantity. Let them know that in this task, they will explore the prices for individual tickets versus a package price.

Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students complete the table and plot the points in Problem 1, provide them with a pre-completed table and graph. Have them begin the activity with the text preceding Problem 2.

Math Language Development

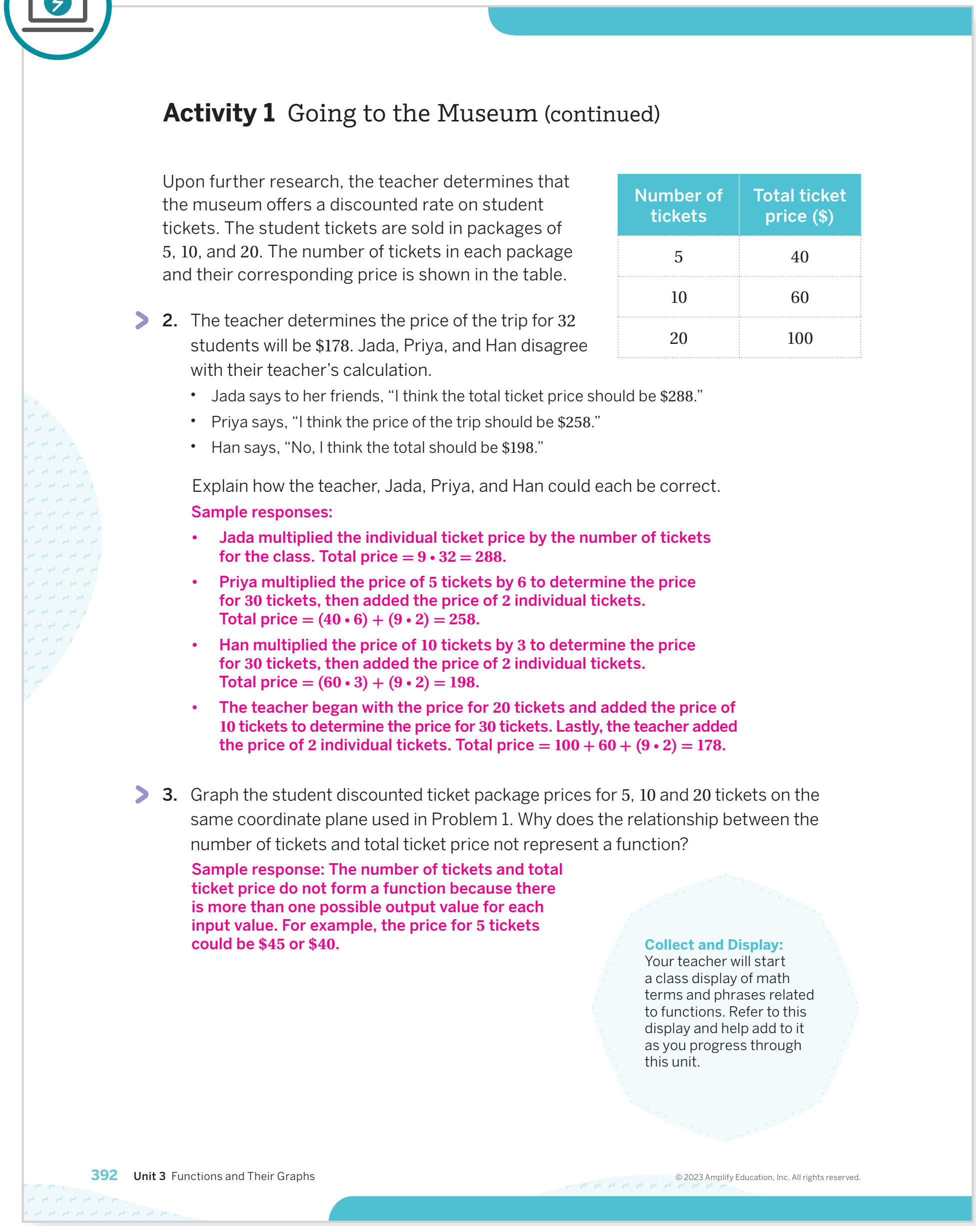
MLR2: Collect and Display

Start a class display of math terms and phrases related to functions that students can refer to for the remainder of this unit. For this lesson, begin the display by writing the term *function*, along with its definition. Include a visual example of a table in which the relationship is a function and one in which the relationship is not a function.

Pairs | 10 min

Activity 1 Going to the Museum (continued)

Students analyze a situation using a table, verbal description, and graph to determine whether a relationship is a function. They critique the reasoning of others to solve a problem.



Activity 1 Going to the Museum (continued)

Upon further research, the teacher determines that the museum offers a discounted rate on student tickets. The student tickets are sold in packages of 5, 10, and 20. The number of tickets in each package and their corresponding price is shown in the table.

Number of tickets	Total ticket price ($)
5	40
10	60
20	100

2. The teacher determines the price of the trip for 32 students will be $178. Jada, Priya, and Han disagree with their teacher's calculation.
   - Jada says to her friends, "I think the total ticket price should be $288."
   - Priya says, "I think the price of the trip should be $258."
   - Han says, "No, I think the total should be $198."

   Explain how the teacher, Jada, Priya, and Han could each be correct.

   Sample responses:
   - **Jada multiplied the individual ticket price by the number of tickets for the class. Total price $= 9 \cdot 32 = 288$.**
   - **Priya multiplied the price of 5 tickets by 6 to determine the price for 30 tickets, then added the price of 2 individual tickets. Total price $= (40 \cdot 6) + (9 \cdot 2) = 258$.**
   - **Han multiplied the price of 10 tickets by 3 to determine the price for 30 tickets, then added the price of 2 individual tickets. Total price $= (60 \cdot 3) + (9 \cdot 2) = 198$.**
   - **The teacher began with the price for 20 tickets and added the price of 10 tickets to determine the price for 30 tickets. Lastly, the teacher added the price of 2 individual tickets. Total price $= 100 + 60 + (9 \cdot 2) = 178$.**

3. Graph the student discounted ticket package prices for 5, 10 and 20 tickets on the same coordinate plane used in Problem 1. Why does the relationship between the number of tickets and total ticket price not represent a function?

   Sample response: The number of tickets and total ticket price do not form a function because there is more than one possible output value for each input value. For example, the price for 5 tickets could be $45 or $40.

Collect and Display: Your teacher will start a class display of math terms and phrases related to functions. Refer to this display and help add to it as you progress through this unit.

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have student pairs share their responses and thinking. Select and sequence student pairs that used productive strategies.

Highlight that there is more than one possible price, or output, for an input of 32 tickets, so the relationship is not a function.

Ask:

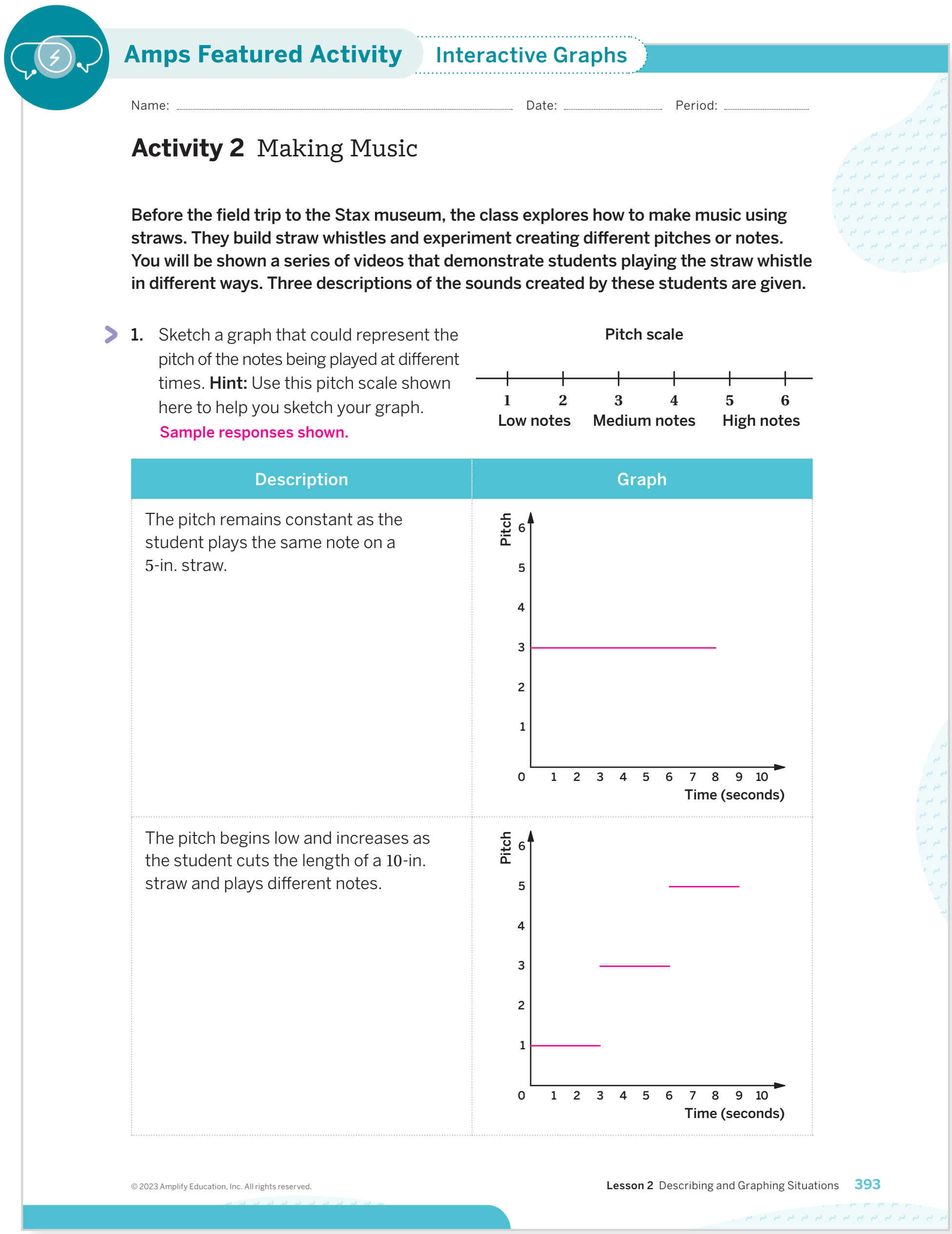
- "What do the points (5, 45) and (5, 40) represent in this context?" Purchasing 5 tickets could cost $45 or $40.
- "How does the graph show that the relationship between the quantities is not a function?" There are two output values for some of the input values.

Pairs | 15 min

Activity 2 Making Music

Students sketch functions to model verbal descriptions of how the pitch of a sound changes over time.

Amps Featured Activity Interactive Graphs



Name: Date: Period:

Activity 2 Making Music

Before the field trip to the Stax museum, the class explores how to make music using straws. They build straw whistles and experiment creating different pitches or notes. You will be shown a series of videos that demonstrate students playing the straw whistle in different ways. Three descriptions of the sounds created by these students are given.

1. Sketch a graph that could represent the pitch of the notes being played at different times. **Hint:** Use this pitch scale shown here to help you sketch your graph.
   Sample responses shown.

Description	Graph
The pitch remains constant as the student plays the same note on a 5-in. straw.	
The pitch begins low and increases as the student cuts the length of a 10-in. straw and plays different notes.	

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the scenario together as a class and display a video of students playing straw whistles. Explain how the pitch scale corresponds to musical notes. Have students work independently for a few minutes on each description before sharing their thoughts with a partner.

2 Monitor

Help students get started by suggesting students sing to model how pitch can change over time.

Look for points of confusion:

- **Drawing an increasing or decreasing graph for a constant pitch.** Ask students to graph consecutive points and describe how the pitch changes over time.
- **Graphing points instead of line segments.** Show students the video again and ask them to identify how long each pitch is playing. Ask, "What do you think a point would sound like?"

Look for productive strategies:

- Noticing the input is labeled along the horizontal axis, and the output is labeled along the vertical axis.
- Referring to the slope and associating it with line direction.
- Describing the relationship between pitch and time.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can move a slider to see how the pitch changes when a person plays an instrument. This will allow them to experiment with changing the dependent variable and see the effects on the graph.

Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students sketch the graphs, provide a set of pre-drawn graphs and have students match them to their corresponding descriptions.

Math Language Development

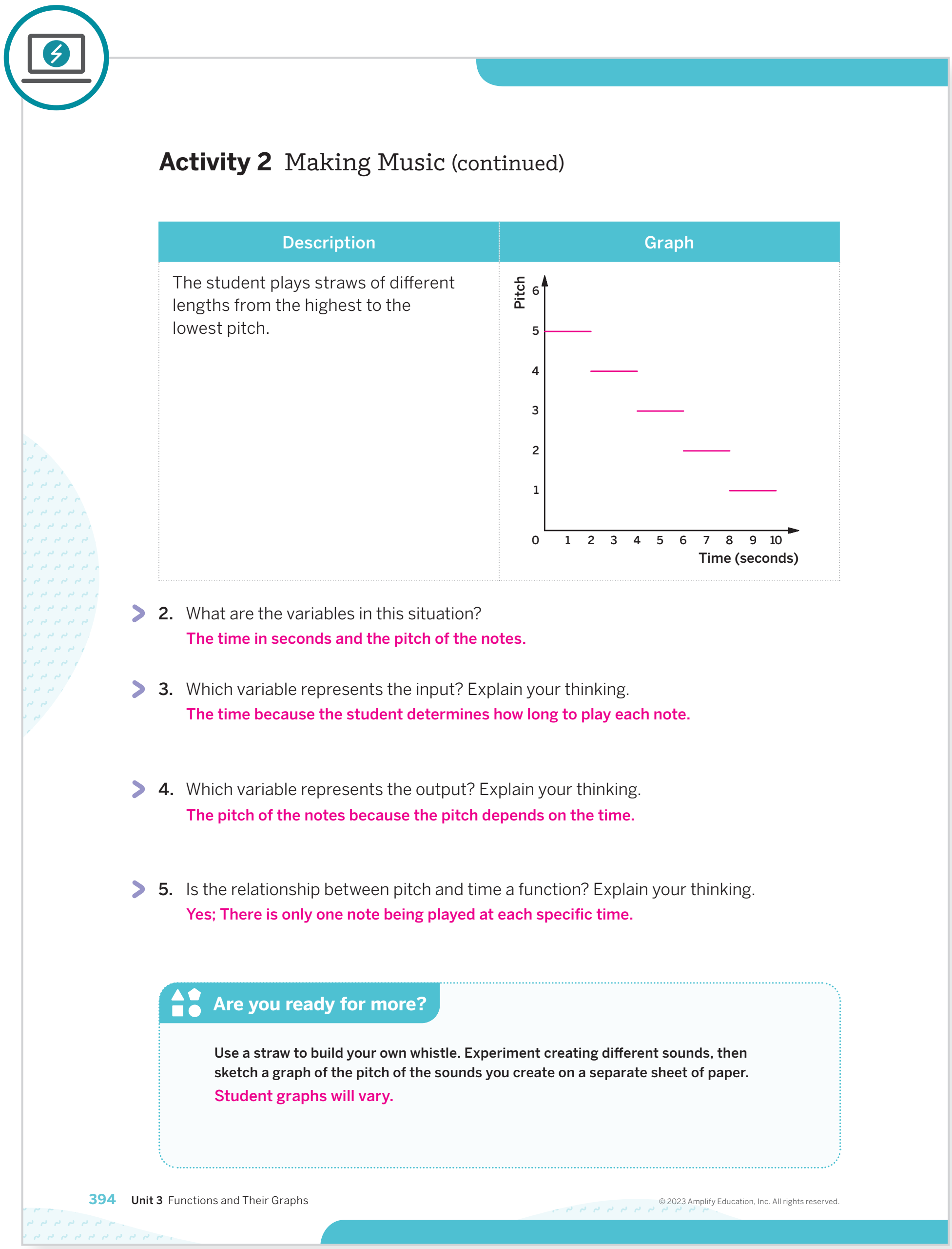
MLR2: Collect and Display

While students work and during the Connect, listen for and collect the language they use to make the connections between verbal descriptions and graphs. Add these words and phrases to the visual display you started in the previous activity. Include sample diagrams to illustrate words such as *constant, increases, decreases, input,* and *output*.

Pairs | 15 min

Activity 2 Making Music (continued)

Students sketch functions to model verbal descriptions of how the pitch of a sound changes over time.



Activity 2 Making Music (continued)

Description	Graph
The student plays straws of different lengths from the highest to the lowest pitch.	

2. What are the variables in this situation?
The time in seconds and the pitch of the notes.

3. Which variable represents the input? Explain your thinking.
The time because the student determines how long to play each note.

4. Which variable represents the output? Explain your thinking.
The pitch of the notes because the pitch depends on the time.

5. Is the relationship between pitch and time a function? Explain your thinking.
Yes; There is only one note being played at each specific time.

Are you ready for more?

Use a straw to build your own whistle. Experiment creating different sounds, then sketch a graph of the pitch of the sounds you create on a separate sheet of paper.
Student graphs will vary.

394 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have pairs of students share their graphs for each description with the class.

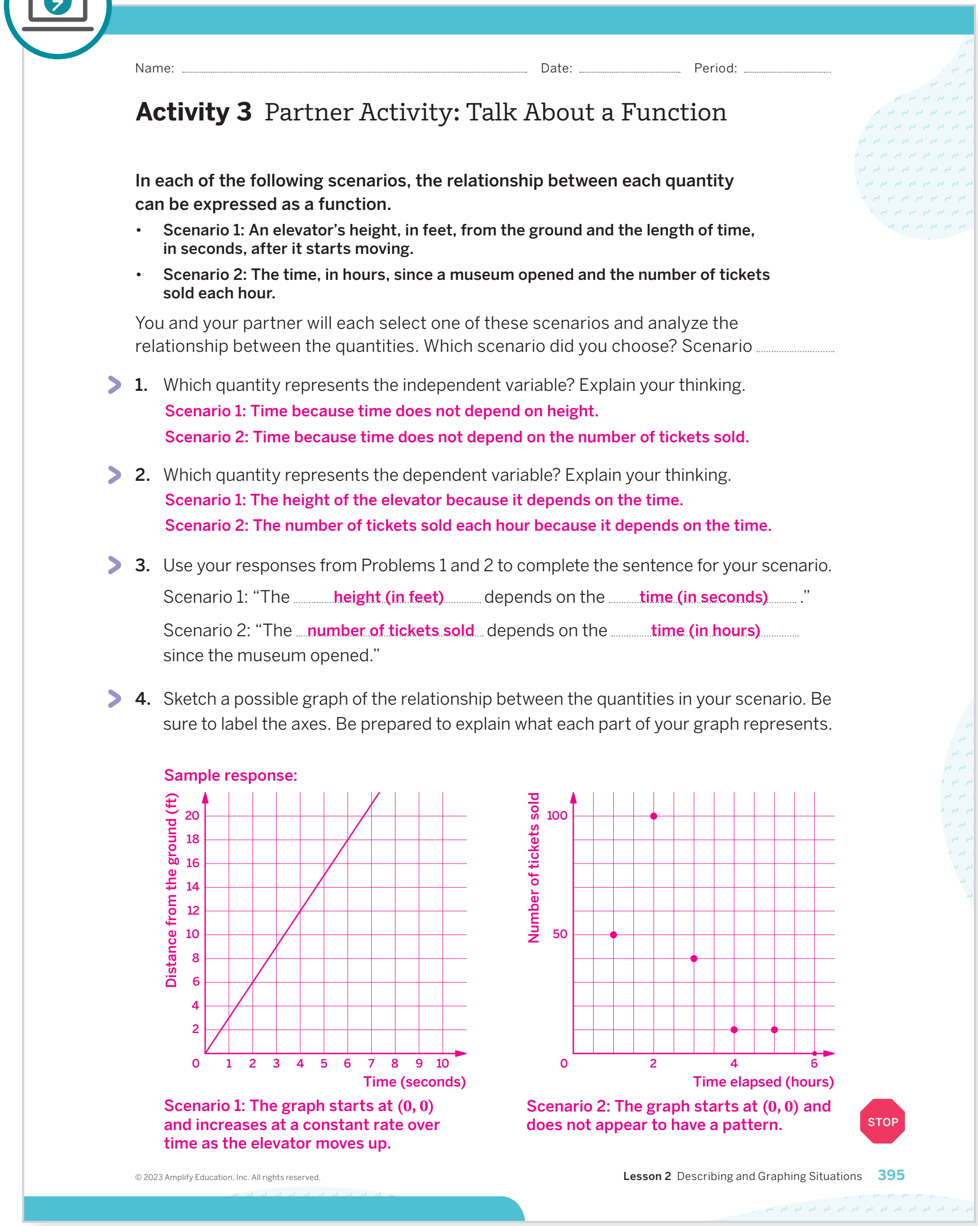
Display an interactive graph that shows how a pitch changes when a student plays different straw whistles.

Highlight that one way to represent and visualize a function is with a graph.

Pairs | 10 min

Activity 3 Partner Activity: Talk About a Function

Students determine independent and dependent variables to model verbal descriptions of real-world scenarios and represent the relationships graphically.



Name: .... Date: .... Period: ....

Activity 3 Partner Activity: Talk About a Function

In each of the following scenarios, the relationship between each quantity can be expressed as a function.

- **Scenario 1: An elevator's height, in feet, from the ground and the length of time, in seconds, after it starts moving.**
- **Scenario 2: The time, in hours, since a museum opened and the number of tickets sold each hour.**

You and your partner will each select one of these scenarios and analyze the relationship between the quantities. Which scenario did you choose? Scenario ....

1. Which quantity represents the independent variable? Explain your thinking.
   Scenario 1: Time because time does not depend on height.
   Scenario 2: Time because time does not depend on the number of tickets sold.

2. Which quantity represents the dependent variable? Explain your thinking.
   Scenario 1: The height of the elevator because it depends on the time.
   Scenario 2: The number of tickets sold each hour because it depends on the time.

3. Use your responses from Problems 1 and 2 to complete the sentence for your scenario.

   Scenario 1: "The **height (in feet)** depends on the **time (in seconds)**."

   Scenario 2: "The **number of tickets sold** depends on the **time (in hours)** since the museum opened."

4. Sketch a possible graph of the relationship between the quantities in your scenario. Be sure to label the axes. Be prepared to explain what each part of your graph represents.

Sample response:

Scenario 1: The graph starts at $(0, 0)$ and increases at a constant rate over time as the elevator moves up.

Scenario 2: The graph starts at $(0, 0)$ and does not appear to have a pattern.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 2 Describing and Graphing Situations 395

1 Launch

Say, "You and your partner will each choose a scenario to analyze." Have students work independently before sharing their thinking with their partner.

2 Monitor

Help students get started by asking them which quantity depends on the other in each scenario.

Look for points of confusion:

- **Misrepresenting the variables or mislabeling the axes of the graphs.** Remind students that the input value is the independent variable labeled on the horizontal axis while the output value is the dependent variable labeled on the vertical axis.
- **Using a scale that is not realistic.** Ask students to describe an interval of reasonable values for the input and output.

Look for productive strategies:

- Plotting specific pairs of input and output values.
- Determining whether or not it makes sense to connect the points.

3 Connect

Have individual students share their scenario, display their graph, and explain what each part of the graph represents.

Highlight how the graphs describe the change in the dependent variable over time.

Ask, "As the independent variable increases, does the dependent variable increase, decrease, or stay the same?" For Scenario 1, it depends on whether the elevator is going up or going down. For Scenario 2, it is reasonable to assume that the number of tickets sold decreases in the afternoon.

Differentiated Support

Accessibility: *Activate Prior Knowledge, Clarify Vocabulary and Symbols*

Remind students they learned about independent and dependent variables in middle school. Review with them what these terms mean by asking:

- "What does it mean to be independent?"
- "What does it mean to be dependent upon someone else or something else?"
- "How can you use these everyday meanings to help you remember what an independent or dependent variable is?"

Remind students that the independent variable is typically labeled along the horizontal axis of a graph, while the dependent variable is typically labeled along the vertical axis of a graph.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students respond to the Ask question, focus their attention on interpreting the graphs they drew within the context of the scenario. Display these sentence frames to help organize their thinking:

- "In Scenario 1, as the independent variable increases, the dependent variable ______. This makes sense because . . ."
- "In Scenario 2, as the independent variable increases, the dependent variable ______. This makes sense because . . ."

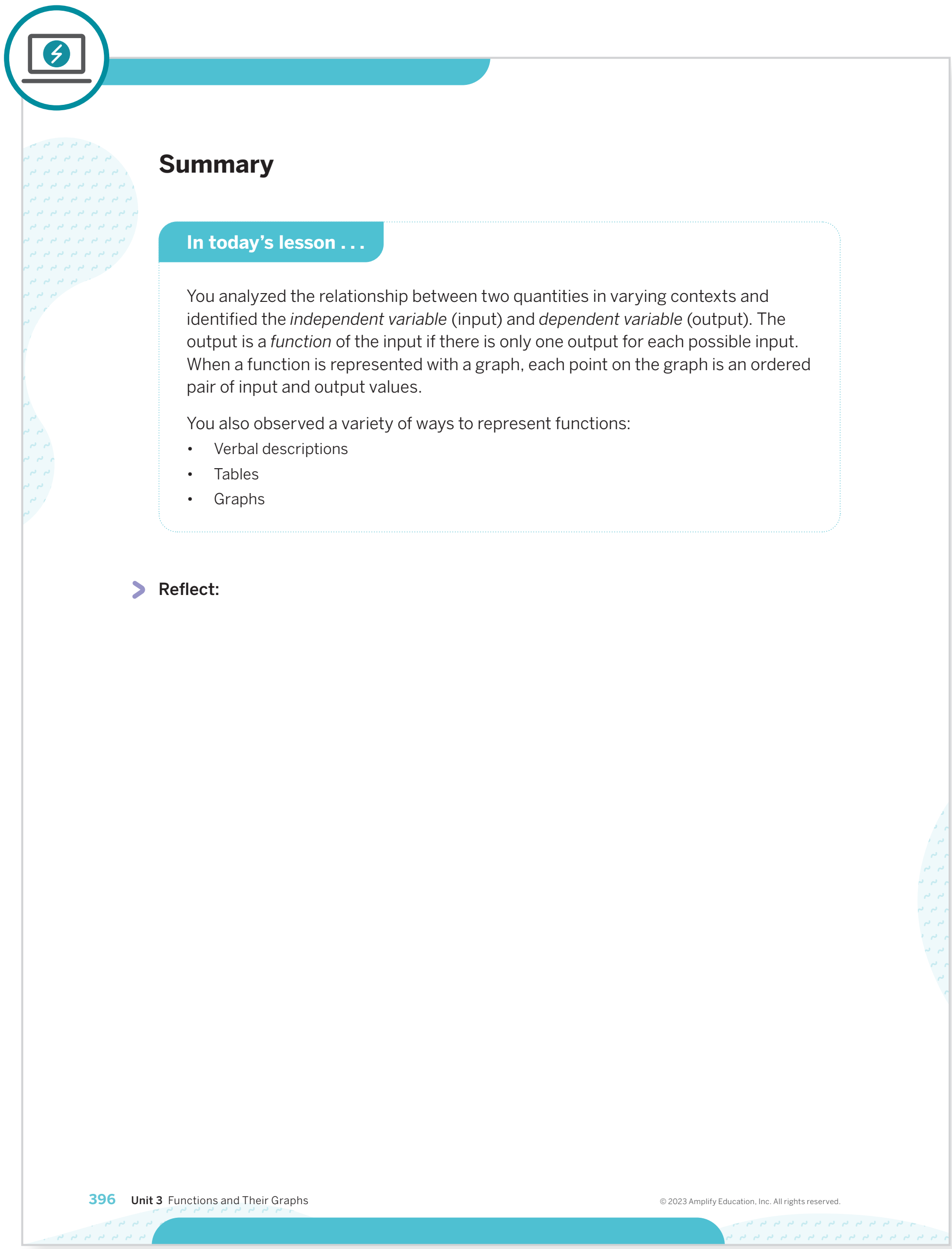
English Learners

Have students color code the independent variables in each scenario and graph labels in one color and the dependent variables in another color.

Whole Class | 5 min

Summary

Review and synthesize relationships in context and represent functions with words and graphs.



Summary

In today's lesson . . .

You analyzed the relationship between two quantities in varying contexts and identified the *independent variable* (input) and *dependent variable* (output). The output is a *function* of the input if there is only one output for each possible input. When a function is represented with a graph, each point on the graph is an ordered pair of input and output values.

You also observed a variety of ways to represent functions:

- Verbal descriptions
- Tables
- Graphs

Reflect:

396 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the graphs from Activity 2.

Highlight that it is important to identify the input and output variables in context to determine how one depends on the other. When connecting graphs to descriptions of functions, it is helpful to plot points or make tables. Functions are important relationships that are often used to model problems and make predictions.

Ask:

- "Can time be a function of pitch?" No; Time does not depend on pitch, so it cannot be a function of pitch.
- "Why are relationships represented in multiple ways?" There are advantages to tables, graphs, and verbal descriptions when working with functions.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

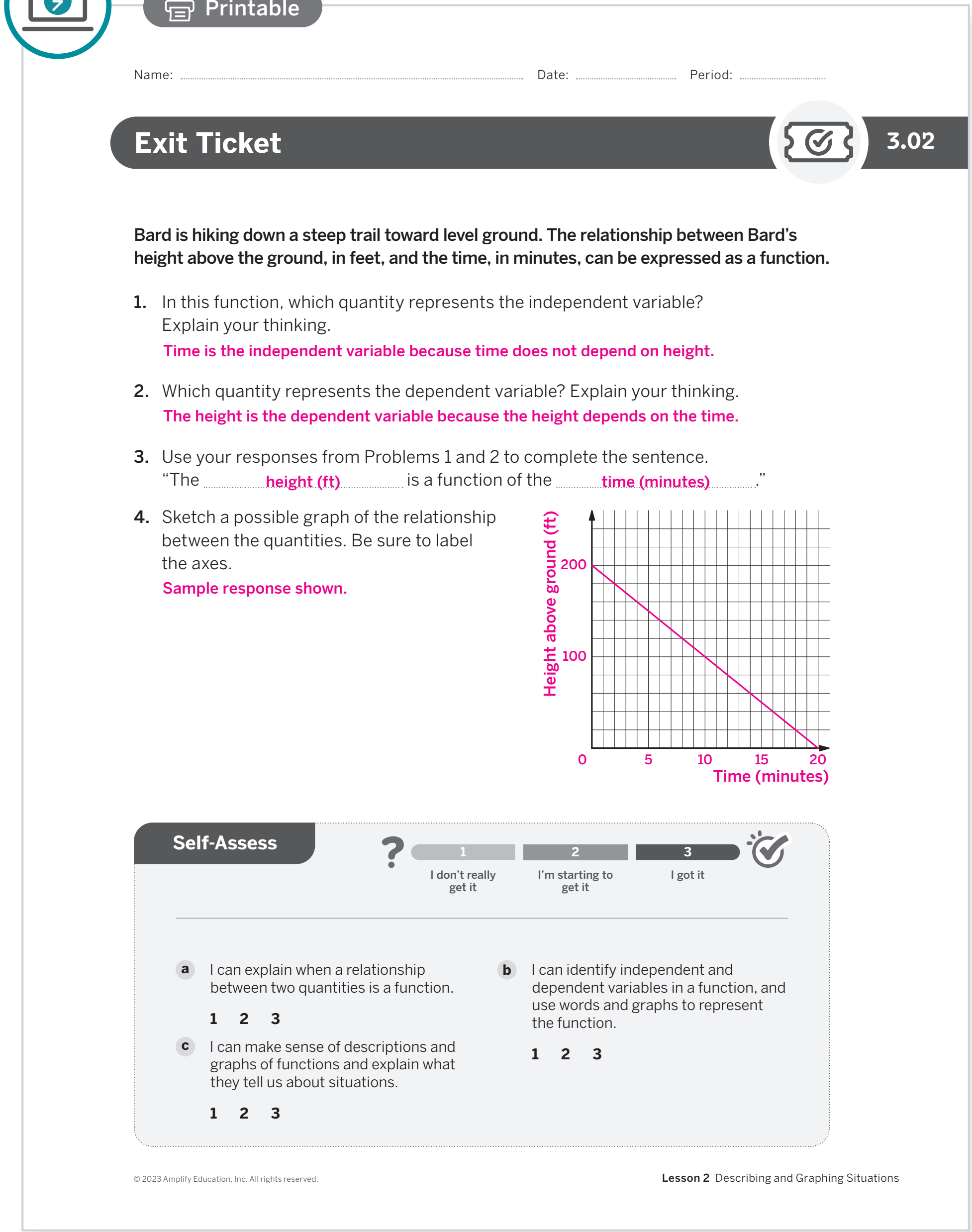
- "Why should functions be analyzed graphically?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of functions by describing a relationship that represents real-world quantities.

Printable



Name: ______________ Date: ______________ Period: ______________

Exit Ticket 3.02

Bard is hiking down a steep trail toward level ground. The relationship between Bard's height above the ground, in feet, and the time, in minutes, can be expressed as a function.

1. In this function, which quantity represents the independent variable? Explain your thinking.
   Time is the independent variable because time does not depend on height.
2. Which quantity represents the dependent variable? Explain your thinking.
   The height is the dependent variable because the height depends on the time.
3. Use your responses from Problems 1 and 2 to complete the sentence.
   "The **height (ft)** is a function of the **time (minutes)**."
4. Sketch a possible graph of the relationship between the quantities. Be sure to label the axes.
   Sample response shown.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can explain when a relationship between two quantities is a function.
1 2 3

b. I can identify independent and dependent variables in a function, and use words and graphs to represent the function.
1 2 3

c. I can make sense of descriptions and graphs of functions and explain what they tell us about situations.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 2 Describing and Graphing Situations

Success looks like . . .

- **Goal:** Understanding that a relationship between two quantities is a function if there is only one possible output for each input.
- **Language Goal:** Interpreting descriptions and graphs of functions in context. **(Reading and Writing)**
- **Language Goal:** Using words and graphs to represent relationships that are functions, including identifying the independent and dependent variables. **(Reading and Writing)**
  - » Determining the independent and dependent variables of the function from the context in Problems 1 and 2.

Suggested next steps

If students cannot identify independent and dependent variables in Problems 1 and 2, consider:

- Making their thinking visible by drawing arrows to map specific input values to their corresponding output value.
- Asking, "Which quantity depends on the other?"
- Assigning Practice Problem 1a.

If students cannot relate a function's graph to its description in Problem 3, consider:

- Asking, "Which axis represents the input and which axis represents the output?"
- Making a table or set of ordered pairs in which each point on the graph is a specific pair of input and output values.
- Reviewing Activity 3.
- Assigning Practice Problem 1.

Professional Learning

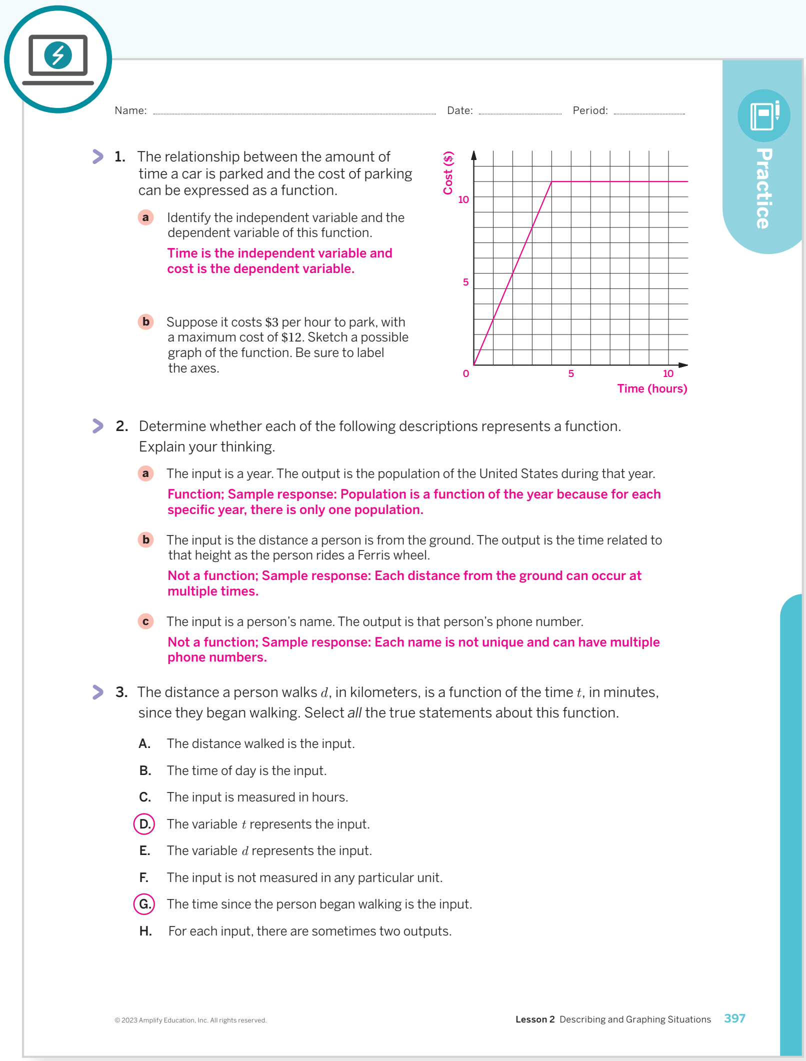
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What challenges did students encounter as they worked on Activity 3? How did they work through them? What might you change for the next time you teach this lesson?
- In this lesson, students continued to develop the concept of a function. How will that support a more formal study of function notation?

Practice

Independent

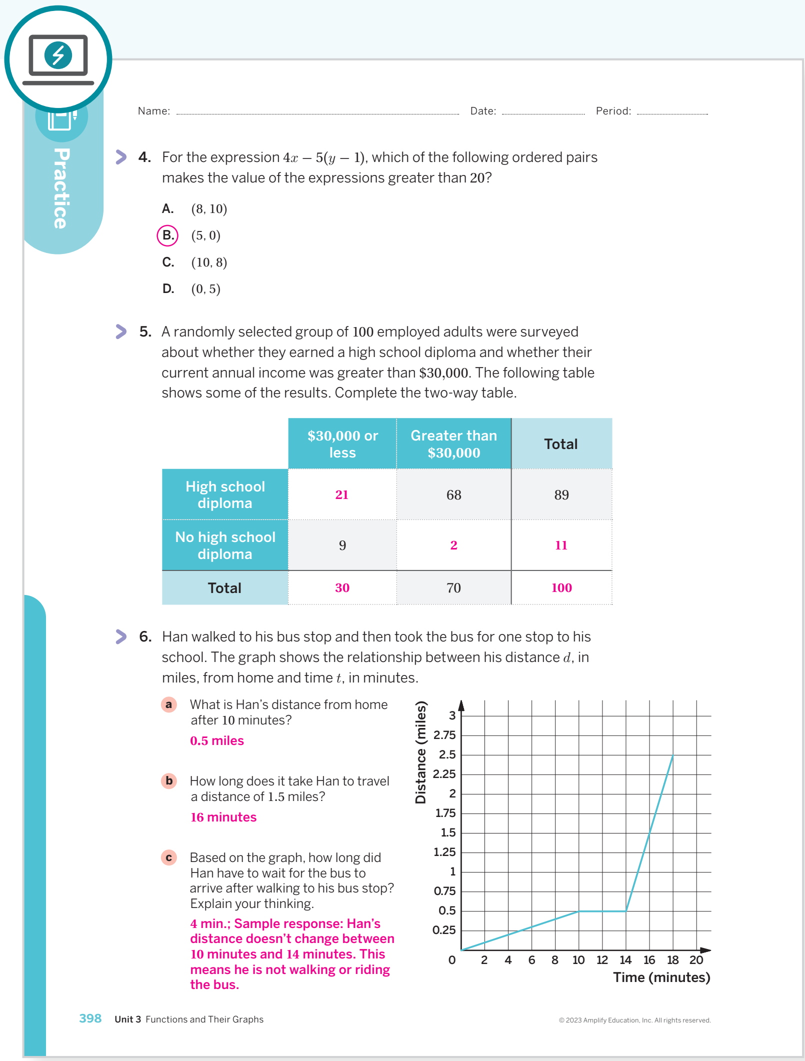


Name: Date: Period:

1. The relationship between the amount of time a car is parked and the cost of parking can be expressed as a function.
   - **a** Identify the independent variable and the dependent variable of this function.
     Time is the independent variable and cost is the dependent variable.
   - **b** Suppose it costs $3 per hour to park, with a maximum cost of $12. Sketch a possible graph of the function. Be sure to label the axes.

2. Determine whether each of the following descriptions represents a function. Explain your thinking.
   - **a** The input is a year. The output is the population of the United States during that year.
     Function; Sample response: Population is a function of the year because for each specific year, there is only one population.
   - **b** The input is the distance a person is from the ground. The output is the time related to that height as the person rides a Ferris wheel.
     Not a function; Sample response: Each distance from the ground can occur at multiple times.
   - **c** The input is a person's name. The output is that person's phone number.
     Not a function; Sample response: Each name is not unique and can have multiple phone numbers.

3. The distance a person walks d, in kilometers, is a function of the time t, in minutes, since they began walking. Select *all* the true statements about this function.
   - A. The distance walked is the input.
   - B. The time of day is the input.
   - C. The input is measured in hours.
   - [✓ D.] The variable t represents the input.
   - E. The variable d represents the input.
   - F. The input is not measured in any particular unit.
   - [✓ G.] The time since the person began walking is the input.
   - H. For each input, there are sometimes two outputs.



Name: Date: Period:

4. For the expression $4x - 5(y - 1)$, which of the following ordered pairs makes the value of the expressions greater than 20?
   - A. $(8, 10)$
   - [✓ B.] $(5, 0)$
   - C. $(10, 8)$
   - D. $(0, 5)$

5. A randomly selected group of 100 employed adults were surveyed about whether they earned a high school diploma and whether their current annual income was greater than $30,000. The following table shows some of the results. Complete the two-way table.

	$30,000 or less	Greater than $30,000	Total
High school diploma	**21**	68	89
No high school diploma	9	**2**	**11**
Total	**30**	70	**100**

6. Han walked to his bus stop and then took the bus for one stop to his school. The graph shows the relationship between his distance d, in miles, from home and time t, in minutes.
   - **a** What is Han's distance from home after 10 minutes?
     0.5 miles
   - **b** How long does it take Han to travel a distance of 1.5 miles?
     16 minutes
   - **c** Based on the graph, how long did Han have to wait for the bus to arrive after walking to his bus stop? Explain your thinking.
     4 min.; Sample response: Han's distance doesn't change between 10 minutes and 14 minutes. This means he is not walking or riding the bus.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	1
	3	Activity 3	2
Spiral	4	Unit 1 Lesson 14	2
	5	Unit 2 Lesson 15	2
Formative	6	Unit 3 Lesson 3	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Function Notation

Let's explore a more efficient way to refer to and and communicate about functions.

Focus

Goals

1. **Language Goal:** Interpret statements that use function notation and explain their meaning in terms of a situation. **(Speaking and Listening, Writing)**
2. Understand that function notation is a succinct way to name a function and specify its input and output values.
3. Use function notation to express functions with specific input and output values.

Coherence

Today

Students are introduced to function notation as a way to succinctly name a function and communicate information about its input and output variables in specific situations. They interpret function notation in terms of the quantities in a situation and use function notation to represent simple statements about a function, prompting students to reason quantitatively and abstractly.

Previously

In Lesson 2, students reviewed independent and dependent variables and sketched functions given a context.

Coming Soon

In Lesson 4, students will further their understanding of function notation by making informal connections to graphs and verbal descriptions of functions in real-world situations.

Rigor

- Students develop **conceptual understanding** of function notation by recognizing that it is a succinct way to describe a function at specific input and output values.
- Students develop **procedural fluency** interpreting and writing function notation when a function is represented by a table, graph, or verbal description.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	10 min	10 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Function Notation*

Math Language Development

New words

- ***function notation***

Review words

- *function*

Amps | **Featured Activity**

Activity 1
Toggle Between Graphs

Students can highlight which graph they want to study by toggling between buses.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might be tempted to disregard the structure of this new notation because they are unfamiliar with it and it makes them uncomfortable. Help students motivate themselves to learn and understand the notation by explaining that it will definitely be useful in future mathematics. For those who like efficiency, explain that the notation minimizes how much they will need to write in some cases.

Modifications to Pacing

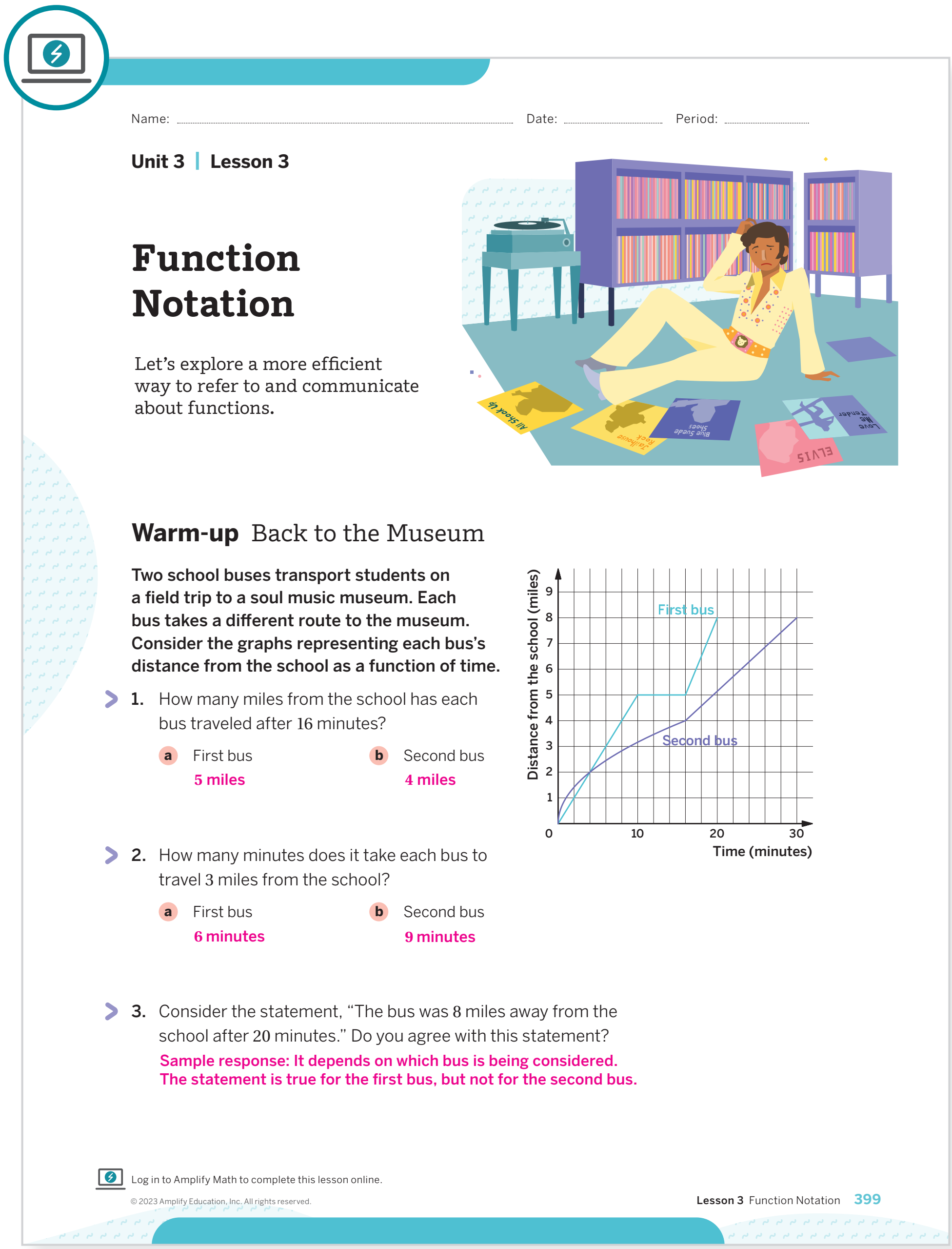
You may want to consider this additional modification if you are short on time.

- **Activity 2** may be omitted.

Independent | 10 min

Warm-up Back to the Museum

Students use a graph of two functions to examine an unclear statement to develop a need for a consistent and concise way to name a function.



Name: ______________________ Date: __________ Period: __________

Unit 3 | Lesson 3

Function Notation

Let's explore a more efficient way to refer to and communicate about functions.

Warm-up Back to the Museum

Two school buses transport students on a field trip to a soul music museum. Each bus takes a different route to the museum. Consider the graphs representing each bus's distance from the school as a function of time.

1. How many miles from the school has each bus traveled after 16 minutes?

   a First bus **5 miles**

   b Second bus **4 miles**

2. How many minutes does it take each bus to travel 3 miles from the school?

   a First bus **6 minutes**

   b Second bus **9 minutes**

3. Consider the statement, "The bus was 8 miles away from the school after 20 minutes." Do you agree with this statement?

   Sample response: It depends on which bus is being considered. The statement is true for the first bus, but not for the second bus.

Log in to Amplify Math to complete this lesson online.

©2023 Amplify Education, Inc. All rights reserved.

Lesson 3 Function Notation 399

1 Launch

Have students work independently before sharing their thinking with a partner.

2 Monitor

Help students get started by having them identify specific input and output values on each graph.

Look for points of confusion:

- **Interchanging the independent and dependent variables.** Ask, "Which axis label represents the input? The output?"
- **Having difficulty determining which bus is being referred to in Problem 3.** Encourage students to record their own interpretation.

Look for productive strategies:

- Noticing the axes labels and identifying the quantities graphed.

3 Connect

Display each graph. Discuss Problems 1 and 2. For Problem 3, ask the question for each bus.

Have students share their responses to Problem 3. Select and sequence those providing responses for one or both buses.

Highlight that Problem 3 is unclear because it does not specify which bus (function) is being referenced. When comparing and discussing two or more functions, it is helpful to use concise symbols or consistent names.

Ask, "What information is needed in order for everyone to have the same answer to Problem 3?" Specifying which bus is being referenced.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their responses to Problem 3, draw attention to the phrase "the bus" and ask students what that phrase means where there is more than one bus in the scenario. Ask students to brainstorm different ways they could refer to each bus so that there is no ambiguity. Sample responses: First Bus, Second Bus, Bus 1, Bus 2, Bus A, Bus B, etc.

Power-up

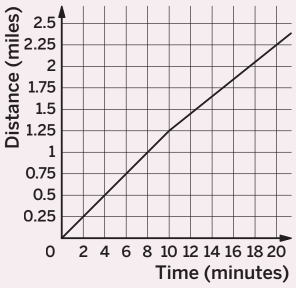
To power up students' ability to interpret a graph, have students complete:

Priya's fitness tracker created a graph based on her last run. What does the point $(4, 0.5)$ represent on the graph?

A. After 0.5 minutes, Priya ran 4 miles.

B. Priya finished half of her run in 4 minutes.

[✓ **C.**] Priya ran 0.5 miles in 4 minutes.



Use: Before the Warm-up

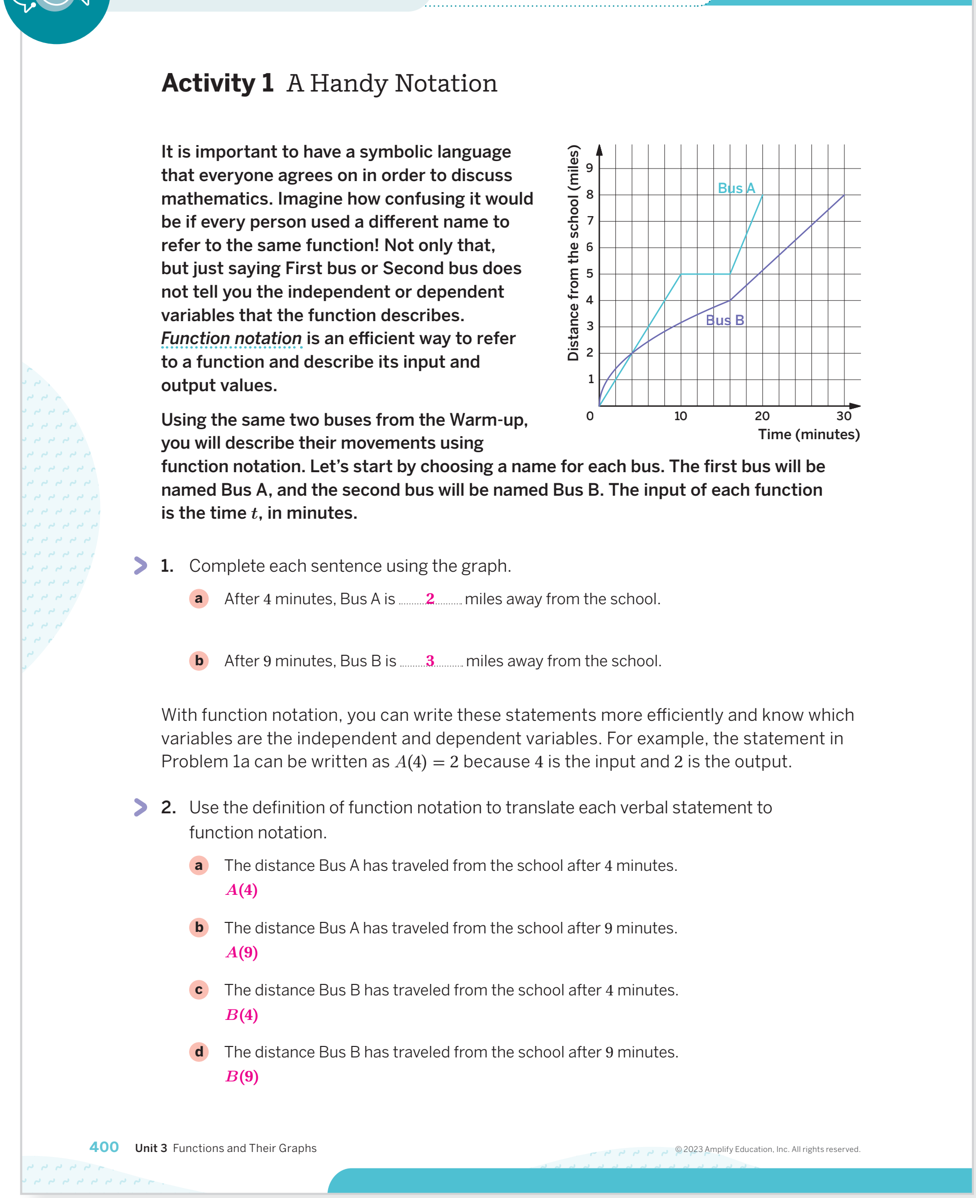
Informed by: Performance on Lesson 2, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 4

Pairs | 15 min

Activity 1 A Handy Notation

Students see the need to decontextualize descriptions of functions to be more concise and are introduced to function notation.

Amps Featured Activity Toggle Between Graphs



Activity 1 A Handy Notation

It is important to have a symbolic language that everyone agrees on in order to discuss mathematics. Imagine how confusing it would be if every person used a different name to refer to the same function! Not only that, but just saying First bus or Second bus does not tell you the independent or dependent variables that the function describes. *Function notation* is an efficient way to refer to a function and describe its input and output values.

Using the same two buses from the Warm-up, you will describe their movements using function notation. Let's start by choosing a name for each bus. The first bus will be named Bus A, and the second bus will be named Bus B. The input of each function is the time t, in minutes.

1. Complete each sentence using the graph.
   - a After 4 minutes, Bus A is 2 miles away from the school.
   - b After 9 minutes, Bus B is 3 miles away from the school.

With function notation, you can write these statements more efficiently and know which variables are the independent and dependent variables. For example, the statement in Problem 1a can be written as $A(4) = 2$ because 4 is the input and 2 is the output.

2. Use the definition of function notation to translate each verbal statement to function notation.
   - a The distance Bus A has traveled from the school after 4 minutes.
     $A(4)$
   - b The distance Bus A has traveled from the school after 9 minutes.
     $A(9)$
   - c The distance Bus B has traveled from the school after 4 minutes.
     $B(4)$
   - d The distance Bus B has traveled from the school after 9 minutes.
     $B(9)$

400 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the narrative as a class and have students complete Problem 1. Display the Anchor Chart PDF, *Function Notation*. Have students work independently for each of the remaining problems before sharing their thinking with a partner.

2 Monitor

Help students get started by modeling how to write each bus' function using function notation. First bus: $A(t)$ and Second bus: $B(t)$.

Look for points of confusion:

- **Confusing "$A(t)$" for A times the variable t.** Using the Anchor Chart PDF, *Function Notation*, emphasize that t is a placeholder representing the input value to the function named "A".
- **Confusing the meaning of the equal sign in Problem 5.** Explain that $A(2) = 1$ means that $A(2)$ and 1 are equivalent and both represent the output value, or the distance of Bus A from the school.

Look for productive strategies:

- Labeling each component of a statement written in function notation.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can view an animation of a bus traveling along a street, while the graph of the time and distance from school are simultaneously displayed. This will support students' conceptual understanding of the graphical representation of this relationship.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on completing Problems 1, 2a, 2c, 3a, 4a, and 5.

Math Language Development

MLR3: Critique, Correct, Clarify

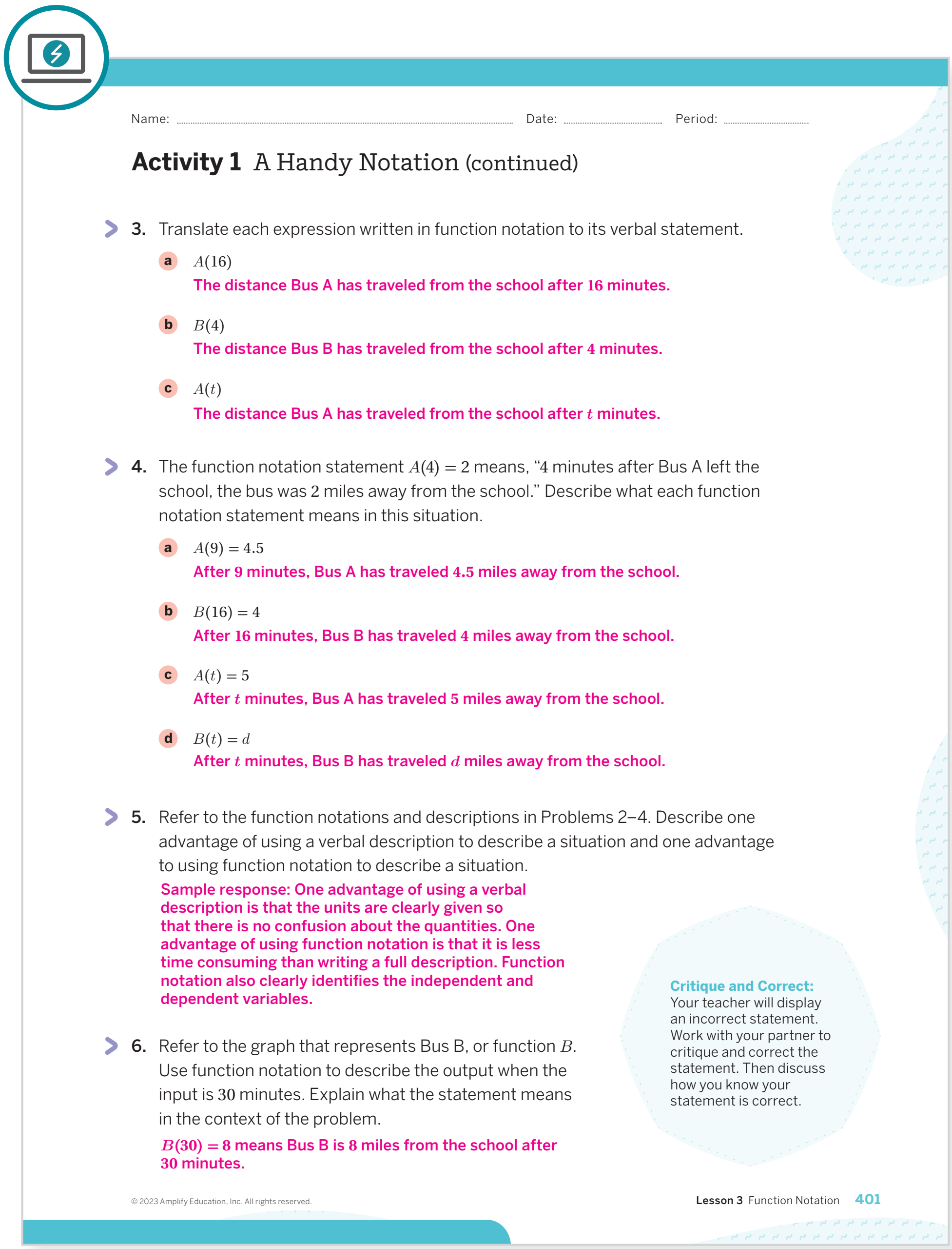
During the Connect, display an incorrect statement written in function notation, such as "$B(4) = 16$." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking."
  Sample response: I disagree because $B(4)$ means the distance Bus B has traveled after 4 minutes, which is not 16 miles. This statement transposed the values 4 and 16
- ***Correct:*** "Write a corrected statement that is now true."
  Sample response: $B(16) = 4$
- ***Clarify:*** "How did you correct the statement? How do you know that the statement is now true?"

Pairs | 15 min

Activity 1 A Handy Notation (continued)

Students see the need to decontextualize descriptions of functions to be more concise and are introduced to function notation.



Name: ______________________ Date: __________ Period: __________

Activity 1 A Handy Notation (continued)

3. Translate each expression written in function notation to its verbal statement.
   - a $A(16)$
     The distance Bus A has traveled from the school after 16 minutes.
   - b $B(4)$
     The distance Bus B has traveled from the school after 4 minutes.
   - c $A(t)$
     The distance Bus A has traveled from the school after t minutes.

4. The function notation statement $A(4) = 2$ means, "4 minutes after Bus A left the school, the bus was 2 miles away from the school." Describe what each function notation statement means in this situation.
   - a $A(9) = 4.5$
     After 9 minutes, Bus A has traveled 4.5 miles away from the school.
   - b $B(16) = 4$
     After 16 minutes, Bus B has traveled 4 miles away from the school.
   - c $A(t) = 5$
     After t minutes, Bus A has traveled 5 miles away from the school.
   - d $B(t) = d$
     After t minutes, Bus B has traveled d miles away from the school.

5. Refer to the function notations and descriptions in Problems 2–4. Describe one advantage of using a verbal description to describe a situation and one advantage to using function notation to describe a situation.
   Sample response: One advantage of using a verbal description is that the units are clearly given so that there is no confusion about the quantities. One advantage of using function notation is that it is less time consuming than writing a full description. Function notation also clearly identifies the independent and dependent variables.

6. Refer to the graph that represents Bus B, or function B. Use function notation to describe the output when the input is 30 minutes. Explain what the statement means in the context of the problem.
   $B(30) = 8$ means Bus B is 8 miles from the school after 30 minutes.

Critique and Correct: Your teacher will display an incorrect statement. Work with your partner to critique and correct the statement. Then discuss how you know your statement is correct.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 3 Function Notation 401

3 Connect

Display the Anchor Chart PDF, *Function Notation*. Engage in choral response to have students practice speaking function notation, e.g. $A(t)$, A is a function of t.

Have student pairs share their response to Problems 5–6. Record the corresponding points on the graph.

Highlight that the notation $f(x)$ is read "f of x." It tells us that "f" is the name of the function, "x" is the input of the function, and "$f(x)$" is the output or the value of the function when the input is "x." The statement "$g(t) = d$" is read "g of t is equal to d." It tells us that "g" is the name of the function and "t" is the input. It also tells us that "$g(t)$" is the output or the value of the function at "t", and has the same value as "d."

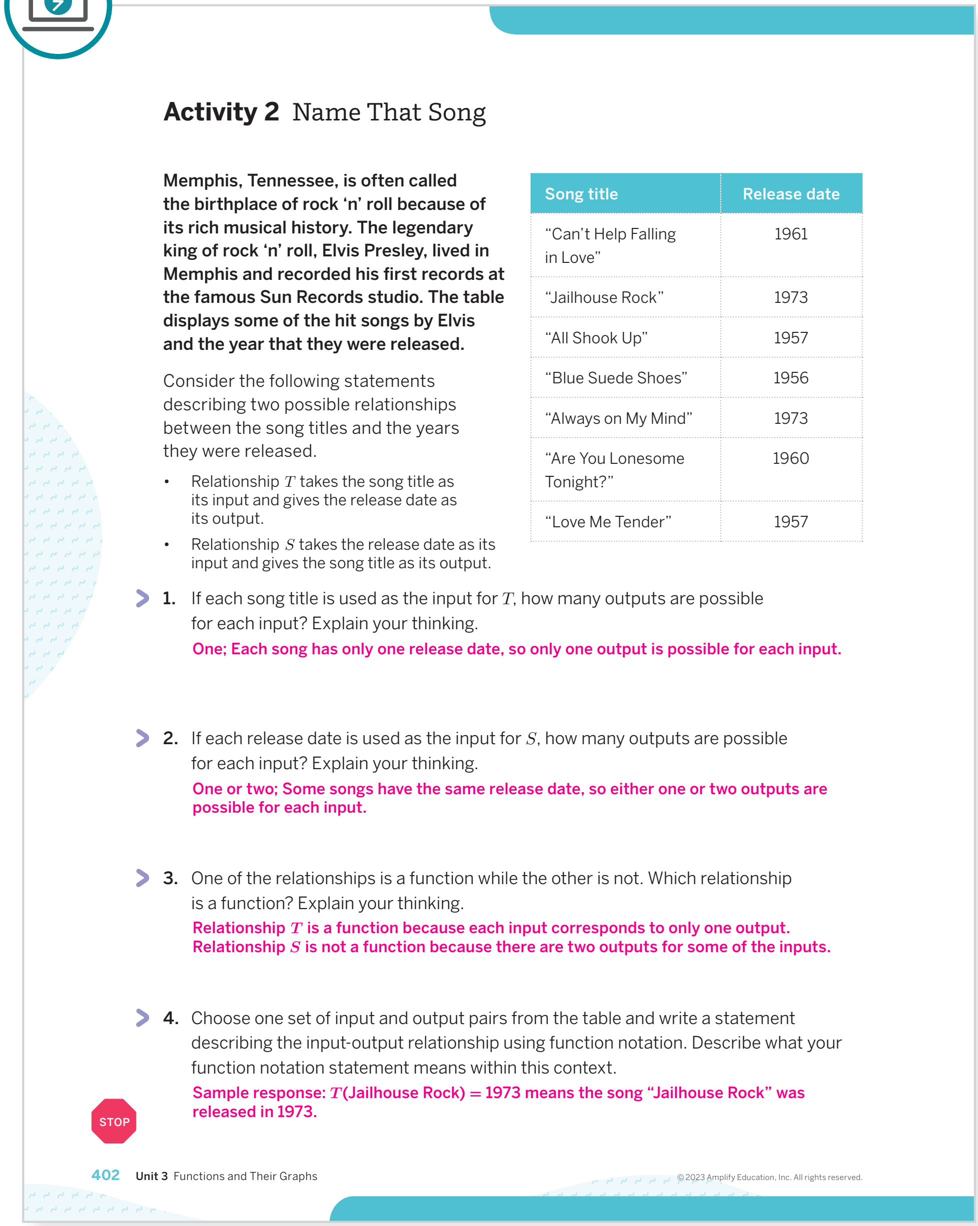
Define the term ***function notation*** as a consistent and concise way to name a function and describe its input and output values.

Ask, "What is the difference between $A(10)$ and $A(10) = 5$?" $A(10)$ represents the output of function A when the input is 10 but does not specify its value. $A(10) = 5$ specifies that when the input of function A is 10, the output is 5.

Pairs | 10 min

Activity 2 Name That Song

Students construct and interpret non-numerical function notation statements within context and connect the structure of function notation to the definition of a function.



Activity 2 Name That Song

Memphis, Tennessee, is often called the birthplace of rock 'n' roll because of its rich musical history. The legendary king of rock 'n' roll, Elvis Presley, lived in Memphis and recorded his first records at the famous Sun Records studio. The table displays some of the hit songs by Elvis and the year that they were released.

Song title	Release date
"Can't Help Falling in Love"	1961
"Jailhouse Rock"	1973
"All Shook Up"	1957
"Blue Suede Shoes"	1956
"Always on My Mind"	1973
"Are You Lonesome Tonight?"	1960
"Love Me Tender"	1957

Consider the following statements describing two possible relationships between the song titles and the years they were released.

- Relationship T takes the song title as its input and gives the release date as its output.
- Relationship S takes the release date as its input and gives the song title as its output.

1. If each song title is used as the input for T, how many outputs are possible for each input? Explain your thinking.
   One; Each song has only one release date, so only one output is possible for each input.

2. If each release date is used as the input for S, how many outputs are possible for each input? Explain your thinking.
   One or two; Some songs have the same release date, so either one or two outputs are possible for each input.

3. One of the relationships is a function while the other is not. Which relationship is a function? Explain your thinking.
   Relationship T is a function because each input corresponds to only one output. Relationship S is not a function because there are two outputs for some of the inputs.

4. Choose one set of input and output pairs from the table and write a statement describing the input-output relationship using function notation. Describe what your function notation statement means within this context.
   Sample response: T(Jailhouse Rock) = 1973 means the song "Jailhouse Rock" was released in 1973.

STOP

402 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have students work independently before sharing their thinking with a partner.

2 Monitor

Help students get started by activating prior knowledge. Ask, "What makes a relationship between two variables a function?" Sample response: There should be a unique output value for each input value.

Look for points of confusion:

- **Thinking that two different song titles cannot have the same year.** Remind students that each input value must have only one output value but that this output value can be repeated.

Look for productive strategies:

- Making their thinking visible by drawing arrows to map a specific input value to its corresponding output value.
- Writing the input and output pairs in the table as ordered pairs.

3 Connect

Display the table.

Have student pairs share their thinking for why S is not a function.

Highlight that even if a relationship is not a function, students may want to understand the relationship between the input and output values. Ask, "How many songs did Elvis release each year during the height of his popularity?"

Ask, "Is it possible to use function notation to describe the relationship S that takes the release date as its input and gives the song title as the output?" Sample response: No, because S is not a function.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students annotate each different song title in the table as Song A, Song B, Song C, etc. Consider displaying or providing a table similar to the following and ask them to complete it to help them visualize Relationships T and S.

Relationship T	
Input	Output
Song A	1961

Relationship S	
Input	Output
1961	Song A

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

Read 1: Students should understand that the table gives the release dates for different song titles.

Read 2: Ask students to identify the differences between Relationship T and Relationship S.

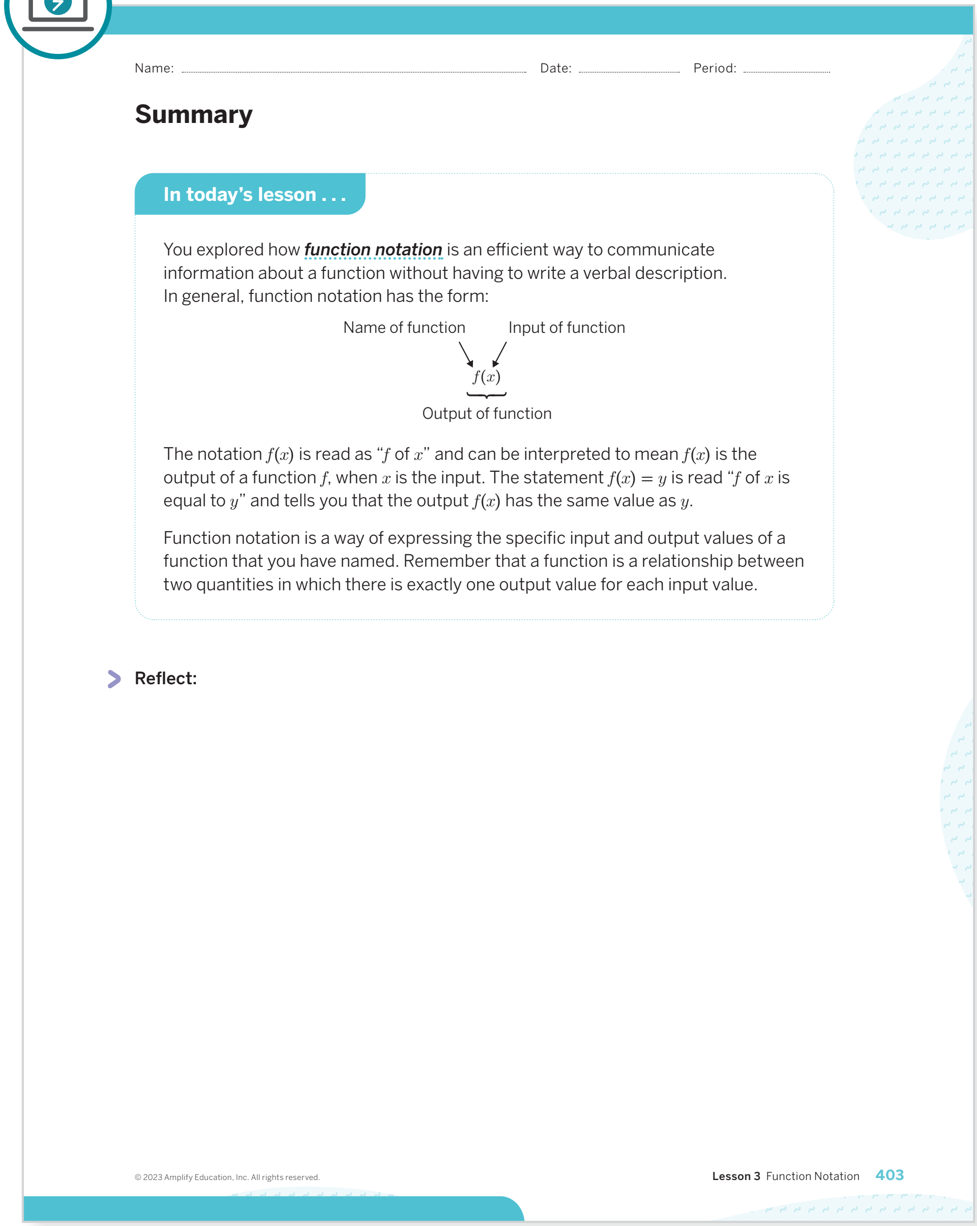
Read 3: Ask students to preview Problems 1 and 2 and brainstorm strategies they could use to complete them.

English Learners

Instead of using words to represent the song titles, have students label each song title with a different letter, such as Song A, Song B, Song C, etc.

Summary

Review and synthesize using function notation and summarize the process of interpreting and writing statements in function notation.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored how ***function notation*** is an efficient way to communicate information about a function without having to write a verbal description. In general, function notation has the form:

Name of function   Input of function

$$\underbrace{f(x)}_{\text{Output of function}}$$

The notation $f(x)$ is read as "f of x" and can be interpreted to mean $f(x)$ is the output of a function f, when x is the input. The statement $f(x) = y$ is read "f of x is equal to y" and tells you that the output $f(x)$ has the same value as y.

Function notation is a way of expressing the specific input and output values of a function that you have named. Remember that a function is a relationship between two quantities in which there is exactly one output value for each input value.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 3 Function Notation 403

Synthesize

Display the Anchor Chart PDF, *Function Notation*.

Highlight that it is important to identify the input and output variables to determine how to express information about a function using function notation. Function notation serves as a placeholder for expressing a function and its input values.

Formalize vocabulary: ***function notation***

Ask, "What is the difference between $f(x)$ and $f(x) = y$?" $f(x)$ represents the output even though the value is not stated, and $f(x) = y$ assigns the output a value of y.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What are the advantages of using function notation?"

Math Language Development

MLR2: Collect and Display

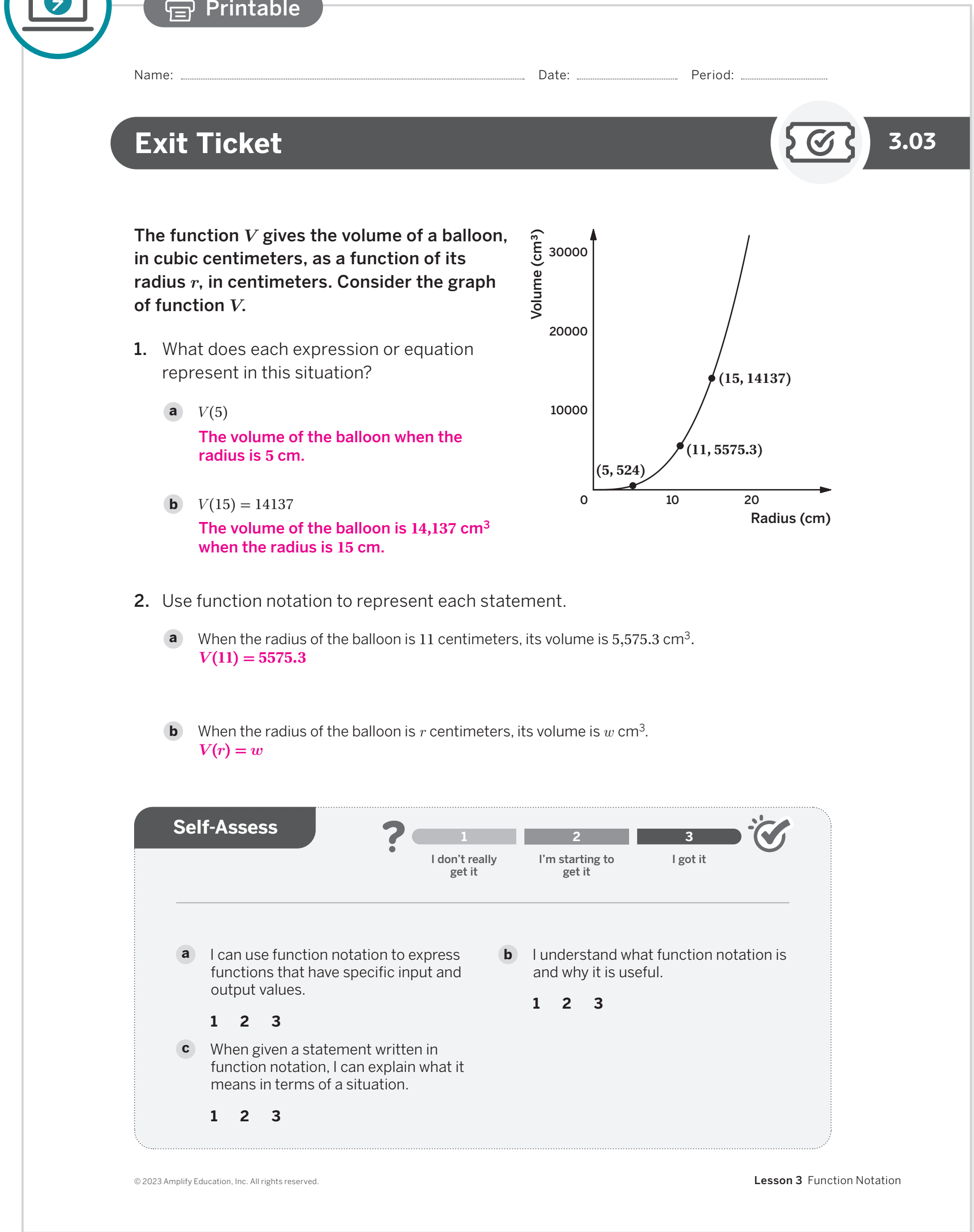
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the term *function notation* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of the structure of function notation by constructing and interpreting function notation statements within a real-world context.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 3.03

The function V gives the volume of a balloon, in cubic centimeters, as a function of its radius r, in centimeters. Consider the graph of function V.

1. What does each expression or equation represent in this situation?

   a. $V(5)$

   The volume of the balloon when the radius is 5 cm.

   b. $V(15) = 14137$

   The volume of the balloon is 14,137 cm^3 when the radius is 15 cm.

2. Use function notation to represent each statement.

   a. When the radius of the balloon is 11 centimeters, its volume is 5,575.3 cm^3.

   $V(11) = 5575.3$

   b. When the radius of the balloon is r centimeters, its volume is w cm^3.

   $V(r) = w$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I can use function notation to express functions that have specific input and output values.

1 2 3

b. I understand what function notation is and why it is useful.

1 2 3

c. When given a statement written in function notation, I can explain what it means in terms of a situation.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 3 Function Notation

Success looks like . . .

- **Language Goal:** Interpreting statements that use function notation and explaining their meaning in terms of a situation. **(Speaking and Listening, Writing)**
  - » Explaining the meanings of the function expression and function equation in Problem 1.
- **Goal:** Understanding that function notation is a succinct way to name a function and specify its input and output values.
- **Goal:** Using function notation to express functions with specific inputs and outputs.
  - » Expressing each specific input and output pair using function notation in Problem 2.

Suggested next steps

If students confuse the meaning of the expression or equation in Problem 1, consider:

- Reviewing examples from Activity 2.
- Assigning Practice Problem 3.

If students do not use function notation correctly in Problem 2, consider:

- Displaying the Anchor Chart PDF, *Function Notation*.
- Asking, "What are the input value and the output value in this statement?"
- Asking, "What is the name of the function?"
- Assigning Practice Problem 1.

Professional Learning

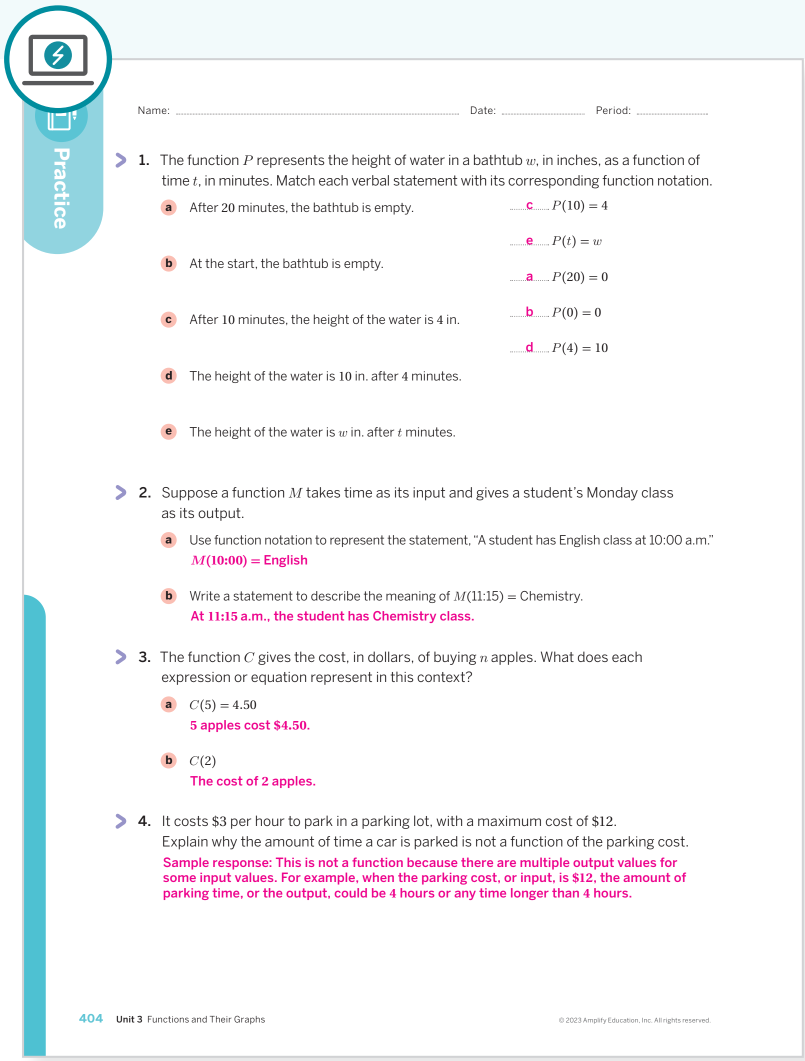
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and what didn't work today? In what ways have your students gotten better at reasoning abstractly and quantitatively?
- Have you changed any ideas you used to have about function notation as a result of today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. The function P represents the height of water in a bathtub w, in inches, as a function of time t, in minutes. Match each verbal statement with its corresponding function notation.

   a. After 20 minutes, the bathtub is empty.
   b. At the start, the bathtub is empty.
   c. After 10 minutes, the height of the water is 4 in.
   d. The height of the water is 10 in. after 4 minutes.
   e. The height of the water is w in. after t minutes.

   c $P(10) = 4$

   e $P(t) = w$

   a $P(20) = 0$

   b $P(0) = 0$

   d $P(4) = 10$

2. Suppose a function M takes time as its input and gives a student's Monday class as its output.

   a. Use function notation to represent the statement, "A student has English class at 10:00 a.m."
   M(10:00) = English
   b. Write a statement to describe the meaning of M(11:15) = Chemistry.
   At 11:15 a.m., the student has Chemistry class.

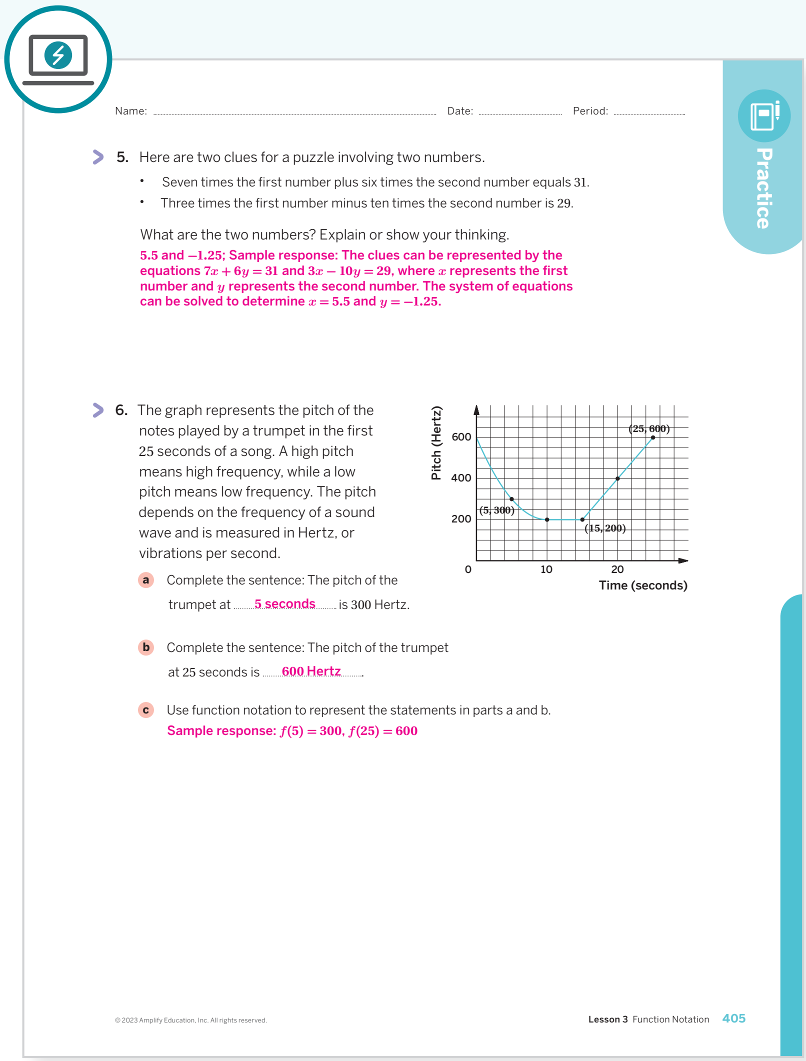
3. The function C gives the cost, in dollars, of buying n apples. What does each expression or equation represent in this context?

   a. $C(5) = 4.50$
   5 apples cost $4.50.
   b. $C(2)$
   The cost of 2 apples.

4. It costs $3 per hour to park in a parking lot, with a maximum cost of $12. Explain why the amount of time a car is parked is not a function of the parking cost.
   Sample response: This is not a function because there are multiple output values for some input values. For example, when the parking cost, or input, is $12, the amount of parking time, or the output, could be 4 hours or any time longer than 4 hours.

404 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: Date: Period:

5. Here are two clues for a puzzle involving two numbers.
   - Seven times the first number plus six times the second number equals 31.
   - Three times the first number minus ten times the second number is 29.

   What are the two numbers? Explain or show your thinking.
   5.5 and −1.25; Sample response: The clues can be represented by the equations $7x + 6y = 31$ and $3x - 10y = 29$, where x represents the first number and y represents the second number. The system of equations can be solved to determine $x = 5.5$ and $y = -1.25$.

6. The graph represents the pitch of the notes played by a trumpet in the first 25 seconds of a song. A high pitch means high frequency, while a low pitch means low frequency. The pitch depends on the frequency of a sound wave and is measured in Hertz, or vibrations per second.

   a. Complete the sentence: The pitch of the trumpet at **5 seconds** is 300 Hertz.
   b. Complete the sentence: The pitch of the trumpet at 25 seconds is **600 Hertz**.
   c. Use function notation to represent the statements in parts a and b.
   Sample response: $f(5) = 300$, $f(25) = 600$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 3 Function Notation 405

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 3	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 2	2
	5	Unit 1 Lesson 21	3
Formative ⚡	6	Unit 3 Lesson 4	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 3 | **LESSON 4**

Interpreting and Using Function Notation

Let's explore how to use function notation to describe quantities in a graph.

Focus

Goals

1. **Language Goal:** Describe connections between statements that use function notation and a graph of the function. **(Speaking and Listening, Writing)**
2. **Language Goal:** Interpret statements that use function notation and explain their meaning in terms of a situation. **(Speaking and Listening, Writing)**
3. Sketch a graph of a function given statements in function notation.

Coherence

Today

Students interpret symbolic statements in function notation and reason about inequalities such as $f(a) > f(b)$ in terms of a situation. Students attend to precision and use information in function notation to sketch a possible graph of a function where each point has the coordinates $(x, f(x))$.

Previously

In Lesson 3, students used function notation to interpret statements and communicate information about the relationship between quantities in specific situations.

Coming Soon

In Lessons 5, students will learn how to use function notation to describe the rule of a function.

Rigor

- Students continue to build **conceptual understanding** of function notation by interpreting symbolic statements and inequalities written in function notation.
- Students develop **procedural fluency** graphing a function when coordinate points are written in function notation.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Function Notation*
- Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning*
- rulers

Math Language Development

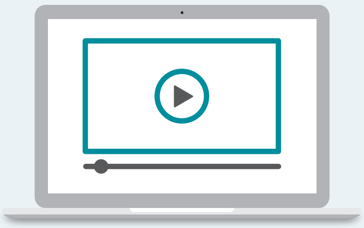
Review words

- *function*
- *function notation*

Amps | Featured Activity

Activity 1
See Student Thinking

Students compare values of a function based on its graph and justify their thinking, which you can see in real time.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Function notation is a mathematical way to keep quantitative information organized. The notation itself is very abstract and might remind students of using parentheses to represent multiplication. However, what it stands for can be quantified and used to compare function values for different values of the independent variable or to plot ordered pairs on a coordinate plane in order to graph the function.

Modifications to Pacing

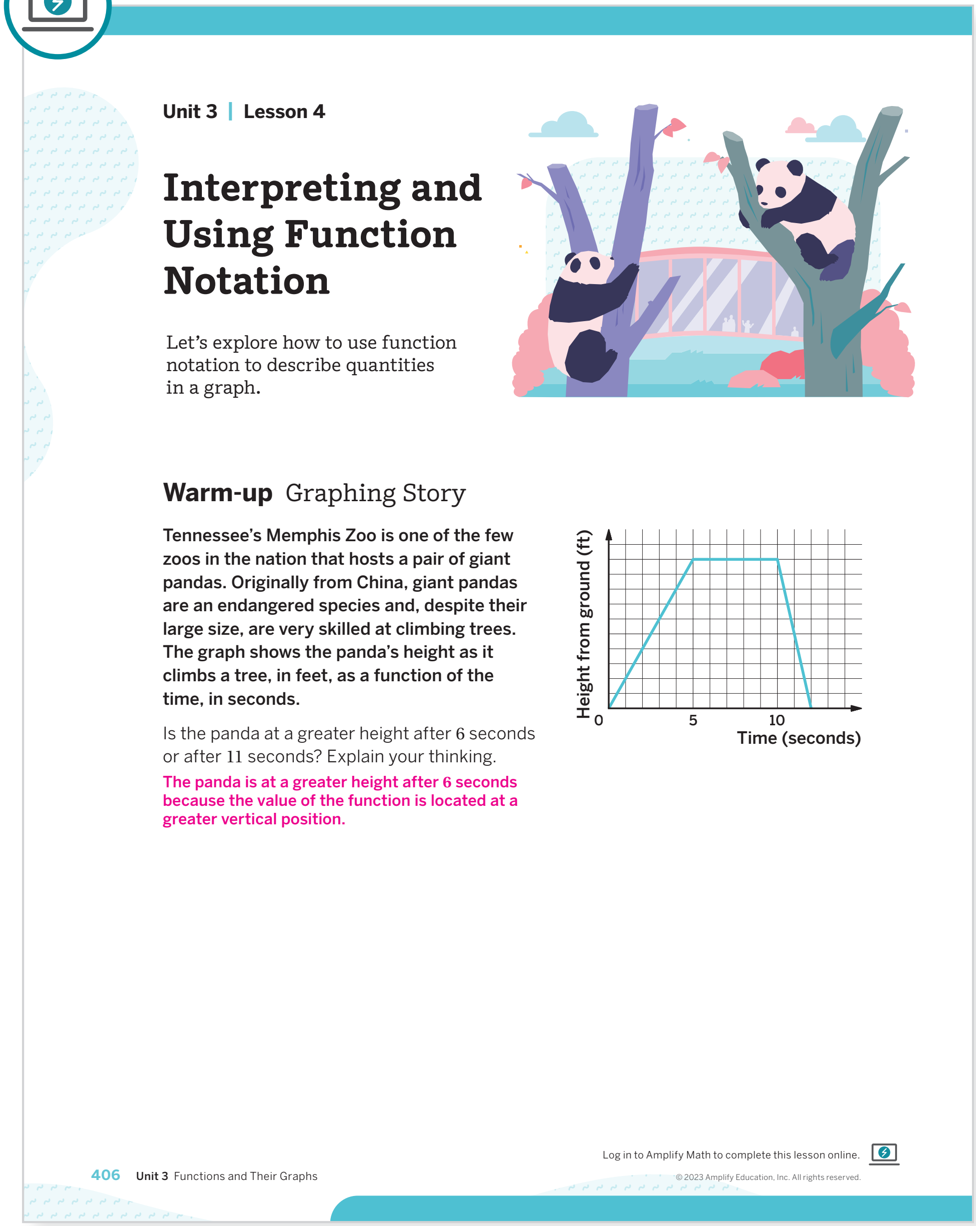
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up** and **Activity 1**, provide a number scale for the vertical axis.
- In **Activity 2**, omit part c in Problems 1 and 2.

Independent | 5 min

Warm-up Graphing Story

Students attend to precision when using and interpreting function notation within context, being careful to identify the independent and dependent variables.



Unit 3 | Lesson 4

Interpreting and Using Function Notation

Let's explore how to use function notation to describe quantities in a graph.

Warm-up Graphing Story

Tennessee's Memphis Zoo is one of the few zoos in the nation that hosts a pair of giant pandas. Originally from China, giant pandas are an endangered species and, despite their large size, are very skilled at climbing trees. The graph shows the panda's height as it climbs a tree, in feet, as a function of the time, in seconds.

Is the panda at a greater height after 6 seconds or after 11 seconds? Explain your thinking.

The panda is at a greater height after 6 seconds because the value of the function is located at a greater vertical position.

Log in to Amplify Math to complete this lesson online.

406 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the narrative together as a class. Have students work independently before sharing their thinking with a partner.

2 Monitor

Help students get started by asking what each point on the graph represents in the context of the problem.

Look for points of confusion:

- **Struggling to compare points on the graph when the output values are unknown.** Ask students how they can determine the output values if they are given the input values.

Look for productive strategies:

- Noticing the axes labels and identifying the quantities graphed.
- Plotting points on the graph and describing what they represent in the context of the problem.
- Comparing the vertical values of the points on the graph at different times.

3 Connect

Display the graph.

Have students share their responses.

Highlight that points with greater y-values are at greater vertical positions on the graph.

Ask, "What time is the panda farthest away from the ground?" Between 5 to 10 seconds.

Math Language Development

MLR8: Discussion Supports

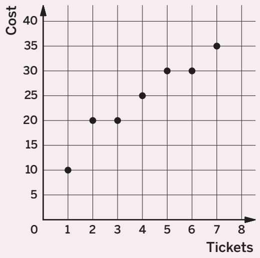
During the Connect, as students share their responses, provide the following sentence frame for them to use to help them organize their thinking.

"The panda is at a greater height after _____ seconds because . . ."

Ask students to explain why the vertical axis did not need to be labeled with a numerical scale.

Power-up

To power up students' ability to use function notation to describe values in graphs, have students complete:



The graph models the cost c for t tickets. Determine which of the following function values match coordinate pairs on the graph. Select *all* that apply.

[✓ A.] $C(1) = 10$

B. $C(10) = 1$

C. $C(4) = 20$

[✓ D.] $C(6) = 30$

E. $C(35) = 7$

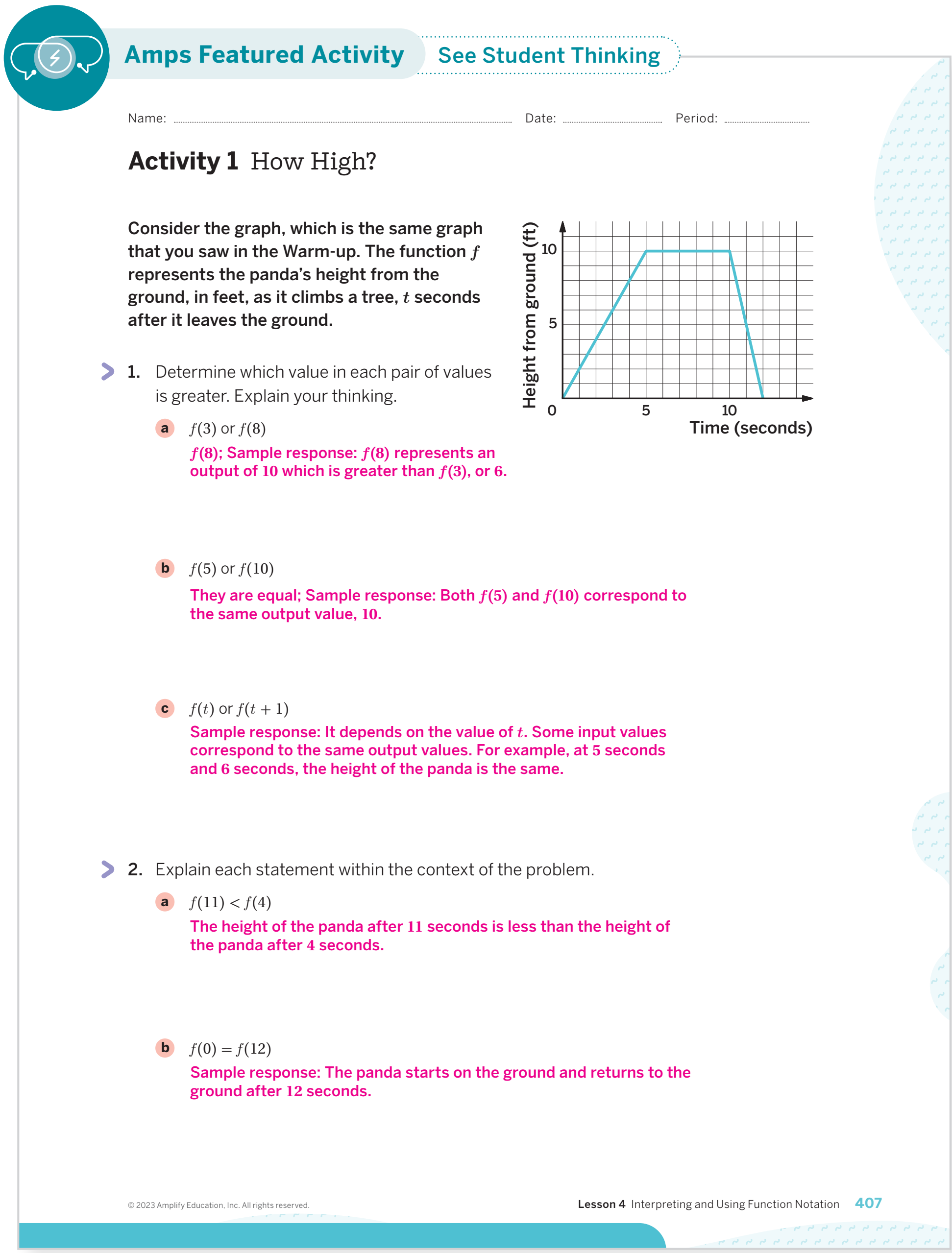
Use: Before Activity 1

Informed by: Performance on Lesson 3, Practice Problem 6

Pairs | 10 min

Activity 1 How High?

Students attend to precision when using and interpreting function notation within context, being careful to identify the independent and dependent variables.



Amps Featured Activity See Student Thinking

Name: ______ Date: ______ Period: ______

Activity 1 How High?

Consider the graph, which is the same graph that you saw in the Warm-up. The function f represents the panda's height from the ground, in feet, as it climbs a tree, t seconds after it leaves the ground.

1. Determine which value in each pair of values is greater. Explain your thinking.
   - a $f(3)$ or $f(8)$
     $f(8)$; Sample response: $f(8)$ represents an output of 10 which is greater than $f(3)$, or 6.
   - b $f(5)$ or $f(10)$
     They are equal; Sample response: Both $f(5)$ and $f(10)$ correspond to the same output value, 10.
   - c $f(t)$ or $f(t+1)$
     Sample response: It depends on the value of t. Some input values correspond to the same output values. For example, at 5 seconds and 6 seconds, the height of the panda is the same.

2. Explain each statement within the context of the problem.
   - a $f(11) < f(4)$
     The height of the panda after 11 seconds is less than the height of the panda after 4 seconds.
   - b $f(0) = f(12)$
     Sample response: The panda starts on the ground and returns to the ground after 12 seconds.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Interpreting and Using Function Notation 407

1 Launch

Read the scenario together as a class. Have students work independently before sharing their thinking with a partner.

2 Monitor

Help students get started by asking, "What do you notice about the graph?"

Look for points of confusion:

- **Having difficulty interpreting function notation in Problem 1c.** Have students make a table of values for different times and heights.
- **Struggling to interpret inequalities written in function notation.** Ask students what quantities are being compared in the context of the problem.

Look for productive strategies:

- Using the shape of the graph to reason about function values.
- Referring to the input and output as time and height.
- Recognizing the response to Problem 1c depends on the value of t, and using specific input values to justify their response.

3 Connect

Display the graph.

Have student pairs share how they compared each pair of output values. Select and sequence students who use concrete to more abstract reasoning in their response to Problem 1c.

Highlight that the coordinates of each point on the graph of this function are given by the ordered pairs $(t, f(t))$ or (time, height).

Ask, "What are the coordinates of a point when the panda is climbing up the tree? Down the tree? Staying at the same height?" Sample response: Up: $(2, f(2))$; Down: $(11, f(11))$; Same height: $(6, f(6))$.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can view an animation of a panda climbing a tree, while the graph of the panda's height and time are simultaneously displayed. This will support students' understanding of the graphical representation of this relationship.

Accessibility: *Guide Processing and Visualization*

Display the Anchor Chart PDF, *Function Notation* for students to reference as they complete this activity. Ask students what they notice about the vertical axis of the graph. It is not labeled with a numerical scale. Ask them if they need to know the numerical scale to complete the activity.

Extension: *Math Enrichment*

Have students determine whether each of these statements is true or false and have them explain their thinking.

$f(6) = f(8)$ True, the panda is not climbing or descending.

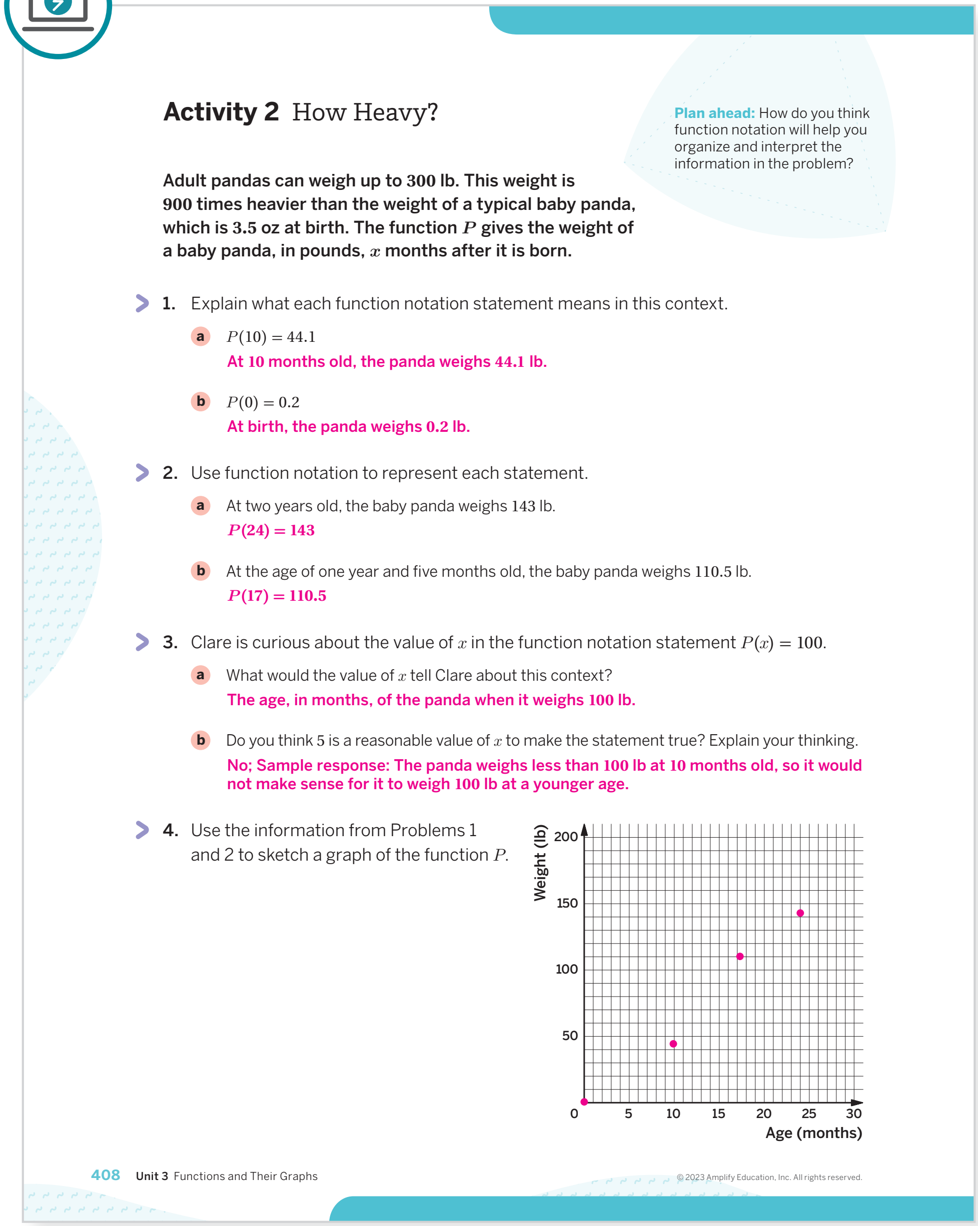
$f(13) < 0$ False. We do not know what happens after 12 seconds, but it is not likely that the panda is underground.

$f(t+1) > f(t)$ False, this is only true when the function is increasing.

Pairs | 10 min

Activity 2 How Heavy?

Students construct and interpret function notation statements within context and create a graph to represent the relationship.



Activity 2 How Heavy?

Plan ahead: How do you think function notation will help you organize and interpret the information in the problem?

Adult pandas can weigh up to 300 lb. This weight is 900 times heavier than the weight of a typical baby panda, which is 3.5 oz at birth. The function P gives the weight of a baby panda, in pounds, x months after it is born.

1. Explain what each function notation statement means in this context.
   - a $P(10) = 44.1$
     At 10 months old, the panda weighs 44.1 lb.
   - b $P(0) = 0.2$
     At birth, the panda weighs 0.2 lb.
2. Use function notation to represent each statement.
   - a At two years old, the baby panda weighs 143 lb.
     $P(24) = 143$
   - b At the age of one year and five months old, the baby panda weighs 110.5 lb.
     $P(17) = 110.5$
3. Clare is curious about the value of x in the function notation statement $P(x) = 100$.
   - a What would the value of x tell Clare about this context?
     The age, in months, of the panda when it weighs 100 lb.
   - b Do you think 5 is a reasonable value of x to make the statement true? Explain your thinking.
     No; Sample response: The panda weighs less than 100 lb at 10 months old, so it would not make sense for it to weigh 100 lb at a younger age.
4. Use the information from Problems 1 and 2 to sketch a graph of the function P.

408 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the narrative as a class. Have students complete the problems independently before sharing their thoughts with a partner.

2 Monitor

Help students get started by asking them to identify the input and output of this function and the units in which each variable is measured.

Look for points of confusion:

- **Having difficulty converting years to months.** Model converting years to months.
- **Struggling to sketch a graph of the situation.** Ask students how they can use their responses to Problems 1 and 2 to help them determine points on the graph.

Look for productive strategies:

- Plotting points on the graph and explaining what they represent in the context of the problem.
- Making a table of input and output values by translating the statements written in function notation.

3 Connect

Have students display their graphs and share their interpretations. Make sure they interpret and articulate statements such as $P(x) = 100$ in complete sentences.

Highlight that the coordinate of each point on the graph is $(x, P(x))$. The graph of function P can be drawn in different ways, but it makes sense for the panda's weight to increase over time when it is young.

Ask, "Why does it make sense to label the horizontal axis in months instead of years?" If the scale of the horizontal axis were in years, it would be challenging to read the age of a panda in fractions of a year.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Draw students' attention to the fact that the panda's weight at birth is given in ounces. Provide the panda's weight in pounds, approximately 0.2 lb, or provide the conversion rate. Continue to display the Anchor Chart PDF, *Function Notation*.

Accessibility: *Guide Processing and Visualization*

In Problem 4, suggest that students create a table of input values and corresponding output values to help them graph the relationship.

Time (months)	Weight (lb)
10	44.1

Math Language Development

MLR7: Compare and Connect

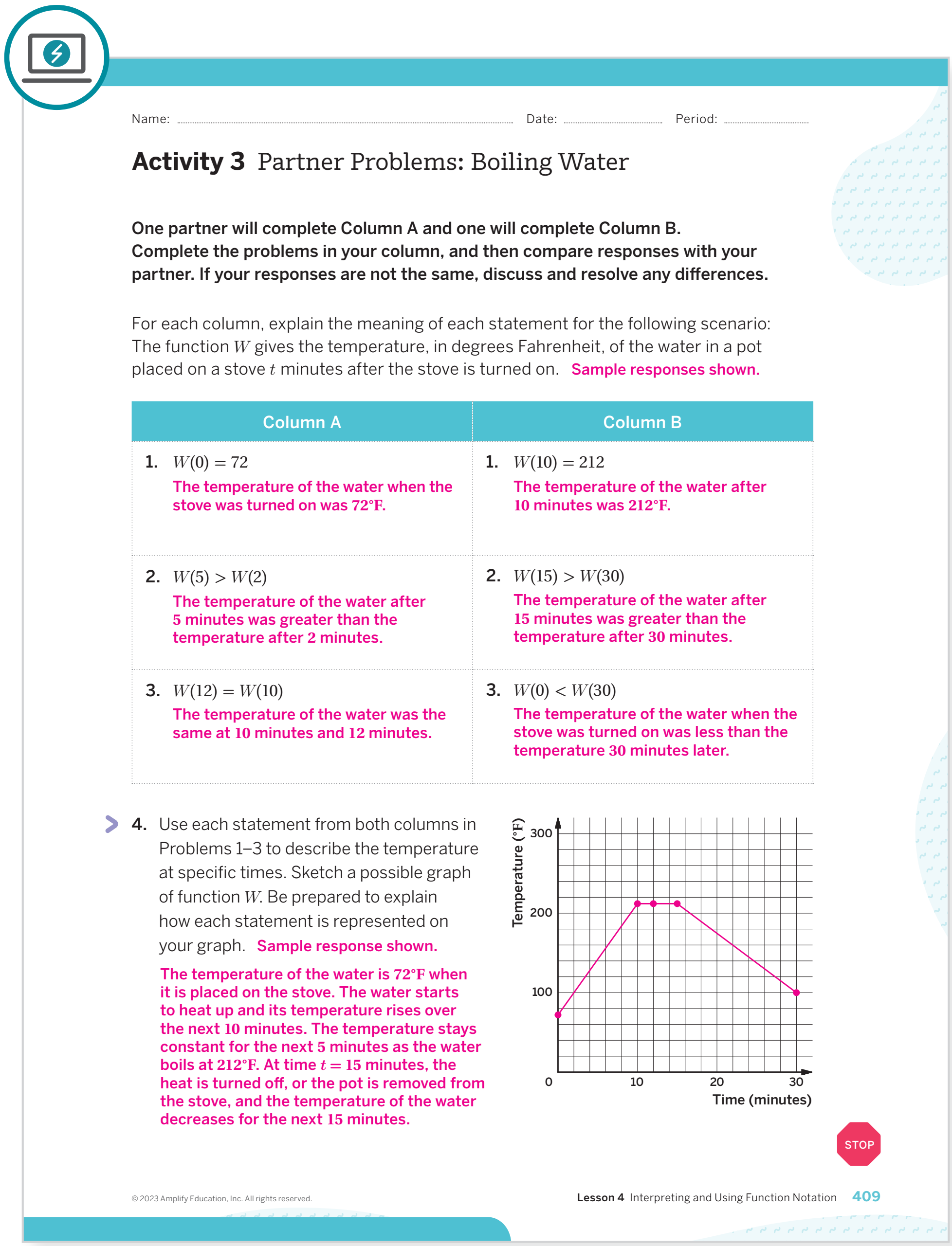
During the Connect, as students share their graphs and interpretations, focus their attention on how the graphs show information provided by the function notation statements and the connection between an ordered pair and function notation. Consider displaying the following:

Function notation	Ordered pair	Input, Output
$P(10) = 44.1$	$(10, 44.1)$	$10, 44.1$
$P(x) =$ _____	$(x, (P(x))$	$x, P(x)$

Pairs | 15 min

Activity 3 Partner Problems: Boiling Water

Students interpret function notation statements within context to compare output values of a function and discuss and resolve any disagreements with their partner.



Name: ______________ Date: ______ Period: ______

Activity 3 Partner Problems: Boiling Water

One partner will complete Column A and one will complete Column B. Complete the problems in your column, and then compare responses with your partner. If your responses are not the same, discuss and resolve any differences.

For each column, explain the meaning of each statement for the following scenario: The function W gives the temperature, in degrees Fahrenheit, of the water in a pot placed on a stove t minutes after the stove is turned on. Sample responses shown.

Column A	Column B
1. $W(0) = 72$ The temperature of the water when the stove was turned on was 72°F.	1. $W(10) = 212$ The temperature of the water after 10 minutes was 212°F.
2. $W(5) > W(2)$ The temperature of the water after 5 minutes was greater than the temperature after 2 minutes.	2. $W(15) > W(30)$ The temperature of the water after 15 minutes was greater than the temperature after 30 minutes.
3. $W(12) = W(10)$ The temperature of the water was the same at 10 minutes and 12 minutes.	3. $W(0) < W(30)$ The temperature of the water when the stove was turned on was less than the temperature 30 minutes later.

4. Use each statement from both columns in Problems 1–3 to describe the temperature at specific times. Sketch a possible graph of function W. Be prepared to explain how each statement is represented on your graph. Sample response shown.

The temperature of the water is 72°F when it is placed on the stove. The water starts to heat up and its temperature rises over the next 10 minutes. The temperature stays constant for the next 5 minutes as the water boils at 212°F. At time $t = 15$ minutes, the heat is turned off, or the pot is removed from the stove, and the temperature of the water decreases for the next 15 minutes.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Interpreting and Using Function Notation 409

1 Launch

Read the narrative together as a class. Say, "You and your partner will each select a column to complete. You will then share your thinking with your partner." Conduct the ***Partner Problems*** routine, having students work independently before sharing. Provide access to rulers so for Problem 4.

2 Monitor

Help students get started by asking them to identify the input and output of this function and the units in which each variable is measured.

Look for points of confusion:

- **Incorrectly translating the inequalities to a verbal description.** Ask students what quantities are being compared in the context of the problem.
- **Struggling to plot points on the graph when the output value is unknown.** Suggest students create their own vertical number scale to compare temperatures at different times.
- **Not recognizing that there are many possible graphs.** Tell students that there is more than one correct response and that they should graph their interpretation.

Look for productive strategies:

- Using complete sentences to translate symbolic statements in the context of the problem.
- Explaining how different points on their graph make the inequalities true.

3 Connect

Have student pairs share their graphs and explain how the statements written in function notation relate to the graph. Select partners whose graphs look different, but are both correct.

Highlight the connection between the statements in function notation and what they represent in the context of the problem.

Ask, "Why might it be true that $W(15) > W(30)$?" The heat was turned off after 15 minutes and the pot of water was taken off the stove.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Instead of asking students to write verbal descriptions, have them orally explain the meaning of each function notation statement in Problems 1–3.

Accessibility: *Guide Processing and Visualization*

Consider demonstrating how to write the ordered pair represented by the first function notation statement, $W(0) = 72$, as $(0, 72)$.

Extension: *Math Enrichment*

Tell students that water boils at 212°F. Have them interpret the graph they created in Problem 4 to describe the time interval at which the water on the stove was boiling. Answers may vary.

Math Language Development

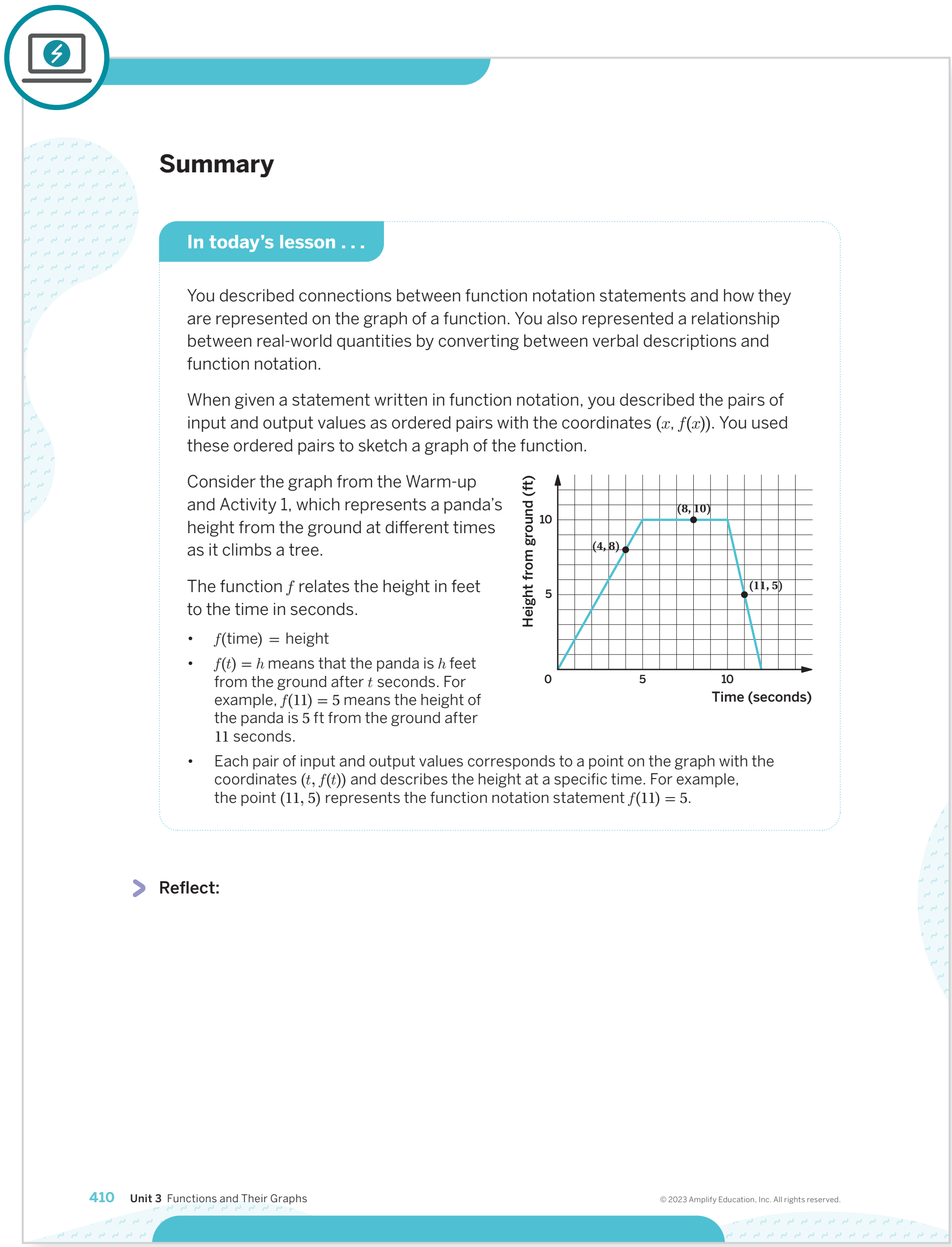
MLR8: Discussion Supports

While students work, display the Anchor Chart PDF, *Sentence Stems, Partner and Group Questioning* to support student discussion. Encourage students to borrow phrases from the anchor chart and to respectfully challenge each other's reasoning when they disagree.

During the Connect, as students share their graphs, have them describe in words what is happening to the pot of water as time passes. Encourage the use of mathematical vocabulary, such as *increases*, *remains constant*, *decreases*, etc.

Summary

Review and synthesize the process of analyzing and sketching functions in context to explain connections between graphs, verbal descriptions, and function notation.



Summary

In today's lesson . . .

You described connections between function notation statements and how they are represented on the graph of a function. You also represented a relationship between real-world quantities by converting between verbal descriptions and function notation.

When given a statement written in function notation, you described the pairs of input and output values as ordered pairs with the coordinates $(x, f(x))$. You used these ordered pairs to sketch a graph of the function.

Consider the graph from the Warm-up and Activity 1, which represents a panda's height from the ground at different times as it climbs a tree.

The function f relates the height in feet to the time in seconds.

- $f(\text{time}) = \text{height}$
- $f(t) = h$ means that the panda is h feet from the ground after t seconds. For example, $f(11) = 5$ means the height of the panda is 5 ft from the ground after 11 seconds.
- Each pair of input and output values corresponds to a point on the graph with the coordinates $(t, f(t))$ and describes the height at a specific time. For example, the point $(11, 5)$ represents the function notation statement $f(11) = 5$.

Reflect:

410 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the graph from Activity 1 and label specific points to emphasize different features of the graph.

Have students share how they can interpret information given by a graph, a verbal description, and statements written in function notation.

Highlight the ways different representations of functions are related. For example, the point $(8, 10)$ on the graph can be expressed as $f(8) = 10$, and means that the panda is at a distance of 10 ft from the ground after 8 seconds.

Ask, "What are some advantages of a function represented by a graph?" A graph can visually describe a relationship between quantities.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

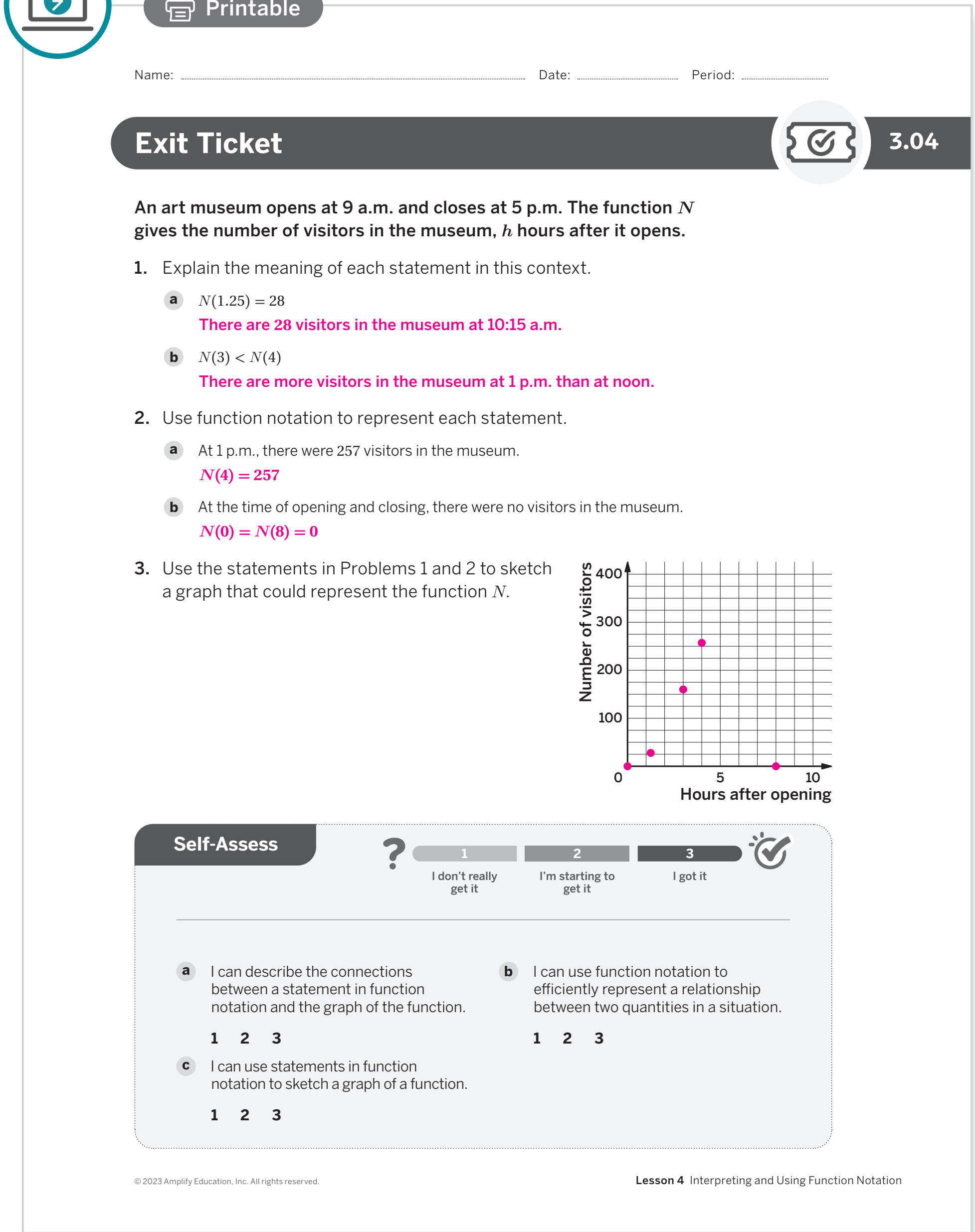
- "How can you compare multiple representations of functions?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding of function notation by interpreting and sketching a graph of a function that represents a real-world situation.

Printable



Name: ______________ Date: ________ Period: ________

Exit Ticket 3.04

An art museum opens at 9 a.m. and closes at 5 p.m. The function N gives the number of visitors in the museum, h hours after it opens.

1. Explain the meaning of each statement in this context.
   - a $N(1.25) = 28$
     There are 28 visitors in the museum at 10:15 a.m.
   - b $N(3) < N(4)$
     There are more visitors in the museum at 1 p.m. than at noon.
2. Use function notation to represent each statement.
   - a At 1 p.m., there were 257 visitors in the museum.
     $\mathbf{N(4) = 257}$
   - b At the time of opening and closing, there were no visitors in the museum.
     $\mathbf{N(0) = N(8) = 0}$
3. Use the statements in Problems 1 and 2 to sketch a graph that could represent the function N.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can describe the connections between a statement in function notation and the graph of the function.
1 2 3

b I can use function notation to efficiently represent a relationship between two quantities in a situation.
1 2 3

c I can use statements in function notation to sketch a graph of a function.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Interpreting and Using Function Notation

Success looks like . . .

- **Language Goal:** Describing connections between statements that use function notation and a graph of the function. **(Speaking and Listening, Writing)**
  - » Representing each statement with function notation in Problem 2.
- **Language Goal:** Interpreting statements that use function notation and explaining their meaning in terms of a situation. **(Speaking and Listening, Writing)**
- **Goal:** Sketching a graph of a function given statements in function notation.
  - » Sketching a graph of the function representing hours after opening and the number of visitors in Problem 3.

Suggested next steps

If students are confused about interpreting or writing statements in function notation in Problems 1 and 2, consider:

- Reviewing examples from Activity 3 or assigning Practice Problem 2.
- Displaying the Anchor Chart PDF, *Function Notation* and describing how each component of the statement relates to the situation.

If students cannot sketch a graph of the function, consider:

- Reviewing the example in the Summary.
- Assigning Practice Problem 4.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What resources did students use as they worked on Activity 1? Which resources were especially helpful?
- What challenges did students encounter as they worked on Activity 3? How did they work through them? What might you change for the next time you teach this lesson?

Math Language Development

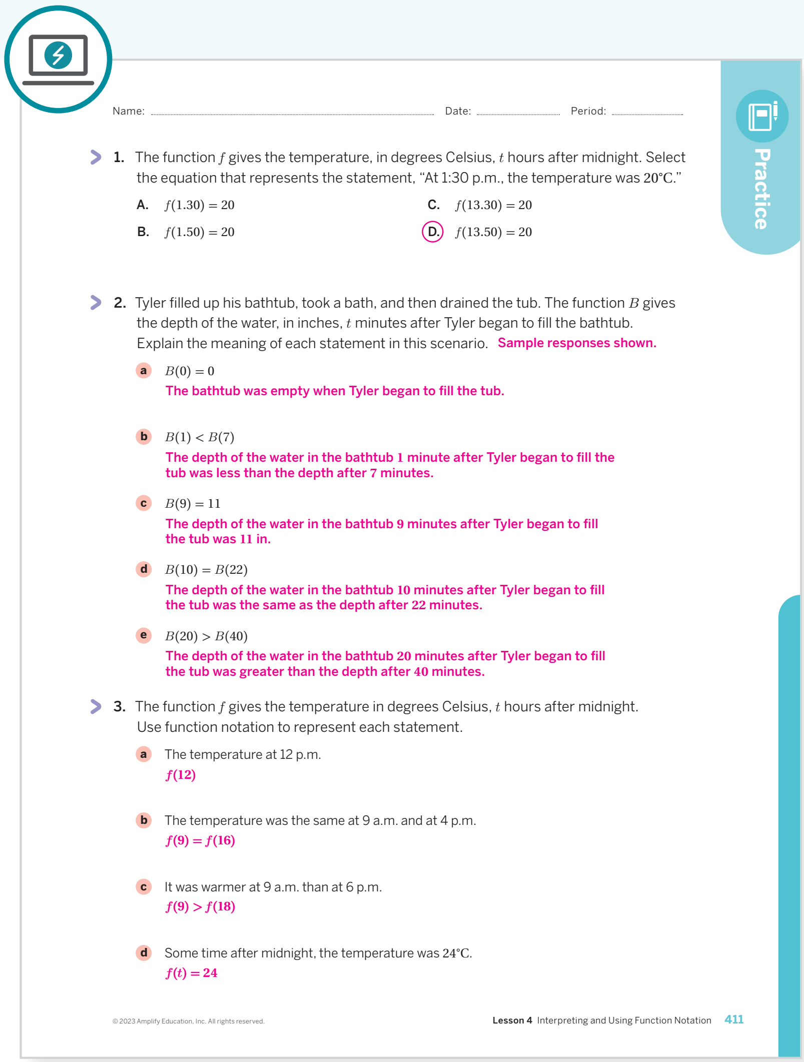
Language Goal: Interpreting statements that use function notation and explaining their meaning in terms of a situation.

Reflect on students' language development toward this goal.

- How have students progressed in their explanations of the meaning of function notation so far in this unit? Are they using language such as "_____ is a function of _____"?
- How did using the language routines in this lesson help students interpret the structure of function notation in order to be able to interpret the notation in context?

Practice

Independent



Name: Date: Period:

Practice

1. The function f gives the temperature, in degrees Celsius, t hours after midnight. Select the equation that represents the statement, “At 1:30 p.m., the temperature was $20°C$.”

   A. $f(1.30) = 20$
   C. $f(13.30) = 20$
   B. $f(1.50) = 20$
   [✓ D.] $f(13.50) = 20$

2. Tyler filled up his bathtub, took a bath, and then drained the tub. The function B gives the depth of the water, in inches, t minutes after Tyler began to fill the bathtub. Explain the meaning of each statement in this scenario. **Sample responses shown.**

   a. $B(0) = 0$
   The bathtub was empty when Tyler began to fill the tub.

   b. $B(1) < B(7)$
   The depth of the water in the bathtub 1 minute after Tyler began to fill the tub was less than the depth after 7 minutes.

   c. $B(9) = 11$
   The depth of the water in the bathtub 9 minutes after Tyler began to fill the tub was 11 in.

   d. $B(10) = B(22)$
   The depth of the water in the bathtub 10 minutes after Tyler began to fill the tub was the same as the depth after 22 minutes.

   e. $B(20) > B(40)$
   The depth of the water in the bathtub 20 minutes after Tyler began to fill the tub was greater than the depth after 40 minutes.

3. The function f gives the temperature in degrees Celsius, t hours after midnight. Use function notation to represent each statement.

   a. The temperature at 12 p.m.
   $f(12)$

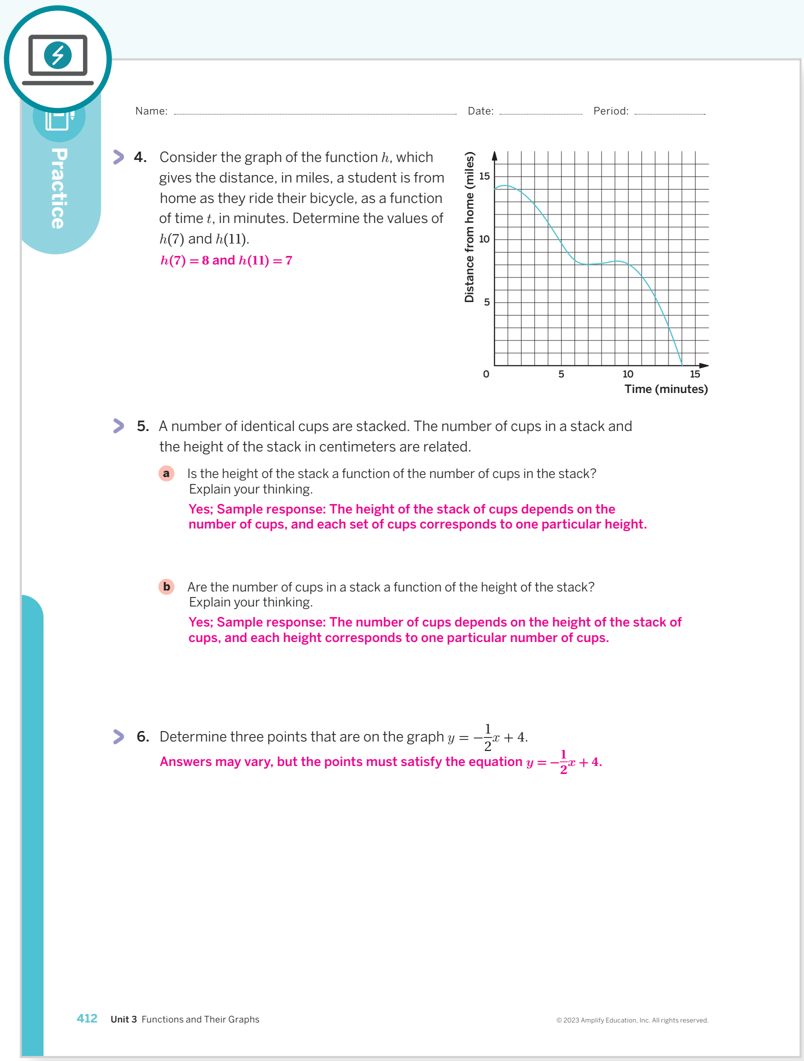
   b. The temperature was the same at 9 a.m. and at 4 p.m.
   $f(9) = f(16)$

   c. It was warmer at 9 a.m. than at 6 p.m.
   $f(9) > f(18)$

   d. Some time after midnight, the temperature was $24°C$.
   $f(t) = 24$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 4 Interpreting and Using Function Notation 411



Name: Date: Period:

Practice

4. Consider the graph of the function h, which gives the distance, in miles, a student is from home as they ride their bicycle, as a function of time t, in minutes. Determine the values of $h(7)$ and $h(11)$.
   $h(7) = 8$ and $h(11) = 7$

5. A number of identical cups are stacked. The number of cups in a stack and the height of the stack in centimeters are related.

   a. Is the height of the stack a function of the number of cups in the stack? Explain your thinking.
   Yes; Sample response: The height of the stack of cups depends on the number of cups, and each set of cups corresponds to one particular height.

   b. Are the number of cups in a stack a function of the height of the stack? Explain your thinking.
   Yes; Sample response: The number of cups depends on the height of the stack of cups, and each height corresponds to one particular number of cups.

6. Determine three points that are on the graph $y = -\frac{1}{2}x + 4$.
   Answers may vary, but the points must satisfy the equation $y = -\frac{1}{2}x + 4$.

412 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 3	2
	3	Activity 3	2
Spiral	4	Unit 3 Lesson 3	2
	5	Unit 3 Lesson 2	2
Formative ⚡	6	Unit 3 Lesson 5	2

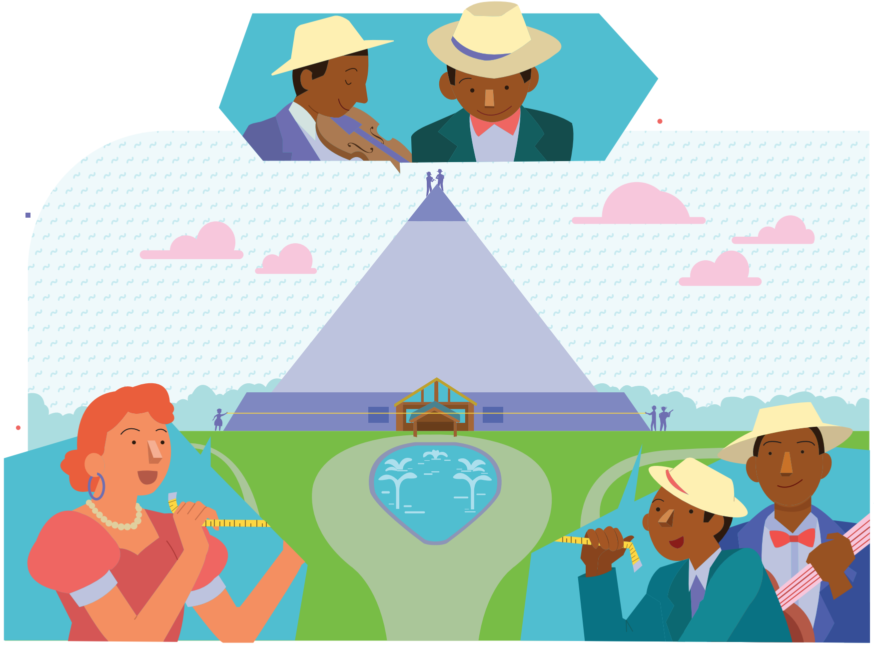
⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 3 | **LESSON 5**

Using Function Notation to Describe Rules (Part 1)



Let’s explore how to use function notation to write equations that represent function rules.

Focus

Goals

1. **Goal:** Create tables and graphs to represent a function given statements in function notation.
2. **Goal:** Interpret rules of functions that are expressed using function notation.
3. **Goal:** Use function notation to write equations that represent rules of functions.

Coherence

Today

Students use function notation to express the rule of a function and use structure to make connections between verbal and algebraic representations of the function. They also analyze real-world scenarios and use tables, graphs, and equations to solve problems.

Previously

In Lesson 4, students interpreted and wrote statements in function notation to represent relationships between different output values. They used this information to analyze and create graphs of a function.

Coming Soon

In Lesson 6, students will use equations in function notation to determine missing input or output values.

Rigor

- Students develop **procedural fluency** writing equations using function notation.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Independent	Small Groups	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one card per student
- Activity 1 PDF (answers)
- Anchor Chart PDF, *Sentence Stems, Notice and Wonder* (as needed)
- Instructional Routine PDF, *Jigsaw: Instructions*

Math Language Development

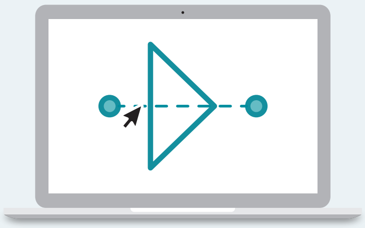
Review words

- *function*
- *function notation*

Amps | **Featured Activity**

Activity 2
Sketching Graphs

Students sketch how the area of the base of a pyramid changes with side length, and you can overlay these sketches in real time.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might lack the discipline to draw connections between the different representations of a function, but structure of function notation does just that. Through the structure of function notation, students can see how the verbal description and the algebraic representation of a function align.

Modifications to Pacing

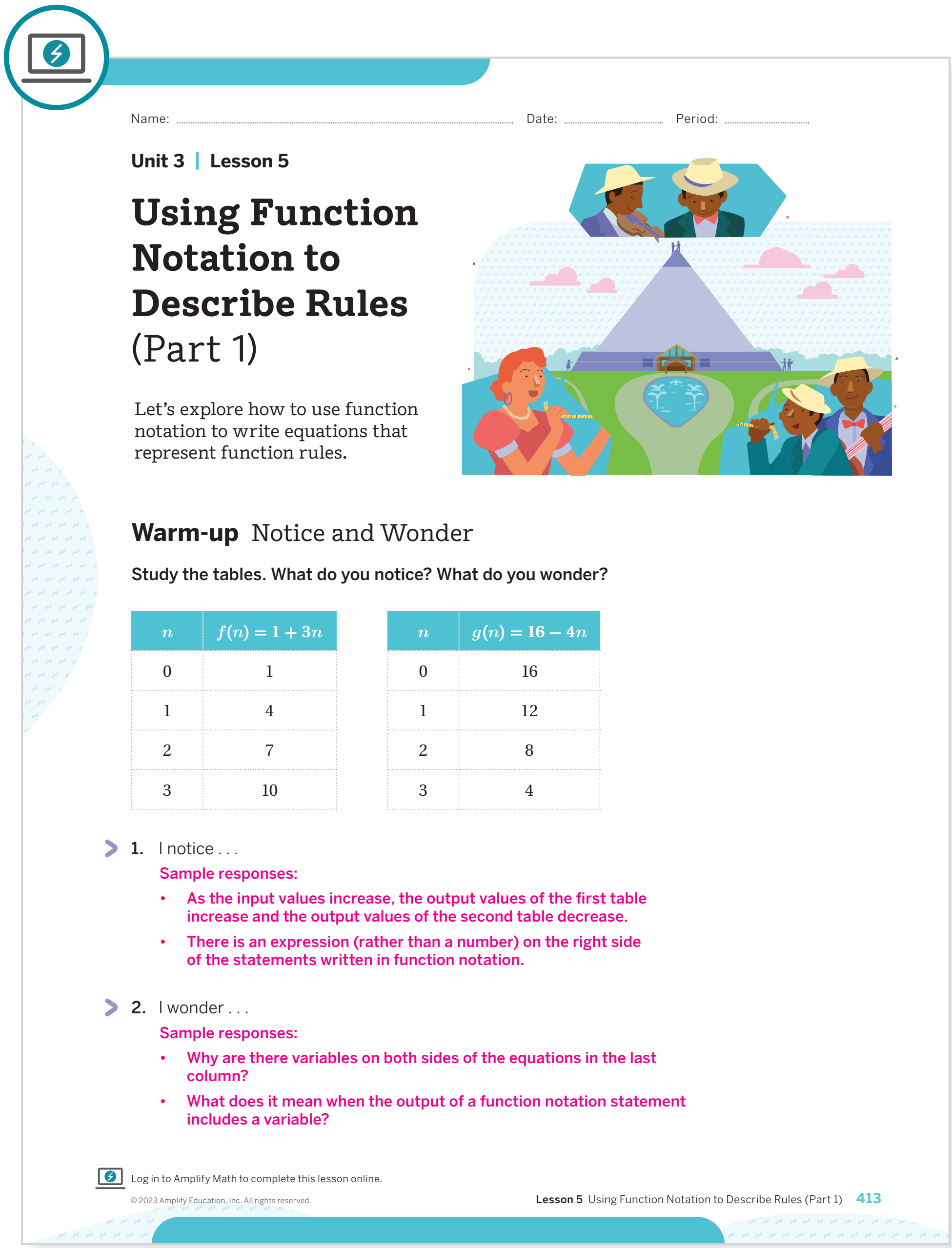
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, instead of grouping students into a new group, discuss observations as a whole class.
- Omit Problem 1 in **Activity 2**.

Independent | 5 min

Warm-up Notice and Wonder

Students analyze two input-output tables to notice the relationship between the input and output values from the function notation.



Name: ____________ Date: ________ Period: ________

Unit 3 | Lesson 5

Using Function Notation to Describe Rules (Part 1)

Let's explore how to use function notation to write equations that represent function rules.

Warm-up Notice and Wonder

Study the tables. What do you notice? What do you wonder?

n	$f(n) = 1 + 3n$
0	1
1	4
2	7
3	10

n	$g(n) = 16 - 4n$
0	16
1	12
2	8
3	4

1. I notice . . .

   Sample responses:

   - **As the input values increase, the output values of the first table increase and the output values of the second table decrease.**
   - **There is an expression (rather than a number) on the right side of the statements written in function notation.**

2. I wonder . . .

   Sample responses:

   - **Why are there variables on both sides of the equations in the last column?**
   - **What does it mean when the output of a function notation statement includes a variable?**

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Give the students a minute of think-time to study the table. Have students complete the problems independently and tell them there are no incorrect responses. Conduct the ***Notice and Wonder*** routine.

2 Monitor

Help students get started by reminding them that a relationship between variables can sometimes be expressed as an equation.

Look for points of confusion:

- **Struggling to interpret the function notation.** Remind students how the input and output are represented in function notation.

Look for productive strategies:

- Substituting the values of n into each equation to get the values in the second column.
- Annotating the equation of the function to show that x and y correspond to n and $g(n)$.

3 Connect

Display the tables.

Have students share what they notice and wonder. Record and display students' thinking for each corresponding table.

Highlight that the columns representing the output correspond to the expression describing each function evaluated at the different input values. Say, "$f(n)$ and $g(n)$ represent an output and can also be expressed as $f(n) = y$ and $g(n) = y$."

Ask, "Is there anything on the list of responses that you are wondering about?" Encourage students to ask for clarification or identify any contradicting information.

Math Language Development

MLR5: Co-craft Questions

After students complete Problems 1 and 2, have them meet with a partner to write 2–3 mathematical questions about the values shown in the table. The collaboration will help them consider other aspects of the relationships in the table they might not have considered on their own.

English Learners

Display or provide the Anchor Chart PDF, *Sentence Stems, Notice and Wonder* to support students in structuring their responses. Allow students to rehearse what they will say before sharing with the whole class.

Power-up

To power up students' ability to determine whether a coordinate pair is on the graph of a linear function when given an equation, have students complete:

Recall that a coordinate pair is on the graph of an equation if, when the values of the coordinate pair are substituted for x and y, the equation is true.

Use substitution to determine whether (2, 7) is on the graph of the function $y = 2x + 3$.

Yes; $7 = 2(2) + 3$

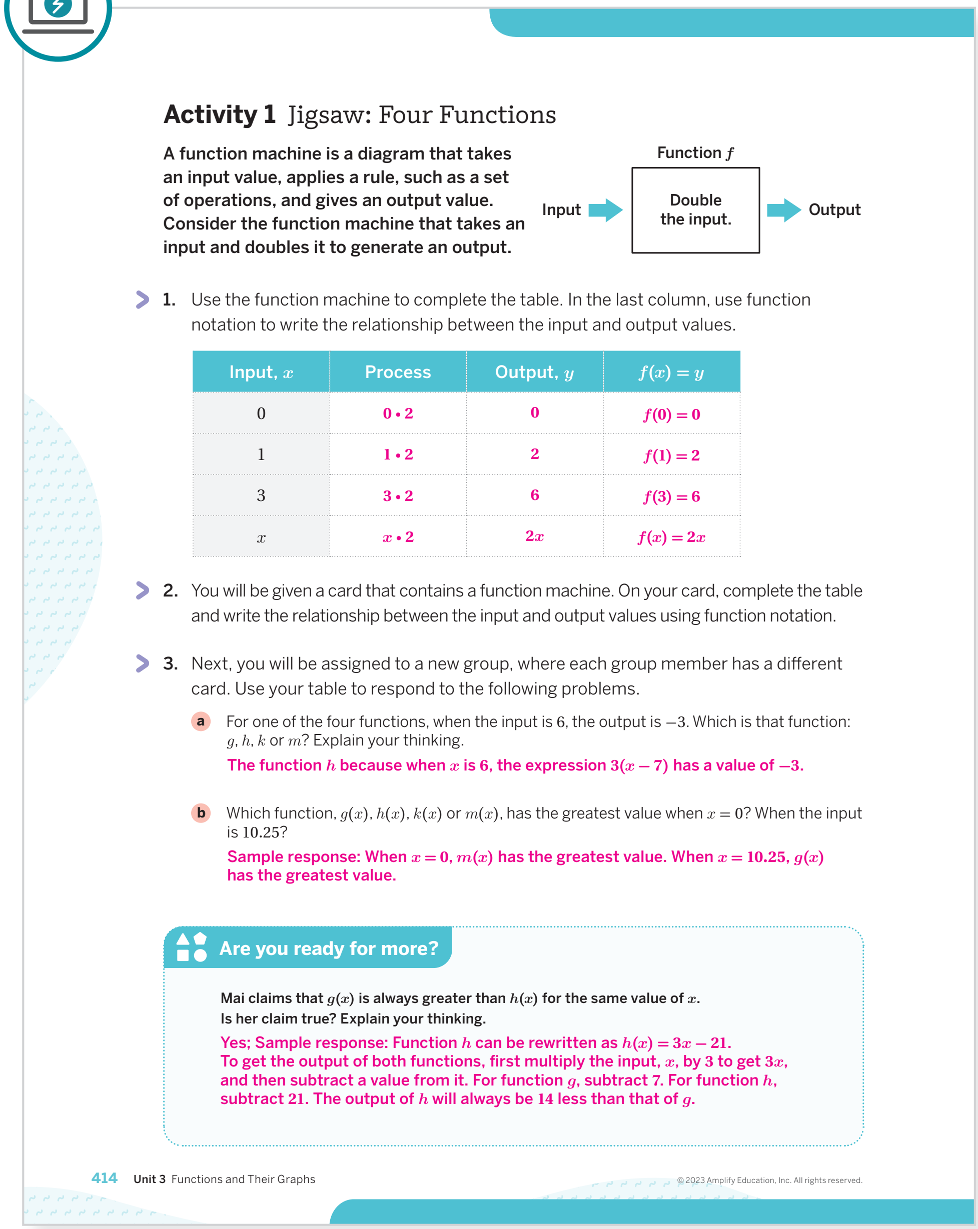
Use: Before Activity 1

Informed by: Performance on Lesson 4, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 3

Small groups | 20 min

Activity 1 Jigsaw: Four Functions

Students make use of structure to write an algebraic equation in function notation to define a function.



Activity 1 Jigsaw: Four Functions

A function machine is a diagram that takes an input value, applies a rule, such as a set of operations, and gives an output value. Consider the function machine that takes an input and doubles it to generate an output.

Function f

Input → Double the input. → Output

1. Use the function machine to complete the table. In the last column, use function notation to write the relationship between the input and output values.

Input, x	Process	Output, y	$f(x) = y$
0	$0 \cdot 2$	0	$f(0) = 0$
1	$1 \cdot 2$	2	$f(1) = 2$
3	$3 \cdot 2$	6	$f(3) = 6$
x	$x \cdot 2$	$2x$	$f(x) = 2x$

2. You will be given a card that contains a function machine. On your card, complete the table and write the relationship between the input and output values using function notation.

3. Next, you will be assigned to a new group, where each group member has a different card. Use your table to respond to the following problems.

   a. For one of the four functions, when the input is 6, the output is -3. Which is that function: g, h, k or m? Explain your thinking.

   The function h because when x is 6, the expression $3(x - 7)$ has a value of -3.

   b. Which function, $g(x)$, $h(x)$, $k(x)$ or $m(x)$, has the greatest value when $x = 0$? When the input is 10.25?

   Sample response: When $x = 0$, $m(x)$ has the greatest value. When $x = 10.25$, $g(x)$ has the greatest value.

Are you ready for more?

Mai claims that $g(x)$ is always greater than $h(x)$ for the same value of x. Is her claim true? Explain your thinking.

Yes; Sample response: Function h can be rewritten as $h(x) = 3x - 21$. To get the output of both functions, first multiply the input, x, by 3 to get $3x$, and then subtract a value from it. For function g, subtract 7. For function h, subtract 21. The output of h will always be 14 less than that of g.

414 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Give students time to complete the table independently before having a whole class discussion. Distribute the pre-cut cards from the Activity 1 PDF so that each student in a group receives the same card. Conduct the ***Jigsaw*** routine and regroup the students so that each student in the group has a different card.

2 Monitor

Help students get started by annotating the last column in the table, writing "Input" for x and "Output" for y.

Look for points of confusion:

- **Having difficulty expressing a relationship as a function.** Have students translate the verbal description of a function into an equation written in function notation.
- **Writing incorrect equations for the output of a function.** Encourage students to substitute an input value and output value to check that the point makes the equation true.

Look for productive strategies:

- Recognizing a pattern in the tables and generalizing a rule to relate the input values to the output values.
- Substituting the input values into the function to determine and compare the output values.

3 Connect

Display the four tables with numerical solutions and record student responses for the last row.

Have groups of students share their generalized equations and any connections between their tables or verbal descriptions.

Highlight explanations for Problem 3 that mention evaluating each function at a different input to see which generates the correct output.

Ask, "How did you determine your responses for Problems 1 and 2?"

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them omit the first row on each table for Cards 1–4.

Accessibility: *Activate Prior Knowledge, Guide Processing and Visualization*

Remind students they worked with function machines in Grade 8. Consider demonstrating how the function machine shown produces an output, based on the function rule and an input value. Ask students to volunteer a sample input value.

Math Language Development

MLR8: Discussion Supports

During the Connect, draw students' attention to connections across representations (verbal description, table of values, and generalized equation written). Consider asking the following for Problem 1:

- "Where do you see 'double the input' in the table?" Sample response: 0 is double 0, 2 is double 1, and 6 is double 3.
- "Where do you see 'double the input' in the equation written in function notation?" Sample response: $2x$ is double x.

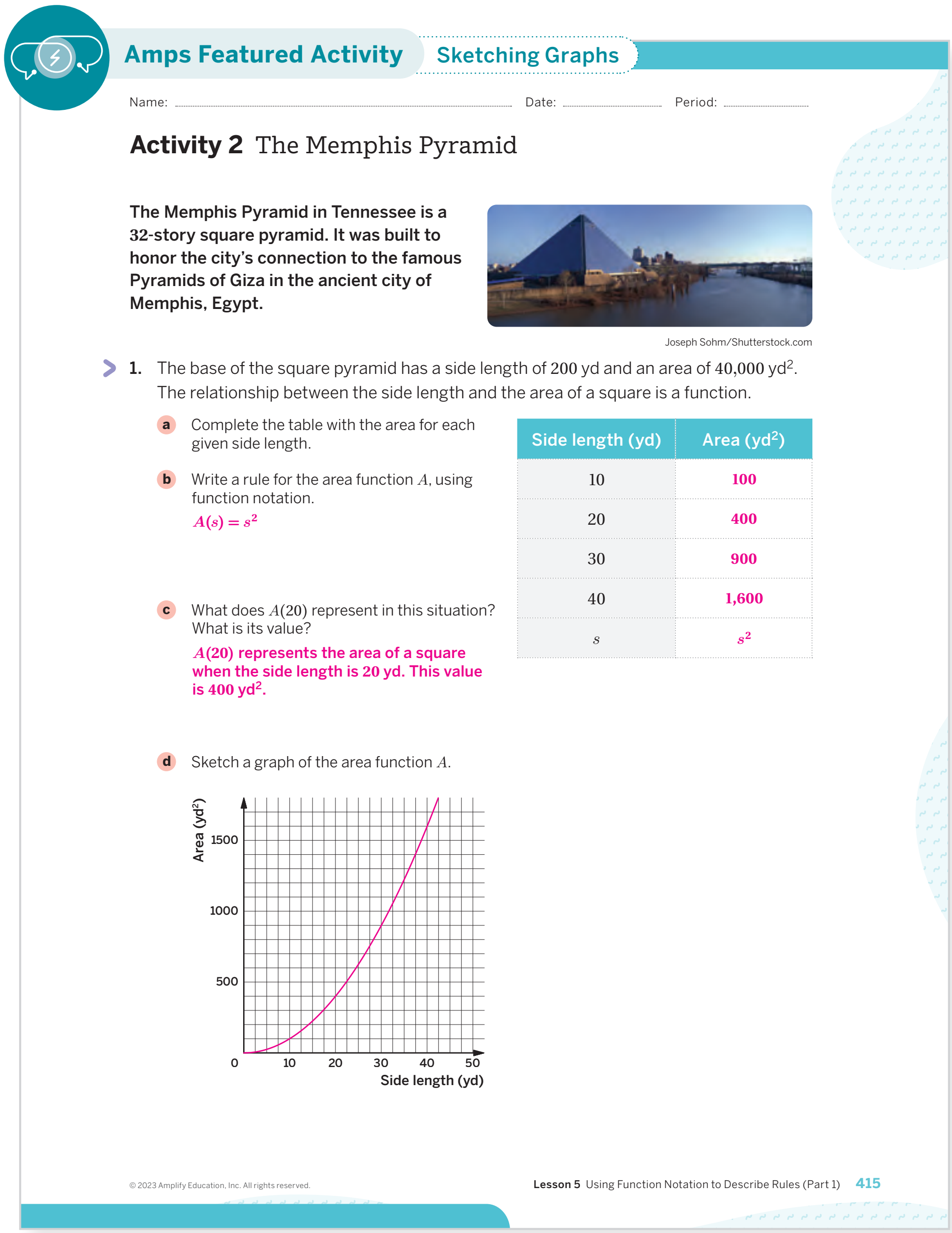
English Learners

Annotate the phrase "double the input" with other phrases, such as "multiply by 2" to reinforce different ways of expressing the same relationship.

Pairs | 15 min

Activity 2 The Memphis Pyramid

Students analyze geometric relationships to construct and interpret an equation written in function notation.



Amps Featured Activity Sketching Graphs

Name: Date: Period:

Activity 2 The Memphis Pyramid

The Memphis Pyramid in Tennessee is a 32-story square pyramid. It was built to honor the city's connection to the famous Pyramids of Giza in the ancient city of Memphis, Egypt.

Joseph Sohm/Shutterstock.com

1. The base of the square pyramid has a side length of 200 yd and an area of 40,000 yd^2. The relationship between the side length and the area of a square is a function.

   a. Complete the table with the area for each given side length.

   b. Write a rule for the area function A, using function notation.
   $A(s) = s^2$

   c. What does $A(20)$ represent in this situation? What is its value?
   $A(20)$ represents the area of a square when the side length is 20 yd. This value is 400 yd^2.

   d. Sketch a graph of the area function A.

Side length (yd)	Area (yd^2)
10	100
20	400
30	900
40	1,600
s	s^2

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 5 Using Function Notation to Describe Rules (Part 1) 415

1 Launch

Arrange students in pairs and read the narrative for Problem 1 together as a class. Have students work independently before sharing their thinking with their partner. Repeat for Problem 2.

2 Monitor

Help students get started by asking, "What is the difference between the area and the perimeter of a rectangle?"

Look for points of confusion:

- **Having difficulty generalizing a pattern to write an equation to represent the relationship.** Have students write a statement to express the relationship and then translate it into an equation.

Look for productive strategies:

- Looking for patterns within the table of values to determine a relationship.
- Annotating statements about the relationship between the quantities to represent the components of function notation.

Activity 2 continued ›

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they have ever seen photos of the Pyramids of Giza in Egypt, or even the Memphis Pyramid in Tennessee. Ask them to describe what the phrase "square pyramid" tells them. The shape of the base of the pyramid is a square.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can change the length of a rectangular playground to determine possible perimeters when the width remains fixed.

Extension: Math Enrichment

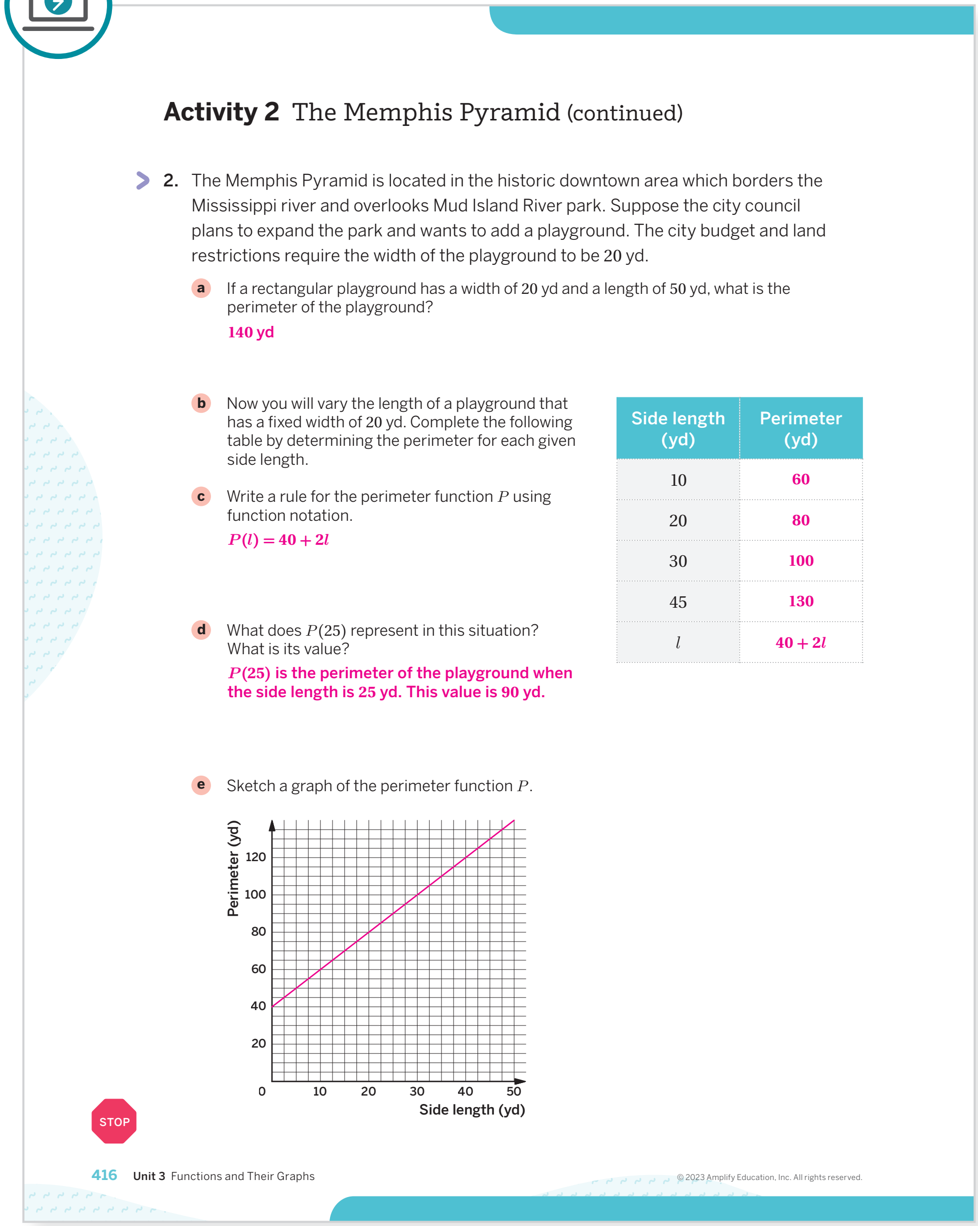
Ask students to analyze the area and perimeter graphs and respond to the following questions:

- "What is $A(0)$? Does this value make sense within this context? Why or why not?" $A(0) = 0$; This value makes sense within this context because if there was no length (0 yd), there would be no area (0 yd^2).
- "What is $P(0)$? Does this value make sense within this context? Why or why not?" $P(0) = 40$; This value does not make sense within this context because if there was no length (0 yd), the rectangle could not exist and the perimeter would not exist.

Pairs | 15 min

Activity 2 The Memphis Pyramid (continued)

Students analyze geometric relationships to construct and interpret an equation written in function notation.



Activity 2 The Memphis Pyramid (continued)

2. The Memphis Pyramid is located in the historic downtown area which borders the Mississippi river and overlooks Mud Island River park. Suppose the city council plans to expand the park and wants to add a playground. The city budget and land restrictions require the width of the playground to be 20 yd.

a. If a rectangular playground has a width of 20 yd and a length of 50 yd, what is the perimeter of the playground?

140 yd

b. Now you will vary the length of a playground that has a fixed width of 20 yd. Complete the following table by determining the perimeter for each given side length.

Side length (yd)	Perimeter (yd)
10	60
20	80
30	100
45	130
l	$40 + 2l$

c. Write a rule for the perimeter function P using function notation.

$P(l) = 40 + 2l$

d. What does $P(25)$ represent in this situation? What is its value?

$P(25)$ is the perimeter of the playground when the side length is 25 yd. This value is 90 yd.

e. Sketch a graph of the perimeter function P.

STOP

416 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have students share their responses to Problem 2. Select and sequence pairs to discuss how they determined the rule for the perimeter function P. Record the equations that students share and display for the remainder of the lesson.

Highlight that the equations displayed are equivalent and all represent a linear relationship. Show this is true by substituting the values from the table into each equation.

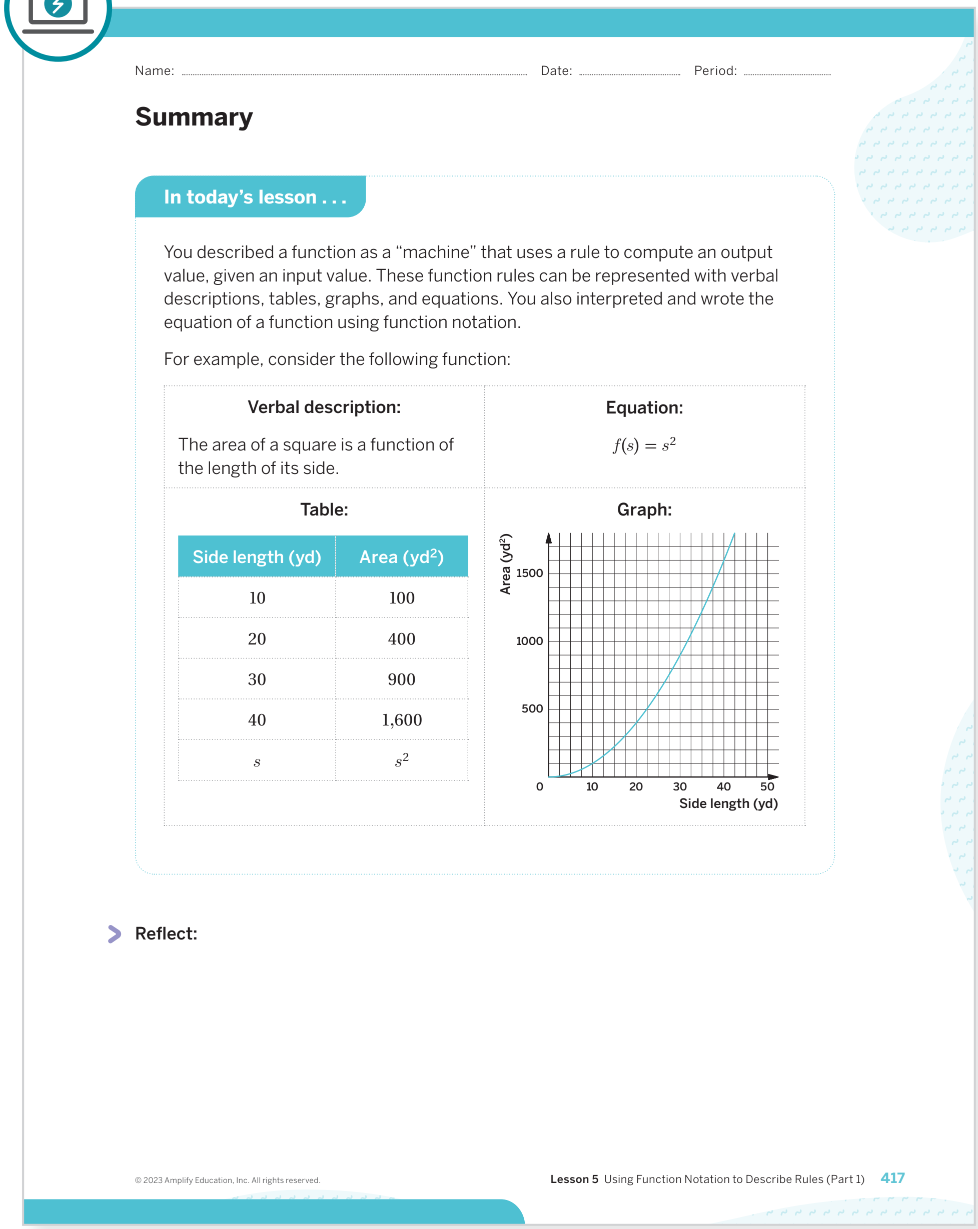
Ask:

- "Do these equations all represent the same function? Explain your thinking." Yes; Substituting the same input value into each equation all yield the same output value.
- "Is the relationship between perimeter and side length linear? Explain your thinking." Yes because the graph is a line.
- "What terms or coefficients in the equation $P(\ell) = 2\ell + 60$ represent the slope and vertical intercept?" 2 is the slope and 60 is the vertical intercept.

Whole Class | 5 min

Summary

Review and synthesize different ways to represent and understand functions by making connections between tables, equations, graphs and statements written in function notation.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You described a function as a "machine" that uses a rule to compute an output value, given an input value. These function rules can be represented with verbal descriptions, tables, graphs, and equations. You also interpreted and wrote the equation of a function using function notation.

For example, consider the following function:

Verbal description:

The area of a square is a function of the length of its side.

Equation:

$f(s) = s^2$

Table:

Side length (yd)	Area (yd^2)
10	100
20	400
30	900
40	1,600
s	s^2

Graph:

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 5 Using Function Notation to Describe Rules (Part 1) 417

Synthesize

Display the table and graph from Problem 1 in Activity 2.

Have students share how to interpret the equation when it is written in function notation.

Highlight that functions define the relationship between quantities with a rule. Rules can be expressed in different but equivalent ways.

Ask, "How can you graph a function described with an equation?"

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

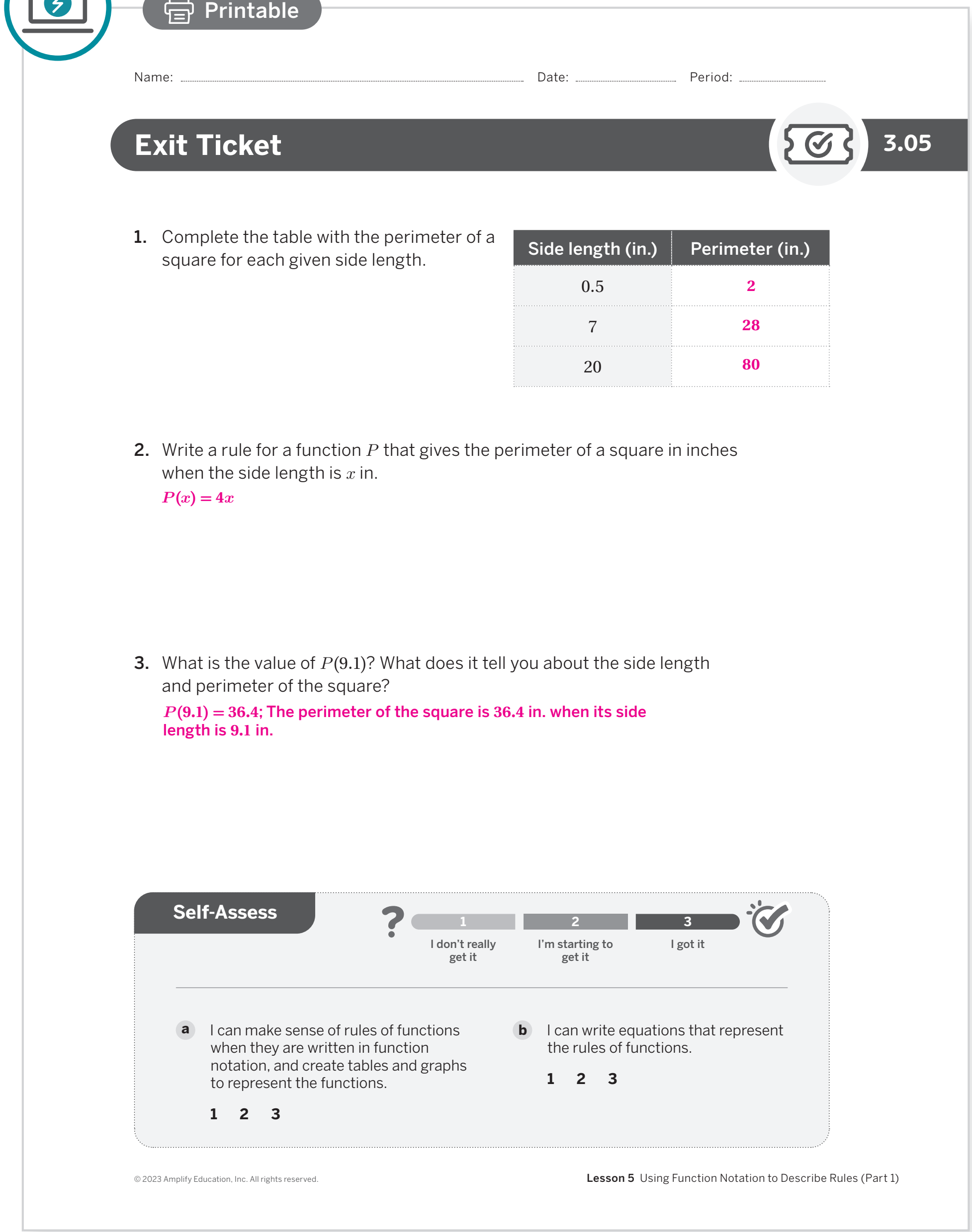
- "How do equations, written using function notation, help you determine the output value of functions at different input values?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by constructing and interpreting equations written in function notation that model a real-world context.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.05

1. Complete the table with the perimeter of a square for each given side length.

Side length (in.)	Perimeter (in.)
0.5	2
7	28
20	80

2. Write a rule for a function P that gives the perimeter of a square in inches when the side length is x in.

$P(x) = 4x$

3. What is the value of $P(9.1)$? What does it tell you about the side length and perimeter of the square?

$P(9.1) = 36.4$; The perimeter of the square is 36.4 in. when its side length is 9.1 in.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can make sense of rules of functions when they are written in function notation, and create tables and graphs to represent the functions.

1 2 3

b I can write equations that represent the rules of functions.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 5 Using Function Notation to Describe Rules (Part 1)

Success looks like . . .

- **Goal:** Creating tables and graphs to represent a function given statements in function notation.
- **Goal:** Interpreting rules of functions that are expressed using function notation. **(Speaking, Listening and Writing)**
  - » Interpreting the value of $P(9.1)$ in Problem 3.
- **Goal:** Using function notation to write equations that represent rules of functions.
  - » Writing the rule for the function P in Problem 2.

Suggested next steps

If students cannot write a function equation in Problem 2, consider:

- Reviewing generalizing patterns from Activity 2.
- Assigning Practice Problem 2.

If students cannot interpret function notation in Problem 3, consider:

- Reviewing how to interpret function notation in context from the summary of Lesson 4.
- Assigning Practice Problem 4.

Professional Learning

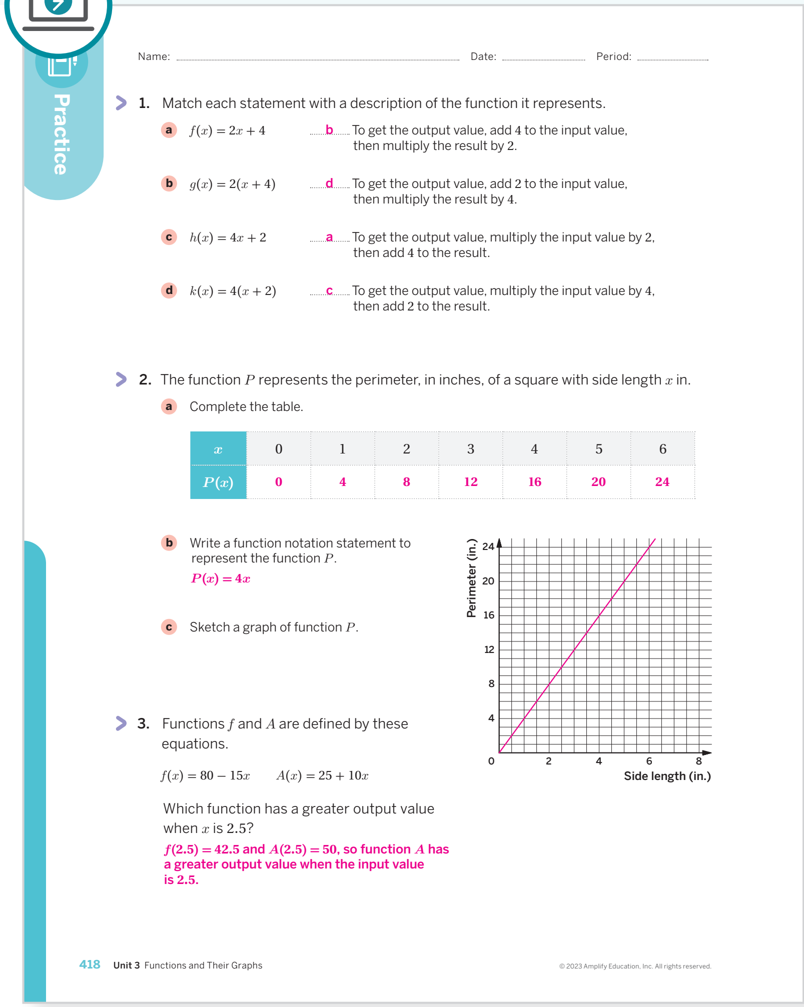
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Knowing where students need to be by the end of this unit, how did writing equations for functions influence that future goal?
- How did students look for and use structure today? What might you change for the next time you teach this lesson?

Practice

Independent



Name: .... Date: .... Period: ....

1. Match each statement with a description of the function it represents.

a $f(x) = 2x + 4$ ..b.. To get the output value, add 4 to the input value, then multiply the result by 2.

b $g(x) = 2(x + 4)$ ..d.. To get the output value, add 2 to the input value, then multiply the result by 4.

c $h(x) = 4x + 2$ ..a.. To get the output value, multiply the input value by 2, then add 4 to the result.

d $k(x) = 4(x + 2)$ ..c.. To get the output value, multiply the input value by 4, then add 2 to the result.

2. The function P represents the perimeter, in inches, of a square with side length x in.

a Complete the table.

x	0	1	2	3	4	5	6
$P(x)$	0	4	8	12	16	20	24

b Write a function notation statement to represent the function P.

$P(x) = 4x$

c Sketch a graph of function P.

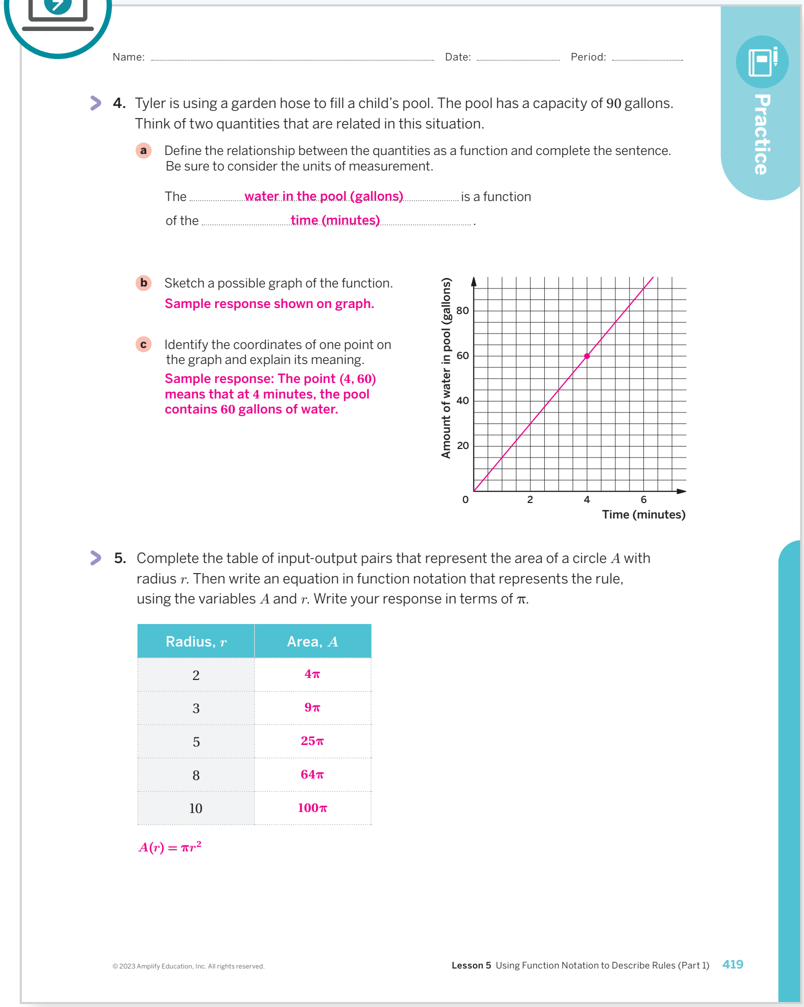
3. Functions f and A are defined by these equations.

$f(x) = 80 - 15x$ $A(x) = 25 + 10x$

Which function has a greater output value when x is 2.5?

$f(2.5) = 42.5$ and $A(2.5) = 50$, so function A has a greater output value when the input value is 2.5.

418 **Unit 3** Functions and Their Graphs © 2023 Amplify Education, Inc. All rights reserved.



Name: .... Date: .... Period: ....

4. Tyler is using a garden hose to fill a child's pool. The pool has a capacity of 90 gallons. Think of two quantities that are related in this situation.

a Define the relationship between the quantities as a function and complete the sentence. Be sure to consider the units of measurement.

The ....**water in the pool (gallons)**.... is a function

of the ....**time (minutes)**.... .

b Sketch a possible graph of the function.

Sample response shown on graph.

c Identify the coordinates of one point on the graph and explain its meaning.

Sample response: The point $(4, 60)$ means that at 4 minutes, the pool contains 60 gallons of water.

5. Complete the table of input-output pairs that represent the area of a circle A with radius r. Then write an equation in function notation that represents the rule, using the variables A and r. Write your response in terms of π.

Radius, r	Area, A
2	4π
3	9π
5	25π
8	64π
10	100π

$A(r) = \pi r^2$

© 2023 Amplify Education, Inc. All rights reserved. **Lesson 5** Using Function Notation to Describe Rules (Part 1) 419

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 4	2
Formative ⚡	5	Unit 3 Lesson 6	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 3 | **LESSON 6**

Using Function Notation to Describe Rules (Part 2)

Let's explore different ways to determine the input value of a function, given its output value, and vice versa.

Focus

Goals

1. Evaluate functions and solve equations given in function notation, either by graphing or by reasoning algebraically.
2. Understand a linear function as a function whose output changes at a constant rate and whose graph is a line.
3. Use technology to graph and evaluate functions given in function notation.

Coherence

Today

Students solve problems as they interpret function notation statements and graphs of functions and contextualize the solutions within the context of the real-world problems the functions represent. Students will use graphing technology to graph equations of functions written in function notation and determine missing input and output values. They also revisit the definition for linear functions which were defined in Grade 8. The definition is updated in this course to incorporate new mathematical understandings.

Previously

In Lesson 5, students wrote equations of functions with function notation and used different representations of functions to analyze real-world scenarios.

Coming Soon

In Lesson 7, students will learn new vocabulary to describe important features of graphs of functions.

Rigor

- Students develop **procedural fluency** with function notation by evaluating and solving equations written in function notation.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2 (optional)	Summary	Exit Ticket
10 min	25 min	20 min	10 min	5 min
Independent	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- graphing technology
- rulers

Math Language Development

New words

- ***linear function***

Review words

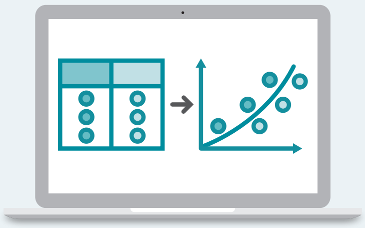
- *function*
- *function notation*
- *rate of change*
- *slope*
- *slope-intercept form*
- *y-intercept*

Amps | Featured Activity

Activity 1

Using Work From Previous Slides

In later slides, students can build on their work from previous slides to help them create a graph.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might always choose to use technology such as the graphing calculator when working with functions. Discuss the pros and cons of using technology and ask students to describe how they will decide whether they need a calculator or not. Explain that sometimes the use of a calculator is less efficient because the analysis could be done quickly using mental math.

Modifications to Pacing

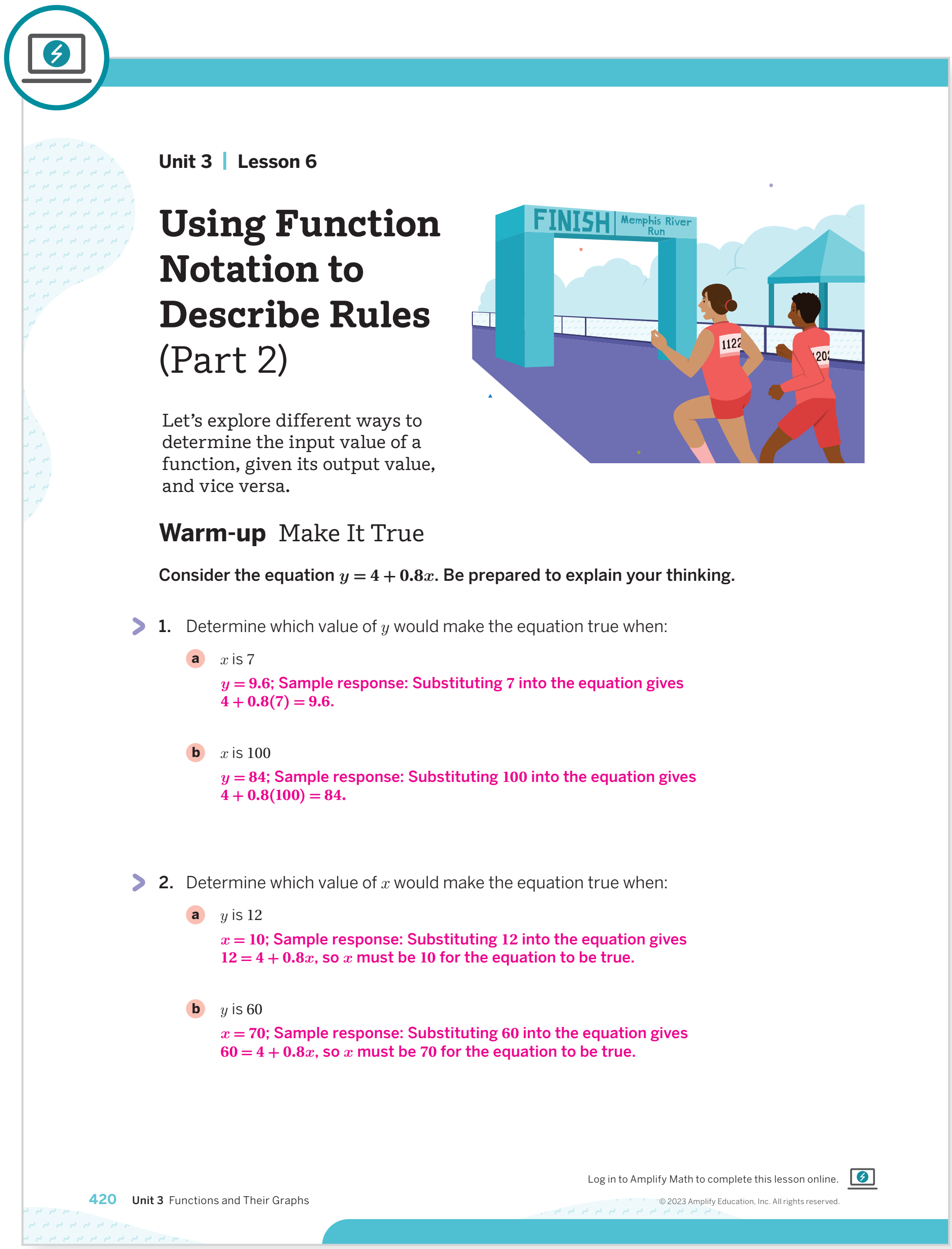
You may want to consider these additional modifications if you are short on time.

- Omit Problems 1b and 2b in the **Warm-up**.
- Optional **Activity 2** may be omitted.

Independent | 10 min

Warm-up Make It True

Students review how to algebraically determine solutions of two-variable equations to prepare for working with equations in function notation.


Unit 3 | Lesson 6

Using Function Notation to Describe Rules (Part 2)

Let's explore different ways to determine the input value of a function, given its output value, and vice versa.

Warm-up Make It True

Consider the equation $y = 4 + 0.8x$. Be prepared to explain your thinking.

1. Determine which value of y would make the equation true when:
   - a x is 7

     $y = 9.6$; Sample response: Substituting 7 into the equation gives $4 + 0.8(7) = 9.6$.
   - b x is 100

     $y = 84$; Sample response: Substituting 100 into the equation gives $4 + 0.8(100) = 84$.
2. Determine which value of x would make the equation true when:
   - a y is 12

     $x = 10$; Sample response: Substituting 12 into the equation gives $12 = 4 + 0.8x$, so x must be 10 for the equation to be true.
   - b y is 60

     $x = 70$; Sample response: Substituting 60 into the equation gives $60 = 4 + 0.8x$, so x must be 70 for the equation to be true.

Log in to Amplify Math to complete this lesson online.

420 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Have students work independently before sharing their thinking with a partner.

2 Monitor

Help students get started by activating prior knowledge about how to solve for one variable (x or y) if they are given the value of the other.

Look for points of confusion:

- **Struggling to determine the value of x when given y.** Remind students that the solutions to an equation in two variables are the values of x and y that make the equation true.

Look for productive strategies:

- Substituting the given values for x or y into the equation and solving for the missing variable.
- Graphing the equation and recognizing that points on the graph are solutions to the equation.

3 Connect

Have students share their thinking and the strategies they used to determine solutions to the equation.

Highlight that students can use different strategies to determine the values of x or y.

Ask, "What do the ordered pairs $(7, 9.6)$, $(100, 84)$, $(10, 12)$ and $(70, 60)$ represent?" Sample response: These represent solutions to the equation and would also be the coordinates of points that fall on the line when this equation is graphed on the coordinate plane.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they have worked with equations and solutions to equations in prior grades and in prior units. Consider displaying a sample equation solved for y and its equivalent equation solved for x. For example, display the following two equations to help activate students' prior knowledge with writing equivalent equations to isolate for either variable.

$$y = 2x + 3$$

$$x = \frac{y - 3}{2}$$

Power-up

To power up students' ability to complete an input-output table for a given relationship, have students complete:

Recall that the relationship between the radius of a circle and its circumference is $C = 2\pi r$.

Complete the table of input-output pairs for the circumference C of a circle with radius r.

Radius, r	Circumference, C
2	4π
4	8π
5	10π

Use: Before Activity 1

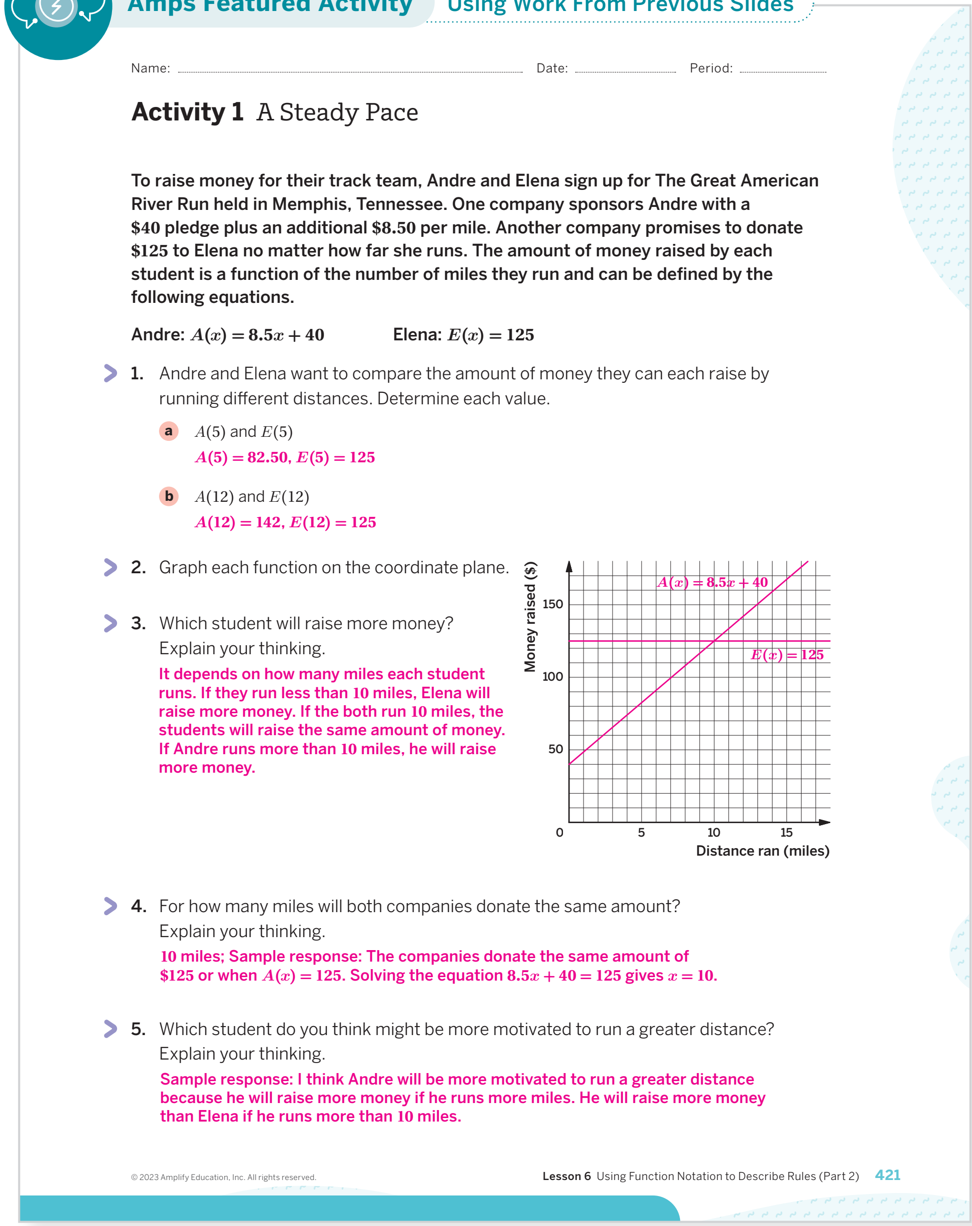
Informed by: Performance on Lesson 5, Practice Problem 5

Pairs | 25 min

Activity 1 A Steady Pace

Students analyze and compare two functions representing a real-world context in order to solve problems and intepret the solutions within the context.

Amps Featured Activity Using Work From Previous Slides



Name: Date: Period:

Activity 1 A Steady Pace

To raise money for their track team, Andre and Elena sign up for The Great American River Run held in Memphis, Tennessee. One company sponsors Andre with a $\$40$ pledge plus an additional $\$8.50$ per mile. Another company promises to donate $\$125$ to Elena no matter how far she runs. The amount of money raised by each student is a function of the number of miles they run and can be defined by the following equations.

Andre: $A(x) = 8.5x + 40$ **Elena:** $E(x) = 125$

1. Andre and Elena want to compare the amount of money they can each raise by running different distances. Determine each value.
   - a $A(5)$ and $E(5)$
     $A(5) = 82.50, E(5) = 125$
   - b $A(12)$ and $E(12)$
     $A(12) = 142, E(12) = 125$
2. Graph each function on the coordinate plane.
3. Which student will raise more money? Explain your thinking.
   It depends on how many miles each student runs. If they run less than 10 miles, Elena will raise more money. If the both run 10 miles, the students will raise the same amount of money. If Andre runs more than 10 miles, he will raise more money.

4. For how many miles will both companies donate the same amount? Explain your thinking.
   10 miles; Sample response: The companies donate the same amount of $\$125$ or when $A(x) = 125$. Solving the equation $8.5x + 40 = 125$ gives $x = 10$.
5. Which student do you think might be more motivated to run a greater distance? Explain your thinking.
   Sample response: I think Andre will be more motivated to run a greater distance because he will raise more money if he runs more miles. He will raise more money than Elena if he runs more than 10 miles.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 6 Using Function Notation to Describe Rules (Part 2) 421

1 Launch

Read the narrative together as a class. Have students work independently before sharing their thinking with a partner. Provide access to rulers.

2 Monitor

Help students get started by asking them to identify the input and output of the function equations and the units in which each variable is measured.

Look for points of confusion:

- **Not recognizing that $E(x)$ is a function.** Ask students if there is only one output value for each input value.
- **Using the *guess-and-check* strategy.** Ask students if there is a more efficient strategy to use.

Look for productive strategies:

- Analyzing the graph to identify the point of intersection of the lines or the x value when $y = 125$.
- Solving the equation $8.5x + 40 = 125$ algebraically.

3 Connect

Display the graph of the two functions.

Have student pairs share their responses to Problems 3 and 4 and record their strategies on the graph. Select and sequence students from least to most productive strategies.

Highlight that the equations represent linear relationships and that there are different ways to determine unknown input and output values of linear functions.

Define the term ***linear function*** as a function that has a constant rate of change.

Ask, "Was it more straightforward to use the graph or the equation to determine output values?"

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can retain their work from earlier slides in the activity to help them create their graph.

Extension: *Math Enrichment*

In this activity, one line had a positive slope and the other line had a slope of zero. Ask students whether it would make sense for a line to have a negative slope in this context, and have them explain their thinking. Sample response: No, a negative slope would not make sense because that would represent a student losing donations the more miles they run.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

Read 1: Students should understand that there are two different ways Andre and Elena are raising money for the run.

Read 2: Ask students to name given quantities and relationships, such as Andre will raise $\$40$ plus $\$8.50$ per mile.

Read 3: Ask students to preview Problem 3 and brainstorm how equations and graphs can help them determine the answer to this question.

English Learners

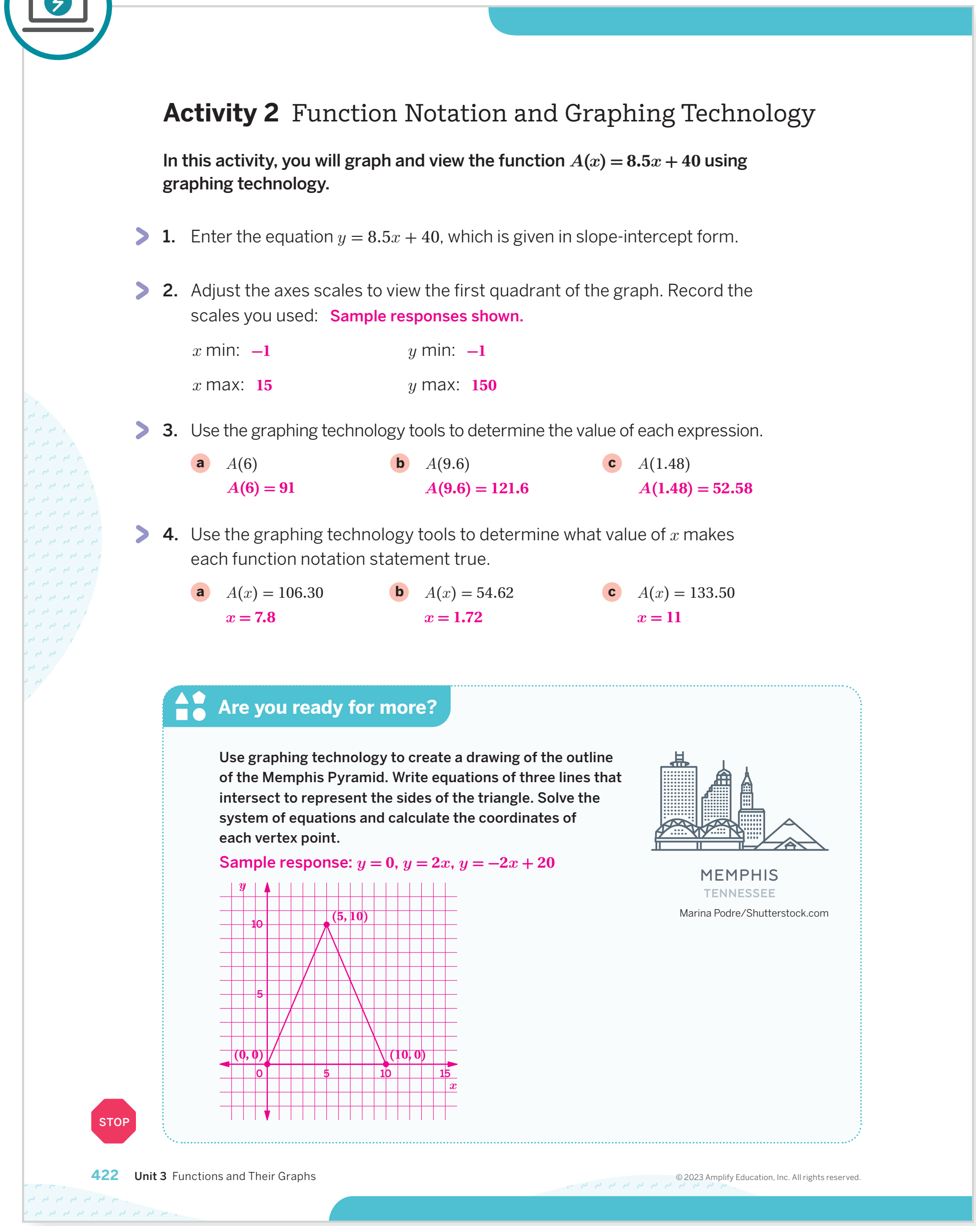
Highlight the phrase "no matter how far she runs" in the text and show that corresponds to the horizontal line.

Optional

Independent | 20 min

Activity 2 Function Notation and Graphing Technology

Students experiment with graphing technology to adjust and choose appropriate axes scales to view the graph of a function.



Activity 2 Function Notation and Graphing Technology

In this activity, you will graph and view the function $A(x) = 8.5x + 40$ using graphing technology.

1. Enter the equation $y = 8.5x + 40$, which is given in slope-intercept form.

2. Adjust the axes scales to view the first quadrant of the graph. Record the scales you used: **Sample responses shown.**

x min: -1 y min: -1

x max: 15 y max: 150

3. Use the graphing technology tools to determine the value of each expression.

a $A(6)$ $A(6) = 91$	**b** $A(9.6)$ $A(9.6) = 121.6$	**c** $A(1.48)$ $A(1.48) = 52.58$

4. Use the graphing technology tools to determine what value of x makes each function notation statement true.

a $A(x) = 106.30$ $x = 7.8$	**b** $A(x) = 54.62$ $x = 1.72$	**c** $A(x) = 133.50$ $x = 11$

Are you ready for more?

Use graphing technology to create a drawing of the outline of the Memphis Pyramid. Write equations of three lines that intersect to represent the sides of the triangle. Solve the system of equations and calculate the coordinates of each vertex point.

Sample response: $y = 0$, $y = 2x$, $y = -2x + 20$

Marina Podre/Shutterstock.com

422 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Provide access to graphing technology. Show the graph of $y = 8.5x + 40$ and ask students to identify one ordered pair on the line.

2 Monitor

Help students get started by providing a tutorial or tour of the graphing technology being used.

Look for points of confusion:

- **Having difficulty adjusting the settings or using the features.** Provide written instructions with images illustrating appropriate keystrokes.

Look for productive strategies:

- Adjusting the axes scales as needed to better view the graph.

3 Connect

Have students share their thinking about using graphing technology. Ask, "Do you prefer using graphing technology or graphing by hand? Explain your preference."

Highlight that students will frequently be using graphing technology throughout the course to create graphs and analyze functions.

Ask, "What are some advantages and disadvantages of using graphing technology when determining the input and output values of a function?"

Sample response:

- Advantages: I can adjust the axes scales quickly. The ordered pairs (x, y) are displayed as I move along the graph. I can toggle back and forth to the table.
- Disadvantages: The window is small and I can only view part of the graph at one time. I can only estimate the values of x and y unless they are exact numbers.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Depending on the type of graphing technology your students are using, provide a graphing technology cheat sheet that includes images and keystrokes for how to enter an equation, adjust the axes scales, and determine the location of specific values along the graph.

Accessibility: *Clarify Vocabulary and Symbols*

Be sure students understand that even though the function is given as $A(x)$, entering the equation into graphing technology usually requires representing the output values with the variable y. This is why the equation in Problem 1 is written in $y =$ form.

Extension: *Math Enrichment*

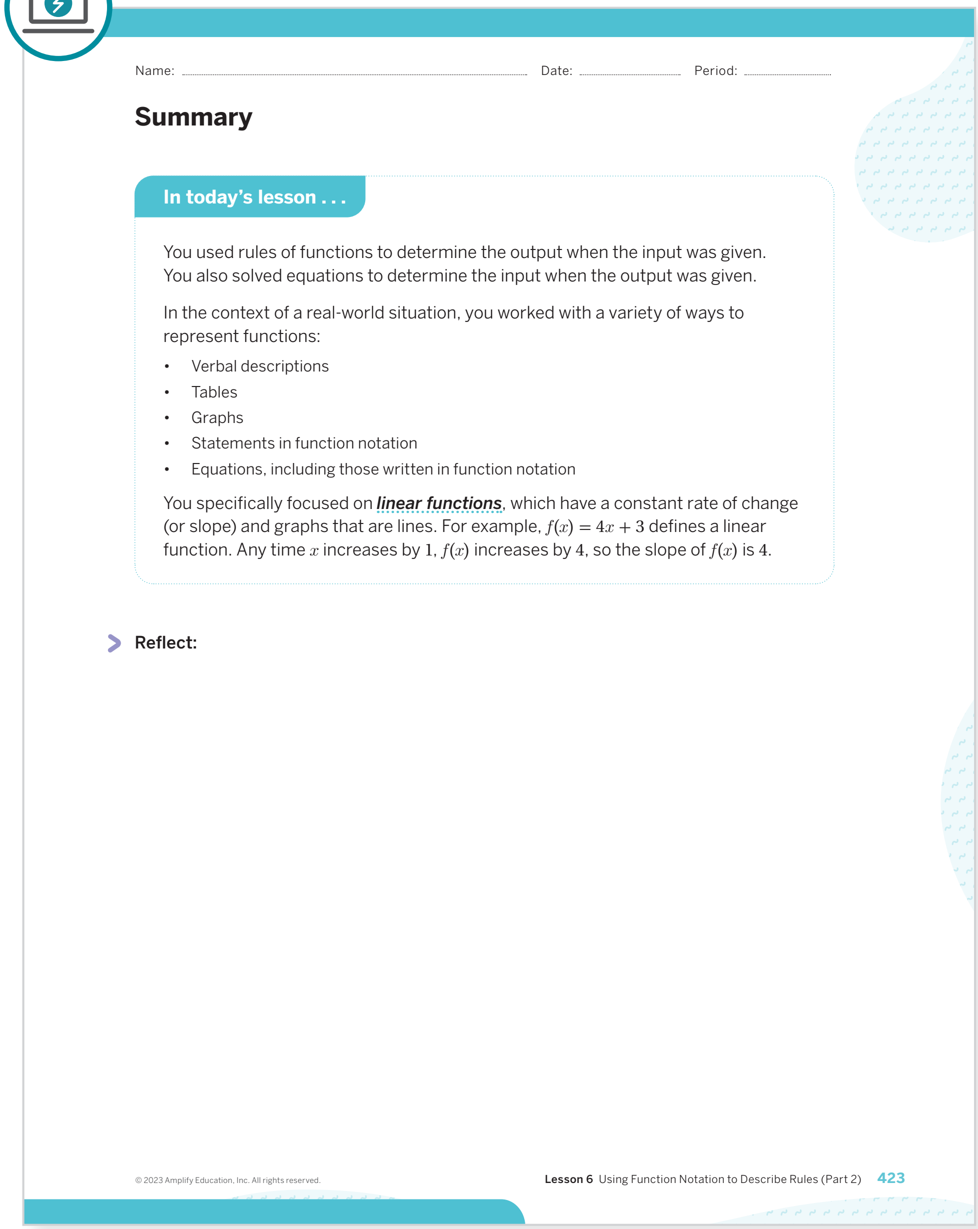
As a follow-up to the Are you ready for more? problem, ask students to verbally describe the interval of x-values and y-values, for each equation, that make up the outline of the pyramid.

$y = 0$	The values of x go from 0 to 10. The value of y is always 0.
$y = 2x$	The values of x go from 0 to 5. The values of y go from 0 to 10.
$y = -2x + 20$	The values of x go from 5 to 10. The values of y go from 10 to 0 as the values of x go from 5 to 10.

Whole Class | 10 min

Summary

Review and synthesize algebraic and graphical representations of equations written in function notation, and how they can be used to determine missing input and output values.



Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You used rules of functions to determine the output when the input was given. You also solved equations to determine the input when the output was given.

In the context of a real-world situation, you worked with a variety of ways to represent functions:

- Verbal descriptions
- Tables
- Graphs
- Statements in function notation
- Equations, including those written in function notation

You specifically focused on ***linear functions***, which have a constant rate of change (or slope) and graphs that are lines. For example, $f(x) = 4x + 3$ defines a linear function. Any time x increases by 1, $f(x)$ increases by 4, so the slope of $f(x)$ is 4.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the equations and graph from Activity 1.

Highlight that knowing the rule that defines a function can be very useful, especially when it is written in function notation. It is useful when determining the input value when the output value is known, creating a table of values, and when making a graph of the function.

Ask, “Which method did you prefer to use when solving equations written in function notation?”

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

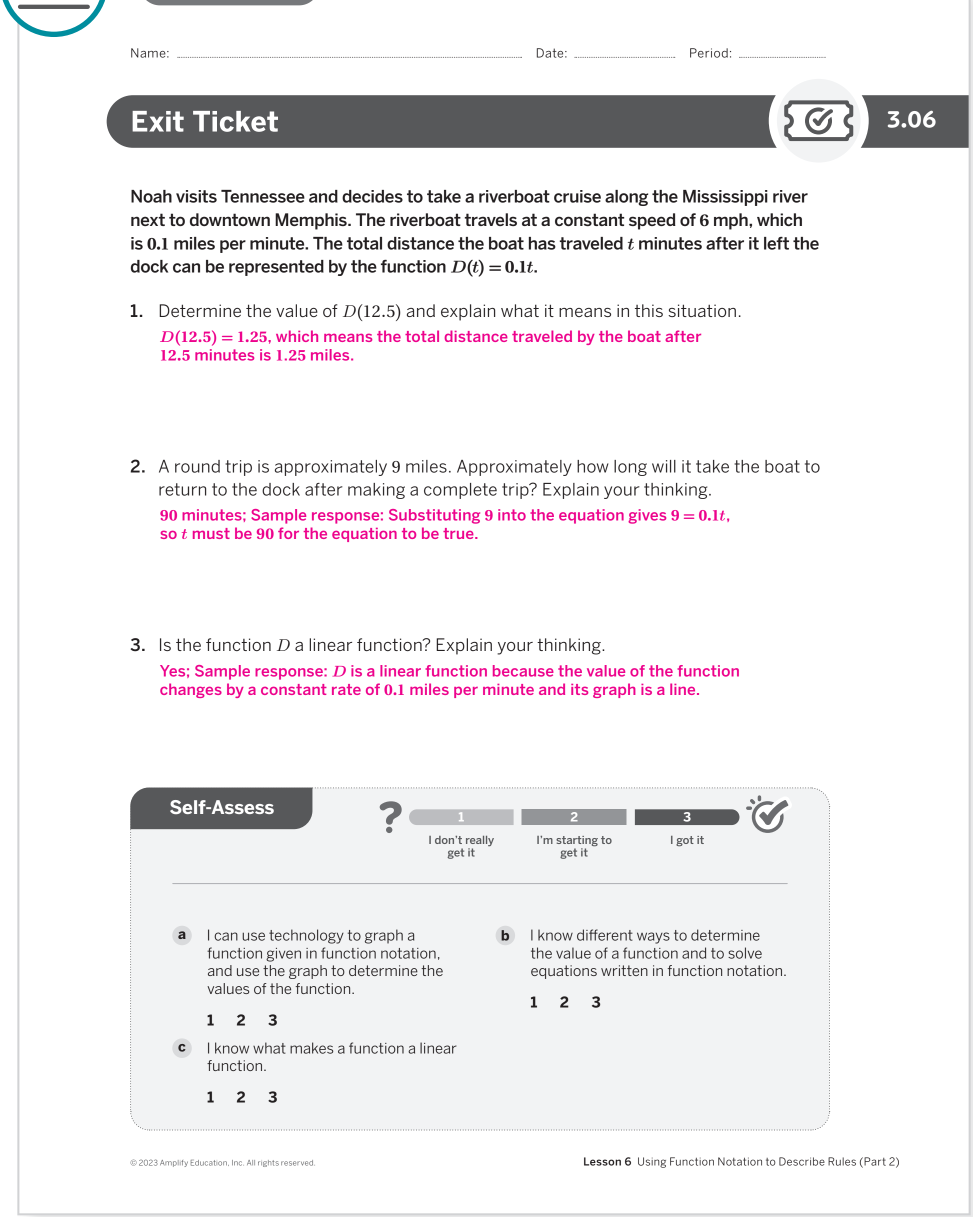
- “How do equations written in function notation help you determine the input value of functions at different output values?”

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by interpreting function notation in context, use the function to solve a problem, and precisely communicating whether the given relationship is a function.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket 3.06

Noah visits Tennessee and decides to take a riverboat cruise along the Mississippi river next to downtown Memphis. The riverboat travels at a constant speed of 6 mph, which is 0.1 miles per minute. The total distance the boat has traveled t minutes after it left the dock can be represented by the function $D(t) = 0.1t$.

1. Determine the value of $D(12.5)$ and explain what it means in this situation.

   $D(12.5) = 1.25$, which means the total distance traveled by the boat after 12.5 minutes is 1.25 miles.

2. A round trip is approximately 9 miles. Approximately how long will it take the boat to return to the dock after making a complete trip? Explain your thinking.

   90 minutes; Sample response: Substituting 9 into the equation gives $9 = 0.1t$, so t must be 90 for the equation to be true.

3. Is the function D a linear function? Explain your thinking.

   Yes; Sample response: D is a linear function because the value of the function changes by a constant rate of 0.1 miles per minute and its graph is a line.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can use technology to graph a function given in function notation, and use the graph to determine the values of the function.

1 2 3

b. I know different ways to determine the value of a function and to solve equations written in function notation.

1 2 3

c. I know what makes a function a linear function.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 6 Using Function Notation to Describe Rules (Part 2)

Success looks like . . .

- **Goal:** Evaluating functions and solving equations given in function notation, either by graphing or by reasoning algebraically.
  - » Determining the value of $D(12.5)$ in Problem 1.
- **Goal:** Understanding a linear function as a function whose output changes at a constant rate and whose graph is a line.
- **Goal:** Using technology to graph and evaluate functions given in function notation.

Suggested next steps

If students cannot evaluate a function and explain what it represents, consider:

- Reviewing Problem 1 from Activity 1.
- Assigning Practice Problem 3a.

If students cannot solve a function equation in Problem 2, consider:

- Reviewing strategies from the Warm-up.
- Assigning Practice Problem 3b.

If students cannot identify a linear function in Problem 3, consider:

- Reviewing the definition of linear function in the Summary.

Professional Learning

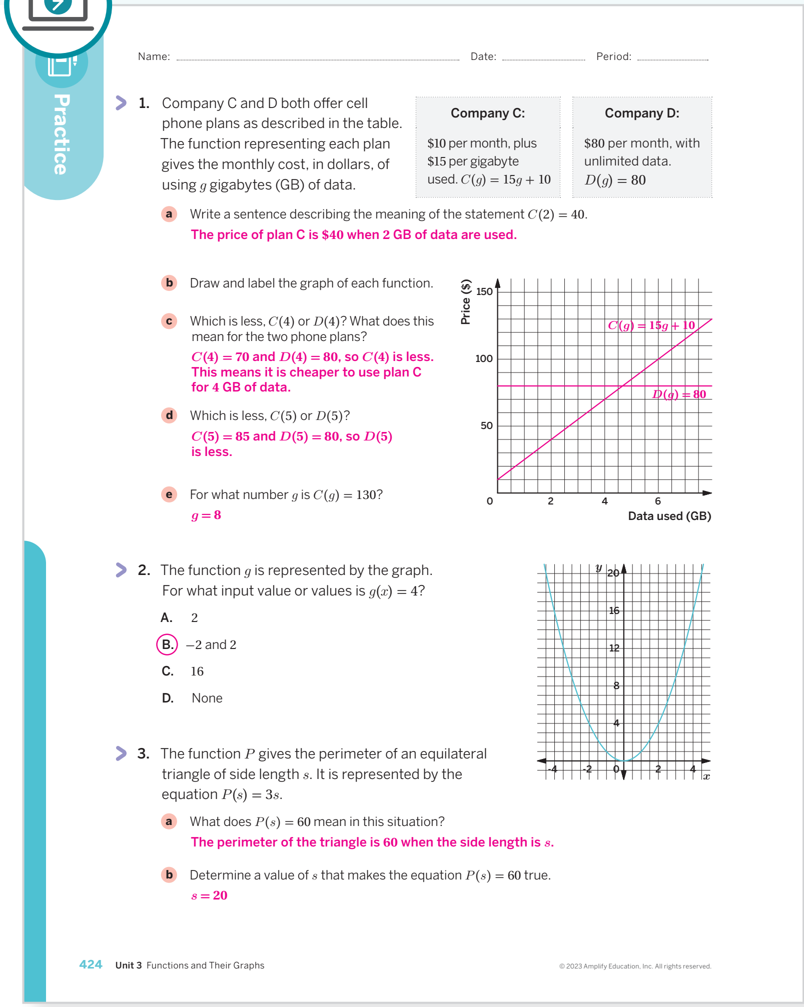
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What did you see in the way some students approached solving equations in function notation that you would like other students to try?
- The focus of this lesson was evaluating and solving equations in function notation. How did solving equations in function notation go? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Company C and D both offer cell phone plans as described in the table. The function representing each plan gives the monthly cost, in dollars, of using g gigabytes (GB) of data.

Company C:	Company D:
\$10 per month, plus \$15 per gigabyte used. $C(g) = 15g + 10$	\$80 per month, with unlimited data. $D(g) = 80$

a. Write a sentence describing the meaning of the statement $C(2) = 40$.
The price of plan C is \$40 when 2 GB of data are used.

b. Draw and label the graph of each function.

c. Which is less, $C(4)$ or $D(4)$? What does this mean for the two phone plans?
$C(4) = 70$ and $D(4) = 80$, so $C(4)$ is less. This means it is cheaper to use plan C for 4 GB of data.

d. Which is less, $C(5)$ or $D(5)$?
$C(5) = 85$ and $D(5) = 80$, so $D(5)$ is less.

e. For what number g is $C(g) = 130$?
$g = 8$

2. The function g is represented by the graph. For what input value or values is $g(x) = 4$?

A. 2

[✓ B.] −2 and 2

C. 16

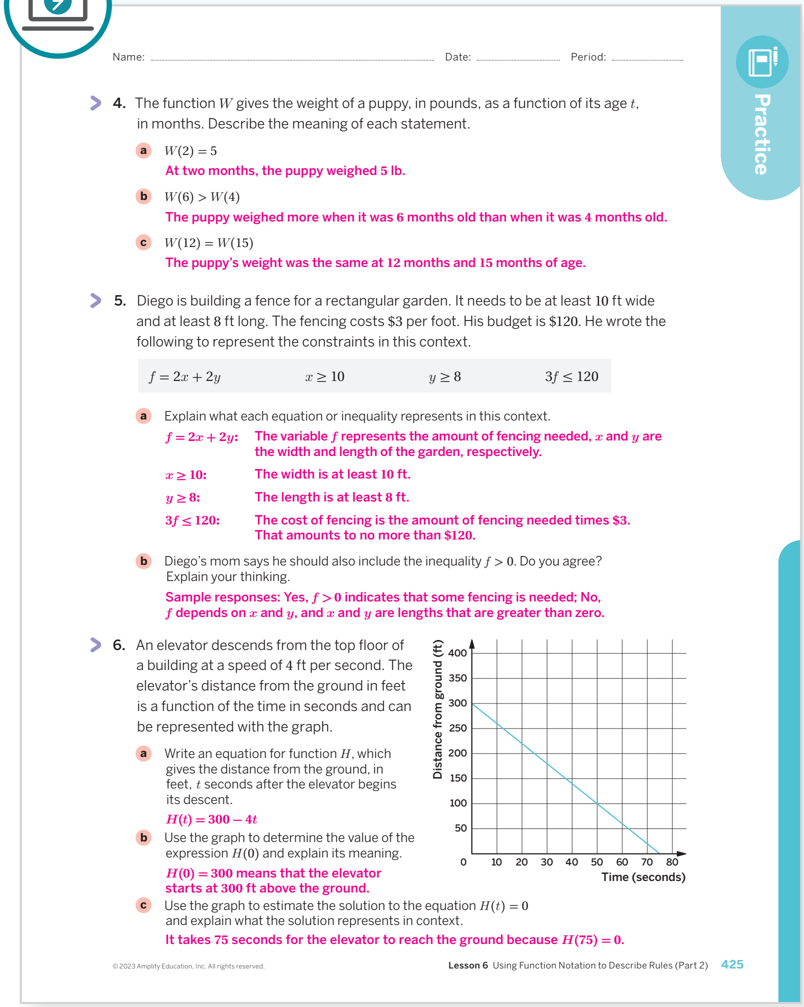
D. None

3. The function P gives the perimeter of an equilateral triangle of side length s. It is represented by the equation $P(s) = 3s$.

a. What does $P(s) = 60$ mean in this situation?
The perimeter of the triangle is 60 when the side length is s.

b. Determine a value of s that makes the equation $P(s) = 60$ true.
$s = 20$

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: Date: Period:

4. The function W gives the weight of a puppy, in pounds, as a function of its age t, in months. Describe the meaning of each statement.

a. $W(2) = 5$
At two months, the puppy weighed 5 lb.

b. $W(6) > W(4)$
The puppy weighed more when it was 6 months old than when it was 4 months old.

c. $W(12) = W(15)$
The puppy's weight was the same at 12 months and 15 months of age.

5. Diego is building a fence for a rectangular garden. It needs to be at least 10 ft wide and at least 8 ft long. The fencing costs \$3 per foot. His budget is \$120. He wrote the following to represent the constraints in this context.

$f = 2x + 2y$ $x \geq 10$ $y \geq 8$ $3f \leq 120$

a. Explain what each equation or inequality represents in this context.

$f = 2x + 2y$:	**The variable f represents the amount of fencing needed, x and y are the width and length of the garden, respectively.**
$x \geq 10$:	**The width is at least 10 ft.**
$y \geq 8$:	**The length is at least 8 ft.**
$3f \leq 120$:	**The cost of fencing is the amount of fencing needed times \$3. That amounts to no more than \$120.**

b. Diego's mom says he should also include the inequality $f > 0$. Do you agree? Explain your thinking.
Sample responses: Yes, $f > 0$ indicates that some fencing is needed; No, f depends on x and y, and x and y are lengths that are greater than zero.

6. An elevator descends from the top floor of a building at a speed of 4 ft per second. The elevator's distance from the ground in feet is a function of the time in seconds and can be represented with the graph.

a. Write an equation for function H, which gives the distance from the ground, in feet, t seconds after the elevator begins its descent.
$H(t) = 300 - 4t$

b. Use the graph to determine the value of the expression $H(0)$ and explain its meaning.
$H(0) = 300$ means that the elevator starts at 300 ft above the ground.

c. Use the graph to estimate the solution to the equation $H(t) = 0$ and explain what the solution represents in context.
It takes 75 seconds for the elevator to reach the ground because $H(75) = 0$.

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 4	2
	5	Unit 1 Lesson 5	2
Formative	6	Unit 3 Lesson 7	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 2

Whole Class

Analyzing and Creating Graphs of Functions

In this Sub-Unit, students use new language to describe key features of graphs and create graphs of music situations west of the Mississippi river.

SUB-UNIT

2 Analyzing and Creating Graphs of Functions

Narrative Connections

What's the function of a jazz solo?

Jazz has its roots in the history and culture of New Orleans, Louisiana. Its distinctive sound draws from numerous threads of New Orleans culture.

In the 18th century, enslaved Africans held dance and music gatherings in what is now known as Congo Square. West African and Caribbean drumming and call-and-response chanting were combined with the instruments of European colonizers: guitars, trumpets, and pianos.

It is from this blend of cultures that jazz was born. One of jazz music's most defining characteristics is its improvisation. In a band, each musician might take up the main theme of a song and riff on it, weaving in and out of each other's lines. A soloist such as the New Orleans-born trumpeter Louis Armstrong can use a song's chords to invent a brand new melody on the spot.

In each case, the musician takes an input — a chord progression, a theme, or a melody — then applies knowledge, creativity, and skill to transform it into something new. And just as a solo can be transcribed into sheet music, a function can also be graphed in order to be analyzed and understood.

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore creating and analyzing graphs of functions — within the context of music and the city of New Orleans — in the following places:

- **Lesson 7, Activity 3:** Jazz and Heritage Festival
- **Lesson 9, Activity 2:** Traffic in New Orleans
- **Lesson 10, Activities 1–2:** Reasonable Inputs, Card Sort: Do the Input Values Make Sense?
- **Lesson 13, Activities 1–3:** Changing Pitch, Comparing Scenarios, The New Orleans Skyline

UNIT 3 | **LESSON 7**

Features of Graphs

Let's determine important features of graphs of functions.

Focus

Goals

1. **Language Goal:** Determine important features of graphs of functions and explain what they mean in the situations represented. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Understand and use the terms *horizontal intercept*, *vertical intercept*, *maximum*, and *minimum* when talking about functions and their graphs. **(Speaking and Listening, Reading and Writing)**

Coherence

Today

Students build the need for important vocabulary when describing characteristics of functions. In the first activity, students work to describe the graph of a scenario in detail, and then have their partner sketch a graph based on their description. This drives the need for precise vocabulary for the features of functions that students continue to build on in the second activity. In Activity 3, students are given vocabulary for key features and equations associated with a scenario and match these with statements describing the situation.

Previously

In Lessons 2–6, students described and graphed situations and learned how to use and interpret function notation.

Coming Soon

In Lessons 8 and 9, students will learn about how to scale graphs and about discrete situations and average rate of change.

Rigor

- Students build **procedural fluency** defining and using vocabulary to refer to key features of graphs.
- Students **apply** their understanding of key features of graphs in context.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	10 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per pair

Math Language Development

New words

- *global maximum**
- *global minimum**
- *local maximum**
- *local minimum**

Review words

- *decreasing*
- *function*
- *horizontal intercept*
- *increasing*
- *vertical intercept*

* Students may be familiar with the everyday use of the terms *global* and *local*. Be ready to address how the everyday meanings are similar to the mathematical meanings as they refer to points on the graph of a function.

Amps | Featured Activity

Activity 3
Interactive Jazz and Heritage Festival Graph

Students are given expressions, verbal descriptions, and mathematical vocabulary that refer to features of a graph and are able to drag and match each item.

Building Math Identity and Community

Connecting to Mathematical Practices

As students describe key features of the graphs of the functions in Activities 1–3, encourage their developing use of precise mathematical terms. Allow them to describe the features of the graphs in their own words, especially at first, and model the mathematical vocabulary to help them make connections and develop their mathematical language skills.

Modifications to Pacing

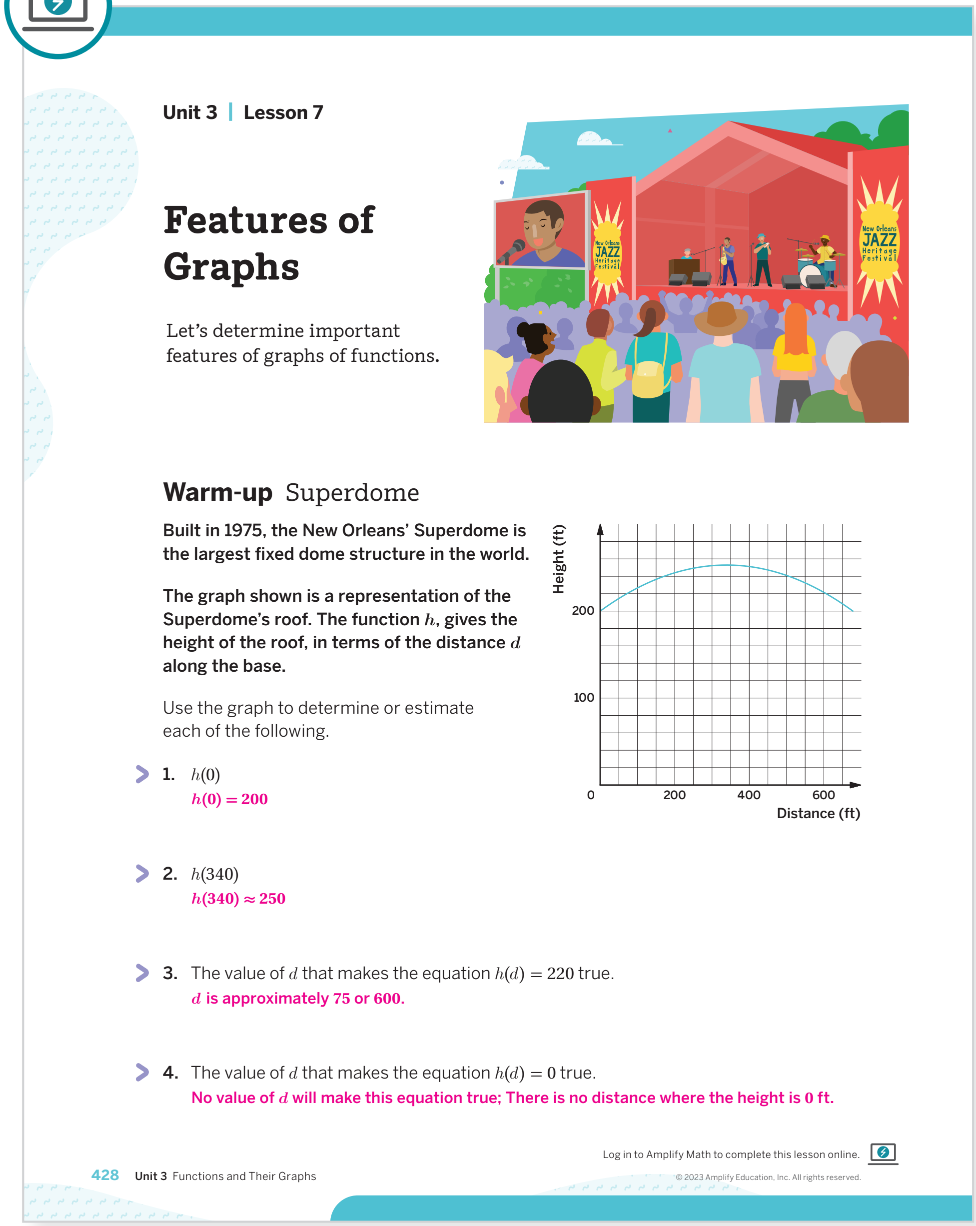
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 4 may be omitted.
- In **Activity 1**, Problem 3 may be omitted.
- In **Activity 3**, Problem 2 may be omitted.

Independent | 5 min

Warm-up Superdome

Students determine solutions to statements in function notation to prepare to use precise vocabulary when describing features of graphs.



Unit 3 | Lesson 7

Features of Graphs

Let's determine important features of graphs of functions.

Warm-up Superdome

Built in 1975, the New Orleans' Superdome is the largest fixed dome structure in the world.

The graph shown is a representation of the Superdome's roof. The function h, gives the height of the roof, in terms of the distance d along the base.

Use the graph to determine or estimate each of the following.

1. $h(0)$
   $h(0) = 200$

2. $h(340)$
   $h(340) \approx 250$

3. The value of d that makes the equation $h(d) = 220$ true.
   d is approximately 75 or 600.

4. The value of d that makes the equation $h(d) = 0$ true.
   No value of d will make this equation true; There is no distance where the height is 0 ft.

Log in to Amplify Math to complete this lesson online.

428 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Activate students' prior knowledge by asking, "What do you know about the New Orleans' Superdome?"

2 Monitor

Help students get started by having students label ordered pairs on the graph.

Look for points of confusion:

- **Confusing the input with the output and vice versa in function notation.** Have students identify the independent and dependent variables. Ask, "Which variable represents the input? The output?"

Look for productive strategies:

- Using the scale of the graph to estimate values.
- Recognizing when solutions will not be possible given context.

3 Connect

Display the graph of the Superdome.

Have individual students share their solutions to Problems 1–4.

Ask, "What might be some precise words you could use to refer to the value in Problem 1?"
Sample responses:

- Starting point or initial value
- Vertical intercept
- h-intercept, or y-intercept

Highlight that many graphs contain features that are important and distinct when compared to other features of the graph. Sometimes, it is not possible to identify these features because not all graphs contain all possible features.

Math Language Development

MLR2: Collect and Display

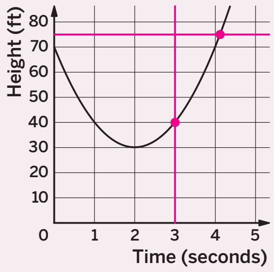
During the Connect, as students respond to the Ask question, add the language students use to describe the value in Problem 1 to the class display. Add a graph to the class display and annotate it with those words, including *initial value*, *starting point*, *vertical intercept*, and *y-intercept*. Emphasize that if the dependent variable is a different letter, that letter can be used to describe the intercept, e.g., h-intercept.

Power-up

To power up students' ability to use a graph to determine a value of an expression or solution to an equation in function notation, have students complete:

The graph models the height H of a bird, in feet, after t seconds.

1. Determine the value of the expression $H(3)$ by drawing a vertical line intersecting with 3 on the x-axis.
   $H(3) = 40$
2. Estimate the value of t when $H(t) = 75$ by drawing a horizontal line intersecting with 75 on the y-axis.
   $t \approx 4.25$



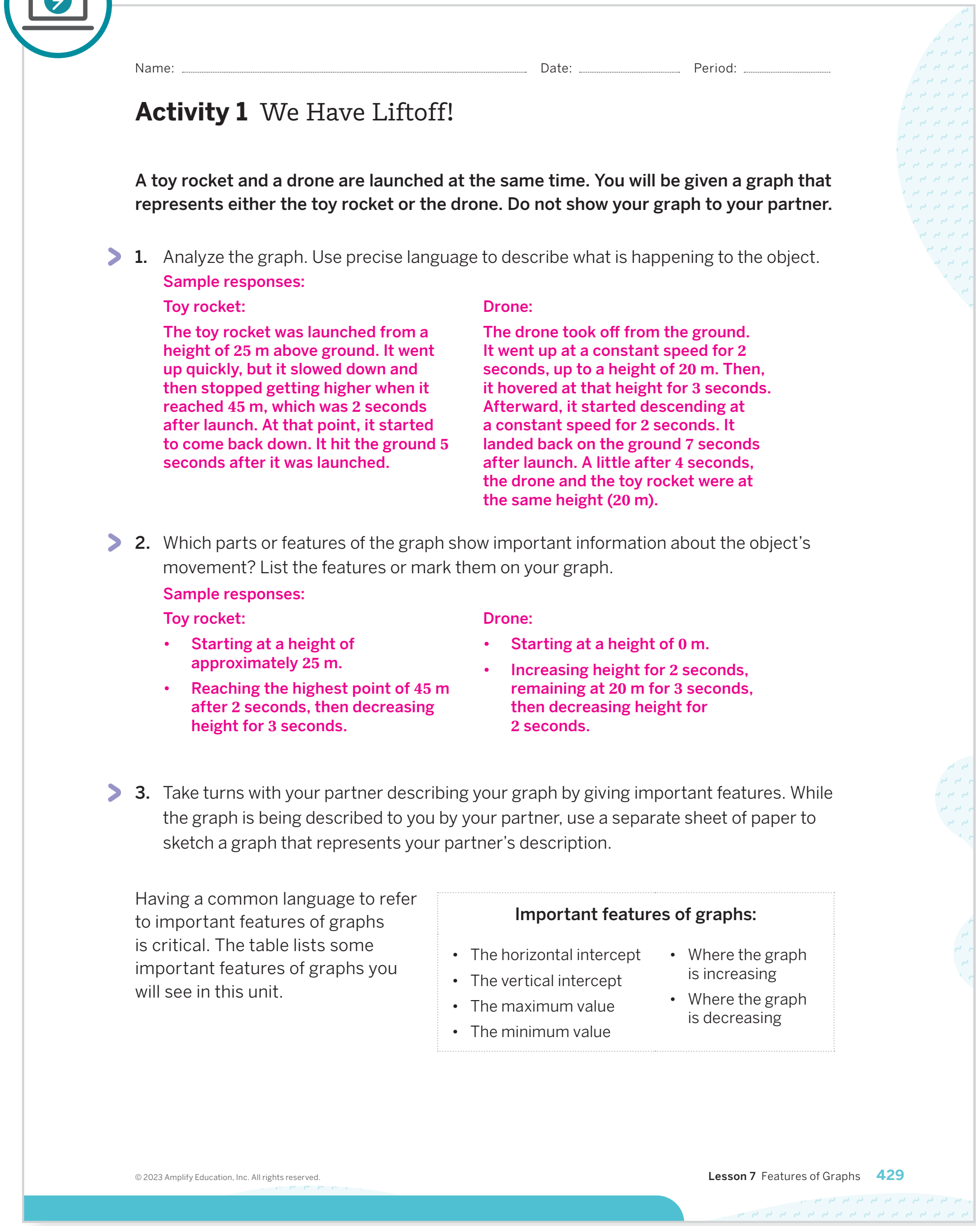
Use: Before the Warm-up

Informed by: Performance on Lesson 6, Practice Problem 6

Pairs | 10 min

Activity 1 We Have Liftoff!

Students analyze and describe two graphs to develop the need for common vocabulary for describing features of graphs.



Name: ______ Date: ______ Period: ______

Activity 1 We Have Liftoff!

A toy rocket and a drone are launched at the same time. You will be given a graph that represents either the toy rocket or the drone. Do not show your graph to your partner.

1. Analyze the graph. Use precise language to describe what is happening to the object.

Sample responses:

Toy rocket:	Drone:
The toy rocket was launched from a height of 25 m above ground. It went up quickly, but it slowed down and then stopped getting higher when it reached 45 m, which was 2 seconds after launch. At that point, it started to come back down. It hit the ground 5 seconds after it was launched.	The drone took off from the ground. It went up at a constant speed for 2 seconds, up to a height of 20 m. Then, it hovered at that height for 3 seconds. Afterward, it started descending at a constant speed for 2 seconds. It landed back on the ground 7 seconds after launch. A little after 4 seconds, the drone and the toy rocket were at the same height (20 m).

2. Which parts or features of the graph show important information about the object's movement? List the features or mark them on your graph.

Sample responses:

Toy rocket:	Drone:
• Starting at a height of approximately 25 m. • Reaching the highest point of 45 m after 2 seconds, then decreasing height for 3 seconds.	• Starting at a height of 0 m. • Increasing height for 2 seconds, remaining at 20 m for 3 seconds, then decreasing height for 2 seconds.

3. Take turns with your partner describing your graph by giving important features. While the graph is being described to you by your partner, use a separate sheet of paper to sketch a graph that represents your partner's description.

Having a common language to refer to important features of graphs is critical. The table lists some important features of graphs you will see in this unit.

Important features of graphs:

- The horizontal intercept
- The vertical intercept
- The maximum value
- The minimum value
- Where the graph is increasing
- Where the graph is decreasing

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in pairs. Distribute the Activity 1 PDF so each partner receives one of the two graphs.

2 Monitor

Help students get started by asking, "How can you use the axes labels to interpret the change in height for each object?"

Look for points of confusion:

- **Giving vague descriptions of the change in height over time.** Have students label key points on the graph. Ask, "How can you use numerical values to precisely describe the change in height over time?"

Look for productive strategies:

- Using specific values to describe the height of the toy rocket/drone.
- Using specific time intervals to describe how the height of the object is changing.
- Specifying the starting height, maximum height, or ending height, and times at which these features occur.

3 Connect

Display the graphs representing the toy rocket and drone.

Have pairs of students share their descriptions of each scenario. Select and sequence student responses describing how the height changes, detailing specific changes in height, and determining values of important features.

Ask, "What are some descriptive words that can be used when referring to features of graphs?" Sample response: Increasing, decreasing, horizontal intercept, vertical intercept, maximum, minimum.

Highlight that these words help communicate precisely about specific features so that there is no confusion.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

As students respond to Problems 1–2, have them refer to the table at the bottom of their Student Edition page that lists some important features of graphs. They may choose to describe some of those features as they respond to Problems 1–2, or they may choose to describe what is happening in the graphs using their own words.

Extension: *Math Enrichment*

Have students study both graphs and write two statements in function notation that are true and one statement that is false. Each statement should compare the height of the toy rocket to the height of the drone. A sample true statement is $r(2) > d(2)$, where r represents the height of the rocket and d represents the height of the drone.

Math Language Development

MLR7: Compare and Connect

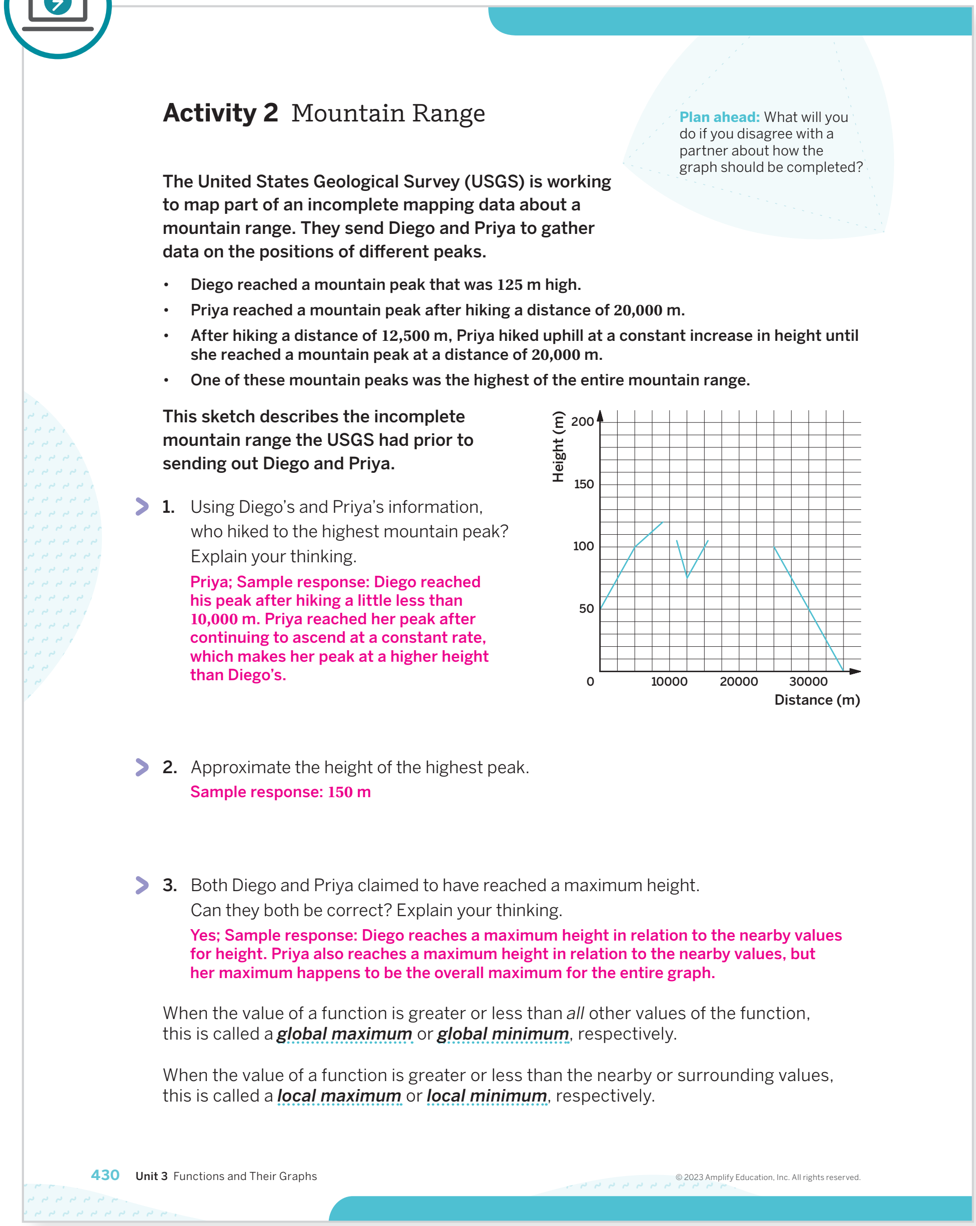
During the Connect, as students share their descriptions for each object and respond to the Ask question, draw attention to how these descriptive words compare between the two graphs. For example, the toy rocket reaches a maximum height at one particular time, while the drone maintains its maximum height for several seconds.

Note: Add the important features of graphs noted in the table in the Student Edition to the class display. Include visual examples, if possible.

Pairs | 15 min

Activity 2 Mountain Range

Students analyze key features of graphs and are introduced to the mathematical vocabulary used to describe these features.



Activity 2 Mountain Range

Plan ahead: What will you do if you disagree with a partner about how the graph should be completed?

The United States Geological Survey (USGS) is working to map part of an incomplete mapping data about a mountain range. They send Diego and Priya to gather data on the positions of different peaks.

- **Diego reached a mountain peak that was 125 m high.**
- **Priya reached a mountain peak after hiking a distance of 20,000 m.**
- **After hiking a distance of 12,500 m, Priya hiked uphill at a constant increase in height until she reached a mountain peak at a distance of 20,000 m.**
- **One of these mountain peaks was the highest of the entire mountain range.**

This sketch describes the incomplete mountain range the USGS had prior to sending out Diego and Priya.

1. Using Diego's and Priya's information, who hiked to the highest mountain peak? Explain your thinking.
   Priya; Sample response: Diego reached his peak after hiking a little less than 10,000 m. Priya reached her peak after continuing to ascend at a constant rate, which makes her peak at a higher height than Diego's.

2. Approximate the height of the highest peak.
   Sample response: 150 m

3. Both Diego and Priya claimed to have reached a maximum height. Can they both be correct? Explain your thinking.
   Yes; Sample response: Diego reaches a maximum height in relation to the nearby values for height. Priya also reaches a maximum height in relation to the nearby values, but her maximum happens to be the overall maximum for the entire graph.

When the value of a function is greater or less than *all* other values of the function, this is called a ***global maximum*** or ***global minimum***, respectively.

When the value of a function is greater or less than the nearby or surrounding values, this is called a ***local maximum*** or ***local minimum***, respectively.

430 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the narrative together as a class. Draw student attention to the missing portions of the given graph.

2 Monitor

Help students get started by having them make conjectures about what the incomplete portions of the graph could look like.

Look for points of confusion:

- **Thinking there can only be one maximum value.** Cover up the higher peak represented on the graph and ask, "If this was the only information you had been given, would this peak be a maximum? Why?"

Look for productive strategies:

- Visually estimating the slope of each line segment to continue extending the incomplete segments.
- Recognizing there could be more than one maximum value depending on the interval.

3 Connect

Display the sketch of the incomplete mountain range.

Have pairs of students share their responses.

Define:

- *global maximum*
- *global minimum*
- *local maximum*
- *local minimum*

Ask, "How do you know that Priya's peak was the global maximum?" Sample response: Because one of the two peaks was the highest, and Priya reached the higher peak, her peak must be the global maximum.

Highlight that some graphs of functions do not show the entire function, so a global maximum might occur somewhere outside of the viewable boundaries.

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols, Activate Background Knowledge*

At the end of the activity, ask students what words or phrases they have seen or used before that relate to the terms local or global. Sample responses: local community, local news, local weather, global warming

Ask:

- "How might local news compare to global news?" Sample response: Local news is confined to a smaller area and global news would represent news from around the entire world.
- "How might a local maximum on the graph of a function compare to a global maximum?" Sample response: A local maximum is confined to a smaller area and the global maximum is the maximum of the entire function.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

Read 1: Students should understand that Diego and Priya both hiked different peaks in the mountain range.

Read 2: Ask students to name given quantities and relationships, such as the height of the peak that Diego reached, 125 m.

Read 3: Ask students to preview Problem 1 and brainstorm possible strategies they could use to solve the problem.

English Learners

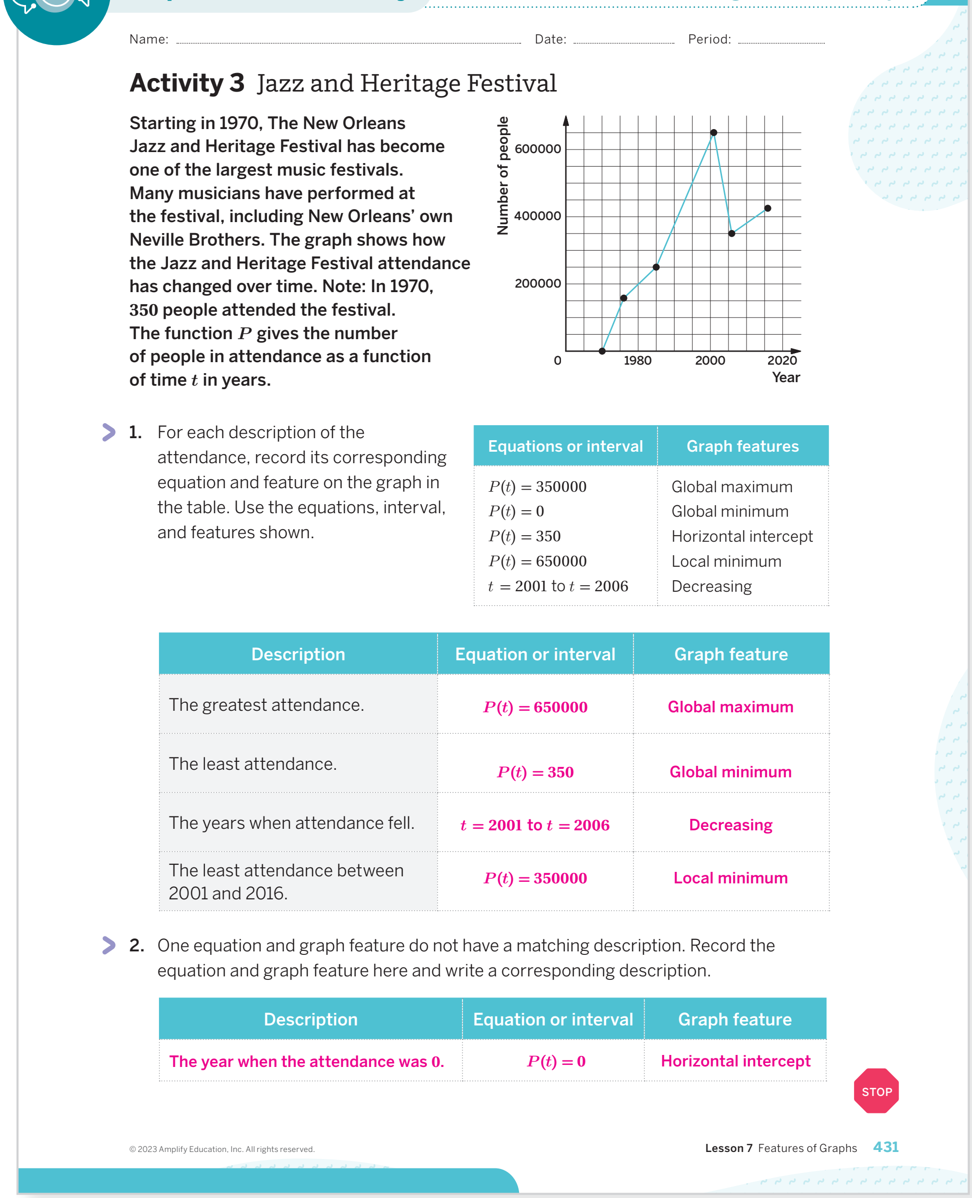
Highlight the term "uphill" and let students know this means that Priya continued hiking up the mountain.

Pairs | 10 min

Activity 3 Jazz and Heritage Festival

Students make connections between verbal descriptions, equations, and vocabulary to apply their understanding of mathematical vocabulary.

Amps Featured Activity Interactive Jazz and Heritage Festival Graph



Name: ______ Date: ______ Period: ______

Activity 3 Jazz and Heritage Festival

Starting in 1970, The New Orleans Jazz and Heritage Festival has become one of the largest music festivals. Many musicians have performed at the festival, including New Orleans' own Neville Brothers. The graph shows how the Jazz and Heritage Festival attendance has changed over time. Note: In 1970, 350 people attended the festival. The function P gives the number of people in attendance as a function of time t in years.

1. For each description of the attendance, record its corresponding equation and feature on the graph in the table. Use the equations, interval, and features shown.

Equations or interval	Graph features
$P(t) = 350000$ $P(t) = 0$ $P(t) = 350$ $P(t) = 650000$ $t = 2001$ to $t = 2006$	Global maximum Global minimum Horizontal intercept Local minimum Decreasing

Description	Equation or interval	Graph feature
The greatest attendance.	$P(t) = 650000$	**Global maximum**
The least attendance.	$P(t) = 350$	**Global minimum**
The years when attendance fell.	$t = 2001$ **to** $t = 2006$	**Decreasing**
The least attendance between 2001 and 2016.	$P(t) = 350000$	**Local minimum**

2. One equation and graph feature do not have a matching description. Record the equation and graph feature here and write a corresponding description.

Description	Equation or interval	Graph feature
The year when the attendance was 0.	$P(t) = 0$	**Horizontal intercept**

STOP

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the narrative aloud. Give students two minutes of quiet think-time before working with their partner.

2 Monitor

Help students get started by labeling points on the graph using the given equations.

Look for points of confusion:

- **Thinking $P(t) = 0$ must be the global minimum.** Ask, "What does $P(t) = 0$ mean in context?"
- **Thinking that the interval of years in which the attendance fell represents the local minimum.** Have students highlight the graph during the years in which the attendance fell and ask, "Does a single value of attendance represent this interval? Why or why not?"

Look for productive strategies:

- Labeling coordinate points and graph features on the graph.
- Recognizing $P(t) = 0$ has no solution because there was not a year when 0 people were in attendance.

3 Connect

Display the graph, descriptions, equations and interval, and graph features.

Have pairs of students share how they determined which equation, interval, and graph feature matched with each description.

Ask, "Does $P(t) = 0$ make sense in this context? Why or why not?" No, because there was no year where 0 people were in attendance.

Highlight that this scenario is an example in which some graph features occur, but not all make sense in context. Reading the labels on a graph and labeling important coordinate points can be helpful in identifying graph features.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and suggest that students circle the point on the graph that corresponds with each graph feature represented in the table in Problem 1. For example, circle the highest point on the graph in one color and circle the words "Global maximum" in the table in the same color.

Extension: *Math Enrichment*

Have students estimate a different time interval in which the local minimum or local maximum is different from the global minimum or global maximum. Answers will vary.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their matches, draw connections between the verbal description and the corresponding graph feature. Ask:

- "What clues in the verbal description indicate the global minimum?" Least attendance (without a time interval)
- "What clues in the verbal description indicate a local minimum?" Least attendance within a certain time interval.

English Learners

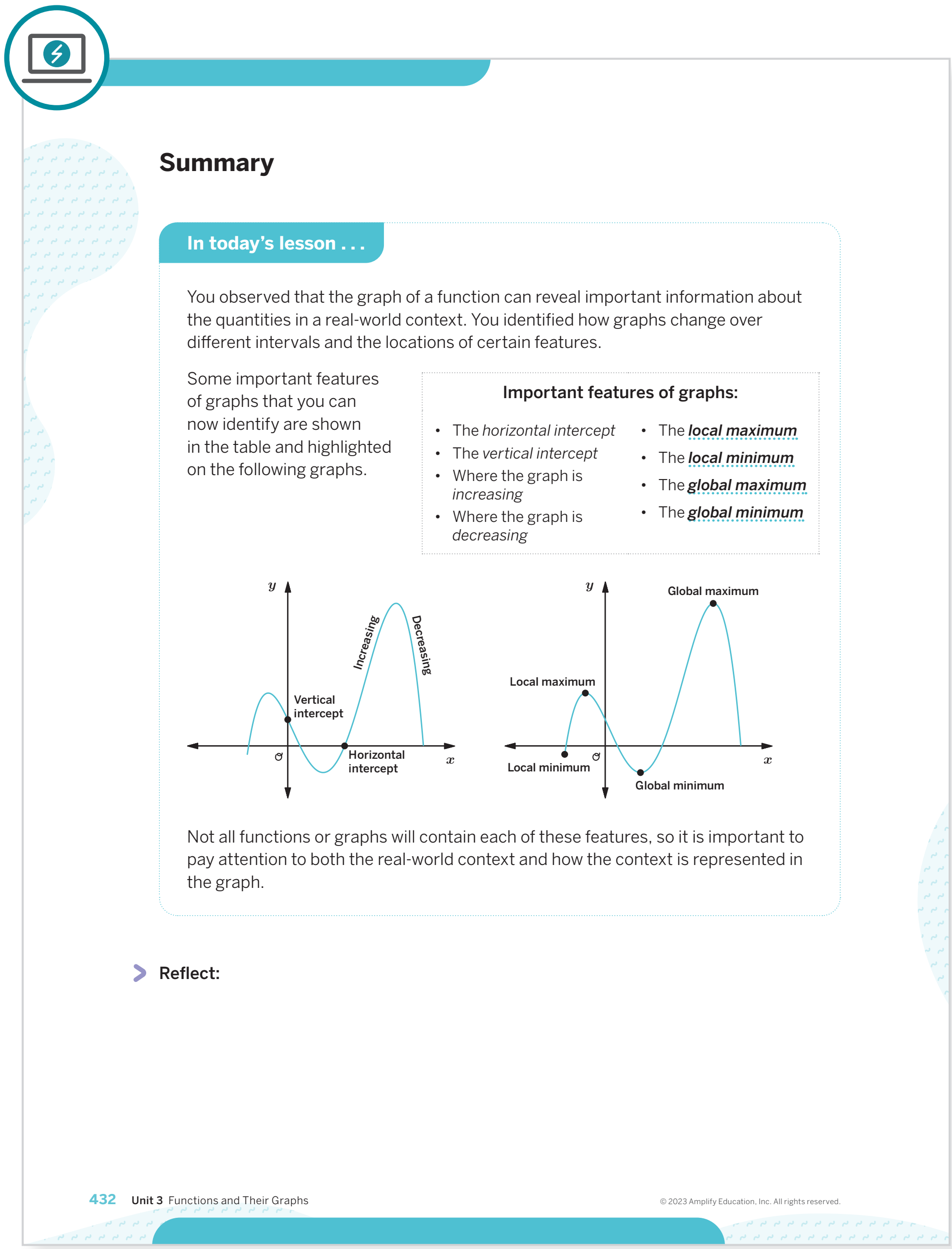
As students share their matches, display these sentence frames for all to see:

- "_____ matches _____ because . . ."
- "I noticed _____ , so I matched . . ."

Whole Class | 5 min

Summary

Review and synthesize key features of the graphs of functions, emphasizing the mathematical vocabulary used to describe these features.



Summary

In today's lesson . . .

You observed that the graph of a function can reveal important information about the quantities in a real-world context. You identified how graphs change over different intervals and the locations of certain features.

Some important features of graphs that you can now identify are shown in the table and highlighted on the following graphs.

Important features of graphs:

- The *horizontal intercept*
- The *vertical intercept*
- Where the graph is *increasing*
- Where the graph is *decreasing*
- The ***local maximum***
- The ***local minimum***
- The ***global maximum***
- The ***global minimum***

Not all functions or graphs will contain each of these features, so it is important to pay attention to both the real-world context and how the context is represented in the graph.

> Reflect:

432 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

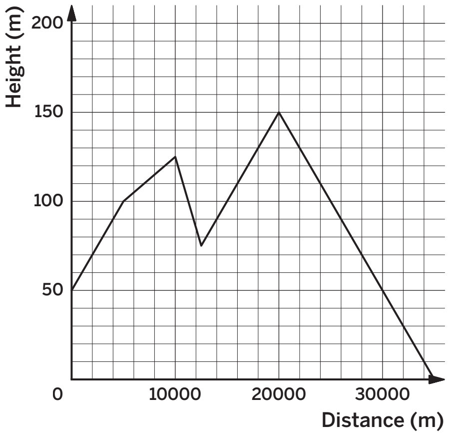
Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the terms *global maximum, global minimum, local maximum,* and *local minimum* that were added to the display during the lesson.

Synthesize

Display the following completed graph of the mountain range from Activity 2.



Ask, "What key features can you identify from the graph of the mountain range?"

Have students share key features of the graph of the mountain range. Select and sequence student responses using specific values and vocabulary referencing key features of graphs.

Highlight that graphs of functions can be vastly different, but the language used to refer to key features of graphs should be the same to avoid confusion. Paying attention to context and details of a graph can reveal what features are shown.

Formalize vocabulary:

- *global maximum*
- *global minimum*
- *local maximum*
- *local minimum*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

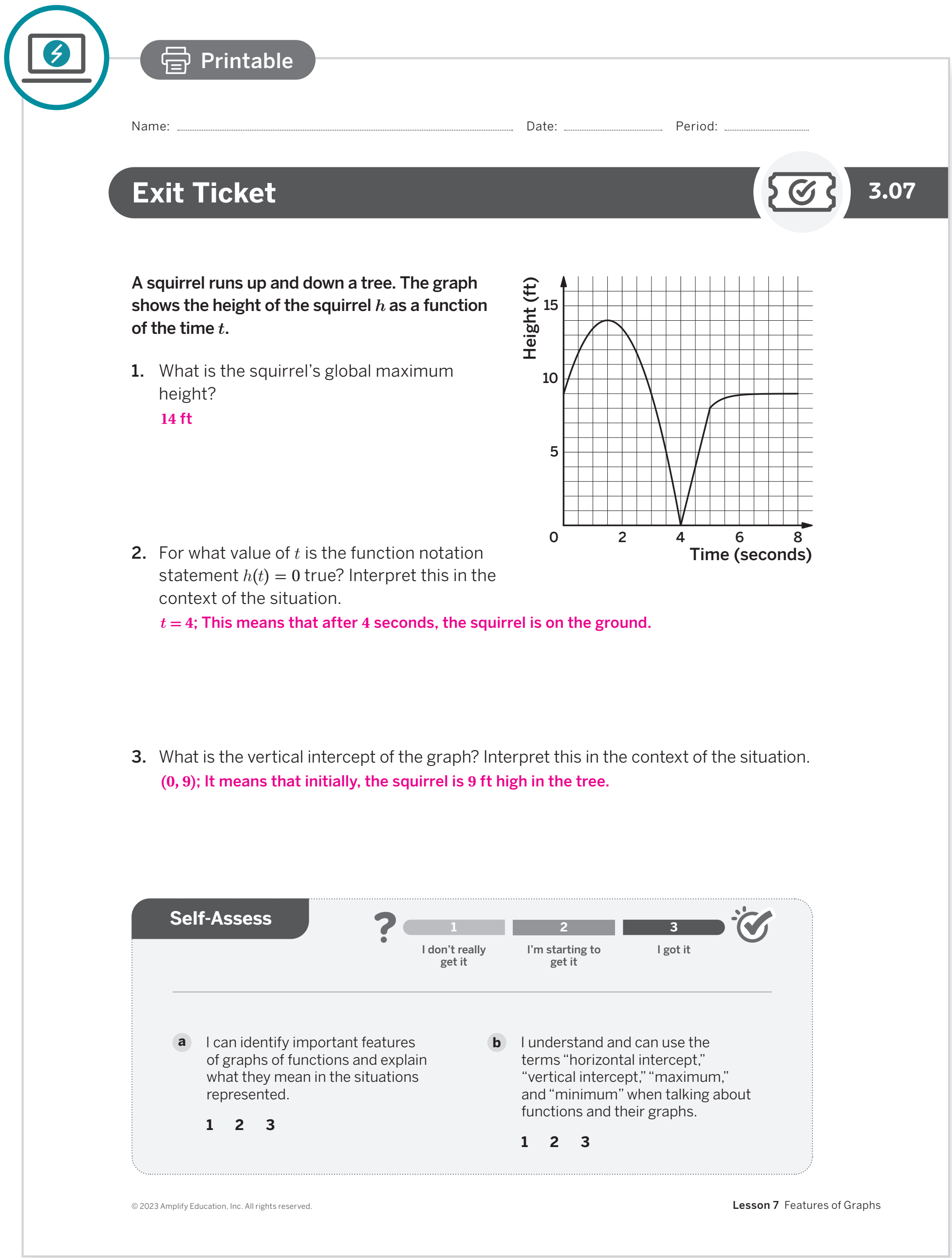
- "Why is it important to have precise vocabulary when referring to the features of graphs?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by describing and interpreting key features of a graph using precise vocabulary and interpreting a function notation statement in context.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.07

A squirrel runs up and down a tree. The graph shows the height of the squirrel h as a function of the time t.

1. What is the squirrel's global maximum height?
   14 ft
2. For what value of t is the function notation statement $h(t) = 0$ true? Interpret this in the context of the situation.
   $t = 4$; This means that after 4 seconds, the squirrel is on the ground.
3. What is the vertical intercept of the graph? Interpret this in the context of the situation.
   $(0, 9)$; It means that initially, the squirrel is 9 ft high in the tree.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can identify important features of graphs of functions and explain what they mean in the situations represented.

1 2 3

b I understand and can use the terms "horizontal intercept," "vertical intercept," "maximum," and "minimum" when talking about functions and their graphs.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 7 Features of Graphs

Success looks like . . .

- **Language Goal:** Determining important features of graphs of functions and explaining what they mean in the situations represented. **(Speaking and Listening, Reading and Writing)**
  - » Determining the value for which $h(t) = 0$ and explaining its meaning in relation to the height of the squirrel in Problem 2.
- **Language Goal:** Understanding and using the terms *horizontal intercept*, *vertical intercept*, *maximum*, and *minimum* when talking about functions and their graphs. **(Speaking and Listening, Reading and Writing)**
  - » Explaining the meaning of the vertical intercept of the graph in Problem 3.

Suggested next steps

If students identify the incorrect global maximum height in Problem 1, consider:

- Reviewing vocabulary from Activity 1.
- Assigning Practice Problem 1.
- Asking, "What is the difference between a global and local maximum?"

If students determine and interpret the incorrect value for t in Problem 2, consider:

- Reviewing identifying important graph features from Activity 3.
- Assigning Practice Problem 3.
- Asking, "What does the 0 in $h(t) = 0$ refer to?"

If students determine and interpret the incorrect value for the vertical intercept in Problem 3, consider:

- Reviewing vocabulary from Activity 1.
- Assigning Practice Problem 2.
- Asking, "How much time has passed at the vertical intercept?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The focus of this lesson was key features of graphs. How did it go?
- Did students find Activity 1 or Activity 3 more engaging today? Why do you think that is? What might you change for the next time you teach this lesson?

Math Language Development

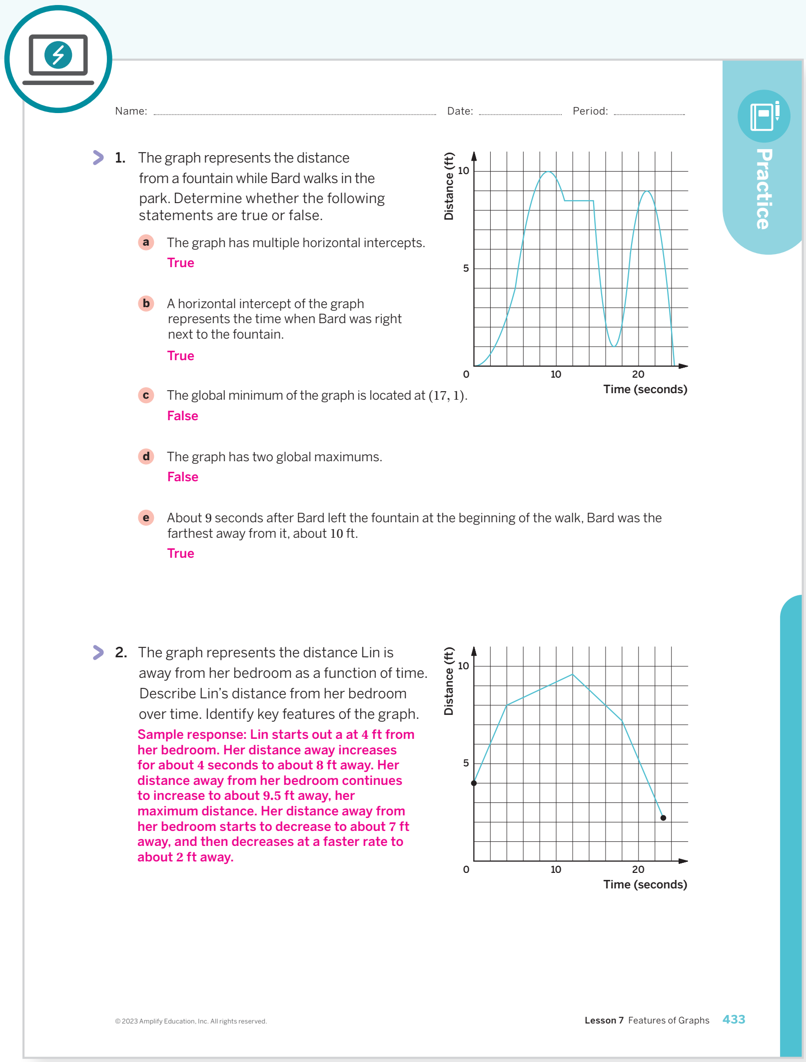
Language Goal: Understanding and using the terms *horizontal intercept, vertical intercept, maximum,* and *minimum* when talking about functions and their graphs.

Reflect on students' language development toward this goal.

- Are students gaining comfort using these terms as they describe the key features of the graphs of functions? Are they able to explain the difference between a global extreme value and a local extreme value?
- How did using the language routines in this lesson help students practice using these terms? Would you change anything the next time you use these routines?

Practice

Independent



Practice

Name: .... Date: .... Period: ....

1. The graph represents the distance from a fountain while Bard walks in the park. Determine whether the following statements are true or false.

   a. The graph has multiple horizontal intercepts.
   True

   b. A horizontal intercept of the graph represents the time when Bard was right next to the fountain.
   True

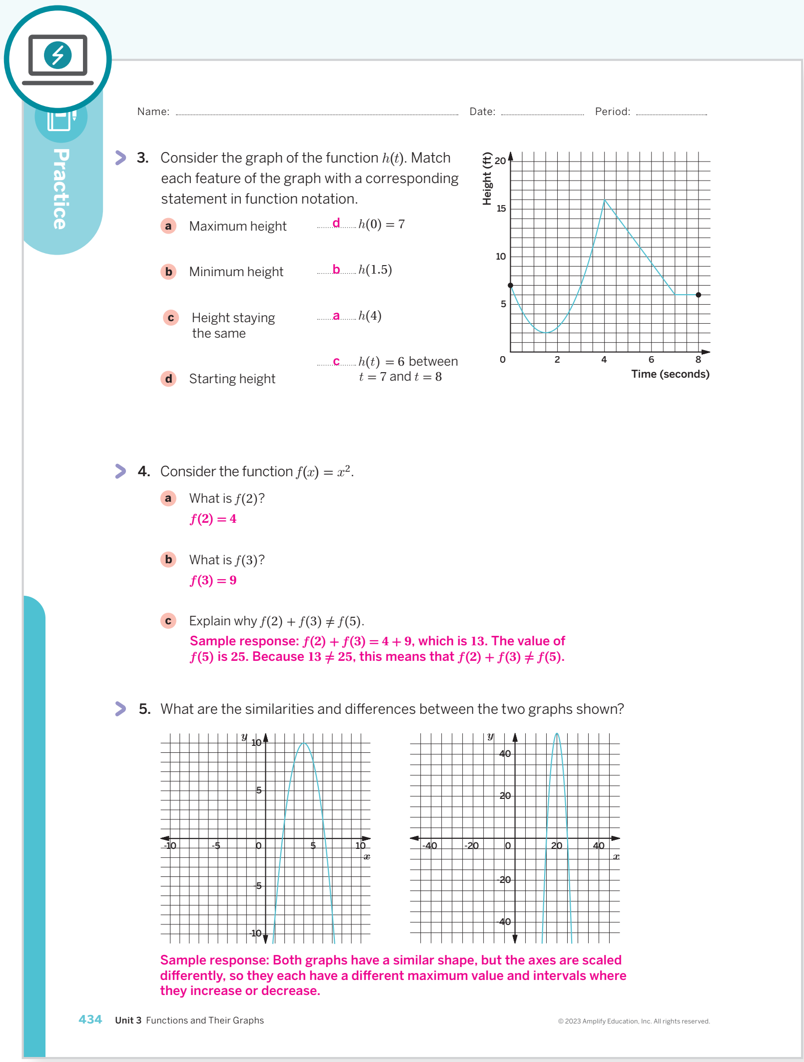
   c. The global minimum of the graph is located at $(17, 1)$.
   False

   d. The graph has two global maximums.
   False

   e. About 9 seconds after Bard left the fountain at the beginning of the walk, Bard was the farthest away from it, about 10 ft.
   True

2. The graph represents the distance Lin is away from her bedroom as a function of time. Describe Lin's distance from her bedroom over time. Identify key features of the graph.
Sample response: Lin starts out a at 4 ft from her bedroom. Her distance away increases for about 4 seconds to about 8 ft away. Her distance away from her bedroom continues to increase to about 9.5 ft away, her maximum distance. Her distance away from her bedroom starts to decrease to about 7 ft away, and then decreases at a faster rate to about 2 ft away.

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: .... Date: .... Period: ....

3. Consider the graph of the function $h(t)$. Match each feature of the graph with a corresponding statement in function notation.

a	Maximum height	d	$h(0) = 7$
b	Minimum height	b	$h(1.5)$
c	Height staying the same	a	$h(4)$
d	Starting height	c	$h(t) = 6$ between $t = 7$ and $t = 8$

4. Consider the function $f(x) = x^2$.

   a. What is $f(2)$?
   $f(2) = 4$

   b. What is $f(3)$?
   $f(3) = 9$

   c. Explain why $f(2) + f(3) \neq f(5)$.
   Sample response: $f(2) + f(3) = 4 + 9$, which is 13. The value of $f(5)$ is 25. Because $13 \neq 25$, this means that $f(2) + f(3) \neq f(5)$.

5. What are the similarities and differences between the two graphs shown?

Sample response: Both graphs have a similar shape, but the axes are scaled differently, so they each have a different maximum value and intervals where they increase or decrease.

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	1
	2	Activity 1	3
	3	Activity 3	2
Spiral	4	Unit 3 Lesson 5	2
Formative ⚡	5	Unit 3 Lesson 8	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

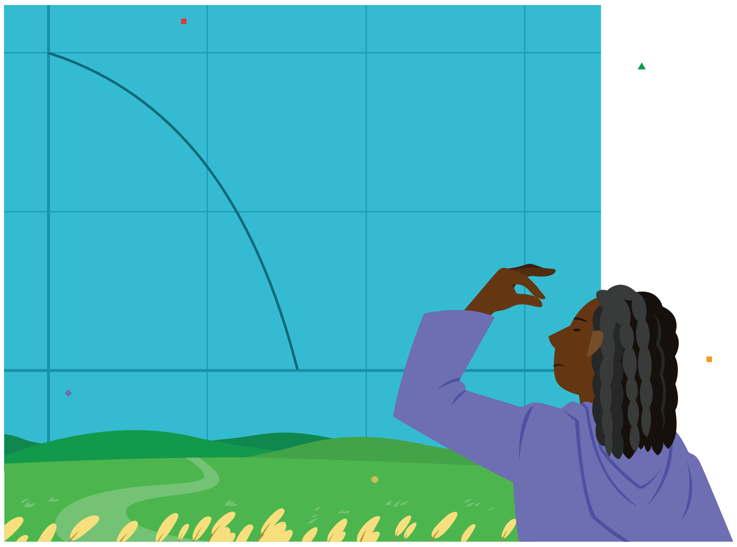
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 3 | **LESSON 8**

Understanding Scale

Let's interpret the scale of a graph and determine whether the points on a graph should be connected.



Focus

Goals

1. Understand how the scale of the horizontal and vertical axes affects the graph.
2. **Language Goal:** Determine whether a scenario is discrete and explain why. **(Reading and Writing)**

Coherence

Today

Students determine how the scale of axes can affect the graph. In the Warm-up, students compare two graphs that look similar, but whose axes are different. They expand on these graphs in Activity 1 when they are given context and asked to interpret the differences in each graph, paying attention to detail, and asked to reason about why one graph is connected and the other is not. Students then formalize their understanding of discrete scenarios through a card sort, identifying situations and their corresponding graphs as being discrete or not.

Previously

In Lesson 7, students learned formal vocabulary to refer to the key features of the graphs of functions.

Coming Soon

In Lesson 9, students will learn about how to determine and interpret the average rate of change of functions.

Rigor

- Students build **conceptual understanding** of scaling graphs and what discrete means.
- Students build **procedural fluency** interpreting the scale of graphs and identifying scenarios as discrete or not.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	20 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Are you ready for more?*
- Activity 1 PDF, *Are you ready for more?* (answers)
- Activity 2 PDF, pre-cut cards, one set per pair
- Anchor Chart PDF, *Sentence Stems, Notice and Wonder* (as needed)

Math Language Development

New words

- ***discrete***

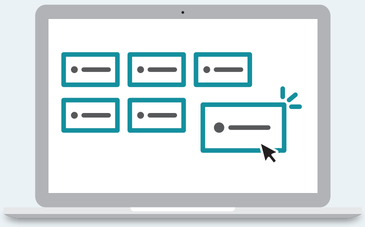
Review words

- *decreasing*
- *global maximum*
- *global minimum*
- *horizontal intercept*
- *increasing*
- *local maximum*
- *local minimum*
- *vertical intercept*

Amps | **Featured Activity**

Activity 2
Digital Card Sort

Students match scenario cards with graph cards by dragging and connecting them on screen.



Building Math Identity and Community

Connecting to Mathematical Practices

Some students might not understand the context of a graph enough to analyze the precise concepts such as whether a value makes sense for it or how the scale contributes. As students work with others to analyze graphs, they need to be able to take on different perspectives. While the answers might be obvious to them, their partner might have other thoughts, which should be considered and discussed, while showing respect towards the person.

Modifications to Pacing

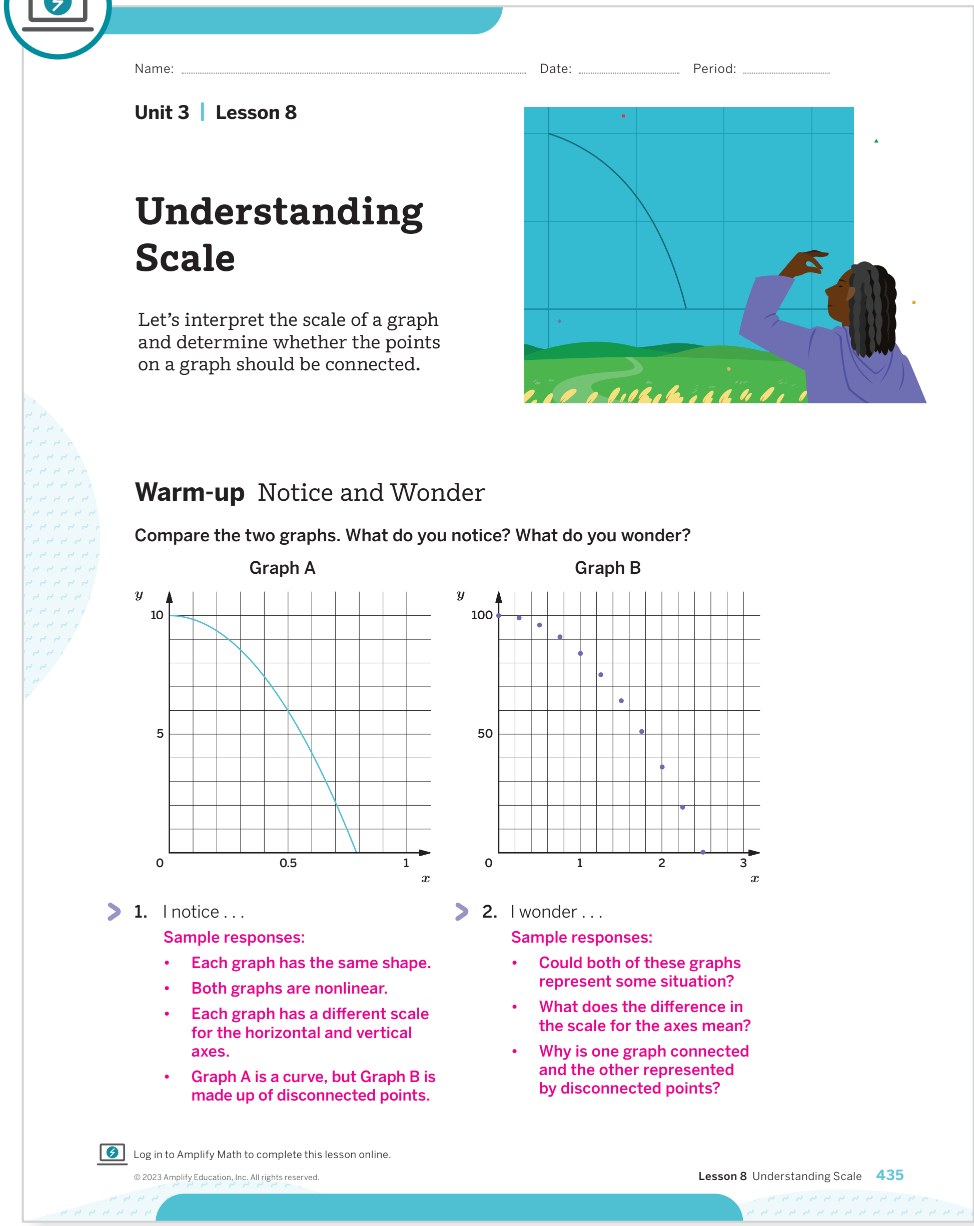
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 3 may be omitted.
- In **Activity 2**, Problem 3 may be omitted.

Pairs | 5 min

Warm-up Notice and Wonder

Students notice differences in the scale and type of graph shown to infer reasons as to why such differences make sense.



Name: ______ Date: ______ Period: ______

Unit 3 | Lesson 8

Understanding Scale

Let's interpret the scale of a graph and determine whether the points on a graph should be connected.

Warm-up Notice and Wonder

Compare the two graphs. What do you notice? What do you wonder?

1. I notice . . .
   Sample responses:
   - Each graph has the same shape.
   - Both graphs are nonlinear.
   - Each graph has a different scale for the horizontal and vertical axes.
   - Graph A is a curve, but Graph B is made up of disconnected points.

2. I wonder . . .
   Sample responses:
   - Could both of these graphs represent some situation?
   - What does the difference in the scale for the axes mean?
   - Why is one graph connected and the other represented by disconnected points?

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 8 Understanding Scale 435

1 Launch

Arrange students in pairs and conduct the *Notice and Wonder* routine. Give students a minute of independent think time, and a minute to discuss what they notice and wonder with their partner.

2 Monitor

Help students get started by asking, "What is similar or different about the two graphs?"

Look for points of confusion:

- **Thinking both graphs have the same vertical and horizontal intercept.** Ask, "What value does each tick mark on the graph represent? How do you know?"

Look for productive strategies:

- Noticing both graphs have a similar shape, but one is connected and the other is not.
- Noticing the difference in scale between the graphs.
- Wondering if the difference between the graphs is due to both representing different situations.

3 Connect

Display Graph A and Graph B.

Have pairs of students share what they notice and wonder. Select and sequence student responses noticing if the graph is connected, the scale of each graph, and wondering if each graph represents a scenario.

Highlight that the difference in the scale of each graph, along with the difference in whether or not the graph is connected, are details that can represent different scenarios and show key features of graphs.

Ask, "How would Graph A look different if the horizontal axis scale went from 0 to 0.5?" Sample response: The horizontal intercept would not be shown.

Math Language Development

MLR5: Co-craft Questions

After students complete Problems 1 and 2, have them meet with a partner to write 2–3 mathematical questions about Graphs A and B. The collaboration will help them consider other aspects of the graphs they may not have considered on their own.

English Learners

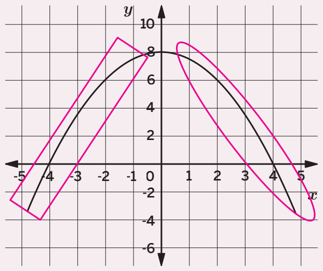
Display or provide the Anchor Chart PDF, *Sentence Stems, Notice and Wonder* to support students in structuring their responses. Allow students to rehearse what they will say before sharing with the whole class.

Power-up

To power up students' ability to identify key features of a graph, have students complete:

Describe each key feature of the graph.

1. What is the scale of the x-axis? 1
2. What is the scale of the y-axis? 2
3. Box the portion of the graph that is increasing.
4. Circle the portion of the graph that is decreasing.



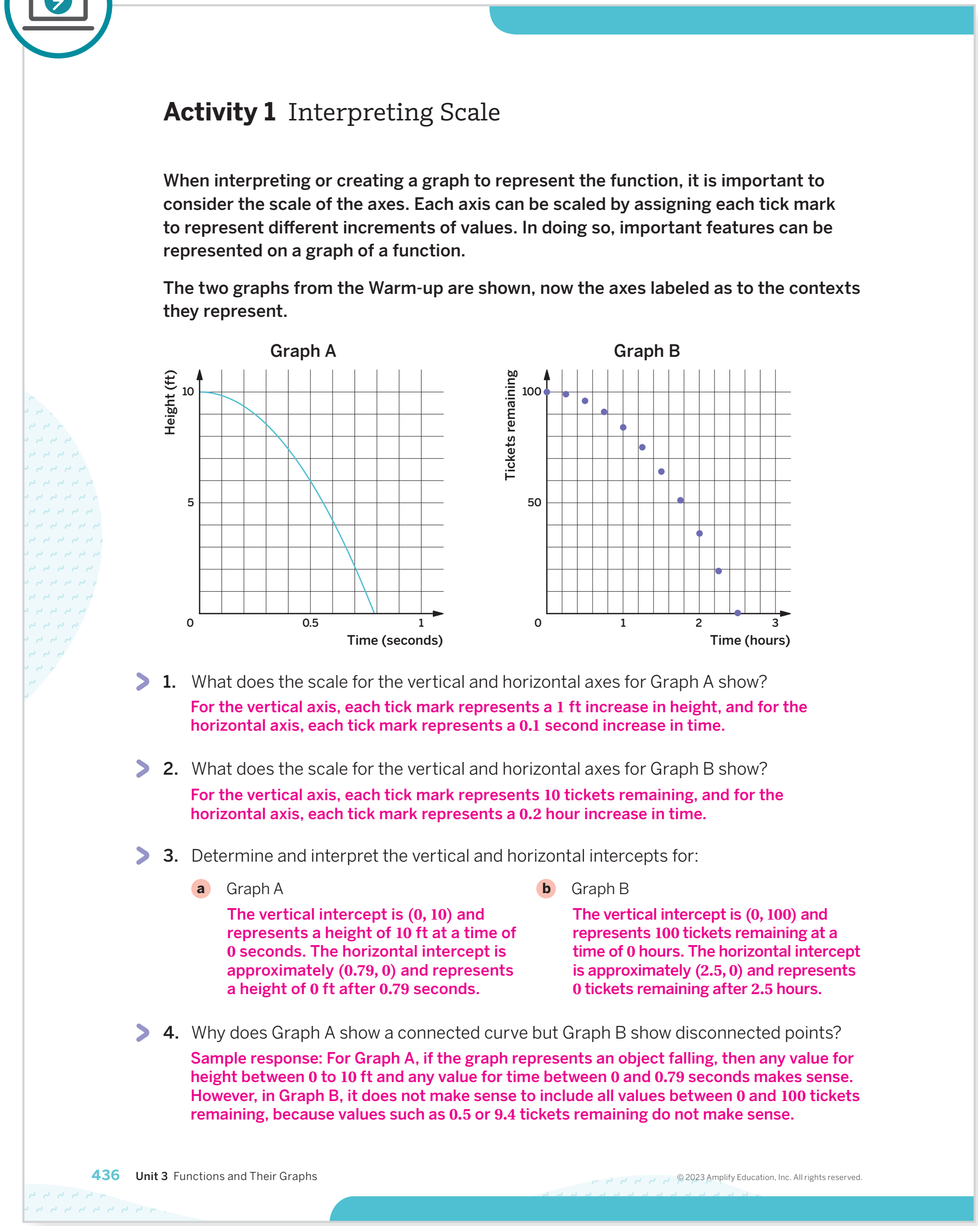
Use: Before the Warm-up

Informed by: Performance on Lesson 7, Practice Problem 5

Pairs | 15 min

Activity 1 Interpreting Scale

Students analyze an added context of the graphs from the Warm-up to understand why differences in the scale and type of graph shown is important.



Activity 1 Interpreting Scale

When interpreting or creating a graph to represent the function, it is important to consider the scale of the axes. Each axis can be scaled by assigning each tick mark to represent different increments of values. In doing so, important features can be represented on a graph of a function.

The two graphs from the Warm-up are shown, now the axes labeled as to the contexts they represent.

1. What does the scale for the vertical and horizontal axes for Graph A show?

   For the vertical axis, each tick mark represents a 1 ft increase in height, and for the horizontal axis, each tick mark represents a 0.1 second increase in time.

2. What does the scale for the vertical and horizontal axes for Graph B show?

   For the vertical axis, each tick mark represents 10 tickets remaining, and for the horizontal axis, each tick mark represents a 0.2 hour increase in time.

3. Determine and interpret the vertical and horizontal intercepts for:

   a. Graph A

   The vertical intercept is $(0, 10)$ and represents a height of 10 ft at a time of 0 seconds. The horizontal intercept is approximately $(0.79, 0)$ and represents a height of 0 ft after 0.79 seconds.

   b. Graph B

   The vertical intercept is $(0, 100)$ and represents 100 tickets remaining at a time of 0 hours. The horizontal intercept is approximately $(2.5, 0)$ and represents 0 tickets remaining after 2.5 hours.

4. Why does Graph A show a connected curve but Graph B show disconnected points?

   Sample response: For Graph A, if the graph represents an object falling, then any value for height between 0 to 10 ft and any value for time between 0 and 0.79 seconds makes sense. However, in Graph B, it does not make sense to include all values between 0 and 100 tickets remaining, because values such as 0.5 or 9.4 tickets remaining do not make sense.

436 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Students remain in pairs. Read the prompt as a whole class.

2 Monitor

Help students get started by asking, "How can you use the given values on the axes to determine what each unlabeled tick mark represents?"

Look for points of confusion:

- **Thinking each tick mark can represent a different increment in value.** Have students divide a given value on an axis by the number of tick marks between that number and 0.
- **Thinking Graph B is not connected because time is measured in hours.** Ask, "What variable is being measured on the vertical axis? What are the values that make sense for this variable?"

Look for productive strategies:

- Identifying the scale of each graph and using it to explain why Graph A is connected and Graph B is not.

3 Connect

Display Graphs A and B.

Have pairs of students share their interpretation of scale for each axis of each graph, interpretations for the intercepts, and why Graph A is connected but Graph B is not.

Highlight that the scale of a graph is important to consider when analyzing key features of graphs in order to properly interpret different values.

Ask, "What might be some other situations that have graphs that are not connected?" Sample response: Number of people, number of seats in a stadium, number of records sold by a musician.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

As students respond to Problems 1 and 2, consider displaying sentence frames they can use to complete, such as:

- For the vertical axis, each tick mark represents . . .
- For the horizontal axis, each tick mark represents . . .

Extension: *Math Enrichment*

Have students complete the problem on the *Are you ready for more?* PDF, in which they will choose an appropriate scale and create a graph to represent a given context.

Math Language Development

MLR7: Compare and Connect

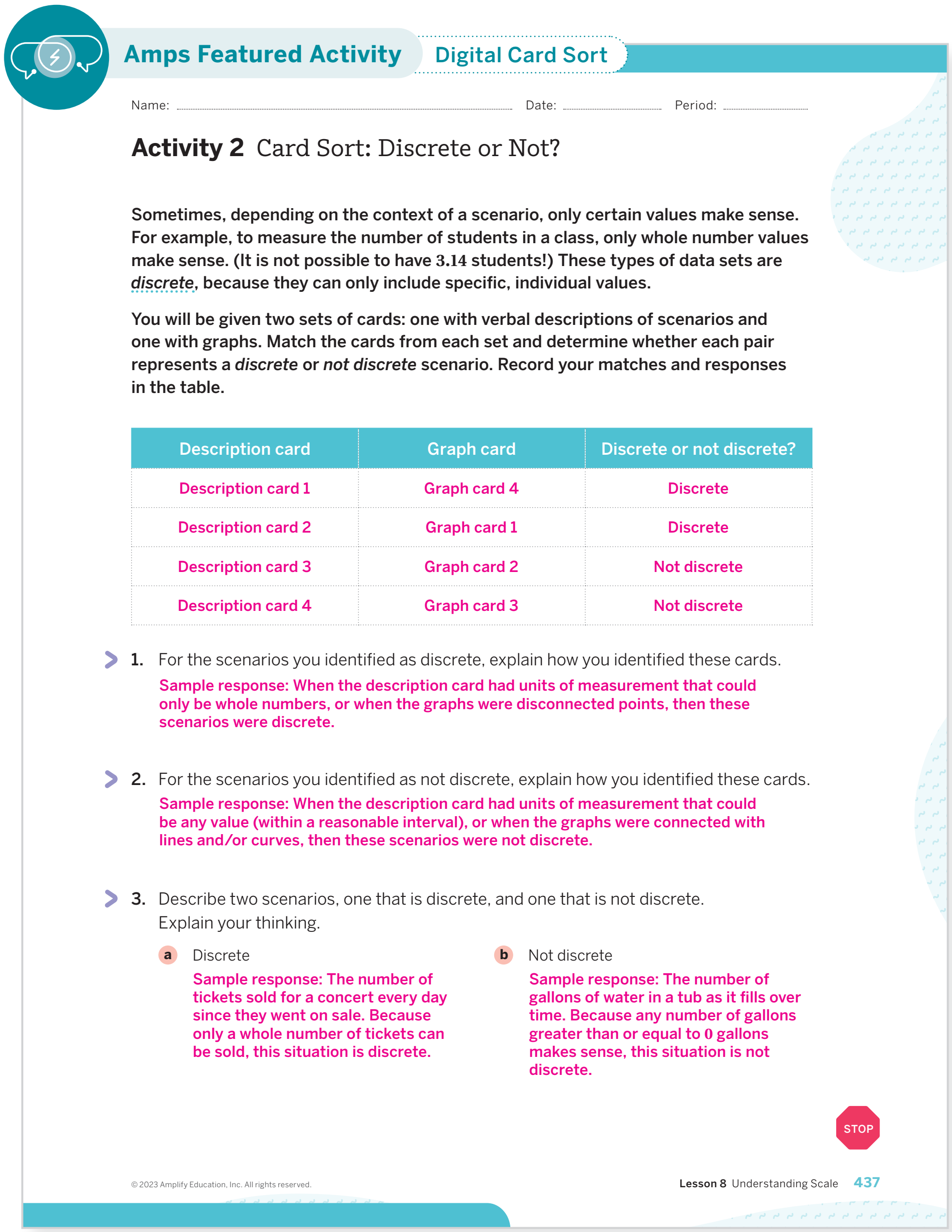
During the Connect, as students share why Graph A is connected, but Graph B is not, listen for and amplify the language they use to describe each graph. Consider displaying these terms or adding them to the class display.

Graph A	Graph B
Connected Smooth curve	Disconnected Not connected Individual points Separate points

Pairs | 20 min

Activity 2 Card Sort: Discrete or Not?

Students match description and graph cards and identify them as discrete or not to build understanding of when situations are discrete.



Amps Featured Activity Digital Card Sort

Name: ______ Date: ______ Period: ______

Activity 2 Card Sort: Discrete or Not?

Sometimes, depending on the context of a scenario, only certain values make sense. For example, to measure the number of students in a class, only whole number values make sense. (It is not possible to have 3.14 students!) These types of data sets are *discrete*, because they can only include specific, individual values.

You will be given two sets of cards: one with verbal descriptions of scenarios and one with graphs. Match the cards from each set and determine whether each pair represents a *discrete* or *not discrete* scenario. Record your matches and responses in the table.

Description card	Graph card	Discrete or not discrete?
Description card 1	Graph card 4	Discrete
Description card 2	Graph card 1	Discrete
Description card 3	Graph card 2	Not discrete
Description card 4	Graph card 3	Not discrete

1. For the scenarios you identified as discrete, explain how you identified these cards.
Sample response: When the description card had units of measurement that could only be whole numbers, or when the graphs were disconnected points, then these scenarios were discrete.

2. For the scenarios you identified as not discrete, explain how you identified these cards.
Sample response: When the description card had units of measurement that could be any value (within a reasonable interval), or when the graphs were connected with lines and/or curves, then these scenarios were not discrete.

3. Describe two scenarios, one that is discrete, and one that is not discrete. Explain your thinking.

a Discrete
Sample response: The number of tickets sold for a concert every day since they went on sale. Because only a whole number of tickets can be sold, this situation is discrete.

b Not discrete
Sample response: The number of gallons of water in a tub as it fills over time. Because any number of gallons greater than or equal to 0 gallons makes sense, this situation is not discrete.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 8 Understanding Scale 437

1 Launch

Read the opening paragraph as a class. Distribute the pre-cut cards from the Activity 2 PDF to each student pair and conduct the *Card Sort* routine.

2 Monitor

Help students get started by having them list values that do and do not make sense for each description card.

Look for points of confusion:

- **Matching a discrete description with a non-discrete graph.** Have students identify the independent and dependent variables and ask, "What values make sense for each variable? Why?"
- **Mixing up matching the graphs of the two discrete scenarios or the two non-discrete scenarios.** Have students identify key feature vocabulary in each description card.

Look for productive strategies:

- Using key feature vocabulary such as *increasing* or *decreasing* to match the description with a graph.
- Determining if the scale of each graph matches and makes sense with the given description.
- Justifying reasons why descriptions and graphs are discrete or not, by determining what values make sense in context.

3 Connect

Define the term *discrete*.

Display the four description cards and graph cards.

Have pairs of students share their matching cards, whether or not they were discrete, and how they identified each card.

Highlight that asking themselves what values make sense for both the independent and dependent variable can help determine if a situation is discrete or not.

Have pairs of students share scenarios that they identified as discrete or not discrete.

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Demonstrate how to determine whether a graph shows a *discrete* data set. Have students refer back to Graphs A and B from Activity 1. Tell students that Graph B shows a discrete data set, because only individual points are plotted. Graph A shows a data set that is not discrete.

Extension: *Math Enrichment*

Tell students that while in this unit, they will use the terms *discrete* and *not discrete*, there is another term that mathematicians use to describe data sets that are not discrete. That term is *continuous*. Ask students to think of other terms that are similar to help them understand the mathematical meaning.
Sample responses: continuing, continuity

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the text on each Description card. A sample routine is shown for Description card 1.

Read 1: Students should understand the general scenario of an online music streaming service keeping records of the number of new users.

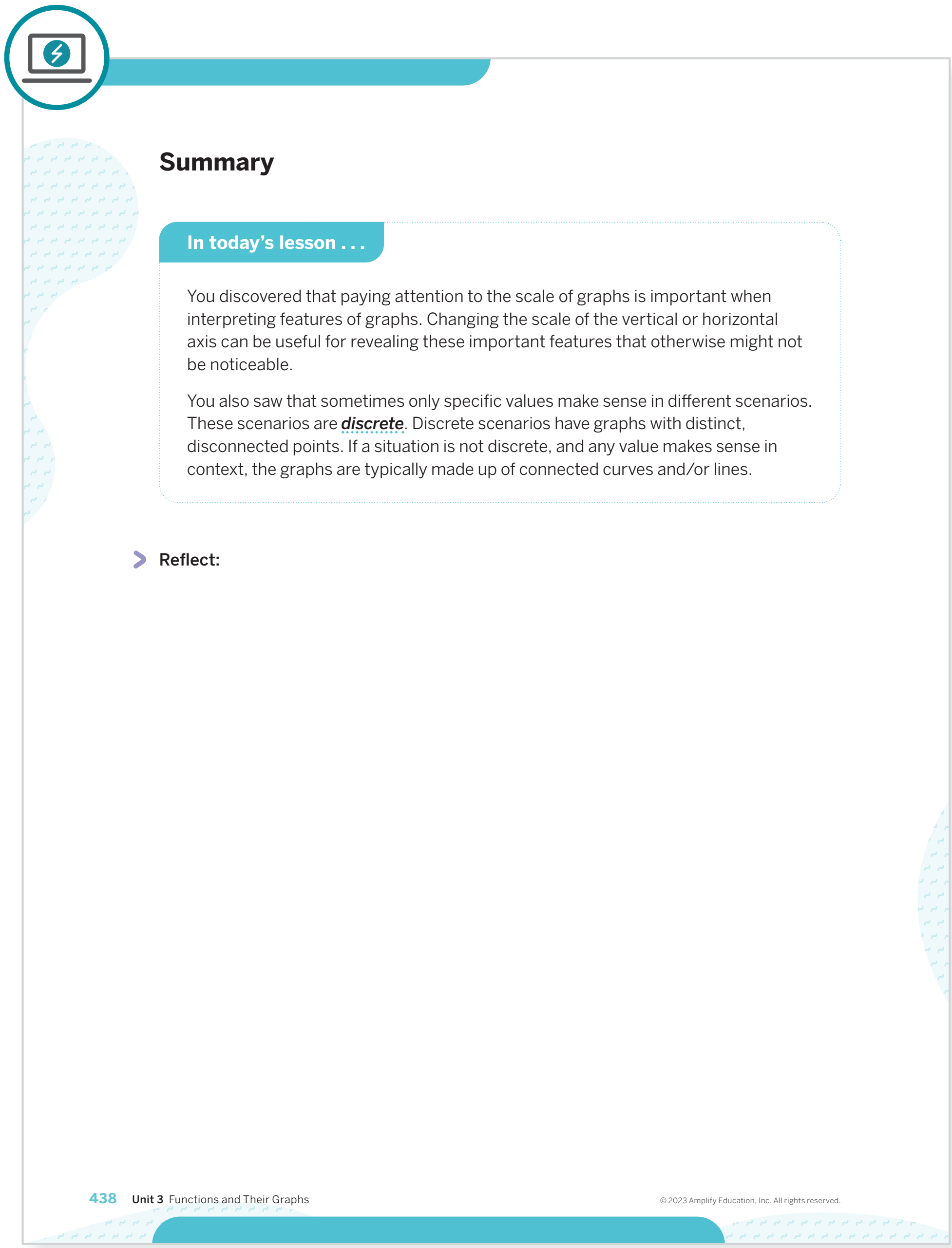
Read 2: Ask students to identify the independent variable (years) and dependent variable (number of new users).

Read 3: Ask students to ask themselves, "Can there be fractional or decimal values for the number of new users or years?"

Whole Class | 5 min

Summary

Review and synthesize the scale of graphs and whether or not they are discrete.



Summary

In today's lesson . . .

You discovered that paying attention to the scale of graphs is important when interpreting features of graphs. Changing the scale of the vertical or horizontal axis can be useful for revealing these important features that otherwise might not be noticeable.

You also saw that sometimes only specific values make sense in different scenarios. These scenarios are ***discrete***. Discrete scenarios have graphs with distinct, disconnected points. If a situation is not discrete, and any value makes sense in context, the graphs are typically made up of connected curves and/or lines.

> Reflect:

438 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

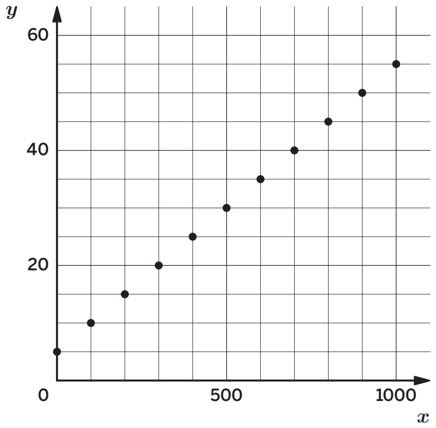
Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the term *discrete* that were added to the display during the lesson.

Synthesize

Display the following graph.



Ask:

- "What is the scale of each axis?" Each tick mark on the horizontal axis represents 100 units and each tick mark on the vertical axis represents 5 units.
- "Is the graph discrete or not? Explain your thinking." The graph is discrete, because it is made up of distinct disconnected points.

Have students share their responses to both questions.

Highlight that the scale of a graph can be changed to show key features and help determine whether it is discrete depending on values that make sense in context.

Formalize vocabulary: ***discrete***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

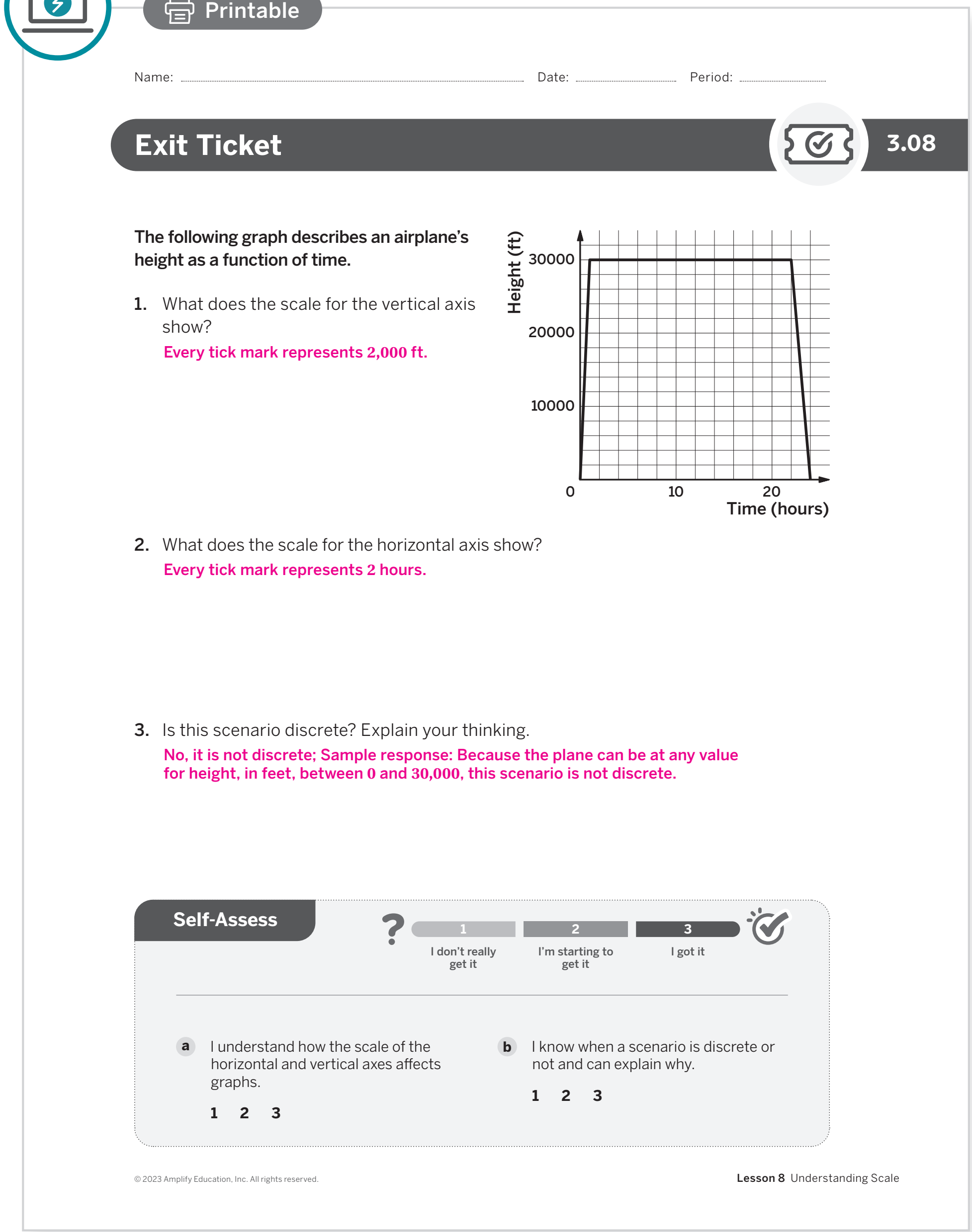
- "Why is an appropriate scale for each axis on a graph important?"
- "How can you determine whether a situation is discrete or not?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the scale of a graph and whether it is discrete.

Printable



Name: Date: Period:

Exit Ticket 3.08

The following graph describes an airplane's height as a function of time.

1. What does the scale for the vertical axis show?
   Every tick mark represents 2,000 ft.

2. What does the scale for the horizontal axis show?
   Every tick mark represents 2 hours.

3. Is this scenario discrete? Explain your thinking.
   No, it is not discrete; Sample response: Because the plane can be at any value for height, in feet, between 0 and 30,000, this scenario is not discrete.

Self-Assess

1 I don't really get it

2 I'm starting to get it

3 I got it

a I understand how the scale of the horizontal and vertical axes affects graphs.
1 2 3

b I know when a scenario is discrete or not and can explain why.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 8 Understanding Scale

Success looks like . . .

- **Goal:** Understanding how the scale of the horizontal and vertical axes affects the graph.
- **Language Goal:** Determining whether a scenario is discrete and explaining why. **(Reading and Writing)**
  - » Determining whether the airplane's height as a function of time is discrete in Problem 3.

Suggested next steps

If students interpret the scale of the axes incorrectly in Problems 1 or 2:

- Reviewing interpreting scale from Activity 1.
- Assigning Practice Problem 1.
- Asking, "How can you use the given values to determine what each tick mark represents on each axis?"

If students determine the scenario is discrete in Problem 3:

- Reviewing whether situations are discrete from Activity 2.
- Assigning Practice Problem 3.
- Asking, "What values make sense for the independent and dependent variables? Why?"

Professional Learning

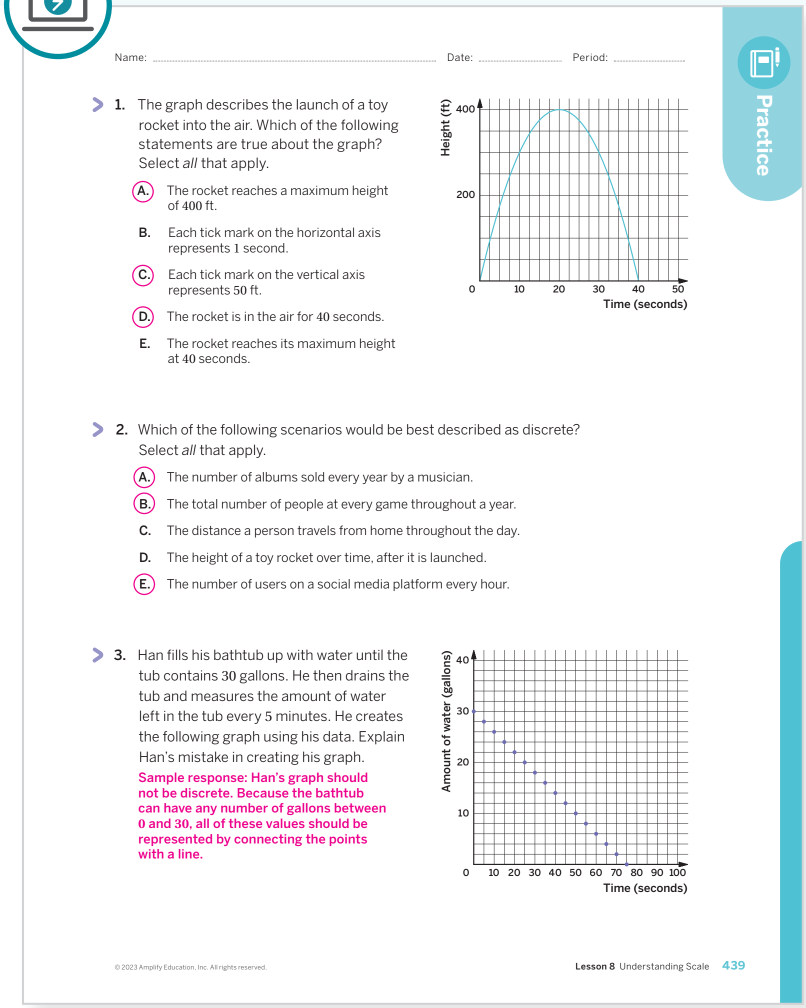
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What routines enabled all students to do math in today's lesson?
- In what ways did the card sort in Activity 2 go as planned? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. The graph describes the launch of a toy rocket into the air. Which of the following statements are true about the graph? Select *all* that apply.
   - [✓ A.] The rocket reaches a maximum height of 400 ft.
   - B. Each tick mark on the horizontal axis represents 1 second.
   - [✓ C.] Each tick mark on the vertical axis represents 50 ft.
   - [✓ D.] The rocket is in the air for 40 seconds.
   - E. The rocket reaches its maximum height at 40 seconds.

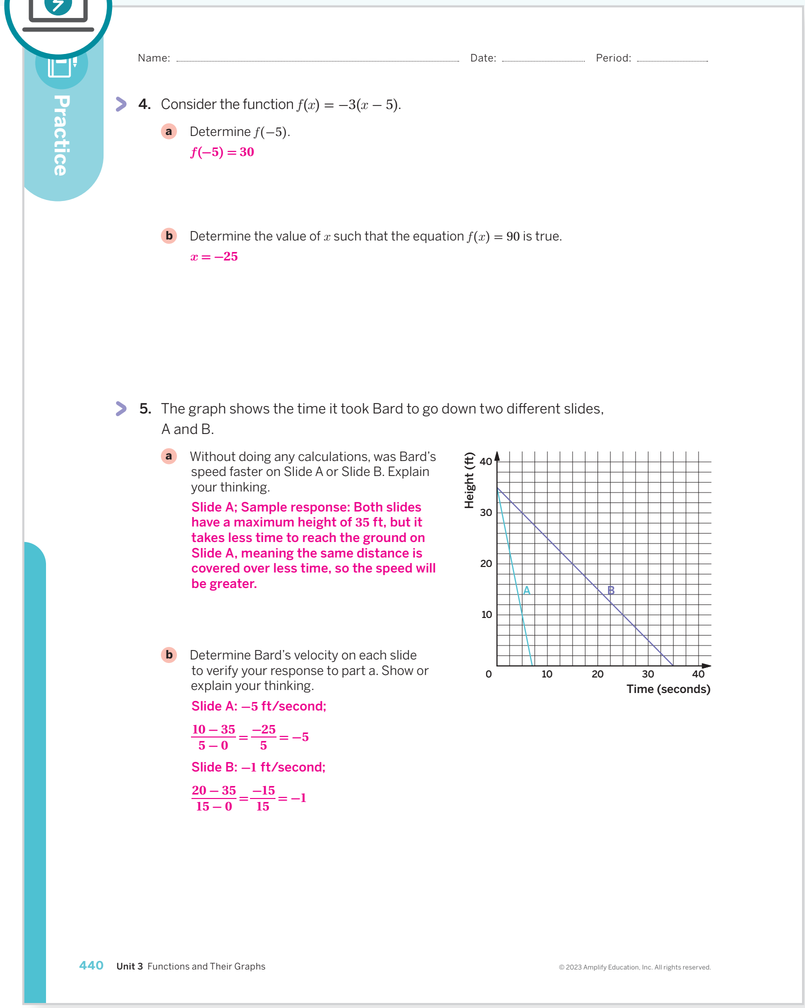
2. Which of the following scenarios would be best described as discrete? Select *all* that apply.
   - [✓ A.] The number of albums sold every year by a musician.
   - [✓ B.] The total number of people at every game throughout a year.
   - C. The distance a person travels from home throughout the day.
   - D. The height of a toy rocket over time, after it is launched.
   - [✓ E.] The number of users on a social media platform every hour.

3. Han fills his bathtub up with water until the tub contains 30 gallons. He then drains the tub and measures the amount of water left in the tub every 5 minutes. He creates the following graph using his data. Explain Han's mistake in creating his graph.

   Sample response: Han's graph should not be discrete. Because the bathtub can have any number of gallons between 0 and 30, all of these values should be represented by connecting the points with a line.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 8 Understanding Scale 439



Name: Date: Period:

4. Consider the function $f(x) = -3(x - 5)$.
   - a Determine $f(-5)$.
     $f(-5) = 30$
   - b Determine the value of x such that the equation $f(x) = 90$ is true.
     $x = -25$

5. The graph shows the time it took Bard to go down two different slides, A and B.
   - a Without doing any calculations, was Bard's speed faster on Slide A or Slide B. Explain your thinking.
     Slide A; Sample response: Both slides have a maximum height of 35 ft, but it takes less time to reach the ground on Slide A, meaning the same distance is covered over less time, so the speed will be greater.
   - b Determine Bard's velocity on each slide to verify your response to part a. Show or explain your thinking.
     Slide A: –5 ft/second;
     $$\frac{10-35}{5-0} = \frac{-25}{5} = -5$$
     Slide B: –1 ft/second;
     $$\frac{20-35}{15-0} = \frac{-15}{15} = -1$$

440 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 5	1
Formative ⚡	5	Unit 3 Lesson 9	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 3 | **LESSON 9**

How Do Graphs Change?

Let's determine how to calculate and interpret the average rate of change.

Focus

Goals

1. **Language Goal:** Understand the meaning of the term *average rate of change*. **(Speaking and Listening, Reading and Writing)**
2. Determine the average rate of change of a function between two points.

Coherence

Today

Students begin by evaluating statements made about the change in temperature over two different time intervals to begin thinking about what it means to determine rates of change on different intervals. Then, students are formally introduced to the average rate of change by expanding on the scenario presented in the Warm-up, determining the average rate of change in temperature across different intervals. Finally, students apply their understanding of the average rate of change in the context of traffic in New Orleans.

Previously

In Lesson 7, students learned formal vocabulary used to refer to key features of graphs, such as increasing and decreasing.

Coming Soon

In Lessons 10 and 11, students will learn about reasonable input and output values for functions, connecting this idea to domain and range.

Rigor

- Students build **procedural fluency** determining the average rate of change.
- Students **apply** their understanding by interpreting the average rate of change in context.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	15 min	15 min	5 min	10 min
Independent	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

New words

- *average rate of change*

Amps | Featured Activity

Activity 2
Animation of Traffic in New Orleans

Students observe an animation of a particle tracing a graph to see the change in traffic over time to help them visualize the scenario.

Building Math Identity and Community

Connecting to Mathematical Practices

During Activity 1, students use the average rate of change to reflect back on Tyler's and Mai's statements from the Warm-up. Remind them that their original responses from the Warm-up were made before they were provided with more temperature data and before they explored the average rate of change. Now that they have more informed, they can make more precise statements about how the temperature changed.

Modifications to Pacing

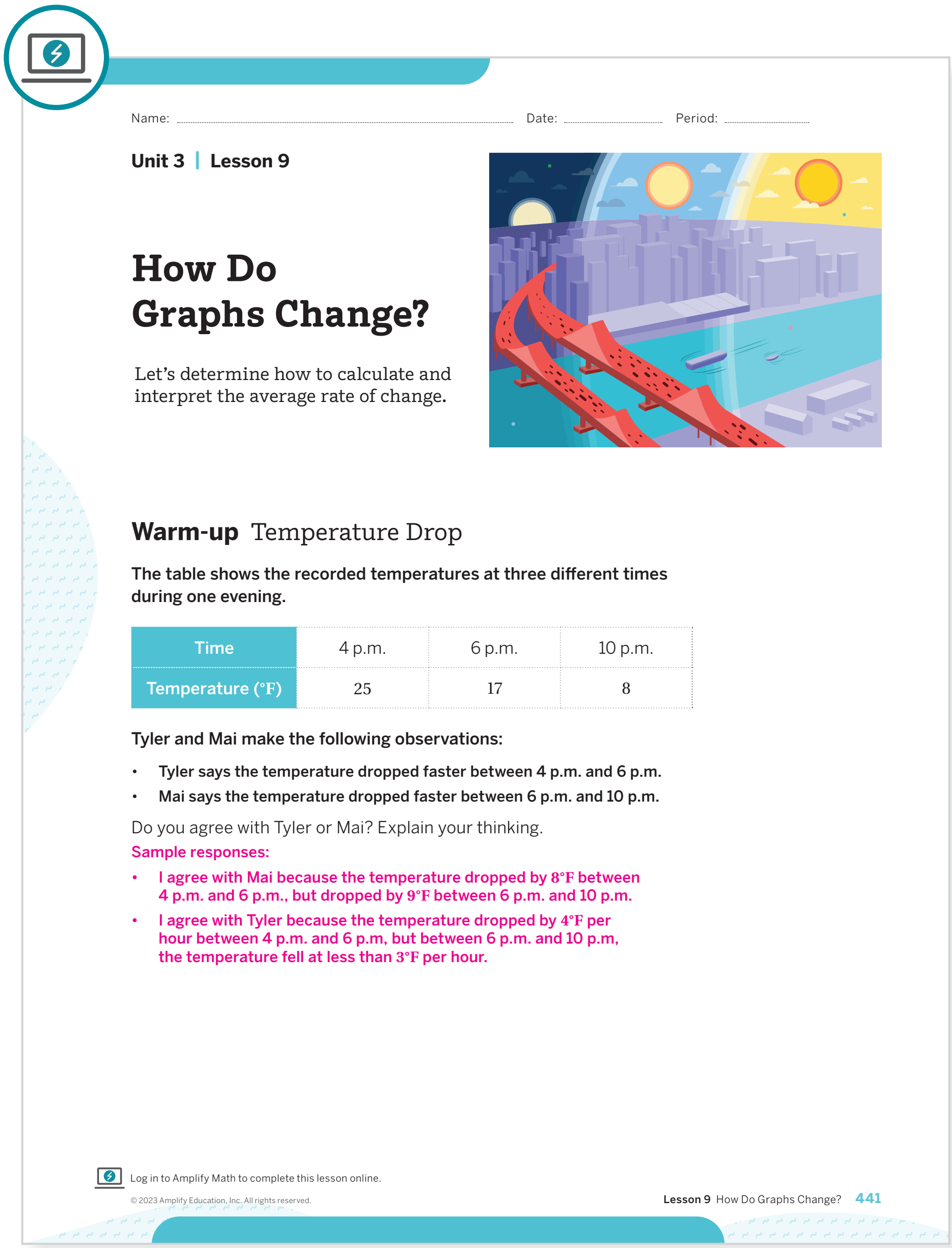
You may want to consider these additional modifications if you are short on time.

- In **Activity 1,** Problem 1c may be omitted.
- In **Activity 2,** Problem 2 may be omitted.

Independent | 5 min

Warm-up Temperature Drop

Students evaluate two statements on changing temperature to prepare for thinking about rates of change over different intervals.



Name: ____________ Date: ________ Period: ________

Unit 3 | Lesson 9

How Do Graphs Change?

Let's determine how to calculate and interpret the average rate of change.

Warm-up Temperature Drop

The table shows the recorded temperatures at three different times during one evening.

Time	4 p.m.	6 p.m.	10 p.m.
Temperature (°F)	25	17	8

Tyler and Mai make the following observations:

- Tyler says the temperature dropped faster between 4 p.m. and 6 p.m.
- Mai says the temperature dropped faster between 6 p.m. and 10 p.m.

Do you agree with Tyler or Mai? Explain your thinking.

Sample responses:

- I agree with Mai because the temperature dropped by 8°F between 4 p.m. and 6 p.m., but dropped by 9°F between 6 p.m. and 10 p.m.
- I agree with Tyler because the temperature dropped by 4°F per hour between 4 p.m. and 6 p.m, but between 6 p.m. and 10 p.m, the temperature fell at less than 3°F per hour.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 How Do Graphs Change? 441

1 Launch

Read the observations together as a class. Use the *Poll the Class* routine to see whose observation they initially agree with before responding to the problem.

2 Monitor

Help students get started by having them create a sketch of a scatter plot with the listed values from the table.

Look for points of confusion:

- **Not understanding what "faster" means in the context.** Encourage students to interpret "faster" using the variables presented in the table.

Look for productive strategies:

- Determining the change in temperature between time intervals.
- Determining the rate of change, or slope, between the two time intervals.
- Interpreting the rate of change in the context of temperature and time.

3 Connect

Display the table of values and observations.

Have individual students share which student, Tyler or Mai, they agree with and their thinking. Record and display student thinking.

Ask, "Has anyone changed their mind? Why?"

Highlight that the two intervals of time, between 4 p.m. to 6 p.m. and 6 p.m. to 10 p.m., are not equal, which is important to note when determining how the temperature is changing.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest students create a quick sketch of the points plotted on a coordinate plane to help with their thinking. If students use this approach, ask them during the Connect how they decided which variable was the independent variable and which was the dependent variable.

Power-up

To power up students' ability to determine slope from a graph, have students complete:

Recall that the slope between two points (x_1, y_1) and (x_2, y_2) can be calculated using the formula $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$.

1. Determine the slope between the points $(4, 25)$ and $(6, 17)$.

   -4

2. Determine the slope between the points $(6, 17)$ and $(10, 8)$.

   -2.25

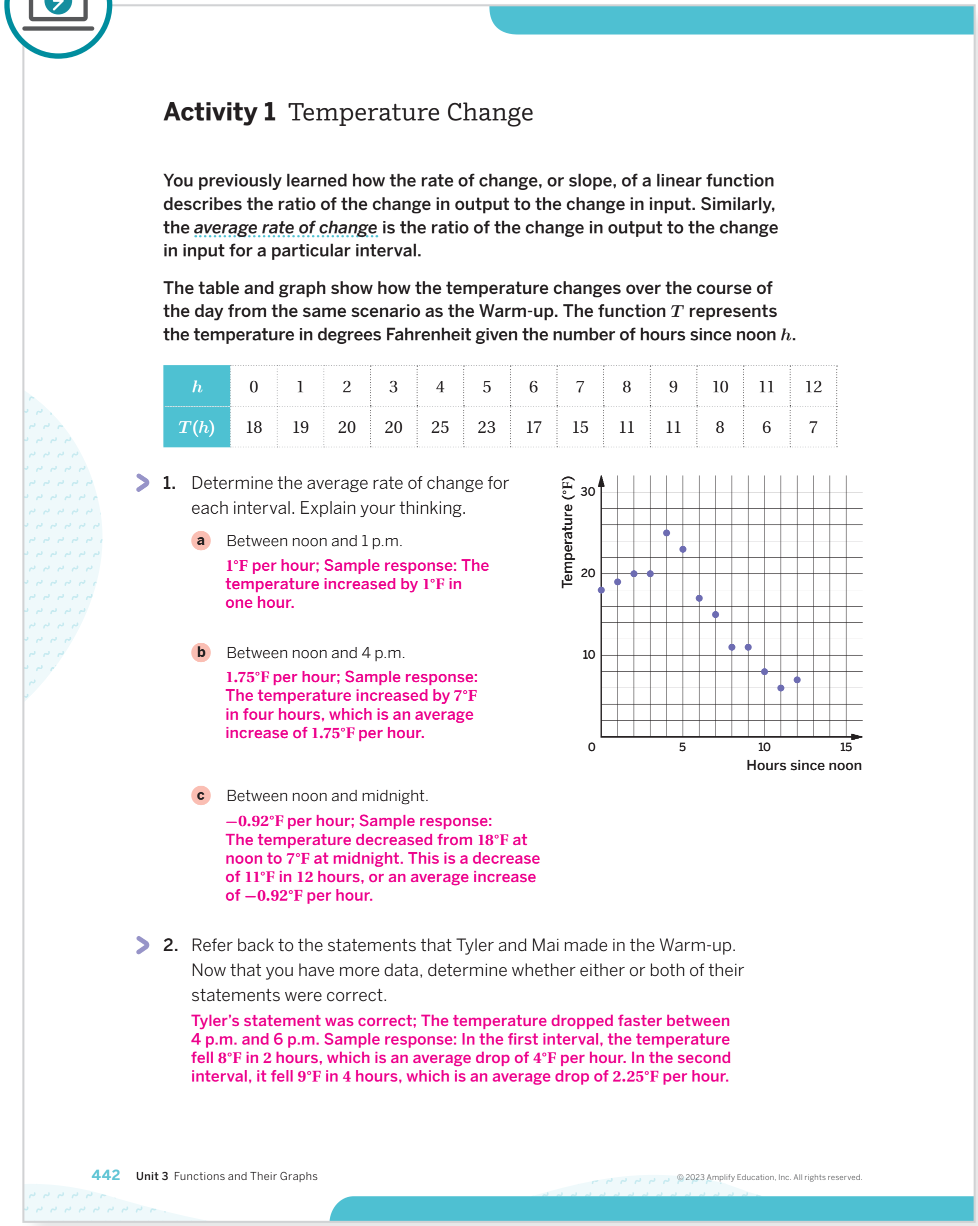
Use: Before the Warm-up

Informed by: Performance on Lesson 8, Practice Problem 5 and Pre-Unit Readiness Assessment, Problem 5

Small Groups | 15 min

Activity 1 Temperature Change

Students determine how temperature changes over different time intervals to build an understanding of the average rate of change.



Activity 1 Temperature Change

You previously learned how the rate of change, or slope, of a linear function describes the ratio of the change in output to the change in input. Similarly, the *average rate of change* is the ratio of the change in output to the change in input for a particular interval.

The table and graph show how the temperature changes over the course of the day from the same scenario as the Warm-up. The function T represents the temperature in degrees Fahrenheit given the number of hours since noon h.

h	0	1	2	3	4	5	6	7	8	9	10	11	12
$T(h)$	18	19	20	20	25	23	17	15	11	11	8	6	7

1. Determine the average rate of change for each interval. Explain your thinking.
   - **a** Between noon and 1 p.m.
     1°F per hour; Sample response: The temperature increased by 1°F in one hour.
   - **b** Between noon and 4 p.m.
     1.75°F per hour; Sample response: The temperature increased by 7°F in four hours, which is an average increase of 1.75°F per hour.
   - **c** Between noon and midnight.
     —0.92°F per hour; Sample response: The temperature decreased from 18°F at noon to 7°F at midnight. This is a decrease of 11°F in 12 hours, or an average increase of —0.92°F per hour.
2. Refer back to the statements that Tyler and Mai made in the Warm-up. Now that you have more data, determine whether either or both of their statements were correct.
   Tyler's statement was correct; The temperature dropped faster between 4 p.m. and 6 p.m. Sample response: In the first interval, the temperature fell 8°F in 2 hours, which is an average drop of 4°F per hour. In the second interval, it fell 9°F in 4 hours, which is an average drop of 2.25°F per hour.

442 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in pairs. Read the narrative as a class. Pause for a class discussion after students complete Problem 1.

2 Monitor

Help students get started by having them identify which variable represents the output and which variable represents the input.

Look for points of confusion:

- **Calculating the rate of change between every input value on an interval.** Ask, "How is determining the average rate of change different from rate of change or slope?"

Look for productive strategies:

- Connecting endpoints of an interval with a line segment on the graph and determining the slope.
- Determining average rate of change as a ratio using values from endpoints of the interval.
- Interpreting the average rate of change in the context of temperature and time.

3 Connect

Display the table of values and the graph.

Have pairs of students share how they determined the average rate of change for each interval, and whether Tyler or Mai was correct, and their thinking.

Define the ***average rate of change***.

Highlight that the average rate of change is a way to quantify changes over a particular interval without worrying about the smaller changes in between.

Ask, "How is the average rate of change different from slope?" Sample response: The slope describes the rate of change of a linear function, which remains the same on all intervals. The average rate of change can be used to determine how any function changes across any interval.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they learned about the rate of change for linear relationships in middle school. Consider showing the graph of a line and demonstrate how to determine the slope of the line by determining the ratio of the change in vertical distance to the change in horizontal distance between two points. In order words, this is the ratio of the change in output to the change in input.

Extension: *Math Enrichment*

Have students complete the following problem: Over what interval did the temperature decrease the most rapidly? Increase the most rapidly? Decreased the most rapidly between 5 p.m. and 6 p.m. and increased the most rapidly between 3 p.m. and 4 p.m.

Math Language Development

MLR7: Compare and Connect

Before the Connect, invite students to create a visual display that shows how they determined the average rates of change for the specified intervals in Problem 1. Students should consider how to represent their strategy so that other students will be able to understand their solution method.

English Learners

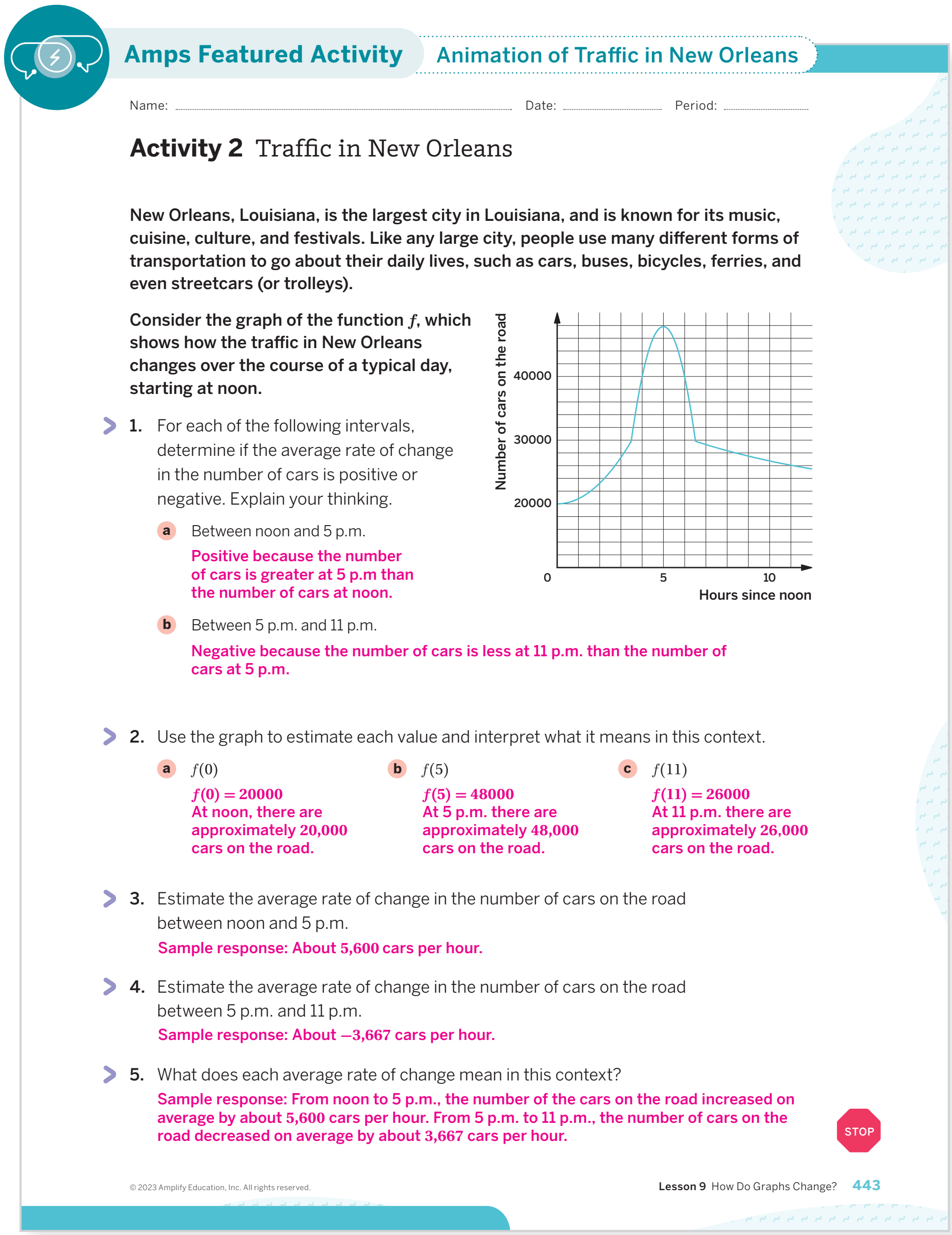
Consider partnering students who speak the same primary language, in order to support their use of developing mathematical language.

Small Groups | 15 min

Activity 2 Traffic in New Orleans

Students analyze how traffic changes over time to apply their understanding of the average rate of change in context.

Amps Featured Activity Animation of Traffic in New Orleans



Name: ______ Date: ______ Period: ______

Activity 2 Traffic in New Orleans

New Orleans, Louisiana, is the largest city in Louisiana, and is known for its music, cuisine, culture, and festivals. Like any large city, people use many different forms of transportation to go about their daily lives, such as cars, buses, bicycles, ferries, and even streetcars (or trolleys).

Consider the graph of the function f, which shows how the traffic in New Orleans changes over the course of a typical day, starting at noon.

1. For each of the following intervals, determine if the average rate of change in the number of cars is positive or negative. Explain your thinking.
   - a Between noon and 5 p.m.
     Positive because the number of cars is greater at 5 p.m than the number of cars at noon.
   - b Between 5 p.m. and 11 p.m.
     Negative because the number of cars is less at 11 p.m. than the number of cars at 5 p.m.

2. Use the graph to estimate each value and interpret what it means in this context.
   - a $f(0)$
     $f(0) = 20000$
     At noon, there are approximately 20,000 cars on the road.
   - b $f(5)$
     $f(5) = 48000$
     At 5 p.m. there are approximately 48,000 cars on the road.
   - c $f(11)$
     $f(11) = 26000$
     At 11 p.m. there are approximately 26,000 cars on the road.

3. Estimate the average rate of change in the number of cars on the road between noon and 5 p.m.
   Sample response: About 5,600 cars per hour.

4. Estimate the average rate of change in the number of cars on the road between 5 p.m. and 11 p.m.
   Sample response: About −3,667 cars per hour.

5. What does each average rate of change mean in this context?
   Sample response: From noon to 5 p.m., the number of the cars on the road increased on average by about 5,600 cars per hour. From 5 p.m. to 11 p.m., the number of cars on the road decreased on average by about 3,667 cars per hour.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 How Do Graphs Change? 443

1 Launch

Read the narrative as a class. Activate prior knowledge by asking, "Why might traffic in New Orleans increase or decrease throughout the day?"

2 Monitor

Help students get started by having them estimate values for the endpoints of each interval and label them on the graph.

Look for points of confusion:

- **Calculating the average rate of change in Problem 1.** Have students connect the endpoints of the interval with a line. Ask, "Without performing any calculations, what trend does this line show?"
- **Misinterpreting the average rate of change in context.** Have students identify the independent and dependent variables.

Look for productive strategies:

- Recognizing the average rate of change is positive/negative when a graph is increasing/decreasing.
- Using the scale to estimate output values when calculating the average rate of change.
- Interpreting the values determined for average rate of change as an increase or decrease in cars on the road.

3 Connect

Display the graph of the traffic in New Orleans.

Have pairs of students share how they determined and interpreted the average rate of change over different intervals.

Ask, "Why did some of you arrive at slightly different values for the average rate of change?" Sample response: When analyzing the graph, values could be estimated differently.

Highlight that paying close attention to the scale of a graph is important when determining the average rate of change in context because values must sometimes be estimated.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students what time periods throughout the day typically have greater traffic on the road than other time periods. Sample responses: the morning rush (to work), lunch, the evening rush (from work).

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can view an animation that shows the change in traffic over time. This will support them in visualizing the scenario in the activity.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their responses to Problem 1, draw connections between the terms *increasing* and *decreasing* intervals and whether the average rate of change is positive or negative for those intervals.

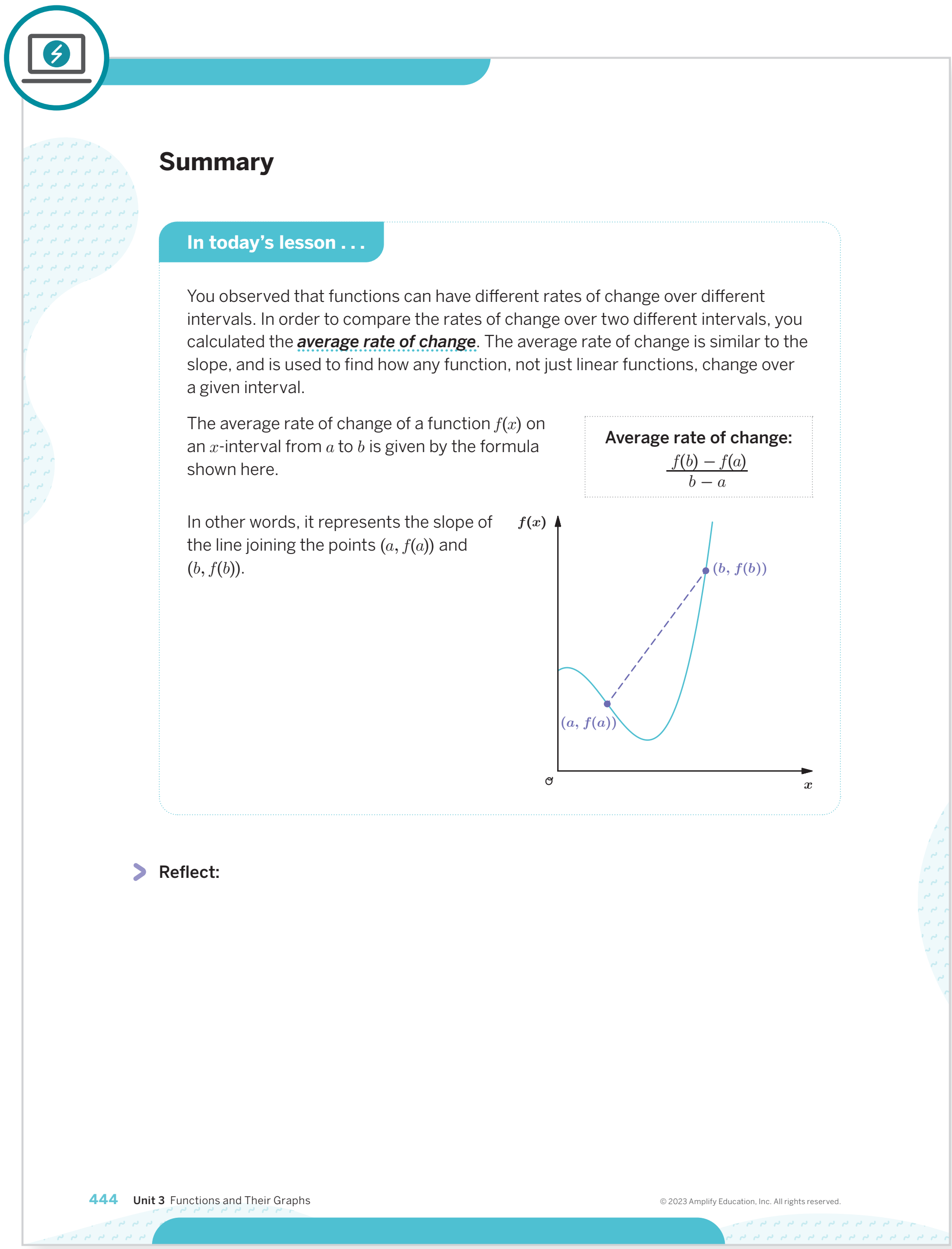
English Learners

Annotate the graph with the terms *increasing* and *positive average rate of change* for the interval from noon to 5 p.m. Then annotate the graph with the terms *decreasing* and *negative average rate of change* for the interval from 5 p.m. to midnight.

Whole Class | 5 min

Summary

Review and synthesize how to determine and interpret the average rate of change.



Summary

In today's lesson . . .

You observed that functions can have different rates of change over different intervals. In order to compare the rates of change over two different intervals, you calculated the ***average rate of change***. The average rate of change is similar to the slope, and is used to find how any function, not just linear functions, change over a given interval.

The average rate of change of a function $f(x)$ on an x-interval from a to b is given by the formula shown here.

Average rate of change: $\frac{f(b) - f(a)}{b - a}$

In other words, it represents the slope of the line joining the points $(a, f(a))$ and $(b, f(b))$.

Reflect:

444 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

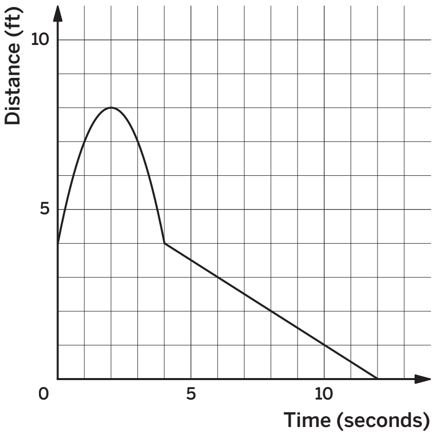
Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the term *average rate of change* that were added to the display during the lesson.

Synthesize

Display the following graph.



Ask, "What is the average rate of change between 0 seconds and 6 seconds?"

$-\frac{1}{6}$ ft per second

Have students share how they calculated the average rate of change for this interval and what it means within the context of the scenario.

Highlight that similar to slope, the average rate of change is determined by finding the ratio of outputs to inputs, but describes changes over any interval for both linear and nonlinear functions.

Formalize vocabulary: ***average rate of change***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

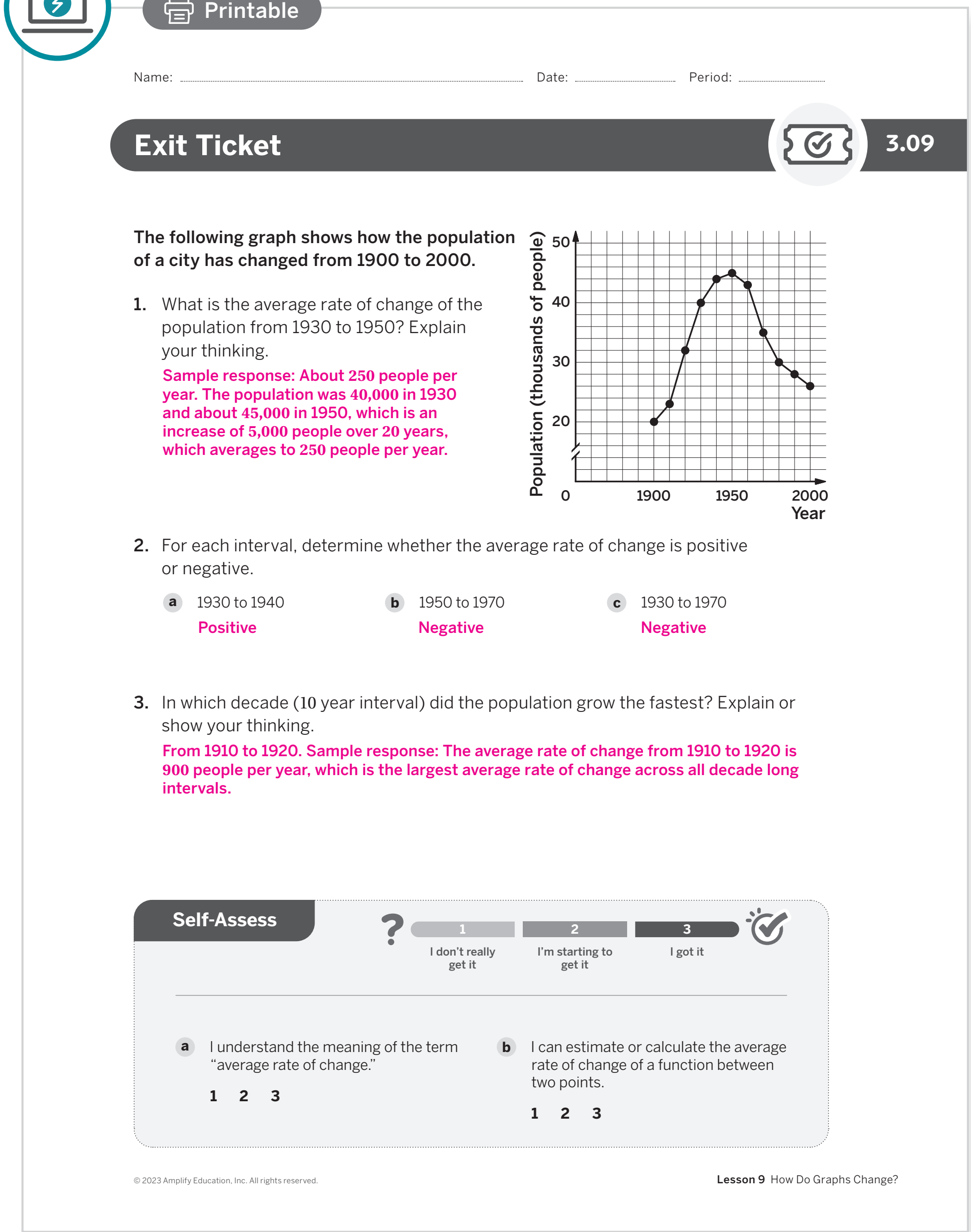
- "How is the average rate of change different from calculating slope?"

Independent | 10 min

Exit Ticket

Students demonstrate their understanding by determining and interpreting the average rate of change of a function over a specified interval within the context of a scenario.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.09

The following graph shows how the population of a city has changed from 1900 to 2000.

1. What is the average rate of change of the population from 1930 to 1950? Explain your thinking.
   Sample response: About 250 people per year. The population was 40,000 in 1930 and about 45,000 in 1950, which is an increase of 5,000 people over 20 years, which averages to 250 people per year.

2. For each interval, determine whether the average rate of change is positive or negative.
   - a 1930 to 1940 Positive
   - b 1950 to 1970 Negative
   - c 1930 to 1970 Negative

3. In which decade (10 year interval) did the population grow the fastest? Explain or show your thinking.
   From 1910 to 1920. Sample response: The average rate of change from 1910 to 1920 is 900 people per year, which is the largest average rate of change across all decade long intervals.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a I understand the meaning of the term "average rate of change."
1 2 3

b I can estimate or calculate the average rate of change of a function between two points.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 How Do Graphs Change?

Success looks like . . .

- **Language Goal:** Understanding the meaning of the term *average rate of change*. **(Speaking and Listening, Reading and Writing)**
- **Goal:** Determining the average rate of change of a function between two points.
  - » Determining the average rate of change from 1930 to 1960 in Problem 1.

Suggested next steps

If students incorrectly determine or interpret the average rate of change in Problem 1, consider:

- Reviewing how to determine and interpret average rate of change from Activity 1.
- Assigning Practice Problem 3.
- Asking, "What are the values for the endpoints of the interval from 1930 to 1950?"

If students incorrectly determine whether the average rate of change is positive or negative in Problem 2, consider:

- Reviewing how to determine if the average rate of change is positive or negative from Activity 2.
- Assigning Practice Problem 1.
- Asking, "What key feature means the same as the average rate of change being positive or negative?"

If students incorrectly determine the interval in which population grew the fastest in Problem 3, consider:

- Reviewing comparing average rates of change from Activity 1.
- Assigning Practice Problem 2.
- Asking, "Are there any intervals you know where the population was not growing? How do you know?"

Professional Learning

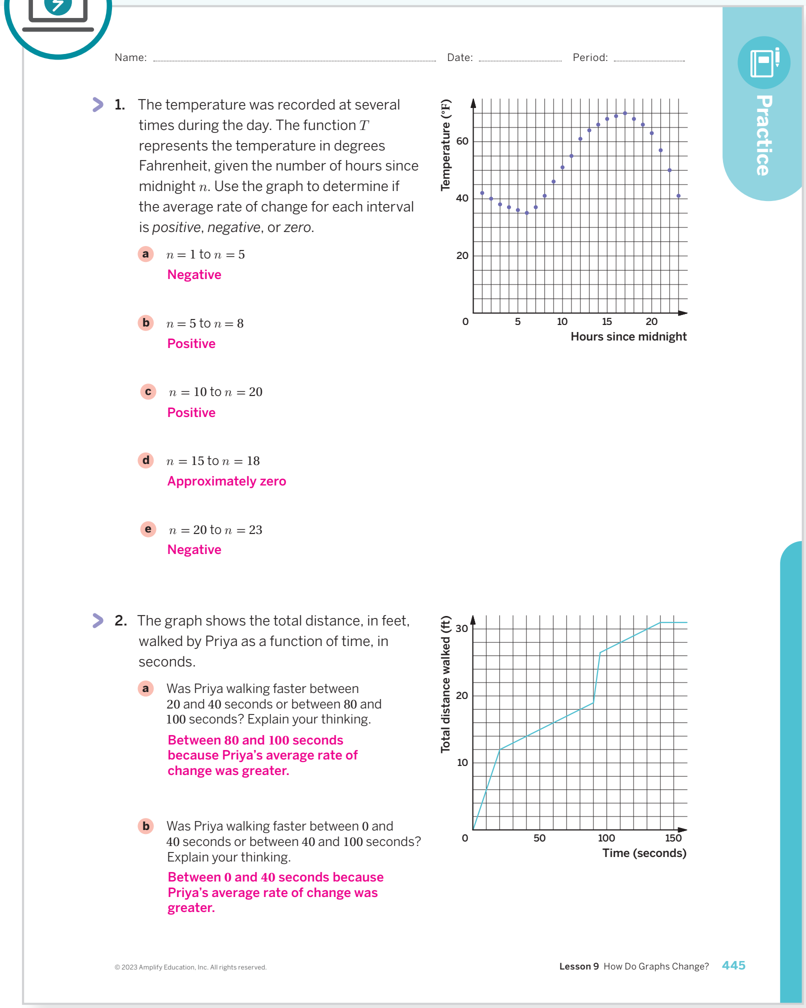
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In earlier lessons, students learned about increasing and decreasing graphs. How did that support understanding of the average rate of change? What might you change for the next time you teach this lesson?
- In Activity 1, you used intentional grouping with MLR7 to pair students who speak the same primary language. What effect did this grouping strategy have on students' use of developing mathematical language? Would you change anything the next time you use this routine?

Practice

Independent



Name: ________ Date: ________ Period: ________

Practice

1. The temperature was recorded at several times during the day. The function T represents the temperature in degrees Fahrenheit, given the number of hours since midnight n. Use the graph to determine if the average rate of change for each interval is *positive*, *negative*, or *zero*.

   a. $n = 1$ to $n = 5$
   Negative

   b. $n = 5$ to $n = 8$
   Positive

   c. $n = 10$ to $n = 20$
   Positive

   d. $n = 15$ to $n = 18$
   Approximately zero

   e. $n = 20$ to $n = 23$
   Negative

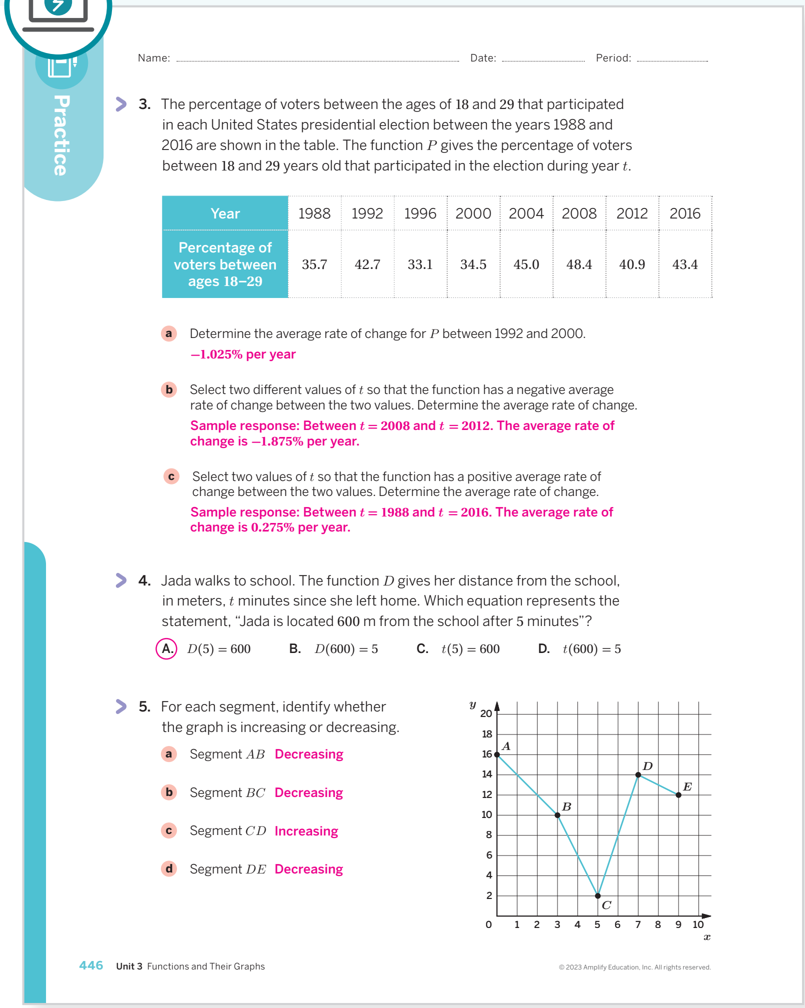
2. The graph shows the total distance, in feet, walked by Priya as a function of time, in seconds.

   a. Was Priya walking faster between 20 and 40 seconds or between 80 and 100 seconds? Explain your thinking.
   Between 80 and 100 seconds because Priya's average rate of change was greater.

   b. Was Priya walking faster between 0 and 40 seconds or between 40 and 100 seconds? Explain your thinking.
   Between 0 and 40 seconds because Priya's average rate of change was greater.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 9 How Do Graphs Change? 445



Name: ________ Date: ________ Period: ________

Practice

3. The percentage of voters between the ages of 18 and 29 that participated in each United States presidential election between the years 1988 and 2016 are shown in the table. The function P gives the percentage of voters between 18 and 29 years old that participated in the election during year t.

Year	1988	1992	1996	2000	2004	2008	2012	2016
Percentage of voters between ages 18–29	35.7	42.7	33.1	34.5	45.0	48.4	40.9	43.4

   a. Determine the average rate of change for P between 1992 and 2000.
   −1.025% per year

   b. Select two different values of t so that the function has a negative average rate of change between the two values. Determine the average rate of change.
   Sample response: Between $t = 2008$ and $t = 2012$. The average rate of change is −1.875% per year.

   c. Select two values of t so that the function has a positive average rate of change between the two values. Determine the average rate of change.
   Sample response: Between $t = 1988$ and $t = 2016$. The average rate of change is 0.275% per year.

4. Jada walks to school. The function D gives her distance from the school, in meters, t minutes since she left home. Which equation represents the statement, "Jada is located 600 m from the school after 5 minutes"?

   [✓ A.] $D(5) = 600$ B. $D(600) = 5$ C. $t(5) = 600$ D. $t(600) = 5$

5. For each segment, identify whether the graph is increasing or decreasing.

   a. Segment AB **Decreasing**

   b. Segment BC **Decreasing**

   c. Segment CD **Increasing**

   d. Segment DE **Decreasing**

446 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	1
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 4	1
Formative ⚡	5	Unit 3 Lesson 10	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 3 | **LESSON 10**

Where Are Functions Changing?

Let's determine the reasonable input and output values of a function.

Focus

Goal

1. **Language Goal:** Determine reasonable input and output values for a function given a description of a situation. **(Reading and Writing)**

Coherence

Today

Students make sense of different situations by determining what are reasonable input and output values for a given the context of a given scenario. Students describe in words what some reasonable input values are for two different scenarios, and then attend to precision when asked to specifically identify certain input values as reasonable or unreasonable for different scenarios. Students then identify output values as reasonable or unreasonable for scenarios they examined in the previous activity. Domain and range are not defined in this lesson.

Previously

In Lesson 9, students determined how a function changes over specified intervals by determining the average rate of change.

Coming Soon

In Lesson 11, students will formally define *domain* and *range* and explore how interval notation can be used to describe a function's domain and range.

Rigor

- Students build **conceptual understanding** of domain and range by exploring the reasonableness of input and output values in different contexts.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	10 min	5 min	5 min
Independent	Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one set per pair

Math Language Development

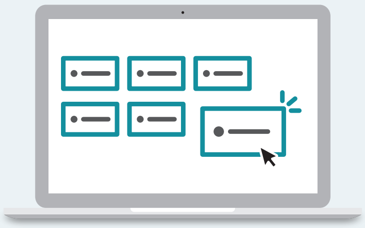
Review words

- *function*
- *input*
- *output*

Amps | Featured Activity

Activity 2
Digitally Matching Possible Input Values

Students are able to digitally create matches between the possible input values for each of the four scenarios.



Building Math Identity and Community

Connecting to Mathematical Practices

When dealing with great precision, some students struggle to be able to motivate themselves to care about all of the details. It is just this level of precision, however, that makes these graphical mathematical models useful. Before students set their minds to determining whether or not the inputs are reasonable, encourage them to write a peer a motivational note to read throughout the activity when they feel themselves being mentally pulled away from the task.

Modifications to Pacing

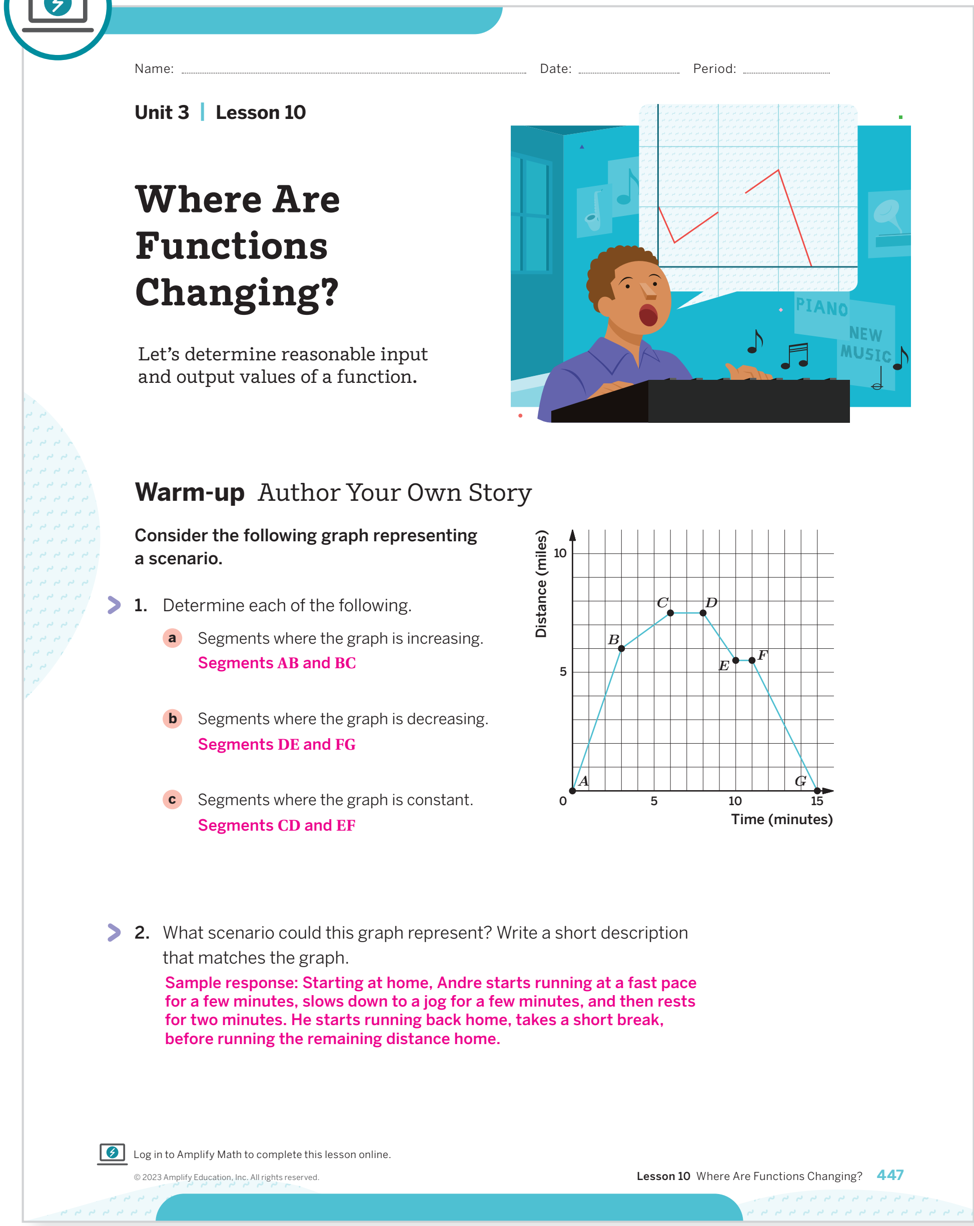
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 1 may be omitted.
- In **Activity 1**, have groups of students complete one scenario then present each scenario during the Connect.
- In **Activity 2**, Problem 4 may be omitted.

Independent | 5 min

Warm-up Author Your Own Story

Students analyze changes in a graph to make sense of important characteristics and use them to write a story.



Name: ______ Date: ______ Period: ______

Unit 3 | Lesson 10

Where Are Functions Changing?

Let's determine reasonable input and output values of a function.

Warm-up Author Your Own Story

Consider the following graph representing a scenario.

1. Determine each of the following.
   - a Segments where the graph is increasing.
     Segments AB and BC
   - b Segments where the graph is decreasing.
     Segments DE and FG
   - c Segments where the graph is constant.
     Segments CD and EF

2. What scenario could this graph represent? Write a short description that matches the graph.
   Sample response: Starting at home, Andre starts running at a fast pace for a few minutes, slows down to a jog for a few minutes, and then rests for two minutes. He starts running back home, takes a short break, before running the remaining distance home.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Where Are Functions Changing? 447

① Launch

Describe a story that could represent the graph. Then have students work independently to author their own story.

② Monitor

Help students get started by asking, "On a graph of a function, what is the difference between increasing, decreasing, and constant?"

Look for points of confusion:

- **Thinking all segments are constant because they are all linear.** Ask, "If part of a graph is constant, what is its average rate of change?"
- **Having difficulty writing a story to match the graph.** Ask, "What could be happening at each line segment?"

Look for productive strategies:

- Describing change in distance relative to time, for example, 4 miles in 3 minutes.
- Describing the slope of segments as being positive, negative, or zero.
- Recognizing changes in distance as moving away or towards the initial starting point.
- Connecting changes in slope as changes in velocity/speed.

③ Connect

Display the graph of the scenario.

Have individual students share their stories. Select and sequence students using key terminology.

Highlight that segments on a graph will not always be labeled, and sometimes there is a need to be specific about where changes to a function's graph occur.

Ask, "If each segment of the graph were not labeled, how could you describe where the changes occur?"

Math Language Development

MLR7: Compare and Connect

Before having individual students share their stories with the class during the Connect, ask students to share their stories with a partner. Encourage partners to press for details about how their stories connect to the graphs. Ask partners to discuss the following question: "What words or phrases in your story represent where the graph is increasing? Decreasing? Constant?"

English Learners

As students share their stories with the class, use gestures to connect students' stories to the graphs. For example, as you say the term *constant*, hold your arm horizontally to illustrate a horizontal line segment.

Power-up

To power up students' ability to identify increasing or decreasing line segments on a graph, have students complete:

Determine which statements about the graph are true. Select *all* that apply.

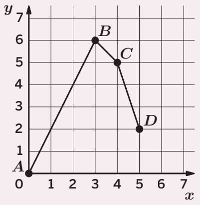
A. Two segments on the graph are increasing and one segment is decreasing.

[✓ **B.**] Segment AB is increasing.

[✓ **C.**] Segment BC is decreasing.

[✓ **D.**] Two segments on the graph are decreasing and one segment is increasing.

E. Segment CD is increasing.



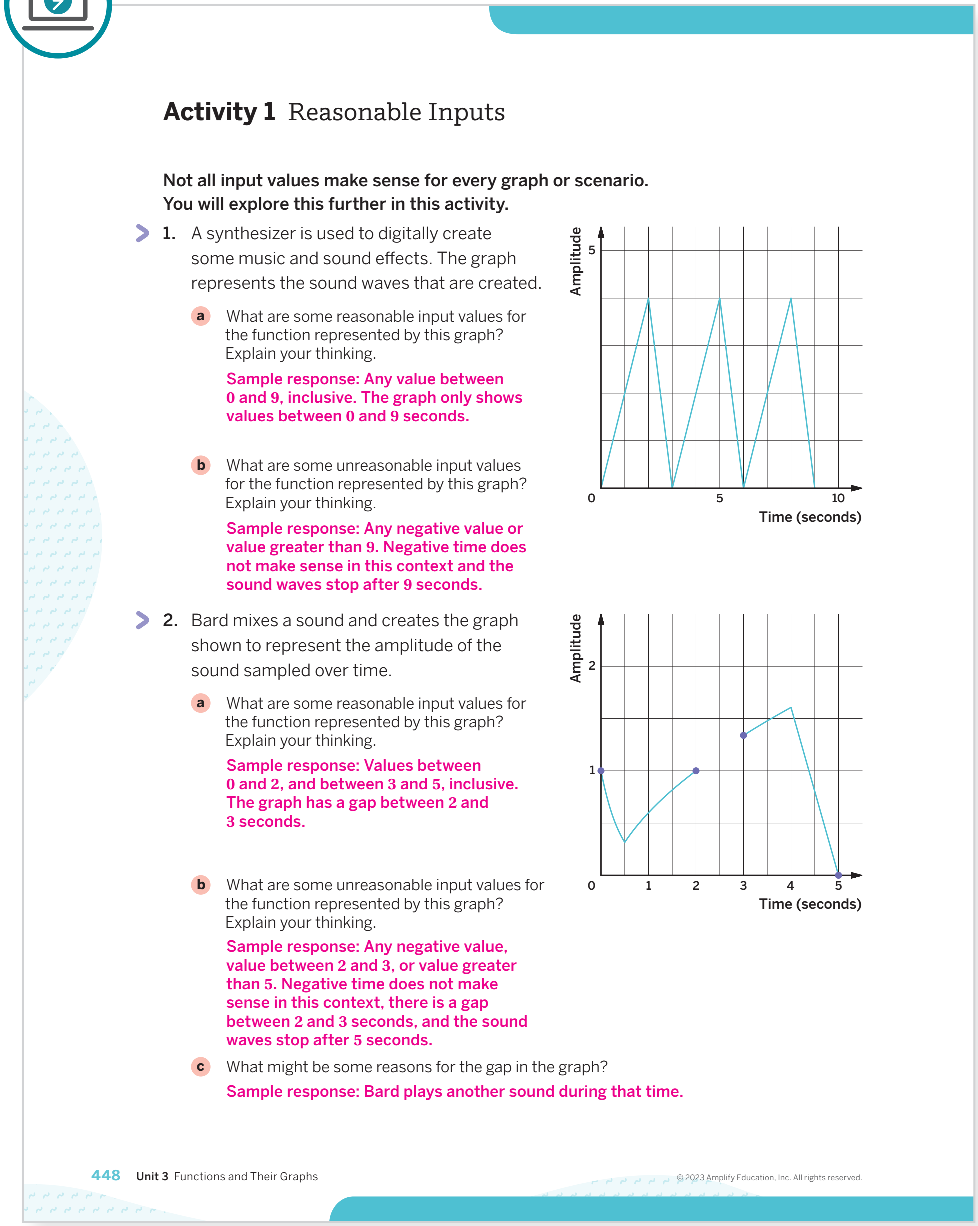
Use: Before the Warm-up

Informed by: Performance on Lesson 9, Practice Problem 5 and Exit Ticket

Small Groups | 10 min

Activity 1 Reasonable Inputs

Students determine reasonable input values for graphs of different scenarios to graphically reason why some input values make sense and others do not.



Activity 1 Reasonable Inputs

Not all input values make sense for every graph or scenario. You will explore this further in this activity.

1. A synthesizer is used to digitally create some music and sound effects. The graph represents the sound waves that are created.

a. What are some reasonable input values for the function represented by this graph? Explain your thinking.

Sample response: Any value between 0 and 9, inclusive. The graph only shows values between 0 and 9 seconds.

b. What are some unreasonable input values for the function represented by this graph? Explain your thinking.

Sample response: Any negative value or value greater than 9. Negative time does not make sense in this context and the sound waves stop after 9 seconds.

2. Bard mixes a sound and creates the graph shown to represent the amplitude of the sound sampled over time.

a. What are some reasonable input values for the function represented by this graph? Explain your thinking.

Sample response: Values between 0 and 2, and between 3 and 5, inclusive. The graph has a gap between 2 and 3 seconds.

b. What are some unreasonable input values for the function represented by this graph? Explain your thinking.

Sample response: Any negative value, value between 2 and 3, or value greater than 5. Negative time does not make sense in this context, there is a gap between 2 and 3 seconds, and the sound waves stop after 5 seconds.

c. What might be some reasons for the gap in the graph?

Sample response: Bard plays another sound during that time.

448 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Tell students they will now analyze two graphs to determine reasonable input values. Explain what “amplitude” means by relating high and low points to loudness.

2 Monitor

Help students get started by having them label key features on the graphs.

Look for points of confusion:

- **Confusing the possible output values for the input values.** Have students identify the independent and dependent variables. Then ask, “Do your chosen values still make sense?”

Look for productive strategies:

- Recognizing each endpoint as bounds for the input values.
- Recognizing the gap in the graph in Problem 2 means those values are excluded as inputs.
- Reasoning about time and amplitude to explain why certain values are unreasonable inputs.

3 Connect

Display the graphs of each scenario.

Have groups of students share their chosen input values along with their respective explanations.

Highlight that in many situations, time is not negative. Because time moves forward, negative values for time do not make sense because they occur before an event has taken place.

Ask, “What are some possible situations where negative input values would make sense?” Sample response: If the input is time since noon, or years after 2010, then negative values would represent time before noon or years before 2010.

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

In previous lessons, students explored the graph of the *pitch* of a sound as it relates to time. In this activity, students explore the graph of the *amplitude* of a sound over time. Tell students that amplitude represents the level of loudness of a sound. Consider playing sounds of two different pitches versus two different amplitudes to illustrate the difference between these terms.

Extension: *Math Enrichment*

Ask students to determine the slope of each increasing and decreasing line segment in Problem 1 and describe what they notice. The slope for each increasing line segment is 2. The slope for each decreasing line segment is –4. The slopes alternate.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share their responses, listen for and amplify responses that use precise mathematical vocabulary, such as whether to include or not include the endpoints, the meaning of the gap in Problem 2, and how to interpret reasonable input values based on the graph.

English Learners

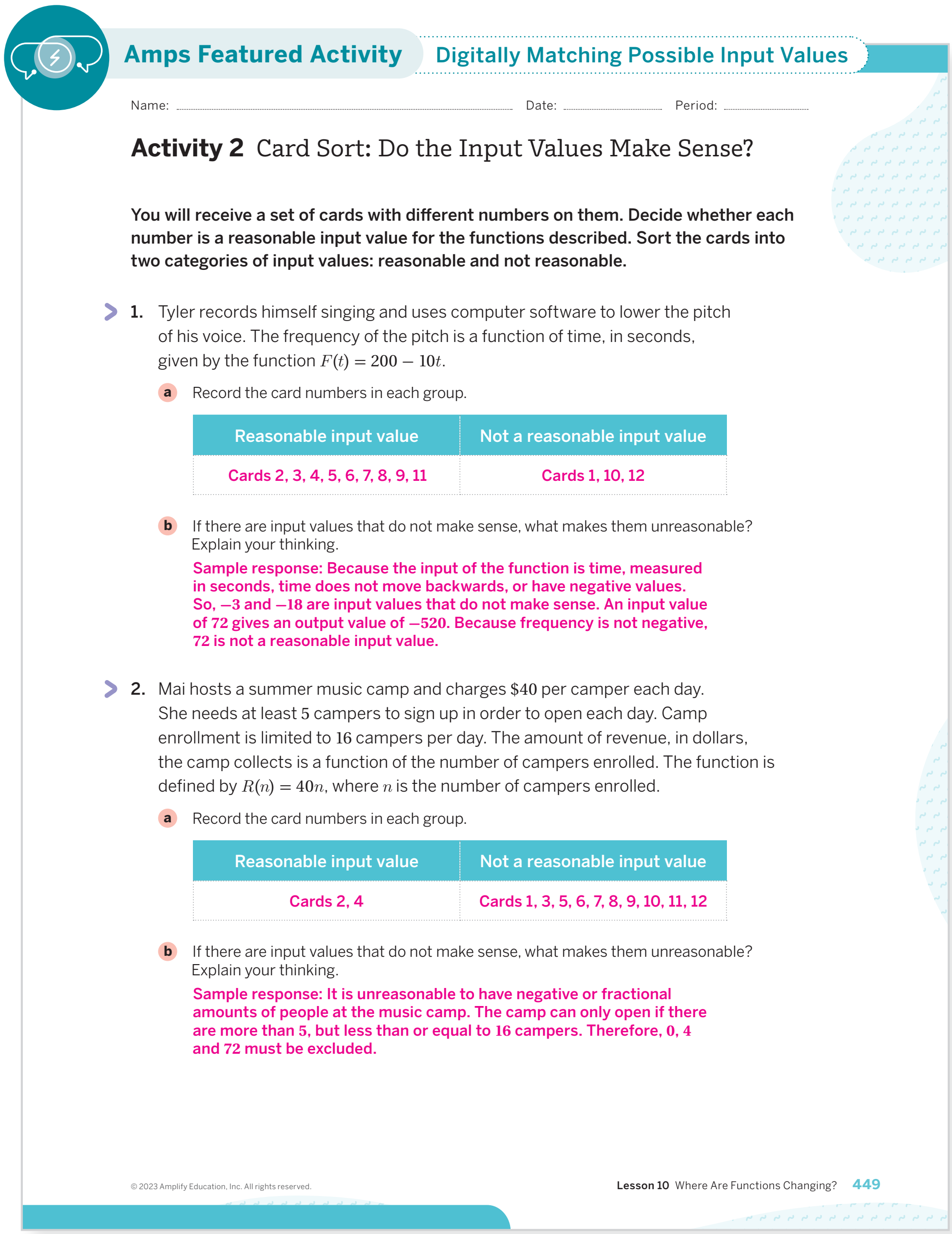
Reinforce the meaning of the terms *reasonable* and *unreasonable* by placing a checkmark next to reasonable input values and crossing out unreasonable input values. Annotate these with the terms *reasonable* and *unreasonable*.

Small Groups | 15 min

Activity 2 Card Sort: Do the Input Values Make Sense?

Students consider different scenarios to reason about input values that do or do not make sense in the context of each scenario.

Amps Featured Activity Digitally Matching Possible Input Values



Name: ______ Date: ______ Period: ______

Activity 2 Card Sort: Do the Input Values Make Sense?

You will receive a set of cards with different numbers on them. Decide whether each number is a reasonable input value for the functions described. Sort the cards into two categories of input values: reasonable and not reasonable.

1. Tyler records himself singing and uses computer software to lower the pitch of his voice. The frequency of the pitch is a function of time, in seconds, given by the function $F(t) = 200 - 10t$.

a. Record the card numbers in each group.

Reasonable input value	Not a reasonable input value
Cards 2, 3, 4, 5, 6, 7, 8, 9, 11	Cards 1, 10, 12

b. If there are input values that do not make sense, what makes them unreasonable? Explain your thinking.

Sample response: Because the input of the function is time, measured in seconds, time does not move backwards, or have negative values. So, -3 and -18 are input values that do not make sense. An input value of 72 gives an output value of -520. Because frequency is not negative, 72 is not a reasonable input value.

2. Mai hosts a summer music camp and charges \$40 per camper each day. She needs at least 5 campers to sign up in order to open each day. Camp enrollment is limited to 16 campers per day. The amount of revenue, in dollars, the camp collects is a function of the number of campers enrolled. The function is defined by $R(n) = 40n$, where n is the number of campers enrolled.

a. Record the card numbers in each group.

Reasonable input value	Not a reasonable input value
Cards 2, 4	Cards 1, 3, 5, 6, 7, 8, 9, 10, 11, 12

b. If there are input values that do not make sense, what makes them unreasonable? Explain your thinking.

Sample response: It is unreasonable to have negative or fractional amounts of people at the music camp. The camp can only open if there are more than 5, but less than or equal to 16 campers. Therefore, 0, 4 and 72 must be excluded.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Where Are Functions Changing? 449

1 Launch

Keep students in groups and distribute the pre-cut cards from the Activity 2 PDF to each group and conduct the *Card Sort* routine. Say, "Each card contains a different number. For each scenario, sort the cards into two categories of input values: reasonable and not reasonable."

2 Monitor

Help students get started by using the *Three Reads* routine as described in the Math Language Development section.

Look for points of confusion:

- **For Problems 1 and 2, thinking all positive whole number values make sense.** Ask, "For an input of 72, can you estimate what an output value might be? Does that make sense given the context or constraints?"
- **Having difficulty making sense of an exponential function.** Have students identify the independent and dependent variables.

Look for productive strategies:

- Recognizing that negative values do not make sense if the input is time or people.
- Recognizing that the length of a square cannot be negative.
- Recognizing that some whole number values are outside the constraints of the context.
- Substituting whole number values into a function to determine whether the output value makes sense in context.

Activity 2 continued >

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students first sort the cards into three categories:

- Positive whole numbers
- Fractions and decimals
- Negative values

Then have them sort each category, one at a time, as to whether the values are reasonable or not reasonable as input values for each function.

Extension: *Math Enrichment*

Ask students how they could alter the scenario in Problem 3 so that Card 6 (4) is now a reasonable input value. Sample response: The camp can open if at least 4 (not 5) campers sign up each day.

Math Language Development

MLR6: Three Reads

Have students read each scenario three times to help make sense of the quantities and relationships. A sample routine is shown for Problem 1.

Read 1: Students should understand the function relates pitch and time.

Read 2: Ask students to identify the independent variable (time) and dependent variable (pitch).

Read 3: Ask students to ask themselves, "Can there be fractional, decimal, or negative values of the independent variable (time)?"

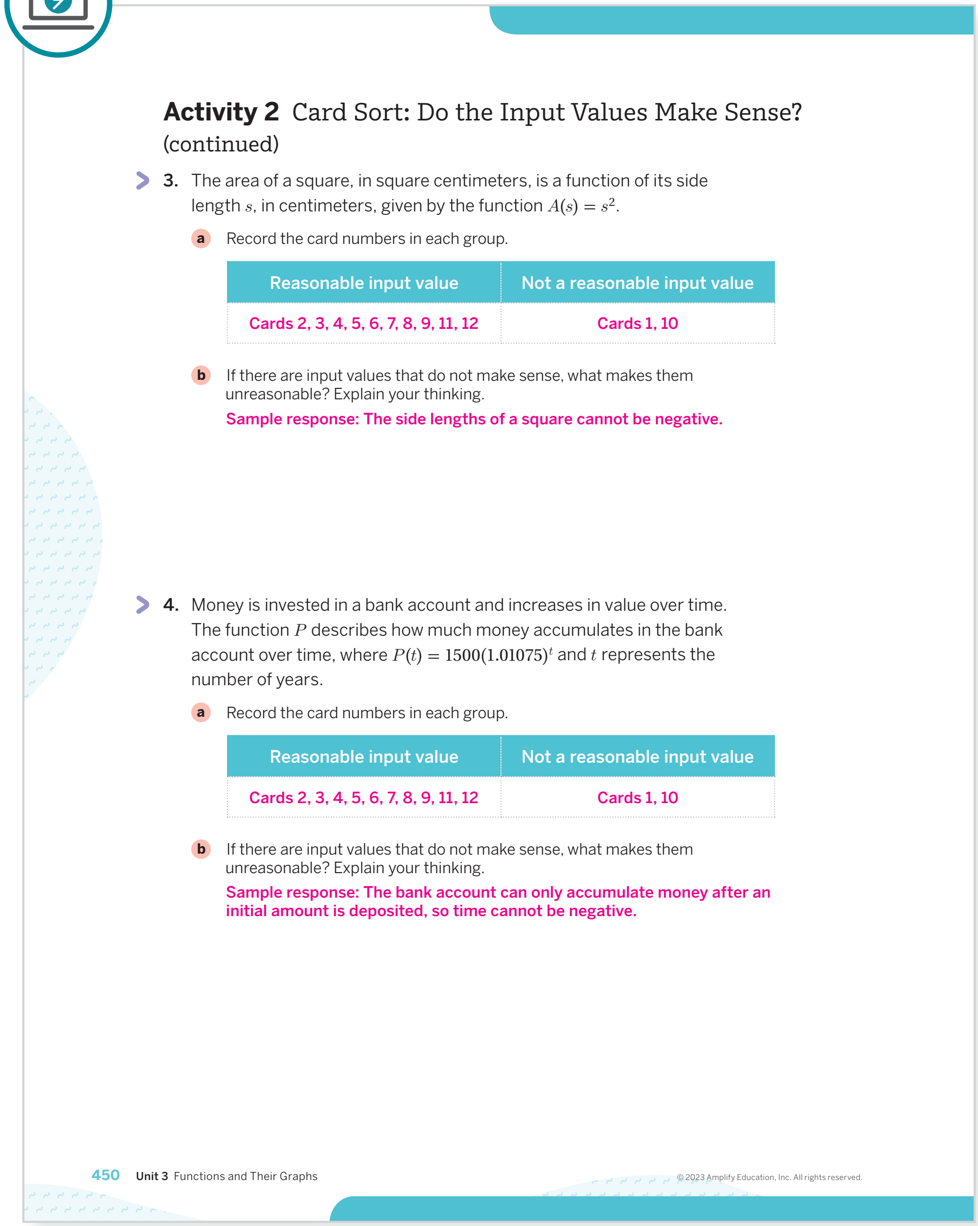
English Learners

Annotate the independent variables in each scenario and write the term *input values* next to them.

Activity 2 Card Sort: Do the Input Values Make Sense? (continued)

Small Groups | 15 min

Students consider different scenarios to reason about input values that do or do not make sense in the context of each scenario.



Activity 2 Card Sort: Do the Input Values Make Sense? (continued)

3. The area of a square, in square centimeters, is a function of its side length s, in centimeters, given by the function $A(s) = s^2$.

a Record the card numbers in each group.

Reasonable input value	Not a reasonable input value
Cards 2, 3, 4, 5, 6, 7, 8, 9, 11, 12	Cards 1, 10

b If there are input values that do not make sense, what makes them unreasonable? Explain your thinking.

Sample response: The side lengths of a square cannot be negative.

4. Money is invested in a bank account and increases in value over time. The function P describes how much money accumulates in the bank account over time, where $P(t) = 1500(1.01075)^t$ and t represents the number of years.

a Record the card numbers in each group.

Reasonable input value	Not a reasonable input value
Cards 2, 3, 4, 5, 6, 7, 8, 9, 11, 12	Cards 1, 10

b If there are input values that do not make sense, what makes them unreasonable? Explain your thinking.

Sample response: The bank account can only accumulate money after an initial amount is deposited, so time cannot be negative.

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the four scenarios and the Activity 2 PDF.

Have groups of students share which input values made sense and which did not for each scenario and why. Select and sequence student responses sorting negative or fractional values, using the scenario's context to sort, and substituting certain values.

Highlight that when a graphical representation is not provided, it is important to read the context to determine if input values make sense. It may not be immediately obvious that some input values do not make sense, but by substituting these values into the function, students will observe value(s) that do not make sense.

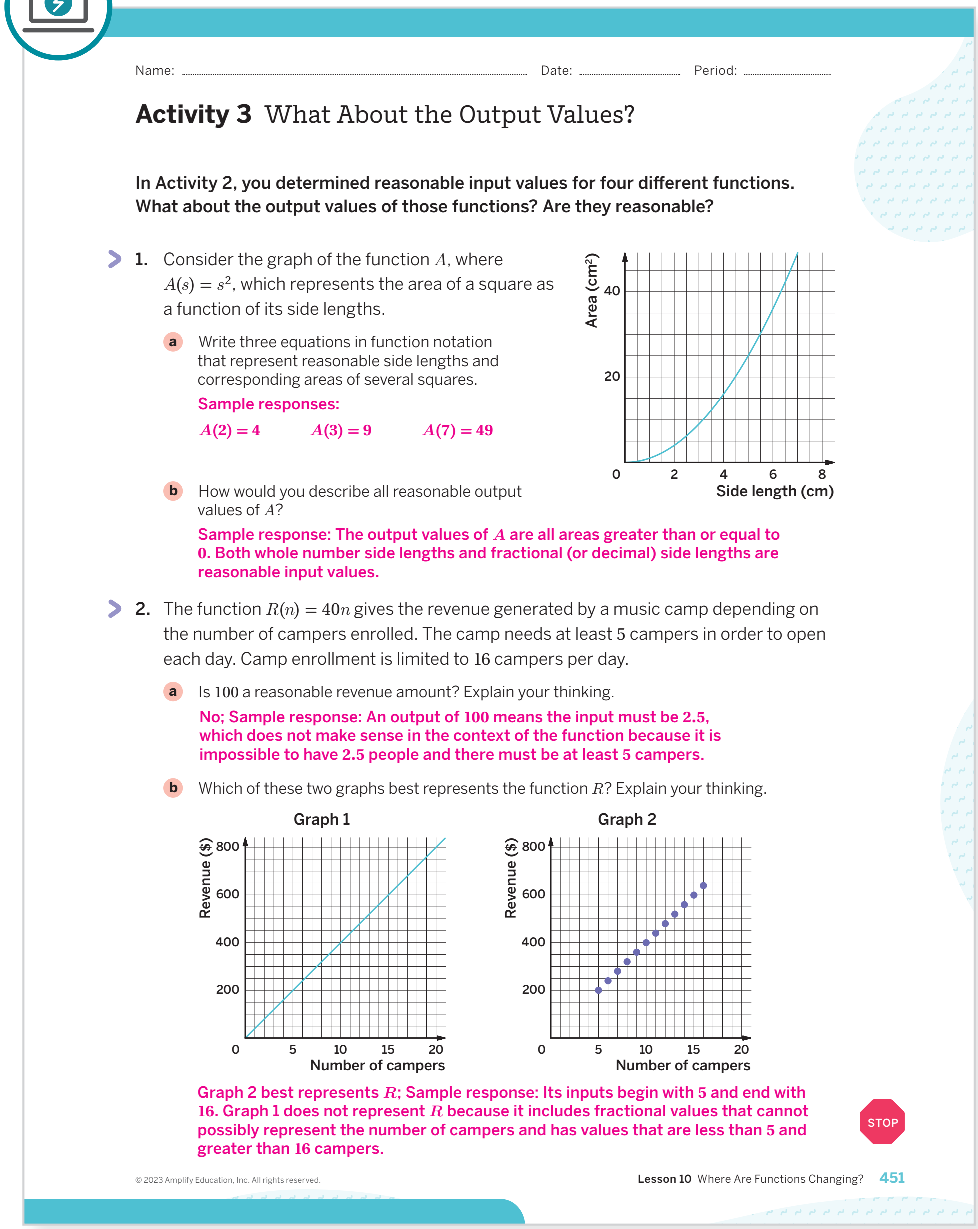
Ask, "How could you determine if an output value makes sense in a scenario?"

Note: Some students may say that Card 7 (0) is not a reasonable input value for the scenarios in Problem 4. Ask them to construct an argument defending their reasoning. For example, some students may say that some banks require a minimum amount to be present in an account or that the text states that money is invested, implying that 0 might not be a reasonable value.

Small Groups | 10 min

Activity 3 What About the Output Values?

Students determine possible output values for two scenarios from the previous activity to reason about why some output values do or do not make sense.



Name: ________ Date: ________ Period: ________

Activity 3 What About the Output Values?

In Activity 2, you determined reasonable input values for four different functions. What about the output values of those functions? Are they reasonable?

1. Consider the graph of the function A, where $A(s) = s^2$, which represents the area of a square as a function of its side lengths.

   a. Write three equations in function notation that represent reasonable side lengths and corresponding areas of several squares.

   Sample responses:

   $A(2) = 4$ $A(3) = 9$ $A(7) = 49$

   b. How would you describe all reasonable output values of A?

   Sample response: The output values of A are all areas greater than or equal to 0. Both whole number side lengths and fractional (or decimal) side lengths are reasonable input values.

2. The function $R(n) = 40n$ gives the revenue generated by a music camp depending on the number of campers enrolled. The camp needs at least 5 campers in order to open each day. Camp enrollment is limited to 16 campers per day.

   a. Is 100 a reasonable revenue amount? Explain your thinking.

   No; Sample response: An output of 100 means the input must be 2.5, which does not make sense in the context of the function because it is impossible to have 2.5 people and there must be at least 5 campers.

   b. Which of these two graphs best represents the function R? Explain your thinking.

Graph 2 best represents R; Sample response: Its inputs begin with 5 and end with 16. Graph 1 does not represent R because it includes fractional values that cannot possibly represent the number of campers and has values that are less than 5 and greater than 16 campers.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Where Are Functions Changing? 451

1 Launch

Say, "You have determined whether input values make sense in a context, now you will examine what output values make sense."

2 Monitor

Help students get started by asking, "What notation have you learned about that could help express input-output pairs?"

Look for points of confusion:

- **Describing the output values in Problem 1 using only visible values on the graph.** Ask, "What is the longest possible side length of a square? What is the greatest possible area?"
- **Having difficulty determining the graph that best represents the function R.** Have students re-read Problem 2 to relate given values to Graphs 1 and 2.

Look for productive strategies:

- Using function notation to describe input-output pairs.
- Recognizing the function R must be discrete because all input values do not make sense in context.

3 Connect

Display the graph of Problem 1 and the two possible graphs for Problem 2.

Have groups of students share their descriptions of possible output values, whether 100 is a reasonable output value, and which graph best represents the function R.

Highlight that the output values of function A are all values that are greater than or equal to 0, and that the possible output values for the function R are whole number multiples of 40 between 200 and 640.

Ask, "Is there a more efficient way to write the input and output values of a function?"

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Display the function from Problem 1, $A(s) = s^2$. Ask students to identify the input. Then ask them to identify the output. The input is s, the side length of a square. The output is $A(s)$, the area of the square. The output is also given by s^2.

Extension: *Math Enrichment*

Have students complete the following problem:

Another music camp generates revenue given by the function $P(n) = 100 + 30n$. For how many campers will the revenue for both camps be the same? What is that revenue? For 10 campers, both camps will generate a revenue of $400.

Math Language Development

MLR3: Critique, Correct, Clarify

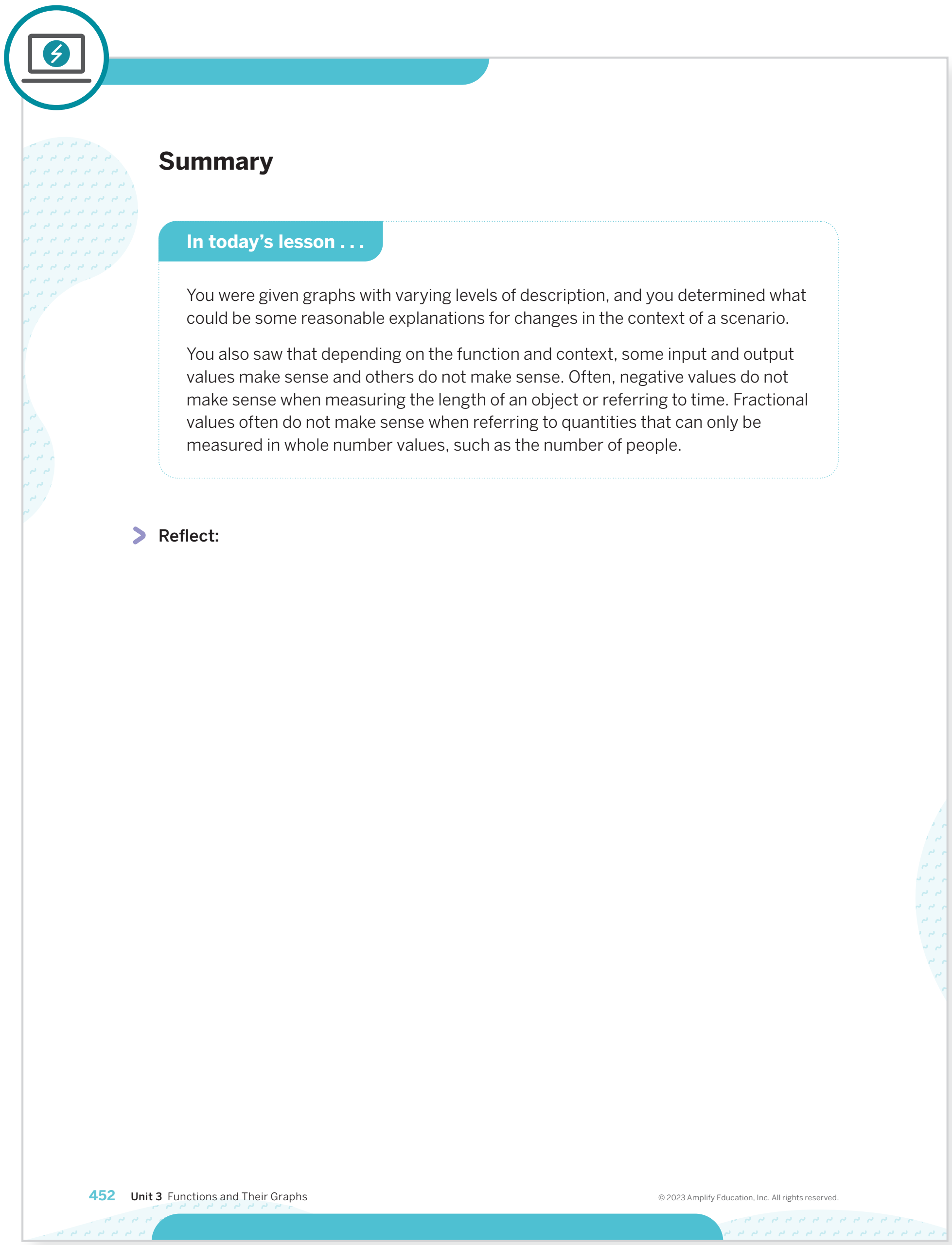
Display the incorrect statement, "The reasonable output values of function A are values from 0 to 50 because the graph only reaches up to 50 on the vertical axis." Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking." Sample response: I disagree because the graph will continue past 50. A square could have a side length of 16 cm, for example, with an area of 256 cm^2.
- ***Correct:*** "Write a corrected statement that is now true." Sample response: The reasonable output values of function A are all numbers greater than or equal to 0.
- ***Clarify:*** "How did you correct the statement? How do you know that the statement is now true?"

Whole Class | 5 min

Summary

Review and synthesize reasonable input and output values for different functions.



Summary

In today's lesson . . .

You were given graphs with varying levels of description, and you determined what could be some reasonable explanations for changes in the context of a scenario.

You also saw that depending on the function and context, some input and output values make sense and others do not make sense. Often, negative values do not make sense when measuring the length of an object or referring to time. Fractional values often do not make sense when referring to quantities that can only be measured in whole number values, such as the number of people.

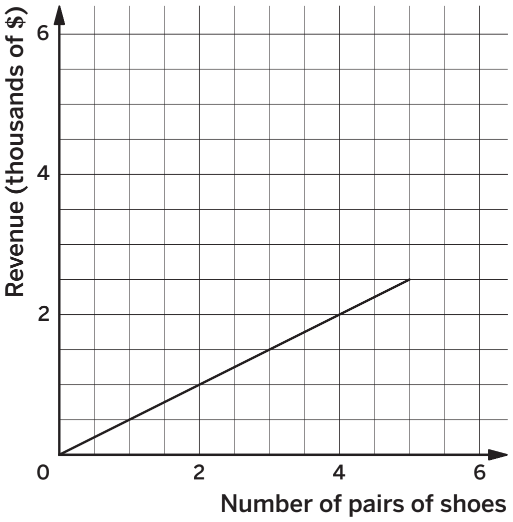
Reflect:

452 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the following graph with the scenario: "Kiran is selling 5 pairs of collectible shoes online. The graph represents the revenue generated from the number of pairs of shoes he sells."



Have students share reasonable input and output values for the scenario presented on the graph, and if the graph makes sense in the context of the scenario.

Highlight that many functions have reasonable input and output values that only contain certain values. This most often happens in scenarios where either only positive values make sense or whole number values make sense.

Ask, "What are some scenarios where the possible input and output values are limited by the context?" Sample response: Number of customers at a store and profit, the time it takes for a pot of water to boil, or the area of a triangle.

Reflect

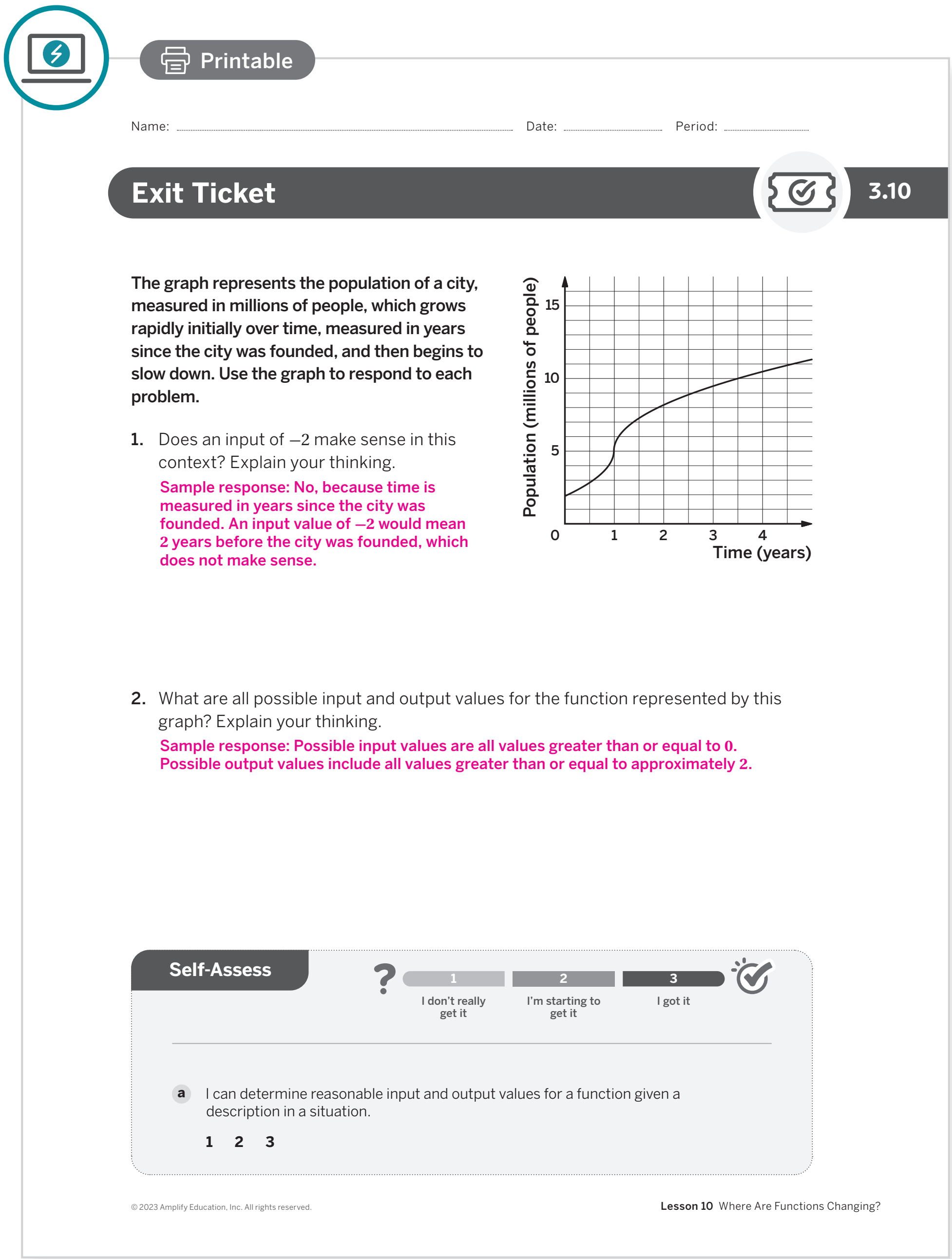
After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Why are not all values considered possible input and output values for all functions?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining reasonable input and output values for a given scenario.



Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 3.10

The graph represents the population of a city, measured in millions of people, which grows rapidly initially over time, measured in years since the city was founded, and then begins to slow down. Use the graph to respond to each problem.

1. Does an input of –2 make sense in this context? Explain your thinking.
   Sample response: No, because time is measured in years since the city was founded. An input value of –2 would mean 2 years before the city was founded, which does not make sense.

2. What are all possible input and output values for the function represented by this graph? Explain your thinking.
   Sample response: Possible input values are all values greater than or equal to 0. Possible output values include all values greater than or equal to approximately 2.

Self-Assess

a I can determine reasonable input and output values for a function given a description in a situation.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 10 Where Are Functions Changing?

Success looks like . . .

- **Language Goal:** Determining reasonable input and output values for a function given a description of a situation. **(Reading and Writing)**
  - » Determining all possible input and output values of the function represented by the graph in Problem 2.

Suggested next steps

If students choose unreasonable input values for Problem 1, consider:

- Reviewing reasonable input values from Activity 2.
- Assigning Practice Problem 1.
- Asking, "How can you use the context to determine what input values make sense?"

If students incorrectly describe all possible input and output values for Problem 2, consider:

- Reviewing describing input and output values from Activity 3.
- Assigning Practice Problem 3.
- Asking, "What values make sense for time and population?"

Professional Learning

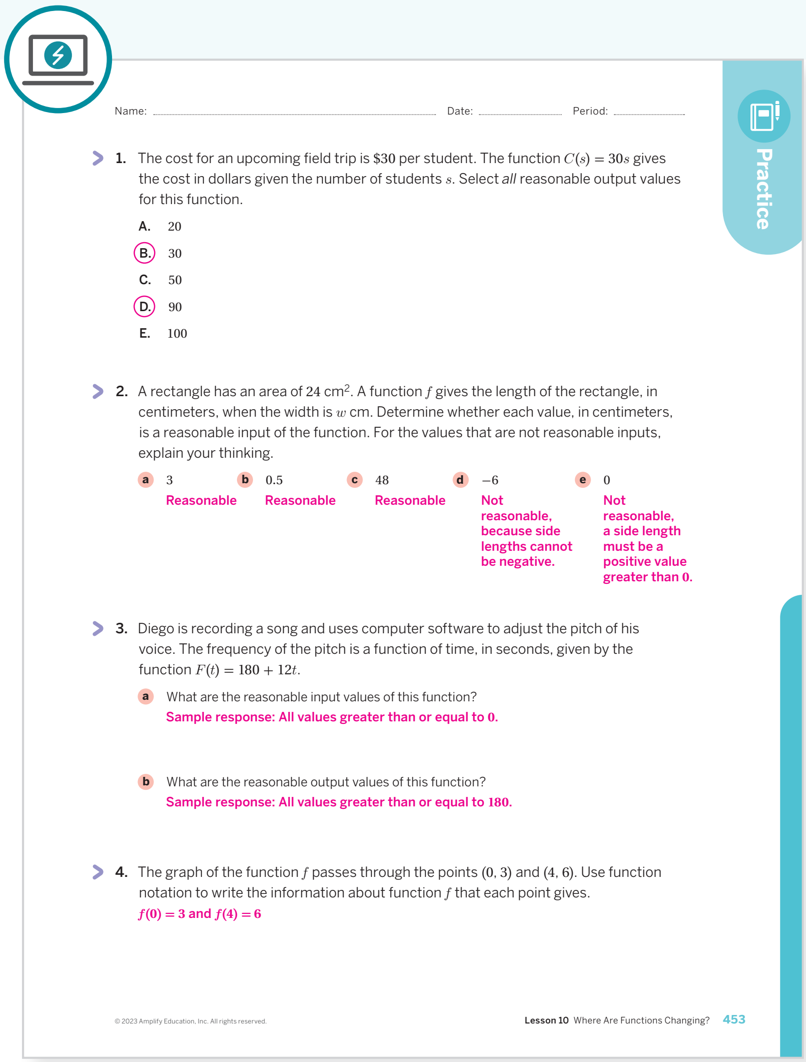
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? During the discussion about input values that make sense in Activity 2, how did you encourage each student to share their understanding?
- The focus of this lesson was reasonable inputs and outputs. How did it go? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

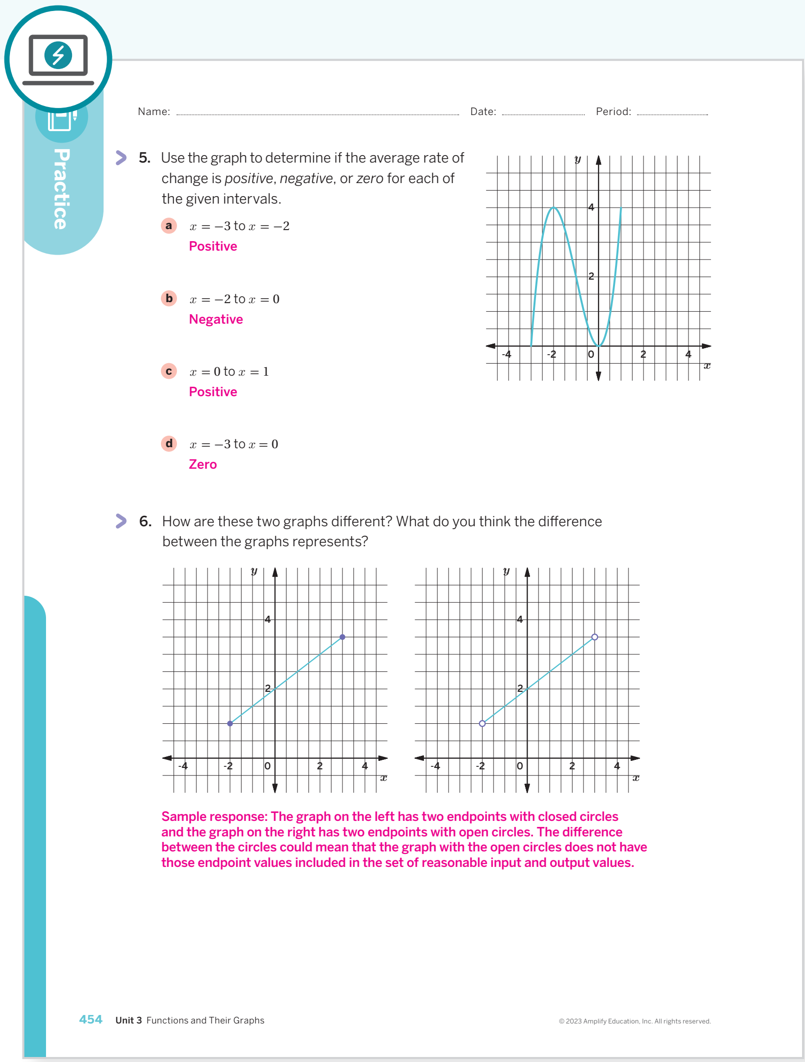
1. The cost for an upcoming field trip is \$30 per student. The function $C(s) = 30s$ gives the cost in dollars given the number of students s. Select *all* reasonable output values for this function.
   - A. 20
   - [✓ B.] 30
   - C. 50
   - [✓ D.] 90
   - E. 100

2. A rectangle has an area of 24 cm^2. A function f gives the length of the rectangle, in centimeters, when the width is w cm. Determine whether each value, in centimeters, is a reasonable input of the function. For the values that are not reasonable inputs, explain your thinking.
   - a 3 **Reasonable**
   - b 0.5 **Reasonable**
   - c 48 **Reasonable**
   - d -6 **Not reasonable, because side lengths cannot be negative.**
   - e 0 **Not reasonable, a side length must be a positive value greater than 0.**

3. Diego is recording a song and uses computer software to adjust the pitch of his voice. The frequency of the pitch is a function of time, in seconds, given by the function $F(t) = 180 + 12t$.
   - a What are the reasonable input values of this function?
     Sample response: All values greater than or equal to 0.
   - b What are the reasonable output values of this function?
     Sample response: All values greater than or equal to 180.

4. The graph of the function f passes through the points $(0, 3)$ and $(4, 6)$. Use function notation to write the information about function f that each point gives.
   $f(0) = 3$ and $f(4) = 6$

© 2023 Amplify Education, Inc. All rights reserved. **Lesson 10** Where Are Functions Changing? 453



Name: Date: Period:

5. Use the graph to determine if the average rate of change is *positive*, *negative*, or *zero* for each of the given intervals.
   - a $x = -3$ to $x = -2$
     Positive
   - b $x = -2$ to $x = 0$
     Negative
   - c $x = 0$ to $x = 1$
     Positive
   - d $x = -3$ to $x = 0$
     Zero

6. How are these two graphs different? What do you think the difference between the graphs represents?

   Sample response: The graph on the left has two endpoints with closed circles and the graph on the right has two endpoints with open circles. The difference between the circles could mean that the graph with the open circles does not have those endpoint values included in the set of reasonable input and output values.

454 **Unit 3** Functions and Their Graphs © 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	1
	2	Activity 2	2
	3	Activity 3	2
Spiral	4	Unit 3 Lesson 4	1
	5	Unit 3 Lesson 9	2
Formative ⚡	6	Unit 3 Lesson 11	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 3 | **LESSON 11**

Domain and Range

Let's represent the input and output values of a function using interval notation.

Focus

Goals

1. **Language Goal:** Understand what is meant by the terms *domain* and *range*. **(Speaking and Listening, Reading and Writing)**
2. Understand how to represent values that continue forever without bound.
3. Use interval notation to represent domain and range of a function.

Rigor

- Students build **conceptual understanding** of the concept of infinity and bounded and unbounded intervals.
- Students develop **procedural fluency** by representing the domain and range of functions using interval notation.

Coherence

Today

Students formalize their understanding of inputs and outputs as *domain* and *range*. They describe the domain and range of graphs of different continuous (not discrete) or discrete functions using words, lists, inequalities, and interval notation. Students are introduced to the concept of infinity and its symbol.

Previously

In Lesson 10, students were informally introduced to domain and range by determining reasonable input and output values for functions given a context.

Coming Soon

In Lesson 12, students will be precisely interpreting key features of graphs of functions that represent different scenarios.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Critiquing*
- Anchor Chart PDF, *Interval Notation*

Math Language Development

New words

- ***domain***
- ***infinite (infinity, ∞)***
- ***interval notation***
- ***range***

Review words

- *function*
- *input*
- *output*

Amps | Featured Activity

Activity 3
Interactive Interval Notation

Students are able to interact with the graphs of functions with restricted domains and ranges in order to see how the interval notation changes in real time.

Building Math Identity and Community

Connecting to Mathematical Practices

The quantitative reasoning required of students as they analyze the domain and range of a function reminds students to be focused on more than just the task, but the meaning of each scenario. To avoid impulsive decisions, students need to focus on staying the course as they consider what values make sense for a function. Because this is where mathematics meets the real-world, students should look for ways to appreciate the ways mathematics provides accurate models for everyday life events.

Modifications to Pacing

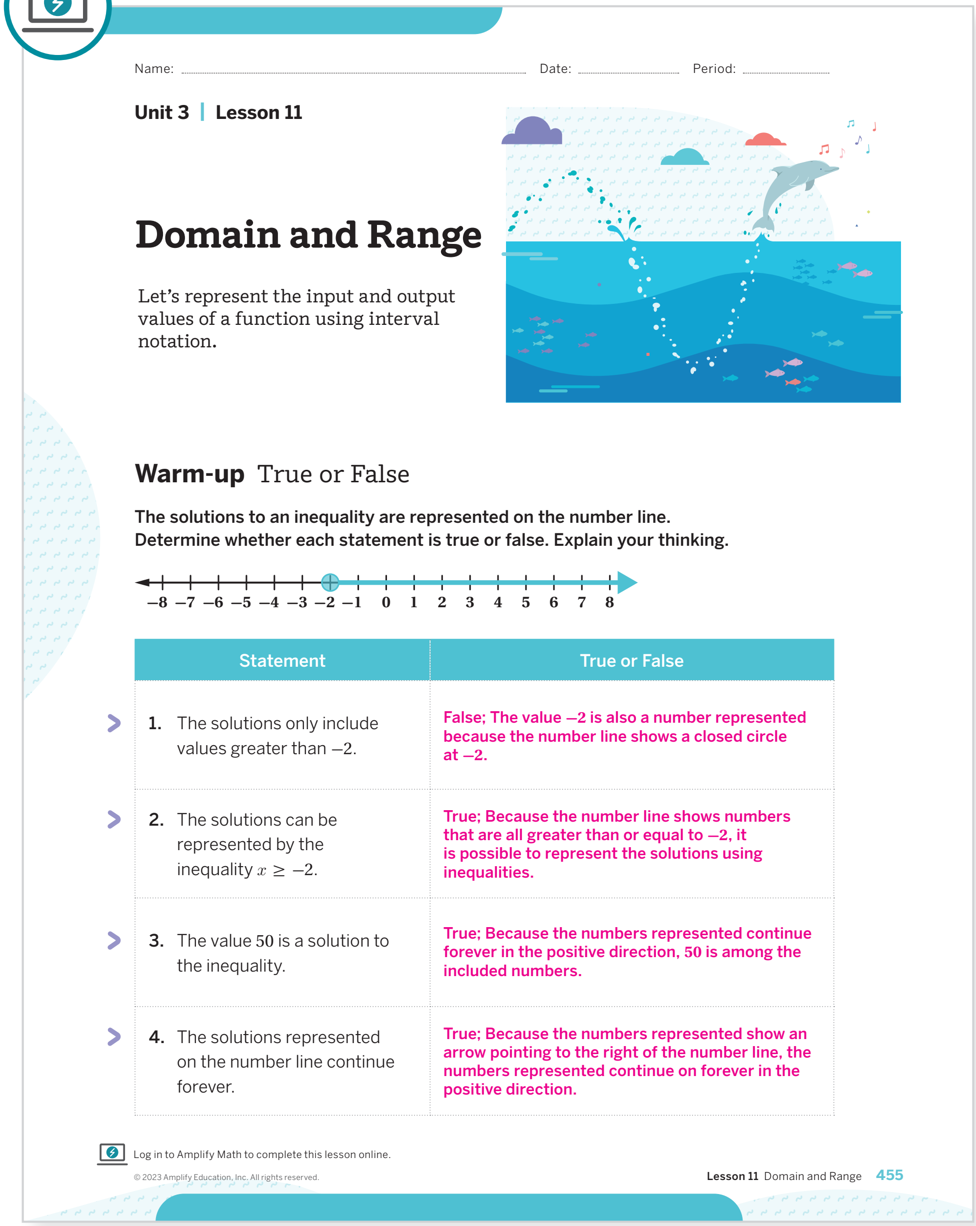
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 4 may be omitted.
- In **Activity 2**, Problem 1 may be omitted.

Independent | 5 min

Warm-up True or False

Students determine the validity of statements based on solutions presented on a number line to recall interpreting inequality solutions and grapple with infinity.



Name: ______ Date: ______ Period: ______

Unit 3 | **Lesson 11**

Domain and Range

Let's represent the input and output values of a function using interval notation.

Warm-up True or False

The solutions to an inequality are represented on the number line. Determine whether each statement is true or false. Explain your thinking.

Statement	True or False
1. The solutions only include values greater than –2.	**False; The value –2 is also a number represented because the number line shows a closed circle at –2.**
2. The solutions can be represented by the inequality $x \geq -2$.	**True; Because the number line shows numbers that are all greater than or equal to –2, it is possible to represent the solutions using inequalities.**
3. The value 50 is a solution to the inequality.	**True; Because the numbers represented continue forever in the positive direction, 50 is among the included numbers.**
4. The solutions represented on the number line continue forever.	**True; Because the numbers represented show an arrow pointing to the right of the number line, the numbers represented continue on forever in the positive direction.**

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 11 Domain and Range 455

① Launch

Display each statement, one at a time. Give students a minute of think-time, then use the *Poll the Class* routine for each statement before they write their reasoning.

② Monitor

Help students get started by asking, "What information does the endpoint and direction of the arrow provide about the included values on the number line?"

Look for points of confusion:

- **Not including –2 as an included value.** Review open and closed notation on a number line.
- **Thinking included values stop at 8.** Ask, "What does the arrow at the end of the graph represent?"

Look for productive strategies:

- Recognizing –2 is included in the solution.
- Understanding that any value greater than or equal to –2 is a solution.

③ Connect

Display the number line.

Have individual students share their thinking for each statement.

Highlight that the values represented on the number line can be written using words or symbols, such as inequalities, if there are points or rays. Because the values continue on forever, any number greater than or equal to –2 is included.

Ask, "What are the pros and cons of using symbols instead of words when describing represented values?" Sample response: Verbal descriptions are time consuming, but do not require interpreting symbols. Using symbols, such as inequalities, are time efficient, but you must know what each symbol means.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display the Anchor Chart PDF, *Sentence Stems, Critiquing* as students share their responses and explanations for each statement. Ask other students to support, critique, or ask clarifying questions of the explanations and reasoning shared.

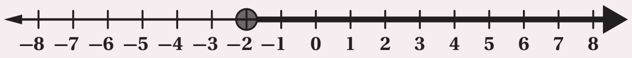
English Learners

Annotate the number line by writing the phrase closed circle and *includes 2* above the closed circle for 2. Then annotate the number line by writing *continues forever* where the arrow is shaded on the right.

Power-up

To power up students' ability to represent boundary values on graphs, have students complete:

Use the number line to complete the following problems.



1. Select *all* solutions to the values represented on the number line.

[✓ **A.**] 4 **B.** –6 [✓ **C.**] 24 [✓ **D.**] 99 **E.** –10

2. Which of the following inequalities represents the values on the number line?

A. $x > -2$ **B.** $x < -2$ [✓ **C.**] $x \geq -2$ **D.** $x \leq -2$

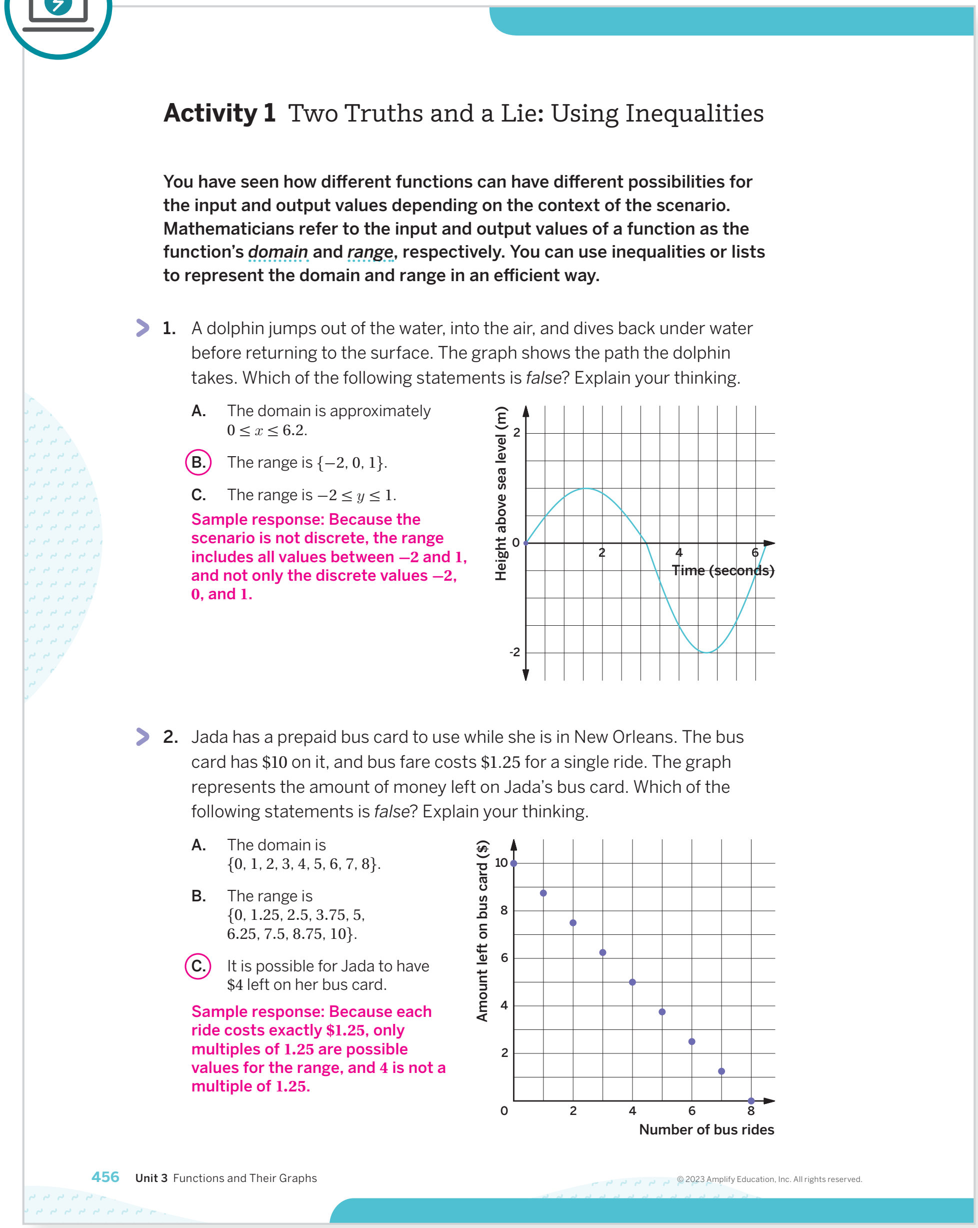
Use: Before the Warm-up

Informed by: Performance on Lesson 10, Practice Problem 6

Pairs | 10 min

Activity 1 Two Truths and a Lie: Using Inequalities

Students determine the validity of statements about domain and range to see how inequalities and lists are used to represent domain and range.



Activity 1 Two Truths and a Lie: Using Inequalities

You have seen how different functions can have different possibilities for the input and output values depending on the context of the scenario. Mathematicians refer to the input and output values of a function as the function's *domain* and *range*, respectively. You can use inequalities or lists to represent the domain and range in an efficient way.

1. A dolphin jumps out of the water, into the air, and dives back under water before returning to the surface. The graph shows the path the dolphin takes. Which of the following statements is *false*? Explain your thinking.

   A. The domain is approximately $0 \le x \le 6.2$.

   [✓ B.] The range is $\{-2, 0, 1\}$.

   C. The range is $-2 \le y \le 1$.

   Sample response: Because the scenario is not discrete, the range includes all values between -2 and 1, and not only the discrete values -2, 0, and 1.

2. Jada has a prepaid bus card to use while she is in New Orleans. The bus card has $10 on it, and bus fare costs $1.25 for a single ride. The graph represents the amount of money left on Jada's bus card. Which of the following statements is *false*? Explain your thinking.

   A. The domain is $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$.

   B. The range is $\{0, 1.25, 2.5, 3.75, 5, 6.25, 7.5, 8.75, 10\}$.

   [✓ C.] It is possible for Jada to have $4 left on her bus card.

   Sample response: Because each ride costs exactly $1.25, only multiples of 1.25 are possible values for the range, and 4 is not a multiple of 1.25.

456 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the introduction together and introduce the terms *domain* and *range*. Tell students they will explore different types of notation to represent the domain and range.

2 Monitor

Help students get started by asking them first to make sense of each graph.

Look for points of confusion:

- **Thinking it is possible for Jada to have $4 left on her bus card.** Ask, "In order for Jada to have $4 left, how many times must she have ridden the bus?"

Look for productive strategies:

- Using verbal descriptions to make sense of domain and range.
- In Problem 1, recognizing the range can be an integer from -2 to 1, due to the context.
- Recognizing having $4 left on the bus card is impossible because it would correspond with 4.8 rides, which is impossible in the given context.

3 Connect

Display both graphs of the scenarios.

Have pairs of students share which statement was false in each scenario and their explanations.

Define the terms ***domain*** and ***range***.

Highlight that when a graph is connected with a line or curve, inequalities can be used to represent all possible values in the domain and range. When a graph is not connected, and only some values make sense, the values should be listed in set notation.

Ask, "Could you have determined the domain and range in Problem 2 without a graph? Explain your thinking." Sample response: Yes, I could make a table to represent the amount of money on the bus card Jada starts with and the amount left after each bus ride.

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

During the Launch, display the following to help students make connections between the input and output of a function and its domain and range.

Input ↔ Domain	Output ↔ Range

Extension: *Math Enrichment*

Have students write the equation of the function that could represent the amount of money Jada has left on her bus voucher in Problem 2. Then ask them if the function is discrete or not discrete and explain their thinking. Sample response: $B(r) = 10 - 1.25r$; The function is discrete because the points are disconnected.

Math Language Development

MLR7: Compare and Connect

During the Connect, have pairs of students compare the graphs and each of the representations used to describe the domain or range. Ask:

- "What do you notice about the graph in Problem 1 compared to the graph in Problem 2?" Listen for students who use the terms *discrete* and *not discrete*.
- "What do you notice about the notations that describe the domain and range of a discrete graph? The graph that is not discrete?"

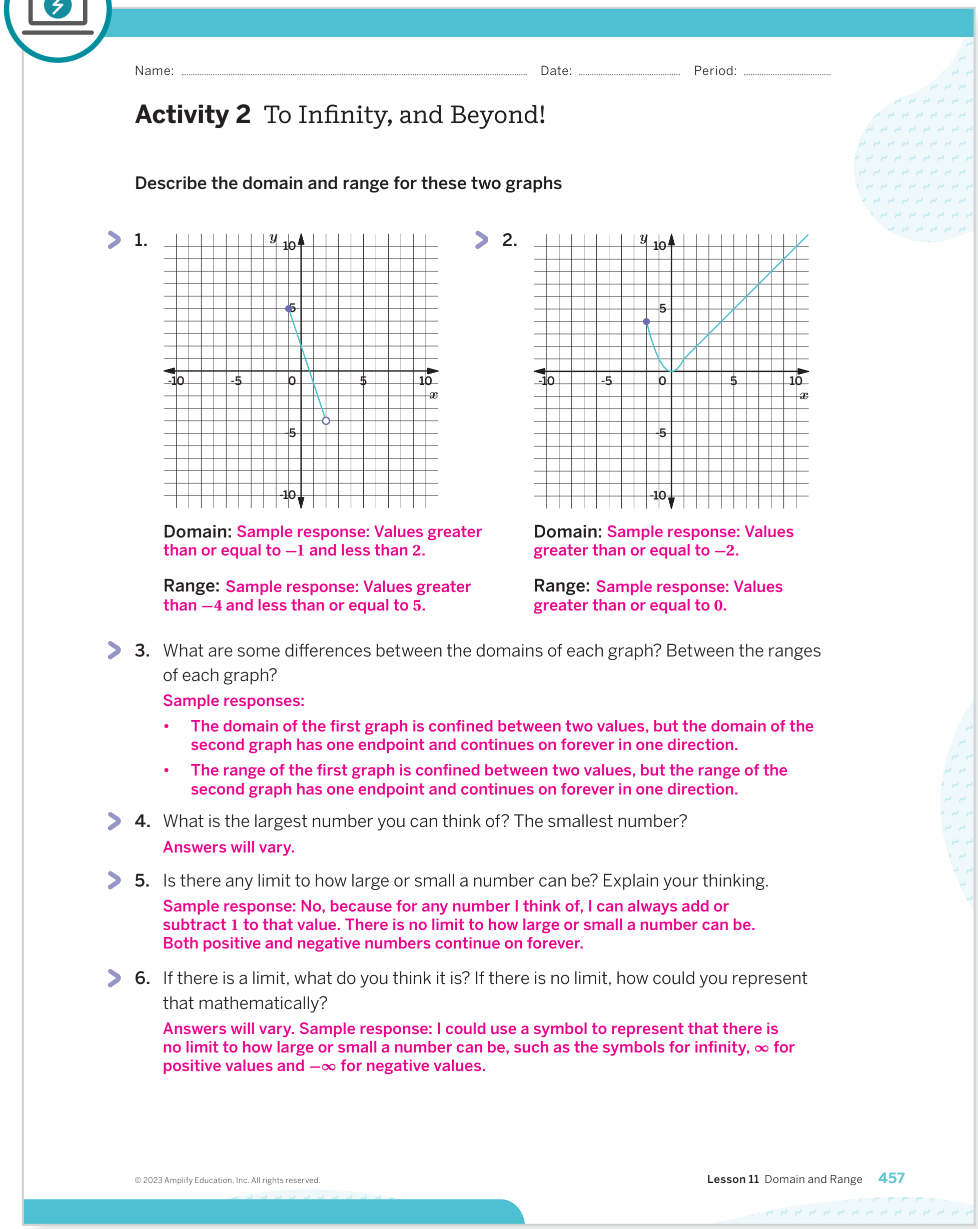
English Learners

Encourage students to refer to and use language from the class display to support their use of appropriate mathematical language.

Pairs | 10 min

Activity 2 To Infinity, and Beyond!

Students describe the domain and range of functions and infer about the bounds of numbers to drive the need for a notation that describes the concept of infinity.



Name: ____________ Date: ________ Period: ________

Activity 2 To Infinity, and Beyond!

Describe the domain and range for these two graphs

1.

Domain: Sample response: Values greater than or equal to -1 and less than 2.

Range: Sample response: Values greater than -4 and less than or equal to 5.

2.

Domain: Sample response: Values greater than or equal to -2.

Range: Sample response: Values greater than or equal to 0.

3. What are some differences between the domains of each graph? Between the ranges of each graph?

   Sample responses:

   - The domain of the first graph is confined between two values, but the domain of the second graph has one endpoint and continues on forever in one direction.
   - The range of the first graph is confined between two values, but the range of the second graph has one endpoint and continues on forever in one direction.

4. What is the largest number you can think of? The smallest number?

   Answers will vary.

5. Is there any limit to how large or small a number can be? Explain your thinking.

   Sample response: No, because for any number I think of, I can always add or subtract 1 to that value. There is no limit to how large or small a number can be. Both positive and negative numbers continue on forever.

6. If there is a limit, what do you think it is? If there is no limit, how could you represent that mathematically?

   Answers will vary. Sample response: I could use a symbol to represent that there is no limit to how large or small a number can be, such as the symbols for infinity, ∞ for positive values and $-\infty$ for negative values.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Give students 5 minutes to complete Problems 1–3 before a class discussion. Then have students complete the remaining problems.

2 Monitor

Help students get started by having them label endpoints or other important values on each graph.

Look for points of confusion:

- **Describing domain using y values or range using x values.** Ask, "What are other words or phrases that mean the same as domain and range?"
- **Thinking the limit to how "small" a number can be are positive numbers close to 0.** Have students draw a number line, with 0 in the center, and ask, "Which numbers are smaller than 0? Why?"

Look for productive strategies:

- Using the x-values of endpoints to represent the bounds of the domain and y-values for range.
- Recognizing the domain and range of the second graph continues forever.
- Recognizing there is no limit to how large positive numbers can be or how small negative numbers can be.

3 Connect

Have pairs of students share their responses to Problems 4–6.

Display a number line and the symbols $-\infty$ and ∞.

Ask, "What do you think each symbol represents?"

Define the term ***infinite (infinity, ∞)***.

Highlight that $-\infty$ and ∞ represent the concept that numbers continue on forever in the positive and negative direction. It is not possible to "arrive" at $-\infty$ and ∞ on a number line because they are not numbers, but symbols indicating that numbers are endless.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest students use colored pencils to draw vertical lines from each endpoint to the horizontal axis and a horizontal line to the vertical axis to help them connect these boundary values to the domain and range. Alternatively, suggest they use their pencil to trace (without actually drawing) these lines.

Extension: *Math Enrichment*

Ask students to determine how many values are between the numbers 1 and 2 and to explain their thinking. Sample response: An infinite number of values. For any two rational numbers between 1 and 2, there will always be more rational numbers between them.

Math Language Development

MLR2: Collect and Display

During the Connect, listen for and collect vocabulary, gestures, and diagrams students use to describe the concepts of *domain*, *range*, and *limits*. Add the term *infinity (infinite)* and the symbols for positive infinity and negative infinity to the class display. Add the terms *forever* and *endless* to the class display next to the term infinity.

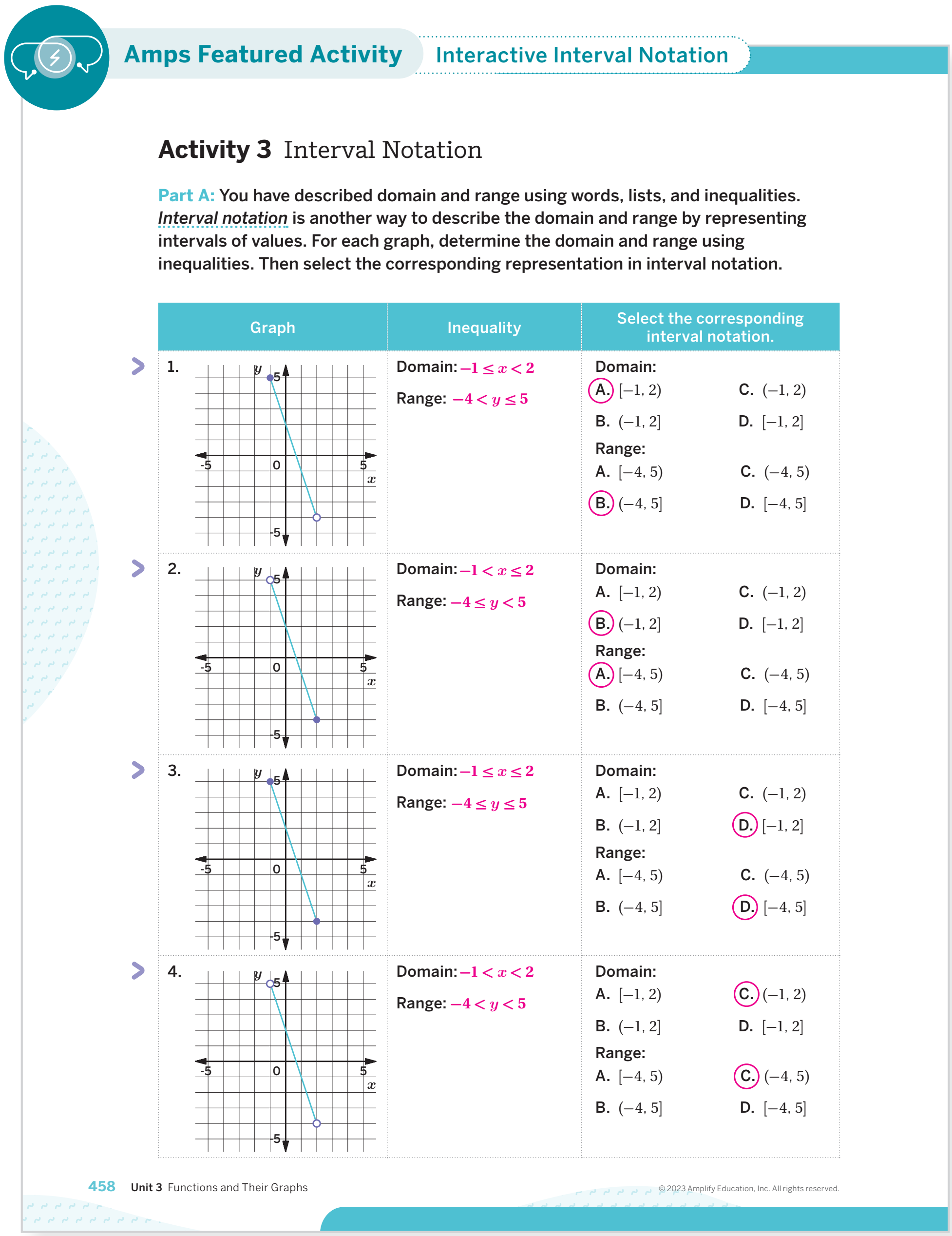
English Learners

Include visual displays of what infinity means, such as a number line that is shaded in one or both directions. Annotate the shaded part with the term *infinity* and the phrase *goes on forever*.

Pairs | 15 min

Activity 3 Interval Notation

Students build procedural fluency with using interval notation to represent the domain and range of functions, attending to precision as to whether the endpoints are included.



Amps Featured Activity Interactive Interval Notation

Activity 3 Interval Notation

Part A: You have described domain and range using words, lists, and inequalities. *Interval notation* is another way to describe the domain and range by representing intervals of values. For each graph, determine the domain and range using inequalities. Then select the corresponding representation in interval notation.

Graph	Inequality	Select the corresponding interval notation.
1.	Domain: $-1 \le x < 2$ Range: $-4 < y \le 5$	Domain: [✓ A.] $[-1, 2)$ C. $(-1, 2)$ B. $(-1, 2]$ D. $[-1, 2]$ Range: A. $[-4, 5)$ C. $(-4, 5)$ [✓ B.] $(-4, 5]$ D. $[-4, 5]$
2.	Domain: $-1 < x \le 2$ Range: $-4 \le y < 5$	Domain: A. $[-1, 2)$ C. $(-1, 2)$ [✓ B.] $(-1, 2]$ D. $[-1, 2]$ Range: [✓ A.] $[-4, 5)$ C. $(-4, 5)$ B. $(-4, 5]$ D. $[-4, 5]$
3.	Domain: $-1 \le x \le 2$ Range: $-4 \le y \le 5$	Domain: A. $[-1, 2)$ C. $(-1, 2)$ B. $(-1, 2]$ [✓ D.] $[-1, 2]$ Range: A. $[-4, 5)$ C. $(-4, 5)$ B. $(-4, 5]$ [✓ D.] $[-4, 5]$
4.	Domain: $-1 < x < 2$ Range: $-4 < y < 5$	Domain: A. $[-1, 2)$ [✓ C.] $(-1, 2)$ B. $(-1, 2]$ D. $[-1, 2]$ Range: A. $[-4, 5)$ [✓ C.] $(-4, 5)$ B. $(-4, 5]$ D. $[-4, 5]$

458 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the introduction together and introduce the term *interval notation*. Tell students they will explore how inequalities representing the domain and range relate to interval notation.

2 Monitor

Help students get started by asking, "What is the relationship between the types of endpoints, inequality symbols, and interval notation symbols used?"

Look for points of confusion:

- **Thinking interval notation represents an ordered pair.** Remind students that domain and range represent possible input and output values, not a single point.
- **Switching the lower and upper bound values when writing interval notation.** Have students consider a number line and how numbers are typically organized and represented.
- **Using a bracket when a graph continues forever.** Remind students that a bracket should only be used when the graph has an endpoint.

Look for productive strategies:

- Recognizing "["or"]" can never be used when a graph has no endpoint.
- Drawing connections between the types of endpoints used, the inequality symbols used, and the symbols used for interval notation.
- Using interval notation to describe the domain and range and where the function is changing in Problems 5–8.

Activity 3 continued ›

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Display the Anchor Chart PDF, *Interval Notation* for students to reference as they progress through the activity.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can digitally interact with the graphs of functions that have restricted domains and ranges. This will support their visualization of how the interval notation changes in real time to match restricted domain and range.

Math Language Development

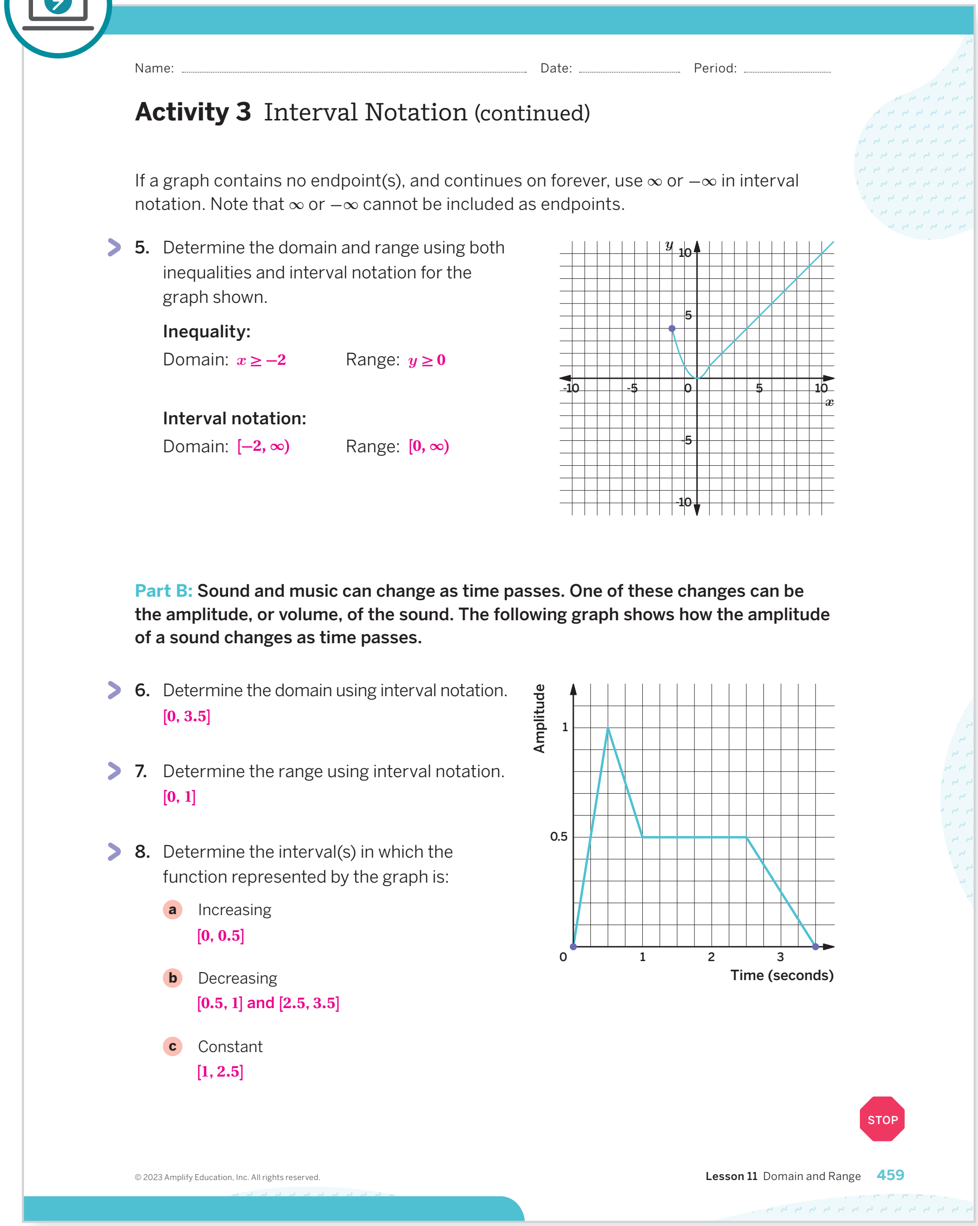
MLR7: Compare and Connect

During the Connect, as students share the connections between interval notation, inequality notation, and the types of endpoints represented, display the Anchor Chart PDF, *Interval Notation*. Annotate the Anchor Chart by adding a table similar to the following.

Endpoint	Inequality symbols	Interval notation
Closed circle	≤ or ≥	Brackets [or]
Open circle	< or >	Parentheses (or)
No endpoint (infinity)	∞ or –∞	Parentheses (or)

Activity 3 Interval Notation (continued)

Students build procedural fluency with using interval notation to represent the domain and range of functions, attending to precision as to whether the endpoints are included.



Name: ________ Date: ________ Period: ________

Activity 3 Interval Notation (continued)

If a graph contains no endpoint(s), and continues on forever, use ∞ or −∞ in interval notation. Note that ∞ or −∞ cannot be included as endpoints.

5. Determine the domain and range using both inequalities and interval notation for the graph shown.

Inequality:

Domain: $x \geq -2$ Range: $y \geq 0$

Interval notation:

Domain: $[-2, \infty)$ Range: $[0, \infty)$

Part B: Sound and music can change as time passes. One of these changes can be the amplitude, or volume, of the sound. The following graph shows how the amplitude of a sound changes as time passes.

6. Determine the domain using interval notation.
   $[0, 3.5]$

7. Determine the range using interval notation.
   $[0, 1]$

8. Determine the interval(s) in which the function represented by the graph is:
   - a Increasing
     $[0, 0.5]$
   - b Decreasing
     $[0.5, 1]$ and $[2.5, 3.5]$
   - c Constant
     $[1, 2.5]$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 11 Domain and Range 459

3 Connect

Display the graphs from Problems 1–4.

Ask, "How did you determine which interval notation representation corresponds with the domain and range in each graph?"

Have pairs of students share the connections between interval notation, inequality notation, and the type of endpoints represented.

Define the term ***interval notation***.

Highlight that interval notation is an efficient way to communicate domain, range, or where important characteristics of the graph of a function occur. If an endpoint is included, brackets must be used. If an endpoint is not included, parentheses are used. Infinity cannot be considered an endpoint, so parentheses are always used when values continue to infinity or negative infinity.

Note: For increasing, decreasing, and constant intervals, either brackets or parentheses are acceptable to use as long as the endpoints are part of the domain. Suggest students use either brackets or parentheses consistently when writing interval notation for increasing, decreasing, and constant intervals.

Ask, "What are some pros and cons to using interval notation compared to using inequalities to represent domain and range?" Sample response: Some pros are that it can be quicker to write and there are only two symbols to remember instead of four. Some cons are that the infinity symbol needs to be written for intervals that continue forever, and it does not explicitly state variables in the notation.

Whole Class | 5 min

Summary

Review and synthesize how the terms *domain* and *range* precisely represent the input and output of a function, and how interval notation can be used to represent the domain and range of a function.

Summary

In today's lesson . . .

You explored the common mathematical language that is used to communicate about input and output values; ***domain*** and ***range***. The domain of a function is the set of all possible input values. The range of a function is the set of all possible output values.

When values continue on forever without end, these values are ***infinite***.

- For values that continue on forever in the positive direction, use the infinity symbol, ∞.
- For values that continue on forever in the negative direction, use the negative infinity symbol, −∞.

Inequalities can be used to represent domain and range, as they can help describe domain and range more efficiently than writing a description.

You also saw that in addition to inequalities, ***interval notation*** can be used to represent the domain and range of a function in another way. Interval notation has its own efficiencies for expressing intervals of values, and is used widely among mathematicians and scientists.

- Parentheses indicate the beginnings or endings of intervals in which the endpoint values are not included. For example, (−∞, 5) represents the interval from negative infinity all the way up to 5, but does not include the value 5.
- Brackets indicate the beginnings or endings of intervals in which the endpoint values are included. For example, (−∞, 5] represents the interval from negative infinity all the way up to 5, and includes the value 5.

Reflect:

460 Unit 3 Functions and Their Graphs © 2023 Amplify Education, Inc. All rights reserved.

Math Language Development

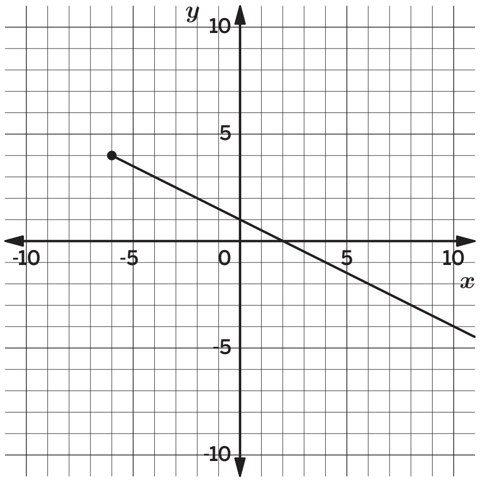
MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the terms *domain*, *infinite* (*infinity*, ∞), *interval notation*, and *range* that were added to the display during the lesson.

Emphasize the difference between the term *range* as a statistical measure that students have previously learned about and the *range* of a function.

Synthesize

Display the following graph.



Ask, "How could you represent the domain and range using inequalities? Interval notation?"

Have students share the domain and range of the graph using inequalities and interval notation.

Highlight that the domain and range of functions can be represented using interval notation. Domain and range represent the regular terminology used to refer to the possible input and output values of a function, and interval notation is used widely throughout mathematics to not only communicate the domain and range, but also where key characteristics of a function occur.

Formalize vocabulary:

- *domain*
- *infinite (infinity, ∞)*
- *interval notation*
- *range*

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

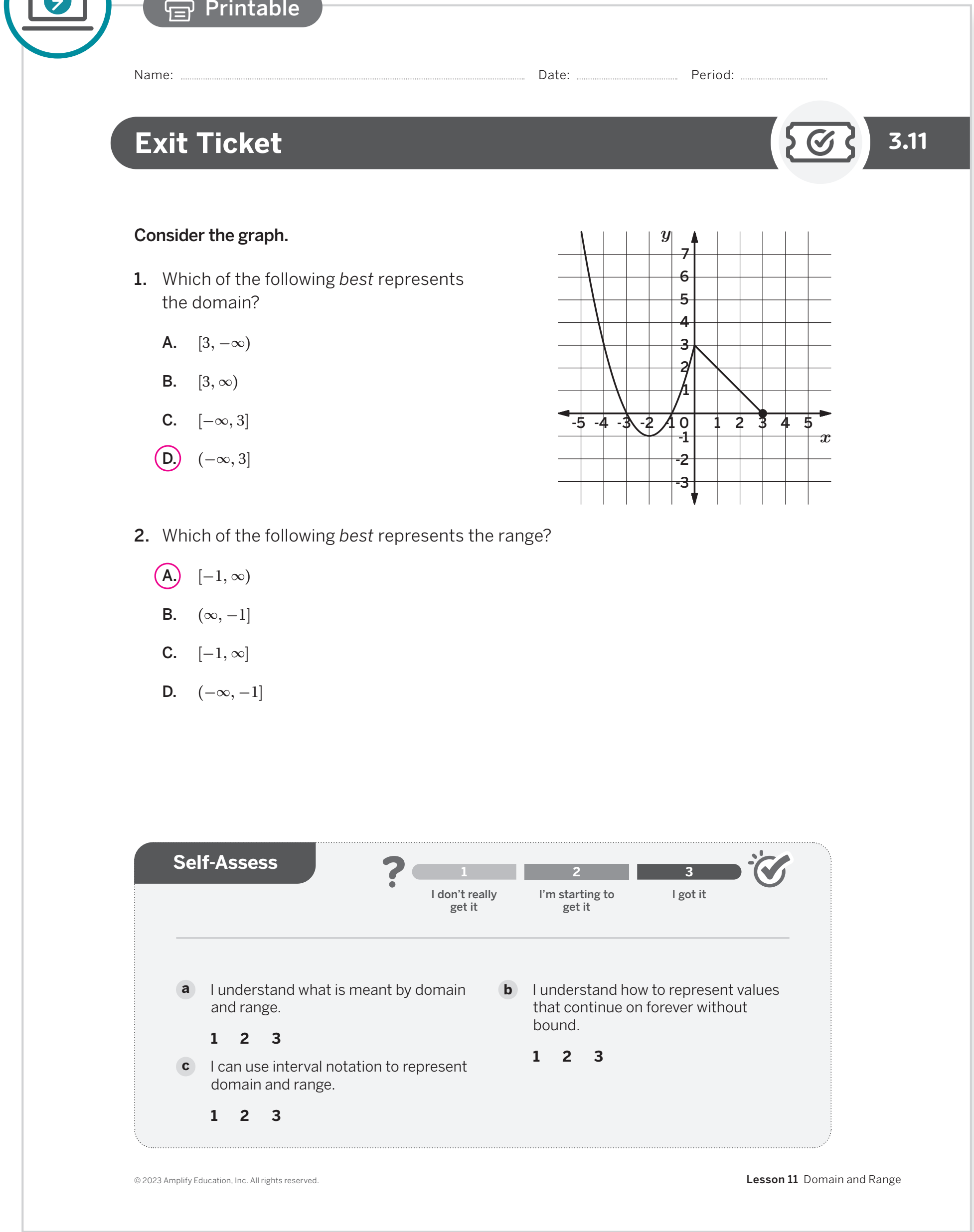
- "What are domain and range?"
- "What is interval notation and how is it used?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by determining the domain and range of a function using interval notation, attending to precision as to whether the endpoints are included.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.11

Consider the graph.

1. Which of the following *best* represents the domain?

   A. $[3, -\infty)$

   B. $[3, \infty)$

   C. $[-\infty, 3]$

   [✓ D.] $(-\infty, 3]$

2. Which of the following *best* represents the range?

   [✓ A.] $[-1, \infty)$

   B. $(\infty, -1]$

   C. $[-1, \infty]$

   D. $(-\infty, -1]$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I understand what is meant by domain and range.
1 2 3

b I understand how to represent values that continue on forever without bound.
1 2 3

c I can use interval notation to represent domain and range.
1 2 3

©2023 Amplify Education, Inc. All rights reserved.

Lesson 11 Domain and Range

Success looks like . . .

- **Language Goal:** Understanding what is meant by the terms *domain* and *range*. **(Speaking and Listening, Reading and Writing)**
- **Goal:** Understanding how to represent values that continue on forever without bound.
- **Goal:** Using interval notation to represent domain and range.
  - » Identifying the domain and range of the function shown in the graph in Problems 1 and 2.

Suggested next steps

If students select the incorrect interval in Problem 1, consider:

- Reviewing interval notation from Activity 3.
- Assigning Practice Problem 3.
- Asking, "What are other words or phrases that mean the same thing as *domain*?"

If students select the incorrect interval in Problem 2, consider:

- Reviewing interval notation from Activity 3.
- Assigning Practice Problem 1.
- Asking, "What are other words or phrases that mean the same thing as *range*?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways have your students gotten better at representing possible input and output values for a function?
- How did describing domain and range in writing set students up to develop understanding of interval notation? What might you change for the next time you teach this lesson?

Math Language Development

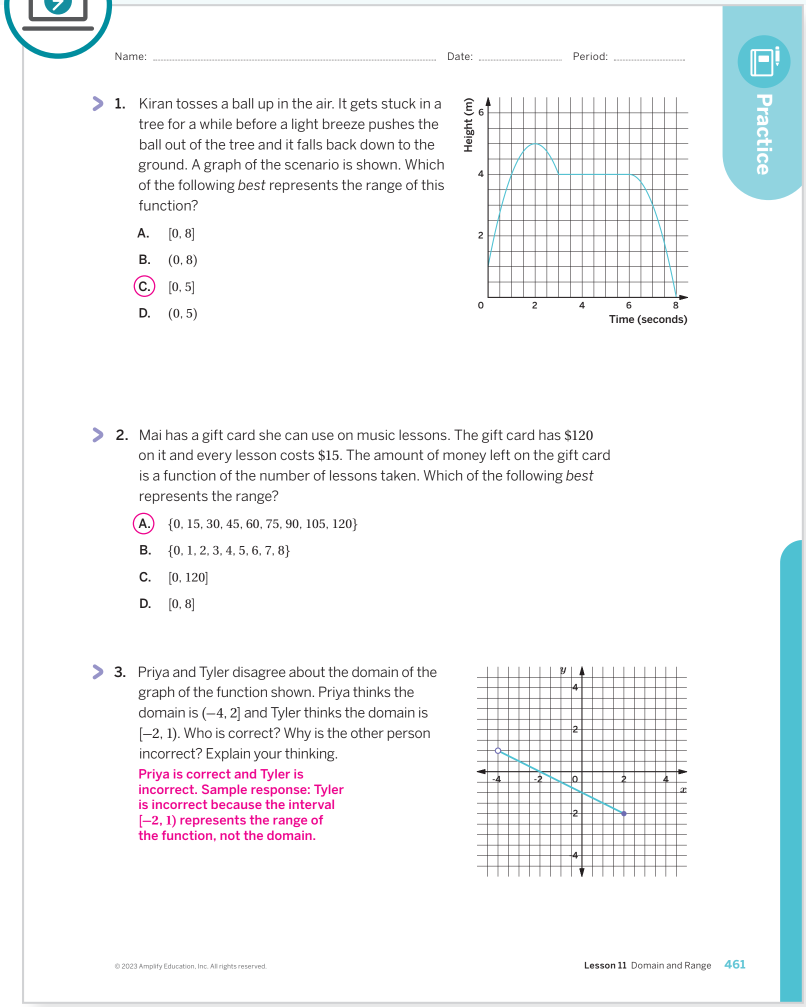
Language Goal: Understanding what is meant by the terms *domain* and *range*.

Reflect on students' language development toward this goal.

- Are students gaining comfort using and interpreting the domain and range of functions?
- How did using the language routines in this lesson help students practice using these terms? Would you change anything the next time you use these routines?

Practice

Independent



Name: Date: Period:

1. Kiran tosses a ball up in the air. It gets stuck in a tree for a while before a light breeze pushes the ball out of the tree and it falls back down to the ground. A graph of the scenario is shown. Which of the following *best* represents the range of this function?

- A. $[0, 8]$
- B. $(0, 8)$
- [✓ C.] $[0, 5]$
- D. $(0, 5)$

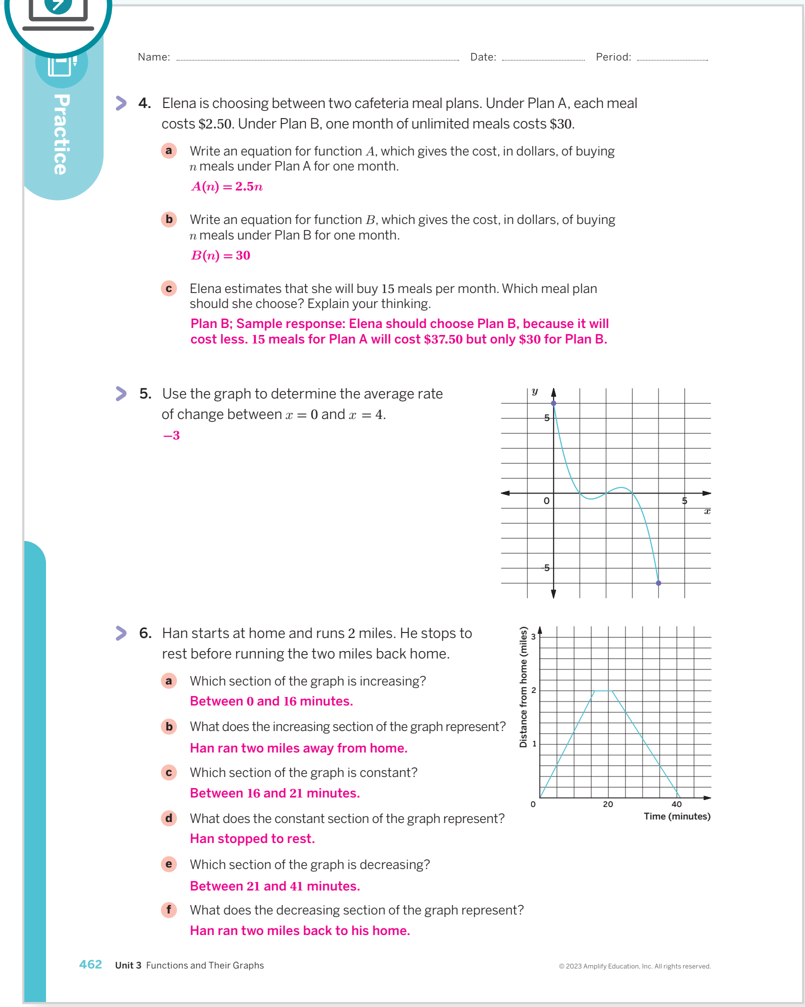
2. Mai has a gift card she can use on music lessons. The gift card has $120 on it and every lesson costs $15. The amount of money left on the gift card is a function of the number of lessons taken. Which of the following *best* represents the range?

- [✓ A.] $\{0, 15, 30, 45, 60, 75, 90, 105, 120\}$
- B. $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
- C. $[0, 120]$
- D. $[0, 8]$

3. Priya and Tyler disagree about the domain of the graph of the function shown. Priya thinks the domain is $(-4, 2]$ and Tyler thinks the domain is $[-2, 1)$. Who is correct? Why is the other person incorrect? Explain your thinking.

Priya is correct and Tyler is incorrect. Sample response: Tyler is incorrect because the interval $[-2, 1)$ represents the range of the function, not the domain.

© 2023 Amplify Education, Inc. All rights reserved.



Name: Date: Period:

4. Elena is choosing between two cafeteria meal plans. Under Plan A, each meal costs $2.50. Under Plan B, one month of unlimited meals costs $30.

   a. Write an equation for function A, which gives the cost, in dollars, of buying n meals under Plan A for one month.

   $A(n) = 2.5n$

   b. Write an equation for function B, which gives the cost, in dollars, of buying n meals under Plan B for one month.

   $B(n) = 30$

   c. Elena estimates that she will buy 15 meals per month. Which meal plan should she choose? Explain your thinking.

   Plan B; Sample response: Elena should choose Plan B, because it will cost less. 15 meals for Plan A will cost $37.50 but only $30 for Plan B.

5. Use the graph to determine the average rate of change between $x = 0$ and $x = 4$.

   -3

6. Han starts at home and runs 2 miles. He stops to rest before running the two miles back home.

   a. Which section of the graph is increasing?

   Between 0 and 16 minutes.

   b. What does the increasing section of the graph represent?

   Han ran two miles away from home.

   c. Which section of the graph is constant?

   Between 16 and 21 minutes.

   d. What does the constant section of the graph represent?

   Han stopped to rest.

   e. Which section of the graph is decreasing?

   Between 21 and 41 minutes.

   f. What does the decreasing section of the graph represent?

   Han ran two miles back to his home.

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 3	1
	2	Activity 2	1
	3	Activity 3	2
Spiral	4	Unit 3 Lesson 5	2
	5	Unit 3 Lesson 9	1
Formative	6	Unit 3 Lesson 12	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 3 | **LESSON 12**

Interpreting Graphs

Let's describe and interpret important features of graphs.

Focus

Goals

1. **Language Goal:** Determine and interpret important features of a graph in context. **(Reading and Writing)**
2. **Language Goal:** Interpret the average rate of change of a graph given a context. **(Reading and Writing)**

Coherence

Today

Students apply their understanding of important features of functions to interpret graphs representing functions of various real-world scenarios. Information is presented to students in the form of graphs, but compared to prior activities students have done, the scenarios presented in this lesson are more complex, and require students to persevere in sense making and problem solving.

Previously

In Lessons 7–11, students learned about important features of graphs, average rate of change, and domain and range.

Coming Soon

In Lesson 13, students will create graphs of functions.

Rigor

- Students build **fluency** using mathematical language to communicate the important features, domain, range, and average rate of change of graphs.
- Students **apply** their understanding of important features of graphs in different contexts.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	15 min	5 min	5 min
Pairs	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Activity 1 PDF
- Anchor Chart PDF, *Sentence Stems, Which One Doesn't Belong?* (as needed)

Math Language Development

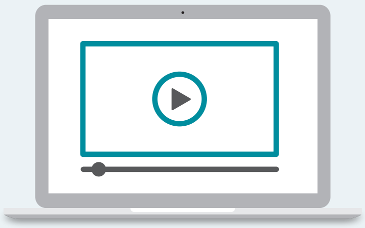
Review words

- *average rate of change*
- *decreasing*
- *domain*
- *increasing*
- *range*

Amps | Featured Activity

Activity 1
Toggle Between Graphs

Students can choose which graph they want to view, as they describe how it relates to the motion of a flag.



Amps
POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may struggle to embrace their strengths as they try to make sense of new scenarios. Even for places or events that they have never experienced, students can apply a growth mindset. They might not fully understand or appreciate the application of the mathematics yet, but as they continue to work with functions, they will better be able to make sense of problems when they arise.

Modifications to Pacing

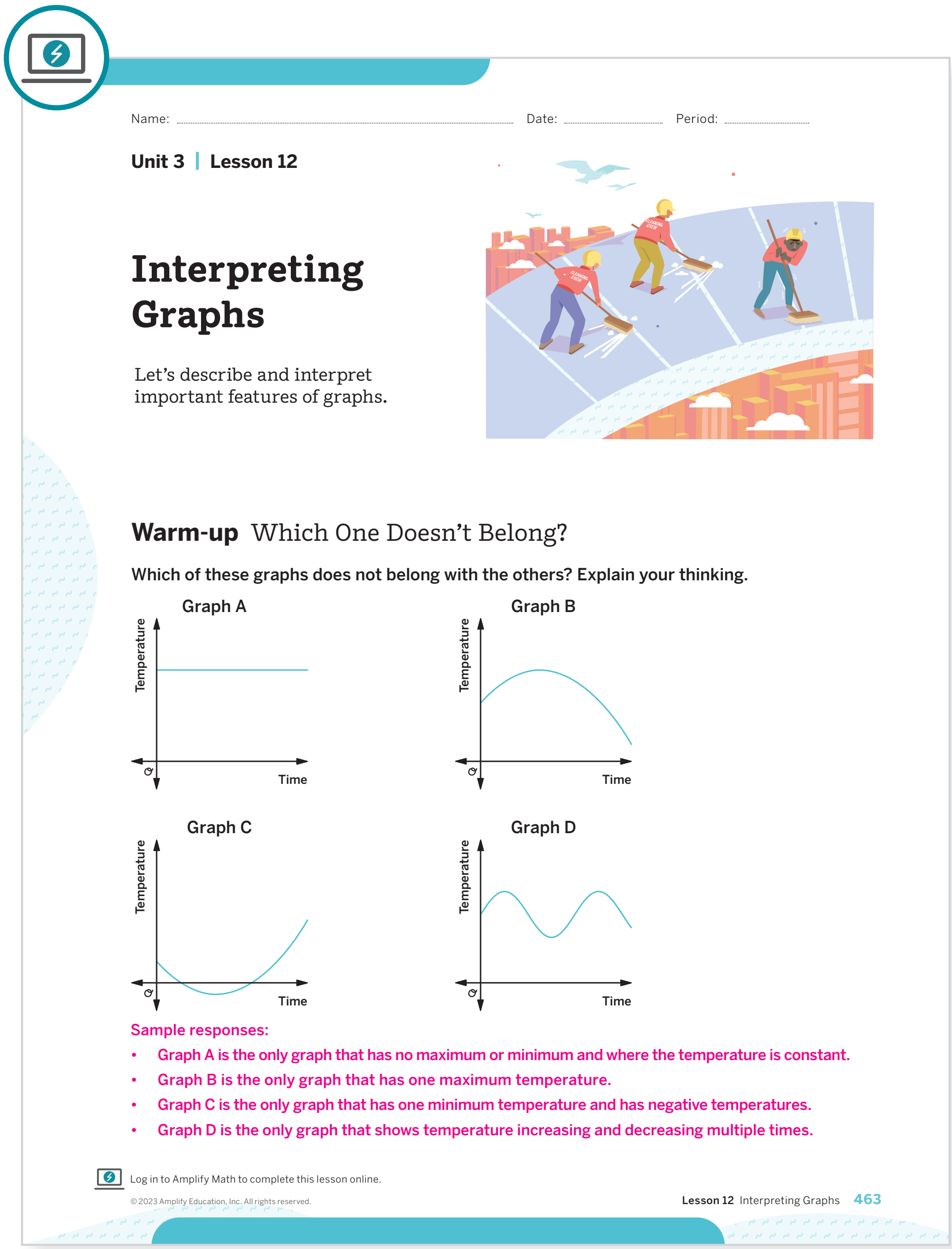
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, Graphs A and D may be omitted.

Pairs | 5 min

Warm-up Which One Doesn't Belong?

Students compare graphs of temperature change over time to compare differences in the important features of each graph using mathematical language.



Name: ______ Date: ______ Period: ______

Unit 3 | Lesson 12

Interpreting Graphs

Let's describe and interpret important features of graphs.

Warm-up Which One Doesn't Belong?

Which of these graphs does not belong with the others? Explain your thinking.

Sample responses:

- Graph A is the only graph that has no maximum or minimum and where the temperature is constant.
- Graph B is the only graph that has one maximum temperature.
- Graph C is the only graph that has one minimum temperature and has negative temperatures.
- Graph D is the only graph that shows temperature increasing and decreasing multiple times.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 12 Interpreting Graphs 463

1 Launch

Arrange students in pairs and conduct the ***Which One Doesn't Belong*** routine. Give students a minute of independent think-time before sharing their thoughts with their partner.

2 Monitor

Help students get started by asking, "What are some key differences between each graph?"

Look for points of confusion:

- **Misusing mathematical terminology.** Have students label the graph with key features and then re-state in a sentence using appropriate language.

Look for productive strategies:

- Using appropriate mathematical vocabulary.
- Describing how the temperature changes over time.

3 Connect

Display the four graphs of temperature and time.

Have pairs of students share their thinking for which graph does not belong. Select and sequence student responses using appropriate math vocabulary and describing how the temperature changes over time.

Highlight that each graph could represent a scenario where temperature is changing. Even though the exact function that describes each graph may not be known, it is still possible to compare important features and analyze the graphs.

Ask, "What other features or interpretations could you make about these graphs? What would you need to know?" Sample response: I could compare the average rate of change, domain, or range. I would need to know the scale of the axes before determining this information.

Math Language Development

MLR2: Collect and Display

Circulate as partners discuss which graph doesn't belong. Listen for students' use of their developing math language as they describe which graph doesn't belong, such as *minimum*, *maximum*, *constant*, *increasing*, *negative*, *decreasing*, etc. Add this language to the class display.

English Learners

Display or provide the Anchor Chart PDF, *Sentence Stems, Which One Doesn't Belong?* to support students in structuring their responses. Allow students to rehearse what they will say before sharing with the whole class.

Power-up

To power up students' ability to identify key features of a scenario on a graph:

Provide students with a copy of the Power-up PDF.

Use: Before Activity 1

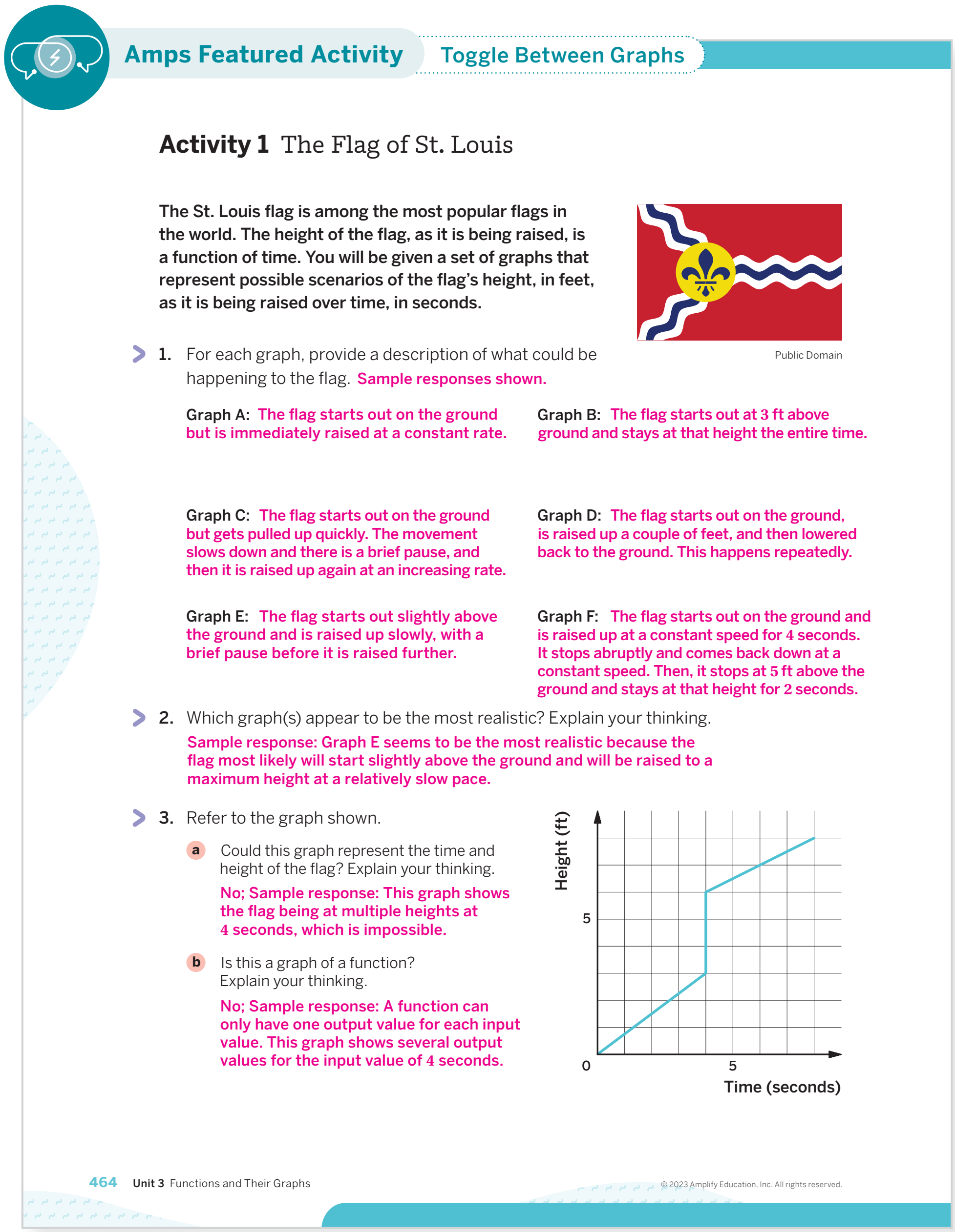
Informed by: Performance on Lesson 11, Practice Problem 6

Pairs | 20 min

Activity 1 The Flag of St. Louis

Students make sense of complex graphs representing raising a flag and interpret key features of the graph within context to reason about realistic scenarios.

Amps Featured Activity Toggle Between Graphs



Activity 1 The Flag of St. Louis

The St. Louis flag is among the most popular flags in the world. The height of the flag, as it is being raised, is a function of time. You will be given a set of graphs that represent possible scenarios of the flag's height, in feet, as it is being raised over time, in seconds.

Public Domain

1. For each graph, provide a description of what could be happening to the flag. **Sample responses shown.**

   Graph A: **The flag starts out on the ground but is immediately raised at a constant rate.**

   Graph B: **The flag starts out at 3 ft above ground and stays at that height the entire time.**

   Graph C: **The flag starts out on the ground but gets pulled up quickly. The movement slows down and there is a brief pause, and then it is raised up again at an increasing rate.**

   Graph D: **The flag starts out on the ground, is raised up a couple of feet, and then lowered back to the ground. This happens repeatedly.**

   Graph E: **The flag starts out slightly above the ground and is raised up slowly, with a brief pause before it is raised further.**

   Graph F: **The flag starts out on the ground and is raised up at a constant speed for 4 seconds. It stops abruptly and comes back down at a constant speed. Then, it stops at 5 ft above the ground and stays at that height for 2 seconds.**

2. Which graph(s) appear to be the most realistic? Explain your thinking.

   Sample response: Graph E seems to be the most realistic because the flag most likely will start slightly above the ground and will be raised to a maximum height at a relatively slow pace.

3. Refer to the graph shown.

   a. Could this graph represent the time and height of the flag? Explain your thinking.

   No; Sample response: This graph shows the flag being at multiple heights at 4 seconds, which is impossible.

   b. Is this a graph of a function? Explain your thinking.

   No; Sample response: A function can only have one output value for each input value. This graph shows several output values for the input value of 4 seconds.

464 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Activate prior knowledge by asking, "What does a flag being raised on a flagpole look like?" Distribute the Activity 1 PDF to each student pair.

2 Monitor

Help students get started by having them label key features on each graph.

Look for points of confusion:

- **Thinking each graph represents the vertical and horizontal position of the flag.** Ask, "Because flags are on a flagpole, which direction(s) can they move?"
- **Thinking a vertical segment represents moving vertically on the flagpole.** Remind students that one axis represents time. Ask, "How much time passes given the vertical segment? What does it mean for the height of the flag?"

Look for productive strategies:

- Describing the vertical intercept as the starting height.
- Describing a maximum or minimum in the flag's height.
- Using the average rate of change to describe how fast the flag is moving up or down.
- Recognizing a vertical segment represents an impossible scenario in the context of raising a flag.

3 Connect

Display Graphs A–F.

Have pairs of students share their descriptions of each graph and their responses to Problem 3.

Highlight that when determining whether a graph is realistic, it is helpful to use math vocabulary to describe key features of the graphs representing each scenario. A vertical segment would represent the flag being at multiple heights at one time, which is impossible.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can view an animation of a flag moving and have the ability to control the playback. This will help them visualize the movement of the flag.

Accessibility: *Vary Demands to Optimize Challenge*

Instead of asking students to write the verbal descriptions in Problem 1, allow them to jot down notes and observations and orally describe what would happen to the flag for each graph.

Math Language Development

MLR7: Compare and Connect

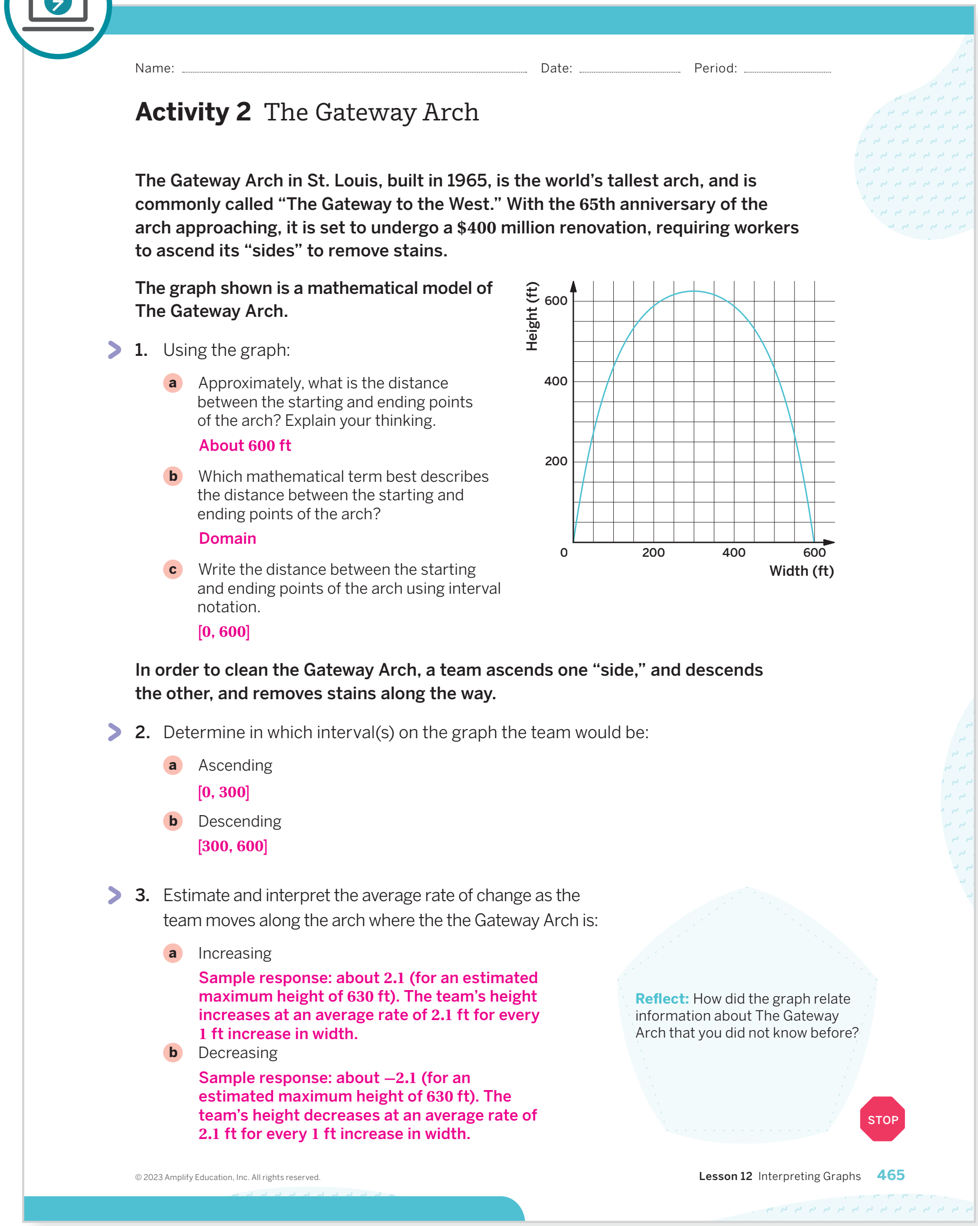
During the Connect, as students share their descriptions for each graph and their response to Problem 3, draw their attention to what is similar and different about the graphs, especially as it relates to the movement of the flag in context. Ask:

- "What would a graph that is constant mean?"
- "What would a graph with a slanted line segment mean?"
- "What would a graph with a vertical line segment mean?"
- "What would a graph that is not composed of straight line segments mean?"

Pairs | 15 min

Activity 2 The Gateway Arch

Students make sense of a complex graph of the Gateway Arch and interpret key features of the graph within context.



Name: ________ Date: ________ Period: ________

Activity 2 The Gateway Arch

The Gateway Arch in St. Louis, built in 1965, is the world's tallest arch, and is commonly called "The Gateway to the West." With the 65th anniversary of the arch approaching, it is set to undergo a $400 million renovation, requiring workers to ascend its "sides" to remove stains.

The graph shown is a mathematical model of The Gateway Arch.

1. Using the graph:
   - **a** Approximately, what is the distance between the starting and ending points of the arch? Explain your thinking.
     About 600 ft
   - **b** Which mathematical term best describes the distance between the starting and ending points of the arch?
     Domain
   - **c** Write the distance between the starting and ending points of the arch using interval notation.
     [0, 600]

In order to clean the Gateway Arch, a team ascends one "side," and descends the other, and removes stains along the way.

2. Determine in which interval(s) on the graph the team would be:
   - **a** Ascending
     [0, 300]
   - **b** Descending
     [300, 600]

3. Estimate and interpret the average rate of change as the team moves along the arch where the the Gateway Arch is:
   - **a** Increasing
     Sample response: about 2.1 (for an estimated maximum height of 630 ft). The team's height increases at an average rate of 2.1 ft for every 1 ft increase in width.
   - **b** Decreasing
     Sample response: about –2.1 (for an estimated maximum height of 630 ft). The team's height decreases at an average rate of 2.1 ft for every 1 ft increase in width.

Reflect: How did the graph relate information about The Gateway Arch that you did not know before?

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 12 Interpreting Graphs 465

1 Launch

Activate prior knowledge by asking, "How many of you have seen the Gateway Arch? Who has visited?"

2 Monitor

Help students get started by asking, "What are some words or phrases that you can use to refer to the horizontal axis?"

Look for points of confusion:

- **Switching the order of the values for the domain.** Have students write their domain in inequality notation or with a verbal description and ask if their interval matches.
- **Interpreting the average rate of change as a 2.1 ft increase in width for every 1 ft increase in height.** Ask students to label their work with units and whether the values correspond with the width or height.

Look for productive strategies:

- Recognizing the domain is referring to the width of the arch.
- Interpreting the average rate of change as a change in the height relative to the width.

3 Connect

Display the graph of the Gateway arch.

Have pairs of students share the key features they identified in Problems 1–3 along with their interpretations.

Highlight that it is important to pay attention to the scale and units of both axes because estimates might need to be made when determining key features.

Ask, "What did you notice about the average rate of change where the Gateway Arch is increasing and decreasing?" Sample response: The average rates of change were opposite of each other in value.

Differentiated Support

Accessibility: *Activate Background Knowledge, Guide Processing and Visualization*

Show an image of the Gateway Arch in St. Louis to help students visualize this structure. Consider showing images of other arches, such as the Arc de Triomphe in Paris, France, or the Washington Square Arch in New York City. Ask students to describe the shape of an arch in their own words. Sample response: An arch is a curve that increases to a maximum value and then decreases.

Extension: *Math Enrichment*

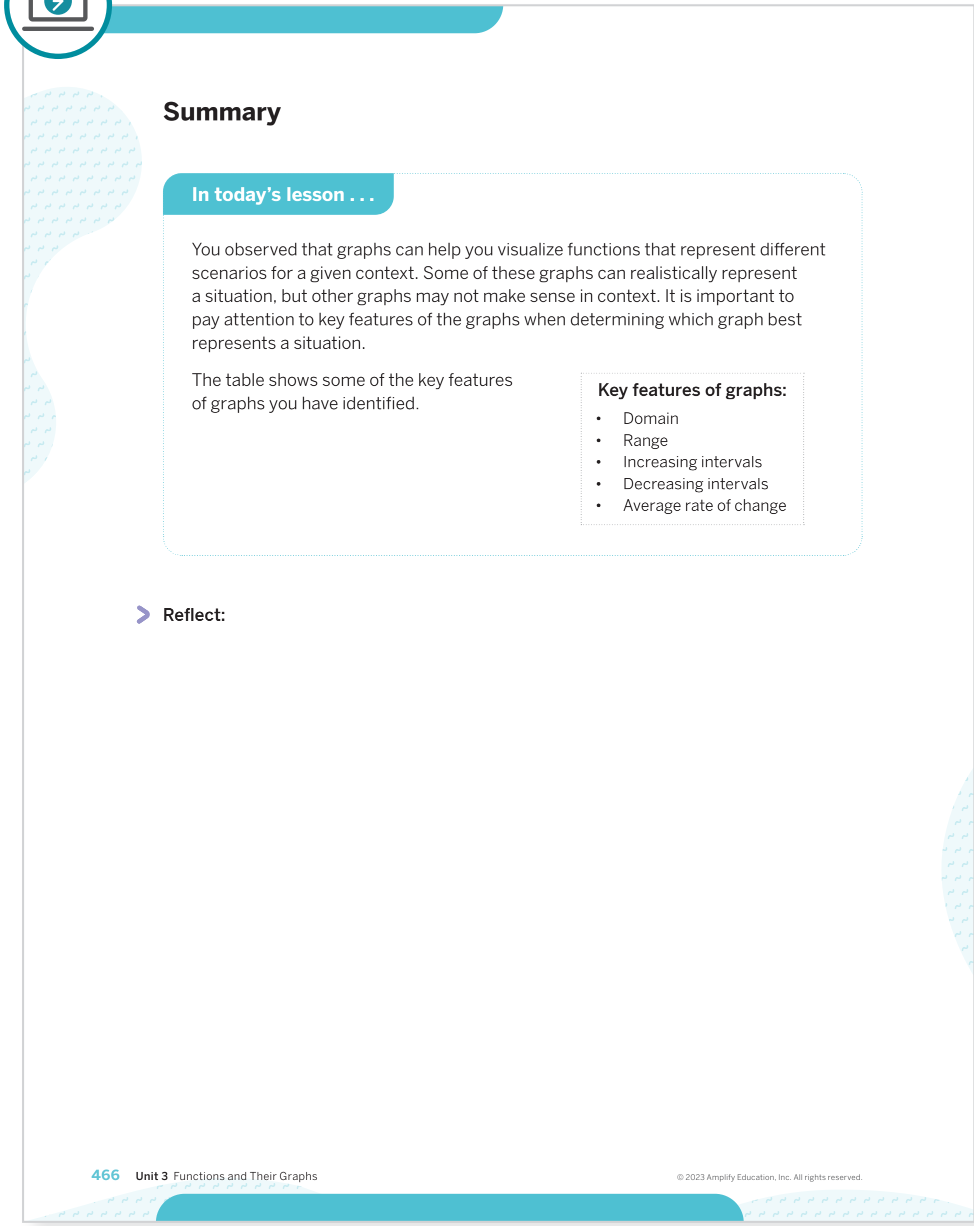
Have students complete the following problems as an extension to Problem 3: Sample responses are shown, using an estimated value for the maximum height of the arch of 630 ft.

- What is the equation of the line that connects the leftmost horizontal intercept of the graph with the maximum value? $y = 2.1x$
- What is the equation of the line that connects the maximum value with the rightmost horizontal intercept of the graph? $y = -2.1x + 1260$, (or equivalent)
- What do you notice about these equations compared to the average rate of change for each interval in Problem 3? The slope of these lines are the same as the average rates of change. For the increasing interval, the slope is positive. For the decreasing interval, the slope is negative.

Whole Class | 5 min

Summary

Review and synthesize how to interpret key features of graphs within context.



Summary

In today's lesson . . .

You observed that graphs can help you visualize functions that represent different scenarios for a given context. Some of these graphs can realistically represent a situation, but other graphs may not make sense in context. It is important to pay attention to key features of the graphs when determining which graph best represents a situation.

The table shows some of the key features of graphs you have identified.

Key features of graphs:

- Domain
- Range
- Increasing intervals
- Decreasing intervals
- Average rate of change

Reflect:

466 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display one of the graphs from Activity 1.

Have students share key features and interpretations using the graph.

Highlight that interpreting key features of graphs involves paying attention to the context, units being used, and scale of the axes. These key features can include: domain, range, increasing/decreasing intervals, average rate of change, maximum, and minimum.

Ask, "Why do vertical lines often not make sense in the context of a scenario?" Sample response: Because vertical lines do not represent a function, interpreting them in context can lead to impossible explanations.

Reflect

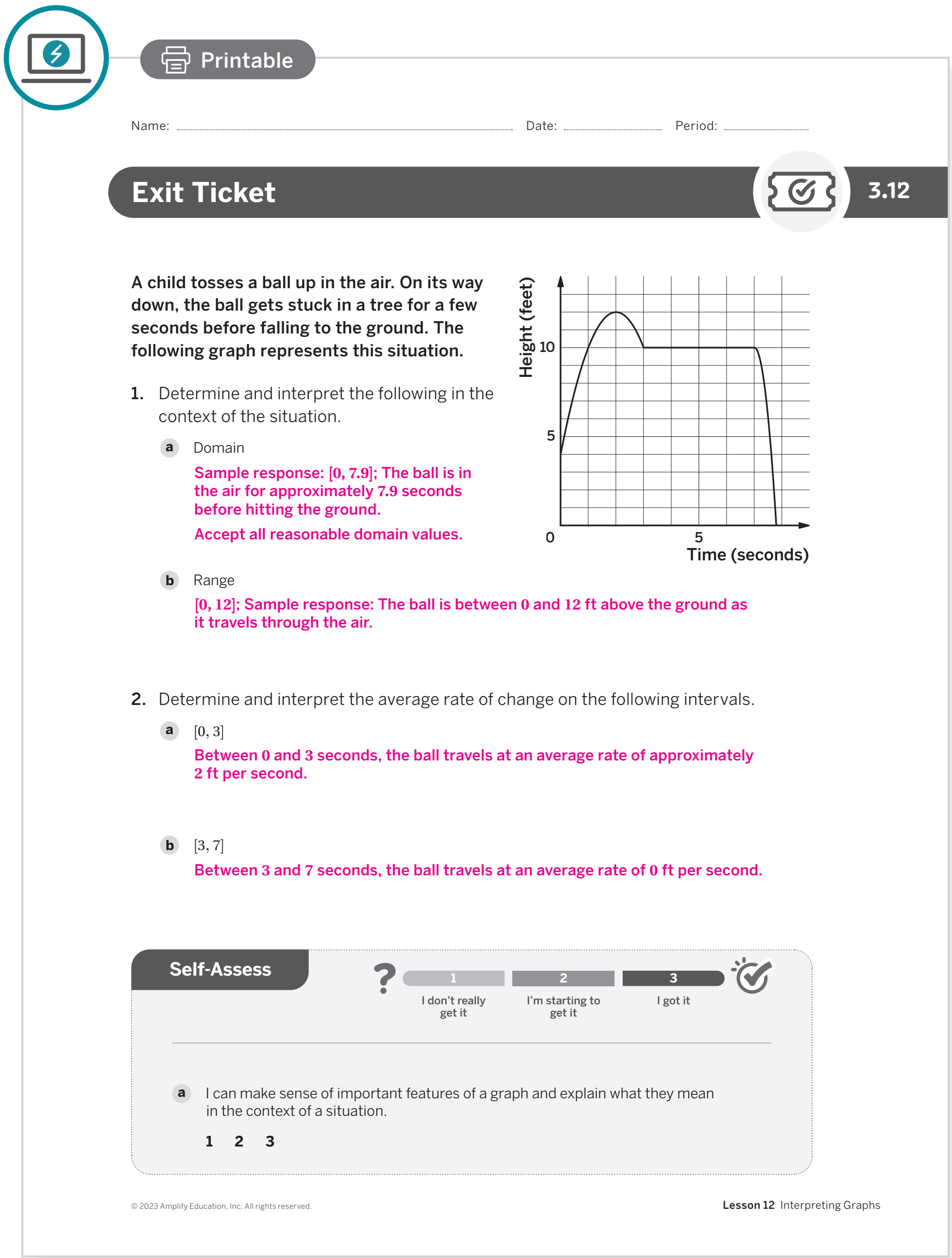
After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Given a graph, how can you interpret key features in the context of a scenario?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by making sense of a complex graph and interpreting key features of the graph within context.



Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 3.12

A child tosses a ball up in the air. On its way down, the ball gets stuck in a tree for a few seconds before falling to the ground. The following graph represents this situation.

1. Determine and interpret the following in the context of the situation.
   - **a** Domain
     Sample response: [0, 7.9]; The ball is in the air for approximately 7.9 seconds before hitting the ground.
     Accept all reasonable domain values.
   - **b** Range
     [0, 12]; Sample response: The ball is between 0 and 12 ft above the ground as it travels through the air.
2. Determine and interpret the average rate of change on the following intervals.
   - **a** [0, 3]
     Between 0 and 3 seconds, the ball travels at an average rate of approximately 2 ft per second.
   - **b** [3, 7]
     Between 3 and 7 seconds, the ball travels at an average rate of 0 ft per second.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can make sense of important features of a graph and explain what they mean in the context of a situation.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Success looks like . . .

- **Language Goal:** Determining and interpreting important features of a graph in context. **(Reading and Writing)**
  - » Identifying and interpreting the domain and range in Problem 1.
- **Language Goal:** Interpreting the average rate of change of a graph given a context. **(Reading and Writing)**
  - » Interpreting the average rate of change on the given intervals in Problem 2.

Suggested next steps

If students incorrectly determine and/or interpret the domain and range in Problem 1, consider:

- Reviewing identifying key features from Activity 2.
- Assigning Practice Problem 2.
- Asking, "What are other words or phrases that mean the same thing as domain and range?"

If students incorrectly determine and/or interpret the average rate of change in Problem 2, consider:

- Reviewing identifying the average rate of change from Activity 2.
- Assigning Practice Problem 2.
- Asking, "What coordinate points could help in determining the average rate of change?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Did students find Activity 1 or Activity 2 more engaging today? Why do you think that is?
- Which groups of students did and didn't have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Math Language Development

Language Goal: Determining and interpreting important features of a graph in context.

Reflect on students' language development toward this goal.

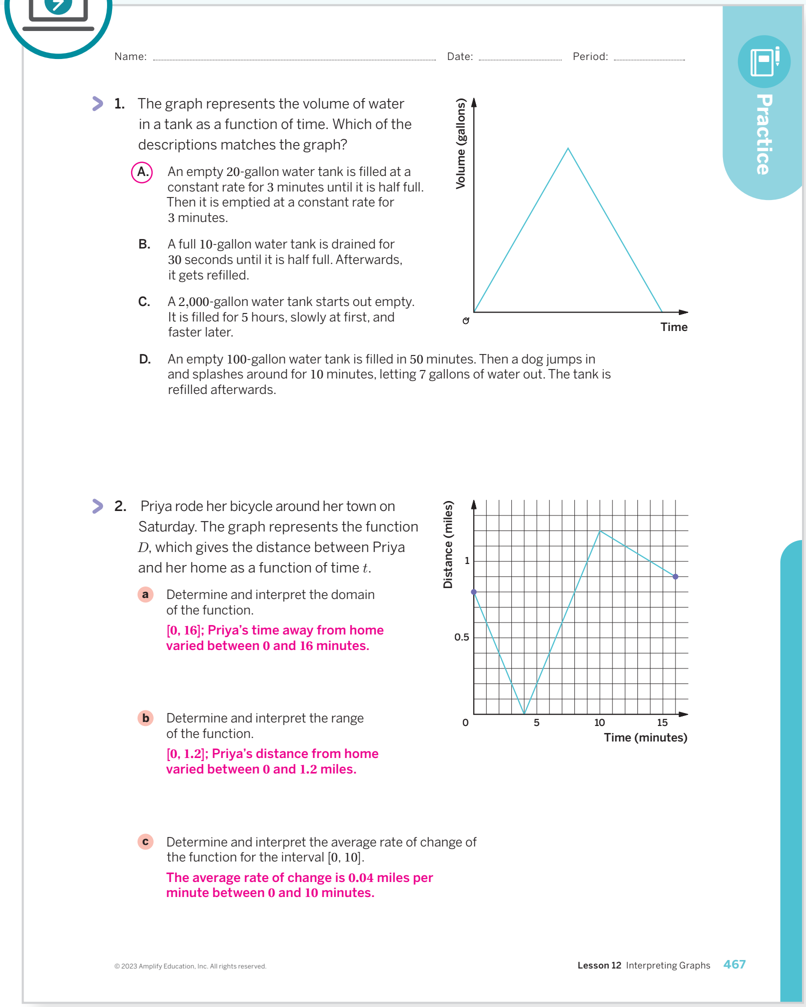
- How did students begin describing key features of the graphs of functions earlier in this unit? How have they progressed in their descriptions and how can you help them be more precise?

Sample descriptions:

Emerging	Expanding
The graph goes from 0 to a little less than 8.	The domain is approximately [0, 7.9], which represents the time the ball is in the air.

Practice

Independent



Practice

Name: ______________________ Date: __________ Period: __________

1. The graph represents the volume of water in a tank as a function of time. Which of the descriptions matches the graph?

   [✓ A.] An empty 20-gallon water tank is filled at a constant rate for 3 minutes until it is half full. Then it is emptied at a constant rate for 3 minutes.

   B. A full 10-gallon water tank is drained for 30 seconds until it is half full. Afterwards, it gets refilled.

   C. A 2,000-gallon water tank starts out empty. It is filled for 5 hours, slowly at first, and faster later.

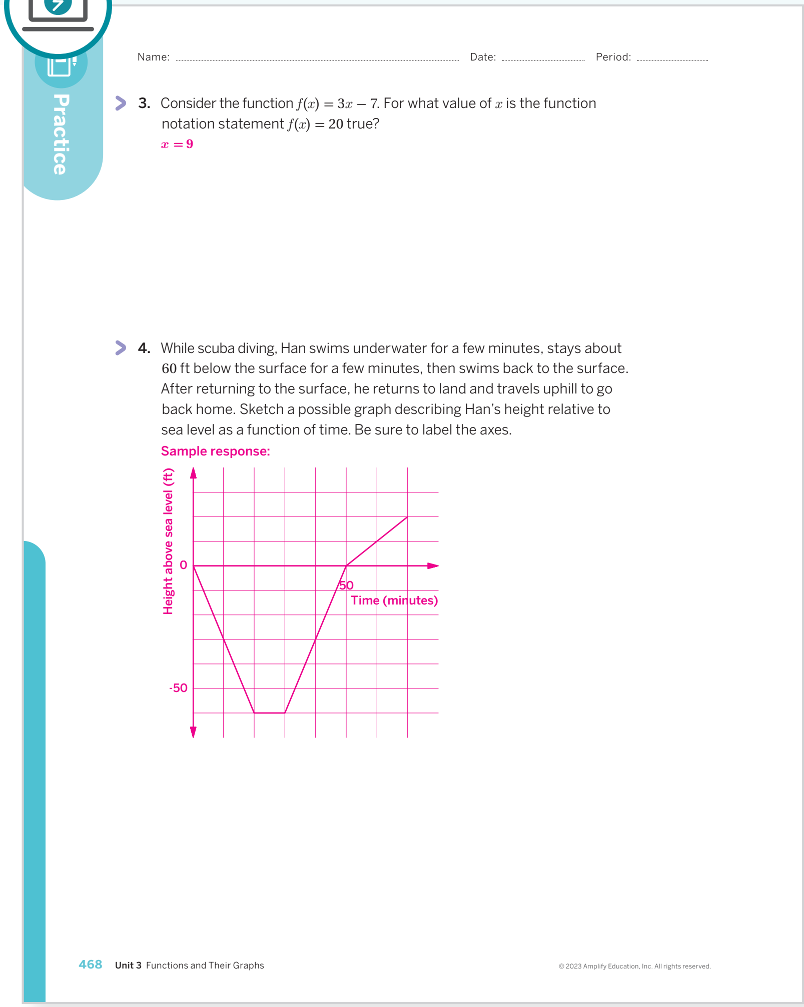
   D. An empty 100-gallon water tank is filled in 50 minutes. Then a dog jumps in and splashes around for 10 minutes, letting 7 gallons of water out. The tank is refilled afterwards.

2. Priya rode her bicycle around her town on Saturday. The graph represents the function D, which gives the distance between Priya and her home as a function of time t.

   a. Determine and interpret the domain of the function.
   [0, 16]; Priya's time away from home varied between 0 and 16 minutes.

   b. Determine and interpret the range of the function.
   [0, 1.2]; Priya's distance from home varied between 0 and 1.2 miles.

   c. Determine and interpret the average rate of change of the function for the interval [0, 10].
   The average rate of change is 0.04 miles per minute between 0 and 10 minutes.



Practice

Name: ______________________ Date: __________ Period: __________

3. Consider the function $f(x) = 3x - 7$. For what value of x is the function notation statement $f(x) = 20$ true?
   $x = 9$

4. While scuba diving, Han swims underwater for a few minutes, stays about 60 ft below the surface for a few minutes, then swims back to the surface. After returning to the surface, he returns to land and travels uphill to go back home. Sketch a possible graph describing Han's height relative to sea level as a function of time. Be sure to label the axes.
   Sample response:

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 2	2
Spiral	3	Unit 3 Lesson 6	1
Formative	4	Unit 3 Lesson 13	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

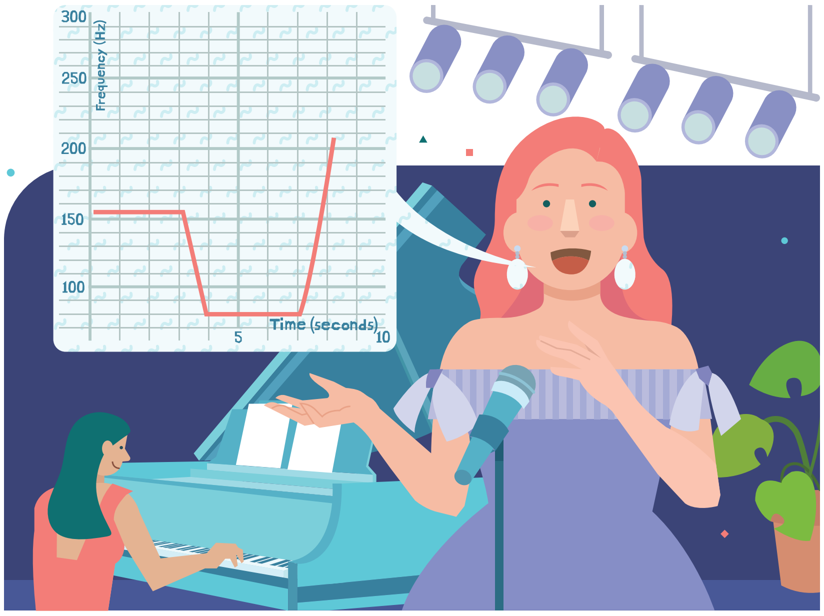
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 3 | **LESSON 13**

Creating Graphs of Functions

Let's create graphs of functions to represent real-world contexts and highlight their important features.



Focus

Goals

1. **Language Goal:** Sketch a graph that shows important features of a function that represents a situation. **(Reading and Writing)**

Coherence

Today

Students are presented with descriptions of situations and sketch graphs to accurately match those descriptions. Students start by sketching two graphs, one realistic and one unrealistic, to represent a ball bouncing. Then, students are given a verbal description with key information, which they use to sketch a graph, and identify key features. They compare the key features of the graph they sketched to a given graph, and finally, students use the New Orleans skyline to sketch two buildings from a description.

Previously

In Lesson 12, students determined and interpreted key features of graphs representing scenarios.

Coming Soon

In Lesson 14, students interpret graphs or rules of piecewise functions.

Rigor

- Students **apply** their understanding of key features of functions in order to sketch graphs.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	10 min	15 min	5 min	5 min
Pairs	Small Groups	Small Groups	Small Groups	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

Review words

- *average rate of change*
- *decreasing*
- *domain*
- *increasing*
- *range*

Amps | Featured Activity

Activity 3

Digitally Sketching the New Orleans Skyline

Given a description of two buildings in New Orleans, students create a sketch to represent these buildings. You can overlay student responses to provide immediate feedback.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students might lack self-confidence as they begin to try to compare functions because the key features are relatively new learning for them. Encourage students to get organized and make a list of key features that they need to look for in graphs. Then they can identify and compare each feature, one at a time, without forgetting any of them.

Modifications to Pacing

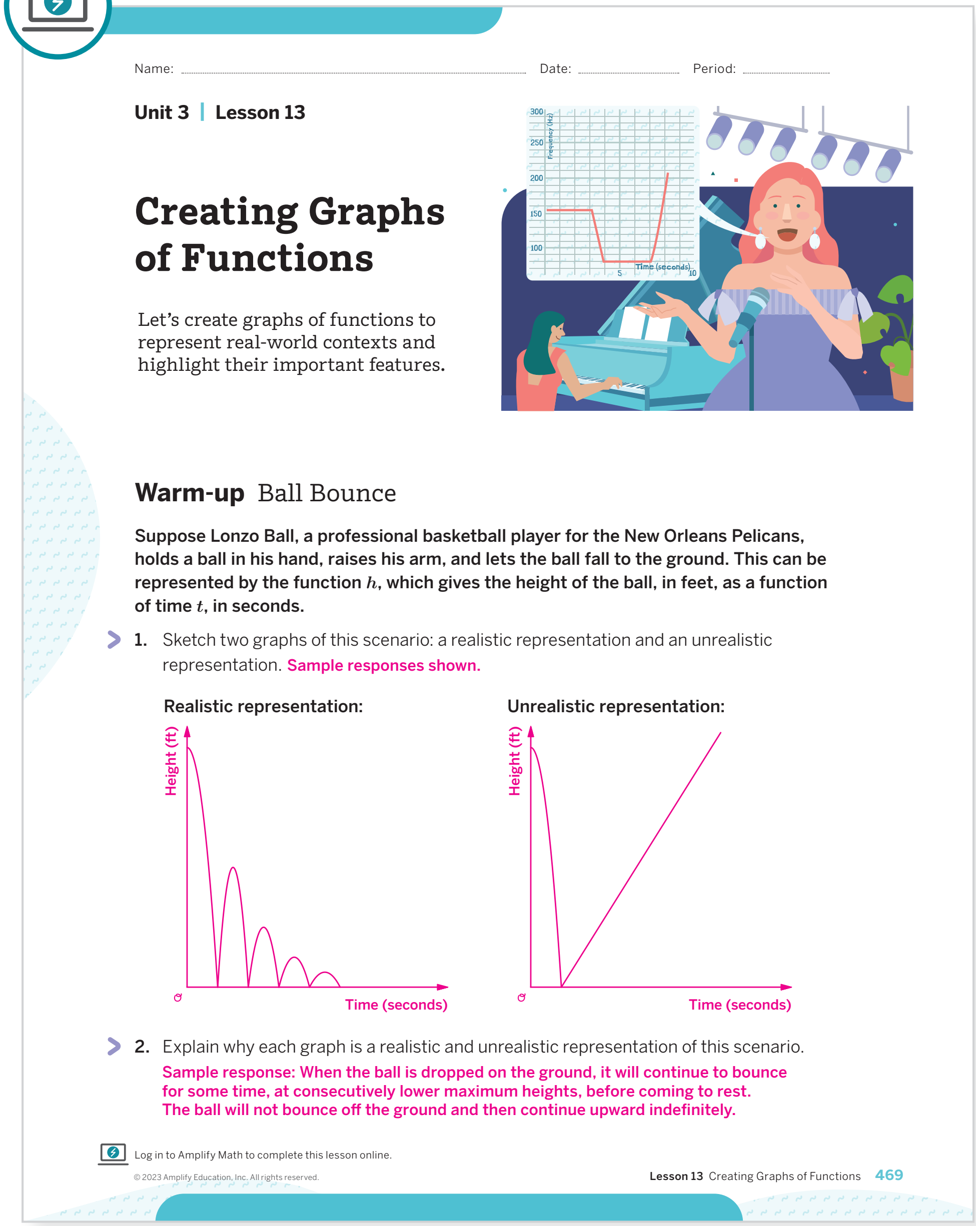
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problems 1 and 2 may be omitted.
- In **Activity 3**, Problem 2 may be omitted.

Pairs | 5 min

Warm-up Ball Bounce

Students sketch realistic and unrealistic graphs that could or could not model a real-world scenario to prepare them for sketching graphs to model real-world scenarios.



Name: ______________ Date: ________ Period: ________

Unit 3 | Lesson 13

Creating Graphs of Functions

Let's create graphs of functions to represent real-world contexts and highlight their important features.

Warm-up Ball Bounce

Suppose Lonzo Ball, a professional basketball player for the New Orleans Pelicans, holds a ball in his hand, raises his arm, and lets the ball fall to the ground. This can be represented by the function h, which gives the height of the ball, in feet, as a function of time t, in seconds.

1. Sketch two graphs of this scenario: a realistic representation and an unrealistic representation. **Sample responses shown.**

Realistic representation:

Unrealistic representation:

2. Explain why each graph is a realistic and unrealistic representation of this scenario. **Sample response: When the ball is dropped on the ground, it will continue to bounce for some time, at consecutively lower maximum heights, before coming to rest. The ball will not bounce off the ground and then continue upward indefinitely.**

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in pairs. Have students respond to Problems 1 and 2 independently before sharing their responses with their partner.

2 Monitor

Help students get started by having them verbally describe what a bouncing ball would look like.

Look for points of confusion:

- **Having difficulty sketching a realistic representation.** Ask, "What does the word 'bounce' mean? What would this look like graphically?"

Look for productive strategies:

- For the realistic graph, showing the ball hitting the ground and bouncing at consecutively lower local maximum heights.
- For the unrealistic graph, showing the ball flying in a loop, through the air indefinitely, or bouncing to higher heights than the initial point from which it was dropped.

3 Connect

Have pairs of students share their sketches for the realistic and unrealistic graphs and explanations from Problem 2.

Display student sketches for both the realistic and unrealistic graphs.

Highlight that sketching graphs that represent real-world scenarios often involves making sense of the scenario before beginning a sketch. Thinking about what will and will not make sense can help evaluate one's sketch of a scenario.

Ask, "How might the graphs change if instead of a basketball, a brick was dropped on the ground?" Sample response: The brick would not bounce at consecutively lower heights, but hit the ground once and remain there.

Differentiated Support

Accessibility: *Activate Background Knowledge, Guide Processing and Visualization*

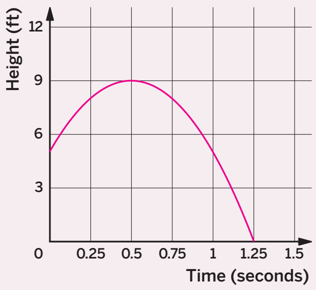
Ask students if they are interested in the sport of basketball and if so, have them name their favorite sports team or player. Ask a student volunteer to demonstrate the motion described in the introductory text. Consider providing them with a basketball or other type of bouncing ball.

Power-up

To power up students' ability to relate scenarios to key features on a graph, have students complete:

Sketch a graph that represents the height of a ball that is tossed into the air with an initial height of 5 ft and reaches a maximum height of 9 ft before falling to the ground.

Sample response shown. The graph should be increasing from 5 ft to 9 ft and then decreasing to 0 ft.



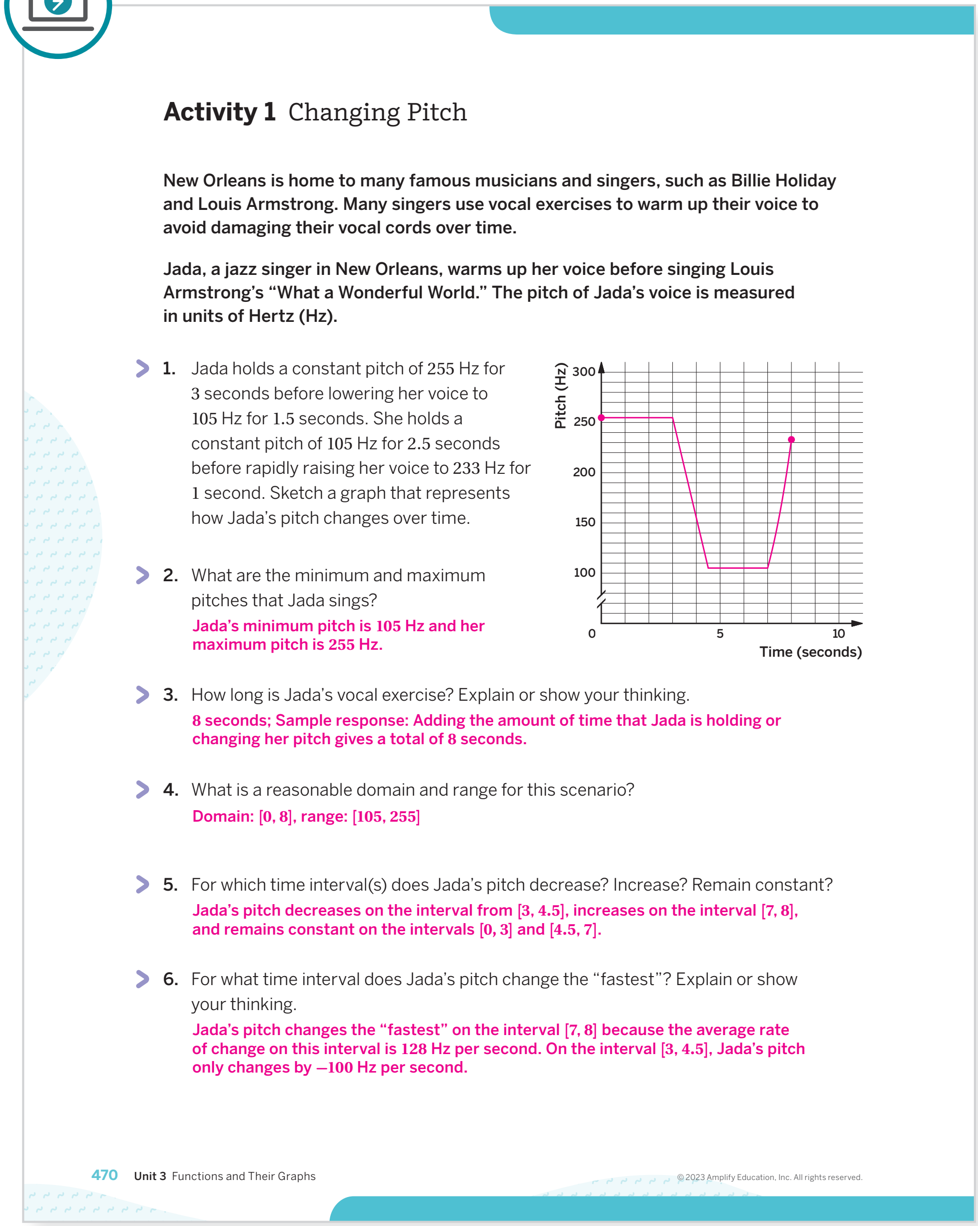
Use: Before the Warm-up

Informed by: Performance on Lesson 12, Practice Problem 4

Small groups | 10 min

Activity 1 Changing Pitch

Students sketch a graph to model a real-world scenario, identify key features, and attend to precision when determining a reasonable domain and range.



Activity 1 Changing Pitch

New Orleans is home to many famous musicians and singers, such as Billie Holiday and Louis Armstrong. Many singers use vocal exercises to warm up their voice to avoid damaging their vocal cords over time.

Jada, a jazz singer in New Orleans, warms up her voice before singing Louis Armstrong's "What a Wonderful World." The pitch of Jada's voice is measured in units of Hertz (Hz).

1. Jada holds a constant pitch of 255 Hz for 3 seconds before lowering her voice to 105 Hz for 1.5 seconds. She holds a constant pitch of 105 Hz for 2.5 seconds before rapidly raising her voice to 233 Hz for 1 second. Sketch a graph that represents how Jada's pitch changes over time.

2. What are the minimum and maximum pitches that Jada sings?
   Jada's minimum pitch is 105 Hz and her maximum pitch is 255 Hz.

3. How long is Jada's vocal exercise? Explain or show your thinking.
   8 seconds; Sample response: Adding the amount of time that Jada is holding or changing her pitch gives a total of 8 seconds.

4. What is a reasonable domain and range for this scenario?
   Domain: $[0, 8]$, range: $[105, 255]$

5. For which time interval(s) does Jada's pitch decrease? Increase? Remain constant?
   Jada's pitch decreases on the interval from $[3, 4.5]$, increases on the interval $[7, 8]$, and remains constant on the intervals $[0, 3]$ and $[4.5, 7]$.

6. For what time interval does Jada's pitch change the "fastest"? Explain or show your thinking.
   Jada's pitch changes the "fastest" on the interval $[7, 8]$ because the average rate of change on this interval is 128 Hz per second. On the interval $[3, 4.5]$, Jada's pitch only changes by −100 Hz per second.

470 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Activate students' prior knowledge by asking, "Who has heard the song, 'What a Wonderful World,' by Louis Armstrong?"

2 Monitor

Help students get started by asking, "What would the words constant, lowering, or raising look like graphically?"

Look for points of confusion:

- **Creating a discrete graph instead of a graph that is not discrete.** Ask, "What does discrete mean? Does it make sense in this context?"
- **Creating a graph with gaps.** Ask, "What would a gap in the graph represent? Is that described in the scenario?"

Look for productive strategies:

- Creating horizontal lines to represent specific constant pitch values.
- Creating increasing or decreasing lines or curves when the pitch lowers or raises.
- Representing constant pitch or changes in pitch for the appropriately described time intervals.
- Using given values from Problem 1 to identify and determine key features of the graph in Problems 2–6.

3 Connect

Have groups of students share the graph they sketched in Problem 1 and key features they identified in Problems 2–6.

Display student graphs from Problem 1.

Highlight that paying close attention to scale, given values, and keywords is essential for creating an accurate graph. Knowing that time represents the independent variable and frequency the dependent variable is important when determining the maximum, minimum, domain, range, and average rate of change.

Ask, "What would a 'pause' in Jada's Warm-up look like graphically?" Sample response: There would be a gap, or break, in the graph between two segments or curves.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students use hand gestures to illustrate how the pitch changes over time and then translate their hand gestures to the graph.

Extension: *Interdisciplinary Connections*

Tell students that all sound travels as sound waves. For different pitches, sound waves have different frequencies and 1 Hertz is equivalent to 1 vibration per second. Higher pitches have greater frequencies than lower pitches. Consider showing students a graph of two sound waves to illustrate what the graph of a higher frequency (higher pitch) sound wave looks like than a lower frequency (lower pitch). **(Science)**

Math Language Development

MLR6: Three Reads

Have students read the text in Problem 1 three times to help them make sense of the information provided.

Read 1: Students should understand that as Jada sings, her pitch varies.

Read 2: Ask students to name the given quantities and relationships, such as Jada holds a constant pitch of 255 Hz for 3 seconds.

Read 3: Ask students to brainstorm strategies for how they will sketch the graph that shows how Jada's pitch changes over time

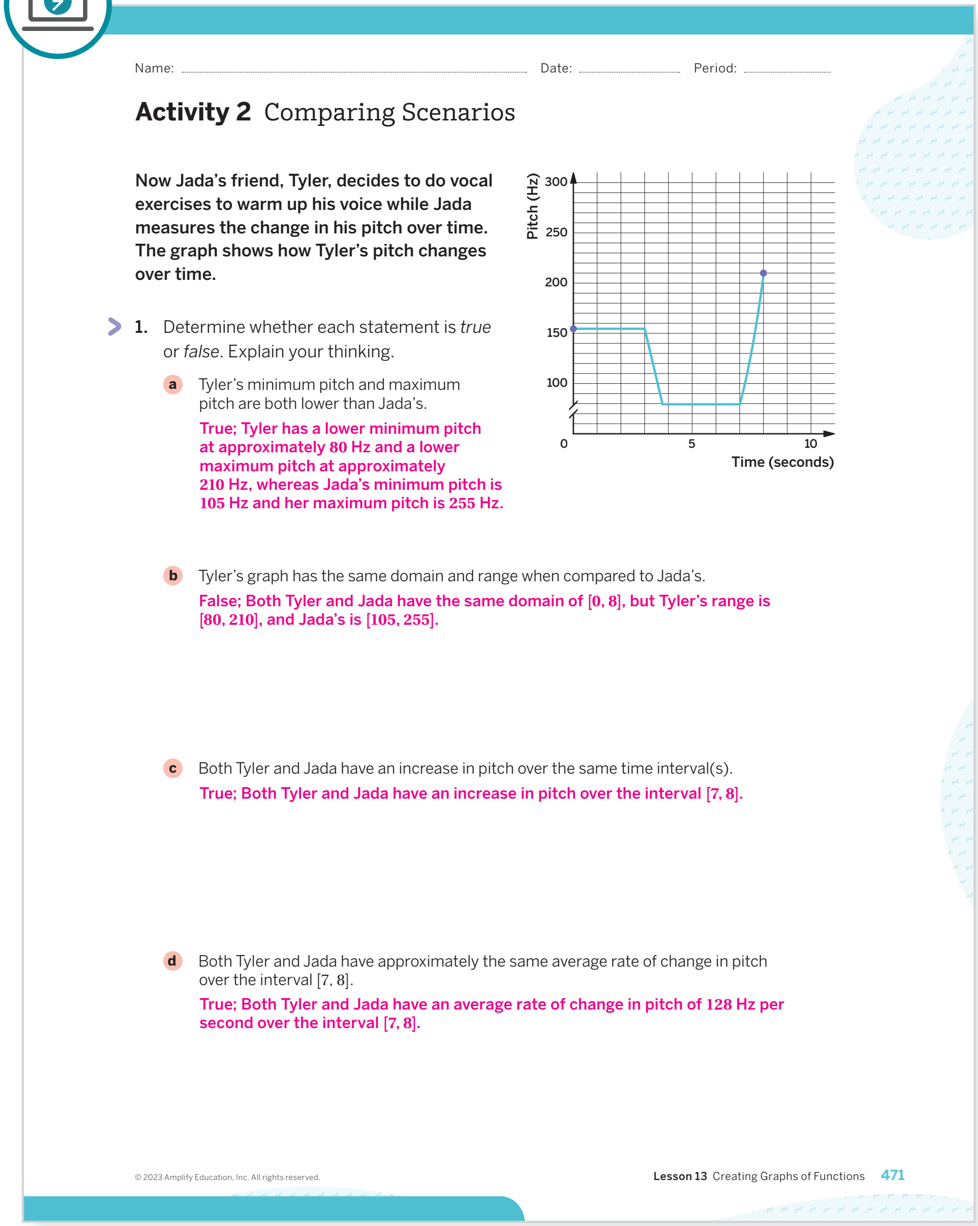
English Learners

Annotate the phrase "rapidly raising" with the phrase *increases at a fast rate* to help students connect their meanings.

Small groups | 10 min

Activity 2 Comparing Scenarios

Students compare graphs of two functions and use their developing mathematical vocabulary to compare key features of both graphs.



Name: Date: Period:

Activity 2 Comparing Scenarios

Now Jada's friend, Tyler, decides to do vocal exercises to warm up his voice while Jada measures the change in his pitch over time. The graph shows how Tyler's pitch changes over time.

1. Determine whether each statement is *true* or *false*. Explain your thinking.

   a Tyler's minimum pitch and maximum pitch are both lower than Jada's.

   True; Tyler has a lower minimum pitch at approximately 80 Hz and a lower maximum pitch at approximately 210 Hz, whereas Jada's minimum pitch is 105 Hz and her maximum pitch is 255 Hz.

   b Tyler's graph has the same domain and range when compared to Jada's.

   False; Both Tyler and Jada have the same domain of $[0, 8]$, but Tyler's range is $[80, 210]$, and Jada's is $[105, 255]$.

   c Both Tyler and Jada have an increase in pitch over the same time interval(s).

   True; Both Tyler and Jada have an increase in pitch over the interval $[7, 8]$.

   d Both Tyler and Jada have approximately the same average rate of change in pitch over the interval $[7, 8]$.

   True; Both Tyler and Jada have an average rate of change in pitch of 128 Hz per second over the interval $[7, 8]$.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Students remain in small groups. Have students work individually on Problem 1 before comparing with their groups.

2 Monitor

Help students get started by having them identify key features on Tyler's graph.

Look for points of confusion:

- **Thinking the domain and range for Tyler and Jada are the same.** Ask, "What is the dependent variable? How did the maximum and minimum relate to the range in Activity 1?"

Look for productive strategies:

- Using the maximum and minimum values to compare the range between the two graphs.
- Sketching Jada's graph on Tyler's graph to easily reveal similarities and differences between the two scenarios.

3 Connect

Display the graphs for both Tyler and Jada.

Have groups of students share their responses and thinking for Problem 1.

Highlight that both graphs look similar, but do contain key features that are slightly different. Labeling points or values on each graph can help see these differences or similarities.

Ask, "What could be a reason why the two graphs look so similar, but have a different range?" Sample response: Different people have natural differences in pitch, some are higher and some lower.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider making copies of Tyler's and Jada's graph on the same sheet of paper so that students can compare them side-by-side.

Extension: *Math Enrichment*

Have students write the equations of the lines representing any four pieces of Tyler's graph. For each equation, have them describe the domain for that interval on the graph.

$y = 155$	Domain: $[0, 3]$
$y = -100x + 455$	Domain: $[3, 3.7]$
$y = 80$	Domain: $[3.7, 7]$

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display the false statement in Problem 1b.

Ask:

Critique: "Is any part of this statement correct, or is all of it incorrect?" Part of the statement is correct. The graphs have the same domain.

Correct: "How could you alter this statement so that it is correct?" Sample response: Tyler's graph has the same domain as Jada's graph, but a different range.

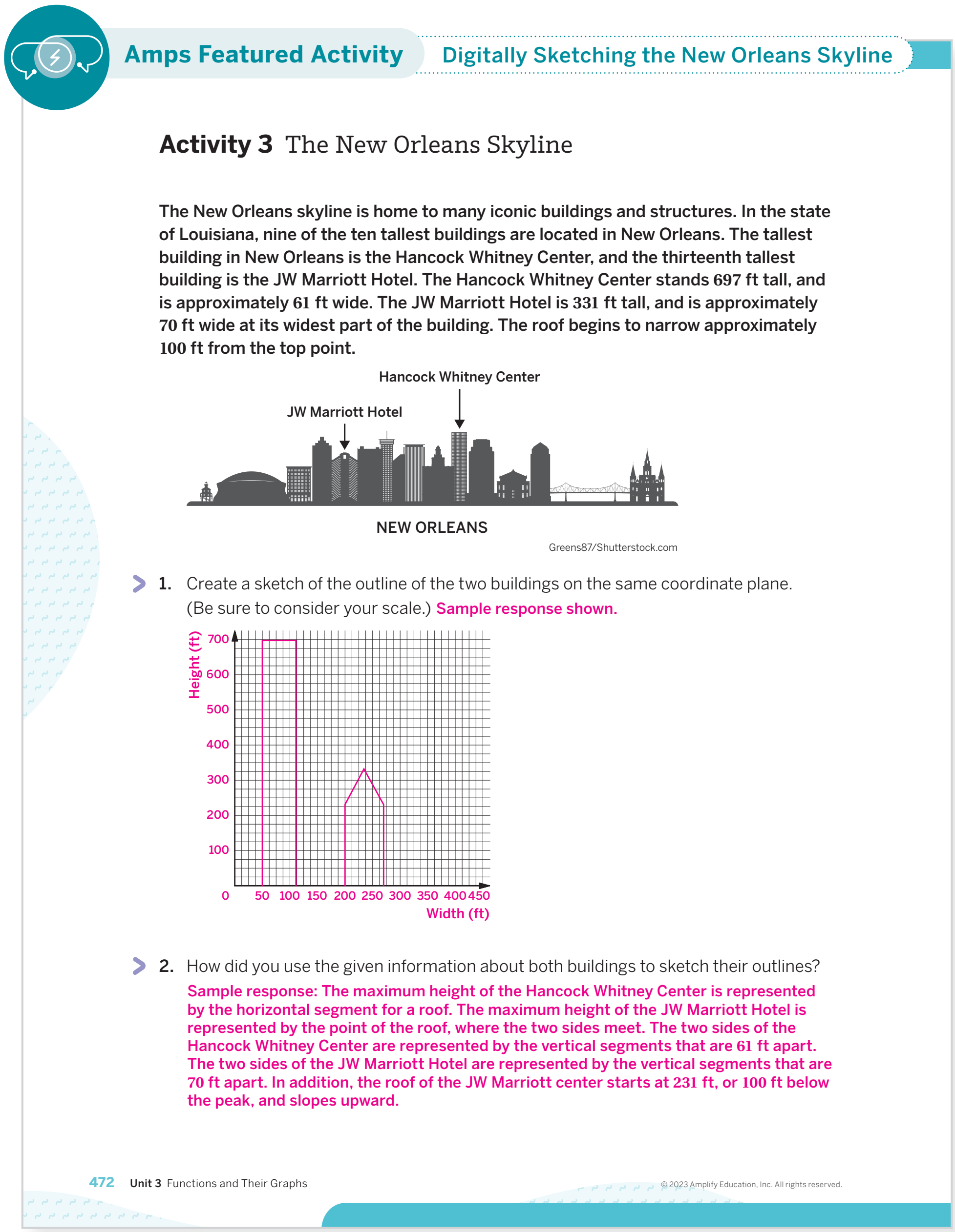
Clarify: "How could you add to the statement you wrote to provide more detail?" Sample response: I could give the actual domains for each graph showing they are the same. I could give the ranges for each graph, showing they are different.

Small groups | 15 min

Activity 3 The New Orleans Skyline

Students sketch a graph to model the New Orleans skyline to apply their understanding of key features of graphs, using precise mathematical vocabulary.

Amps Featured Activity Digitally Sketching the New Orleans Skyline



Activity 3 The New Orleans Skyline

The New Orleans skyline is home to many iconic buildings and structures. In the state of Louisiana, nine of the ten tallest buildings are located in New Orleans. The tallest building in New Orleans is the Hancock Whitney Center, and the thirteenth tallest building is the JW Marriott Hotel. The Hancock Whitney Center stands 697 ft tall, and is approximately 61 ft wide. The JW Marriott Hotel is 331 ft tall, and is approximately 70 ft wide at its widest part of the building. The roof begins to narrow approximately 100 ft from the top point.

Greens87/Shutterstock.com

1. Create a sketch of the outline of the two buildings on the same coordinate plane. (Be sure to consider your scale.) Sample response shown.

2. How did you use the given information about both buildings to sketch their outlines? Sample response: The maximum height of the Hancock Whitney Center is represented by the horizontal segment for a roof. The maximum height of the JW Marriott Hotel is represented by the point of the roof, where the two sides meet. The two sides of the Hancock Whitney Center are represented by the vertical segments that are 61 ft apart. The two sides of the JW Marriott Hotel are represented by the vertical segments that are 70 ft apart. In addition, the roof of the JW Marriott center starts at 231 ft, or 100 ft below the peak, and slopes upward.

472 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Activate prior knowledge by asking, “Who has traveled to New Orleans? What are some notable places?”

2 Monitor

Help students get started by having them create a scale for their vertical and horizontal axes using the given information.

Look for points of confusion:

- **Creating a horizontal axis scale that will not fit both buildings.** Have students observe the given skyline and ask, “How can you change the scale to show distance between the buildings?”
- **Thinking the range of the JW Marriott Hotel roof is a single value.** Ask, “Where does the roof start? Where does it end?”

Look for productive strategies:

- Choosing a vertical scale over 700 ft and a horizontal scale over 200 ft.
- Recognizing that the width of the building will give the domain.
- Recognizing the range will be a single value at the maximum height for the Hancock Whitney Center.
- Recognizing that range for the roof of the JW Marriott Hotel will be an interval.

Activity 3 continued >

Differentiated Support

Accessibility: *Activate Background Knowledge, Guide Processing and Visualization*

Consider showing some photos of the Hancock Whitney Center and the JW Marriot Hotel in New Orleans to give students some visual perspective for how the two buildings compare.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can create digital sketches of the skyline represented by two buildings. You can overlay student responses to provide immediate feedback.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

Read 1: Students should understand that the heights and widths of two buildings in New Orleans are given.

Read 2: Ask students to name the given heights and widths, such as the Hancock Whitney Center is 697 ft tall and about 61 ft wide.

Read 3: Ask students to brainstorm strategies for how they will sketch the outline of the two buildings in Problem 1.

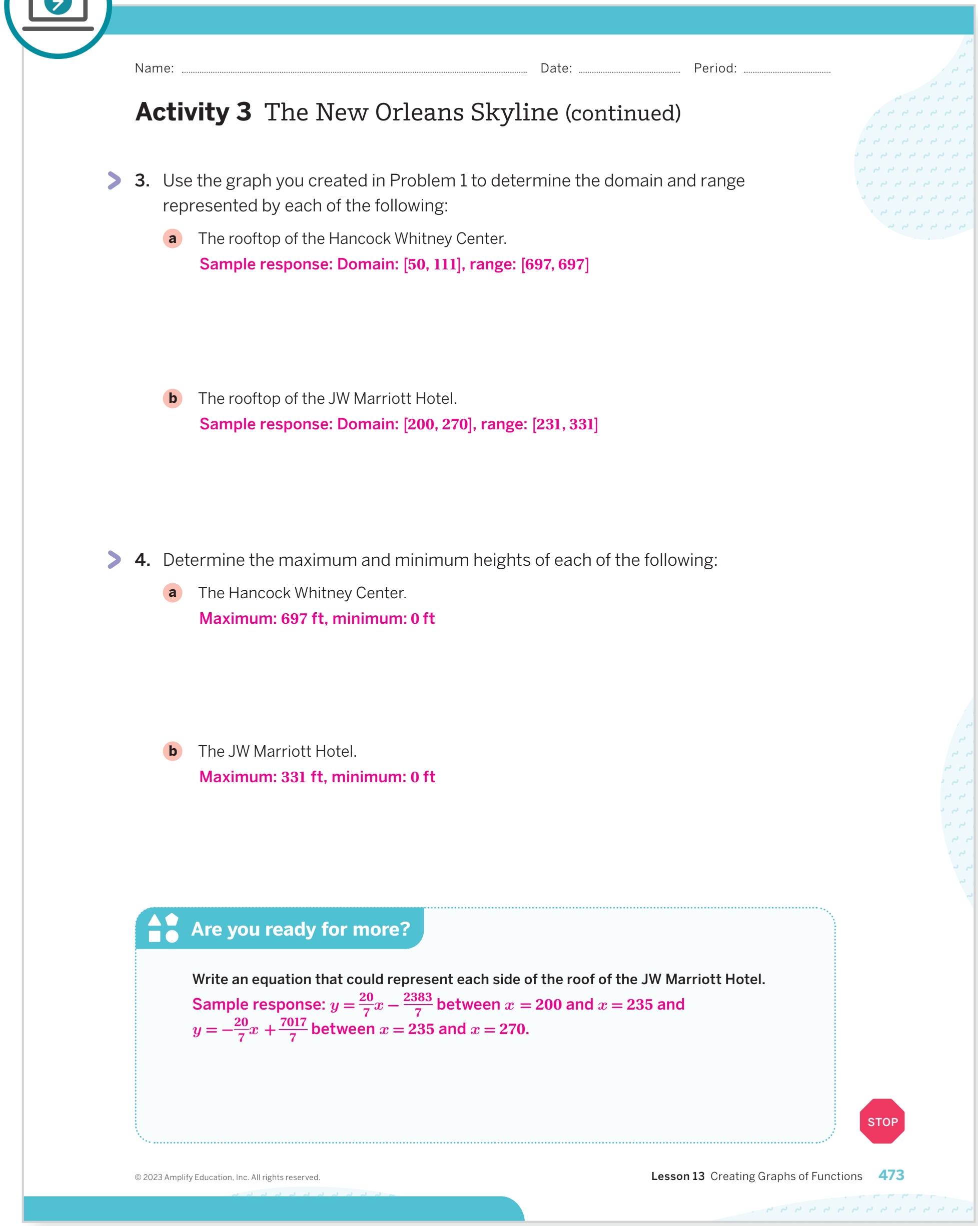
English Learners

Use hand gestures to illustrate what the phrase “the roof begins to narrow” means.

Small groups | 15 min

Activity 3 The New Orleans Skyline (continued)

Students sketch a graph to model the New Orleans skyline to apply their understanding of key features of graphs, using precise mathematical vocabulary.



Name: ____________ Date: ________ Period: ________

Activity 3 The New Orleans Skyline (continued)

3. Use the graph you created in Problem 1 to determine the domain and range represented by each of the following:
   - a The rooftop of the Hancock Whitney Center.
     Sample response: Domain: [50, 111], range: [697, 697]
   - b The rooftop of the JW Marriott Hotel.
     Sample response: Domain: [200, 270], range: [231, 331]

4. Determine the maximum and minimum heights of each of the following:
   - a The Hancock Whitney Center.
     Maximum: 697 ft, minimum: 0 ft
   - b The JW Marriott Hotel.
     Maximum: 331 ft, minimum: 0 ft

Are you ready for more?

Write an equation that could represent each side of the roof of the JW Marriott Hotel.

Sample response: $y = \frac{20}{7}x - \frac{2383}{7}$ **between** $x = 200$ **and** $x = 235$ **and** $y = -\frac{20}{7}x + \frac{7017}{7}$ **between** $x = 235$ **and** $x = 270$.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have groups of students share the graphs they created to represent both buildings, how they created the graphs, and the important features identified in Problems 3 and 4.

Display student graphs from Problem 1.

Highlight that the domain can differ depending on the scale chosen and how far apart the two buildings were placed. The scale might cause some graphs of the buildings to look different, but the range, maximum, and minimum should all be the same.

Ask, "Why is the range for the roof of the Hancock Whitney center a single value?" Sample response: The roof is a horizontal line, this means it only has one value for y, which is 697.

Summary

Review and synthesize creating graphs to represent important features of functions.

Summary

In today's lesson . . .

You created graphs of functions given their descriptions. This included determining what a reasonable sketch of a graph could look like, and then turning that sketch into a graph.

It is important to take into account key features of graphs when sketching and comparing them, such as the ones shown in the table.

Key features of graphs:

- The scale of each axis.
- The domain and range.
- The intervals for which the function is increasing and decreasing.
- The minimum and maximum values of the function.
- The average rate of change over specified intervals.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the following scenario: "Jada holds a constant pitch of 240 Hz for 2 seconds before lowering her voice to 140 Hz for 3 seconds. She holds this pitch for 2 seconds before rapidly raising her voice to 250 Hz for 1 second."

Have students share strategies they would use to graph the above scenario and how they would identify key features.

Highlight that sketching a graph from a verbal description involves attending to precision. In previous lessons and grades, sketches that showed some detail were acceptable, but now, exact values and descriptions are often given, and the graph that is created must meet the criteria.

Ask, "What key non-numerical phrases in the scenario described can help you create a graph?" Constant, lowering, and raising.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

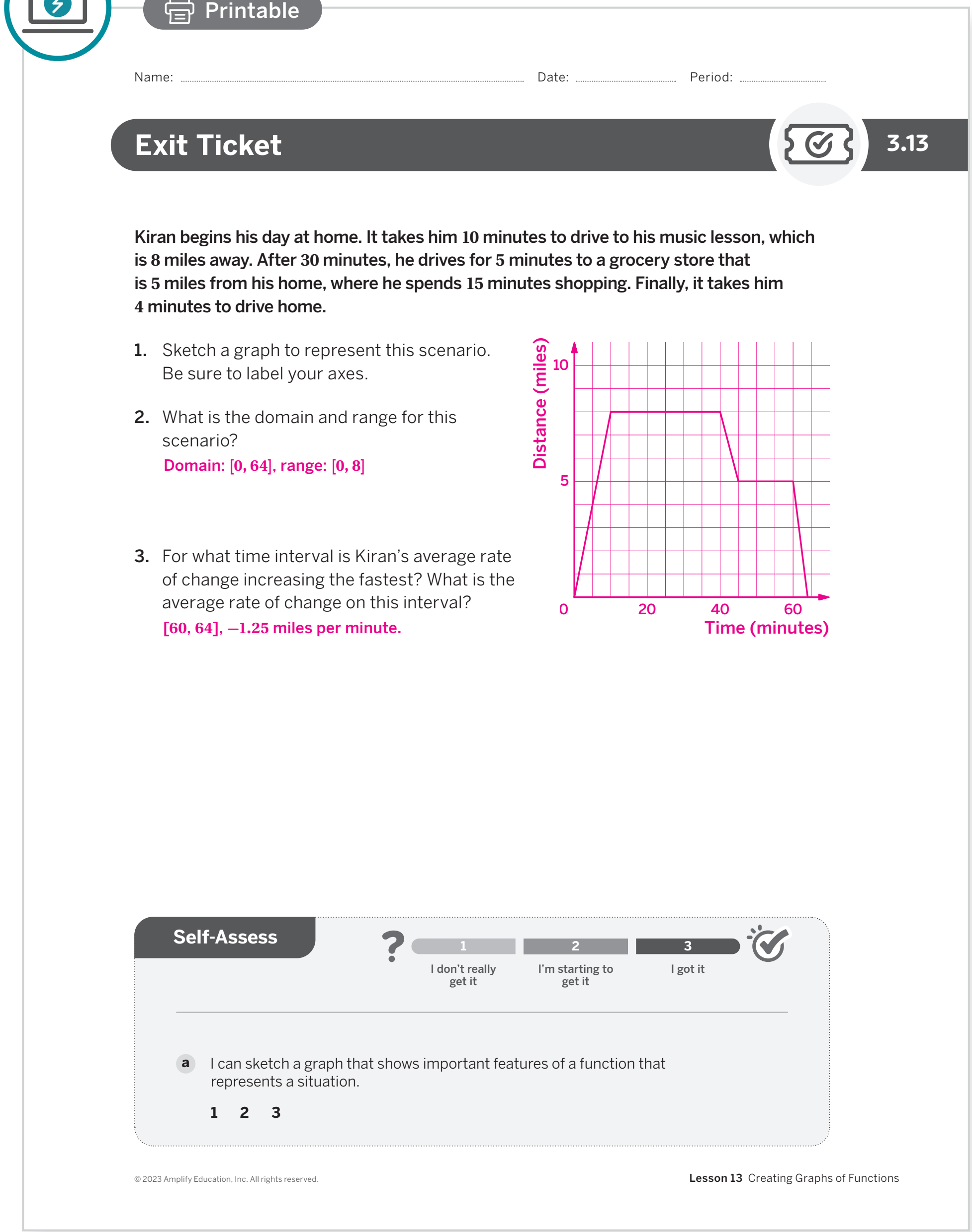
- "Why is paying attention to small details in a verbal description important when creating a graph?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by sketching a graph to model a real-world scenario and attending to precision as they represent the domain and range.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket 3.13

Kiran begins his day at home. It takes him 10 minutes to drive to his music lesson, which is 8 miles away. After 30 minutes, he drives for 5 minutes to a grocery store that is 5 miles from his home, where he spends 15 minutes shopping. Finally, it takes him 4 minutes to drive home.

1. Sketch a graph to represent this scenario. Be sure to label your axes.
2. What is the domain and range for this scenario?
   Domain: [0, 64], range: [0, 8]
3. For what time interval is Kiran's average rate of change increasing the fastest? What is the average rate of change on this interval?
   [60, 64], –1.25 miles per minute.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can sketch a graph that shows important features of a function that represents a situation.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Success looks like . . .

- **Language Goal:** Sketching a graph that shows important features of a function that represents a situation. **(Reading and Writing)**
  - » Sketching the graph to represent Kiran's travels in Problem 1.

Suggested next steps

If students create an inaccurate sketch in Problem 1, consider:

- Reviewing sketching graphs from Activity 1.
- Assigning Practice Problem 1.
- Asking, "What words or phrases tell you where Kiran is in relation to his home?"

If students determine the incorrect domain and range in Problem 2, consider:

- Reviewing identifying key features from Activity 1.
- Assigning Practice Problem 2.
- Asking, "How can intercepts, maximum, and minimum help determine domain and range?"

If students determine the incorrect interval and average rate of change in Problem 3, consider:

- Reviewing how to find the average rate of change from Activity 1.
- Assigning Practice Problem 2.
- Asking, "What would a line segment look like when Kiran is moving the fastest? The slowest?"

Professional Learning

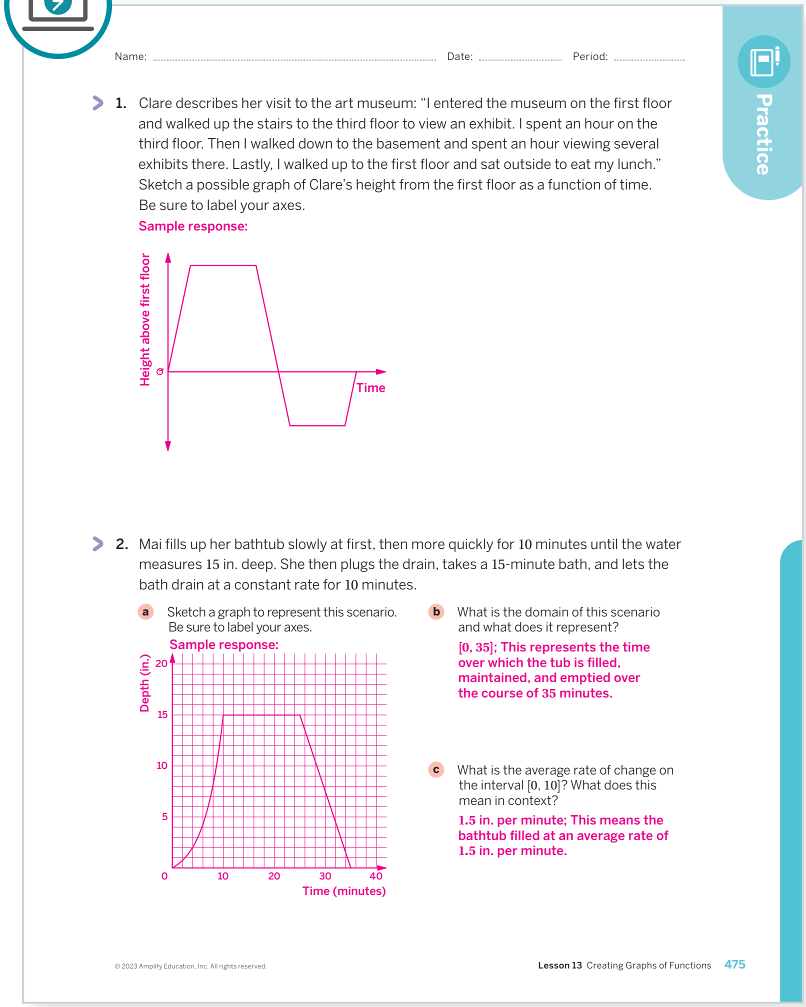
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students worked on creating the New Orleans skyline?
- In earlier lessons, students interpreted graphs of functions. How did that support creating graphs of functions? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Clare describes her visit to the art museum: "I entered the museum on the first floor and walked up the stairs to the third floor to view an exhibit. I spent an hour on the third floor. Then I walked down to the basement and spent an hour viewing several exhibits there. Lastly, I walked up to the first floor and sat outside to eat my lunch." Sketch a possible graph of Clare's height from the first floor as a function of time. Be sure to label your axes.

   Sample response:

2. Mai fills up her bathtub slowly at first, then more quickly for 10 minutes until the water measures 15 in. deep. She then plugs the drain, takes a 15-minute bath, and lets the bath drain at a constant rate for 10 minutes.

   a. Sketch a graph to represent this scenario. Be sure to label your axes.

   Sample response:

   b. What is the domain of this scenario and what does it represent?

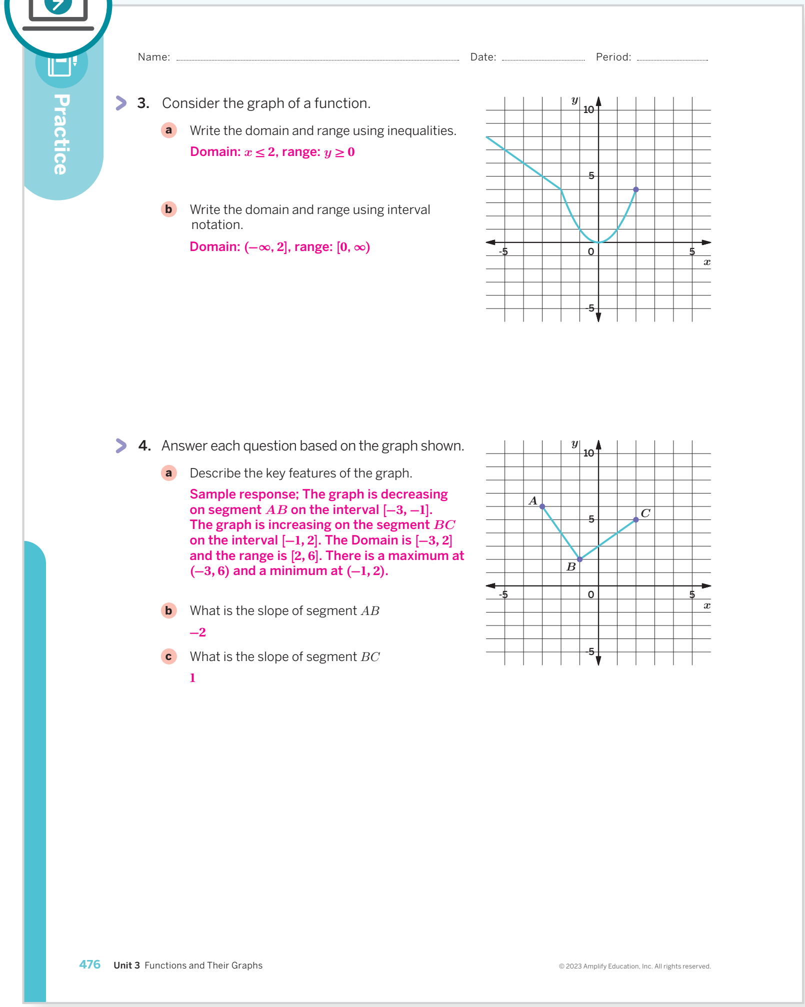
   $[0, 35]$; This represents the time over which the tub is filled, maintained, and emptied over the course of 35 minutes.

   c. What is the average rate of change on the interval $[0, 10]$? What does this mean in context?

   1.5 in. per minute; This means the bathtub filled at an average rate of 1.5 in. per minute.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 13 Creating Graphs of Functions 475



Name: Date: Period:

3. Consider the graph of a function.

   a. Write the domain and range using inequalities.

   Domain: $x \leq 2$, range: $y \geq 0$

   b. Write the domain and range using interval notation.

   Domain: $(-\infty, 2]$, range: $[0, \infty)$

4. Answer each question based on the graph shown.

   a. Describe the key features of the graph.

   Sample response; The graph is decreasing on segment AB on the interval $[-3, -1]$. The graph is increasing on the segment BC on the interval $[-1, 2]$. The Domain is $[-3, 2]$ and the range is $[2, 6]$. There is a maximum at $(-3, 6)$ and a minimum at $(-1, 2)$.

   b. What is the slope of segment AB

   -2

   c. What is the slope of segment BC

   1

476 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	1
	2	Activity 1	2
Spiral	3	Unit 3 Lesson 11	2
Formative ⚡	4	Unit 3 Lesson 14	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Mathematical Modeling Prompt

At any point during the course and after completing this lesson, have students work in pairs or small groups on this unit's Mathematical Modeling Prompt, *Own a Car or Not?*, which is available in the **Algebra 1 Additional Practice**.

Sub-Unit 3

Piecewise Functions

Whole Class

In this Sub-Unit, students create, graph, interpret, and analyze piecewise and absolute value functions, and relate them to the music of Atlanta.

SUB-UNIT

3 Piecewise Functions

Narrative Connections

Where did the world meet soul?

Ray Charles sat down with the executives of Atlantic Records at The Royal Peacock club. Charles had been on the road, opening for other artists. For the last three years, the 24-year-old musician had spent his career imitating the jazz style of singers like Nat "King" Cole — but now he wanted something different. Solemnly, Charles took the stage, and launched into a new song, "I Got a Woman." It was a fiery, gospel-inflected blues/jazz fusion whose driving rhythms were something many executives had never heard before.

And so, on a November day in 1954, in a club in the Sweet Auburn district of Atlanta, Ray Charles introduced the world to soul music. The Sweet Auburn neighborhood of Atlanta is full of stories of Black cultural excellence. Miles Davis, B.B. King, Nina Simone, Sam Cooke, and Gladys Knight are just a handful of the artists that have played in Sweet Auburn.

The neighborhood was formed in the early 1900s, when many Black-owned businesses relocated from Atlanta's downtown area to Auburn Avenue. The area became home to the Ebenezer Baptist Church, where Martin Luther King was pastor, as well as one of the earliest and most influential Black-owned newspapers, the Atlanta Daily World. Over time, the neighborhood, and the institutions within it, transformed Atlanta into a hub for culture and civil rights.

The next time you listen to "I Got a Woman," pay special attention to the saxophone solo in the middle. If you graphed the solo, what would it look like? Whether you're plotting the notes or the rhythm, your graph will probably have a lot in common with the piecewise functions you'll encounter in the next few lessons.

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore piecewise and absolute value functions — within the context of music and the city of Atlanta — in the following places:

- **Lesson 14, Activities 1–2:** Digital Music Subscription, The Cost of Connectivity
- **Lesson 15, Activities 1–2:** The Pieces of Sound, Creating a Melody
- **Lesson 17, Activity 3:** The Atlanta Skyline Continued

UNIT 3 | **LESSON 14**

Piecewise Functions
(Part 1)

Let's look at functions that are defined in pieces.

Focus

Goals

1. **Language Goal:** Interpret a graph of a piecewise function or the rules given in function notation, and explain the rules in terms of a situation. **(Speaking and Listening, Reading and Writing)**
2. Sketch a graph that represents the rules of a piecewise function, paying special attention to the endpoints of each interval.
3. Understand a piecewise function as a function defined by different rules for different intervals of the domain.

Coherence

Today

Students are introduced to a piecewise-defined function by exploring different real-world contexts in which multiple rules apply. They interpret and evaluate the function using its rules, graph the function, and interpret values within the given contexts. Specifically, they pay close attention to the boundary points, where one rule ends and another begins, and interpret the notation of piecewise functions.

Previously

In Lesson 13, students sketched and interpreted graphs of functions representing situations using the key features of each function.

Coming Soon

In Lesson 15, students will graph and interpret piecewise functions in the context of a sound's pitch and to design part of Atlanta's skyline.

Rigor

- Students build **conceptual understanding** of piecewise functions as pieces defined by rules.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps*

Math Language Development

New words

- ***piecewise function***
- ***step function***

Review words

- *domain*
- *function*
- *range*

Amps | Featured Activity

Activity 1
Graphing the Cost of Music

Students graph a piecewise function to model the cost of a digital music subscription. They then check symbolic representations of the piecewise function against their graph to determine the accuracy of these functions.

Building Math Identity and Community

Connecting to Mathematical Practices

The boundaries of a piecewise function need to be emphasized because that is where the rules of the function begin to change. Apply this to the structure of decision making for students. Explain that in order to make good decisions, one must have a good sense of boundaries. Students must consider how the boundary changes are portrayed in the graph. Similarly, they must determine how to handle those boundary edges in their own lives.

Modifications to Pacing

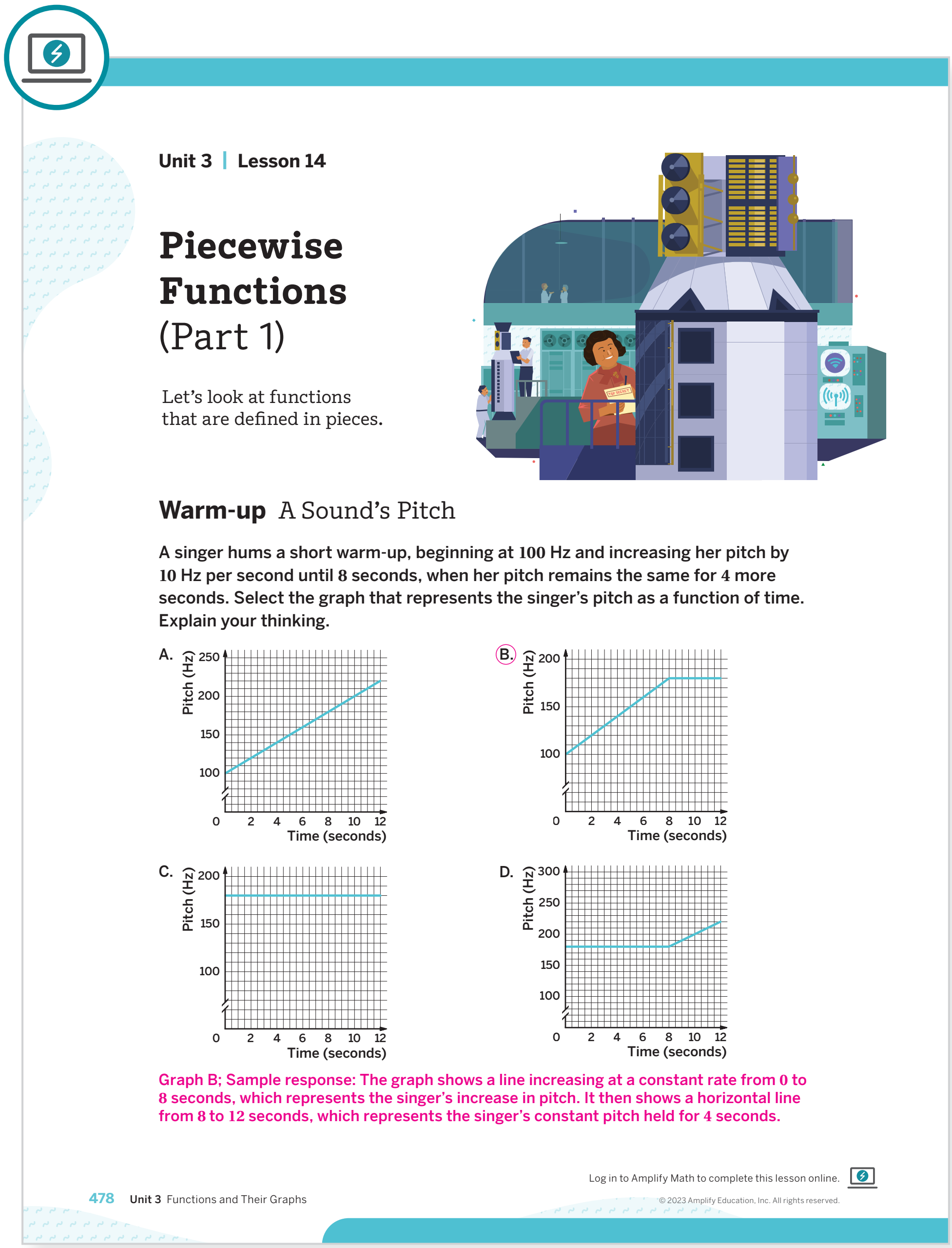
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Graphs A or B may be omitted from the choices.
- In **Activity 1**, have half the class examine Tyler's function, half the class examine Mai's function, then compare.

Pairs | 10 min

Warm-up A Sound's Pitch

Students identify a graph that represents a sound's pitch by noticing that a graph of such a function has different features for different parts of the domain.



Unit 3 | Lesson 14

Piecewise Functions (Part 1)

Let's look at functions that are defined in pieces.

Warm-up A Sound's Pitch

A singer hums a short warm-up, beginning at 100 Hz and increasing her pitch by 10 Hz per second until 8 seconds, when her pitch remains the same for 4 more seconds. Select the graph that represents the singer's pitch as a function of time. Explain your thinking.

Graph B; Sample response: The graph shows a line increasing at a constant rate from 0 to 8 seconds, which represents the singer's increase in pitch. It then shows a horizontal line from 8 to 12 seconds, which represents the singer's constant pitch held for 4 seconds.

Log in to Amplify Math to complete this lesson online.

478 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in pairs. Read the narrative together as a class. Then have students describe the pitch of the sound represented by each graph before selecting a graph.

2 Monitor

Help students get started by having them identify the starting pitch and increasing segments on each graph.

Look for points of confusion:

- **Using only one segment of the pitch's description to identify a graph.** Have students describe the different segments of the pitch and determine which graph reflects these pieces.

Look for productive strategies:

- Annotating the initial pitch and the rate of change of each piece on each graph.

3 Connect

Have pairs of students share the graph they selected and their reasoning.

Ask, "When does the graph of the pitch change? How is this graph reflected in the description?" The pitch changes at 8 seconds when it goes from increasing to constant.

Define a ***piecewise function*** as a function in which different rules are applied to different input values to determine the output values.

Highlight that the relationship between the pitch and time is an example of a piecewise function. In this case, the two pieces of the graph correspond to the two rules.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share which graph they selected and their reasoning, draw their attention to the similarities and differences among the four graphs. Ask:

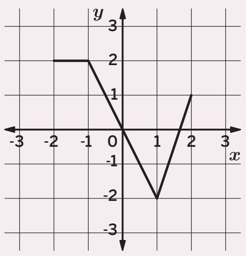
- "How are all of the graphs similar?" Sample response: They all are composed of straight line segments.
- "How are Graphs B and D different?" Both are composed of more than one line segment.

As you define the term *piecewise function*, annotate Graphs B and D with the term *piecewise* and say, "Each graph is composed of different *pieces*."

Power-up

To power up students' ability to identify and describe key features of a graph, have students complete:

Determine the interval on which the graph is:



a. Increasing
[1, 2]

b. Decreasing
[−1, 1]

c. Constant
[−2, −1]

Use: Before Activity 1

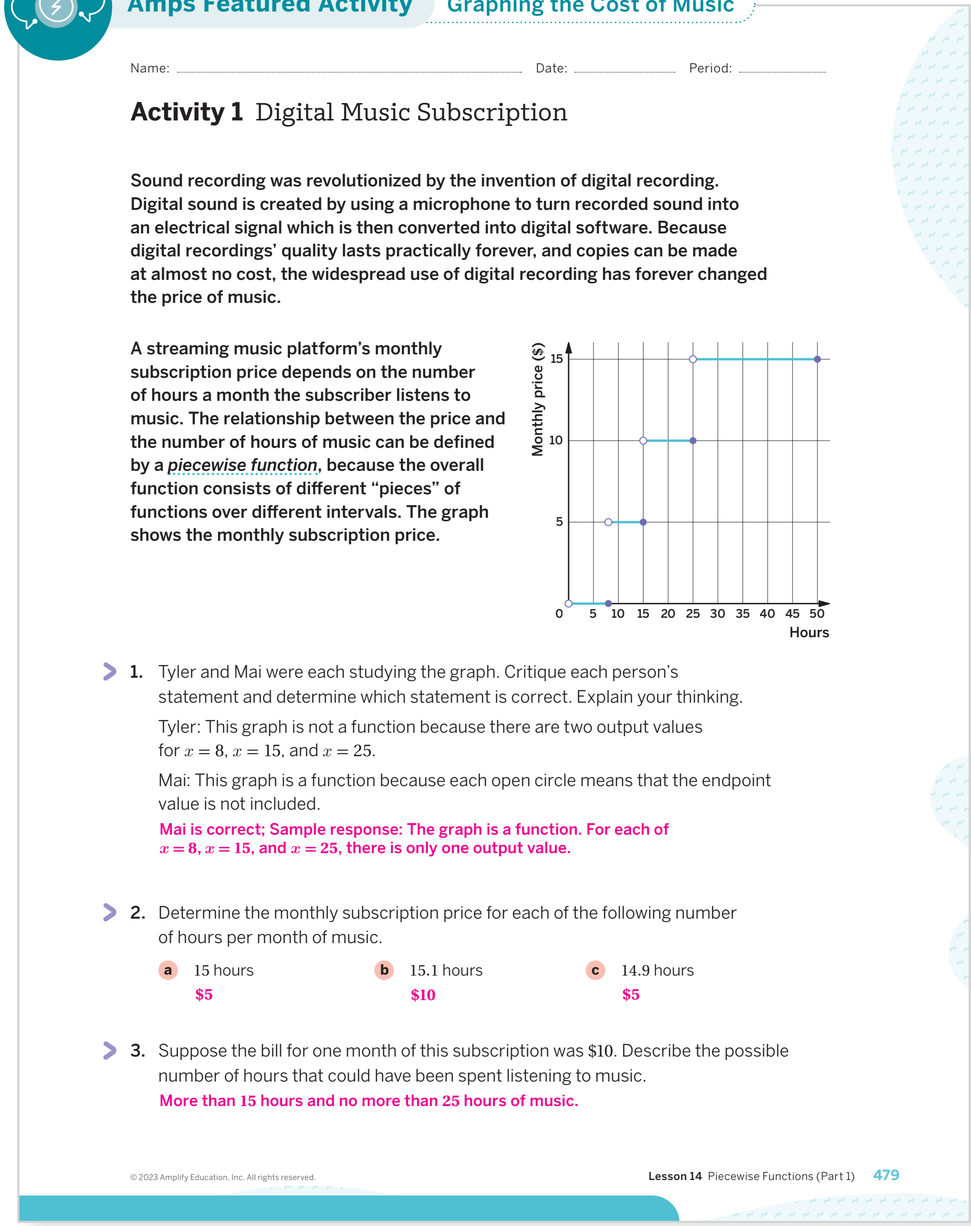
Informed by: Performance on Lesson 13, Practice Problem 4

Pairs | 15 min

Activity 1 Digital Music Subscription

Students critique the reasoning of others by attending to precision about boundary values and key features as they analyze the graph of a piecewise-defined function.

Amps Featured Activity Graphing the Cost of Music



Name: ______ Date: ______ Period: ______

Activity 1 Digital Music Subscription

Sound recording was revolutionized by the invention of digital recording. Digital sound is created by using a microphone to turn recorded sound into an electrical signal which is then converted into digital software. Because digital recordings' quality lasts practically forever, and copies can be made at almost no cost, the widespread use of digital recording has forever changed the price of music.

A streaming music platform's monthly subscription price depends on the number of hours a month the subscriber listens to music. The relationship between the price and the number of hours of music can be defined by a *piecewise function*, because the overall function consists of different "pieces" of functions over different intervals. The graph shows the monthly subscription price.

1. Tyler and Mai were each studying the graph. Critique each person's statement and determine which statement is correct. Explain your thinking.

   Tyler: This graph is not a function because there are two output values for $x = 8$, $x = 15$, and $x = 25$.

   Mai: This graph is a function because each open circle means that the endpoint value is not included.

   Mai is correct; Sample response: The graph is a function. For each of $x = 8$, $x = 15$, and $x = 25$, there is only one output value.

2. Determine the monthly subscription price for each of the following number of hours per month of music.

a 15 hours	b 15.1 hours	c 14.9 hours
$5	**$10**	**$5**

3. Suppose the bill for one month of this subscription was $10. Describe the possible number of hours that could have been spent listening to music.

   More than 15 hours and no more than 25 hours of music.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Students remain in pairs. Display the graph in the activity and have students share what they notice and wonder. Give students a minute of think-time for the first two problems and then time to discuss their responses with their partner.

2 Monitor

Help students get started by having them plot the points on the graph for each number of hours given in Problem 1.

Look for points of confusion:

- **Using the coordinates of an open circle to define the function.** Ask, "How did you use open and closed circles on a number line when graphing an inequality in one variable?"
- **Misusing the open and closed circle to identify the errors in the given piecewise functions.** Ask, "How are the inequalities $x \geq 0$ and $x > 0$ represented differently on a number line? How can you use the open and closed circle to determine the inequalities used in the piecewise function?"

Look for productive strategies:

- Using the graph to write the piecewise function that models the scenario.
- Annotating the segments of the graph with domain intervals and function values.
- Determining the domain and range of the entire function and of each piece.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can digitally graph a piecewise function to model the cost of the music subscription.

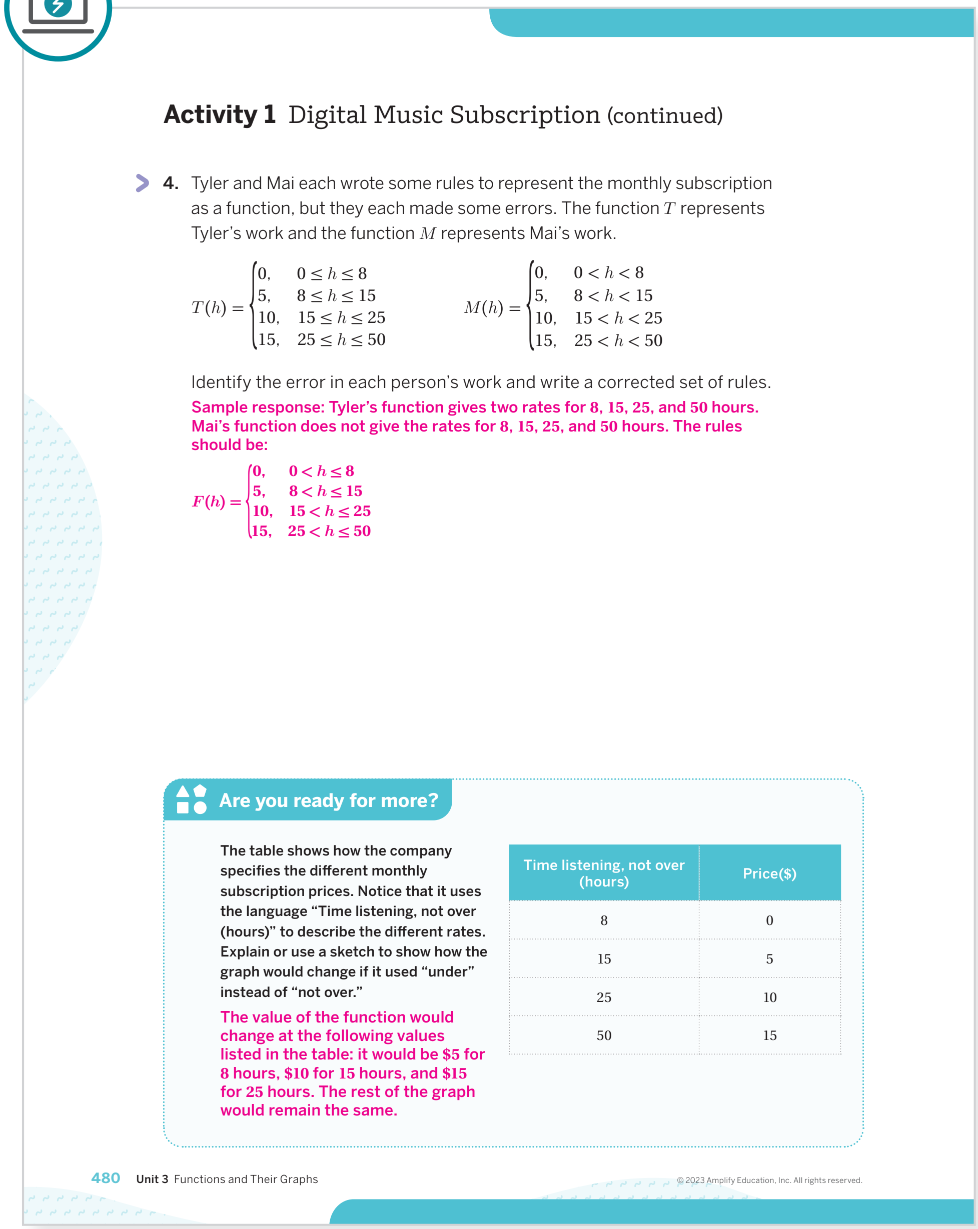
Accessibility: *Guide Processing and Visualization*

As students complete Problem 4, provide access to colored pencils and have students color code each piece of the graph with the corrected rule they wrote that represents that piece. Suggest they annotate each piece of the graph with the rule that represents it to help them connect the different representations.

Pairs | 15 min

Activity 1 Digital Music Subscription (continued)

Students critique the reasoning of others by attending to precision about boundary values and key features as they analyze the graph of a piecewise-defined function.



Activity 1 Digital Music Subscription (continued)

4. Tyler and Mai each wrote some rules to represent the monthly subscription as a function, but they each made some errors. The function T represents Tyler's work and the function M represents Mai's work.

$$T(h) = \begin{cases} 0, & 0 \le h \le 8 \\ 5, & 8 \le h \le 15 \\ 10, & 15 \le h \le 25 \\ 15, & 25 \le h \le 50 \end{cases} \qquad M(h) = \begin{cases} 0, & 0 < h < 8 \\ 5, & 8 < h < 15 \\ 10, & 15 < h < 25 \\ 15, & 25 < h < 50 \end{cases}$$

Identify the error in each person's work and write a corrected set of rules.

Sample response: Tyler's function gives two rates for 8, 15, 25, and 50 hours. Mai's function does not give the rates for 8, 15, 25, and 50 hours. The rules should be:

$$F(h) = \begin{cases} 0, & 0 < h \le 8 \\ 5, & 8 < h \le 15 \\ 10, & 15 < h \le 25 \\ 15, & 25 < h \le 50 \end{cases}$$

Are you ready for more?

The table shows how the company specifies the different monthly subscription prices. Notice that it uses the language "Time listening, not over (hours)" to describe the different rates. Explain or use a sketch to show how the graph would change if it used "under" instead of "not over."

Time listening, not over (hours)	Price($)
8	0
15	5
25	10
50	15

The value of the function would change at the following values listed in the table: it would be $5 for 8 hours, $10 for 15 hours, and $15 for 25 hours. The rest of the graph would remain the same.

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the graph of the function.

Have individual students share their analysis of Tyler's and Mai's work.

Highlight that both Tyler and Mai made an error in the inequality symbols they used. The function can only be defined by one piece over the domain, and should reflect the entire domain of the graph.

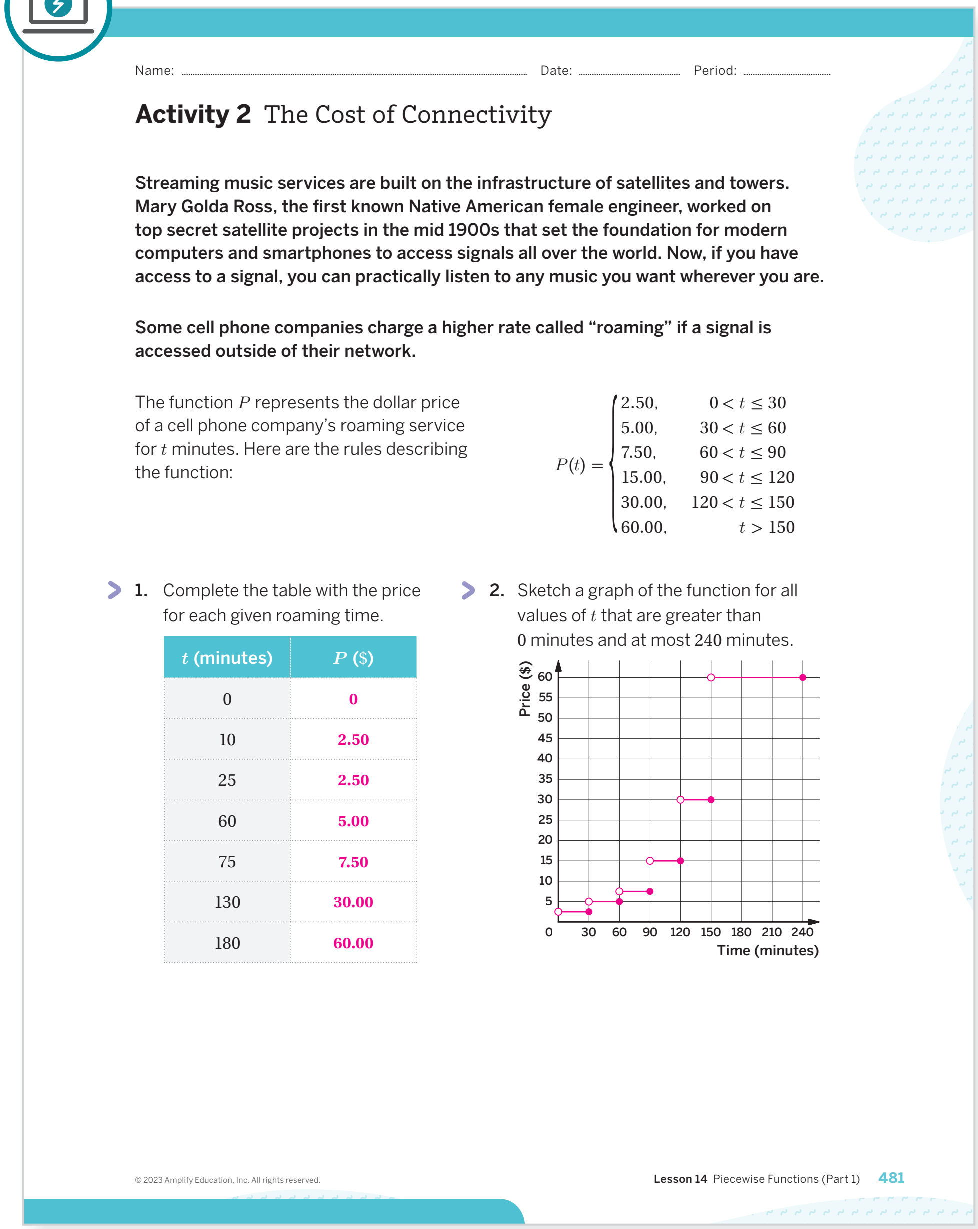
Ask:

- "How can you determine whether to use $<$ or $\le$ by looking at the graph?" A point with an open circle means the endpoint value is not included, but values that are greater than or less than, depending on the graph, are included. So, an open circle corresponds to $<$ or $>$. A point with a solid or closed circle means the endpoint value is included, so it corresponds to $\le$ or $\ge$.
- "By looking at the graph, how can you determine how many rules should be included in the function?" Each piece of the graph should be represented by a rule. Because there are four pieces of the graph, there should be four rules.

Independent | 15 min

Activity 2 The Cost of Connectivity

Students create and connect multiple representations of a piecewise function and interpret the function within the given context and attend to precision as they represent the domain and range.



Name: ______ Date: ______ Period: ______

Activity 2 The Cost of Connectivity

Streaming music services are built on the infrastructure of satellites and towers. Mary Golda Ross, the first known Native American female engineer, worked on top secret satellite projects in the mid 1900s that set the foundation for modern computers and smartphones to access signals all over the world. Now, if you have access to a signal, you can practically listen to any music you want wherever you are.

Some cell phone companies charge a higher rate called "roaming" if a signal is accessed outside of their network.

The function P represents the dollar price of a cell phone company's roaming service for t minutes. Here are the rules describing the function:

$$P(t) = \begin{cases} 2.50, & 0 < t \leq 30 \\ 5.00, & 30 < t \leq 60 \\ 7.50, & 60 < t \leq 90 \\ 15.00, & 90 < t \leq 120 \\ 30.00, & 120 < t \leq 150 \\ 60.00, & t > 150 \end{cases}$$

1. Complete the table with the price for each given roaming time.

t (minutes)	P ($)
0	0
10	2.50
25	2.50
60	5.00
75	7.50
130	30.00
180	60.00

2. Sketch a graph of the function for all values of t that are greater than 0 minutes and at most 240 minutes.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 14 Piecewise Functions (Part 1) 481

1 Launch

Display the function P. Draw students' attention to the two parts (separated by a comma) in each rule or each case. Highlight that the first part of each rule represents an output value and the second part specifies a set of input values.

2 Monitor

Help students get started by having them first plot the endpoints of each piece.

Look for points of confusion:

- **Interchanging open and closed circles in their graph.** Have students review how the inequality symbols are reflected in the graph in Activity 1.
- **Specifying two output values for the same input value in their description.** Remind students that for a function, every input value can only have one output value. Have students review their description in Problem 3. Ask, "Does it make sense for equal amounts of time spent roaming to have more than one price?"

Look for productive strategies:

- Sketching each line segment first, and then going back through their graph to determine if the endpoints are open or closed circles.
- Checking that their table of values and graph reflect one another.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization, Vary Demands to Optimize Challenge*

Ask students to complete the table in Problem 1. Then demonstrate how to graph the first two line segments in Problem 2, using open and closed circles. Ask students to complete the graph.

Extension: *Math Enrichment*

Tell students that one of the most famous step functions is called the Greatest Integer Function. This function takes x as its input and gives "the greatest integer less than or equal to x" as its output. Have students generate input and output values according to the Greatest Integer Function and draw a sketch of its graph.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students write a draft response to Problem 3, have them meet with 2–3 partners to give and receive feedback. Encourage reviewers to ask clarifying questions such as:

- "Does the response include how the price changes over time?"
- "Does the response include specific quantities or relationships and are those described accurately?"

Have students use the feedback to improve their responses.

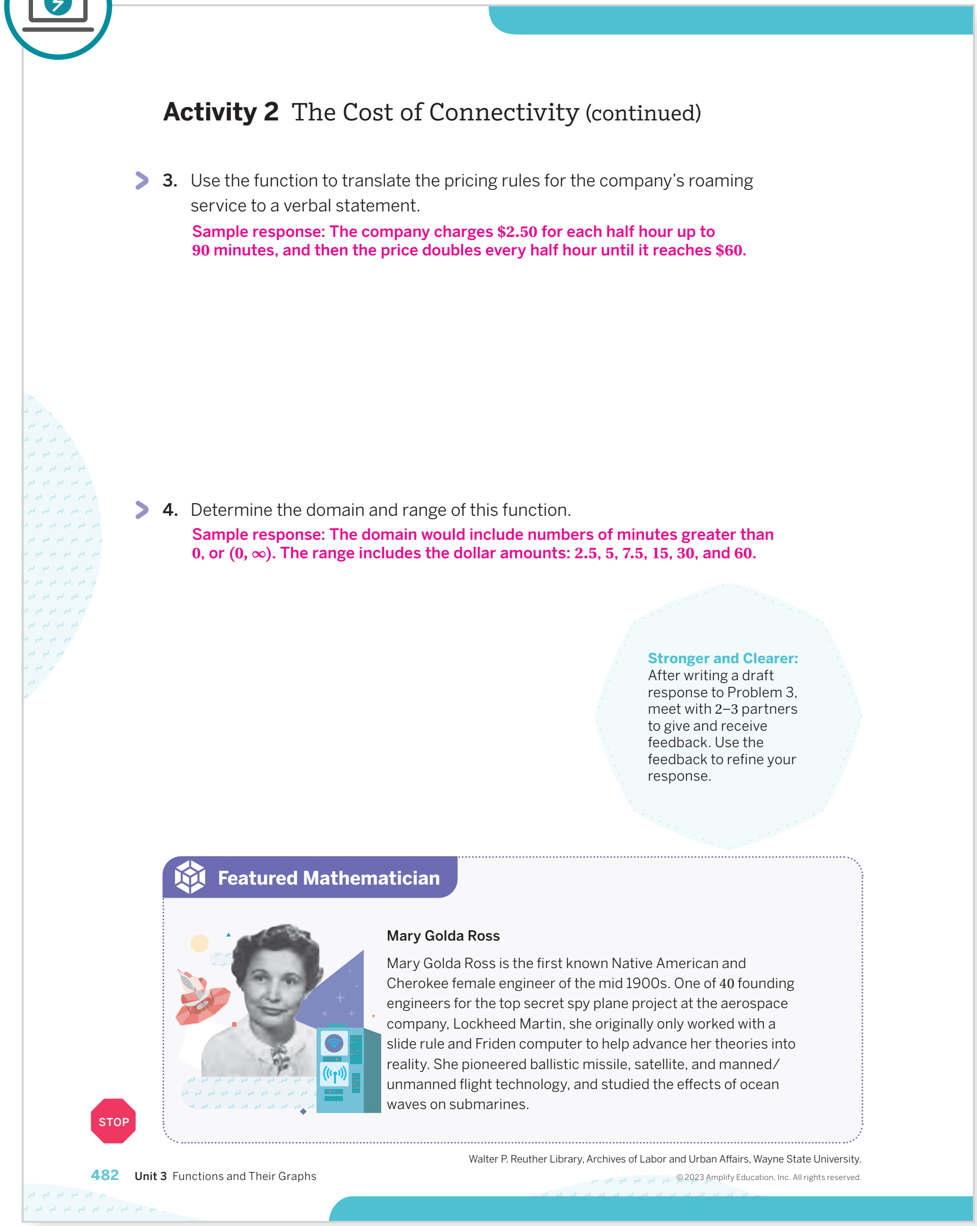
English Learners

Consider pairing students with different levels of English language proficiencies, so that students at various proficiency levels can interact together and hear a variety of perspectives.

Independent | 15 min

Activity 2 The Cost of Connectivity (continued)

Students create and connect multiple representations of a piecewise function and interpret the function within the given context and attend to precision as they represent the domain and range.



Activity 2 The Cost of Connectivity (continued)

3. Use the function to translate the pricing rules for the company's roaming service to a verbal statement.
   Sample response: The company charges \$2.50 for each half hour up to 90 minutes, and then the price doubles every half hour until it reaches \$60.

4. Determine the domain and range of this function.
   Sample response: The domain would include numbers of minutes greater than 0, or $(0, \infty)$. The range includes the dollar amounts: 2.5, 5, 7.5, 15, 30, and 60.

Stronger and Clearer: After writing a draft response to Problem 3, meet with 2–3 partners to give and receive feedback. Use the feedback to refine your response.

Featured Mathematician

Mary Golda Ross

Mary Golda Ross is the first known Native American and Cherokee female engineer of the mid 1900s. One of 40 founding engineers for the top secret spy plane project at the aerospace company, Lockheed Martin, she originally only worked with a slide rule and Friden computer to help advance her theories into reality. She pioneered ballistic missile, satellite, and manned/unmanned flight technology, and studied the effects of ocean waves on submarines.

STOP

Walter P. Reuther Library, Archives of Labor and Urban Affairs, Wayne State University.

482 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the graph and equation of the function.

Have individual students share their descriptions of the pricing rules for the company's roaming service.

Define a ***step function*** as a piecewise function in which the pieces represent constant values.

Highlight that the domain contains all values greater than 0. The range is defined by the constant values of each piece, so the range does not contain all values from 2.50 to 60. Piecewise functions may have connecting pieces with no breaks in the graph, or there may be breaks in the domain or range.

Ask:

- "Is P a function of t? How do you know?" Yes; The price is determined by the amount of time spent roaming. Every amount of time spent roaming of identical length will cost the same amount.
- "Is t a function of P? How do you know?" No; Knowing how much someone paid does not indicate the amount of time they spent roaming.

Featured Mathematician

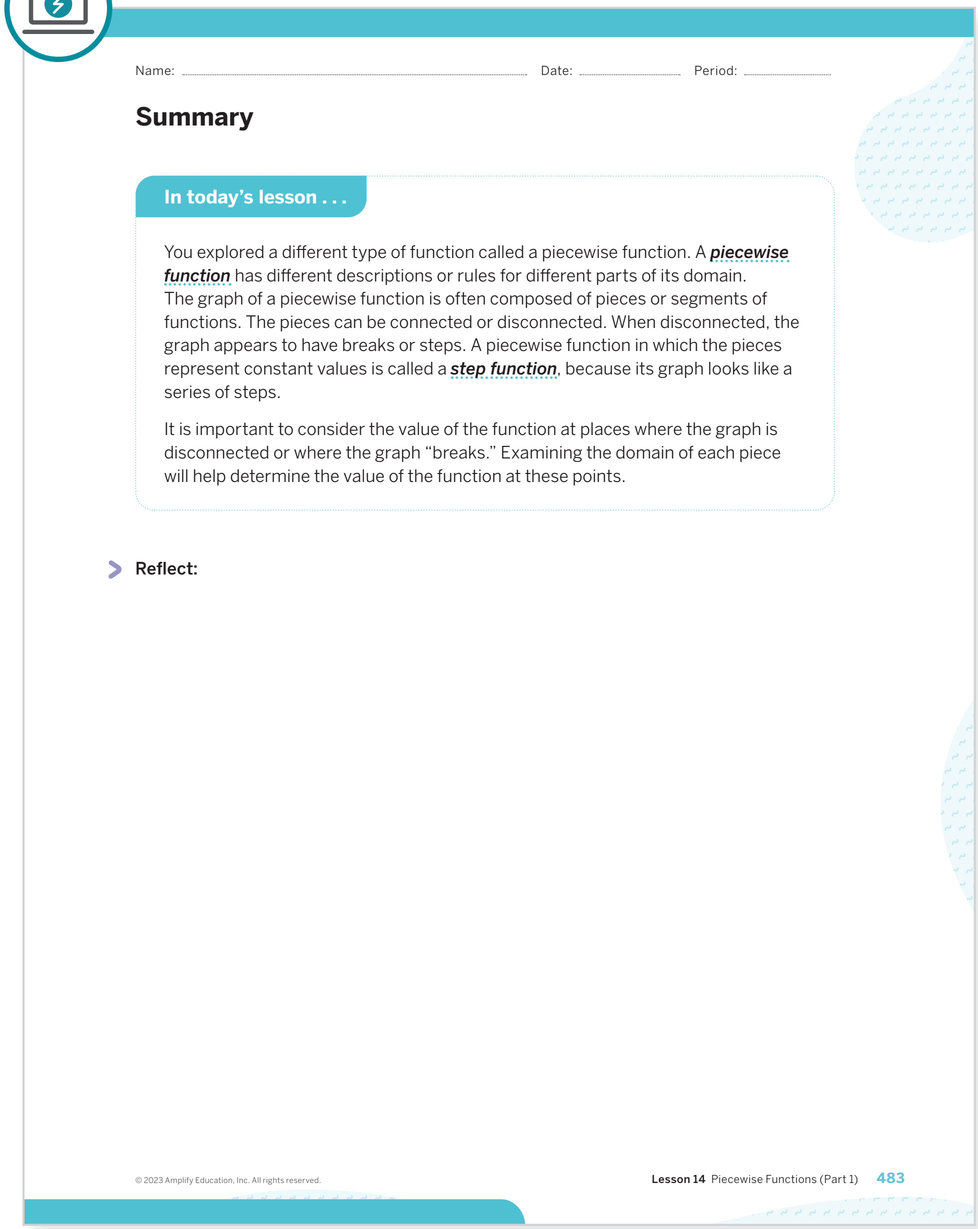
Mary Golda Ross

Have students read about featured mathematician Mary Golda Ross, who pioneered ballistic missile, satellite, and manned/unmanned flight technology and studied the effects of ocean waves on submarines.

Whole Class | 5 min

Summary

Review and synthesize how a piecewise function is composed of different "pieces" of functions and what it means when the pieces are connected or disconnected.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored a different type of function called a piecewise function. A ***piecewise function*** has different descriptions or rules for different parts of its domain. The graph of a piecewise function is often composed of pieces or segments of functions. The pieces can be connected or disconnected. When disconnected, the graph appears to have breaks or steps. A piecewise function in which the pieces represent constant values is called a ***step function***, because its graph looks like a series of steps.

It is important to consider the value of the function at places where the graph is disconnected or where the graph "breaks." Examining the domain of each piece will help determine the value of the function at these points.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 14 Piecewise Functions (Part 1) 483

Synthesize

Display the piecewise function and graph from Activity 2.

Have students share their strategies for graphing a piecewise function.

Highlight that piecewise functions consist of different rules that exist over a specified domain of values. The pieces may or may not intersect.

Formalize vocabulary:

- *piecewise function*
- *step function*

Ask:

- "How can you determine where each piece starts and ends?" I can use the rules given to determine the domain of each piece.
- "What does an open circle indicate? A closed circle?" An open circle indicates that the piece of the function is not defined for the given input value, while a closed circle indicates that the piece of the function is defined for the given input value.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Why are these functions called piecewise functions?"
- "What are the characteristics of a step function?"

Math Language Development

MLR2: Collect and Display

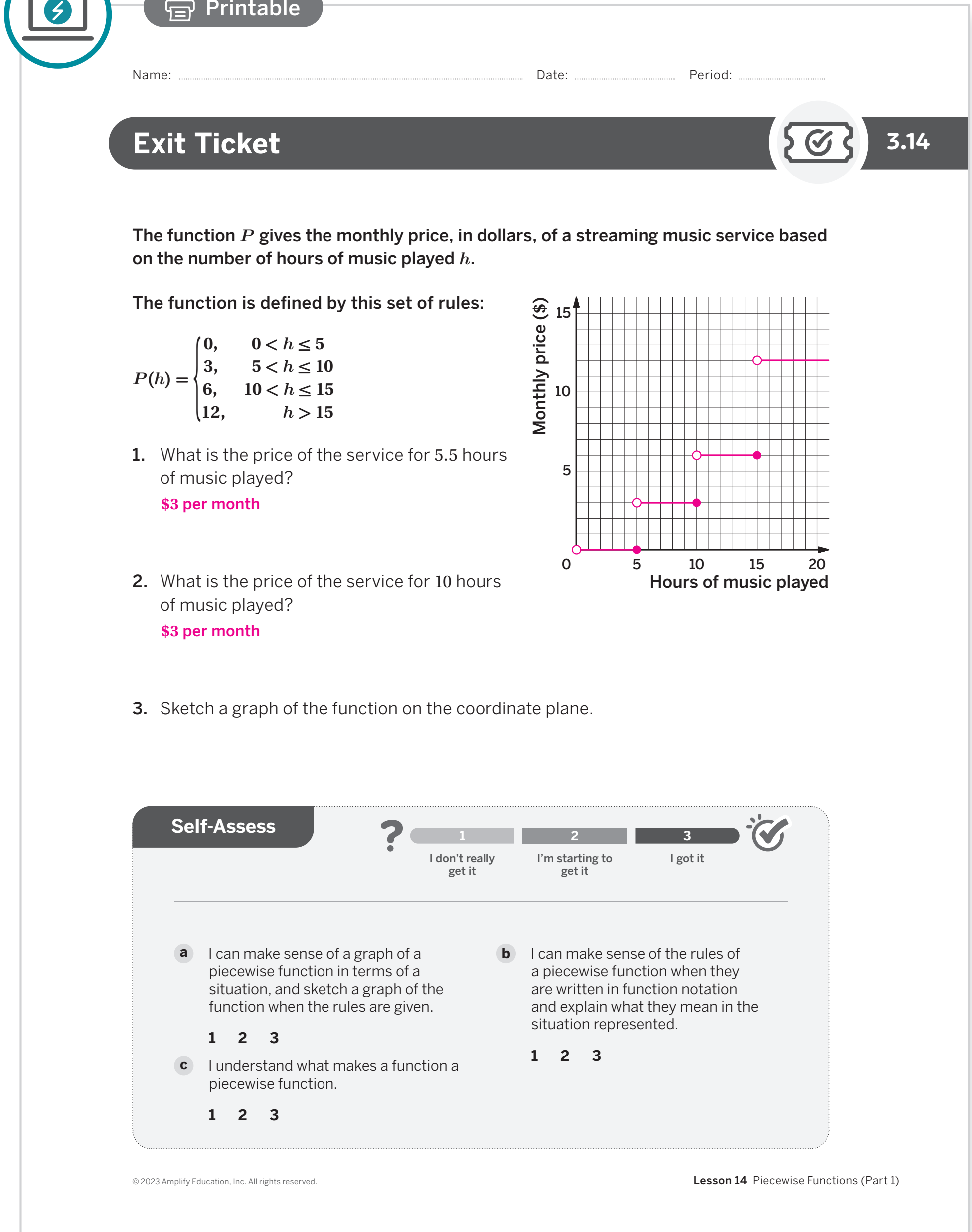
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the terms *piecewise function* and *step function* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by evaluating and graphing a piecewise function.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.14

The function P gives the monthly price, in dollars, of a streaming music service based on the number of hours of music played h.

The function is defined by this set of rules:

$$P(h) = \begin{cases} 0, & 0 < h \leq 5 \\ 3, & 5 < h \leq 10 \\ 6, & 10 < h \leq 15 \\ 12, & h > 15 \end{cases}$$

1. What is the price of the service for 5.5 hours of music played?

   $3 per month

2. What is the price of the service for 10 hours of music played?

   $3 per month

3. Sketch a graph of the function on the coordinate plane.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can make sense of a graph of a piecewise function in terms of a situation, and sketch a graph of the function when the rules are given.

1 2 3

b I can make sense of the rules of a piecewise function when they are written in function notation and explain what they mean in the situation represented.

1 2 3

c I understand what makes a function a piecewise function.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 14 Piecewise Functions (Part 1)

Success looks like . . .

- **Language Goal:** Interpreting a graph of a piecewise function or the rules given in function notation, and explaining the rules in terms of a situation. **(Speaking and Listening, Reading and Writing)**
- **Goal:** Sketching a graph that represents the rules of a piecewise function, paying special attention to the endpoints of each interval.
  - » Sketching the graph of the function P in Problem 3.
- **Goal:** Understanding a piecewise function as a function defined by different rules for different intervals of the domain.
  - » Evaluating the function P for a given number of hours on the correct interval of the domain in Problems 1 and 2.

Suggested next steps

If students inaccurately evaluate the function in Problems 1 and 2, consider:

- Reviewing using the rules of piecewise functions to evaluate the function in Activity 2.
- Assigning Practice Problem 1.
- Asking, "Which piece represents the price for the given number of hours? How can you use the inequality symbol to determine which piece to use?"

If students inaccurately graph the piecewise function for Problem 3, consider:

- Reviewing graphing a piecewise function in Activity 2.
- Assigning Practice Problems 1 and 3.
- Asking, "How can you determine where the endpoints are located and whether they are open or closed?"

Professional Learning

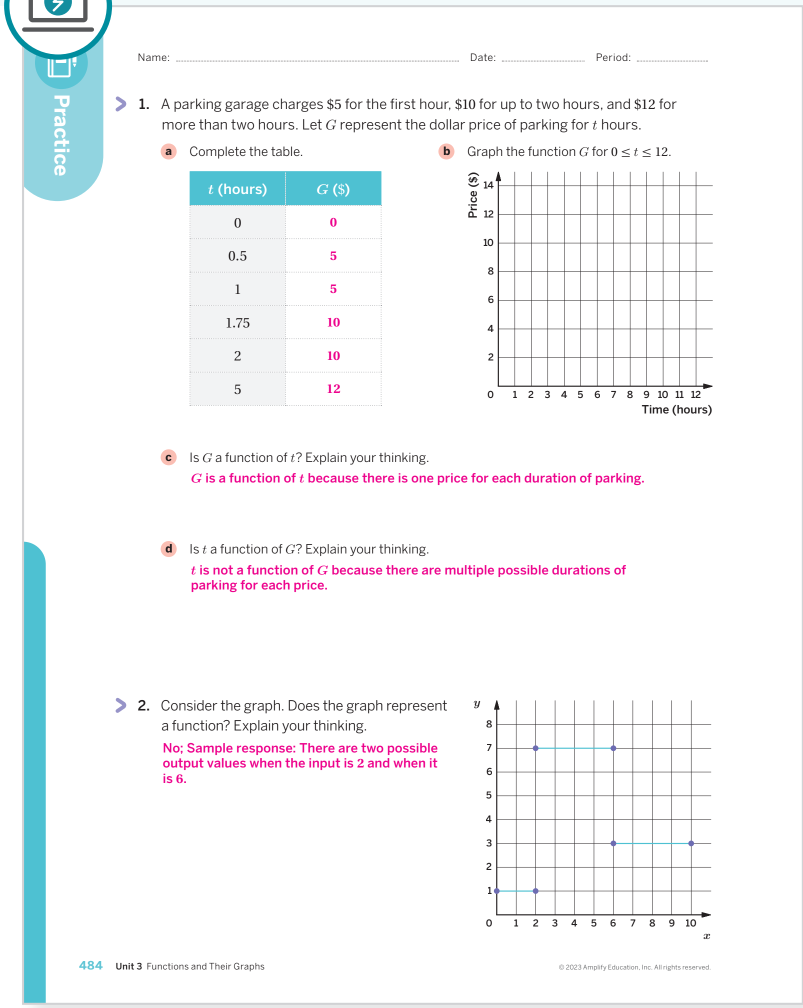
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- In Activity 2, you used intentional grouping with MLR1 to pair students with various levels of English language proficiencies. What effect did this grouping strategy have on students' conversations and revisions? Would you change anything the next time you use MLR1?
- In this lesson, students graphed linear piecewise functions. How will that support graphing the absolute value function? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. A parking garage charges $5 for the first hour, $10 for up to two hours, and $12 for more than two hours. Let G represent the dollar price of parking for t hours.

   a Complete the table.

t (hours)	G ($)
0	**0**
0.5	**5**
1	**5**
1.75	**10**
2	**10**
5	**12**

   b Graph the function G for $0 \le t \le 12$.

   c Is G a function of t? Explain your thinking.

   G is a function of t because there is one price for each duration of parking.

   d Is t a function of G? Explain your thinking.

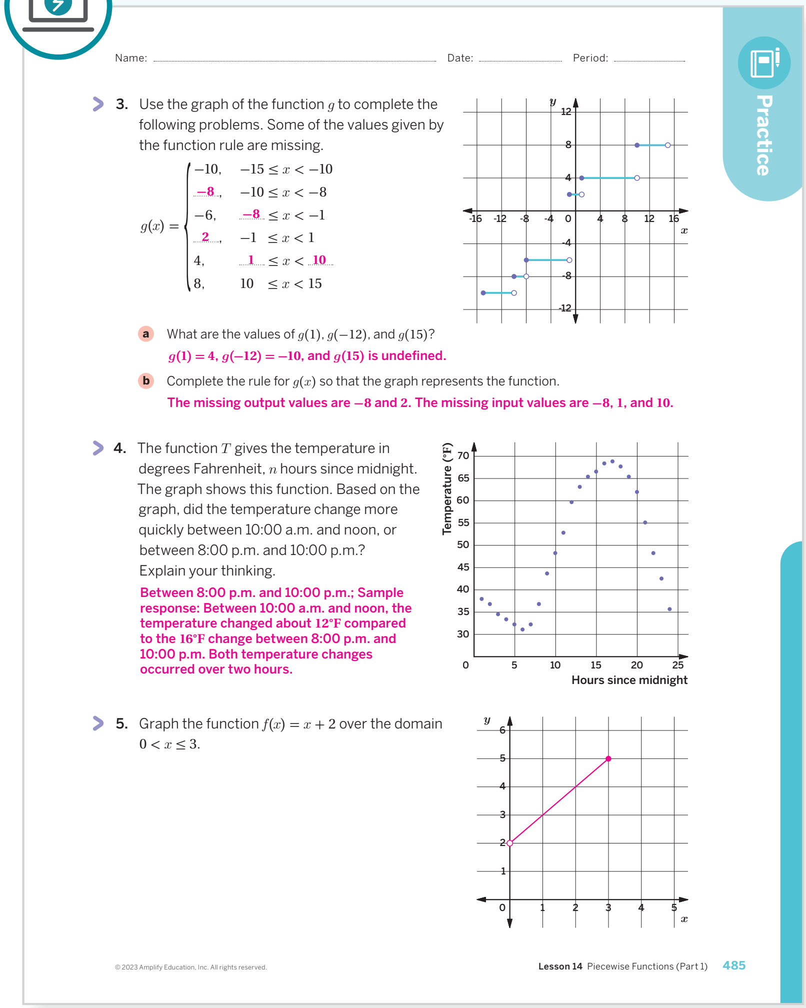
   t is not a function of G because there are multiple possible durations of parking for each price.

2. Consider the graph. Does the graph represent a function? Explain your thinking.

   No; Sample response: There are two possible output values when the input is 2 and when it is 6.

484 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.



Name: Date: Period:

Practice

3. Use the graph of the function g to complete the following problems. Some of the values given by the function rule are missing.

$$g(x) = \begin{cases} -10, & -15 \le x < -10 \\ \underline{-8}, & -10 \le x < -8 \\ -6, & \underline{-8} \le x < -1 \\ \underline{2}, & -1 \le x < 1 \\ 4, & \underline{1} \le x < \underline{10} \\ 8, & 10 \le x < 15 \end{cases}$$

   a What are the values of $g(1)$, $g(-12)$, and $g(15)$?

   $g(1) = 4$, $g(-12) = -10$, and $g(15)$ is undefined.

   b Complete the rule for $g(x)$ so that the graph represents the function.

   The missing output values are −8 and 2. The missing input values are −8, 1, and 10.

4. The function T gives the temperature in degrees Fahrenheit, n hours since midnight. The graph shows this function. Based on the graph, did the temperature change more quickly between 10:00 a.m. and noon, or between 8:00 p.m. and 10:00 p.m.? Explain your thinking.

   Between 8:00 p.m. and 10:00 p.m.; Sample response: Between 10:00 a.m. and noon, the temperature changed about 12°F compared to the 16°F change between 8:00 p.m. and 10:00 p.m. Both temperature changes occurred over two hours.

5. Graph the function $f(x) = x + 2$ over the domain $0 < x \le 3$.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 14 Piecewise Functions (Part 1) 485

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 9	2
Formative ⚡	5	Unit 3 Lesson 15	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Optional

UNIT 3 | **LESSON 15**

Piecewise Functions
(Part 2)

Let’s see how piecewise functions can represent design and sound.

Focus

Goals

1. **Language Goal:** Interpret a graph of a piecewise function or the rules given in function notation, and explain the rules in terms of a situation. **(Speaking and Listening, Reading and Writing)**
2. Sketch a graph that represents the rules of a piecewise function, paying special attention to the endpoints of each interval.
3. Understand a piecewise function as a function defined by different rules for different intervals of the domain.

Coherence

Today

Students graph and interpret piecewise functions that model sound. They describe how changes in pitch are reflected in the graph and function notation of piecewise functions. Students then adjust a piecewise function’s expressions and domain to eliminate breaks and jumps in the sound it represents.

Previously

In Lesson 14, students interpreted and graphed piecewise functions.

Coming Soon

In Lesson 16, students will be introduced to the absolute value function as a type of distance function.

Rigor

- Students build **conceptual understanding** of different representations of piecewise functions.
- Students **apply** piecewise functions within the context of music’s pitch to create a melody with no breaks or jumps.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	10 min	20 min	5 min	5 min
Independent	Whole Class	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Power-up PDF (as needed)
- Power-up PDF (answers)
- Anchor Chart PDF, *Sentence Stems, Describing My Thinking*
- rulers

Math Language Development

Review words

- *domain*
- *function*
- *piecewise function*

Amps | Featured Activity

Activity 2
Interactive Graphs

Students change pieces of a piecewise function to eliminate breaks and jumps in a melody by adjusting the expression and the domain of the piece. Students check that their changes to the piecewise function eliminate breaks or jumps by hearing the melody the graph represents.

Building Math Identity and Community

Connecting to Mathematical Practices

Students might become stressed as they determine how to rewrite a piecewise function to eliminate breaks and jumps. Work with students on ways to control their stress levels. Begin by reviewing what they already know about piecewise functions and then add a little at a time to reassure them that they can combine pieces of sound to create a melody.

Modifications to Pacing

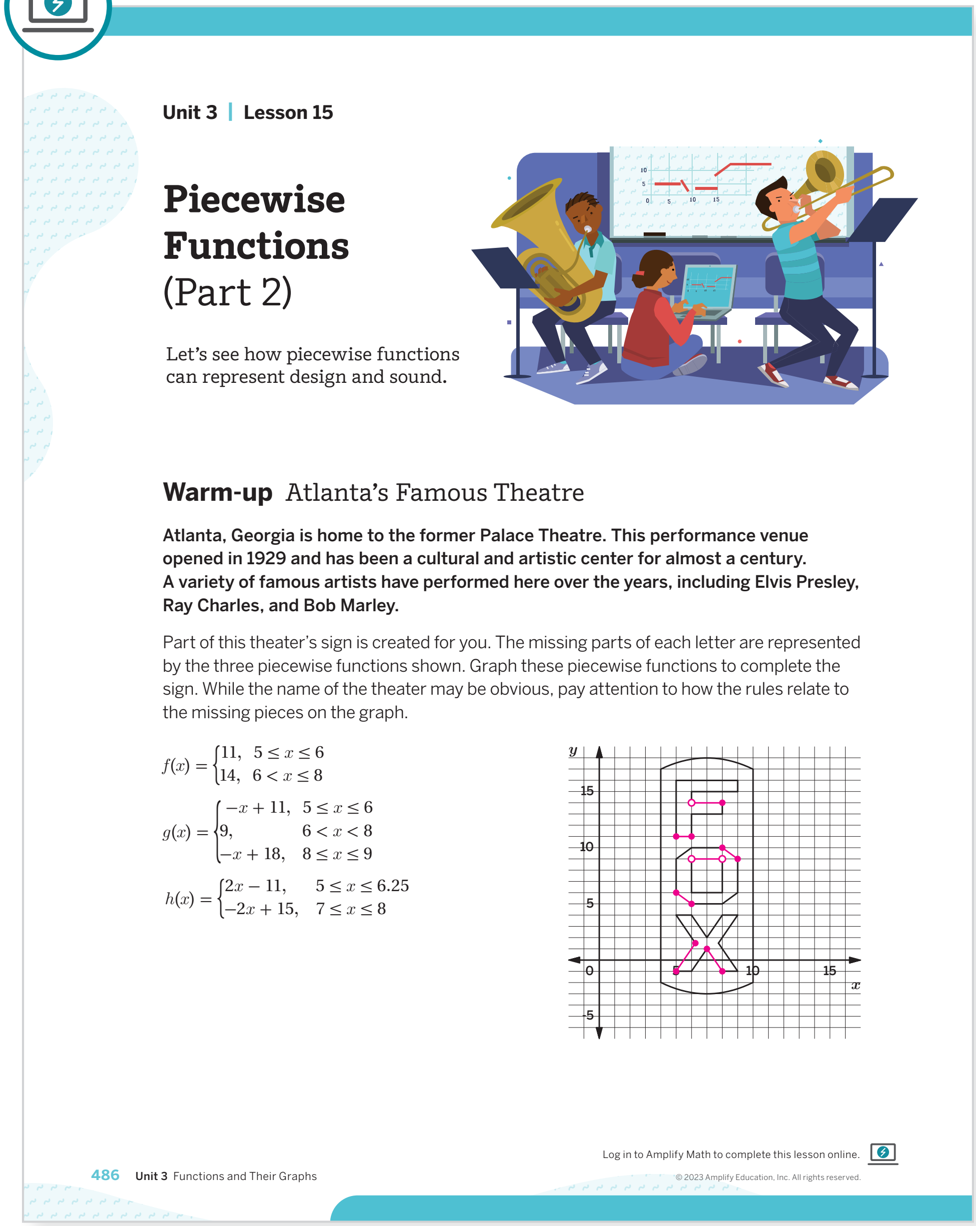
You may want to consider this additional modification if you are short on time.

- In **Activity 2** Problem 4, have students write a piecewise function so that only the first four pieces have no breaks or jumps in the recording.

Independent | 10 min

Warm-up Atlanta’s Famous Theatre

Students graph piecewise functions to complete a theaters sign to see how precise graphing of these functions can be used in design.



Unit 3 | Lesson 15

Piecewise Functions (Part 2)

Let’s see how piecewise functions can represent design and sound.

Warm-up Atlanta’s Famous Theatre

Atlanta, Georgia is home to the former Palace Theatre. This performance venue opened in 1929 and has been a cultural and artistic center for almost a century. A variety of famous artists have performed here over the years, including Elvis Presley, Ray Charles, and Bob Marley.

Part of this theater’s sign is created for you. The missing parts of each letter are represented by the three piecewise functions shown. Graph these piecewise functions to complete the sign. While the name of the theater may be obvious, pay attention to how the rules relate to the missing pieces on the graph.

$$f(x) = \begin{cases} 11, & 5 \le x \le 6 \\ 14, & 6 < x \le 8 \end{cases}$$

$$g(x) = \begin{cases} -x + 11, & 5 \le x \le 6 \\ 9, & 6 < x < 8 \\ -x + 18, & 8 \le x \le 9 \end{cases}$$

$$h(x) = \begin{cases} 2x - 11, & 5 \le x \le 6.25 \\ -2x + 15, & 7 \le x \le 8 \end{cases}$$

Log in to Amplify Math to complete this lesson online.

486 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the narrative together as a class. Highlight that the missing parts of each letter are represented by the three piecewise functions. Provide access to rulers.

2 Monitor

Help students get started by having them first plot the endpoints of each piece.

Look for points of confusion:

- **Inaccurately graphing or excluding endpoints of each piece.** Ask, “How does the inequality symbol used in the domain of each piece affect the graph?”

Look for productive strategies:

- Graphing the y-intercept and using the slope to sketch each piece.

3 Connect

Display the completed design.

Highlight that by extending each side of the design that students graphed using a ruler or straightedge, they can determine its intercepts and use these to check the accuracy of their line.

Ask:

- “How would vertical lines be represented in a piecewise function?” They cannot be represented because vertical lines are not functions.
- “Could you combine all the piecewise functions together into one piecewise function?” No, because some of the pieces are defined over the same domain. If they were all combined, the relationship would not be a function.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide access to colored pencils and have students color code the rule with the missing line segment on the graph. This will help them make connections between the graph and the corresponding rule of the piecewise function.

Power-up

To power up students’ ability to graph a linear function over a given domain:

Provide students with a copy of the Power-up PDF.

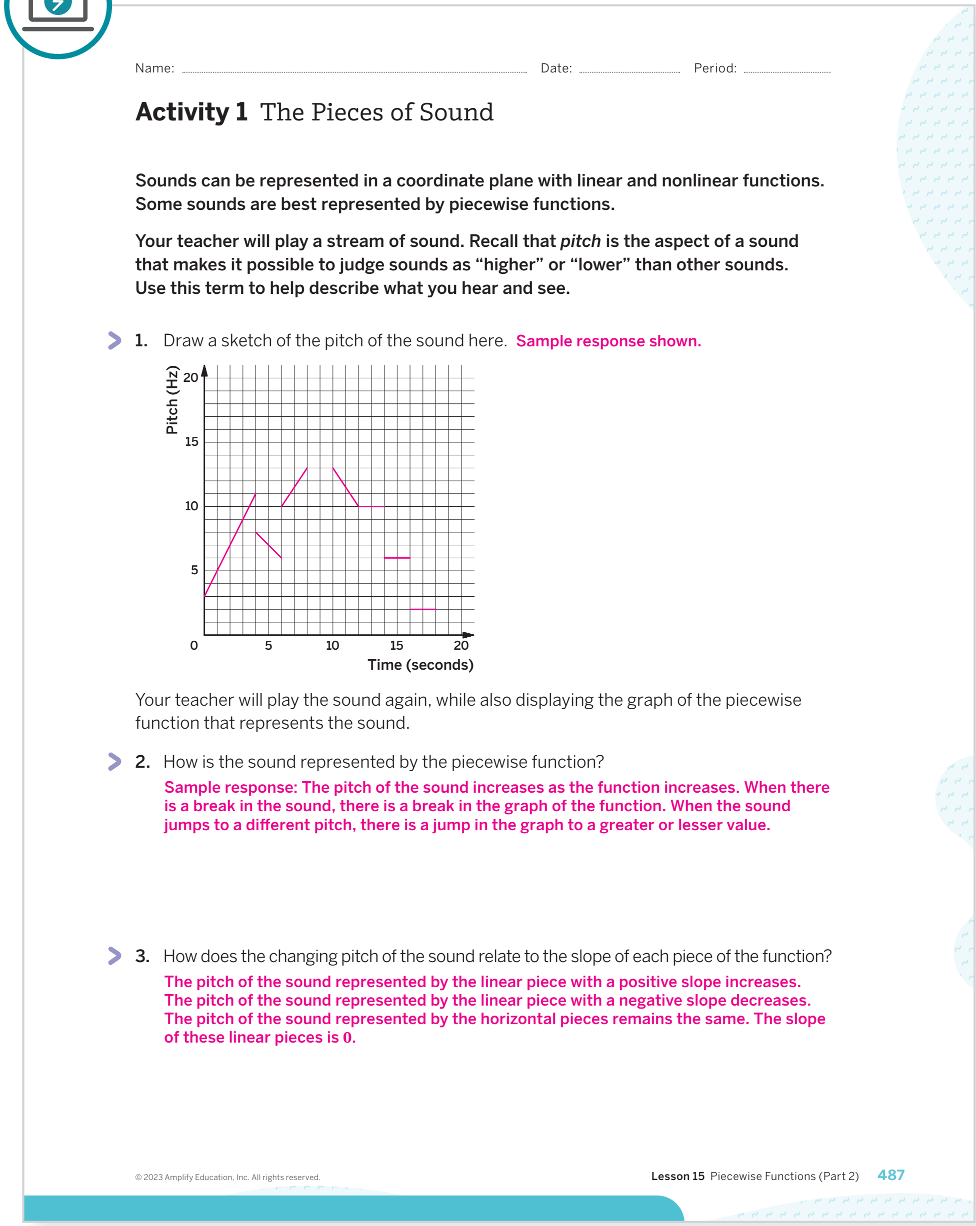
Use: Before the Warm-up

Informed by: Performance on Lesson 14, Practice Problem 5

Whole Class | 10 min

Activity 1 The Pieces of Sound

Students draw a sketch of the piecewise function modeling the pitch of a sound to connect the changing pitch to the slope of each piece of the function.



Name: Date: Period:

Activity 1 The Pieces of Sound

Sounds can be represented in a coordinate plane with linear and nonlinear functions. Some sounds are best represented by piecewise functions.

Your teacher will play a stream of sound. Recall that *pitch* is the aspect of a sound that makes it possible to judge sounds as “higher” or “lower” than other sounds. Use this term to help describe what you hear and see.

1. Draw a sketch of the pitch of the sound here. **Sample response shown.**

Your teacher will play the sound again, while also displaying the graph of the piecewise function that represents the sound.

2. How is the sound represented by the piecewise function?
   Sample response: The pitch of the sound increases as the function increases. When there is a break in the sound, there is a break in the graph of the function. When the sound jumps to a different pitch, there is a jump in the graph to a greater or lesser value.

3. How does the changing pitch of the sound relate to the slope of each piece of the function?
   The pitch of the sound represented by the linear piece with a positive slope increases. The pitch of the sound represented by the linear piece with a negative slope decreases. The pitch of the sound represented by the horizontal pieces remains the same. The slope of these linear pieces is 0.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 15 Piecewise Functions (Part 2) 487

1 Launch

Read the narrative aloud. Have students give examples of sounds with low or high pitches. Ask them to preview Problems 1–3 before listening to the recording.

2 Monitor

Help students get started by asking them to think about how the pitch changes as the sound is played.

Look for points of confusion:

- **Having difficulty hearing the differences or change in pitch.** Replay sections of the graph that involve these transitions. Consider also showing students the graph of the piecewise function instead of having them translate what they hear.

Look for productive strategies:

- Using a piece’s rate of change, maximum, minimum, and slope to describe its pitch.

3 Connect

Have pairs of students share their thinking for Problems 1–3.

Ask, “How is the sound of the linear pieces with a positive slope different from the linear pieces with a negative slope?” The linear pieces with a positive slope have an increasing pitch, while the linear pieces with a negative slope have a decreasing pitch.

Highlight that because the graph represents the change in pitch over time, the slope of each linear piece of the graph represents how the pitch increases, decreases, or remains constant.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students use hand gestures to illustrate how the pitch changes over time and then translate their hand gestures to the graph.

Accessibility: *Vary Demands to Optimize Challenge*

Instead of having students translate the sound to the graph, consider playing the sound and providing students with the graph that represents the sound. Have them begin the activity with Problem 2.

Extension: *Math Enrichment*

Have students complete the following problem:
A sound first starts at a constant pitch. Then it decreases at a relatively slow and steady rate for a while until it begins to increase at a very fast and steady rate. Construct the rules for a possible piecewise function to model the pitch over time. Sample response:

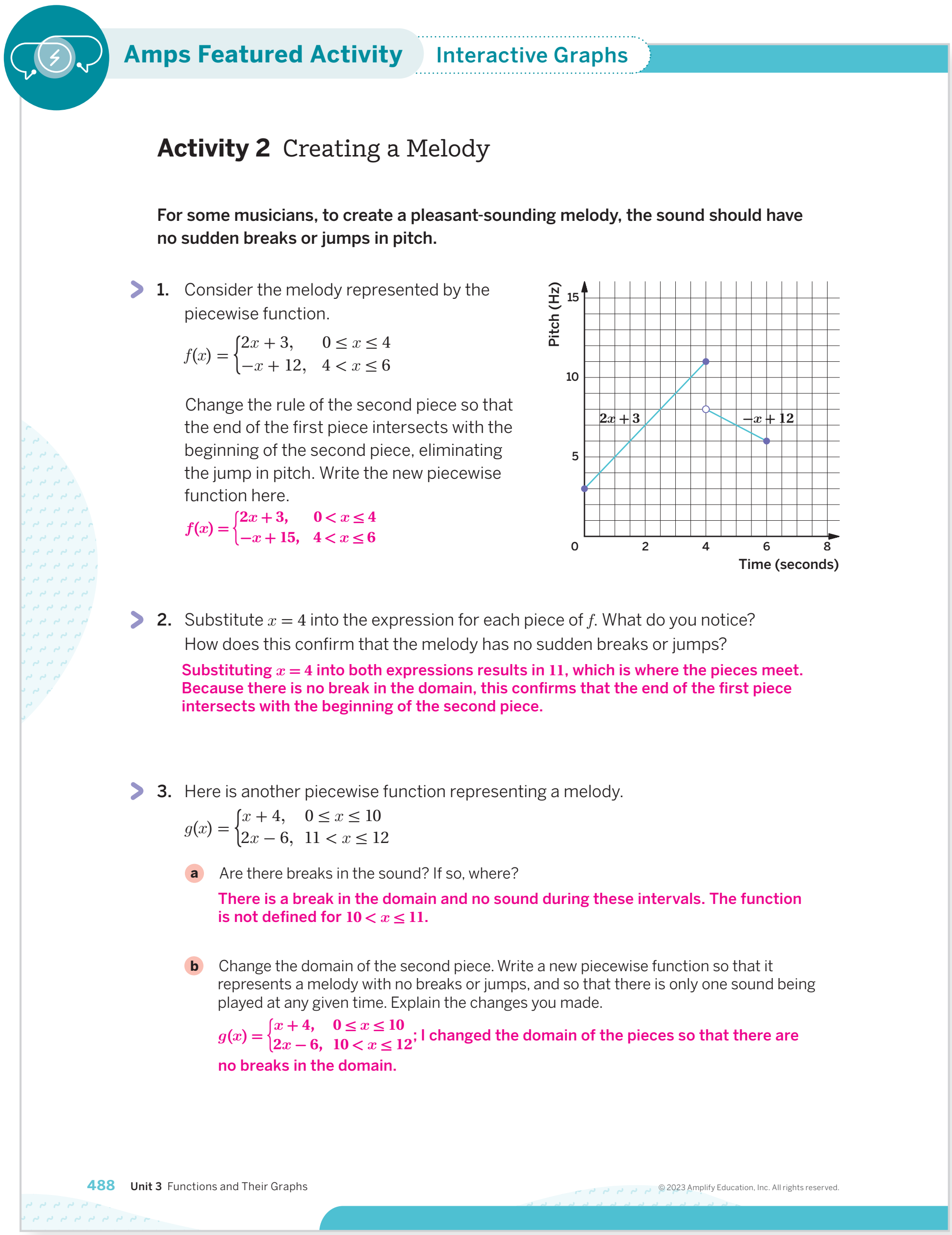
$$f(x) = \begin{cases} 12, & 0 \leq x < 3 \\ -x + 15, & 3 \leq x < 8 \\ 5x - 33, & 8 \leq x < 15 \end{cases}$$

Independent | 20 min

Activity 2 Creating a Melody

Students adjust a piecewise function to eliminate breaks or jumps in its graph to understand how a change in the vertical intercept and domain affects the graph.

Amps Featured Activity Interactive Graphs



Activity 2 Creating a Melody

For some musicians, to create a pleasant-sounding melody, the sound should have no sudden breaks or jumps in pitch.

1. Consider the melody represented by the piecewise function.

$$f(x) = \begin{cases} 2x + 3, & 0 \leq x \leq 4 \\ -x + 12, & 4 < x \leq 6 \end{cases}$$

Change the rule of the second piece so that the end of the first piece intersects with the beginning of the second piece, eliminating the jump in pitch. Write the new piecewise function here.

$$f(x) = \begin{cases} 2x + 3, & 0 < x \leq 4 \\ -x + 15, & 4 < x \leq 6 \end{cases}$$

2. Substitute $x = 4$ into the expression for each piece of f. What do you notice? How does this confirm that the melody has no sudden breaks or jumps?

Substituting $x = 4$ into both expressions results in 11, which is where the pieces meet. Because there is no break in the domain, this confirms that the end of the first piece intersects with the beginning of the second piece.

3. Here is another piecewise function representing a melody.

$$g(x) = \begin{cases} x + 4, & 0 \leq x \leq 10 \\ 2x - 6, & 11 < x \leq 12 \end{cases}$$

a Are there breaks in the sound? If so, where?

There is a break in the domain and no sound during these intervals. The function is not defined for $10 < x \leq 11$.

b Change the domain of the second piece. Write a new piecewise function so that it represents a melody with no breaks or jumps, and so that there is only one sound being played at any given time. Explain the changes you made.

$g(x) = \begin{cases} x + 4, & 0 \leq x \leq 10 \\ 2x - 6, & 10 < x \leq 12 \end{cases}$; I changed the domain of the pieces so that there are no breaks in the domain.

488 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display the graph from Activity 1 and play the sound represented by the graph. Have students explain what a jump or break in the melody sounds like, and how these jumps and breaks are represented on the graph. Ask, "What would the graph of a melody with no jumps or breaks look like?"

2 Monitor

Help students get started by having them make a simple sketch of a piecewise graph and humming the sound the graph represents.

Look for points of confusion:

- **Having difficulty understanding how changing the vertical intercept affects the graph.** Have students graph $y = -x + 11$ and $y = x + 13$. Ask, "What changed and what remained the same in each equation? What changed and what remained the same in each graph?"
- **Having overlapping or breaks in the domain of the piecewise function.** Have students determine which piece defines the function at $x = 10$ in Problem 3b. Ask, "What does the domain of a piecewise function with no breaks look like?"

Look for productive strategies:

- Sketching the graph of the piecewise function with no breaks or jumps to help write its corresponding piecewise function.
- Annotating each piece on the graph with key features such as its domain, slope, and vertical intercept to help write the piecewise function.

Activity 2 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can digitally change pieces of a piecewise function to eliminate breaks and jumps in a melody by adjusting the expression and domain representing that piece. They can check their accuracy by hearing the melody the graph represents.

Accessibility: *Guide Processing and Visualization*

Provide access to graph paper and suggest that students draw a sketch of the graph that would eliminate the jump in pitch for Problem 1 before they write the rules for the new piecewise function.

Math Language Development

MLR7: Compare and Connect

During the Connect, display the original graph from Problem 4 and sample students' graphs they drew in Problem 4b. Highlight the connections between how the original graph shows breaks or jumps and how the equations for the piecewise function were altered so that the graph in Problem 4b does not have any breaks or jumps. As students share how they changed each piece, listen for and amplify mathematical language, such as "I altered the vertical intercept" or "I changed the boundary value."

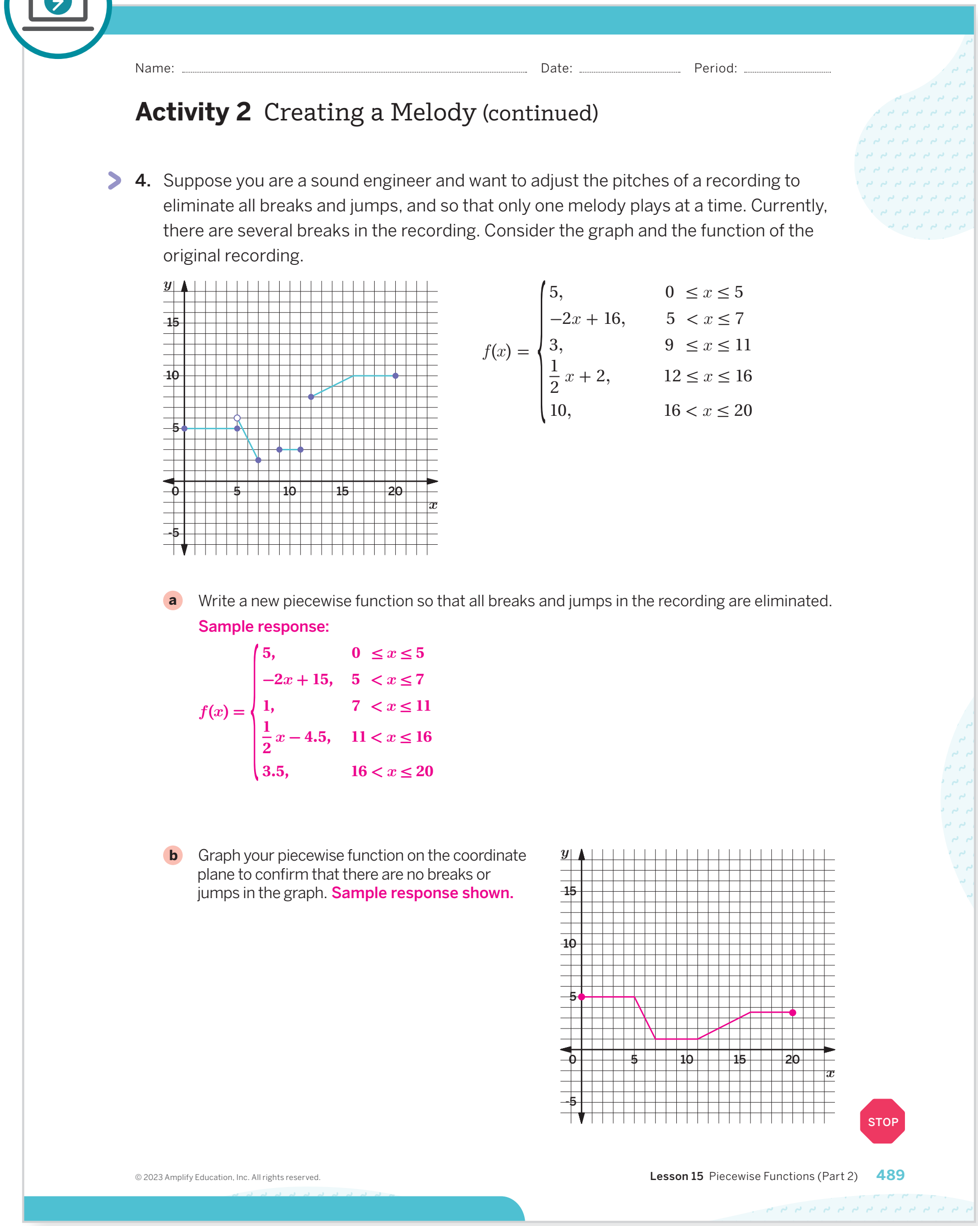
English Learners

Annotate the locations of the graphs, or use gestures such as pointing, when you use the terms *breaks* or *jumps* in the graph.

Independent | 20 min

Activity 2 Creating a Melody (continued)

Students adjust a piecewise function to eliminate breaks or jumps in its graph to understand how a change in the vertical intercept and domain affects the graph.



Name: ______ Date: ______ Period: ______

Activity 2 Creating a Melody (continued)

4. Suppose you are a sound engineer and want to adjust the pitches of a recording to eliminate all breaks and jumps, and so that only one melody plays at a time. Currently, there are several breaks in the recording. Consider the graph and the function of the original recording.

$$f(x) = \begin{cases} 5, & 0 \leq x \leq 5 \\ -2x + 16, & 5 < x \leq 7 \\ 3, & 9 \leq x \leq 11 \\ \frac{1}{2}x + 2, & 12 \leq x \leq 16 \\ 10, & 16 < x \leq 20 \end{cases}$$

a Write a new piecewise function so that all breaks and jumps in the recording are eliminated.

Sample response:

$$f(x) = \begin{cases} 5, & 0 \leq x \leq 5 \\ -2x + 15, & 5 < x \leq 7 \\ 1, & 7 < x \leq 11 \\ \frac{1}{2}x - 4.5, & 11 < x \leq 16 \\ 3.5, & 16 < x \leq 20 \end{cases}$$

b Graph your piecewise function on the coordinate plane to confirm that there are no breaks or jumps in the graph. **Sample response shown.**

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 15 Piecewise Functions (Part 2) 489

3 Connect

Have individual students share their function they created for Problem 4a, and how they changed each piece to eliminate the breaks and jumps.

Display a graph of a melody with no breaks or jumps and its corresponding piecewise function from Problem 4.

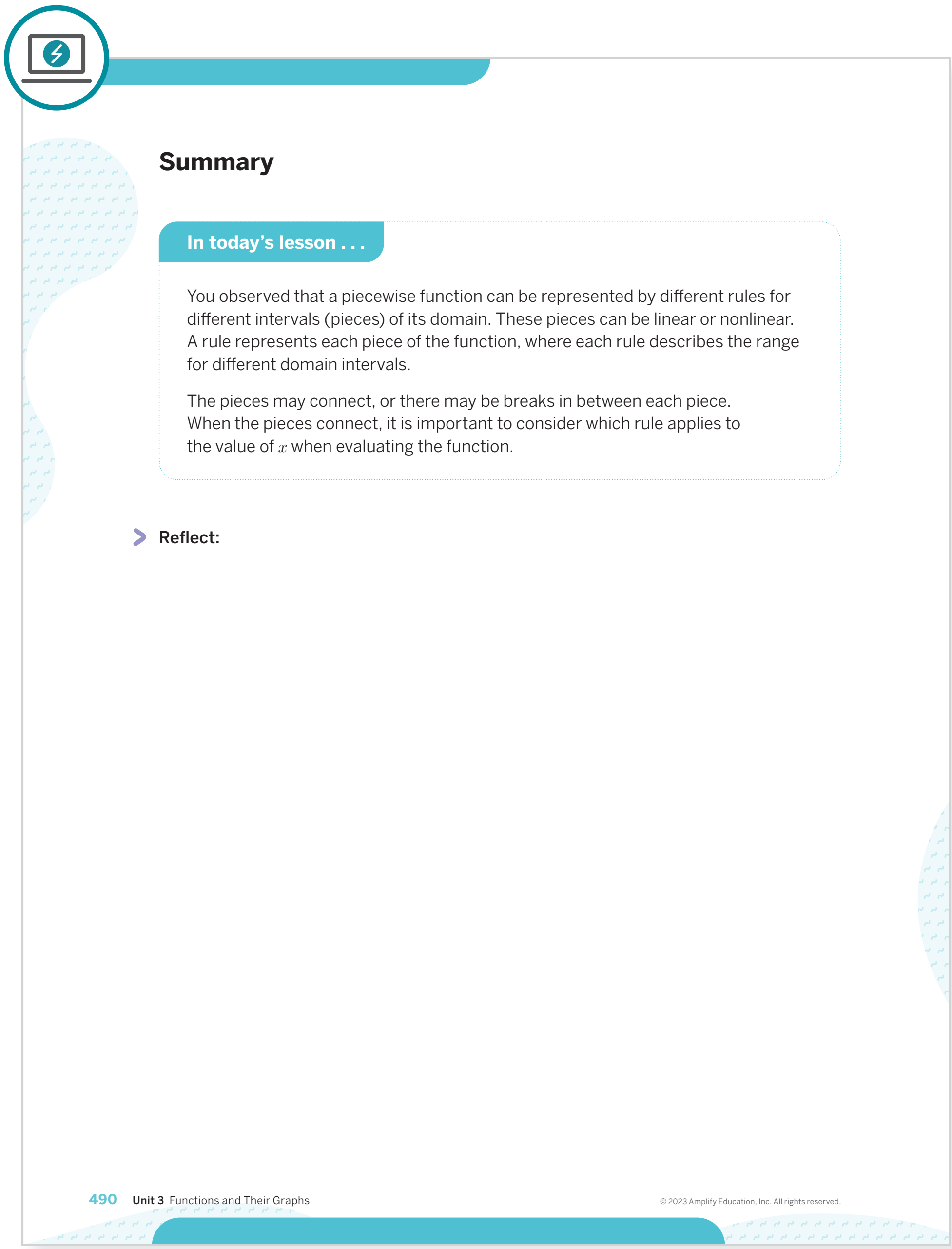
Ask:

- "How does changing the vertical intercept of a linear function move its graph?" Increasing the vertical intercept of a linear function moves its graph up, and decreasing it moves the graph down.
- "How does changing the domain of a linear function change the graph of a linear function?" Changing the domain of a piece extends or restricts the graph to the left or right.

Highlight that for there to be no breaks or jumps in the graph of a piecewise function, two adjacent pieces must meet at the same point. Only one piece actually contains this point. Students can determine if pieces' endpoints intersect by substituting the x-coordinate of the intersection point into each piece's expressions to see whether each results in the same y-coordinate of the point.

Summary

Review and synthesize using piecewise functions in design and to describe sound.



Summary

In today's lesson . . .

You observed that a piecewise function can be represented by different rules for different intervals (pieces) of its domain. These pieces can be linear or nonlinear. A rule represents each piece of the function, where each rule describes the range for different domain intervals.

The pieces may connect, or there may be breaks in between each piece. When the pieces connect, it is important to consider which rule applies to the value of x when evaluating the function.

Reflect:

490 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the graph from Activity 1 while playing the sound represented by the graph.

Have students share how a piecewise function could create a melody with no jumps or breaks.

Highlight that to ensure no breaks in design or sounds, the piecewise function needs to be defined over the entire domain of the design or sound, which means paying close attention to the inequality symbols used.

Ask, "Could two voices singing different pitches at once be represented by a piecewise function?" No, this would not be a function. The voices would need to be represented by two different piecewise functions.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

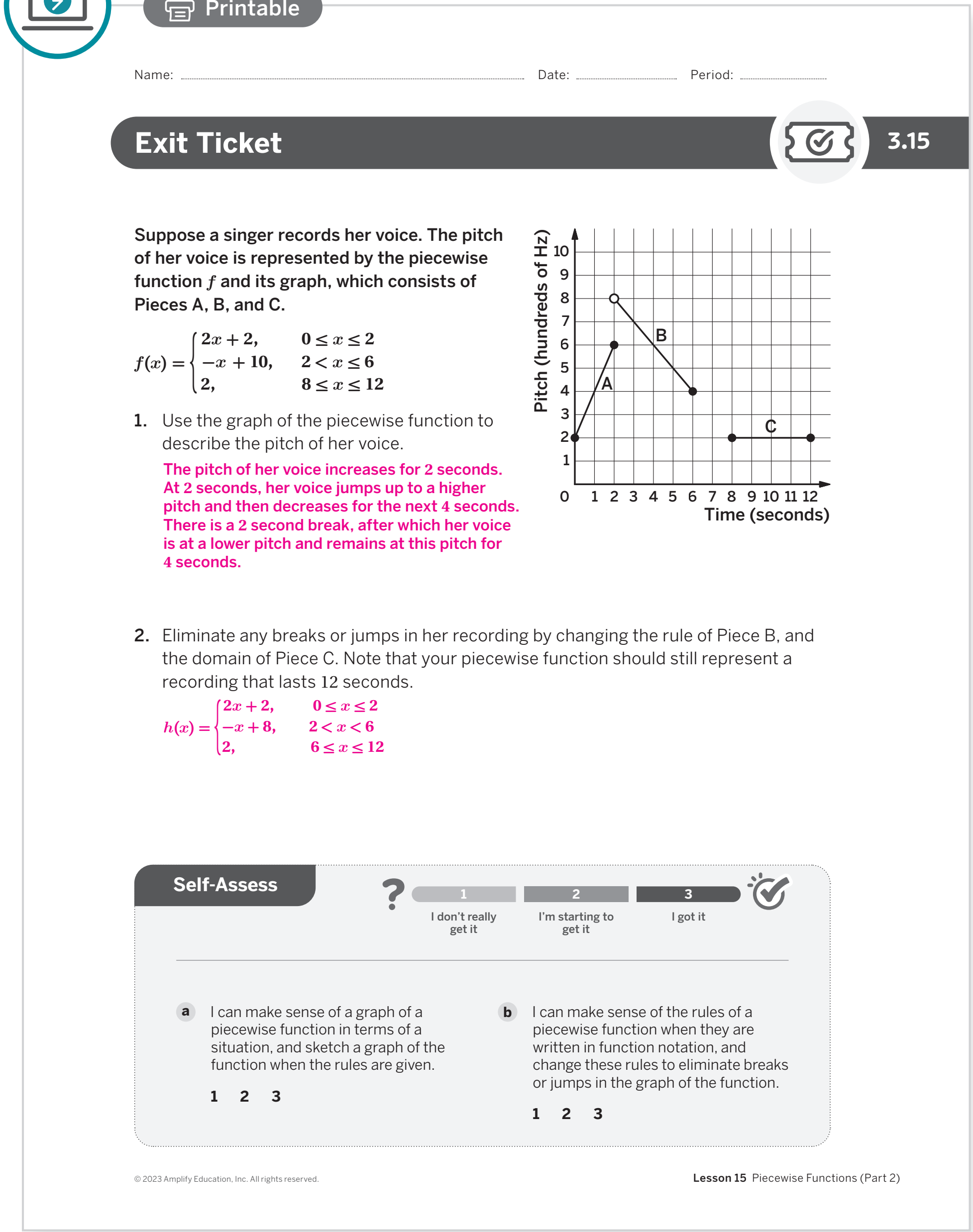
- "What would a graph of the sounds from multiple instruments being played at different times look like?"
- "How are the breaks and jumps in a melody represented in a piecewise function?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by intepreting the graph of a piecewise function within a given context and constructing a new piecewise function to eliminate any breaks or jumps.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

3.15

Suppose a singer records her voice. The pitch of her voice is represented by the piecewise function f and its graph, which consists of Pieces A, B, and C.

$$f(x) = \begin{cases} 2x + 2, & 0 \leq x \leq 2 \\ -x + 10, & 2 < x \leq 6 \\ 2, & 8 \leq x \leq 12 \end{cases}$$

1. Use the graph of the piecewise function to describe the pitch of her voice.

   The pitch of her voice increases for 2 seconds. At 2 seconds, her voice jumps up to a higher pitch and then decreases for the next 4 seconds. There is a 2 second break, after which her voice is at a lower pitch and remains at this pitch for 4 seconds.

2. Eliminate any breaks or jumps in her recording by changing the rule of Piece B, and the domain of Piece C. Note that your piecewise function should still represent a recording that lasts 12 seconds.

$$h(x) = \begin{cases} 2x + 2, & 0 \leq x \leq 2 \\ -x + 8, & 2 < x < 6 \\ 2, & 6 \leq x \leq 12 \end{cases}$$

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it ✓

a. I can make sense of a graph of a piecewise function in terms of a situation, and sketch a graph of the function when the rules are given.

1 2 3

b. I can make sense of the rules of a piecewise function when they are written in function notation, and change these rules to eliminate breaks or jumps in the graph of the function.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Success looks like . . .

- **Language Goal:** Interpreting a graph of a piecewise function or the rules given in function notation, and explaining the rules in terms of a situation. **(Speaking and Listening, Reading and Writing)**
  - » Interpreting the graph in terms of the singer's voice in Problem 1.
- **Goal:** Sketching a graph that represents the rules of a piecewise function, paying special attention to the endpoints of each interval.
- **Goal:** Understanding a piecewise function as a function defined by different rules for different intervals of the domain.

Suggested next steps

If students inaccurately or vaguely describe the pitch in Problem 1, consider:

- Reviewing how to use the graph to describe the pitch in Activity 1.
- Assigning Practice Problems 2 and 3.
- Asking, "What does an increasing, decreasing, or constant function sound like?"

If students inaccurately change the piecewise function to eliminate the jump in the sound in Problem 2, consider:

- Reviewing how to change the vertical intercept of a line to eliminate a jump in the piecewise function graph in Activity 2.
- Assigning Practice Problems 2 and 3.
- Asking, "Where do the pieces need to meet to eliminate the break between these pieces?"

If students inaccurately change the domain of pieces in the piecewise function to eliminate breaks in sound in Problem 2, consider:

- Reviewing changing the domain of pieces to eliminate breaks in the graph in Activity 2.
- Assigning Practice Problems 2 and 3.

Professional Learning

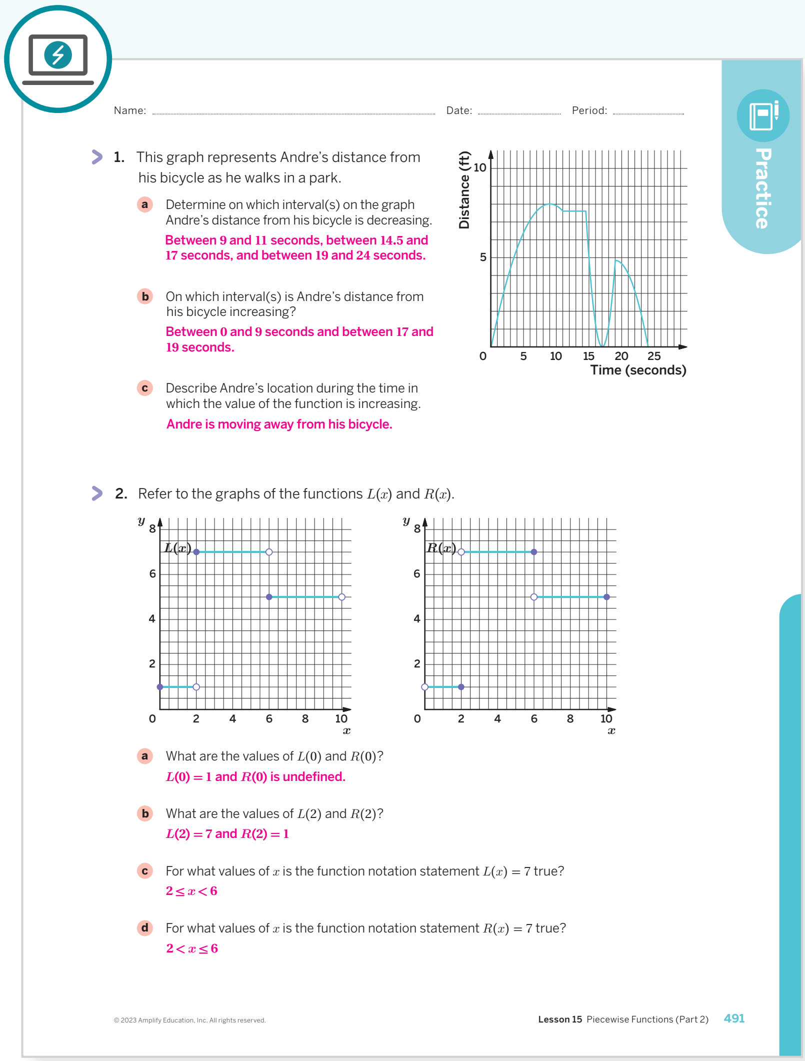
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What different ways did students approach adjusting the piecewise functions to make the melody? What does that tell you about similarities and differences among your students?
- How did describing how pitch is represented in the graph set them up to eliminate breaks and jumps in the sound in Activity 2? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

1. This graph represents Andre's distance from his bicycle as he walks in a park.

   a. Determine on which interval(s) on the graph Andre's distance from his bicycle is decreasing.

   Between 9 and 11 seconds, between 14.5 and 17 seconds, and between 19 and 24 seconds.

   b. On which interval(s) is Andre's distance from his bicycle increasing?

   Between 0 and 9 seconds and between 17 and 19 seconds.

   c. Describe Andre's location during the time in which the value of the function is increasing.

   Andre is moving away from his bicycle.

2. Refer to the graphs of the functions $L(x)$ and $R(x)$.

   a. What are the values of $L(0)$ and $R(0)$?

   $L(0) = 1$ and $R(0)$ is undefined.

   b. What are the values of $L(2)$ and $R(2)$?

   $L(2) = 7$ and $R(2) = 1$

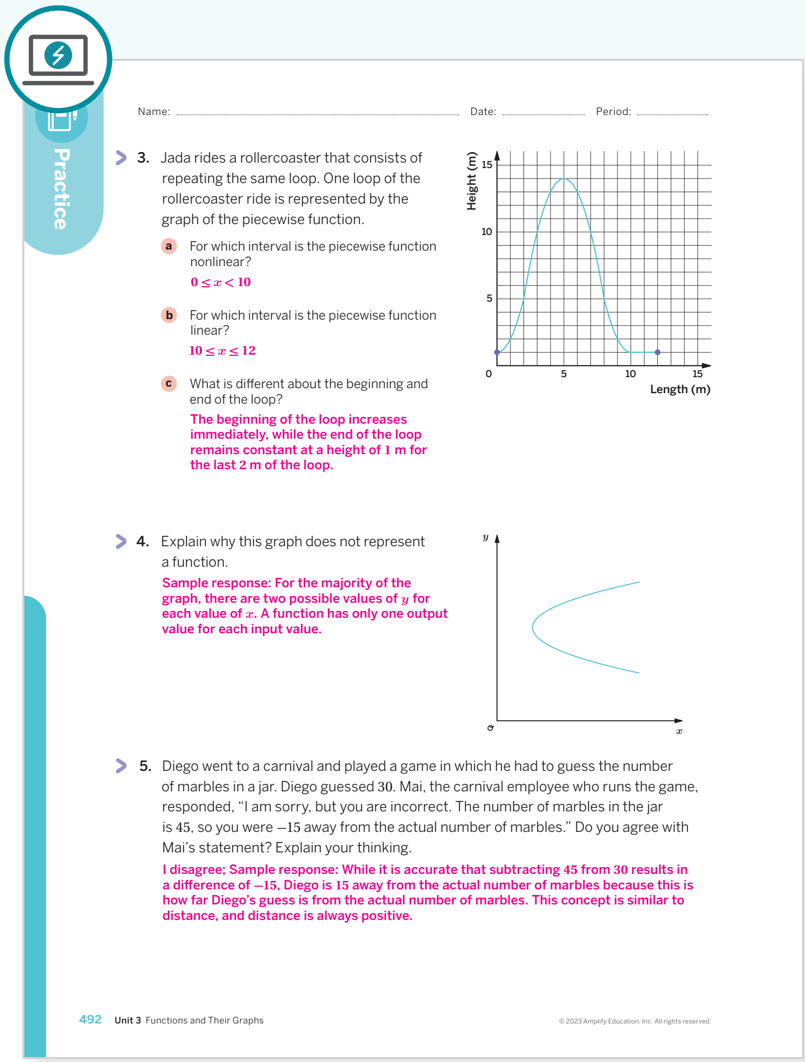
   c. For what values of x is the function notation statement $L(x) = 7$ true?

   $2 \leq x < 6$

   d. For what values of x is the function notation statement $R(x) = 7$ true?

   $2 < x \leq 6$

© 2023 Amplify Education, Inc. All rights reserved.



Name: ______ Date: ______ Period: ______

3. Jada rides a rollercoaster that consists of repeating the same loop. One loop of the rollercoaster ride is represented by the graph of the piecewise function.

   a. For which interval is the piecewise function nonlinear?

   $0 \leq x < 10$

   b. For which interval is the piecewise function linear?

   $10 \leq x \leq 12$

   c. What is different about the beginning and end of the loop?

   The beginning of the loop increases immediately, while the end of the loop remains constant at a height of 1 m for the last 2 m of the loop.

4. Explain why this graph does not represent a function.

   Sample response: For the majority of the graph, there are two possible values of y for each value of x. A function has only one output value for each input value.

5. Diego went to a carnival and played a game in which he had to guess the number of marbles in a jar. Diego guessed 30. Mai, the carnival employee who runs the game, responded, "I am sorry, but you are incorrect. The number of marbles in the jar is 45, so you were –15 away from the actual number of marbles." Do you agree with Mai's statement? Explain your thinking.

   I disagree; Sample response: While it is accurate that subtracting 45 from 30 results in a difference of –15, Diego is 15 away from the actual number of marbles because this is how far Diego's guess is from the actual number of marbles. This concept is similar to distance, and distance is always positive.

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 3	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 9	2
Formative ⚡	5	Unit 3 Lesson 16	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Mathematical Modeling Prompt

At any point during the course and after completing this lesson, have students work in pairs or small groups on this unit's Mathematical Modeling Prompt, *RGB Display*, which is available in the **Algebra 1 Additional Practice**.

Another Function?

Let's make some guesses and see how close they are to actual values.

Focus

Goals

1. **Language Goal:** Analyze and describe features of a scatter plot that relates guesses and absolute errors. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Generalize the relationship between guesses and absolute errors. **(Reading and Writing)**
3. Given a set of numerical guesses and a target number, calculate absolute errors and create a scatter plot of the data.

Coherence

Today

Students experience the idea of a distance function in the context of guessing the number of objects in a container, calculating the absolute guessing errors of the guesses, and plotting the absolute guessing errors as a function of the guesses.

Previously

Students analyzed and graphed linear and nonlinear piecewise functions in Lessons 14 and 15.

Coming Soon

Students will define and graph the absolute value function and translations of the absolute value function in Lesson 17.

Rigor

- Students build **conceptual understanding** of the absolute value function through calculating and graphing absolute guessing errors.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	15 min	15 min	5 min	5 min
Whole Class	Whole Class	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, (as needed)
- Anchor Chart PDF, *Sentence Stems, Describing My Thinking*
- graphing technology
- transparent jar with 30–50 small objects

Math Language Development

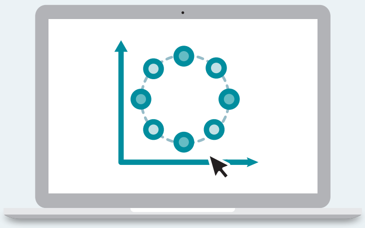
Review words

- *absolute value*
- *domain*
- *piecewise function*
- *range*

Amps | Featured Activity

Activity 1
Compiling Guesses

Students plot the absolute guessing errors from their table in the Warm-up. All students' plotted points are then compiled onto a class graph that students use to make observations of key features.



Building Math Identity and Community

Connecting to Mathematical Practices

Students might be hesitant to make a guess because they fear being completely wrong. By working with the structure of a scatter plot, students can see that their errors contribute to the graph and have mathematical meaning. Encourage students to have the self-confidence to make a guess, recognize the mathematical error, and then work with it. Taking that chance and learning from mistakes are skills that will benefit them in mathematics and real life.

Modifications to Pacing

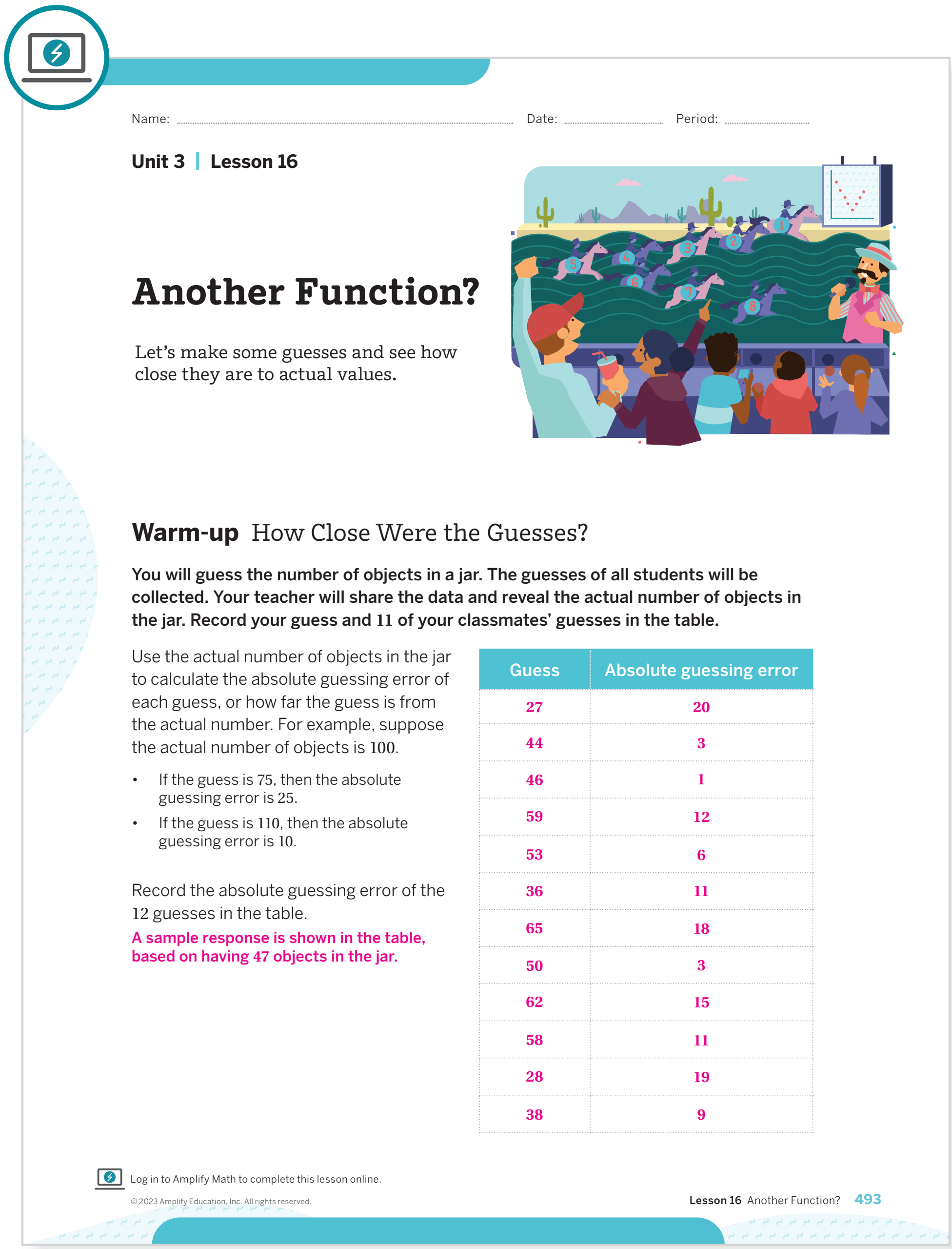
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, limit the number of absolute guessing errors calculated.
- In **Activity 1**, have students make and share their observations verbally.

Whole Class | 10 min

Warm-up How Close Were the Guesses?

Students compute absolute guessing errors using class data to informally experience the values of a distance function.



Name: ______ Date: ______ Period: ______

Unit 3 | Lesson 16

Another Function?

Let's make some guesses and see how close they are to actual values.

Warm-up How Close Were the Guesses?

You will guess the number of objects in a jar. The guesses of all students will be collected. Your teacher will share the data and reveal the actual number of objects in the jar. Record your guess and 11 of your classmates' guesses in the table.

Use the actual number of objects in the jar to calculate the absolute guessing error of each guess, or how far the guess is from the actual number. For example, suppose the actual number of objects is 100.

- If the guess is 75, then the absolute guessing error is 25.
- If the guess is 110, then the absolute guessing error is 10.

Record the absolute guessing error of the 12 guesses in the table.

A sample response is shown in the table, based on having 47 objects in the jar.

Guess	Absolute guessing error
27	20
44	3
46	1
59	12
53	6
36	11
65	18
50	3
62	15
58	11
28	19
38	9

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 16 Another Function? 493

1 Launch

Display a transparent jar that contains about 30–50 small objects, or display a picture of such a jar. Ask students to guess the number of objects in the jar. Collect their guesses and compile them in a table to display to the whole class. After all data is recorded, reveal the actual number of objects in the jar.

2 Monitor

Help students get started by helping them calculate the absolute guessing error for one value from the class data.

Look for points of confusion:

- **Recording negative values for the absolute guessing error.** Have students revisit the examples given, and rephrase the question by asking, "How far away is the value from the actual value?"

Look for productive strategies:

- Arranging guesses from least to greatest in their table.

3 Connect

Display a completed table of all of the class' data.

Have individual students share their observations about the absolute guessing errors they found.

Highlight that the absolute guessing errors are all positive because they represent the distance of each guess from the actual value.

Ask, "What does an absolute guessing error of 0 represent?" This represents guessing the actual number of objects.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider providing the Warm-up PDF to help students organize their thinking as they determine the absolute guessing error. They can use the table provided on the PDF, which scaffolds the process for determining the absolute guessing error.

Power-up

To power up students' ability to calculate absolute error, have students complete:

Recall that the *absolute error* is the distance a guess or estimate is from the actual value. Determine the absolute error given each guess and actual value.

a. Guess: 4 Actual value: 4 0
b. Guess: –2 Actual value: –10 8
c. Guess: 97 Actual value: 103 6

Use: Before the Warm-up

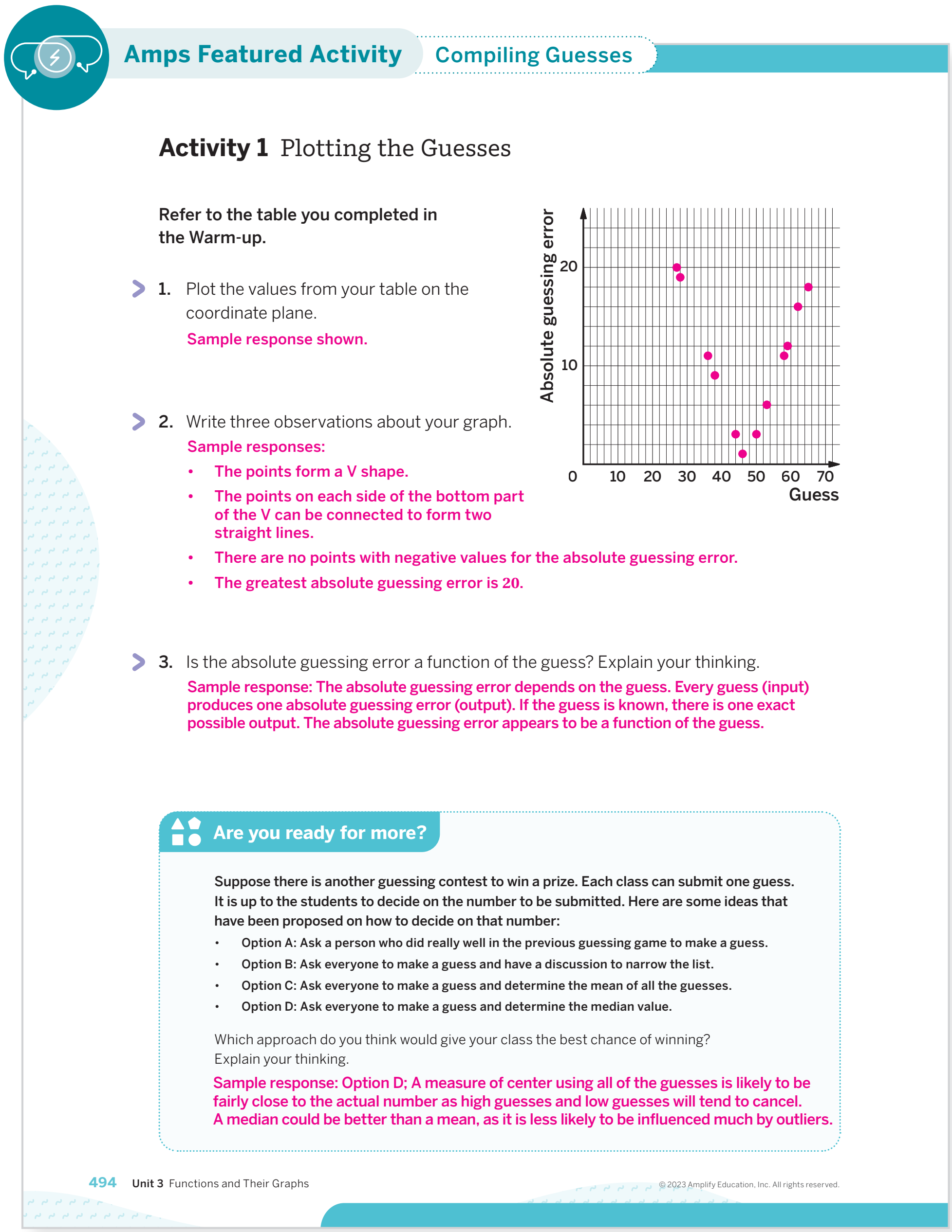
Informed by: Performance on Lesson 15, Practice Problem 5

Whole Class | 15 min

Activity 1 Plotting the Guesses

Students graph the absolute guessing error data to make observations about the structure of the graph and to determine whether the relationship is a function, justifying their conclusion using mathematical language.

Amps Featured Activity Compiling Guesses



Activity 1 Plotting the Guesses

Refer to the table you completed in the Warm-up.

1. Plot the values from your table on the coordinate plane.
   Sample response shown.

2. Write three observations about your graph.
   Sample responses:
   - **The points form a V shape.**
   - **The points on each side of the bottom part of the V can be connected to form two straight lines.**
   - **There are no points with negative values for the absolute guessing error.**
   - **The greatest absolute guessing error is 20.**

3. Is the absolute guessing error a function of the guess? Explain your thinking.
   Sample response: The absolute guessing error depends on the guess. Every guess (input) produces one absolute guessing error (output). If the guess is known, there is one exact possible output. The absolute guessing error appears to be a function of the guess.

Are you ready for more?

Suppose there is another guessing contest to win a prize. Each class can submit one guess. It is up to the students to decide on the number to be submitted. Here are some ideas that have been proposed on how to decide on that number:

- **Option A: Ask a person who did really well in the previous guessing game to make a guess.**
- **Option B: Ask everyone to make a guess and have a discussion to narrow the list.**
- **Option C: Ask everyone to make a guess and determine the mean of all the guesses.**
- **Option D: Ask everyone to make a guess and determine the median value.**

Which approach do you think would give your class the best chance of winning? Explain your thinking.

Sample response: Option D; A measure of center using all of the guesses is likely to be fairly close to the actual number as high guesses and low guesses will tend to cancel. A median could be better than a mean, as it is less likely to be influenced much by outliers.

494 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

① Launch

Have students use their data from the Warm-up to create a scatter plot and respond to Problems 2 and 3 before compiling the class data from all guesses onto a scatter plot.

② Monitor

Help students get started by helping them plot one data point from the class data.

Look for points of confusion:

- **Thinking that the graph makes a "U" or curve shape.** Have students plot an ordered pair with an absolute guessing error of zero, and connect the points with two line segments.

Look for productive strategies:

- Confirming that each part of their scatter plot aligns along a line to check the accuracy of their points.

③ Connect

Display a class scatter plot of the data from all guesses.

Have individual students share their observations and something they wonder.

Ask, "Where do the guesses with the greatest absolute error appear on the scatter plot? What about the guesses with the least absolute error?" Sample response: The guesses with the greatest absolute error are far away from 47. The guesses with the least absolute error are close to (47, 0).

Highlight that the absolute guessing error is a function of the guess because there is only one possible absolute guessing error for each guess.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can plot their absolute guessing errors from the table in the Warm-up. You can compile all of the students' points onto a class graph to help students make connections and observations.

Accessibility: *Vary Demands to Optimize Challenge*

Provide a pre-completed graph for students to analyze instead of having them do the actual plotting of the points. Have them begin the activity with Problem 2.

Math Language Development

MLR8: Discussion Supports

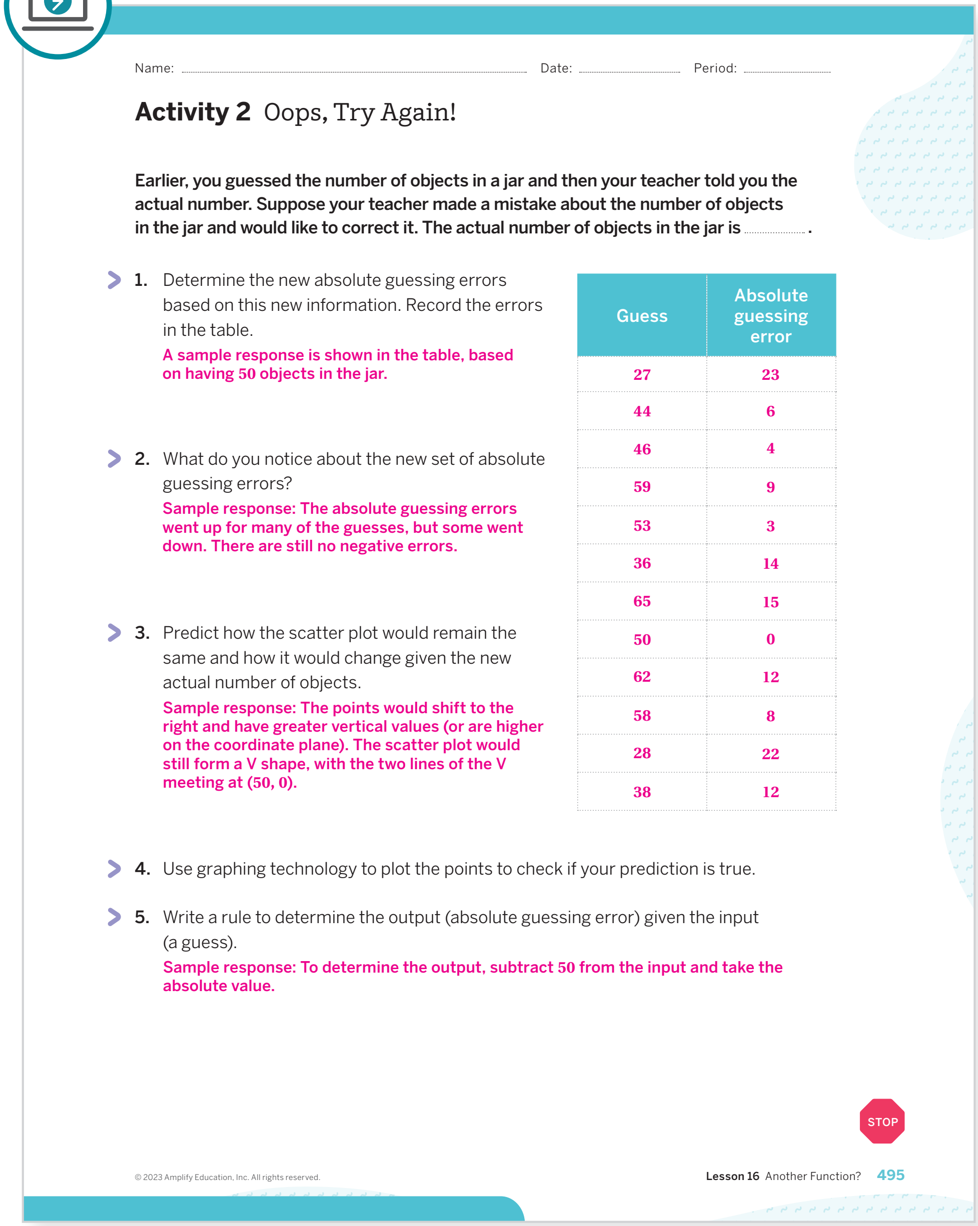
During the Connect, ask students to share their responses to Problem 3. Probe for student understanding by asking these follow-up questions:

- "Does it make sense that there is only one absolute guessing error for each guess? Explain your thinking."
- "What if the input was the absolute guessing error and the output was the guess. Would the guess be a function of the absolute guessing error? Explaining your thinking." No, because one absolute guessing error may have multiple guesses since guesses can be higher or lower than the actual value (with equal absolute guessing errors).

Independent | 15 min

Activity 2 Oops, Try Again!

Students see how changing the target number changes the absolute guessing errors and informally explore translations of the graph of the data, paying attention to the structure of the graphs.



Name: ______________ Date: __________ Period: __________

Activity 2 Oops, Try Again!

Earlier, you guessed the number of objects in a jar and then your teacher told you the actual number. Suppose your teacher made a mistake about the number of objects in the jar and would like to correct it. The actual number of objects in the jar is ________.

1. Determine the new absolute guessing errors based on this new information. Record the errors in the table.
   A sample response is shown in the table, based on having 50 objects in the jar.

Guess	Absolute guessing error
27	23
44	6
46	4
59	9
53	3
36	14
65	15
50	0
62	12
58	8
28	22
38	12

2. What do you notice about the new set of absolute guessing errors?
   Sample response: The absolute guessing errors went up for many of the guesses, but some went down. There are still no negative errors.

3. Predict how the scatter plot would remain the same and how it would change given the new actual number of objects.
   Sample response: The points would shift to the right and have greater vertical values (or are higher on the coordinate plane). The scatter plot would still form a V shape, with the two lines of the V meeting at (50, 0).

4. Use graphing technology to plot the points to check if your prediction is true.

5. Write a rule to determine the output (absolute guessing error) given the input (a guess).
   Sample response: To determine the output, subtract 50 from the input and take the absolute value.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Provide access to graphing technology. Read the scenario together as a class and have students share their thoughts about how the values of the absolute guessing errors will change.

2 Monitor

Help students get started by referring back to the Warm-up to review calculating the absolute guessing error.

Look for points of confusion:

- **Thinking that each point only translates to the right.** Have students plot their new data on the scatter plot from Activity 1. Have them compare the two scatter plots, and explain the shift of a point for one specific guess.

Look for productive strategies:

- Plotting points of the scatter plot on the same coordinate plane in Activity 1 to compare values.

3 Connect

Display both scatter plots on the same coordinate plane.

Have individual students share their observations of the new scatter plot.

Highlight that the scatter plot is still in a "V" shape, with the two lines formed by the points each having the same slope.

Ask, "Why did the points to the right of $(50, 0)$ shift up, but the points to the left of $(50, 0)$ shift down?" Because the actual number of objects increased, guesses greater than the new actual value are now closer to the new actual value, while values less than the new actual value are now farther away.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students plot their data on the same scatter plot from Activity 1, but using a different color, to more easily compare the new data set to the original data set.

Extension: *Math Enrichment*

Have students describe a scenario in which the points in the scatter plot would all shift to the left by the same amount and all shift up by the same amount. The actual value decreases and all of the guesses increase by the same amount.

Math Language Development

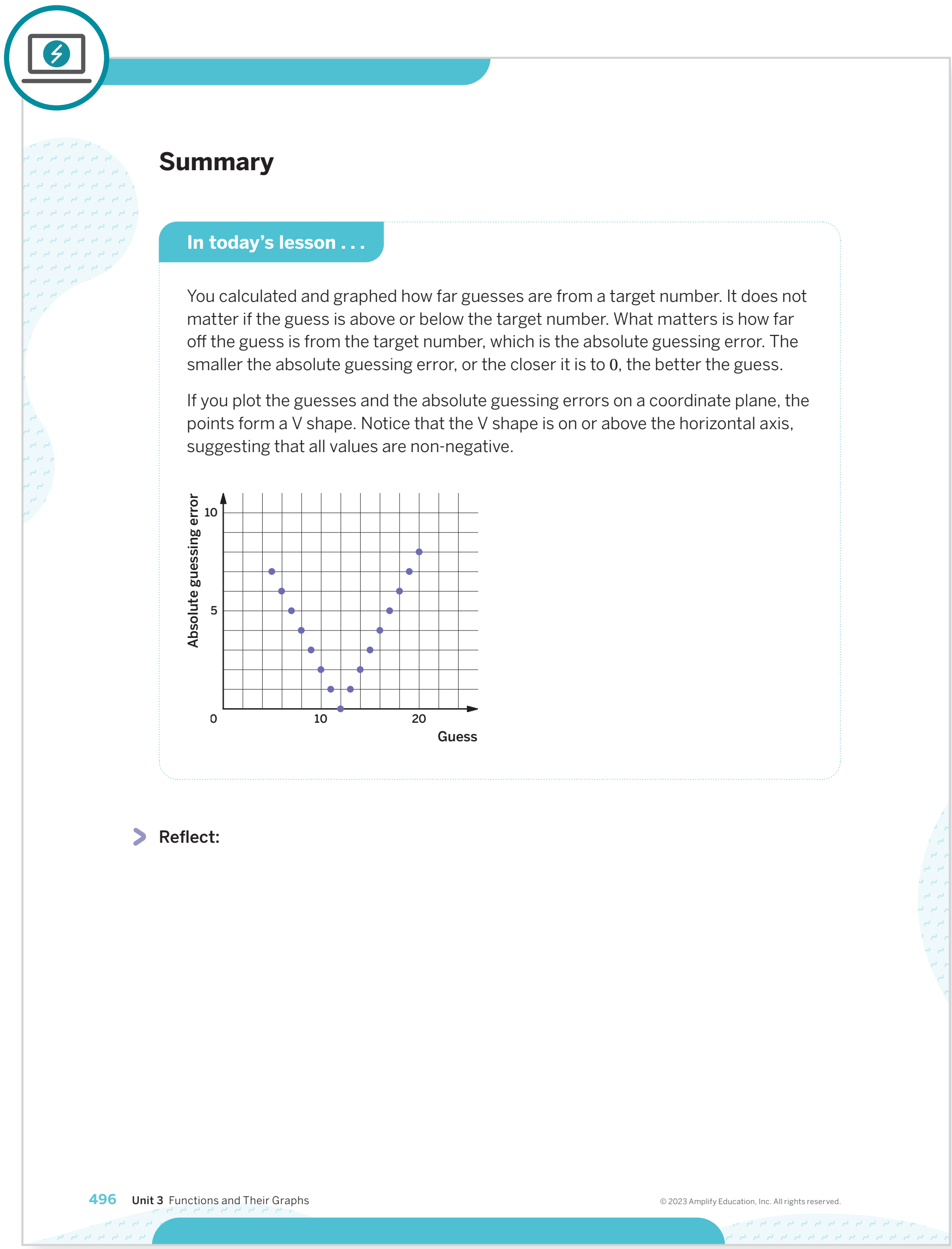
MLR2: Collect and Display

Before the Connect, have students share their observations of the new data set with a partner. Circulate and listen for the phrases students use to describe the new data set, such as "form a V," "no negative numbers," and "shift to the right." Add these phrases to the class display and encourage students to use these phrases to refine their observations during the whole-class discussion.

Whole Class | 5 min

Summary

Review and synthesize how the concept of distance is used to create a function of absolute guessing errors.



Summary

In today's lesson . . .

You calculated and graphed how far guesses are from a target number. It does not matter if the guess is above or below the target number. What matters is how far off the guess is from the target number, which is the absolute guessing error. The smaller the absolute guessing error, or the closer it is to 0, the better the guess.

If you plot the guesses and the absolute guessing errors on a coordinate plane, the points form a V shape. Notice that the V shape is on or above the horizontal axis, suggesting that all values are non-negative.

Reflect:

496 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the scatter plot of the students' guesses and the absolute guessing errors from Activity 1.

Have students share why absolute guessing error values cannot be negative.

Highlight that an over or underestimate of 5 will both result in an absolute guessing error of 5. What matters is the distance from the actual number, not whether the guess is above or below the actual number.

Ask:

- "What affects the size of the absolute guessing error?" The farther a guess is from the actual number, the greater the absolute guessing error. The closer it is to the actual number, the smaller the absolute guessing error.
- "Is the absolute guessing error a function of the guess? Why or why not?" Yes; For every guess, there is one absolute guessing error.
- "Is there a rule you can write to define the relationship between the input and output values of this function?" The output value is the distance of a guess from the actual number of objects, which I can express as a difference: output = input – actual number.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

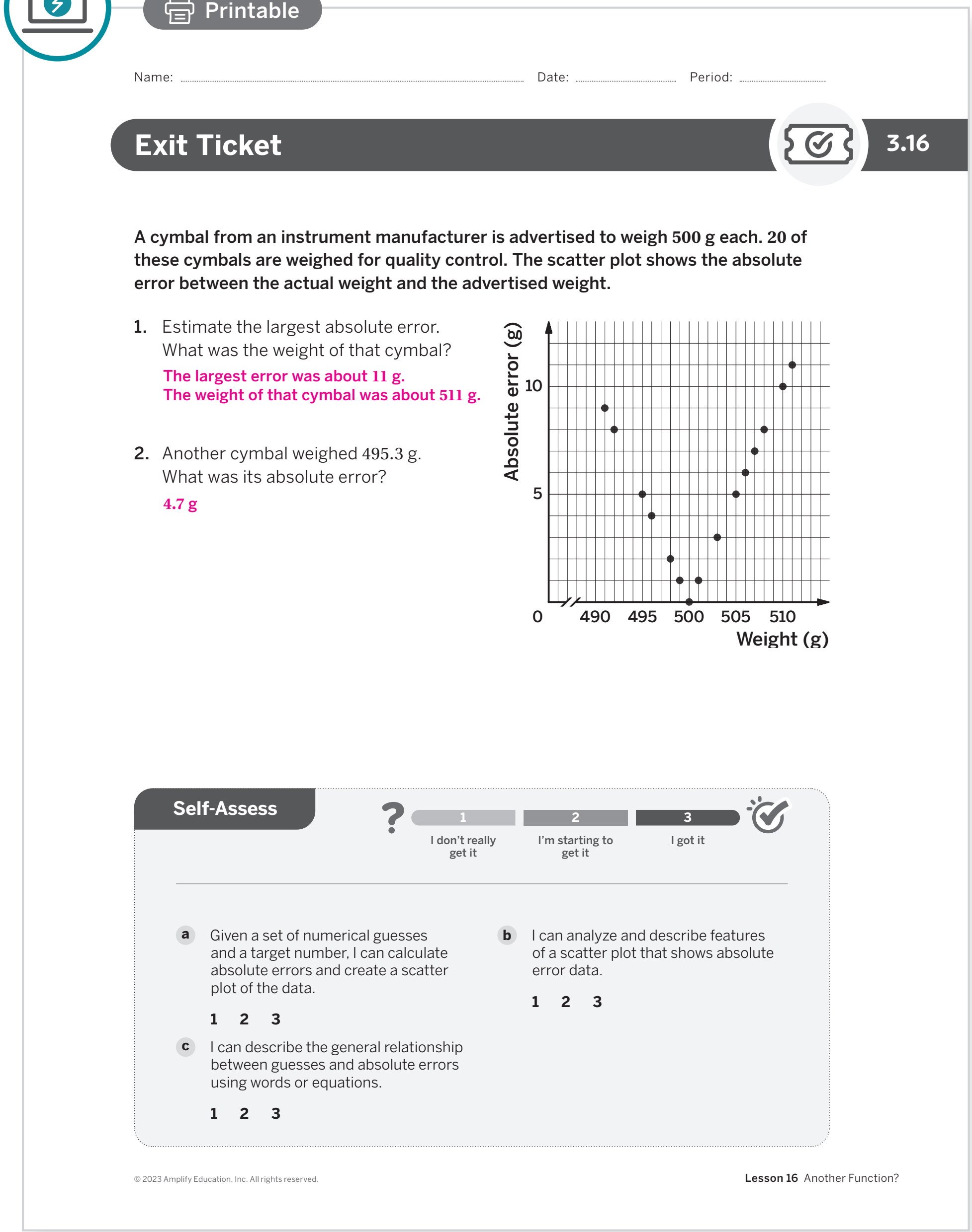
- "Why are all absolute guessing error values positive?"
- "How does a change in the actual value change the scatter plot?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by calculating and interpreting the graph of absolute errors.

Printable



Name: ______________________ Date: ____________ Period: ____________

Exit Ticket 3.16

A cymbal from an instrument manufacturer is advertised to weigh 500 g each. 20 of these cymbals are weighed for quality control. The scatter plot shows the absolute error between the actual weight and the advertised weight.

1. Estimate the largest absolute error. What was the weight of that cymbal?
   The largest error was about 11 g. The weight of that cymbal was about 511 g.

2. Another cymbal weighed 495.3 g. What was its absolute error?
   4.7 g

Self-Assess

a. Given a set of numerical guesses and a target number, I can calculate absolute errors and create a scatter plot of the data.
   1 2 3

b. I can analyze and describe features of a scatter plot that shows absolute error data.
   1 2 3

c. I can describe the general relationship between guesses and absolute errors using words or equations.
   1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 16 Another Function?

Success looks like . . .

- **Language Goal:** Analyzing and describing features of a scatter plot that relates guesses and absolute errors. **(Reading and Writing)**
  - » Determining values related to an absolute error from the scatter plot in Problems 1 and 2.
- **Language Goal:** Generalizing the relationship between guesses and absolute errors. **(Reading and Writing)**
- **Goal:** Given a set of numerical guesses and a target number, calculating absolute errors and creating a scatter plot of the data.

Suggested next steps

If students inaccurately calculate the weight of the cymbal in Problem 1, consider:

- Reviewing how to read and use the scatter plot to determine the value of a guess in Activity 1.
- Assigning Practice Problems 1–3.
- Asking, "How could you use the absolute error and advertised weight to determine the weight of a cymbal?"

If students inaccurately calculate the absolute error in Problem 2, consider:

- Reviewing how to calculate the absolute guessing error from the Warm-up.
- Assigning Practice Problems 1–3.
- Asking, "What is the meaning of the phrase guessing error?"

Professional Learning

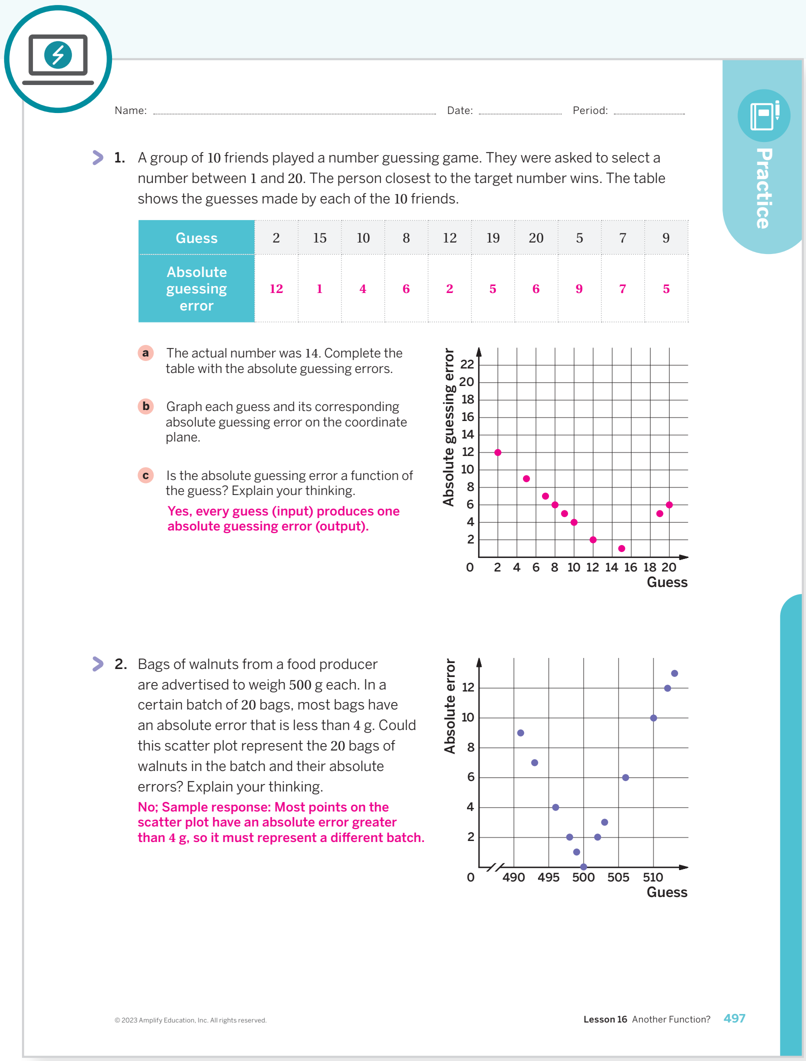
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective?
- How did calculating and plotting absolute guessing errors set students up to develop the definition of the absolute value function? What might you change for the next time you teach this lesson?

Practice

Independent



Name: .... Date: .... Period: ....

1. A group of 10 friends played a number guessing game. They were asked to select a number between 1 and 20. The person closest to the target number wins. The table shows the guesses made by each of the 10 friends.

Guess	2	15	10	8	12	19	20	5	7	9
Absolute guessing error	12	1	4	6	2	5	6	9	7	5

a. The actual number was 14. Complete the table with the absolute guessing errors.

b. Graph each guess and its corresponding absolute guessing error on the coordinate plane.

c. Is the absolute guessing error a function of the guess? Explain your thinking.

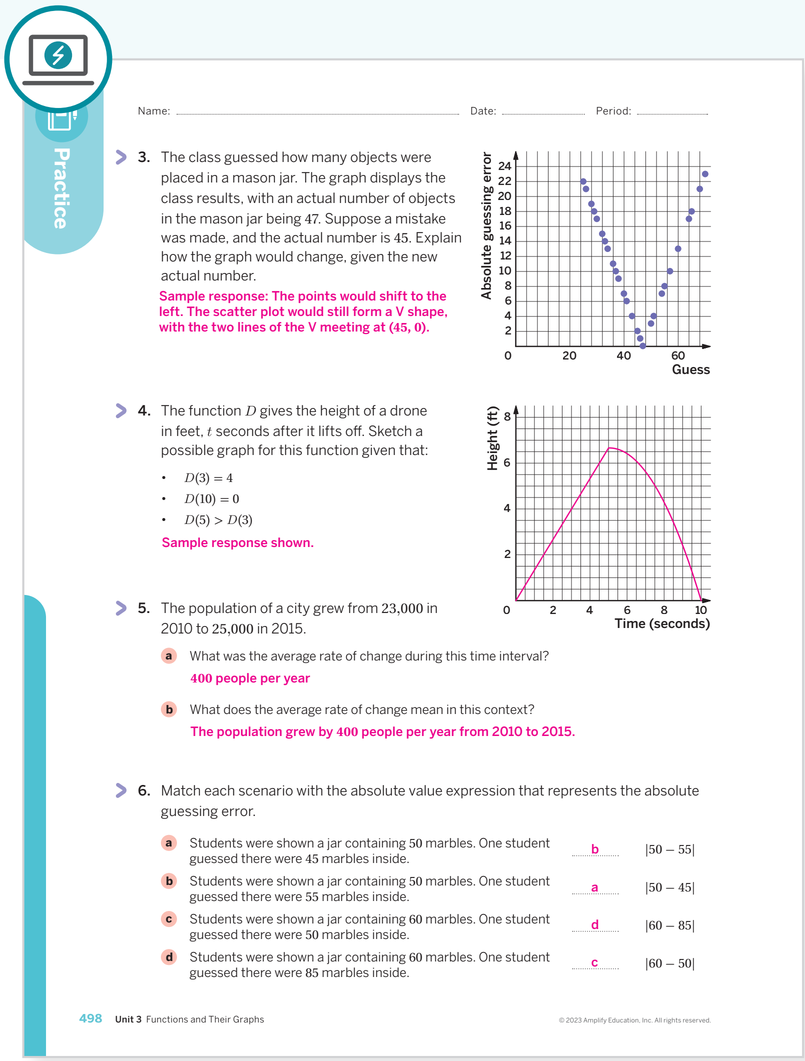
Yes, every guess (input) produces one absolute guessing error (output).

2. Bags of walnuts from a food producer are advertised to weigh 500 g each. In a certain batch of 20 bags, most bags have an absolute error that is less than 4 g. Could this scatter plot represent the 20 bags of walnuts in the batch and their absolute errors? Explain your thinking.

No; Sample response: Most points on the scatter plot have an absolute error greater than 4 g, so it must represent a different batch.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 16 Another Function? 497



Name: .... Date: .... Period: ....

3. The class guessed how many objects were placed in a mason jar. The graph displays the class results, with an actual number of objects in the mason jar being 47. Suppose a mistake was made, and the actual number is 45. Explain how the graph would change, given the new actual number.

Sample response: The points would shift to the left. The scatter plot would still form a V shape, with the two lines of the V meeting at (45, 0).

4. The function D gives the height of a drone in feet, t seconds after it lifts off. Sketch a possible graph for this function given that:

- $D(3) = 4$
- $D(10) = 0$
- $D(5) > D(3)$

Sample response shown.

5. The population of a city grew from 23,000 in 2010 to 25,000 in 2015.

a. What was the average rate of change during this time interval?

400 people per year

b. What does the average rate of change mean in this context?

The population grew by 400 people per year from 2010 to 2015.

6. Match each scenario with the absolute value expression that represents the absolute guessing error.

a. Students were shown a jar containing 50 marbles. One student guessed there were 45 marbles inside. **b** $|50 - 55|$

b. Students were shown a jar containing 50 marbles. One student guessed there were 55 marbles inside. **a** $|50 - 45|$

c. Students were shown a jar containing 60 marbles. One student guessed there were 50 marbles inside. **d** $|60 - 85|$

d. Students were shown a jar containing 60 marbles. One student guessed there were 85 marbles inside. **c** $|60 - 50|$

498 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 4	2
	5	Unit 3 Lesson 7	2
Formative ⚡	6	Unit 3 Lesson 17	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 3 | **LESSON 17**

Absolute Value Functions

Let's investigate distance as a function.

Focus

Goals

1. **Language Goal:** Analyze and describe the effects of adding a constant term to an expression defining an absolute value function. **(Speaking and Listening, Reading and Writing)**
2. **Language Goal:** Define an absolute value function in terms of the distance of the input value from 0. **(Reading and Writing)**
3. **Language Goal:** Interpret an absolute value function described in words or in function notation, and create a table of values and a graph to represent the function. **(Reading and Writing)**

Coherence

Today

Students define the absolute value function in terms of the distance of a number from 0 on the number line. They also see that, because the graph of the absolute value function is composed of two linear pieces that form a "V" shape, the same function can also be defined as a piecewise function. Students also encounter graphs of functions that have been translated horizontally or vertically and make sense of how these translations are represented in the function.

Previously

In Lesson 16, students computed and plotted absolute errors of a set of data to explore the absolute value function, recognizing that each absolute error is a distance from a target number.

Coming Soon

In Lesson 18, students are introduced to inverse functions in the context of encoding and decoding messages.

Rigor

- Students build **conceptual understanding** of transformations of the absolute value function.
- Students graph absolute value functions to build **procedural fluency**.
- Students **apply** absolute value functions to a skyline design.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	20 min	15 min	5 min	5 min
Independent	Pairs	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF, *How Close Were the Guesses*, (from Lesson 16, as needed)
- graphing technology

Math Language Development

New words

- ***absolute value function***
- ***vertex***

Review words

- *absolute value*
- *domain*
- *piecewise function*
- *range*

Amps | Featured Activity

Activity 3
Graphing a Skyline

Students use the outline of the Atlanta skyline to examine how constants in an absolute value function affect the position and shape of the graph.

Building Math Identity and Community

Connecting to Mathematical Practices

The additional type of function, an absolute value function, might begin to overwhelm students who are just beginning to feel confident with linear and piecewise functions. Point out that the structure of an absolute value function is like a piecewise function made of two linear functions. Have students create a chart where they list the types of functions that they know and the key features of each. Seeing commonalities will help reduce the stress that comes from new material.

Modifications to Pacing

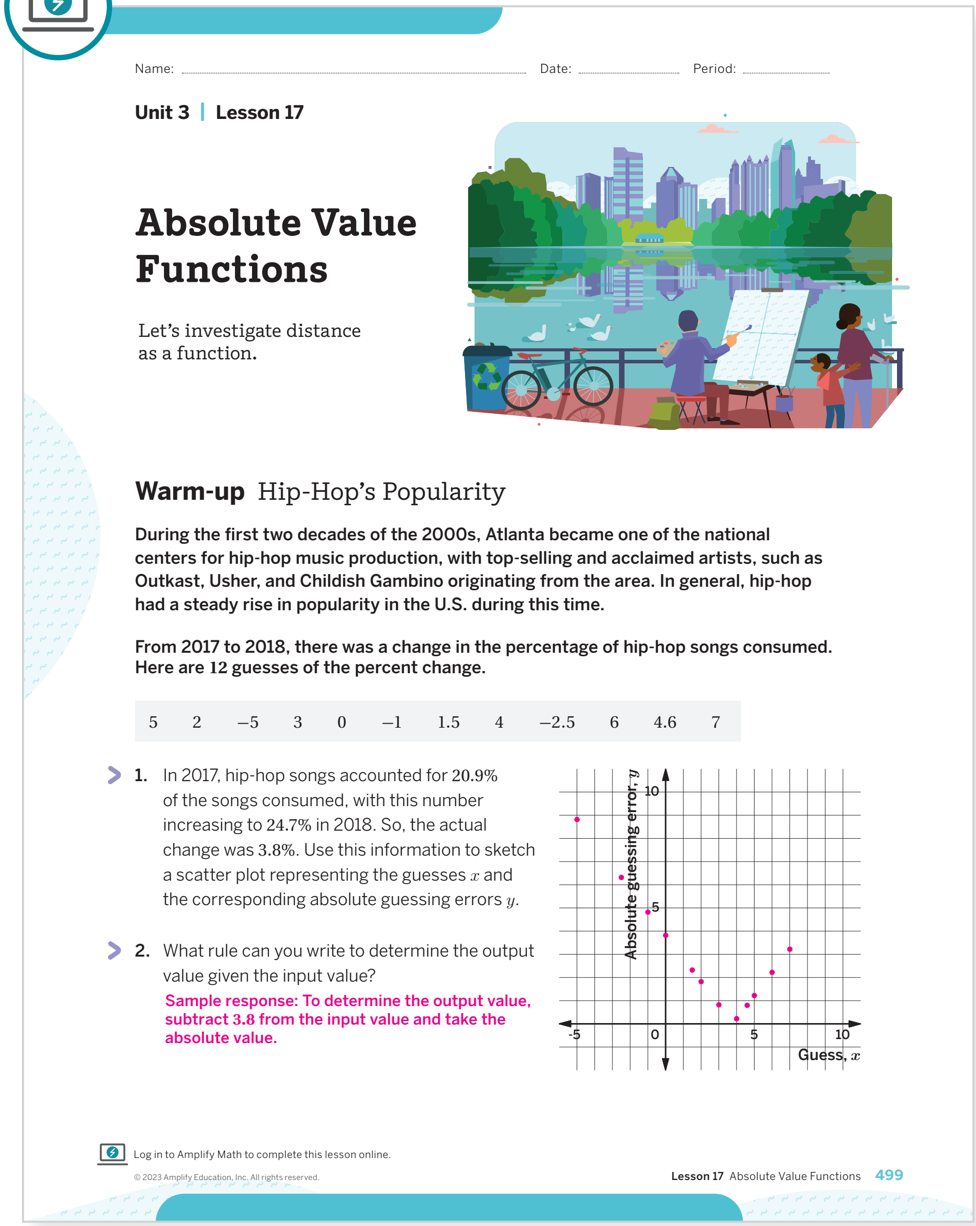
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 1 may be omitted. Provide the completed scatter plot.
- In **Activity 1**, provide half of the table already completed.
- Optional **Activity 3** may be omitted.

Independent | 5 min

Warm-up Hip-Hop's Popularity

Students informally describe the absolute value function as they explain a rule to determine the corresponding absolute guessing error when given a guess.



Name: ______________________ Date: ____________ Period: ____________

Unit 3 | **Lesson 17**

Absolute Value Functions

Let's investigate distance as a function.

Warm-up Hip-Hop's Popularity

During the first two decades of the 2000s, Atlanta became one of the national centers for hip-hop music production, with top-selling and acclaimed artists, such as Outkast, Usher, and Childish Gambino originating from the area. In general, hip-hop had a steady rise in popularity in the U.S. during this time.

From 2017 to 2018, there was a change in the percentage of hip-hop songs consumed. Here are 12 guesses of the percent change.

5	2	-5	3	0	-1	1.5	4	-2.5	6	4.6	7

1. In 2017, hip-hop songs accounted for 20.9% of the songs consumed, with this number increasing to 24.7% in 2018. So, the actual change was 3.8%. Use this information to sketch a scatter plot representing the guesses x and the corresponding absolute guessing errors y.

2. What rule can you write to determine the output value given the input value?
   Sample response: To determine the output value, subtract 3.8 from the input value and take the absolute value.

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 17 Absolute Value Functions 499

1 Launch

Read the narrative together. Highlight that the actual change and the guesses given will be used to determine the absolute guessing error, as they did in the previous lesson.

2 Monitor

Help students get started by calculating the guessing error for the first value.

Look for points of confusion:

- **Using negative values for the absolute guessing error.** Explain how 3 and 4.6 both have an absolute guessing error of 0.8 because they both are 0.8 units away from the actual value, 3.8. Have students come up with their own set of values with the same absolute guessing error.

Look for productive strategies:

- Using the symmetry of the sketch to check that each point is plotted accurately.

3 Connect

Have individual students share the rule they wrote in Problem 2.

Highlight that guesses that are the same distance away from the guess will have the same absolute guessing error, which is the reason the scatter plot is symmetric.

Ask, "How is the scatter plot for this data similar and different from the scatter plot for the absolute guessing errors from an earlier lesson?" They are similar because the points still form a "V" shape and there are still no negative values of y. They are different because the two parts of the "V" now meet at 3.8.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider providing additional copies of the Warm-up PDF, *How Close Were the Guesses*, from Lesson 16, to help students organize their thinking as they determine the absolute guessing error. They can use the table provided on the PDF, which scaffolds the process for determining the absolute guessing error.

Power-up

To power up students' ability to determine the absolute value, have students complete:

Recall that *absolute value* means the distance from zero. Determine whether each statement is *true* or *false*.

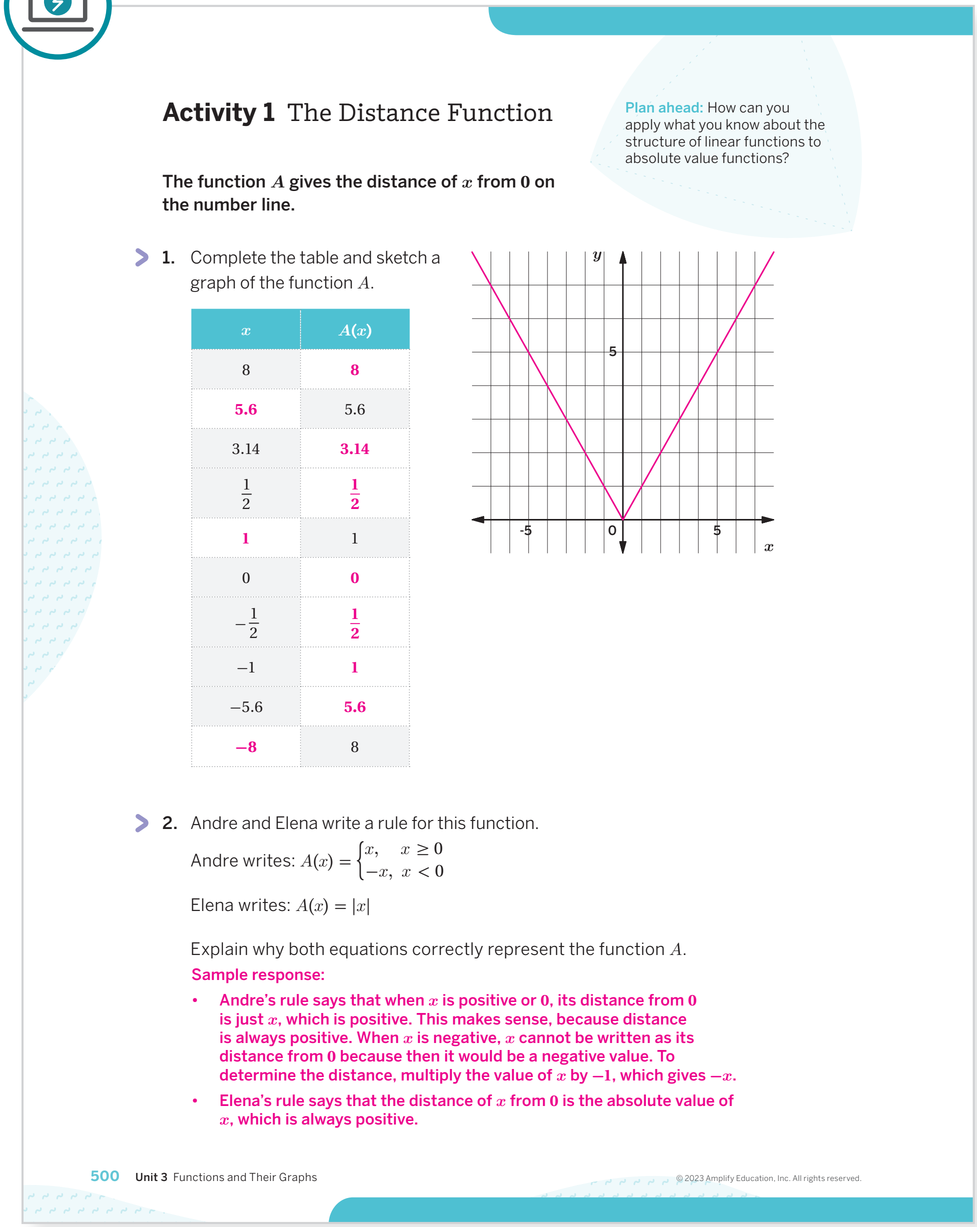
1. $|-5| = \frac{1}{5}$ False
2. $|-5| = 5$ True
3. $|-5| = -5$ False
4. $|-5| = |5|$ True
5. $|5| = 5$ True

Use: Before the Warm-up
Informed by: Performance on Lesson 16, Practice Problem 6

Pairs | 15 min

Activity 1 The Distance Function

Students examine and compare the absolute value function to a piecewise function that takes an input value and gives its distance from the origin as the output.



Activity 1 The Distance Function

Plan ahead: How can you apply what you know about the structure of linear functions to absolute value functions?

The function A gives the distance of x from 0 on the number line.

1. Complete the table and sketch a graph of the function A.

x	$A(x)$
8	**8**
5.6	5.6
3.14	**3.14**
$\frac{1}{2}$	**$\frac{1}{2}$**
1	1
0	**0**
$-\frac{1}{2}$	**$\frac{1}{2}$**
-1	**1**
-5.6	**5.6**
-8	8

2. Andre and Elena write a rule for this function.

Andre writes: $A(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Elena writes: $A(x) = |x|$

Explain why both equations correctly represent the function A.

Sample response:

- **Andre's rule says that when x is positive or 0, its distance from 0 is just x, which is positive. This makes sense, because distance is always positive. When x is negative, x cannot be written as its distance from 0 because then it would be a negative value. To determine the distance, multiply the value of x by -1, which gives $-x$.**
- **Elena's rule says that the distance of x from 0 is the absolute value of x, which is always positive.**

500 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Give students a few minutes of independent work time, then time to share their responses with their partner. Follow with a whole-class discussion.

2 Monitor

Help students get started by providing the function value when $x = -1$, and explaining how this value is a distance of 1 from 0.

Look for points of confusion:

- **Interpreting $-x$ in the piecewise function as negative output values.** Have students test negative x values in the expression to see that the output value is positive.

Look for productive strategies:

- Graphing the piecewise function over the sketch to determine if they represent the same function.

3 Connect

Have individual students share their graph of function A.

Define the ***absolute value function*** as a function that gives the distance of an input value from a certain value.

Highlight that the graph of function A is a "V" shape with the two lines meeting at the point $(0, 0)$, which is the minimum of the graph. This point is called the vertex of the graph.

Define the term ***vertex*** as the point where the graph changes direction.

Ask, "Why is the absolute value function also considered a piecewise function?" Because it can be represented as a piecewise function with more than one rule to be applied to different parts of the domain to get the output value.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

For Problem 2, consider providing students with a table (or suggest they create their own) that they can use to organize their thinking as they analyze Andre's and Elena's function rules.

Extension: *Math Enrichment*

Have students complete the following problem:
An absolute value function determines the distance from the point $(2, 0)$, rather than from the origin. Write a piecewise function that could represent this absolute value function. $f(x) = \begin{cases} x - 2, & x \geq 2 \\ -(x - 2), & x < 2 \end{cases}$

Math Language Development

MLR1: Stronger and Clearer Each Time

After students write a draft response to Problem 2, have them meet with 2–3 partners to give and receive feedback. Encourage reviewers to ask clarifying questions such as:

- "Does the response include how *both* of the rules correctly describe the function?"
- "Does the response include discussion of how both rules show that the distance is always positive?"

Have students use the feedback to improve their responses.

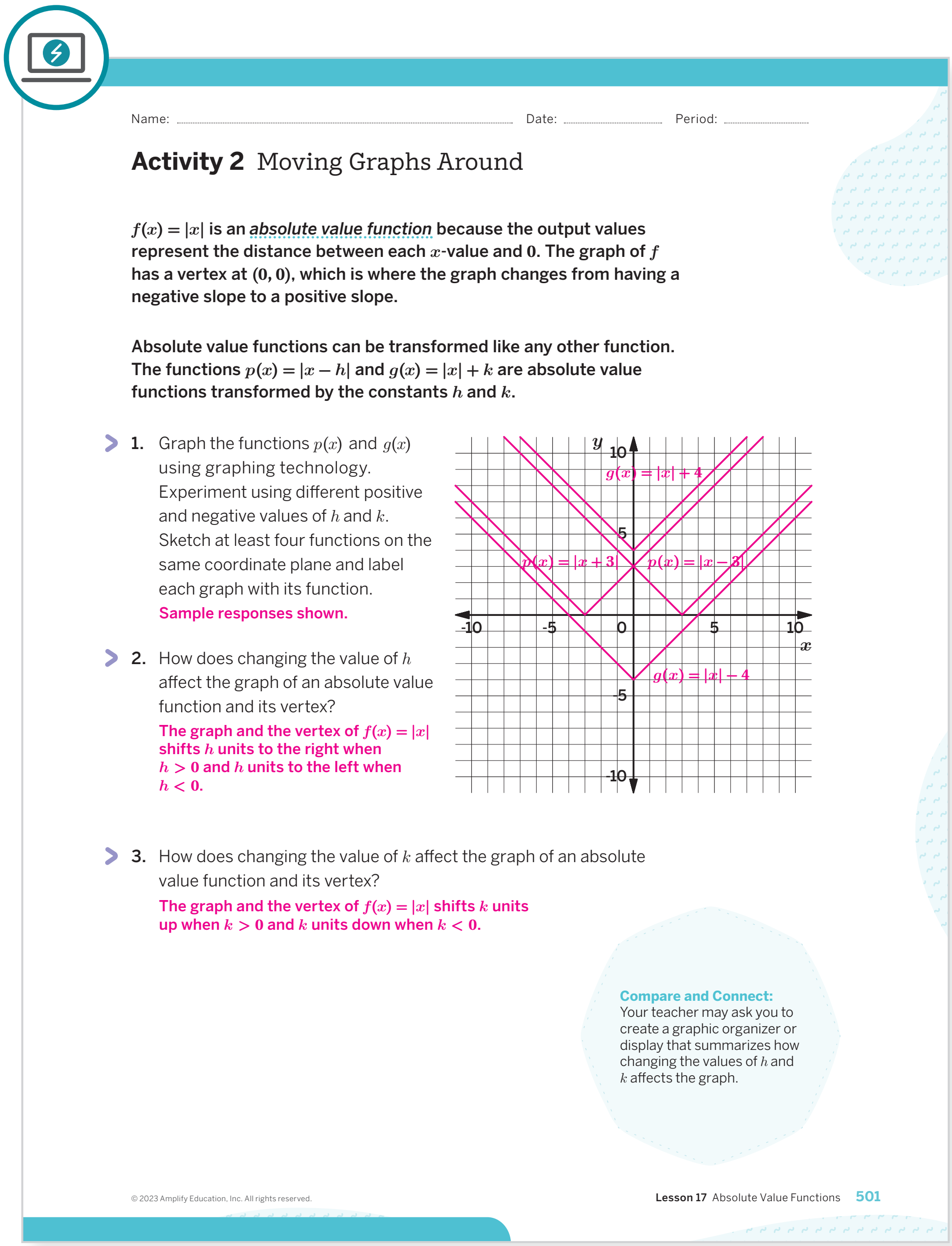
English Learners

Allow students who speak the same primary language to provide feedback to each other.

Pairs | 20 min

Activity 2 Moving Graphs Around

Students examine and compare the structure of the absolute value function to a piecewise function that takes an input value and gives its distance from the origin as the output.



Name: ________ Date: ________ Period: ________

Activity 2 Moving Graphs Around

$f(x) = |x|$ is an *absolute value function* because the output values represent the distance between each x-value and 0. The graph of f has a vertex at (0, 0), which is where the graph changes from having a negative slope to a positive slope.

Absolute value functions can be transformed like any other function. The functions $p(x) = |x - h|$ and $g(x) = |x| + k$ are absolute value functions transformed by the constants h and k.

1. Graph the functions $p(x)$ and $g(x)$ using graphing technology. Experiment using different positive and negative values of h and k. Sketch at least four functions on the same coordinate plane and label each graph with its function.
   Sample responses shown.

2. How does changing the value of h affect the graph of an absolute value function and its vertex?
   The graph and the vertex of $f(x) = |x|$ shifts h units to the right when $h > 0$ and h units to the left when $h < 0$.

3. How does changing the value of k affect the graph of an absolute value function and its vertex?
   The graph and the vertex of $f(x) = |x|$ shifts k units up when $k > 0$ and k units down when $k < 0$.

Compare and Connect: Your teacher may ask you to create a graphic organizer or display that summarizes how changing the values of h and k affects the graph.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 17 Absolute Value Functions 501

1 Launch

Students remain in pairs. Read the narrative aloud. Choose values of h and k and show students a graph of each absolute value function.

2 Monitor

Help students get started by providing them with a value of h and a value of k and their corresponding functions to graph.

Look for points of confusion

- **Thinking that a negative value of h moves the graph to the right.** Have students graph multiple absolute value functions with negative values of h to see how each graph moves further to the left as h decreases.
- **Thinking that the output of an absolute value function can never be negative.** Have students make a table of values for $f(x) = |x|$, then have them subtract the value of k from each output value to see how some output values become negative.

Look for productive strategies:

- Using opposite values for h and k to determine the effects on the graph.
- Identifying the vertex of the absolute value functions to match to a graph.

Activity 2 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with sample values of h and k to use in Problem 1, such as the following.

$g(x) = |x| - 4$, $g(x) = |x| + 4$
$h(x) = |x - 3|$, $h(x) = |x + 3|$

Suggest that students use colored pencils to color-code each equation they write and its corresponding graph.

Math Language Development

MLR7: Compare and Connect

Have students create a visual display that summarizes how changing the values of h and k affects the graph. Have them add notes or details to their displays to help communicate their thinking. Begin the Connect discussion by selecting the creators to share their displays with the class.

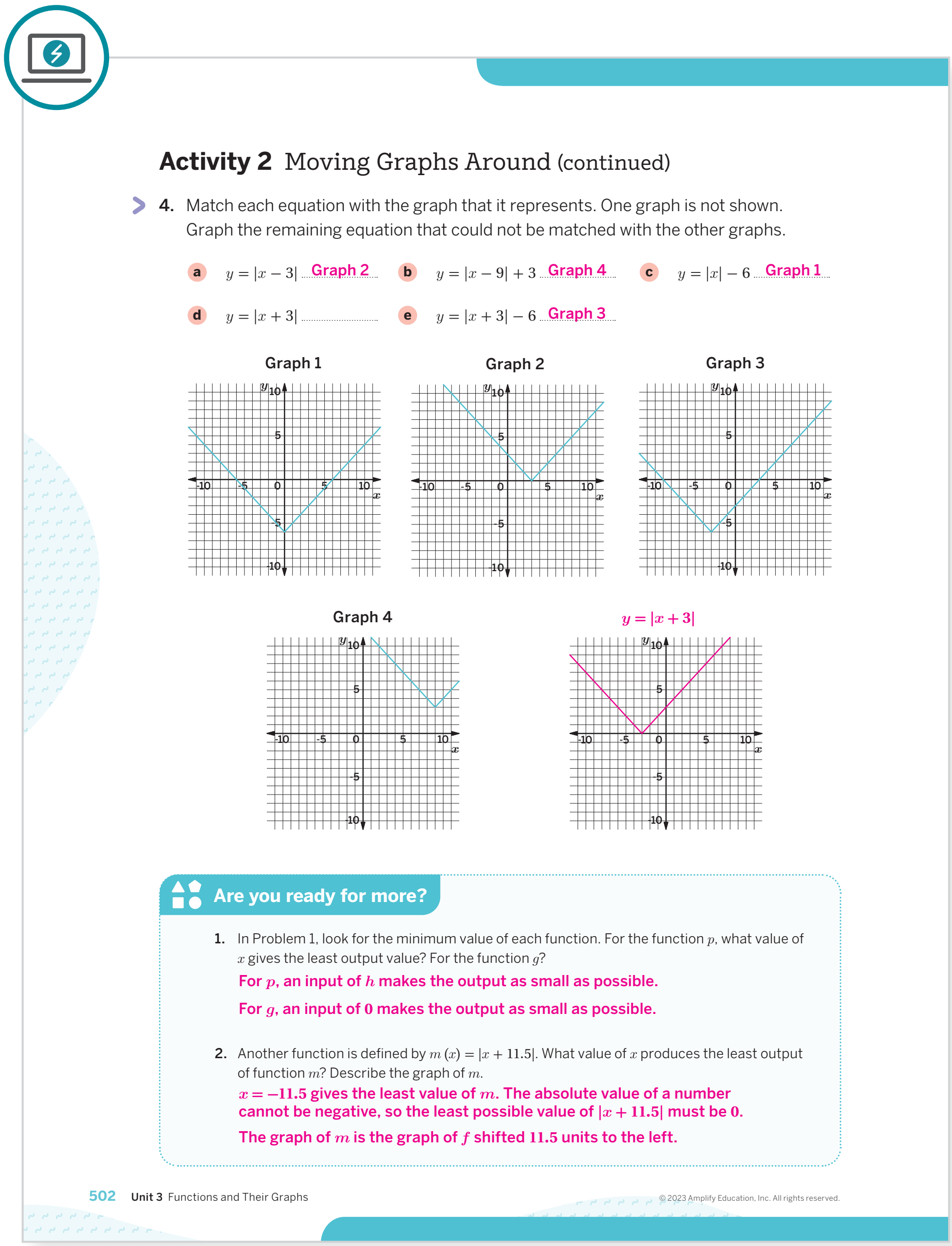
English Learners

As students share their displays, use gestures and pointing to emphasize key features of the graph and to help students connect the language used to the visual display.

Pairs | 20 min

Activity 2 Moving Graphs Around (continued)

Students examine and compare the structure of the absolute value function to a piecewise function that takes an input value and gives its distance from the origin as the output.



Activity 2 Moving Graphs Around (continued)

4. Match each equation with the graph that it represents. One graph is not shown. Graph the remaining equation that could not be matched with the other graphs.

a $y = |x - 3|$ Graph 2

b $y = |x - 9| + 3$ Graph 4

c $y = |x| - 6$ Graph 1

d $y = |x + 3|$

e $y = |x + 3| - 6$ Graph 3

Are you ready for more?

1. In Problem 1, look for the minimum value of each function. For the function p, what value of x gives the least output value? For the function g?

   For p, an input of h makes the output as small as possible.

   For g, an input of 0 makes the output as small as possible.

2. Another function is defined by $m\,(x) = |x + 11.5|$. What value of x produces the least output of function m? Describe the graph of m.

   $x = -11.5$ gives the least value of m. The absolute value of a number cannot be negative, so the least possible value of $|x + 11.5|$ must be 0.

   The graph of m is the graph of f shifted 11.5 units to the left.

502 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Display the correct matches of functions and graphs for Problem 4.

Have individual students share their explanation of how values of h and k affect the graph and vertex of the absolute value function.

Highlight that because h moves the graph left and right, and k up and down, the values of these constants help determine the coordinates of the vertex. The vertex of the absolute value function is (h, k).

- "For the absolute value function $h(x) = |x - 2|$, what is the piecewise function that also defines h?"

  $$h(x) = \begin{cases} x - 2, & x \geq 2 \\ -(x - 2), & x < 2 \end{cases}$$

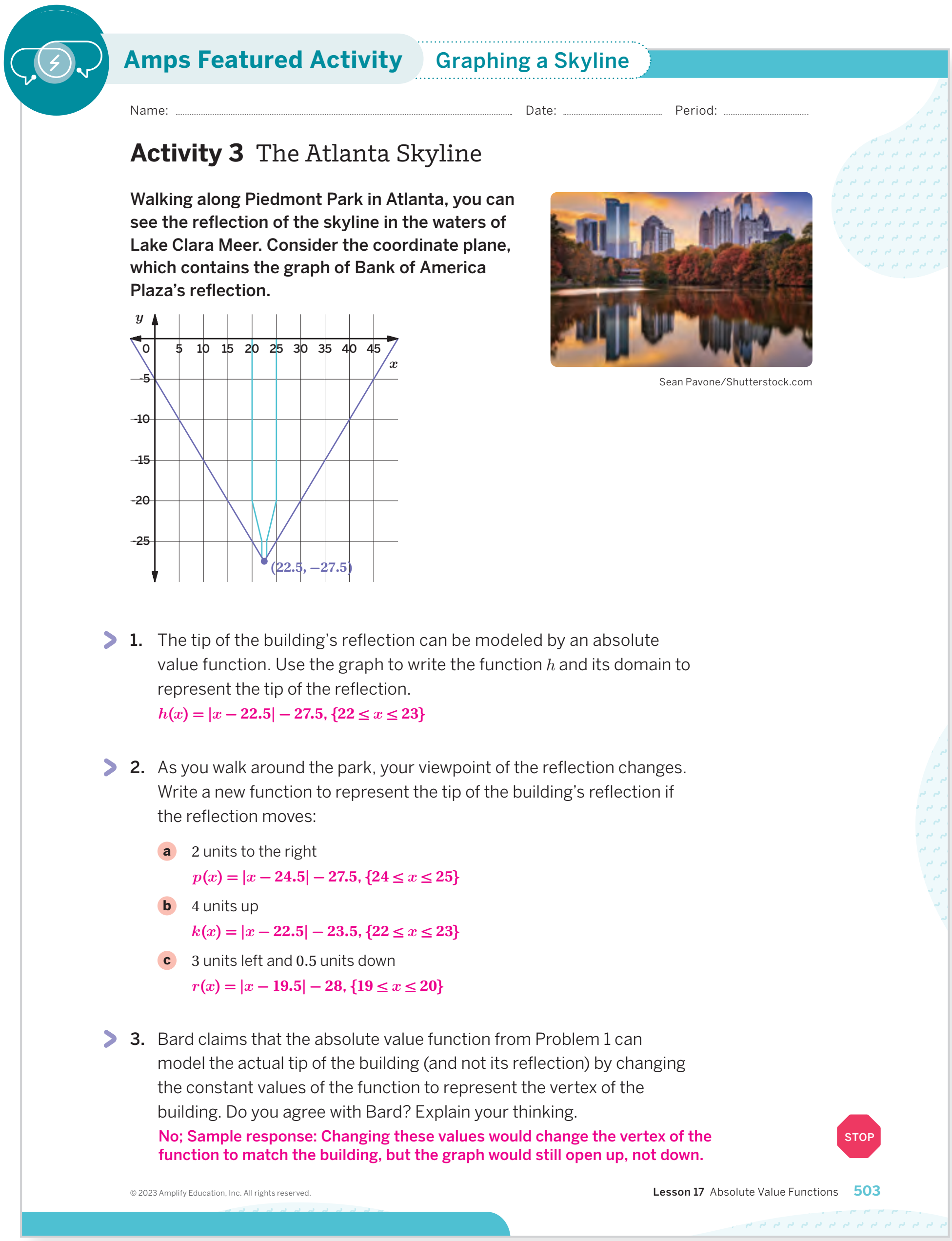
- "Why does subtracting 2 from x move the graph 2 units to the right, and why does adding 2 to x moves it 2 units to the left?" Subtracting 2 from x means determining the distance of a value of x from 2, moving all the original points of $f(x) = |x|$ 2 units to the right. Adding 2 to x means determining the distance of a value of x from –2, moving all the original points of $f(x) = |x|$ 2 units to the left.

Optional

Independent | 15 min

Activity 3 The Atlanta Skyline

Students construct a function to model the Atlanta skyline, attending to precision identifying the domain, and making use of structure as they write new functions representing different viewpoints.



Amps Featured Activity Graphing a Skyline

Name: ______ Date: ______ Period: ______

Activity 3 The Atlanta Skyline

Walking along Piedmont Park in Atlanta, you can see the reflection of the skyline in the waters of Lake Clara Meer. Consider the coordinate plane, which contains the graph of Bank of America Plaza's reflection.

Sean Pavone/Shutterstock.com

1. The tip of the building's reflection can be modeled by an absolute value function. Use the graph to write the function h and its domain to represent the tip of the reflection.
$h(x) = |x - 22.5| - 27.5, \{22 \leq x \leq 23\}$

2. As you walk around the park, your viewpoint of the reflection changes. Write a new function to represent the tip of the building's reflection if the reflection moves:
   - a 2 units to the right
   $p(x) = |x - 24.5| - 27.5, \{24 \leq x \leq 25\}$
   - b 4 units up
   $k(x) = |x - 22.5| - 23.5, \{22 \leq x \leq 23\}$
   - c 3 units left and 0.5 units down
   $r(x) = |x - 19.5| - 28, \{19 \leq x \leq 20\}$

3. Bard claims that the absolute value function from Problem 1 can model the actual tip of the building (and not its reflection) by changing the constant values of the function to represent the vertex of the building. Do you agree with Bard? Explain your thinking.
No; Sample response: Changing these values would change the vertex of the function to match the building, but the graph would still open up, not down.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 17 Absolute Value Functions 503

1 Launch

Read the narrative together as a class. Identify the buildings used in the activity in the skyline. Ask students where they think an absolute value function could be used when graphing the skyline.

2 Monitor

Help students get started by having them mark on the graph the values of x that contain the domain of the absolute value function used.

Look for points of confusion:

- **Misusing a negative coefficient for their absolute value functions.** Have students graph $f(x) = |x|$ and $f(x) = -|x|$ and ask, "How does the negative sign affect the graph?"
- **Using an incorrect constant value for the absolute value function.** Have students graph their absolute value function to see how this value changes the vertex.

Look for productive strategies:

- Annotating the coordinates of the vertex of each piece on the graph to write the absolute value functions.
- Entering all functions into their graphing technology to check the accuracy of their functions.

3 Connect

Display the functions and their domains.

Have individual students share how they determined the constant values used in each absolute value function.

Highlight that the constant values help to determine the vertex of the absolute value function, and the coefficient helps to determine the slope of each piece of the absolute value function.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can use a digital outline of the Atlanta skyline to examine how constants in an absolute value function affect the position and shape of the graph.

Accessibility: *Guide Processing and Visualization*

Have students rotate their Student Edition upside down so that they can visualize the outline of the building before it is reflected in the lake.

Math Language Development

MLR7: Compare and Connect

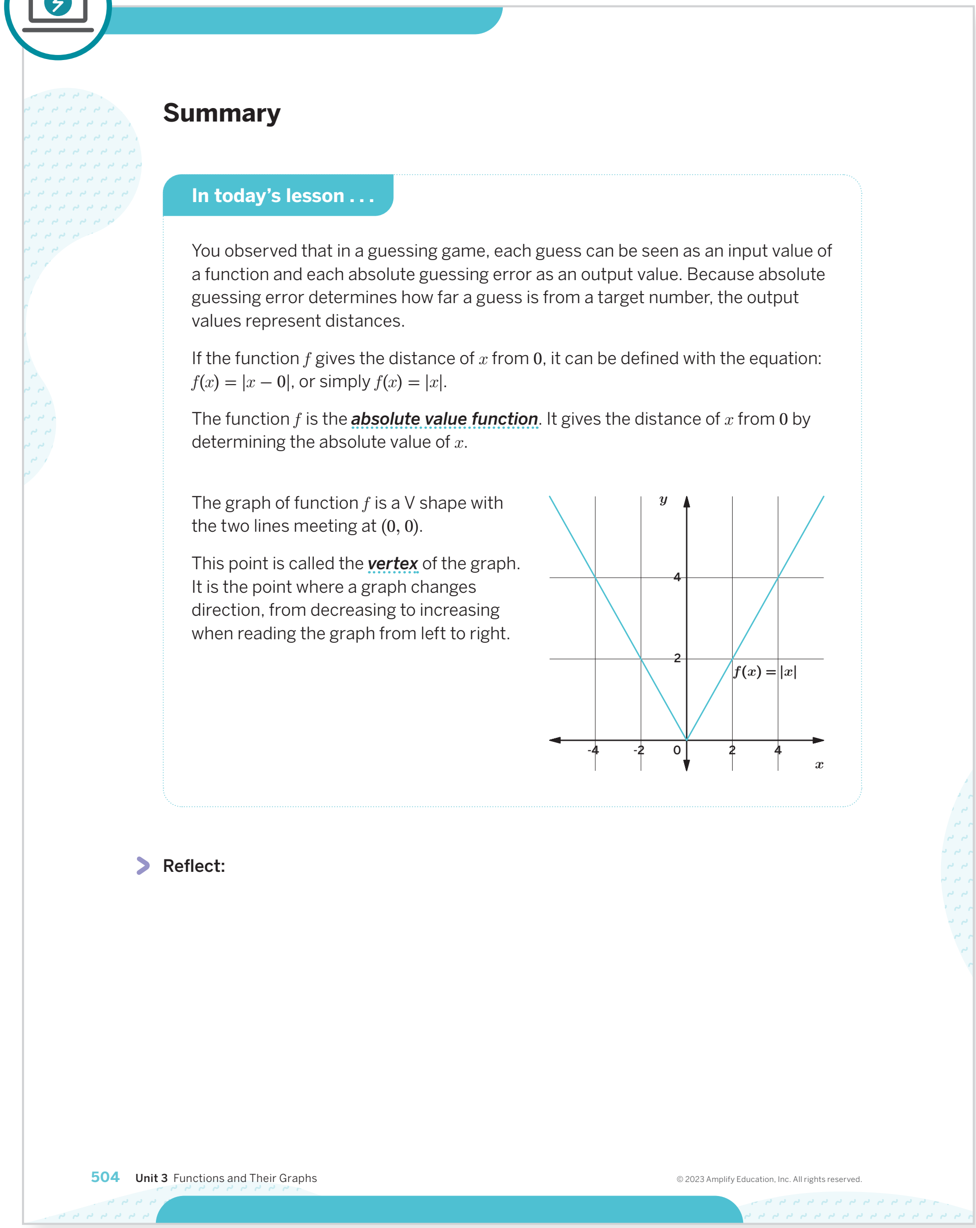
During the Connect, as students share how they determine the constant values and coefficients for each absolute value function, ask these questions:

- "Why does a negative coefficient cause an absolute value function to open downward?" Multiplying each output value by a negative value gives the opposite value.
- "What are the similarities and differences of $f(x) = -|x| + 3$ and $g(x) = -|x + 3|$?" Both functions open downward, but $f(x)$ has a vertex of $(0, 3)$ and $g(x)$ has a vertex of $(-3, 0)$.

Whole Class | 5 min

Summary

Review and synthesize why the absolute value function is considered both a distance function and a piecewise function.



Summary

In today's lesson . . .

You observed that in a guessing game, each guess can be seen as an input value of a function and each absolute guessing error as an output value. Because absolute guessing error determines how far a guess is from a target number, the output values represent distances.

If the function f gives the distance of x from 0, it can be defined with the equation: $f(x) = |x - 0|$, or simply $f(x) = |x|$.

The function f is the ***absolute value function***. It gives the distance of x from 0 by determining the absolute value of x.

The graph of function f is a V shape with the two lines meeting at $(0, 0)$.

This point is called the ***vertex*** of the graph. It is the point where a graph changes direction, from decreasing to increasing when reading the graph from left to right.

Reflect:

504 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the graph of $f(x) = |x|$, the table of values, and the piecewise function from Activity 1.

Have students share why the absolute value function can be viewed as a distance function.

Ask:

- "Suppose you know that $f(x)$ is 4. How can you determine what value or values of x would give an output of 4?" I can look at numbers that are 4 units from 0. There are two numbers that meet this requirement: 4 and -4.
- "How can you use the piecewise function to determine the output value at $x = -5$?" Because -5 is less than 0, I use the rule for $x < 0$ and determine the value of $f(-5)$, which gives $-(-5)$, or 5.

Highlight that the pieces of a piecewise function that represent the absolute value function depends on the vertex and the slope of each piece of the absolute value function.

Formalize vocabulary:

- ***absolute value function***
- ***vertex***

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is the vertex of the graph of an absolute value function reflected in the function?"
- "Why is the absolute value function a distance function?"

Math Language Development

MLR2: Collect and Display

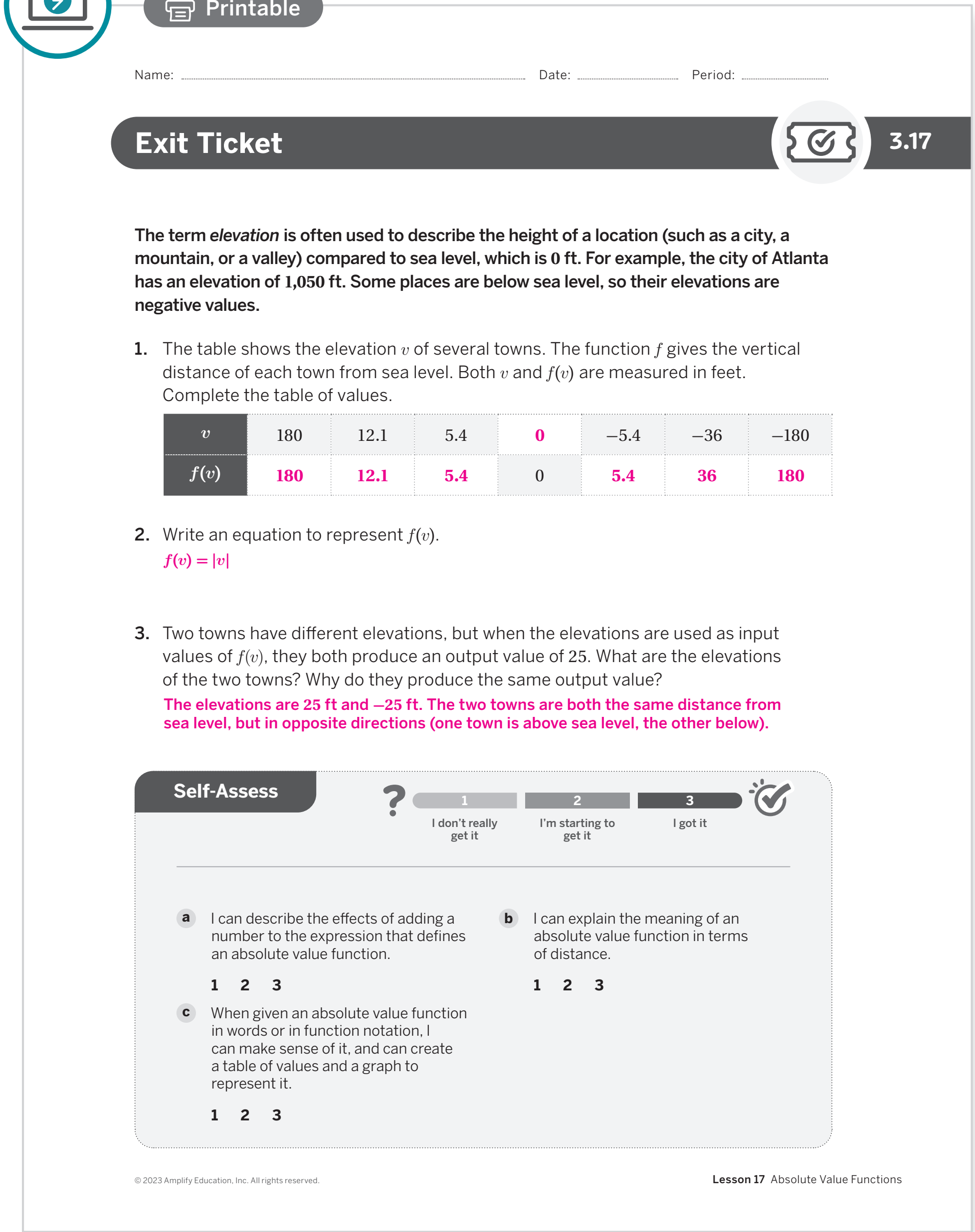
As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the terms *absolute value function* and *vertex* that were added to the display during the lesson.

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by constructing an absolute value function to represent a real-world scenario.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.17

The term *elevation* is often used to describe the height of a location (such as a city, a mountain, or a valley) compared to sea level, which is 0 ft. For example, the city of Atlanta has an elevation of 1,050 ft. Some places are below sea level, so their elevations are negative values.

1. The table shows the elevation v of several towns. The function f gives the vertical distance of each town from sea level. Both v and $f(v)$ are measured in feet. Complete the table of values.

v	180	12.1	5.4	**0**	–5.4	–36	–180
$f(v)$	**180**	**12.1**	**5.4**	0	**5.4**	**36**	**180**

2. Write an equation to represent $f(v)$.
   $f(v) = |v|$

3. Two towns have different elevations, but when the elevations are used as input values of $f(v)$, they both produce an output value of 25. What are the elevations of the two towns? Why do they produce the same output value?
   The elevations are 25 ft and –25 ft. The two towns are both the same distance from sea level, but in opposite directions (one town is above sea level, the other below).

Self-Assess

?
1 I don't really get it
2 I'm starting to get it
3 I got it

a I can describe the effects of adding a number to the expression that defines an absolute value function.
1 2 3

b I can explain the meaning of an absolute value function in terms of distance.
1 2 3

c When given an absolute value function in words or in function notation, I can make sense of it, and can create a table of values and a graph to represent it.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? The focus of this lesson was the absolute value function as a distance function and piecewise function. How did the focus go?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Success looks like . . .

- **Language Goal:** Analyzing and describing the effects of adding a constant term to an expression defining an absolute value function. **(Speaking and Listening, Reading and Writing)**
- **Language Goal:** Defining an absolute value function in terms of the distance of the input from 0. **(Reading and Writing)**
- **Language Goal:** Interpreting an absolute value function described in words or in function notation, and creating a table of values and a graph to represent the function. **(Reading and Writing)**
  - » Completing the table of vertical distance in Problem 1.

Suggested next steps

If students inaccurately complete the table in Problem 1, consider:

- Reviewing strategies for completing the table from Activity 1.
- Assigning Practice Problem 1.
- Asking, "If a location is 5 ft below you, what is your distance from this location?"

If students inaccurately write the function in Problem 2, consider:

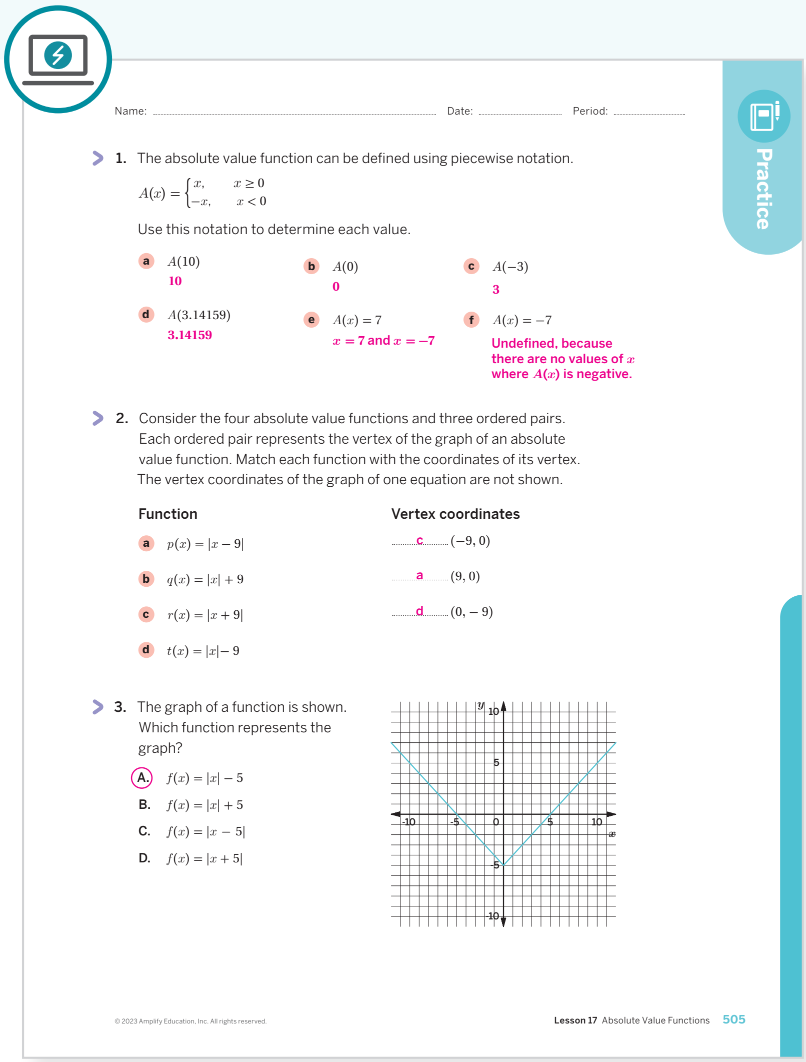
- Reviewing why the absolute value function reflects the table from Activity 1.
- Assigning Practice Problems 1 and 2.
- Asking, "What is unique about all the function values in the table? How could this be reflected in the function?"

If students give an inaccurate value or explanation in Problem 2, consider:

- Reviewing how opposite values have the same function value in Activity 1.
- Assigning Practice Problems 1 and 2.
- Asking, "What does it mean for a town to have a function value of 25?"

Practice

Independent



Name: Date: Period:

1. The absolute value function can be defined using piecewise notation.

$$A(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Use this notation to determine each value.

a $A(10)$
10

b $A(0)$
0

c $A(-3)$
3

d $A(3.14159)$
3.14159

e $A(x) = 7$
$x = 7$ and $x = -7$

f $A(x) = -7$
Undefined, because there are no values of x where $A(x)$ is negative.

2. Consider the four absolute value functions and three ordered pairs. Each ordered pair represents the vertex of the graph of an absolute value function. Match each function with the coordinates of its vertex. The vertex coordinates of the graph of one equation are not shown.

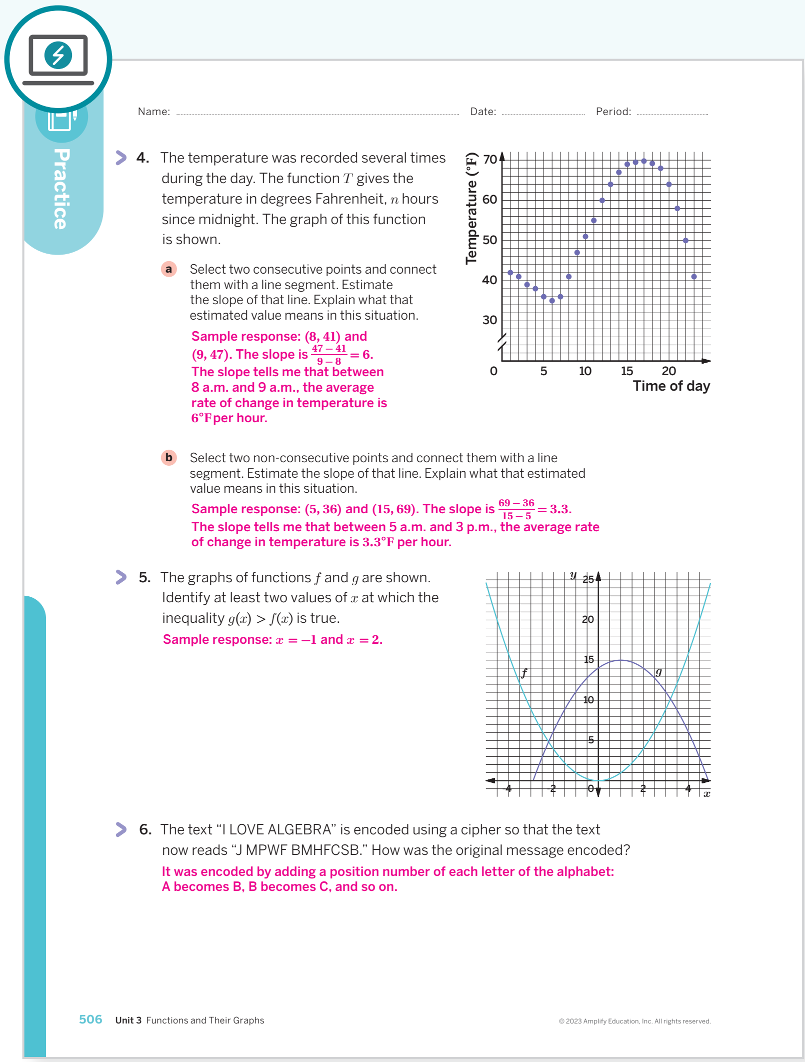
Function	Vertex coordinates
a $p(x) = \lvert x - 9\rvert$	**c** $(-9, 0)$
b $q(x) = \lvert x\rvert + 9$	**a** $(9, 0)$
c $r(x) = \lvert x + 9\rvert$	**d** $(0, -9)$
d $t(x) = \lvert x\rvert - 9$	

3. The graph of a function is shown. Which function represents the graph?

[✓ A.] $f(x) = |x| - 5$
B. $f(x) = |x| + 5$
C. $f(x) = |x - 5|$
D. $f(x) = |x + 5|$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 17 Absolute Value Functions 505



Name: Date: Period:

4. The temperature was recorded several times during the day. The function T gives the temperature in degrees Fahrenheit, n hours since midnight. The graph of this function is shown.

a Select two consecutive points and connect them with a line segment. Estimate the slope of that line. Explain what that estimated value means in this situation.

Sample response: $(8, 41)$ and $(9, 47)$. The slope is $\frac{47-41}{9-8} = 6$. The slope tells me that between 8 a.m. and 9 a.m., the average rate of change in temperature is $6°\text{F}$ per hour.

b Select two non-consecutive points and connect them with a line segment. Estimate the slope of that line. Explain what that estimated value means in this situation.

Sample response: $(5, 36)$ and $(15, 69)$. The slope is $\frac{69-36}{15-5} = 3.3$. The slope tells me that between 5 a.m. and 3 p.m., the average rate of change in temperature is $3.3°\text{F}$ per hour.

5. The graphs of functions f and g are shown. Identify at least two values of x at which the inequality $g(x) > f(x)$ is true.

Sample response: $x = -1$ and $x = 2$.

6. The text "I LOVE ALGEBRA" is encoded using a cipher so that the text now reads "J MPWF BMHFCSB." How was the original message encoded?

It was encoded by adding a position number of each letter of the alphabet: A becomes B, B becomes C, and so on.

506 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 8	2
	5	Unit 3 Lesson 10	2
Formative ⚡	6	Unit 3 Lesson 18	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Sub-Unit 4

Inverses of Functions

Whole Class

In this Sub-Unit, students are introduced to the inverse of a linear function, as they explore their last cityscape of Chicago.

SUB-UNIT

4 Inverses of Functions

Narrative Connections

How do you get Sunday shoppers to hear your song?

For much of the 20th century, Maxwell Street in Chicago was a lively outdoor marketplace. Immigrants from various countries in Europe sold goods from pushcarts and stalls. Anything you wanted to buy, from produce and pants, to pots and pans, could be found on Maxwell Street. There was also one unusual thing on offer: live blues music.

African-American blues musicians who had migrated to Chicago from the South came to Maxwell Street to play for the passing crowds in exchange for tips. Great musicians such as Muddy Waters and Howlin' Wolf could, at various times, be found busking for the crowd of a Sunday afternoon.

These artists faced an unusual challenge, playing not in a club or theater, but on a busy street in broad daylight, alongside vendors hawking and cars honking. How did they cut through the din and the chaos to make their music heard?

For many of them, the answer was to plug in their instruments and amplify them.

As a result, Chicago is known for its electric blues, and many credit the noise on Maxwell Street as one of the factors leading blues artists to switch from acoustic instruments to electrified, amplified ones. You can think of the acoustic sound as an input and the amplified sound as an output. But what if you heard the amplified sound and wanted to know what the acoustic version sounded like? For that, you'd need inverses of functions, which you'll encounter in these next few lessons.

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually. Students continue to explore inverses of functions — within the context of music and the city of Chicago — in the following places:

- **Lesson 18, Activity 2:** Exchange Rates
- **Lesson 19, Activity 1:** Chicago's Cold Weather Blues
- **Lesson 20, Activities 1–3:** Another Look at the Great Migration, The Great Migration From the South, The Electric Guitar

UNIT 3 | **LESSON 18**

Inverses of Functions

Let’s explore what happens when the input and output values trade places.

Focus

Goals

1. **Language Goal:** Given a linear function in context, describe its inverse. **(Speaking and Listening, Reading and Writing)**
2. Recognize that if a function takes a as its input and gives b as its output, its inverse takes b as its input and gives a as the output.
3. Understand that the inverse of a linear function can be determined by reversing the process that defines the initial function.

Coherence

Today

Students are introduced to the inverse of a function as they use and create ciphers to encode and decode messages and exchanging currency. They describe and determine the inverse of a linear function as they work forward and backward and perform calculations with numerical values to switch input and output values.

Previously

In Lesson 17, students graphed and wrote absolute value functions.

Coming Soon

In Lesson 19, students determine and interpret the inverses of functions within real-world contexts.

Rigor

- Students build **conceptual understanding** of what the inverse of a function means.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	20 min	10 min	5 min	5 min
Independent	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Warm-up PDF (as needed)
- Activity 1 PDF, *Are you ready for more?*
- Activity 1 PDF, *Are you ready for more?* (answers)
- four-function or scientific calculators

Math Language Development

New words

- ***inverse of a function***

Review words

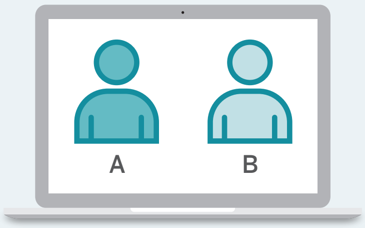
- *function*

Amps | Featured Activity

Activity 1

Using Ciphers

Students create their own cipher to encode text. Other students then determine the cipher and decode messages.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

When students work in pairs, they should apply their communication and listening skills as they agree or disagree with the claims. Tell them that while partners need not agree, they do need to be able to justify their thinking with good and valid mathematics. Also urge them to act maturely as they help each other determine causation or correlation.

Modifications to Pacing

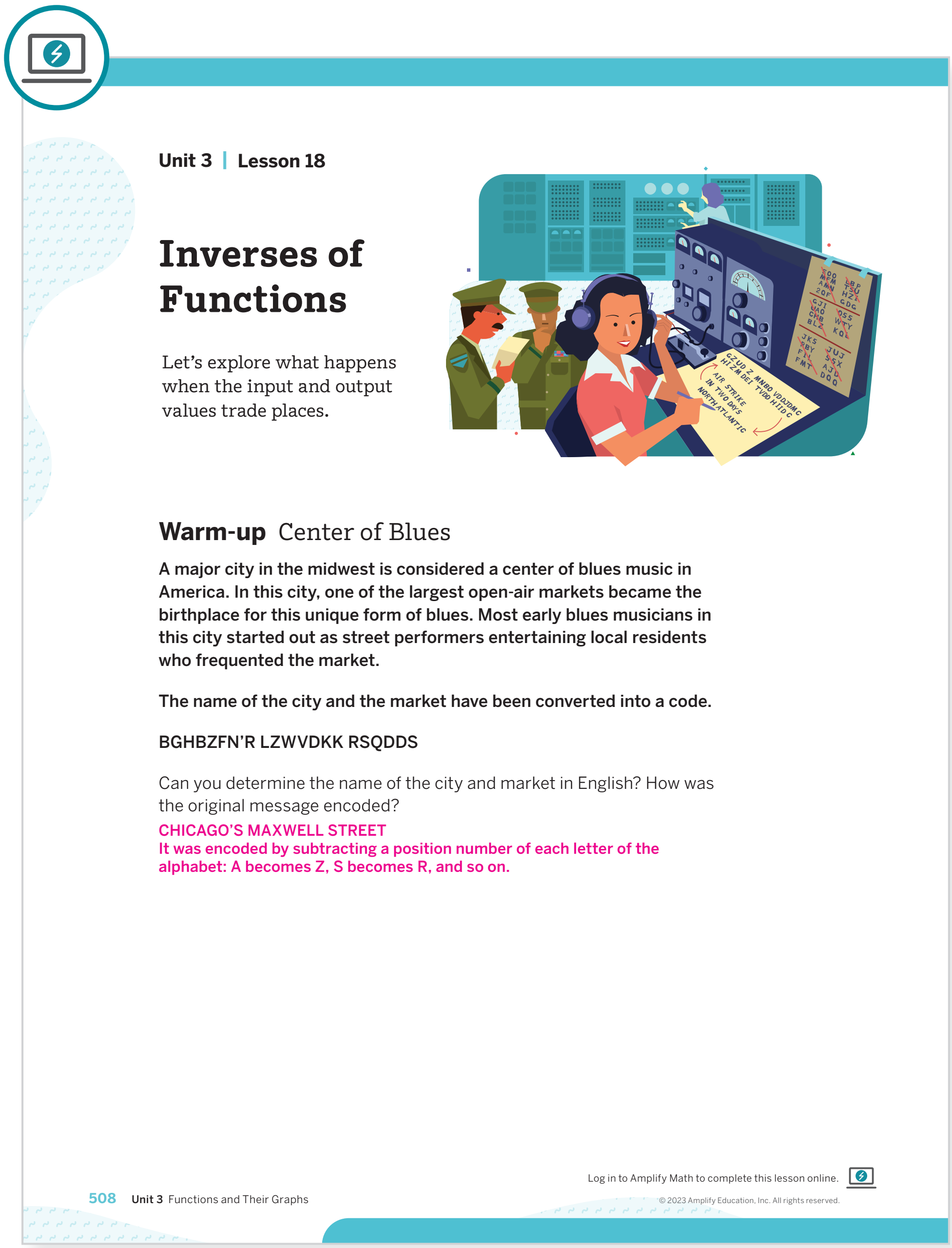
You may want to consider this additional modification if you are short on time.

- In **Activity 1**, the exchange of encoded messages can be omitted.

Independent | 10 min

Warm-up Center of Blues

Students decode a message to think about reversing the process that defines a function and about using outputs as inputs.



Unit 3 | Lesson 18

Inverses of Functions

Let's explore what happens when the input and output values trade places.

Warm-up Center of Blues

A major city in the midwest is considered a center of blues music in America. In this city, one of the largest open-air markets became the birthplace for this unique form of blues. Most early blues musicians in this city started out as street performers entertaining local residents who frequented the market.

The name of the city and the market have been converted into a code.

BGHBZFN'R LZWVDKK RSQDDS

Can you determine the name of the city and market in English? How was the original message encoded?

CHICAGO'S MAXWELL STREET

It was encoded by subtracting a position number of each letter of the alphabet: A becomes Z, S becomes R, and so on.

Log in to Amplify Math to complete this lesson online.

508 Unit 3 Functions and Their Graphs

©2023 Amplify Education, Inc. All rights reserved.

1 Launch

Read the narrative together. Give students examples of ciphers, such as multiplying the original position number of each letter of the alphabet by a value or adding or subtracting a number to the position number.

2 Monitor

Help students get started by giving a specific example of how a cipher is used to encode a letter.

Look for points of confusion:

- **Using a cipher to encode the given words rather than using the reverse operations to decode.** Have students write out the alphabet and label which letters are encoded and which are decoded.

Look for productive strategies:

- Writing the letters of the alphabet and their encoded letters to test a cipher.

3 Connect

Have individual students share the ciphers they tried, their findings, and how they went about decoding the words.

Display the cipher.

Ask, "What do you notice about the operations involved in encoding and decoding the message?" They are opposites of each other.

Highlight that the inputs are the letters of the alphabet in the normal order, and using the cipher, the outputs are the letters of the encoded words. To determine the original letters, the outputs and inputs switch. The students can take an encoded letter, use the reverse of the cipher, and find the decoded letter.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Have students use the Warm-up PDF to help them organize their thinking as they experiment with different ciphers to test their validity. If students are struggling, provide a clue using a letter that is not in the coded message, such as "the letter N in the message becomes M in the code" and have students determine the rule for the cipher and decode the message.

Power-up

To power up students' ability to use a cipher to encode a message, have students complete:

A 'shift of 3 forward' to the position of the letters in the alphabet would make each letter encoded as follows:

Plain text	A	B	C	D	E	F	G	H	I	J	K	L	M
Encoded text	D	E	F	G	H	I	J	K	L	M	N	O	P

Encode the following letters using the same cipher: (a "shift of 3 forward"):

Plain text	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Encoded text	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

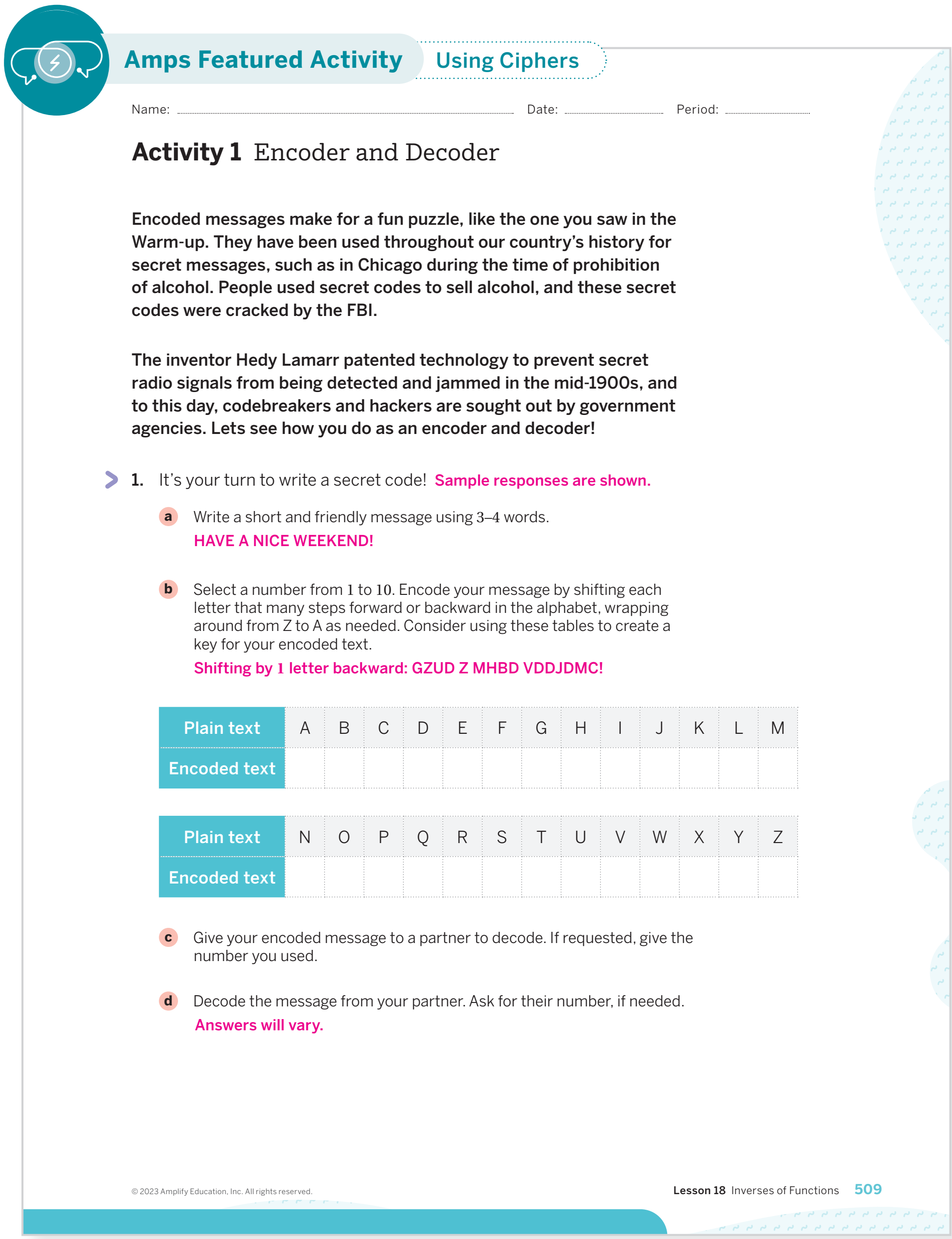
Use: Before the Warm-up

Informed by: Performance on Lesson 17, Practice Problem 6

Pairs | 20 min

Activity 1 Encoder and Decoder

Students write a function for their own cipher and determine a method to decode messages to write the inverse of a function.



Amps Featured Activity Using Ciphers

Name: ______ Date: ______ Period: ______

Activity 1 Encoder and Decoder

Encoded messages make for a fun puzzle, like the one you saw in the Warm-up. They have been used throughout our country's history for secret messages, such as in Chicago during the time of prohibition of alcohol. People used secret codes to sell alcohol, and these secret codes were cracked by the FBI.

The inventor Hedy Lamarr patented technology to prevent secret radio signals from being detected and jammed in the mid-1900s, and to this day, codebreakers and hackers are sought out by government agencies. Lets see how you do as an encoder and decoder!

1. It's your turn to write a secret code! Sample responses are shown.

   a. Write a short and friendly message using 3–4 words.
   HAVE A NICE WEEKEND!

   b. Select a number from 1 to 10. Encode your message by shifting each letter that many steps forward or backward in the alphabet, wrapping around from Z to A as needed. Consider using these tables to create a key for your encoded text.
   Shifting by 1 letter backward: GZUD Z MHBD VDDJDMC!

Plain text	A	B	C	D	E	F	G	H	I	J	K	L	M
Encoded text													

Plain text	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Encoded text													

   c. Give your encoded message to a partner to decode. If requested, give the number you used.

   d. Decode the message from your partner. Ask for their number, if needed.
   Answers will vary.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Arrange students in pairs. Read the narrative together. Tell students that they are to use a shift cipher to encode a short secret message, exchange it with their partner, then decode their partner's secret message.

2 Monitor

Help students get started by having them use the Warm-up cipher as a model to determine a cipher to use.

Look for points of confusion:

- **Thinking the equation used for encoding a message is also used for decoding (Problem 2).** Have students check their equation by substituting the encoded letter into the equation to see if it produces the original letter.
- **Struggling to determine the equation used to decode a message through using the table of letters (Problem 2).** Ask, "How can you use the equation for encoding to determine an equation where m is isolated?"

Look for productive strategies:

- Annotating the table of letters with the relationship for encoding and decoding.
- Rewriting an equation so that the other variable is isolated and checking it against the original equation to check its accuracy.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Consider providing students a message they can use in Problem 1 instead of having them create their own.

Extension: *Math Enrichment*

Have students complete the *Are you ready for more?* PDF, in which they will use an equation they wrote to encode letters to plot the input and output ordered pairs and analyze the resulting graph.

Math Language Development

MLR2: Collect and Display

During the Connect, collect the language students use to share their equations that represent the encoding and decoding of a message, and how they are similar and different. Write students' words and phrases on a display. Be sure to emphasize words and phrases such as "undo" and "reverse." When defining the terms *inverse of a function*, use student language from the display.

English Learners

Add illustrations that show what it means to "undo" or "reverse" the input-output pairs. For example, display the tables shown.

Original function

Input	Output
2	3

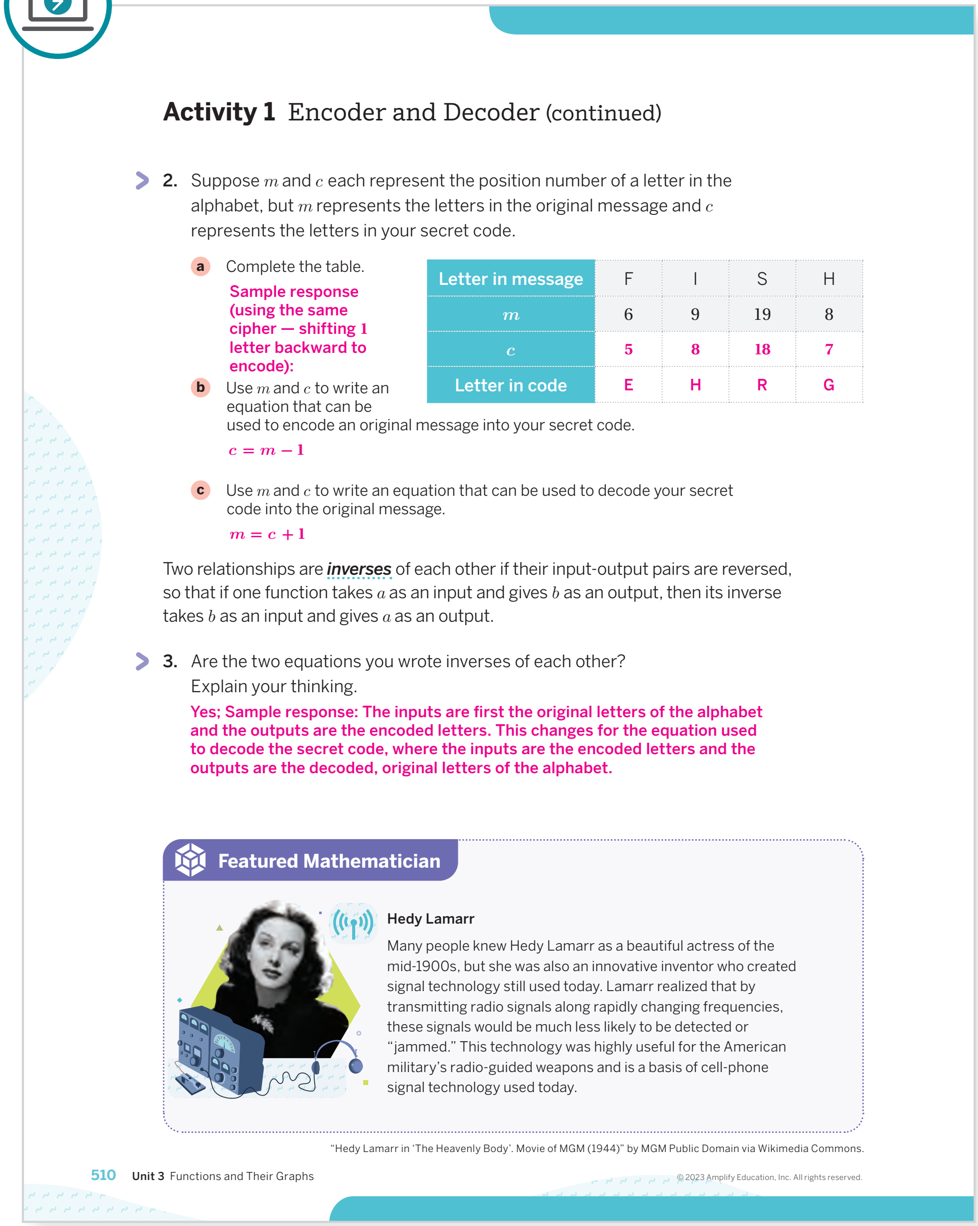
Inverse ("undo", "reverse")

Input	Output
3	2

Pairs | 20 min

Activity 1 Encoder and Decoder (continued)

Students write a function for their own cipher and determine a method to decode messages to write the inverse of a function.



Activity 1 Encoder and Decoder (continued)

2. Suppose m and c each represent the position number of a letter in the alphabet, but m represents the letters in the original message and c represents the letters in your secret code.

   a. Complete the table.

   Sample response (using the same cipher — shifting 1 letter backward to encode):

Letter in message	F	I	S	H
m	6	9	19	8
c	5	8	18	7
Letter in code	E	H	R	G

   b. Use m and c to write an equation that can be used to encode an original message into your secret code.

   $c = m - 1$

   c. Use m and c to write an equation that can be used to decode your secret code into the original message.

   $m = c + 1$

Two relationships are ***inverses*** of each other if their input-output pairs are reversed, so that if one function takes a as an input and gives b as an output, then its inverse takes b as an input and gives a as an output.

3. Are the two equations you wrote inverses of each other? Explain your thinking.

   Yes; Sample response: The inputs are first the original letters of the alphabet and the outputs are the encoded letters. This changes for the equation used to decode the secret code, where the inputs are the encoded letters and the outputs are the decoded, original letters of the alphabet.

Featured Mathematician

Hedy Lamarr

Many people knew Hedy Lamarr as a beautiful actress of the mid-1900s, but she was also an innovative inventor who created signal technology still used today. Lamarr realized that by transmitting radio signals along rapidly changing frequencies, these signals would be much less likely to be detected or "jammed." This technology was highly useful for the American military's radio-guided weapons and is a basis of cell-phone signal technology used today.

"Hedy Lamarr in 'The Heavenly Body'. Movie of MGM (1944)" by MGM Public Domain via Wikimedia Commons.

510 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

3 Connect

Have individual students share the equations they wrote for Problems 2b and 2c.

Ask:

- "Can the process of encoding a message be thought of as a function? Why or why not?" Yes; For every input letter, there is only one possible output letter. The plain-text letters are the inputs. The ciphered letters are the outputs.
- "Can the process of decoding a secret code be thought of as a function? Why or why not?" Yes; Every coded letter used as an input has only one output. The ciphered letters are the inputs. The plain-text letters are the outputs.

Define the term ***inverse of a function*** as the relationship whose input-output pairs are reversed, related to the original function. If one function takes a as its input and gives b as an output, then its inverse takes b as its input and gives a as an output.

Highlight that if the rule for encoding is a function, then the rule for decoding is its inverse.

Note: In this lesson, both the original equation and its inverse were both functions. In a future lesson, students will learn that not all inverses of functions are functions themselves.

Featured Mathematician

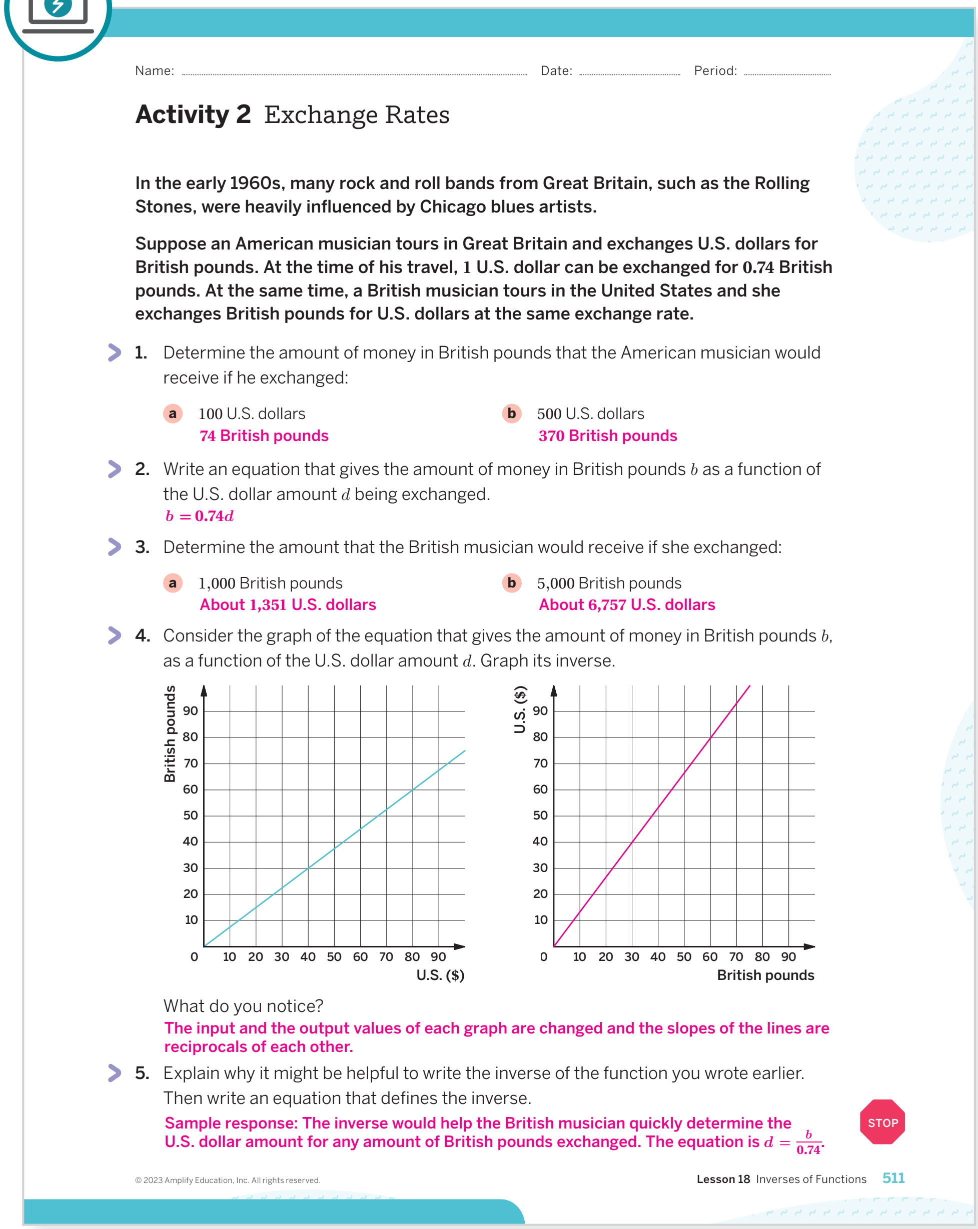
Hedy Lamarr

Have students read about featured Mathematician Hedy Lamarr, who invented methods in radio transmissions to make the signals less likely to be detected or jammed.

Independent | 10 min

Activity 2 Exchange Rates

Students construct a function to model an exchange rate context and interpret the inverse of the function in the given context.



Name: ______ Date: ______ Period: ______

Activity 2 Exchange Rates

In the early 1960s, many rock and roll bands from Great Britain, such as the Rolling Stones, were heavily influenced by Chicago blues artists.

Suppose an American musician tours in Great Britain and exchanges U.S. dollars for British pounds. At the time of his travel, 1 U.S. dollar can be exchanged for 0.74 British pounds. At the same time, a British musician tours in the United States and she exchanges British pounds for U.S. dollars at the same exchange rate.

1. Determine the amount of money in British pounds that the American musician would receive if he exchanged:

   a. 100 U.S. dollars
   74 British pounds

   b. 500 U.S. dollars
   370 British pounds

2. Write an equation that gives the amount of money in British pounds b as a function of the U.S. dollar amount d being exchanged.
   $b = 0.74d$

3. Determine the amount that the British musician would receive if she exchanged:

   a. 1,000 British pounds
   About 1,351 U.S. dollars

   b. 5,000 British pounds
   About 6,757 U.S. dollars

4. Consider the graph of the equation that gives the amount of money in British pounds b, as a function of the U.S. dollar amount d. Graph its inverse.

   What do you notice?
   The input and the output values of each graph are changed and the slopes of the lines are reciprocals of each other.

5. Explain why it might be helpful to write the inverse of the function you wrote earlier. Then write an equation that defines the inverse.
   Sample response: The inverse would help the British musician quickly determine the U.S. dollar amount for any amount of British pounds exchanged. The equation is $d = \frac{b}{0.74}$.

STOP

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Use the *Three Reads* routine to review the narrative before having students work independently. Provide access to scientific or four-function calculators.

2 Monitor

Help students get started by having them determine how many U.S. dollars are equivalent to one British pound.

Look for points of confusion:

- **Multiplying the British pounds by 0.74 to convert to U.S. dollars.** Have students check this method if the exchange rate was 1 U.S. dollar to 2, 3, or 4 British pounds.

Look for productive strategies

- Creating a table of values to check or help write the equations of the exchange rates.

3 Connect

Have individual students share the two equations they wrote in Problems 2 and 5.

Ask, "How did you determine the equation that represents the inverse of the function?" By reversing the steps used to determine the amount in British pounds when I know the U.S. dollar amount. I solved the equation for d.

Highlight that the inverse of the function contains the reverse operation as the original function. On the graph, the slopes are reciprocals and the axes are reversed.

Differentiated Support

Accessibility: *Guide Processing and Visualization, Activate Background Knowledge*

Ask students what they know about exchange rates. Consider showing today's exchange rate between the U.S. dollar and the British pound, and consider showing images of what British currency looks like. This will help students visualize the context of this activity.

Extension: *Math Enrichment*

Display today's exchange rate between U.S. dollars and British pounds. Ask students to write two equations that represent this exchange rate, similar to the equations they wrote in this activity.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.
Read 1: Students should understand that two musicians are traveling, one from Great Britain to the U.S., and the other from the U.S. to Great Britain. Both musicians want to exchange currency.
Read 2: Ask students to name given quantities or relationships, such as 1 U.S. dollar is equivalent to 0.74 British pounds.
Read 3: Ask students to brainstorm strategies for how they will respond to Problem 1.

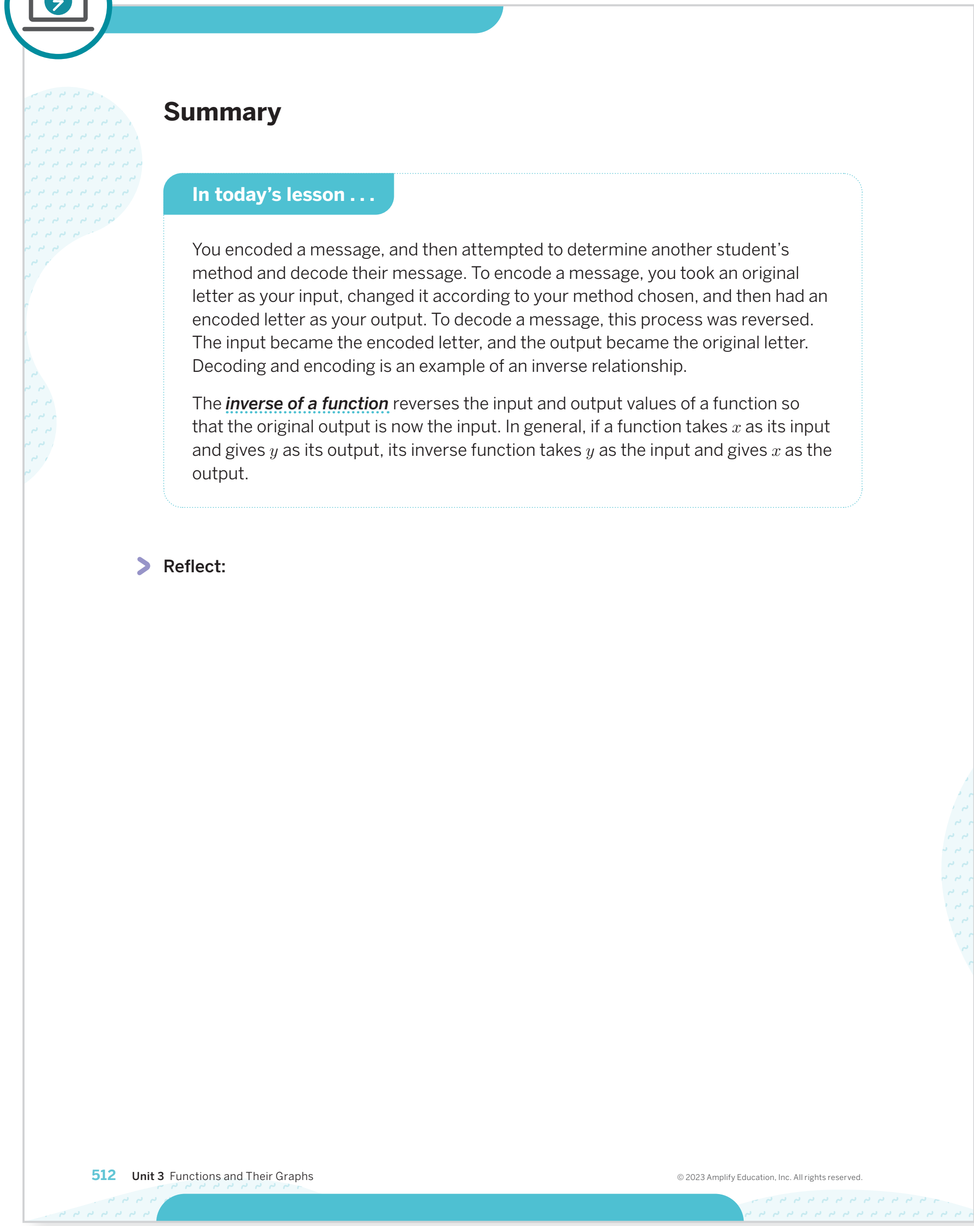
English Learners

Annotate the introductory text by writing "1 U.S. dollar = 0.74 British pounds".

Whole Class | 5 min

Summary

Review and synthesize how the inverse of a function reverses the input and output values of a function.



Summary

In today's lesson . . .

You encoded a message, and then attempted to determine another student's method and decode their message. To encode a message, you took an original letter as your input, changed it according to your method chosen, and then had an encoded letter as your output. To decode a message, this process was reversed. The input became the encoded letter, and the output became the original letter. Decoding and encoding is an example of an inverse relationship.

The ***inverse of a function*** reverses the input and output values of a function so that the original output is now the input. In general, if a function takes x as its input and gives y as its output, its inverse function takes y as the input and gives x as the output.

Reflect:

512 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display The equation and its inverse from Activity 2.

Have students share the definition of the inverse of a function in their own words.

Formalize vocabulary: *inverse of a function*

Ask:

- "How can you determine the equation that represents the inverse of a function?" I can reverse the process and operations in the equation representing the function and solve for the other variable.
- "Why might it be helpful to write the inverse equation?" Sometimes I know the original output values and am trying to determine the corresponding input values. The inverse equation can help make this process more efficient.

Highlight that input-output pairs of the inverse of a function are reversed, which is demonstrated by solving for the other variable in the original equation.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What does it mean for a function to have an inverse?"

Math Language Development

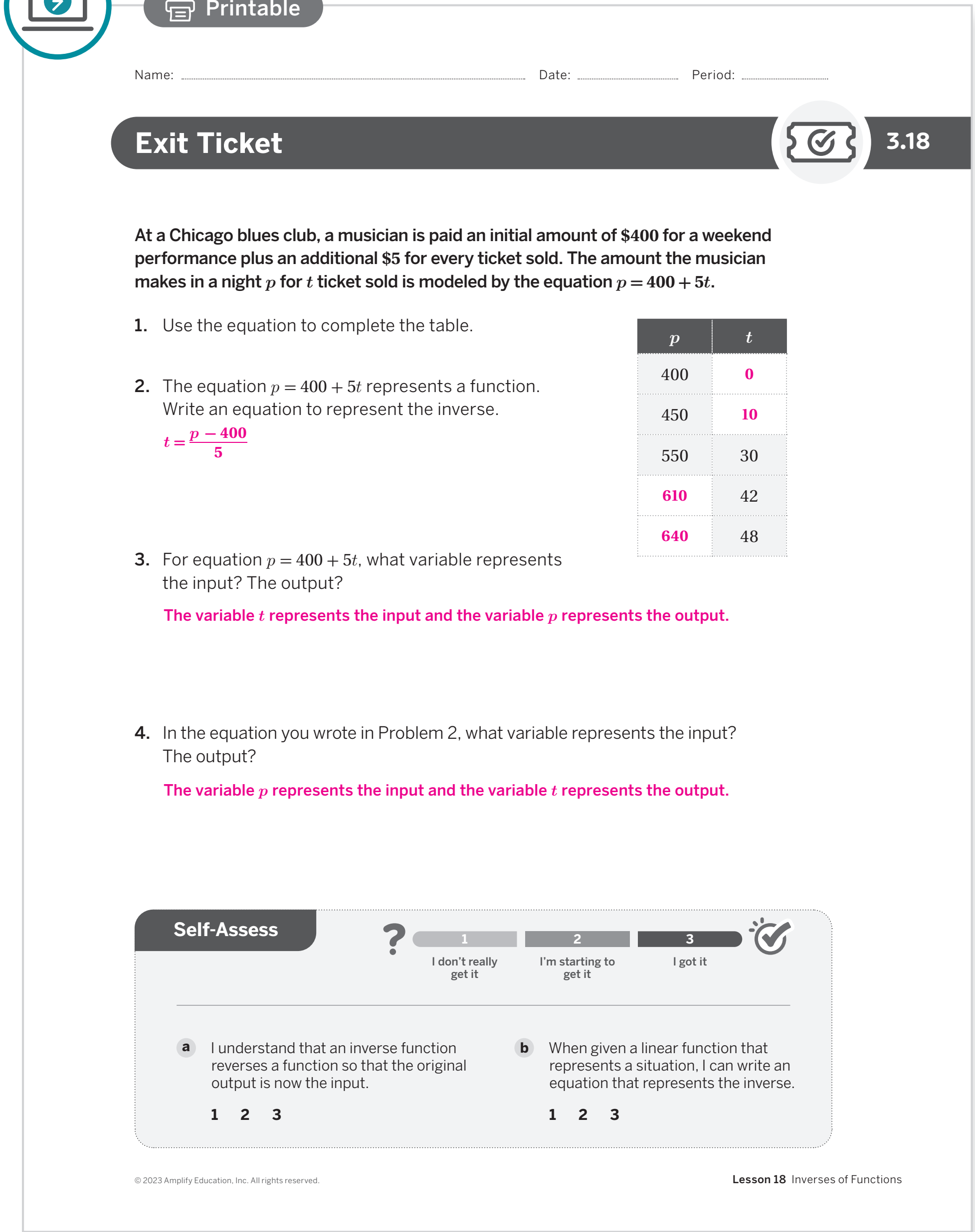
MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display for this unit that you started in Lesson 2. Ask them to review and reflect on any terms and phrases related to the term *inverse of a function* that were added to the display during the lesson.

Exit Ticket

Students determine the equation that represents the inverse of a given linear equation and interpret the input and output values for the given equation and its inverse within the given context.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.18

At a Chicago blues club, a musician is paid an initial amount of $400 for a weekend performance plus an additional $5 for every ticket sold. The amount the musician makes in a night p for t ticket sold is modeled by the equation $p = 400 + 5t$.

1. Use the equation to complete the table.

p	t
400	0
450	10
550	30
610	42
640	48

2. The equation $p = 400 + 5t$ represents a function. Write an equation to represent the inverse.

$t = \frac{p - 400}{5}$

3. For equation $p = 400 + 5t$, what variable represents the input? The output?

The variable t represents the input and the variable p represents the output.

4. In the equation you wrote in Problem 2, what variable represents the input? The output?

The variable p represents the input and the variable t represents the output.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I understand that an inverse function reverses a function so that the original output is now the input.

1 2 3

b. When given a linear function that represents a situation, I can write an equation that represents the inverse.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 18 Inverses of Functions

Success looks like . . .

- **Language Goal:** Given a linear function in context, describing its inverse. **(Speaking and Listening, Reading and Writing)**
  - » Writing the equation for the inverse of function p in Problem 2.
- **Goal:** Recognizing that if a function takes a as its input and gives b as its output, its inverse takes b as its input and gives a as the output.
  - » Identifying the independent and dependent variables of the inverse function of p in Problem 4.
- **Goal:** Understanding that the inverse of a linear function can be determined by reversing the process that defines the initial function.

Suggested next steps

If students inaccurately complete the table of values in Problem 1, consider:

- Reviewing using an equation to determine values from Activity 2.
- Assigning Practice Problems 1 and 2.
- Asking, "How can you solve for t when given a value of p?"

If students inaccurately determine the inverse in Problem 2, consider:

- Reviewing determining an inverse of an equation from Activity 2.
- Assigning Practice Problems 1–3.
- Asking, "Which variable do you isolate to determine the inverse of the original equation?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did the encoding and decoding activity support students in developing conceptual understanding of inverses of functions?
- Thinking about the questions you asked students today and what the students said or did as a result of the questions, which question was the most effective? What might you change for the next time you teach this lesson?

Math Language Development

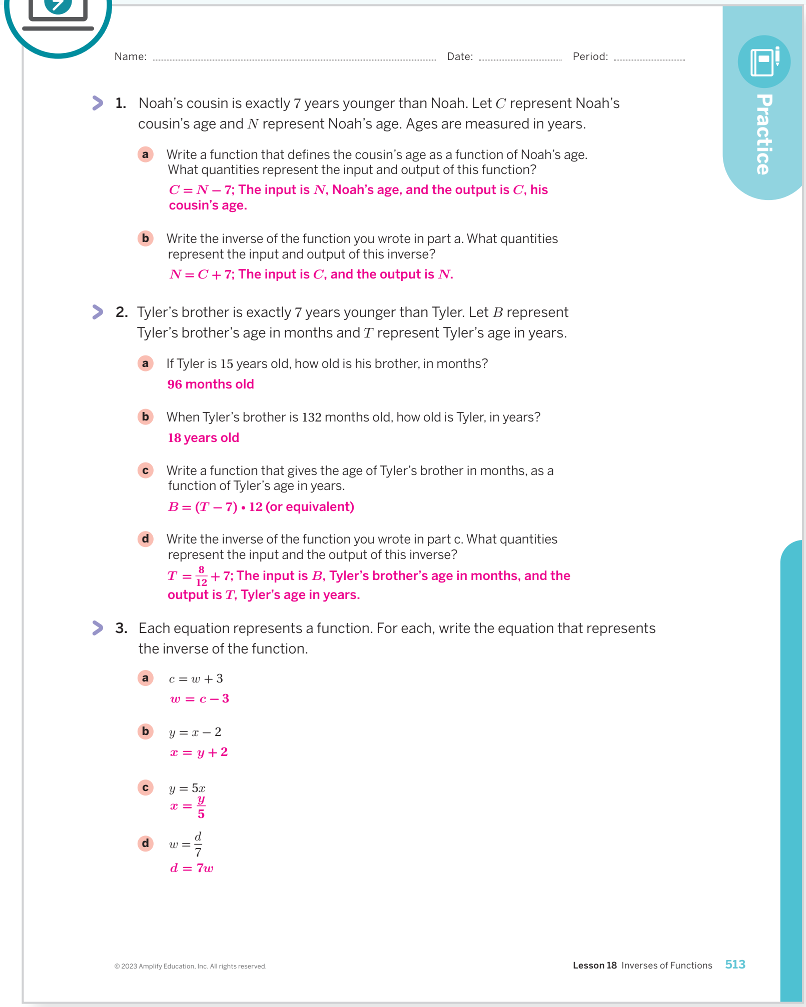
Language Goal: Given a linear function in context, describing its inverse.

Reflect on students' language development toward this goal.

- How did using the *Collect and Display* routine in Activity 1 help students understand the relationship between a function and its inverse?
- Do students' responses to Problems 3 and 4 of the Exit Ticket demonstrate they can describe the input and output of a function and its inverse? What other strategies can you use to support their descriptions?

Practice

Independent



Name: ________ Date: ________ Period: ________

Practice

1. Noah's cousin is exactly 7 years younger than Noah. Let C represent Noah's cousin's age and N represent Noah's age. Ages are measured in years.

   a Write a function that defines the cousin's age as a function of Noah's age. What quantities represent the input and output of this function?

   $C = N - 7$; The input is N, Noah's age, and the output is C, his cousin's age.

   b Write the inverse of the function you wrote in part a. What quantities represent the input and output of this inverse?

   $N = C + 7$; The input is C, and the output is N.

2. Tyler's brother is exactly 7 years younger than Tyler. Let B represent Tyler's brother's age in months and T represent Tyler's age in years.

   a If Tyler is 15 years old, how old is his brother, in months?

   96 months old

   b When Tyler's brother is 132 months old, how old is Tyler, in years?

   18 years old

   c Write a function that gives the age of Tyler's brother in months, as a function of Tyler's age in years.

   $B = (T - 7) \cdot 12$ (or equivalent)

   d Write the inverse of the function you wrote in part c. What quantities represent the input and the output of this inverse?

   $T = \frac{8}{12} + 7$; The input is B, Tyler's brother's age in months, and the output is T, Tyler's age in years.

3. Each equation represents a function. For each, write the equation that represents the inverse of the function.

   a $c = w + 3$

   $w = c - 3$

   b $y = x - 2$

   $x = y + 2$

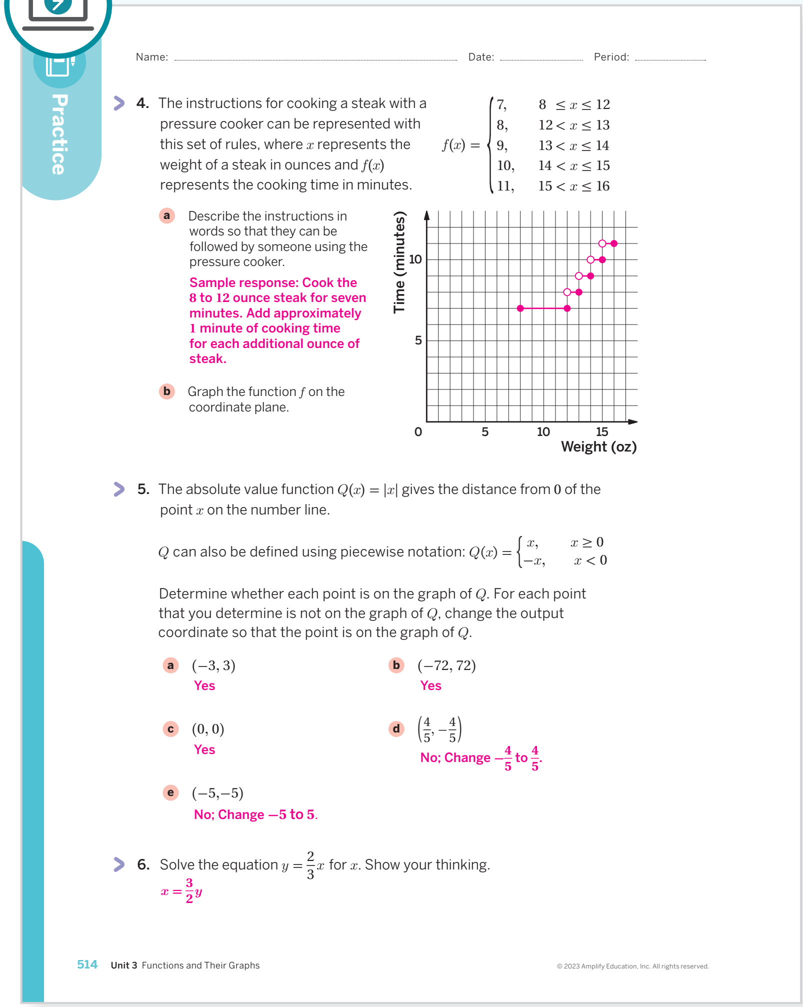
   c $y = 5x$

   $x = \frac{y}{5}$

   d $w = \frac{d}{7}$

   $d = 7w$

© 2023 Amplify Education, Inc. All rights reserved.



Name: ________ Date: ________ Period: ________

Practice

4. The instructions for cooking a steak with a pressure cooker can be represented with this set of rules, where x represents the weight of a steak in ounces and $f(x)$ represents the cooking time in minutes.

$$f(x) = \begin{cases} 7, & 8 \leq x \leq 12 \\ 8, & 12 < x \leq 13 \\ 9, & 13 < x \leq 14 \\ 10, & 14 < x \leq 15 \\ 11, & 15 < x \leq 16 \end{cases}$$

   a Describe the instructions in words so that they can be followed by someone using the pressure cooker.

   Sample response: Cook the 8 to 12 ounce steak for seven minutes. Add approximately 1 minute of cooking time for each additional ounce of steak.

   b Graph the function f on the coordinate plane.

5. The absolute value function $Q(x) = |x|$ gives the distance from 0 of the point x on the number line.

   Q can also be defined using piecewise notation: $Q(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

   Determine whether each point is on the graph of Q. For each point that you determine is not on the graph of Q, change the output coordinate so that the point is on the graph of Q.

   a $(-3, 3)$

   Yes

   b $(-72, 72)$

   Yes

   c $(0, 0)$

   Yes

   d $\left(\frac{4}{5}, -\frac{4}{5}\right)$

   No; Change $-\frac{4}{5}$ to $\frac{4}{5}$.

   e $(-5, -5)$

   No; Change −5 to 5.

6. Solve the equation $y = \frac{2}{3}x$ for x. Show your thinking.

   $x = \frac{3}{2}y$

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 1	2
	3	Activity 2	2
Spiral	4	Unit 3 Lesson 14	2
	5	Unit 3 Lesson 16	2
Formative ⚡	6	Unit 3 Lesson 19	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

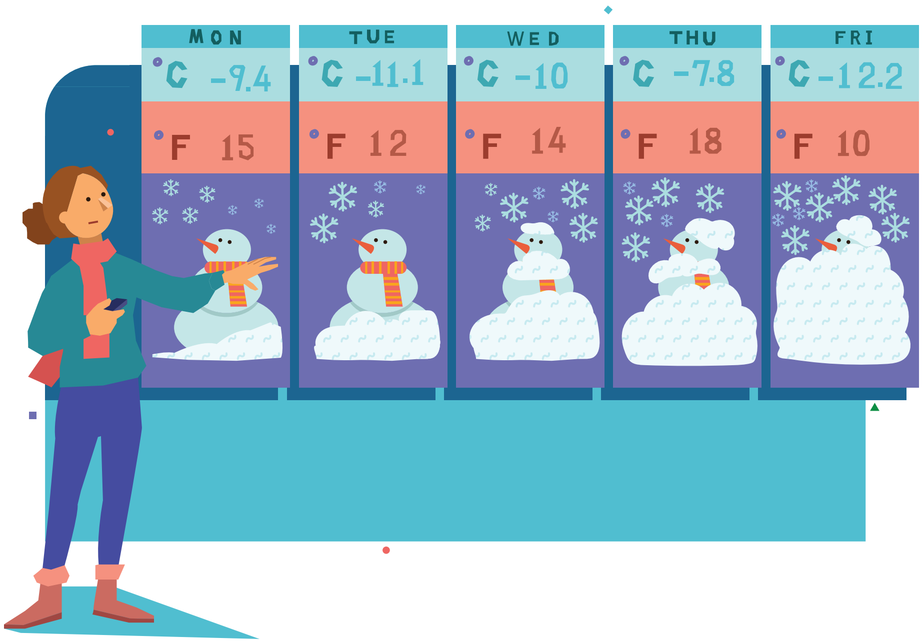
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 3 | **LESSON 19**

Finding and Interpreting Inverses of Functions

Let's determine the inverses of linear functions.



Focus

Goals

1. Write the equation that represents the inverse of a function by solving the equation that represents the function for the input variable.
2. **Language Goal:** Interpret the inverse of a function in terms of the quantities in a situation. **(Speaking and Listening, Writing)**

Coherence

Today

Students write inverses for functions that are defined using multiple operations, recognizing that the process is comparable to their earlier work of solving for a variable. Students also interpret the inverse functions and their domain and range in terms of situations.

Previously

In Lesson 18, students wrote inverses for functions where the input and output were related by one operation.

Coming Soon

In Lesson 20, students will write the inverses of functions to analyze and solve problems in context.

Rigor

- Students build **procedural fluency** of finding the inverse of linear functions.
- Students **apply** the inverse of functions to interpret their meaning in context.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	10 min	5 min	5 min
Independent	Independent	Pairs	Pairs	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF, pre-cut cards, one per pair
- Activity 2 PDF (answers)
- Instructional Routine PDF, *Info Gap: Instructions*
- Instructional Routine PDF, *Info Gap: Types of Questioning*
- four-function calculators

Math Language Development

Review words

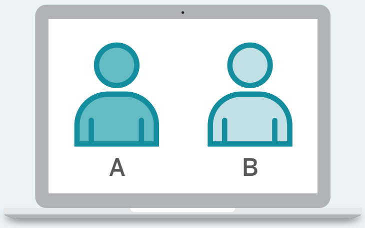
- *domain*
- *inverse functions*
- *function*
- *range*

Amps | Featured Activity

Activity 2

Digital Collaboration

Students are paired to determine and request the information needed to understand the relationship of a function and its inverse.



Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

As students gather the information that they need to write and interpret the inverses of functions, the quantitative and abstract thinking required might cause their thoughts to wander. The intensity and focus required for the activity could lead students to be distracted and distance instead of leaning into the activity and conquering the new material. Before the activity, have students set a goal that will help them do their best throughout the activity.

Modifications to Pacing

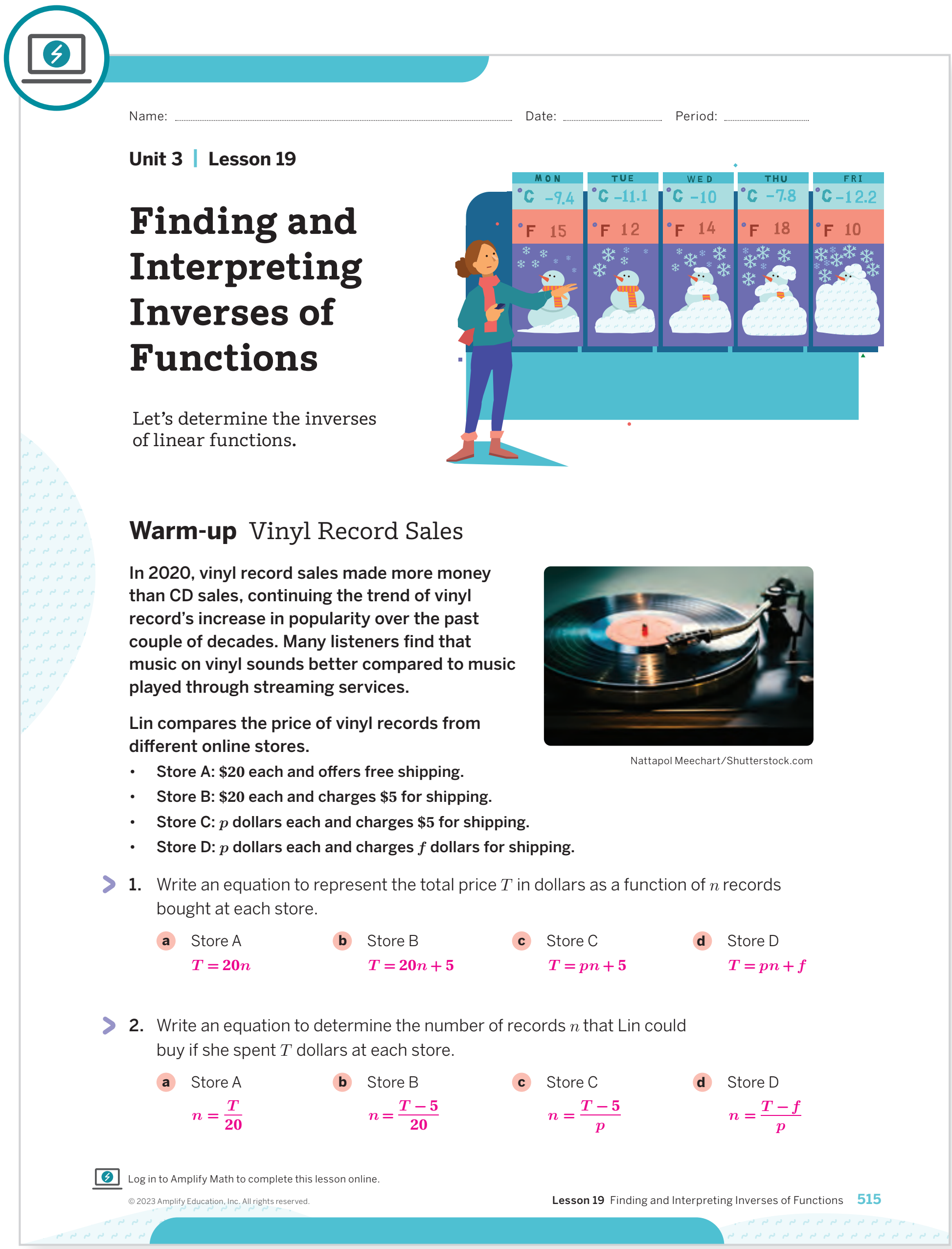
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Store A may be omitted.
- In **Activity 3**, provide students with the equation for Problem 1.

Independent | 5 min

Warm-up Vinyl Records Sales

Students construct functions with two operations to model given scenarios, preparing them to construct inverses of functions in the upcoming activities.



Name: ______ Date: ______ Period: ______

Unit 3 | Lesson 19

Finding and Interpreting Inverses of Functions

Let's determine the inverses of linear functions.

Warm-up Vinyl Record Sales

In 2020, vinyl record sales made more money than CD sales, continuing the trend of vinyl record's increase in popularity over the past couple of decades. Many listeners find that music on vinyl sounds better compared to music played through streaming services.

Lin compares the price of vinyl records from different online stores.

- **Store A: $20 each and offers free shipping.**
- **Store B: $20 each and charges $5 for shipping.**
- **Store C: p dollars each and charges $5 for shipping.**
- **Store D: p dollars each and charges f dollars for shipping.**

Nattapol Meechart/Shutterstock.com

1. Write an equation to represent the total price T in dollars as a function of n records bought at each store.

a Store A	b Store B	c Store C	d Store D
$T = 20n$	$T = 20n + 5$	$T = pn + 5$	$T = pn + f$

2. Write an equation to determine the number of records n that Lin could buy if she spent T dollars at each store.

a Store A	b Store B	c Store C	d Store D
$n = \frac{T}{20}$	$n = \frac{T-5}{20}$	$n = \frac{T-5}{p}$	$n = \frac{T-f}{p}$

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 19 Finding and Interpreting Inverses of Functions 515

1 Launch

Use the *Three Reads* routine as described in the Math Language Development section. Have students share similarities and differences they notice between the descriptions.

2 Monitor

Help students get started by calculating the price of buying 1 and 2 records from each store.

Look for points of confusion:

- **Interchanging the price per record and shipping price in the equations.** Have students check their equations for the price of purchasing 10 records. Ask, "Which value remains constant no matter the number of records purchased?"

Look for productive strategies:

- Substituting input-output pairs into both sets of equations to determine the accuracy of their equations in Problem 2.

3 Connect

Have individual students share their equations and their method for determining their equations in Problem 2.

Highlight that writing the equations in Problem 2 involves undoing each operation in the corresponding equation in Problem 1 and doing so in reverse order.

Ask, "Is each equation in Problem 2 the inverse of the corresponding equation in Problem 1? Explain your thinking." Yes; Each isolated variable is now an output, while previously it was an input. The variable that was previously isolated is now an input.

Math Language Development

MLR6: Three Reads

Use this routine to help students make sense of the introductory text.

Read 1: Students should understand that there are four different stores that charge different prices for vinyl records.

Read 2: Ask students to name given quantities or relationships, such as Store B charges $20 for each record and $5 for shipping.

Read 3: Ask students to brainstorm strategies for how they can write the equations in Problems 1 and 2.

English Learners

Annotate the introductory text by highlighting key words and phrases, such as "each" and "free shipping."

Power-up

To power up students' ability to write a linear equation to represent a verbal description of two quantities, have students complete:

Mai has $250 in her bank account, she spends $4.50 each day at lunch. Determine which equation represents the amount of money m she has after d days.

A. $d = -4.50m + 250$

[✓ B.] $m = 250 - 4.50d$

C. $m = 250 + 4.50d$

D. $m = 250d - 4.50$

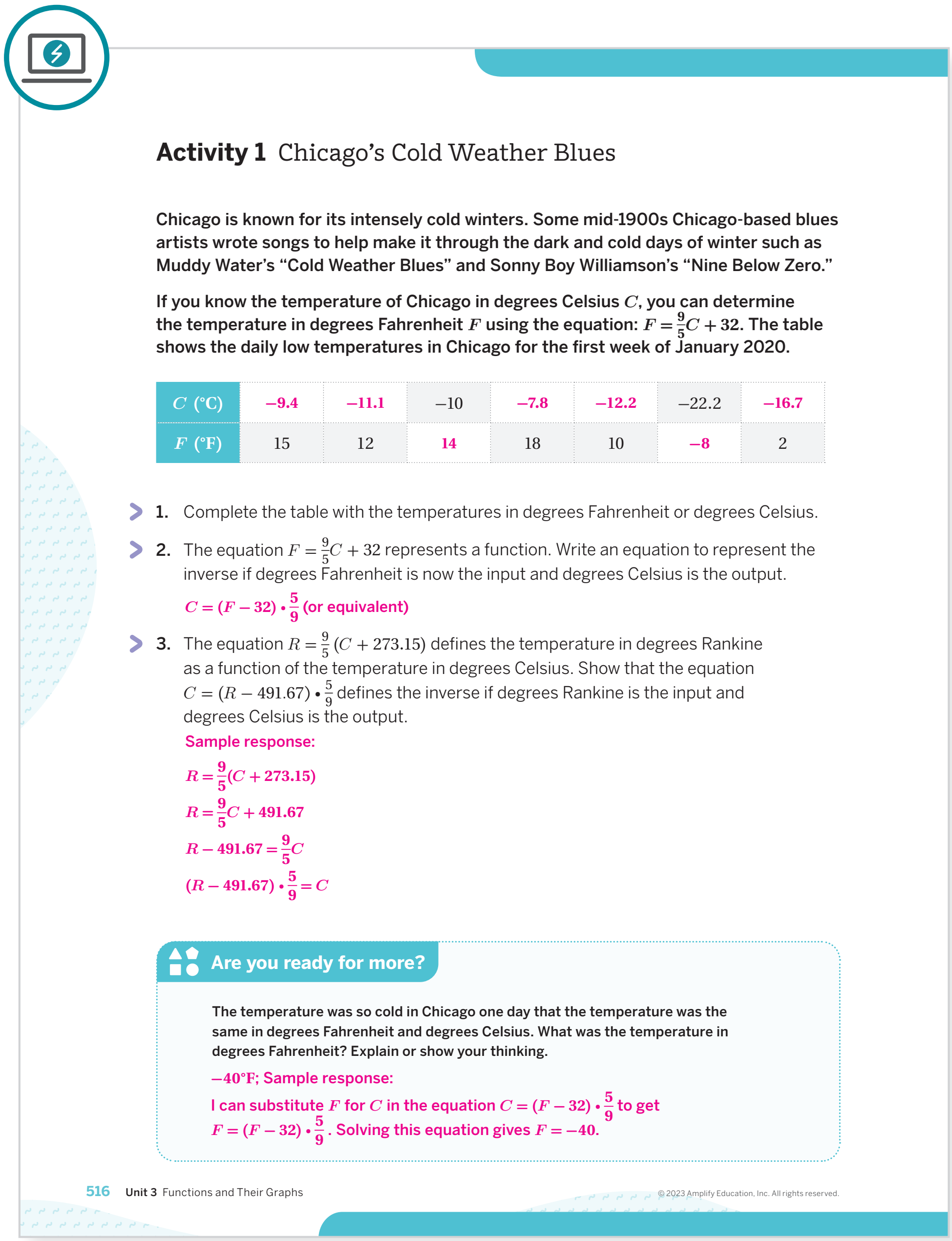
Use: Before the Warm-up

Informed by: Performance on Lesson 18, Practice Problem 6 and Pre-Unit Readiness Assessment, Problem 6

Independent | 10 min

Activity 1 Chicago's Cold Weather Blues

Students study the structure of an equation and its inverse to show how the inverse reverses the independent and dependent variables of the original equation.



Activity 1 Chicago's Cold Weather Blues

Chicago is known for its intensely cold winters. Some mid-1900s Chicago-based blues artists wrote songs to help make it through the dark and cold days of winter such as Muddy Water's "Cold Weather Blues" and Sonny Boy Williamson's "Nine Below Zero."

If you know the temperature of Chicago in degrees Celsius C, you can determine the temperature in degrees Fahrenheit F using the equation: $F = \frac{9}{5}C + 32$. The table shows the daily low temperatures in Chicago for the first week of January 2020.

C (°C)	−9.4	−11.1	−10	−7.8	−12.2	−22.2	−16.7
F (°F)	15	12	14	18	10	−8	2

1. Complete the table with the temperatures in degrees Fahrenheit or degrees Celsius.
2. The equation $F = \frac{9}{5}C + 32$ represents a function. Write an equation to represent the inverse if degrees Fahrenheit is now the input and degrees Celsius is the output.

   $C = (F - 32) \cdot \frac{5}{9}$ (or equivalent)
3. The equation $R = \frac{9}{5}(C + 273.15)$ defines the temperature in degrees Rankine as a function of the temperature in degrees Celsius. Show that the equation $C = (R - 491.67) \cdot \frac{5}{9}$ defines the inverse if degrees Rankine is the input and degrees Celsius is the output.

   Sample response:

   $R = \frac{9}{5}(C + 273.15)$

   $R = \frac{9}{5}C + 491.67$

   $R - 491.67 = \frac{9}{5}C$

   $(R - 491.67) \cdot \frac{5}{9} = C$

Are you ready for more?

The temperature was so cold in Chicago one day that the temperature was the same in degrees Fahrenheit and degrees Celsius. What was the temperature in degrees Fahrenheit? Explain or show your thinking.

−40°F; Sample response:

I can substitute F for C in the equation $C = (F - 32) \cdot \frac{5}{9}$ to get $F = (F - 32) \cdot \frac{5}{9}$. Solving this equation gives $F = -40$.

516 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Explain to students that the Rankine scale is a temperature scale that is sometimes used in engineering systems, typically alongside measurements in Fahrenheit. Provide access to four-function calculators.

2 Monitor

Help students get started by showing them how to determine a temperature in Celsius when it is given in Fahrenheit.

Look for points of confusion:

- **Struggling to solve for C.** Have students think of $\frac{9}{5}C$ as $9 \cdot \frac{1}{5}C$ and then undo one operation at a time.

Look for productive strategies:

- Solving the equation that represents the inverse for the other variable to see if it results in the original given function.

3 Connect

Have individual students share how they checked their inverse equation.

Highlight that to verify that the two equations in Problem 3 are inverses, students can solve the first equation for C or solve the second equation for R and see if they match the other equation.

Ask, "What can each inverse equation be used to determine?" The first can be used to determine the temperature in degrees Celsius if the temperature in degrees Fahrenheit is given. The second can be used to determine the temperature in degrees Celsius if the temperature in degrees Rankine is given.

Differentiated Support

Accessibility: *Activate Background Knowledge*

Ask students if they are familiar with different temperature scales, such as Fahrenheit, Celsius, Rankine, or even Kelvin. Students from other countries may be more familiar with Celsius scales than Fahrenheit.

Extension: *Math Enrichment*

Tell students there is one temperature at which the number of degrees are equal for Fahrenheit and Celsius. Have students determine this temperature. −40 degrees

Math Language Development

MLR7: Compare and Connect

While students complete Problem 3, ask them to create a display that shows their strategy for verifying the inverse equation. Their display should be neat and organized so that others can follow it. Give students time during the Connect to analyze and compare the strategies of at least 2 other displays. Ask students to share how the strategies were similar or different. For example:

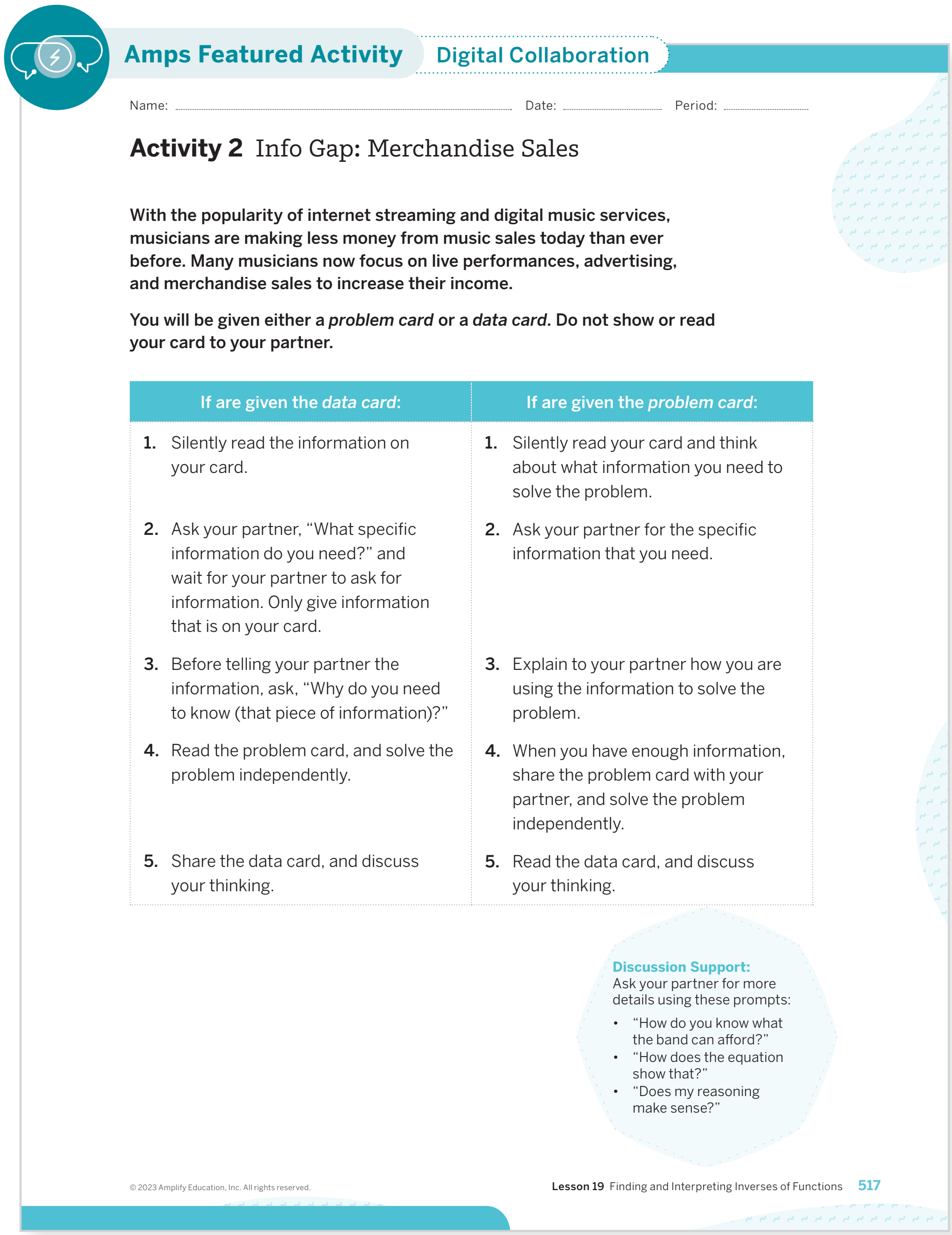
- Some students may first use the Distributive Property and then subtract 491.67 from each side.
- Other students may first multiply each side by $\frac{5}{9}$ to eliminate the fraction.
- Students could also multiply each side by 5 to eliminate the fraction and then use the Distributive Property or divide by 9.

Pairs | 15 min

Activity 2 Info Gap: Merchandise Sales

Students participate in an Info Gap to build their communication skills and understanding of the inverses of functions.

Amps Featured Activity Digital Collaboration



Name: ______ Date: ______ Period: ______

Activity 2 Info Gap: Merchandise Sales

With the popularity of internet streaming and digital music services, musicians are making less money from music sales today than ever before. Many musicians now focus on live performances, advertising, and merchandise sales to increase their income.

You will be given either a *problem card* or a *data card*. Do not show or read your card to your partner.

If are given the *data card*:	If are given the *problem card*:
1. Silently read the information on your card.	**1.** Silently read your card and think about what information you need to solve the problem.
2. Ask your partner, "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card.	**2.** Ask your partner for the specific information that you need.
3. Before telling your partner the information, ask, "Why do you need to know (that piece of information)?"	**3.** Explain to your partner how you are using the information to solve the problem.
4. Read the problem card, and solve the problem independently.	**4.** When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Share the data card, and discuss your thinking.	**5.** Read the data card, and discuss your thinking.

Discussion Support:
Ask your partner for more details using these prompts:
- "How do you know what the band can afford?"
- "How does the equation show that?"
- "Does my reasoning make sense?"

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 19 Finding and Interpreting Inverses of Functions 517

1 Launch

Display the Instructional Routine PDF, *Info Gap: Instructions* and review the ***Info Gap*** routine. Consider demonstrating the routine if students are unfamiliar. Distribute the pre-cut cards from the Activity 2 PDF to each student pair.

2 Monitor

Help students get started by practicing the instructional routine with them.

Look for points of confusion:

- **Subtracting the discount from the equation in Problem Card 2.** Have students rewrite their equation to solve for n to see if this version of their equation is accurate.

Look for productive strategies:

- Substituting values into their equations to check its accuracy.

3 Connect

Have pairs of students share what questions they asked each other to help determine the equation.

Display the correct equations.

Highlight that students can check to see if the equations are inverses of each other by isolating each equation for the other variable.

Ask, "Do both equations define functions? Explain your thinking." Yes; For each price per poster, there is only one possible number of posters that can be bought. For each number of posters that the band buys, there is only one possible original price.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Display Problem Card 1. Use a think-aloud to model Steps 1 and 2 as if you were the recipient of that card. Consider using the following during the think-aloud.

- "I need to determine the number of posters the band can afford. But I don't know the regular price for one poster. I think I should ask for the regular price"
- "The text mentions a discount, but I don't know what that is. I think I should ask for the discount."
- "To know how many posters the band can afford, I need to know their budget."

Math Language Development

MLR8: Discussion Supports

While students work, encourage them to use the prompts provided in the Student Edition to ask their partner for more details. Display or provide the Instructional Routine PDF, *Info Gap: Types of Questioning* to support students in their discussions with their partner.

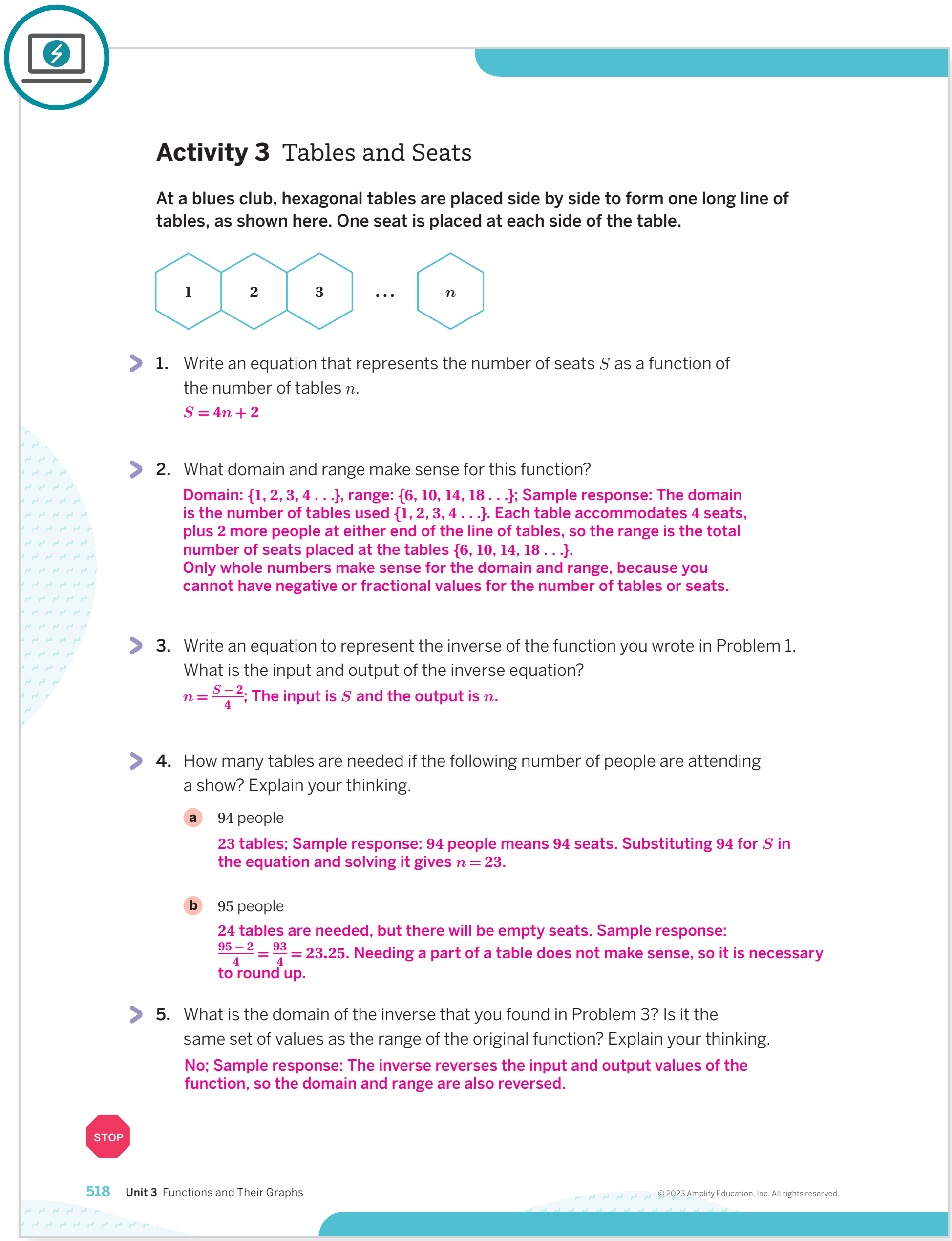
English Learners

Have students highlight or circle key words and phrases in the text, such as "depends on," "available," "each poster," and "discount." Annotate the term "available" with *budget* to help make this connection.

Pairs | 10 min

Activity 3 Tables and Seats

Students construct equations to represent a given context, use the equation's structure to construct the inverse equation, and interpret the equations within the given context.



Activity 3 Tables and Seats

At a blues club, hexagonal tables are placed side by side to form one long line of tables, as shown here. One seat is placed at each side of the table.

1. Write an equation that represents the number of seats S as a function of the number of tables n.

   $S = 4n + 2$

2. What domain and range make sense for this function?

   Domain: {1, 2, 3, 4 . . .}, range: {6, 10, 14, 18 . . .}; Sample response: The domain is the number of tables used {1, 2, 3, 4 . . .}. Each table accommodates 4 seats, plus 2 more people at either end of the line of tables, so the range is the total number of seats placed at the tables {6, 10, 14, 18 . . .}.
   Only whole numbers make sense for the domain and range, because you cannot have negative or fractional values for the number of tables or seats.

3. Write an equation to represent the inverse of the function you wrote in Problem 1. What is the input and output of the inverse equation?

   $n = \frac{S-2}{4}$**; The input is S and the output is n.**

4. How many tables are needed if the following number of people are attending a show? Explain your thinking.

   a. 94 people

   23 tables; Sample response: 94 people means 94 seats. Substituting 94 for S in the equation and solving it gives $n = 23$.

   b. 95 people

   24 tables are needed, but there will be empty seats. Sample response: $\frac{95-2}{4} = \frac{93}{4} = 23.25$. Needing a part of a table does not make sense, so it is necessary to round up.

5. What is the domain of the inverse that you found in Problem 3? Is it the same set of values as the range of the original function? Explain your thinking.

   No; Sample response: The inverse reverses the input and output values of the function, so the domain and range are also reversed.

STOP

518 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Show the diagram and clarify where seats can be placed along the tables.

2 Monitor

Help students get started by having them list the number of seats for 1, 2, 3, and 4 tables.

Look for points of confusion:

- **Using $n \geq 1$ and $S \geq 6$ as the domain and range.** Have students substitute values of n into their equation that are not whole numbers. Ask, "Do these values make sense? What type of numbers make sense for this scenario?"

Look for productive strategies:

- Annotating and extending the diagram of tables and seats to help determine the equation and the domain and range.

3 Connect

Display the equation, its inverse, and the diagram.

Highlight that just as solutions to equations need to be interpreted in context, so do the domain and range of a function and its inverse. Partial tables and seats would not make sense in this context, so the domain and range only consists of values that are whole numbers.

Ask, "Why is the domain and range not the same for the inverse as the original function?" The inverse reverses the input-output pairs, so the domain and range are also reversed.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest students first make sense of the diagram before beginning Problem 1. Have them annotate where the seats can be placed in the diagram. Then suggest they create a table, similar to the following, that shows the number of seats for the first three tables. Encourage them to draw diagrams of each set of tables to help with their thinking.

Table	1	2	4
Total number of seats	6	10	14

Math Language Development

MLR7: Compare and Connect

While students work, display the following to help them remember the connection between the input/output of a function and its domain/range.

Input ↔ Domain

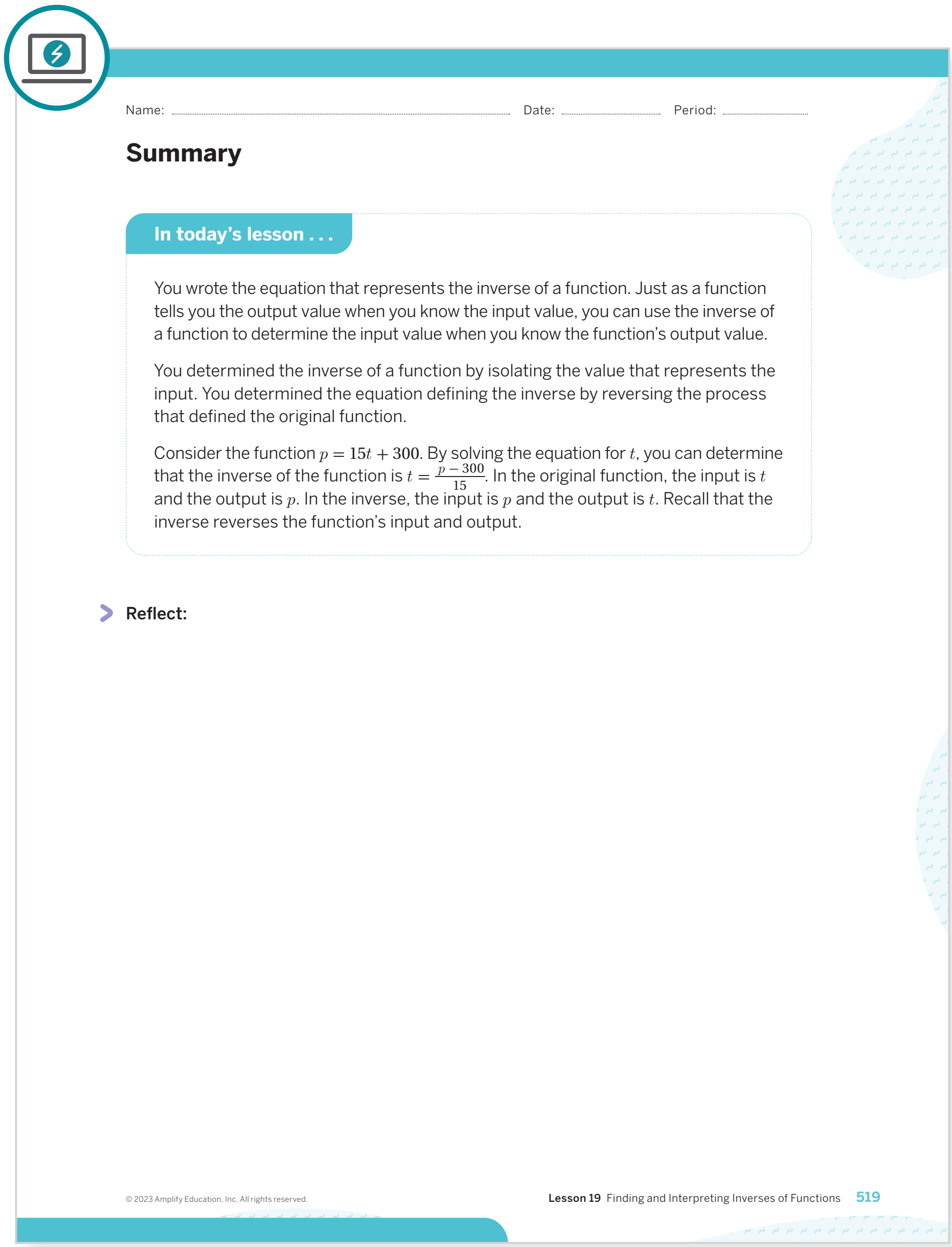
Output ↔ Range

During the Connect, draw students' attention to the fact that the domain and range of a function and its inverse are reversed because the input and output values are reversed. Emphasize the need to interpret the domain and range within context, as only whole numbers make sense for the number of tables as well as the number of seats.

Whole Class | 5 min

Summary

Review and synthesize how to determine the inverse of a function by reversing the process and operations that define the original function.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You wrote the equation that represents the inverse of a function. Just as a function tells you the output value when you know the input value, you can use the inverse of a function to determine the input value when you know the function's output value.

You determined the inverse of a function by isolating the value that represents the input. You determined the equation defining the inverse by reversing the process that defined the original function.

Consider the function $p = 15t + 300$. By solving the equation for t, you can determine that the inverse of the function is $t = \frac{p - 300}{15}$. In the original function, the input is t and the output is p. In the inverse, the input is p and the output is t. Recall that the inverse reverses the function's input and output.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the first equation and its inverse from Activity 1.

Have students share the domain and range of the function and its inverse.

Ask, "When is each equation more useful?" The first equation is more useful when the temperature in degrees Celsius is given and I want to determine the equivalent temperature in degrees Fahrenheit. The second equation is more useful when the temperature in degrees Fahrenheit is given and I want to determine the temperature in degrees Celsius.

Highlight that determining the equation that represents the inverse of a function is particularly useful when students have multiple output values for which they wish to determine the corresponding input values. Evaluating a function at a value (using the equation for the inverse) is much more efficient than solving an equation (using the original equation).

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

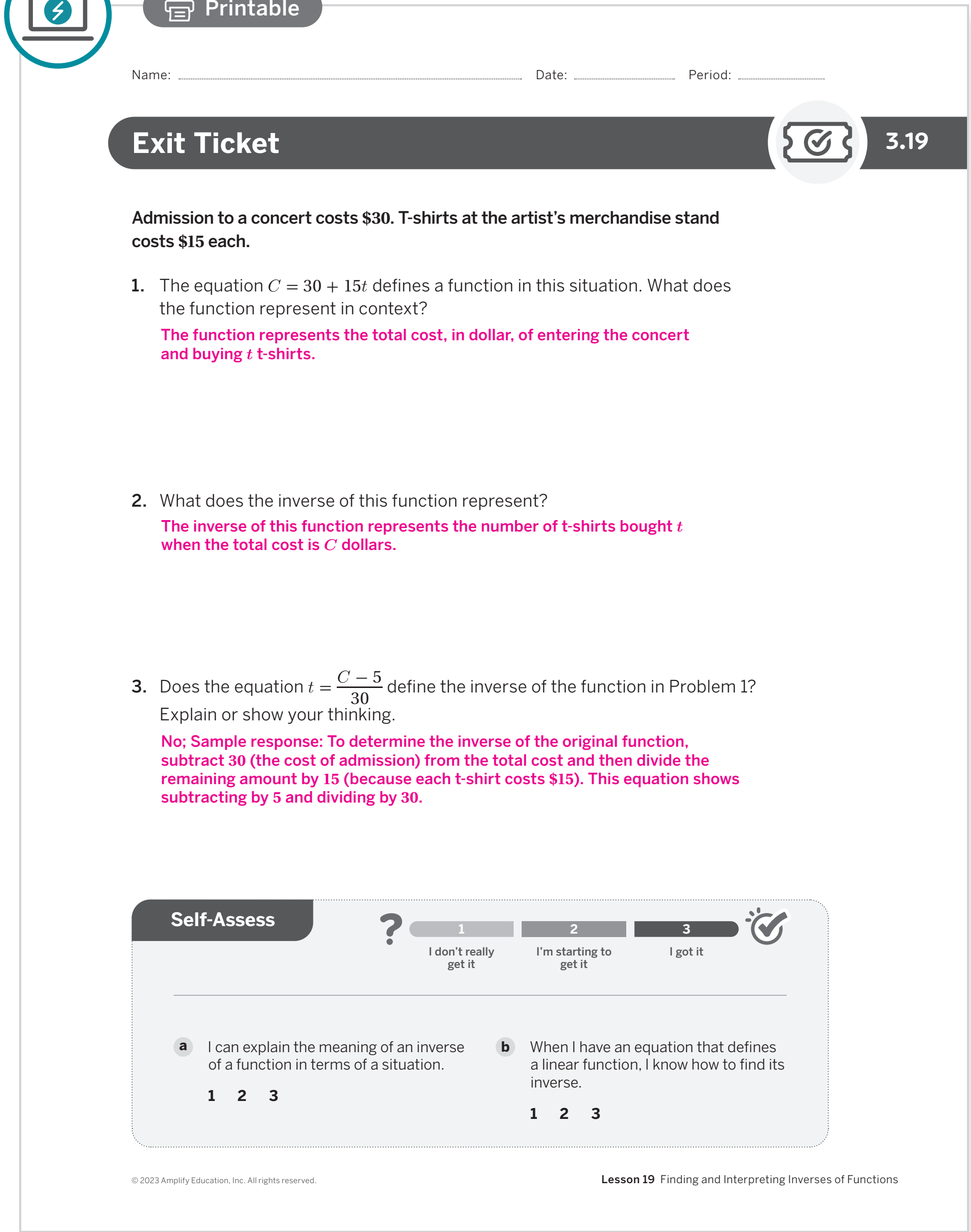
- "Why might it be helpful to determine the equation that represents the inverse of a function?"

Independent | 5 min

Exit Ticket

Students demonstrate interpreting an equation and its inverse within a real-world context and using the structure of equations to determine whether a given equation represents the inverse.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.19

Admission to a concert costs $30. T-shirts at the artist's merchandise stand costs $15 each.

1. The equation $C = 30 + 15t$ defines a function in this situation. What does the function represent in context?
   The function represents the total cost, in dollar, of entering the concert and buying t t-shirts.

2. What does the inverse of this function represent?
   The inverse of this function represents the number of t-shirts bought t when the total cost is C dollars.

3. Does the equation $t = \frac{C-5}{30}$ define the inverse of the function in Problem 1? Explain or show your thinking.
   No; Sample response: To determine the inverse of the original function, subtract 30 (the cost of admission) from the total cost and then divide the remaining amount by 15 (because each t-shirt costs $15). This equation shows subtracting by 5 and dividing by 30.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I can explain the meaning of an inverse of a function in terms of a situation.

1 2 3

b When I have an equation that defines a linear function, I know how to find its inverse.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 19 Finding and Interpreting Inverses of Functions

Success looks like . . .

- **Goal:** Writing the equation that represents the inverse of a function by solving the equation that represents the function for the input variable.
- **Language Goal:** Interpreting the inverse of a function in terms of the quantities in a situation. **(Speaking and Listening, Writing)**
  - » Explaining the meaning of the inverse of the function C in Problem 2.

Suggested next steps

If students explanations are vague or incorrect in Problems 1 and 2, consider:

- Reviewing interpreting the functions and their inverse in Activities 1 and 3.
- Assigning Practice Problems 1 and 2.
- Asking, "What does each variable and value in the equation represent?"

If students explanation is vague or incorrect in Problem 3, consider:

- Reviewing determining the inverses of the given functions in Activity 1.
- Assigning Practice Problems 1 and 2.
- Asking, "How could you check to see if this equation is the inverse?"

Professional Learning

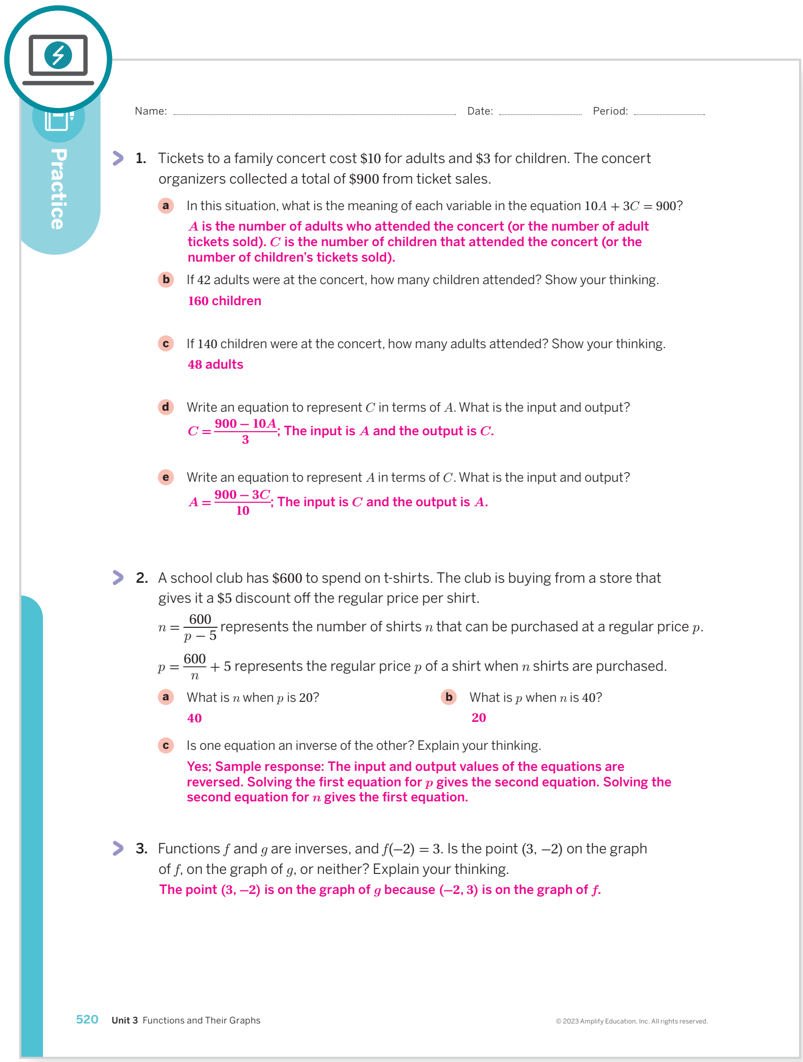
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways have your students improved in writing the equation that represents the inverse of a function?
- What different ways did students approach checking the accuracy of their inverse equations? What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ______ Date: ______ Period: ______

1. Tickets to a family concert cost $10 for adults and $3 for children. The concert organizers collected a total of $900 from ticket sales.
   - a. In this situation, what is the meaning of each variable in the equation $10A + 3C = 900$?
     A is the number of adults who attended the concert (or the number of adult tickets sold). C is the number of children that attended the concert (or the number of children's tickets sold).
   - b. If 42 adults were at the concert, how many children attended? Show your thinking.
     160 children
   - c. If 140 children were at the concert, how many adults attended? Show your thinking.
     48 adults
   - d. Write an equation to represent C in terms of A. What is the input and output?
     $C = \frac{900 - 10A}{3}$; The input is A and the output is C.
   - e. Write an equation to represent A in terms of C. What is the input and output?
     $A = \frac{900 - 3C}{10}$; The input is C and the output is A.

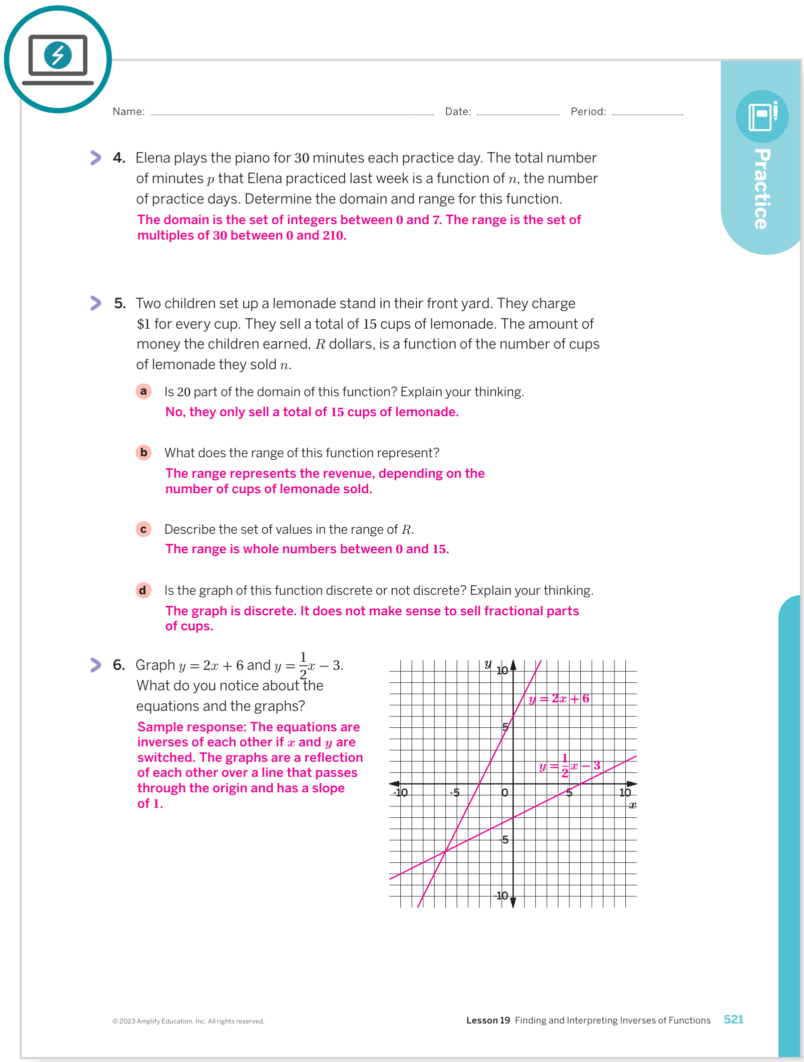
2. A school club has $600 to spend on t-shirts. The club is buying from a store that gives it a $5 discount off the regular price per shirt.

   $n = \frac{600}{p - 5}$ represents the number of shirts n that can be purchased at a regular price p.

   $p = \frac{600}{n} + 5$ represents the regular price p of a shirt when n shirts are purchased.
   - a. What is n when p is 20?
     40
   - b. What is p when n is 40?
     20
   - c. Is one equation an inverse of the other? Explain your thinking.
     Yes; Sample response: The input and output values of the equations are reversed. Solving the first equation for p gives the second equation. Solving the second equation for n gives the first equation.

3. Functions f and g are inverses, and $f(-2) = 3$. Is the point $(3, -2)$ on the graph of f, on the graph of g, or neither? Explain your thinking.
   The point $(3, -2)$ is on the graph of g because $(-2, 3)$ is on the graph of f.

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ______ Date: ______ Period: ______

4. Elena plays the piano for 30 minutes each practice day. The total number of minutes p that Elena practiced last week is a function of n, the number of practice days. Determine the domain and range for this function.
   The domain is the set of integers between 0 and 7. The range is the set of multiples of 30 between 0 and 210.

5. Two children set up a lemonade stand in their front yard. They charge $1 for every cup. They sell a total of 15 cups of lemonade. The amount of money the children earned, R dollars, is a function of the number of cups of lemonade they sold n.
   - a. Is 20 part of the domain of this function? Explain your thinking.
     No, they only sell a total of 15 cups of lemonade.
   - b. What does the range of this function represent?
     The range represents the revenue, depending on the number of cups of lemonade sold.
   - c. Describe the set of values in the range of R.
     The range is whole numbers between 0 and 15.
   - d. Is the graph of this function discrete or not discrete? Explain your thinking.
     The graph is discrete. It does not make sense to sell fractional parts of cups.

6. Graph $y = 2x + 6$ and $y = \frac{1}{2}x - 3$. What do you notice about the equations and the graphs?
   Sample response: The equations are inverses of each other if x and y are switched. The graphs are a reflection of each other over a line that passes through the origin and has a slope of 1.

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
	2	Activity 2	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 11	2
	5	Unit 3 Lesson 11	2
Formative ⚡	6	Unit 3 Lesson 20	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

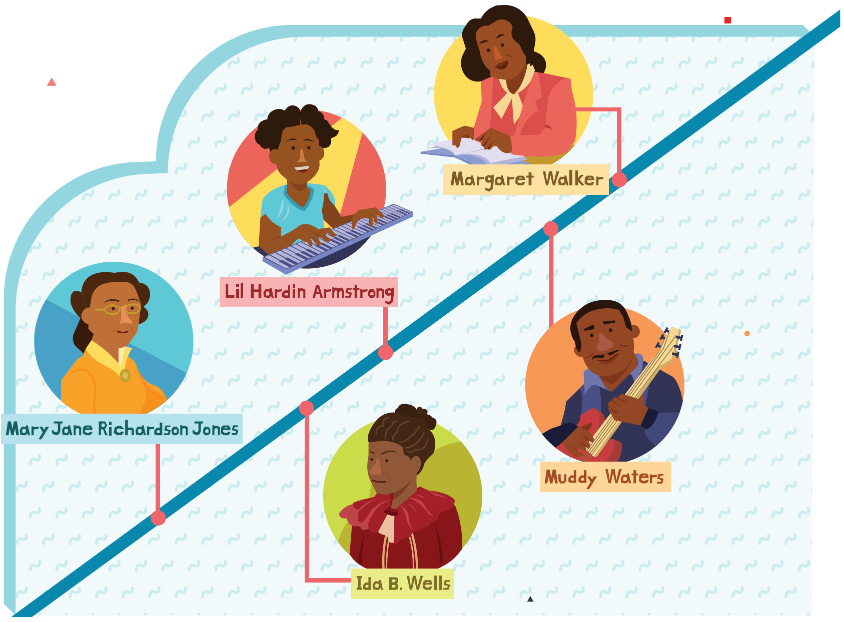
Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 3 | **LESSON 20**

Writing Inverses of Functions to Solve Problems

Let's use inverses of functions to solve real-world problems.



Focus

Goals

1. Determine the inverse of a linear function given in function notation.
2. **Language Goal:** Construct a linear function and its inverse to model data and solve problems, and interpret the function and its inverse within context. **(Reading and Writing)**

Coherence

Today

Students continue to expand their capacity to work with inverses of functions. They find and interpret inverses of linear functions in various contexts and determine which equation is more efficient to use when determining certain values.

Previously

In Lesson 19, students wrote the equations that represent inverses for functions that are defined using multiple operations.

Coming Soon

In Lesson 21, students will explore the relationship between the graph of a function and its inverse and realize the graph of a function and its inverse are reflections across the line $y = x$, if graphed using the same axes labels.

Rigor

- Students build **procedural fluency** of determining the inverses of linear functions.
- Students **apply** the inverses of functions to determine values and solve real-world problems, including the Great Migration.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	10 min	5 min	5 min
Pairs	Independent	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Anchor Chart PDF, *Function Notation*
- graphing technology

Math Language Development

Review words

- *function*
- *input*
- *inverse functions*
- *output*

Amps | **Featured Activity**

Activity 2
Finding a Line of Fit

Students write a line of fit of population data represented on a coordinate plane. Students then validate the accuracy of their line of fit.

Building Math Identity and Community

Connecting to Mathematical Practices

By needing to write the equations that represent the inverses of functions to solve problems, students will begin to recognize their strengths as well as their limitations. When the theoretical concept becomes practical, students might need to have the confidence to ask for help so that any gap in their understanding can be filled in. Remind them that it is ok not to understand something . . . yet.

Modifications to Pacing

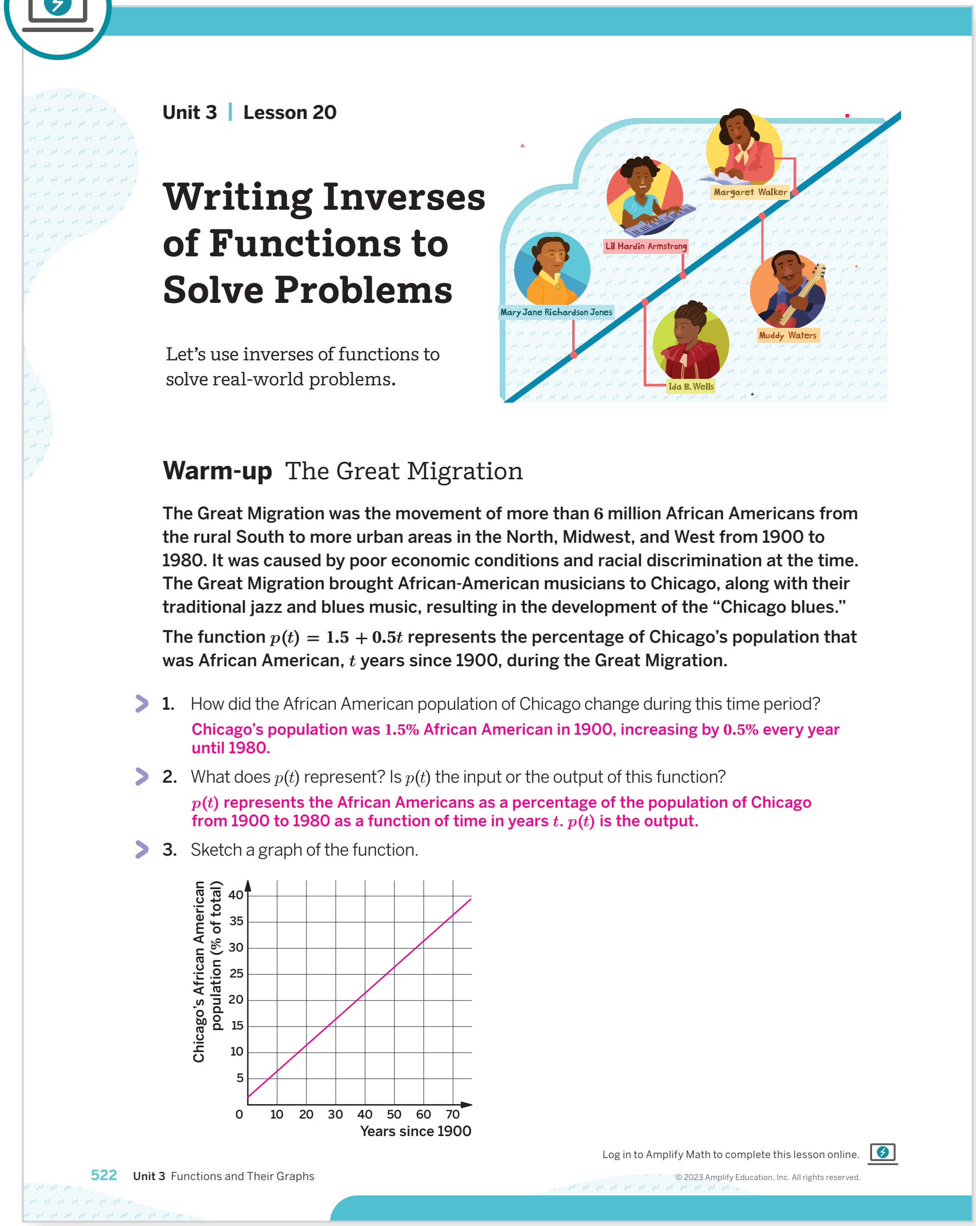
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Problem 3 may be omitted.
- In **Activity 2**, Problems 2 and 3 may be omitted.

Pairs | 5 min

Warm-up The Great Migration

Students interpret statements in function notation and sketch a graph of a function to prepare them to determine and interpret the inverse of a function within the context of the Great Migration.



Unit 3 | Lesson 20

Writing Inverses of Functions to Solve Problems

Let's use inverses of functions to solve real-world problems.

Warm-up The Great Migration

The Great Migration was the movement of more than 6 million African Americans from the rural South to more urban areas in the North, Midwest, and West from 1900 to 1980. It was caused by poor economic conditions and racial discrimination at the time. The Great Migration brought African-American musicians to Chicago, along with their traditional jazz and blues music, resulting in the development of the "Chicago blues."

The function $p(t) = 1.5 + 0.5t$ represents the percentage of Chicago's population that was African American, t years since 1900, during the Great Migration.

1. How did the African American population of Chicago change during this time period?

   Chicago's population was 1.5% African American in 1900, increasing by 0.5% every year until 1980.

2. What does $p(t)$ represent? Is $p(t)$ the input or the output of this function?

   $p(t)$ represents the African Americans as a percentage of the population of Chicago from 1900 to 1980 as a function of time in years t. $p(t)$ is the output.

3. Sketch a graph of the function.

Log in to Amplify Math to complete this lesson online.

522 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Ask students if they have heard of or previously learned about the Great Migration. Read the introduction together as a class. Have students discuss the meanings of the input and output values of the function with their partner, before completing the problems independently.

2 Monitor

Help students get started by having them identify the initial percentage of the population and how it changed each year.

Look for points of confusion:

- **Misinterpreting the meaning of $p(t)$.** Have students annotate the description of the function in the narrative, highlighting key words or phrases.

Look for productive strategies:

- Using the vertical intercept and slope to graph the function.

3 Connect

Display the function and its graph.

Have individual students share their description and interpretation of the function.

Highlight that the input t is the number of years since 1900, and the output $p(t)$ is the percentage of the population of Chicago that was African American during year t.

Ask, "Was Chicago's African American population increasing or decreasing during this time period? Explain your thinking." The African American population was also increasing because the function's rate of change is positive.

Power-up

To power up students' ability to graph a linear function, have students complete:

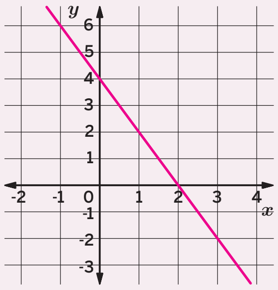
Recall that when linear equations are written of the form $y = mx + b$, m represents the slope and b represents the y-intercept $(0, b)$.

Complete each problem for the equation $y = -2x + 4$.

a. What are the coordinates of the y-intercept? **$(0, 4)$**

b. What is the slope? **-2**

c. Graph the function.



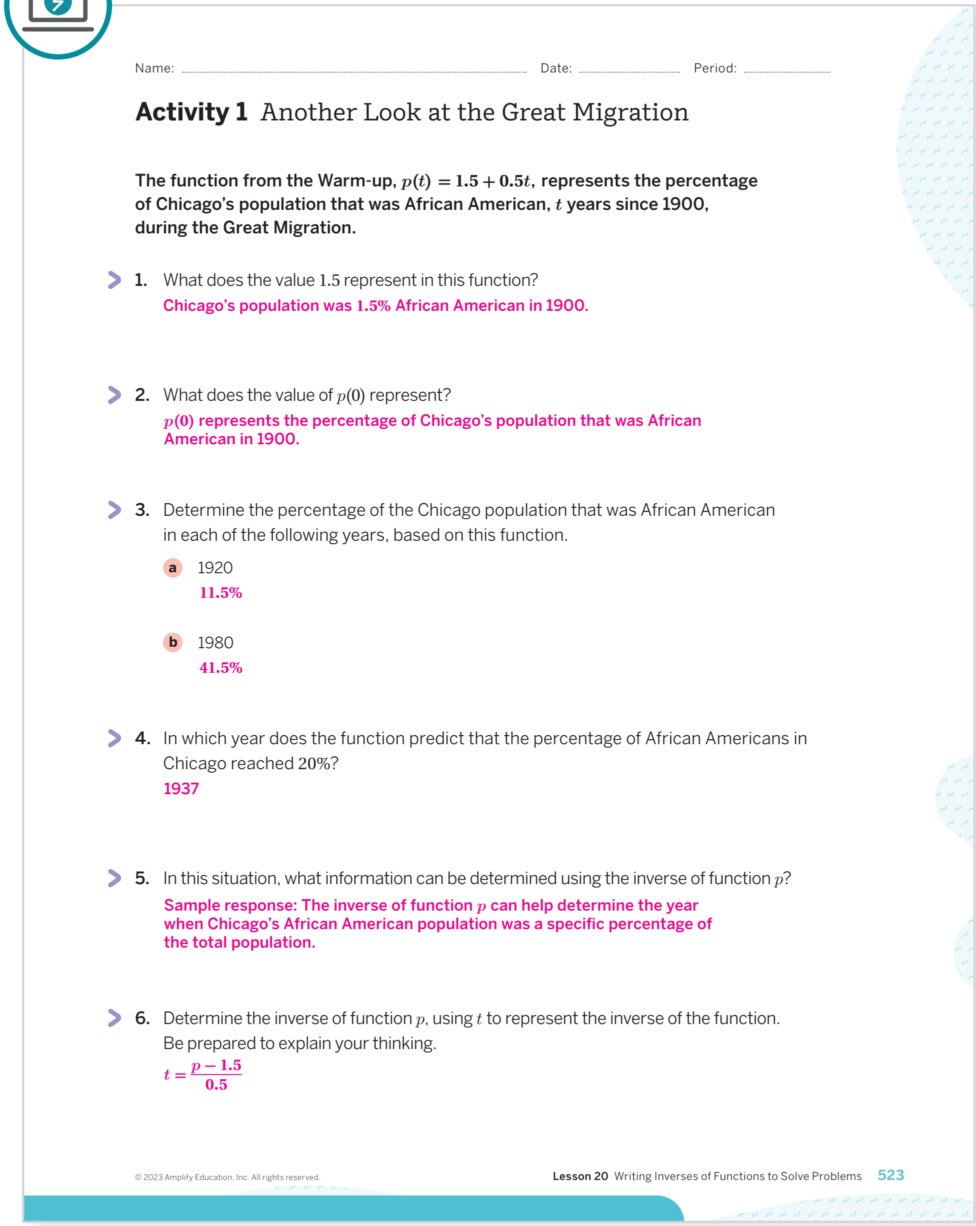
Use: Before the Warm-up

Informed by: Performance on Lesson 19, Practice Problem 6

Independent | 10 min

Activity 1 Another Look at the Great Migration

Students continue studying the Great Migration, constructing the inverse of the function from the Warm-up and interpreting it in context.



Name: __________ Date: __________ Period: __________

Activity 1 Another Look at the Great Migration

The function from the Warm-up, $p(t) = 1.5 + 0.5t$, represents the percentage of Chicago's population that was African American, t years since 1900, during the Great Migration.

1. What does the value 1.5 represent in this function?
   Chicago's population was 1.5% African American in 1900.

2. What does the value of $p(0)$ represent?
   $p(0)$ represents the percentage of Chicago's population that was African American in 1900.

3. Determine the percentage of the Chicago population that was African American in each of the following years, based on this function.
   - a 1920
     11.5%
   - b 1980
     41.5%

4. In which year does the function predict that the percentage of African Americans in Chicago reached 20%?
   1937

5. In this situation, what information can be determined using the inverse of function p?
   Sample response: The inverse of function p can help determine the year when Chicago's African American population was a specific percentage of the total population.

6. Determine the inverse of function p, using t to represent the inverse of the function. Be prepared to explain your thinking.
   $t = \frac{p - 1.5}{0.5}$

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Writing Inverses of Functions to Solve Problems 523

1 Launch

Highlight that the given function is the same one from the Warm-up. Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by having them use the graph from the Warm-up to determine values of the function.

Look for points of confusion:

- **Vaguely explaining the use of the inverse.** Have students determine what the input and output values of the inverse represent.

Look for productive strategies:

- Checking the accuracy of their inverse equation by substituting input-output pairs from previous problems.

3 Connect

Have individual students share their equation and interpretation of the inverse.

Highlight that the inverse is represented as an equation that gives t in terms of p because the input of the inverse is a percentage of Chicago's African American population and can help to determine the year associated with this percentage.

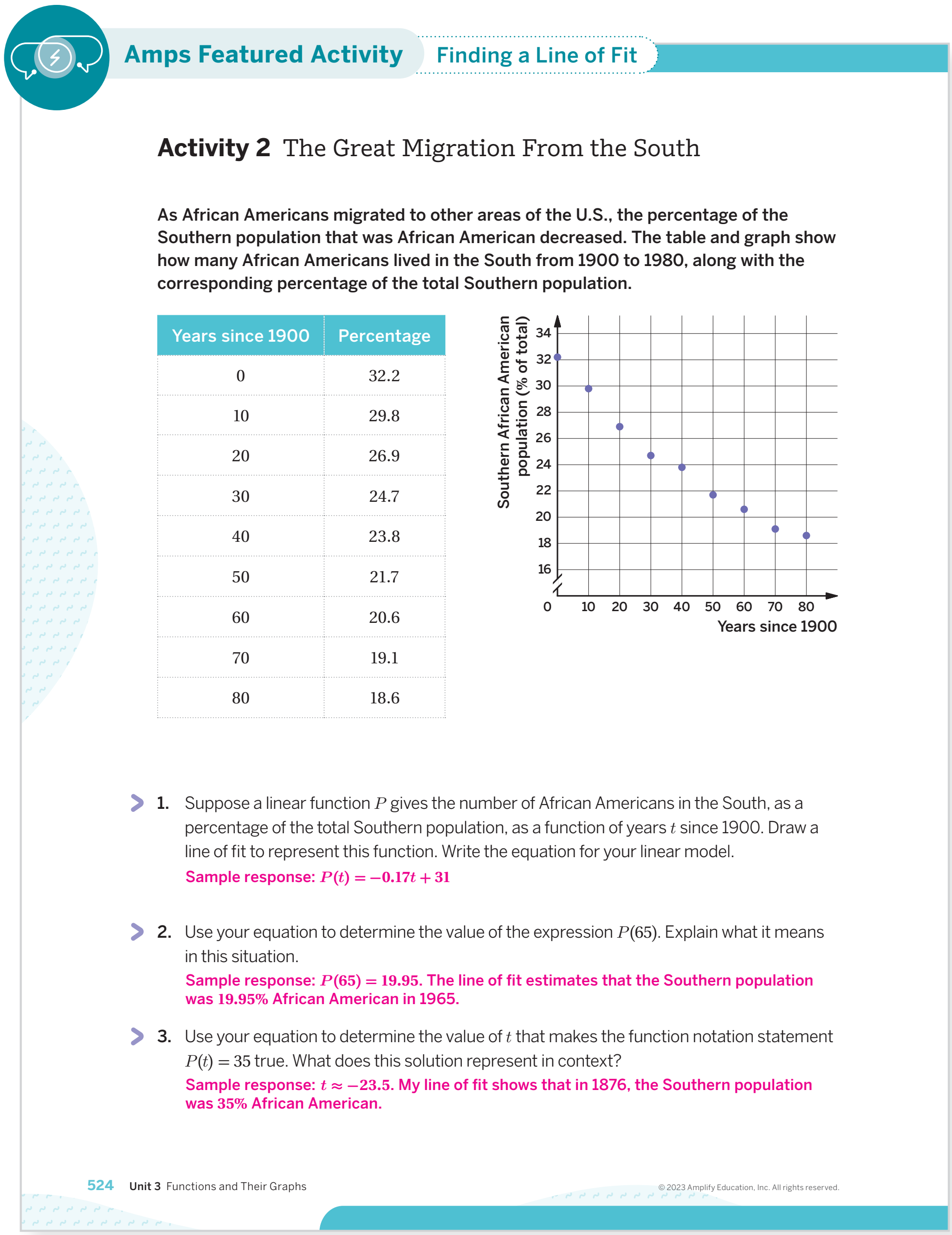
Ask, "Which equation would you choose to use if you wanted to determine the year in which the African American population in Chicago was a certain percentage? Why?"
Sample response: I would use the inverse equation because it is already solved for the variable t, in terms of p.

Pairs | 15 min

Activity 2 The Great Migration From the South

Students determine a linear function that models given data about the Great Migration and are motivated to determine the inverse of the function to more efficiently answer questions within context.

Amps Featured Activity Finding a Line of Fit



Activity 2 The Great Migration From the South

As African Americans migrated to other areas of the U.S., the percentage of the Southern population that was African American decreased. The table and graph show how many African Americans lived in the South from 1900 to 1980, along with the corresponding percentage of the total Southern population.

Years since 1900	Percentage
0	32.2
10	29.8
20	26.9
30	24.7
40	23.8
50	21.7
60	20.6
70	19.1
80	18.6

1. Suppose a linear function P gives the number of African Americans in the South, as a percentage of the total Southern population, as a function of years t since 1900. Draw a line of fit to represent this function. Write the equation for your linear model.
   Sample response: $P(t) = -0.17t + 31$

2. Use your equation to determine the value of the expression $P(65)$. Explain what it means in this situation.
   Sample response: $P(65) = 19.95$. The line of fit estimates that the Southern population was 19.95% African American in 1965.

3. Use your equation to determine the value of t that makes the function notation statement $P(t) = 35$ true. What does this solution represent in context?
   Sample response: $t \approx -23.5$. My line of fit shows that in 1876, the Southern population was 35% African American.

524 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Provide access to graphing technology. Explain to students that they may sketch a line of fit or use graphing technology to determine a line of fit.

2 Monitor

Help students get started by helping them sketch a line of fit that runs through the data so that as many points as possible are close to the line.

Look for points of confusion:

- **Having difficulty creating an equation for their line of fit.** Have students identify the vertical intercept and the slope to help them create their equation.
- **Not using their original function to determine an equation in Problem 4.** Ask, "How could you create an equation so that the percentage of the Southern population that is African American is the input of the equation?"

Look for productive strategies:

- Checking the equation of their line of fit by evaluating the equation for various years and comparing these input-output pairs with the graph of the line of fit and the scatter plot.
- Using their inverse equation to determine or check the value they found in Problem 3.

Activity 1 continued ›

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Students have previously learned to determine a line of fit for data on a scatter plot. Consider demonstrating, or ask a student volunteer to demonstrate, how to use graphing technology to determine a line of fit.

Extension: *Math Enrichment*

Have students use graphing technology to graph the function and its inverse on the same coordinate plane, where the horizontal axis represents the time and the vertical axis represents the population. Have them describe what they notice. The graphs are a reflection of each other over a line that passes through the origin and has a slope of 1.

Math Language Development

MLR8: Discussion Supports — Revoicing

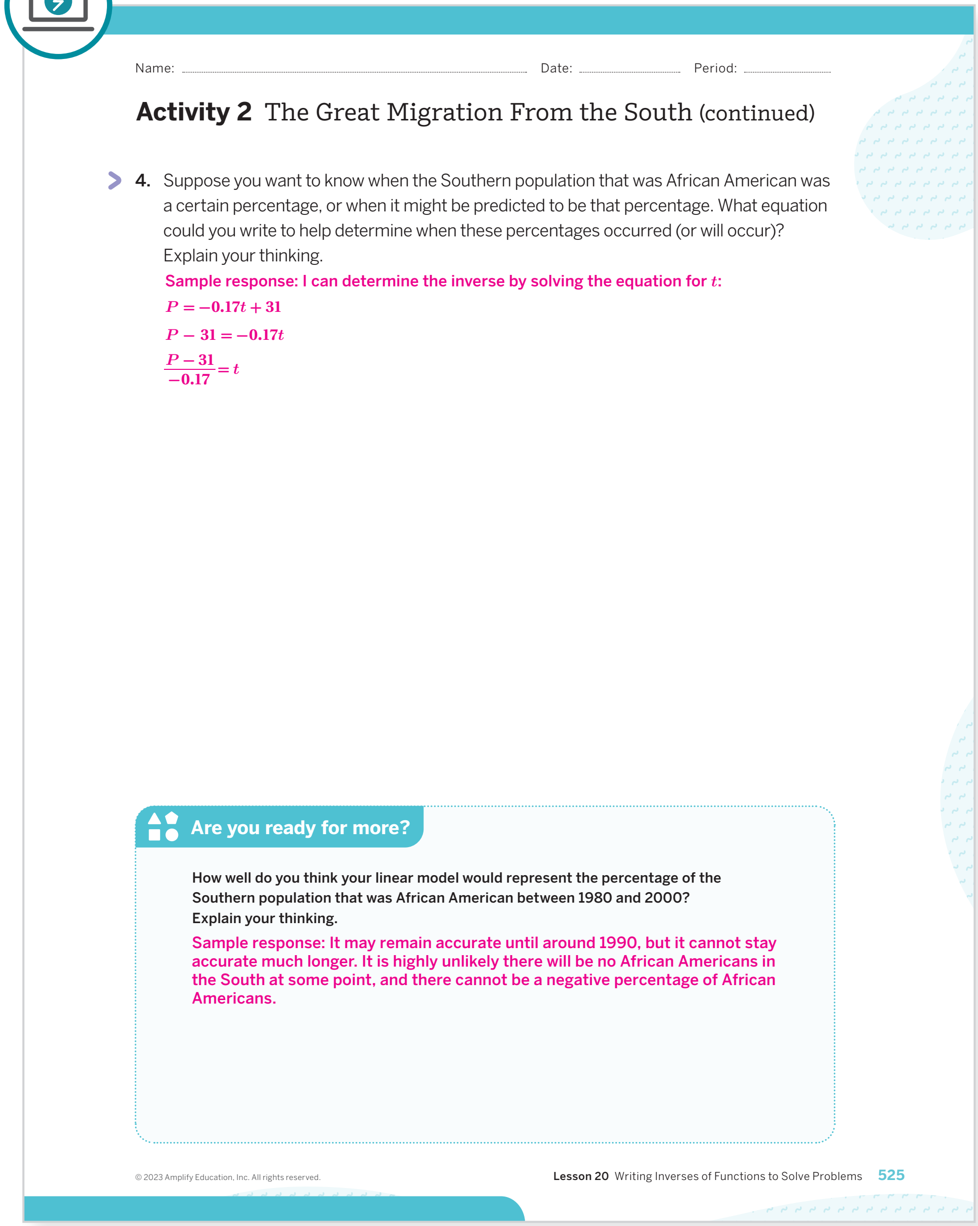
During the Connect, as students respond to the Ask questions, listen for evidence of their developing math language, particularly their comfort level with using the terms *input, output, domain, range, function,* and *inverse*. Revoice their statements using correct mathematical vocabulary to help reinforce these terms.

Some students may use the term *inverse function*, even though they have not yet determined whether the inverse is actually a function, which they will do in upcoming lessons.

Pairs | 15 min

Activity 2 The Great Migration From the South (continued)

Students determine a linear function that models given data about the Great Migration and are motivated to determine the inverse of the function to more efficiently answer questions within context.



Name: ________ Date: ________ Period: ________

Activity 2 The Great Migration From the South (continued)

4. Suppose you want to know when the Southern population that was African American was a certain percentage, or when it might be predicted to be that percentage. What equation could you write to help determine when these percentages occurred (or will occur)? Explain your thinking.

Sample response: I can determine the inverse by solving the equation for t:

$P = -0.17t + 31$

$P - 31 = -0.17t$

$\frac{P - 31}{-0.17} = t$

Are you ready for more?

How well do you think your linear model would represent the percentage of the Southern population that was African American between 1980 and 2000? Explain your thinking.

Sample response: It may remain accurate until around 1990, but it cannot stay accurate much longer. It is highly unlikely there will be no African Americans in the South at some point, and there cannot be a negative percentage of African Americans.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Writing Inverses of Functions to Solve Problems 525

3 Connect

Display the scatter plot with a line of fit.

Have individual students share how they wrote an equation to model the relationship in the data.

Highlight that depending on the value given and the value students are attempting to determine, the function or its inverse may be more efficient. Either one could be used to determine any input-output pair, but additional steps would be needed if the less efficient equation is used.

Ask:

- "Which equation would you use to complete Problem 3? Explain your thinking." I would use the inverse of P. The input for this equation is the percentage of the Southern population that is African American, so I could substitute this value into the inverse equation and simplify the expression on one side of the equation to determine the value of t.
- "What is an example of a function where both the function and its inverse are both equally efficient to use no matter whether input or output values are given?" Sample response: $y = x$.

Fostering Diverse Thinking

Individuals in the Great Migration

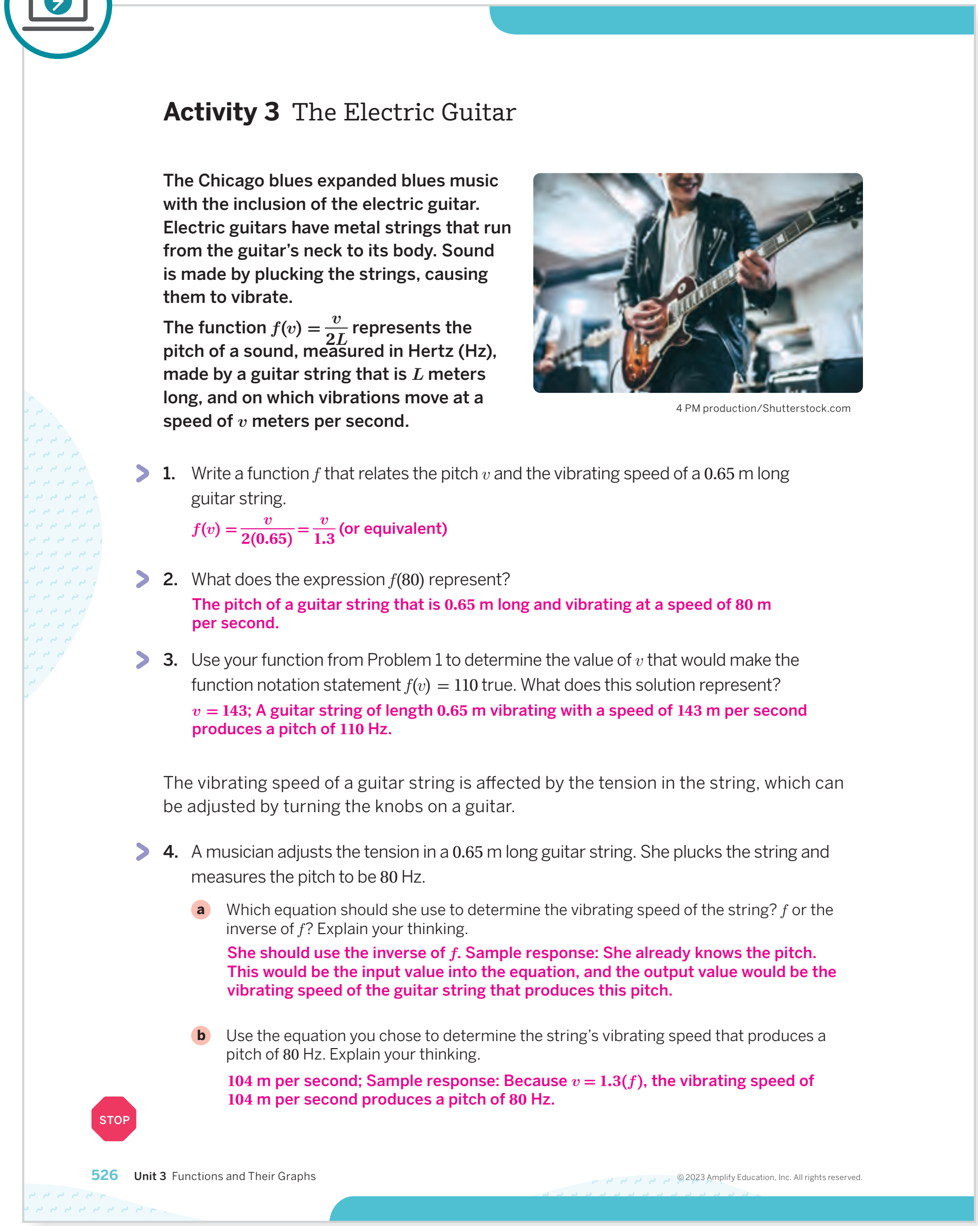
Highlight that the graph in the activity is not quite linear. Ask students when the Great Migration might have peaked. Then have them research individuals who were part of the Great Migration, and when they migrated. Such individuals might include:

- Mallie Robinson, who, as a single mother, took her and her five children from Georgia to California in 1920. Her son would later grow up to break the color barrier in Major League Baseball.
- Tom Bradley, the first Black mayor of Los Angeles, California. His parents were sharecroppers in Texas and migrated in the 1920s, when he was just 7 years old.
- Muddy Waters, who migrated in 1943 from Mississippi to Chicago and became an American blues singer and songwriter.

Independent | 10 min

Activity 3 The Electric Guitar

Students construct a function to model the sound made by a guitar string, answer questions about the function within the context, and determine whether the function or its inverse is more efficient.



Activity 3 The Electric Guitar

The Chicago blues expanded blues music with the inclusion of the electric guitar. Electric guitars have metal strings that run from the guitar's neck to its body. Sound is made by plucking the strings, causing them to vibrate.

The function $f(v) = \frac{v}{2L}$ represents the pitch of a sound, measured in Hertz (Hz), made by a guitar string that is L meters long, and on which vibrations move at a speed of v meters per second.

4 PM production/Shutterstock.com

1. Write a function f that relates the pitch v and the vibrating speed of a 0.65 m long guitar string.
   $f(v) = \frac{v}{2(0.65)} = \frac{v}{1.3}$ **(or equivalent)**
2. What does the expression $f(80)$ represent?
   The pitch of a guitar string that is 0.65 m long and vibrating at a speed of 80 m per second.
3. Use your function from Problem 1 to determine the value of v that would make the function notation statement $f(v) = 110$ true. What does this solution represent?
   $v = 143$; **A guitar string of length 0.65 m vibrating with a speed of 143 m per second produces a pitch of 110 Hz.**

The vibrating speed of a guitar string is affected by the tension in the string, which can be adjusted by turning the knobs on a guitar.

4. A musician adjusts the tension in a 0.65 m long guitar string. She plucks the string and measures the pitch to be 80 Hz.
   a. Which equation should she use to determine the vibrating speed of the string? f or the inverse of f? Explain your thinking.
      She should use the inverse of f. Sample response: She already knows the pitch. This would be the input value into the equation, and the output value would be the vibrating speed of the guitar string that produces this pitch.
   b. Use the equation you chose to determine the string's vibrating speed that produces a pitch of 80 Hz. Explain your thinking.
      104 m per second; Sample response: Because $v = 1.3(f)$, the vibrating speed of 104 m per second produces a pitch of 80 Hz.

STOP

526 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display the narrative and the function. Ask students what is different about this equation from earlier equations. Highlight that there are three variables, emphasizing that students will use a string length of 0.65 m for all problems.

2 Monitor

Help students get started by highlighting that $L = 0.65$ can be considered a constant in this activity and replaces L in the function $f(v)$.

Look for points of confusion:

- **Substituting 0.65 for v or $f(v)$.** Ask, "What is the input and output of $f(v)$? Does the guitar string length change?"

Look for productive strategies:

- Checking the value they found in Problem 3 using the inverse of $f(v)$ by substituting their solution and $L = 0.65$ into the function $f(v)$.

3 Connect

Display the function $f(v)$ and its inverse.

Have individual students share their thinking for Problem 4.

Highlight that in this scenario, L is considered a constant and students can determine the inverse of $f(v)$ by solving the equation for v.

Ask, "How would the function change if you are given a vibrating speed, and the pitch is now only a function of L?" The function would change to $f(L)$, because L is now the input variable.

Differentiated Support

Accessibility: ***Guide Processing and Visualization***

Display the original function and demonstrate to students how to write the function, given the length of the guitar string of 0.65 m in Problem 1.

Extension: ***Math Enrichment***

Have students write the equation that represents the length (in meters) of the guitar string L if the pitch of the sound is p and the speed of the vibrations (in meters per second) is v.

$L = \frac{v}{2p}$ (or equivalent)

Math Language Development

MLR5: Co-craft Questions

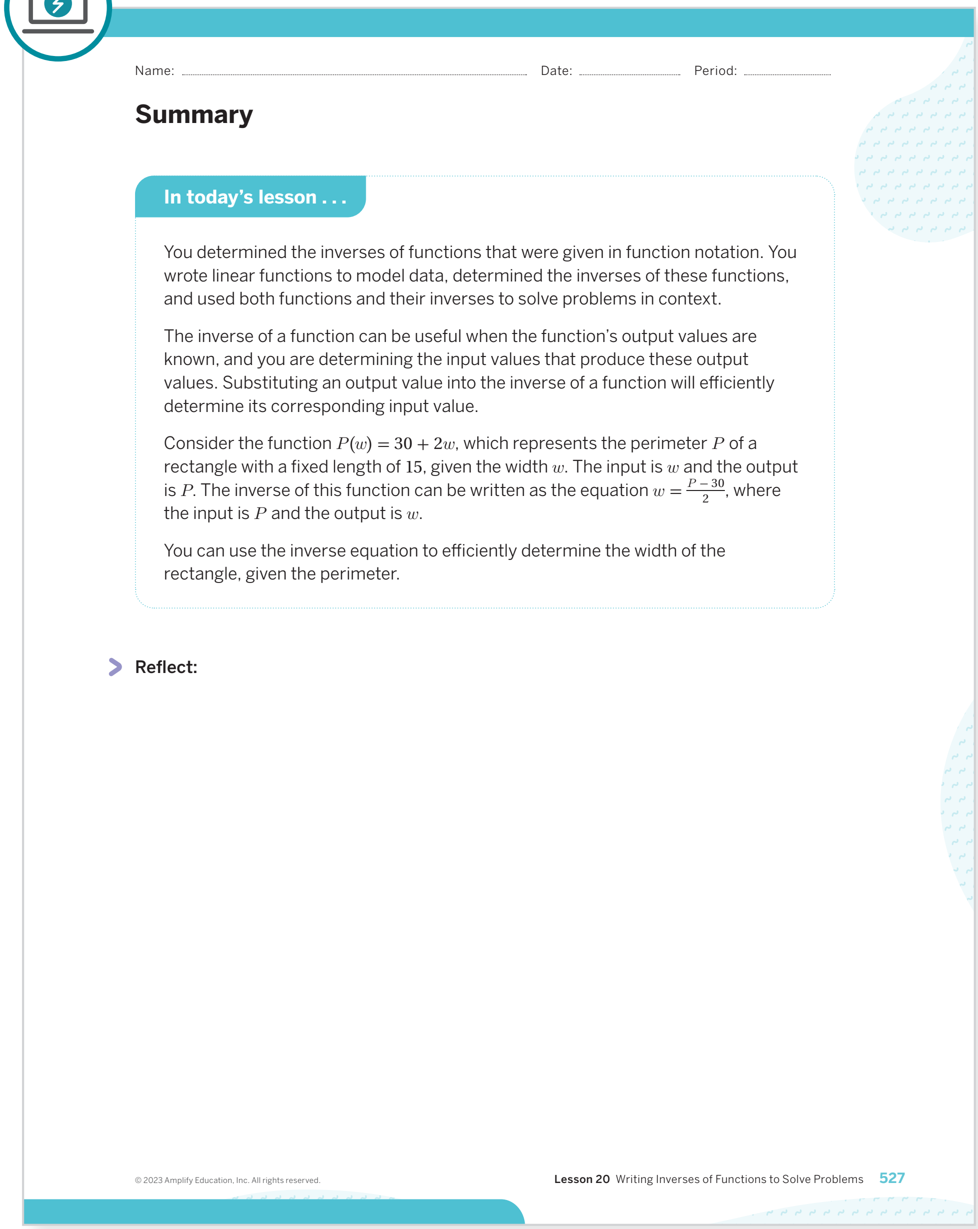
During the Launch, display only the introductory text and the given function. Ask students to work with a partner to write 2 – 3 mathematical questions they have about this scenario or function. Ask volunteers to share their questions with the class before students begin the activity. Some sample questions could be:

- "Why are there three variables in this function?"
- "What are the three variables in this function?"
- "What does this function mean? Why is the pitch dependent on two different input values?"

Whole Class | 5 min

Summary

Review and synthesize how the inverses of functions and be constructed to model and solve real-world problems.



Name: Date: Period:

Summary

In today's lesson . . .

You determined the inverses of functions that were given in function notation. You wrote linear functions to model data, determined the inverses of these functions, and used both functions and their inverses to solve problems in context.

The inverse of a function can be useful when the function's output values are known, and you are determining the input values that produce these output values. Substituting an output value into the inverse of a function will efficiently determine its corresponding input value.

Consider the function $P(w) = 30 + 2w$, which represents the perimeter P of a rectangle with a fixed length of 15, given the width w. The input is w and the output is P. The inverse of this function can be written as the equation $w = \frac{P - 30}{2}$, where the input is P and the output is w.

You can use the inverse equation to efficiently determine the width of the rectangle, given the perimeter.

Reflect:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Writing Inverses of Functions to Solve Problems 527

Synthesize

Display the function and its inverse from Activity 1, as well as the graph of the function and the graph of its inverse.

Have students share what the output values of the inverse represent in this context.

Highlight that a function and its inverse represent the same relationship between two sets of values. While the input-output pairs switch for a function and its inverse, the same relationship still holds true.

Ask, "When is it helpful to determine and use the inverse of a function?" When several output values of the original function are given and we want to determine the corresponding input values of the original function.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

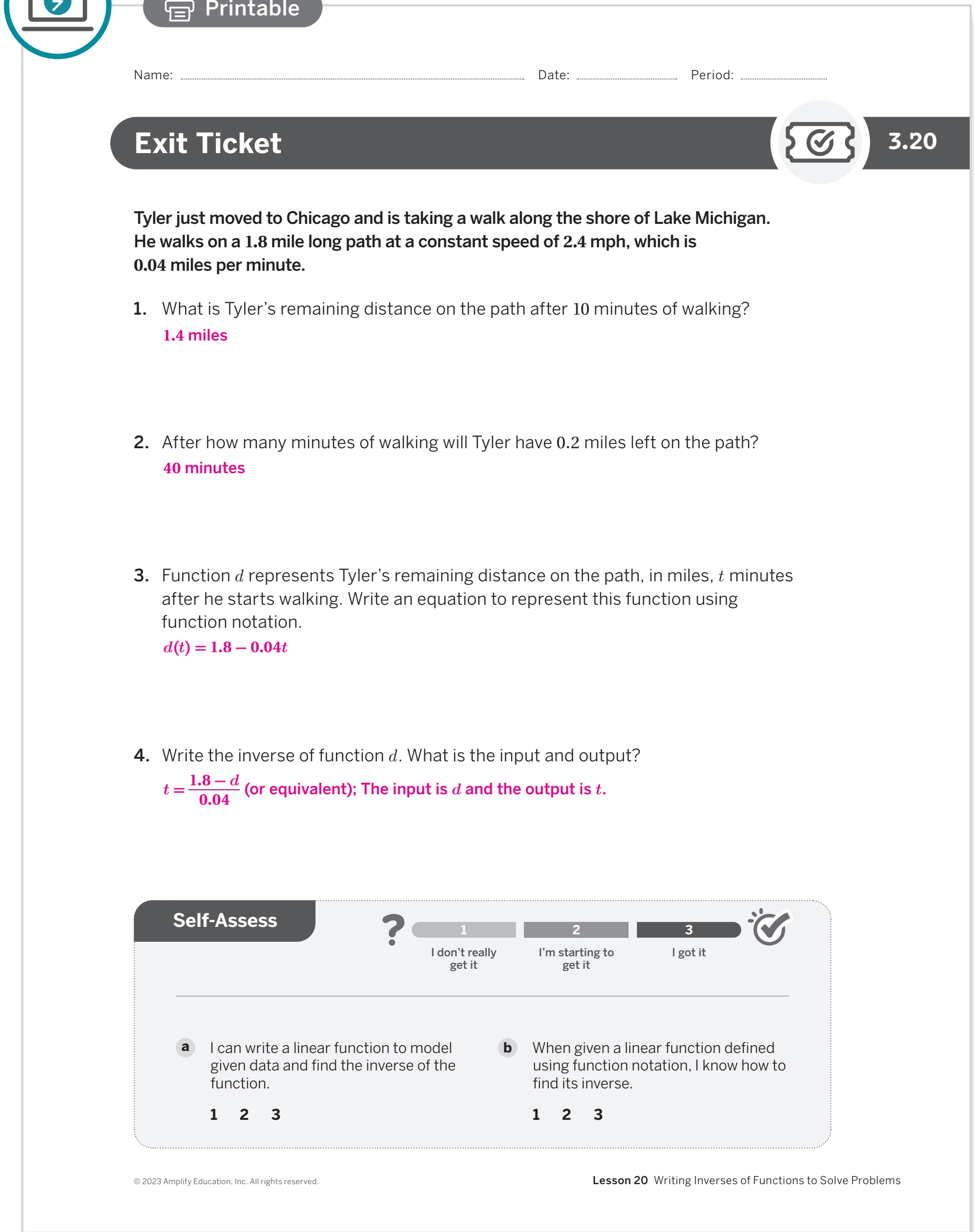
- "Why is it helpful to determine the inverse of a function when working with real-world problems"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by constructing a function to model a given context, use the function to solve problems, and interpret the inverse of the function in context.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket 3.20

Tyler just moved to Chicago and is taking a walk along the shore of Lake Michigan. He walks on a 1.8 mile long path at a constant speed of 2.4 mph, which is 0.04 miles per minute.

1. What is Tyler's remaining distance on the path after 10 minutes of walking?
   1.4 miles

2. After how many minutes of walking will Tyler have 0.2 miles left on the path?
   40 minutes

3. Function d represents Tyler's remaining distance on the path, in miles, t minutes after he starts walking. Write an equation to represent this function using function notation.
   $d(t) = 1.8 - 0.04t$

4. Write the inverse of function d. What is the input and output?
   $t = \frac{1.8 - d}{0.04}$ **(or equivalent); The input is d and the output is t.**

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can write a linear function to model given data and find the inverse of the function.
1 2 3

b. When given a linear function defined using function notation, I know how to find its inverse.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Writing Inverses of Functions to Solve Problems

Success looks like . . .

- **Goal:** Determining the inverse of a linear function given in function notation.
- **Language Goal:** Constructing a linear function and its inverse to model data and solve problems, and interpreting the function and its inverse within context. **(Reading and Writing)**.
  - » Determining function d and its inverse in Problems 3 and 4.

Suggested next steps

If students' values are inaccurate for Problems 1 and 2, consider:

- Reviewing determining input and output values from Activity 1.
- Assigning Practice Problems 1 and 2.
- Asking, "Which rate in the problem would be helpful to determine this value?"

If students write an incorrect equation for Problem 3, consider:

- Reviewing the meaning of the values in the given function from Activity 1.
- Assigning Practice Problems 1 and 2.
- Asking, "What is the initial distance Tyler has to cover? How quickly is this distance decreasing?"

If students write an incorrect inverse equation or incorrectly describe the input and output for Problem 4, consider:

- Reviewing how to determine the equation of the inverse of a function and its use and meaning from Activities 2 and 3.
- Assigning Practice Problems 1–3.
- Asking, "How is the inverse of a function different from the function itself?"

Professional Learning

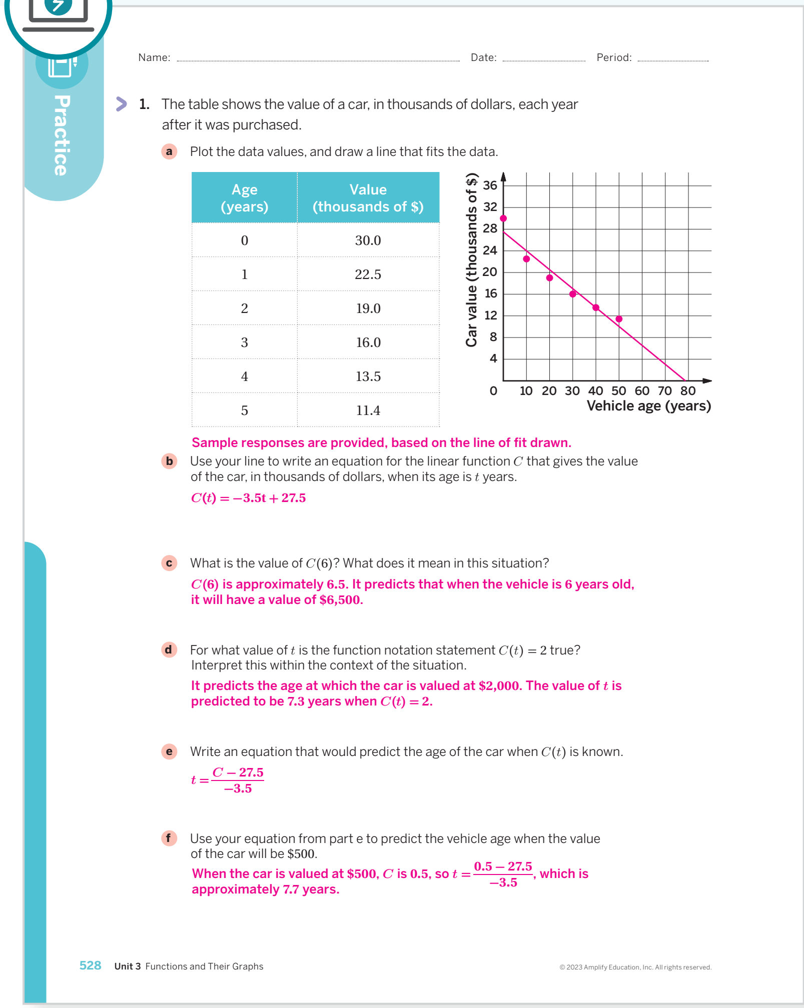
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? When you compare and contrast today's work with work students did earlier this year on interpreting the meaning of a function or equation, what similarities and differences do you see?
- During the discussion about students' interpretation of the inverse of a function, how did you encourage each student to share their understandings? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

1. The table shows the value of a car, in thousands of dollars, each year after it was purchased.

   a. Plot the data values, and draw a line that fits the data.

Age (years)	Value (thousands of $)
0	30.0
1	22.5
2	19.0
3	16.0
4	13.5
5	11.4

Sample responses are provided, based on the line of fit drawn.

   b. Use your line to write an equation for the linear function C that gives the value of the car, in thousands of dollars, when its age is t years.

   $C(t) = -3.5t + 27.5$

   c. What is the value of $C(6)$? What does it mean in this situation?

   $C(6)$ is approximately 6.5. It predicts that when the vehicle is 6 years old, it will have a value of $6,500.

   d. For what value of t is the function notation statement $C(t) = 2$ true? Interpret this within the context of the situation.

   It predicts the age at which the car is valued at $2,000. The value of t is predicted to be 7.3 years when $C(t) = 2$.

   e. Write an equation that would predict the age of the car when $C(t)$ is known.

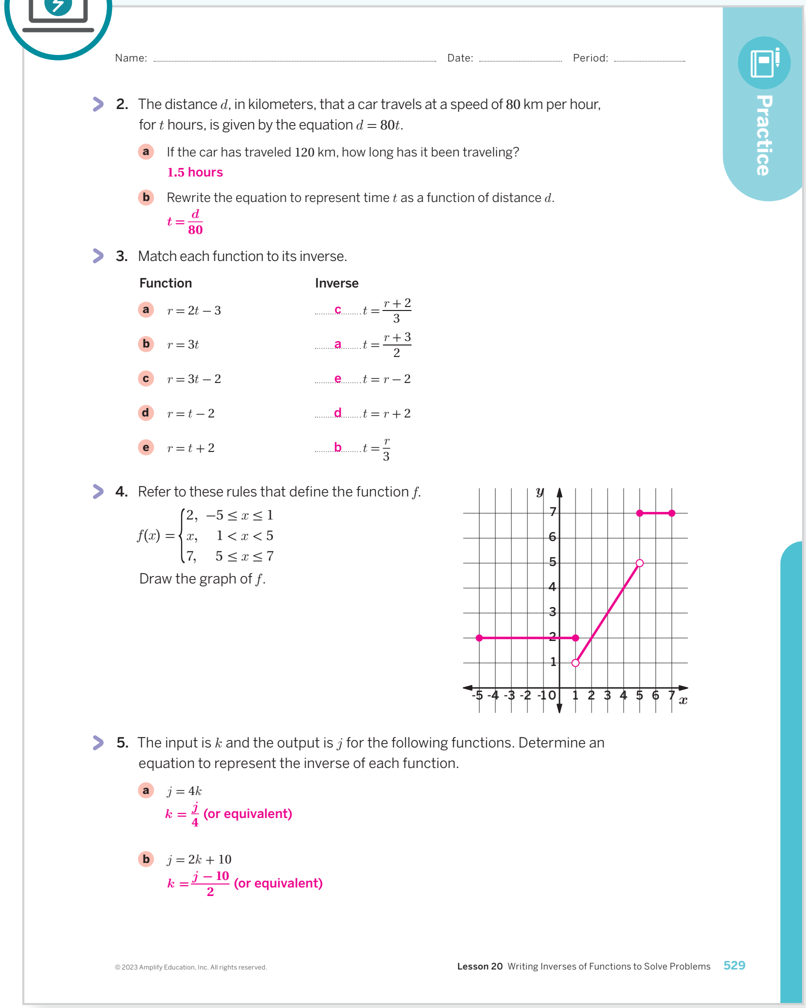
   $t = \frac{C - 27.5}{-3.5}$

   f. Use your equation from part e to predict the vehicle age when the value of the car will be $500.

   When the car is valued at $500, C is 0.5, so $t = \frac{0.5 - 27.5}{-3.5}$, which is approximately 7.7 years.

528 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.



Practice

Name: ________ Date: ________ Period: ________

2. The distance d, in kilometers, that a car travels at a speed of 80 km per hour, for t hours, is given by the equation $d = 80t$.

   a. If the car has traveled 120 km, how long has it been traveling?

   1.5 hours

   b. Rewrite the equation to represent time t as a function of distance d.

   $t = \frac{d}{80}$

3. Match each function to its inverse.

Function	Inverse
a. $r = 2t - 3$	c $t = \frac{r+2}{3}$
b. $r = 3t$	a $t = \frac{r+3}{2}$
c. $r = 3t - 2$	e $t = r - 2$
d. $r = t - 2$	d $t = r + 2$
e. $r = t + 2$	b $t = \frac{r}{3}$

4. Refer to these rules that define the function f.

$$f(x) = \begin{cases} 2, & -5 \le x \le 1 \\ x, & 1 < x < 5 \\ 7, & 5 \le x \le 7 \end{cases}$$

   Draw the graph of f.

5. The input is k and the output is j for the following functions. Determine an equation to represent the inverse of each function.

   a. $j = 4k$

   $k = \frac{j}{4}$ **(or equivalent)**

   b. $j = 2k + 10$

   $k = \frac{j - 10}{2}$ **(or equivalent)**

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 20 Writing Inverses of Functions to Solve Problems 529

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 1	2
	3	Activity 1	2
Spiral	4	Unit 3 Lesson 16	2
Formative ⚡	5	Unit 3 Lesson 21	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 3 | **LESSON 21**

Graphing Inverses of Functions

Let’s examine the relationship between the graph of a function and its inverse.

Focus

Goals

1. Graph the inverse of a linear function.
2. Understand that the graph of the inverse of a function can be found by reflecting the function’s graph across the line $y = x$.

Coherence

Today

Students notice that the table of values for a function and its inverse are reversed. They use these tables to construct a graph of the inverse to see that the graph of a function and its inverse are reflections of each other across the line $y = x$. They then use this understanding to graph the inverse of nonlinear graphs.

Previously

In Lesson 20, students determined and interpreted inverses of linear functions in various contexts.

Coming Soon

In Lesson 22, the capstone lesson, students will create piecewise functions using descriptions of their key features and parts of their graphs.

Rigor

- Students build **conceptual understanding** of the relationship between the graph of a function and the graph of the inverse of the function.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	10 min	15 min	10 min	5 min	5 min
Independent	Independent	Pairs	Independent	Whole Class	Independent

Amps powered by desmos | Activity and Presentation Slides

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF
- Anchor Chart PDF, *Sentence Stems, Describing My Thinking* (as needed)
- Anchor Chart PDF, *Sentence Stems, Notice and Wonder* (as needed)
- graphing technology

Math Language Development

Review words

- *function*
- *input*
- *inverse functions*
- *nonlinear function*
- *output*

Amps | Featured Activity

Activity 3
Interactive Graphs

Students create the graph of the inverse of a function given the graph of a function.

Building Math Identity and Community

Connecting to Mathematical Practices

Graphing the inverses of functions might seem like a difficult task, but as students analyze the structure of the graphs, the task will become more manageable. By seeing the inverse as a reflection of the function, they can use the structure of the coordinate plane to create its graph. While this is an optional lesson, it is a good lesson for students to use as an academic goal.

Modifications to Pacing

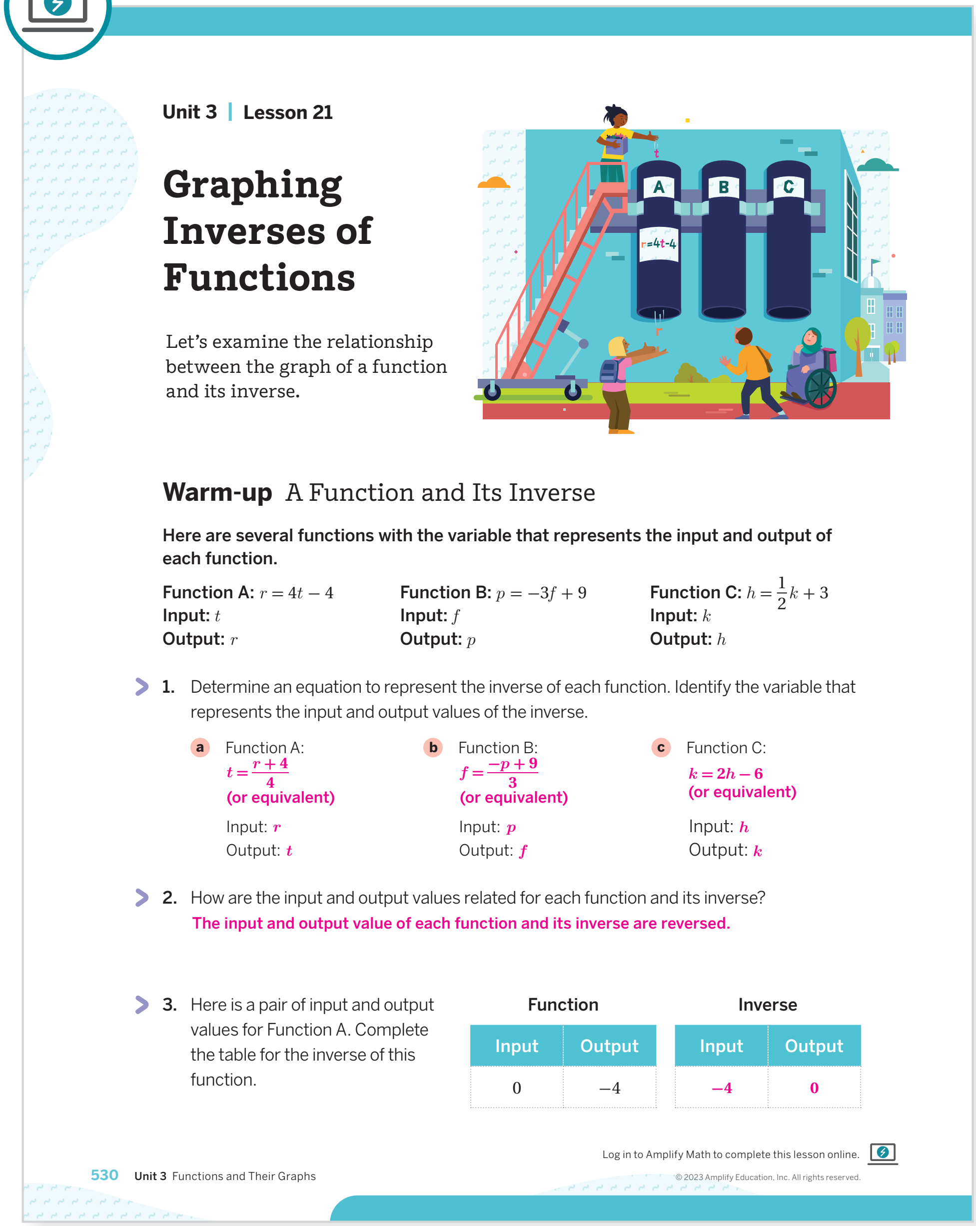
You may want to consider these additional modifications if you are short on time.

- In the **Warm-up**, Function B can be omitted.
- In **Activity 1**, have students only complete tables for Functions A and B, and then only assign these functions in **Activity 2**.
- **Activity 3** may be omitted.

Independent | 5 min

Warm-up A Function and Its Inverse

Students determine the equations that represent the inverses of three functions and describe their input-output pairs to prepare them for examining the relationship between functions and their inverses more closely.



Unit 3 | Lesson 21

Graphing Inverses of Functions

Let's examine the relationship between the graph of a function and its inverse.

Warm-up A Function and Its Inverse

Here are several functions with the variable that represents the input and output of each function.

Function A: $r = 4t - 4$
Input: t
Output: r

Function B: $p = -3f + 9$
Input: f
Output: p

Function C: $h = \frac{1}{2}k + 3$
Input: k
Output: h

1. Determine an equation to represent the inverse of each function. Identify the variable that represents the input and output values of the inverse.

   a. Function A:
   $t = \frac{r + 4}{4}$
   (or equivalent)
   Input: r
   Output: t

   b. Function B:
   $f = \frac{-p + 9}{3}$
   (or equivalent)
   Input: p
   Output: f

   c. Function C:
   $k = 2h - 6$
   (or equivalent)
   Input: h
   Output: k

2. How are the input and output values related for each function and its inverse?
   The input and output value of each function and its inverse are reversed.

3. Here is a pair of input and output values for Function A. Complete the table for the inverse of this function.

Function		Inverse	
Input	Output	Input	Output
0	-4	-4	0

Log in to Amplify Math to complete this lesson online.

530 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Say, "A function and its inverse have a unique relationship, and today, you will explore how that relationship is represented on a graph." Set an expectation for the amount of time students will have to work individually on the activity.

2 Monitor

Help students get started by reminding them that to determine the equation of the inverse, they need to isolate the other variable.

Look for points of confusion:

- **Leaving the input-output pair in the table unchanged.** Ask, "What is the advantage of using the equation of the inverse? What value would you substitute into the equation of the inverse?"

Look for productive strategies:

- Checking the input-output values from the tables to confirm their description of their relationship.

3 Connect

Display both tables.

Have individual students share their equations of each inverse and their description of the relationship from Problem 2.

Highlight that the variables that represent the input and the output are reversed from the original function. This is illustrated by the values in the tables also being reversed.

Ask, "How does the equation for the inverse of a function reflect the reversal of input-output pairs?" The variable that represented the input for the function is isolated and solved for in the inverse equation.

Power-up

To power up students' ability to rewrite an equation as its inverse, have students complete:

Recall that *inverse equations* have input and output values that are reversed. Rewrite the equation $g = \frac{1}{3}h - 6$ so that g is the input value and h is the output value.

$h = 3(g + 6)$ or equivalent

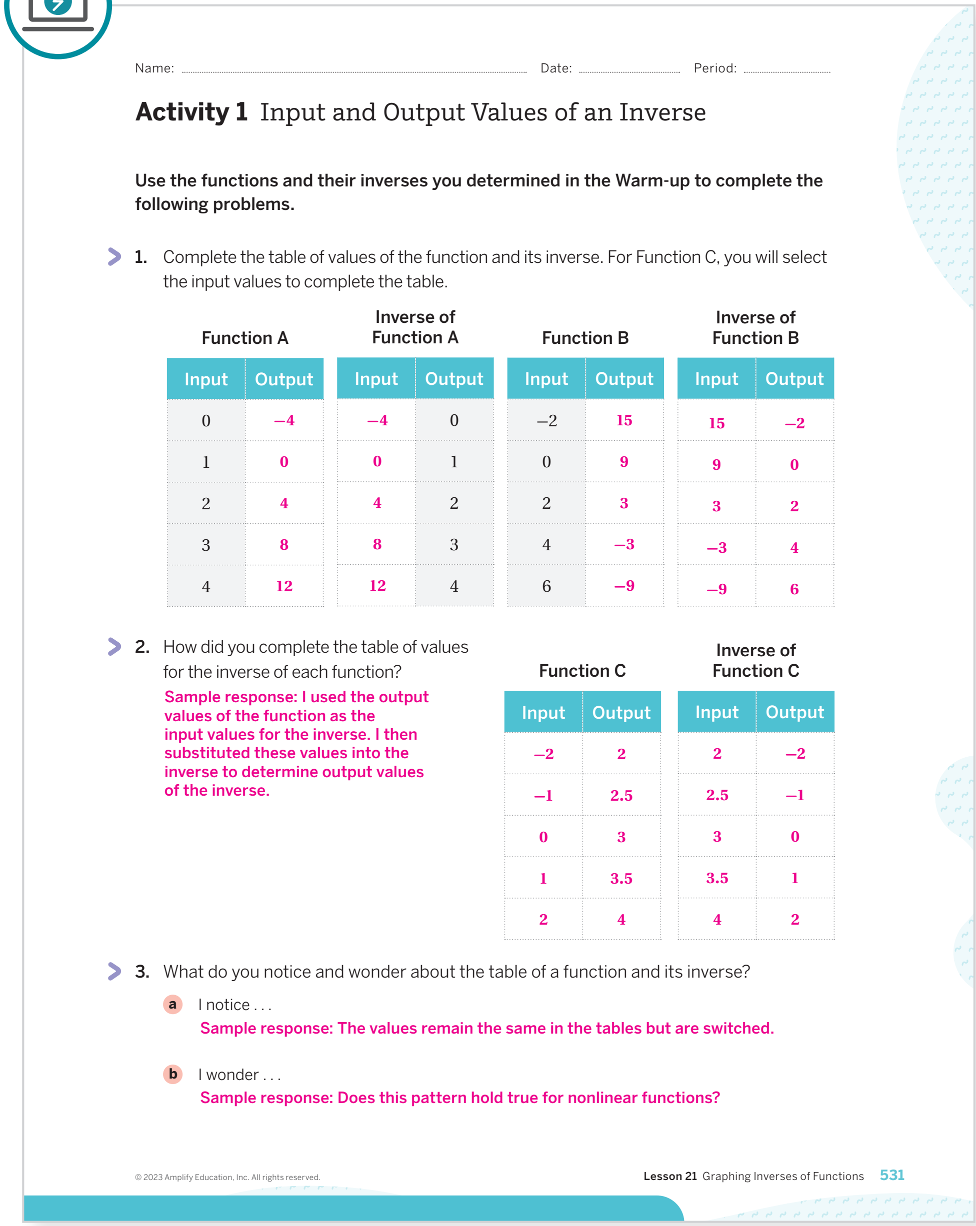
Use: Before the Warm-up

Informed by: Performance on Lesson 20, Practice Problem 5

Independent | 10 min

Activity 1 Input and Output Values of an Inverse

Students solidify their understanding of the relationship between input-output pairs of a function and its inverse by completing tables of values.



Name: ______ Date: ______ Period: ______

Activity 1 Input and Output Values of an Inverse

Use the functions and their inverses you determined in the Warm-up to complete the following problems.

1. Complete the table of values of the function and its inverse. For Function C, you will select the input values to complete the table.

Function A		Inverse of Function A		Function B		Inverse of Function B	
Input	Output	Input	Output	Input	Output	Input	Output
0	−4	−4	0	−2	15	15	−2
1	0	0	1	0	9	9	0
2	4	4	2	2	3	3	2
3	8	8	3	4	−3	−3	4
4	12	12	4	6	−9	−9	6

2. How did you complete the table of values for the inverse of each function?
Sample response: I used the output values of the function as the input values for the inverse. I then substituted these values into the inverse to determine output values of the inverse.

Function C		Inverse of Function C	
Input	Output	Input	Output
−2	2	2	−2
−1	2.5	2.5	−1
0	3	3	0
1	3.5	3.5	1
2	4	4	2

3. What do you notice and wonder about the table of a function and its inverse?
   - a I notice . . .
     Sample response: The values remain the same in the tables but are switched.
   - b I wonder . . .
     Sample response: Does this pattern hold true for nonlinear functions?

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display both completed tables from the Warm-up. Say, "Now you will determine if the relationship you determined in the Warm-up holds true for all input-output pairs of a function and its inverse."

2 Monitor

Help students get started by asking, "What do you notice about the completed values in the tables of Function A and its inverse?"

Look for points of confusion:

- **Completing the inverse tables by substituting values in for the original input variable in the function.** Have students pause after attempting this strategy a few times and ask what pattern they notice between the input-output pairs of the function and its inverse.

Look for productive strategies:

- Completing the table of values using the relationship they found in the Warm-up.

3 Connect

Display the completed tables for Function C and its inverse.

Have individual students share their strategies for completing the tables, and what they notice and wonder.

Highlight that the variables are not reversed in the equation of the function and its inverse, but what they represent, the input and output, are reversed.

Ask, "How could you check the accuracy of the table of values for each inverse?" I can substitute input-output pairs from the table into the equation.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a completed table of values for Function A (including its inverse) to use as a reference when completing the tables for Function B. For Function C, provide a sample set of input and output pairs that students could use if they do not want to create their own.

Extension: *Math Enrichment*

Have students complete a table of input-output pairs for the function $A = s^2$ and its inverse. Answers will vary.

Math Language Development

MLR7: Compare and Connect

Before the Connect, ask students to share their tables and strategies for completing the tables with a partner. Have them discuss their observations for Problem 3. Then ask student volunteers to share the strategies and observations with the whole class.

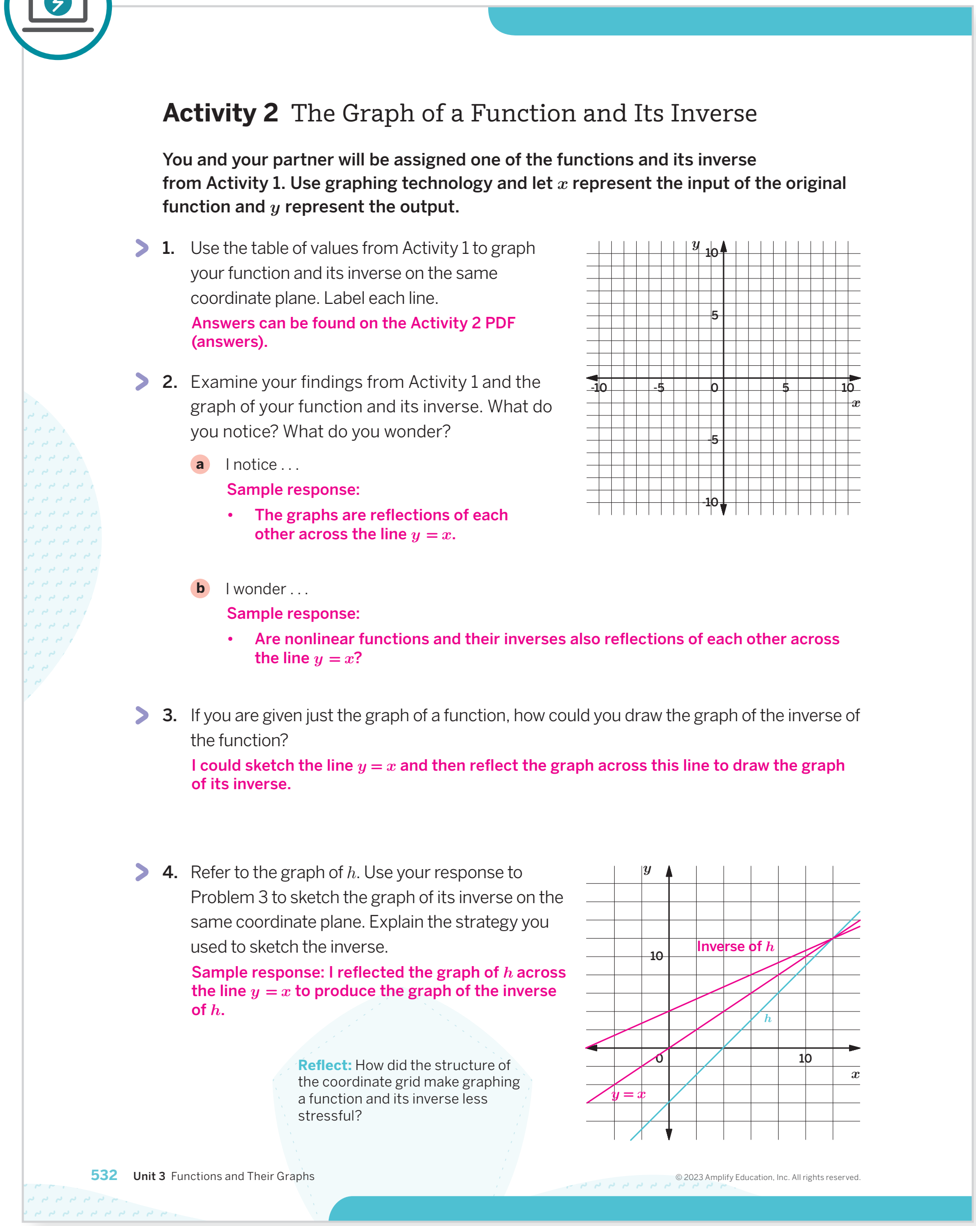
English Learners

Support discussion as partners explain their thinking by displaying or providing the Anchor Chart PDF, *Sentence Stems, Describing My Thinking*.

Pairs | 15 min

Activity 2 The Graph of a Function and Its Inverse

Students examine the graph of a function and its inverse on the same coordinate plane to describe the relationship between these two graphs.



Activity 2 The Graph of a Function and Its Inverse

You and your partner will be assigned one of the functions and its inverse from Activity 1. Use graphing technology and let x represent the input of the original function and y represent the output.

1. Use the table of values from Activity 1 to graph your function and its inverse on the same coordinate plane. Label each line.
   Answers can be found on the Activity 2 PDF (answers).

2. Examine your findings from Activity 1 and the graph of your function and its inverse. What do you notice? What do you wonder?
   - a I notice . . .
     Sample response:
     - **The graphs are reflections of each other across the line $y = x$.**
   - b I wonder . . .
     Sample response:
     - **Are nonlinear functions and their inverses also reflections of each other across the line $y = x$?**

3. If you are given just the graph of a function, how could you draw the graph of the inverse of the function?
   I could sketch the line $y = x$ and then reflect the graph across this line to draw the graph of its inverse.

4. Refer to the graph of h. Use your response to Problem 3 to sketch the graph of its inverse on the same coordinate plane. Explain the strategy you used to sketch the inverse.
   Sample response: I reflected the graph of h across the line $y = x$ to produce the graph of the inverse of h.

Reflect: How did the structure of the coordinate grid make graphing a function and its inverse less stressful?

532 Unit 3 Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Assign each pair of students a function from Activity 1 to graph. Give several minutes to complete Problems 1 and 2, conducting the *Notice and Wonder* routine for Problem 2. Pause for students to share what their responses before completing Problems 3 and 4.

2 Monitor

Help students get started by highlighting that the horizontal axis represents the input values and the vertical axis represents the output values.

Look for points of confusion:

- **Having difficulty describing how to use the graph of a function to draw the graph of its inverse.** Have students focus on two corresponding points and try to describe the relationship between these two points without using their coordinates.

Look for productive strategies:

- Checking possible lines of reflection by sketching them onto the coordinate plane.

3 Connect

Display the Activity 2 PDF.

Have pairs of students share their strategy they determined for Problem 3.

Highlight that the graph of the inverse is a reflection of the function across the line $y = x$ because this reflection switches the coordinates of all points of the function, which is the same as switching input-output pairs.

Ask, "When a point that lies on the line $y = x$ is reflected across this line, what happens to the coordinates of the point?" The coordinates of the point remain unchanged.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, or provide access to other graphing technology, in which they can graph their function and its inverse on the same coordinate plane. If they do not see that the graphs are reflections of each other across the line $y = x$, suggest they graph this line to help them make the connection.

Extension: *Math Enrichment*

Ask students to explain why the line $y = x$ is the line of reflection and not some other line. The inverse reverses the input and output pairs, so if x represents the input of the original function, x now represents the output of the original function. In other words, $y = x$.

Math Language Development

MLR2: Collect and Display

As students share what they notice and wonder, collect the language they use to discuss the graphs and their inverses and add this language to the class display. Be sure to add phrases such as "the graph of a function and its inverse are reflections across the line $y = x$."

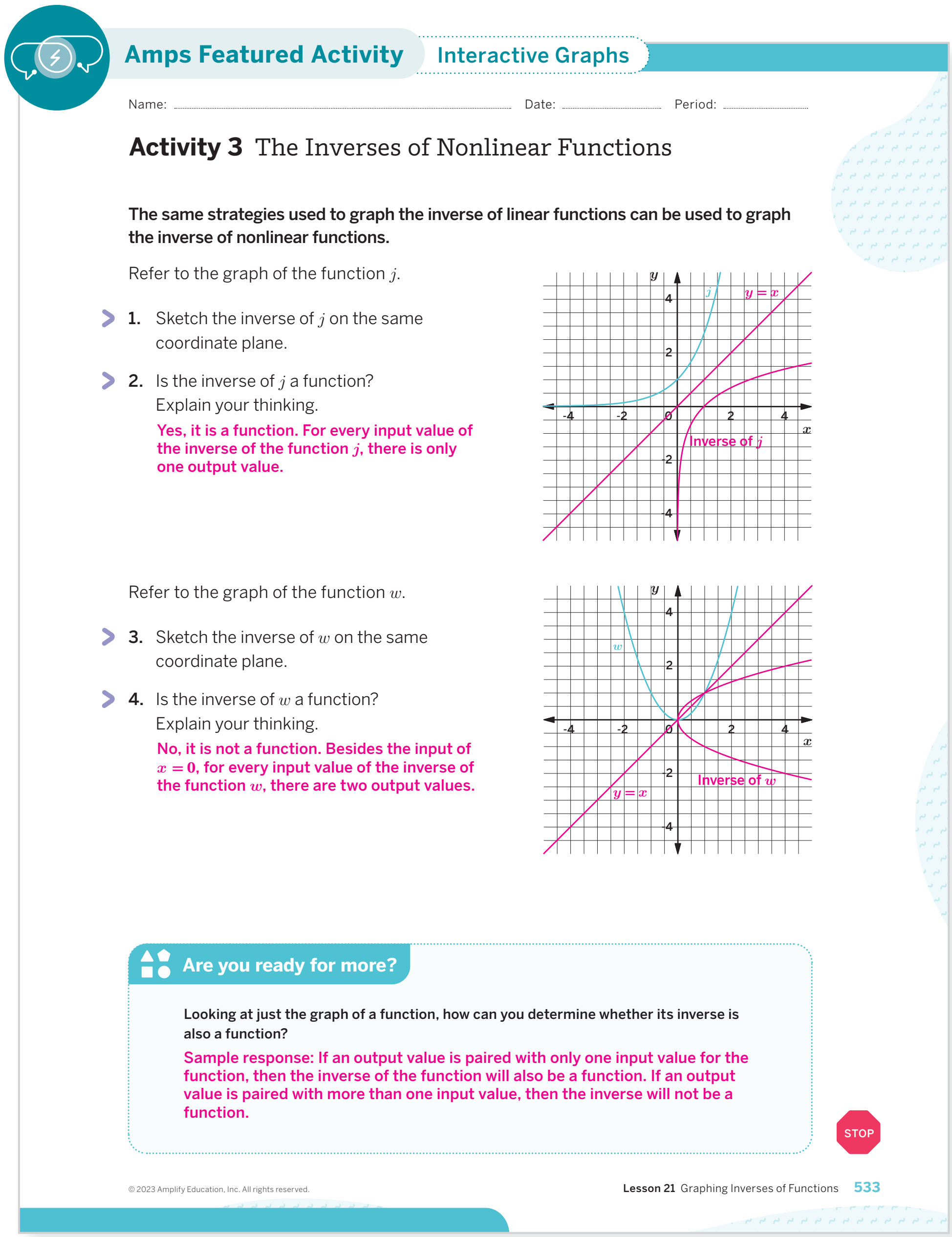
English Learners

After students complete Problem 1 and 2, display or provide the Anchor Chart PDF, *Sentence Stems, Notice and Wonder* to support students in describing what they notice and wonder about the graph of a function and its inverse.

Independent | 10 min

Activity 3 The Inverses of Nonlinear Functions

Students reflect the graph of a nonlinear function across the line $y = x$ to sketch the graph of its inverse.



Amps Featured Activity Interactive Graphs

Name: .... Date: .... Period: ....

Activity 3 The Inverses of Nonlinear Functions

The same strategies used to graph the inverse of linear functions can be used to graph the inverse of nonlinear functions.

Refer to the graph of the function j.

1. Sketch the inverse of j on the same coordinate plane.
2. Is the inverse of j a function? Explain your thinking.
   Yes, it is a function. For every input value of the inverse of the function j, there is only one output value.

Refer to the graph of the function w.

3. Sketch the inverse of w on the same coordinate plane.
4. Is the inverse of w a function? Explain your thinking.
   No, it is not a function. Besides the input of $x = 0$, for every input value of the inverse of the function w, there are two output values.

Are you ready for more?

Looking at just the graph of a function, how can you determine whether its inverse is also a function?

Sample response: If an output value is paired with only one input value for the function, then the inverse of the function will also be a function. If an output value is paired with more than one input value, then the inverse will not be a function.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 21 Graphing Inverses of Functions 533

1 Launch

Say, “You can graph the inverses of nonlinear functions using the same strategies for graphing the inverses of linear functions.” Have students *Turn and Talk* and share how to approach graphing the inverse of a function, given the graph of a function.

2 Monitor

Help students get started by having them sketch the graph of $y = x$.

Look for points of confusion:

- **Overlapping graphs in Problem 2.** Have students determine the coordinates of points along the graph of the function. Have them switch the coordinates to determine points to plot along the graph of the inverse.

Look for productive strategies:

- Plotting a few critical points, such as intercepts, of both the function and its inverse, to make an accurate sketch of each.

3 Connect

Display the graph of the function w, $y = x$, and its inverse.

Have individual students share how they graphed the inverse of w.

Highlight that it could be helpful to reflect a few specific points across the line $y = x$ to help accurately sketch the graph of the inverse.

Ask, “How do you know if the graph of the inverse of a function is also a function?” I can determine if every input value only has one output value.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can digitally sketch the inverse of the graph of a function. They are able to check the accuracy of their graph by observing a reflection of the function’s graph across the line $y = x$.

Accessibility: *Guide Processing and Visualization*

Provide students with blank tables that they can use to generate input and output pairs of functions j and w to help them graph the inverse. Alternatively, suggest students draw the line $y = x$ and then fold their paper across that line to sketch the graph of the inverse. It may be helpful to provide separate sheets of graph paper for this use.

Math Language Development

MLR7: Compare and Connect

During the Launch, as students *Turn and Talk* to discuss possible strategies for sketching the graph of each function’s inverse, circulate and listen to the language they use, such as “reflect the function across the line $y = x$” or “create a table of values.” Draw attention to these different strategies during the Connect.

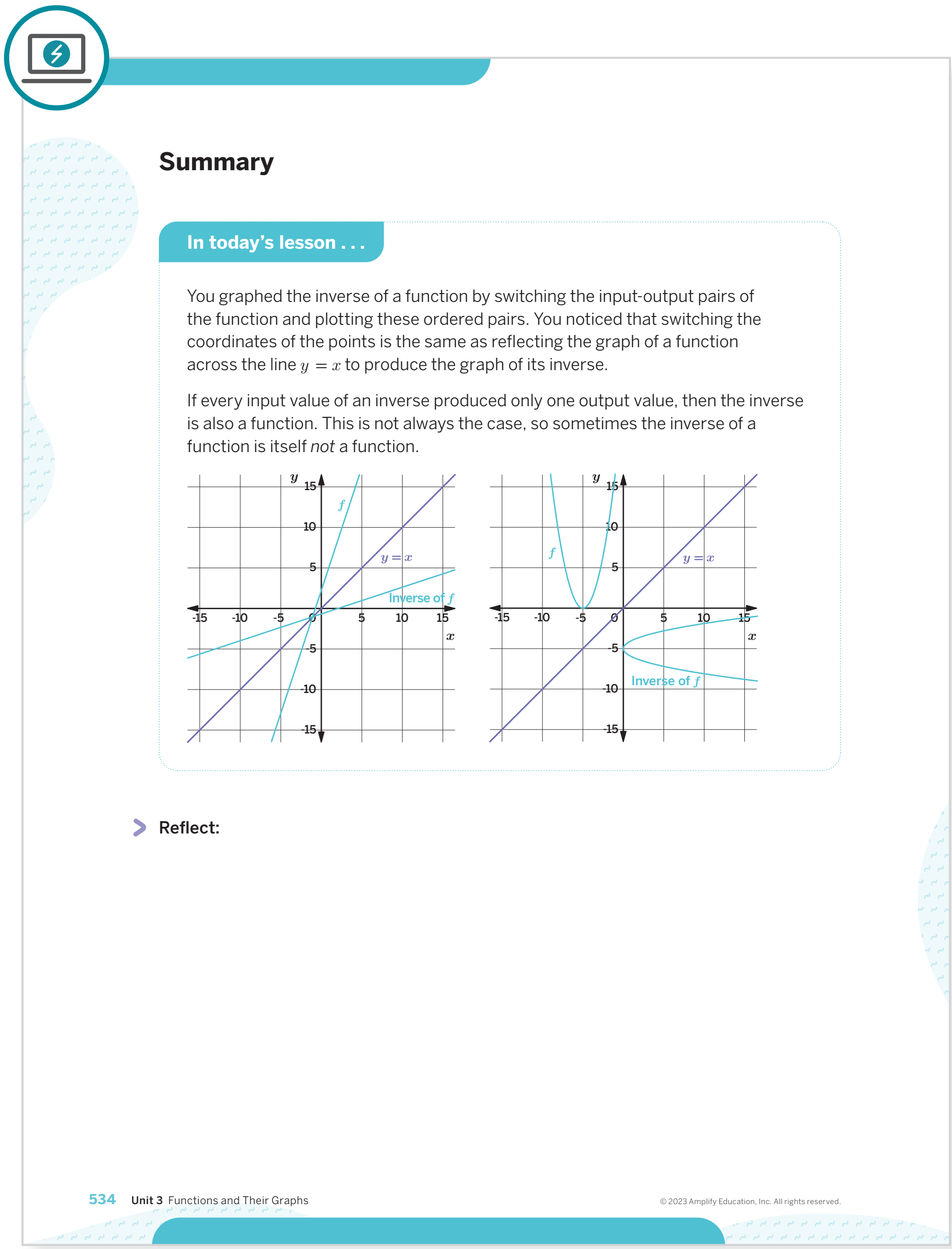
English Learners

Support discussion by displaying or providing the Anchor Chart PDF, *Sentence Stems, Describing My Thinking*.

Whole Class | 5 min

Summary

Review and synthesize strategies that can be used for graphing the inverse of a function.



Summary

In today's lesson . . .

You graphed the inverse of a function by switching the input-output pairs of the function and plotting these ordered pairs. You noticed that switching the coordinates of the points is the same as reflecting the graph of a function across the line $y = x$ to produce the graph of its inverse.

If every input value of an inverse produced only one output value, then the inverse is also a function. This is not always the case, so sometimes the inverse of a function is itself *not* a function.

Reflect:

534 Unit 3 Functions and Their Graphs

©2023 Amplify Education, Inc. All rights reserved.

Synthesize

Display the graph of function h, the line $y = x$, and its inverse from Activity 2, Problem 4.

Have students share how to graph the inverse of a linear function, given the graph of the function.

Highlight that it is possible to reflect the graph of a function across the line $y = x$ to produce the graph of its inverse. Students can also use a table of values, and switch the input-output pairs to produce the coordinates of points along the inverse of the function, and graph these points to accurately sketch the inverse.

Ask:

- "Where will the graph of a linear function and its inverse intersect?"

  They intersect at a point along the line $y = x$.

- "What is unique about the slope of a linear function and the slope of its inverse?"

  The slopes are reciprocals of each other.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

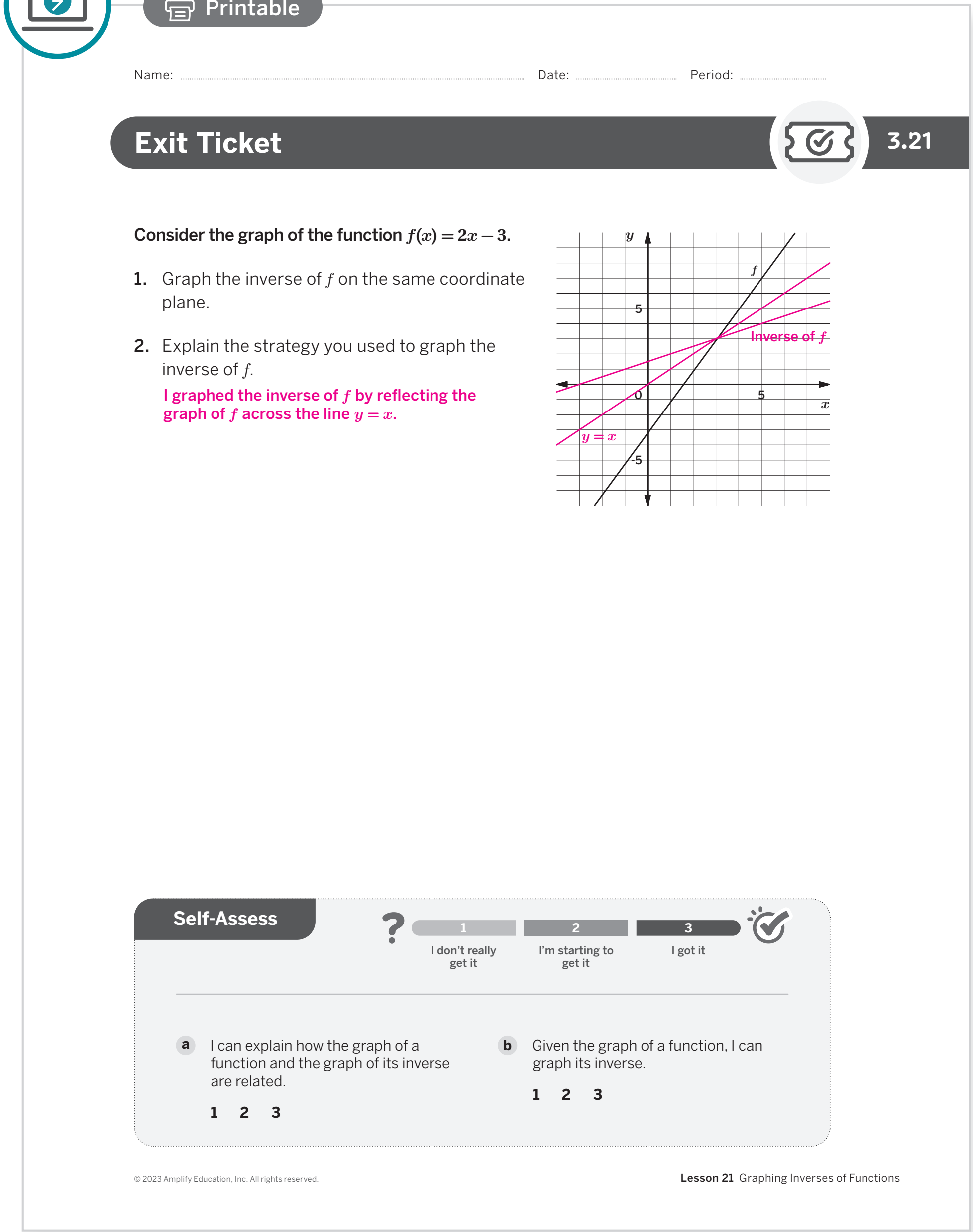
- "Why is it possible to reflect the graph of a function across the line $y = x$ to produce the graph of the inverse of the function?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by graphing the inverse of a function, given the graph of the function, and describing how the graphs are related.

Printable



Name: ____________ Date: ________ Period: ________

Exit Ticket 3.21

Consider the graph of the function $f(x) = 2x - 3$.

1. Graph the inverse of f on the same coordinate plane.
2. Explain the strategy you used to graph the inverse of f.

   I graphed the inverse of f by reflecting the graph of f across the line $y = x$.

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a. I can explain how the graph of a function and the graph of its inverse are related.

1 2 3

b. Given the graph of a function, I can graph its inverse.

1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 21 Graphing Inverses of Functions

Success looks like . . .

- **Goal:** Graphing the inverse of a linear function.
  - » Graphing the inverse of f on the coordinate plane.
- **Goal:** Understanding that the graph of the inverse of a function can be found by reflecting the function's graph across the line $y = x$.
  - » Explaining that the the inverse of f was graphed by reflection the graph of f access the line $y = x$.

Suggested next steps

If students inaccurately sketch the graph of the inverse of f for Problem 1, consider:

- Reviewing using the graph of a function and the line $y = x$ to sketch a graph of the inverse of the function from Activity 2.
- Assigning Practice Problems 2 and 3.
- Asking, "What other lines could be helpful to sketch first before sketching the graph of the inverse of function f?"

If students give a vague or inaccurately explanation for Problem 2, consider:

- Reviewing how the graph of a function and its inverse are related from Activity 3.
- Assigning Practice Problems 1–3.
- Asking, "How are the input-output values of a function and its inverse related? How is this relationship represented in the graph of a function and its inverse?"

Professional Learning

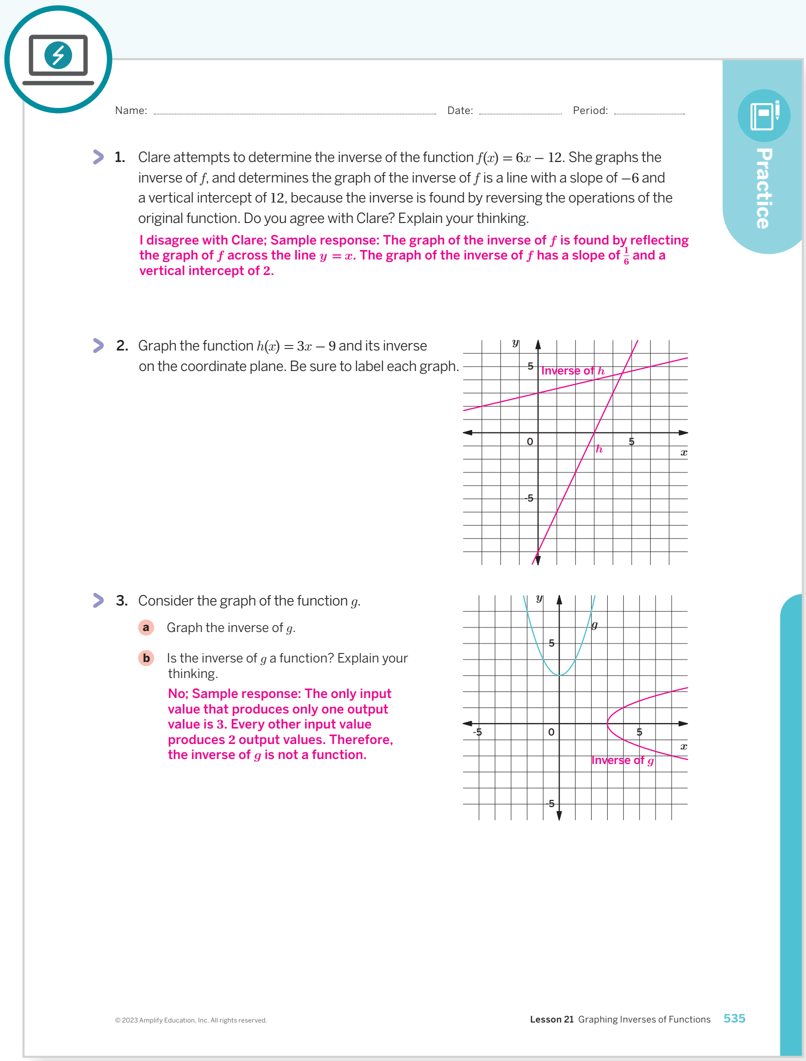
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? How did the ***Notice and Wonder*** routine support students in making connections between the graph of a function and its inverse?
- What challenges did students encounter as they worked on graphing the inverse of a function with precision? How did they work through them? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: Date: Period:

1. Clare attempts to determine the inverse of the function $f(x) = 6x - 12$. She graphs the inverse of f, and determines the graph of the inverse of f is a line with a slope of -6 and a vertical intercept of 12, because the inverse is found by reversing the operations of the original function. Do you agree with Clare? Explain your thinking.

I disagree with Clare; Sample response: The graph of the inverse of f is found by reflecting the graph of f across the line $y = x$. The graph of the inverse of f has a slope of $\frac{1}{6}$ and a vertical intercept of 2.

2. Graph the function $h(x) = 3x - 9$ and its inverse on the coordinate plane. Be sure to label each graph.

3. Consider the graph of the function g.

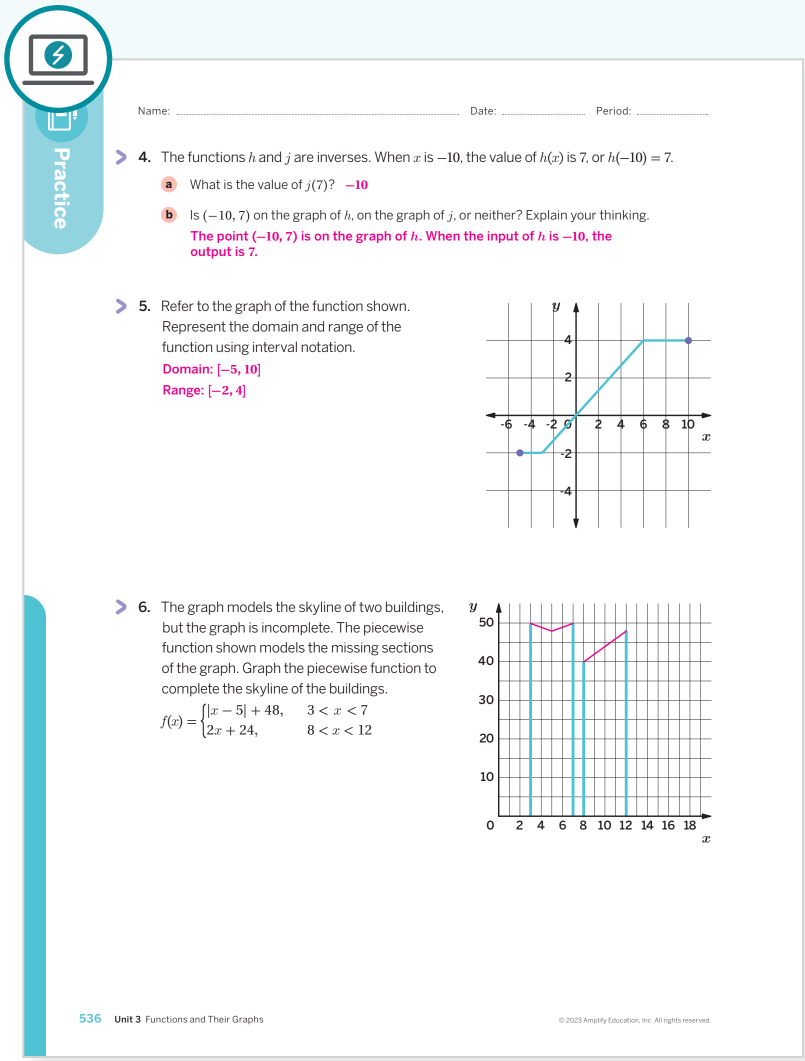
a. Graph the inverse of g.

b. Is the inverse of g a function? Explain your thinking.

No; Sample response: The only input value that produces only one output value is 3. Every other input value produces 2 output values. Therefore, the inverse of g is not a function.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 21 Graphing Inverses of Functions 535



Practice

Name: Date: Period:

4. The functions h and j are inverses. When x is -10, the value of $h(x)$ is 7, or $h(-10) = 7$.

a. What is the value of $j(7)$? **-10**

b. Is $(-10, 7)$ on the graph of h, on the graph of j, or neither? Explain your thinking.

The point $(-10, 7)$ is on the graph of h. When the input of h is -10, the output is 7.

5. Refer to the graph of the function shown. Represent the domain and range of the function using interval notation.

Domain: $[-5, 10]$

Range: $[-2, 4]$

6. The graph models the skyline of two buildings, but the graph is incomplete. The piecewise function shown models the missing sections of the graph. Graph the piecewise function to complete the skyline of the buildings.

$$f(x) = \begin{cases} |x - 5| + 48, & 3 < x < 7 \\ 2x + 24, & 8 < x < 12 \end{cases}$$

536 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 2	2
	2	Activity 2	2
	3	Activity 3	2
Spiral	4	Unit 3 Lesson 22	2
	5	Unit 3 Lesson 9	1
Formative ⚡	6	Unit 3 Lesson 22	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

UNIT 3 | **LESSON 22 – CAPSTONE**

Freerunning Functions

Let's create piecewise functions using descriptions of their key features and parts of their graphs.

Focus

Goals

1. **Language Goal:** Identify and use key features of a graph — global and local maximums and minimums, and the intervals when the function is increasing, decreasing, or constant — to create a piecewise function. **(Speaking and Listening, Reading and Writing)**
2. Given a graph of a function, estimate or calculate the average rate of change over a specified interval.
3. **Language Goal:** Understand a piecewise function as a function defined by different rules for different intervals of the domain.

Coherence

Today

Students create piecewise functions that model the path of a freerunner and identify key features of the piecewise function. They also use given key features to accurately write a symbolic representation and sketch a graph of a piecewise function to meet the given criteria.

Previously

In Lesson 21, students examined the relationship between the graph of a function and its inverse and used this relationship to graph the inverses of functions.

Coming Soon

In Unit 4, students will study exponential functions and how they compare to other types of functions.

Rigor

- Students **apply** their understanding of key features of functions to create piecewise functions that meet given criteria.

Pacing Guide

Suggested Total Lesson Time ~**50 min**

Warm-up	Activity 1	Activity 2	Activity 3 (optional)	Summary	Exit Ticket
5 min	15 min	20 min	15 min	5 min	5 min
Independent	Pairs	Independent	Pairs	Whole Class	Independent

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (as needed)
- Anchor Chart PDF, *Sentence Stems, Explaining My Steps* (as needed)
- Anchor Chart PDF, *Sentence Stems, Describing My Thinking* (as needed)
- Anchor Chart PDF, Sentence Stems, *Partner and Group Questioning* (as needed)
- rulers

Math Language Development

Review words

- *absolute value function*
- *average rate of change*
- *decreasing*
- *domain*
- *global maximum*
- *global minimum*
- *increasing*
- *local maximum*
- *local minimum*
- *piecewise function*
- *range*

Amps | Featured Activity

Activity 2

Mapping the Path of a Freerunner

Students validate that their piecewise function meets given criteria by having a freerunner run the path created by their function. If the freerunner fails to make it through the course, students are able to examine and adjust their piecewise function to accurately reflect the given criteria.

Amps POWERED BY desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students will translate a real-life scenario into a graphical model using piecewise functions. Explain that the graph tells the story and that key features of the graph provide details that can be used to analyze the situation. Similarly, as they learn to recognize their own emotions and the impact their emotions have on their behaviors, students might try to create a chart or graph that tracks the data. By looking at this chart or graph, they might gain some insight about how to work through their emotions productively.

Modifications to Pacing

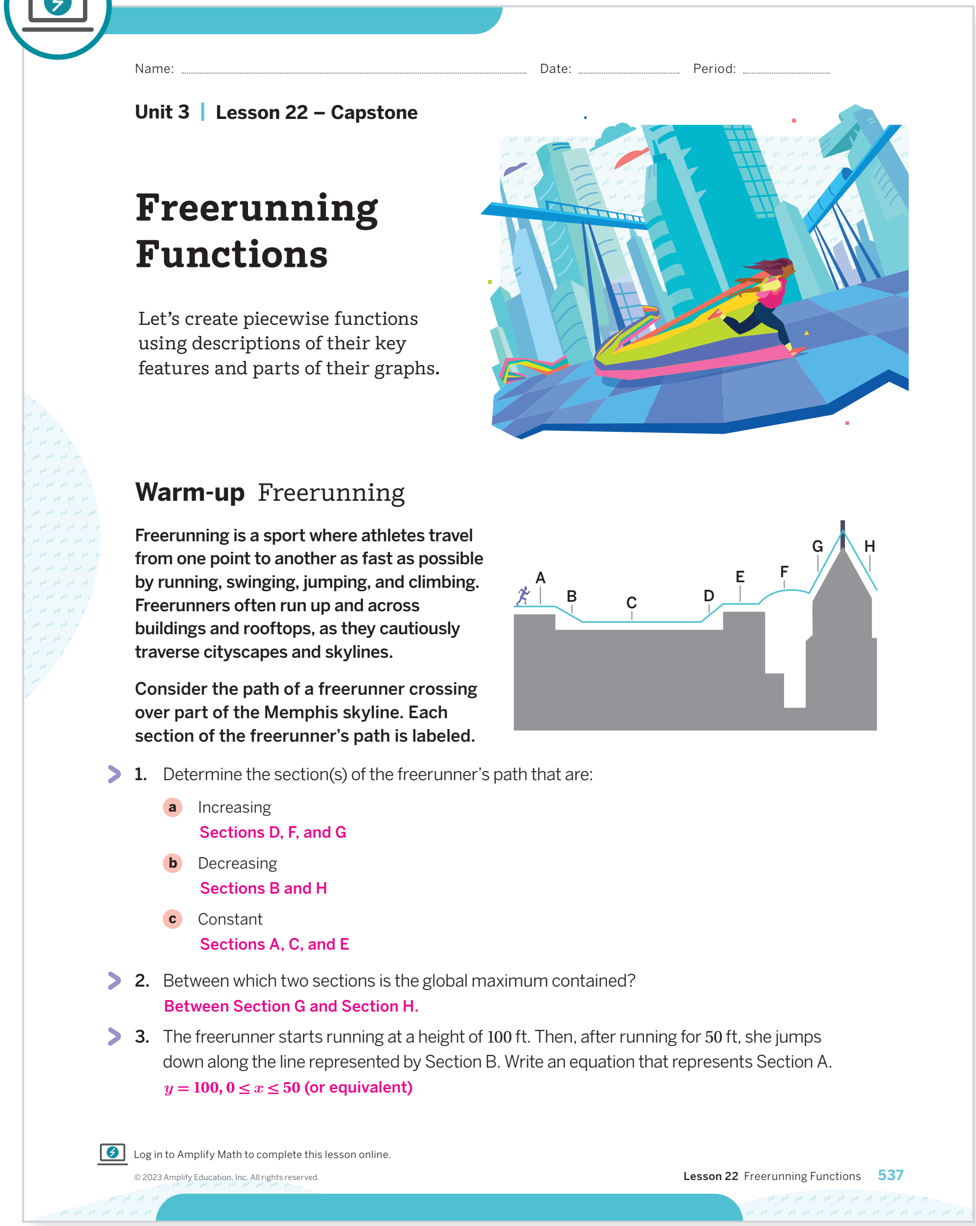
You may want to consider these additional modifications if you are short on time.

- In **Activity 1**, Problem 5 may be omitted.
- In **Activity 2**, some of the constant intervals may be deleted so students have less criteria to meet.
- Optional **Activity 3** may be omitted.

Independent | 5 min

Warm-up Freerunning

Students identify sections of a path that represent key features to informally identify intervals of a piecewise function that contain these features.



Name: ______ Date: ______ Period: ______

Unit 3 | **Lesson 22 – Capstone**

Freerunning Functions

Let's create piecewise functions using descriptions of their key features and parts of their graphs.

Warm-up Freerunning

Freerunning is a sport where athletes travel from one point to another as fast as possible by running, swinging, jumping, and climbing. Freerunners often run up and across buildings and rooftops, as they cautiously traverse cityscapes and skylines.

Consider the path of a freerunner crossing over part of the Memphis skyline. Each section of the freerunner's path is labeled.

1. Determine the section(s) of the freerunner's path that are:
   - a Increasing
     Sections D, F, and G
   - b Decreasing
     Sections B and H
   - c Constant
     Sections A, C, and E
2. Between which two sections is the global maximum contained?
   Between Section G and Section H.
3. The freerunner starts running at a height of 100 ft. Then, after running for 50 ft, she jumps down along the line represented by Section B. Write an equation that represents Section A.
   $y = 100, 0 \leq x \leq 50$ **(or equivalent)**

Log in to Amplify Math to complete this lesson online.

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 Freerunning Functions 537

1 Launch

Read the narrative together. Have students *Turn and Talk* to describe the path of the freerunner in their own words. Have students share their descriptions before working independently.

2 Monitor

Help students get started by asking, "What do an increasing, decreasing, and constant graph look like?"

Look for points of confusion:

- **Excluding parts of Section F in the increasing and decreasing lists.** Ask, "Using the terms 'increasing' and 'decreasing,' how would you describe Section F?"

Look for productive strategies:

- Annotating the graph with the intervals of increasing, decreasing, and constant, and other key features.

3 Connect

Display the graph.

Highlight that it is possible to identify nonlinear sections, such as Section F, as "increasing" because the output values increase as the input values increase.

Ask, "What is the difference between a local and global maximum of a graph?" A local maximum is the maximum with a restricted interval of a graph, while the global maximum is the maximum of the entire graph.

Have individual students share how they used the information in Problem 3 to write a function and its domain.

Math Language Development

MLR7: Compare and Connect

During the Connect, ask students to share their responses to the Warm-up with a partner. Circulate and listen to the language they use. Amplify mathematical language used, such as *increasing, decreasing, constant, linear, nonlinear, global maximum,* and *local maximum*.

English Learners

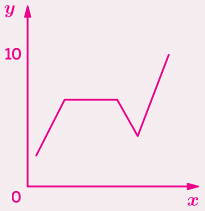
Support discussion by displaying or providing the Anchor Chart PDF, *Sentence Stems, Describing My Thinking*.

Power-up

To power up students' ability to graph piecewise functions, have students complete:

Make a sketch of a graph that meets the following criteria.

a. The first section is increasing.
b. The second section is constant.
c. The third section is decreasing.
d. The final section is also increasing, and reaches a maximum of 10. Sample response shown.



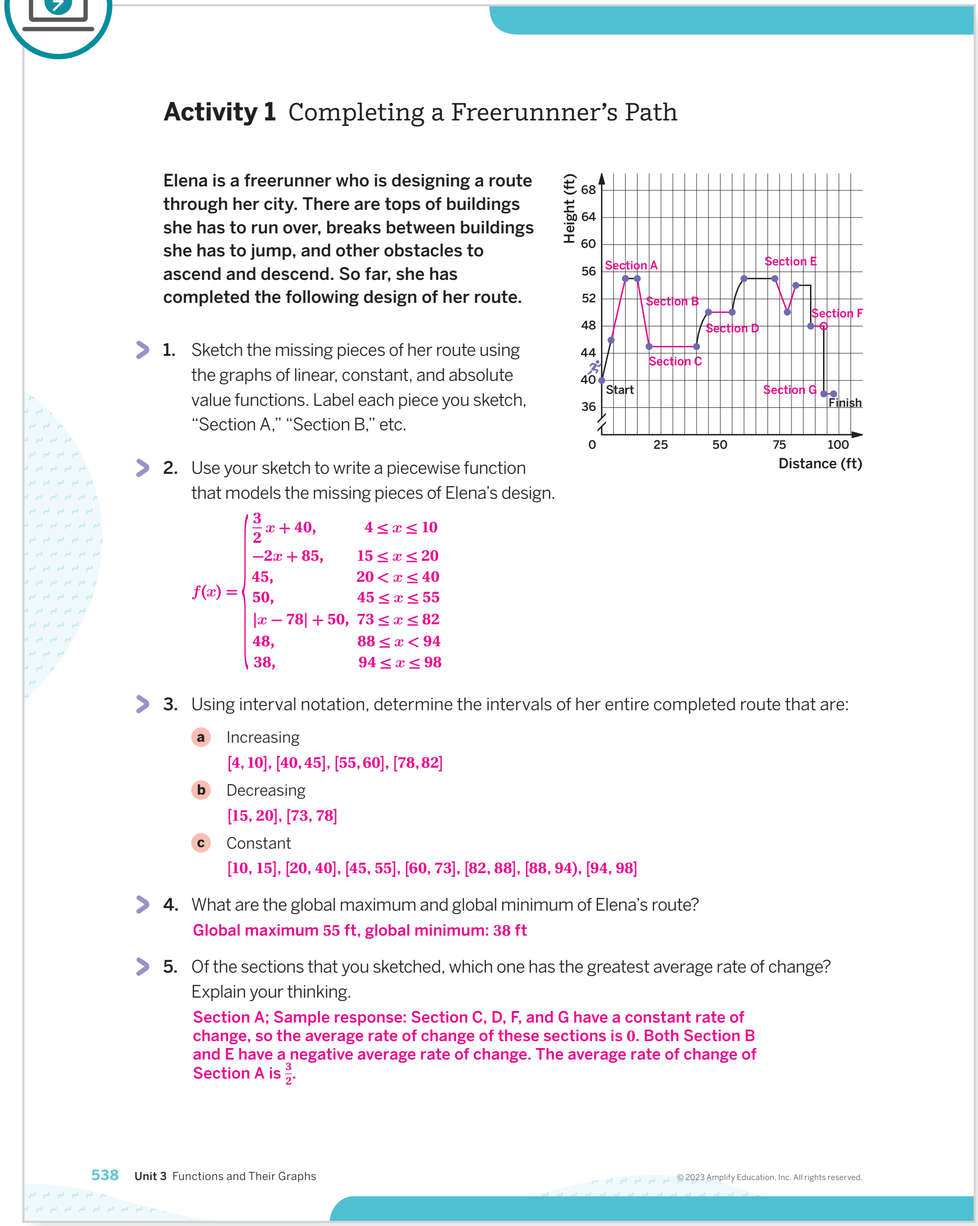
Use: Before Activity 1

Informed by: Performance on Lesson 21, Practice Problem 6

Pairs | 15 min

Activity 1 Completing a Freerunnner's Path

Students construct a function to model a freerunner's path and attend to precision as they use interval notation to identify key features of the graph of the function.



Activity 1 Completing a Freerunnner's Path

Elena is a freerunner who is designing a route through her city. There are tops of buildings she has to run over, breaks between buildings she has to jump, and other obstacles to ascend and descend. So far, she has completed the following design of her route.

1. Sketch the missing pieces of her route using the graphs of linear, constant, and absolute value functions. Label each piece you sketch, "Section A," "Section B," etc.

2. Use your sketch to write a piecewise function that models the missing pieces of Elena's design.

$$f(x) = \begin{cases} \frac{3}{2}x + 40, & 4 \le x \le 10 \\ -2x + 85, & 15 \le x \le 20 \\ 45, & 20 < x \le 40 \\ 50, & 45 \le x \le 55 \\ |x - 78| + 50, & 73 \le x \le 82 \\ 48, & 88 \le x < 94 \\ 38, & 94 \le x \le 98 \end{cases}$$

3. Using interval notation, determine the intervals of her entire completed route that are:
   - a Increasing
     $[4, 10], [40, 45], [55, 60], [78, 82]$
   - b Decreasing
     $[15, 20], [73, 78]$
   - c Constant
     $[10, 15], [20, 40], [45, 55], [60, 73], [82, 88], [88, 94), [94, 98]$

4. What are the global maximum and global minimum of Elena's route?
   Global maximum 55 ft, global minimum: 38 ft

5. Of the sections that you sketched, which one has the greatest average rate of change? Explain your thinking.
   Section A; Sample response: Section C, D, F, and G have a constant rate of change, so the average rate of change of these sections is 0. Both Section B and E have a negative average rate of change. The average rate of change of Section A is $\frac{3}{2}$.

538 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Display the graph. Say, "You will complete the graph using only linear, constant, and absolute value functions." Provide access to rulers.

2 Monitor

Help students get started by having them use a ruler or straightedge to complete the graph.

Look for points of confusion:

- **Incorrectly identifying the domain of each piece.** Have students annotate the coordinates of the endpoints of each piece they sketch on the graph to help determine the domain of each piece.
- **Having difficulty writing the equation that models the Sections A and B.** Have students extend each line so that they each intersect the vertical axis. Have them use the vertical intercept and slope of each line to help write an equation for each.

Look for productive strategies:

- Using an absolute value function to represent Section E.

3 Connect

Display the completed graph.

Have pairs of students share their piecewise function, asking for students who have different symbolic representations to share.

Highlight that the piecewise function can only be defined by one piece for any input value, so there can be no overlap in the intervals of the domain of each piece.

Ask, "What different ways can Section E be represented in the piecewise function?" Section E can be represented with an absolute value function or it could be represented by two pieces, each being a linear function.

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

Provide students with half of the function rules and half of the domain intervals for the piecewise function. Have them complete the missing information using their sketch.

Extension: *Math Enrichment*

Ask students to determine whether the vertical pieces of the path could be incorporated into the symbolic representation of the piecewise function and explain their thinking. No; Sample response: Vertical lines are not functions because for the one input value, there are an infinite number of output values. These cannot be represented in the piecewise function.

Math Language Development

MLR8: Discussion Supports

Before the Connect, display or provide the Anchor Chart PDF, *Sentence Stems, Explaining My Steps* to support students when they explain how they created the symbolic representation of their piecewise function. Amplify the use of mathematical language, such as *piecewise function, interval, domain, rule, linear, slope, absolute value, constant, increasing,* or *decreasing*.

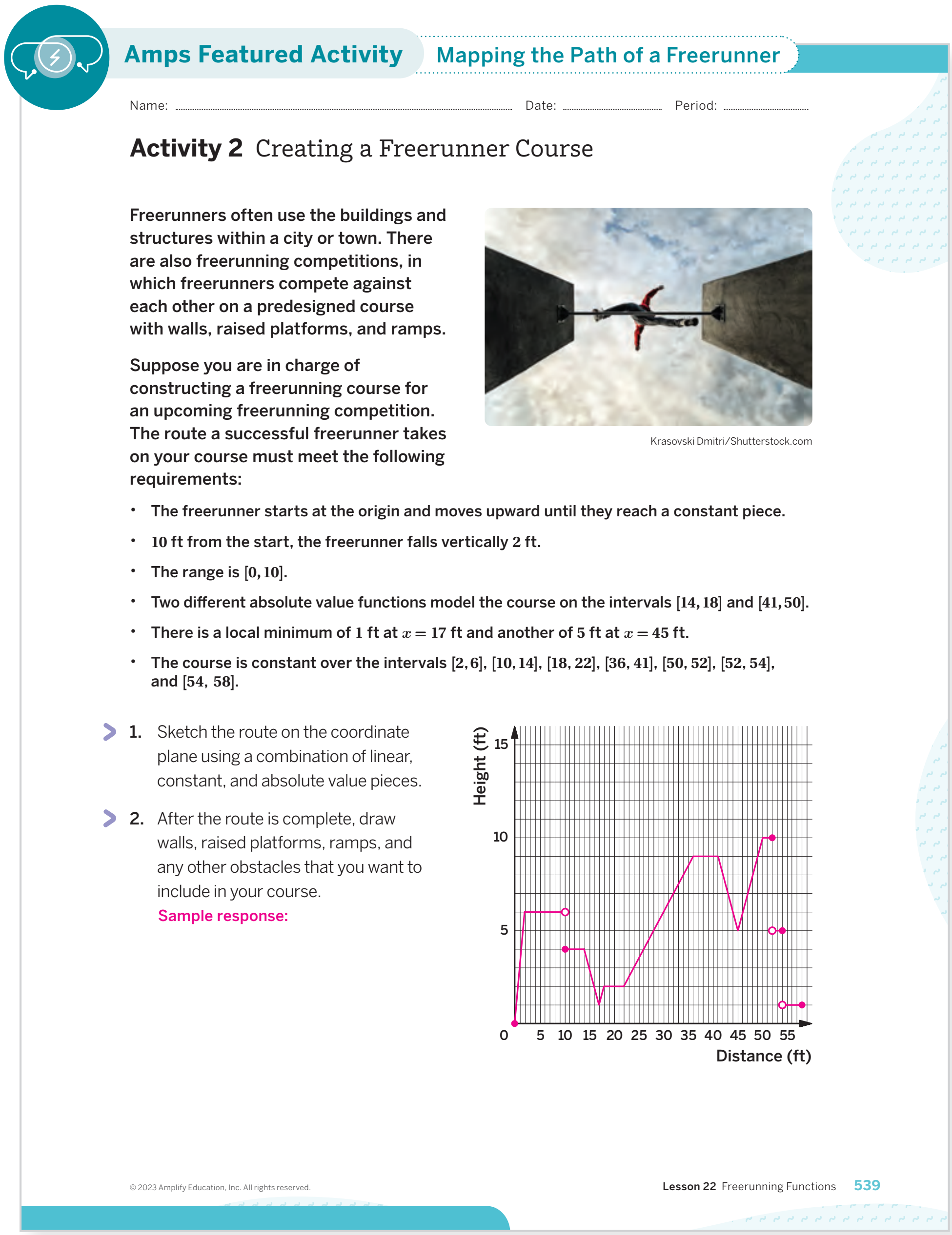
English Learners

Allow students to rehearse what they will say before sharing with the whole class.

Independent | 20 min

Activity 2 Creating a Freerunner Course

Students construct a function to model a freerunner course, carefully adhering to the given requirements.



Amps Featured Activity Mapping the Path of a Freerunner

Name: ______ Date: ______ Period: ______

Activity 2 Creating a Freerunner Course

Freerunners often use the buildings and structures within a city or town. There are also freerunning competitions, in which freerunners compete against each other on a predesigned course with walls, raised platforms, and ramps.

Suppose you are in charge of constructing a freerunning course for an upcoming freerunning competition. The route a successful freerunner takes on your course must meet the following requirements:

Krasovski Dmitri/Shutterstock.com

- The freerunner starts at the origin and moves upward until they reach a constant piece.
- 10 ft from the start, the freerunner falls vertically 2 ft.
- The range is $[0, 10]$.
- Two different absolute value functions model the course on the intervals $[14, 18]$ and $[41, 50]$.
- There is a local minimum of 1 ft at $x = 17$ ft and another of 5 ft at $x = 45$ ft.
- The course is constant over the intervals $[2, 6]$, $[10, 14]$, $[18, 22]$, $[36, 41]$, $[50, 52]$, $[52, 54]$, and $[54,\ 58]$.

1. Sketch the route on the coordinate plane using a combination of linear, constant, and absolute value pieces.

2. After the route is complete, draw walls, raised platforms, ramps, and any other obstacles that you want to include in your course.
Sample response:

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 Freerunning Functions 539

① Launch

Show the completed graph from Activity 1 and say, "Now you will design your own freerunner course and represent it graphically as you did in Activity 1." Read the narrative and the description of the key features together.

② Monitor

Help students get started by suggesting they first plot the starting point and the given local minimums.

Look for points of confusion:

- **Having difficulty graphing the absolute value pieces.** "What other points can you plot from another listed key feature? What is the slope of each line segment that constructs the absolute value function?"

Look for productive strategies:

- Annotating the horizontal axis with the intervals the function is constant, and the vertical axis with the range.

③ Connect

Have individual students share their graph of their course.

Highlight that a point at the origin and the local minimum can be plotted from the description. The absolute value function pieces could then be drawn because both pieces have vertices that are the points that represent the local minimums.

Ask, "What other descriptions could you give of the graph using a key feature?"
Sample response: The graph decreases on the intervals of $[14, 17]$ and $[41, 45]$.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can digitally create a function and watch a freerunner run the path created by their function. This provides them with immediate feedback and visual validation that their piecewise function meets the given criteria. Students can digitally adjust their function as needed.

Math Language Development

MLR1: Stronger and Clearer Each Time

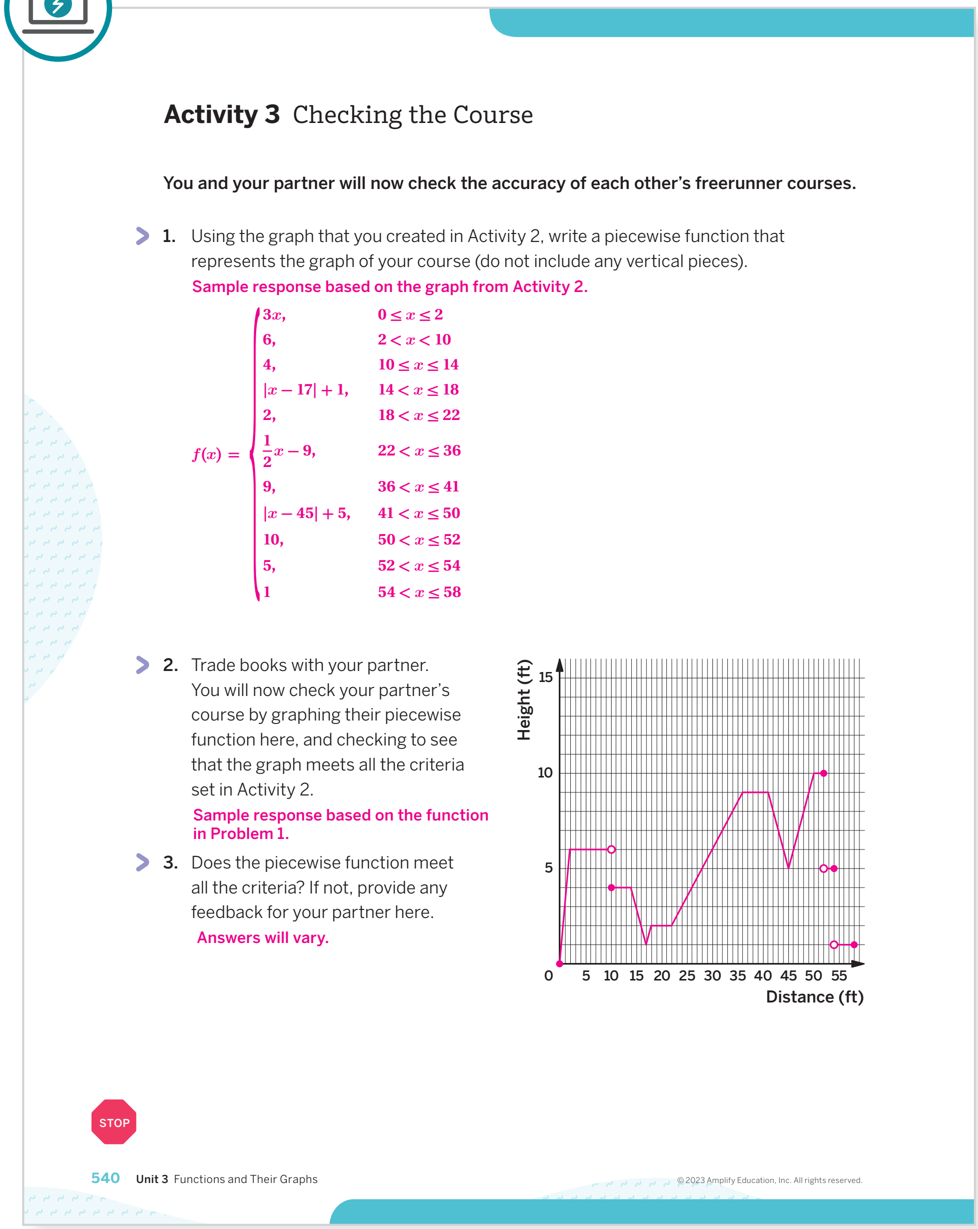
Use this routine to strengthen and refine students' sketches of their graphs. Give students time to sketch their graphs before meeting with 2–3 partners to receive feedback on their sketches. Students should then use the feedback to refine and improve their initial sketches. Provide them with additional pieces of graph paper or copies of the blank graph shown in the Student Edition for them to use to refine their sketches.

Optional

Pairs | 15 min

Activity 3 Checking the Course

Students construct a piecewise function to model their freerunner course from Activity 2 and verify their graph meets all of the given requirements.



Activity 3 Checking the Course

You and your partner will now check the accuracy of each other's freerunner courses.

1. Using the graph that you created in Activity 2, write a piecewise function that represents the graph of your course (do not include any vertical pieces).
   Sample response based on the graph from Activity 2.

$$f(x) = \begin{cases} 3x, & 0 \leq x \leq 2 \\ 6, & 2 < x < 10 \\ 4, & 10 \leq x \leq 14 \\ |x - 17| + 1, & 14 < x \leq 18 \\ 2, & 18 < x \leq 22 \\ \frac{1}{2}x - 9, & 22 < x \leq 36 \\ 9, & 36 < x \leq 41 \\ |x - 45| + 5, & 41 < x \leq 50 \\ 10, & 50 < x \leq 52 \\ 5, & 52 < x \leq 54 \\ 1 & 54 < x \leq 58 \end{cases}$$

2. Trade books with your partner. You will now check your partner's course by graphing their piecewise function here, and checking to see that the graph meets all the criteria set in Activity 2.
   Sample response based on the function in Problem 1.
3. Does the piecewise function meet all the criteria? If not, provide any feedback for your partner here.
   Answers will vary.

© 2023 Amplify Education, Inc. All rights reserved.

1 Launch

Review the instructions of the activity together as a class. Give students time to first work independently on Problem 1 before having them trade books with their partner to complete Problems 2 and 3.

2 Monitor

Help students get started by having them annotate their graph with whether each piece will be represented by a linear, constant, or absolute value function.

Look for points of confusion:

- **Having difficulty determining if a piece includes the endpoints.** Have students plot an open or closed circle at the endpoints of each piece. For pieces that intersect, have them decide which piece contains the point of intersection.

Look for productive strategies:

- Annotating the start and end of the domain of each piece along the horizontal axis.

3 Connect

Have pairs of students share any similarities and differences they found in each other's graph and function.

Highlight that students can check to see if their piecewise function only has one output value for every input value by examining the start and end value of the domain of each piece. They should check the inequality symbols to make sure these intervals do not overlap.

Ask, "Which pieces could have been graphed differently and still have met the given key features?" The last three constant pieces could have been constructed differently.

Differentiated Support

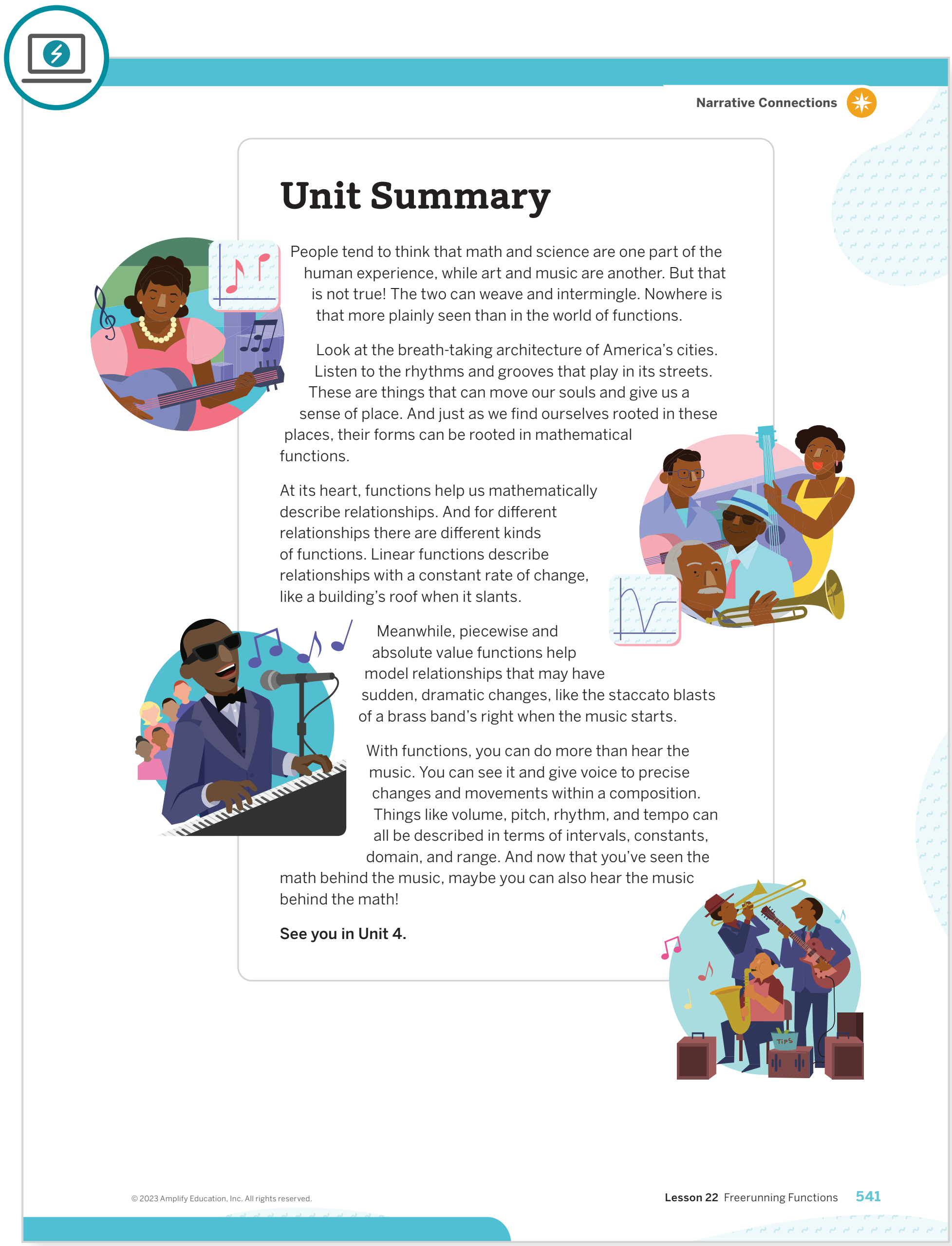
Extension: ***Math Enrichment***

Ask students how might their sketch change if the local minimum values of the course were not given. Sample response: The vertices of the absolute value function pieces could change. This would affect all other pieces, except the last three constant pieces.

Whole Class | 5 min

Unit Summary

Review and synthesize the concepts of this unit and how students deepened their understanding of functions from Grade 8.



Narrative Connections

Unit Summary

People tend to think that math and science are one part of the human experience, while art and music are another. But that is not true! The two can weave and intermingle. Nowhere is that more plainly seen than in the world of functions.

Look at the breath-taking architecture of America's cities. Listen to the rhythms and grooves that play in its streets. These are things that can move our souls and give us a sense of place. And just as we find ourselves rooted in these places, their forms can be rooted in mathematical functions.

At its heart, functions help us mathematically describe relationships. And for different relationships there are different kinds of functions. Linear functions describe relationships with a constant rate of change, like a building's roof when it slants.

Meanwhile, piecewise and absolute value functions help model relationships that may have sudden, dramatic changes, like the staccato blasts of a brass band's right when the music starts.

With functions, you can do more than hear the music. You can see it and give voice to precise changes and movements within a composition. Things like volume, pitch, rhythm, and tempo can all be described in terms of intervals, constants, domain, and range. And now that you've seen the math behind the music, maybe you can also hear the music behind the math!

See you in Unit 4.

© 2023 Amplify Education, Inc. All rights reserved.

Narrative Connections

Read the narrative aloud as a class or have students read it individually.

Synthesize

Display the completed graph and the piecewise function from Activity 1.

Ask, "What key features of the function can be determined using the graph?" The intervals of increasing, decreasing, and constant. The local and global maximum and minimum, the vertical and horizontal intercepts, and the average rate of change over a given interval.

Have students share how these key features help to sketch a graph of a function when no graph is given.

Highlight that it is helpful to have a sketch of the graph of the function to help write the symbolic representation of the function. Once given key features are used to sketch the graph, then it is possible to determine other helpful features of each piece of the graph, such as slope, domain, intercepts, and vertices for absolute value functions, which help to write the symbolic representation of the piecewise function.

Reflect

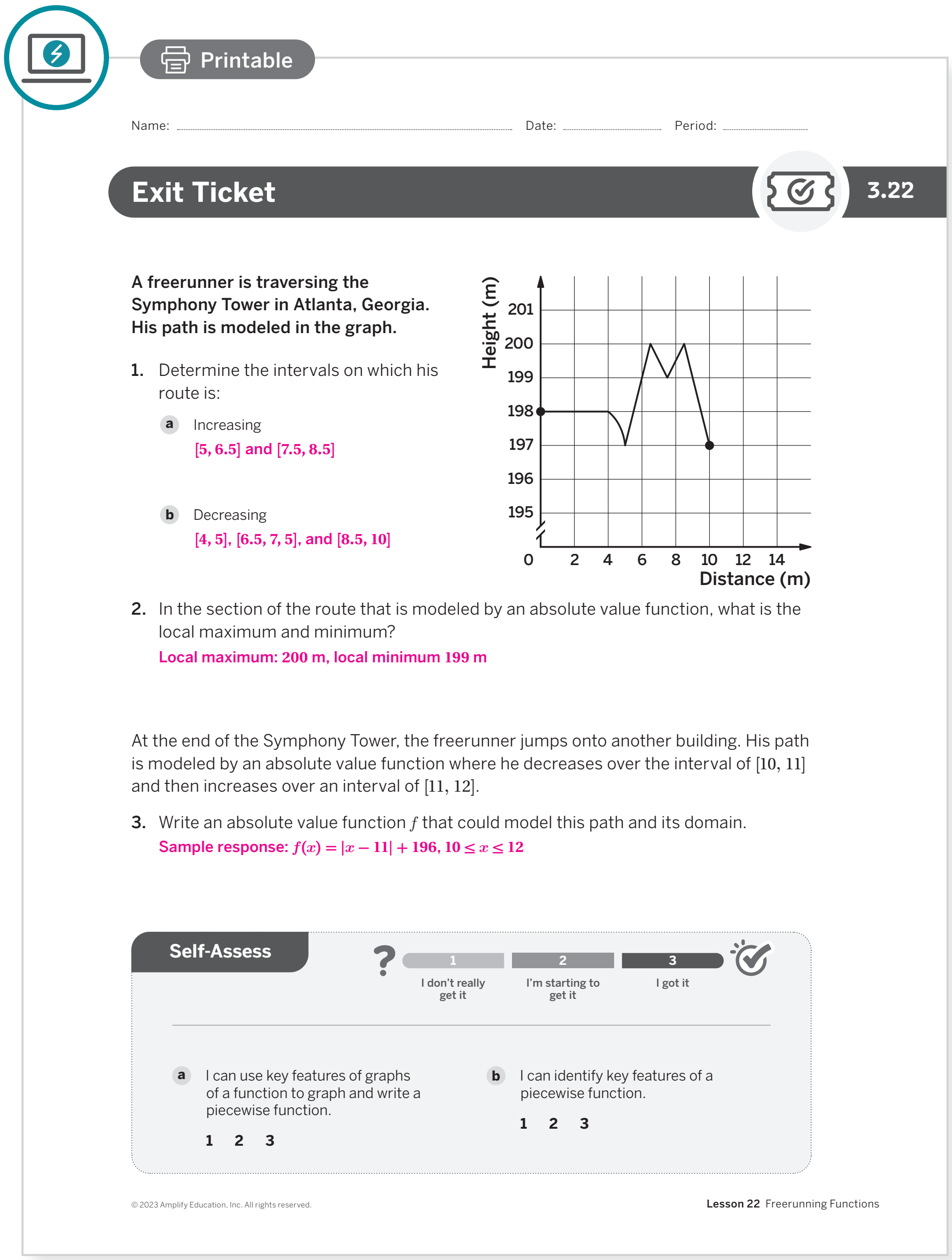
After synthesizing the concepts of this unit, allow students a few moments for reflection around the concepts of the unit. To help them engage in meaningful reflection, consider asking:

- "Did anything surprise you while reading the narratives of this unit?"
- "Is there anything more you would like to learn about these topics? What are some steps you can take to learn more?"

Independent | 5 min

Exit Ticket

Students demonstrate their understanding by attending to precision as they identify key features and construct a function to model a freerunner's path.



Printable

Name: ______ Date: ______ Period: ______

Exit Ticket 3.22

A freerunner is traversing the Symphony Tower in Atlanta, Georgia. His path is modeled in the graph.

1. Determine the intervals on which his route is:
   - a Increasing
     [5, 6.5] and [7.5, 8.5]
   - b Decreasing
     [4, 5], [6.5, 7, 5], and [8.5, 10]
2. In the section of the route that is modeled by an absolute value function, what is the local maximum and minimum?
   Local maximum: 200 m, local minimum 199 m

At the end of the Symphony Tower, the freerunner jumps onto another building. His path is modeled by an absolute value function where he decreases over the interval of $[10, 11]$ and then increases over an interval of $[11, 12]$.

3. Write an absolute value function f that could model this path and its domain.
   Sample response: $f(x) = |x - 11| + 196, 10 \leq x \leq 12$

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can use key features of graphs of a function to graph and write a piecewise function.
1 2 3

b I can identify key features of a piecewise function.
1 2 3

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 Freerunning Functions

Success looks like . . .

- **Language Goal:** Identifying and using key features of a graph — global and local maximums and minimums, and the intervals when the function is increasing, decreasing, or constant — to create a piecewise function. **(Speaking and Listening, Reading and Writing)**
  - » Determining the intervals on which the function is increasing and decreasing in Problem 1.
- **Goal:** Given a graph of a function, calculating the average rate of change over a specified interval.
- **Language Goal:** Understanding a piecewise function as a function defined by different rules for different intervals of the domain. **(Reading and Writing)**

Suggested next steps

If students incorrectly list the intervals of increasing and decreasing in Problem 1, consider:

- Reviewing identifying intervals of increasing and decreasing from Activity 1.
- Assigning Practice Problem 1.
- Asking, "At what value of x does the path start increasing and decreasing?"

If students inaccurately identify the local minimum and maximum in Problem 2, consider:

- Reviewing using the local maximum and minimum in Activity 2.
- Assigning Practice Problem 1.

If students inaccurately write the function and its domain in Problem 3, consider:

- Writing a piecewise function from a description of the graph in Activity 1.
- Assigning Practice Problem 1.

Professional Learning

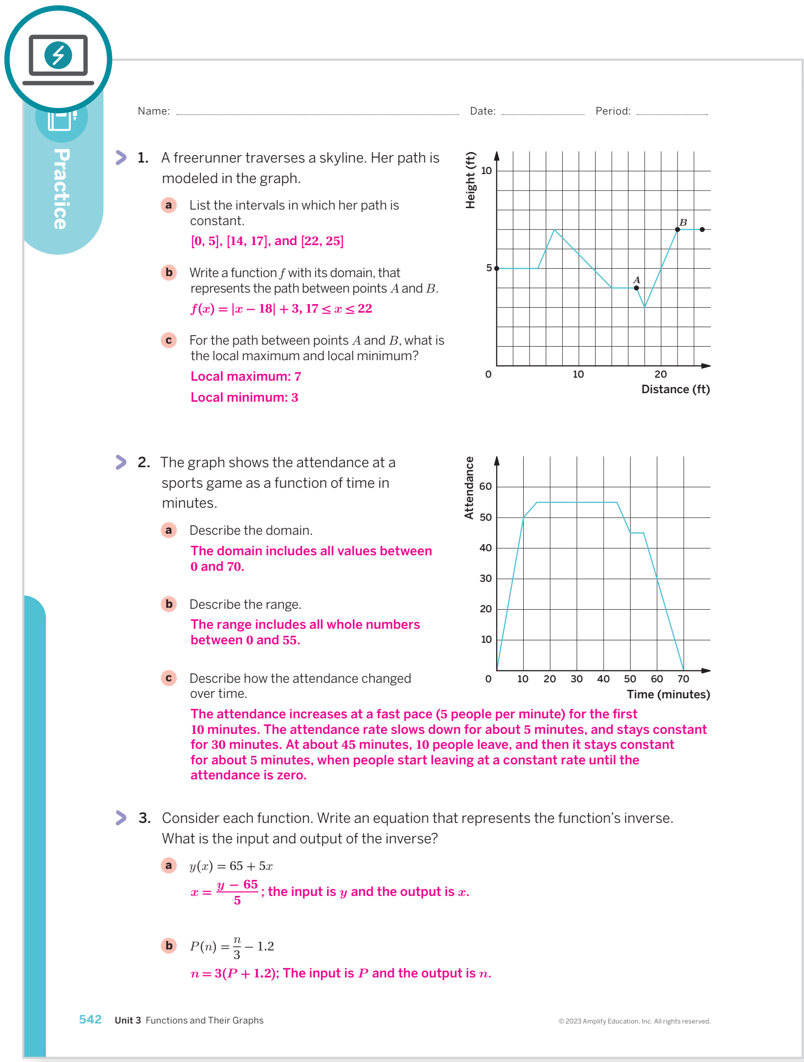
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? When you compare and contrast today's work with work students did earlier this year on interpreting key features of graphs and graphing piecewise functions, what similarities and differences do you see?
- What different ways did students approach creating the path of the freerunner to match the given criteria? What does that tell you about similarities and differences among your students? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ______ Date: ______ Period: ______

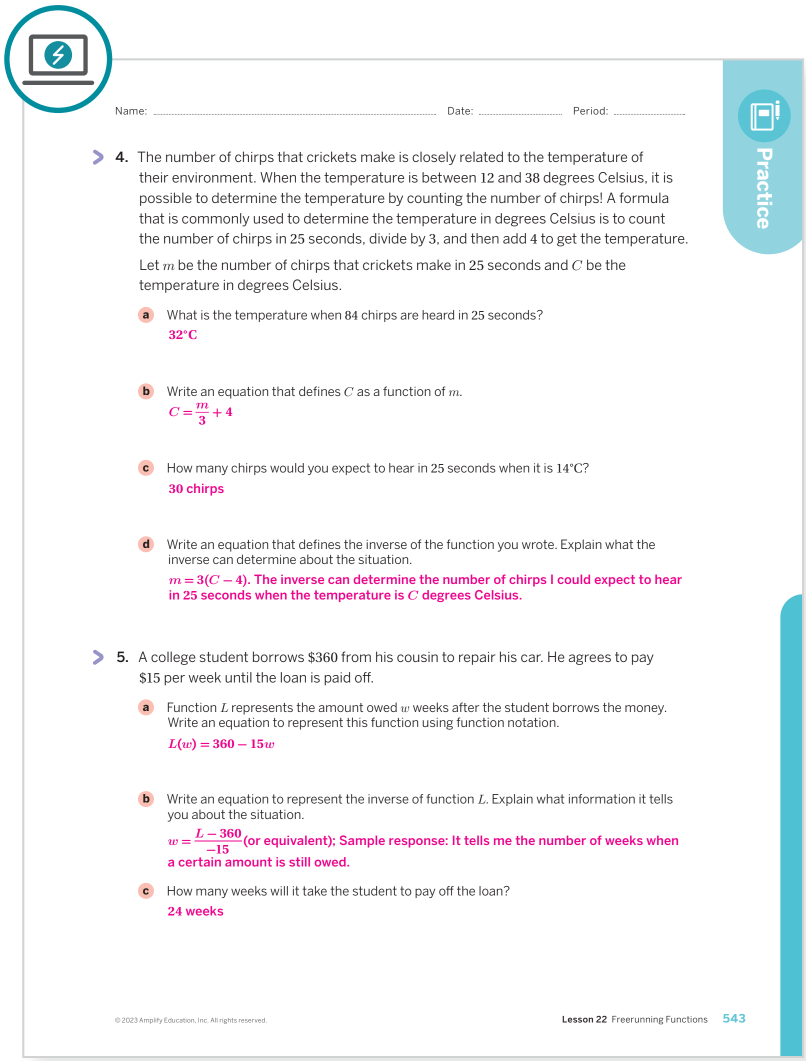
1. A freerunner traverses a skyline. Her path is modeled in the graph.
   - a List the intervals in which her path is constant.
     [0, 5], [14, 17], and [22, 25]
   - b Write a function f with its domain, that represents the path between points A and B.
     $f(x) = |x - 18| + 3, 17 \le x \le 22$
   - c For the path between points A and B, what is the local maximum and local minimum?
     Local maximum: 7
     Local minimum: 3

2. The graph shows the attendance at a sports game as a function of time in minutes.
   - a Describe the domain.
     The domain includes all values between 0 and 70.
   - b Describe the range.
     The range includes all whole numbers between 0 and 55.
   - c Describe how the attendance changed over time.
     The attendance increases at a fast pace (5 people per minute) for the first 10 minutes. The attendance rate slows down for about 5 minutes, and stays constant for 30 minutes. At about 45 minutes, 10 people leave, and then it stays constant for about 5 minutes, when people start leaving at a constant rate until the attendance is zero.

3. Consider each function. Write an equation that represents the function's inverse. What is the input and output of the inverse?
   - a $y(x) = 65 + 5x$
     $x = \frac{y - 65}{5}$; the input is y and the output is x.
   - b $P(n) = \frac{n}{3} - 1.2$
     $n = 3(P + 1.2)$; The input is P and the output is n.

542 **Unit 3** Functions and Their Graphs

© 2023 Amplify Education, Inc. All rights reserved.



Name: ______ Date: ______ Period: ______

4. The number of chirps that crickets make is closely related to the temperature of their environment. When the temperature is between 12 and 38 degrees Celsius, it is possible to determine the temperature by counting the number of chirps! A formula that is commonly used to determine the temperature in degrees Celsius is to count the number of chirps in 25 seconds, divide by 3, and then add 4 to get the temperature.

   Let m be the number of chirps that crickets make in 25 seconds and C be the temperature in degrees Celsius.
   - a What is the temperature when 84 chirps are heard in 25 seconds?
     32°C
   - b Write an equation that defines C as a function of m.
     $C = \frac{m}{3} + 4$
   - c How many chirps would you expect to hear in 25 seconds when it is 14°C?
     30 chirps
   - d Write an equation that defines the inverse of the function you wrote. Explain what the inverse can determine about the situation.
     $m = 3(C - 4)$. The inverse can determine the number of chirps I could expect to hear in 25 seconds when the temperature is C degrees Celsius.

5. A college student borrows $360 from his cousin to repair his car. He agrees to pay $15 per week until the loan is paid off.
   - a Function L represents the amount owed w weeks after the student borrows the money. Write an equation to represent this function using function notation.
     $L(w) = 360 - 15w$
   - b Write an equation to represent the inverse of function L. Explain what information it tells you about the situation.
     $w = \frac{L - 360}{-15}$(or equivalent); Sample response: It tells me the number of weeks when a certain amount is still owed.
   - c How many weeks will it take the student to pay off the loan?
     24 weeks

© 2023 Amplify Education, Inc. All rights reserved.

Lesson 22 Freerunning Functions 543

Practice Problem Analysis

Type	Problem	Refer to	DOK
On-lesson	1	Activity 1	2
Spiral	2	Unit 3 Lesson 11	2
	3	Unit 3 Lesson 20	2
	4	Unit 3 Lesson 19	2
	5	Unit 3 Lesson 20	2

Additional Practice Available

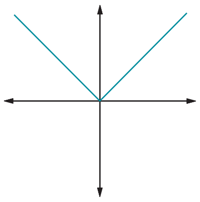
For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

Glossary/Glosario

English

A

absolute value function A function whose output value is the distance of its input value from 0. In other words, the absolute value function is a piecewise function that takes negative input values and makes them positive.

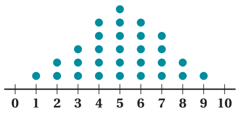


association When a change in one variable suggests another may change as well, the variables have an *association* and are said to be *associated* with one another.

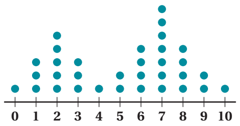
average rate of change The ratio of the change in the outputs to the change in the inputs, for a given interval of a function.

B

bell shaped A distribution that looks like a bell, with most of the data near the center and fewer points farther from the center, is called *bell shaped*.



bimodal A distribution with two distinct peaks is called *bimodal*.

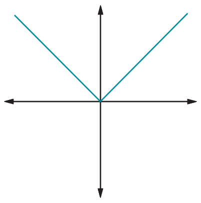


boundary line The line that represents the boundary between the region containing solutions and the region containing non-solutions for an inequality.

Español

A

función de valor absoluto Función cuya salida es la distancia entre su entrada y 0. En otras palabras, la función de valor absoluto es una función definida a trozos que toma entradas negativas y las hace positivas.

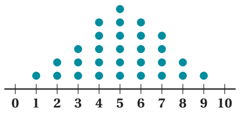


asociación Cuando un cambio en una variable sugiere que otra también podría cambiar, las variables tienen una *asociación* y están *asociadas* entre sí.

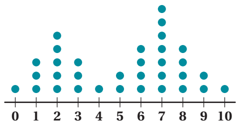
tasa de cambio promedio Razón entre el cambio de las salidas y el cambio de las entradas para un determinado intervalo de una función.

B

acampanada Una distribución que asemeja a una campana, con la mayoría de los datos cerca del centro y una menor cantidad de puntos más lejos del centro, es llamada *acampanada*.



bimodal Una distribución con dos picos distintivos es llamada *bimodal*.



línea límite Línea que representa el límite entre la región que contiene soluciones a una desigualdad y la región que contiene no-soluciones.

© 2023 Amplify Education, Inc. All rights reserved.

Glossary/Glosario

C

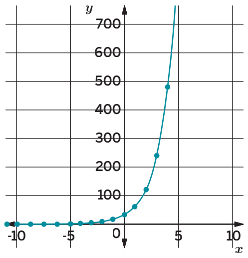
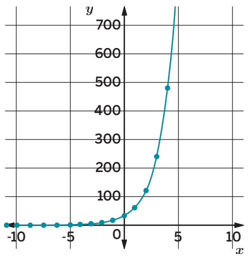
English	Español
categorical variable A variable that can be partitioned into groups or categories.	**variable categórica** Variable que puede partirse en grupos o categorías.
causation When a change in one variable is shown, through careful experimentation, to cause a change in another variable.	**causalidad** Cuando se muestra que un cambio en una variable causa un cambio en otra variable, a través de cuidadosa experimentación.
common difference The difference between two consecutive terms in a linear pattern.	**diferencia común** Diferencia entre dos términos consecutivos de un patrón lineal.
common factor The factor by which each term is multiplied to generate an exponential pattern.	**factor común** Factor por el cual multiplicamos cada término para generar un patrón exponencial.
commutative property Changing the order in which numbers are either added or multiplied does not change the value of the sum or product.	**propiedad conmutativa** Cambiar el orden en que los números se suman o multiplican no cambia el valor de la suma o el producto.
completing the square Completing the square in a quadratic expression means transforming it into the form $a(x - h)^2 + k$.	**completar el cuadrado** Completar el cuadrado en una expresión cuadrática significa transformarla en la forma $a(x - h)^2 + k$.
compounding (interest) When interest itself earns further interest, it is said to be compounded, or applied to itself multiple times.	**(interés) compuesto** Cuando el interés genera más interés, se dice que es compuesto, o que se aplica a sí mismo múltiples veces.
constraint A limitation on the possible values of variables, often expressed by equations or inequalities. For example, distance above the ground d might be constrained to be non-negative: $d \geq 0$.	**limitación** Restricción de los posibles valores de las variables, usualmente expresada por ecuaciones o desigualdades. Por ejemplo, la distancia desde el suelo d puede ser limitada a ser no negativa: $d \geq 0$.
correlation coefficient A value that describes the strength and direction of a linear association between two variables. Strong positive associations have correlation coefficients close to 1, strong negative associations have correlation coefficients close to -1, and weak associations have correlation coefficients close to 0.	**coeficiente de correlación** Valor que describe la fuerza y dirección de una asociación lineal entre dos variables. Asociaciones positivas fuertes tienen coeficientes de correlación cercanos a 1, mientras que asociaciones negativas fuertes tienen coeficientes de correlación cercanos a -1, y asociaciones débiles tienen coeficientes de correlación cercanos a 0.

D

English	Español
decay factor A common factor in an exponential pattern that is between 0 and 1.	**factor de decaimiento** Factor común en un patrón exponencial que se encuentra entre 0 y 1.
difference of squares Two squared terms that are separated by a subtraction sign.	**diferencia de cuadrados** Dos términos al cuadrado que están separados por un signo de resta.
discrete Separate and distinct values or points.	**discreto** Valores o puntos separados y distintivos.
discriminant For a quadratic equation of the form $ax^2 + bx + c = 0$, the discriminant is $b^2 - 4ac$.	**discriminante** Para una ecuación cuadrática de la forma $ax^2 + bx + c = 0$, el discriminante es $b^2 - 4ac$.
domain The set of all of possible input values for a given function.	**dominio** Conjunto de todos los posibles valores de entrada para una determinada función.

© 2023 Amplify Education, Inc. All rights reserved.

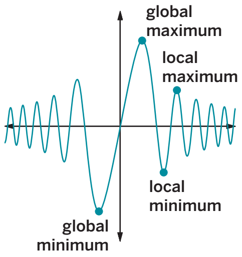
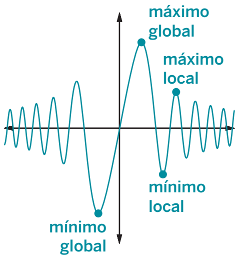
E

English	Español
effective rate The actual interest amount earned over a year, taking into account the interest payment.	**tasa efectiva** Monto del interés real ganado en un año, después de tomar en cuenta el pago de intereses.
elimination The removal of a variable from a system of equations by adding or subtracting equations.	**eliminación** Anulación de una variable de un sistema de ecuaciones por medio de la suma o resta de ecuaciones.
equivalent equations Equations that have the same solution or solutions.	**ecuaciones equivalentes** Ecuaciones que tienen la misma solución o soluciones.
equivalent systems Systems of equations that have the exact same solution or solutions.	**sistemas equivalentes** Sistemas de ecuaciones que tienen exactamente la misma solución o soluciones.
exponential (growth) Describes a change characterized by the repeated multiplication of a common factor. 	**(crecimiento) exponencial** Describe un cambio caracterizado por la multiplicación repetida de un factor común. 
exponential function A one-to-one relationship in which a constant is raised to a variable power.	**función exponencial** Relación uno a uno en la cual una constante se eleva a una potencia variable.

F

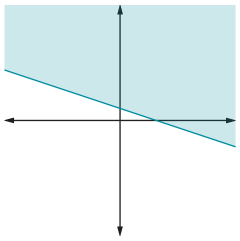
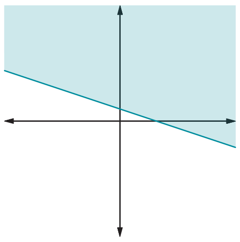
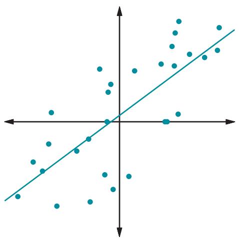
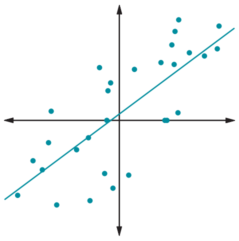
English	Español
factored form (of a quadratic expression) A quadratic expression that is written as the product of a constant and two linear factors is said to be in factored form.	**forma factorizada (de una expresión cuadrática)** Una expresión cuadrática escrita como el producto de una constante multiplicada por dos factores lineales se considera que está en forma factorizada.
first difference The difference between two consecutive dependent terms for a function.	**primera diferencia** Diferencia entre dos términos dependientes y consecutivos de una función.
function notation A way of writing the output of a named function. For example, if the function f has an input x, then $f(x)$ denotes the corresponding output.	**notación de función** Forma de escribir la salida de una determinada función. Por ejemplo, si la función f tiene una entrada x, entonces $f(x)$ denota la salida correspondiente.

G

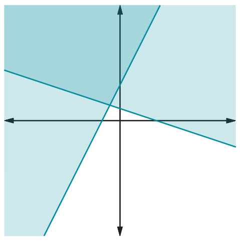
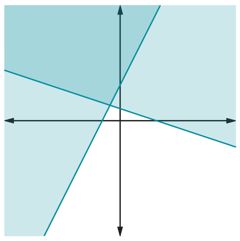
English	Español
global maximum The greatest value of a function over its entire domain. 	**máximo global** El mayor valor de una función por sobre la totalidad de su dominio. 
global minimum The least value of a function over its entire domain.	**mínimo global** El menor valor de una función por sobre la totalidad de su dominio.
growth factor The common factor that is multiplied over equal intervals in an exponential pattern. In exponential functions of the form $f(x) = a \cdot (1 + r)^x$, the growth factor is $1 + r$.	**factor de crecimiento** Factor común que es multiplicado en intervalos iguales como parte de un patrón exponencial. En funciones exponenciales de la forma $f(x) = a \cdot (1 + r)^x$, el factor de crecimiento es $1 + r$.
growth rate The percent change of an exponential function. In exponential functions of the form $f(x) = a \cdot (1 + r)^x$, the growth rate is r.	**tasa de crecimiento** El cambio porcentual de una función exponencial. En funciones exponenciales de la forma $f(x) = a \cdot (1 + r)^x$, la tasa de crecimiento es r.

© 2023 Amplify Education, Inc. All rights reserved.

Glossary/Glosario

English	Español
H	
half-plane The set of points in the coordinate plane on one side of a boundary line. 	**medio plano** Conjunto de puntos en el plano de coordenadas que está a un solo lado de una línea límite. 
I	
index fund An investment fund constructed to track segments of a financial market.	**fondo indexado** Fondo de inversiones elaborado para seguir segmentos de un mercado financiero.
infinity A boundless value, greater than that of any real number.	**infinito** Valor ilimitado, mayor que el valor de cualquier número real.
interest A percentage of the principal that is paid or owed over a specific amount of time.	**interés** Un porcentaje del principal que se paga o debe durante un periodo de tiempo específico.
interval notation A way to represent a set of numbers using parentheses and brackets. For example, the interval $(3, 5]$ represents all the values greater than 3 and less than or equal to 5.	**notación de intervalo** Forma de representar un conjunto de números por medio de paréntesis y corchetes. Por ejemplo, el intervalo $(3, 5]$ representa todos los valores mayores que 3 y menores o iguales que 5.
inverse of a function The inverse of a function is created by reversing all of the function's input-output pairs. It can be determined by reversing the process that defined the original function.	**inverso de una función** El inverso de una función es creado al revertir todos los pares entrada-salida de la función. Se le puede determinar revirtiendo el proceso que definió a la función original.
L	
line of best fit The linear model that has the smallest possible sum of the squares of the residuals. 	**línea de ajuste óptimo** Modelo lineal que tiene la menor suma posible de los cuadrados de los residuos. 
linear function A function with a constant rate of change.	**función lineal** Función con una tasa de cambio constante.
local maximum The value of a function that is greater than the nearby or surrounding values of the function.	**máximo local** Valor de una función que es mayor a los valores cercanos o circundantes de la función.
local minimum The value of a function that is less than the nearby or surrounding values of the function.	**mínimo local** Valor de una función que es menor a los valores cercanos o circundantes de la función.
M	
monic quadratic An expression of the form $x^2 + bx + c$, where the coefficient of the x^2 term is 1.	**ecuación cuadrática mónica** Expresión de la forma $x^2 + bx + c$, en la cual el coeficiente del término x^2 es 1.
N	
nominal rate The stated or published rate.	**tasa nominal** Tasa declarada o publicada.
non-monic quadratic An expression of the form $ax^2 + bx + c$, where a does not equal 1 or 0.	**ecuación cuadrática no mónica** Expresión de la forma $ax^2 + bx + c$, en la cual a no es igual a 1 o 0.
nonlinear relationship A relationship between two quantities in which there is no constant rate of change.	**relación no lineal** Una relación entre dos cantidades que no tiene una tasa de cambio constante.

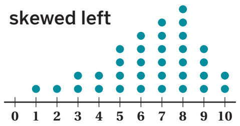
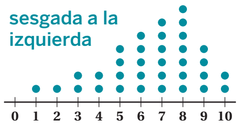
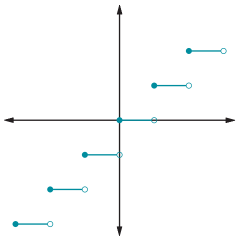
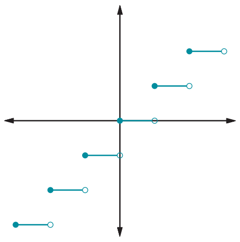
© 2023 Amplify Education, Inc. All rights reserved.

English	Español
O	**O**
overlap of graphs of inequalities The set of points that satisfy two or more inequalties. 	**superposición de gráficas de desigualdades** Conjunto de puntos que satisfacen dos o más desigualdades. 
P	**P**
piecewise function A function defined using different expressions for different intervals in its domain.	**función definida a trozos** Función definida por el uso de diferentes expresiones para diferentes intervalos de su dominio.
plus-or-minus symbol A symbol used to represent both the positive and negative of a number ($\pm$).	**símbolo de más menos** Usado para representar tanto el positivo como el negativo de un número ($\pm$).
principal Initial amount of a loan, investment, or deposit.	**principal** Monto inicial de un préstamo, inversión o depósito.
Q	**Q**
quadratic equation An equation in which the highest power of the variable is 2. Also called an equation of the second degree.	**ecuación cuadrática** Ecuación en la cual la potencia más alta de la variable es 2. También se llama ecuación de segundo grado.
Quadratic Formula The formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ that gives the solutions to the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$.	**Fórmula cuadrática** Fórmula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ que provee las soluciones de la ecuación cuadrática $ax^2 + bx + c = 0$, en la cual $a \neq 0$.
quadratic function A function in which the output is given by a quadratic expression.	**función cuadrática** Función en la cual la salida está dada por una expresión cuadrática.
R	**R**
range In algebra, a function's *range* is the set of all possible output values for the function. In statistics, the *range* of a data distribution is the difference between the maximum and minimum occurring values.	**rango** En algebra, el *rango* de una función es el conjunto de todos los posibles valores de salida de la función. En estadística, el *rango* de una distribución de datos es la diferencia entre los valores máximo y mínimo existentes.
relative frequency table A two-way table that shows the proportion of each value — expressed as fractions, decimals, or percentages — compared to the total in each row, column or in the entire table.	**tabla de frecuencia relativa** Tabla de doble entrada que muestra la proporción de cada valor (expresada como fracciones, decimales o porcentajes), en comparación con el total de cada fila, columna o con toda la tabla.
residual The difference between the actual y-coordinate and y-coordinate predicted by a model, given the x-coordinate.	**residuo** Diferencia entre la coordenada y real y la coordenada y pronosticada por un modelo, dada la coordenada x.
revenue The income generated from selling of a product or service.	**ingreso** Entrada de dinero generada por la venta de un producto o servicio.

© 2023 Amplify Education, Inc. All rights reserved.

Glossary/Glosario

S

English	Español
second difference The difference between two consecutive first differences.	**segunda diferencia** Diferencia entre dos primeras diferencias consecutivas.
skewed A distribution with a long tail, where data extends far away from the center, is called *skewed*. 	**sesgada** Una distribución de cola larga, en la cual los datos se extienden en dirección opuesta al centro, se conoce como *sesgada*. 
solution set The set of all values that satisfy an equation or inequality.	**conjunto de soluciones** Conjunto de todos los valores que satisfacen una ecuación o una desigualdad.
square expression An expression that represents the product of two identical expressions.	**expresión cuadrada** Expresión que representa el producto de dos expresiones idénticas.
standard deviation A commonly used measure of variability. It is the square root of the average of the squares of the distances between data values and the mean.	**desviación estándar** Medida de variabilidad de uso común. Se trata de la raíz cuadrada del promedio de las distancias elevadas al cuadrado entre los valores de los datos y la media.
standard form (of a quadratic expression) The standard form of a quadratic expression in x is $Ax^2 + Bx + C$, where A, B, and C are constants, and $A \neq 0$.	**forma estándar (de una expresión cuadrática)** La forma estándar de una expresión cuadrática en x es $Ax^2 + Bx + C$, en la cual A, B y C son constantes, y $A \neq 0$.
step function A piecewise function whose pieces are all constant values. 	**función escalonada** Función definida a trozos, cuyos trozos son todos valores constantes. 
system of linear inequalities Two or more inequalities that represent the constraints in the same situation.	**sistema de desigualdades lineales** Dos o más desigualdades que representan las limitaciones en la misma situación.

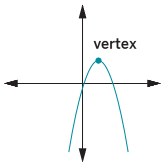
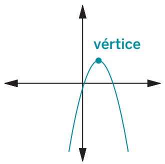
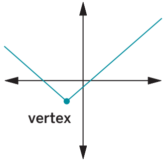
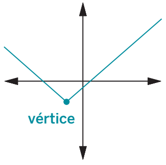
T

English	Español
two-way table A table that organizes categorical data into cells. The categories do not overlap, so that each data value is recorded in exactly one cell.	**tabla de doble entrada** Tabla que organiza datos categóricos en celdas. Las categorías no se superponen, de manera que el valor de cada dato es registrado exactamente en una sola celda.

U

English	Español
uniform A distribution in which data is evenly distributed throughout the range is called *uniform*.	**uniforme** Distribución en la cual los datos son distribuidos de manera regular a través del rango.

V

English	Español
vertex (of a graph) The *vertex* of the graph of a quadratic function or of an absolute value function is the point where the graph changes from increasing to decreasing or vice versa. It is the highest or lowest point on the graph. 	**vértice (de una gráfica)** El *vértice* de la gráfica de una función cuadrática o de una función de valor absoluto es el punto en que la tendencia de la gráfica cambia de aumentar a disminuir o viceversa. Es el punto más alto o más bajo de la gráfica. 
vertex form An equation of the form $y = a(x - h)^2 + k$ where (h, k) represents the coordinates of the vertex of a quadratic function. 	**forma de vértice** Ecuación de la forma $y = a(x - h)^2 + k$, en la cual (h, k) representa las coordenadas del vértice de una función cuadrática. 

Z

English	Español
Zero Product Principle This principle states that $a \cdot b = 0$, if and only if $a = 0$ or $b = 0$.	**Principio de producto cero** Este principio establece que $a \cdot b = 0$ si y solo si $a = 0$ o $b = 0$.
zeros (of a function) The values at which the function is zero.	**ceros (de una función)** Valores para los cuales la función es cero.

© 2023 Amplify Education, Inc. All rights reserved.

Index

© 2023 Amplify Education, Inc. All rights reserved.

Index

© 2023 Amplify Education, Inc. All rights reserved.

© 2023 Amplify Education, Inc. All rights reserved.

Index

© 2023 Amplify Education, Inc. All rights reserved.

© 2023 Amplify Education, Inc. All rights reserved.

Index

© 2023 Amplify Education, Inc. All rights reserved.

© 2023 Amplify Education, Inc. All rights reserved.

Index

© 2023 Amplify Education, Inc. All rights reserved.

© 2023 Amplify Education, Inc. All rights reserved.

Index

© 2023 Amplify Education, Inc. All rights reserved.